ESTIMATING PEAK ACCELERATIONS

FOR PACKAGE CUSHIONING DESIGN

by

David Martin

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ABSTRACT

Conventional packaging design uses curves of peak acceleration experienced by a mass nested in a compliant material, dropped from different heights, as the basis for choosing the amount and type of cushioning to protect items during shipment. These curves are usually obtained by performing a series of drop tests. A method is presented for estimating the peak acceleration modeling the impact as a one-dimensional wave propagation problem in the cushioning material. The method is implemented in a computer program written in the CBASIC language. Preliminary results indicate that the method is promising, with further work required to determine how to represent variation in the elastic properties of the material; additional work to define the range of applicability of the method is also needed. Sample results and military packaging requirements are summarized.

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Introduction

Almost all types of manufactured goods must be packaged to protect them from the loads and hazards during shipping and storage from the manufacturer to the end user. Environments to which an item is exposed generally include temperature, moisture, sunlight, shock and vibration. Packaging to provide protection takes the form of insulation, vapor barriers, opaque containers, and cushioning. The shock and vibration environments are considered here.

The shock and vibration loads which might need to be considered in a practical packaging problem can vary widely. However it has become customary to design the cushioning to provide protection of the item when dropped onto a rigid surface (ie. a concrete floor) from a specified height. The design is then checked for adequacy with respect to other considerations such as other types of drops; corner and edge drops, random and sine vibration, and compression of the cushioning material.

The conventional design method is described in References 5 and 7. To use the method, knowledge of the height of the drop and of the fragility of the item to be protected, are required. An item's fragility must be given in terms of the maximum linear acceleration which the item may be exposed to without sustaining damage. Fragility is a quantity which is usually poorly defined. In the case of inexpensive, simple items, tests may be performed to establish the fragility. As the value of the item increases, and as the number of samples available for destructive test decreases, rules of thumb based on experience with similar equipment must be used. In some cases the level of protection may be specified by the customer.

Typical military specifications for preservation and packaging provide definitions of the range of drop heights over which an item must be protected. The maximum drop height from which the item must be protected depends on its weight and on the dimensions of the item as packaged. The range of weights and dimensions which are considered appropriate for one-man and two-man carries, and for machine lifts are shown in Table I. It can be seen that the smaller and lighter a package is, the greater the height from which it may be dropped, and the greater the acceleration from which it must be protected.

Table I: Drop Heights for Packaged Items

Package Gross Weight (lb)	Type of Handling	Design Drop Height (in)
0 to 20	manual	30
20 to 40	manual	26
40 to 60	manual	24
60 to 80	manual	18
80 to 100	manual	15
100 to 150	mechanical	12
150 to 250	mechanical	10
250 and up	mechanical	8

Once the drop height and fragility are known or specified, the material and thickness of the cushioning may be selected using the peak acceleration vs. static stress curves, obtained for each material and drop height, such as those in Ref. 7. A family of curves with material thickness as a parameter are plotted on one graph. A material and thickness is chosen which will provide a peak acceleration less than the fragility. An allowance may be added to the thickness to assure that adequate protection is provided after settling or creep of the cushioning material has occurred. Peak Acceleration data is obtained by performing drop tests on samples of the cushioning material, using a standard test procedure such as Ref. 8.

In this thesis a model which describes the behaviour of cushioning material during a flat drop is developed by assuming that the material behaves as a one-dimensional bar which conducts a longitudinal plane stress wave along its length. A plane stress wave is generated at the instant at which the package contacts the rigid floor. If the cushioning material is assumed to decelerate instantaneously at contact (ie. that it has zero mass), while the object is moving with the velocity attained as a result of the drop, the cushioning material can be represented as a one dimensional bar, as shown in Figure 1. The left end of the bar (struck end) is impacted by the object to be protected (mass). The right end of the bar (fixed end) is bounded by the face of the container. The model developed allows curves of peak acceleration versus static loading of the cushioning material to be estimated from calculations based on static material properties rather than by a series of time-consuming dynamic tests.

Impact of a Bar By a Finite Mass: Standard Solution

A differential element of a bar, with cross sectional area A, is shown in Figure 2. Since the bar is one-dimensional, only the σ_x stress needs to be considered. The stress on the left face of the element is σ, while the stress on the face a distance dx to the right is $\sigma+(\partial\sigma/\partial x)dx$. The equation of motion of the element is

$$\frac{\partial \sigma}{\partial x}\, dx\, A = \frac{\partial^2 u}{\partial t^2}\, \rho\, dx\, A. \qquad (1)$$

Rearrangement and division of both sides by dx A, results in

$$\frac{\partial \sigma}{\partial x} = \rho\, \frac{\partial^2 u}{\partial t^2}\, . \qquad (2)$$

As the bar is in simple uniaxial strain, the stress at a given point along the length of the bar is determined by the axial strain, $\partial u/\partial x$ at that location. The axial strain is a function of x and t only. Therefore the stress in the bar is given by

$$\sigma = E\, \frac{\partial u}{\partial x}\, . \qquad (3)$$

Combining and rearranging Eqs. 2 and 3 results in the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2\, \frac{\partial^2 u}{\partial x^2}\, . \qquad (4)$$

where

$$c = \left(\frac{E}{\rho} \right)^{1/2} \qquad (5)$$

is the speed of sound in the bar material. The general solution to Eq. 5 is (Ref. 2)

$$u = F(x-ct) + G(x+ct) \; . \qquad (6)$$

The function F is a wave travelling to the right; the function G is a similar wave travelling to the left. Any type of longitudinal stress wave within the bar may be represented as the superposition of appropriate functions F and G.

At the free end of a bar the stress must be zero, but the displacement may be finite. The effect of a free end on the reflection of a stress wave may be found by considering the incident pulse to be

$$u_1 = F(x-ct) \; , \qquad (7)$$

and the reflected pulse to be

$$u_2 = G(x+ct) \; , \qquad (8)$$

then the stresses produced by the two waves will each be

$$E \frac{\partial u_1}{\partial x} \quad \text{and} \quad E \frac{\partial u_2}{\partial x} \; , \qquad (9)$$

and the resultant stress is

$$E \left(\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial x} \right) = E \, [F'(x-ct) + G'(x+ct)] . \qquad (10)$$

If x=0 at the free end of the bar, then the condition that the bar is stress free is

$$0 = F'(x-ct) + G'(x+ct), \qquad (11)$$

where the prime indicates differentiation with respect to the argument of the functions F and G. From this it is evident that a stress wave is reflected from a free end with the same shape, but with a change in sign.

When a stress wave is reflected from a fixed end, the velocity displacement must be zero. Measuring x from the fixed end of the bar,

$$0 = -cF'(0-ct) + cG'(0+ct) \qquad (12)$$

It is apparent that the velocities are equal and opposite, and that the stresses have the same sign and magnitude. Therefore the resultant stress upon reflection from a fixed end is twice that of the incident stress.

A rigid object striking the free end of a bar with the other end fixed is shown in Figure 1. Taking m as the mass per unit area of bar cross section, v_0 as the velocity of the object at impact, and t=0 as the time of impact, the impact generates a compressive stress at the struck end of

$$\sigma_0 = v_0 (E\rho)^{1/2} . \qquad (13)$$

The velocity of the mass at impact is found from conservation of energy applied to the gravitational potential, and the kinetic energy of the object,

$$\tfrac{1}{2} m v_0^2 = m g_0 h. \qquad (14)$$

Solving Eq. 14 for v_0 as a function of the drop height, h gives

$$v_0 = (2 g_0 h)^{1/2}. \qquad (15)$$

The equation of motion of the rigid object may be found by summing the forces acting on the object.

$$m \frac{dv}{dt} + \sigma = 0. \qquad (16)$$

Substituting the wave front stress for the velocity of the struck end into Eq. 16 gives

$$\frac{m}{(E\rho)^{1/2}} \frac{d\sigma}{dt} + \sigma = 0. \qquad (17)$$

The solution to this first order differential equation for the contact stress is

$$\sigma = \sigma_0 e^{-(t\sqrt{E\rho/M})}. \qquad (18)$$

Equation 18 is valid as long as $t < 2L/c$, that is until the front of stress wave returns to the struck end, and is reflected. When the compression wave front returns to the struck end of the bar, it is reflected as if the end of the bar were fixed, because of the relatively high inertia of the object at the struck end. At the instant of reflection, the compressive stress at the wave front increases suddenly by $2\sigma_0$. This sudden increase in the stress at the wave front occurs every time the wave front returns to the struck end of the bar, at time intervals of $2L/c$. It is this increase in the compressive stress between the bar and the object which causes the increase in the acceleration of the object, since the force acting on the object increases with each reflection from the mass.

