

AN OPTION PRICING MODEL FOR R&D PROJECTS

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ABSTRACT

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**Submitted to the Alfred P. Sloan School of Management
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Technology continuously evolves, improving old products, creating new ones, and generating new industries. The profitable life of a product, or product life cycle, can be as short as eighteen months in some industries. The second industrial revolution -- spawned by the electronic computer; computer aided design and engineering; and flexible, automated manufacturing systems -- will increase the rate at which technological innovations come to the market. In a world of increasing technological change, corporate research and development (R&D) is a vital, value-adding strategic function when coupled to marketing and manufacturing. Determining the value added by R&D and the optimal time to bring innovations to the marketplace in a world of technology induced market uncertainty will be a problem confronting all corporations.

A simple analytic option pricing formula suitable for estimating the option value of an R&D project is obtainable by using an optimal stopping formulation. The option pricing model displays characteristics similar to common stock options. Although the formula is limited, it reveals the value of applying a more general numerical formulation to the management of R&D projects.

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TABLE OF CONTENTS

Introduction	4
Valuing the Factory	9
Option Pricing Model	14
Application, Limitations, and Conclusion	18
Appendix A	24

CHAPTER 1: INTRODUCTION

Technology continuously evolves, improving old products, creating new ones, and generating new industries. The profitable life of a product, or product life cycle, can be as short as eighteen months in some industries.¹ The second industrial revolution -- spawned by the electronic computer; computer aided design and engineering; and flexible, automated manufacturing systems -- will increase the rate at which technological innovations come to the market. In a world of increasing technological change, corporate research and development (R&D) is a vital, value-adding strategic function when coupled to marketing and manufacturing.² Determining the value added by R&D and the optimal time to bring innovations to the marketplace in a world of technology induced market uncertainty is a problem confronting all corporations.

Determining the value added by R&D is a difficult problem because it provides future business opportunities rather than immediate returns on investment (ROI). Capital budgeting techniques typically used to value a business opportunity (a project) assume that an investment decision is an immediate result of the valuation. These techniques are well suited to decisions requiring immediate action, such as a decision to replace existing obsolete, inefficient manufacturing equipment, and are expected to produce immediate profits. Data influencing the capital budgeting decisions are quantifiable because similar projects usually

¹ Mitchell, Graham, private communication.

² Ansoff, H. I., and Stewart, J. M., "Strategies for a Technology-Based Business," Harvard Business Review, November-December 1967, pp 71-83.

exist. R&D projects usually offer business opportunities that will occur in the future, and applying existing capital budgeting techniques to the valuation of R&D projects is erroneous.

In order to understand the causes of evaluation error, let us compare replacing manufacturing equipment to developing new manufacturing equipment. A choice of one of these proposals requires comparison of existing machinery to nonexisting machinery. An evaluation of a proposal to replace manufacturing equipment uses information and costs related to existing machinery. The proposal to develop new manufacturing equipment uses estimates of the expected outcome of the development project. Evaluating the development proposal on the basis of existing technology and costs ignores the potential added benefit of an uncertain outcome, -- the development project may produce superior equipment.

This situation is analogous to a choice between R&D projects and current business opportunities. Evaluating a business opportunity resulting from an R&D project with current data also ignores the potential benefits of an uncertain outcome, -- a financially unsound current opportunity may become financially attractive as a result of R&D. This thesis examines economic and financial option pricing models that may provide a value for R&D projects.

Option Pricing Models

Let us first consider the similarities between R&D projects and call options on common stocks.³ Call options give their owners the right to buy (exercise) common stock at prespecified costs (striking price) on or before prespecified dates (expiration date). R&D gives corporations call options on business opportunities resulting from new technologies. The decision to launch a new product is similar to exercising a call option. In both situations, a rational decision maker will only exercise an option if it is in his best interest to do so. This means that the value of the stock (or product) purchased by exercising the option must exceed its striking price, -- a decision rule identical to "invest if the net present value is positive. A call option prior to expiration has a positive value even if the current stock price is less than the exercise price. It has a positive value because the stock price may be greater than the exercise price on the expiration date. For a similar reason, firms conduct R&D on innovative products even though they would produce losses if manufactured today; the value of producing one of these items may someday exceed investment cost. Both product and stock values change unpredictably, and therefore, options to launch a product and to buy a common stock are valuable. Option pricing models value the opportunity to make future investment decisions.

