Sloane's tables of point configurations on spheres

maintained by Henry Cohn

These tables of point configurations on spheres were created by N. J. A. Sloane based on joint work with R. H. Hardin, W. D. Smith, and others. Sloane has since retired from AT&T Labs, and I have taken over maintaining the tables. Please let me know if you can improve on any of these records.

The table of packings that was previously part of these tables has now been superseded by the table at `https://hdl.handle.net/1721.1/153543`.

On this page there are tables of coverings, designs, maximal volume arrangements, and minimal-energy hard-sphere clusters, as well as packings, coverings, and maximal volume arrangements with icosahedral symmetry. The original tables also included tables of minimizers for the Coulomb and Lennard-Jones potentials, but those tables have been superseded by the Cambridge Energy Landscape Database.

All numbers in these tables are rounded in the worse direction. For example, minimal distances in packings are rounded down, while covering radii are rounded up.

The file `SloaneData.tar` is a tar file of all these configurations. The file names are as specified below for each individual table, and the files list one point per line, with coordinates separated by commas.

Contents

1. Covering by N spherical caps in 3 dimensions

These calculations were done by R. H. Hardin, N. J. A. Sloane, and W. D. Smith in 1994. The corresponding files in the data set are named `cov3-`N`.txt`.

N	Covering radius
4	70.5287794°
5	63.4349489°
6	54.7356104°
7	51.0265527°
8	48.1395291°
9	45.8788879°
10	42.3078267°
11	41.4271960°
12	37.3773682°
13	37.0685428°
14	34.9379270°
15	34.0399002°
16	32.8988128°
17	32.0929329°
18	31.0131719°
19	30.3686749°
20	29.6230959°

21	28.8244769°
22	27.8100588°
23	27.4818687°
24	26.8126364°
25	26.3287855°
26	25.8449223°
27	25.2509549°
28	24.6589490°
29	24.3683987°
30	23.8787580°
31	23.6119921°
32	22.6904805°
33	22.5905117°
34	22.3314637°
35	22.0725569°
36	21.6994390°
37	21.3100299°
38	21.0698585°
39	20.8511245°
40	20.4721354°
41	20.3177153°
42	20.0480918°
43	19.8428334°
44	19.6375705°
45	19.4207407°
46	19.1586113°
47	18.9924594°
48	18.6892566°
49	18.5926797°
50	18.3000226°
51	18.1990012°
52	18.0544758°
53	17.8845735°
54	17.6791448°
55	17.5222393°
56	17.3501140°
57	17.1758476°
58	17.0199611°
59	16.9034031°
60	16.7719330°
61	16.6391846°
62	16.4906597°
63	16.3679364°
64	16.1940191°
65	16.1114062°
66	15.9550615°
67	15.8581808°
68	15.7236959°
69	15.5950401°
70	15.4951288°
71	15.3918905°

72	15.1445321°
73	15.1164438°
74	15.0311866°
75	14.9454278°
76	14.8539208°
77	14.7449905°
78	14.6550578°
79	14.5627674°
80	14.4503043°
81	14.3767803°
82	14.2863118°
83	14.2239571°
84	14.1157902°
85	14.0452618°
86	13.9626271°
87	13.8849703°
88	13.7904978°
89	13.7120949°
90	13.6208737°
91	13.5633748°
92	13.4878634°
93	13.4258226°
94	13.3486302°
95	13.2858097°
96	13.2112887°
97	13.1421391°
98	13.0644481°
99	12.9972792°
100	12.9360973°
101	12.8693268°
102	12.8065481°
103	12.7396985°
104	12.6710008°
105	12.6206479°
106	12.5580705°
107	12.4984677°
108	12.4268412°
109	12.3823879°
110	12.3000527°
111	12.2463575°
112	12.1906904°
113	12.1475148°
114	12.0965651°
115	12.0509312°
116	11.9886435°
117	11.9433898°
118	11.8858176°
119	11.8437245°
120	11.7866487°
121	11.7339187°
122	11.6770715°

123	11.6364348°
124	11.5887834°
125	11.5384928°
126	11.4894028°
127	11.4535231°
128	11.4068507°
129	11.3563380°
130	11.3165625°

2. N-point spherical t-designs in 3 dimensions

This data accompanies the paper *McLaren's improved snub cube and other new spherical designs in three dimensions* by R. H. Hardin and N. J. A. Sloane (Discrete Comput. Geom. **15** (1996), 429–441, doi:10.1007/BF02711518, arXiv:math/0207211). The corresponding files in the data set are named `des3-N-t.txt`.

The following table, which is essentially Table 1 from the paper, gives the putatively optimal design strength t for N points on a sphere in three dimensions. See the paper for references and explanations.

