INTACT STABILITY STUDY

PROGRAMMED FOR A

DIGITAL COMPUTER

by

CHARLES WILLIAM ADAMS

S.B., Massachusetts Institute of Technology

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Signature of Author . . . Signature redacted . . .

Department of Mathematics,
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Certified by . . . Signature redacted . . .

Prof. Philip Franklin,
Thesis Supervisor

. . . . Signature redacted . . .

Chairman, Department Committee
on Graduate Students

Math
Thesis
1949

ABSTRACT

Large-scale, high-speed electronic digital computers are at the present time under rapid development in several laboratories. Unless the mathematics of specific applications are worked out in detail before these computers become working realities, there will be an unnecessary delay between the day the first computer is completed and the day the first useable computation is performed.

In this thesis a single practical problem, naval intact stability study, is reduced to a strictly numerical procedure and is programmed in detail for the Whirlwind Computer under development by the M.I.T. Servomechanisms Laboratory.

Apart from the problem of intact stability study in which the program can be of specific use, the work presented demonstrates that a digital computer is capable of performing not only problems which are inherently numerical, such as the first half of an intact stability study, but also problems which, like the second half of an intact stability study, are by their nature graphical (analog) and are not well suited for digital procedures.

It is further demonstrated that even with the limited prototype memory ability of the Whirlwind Computer, problems of the magnitude of intact stability study requiring twenty to thirty

man-days of machine-assisted hand computation can be reduced to a very few man-hours of data preparation and a few computer-seconds of computation.

ACKNOWLEDGEMENTS

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CONTENTS

ILLUSTRATIONS

I - Introduction

The rapid present-day development of large-scale, high-speed electronic digital computers has given rise to a need for investigating the ways in which solutions of engineering and scientific problems can be set up to be carried out by a digital computer. Investigations have been and are being made to determine the most suitable general numerical methods of the solutions of systems of algebraic equations, differential equations, integral equations, and the like. A general procedure for attacking a given problem having been decided upon, it is then necessary that the solution be set up in detail (i.e., programmed) using the order code designed for a particular computer.

The importance of writing a program for a specific problem lies largely in the fact that few such programs have yet been written. The programs which have been written are for the most part concerned with general numerical procedures or with portions of particular problems, usually without any regard for the size of the numbers involved. Most present-day computers are so-called fixed point machines in which the decimal point remains fixed, so that if the numbers resulting from intermediate computations become small, the significant digits can all be lost "off the right-hand end", while if they become too large, an overflow alarm will stop the computer entirely. The problem of keeping the significant digits "centered" in the computer throughout the computation is usually referred to as the "scale factor problem". The techniques used to handle scale factor in a specific problem, although immediately applicable only to that problem itself, shed light on a general procedure which may be applicable to whole classes of problems.

The present thesis proposes a detailed program by which the Whirlwind computer being developed by the M.I.T. Servomechanisms Laboratory can be directed to carry out a specific, practical problem, an intact stability study. The numerical-graphical procedure currently used by the U.S. Navy's Bureau of Ships in performing such a study has been used as a basis for formulating a strictly numerical procedure which has been programmed for the Whirlwind Computer using the order code already established for that computer currently under construction at M.I.T.

The intact stability study seems to be a very suitable specific problem from a number of points of view. In the first place, the computation is one that is now carried out frequently by hand, so that it is a practical problem to set up for the computer. Since it has been carried out regularly, the numerical procedure is well established and the magnitudes of the numbers involved are well known. Furthermore, it is a problem which is not particularly well adapted for digital computation but is much more easily handled by graphical (i.e., analog) methods. Consequently in programming it for a digital computer, one makes a digital machine compete with analog equipment under conditions most adverse to the digital computer.

II - Intact Stability Study

Desired results

The set of calculations by which the so-called curves of form showing the stability characteristics of a ship are computed from the shape of the hull of the ship is called an "intact stability study", the term "intact" being used to differentiate the study from a "damaged" study, in which various types of hull damage (e.g., flooded compartments arising through accidents or combat) are considered.

"Curves of form", strictly speaking, include two sets of plotted curves, the first set of which shows the following quantities, all plotted against the draft, or depth of keel amidships:

Δ = displacement in salt water, measured in tons

Δ_F = displacement in fresh water, measured in tons

KB = vertical position of the center of buoyancy, measured in feet above the base line of the ship

$\otimes$B = longitudinal position of the center of buoyancy, measured in feet forward (positive) or aft (negative) of the midship station

T/in. = tons of displacement per inch of immersion in salt water

$\otimes$F = position of the center of flotation, measured in feet forward (positive) or aft (negative) of the midship station

KM = position of the initial transverse metacenter, measured in feet above the base line of the ship

MT_1 = moment required to change the trim of the ship by one inch, measured in foot tons

The results of a typical study, supplied by the Bureau of Ships, illustrate these curves of form (Fig. 1).

The second set of curves are called "cross curves of stability", and while the cross curves are really part of the curves of form, convenience and common practice permit the cross curves to be considered a separate entity, thereby allowing the use of the title "curves of form" to describe only the set of curves listed above. The cross curves (shown in Fig. 2) show the righting arm (OZ) as a function of displacement for a set of given angles of inclination (roll angle or heel) of the ship. As now carried out by the Navy, cross curves are plotted for angles of inclination at intervals of 10° between angles of 10° and 80° inclusive.

Source of data

Both curves of form and cross curves are functions solely of the shape of the hull of the ship in question. The shape of a hull is given in a lines plan, which is in reality a set of plan views taken at different heights (waterplanes), a set of side elevation views taken at different widths (buttocks) from the centerline, and a set of front elevation views taken at different lengths (stations) along the length of the ship. For the purpose of carrying out the numerical integrations of an intact stability study by digital methods, it is necessary only to know the coordinates of the hull for a selected grid of points. Thus, knowledge of the breadth of the ship at a set of different stations along the length of the ship and at a set of different waterplanes along the height of the ship is all that is necessary in the way of data. Such a set of breadths are readily available on the front elevation view, known as the body plan, which shows cross sections of the ship at various, usually equally spaced stations along the length of the ship. Thus the set of points taken from the body plan (such as is shown in Fig. 3) is the only data needed for the intact stability study. However, in the procedure currently used for hand calculation, much of the integration is done with mechanical integraphs so that the body plan itself is essential. It should be noted that the line labelled "deck" in Fig. 3 is really only the deck line amidships, and that there is a separate deck line implied for each station. Actually, also, the deck usually has camber rather than being flat as shown in this typical study.

Calculation

A procedure for calculating the desired quantities has been worked out in detail by the Bureau of Ships and set up in four computation forms, called Tabulations #1, #2, #3, and #4, examples of which are reproduced here as Figures 4, 5, 6, and 7. In Tabulation #1, the area (A), the moment of the area taken about the midship station (M), and the moments of inertia of the area taken about the midship station ($I_{⊠}$) and about the centerline (I_{CL}) are calculated for each desired waterplane, i.e., at the various heights h, by straightforward numerical integration. The integration is done by the Tchebycheff five-point rule, used for ten stations, which involves simple addition of the breadths (times suitable moment arms) at stations chosen at the proper intervals (called Tchebycheff stations). Since the stations normally taken for a body plan are equally spaced, it is necessary to lift a new body plan from the ship's lines before this Tchebycheff integration can be applied.

In the tabulation, t is the distance of the Tchebycheff station from the midship station of the ship, measured as a fraction of the length of the ship, b/2 is the half-breadth of the ship at the given station and waterplane, and L is the length of the ship, measured between the (arbitrarily chosen) forward and after perpendiculars (corresponding to the first and last stations, shown as FP and AP on the body plan). Since portions of the bow and stern project beyond the forward and after perpendiculars, separate rough calculations of the A, M, $I_{\boxtimes}$, and I_{CL} must be carried out for these appendages. This is done by determining the shape and extent of the projection and integrating by hand and by eye. The results of this separate approximation work are entered in the boxes labelled "Ford. F.P." and "Aft A.P.", and, as may be seen in the typical study, the contribution of the appendages is relatively small compared to that of the rest of the ship, even though the effect on M and $I_{\boxtimes}$ is heightened by large moment arms.

Tabulation #2 provides for finding volume integrals corresponding to the volume (V) and moments of volume taken about the base plane (M_V) and about the plane of the midship station (M_I). The area of cross section (a) of the ship up to each waterplane at the Tchebycheff stations is found by use of a mechanical area and moment integraph, while the moment about the base of the ship (m) is found directly at the same time. The factors c' and c" are the integraph constants for area and moment respectively, while s is the scale factor of the drawing, in feet of ship per inch of plan. The integration over the length of the ship to obtain the volume integrals is again done by Tchebycheff integration. The contribution of the appendages is calculated and added in as before.

Tabulation #3, integration of stations in inclined positions, again depends upon the use of an area and moment integraph, this time to obtain the area (a) under a rotated waterline at a given station and the moment of the area (m_t) about a rotated vertical line. The volume integration is by the Tchebycheff rule and the volume integrals are called V_I and M_t respectively. Although space is provided for entering appendage effects, the appendages are seldom considered in this phase since the difficulties are great for the increased accuracy obtained. In order to visualize the inclined positions properly, one must imagine the ship inclined in the water by some angle (in this case, angles taken at intervals of 10^o). The calculation is done in practice by drawing a vertical line and a set of horizontal waterlines on transparent paper, rotating the ship body plan underneath this waterline sheet, and running the integraph around the area bounded by the desired station and the desired waterline. In Figure 8, the lines labelled $W_1 L_1$, $W_2 L_2$, $W_3 L_3$, $W_4 L_4$ are the desired waterlines, and the ship section has been rotated about the fixed point KG which is chosen at an arbitrary height on the vertical centerline. The distance from the base line of the unrotated body plan to the assumed KG is listed as KO on Tabulation #3, and will be known as h_{CG} in the discussion of the program later.

Tabulation #4 permits the calculation of the final results. The values which are labelled "plot" are of course those which make up the curves of form, while the other values, BM, I_o, and BM, , as well as the quantities marked in by hand, are intermediate results needed to obtain the final plotted results but of little value themselves.

Importance of Intact Stability Study

Representing the Stability Section of the Navy's Bureau of Ships, Charles L. Wright, Jr., who was the originator of the problem of applying high-speed digital computation to intact stability studies and who made available the typical study most of which is reproduced here, stated in a letter to the writer dated September 28, 1948, that "Intact stability is a fundamental factor in the seaworthiness of a vessel and must be carefully studied for each new design. Studies are made during the development of the design and are often repeated when the final lines are lifted in the shipyard mold loft. If studies could be carried out in such a short period of time as is contemplated for the Whirlwind Computer, several studies might also be made during the preliminary stages of each design to replace the approximations that are now used. New studies are also made whenever extensive alterations are made in the watertight portions of the hulls. Stability studies are made, not only for naval vessels, but for practically every type of craft including pleasure boats and merchant ships. Probably more of these studies are made by small boat designers than by the Navy, since there are many more different shapes of hulls in the small boat field."

Furthermore, properly applied digital computation should give as much accuracy as is desired, although accuracy beyond three decimal digits is hardly worth while. Again according to C. L. Wright, Jr., "The accuracy of all curves of form" (particularly cross curves) "would be appreciably increased if values could be obtained for additional waterlines more closely spaced. Similarly, values at smaller increments of angles of heel would improve the accuracy of the cross curves of stability."

An intact stability study of the type considered here requires from twenty to thirty man-days of work. For a high-speed digital computer a similar study could be performed in a matter of seconds, and taking closer waterlines and smaller angles of increment would lengthen the time by only a few seconds. Thus use of a computer would not only save a considerable amount of labor now expended on stability studies, but would permit more frequent and more accurate studies to be performed. In fact, the principal objection to the use of a high-speed computer for problems of this type is that the computing time is so short that the seconds or minutes of computer time required to prepare the computer to solve the problem can hardly be justified. Such considerations suggest that a possible application of a computer to stability study might be to carry out a trial-and-error synthesis. That is, with the high speed of an electronic computer it would be possible to try many hundreds or thousands of variations of hull shape in order to select the one with the best stability characteristics of the group.

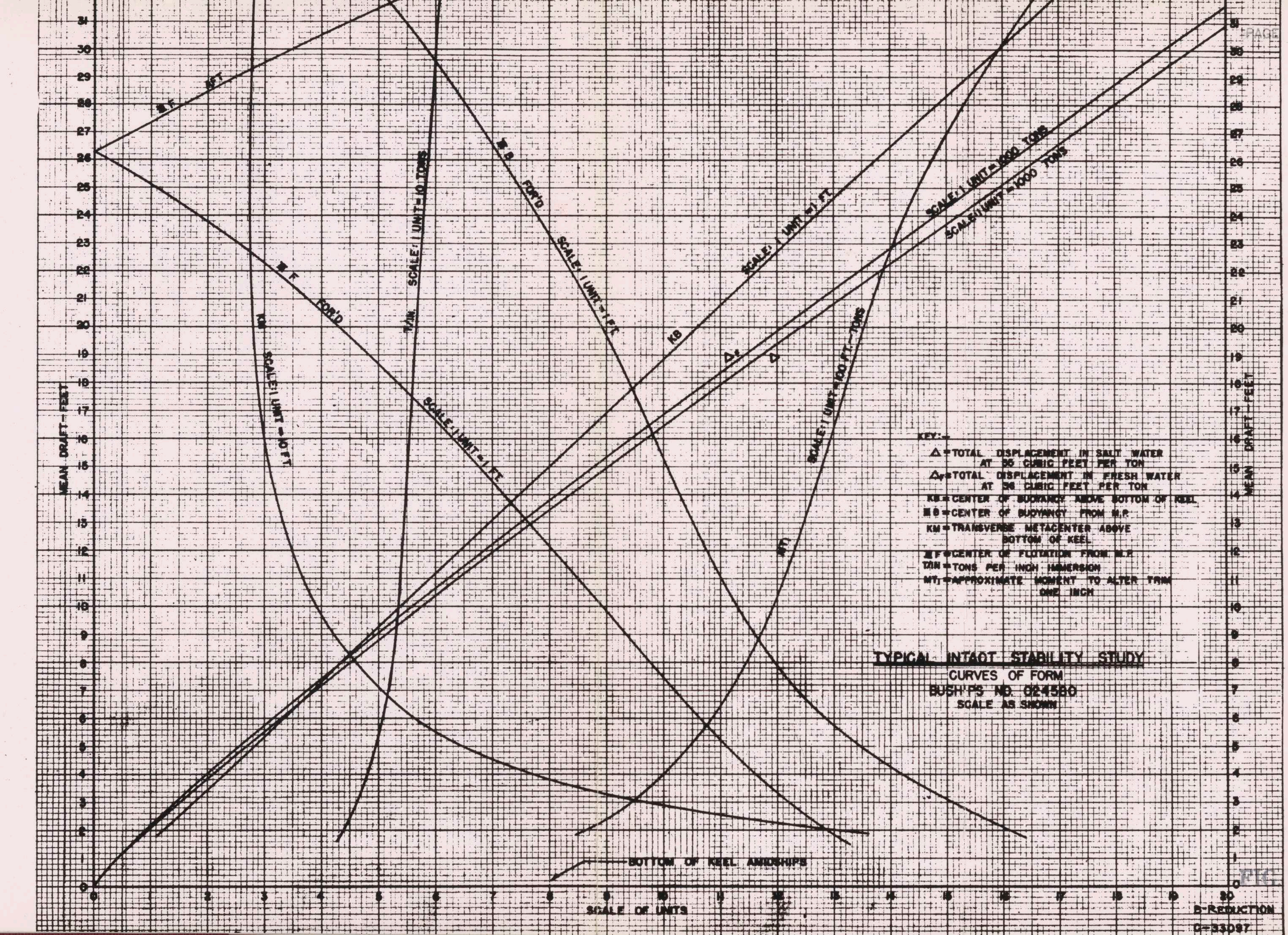

MEAN DRAFT—FEET
SCALE OF UNITS
KEY:—
Δ = TOTAL DISPLACEMENT IN SALT WATER AT 35 CUBIC FEET PER TON
Δf = TOTAL DISPLACEMENT IN FRESH WATER AT 36 CUBIC FEET PER TON
KB = CENTER OF BUOYANCY ABOVE BOTTOM OF KEEL
⊠B = CENTER OF BUOYANCY FROM M.P.
KM = TRANSVERSE METACENTER ABOVE BOTTOM OF KEEL
⊠F = CENTER OF FLOTATION FROM M.P.
T/IN = TONS PER INCH IMMERSION
MT1 = APPROXIMATE MOMENT TO ALTER TRIM ONE INCH
TYPICAL INTACT STABILITY STUDY
CURVES OF FORM
BUSHIPS NO. 024580
SCALE AS SHOWN
KM SCALE: 1 UNIT = 10 FT.
T/IN. SCALE: 1 UNIT = 10 TONS
⊠B FOR'D SCALE: 1 UNIT = 1 FT.
⊠F FOR'D SCALE: 1 UNIT = 1 FT.
KB SCALE: 1 UNIT = 1 FT.
SCALE: 1 UNIT = 1000 TONS
SCALE: 1 UNIT = 1000 TONS
MT1 SCALE: 1 UNIT = 100 FT.-TONS
BOTTOM OF KEEL AMIDSHIPS
B-REDUCTION
C-33097

FIG. 1

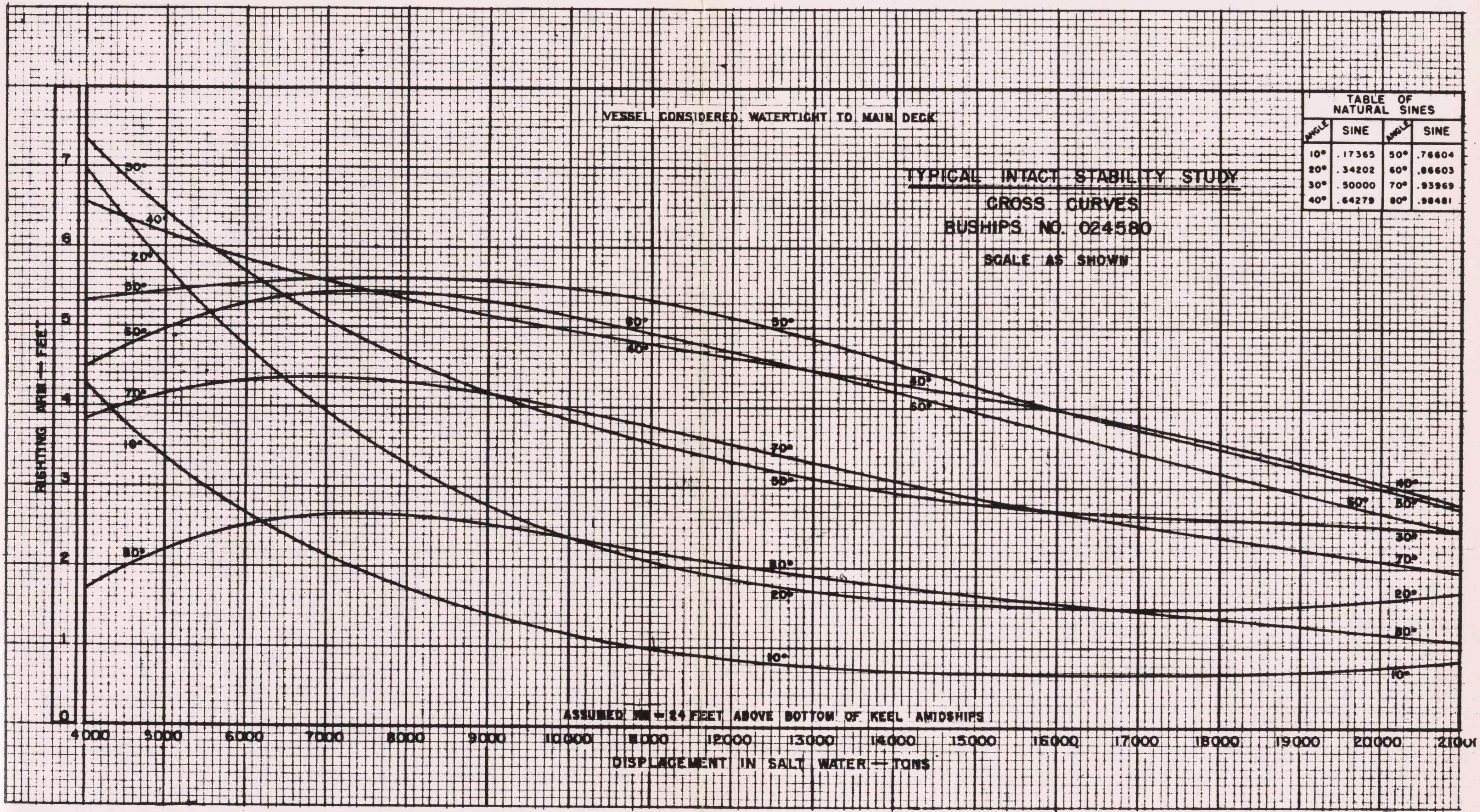

VESSEL CONSIDERED WATERTIGHT TO MAIN DECK
TYPICAL INTACT STABILITY STUDY
CROSS CURVES
BUSHIPS NO. 024580
SCALE AS SHOWN
TABLE OF NATURAL SINES
ANGLE SINE ANGLE SINE
10° .17365 50° .76604
20° .34202 60° .86603
30° .50000 70° .93969
40° .64279 80° .98481
RIGHTING ARM — FEET
0 1 2 3 4 5 6 7
10° 20° 30° 40° 50° 60° 70° 80°
FEET ABOVE BOTTOM OF KEEL AMIDSHIPS
4000 5000 6000 7000 8000 9000 10000 11000 12000 13000 14000 15000 16000 17000 18000 19000 20000 21000
DISPLACEMENT IN SALT WATER — TONS

FIG. 2

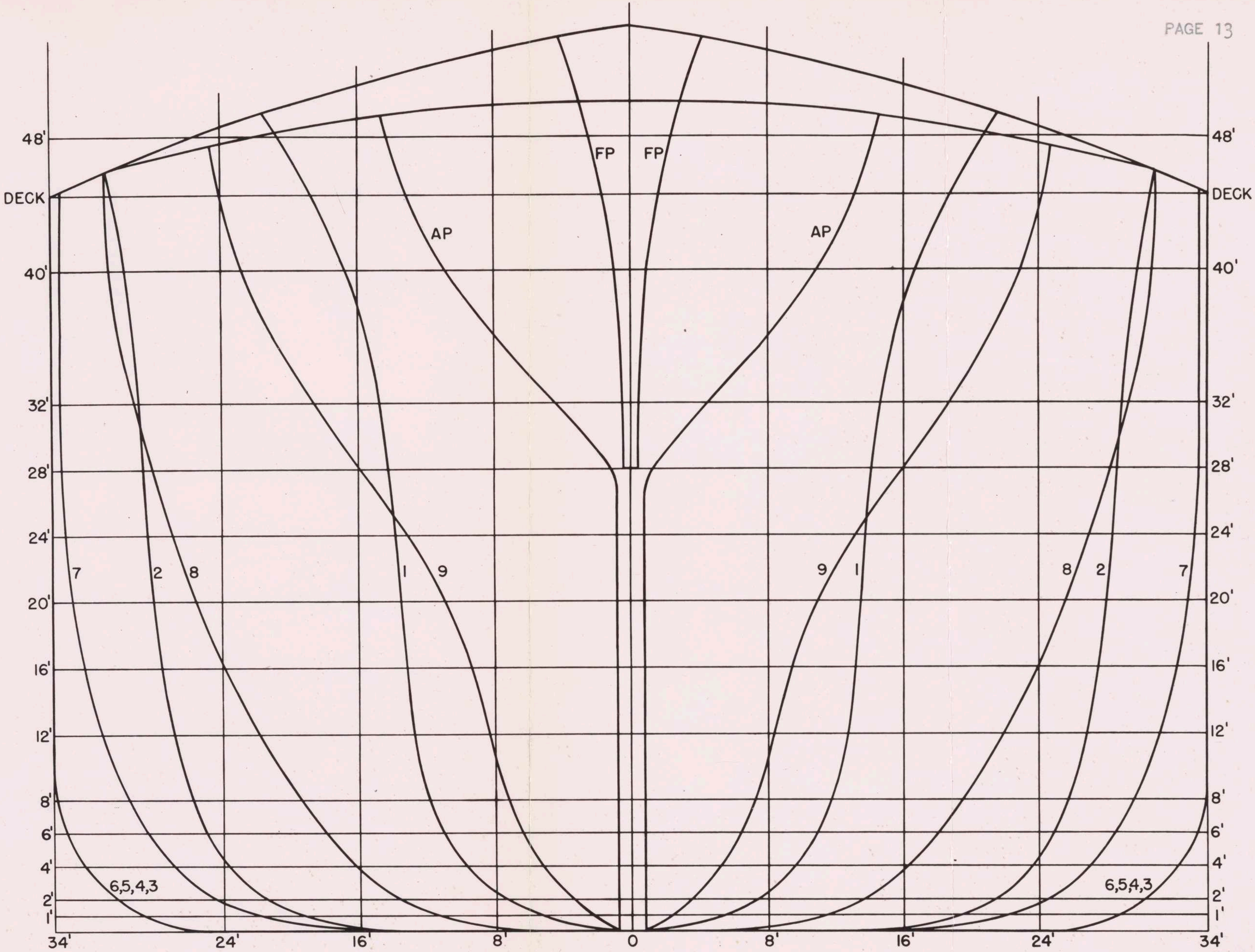


FIG. 3
TYPICAL INTACT STABILITY STUDY
BODY PLAN-FROM BUSHIPS NO. 021613
SCALE 1/4" = 1'

U.S.S. TYPICAL STUDY
CALCULATED BY: J.W.W.
CHECKED BY:

INTACT STABILITY CALCULATIONS

TABULATION #1

INTEGRATION OF WATERPLANES

L = 480 FT.

