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Citation: Cherapanamjeri, Yeshwanth, Daskalakis, Constantinos, Ilyas, Andrew and Zampetakis, Manolis. 2022. "Estimation of Standard Auction Models."

As Published: <https://doi.org/10.1145/3490486.3538284>

Publisher: ACM|Proceedings of the 23rd ACM Conference on Economics and Computation

Persistent URL: <https://hdl.handle.net/1721.1/146420>

Version: Final published version: final published article, as it appeared in a journal, conference proceedings, or other formally published context

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Estimation of Standard Auction Models

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We provide efficient estimation methods for first- and second-price auctions under independent (asymmetric) private values and partial observability. Given a finite set of observations, each comprising the identity of the winner and the price they paid in a sequence of identical auctions, we provide algorithms for non-parametrically estimating the bid distribution of each bidder, as well as their value distributions under equilibrium assumptions. We provide finite-sample estimation bounds which are uniform in that their error rates do not depend on the bid/value distributions being estimated. Our estimation guarantees advance a body of work in Econometrics wherein only identification results have been obtained (e.g. [1, 2]), unless the setting is symmetric (e.g. [8, 9]), parametric (e.g. [3]), or all bids are observable (e.g. [6]). Our guarantees also provide computationally and statistically effective alternatives to classical techniques from reliability theory [7]. Finally, our results are immediately applicable to Dutch and English auctions.

The full version of this work can be found at: <https://arxiv.org/abs/2205.02060>

CCS Concepts: • **Mathematics of computing** → **Nonparametric statistics**; *Partial differential equations*; • **Computing methodologies** → **Supervised learning**; • **Applied computing** → **Economics**.

Additional Key Words and Phrases: econometrics, auctions, order statistics, fixed-point analysis

ACM Reference Format:

Yeshwanth Cherapanamjeri, Constantinos Daskalakis, Andrew Ilyas, and Manolis Zampetakis. 2022. Estimation of Standard Auction Models. In *Proceedings of the 23rd ACM Conference on Economics and Computation (EC '22), July 11–15, 2022, Boulder, CO, USA*. ACM, New York, NY, USA, 2 pages. <https://doi.org/10.1145/3490486.3538284>

1 INTRODUCTION

Estimating value and/or bid distributions from an observed sequence of auctions is a fundamental challenge in Econometrics with direct practical applications. For example, these fundamentals allow one to analyze the performance of an auction and make counterfactual predictions. The difficulty lies in the format of the auctions and the structure of the observed information, as well as how the fundamentals of bidders are interrelated and vary across the sequence of observations.

In this paper, we study a basic version of the afore-described estimation challenge, wherein the auction format and the bidder distributions stay fixed across observations, and the bidders have independent private values (which are independently resampled across different observations). What will make our problem challenging is that (i) our bidders are *ex ante* asymmetric, drawing their independent private values from different distributions; (ii) we will make no parametric assumptions about these distributions; and (iii) we will only be observing the identity of the winner and the price they paid but not the losing bids. Under this model, we focus on first- and second-price auctions, with our results automatically extending to Dutch and English auctions.

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EC '22, July 11–15, 2022, Boulder, CO, USA.

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ACM ISBN 978-1-4503-9150-4/22/07.

<https://doi.org/10.1145/3490486.3538284>

In the above settings, we give computationally and sample efficient methods for estimating all agents' bid distributions and (under equilibrium assumptions) value distributions:

First-price auctions. For first-price auctions, we provide finite-sample estimation guarantees under Lévy, Kolmogorov and Total Variation distances with minimal assumptions. Under a lower bound on the effective densities of the bid distributions (although we actually do not need existence of densities), we show that the bid distributions can be estimated to within ε in Lévy distance, using $1/\varepsilon^{O(k)}$ samples, where k is the number of bidders. We then show that the exponential dependence on k is necessary, and that Lévy distance cannot be strengthened to Kolmogorov distance. In the effective support regime (i.e the part of the support likely to be observed [4]), we sidestep the exponential sample dependence on k and remove the density lower bound assumption to estimate the bid distributions to within ε in Kolmogorov distance on their effective supports. Finally, under Lipschitzness assumptions on the densities of the bid distributions, we improve the latter to Total Variation distance. Our sample requirements for estimation over the effective supports in either Kolmogorov or TV distance are dramatically improved to *logarithmic* in the number of bidders and benign in $1/\varepsilon$. Finally, assuming that bidders use Bayesian Nash equilibrium strategies, we show that bidders' value distributions can be estimated over their full and, effective supports with similar sample sizes as those needed for the estimation of bid distributions.

All of our algorithms run in polynomial time in their sample sizes, and all our estimation error bounds are uniform in that they do not depend on the bid/value distributions being estimated. It is also important to note that we estimate the value distributions in Lévy distance which suffices for performing counterfactual predictions about revenue from alternative auctions [5].

Second-price auctions. For second-price auctions, we establish that bid distributions can be estimated to within ε in Kolmogorov distance over their entire supports assuming upper and lower bounds on their density functions. Again the sample complexity scales as $1/\varepsilon^{O(k)}$. This result involves overcoming major technical challenges, requiring a computationally and statistically effective, fixed point computation alternative to Meilijson [7]'s method. In an effective support setting similar to that proposed by [4], we show that bid distributions can be estimated to within ε in Kolmogorov distance over their effective supports, using a sample size that is polynomial in both $1/\varepsilon$ and k (translatable into a Total Variation guarantee under Lipschitzness of the densities) sidestepping the exponential dependence. Assuming that the bidders bid according to the truthful bidding equilibrium, our results automatically translate to accurate estimation of value distributions.

To our knowledge, ours are the first finite-sample non-parametric guarantees for this problem.

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