FRAME-TO-FRAME EXTRAPOLATION OF TELEVISION FIELDS

by

JAMES JOSEPH KELLEHER JR.

SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
June, 1960

Signature of Author Signature redacted
Department of Electrical Engineering, May 21, 1960

Certified by Signature redacted
Thesis Supervisor

Accepted by Signature redacted
Chairman, Departmental Committee on Graduate Students

FRAME-TO-FRAME EXTRAPOLATION OF TELEVISION FIELDS

by

JAMES JOSEPH KELLEHER JR.

Submitted to the Department of Electrical Engineering on May 21, 1960 in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

This paper describes a procedure for the frame-to-frame extrapolation of television fields taken from a motion sequence. From two know frames of a motion sequence it is desired to estimate the motion that has occurred between frames and to predict the next picture of the series. To do this a rather limited model of the motion is adopted whereby corresponding areas of the two frames are assumed to be identical except for a shift of position. Under this assumption the process attempts to solve for the horizontal and vertical components of the motional displacement at each point in the picture.

This solution is carried out by expressing the second frame in terms of the first frame and the unknown motion by means of a Taylor series approximation. The resulting expressions are then used to derive linear equations that can be solved for the unknowns, and the extrapolation itself is carried out by repeating the motion that has been calculated by this method. A similar procedure is used to interpolate a picture between two frames of a motion sequence.

Two processes are considered, one using a first order Taylor series expansion and the other for a second order approximation. An experimental evaluation has been carried out for both processes by simulation on an IBM 704 digital computer. Results were obtained for frames taken from a real motion sequence as well as for artificial sequences in which the assumed model of the motion was known to be valid. In all of the results only horizontal motion was considered so that the effect of processing the motion field itself could be observed.

Thesis Supervisor: Peter Elias
Title: Associate Professor of Electrical Engineering

ACKNOWLEDGEMENTS

The work reported in this paper was carried out as part of a cooperative course plant assignment in the visual and acoustics research department of the Bell Telephone Laboratories in Murray Hill, New Jersey. The author would like to express his gratitude to all the members of that department, particularly to H. S. McDonald and P. P. Giordano. Special thanks are due to W. A. Anderson of the visual research department, who was of inestimable value during the course of this investigation.

TABLE OF CONTENTS

FIGURES

I. Introduction

Modern communication theory has shown that the channel capacity required to transmit uncoded signals in general is far greater than the actual rate of information flow. This difference in rate is due to redundant or superfluous information in the message (1,2). Redundancy is perhaps most evident in television signals where there is usually very little difference between successive scanning lines in a given picture and where each frame of a motion sequence is very much like the next. Since the redundancy is so large video communication requires a relatively large bandwidth, making long distance communication by cable very expensive. For example, the binary PCM transmission of conventional television, consisting of 525 scanning lines with about 500 resolvable points per line and a rate of thirty frames per second, requires a bandwidth of 25 megacycles if one allows 128 brightness levels for each point or seven binary digits per sample (3).

Upper bounds on the source rate have been calculated from statistical data for various pictures, and the results indicate a rate of two to three bits per samples, less than one-half of the channel capacity mentioned above (4,5). However, these results are based on the short-term, local statistics in the pictures and include neither the more removed high order probabilities of a given frame nor the correlation between frames of a motion sequence. Thus, the actual source rate is undoubtedly lower than the results of these calculations.

As a result many experiments have been proposed and carried out in an attempt to reduce the required channel capacity by efficient coding. These have included point-to-point and line-to-line prediction (6), run-length coding (7,8), and piecewise linear approximations to the video waveform (9). These methods have resulted in picture transmission of reasonable quality with one to three bits per sample. Run-length coding and piecewise linear approximation have as yet been applied only in the horizontal direction, and presumably a further saving would result if these methods were applied in the vertical direction as well. Other systems have been developed to save bandwidth by taking advantage of the characteristics and limitations of the observer, the ultimate receiver of the picture (10,11). It is often possible to relax the requirements on the channel by removing or misinterpreting information to which the observer is not sensitive (12,13).

In any case, all of the bandwidth reduction schemes mentioned above have dealt only with information within a given frame, and thus far no work, at least none of an experimental nature, has dealt with the processing of information between frames of a motion sequence. This paper describes a procedure for the frame-to-frame extrapolation of television pictures, that is, from two known frames of a motion sequence it is desired to determine the motion that has taken place between frames and to predict the third picture of the series. To describe the relationship between two such frames a rather limited model of the motion is adopted whereby the corresponding areas in the two frames are assumed to be identical except for a shift of position. Under this assumption the process attempts to solve for the horizontal and vertical components of the motional displacement at each point in the picture.

To determine the displacement at a given point the second frame is expanded in a Taylor series approximation about this point in terms of the first frame and the unknown motion. The resulting expressions are then used to derive linear equations that can be solved for the unknowns, and this calculated description of the motion is applied as an operator on the second frame to produce a prediction for the third frame of the series. The same process is also used to interpolate a picture between two frames of a motion sequence. Two different processes are used, one for a first order Taylor approximation and another for a second order expansion.

II. Representation of Motion

For these processes the relationship between two frames of a motion sequence is assumed to be a simple shift, where the magnitude and direction of the shift may vary from point to point. Thus, the two frames can be expressed as

(2.1) first frame: $f_a(x,y) = f(x,y)$

(2.2) second frame: $f_b(x,y) = f(x+u,y+v)$

Here $f_a(x,y)$ and $f_b(x,y)$ represent the brightness at the point (x,y) in the first and second frames. The motion is described by u and v, the horizontal and vertical components of the motional displacement at (x,y), which are themselves functions of x and y.

This characterization of the motion does not account for sequences in which picture material is hidden or revealed by a moving object. Moreover, the use of this model for the portrayal of motion implicitly assumes that shadows or changes in illumination induced by the motion do not change the brightness levels in corresponding areas of the two frames.

III. First Order Process

This process uses a first order Taylor series approximation to derive linear equations in u and v. First, $f_b(x,y)$ is expanded in terms of $f_a(x,y)$ at each point (i,j), so that every point is associated with a linear equation for the unknowns as shown below.

(3.1) $f_b(i,j) = f_x(i,j)\cdot u + f_y(i,j)\cdot v + f_a(i,j)$

Here $f_x(i,j)$ and $f_y(i,j)$ are the first partial derivatives of $f_a(x,y)$ at the point (i,j).

(3.2) $a\cdot u + b\cdot v = c$

where $a = f_x(i,j)$

$b = f_y(i,j)$

$c = f_b(i,j) - f_a(i,j)$

To solve for u and v another equation, corresponding to a point near (i,j), is determined and solved in conjunction with the equation for the point (i,j). The two equations

(3.3a) $a_i u + b_i v = c_i$

(3.3b) $a_j u + b_j v = c_j$

have the solution

(3.4a) $u = \dfrac{c_i b_j - c_j b_i}{a_i b_j - a_j b_i}$

(3.4b) $v = \dfrac{a_i c_j - a_j c_i}{a_i b_j - a_j b_i}$

provided that the determinant, $a_i b_j - a_j b_i$, does not vanish. If the determinant is zero then the equation for the point (i,j) is solved in conjunction with the equation for some other point near (i,j), and so on.

