FORCE FITS.

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OBJECT AND NECESSITY.

The nature of machines and built up objects is such as to call for the rigid frame for certain parts and moving ones for others; but owing to the complicated forms of such machines, it is not always possible to have in a single piece all the parts which are meant to be rigid ones. Some parts demand far greater strength while in other parts comparatively less strength will suffice. Besides, according to the nature of the work, the machinery is intended to perform, some portions ought to be heavier than others and not requiring so much strength. As the cost of the machine depends on the strength of the pieces employed to build it as well as on its weight, it becomes necessary to build up the machine parts in such a way as to be very economical, while at the same time the machine must be perfectly fit to perform the duty for which it is intended. Thus the building up of different parts into a rigid whole is not only a matter of economy but of necessity as well.

This object is secured in various ways. There are key-way and shaft coupling, the screw and bolt connection, shrinkage and FORCE FITS, and so on. Each has its own filled up utility and does the work best with economy when judiciously selected.

FORCE FITS are generally employed where power is to be transmitted, e.g. crank-pins, cranks, wheels and axles of engines and cars. They practically replace the duty of key-way connection in shafts and have the advantage over key-way connection in that they do not lower the strength of the piece in having a portion cut off. In key-way connection the entire power to be transmitted is practically borne by the key only. Sometimes combined force and key-way fitting is secured by giving a small allowance to the shaft while the key is inserted in the slot cut in the hub and shaft. Besides in key-way fitting has the great disadvantage that in case of loose fitting the connective piece begins to wobble. This wobbling develops great centrifugal force in heavier ~~pulling~~ pulley.

Shrinkage fit is a kind of force fit, where the necessary amount of force to bring about the union of the two pieces is obtained by the ~~pulling~~ cooling of the piece to the air temperature. Two pieces are finished with ~~equal~~ required diameters with the given allowance for the inner one, while the outer one is heated, - thereby it expands, and then it shrinks into the position over the inner one. Now the cooling contracts it and secures the necessary tightening effect.

No doubt, Force Fits develop at the surfaces of contact and internal stress in the metal, but if this stress be kept within elastic limits it will not deteriorate the strength of the metals while they will perform all the duties required of them.

In Force Fits the diameter of a piece to be forced in a bore or whole is generally kept a little larger than that of the bore. The excess of the former over the latter is called allowance,- generally 1/1000 of an inch per inch of diameter. After the piece is forced in there is a mutual distortion of the metals equal to this allowance - strain. This strain develops a stress in the metal in contact. The amount of the stress that is produced is the object of the study in this thesis, so as to be able to regulate the strain in such a manner as never to go beyond the elastic limit.

(II) Basis on Which Such A Stress and Strain can be Realized Is By Mathematical Calculation.

These calculations are based Hook's Laws, i.e., ~~a~~ the stress is proportional to ~~a~~ the strain. It also assumes that the effect of temperature is negligible, and that the initial state of no strain differs so little from final strain that ~~a~~ square and cube of this strain can be neglected; and lastly, there is no permament set in the metal.

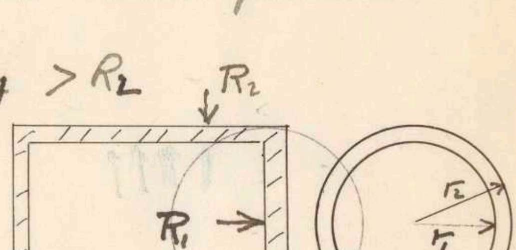
Major Birnie, following the method of Lamé, had developed a theory of stress which with radial strain in case of shrunk fit in the gun construction. He assumed that there is no longitudinal stress along the axis of the bore, and found that the greatest stress is the Hoop tension, or circumferential tension at the inner surface of the outer ring. This shrunk fit decrease the external diameter of the inner ring and increases the internal diameter of the outer ring. In Force Fits the same thing happens, so his formula for Hoop tension is applicable to Force Fits provided we take into consideration the different metals we are dealing with. I followed his method of calculating ~~a~~ the stress. From that stress calculation got this formula of Force Fits as was given by Professors Haven and Swett. The calculations are as follows:

Deduction of Birnie's Formula.

Following the train of reasoning of Lamé as is given by Merriman in his book on Mechanics of Materials. the equation of Equilibrium of a small part of the body in the body of the tube is derived as follows:

The tube is supposed to be closed at both ends + subjected to an internal pressure R_1 + an external pressure R_2. + the point in question is supposed to be so far away from the ends of the tubes as to be unaffected by their influences.

Let s_0 be the longitudinal stress produced due to the pressures R_1 + R_2, $R_1 > R_2$



Then for Equilibrium.

$$s_0 \pi (r_2^2 - r_1^2) = R_2 \pi r_2^2 - R_1 \pi r_1^2$$

$$\therefore s_0 = \frac{R_2 r_2^2 - R_1 r_1^2}{r_2^2 - r_1^2} = \frac{R_1 r_1^2 - R_2 r_2^2}{r_2^2 - r_1^2} \quad \text{when } R_1 > R_2$$

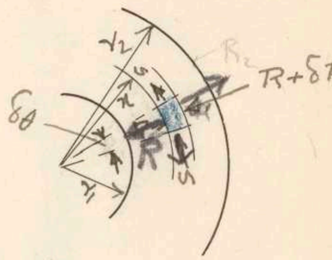
T = Long. Stress,

E =

Let e_0 = longitudinal elongation

Then $e_0 = \frac{T}{E} = (S_0 - \lambda S - \lambda R)/E$

where S + R are hoop tension + radial tension at the point in question.



But e_0 + S_0 are constants (assumption)

$$\therefore \quad R + S = \text{const} = 2C_1 \quad \ldots \quad (1)$$

Take the equilibrium of the rectangle abc in cross section + of depth unity in length at right angles to the plane of the paper.

then $$(R+\delta R)(x+\delta x)\,\delta\theta - Rx\,\delta\theta = S\,\delta x \sin\delta\theta$$

$$R\,\delta x + x\,\delta R = S\cdot\delta x \quad \text{-} \ldots (2)$$

neglecting small quantity of 2nd order.

$$\therefore \quad R + x\frac{\delta R}{\delta x} = S$$

$$= 2C_1 - R \qquad \text{from Eq. 1}$$

ie, $$\frac{\delta R}{\delta x} + \frac{2R}{x} = \frac{2C_1}{x}$$

Multiplying by x^2

$$\frac{\partial (R.x^2)}{\delta x} = 2C_1 x$$

$$\therefore \quad Rx^2 = C_1 x^2 - C_2$$

$$R = C_1 - \frac{C_2}{x^2} \quad \text{- - - -} \quad (3)$$

3

NB This sign in $R = C_1 - \frac{C_2}{X^2}$ should) be $R = C_1 + \frac{C_2}{x^2}$ but Merriman has kept it negative & so I took it to be so knowing C_2 itself might be negative.

$$\& \quad S = 2C_1 - R = C_1 + \frac{C_2}{x^2} \quad \cdots 4.$$

To find the consts. C_1 & C_2

$R = -R_1$ when $x = r_1$ then $-R_1 = C_1 - \frac{C_2}{r_1^2}$

$R = -R_2$ " $x = r_2$, $-R_2 = C_1 - \frac{C_2}{r_2^2}$

$$\therefore R_1 - R_2 = \cancel{C_1} - \cancel{C_1} + C_2\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) \text{ From Eq. 3.}$$

$$= C_2 \frac{r_2^2 - r_1^2}{r_1^2 \cdot r_2^2}$$

$$\therefore C_2 = \frac{r_1^2 r_2^2 (R_1 - R_2)}{r_2^2 - r_1^2} \quad \cdots 5.$$

$$\& \quad C_1 = \frac{C_2}{r_1^2} - R_1$$

$$= \frac{r_2^2 (R_1 - R_2)}{r_2^2 - r_1^2} - R_1$$

$$= \frac{R_1 r_1^2 - R_2 r_2^2}{r_2^2 - r_1^2} \quad \cdots\cdots 6.$$

$$\therefore R = C_1 - \frac{C_2}{x^2}$$

$$= \frac{R_1 r_1^2 - R_2 r_2^2}{r_2^2 - r_1^2} - \frac{r_1^2 r_2^2 (R_1 - R_2)}{\cancel{x^2}\,(r_2^2 - r_1^2)}$$

$$= \frac{1}{r_2^2 - r_1^2}\left\{R_1 r_1^2 - R_2 r_2^2 - \frac{r_1^2 r_2^2}{x}(R_1 - R_2)\right\} \cdots 7$$

$$S = \frac{1}{r_2^2 - r_1^2}\left\{ R_1 r_1^2 - R_2 r_2^2 + \frac{r_1^2 r_2^2}{x^2}(R_1 - R_2)\right\} \cdots 8$$

Birnie's Formula. tangential unit stress

$T = S - \lambda R.$ since $s_0 = 0$

as there is no force in the direction of the axis.