STA.	h_1 = 2 FT. b/2	$(b/2)^3$	t	(b/2)t	$(b/2)t^2$	h_2 = 4 FT. b/2	$(b/2)^3$	t	(b/2)t	$(b/2)t^2$
1	2.57	20	+0.458	1.177	.539	3.26	40	+0.458	1.493	.684
2	14.82	3260	+0.344	5.098	1.754	17.41	5280	+0.344	5.989	2.060
3	27.56	20930	+0.250	6.890	1.722	29.46	25570	+0.250	7.365	1.841
4	30.10	27270	+0.156	4.696	.732	32.14	33200	+0.156	5.014	.782
5	30.10	27270	+0.042	1.264	.053	32.14	33200	+0.042	1.350	.057
6	30.10	27270	−0.042	−1.264	.053	32.14	33200	−0.042	−1.350	.057
7	27.83	21560	−0.156	−4.342	.677	30.19	27520	−0.156	−4.710	.735
8	19.45	7360	−0.250	−4.862	1.216	22.12	10820	−0.250	−5.530	1.382
9	8.34	580	−0.344	−2.869	.987	10.93	1310	−0.344	−3.760	1.293
10	1.34	0	−0.458	− .614	.281	1.63	0	−0.458	− .746	.342
Σ	192.21	135520		5.174	8.014	211.42	170140		5.115	9.233

STA.	h_3 = 6 FT. b/2	$(b/2)^3$	t	(b/2)t	$(b/2)t^2$	h_4 = 8 FT. b/2	$(b/2)^3$	t	(b/2)t	$(b/2)t^2$	h_5 = 12 FT. b/2	$(b/2)^3$	t	(b/2)t	$(b/2)t^2$
1	3.75	50	+0.458	1.718	.787	4.06	70	+0.458	1.860	.852	4.46	90	+0.458	2.043	−.936
2	18.96	6820	+0.344	6.522	2.244	19.99	7990	+0.344	6.877	2.366	21.20	9530	+0.344	7.293	2.509
3	30.48	28320	+0.250	7.620	1.905	31.14	30200	+0.250	7.785	1.946	31.75	32010	+0.250	7.938	1.984
4	33.19	36560	+0.156	5.178	.808	33.74	38410	+0.156	5.263	.821	34.00	39300	+0.156	5.304	.827
5	33.19	36560	+0.042	1.394	.058	33.74	38410	+0.042	1.417	.060	34.00	39300	+0.042	1.428	.060
6	33.19	36560	−0.042	−1.394	.058	33.74	38410	−0.042	−1.417	.060	34.00	39300	−0.042	−1.428	.060
7	31.48	31200	−0.156	−4.911	.766	32.25	33540	−0.156	−5.031	.785	33.08	36200	−0.156	−5.161	.805
8	23.82	13520	−0.250	−5.955	1.489	25.11	15830	−0.250	−6.278	1.569	27.20	20120	−0.250	−6.800	1.700
9	12.57	1990	−0.344	−4.324	1.488	13.85	2660	−0.344	−4.764	1.639	15.98	4080	−0.344	−5.497	1.891
10	1.97	10	−0.458	− .902	.413	2.18	10	−0.458	− .998	.457	2.42	10	−0.458	−1.108	.508
Σ	222.60	191590		4.946	10.016	229.80	205530		4.714	10.555	238.09	219940		4.012	11.280

A $(2/10)L\,[\Sigma(b/2)] = 96[\Sigma(b/2)]$

M $(2/10)L^2\,[\Sigma(b/2)t] = 46080[\Sigma(b/2)t]$

$I_℄$ $(2/30)L\,[\Sigma(b/2)^3] = 32[\Sigma(b/2)^3]$

I_{III} $(2/10)L^3\,[\Sigma(b/2)t^2] = 22{,}118{,}400[\Sigma(b/2)t^2]$

	h_1 A	M	$I_℄$	I_{III}	h_2 A	M	$I_℄$	I_{III}	h_3 A	M	$I_℄$	I_{III}	h_4 A	M	$I_℄$	I_{III}	h_5 A	M	$I_℄$	I_{III}
FORD. F.P.																				
BETW. PERPS.																				
AFT A.P.																				
TOTAL	18,452	238,400	4,336,000	177,260,000	20,296	235,700	5,444,000	204,200,000	21,370	227,900	6,131,000	221,500000	22,061	217,200	6,576,600	233,500,000	22,857	184,870	7,038,100	249,500,000

STA.	h_6 = 16 FT. b/2	$(b/2)^3$	t	(b/2)t	$(b/2)t^2$	h_7 = 20 FT. b/2	$(b/2)^3$	t	(b/2)t	$(b/2)t^2$
1	4.70	100	+0.458	2.153	.986	4.90	120	+0.458	2.244	1.028
2	21.86	10450	+0.344	7.520	2.587	22.29	11080	+0.344	7.668	2.638
3	32.10	33080	+0.250	8.025	2.006	32.34	33820	+0.250	8.085	2.021
4	34.00	39300	+0.156	5.304	.827	34.00	39300	+0.156	5.304	.827
5	34.00	39300	+0.042	1.428	.060	34.00	39300	+0.042	1.428	.060
6	34.00	39300	−0.042	−1.428	.060	34.00	39300	−0.042	−1.428	.060
7	33.56	37800	−0.156	−5.235	.817	33.88	38890	−0.156	−5.285	.824
8	28.92	24190	−0.250	−7.230	1.808	30.13	27350	−0.250	−7.532	1.883
9	17.90	5740	−0.344	−6.158	2.118	19.71	7660	−0.344	−6.780	2.332
10	2.74	20	−0.458	−1.255	.575	3.20	30	−0.458	−1.466	.671
Σ	243.78	229280		3.124	11.844	248.45	236850		2.238	12.344

STA.	h_8 = 24 FT. b/2	$(b/2)^3$	t	(b/2)t	$(b/2)t^2$	h_9 = 28 FT. b/2	$(b/2)^3$	t	(b/2)t	$(b/2)t^2$	h_{10} = 32 FT. b/2	$(b/2)^3$	t	(b/2)t	$(b/2)t^2$
1	5.16	140	+0.458	2.363	1.082	5.45	160	+0.458	2.496	1.143	5.86	200	+0.458	2.684	1.229
2	22.59	11530	+0.344	7.771	2.673	22.86	11950	+0.344	7.864	2.705	23.24	12550	+0.344	7.995	2.750
3	32.56	34520	+0.250	8.140	2.035	32.68	34900	+0.250	8.170	2.042	32.81	35320	+0.250	8.202	2.051
4	34.00	39300	+0.156	5.304	.827	34.00	39300	+0.156	5.304	.827	34.00	39300	+0.156	5.304	.827
5	34.00	39300	+0.042	1.428	.060	34.00	39300	+0.042	1.428	.060	34.00	39300	+0.042	1.428	.060
6	34.00	39300	−0.042	−1.428	.060	34.00	39300	−0.042	−1.428	.060	34.00	39300	−0.042	−1.428	.060
7	33.89	38920	−0.156	−5.287	.825	33.90	38960	−0.156	−5.288	.825	33.92	39030	−0.156	−5.292	.826
8	31.02	29850	−0.250	−7.755	1.939	31.60	31540	−0.250	−7.900	1.975	32.09	33040	−0.250	−8.022	2.006
9	21.62	10110	−0.344	−7.437	2.558	23.41	12830	−0.344	−8.053	2.770	25.05	15720	−0.344	−8.617	2.964
10	4.64	100	−0.458	−2.125	.973	7.46	420	−0.458	−3.417	1.565	10.85	1280	−0.458	−4.969	2.276
Σ	253.48	243070		.974	13.032	259.36	248660		− .824	13.972	265.82	255040		−2.715	15.049

A $(2/10)L\,[\Sigma(b/2)]$

M $(2/10)L^2\,[\Sigma(b/2)t]$

$I_℄$ $(2/30)L\,[\Sigma(b/2)^3]$

I_{III} $(2/10)L^3\,[\Sigma(b/2)t^2]$

	h_6 A	M	$I_℄$	I_{III}	h_7 A	M	$I_℄$	I_{III}	h_8 A	M	$I_℄$	I_{III}	h_9 A	M	$I_℄$	I_{III}	h_{10} A	M	$I_℄$	I_{III}
FORD. F.P.																	1	150	Negl.	
BETW. PERPS.													24,899	−38,000	7,957,100	309,000,000	25,519	−125,110	8,161,300	332,900,000
AFT A.P.													6	− 1,500	Negl.	400,000	59	− 14,340	300	3,500,000
TOTAL	23,403	143,950	7,337,000	262,000,000	23,851	103,130	7,579,200	273,000,000	24,334	44,900	7,778,200	288,300,000	24,905	−39,500	7,957,100	309,400,000	25,579	−139,300	8,161,600	336,400,000

K.H.C.

FIG. 4

B-33368

U.S.S. TYPICAL STUDY
CALCULATED BY J. W. W.
CHECKED BY

INTACT STABILITY CALCULATIONS

TABULATION #2

INTEGRATION OF STATIONS IN UPRIGHT POSITION

L = 480 FT.

STA.	h_1 = 2 FT. a	m	t	at	h_2 = 4 FT. a	m	t	at
1	13	1	+0.458	6.0	30	5	+0.458	13.7
2	79	7	+0.344	27.2	176	25	+0.344	60.5
3	163	11	+0.250	40.8	336	48	+0.250	84.0
4	177	13	+0.156	27.6	369	52	+0.156	57.6
5	177	13	+0.042	7.4	369	52	+0.042	15.5
6	177	13	-0.042	- 7.4	369	52	-0.042	-15.5
7	150	11	-0.156	-23.4	336	47	-0.156	-52.4
8	104	8	-0.250	-26.0	236	33	-0.250	-59.0
9	36	3	-0.344	-12.4	94	14	-0.344	-32.3
10	7	1	-0.458	- 3.2	19	3	-0.458	- 8.7
Σ	1083	81		36.6	2334	331		63.4

STA.	h_3 = 6 FT. a	m	t	at	h_4 = 8 FT. a	m	t	at	h_5 = 12 FT. a	m	t	at
1	53	12	+0.458	24.3	77	23	+0.458	35.3	129	57	+0.458	59.1
2	293	64	+0.344	100.8	410	116	+0.344	141.0	668	278	+0.344	229.8
3	528	109	+0.250	132.0	719	192	+0.250	179.8	1110	438	+0.250	277.5
4	582	117	+0.156	90.8	784	208	+0.156	122.3	1204	472	+0.156	187.8
5	582	117	+0.042	24.4	784	208	+0.042	32.9	1204	472	+0.042	50.6
6	582	117	-0.042	- 24.4	784	208	-0.042	- 32.9	1204	472	-0.042	- 50.6
7	534	108	-0.156	- 83.3	728	197	-0.156	-113.6	1123	451	-0.156	-175.2
8	379	80	-0.250	- 94.8	530	147	-0.250	-132.5	852	352	-0.250	-213.0
9	165	40	-0.344	- 56.8	251	75	-0.344	- 86.3	439	190	-0.344	-151.0
10	29	7	-0.458	- 13.3	40	13	-0.458	- 18.3	71	31	-0.458	- 32.5
Σ	3727	771		99.9	5107	1387		127.7	8004	3213		182.5

V: (1/10)c' s^2 L [Σa] = $\frac{1}{10}$x .04x4^2x480x[Σa] = 30.72 x[Σa]

M_V: (1/10)c'' s^3 L [Σm] = $\frac{1}{10}$x .16x4^3x480x[Σm] = 491.52[Σm]

M_I: (1/10)c' s^2 L^2 [Σat] = $\frac{1}{10}$x .04x4^2x480^2x[Σat] = 14,746[Σat]

	h_1 V	M_V	M_I	h_2 V	M_V	M_I	h_3 V	M_V	M_I	h_4 V	M_V	M_I	h_5 V	M_V	M_I
FORD. F.P.															
BETW. PERPS.															
AFT A.P.															
TOTAL	33,300	40,000	540,000	71,700	162,700	935,000	114,490	379,000	1,473,000	156,890	682,000	1,883,000	245,900	1,579,300	2,690,000

STA.	h_6 = 16 FT. a	m	t	at	h_7 = 20 FT. a	m	t	at
1	186	107	+0.458	85.2	247	176	+0.458	113.1
2	934	525	+0.344	321.3	1220	832	+0.344	419.7
3	1513	790	+0.250	378.2	1920	1253	+0.250	480.0
4	1628	847	+0.156	254.0	2061	1327	+0.156	321.5
5	1628	847	+0.042	68.4	2061	1327	+0.042	86.6
6	1628	847	-0.042	- 68.4	2061	1327	-0.042	- 86.6
7	1547	819	-0.156	-241.3	1978	1298	-0.156	-308.6
8	1206	663	-0.250	-301.5	1581	1085	-0.250	-395.2
9	650	380	-0.344	-223.6	885	650	-0.344	-304.4
10	103	58	-0.458	- 47.2	136	96	-0.458	- 62.3
Σ	11023	5883		225.1	14150	9371		263.8

STA.	h_8 = 24 FT. a	m	t	at	h_9 = 28 FT. a	m	t	at	h_{10} = 32 FT. a	m	t	at
1	310	264	+0.458	142.0	373	373	+0.458	170.8	447	504	+0.458	204.7
2	1503	1225	+0.344	517.0	1778	1683	+0.344	611.6	2068	2228	+0.344	711.4
3	2325	1819	+0.250	581.2	2723	2476	+0.250	680.8	3121	3239	+0.250	780.2
4	2489	1928	+0.156	388.3	2905	2616	+0.156	453.2	3332	3413	+0.156	519.8
5	2489	1928	+0.042	104.5	2905	2616	+0.042	122.0	3332	3413	+0.042	139.9
6	2489	1928	-0.042	-104.5	2905	2616	-0.042	-122.0	3332	3413	-0.042	-139.9
7	2398	1893	-0.156	-374.1	2822	2582	-0.156	-440.2	3247	3381	-0.156	-506.5
8	1967	1619	-0.250	-491.8	2352	2258	-0.250	-588.0	2752	3007	-0.250	-688.0
9	1149	1014	-0.344	-395.3	1425	1471	-0.344	-490.2	1728	2043	-0.344	-594.4
10	186	167	-0.458	- 85.2	261	292	-0.458	-119.5	376	513	-0.458	-172.2
Σ	17305	13785		281.9	20449	18983		278.5	23735	25154		255.0

V: (1/10)c' s^2 L [Σa]

M_V: (1/10)c'' s^3 L [Σm]

M_I: (1/10)c' s^2 L^2 [Σat]

	h_6 V	M_V	M_I	h_7 V	M_V	M_I	h_8 V	M_V	M_I	h_9 V	M_V	M_I	h_{10} V	M_V	M_I
FORD. F.P.															0
BETW. PERPS.										628,190	9,331,000	4,107,000	729,140	12,363,700	3,760,000
AFT A.P.										0	0	-2,000	120	3,600	-29,000
TOTAL	338,600	2,892,000	3,319,000	434,700	4,606,000	3,890,000	531,600	6,776,000	4,157,000	628,190	9,331,000	4,105,000	729,260	12,367,300	3,731,000

FIG. 5

K.H.C.

B-33369

U.S.S. TYPICAL STUDY
CALCULATED BY K. H. C.
CHECKED BY

INTACT STABILITY CALCULATIONS

TABULATION # 3

INTEGRATION OF STATIONS IN INCLINED POSITIONS

KO = 24 FT.
L = 480 FT.

8' W_1L_1

STA.	10° a	10° m_t	20° a	20° m_t	30° a	30° m_t	40° a	40° m_t	50° a	50° m_t	60° a	60° m_t	70° a	70° m_t	80° a	80° m_t
1																
2																
3																
4																
5																
6																
7																
8																
9																
10																
Σ	5081	1209	4975	2023	5177	2205	5494	2138	6064	2081	6736	2225	7352	2004	7662	1267
V_1																
M_t																
	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t
FORD. F.P.																
BETW PERPS.																
AFT A.P.																
TOTAL	156K	594K	152K	994K	159K	1,083K	168K	1,050K	186K	1,022K	206K	1,093K	225K	985K	235K	623K

16' W_2L_2

STA.	10° a	10° m_t	20° a	20° m_t	30° a	30° m_t	40° a	40° m_t	50° a	50° m_t	60° a	60° m_t	70° a	70° m_t	80° a	80° m_t
1																
2																
3																
4																
5																
6																
7																
8																
9																
10																
Σ	11028	830	10809	1725	10700	2691	10983	3443	11641	3979	11990	3754	12237	2928	12386	1711
V_1																
M_t																
	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t
FORD. F.P.																
BETW PERPS.																
AFT A.P.																
TOTAL	338K	408K	332K	848K	328K	1,322K	337K	1,692K	357K	1,955K	368K	1,845K	375K	1,439K	380K	841K

$\frac{480}{10} \times 0.04 \times 4^2 \times [\Sigma a] = 30.72 \times [\Sigma a]$

$\frac{480}{10} \times 0.16 \times 4^3 \times [\Sigma m_t] = 491.52 \times [\Sigma m_t]$

V_1: $(L/10)c's^2[\Sigma a]$

M_t: $(L/10)c''s^3[\Sigma m_t]$

24' W_3L_3

STA.	10° a	10° m_t	20° a	20° m_t	30° a	30° m_t	40° a	40° m_t	50° a	50° m_t	60° a	60° m_t	70° a	70° m_t	80° a	80° m_t
1																
2																
3																
4																
5																
6																
7																
8																
9																
10																
Σ	17278	739	17349	1633	17556	3032	17736	4503	17852	4758	17797	4213	17819	3084	17835	1786
V_1																
M_t																
	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t
FORD F.P.																
BETW PERPS																
AFT A.P.																
TOTAL	531K	363K	533K	803K	539K	1,490K	544K	2,213K	548K	2,339K	546K	2,071K	547K	1,515K	547K	878K

32' W_4L_4 / **SUBMERGED**

STA.	10° a	10° m_t	20° a	20° m_t	30° a	30° m_t	40° a	40° m_t	50° a	50° m_t	60° a	60° m_t	70° a	70° m_t	80° a	80° m_t	SUBMERGED 90° a	SUBMERGED 90° m_t
1																		
2																		
3																		
4																		
5																		
6																		
7																		
8																		
9																		
10																		
Σ	23873	1241	24304	2649	24571	3699	24289	4110	24001	4099	23514	3764	23321	2951	23169	1700	35462	758
V_1																		
M_t																		
	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t	V_1	M_t
FORD F.P.																		
BETW PERPS																		
AFT A.P.																		
TOTAL	733K	610K	746K	1,302K	737K	1,818K	746K	2,020K	737K	2,015K	722K	1,850K	716K	1,450K	711K	836K	1,089K	373K

FIG. 6

K.H.C.

B-33370

U.S.S. TYPICAL STUDY
CALCULATED BY J. W. W. and K. H. C.
CHECKED BY

INTACT STABILITY CALCULATIONS
TABULATION #4
FINAL RESULTS

L = 480 FT.