If no pair of points in a given elementary region around (i,j) results in a non-zero determinant then there is no solution for the displacement at this point. If this is the case then

(3.5) $a_i b_j = a_j b_i$

for all points in the region. This means that the direction of the gradient of $f_a(x,y)$ is the same throughout the region, implying that the contours of constant amplitude are parallel lines. However, in this situation any motion perpendicular to the gradient would not be noticed by an observer, for such a displacement would be parallel to the lines of constant brightness. So we need to solve only for the component of motion in the direction of the gradient. If we denote the true displacement by the vector

$$(3.6)\quad \vec{r} = u \cdot \vec{i}_x + v \cdot \vec{i}_y$$

and the gradient of $f_a(x,y)$ at (i,j) by

$$(3.7)\quad \vec{g} = a \cdot \vec{i}_x + b \cdot \vec{i}_y$$

then we have

$$(3.8)\quad (\vec{r},\vec{g}) = a \cdot u + b \cdot v = f_b(i,j) - f_a(i,j) = c$$

where $(\vec{r},\vec{g})$ is the scalar product of the two vectors. Now the component of motion parallel to the gradient will be written as

$$(3.9)\quad \vec{r}' = u' \cdot \vec{i}_x + v' \cdot \vec{i}_y$$

and is given by

$$(3.10)\quad \vec{r}' = (|\vec{r}| \cdot \cos(Z)) \cdot \frac{\vec{g}}{|\vec{g}|}$$

where Z is the angle between $\vec{r}$ and $\vec{g}$.

$$(3.11)\quad \vec{r}' = \frac{(\vec{r},\vec{g})}{|\vec{g}|} \cdot \frac{\vec{g}}{|\vec{g}|} = \frac{(\vec{r},\vec{g})}{|\vec{g}|^2} \cdot \vec{g}$$

$$(3.12)\quad \vec{r}' = \frac{c}{|\vec{g}|^2} \cdot \vec{g}$$

$$(3.13)\quad \vec{r}' = \frac{c}{(a^2+b^2)} \cdot (a \cdot \vec{i}_x + b \cdot \vec{i}_y)$$

So the modified solution is given by

$$(3.14a) \quad u' = \frac{a \cdot c}{a^2+b^2}$$

$$(3.14b) \quad v' = \frac{b \cdot c}{a^2+b^2}$$

A flow diagram for this procedure is shown in Figure 1. First the size of the elementary region is set and pairs of equations are determined for the points in this region. To conserve computation not all of the points in the region are used, and not all of the possible pairings of those points used are taken. The points used in the computation for elementary regions of 7 x 7 and 5 x 5 points are shown in Figure 2. Such points are taken two at a time and a determinant, $a_i b_j - a_j b_i$, is calculated for each pair. Then a count is made of the number of pairs associated with a determinant greater in magnitude than a threshold T_1. If the count is greater than a decision level M, these pairs are solved according to (3.4), and the average of the solutions is taken as the displacement.

If the count is less than or equal to M, it is assumed that the direction of the gradient in the first frame is changing too slowly in the region for the solutions as given by (3.4) to have any meaning. The routine then proceeds to calculate the component of the motion parallel to the gradient. A similar procedure is followed here, taking the points one at a time and counting the number of points for which the quantity $(a_i^2-b_i^2)$ is greater than a second threshold T_2. If this count is greater than another decision level N, the solutions are taken according to (3.14) and averaged to determine the displacement. If the count is less than or equal to N, the displacement is assumed to be zero.

The effect of the process, in one dimension, on a displaced ramp is shown in Figure 3. This function, which was used for testing, is described by

$$(3.15a)\quad f_a(x) = \begin{cases} 0 & x \leqq 10 \\ 15(x-10) & 10 \leqq x \leqq 50 \\ 600 & 50 \leqq x \end{cases}$$

$$(3.15b)\quad f_b(x) = \begin{cases} 0 & x \leqq 15 \\ 15(x-15) & 15 \leqq x \leqq 55 \\ 600 & 55 \leqq x \end{cases}$$

For this function the maximum error from the desired prediction occurs when the ramp is leveling off. In this region the first frame is changing relatively slowly, so that the corresponding determinants are small, while the difference between the two frames is still large. Thus, the calculated displacement in this region is larger than the actual displacement.

IV. Second Order Process

If a second order Taylor series approximation is made at the point (i,j), the resulting expression is

$$(4.1)\quad f_b(i,j) = au^2 + buv + cv^2 + du + ev + f_a(i,j)$$

where $a = 1/2(f_{xx}(i,j))$

$b = f_{xy}(i,j) = f_{yx}(i,j)$

$c = 1/2(f_{yy}(i,j))$

$d = f_x(i,j)$

$e = f_y(i,j)$

Now consider another point close to (i,j) denoting this point by (i+h,j+k). Under the assumption that the motional displacement is a slowly changing function of position the displacement at (i+h,j+k) should be approximately equal to the displacement at (i,j) and we can write

$$(4.2) \quad f_b(i+h,j+k) = a(u+h)^2 + b(u+h)(v+k) + c(v+k)^2 + d(u+h) + e(v+k) + f_a(i,j)$$

Subtracting $f_b(i,j)$ from $f_b(i+h,j+k)$

$$(4.3) \quad f_b(i+h,j+k) - f_b(i,j) = a(2uh + h^2) + b(uk + vh + hk) + c(2ck + k^2) + dh + ek$$

or $$(4.4) \quad (2ah + bk)u + (bh + 2ck)v = M(h,k)$$

where

$$(4.5) \quad M(h,k) = f_b(i+h,j+k) - f_b(i,j) - ah^2 - bhk - ck^2 - dh - ck$$

Thus, we have a linear equation for u and v, and if this is done for two points, $(i+h_1,j+k_1)$ and $(i+h_2,j+k_2)$, we have the two equations

$$(4.6a) \quad (2ah_1+bk_1)u + (bh_1+2ck_1) = M(h_1,k_1)$$

$$(4.6b) \quad (2ah_2+bk_2)u + (bh_2+2ck_2) = M(h_2,k_2)$$

The solution to these is given by

$$(4.7a) \quad u = \frac{(bh_2+2ck_2)\cdot M(h_1,k_1) - (bh_1+2ck_1)\cdot M(h_2,k_2)}{(h_1k_2-h_2k_1)\cdot(4ac-b^2)}$$

$$(4.7b) \quad v = \frac{(2ah_1+bk_1)\cdot M(h_2,k_2) - (2ah_2+bk_2)\cdot M(h_1,k_1)}{(h_1k_2-h_2k_1)\cdot(4ac-b^2)}$$

provided that the determinant does not vanish, or provided that the following conditions are satisfied.

(4.8a) $h_1k_2 \neq h_2k_1$

(4.8b) $4ac \neq b^2$

The first of these, which must be observed at all times if Equations (4.6) are to be independent, is a condition on the points chosen and requires that the three points, $(i+h_1, j+k_1)$, (i,j), $(i+h_2, j+k_2)$, must not be collinear. The second is expressed in terms of quantities derived from $f_a(x,y)$, which cannot be controlled, so that no solution exists if

(4.9a) $4ac = b^2$

or (4.9b) $(f_{xx})\circ(f_{yy}) = (f_{xy})^2$

But in this case the contours of constant amplitude are parallel lines since the solution to the partial differential equation of (4.9b) is of the form

(4.10) $f(x,y) = g(Ax+By)$

where A and B are constants. This situation is the same as that described in the previous section and we need to solve only for motion parallel to the gradient, which will be given by the equations

$$(4.11a) \quad (2ah_1+bk_1)u' + (bh_1+2ck_1)v' = M(h_1,k_1)$$

$$(4.11b) \quad eu' - dv' = 0$$

so that the modified solution is

$$(4.12a) \quad u' = \frac{M(h_1,k_1)\cdot d}{(2ah_1+bk_1)d + (bh_1+2ck_1)e}$$

$$(4.12b) \quad v' = \frac{M(h_1,k_1)\cdot e}{(2ah_1+bk_1)d + (bh_1+2ck_1)e}$$

A flow diagram for this process is shown in Figure 4 for the particular case $(h_1=k_2=1, h_2=k_1=0)$. The organization of this procedure is quite similar to that for the first order routine except that no counting or averaging is done.

