MODELLING AND CONTROL OF AN AUTOMATIC TRANSMISSION

by

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ABSTRACT

A mathematical model was derived for simulating the Hydramatic 440 automatic transmission. A closed-loop controller for the 1-2 shift was then developed to produce a smooth ouput shaft torque.

Comparisons of the open-loop and closed-loop simulations show that the jerk associated with current automatic powered upshifts can be eliminated via closed-loop control of the engine torque and the clutch pressures during the shift.

Thesis Supervisor: J.K. Hedrick

Title: Professor of Mechanical Engineering

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1
Introduction

To date, very little work has been done in controlling the total powertrain of an automobile during the shift in an automatic transmission. The objective of this study is to derive a mathematical model of an automatic transmission and then develop a closed-loop control algorithm to produce a smooth output shaft torque by modulating engine torque and the clutch pressures.

The system equations for the Hydramatic 440 automatic transmission are derived in Chapter 2. The objective of the model developed is for both simulation and control of a transmission shift. The equations are derived using a bond graph approach. This approach could easily be extended to other configurations of the planetary gear sets and clutches of the automatic transmission.

The model derived is simulated in Chapter 3 using the current open-loop controls for the Hydramatic 440 transmission. The transients which occur during the first to second gear powered upshift are then analyzed.

A closed-loop controller is developed in Chapter 4 with the intent of obtaining a smooth shift. The control inputs are taken as engine torque and the clutch torques.

Recommendations for future work are then given in Chapter 5.

2

Transmission Modelling

In this section, the system equations for simulating the Hydramatic 440 automatic transmission are derived. The block diagram for the engine/transmission/driveline system is shown below in Figure 3.1.

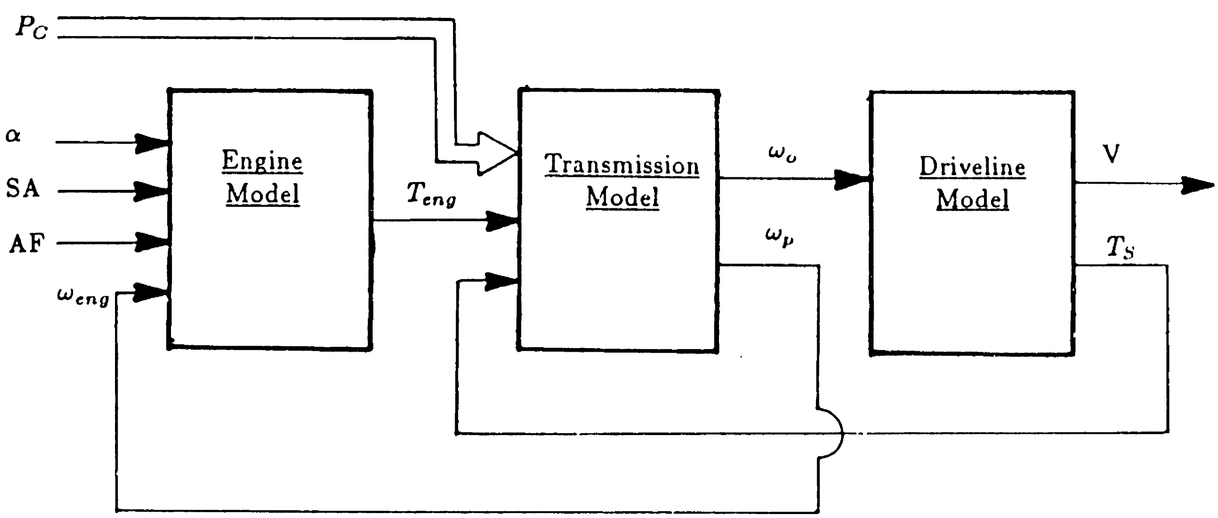


Figure 2.1

2.1 Engine Model

The engine is modelled as a torque-producing device with one inertia. Engine torque is taken as a function of the throttle angle (α), spark advance (SA), air-fuel ratio (AF) and engine speed (ω_e). The system equations describing the engine are not included in this report.

2.2 Transmission Model

The schematic diagram for the Hydramatic 440 automatic transmission is shown below in Figure 2.2.1. The clutch schedule and the overall transmission gear ratio, including the final drive gear ratio of 2.84, is shown in Table 2.2.1 for each gear range.

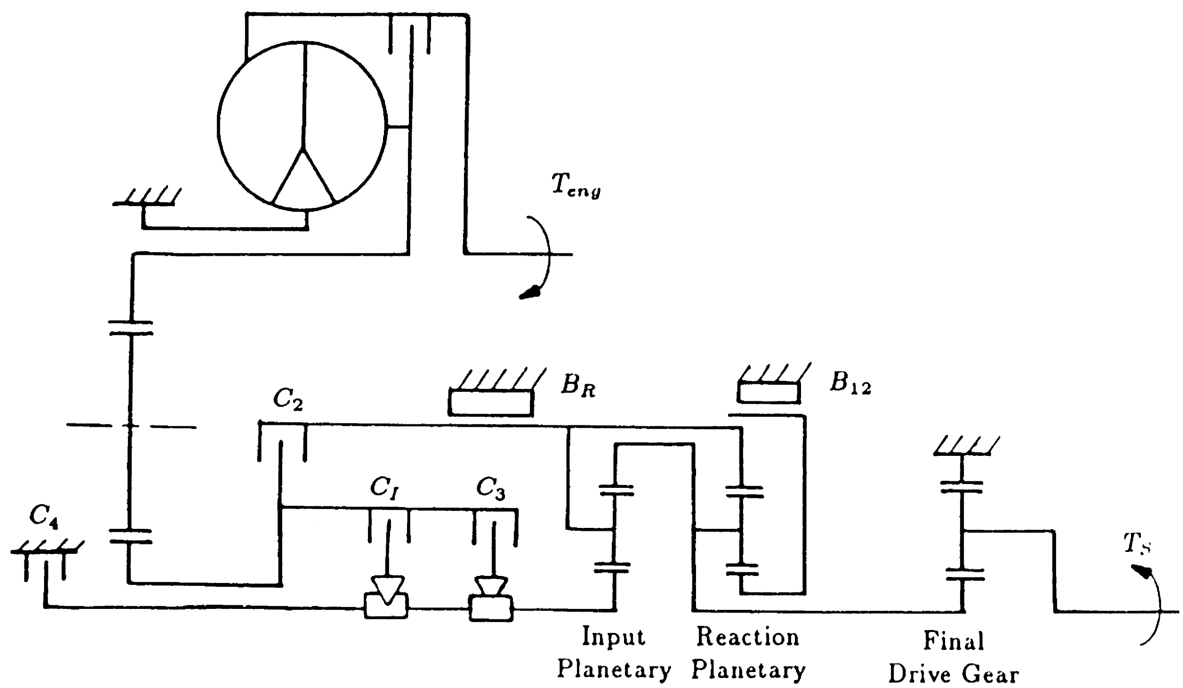


Figure 2.2.1

Range	Clutches Engaged						Gear Ratio
	C_i	C_2	C_3	C_4	B_{12}	B_R	
1^{st}	x				x		8.32
2^{nd}	⊗	x			x		4.46
3^{rd}		x	x				2.84
4^{th}		x	⊗	x			1.99
Rev	x					x	6.76

x ~ clutch on ⊗~ sprag over-running

Table 2.2.1

Note that the input clutch, C_i, and third clutch, C_3, are connected to one-way sprags.

The transmission model consists of five lumped inertias, four range clutches, two bands, two sprags, an ideal compound planetary gear set, and a static torque converter model with a lock-up clutch. The transmission parameters corresponding to the Hydramatic 440 are included in Appendix I. The inputs to the transmission are engine torque, output shaft torque, and the clutch pressures. The transmission outputs are engine speed and transmission output speed.

The system equations describing the mechanics of the transmission are derived next in sections 2.2.1-4, followed by the torque converter equations in section 2.2.5. The system equations are derived using a bond graph approach.

2.2.1 Single Planetary Gear Set

A single planetary, or epicyclic, gear set and its schematic representation are shown below in Figure 2.2.1.1. Note that only half of the schematic representation is shown for convenience.

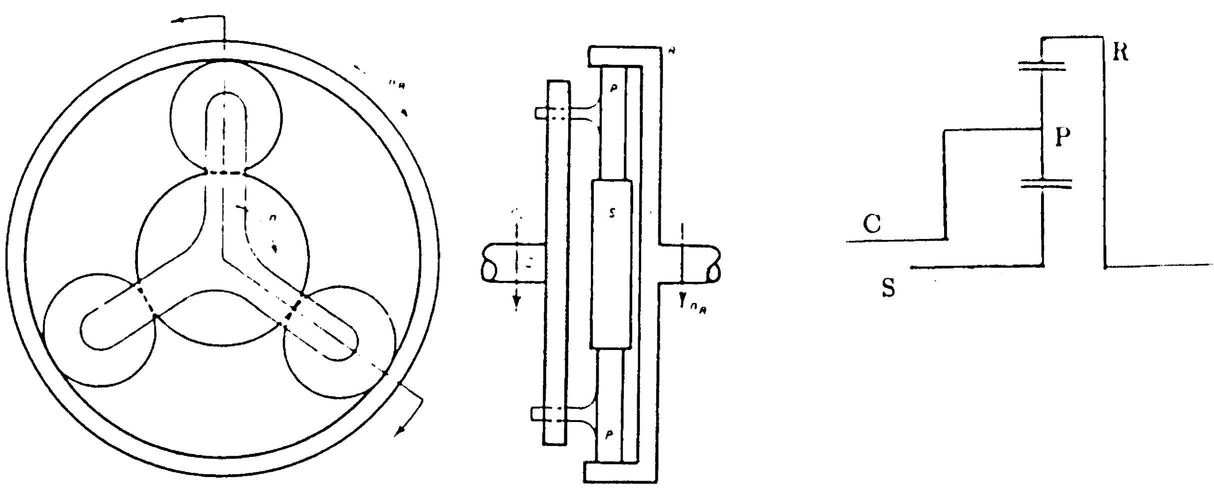


Figure 2.2.1.1

Assuming an ideal planetary gear set, i.e. no backlash or compliance between gears, the speed and torque relationships are

$$\omega_C = \left(\frac{N_S}{N_S + N_R}\right)\omega_S + \left(\frac{N_R}{N_S + N_R}\right)\omega_R \tag{2.2.1.1}$$

$$T_S = \left(\frac{N_S}{N_S + N_R}\right)T_C \tag{2.2.1.2}$$

$$T_R = \left(\frac{N_R}{N_S + N_R}\right)T_C \tag{2.2.1.3}$$

where

$\omega_{C,S,R}$ denotes the speed of the carrier, sun, and ring gear, respectively,

$T_{C,S,R}$ denotes the torque of the carrier, sun, and ring gear, respectively, and

$N_{S,R}$ denotes the number of teeth on the sun and ring gears, respectively

Using these equations as definitions, the bond graph for a planetary gear set becomes

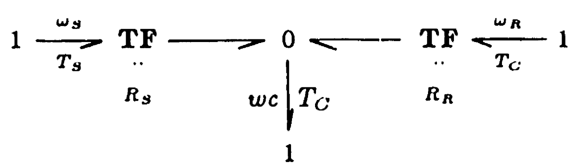


Figure 2.2.1.2

where

$$R_S = \frac{N_S}{N_S + N_R} \tag{2.2.1.4}$$

$$R_R = \frac{N_R}{N_S + N_R}. \tag{2.2.1.5}$$

Note that the sign convention for the bond graph has already been determined by the defining equations; therefore, the direction of the arrows is not arbitrary and must agree with equations 2.2.1.1-3.

2.2.2 Compound Planetary Gear Set

The compound planetary gear set for the Hydramatic 440 transmission is shown below in Figure 2.2.2.1.

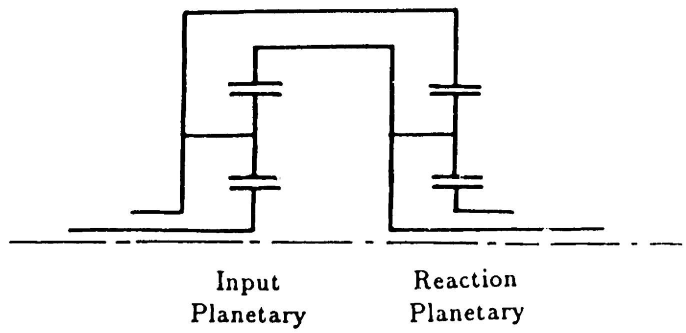


Figure 2.2.2.1

Note that the compound planetary is formed by connecting the input carrier to the reaction ring gear, and the input ring gear to the reaction carrier. Its bond graph is therefore formed by connecting the input and reaction planetary gear set bond graphs in this way, i.e., connecting the input carrier speed, ω_{CI}, to the reaction ring gear speed, ω_{RR}, and the input ring gear speed, ω_{RI}, to the reaction carrier speed, ω_{CR}. This is done below in Figure 2.2.2.2. Note that the subscript '*I*' following each speed and transformer modulus corresponds to the input planetary and the subscript '*R*' corresponds to the reaction planetary.

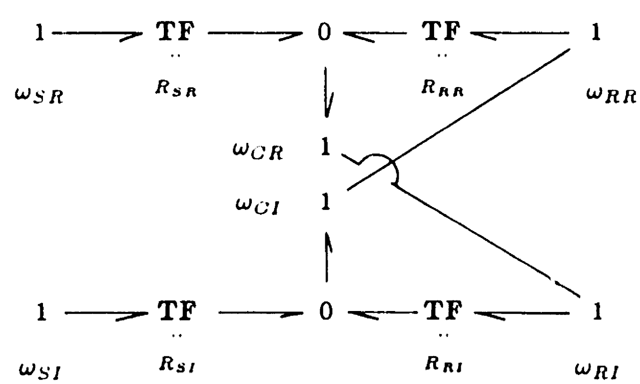


Figure 2.2.2.2

This bond graph can then be reduced to yield

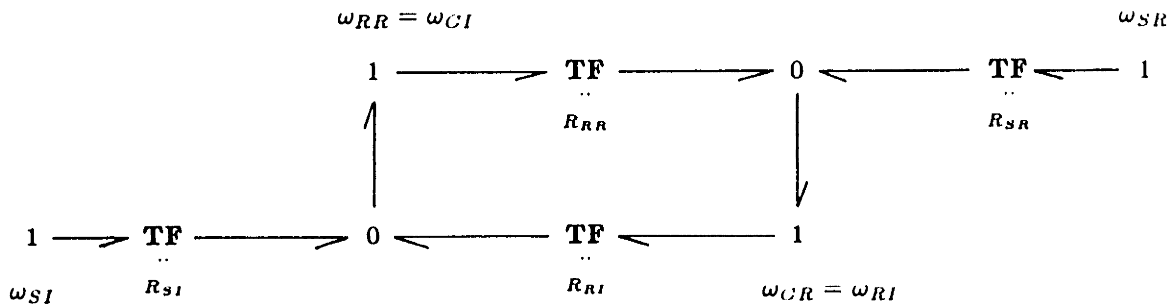


Figure 2.2.2.3

Figure 2.2.2.3 thus depicts the bond graph for the compound planetary gear set in the Hydramatic 440 transmission. Note that this bond graph contains a ring structure within it which will create algebraic loops when the system equations are solved. The bond graph can, however, be further simplified using multiport analysis so that algebraic loops do not exist.

The multiport junction for the compound planetary gear set is shown below in Figure 2.2.2.4. Note the junction is termed **PL** to denote a planetary gear set.

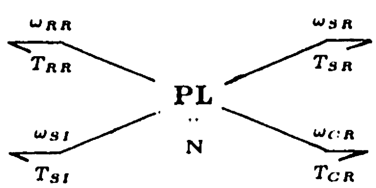


Figure 2.2.2.4

The modulus for the multiport, **N**, is found from the system equations for the compound planetary gear set and is dependent on the causality, or input/output structure, defined for the multiport junction. Note that causality for the multiport will be determined when the entire bond graph for the transmission has been made.

The system equations for the input and reaction planetary gear sets are

$$\omega_{CI} = R_{SI}\omega_{SI} + R_{RI}\omega_{RI} \tag{2.2.2.1}$$

$$\omega_{CR} = R_{SR}\omega_{SR} + R_{RR}\omega_{RR} \tag{2.2.2.2}$$

and the kinematic constraint equations for the compound planetary arrangemnent are

$$\omega_{CI} = \omega_{RR} \tag{2.2.2.3}$$

$$\omega_{RI} = \omega_{CR}. \tag{2.2.2.4}$$

Substituting equations 2.2.2.3-4 into equations 2.2.2.1-2 yields the following two equations in four unknowns:

$$\omega_{RR} = R_{SI}\omega_{SI} + R_{RI}\omega_{CR} \tag{2.2.2.5}$$

$$\omega_{CR} = R_{SR}\omega_{SR} + R_{RR}\omega_{RR}. \tag{2.2.2.6}$$

Equations 2.2.2.5-6 therefore show that the multiport consists of two independent speeds and two dependent speeds. Once the causality for the multiport junction has been determined, equations 2.2.2.5-6 must then be solved simultaneously to determine the multiport modulus **N**. This has been done in Appendix II for all causal arrangements for the multiport junction.

For the causality defined by

$$\begin{pmatrix} \omega_{SR} \\ \omega_{RR} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \omega_{SI} \\ \omega_{CR} \end{pmatrix}, \tag{2.2.2.7}$$

the multiport modulus **A** is found in Appendix II to be

$$\mathbf{A} = \frac{1}{R_{SR}} \begin{bmatrix} -R_{SI}R_{RR} & 1 - R_{RI}R_{RR} \\ R_{SI}R_{SR} & R_{RI}R_{SR} \end{bmatrix} \tag{2.2.2.8}$$

and the corresponding bond graph is shown below in Figure 2.2.2.5.

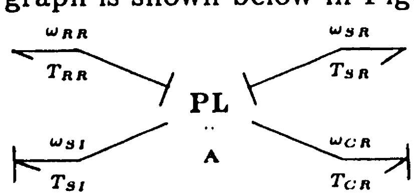


Figure 2.2.2.5

Note that the general multiport modulus, **N**, has been changed to correspond to the specific multiport modulus, **A**, for this causal arrangement as found in Appendix II.

The dependent torques for this multiport junction are found using the conservation of power; i.e., the sum of the power around the multiport must equal zero. This is true because the multiport consists only of interconnected transformers and has no energy storage or dissipative elements. For the sign convention defined by Figure 2.2.2.5, the conservation of power yields

$$\begin{pmatrix} \omega_{SI} & \omega_{CR} \end{pmatrix} \begin{pmatrix} T_{SI} \\ T_{CR} \end{pmatrix} + \begin{pmatrix} \omega_{SR} & \omega_{RR} \end{pmatrix} \begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix} = 0. \tag{2.2.2.9}$$

Rearranging and substituting equation 2.2.2.7 yields

$$\begin{pmatrix} \omega_{SI} & \omega_{CR} \end{pmatrix} \begin{pmatrix} T_{SI} \\ T_{CR} \end{pmatrix} = -\begin{pmatrix} \omega_{SI} & \omega_{CR} \end{pmatrix} \mathbf{A}^T \begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix} \tag{2.2.2.10}$$

which implies that the dependent torques are defined by

$$\begin{pmatrix} T_{SI} \\ T_{CR} \end{pmatrix} = -\mathbf{A}^T \begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix}. \tag{2.2.2.11}$$

2.2.3 Bond Graph for the Hydramatic 440 Transmission

Referring to the transmission schematic shown in Figure 2.2.1, the bond graph of the transmission can now be found.

The lumped inertias of the planetary gear set are first added to the multiport junction of Figure 2.2.2.4. This is done below in Figure 2.2.3.1.

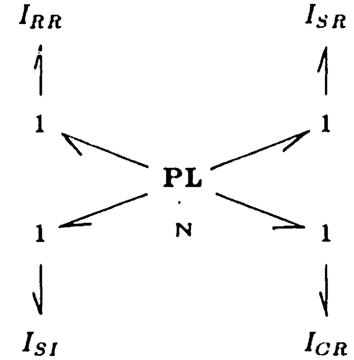


Figure 2.2.3.1

Note that each inertia is a lumped sum of all inertias rotating at the respective speed, i.e.

$I_{SI} \sim$ inertia of the input sun gear, two sprag clutches, and fourth clutch plate,

$I_{RR} \sim$ inertia of the reaction ring gear, input carrier, and reverse reaction drum,

$I_{SR} \sim$ inertia of the reaction sun gear, and

$I_{CR} \sim$ inertia of the reaction carrier,input ring gear, and final drive gear.

The stationary clutch torques are next added to the bond graph. The clutch torques are calculated from equation 2.2.3.1 for plate clutches and from equation 2.2.3.2 for band clutches.†

$$T_C = P \cdot \mu(\omega_C) \cdot A_C \cdot R_C \cdot Sgn(\omega_C) \tag{2.2.3.1}$$

$$T_B = P \cdot A_B \cdot R_B \cdot (e^{\mu(\omega_B) \cdot \theta_B} - 1) \cdot Sgn(\omega_B) \tag{2.2.3.2}$$

where

$$\begin{aligned} P &\equiv \text{clutch pressure} \\ \mu(\omega) &\equiv \text{coefficient of friction‡} \\ &= \begin{cases} 0.1545, & \omega = 0; \\ 0.0631 + 0.0504e^{-0.033|\omega|}, & \text{otherwise} \end{cases} \\ A_{C,B} &\equiv \text{clutch plate/band piston area,} \\ R_{C,B} &\equiv \text{clutch plate/band effective radius,} \\ \theta_B &\equiv \text{band wrap angle, and} \\ Sgn(\omega_{C,B}) &\equiv \text{sign of the clutch/band slip speed,} \end{aligned}$$

and the clutch slip speeds are

$$\omega_{C4} = \omega_{SI} \tag{2.2.3.3}$$

$$\omega_{B12} = \omega_{SR} \tag{2.2.3.4}$$

$$\omega_{BR} = \omega_{RR}. \tag{2.2.3.5}$$

† A.D. Deutschman, *Machine Design; Theory and Practice*, Macmillan Publishing Co., 1975, pp 682-5, 697-8.

‡ private communication with General Motors Research Labs, Warren MI

$$\omega_{C4} = \omega_{SI} \tag{2.2.3.3}$$

$$\omega_{B12} = \omega_{SR} \tag{2.2.3.4}$$

$$\omega_{BR} = \omega_{RR}. \tag{2.2.3.5}$$

Note the sign of the torques is equal to the sign of the slip across the clutch.

Figure 2.2.3.2 shows the bond graph with the stationary clutch torques T_{CR}, T_{B12}, and T_{BR} added.

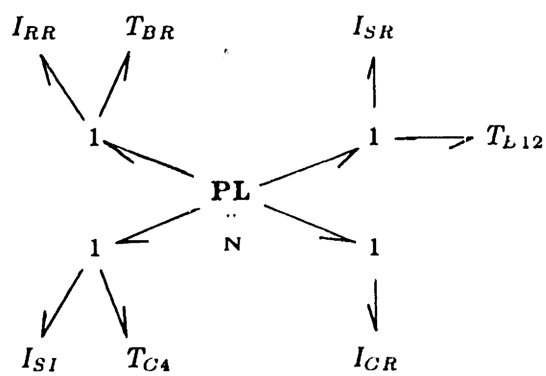


Figure 2.2.3.2

Note that the sign convention for the stationary clutch torques on the bond graph agrees with equations 2.2.3.1-5 in that the stationary clutch torque applied to the respective inertia opposes its motion.

The rotating clutch torques, turbine torque, turbine inertia, final drive gear ratio and shaft torque are next added to the bond graph, which is shown in Figure 2.2.3.3. Note that turbine inertia is a lumped sum of all the inertias rotating at turbine speed; i.e., the turbine inertia, chain assembly, and the input and third clutch housings. This yields the complete bond graph for the mechanics of the Hydramatic 440 transmission.

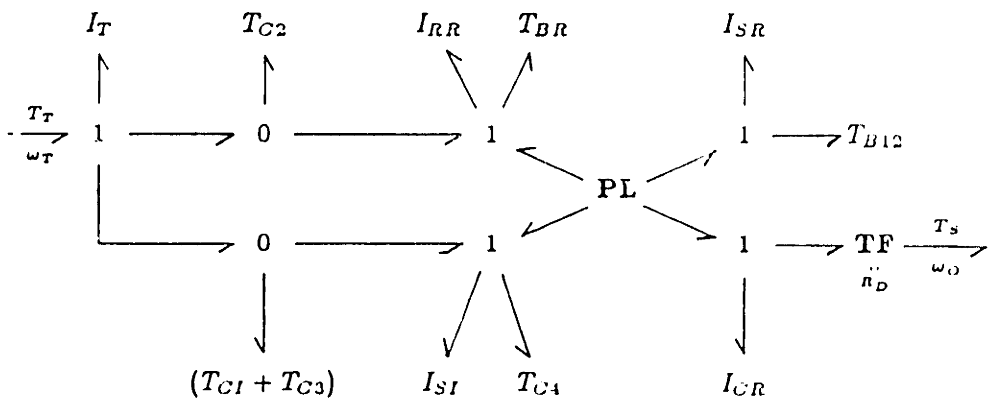


Figure 2.2.3.3

The rotating clutch torques are calculated using equation 2.2.3.1 and the clutch slip speeds shown below.

$$\omega_{CI} = \omega_{C3} = \omega_T - \omega_{SI} \tag{2.2.3.6}$$

$$\omega_{C2} = \omega_T - \omega_{RR}. \tag{2.2.3.7}$$

Note equations 2.2.3.6-7 agree with the sign convention chosen in the bond graph in that a positive clutch slip speed decelerates the turbine inertia and accelerates the planetary inertia, and vice versa.

The transformer modulus for the final drive gear, R_D, is defined by the equation

$$\omega_o = R_D \omega_{CR}, \tag{2.2.3.8}$$

thus, R_D is equal to the inverse of the final drive gear ratio.

