MONTE CARLO SIMULATION FOR THE PRICING OF GNMA SECURITIES

by

ROBERT J. FETTER

S. M., Massachusetts Institute of Technology (1978)
S. B., Massachusetts Institute of Technology (1978)

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Signature redacted

Signature of Author: ________________
Alfred P. Sloan School of Management
May 16, 1986

Signature redacted

Certified by: ________________
Chi-fu Huang
Thesis Supervisor

Signature redacted

Accepted by: ________________
Jeffrey A. Barks
Associate Dean, Master's and Bachelor's Programs

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ABSTRACT

The valuation of mortgage pass-through securities is complicated due to the uncertain future cash flow derived from the mortgage pool. In the past, investors have used simplistic models such as twelve year average life and FHA mortality statistics to predict future cash flows. These models were developed in the era of stable upwardly rising interest rates of the 1960s and 1970s. In the past few years, mortgage pass-through securities have become a major component of the fixed income market. This is due to the high yields which investors have realized from these securities.

In this thesis, I develop a model to predict prepayment rates using a monte carlo simulation of future interest rate dynamics. In this model, there are two types of prepayment, "optimal" prepayment which occurs when the home owner can refinance his existing mortgage as a positive net present value project, and "suboptimal" prepayment which is due to moving, disaster, default, etc. Optimal prepayment is simulated using an implied forward rate model for future interest rates. Suboptimal prepayment is simulated using FHA statistics.

The results of the simulation show that deep discount GNMA coupons are priced using FHA prepayment statistics. In the current and premium coupon GNMAs, the possibility of optimal prepayment becomes high enough to become a major driving force in the prepayment profile. The new model does a better job in valuing the higher coupon GNMAs since it explicitly values the compound call option feature imbedded in the GNMA mortgage pool.

Thesis Supervisor: Dr. Chi-fu Huang

Title: Assistant Professor of Finance

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Chapter One

Introduction

Over the past decade, mortgage pass-through securities have become a major component of the fixed income market. The large majority of these securities have been issued under the auspices of a United State's government agency, Government National Mortgage Association (GNMA, nicknamed "Ginnie Mae"), and/or pseudo-agencies, Federal Home Loan Mortgage Corporation (FHLMC, nicknamed "Freddie Mac") and Federal National Mortgage Association (FNMA, nicknamed "Fannie Mae"). While all mortgage pass-through securities have the same basic structure, the details of the securities can differ substantially. [1] Some of these differences are related to: (1) the nature of the underlying component mortgages in the pool, (2) the method for determining and distributing payments to investors, and (3) the guarantees on the securitiy and the underlying mortgages.

In this thesis, I concentrate my analysis on GNMA mortgage securities. This is because GNMA securities have consistently constituted more than 75 percent of all outstanding pass-through securities. As of the end of 1985, there were $171 billion of securitized single-family mortgage pools issued by GNMA. [2] Appendix A contains a detailed description of a GNMA pass-through security.

A mortgage pass-through security is a collection, or "pool," of mortgages in which the monthly cash flows generated by the underlying component mortgages are distributed to the owners of the security in a pro rata fashion. If the underlying component mortgages in the pool meet GNMA specifications, the originator can obtain a commitment from GNMA to guarantee the timely payment of interest and principal. This loan guarantee by GNMA is considered a general obligation of the United States government and is backed by its full faith and credit. Each mortgage pool is serviced by its originator, or another servicer, who is responsible for managing the monthly cash flow distribution from the pool to the security holders. The servicer receives a fee of 1/2 of one percent of the outstanding mortgage pool principal as a gross fee. The servicer pays .06 percent from this fee to GNMA for the loan guarantee. GNMA securities are the only pass-through which carry a loan guarantee backed by the United States government. This is the main reason for the overwhelming popularity of GNMA pass-throughs.

Individual mortgages must be securitied in their first year to be eligible for inclusion in a GNMA pool. They are typically thirty year mortgages. Pools are issued with a minimum total principal of $1 million. In general, GNMA pools to contain 50 or fewer mortgages from a single originator and tend to be fairly regionalized. Geographic distribution is important because of its effect on prepayment likelihood. Prepayment is the early payment of outstanding principal. Since different parts of the country have different demographic characteristics, the regional nature of GNMA pools can lead to wide disparities in the prepayment behavior of otherwise similar GNMA pools.

Mortgage pass-through securities are considered to be one of the most complicated fixed income securities that is actively traded. The complexity is related to the uncertainty in future cash flows due to the possibility of prepayment. Historically, pass-through securities have enjoyed an average yield spread of 130 basis points over comparable treasuries. [3] This spread is related to the risk premium investors demand for holding these securities. The major factor in the risk of holding pass-throughs is the prepayment risk which is unique to pass-throughs. The realized yield of a pass-through can be extremely sensitive to the actual future prepayment rate. Since it is difficult to accurately predict the future prepayment profile, pass-throughs have a positive yield spread to compensate investors for this uncertainty.

This uncertainty in prepayments led the First Boston Corporation to develop a new derivative security called the collateralized mortgage obligation (CMO) in June, 1983. [4] CMOs are bonds that are collateralized by whole loan mortgages or mortgage pass-through securities. CMOs are structured as a series of sub-pools called traunches. The key difference between pass-throughs and CMOs is in the treatment of distribution of prepayment. In a pass-through, all holders receive their pro rata share of any prepayment. For CMOs all cash flow due to prepayment is allocated first to the holders of the first traunche until it is completely retired. Following retirement of this class, the next traunche in the sequence then becomes the exclusive recipient of principal. This sequential process continues until all the traunches of the CMO are retired. Only one traunche at a time receives prepayment cash flow. The other traunches receive semi-annual interest payments similar to a standard bond. The innovation of the CMO is to break a mortgage pass-through security into short, medium, and long maturity securities. The longer traunches provide investors with a measure of call protection. The CMO structure appeals to investors who want to participate in the mortgage securities market, but do not want to bear the full prepayment and reinvestment risk that pass-through securities carry.

The objective of this thesis is to examine improved means for estimating prepayment and the corresponding effect on the yield of the pass-through security. The role of prepayment and its estimation is discussed in Chapter Two. Chapter Three describes the model that I have developed and the results are critiqued in Chapter Four. Finally, Chapter five contains some conclusions and recommendations for extending the model.

The fundamental differences of a pass-through from a standard bond have proven to be a challenge to investors in determining relative value of the two securities. To quote Martin Liebowitz of Salomon Brothers [5]: "As always, that which constitutes a special problem for some investors will create a special opportunity for others. Over the past several years, this special opportunity in the mortgage securities market has often expressed itself in the very concrete form of a significant yield advantage." I hope that my thesis results will help investors to better understand mortgage securities and provide a new means for predicting prepayment profiles and their impact on pass-through yields.

Chapter Two

Prepayment and its Estimation

To evaluate a mortgage pass-through security as an investment, it is necessary to estimate the cash flows expected from the underlying component mortgages. If no prepayment of principal occurs, then the future cash flows will be known with certainty. However, the possibility of prepayment can alter the cash flow pattern of the mortgage pool. The uncertain future cash flow makes mortgage pass-through securities difficult to value. This has led to a number of ways for estimating prepayment and assessing its impact on the yield of the mortgage security.

A prepayment occurs whenever a monthly mortgage payment is made in excess of the amount actually due. The excess payment is applied toward reducing the outstanding principal of the loan and therefore serves to complete the loan earlier than its original maturity date. Usually a prepayment means that the entire outstanding principal is paid off and the mortgage is terminated. There are several possible causes for prepayment [1], including:

1) Sale of the property - the original property owner repays the outstanding mortgage when he sells his house. Some mortgages are assumable, so it is possible that the sale of the house does not cause prepayment.

2) Refinancing - if interest rates fall, it may

be possible for the homeowner to take out a new mortgage at a more favorable interest rate and use the proceeds to retire his original mortgage.

3) Disaster - in the event the property is destroyed by fire, flood, or other disaster, insurance proceeds will pay off the mortgage. If the home owner dies, life insurance proceeds might be used to pay off the existing mortgage.

In addition to the above causes of prepayment, for fully insured mortgage pass-through securities, if one of the underlying mortgages defaults, then the loan guarantor is obligated to pay the outstanding principal balance. For GNMA pass-throughs, the underlying mortgages are fully insured by the Federal Housing Administration (FHA).

Some mortgage lenders impose prepayment penalties to discourage home owners from prepaying their mortgages. This helps to insure the mortgage lender from prepayments due to refinancing when interest rates drop. However, the issue of prepayment penalties is not relevent for GNMA securities since the mortgages comprising GNMA pools do not permit prepayment penalties to be charged.

One of the early standards for determining the yield of a mortgage pass-through was to assume all the underlying mortgages had a twelve year average life. [5] Under this scenario, the mortgage would prepay the outstanding principal in a lump sum at the end of the twelfth year. As mortgages pools were constructed and sold as pass-through securities, this twelve year average life convention became the standard for determining yield quotations. For investors, this meant that the mortgage pool's cash flow would consist of uniform annual payments for twelve years with a lump sum balloon payment at the end of the twelfth year. The application of this idea of all mortgages in a pool prepaying after the twelfth year is not very realistic. This conjures up visions of whole neighborhoods getting up in unison and moving after twelve years.

As an improvement to the twelve year average life assumption, investors turned to statistics released by the Department of Housing and Urban Development on prepayment rates for FHA insured mortgages. The FHA has compiled a historical record of the performance of FHA insured home mortgage loans. [6] The data is used by the FHA to determine the adequacy of the reserves of the insurance funds used to back home mortgages. The FHA has been updating these statistics every one to two years.

The result of these studies is the FHA experience series which represents the average yearly prepayment and default experience for FHA insured mortgages. Since the FHA is acting as a loan guarantor, a default on a FHA insured loan obligates the FHA to step in and make a timely payment of the outstanding principal balance of the mortgage in default. Therefore, a summation of the terminations due to prepayment and due to default gives the overall expected prepayment profile for FHA insured mortgages over their thirty year life.

The FHA experience series is presented in the form of a mortality table since the unscheduled payment of outstanding principal can, and regularly does, result in the early "death" of an individual mortgage. If the mortgage is part of a pool, then the mortality statistics applied to the underlying component mortgages can be used to estimate the prepayment profile for the entire pool. Clearly, for a specific mortgage pool the exact prepayment profile cannot be predicted exactly from the FHA statistics. However, the FHA experience series can be used as a yardstick for measuring future prepayment experience.

The FHA experience series takes the form of thirty "survivorship balances," representing the percentage of an original pool of mortgages that is expected to be "alive" at the start of each year of the mortgage term. The FHA has released four experience series to date. [6,7] The "survivorship balances" for the four series are given in Table 1. A plot of the survivor profiles is given in Figure 1.