V. Picture Format

A study of these processes has been carried out through simulation routines on an IBM 704 digital computer. The pictures themselves are recorded on computer tapes by the slow scan video system to be described (14). First the video signal is obtained from a flying spot scanner operating on positive transparencies mounted in 2-inch slides. This signal is then filtered through a 2.5-kc low-pass filter and sampled at the Nyquist rate of 5-kc. The samples are quantized uniformly into 1024 levels (10 binary digits per sample) and recorded on magnetic tape. The frame time is 2.4 seconds and the line time 24 milliseconds, so each picture consists of 100 lines of 120 samples each.

Thus, a picture is a rectangular array of 12,000 samples, corresponding to about 1/25 of a conventional picture. Of the 100 lines the first serves as vertical sync and blanking and fifteen samples in every other line serve as horizontal sync and blanking. Black level is set at a sample value of 1024, white at zero, and sync at 1500 as shown in Figure 5.

After recording, successive frames are separated by an end-of-file gap and each file is divided into 25 tape records, each containing four lines of television samples. Each sample is made up of two tape characters, so a 704 word on tape consists of three samples. Input-output routines break each word up into its samples and stores them in a 12,000 word buffer of core memory when a tape is read, while this process is reversed when a tape is written by the computer. All the programs for the extrapolation processes were written in FORTRAN, while some of the shorter routines, such as those for averaging, shifting, and tape correction, were programmed in SAP.

VI. Approximation to Derivatives

As might be expected, finite differences were used to approximate the partial derivatives. As a first approximation the average of the first foreward and first backward differences was taken as the first derivative, and the second derivative was represented by the second difference. So if the picture is specified by the sample matrix shown in Figure 6, the expressions for the partial derivatives are

$$(6.1)\quad f_x(i,j) = 1/2\,[f(i+1,j) - f(i-1,j)]$$

$$(6.2)\quad f_y(i,j) = 1/2\,[f(i,j-1) - f(i,j+1)]$$

$$(6.3)\quad f_{xx}(i,j) = f(i+1,j) - 2\,f(i,j) + f(i-1,j)$$

$$(6.4)\quad f_{yy}(i,j) = f(i,j-1) - 2\,f(i,j) + f(i,j+1)$$

$$(6.5)\quad f_{xy}(i,j) = 1/4[f(i+1,j-1) - f(i+1,j+1) - f(i-1,j-1) + f(i-1,j+1)]$$

The use of polynomial expressions and other interpolation functions to approximate the partial derivatives was considered but was not attempted because of the greater computation time involved. For example a second order polynomial would give the same results as (6.1) through (6.5) except that the expression for $f_{xy}(i,j)$ would not be as accurate. Thus, a polynomial of at least third order is required to improve on these approximations.

VII. Extrapolation and Interpolation

After u and v have been determined at (i,j) we have

$$(7.1)\quad f_b(i,j) = f_a(i+u,j-v)$$

and if the motion is repeated then

$$(7.2)\quad f_c(i,j) = f_b(i+u,j-v)$$

where $f_c(i,j)$ represents the third frame of the series. Now let m be the closest integer to i+u and let n be the integer nearest j-v, so that (m,n) is the point with integral coordinates closest to (i+u,j-v). Then $f_c(i,j)$ is approximated by

$$(7.3)\quad f_c(i,j) = f_b(m,n)$$

For interpolation we have

$$(7.4)\quad f_{ab}(i,j) = f_b(i+u/2,j-v/2)$$

where $f_{ab}(i,j)$ is the frame that is to be interpolated between $f_a(i,j)$ and $f_b(i,j)$. Then an identical procedure is applied to express $f_{ab}(i,j)$ in the form of (7.3).

VIII. Actual Procedure

In the experimental evaluation of these processes on an IBM 704 digital computer the motional field was not calculated from the original pictures but from a smoothed or filtered version of these pictures. Since the originals are sampled at the minimum rate, the Nyquist frequency, one would not expect the finite difference approximations to the derivatives to be very accurate. Moreover, the higher order derivatives in the original picture may be so large that the Taylor approximation has no meaning.

As a result the calculations are carried out on the pictures after a smoothing operation is carried out. The smoothing, which operates in two dimensions, is done in the computer by successive applications of an eight-point averaging routine in which each sample is replaced by the arithmetic mean of that sample's eight nearest neighbors. The result is a video signal that is oversampled, thereby decreasing the high order derivatives and improving the accuracy of the difference approximations. The extrapolation itself, however, since it is trying to predict a detailed version of the third frame, is applied to the original detailed version of the second frame.

In addition, the process is limited to horizontal motion, without considering the vertical components. This makes it possible to write the horizontal motion field on tape, in the normal television format, so that processing routines may be applied to the motion field itself.

IX. Results

The picture material used in the processes is shown in Figure 7. Here (a) and (b) are the originals while (c) and (d) are the corresponding filtered versions. In this case the filtering is due to five successive applications of the eight-point averaging routine, so each sample in a filtered frame is a weighted sum of 121 samples in the corresponding original. The test chart (e) is shown to illustrate the quality of the recording and playback system.

The processes were run on three different motion sequences, two of which were artificial. In these two sequences the motion was created in the computer by shifting an entire frame horizontally, once for a shift of two Nyquist intervals and again for a shift of four intervals. Thus, in these two cases the assumed model for the motion is known to be valid throughout the frame. Other runs were made on frames (a) and (b) of Figure 7, which were taken from an actual motion sequence. For these pictures the model of the motion is not valid in a large part of the frame because of brightness changes induced by the motion.

In all of the runs of the first order processes the two thresholds T_1 and T_2, as illustrated in Figure 1, were both set at a value of 100. The two decision levels, M and N, varied with the size of the elementary region used in the calculations. For the 5x5 region, in which six pairs were taken from nine points, M and N were 3 and 6 respectively, while for the 7x7 region, using twelve pairs from sixteen points, they were set at 6 and 10. In the second order routines, as illustrated in Figure 4, two cases were considered, namely $(h_1=k_2=1, h_2=k_1=0)$ and $(h_1=k_2=2, h_2=k_1=0)$.

Figure 8 shows the results of the first order processes operating on the artificial sequence for which the horizontal displacement was two Nyquist intervals. (At this speed an object would traverse a conventional television screen in about eight seconds.) Pictures are shown for both the extrapolation and the interpolation. The effect of filtering the horizontal motion field is shown, where the filtering is the result of two applications of the eight-point averaging routine. The filtering here is quite beneficial as far as a subjective evaluation of the result is concerned. This is because the filtering increases the horizontal and vertical correlation in the motion field, thereby increasing the correlation in the calculated picture. The superiority of the use of the 7x7 region over the use of the 5x5 is evidently due to the fact that the former uses more points in the calculations, resulting in a higher correlation in the motion field.

Figure 9 illustrates the effect of the second order processes, using the two cases mentioned above, on the same motion sequence. The $(h_1=k_2=1, h_2=k_1=0)$ routine is more satisfactory than the other, as one would expect from the development, although the difference between the two is not very great. Again the quality of the results is greatly improved by the filtering of the motion field.

Figure 10 again shows results obtained from the first order routines, but in this case the processes are operating on the artificial sequence for which the horizontal displacement is four Nyquist intervals. Figure 11 shows the effect of the second order routines on the same sequence. Clearly the processes deteriorate rapidly as the magnitude of the motional displacement increases, and the improvements brought about by the smoothing of the motion field is more apparent than in the previous results.