N	t	Group	Order	Orbits	Description
1	0	∞	∞	1	single point
2	1	∞	∞	2	2 antipodal points
3	1	[2,3]	12	3	equilateral triangle
4	2	[3,3]	24	4	regular tetrahedron
5	1	[2,3]	12	3+2	triangular bipyramid
6	3	[3,4]	48	6	regular octahedron
7	2	[3]	6	3^2+1	
8	3	[3,4]	48	8	cube
9	2	[2,3]	12	6+3	triangular biprism
10	3	$[2^+,10]$	20	10	pentagonal prism
11	3	$[2,3]^+$	6	6+3+2	
12	5	[3,5]	120	12	regular icosahedron
13	3	[4]	8	4^3+1	
14	4	$[2,3]^+$	6	6^2+2	
15	3	[2,5]	20	10+5	
16	5	$[3,3]^+$	12	12+4	hexakis truncated tetrahedron
17	4	$[2,3]^+$	6	6^2+3+2	
18	5	$[2^+,6]$	12	12+6	
19	4	[3]	6	6^2+3^2+1	
20	5	[3,5]	120	20	regular dodecahedron
21	4	[2,3]	12	12+6+3	
22	5	$[2^+,10]$	20	10^2+2	
23	5	$[2,3]^+$	6	6^3+3+2	
24	7	$[3,4]^+$	24	24	improved snub cube
25	5	$[2,5]^+$	10	10^2+5	
26	6	$[2,3]^+$	6	6^4+2	
27	5	[2,3]	12	12^2+3	
28	6	$[2^+,4]$	8	8^3+4	
29	6	$[2]^+$	2	2^{14}+1	
30	7	$[3,4]^+$	24	24+6	tetrakis snub cube
31	6	$[5]^+$	5	5^6+1	
32	7	$[3,4]^+$	24	24+8	snub cube + cube

33	6	$[2,3]^+$	6		
34	7	$[2,4]^+$	8		
35	6	$[2,5]^+$	10	10^3+5	
36	8	$[3,3]^+$	12	12^3	3 snub tetrahedra
37	7	$[3]^+$	3		
38	7	$[3,4]^+$	24	24+8+6	
39	7	$[2,3]^+$	6		
40	8	$[3,3]^+$	12	12^3+4	
41	7	$[2,3]^+$	6		
42	8	$[2,4]^+$	8		
43	7	$[6]^+$	6		
44	8	$[3,3]^+$	12	12^3+4^2	
45	8	$[2]^+$	2		
46	8	$[2,4]^+$	8		
47	8	$[2,3]^+$	6		
48	9	$[3,4]^+$	24	24^2	two snub cubes
49	8	$[4]^+$	4		
50	9	$[2,6]^+$	12	12^4+2	
51	8	$[2,3]^+$	6		
52	9	$[3,3]^+$	12	12^4+4	
53	8	$[2,3]^+$	6		
54	9	$[3,4]^+$	24	24^2+6	
55	9	$[2]^+$	2		
56	9	$[3^+,4]$	24	24^2+8	
57	9	$[2,3]^+$	6		
58	9	$[2,4]^+$	8		
59	9	$[2,3]^+$	6		
60	10	$[3,3]^+$	12	12^5	5 snub tetrahedra
61	9	$[6]^+$	6		
62	10	$[2,3]^+$	6		
63	9	$[2,7]^+$	14	14^4+7	
64	10	$[3,3]^+$	12	12^5+4	
65	10	$[2]^+$	2		
66	10	$[2,4]^+$	8		
67	10	$[2]^+$	2		
68	10	$[2^+,4]$	8		
69	10	$[4]^+$	4		
70	11	$[2,5]^+$	10	10^7	
71	10	$[2,3^+]$	6		
72	11	$[3,5]^+$	60	60+12	pentakis truncated icosahedron
73	10	$[4]^+$	4		
74	11	$[2,6]^+$	12	12^6+2	
75	11	$[2]^+$	2		
76	11	$[3,3]^+$	12	12^6+4	
77	11	$[4]^+$	4		
78	11	$[3,4]^+$	24	24^3+6	
79	11	$[2]^+$	2		
80	11	$[3,5]^+$	60	60+20	hexakis truncated icosahedron
81	11	$[4]^+$	4		
82	11	$[2^+,10^+]$	10	10^8+2	
83	11	$[2,3]^+$	6		

84	12	[3,3]$^+$	12	12^7	7 snub tetrahedra
85	11	[2,5]$^+$	10		
86	12	[2,2]$^+$	4		
87	12	[1]$^+$	1		
88	12	[3,3]$^+$	12	12^7+4	
89	12	[2]$^+$	2		
90	12	[2,4]$^+$	8		
91	12	[2]$^+$	2		
92	12	[3,3]$^+$	12	12^7+4^2	
93	12	[4]$^+$	4		
94	13	[2^+,2^+]	2		
95	12	[2]$^+$	2		
96	13	[3,3]$^+$	12	12^8	8 snub tetrahedra
97	12	[4]$^+$	4		
98	13	[2,4]$^+$	8		
99	12	[2]	4		
100	13	[3,3]$^+$	12	12^8+4	

3. Maximal volume arrangements of N points on a unit sphere in 3 dimensions

These calculations were done by R. H. Hardin, N. J. A. Sloane, and W. D. Smith in 1994. The corresponding files in the data set are named `maxvol3-`N`.txt`.