The general expression for the stress at the struck end during any interval $nT<t<(n+1)T$ is

$$\sigma = s_n(t) + s_{n-1}(t-T). \quad (19)$$

where T, the time between reflections at the struck end is

$$T = \frac{2L}{c}. \quad (20)$$

By using Eqs. 18 and 19, the stress at the struck end for the interval after the first reflection may be expressed as (Ref. 3)

$$s_1 = s_0 + \sigma_0 e^{-2\alpha[(t/T)-1]} \left[1+4\alpha\left(1-\frac{t}{T}\right)\right]. \quad (21)$$

and for the interval after the second reflection as

$$s_2 = s_1 + \sigma_0 e_0^{-2\alpha[(t/T)-1]} \left[1+8\alpha\left(2-\frac{t}{T}\right)+8\alpha^2\left(2-\frac{t}{T}\right)^2\right]. \quad (22)$$

Ref. 1 gives as a compact expression for this recursive relation,

$$s_n = \sigma_0 \sum_{m=0}^{m=n} e z_m \sum_{h=0}^{h=m} \frac{m!(2z_m)^h}{(h!)^2(m-h)!}, \quad (23)$$

where z_m is a function defined by

$$z_m = 2\alpha\left(m-\frac{t}{T}\right). \quad (24)$$

In the next section an approximate analysis of a plane longitudinal stress wave in a bar will be developed for estimating the peak acceleration of a packaged item.

Impact of a Bar By a Finite Mass: Cushioning Solution

For the cushioning problem, the analysis proceeds parallel to that derived above, with several exceptions. First, large strains will be allowed. I will assume that over the interval between reflections, the elastic modulus and the density of the bar are constant, and equal to their value at the instant of reflection. At each reflection the elastic modulus and the density of the bar material will be determined as a function of the strain at that time. Thus the sonic velocity, and the duration of the interval between reflections changes as the bar is compressed. and its properties change. I will also assume that the bar material will remain elastic, although it may show highly non-linear behaviour.

Referring to Figure 2, equilibrium for a differential element of the bar requires that

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \qquad (2)$$

and the stress-strain relation for the bar material is

$$\sigma = E \frac{\partial u}{\partial x}. \qquad (3)$$

The one-dimensional wave equation is found by combining Eqs. 2 and 3, as before

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad (4)$$

where the sonic velocity within the bar is defined as

$$c = \left(\frac{E}{\rho} \right)^{1/2} . \qquad (5)$$

is the speed of sound in the bar material. The general solution to Eq. 4 is again,

$$u = F(x-ct) + G(x+ct) . \qquad (6)$$

The function F is a wave travelling to the right; the function G is a similar wave travelling to the left. By differentiating the functions F and G with respect to the argument of each function, and using Eq. (3), the stress in the bar may be expressed in terms of the wave function derivatives,

$$\sigma = EF'(x-ct) + EG'(x+ct). \qquad (25)$$

Similarly Eq. 6 may be differentiated with respect to time to find the velocity of points along the length of the bar,

$$\frac{\partial u}{\partial t} = -cF'(x-ct) + cG'(x+ct). \qquad (26)$$

The boundary conditions along the bar remain the same as previously. At x=L, $\partial u/\partial t = 0$, which implies that at all times

$$F'(L-ct) = G'(L+ct). \qquad (27)$$

This is guaranteed if the stress in the bar are viewed as being the superposition of two waves as shown in Figure 3, which shows the actual bar between $0 \leq x \leq L$, and the imaginary bar between $L \leq x \leq 2L$. Since both the bar, and the wave functions are symmetric about x=L, that point will remain fixed, and represents the restrained end of the actual bar. The functions F' and G' differ in phase by an amount equal to twice the length of the actual bar,

$$G'(x+ct) = F'(2L-x-ct). \qquad (28)$$

Evaluation of Eq. 28 at x=L verifies that

$$G'(L+ct) = F'(2L-L-ct). \qquad (29)$$

Equation 28 allows rewriting Eqs. 25 and 26 as

$$\sigma = EF'(x-ct) + EF'(2L-x-ct), \qquad (30)$$

$$\frac{\partial u}{\partial t} = -cF'(x-ct) + cF'(2L-x-ct). \qquad (31)$$

Since the stress and the velocities are the quantities of primary interest, an additional notation change will be made to Eqs. 30 and 31 to delete the primes by using (Ref. 4)

$$F'(x-ct) = f(x-ct), \qquad (32)$$

which allows writing Eqs. 30 and 31 as

$$\sigma = Ef(x-ct) + Ef(2L-x-ct), \qquad (33)$$

$$\frac{\partial u}{\partial t} = -cf(x-ct) + cf(2L-x-ct). \qquad (34)$$

Equations 33 and 34 are valid for times $0 \leqslant t \leqslant 2L/c$, and for over the length $0 \leqslant x \leqslant L$. Using Eqs. 13 and 15,the initial conditions for stress and velocity at x=0 for t=0 are

$$s(t) = \sigma(0,t) = s_0 = v_0(E\rho)^{1/2}, \qquad (35)$$

$$v(t) = \frac{\partial u}{\partial x}(0,t) = v_0 = (2g_0 h)^{1/2}. \qquad (36)$$

Since for ct<2L,

$$f(2L-0-ct) = f(2L-ct) = 0 , \qquad (37)$$

only the first terms on the right side of Eqs. 33 and 34 are non-zero, so the initial conditions become

$$s_0(t) = Ef_0(x-ct), \qquad (38)$$

and

$$v_0(t) = -cf_0(x-ct). \qquad (39)$$

Next consider the time interval after the stress wave front has reflected off of the mass for the first time, or $2L/c \leq t \leq 4L/c$. The stress will be given by

$$\sigma = Ef_0(x-ct) + Ef_0(2L-x-ct) + Ef(x-ct) , \qquad (40)$$

where the last term of Eq. 40 represents the new wave travelling to the right after reflection off of the mass. This allows the definition of an expression for the total rightward travelling wave as

$$f_1(x-ct) = Ef_0(x-ct) + Ef\ (x-ct). \qquad (41)$$

Substitution of Eq. 41 into Eq. 40, and into a similar expression for the velocity of the struck end, will give the stress as

$$s_1(t) = Ef_0(2L-ct) + Ef_1(-ct) , \qquad (42)$$

and the velocity as,

$$v_1(t) = cf_0(2L-ct) - cf_1(-ct). \qquad (43)$$

The travel time, $T = 2L/c$, is the time which it takes for the stress wave front to travel from the struck end, down to the fixed end, and back to the fixed end. A pattern is seen to be emerging from Eqs. 38, 39, 42 and 43 for expressions for the stress and velocity at the struck end. For the interval after the j^{th} reflection from the struck end,

$$t_j \leq t \leq t_j + \frac{2L_j}{c_j} , \qquad (44)$$

the time since the initial impact being given by

$$t_j = t_{j-1} + T_{j-1} . \qquad (45)$$

The expressions for the stress and velocity are

$$s_j(t) = Ef_j(t) + Ef_{j-1}(t-T) \qquad (46)$$

and

$$v_j(t) = -cf_j(t) + cf_{j-1}(t-T). \qquad (47)$$

The next step is to modify Eqs. 45 through 47 to allow the length of the bar to change, and finally to allow the density and elastic modulus to change also. Properties are allowed to vary as a function of the average strain in the bar. They will be updated on each reflection from the struck end of the bar, and will be treated as if constant until the next such reflection.

In terms of the displacement of the struck end of the bar, the average strain in the bar at the time of the j^{th} reflection is

$$\epsilon_j = \frac{-u_j}{L_0} , \qquad (48)$$

where u_J is the displacement of the struck end, and the length is

$$L_J = L_O(1+\epsilon_J). \qquad (49)$$

The travel time changes due in part to the shortening of the bar, but also due to change in the effective density of the bar material, and to the change in the elastic modulus since the strains are large. All of these changes need to be considered in developing a realistic model of cushioning material behaviour. Since the mass of the bar is conserved, the density can be shown to be

$$\rho_J = \frac{\rho_O}{(1-\epsilon_J)} . \qquad (50)$$

The elastic modulus, E_J, can be found as function of the strain by a table look-up procedure; or by fitting a curve to experimental data for a particular material, and evaluating that expression for the strain ϵ_J.