The three most recent series, 1984 - 1986, include only mortgages originated since 1970. Also, the mortgages originated since 1980 are given extra weight in the

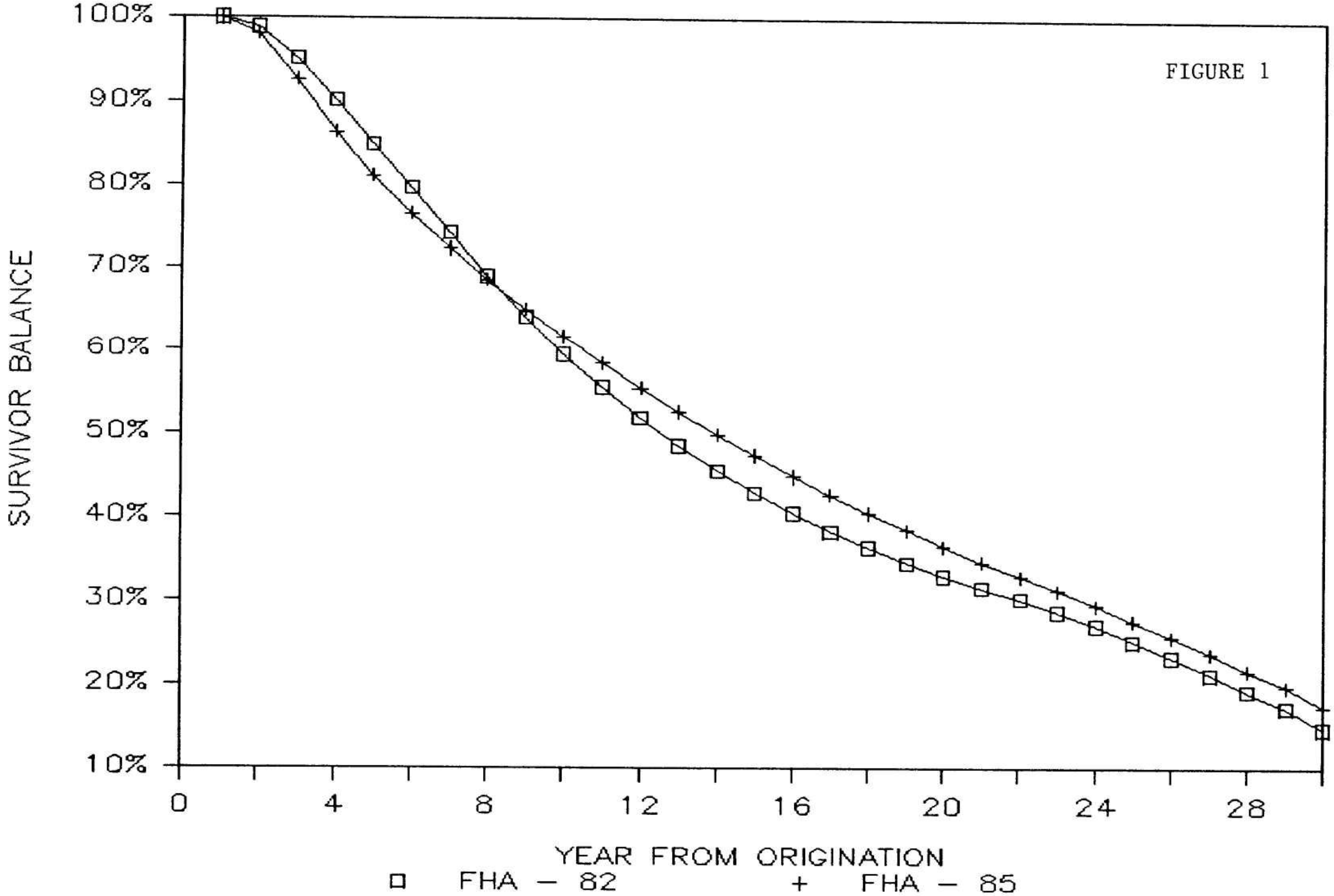
TABLE 1

FHA SURVIVORSHIP BALANCES

LIVING AT BEGINNING OF YEAR

YEAR	FHA - 82	FHA - 84	FHA - 85	FHA - 86
1	1.00000	1.00000	1.00000	1.00000
2	0.98874	0.97915	0.98144	0.98146
3	0.95146	0.92239	0.92348	0.92646
4	0.90224	0.86447	0.86451	0.86430
5	0.85050	0.81413	0.81613	0.81249
6	0.79771	0.76596	0.77180	0.76642
7	0.74343	0.71763	0.72650	0.72470
8	0.69050	0.67280	0.68202	0.68622
9	0.64055	0.62974	0.64309	0.65022
10	0.59573	0.58873	0.60685	0.61652
11	0.55591	0.55803	0.57304	0.58488
12	0.51926	0.53229	0.54133	0.55490
13	0.48590	0.50865	0.51158	0.52651
14	0.45620	0.48572	0.48352	0.49963
15	0.42948	0.46325	0.45706	0.47416
16	0.40544	0.44152	0.43211	0.45004
17	0.38378	0.42050	0.40856	0.42718
18	0.36419	0.40036	0.38635	0.40553
19	0.34641	0.38121	0.36539	0.38501
20	0.33029	0.36312	0.34561	0.36557
21	0.31578	0.34613	0.32694	0.34715
22	0.30279	0.33023	0.30932	0.32968
23	0.28775	0.31267	0.29085	0.31313
24	0.27100	0.29378	0.27181	0.29534
25	0.25277	0.27391	0.25244	0.27663
26	0.23343	0.25341	0.23302	0.25731
27	0.21343	0.23263	0.21377	0.23767
28	0.19318	0.21191	0.19490	0.21801
29	0.17310	0.19155	0.17661	0.19858
30	0.14806	0.16538	0.15601	0.17357

FHA SURVIVORSHIP EXPERIENCE



averaging process. In contrast, the 1982 series was based on mortgage data from 1957 - 1981 with each mortgage weighted equally. The change in the analysis of the statistical data was prompted to combat obvious weaknesses inherent in the 1982 series. [1] The biggest weakness stems from averaging the terminations of mortgages with different coupons which were originated during different economic environments. The higher coupon mortgages originated in the 1980's have a dramatically different termination behavior than the lower coupon mortgages of the 1960 - 1970's. Another problem in the survivorship balances is that they are based on fewer than thirty years of data. For the updated 1984 - 1986 series', these profiles are based on at most sixteen years of termination data. This means that almost half of the series is based on extrapolation. Also, with the FHA statistics being updated regularly, investors using these statistics have to continually re-evaluate the mortgage pass-through securities using the newly published statistics. If there are major shifts in the survivorship balances, this could lead to significant, and at times painful, changes to the values of securities in his portfolio.

The FHA prepayment model is a significant improvement over the twelve year average life model. It takes into account the actual age of the pool's component mortgages and gives a prepayment distribution based on a historical precedent. Use of FHA statistics has been criticized because of its basis on historical data. This implies that future prepayment behavior will mirror the past. The FHA statistics will fail to quickly pick up on changes in demographics and interest rate volatility since it will take a long time for these trends to have a significant impact in the averaging process which goes into generating the updated mortality statistics. With the dramatic swings in interest rates setting off significant refinancing activity, FHA statistics have recently been underestimating prepayment rates of premium coupon GNMA pools. Each individual pool has its own unique prepayment character. Pools from certain parts of the United States, such as California, tend to prepay quicker than other pools. These pools are referred to as "fast" pools. Other pools from places such as Maine may have a much slower prepayment experience and are known as "slow" pools. Investors characterize these unique pools using multiples of FHA experience. For instance, a fast pool may be described as prepaying at 200% FHA to indicate prepayments running at twice of FHA experience. Slow pools might be described as behaving at 50% FHA to indicate prepayments were half of what was expected.

Whereas the twelve year average life and the FHA models try to predict prepayments, a number of newer prepayment measurements have come into use for evaluating the impact of a given prepayment rate on the pass-through's yield. These models are called Single Monthly Mortality (SMM) [1] and Conditional Prepayment Rate (CPR). [8] The SMM rate of a mortgage pool is the percentage of outstanding mortgages assumed to terminate each month. Unlike the FHA model, the SMM model assumes that a pool of mortgages will prepay at a fixed percentage rate, regardless of the age of the mortgages. The CPR rate is similar to the SMM measurement in that it assumes a percentage of the outstanding principal of the pool will prepay in future periods. However, the CPR rate is set by the pool's actual prepayment experience for the previous period. Therefore, CPR extrapolates the most recent prepayment experience over the remaining life of the pool. Both SMM and CPR are appealing in their simplicity for calculating future cash flows of mortgage pools, however, they should in no way be considered predictive models for future prepayment rates.

Finally, the last prepayment model of note is the PSA Standard Prepayment Model. [9] This model was developed by Public Securities Association (PSA) to address the problem of numerous competing prepayment models in general with emphasis on prepayment estimation for collateralized mortgage obligations (CMO). While the PSA model will be used initially to construct CMO yield tables, First Boston Corporation sees the PSA model eventually replacing the already mentioned models for analyzing all mortgage pass-through securities. The PSA model is a compromise of sorts between the FHA and the SMM models. It tries to increase prepayment rates for the early years of a new mortgage similar to FHA, but, in later years leveling off at a constant prepayment rate, akin to the SMM model. Table 2 gives a comparison of the PSA prepayment rates with the 1982 and 1986 FHA statistics. The PSA prepayment percentages are constructed assuming a .2% annual prepayment in the first month, .4% in the second month, .6% in the third month, and so on until in months 30 and beyond, the mortgage will prepay at an annual rate of 6%. Figure 2 shows graphically how PSA compares with FHA statistics. It is interesting to note that FHA statistics predict faster prepayments initially (years 1 thru 10) with PSA faster in the remaining 20 years. We will have to wait to see if indeed the PSA model can restore some sense of order to the pass-through prepayment estimation field.