2.2.4 System Equations for the Hydramatic 440 Transmission

Before the system equations can be found from the bond graph, the causal assignment must be made. The inputs to the mechanical section of the transmission are turbine torque from the torque converter, output shaft torque, and the clutch pressures. The causality of each clutch torque will depend on its mode of operation. If the clutch is slipping, then the clutch is a torque source and the torque is found from equations 2.2.3.1-7. If, however, the clutch is locked up, then the clutch is a speed source and the clutch slip speed is zero. Note when the clutch is in locked-up mode, the bond graph determines the reaction torque of the clutch.

In Appendix III, this causal assignment has been given to the bond graph of Figure 2.2.3.3 for each mode of transmission's operation, where the mode of operation is determined by the clutches defined locked up. Each mode of operation in Appendix III begins with the complete bond graph showing the causal assignment. Next, the kinematic constraints for all locked-up clutches are given, followed by the equations describing the multiport planetary junction based on the causality. The transmission system equations are then derived including the reaction torques for all locked-up clutches.

The computer program for simulating the transmission must have the necessary logic to determine the mode of operation for the transmission; or, equivalently, it must determine which clutches are locked up. The required logic for this is outlined below.

For a locked-up clutch, the magnitude of the clutch torque as defined by equations 2.2.3.1-2 must be greater than the clutch reaction torque as defined by the system equations in Appendix III. If this is not true, then the clutch will begin slipping. The causality of the clutch will therefore change from a speed source to a torque source whose torque is found from equations 2.2.3.1-7. Note that for these equations, the sign of the clutch torque is set equal to the sign of the slip across the clutch. However, when the clutch breaks from lock-up mode, the slip speed is zero and thus its sign is undetermined. Therefore, when a locked-up clutch breaks loose, the sign of the clutch torque as defined by equations 3.2.3.1-2 must be initially set to the sign of its reaction torque.

A slipping clutch can only go into lock-up mode when the clutch slip speed numerically passes through zero. When this happens, the simulation program calculates the system equations corresponding to the clutch in its locked-up mode and the above test for a locked-up clutch is made to check that the clutch would stay locked up. Note that when a slipping clutch does lock up, its causality changes from a torque source to a speed source whose slip speed is zero.

The sprags found on the input clutch, C_i, and third clutch, C_3, can only transmit torque in one direction. The input clutch sprag transmits a positive torque. If the torque through this clutch goes negative, then its clutch capacity as defined by equations 3.2.3.1-2 is set to zero. Note that the sign of the torque for a slipping clutch is determined by its slip speed; whereas the sign for a locked-up clutch is determined by its reaction torque. Third clutch is equivalent except that its sprag transmits a negative torque.

2.2.5 Torque Converter Equations

The torque converter is described by a two-port junction, **TC**, as shown by Figure 2.2.4.1, with pump and turbine torque modelled as a function of pump and turbine speed.†

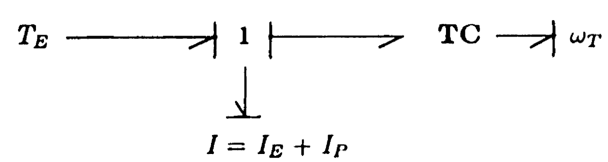


Figure 2.2.4.1

Note that the engine and pump inertia are lumped together.

The describing equations for the pump torque, T_P, and turbine torque, T_T, depend upon the mode of operation for the torque converter. For the converter mode, the pump and turbine torques are‡

$$T_P = 3.4325 \times 10^{-3}\omega_P^2 + 2.2210 \times 10^{-3}\omega_P\omega_T - 4.6041 \times 10^{-3}\omega_T^2 \tag{2.2.4.1}$$

$$T_T = 5.7656 \times 10^{-3}\omega_P^2 + 3.107 \times 10^{-4}\omega_P\omega_T - 5.4323 \times 10^{-3}\omega_T^2. \tag{2.2.4.2}$$

When in converter mode, the torque converter acts as a torque multiplier, i.e., turbine torque is greater than pump torque. However, when the turbine torque in

† A.J. Kotwicki, *Dynamic Models for Torque Converter Equipped Vehicles*, SAE #820393.

‡ private communication with General Motors Research Labs, Warren MI.

converter mode is calculated to be less than the pump torque, then the torque converter has changed to a fluid coupling mode and the pump and turbine torques are defined by†

$$T_P = T_T = -6.7644 \times 10^{-3}\omega_P^2 + 3.20084 \times 10^{-2}\omega_P\omega_T - 2.52441 \times 10^{-2}\omega_T^2. \tag{2.2.4.3}$$

The differential equation for the engine/pump speed, ω_P, is then

$$\dot{\omega}_P = \frac{1}{I_E + I_P}(T_E - T_P) \tag{2.2.4.4}$$

where T_E is the engine torque.

† private communication with General Motors Research Labs, Warren MI.

2.3 Driveline Model

The driveline is modelled as a linear shaft of spring rate k connected to a vehicle inertia with a load torque, T_L, due to wind and rolling resistance. The input to the driveline is the transmission output speed, ω_o. The schematic representation for the driveline is shown in Figure 2.3.1 and the corresponding bond graph is shown in Figure 2.3.2.

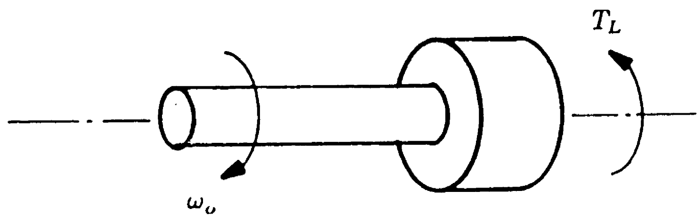


Figure 2.3.1

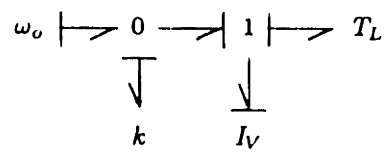


Figure 2.3.2

For a vehicle of mass m and tire radius r, the equivalent vehicle inertia is

$$I_V = mr^2. \tag{2.3.1}$$

The load torque due to wind and rolling resistance is modelled as†

$$T_L = (158.2 + 4.479 \times 10^{-2} V_{kph}^2) r, \tag{2.3.2}$$

where V_{kph} is the vehicle velocity in kilometers per hour. The system equations for the driveline are then

$$T_S = k(\omega_o - \omega_V) \tag{2.3.3}$$

$$\dot{\omega}_V = \frac{1}{I_V}(T_S - T_L), \tag{2.3.4}$$

where ω_o is defined by equation 2.2.3.8.

† private communication with General Motors Research Labs, Warren MI.

2.4 Conclusions on the System Model

The transmission/driveline is modelled as a 4-6 state system depending on the number of clutches locked up in the transmission. Table 2.4.1 lists the states for each mode of operation.

Mode	Number of locked clutches	States
$1^{st} - 4^{th}$ gear	2	$\omega_P\ \omega_T\ T_S\ \omega_V$
shifting	1	$\omega_P\ \omega_T\ \omega_{CR}\ T_S\ \omega_V$
shifting	0	$\omega_P\ \omega_T\ \omega_{CR}\ \omega_{SI}\ T_S\ \omega_V$

Table 2.4.1

Note that whenever a locked clutch breaks free, a new state is added to the system.

As described in Section 2.2.3 and Appendix III, ten separate sets of system equations corresponding to each mode of operation for the transmission along with the necessary logic to determine the mode of operation were derived to simulate the tramsmission. A simpler method for simulation would be to always describe each clutch as a torque source. The transmission could then be simulated using the equations found in Appendix III.1, *All Clutches Slipping*, since these model all clutches as torque sources. The simulation would then be done with only one set of system equations. Note that a locked-up clutch can be modelled as a torque source by describing it as either a spring or a viscous damper.

Although this method would be simpler for simulation, it also would complicate the control problem by adding more states. Since the ultimate objective of this paper is closed loop control, the system equations for each mode of operation were derived to simplify the control problem.

The system equations describing the transmission were derived using a bond graph approach; however, several other methods could have been used. These include free body analysis, the lever analogy†, and Lagrangian analysis.

Bond graph modelling has the advantages that it

- shows physical connections and power flow through the drivetrain;
- is in a modular form;
- allows use of a more complex planetary gear set, i.e., compliance, backlash, etc.; and
- computer programs exist which will simulate bond graphs and can be used for analysis.

Free body analysis becomes complicated in analyzing the compound planetary gear set for a transmission. Note, however, that the analysis presented in Appendix III can be followed by one not familiar with bond graphs by considering the free body analysis for each equation. The advantage bond graphs have over free body analysis is that the bond graph is literally a road map for finding the system equations. Also, the planetary gear set is modelled as a multiport junction using bond graphs. This allows the dependent inertias to be easily reflected through the planetary and lumped into the independent inertias.

The lever analogy transforms the rotational elements of the planetary gear set into a lever. Torques and angular speeds of the planetary are then represented by forces and linear velocities on the lever. Rotational inertias of the planetary can also be transformed into linear inertias and added to the lever. Thus, the lever analogy can be used to solve for the system equations of the transmission by solving the system equations for a lever. For range shifts characterized by a stationary clutch staying locked up throughout the shift, the corresponding lever is one pinned to ground at the reaction point. The system equations for this lever are then simply found by summing the moments about the reaction point. However, for all other shifts, the lever will be in both rotation and translation. This corresponds to two degrees of freedom for the lever and is more difficult to solve.

† *The Lever Analogy: A New Tool in Transmission Analysis*, H. L. Benford and M. B. Leising, SAE paper 810102.

3

Open–Loop Simulation

The overall engine/transmission/driveline system is simulated using the model derived in Section 2 and transmission parameters listed in Appendix I. The engine model for the simulation corresponds to a 3.8 liter carbureted engine with a peak torque of 262 Nt·m at 2000 rpm.

At the start of each time step, a check is performed to determine the mode of operation for the transmission as described in Section 2.2.4. The corresponding system equations from Appendix III are then evaluated using a fourth-order Runga-Kutta integration method. The engine model is warmed up for the first three seconds of simulation time at a 12° throttle angle with engine speed starting at 800 rpm. After three seconds, the throttle angle is set to 70° and the shift points in terms of vehicle velocity are taken as 32.3, 52.8, and 70.5 mph, for the 1-2, 2-3, and 3-4 shifts, respectively. The pressure profile for the on-coming clutch at each shift point is shown below in Figure 3.1.

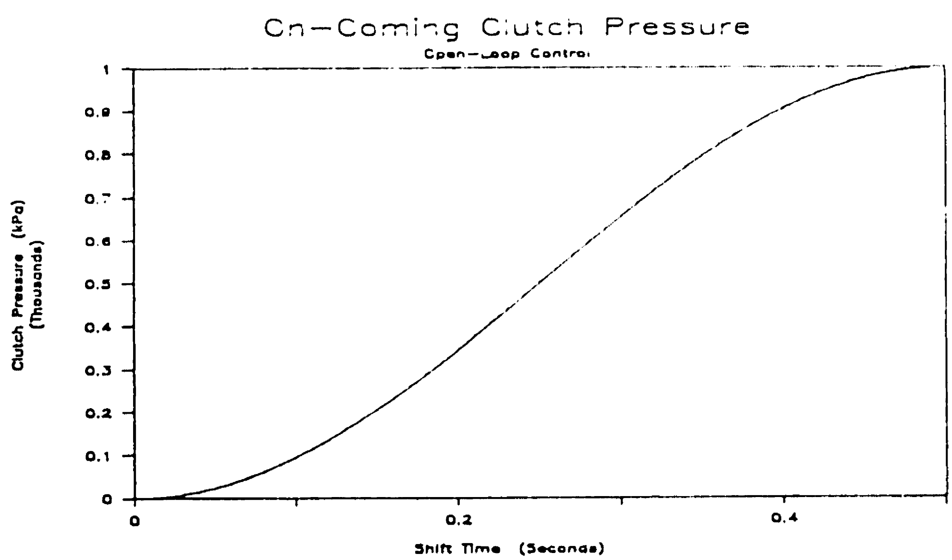


Figure 3.1

When the reaction torque of the off-going clutch goes to zero, then its clutch pressure is set to zero. Note that the off-going clutch pressure profile only effects the 2-3 shift because the input and third clutch sprags will overrun during the 1-2 and 3-4 shifts.

The results of the simulation for a vehicle running from first to fourth gear are shown in Figures 3.2-6. Each shift point is characterized by a drop in both engine and turbine speeds as shown in Figure 3.2; an increase in engine, pump, and turbine torques as shown in Figures 3.3-4; and a sharp peak in both the output shaft torque and vehicle acceleration as shown in Figures 3.5-6. Note that a high slip across the torque converter results in a large torque multiplication ratio between the turbine and pump torques.

During each shift, the clutch energy of the on-coming clutch is calculated by

$$E = \int_{t_o}^{t_f} T_C \omega_C dt, \tag{3.1}$$

where t_o denotes the start of the shift and t_f denotes the end, T_C is the clutch torque and ω_C is the clutch slip speed. The clutch energy corresponding to each shift for the simulation are given below in Table 3.1. Note that the clutch energy arises from the heat generated between the sliding friction surfaces; therefore, high clutch energies will generally result in a low clutch life.

Shift	On-Coming Clutch	Clutch Energy (kJ)
$1 \rightarrow 2$	C_2	15.9
$2 \rightarrow 3$	C_3	7.61
$3 \rightarrow 4$	C_4	5.04

Table 3.1

Engine, Turbine and Vehicle Speeds

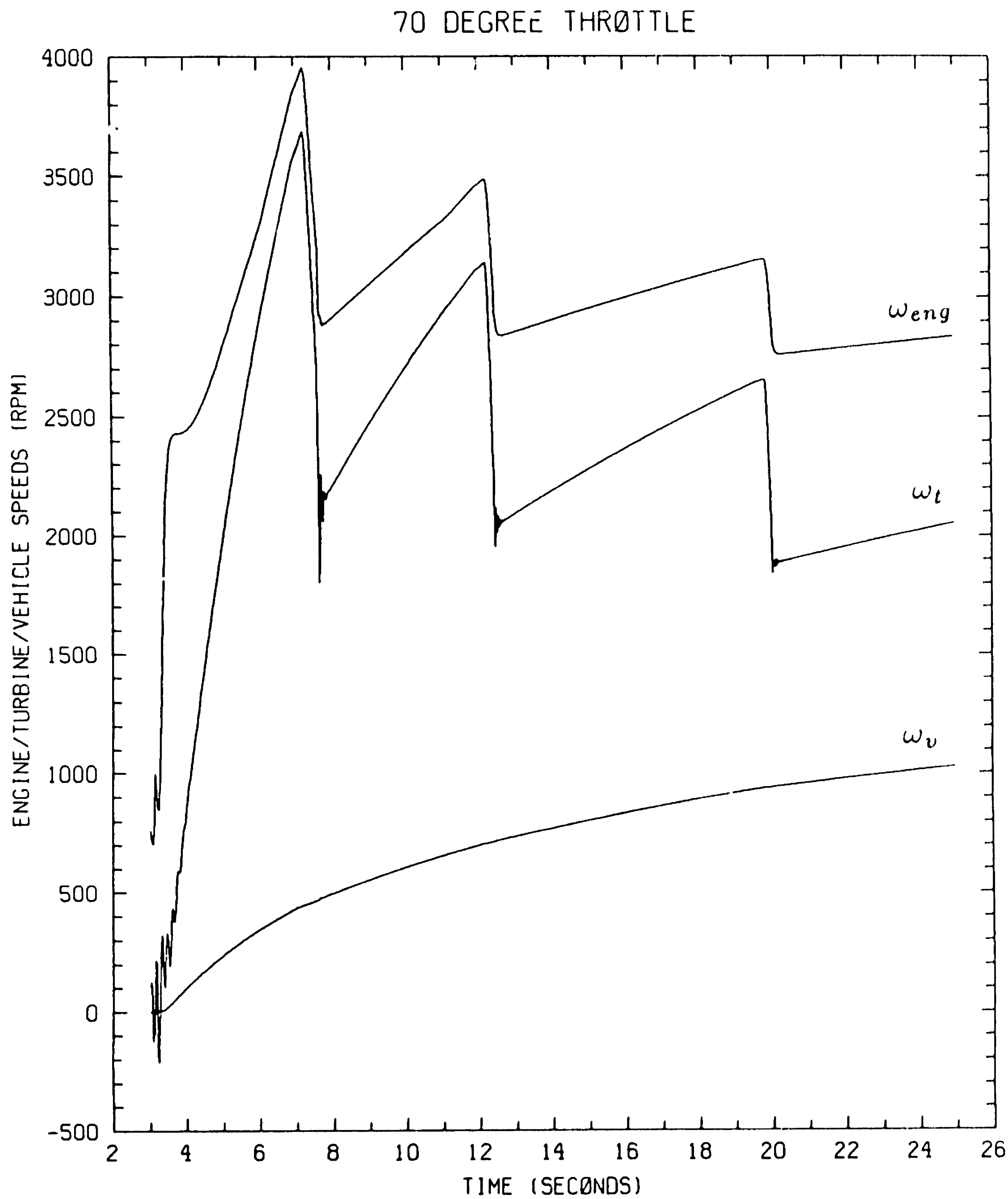


Figure 3.2

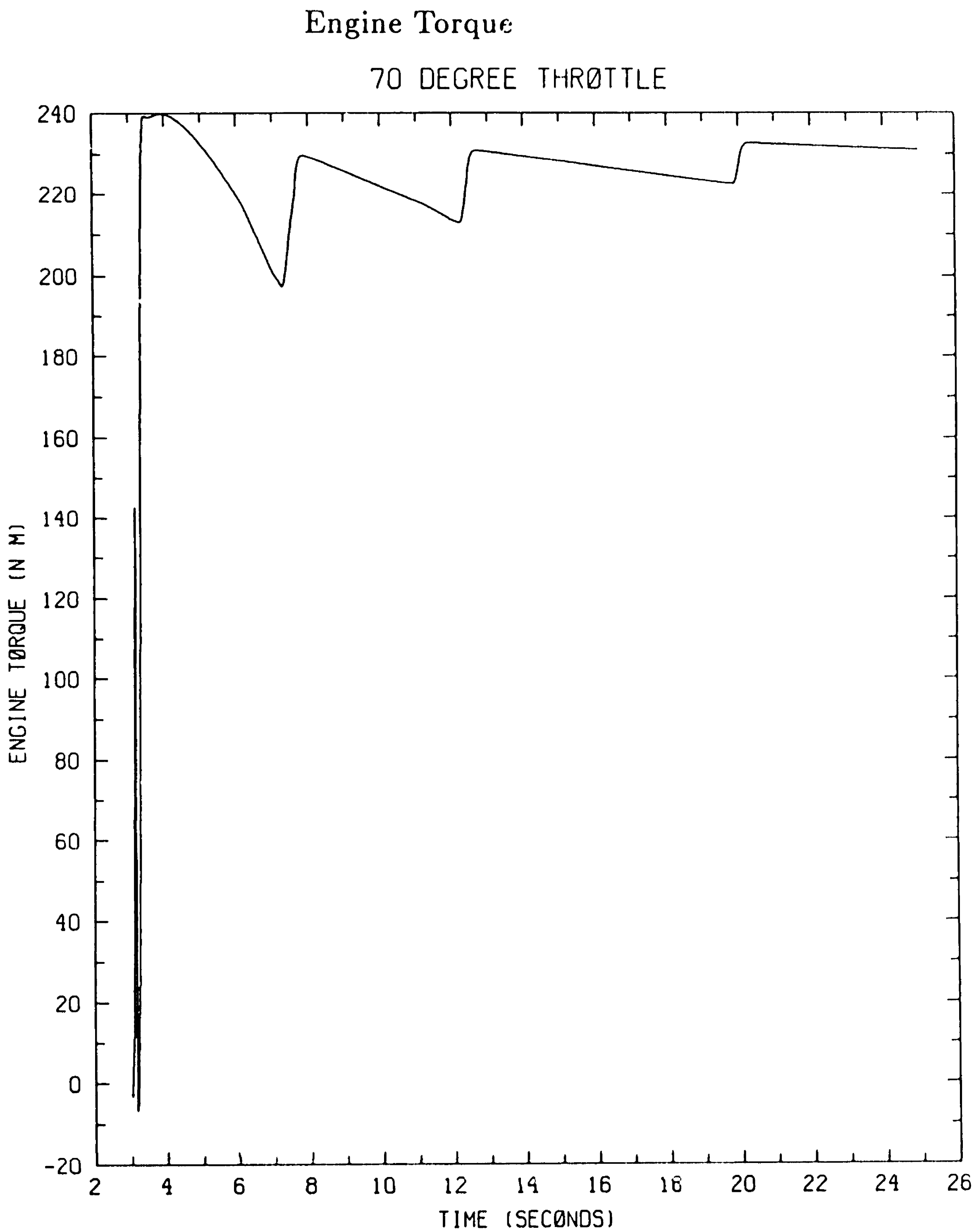

Engine Torque
70 DEGREE THRØTTLE
ENGINE TØRQUE (N M)
240
220
200
180
160
140
120
100
80
60
40
20
0
-20
2
4
6
8
10
12
14
16
18
20
22
24
26
TIME (SECØNDS)

Figure 3.3

Converter Torques

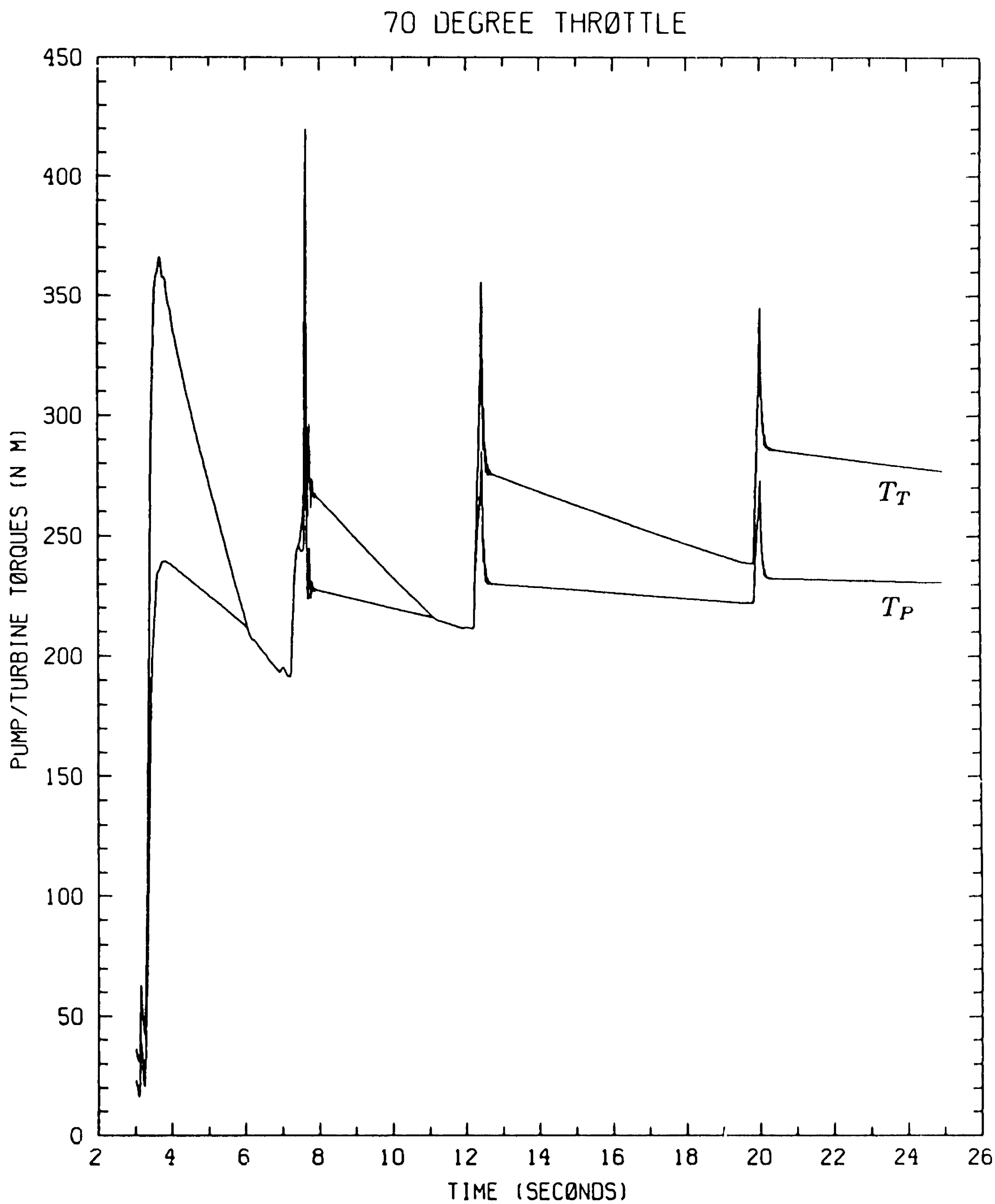


Figure 3.4

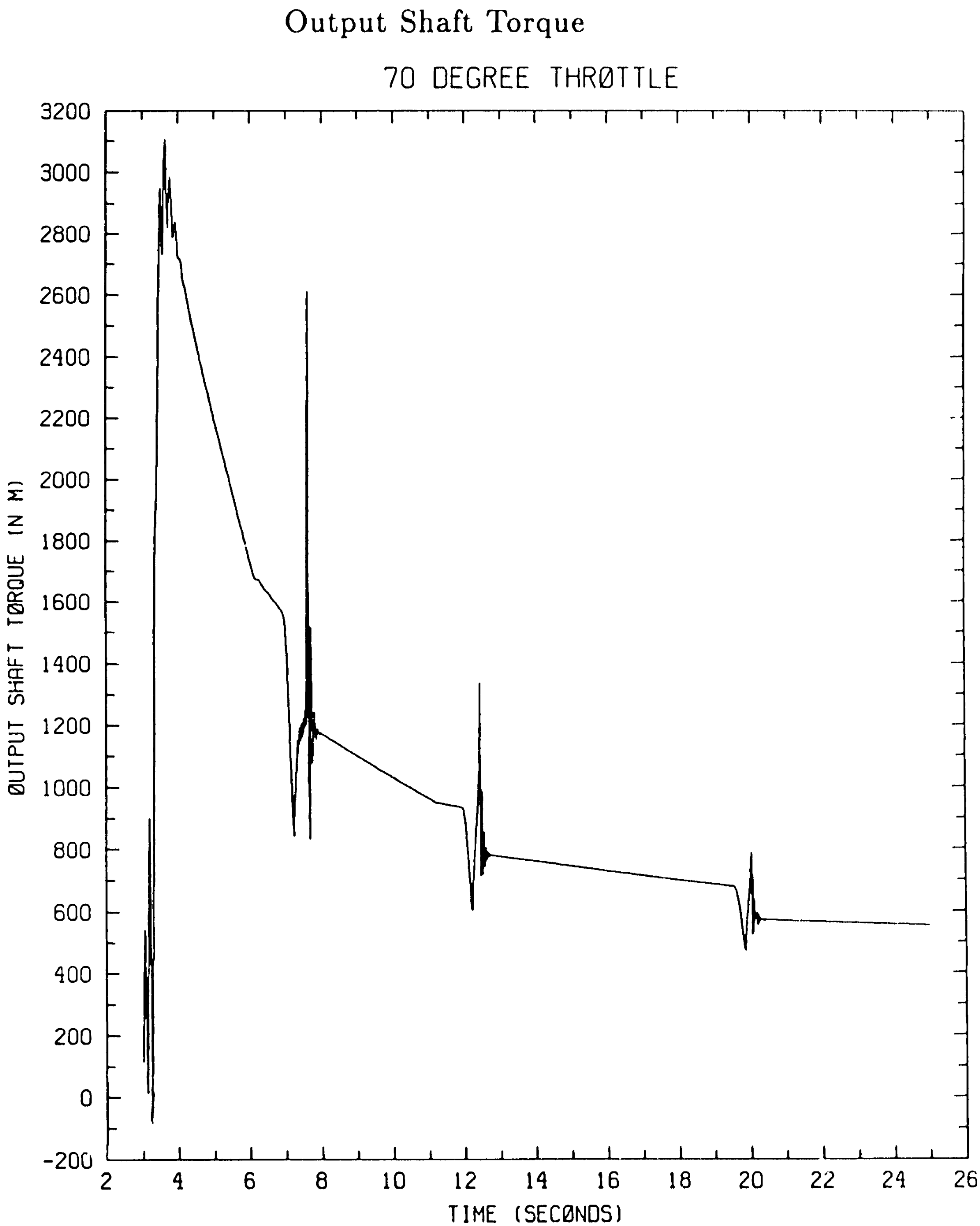

Output Shaft Torque
70 DEGREE THRØTTLE
ØUTPUT SHAFT TØRQUE (N M)
3200
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
-200
2
4
6
8
10
12
14
16
18
20
22
24
26
TIME (SECØNDS)