A change in the sonic velocity results form the changes occurring in the density and elastic modulus. The sonic velocity is given by

$$c_J = \left(\frac{E_J}{\rho_J} \right)^{1/2}. \qquad (51)$$

The form of the stress function still remains to be chosen. Since Eq. 18 indicates that the stress at the struck end decays exponentially, expanding the exponential in a power series and retaining only the first two terms suggests (Ref. 9)

$$f_j(t) = a_j - b_j(t-t_j) \ , \qquad (52)$$

where f is a function of time, and a_j and b_j are constants to be determined. Referring to the mass as shown in Figure 1, the equation of motion for the mass is

$$M \frac{dv}{dt} = A\ s \ . \qquad (53)$$

Integrating Eq. 55 over any time interval between successive reflections from opposite ends of the bar, and substituting Eq. 45 gives

$$\int_{t_j}^{t_{j+1}} \frac{dv}{dt}\, dt = \frac{AE}{M} \int_{t_j}^{t_{j+1}} [f_j(t)+f_{j-1}(t-T_{j-1})]\ dt \qquad (54)$$

Following the form of Eq. 52, the stress function for the previous interval at the same time, t, is

$$f_{j-1}(t) = a_{j-1} - b_{j-1}(t-t_{j-1}) \ . \qquad (55)$$

At one time interval earlier, Eq. 55 can be written, letting $t = t-T^{j-1}$,

$$f_{j-1}(t-T_{j-1}) = a_{j-1} - b_{j-1}(t-(t_{j-1}+T_{j-1})\ ). \qquad (56)$$

Since $t_j = t_{j-1}+T_{j-1}$, Eq. 55 at the time one interval earlier can also be expressed as

$$f_{j-1}(t-T_{j-1}) = a_{j-1} - b_{j-1}(t-t_j) \ . \qquad (57)$$

Carrying out the integration on the left side of Eq. 54, and substituting Eq. 57 in the left side results in

$$v_j(t_j)-v_{j-1}(t_{j-1}) = \frac{A_j E_j}{M} \int_{t_j}^{t_{j+1}} [(a_j+a_{j-1})-(b_j+b_{j-1})(t-t_j)]\, dt. \tag{58}$$

The left side of Eq. 58 can be written in terms of the velocities using Eqs. 47, 52, and 57, and after simplifying, as

$$v_j(t_j)-v_{j-1}(t_{j-1}) = c_j(b_j-b_{j-1})T_j \ . \tag{59}$$

The factor before the integral on the right side of Eq. 58 can be written:

$$\frac{A_j E_j}{M} = \frac{2mc_j}{M\ T_j} = 2\alpha \frac{c_j}{T_j} \tag{60}$$

α is the ratio of the mass of the bar to the mass of the object striking the bar, and it is a constant which is independent of the elastic modulus, and of the way in which any of the properties of the bar change with strain. When α is much smaller than one, the stress will change almost linearly along the length of the bar, which justifies using Eq. 52 to replace the exponential in Eq. 18. Substituting Eq. 60 for the factor in front of the integral symbol in Eq. 58, and integrating provides the first of the two equations necessary to solve for a_j and b_j. In terms of their previous values

$$T(b_j-b_{j-1}) = 2\alpha\ [(a_j+a_{j-1})-(b_j+b_{j-1})\frac{T_j}{2}] \tag{61}$$

The second equation is provided by considering the $2\sigma_0$ jump in the stress at the struck end that occurs each time the wave front is reflected from the object, for the second and later reflections,

$$s_j(t_j) = s_{j-1}(t_{j-1}) + 2\sigma_0 \qquad (62)$$

Substituting Eqs. 46, 55 and 57 into Eq. 62 gives

$$E_j a_j + E_j a_{j-1} = E_{j-1}[(a_{j-1}+a_{j-2})-(b_{j-1}+b_{j-2})T_{j-1}] + 2\sigma \ . \qquad (63)$$

Solving Eqs. 61 and 63 simultaneously for a_j and b_j results in

$$a_j = \frac{a_{j-1}}{E_j}(E_{j-1}-E_j) - \frac{E_{j-1}}{E_j}(b_{j-1}+b_{j-2})T_{j-1} + \frac{E_{j-1}a_{j-2}}{E_j} + \frac{2\sigma_0}{E_j} \qquad (64)$$

and

$$b_j = \frac{[T_j(1-\alpha)b_{j-1}+2\alpha(a_j+a_{j-1})]}{T(1+\alpha)} \ . \qquad (65)$$

For time less than 0, all of the a and b coefficients are equal to zero. This allows a_0 and b_0 to be determined in terms of the other parameters at t = 0.

$$a_0 = \frac{-v_0}{c_0} \qquad (66)$$

$$b_0 = \frac{2\alpha a_0}{(1+\alpha)T_0} \qquad (67)$$

When the reflection is at the struck end the stress at the same end is, in terms of a_j and b_j,

$$s_j = E_j(a_j+a_{j-1}-(b_j+b_{j-1})T_{j-1}) + 2\sigma_0 . \qquad (68)$$

The displacement of the struck end of the bar can also be written in terms of the linearized stress function coefficients, since

$$\frac{du_j(t)}{dt} = v_j(t) \quad . \qquad (69)$$

Eq. 58 can be solved for v_j and substituted in the right side of Eq. 69.

$$v_j(t_j) = v_j(t_{j-1}) + c_j(b_j-b_{j-1})T_j \quad . \qquad (70)$$

substituting Eq. 70 in the right side of Eq. 69, integrating, and solving for $u^j(t^j)$ gives

$$u_j(t_j) = u_j(t_{t-1})-c_{j-1}T_{j-1} \left[(a_{j-1}-a_{j-2})-(b_{j-1}-b_{j-2})\frac{T_{j-1}}{2}\right] \quad . \qquad (71)$$

The acceleration of the mass is found from

$$A_j = \frac{s_j}{W} \quad . \qquad (72)$$

where W is the static stress.

Discussion of results

To numerically test the model of cushioning material described above, a computer program was written which calculates the time history of a cushioned item to obtain the peak acceleration. The program KUSHION is listed and described in Appendices B and C.

The program was run with materials of constant elastic modulus. The curves shown on Figs. 4 and 5 represent values for E of 1.0 and 2.5 psi, respectively with a drop height of 12 in. Only the portion of the curves to the left of the minimum were obtained. A minimum point should be reached at some static stress, is shown in Fig. 6 for a typical cushioning material from Ref. 7. The Design Curves obtained using the modulus of a real material, one that varied with strain, caused erratic changes in the stress at the struck end, and in the velocity of the object. Values of the elastic modulus used represent approximate average values of polyurethane ether and polyurethane ester (Ref. 7).

There are three conditions for terminating a time history in the program:

1. Stress at the struck end is greater than or equal to zero.
2. Magnitude of the object's velocity is greater than or equal to it's initial velocity.
3. The strain is outside the range over which the modulus is defined.

Using a constant modulus all of the time histories terminated due to a strain greater than zero. The first two conditions indicate that the object has separated from the the bar. The stress and the velocity would be zero if the instant of separation coincided exactly with the instant of reflection of the stress wave front. Since this will not generally occur, values indicating a tensile stress or a gain in kinetic energy of the mass are possible. The third condition could occur if a positive strain or a strain larger than the maximum for which the elastic modulus is defined were calculated. A positive strain would indicate separation from the bar. A strain larger than the maximum for which the elastic modulus is defined indicates that the cushioning material has bottomed out, with the stiffness not being defined beyond that point.

Conclusion

This method of estimating peak accelerations for packaging design appears to have potential as a supplement to drop testing. Further work is necessary, especially to determine the the appropriate form of representation of the elastic modulus, and extension of the method to correctly handle the change in the elastic modulus is necessary. Performing measurements of the static and dynamic properties of an actual cushioning material, with the objective of verifying the method is recommended.

Further analysis of the method as presented is also needed. Areas in which additional effort should be made include:

1. Considering the Poisson effect on the stress at the struck end.
3. Consider updating the velocity, stress, etc. at the instant of reflection from both ends of the bar.

Development of a computer program based on this method to choose the type and thickness of cushioning material, when the drop height, static stress, and fragility are defined should be straightforward after the points raised above are resolved. Adding the capability to handle buckling and creep of the cushioning material would help in optimization of the size and cost of packaging fragile items. Extension of the wave propagation model of cushioning material behaviour to an item subject to sinusoidal vibration also appears to be worthwhile, where the item would be cushioned by the same thicknesses and type of cushioning on opposite faces.