N	Volume
4	0.5132002
5	0.8660254
6	1.3333333
7	1.5850941
8	1.8157161
9	2.0437501
10	2.2187111
11	2.3546344
12	2.5361507
13	2.6128341
14	2.7209778
15	2.8043793
16	2.8864553
17	2.9475229
18	3.0096132
19	3.0632162
20	3.1185387
21	3.1644416
22	3.2082399
23	3.2469420
24	3.2839952
25	3.3162635
26	3.3493598
27	3.3804160
28	3.4073797
29	3.4309531

30	3.4551257
31	3.4798018
32	3.5048740
33	3.5193039
34	3.5381696
35	3.5550943
36	3.5724490
37	3.5900114
38	3.6049983
39	3.6202729
40	3.6341303
41	3.6469596
42	3.6593520
43	3.6709063
44	3.6825759
45	3.6925967
46	3.7025257
47	3.7126857
48	3.7228853
49	3.7314376
50	3.7409408
51	3.7492845
52	3.7571646
53	3.7652096
54	3.7728949
55	3.7802724
56	3.7872202
57	3.7942474
58	3.8008292
59	3.8072864
60	3.8138351
61	3.8194385
62	3.8252192
63	3.8311172
64	3.8366352
65	3.8416863
66	3.8468056
67	3.8522928
68	3.8569277
69	3.8615737
70	3.8661887
71	3.8707190
72	3.8757470
73	3.8792822
74	3.8831655
75	3.8873096
76	3.8912264
77	3.8954100
78	3.8992201
79	3.9025484
80	3.9058168

81	3.9091706
82	3.9125326
83	3.9158307
84	3.9190545
85	3.9222465
86	3.9253555
87	3.9283380
88	3.9313639
89	3.9341831
90	3.9369908
91	3.9396920
92	3.9424750
93	3.9449378
94	3.9475765
95	3.9500387
96	3.9524804
97	3.9548457
98	3.9572867
99	3.9595009
100	3.9617533
101	3.9639988
102	3.9660958
103	3.9682394
104	3.9703323
105	3.9723428
106	3.9743768
107	3.9763828
108	3.9783768
109	3.9802460
110	3.9821639
111	3.9840969
112	3.9858978
113	3.9876200
114	3.9893313
115	3.9910376
116	3.9927304
117	3.9943534
118	3.9959896
119	3.9976502
120	3.9992018
121	4.0008552
122	4.0025594
123	4.0038205
124	4.0052267
125	4.0067521
126	4.0081066
127	4.0096407
128	4.0109471
129	4.0123215
130	4.0136555

4. Minimal-energy clusters of N hard spheres in n dimensions

Here the problem is how to arrange N non-overlapping unit spheres so as to minimize the second moment about their centroid. This data accompanies the paper *Minimal-energy clusters of hard spheres* by N. J. A. Sloane, R. H. Hardin, T. D. S. Duff, and J. H. Conway (Discrete Comput. Geom. **14** (1995), 237–259, doi:10.1007/BF02570704). The corresponding files in the data set are named `clust`n-N`.txt`.

n	N	Moment
3	4	6.0000000
3	5	9.3333334
3	6	12.0000000
3	7	16.6832816
3	8	21.1566742
3	9	25.8989795
3	10	31.8278963
3	11	37.8346428
3	12	42.8162588
3	13	47.7012169
3	14	54.8783069
3	15	62.1070790
3	16	69.7925796
3	17	78.1282260
3	18	86.3012403
3	19	95.1283746
3	20	105.0434487
3	21	114.2222223
3	22	122.4848485
3	23	131.7681160
3	24	141.2777778
3	25	151.6266667
3	26	161.3333334
3	27	172.8888889
3	28	183.7619048
3	29	193.4559387
3	30	205.7135803
3	31	217.3094385
3	32	229.3750001
3	33	240.8305275
3	34	253.1764706
3	35	265.6380953
3	36	277.8518519
3	37	289.7297298
3	38	300.0000000
3	39	317.5384616
3	40	334.3000001
3	41	349.2032521
3	42	364.0952381
3	43	378.5116280
3	44	392.6060607
3	45	408.7111112

3	46	422.9565218
3	47	440.0000000
3	48	456.4166667
3	49	473.1700681
3	50	489.3333334
3	51	504.8714597
3	52	520.6495727
3	53	537.0398323
3	54	553.6378601
3	55	568.8969697
3	56	586.1904762
3	57	603.1578948
3	58	619.7471265
3	59	634.6666667
3	60	656.0814815
3	61	676.8597450
3	62	697.9139785
3	63	717.1428572
3	64	735.8333334
3	65	757.0666667
3	66	778.0942761
3	67	799.2968491
3	68	820.2360204
3	69	840.7149759
3	70	861.9492064
3	71	882.2159625
3	72	902.6666667
3	73	924.1095891
3	74	944.1621622
3	75	964.1600001
3	76	986.3157895
3	77	1008.1038962
3	78	1031.0978158
3	79	1052.7594937
3	80	1073.3333334
3	81	1094.4032922
3	82	1115.7398375
3	83	1137.3333334
3	84	1161.0476191
3	85	1185.3333334
3	86	1210.3720931
3	87	1235.0191571
3	88	1260.1969698
3	89	1285.7078652
3	90	1309.6711285
3	91	1334.9368293
3	92	1360.2940080
3	93	1385.5592027
3	94	1411.6933350
3	95	1436.7937623
3	96	1461.1296297

