CREDIT RATIONING IN GENERAL EQUILIBRIUM

by

WILLIAM BERKELEY ENGLISH

B.A., Yale University
(1982)

Submitted to the Department of
Economics
in Partial Fulfillment of the
Requirements for the Degree of

DOCTOR OF PHILOSOPHY

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 1986

(c) Copyright William B. English 1986

Signature of Author_____
                        Department of Economics
                        August 11, 1986

Certified by_____
                        Stanley Fischer
                        Thesis Supervisor

Accepted by_____
                        Richard S. Eckaus
                        Departmental Graduate Committee

CREDIT RATIONING IN

GENERAL EQUILIBRIUM

by

WILLIAM BERKELEY ENGLISH

Submitted to the Department of Economics
on August 11, 1986 in partial fulfillment
of the requirements for the Degree of
Doctor of Philosophy in Economics

ABSTRACT

This thesis addresses theoretical and empirical aspects of credit rationing. The first two chapters analyze the general equilibrium effects of credit rationing. The models used have a simple overlapping generations structure in which banks serve as intermediaries between borrowers and lenders. Money is held by banks to satisfy a reserve requirement. In the first chapter, borrowers are constrained in the amount of credit they can obtain at a given interest rate, while in the second chapter some would-be borrowers are unable to obtain any credit at all. The rationing in the first chapter occurs because loans become riskier as they grow larger. In the second chapter, rationing occurs in the credit market because of the adverse selection effects of high interest rates.

These chapters focus on two sets of questions. First, positive issues are considered. The characteristics of the steady-states of the models are examined, as well as the effects on these steady-states of changes in the required reserve ratio, the rate of growth of high-powered money, and the level of government debt. Second, normative results are discussed. Optimal government policies are derived for each model. Then the social welfare effects of the credit rationing are studied. In addition, the thesis examines the robustness of the models to changes in assumptions.

The third chapter tests the empirical evidence for rationing of a particular type. Data on over 600 United States manufacturing firms are used. The data generally support the conclusion that firms are constrained in their borrowing. At least for the firms in the sample, however, the constraint does not appear to be of critical importance.

Thesis Supervisor: Stanley Fischer
           Title: Professor of Economics

TABLE OF CONTENTS

ACKNOWLEDGMENTS

There are many people I would like to thank for their help while I was working on this thesis. First, I thank Stan Fischer, my primary thesis advisor. Stan donated a great deal of his time to my thesis, reading several drafts of each of the chapters. His encouragement and advice were invaluable, and his efforts improved the thesis immeasurably. Second, I thank Olivier Blanchard, my second reader. Olivier read and commented on each of the chapters at least twice, and supplied me with many perceptive comments.

In addition, I thank many of my fellow students-- particularly those in the informal monetary economics lunch group. David Romer gave me many useful comments on an early draft of the second chapter. David Wilcox and I had many useful discussions on these and other topics. Hide Ichimura checked some of my algebra and made many useful suggestions.

The empirical work in the final chapter was greatly improved by discussions with Danny Quah, Glenn Sueyoshi, and Eric Moran. Glenn and Brad Reiff were both generous with their time when I had problems with the computers.

My understanding of rationing was aided by many conversations with my father, who initially interested me in the subject, and also by discussions with bankers at the Connnecticut Bank and Trust Company, Bank of New England, and Bankers' Trust Company. In particular, Scott Reynolds of Bankers' Trust was very helpful.

Finally, I would like to thank my family and friends for their kindness and assistance during the writing of this thesis. Some of these require special mention. My parents, Jim and Isabelle English gave me unwavering support--both emotional and financial--throughout my graduate school career. Tracey Funari's kindness and patience made writing a thesis much pleasanter than it otherwise would have been. Finally, Steve Herzenberg's generosity and enthusiasm made him an excellent roommate and friend.

INTRODUCTION

The general equilibrium view of monetary policy is now
well established. In this view, government interventions in
asset markets have their effects in an indirect manner.
Initially, the government changes the supplies of assets
held by the public and asset prices then adjust to
equilibrate the asset markets (Patinkin, 1964). In
general, this process has an effect on Tobin's q, and as a
result capital is either accumulated or decumulated. The
new long-run equilibrium is characterized by either a larger
or a smaller stock of real capital (Tobin, 1969; Modigliani,
1963).

In this thesis I focus on a criticism of the general
equilibrium monetary model following from the "availability
doctrine" of the early 1950's (Roosa, 1951). Adherents of
this view believe that it is not interest rates, but the
availability of credit that affects the real economy. One
way to interpret this view is to assume that market
imperfections cause the credit market to be characterized by

excess demand in equilibrium. Since asset prices (in this case the interest rate on loans) do not adjust to equate supply and demand, they may not respond in the expected way to government engineered changes in asset supplies. Instead, in this view, changes in asset supplies make themselves felt as rationing constraints are relaxed, and quantities adjust directly (Okun, 1963).

The problem with this view is that it is hard to explain why the excess demand persists. Several authors assume the existence of institutional constraints that keep the credit market from equilibrating. For example, Modigliani (1963) considers a case in which the loan interest rate is determined by "institutional forces," or is slow to adjust to changing market conditions. In this case, rationing must occur if there is excess demand for credit at the institutionally determined rate. Similarly, Freimer and Gordon (1965) argue that banks charge a "conventional" rate on loans. Finally, Jaffee and Modigliani (1969) hypothesize that banks have to charge borrowers of different levels of risk the same loan rate due to institutional constraints. The result of this constraint may again be rationing.

More recently, several authors have developed partial equilibrium models in which rationing by lenders is optimal. Keeton (1979), divides these models into two types. In the case of "type one" rationing, borrowers receive credit at the prevailing interest rate, but do not obtain as much

credit as they would choose at that rate. On the other
hand, in "type two" rationing, some potential borrowers
obtain loans while other, identical, potential borrowers do
not.

In order to understand how these results are obtained,
one must first remember the situation in a riskless
competetive loan market. In such a market, loans are repaid
with certainty. The supply curve facing a given borrower is
horizontal at the interest rate determined by the aggregate
supply of and demand for funds. (See Figures 1 and 2) The
quantity demanded by the borrower, L^*, is given by the
intersection of this supply curve and his demand curve. The
lender's return on this loan is simply the interest rate
charged, r^*.

The imposition of a usury law or an institutionally
determined loan rate would, if binding, cause rationing.
Whether the rationing would take the type one or type two
form is indeterminate. Aggregate demand for credit could be
limited either by lending each borrower less than he would
choose, or by excluding some particular borrowers from the
credit market.

In order to induce rationing endogenously, some changes
must be made to this simple model. Typically it is assumed,
first, that borrowers may default, and, second, that when
default occurs there is limited liability. As a result,
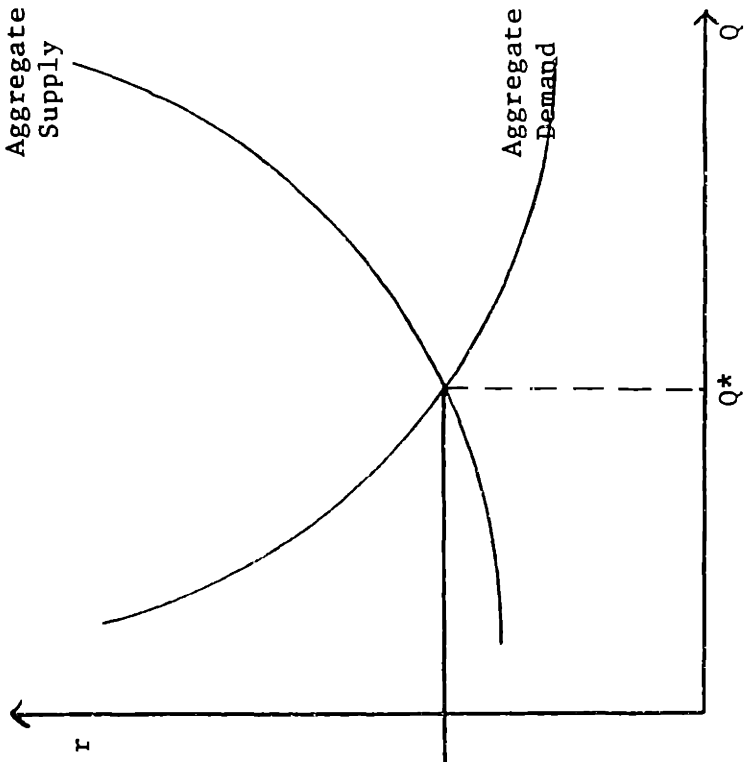there is some probability that the lender will have to

Aggregate
Supply

Aggregate
Demand

r

Q*   Q

FIGURE 2

Aggregate Market

Individual's
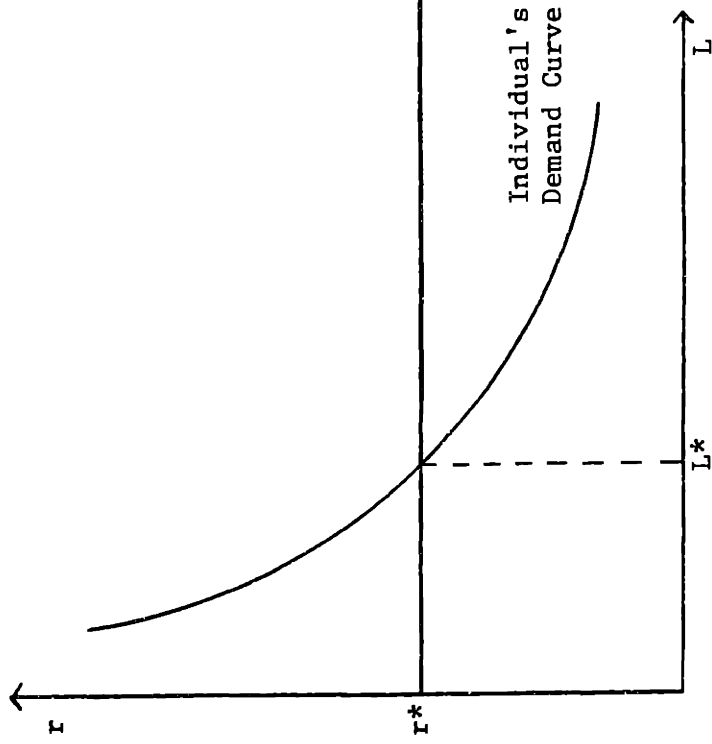Demand Curve

r*

L*   L

FIGURE 1

Individual Borrower

absorb a loss. By themselves, these two changes do not necessarily lead to rationing. The rationing result depends on the modeling of the probability of default. Keeton (1979) and Jaffee and Russell (1976) both develop models in which the probability of default is an increasing function of the loan size. In addition, the expected return to the lender on a loan if default does occur is a decreasing function of loan size. As a result, lenders must charge higher interst rates on larger loans in order to maintain a given average rate of return. If lenders are assumed to be competitive and make zero profits, then the "supply curve" to the individual is the lender's zero-profit curve. This curve is upward sloping, but the slope is not due to monopsony power of the borrower because the borrower is assumed to be small relative to the market. Instead, the slope is due to the assumed relationship between the probability of default and the loan size.

Nevertheless, the solution looks much like the solution to a monopsonist's problem. The borrower maximizes his utility subject to the the lender's zero-profit constraint. The result is a smaller loan size and lower interst rate than one finds at the intersection of the supply and demand curves. (See Figure 3.) Clearly, the borrower would like to obtain a larger loan at the same loan interest rate, but he cannot do so. It is in this sense that there is rationing in the model.

In contrast, models of type two rationing, such as those by Keeton (1979) and Stiglitz and Weiss (1981), depend on a positive relationship between the probability of default and the loan interest rate, rather than the loan size. Such a relationship can be based on either the adverse incentive or adverse selection effects of high interest rates. It is simplest to see this result if the loan size is fixed.[1] In this case one can plot the lender's rate of return, ρ, as a function of the loan interest rate, r. In the riskless loan market, this function is simply a 45-degree line. In the case of type two rationing, however, the increasing probability default means that this function is concave and always lies below the 45-degree line. If this function has a unique maximum at a loan interest rate, r*, below the rate at which supply and demand are equalized, r_m, then lenders will not choose to raise the rate beyond r* in order to clear the market. As a result, some borrowers will be rationed.

In short, models of type one rationing are based on the fact that credit is a non-homogenious good. In other words, lending a thousand people a dollar is not the same as lending one of them a thousand dollars. On the other hand, models of type two rationing are based on the observation that the "quality" of a loan--i.e. its probability of default--may depend on its price, the loan interest rate.

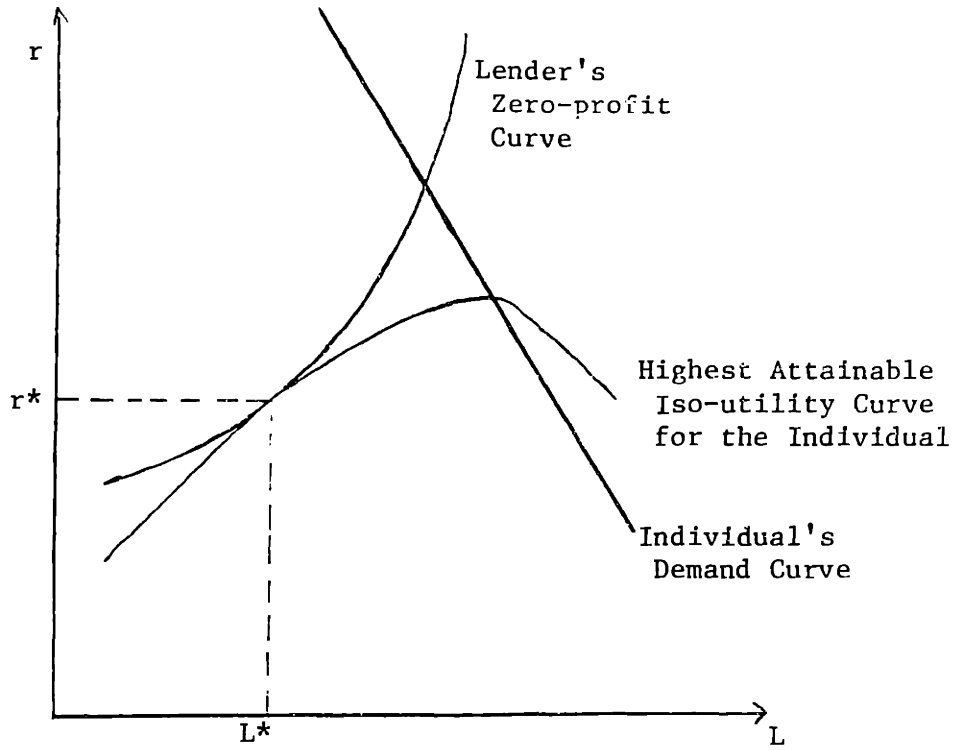Unfortunately, these microeconomic models of rationing
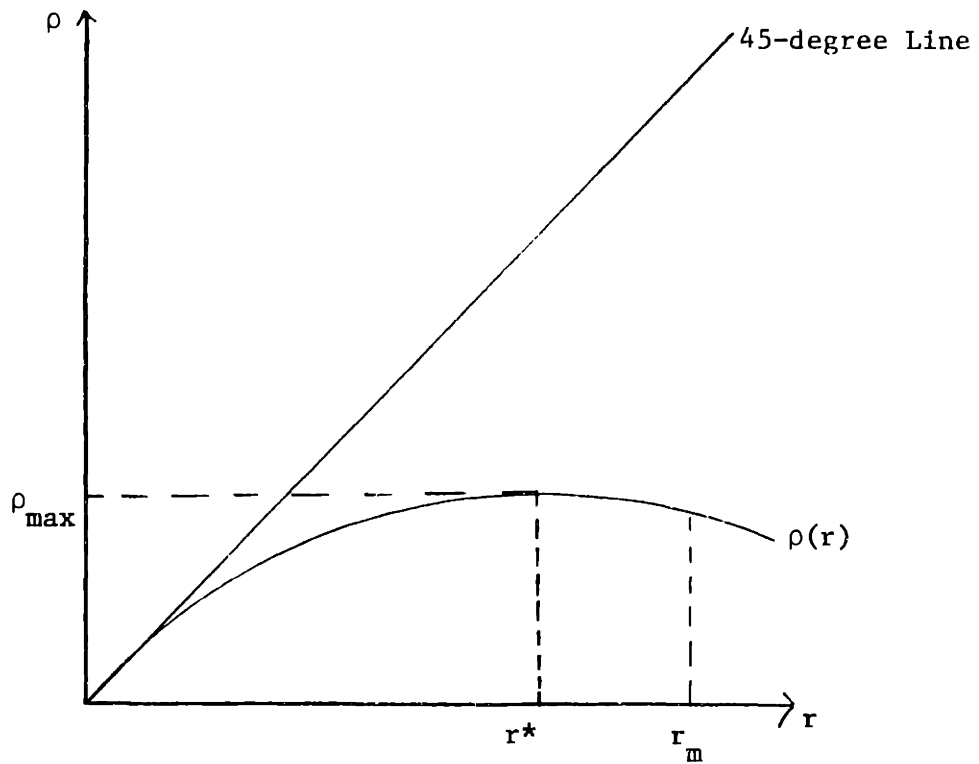
FIGURE 3

Type One Rationing

FIGURE 4

Type Two Rationing

have not generally been integrated into macroeconomic models. Instead, macroeconomic models that consider the effects of rationing generally assume that interest rates are fixed exogenously, as in Modigliani (1963), or that they can be ignored, as in Blinder (1985). One exception is a paper by Bruce Smith (Smith, 1983). He constructs a simple general equilibrium model in which type one rationing occurs endogenously. However, unlike the models in this thesis, his model has no production and no financial structure. As a result, the policy implications of his work are not clear.

In the first two chapters of this thesis I construct simple general equilibrium models characterized by rationing in the credit market. I focus on the steady-state, welfare and efficiency aspects of rationing. Thus this paper gives a long-run, general equilibrium, view of the impact of credit rationing on monetary policy and credit market regulation, whereas recent papers by Blinder and Stiglitz (1983) and Blinder (1985) focus on rationing as a vehicle for the expansionary and contractionary influences of monetary policy in the short-run. Furthermore, because I build up my models from assumptions about tastes and technology, I can be sure that my macroeconomic results are consistent with the micro-foundations of the credit rationing. Finally, I can compare the effects of the two types of rationing.

In Chapter One I study the effects of type one

rationing, while type two rationing is considered in Chapter Two. In both chapters the model used is a simple overlapping generations model. I impose on the models a simple financial structure in which banks intermediate between borrowers and lenders. The government supplies high-powered money to satisfy a reserve requirement and also issues government debt. Following Romer (1985) I use a simple storage technology, rather than a production function including capital and labor, to model investment opportunities. In both cases, rationing arises endogenously given the tastes of the agents and the storage technology.

The analysis in the chapters proceeds in two directions. First, I consider positive issues: what does the steady-state of the economy look like? What are the effects on the steady-state of changes in government policy? (The government policy variables in the models are the required reserve ratio, the rate of growth of high-powered money, and the level of government debt.) Second, since the models are based on tastes and technology, I can analyze welfare issues. In particular, I derive the policies that the government should persue in order to maximize social welfare, and consider the welfare implications of the rationing.

The results presented in these two chapters show that the two types of rationing have different effects on the economy, and both differ significantly from models with

fixed exogenous interest rates. In the case of type one rationing, the economy is very similar to one without rationing. The loan interest rate, however, may move in perverse ways in response to government policy shifts. For example, the loan interest rate may fall in response to an increase in government debt--in spite of the fact that the increase in government debt crowds out investment. As a result, if the monetary authorities are using the loan rate to asses the effects of policy, they may misunderstand what is going on in the economy.

On the other hand, in economies with type two rationing, changes in the level of government debt have no effect on the loan interest rate. Changes in the required reserve ratio or the rate of growth of high-powered money, however, do affect the loan interest rate. Thus looking at the loan interest rate may again be misleading for policy makers.

The welfare implications of rationing also differ depending on its type. Type one rationing does not imply that the economy cannot attain a first-best allocation of resources. This outcome occurs if the government manipulates the level of government debt in order to attain dynamic efficiency. In contrast, with type two rationing the first-best outcome cannot be attained. It is shown in Chapter Two, however, that the rationing can increase social welfare relative to an alternate regime without rationing. As a result, the government may want to induce rationing by

imposing a usury law if it does not occur endogenously.[2]

These results show that the existence of rationing in an economy should be of concern to policy makers. Thus it is suprising that more empirical work to test the practical importance of rationing has not been done. In Chapter Three I implement an empirical test for rationing. Before I can consider testing for credit rationing, however, I first have to decide what type of rationing to test for. Empirical testing of type two rationing is very difficult for two reasons. First, the econometrician requires data on all would-be borrowers, including those who do not receive credit. Unfortunatly, data appears to be available only on loans that are actually made. Second, the econometrician must be certain that there is no unobserved variable determiniaing which agents receive credit and which do not.

A test of type one rationing, on the other hand, presents no theoretical difficulties, and it is this type of rationing that I test for in Chapter Three. Using financial data on over 600 large United States manufacturing firms, I estimate the relation between the loan interest rate charged a firm and its level of financial leverage. For the firms in the sample, increases in leverage increase the loan interest rate in a statistically significant way, although the size of the increase is small. These results suggest that type one rationing exists, but also that it is not a vastly important phenomenon--at least for the firms in the

sample.

The weakness of these results should not come as a great suprise. The firms in my sample are generally large and well known, averageing over 2 billion dollars in assets (1982 dollars) and being traded primarily on the New York and American Stock Exchanges. Because rationing depends on the firm's probability of default, these firms are probably an unlikely group to be seriously affected by credit constraints.

These results do not imply that small businesses and individual consumers are not affected by credit rationing.[3] In fact, anecdotal evidence suggests that small businesses do indeed face rationing at least on a cyclical basis, although empirical support for this claim is lacking.[4] On the other hand, there are empirical results that support the claim that consumers are credit constrained. In particular, Hall and Mishkin (1983) find that panel data on income and consumption are consistent with both type one and type two rationing. They write, "The negative relation between the lagged change in income and the current change in consumption is consistent with constrained consumption behavior for about 20 percent of consumption. We are able to distinguish this symptom of inability (or unwillingness) to borrow and lend from the type of behavior characteristic of consumers who simply face high effective interest rates. The data show signs of both influences." (Hall and Mishkin,

p.480)

Nonetheless, their results only suggest rationing--they could, for example, be due to irrationality on the part of consumers. Recently two authors have attempted more direct tests of type one rationing of consumers. Paxton (1986) estimates reduced form asset demand equations for housholds that face increasing interest rates when they increase their debt and who may be constrained by usury ceilings. Moran (1986), on the other hand, tries to estimate simultaneously the consumption Euler equation and a credit supply equation, imposing cross equation restrictions. The results in both papers are inconclusive, although Moran finds evidence that members of minority groups are unable to obtain as much credit as they would choose.

Macroeconomists have long been interested in the possibility of credit rationing. This interest is not suprising given the substantial policy implications rationing would have. Nonetheless, they have shied away from constructing macroeconomic models with rationing based on explicit microeconomic foundations, and also from the empirical work needed to test the practical importance of rationing. This thesis is intended as a start in both of these directions.

NOTES

1. Although this assumption is not needed. What is required is that there exist an optimal interest rate and loan size that maximizes the return to the lender. See the discussion in Keeton, 1979, Chapter 3.