An option pricing model determines the value of an option by using the characteristics of the underlying asset, the cost to buy the asset on exercise, interest rates, and time to expiration. For example, let us assume that Company X has a product that is a technical innovation that no one else sells today. The

³ Cox, J. C., and Rubinstein, M., Options Markets, Prentice Hall, New Jersey, © 1985, pp 1-62.

company wants to know the value of its option to the business opportunity provided by continued research on their product and the optimal time to launch the product.⁴ The underlying asset for the option to launch the product is the factory manufacturing it. Chapter 2 discusses the valuation of such a factory.

Company X's option to launch their product is infinitely lived. The Science Policy Research Unit examined characteristic factors affecting the commercial success of technical innovations.⁵ This study discovered that time limits on innovative R&D projects did not differentiate between commercial success and failure. The factors that did differentiate between success and failure include satisfying customer needs and developing manufacturing capacity. These factors imply that a decision to launch the innovative product depends upon the degree of demand and the ability to manufacture the product. Therefore, Company X needs a decision rule that considers market demand and manufacturing and is independent of time. An extension of the option pricing model derived by Mc Donald, R., and Siegel D for an infinitely lived option provides such a decision rule.⁶

Let us also assume the Company X can estimate the current cost of building a profit maximizing factory but may be unable to accurately predict the cost of a future one. Chapter 3 discusses the option pricing model and decision rule that allow for variable investment cost. There are two steps in the decision process;

⁴ We consider the cost to build a factory as a part of the costs to launch a new product.

⁵ Freeman, Christopher, Economics of the Industrial Revolution, The MIT Press, Massachusetts, © 1982, pp 107-130.

⁶ Mc Donald, R., and Siegel D., "The Value of Waiting to Invest," Quarterly Journal of Economics, November 1986, pp 707-727.

Company X determines the current factory value and then values R&D. Chapter 4 provides a numerical example describing the application of this option pricing model and a discussion of its limitations. This numerical example and the assumptions required to develop the option pricing model indicate that more work is required to apply it to R&D project management. Chapter 4 also concludes this thesis.

CHAPTER 2: VALUING THE FACTORY

In order to make a rational, optimal decision to exercise an option to launch an innovative product, Company X requires an estimate of the value of a factory manufacturing their product. This valuation is difficult because a similar factory may not exist. This chapter develops a project model to value a factory for an innovative product. The project model considers variable factory productivity and so is an extension of the model derived by Mc Donald and Siegel.⁷ Their model is a special case of this more general one. One aspect of R&D uncertainty is the production technology that will eventually manufacture the product. R&D provides the time to find the best manufacturing technology for a product. Once a factory is built, its manufacturing technology is less flexible. This model assumes that the productivity of a factory remains constant over its life. The project model also provides a measure of the financial risk required to determine a cost of capital. The profits estimated by the project model and the risk adjusted cost of capital determine the current factory value. This chapter concludes with a description of the factory valuation.

The Project Model

The commercial success of a new product is a consequence of market demand, which grows with a degree of randomness or stochastic behavior. The random nature of market demand is quite understandable. Variations in economic conditions cause unexplainable random shocks to the demand for a

⁷ Mc Donald, R. and Siegel, D. *op. cit.*, pp 722-724.