3	97	1485.0624921
3	98	1508.8052407
3	99	1532.2299539
4	5	8.0000000
4	6	11.0000000
4	7	13.7142858
4	8	16.0000000
4	9	20.6075097
4	10	24.0000000
4	11	27.8080495
4	12	31.5497036
4	13	36.0832338
4	14	40.2135514
4	15	44.5974258
4	16	49.5232044
4	17	55.1043519
4	18	60.6925988
4	19	65.9112023
4	20	69.9411255
4	21	76.8885860
4	22	82.8375730
4	23	87.4782609
4	24	91.8333334
4	25	96.0000000
4	26	103.6923077
4	27	111.1111112
4	28	118.2462885
4	29	124.7058825
4	30	131.1804111
4	31	138.6265098
4	32	145.0449556

5. Packings of N points on a sphere in 3 dimensions with icosahedral symmetry

These calculations were done by R. H. Hardin, N. J. A. Sloane, and W. D. Smith between 1994 and 2000. The corresponding files in the data set are named `ipack3-`N`.txt`.

The configurations in this table are all invariant under the rotational symmetries of the regular icosahedron. Specifically, they are invariant under the rotations $(x, y, z) \mapsto (-x, -y, z)$, $(x, y, z) \mapsto (y, z, x)$, and $(x, y, z) \mapsto (\alpha x - \beta y + z/2, \beta x + y/2 + \alpha z, -x/2 + \alpha y + \beta z)$, where $\alpha = (\sqrt{5}-1)/4$ and $\beta = (\sqrt{5}+1)/4$. To save space, the file contains only one point in each orbit. The only points that occur in these files with nontrivial stabilizers are $(1, 0, 0)$, $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, and $(\sqrt{50 - 10\sqrt{5}}/10, \sqrt{50 + 10\sqrt{5}}/10, 0)$.

N	Minimal angle
60	26.8212672°
72	24.8397619°
80	23.2968305°
90	20.1626222°
92	21.3565202°
102	19.3482659°

110	20.1036108°
120	19.3240199°
122	18.7125471°
132	18.3665154°
140	16.5945955°
150	17.1075770°
152	16.2248754°
162	16.1321920°
170	14.8456316°
180	15.8187591°
182	14.5150376°
192	15.1786631°
200	14.9957661°
210	13.9016164°
212	14.4686011°
222	13.7488481°
230	13.1414711°
240	13.5582065°
242	12.9608113°
252	13.0148857°
260	13.1183200°
270	12.9369929°
272	12.6325892°
282	12.4413806°
290	12.0198028°
300	12.2532100°
302	12.0021534°
312	11.9373561°
320	11.5741321°
330	11.2504042°
332	11.4927363°
342	10.9548798°
350	11.1860732°
360	11.2024757°
362	10.8533792°
372	10.9237102°
380	10.8967748°
390	10.3507709°
392	10.5881051°
402	10.2554418°
410	10.3359904°
420	10.3400850°
422	10.2459522°
432	10.1529059°
440	9.8001885°
450	9.8979242°
452	9.7202420°
462	9.8320464°
470	9.7349506°
480	9.6937434°
482	9.4755147°

492	9.4643776°
500	9.4212804°
510	9.4031989°
512	9.3524847°
522	9.1886038°
530	8.9230077°
540	9.0891735°
542	8.8274874°
552	9.0239937°
560	8.9487383°
570	8.8221122°
572	8.8683528°
582	8.7646412°
590	8.5374731°
600	8.6569116°
602	8.3885665°
612	8.5743273°
620	8.5261743°
630	8.3579318°
632	8.3681410°
642	8.2723693°
650	8.0977578°
660	8.2592976°
662	8.0367323°
672	8.1831041°
680	8.1384497°
690	8.0782820°
692	8.0851606°
702	8.0248120°
710	7.8671044°
720	7.9075339°
722	7.8333858°
732	7.8524502°
740	7.7826079°
750	7.7467383°
752	7.7197487°
762	7.6026191°
770	7.6483478°
780	7.6032035°
782	7.5941387°
792	7.5075987°
800	7.4461115°
810	7.3595689°
812	7.4175884°
822	7.2674146°
830	7.3595689°
840	7.3402581°
842	7.2392374°
852	7.2955763°
860	7.2520937°
870	7.1502922°