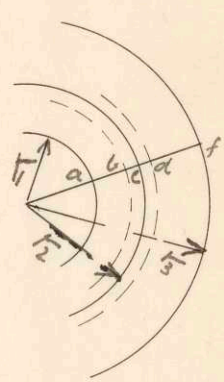
Taking $\lambda = \frac{1}{3}$

$$T = S - \tfrac{1}{3} R.$$

$$= \frac{1}{3(r_2^2 - r_1^2)}\left\{ 2(R_1 r_1^2 - R_2 r_2^2) + 4\frac{r_1^2 r_2^2}{x^2}(R_1 - R_2)\right\} \cdots 9.$$

The application of this formula to Force Fits.

suppose the tube ab is forced inside hoop ef there will be contraction of the outside radius of the inner tube & an expansion of the inside radius of the outer tube.



Let $e_2 = bc =$ contraction & $e_2' = cd =$ expansion

Then the total shrinkage

$$e = e_2 + e_2'$$

Let T_2 be the tangential compression producing e_2 + T_2' tangential tension producing e_2'

then
$$e_2 = \frac{T_2 r_2}{\varepsilon r_2}$$
$$e_2' = \frac{T_2' r_2}{\varepsilon' r_2}$$
$$\therefore e = \frac{T_2 r_2}{\varepsilon r_2} + \frac{T_2' r_2}{\varepsilon' r_2}$$

Applying it to the case of Force fits let

$\varepsilon = \varepsilon_h$ = modulus of Elasticity of hub.

$\varepsilon' = \varepsilon_s$ = " " " shaft.

then
$$e = \frac{T_2 r_2}{\varepsilon_s r_2} + \frac{T_2' r_2}{\varepsilon_h \cdot r_2}$$

Putting for the Shaft $R_1 = 0$

+ $x = r_2$ in the Eqn no. 9.

$$T_2 = \frac{R_2 (4r_1^2 + 2r_2^2)}{3(r_2^2 - r_1^2)}$$

To find T_2' put $r_3' = r_2$. $r_2' = r_1$, $R_2' = R_1$

+ $R_2 = R_3' = 0$

then
$$T_2' = \frac{R_2 (2r_2^2 + 4r_3^2)}{3(r_3^2 - r_2^2)}$$

Then $e = \frac{T_2 r_2}{\varepsilon_s r_2} + \frac{T_2' r_2}{\varepsilon_h r_2}$

$$= \frac{R_2(4r_1^2 + 2r_2^2)\cdot r_2^2}{3\varepsilon_s(r_2^2 - r_1^2)} + \frac{R_2(2r_2^2 + 4r_3^2)\cdot r_2^2}{3\varepsilon_h(r_3^2 - r_2^2)}$$

In the case of the shaft $r_1 = 0$

$$\therefore \frac{e}{r_2} = \frac{R_2 \cdot 2r_2^2}{3\varepsilon_s(r_2^2 - \cancel{r_1^2})} + \frac{R_2(2r_2^2 + 4r_3^2)}{3\varepsilon_h(r_3^2 - r_2^2)}$$

$$= \frac{2}{3} R_2 \left\{ \frac{\varepsilon_h(r_3^2 - r_2^2) + \varepsilon_s(2r_2^2 + 4r_3^2)}{\varepsilon_s \varepsilon_h (r_3^2 - r_2^2)} \right\}$$

$$\therefore R_2 = \frac{3\varepsilon_h \varepsilon_s e (r_3^2 - r_2^2)}{2\{\varepsilon_h(r_3^2 - r_2^2) + \varepsilon_s(2r_2^2 + 4r_3^2)\} r_2^2}$$

$$\therefore T_2' = \frac{R_2(2r_2^2 + 4r_3^2)}{3(r_3^2 - r_2^2)}$$

$$= \frac{3\varepsilon_h \cdot \varepsilon_s \cdot e(r_3^2 - r_2^2)(2r_2^2 + 4r_3^2)}{3(r_3^2 - r_2^2) \times 2\{\varepsilon_h(r_3^2 - r_2^2) + \varepsilon_s(2r_2^2 + 4r_3^2)\} r_2^2}$$

$$= \frac{\varepsilon_h \varepsilon_s e (r_2^2 + 2r_3^2)}{r_2\{\varepsilon_h(r_3^2 - r_2^2) + \varepsilon_s 2(r_2^2 + 2r_3^2)}$$

If $A = \frac{2}{3}\frac{(r_2^2 + 2r_3^2)}{(r_3^2 - r_2^2)}$

then $r_2^2 + 2r_3^2 = \frac{3}{2} A (r_3^2 - r_2^2)$

$$\therefore T_2' = \frac{3\varepsilon_h \cdot \varepsilon_s e A}{r_2\{3\varepsilon_s A + 2\varepsilon_h\}}$$

Having a mind to compare how this old method of procedure in working a stress and strain relation tallies with the comparatively recent methods as given in Love's books on Theory of Elasticity, I tried to work out the same formula on the basis of reasoning followed by Love in his book.

The manner of procedure is as follows, and the notations used are those used by Love. The formula obtained by this method does not agree with that obtained by Birnie's process. Though they have the same general appearance, there are some numerical discrepancies, and it remains to be seen which comes nearer to the truth as verified by experiment.

Using cylindrical coördinates since the displacement is the same in all planes passing through the z axis and lies in those planes.

Notation from Love's Theory of Elasticity Page 140

e $\quad e_{rr} = \frac{\delta u_r}{\delta r}$ = strain along radius r

$e_{\theta\theta} = \frac{1}{r}\frac{\delta u_\theta}{\delta\theta} + \frac{u_r}{r} = \frac{u_r}{r}$ for $u_\theta = 0$ displacement ⊥ to r

$e_{zz} = \frac{\delta u_z}{\delta z} = 0$ displacement in direction of axis z

take $u_r = U$

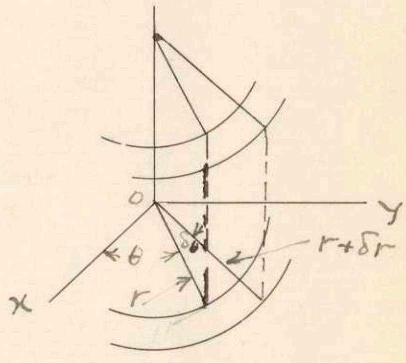
$$\Delta = \frac{\delta U}{\delta r} + \frac{U}{r}$$

$$\omega_r = \omega_z = \omega_\theta = 0$$

$$2\omega_\theta = \frac{\delta U}{\delta z} - \frac{\delta w}{\delta r} = 0$$

$$2\omega_r = \frac{1}{r}\frac{w_z}{\delta\theta} - \frac{\delta u_\theta}{\delta z} = 0$$

similarly $\omega_z = 0$



Equation of Equilibrium where there is no body force Love - Page 140.