product. For this discussion we shall assume the firm faces an inverse demand curve as given by,

$$P_t = \Theta_t Q_t^{-1/\eta}, \quad (2.1)$$

where P_t and Q_t are the price and quantity demanded at time t ; η is the price elasticity of demand; and Θ_t is the demand shift parameter expressing the stochastic process,

$$d\Theta_t / \Theta_t = \mu_\theta dt + \sigma_\theta dz_1. \quad (2.2)$$

The factory production function, that is the relationship of capital and labor dedicated to manufacturing, also behaves stochastically. Increased capital productivity reduces the labor required to obtain a specific level of production and therefore, reduces product cost. A reduction in cost allows the company to lower product price and increase the quantity demanded. We assume the factory produces according to a Cobb-Douglas production function,

$$Q_t = \Psi_t K^\alpha L_t^\beta, \quad (2.3)$$

where K is a fixed quantity of capital and Ψ_t is the productivity factor. Production behaves stochastically according to

$$d\Psi_t / \Psi_t = \mu_\psi dt + \sigma_\psi dz_2(t). \quad (2.4)$$

where μ_ψ is the expected growth rate productivity, and σ_ψ is its volatility.

We shall assume that labor is the only cost of production. The profits, Π_t , are revenues less labor costs at each time t ,

$$\Pi_t = P_t Q_t - wL_t, \quad (2.5)$$

where w is a constant wage rate. The firm selects a quantity of labor at each time t to maximize the profit of the factory by solving a mathematical program with labor as the control variable,

$$\Pi_t^* = \{\Theta_t \Psi_t^{(1-1/\eta)}\}^{(1/1-b)} K^{(a/1-b)} w^{(-b/1-b)} [b^{(b/1-b)} - b^{(1/1-b)}], \quad (2.6)$$

where $a = \alpha(1-1/\eta)$ and $b = \beta(1-1/\eta)$. Equation 2.6 may be simplified with the following definitions:

$$\Pi_0^* = K^{(a/1-b)} w^{(-b/1-b)} [b^{(b/1-b)} - b^{(1/1-b)}] \quad (2.7)$$

$$g = 1/1-b; \text{ and} \quad (2.8)$$

$$h = 1-1/\eta. \quad (2.9)$$

With these definitions we have,

$$\Pi_t^* = \{\Theta_t \Psi_t^h\}^g \Pi_0^*. \quad (2.10)$$

By applying Ito's Lemma to equation 2.10 and using the definitions in equations 2.2 and 2.4, we obtain the expected growth rate of profit,⁸

$$\mu_\Pi = g(\mu_\Theta + h \mu_\Psi) + \{g(g-1)\sigma_\Theta^2 + gh(gh-1)\sigma_\Psi^2 + 2g^2h\sigma_{\Theta\Psi}\}/2, \quad (2.11)$$

and its components of volatility,

$$\sigma_{\Pi 1} = g\sigma_\Theta, \quad (2.12)$$

and

$$\sigma_{\Pi 2} = gh\sigma_\Psi. \quad (2.13)$$

The term $\sigma_{\Theta\Psi}$ is the covariance between demand and productivity. The volatility of profit is

$$\sigma_\Pi = \{g^2\sigma_\Theta^2 + g^2h^2\sigma_\Psi^2 + 2g^2h\sigma_{\Theta\Psi}\}^{1/2} \quad (2.14)$$

with the assumption that g , h , σ_Θ , σ_Ψ , and $\sigma_{\Theta\Psi}$ are constants. From the definition of a correlation matrix, we may write the correlation of profit to the market portfolio as,

⁸ Harrison, J. M., Brownian Motion and Stochastic Flow Systems, John Wiley and Sons, Inc., New York, © 1985, pp 54-79.

$$\rho_{\Pi R_m} = \rho_{\theta R_m} + \rho_{\psi R_m}, \quad (2.15)$$

where $\rho_{\theta R_m}$ and $\rho_{\psi R_m}$ are the correlations of the market portfolio to demand and productivity.^{9,10} The simpler case of Mc Donald and Siegel is obtained by setting the terms in equation 2.4 to zero. In this situation the productivity factor is a constant equal to one. The model now supplies the information required to value the factory.