872	7.2072109°
882	7.1111613°
890	7.0153910°
900	7.0811605°
902	6.9669822°
912	7.0412669°
920	6.9827864°
930	6.9415928°
932	6.8860230°
942	6.8934557°
950	6.8599238°
960	6.8589310°
962	6.8404101°
972	6.8160292°
980	6.7614327°
990	6.7273440°
992	6.7392799°
1002	6.7087346°
1010	6.6970014°
1020	6.6681391°
1022	6.6551890°
1032	6.6322336°
1040	6.5790862°
1050	6.5696312°
1052	6.5564410°
1062	6.5293185°
1070	6.4531935°
1082	6.3685190°
1112	6.3795686°
1172	6.2225445°
1232	6.0709331°
1292	5.8881818°
1352	5.8000303°
1412	5.6532545°
1472	5.5111948°
1532	5.4284268°
1592	5.3543572°
1652	5.2413674°
1712	5.1496716°
1772	5.0690366°
1832	4.9758495°
1892	4.9121805°
1952	4.8193865°
2012	4.7537428°
2040	4.7169581°
2052	4.7151856°
2060	4.6994245°
2072	4.6955630°
4112	3.3376264°
8192	2.3700355°
32762	1.1860276°

33002	1.1806397°

6. Coverings by N spherical caps in 3 dimensions with icosahedral symmetry

These calculations were done by R. H. Hardin, N. J. A. Sloane, and W. D. Smith between 1994 and 2000. The corresponding files in the data set are named `icov3-`N`-`i`-`j`.txt`.

The icosahedral symmetry is the same as in the previous table. The parameters i and j indicate that vertices of degree 5 are connected by i steps out followed by$j$ steps up along edges, in which case $N = 2+10(i^2+ij+j^2)$.

N	Covering radius	i	j
72	15.1445321°	2	1
92	13.6762972°	3	0
122	11.6856375°	2	2
132	11.2165932°	3	1
162	10.1934606°	4	0
192	9.2462137°	3	2
212	8.8386535°	4	1
252	8.1277102°	5	0
272	7.7606646°	3	3
282	7.6178900°	4	2
312	7.2722947°	5	1
362	6.7588615°	6	0
372	6.6225829°	4	3
392	6.4589656°	5	2
432	6.1703397°	6	1
482	5.8140630°	4	4
492	5.7529244°	5	3
492	5.7847759°	7	0
522	5.5957473°	6	2
572	5.3553545°	7	1
612	5.1546872°	5	4
632	5.0748888°	6	3
642	5.0561151°	8	0
672	4.9297975°	7	2
732	4.7290786°	8	1
752	4.6482959°	5	5
762	4.6172199°	6	4
792	4.5330294°	7	3
812	4.4904796°	9	0
842	4.4021246°	8	2
912	4.2185580°	6	5
912	4.2331691°	9	1
932	4.1742395°	7	4
972	4.0913749°	8	3
1002	4.0386573°	10	0
1032	3.9745099°	9	2
1082	3.8718920°	6	6
1092	3.8539391°	7	5
1112	3.8309534°	10	1
1122	3.8041789°	8	4

1172	3.7253076°	9	3
1212	3.6694362°	11	0
1242	3.6214222°	10	2
1272	3.5697256°	7	6
1292	3.5426550°	8	5
1332	3.4912417°	9	4
1332	3.4982723°	11	1
1392	3.4175778°	10	3
1442	3.3620625°	12	0
1472	3.3176954°	7	7
1472	3.3252106°	11	2
1482	3.3064162°	8	6
1512	3.2746915°	9	5
1562	3.2237309°	10	4
1572	3.2185863°	12	1
1632	3.1556162°	11	3
1692	3.0937120°	8	7
1692	3.1021980°	13	0
1712	3.0760190°	9	6
1722	3.0733098°	12	2
1752	3.0420475°	10	5
1812	2.9927803°	11	4
1832	2.9802004°	13	1
1892	2.9301478°	12	3
1922	2.9022603°	8	8
1932	2.8947016°	9	7
1962	2.8732860°	10	6
1962	2.8796186°	14	0
1992	2.8565576°	13	2
2012	2.8385726°	11	5
2082	2.7916556°	12	4
2112	2.7746165°	14	1
2172	2.7296149°	9	8
2172	2.7341970°	13	3
2192	2.7174066°	10	7
2232	2.6938502°	11	6
2252	2.6868374°	15	0
2282	2.6681381°	14	2
2292	2.6593857°	12	5
2372	2.6151078°	13	4
2412	2.5955134°	15	1
2432	2.5792743°	9	9
2442	2.5739685°	10	8
2472	2.5588419°	11	7
2472	2.5624150°	14	3
2522	2.5341872°	12	6
2562	2.5182463°	16	0
2592	2.5005827°	13	5
2592	2.5028762°	15	2
2682	2.4590127°	14	4
2712	2.4421456°	10	9