	VALUE	FORMULA	h_1 = 2 FT.	h_2 = 4 FT.	h_3 = 6 FT.	h_4 = 8 FT.	h_5 = 12 FT.	h_6 = 16 FT.	h_7 = 20 FT.	h_8 = 24 FT.	h_9 = 28 FT.	h_{10} = 32 FT.
PLOT	Δ	$\frac{TOTAL\ V}{35}$	940	2050	3270	4480	7030	9690	12430	15200	17950	20830
PLOT	Δ_F	$\frac{TOTAL\ V}{36}$	920	1990	3180	4360	6830	9410	12070	14770	17450	20260
PLOT	KB	$\frac{TOTAL\ M_V}{TOTAL\ V}$	1.20	2.27	3.31	4.35	6.422	8.541	10.596	12.746	14.854	16.960
PLOT	⊠B	$\pm \frac{TOTAL\ M_L}{TOTAL\ V}$	16.23	13.04	12.86	12.00	10.94	9.80	8.95	7.82	6.53	5.12
PLOT	T/IN.	$\frac{TOTAL\ A}{420}$	44.	48.3	50.9	52.5	54.4	55.7	56.8	57.9	59.3	60.9
PLOT	⊠F	$\pm \frac{TOTAL\ M}{TOTAL\ A}$	12.92	11.61	10.66	9.84	8.09	6.15	4.32	1.84	-1.58	-5.45
	BM	$\frac{TOTAL\ I_t}{TOTAL\ V}$	130.2	75.93	53.55	41.92	28.62	21.67	17.435	14.635	12.667	11.192
PLOT	KM	KB + BM	131.4	78.20	56.86	46.27	35.05	30.21	28.03	27.38	27.52	28.15
	I_o	TOT. $I_{⊠}$ − (TOT. A)(⊠F)^2	174 200 000	201 000 000	220 000 000	232 000 000	249 000 000	261 000 000	273 000 000	288 000 000	309 000 000	335 000 000
	BM_L	$\frac{I_o}{TOTAL\ V}$	5230	2800	1922	1479	1013	771	628	542	492	459
PLOT	MT_1	$\frac{\Delta (BM_L)}{12 \cdot L}$	850	1000	1090	1150	1240	1300	1360	1430	1530	1660
		⊠ F^2	166.9	134.8	113.6	96.8	65.4	37.8	18.66	3.4	2.5	29.7
		(Tot. A) (⊠F)^2	3 100 000	3 000 000	2 000 000	2 000 000	1 000 000	1 000 000	0	0	0	1 000 000

KO = 24'

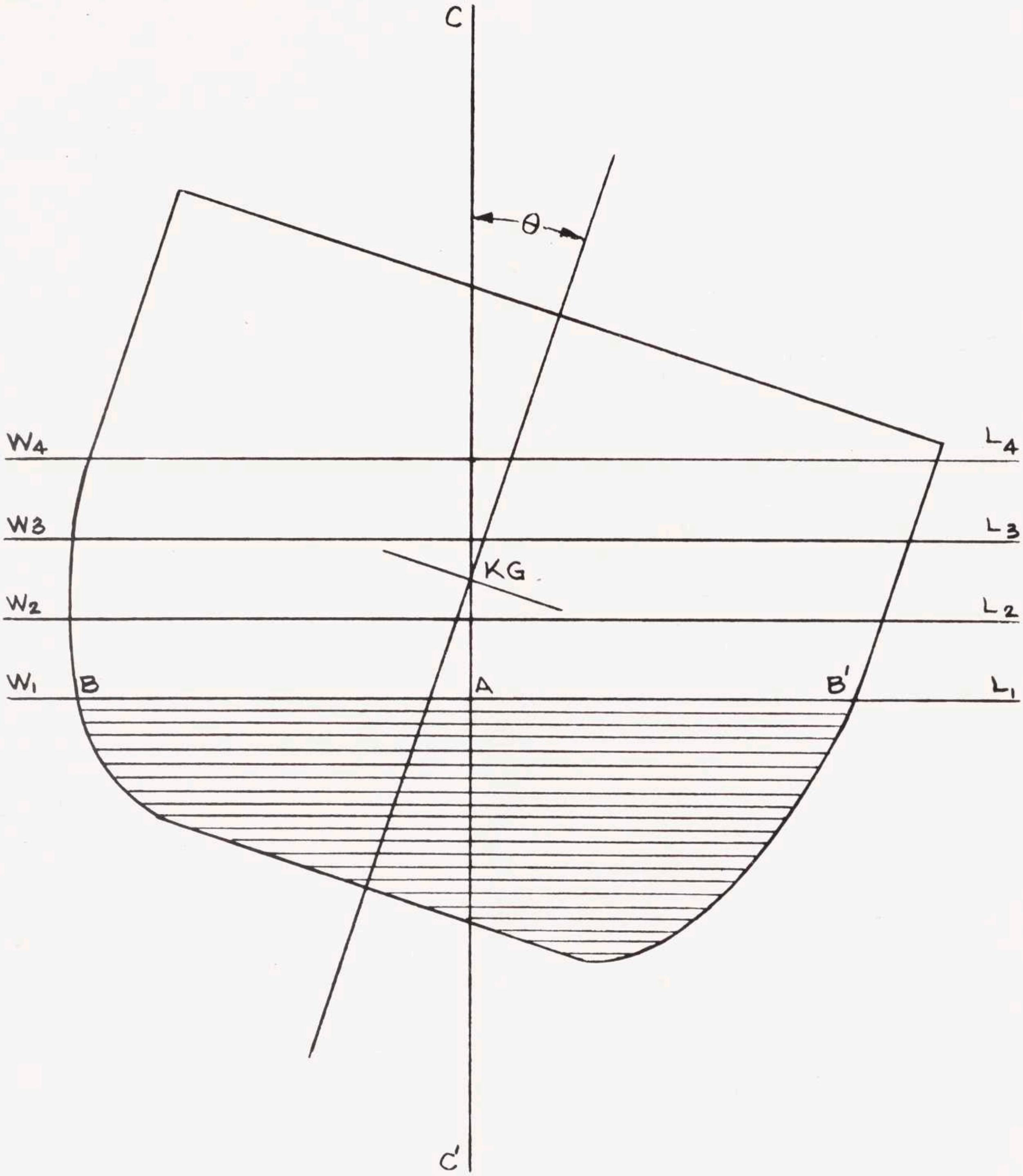
			10°		20°		30°		40°		50°		60°		70°		80°		90°	
			Δ	OZ	Δ	OZ	Δ	OZ	Δ	OZ	Δ	OZ	Δ	OZ	Δ	OZ	Δ	OZ	Δ	OZ
PLOT	LOW POINT	DIRECT MEASUREMENT	0	21.2'		18.1'		14.3'		9.6'		4.6'		0.4'		-5.3'		-10.1'		
PLOT	W_1L_1 8'	$\frac{TOTAL\ V_1}{35}$ / $\frac{TOTAL\ M_1}{TOTAL\ V_1}$	4460	3.8'	4370	6.51'	4540	6.81'	4820	6.23'	5320	5.49'	5910	5.29'	6450	4.36'	6720	2.6'		
PLOT	$W_2L_2$16'	$\frac{TOTAL\ V_1}{35}$ / $\frac{TOTAL\ M_1}{TOTAL\ V_1}$	9680	1.20'	9490	2.6'	9390	4.02'	9640	5.02'	10220	5.47'	10520	5.01'	10740	3.83'	10870	2.2'		
PLOT	$W_3L_3$24'	$\frac{TOTAL\ V_1}{35}$ / $\frac{TOTAL\ M_1}{TOTAL\ V_1}$	15170	0.68'	15230	1.51'	15410	2.76'	15570	4.06'	15670	4.08'	15620	3.79'	15640	2.77'	15650	1.60'	15560	0.34'
PLOT	$W_4L_4$32'	$\frac{TOTAL\ V_1}{35}$ / $\frac{TOTAL\ M_1}{TOTAL\ V_1}$	20950	0.83'	21330	1.74'	21060	2.47'	21320	2.71'	21070	2.73'	20640	2.56'	20470	2.02'	20340	1.17'		
PLOT	SUBMGD.	$\frac{TOTAL\ V_1}{35}$ / SIN θ · $\frac{TOTAL\ M_1}{TOTAL\ V_1}$	31130	0.06'	31130	0.12'	31130	0.17'	3113	0.22'	31130	0.26'	31130	0.29'	31130	0.32'	31130	0.33'	31130	0.34'

K.H.C.

FIG. 7

B-33371

A-33250



ROTATED WATERLINES
ON A SHIP SECTION

FIG. 8

III - The Whirlwind Computer

General description

Generally speaking, the device commonly referred to as Whirlwind One, or simply WWI, being developed by the ONR-sponsored Project Whirlwind at M.I.T. is a large-scale, high-speed electronic digital computer. The term computer is used here to mean a device capable of performing the basic arithmetic operations characteristic of a calculating machine according to some predetermined plan, so that a computer may be thought of as an automatic sequence controlled calculator. A digital computer is one which operates by the use of numbers (digital quantities), as in a desk calculating machine, rather than by the use of shaft rotations, rod positions, or other analog quantities, as in a slide rule, a planimeter, or a differential analyzer. The term electronic implies that the computer is constructed for the most part out of electron tubes rather than out of shafts, gears, relays, motors, and such mechanical and electrical devices. The adjectives high-speed and large-scale are of course purely relative and are used here to mean that in point of memory capacity and operating speed the Whirlwind Computer is nearly as large and as rapid as any computer built or being built in 1948.

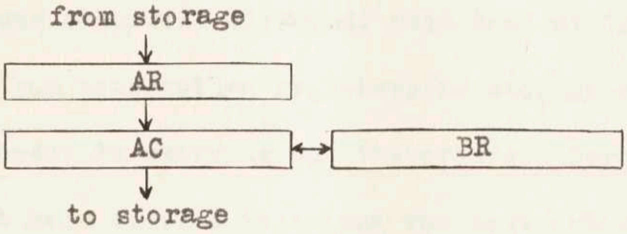
In particular, WWI (having a multiplication time of about fifty millionths of a second) is actually intended to be of the order of half a million times as rapid as a person using an ordinary desk calculator. Although it is intended as a prototype for a larger computer to be built later, WWI has a high-speed memory capacity of 2048 words, each word containing sixteen binary (or 4.8 decimal) digits, for a total capacity of about 10000 decimal digits. Whirlwind I uses the pure binary number scale in its operation. Like most other present-day computers, it is a fixed-point computer in which the binary (analogous to decimal) point remains at the left-hand end of every number, so that a number of magnitude greater than or equal to one is not admissible in the computer.

Design and nomenclature

Any electronic circuit which has two and only two mutually exclusive possible states (for instance, a switch which can be open or closed, or a voltage polarity which can be positive or negative) can be used to "store" a digit, and for that reason can be called a "digit space". The set of digit spaces in which a binary number is stored is called a "register". Generally speaking, all registers in a computer are of the same length, i.e., have the same number of digit spaces. In Whirlwind I the registers have 16 digit spaces and hence store a 15 binary digit number and a sign digit.

Whirlwind I has four logically independent elements which when suitably integrated make up the computer. Each of these elements has a direct analogue in ordinary manual computation arrangements, the desk calculator which performs the arithmetic operations corresponding to the "arithmetic element", the operator who supplies numbers to the calculator together with instructions of what operations to perform (e.g., add, multiply, divide) corresponding to "control", the form on which the instructions ("program") are printed and on which the intermediate results are to be written corresponding to "storage", and the paper on which the data and results are entered corresponding to "input and output".

The arithmetic element is composed of three 16 digit registers, called the A-Register (AR), the Accumulator (AC), and the B-Register (BR) which may be thought of in this pattern:



Numbers can be transmitted from storage to AR, and the contents of AR can be added or subtracted into whatever is in AC. Numbers can be transmitted from AC to storage, and AC can be made to shift its contents to the left or to the right, the contents of each digit space (except the leftmost or sign digit) moving into the digit next to it on the right or left. The contents of BR is always shifted simultaneously with that of AC and the leftmost digit of BR is connected as if it were on the right of the rightmost digit of AC.

In order to carry out multiplications, divisions, and multiple shift operations, some circuitry known as "arithmetic control" is provided. This control directs the arithmetic element through the necessary repeated additions, subtractions, and shifts to perform multiplication, division, and shifts.

Storage is composed of $2048 = 2^{11}$ identical storage registers which are numbered consecutively from 0 through 2047. The eleven digit binary number corresponding to a particular register is called its "address". It might seem that a distinction should be made between the storage used to store the orders or program and the storage used to store numbers, but such a distinction is in fact unnecessary and any storage registers can be used to store either an order or a number.

Unless otherwise directed, main control in Whirlwind I selects orders from consecutive registers in storage and directs the arithmetic element in carrying out the orders. Certain orders (sp, cp) can direct main control to select the next order not from the next register in succession, but from a new register anywhere in storage.

An order is a 16 digit number which is interpreted in two parts as shown:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Operation					Address										

The eleven digit address section is generally used to select the storage register from which a number to be operated on is to be taken or in which a number to be stored is to be stored. If, however, the operation called for is a shift, the address section is merely an indication of the number of single-step shifts to be made, while if the operation is intended to direct main control, the address section gives the address from which the next order is to be taken.

WWI operations

The five digits assigned to the operation section of an order give the possibility of indicating one of $2^5 = 32$ different operations. For convenience in writing, these different operations are each written as a pair of lower case English letters. The letters, or initials, are chosen with the intention of suggesting the operation. In starting any process, it is usually necessary to clear AC and transfer a new number from storage to AC. Therefore, an operation may be chosen with which to clear AC and transfer to AC either the positive or the negative of the value of the contents of the storage register indicated in the address section of the order:

ca - clear and add

cs - clear and subtract

To add or subtract the contents of a second register to or from the number in AC there are the operations:

ad - add

su - subtract

In order to store a result:

ts - transfer to storage

In order to multiply or divide, the number to be multiplied or divided is first put in AC by means of the ca (or cs) operation and use is made of one of the following operations, the address section of the order indicating the multiplier or divisor:

mr - multiply and roundoff

mh - multiply and hold full product

dv - divide

When two 15 digit numbers are multiplied together, a 30 digit product is formed, and in WWI the first, or significant 15 digits appear in AC, the last 15 in BR, and these last 15 may be correctly rounded off, by mr, or left intact, by mh. Because of electronic peculiarities, the quotient after a division appears in BR and must be shifted into AC by use of a separate shift left operation. As has been mentioned, the address section of a shift order indicates the number of shift steps to be made, the operations being:

sr - shift right

sl - shift left

The operation which causes main control to select the next order from the storage register whose address appears in the address section of the order is

sp - subprogram

In order to repeat a process a definite number of times (for instance, until an iterative process has converged satisfactorily), it is necessary to be able to subprogram or not depending on the size of some number. Hence, the operation

cp - condition subprogram

acts like the subprogram operation if the number in AC is positive and does nothing if the contents of AC is negative or zero.

In order to change the address section of an order without affecting the operation section, there is an operation by which only the eleven digits corresponding to the address section are transferred to the storage register:

td - transfer digits

Placing a new address into AC prior to using a td order may be accomplished by having the address stored separately somewhere and using a ca order to get it to AC. Storing the new address separately can be avoided by use of a special operation (which is really intended as part of a specialized procedure called "automatic subprogramming"):

as - automatic setup

Part of the result of the as operation is the transfer to AC of the entire as order itself, thus in particular putting the address section of the as order into AC, whence the td operation transfers it to storage. Thus to insert the new address 678 into the address section of the order in register 1284, it is necessary only to order as 678 followed by td 1284.

In many cases (for instance, when keeping count in a multiple step program) it is desirable to be able to increase the contents of a storage register by one in the right-hand end. A single operation by which this increase can be accomplished and the negative of the increased number left in AC (for convenience in applying the cp operation immediately afterwards) is the operation:

ao - add one

While other rather specialized operations planned for WWI and necessary input and output operations not yet designed in detail have been omitted, the discussion above includes all the operations used in the program which follows. The operations used are listed alphabetically and described more precisely below:

ad x (add)

Add the contents of register x into whatever is in AC.

ao x (add one)

Add 2^{-15} to the contents of register x (addition actually performed in AC) and put the negative of the increased quantity into AC.

as x (automatic setup)

Clear AC and transfer the order as x itself to AC.

ca x (clear and add)

Clear AC and add the contents of register x into it.

cp x (conditional program)

If the number in AC is positive, take the next order from register number x.

cs x (clear and subtract)

Clear AC and subtract the contents of register x into it.

dv x (divide)

Divide the contents of AC by whatever is in register x.

mh x (multiply and hold full product)

Multiply the contents of register x by whatever is in AC, leave the full product in AC and BR.

mr x (multiply and round off)

Multiply the contents of register x by whatever is in AC and round off the result to one register length.

sl x (shift left)

Shift the contents of AC and BR to the left x times and round off the result to one register length.

sp x (subprogram)

Take the next order from register number x.

sr x (shift right)

Shift the contents of AC and BR to the right x times and round off the result to one register length.

su x (subtract)

Subtract the contents of register x from whatever is already in AC.

td x (transfer digits)

Transfer the right-hand eleven digits in AC to the right-hand eleven digits of register x.

ts x (transfer to storage)

Transfer the contents of AC to register x.

IV - Development of the General Procedure

In general, the tabulation forms of the typical study (Figures 4, 5, 6, 7) provide a valid basis for setting up a procedure for a computer. However, in order to reduce the amount of temporary storage space needed during the computation, the final results of Tabulation #4 are calculated and stored as soon as the necessary quantities become known rather than storing all the intermediate results and collecting them all at the end. Thus Tabulation #4 is absorbed into various parts of the program and loses its identity.

<u>Integration of waterplanes</u>

The integrations to determine area, moment, and moments of inertia of the various waterplanes are strictly numerical procedures which can be adopted directly. Since use of Tchebycheff integration requires a special set of half-breadths to be taken at the Tchebycheff stations while Simpson's rule would permit use of equally-spaced stations, and since Simpson's rule is of nearly the same accuracy and calculational simplicity (especially since the factors 1, 4, 2, 4,.. .., 1 are all "round" binary numbers) as Tchebycheff's rule, the program here uses Simpson's rule integration. Following the pattern of Tabulation #1, the half-breadths corresponding to the first waterplane (h_1) are chosen. Referring to Flow Diagram I (Figure 12), in which the blocks 1. through 31. correspond to Tabulation #1, the blocks 1. through 3. represent the selection of the half-breadths. A more detailed discussion of the individual blocks is given, together with the specific orders, in Chapter VI. After the proper half-breadths have been chosen, the Simpson's rule summation is performed (block 5.) but since the same summation must be repeated in determining M, I_{CL}, and $I_{\boxtimes}$, block 5. is actually a subprogram which is used four times.

The appendage area must be calculated and added to the integral just formed (blocks 7. and 8.). The appendages can be considered to be approximately triangles, parabolas, or rectangles, depending on the ship. In any case, the base of the triangle, parabola, or rectangle is the breadth of the ship at the forward (or aft) perpendicular, and if the height of the figure (i.e., the length of the projection fore or aft) is known, the area, moment, I_{CL}, and $I_{\boxtimes}$ can all be calculated by elementary methods. It turns out that the following formulas apply, where b is the breadth at the end perpendicular, d the length of the projection beyond the end perpendicular, and L the length of ship between end perpendiculars:

$$A = b \cdot d \cdot (1, \tfrac{2}{3}, \tfrac{1}{2})$$

$$M = \left(\frac{L}{2} + (\tfrac{1}{2}, \tfrac{2}{5}, \tfrac{1}{3})d\right) \cdot A$$

$$I_{CL} = (\tfrac{1}{3}, \tfrac{1}{5}, \tfrac{1}{6})b^2 \cdot A$$

$$I_{\boxtimes} = \left(\frac{L^2}{4} + (\tfrac{1}{3}, \tfrac{8}{35}, \tfrac{1}{6})d^2\right) \cdot A$$

The three alternative constants enclosed in parentheses are the factors to be used if the shape of the appendage most nearly approximates a rectangle, a parabola, or a triangle, respectively.

Once the total area is known, the value of $T/IN. = \frac{A}{420}$ is calculated directly (block 9.). Then the cubes of the breadths are formed and the value of I_{CL} is calculated and stored complete with the contribution of the appendages (blocks 10. to 15.). Forming the products of breadth times distance from midship station (blocks 16. to 18.), carrying out the same integration program (block 5.), calculating and adding on the appendage moment (blocks 20.and 21.), and dividing by the area already determined yields the value of $\otimes F$ (block 22.). A similar process (blocks 23. to 28.) using the square of the moment arm, results in the value of $I_{\otimes}$, and MT_1 is formed directly as indicated in block 29. Block 30. is used to prepare the program for the next waterplane, and block 31. checks to see whether all the waterplanes have been completed before starting the cycle over again.

Integration of volume in upright position

After the waterplanes have been completed, the program proceeds to the equivalent of Tabulation #2. Integrating over the width and height of the submerged portion of each station which, in the hand computation, is done with a mechanical integraph, and then forming the double integral over the length of the ship is exactly equivalent to integrating over the length and width of each submerged waterplane and then forming the double integral over the height. Thus,

$$V = \int (\int b\,dh)d\ell = \int (\int b\,d\ell)dh$$

$$M_V = \int (\int bh\,dh)d\ell = \int (\int b\,d\ell)h\,dh$$

$$M_1 = \int (\int b\,dh)\ell\,d\ell = \int (\int b\ell\,d\ell)dh$$

It may be seen that the first expressions are the one used in Tabulation #2. However, the second expressions are much more convenient from the point of view of the computer, for the integrals $\int b d\ell$ and $\int b\ell d\ell$ are simply A and M from Tabulation #1, and the integration over h is a straightforward process numerically. An added advantage is that the appendages, which have already been considered in calculating A and M, take care of themselves in the volume integration.

The only problem which arises is that of picking the waterplanes so that the normal Simpson rule which requires an odd number of equally spaced intervals can be used. It is not reasonable to require that all waterplanes be equally spaced, for since most ships have much more curvature (shape) near the base than higher up, it is both reasonable and common to take waterplanes closer together near the base than higher up. The normal procedure is to take waterplanes every 2, 4, or 8 feet from the base to above the greatest expected draft of the ship and then to subdivide near the base, so that in the typical study, for instance, planes are taken at 0, 2, 4, 6, 8, 12, 16, 20, 24, 28, and 32 feet. An assumption which simplifies the programming considerably without placing any undue restriction on the generality of the treatment is simply that

1) the interval between any two waterplanes is either exactly equal to or exactly twice the interval immediately below it.

2) every interval has at least one other interval equal to it.

3) the third interval is twice the second interval.

In other words, the relative spacing of waterplanes can be 1, 1, 2, 2, 4, 4, 4, 8, 8 but not 1, 1, 2, 2, 3, 3, etc. nor 1, 1, 2, 4, 4, nor 1, 1, 1, 1, 2, 2, etc.

Under the assumption just made it is possible to define an integration procedure as follows:

1) Disregard the integral up to the first waterplane. This interval is usually at such a shallow draft that it has no significance in the curves of form, and in any case the bottom interval can always be subdivided if necessary.

2) Since the first and second intervals are equal (this follows from the stated assumption without being explicitly required), Simpson's rule can be used to integrate up to the second waterplane. It should be noted that Simpson's rule always takes two intervals at a time, weighting the three points at 1, 4, 1 respectively, and that for a given interval length the rule is no less accurate for three points than for eleven.

3) Lump the first and second intervals together as one interval, so that their sum equals the third interval, and use Simpson's rule again to integrate up to third waterplane. That is, use the base, second, and third waterplanes, omitting the first.

4) The fourth interval equals the third, so that Simpson's rule can be used to integrate from the second to the fourth waterplane, and to this can be added

the integral up to the second waterplane, already obtained, yielding the integral up to fourth.

5) Proceed according to the following inductive plan: If the i^{th} interval is equal to the $i-1^{th}$ interval, use Simpson's rule directly and add the integral obtained to the integral up to the $i-2^{th}$ waterplane. If the i^{th} interval is twice the $i-1^{th}$ interval, lump the $i-1^{th}$ interval with the $i-2^{th}$ interval, apply Simpson's rule to the waterplanes i-3, i-1, i, and add the integral obtained to the integral up to the $i-3^{th}$ waterplane.

In programming the process, the values of the intervals are calculated ($\Delta h_i = h_i - h_{i-1}$) and stored for all values of i, and the products of area times height are formed and stored incidentally to the formation of the intervals (blocks 32. to 35. on Flow Diagram I).