2. This is not a new observation. As Professor Michner of the University of Virginia pointed out to me, Adam Smith wrote:

"The legal rate [i.e. the usury ceiling], it is to be observed, though it ought to be somewhat above, ought not to be much above the lowest market rate. If the legal rate of interest...was fixed so high as eight or ten per cent., the greater part of the money which was to be lent, would be lent to prodigals and projectors, who alone would be willing to give this high interest. Sober people...would not venture into the conpetition. A great part of the capital of the country would thus be kept out of the hands which were most likely to make a profitable and advantageous use of it, and thrown into those which were most likely to waste and destroy it." (The Wealth of Nations, II.iv.15)

3. I leave aside the work of Jaffee on the housing market (Jaffee, 1971; Jaffee and Rosen, 1979). The rationing results in these papers were based on the effects of the ceilings Regulation Q imposed on deposit interest rates, and those ceilings are no longer in effect. Nonetheless, it is not obvious that the Regulation Q ceilings should have induced rationing. Instead, one would have expected them to cause spikes in the mortgage loan rate during periods of disintermediation.

4. For example, see the discussion in Hodgman (1963) pp.155-158.

REFERENCES

Blinder, Alan S. "Credit Rationing and Effective Supply Failures." National Bureau of Economic Research. Working Paper No. 1619. 1985.

_____, and Stiglitz, Joseph E. "Money, Credit Constraints, and Economic Activity." American Economic Review 72 (March 1983): 297-302.

Freimer, Marshall and Gordon, Myron J. "Why Bankers Ration Credit." Quarterly Journal of Economics 79 (August 1965): 397-416.

Hall, Robert E. and Mishkin, Frederic S. "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households." Econometrica 50 (March 1982): 461-481.

Hodgman, Donald R. Commercial Bank Loan and Investment Policy. Champaign Illinois: Bureau of Economics and Business Research, 1963.

Jaffee, Dwight M. Credit Rationing and the Commercial Loan Market; an econometric study of the structure of the commercial loan market. New York: Wiley, 1971.

_____, and Modigliani, Franco. "A Theory and Test of Credit Rationing." American Economic Review 59 (December, 1969): 850-872.

_____, and Rosen, Kenneth T. "Mortgage Credit Availability and Residential Construction." Brookings Papers on Economic Activity (July 1979): 333-376.

_____, and Russell, Thomas. "Imperfect Information, Uncertainty, and Credit Rationing." Quarterly Journal of Economics 90 (November 1976): 651-666.

Keeton, William. Equilibrium Credit Rationing. MIT Doctoral Thesis, 1976. Also, New York: Garland Press, 1979. Page References are to the MIT Thesis.

Modigliani, Franco. "The Monetary Mechanism and Its Interaction With Real Phenomena." Review of Economics and Statistics 45 (February 1963): s79-s107.

Moran W. Eric. "Direct Tests of Liquidity Constraints." Mimeo. MIT, 1986.

Okun, Arthur. "Comment." (Milton Friedman and Anna J. Schwartz: "Money and Business Cycles.") Review of Economics and Statistics 45 (February 1963): s72.

Patinkin, Don. Money, Interest Rates, and Prices: An Integration of Monetary and Value Theory. New York: Harper and Row, 1965.

Paxton, Christina. "Credit Market Regulations and Asset Composition." Mimeo. Columbia University, 1986.

Romer, David. "Financial Intermediation, Reserve Requirements, and Inside Money: A General Equilibrium Analysis." Journal of Monetary Economics 16 (September 1985): 175-194.

Roosa, Robert V. Money Trade and Economic Growth, in honor of John H. Williams. New York: Macmillan, 1951.1951.

Smith, Bruce. "Limited Information, Credit Rationing, and Optimal Government Lending Policy." American Economic Review 73 (June 1983): 305-318.

Stiglitz, Joseph E. and Weiss, Andrew. "Credit Rationing in Models with Imperfect Information." American Economic Review 71 (June 1981): 393-410.

Tobin, James. "A General Equilibrium Approach to Monetary Theory." Journal of Money, Credit, and Banking 1 (February 1969): 15-29.

CHAPTER ONE

CREDIT CONSTRAINTS IN GENERAL EQUILIBRIUM

Introduction

In this chapter I embed a credit market with type one rationing in a general equlibrium model with money. The rationing model I employ is similar to those developed in Jaffee and Russell (1976) and Keeton (1979). As noted in the introduction, in these models individual loans become riskier as they get larger. As a result, if the loan market is competitive, borrowers find that they have to pay a higher interest rate in order to increase the size of their loan. In the resulting equilibrium borrowers would choose to borrow more at the agreed upon interest rate.

The general equilibrium model presented is an overlapping generations model on which I have imposed a simple financial structure, similar to that of Romer's "pure banking economy."(Romer, 1985) There is a banking sector that intermediates between borrowers and lenders, and money serves only as the reserves of the banks. Unlike Romer's model, the returns on investment in this model are uncertain.

The model allows me to investigate the steady state effects on interest rates and income of changes in

government policy variables. The shifts I consider are changes in the rate of growth of high-powered money, the required reserve ratio, and the real per-capita quantity of government debt. These comparative steady state results correspond to the effects of an unanticipated, permanent, change in the parameters (leaving asside the effects on the initial old). The general equilibrium nature of the model also allows me to consider what government policies maximize social welfare.

In addition I note that deposits would be dominated if costless private lending were possible. By introducing a costly technology for private loans, I am able to construct a model in which bank lending and a bond market coexist. Using this more complex model, I show that the results obtained with the simple model are robust to this change.

The first section of the paper presents a partial equilibrium version of the rationing model that I use. The second section presents the general equilibrium model and displays its steady-state conditions. The third section presents the effects on the steady state of the model of changes in government policy variables, and the fourth section addresses wealfare issues. The more complex version of the model is discussed in Section V. A final section summarizes the results of the chapter.

I. The Credit Market

I begin by considering a partial equilibrium model of borrowing and lending in an economy with risky investment projects. The model is a simplified version of Keeton's model of type one rationing (Keeton, 1979, Chapter 1).

Individuals. Consider an individual who has a quantity of capital, W. He can borrow additional funds from a bank if he chooses to do so, and then invests in a project. The project succeeds with a probability $P(I)$, where I is the size of the project. It fails with a probability $1-P(I)$. The project returns $(1+r)I$ when successful, and it returns zero when it fails. Thus the expected return on a project of size I is:

$$R(I) = P(I)(1+r)I$$

I assume that the expected return is a twice continuously differentiable function of project size. I also assume that it rises with the size of the project, but at a diminishing rate:

$$R'(I) > 0$$

$$R''(I) < 0$$

In order to make the demand for loans well-behaved, I also

assume that:

$$R'(W) > 1+r^d$$

and:

$$R'(\hat{I}) = 1+r^d$$
$$\text{with } \hat{I} < \infty$$

where r^d is the interest rate banks pay on deposits.

Banks. I assume that banks are risk-neutral, competitive intermediaries (or are so large that the risk from an individual loan is completely diversified). Banks' intermediation services are assumed to be costless. Because banking is competitive, a bank's rate of return on each loan must equal the rate it pays on deposits. This implies that for each loan size, L, the loan rate, r^1, must satisfy:

(1) $(1+r^d) = P(W+L)(1+r^1)$

If the borrower is trying to maximize his expected second-period profits from his project, then his problem can be written as:

$$\text{Maximize: } R(W+L) - P(W+L)(1+r^1)L$$
$$L,r^1$$

$$\text{Subject to: } (1+r^d) = P(W+L)(1+r^1)$$

The bank's zero profit curve is plotted in Figure 1. It is the bank's supply curve of funds to the borrower. Because projects get riskier as they get larger, the supply curve slopes upward. The borrowers problem, then, looks similar to that of a monopsonist. It is not the same because there are many borrowers, each assumed small with
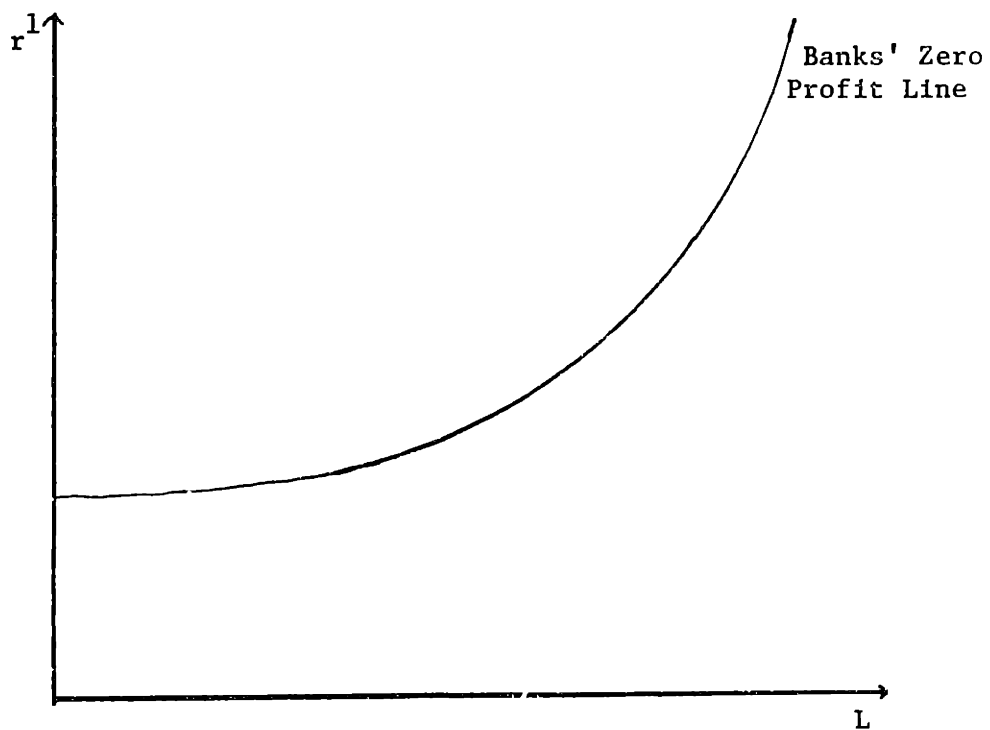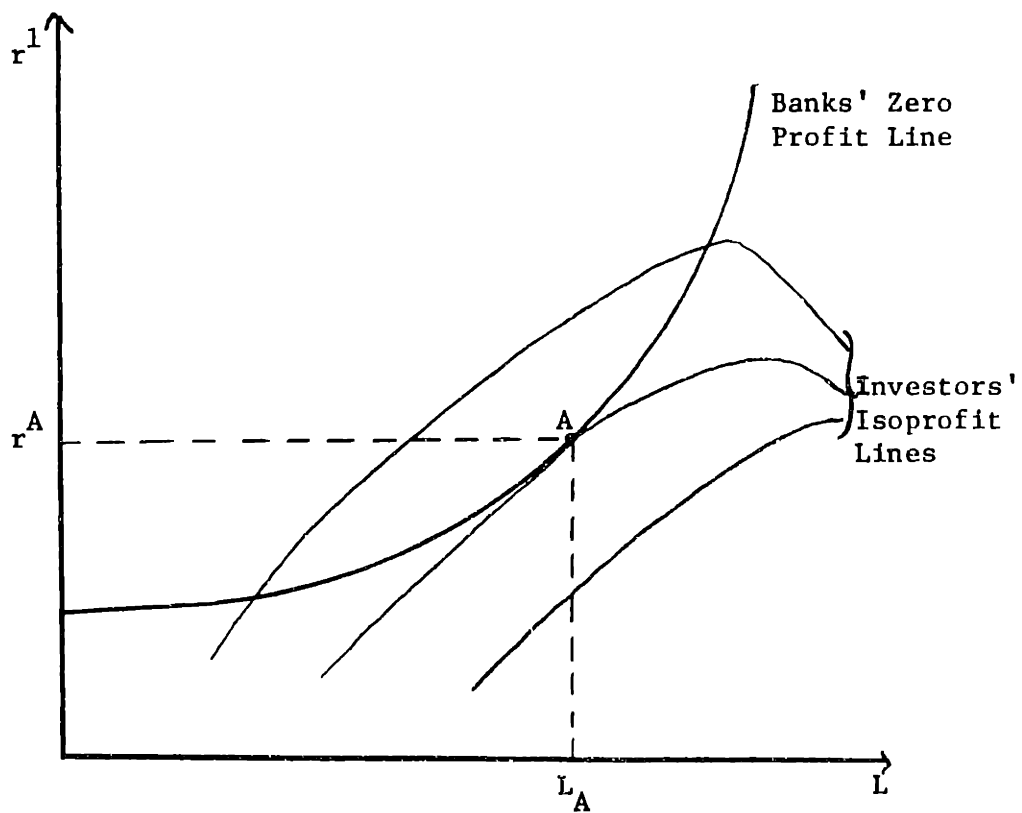
FIGURE 1

FIGURE 2

respect to the bank. (In fact, as Keeton (1979, Chapter 1) points out, credit is a non-homogeneous good in this case.)

Nonetheless, the solution to the borrower's optimization problem is like that of a monopsonist. The first order conditions are the constraint, and, after substituting for the Lagrange multiplier:

(2) $R'(W+L) = (1+r^d)$

In words, the marginal expected return on investment must equal the bank's opportunity cost of funds. The second-order condition for a maximum is:

$$R''(W+L) < 0$$

This condition is satisfied given our assumptions about $R(I)$. The solution is displayed graphically in Figure 2. The investor chooses the highest isoprofit line that he can attain given the bank's zero profit condition. Thus his choice is the isoprofit curve tangent to the bank's zero-profit line, and the contract is chosen at point A.

There are two characteristics of this equilibrium worth noting. First, the borrower does not choose the size of his loan given the loan interest rate, instead, he chooses both the size of the loan and the loan rate subject to the bank's zero profit condition. Thus, both the size of the loan and the loan interest rate are functions of the bank's cost of funds, r^d. As a result, the investor does not receive the quantity of funds he would choose at the contracted loan rate. The investor's demand curve is the

solution of:

$$\text{Maximize: } R(W+L) - P(W+L)(1+r^1)L$$
$$L$$

The first-order condition for this unconstrained problem is:

$$R'(W+L) - P(W+L)(1+r^1) - P'(W+L)(1+r^1)L = 0$$

This condition implies that the demand curve intersects the investor's isoprofit lines at the point at which their slope is zero. In Figure 3 the demand curve has been added to the diagram in Figure 2. It is clear that a contract at point B, the intersection of the supply and demand curves, is strictly worse for the borrower than the optimal solution at point A. It is also clear that, given the interest rate at point A, the borrower's demand for credit would be at point C. It is in this sense that there is rationing in the credit market.[1]

Second, from equation (2) and the assumptions about $R(I)$, one can see that there is an optimal project size, $I^*(r^d)$, at which the marginal expected return on a project is just equal to the bank's opportunity cost of funds, $(1+r^d)$:

$$I^*(r^d) = R'^{-1}(1+r^d)$$

Given the assumed form of $R(I)$, $I^*(r^d)$ is not dependent on W. Thus the borrower's optimal loan size can be written as:

$$L = I^*(r^d) - W$$
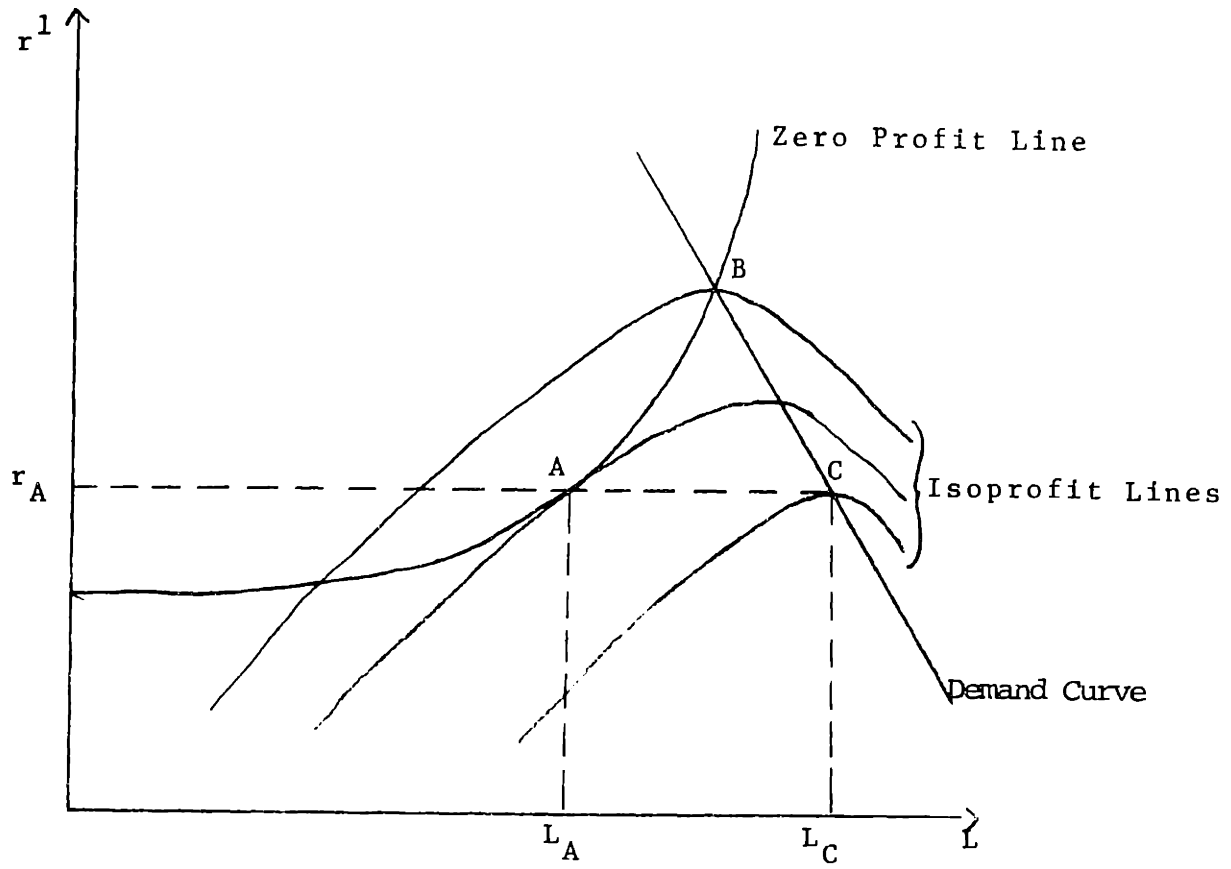
FIGURE 3

II. A General Equilibrium Model

The model that I use in this paper is similar to that used by Romer in his general equilibrium analysis of credit market regulation (Romer, 1985). The difference between the two models is that, following the partial equilibrium model from Section I, I make the results of investment uncertain.

Individuals. People live for two periods. When they are young they are endowed with an amount, W, of the single good in the economy. I assume that f(W), the distribution of W, is continuous from $\underline{W}$ to $\bar{W}$. Individuals are risk-neutral, and choose to consume only when old. They have two ways to transfer wealth from one period to the next. First, they can invest in a risky project like those described in the previous section. Second, they can make a deposit at a bank and receive a return $(1+r^d)$. I still assume that:[2]

$$R'(I) > 0$$

$$R''(I) < 0$$

and also that:

$$R'(\bar{W}) = 0$$

Banks. As before, banks are competitive, risk-neutral, zero-profit institutions. They accept deposits from individuals and invest these funds in two assets: currency, which they hold to satisfy a required reserve ratio; and loans to individu_ls. The only role for currency in the model is as reserves for the banks. No loans directly from one person to another are allowed (until Section V).

Government. The government has three roles in the model. First, it supplies currency to the banks. Second, it supplies bonds, paying an interest rate r^b, to individuals. Third, it takes the resources that it receives from these activities and spends them. The government must generate a non-negative level of revenues because it does not produce anything itself.

The balance sheets of the different groups are summarized in Table 1. `

TABLE 1

Asset/Liability	Individuals	Banks	Government
Deposits/Bonds	D	-(D-B)	-B
Loans	-L	L	
Currency		ϕ(D-B)	-H
Investments	W_m+L-D		

Where the following are real values per member of the young cohort:

D = individuals' holdings of financial assets--i.e. deposits and bonds.

B = individuals' holdings of government bonds.

L = the quantity of loans to individuals.

W_m = the average endowment size.

I assume that nominal interest rates are positive, and so banks do not choose to hold excess reserves, and individuals do not want to hold currency.

I also define the following:

h = the rate of growth of the nominal money stock.

n = the rate of growth of the population.

π = the inflation rate.

ϕ = the required reserve ratio.

g = the level of government spending.

i^d, i^l, i^b = the nominal interest rate on deposits, loans, and government bonds respectively.

r^d, r^l, r^b = the real interest rates on deposits, loans, and government bonds respectively.

The Loan Market. Because loans are risky, r^d is less than r^l. Thus risk-neutral borrowers will not choose to both borrow and hold deposits, as such a strategy would reduce their expected second period incomes. Also, since deposits and government bonds have the same characteristics, they both yield the same return. As a result, the level of

financial assets an individual chooses to hold is determined by r^d. Again, define $I^*(r^d)$ to be the investment size at which:

$$R'(I^*(r^d)) = (1+r^d)$$

Individuals endowed with more then $I^*(r^d)$ choose to save the amount of their endowment over that level in the form of deposits or bonds, thus the demand for financial assets is given by:

$$(3) \qquad D(r^d) = \int_{I^*(r^d)}^{\bar{W}} (W - I^*(r^d))f(W)dW$$

Similarly, those with small endowments will borrow until their project reaches a critical size, $I^*(\rho)$, at which the marginal expected return on investment is just equal to $1+\rho$, the expected return on loans that banks require in order to pay r^d on their deposits. We can derive the value of ρ from the banks' zero profit condition:

$$(1+r^d) = (1-\phi)(1+\rho) + \phi\frac{1}{1+\pi}$$

so,

$$(1+\rho) = \frac{1}{1-\phi}(1+r^d) - \frac{\phi}{1-\phi}\frac{1}{1+\pi}$$

Note that because of the required reserve ratio, $I^*(r^d)$ will not generally equal $I^*(\rho)$.[3] The quantity of loans is

given by:

$$(4) \qquad L(\rho) = \int_{\underline{W}}^{I^*(\rho)} (I^*(\rho) - W)f(W)dW$$

Notice that the quantity of loans is, through ρ, a function of the deposit interest rate, the inflation rate, and the required reserve ratio, rather than of the loan rate. This result is due to the fact that the borrower chooses the loan rate and amount constrained only by the bank's zero profit condition.

Steady State Conditions. The steady state conditions for this economy are easily derived. First, the inflation rate is determined by the rate of growth of high-powered money and the rate of growth of the population:

$$(5) \qquad 1+\pi = \frac{1+h}{1+n}$$

Second, since I assume that banks make zero profits, and also that intermediation is costless, the following condition must hold:

$$(6) \qquad 1+i^d = (1-\phi)P(I^*(\rho))(1+i^l) + \phi$$

In words, the nominal rate on deposits must equal the average rate banks earn on their assets. It is more useful to write this equation in terms of real interest rates:

$$(6') \qquad 1+r^d = (1-\phi)P(I^*(\rho))(1+r^l) + \phi \frac{1}{1+\pi}$$

Third, the banking sector balance sheet shows that:

(7) $L(\rho) = (1-\phi)(D(r^d)-B)$

and,

(8) $H = \phi(D(r^d)-B)$

Note that banks would choose to hold excess reserves if nominal interest rates were negative. Thus, conditions (7) and (8) would not hold if I did not assume that nominal rates were positive.