Cylin Coord

$$(\lambda + 2\mu)\frac{\delta\Delta}{\delta r} = 0 \qquad \text{xx}$$

ei. $\frac{\delta}{\delta r}\left(\frac{\delta U}{\delta r} + \frac{U}{r}\right) = 0$

$$\frac{\delta U}{\delta r} + \frac{U}{r} = A \quad (A = \text{constant of integration})$$

multiplying by r.

$$r\frac{\delta U}{\delta r} + U = Ar$$

or $\frac{\delta}{\delta r}(rU) = Ar$

integrating $\quad rU = \frac{Ar^2}{2} + B$

$$U = \frac{Ar}{2} + \frac{B}{r} = A'r + \frac{B}{r} = \text{total radial strain}$$

xx Eq. 52. $\quad (\lambda + 2\mu)\frac{\delta\Delta}{\delta r} + 2\mu\frac{\delta\omega_\theta}{\delta z} + \rho F_r = \rho f_r$

F_r = body force = 0. f_r = acceleration in the particle = 0 stress in terms of strain

$\widehat{rr} = \lambda\Delta + 2\mu e_{rr}$ stress in direction of R

$= \lambda\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right) + 2\mu\frac{\partial U}{\partial r}$

$= (\lambda + 2\mu)\frac{\partial U}{\partial r} + \lambda\frac{U}{r}$

$\widehat{\theta\theta} = \lambda\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right) + 2\mu\frac{U}{r}$ stress $\perp^r$ to R Hoop tension

$= \lambda\frac{\partial U}{\partial r} + (\lambda + 2\mu)\frac{U}{r}$

$\widehat{rr} = (\lambda + 2\mu)\left(A - \frac{B}{r^2}\right) + \lambda\left(A + \frac{B}{r^2}\right)$ since $U = Ar + \frac{B}{r}$

$= 2(\lambda + \mu)A - 2\mu\frac{B}{r^2}$

$\widehat{\theta\theta} = \lambda\left(A - \frac{B}{r^2}\right) + (\lambda + 2\mu)\left(A + \frac{B}{r^2}\right)$

$= 2(\lambda + \mu)A + 2\mu\frac{B}{r^2}$

P_0 = pressure outside the hub – negligible

P_2 = " inside " "

(i) Take the case of the value r between r_1 and r_0

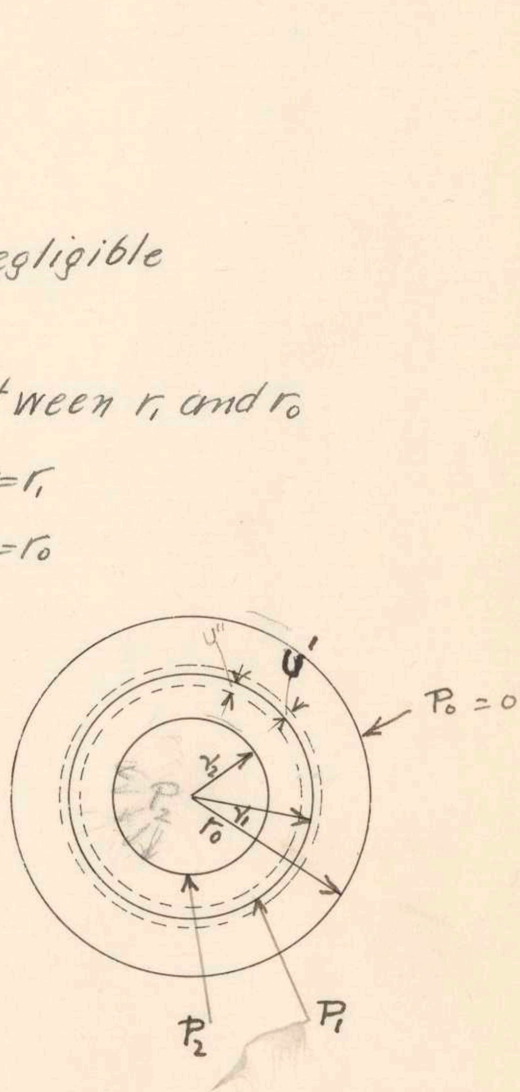
$\widehat{rr} = -P_1 = 2(\lambda + \mu)A - 2\mu\frac{B}{r_1^2}$ when $r = r_1$

$\widehat{rr} = 0 = 2(\lambda + \mu)A - 2\mu\frac{B}{r_0^2}$ when $r = r_0$

ei. $2\mu B\left(\frac{1}{r_1^2} - \frac{1}{r_0^2}\right) = P$

$B = \frac{P}{2\mu}\frac{r_1^2 r_0^2}{r_0^2 - r_1^2}$

$A = \frac{P}{2(\lambda + \mu)} \cdot \frac{r_1^2}{r_0^2 - r_1^2}$



$\theta\theta = 2(\lambda + \mu)A + 2\mu\frac{B}{r^2}$

$= \frac{P_1 r_1^2}{r_0^2 - r_1^2} + \frac{P_1 r_0^2 r_1^2}{(r_0^2 - r_1^2)r^2}$

(ii) If $P_2 = 0$ and $r_1 > r > r_2$

$-P_1 = 2(\lambda + \mu)A' - 2\mu\frac{B'}{r_1^2}$ when $r = r_1$

$0 = 2(\lambda + \mu)A' - 2\mu\frac{B'}{r_2}$ when $r = r_2$

$$2\mu B'\left(\frac{1}{r_1^2}-\frac{1}{r_2^2}\right)=P_1$$

$$\text{or } B' = \frac{P_1 r_2^2 r_1^2}{(r_2^2-r_1^2)2\mu}$$

$$= \frac{-P_1 r_1^2 r_2^2}{2\mu(r_1^2-r_2^2)} = 0 \quad \text{when } r_2 = 0$$

$$A' = -\frac{P_1 r_1^2}{(r_1^2-r_2^2)2(\lambda+\mu)} = \frac{-P_1}{2(\lambda+\mu)} \quad \text{when } r_2 = 0$$

$$\widehat{\theta\theta} = -\frac{P_1 r_1^2(r^2+r_2^2)}{r^2(r_1^2-r_2^2)}$$

(iii) If P_2 be not equal to zero there will be an aditional stress and strain due to P_2. Taking its effect between the limits r_2 and r_0 $\quad r_0 > r > r_2$

$$0 = (\lambda+2\mu)A'' - (2\mu)\frac{B''}{r_0^2} \quad \text{when } r = r_0$$

$$-P_2 = (\lambda+2\mu)A'' - (2\mu)\frac{B''}{r_2^2} \quad \text{when } r = r_2$$

$$2\mu B''\left(\frac{1}{r_2^2}-\frac{1}{r_0^2}\right) = P$$

$$B'' = \frac{P}{2\mu}\frac{r_0^2 r_2^2}{(r_0^2-r_2^2)} = 0 \quad \text{when } r_2 = 0$$

$$A'' = \frac{r_2 P_2}{2(\lambda+\mu)} = 0 \quad \text{when } r_2 = 0$$

This shows that the effect of P_2 on the radial displacement $= 0$ when $r_2 = 0$.

Case i

$$\theta\theta = \frac{P_1 r_1^2(r^2+r_0^2)}{r^2(r_0^2+r)} = \frac{P_1 r_1^2(r_1^2+r_0^2)}{r_1^2(r_0^2-r_1^2)} \quad \text{when } r = r_1$$

$$U = Ar + \frac{B}{r}$$

$$U' = Ar_1 + \frac{B}{r_1} \quad \text{when } r = r_1$$

$$= \frac{P_1}{2(\lambda+\mu)}\frac{r_1^3}{(r_0^2-r_1^2)} + \frac{P}{2\mu}\frac{r_1^2 r_0^2}{r_1(r_0^2-r_1^2)}$$

replacing the value of A and B from (i)

In case ii

$$v'' = A'r_1 + \frac{B'}{r_1} = A'r_1 \quad \text{for } B' = 0 \text{ when } r_2 = 0$$

$$= -\frac{P_1 r_1}{2(\lambda + \mu)} \quad \text{replacing value of } A' \text{ from case ii}$$

$$v' - v'' = e = \frac{P_1}{2(\lambda+\mu)} \frac{r_1^3}{(r_0^2 - r_1^2)} + \frac{P_1 r_1 r_0^2}{2\mu(r_0^2 - r_1^2)} + \frac{P_1 r_1}{2(\lambda' + \mu')}$$

For the radius of the shaft decreases and the inside radius of the hub increases

$\therefore e =$ total allowance or displacement in the metal

Love page 123.

$$\lambda = \frac{E\sigma}{(1+\sigma)(1-2\sigma)} \qquad \text{taking } \sigma = \tfrac{1}{3} \text{ (Poisson's ratio)}$$

$$= \frac{E \cdot \frac{1}{3}}{\frac{4}{3} \cdot \frac{1}{3}} = \frac{3E}{4}$$

$$2\mu = \frac{E}{1+\sigma} = \frac{3E}{4}$$

$$2(\lambda+\mu) = \frac{3E}{2} + \frac{3E}{4} = \frac{9E}{4}$$

$$\therefore e = \frac{P_1 r_1^3}{\frac{9}{4}E(r_0^2 - r_1^2)} + \frac{P_1 r_1 r_0^2}{\frac{3E}{4}(r_0^2 - r_1^2)} + \frac{P_1 r_1}{\frac{9E'}{4}}$$

$$= \frac{4P_1\left\{(r_1^3 + 3r_1 r_0^2)E' + r_1 E(r_0^2 - r_1^2)\right\}}{9EE'(r_0^2 - r_1^2)}$$