Then step 5 of the procedure outlined requires that the three integrals desired (V, M_V, and M_I) be calculated stepwise for three points at a time. In the program the integrals are formed in blocks 39. and 40., 44. and 45., and 48. (48. is logically two blocks), and are added to the integral up to the bottom point of the three points in 41.,46., and 49. The various desired results (Δ, Δ_F, [illegible]B, KB, KM) are calculated in 42., 43., 47., 50., and 51. Block 52. then increases the various addresses in the program so that the $i+1^{th}$ waterplane is considered. Now if the $i+1^{th}$ interval is greater than (i.e., double) the i^{th} interval, the storage addresses in the integration blocks (39., 44., 48.) of the bottom point of the three points should be left unchanged, as should the storage addresses from which are taken the integrals up to the bottom point. Therefore these so-called (i-m) addresses are not changed in 52., and since on the next cycle the result of the comparison in block 36. is positive, they are not changed at all but are left at i-2. If, however, the $i+1^{th}$ and i^{th} intervals are equal, the bottom point addresses should be increased to be $i-1^{th}$ addresses. But suppose that on the cycle before, the i^{th} cycle, the bottom addresses were left unchanged (because the i^{th} interval was greater than the $i-1^{th}$). Then the bottom point addresses are set at the $i-3^{th}$ plane, and must be increased by two. Therefore a second comparison (block 37.) determines whether or not the i^{th} interval is greater than the $i-1^{th}$ interval. Block 38. causes the bottom point addresses to be increased by one in either case, and block 54. increases them a second time if necessary. Block 53., at the end, simply determines whether all waterplanes have been done, and, if not, causes the cycle to be repeated. The detailed arrangement of the initial values in the program assures the initial procedure for the first three waterplanes is actually performed as required by steps 1 through 4.

<u>Integration in inclined positions</u>

As has been seen, the cross curves of stability (Fig. 2) are obtained by imagining the ship rotated in the water through a set of angles θ about a longitudinal axis through some chosen center of gravity and determining the submerged volume and moment of volume about the original vertical centerline below each of a set of chosen waterplanes. In practice this computation is accomplished with a mechanical integraph by determining for each station the area and moment of area below each of a set of rotated waterlines and then integrating numerically over the length of the ship to obtain volume and moment of volume. The configuration involved is shown in Figure 8. Use of an integraph which gives area and moment simultaneously makes it a simple task to measure the area of the shaded portion in that figure and the moment of that area about the centerline CC'.

Reducing this computation to a strictly numerical procedure does not appear to be simple. The process of integration would not differ from that used in the preceding work if it were possible to obtain the perpendicular distances from the rotated centerline to the ship hull at any given height on the centerline (i.e., the lengths of AB and AB' on Fig. 8 and of all the equivalent lines parallel to BB' and at uniform distances from BB'). This problem amounts to finding the intersection of a straight line with a curve (the ship section) defined by a set of discrete points (i.e., the half breadths at given vertical heights). It is comparatively easy to set up an iterative scheme to determine the intersection. For instance, as a first guess, the point in the rotated waterline which is directly below the intersection of the unrotated waterline with the ship section can be taken. Thus if the hull were vertical in the region in question, the first guess would be the solution. As illustrated in Figure 9, the first guess is the point E. This point is readily determined, for AD is the half-breadth at A, AB is found from multiplying the distance from A to CG by $\tan\frac{\theta}{2}$, AB + AD = BD, and DE = BD $\tan\theta$. Knowing DE, the vertical height of V can be determined and by interpolation between the given half-breadths the length of VF is found. Then FE = JE - JF and GF = FE $\tan\theta$, and so on.

However, difficulties arise when the number of different alternative circumstances are considered. If one chooses a point on CC' and looks for the intersection with the ship of the line perpendicular to CC' at that point, one might expect to find two intersections. However, there are always lines with no intersection; there are occasionally lines with four intersections (see, for example, WL_2 in Fig. 11); and lines may accidentally be chosen with one or three intersections (i.e., tangent to the hull). These several possibilities, combined with the cases in which the line intersects the ship at the deck or through the flat part of the keel, make the number of alternative procedures necessary to find a given intersection so great as to be almost prohibitive.

If sufficient attention were given to the alternative procedures necessary to determine how many intersections to expect, the iterative method suggested might be developed into a useable procedure. It is not used because a second procedure, perhaps less elegant, but with no problem of alternatives is thought to be more suitable.

The second procedure, which has been programmed for the computer, makes a slightly different attack on the problem. Instead of a point on the centerline being chosen and the perpendicular distance from it to the ship section being determined, a point on the ship section is chosen and the perpendicular distance to the centerline as well as the position of the point of intersection on the centerline are determined. Since the process is simply a rotation of axes, these distances are extremely easy to compute, as can be seen in Figure 10. While none of the problems involving alternative procedures appear here, a new difficulty arises in that the intervals along the waterline (along which the integration is to be done) are not equally spaced. This is shown in Figure 11. Two alternatives suggest themselves: interpolate to find the values of d at equal intervals along the centerline, or integrate directly over the unequal intervals. Linear interpolation followed by high order integration is no improvement over linear (i.e., trapezoidal) integration. High order interpolation followed by high order integration or direct high order integration would yield satisfactory results, but the difficulties involved in programming the integration or the interpolation are great. Consequently, the procedure followed here has been to take the points relatively close together and to use trapezoidal integration.

Since the waterplanes for which the breadths are known are probably a little too far apart to guarantee reasonable results from trapezoidal integration, it is necessary to interpolate for points between the given points. Some definite number of points may as well be found between each pair of given points, for the given points are presumably chosen closer together where the curvature is great than where it is small. A compromise must be made in choosing how many points to interpolate between each pair of given points, for increasing the number decreases the truncation error but increases the roundoff error. In the program as written, three equally spaced points have been found between the given points.

The interpolation has been programmed using a specialized form of the general Lagrange polynomial interpolation formula, which expresses the unique polynomial of degree M-1 which assumes M given values $p_1, \ldots, p_M$ at M given places $x_1, \ldots, x_M$, respectively:

$$P(x) = \sum_{i=1}^{M} p_i \cdot \frac{\prod_{j=1}^{M}(x-x_j)}{\prod_{j=1}^{M}(x_i-x_j)} \qquad \text{(with } j \neq i\text{)}$$

Since third order interpolation is satisfactory for curves like ship lines which are probably not much worse than second degree, only four points (x_1, p_1; x_2, p_2; x_3, p_3; x_4, p_4) are used. Then the Lagrange formula specializes to:

$$P(x) = p_1 \cdot \frac{(x-x_2)\cdot(x-x_3)\cdot(x-x_4)}{(x_1-x_2)\cdot(x_1-x_3)\cdot(x_1-x_4)} + p_2 \cdot \frac{(x-x_1)\cdot(x-x_3)\cdot(x-x_4)}{(x_2-x_1)\cdot(x_2-x_3)\cdot(x_2-x_4)}$$
$$+ p_3 \cdot \frac{(x-x_1)\cdot(x-x_2)\cdot(x-x_4)}{(x_3-x_1)\cdot(x_3-x_2)\cdot(x_3-x_4)} + p_4 \cdot \frac{(x-x_1)\cdot(x-x_2)\cdot(x-x_3)}{(x_4-x_1)\cdot(x_4-x_2)\cdot(x_4-x_3)}$$

From the assumptions about the spacing made on page 31, it follows that, if the x's are the waterplane heights and $x_1 > x_2 > x_3 > x_4$, only the following relative values are possible:

a) $(x_2-x_1)=(x_3-x_2)=(x_4-x_3)$

b) $\frac{1}{2}(x_2-x_1)=(x_3-x_2)=(x_4-x_3)$

or c) $(x_2-x_1)=(x_3-x_2)=2(x_4-x_3)$

Furthermore, since only three values of x are to be calculated between x_2 and x_3, it is apparent (remembering that $x_2 > x_3$) that $x = x_2 - (\frac{1}{4}, \frac{2}{4} \text{ or } \frac{3}{4})(x_2 - x_3)$. Consequently it is possible to calculate all possible values of the coefficients in the Lagrange formula once and for all, obtaining three sets of four coefficients for each case a, b, and c, or 36 coefficients in all.

These coefficients are only useful in obtaining points in the middle interval of the four points. By shifting the four points along, the middle interval can be made to coincide with every interval between waterplanes except the first and last. However, it is not unreasonable to require that the first three intervals immediately below the deck line be equal, so that condition (a) holds, and $x = x_1 - (\frac{1}{4}, \frac{2}{4} \text{ or } \frac{3}{4})(x_1 - x_2)$ where x_1 is the height of the waterplane just below the deck line. At the base of the ship, condition (b) holds and $x = x_3 - (\frac{1}{4}, \frac{2}{3} \text{ or } \frac{3}{4})(x_3 - x_4)$ where x_4 is the base of the ship. Thus two more sets of coefficients are needed for initial and final interpolations. For use in the program here these coefficients are stored in five blocks of storage, labelled $K_{I,\ II,\ III,\ IV,\ \text{or}\ V}\ \ell, m$, where the Roman numerals correspond to the initial, three intermediate (a, b, and c), and final cases respectively, ℓ indicates which of the three values of x is involved, and m indicates to which of the four points x_1, x_2, x_3 or x_4 the coefficient corresponds. Specifically $K_{II}1, 3$ is the third coefficient to be used in calculating the point $x_2 - \frac{1}{4}(x_2 - x_3)$ when all three intervals are equal.

Another special case occurs at the top of the ship, since the deck height need not correspond to any given waterplane. It has been assumed that if the p^{th} waterplane is the lowest given waterplane above the deck of the ship at the j^{th} station, that the register Bj,p will contain the breadth at the j^{th} station at the deck (since the p^{th} waterplane is above the deck, the breadth at the p^{th} waterplane would be meaningless). No points are interpolated between the deck and the $p-1^{th}$ waterplane.

The program for forming a set of breadths at intervals one fourth as large as the given intervals is illustrated in Flow Diagram II. After restoring and indexing addresses in preparation for repeating the cycle (block 55.), the deck height at the j^{th} station is compared with all the waterplane heights from the base up until the p^{th} waterplane is found (56., 57.). The breadth at the deck is stored (58.) as the first new breadth, the height of the deck above the point KG is found by subtracting h_{CG} from the deck height (59.), and the proper addresses of the breadth, height and coefficients for the first interpolation are placed in the orders involved (58. to 61.).

The general interpolation scheme first stores the given breadth (62.) and height (63.), (initially p-i=1), then calculates the actual height of the interval (64.), forms and stores each interpolated breadth and the corresponding height (65. to 69.) until block 67. finds that all three interpolated points have been calculated. The interpolation is then repeated with modified result-storage addresses (72.), but for the second interval the same points are used, as for the first, so flow is directed (60.) around the block (77.) which would change the points, and block 71. is used to redirect the flow for subsequent cycles. In preparing for the second interval, the Lagrange coefficients should be changed from K_I to K_{II}, and to accomplish this change the flow diagram should show block 76. immediately preceding block 70. rather than in the position shown. This change is readily made in the program orders.

To determine when the base of the ship is being neared, comparison block 73. is used. If the base has not been reached, the blocks 74. to 79. are used to select the proper Lagrange coefficients for the next cycle. The final interpolation is carried out when 73. gives a positive and 80. a negative result, 81. supplying the final coefficients. After the final interpolation, block 80. gives a positive result. The breadth and height at the base are stored as the final pair of points (82.) and the interpolation is complete.

The actual trapezoidal integration must now be performed, following Flow Diagram III. In Figure 11, a not very typical but illustrative ship section is shown rotated counterclockwise through an angle θ from the stationary vertical CC'. The point A and all points on the right-hand side have moved from the original coordinates $\frac{b}{2}$, w to $\frac{b}{2}\cos\theta - w\sin\theta$, $\frac{b}{2}\sin\theta + w\cos\theta$; similarly G and all points on the left-hand side have moved from $-\frac{b}{2}$, w to $-\frac{b}{2}\cos\theta - w\sin\theta$, $-\frac{b}{2}\sin\theta + w\cos\theta$. A direct procedure to follow in the integration is to assume some one station j and some angle θ_k and to start at point A (block 83.), then move across the deck to point G and consider as the first trapezoid the figure bounded by the deck (AG), line

CC', and the perpendiculars from A and G to CC'. (Such a step essentially assumes a flat deck. Most decks actually have an appreciable camber which could, and should, be taken into account.) Block 83. is used to calculate the rotated coordinates on the right-hand side of the ship, block 86. is used for the left side. Normal flow from 83. will be to 87., but in crossing the deck the flow proceeds from 83. to 86. under instructions from 84. The formulas shown in 83. and 86. are incorrect.

Since the desired result is the integral up to each of several desired waterlines (assume that the desired waterlines, which in the typical study are 8', 16', 24', and 32', are stored measured from CG in a block of registers labelled WL). It is necessary to break the trapezoid up into the intervals defined by the desired waterlines. Since point A is in all likelihood the highest point on the ship and is probably above the highest desired waterline, it is necessary only to determine whether G is below a desired waterline. Therefore, after the new coordinates u_1, t_1, and u_2, t_2 of A and G have been found, the vertical distance from A to G is formed (87.) for use later and the height of G is compared (88.) with the height of the highest desired waterline. If G is above the waterline, it is compared (89.) with the next higher waterline (an impossibly high value is stored in WLq_o as the height of the waterline above the highest one desired, so that in this case G must be below the next higher). If A and G are thus found to be in the same interval, the proper address is sent ahead to block 96. by block 90. and the area and the moment of the area of the trapezoid are formed, multiplied by the Simpson rule coefficient 1, 2 or 4 (c_j) corresponding to the j^{th} station and added to the previous values from the j^{th} and all lesser stations found in the same interval (between the same waterplanes) by blocks 92. to 96.

However, if G is below the first desired waterline below A, then the height of the desired waterline and the breadth at that waterline (found by linear interpolation between A and G) are found (97. to 100.) and are used as the end point of the trapezoid beginning at A and also as the initial point of the trapezoid ending at G (i.e., the A to G trapezoid is divided into two by the waterline). Then the same trapezoidal integration 92. to 96. is performed as before, only now (on account of block 97.) the flow proceeds to 107. where the next lower interval, q-1, containing the new initial point, is substituted for the previous interval q. Then the same comparison sequence, beginning at 87., is recommenced. If, on the other hand, G had been above the interval containing A (not possible actually for G but quite possible, say, in the region between G and F, and quite likely when the integration comes back up the right-hand side between D and A) the same insertion of the proper waterline between A and G would have been performed in 102., 99., and 100.; and 101. would have directed flow from 96. to 108. where the next higher interval, q+1, containing the new point, is substituted for the previous interval q. After 108. the same comparison procedure beginning with 87. is repeated. It is possible, and in Figure 11 it is in fact the case, that the interval between A and G will be divided by more than one waterline, but in any case these divisions will obviously be handled correctly and eventually the point G will be reached. Then G becomes the initial point (103. and 105.) and the next lower point on the left side is converted to rotated coordinates (86.) and becomes the end point, unless block 106. finds that the base of the ship has been reached.

If the base has been reached, the point D at the base on the right-hand side is chosen next (block 109. putting the proper addresses in 83. and restoring 86.), thus integrating the trapezoid bounded by the base ED, and the integration proceeds from there up the right-hand side. Since the integration will henceforth be up rather than down the ship's side, block 110. causes flow to proceed henceforth from 104. to 111. which in turn will check to see whether the top (point A) has been reached, and, if not, will allow 121. to step 83. up to the next point and continue the process.

When A is finally reached, the entire station has been integrated over and the flow-directing blocks 84. and 104. are restored to their initial values (block 112.). Block 113. checks to see whether all the angles have been considered for this station and, if not, allows 122. to index the whole cycle up to take the next desired angle θ_{k+1}.

When all the angles have been completed, the program is restored to the values for θ_1 again (114.), and 115. checks to see whether all the stations have been completed. If not, the proper change is made in the Simpson coefficients (123. to 127.) and flow is directed back to the beginning of Flow Diagram II to interpolate the breadths at the next station and repeat the integrations for the next station.

When all the stations have been completed, the integrals between intervals should be summed from the bottom up (116. to 118.) for all angles (119., 120.) yielding the integrals up to the desired waterlines. From consideration of scale factor it might be more satisfactory to combine this summation with the output program, since two or three digits of accuracy might be saved. In any case, the results of Tabulation #3 have been obtained and, save for an output program to print or plot the data, the program is complete.

The procedure used automatically takes care of the sign of the moment and yields the correct area and moment regardless of multiple intersections such as occur with WL_2 in Figure 11. For example, the trapezoid from A to G has a negative height, negative length along CC', and negative moment arm, yielding positive area and negative moment. From G to H the length along CC' is positive, so the area is negative and the moment positive, thereby subtracting out the shaded area under GH. Between H and F the length along CC' is again negative and the area and moment under HF is added correctly. From F to J the height and moment arm are positive, the length remaining negative, so that negative area and negative moment both result, effectively subtracting the shaded area above FJ from the integral. From J around the base to B, area and moment are properly positive, while from B to A area is negative and moment positive, thereby subtracting that area from the total.

