

MIT Open Access Articles

Study of SRCs in neutron-rich nuclei with inverse kinematics measurements

The MIT Faculty has made this article openly available. *Please share* how this access benefits you. Your story matters.

Citation: The European Physical Journal A. 2023 Jun 12;59(6):128

As Published: https://doi.org/10.1140/epja/s10050-023-01028-1

Publisher: Springer Berlin Heidelberg

Persistent URL: https://hdl.handle.net/1721.1/150922

Version: Final published version: final published article, as it appeared in a journal, conference

proceedings, or other formally published context

Terms of use: Creative Commons Attribution





Review

Study of SRCs in neutron-rich nuclei with inverse kinematics measurements

Julian Kahlbow^{1,2,a}

¹ Massachusetts Institute of Technology, Cambridge, MA 02139, USA

² School of Physics and Astronomy, Tel Aviv University, Tel Aviv 6997801, Israel

Received: 14 February 2023 / Accepted: 4 May 2023 © The Author(s) 2023 Communicated by Patrizia Rossi

Abstract Experiments in inverse kinematics are a novel approach to study short-range correlations in atomic nuclei. Such fully exclusive experiments offer unique advantages over normal kinematics as they allow to study the structure of short-lived radioactive-nuclei using quasi-free scattering reactions and measure the complete final state directly. In pilot experiments at JINR using a ¹²C beam at 4 GeV/c/nucleon on a proton target, we show that challenges posed by final-state interactions can be overcome taking advantage of post-selection. This opens a new research path to study the dynamics of high-momentum nucleons in extremely asymmetric nuclei.

1 Introduction

Short-range correlations (SRCs) have been measured and studied extensively over the last two decades using primarily electron scattering [1], as is being discussed in this collection as well. In these experiments, the SRC pair is probed in quasi-elastic (QE) scattering conditions induced by large momentum transfer. Following the break up of the pair, the scattered electron is detected possibly in coincidence with the nucleon(s) of the pair in (e, e'pN) reaction channels. Specific kinematic selections allow to suppress the otherwise large final-state interactions (FSI) and identify SRCs unambiguously in the high-momentum tail of the nucleon momentum distribution [2–5]. To disentangle the reaction mechanism from the nuclear ground-state distributions, the experimental results are often compared to theory predictions that account for, in a factorized approach, the pair physics and reaction theory [4]. Calculations in the framework of the Generalized Contact Formalism reproduce experimental results well [6,7].

Published online: 12 June 2023

Results using different experimental probes must result from the same ground-state distributions independently of the reaction mechanism in terms of universality and probe independence. One of the first experiments that observed correlated nucleons at high momentum was performed at the EVA setup at Brookhaven National Laboratory using high-energy proton beams and measuring the knock-out protons and recoil neutrons [8,9]. The results also lead to the indication of proton-neutron pair dominance which has later been found in electron scattering to be a SRC characteristic over a large mass range [5,10].

This article will discuss a new generation of experiments in which nuclear beams scatter off proton targets. This kind of inverse kinematics reactions offer unique advantages to measure all the reaction particles in coincidence and extend experiments to most neutron-rich short-lived nuclei.

2 SRC in asymmetric nuclei

Knowing that neutron-proton pairs (spin S=1) dominate over the isospin-like pairs in light to heavy nuclei (12 C to 208 Pb) raises the question about the nucleon dynamics in increasingly neutron-rich nuclear environments. Electron-scattering experiments measured quasi-elastic proton- and neutron-knockout reactions (e, e'N) with the CLAS detector in Hall B at Jefferson Lab on various stable nuclear targets (C, Al, Fe, Pb) to quantify the relative fractions of high-momentum nucleons covering neutron excess up to $N/Z \lesssim 1.5$. results by Duer et al. from Ref. [11] are discussed in the following.

Firstly, the results confirm that the neutron/proton abundance increases essentially linearly with neutron excess N/Z in the low-momentum regime $(p < k_F)$ as expected for fairly mean-field nucleons. In contrast, the neutron/proton abundance of high-momentum $(p > k_F)$ SRC nucleons



^a e-mail: jkahlbow@mit.edu (corresponding author)