While prepayment estimation is complicated, there are a plethora of available models. This evolution of prepayment models serves to underscore the importance of accurate future cash flow projections in the valuation of pass-through securities. While there are many valid complaints against using FHA experience, it still remains the most widely used estimator of future prepayment profiles. In the next chapter describing my model for valuing GNMA securities, I use FHA experience incorporating additional interest rate sensitivity for estimating future prepayment

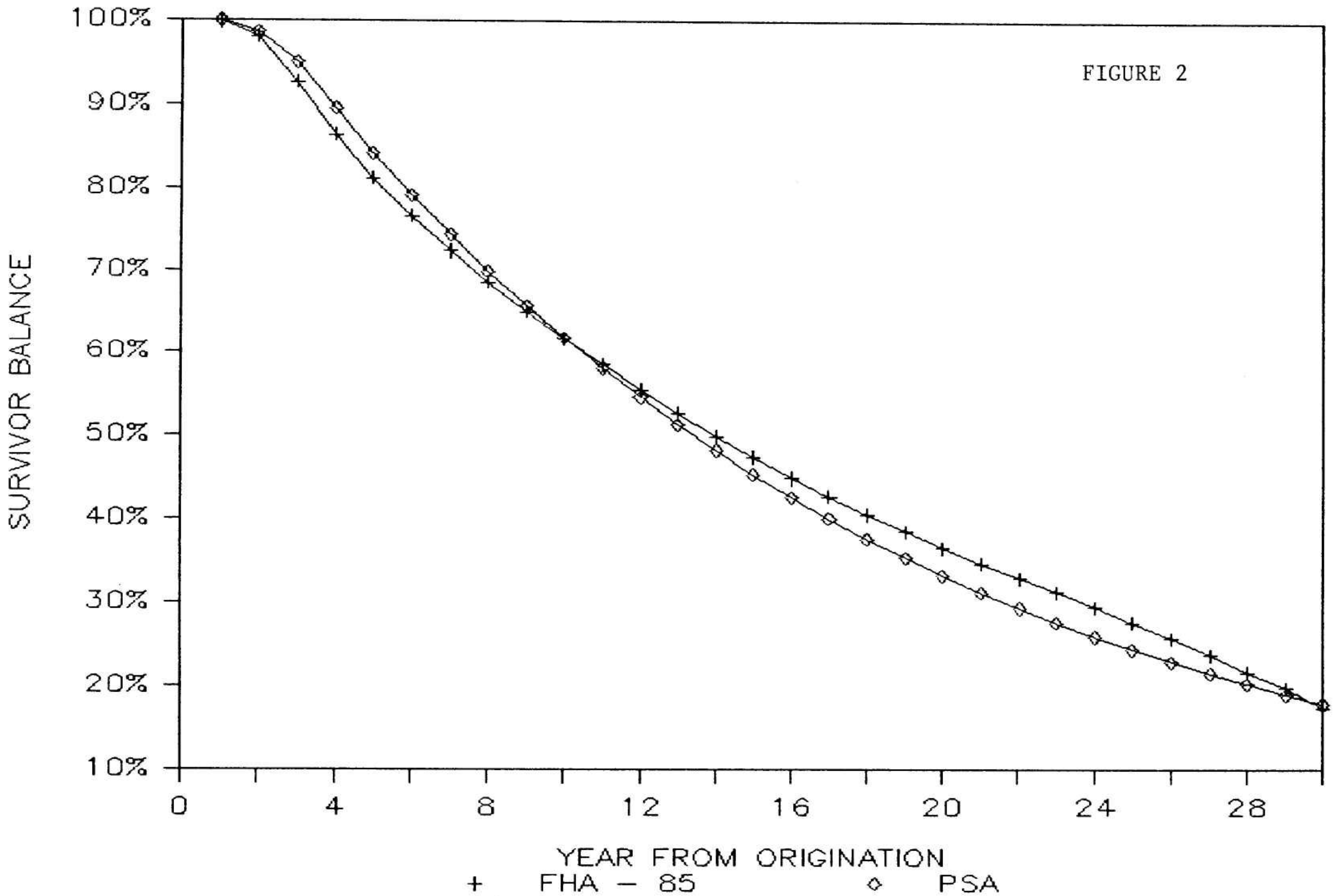
TABLE 2

COMPARISON OF FHA AND PSA EXPERIENCE

LIVING AT BEGINNING OF YEAR

YEAR	FHA - 82	FHA - 86	PSA
1	1.00000	1.00000	1.00000
2	0.98874	0.98146	0.98698
3	0.95146	0.92646	0.95043
4	0.90224	0.86430	0.89578
5	0.85050	0.81249	0.84203
6	0.79771	0.76642	0.79151
7	0.74343	0.72470	0.74402
8	0.69050	0.68622	0.69938
9	0.64055	0.65022	0.65742
10	0.59573	0.61652	0.61797
11	0.55591	0.58488	0.58089
12	0.51926	0.55490	0.54604
13	0.48590	0.52651	0.51328
14	0.45620	0.49963	0.48248
15	0.42948	0.47416	0.45353
16	0.40544	0.45004	0.42632
17	0.38378	0.42718	0.40074
18	0.36419	0.40553	0.37670
19	0.34641	0.38501	0.35409
20	0.33029	0.36557	0.33285
21	0.31578	0.34715	0.31288
22	0.30279	0.32968	0.29410
23	0.28775	0.31313	0.27646
24	0.27100	0.29534	0.25987
25	0.25277	0.27663	0.24428
26	0.23343	0.25731	0.22962
27	0.21343	0.23767	0.21584
28	0.19318	0.21801	0.20289
29	0.17310	0.19858	0.19072
30	0.14806	0.17357	0.17928

FHA AND PSA SURVIVORSHIP EXPERIENCE



rates. I feel the FHA statistics adequately reflect the demographic features of our more mobile society and the random effects of catastrophic losses. However, the FHA statistics do not incorporate the refinancing aspects brought into the spotlight by the increased interest rate volatility of the past few years.

Chapter Three

Modeling of GNMA Prices

III.1 Background

There is considerable interest among portfolio managers, financial analysts, security dealers, etc. on the pricing and investment performance of GNMA securities. To be able to accurately price a GNMA, one needs to make a prediction on the future prepayment profile to determine the cash flows. From the previous chapter, people have developed simplistic models, such as twelve year average life and FHA experience, for estimating the prepayment profile to determine the yield of a GNMA. A more complicated analysis tries to value GNMA's as callable bonds.

Dunn and McConnell have presented a pricing model for GNMA securities based on the general model for pricing interest contingent claims. [10] In their model, they treat GNMA's as callable loans with the home owner (mortgagor) able to call his loan if he can refinance his existing mortgage with a similar new loan that has a lower contract interest rate. This is referred to as an "optimal" prepayment policy. They also notice that mortgagors, in practice, will call their loans even when it is not optimal, ie the prevailing market interest rate is above their current mortgage interest rate. This "suboptimal" prepayment mechanism is usually due to: (1) the mortgagor moving with no assumption of his existing loan, (2) the house is refinanced to remove accrued equity, (3) the mortgagor defaults forcing the guarantor to pay off the mortgage, and (4) the house is destroyed by disaster (fire, flood, etc.). The price of a GNMA should reflect the possible occurance of both optimal and suboptimal prepayment.

To model suboptimal prepayment, Dunn and McConnell incorporate a poisson process into the contingent claim differential equation. They use a mean reverting stationary Markov process for the interest rate. This implies that only the short term interest rate is important for valuing the call option feature of the GNMA mortgage. They set the poisson process to simulate 100% FHA experience to model the suboptimal prepayment.

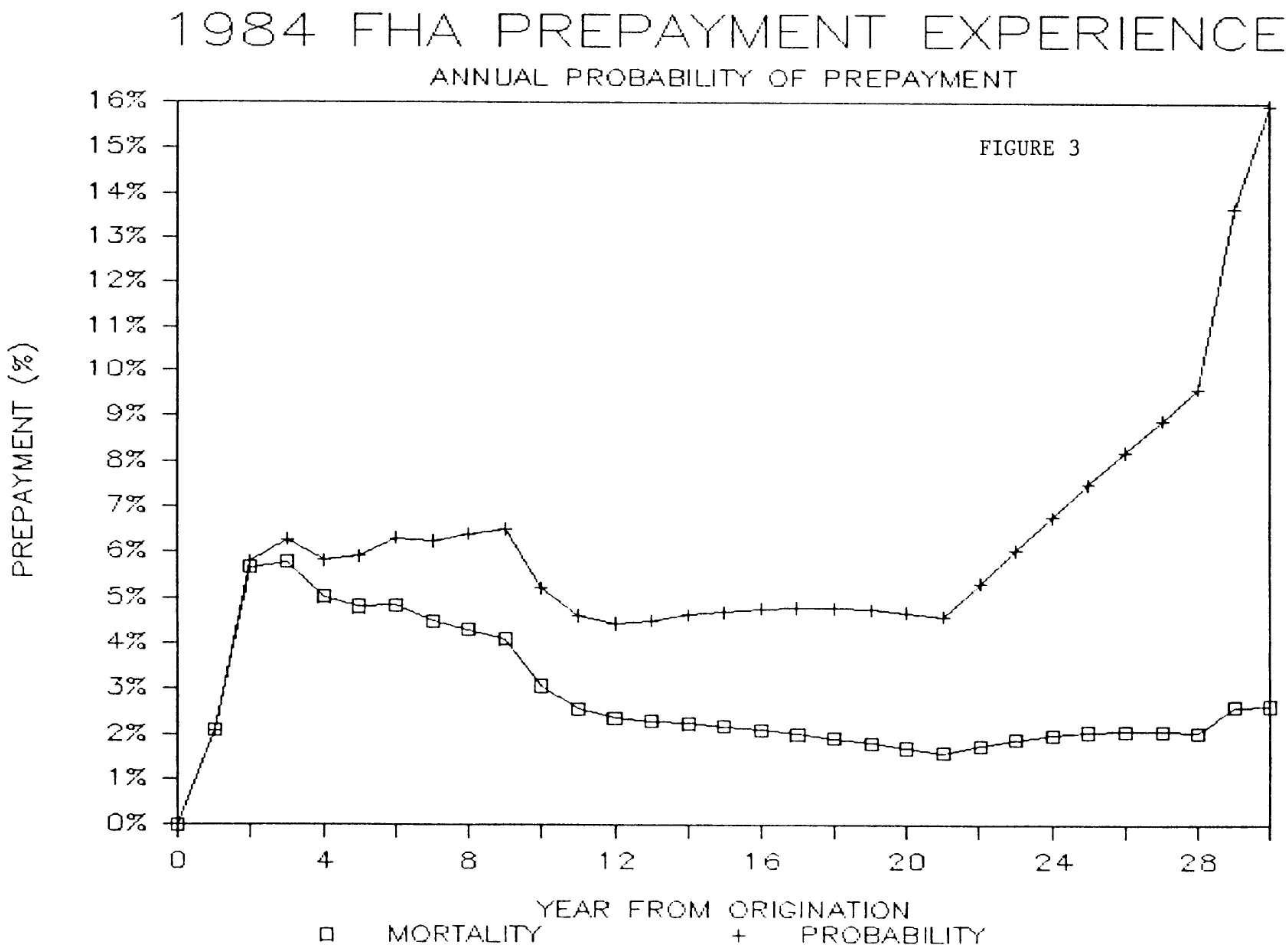
Dunn and McConnell use this model to compare GNMA's to callable/non-callable amortizing/non-amortizing bonds. Their results show that amortization and prepayment increase the price of the GNMA, while, the callability feature decreases the price. The term to maturity affects the magnitude of these features. When the term to maturity is long, the callability outweighs the amortization and prepayment effects and for shorter maturities, the amortization feature has the largest impact.

