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A CORRELATION FOR RELATING THE FRICTION FACTOR
IN CURVED PIPE TO THE FRICTION FACTOR IN A
STRAIGHT LENGTH OF PIPE

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Professor Robert C. Reid
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Dear Professor Reid,

During the recent months, I have performed an experimental investigation on my thesis topic - the deriving of a generalized correlation for relating the friction factor in curved pipe to the friction factor in a straight length of pipe. In this report, I am presenting the background of the topic and the results of my investigation.

Very truly yours,

Signature redacted

LAWRENCE R. KRAVITZ

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I. SUMMARY

The object of this thesis was the establishing of a generalized correlation relating the friction factor of curved pipe to the friction factor of a straight length of pipe. Although an empirical correlation has already been established for both streamline and turbulent fluid flow in pipe coils, these correlations are restricted to helices with closely spaced turns. After verifying the applicability of the established correlations to coils with closely wound turns, this investigator intended to increase the pitch in the turns and note the effect on the correlations.

No results were obtained. For the streamline flow of water in his straight test section of high pressure rubber hose, this investigator obtained pressure losses greater than anticipated for viscous fluid flow in pipes. The data for turbulent flow gave also for particular Reynolds Numbers values of friction factors greater than expected for smooth tubing. Furthermore, the pressure loss in a helix for a distinct value of the Reynolds Number during turbulent flow was usually the same or less than the value in the straight test section - for this helix, a value 35% greater was anticipated.

This investigator believes the source of the error lies in the use of glass tees to measure the pressure differentials across the test section. These tees caused an

unaccountable pressure loss in the test section; they evidently masked the effect on the friction factor when the tubing was wrapped into a helix. An alternative to the use of the glass tees is recommended - a small hole may be punctured in the wall of the rubber hose and a thin hypodermic needle, the manometer tap, may be inserted flush with the inside wall of the tubing. This technique would avoid inducing any turbulence into the flow because of an obstruction in the path of the fluid.

II. INTRODUCTION

The application of pipe coils in chemical engineering processes appears in heat exchangers. A correlation for estimating the friction factor in a curved pipe would lead, by application of the Bernoulli Equation, to an estimation of the pressure loss in a pipe coil. This information would be useful for predicting power requirements for pumping and suitable pipe and coil sizes.

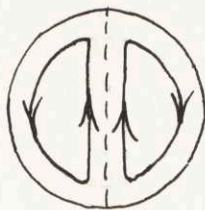
The pressure drop in pipes is expressed by the Bernoulli Equation

$$-\int v \, dP = \frac{\Delta U^2}{2g_c} + \frac{g\Delta X}{g_c} + F_R \quad 1$$

The quantities $\frac{\Delta U^2}{2g_c}$ and $\frac{g\Delta X}{g_c}$ are state functions, independent of the path. The fluid friction loss, F_R , however, is dependent upon the path of the fluid flow. The value of F_R is calculated from the Fanning Equation for fluid flow in pipes. Experimental investigation by White for both streamline and turbulent fluid flow has revealed that when a length of straight tubing is wound into a helix, the frictional loss for the same Reynolds Number is increased (8). This effect arises from an increase in the friction factor.

Taylor examined the flow lines of streamline flow through curved pipes using color bands (7). The figure below is a cross-section by a vertical axial plane of the

upper section of one of the coils of a helix laying in a horizontal plane. The flow is toward the axis of the helix



near the wall and away from it in the center. This spiraling motion in the fluid results from the radial pressure gradients arising from the centrifugal action induced by the helix. Because of the radial component in velocity of the fluid molecules during flow through a curved pipe and the longer path per unit length of pipe traveled by each molecule, a higher pressure drop in a helix is anticipated.

Dean, in an analysis of the fluid motion during streamline flow, showed that a relationship, C_s , equal to the ratio of the friction factor in the curved pipe to that in the straight, $\frac{f_c}{f_s}$, is a function of $Re \sqrt{\frac{d}{D}}$ where d is the inner diameter of the tubing and D is the mean diameter of the helix (2). White verified this relationship experimentally. White's results could be expressed as

$$(C_s)^{-1} = \left(\frac{f_c}{f_s}\right)^{-1} = 1 - \left[1 - \left(\frac{11.6}{Re \sqrt{\frac{d}{D}}}\right)^{0.45}\right]^{\frac{1}{0.45}} \quad 2$$

(For a graphical presentation of the relationship, refer to Figure 1). Streamline fluid flow exists for values of f_c greater than 0.009; for smaller values of f_c , the fluid flow