Figure 12 demonstrates the results of applying the first order routines to frames from a real motion sequence. The previous comments are still valid, but in this case the effect of brightness changes between frames is illustrated. Figure 13 shows the corresponding results for the second order processes. These routines are not affected by the brightness changes as seriously as is the first order process. This is because the difference term, $f_b(i,j)-f_a(i,j)$, is cancelled out when the second order approximation is reduced to derive linear equations for the unknown motion, so that in this routine the changes in the brightness levels have an indirect effect on the solutions. However, in the first order process this difference term does not cancel out, and the solutions depend directly on these terms. Thus, if there is an appreciable change in the brightness levels between the two frames, as in this last sequence, the resulting errors will be greater for the first order process than for the other.

X. Conclusions

The results of the processes, when evaluated by a subjective appraisal, are reasonably good for picture sequences in which the model of the motion is valid and where the magnitude of the motion is small. Such results are shown in Figures 8 and 9, where the horizontal displacement is two Nyquist intervals. For this sequence the motion is accurately determined by the routines, and little distortion of the scene is produced. The quality of the results is substantially improved by the application of a filtering routine to the motional field.

However, as the magnitude of the motion is increased, the quality of the results deteriorate and serious distortions of the moving object are introduced. Moreover, a great deal of distortion occurs when there are differences in the brightness levels of the two frames, particularly from the first order process. Filtering the motion is again beneficial, but the distortions are still quite conspicuous. In addition, it must be remembered that these results deal only with horizontal displacements, and it is evident that further distortion would result if vertical motion were also considered.

Of course, one cannot say that these processes would effect an acceptable portrayal of motion, for a real time experiment is required to decide this. Nevertheless, judging from Figures 10 and 11 one would not expect this to be the case. The displacement of four Nyquist intervals for these figures is not a very large velocity as far as conventional television is concerned, and yet very serious distortions are introduced by the extrapolation routines. These distortions would almost certainly be objected to by a human observer, as would those of Figures 12 and 13.

Thus, it seems that only relatively small displacements can be handled acceptably by considering just the information in small, local areas of a frame. To deal successfully with larger motion more extensive regions of the picture must be considered in an attempt to identify correspondences in intensity patterns between frames of a motion sequence. This approach is equivalent to the problem of object identification and it is felt that a satisfactory extrapolation process must incorporate a solution to this problem.

In future motion studies it will be desirable, if not actually necessary, to know the relationships between the object velocity and the fidelity criterion of the observer. For example, one would expect that visual accuity would decrease as the rate of motion increases, and it may be possible to take advantage of this characteristic in a motion extrapolation routine. At high velocities the observer might tolerate distortion of the object as well as a loss of detail and this could be of some use. So in conclusion the author would like to recommend that studies be undertaken to determine the fidelity criterion as a function of the object velocity. The results of such an experiment would then determine the conditions that an extrapolation process would have to satisfy.

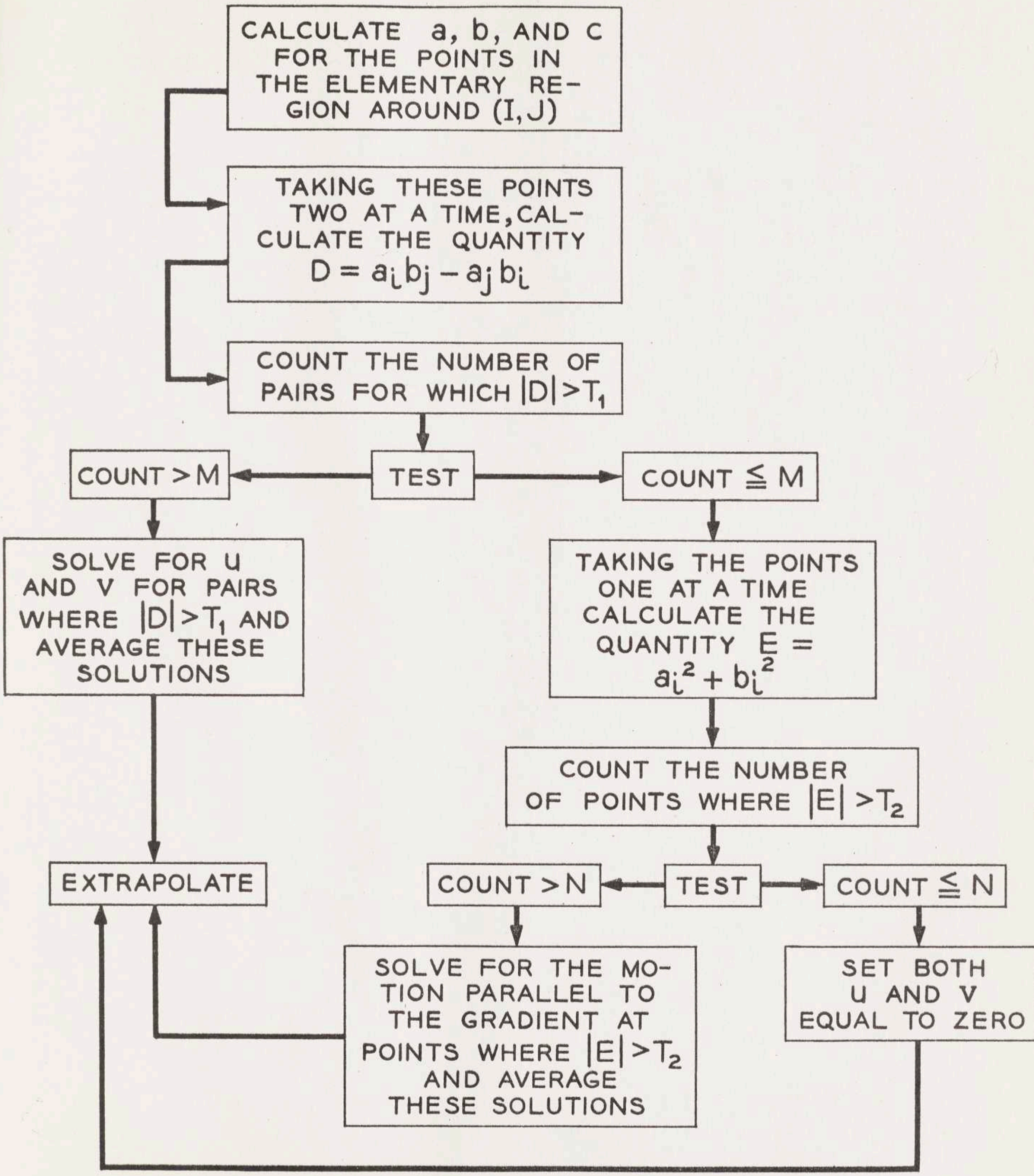
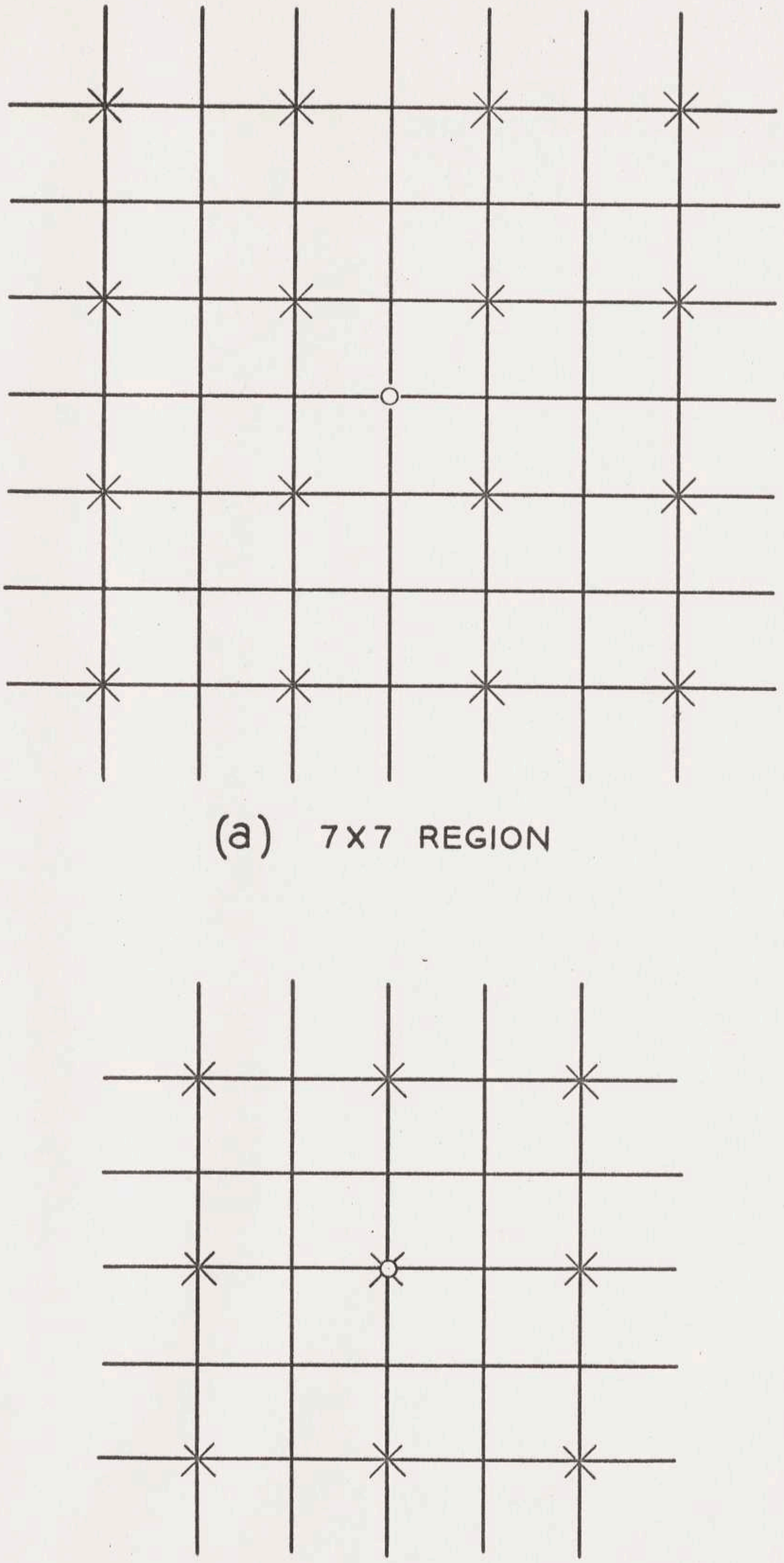