2732	2.4333803°	11	8
2732	2.4380930°	16	1
2772	2.4163918°	12	7
2792	2.4106549°	15	3
2832	2.3913880°	13	6
2892	2.3695615°	17	0
2912	2.3590016°	14	5
2922	2.3567762°	16	2
3002	2.3209697°	10	10
3012	2.3170977°	11	9
3012	2.3200992°	15	4
3042	2.3060231°	12	8
3072	2.2986486°	17	1
3092	2.2879060°	13	7
3132	2.2756547°	16	3
3162	2.2630603°	14	6
3242	2.2374539°	18	0
3252	2.2320905°	15	5
3272	2.2267054°	17	2
3312	2.2094283°	11	10
3332	2.2029150°	12	9
3362	2.1957349°	16	4
3372	2.1902680°	13	8
3432	2.1715799°	14	7
3432	2.1742716°	18	1
3492	2.1548139°	17	3
3512	2.1472263°	15	6
3612	2.1177507°	16	5
3612	2.1192978°	19	0
3632	2.1096869°	11	11
3642	2.1067781°	12	10
3642	2.1101757°	18	2
3672	2.0984265°	13	9
3722	2.0847360°	14	8
3732	2.0837934°	17	4
3792	2.0658732°	15	7
3812	2.0626479°	19	1
3872	2.0460388°	18	3
3882	2.0422177°	16	6
3972	2.0171859°	12	11
3992	2.0122181°	13	10
3992	2.0142570°	17	5
4002	2.0129941°	20	0
4032	2.0025533°	14	9
4032	2.0051864°	19	2
4092	1.9882287°	15	8
4122	1.9825340°	18	4
4172	1.9694745°	16	7
4212	1.9619138°	20	1
4272	1.9466480°	17	6
4272	1.9476248°	19	3

4322	1.9336582°	12	12
4332	1.9314152°	13	11
4362	1.9249619°	14	10
4392	1.9201740°	18	5
4412	1.9143683°	15	9
4412	1.9168447°	21	0
4442	1.9101109°	20	2
4482	1.8997239°	16	8
4532	1.8905196°	19	4
4572	1.8812746°	17	7
4632	1.8705515°	21	1
4682	1.8593435°	18	6
4692	1.8557082°	13	12
4692	1.8581708°	20	3
4712	1.8518305°	14	11
4752	1.8442783°	15	10
4812	1.8330665°	16	9
4812	1.8343044°	19	5
4842	1.8294610°	22	0
4872	1.8236130°	21	2
4892	1.8183331°	17	8
4962	1.8065577°	20	4
4992	1.8003168°	18	7
5072	1.7847401°	13	13
5072	1.7873124°	22	1
5082	1.7829755°	14	12
5112	1.7778856°	15	11
5112	1.7793104°	19	6
5132	1.7765152°	21	3
5162	1.7695237°	16	10
5232	1.7579340°	17	9
5252	1.7556398°	20	5
5292	1.7496967°	23	0
5322	1.7432792°	18	8
5322	1.7445857°	22	2
5412	1.7296485°	21	4
5432	1.7257819°	19	7
5472	1.7181595°	14	13
5492	1.7150753°	15	12
5532	1.7090632°	16	11
5532	1.7111601°	23	1
5562	1.7057031°	20	6
5592	1.7001244°	17	10
5592	1.7016879°	22	3
5672	1.6883459°	18	9
5712	1.6833265°	21	5
5762	1.6765969°	24	0
5772	1.6738914°	19	8
5792	1.6721041°	23	2
5882	1.6571176°	14	14
5882	1.6589495°	22	4

5892	1.6557038°	15	13
5892	1.6569650°	20	7
5922	1.6516178°	16	12
5972	1.6449022°	17	11
6012	1.6412272°	24	1
6032	1.6377979°	21	6
6042	1.6355778°	18	10
6072	1.6328722°	23	3
6132	1.6237532°	19	9
6192	1.6166391°	22	5
6242	1.6095836°	20	8
6252	1.6093598°	25	0
6282	1.6053896°	24	2
6312	1.5995891°	15	14
6332	1.5970953°	16	13
6372	1.5922305°	17	12
6372	1.5932548°	21	7
6372	1.5937456°	23	4
6432	1.5849924°	18	11
6512	1.5754336°	19	10
6512	1.5767821°	25	1
6522	1.5749732°	22	6
6572	1.5693756°	24	3
6612	1.5636680°	20	9
6692	1.5549559°	23	5
6732	1.5498412°	21	8
6752	1.5465279°	15	15
6762	1.5453780°	16	14
6762	1.5473074°	26	0
6792	1.5420485°	17	13
6792	1.5437817°	25	2
6842	1.5365747°	18	12
6872	1.5341218°	22	7
6882	1.5334263°	24	4
6912	1.5289631°	19	11
7002	1.5192880°	20	10
7682	1.4497746°	16	16
8192	1.4038820°	18	15
8192	1.4053004°	27	3
8672	1.3644140°	17	17
9722	1.2885458°	18	18
10832	1.2206701°	19	19
12002	1.1595872°	20	20
13232	1.1043260°	21	21
14522	1.0540920°	22	22
15872	1.0082292°	23	23
17282	0.9661909°	24	24
18752	0.9275176°	25	25
20282	0.8918211°	26	26
21872	0.8587704°	27	27
23522	0.8280817°	28	28