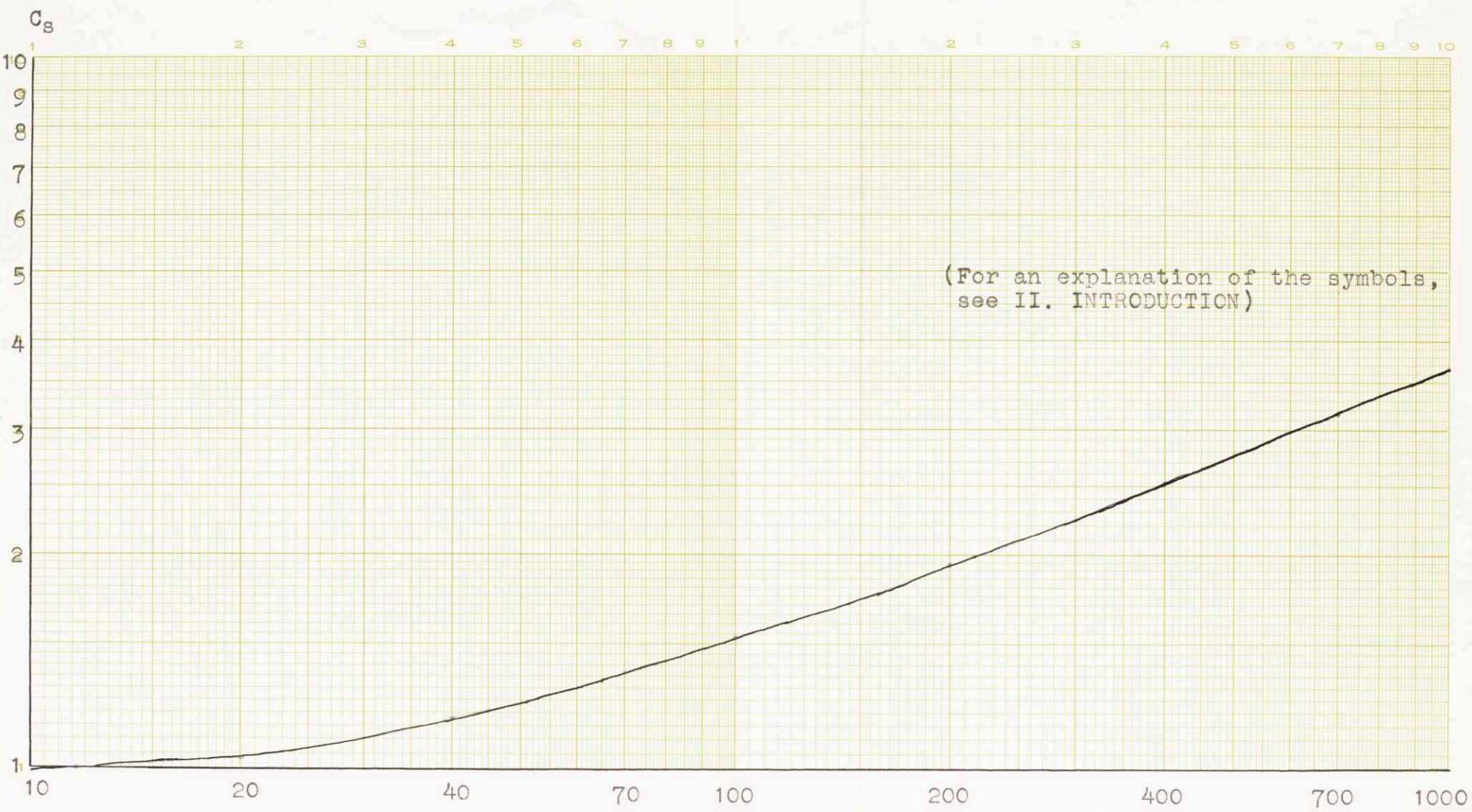


Figure 1

A plot of the White Correlation relating the friction factors of curved and straight pipe during streamline flow in pipe coils

$$Re \sqrt{\frac{d}{D}}$$

is in the turbulent region where the rule is no longer applicable.

Streamline motion continues for higher Reynolds Numbers in curved pipe than in straight pipe. An analysis of Taylor's and White's results reveals that the sharper the curvature of the helix, the higher will be the Reynolds Number for the continuance of streamline motion. Furthermore, the transition from streamline to turbulent fluid flow is not as marked for curved pipe as it is for straight pipe. Dean accounts for this by considering the motion of the fluid in each situation: turbulence in straight pipe is accompanied by the lateral movement of fluid (swirls and eddies) which implies a loss of energy that has no counterpart in viscous flow. In a curved pipe, lateral movement of the fluid is occurring before the critical velocity is reached; it probably increases when turbulence sets in but there will not be a rapid change in the friction factor of the curved pipe (3).

No theoretical correlation for the relationship of f_c and f_s during turbulent fluid flow has been proposed. Cremer and Davies mention that an empirical relationship of the friction factor of the curved pipe to that of the straight for turbulent fluid flow in pipe coils has been derived (1). This relationship is expressed as

$$C_t = \frac{f_c}{f_s} = \exp \frac{2\pi d}{D}$$

3

(For a graphical presentation of the relationship, refer to Figure 2). The author of this thesis, however, endeavoring to learn the conditions under which the investigation was performed and the range of Reynolds Numbers encompassed by the correlation, was unable to locate the original paper.