The Dunn and McConnell paper is noteworthy because it is the first analysis which attempts to explicitly value the call option of the GNMA security. Their analysis suffers from the assumption of optimal call policy thereby, not allowing the GNMA to be priced above par. It may not be realistic to ignore transaction costs and their affect on optimal call policy. Since new mortgagors typically pay a number of points upfront at the origination of a new mortgage, it is possible for the GNMA to be selling above par before it is optimal to refinance the remaining principal. The single parameter interest rate model implies that all treasury bonds are prefectly correlated and changes in the short term rate are only due to stochastic variables. There have been papers published which analyze bonds using a more complicated two parameter model for interest rates. [11] These models have both short and long rates following stochastic processes with the short rate following the long rate. However, I have not seen this analysis applied to the valuation of GNMA's.

III.2 The Implied Forward Rate Model for Estimation of Prepayments

The main intent of my model is to predict the future prepayment profiles for GNMA securities. In the simulation, I allow for the possibility of both optimal and suboptimal prepayments. An optimal prepayment occurs when the home owner can refinance his existing mortgage at a sufficiently low enough rate to make the refinancing a positive net present value project. This takes into account the origination points he will have to pay upfront to refinance his loan. Along with the possibility of optimal refinancing, it is important to provide a mechanism for simulating suboptimal prepayments. To model this, I use the FHA statistics since these reflect the historical pattern of demographic behavior and random nature of disasters which drive the suboptimal prepayment process. Figure 3 shows the 1984 FHA mortality statistics along with the probability of prepayment based on these statistics. The FHA statistics are based on the percent of the original pool that has prepaid during a given year. The probability of prepayment is based on the percentage of existing mortgages alive at the start of the year that prepay during that given year. The suboptimal prepayment process is simulated using a random variable generated from a uniform {0,1} distribution. If the random variable is less than the FHA probability, then the mortgage is assumed to prepay in that year. While it is true that FHA statistics include terminations due to optimal refinancing, the 1984 statistics which were collected during the 1970 - 1984 period, a period of mostly rising interest rates, the terminations due to refinancing will be minimized. Therefore, the prepayment pattern derived from these statistics will reflect the suboptimal prepayment character.

To estimate the optimal prepayments due to refinancing, I needed to be able to simulate the interest rate dynamics of shifts in the future term structure. To do this, I used


1984 FHA PREPAYMENT EXPERIENCE
ANNUAL PROBABILITY OF PREPAYMENT
FIGURE 3
PREPAYMENT (%)
16%
15%
14%
13%
12%
11%
10%
9%
8%
7%
6%
5%
4%
3%
2%
1%
0%
0
4
8
12
16
20
24
28
YEAR FROM ORIGINATION
□ MORTALITY
+ PROBABILITY

the Salomon Brothers' implied forward rate model which they developed for analyzing adjustable rate mortgages. [12,13] In this model, the implied one year forward rates for the next thirty years are derived from the present term structure of interest rates. The procedure for simulating future one year treasury rates is done using the equation:

$$\ln R_i = \ln R_{i-1} + M_i + U_i \qquad (1)$$
$$\text{for } i = 1, 2, \ldots, 29$$

This simulation is based on the current one year rate, R_0. The M_i's are constants calculated so that the price of treasury securities will have a zero spread off of the present term structure. This is equivalent to:

$$p(0,j) = \prod_{i=0}^{j-1} \frac{1}{(1 + R_i)} \qquad (2)$$

Where $p(0,j)$ is the price of a pure discount treasury bond maturing in j years, as implied by the current treasury curve.

The U_i's are independent normal random variables with a zero mean and a standard deviation equal to the volatility of one year treasury yields.

The simulation gives a new series of thirty one-year implied forward interest rates. To determine if the mortgage will refinance, the new mortgage rate is calculated by adding the present spread of mortgage rates over the thirty year treasury rate to the new thirty year rate implied by the simulation. The new mortgage would refinance the outstanding principal of the existing mortgage amortized over thirty years. If the present value of the payments from the new mortgage discounted at the simulated forward rates plus an upfront origination fee (2 to 5 points) is less than the present value of the discounted present mortgage payments, then an optimal prepayment is assumed to have occurred and the mortgage terminates. If no prepayment occurs, then the annual cash flow is just the annual mortgage payment from the original mortgage. Note, this simulation is based on annual instead of monthly payments. This simplification was done to reduce the number of calculations that go into the simulation.

If no prepayment has occurred, either optimally or suboptimally, then the simulation continues into the next year of the mortgage and the calculations are repeated. The FHA probability is updated to reflect the aging of the mortgage. The refinancing calculation uses the next one year forward rate as the R_0 seed for generating a new thirty year treasury rate needed to calculate the new refinancing interest rate. If no prepayments occur in the first twenty nine years of the mortgage, then the mortgage is assumed to go to maturity.

The monte carlo simulation [14] is repeated 10,000 times to give a prepayment probability for each year of the mortgage life. The prepayment profile is used to generate an average cash flow for the mortgage. These cash flows are discounted using the present term structure to give a price for the GNMA security.

The price of the GNMA security from the simulation is compared with the market price to determine the yield spread over treasuries. Typically, GNMA's are priced to yield a positive spread over treasuries. A second calculation is done to add an effective margin (measured in basis points) to the treasury rates used in discounting the annual cash flows until the simulation price matches the market price.

An investor buying the GNMA will pay the market price and expect to receive the simulated annual cash flows. An internal rate of return calculation gives the cash flow yield of the GNMA. From the cash flows and cash flow yield, a modified duration can be calculated to measure the interest rate sensitivity of the GNMA. This duration can be used to compare GNMA's to treasury securities, however, the investor has to be careful of this measurement. This is because interest rate fluctuations can have major effects on the cash flows due to changes in the prepayment profile. Recently, many Wall Street firms have been losing money trading GNMAs in the most recent bond rally. [15] This is because they had hedged their GNMA positions with treasuries. As interest rates fell, the surge in prepayments due to refinancing have kept GNMA prices from appreciating relative to the shorted treasury securities.

This simulation is an attempt to explicitly value the call option of the GNMA security. This option instead of being a single call option is in reality a series of twenty nine european call options with expiration dates at the end of each year of the mortgage life. The exercise of any one of the options preclude exercise of any other future call option. The value of these compound options can be determined by looking at the difference in price between a thirty year non-callable mortgage and the simulation price of the GNMA. The value of this option is affected by the volatility of the one year interest rates and the time to expiration. For discount mortgages, exercise of the option is not optimal and will raise the price of the GNMA relative to the non-callable mortgage. For at par and premium mortgages, the price of the GNMA will be below the non-callable mortgage since the call option will have a positive value.

Chapter Four

Analysis of Results

IV.1 Background on Data Collection

The monte carlo simulation results use data collected for October 12, 1984. The reason why this date was chosen was that all the relevant data on the yield curve, volatility of interest rates, and GNMA prices were readily available in the open literature. [12,13,16] The treasury yield curve is presented in Figure 4. From the yield curve, the one year implied forward rates can be determined, see Table 3. These known forward rates are used to calculate the constants, M_i's, in equation (1). Also, the present one year forward rate, 11.04% is used as the R_0 seed in the interest rate dynamic simulation.

Since I am trying to calculate the value of the call option imbedded in the GNMA security, the volatility of the one year treasury rate is needed. As is generally the case in option valuation, the greater the volatility of interest rates, the higher the value of the call option. [17] The historical volatility of the one year treasury rate for forty day periods during the past year is shown in Figure 5. The volatility has varied from 8% to 16% with an average of 12% for the year. [13] The volatility measurement is used as the standard deviation of the non-uniform normal distribution of the random variable, U_i, in equation (1). The higher the volatility, the more drastic the movement in the

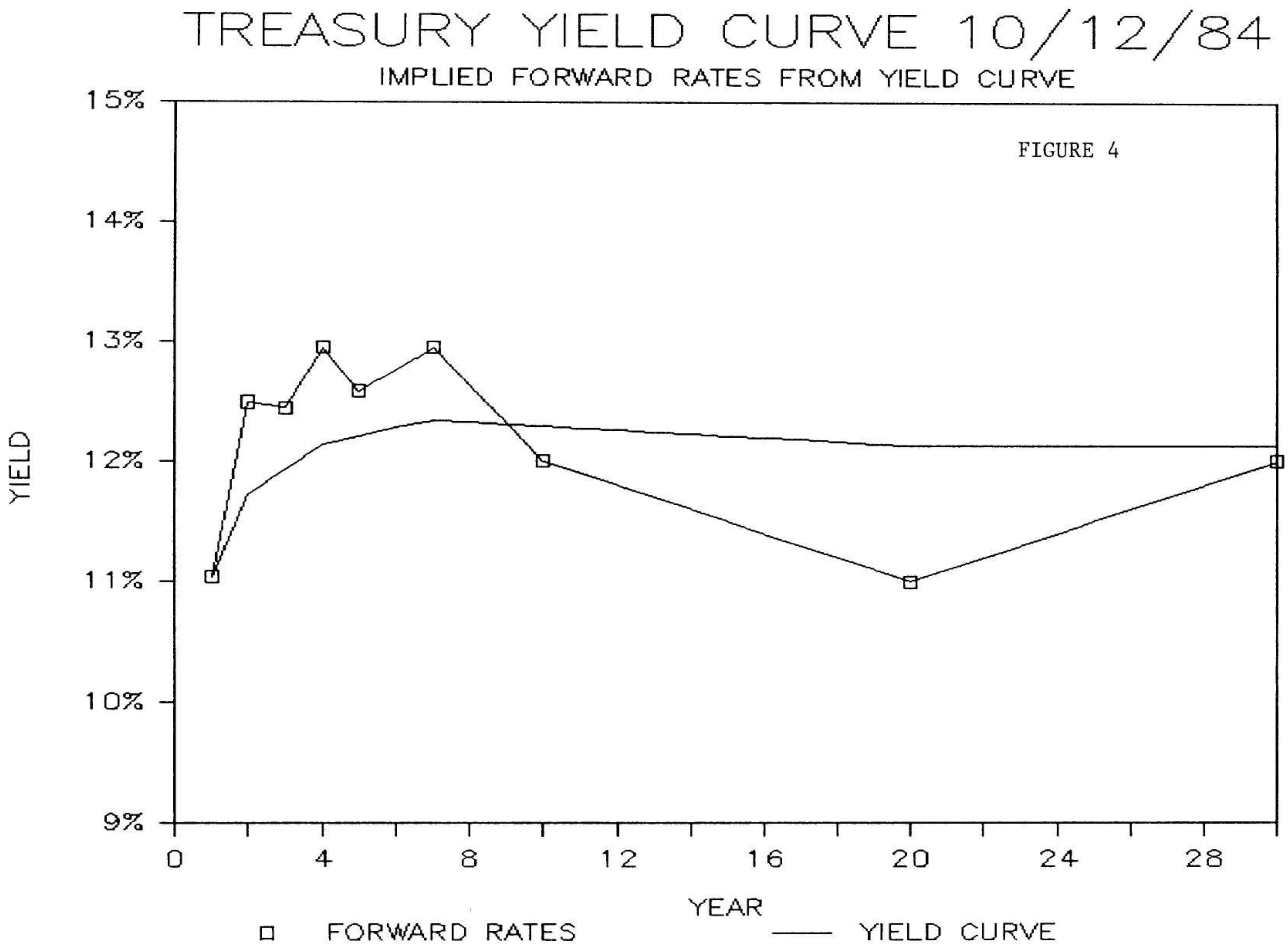

TREASURY YIELD CURVE 10/12/84
IMPLIED FORWARD RATES FROM YIELD CURVE
FIGURE 4
YIELD
15%
14%
13%
12%
11%
10%
9%
0
4
8
12
16
20
24
28
YEAR
FORWARD RATES
YIELD CURVE