128 Page 2 of 6 Eur. Phys. J. A (2023) 59:128

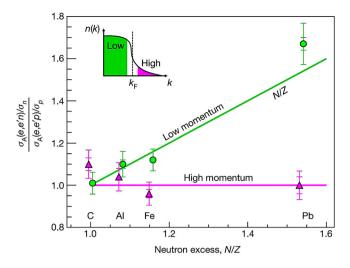
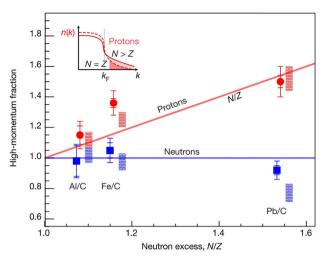


Fig. 1 (left) Reduced cross-section ratio for different target nuclei, $[\sigma_A(e,e'n)/\sigma_n]/[\sigma_A(e,e'p)/\sigma_p]$, for low-momentum (green circles) and high-momentum (purple triangles) events. (right) Red circles denote the double ratio of the number of (e,e'p) high-momentum proton events to low-momentum proton events for nucleus A relative to carbon. Blue

stays constant indicating that pair correlations prevail, see Fig. 1 (left). Looking separately at the high-momentum fraction for protons and neutrons for the heavy nuclei relative to ¹²C shows that the fraction of the minority species, the protons, to increase which can be described by a linear trend with N/Z, while the neutron fraction saturates, Fig. 1 (right). To our current understanding, this observation indicates that high-momentum protons become more correlated as the SRC neutron-proton pair fraction remains essentially constant even in neutron-rich nuclei, as is being predicted in phenomenological and ab-initio calculations as well [12,13]. In other words, the more neutron-rich a nucleus is, the larger is the average kinetic energy of the protons and momentum-sharing inversion seems to appear in heavier nuclei, which possibly has consequences on the properties of neutron stars [10, 14].

This study was performed with a limited amount of stable targets for which an increase in neutron number goes along with an increase in isotope number and large nuclear mass difference. Additional experiments that are sensitive to nuclear structure details and shells are being performed adding nuclei with N/Z between 1.2 and 1.4, for example on B and Be isotopes as well as Ca. In order to disentangle the impact of increasing neutron number from proton number, semi-exclusive experiments are studying 40 Ca and 48 Ca with Z=20, and 20 and 28 neutrons, respectively. Results from inclusive measurements on Ca showed evidence of np dominance by making use of the isospin structure of the target [15]. Systematic studies along isotopic chains towards very neutron-rich short-lived nuclei cannot be performed in conventional electron scattering. New experiments that make



squares with error bars show the same for neutron events. Red and blue rectangles show the range of predictions of a phenomenological *np*-dominance model for proton and neutron ratios, respectively. Figures taken from [11]

the first step in this direction are discussed in the following section.

3 Nuclear structure studies in inverse kinematics

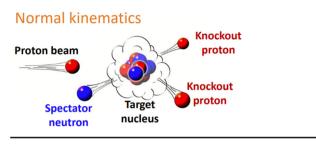
All scattering experiments with electron probes and in normal kinematics are limited to stable targets which eventually limits the reachable isospin asymmetry (N/Z). To overcome this obstacle, three changes must be made: (i) experiments are performed in inverse kinematics in which the nuclei of interest form the particle beam, (ii) the target must be stable, suitable are liquid-hydrogen (LH₂) targets for proton-induced reactions, (iii) the (high-energy) radioactive-ion beam must be available at sufficient intensity.

Scattering experiments in inverse kinematics have become a standard tool in low-energy nuclear-structure physics [16]. The sketch in Fig. 2 depicts the change in kinematics (in the SRC case). Facilities like GSI-FAIR (Germany), RIBF-RIKEN (Japan), and FRIB (US) use fast radioactive-ion (RI) beams to for example study single-particle nuclear structure in proton-induced nucleon knockout reactions of extremely neutron-rich nuclei at and around the neutron dripline [17–19].

The use of proton probes allows for a quasi-free reaction in which the incident proton scatters with the struck nucleon in a single interaction (impulse approximation) under large energy and momentum transfer, while the residual nucleus acts as spectator, and both nucleons leave the nuclear environment without re-interaction [20–22]. Multi-step FSI cause attenuation and lead to reduced transparency and distortion



Eur. Phys. J. A (2023) 59:128 Page 3 of 6 128



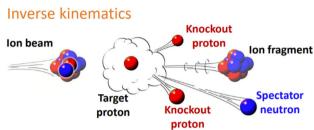


Fig. 2 Schematic of an SRC-pair break-up reaction in normal kinematics (top) and inverse kinematics (bottom) using proton probe

of the reaction channel of interest, but can experimentally be rejected [23,24]. Theoretically the (coherent) multiple scattering can be described in terms of the distorted wave impulse approximation (DWIA), medium modifications of the nucleon-nucleon interaction have to be taken into account as well [20]. Experimentally, the characteristic kinematical correlations between the protons and fragment can be observed to help identify the reaction [17]. This mechanism has been introduced theoretically to SRC studies using a factorized approach in the quasi-free reaction assumption, relating the close-proximity pair to two-nucleon knockout cross sections [4,25].