Figure 3.5

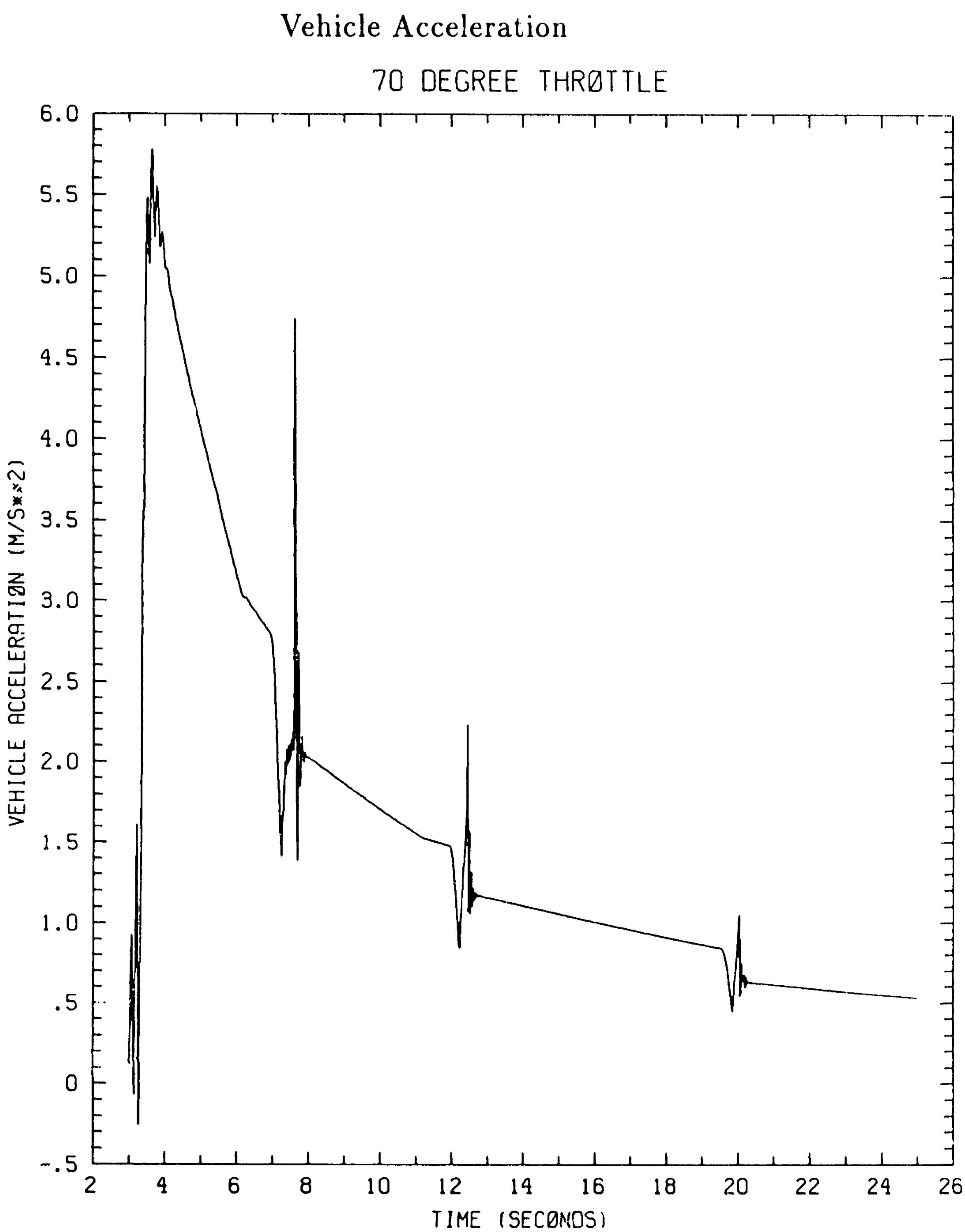

Vehicle Acceleration
70 DEGREE THRØTTLE
VEHICLE ACCELERATIØN (M/S**2)
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
.5
0
-.5
2
4
6
8
10
12
14
16
18
20
22
24
26
TIME (SECØNDS)

Figure 3.6

3.1 Analysis of the Open–Loop Simulation

The characteristics of a powered-on upshift are further analized by studing the first to second gear shift from the previous simulation.

An upshift consists of two basic parts, the torque phase and the inertia phase. The shift begins with the torque phase, which is characterized by a drop in the output shaft torque, and ends with the inertia phase, which is characterized by turbine speed changing to its new synchronous value.

During the torque phase of the 1-2 shift, it is seen from Figure 3.1.3 that the effect of increasing the clutch torque of the on-coming clutch, C_2, is to reduce the reaction torque of the off-going clutch, C_i. When the reaction torque of the input clutch goes to zero, the input sprag begins to overrun and the inertia phase of the shift begins. The increasing clutch torque of the on-coming clutch then causes the turbine inertia to decelerate to its new synchronous speed as noted in Figure 3.1.1. Note that although the on-coming clutch torque is increasing during the torque phase, the turbine speed continues to increase; whereas during the inertia phase, turbine speed drops. This is because an increase in the on-coming clutch torque during the torque phase results in a decrease in the reaction clutch torque and thus the net torque acting on the turbine inertia remains unchanged. However, during the inertia phase, an increase in the on-coming clutch torque results in a decrease in the net torque acting on the turbine inertia and the turbine speed decelerates. The decrease in turbine speed results in a higher slip across the converter which yields a higher pump torque as noted in Figure 3.1.2. This then causes the engine inertia to also decelerate.

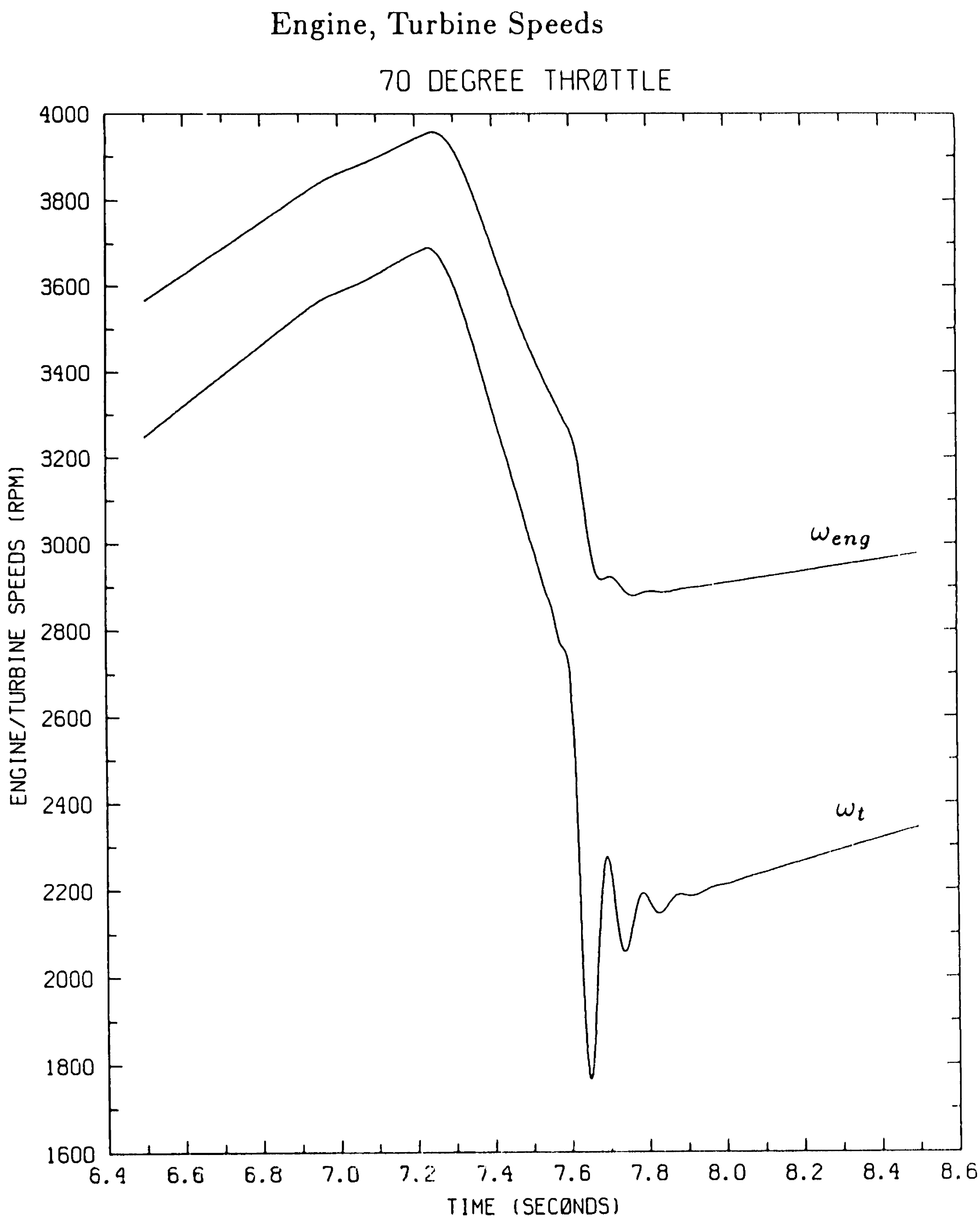

Engine, Turbine Speeds
70 DEGREE THRØTTLE
4000
3800
3600
3400
3200
3000
2800
2600
2400
2200
2000
1800
1600
6.4
6.6
6.8
7.0
7.2
7.4
7.6
7.8
8.0
8.2
8.4
8.6
TIME (SECØNDS)
ENGINE/TURBINE SPEEDS (RPM)
ω_{eng}
ω_t

Figure 3.1.1

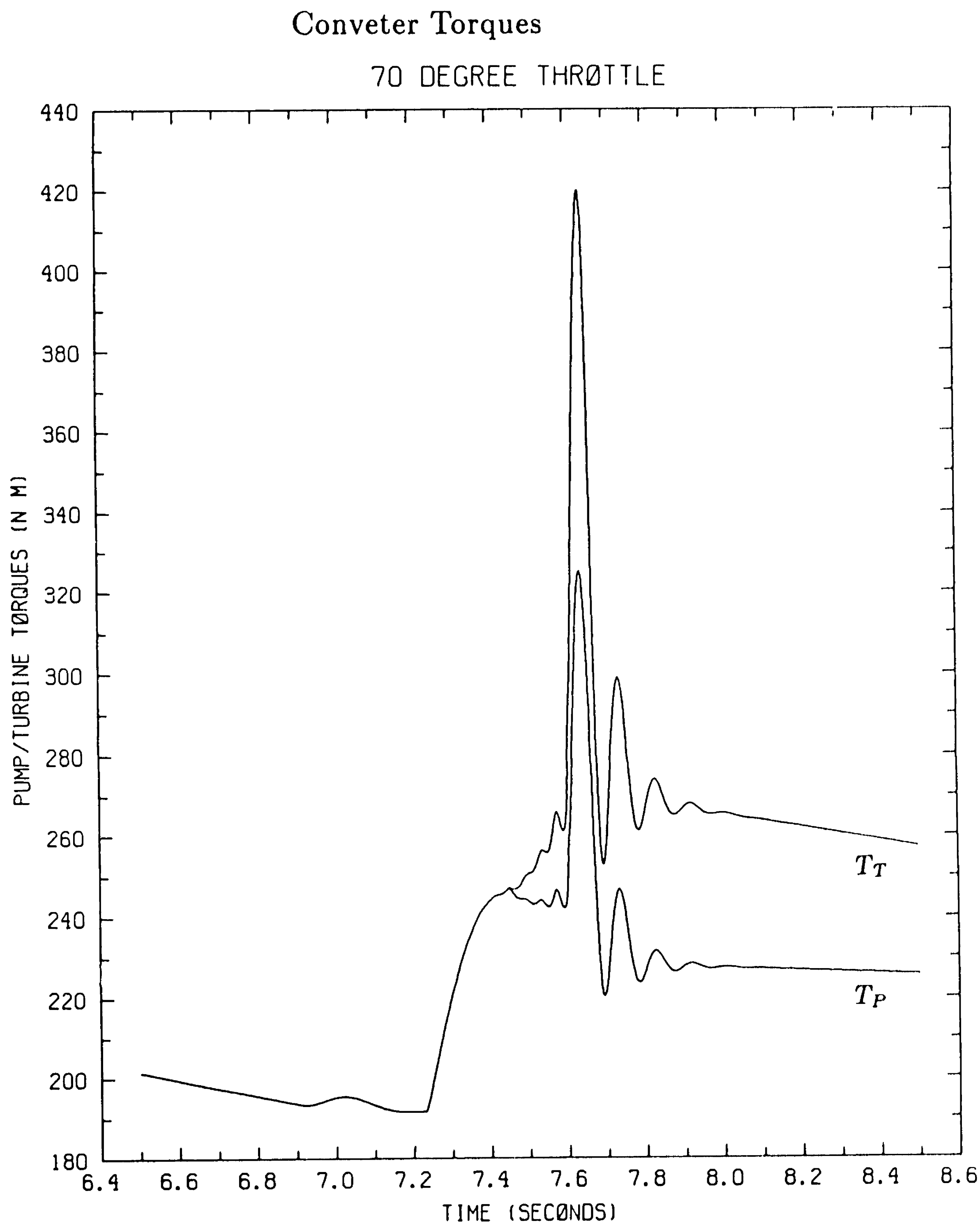

Conveter Torques
70 DEGREE THRØTTLE
PUMP/TURBINE TØRQUES (N M)
440
420
400
380
360
340
320
300
280
260
240
220
200
180
6.4
6.6
6.8
7.0
7.2
7.4
7.6
7.8
8.0
8.2
8.4
8.6
TIME (SECØNDS)
T_T
T_P

Figure 3.1.2

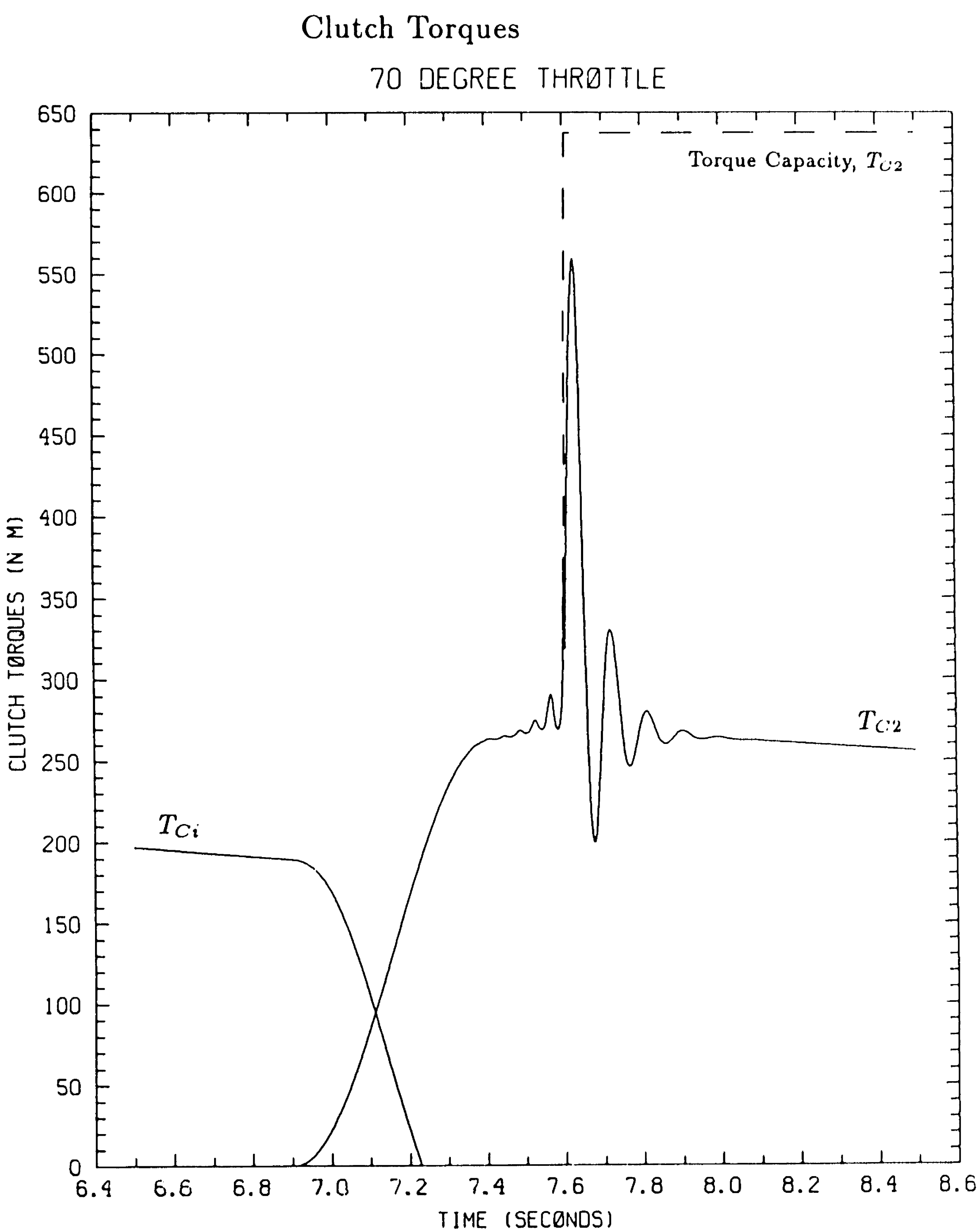

Clutch Torques
70 DEGREE THRØTTLE
Torque Capacity, T_{C2}
T_{C2}
T_{Ci}
CLUTCH TØRQUES (N M)
TIME (SECØNDS)
0
50
100
150
200
250
300
350
400
450
500
550
600
650
6.4
6.6
6.8
7.0
7.2
7.4
7.6
7.8
8.0
8.2
8.4
8.6

Figure 3.1.3

Note that during the torque phase of the 1-2 shift, the transmission is still in first gear and the on-coming clutch acts as a brake. This is the reason for the decrease in the output shaft torque as seen in Figure 3.1.4. After the torque phase, the output shaft torque begins to oscillate. For the inertia phase, the resonant frequency of the output shaft is seen to be

$$\begin{aligned} f &= \frac{9 \text{ cycles}}{(7.575 - 7.225) \text{ sec}} = 25.71 \text{ Hz} \\ &= 161.6 \text{ rad/s} \end{aligned}$$

and in second gear, the resonant frequency is

$$\begin{aligned} f &= \frac{4 \text{ cycles}}{(7.975 - 7.625) \text{ sec}} = 11.43 \text{ Hz} \\ &= 71.81 \text{ rad/s} \end{aligned}$$

These resonant frequencies can be found analytically by calculating the natural frequency of the output shaft connected between the transmission inertia and ground. Note the shaft can be considered as connected to ground because the vehicle inertia is so large that its speed changes are negligible during the shift. For this type of mass-spring system, the resonant frequency, ω_R, is found to be

$$\omega_R = \sqrt{\frac{k}{I_{EQ} G^2}},$$

where k is the shaft spring rate, I_{EQ} is the equivalent transmission inertia found from the system equations in Appendex III, and G is the corresponding gear ratio between the inertia and the shaft. For the inertia phase, the equivalent inertia is found from equations III.5.11 to be

$$\begin{aligned} I_{EQ} &= I_{CR} + I_{SI}(c_{22})^2 \\ &= 0.0367 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

and the corresponding gear ratio is that of the final drive gear equal to 2.84. For a spring rate fo $k = 7625$ Nt·m/rad, the resonant frequency during the inertia phase is calculated to be

$$\begin{aligned} \omega_R &= \sqrt{\frac{7625}{(0.0367)(2.84)^2}} \\ &= 160.5 \text{ rad/s} \end{aligned}$$

which agrees with the simulation results found earlier.

During second gear, the equivalent inertia is found from equation III.7.11 to be

$$I_{EQ} = 0.0712 \text{ kg}\cdot\text{m}^2$$

and the corresponding gear ratio is that of second gear equal to 4.46. Thus, the resonant frequency during second gear is calculated to be

$$\omega_R = \sqrt{\frac{7625}{(0.0712)(4.46)^2}}$$
$$= 73.37 \text{ rad/s}$$

which again agrees with the simulation results found earlier.

The torsionals in the output shaft torque during second gear damp out much faster than in the inertia phase of the 1-2 shift. This is because the shaft is connected directly to the torque converter in second gear; whereas during the inertia phase, the slipping clutch decouples the output shaft from the torque converter. Thus, it is seen that the torque converter provides good damping characteristics to the drivetrain.

At the end of the shift, the output shaft torque has a large transient as shown in Figure 3.1.4 which results in a similar transient in the vehicle acceleration as shown in Figure 3.1.5. To a passenger of the automobile, this rapid change in the vehicle acceleration is felt as a jerk. The cause of this rapid torque change can be determined through Figures 3.1.6-8. Figure 3.1.6 is a plot of ω_T and $\omega_{RR} = 1.57\omega_{CR}$. The difference between these two curves yields the slip across the second clutch, C_2, as defined by equation 2.2.3.7. Thus, when the two curves meet, the shift is over and the transmission is in second gear. The time derivative of the shaft torque, $\dot{T}_S$, is plotted in Figure 3.1.7 and shaft torque is plotted in Figure 3.1.8. From Section 2, the system equation for shaft torque is given by

$$\dot{T}_S = 7625(R_D\omega_{CR} - \omega_V), \tag{3.1.1}$$

As seen by Figure 3.1.6, ω_{CR} increases as the shift is ending. This increase in ω_{CR} is due to an increase in T_{C2}, which is a result of the coefficient of friction increasing as the clutch slip speed goes to zero. From equation 3.1.1, the increase in ω_{CR} at the end of the shift also yields the increase in $\dot{T}_S$ as seen in Figure 3.1.7. It is this high positive value of $\dot{T}_S$ at the end of the shift which therefore results in the large peak in shaft torque. Note also that the value of the shaft torque when the shift is completed is less than 1500 Nt·m and that the peak value in shaft torque of 2700 Nt·m occurs when the transmission is in second gear.

If the clutch capacity of the on-coming clutch were limited such that ω_{CR} did not increase as shown in Figure 3.1.6, then the output shaft torque transient of Figure 3.1.4 would not be as large. However, limiting the clutch capacity as described above will also increase the shift time and thus the energy absorbed by the on-coming clutch during the shift. Conversely, a fast shift results in low clutch energies; however, this also requires the vehicle to absorb the change in energy needed to decelerate the turbine inertia to its new synchronous speed. This is what is seen in Figures 3.1.6-8; i.e., the high clutch torque quickly increases the speed of the small inertia on ω_{CR} and the shift is completed. However, the sudden increase in ω_{CR} also causes an increase in the shaft torque and the vehicle must absorb the change in energy required to decelerate the turbine inertia to its new synchronous value. This change in turbine speed can be specifically noted in Figure 3.1.1 where turbine speed is approximately 2800 rpm when the shift is completed and then immediately drops to 2200 rpm.