TABLE 3

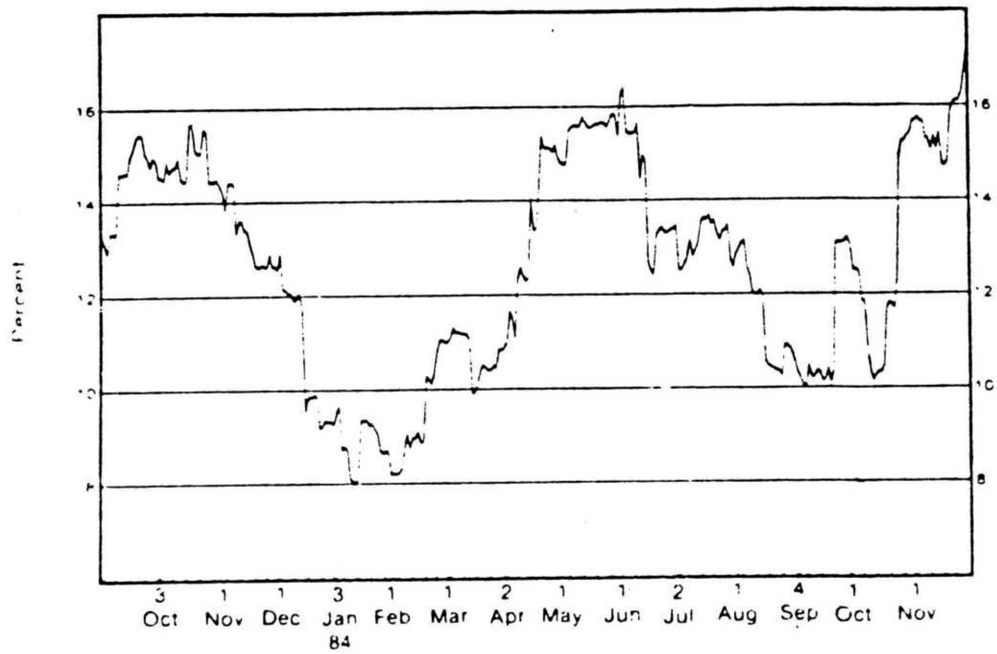
TREASURY YIELD CURVE AND

IMPLIED FORWARD ONE YEAR RATES

10/12/84

YEAR	TREASURY YIELD	FORWARD RATE
1	11.04%	11.04%
2	11.72%	12.49%
3	11.93%	12.44%
4	12.14^	12.95%
5	12.21%	12.58%
7	12.34%	12.95%
10	12.29%	12.00%
20	12.13%	11.00%
30	12.13%	12.00%

Figure 5 VOLATILITY OF ONE YEAR INTEREST RATES
(reproduced from reference 13)



simulated one year forward rates. This makes it more likely that the call option will be exercised due to an optimal prepayment because of advantageous refinancing. This will tend to lower the price of mortgages selling above par (ie, selling at a premium) and raise the price of mortgages selling at a discount.

The market prices of the actively traded GNMA securities were found in the Wall Street Journal. [16] In general, the prices quoted in the Journal are based on one major trade for a given security for that day. They are not necessarily the closing prices. The prices of the GNMA securities are given in Table 4. Also included in Table 4 is the average seasoning of the various GNMA coupons. [18] This is important in the application of FHA statistics for estimating prepayment since the FHA experience depends on the age of the mortgage pool.

IV.2 Discussion of Results

The pricing of GNMA securities if the prepayment profile is known is equivalent to finding the present value of a series of cash flows. When the series of cash flows is dependent upon the exercise of a call option, then a simulation approach is needed to determine when that option will be exercised thus initiating a prepayment. If one assumes the GNMA securities adher to a twelve year average lifetime behavior, then the prices of the GNMA coupons are given in Table 5. These prices are visually compared to market

TABLE 4

MARKET PRICES OF GNMA SECURITIES

10/12/84

GNMA COUPON	PRICE (IN 1/32s)	YEARS SEASONED
8	71.3	8
9	77.19	6
9.5	80.0	5
10	82.17	4
11	88.10	3
11.5	91.1	2
12	93.29	1
12.5	96.19	2
13	98.24	2
13.5	100.26	2
14	103.1	3
15	105.27	3
16	109.25	3

TABLE 5

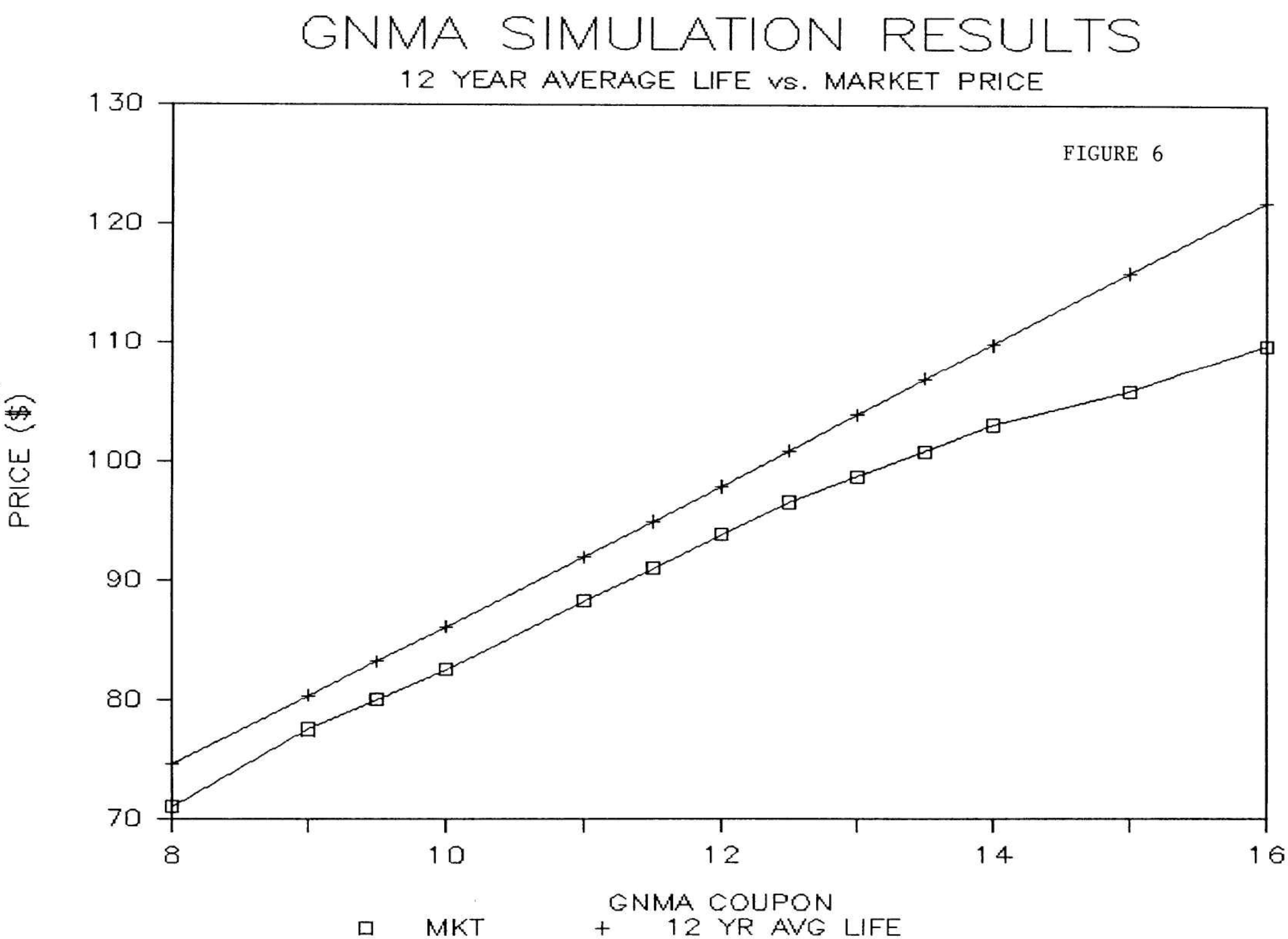
PRICING OF GNMA SECURITIES

ASSUMING A TWELVE YEAR AVERAGE LIFETIME

GNMA COUPON	PRICE	YIELD SPREAD (BP)	MODIFIED DURATION
8	74.68	80	6.92
9	80.36	58	6.86
9.5	83.24	66	6.81
10	86.14	71	6.76
11	91.99	68	6.68
11.5	94.94	71	6.64
12	97.90	71	6.61
12.5	100.88	74	6.57
13	103.87	87	6.52
13.5	106.87	101	6.46
14	109.87	112	6.42
15	115.91	161	6.29
16	121.97	190	6.20

prices in Figure 6. As expected, the twelve year average lifetime concept over-estimates the price of the premium GNMAs. This implies that the market does not expect to receive the high coupon cash flows for a full twelve years. From Figure 6, the slope of the line of twelve year average lifetime prices does not parallel the market price line. This can be shown by the variation in the yield spread from Table 5. The yield spread is the incremental amount that has to be added to the treasury yields in the discounting process of the future cash flows to make the present value of them equal to the market price.