FIG. 1 FLOW CHART FOR FIRST ORDER PROCESS



(a) 7X7 REGION

(b) 5X5 REGION

FIG. 2 POINTS USED FOR CALCULATIONS IN ELEMENTARY AREAS FOR FIRST ORDER PROCESS

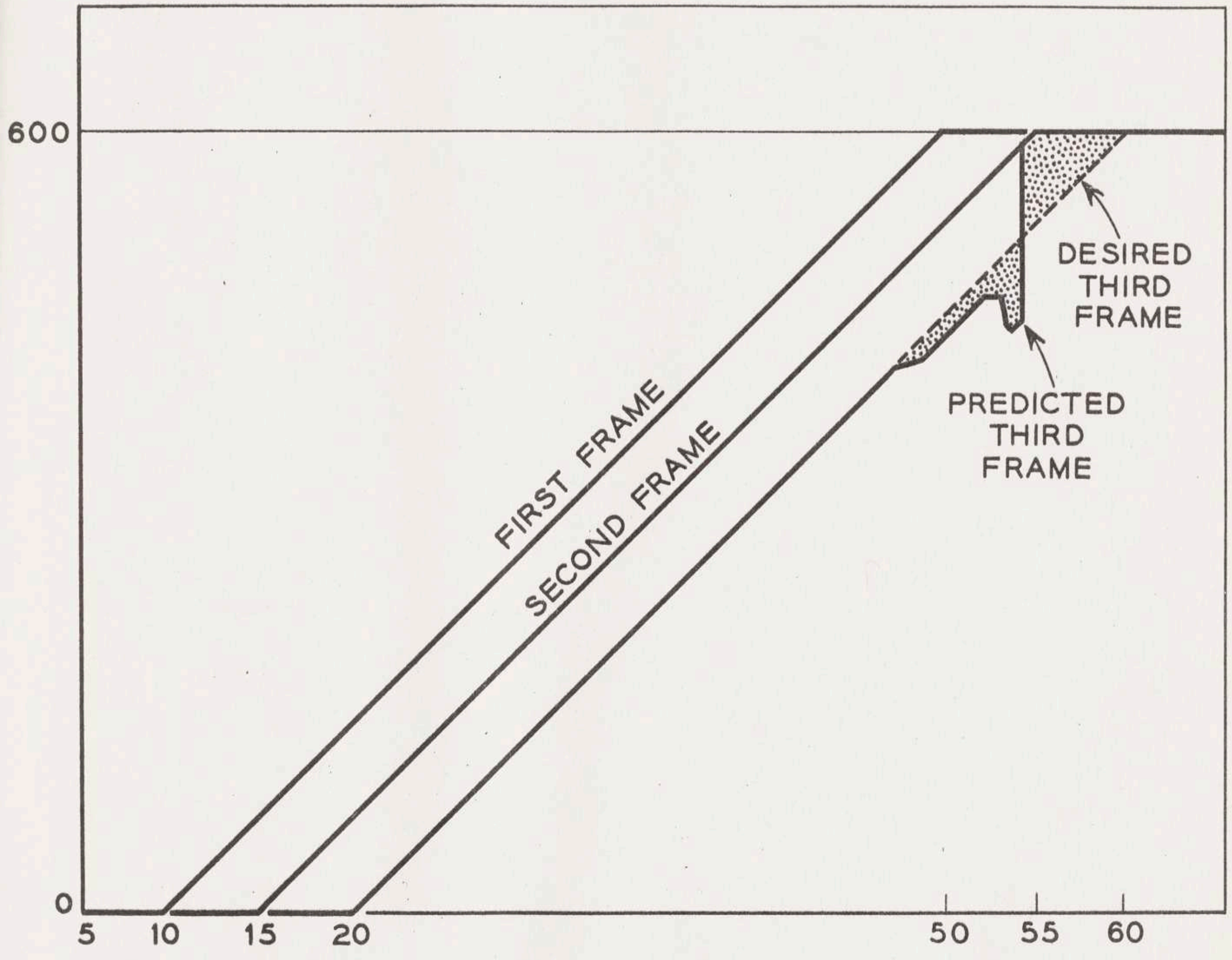