Another unique advantage of inverse kinematics in contrast to normal kinematics is the coincident measurement of all reaction particles, including the heavy fragment after the reaction to determine unambiguously the reaction final state. Due to the Lorentz boost beam-like particles will be focused in forward direction, a large phase space is mapped into a small acceptance in beam direction to be covered by detector systems.

To probe SRCs, separate the reaction scale, and provide quasi-free scattering conditions, experiments require large momentum transfer that cannot be reached with beam energies smaller than a few-hundred MeV/nucleon as usually used in the mentioned nuclear structure studies. A pilot experiment in inverse kinematics with (stable) ¹²C beam at 4 GeV/c/nucleon was performed at the JINR (Russia) to prove the reaction mechanism in QE proton knockout reaction. In the following, details of Ref. [26] are being discussed, starting with investigations of the kinematics in the single-nucleon removal reactions.

In the experiment at the BM@N beamline, the beam that was provided by the Nuclotron accelerator impinges on a

 $30\,\mathrm{cm}$ LH₂ target, the scattered protons from the relevant (p,2p) reactions are measured with a two-arm time-of-flight spectrometer. The BM@N setup had been further upgraded with multiple energy-loss and position sensitive detectors to accommodate for the identification and momentum determination of the heavy fragment after the reaction based on its bending in a large open dipole magnet. A sketch of the setup is shown in Fig. 3.

The QE nucleon knockout reaction contributes only a small fraction to the interaction cross section, especially those reactions at largest momentum transfer at 90° centerof-mass scattering. In addition, initial- and final-state interactions lead to multi-step processes that – depending on their kinematics - lead to a reduction in the OE cross-section (attenuation) or kinematical distortion of the reconstructed single-nucleon ground-state properties. Taking advantage of post-selection by tagging the coincident reaction fragment and single proton removal in the reference reaction 12 C $(p, 2p)^{11}$ B, we show that multi-step processes leading to distortions and fragment break-up are suppressed allowing for the identification of unperturbed single-step QE nucleon knockout reactions. Further confirmation comes from the fact that the A-1 fragment and reconstructed knockout proton share an opening angle of close to 180°, indicating that the fragment carries the recoil momentum of the knocked-out proton. The undisturbed measurement allows here to extract the ground-state missing-momentum distribution of p-shell nucleons in ¹²C, see Fig. 4. Comparisons to Glauber calculations, explicitly not including distortion FSI effects, show remarkable agreement with data. In contrast, the inclusive data from all ${}^{12}C(p, 2p)$ reactions show an extended tail caused by FSI. It should be emphasized that the ¹¹B fragment momentum distribution shows the same shape independent of quasi-elastic or inelastic event selection, meaning that the fragment kinematics is not affected by proton FSI. This is the confirmation, at high energies, that measuring the fragment momentum is equivalent to the measurement of the knockout-nucleon momentum distribution.

Studying nucleon knockout reactions in inverse kinematics at high energies with proton probes establishes a clean reaction mechanism in which FSI can be suppressed and ground-state momentum distributions be extracted. This result paves the way for SRC studies in inverse kinematics as is being discussed in the following section.

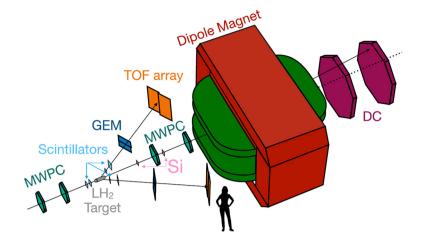
4 SRC pilot experiments at JINR and GSI-FAIR

On the way to studies of SRCs in short-lived nuclei, we performed the pilot experiment at JINR using a ¹²C beam as described in the previous section [26]. Under the same conditions we searched for signatures of SRC protons in coin-



128 Page 4 of 6 Eur. Phys. J. A (2023) 59:128

Fig. 3 Illustration of the SRC experimental setup at the BM@N beamline at JINR. A high-energy ¹²C beam is incident from the left on a liquid hydrogen target. The (p, 2p) reaction is measured under large opening angle using a two-arms time-of-flight spectrometer. The residual nuclear fragment is measured using a magnetic spectrometer downstream the target