Valuation of the Factory

With the definition for the stochastic profit function, the probability density distribution of $\ln(\Pi_t^*)$ is normal with a mean of $\{\mu_{\Pi} t\}$ and variance of $\{\sigma^2_{\Pi} t\}$. These factors provide the information determining the distribution of profits received in the future. From the assumptions of Weiner processes, the evaluation of the future profits will change as we collect more information over time. Therefore, the correct procedure discounts cash flows at the risk adjusted discount rate for all time. In order to perform the valuation, we need the expected value of a lognormally distributed variable y which is,¹¹

$$E(y) = \exp\{\mu_{\ln y} + \sigma^2_{\ln y}/2\}. \quad (2.16)$$

The intertemporal discounting formula is

$$V = \int_t \exp\{- (r_f + \beta_{\Pi}(r_M - r_f) t)\} E(\Pi_t^*) dt, \quad (2.17)$$

⁹ The Standard and Poor's 500 Composite Index of common stocks is the proxy for the market portfolio.

¹⁰ Morrison, Donald F., Multivariate Statistical Methods, Mc Graw-Hill Book Co., New York, © 1967, pp 75 -80.

¹¹ Rubinstein, M., "The Valuation of Uncertain Income Streams and the Pricing of Options," Bell Journal of Economics and Management Science, 7 2 (Autumn 1976), pp 407-425.

where

$$\beta_{\Pi} = \rho_{\Pi R_M} \sigma_{\Pi} / \sigma_{R_M}, \quad (2.18)$$

and r_f and r_M are the riskfree discount rate and the return on the market portfolio.^{12,13} Since the stochastic profit function is lognormally distributed with mean of $\{\mu_{\Pi}t\}$ and a variance of $\{\sigma^2_{\Pi}t\}$, the expected value of profit at time t is,

$$E(\Pi_t) = \Pi_0 \exp \{(\mu_{\Pi} + \sigma^2_{\Pi}/2) t\}, \quad (2.19)$$

and, the present value of the profit stream is (for an infinitely lived factory),

$$V_0 = \Pi_0 / ((r_f + \beta_{\Pi}(r_M - r_f)) - (\mu_{\Pi} + \sigma^2_{\Pi}/2)), \quad (2.20)$$

with the condition that

$$(\mu_{\Pi} + \sigma^2_{\Pi}/2) < (r_f + \beta_{\Pi}(r_M - r_f)). \quad (2.21)$$

Equation 2.20 reduces to the Williams - Gordon perpetual growth model in a world of certainty ($\beta_{\Pi} = 0$).

The valuation a factory at anytime t assumes that the productivity factor remains constant over its life. This condition simply means that both the growth rate and volatility of the productivity factor will be zero and the factor will be some constant. The factory productivity will change as R&D progresses. At each time the productivity factor will be different and vary according to the description in equation 2.4. Therefore the value of the factory will be lognormally distributed with a mean drift of $\{\mu_{\Pi}t\}$ and a variance of $\{\sigma^2_{\Pi}t\}$.

¹² Breeden, D. T., and Litzenberger, R. H., "Prices of State-contingent Claims implicit in Options Prices," Journal of Business, 51 4 (1978), pp 621-651.

¹³ Brealey, R. and Myers, S., The Principles of Corporate Finance, Second Edition, Mc Graw Hill Book Co., New York, © 1984, pp 117-188.

CHAPTER 3: OPTION PRICING MODEL

The previous chapter estimated the value of the factory which is the underlying asset of the implied option to launch a innovative product. The option pricing model discussed in this chapter provides a value of R&D and the optimal time to launch the new product. The option pricing model derived by Mc Donald and Siegel includes uncertainty in the cost to launch a new product.¹⁴ In order to develop this option pricing model, we must assume that portfolios of securities exist that are perfectly correlated to the stochastic variations in factory value, investment cost, present value of R&D costs and option value. If no such portfolios exist, we assume that they can be constructed from existing securities. These portfolios will provide the risk adjusted costs of capital required by the option pricing model.

Effect of R&D on Factory Value

The factory valuation model implicitly assumes that productivity is constant throughout the factory life. Such an assumption regarding R&D of a innovative product excludes an important reason to continue R&D.^{15,16} Technology evolves by the process of continuous innovation. This process reveals more efficient techniques -- technology -- to produce an item. Excluding this effect assumes that

¹⁴ Mc Donald, R. and Siegel, D., *op. cit.*, pp 711-726.