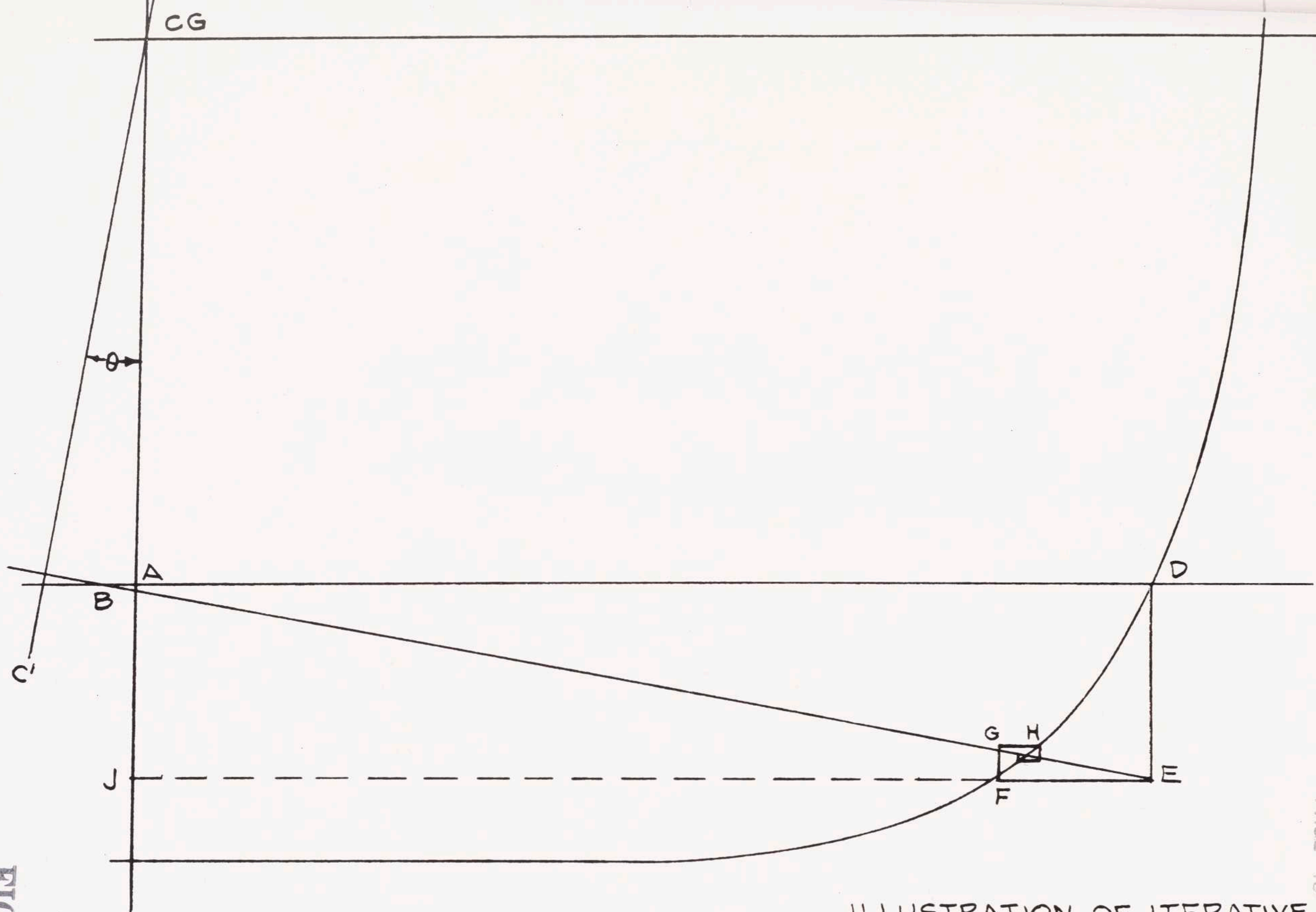


FIG. 9

ILLUSTRATION OF ITERATIVE METHOD TO DETERMINE INTERSECTION OF ROTATED WATERLINE WITH SHIP SECTION

A-33252

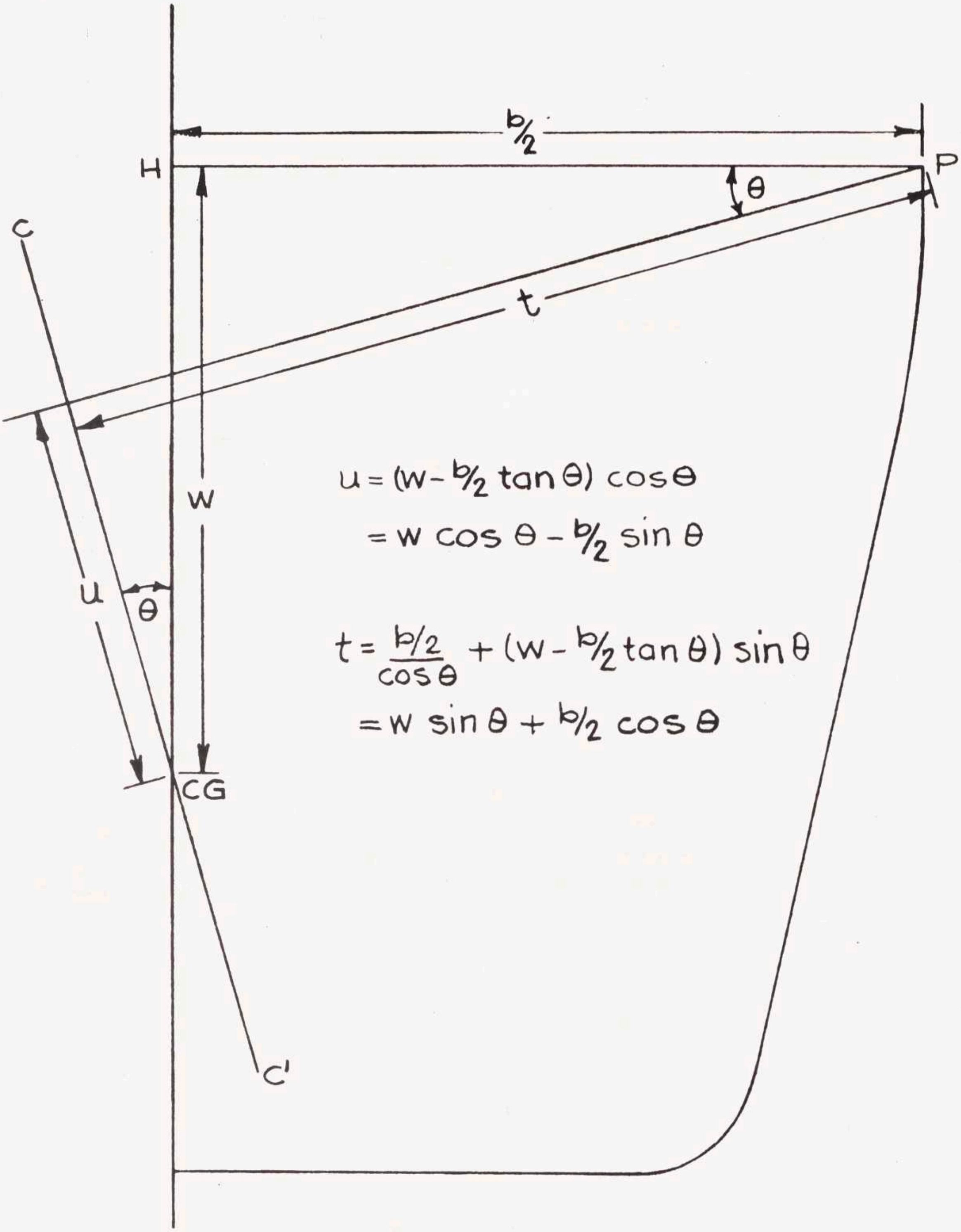


FIG. 10

DETERMINATION OF THE ROTATED COORDINATES OF A GIVEN POINT P ON A SHIP SECTION

A-33253

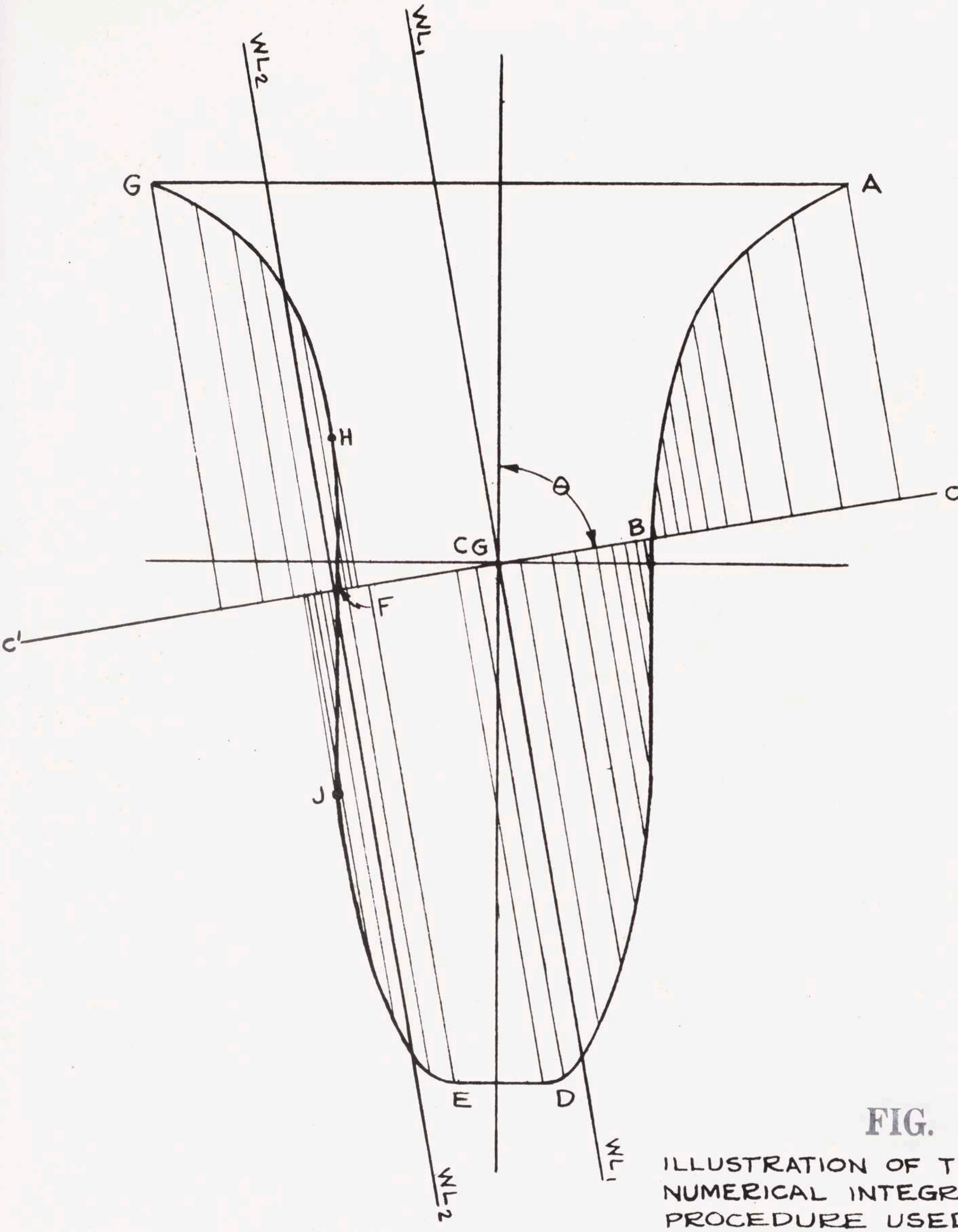


FIG. 11

ILLUSTRATION OF THE NUMERICAL INTEGRATION PROCEDURE USED TO INTEGRATE UP TO A ROTATED WATERLINE ON A SHIP SECTION

V - Program Number Storage

Division into groups

The numbers which are stored in connection with any problem are generally data, constants, intermediate results or results. In practice the storage addresses assigned to all these numbers will simply be consecutive register numbers, but for convenience in writing and describing the program, the numbers have been divided into blocks. All numbers in any block have a logical association with one another, and each number is designated by a capital letter, or pair of letters, chosen to suggest the values stored in the block, followed by an arabic numeral, or pair of numerals denoting the position of the number within the block. Thus, the breadth of the ship at the 4^{th} waterplane and 6^{th} station is stored in register number B6,4, while the value of position of the center of flotation at the i^{th} waterplane is stored, when calculated, in Fi. The storage registers needed in each of the three flow diagrams are listed on the respective diagrams.

Arbitrary choice of some of the parameters

On the flow diagrams many quantities have been represented symbolically which in practice should be assigned some definite value once and for all. For example, the number of stations to be used is called j_o, but there is no need to have the value of j_o differ from ship to ship. If eleven stations, say, are good enough for one ship, eleven stations are good enough for all ships. Indeed, to change j_o as the program stands involves considerable rewriting, and while the program could be modified to allow j_o to vary from ship to ship, the complications involved, although not great, do not seem to be worth while. The choice of j_o is in practice restricted to choosing between the two values most commonly used in design practice, eleven and twenty-one. At first glance, use of twenty-one would seem to be preferable, for it would improve the basic accuracy of the numerical integrations. However, there are several arguments in favor of using only eleven stations: the computation is reduced appreciably (in particular using eleven stations halves the time for the inclined position integration which requires most of the computing time anyway);the roundoff error is reduced; the truncation error is not much increased because ships lines are generally so smooth that the accuracy is as great as is needed; and, most important, the amount of data to be transmitted to the computer is halved.

The program written here therefore takes j_o equal to eleven. A value of twenty has been assumed for i_o, allowing that breadths be given for a maximum of twenty waterplanes above the base plane. The number of waterplanes actually used (n) need not be the same from ship to ship. The sines and cosines of the angles for which cross curves are desired could be left unassigned, because by simply changing them, the cross curves will be calculated for new angles. However, C. L. Wright, Jr., suggests that these angles be chosen once and for all as 1°, 2°, 3°, 4°, 5°, 7°, 10°, 15°, 20°, and at intervals of ten degrees to 80°. All such choices are advantageous because they reduce the amount of effort and time involved in preparing and transmitting data.

For the same reason it might be worth while to choose the waterlines for which values are to be obtained in the inclined position integration. Such a choice is not as arbitrary as it might seem because the actual choices determine the position of points (v,m) on the cross curves only indirectly, both v and m being calculated quantities.

Scale factor requirements on data

To meet the requirement that numbers in WWI be less than one in magnitude, some scale factor conventions have to be agreed upon. It turns out to be convenient to require data with two different scale factors chosen as follows:

Breadths and heights are expressed in feet times 2^{-q} where q is chosen so that the maximum breadth is greater than or equal to $\frac{1}{8}$ and less than $\frac{1}{4}$. It has been assumed that the maximum height from keel to weather deck is less than one-half of the maximum breadth (beam). This means that the breadth is less than 2^{q-2}, and the height is less than 2^{q-3}.

The length of the ship in feet is first divided by thirty, and the resultant quantity is multiplied by 2^{-p}, where p is chosen so that the quantity is less than one. Since $\frac{L}{30} \cdot 2^{-p} < 1$, it follows that $L < 30 \cdot 2^{p} < 2^{p+5}$. The appendage projections are expressed in feet times 2^{-p}.

Neither of these requirements will actually have to be considered in supplying the data initially, for the input conversion will be able to make the proper adjustments on raw data, but in the program here, which has no input or output program (final design of that part of the computer not having been completed), the assumptions are necessary.

Contents of storage register blocks C, D, E, K, and L

constants	C 1)	$21 \cdot 2^{-15}$
	2)	ts F12
	3)	2^{-5}
	4)	$\frac{2^{8}}{420}$
	5)	$209 \cdot 2^{-15}$
	6)	$\frac{2}{3}$
	7)	ts 0
	8)	ca 0
	9)	$4 \cdot 2^{-15}$
	10)	2^{-2}
	11)	$\frac{2^{10}}{3150}$
	12)	$\frac{2^{5}}{35}$
	13)	$\frac{2^{5}}{36}$
	21)	$21 \cdot 2^{-15}$
	22)	$1 \cdot 2^{-15}$
	23)	0
	24)	$2 \cdot 2^{-15}$
	25)	ca H3
	26)	$4 \cdot 2^{-15}$

C 31) ca 0

32) ts 0

33) $1 \cdot 2^{-15}$

34) ca X1

35) mr θ15

36) su DH11

37) $17 \cdot 2^{-15} (=q_o)$

38) ca V15,16 (=ca Vk_o, q_o-1)

39) sr 1

---

all D registers are temporary storage

D 1) temporary

2) a_i

3) forward appendage projection

4) forward appendage area

5) after appendage projection

6) after appendage area

7) $\frac{L}{2} \cdot 2^{-p-4}$

8) I_{CL}

9) m_i

10) I[illegible]

11) $\frac{m_i}{a_i}$

12) $L^2 \cdot 2^{-2p-10}$

13) I[illegible]

14) v

15) KB

D 21) $(B1,0 - H0 + 21j) \cdot 2^{-15}$

22) $h_i - h_{CG}$

23) Δw_i

24) ℓ

25) partial sum of interpolants

31) temporary

32) temporary

33) u_ℓ

34) t_ℓ

35) $u_{\ell-1}$

36) $t_{\ell-1}$

37) Δu_ℓ

38) Σt_ℓ

39) $u_{\ell'}$

40) $t_{\ell'}$

41) $\Delta u_{\ell'}$

42) $- q_o$

E 1) $\frac{L}{30} \cdot 2^{-p}$ ship length

2) $q - p$ difference between scale factors

3) $\frac{1}{2}$ forward area

4) $\frac{2}{3}$ aft area

5) $\frac{1}{6}$ forward I_{CL}

6) $\frac{1}{5}$ aft I_{CL}

7) $\frac{2}{3}$ forward moment × 2

8) $\frac{4}{5}$ aft moment × 2

(Items 3–8: appendage shape factors--values given are for triangular bow and parabolic stern)

E	9)	$\frac{1}{3}$	forward $I_{[illegible]}$ 2
	10)	$\frac{16}{35}$	aft $I_{[illegible]}$ 2
	11)	An+1	
	12)	h_{CG}	2^{-q}

Specialized Lagrange coefficients:

initial interval, with 1,1,1 spacing

K_I (all values $\times\frac{1}{2}$)

1,1)	77/128		
2,1)		5/16	
3,1)			15/128
1,2)	77/128		
2,2)		15/16	
3,2)			135/128
1,3)	-33/128		
2,3)		-5/16	
3,3)			-27/128
1,4)	7/128		
2,4)		1/16	
3,4)			5/128

intermediate interval, with 1,1,1 spacing

K_{II} (all values $\times\frac{1}{2}$)

1,1)	-7/128		
2,1)		-1/16	
3,1)			-5/128
1,2)	105/128		
2,2)		9/16	
3,2)			35/128
1,3)	35/128		
2,3)		9/16	
3,3)			105/128
1,4)	-5/128		
2,4)		-1/16	
3,4)			-7/128

intermediate interval, with 2,1,1 spacing

K_{III} (all values $\times\frac{1}{2}$)

1,1)	-7/512		
2,1)		-1/64	
3,1)			-5/512
1,2)	378/512		
2,2)		30/64	
3,2)			110/512
1,3)	168/512		
2,3)		40/64	
3,3)			440/512
1,4)	-27/512		
2,4)		-5/64	
3,4)			-33/512

intermediate interval, with 1,1,$\frac{1}{2}$ spacing	K_{IV}	1,1)	-3/64		
		2,1)		-1/20	
		3,1)			-9/320
		1,2)	50/64		
		2,2)		10/20	
		3,2)			70/320
		1,3)	25/64		
		2,3)		15/20	
		3,3)			315/320
		1,4)	-8/64		
		2,4)		-4/20	
		3,4)			-56/320

(all K_{IV} values $\times\frac{1}{2}$)

	K_V	1,1)	5/512		
		2,1)		3/192	
		3,1)			7/512
		1,2)	-78/512		
		2,2)		-42/192	
		3,2)			-90/512
		1,3)	520/512		
		2,3)		168/192	
		3,3)			280/512
		1,4)	65/512		
		2,4)		63/192	
		3,4)			315/512

(all K_V values $\times\frac{1}{2}$)

---

relative distance from midship station to station j.	L	1)	$15 \cdot 2^{-4}$
		2)	$12 \cdot 2^{-4}$
		3)	$9 \cdot 2^{-4}$
		4)	$6 \cdot 2^{-4}$
		5)	$3 \cdot 2^{-4}$
		6)	$0 \cdot 2^{-4}$
		7)	$-3 \cdot 2^{-4}$
		8)	$-6 \cdot 2^{-4}$
		9)	$-9 \cdot 2^{-4}$
		10)	$-12 \cdot 2^{-4}$
		11)	$-15 \cdot 2^{-4}$

VI - Program Orders

The orders which make up the program, 758 in all, have been arranged in blocks corresponding to the blocks of the flow diagrams (Figures 12, 13, and 14), and a discussion has been placed beside each block in an attempt to show generally the purpose of that block. The single lines are intended simply to help separate the blocks, while the double lines indicate a logical break in the flow.

To carry out integrations over some one waterplane starting at the base, the breadths at each station for the chosen waterplane are stored in a block of storage registers assigned for the purpose. In this manner the breadth at any station is at a definite storage address regardless of which waterplane is being considered, and the addresses of the breadths appearing in many orders throughout the succeeding blocks do not have to be changed when the waterplane is changed. Rather than to store a separate pair of orders to transfer each of the breadths, a short cycle is used, in which only one breadth is transferred and then the orders are modified, another transfer carried out, and so on.

```
1.01  ca  B1,0
 .02  ts  F1
```

The addresses of the orders in block 1. are each modified. The separation (i_0+1) between B1,0 and B2,0 has been set at 21 registers, so $21 \cdot 2^{-15}$ is stored in C1.

```
2.01  ca  1.01
 .02  ad  C1
 .03  ts  1.01
 .04  ao  1.02
```

Since 11 stations will be assumed for this program, when F11 has been filled the small cycle will be complete. The ao order in 2.04 will leave the negative of the contents of 1.02 in AC, and when ts F12 from C2 is added the quantity 12-(j+1), or 11-j will be in AC and will be positive until the process is

```
3.01  ad  C2
 .02  cp  1.01
```

complete. Therefore, the cycle is repeated if the quantity in AC is positive.

When block 5. has been passed, the flow must be directed along one of four paths. Hence the proper address (7.01) is sent ahead to the sp order in 6.01

```
4.01  as  7.01
 .02  td  6.01
```

It is quite possible to carry out a Simpson's rule integration by using a cyclic process. However, close inspection discloses that if, in the integrations in which some contributions are negative and others positive, the terms are combined in a chosen order rather than straight from front to back, as they would be if a cyclic process were used, scale factors can be held at closer tolerances than would otherwise be possible. These integrals, being the difference of nearly equal fore and aft moments, are particularly vulnerable in any case. The order in which the terms of the sum are combined here was chosen to improve the scale factors as much as possible in the integration of moment and moment of inertia about the midship station. Notice that if the magnitude of the contents of Fj is less than $\frac{1}{4}$ for all j, the summation goes through regardless of

```
5.01  ca  F1
 .02  ad  F11
 .03  sr  2
 .04  ad  F2
 .05  ad  F10
 .06  ad  F6
 .07  sr  1
 .08  ts  D1
 .09  ca  F3
 .10  ad  F9
 .11  ad  F5
 .12  ad  F7
 .13  sr  1
 .14  ad  F4
 .15  ad  F8
 .16  sr  1
 .17  ad  D1
```

the sign. When the moment integral is to be determined, the registers F1, 2, ..., 11 will contain, respectively, at most (positive maximum) 15, 12, 9, 6, 3, 0, 0, 0, 0, 0, 0, each multiplied by 2^{-4} or $\frac{1}{16}$. The situation arises because the breadths are less than $\frac{1}{4}$, and each of them is multiplied by $\frac{3}{4}$ times a moment arm of 5, 4, 3, 2, 1, 0, -1, -2, -3, -4, -5 respectively. By carrying these values through the calculation as programmed, it will be seen that the contents of AC can never exceed one. A similar analysis can be carried out for the moment of inertia integration. A factor of 2^{-3} is introduced in performing the summation.	5.18	mr	E1

The single sp order which directs the flow to the proper place receives different addresses from blocks 4., 13., 19., and 26.	6.01	sp	--

---

Before the appendage area can be calculated, the	7.01	ts	D2
result of the summation done in 5. must be removed	.02	ca	E2
from AC. Since the length of the projection for-	.03	td	7.07
ward of F.P. for the i^{th} waterplane is stored in	.04	td	7.15
$J_F i$, the projection aft of A.P. is in $J_A i$ and the	.05	ca	$J_F 0$
shape factors for the fore and aft areas are in E3	.06	mh	C3
and E4 respectively, the appendage area is easily	.07	sl	--

computed. The scale factor for the appendages must of course be made to coincide with that of the main area. The area has a factor of 2^{-q} introduced from the breadth, 2^{-p} from the length, and 2^{-3} from the Simpson summation, while the appendage area will have 2^{-q} from the breadth and 2^{-q} from the length, so the appendage area must be multiplied by 2^{q-p-3}. Now, q-p must be positive. (The fact that $b \cdot 2^{-q} < \frac{1}{4}$ while $\frac{L}{30} \cdot 2^{-p} \geqq \frac{1}{2}$ yields $\frac{L}{30} \cdot 2^{-p} > 2b \cdot 2^{-q}$, or $b < \frac{2^{q-p-1}}{30} \cdot L$ and it is reasonable to assume $b > \frac{1}{50} \cdot L$, so $\frac{2^{q-p-1}}{30} > \frac{1}{50}$, and therefore $q-p-1 > -1$, or $q-p > o$.) The quantity q-p (which is data) is stored in E2, where it is available to control shifting the appendage area, and since q-p is positive, the shift is to the left. It is desirable to have the contents of $J_F i$, multiplied by 2^{q-p-5}, stored in a temporary register for use later in calculating the moment appendage. Therefore, this value is obtained first. The factor 2^{-5} is later corrected to 2^{-3} by a shift left. The 2^{-5} is introduced by multiplying instead of by shifting to prevent losing figures through roundoff. It is permissible to add the appendage area directly to the main area if the appendage area is always less than $\frac{1}{15}$th of the main area. (The main area is less than

7.08	ts	D3
.09	mr	E3
.10	mh	F1
.11	sl	2
.12	ts	D4
.13	ca	$J_A 0$
.14	mh	C3
.15	sl	—
.16	ts	D5
.17	mr	E4
.18	mh	F11
.19	sl	2
.20	ts	D6
.21	ad	D4
.22	ad	D2

the maximum length times the maximum breadth times the scale factor 2^{-q-p-3}, hence $a < (30 \cdot 2^{p})(2^{q-2})(2^{-q-p-3}) = 30 \cdot 2^{-5} = \frac{15}{16}$.)

For convenience in forming sums later, the area is shifted right once before being stored for use in the volume integration. Since the area is needed again in block 22., it is also stored in a register whose address is changed between cycles. The scale factor is 2^{-q-p-4}.	8.01 sr 1 .02 ts A0 .03 ts D2
The quantity tons per inch is easily formed and stored, with a total scale factor of 2^{-q-p+4}, by multiplying by $\frac{2^8}{420}$ from C4.	9.01 mr C4 .02 ts T0
The j^{th} breadth is cubed and, since the breadth is less than 2^{-2} and its cube therefore less than 2^{-6}, a factor of 2^4 is introduced. The net result is still less than 2^{-2}.	10.01 ca F1 .02 sl 2 .03 ts D1 .04 mr D1 .05 mh D1 .06 sr 2 .07 ts F1
The j index in 10. is increased by one.	11.01 ao 10.01 .02 ao 10.07

The quantity $j_0 - j$ is checked, as in 3., and the cycle is repeated if necessary.	12.01 ad C2 .02 cp 10.01
The proper address is sent ahead to 6., making use of the td 6.01 already stored in 4.02, and after 4.02 the flow proceeds to 5.	13.01 as 14.01 .02 sp 4.02
The shape factor is stored in E5 and E6, and multiplication by the projection from D3 and by the cube of the breadth from F1 yields the appendage moment of inertia about the centerline. The main integral has a scale factor of $2^{-3q+4-p-3}$ and the appendage has a factor $2^{-3q+4-p-5}$, so that the appendage must be multiplied by 2^2. The total factor is $2^{-3q-p+1}$.	14.01 ts D8 .02 ca E5 .03 mr F1 .04 mh D3 .05 sl 2 .06 ts D1 .07 ca E6 .08 mr F11 .09 mh D5 .10 sl 2 .11 ad D1 .12 ad D8
The result is stored.	15.01 ts I_{CL}0
To prepare to integrate the moment of area, the j^{th} breadth times 2^2 times the j^{th} moment arm is stored in Fj.	16.01 ca B1,0 .02 sl 2 .03 mr L1 .04 ts F1

The j index in 16. is increased by one, as in 2.	17.01	ca	16.01
	.02	ad	C1
	.03	ts	16.01
	.04	ao	16.03
	.05	ao	16.04
The quantity $j_0 - j$ is checked, as in 3., and the cycle is repeated if necessary.	18.01	ad	C2
	.02	cp	16.01
The proper address is sent ahead to 6., using the td 6.01 order already in 4.02 and the flow proceeds to 5.	19.01	as	20.01
	.02	sp	4.02
The moment integral is multiplied by $\frac{L}{30} \cdot 2^{-p+2}$ to convert the relative distance factors which were taken from Lj to actual distances. The appendage moment equals the appendage area multiplied by the moment arm, and the moment arm is $\frac{L}{2}$ plus the projection times a suitable shape factor. The appendage area is stored in D4 and D6, the projection in D3 and D5, twice the shape factor in E7 and E8, and the value $\frac{L}{2}$ results from multiplying $\frac{L}{30} \cdot 2^{-p}$ stored in E1 by $15 \cdot 2^{-4}$ stored in L1. The scale factor will be 2^{-p-4}. The projection has a factor of 2^{-p-5} and the factor of 2 in the shape factor	20.01	mh	E1
	.02	sl	2
	.03	ts	D9
	.04	ca	L1
	.05	mr	E1
	.06	ts	D7
	.07	ca	E7
	.08	mr	D3
	.09	ad	D7
	.10	mh	D4
	.11	sl	4
	.12	ts	D10

gives the product also a factor of 2^{-p-4}. The	20.13	ca	E8
appendage scale factor, allowing for the 2^{-q-p-3}	.14	mr	D5
in the appendage area, is $2^{-q-2p-7}$. To match this	.15	ad	D7
with the $2^{-q+2-4-2p-3+2}$ belonging to the main inte-	.16	mh	D6
gral, the appendage is multiplied by 2^{4}. The total	.17	sl	4
scale factor is then $2^{-q-2p-3}$, which still leaves	.18	ad	D10
the moment less than $\frac{1}{2}$. (The moment must be less	.19	ad	D9
than 2^{p-1} times the area, or the center of flotation			
would be outside the ship. Since $m < 2^{p-1}a$, it fol-			
lows that $2^{-q-2p-3} \cdot m < 2^{-q-p-4} \cdot a < \frac{1}{2}$.)			