Equation (7) is more intuitive if rewritten as:

(7') $B + \frac{1}{1-\phi}L(\rho) = D(r^d)$

Now the left-hand side of Equation (7') is the demand for savings by the government, B, and by banks to fund their loans. I call $1/(1-\phi)*L(\rho)$ the indirect demand for funds by borrowers. The right-hand side of the equation is simply the amount of funds supplied by savers.

Finally, the goods market must clear. This means that the government must spend all of its revenues. The government gets resources from two sources: seigniorage and bonds. The amount obtained from seigniorage is positive only if the rate of growth of the money stock is positive, while the revenues obtained from the stock of government bonds are positive only if the interest rate paid on them is

less than the rate of population growth:

$$(9) \qquad g = H \cdot \frac{h}{1+h} + B \cdot \frac{n-r^d}{1+n}$$

As noted earlier, the government cannot produce goods, hence:

$$(9') \qquad g \geq 0$$

Equilibrium Interest Rates. Holding ϕ and π constant, ρ is a function of r^d, and so both the demand for funds and the quantity of savings available are functions of the deposit interest rate. Note, however, that $1/(1-\phi)*L(\rho(r^d))$ is not a demand curve in the ordinary sense. nstead, it is the amount of funds required by banks to satisfy the level of loans borrowers choose given the banks' cost of funds at a deposit rate of r^d, a required reserve ratio of ϕ, and a rate of growth of high-powered money of h. Thus the quantity "demanded" responds to changes in the supply side of the market. Similarly, the loan rate, r^1, is chosen by borrowers in response to variables affecting the supply of funds to the banking system.

Nonetheless, one can plot the supply of and demand for savings on the same axies in order to find the equilibrium level of the deposit rate, the quantity of loans made, and the level of financial assets held by savers. Then, by using the banks' zero profit condition and the definition of $I^*(\rho)$, one can obtain the loan rate. The equilibrium in the credit market is shown in Figure 4. The deposit rate at

which equation (7') holds is denoted r_A^d. The equilibrium

level of financial assets per-capita is $D(r_A^d)$.
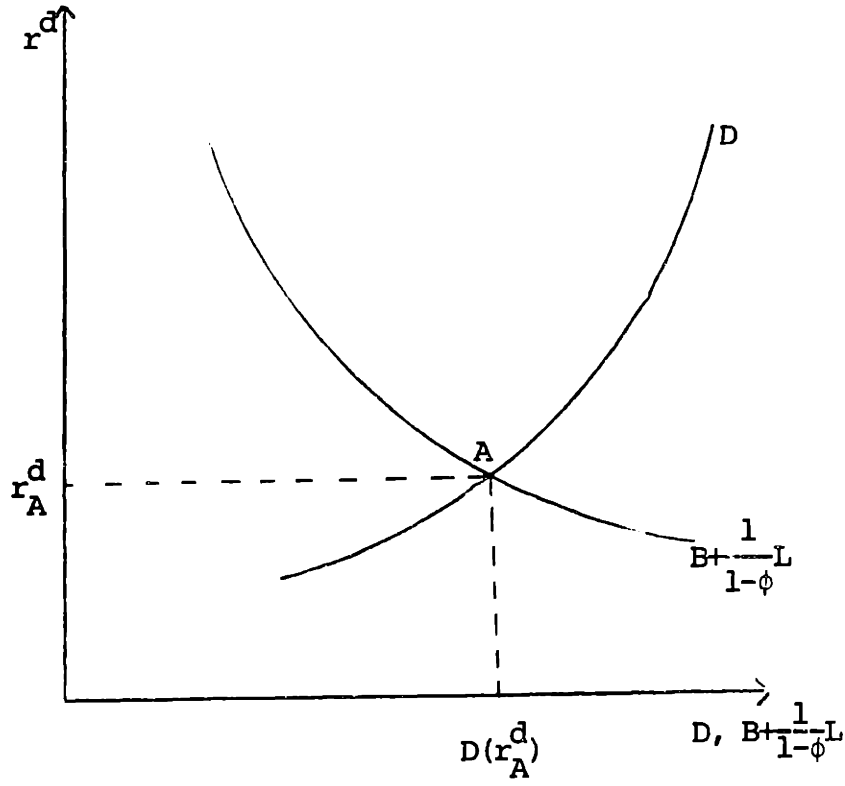
FIGURE 4

III. The Effect of Changes in Policy Variables.

In this section I compare different steady states of the model. In particular, I consider the effects on the steady state of changes in the government policy variables: h, the rate of growth of high-powered money; ϕ, the required reserve ratio; and B, the per-capita level of government debt. I reserve for the next section the effects of these changes on aggregate consumption, and look only at their effects on equilibrium interest rates and government revenues.

Change in the rate of money growth. A rise in h, the rate of growth of high-powered money, causes a rise in the steady-state inflation rate, π. The increase in inflation causes a rise in ρ, the opportunity cost of loanable funds for any given deposit rate. This rise in ρ leads borrowers to scale down the size of their loans, which leads in turn to a decline in the per-capita quantity of loans demanded at a given r^d. On the other hand, the rise in h does not cause a shift in $D(r^d)$. The new steady-state is shown in Figure 5. The new deposit rate, r_B^d, is below the original

one. The new equlibrium level of deposits is therefore lower than before, and the same is true of the equilibrium level of loans.

In words, the increase in π caused by the rise in h causes a decline in the return banks earn on their reserves. Thus, for a given r^d and φ, the banks' return on loans must rise. This increase can be accomplished either by raising r^1 or by reducing the size of loans (and thereby making them safer). Unless the demand for loans is entirly inelastic, the result will be a smaller loan size.[4] Thus banks will reduce r^d to equilibrate the level of deposits to the level of loans.

The results so far are similar to those found by Romer (1985). In this model, however, the effect of the change in h on the loan rate, r^1, is ambiguous, whereas in Romer's model the loan rate must rise. The ambiguity here is due to the relation between loan risk and loan size. If the elasticity of optimal loan size with respect to the deposit rate is very large, then a rise in h may cause a big fall in the size of loans. But this fall can reduce the risk of default so much that r^1 actually declines.[5]

An example of this counter-intuitive result is shown in Figure 6. The original equilibrium is at point A. A rise in h, by reducing the bank's return on its reserves, raises its zero profit curve. The new equilibrium point is at B. As shown in the diagram, it is possible for the new loan
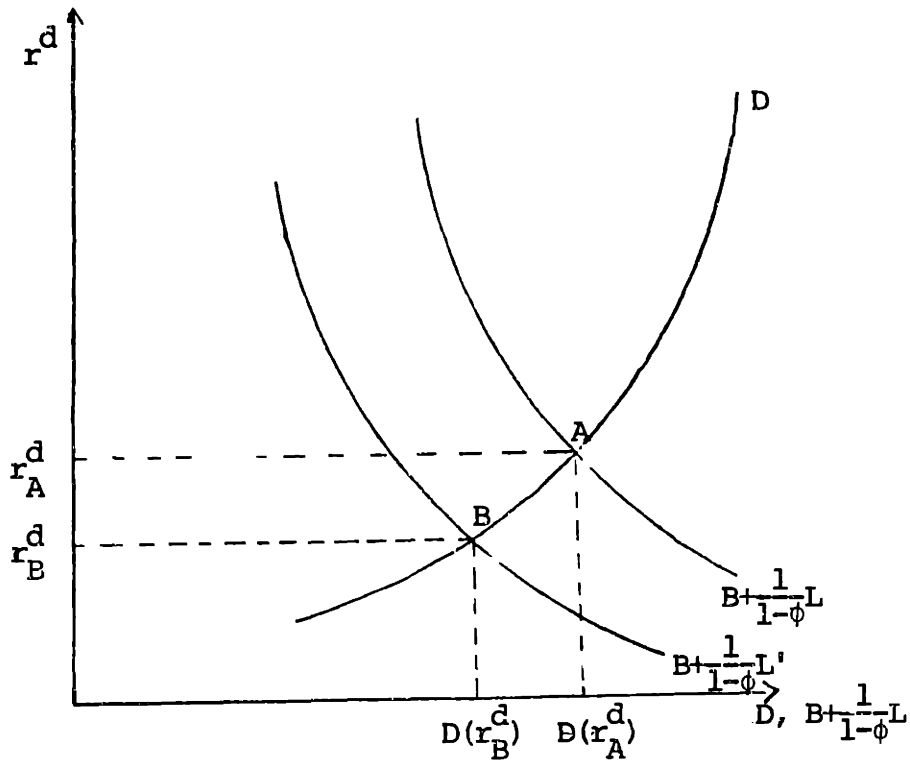
FIGURE 5

Effects of an Increase in h
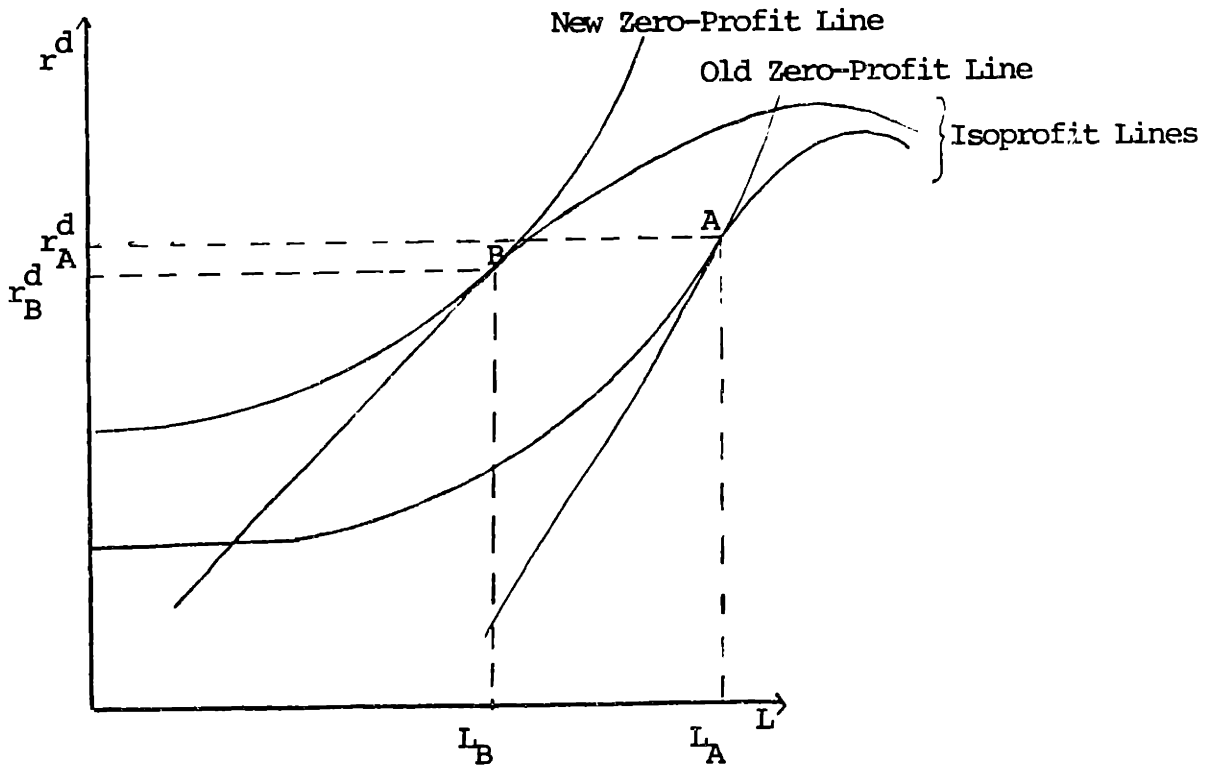
FIGURE 6

rate to be below the old. Algebraically, one can show that r^1 falls if:

$$-1 - \frac{r-r^1}{1+r} > \eta > -2$$

where,

$$\eta = \frac{P''(I)}{P'(I)} I$$

On the other hand, given our assumptions about $R(I)$, it is not possible for the new equilibrium to be at a larger loan size and higher loan rate.

Finally, a rise in h has as ambiguous effect on g, the level of government revenues. We can write the derivative as:

$$(10) \quad \frac{dg}{dh} = H - \frac{1}{(1+h)^2} + \phi D'(r^d)\frac{h}{1+h}\frac{dr^d}{dh} - B\frac{1}{1+n}\frac{dr^d}{dh}$$

The first term of this expression shows that at a given real level of high-powered money, H, an increase in h causes a rise in seigniorage. The second term takes account of the reduced holdings of financial assets caused by the fall in r^d. This decline reduces the equilibrium level of high-powered money and therefore reduces seigniorage if h is greater than zero. The final term takes account of the positive effect that the decline in the deposit rate has on net government revenues due to the decline in debt service payments. The second term is positive if h is negative, and small and negative if h is small and positive. Thus, the overall effect of a rise in h on government revenues is

positive for negative or small positive levels of h.

Change in the required reserve ratio. An increase in ϕ, the required reserve ratio forces banks to hold a larger portion of their assets as non-interest-bearing reserves. Thus the increase causes a rise in the banks' cost of loanable funds, ρ. As in the case of a rise in h, the result is a decline in the size of loans chosen by borrowers at a given deposit rate. On the other hand, the increase in ϕ increases the amount of savings required to fund a loan of a given size because a smaller portion of deposited funds are being loaned out by banks. Thus, while the quantity of loans clearly falls as a result of the change the demand curve can shift in either direction. As a result, the effect on r^d, and therefore on $D(r^d)$, is ambiguous. In particular:

$$(11) \quad \frac{dr^d}{d\phi} = \left\{ 1 - \frac{L'(\rho)}{L(\rho)} \rho \frac{\partial \rho}{\partial (1-\phi)} \frac{(1-\phi)}{\rho} \right\} \frac{L(\rho)}{(1-\phi)^2 D'(r^d) - L'(\rho)}$$

This expression is positive if the elasticity of loan demand with respect to ρ is larger than the inverse of the elasticity of ρ with respect to $(1-\phi)$--i.e. if the term in brackets is positive. In models such as Romer's, a rise in the required reserve ratio causes an increase in the loan rate (Romer, p.183). In this model, however, the banks' return on loans must rise, but r^1 need not do so. As in the case of a rise in h, if the size of loans declines by a large enough amount, r^1 may actually fall.

The effect of a rise in ϕ on government revenue is the sum of three effects:

$$(12) \qquad \frac{dg}{d\phi} = (D(r^d)-B)\frac{h}{1+h} + \phi D'(r^d)\frac{h}{1+h}\frac{dr^d}{d\phi} - B\frac{1}{1+n}\frac{dr^d}{d\phi}$$

The first term is the direct effect of a rise in ϕ on government seigniorage. All else equal, as the required reserve ratio rises, the quantity of high-powered money does as well, thereby increasing seigniorage so long as h is positive. The second term is the indirect effect on seigniorage caused by the change in the deposit rate and the consequent change in the steady-state quantity of high-powered money. The final term takes account of the effect the change in r^d has on government debt service. The last two terms are of opposite sign, and the overall effect is ambiguous.

Change in the level of government debt. Given r^d, an increase in B, the level of government debt per-capita, does not affect ρ, and so has no effect on the size of loans. Instead the change shifts the demand curve directly. The effect of this change is clearly to raise r^d and $D(r^d)$, as shown in Figure 8. The rise in r^d induced by this change reduces the size of loans and investment.

As in the two cases above, the effect of this change on r^1 is not clear because, at the new equilibrium loan size, loans will be safer. Thus the usual result--that r^1 rises-- again does not follow in this case.[6]
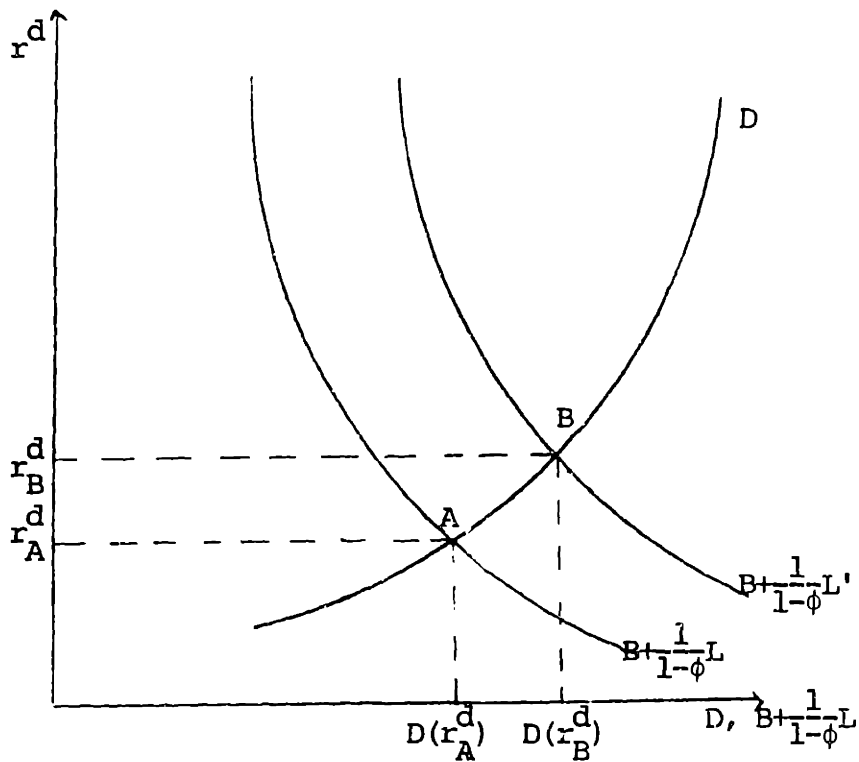
FIGURE 7

Effect of an Increase in ϕ --case with $\frac{dr^d}{d\phi} > 0$
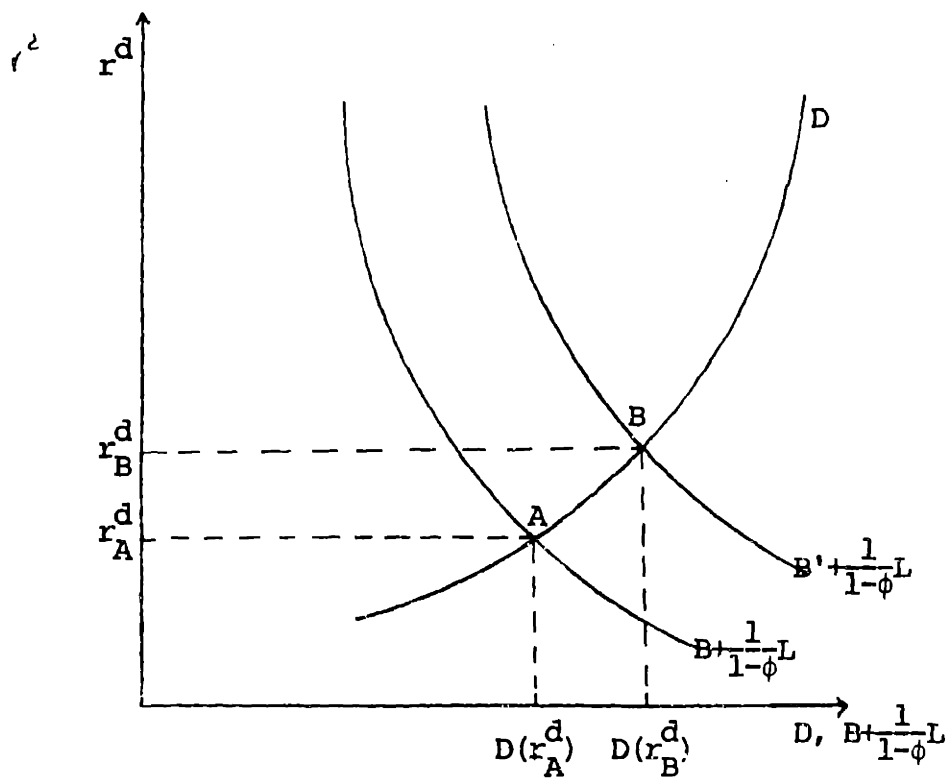
FIGURE 8

Effect of an Increase in B

The effect of an increase in B on the level of
government revenues is again the sum of three terms:

$$(13) \qquad \frac{dg}{dB} = -\frac{\phi}{1-\phi} L'(\rho) \frac{\partial \rho}{\partial r^d} \frac{\partial r^d}{\partial B} \frac{h}{1+h} + \frac{n-r^d}{1+n} - B \frac{1}{1+n} \frac{dr^d}{dB}$$

The first term takes account of the change in seigniorage
due to the reduced level of loans and consequent fall in the
equilibrium level of high-powered money. It is
unambiguously negative if h is greater than zero. The
second term is the direct effect of an increase in B on
government revenues, and it can be positive or negative.
The final term takes account of the revenues lost due to the
higher interest payments on the stock of government debt.
The total effect is, again, of ambiguous sign.

The investigation of comparative steady-states suggests
an important result: the loan rate may not serve well as an
indicator of what is happening in the money market. In
particular, a tighter market for loans (as measured by the
quantity of lending, or equivalently by the expected return
on investment) may not be characterized by higher loan
interest rates. The reason for this complication is that
changes in the size of loans cause a change in the riskiness
of the loan. Thus policy makers can do better by focusing
on a risk-free rate, such as the deposit rate, or, more
directly, on the quantity of lending, when assesing the
effects of asset market interventions on the real economy.

(BLANK PAGE)

IV. Welfare Issues

In this section I consider the problem of a central planner attempting to maximize social welfare. If the planner assumes that all individuals have identical linear utility functions, then this problem is equivalent to maximizing expected aggregate consumption. If the planner refunds government revenues to each period's old generation, then consumption per member of the old cohort is:

$$(14) \qquad C = (1+r^d)D(r^d) + P(I^*(r^d))(1+r)I^*(r^d)\int_{I^*(r^d)}^{\bar{W}} f(W)dW$$

$$+ \int_{I^*(\rho)}^{I^*(r^d)} P(W)(1+r)Wf(W)dW$$

$$+ P(I^*(r^d))\int_{\underline{W}}^{I^*(\rho)} \left\{(1+r)I^*(\rho) - (1+r^1)(I^*(\rho) - W)\right\}f(W)dW$$

$$+ (1+n)g$$

The first term is the second-period return on the deposits of the old generation. The second, third, and fourth terms are the returns on investment, less loan repayments, of those with high, medium, and low endowments respectively.

member of the old cohort. This expression can be rewritten as:

$$C = R(I^*(r^d))[1-F(I^*(r^d))] + \int_{I^*(\rho)}^{I^*(r^d)} R(W)f(W)dW$$

$$+ R(I^*(\rho))F(I^*(\rho)) + (1+n)H + (1+n)B$$

where F(W) is the cumulative distribution function for W. The first three terms are the returns on investment of high, medium, and low endowment individuals. The last two terms are the income that accrues to the old cohort due to the "sale" of high-powered money and bonds to the young cohort. It is convenient to write the last two terms as:

$$(1+n)\phi D(r^d) + (1+n)(1-\phi)B$$

The central planner's problem is:

$$\begin{array}{c} \text{Maximize C} \\ \phi, h, B \end{array}$$

Remembering the definitions of $I^*(r^d)$, $I^*(\rho)$, and ρ; one can write the following first order conditions:

(16) $$\xi\frac{dr^d}{d\phi} + R'(I^*(\rho))L'(\rho)\frac{\partial\rho}{\partial\phi} + (1+n)[D(r^d)-B] = 0$$

(17) $$\xi\frac{dr^d}{dh} + R'(I^*(\rho))L'(\rho)\frac{\partial\rho}{\partial h} = 0$$

(18) $$\xi\frac{dr^d}{dB} + (1+n)(1-\phi) = 0$$

where,

$$\xi = R'(I^*(\rho))L'(\rho)\frac{\partial\rho}{\partial r^d} + [(1+n)\phi - R'(I^*(r^d))]D'(r^d)$$

From these first order conditions it is straightforward to show the following:

(19) $\quad R'(I^*(\rho)) = R'(I^*(r^d)) = 1+r^d = 1+n$

(20) $\quad P(I^*(\rho))(1+r^l) = 1+n$

(21) $\quad h = 0$

Equation (19) shows that in order to maximize aggregate steady-state consumption, the central planner should choose ϕ and B so as to make the marginal return on investment equal to the rate of growth of the population--the familiar dynamic efficiency result from Diamond (1965). Equation (20) again suggests that policy makers should consider the risk-free rate or take the level of risk into account when formulating policy. For example, if the government set the loan interest rate equal to the rate of growth of the population, then the expected return on investment would be lower than the rate of population growth. As a result, the economy would be inefficient. Finally, equation (21) implies the usual optimal money supply rule result: that thereturn on currency should be equal to the return on capital.