The pricing of GNMAs assuming FHA prepayment experience can also be done without the use of simulation. Since the FHA statistics give you the probabillity of a prepayment occurring in a given year, if the pool is sufficiently large, the prepayment profile should converge to the FHA experience curve. The pricing of GNMAs based on FHA prepayment is given in Table 6. These results are also shown in Figure 7. From Table 6, it is clear again that the premium coupons are expected to prepay faster than 100% FHA. However, in the discount coupons, the FHA prepayment prices show a fairly constant spread over treasuries. Also included in Figure 7 is the FHA price line shifted downward to coincide with the market price line. In the discount coupon range, the slopes of the two lines are nearly identical. This indicates that investors are basing their pricing on



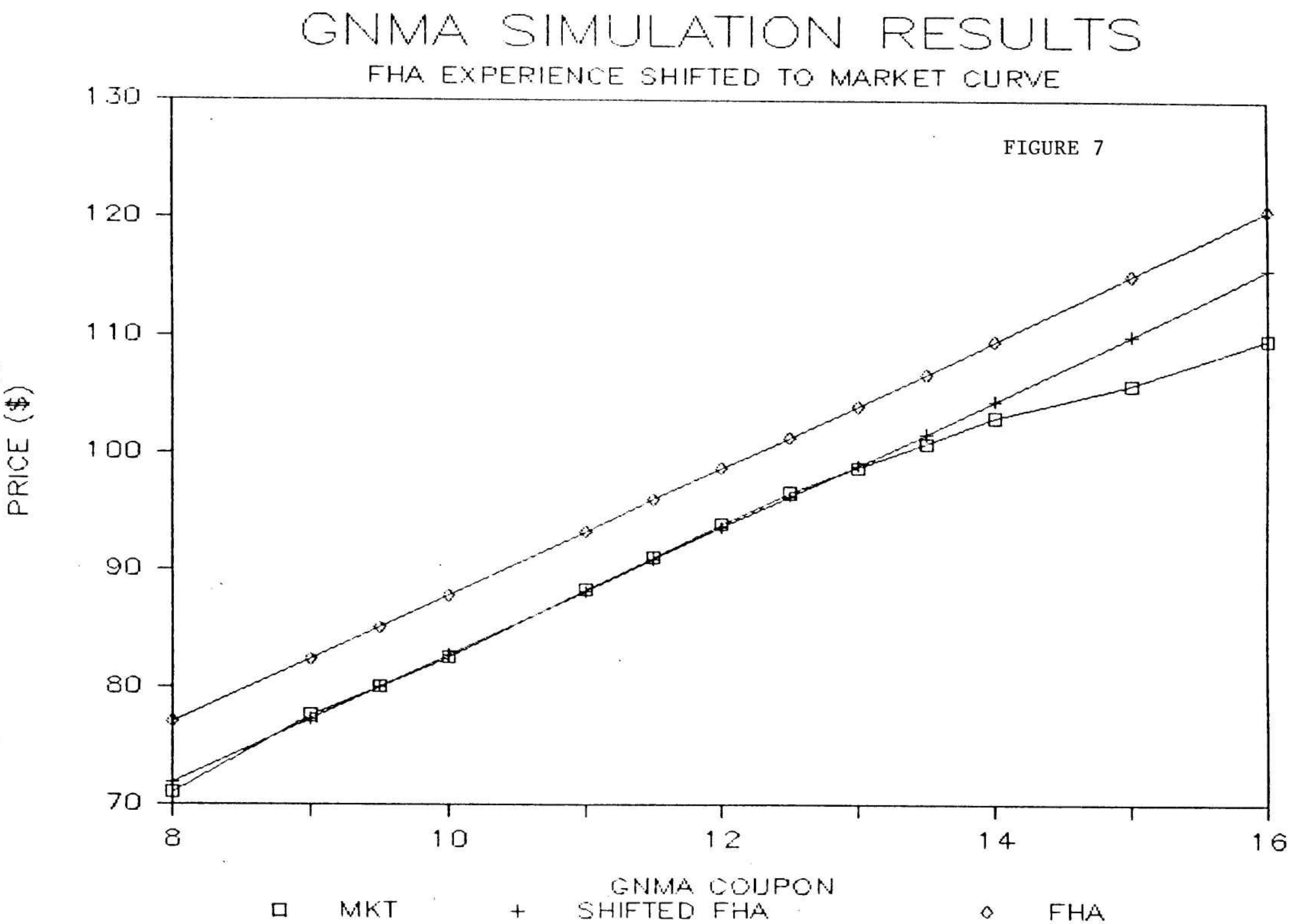
GNMA SIMULATION RESULTS
12 YEAR AVERAGE LIFE vs. MARKET PRICE
FIGURE 6
PRICE ($)
130
120
110
100
90
80
70
8
10
12
14
16
GNMA COUPON
MKT
12 YR AVG LIFE

TABLE 6

PRICING OF GNMA SECURITIES

ASSUMING FHA EXPERIENCE FOR PREPAYMENT

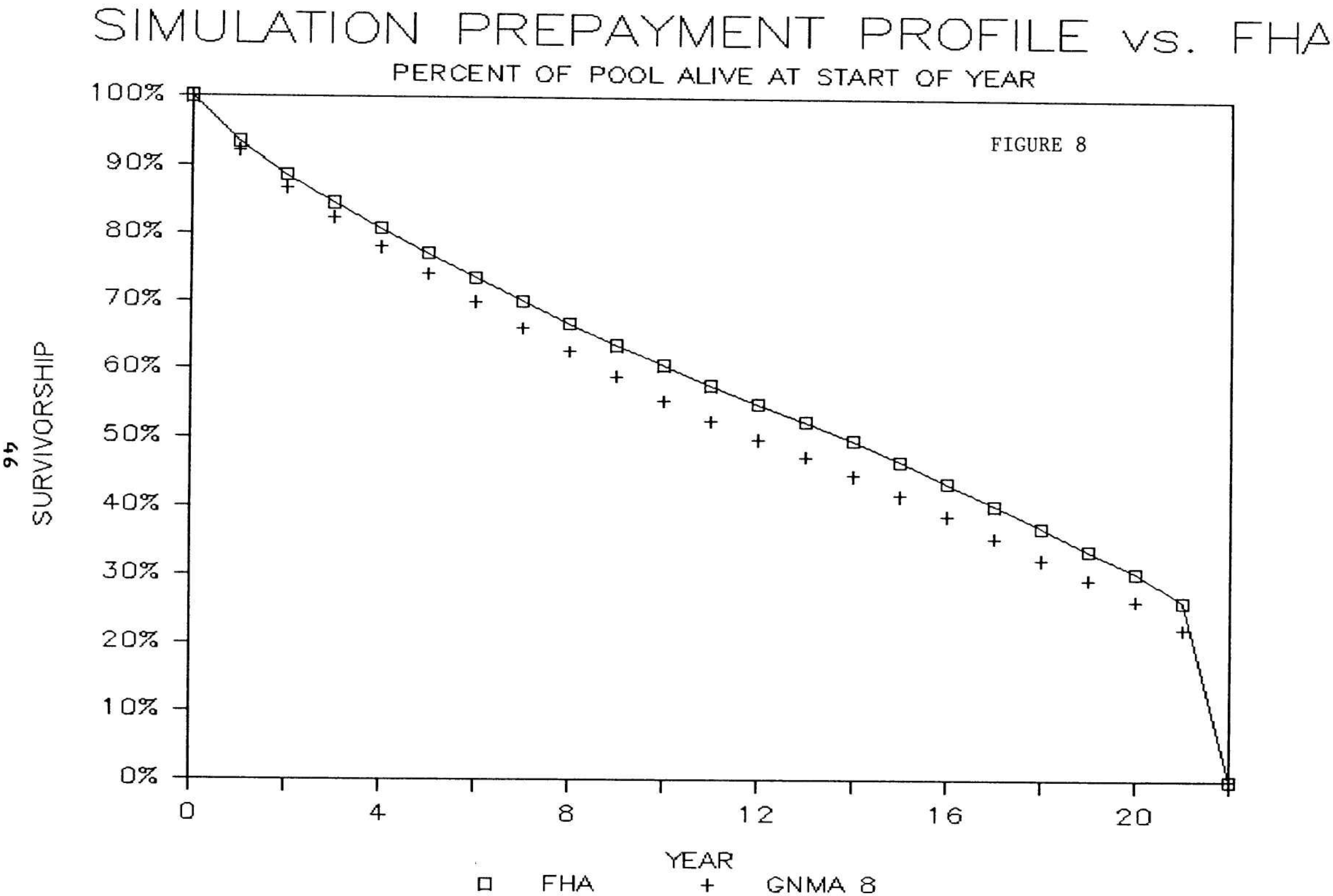
GNMA COUPON	PRICE	YIELD SPREAD (BP)	MODIFIED DURATION
8	77.07	152	5.86
9	82.36	112	5.90
9.5	85.10	117	5.85
10	87.83	118	5.82
11	93.23	103	5.84
11.5	96.00	102	5.75
12	98.68	96	5.70
12.5	101.33	92	5.80
13	104.02	100	5.79
13.5	106.73	109	5.78
14	109.59	116	5.87
15	115.16	160	5.79
16	120.78	182	5.76

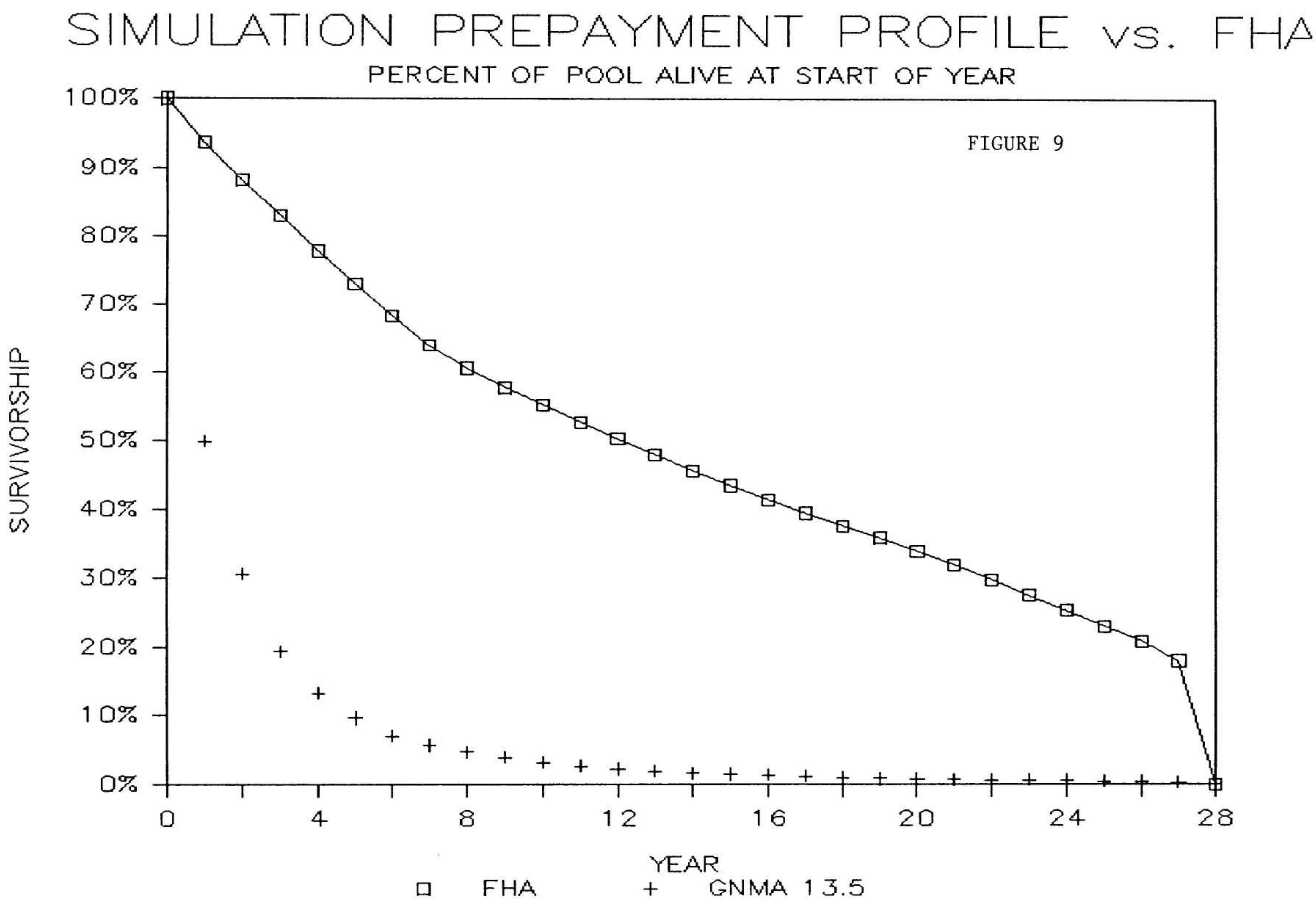

GNMA SIMULATION RESULTS
FHA EXPERIENCE SHIFTED TO MARKET CURVE
FIGURE 7
PRICE ($)
130
120
110
100
90
80
70
8
10
12
14
16
GNMA COUPON
MKT
SHIFTED FHA
FHA

FHA experience. This assumption is reasonable if investors do not expect a strong bond rally which could increase prepayments above FHA due to the optimal refinancing possibility. The Lotus spreadsheets used to calculate these prices are given in Appendix C.