FIG. 3 EFFECT OF FIRST ORDER EXTRAPOLATION ON A DISPLACED RAMP

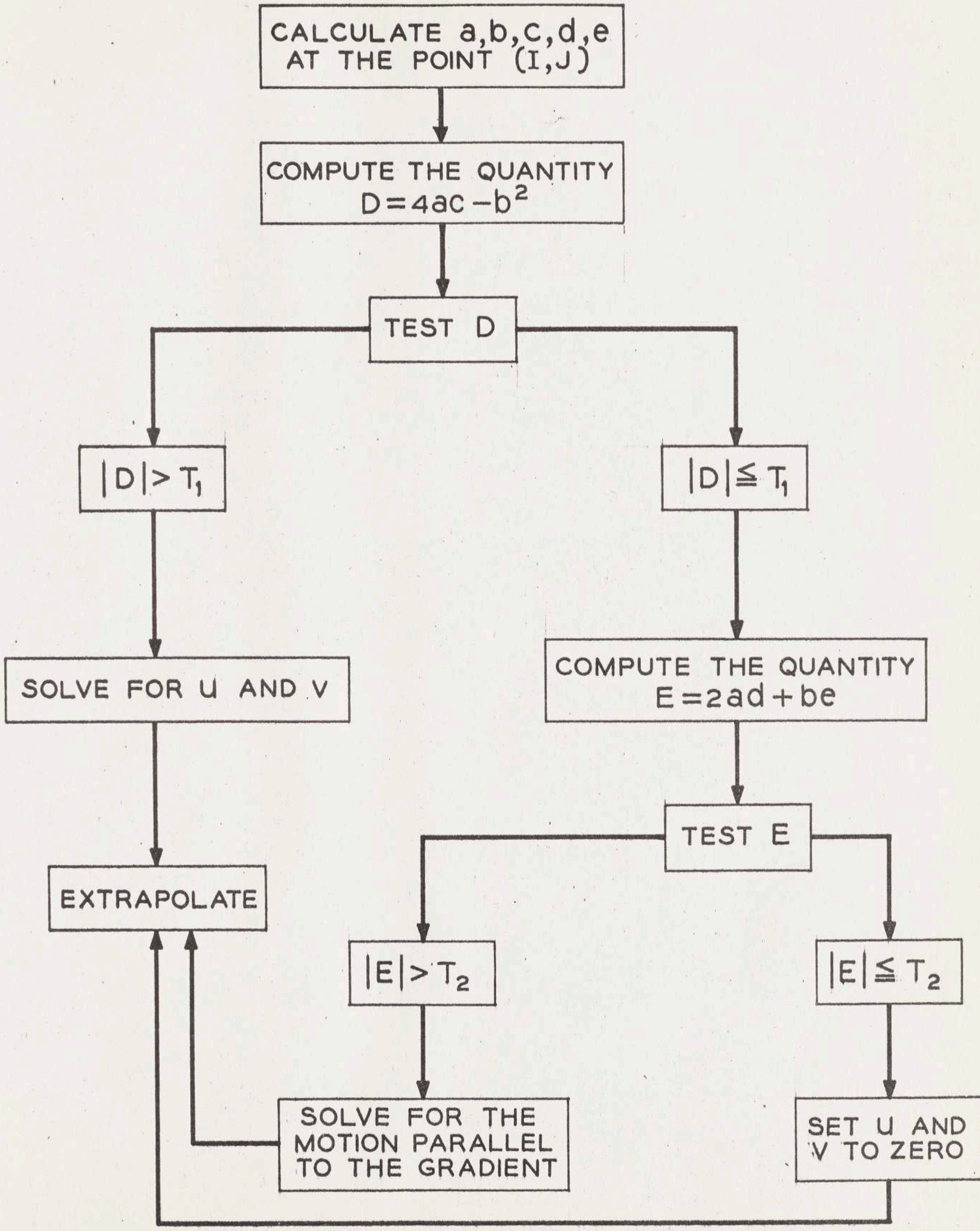
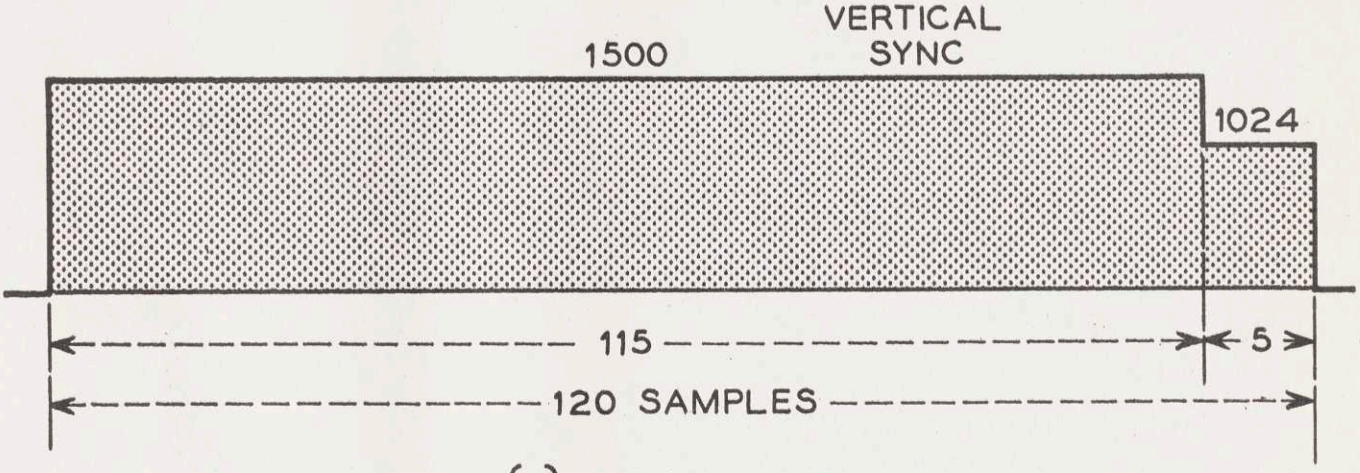
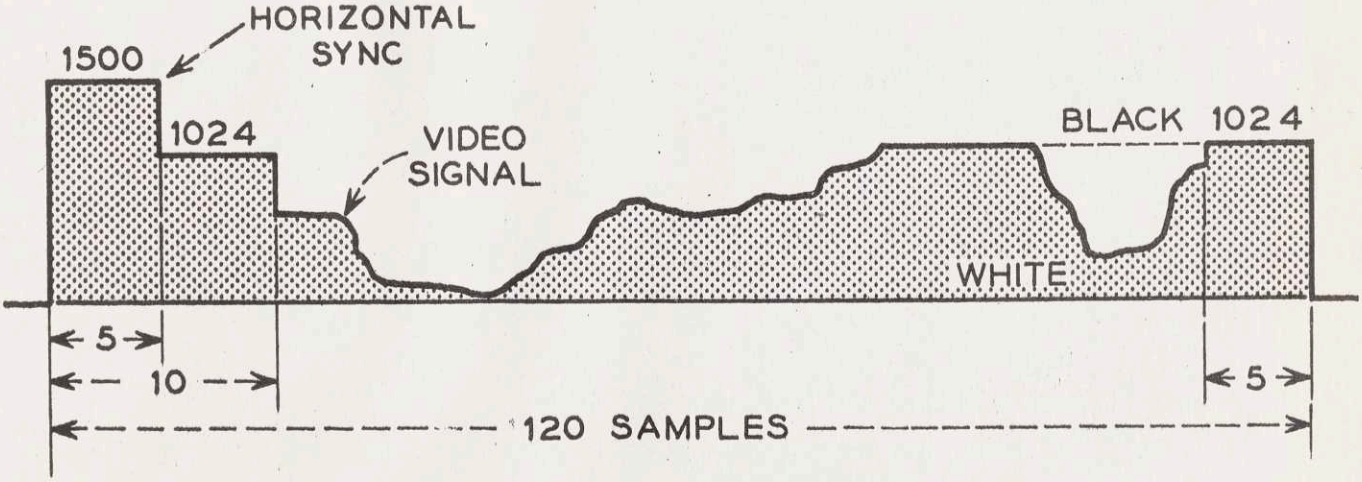