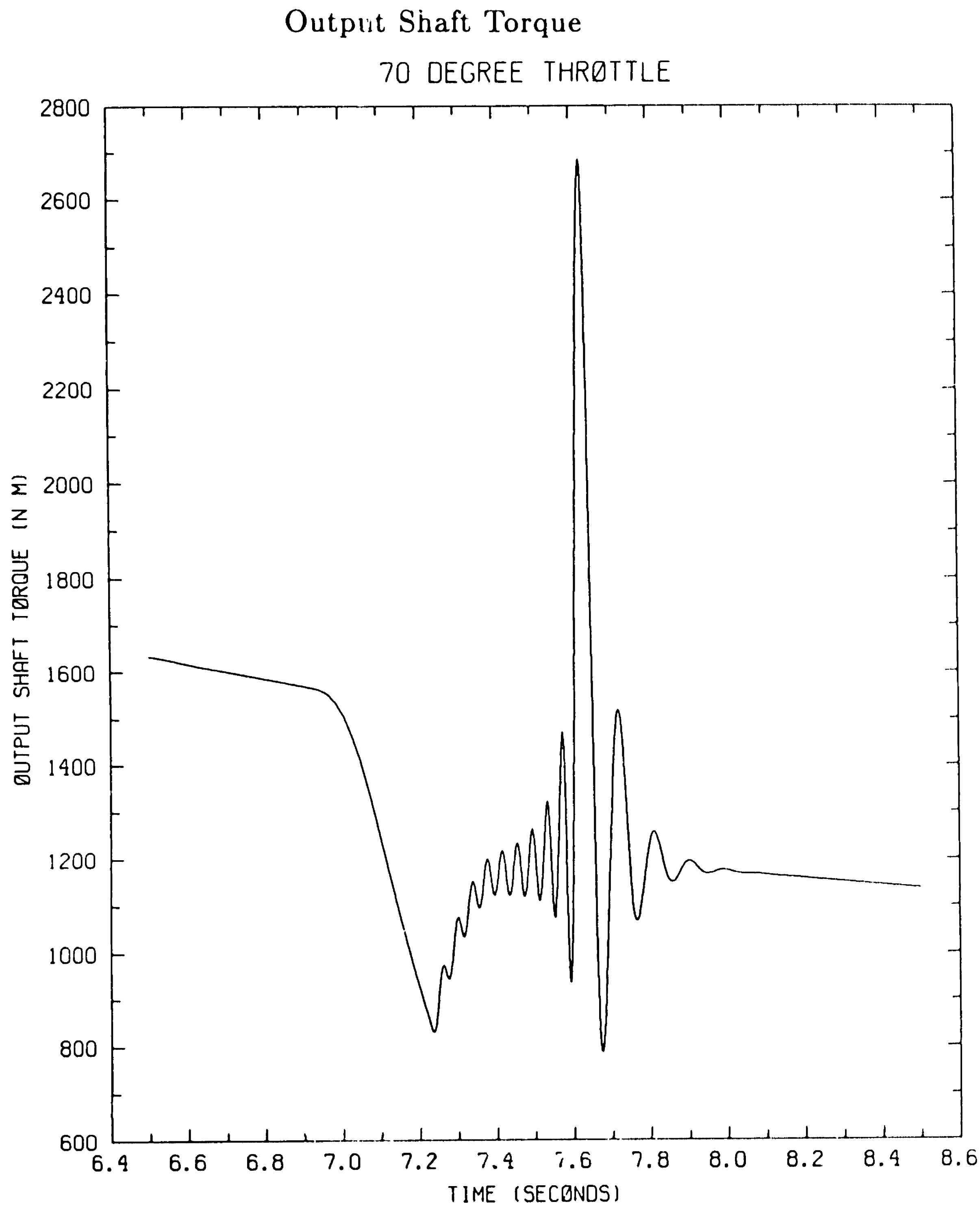

Output Shaft Torque
70 DEGREE THRØTTLE
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
ØUTPUT SHAFT TØRQUE (N M)
6.4
6.6
6.8
7.0
7.2
7.4
7.6
7.8
8.0
8.2
8.4
8.6
TIME (SECØNDS)

Figure 3.1.4

Vehicle Acceleration

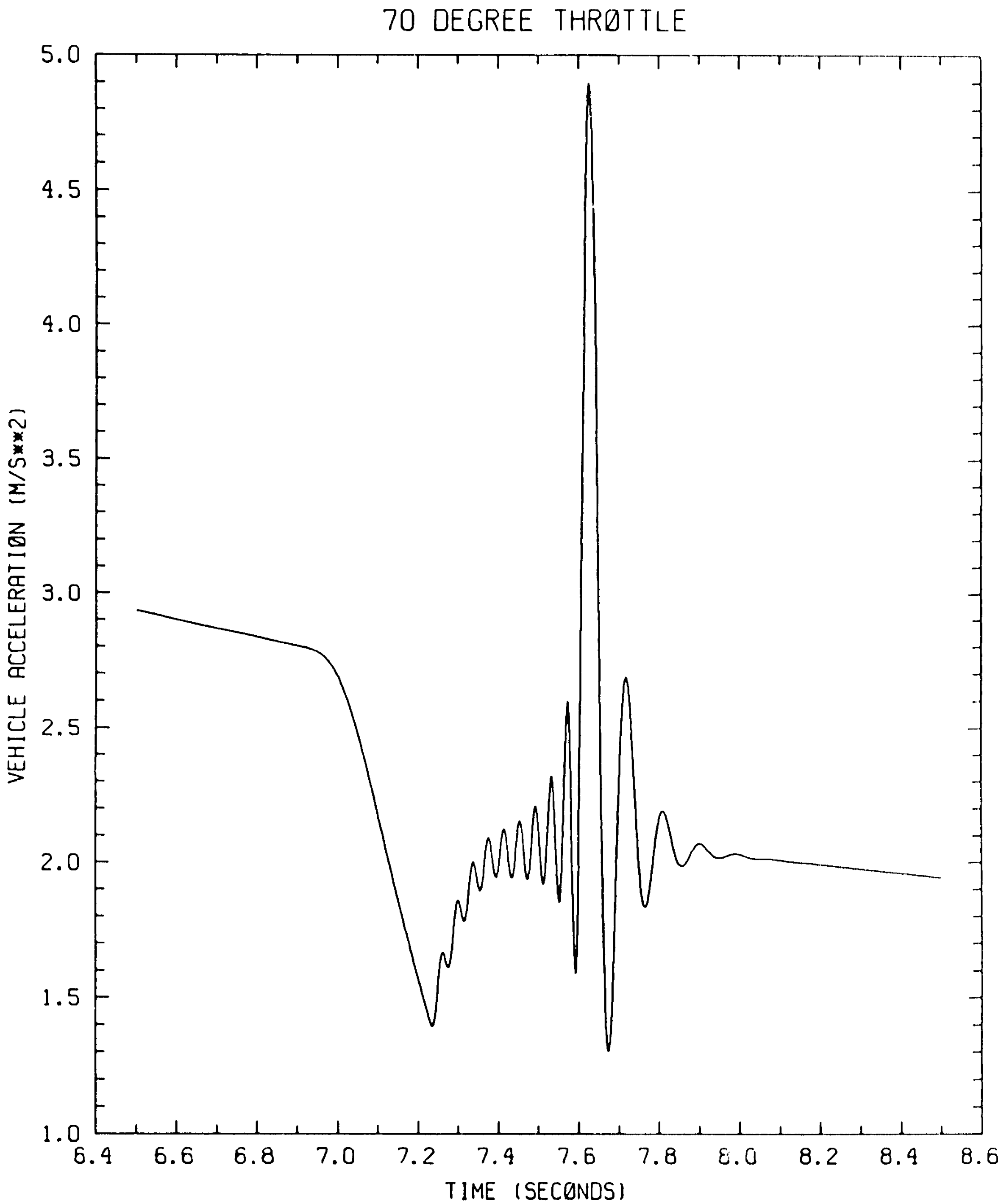


Figure 3.1.5

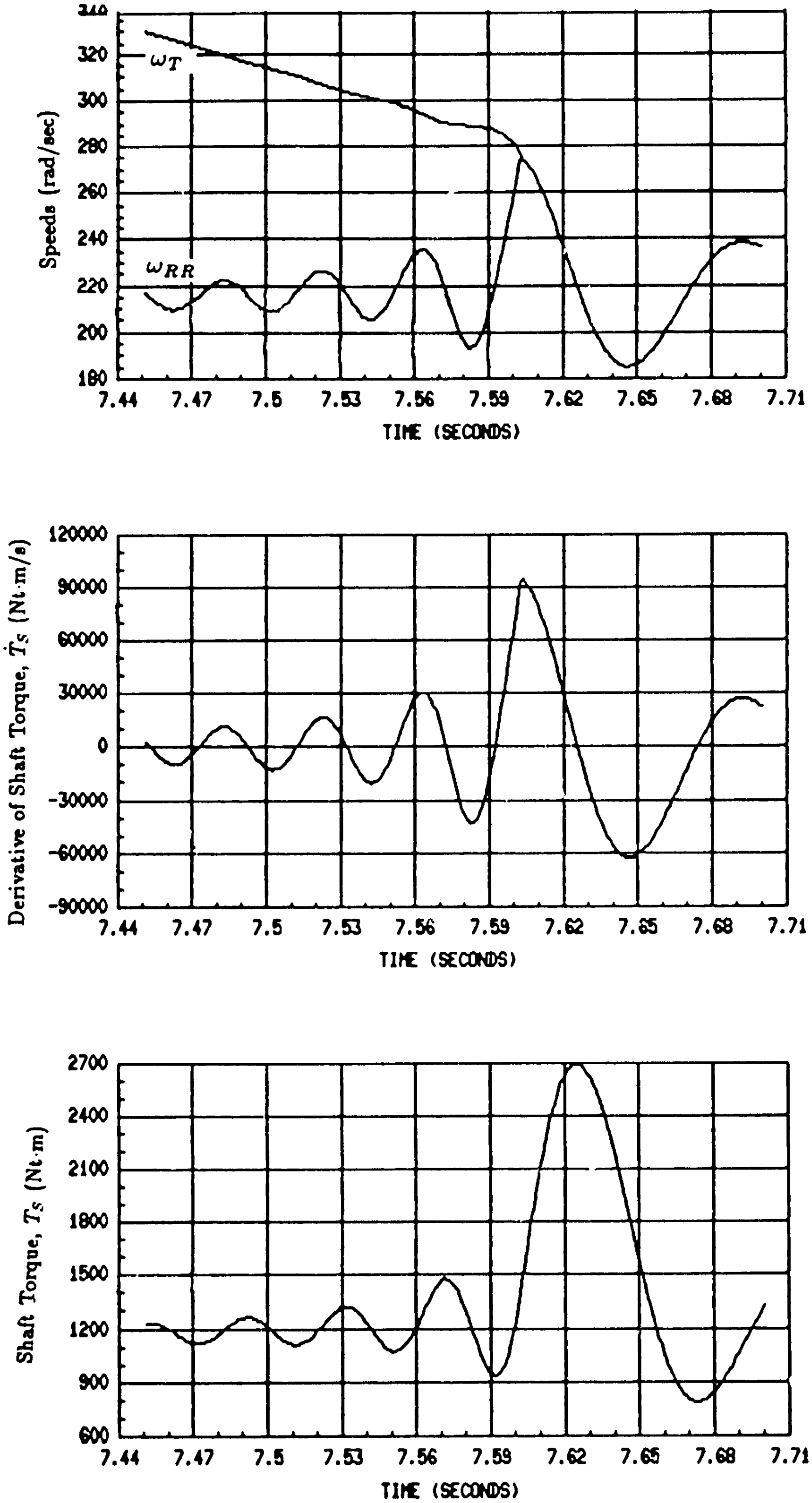

Speeds (rad/sec)
320
300
280
260
240
220
200
180
ωT
ωRR
7.44 7.47 7.5 7.53 7.56 7.59 7.62 7.65 7.68 7.71
TIME (SECONDS)
Derivative of Shaft Torque, ṪS (Nt·m/s)
120000
90000
60000
30000
0
-30000
-60000
-90000
7.44 7.47 7.5 7.53 7.56 7.59 7.62 7.65 7.68 7.71
TIME (SECONDS)
Shaft Torque, TS (Nt·m)
2700
2400
2100
1800
1500
1200
900
600
7.44 7.47 7.5 7.53 7.56 7.59 7.62 7.65 7.68 7.71
TIME (SECONDS)

Figure 3.1.6-8

4

Closed–Loop Control

Typical shift transients for current automatic transmissions have been characterized in Section 3. Because the control for these shifts are performed open-loop, the output shaft torque has undesirable oscillations (reference Figure 3.1.4, Output Shaft Torque) which are transmitted directly to the passengers of the automobile (reference Figure 3.1.5, Vehicle Acceleration).

In this section, a closed-loop controller is designed with the specific objectives of:

1) producing a smooth shift; i.e., the output shaft torque does not oscillate; and

2) the shift time is to be of the same duration as the open-loop case.

Furthermore, the closed-loop controller is designed with the intent of obtaining a "perfect shift"; meaning that the output shaft torque is to follow exactly some desired torque trajectory and that the shift time is to be of some set duration. Although the control inputs to achieve a perfect shift may not be practically realizable, this design philosophy is used to determine what the control inputs must be to achieve the most optimal shift. Afterwards, sub-optimal shifts may be evaluated to yield a set of realizable inputs that may be applied to an actual vehicle.

4.1 Control Design

To meet the control objectives set in the previous section, the controller is designed using Sliding Mode Control,† which is a control technique for nonlinear systems and is applicable to tracking a changing reference signal. The principles of Sliding Mode Control Theory are discussed further in Appendix IV.

The control design is broken up into two sections; Section 4.1.1 considers the torque phase and section 4.1.2 considers the inertia phase. Only the control equations for the first to second gear powered upshift will be developed. The torque converter will be modelled as in lockup mode throughout the shift. This is done by lumping the engine and pump inertias into the turbine inertia. The input torque for the equations of Appendix III is then engine torque instead of the turbine torque from the torque converter. The reasons for locking the torque converter are discussed later in Section 4.3 when they will become more apparent.

4.1.1 Torque Phase

As seen in Section 3.1, the torque phase for a current automatic transmission is characterized by a droop in the output shaft torque. This was seen to happen due to the braking effect of the on-coming clutch. To yield a smooth output torque during the torque phase, it is therefore apparent that engine torque must increase as the on-coming clutch torque increases. Thus, during the torque phase, the on-coming clutch will be applied at its maximum rate and the controller will modulate engine torque to produce the desired shaft torque. By applying the on-coming clutch at its maximum rate, the torque phase will be of the same time duration as the open-loop case and will end as fast as possible.

For the output shaft torque, T_S, to follow some desired shaft torque, T_{SD}, define the sliding surface, S_1, as

† J.J.E. Slotine, *International Journal of Control*, 1984, Vol. 40, No. 2,pp. 421-434

$$S_1 = \dot{\epsilon}_1 + \lambda_1 \epsilon_1 \tag{4.1.1.1}$$

where the error, ϵ_1, is defined by

$$\epsilon_1 = T_S - T_{SD} \tag{4.1.1.2}$$

$$\dot{\epsilon}_1 = \dot{T}_S - \dot{T}_{SD}. \tag{4.1.1.3}$$

If a controller can be found which causes $S_1 \rightarrow 0$, then this implies

$$\dot{\epsilon}_1 = -\lambda_1 \epsilon_1. \tag{4.1.1.4}$$

Equation 4.1.1.4 will then define the dynamics of the error, ϵ_1; i.e., if a disturbance causes $T_S \neq T_{SD}$, then the response of $\epsilon_1 \rightarrow 0$ is determined by equation 4.1.1.4. Note that for large λ_1, ϵ_1 will go to zero quickly.

To ensure $S_1 \rightarrow 0$, define a Lyapunov function, V_1, as

$$V_1 = \frac{1}{2} S_1^2. \tag{4.1.1.5}$$

Then S_1 will be guaranteed to go to zero if the relation

$$\dot{V}_1 = S_1 \dot{S}_1 < 0 \tag{4.1.1.6}$$

is satisfied. Thus, choose $\dot{S}_1$ to be defined by

$$\dot{S}_1 = -\eta_1 Sat(S_1) \tag{4.1.1.7}$$

where

$$Sat(S_1) = \begin{cases} S_1/\phi_1, & S_1 < \phi_1; \\ Sgn(S_1), & S_1 \geq \phi_1; \end{cases} \tag{4.1.1.8}$$

ϕ_1 is the boundary layer as presented by Slotine, and η_1 is chosen as described in Appendix IV based on modelling errors.

Differentiating equation 4.1.1.1 and substituting into equation 4.1.1.7 yields

$$\ddot{\epsilon}_1 + \lambda_1 \dot{\epsilon}_1 = -\eta_1 Sat(S_1) \qquad (4.1.1.9)$$

or

$$\ddot{T}_S = \ddot{T}_{SD} - \lambda_1 \dot{\epsilon}_1 - \eta_1 Sat(S_1) \qquad (4.1.1.10)$$

The system equation for the shaft torque was found in Section 2 to be

$$\dot{T}_S = 7625(R_D \omega_{CR} - \omega_V) \qquad (4.1.1.11)$$

Differentiating equation 4.1.1.11 and substituting into equation 4.1.1.10 yields

$$7625(R_D \dot{\omega}_{CR} - \dot{\omega}_V) = \ddot{T}_{SD} - \lambda_1 \dot{\epsilon}_1 - \eta_1 Sat(S_1) \qquad (4.1.1.12)$$

From Appendix III.7, $\dot{\omega}_{CR}$ is found for first gear with the torque converter locked up to be

$$\begin{aligned} \dot{\omega}_{CR} &= \frac{1}{2.92} \dot{\omega}_T \\ &= \frac{1}{2.92}(4.8558 T_{eng} - 2.2498 T_{C2} - 1.6624 R_D T_S) \end{aligned} \qquad (4.1.1.13)$$

Substituting equation 4.1.1.13 into equation 4.1.1.12 and solving for T_{eng} yields

$$\begin{aligned} T_{eng} = \frac{1}{4.8558} \Bigg\{ & 2.2498 T_{C2} + 1.6624 R_D T_S \\ & + \frac{2.92}{R_D} \left[\dot{\omega}_V + \frac{1}{7625} \left(\ddot{T}_{SD} - \lambda_1 \dot{\epsilon}_1 - \eta_1 Sat(S_1) \right) \right] \Bigg\} \end{aligned} \qquad (4.1.1.14)$$

Thus, equation 4.1.1.14 determines the control law for engine torque such that T_S follows T_{SD} during the torque phase of the 1-2 shift. Note that T_{C2} is to be applied at its maximum rate as discussed in Section 4.1.1.

For T_S to exactly follow T_{SD}, then T_{SD} must be chosen to satisfy the system equations. The system equation for shaft torque is defined by equation 4.1.1.11. Note that since neither ω_{CR} nor ω_V can be made to change instantaneously, the desired shaft torque must be chosen such that T_{SD} and $\dot{T}_{SD}$ are both continuous to satisfy equation 4.1.1.11. This implies that at the start of the torque phase, the desired shaft torque must be chosen with the boundary constraints

$$T_{SD}\big|_{t=t_0} = T_S\big|_{t=t_0^-} \tag{4.1.1.15}$$

and

$$\dot{T}_{SD}\big|_{t=t_0} = \dot{T}_S\big|_{t=t_0^-} \tag{4.1.1.16}$$

Differentiating equation 4.1.1.11 with respect to time yields

$$\ddot{T}_S = 7625(R_D\dot{\omega}_{CR} - \dot{\omega}_V) \tag{4.1.1.17}$$

Since $\dot{\omega}_{CR}$ is a function of the control input, T_{eng}, an instantaneous change in $\ddot{T}_{SD}$ would therefore require an instantaneous change in engine torque. If engine torque is limited to be continuous, then the desired shaft torque must also be chosen such that $\ddot{T}_{SD}$ is continuous. At the start of the inertia phase, the desired shaft torque must therefore be defined by

$$\ddot{T}_{SD}\big|_{t=t_0} = \ddot{T}_S\big|_{t=t_0^-} \tag{4.1.1.18}$$

to yield a continuous control law for T_{eng}.

To understand why this control law yields the desired shaft torque, substitute equation 4.1.1.14 into equation 4.1.1.13. This yields

$$\dot{\omega}_{CR} = \frac{1}{R_D}\left[\dot{\omega}_V + \frac{1}{7625}\left(\ddot{T}_{SD} - \lambda_1\dot{\epsilon}_1 - \eta_1 Sat(S_1)\right)\right] \tag{4.1.1.19}$$

Substituting equation 4.1.1.19 into equation 4.1.1.17 then yields

$$\ddot{T}_S = \ddot{T}_{SD} - \lambda_1 \dot{\epsilon}_1 - \eta_1 Sat(S_1) \qquad (4.1.1.20)$$

Thus, given the initial conditions defined by equations 4.1.1.15 &16, T_S will follow T_{SD}. Note that in the presence of initial condition errors, modelling errors, and unknown disturbances, $\lambda_1 \dot{\epsilon}_1$ and $\eta_1 Sat(S_1)$ are stability terms which cause $T_S \rightarrow T_{SD}$.

4.1.2 Inertia Phase

For the inertia phase, the control objectives are to produce a smooth output shaft torque as before and to decrease the turbine speed to its new synchronous speed. For current automatic transmissions, the clutch torque of the on-coming clutch is set high to decelerate the turbine inertia. It was seen in Section 3.1 that this high clutch torque also gives rise to the large transients in the output shaft torque. However, the turbine inertia can also be decelerated by controlling the engine torque which then allows the output shaft torque to be controlled by modulating the on-coming clutch pressure.

For the inertia phase of the 1-2 shift, the system equations with the torque converter locked are found from Appendix III.5 to be

$$\dot{\omega}_T = 4.9593(T_{eng} - T_{C2}) \qquad (4.1.2.1)$$

$$\dot{\omega}_{CR} = 42.7438 T_{C2} - 27.2663 R_D T_S \qquad (4.1.2.2)$$

Using the results of Section 4.1.1, the control law for tracking the desired output shaft torque is found by substituting equation 4.1.2.2 into equation 4.1.1.12 and solving for the second clutch torque, T_{C2}. This is done below in equation 4.1.2.3.

$$T_{C2} = \frac{1}{42.7438} \Bigg\{ 27.2663 R_D T_S + \frac{1}{R_D} \left[\dot{\omega}_V + \frac{1}{7625} \left(\ddot{T}_{SD} - \lambda_1 \dot{\epsilon}_1 - \eta_1 Sat(S_1) \right) \right] \Bigg\} \qquad (4.1.2.3)$$

Note that substitution of equation 4.1.2.3 into equation 4.1.2.2 again yields equation 4.1.1.19; thus, T_S will follow T_{SD} given the constraint equations 4.1.1.15 &16 and T_{C2} will be continuous at $t = t_0$ given the constraint equation 4.1.1.18, where t_0 now defines the start of the inertia phase.

The control law for engine torque will now be developed to decelerate the turbine inertia from its first gear synchronous speed of

$$\omega_T = 2.92\omega_{CR} \tag{4.1.2.4}$$

to its second gear synchronous speed of

$$\omega_T = 1.57\omega_{CR}. \tag{4.1.2.5}$$

Note that when the turbine speed reaches its new synchronous value, the shift is completed.

Define the change in turbine speed, $\delta\omega_t$, as

$$\delta\omega_t = \omega_T - 1.57\omega_{CR}. \tag{4.1.2.6}$$

Thus, $\delta\omega_t$ determines the required change in turbine speed for the shift to be completed. For $\delta\omega_t$ to follow some desired trajectory, $\delta\omega_{t_D}$, define the sliding surface, S_2, as

$$S_2 = \epsilon_2 + \lambda_2 \int \epsilon_2 \tag{4.1.2.7}$$

where the error, ϵ_2, is defined by

$$\epsilon_2 = \delta\omega_t - \delta\omega_{t_D} \tag{4.1.2.8}$$

If a controller can be found which causes $\dot{S}_2 \to 0$, then this implies

$$\dot{\epsilon}_2 = -\lambda_2 \epsilon_2 \qquad (4.1.2.9)$$

and thus the error dynamics will be determined by λ_2.

To ensure $S_2 \rightarrow 0$, which then implies $\dot{S}_2 \rightarrow 0$, choose $\dot{S}_2$ as before in equation 4.1.1.7. Differentiating equation 4.1.2.7 and substituting into equation 4.1.1.7 yields

$$\dot{\epsilon}_2 + \lambda_2 \epsilon_2 = -\eta_2 Sat(S_2), \qquad (4.1.2.10)$$

where, from equations 4.1.2.1, 6, &8,

$$\begin{aligned} \dot{\epsilon}_2 &= \delta\dot{\omega}_t - \delta\dot{\omega}_{t_D} \\ &= \dot{\omega}_T - 1.57\dot{\omega}_{CR} - \delta\dot{\omega}_{t_D} \\ &= 4.9593(T_{eng} - T_{C2}) - 1.57\dot{\omega}_{CR} - \delta\dot{\omega}_{t_D}. \end{aligned} \qquad (4.1.2.11)$$

Substituting equation 4.1.2.11 into 4.1.2.10 and solving for T_{eng} yields

$$T_{eng} = T_{C2} + \frac{1}{4.9593}\left(1.57\dot{\omega}_{CR} + \delta\dot{\omega}_{t_D} - \lambda_2\epsilon_2 - \eta_2 Sat(S_2)\right). \qquad (4.1.2.12)$$

Thus, equation 4.1.2.12 determines the control law for engine torque such that $\delta\omega_t$ follows $\delta\omega_{t_D}$ during the inertia phase of the 1-2 shift.