V. More Complex Financial Structure

One problem with the model presented thus far is that the banks would be dominated by a bond market, because by using bonds to lend among themselves, agents could avoid the required reserve "tax." In this section I relax the assumption, which I have maintained until now, that direct loans between agents are not possible.

There are three possible ways to add a bond market to the model without completely eliminating the banking sector. First, one can assume that banks exist because they solve an information problem that bond markets cannot solve (Bernanke and Gertler, 1985). Second, one can impose that the two types of credit have different cost structures. For example, if there were a substantial fixed cost of using the bond market, then those who want to borrow small amounts would choose to borrow from the bank, even if in so doing they incur a proportional tax.

Finally, following Romer, one could assume that deposits provide transactions services. In this case, banks need not pay a return on deposits that is competetive with the return on private loans (Romer, 1986). As Fama has pointed out,

however, the rate paid on certificates of deposit--banks'

marginal source of funds--is virtually identical to that

paid on commercial paper (Fama,1985). Thus borrowers from

banks, not depositors, bear the incidence of the tax.

In this section I consider the effect of adding a bond

market with a fixed entry cost--the second strategy just

mentioned--to the model presented earlier. In particular, I

allow borrowers to borrow directly from other individuals in

a bond market by paying a fixed cost of k units of the

good.[7] I assume that the k units can be borrowed if the

agent's endowment is less than k. Because individuals are

risk neutral, the expected return on these bonds is the same

as the return on bank deposits, r^d.

Partial Equilibrium. The potential borrower faces a

more complex problem in this case. It is:

$$\text{Max: } R(W+L+BC-j^bk) - (1+\rho)L - (1+r^d)BC - j^bk(1+r^d)$$
$$\text{L,BC}$$

where L is the size of the bank loan, BC is the amount of

credit obtained in the bond market, and j^b is one if the

bond market is used and zero if it is not. Notice that I

have already substituted the bank zero profit condition and

the equality of the expected return on bonds to the deposit

rate into the problem.

Clearly, since r^d is less than ρ, borrowers will use

either the bond market or bank credit, but never both. If

an agent were to use both, a reduction in bank borrowing and

an increase in bond borrowing would raise his expected second period income. Thus agents will choose the better of the following alternatives:

$$L = I^*(\rho) - W, \quad BC = 0$$

or,

$$BC = I^*(r^d) - W + k, \quad L = 0$$

Notice that in each case the marginal expected return on investment is equal to the cost of funds.

For large enough values of k, no individual will ever choose to use the bond market, regardless of the value of his endowment. Conversely, for small values of k, bank credit will never be chosen. I focus here on values of k that allow either type of borrowing to be used, depending on the borrowers endowment size. As one would expect, agents with small endowments find it worthwhile to pay the entrance fee and get the lower bond market interest rate on a large loan, while agents with larger endowments borrow a smaller amount at the higher interest rates charged by banks. I define W^* to be the endowment size at which agents are just indifferent between the two alternatives. Thus at W^*:

$$R(I^*(\rho)) - (1+\rho)(I^*(\rho) - W^*)$$
$$= R(I^*(r^d)) - (1+r^d)(I^*(r^d) - W^* + k)$$

so,

$$W^* = -\frac{1}{\rho - r^d}\left\{R(I^*(r^d)) - R(I^*(\rho)) - (1+r^d)(I^*(r^d) + k) + (1+\rho)I^*(\rho)\right\}$$

Thus the demand for credit in the bond market and the demand for loans are given by:

$$(22) \qquad BC(r^d,\rho) = \int_{\underline{W}}^{W^*(r^d,\rho)} (I^*(r^d) - W + k)f(W)dW$$

and,

$$(23) \qquad L(r^d,\rho) = \int_{W^*(r^d,\rho)}^{I^*(\rho)} (I^*(\rho) - W)f(W)dW$$

General Equilibrium. The balance sheets of the agents now look as follows:

TABLE 2

Asset/Liability	Individuals	Banks	Government
Deposits/Bonds	D-BC	-(D-BC-B)	-B
Loans	-L	L	
Currency		ϕ(D-BC-B)	-H
Investments	W_m+L-D+BC-k_m		

Most of the variables in Table 2 are the same as those in Table 1. I have added two new variables. First, k_m, the per capita cost of using the bond market. Second, BC, the per capita amount borrowed in the bond market. Hence, D-BC is the net holdings of financial assets by individuals. The amount of deposits at banks is clearly the net financial assets of individuals less their holdings of government bonds.

With the changes in the model, equations (7) and (8) also change:

(7'') $\qquad L(\rho,r^d) = (1-\phi)(D(r^d)-B-BC(\rho,r^d))$

(8') $\qquad H = \phi(D(r^d)-BC-B)$

Both of these equations are results of the banks' balance sheet identity and the assumption that nominal interest rates are positive—i.e. banks do not choose to hold excess reserves. As before, it will prove simpler to rewrite equation (7'') as:

(7''') $\qquad B + BC(\rho,r^d) + \frac{1}{1-\phi}L(\rho,r^d) = D(r^d)$

Again the left-hand side is the demand for savings, and the right-hand side is the supply of savings. The $1/(1-\phi)$ term accounts for the fact that it takes $1/(1-\phi)$ dollars of deposits to fund a dollar of loans. $1/(1-\phi)*L(\rho,r^d)$ is again the indirect demand for funds by bank borrowers.

The other steady-state conditions for the economy remain unchanged.

<u>Equilibrium Interest Rates</u>. As noted earlier, for constant values of ϕ and π, the banks' zero profit condition implies a linear relationship between ρ and r^d:

$$1+r^d = (1-\phi)(1+\rho) + \phi\frac{1}{1+\pi}$$

By substituting this relationship into equation (7'''), I again make both the supply of and the demand for funds be

functions of a single interest rate, r^d. The curves are shown in Figure 9. It is not obvious that the demand for funds is necessarily a decreasing function of r^d, but this is the case. If X is the demand for funds, then its derivative is:

$$X = B + BC(\rho(r^d), r^d) + \frac{1}{1-\phi}L(\rho(r^d), r^d)$$

$$\frac{dX}{dr^d} = \frac{\partial BC}{\partial \rho} \frac{d\rho}{dr^d} + \frac{\partial BC}{\partial r^d} + \frac{1}{1-\phi}\left\{\frac{\partial L}{\partial \rho} \frac{d\rho}{dr^d} + \frac{\partial L}{\partial r^d}\right\}$$

or,

$$= \int_{\underline{W}}^{W^*(\rho, r^d)} I^{*'}(r^d)f(W)dW + \frac{1}{(1-\phi)^2}\int_{W^*(\rho, r^d)}^{I^*(\rho)} I^{*'}(\rho)f(W)dW$$

$$+ \left[I^*(r^d) + k - W^*(\rho, r^d) - \frac{1}{1-\phi}(I^*(\rho) - W^*(\rho, r^d))\right] \frac{dW^*}{dr^d}$$

The first two terms account for the effect of higher interest rates on the size of loans and bond issues. Both are negative. The third term accounts for the fact that borrowers with endowment size W^* borrow different amounts depending on whether they borrow from banks or from the bond market. Thus shifts in W^* will change the level of demand by shifting some of these marginal borrowers from banks to the bond market, or visa versa. The change in W^* is given by:

$$\frac{dW^*}{dr^d} = -\left\{I^*(r^d) + k - W^* - \frac{1}{1-\phi}(I^*(\rho) - W^*)\right\} \frac{1}{\rho - r^d}$$
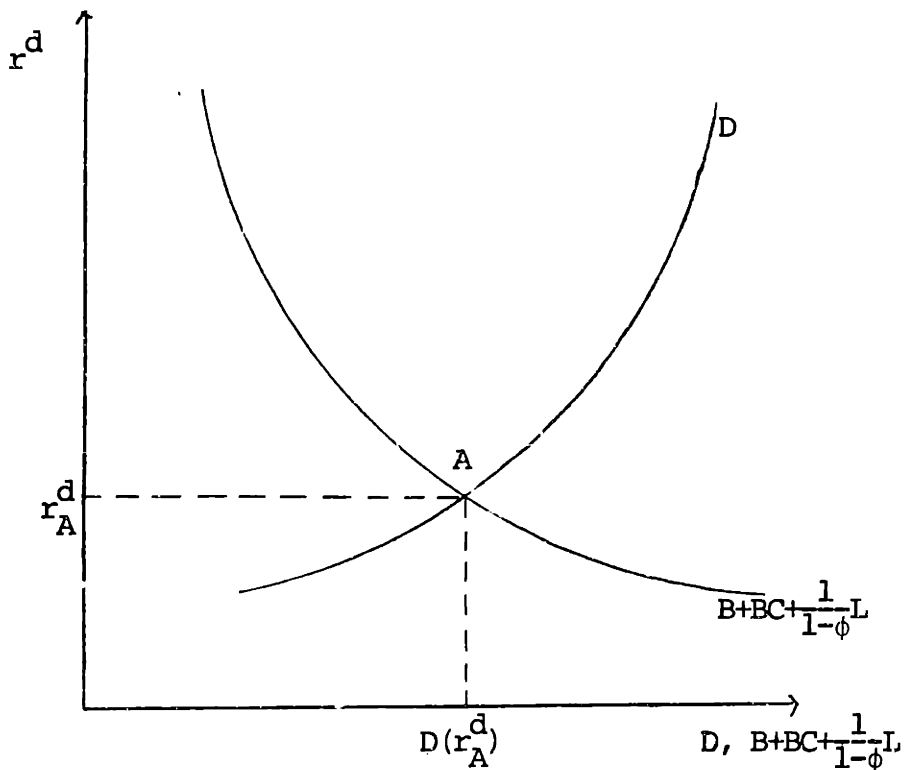
FIGURE 9
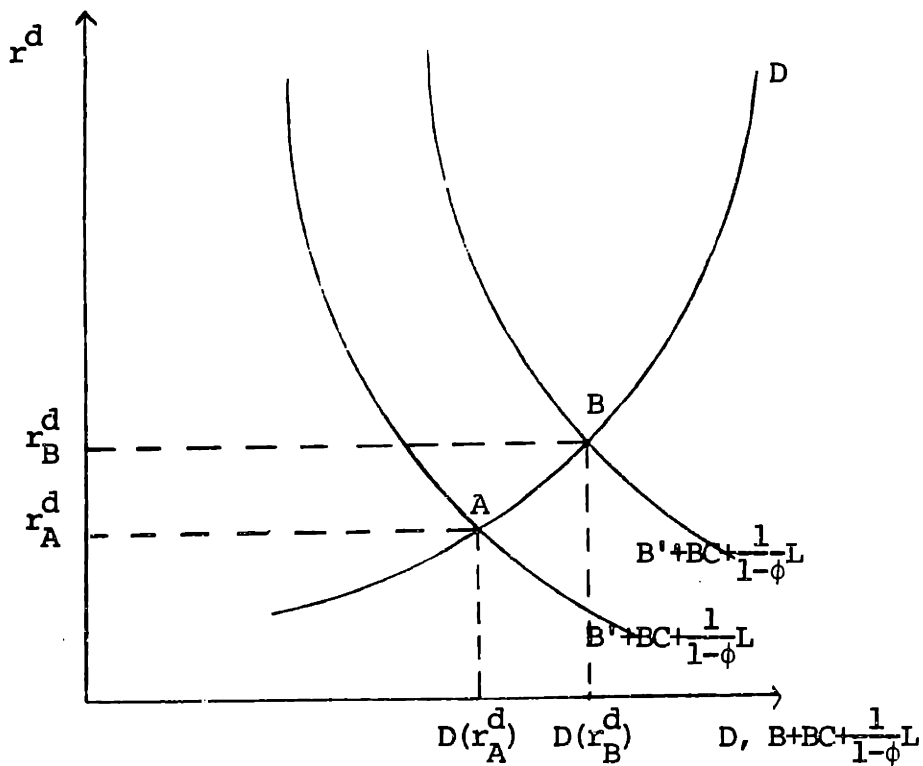
FIGURE 10

Effect of an Increase in B

Notice that the bracketed term is the same as the one in the third term of dX/dr^d. As a result, the third term is negative, and so the demand for funds does indeed fall as r^d rises.

Effects of Changes in Policy Variables. The effect of a change in the level of government debt, B, is obvious. As in the simpler case, demand rises for each level of r^d, while supply remains the same. Thus, r^d rises and savings increases. Because ρ rises with r^d, investment declines. (See Figure 10)

The effects of changes in the rate of growth of high-powered money, h, and the required reserve ratio, ϕ, are not clear. An increase in h raises the steady-state inflation rate. This change raises ρ for a given level of ϕ and r^d. This change has two effects. First, the rise in ρ reduces loan demand by those borrowing from banks. Second, the change in ρ causes a rise in W^*. Not surprisingly, some individuals who were using banks now find the bond market preferable. The sum of these two effects is negative if:

(25) $I^*(r^d) + k - W^* < \frac{1}{1-\phi}(I^*(\rho) - W^*)$

In words, the indirect demand for funds by those with endowments of W^* borrowing from banks is larger than the demand for funds by those borrowing in the bond market.

The reason for this result is clear. The rise in ρ causes a fall in demand by those borrowing from banks. If

it also causes a fall in demand by those who shift from banks to the bond market, then the overall effect is negative. If the inequality in (25) does not hold, then the result is ambiguous.

The effect of a rise in ϕ is even more uncertain. For given levels of h and r^d, a rise in ϕ causes a rise in ρ. Thus both of the effects of a change in h are also caused by a change in ϕ. To these the change in ϕ adds another effect. When ϕ rises the indirect demand for funds, given L, rises because banks must hold additional reserves. All else equal, this shift would cause an increase in demand. As a result, the net effect is not clear regardless of the condition in (25).

Loan Interest Rates. As in the simpler model, the rate charged on a loan must take account of the changes in the probability of default. As a result, there is no necessary correspondence between the loan and bond interest rates and ρ and r^d. In particular, perverse changes in the loan rates are still possible.

Welfare Results. In spite of the more complex financial structure, the welfare results from Section IV still hold here. The only difference is that now the government should reduce the required reserve ratio and the rate of growth of high-powered money in order to eliminate bond market borrowing by individuals. This policy raises welfare

because it eliminates the dead-weight loss associated with the cost of entering the bond market. Clearly, if I assumed that bank intermediation was not costless, then this result would change. Because there are no externalities, if the government removed the tax on deposits--i.e. set the required reserve ratio equal to zero--then the market would choose the optimal mix of bond market and bank lending. In any case, it is still optimal for the government to use its debt to equate the expected return on investment to the rate of growth of the population in order to achieve dynamic efficiency.

VI. Summary

In this chapter I consider the effect credit rationing has on the steady states of a simple general equilibrium model. I also examine the effects of changes in the reserve requirement, the rate of growth of the money supply, and the level of government debt on these steady states. Finally, I investigate the issue of optimal government policy in such an economy.

The model has a two-period overlapping generations structure. A simple financial structure is imposed, with banks serving as the only intermediaries between borrowers and lenders. The only role for money in the model is as reserves held by the banks. Type one credit rationing results from the fact that investment projects become riskier as they become larger.

The paper concentrates on steady-state equilibria. I find that changes in government policy variables have effects on the deposit rate and quantity of lending much like those in a model without any rationing in the credit market, such as Romer's (Romer, 1985). The effects on the loan rate, however, can differ considerably. The reason for

the difference is that in this model policies that cause a reduction in loan size also make projects safer. As a result, the loan rate can actually fall in the face of tighter monetary policy. This result implies that the government should be wary of using the loan rate to gauge the effectiveness of monetary policy. Instead, a risk-free rate or the quantity of credit extended should be used.

The solution to the problem of a central planner maximizing social welfare is reminiscent of Diamond's result (Diamond, 1965). The planner should choose the required reserve ratio and level of government debt so as to equate the return on investment and the rate of population growth. In addition, the rate of growth of high-powered money should be set to zero. This action makes the return on holding currency equal to the rate of population growth, hence it eliminates the opportunity cost of holding money. It is also noted that the risk-free rate and not the loan rate should be equated to the rate of population growth in order to attain dynamic efficiency.

I then consider an economy in which loans can be made directly between agents. The banking sector is not crowded out completely because there is a fixed cost of using the "bond market." The results of this more complex model are similar to those of the simpler model. In particular, the correct interest rate to use in assessing the effects of monetary policy is still the risk-free rate. As I show in

the next chapter, this simple result does not carry over to a model with type two rationing.

NOTES

1. This result can be put in terms of Tobin's q. At point A the marginal q for the investor is greater than 1, but the marginal social q is equal to 1.

2. I need to make an additional assumption in order to be sure that the banks are used. They will be used if the person with the largest endowment can get a higher return by depositing a marginal piece of his endowment rather than by investing it himself. This condition amounts to:

$$(1-\phi)R'(\underline{W}) + \phi\frac{1}{1+\pi} > 0$$

because the bank's loan of the marginal amount will go to the individual with the smallest endowment.

3. More precisely, the two will be equal only when the rate of deflation is equal to the return on loans--i.e. when the nominal interest rate is zero. Note also that costs of intermediation or bankruptcy costs would make the two unequal.

4. Since $R'(I)$ is continuous, demand cannot be inelastic except at a point.

5. This fact was noted by Keeton in a partial equilibrium setting (Keeton, 1979, Chapter 1).

6. See Romer, 1986, p.185. The fact that crowding out occurs without necessarily raising the loan rate is somewhat like the result I show in Chapter 2. Note, however, that the deposit rate does rise as does ρ. This will not be the case there.

7. This is an overstatement, of course. The results should not be greatly changed if one assumes bond market borrowing is characterized by subtantial increasing returns to scale.

REFERENCES

Bernanke, Ben and Gertler, Mark. "Banking in General Equilibrium." Mimeo., University of Wisconsin, May 1985.

Diamond, Peter A. "National Debt in a Neoclassical Growth Model." American Economic Review 55 (December 1965): 1126-1150.

Fama, Eugene F. "What's Different about Banks?" Journal of Monetary Economics 15 (January 1985): 29-40

Jaffee, Dwight M. and Russell, Thomas. "Imperfect Information, Uncertainty, and Credit Rationing." Quarterly Journal of Economics 90 (November 1976): 651-666.

Keeton, William. Equilibrium Credit Rationing. New York: Garland Press, 1979.

Romer, David. "Financial Intermediation, Reserve Requirements, and Inside Money: A General Equilibrium Analysis." Journal of Monetary Economics 16 (September 1985): 175-194.

Smith, Bruce. "Limited Information, Credit Rationing, and Optimal Government Lending Policy." American Economic Review 73 (June 1983): 305-318.

CHAPTER TWO

CREDIT RATIONING IN GENERAL EQUILIBRIUM

Introduction

In this chapter I consider the general equilibrium implications of type two rationing. Two models of type two rationing are proposed by Keeton (1979), and Stiglitz and Weiss (1981). Keeton uses a principal-agent model to show that high interest rates can have an adverse effect on the probability of repayment. If this effect is strong enough, and the size of the loan is fixed, then the competitive equilibrium may be at an interest rate that does not clear the market for loans. This result obtains because an increase in the loan rate would cause such a large fall in the probability of repayment that the lender's profits would actually fall. Similarly, Stiglitz and Weiss show that as a lender raises the loan interest rate, the pool of would-be borrowers becomes riskier. Again, if this effect is strong enough, then rationing results.

The fact that lenders may choose to ration credit rather than raise interest rates in order to clear the market raises three interesting issues. First, rationing may alter the way government asset market interventions affect the economy in a general equilibrium setting. In

particular, one would like to know how changes in the required reserve ratio, the rate of growth of high-powered money, and the per-capita level of government debt affect an economy with this sort of rationing. These results could then be compared to those in a model, like that in Romer (1985), that does not have rationing. Second, it is not clear what government policies are optimal in an economy with credit rationing. Third, one would like to know the welfare implications of the rationing. As will be noted below, the expected adverse effects of rationing need not occur if the rationing is chosen by optimizing lenders.

In order to study these three issues, I embed a model of type two rationing in a simple overlapping generations general equilibrium model similar to the one used in the last chapter. Again, I impose a simple financial structure on the model. There is a banking sector that intermediates between borrowers and lenders, and money serves only as reserves for the banks. As in the last chapter, the returns on investment are uncertain, but now I assume that borrowers are non-homogeneous. These changes cause banks to ration credit because of the adverse selection effects of higher interest rates, as in Stiglitz and Weiss.

The first section of the paper presents the model. First a version without money is considered in order to show why rationing occurs in the loan market. Then I add money to the model and characterize the steady-state. The second

section investigates the effects on the steady-state of varying the required reserve ratio, the rate of growth of high powered money, and the real level of government debt. The third section contains an analysis of government policy and the welfare effects of the rationing. The fourth section considers some generalizations of the model, and a final section summarizes the results.

I. The Model

A. The model without money

I first consider the model without the complication of money in order to display more clearly the source of the banks' rationing behavior.

Individuals. People are risk-neutral; they are endowed with W when they are young. They desire to consume only when old. They have two ways to save for the next period. First, people can deposit their endowment in a bank and receive $(1+r^d)W$ next period. Second, they can take out a loan from a bank and invest their endowment plus the amount of the loan in a project. The projects have an uncertain return. I assume that endowments cannot be stored.

I assume that equity contracts are not available. If such contracts were available, then the imperfections in the credit market examined here could be avoided. Endogenizing the lack of an equity market poses no technical difficulties, but would make the exposition less clear. For example, if the outcome of investment projects depended on the investor's level of effort, then I could introduce

results from the share-cropping literature to show that equity contracts are also inefficient (Stiglitz, 1974). As a result, debt contracts could be used even though imperfect. More simply, one could hypothesize that investors can lie about the outcome of their projects. If a declaration of bankruptcy allows the lender to "check the books," while declaring a bad state under an equity contract does not give the equity holder the same right, then debt would have a natural advantage over equity.

I show in Section IV that projects in this model can be quite complex, but for expositional purposes I use a particularly simple arrangement here. Projects have a fixed size and are of one of two types. People borrow W from a bank at a rate r^1 and invest 2W in a project. People are in one of two classes: those with high-risk projects and those with low-risk ones, each with probability one half. An agent's class is not observable, although agents are assumed to know their own class. High risk projects succeed with a probability P_h and fail with a probability $1-P_h$. Individual projects are independent. When they succeed the projects yield $W+W(1+r_h)$. When they fail they yield W, i.e., the investor pays back the loan, but without interest. Similarly with low-risk individuals' projects, which have a probability of success of P_l ($P_l > P_h$) and which yield $W+W(1+r_l)$ if successful.