High pressure rubber hose will be used to facilitate experimental work in this project. After the relationship of the friction factor to the Reynolds Number for a straight test section of the rubber tubing using water as a fluid is determined, the tubing will be wound into helices of different diameters. For each helix data will be obtained relating the friction factor of the helix to the Reynolds Number for both streamline and turbulent fluid flow. White's empirical correlation for streamline flow can then be verified. The applicability of the correlation for turbulent flow in pipe coils proposed by Cremer can be determined.

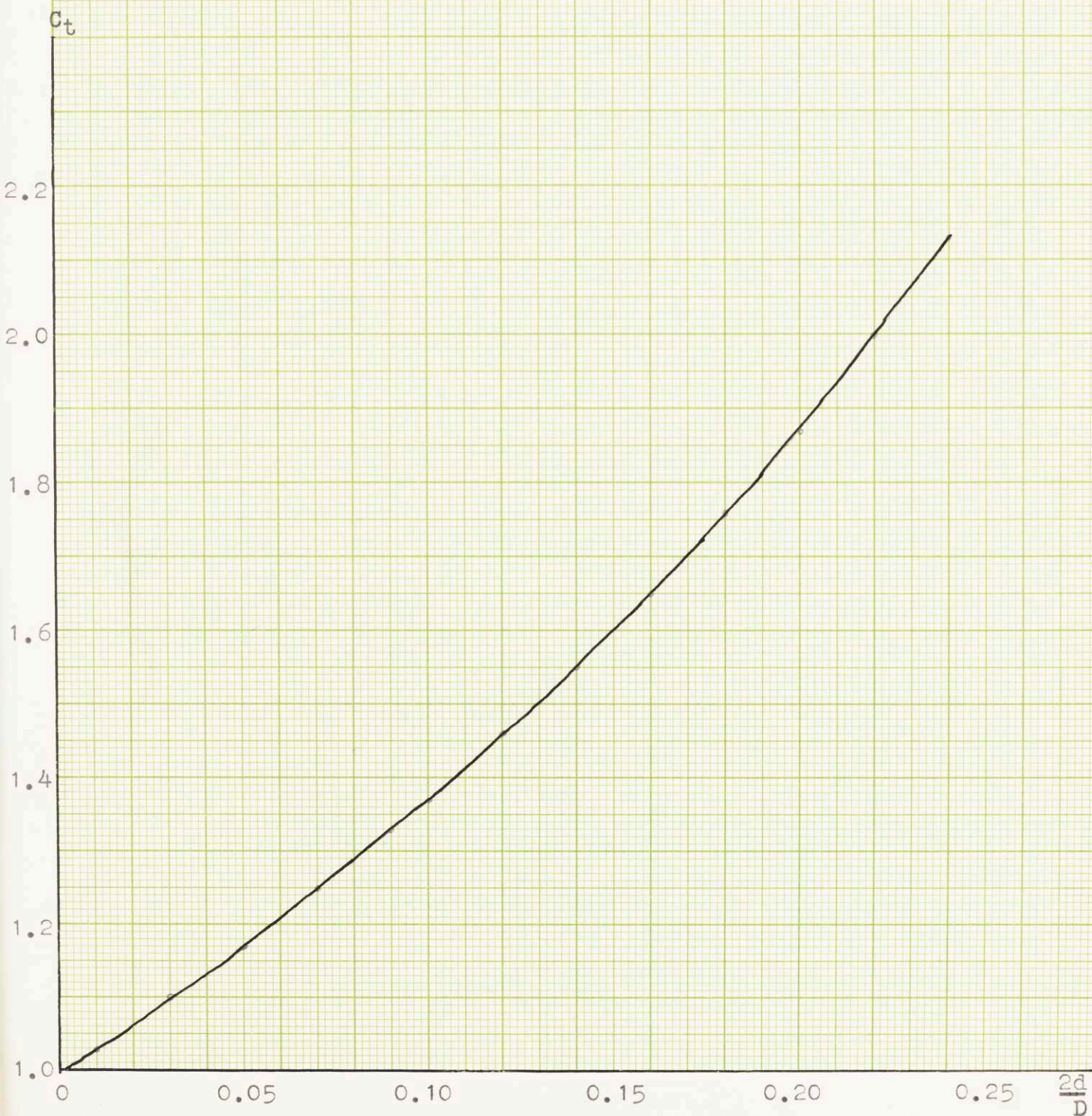
The previous research into this problem failed to note the effect of the pitch of the turns in the helix upon the correlations. Only Eustice gave information about the pitch of the tubing in his work - the turns had a pitch of eight degrees (4). Although the other researchers do not mention the pitch in their helices, this investigator is inclined to believe that since they compared their results with Eustice's, their coils too must have been closely wound.

It appears evident that as the turns in the helix are spread apart increasing the pitch, the frictional loss per

Figure 2

A plot of the Cremer and Davies Correlation relating the friction factors of curved and straight pipe during turbulent fluid flow in pipe coils

(For an explanation of the symbols, see II. INTRODUCTION)



unit length of the tubing will decrease. In the limit, the pitch is ninety degrees - the helix has now lost its identity and the tubing is stretched out in a straight line. The plot of the friction factor versus the Reynolds Number for the curved pipe will now be coincident with the plot of the straight test section.