For $\delta\omega_t$ to exactly track $\delta\omega_{t_D}$, then $\delta\omega_{t_D}$ must be chosen to satisfy the system equations. Since $\delta\omega_t$ cannot be made to change instantaneously, then $\delta\omega_{t_D}$ must be chosen to be continuous. Thus, for t_1 defining the start of the inertia phase and t_f defining the end, this implies

$$\delta\omega_{t_D}\big|_{t=t_1} = \delta\omega_t\big|_{t=t_1^-} \qquad (4.1.2.13)$$

and by definition of $\delta\omega_t$,

$$\begin{aligned}\delta\omega_{t_D}\big|_{t=t_f} &= \delta\omega_t\big|_{t=t_f^+} \\ &= 0 \end{aligned} \qquad (4.1.2.14)$$

Note also from equation 4.1.2.12 that an instantaneous change in $\delta\dot{\omega}_{t_D}$ will cause engine torque to change instantaneously. Thus, if engine torque is limited to be continuous, then $\delta\dot{\omega}_{t_D}$ must be chosen to be continuous. To determine the boundary constraints on $\delta\dot{\omega}_{t_D}$, differentiate equation 4.1.2.6. This yields

$$\delta\dot{\omega}_t = \dot{\omega}_T - 1.57\dot{\omega}_{CR}. \qquad (4.1.2.15)$$

Noting that ω_T and ω_{CR} are related by equation 4.1.2.4 at the start of the inertia phase and by equation 4.1.2.5 at the end, this then implies

$$\begin{aligned}\delta\dot{\omega}_t\big|_{t=t_1^-} &= \left(\dot{\omega}_T - 1.57\left(\frac{1}{2.92}\dot{\omega}_T\right)\right)\Bigg|_{t=t_1^-} \\ &= 0.462\dot{\omega}_T\big|_{t=t_1^-} \end{aligned} \qquad (4.1.2.16)$$

and

$$\begin{aligned}\delta\dot{\omega}_t\big|_{t=t_f^+} &= \left(\dot{\omega}_T - 1.57\left(\frac{1}{1.57}\dot{\omega}_T\right)\right)\Bigg|_{t=t_f^+} \\ &= 0 \end{aligned} \qquad (4.1.2.17)$$

Thus, $\delta\omega_{t_D}$ must be chosen such that

$$\delta\dot{\omega}_{t_D}\big|_{t=t_1} = 0.462\dot{\omega}_T\big|_{t=t_1^-} \qquad (4.1.2.18)$$

and

$$\delta\dot{\omega}_{t_D}\Big|_{t=t_f} = 0 \tag{4.1.2.19}$$

to yield a continuous control law. Note that if $\delta\omega_{t_D}$ does not satisfy equation 4.1.2.19, then engine torque must change instantaneously at the start of second gear to yield a smooth output shaft torque.

To understand why the sliding mode controller causes $\delta\omega_t$ to track $\delta\omega_{t_D}$, substitute the control law, equation 4.1.2.12, into equation 4.1.2.1. This yields

$$\dot{\omega}_T = 1.57\dot{\omega}_{CR} + \delta\dot{\omega}_{t_D} - \lambda_2\epsilon_2 - \eta_2 Sat(S_2) \tag{4.1.2.20}$$

or, using equation 4.1.2.6,

$$\delta\dot{\omega}_t = \delta\dot{\omega}_{t_D} - \lambda_2\epsilon_2 - \eta_2 Sat(S_2) \tag{4.1.2.21}$$

Thus, given the initial condition defined by equation 4.1.2.13, $\delta\omega_t$ will follow $\delta\omega_{t_D}$ and the terms $\lambda_2\epsilon_2$ and $\eta_2 Sat(S_2)$ provide robustness to initial condition errors, unmodelled dynamics, and unknown disturbances.

4.2 Closed–Loop Simulation

The first to second gear shift is simulated using the control design of Section 4.1. The initial conditions for the 1-2 shift are listed in Table 4.2.1 and correspond to those from the open-loop case presented in Section 3 for a 70° throttle angle. Note, however, that in the open-loop case, the torque converter is slipping; whereas in the closed-loop case, it has been defined to be locked up. Thus, the initial engine/turbine speed in the closed-loop simulation corresponds to the turbine speed from the open-loop simulation.

t_0	6.900 sec
$\omega_T(t_0)$	369.71 rad/sec
$\omega_{CR}(t_0)$	126.61 rad/sec
$T_S(t_0)$	1569.8 Nt·m
$\dot{T}_S(t_0)$	-121.4 Nt·m/s
$\ddot{T}_S(t_0)$	278.5 Nt·m/s^2
$\omega_V(t_0)$	44.596 rad/sec

Table 4.2.1

During the torque phase of the 1-2 shift, the control objective is to apply the second clutch at some maximum rate and then modulate engine torque such that shaft torque follows some desired trajectory. For the torque phase of this simulation, the second clutch torque is applied at a rate of 1000 Nt·m/s and the desired shaft torque is chosen such that its second derivative is constant. Then to satisfy the constraints of equations 4.1.1.15, 16, &18, the desired shaft torque is defined as

$$\ddot{T}_{SD}(t) = \ddot{T}_S(t_0) = 278.5 \tag{4.2.1}$$

$$\dot{T}_{SD}(t_0) = \dot{T}_S(t_0) = -121.4 \tag{4.2.2}$$

and

$$T_{SD}(t_0) = T_S(t_0) = 1569.8 \tag{4.2.3}$$

where t_0 defines the start of the torque phase.

When the reaction torque of the input clutch goes to zero, the input clutch sprag starts to overrun and the inertia phase of the shift begins. During the inertia phase, the desired change in turbine torque is constrained by equations 4.1.2.13-14 and 4.1.2.18-19. A convenient function to satisfy these constriants is defined as

$$\delta\omega_{t_D}(t) = \begin{cases} \frac{\delta\omega_t(t_1)}{2}\left[1 + cos\theta(t)\right], & \theta(t) < \pi; \\ 0, & \text{otherwise}, \end{cases} \tag{4.2.4}$$

$$\theta(t) = \pi\frac{(t - t_1)}{\Delta} + \varphi$$

where, t_1 defines the start of the inertia phase, the amplitude $\delta\omega_t(t_1)$ is chosen to satisfy equation 4.1.2.13, the phase φ is chosen to satisfy equation 4.1.2.18, and the frequency π/Δ is chosen to determine the shift time, i.e., when $\delta\omega_{t_D}$ goes to zero. Note that after $\delta\omega_{t_D}$ goes to zero, e.g., when

$$\theta(t) =\geq \pi,$$

then $\delta\omega_{t_D}$ is set to zero which thus satisfies equations 4.1.2.14 &17.

The desired shaft torque for the inertia phase is chosen such that $\dot{T}_{SD}$ will go to zero in order to yield a constant output shaft torque at the end of the shift. This is done by choosing

$$\ddot{T}_{SD}(t) = \begin{cases} \ddot{T}_S(t_1)\left(-\frac{(t-t_1)}{T} + 1\right), & (t - t_1) < T \\ 0, & \text{otherwise}, \end{cases} \tag{4.2.5}$$

$$T = \frac{2\dot{T}_S(t_1)}{T_S(t_1)}$$

$$\dot{T}_{SD}(t_1) = \dot{T}_S(t_1) \tag{4.2.6}$$

$$T_{SD}(t_1) = T_S(t_1) \tag{4.2.7}$$

Note that the choice of T defined by equation 4.2.5 will cause $\dot{T}_{SD}$ and T_{SD} to go to zero in T seconds; thus, the desired shaft torque will be constant after T seconds.

After simulating the torque phase, the initial conditions for the inertia phase are given below in Table 4.2.2.

t_1	7.248 sec
$\omega_T(t_1)$	394.72 rad/sec
$\omega_{CR}(t_1)$	135.18 rad/sec
$T_S(t_1)$	1544.4 Nt·m
$\dot{T}_S(t_1)$	-24.5 Nt·m/s
$\ddot{T}_S(t_1)$	278.5 Nt·m/s^2
$\omega_V(t_1)$	47.600 rad/sec

Table 4.2.2

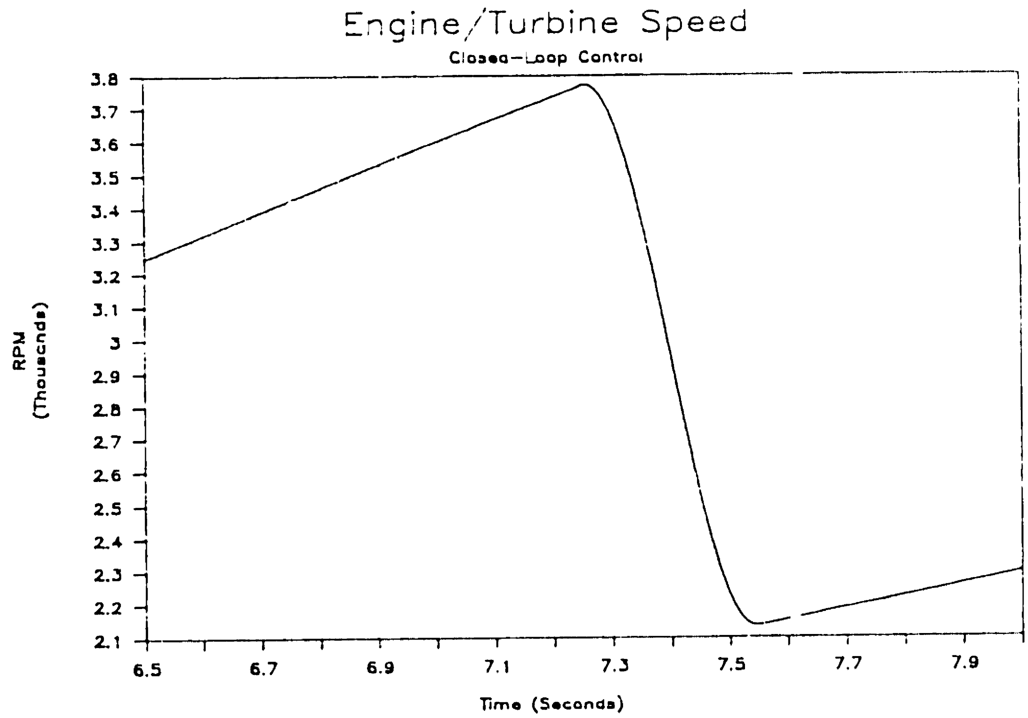
For a shift time of $(t - t_1) = 0.3$ seconds, set

$$\varphi = -0.0341 \text{ rad}$$

$$\Delta = 0.2968 \text{ sec}$$

to satisfy equation 4.1.2.18.

The inertia phase is simulated using the above parameters. The shift is completed at $t = 7.548$ seconds, where engine torque is then held constant through second gear. The results of the 1-2 shift are given in Figures 4.2.1-6.


Engine/Turbine Speed
Closed-Loop Control
RPM (Thousands)
3.8
3.7
3.6
3.5
3.4
3.3
3.2
3.1
3
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
6.5
6.7
6.9
7.1
7.3
7.5
7.7
7.9
Time (Seconds)

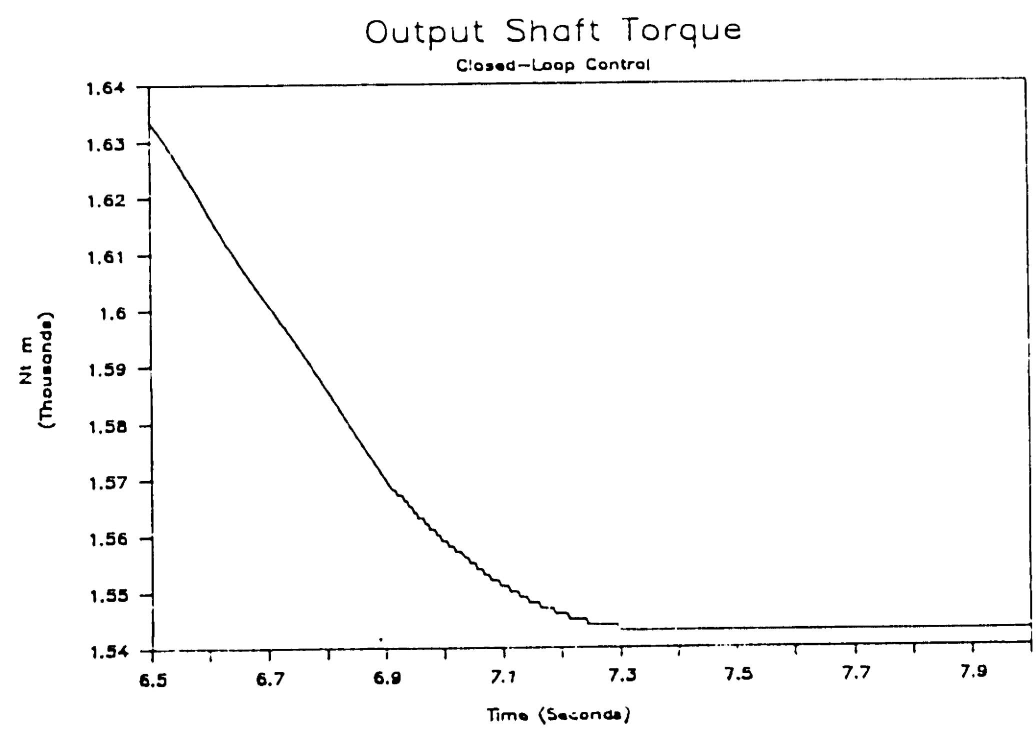
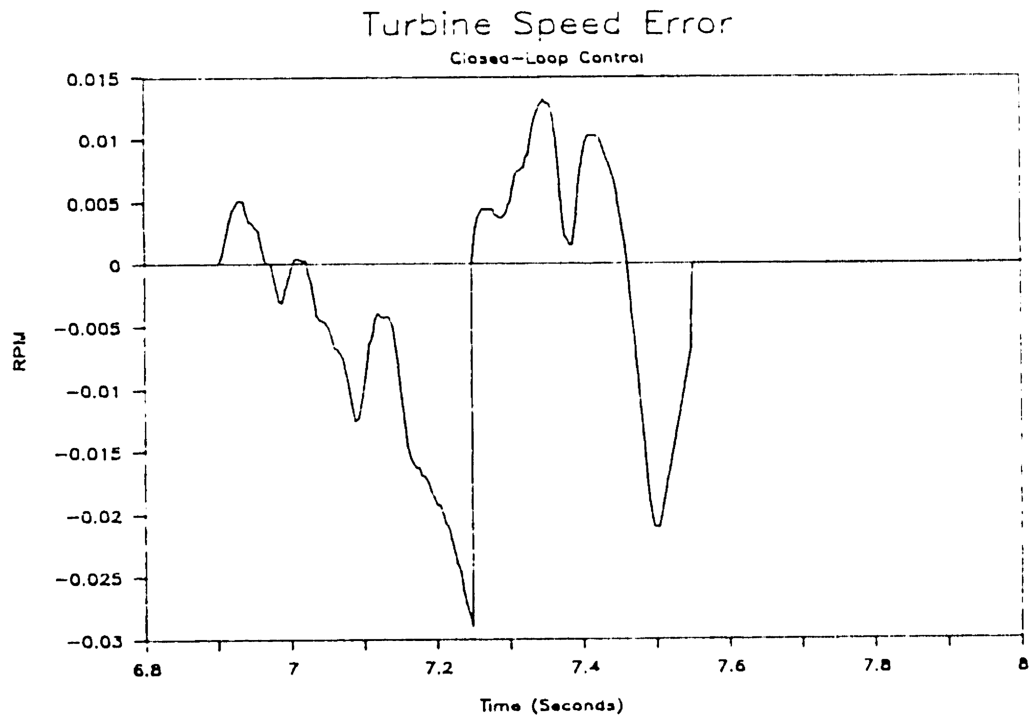

Output Shaft Torque
Closed-Loop Control
Nt m (Thousands)
1.64
1.63
1.62
1.61
1.6
1.59
1.58
1.57
1.56
1.55
1.54
6.5
6.7
6.9
7.1
7.3
7.5
7.7
7.9
Time (Seconds)

Figure 4.2.1-2


Turbine Speed Error
Closed-Loop Control
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
RPM
6.8
7
7.2
7.4
7.6
7.8
8
Time (Seconds)

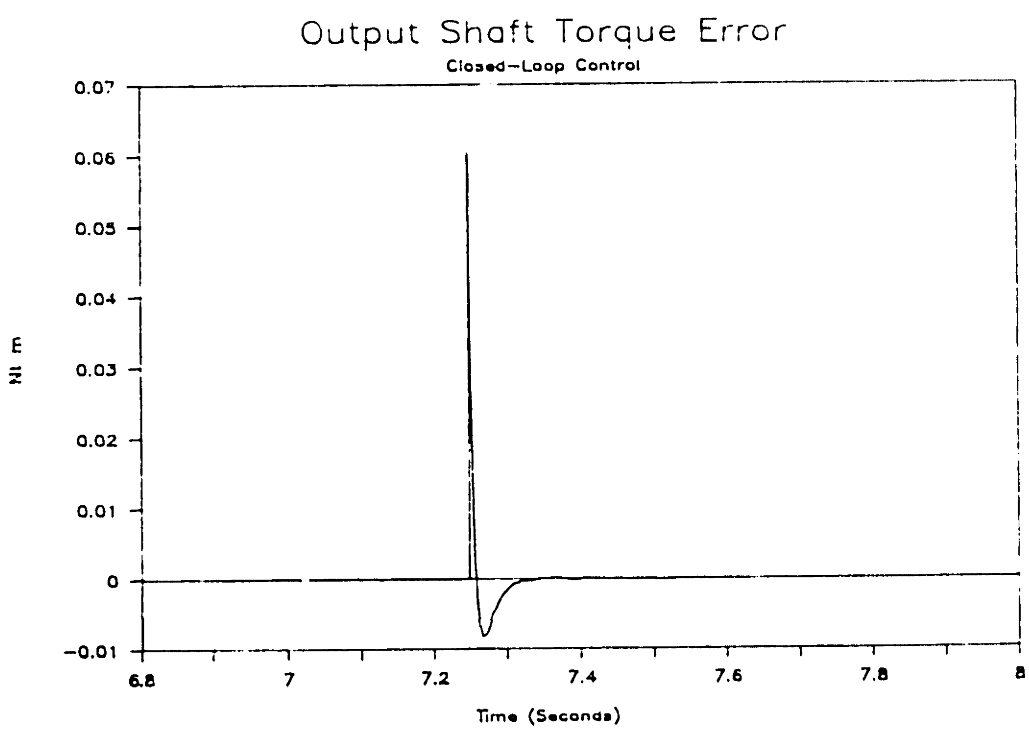
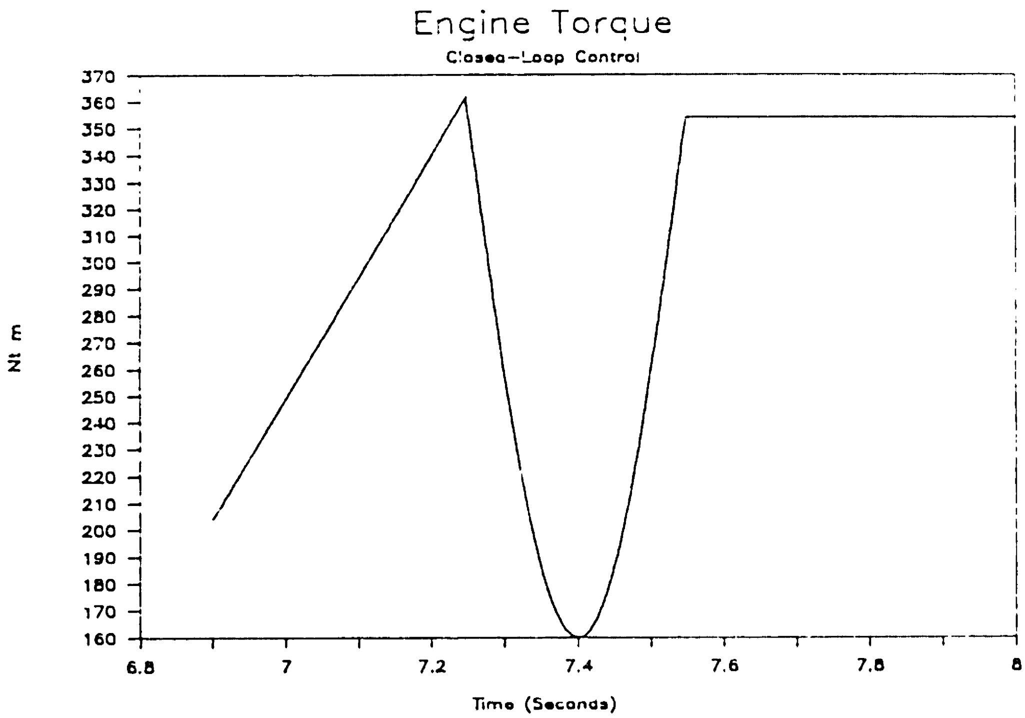

Output Shaft Torque Error
Closed-Loop Control
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
Nt m
6.8
7
7.2
7.4
7.6
7.8
8
Time (Seconds)

Figure 4.2.3-4


Engine Torque
Closed-Loop Control
370
360
350
340
330
320
310
300
290
280
270
260
250
240
230
220
210
200
190
180
170
160
Nt m
6.8
7
7.2
7.4
7.6
7.8
8
Time (Seconds)

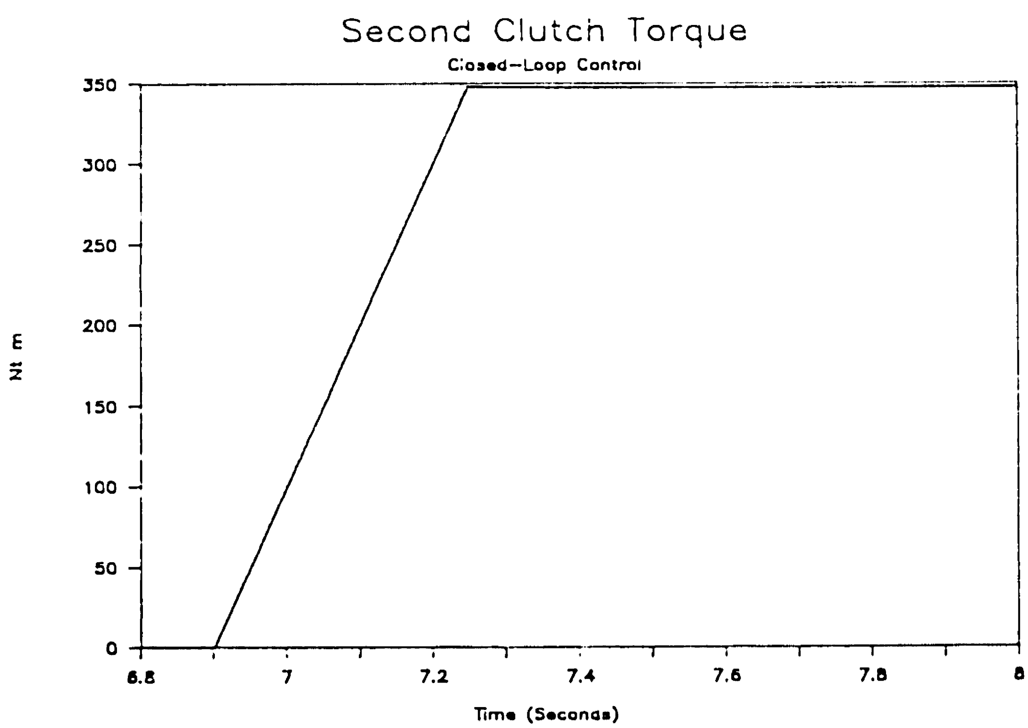

Second Clutch Torque
Closed-Loop Control
350
300
250
200
150
100
50
0
Nt m
6.8
7
7.2
7.4
7.6
7.8
8
Time (Seconds)

Figure 4.2.5-6

4.3 Analysis of the Closed–Loop Simulation

The turbine speed, ω_T, and shaft torque, T_S, are plotted in Figures 4.2.1-2 for the closed-loop controller and their respective error functions, ϵ_1 and ϵ_2, are plotted in Figures 4.2.3-4, where ϵ_1 is defined by equation 4.1.1.2 and ϵ_2 is defined by equation 4.1.2.8. The errors shown by ϵ_1 and ϵ_2 during the simulation are attributed to the finite machine precision of the computer. The control inputs T_{eng} and T_{C2} are then plotted in Figures 4.2.5-6, respectively.

As expected, engine torque increases during the torque phase to negate the braking effect of T_{C2} and then decreases during the inertia phase to cause the turbine inertia to decelerate to its new synchronous speed. At the end of the inertia phase, engine torque increases due to the constraint imposed by equation 4.1.2.19. Note again that if this constraint is not satisfied, then engine torque will be required to instantaneously change at the end of the shift to yield a smooth output shaft torque at the start of second gear. It is also seen that engine torque at the end of the torque phase is approximately equal to the engine torque in second gear. Note that at the end of the torque phase, the reaction torque of the input clutch is zero. By neglecting inertial torques, then second clutch torque at the end of the torque phase is equal to engine torque. Similary, the reaction torque of the second clutch is equal to engine torque in second gear. Noting that the output shaft torque in both cases is equal to the second clutch torque times the second gear ratio, then engine torque must be the same just before and after the inertia phase if the output shaft torque is to be constant throughout the shift.

During the inertia phase, the changes in the second clutch torque, T_{C2}, required to yield a smooth output torque are very small. This is because the desired output shaft torque is nearly constant. Note, however, that although the second clutch torque is nearly constant, the clutch pressure will be required to decrease at the end of the shift due to the increase in the coefficient of friction as the clutch slip speed goes to zero.

It was stated at the beginning of Section 4 that the controller was to be designed with the torque converter locked up. There are two reasons for this. The first is that if the torque converter is slipping, then turbine torque would be required to follow a profile similar to that shown in Figure 4.2.5 for engine torque. From Section 2.2.5, it is seen that the torque converter is modelled as a nonlinear damper and thus turbine torque is a function of engine speed. Torque changes in the turbine torque similar to that found in Figure 4.2.5 would therefore require rapid changes in engine speed, which implies larger transients in engine torque than when the converter is locked up. The second reason for locking up the torque converter is that it is modelled using steady-state equations. Specifically, the equations for pump and turbine torque described in Section 2.2.5 consider the fluid flow in the torque converter to be at steady-state. For the fast transients that occur during a shift, this assumption may lead to large errors in predicting the pump and turbine torques. Note that Sliding Mode Theory can be applied to a transmission with the torque converter slipping and the modelling errors associated with the torque converter could be added to η to yield a robust control design. However, the cost of using a torque converter model with large errors will be even higher control efforts in engine torque.