I assume that:

$$P_h(1+r_h) = P_1(1+r_1) = 1+r^*$$

In other words, the expected value of high and low-risk projects before the payment of interest is the same.

Taking interest payments into account, if a project is successful, then investors receive:

$$W + W(1+r_i) - (1+r^1)W \qquad\qquad i = h, 1$$

If the project is not successful, then investors go bankrupt and pay the entire gross return, W, to the bank.

Because individuals are risk-neutral, they care only about their expected return or:

$$P_i(W+(1+r_i)W - (1+r^1)W) + (1-P_i)\ 0 \qquad i = h, 1$$

$$= P_i((1+r_i)W - r^1 W)$$

$$= ((1+r^*) - P_i r^1)W$$

$$\equiv (1+\rho^1)W$$

Notice that the expected return rises as P_i falls--i.e., the expected return to the investor after payment of interest is higher for the riskier projects. The reason for this result is that the expected interest payments are lower because the loan is repaid less often.

Banks. Banks are risk-neutral intermediaries (or are so large that the risk from an individual borrower is completely diversified). No loans directly from one individual to another are allowed. Banks are assumed competitive and make zero profits. Their intermediation services are assumed costless. As noted above, they cannot

tell if individuals are high- or low-risk. A bank's return on a loan to an investor is:

$$P_i(1+r^1)W + (1-P_i)W(1-c) \qquad i = H, L$$

where c is the cost (to the bank) of bankruptcy as a percentage of the loan.[1] Notice that the return on loans is lower for loans to high-risk individuals (at a given r^1).

The market for loans. In order to study the equilibrium in the credit market, I begin by looking at the combinations of deposit rates and loan rates that leave borrowers just indifferent between depositing their endowments and borrowing in order to invest in a project. Low-risk borrowers get a return on investment of:

$$\rho^1 = r^* - P_1 r^1$$

Because they are risk neutral, they are indifferent between their portfolio alternatives if this rate of return is equal to that available on deposits and bonds. Thus their indifference locus is given by:

$$r^d = r^* - P_1 r^1$$

where r^d is the rate paid by banks on deposits. Similarly, high-risk borrowers have an indifference locus given by:

$$r^d = r^* - P_h r^1$$

These lines are plotted in Figure 1.

Because the banks are assumed to make zero profits, the deposit rate is equal to the return on their loan portfolios. Thus one must consider the expected return banks make on loans. At low loan interest rates, borrowing
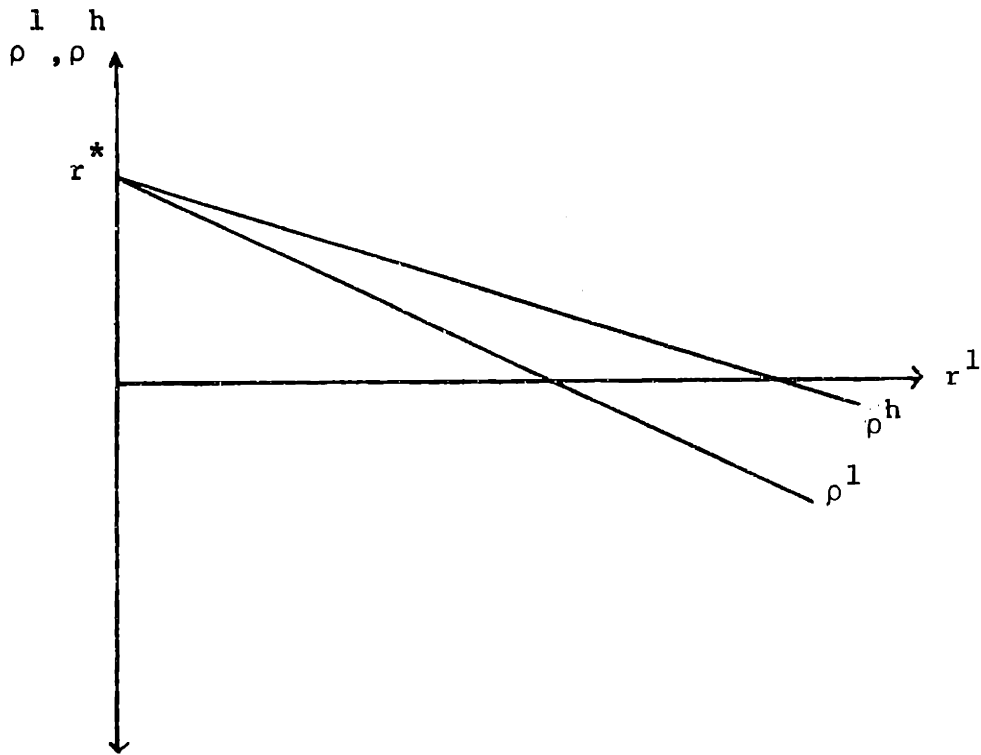
FIGURE 1
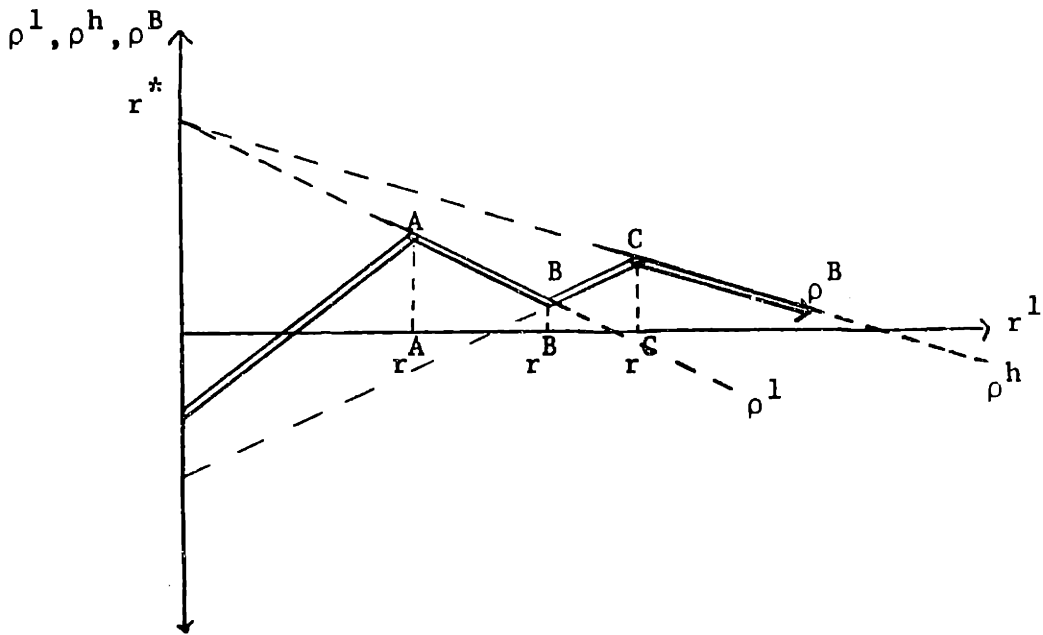
FIGURE 2

and investing is profitable for both high and low-risk individuals, hence there is excess demand for loans. As a result, the banks make loans to half the population, forcing the other half to deposit their endowments as their next best alternative. Their expected return on loans is:

$$\rho^B = \bar{P}r^l - (1-\bar{P})c$$

where $\bar{P} = (P_l + P_h)/2$. In other words, the banks expect to make loans to an equal mix of both types. This line is plotted in Figure 2 for loan rates between zero and r^a.

If, however, banks charge a loan rate above r^a, low-risk borrowers would prefer to deposit their endowments. Thus if the banks offer a loan rate between r^a and r^b, arbitrage by low risk agents will ensure the equality of ρ^B and ρ^l y decreasing the proportion of the banks' borrowers that are low-risk. (See Figure 2.)

At a loan rate of r^b, all low-risk individuals deposit their endowments. Therefore, additional rises in the loan rate cannot cause more adverse selection. As a result, the banks return on loans made at rates between r^b and r^c is given by:

$$\rho^B = P_h r^l - (1-P_h)c$$

This line is plotted in Figure 2. Since in this case the low-risk half of the population makes deposits and the high-risk half borrows, the credit market clears at any rate in this interval.

Finally, if the banks loan funds at a rate above r^c,

their rate of return on loans exceeds the rate of return on investment for high-risk people. As a result, some high-risk people deposit their endowment, causing the banks to hold excess reserves and thereby reducing the rate paid on deposits to the rate of return high-risk people get by borrowing and investing. (See Figure 2.)

Profit maximizing banks will charge the loan rate that maximizes the rate of return on their loan portfolio. Thus, there are two possible competitive equilibria in this model, points A and C. I show below that for sufficiently high bankruptcy costs the banks' return at r^a is higher than that at r^c. If in this case the economy is at point C, then any bank could make loans at r^a and pay depositors a rate of return equal to that earned by the banks loaning at r^c. This strategy would result in strictly positive profits for the bank, and so the equilibrium would move to point A. In other words, point A is the unique Nash equilibrium in this case. At point A, of course, all individuals would like to borrow. Since the banks can only lend to half of the population, while the other half must supply the deposits, banks have to ration credit. Thus the macro equilibrium is in this case characterized by rationing in the credit market.

B. The model with money

As in the case above, banks accept deposits and make loans, only now they have to hold part of their deposits as

reserves.[2] In the case I consider here, money serves only as reserves for the banks. (Although it is noted in Section IV that a particular type of transactions demand for money does not greatly change the results presented here.) In addition to supplying reserves to the banks, the government can sell bonds to the younger generation. Thus, individuals now choose between holding government bonds, depositing their endowment in the bank, or borrowing from the bank and investing in their project.

The balance sheets of the different groups are similar to those in the last chapter. They are summarized in Table 1.

TABLE 1

Asset/Liability	Individuals	Banks	Government
Deposits/Bonds	D	-(D-B)	-B
Loans	-L	L	
Currency		$\phi(D-B)$	-H
Investments	W+L-D		

Where the following are real values per member of the young cohort:

D = individuals' holdings of financial assets--i.e. deposits and bonds.

B = individuals' holdings of government bonds.

H = the stock of currency (high-powered money).

L = the quantity of loans individuals receive from banks.

I assume that nominal interest rates are positive, and so banks do not choose to hold excess reserves, and individuals do not want to hold currency.

I also define the following:

h = the rate of growth of the nominal money stock.

n = the rate of growth of the population.

π = the inflation rate.

ϕ = the required reserve ratio.

g = the level of government spending.

i^d, i^l, i^b = the nominal interest rate on deposits, loans, and government bonds respectively.

r^d, r^l, r^b = the real interest rates on deposits, loans, and government bonds respectively.   .

ι^B, ρ^B = the nominal and real expected rates of return on loans respectively.

ρ^h, ρ^l = the real expected rate of return on high- and low-risk investments respectively.

c = the cost to the bank of bankruptcy, as a percentage of the loan amount.

The probabilities are the same as in the case without money.

Steady state conditions. First, inflation in steady-state is determined by the rate of growth of the money stock

and that of the population:

(1) $1+\pi = \dfrac{1+h}{1+n}$

Second, I assume that banks make zero profits:

(2) $i^d = (1-\phi)\iota^B$

Third, because bonds and deposits have exactly the same characteristics, their rates of return must be the same:

(3) $i^d = i^b$

Fourth, from the banks' balance sheet identity and the assumption of positive nominal interest rates, I know that:

(4) $L = (1-\phi)(D-B)$

But individuals must be willing to take out these loans, so:

(4.1) $L \leq L^*(r^d, r^1)$

where L^* is the level of loans borrowers would choose given the deposit and loan rates.

Fifth, because required reserves are the only demand for high-powered money,

(5) $H = \phi(D-B)$

Sixth, from the facts that the quantity of investment is W+L-D and that loans finance half of investment, I know:

(6) $D = W-L$

Seventh, the goods market must clear. The total resources available are the output from last period's investment plus this period's endowment. In per capita

quantities this is:

$$\frac{1}{1+n}L(2+r^*-(1-P^*)c) + W$$

Where $P^* = \bar{P}$ if there is rationing and P_h if there is not.
The total uses of resources are the consumption of the old,
the investment of the young, and government spending:

$$\frac{1}{1+n}(L(1+r^*-r^1 P^*)+D(1+r^d)) + 2L + g$$

Equating these two expressions, and substituting $W = D+L$,
one obtains after some reorganization:

$$L\ \frac{1+r^1 P^*-c(1-P^*)}{1+n} + D = L +D\ \frac{1+r^d}{1+n} + g$$

The left-hand side of this equality is the quantity of
funds supplied to the banking system: repayments of loans
from the old cohort, and new deposits from the young cohort.
The first two terms on the right-hand side are the banks'
loans to the young and repayments of deposits to the old.
The final term is the level of government spending.

This expression can be rewritten using equations (1) to
(6) as:

(7) $$g = H\ \frac{h}{1+h} +B\ \frac{n-r^d}{1+n}$$

This equation makes sense. The first term is simply the
government's seignorage from the stock of high-powered
money. The second term indicates that the government gains
revenue from its bonds if the rate of interest it must pay
on them is less than the rate of growth of the population

and visa versa.

In addition, because the government cannot produce anything, it must obtain a positive amount of revenue. Hence:

(7') $g \geq 0$

Equilibrium. What determines if the equilibrium is one with rationing or without? Alternatively, if one cannot use market clearing to determine the interest rates, then how does one tell what they are? As in the case without money, if the banks can get a higher return on their loan portfolio by rationing credit than by loaning at the market clearing rate, then they will do so. Thus, one must look at the interest rates that the banks could choose. First, note that equation (2) can be rewritten as:

(2') $r^d = (1-\phi)\rho^B - \phi\frac{h-n}{1+h}$

If the banks choose to ration credit, then their return on loans is given by:

$$\rho^b = \bar{P}r^1 - (1-\bar{P})c$$

Again, define r^a to be the highest loan rate that the banks can charge without inducing some adverse selection (i.e. the profit maximizing rate under rationing). This rate is the one at which the return on investment for low-risk individuals is the same as the rate they could obtain by

depositing their endowment at a bank, or:

$$r^* - P_1 r^1 = (1-\phi)(\bar{P}r^1 - (1-\bar{P})c) - \phi\frac{h-n}{1+h}$$

which means that:

$$(8) \qquad r^a = \frac{1}{P_1 + (1-\phi)\bar{P}} \left\{ r^* + (1-\phi)(1-\bar{P})c + \phi\frac{h-n}{1+h} \right\}$$

Similarly, I define r^c to be the loan rate at which high-risk borrowers are indifferent between holding deposits and investing. This rate is the market clearing rate that maximizes the banks' return. At this rate:

$$r^* - P_h r^1 = (1-\phi)(P_h r^1 - (1-P_h)c) - \phi\frac{h-n}{1+h}$$

so r^c is:

$$(8') \qquad r^c = \frac{1}{(2-\phi)P_h} \left\{ r^* + (1-\phi)(1-P_h)c + \phi\frac{h-n}{1+h} \right\}$$

In order for banks to choose to ration, the return on their loan portfolios at a loan rate of r^a must be larger than at a rate of r^c. This will be the case if:

$$(9) \qquad c > \frac{r^* + \phi\frac{h-n}{1+h}}{(1-\phi)(P_1 + P_h - 1) + P_1} \equiv \underline{c}$$

In order for the banks to be willing to make loans, however, they must be making a non-negative return on them.

This condition implies that:

$$(10) \qquad c \leq \frac{\bar{P}\left\{r^* + \phi\frac{h-n}{1+h}\right\}}{P\,(1-\bar{P})} \equiv \bar{c}$$

Both (9) and (10) can hold only if $\bar{P}$ is greater than a half. This condition on $\bar{P}$ is somewhat intuitive: if the average probability of success is very low then the size of the bankruptcy costs required to make rationing the preferred alternative is also large enough (and paid often enough) to make loaning money unprofitable.

Thus the addition of money to the model does not change the fundamental rationing result. If bankruptcy costs are sufficiently large, then banks do not use the interest rate to clear the market. If they did so, they would earn a lower return on their loan portfolios. As before, the reduced return is due to the adverse effect that a rise in interest rates would have on the riskiness of loans. In particular, if the banks chose to clear the credit market by raising the loan interest rate, then the loss due to a larger number of defaults and the consequent decrease in repayments and increase in bankruptcy costs would more than offset the increase in interest payments from those with successful projects. As a result, banks choose to make loans at a rate that is low enough to keep all individuals in the market for loans. Because it is not possible for all people to borrow, however, the banks must ration loans to

part of the population. The remainder choose to deposit their endowment because it is their next best alternative.

II. Effects of Changes in Policy Variables

Changes in the rate of growth of high-powered money, h, and the required reserve ratio, ϕ, can change $\underline{c}$ and so shift the economy from a rationed to an unrationed regime. This possibility is taken up in the next section. In this section I consider different steady states with rationing. The equilibrium loan rate, r^a, and deposit rate, r^d, are determined by the intersection of two loci: The banks' zero profit locus, and the curve along which low-risk individuals are just indifferent between depositing their endowment and borrowing and investing.

The zero profit condition is:

$$r^d = (1-\phi)\rho^B - \phi\frac{h-n}{1+h}$$

$$= (1-\phi)(\bar{P}r^1-(1-\bar{P})c) - \phi\frac{h-n}{1+h}$$

$$(11) \qquad r^d = (1-\phi)\bar{P}r^1 - (1-\phi)(1-\bar{P})c - \phi\frac{h-n}{1+h}$$

The indifference locus is:

$$r^d = \rho^1$$

$$(12) \qquad r^d = r^* - P_1 r^1$$

The two curves are plotted in Figure 3.
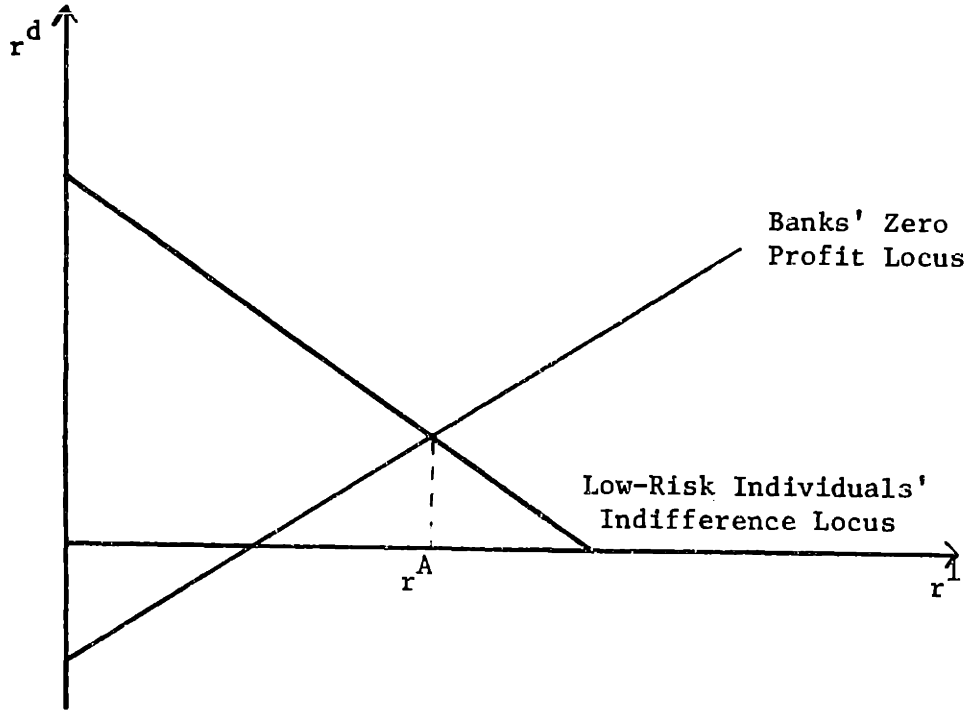
FIGURE 3
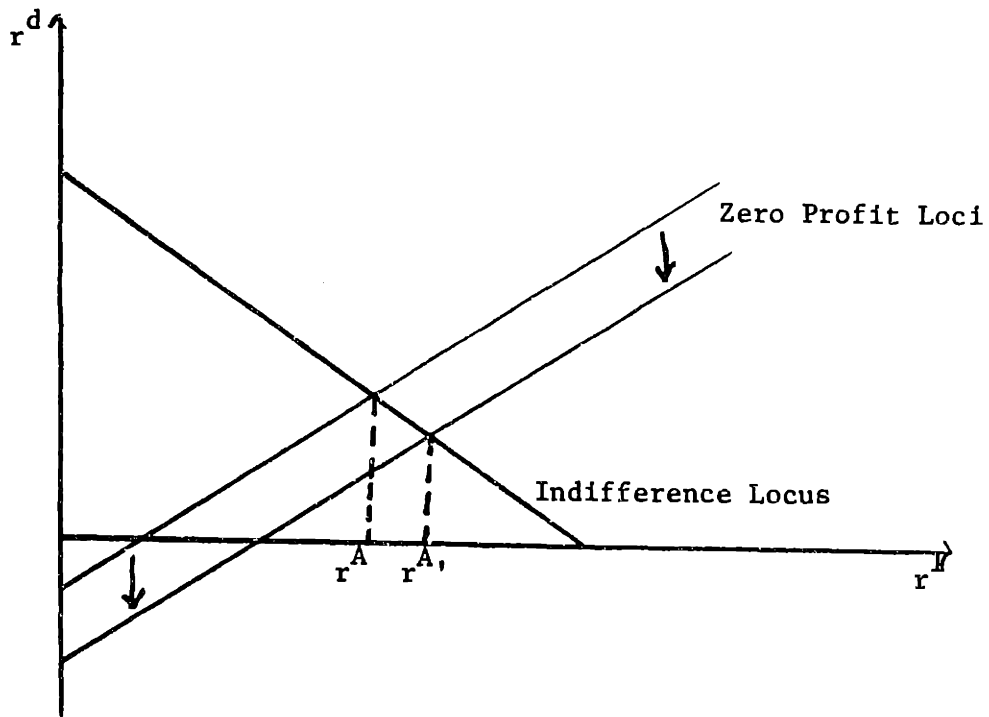
FIGURE 4

The Effect of an increase in the Rate
of Growth of High-Powered Money

Increase in the rate of money growth. When the rate of
growth of high-powered money, h, rises, the low-risk
individuals' indifference curve does not shift. The zero
profit locus has the same slope as before, but its intercept
with the r^d axis shifts down. The reason for this shift is
clear: as h rises, the banks' profits are reduced because
inflation rises (or deflation falls), reducing the return
the banks get on their reserves. Thus, for a given r^1, r^d
will fall. As a result, the loan rate unambiguously rises,
and the deposit rate falls. (See Figure 4.) It can be
shown, however, that the nominal rate on deposits, i^d,
rises, but not by enough to offset the increased inflation.
Thus, Keeton's partial equilibrium result, showing that
expansionary monetary policy leaves interest rates unchanged
but increases the number of loans when there is rationing,
does not apply in this long-run general equilibrium setting
(Keeton, 1976, pp.176-77). Romer finds a result similar to
this one in his general equilibrium model without credit
rationing (Romer, 1985, p.185).