25232	0.7995107°	29	29
27002	0.7728454°	30	30
28832	0.7479014°	31	31
30722	0.7245172°	32	32
32672	0.7025509°	33	33
34682	0.6818774°	34	34
36752	0.6623857°	35	35
38882	0.6439775°	36	36
41072	0.6265647°	37	37
43322	0.6100687°	38	38
45632	0.5944191°	39	39
48002	0.5795523°	40	40
50432	0.5654110°	41	41
52922	0.5519434°	42	42
55472	0.5391023°	43	43
58082	0.5268452°	44	44
60752	0.5151331°	45	45
63482	0.5039304°	46	46
66272	0.4932045°	47	47
69122	0.4829370°	48	48
72032	0.4730771°	49	49
75002	0.4636117°	50	50
78032	0.4545177°	51	51

7. Maximal volume arrangements of N points on a unit sphere in 3 dimensions with icosahedral symmetry

These calculations were done by R. H. Hardin, N. J. A. Sloane, and W. D. Smith between 1994 and 2000. The corresponding files in the data set are named `imaxvol3-`N`-`i`-`j`.txt`.

The icosahedral symmetry and parameters i and j are the same as above.

N	Volume	i	j
72	3.8757470	2	1
92	3.9424508	3	0
122	4.0025594	2	2
132	4.0164658	3	1
162	4.0480801	4	0
192	4.0699821	3	2
212	4.0810999	4	1
252	4.0981035	5	0
272	4.1047698	3	3
282	4.1077306	4	2
312	4.1154862	5	1
362	4.1255771	6	0
372	4.1272837	4	3
392	4.1304086	5	2
432	4.1357966	6	1
482	4.1412882	4	4
492	4.1422444	7	0
492	4.1422503	5	3

522	4.1449171	6	2
572	4.1487434	7	1
612	4.1513604	5	4
632	4.1525418	6	3
642	4.1531026	8	0
672	4.1546948	7	2
732	4.1574849	8	1
752	4.1583190	5	5
762	4.1587181	6	4
792	4.1598550	7	3
812	4.1605652	9	0
842	4.1615704	8	2
912	4.1636570	9	1
912	4.1636588	6	5
932	4.1641972	7	4
972	4.1652077	8	3
1002	4.1659123	10	0
1032	4.1665771	9	2
1082	4.1676037	6	6
1092	4.1677975	7	5
1112	4.1681736	10	1
1122	4.1683580	8	4
1172	4.1692287	9	3
1212	4.1698733	11	0
1242	4.1703301	10	2
1272	4.1707659	7	6
1292	4.1710446	8	5
1332	4.1715764	11	1
1332	4.1715769	9	4
1392	4.1723182	10	3
1442	4.1728887	12	0
1472	4.1732127	11	2
1472	4.1732133	7	7
1482	4.1733183	8	6
1512	4.1736250	9	5
1562	4.1741100	10	4
1572	4.1742030	12	1
1632	4.1747392	11	3
1692	4.1752370	13	0
1692	4.1752375	8	7
1712	4.1753957	9	6
1722	4.1754731	12	2
1752	4.1757013	10	5
1812	4.1761344	11	4
1832	4.1762722	13	1
1892	4.1766691	12	3
1922	4.1768586	8	8
1932	4.1769203	9	7
1962	4.1771013	14	0
1962	4.1771016	10	6
1992	4.1772773	13	2

2012	4.1773919	11	5
2082	4.1777749	12	4
2112	4.1779311	14	1
2172	4.1782311	13	3
2172	4.1782313	9	8
2192	4.1783276	10	7
2232	4.1785150	11	6
2252	4.1786060	15	0
2282	4.1787398	14	2
2292	4.1787838	12	5
2372	4.1791211	13	4
2412	4.1792813	15	1
2432	4.1793597	9	9
2442	4.1793983	10	8
2472	4.1795121	14	3
2472	4.1795122	11	7
2522	4.1796961	12	6
2562	4.1798378	16	0
2592	4.1799415	15	2
2592	4.1799416	13	5
2682	4.1802384	14	4
2712	4.1803331	10	9
2732	4.1803947	16	1
2732	4.1803949	11	8
2772	4.1805160	12	7
2792	4.1805752	15	3
2832	4.1806912	13	6
2892	4.1808591	17	0
2912	4.1809136	14	5
2922	4.1809405	16	2
3002	4.1811498	10	10
3012	4.1811750	15	4
3012	4.1811752	11	9
3042	4.1812502	12	8
3072	4.1813237	17	1
3092	4.1813721	13	7
3132	4.1814667	16	3
3162	4.1815362	14	6
3242	4.1817151	18	0
3252	4.1817369	15	5
3272	4.1817800	17	2
3312	4.1818647	11	10
3332	4.1819063	12	9
3362	4.1819676	16	4
3372	4.1819879	13	8
3432	4.1821067	18	1
3432	4.1821068	14	7
3492	4.1822215	17	3
3512	4.1822590	15	6
3612	4.1824397	19	0
3612	4.1824397	16	5