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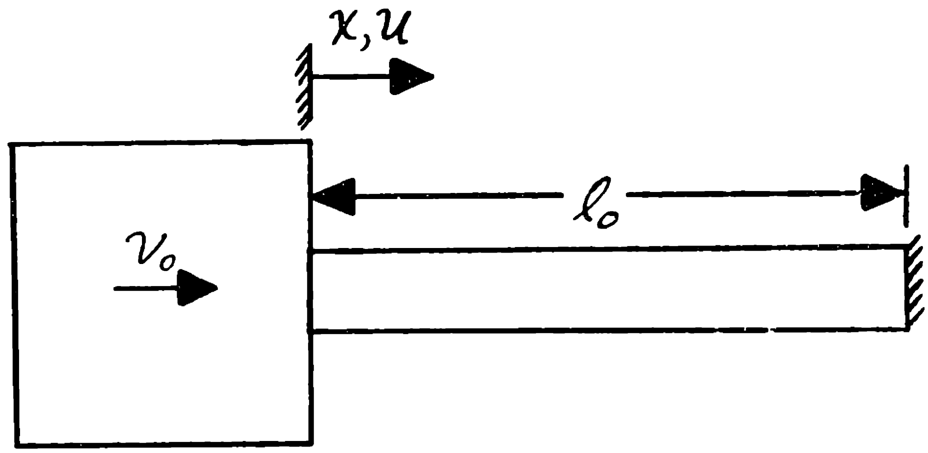