Changing the required reserve ratio. The eff ct of a
change in the required reserve ratio, ϕ, is best seen by
looking at the equilibrium in terms of the nominal interest
rates. The zero profit locus can be written as:

$$i^d = (1-\phi) \iota^B$$

$$(11') \qquad i^d = (1-\phi)\bar{P}i^1 + (1-\phi)(1-\bar{P})(\pi - c(1+\pi))$$

The low-risk individuals' indifference curve is:

(12') $i^d = i^* - P_1(i^1 - \pi)$

where

$$(1+i^*) = (1+r^*)(1+\pi)$$

These two loci are shown in Figure 5.

A rise in the required reserve ratio causes no change in the indifference locus. The increase in the reserve ratio, however, rotates the zero profit line around the point at which it crosses the i^1 axis. The reason for this change is clear. As banks hold more reserves, they receive a lower nominal return on their assets, thus the nominal interest they can pay on deposits declines (see Figure 6).

As a result of the change in ϕ the nominal loan rate rises and the nominal deposit rate falls. Because a change in ϕ does not change the steady-state inflation rate, the same is true for the real loan rate and the real deposit rate. This result contradicts Fama's partial equilibrium result for a non-rationing regime, which is that r^d falls and r^1 remains unchanged (Fama, 1980). Romer finds in his general equilibrium model without rationing that r^1 rises and r^d rises or falls depending on whether deposits are inelastically or elastically supplied by savers. This result is consistent with his because in this case the banking system obtains more deposits simply by rationing more would-be borrowers. The people denied credit deposit their endowment because it is their best remaining alternative.
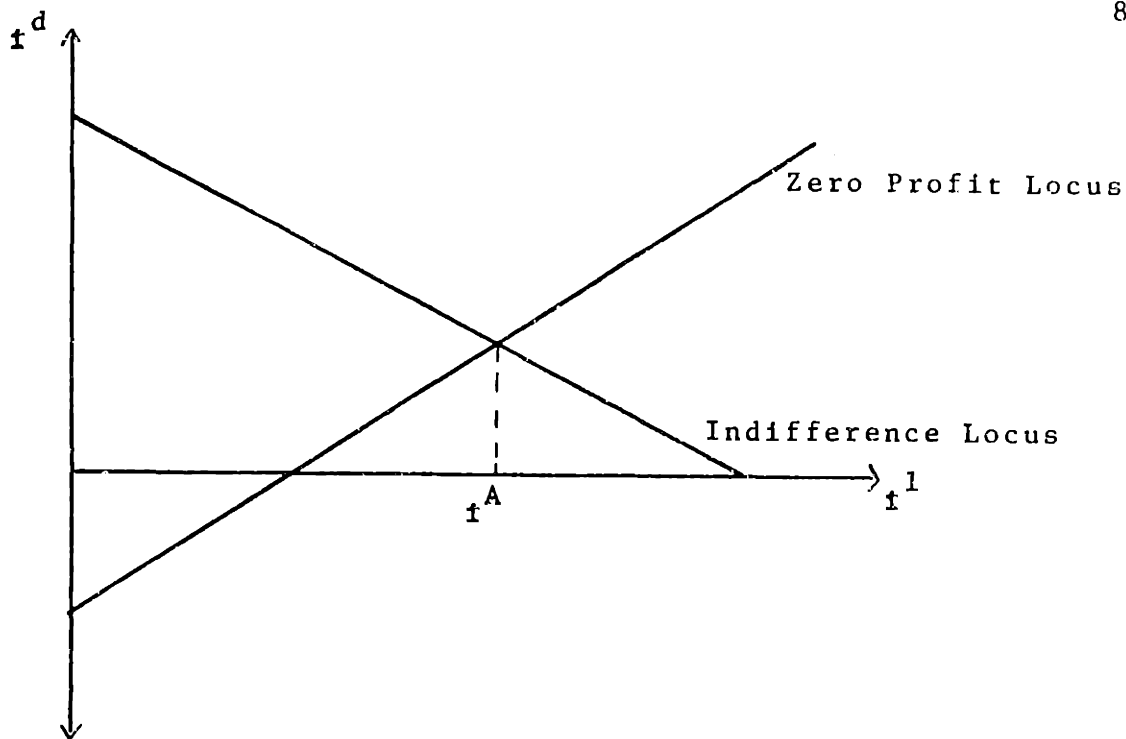
FIGURE 5
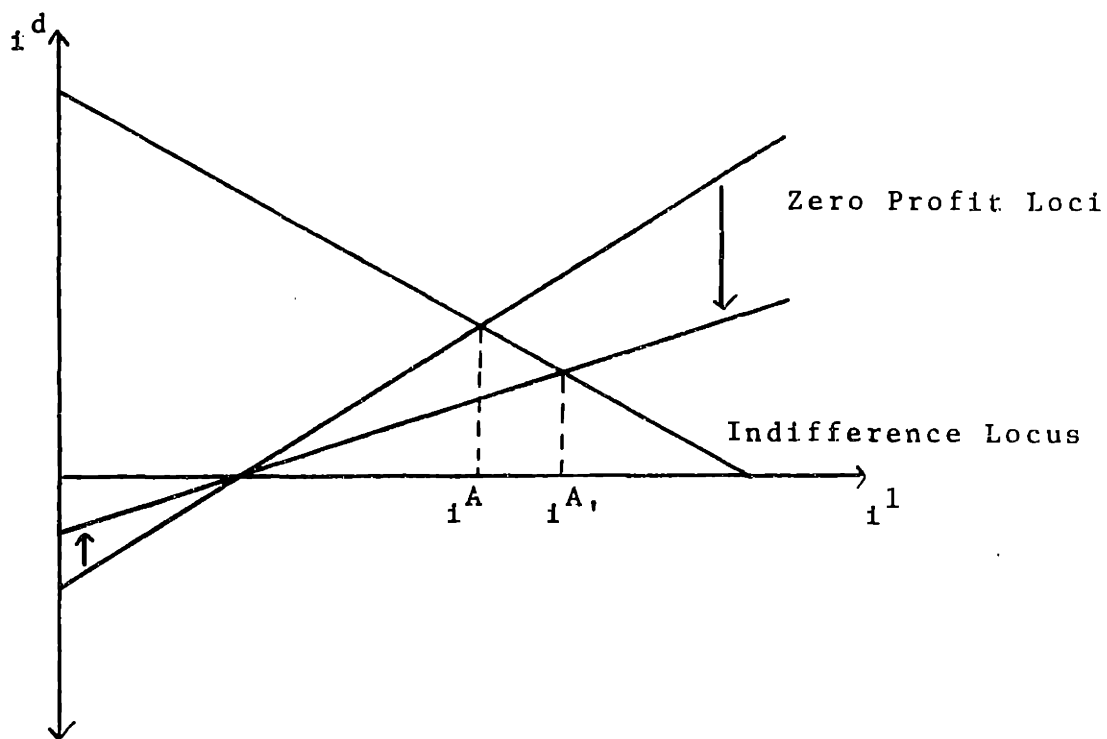
FIGURE 6

The Effect of an Increase in the
Required Reserve Ratio

It is a suprise, perhaps, that the loan interest rate shifts in response to government policy changes. For example, if the rationing were caused by the imposition of an arbitrary "conventional" loan interest rate, or a usury ceiling, then the loan rate would not respond to changes in ϕ or h. In the case considered here, since banks do not find it worthwhile to ra se r^l before the change in government policy, it is worth noting why they do choose to do so after the change. An increase in the required reserve ratio or the rate of growth of high-powered money increases the "tax" on deposits. As a result, r^d falls for any given r^l. Now r^l is set at the rate that keeps low-risk individuals just indifferent between depositing and borrowing and investing. Since the change in government policy makes deposits a less profitable alternative, banks can raise r^l until low-risk individuals are just indifferent again without fear of inducing adverse selection. As a result, so long as the rationing is endogenous, the increased tax on deposits falls on both sides of the market.

<u>Changing the level of government debt</u>. A change in the per-capita quantity of real bonds sold by the government, B, has no effect on either curve. As a result both nominal and real interest rates do not change. Given the above discussion, it is easy to see why. An increase in B does not change r^d for a given r^l. Thus deposits do not become a less profitable alternative for low-risk borrowers, and

banks cannot increase the loan interest rate without inducing adverse selection.

From equations (4) and (5) one can show that:

(13) $L = \frac{1-\phi}{2-\phi}(W-B)$

Thus a one dollar increase in government debt decreases lending by $(1-\phi)/(2-\phi)$ dollars. The effect on investment is a reduction of $2(1-\phi)/(2-\phi)$. The effect on investment is less than one for one because, although the increased government borrowing crowds out bank deposits, some of the deposited funds would have been held as reserves and not loaned out.

The crowding out is accomplished without affecting interest rates because, as noted above, the demand for financial assets (deposits and bonds) is determined by the rationing policy of banks, not interest rates. Thus an increase in B reduces the supply of deposits to banks and causes increased rationing, just at a rise in ϕ did. As in that case, the increased rationing causes increased demand for financial assets (bonds and deposits) as those denied credit invest their endowments. Unlike that case, however, there is no change in r^d because the banks' return on assets (loans and reserves) is the same as before the change in B. Only the levels of the banks' assets and liabilities have changed.[3]

This result differs from that of Romer. He finds that

increases in government debt cause crowding out via a higher interest rate. The only way an increase in B can fail to raise interest rates in his model is if the supply of deposits is completely elastic at the going interest rate. In this case, however, there is no crowding out of investment due to the increase in government debt.

The fact that the government can increase its debt without raising interest rates raises an interesting question: how high can the debt rise? So long as ϕ and h are at levels where the interest rate on deposits and bonds is below the rate of growth of the population (so that government revenues remain positive), the government can borrow the entire savings of the young cohort. As shown in Section IV, this result remains true even when some agents choose to consume in both periods, although in that case the savings of the young is not equal to their entire endowment.

This result also has an important implication for policy makers. In Chapter One I note that type one rationing implies that the loan interest rate should not be used as a guide to the effects of monetary policy because it can move in perverse directions in response to policy changes. In that case, however, the deposit rate continues to move in the expected manner. In the case of type two rationing, it is clear that no interest rate can serve as a guide to policy makers. Changes in the level of government debt can have large effects on the steady-state of

economy and yet have no effect at all on interest rates. As a result, policy makers must look directly at the quantity of lending or the quantity of investment in order to assess the effects of policy.

III. Government Policy

In this section I consider what a central planner should do in order to maximize social welfare. If the planner assumes that all individuals have identical linear utility functions, then this problem amounts to maximizing expected aggregate consumption. I also assume that the government pays out its revenues to the old generation. Thus, aggregate consumption is given by:

$$C = (1+r^d)D + (1+r^* - \bar{P}r^1)L + (1+n)g$$

The first term is the second period income of those who deposit their endowments; the second term is the expected second period income of those who borrow and invest; the final term is the level of government revenues per member of the old cohort. This expression can be rewritten as:

(14) $C = (2+r^* - (1-\bar{P})c)L + (1+n)H + (1+n)B$

Written this way, second period consuption is simply the return on investments plus the returns accruing to the old cohort due to the "sale" of currency and bonds to the younger cohort. Finally, using equations (4), (6) and (13),

one can write this as:

$$(15) \quad C = (2+r^*-(1-\bar{P})c)\frac{1-\phi}{2-\phi}(W-B) + (1+n)\frac{\phi}{2-\phi}(W-B) + (1+n)B$$

If there is no rationing, then aggregate consumption is:

$$C' = (2+r^*-(1-P_h)c)\frac{1-\phi}{2-\phi}(W-B) + (1+n)\frac{\phi}{2-\phi}(W-B) + (1+n)B$$

The change in per capita consumption due to rationing is:

$$C-C' = (\bar{P}-P_h)c\frac{1-\phi}{2-\phi}(W-B)$$

Thus if $c>0$, it is always better to have rationing than not to have it. The reason for this simple result is that at the aggregate level the only difference between the two regimes is that the bankruptcy costs are paid less often when there is rationing. In fact, the difference in consumption can be rewritten as:

$$C-C' = [(1-\bar{P})-(1-P_h)]cL$$

Which is simply the difference in the probability of default times the loss associated with default.[4] Put another way, the reason for the welfare loss is that in the unrationed equlibrium the quality of the investments is lower: the individuals who receive loans have projects that have a lower expected return once bankruptcy costs are taken into account.

Ordinarily, one might expect that the quantity of loans would differ accross the two equilibria. This is not the case because individuals do not want to consume when young, and cannot invest without obtaining a loan because projects

are of a fixed size. Thus no quantity adjustments occur from one equilibrium to the other. It is certainly the case that different individuals receive loans in the two cases, but those who do not obtain loans deposit their endowments in either case.

Another result of the assumption that individuals do not want to consume when young is that h, the rate of growth of high-powered money, has no effect on welfare. In most models with money, changes in h shift individuals' intertemporal trade-offs and so cause distortions. This effect does not appear in this model because individuals are at a corner solution, saving their entire income, and so changes in interest rates have no effect on their behavior. In Section IV it is shown that if one does not make this assumption, then a change in h, by changing real interest rates, causes intertemporal substitution of consumption.

Maximization. A central planner can set the rate of growth of high-powered money, h, the required reserve ratio, ϕ, and the per capita level of government debt, B, so as to maximize aggregate consumption. Because h has no effect on consumption, however, the planner has only two policy instruments, ϕ and B.

Consider the derivatives of aggregate consumption with

respect to ϕ and B:

$$\frac{\partial C}{\partial \phi} = \frac{2}{(2-\phi)^2}\left[(1+n) - \frac{2+r^*-(1-\bar{P})c}{2}\right] \quad (W-B)$$

and,

$$\frac{\partial C}{\partial B} = \frac{2(1-\phi)}{2-\phi}\left[(1+n) - \frac{2+r^*-(1-\bar{P})c}{2}\right]$$

It is clear that if

$$\frac{1}{2}(2+r^*-(1-\bar{P})c) > (1+n)$$

then both derivatives are negative for $\phi \in (0,1)$ and $B \in (0,W)$, and so the optimal policy is $\phi=0$, and $B=0$. On the other hand, if the above inequality is reversed, then a policy of $\phi=1$ or $B=W$ is called for.[5]

This result is similar to Diamond's dynamic efficiency result (Diamond, 1965). The inequality can be rewritten as:

$$\frac{1}{2}[(1+\bar{P}r^1-(1-\bar{P})c) + (1+r^*-\bar{P}r^1)] > (1+n)$$

But the first term on the left-hand side of this expression is just the banks' expected return on a loan of W, and the second term is the average borrowers' return on his investment of W. Thus the condition states that if the total expected social return on investment is higher than the rate of population growth, then steady-state per capita consumption is maximized by eliminating intergenerational trade--i.e. eliminating both currency and government bonds. On the other hand, if the population's growth rate is higher than the social return on investment, then government debt, fiat money, or a social security system will lead to higher

consumption (Diamond's inefficient case).

Notice that the rate of return to be equated to the rate of growth of the population does not correspond to the return on any asset in the economy. This result should not be confused with that of Blanchard (1985). He finds that if an economy does not have a riskless asset, then dynamic efficiency corresponds to equating the rate of population growth to the return on a riskless asset introduced at the margin. In the model considered here, however, there are riskless assets--deposits and government bonds, but equating the rate of return on these assets to the rate of growth of the population does not ensure dynamic efficency. Instead, the result in this chapter is due to the type two rationing in the model. Because of the rationing, investors do not equate the marginal return on investment to the marginal opportunity cost of funds as in Chapter One.

Interest rate ceilings. If the rate of return on investment is higher than the rate of growth of the population, then the central planner should set $\phi=B=0$. This policy, however, may have a serious consequence. In Section II I noted that rationing only occurs if bankruptcy costs, c, are larger than a minimum level, $\underline{c}$:

$$\underline{c} = \frac{r^* + \phi\frac{h-n}{1+h}}{(1-\phi)(P_1+P_h-1)+P_1}$$

But with $\phi=0$, this becomes:

$$c \geq \frac{r^*}{2P_1+P_h-1}$$

Thus for values of c less than $r^*/(2P_1+P_h-1)$ but greater than zero, credit rationing raises aggregate consumption, but banks do not choose to ration it. How can this be? In the unrationed equilibrium, all borrowers are just at the margin--indifferent between borrowing and investing and depositing their endowments. On the other hand, in the rationed equilibrium only the low-risk borrowers are on the margin. Thus, if high-risk borrowers are far riskier than low-risk ones, they are far better off borrowing and investing than depositing. As a result, however, they are very unprofitable to lend to in the rationed equilibrium. Thus, if the cost of bankruptcy is low enough, banks may prefer to lend only to the risky borrowers, but at a very high interest rate.[6]

In such situations, the central planner has to use some other policy implement in order to maximize consumption. For example, if the planner can impose a ceiling on the loan rate, then consumption can still be maximized. To do so, the planner must set the ceiling at a rate r^u, $r^a \leq r^u < r^c$, such that the banks' return on loans at r^u is less than that at r^a. This rule would cause the banks to choose the rationed equilibrium rather than the unrationed equilibrium at r^u in order to maximize their

expected return on loans.

In short, this model suggests that, in the presence of uncertainty, usury laws can increase welfare.[7] As in the case without uncertainty, a binding loan rate ceiling induces rationing in the market for loans. (Although in this model, of course, the loan rate after the usury law is imposed may be lower than the ceiling rate stipulated in the law.) In the certainty case, those who borrow at a given rate are the individuals who can earn the highest returns with their projects. As a result, rationing, because it implies that the interest rate cannot be used to select those with the best projects, causes a social loss unless some alternate sorting mechanism is available.

In this case, however, rationing does not imply a misallocation of resources and consequent decline in welfare. In this model some borrowers are willing to pay a higher interest rate, not because their projects are more productive, but because they are more risky. Thus, rationing causes an improved allocation of resources because it leads to a lower-risk pool of borrowers receiving loans. As a result, bankruptcy costs are paid less often and aggregate consumption is increased.[8] Moreover, the lower deposit rate caused by the usury law does not cause a reduction in the supply of deposits (and therefore loans) because, as noted earlier, individuals who do not receive loans have no real alternative to depositing their

endowments.   This is not to say that rationing le: ls to a completely efficient allocation of resources. Complete efficiency would require that only low-risk individuals receive loans, and this is clearly not the case.[9]

IV. Generalizations

The model in Section I includes four rather strong assumptions: first, that there are only two types of individuals; second, that all projects are of a fixed size; third, that saving does not respond to interest rates because individuals do not choose to consume when young; and fourth, that there is no transactions demand for currency. This section sketches, without proofs, the effects of relaxing these assumptions.[10]

Distribution of individuals. The results of the model are essentially unchanged if the riskiness of investors' projects is uniformly distributed from those of the lowest risk, P_1, to those of the highest risk, P_h. In this case, interest rates in the rationed equilibrium leave the lowest-risk individual just indifferent between borrowing and investing and depositing his endowment. In the unrationed equilibrium, interest rates are set to insure that only the number of investors who can be funded choose to borrow, and all others choose not to do so.[11]

In fact, similar results should apply in any case where the maximum rate of return to the lender occurs at a

loan rate below the market clearing rate.[12] The only new worry would be that government intervention might shift the market clearing rate to a level below the return-maximizing rate. In that case, the rationing would cease.

Project size. If projects are not of a fixed size, then banks may be able to reduce the riskiness of their loans by making them smaller--i.e. by decreasing the debt-equity ratio of the projects. If banks can do so, however, then the type of rationing presented here (in which some would-be borrowers receive credit and others do not) is not generally obtained. There are at least two reasons (other than a fixed project size) why banks may not make this adjustment. First, at small project sizes there may be increasing returns to scale. As a result, smaller loans may be more, not less, risky. (See Stiglitz and Weiss, 1981, p.402). Second, banks may have a fixed cost of making a loan. As Keeton shows, such costs can more than offset the effect on bank profits of the reduction in risk associated with a smaller loan size (Keeton, 1979, Chapter 3). Thus, in either of these two cases, results similar to those presented here should continue to hold.[13]

On the other hand, if banks can avoid the adverse consequences of higher loan interest rates by reducing loan size, then type one rationing results. As I have shown, the implications of that sort of rationing are different than those of type two rationing.

Interest sensitivity of savings. Many of the results in Sections II and III depend on the non-response of savings to the interest rate. This non-response is due in part to the fixed size of projects discussed above, but also to the fact that individuals consume only when old. The model results, of course, would be similar so long as the interest elasticity of savings were small. Given the generally small elasticities found in empirical studies, the assumption of a zero elasticity may not be far off the mark. Nonetheless, it is interesting to consider the effects of a non-zero elasticity.

For example, one could add a third type of individual to the model: these individuals receive an endowment in the first period of their lives and consume part of it, depositing the rest in a bank for use in the second period. They have no projects in which they can invest.

Adding these individuals makes the results found in Section II more complex. In particular, changes in the rate of growth of high-powered money, h, and the required reserve ratio, ϕ, which before served only to change interest rates, now change the levels of savings and investment as well. If I assume that savings responds positivly to interest rates, then increases in h or ϕ would cause a decline in economic activity. On the other hand changes in the level of government debt, because they have no effect on interest rates, have the same effect as found earlier. The

government is again able to borrow the entire savings of the young cohort. This quantity is less than the endowment of the young because the new type of agent chooses to consume some of his endowment.

With a non-zero interest elasticity of savings some of the welfare results in Section III no longer hold. The problem is that one cannot simply look at aggregate consumption as an index of social welfare. Instead one must now consider the utility of the new type of individual as well as the aggregate consumption of the high- and low-risk individuals. Nonetheless, if banks choose to ration credit, then it is still welfare improving. High- and low-risk individuals are better off for the same reasons given in Section III, and the pure savers are better off because the deposit rate is higher in the rationed equilibrium.[14] On the other hand, if the level of bankruptcy costs is below the threshold level, $\underline{c}$, then banks will not choose to ration credit. In this situation using a usury law to force rationing is not Pareto-improving because the new individuals, who consume in both periods, are made worse off by the resulting fall in the deposit rate.

Similarly, if banks choose to ration credit, optimal government policy may not be the same as in Section III. The problem is that the optimization rule developed there only takes account of the consumption of high- and low-risk individuals, but does not maximize the deposit rate. Thus,

following this rule may not be pareto-superior to a class of other government policies in which high- and low-risk individuals are worse off than under the policy from Section III, but those who consume in both periods are better off because they obtain a higher deposit rate.

Transactions demand for currency. The final important assumption in the model is that currency serves only as the required reserves of the banks. This assumption can be relaxed without greatly changing the results of the model.

First, however, consider the effect of imposing a Clower-type constraint that forces individuals to hold currency proportional to their expected second period income. Such a constraint would destroy the rationing result. High-risk borrowers have a higher expected second period income than low-risk ones because their projects are more profitable. As a result, high-risk borrowers would give themselves away by holding more currency than low-risk borrowers (and by demanding larger loans).

A simple alternative is to make the banks hold excess reserves of currency in proportion to next periods' expected aggregate consumption (assuming that the excess reserves do not raise the reserve ratio above one). The effect of such a change on the results in Sections II and III is small assuming that the increase in reserves is small. In Section II I noted that an increase in the rate of growth of high-powered money, h, raises the loan rate and lowers the

deposit rate, as well as causing a rise in the inflation rate. With the new constraint, the rise in inflation forces banks to hold more currency. This increase in reserves, in turn, reinforces the interest rate movements caused by the increase in h. I also found in section II that a change in the required reserve ratio, ϕ, has no effect on steady-state inflation. It does, however, affect aggregate consumption. If one assumes that the social return on investment is higher than the rate of growth of the population, then a rise in ϕ reduces aggregate consumption. This decline in consumption means that, with the new constraint, banks will hold fewer excess reserves, partially offsetting the rise in ϕ. The final result in Section II was that a rise in the level of government debt, B, causes no change in interest rates, but reduces aggregate consumption if the return on intergenerational trade is below that on investment. With the Clower-type constraint, a rise in B causes a fall in excess reserves which reduces the effect of the rise in B on consumption. An additional effect is to lower the loan rate and raise the deposit rate.