After the correlations for the streamline and the turbulent fluid flow have been verified, the pitch in the turns of a helix will be changed and the effect on the correlations noted. The intention of this work is to generalize the correlations which have already been established.

III. PROCEDURE

The pressure loss in the test section of high pressure rubber hose was measured by a mercury, a carbon tetrachloride, or an inverted toluene manometer depending upon the magnitude of the pressure difference. The leads of tygon tubing extended to the manometers from the glass tees inserted at each end of the test section. The author anticipated the loss due to the tees to be negligible.

The glass tees were placed on the same horizontal plane; the tap water used in the project was considered incompressible. Hence Equation 1 reduces to

$$- \int v \, dP = F_R \quad 4$$

$$\text{But } - \int v \, dP = \frac{1}{\rho} (\rho_{\text{manometer fluid}} - \rho) \frac{g}{g_c} H \quad 5$$

The quantity, X , is the displacement of the manometer fluid. The frictional loss, F_R , is expressed by the Fanning Equation for fluid flow in pipes

$$F_R = \frac{32 f L w^2}{g_c \pi^2 d^5 \rho^2} \quad 6$$

where w is the mass flow rate and L is the length of the tubing. Hence

$$\frac{1}{\rho} (\rho_{\text{manometer fluid}} - \rho) \frac{g}{g_c} H = \frac{32 f L w^2}{g_c \pi^2 d^5 \rho^2} \quad 7$$

Data was collected relating the friction factor to the

Reynolds Number for the flow through the hose.

When the author observed that the results were not conforming with those expected, he regarded the loss due to the tees as appreciable and tried two methods for calculating this loss. The first was the insertion of two more tees at the downstream end of the test section. A length of hose, about an inch long, separated each of the tees. The pressure loss between the upstream tee and the first downstream tee was considered the loss for the test section and one tee; the loss between the upstream tee and the second downstream tee was regarded as the loss for the test section and two tees; etc. These losses were plotted and extrapolated to zero tees. However, these results did not correspond with those expected.

The second method was the repeating of the runs for a shorter test section. The pressure losses were plotted as a function of the length of the test section and extrapolated to zero length to give the loss due to the tees. This method too failed to correct the results for the smooth rubber tubing.

IV. RESULTS

The results of the project are presented in Figures three to six.

Figure 3 is a plot of the Fanning friction factor vs the Reynolds Number for streamline fluid flow in the straight test section. This graph and the subsequent Figures four and five present the result of correcting the data for the frictional loss from the one tee influencing the fluid flow in the test section. This was evaluated by an extrapolation of the pressure loss to zero tees.

A plot of the Fanning friction factor vs the Reynolds Number for turbulent fluid flow in the straight test section is presented in Figure 4.

Figure 6 is also a plot of the friction factor vs the Reynolds Number for turbulent flow in the straight test section. But this plot presents the result of deducting the pressure loss of the tee influencing the flow of water through the section, evaluated by extrapolating the pressure loss to zero length.

Figure 5 is a plot of the friction factor, Reynolds Number relationship for the turbulent fluid flow in a helix.

f

0.1

0.07

0.04

0.02

0.01

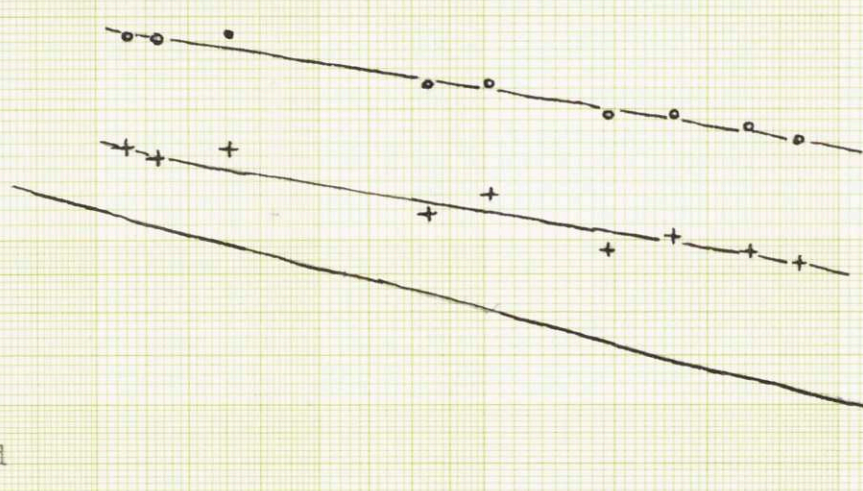
0.007

0.004

0.002

Figure 4

A plot of the Fanning friction factor vs the Reynolds Number for turbulent fluid flow in the straight test section using the method of extrapolating to zero tees to determine the frictional loss from one tee



Legend

—○—○—

The curve for the uncorrected data

—+—+—

The curve for the data corrected for the frictional loss from one tee using the technique of extrapolating to zero tees

————

The curve for the expected results

4000

10000

40000

105000

Re

f

Figure 5

A plot of the Fanning friction factor vs the Reynolds Number for turbulent fluid flow in the helical test section