The moment is stored.	21.01	ts	MO
	.02	ts	D1

The center of flotation is formed and stored. The	22.01	dv	D2
moment is less than the area as discussed under	.02	sl	15
block 20., so the division is permissible. The or-	.03	ts	FO
der sl 15 must follow the divide operation to shift	.04	ts	D11
the result into AC. The total scale factor is			
$\frac{2^{-q-2p-3}}{2^{-q-p-4}} = 2^{-p+1}$.			

The breadth times moment arm stored in Fj is multi-	23.01	ca	F1
plied again by the moment arm to prepare to find	.02	mr	L1
the moment of inertia about the midshif station.	.03	sr	1
The maximum values of the breadth times the square	.04	ts	F1
of moment arm are such that a factor of $\frac{1}{2}$ must be			

introduced to permit integration to be carried out according to the plan followed in block 5., hence the sr 1 order. The scale factor of the breadth times the square of the moment arm is $2^{-q+2-4-4-1}$.

The j index in 23. is increased by one.	24.01	ao	23.01
	.02	ao	23.02
	.03	ao	23.04

The quantity $j_o - j$ is checked, as in 3., and the cycle is repeated if necessary.	25.01	ad	C2
	.02	cp	23.01

The proper address is sent ahead to 6., using the td order in 4.02, and flow proceeds to 5.	26.01	as	27.01
	.02	sp	4.02

The result of the summation is multiplied by $(\frac{L}{30})^2$ in order to convert the relative moment arm (squared) into actual moment arm. The moment of inertia has the scale factor $2^{-q+2-4-4-1-3-p-2p} = 2^{-q-3p-10}$. The appendage has two terms, namely $\frac{L^2}{4}$ ($\frac{L}{2}$ is still stored in D7 with a scale factor of 2^{-p-4}) and the shape factor (stored with a factor of 2) multiplied by the appendage projection squared (a factor of $2^{2(-p-5)}$), so a factor of 2 is introduced

27.01	mr	E1
.02	mr	E1
.03	ts	D10
.04	ca	D7
.05	mr	D7
.06	ts	D12
.07	ca	D3
.08	mh	D3
.09	sl	1

in the latter term and both terms are multiplied	27.10	mr	E9
by the appendage area (with a scale factor 2^{-q-p-3}).	.11	ad	D12
When the appendage scale factor is corrected by	.12	mh	D4
multiplying by 2, the appendage scale factor be-	.13	sl	1
comes $2^{-q-3p-10}$.	.14	ts	D1
	.15	ca	D5
	.16	mh	D5
	.17	sl	1
	.18	mr	E10
	.19	ad	D12
	.20	mh	D6
	.21	sl	1
	.22	ad	D1
	.23	ad	D10

The quantity $I_{xx}i$ is stored temporarily.	28.01	ts	D10

The scale of $\frac{(m_i)^2}{a_i}$, formed by multiplying $\frac{m_i}{a_i}$	29.01	cs	D11
from D11 by m_i from D9, is $2^{-p+1-q-2p-3} =$	.02	mh	D9
$2^{-q-3p-2}$ while the scale of I_{xx}, from D10, is	.03	sr	8
$2^{-q-3p-10}$. Consequently, $\frac{(m_i)^2}{a_i}$ is multiplied	.04	ad	D10
by 2^{-8} by shifting, and I_{xx} is added (since both	.05	ts	D10
terms are positive their difference cannot ex-	.06	ca	C11
ceed one). The factor $\frac{1}{420\ L}$ is obtained by di-	.07	dv	E1
viding $\frac{L}{30} \cdot 2^{-p}$ (stored in E1 and known to be $\geq \frac{1}{2}$)	.08	sl	15

into $\frac{2^{10}}{3150}$ (stored in C11 and obviously $< \frac{1}{2}$), forming $\frac{30}{3150\ L} \cdot 2^{p+10}$ or $\frac{2^{p+10}}{420\ L}$. The product of this quantity and the difference formed above is the moment to change trim one inch, with a scale factor of $2^{-q-3p-10+p+12} = 2^{-q-2p+2}$.

29.09	mr	D10
.10	ts	MT 0

In repeating the cycle, addresses referring to stations which have been increased during the cycle must be restored, and addresses referring to waterplanes must be increased to the next waterplane. In the case of Bj,i, the address must be changed from B 11,i to B 1,i+1 making use of the relation (B 11,i) - 210+1 = (B 1,i) + 1 = (B 1,i+1), so the constant 209 is stored in C 5 and is subtracted from B 11,i.

30.01	ca	1.01
.02	su	C5
.03	td	1.01
.04	td	16.01
.05	as	F1
.06	td	1.02
.07	td	10.01
.08	td	10.07
.09	td	16.04
.10	td	23.01
.11	td	23.04
.12	as	L1
.13	td	16.03
.14	td	23.02
.15	ao	7.05
.16	ao	7.13
.17	ao	9.02
.18	ao	15.01
.19	ao	21.01

	30.20	ao	22.03
	.21	ao	29.10
	.22	ao	8.02

To determine whether or not to repeat the cycle, integrating over the next higher waterplane, it is necessary to determine whether the last waterplane has been done or not. The quantity -(ts Ai+1) is in AC as a result of 30.22. To this is added An+1 and then ts o, leaving the quantity n-i in AC. If this quantity is positive, the cycle is repeated. An is needed in 35. and 53. also; otherwise ts(An+1) would have been stored in E11 instead of simply An+1.	31.01	ad	E11
	.02	ad	C7
	.03	cp	1.01

The distance between two successive waterplanes is formed and stored, starting with the bottom two. The factor $\frac{2}{3}$, stored in C6, called for by Simpson's rule, is multiplied in at this time for convenience.	32.01	ca	H1
	.02	su	HO
	.03	mr	C6
	.04	ts	DH1

The i^{th} waterplane area is multiplied by its height (moment arm) and the product is stored. By storing a zero in GO the base waterplane is automatically taken care of. Since the contents of Hi is always less than 2^{-3}, a factor of 2^3 is introduced.	33.01	ca	A1
	.02	mh	H1
	.03	sl	3
	.04	ts	G1

The two blocks above must be repeated until all waterplanes have been covered. Therefore, addresses referring to waterplanes must be increased	34.01 ao 32.01 .02 ao 32.02 .03 ao 32.04 .04 ao 33.03 .05 ao 33.02 .06 ao 33.01
The quantity -(ca Ai+1) is in Ac as a result of 34.06. The operation code portion is removed, by adding ca 0 from C8, and the quantity An+1 stored in E11 is added, forming n-i. If the quantity is positive the cycle is repeated.	35.01 ad C8 .02 ad E11 .03 cp 32.01
Two successive values of Δh are subtracted to determine whether or not the interval length has changed. A small quantity stored in C9 is subtracted to prevent roundoff in the initial values of h and the formed values of Δh from making a zero result have a positive remainder and appear positive. If the higher interval is larger than the lower one, it follows that the bottom point of the three point integral to be formed should remain at the value it had during the last inetgration, whereas if the two intervals are equal, the bottom point is to be moved up one or two notches.	36.01 ca ΔH2 .02 su ΔH1 .03 su C9 .04 cp 39.01

Whether to notch the bottom point up once or twice depends on whether the bottom point was formerly the first point below the middle point, or whether it was two points below the middle point. This is determined by finding whether the two intervals below the top one are equal (in which case the first case obtains) or unequal. Initially, the question which this block asks is meaningless, since there is no Δh_0. Therefore, zero is stored for ΔH0 so that the comparison will always be positive.

```
37.01  ca  ΔH1
  .02  su  ΔH0
  .03  su  C9
  .04  cp  54.01
```

The bottom point is moved up one notch.

```
38.01  ao  39.01
  .02  ao  41.01
  .03  ao  44.01
  .04  ao  46.01
  .05  ao  48.01
  .06  ao  49.01
```

Since the waterplane areas stored in Ai are all less than $\frac{1}{2}$ (see block 8.) the integration can be carried out directly. The initial three points are 0, 1, and 2, but the address in 39.01 will be increased twice (since the initial comparison in block 37. will be positive) so that the correct initial address is the apparently meaningless A(-2). A scale factor of 2^{-2} is introduced in forming the sum.

```
39.01  ca  A(-2)
  .02  ad  A2
  .03  sr  2
  .04  ad  A1
```

The area sum is multiplied by the Simpson coefficient $2\frac{h_i}{3} \cdot 2^{-q}$ giving a total scale factor of $2^{(-q-p-4)-2+1-q}$ or $2^{-2q-p-5}$. Since the total volume of the ship is less than the maximum breadth times the maximum length times the maximum height of the ship, or $2^{(q-2)+(p+5)+(q-3)}$ or 2^{2q+p}, the volume may be multiplied by 2^5, giving a total factor of 2^{-2q-p}.

40.01	mh	ΔH2
.02	sl	5

The volume up to the bottom point is added to the volume resulting from the integration. The initial contribution must be added to the volume up to the base which of course is zero, and since V(0) would logically contain the volume up to the base, V(0) contains 0. However, before the present block is reached, the address in 40.02 will have been increased twice (as a result of block 37.) so that V(-2) is the proper initial address. It is assumed that the third and fourth intervals are equal and that each is twice as large as either of the bottom two, which are equal. Hence, on the second time through the cycle, the comparison in 36. will be positive, and on the third time, 36. will be negative and 37. positive. The contents of V1, supposedly the integral up to first waterplane above the base,

41.01	ad	V(-2)
.02	ts	V2
.03	ts	D14

cannot be calculated by Simpson's rule, but the contents of V1 is immaterial since the restrictions just mentioned prevent the contents of V1 from ever being used.

The volume (still in AC) is multiplied by $\frac{2^5}{35}$ from C12, forming the tons of displacement in salt water. The factor 35 is exact, since a displacement ton is by definition 35 cubic feet. The scale factor is $2^{-2q-p+5}$.

```
42.01  mr  C12
  .02  ts  Δ2
```

The fresh water displacement is found by multiplying the volume by $\frac{2^5}{36}$ from C13.

```
43.01  ca  D14
  .02  mr  C13
  .03  ts  Δ_F2
```

The moment is less than $\frac{1}{2}$ (see discussion under block 20.) so that the sum can be formed directly, introducing a factor of 2^{-2}. The initial addresses are chosen as in 39.

```
44.01  ca  M(-2)
  .02  ad  M2
  .03  sr  2
  .04  ad  M1
```

The moment sum is multiplied by the Simpson coefficient $2\frac{h_i}{3}$, giving a total scale factor of $2^{-q-2p-3-2+1-q} = 2^{-2q-2p-4}$. Now when M_1 is divided by the volume, the result is MB (horizontal distance from the center of bouyancy to the midship station),

```
45.01  mh  ΔH2
  .02  sl  1
```

and this is not likely to be greater than $\frac{8}{30}$ the length of the ship, or 2^{p+3}. The scale factor associated with the volume is 2^{-2q-p}. Therefore, a factor of 2 can be introduced into M_l, giving a total scale factor for LB (when formed) of 2^{-p-3}.

The value of M_l up to the bottom point is added to the quantity resulting from the integration. For a discussion of the initial values, see block 41.

46.01	ad	$M_l(-2)$
.02	ts	M_l2

The division by the volume can safely be carried out (see the discussion under 45.), and the result is stored as indicated, with scale factor 2^{-p-3}.

47.01	dv	D14
.02	sl	15
.03	ts	LB2

The product of area times height of the i^{th} waterplane (with a scale factor of $2^{-2q-p-1}$) is in Gi as a result of 33. The area was less than $\frac{1}{2}$ and hence these terms are less than $\frac{1}{2}$ and can be summed directly. The result is multiplied by the Simpson coefficient $2\frac{h_i}{3}$. This moment M_v, when divided by the volume, yields KB, the height of the center of bouyancy above the base, a quantity which cannot be greater than the height of the ship, or 2^{q-3}. The factor associated with M_v, before shifting left, is

48.01	ca	G(-2)
.02	ad	G2
.03	sr	2
.04	ad	G1
.05	mh	ΔH2
.06	sl	5

$2^{-2q-p-1-2+1-q} = 2^{-3q-p-2}$ and after division by the volume the factor would be $2^{-3q-p-2-(-2q-p)} = 2^{-q-2}$. Hence M_V can safely be increased by 2^5, yielding a total scale factor for KB of 2^{-q+3} when KB is formed later. Initial addresses are chosen as in 39.

The value of M_V up to the bottom point is added	49.01	ad	$M_V(-2)$
to the quantity resulting from the integration.	.02	ts	$M_V 2$
for a discussion of the initial values, see block 41.			

The division by the volume can safely be carried	50.01	dv	D14
out (see discussion under block 48.) and the re-	.02	sl	15
sult is stored, with scale factor 2^{-q+3}.	.03	ts	KB2

A reasonable maximum of $BM = \frac{I_{CL}}{V}$ is about 7 times	51.01	sr	3
the height of the ship, or $\frac{7}{8} \cdot 2^q$. The result of	.02	ts	D15
the division, without shifting, would have a scale	.03	ca	$I_{CL}2$
factor of $2^{-3q-p+1-(-2q-p)} = 2^{q+1}$, so that a fac-	.04	sr	1
tor of 2^{-1} must be introduced into I_{CL} before di-	.05	dv	D14
vision. In order that KB (which is in AC as a re-	.06	sl	15
sult of block 50.) can be added to BM, KB is multi-	.07	ad	D15
plied by 2^{-3}, giving it the proper scale factor and	.08	ts	KM2
at the same time assuring that it is less than $\frac{1}{8}$ so			

it may safely be added to the $\frac{7}{8}$ which is the maximum of BM. The sum is the quantity KM, with a scale factor 2^{-q}.

The various addresses which depend on the water-plane chosen are increased by one in preparation for the next cycle.	52.01	ao	36.01
	.02	ao	36.02
	.03	ao	37.01
	.04	ao	37.02
	.05	ao	39.04
	.06	ao	40.01
	.07	ao	41.02
	.08	ao	42.02
	.09	ao	43.03
	.10	ao	44.02
	.11	ao	44.04
	.12	ao	45.01
	.13	ao	46.02
	.14	ao	47.03
	.15	ao	48.02
	.16	ao	48.04
	.17	ao	49.02
	.18	ao	50.03
	.19	ao	51.03
	.20	ao	51.08
	.21	ao	39.02

Exactly as in 35. the quantity n-i is formed, and	53.01	ad	C8
if the quantity is positive, the cycle is repeated.	.02	ad	E11
If negative, the program proceeds to block 55., on	.03	cp	36.01
Flow Diagram II.	.04	sp	55.01

The orders appearing in block 38. are duplicated	54.01	ao	39.01
here, and flow is returned to 38. It would be pos-	.02	ao	41.01
sible to use this block to direct the flow through	.03	ao	44.01
38. twice, avoiding a repetition of orders here,	.04	ao	46.01
but the number of "red tape" orders which this pro-	.05	ao	48.01
cedure would entail is only one fewer than the	.06	ao	49.01
number of duplicated orders.	.07	sp	38.01

The addresses of the temporary storage registers	55.01	as	H1
X and W are changed throughout the program which	.02	td	56.01
follows, and these must be restored to their ini-	.03	as	X2
tial values before starting the program. Certain	.04	td	62.02
addresses which refer to the particular station	.05	as	W2
involved must be increased to provide for pro-	.06	td	63.05
gressing from one station to the next.	.07	as	X3
	.08	td	66.01
	.09	as	W3
	.10	td	69.05
	.11	ao	56.02
	.12	ao	59.15

To determine the position of the deck at the j^{th} station relative to the given waterplanes, the difference between the height of the i^{th} waterplane (initial value coming from 55.) and the height of the deck, given in DHj (j is increased by 55. so the correct initial j is 0), is formed. If the result is positive, the waterplane is above the deck, and the program proceeds to 58.

56.01	ca	--
.02	su	DH0
.03	cp	58.01

If the result of the above was negative, 56.01 is increased to the next higher waterplane and the comparison is repeated.

57.01	ao	56.01
.02	sp	56.01

D21 contains initially the number (B1,0 - H0 - 20) 2^{-15}. Its contents is increased by 2 from C21, making it contain always B1,0 - H0 + 2(j-1). To this quantity is added the value of Hp, which is the address of the height of the first waterplane above the deck, determined just above. This gives the address Bj,p in AC (together with the ca operation code symbol which was in 56.01 but which is not significant). The address is sent to 58.16 where it is used to send the actual ship breadth (which is stored in Bj,p) to X1 for use in the integration program later. The address

58.01	ca	C21
.02	ad	D21
.03	ts	D21
.04	ad	56.01
.05	td	58.16
.06	su	C22
.07	td	62.01
.08	td	65.01
.09	su	C22
.10	td	65.04
.11	su	C22
.12	td	65.08

then is reduced by one from C22 repeatedly, and	58.13	su	C22
the new addresses, Bj,p-1; Bj,p-2; etc. are sent	.14	td	65.12
to the parts of the program where they will be	.15	ca	--
needed to initiate the interpolation procedure.	.16	ts	X1

The address of the height of the waterplane im-	59.01	ca	56.01
mediately below the deck is formed by subtract-	.02	su	C22
ing 1 from Hp, formed in 56., and is sent to parts	.03	td	63.01
of the program where it will be needed subsequently.	.04	td	64.01
Similarly, addresses of the next lower heights are	.05	td	74.03
formed and sent ahead. The value of h_{CG} is sub-	.06	su	C22
tracted from the actual deck height, stored in DHj	.07	td	64.02
(again 0 is the proper initial value since j is	.08	td	74.01
increased by 55.), the result is multiplied by 2^2	.09	td	75.03
and sent to W1 for use in the integration program	.10	su	C22
later.	.11	td	74.04
	.12	td	75.01
	.13	su	C22
	.14	td	75.04
	.15	ca	DH0
	.16	su	E21
	.17	sl	2
	.18	ts	W1

The address 71.01 is sent to 70.01 to direct the flow properly, since this will be the first traversal of the interpolation program and special procedures must be used for interpolating between the initial two points.	60.01	as	71.01
	.02	td	70.01

The first interpolation must be carried using the initial interpolation coefficients stored in the block of registers $I_I \ell,m$ so that $K_I 1,m$ for m=1, 2, 3, 4, is sent to the 65. group where the interpolation is actually done. It is as simple to use the <u>as</u> operation repeatedly as to add 3 to $K_I 1,m$ to form $K_I 1, m+1$.	61.01	as	$K_I 1,1$
	.02	td	65.02
	.03	as	$K_I 1,2$
	.04	td	65.05
	.05	as	$K_I 1,3$
	.06	td	65.09
	.07	as	$K_I 1,4$
	.08	td	65.13

The breadth is known (need not be interpolated for) at every fourth waterplane, but the value must be stored in its proper place in the X block of registers for use in the integration later. The addresses come initially from 55. and 59., and afterwards from 72.	62.01	ca	--
	.02	ts	--

The height corresponding to the known breadth above minus the distance from the base to the assumed KG (i.e., $h_i - h_{CG}$) is formed, multiplied	63.01	ca	--
	.02	su	E1
	.03	sl	2

by 4 by shifting, and stored. Since the distance	63.04	ts	D22
from KG to the waterplane is also needed later it	.05	ts	--
is stored also in a temporary register. The ad-			
dresses come from 55. and 59. initially and from			
72. afterwards.			

The addresses of the i^{th} and $i-1^{th}$ waterplane	64.01	ca	--
heights have been sent initially from 59. and	.02	su	--
later from 72., so that Δw_i, the difference	.03	ts	D23
between the i^{th} and $i-1^{th}$ heights, is formed	.04	ca	C23
directly and stored in a temporary register for	.05	ts	D24
use in forming the heights at the interpolated			
points. The value 0 (not 1, for a reason given			
below at 67.) is stored temporarily for use as			
the index ℓ in counting the number of interpo-			
lations made between two adjacent points.			

The actual interpolation is done straightforwardly	65.01	ca	--
by forming the sum of the products of the four	.02	mr	--
known breadths, the addresses of which were sent	.03	ts	D25
from 58. initially and later changed by 77., with	.04	ca	--
the predetermined Lagrange interpolation coef-	.05	mr	--
ficients, the addresses of which were supplied	.06	ad	D25
initially by 61., later by 76., 78., or 79., and	.07	ts	D25
finally by 81. The accumulated product is possible	.08	ca	--

since the coefficients are less than one, and	65.09	mr	--
the breadths less than $\frac{1}{4}$ so that the sum of	.10	ad	D25
four products is less than one. The result is	.11	ts	D25
shifted left once to compensate for the factor	.12	ca	--
of $\frac{1}{2}$ in the coefficients and left in the ac-	.13	mr	--
cumulator momentarily.	.14	ad	D25
	.15	sl	1

Logically important because the index i is	66.01	ts	--
changed here but not in 65. after the first			
and before the last interpolation, the block			
is coded quite simply, the address coming from			
55. initially and being corrected from 72.			

The quantity ℓ-2 is formed by the devious route	67.01	ao	D24
of forming (ℓ-1)+1-2, since it is desirable to	.02	ca	D24
keep ℓ-1 in storage so that after the index has	.03	su	C24
already increased the numerical value ℓ rather	.04	cp	76.01
than ℓ+1 will be available for use in 69. A			
positive result in 67. directs flow to 76., rather			
than to 70. as the flow diagram indicates, to take			
care of the initial case (see page 41, line 1).			