FIGURE 1: MASS IMPACTING BAR

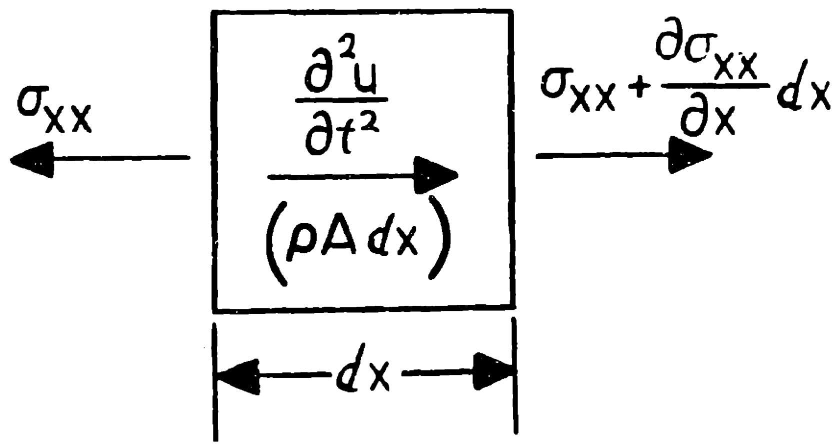

FIGURE2: DIFFERENTIAL ELEMENT OF BAR

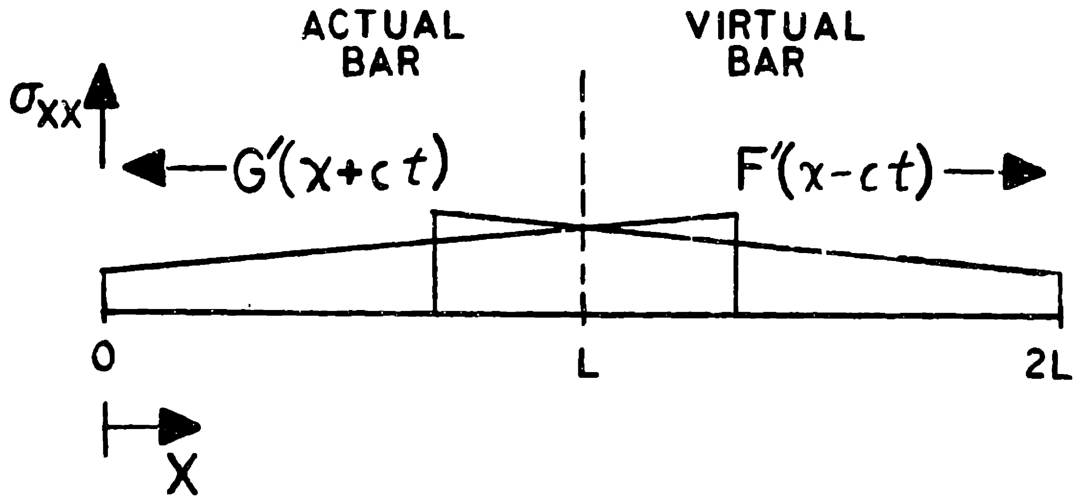

FIGURE 3: SUPERPOSITION OF STRESS WAVES

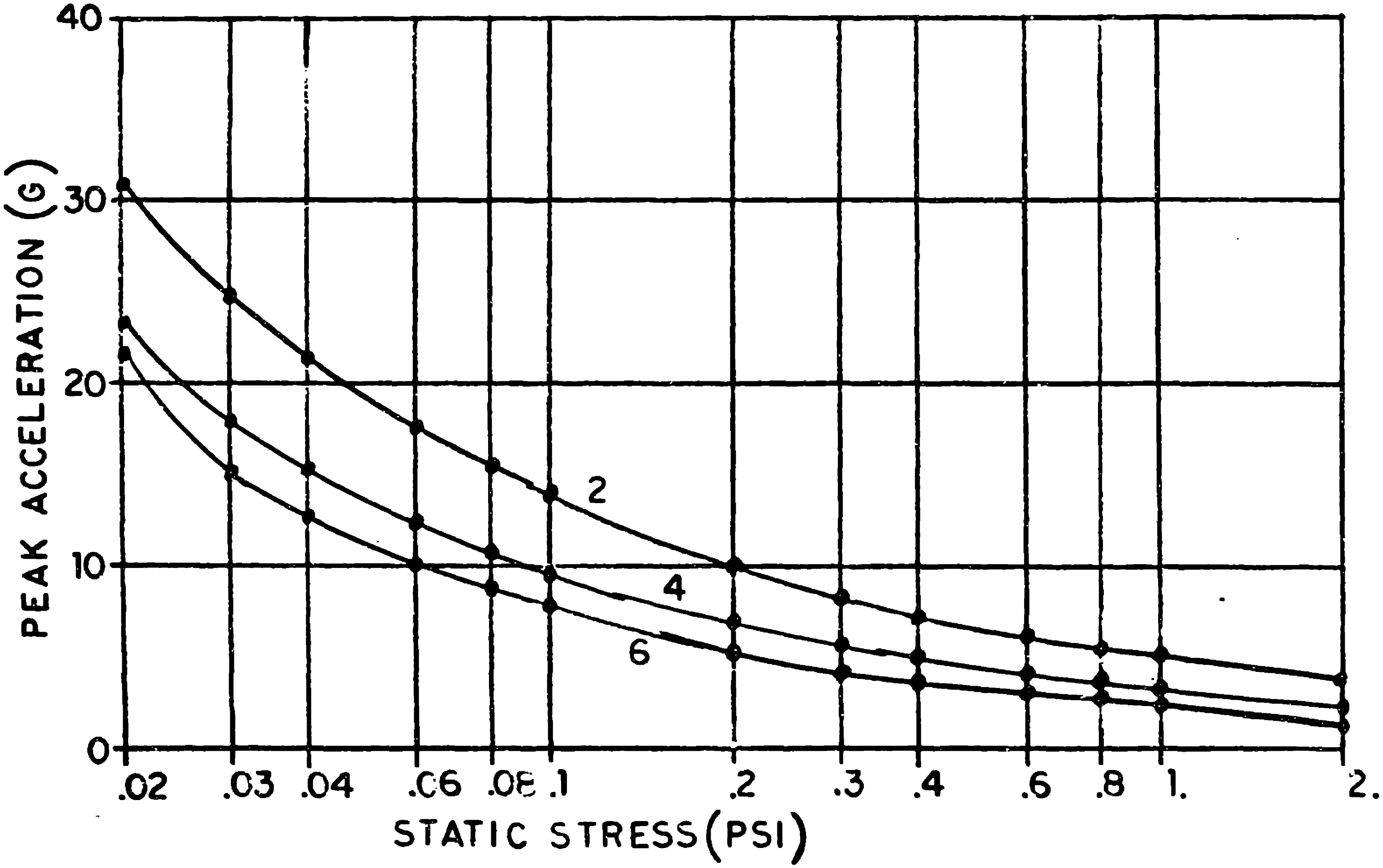

FIGURE 4: CUSHIONING DESIGN CURVE, MODULUS=1 PSI, 1.5 PCF, 12 IN DROP

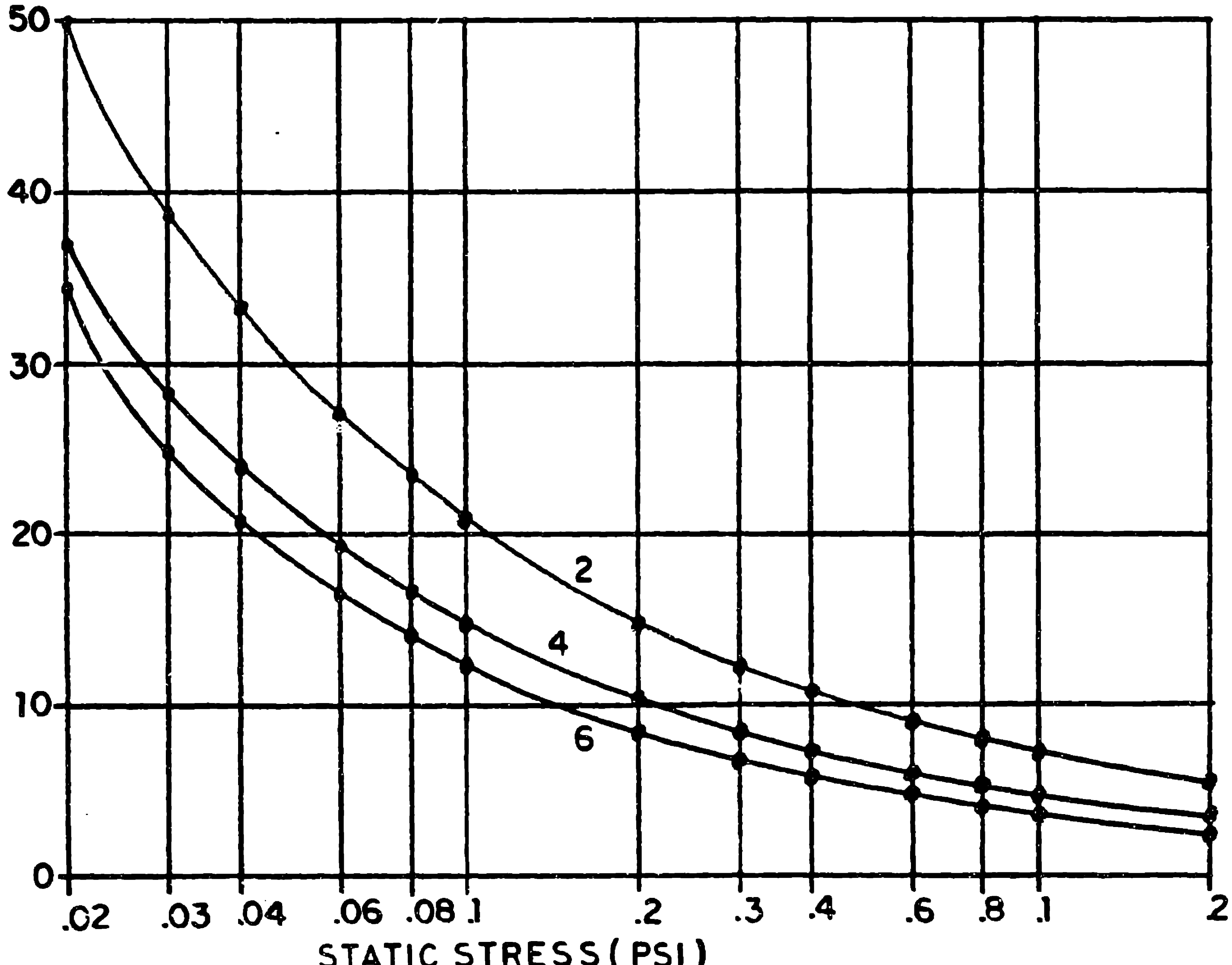

FIGURE 5: CUSHIONING DESIGN CURVE, MODULUS=2.5 PSI,1.5 PCF, 12 IN DROP

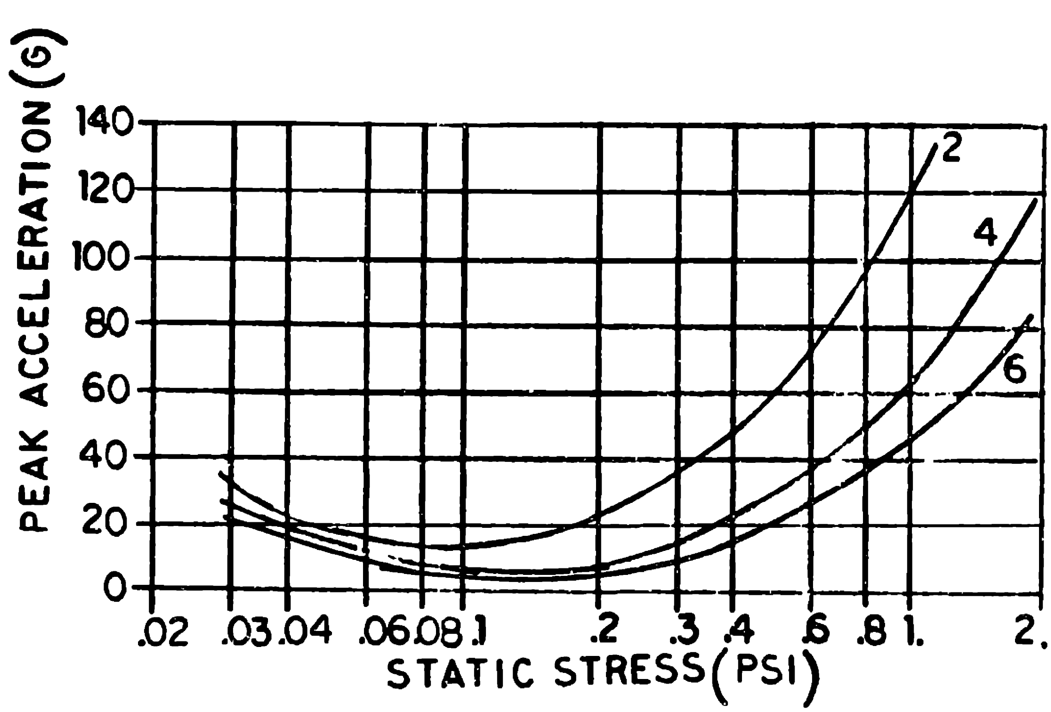

FIGURE 6: CUSHIONING DESIGN CURVE, POLYURETHANE ETHER, 1.5 PCF, 12 IN DROP

APPENDIX A

COMPUTER PROGRAM "KUSHION": LISTING AND CROSS REFERENCE

CBASIC LISTING OF PROGRAM KUSHION

```
        REM ----- KUSHION -----
        REM PROPERTIES VARIABLE, LENGTH VARIABLE
        REM STRESS VARIES ALONG LENGTH (LINEARIZED
        EXPONENTIAL)
        REM UPDATES ON EVERY REFLECTION
        REM (LINEAR APPROXIMATION TO EXPONENTIAL DECAY OF
        STRESS ALONG LENGTH)
        REM NOTE: REM FOLLOWED BY TEXT IS AN ORDINARY REMARK
        STATEMENT
                 REM* IS A PRINT STATEMENT FOR DEBUGGING
        WHICH HAS BEEN
                 TURNED OFF.
        REM  BY D. MARTIN
        VERSION$ = "86.06.06.2": REM
        YEAR.MONTH.DAY.COMPILATION
        REM EPJ - STRAIN
        DEF FN.EPJ(U) = -U/LO
        REM RHO - MASS DENSITY OF BAR
        DEF FN.RHO(EP)=RHOO/(1+0.5*EP)
        REM C - SPEED OF SOUND IN BAR
        DEF FN.C(E,RHO) = SQR(E/RHO)
    90  GO = 386.4
        RESTORE
        DIM WEIGHT(13)
        DATA .02,.03,.04,.06,.08,.1,.2,.3,.4,.6,.8,1.,2.
        FOR I% = 1 TO 13: READ WEIGHT(I%): NEXT I%
        REM GO - GRAVITATIONAL ACCELERATION (IN/SEC^2)
        REM MU - BAR WEIGHT DENSITY (LB/FT^3)
        MU = 1.0
        REM AC - CONTACT AREA (IN^2)
        AC = 1
        REM W - OBJECT WEIGHT (LB)
        REM MU - BAR WEIGHT DENSITY (LB/FT^3)
        DIM LBAR(6)
        DATA 1,2,3,4,5,6
        FOR I% = 1 TO 6: READ LBAR(I%): NEXT I%
        REM SJ - CONTACT STRESS AT STRUCK END (LB/IN^2)
        OPEN "B:DEBUG" AS 1
        OPEN "B:RESULTS" AS 3
        PRINT CHR$(26)
        PRINT: PRINT ,"KUSHION": PRINT ,"VERSION: ";VERSION$:
```

```
      REM INPUT DATA
      REM H - DROP HEIGHT (IN)
      INPUT "ENTER DROP HEIGHT (IN)";H
      REM STORE ELASTIC MODULUS DATA IN ARRAY SESA
      DIM SESA(1,20)
      INPUT "ENTER MATERIAL PROPERTY DATA FILE
      NAME:";MATFIL$
      OPEN "B:"+MATFIL$+".DAT" AS 2
      IF END #2 THEN 400
      READ #2;MATNAME$,MU,NPOINTS%
REM* PRINT MATNAME$,MU
REM* PRINT "READING STRAIN"
      FOR I%=0 TO NPOINTS%-1: READ #2;SESA(0,I%): NEXT I%
REM* PRINT "READING MODULUS"
      FOR I%=0 TO NPOINTS%-1: READ #2;SESA(1,I%): NEXT I%
REM* PRINT "STRAINS AS READ IN"
FOR I%=0 TO NPOINTS%-1: NEXT I%: REM* PRINT I%,SESA(0,I%)
REM* PRINT "MODULI AS READ IN"
FOR I%=0 TO NPOINTS%-1: NEXT I%: REM* PRINT I%,SESA(1,I%)
400 CLOSE 2
      EPLIM = -SESA(1,NPOINTS%-1)
      PRINT "MATERIAL:";MATNAME$
      PRINT "DENSITY = ";MU;" lb/ft^3."
      PRINT "STRAIN","MODULUS"
      FOR I%=0 TO NPOINTS%
      PRINT SESA(0,I%),SESA(1,I%)
      NEXT I%
      FOR ILBAR% = 6 TO 1 STEP -1: REM STEP THROUGH
      THICKNESSES
      FOR IWEIGHT% = 1 TO 13 STEP 1: REM STEP THROUGH
      STATIC LOADING
      REM* IWEIGHT% = 12
      W = WEIGHT(IWEIGHT%)
      REM M - OBJECT MASS PER UNIT AREA OF CONTACT
      M = W/(AC*GO)
      REM ECHO INPUT DATA AND COMPUTED CONSTANTS AND
      INITIAL CONDITIONS
      REM INITIALIZE VARIABLES
      REM J% - REFLECTIONS FROM STRUCK END
      J% =0
      REM UJ - DISPLACEMENT OF STRUCK END
      UJ = 0
      REM VO - INITIAL VELOCITY
      VO = SQR(2*GO*H): VJ = VO
      VSGNJM1% = SGN(VJ)
      REM ACCEL - ACCELERATION OF MASS AND STRUCK END
      ACCEL = 0
      REM LO - FREE LENGTH OF BAR (IN)
      LO = LBAR(ILBAR%): LJ = LO
      EPJ = FN.EPJ(UJ)
      RHOO = MU/(GO*1728): RHOJ = FN.RHO(EPJ)
      REM ALPHA - RATIO OF BAR MASS TO OBJECT MASS