FIG 4 FLOW CHART FOR SECOND ORDER
WITH $h_1 = k_2 = 1, h_2 = k_1 = 0$



(a) FIRST LINE



(b) ALL OTHER LINES

FIG. 5 PICTURE FORMAT

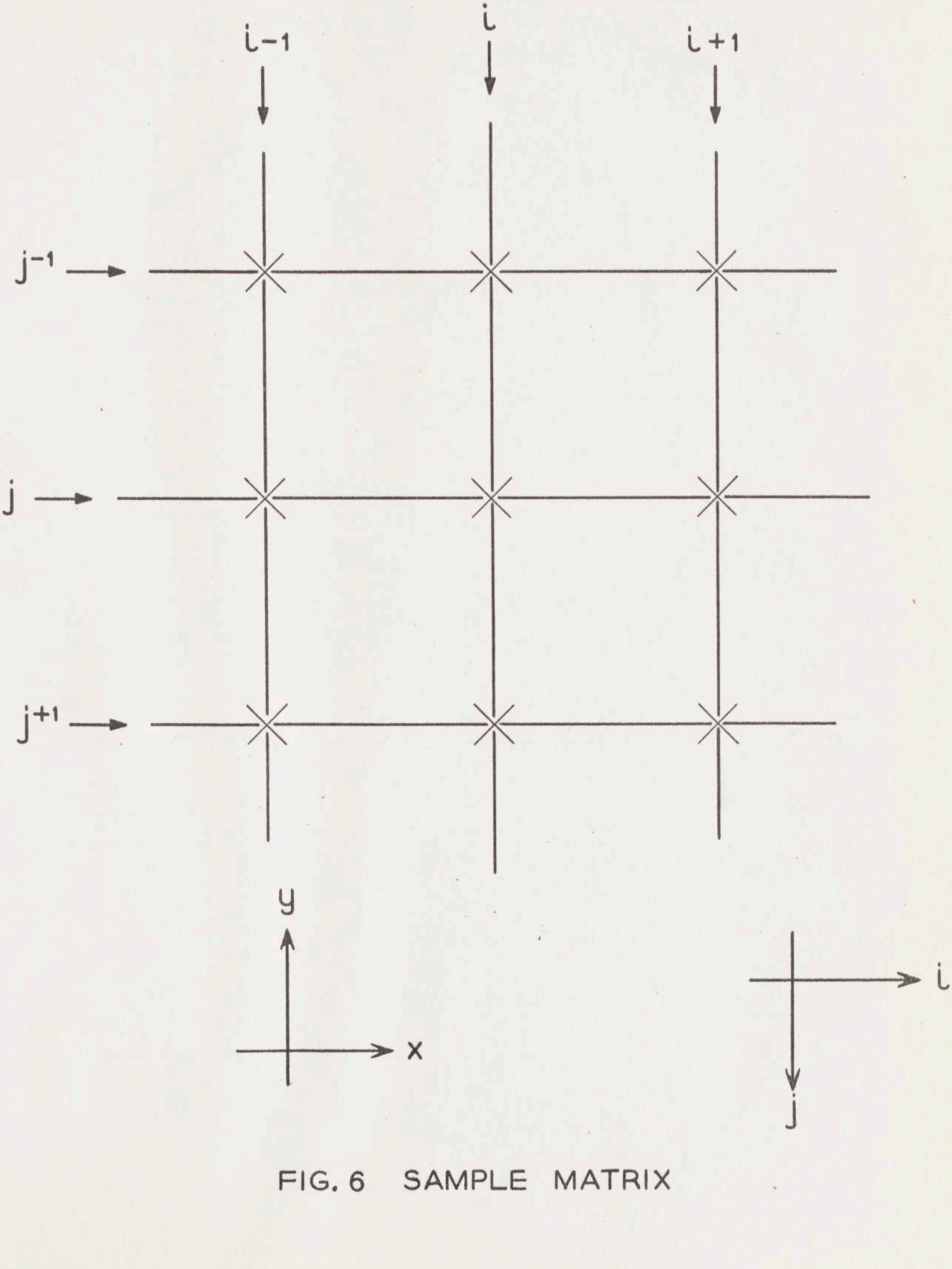
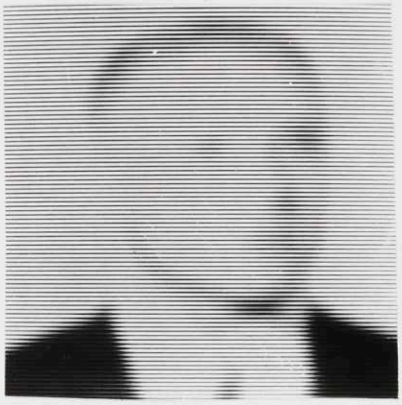


FIG. 6 SAMPLE MATRIX

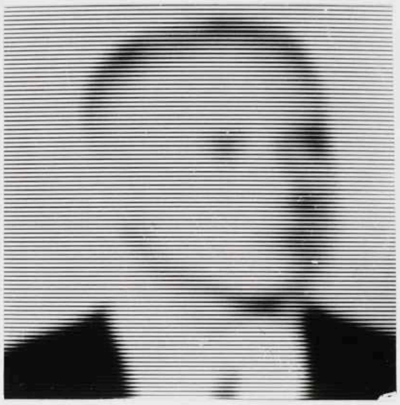
(a)

(b)

Originals

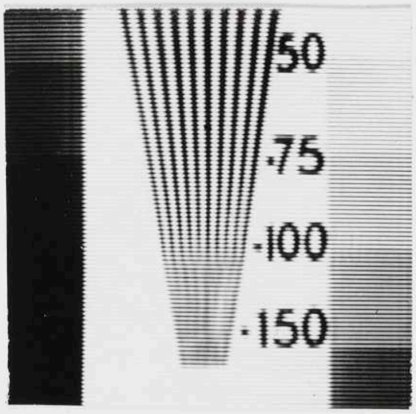


(c)



(d)

Filtered



(e) Test Chart

Figure 7. Picture Material

Figure 8. Effect of First Order Processes on Artificial Displacement of Two Nyquist Intervals

Interpolation Extrapolation

$h_1=k_2=2$
$h_2=k_1=0$

$h_1=k_2=1$
$h_2=k_1=0$

$h_1=k_2=2$
$h_2=k_1=0$
motion filtered

$h_1=k_2=1$
$h_2=k_1=0$
motion filtered

Figure 9. Effect of Second Order Processes on Artificial Displacement of Two Nyquist Intervals

Figure 10. Effect of First Order Processes on Artificial Displacement of Four Nyquist Intervals

Figure 11. Effect of Second Order Processes on Artificial Displacement of Four Nyquist Intervals