The results of the monte carlo simulations gave a prepayment profile for the GNMA coupons based on optimal prepayment due to refinancing and sub-optimal prepayment based on FHA experience. To give a flavor for the predicted prepayment profiles, Figures 8, 9, and 10 show the survivorship balances for the GNMA 8, 13.5, and 16 simulations. The GNMA 8 profile closely follows the FHA experience. There is a small amount of additional prepayment due to optimal refinancing even for this deep discount coupon. This is just a reflection on the 12% volatility of the simulated one year rates. The GNMA 13.5 profile shows that even the at par coupon can expect a substantial amount of prepayment due to optimal refinancing. The optimal prepayment is expected to terminate an additional 43% of the pool over FHA alone in the next year. The prepayment profile of the GNMA 16 shows an even steeper decline in the survivor balances, with a substantial portion of the terminations due to optimal refinancing. The FHA profiles vary slightly in the three figures due to the difference in seasoning of the three coupons.

The predicted prices and yield spreads are given in


SIMULATION PREPAYMENT PROFILE vs. FHA
PERCENT OF POOL ALIVE AT START OF YEAR
FIGURE 8
SURVIVORSHIP
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0
4
8
12
16
20
YEAR
FHA
GNMA 8


SIMULATION PREPAYMENT PROFILE vs. FHA
PERCENT OF POOL ALIVE AT START OF YEAR
FIGURE 9
SURVIVORSHIP
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0
4
8
12
16
20
24
28
YEAR
FHA
GNMA 13.5

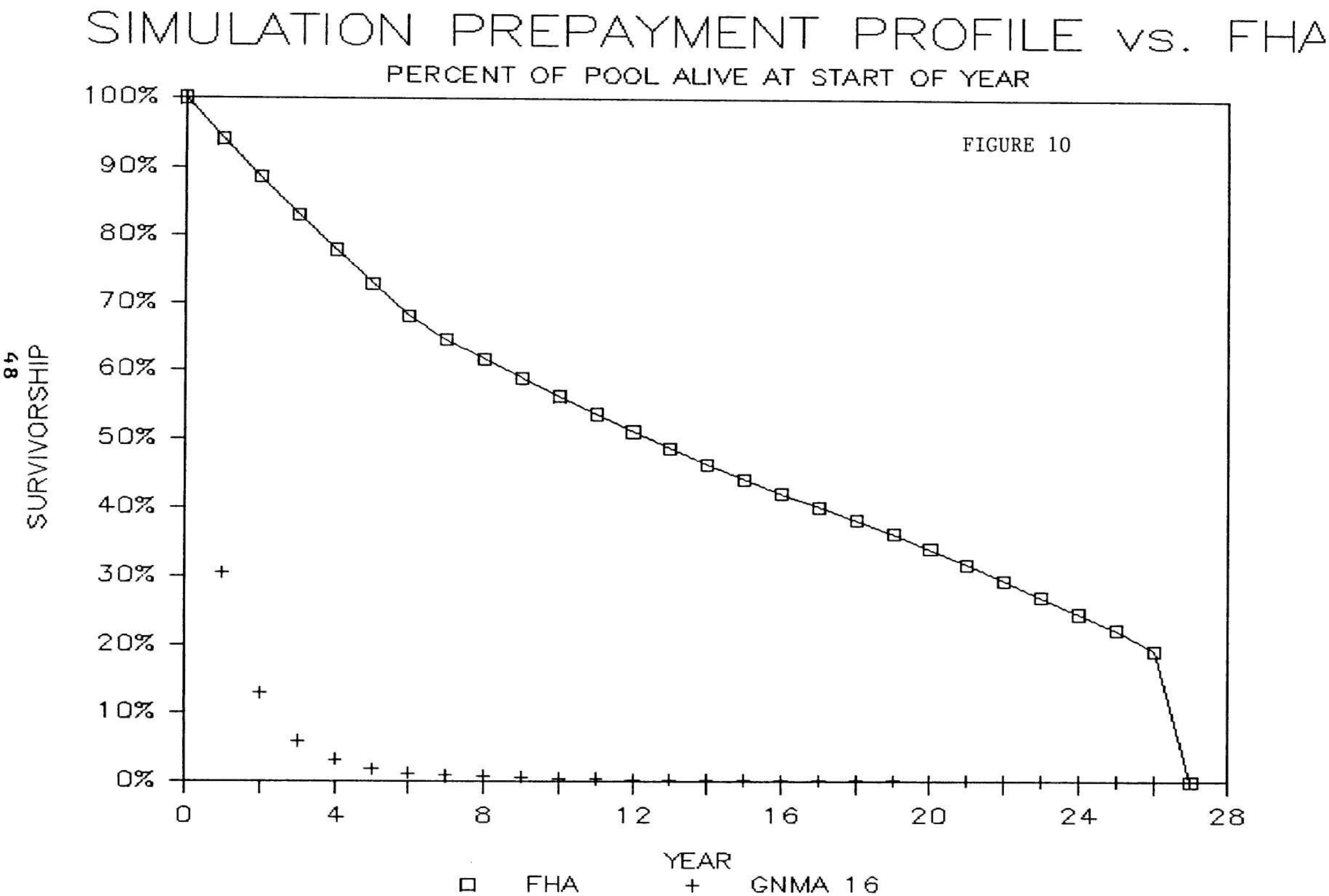

SIMULATION PREPAYMENT PROFILE vs. FHA
PERCENT OF POOL ALIVE AT START OF YEAR
FIGURE 10
SURVIVORSHIP
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0
4
8
12
16
20
24
28
YEAR
FHA
GNMA 16

Table 7 and shown in Figure 11. The simulation results overestimate the price of the discount coupons with yield spreads of 250 to 300 basis points. This is in part due to the additional prepayments generated by the optimal refinancing possibility from the simulation. Prepayments in a discount security will raise its price since the investor will receive the outstanding balance of the principal sooner than it the mortgage went to term. The simulation prepayment profile results are given in Appendix B. The Lotus spreadsheet results to determine the yield spread are given in Appendix C.

In the current coupon range (the GNMAs selling at or near par), the simulation results pick up the curvature in the market price line. This curvature is a reflection of the market's belief that the possibility of optimal prepayment is high enough to include it in the pricing estimate. From Table 7, the yield spread of the current coupon GNMA (GNMA 13.5) is 124 basis points. This is in good agreement with the historical spreads of the current coupons over treasuries [3], see Figure 12.

In the premium GNMAs, the simulation underprices the security due to an over-estimate of the prepayment rate. This over-estimation leads to a predicted yield spread which is negative, see Figure 13. This simulation assumes that in the refinancing present value calculation, that the home owner pays 2 points upfront as an origination cost for his

TABLE 7

PRICING OF GNMA SECURITIES

ASSUMING FHA AND OPTIMAL REFINANCING FOR PREPAYMENT

GNMA COUPON	PRICE	YIELD SPREAD (BP)	MODIFIED DURATION
8	79.89	258	4.96
9	85.96	249	4.52
9.5	88.97	279	4.19
10	91.86	309	3.80
11	96.62	315	3.14
11.5	98.50	310	2.82
12	100.00	270	2.59
12.5	101.23	217	2.42
13	102.27	177	2.22
13.5	103.13	124	2.06
14	103.85	46	1.92
15	105.00	-53	1.68
16	105.91	-266	1.51