¹⁵ Skinner, W., "Manufacturing - Missing Link in Corporate Strategy," Harvard Business Review," May-June 1969, pp 136-144.

¹⁶ Kantrow, A. M., "The Strategy Technology Connection," Harvard Business Review," July-August 1980, pp 6-21.

waiting for market demand is the only source of option value. Technology that increases the productivity of capital for an innovation reduces its cost and increases value. This effect was revealed in Chapter 2 by allowing the Cobb-Douglas production function to behave stochastically. The factory value will vary as a result of R&D as

$$dV_t / V_t = \mu_V dt + \sigma_V dz_3 \quad (3.1)$$

where μ_V is the expected growth rate of factory value and σ is its volatility.¹⁷ The stochastic variation of the factory value has a correlation to the market portfolio of

$$\rho_{VRm} = \rho_{\Psi Rm} + \rho_{\theta Rm} \quad (3.2)$$

and variance of

$$\sigma_V^2 = (g\sigma_\theta)^2 + (gh\sigma_\Psi)^2 + 2g^2h\sigma_{\theta\Psi}. \quad (3.3)$$

Valuing the R&D Option

An option pricing model for R&D differs from the Black-Scholes stock option pricing formula in several ways.^{18,19} R&D is similar to a capital asset because both are not traded or sold frequently, a consequence of limited secondary markets. Investors have more knowledge about security values than of R&D because active trading of securities determines market prices. Moreover, corporations know more about their assets than the average investor because understanding the complex integration of their technologies requires a high

degree of specialization. As a result of these differences, the Black-Scholes

¹⁷ Wiener process z_3 is a combination of processes z_1 and z_2 with the assumption that g , h , σ_θ , σ_Ψ , and $\sigma_{\theta\Psi}$ are constants.

¹⁸ Black, F. and Scholes, M., "The Pricing of Options and Corporate Liabilities," Journal of Political Economy, 81 (May-June 1973), pp 637-659.

¹⁹ Merton, R. C., "Theory of Rational Option Pricing," Bell Journal of Economics and Management, 4 (Spring 1973), pp 141-183.

formula is unsuitable to value options on capital assets or R&D options. The option pricing model discussed in this section is an optimal stopping formulation providing an option value and a decision rule.

An optimal stopping formula determines the conditions under which the opportunity cost of delaying investment exceeds the benefit of continuing R&D. The option value of R&D reaches its maximum at the optimal stopping condition. If investment occurs at this point, the firm receives the factory value, V_t , less investment cost, I_t , without losing any R&D option value. The optimal stopping formulation determines a schedule of values, C^*_t , for all times t such that the firm continues R&D if $V_t/I_t < C^*_t$, otherwise it builds the factory. Since we accept the Science Policy Research Unit observation of the unimportance of time limits, the schedule of C^*_t is independent of time, that is $C^*_t = C^*$.

The investment cost for the factory at the optimal exercise point is uncertain because of technological and economic uncertainties. Therefore, investment cost behaves stochastically as

$$dI_t / I_t = \mu_I dt + \sigma_I dz_4, \quad (3.4)$$

with correlation ρ_{IRm} to the market portfolio and covariance σ_{VI} with the factory value. With these conditions, the optimal stopping formulation as derived by McDonald and Siegel give the option value for R&D, X , as

$$X = (C^* - 1) I_0 [(V_0 / I_0) / C^*]^\epsilon, \quad (3.5)$$

where I_0 and V_0 are the current estimates of investment cost and factory value.