It must be noted that values for η, ϕ, and λ were not given for the simulation even though they are parameters in the control laws. This is because initial condition errors, modelling errors, and unknown disturbances were not added to the system. For this case, the values of η, ϕ, and λ will not have an effect on the simulation. Specifically, for the conditions stated above, both ϵ and S will be identically equal to zero and thus the terms $\lambda\epsilon$ and $Sat(S)$ will be equal to zero.

4.4 Conclusions on Closed–Loop Control

As shown by both Figures 4.2.2 &4, the output shaft torque follows the desired shaft torque. However, the desired shaft torque should be chosen with considerations given to the requirements it will have on engine torque. For example, note that the desired shaft torque chosen implies that the shift point corresponds to where the tractive effort curves for first gear and second gear cross, i.e., when the output shaft torque in first gear equals the output shaft torque in second gear. Neglecting inertial torques, this implies that engine torque must increase by a factor of G_1/G_2 after the upshift due to the lower gear ratio, where G is the gear ratio. Also, the first derivative on the desired shaft torque was chosen to go to zero at the end of the shift to yield a constant output torque in second gear. However, as seen in Figures 3.2-3, engine torque for a real engine decreases as speed increases for a constant throttle angle. Thus, the derivative on the desired shaft torque at the end of the shift should correspond to the derivative of engine torque in the new gear range.

For practical reasons, it may not be possible for engine torque to increase during the torque phase of the shift. If this is the case, then the output shaft torque will drop during the inertia phase due to the braking effect of the on-coming clutch. The control objective would then be to specify the desired shaft torque beginning at the start of the inertia phase.

Another problem that concerns obtaining a smooth shift is that the driver will often force an upshift; meaning that the driver will start the vehicle with a high throttle angle to accelerate to some desired velocity and then back off on the throttle when this velocity is reached. This new throttle angle typically causes the transmission to shift. The control problem is then again one of selecting the desired shaft torque trajectory for the shift which the driver is expecting.

5
Summary

The objectives set out in the introduction of this report were satisfied; bond graphs were found to be a convenient way to describe a planetary gear set in order to develop the system equations for an automatic transmission and Sliding Mode Theory determined the control requirements on engine torque and the clutch pressures to yield a smooth shift. The remaining discussion will then give recommendations for future work in the area of automatic transmission modelling and control.

The simulated shift transients shown in Section 3.1 should be compared to some actual experimental data to validate the overall transmission model. For torque converter equipped vehicles, the validity associated with using a steady-state torque converter model should be investigated. Specifically, with the present steady-state torque converter model, changes in pump and turbine speeds yield immediate changes in the pump and turbine torque. However, for a real torque converter, changes in pump and turbine speeds require the fluid in the torque converter to accelerate. If the inertial effects of the fluid are not negligible, then a delay will be present in the changes associated with the pump and turbine torques. For the open-loop simulations presented in Chapter 3, this means that the transients in the pump and turbine torques shown in Figure 3.4 may be attenuated.

In the area of closed–loop control, work should be done to determine the limitations of a real engine when chosing the desired output shaft torque trajectory during a shift. For the desired torque trajectory chosen in Section 4.2, it was seen that engine torque must increase by a factor of G_1/G_2 after the shift for a constant output shaft torque, where G refers to the transmission gear ratio. If this is not possible, then the engine torque after the shift must at least be known so that a smooth ouput shaft torque trajectory can be specified. Note that the final ouput torque should then be set equal to engine torque at the end of the shift times the new gear ratio. Once a torque trajectory is given, the requirements on the pressure transducers for the clutches can be determined. Note that for the output shaft torque trajectory chosen in Section 4.3, the clutch torque is nearly constant throughout the inertia phase of the shift. The clutch pressure will then be required to decrease at the end of the shift due to the decrease in the clutch coefficient of friction as the clutch slip speed goes to zero.

Implimetation of the control laws, equations 4.1.1.14, 4.1.2.3, and 4.1.2.12, require measurements of ω_T, ω_{CR}, ω_V, T_S, and $\dot{T}_S$. If transducers to measure T_S and $\dot{T}_S$ are not practical, then these measurements can be produced synthetically by developing an observer for them.

Parametric studies will aid in determining the requirements on the above actuators and sensors. In addition, values of η, ϕ, and λ should be determined which will yield a robust control design in the presence of the possible initial condition errors, modelling errors, and unknown disturbances.

Appendix I

The parameters for the Hydramatic 440 automatic transmission are listed below.

Inertia values ($kg \cdot m^2$):

$I_E = 0.087$	Engine inertia
$I_P = 0.058$	Pump inertia
$I_T = 0.05623$	Lumped torque converter turbine inertia
$I_{SI} = 0.00102$	Lumped input planetary sun gear inertia
$I_{RR} = 0.00902$	Lumped reaction planetary ring gear inertia
$I_{SR} = 0.00452$	Lumped reaction planetary sun gear inertia
$I_{CR} = 0.005806$	Lumped reaction planetary carrier inertia

Planetary gear set transformer moduli:

$R_{SI} = 0.2955$	Input planetary sun gear transformer modulus
$R_{RI} = 0.7045$	Input planetary ring gear transformer modulus
$R_{SR} = 0.3621$	Reaction planetary sun gear transformer modulus
$R_{RR} = 0.6379$	Reaction planetary ring gear transformer modulus
$R_D = 0.3521$	Final drive gear transformer modulus

Plate clutch parameters:

	Number of surfaces	Effective radius of plates (m)	Clutch piston area (m^2)
Input	8	0.052	0.0070
Second	10	0.055	0.0075
Third	8	0.050	0.0059
Fourth	2	0.055	0.0078

Band clutch parameters:

	Wrap angle (radians)	Effective radius of drum (m)	Band piston area (m^2)
1-2	12.0	0.069	0.0043
Reverse	6.0	0.075	0.0044

Vehical parameters:

$m = 1644\ kg$	Vehical mass
$r = 0.3214\ m$	Tire radius

Appendix II

The compound planetary gear set for the Hydramatic 440 transmission can be described using bond graphs by a multiport junction as shown below.

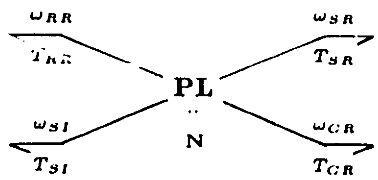


Figure II.1

The equations describing this multiport junction were derived in section 3.2.2 and are repeated here for convenience.

$$\omega_{RR} = R_{SI}\omega_{SI} + R_{RI}\omega_{CR} \tag{II.1}$$

$$\omega_{CR} = R_{SR}\omega_{SR} + R_{RR}\omega_{RR}. \tag{II.2}$$

These equations are then solved simultaniously for all causal arrangements, or input/output structures, for the above multiport and the matrices defining the multiport modulus, **N**, are given below.

For

$$\begin{pmatrix}\omega_{SR}\\ \omega_{RR}\end{pmatrix} = \mathbf{A}\begin{pmatrix}\omega_{SI}\\ \omega_{CR}\end{pmatrix}, \qquad \mathbf{A} = \frac{1}{R_{SR}}\begin{bmatrix}-R_{SI}R_{RR} & 1 - R_{RI}R_{RR}\\ R_{SI}R_{SR} & R_{RI}R_{SR}\end{bmatrix} \tag{II.3}$$

$$\begin{pmatrix}\omega_{SI}\\ \omega_{SR}\end{pmatrix} = \mathbf{B}\begin{pmatrix}\omega_{RR}\\ \omega_{CR}\end{pmatrix}, \qquad \mathbf{B} = \frac{1}{R_{SI}R_{SR}}\begin{bmatrix}R_{SR} & -R_{RI}R_{SR}\\ -R_{SI}R_{RR} & R_{SI}\end{bmatrix} \tag{II.4}$$

$$\begin{pmatrix}\omega_{RR}\\ \omega_{SI}\end{pmatrix} = \mathbf{C}\begin{pmatrix}\omega_{SR}\\ \omega_{CR}\end{pmatrix}, \qquad \mathbf{C} = \frac{1}{R_{SI}R_{RR}}\begin{bmatrix}-R_{SI}R_{SR} & R_{SI}\\ -R_{SR} & 1 - R_{RI}R_{RR}\end{bmatrix} \tag{II.5}$$

$$\begin{pmatrix}\omega_{RR}\\ \omega_{CR}\end{pmatrix} = \mathbf{D}\begin{pmatrix}\omega_{SI}\\ \omega_{SR}\end{pmatrix}, \qquad \mathbf{D} = \frac{1}{1-R_{RI}R_{RR}}\begin{bmatrix}R_{SI} & R_{RI}R_{SR}\\ R_{SI}R_{RR} & R_{SR}\end{bmatrix} \tag{II.6}$$

$$\begin{pmatrix}\omega_{SI}\\ \omega_{CR}\end{pmatrix} = \mathbf{E}\begin{pmatrix}\omega_{RR}\\ \omega_{SR}\end{pmatrix}, \qquad \mathbf{E} = \frac{1}{R_{SI}}\begin{bmatrix}1 - R_{RI}R_{RR} & -R_{RI}R_{SR}\\ R_{SI}R_{RR} & R_{SI}R_{SR}\end{bmatrix} \tag{II.7}$$

$$\begin{pmatrix}\omega_{SR}\\ \omega_{CR}\end{pmatrix} = \mathbf{F}\begin{pmatrix}\omega_{RR}\\ \omega_{SI}\end{pmatrix}, \qquad \mathbf{F} = \frac{1}{R_{RI}R_{SR}}\begin{bmatrix}1 - R_{RI}R_{RR} & -R_{SI}\\ R_{SR} & -R_{SI}R_{SR}\end{bmatrix} \tag{II.8}$$

Appendix III

The system equations for the mechanical section of the Hydramatic 440 automatic transmission are derived in this appendix.

The inputs to the transmission are:

- torque converter turbine torque, T_T,
- output shaft torque, T_S, and
- the clutch torques, $T_{CI}-T_{B12}$.

Note that the clutch torques are computed from equations 3.2.3.1-7.

The outputs from the transmission model are:

- turbine speed, ω_T,
- transmission output speed, ω_o.

Note that the transmission output speed is computed from the reaction carrier speed, ω_{CR}, using equation 3.2.3.8.

III.1 All Clutches Slipping

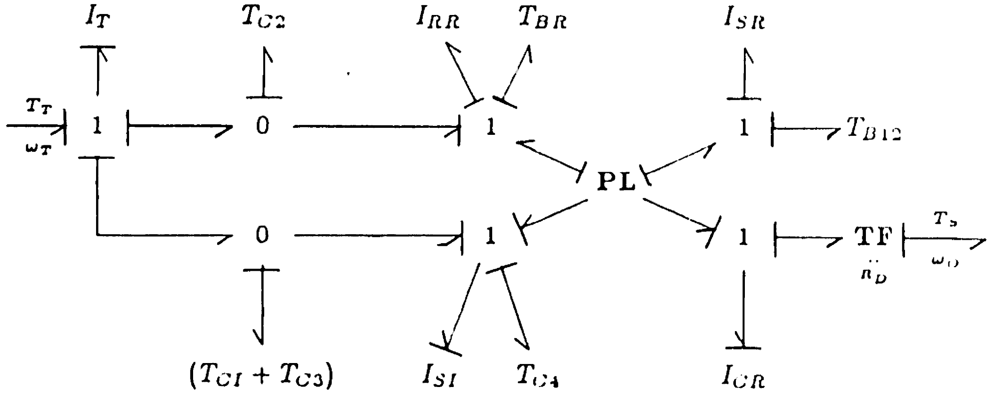


Figure III.1.1

Given

$$\begin{pmatrix} \omega_{SR} \\ \omega_{RR} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \omega_{SI} \\ \omega_{CR} \end{pmatrix} \qquad \text{(III.1.1)}$$

then from the conservation of power

$$\begin{pmatrix} T_{SI} \\ T_{CR} \end{pmatrix} = -\mathbf{A}^{\mathrm{T}} \begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix}. \qquad \text{(III.1.2)}$$

The system equations from the bond graph are

$$\dot{\omega}_T I_T = T_T - T_{C2} - (T_{CI} - T_{C3}) \qquad \text{(III.1.3)}$$
$$\dot{\omega}_{SI} I_{SI} = (T_{CI} + T_{C3}) - T_{C4} + T_{SI} \qquad \text{(III.1.4)}$$
$$\dot{\omega}_{CR} I_{CR} = -T_S R_D + T_{CR} \qquad \text{(III.1.5)}$$

where

$$T_{SR} = I_{SR}\dot{\omega}_{SR} + T_{B12} \qquad \text{(III.1.6)}$$
$$T_{RR} = I_{RR}\dot{\omega}_{RR} + T_{BR} - T_{C2}. \qquad \text{(III.1.7)}$$

Writing equations III.1.(4-7) in matrix form,

$$\begin{bmatrix} I_{SI} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{SI} \\ \dot{\omega}_{CR} \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \vec{\mathbf{T}} + \begin{pmatrix} T_{SI} \\ T_{CR} \end{pmatrix}$$
$$= \begin{bmatrix} 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \vec{\mathbf{T}} - \mathbf{A}^{\mathrm{T}} \begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix} \qquad \text{(III.1.8)}$$

and

$$\begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix} = \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{SR} \\ \dot{\omega}_{RR} \end{pmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \vec{\mathbf{T}} \qquad \text{(III.1.9)}$$

where

$$\vec{\mathbf{T}} = [\, T_T \quad T_{CI} \quad T_{C2} \quad T_{C3} \quad T_{C4} \quad T_{B12} \quad T_{BR} \quad T_S R_D \,]^T . \qquad \text{(III.1.10)}$$

Substituting equation III.1.9 into III.1.8 and rearranging yields

$$\left\{ \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{CR} \end{bmatrix} + \mathbf{A}^{\mathrm{T}} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \mathbf{A} \right\} \begin{pmatrix} \omega_{SI} \\ \omega_{CR} \end{pmatrix} =$$

$$\left\{ \begin{bmatrix} 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} - \mathbf{A}^{\mathrm{T}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \right\} \vec{\mathbf{T}}. \qquad \text{(III.1.11)}$$

By defining

$$\underline{\overline{\mathbf{I_{EQ}}}} = \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{CR} \end{bmatrix} + \mathbf{A}^{\mathrm{T}} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \mathbf{A} \qquad \text{(III.1.12)}$$

and evaluating equation III.1.11, the system equations reduce to

$$\dot{\omega}_T = \frac{1}{I_T}(T_T - T_{CI} - T_{C3} - T_{C2}) \qquad \text{(III.1.13)}$$

and

$$\begin{pmatrix} \dot{\omega}_{SI} \\ \dot{\omega}_{CR} \end{pmatrix} = \left(\underline{\overline{\mathbf{I_{EQ}}}}\right)^{-1} \begin{bmatrix} 0 & 1 & a_{21} & 1 & -1 & -a_{11} & -a_{21} & 0 \\ 0 & 0 & a_{22} & 0 & 0 & -a_{12} & -a_{22} & -1 \end{bmatrix} \vec{\mathbf{T}}. \qquad \text{(III.1.14)}$$

III.2 C_I or C_3 Locked

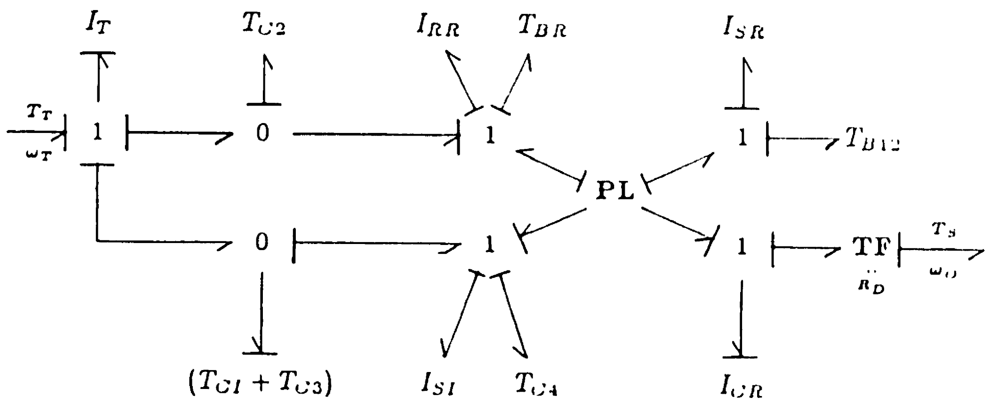


Figure III.2.1

Given

$$\omega_{SI} = \omega_T \tag{III.2.1}$$

$$\begin{pmatrix} \omega_{SR} \\ \omega_{RR} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \omega_{SI} \\ \omega_{CR} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \omega_T \\ \omega_{CR} \end{pmatrix} \tag{III.2.2}$$

then from the conservation of power

$$\begin{pmatrix} T_{SI} \\ T_{CR} \end{pmatrix} = -\mathbf{A}^{\mathrm{T}} \begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix}. \tag{III.2.3}$$

The system equations from the bond graph are

$$\dot{\omega}_T I_T = T_T - T_{C2} - (\dot{\omega}_{SI} I_{SI} + T_{C4} - T_{SI}) \tag{III.2.4}$$

$$\dot{\omega}_{CR} I_{CR} = T_{CR} - T_S R_D \tag{III.2.5}$$

where

$$T_{SR} = \dot{\omega}_{SR} I_{SR} + T_{B12} \tag{III.2.6}$$

$$T_{RR} = \dot{\omega}_{RR} I_{RR} + T_{BR} - T_{C2}. \tag{III.2.7}$$

Rearranging and writing equations III.2.(4-7) in matrix form,

$$\begin{aligned} \begin{bmatrix} (I_T + I_{SI}) & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} &= \begin{bmatrix} 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \vec{\mathbf{T}} + \begin{pmatrix} T_{SI} \\ T_{CR} \end{pmatrix} \\ &= \begin{bmatrix} 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \vec{\mathbf{T}} - \mathbf{A}^{\mathrm{T}} \begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix} \end{aligned} \tag{III.2.8}$$

and

$$\begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix} = \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{SR} \\ \dot{\omega}_{RR} \end{pmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \vec{\mathbf{T}} \tag{III.2.9}$$

where $\vec{\mathbf{T}}$ is defined by equation III.1.10.

Substituting equation III.2.9 into III.2.8 and rearranging yields

$$\left\{ \begin{bmatrix} (I_T + I_{SI}) & 0 \\ 0 & I_{CR} \end{bmatrix} + \mathbf{A}^{\mathrm{T}} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \mathbf{A} \right\} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} =$$

$$\left\{ \begin{bmatrix} 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} - \mathbf{A}^{\mathrm{T}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \right\} \vec{\mathbf{T}}. \quad \text{(III.2.10)}$$

By defining

$$\underline{\overline{\mathbf{I_{EQ1}}}} = \begin{bmatrix} (I_T + I_{SI}) & 0 \\ 0 & I_{CR} \end{bmatrix} + \mathbf{A}^{\mathrm{T}} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \mathbf{A} \quad \text{(III.2.11)}$$

and evaluating equation III.2.10, the system equations reduce to

$$\begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} = \left(\underline{\overline{\mathbf{I_{EQ1}}}}\right)^{-1} \begin{bmatrix} 1 & 0 & (a_{21}-1) & 0 & -1 & -a_{11} & -a_{21} & 0 \\ 0 & 0 & a_{22} & 0 & 0 & -a_{12} & -a_{22} & -1 \end{bmatrix} \vec{\mathbf{T}}. \quad \text{(III.2.12)}$$

The clutch reaction torque, $RT_{CI/3}$, is

$$\begin{aligned} RT_{CI/3} &= I_{SI}\dot{\omega}_{SI} + T_{C4} - T_{SI} \\ &= I_{SI}\dot{\omega}_T + T_{C4} + \begin{bmatrix} a_{11} & a_{21} \end{bmatrix} \begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix}. \end{aligned} \quad \text{(III.2.13)}$$

Writing in matrix form and substituting equation III.2.9 for $\begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix}$ yields

$$\begin{aligned} RT_{CI/3} = &\begin{bmatrix} I_{SI} & 0 \end{bmatrix} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} \\ &+ \begin{bmatrix} a_{11} & a_{21} \end{bmatrix} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \mathbf{A} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} \\ &+ \begin{bmatrix} a_{11} & a_{21} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \vec{\mathbf{T}}. \end{aligned} \quad \text{(III.2.14)}$$

Defining

$$\underline{\overline{\mathbf{I_{EQ2}}}} = \begin{bmatrix} I_{SI} & 0 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{21} \end{bmatrix} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \mathbf{A} \quad \text{(III.2.15)}$$

and substituting equation III.2.12 for $\begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix}$ yields

$$\begin{aligned} RT_{CI/3} = \Bigg\{ &\left(\underline{\overline{\mathbf{I_{EQ2}}}}\right) \left(\underline{\overline{\mathbf{I_{EQ1}}}}\right)^{-1} \begin{bmatrix} 1 & 0 & (a_{21}-1) & 0 & -1 & -a_{11} & -a_{21} & 0 \\ 0 & 0 & a_{22} & 0 & 0 & -a_{12} & -a_{22} & -1 \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 & -a_{21} & 0 & 1 & a_{11} & a_{21} & 0 \end{bmatrix} \Bigg\} \vec{\mathbf{T}}. \end{aligned} \quad \text{(III.2.16)}$$

III.3 C_2 Locked

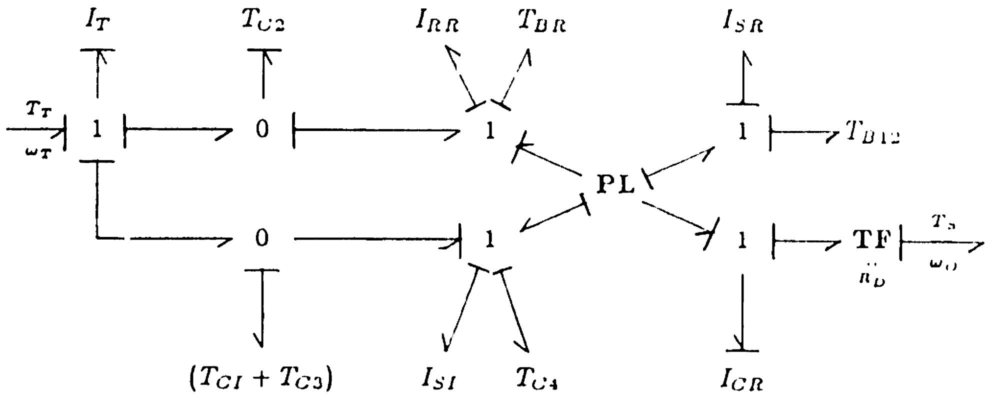


Figure III.3.1

Given

$$\omega_{RR} = \omega_T \tag{III.3.1}$$

$$\begin{pmatrix} \omega_{SI} \\ \omega_{SR} \end{pmatrix} = \mathbf{B} \begin{pmatrix} \omega_{RR} \\ \omega_{CR} \end{pmatrix} = \mathbf{B} \begin{pmatrix} \omega_T \\ \omega_{CR} \end{pmatrix} \tag{III.3.2}$$

then from the conservation of power

$$\begin{pmatrix} T_{RR} \\ T_{CR} \end{pmatrix} = -\mathbf{B}^{\mathbf{T}} \begin{pmatrix} T_{SI} \\ T_{SR} \end{pmatrix}. \tag{III.3.3}$$

The system equations from the bond graph are

$$\dot{\omega}_T I_T = T_T - (I_{RR}\dot{\omega}_{RR} + T_{BR} - T_{RR}) - (T_{CI} + T_{C3}) \tag{III.3.4}$$

$$\dot{\omega}_{CR} I_{CR} = T_{CR} - T_S R_D \tag{III.3.5}$$

where

$$T_{SI} = \dot{\omega}_{SI} I_{SI} + T_{C4} - (T_{CI} + T_{C3}) \tag{III.3.6}$$

$$T_{SR} = \dot{\omega}_{SR} I_{SR} + T_{B12}. \tag{III.3.7}$$

Rearranging and writing equations III.3.(4-7) in matrix form,

$$\begin{bmatrix} (I_T + I_{RR}) & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} = \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \vec{\mathbf{T}} + \begin{pmatrix} T_{RR} \\ T_{CR} \end{pmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \vec{\mathbf{T}} - \mathbf{B}^{\mathbf{T}} \begin{pmatrix} T_{SI} \\ T_{SR} \end{pmatrix} \tag{III.3.8}$$

and

$$\begin{pmatrix} T_{SI} \\ T_{SR} \end{pmatrix} = \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{SR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{SI} \\ \dot{\omega}_{SR} \end{pmatrix} + \begin{bmatrix} 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}}. \qquad \text{(III.3.9)}$$

where $\vec{\mathbf{T}}$ is defined by equation III.1.10.