      ALPHA = (RHOO*LO*AC)/M
      REM E - BAR ELASTIC MODULUS (LB/IN^2)
```

```
 91:     EP = EPJ: GOSUB 2300: EO = E
 92:     EJ = EO: EJM1 = EO
 93:     REM CO - SPEED OF SOUND IN BAR
 94:     CO = FN.C(EO,RHOO): CJ = CO
 95:     REM SIGMAO - WAVE FRONT STRESS
 96:     SIGMAO = -VO*SQR(EO*RHOO): SJ = SIGMAO
 97:     REM TJ - TRAVEL TIME-STRUCK END TO FIXED END TO
         STRUCK END
 98:     TO = 2*LO/CO: TJ = TO
 99: REM* PRINT "J";J,"TJ";TJ,"LJ";LJ,"CJ";CJ,"EJ";EJ,
         "RHOJ";RHOJ
100:     AO = -VO*SQR(RHOO/EO): AJ = AO: AJM1 = O: AJM2 = O
101:     BO - 2*ALPHA*AO/((1+ALPHA)*TO): BJ = BO: BJM1 = O:
         BJM2 = O
102:     REM TSUM - TIME SINCE IMPACT
103:     TSUM = O
104:     PRINT
105:     PRINT "DROP HEIGHT","OBJECT WEIGHT ","CONTACT
         AREA","ALPHA"
106:     PRINT "(in)","(lb)","(in^2)"
107:     PRINT H,W,AC,ALPHA
108: REM* PRINT #1; "DROP HEIGHT","OBJECT WEIGHT ","CONTACT
         AREA","ALPHA"
109: REM* PRINT #1; "(in)","(lb)","(in^2)"
110: REM* PRINT #1; H,W,AC,ALPHA
111:     PRINT
112:     PRINT "WEIGHT DENSITY","MASS DENSITY","ELASTIC
         MODULUS","BAR LENGTH"
113:     PRINT "(lb/ft^3)","(lb(sec/ft^2)^2)","(psi)","(in)"
114:     PRINT MU,RHOO,EO,LO
115: REM* PRINT #1; "WEIGHT DENSITY","MASS DENSITY","ELASTIC
         MODULUS","BAR LENGTH"
116: REM* PRINT #1; "(lb/ft^3)","(lb(sec/ft^2)^2)","(psi)",
          "(in)"
117: REM* PRINT #1; MU,RHOO,EO,LO
118:     PRINT
119:     PRINT "MASS LOADING","SPEED OF SOUND","INITIAL
         VELOCITY"
120:     PRINT "(lb(sec^2)/in^3)","(in/sec)","(in/sec)"
121:     PRINT M,CO,VO
122: REM* PRINT #1; "MASS LOADING","SPEED OF SOUND","INITIAL
         VELOCITY"
123: REM* PRINT #1; "lb(sec^2)/in^3)","(in/sec)","(in/sec)"
124: REM* PRINT #1; M,CO,VO
125:     PRINT
126:     REM - MAX/MIN VALUES
127:     VMIN = VO
128:     TVMIN = O
129:     SJMAX = O
130:     TSJMAX = O
131:     LMIN = LO
132:     TLMIN = O
133:     AMAX = O
134:     TAMAX = O
```

```
     HEAD1$="                        CONTACT    OBJECT       BAR
     OBJECT"
     HEAD2$="   J        TIME     STRESS  VELOCITY    LENGTH
     ACCEL.   STRAIN"
     PRINT USING HEAD1$;
     PRINT USING HEAD2$;
     PRINT
REM* PRINT USING HEAD1$;#1:
REM* PRINT USING HEAD2$;#1:
     FORM$ = " ###   -#.######   -####.##   -####.##
     -####.##   -####.##    "
     +" -#.####"
     REM UPDATE MAXS AND MINS
770 GOSUB 2200
     PRINT USING FORM$;J%,TSUM,SJ,VJ,LJ,ACCEL,EPJ
     J% = J%+1
     UJ = UJ-CJ*TJ*((AJ-AJM1)-(BJ-BJM1)*TJ/2)
     EPJ = FN.EPJ(UJ)
     IF ABS(EPJ)>ABS(EPLIM) THEN GOTO 810
     RHOJ = FN.RHO(EPJ)
     LJ = LO*(1+EPJ)
REM* PRINT #1; "J=",J,"UJ=",UJ,"EPJ=",EPJ,"RHOJ=",RHOJ,
     "VJ=",VJ
     VJM1 = VJ
     VJ = VJ+CJ*(BJ-BJM1)*TJ
     IF VJ = VJM1 THEN GOTO 800
REM*ACCEL = ((VJ-VJM1)/TJ)/GO
     SJM1 = EJ*(AJ+AJM1-(BJ+BJM1)*TJM1)
     REM HANDLE JUMP CONDITIONS
     SJ = SJM1+2*SIGMAO
     ACCEL = SJ/W
REM* PRINT #1; "VJ=",VJ,"ACCEL=",ACCEL,"SJM1=",SJM1,
     "SJ=",SJ,"EJ=",EJ
     EJM1 = EJ
     IF EPJ>O THEN GOTO 780
REM* PRINT "EPJ=";EPJ,"EJ=";EJ
     EP = EPJ: GOSUB 2300: EO = E
     EJ = EO: EJM1 = EO
     IF EO <= -99 THEN GOTO 810:
     ANEXT = (AJ/EJ*(EJM1-EJ))-(EJM1/EJ*(BJ+BJM1)*TJ)
             +(EJM1*AJM1/EJ)+(2*SIGMAO/EJ)
     CJ = FN.C(EJ,RHOJ)
     TJM2 = TJM1
     TJM1 = TJ
     TJ = 2*LJ/CJ
REM* PRINT "J";J,"TJ";TJ,"LJ";LJ,"CJ";CJ,"EJ";EJ,
     "RHOJ";RHOJ
     BNEXT = ((TJ*(1-ALPHA)*BJ)+(2*ALPHA*(ANEXT+AJ)))/
     (TJ*(1+ALPHA))
     AJM2 = AJM1
     AJM1 = AJ
     AJ = ANEXT
     BJM2 = BJM1
     BJM1 = BJ
```

```
     BJ = BNEXT
REM* PRINT #1; "AJ=",AJ,"CJ=",CJ,"TJ=",TJ,"BJ=",BJ,
     "TSUM=",TSUM
     TSUM = TSUM+TJM1
     IF -VJ>VO THEN GOTO 790
     IF (SJ >= 0) THEN GOTO 1000 ELSE GOTO 770
780  PRINT "STRAIN GREATER THAN ZERO.  BAR AT INITIAL
     LENGTH."
     PRINT "RECORD EXTREMES AND CONTINUE WITH NEXT CASE.":
     PRINT
     GOTO 1000