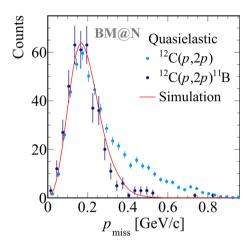


Fig. 4 Measured missing-momentum distribution in the 12 C restframe for the quasi-elastic inclusive 12 C(p, 2p) and exclusive 12 C(p, 2p) 11 B reactions. The data are compared with a PWIA-based simulation of proton knockout from the 12 C p-shell. The 11 B tagging clearly suppresses ISI/FSI distortions at high momenta. Figure adapted from [26]

cidence with $^{10}\mathrm{B}$ and $^{10}\mathrm{Be}$, the A-2 residuals following pn and pp pair break-up, respectively, in $^{12}\mathrm{C}(p,2p)A-2$ reactions at momentum transfers larger $\sim 1.5\,\mathrm{GeV^2}$ [26]. The events are selected based on the (large) opening angle between the knocked-out and scattered protons, small missing energy, proper missing mass, together with large missing momentum $p_{\mathrm{miss}} > 350\,\mathrm{MeV/c}$. The exact selection was guided by simulations using the SRC-GCF model and resulted in the identification of 23 pn-SRC and two pp-SRC pair break-ups. Despite the limited statistics, the pair ratio reflects the np-pair pre-dominance and is in full agreement with predictions based on ab-initio many-body calculations. Contributions from inelastic reactions and from reactions due to mean-field QE scattering followed by FSI are found to be negligible.

In case of a pair breakup in quasi-free scattering, the fragment carries the recoil momentum of the pair, which

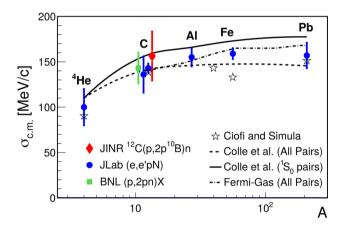


Fig. 5 The width of the SRC pair c.m. momentum distributions extracted from the direct fragment detection in inverse kinematics (red) and from normal-kinematics electron and proton scattering measurements for ¹²C and other nuclei. Figure adapted from [27]

allows to infer the pair center-of-mass momentum directly from the momentum measurement of the A-2 system in inverse kinematics. Done for the first time in this experiment, the obtained Gaussian momentum width (sigma) of $156 \pm 27 \, \text{MeV/c}$ agrees well with previous, but indirect extractions from electron scattering [27], see Fig. 5.

As opposed to meanfield nucleon knockout, where the A-1 system carries the recoil momentum, here the pair nucleon momenta balance each other. This is reflected in the opening angle between the missing momentum and reconstructed nucleon recoil-momentum which peaks towards 180° reflecting a back-to-back emission and confirming a strong correlation of the pair nucleons, see Fig. 6 (left). In contrast, we find the first direct experimental evidence that the pair is scale separated from the rest of the nucleus by the opening angle between the A-2 momentum and the pair relative momentum, Fig. 6 (right). This angular distribution is nearly flat and uncorrelated unlike the previously mentioned nucleon-nucleon angle distribution. The results show a strongly correlated pair while it is only weakly correlated



Eur. Phys. J. A (2023) 59:128 Page 5 of 6 128

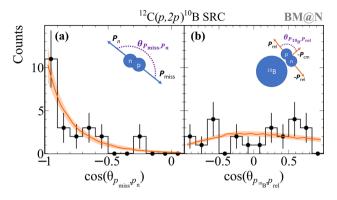


Fig. 6 Angular correlation between the two nucleons in the SRC pair (a) and between the SRC pair c.m. and relative momenta (b). Data are compared with theoretical calculations using the nuclear Generalized Contact Formalism (GCF, orange band) in the 12 C(p, 2p) 10 B reaction, assuming full factorization of SRC wave function from the residual nuclear system. Figure adapted from [26]

with the spectator nucleus. This supports one of the main assumption in our understanding of SRCs and in theories like the GCF, namely a universal scale separation between the pair and the rest of the nucleus.