3632	4.1824748	11	11
3642	4.1824920	18	2
3642	4.1824921	12	10
3672	4.1825435	13	9
3722	4.1826274	14	8
3732	4.1826439	17	4
3792	4.1827412	15	7
3812	4.1827728	19	1
3872	4.1828661	18	3
3882	4.1828813	16	6
3972	4.1830153	12	11
3992	4.1830441	17	5
3992	4.1830442	13	10
4002	4.1830584	20	0
4032	4.1831011	19	2
4032	4.1831012	14	9
4092	4.1831845	15	8
4122	4.1832253	18	4
4172	4.1832920	16	7
4212	4.1833442	20	1
4272	4.1834206	19	3
4272	4.1834207	17	6
4322	4.1834828	12	12
4332	4.1834951	13	11
4362	4.1835315	14	10
4392	4.1835673	18	5
4412	4.1835910	21	0
4412	4.1835911	15	9
4442	4.1836261	20	2
4482	4.1836722	16	8
4532	4.1837286	19	4
4572	4.1837729	17	7
4632	4.1838379	21	1
4682	4.1838908	18	6
4692	4.1839012	20	3
4692	4.1839013	13	12
4712	4.1839220	14	11
4752	4.1839630	15	10
4812	4.1840231	19	5
4812	4.1840232	16	9
4842	4.1840526	22	0
4872	4.1840818	21	2
4892	4.1841011	17	8
4962	4.1841672	20	4
4992	4.1841950	18	7
5072	4.1842674	22	1
5072	4.1842675	13	13
5082	4.1842764	14	12
5112	4.1843028	19	6
5112	4.1843029	15	11
5132	4.1843203	21	3

5162	4.1843463	16	10
5232	4.1844058	17	9
5252	4.1844224	20	5
5292	4.1844554	23	0
5322	4.1844799	22	2
5322	4.1844799	18	8
5412	4.1845515	21	4
5432	4.1845672	19	7
5472	4.1845981	14	13
5492	4.1846133	15	12
5532	4.1846435	23	1
5532	4.1846435	16	11
5562	4.1846658	20	6
5592	4.1846880	22	3
5592	4.1846880	17	10
5672	4.1847458	18	9
5712	4.1847741	21	5
5762	4.1848090	24	0
5772	4.1848159	19	8
5792	4.1848296	23	2
5882	4.1848902	22	4
5882	4.1848902	14	14
5892	4.1848968	20	7
5892	4.1848968	15	13
5922	4.1849166	16	12
5972	4.1849490	17	11
6012	4.1849745	24	1
6032	4.1849872	21	6
6042	4.1849935	18	10
6072	4.1850122	23	3
6132	4.1850492	19	9
6192	4.1850854	22	5
6242	4.1851151	20	8
6252	4.1851210	25	0
6282	4.1851385	24	2
6312	4.1851559	15	14
6332	4.1851673	16	13
6372	4.1851901	23	4
6372	4.1851901	21	7
6372	4.1851901	17	12
6432	4.1852237	18	11
6512	4.1852674	25	1
6512	4.1852675	19	10
6522	4.1852728	22	6
6572	4.1852996	24	3
6612	4.1853207	20	9
6692	4.1853622	23	5
6732	4.1853826	21	8
6752	4.1853927	15	15
6762	4.1853977	26	0
6762	4.1853977	16	14

6792	4.1854126	25	2
6792	4.1854127	17	13
6842	4.1854374	18	12
6872	4.1854520	22	7
6882	4.1854568	24	4
6912	4.1854713	19	11
7002	4.1855140	20	10
7682	4.1858039	16	16
8192	4.1859898	27	3
8192	4.1859898	18	15
8672	4.1861448	17	17
9722	4.1864305	18	18
10832	4.1866723	19	19
12002	4.1868787	20	20
13232	4.1870564	21	21
14522	4.1872104	22	22
15872	4.1873448	23	23
17282	4.1874627	24	24
18752	4.1875667	25	25
20282	4.1876590	26	26
21872	4.1877413	27	27
23522	4.1878148	28	28
25232	4.1878809	29	29
27002	4.1879405	30	30
28832	4.1879945	31	31
30722	4.1880434	32	32
32672	4.1880880	33	33
34682	4.1881287	34	34
36752	4.1881659	35	35
38882	4.1882001	36	36
41072	4.1882316	37	37
43322	4.1882606	38	38
45632	4.1882874	39	39
48002	4.1883122	40	40
50432	4.1883352	41	41
52922	4.1883567	42	42
55472	4.1883766	43	43
58082	4.1883952	44	44
60752	4.1884125	45	45
63482	4.1884288	46	46
66272	4.1884440	47	47
69122	4.1884582	48	48
72032	4.1884717	49	49
75002	4.1884843	50	50
78032	4.1884961	51	51