0.1

0.07

0.04

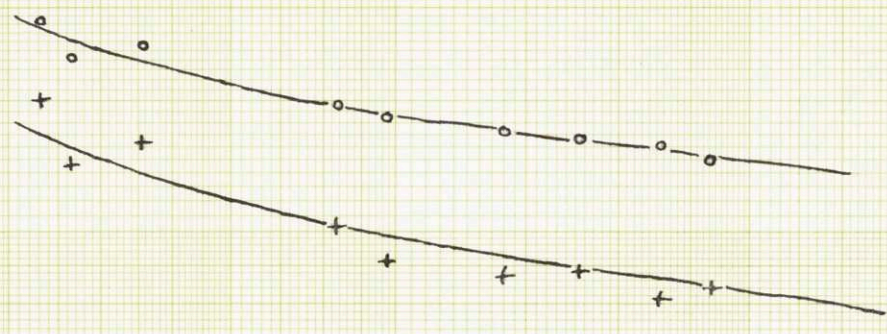
0.02

0.01

0.007

0.004

0.002



Legend

—○—○—

The curve for the uncorrected data

—+—+—

The curve for the data corrected for the frictional loss from one tee using the technique of extrapolating to zero tees

100 2 3 4000 5 6 7 8 9 10 10000 2 3 40000 5 6 7 8 9 10 10⁵ Re

f

0.1

0.07

0.04

0.02

0.01

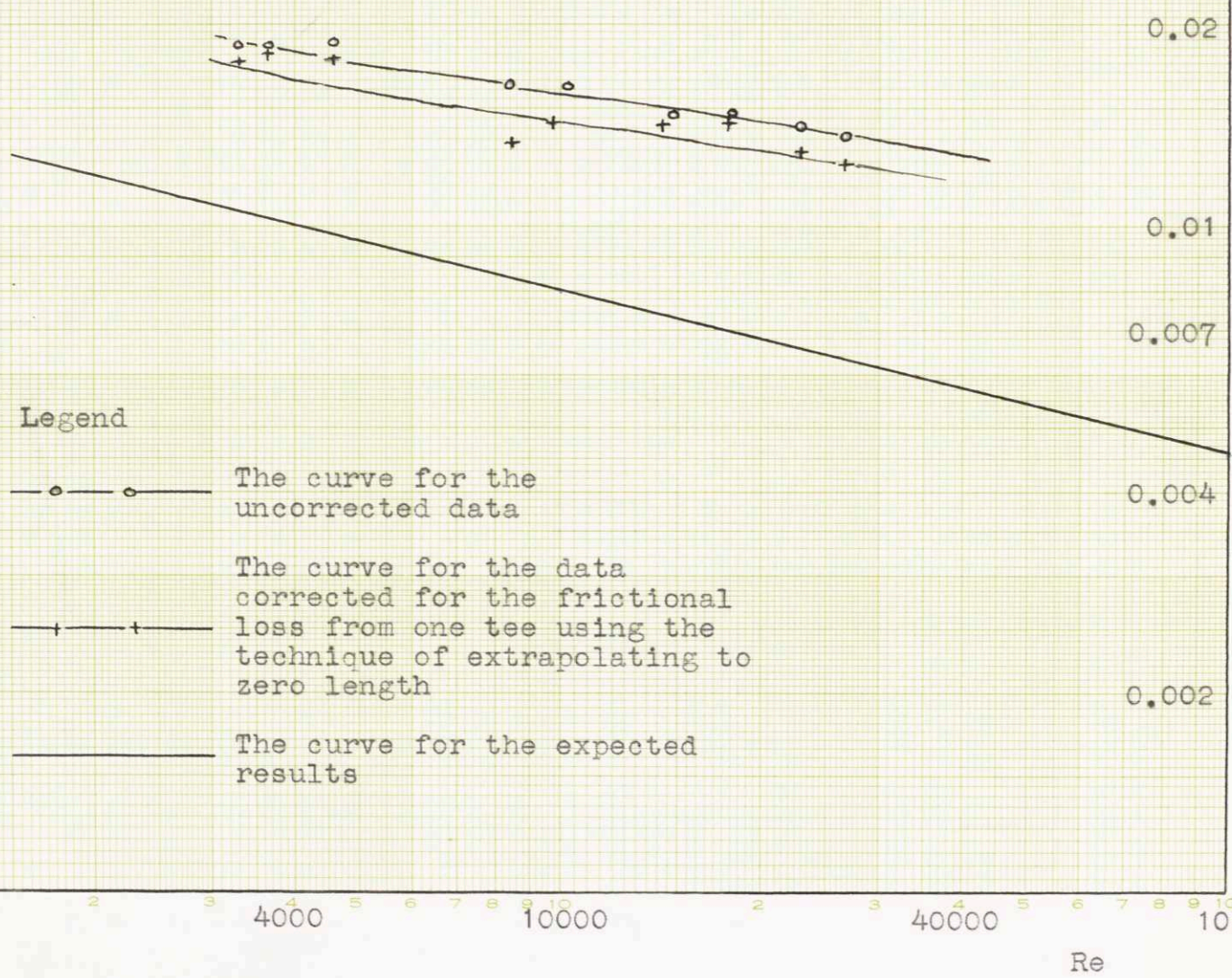
0.007

0.004

0.002

Figure 6

A plot of the Fanning friction factor vs the Reynolds Number for turbulent fluid flow in the straight test section using the method of extrapolating to zero length to determine the frictional loss from one tee