The results of this pilot experiment pave the way for future quantitative SRC studies in inverse kinematics using proton targets. A follow-up experiment with an upgraded setup has been performed at JINR. The first ever experiment studying SRCs in a short-lived neutron-rich nucleus, namely ¹⁶C, has been performed at the R³B setup at GSI-FAIR in May 2022 [28]. The R³B experimental setup is a next-generation and dedicated setup for nuclear structure studies of rareisotope beams in inverse kinematics. The used experimental configuration was similar to that one used at JINR while the scattered protons have been measured in a calorimeter detector surrounding the 5 cm LH₂ target and a silicon-strip detector tracking system. We also employ a large area neutron detector to measure recoil neutrons from SRC pairs as well as decay neutrons that potentially originate from weakly bound residual nuclei as the neutron separation threshold is typically rather small for neutron-rich nuclei. In addition, the granular CsI calorimeter detects prompt gamma-rays following de-excitation of bound-excited states. Altogether, the information from all the detectors in a fully exclusive measurement allow to determine the exact final state after the reaction.

The pilot experiment at GSI-FAIR also sets to determine the kinematical regime in which SRCs can be probed in break-up reactions. As learned from electron scattering, we require reactions with large momentum transfer to probe the pair nucleon in a scale separated regime which translates into beam energies of close to $1\,\text{GeV/nucleon}$ and larger. The experiment at JINR operated well above at $\sim 3.2\,\text{GeV/nucleon}$, but RI beams are usually not produced at such high energies. GSI-FAIR is the unique facil-

ity to provide fast RI beams at similar level, currently up to 1.5 GeV/nucleon (depending on the mass-over-charge ratio). The pilot experiment ran at 1.25 GeV/nucleon and will help determine the kinematical minimum at which SRCs can be probed cleanly.

Experiments in inverse kinematics do not only open the possibility to study short-lived radioactive nuclei but uniquely give access to fragment measurements and the identification of the final state by measuring momenta of all reaction particles. The price to be paid comes with the use of proton probes. Quasi-free scattering has become a valuable tool to study nuclear structure, and by experimentally proving that FSI can be removed by post-selection, it is a tool to study SRCs in break-up reactions. The presented results from the pilot experiment demonstrate the advantages of the reaction kinematics and are a piece towards proving the measurement of SRC ground-state properties independent of the reaction probe.

5 Future research directions

The pilot experiments pave the way for a new branch in SRC studies with new experiments using RI beams and proton targets. The results coming from the first SRC experiments will help to prove probe-independence and universality of SRC ground-state properties. All the results that have been extracted so far agree with results from electron scattering and with predictions by the GCF model. Using proton probes will further help disentangle contributions from structure and reaction physics as for example quasi-free proton scattering probes a different part of the wave function than electron scattering, or neutron-rich nuclei tend to develop neutron skins [20,24]. This might also relate to the probable location of SRCs inside nuclei [13,29].

Taking advantage of the measurement of the final state promises additional research directions. For example, disentangling the spin channels for the same isospin channel will result in different final states and thus allow to determine the spin-structure of the pair by providing detailed structure information [30].

The experiment at GSI-FAIR marks the start of an experimental program to systematically study the dynamics and interactions of SRCs in the nuclear many-body system along isotopic chains. For neutron-rich nuclei, the pair ratio is not expected to globally change significantly while the trend is expected to reverse for neutron-deficient nuclei [25]. However, systematic studies together with final-state tagging will detail that. Even though the neutron-proton asymmetry is even for the most neutron-rich nuclei produced in the lab $(N/Z \leq 3)$ far away from conditions as found in neutron stars, in combination with other inputs and constraints we hope such SRCs studies will contribute to our under-



128 Page 6 of 6 Eur. Phys. J. A (2023) 59:128

standing of cold-dense nuclear matter and nuclear interaction [14,31]. As we are trying to detail the reaction kinematics, new interpretations of SRCs and reactions are discussed. A novel approach fro example embeds SRCs within the energy-density functionals (GRDF) in which np pairs are treated as quasi-deuteron clusters [32]. This suggests to study in a consistent way the equation-of-state and the impact of SRCs at supra-saturation densities as may be relevant for our understanding of neutron-star matter.

Funding Information Open Access funding provided by the MIT Libraries.