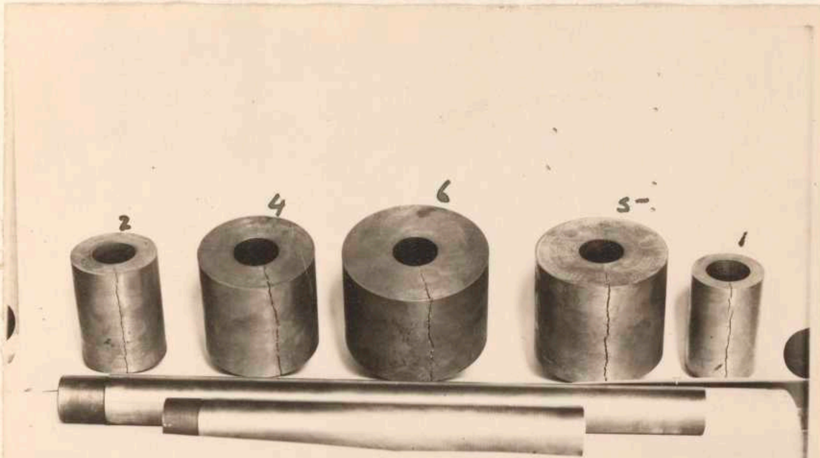
$$P_1 = \frac{9EE'e(r_0^2 - r_1^2)}{4r_1\left\{(r_1^2 + 3r_0^2)E' + E(r_0^2 - r_1^2)\right\}}$$

$$\widehat{\theta\theta} = \frac{P_1(r_1^2 + r_0^2)}{(r_0^2 - r_1^2)}$$

$$= \frac{9EE'e(r_0^2-r_1^2)}{4r_1\left\{(r_1^2+3r_0^2)E'+E(r_0^2-r_1^2)\right\}} \cdot \frac{(r_1^2+r_0^2)}{r_0^2-r_1^2}$$ replacing value of P_1 from preceeding page.

$$= \frac{9}{4} \cdot \frac{EE'e(r_1^2+r_0^2)}{r_1\left\{(r_1^2+3r_0^2)E'+E(r_0^2-r_1^2)\right\}}$$

E' = modulus of elasticity of steel.
E = " " " " cast iron



#3 for T.S. samples

III. The Weight Of Such Calculations To Be Verified By Experiment.

After having got the formulae which expressed the relations of ~~a~~ the stress to that of ~~a~~ the strain or allowance, it remains to be seen how such relations can be verified by experiment. A straight cylindrical shaft cannot be pushed into a straight hole of a hub. It will abrade the metal. So a taper pin with tapering ~~point~~ .06" per foot was selected and a special reamer having the same taper, i.e. .06" per foot, and having the smaller end 1.5" and 6" in length was ordered.

The plan of procedure was this, that 6 cast iron hubs 4" in length and external diameter varying from 2½" to 5" increasing by 1/2", i.e. 2½", 3", 3½", 4", 4½", and 5"; so that the thickness of the metal increases by 1/4 of an inch. These hubs had to be reamed after boring a taper hole with the said reamer. This would give to the hole tapering .06" per ft. having the diameter of the smaller end 1½".

Next a pin of tool steel 12" long had to be hardened in water from a temperature of 1325° F. and tempered in oil after raising it to a temperature of 415 F. so as to impart some tensil strength to the pin. This piece was ground to give a taper of .06" per ft. having the smaller end a diameter of 1.5". So that when the pin was put into the hole of the hub, it came in flush with the smaller end of the hub and bore perfectly well in the hub at all points. Thus, when we should push the pin into the hub it should have the same allowance throughout. Knowing the penetration at the moment of a start and at breaking, we should be able to calculate the allowance. As the tensile strength in that formula is expressed in terms of allowance and known constants, we shall be able to figure out the tensile strength at breaking.

The preparation of the hubs -

The hubs were cast (Cast Iron) with a core of $1\frac{1}{4}$" diameter and having diameters 1/4" larger than given diameters and length $4\frac{1}{2}$". The first put into a chuck and bored 1-3/8" diameter and then put into a mandrel finishing outside; again put into the chuck and bored to the taper hole and afterwards reamed to the above taper.

IV. Experiment Proper.

The object of the experiment is to determine the allowance at which the pieces would burst. To do this we must know the penetration. So, I had to count the gear ratio of the Oilson machine from which the penetration per turn of the wheel was determined; and then, knowing the number of turns of the wheel took from the start to the breaking point, I got the penetration.

It was a compression test. The piece was put on a platform with a central hole to allow the penetrated portion of the pin to pass through. Then the pin was put into the hub and the machine was loaded. When the load on the machine was about 500# or 600# we started to count the revolutions of the wheel and finished counting when the pin cracked. The first pin cracked three hubs of a smaller diameters but it passed through the fourth piece, so a new pin of 20" in length with same taper was made. This pin cracked the rest of the pieces.

Tensile Specimens.

The calculated tensile strength of the first broken piece from the formula gave too high a value so that the broken piece was cut open and four test specimens were prepared from it. Two of them were tried for direct tension only, and after a while it was determined to try the modulous of elasticity.

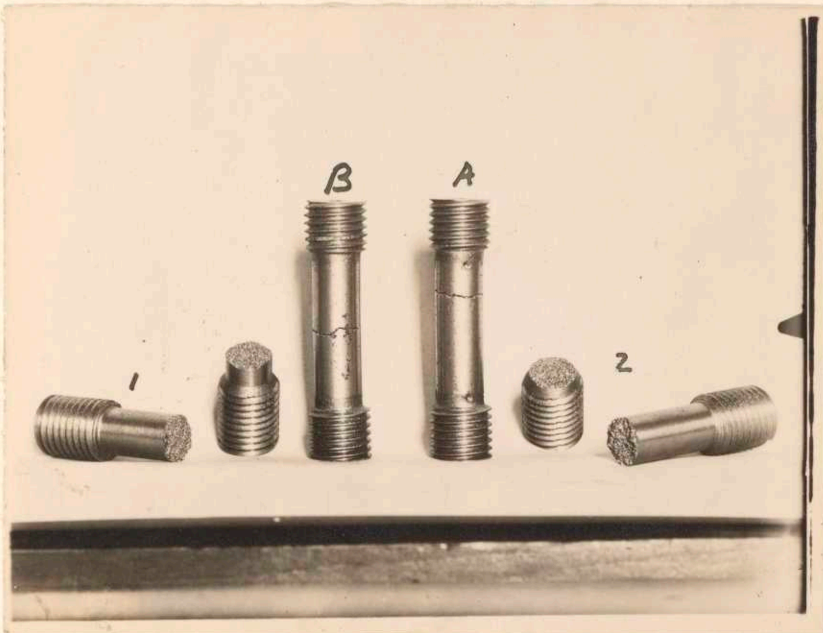
Of the two pieces where modolous of elasticity was to be determined, first piece was tried with micrometer up to 4,500 load and then I used Berry to measure the extension up to the breaking load. In the second piece I used Berry all along up to the breaking point.

The second piece had some blow holes. It broke at about 5,800 lb. load, while the first piece broke at 6,500 lb. load. Now it was left which was to be taken as the modolous of elasticity. If extension at the earlier part of the load be taken, it would give a modolous of elasticity about 8,000,000 per sq. in., while the reading from the latter portion would give only about 37,000 per sq. in. So it was decided to take the entire extension and entire load producing that extension. The modolous of elasticity determined from these values gave 1,880,000. This value of E_h is used in calculating the fibre stress.

The other piece with blow holes gave for E_h 2,900000 per sq. in., but as it broke earlier without reaching its maximum strength it was not taken to calculate the fibre stress of the pieces.