The exponent ϵ is

$$\epsilon = \left\{ \left[\frac{(\delta_I - \delta_V)}{\sigma^2} - 1/2 \right]^2 + 2 \delta_V / \sigma^2 \right\}^{1/2} + \left\{ 1/2 - \left[\frac{(\delta_I - \delta_V)}{\sigma^2} \right] \right\}, \quad (3.6)$$

where the terms δ_I and δ_V are the differences between the expected return of equivalent securities and the expected growth rates as given by,

$$\delta_I = r + \Phi \rho_{IRm} \sigma_I (\mu_I + \sigma_I^2 / 2) \quad (3.7)$$

and²⁰

$$\delta_V = r + \Phi \rho_{VRM} \sigma_V - (\mu_V + \sigma_V^2/2). \quad (3.8)$$

The variance term is,

$$\sigma^2 = \sigma_V^2 + \sigma_I^2 - 2\sigma_{VI}, \quad (3.9)$$

and the optimal boundary is

$$C^* = \epsilon/(\epsilon - 1). \quad (3.10)$$

Appendix A discusses the derivation of these relationships and the option pricing formula.

Equation 3.7 shows the similarities of an R&D option to a common stock option. The option value increases as the value of the underlying asset increases. The value of the R&D option has a positive value even when the factory value is less than its investment cost ($V_0/I_0 < 1$). The option value decreases as the factory investment cost rises (X goes to zero as V_0/I_0 goes to zero). A close examination of the option pricing model also shows that the option value increases as the total volatility increases.

²⁰ The return of an equivalent security is the market rate of return of a security with the same degree of financial risk.

CHAPTER 4: APPLICATION, LIMITATIONS, AND CONCLUSION

The previous chapters developed a method to choose between launching an innovative product and continuing R&D. This chapter presents a simple numerical example and discusses the limitations of the option pricing model proposed in this thesis.

Numerical Example

This section provides a simple numerical example describing an application of the option pricing model to a contrived decision problem. We shall assume that Company X's marketing and engineering departments provide the data in table 4.1. By using equations 2.8, 2.9, 2.11, 2.14, and 2.15, we can calculate the current value of launching a new product (V_0) for μ_ψ and σ_ψ both equal to zero. Equations 2.11 and 2.14 with μ_ψ and σ_ψ equal to their values in the table gives the growth rate of the factory value (μ_V) and its volatility (σ_V). Engineering also provides a current estimation of a factory value and an investment cost (I_0), the expected growth rate of investment cost (μ_I), its volatility (σ_I), and its correlation to the market portfolio (ρ_{IRm}). The factory value and investment cost are for a profit maximizing factory. A financial analyst also supplies the long term riskfree interest rate (r) and the market price of risk (Φ). The results of calculations and this additional data are summarized in table 4.2.

The variance, σ^2 , is computed using equation 3.9. The terms δ_I and δ_V are computed using equations 3.7 and 3.8. With these computed values, we use equations 3.6 and 3.10 to find ϵ and the stopping boundary, C^* . Because the

Growth rate of demand	μ_{θ}	0.020
Volatility of μ_{θ}	σ_{θ}	0.283
Correlation between μ_{θ} and r_M	$\rho_{\theta R_M}$	0.380
Growth rate of productivity	μ_{ψ}	0.035
Volatility of μ_{ψ}	σ_{ψ}	0.460
Correlation between μ_{ψ} and R_M	$\rho_{\psi R_M}$	0.220
Correlation between μ_{θ} and μ_{ψ}	$\rho_{\theta\psi}$	0.050
Price elasticity of demand	η	1.75
Productivity of labor	α	0.640
Productivity of capital	β	0.360
Estimate of profit for first year	Π^*_0	14.234

Table 4.1: Example Data

Current Factory Value	V_0	132.00
Expected Growth Rate of V_0	μ_V	0.0269
Volatility of μ_V	σ_V	0.4670
Correlation between μ_V and R_M	ρ_{VRM}	0.4500
Current Investment Cost	I_0	120.00
Expected Growth Rate of I_0	μ_I	0.0800
Volatility of μ_I	σ_I	0.220
Correlation between μ_I and R_M	ρ_{IRM}	0.3500
Longterm Riskfree Interest Rate	r	0.0100
Market Price of Risk	Φ	1.7306
Correlation Between I_0 and V_0	ρ_{VI}	0.8986

Table 4.2: Results of Calculations and Investment Data

σ^2	0.0818
δ_I	0.0633
δ_V	0.2377
ϵ	5.543
C^*	1.220
V_0/I_0	1.100
X	14.863
$V_0 - I_0$	12.00

Table 4.3 Summary of Option Value Calculations

current ratio of factory value to investment cost is less than C^* , we will continue R&D. The value of the option to launch a new product is calculated using equation 3.5. The option is more valuable than launching the product ($X = 14.863 > (V_0 - I_0) = 12.00$).