Legend

- The curve for the uncorrected data
- +—+— The curve for the data corrected for the frictional loss from one tee using the technique of extrapolating to zero length
- The curve for the expected results

V. DISCUSSION OF RESULTS

Referring to Figure 3, the plot of the friction factor, Reynolds Number relationship for the streamline flow through the test section, one notes that the curve of the uncorrected data approaches an asymptote at low Reynolds Numbers; the asymptotic line is the line expected for streamline flow in circular pipes. However, the inconsistency of the experimental plot with the one expected for the higher Reynolds Numbers had prompted the author to consider the source of any turbulence promotion in the flow - since there was no leaks at the manometer joints nor air bubbles in the leads, the manometer was not at fault.

The author had considered the frictional loss from the tees to be negligible when he designed the system. But they appeared to be the only source of turbulence promotion in the test section. The technique of extrapolating the pressure loss to zero tees to estimate the loss from the one tee influencing the flow in the test section was then applied - the expansion effect of the upstream tee and the contraction loss of the downstream tee was considered the loss from one tee. However, the corrected results did not correspond with the ones anticipated.

Nevertheless, data for a helix was obtained with the intention of applying another method to estimate the frictional loss of the tees in the straight test section and

which would then provide a correlation with the data for the coil - the approximate f_s , Re relationship for the smooth rubber tubing was known.

However, from reference to Figures 4 and 5, the pressure losses in the coil are about the same or less than those in the straight section for the same Reynolds Numbers. From the Cremer and Davies correlation, the author expected a loss 35% greater for the coil than the straight section. Hence, no conclusions about any correlations could be derived from the data.

The attempt to estimate the frictional loss of the tees by an extrapolation of the pressure losses to zero length did not improve the data very much (see Figure 6).

The only conclusion which can be drawn from these results is that an unaccountable frictional loss was being induced by the tees - the only source of turbulence in the system. The tees were concealing any increase in the pressure loss when the hose was wrapped into a helix. For future research into verifying and generalizing the correlations established for fluid flow in pipe coils, the author recommends the use of another method for measuring the pressure in the test section.

The inserting of a thin hypodermic needle through the wall of the hose would avoid placing any obstructions in the path of the fluid. The manometer fluid would, with this device, take longer to reach equilibrium.

VI. CONCLUSIONS AND RECOMMENDATIONS

The author had proposed and used a particular method for measuring the pressure loss in the test section. But subsequent work indicated that this technique was unsuitable.

To avoid introducing unaccountable losses in the test section by the use of glass tees, the author suggests that a small hole be punctured in the wall of the hose and a thin hypodermic needle be inserted flush with the inside wall of the tubing. The manometer fluid, however, would take longer to attain equilibrium before the pressure difference could be measured.

The author concludes from the literature survey that the correlations relating the Fanning friction factor of curved pipe to the friction factor of a straight length of pipe applies to only helices of closely wound turns. Further investigation ought to note the effect on the established correlations by the changing of the pitch in the turns of the helix.

Work should be performed to determine the range of Reynolds Numbers encompassed by the correlation for turbulent fluid flow in pipe coils proposed by Cremer and Davies.

The author suggests that when high static pressures exist in the test section, a hose stronger than tygon tubing

should be used for the manometer leads.

VII. APPENDIX

A. DESCRIPTION OF APPARATUS

High pressure rubber hose was used to facilitate the experimental work in this investigation. Tap water, the flow rate measured by a calibrated orifice, flowed through a three foot stilling length of the hose before entering the test section. Leads of tygon tubing extended from the 12 mm glass tees at the ends of the twenty and one-half foot test section of 0.497 inch id. hose to a mercury, a carbon tetrachloride, and an inverted toluene manometer permitting the measuring of the pressure loss in the section. A length of hose after the downstream tap avoided the introduction of turbulence into the flow in the test section because of downstream bends. The straight test section layed inside of an iron pipe; the helix was wound about a drum.

This investigator attributed the failure to obtain results to the method of measuring the pressure loss in the test section. The author suggests that a thin hypodermic needle be inserted through the wall of the tubing and flush with the inside wall of the hose. This technique avoids introducing the unaccountable turbulence which the glass tees introduced into the flow of the fluid through the test section. However, the system would take longer to reach equilibrium before the pressure difference could be measured.

This investigator noted that during high flow rates of water through his system, there was a high static pressure in the test section. This effect manifested itself in a swelling of the tygon tubing - the manometer leads. The author is not familiar with the strength of tygon tubing but he believes that for very high static pressure in the system, a stronger hose should be substituted for this tubing.

The hose in this project was pliable enough so that if it was wrapped into a helix of a moderate sized diameter, it would retain its circular cross-section. However, in a helix of a small diameter, the hydraulic radius must be applied in the correlations.