The index ℓ is increased in the required places	68.01	ao	65.02
by straightforward application of the add one	.02	ao	65.05

operation.	68.03	ao	65.09
	.04	ao	65.13
	.05	ao	66.01
	.06	ao	69.05

$(-\ell \cdot \Delta w_i)$ is formed by multiplying the value	69.01	cs	D23
of $-\Delta w_i$ by $\ell \cdot 2^{-15}$, holding full product, and	.02	mh	D24
than multiplying the product by 2^{15} by shifting	.03	sl	15
left, and $4(h_i - h_{CG})$ is added. The address to	.04	ad	D22
which the result is sent, in the W block, is	.05	ts	--
provided originally by 55. and later corrected	.06	sp	65.01
by 68. and by 72.			

This block is used only to direct the flow pro-	70.01	sp	--
perly after the initial set of interpolations.			

Following only the initial set of interpolations,	71.01	as	73.01
the program moves from 70. to 71. where a new ad-	.02	td	70.01
dress is sent to 70. in order to redirect the flow			
for later interpolations.			

Substituting i-1 for i involves subtracting one	72.01	ca	62.01
from some of the addresses in 62., 63., and 64.	.02	su	C22
In places where i appears with a minus sign, in	.03	td	62.01
p-i, decreasing i by 1 amounts to increasing the	.04	ca	64.02
given address by four. In blocks 66. and 69.,	.05	td	63.01

the address should apparently be increased by	72.06	td	64.01
four, but since block 68. (by increasing ℓ)	.07	su	C22
has already increased the addressed by two,	.08	td	64.02
they need only be increased by two here, there-	.09	ca	62.02
by substituting i-1 for i and restoring 1 for	.10	ad	C26
ℓ, both at once.	.11	td	62.02
	.12	ca	63.05
	.13	ad	C26
	.14	td	63.05
	.15	ao	66.01
	.16	ao	66.01
	.17	ao	69.05
	.18	ao	69.05
	.19	sp	62.01

The index i has not actually been stored separately,	73.01	ca	C25
but is available in the form ca Hi, in block 63.	.02	su	63.01
The interpolation just completed has covered the	.03	cp	80.01
interval between the i^{th} and the $i-1^{th}$ waterplane.			
Consequently, if i=2, the next interpolation is			
to be the last, while if $i \geqq 3$, the intermediate			
procedure is to be followed. The quantity 3-i,			
which will be negative if $i \geqq 3$, can be formed in-			
directly by forming ca H3 - ca Hi. The quantity			
ca H3 is hence stored as a constant, and the dif-			
ference is easily formed.			

The choice of the proper interpolation coefficients depends on the spacing of the given waterplanes between which the interpolation is to occur. If the interval just completed is larger than the one about to be started, block K_{III} of coefficients is to be used, while if these intervals are equal, the choice depends on the spacing of the interval about to be started compared with next interval below it. Since $(h_i-h_{i-1})-(h_{i-1}-h_{i-2})= -2h_{i-1}+ h_i+ h_{i-2}$, the formation of the difference of the intervals is conveniently carried out as indicated by the program. A small constant is subtracted to make sure that rounding off the values of h when they were initially put in the machine cannot cause a zero result to appear to be positive. If the result is truly positive, the subtracted quantity is too small to have any effect.	74.01	cs	--
	.02	sl	1
	.03	ad	--
	.04	ad	--
	.05	su	C26
	.06	cp	78.01

The next two intervals are compared, as in 74. Since the small negative quantity remained in AC, it need not be subtracted again. Block 76. has been moved to a new position (see block 67.), so flow is directed to 77.	75.01	su	--
	.02	sl	1
	.03	ad	--
	.04	ad	--
	.05	cp	79.01
	.06	sp	77.01

Since all three intervals were equal, the equal interval coefficients in block K_{II} are to be used in the interpolation, and the values of $K_{II}1,m$ for m=1, 2, 3, 4 are sent to 65. As is mentioned under block 67., this block belongs ahead of block 70., so flow is directed to 70.

76.01	as	$K_{II}1,1$
.02	td	65.02
.03	as	$K_{II}1,2$
.04	td	65.05
.05	as	$K_{II}1,3$
.06	td	65.09
.07	as	$K_{II}1,4$
.08	td	65.13
.09	sp	70.01

Decreasing i by one is a laborious but straightforward process of subtracting one from the necessary addresses. By choosing an address already containing i-1 instead of choosing one containing i, the first subtraction can be avoided. From here the flow must always proceed to 72., hence the sp operation at the end.

77.01	ca	65.04
.02	td	65.01
.03	su	C22
.04	td	65.04
.05	su	C22
.06	td	65.08
.07	su	C22
.08	td	65.12
.09	ca	74.01
.10	td	74.03
.11	su	C22
.12	td	74.01
.13	td	75.03
.14	su	C22
.15	td	74.04

	77.16	td	75.01
	.17	su	C22
	.18	td	75.04
	.19	sp	72.01

If the comparison in 74. was positive, the coefficients designed for the spacing 2, 1, 1 must be used in the interpolation, and the values of $K_{III}1,m$ for m=1, 2, 3, 4 are sent to 65. From here the flow must always proceed to 77., hence the sp operation at the end.	78.01	as	$K_{III}1,1$
	.02	td	65.02
	.03	as	$K_{III}1,2$
	.04	td	65.05
	.05	as	$K_{III}1,3$
	.06	td	65.09
	.07	as	$K_{III}1,4$
	..08	td	65.13
	.09	sp	77.01

If the comparison in 75. was positive, the coefficients designed for the spacing 1, 1, $\frac{1}{2}$ must be used in the interpolation, and the values of $K_{IV}1,m$ for m=1, 2, 3, 4 are sent to 65. From here the flow must always proceed to 77., hence the sp operation at the end.	79.01	as	$K_{IV}1,1$
	.02	td	65.02
	.03	as	$K_{IV}1,2$
	.04	td	65.05
	.05	as	$K_{IV}1,3$
	.06	td	65.09
	.07	as	$K_{IV}1,4$
	.08	td	65.13
	.09	sp	77.01

At the beginning of this block AC contains the	80.01	ad	C22
positive quantity 3-i since 80.01 can be reached	.02	cp	82.01
only by the cp operation in 73. Now if i=2, there			
is a final interpolation to be made, while if			
$i < 2$, the interpolations are complete, so that			
2-i must be formed, by simply adding one to 3-i			
already in AC.			

The final interpolation coefficients must be used	81.01	as	$K_V 1,1$
in the interpolation, and the values of $K_V 1,m$	.02	td	65.02
for m=1, 2, 3, 4, are sent to 65. From here the	.03	as	$K_V 1,2$
flow must always proceed to 72., hence the sp	.04	td	65.05
operation at the end.	.05	as	$K_V 1,3$
	.06	td	65.09
	.07	as	$K_V 1,4$
	.08	td	65.13
	.09	sp	72.01

Since the last interpolation has been done, the	82.01	ca	65.12
bottom of the ship has been reached. The values	.02	td	82.09
of breadth and four times the distance from KG	.03	ca	62.02
(which in this case is simply $-4h_{CG}$) must be stored.	.04	ad	C26
The address Bj,0 can be obtained from 65.12, while	.05	td	82.10
X(4p-6) and W(4p-6) are in 62.02 and 63.05 so that	.06	ca	63.05
X(4p-2) and W(4p-2) are easily formed.	.07	ad	C26

82.08	td	82.13
.09	ca	--
.10	ts	--
.11	cs	E21
.12	sl	2
.13	ts	--

From the height of the deck (in W1), and of the subsequent interpolated waterlines (in W2, etc.) and the corresponding breadths (in X1, etc.) it is a simple task to calculate the rotated coordinates of the points along the right-hand side of the ship section.

$$u = \frac{b}{2}\sin\theta + w\cos\theta$$

$$t = \frac{b}{2}\cos\theta - w\sin\theta$$

Since the values of w were given a scale factor of 2^2 when they were formed, the value of $\frac{b}{2}$ must be multiplied by 2^2 to match, hence b is multiplied by 2.

83.01	ca	W1
.02	mr	θ'1
.03	ts	D31
.04	ca	X1
.05	sl	1
.06	mr	θ1
.07	ad	D31
.08	ts	D33
.09	ca	W1
.10	mr	θ1
.11	ts	D32
.12	ca	X1
.13	sl	1
.14	mr	θ'1
.15	su	D32
.16	ts	D34

Before any trapezoidal integration can be carried out, it is necessary to have two points, i.e., to have a point with index $\ell-1$ as well as ℓ, so the initial procedure is different from the succeeding cycles of the program. The sp order will be modified from 85. and restored from 112.

```
84.01  sp  85.01
```

The sp order in 84. is modified as required, and the initial values of u_ℓ and t_ℓ are relocated in storage so that they become $u_{\ell-1}$ and $t_{\ell-1}$.

```
85.01  as  87.01
  .02  td  84.01
  .03  ca  D33
  .04  ts  D35
  .05  ca  D34
  .06  ts  D36
```

In a similar fashion to the procedure used in 83., the coordinates of the points on the left-hand half of the ship section are calculated and stored. In the initial cycle (i.e., when this group is reached from 85. rather than 106.) the first values of u_ℓ and t_ℓ were calculated on the right-hand ship, so that the effect is to span the deck of the ship, which is assumed flat.

$$u = -\frac{b}{2}\sin\theta + w\cos\theta$$

$$t = -\frac{b}{2}\cos\theta - w\sin\theta$$

The factor 2 is introduced into the breadth as in

```
86.01  ca  W1
  .02  mr  θ'1
  .03  ts  D31
  .04  cs  X1
  .05  sl  1
  .06  mr  θ1
  .07  ad  D31
  .08  ts  D33
  .09  ca  W1
  .10  mr  θ1
  .11  ts  D32
```

block 83.	86.12	cs	X1
	.13	sl	1
	.14	mr	Θ'1
	.15	su	D32
	.16	ts	D34

In preparation for the trapezoidal integration (or the possible linear interpolation) to follow, the length Δu_{ℓ} along the vertical CC' between the two is determined directly.	87.01	ca	D33
	.02	su	D35
	.03	ts	D37

Since it is desired to integrate only down to a desired waterline, it is necessary to know whether the new point u_{ℓ} is between the same waterlines as the old point $u_{\ell-1}$ or not. It is first compared with the desired line immediately below $u_{\ell-1}$. Initially $u_{\ell-1}$ is at the deck of the ship on the right-hand (upwards) side, and is consequently nearly (at least) the highest point on the ship. It may reasonably be assumed to be above the highest desired waterline, the value of which is in WLq_o-1. To make group 89. follow 88. properly, the comparison is reversed from that shown in the diagram.	88.01	cs	D33
	.02	ad	WLq_o-1
	.03	cp	97.01

Paralleling the procedure in 88., the value of u_ℓ is compared with the desired waterline immediately above $u_{\ell-1}$. The value stored in WLq_o is higher than u_ℓ can possibly be, namely $1-2^{-15}$.

89.01	cs	WLq_o
.02	su	D33
.03	cp	101.01

Since u_ℓ is in the same range as $u_{\ell-1}$, the appropriate address for such a case is sent ahead to 96.

90.01	as	103.01
.02	td	96.01

To prepare for the trapezoidal integration, the sum of the two breadths $t_\ell + t_{\ell+1}$ is formed and stored, with a factor of $\frac{1}{2}$ introduced since t_ℓ may be greater than $\frac{1}{2}$ and the sum therefore greater than one.

91.01	ca	D34
.02	sr	1
.03	ts	D31
.04	ca	D36
.05	sr	1
.06	ad	D31
.07	ts	D38

The area of the trapezoid is formed from $\frac{1}{2}\Sigma t_\ell \cdot \Delta u_\ell$. By choosing 17 waterlines it seems safe to assume that the area between any two desired waterlines will not exceed $\frac{1}{4}$ of the maximum height since $2b < \frac{1}{2}$ and $4W < \frac{1}{4}$, both t and u must be $< .6$, so that $\frac{1}{4} \cdot (\text{max. } u) \cdot (\text{max. } t) \cdot \frac{1}{8}$. Thus a scale factor of 2^3 could be introduced into the area, but because it is

92.01	ca	D38
.02	mh	D37
.03	sr	2

to be integrated, to form the volume, over 11 stations, with the sum of the Simpson coefficients equal to 30, a scale factor of 2^{-5} is also called for. The factor $\frac{1}{2}$ called for by the trapezoid rule has already been introduced into Σt_{ℓ}. Consequently, the total multiplier is $2^3 \cdot 2^{-5} \cdot c_j = 2^{-2} \cdot c_j$ (c_j being the Simpson coefficient). For the initial case, $c_j=1$, and its later values are supplied from 124., 126., 127. Actually, of course, c_j appears in the number of shifts in the sr order. Since the result is used immediately, it is not stored.

The contribution of the ℓ^{th} trapezoid in the j^{th} station is added to the accumulated total volume between the q^{th} and $(q-1)^{th}$ desired waterlines at the k^{th} angle of inclination.

93.01	ad	Vl, q_o
.02	ts	Vl, q_o

The moment of the area is simply the area times $\frac{1}{2}$ the moment arm, which is in turn $\frac{1}{2} \cdot \Sigma t_{\ell}$. Consequently, the moment $= \frac{1}{2} (\frac{1}{2} \Sigma t_{\ell})^2 \cdot \Delta u_{\ell}$. This product is $< \frac{1}{2} (.6)^2 (.6) \frac{1}{8} < 2^{-6}$ so that a factor of 2^6 could be introduced into the moment of the

94.01	ca	D37
.02	mh	D38
.03	sl	2
.04	mh	D38
.05	sr	2

area, but again a scale factor of 2^{-5} is necessary to allow for the volume of integration, so that the scale factor is 2^1. The multiplier is then $\frac{1}{2}\ 2c_j$. Two shift operations are necessary so that the second one, used for c_j, is the same as that used in 92.

As in 93., the new contribution is added to the moment integral.	95.01	ad	M1,q_o
	.02	ts	M1,q_o

Depending on whether the trapezoid includes a desired waterline as its upper limit, its lower limit or not at all, the program proceeds to 108., 107., or 103. respectively, the addresses having come from 101., 97. or 90.	96.01	sp	--

---

Since the new point has gone below a desired waterline, it is necessary to form the integrals up that waterline first, and then integrate from the waterline to the new point by repeating the cycle. First the proper address must be sent to the sp order in 96. to direct the flow for this case.	97.01	as	107.01
	.02	td	96.01

The desired waterline takes the place of u_ℓ for the time being. The waterline address is modified from 107.

98.01	ca	WLq_0-1
.02	ts	D39

The length of the interval from $u_{\ell-1}$ to the waterline is formed. Linear interpolation yields the value of t at the waterline ($t_{\ell'}$). The comparisons in 88. or 89. leave no possibility that the interval from $u_{\ell-1}$ to the desired waterline is greater than or equal to Δu_ℓ, so the division is permissible. Note that $u_{\ell'}$ is left in AC from 98. Σt_ℓ is formed, with a scale factor of $\frac{1}{2}$ since t_ℓ may be greater than one half and Σt_ℓ could therefore exceed one.

99.01	su	D35
.02	ts	D41
.03	ca	D34
.04	su	D36
.05	ts	D32
.06	ca	D41
.07	dv	D37
.08	sl	15
.09	mr	D32
.10	ad	D36
.11	ts	D40
.12	sr	1
.13	ts	D31
.14	ca	D36
.15	sr	1
.16	ad	D31
.17	ts	D38

The values $u_{\ell'}$ and $t_{\ell'}$ may be stored in place of $u_{\ell-1}$ and $t_{\ell-1}$ since the latter are no longer needed and the former will become $u_{\ell-1}$ and $t_{\ell-1}$ in the

100.01	ca	D39
.02	ts	D35
.03	ca	D40

next cycle. The newly calculated $\Delta u_{\ell'}$ takes	100.04	ts	D36
the place of the earlier Δu_{ℓ} for use im-	.05	ca	D41
mediately in the trapezoidal integrations.	.06	ts	D37
Flow must now proceed to 92., hence the sp	.07	sp	92.01
operation.			

As in 97., since the new point has gone above a	101.01	as	108.01
desired waterline, integrals must now be taken	.02	td	96.01
only up to the waterline, and then integrals be-			
tween the waterline and the new point will be			
formed by another cycle. First the proper address			
must be sent to the sp order in 96. to direct flow			
for this case.			

As in 98., the desired waterline takes the place	102.01	ca	WLq_0
of u_{ℓ} for the time being. The waterline address	.02	ts	D39
is modified by 108. Flow must now proceed to 99.,	.03	sp	99.01
hence the sp operation.			

Having completed one trapezoid, the program must	103.01	ca	D33
prepare for another. The end point u_{ℓ}, t_{ℓ} of	.02	ts	D35
the last trapezoid becomes the initial point	.03	ca	D34
$u_{\ell-1}$, $t_{\ell-1}$ of the trapezoid-to-be.	.04	ts	D36

When this block is reached for the first time,	104.01	sp	105.01

the integration will have gone across the deck and will be ready to start down the left-hand side. In this case the flow should proceed to 105. However, after the left side is completed and the computation goes up the right side, flow should proceed to 111.

The addresses in 86. are modified to permit determination of the rotated coordinates of the next lower point on the left-hand side.	105.01	ao	86.01
	.02	ao	86.04
	.03	ao	86.09
	.04	ao	86.12

To determine whether the bottom of the ship has been reached, the address from which the next interpolated height is to be taken is compared with the address of the height at the base line (obtained from block 82. of Flow Diagram III). Note that the operation codes must also be subtracted out. If the process is complete (next address greater than address at bottom) flow proceeds to 109., while if the bottom has not been reached, flow returns to 86.	106.01	ca	82.13
	.02	su	C31
	.03	ad	C32
	.04	su	86.01
	.05	cp	86.01
	.06	sp	109.01

If the trapezoid included a desired waterline as its lower limit, the new point is in a lower range	107.01	ca	88.02
	.02	td	89.01

than the old point, so that addresses must be changed in 88., 89., 93., 95., 98., and 102. Flow must then return to 87., hence the sp operation.

107.03	td	102.01
.04	su	C33
.05	td	88.02
.06	td	98.01
.07	ca	93.01
.08	su	C33
.09	td	93.01
.10	td	93.02
.11	ca	95.01
.12	su	C33
.13	td	95.01
.14	td	95.02
.15	sp	87.01

Similarly, if the trapezoid included a desired waterline as its upper limit, addresses must be changed as in 107., except that increasing is easier than decreasing by virtue of the ao operation. Flow must then return to 87., hence the sp operation.

108.01	ao	88.02
.02	ao	89.01
.03	ao	93.01
.04	ao	93.02
.05	ao	95.01
.06	ao	95.02
.07	ao	98.01
.08	ao	102.01
.09	sp	87.01

Since the bottom of the ship has been reached on

109.01	ca	82.13

the left-hand side, the next point should be picked at the bottom on the right-hand side. Hence the addresses Wp_o-2 and Xp_o-2 must be sent to 83. In future cycles it will be necessary to have set the addresses in 86. back to the top of the ship, i.e., to W1 and X1.	109.02	td	83.01
	.03	td	83.09
	.04	ca	82.10
	.05	td	83.04
	.06	td	83.12
	.07	as	W1
	.08	td	86.01
	.09	td	86.09
	.10	as	K1
	.11	td	86.04
	.12	td	86.12

Since the integration is starting up the right-hand side, the proper address must be sent to the sp order in 104. to direct future flow.	110.01	as	111.01
	.02	td	104.01
	.03	sp	83.01

To determine whether the top of the ship has been reached on the right-hand side, the address from which the last interpolated breadth was taken is compared with the address of the breadth of the deck.	111.01	ca	83.04
	.02	su	C34
	.03	cp	121.01

The deck having been reached, the flow-directing sp orders in 84. and 104. are set up for a new cycle.	112.01	as	85.01
	.02	td	84.01
	.03	as	105.01

	112.04	td	104.01
To determine whether all the angles have been completed, the address of the last used θ_k is compared with θ_{k_o}.	113.01	ca	C35
	.02	su	83.06
	.03	cp	122.01
The last angle having been completed, the program is reset to the initial angle values for a new cycle.	114.01	as	θ1
	.02	td	83.06
	.03	td	83.10
	.04	td	86.06
	.05	td	86.10
	.06	as	θ'1
	.07	td	83.06
	.08	td	83.14
	.09	td	86.06
	.10	td	86.14
	.11	as	$V1,q_o$
	.12	td	93.01
	.13	td	93.02
	.14	as	$M1,q_o$
	.15	td	95.01
	.16	td	95.02
To determine whether all stations have been completed, appropriate addresses are compared.	115.01	ca	C36
	.02	su	56.02

At this point it should be remembered that Flow Diagram II is a part of the large cycle involved for integrating each station, and the address indicating the station last used is in 56.

115.03	cp	123.01

All of the integrations between waterplanes having been done, the actual values of the integrals up to the various waterplanes must be formed. This involves multiplying each "integral" by the Simpson's rule factor (in this case $\frac{L}{30}$ already stored in E1). The integrals between waterplanes must then be summed to form the integrals up to each waterplane, and a scale factor of 2^{-2} must be introduced before this is permissible. The summation will be done in a double cycle, the first step being to select the integral in the lowest interval, multiply it by the Simpson factor and the scale factor, add to it the integral up to it (in this case the integral up to the bottom, which is of course zero, so that zero is stored in Vk,0 and Mk,0 for all k). (Note that this storage setup is not included in the flow diagram.) The total scale factor associated with V is $2^{2(-q+2)-2-p-2} = 2^{-2q-p}$ which is in accord with volume scale factor for the upright integration and is safe.

116.01	ca	V1,1
.02	mh	E1
.03	sr	2
.04	ad	V1,0
.05	ts	V1,1

The values of moment are summed in a manner exactly analogous to that in 116. The total scale factor is reasonably safe but for absolute assurance another 2^{-3} should be inserted, permitting the moment to be the total maximum volume times $\frac{1}{2}$ the maximum breadth. Even with the factor as it stands, the maximum moment in the typical study would have only 8 binary digits.

117.01	ca	M1,1
.02	mh	E1
.03	sr	3
.04	ad	M1,0
.05	ts	M1,1

The quantity q (index, not scale factor) is not readily available in an address, since the addresses have double indices, and it becomes expedient to store an index equivalent to q in temporary storage D42. Its initial value may as well be $-q_o$ so that if it is increased by one each time it will become zero when the cycle has been completed q_o times, because the ao order leaves the negative of the quantity in AC. The minus sign is desirable. When the index becomes zero, it should be restored to $-q_o$ for the next cycle.

118.01	ao	D42
.02	cp	128.01
.03	cs	C37
.04	ts	D42

The comparison for k_o is simple since the first index q is already stabilized at q_o. Notice that the sign of the comparison differs from that shown in the flow diagram in order to permit 120. to follow 119. directly.

119.01	ca	116.01
.02	su	C38
.03	cp	OUTPUT PROGRAM

The next cycle must be done with k increased by one. Furthermore, q should be restored to one. Only one need be added to each address since $(k,q_o)+1=(k+1,1)$.