Each of these specimens were 4" in length with narrow section about .75" in diameter. The first two pieces were tried for direct tension and readings of load were taken at breaking point. This gave for the tensilar strength of the pieces 14,600 per sq. in. The other two pieces, also, gave the same strength but here the readings of extension and load were taken throughout the entire range. The length between the clamps was 2".

N. B. Piece A in which the extensions were read with the micrometer was released of the load, when the load reached 4,500#, to take the micrometer out, as it was thought not safe to go beyond that load with micrometer on. This brought about a settled state to the piece so that when it was re-loaded to 4,500# it did not show the same extension as before from 4,500# to 5,000#. For this point I took the extension of the other piece which was read with Berry all along from start to finish and which checked well with other readings.


B
A
1
2

Tensil specimens.

To determine direct tension only

Specimen no.	dia.	area.	Breaking Load.	Fibre stress per sq in
1.	·742"	·432 □"	6420 #	14900 #
2	·748	·440"	6350 #	14400

no. 1.

$$f_t = \frac{6420}{\cdot 432} = 14900 \# \text{ per sq.in}$$

no. 2.

$$f_t = \frac{6350}{\cdot 440} = 14400 \quad " \quad "$$

Specimen A.

Load #	Extension reading micrometer 1.	2	Difference no. 1	Difference no. 2.	average	sum of average	Remark.
500	.4195	.4141	.				
1000	.4199	.4144	.0004	.0003	.00035	.00035	
1500	.4203	.4147	.0004	.0003	.00035	.00070	
2000	4207	.4151	.0004	.0004	.0004	.00110	
2500	.210 4210	.4155	.0003	.0004	.00035	.00145	
3000	.4215	.4161	.0005	.0006	.00055	.0020	
3500	.4221	.4167	.0006	.0006	.0006	.0025	
4000	.4228	.4173	.0007	.0006	.00065	.00315	
4500	.4237	.4182	.0009	.0009	.0009	.00405	
4500	xx .0104	.0086					
5000	.0106	.0090	(.0002)	.0004	assumed .0014	.00545	XX. Here load was taken away
5500	.0120	.0116	.0014	.0020	.0014	.00715	to take the
6000	.0140	.0135	.0020	.0015	.0017	.0089	micrometer out
6100	.0156	.0146	.0016	.0011	.00175	.01025	+ Berry was used
6200	.0170	.0152	.0014	.0006 x	.00135	.01155	to read Extension
6300	.0182	.0164	.0012	.0012	.0013	.01275	
6400	.0194	.0176	.0012	.0012	.0012	.01395"	
6500	.0206		.0012		.0012	.01395"	Broke here.

Distance bet clamps = 2"

dia = .755"

area = .4475

$$\varepsilon = \frac{6400 - 500}{.4475 \times \frac{.01395}{2}} = 1880000 \text{ lbs per }\square''$$

Specimen. B.

Load.	Extension Berry 1	2	Diff. no. 1.	Diff n. o. 2.	average of diff.	Sum	
500	.0218"	.0192"	.0	.000			
1000	.0224	.0194	.0006	.0002	.0004	.0004	
1500	.0228	.0196	.0004	.0004	.0004	.0008	
2000	.0231	.0199	.0003	.0003	.0003	.0011	
2500	.0235	.0201	.0004	.0002	.0003	.0014	
3000	.0238	.0205	.0003	.0004	.00035	.00175	
3500	.0242	.0209	.0004	.0004	.0004	.00215	
4000	.0249	.0215	.0007	.0006	.00065	.0028	
4500	.0256	.0225	.0007	.0010	.00085	.00365	
5000	.0270	.0240	.0014	.0015	.00145	.0051	
5500	.0286	.0257	.0016	.0017	.00165	.00675	
5750	.0298	.0273	.0012	.0016	.0014	.00815"	
5800 xx							xx Breaking Load

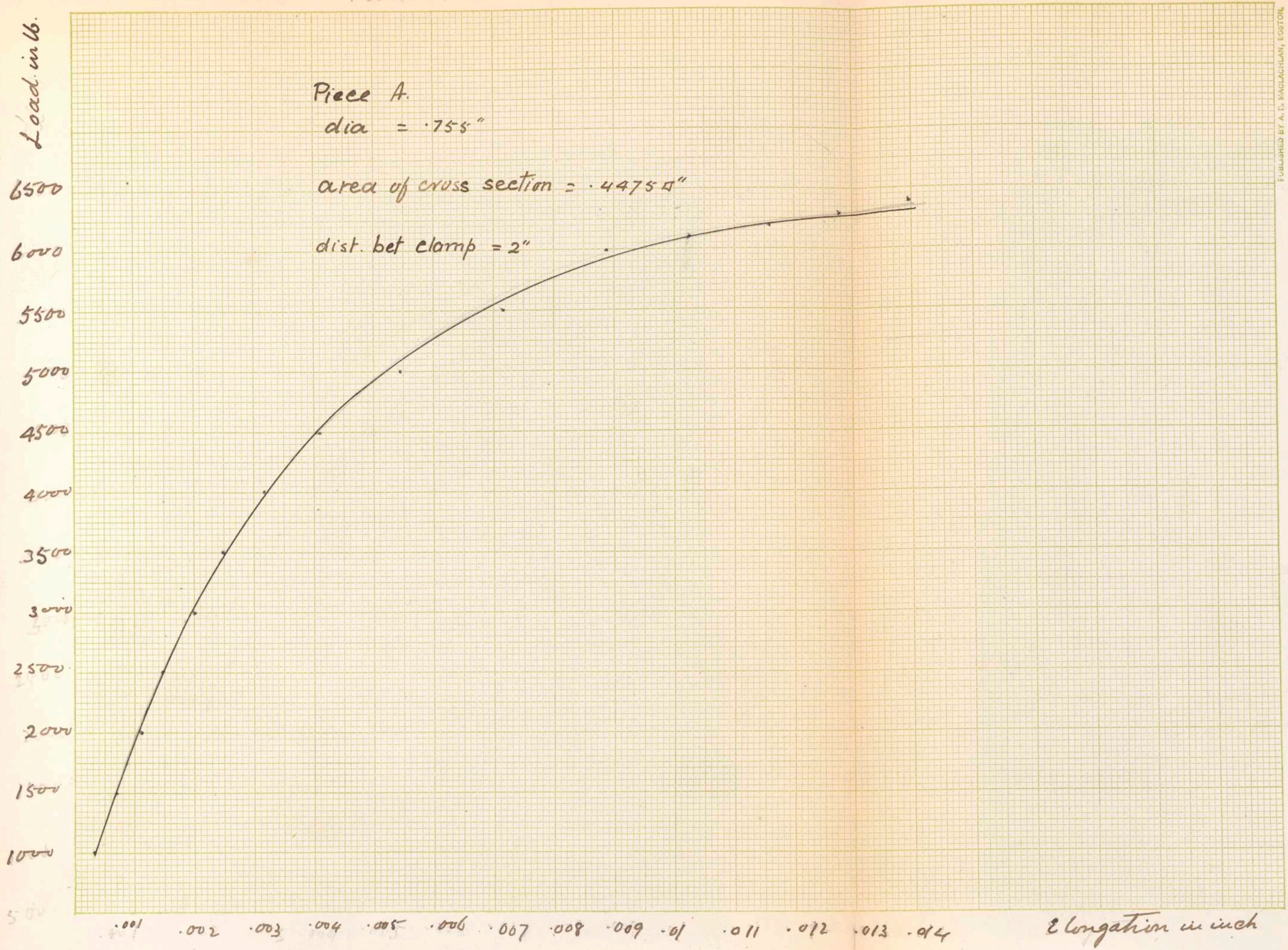
Distance bet clamps. = 2"