Data availability statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: Data shown in this review article can be retrieved upon request from the original publication if available.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

- O. Hen et al., Rev. Mod. Phys. 89, 045002 (2017). https://doi.org/ 10.1103/RevModPhys.89.045002
- C. Ciofi degli Atti, L.P. Kaptari, Phys. Rev. Lett. 95, 052502 (2005). https://doi.org/10.1103/PhysRevLett.95.052502
- R. Cruz-Torres, D. Nguyen, Phys. Rev. Lett. 124, 212501 (2020). https://doi.org/10.1103/PhysRevLett.124.212501
- J.R. Pybus et al., Phys. Lett. B 805, 135429 (2020). https://doi.org/ 10.1016/j.physletb.2020.135429
- M. Duer et al., Phys. Rev. Lett. 122, 172502 (2019). https://doi. org/10.1103/PhysRevLett.122.172502
- R. Weiss et al., Phys. Rev. C 92, 054311 (2015). https://doi.org/10. 1103/PhysRevC.92.054311
- I. Korover et al., Phys. Lett. B 820, 136523 (2021). https://doi.org/ 10.1016/j.physletb.2021.136523

- A. Tang et al., Phys. Rev. Lett. 90, 042301 (2003). https://doi.org/ 10.1103/PhysRevLett.90.042301
- E. Piasetzky et al., Phys. Rev. Lett. 97, 162504 (2006). https://doi. org/10.1103/PhysRevLett.97.162504
- O. Hen et al., Science 346, 614–617 (2014). https://doi.org/10. 1126/science.1256785
- 11. M. Duer et al., Nature **560**, 617–621 (2018). https://doi.org/10. 1038/s41586-018-0400-z
- J. Ryckebusch, W. Cosyn, S. Stevens, C. Casert, J. Nys, Phys. Lett. B 792, 21–28 (2019). https://doi.org/10.1016/j.physletb.2019.03.
- R. Cruz-Torres et al., Nat. Phys. 17, 306–310 (2021). https://doi. org/10.1038/s41567-020-01053-7
- B.-A. Li, B.-J. Cai, L.-W. Chen, J. Xu, Prog. Part. Nucl. Phys. 99, 29–119 (2018). https://doi.org/10.1016/j.ppnp.2018.01.001
- D. Nguyen et al., Phys. Rev. C 102, 064004 (2020). https://doi.org/ 10.1103/PhysRevC.102.064004
- V. Panin, T. Aumann, C.A. Bertulani, Eur. Phys. J. A 57, 103 (2021). https://doi.org/10.1140/epja/s10050-021-00416-9
- V. Panin et al., Phys. Lett. B 753, 204–210 (2016). https://doi.org/ 10.1016/j.physletb.2015.11.082
- L. Atar et al., Phys. Rev. Lett. 120, 052501 (2018). https://doi.org/ 10.1103/PhysRevLett.120.052501
- A. Revel et al., Phys. Rev. Lett. 124, 152502 (2020). https://doi. org/10.1103/PhysRevLett.124.152502
- T. Aumann, C.A. Bertulani, J. Ryckebusch, Phys. Rev. C 88(6), 064610 (2013). https://doi.org/10.1103/PhysRevC.88.064610
- T. Wakasa, K. Ogata, T. Noro, Prog. Part. Nucl. Phys. 96, 32–87 (2017). https://doi.org/10.1016/j.ppnp.2017.06.002
- K. Yoshida, M.C. Atkinson, K. Ogata, W.H. Dickhoff, Phys. Rev. C 105, 014622 (2022). https://doi.org/10.1103/PhysRevC. 105.014622
- M.S. Hussein, R.A. Rego, C.A. Bertulani, Phys. Rep. 201(5), 279– 334 (1991). https://doi.org/10.1016/0370-1573(91)90037-M
- M. Duer et al., Phys. Lett. B 797, 134792 (2019). https://doi.org/ 10.1016/j.physletb.2019.07.039
- S. Stevens, J. Ryckebusch, W. Cosyn, A. Waets, Phys. Lett. B 777, 374–380 (2018). https://doi.org/10.1016/j.physletb.2017.12.045
- M. Patsyuk et al., Nat. Phys. 17, 693 (2021). https://doi.org/10. 1038/s41567-021-01193-4
- E.O. Cohen et al., Phys. Rev. Lett. 121, 092501 (2018). https://doi. org/10.1103/PhysRevLett.121.092501
- 28. S522 experiment proposal. unpublished (2020)
- W. Cosyn, J. Ryckebusch, Phys. Lett. B 820, 136526 (2021). https://doi.org/10.1016/j.physletb.2021.136526
- R. Weiss, A. Lovato, R.B. Wiringa, Phys. Rev. C 106, 054319 (2022). https://doi.org/10.1103/PhysRevC.106.054319
- L. Frankfurt, M. Sargsian, M. Strikman, Int. J. Mod. Phys. A 23(20), 2991–3055 (2008). https://doi.org/10.1142/S0217751X08041207
- 32. S. Burrello, S. Typel, Eur. Phys. J. A **58**, 120 (2022). https://doi. org/10.1140/epja/s10050-022-00765-z