Acquiring Data for the Option Pricing Model

The application of this simple option pricing formula to actual R&D project management requires the accumulation of data from marketing, engineering, research, and manufacturing functions within a corporation. Quantifiable data, especially data on capital productivity, should be readily obtainable from the operation of similar manufacturing technology or a pilot plant. Marketing data and models supply market demand characteristics.. With these data, evaluations of R&D projects are possible.

Another source of data for these calculations is a historical record of factory valuations and investment costs. These data may be from the historical performance of similar R&D efforts. With these data, calculations of correlation coefficients are readily estimable after correcting for time averaging effects. Collecting these data also requires the integration of marketing, engineering, manufacturing, and R&D functions.

Limitations of the Simple Model

The option pricing model developed in this thesis only considers Weiner processes to model the random, observable factors such as demand and productivity. An actual decision may involve a discontinuous jump in demand

because a competitor builds a factory producing a similar product. The development of this simple model also assumed a constant growth rate and volatility parameters that most likely are time dependent. Solution of the optimal stopping problem requires numerical programming techniques to value options in both situations.

The accuracy and availability of data also limits application of this model to actual R&D project management. Accuracy of data may depend on a company's degree of familiarity with the product market and technologies rather than on methods of collecting and analyzing data. The more unfamiliar a company is with either the markets or technologies of an innovative product, the more inaccurate its data are likely to be. Obtaining data from sources with similar experience in markets and technologies relating to the new product becomes more important.

The greatest limitation of this model is that it only considers technical, economic, and financial data. Human factors also must also be included into any decision regarding the management of R&D projects. The blind application of any decision rule based only upon observable, quantifiable data ignores that R&D resources also are the knowledge and creativity of people. This model does not and cannot include these factors into any of the equations contained in this thesis. For this reason, the best application of an option pricing model for R&D projects is for planning technology strategies. This model or one similar to it may provide the tool to determine the new product areas for R&D.

Conclusion

A simple analytic option pricing model suitable for estimating the option value of an R&D project is obtainable by using an optimal stopping formulation. The option pricing model exhibits characteristics similar to common stock options. Applying the method to actual decisions may require numerical solution techniques of the optimal stopping problem. Actual considerations may include growth rate, volatility, and correlation terms as time dependent factors. Inclusion of these factors is beyond the scope of of this analytic model. Although this formula is limited, it reveals the value of applying a more general numerical formulation to the strategic management of R&D.

APPENDIX A: THE OPTIMAL STOPPING PROBLEM

The option pricing model derived by Mc Donald and Siegel is an optimal stopping problem.²¹ This formulation answers the question: When does the expected increase in factory value from continuing R&D become less than the opportunity cost of waiting to build a factory? The answer to the question is the optimal time to stop R&D, hence the name optimal stopping problem. In the case of R&D, we assume that the factory value and its investment cost are stochastic with,

$$dV_t/V_t = \mu_V dt + \sigma_V dz_3, \quad (\text{A.1})$$

and

$$dl_t/l_t = \mu_l dt + \sigma_l dz_4. \quad (\text{A.2})$$

Whenever the firm builds the factory, it receives $V_t - I_t$ of wealth. A schedule of optimal stopping conditions $\{C_t^*\}$ exist with $C_t^* = V_t/l_t$. The option value of the right to make a decision in the future is the discounted expected wealth received in the future,

$$X(T) = E_0\{\exp(-\mu t) [V_t - I_t]\} \quad (\text{A.3})$$

²¹ Mc Donald, R. and Siegel, D., *op. cit.*, pp 711-726.

where μ is a risk adjusted discount rate that is assumed to be known. The optimal stopping problem maximizes the value of the option by finding the optimal stopping conditions or boundaries. In the case of a finite expiration on the option, equation A.3 cannot be solved analytically and requires numerical solution. By including Freeman's observation that time was not a factor influencing commercial success of technical innovations, an expiration date may be dropped from the derivation.