790  PRINT "VELOCITY MAGNITUDE EXCEEDS VO."
     PRINT "RECORD EXTREMES AND CONTINUE WITH NEXT CASE.":
     PRINT
     GOTO 1000
800  PRINT "BAR BOTTOMED OUT."
     PRINT "STATIC LOAD, BAR LENGTH, AND BAR PROPERTIES
     BEYOND RANGE OF MODEL."
     PRINT "CONTINUE WITH NEXT CASE."
     GOTO 1010
810  PRINT "STRAIN EXCEEDS MAXIMUM IN MODULUS-STRAIN
     TABLE."
     PRINT "MAXIMUM ACCELERATION SET TO -9999."
     PRINT "CONTINUING WITH THE NEXT CASE."
     AMAX = -9999
     GOTO 1000
1000 PRINT
      PRINT "MAX STRESS  MIN VELOCITY  MIN LENGTH  MAX
      ACCEL."
      PRINT "(psi)       (in/sec)      (in)          (g)"
      FORM$ = "  -####.##        -####.##     -####.##    -
      ####.##"
      PRINT USING FORM$;SJMAX,VMIN,LMIN,AMAX
      PRINT "TIMES (sec):"
      FORM$ = " -#.######      -#.######    -#.######   -
      #.######   "
      PRINT USING FORM$;TSJMAX,TVMIN,TLMIN,TAMAX
      PRINT #3; LO,W,AMAX
REM   PRINT #1; "MAX STRESS","MIN VELOCITY","MIN
      LENGTH","MAX ACCELERATION"
REM   PRINT #1; "(psi)        (in/sec)      (in)
      (g)"
REM   PRINT #1; SJMAX,VMIN,LMIN,AMAX
REM   PRINT #1; "TIMES (sec):"
REM   PRINT #1; TSJMAX,TVMIN,TLMIN,TAMAX
1010 REM* PRINT "PRESS RETURN TO CONTINUE": TEMP% =
      CONCHAR%
      NEXT IWEIGHT%
      NEXT ILBAR%
      CLOSE 3
      OPEN"B:RESULTS" AS 3
      DIM RESULT (13,6)
      FOR J% = 1 TO 6
      FOR I% = 1 TO 13
```

```
     READ #3; LO,W,AMAX
     FOR ROW% = 1 TO 13
     IF W = WEIGHT(ROW%) THEN RNUM%=ROW%
     NEXT ROW%
     CNUM% = INT(LO)
     RESULT(RNUM%,CNUM%) = -AMAX
     NEXT I%
     NEXT J%
     LPRINTER WIDTH 80
     PRINT ,"KUSHION": PRINT ,"VERSION: ";VERSION$:
     PRINT ,"DROP HEIGHT = ";H;" in"
     PRINT ,MATNAME$
     PRINT ," DENSITY = ";MU;" lb/ft^3"
     PRINT
     PRINT "                          PEAK ACCELERATION
     (g)": PRINT
     PRINT "       STATIC                      THICKNESS
     (in)"
     PRINT "       STRESS    1       2       3       4
     5       6"
     PRINT "       (psi)"
     FORM$ = "          ##.##    ####.## ####.## ####.##
     ####.##"
          + " ####.## ####.##"
     FOR I% = 1 TO 13
     PRINT USING FORM$;WEIGHT(I%),RESULT(I%,1),
     RESULT(I%,2),RESULT(I%,3),
     RESULT(I%,4),RESULT(I%,5),RESULT(I%,6)
     NEXT I%
     CONSOLE
     MORE$ = "N"
     INPUT "ENTER Y FOR ANOTHER CASE; ENTER N TO
     STOP";MORE$
     IF UCASE$(MORE$)="Y" THEN GOTO 90
     CLOSE 1
1999 STOP

2200 REM MAX AND MIN UPDATE SUBROUTINE
     IF ABS(SJ) > ABS(SJMAX) THEN SJMAX = SJ: TSJMAX =
     TSUM: ELSE
     IF ABS(VJ) < ABS(VMIN) THEN VMIN = VJ: TVMIN = TSUM:
     ELSE
     IF LJ < LMIN THEN LMIN = LJ: TLMIN =TSUM: ELSE
     IF ABS(ACCEL) > ABS(AMAX) AND ACCEL < 0 AND (VJ >= 0
     OR
     VSGNJM1% <> SGN(VJ))
     THEN AMAX = ACCEL: TAMAX = TSUM: ELSE
     VSGNJM1% = SGN(VJ)
     RETURN

2300 REM THIS FUNCTION PERFORMS A TABLE LOOKUP AND
     INTERPOLATION
     REM  TO FIND MODULUS AS A FUNCTION OF AVERAGE BAR
     STRAIN
```

```
        ILAST% = -1
        FOR I% = 0 TO 18
   REM* PRINT "I%,ILAST%,EP",I%,ILAST%,EP
        IF (SESA(0,I%)<=-EP) AND (-EP<SESA(0,I%+1)) THEN
        ILAST%=I%
        NEXT I%
             IF ILAST%=-1 THEN  E = -99
             IF ILAST%=-1 THEN  GOTO 10
        PARTX = SESA(0,ILAST%+1)-SESA(0,ILAST%)
   REM* PRINT "SESA(0,ILAST%+1),SESA(0,ILAST%),PARTX",
        SESA(0,ILAST%+1),
        SESA(0,ILAST%),PARTX
        PARTY = SESA(1,ILAST%+1)-SESA(1,ILAST%)
   REM* PRINT "SESA(1,ILAST%+1),SESA(1,ILAST%),PARTY",
        SESA(1,ILAST%+1),
        SESA(1,ILAST%),PARTY
        DX = (-EP - SESA(0,ILAST%))
   REM* PRINT "DX",DX
        IF PARTX <=0 THEN PARTX = 1
        DY = DX * PARTY/PARTX
   REM* PRINT "DY",DY,"SESA(1,ILAST%)",SESA(1,ILAST%)
        E = SESA(1,ILAST%)+DY
   10  RETURN

   9999 END
NO ERRORS DETECTED
CONSTANT AREA:     32
CODE SIZE:       3877
DATA STMT AREA:    60
VARIABLE AREA:    640
```