VII. APPENDIX (Cont.)

B. SUMMARY OF DATA AND CALCULATED VALUES

1. Streamline fluid flow in the straight test section using three tees at the downstream end to account for the loss due to the tees. The quantity H_0 is the pressure loss for the test section and no tees; H_1 is the loss for the test section and one tee; etc.: the superscript ('') indicates a value corrected for the loss due to the tees.

$$L = 20 \text{ ft } 7\frac{1}{2} \text{ ins}$$

$$\text{For runs 113 and 114: } T = 12.8 \text{ }^\circ\text{C}; \mu = 1.21 \text{ cp}$$

$$\text{For runs 115-118: } T = 10.5 \text{ }^\circ\text{C}; \mu = 1.28 \text{ cp}$$

Run	w	Re	H_3	H_2	H_1	H_0	f_s	f'_s
Inches of Toluene								
115	1.02	604	4.55	3.90	3.25	2.60	0.0286	0.0229
118	1.22	726	7.0	5.9	4.9	3.9	0.0299	0.0238
117	1.79	1070	12.0	10.2	8.3	6.5	0.0235	0.0184
113	1.77	1110	13.5	11.9	10.2	8.6	0.0295	0.0249
116	2.56	1520	20.0	17.0	14.0	11.1	0.0193	0.0153
Inches of CCl ₄								
114	3.21	2020	6.94	5.88	4.86	3.69	0.0194	0.0147

2. Turbulent fluid flow in the straight test section using three tees at the downstream end to account for the loss due to the tees.

$$L = 20 \text{ ft } 6\frac{1}{2} \text{ ins}$$

For runs 155-163: $T = 13.5$ °C; $\nu = 1.18$ cp

Run	w	Re	H ₃	H ₂	H ₁	H ₀	f _s	f' _s
Inches of CCl ₄								
155	5.11	3300	18.4	14.6	11.8	8.4	0.0187	0.0133
156	5.62	3630	23.5	18.7	14.6	10.1	0.0186	0.0129
157	7.0	4520	33.9	27.0	21.5	15.2	0.0188	0.0133
Inches of Mercury								
160	13.0	8400	5.25	4.21	3.15	2.10	0.0162	0.0108
161	15.8	10200	7.39	5.85	4.66	3.3	0.0162	0.0115
162	22.8	14700	14.7	11.5	8.8	5.8	0.0147	0.00968
163	28.8	18000	23.3	18.3	14.2	9.7	0.0148	0.0101
158	34.9	22600	32.7	25.6	19.9	13.5	0.0142	0.00961
159	41.0	26500		34.6	26.3	18.0	0.0136	0.00929

3. Turbulent fluid flow in a helical test section using three tees at the downstream end to account for the loss due to the tees. There were six turns in the helix with a half inch overlap into a seventh.

$$L = 20 \text{ ft } 6\frac{1}{2} \text{ ins} \quad D = 14.1 \text{ ins}$$

For runs 145-148: $T = 13.0$ °C; $\nu = 1.20$ cp

For runs 150-154: $T = 13.6$ °C; $\nu = 1.18$ cp

Run	w	Re	H ₃	H ₂	H ₁	H ₀	f _c	f' _c
Inches of CCl ₄								
150	5.11	3300	17.2	14.9	12.1	9.5	0.0191	0.0150
151	5.62	3630	22.6	18.8	14.7	10.6	0.0171	0.0123
152	7.0	4520	31.7	26.6	20.9	15.4	0.0178	0.0131

Run	w	Re	H ₃	H ₂	H ₁	H ₀	f _c	f' _c
Inches of Mercury								
145	13.2	8310	4.84	3.95	2.98	2.04	0.0148	0.0101
146	15.4	9700	6.76	5.25	3.94	2.52	0.0143	0.00915
147	22.2	14000	13.4	10.3	7.8	5.0	0.0136	0.00872
148	28.0	17600	20.7	16.1	12.2	8.0	0.0134	0.00880
153	34.9	22600	31.7	24.3	18.2	11.4	0.0130	0.00811
154	41.0	26500		32.1	24.2	16.3	0.0125	0.00840

4. The pressure loss for the turbulent fluid flow in a shorter straight test section was measured for the same Reynolds Numbers as the flow in the longer section. The values were plotted as a function of the length; the straight line connecting the points was extrapolated to zero length to determine the loss from the tees. This subsection presents the values of the friction factors calculated using this method.

The value of H₀ is the corrected manometer displacement for the longer test section. The shorter test section was 12 ft 1½ ins long.