In this situation the optimal stopping problem may be solved because the boundary, C_1 , is fixed and independent of time. The mathematical program of this problem is,

$$\max [C^*-1]E_0\{I'\exp(-\mu t)\}, \quad (\text{A.4})$$

where $V' = kV$ and $I' = kI$. The expectation of the discounted investment cost is solved by noticing that the expected value will vary stochastically. We let

$$L(V_3, I_3, 0) = E_0\{I'\exp(-\mu t)\}. \quad (\text{A.5})$$

Theorem 7.5 from Malliaris and Brock and the fact that $L_3 = \mu L$ show that L must satisfy the partial differential equation²²,

$$\mu L = \{L_{VV}V^2\sigma^2_V + L_{II}I^2\sigma^2_I + 2L_{VI}VI\sigma_{VI}\}/2 + L_VV\mu_V + L_I I\mu_I. \quad (\text{A.6})$$

The solution is of the form,

$$L = k I^a C^b \quad (\text{A.7})$$

with the following boundary conditions:

$$L = I \text{ when } C = V/I = C^*;$$

and

$$L \text{ goes to zero as } V/I \text{ goes to zero.}$$

The first boundary condition requires $k = C^{*-b}$ and $a=1$. The exponent b is found by rewriting equation A.6 as,

²² Malliaris, A. G., and Brock, W. A., Stochastic Methods in Economics and Finance, North Holland Publishing Co., © 1982, pp 100-101.

$$\mu = b(b-1)\sigma^2/2 + b\mu_V + (1-b)\mu_I, \quad (\text{A.8})$$

where

$$\sigma^2 = \sigma_V^2 + \sigma_I^2 - 2\sigma_{VI}. \quad (\text{A.9})$$

The second boundary condition requires $\mu_I < \mu$ to obtain both a positive and negative root. The positive root, which is the correct one, is given by

$$b = \left\{ \left[\frac{(\mu_V - \mu_I)}{\sigma^2} - 1/2 \right]^2 + 2(\mu - \mu_I/\sigma^2) \right\}^{1/2} + \left\{ 1/2 - \frac{(\mu_V - \mu_I)}{\sigma^2} \right\}. \quad (\text{A.10})$$

The optimal stopping condition C^* is determined from the solution of the mathematical program as,

$$C^* = b/b-1, \quad (\text{A.11})$$

where b is the ϵ in the text.

The risk adjusted cost of capital, μ , is found by applying Ito's Lemma to the option value formula,

$$X = (C^* - 1)I_0[(V_0/I_0)/C^*], \quad (\text{A.12})$$

to obtain

$$dX/X = [b\mu_V + (1-b)\mu_I - b(b-1)\sigma^2/2]dt + b\sigma_V dz_3 + (1-b)\sigma_I dz_4. \quad (\text{A.13})$$

The unanticipated returns in equation A.13 are $b\sigma_V dz_3 + (1-b)\sigma_I dz_4$. The expected rate of return for a security equivalent to X will be the riskfree rate plus the increase compensation for riskiness weighted according to the unanticipated returns in equation A.13. If we define

$$r_I = r + \Phi_{PI} R_m \sigma_I$$

and

$$r_V = r + \Phi_{PV} R_m \sigma_V,$$

and set the risk adjusted discount rate, μ , to the weighted average of r_I and r_V as prescribed by equation A.13, equation A.8 becomes

$$\mu = br_V + (1-b)r_I = b(b-1)\sigma^2/2 + b\mu_V + (1-b)\mu_I. \quad (\text{A.14})$$

We obtain equation 3.6 by defining the terms δ_I and δ_V as the differences between the expected return of equivalent securities and the expected growth rates,

$$\delta_I = r_I - \mu_I .$$

and

$$\delta_V = r_V - \mu_V .$$