Dia = .760

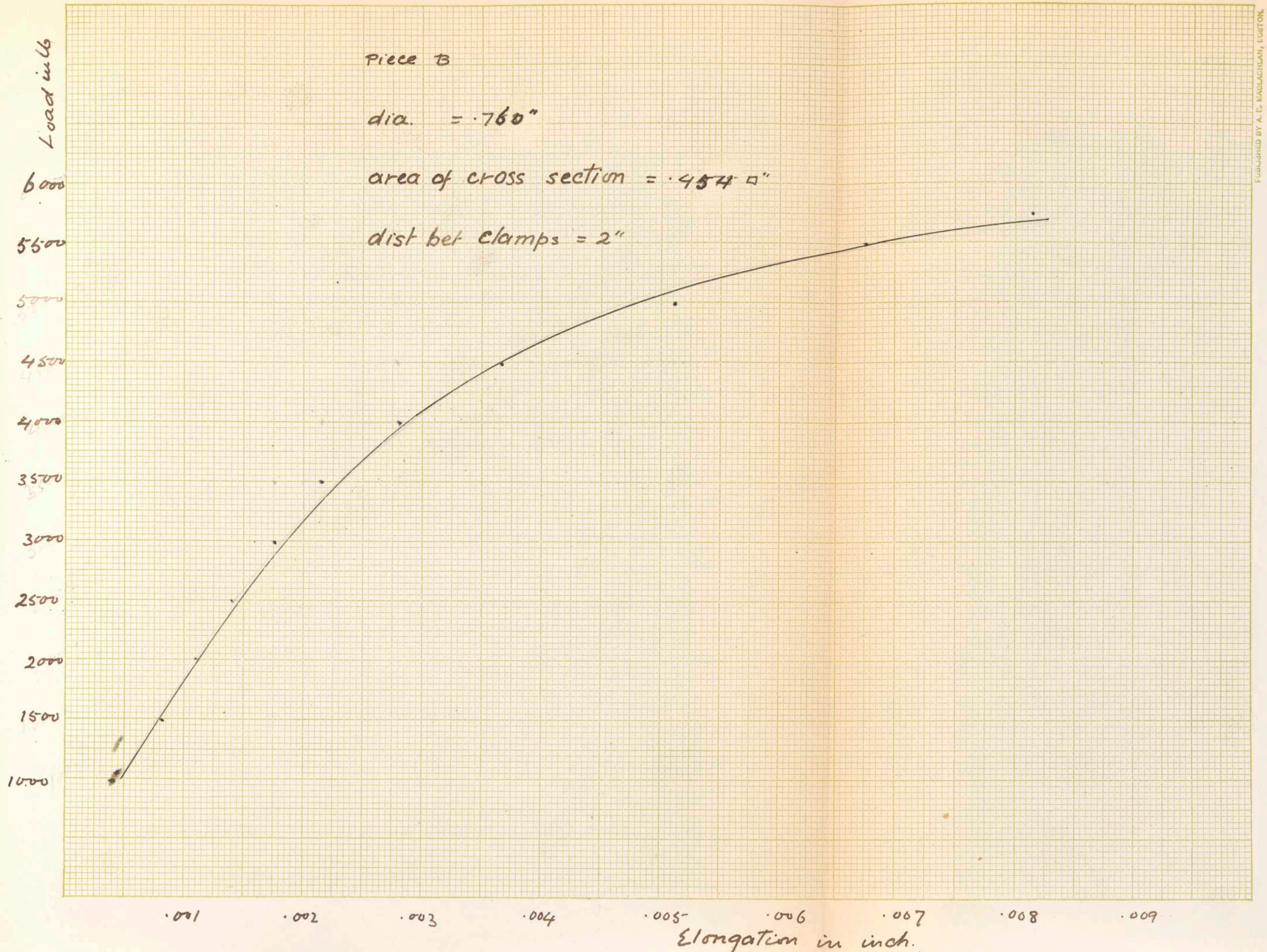
area = .454 □"

$$E = \frac{5750 - 500}{.454 \times \frac{.00815}{2}} = 29{,}000{,}000 \text{ lb per } \square''$$

Piece A

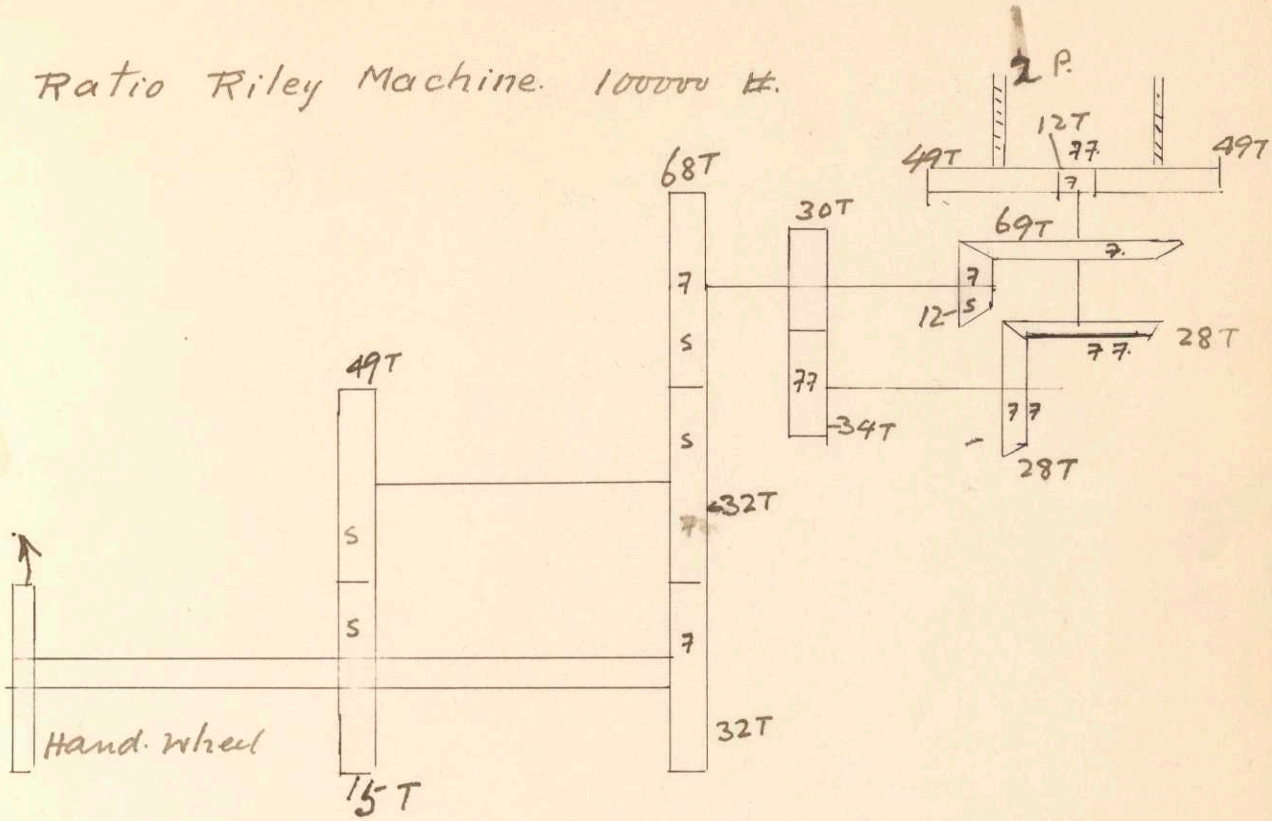


Load in lb.
Piece A.
dia = ·755"
area of cross section = ·4475 □"
dist. bet clamp = 2"
6500
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
·001
·002
·003
·004
·005
·006
·007
·008
·009
·01
·011
·012
·013
·014
Elongation in inch
PUBLISHED BY A. D. MACLACHLAN, BOSTON.



Piece B
dia. = ·760"
area of cross section = ·454 □"
dist bet clamps = 2"
Load in lb
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
·001
·002
·003
·004
·005
·006
·007
·008
·009
Elongation in inch.
PUBLISHED BY A. D. MACLACHLAN, EGSTON

Gear Ratio Riley Machine. 100000 #.



For one turn of Hand Wheel

Feed.