The type of constraint considered here does not affect the welfare results in Section III. Even with a transactions demand for currency, the rationed equilibrium is still preferred to the unrationed one, although the difference between the two may be reduced.[15] Similarly, optimal government policy cannot be reversed by the

constraint. If the return on investment is higher than the rate of growth of the population, then the government still wants to maximize the amount of investment taking place by setting ϕ and B to zero. With the constraint, of course, banks will still hold some reserves even if none are required.[16]

In conclusion, the results derived in Sections II and III are fairly robust to changes in the assumptions of the model. As the assumptions grow more complex, so do the results. Given, however, that the rationing continues to be of the type where some would-be borrowers receive credit while others do not, many of the results remain unchanged.

V. Summary

In this chapter I consider the effect credit rationing has on the steady states of a simple general equilibrium model. I also examine the effects of changes in the reserve requirement, the rate of growth of the money supply, and the level of government debt on these steady states. Finally, I investigate the effects of rationing on social welfare.

In order to study these issues, I develop a simple general equilibrium model charaterized by credit rationing. The model has a two-period overlapping generations structure. A simple financial structure is imposed, with banks serving as the only intermediaries between borrowers and lenders. For most of the paper, the only role for currency in the model is as reserves held by the banks. Credit rationing results from uncertainty on the part of banks as to the riskiness of particular borrowers.

The chapter concentrates on steady-state equilibria. In Section II it is shown that changes in the required reserve ratio and the rate of growth of high-powered money have effects in this model that are similar to those in a model such as Romer's, which does not have credit rationing. In

particular, a rise in either the reserve ratio or the rate
of money growth induces a rise in r^1, the real interest rate
on loans, and a fall in r^d, the real interest rate paid on
deposits. On the other hand, it is also shown that the
government can increase its indebtedness (up to the level of
the young cohorts' endowment) without raising interest
rates. This result holds in spite of the fact that a rise in
the real quantity of government debt still crowds out
investment.

In Section III it is shown that if bankruptcy costs are
strictly positive then social welfare (which in this model
is simply aggregate consumption) is higher in the rationed
equilibrium than in the unrationed equilibrium. The
difference is due to the increased riskiness of the loans
made in the unrationed regime and the resulting increase in
bankruptcy costs. It is also shown that to maximize social
welfare a central planner should choose the required reserve
ratio and the level of government debt in order to attain
dynamic efficiency. Thus, if the social return on
investment is higher than the rate of population growth,
then the the optimal government policy is to have no
intergenerational trade--i.e. no required reserves and no
government debt. In contrast, if the rate of population
growth is larger than the return on investment, then a
reserve ratio of one or government debt equal to the
endowment of the young cohort is called for.

It is also shown, however, that if the government sets both the required reserve ratio and the level of government debt equal to zero in order to maximize consumption, then rationing may not occur. Since aggregate consumption in the unrationed regime is strictly less than that in the rationed regime, the consequence is a serious one. A usury law is proposed as a solution to this policy dilemma. If the government can set a ceiling on the loan rate, then aggregate consumption can be maximized. The ceiling rate is set below the rate prevailing in the unrationed regime, but either equal to or above the rate in the rationed regime. It is set such that banks earn a higher expected return on loans at the rationed equilibrium than by charging the maximum allowable rate. Thus with the usury law, banks choose the rationed equilibrium, as desired.

These results prove to be fairly robust to changes in the assumptions of the model. More general distributions of individuals, variable project size, a non-zero interest elasticity of savings, and a transactions demand for currency are considered. These complexities lead to more complicated but generally similar results.

NOTES

1. The results of the model are unchanged if there are no bankruptcy costs, but high-risk projects have a lower expected return than low-risk ones.

2. Given that reserves serve as a tax on deposits, why would individuals choose to hold deposits rather than make loans directly? Fama (1985) suggests that banks may have an informational advantage that allows them to obtain a higher return on their loans than agents could obtain on their own. As a result, bank deposits can be competetive in spite of the "tax."

   For example, I could add to the model presented here a third type of agent. These individuals live only one period. They receive their endowment, eat it, and then die. Such people would gladly "borrow" and consume the proceeds of the loan, then die without repaying it. If one assumes that banks can tell which potential borrowers are of this type while individuals cannot, then agents may choose to hold "taxed" deposits rather than risk making a loan themselves. This outcome would occur if the loss due to the required reserve ratio were less than the expected loss due to loans to one period lived individuals. So long as no loans are made outside of the banking system, the one period lived agents have no effect on the rest of the economy. As a result, I leave them out of the model presented here.

   Also, see the discussion in Chapter One, Section V.

3. If one reads Keeton's expansionary monetary policy as a reduction in the amount of government debt held by the public, then his partial equilibrium results are correct here as well.

4. Notice that if the bankruptcy cost is a loss only to the bank and not to society (for example, if some borrowers are dishonest and do not report the outcome of the investment correctly if it is not successful), then the two levels of consumption would be the same.

5. The effect of either a policy of $\phi=1$ or B=W would be the same, as one can see from equation (14). If $\phi=1$, then this equation simplifies to:

$$0+(1+n)(W-B)+(1+n)B = (1+n)W$$

and if B=W, one obtains:

$$0+0+(1+n)B = (1+n)W$$

6. In the case where $\phi=1$ or B=W, the existence of rationing is irrelevant because no loans are made. Notice that if $\phi\neq0$, then for any c that is greater than zero but less than

$$[\bar{P}(r^{*}+1)]/[P_1(1-\bar{P})]$$
the government can set h in order to make $\underline{c}$ less than c. This is possible because $\underline{c}$ goes to zero as h is lowered to
$$[1+n]/[1+(r^{*}/\phi)] - 1$$
As h falls to this level, $\bar{c}$ also goes to zero, but it is always greater than $\underline{c}$.

7. Stiglitz and Weiss derive a similar result in a partial equilibrium model in which borrowers choose how risky their project will be. In their case, a usery law causes borrowers to choose safer projects (Stiglitz and Weiss, p.408.).

8. This result is due to the fact that the expected return on high-risk projects, including bankruptcy costs, is less than the expected return on low-risk projects, including bankruptcy costs. Thus rationing can still be optimal even if, not including bankruptcy costs, the expected return on high-risk projects is higher than that on low-risk projects. Of course, if the expected return on high-risk projects is enough higher than that on low-risk projects, then rationing is not optimal in spite of the reduction in bankruptcy costs. In that case, a usury law is not welfare-improving.

9. A bankruptcy tax, imposed on banks if a borrower goes bankrupt, of $r^{*}/(2P_1+P_h-1) - c$ would also induce rationing.

10. I also assume that no loans directly from one individual to another are available. I could relax this assumption in the same way as I did in Chapter One, Section V, the results, however, are not as attractive. Because all loans are of the same size, either the bond market or the banking system must prevail. The only possible equilibria with both types of lending are of the knife-edge variety.

11. This arrangement may, in fact, be an improvement. With just two types of people, not all high-risk people receive loans at the unrationed equilibrium. Thus there is "rationing" of a sort. Those who do not receive credit are indifferent and deposit their endowments instead, and so the "rationing" is not of great interest. With a uniform distribution of types this issue does not arise.

12. The case in which the maximum is above the market clearing rate, but there is a local maximum below that rate, is more complex still. The partial equilibrium result claimed by Stiglitz and Weiss is not generally correct. See Yanelle (1985) for a discussion.

13. As Keeton shows, however, the government may choose to subsidize loans in order to improve the credit market's

allocative efficiency. Such policies are not considered in this paper. (See Keeton, pp 238-40.) Stiglitz and Weiss give other reasons why banks may not be able to protect themselves by limiting loan size in models more complicated than those presented here (Stiglitz and Weiss, 1981, pp.402-408).

14. I assume here that deposits of the new individuals respond positively to a rise in the deposit rate. If this were not the case, then the quantity of loans in the rationed equilibrium would be less than in the unrationed equilibrium. As a result high- and low-risk borrowers might be worse off.

15. The reason for the lack of changes can easily be seen. Assume investment yields a higher social return than intergenerational transfers. Also assume that without the Clower constraint rationing is preferable, but with it the unrationed equilibrium is preferred to the rationed one. The latter assumption implies that with the constraint the reserve ratio is higher in the unrationed equilibrium as well. The return on reserves, however, is lower than the return on investment, and investment in the unrationed equilibrium has a lower social return than that in the rationed equilibrium. Thus aggregate cosumption must be higher in the rationed equilibrium--contradicting our assumption.

What if reserves give a higher return than investment? Then consumption is maximized in both cases by holding all reserves and not lending or investing.

16. For a detailed study of the transactions demand for money in general equilibrium, see Romer, 1984.

REFERENCES

Blanchard, Olivier J. Problem Set for Course 14.462. Mimeo, MIT, 1985.

Diamond, Peter A. "National Debt in a Neoclassical Growth Model." American Economic Review 55 (December 1965): 1126-1150.

Fama, Eugene F. "Banking in the Theory of Finance." Journal of Monetary Economics 6 (January 1980): 39-57.

Keeton, William. Equilibrium Credit Rationing. MIT Doctoral Thesis, 1976. Also, New York: Garland Press, 1979. Page references are to the MIT thesis.

Romer, David. "A Simple General Equilibrium Version of the Baumol-Tobin Model." Mimeo., MIT, 1984.

_____. "Financial Intermediation, Reserve Reqirements, and Inside Money: A General Equilibrium Analysis." Journal of Monetary Economics 16 (September 1985): 175-194.

Stiglitz, Joseph E. "Incentives and Risk Sharing in Sharecropping." Review of Economic Studies 41 (April 1974): 219-255.

_____, and Weiss, Andrew. "Credit Rationing in Markets With Imperfect Information." American Economic Review 71 (June 1981): 393-410.

Yanelle, Marie-Odile. "Bertrand Competition Among Intermediaries." University of Bonn Working Paper No. A-37. 1985.

CHAPTER THREE

THE RELATIONSHIP BETWEEN LOAN INTEREST

RATES AND FIRM LEVERAGE

I. Introduction

Many economists have contributed to the substantial literature on credit rationing. With the exception of work by Dwight Jaffee and his coauthors, however, there has been little study of the empirical validity of rationing models (Jaffee and Modigliani, 1969; Jaffee, 1971; Jaffee and Rosen, 1979). Thus, in spite of the claims made for the importance of credit rationing, for example, in the propagation of monetary shocks and as a reason for the failure of tests of the life-cycle hypothesis, the empirical evidence remains thin. Moreover, the evidence on rationing that is available is based either on aggregate or industry level data. While these results are suggestive, a test based on micro level data would be more convincing.

Before one can consider testing for credit rationing, one must first decide what type of rationing to test for. As noted above, models of rationing can be divided into two types. Type one rationing occurs in models in which individual borrowers must pay a higher interest rate the larger their loan is. Thus borrowers are not rationed, but

feel constrained in the credit market. Type two rationing occurs in models in which some would-be borrowers are unable to obtain credit, but other observationally equivalent agents are able to obtain it.

Empirical testing of type two rationing is very difficult for two reasons. First, the econometrician requires data on all would-be borrowers, including those who do not receive credit. Unfortunately, data appears to be available only on loans that are actually made. Second, the econometrician must be certain that there is no unobserved variable determinining which agents receive credit and which do not. For example, if lenders can observe the quality of a firm's management, then they may not lend to those with bad managers because it is not profitable to do so. If the econometrician cannot observe management quality, however, then he may wrongly identify a firm as being rationed when, in fact, it has been denied credit for a reason.

A test of type one rationing, on the other hand, presents no theoretical difficulties. This sort of rationing implies that the interest rate on a loan is an increasing function of the loan size. In effect, the firm faces an upward sloping supply curve. Thus, testing for type one rationing amounts to simply estimating the supply curve for credit to a particular firm, and testing whether it is upward-sloping.

There are two results from the finance literature that

suggest that type one rationing may be an important phenomenon. The first is contained in the vast literature, initiated by Altman, on the prediction of bankruptcy (Altman, 1968; Zmijewsky, 1984; Mensah, 1984). These writers have generally found that firms that are highly leveraged are more likely to fail than those that are not highly leveraged. For example, in a recent paper Zmijewsky uses a weighted probit model to estimate probabilities of default. He finds that increases in the ratio of debt to assets have substantial and statistically significant effects on the probability of bankruptcy. Mensah finds similar results with a logit model. Thus, if lenders adjust the loan rate in response to changes in the probability of default, then firms that borrow more--and thereby increase their leverage--should be charged a higher interest rate.

The second result from the finance literature that is of interest here is from the work done on modeling bond ratings. In a well-known 1979 article, Kaplan and Urwitz find that the ratio of debt to assets has a significant negative effect on the ratings of both newly-issued and seasoned bonds (Kaplan and Urwitz, 1979). This result holds for other types of borrowers as well. For example, Raman finds that the ratios of debt to population and debt to tax revenues both have significant effects on the ratings of general obligation municipal bonds (Raman, 1982). Because the rate of return on a bond rises as its rating declines,

these results are strong evidence in favor of type one rationing.

In this paper I estimate directly the supply curve of short-term credit for individual firms. I choose short-term debt because the interest rate on short-term debt that is reported by firms is the same as the return earned by the lender. With long term lending, such as bond issues, the rate reported by the firm need not be closely related to the return earned by the present bond holder because of fluctuation in the price of the bond. This issue arrises because the data that I use is collected from the borrowers, and so the actual rate of return to the lender is not known.[1]

In Section II I outline the theoretical model that I use. Section III briefly discusses the data. In Section IV I present and discuss the results of the estimation. In Section V I summarize the results, and make some concluding remarks.

II. Theory

I assume that lenders are risk-neutral. Thus, when a loan is made the following condition must hold:

(1) $E(R^S) = E(\rho)$

where R^S can be interpreted either as the safe rate that is available to the lender or as the opportunity cost of funds, and ρ is the (stochastic) real rate of return on a loan. The expectations operator is required on the left-hand side of the equation because the inflation rate is not known.

So long as the borrower does not go bankrupt, the return on a loan is simply $R^1 = (1+r^1)$, where r^1 is the real interest rate charged on the loan. If failure occurs, then the rate of return is $R^f(X)$, where X is a vector of variables that affect the return to the lender. Finally, the probability of default is given by the function P(X). Thus equation (1) can be rewritten as:

(2) $E(R^S) = E\left\{[1-P(X)]R^1 + P(X)R^f(X)\right\}$

where the expectations operator on the right-hand side is required both because the inflation rate is not known, and

because some variables in X may not be known at the start of the period.[2]

With some reorganization and the introduction of an expectational error term, this equation can be rewritten as:

(2') $R^1 - R^s - P(X)[R^1 - R^f(X)] = \epsilon$

where ϵ is uncorrolated with all information known at the beginning of the period. Given functional forms for $P(X)$ and $R^f(X)$, this model can be estimated using NLIV. The obvious candidates for instruments are the lagged values of the variables in X.

Functional Forms. The functional forms for $P(X)$ and $R^f(X)$ are not clear. Following Mensah, one could hypothesize that bankruptcy follows a logit model, in which case $P(X)$ would be given by:

(3) $P(X) = \dfrac{1}{1 + e^{X\beta}}$

As will be noted below, this specification does not perform well, and so I also consider the linear probability model:

(3') $P(X) = X\beta$

This model, of course, has the unattractive implication that the probability of default need not be greater than zero, nor less than one (See Maddala, 1983). As will be seen in Section IV, this problem arises in our case.

Finally, I assume that the return to the lender if the firm fails is a linear function:[3]

(4) $\qquad R^f = X\gamma$

The Estimated Equation. Putting the pieces together, and adding firm and time subscripts, Equation (2') becomes:

(5) $\qquad (R^1_{it} - R^s_{1t}) - X_{it}\beta(R^1_{it} - X_{it}\gamma) = \epsilon_{it}$

where ϵ_{it} is orthogonal to all information dated t-1 or earlier. If the data set used included data on firms that went bankrupt, then one could estimate Equations (3), (4), and (5) simultaneously, imposing cross-equation restrictions. Unfortunately, the data set that I use has information only on firms that have not gone bankrupt. As a result, I only estimate Equation (5).

Variables in X. In the finance literature noted above, several variables are found to have significant effects on either the probability of default of a firm, or on its bond rating. In particular, the leverage of the firm, its return on assets, its size (i.e. the level of assets), and its level of interest coverage (i.e. the ratio of the sum of interest payments, taxes, and profits to interest payments). In addition, one would expect the level of economic activity in the economy to influence both the probability of default, and the value of the firm if default occurs. The inflation rate may also enter in, especially if Modigliani and Cohn

are correct in their asssertion that investors do not
account for it correctly (Modigliani and Cohn, 1979). Other
variables, such as the ratio of short-term assets to short-
term liabilities, the rate of inventory or receivables
turnover, etc. seem like reasonable candidates for inclusion
in X. Several of these variables were tried, but they did
not add significantly to the results presented below.

III. Data

The data on firms that I use comes from the Compustat annual tape. This tape supplies a great deal of financial information on more than 2000 firms for up to twenty years. Unfortunately, data on the average short term interest rate paid by the firms has only been collected since fiscal year 1977 (i.e. fiscal years ending between June 1977 and May 1978). The final complete year on the tape is fiscal year 1982, while about half of the firms have data for fiscal 1983. Because I use two lags of the variables as instruments, the first year for which I can do estimation is 1979. An additional problem is that some firms began supplying the interest rate data after 1977 or supplied it for only part of the period, reducing the available sample. Furthermore, I use data only on manufacturing firms, as the coefficients that I want to estimate may not be the same in other industries such as banking, insurance, agriculture, trade, etc. A summary profile of the firms in the sample is presented in Table 1, while the variables used in the regressions are presented in Table 2. Some descriptive statistics are shown in Table 3.

TABLE 1

Total Number of Firms On Tape..............2344

Number of Manufacturing Firms..............1157

Number of Manufacturing Firms With
      Interest Rate Data...................647

Total Number of Observations
      (Firms*Years)......................2130

Number by Fiscal Year:
      1983.................................179
      1982.................................473
      1981.................................492
      1980.................................495
      1979.................................491

TABLE 2

R......Average interest rate on short-term borrowing

RCD....The real interest rate on certificates of deposit
      (average of one month rates)

RAS....Real Assets in 100,000,000's of 1982 dollars

ROA....Retern on assets (as a proportion, i.e. .03, not 3%)

INT....The ratio of profits plus taxes plus interest
      payments to interest payments--i.e. interest
      coverage

LEV....The ratio of assets to equity of the firm.

Y......The detrended level of GNP in billions of
      1982 dollars

PI.....One plus the inflation rate as measured
      by the GNP deflator.

TABLE 3

Variable	Mean	Standard Deviation
R	1.072	.046
RCD	1.047	.024
RAS	20.9	62.4
ROA	.043	.048
INT	7.27	11.51
LEV	2.8	1.3
Y	6.99	83.5
PI	1.076	.022
Short-term debt (100's of millions of dollars)	.8	2.5
Nominal loan interest rate	1.153	.045

The construction of the firm-specific variables is straightforward. The level of assets is the figure for the end of the fiscal year divided by the price level as measured by the GNP deflator.[4] The return on assets is the firms' after-tax profits divided by their level of assets. Interest coverage is simply profits plus taxes plus interest payments divided by interest payments. This variable is sometimes referred to as "times interest earned." Leverage is measured as the ratio of assets to equity, although use of a variable such as the debt equity ratio does not greatly change the results. The short-term interest rate is, according to Compustat, the "approximate weighted average interest rate for aggregate short-term borrowings for the reporting year" (Standard and Poors, p.8.88). Compustat

also reports that these short-term borrowing are "usually in the form of credit with banks" (Standard and Poors, p.8.88).

The aggregate data (the level of GNP, the safe interest rate, and the inflation rate) come from the Citibank database. Inflation is measured by the GNP deflator. Because the firms' fiscal years begin in various months, the quarterly data must be weighted to give a fiscal year figure for each firm. The producer price index, which is available monthly, was also tried; the results were similar. I detrend the quarterly GNP series over the period 1974:1 to 1984:4, and used the same weighting to generate the GNP relative to trend variable for each fiscal year. As noted in Table 2, the lender's cost of funds variable is the twelve month average of the monthly CD rate. I also tried the average prime lending rate over the fiscal year, and the rate available on one-year treasury notes at the start of the fiscal year, without obtaining significantly different results.

IV. Estimation

Before considering the estimation of Equation (5), I first note what results should be expected. First, since the business failure rate for the United States has never exceeded 1% since the Great Depression, the probability of default should be quite small for virtually all firms (Zmijewski, 1984, p.59). Second, I expect the probability of default should decline with the size of the firm because of the additional diversification within the firm that size allows. The effect of firm size on the return to lenders given default, R^f, is not clear. Third, the rate of return on assets should reduce the probability of failure and increase R^f by raising the value of firm assets if failure occurs. Fourth, a high level of interest coverage should reduce the probability of default, while its effect on R^f is not clear. Fifth, increased leverage should increase the probability of default and reduce R^f. Sixth, high levels of GNP should reduce the probability of default, and increase the value of the firms assets, rasing R^f. Seventh, high levels of inflation may reduce the probability of default by reducing the burden on the firms of long term debt. On the

other hand, inflation increases the value of payments required early in the term of a loan, and may thereby increase the probability of default by causing cash-flow problems. If Modigliani and Cohn are correct, inflation should also reduce the value of the firms' assets—and thereby R^f—because of market misperceptions.

The correlations of the data are broadly consistent with these expectations. They are presented in Table 4.[5]

TABLE 4

Correlation with R^1

Variable	Correlation
RCD	.57
RAS	.01
ROA	-.12
INT	-.02
LEV	.10
Y	-.48
PI	-.43

The correlations of R^1 with firm-specific variables are suprisingly small. In particular, RAS and INT have negligable correlations, and that of RAS is of the wrong sign. The correlations of LEV and ROA are of the expected sign, and are larger. The correlations with aggregate variables (RCD, Y, and PI) are much larger, suggesting that firm-specific influences are either small, or are poorly measured by the firm-specific variables.

Estimation. As noted above, attempts to estimate
Equation (2') with the logistic form of the probability of
default proved unsuccessful. The estimation routine broke
down because of excessivly large values of the exponential
function. It seems that the data required both a very small
probability of default and a relativly large variance of the
probability of default. This problem becomes clear when
Equation (5) is estimated. Results of estimation of Equation
(5) are presented in Table 5. It is obvious that these
results are quite unsatisfactory. The standard error of the
regression is .046, roughly two thirds of the average loan
rate. The average probability of default, according to
these results, is huge--more than a quarter. Moreover, the
range of the probability of default is from -1.7 to 8.8!
Moreover, the results imply that most of the variation of
the loan rate is explained by variation in the probability
of default, because the return to the lender in the event of
default varies much less, from .54 to 1.4. Finally, only
one variable enters significantly, interest coverage, and it
is of the wrong sign.