120.01	ao	116.01
.02	ao	116.04
.03	ao	116.05
.04	ao	117.01
.05	ao	117.04
.06	ao	117.05
.07	sp	116.01

After 111., the top of the ship having not been reached, the next higher point must be chosen by 83. This involves reducing the addresses by one.

121.01	ca	83.01
.02	su	C33
.03	td	83.01
.04	td	83.09
.05	ca	83.04
.06	su	C33
.07	td	83.04
.08	td	83.12
.09	sp	83.01

After 113., not all angles having been completed, the addresses in 83., 86., 93., 95., must be increased to prepare to take another angle. The addresses Vk,q_o and Mk,q_o must be increased by q_o since $(k,q_o)+q_o=(k+1,q_o)$.

122.01	ao	83.02
.02	ao	83.06
.03	ao	83.10
.04	ao	83.14
.05	ao	86.02
.06	ao	86.06
.07	ao	86.10

	122.08	ao	86.14
	.09	ca	93.01
	.10	ad	C37
	.11	td	93.01
	.12	td	93.02
	.13	ca	95.01
	.14	ad	C37
	.15	td	95.01
	.16	td	95.02
	.17	sp	83.01

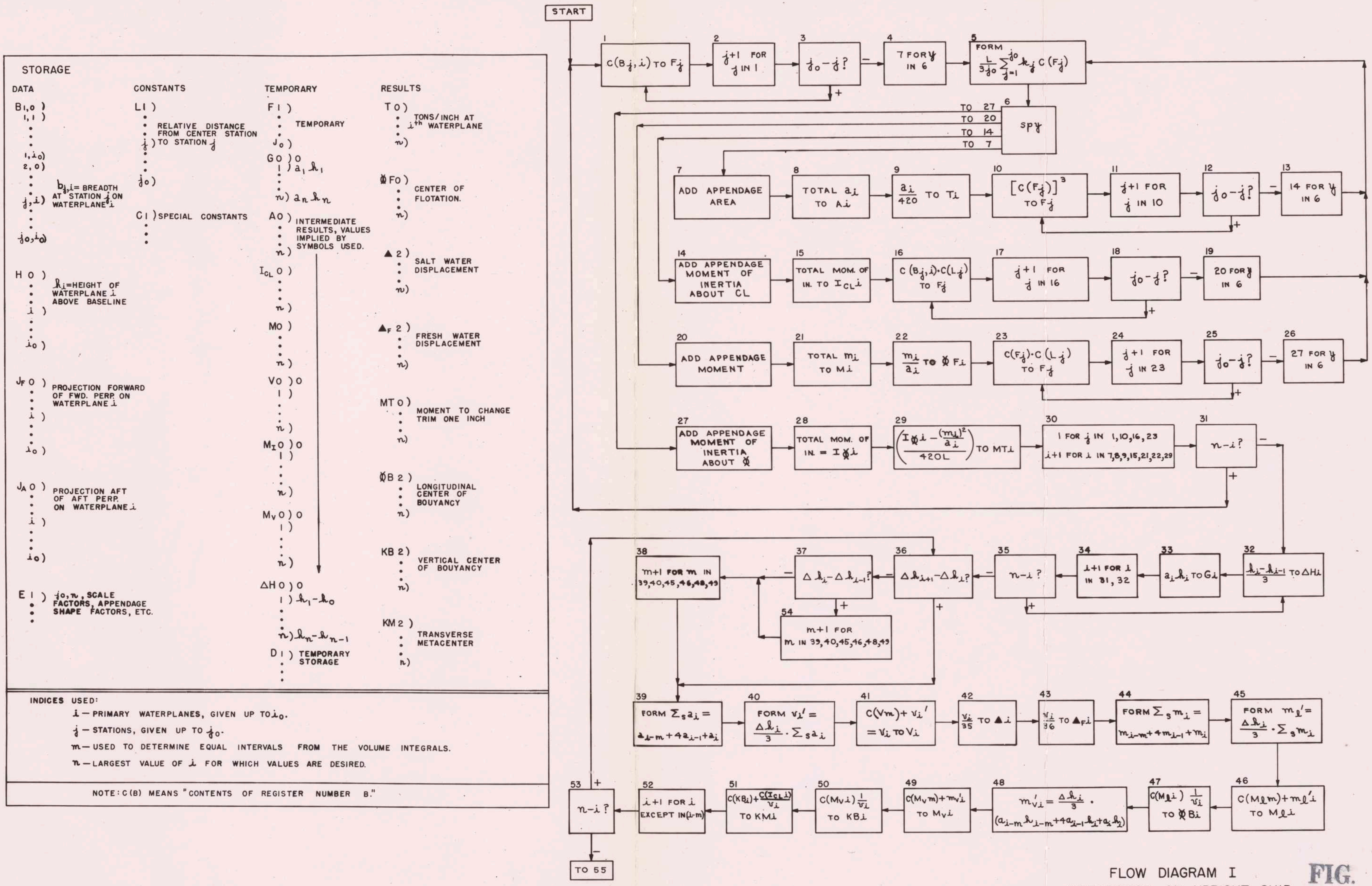
After 115., the quantity j_0-j is in AC. If	123.01	su	C33
j_0-j-1 is negative (zero), the next cycle will	.02	cp	125.01
be the last and the Simpson coefficient should			
be one. Otherwise, the Simpson coefficient will			
be two or four.			

The Simpson coefficient is given its initial	124.01	as	2
value. Flow must now return to 55., hence the	.02	td	92.03
sp order.	.03	td	94.05
	.04	sp	55.01

If the Simpson coefficient was two or one, it	125.01	ca	C40
should become four, and if four, two. To de-	.02	su	92.03
termine what it was, sr 1 is subtracted from	.03	cp	127.01

the sr order. If the coefficient was one (sr 2) or two (sr 1) the result is negative.

The proper number of shifts is put in AC. Since the orders to transfer the digits to the proper places are already in 124., along with the sp 55.01 order, an obvious saving results by ordering sp 124.02.	126.01	as	0
	.02	sp	124.02
Again the proper number of shifts is put in AC and the flow is directed to 124.02.	127.01	as	1
	.02	sp	124.02
After 118., to repeat the cycle with q+1 in place of q, the appropriate addresses are simply increased by one.	128.01	ao	116.01
	.02	ao	116.04
	.03	ao	116.05
	.04	ao	117.01
	.05	ao	117.04
	.06	ao	117.05
	.07	sp	116.01



FLOW DIAGRAM I
INTEGRATION ON UPRIGHT SHIP

FIG. 12

D-33315

STORAGE

DATA

B 1,0)
1,1)
•
•
1,i_o)
2,0)
• $b_{j,i}$ = BREADTH AT STATION j AT WATERPLANE i
•
j_o,i)
•
j_o,i_o)

H 0)
1)
•
i) h_i = HEIGHT OF WATERPLANE i ABOVE BASE LINE
•
i_o)

DH 1)
•
j) DECK HEIGHT AT STATION j
•
j_o)

E 21) h_{cg} = CHOSEN HEIGHT OF KG

CONSTANTS

K_I 1,1)
2,1)
3,1)
1,2)
•
l,m) SPECIALIZED LAGRANGE INTERPOLATION COEFFICIENTS
•
3,4)

K_{II} 1,1)
•
3,4)

K_{III} 1,1)
•
3,4)

K_{IV} 1,1)
•
3,4)

K_V 1,1)
•
3,4)

C 21) SPECIAL CONSTANTS
•
•

TEMPORARY

X 1)
• INTERPOLATED VALUES OF $b_{i,j}$
i)
•
p-2)
•
$4i_o$+1)

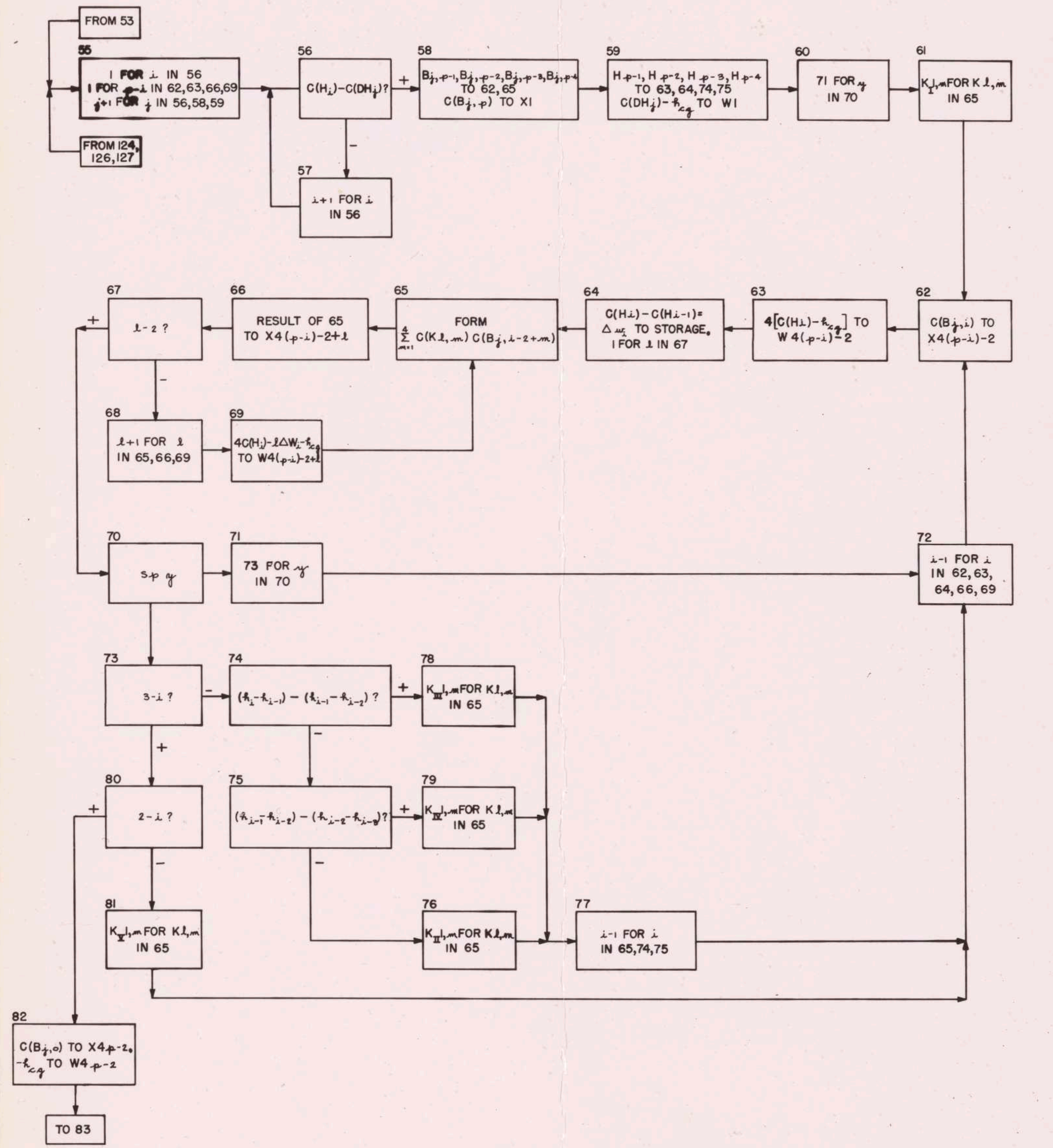
W 1)
•
i) INTERPOLATED VALUES OF $h_i - h_{cg}$
•
p-2)
•
$4i_o$+1)

D 21) INTERMEDIATE RESULTS
•
•

INDICES USED:
i - PRIMARY WATERPLANES
j - STATIONS
l - INTERPOLATED WATERPLANES
m - INTERPOLATION TERM INDEX
p - VALUE OF LEAST i ABOVE DECK

ROMAN NUMERALS:
I INITIAL INTERVAL INTERPOLATION COEFFIENTS
II, III, IV INTERMEDIATE " " , DEPENDING ON SPACING
V FINAL INTERVAL " "

NOTE: C(B) MEANS "CONTENTS OF REGISTER NUMBER B"



FLOW DIAGRAM II
INTERPOLATION OF HALF-BREADTHS

FIG. 13

D-33316

STORAGE

DATA

WL 1) DESIRED WATERLINES, RELATIVE TO KG, POSITIVE UPWARD.
q)
q_o)

θ 1) SIN θ_1 WHERE θ_k = DESIRED ANGLE OF ROTATION.
k) SIN θ_k
k_o) SIN θ_{k_o}

θ' 1) COS θ_1
k) COS θ_k
k_o) COS θ_{k_o}

TEMPORARY

W 1) HEIGHT OF INTERPOLATED WATERLINES, RELATIVE TO KG, POSITIVE UPWARD.
ℓ) C(W1) = DECK HEIGHT
p_o-2)

X 1) INTERPOLATED BREADTHS, CORRESPONDING TO HEIGHTS IN Wℓ
ℓ)
p_o-2)

D 31) TEMPORARY

RESULTS

V 1,1)
1,2) DISPLACED VOLUME TO WATERLINE q WHEN ROTATED TO θ_k
1,q_o)
2,1)
k,q)
k_o,q_o)

M 1,1)
1,2) MOMENT OF DISPLACED VOLUME TO WATERLINE q WHEN ROTATED TO θ_k
1,q_o)
2,1)
k,q)
k_o,q_o)

INDICES USED:
- j – STATIONS
- k – DESIRED ANGULAR ROTATIONS
- ℓ – INTERPOLATED WATERPLANES, INCREASING DOWNWARDS.
- q – DESIRED WATERPLANES
- p_o-2 – TOTAL NUMBER OF INTERPOLATED VALUES, FROM DIAGRAM II.

NOTE: C(B) MEANS "CONTENTS OF REGISTER NUMBER B".

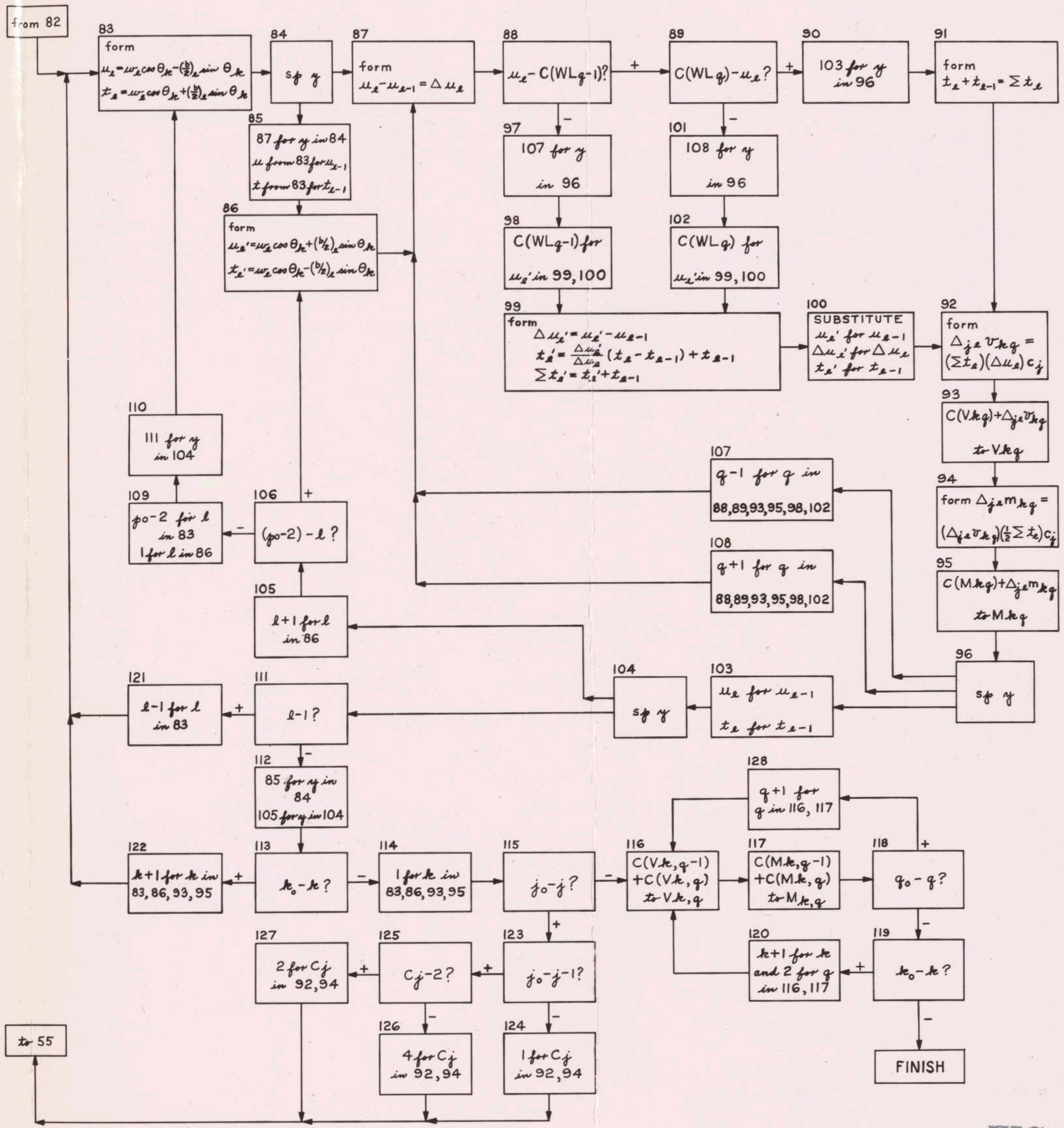


FIG. 14

FLOW DIAGRAM III

INTEGRATION BELOW ROTATED WATERPLANES

D-33317

VII - Summary

Required data and assumptions

In the program as given, the following data are required:

1) The breadth of the ship at eleven equally-spaced stations at i_o waterplanes starting at the base, where the i_o^{th} plane is above the deck of the ship at every station, and the breadth at the deck is given in lieu of the breadth at the waterplane above the deck.
2) The heights of the waterplanes at which the breadths are given.
3) The height of the deck at each station.
4) The length of the projection forward of F.P. and aft of A.P. for every waterplane (zero is the value in most cases).
5) The shape factors for the fore and aft appendages.
6) The height KO to be used in determining cross curves.
7) Optional - The angles and waterplanes to be used in finding cross curves. These could equally well be chosen once and for all, using relative rather than absolute heights for the waterplanes.
8) Optional - The number of waterplanes at which data are given and the number to be used in calculating curves of form. The computer could count the number given, determine the highest waterplane below the

deck at all stations, and calculate curves of form up to that plane.

It has been assumed that the waterplanes for which breadths are given are spaced from the base in the ratio 1,1,2,.. .., 2, 4, ..., $2^{n-1}, 2^n$, ..., $2^n, 2^n, 2^n$.

It has been assumed that the appendages do not contribute more than about 5% to the area and 10% to the moment of inertia about the midship station. These assumptions are easily modified.

Estimate of errors

The validity of Simpson's rule for the integrations in upright position has been tested experimentally on the typical study. The values needed for the curves of form were all calculated and were found to agree well with the values obtained by the conventional method, except at extremely shallow drafts, where percentage differences between results went as high as 10% or more. Neither Simpson's rule nor an integraph is likely to be very accurate at shallow drafts and it would be hard to say which is the more accurate. At more normal drafts, results are better and for the most part the differences between the Simpson rule result and the Tchebycheff result are within the least errors which are expected. C. L. Wright, Jr., states in a letter dated November 9, 1948, that he believes the errors in the conventional method of calculation as illustrated are within the following limits:

"(a) Displacements, tons per inch immersion, and moments to change trim one inch within 0.5%.

"(b) Heights of centers of buoyancy and of metacenters above the keel within 0.2% of the beam of the vessel.

"(c) Distances of centers of buoyancy and of centers of flotation from midships within 0.1% of the length of the vessel.

"(d) Righting Arms within 0.2% of the beam of the vessel."

Computer storage requirement

The program requires 758 orders, and hence 758 storage registers. Data storage, allowing for twenty waterplanes, requires about 320 registers. There are about 130 constants which must be stored. The results will occupy 120 registers for curves of form and 420 for cross curves. Temporary storage as given requires about 250 registers, but by proper assignment, making use of each register as often as possible, this requirement can be reduced to at most 150. The total requirement is then about 1600 registers, well within the 2048 provided in the Whirlwind Computer.

It should be pointed out, however, that the 420 register requirement for cross curves is for points at 16 depths at 15 angles, there being two coordinates for each point. The program can easily be made to give closer spacings of points both in angles and depths, but the process cannot be extended very far without exceeding the storage capacity of the machine.

Computer time requirement

To estimate the time required to perform the program given, it is necessary only to estimate the number of times each block will be gone through in the course of a complete computation. Considering the first Flow Diagram, the total number of orders is the following, where "1. to 31.", for example, means the sum of all the orders in blocks 1. through 31.:

$$j_0(1.\text{ to }3.)+4.+4(5.\text{ to }6.)+7.\text{ to }9.+j_0(10.\text{ to }12.)+13.\text{ to }15.+j_0(16.\text{ to }18.)+19.\text{ to }22.+j_0(23.\text{ to }25.)+26.\text{ to }31.(n+1)+(32.\text{ to }35.)n+(36.\text{ to }53.)(n-1)$$

$$= 39j_0n+308n+39j_0+164.$$

For Flow Diagrams II and III a little more approximation is necessary, since some of the alternative paths depend on the shape of the ship, but a good approximation is:

$$j_0\ 55.+i_0(56.\text{ to }57.)+58.\text{ to }61.+i_0\ 62.\text{ to }64.+4(65.\text{ to }67.)+3(68.\text{ to }69.)+70.\text{ to }78.\ +82+k_0\ 4i_0(83.+84.+86.)+8i_0(87.\text{ to }96.)+q_0(87.+88.+92.\text{ to }102.+107.+108.)+8i_0(103.\text{ to }104.)+4i_0(105.+106.+111.+121.)+109.+110.+112.+113.+122.\ +114.+115.+123.+125.+126+k_0\ q_0(116.\text{ to }118.+128.)+119.+120.$$

$$=468i_0j_0k_0+203i_0j_0+99j_0+20k_0+10k_0$$

If the values as given in the program are substituted, these results yield: $740n + 550$ for the curves of form and

$80000\ i_0 + 5000$ for the cross curves.

With values of 15 for n and 20 for i_0 these become 11600 and 1,600,000 respectively. The total computing time would then be about 30 seconds, with less than $\frac{1}{4}$ second required for the curves of form alone.

BIBLIOGRAPHICAL NOTE

Most of the information used in this thesis was obtained from correspondence with Charles L. Wright, Jr., and from the classified literature of Project Whirlwind. For that reason an attempt was made to reproduce all the necessary information in the body of the thesis in the hope that the sources will not be needed by the reader.

A very helpful standard text on naval architecture is:

Rossell, H. E., and Chapman, H. B., Principles of Naval Architecture - Volume 1, New York, 1939.

A general discussion of programming for digital computers, which includes (in Part II, Volume II) detailed codes for integration and interpolation is:

Goldstone, H. H., and von Neumann, J., Planning and Coding Problems for an Electronic Computing Instrument, Parts I and II, Princeton, N. J., 1946-1948.