Substituting equation III.3.9 into III.3.8 and rearranging yields

$$\left\{ \begin{bmatrix} (I_T + I_{RR}) & 0 \\ 0 & I_{CR} \end{bmatrix} + \mathbf{B}^{\mathbf{T}} \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{SR} \end{bmatrix} \mathbf{B} \right\} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} =$$

$$\left\{ \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} - \mathbf{B}^{\mathbf{T}} \begin{bmatrix} 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \right\} \vec{\mathbf{T}}. \qquad \text{(III.3.10)}$$

By defining

$$\underline{\overline{\mathbf{I_{EQ1}}}} = \begin{bmatrix} (I_T + I_{RR}) & 0 \\ 0 & I_{CR} \end{bmatrix} + \mathbf{B}^{\mathbf{T}} \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{SR} \end{bmatrix} \mathbf{B} \qquad \text{(III.3.11)}$$

and evaluating equation III.3.10, the system equations reduce to

$$\begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} = \left(\underline{\overline{\mathbf{I_{EQ1}}}}\right)^{-1} \begin{bmatrix} 1 & (b_{11}-1) & 0 & (b_{11}-1) & -b_{11} & -b_{21} & -1 & 0 \\ 0 & b_{12} & 0 & b_{12} & -b_{12} & -b_{22} & 0 & -1 \end{bmatrix} \vec{\mathbf{T}}. \qquad \text{(III.3.12)}$$

The clutch reaction torque, RT_{C2}, is

$$\begin{aligned} RT_{C2} &= I_{RR}\dot{\omega}_{RR} + T_{BR} - T_{RR} \\ &= I_{RR}\dot{\omega}_T + T_{BR} + [\, b_{11} \quad b_{21} \,] \begin{pmatrix} T_{SI} \\ T_{SR} \end{pmatrix} \end{aligned} \qquad \text{(III.3.13)}$$

Writing in matrix form and substituting equation III.3.9 for $\begin{pmatrix} T_{SI} \\ T_{SR} \end{pmatrix}$ yields

$$\begin{aligned} RT_{C2} = [\, I_{RR} \quad 0 \,] \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} &+ [\, 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \,] \vec{\mathbf{T}} \\ &+ [\, b_{11} \quad b_{21} \,] \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{SR} \end{bmatrix} \mathbf{B} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} \\ &+ [\, b_{11} \quad b_{21} \,] \begin{bmatrix} 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}}. \end{aligned} \qquad \text{(III.3.14)}$$

Defining

$$\underline{\overline{\mathbf{I_{EQ2}}}} = [\, I_{RR} \quad 0 \,] + [\, b_{11} \quad b_{21} \,] \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{SR} \end{bmatrix} \mathbf{B} \qquad \text{(III.3.15)}$$

and substituting equation III.3.12 for $\begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix}$ yields

$$\begin{aligned} RT_{C2} = \Bigg\{ \left(\underline{\overline{\mathbf{I_{EQ2}}}}\right) \left(\underline{\overline{\mathbf{I_{EQ1}}}}\right)^{-1} &\begin{bmatrix} 1 & (b_{11}-1) & 0 & (b_{11}-1) & -b_{11} & -b_{21} & -1 & 0 \\ 0 & b_{12} & 0 & b_{12} & -b_{12} & -b_{22} & 0 & -1 \end{bmatrix} \\ &+ [\, 0 \quad -b_{11} \quad 0 \quad -b_{11} \quad b_{11} \quad b_{21} \quad 1 \quad 0 \,] \Bigg\} \vec{\mathbf{T}}. \end{aligned} \qquad \text{(III.3.16)}$$

III.4 C Locked

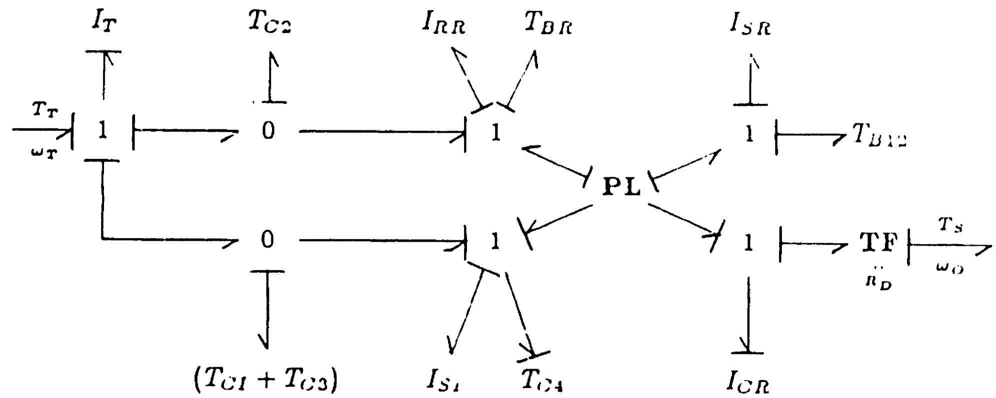


Figure III.4.1

Given

$$\omega_{SI} = 0 \tag{III.4.1}$$

$$\begin{pmatrix} \omega_{SR} \\ \omega_{RR} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \omega_{SI} \\ \omega_{CR} \end{pmatrix} = \mathbf{A} \begin{pmatrix} 0 \\ \omega_{CR} \end{pmatrix} \tag{III.4.2}$$

then from the conservation of power

$$\begin{pmatrix} T_{SI} \\ T_{CR} \end{pmatrix} = -\mathbf{A}^{\mathrm{T}} \begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix}. \tag{III.4.3}$$

The state equations from the bond graph are

$$\dot{\omega}_T I_T = T_T - T_{C2} - (T_{C1} + T_{C3}) \tag{III.4.4}$$

$$\dot{\omega}_{CR} I_{CR} = -T_S R_D + T_{CR} \tag{III.4.5}$$

where

$$T_{SR} = \dot{\omega}_{SR} I_{SR} + T_{B12} \tag{III.4.6}$$

$$T_{RR} = \dot{\omega}_{RR} I_{RR} + T_{BR} - T_{C2}. \tag{III.4.7}$$

Writing equations III.4.(4-7) in matrix form,

$$\begin{bmatrix} I_T & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} = \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \vec{\mathbf{T}} + \begin{pmatrix} 0 \\ T_{CR} \end{pmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \vec{\mathbf{T}} - \begin{bmatrix} 0 & 0 \\ a_{12} & a_{22} \end{bmatrix} \begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix} \tag{III.4.8}$$

and

$$\begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix} = \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{SR} \\ \dot{\omega}_{RR} \end{pmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \vec{\mathbf{T}}$$
$$= \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \begin{bmatrix} 0 & a_{12} \\ 0 & a_{22} \end{bmatrix} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \vec{\mathbf{T}} \qquad \text{(III.4.9)}$$

where $\vec{\mathbf{T}}$ is defined by equation III.1.10.

Substituting equation III.4.9 into III.4.8 and rearranging yields

$$\left\{ \begin{bmatrix} I_T & 0 \\ 0 & I_{CR} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \begin{bmatrix} 0 & a_{12} \\ 0 & a_{22} \end{bmatrix} \right\} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} = \qquad \text{(III.4.10)}$$
$$\left\{ \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \right\} \vec{\mathbf{T}}.$$

By defining

$$\underline{\overline{\mathbf{I_{EQ1}}}} = \begin{bmatrix} I_T & 0 \\ 0 & I_{CR} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \begin{bmatrix} 0 & a_{12} \\ 0 & a_{22} \end{bmatrix} \qquad \text{(III.4.11)}$$

and evaluating equation III.4.10, the system equations reduce to

$$\begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} = \left(\underline{\overline{\mathbf{I_{EQ1}}}}\right)^{-1} \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{22} & 0 & 0 & -a_{12} & -a_{22} & -1 \end{bmatrix} \vec{\mathbf{T}}. \qquad \text{(III.4.12)}$$

The clutch reaction torque, RT_{C4}, is

$$RT_{C4} = T_{CI} + T_{C3} + T_{SI}$$
$$= T_{CI} + T_{C3} - [a_{11} \quad a_{21}] \begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix}. \qquad \text{(III.4.13)}$$

Writing in matrix form and substituting equation III.4.9 for $\begin{pmatrix} T_{SR} \\ T_{RR} \end{pmatrix}$ yields

$$RT_{C4} = [0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0] \vec{\mathbf{T}} - [a_{11} \quad a_{21}] \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \begin{bmatrix} 0 & a_{12} \\ 0 & a_{22} \end{bmatrix} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix}$$
$$- [a_{11} \quad a_{21}] \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \vec{\mathbf{T}}. \qquad \text{(III.4.14)}$$

Defining

$$\underline{\overline{\mathbf{I_{EQ2}}}} = [a_{11} \quad a_{21}] \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{RR} \end{bmatrix} \begin{bmatrix} 0 & a_{12} \\ 0 & a_{22} \end{bmatrix} \qquad \text{(III.4.15)}$$

and substituting equation III.4.12 for $\begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix}$ yields

$$RT_{C4} = \left\{ -\left(\underline{\overline{\mathbf{I_{EQ2}}}}\right) \left(\underline{\overline{\mathbf{I_{EQ1}}}}\right)^{-1} \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{22} & 0 & 0 & -a_{12} & -a_{22} & -1 \end{bmatrix} \right.$$
$$\left. + [0 \quad 1 \quad a_{21} \quad 1 \quad 0 \quad -a_{11} \quad -a_{21} \quad 0] \right\} \vec{\mathbf{T}}. \qquad \text{(III.4.16)}$$

III.5 B_{12} Locked

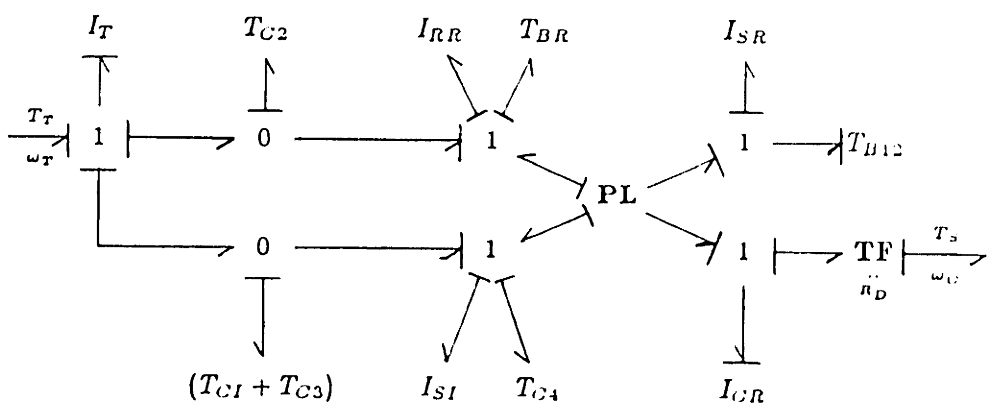


Figure III.5.1

Given

$$\omega_{SR} = 0 \tag{III.5.1}$$

$$\begin{pmatrix} \omega_{RR} \\ \omega_{SI} \end{pmatrix} = \mathbf{C} \begin{pmatrix} \omega_{SR} \\ \omega_{CR} \end{pmatrix} = \mathbf{C} \begin{pmatrix} 0 \\ \omega_{CR} \end{pmatrix} \tag{III.5.2}$$

then from the conservation of power

$$\begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix} = -\mathbf{C}^{\mathrm{T}} \begin{pmatrix} T_{RR} \\ T_{SI} \end{pmatrix}. \tag{III.5.3}$$

The state equations from the bond graph are

$$\dot{\omega}_T I_T = T_T - T_{C2} - (T_{CI} + T_{C3}) \tag{III.5.4}$$

$$\dot{\omega}_{CR} I_{CR} = -T_S R_D + T_{CR} \tag{III.5.5}$$

where

$$T_{RR} = \dot{\omega}_{RR} I_{RR} + T_{BR} - T_{C2} \tag{III.5.6}$$

$$T_{SI} = \dot{\omega}_{SI} I_{SI} + T_{C4} - (T_{CI} + T_{C3}). \tag{III.5.7}$$

Writing equations III.5.(4-7) in matrix form,

$$\begin{bmatrix} I_T & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} = \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \vec{\mathbf{T}} + \begin{pmatrix} 0 \\ T_{CR} \end{pmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \vec{\mathbf{T}} - \begin{bmatrix} 0 & 0 \\ c_{12} & c_{22} \end{bmatrix} \begin{pmatrix} T_{RR} \\ T_{SI} \end{pmatrix} \tag{III.5.8}$$

and

$$\begin{pmatrix} T_{RR} \\ T_{SI} \end{pmatrix} = \begin{bmatrix} I_{RR} & 0 \\ 0 & I_{SI} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{RR} \\ \dot{\omega}_{SI} \end{pmatrix} + \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}}$$
$$= \begin{bmatrix} I_{RR} & 0 \\ 0 & I_{SI} \end{bmatrix} \begin{bmatrix} 0 & c_{12} \\ 0 & c_{22} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{T} \\ \dot{\omega}_{CR} \end{pmatrix} + \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} \quad \text{(III.5.9)}$$

where $\vec{\mathbf{T}}$ is defined by equation III.1.10.

Substituting equation III.5.9 into III.5.8 and rearranging yields

$$\left\{ \begin{bmatrix} I_T & 0 \\ 0 & I_{CR} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ c_{12} & c_{22} \end{bmatrix} \begin{bmatrix} I_{RR} & 0 \\ 0 & I_{SI} \end{bmatrix} \begin{bmatrix} 0 & c_{12} \\ 0 & c_{22} \end{bmatrix} \right\} \begin{pmatrix} \dot{\omega}_{T} \\ \dot{\omega}_{CR} \end{pmatrix} = \quad \text{(III.5.10)}$$
$$\left\{ \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ c_{12} & c_{22} \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \end{bmatrix} \right\} \vec{\mathbf{T}}.$$

By defining

$$\underline{\overline{\mathbf{I_{EQ1}}}} = \begin{bmatrix} I_T & 0 \\ 0 & I_{CR} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ c_{12} & c_{22} \end{bmatrix} \begin{bmatrix} I_{RR} & 0 \\ 0 & I_{SI} \end{bmatrix} \begin{bmatrix} 0 & c_{12} \\ 0 & c_{22} \end{bmatrix} \quad \text{(III.5.11)}$$

and evaluating equation III.5.10, the system equations reduce to

$$\begin{pmatrix} \dot{\omega}_{T} \\ \dot{\omega}_{CR} \end{pmatrix} = \left(\underline{\overline{\mathbf{I_{EQ1}}}}\right)^{-1} \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & c_{22} & c_{12} & c_{22} & -c_{22} & 0 & -c_{12} & -1 \end{bmatrix} \vec{\mathbf{T}}. \quad \text{(III.5.12)}$$

The clutch reaction torque, RT_{B12}, is

$$RT_{B12} = T_{SR}$$
$$= -\begin{bmatrix} c_{11} & c_{21} \end{bmatrix} \begin{pmatrix} T_{RR} \\ T_{SI} \end{pmatrix}. \quad \text{(III.5.13)}$$

Substituting equation III.5.9 for $\begin{pmatrix} T_{RR} \\ T_{SI} \end{pmatrix}$ yields

$$RT_{B12} = -\begin{bmatrix} c_{11} & c_{21} \end{bmatrix} \begin{bmatrix} I_{RR} & 0 \\ 0 & I_{SI} \end{bmatrix} \begin{bmatrix} 0 & c_{12} \\ 0 & c_{22} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{T} \\ \dot{\omega}_{CR} \end{pmatrix}$$
$$-\begin{bmatrix} c_{11} & c_{21} \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}}. \quad \text{(III.5.14)}$$

Defining

$$\underline{\overline{\mathbf{I_{EQ2}}}} = \begin{bmatrix} c_{11} & c_{21} \end{bmatrix} \begin{bmatrix} I_{RR} & 0 \\ 0 & I_{SI} \end{bmatrix} \begin{bmatrix} 0 & c_{12} \\ 0 & c_{22} \end{bmatrix} \quad \text{(III.5.15)}$$

and substituting equation III.5.12 for $\begin{pmatrix} \dot{\omega}_{T} \\ \dot{\omega}_{CR} \end{pmatrix}$ yields

$$RT_{B12} = \left\{ -\left(\underline{\overline{\mathbf{I_{EQ2}}}}\right)\left(\underline{\overline{\mathbf{I_{EQ1}}}}\right)^{-1} \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & c_{22} & c_{12} & c_{22} & -c_{22} & 0 & -c_{12} & -1 \end{bmatrix} \right.$$
$$\left. + \begin{bmatrix} 0 & c_{21} & c_{11} & c_{21} & -c_{21} & 0 & -c_{11} & 0 \end{bmatrix} \right\} \vec{\mathbf{T}}. \quad \text{(III.5.17)}$$

III.6 B_R Locked

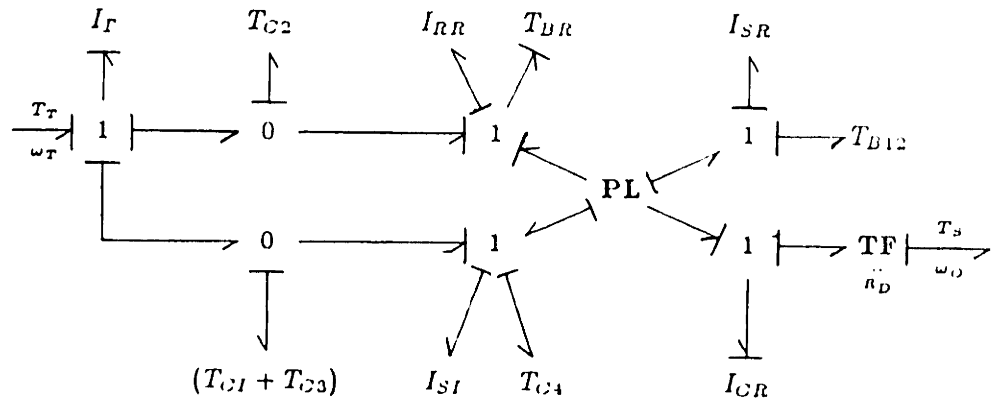


Figure III.6.1

Given

$$\omega_{RR} = 0 \tag{III.6.1}$$

$$\begin{pmatrix} \omega_{SI} \\ \omega_{SR} \end{pmatrix} = \mathbf{B} \begin{pmatrix} \omega_{RR} \\ \omega_{CR} \end{pmatrix} = \mathbf{B} \begin{pmatrix} 0 \\ \omega_{CR} \end{pmatrix} \tag{III.6.2}$$

then from the conservation of power

$$\begin{pmatrix} T_{RR} \\ T_{CR} \end{pmatrix} = -\mathbf{B}^{\mathrm{T}} \begin{pmatrix} T_{SI} \\ T_{SR} \end{pmatrix}. \tag{III.6.3}$$

The system equations from the bond graph are

$$\dot{\omega}_T I_T = T_T - T_{C2} - (T_{CI} + T_{C3}) \tag{III.6.4}$$

$$\dot{\omega}_{CR} I_{CR} = -T_S R_D + T_{CR} \tag{III.6.5}$$

where

$$T_{SI} = \dot{\omega}_{SI} I_{SI} + T_{C4} - (T_{CI} + T_{C3}) \tag{III.6.6}$$

$$T_{SR} = \dot{\omega}_{SR} I_{SR} + T_{B12}. \tag{III.6.7}$$

Writing equations III.6.(4-7) in matrix form,

$$\begin{aligned} \begin{bmatrix} I_T & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} &= \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \vec{\mathbf{T}} + \begin{pmatrix} 0 \\ T_{CR} \end{pmatrix} \\ &= \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \vec{\mathbf{T}} - \begin{bmatrix} 0 & 0 \\ b_{12} & b_{22} \end{bmatrix} \begin{pmatrix} T_{SI} \\ T_{SR} \end{pmatrix} \end{aligned} \tag{III.6.8}$$

and

$$\begin{pmatrix} T_{SI} \\ T_{SR} \end{pmatrix} = \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{SR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{SI} \\ \dot{\omega}_{SR} \end{pmatrix} + \begin{bmatrix} 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}}$$
$$= \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{SR} \end{bmatrix} \begin{bmatrix} 0 & b_{12} \\ 0 & b_{22} \end{bmatrix} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} + \begin{bmatrix} 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} \qquad \text{(III.6.9)}$$

where $\vec{\mathbf{T}}$ is defined by equation III.1.10.

Substituting equation III.6.9 into III.6.8 and rearranging yields

$$\left\{ \begin{bmatrix} I_T & 0 \\ 0 & I_{CR} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{SR} \end{bmatrix} \begin{bmatrix} 0 & b_{12} \\ 0 & b_{22} \end{bmatrix} \right\} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} = \qquad \text{(III.6.10)}$$
$$\left\{ \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \right\} \vec{\mathbf{T}}.$$

By defining

$$\underline{\overline{\mathbf{I_{EQ1}}}} = \begin{bmatrix} I_T & 0 \\ 0 & I_{CR} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{SR} \end{bmatrix} \begin{bmatrix} 0 & b_{12} \\ 0 & b_{22} \end{bmatrix} \qquad \text{(III.6.11)}$$

and evaluating equation III.6.10, the system equations reduce to

$$\begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix} = \left(\underline{\overline{\mathbf{I_{EQ1}}}} \right)^{-1} \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & b_{12} & 0 & b_{12} & -b_{12} & -b_{22} & 0 & -1 \end{bmatrix} \vec{\mathbf{T}}. \qquad \text{(III.6.12)}$$

The clutch reaction torque, RT_{BR}, is

$$RT_{BR} = T_{C2} + T_{RR}$$
$$= T_{C2} - \begin{bmatrix} b_{11} & b_{21} \end{bmatrix} \begin{pmatrix} T_{SI} \\ T_{SR} \end{pmatrix}. \qquad \text{(III.6.13)}$$

Writing in matrix form and substituting equation III.6.9 for $\begin{pmatrix} T_{SI} \\ T_{SR} \end{pmatrix}$ yields

$$RT_{BR} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} - \begin{bmatrix} b_{11} & b_{21} \end{bmatrix} \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{SR} \end{bmatrix} \begin{bmatrix} 0 & b_{12} \\ 0 & b_{22} \end{bmatrix} \begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix}$$
$$- \begin{bmatrix} b_{11} & b_{21} \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}}. \qquad \text{(III.6.14)}$$

Defining

$$\underline{\overline{\mathbf{I_{EQ2}}}} = \begin{bmatrix} b_{11} & b_{21} \end{bmatrix} \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{SR} \end{bmatrix} \begin{bmatrix} 0 & b_{12} \\ 0 & b_{22} \end{bmatrix} \qquad \text{(III.6.15)}$$

and substituting equation III.6.12 for $\begin{pmatrix} \dot{\omega}_T \\ \dot{\omega}_{CR} \end{pmatrix}$ yields

$$RT_{BR} = \Bigg\{ - \left(\underline{\overline{\mathbf{I_{EQ2}}}}\right) \left(\underline{\overline{\mathbf{I_{EQ1}}}}\right)^{-1} \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & b_{12} & 0 & b_{12} & -b_{12} & -b_{22} & 0 & -1 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & b_{11} & 1 & b_{11} & -b_{11} & -b_{21} & 0 & 0 \end{bmatrix} \Bigg\} \vec{\mathbf{T}}. \qquad \text{(III.6.16)}$$

III.7 1st Gear ~ C_I & B_{12} Locked

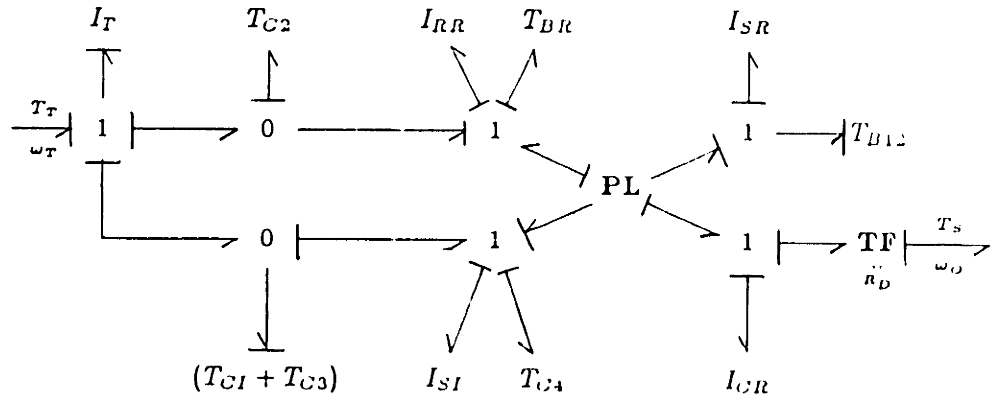


Figure III.7.1

Given

$$\omega_{SI} = \omega_T \tag{III.7.1}$$

$$\omega_{SR} = 0 \tag{III.7.2}$$

$$\begin{pmatrix} \omega_{RR} \\ \omega_{CR} \end{pmatrix} = \mathbf{D} \begin{pmatrix} \omega_{SI} \\ \omega_{SR} \end{pmatrix} = \begin{bmatrix} d_{11} \\ d_{21} \end{bmatrix} \omega_T \tag{III.7.3}$$

then from the conservation of power

$$\begin{pmatrix} T_{SI} \\ T_{SR} \end{pmatrix} = -\mathbf{D}^{\mathbf{T}} \begin{pmatrix} T_{RR} \\ T_{CR} \end{pmatrix}. \tag{III.7.4}$$

The system equations from the bond graph are

$$I_T\dot{\omega}_T = T_T - T_{C2} - (I_{SI}\dot{\omega}_{SI} + T_{C4} - T_{SI}) \tag{III.7.5}$$

where

$$T_{RR} = I_{RR}\dot{\omega}_{RR} + T_{BR} - T_{C2} \tag{III.7.6}$$

$$T_{CR} = I_{CR}\dot{\omega}_{CR} + T_S R_D. \tag{III.7.7}$$

Rearranging and writing equations III.7.(5-7) in matrix form,

$$(I_T + I_{SI})\dot{\omega}_T = [1 \quad 0 \quad -1 \quad 0 \quad -1 \quad 0 \quad 0 \quad 0]\vec{\mathbf{T}} - [d_{11} \quad d_{21}] \begin{pmatrix} T_{RR} \\ T_{CR} \end{pmatrix} \tag{III.7.8}$$

and

$$\begin{pmatrix} T_{RR} \\ T_{CR} \end{pmatrix} = \begin{bmatrix} I_{RR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{RR} \\ \dot{\omega}_{CR} \end{pmatrix} + \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vec{\mathbf{T}} \tag{III.7.9}$$

where $\vec{\mathbf{T}}$ is defined by equation III.1.10.