For runs 155-160: T = 13.5 °C; μ = 1.18 cp

For runs 164-166: T = 13.8 °C; μ = 1.18 cp

For runs 167-175: T = 14.0 °C; μ = 1.17 cp

For L = 20 ft 6 $\frac{1}{2}$ insFor L = 12 ft 1 $\frac{1}{2}$ ins

Run	w	Re	H ₁	Run	w	Re	H ₁	H ₀	f' _s
			Ins of CCl ₄				Ins of CCl ₄	Ins of CCl ₄	
155	5.11	3300	11.7	167	5.06	3300	7.12	11.1	0.0176
156	5.62	3630	14.5	168	5.57	3630	8.65	14.2	0.0181
157	7.0	4520	21.4	169	6.94	4520	12.7	21.0	0.0177
			Ins of Hg				Ins of Hg	Ins of Hg	
160	13.0	8400	3.15	170	12.9	8400	2.10	2.58	0.0133
164	15.1	9750	4.25	171	15.0	9780	2.74	3.71	0.0141
165	21.8	14100	8.50	172	21.6	14100	5.3	7.77	0.0142
166	27.5	17800	12.8	173	27.3	17800	7.9	12.3	0.0141
158	34.9	22600	19.9	174	34.6	22600	12.5	18.1	0.0128
159	41.0	26500	26.3	175	40.6	26400	16.5	24.0	0.0124

VII. APPENDIX (Cont.)

C. SAMPLE CALCULATIONS AND ERROR ANALYSIS

Since no results were obtained, only a discussion of the sources of error will be presented.

1. Sample Calculations

a. Calculation of the Reynolds Number

$$Re = \frac{4w}{\pi d \mu}$$

For run 117

Diameter of tubing $d = 0.497$ ins

Temperature of fluid $T = 10.5$ °C

Mass flow rate $w = 1.79$ $\frac{\text{lbm}}{\text{min}}$

Viscosity of fluid $\mu = 1.28$ cp

$$Re = \frac{(4)(1.79)(12)}{(3.142)(0.497)(1.28)(0.000672)(60)}$$

$$Re = 1070$$

b. Calculation of the friction factor

$$\frac{1}{\rho} (\rho_{\text{manometer fluid}} - \rho) \frac{g}{g_c} H = \frac{32 f L w^2}{g_c \pi^2 d^5 \rho^2} \quad 7$$

For run 117

Diameter of tubing $d = 0.497$ ins

Acceleration of gravity $g = 32.2$ $\frac{\text{ft}}{\text{sec}^2}$

Conversion constant $g_c = 32.2$ $\frac{\text{ft}}{\text{sec}^2} \frac{\text{lbm}}{\text{lbf}}$

Manometer displacement $H = 6.5$ ins of toluene

Length of hose $L = 20.6$ ft

Mass flow rate $w = 1.79 \frac{\text{lbm}}{\text{min}}$

Density of water $\rho = 62.4 \frac{\text{lbm}}{\text{ft}^3}$

Density of manometer fluid $\rho_{\text{tol}} = 54.0 \frac{\text{lbm}}{\text{ft}^3}$

$$f = \frac{(62.4 - 54.0)(32.2)(6.5)(3.142)^2(0.497)^5(62.4)(3600)}{(12)(32)(20.6)(12)^5(1.79)^2}$$

$$f = 0.0184$$

2. Sources of error

The key source of error in the project arose from the use of tees to measure the pressure loss in the test section. An unaccountable frictional loss was induced into the flow of the fluid through the section.

Error was introduced into the measurements because of the necessity for this investigator to average the fluctuations in the flow of the water through the system.

The hose was slightly oval in cross-section. - the value of 0.497 ins for the diameter is the average diameter of the tubing. The hose in the project was pliable enough to retain the same cross-sectional area when wrapped into a helix of a moderate sized diameter. But if the tubing had been wrapped about a smaller drum, the hydraulic radius would have had to been accounted for.

VII. APPENDIX (Cont.)

D. NOMENCLATURE

C_s	The ratio of $\frac{f_c}{f_s}$ for streamline fluid flow	Dimensionless
C_t	The ratio of $\frac{f_c}{f_s}$ for turbulent fluid flow	Dimensionless
d	Inner diameter of the tubing	ins
D	Mean diameter of the helix	ins
f	Fanning friction factor - f_c is the friction factor of curved pipe and f_s , the friction factor of straight pipe; the superscript ('') denotes a value corrected for the loss due to the tees	Dimensionless
F_R	Fluid friction loss	$\frac{\text{ft lbf}}{\text{lbm}}$
g	Acceleration of gravity	$32.2 \frac{\text{ft}}{\text{sec}^2}$
g_c	Conversion constant	$32.2 \frac{\text{lbm}}{\text{lbf}} \frac{\text{ft}}{\text{sec}^2}$
H	Manometer fluid displacement - the subscripts 0, 1, etc. indicate the manometer displacement for no tees, one tee, etc.	ins
L	Length of tubing	ft
dP	Pressure drop in test section	$\frac{\text{lbf}}{\text{in}^2}$
Re	Reynolds Number	Dimensionless
U	Velocity of fluid	$\frac{\text{ft}}{\text{min}}$
v	Specific volume of fluid	$\frac{\text{ft}^3}{\text{lbm}}$
w	Mass flow rate	$\frac{\text{lbm}}{\text{min}}$
X	Vertical displacement of the fluid	ft
ν	Viscosity of fluid	centipoises
ρ	Density of fluid	$\frac{\text{lbm}}{\text{ft}^3}$

VII. APPENDIX (Cont.)

E. LITERATURE CITATIONS

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