1. S $= \frac{15}{49} \times \frac{32}{68} \times \frac{12}{69} \times \frac{12}{49} \times \frac{1}{2} = .00307''$

2. 7 $= \frac{32}{68} \times \frac{12}{69} \times \frac{12}{49} \times \frac{1}{2} = .0100''$

3. 77 $= \frac{32}{68} \times \frac{30}{34} \times \frac{12}{49} \times \frac{1}{2} = .05075''$

Piece no.	Rev. of Hand wheel Feed.			Initial Load	Final Load
	S	F	FF		
3	933	270		700	43 000
4 XX		$73\frac{1}{2}$	$33\frac{3}{4}$	700	35000
5		736		700	33700
6		342	$84\frac{2}{3}$	800	46980

XX This piece was first tried with the small pin but it passed through: so the pin was taken out. But when the other pin was put into it it penetrated ~~about~~ 4" ∴ this 4" was added to the present penetration

piece no. 1 & 2 were broken in the Emery machine.. The distance between the jaws at start + finish gave the penetration

no.	Initial Load	Final Load	Distance bet jaws at start	at finish before breaking	Penetration
1	700	45000	16.94"	13.64"	3.3"
2.	700	42500	16.80"	13.4	3.4"

Piece no.	Thickness of metal	Outside radius of hub	Initial Load #	Final Load #	Penetration of the pin	allowance = e	Value of A	f_t = 16 per □" Birnie's Formula	f_t = # per □" 2nd. Formula
1	$\frac{1}{2}$"	1.25"	700	45000	3.3"	.00823"	2.54	20200	18500
2	$\frac{3}{4}$"	1.5"	700	242500	3.4"	.00850"	1.99	20900	18200
3	1"	1.75"	700	43000	5.56"	.0139"	1.78	34000	28800
4	$1\frac{1}{4}$"	2.00"	700	35000	6.45"	.0161"	1.66	39400	32500
5	$1\frac{1}{2}$"	2.25"	700	33700	7.36"	.0184"	1.58	45000	36500
6	$1\frac{3}{4}$	2.5"	800	46980	7.72"	.0193"	1.52	47000	37800

E_h = modulous of Elasticity of Cast Iron hub = 18800000 # per □". (Exp)

E_s = 32000000, assumed.

r_1 = radius of the pin. = .75"

V.

To find the value of A.

Piece no 1. thickness of metal = $\frac{1}{2}$"

R = inside radius of the hub = ·75"

R_3 = outside " " " "

$$A = \frac{2}{3}\left\{\frac{R^2 + 2R_3^2}{R_3^2 - R^2}\right\}$$

$$= \frac{2}{3}\left\{\frac{(\cdot 75)^2 + 2(1\cdot 25)^2}{(1\cdot 25)^2 - (\cdot 75)^2}\right.$$

$$= \frac{2}{3}\left\{\frac{\cdot 562 + 2 \times 1\cdot 56}{1\cdot 56 - \cdot 562}\right\}$$

$$= 2\cdot 54$$

Piece no. 2. thickness of metal = $\frac{3}{4}$"

R_1 = ·75 + R_3 = 1·5"

$$A = \frac{2}{3}\left\{\frac{\cdot 562 + 2(1\cdot 5)^2}{(1\cdot 5)^2 - \cdot 562}\right\}$$

$$= 1\cdot 99$$

Piece no. 3. thickness of metal 1"

$R = \cdot 75.$ $R_3 = 1\cdot 75''$

$$A = \frac{2}{3}\left\{\frac{(\cdot 75)^2 + 2(1\cdot 75)^2}{(1\cdot 75)^2 - (\cdot 75)^2}\right\}$$

$$= 1\cdot 78.$$

Piece no 4 thickness of metal $1\frac{1}{4}''$

$R_3 = 2''$

$$A = \frac{2}{3}\left\{\frac{(\cdot 75)^2 + 2\times(2)^2}{(2)^2 - (\cdot 75)^2}\right\}$$

$$= 1\cdot 66$$

no. 5. Thickness of metal $= 1\frac{1}{2}''$

$R_3 = 2\cdot 25''$

$$A = \frac{2}{3}\left\{\frac{(\cdot 75)^2 + 2\times(2\cdot 25)^2}{(2\cdot 25)^2 - (\cdot 75)^2}\right\}$$

$$= 1\cdot 58$$

no. 6 Thickness of metal $= 1\frac{3}{4}''$

$R_3 = 2\cdot 5''$

$$A = \frac{2}{3}\left\{\frac{(\cdot 75)^2 + 2\cdot(2\cdot 5)^2}{(2\cdot 5)^2 - (\cdot 75)^2}\right.$$

$$= 1\cdot 52$$

To find the hoop tension. ie., f_t.

Birnie's Formula for hoop tension

$$f_t = \frac{3\,\varepsilon_h\,\varepsilon_s\,e\,A}{R\left\{3\,\varepsilon_s A + 2\varepsilon_h\right\}}$$

ε_h = modulous of Elasticity of hub = 188 0000 pers

ε_s = " " " " shaft. = 32 000000

R = inside hub radius

e = $\frac{\text{allowance}}{2}$ -

Piece no. 1.

$$f_t = \frac{3 \times 1880000 \times 32000000 \times \cdot 00823 \times 2\cdot 54}{\cdot 75\left\{3 \times 32000000 \times 2\cdot 54 + 2 \times 1880000\right\}}$$

$= 20200$ lb per sq. in

no. 2.

$$f_t = \frac{3 \times 1880000 \times 32 \times \cdot 00850 \times 1\cdot 99}{\cdot 75\left\{3 \times 32000000 \times 1\cdot 99 + 2 \times 1880000\right\}}$$

$= 20900$ lb per sq. in

no. 3.

$$f_t = \frac{3 \times 1880000 \times 32000000 \times \cdot 0139 \times 1\cdot 78}{\cdot 75\left(3 \times 32000000 \times 1\cdot 78 + 2 \times 1880000\right)}$$

$= 34000$ lb per sq. in

no. 4.

$$f_t = \frac{3 \times 1880000 \times 32000000 \times \cdot 0161 \times 1\cdot 66}{\cdot 75\,(3 \times 32000000 \times 1\cdot 66 + 2 \times 1880000)}$$

$= 39400$ lb per sq. in.

no. 5.

$$f_t = \frac{3 \times 1880000 \times 32000000 \times \cdot 0184 \times 1\cdot 58}{\cdot 75\,(3 \times 32000000 \times 1\cdot 58 + 2 \times 1880000)}$$

$= 45000$ lb per sq. in.

no. 6.

$$f_t = \frac{3 \times 1880000 \times 32000000 \times \cdot 0193 \times 1\cdot 52}{\cdot 75\,(3 \times 32000000 \times 1\cdot 52 + 2 \times 1880000)}$$

$= 47000$ lb per sq. in

To Calculate the Tensil stress from the 2nd formula.

$$ft = \frac{9\, \Sigma_s \Sigma_h \cdot e\, (r_0^2 + r_1^2)}{4 \times r_1 \left\{ \Sigma_s (r_1^2 + 3 r_0^2) + \Sigma_h (r_0^2 - r_1^2) \right\}}$$

As before

Σ_h = modulous of Elasticity of the hub (Cast Iron)
= 188,0000 determined from Experiment.

Σ_s = " " " " shaft.
= 32 000000 assumed.

$e = \frac{\text{allowance}}{2}$

r_1 = radius of shaft.

r_0 = outside radius of the hub.

specimen no. 1. Thickness of metal = $\frac{1}{2}$"

$r_1 = \cdot 75''$. $r_0 = 1.25''$. $e = .00823''$

$$ft. = \frac{9 \times 1880000 \times 32000000 \times \cdot 00823 \times \{(\cdot 75)^2 + (1.25)^2\}}{4 \times \cdot 75 \{ 32000000 (\overline{\cdot 75}^2 + 3 \times \overline{1.25}^2) + 1880000 (\overline{1.25}^2 - \overline{\cdot 75}^2) \}}$$

$$= \frac{181000 \times 2.145 \times 8.23}{169 + 1.82}$$

= 18500 lb. per sq. in

Piece no 2.. $r_1 = .75''$, $r_0 = 1.5''$

$e = .00850''$

$$f_t = \frac{9\times 1880000\times 32000000\times .0085\,(\overline{.75}^2 + \overline{1.5}^2)}{4\times .75\{32000000\,(\overline{.75}^2 + 3\times\overline{1.5}^2) + (\overline{1.5}^2 - \overline{.75}^2)1880000\}}$$

$= 18200$ lb per sq. in.