Substituting equation III.7.9 into III.7.8 and rearranging yields

$$\left\{ I_T + I_{SI} + \begin{bmatrix} d_{11} & d_{21} \end{bmatrix} \begin{bmatrix} I_{RR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} d_{11} \\ d_{21} \end{bmatrix} \right\} \dot{\omega}_T =$$

$$\left\{ \begin{bmatrix} 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} d_{11} & d_{21} \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right\} \vec{\mathbf{T}}. \quad \text{(III.7.10)}$$

By defining

$$I_{EQ1} = I_T + I_{SI} + \begin{bmatrix} d_{11} & d_{21} \end{bmatrix} \begin{bmatrix} I_{RR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} d_{11} \\ d_{21} \end{bmatrix} \quad \text{(III.7.11)}$$

and evaluating equation III.7.10, the system equations reduce to

$$\dot{\omega}_T = \frac{1}{I_{EQ1}} \begin{bmatrix} 1 & 0 & (d_{11} - 1) & 0 & -1 & 0 & -d_{11} & -d_{21} \end{bmatrix} \vec{\mathbf{T}}. \quad \text{(III.7.12)}$$

The clutch reaction torques, RT_{CI} and RT_{B12}, are

$$\begin{aligned} RT_{CI} &= I_{SI}\dot{\omega}_{SI} + T_{C4} - T_{SI} \\ &= I_{SI}\dot{\omega}_T + T_{C4} - T_{SI} \end{aligned} \quad \text{(III.7.13)}$$

and

$$RT_{B12} = T_{SR}. \quad \text{(III.7.14)}$$

Writing equations III.7.(13-14) in matrix form yields

$$\begin{aligned} \begin{pmatrix} RT_{CI} \\ RT_{B12} \end{pmatrix} &= \begin{bmatrix} I_{SI} \\ 0 \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} + \begin{pmatrix} -T_{SI} \\ T_{SR} \end{pmatrix} \\ &= \begin{bmatrix} I_{SI} \\ 0 \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} + \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} T_{SI} \\ T_{SR} \end{pmatrix} \\ &= \begin{bmatrix} I_{SI} \\ 0 \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} - \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{D}^{\mathbf{T}} \begin{pmatrix} T_{RR} \\ T_{CR} \end{pmatrix}. \end{aligned} \quad \text{(III.7.15)}$$

Substituting equation III.7.9 for $\begin{pmatrix} T_{RR} \\ T_{CR} \end{pmatrix}$ yields

$$\begin{aligned} \begin{pmatrix} RT_{CI} \\ RT_{B12} \end{pmatrix} = \begin{bmatrix} I_{SI} \\ 0 \end{bmatrix} \dot{\omega}_T &+ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} \\ &- \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{D}^{\mathbf{T}} \begin{bmatrix} I_{RR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} d_{11} \\ d_{21} \end{bmatrix} \dot{\omega}_T \\ &- \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{D}^{\mathbf{T}} \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vec{\mathbf{T}}. \end{aligned} \quad \text{(III.7.16)}$$

Defining

$$\underline{\overline{\mathbf{I_{EQ2}}}} = \begin{bmatrix} I_{SI} \\ 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{D}^{\mathrm{T}} \begin{bmatrix} I_{RR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} d_{11} \\ d_{21} \end{bmatrix} \tag{III.7.17}$$

and substituting equation III.7.12 for $\dot{\omega}_T$ yields

$$\begin{pmatrix} RT_{CI} \\ RT_{B12} \end{pmatrix} = \left\{ \frac{1}{I_{EQ1}} \left(\underline{\overline{\mathbf{I_{EQ2}}}} \right) \begin{bmatrix} 1 & 0 & (d_{11}-1) & 0 & -1 & 0 & -d_{11} & -d_{21} \end{bmatrix} \right.$$
$$\left. + \begin{bmatrix} 0 & 0 & -d_{11} & 0 & 1 & 0 & d_{11} & d_{21} \\ 0 & 0 & d_{12} & 0 & 0 & 0 & -d_{12} & -d_{22} \end{bmatrix} \right\} \vec{\mathbf{T}}. \tag{III.7.18}$$

III.8 2nd Gear ~ C_2 & B_{12} Locked

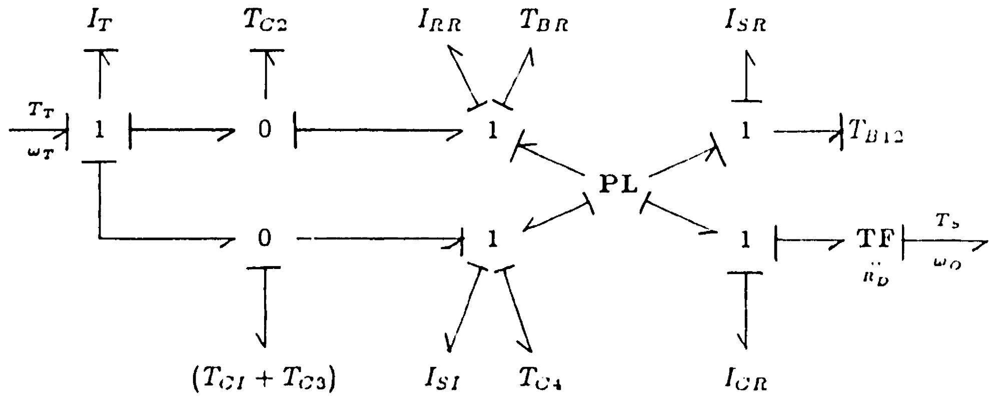


Figure III.8.1

Given

$$\omega_{RR} = \omega_T \tag{III.8.1}$$

$$\omega_{SR} = 0 \tag{III.8.2}$$

$$\begin{pmatrix} \omega_{SI} \\ \omega_{CR} \end{pmatrix} = \mathbf{E} \begin{pmatrix} \omega_{RR} \\ \omega_{SR} \end{pmatrix} = \begin{bmatrix} e_{11} \\ e_{21} \end{bmatrix} \omega_T \tag{III.8.3}$$

then from the conservation of power

$$\begin{pmatrix} T_{RR} \\ T_{SR} \end{pmatrix} = -\mathbf{E}^{\mathrm{T}} \begin{pmatrix} T_{SI} \\ T_{CR} \end{pmatrix}. \tag{III.8.4}$$

The system equations from the bond graph are

$$I_T \dot{\omega}_T = T_T - (I_{RR}\dot{\omega}_{RR} + T_{BR} - T_{RR}) - (T_{CI} + T_{C3}) \tag{III.8.5}$$

where

$$T_{SI} = I_{SI}\dot{\omega}_{SI} + T_{C4} - (T_{CI} + T_{C3}) \tag{III.8.6}$$

$$T_{CR} = I_{CR}\dot{\omega}_{CR} + T_S R_D. \tag{III.8.7}$$

Rearranging and writing equations III.8.(5-7) in matrix form,

$$(I_T + I_{RR})\dot{\omega}_T = [1 \quad -1 \quad 0 \quad -1 \quad 0 \quad 0 \quad -1 \quad 0]\vec{\mathbf{T}} - [e_{11} \quad e_{21}] \begin{pmatrix} T_{SI} \\ T_{CR} \end{pmatrix} \tag{III.8.8}$$

and

$$\begin{pmatrix} T_{SI} \\ T_{CR} \end{pmatrix} = \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{SI} \\ \dot{\omega}_{CR} \end{pmatrix} + \begin{bmatrix} 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vec{\mathbf{T}} \tag{III.8.9}$$

where $\vec{\mathbf{T}}$ is defined by equation III.1.10.

Substituting equation III.8.9 into III.8.8 and rearranging yields

$$\left\{ I_T + I_{RR} + \lfloor e_{11} \quad e_{21} \rfloor \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{21} \end{bmatrix} \right\} \dot{\omega}_T =$$

$$\left\{ \lfloor 1 \quad -1 \quad 0 \quad -1 \quad 0 \quad 0 \quad -1 \quad 0 \rfloor - \lfloor e_{11} \quad e_{21} \rfloor \begin{bmatrix} 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right\} \vec{\mathbf{T}} \tag{III.8.10}$$

By defining

$$I_{EQ1} = I_T + I_{RR} + \lfloor e_{11} \quad e_{21} \rfloor \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{21} \end{bmatrix} \tag{III.8.11}$$

and evaluating equation III.8.10, the system equations reduce to

$$\dot{\omega}_T = \frac{1}{I_{EQ1}} \lfloor 1 \quad (e_{11} - 1) \quad 0 \quad (e_{11} - 1) \quad -e_{11} \quad 0 \quad -1 \quad -e_{21} \rfloor \vec{\mathbf{T}}. \tag{III.8.12}$$

The clutch reaction torques, RT_{C2} and RT_{B12}, are

$$\begin{aligned} RT_{C2} &= I_{RR}\dot{\omega}_{RR} + T_{BR} - T_{RR} \\ &= I_{RR}\dot{\omega}_T + T_{BR} - T_{RR} \end{aligned} \tag{III.8.13}$$

and

$$RT_{B12} = T_{SR}. \tag{III.8.14}$$

Writing equations III.8.(13-14) in matrix form yields

$$\begin{aligned} \begin{pmatrix} RT_{C2} \\ RT_{B12} \end{pmatrix} &= \begin{bmatrix} I_{RR} \\ 0 \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} + \begin{pmatrix} -T_{RR} \\ T_{SR} \end{pmatrix} \\ &= \begin{bmatrix} I_{RR} \\ 0 \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} + \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} T_{RR} \\ T_{SR} \end{pmatrix} \\ &= \begin{bmatrix} I_{RR} \\ 0 \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} - \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{E}^{\mathrm{T}} \begin{pmatrix} T_{SI} \\ T_{CR} \end{pmatrix}. \end{aligned} \tag{III.8.15}$$

Substituting equation III.8.9 for $\begin{pmatrix} T_{RR} \\ T_{CR} \end{pmatrix}$ yields

$$\begin{aligned} \begin{pmatrix} RT_{C2} \\ RT_{B12} \end{pmatrix} = \begin{bmatrix} I_{RR} \\ 0 \end{bmatrix} \dot{\omega}_T &+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} \\ &- \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{E}^{\mathrm{T}} \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{21} \end{bmatrix} \dot{\omega}_T \\ &- \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{E}^{\mathrm{T}} \begin{bmatrix} 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vec{\mathbf{T}}. \end{aligned} \tag{III.8.16}$$

Defining

$$\underline{\overline{\mathbf{I_{EQ2}}}} = \begin{bmatrix} I_{RR} \\ 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{E}^{\mathrm{T}} \begin{bmatrix} I_{SI} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{21} \end{bmatrix} \tag{III.8.17}$$

and substituting equation III.8.12 for $\dot{\omega}_T$ yields

$$\begin{pmatrix} RT_{C2} \\ RT_{B12} \end{pmatrix} = \left\{ \frac{1}{I_{EQ1}} \left(\underline{\overline{\mathbf{I_{EQ2}}}} \right) \left[1 \quad (e_{11} - 1) \quad 0 \quad (e_{11} - 1) \quad -e_{11} \quad 0 \quad -1 \quad -e_{21} \right] \right.$$
$$\left. + \begin{bmatrix} 0 & -e_{11} & 0 & -e_{11} & e_{11} & 0 & 1 & e_{21} \\ 0 & e_{12} & 0 & e_{12} & -e_{12} & 0 & 0 & -e_{22} \end{bmatrix} \right\} \vec{\mathbf{T}}. \tag{III.8.18}$$

III.9 3^{rd} Gear ~ C_2 & C_3 Locked

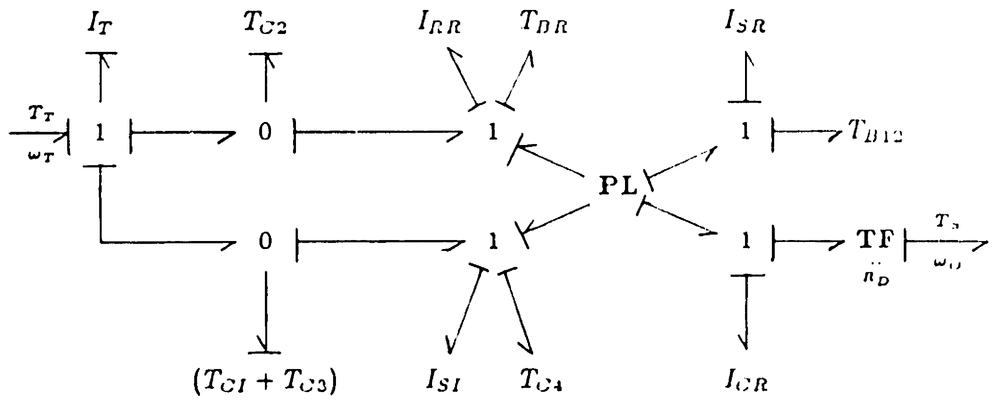


Figure III.9.1

Given

$$\omega_{SI} = \omega_T \tag{III.9.1}$$

$$\omega_{RR} = \omega_T \tag{III.9.2}$$

$$\begin{pmatrix} \omega_{SR} \\ \omega_{CR} \end{pmatrix} = \mathbf{F} \begin{pmatrix} \omega_{RR} \\ \omega_{SI} \end{pmatrix} = \begin{bmatrix} (f_{11} + f_{12}) \\ (f_{21} + f_{22}) \end{bmatrix} \omega_T \tag{III.9.3}$$

then from the conservation of power

$$\begin{pmatrix} T_{RR} \\ T_{SI} \end{pmatrix} = -\mathbf{F}^{\mathrm{T}} \begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix}. \tag{III.9.4}$$

The state equations from the bond graph are

$$I_T \dot{\omega}_T = T_T - (I_{SI}\dot{\omega}_{SI} + T_{C4} - T_{SI}) - (I_{RR}\dot{\omega}_{RR} + T_{BR} - T_{RR}) \tag{III.9.5}$$

where

$$T_{SR} = I_{SR}\dot{\omega}_{SR} + T_{B12} \tag{III.9.6}$$

$$T_{CR} = I_{CR}\dot{\omega}_{CR} + T_S R_D. \tag{III.9.7}$$

Rearranging and writing equations III.9.(5-7) in matrix form,

$$(I_T + I_{SI} + I_{RR})\dot{\omega}_T = [1 \quad 0 \quad 0 \quad 0 \quad -1 \quad 0 \quad -1 \quad 0]\vec{\mathbf{T}} + (T_{RR}) + (T_{SI}). \tag{III.9.8}$$

Or, substituting equation III.9.4

$$
\begin{aligned}
(I_T + I_{SI} + I_{RR})\dot{\omega}_T &= [1 \quad 0 \quad 0 \quad 0 \quad -1 \quad 0 \quad -1 \quad 0]\vec{\mathbf{T}} \\
&\quad - [f_{11} \quad f_{21}] \begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix} - [f_{12} \quad f_{22}] \begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix} \\
&= [1 \quad 0 \quad 0 \quad 0 \quad -1 \quad 0 \quad -1 \quad 0]\vec{\mathbf{T}} \\
&\quad - [(f_{11} + f_{12}) \quad (f_{21} + f_{22})] \begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix}
\end{aligned} \tag{III.9.9}
$$

and

$$
\begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix} = \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{SR} \\ \dot{\omega}_{CR} \end{pmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vec{\mathbf{T}} \tag{III.9.10}
$$

where $\vec{\mathbf{T}}$ is defined by equation III.1.10.

Substituting equation III.9.10 into III.9.9 and rearranging yields

$$
\left\{ I_T + I_{SI} + I_{RR} + [(f_{11} + f_{12}) \quad (f_{21} + f_{22})] \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} (f_{11} + f_{12}) \\ (f_{21} + f_{22}) \end{bmatrix} \right\} \dot{\omega}_T = \tag{III.9.11}
$$

$$
\left\{ [1 \quad 0 \quad 0 \quad 0 \quad -1 \quad 0 \quad -1 \quad 0] - [(f_{11} + f_{12}) \quad (f_{21} + f_{22})] \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right\} \vec{\mathbf{T}}.
$$

By defining

$$
I_{EQ1} = I_T + I_{SI} + I_{RR} + [(f_{11} + f_{12}) \quad (f_{21} + f_{22})] \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} (f_{11} + f_{12}) \\ (f_{21} + f_{22}) \end{bmatrix} \tag{III.9.12}
$$

and evaluating equation III.9.11, the system equations reduce to

$$
\dot{\omega}_T = \frac{1}{I_{EQ1}} [1 \quad 0 \quad 0 \quad 0 \quad -1 \quad -(f_{11} + f_{12}) \quad -1 \quad -(f_{21} + f_{22})] \vec{\mathbf{T}}. \tag{III.9.13}
$$

The clutch reaction torques, RT_{C2} and RT_{C3}, are

$$
\begin{aligned}
RT_{C2} &= I_{RR}\dot{\omega}_{RR} + T_{BR} - T_{RR} \\
&= I_{RR}\dot{\omega}_T + T_{BR} - T_{RR}
\end{aligned} \tag{III.9.14}
$$

and

$$
\begin{aligned}
RT_{C3} &= I_{SI}\dot{\omega}_{SI} + T_{C4} - T_{SI} \\
&= I_{SI}\dot{\omega}_T + T_{C4} - T_{SI}.
\end{aligned} \tag{III.9.15}
$$

Writing equations III.9.(14-15) in matrix form yields

$$\begin{pmatrix} RT_{C2} \\ RT_{C3} \end{pmatrix} = \begin{bmatrix} I_{RR} \\ I_{SI} \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} - \begin{pmatrix} T_{RR} \\ T_{SI} \end{pmatrix}$$

$$= \begin{bmatrix} I_{RR} \\ I_{SI} \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} + \mathbf{F}^{\mathbf{T}} \begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix}. \qquad \text{(III.9.16)}$$

Substituting equation III.9.10 for $\begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix}$ yields

$$\begin{pmatrix} RT_{C2} \\ RT_{C3} \end{pmatrix} = \begin{bmatrix} I_{RR} \\ I_{SI} \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}}$$
$$+ \mathbf{F}^{\mathbf{T}} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} (f_{11} + f_{12}) \\ (f_{21} + f_{22}) \end{bmatrix} \dot{\omega}_T$$
$$+ \mathbf{F}^{\mathbf{T}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vec{\mathbf{T}}. \qquad \text{(III.9.17)}$$

Defining

$$\underline{\overline{\mathbf{I_{EQ2}}}} = \begin{bmatrix} I_{RR} \\ I_{SI} \end{bmatrix} + \mathbf{F}^{\mathbf{T}} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} (f_{11} + f_{12}) \\ (f_{21} + f_{22}) \end{bmatrix} \qquad \text{(III.9.18)}$$

and substituting equation III.9.13 for $\dot{\omega}_T$ yields

$$\begin{pmatrix} RT_{C2} \\ RT_{C3} \end{pmatrix} = \left\{ \frac{1}{I_{EQ1}} \left(\underline{\overline{\mathbf{I_{EQ2}}}} \right) [1 \quad 0 \quad 0 \quad 0 \quad -1 \quad -(f_{11} + f_{12}) \quad -1 \quad -(f_{21} + f_{22})] \right.$$
$$\left. + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & f_{11} & 1 & f_{21} \\ 0 & 0 & 0 & 0 & 1 & f_{12} & 0 & f_{22} \end{bmatrix} \right\} \vec{\mathbf{T}}. \qquad \text{(III.9.19)}$$

III.10 4^{th} Gear ~ C_2 & C_4 Locked

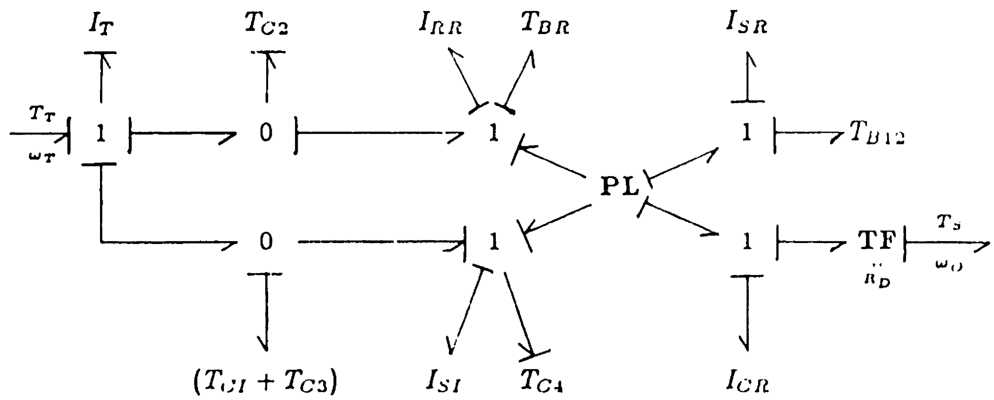


Figure III.10.1

Given

$$\omega_{RR} = \omega_T \tag{III.10.1}$$

$$\omega_{SI} = 0 \tag{III.10.2}$$

$$\begin{pmatrix} \omega_{SR} \\ \omega_{CR} \end{pmatrix} = \mathbf{F} \begin{pmatrix} \omega_{RR} \\ \omega_{SI} \end{pmatrix} = \begin{bmatrix} f_{11} \\ f_{21} \end{bmatrix} \omega_T \tag{III.10.3}$$

then from the conservation of power

$$\begin{pmatrix} T_{RR} \\ T_{SI} \end{pmatrix} = -\mathbf{F}^{\mathrm{T}} \begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix}. \tag{III.10.4}$$

The state equations from the bond graph are

$$I_T \dot{\omega}_T = T_T - (T_{CI} + T_{C3}) - (I_{RR}\dot{\omega}_{RR} + T_{BR} - T_{RR}) \tag{III.10.5}$$

where

$$T_{SR} = I_{SR}\dot{\omega}_{SR} + T_{B12} \tag{III.10.6}$$

$$T_{CR} = I_{CR}\dot{\omega}_{CR} + T_S R_D. \tag{III.10.7}$$

Rearranging and writing equations III.10.(5-7) in matrix form,

$$\begin{aligned} (I_T + I_{RR})\dot{\omega}_T &= [1 \quad -1 \quad 0 \quad -1 \quad 0 \quad 0 \quad -1 \quad 0]\vec{\mathbf{T}} + (T_{RR}) \\ &= [1 \quad -1 \quad 0 \quad -1 \quad 0 \quad 0 \quad -1 \quad 0]\vec{\mathbf{T}} - [f_{11} \quad f_{21}] \begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix} \end{aligned} \tag{III.10.8}$$

and

$$\begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix} = \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{pmatrix} \dot{\omega}_{SR} \\ \dot{\omega}_{CR} \end{pmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vec{\mathbf{T}} \qquad \text{(III.10.9)}$$

where $\vec{\mathbf{T}}$ is defined by equation III.1.10.

Substituting equation III.10.9 into III.10.8 and rearranging yields

$$\left\{ I_T + I_{RR} + \begin{bmatrix} f_{11} & f_{21} \end{bmatrix} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{21} \end{bmatrix} \right\} \dot{\omega}_T =$$

$$\left\{ \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & 0 & -1 & 0 \end{bmatrix} - \begin{bmatrix} f_{11} & f_{21} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right\} \vec{\mathbf{T}}. \qquad \text{(III.10.10)}$$

By defining

$$I_{EQ1} = I_T + I_{RR} + \begin{bmatrix} f_{11} & f_{21} \end{bmatrix} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{21} \end{bmatrix} \qquad \text{(III.10.11)}$$

and evaluating equation III.10.10, the system equations reduce to

$$\dot{\omega}_T = \frac{1}{I_{EQ1}} \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & -f_{11} & -1 & -f_{21} \end{bmatrix} \vec{\mathbf{T}}. \qquad \text{(III.10.12)}$$

The clutch reaction torques, RT_{C2} and RT_{C4}, are

$$\begin{aligned} RT_{C2} &= I_{RR}\dot{\omega}_{RR} + T_{BR} - T_{RR} \\ &= I_{RR}\dot{\omega}_T + T_{BR} - T_{RR} \end{aligned} \qquad \text{(III.10.13)}$$

and

$$RT_{C4} = T_{CI} + T_{C3} + T_{SI}. \qquad \text{(III.10.14)}$$

Writing equations III.10.13 & 14 in matrix form yields

$$\begin{aligned} \begin{pmatrix} RT_{C2} \\ RT_{C4} \end{pmatrix} &= \begin{bmatrix} I_{RR} \\ 0 \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} + \begin{pmatrix} -T_{RR} \\ T_{SI} \end{pmatrix} \\ &= \begin{bmatrix} I_{RR} \\ 0 \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} + \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} T_{RR} \\ T_{SI} \end{pmatrix} \\ &= \begin{bmatrix} I_{RR} \\ 0 \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} - \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{F}^{\mathbf{T}} \begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix}. \end{aligned} \qquad \text{(III.10.15)}$$

Substituting equation III.10.9 for $\begin{pmatrix} T_{SR} \\ T_{CR} \end{pmatrix}$ yields

$$\begin{aligned} \begin{pmatrix} RT_{C2} \\ RT_{C4} \end{pmatrix} = {} & \begin{bmatrix} I_{RR} \\ 0 \end{bmatrix} \dot{\omega}_T + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\mathbf{T}} \\ & - \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{F}^{\mathbf{T}} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{21} \end{bmatrix} \dot{\omega}_T \\ & - \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{F}^{\mathbf{T}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vec{\mathbf{T}}. \end{aligned} \qquad \text{(III.10.16)}$$

Defining

$$\underline{\overline{\mathbf{I_{EQ2}}}} = \begin{bmatrix} I_{RR} \\ 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{F}^{\mathbf{T}} \begin{bmatrix} I_{SR} & 0 \\ 0 & I_{CR} \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{21} \end{bmatrix} \tag{III.10.17}$$

and substituting equation III.10.12 for $\dot{\omega}_T$ yields

$$\begin{pmatrix} RT_{C2} \\ RT_{C3} \end{pmatrix} = \left\{ \frac{1}{I_{EQ1}} \left(\underline{\overline{\mathbf{I_{EQ2}}}} \right) \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & -f_{11} & -1 & -f_{21} \end{bmatrix} \right.$$
$$\left. + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & f_{11} & 1 & f_{21} \\ 0 & 1 & 0 & 1 & 0 & -f_{12} & 0 & -f_{22} \end{bmatrix} \right\} \vec{\mathbf{T}}. \tag{III.10.18}$$