CBASIC CROSS REFERENCE LISTING OF KUSHION

Name	Type	Line No.					
FN.EPJ	FUNCTION	13,	86,	149			
U	PARAMETER	13					
FN.RHO	FUNCTION	16,	87,	151			
EP	PARAMETER	16					
FN.C	FUNCTION	19,	94,	171			
E	PARAMETER	19					
RHO	PARAMETER	19					
AO	GLOBAL	100,	101				
AC	GLOBAL	30,	72,	89,	107		
ACCEL	GLOBAL	83,	146,	161,	259,	261	
AJ	GLOBAL	100,	148,	158,	169,	176,	178,
		179					
AJM1	GLOBAL	100,	148,	158,	170,	177,	178
AJM2	GLOBAL	100,	177				
ALPHA	GLOBAL	89,	101,	107,	176		
AMAX	GLOBAL	133,	200,	206,	210,	224,	229,
		259,	261				
ANEXT	GLOBAL	169,	176,	179			
BO	GLOBAL	101					
BJ	GLOBAL	101,	148,	155,	158,	169,	176,
		181,	182				
BJM1	GLOBAL	101,	148,	155,	158,	169,	180,
		181					
BJM2	GLOBAL	101,	180				
BNEXT	GLOBAL	176,	182				
CO	GLOBAL	94,	98,	121			
CJ	GLOBAL	94,	148,	155,	171,	174	
CNUM%	GLOBAL	228,	229				
DX	GLOBAL	280,	283				
DY	GLOBAL	283,	285				
E	GLOBAL	91,	166,	272,	285		
EO	GLOBAL	91,	92,	94,	96,	100,	114,
		166,	167,	168			
EJ	GLOBAL	92,	158,	163,	167,	169,	170,
		171					
EJM1	GLOBAL	92,	163,	167,	169,	170	
EP	GLOBAL	91,	166,	270,	280		
EPJ	GLOBAL	86,	87,	91,	146,	149,	150,
		151,	152,	164,	166		
EPLIM	GLOBAL	60,	150				
FORM$	GLOBAL	142,	146,	205,	206,	208,	209,
		242,	245				
GO	GLOBAL	21,	72,	80,	87		
H	GLOBAL	43,	80,	107,	234		
HEAD1$	GLOBAL	135,	137				
HEAD2$	GLOBAL	136,	138				
I%	GLOBAL	25,	35,	52,	54,	56,	58,
		64,	65,	66,	223,	230,	244,
		245,	246,	247,	268,	270,	271
ILAST%	GLOBAL	267,	270,	272,	273,	274,	277,

```
                                280,   285
ILBAR%             GLOBAL        67,    85,   218
IWEIGHT%           GLOBAL        68,    70,   217
J%                 GLOBAL        76,   146,   147,   222,   231
LO                 GLOBAL        13,    85,    89,    98,   114,   131,
                                152,   210,   224,   228
LBAR               GLOBAL        33,    35,    85
LJ                 GLOBAL        85,   146,   152,   174,   258
LMIN               GLOBAL       131,   206,   258
M                  GLOBAL        72,    89,   121
MATFIL$            GLOBAL        46,    47
MATNAME$           GLOBAL        49,    61,   235
MORE$              GLOBAL       249,   250,   251
MU                 GLOBAL        28,    49,    62,    87,   114,   236
NPOINTS%           GLOBAL        49,    52,    54,    56,    58,    60,
                                 64
PARTX              GLOBAL       274,   282,   283
PARTY              GLOBAL       277,   283
RESULT             GLOBAL       221,   229,   245,   246
RHOO               GLOBAL        16,    87,    89,    94,    96,   100,
                                114
RHOJ               GLOBAL        87,   151,   171
RNUM%              GLOBAL       226,   229
ROW%               GLOBAL       225,   226,   227
SESA               GLOBAL        45,    52,    54,    60,    65,   270,
                                274,   277,   280,   285
SIGMAO             GLOBAL        96,   160,   170
SJ                 GLOBAL        96,   146,   160,   161,   186,   256
SJM1               GLOBAL       158,   160
SJMAX              GLOBAL       129,   206,   256
TO                 GLOBAL        98,   101
TAMAX              GLOBAL       134,   209,   261
TJ                 GLOBAL        98,   148,   155,   169,   173,   174,
                                176
TJM1               GLOBAL       158,   172,   173,   184
TJM2               GLOBAL       172
TLMIN              GLOBAL       132,   209,   258
TSJMAX             GLOBAL       130,   209,   256
TSUM               GLOBAL       103,   146,   184,   256,   257,   258,
                                261
TVMIN              GLOBAL       128,   209,   257
UJ                 GLOBAL        78,    86,   148,   149
VO                 GLOBAL        80,    96,   100,   121,   127,   185
VERSION$           GLOBAL        10,    40,   233
VJ                 GLOBAL        80,    81,   146,   154,   155,   156,
                                185,   257,   259,   260,   262
VJM1               GLOBAL       154,   156
VMIN               GLOBAL       127,   206,   257
VSGNJM1%           GLOBAL        81,   260,   262
W                  GLOBAL        70,    72,   107,   161,   210,   224,
                                226
WEIGHT             GLOBAL        23,    25,    70,   226,   245
```

APPENDIX B

COMPUTER PROGRAM "KUSHION": PROGRAM DESCRIPTION

KUSHION is a computer program which estimates the peak acceleration of a cushioned item using the method described in this thesis. It is written in CBASIC, using an Osborne I CP/M microcomputer. The program does not use any specific printer control codes, and uses only essential hardware dependent commands, such as CHR$(26) to clear the screen. For a description of the CBASIC language, see Refs. 10 and 11. Reference 11 also contains descriptions of the Osborne I, and of the CP/M operating system. A description of the purpose of statements within the program follows, with statements grouped by similarity of purpose.

Lines	Purpose
1-11	Heading information.
12-20	Define functions for strain, density and sonic velocity.
21-22	Dimension and initialize static stress array. Values chosen cover the range of static stress for which peak acceleration is plotted in the graphs of Ref. 7.
23-32	Initialize variables.
33-36	Dimension and initialize bar length array. Values chosen are those used in the graphs of Ref. 7.
37-40	Open scratch files, clear screen and begin printing to console.
41-46	Enter drop height and material property file name.
47-66	Open material property file, read in data, echo to console.
67-69	Step through range of bar lengths and static stress.
70-72	Initialize constants.
73-103	Initialize variables which are updated at each

reflection.

104-125 Echo to console initial values of variables which are updated at each reflection.

126-134 Initialize variables to record extreme values of velocity, stress, length, and acceleration, and the times at which they occur.

135-143 Print headings for time history listing on console.

144-146 Update extreme values of velocity, stress, length, and acceleration, and the time at which they occur. Print current values of the reflection count, time, velocity, stress, length, and acceleration.

147-153 Update reflection count, displacement, strain, density and length. End time history if the strain exceeds the maximum strain for which the modulus is defined.

154-162 Update velocity, stress, and acceleration at struck end. When the wave front reaches the struck end the stress is incremented by $2\sigma^0$.

163-170 Update the strain. End the time history if the strain is greater than 0. Update the elastic moodulus using a table look-up procedure and the data stored in array SESA. Save value of old coefficient "a" of stress function.

171-175 Update sonic velocity, and interval between reflections.

176-184 Update coefficients "a" and "b" of stress function. Increment time since impact.

185-186 End time history if magnitude of current velocity is greater than the initial velocity, or if the stress at the struck end is greater than zero. These tests check to see if the mass has rebounded free of the bar.

187-189 Print message on console if the stress is greater than zero, indicating that the mass has rebounded free of the bar.

190-192 Print message on console if the magnitude of the current velocity stress is greater than the initial velocity zero, indicating that the mass has rebounded free of the bar.

193-196 Print message on console if the bar has bottomed out, indicated by a displacement of the struck end equal to the initial bar length.

197-201 Print message on console if the strain exceeds the maximum strain for which the modulus is defined. Set the maximum acceleration to -9999.

202-215 Print the summary of extreme values of velocity, stress, length, and acceleration, and the times at which they occur. Print the initial bar length, static stress, and peak acceleration to the scratch file "RESULTS".

216-218 Continue to loop over the range of static stress, and of bar lengths.

219-248 Sort the data in the scratch file "RESULTS", and print a report of the maximum acceleration as a function of static stress and initial bar length.

249-254 Ask if accelerations are to be estimated for another drop height or material. If not then terminate execution.

255-264 Subroutine which updates the extreme values of values of velocity, stress, length, and acceleration, and the times at which they occur.

265-287 Subroutine to interpolate elastic modulus as a function of the current strain, using the data in the array SESA.

288 End of CBASIC source file.