Piece no. 3. $r_1 = .75$, $r_0 = 1.75''$

$e = .0139''$

$$f_t = \frac{9\times 1880000\times 32000000\times .0139\times(\overline{.75}^2 + \overline{1.75}^2)}{4\times .75\{32\times 10^6(\overline{.75}^2 + 3\times\overline{1.75}^2) + (\overline{1.75}^2 - \overline{.75}^2)1.88\times 10^6\}}$$

$= 28800$ lb per □"

Piece no. 4. $r_1 = .75''$, $r_0 = 2''$, $e = .0161''$

$$f_t = \frac{9\times 1.88\times 10^6\times 32\times 10^6\times .0161\times(\overline{.75}^2 + 2^2)}{4\times .75\{32\times 10^6(\overline{.75}^2 + 3\times 2^2) + (2^2 - \overline{.75}^2)1.88\times 10^6}$$

$= 32500$ lb per □"

Piece no. 5. $r_0 = 2.25$, $r_1 = .75$

$e = .0184$

$$f_t = \frac{9\times 1.88\times 10^6\times 32\times 10^6\times .0184\times(\overline{.75}^2 + \overline{2.25}^2)}{4\times .75\times\{32\times 10^6(\overline{.75}^2 + 3\times\overline{2.25}^2) + (\overline{2.25}^2 - \overline{.75}^2)1.88\times 10^6\}}$$

$= 36500$ # per □"

Piece no. 6. $r_1 = \cdot 75$, $r_0 = 2\cdot 5''$

$e = \cdot 0193''$. Thickness of metal = $1\cdot 75''$

$$f_t = \frac{9 \times 1\cdot 88 \times 10^6 \times 32 \times 10^6 \times \cdot 0193\,(\overline{\cdot 75}^2 + \overline{2\cdot 5}^2)}{4 \times \cdot 75 \left\{ 32 \times 10^6 (\overline{\cdot 75}^2 + 3 \times \overline{2\cdot 5}^2) + (\overline{2\cdot 5}^2 - \overline{\cdot 75}^2)\, 1\cdot 88 \times 10^6 \right\}}$$

$= 37800$ lb per $\square''$

VI. Discussion and Conclusion.

An exact agreement between the calculat~~ive~~ed value based on the deduction from formula and the value obtained from test cannot be expected. The closer the one value approaches the other, the better is the soundness of such a formula, and more perfect is the experimental datas. Experimental datas are often vitiated by personal error and instrumental deficiencies. On the other hand, mathematical formulae are deduced based on some kind of hypothesis. The more perfect is the hypothesis and less is the assumption for simplification, the better is the result. But too much refinement is not possible, for the arduous task of simplification becomes so heavy as to make it impossible to arrive at a definite conclusion.

In order to find a relation between strain and stress, it is assumed, 1st, that strain obeys Hook's law, i.e. strain is proportional to stress or linear function of the stress,- This signifies that strain is so small that square, product and cubes of strain can be neglected; 2nd, the effect of temperature due to loading is so small as to be neglected; 3rd, that there is no permanent set, i.e. the body returns to its former estate as soon as the load is removed.

In our case, the materials tested are cast iron and steel. Cast iron is a metal which hardly obeys Hook's law. Of course, the effect of temperature is small enough to be neglected. Besides, in cast iron, any appreciable load always produces a permanent set. It has no elastic limit and its modolous of elasticity is not a fixed quantity within any appreciable range. Besides, in our calculation we took for the modolous of elasticity of that hardened and tempered steel pin to be 32,000,000. As the tensil strength in the formula is the direct product of this quantity, any error in this assumption would affect the calculat~~ive~~ed value of the tensil strength of the cast iron piece tested. As for modolous of elasticity of cast iron, we tested a specimen prepared from the broken piece and determined the modolous of elasticity from the entire elongation produced in this piece by the load from initial to the breaking point. This, of course, is a rough method of handling the difficult~~y~~ies we are confronted with. For the value of E_h is very small as the piece approaches the breaking load. With all these simplifications and approximations, we can hardly expect that the result of our test would agree perfectly well with the value obtained by calculation from the formulae and which are supposed to represent such values.

A glance at the values calculated from the formula (1) by Birnie, and (2) that was deduced from working on Love's theory of elasticity, will show that they came pretty close to actually tested specimens.

For reference, I am giving those values again here:

Specimen No.	1	2	3	4	5	6
Thickness of metal	$\frac{1}{2}$"	$\frac{3}{4}$"	1"	$1\frac{1}{4}$"	$1\frac{1}{2}$"	$1\frac{3}{4}$"
Tensil strength per sq. in.						
1 (Birnie)	20200	20900	34000	39400	45000	47000
2d Formula	18500	18200	28800	32500	36500	37800

Actual tensil strength of cast iron as determined by direct tension is 14,600 per sq. in. This specimen was prepared from the broken hub, consequently cannot be expected to have as much strength as the original metal. So it seems that the calculated tensil strengths from the formulae are not very badly off from what is obtained by actual experiment.

The gradual rise of the tensil strength at breaking load with thicker metal is due to the fact that low rectangular beam sections rupture at a much lower apparent fibre stress than do high ones. This was proved by actual experiment and given in our notes in Machine Design. As rectangles increase in height the tensil strength has to be multiplied by a series of factors varying from 1 to 2 in the case of low to high rectangle to get the modolous of rupture. This is practically verified here, too, though the values from Birnie's formula vary from 20,200 to 47,000, slightly greater than double; the other formula gave the value in case of thick piece exactly double the thinner one.

Comparison of the results as calculated upon the two formulae with the same data shows that the values obtained from the second formula agrees more nearly with the truth of actually tested results than that of Birnie.

The cause of the difference of the two results, due most probably to the manner of simplifying the equation of equilibrium . In Birnie's formula it was started with the assumption that sum of the hoop tension and radial tension was constant, and the equation was simplified from that assumption. Besides, in Equation No. 3 $R_1 = C_1 - \frac{C_2}{X^2}$ (in Merriman) which ought to be $R_1 = C_1 + \frac{C_2}{X^2}$ as was pointed out during the working out of the formula. Love made no such assumption; he first got the equation of equilibrium for all cases and that equation of equilibrium was applied to suit this case. This probably accounts for the difference of results from the two formulae. It is therefore impossible to say which formula is the better; the number of experiments made were only six. From all these, though the second formula seems to give a better result, it rests with after-experiments that are to be made to give a better verdict.

$$R = C_1 + \frac{C_2}{x^2}$$

$$R + S = 2C_1$$

$$S = 2C_1 - R = C_1 - \frac{C_2}{x^2}$$

When $x = r_1 \quad R = -R_1$

$x = r_2 \quad R = -R_2$

$$-R_1 = C_1 + \frac{C_2}{r_1^2}$$

$$-R_2 = C_1 + \frac{C_2}{r_2^2}$$

$$R_1 - R_2 = -\cancel{C_1} - \frac{C_2}{r_1^2} + \cancel{C_1} + \frac{C_2}{r_2^2} = \frac{C_2}{r_2^2} - \frac{C_2}{r_1^2}$$

$$= C_2\left(\frac{r_1^2 - r_2^2}{r_2^2 r_1^2}\right)$$

$$C_2 = \frac{r_1^2 r_2^2 (R_1 - R_2)}{r_1^2 - r_2^2}$$

$$C_1 = -R_1 - \frac{C_2}{r_1^2}$$

$$= -R_1 - \frac{r_2^2 (R_1 - R_2)}{r_1^2 - r_2^2}$$

$$= \frac{-R_1 r_1^2 + R_2 r_2^2}{r_1^2 - r_2^2}$$

$$R = \frac{-R_1 r_1^2 + R_2 r_2^2}{r_1^2 - r_2^2} + \frac{r_1^2 r_2^2}{x^2}\,\frac{(R_1 - R_2)}{(r_1^2 - r_2^2)}$$

$$= \frac{R_2 r_2^2 - R_1 r_1^2 + \frac{r_1^2 r_2^2}{x^2}(R_1 - R_2)}{r_1^2 - r_2^2}$$

$$= \frac{R_1 r_1^2 - R_2 r_2^2 - \frac{r_1^2 r_2^2}{x^2}(R_1 - R_2)}{r_2^2 - r_1^2}$$

$$S = \frac{-R_1 r_1^2 + R_2 r_2^2}{r_1^2 - r_2^2} - \frac{r_1^2 r_2^2}{x^2}\,\frac{(R_1 - R_2)}{r_1^2 - r_2^2}$$

$$= \frac{R_1 r_1^2 - R_2 r_2^2 + \frac{r_1^2 r_2^2}{x^2}(R_1 - R_2)}{r_2^2 - r_1^2}$$