A simple way to limit the variability in P(X) is to
constrain it to be a constant, $\bar{P}$. Estimation of the
resulting equation is presented in Table 6. These results
are also unsatisfactory, as the probability of default (.72)
is far too high. These results can be rationalized. As the
probability of default rises, the parameters are chosen to

TABLE 5

Estimation of Equation (5)

Parameter	Variable	Value	t-statistic
β_0	constant	2.07	.51
β_1	RAS	.274E-2	1.43
β_2	ROA	-2.87	-1.26
β_3	INT	.533E-1	6.16
β_4	LEV	.264E-1	.16
β_5	Y	-.374E-3	-0.23
β_6	PI	-1.90	-0.49
γ_0	constant	.918	1.17
γ_1	RAS	.163E-3	1.51
γ_2	ROA	.424	0.61
γ_3	INT	.840E-3	1.28
γ_4	LEV	-.163E-2	-0.03
γ_5	Y	-.274E-3	-0.99
γ_6	PI	.449E-1	0.06

Observations: 2130

Standard Error: .046

Average P(X):   .265

Average R^f: 1.01

Instruments: First and second lags of all variables.

TABLE 6

Estimation of Equation (5)

--Constant Probability of Default

Parameter	Variable	Value	t-statistic
β_0	constant	.719	58.19
γ_0	constant	1.05	21.72
γ_1	RAS	..473E-5	0.57
γ_2	ROA	.120E-1	0.49
γ_3	INT	-.160E-3	-1.411
γ_4	LEV	-.186E-2	-3.03
γ_5	Y	-.179E-3	-14.69
γ_6	PI	-.870E-2	-0.19

Observations: 2130

Standard Error: .017[*]

Average R^f: 1.04

Instruments: First and second lags of all variables.

[*]The careful reader will note that this is lower than in the unconstrained case. This can be so because this is instrumental variables estimation.

fit R^s rather than R^l. (Notice that if $\bar{P}$ is one, then R^l drops out of the equation altogether.) In the period under consideration, the CD rate has been strongly (negatively) correlated with the level of real GNP. As a result, the residuals in the equation are minimized by choosing a high level of $\bar{P}$, and using the GNP variable to predict the CD rate. One piece of evidence in favor of this view is the substantial coefficient on GNP (with a t-statistic of 14).

Conversly, one could constrain R^f to be a constant, $\bar{R}^f$. The results for such a model are presented in Table 7. Again the model in unsatisfactory. The probability of default ranges from -1.5 to 4.4, and averages .42. In addition, the coefficient on interest coverage continues to be of the wrong sign and significant, as does the coefficient on GNP.

To avoid the problem encountered when P(X) is assumed to be a constant, one can renormalize the equation by dividing by $(1-\bar{P})$. This division is possible because $\bar{P}$ is non-stochastic. The equation can then be written as:

$$(6) \qquad R^l_{it} = \frac{1}{1-\bar{P}}\left\{R^s_{it} - \bar{P}[X_{it}\gamma]\right\} + \frac{\epsilon_{it}}{1-\bar{P}}$$

Equation (6) is linear in variables, and can be estimated by ordinary instrumental variables methods. Estimation of this equation is presented in Table 8.

Again the results do not conform well to the model. The implied probability of default is negative, and it is

TABLE 7

Estimation of Equation (5)

--Constant R^f

Parameter	Variable	Value	t-statistic
β_0	constant	2.37	1.31
β_1	RAS	.243E-3	1.32
β_2	ROA	-2.44	-4.45
β_3	INT	.265E-1	10.00
β_4	LEV	.450E-1	5.15
β_5	Y	.146E-2	3.10
β_6	PI	-1.91	-1.14
γ_0	constant	1.02	676.24

Observations: 2130

Standard Error: .025

Average P(X):   .418

Instruments: First and second lags of all variables.

TABLE 8

Estimation of Equation (6)

Variable	Value	t-statistic
C	.245	2.52
RCD	.896	17.91
RAS	$-.311E-5$	-0.23
ROA	$-.729E-2$	-0.18
INT	.463E-3	2.55
LEV	.530E-2	5.45
Y	$-.540E-4$	-2.45
PI	$-.119$	-1.59

Observations: 2130

Standard Error: .038

R^2:   .33

Instruments: First and second lags of all variables

significantly so. On the other hand, the signs and magnitudes of the other parameters are about what one would expect. While the coefficients on the level of assets and the return on assets are insignificant, the point estimates are of the expected sign, and of reasonable magnitude. The parameter on leverage is positive, as expected, and strongly significant. An increase of .1 in the ratio of assets to equity causes a rise of five basis points in the loan interest rate. Similarly, the level of GNP has a negative and significant effect on the interest rate. As noted above, the sign of the overall effect of inflation on the loan rate is not clear a priori because while inflation reduces the burden of the firms long-term debt, it may also reduce the value of the firm's assets (as hypothesized by Modigliani and Cohn) and cause cash-flow problems for the firm. These results suggest that the first effect dominates, although the coefficient on the inflation rate is not significant.

The one result which is not consistent with the theory is the coefficient on interest coverage. It does not seem sensible that a high level of interest coverage should raise the loan rate. One possible explanation is that interest coverage is correlated with an omitted variable. For example, if lenders have information on the quality of management, then they may demand higher levels of interest coverage from firms with poor management, while also

charging a higher loan rate. If this is the case, of course, then the other coefficients are also biased. After all, if the banks require higher levels of interest coverage from bad managers, they should also require lower levels of leverage.[6]

Another possible explanation for the perverse sign is that some industries have high levels of interest coverage, yet are also intrinsically risky. This possibility can be tested by introducing industry specific dummy variables, or by running the regression industry by industry. In Table 9 six industry groups are displayed. In Table 10 dummy variables for industries one through five are included in the equation, while in Table 11 results are presented for each industry.

TABLE 9

Group	SIC Codes	Industries
1	20-23	Food, Tobacco, Textiles, Apparrel
2	24-27,31,32,39	Lumber, Wood, Furniture, Paper, Publishing, Printing, Leather, Stone, Clay, Glass, Concrete, Other
3	28-30	Chemicals, Petroleum and Coal, Rubber, Plastics
4	33-34	Primary and Fabricated Metal Products except Machinery and Transportation Equipment
5	35,37	Machinery except Electrical Machinery, Transportation Equipment
6	36,38	Electrical Machinery, Instruments

Three of the dummy variables are significant in the regression presented in Table 10. The first group faces

TABLE 10

Estimation of Equation (6)

  --With Industry Dummy Variables

Variable	Value	t-statistic
C	.242	2.50
$DUMMY_1$	-.415E-2	-1.59
$DUMMY_2$	-.669E-2	2.43
$DUMMY_3$	.518E-2	1.99
$DUMMY_4$	.178E-2	0.56
$DUMMY_5$	-.250E-2	-0.96
RCD	.896	18.10
RAS	-.135E-4	-0.99
ROA	-.243E-1	-0.61
INT	.454E-3	2.51
LEV	.482	4.88
Y	-.531E-4	-2.43
PI	-.113	-1.53

Observations: 2130

Standard Error: .038

R^2:   .34

Instruments: First and second lags of all variables.

TABLE 11

Estimation of Equation (6)

  --By Industry

Group:	1	2	3	4	5	6

Variable

	1	2	3	4	5	6
C	0.77	.34E-1	0.20	-0.27	0.62	.55
	(0.62)	(0.22)	(0.62)	(-0.72)	(2.73)	(2.85)
RCD	1.07	0.98	0.92	1.04	0.80	0.68
	(15.9)	(13.47)	(5.62)	(5.40)	(7.04)	(6.86)
RAS	.81E-4	-.30E-4	-.76E-4	.12E-3	.46E-4	.30E-4
	(1.24)	(-0.38)	(-2.77)	(0.67)	(1.76)	(0.95)
ROA	-.84E-1	-0.11	0.21	0.27	-0.13	-0.11
	(-1.39)	(-1.76)	(1.91)	(1.85)	(-1.47)	(-1.63)
INT	.74E-4	.18E-3	.13E-2	.13E-3	.14E-3	.52E-3
	(0.42)	(0.90)	(2.74)	(0.28)	(0.46)	(1.13)
LEV	.29E-2	.46E-2	.13E-1	.59E-2	.12E-2	.36E-2
	(2.23)	(2.40)	(4.37)	(2.26)	(0.38)	(1.86)

Y	-.53E-5	-.25E-4	-.60E-4	-.14E-3	-.50E-5	-.98E-4
	(-0.18)	(-0.81)	(-0.84)	(-1.73)	(-0.94)	(-2.24)
PI	-.12	-.39E-2	-0.13	0.21	-0.36	-0.18
	(-1.26)	(-0.04)	(-0.51)	(0.78)	(-2.02)	(-1.23)
Obs.	358	300	382	209	368	513
S.E.	.020	.021	.054	.045	.037	.036
R^2	.633	.597	.219	.264	.352	.336
D-W	0.95	1.15	1.17	1.12	1.38	1.11

Instruments: First and second lags of all of the variables.
t-statistics in parentheses.

interest rates roughly half a percent less than group six (the arbitrarily chosen base group). The second group pays two thirds of a pecent less, while the third group pays half of a percent more than the base group. Apparently, the industries in groups four and five do not have a significantly different intercept than those in group six.

The individual industry results presented in Table 11 look much like the aggregate results. The coefficient on the CD rate ranges from .68 to 1.07 and is always significant. The coefficient on the leverage term is significant and positive in five of the six cases and ranges between .0029 and .013 in those cases. In two cases GNP is negative and significant. On the other hand, the results for interest coverage, assets, and return on assets remain poor. The level of assets has a positive coefficient that is significant in one case, while the return on assets has such a coefficient in two cases.

On the other hand, the perverse sign on the interest coverage variable is significant in only one group, the third. This lends some support to the hypothesis that the positive sign was due to interest coverage proxying for an industry effect. The sign of the coefficient, however, remains positive in all six groups.

Perhaps suprisingly, given the seeming similarity of the results, a test for equality of slope coefficients across industries rejects equality at conventional significance

levels. This result is probably due to the large size of
the data sample. In addition, the test is not corrected for
possible heteroskedasticity.

An additional problem for the test, and for these linear
regressions, is the exceedingly low values of the Durbin-
Watson statistics reported in Table 11. These low values
are not suprising because there are almost surely firm-
specific effects that are not taken account of in the
regressions. Unfortunatly, the very short panel on each
firm (averaging just three and a half observations) does not
allow estimation of firm-specific dummy variables. The
usual solution to this problem is to estimate the equation
in differences from the firm's mean values of the variables.
To do so in this case is wrong, however, because it mixes
information from different years in the data, and so the
orthogonality conditions required for estimation are
destroyed.

A second possibility is to estimate the model in first
differences of the data:

$$(7) \qquad \Delta R^1_{it} = \Delta X_{it} \beta + \epsilon_{it}$$

This differenced model has the unattractive implication that
there is no long-run relationship between the levels of the
variables.[7] Nonetheless, the results of the estimation are
presented in Table 12. None of the firm-specific variables
enter significantly. The reason for this poor result is

probably that the variation in the right-hand-side variables
is primarily between firms rather than within firms.

Alternatively, I can choose a single observation from
each firm for estimation. A natural way to do so is to
estimate the model for a single fiscal year. The results
for fiscal year 1979 are presented in Table 13.[8] Because of
the small number of observations, the results for individual
industries are very poor. As a result, I have allowed
industry-specific intercepts, but constrain the slope
coefficients to be constant accross industries. The industry
specific results are presented in the Appendix.

The estimates in Table 13 are much like those presented
earlier. The leverage term is again significant and
positive, while both the level of assets and the return on
assets are significant and negative. The counter-intuitive
positive coefficient on interest coverage persists here.
The coefficients on the CD rate, GNP, and inflation are all
measured very imprecisely, probably because of
multicollinearity over such a short period. This is a
particularly serious problem because roughly three-quarters
of the observations are for fiscal years starting in January
or July.

Of course, the aggregate variables can be dropped
entirely if the equation is estimated on data from a single
month. Because more than half of the observations for the
fiscal year come in the month of January, it is the natural

TABLE 12

Estimation of Equation (6)

   --In Differences

Variable	Value	t-statistic
C	.418E-2	2.55
ΔRCD	.829	21.97
ΔRAS	-.173E-3	-0.18
ΔROA	-.997E-1	-1.04
ΔINT	.545	0.12
ΔLEV	-.187E-2	-0.25
ΔY	-.143E-4	-0.69
ΔPI	-.139	-2.37

Observations: 1483

Standard Error: .029

R^2: .314

D-W: 2.35

Instruments: Second and third lags of the levels of all variables.

TABLE 13

Estimation of Equation (6)

--Fiscal Year 1979 and January 1979

Variable	Fiscal 1979	January 1979
C	4.85	1.01
	(1.34)	(82.1)
$DUMMY_1$	$-.285E-2$	$-.135E-2$
	(-0.75)	(-0.18)
$DUMMY_2$	$-.676E-2$	$-.638E-2$
	(-1.73)	(-1.00)
$DUMMY_3$	$.505E-2$	$-.466E-2$
	(1.32)	(-0.08)
$DUMMY_4$	$-.467E-2$	$-.782E-2$
	(-1.03)	(-1.03)
$DUMMY_5$	$-.116E-2$	$-.412E-2$
	(-0.31)	(-0.64)
RCD	0.114	--
	(0.14)	

RAS	-.410E-4	-.485E-4
	(-2.28)	(-2.01)
ROA	-0.182	-0.302
	(-2.76)	(-2.83)
INT	.106E-2	0.261
	(3.52)	(4.62)
LEV	.649E-2	.110E-1
	(3.32)	(3.36)
Y	-.605E-3	--
	(-0.65)	
PI	-3.56	--
	(-1.15)	
Obs.	491	284
S.E	.026	.033
R^2	-.036	-.643

Instruments: First and second lags of all variables.

t-statistics in parentheses.

choice.   The results for estimation on the January data are also presented in Table 13.   They do not differ greatly from those for the whole year.

V.   Concluding Remarks

The arbitrage model of the loan rate presented in Section II is clearly rejected by the data. There are several possible explanations for the rejection. First, there may be imperfect competition among lenders. This possibility is supported by anecdotal evidence, but seems unlikely to have a substantial effect on the firms in this data set. These firms are fairly large and well-known. Thus, if they were very profitable to lend to, either other lenders would discover it and attempt to take their business, or the borrowers would turn to the commercial paper market to obtain credit at fair rates.

A second possible reason for failure is that the loan rate is poorly measured. This problem could have an effect in two ways. First, if R^1 is measured with a great deal of error, the quality of the estimates would be hurt. Given that IV methods are used, and that there is a large data set, however, this problem should not be too extreme. Second, R^1 may not be the "right" price. In particular, if banks provide a bundle of financial services that are priced as a unit, then the apparent profitability of a loan is

immaterial. What matters is the profitability of the whole package. If this is the case, then the estimated model is clearly misspecified.

A final reason for the poor results could be that the lender is risk averse. In this case the equation estimated need not hold. If lenders have mean-variance utility and firm failure probabilities are non-stochastic and independent, then one can derive an equation similar to the arbitrage equation used here. The result, however, is very complex, and the results are poor.[9]

In spite of the failure of the model presented in Section II, a simple linear specification provided some interesting results. First, increases in leverage are associated with increases in the loan interest rate. In particular, an increase of a standard deviation in short-term borrowing causes an increase of .2% in the loan rate for the average firm. Thus, supply curves for credit are significantly, though gently, upward-sloping. This result provides direct evidence for type one rationing, though it casts doubt on the practical importance of this sort of rationing for the majority of firms.

The second result is that the data seem to be characterized by substantial firm-specific and industry-specific effects. As a result, researchers who attempt to estimate probabilities of default, or forecast bond ratings, should test for these effects.

(BLANK PAGE)

APPENDIX

Estimation of Equation (6)

  --By Industry for Fiscal Year 1979

Group:	1	2	3	4	5	6

Variable

	1	2	3	4	5	6
C	8.84	-7.39	0.60	10.04	3.55	4.64
	(1.83)	(-0.95)	(0.10)	(0.81)	(0.40)	(0.47)
RCD	-0.18	0.36	0.42	8.64	-2.02	0.30
	(-0.18)	(0.21)	(0.08)	(1.91)	(-0.49)	(0.14)
RAS	.54E-4	.78E-4	-.83E-4	.69E-4	.45E-4	-.37E-4
	(0.51)	(0.61)	(-2.48)	(0.49)	(1.51)	(-0.45)
ROA	-.99E-1	-.30E-1	-0.18	.73E-1	-0.22	-0.34
	(-0.81)	(-0.29)	(-0.90)	(0.72)	(-1.56)	(-2.54)
INT	.19E-3	-.14E-4	.15E-2	-.12E-3	.58E-3	.26E-2
	(0.37)	(0.06)	(1.73)	(-0.63)	(1.47)	(2.06)
LEV	.29E-2	.38E-2	.71E-2	.48E-2	.20E-2	.84E-2
	(2.23)	(0.99)	(1.16)	(3.24)	(0.41)	(1.17)

Y	-.89E-3	-.11E-2	.68E-5	.12E-1	-.28E-2	-.63E-3
	(-0.89)	(-0.54)	(0.00)	(1.73)	(-0.48)	(-0.27)
PI	-6.92	7.50	*	-17.57	-0.14	-3.55
	(-1.79)	(1.20)		(-1.22)	(-0.01)	(-0.43)
Obs.	83	72	84	52	82	118
S.E.	.016	.016	.035	.016	.021	.037
R^2	.263	.062	-.035	.248	-.047	-.418

Instruments: First and second lags of all of the variables.
t-statistics in parentheses.

* Coefficient not estimated due to singularity of the data.

NOTES

1. There is a substantial literature on the pricing of longer term corporate bonds. These papers have generally focused on the characteristics of the payment stream, and on the particular provisions on the bond (subordination, call provision, etc.). See Foster (1986, pp. 511-12) for a discussion.

2. Of course, one could also write out P and R^f as reduced forms, i.e. in terms of variables known at the start of the period. The likely list of variables in the functions would be extremely large, however.

3. In fact, one would like to constrain this function to be less than R^s because if R^f is greater than R^s then the loan is riskless. One way to impose this constraint is by assuming:

$$(4') \qquad R^f = \frac{1}{1+e^{X\gamma}} R^s$$

The problem with this method is that by setting P(X) equal to one and R^f equal to R^s, one can ensure that the the residuals are exactly zero. I tried imposing:

$$(4'') \qquad R^f = \frac{1}{1+e^{X\gamma}} R^1$$

which is a somewhat weaker condition than (4'). Attempts to estimate the resulting equation failed, however, because of large negative exponents. The data apparently require the exponential term to be nearly zero, but quite variable.

4. One could also consider using the average level of the variables over the fiscal year. Doing so does not greatly change the results.

5. Because of endogenaity, the variables used are fitted values from a linear regression on lagged and doubly lagged values of all of the variables.

6. There is some evidence for this view. For example, Castle finds in a sample of 37 term loan agreements that the number of covenants in the contract is higher for firms with low bond ratings (Castle, 1980). Thus, while firms with a bond rating of Aaa had few, if any, covenants, firms rated B had a minimum of seven. These covenants included required levels of working capital, minimum ratios of current assets to current liabilities, and maximum allowable amounts of

debt. Given the small size of Castle's sample, however, it is hard to know how general this pattern is.

7. An alternative interpretation of this equation may be preferable. If I difference Equation (6), I obtain:

$$(7') \qquad \Delta R^1_{it} = \frac{1}{1-\bar{P}}\left\{\Delta R^s_{it} - \bar{P}(\Delta X_{it}\gamma)\right\} + \frac{\epsilon_{it}-\epsilon_{it-1}}{1-\bar{P}}$$

This error term is still orthogonal to information dated time t-2 and earlier, and so the IV estimates presented in Table 12 are consistent as before (because the instruments used are lagged two periods). In this case, however, the standard errors are not correct.

8. The results for other fiscal years are similar, although those for 1980 and 1983 are somewhat worse. Data for 1980 were presumably affected by the Carter Administration credit controls, while the 1983 sample is small (179 observations).

9. If banks obtain utility from each loan in proportion to its expected return, μ, and the variance of its expected return, σ^2:

$$U = \mu - \phi\sigma^2$$

then one can show that the arbitrage equation becomes:

$$(8) \qquad R^1_{it}-R^s_{it} - \bar{P}(R^1_{it}-R^f_{it}(X)) - \phi P\sigma^2_f - \phi(\bar{P}-\bar{P}^2)(R^1_{it}-R^f_{it}(X))$$
$$+ \phi\sigma^2_\pi(I^{s2}_{it}-(1-\bar{P})I^{12}_{it}) = \epsilon_{it}$$

where I^s and I^1 are the nominal safe and loan interest rates, which are assumed known at the start of the period. σ^2_f is the one period ahead variance of R^1. Similarly, σ^2_π is the variance of $1/(1+\pi)$. $\bar{P}$ is the (non-stochastic) probability of default.

Information dated time t-1 and earlier is still orthogonal to ϵ_{it}, and so estimation is still possible. Attempts to estimate equation (8) were not sucessful because all of the variables other than the nominal interest rates were not significant. Apparently, the data is fit best by using the nominal rates to fit the corresponding real rates. This result is not suprising given the low correlation between the loan interest rate and firm-specific variables.

REFERENCES

Altman, E. I. "Financial Ratios, Discriminant Analysis and Prediction of Corporate Bankruptcy." Journal of Finance 22 (September 1968): 589-610.

Castle, G. R. "Term Lending--A Guide to Negotiating Term Loan Covenants and Other Financial Restrictions." The Journal of Commercial Bank Lending (November, 1980): 26-39.

Foster, George. Financial Statement Analysis. Englewood Cliffs, NJ: Prentice Hall, 1986.

Jaffee, Dwight M. Credit Rationing and the Commercial Loan Market; an econometric study of the structure of the commercial loan market. New York: Wiley, 1971.

_____, and Modigliani, Franco. "A Theory and Test of Credit Rationing." American Economic Review 59 (December, 1969): 850-872.

_____, and Rosen, Kenneth T. "Mortgage Credit Availability and Residentiial Construction." Brookings Papers on Economic Activity (July 1979): 333-376.

Kaplan, Robert S., and Urwitz, Gabriel. "Statistical Models of Bond Ratings: A Methodological Inquiry." Journal of Business 52 (1979): 231-261.

Maddala, G. S. Limited Dependent and Qualitative Variables in Econometrics. New York: Cambridge University Press, 1983.

Mensah, Yaw M. "An Examination of the Stationarity of Multivariate Bankruptcy Models: A Methodological Study." Journal of Accounting Research 22 (Spring 1984): 380-395.

Modigliani, Franco, and Cohn, Richard A. "Inflation, Rational Valuation, and the Market." Financial Analysts Journal 35 (March-April 1979): 24-44.

Raman, K. K. "Financial Reporting and Municipal Bond Ratings." Journal of Accounting, Auditing and Finance (Winter 1982): 144-153.

Zmijewski, Mark E. "Methodological Issues Related to the Estimation of Financial Distress Prediction Models." Journal of Accounting REsearch 22 (Supplement 1984): 59-82.

BIOGRAPHICAL NOTE

William Berkeley English, the son of James and Isabelle English, was born on December 7, 1960 in Hartford, Connecticut. He was raised in West Hartford. He attended the Loomis Chaffee School in Windsor, Conn. from 1974 to 1978. He attended Yale University from 1978 to 1982, receiving a B.A. in Economics and Mathematics. He continued to M.I.T. in 1982 and studied there for four years, receiving his Ph.D. in 1986. He will be an Assistant Professor of Economics at the University of Pennsylvania beginning in the fall of 1986.