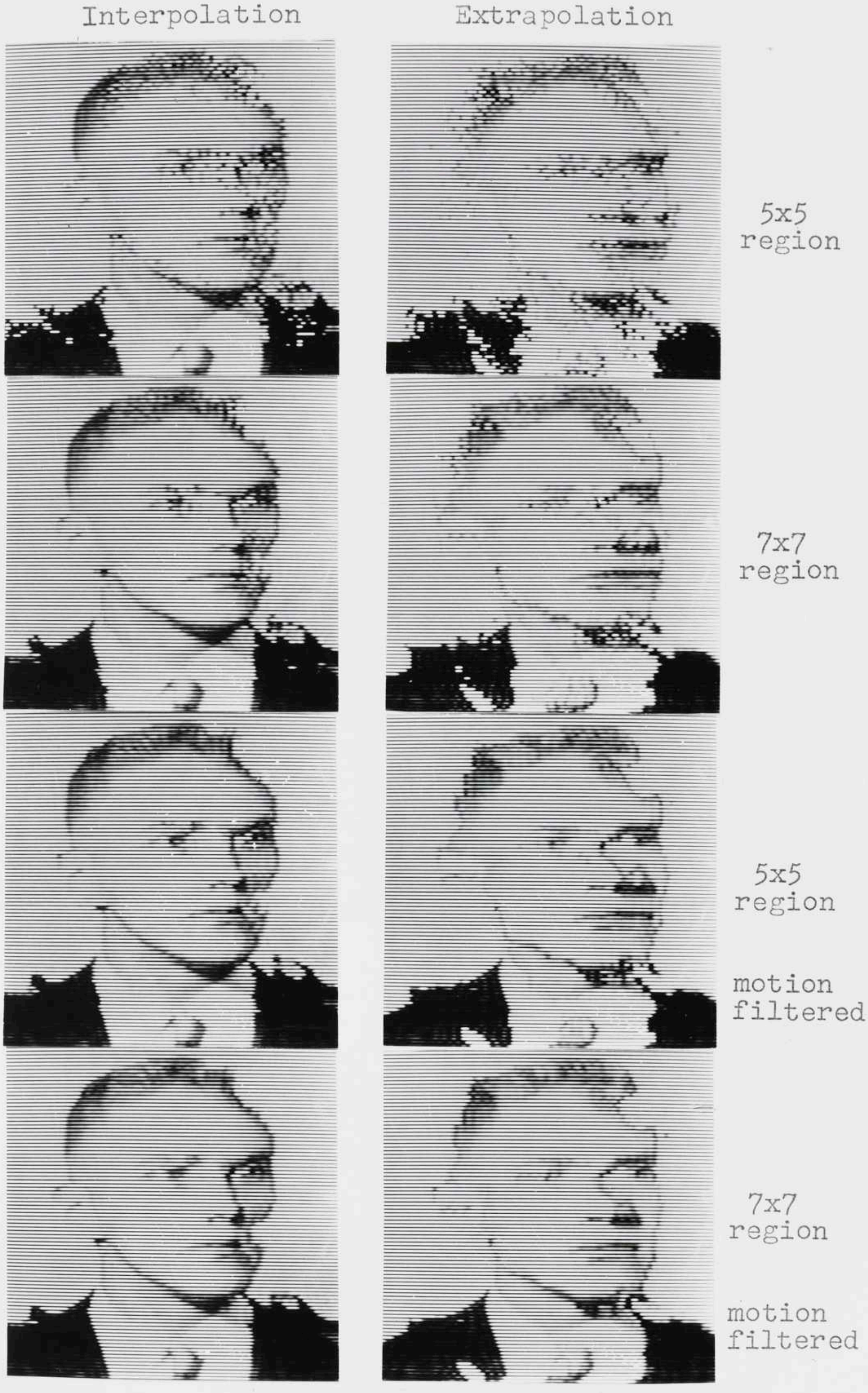


Figure 12, Effect of First Order Processes on a Real Motion Sequence

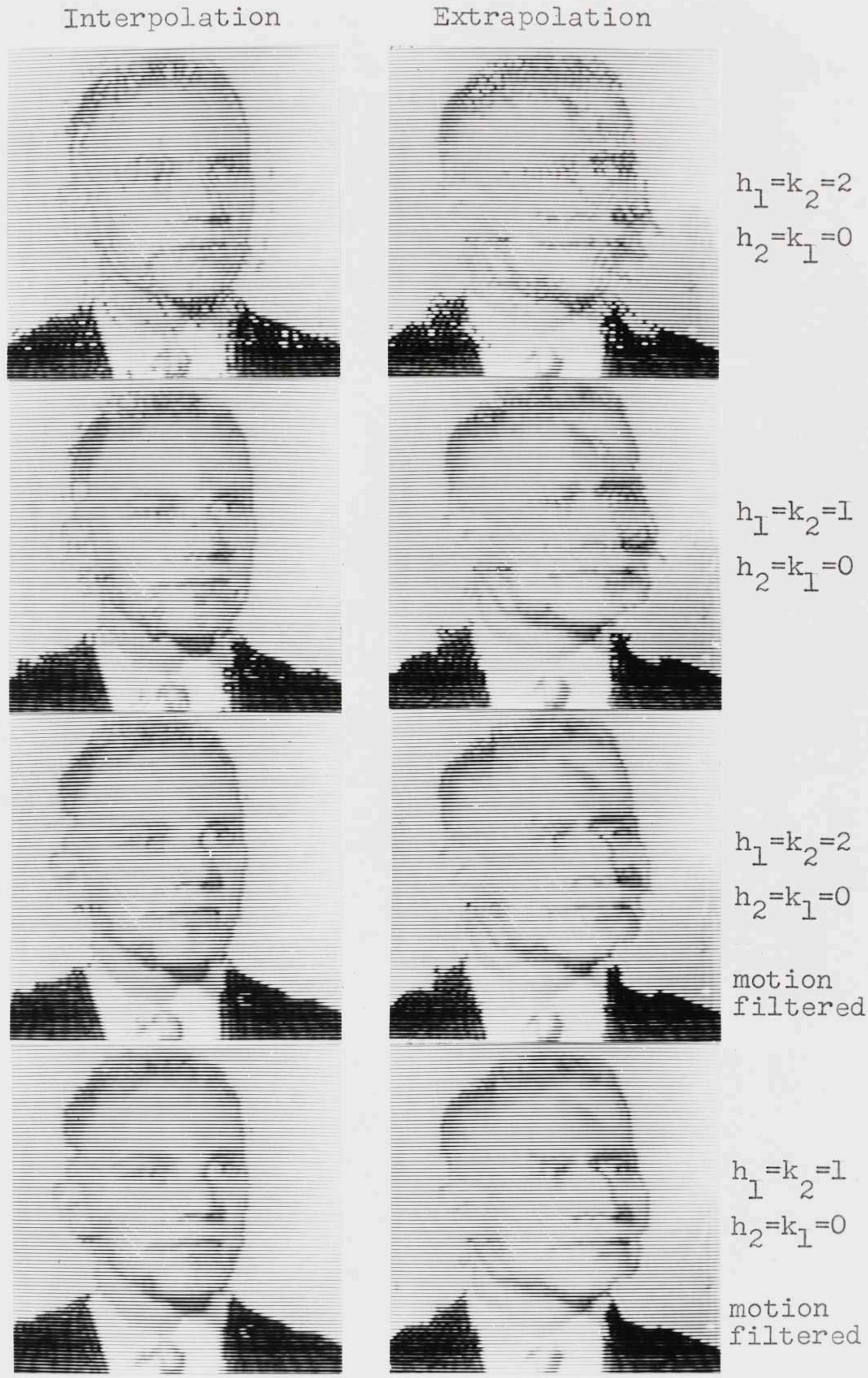


Figure 13. Effect of Second Order Processes on a Real Motion Sequence

References

1. C. E. Shannon, "A Mathematical Theory of Communication," BSTJ, Vol. 29; July 1948 and October 1948
2. C. E. Shannon, "Prediction and Entropy of Printed English," BSTJ, Vol. 36; January 1957
3. W. M. Goodall, "Television by PCM," BSTJ, Vol. 30, January 1951
4. E. R. Kretzmer, "Statistics of Television Signals," BSTJ, Vol. 31, July 1952
5. W. F. Schreiber, "The Measurement of Third-Order Probability Distributions of Television Signals," IRE Transactions on Information Theory, September 1956
6. C. W. Harrison, "Experiments with Linear Prediction in Television," BSTJ, Vol. 31, July 1952
7. W. S. Michel, "Statistical Encoding for Text and Picture Communication," Communications and Electronics, #35, March 1958
8. W. F. Schreiber and C. F. Knapp, "TV Bandwidth Reduction by Digital Encoding," 1958 IRE National Convention Record, Part IV
9. W. A. Youngblood, "Picture Processing," Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., January 1958

10. R. E. Graham, "Communication Theory Applied to Television Coding," Acta Electronica, Vol. 2, Sec. 1-2

11. R. E. Graham, "Subjective Experiments in Visual Communications," 1958 IRE National Convention Record, Part IV

12. E. R. Kretzmer, "Reduced Alphabet Representation of Television Signals," 1956 IRE National Convention Record, Part IV

13. R. E. Graham, "Predictive Quantizing of Television Signals," 1958 IRE WESCON Convention Record, Part IV

14. R. E. Graham, and J. L. Kelly, Jr., "A Computer Simulation Chain for Research on Picture Coding," 1958 IRE WESCON Convention Record, Part IV