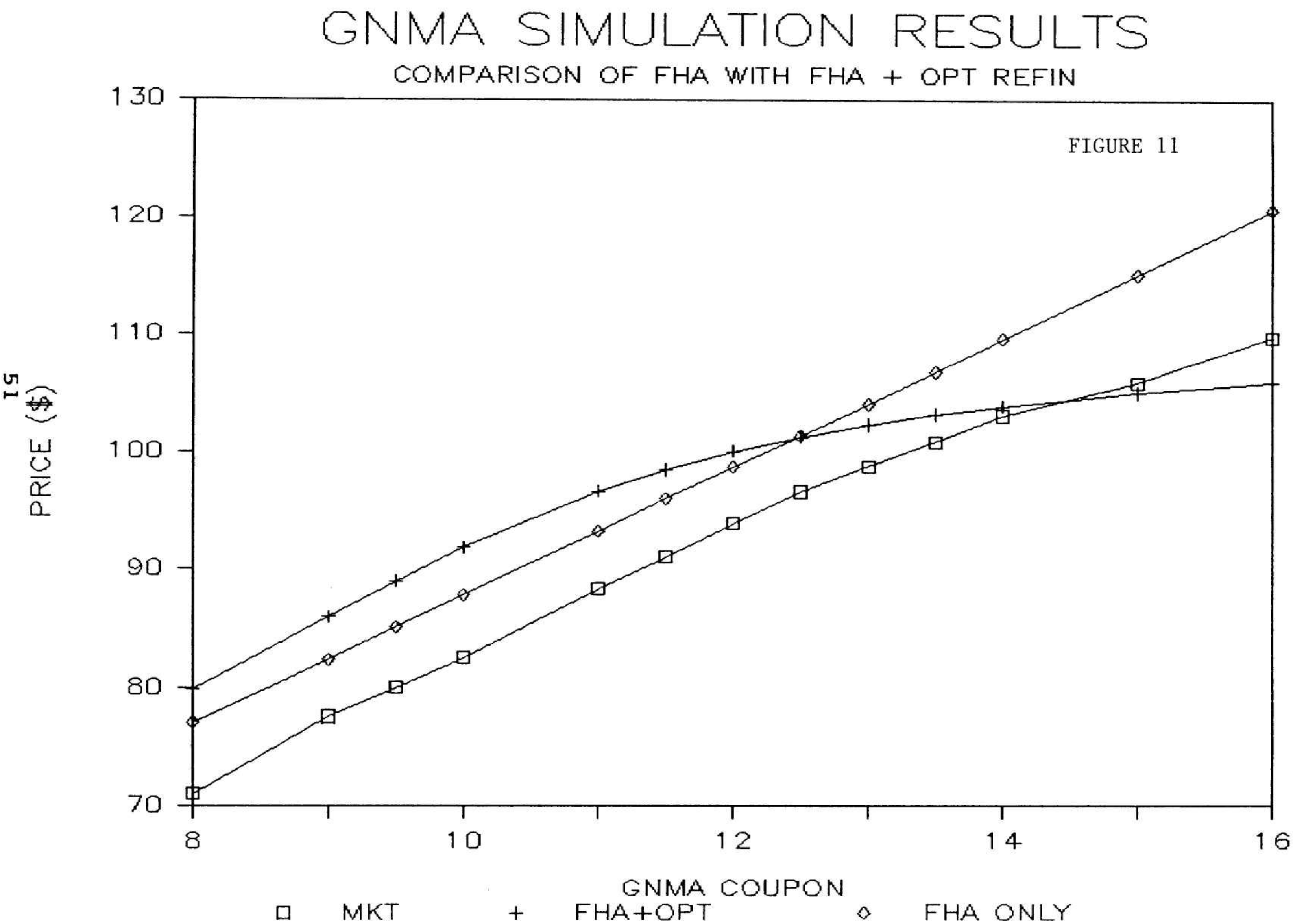
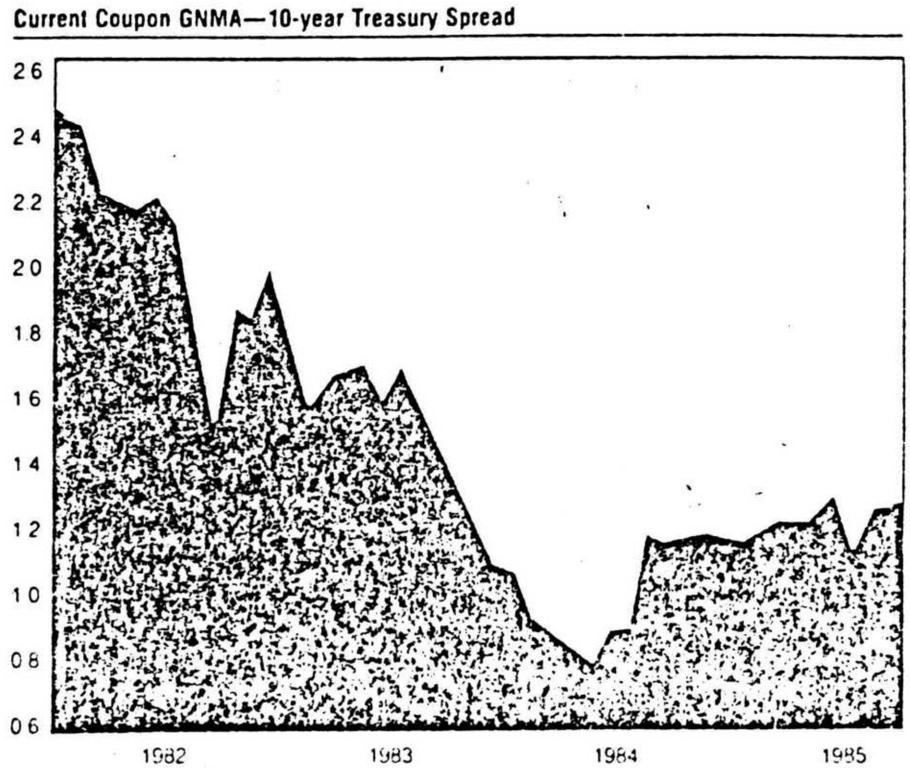
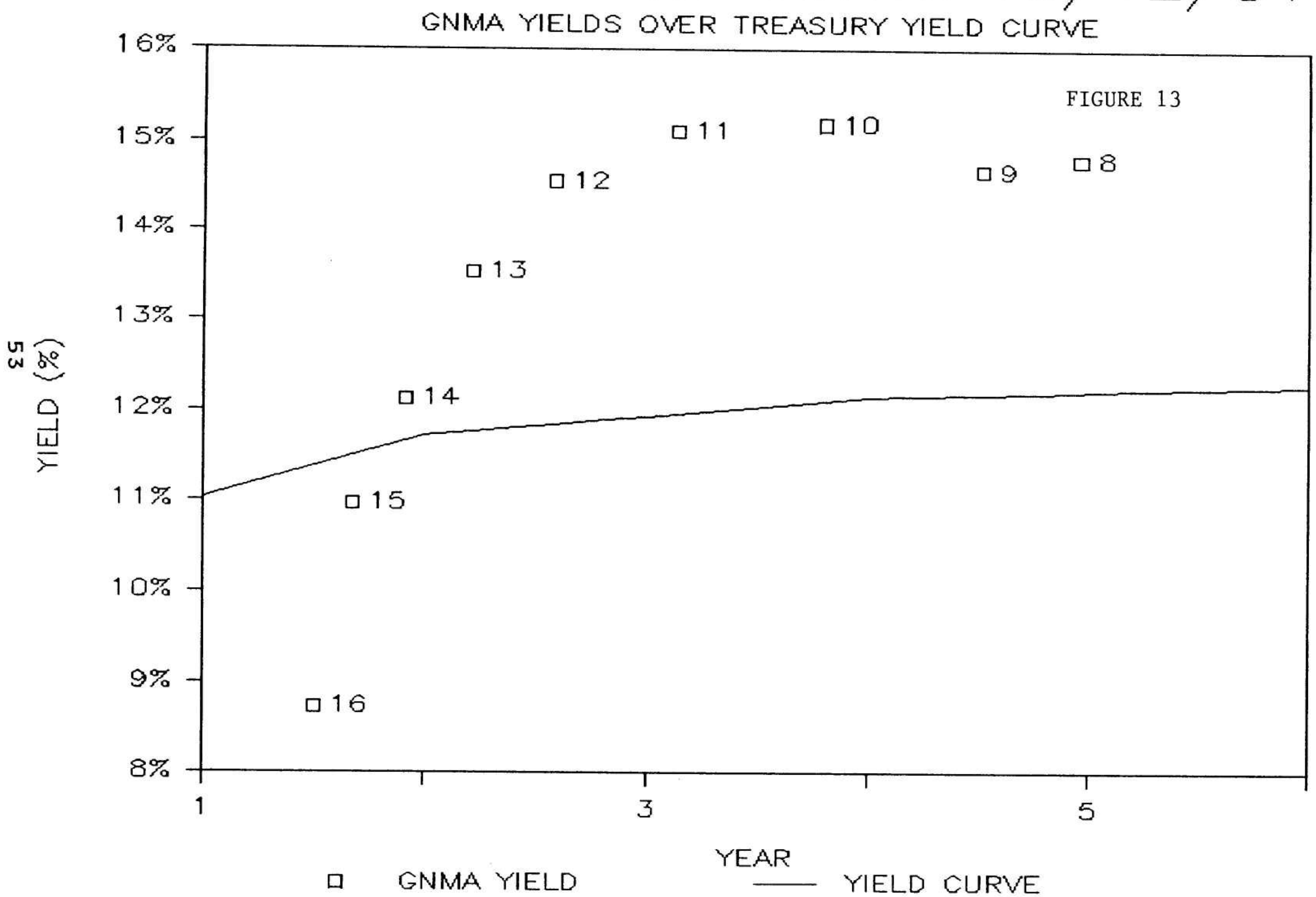

GNMA SIMULATION RESULTS
COMPARISON OF FHA WITH FHA + OPT REFIN
FIGURE 11
PRICE ($)
130
120
110
100
90
80
70
8
10
12
14
16
GNMA COUPON
MKT
FHA+OPT
FHA ONLY

Figure 12 HISTORICAL YIELD SPREADS OF GNMAs OVER TREASURIES
(reproduced from reference 3)



TREASURY YIELD CURVE 10/12/84



new mortgage. Table 8 shows the effect of increasing the points up to 5% of the mortgage principal balance. Since a higher upfront transactions cost makes it less attractive to refinance an existing mortgage, the prices of the discount GNMAs go down while the prices of the premium GNMAs increase.

Looking at the premium GNMA sector a little closer, I wanted to see what the effect of not assuming a 100% certainty of prepayment if it was optimal to refinance would do to the pricing simulation. I investigated at what probability would one have to assume the prepayment would occur at to match the current market price. I assumed a 5 point origination fee for refinancing and a 100 basis point yield spread in the market price. Table 9 shows that one would have to assume a 10% probability of optimal prepayment for the holder of a 14% GNMA. While this seems surprisingly low, looking at Table 6, the results for assuming FHA only prepayment show a 116 basis point spread. To lower this to 100 basis points implies that you want to lower the simulation price slightly so that the reduced discounting (by 16 basis points) will give you the market price of $103.03. To accomplish this requires only a small increase in prepayments due to optimal refinancing. The probabilities for the GNMA 15 and 16's show slightly higher probabilities. This is because the market prices already reflect a high prepayment assumption , so the simulation model assumptions on

TABLE 8

EFFECT OF ORIGINATION POINTS ON SIMULATION

PRICE ASSUMING FHA AND OPTIMAL REFINANCING

GNMA COUPON	2 POINTS	3 POINTS	5 POINTS
8	79.89	79.66	79.35
9	85.96	85.61	85.23
9.5	88.97	88.76	88.35
10	91.86	91.62	91.28
11	96.62	96.49	96.22
11.5	98.50	98.39	98.24
12	100.00	99.93	99.86
12.5	101.23	101.23	101.22
13	102.27	102.29	102.34
13.5	103.13	103.14	103.26
14	103.85	103.90	104.04
15	105.00	105.09	105.26
16	105.91	106.04	106.21

TABLE 9

ESTIMATION OF PROBABILITY OF OPTIMAL

REFINANCING OF PREMIUM GNMAs

GNMA COUPON: 14   MARKET PRICE: 103.03

PROBABILITY	SIMULATION PRICE
10%	103.27

GNMA COUPON: 15   MARKET PRICE: 105.84

PROBABILITY	SIMULATION PRICE
30%	105.92

GNMA COUPON: 16   MARKET PRICE: 109.78

PROBABILITY	SIMULATION PRICE
20%	109.81

ASSUMES: 5 POINTS OF ORIGINATION COSTS
100 BASIS POINT SPREAD OVER TREASURIES

optimal refinancing are more relevant to these securities.

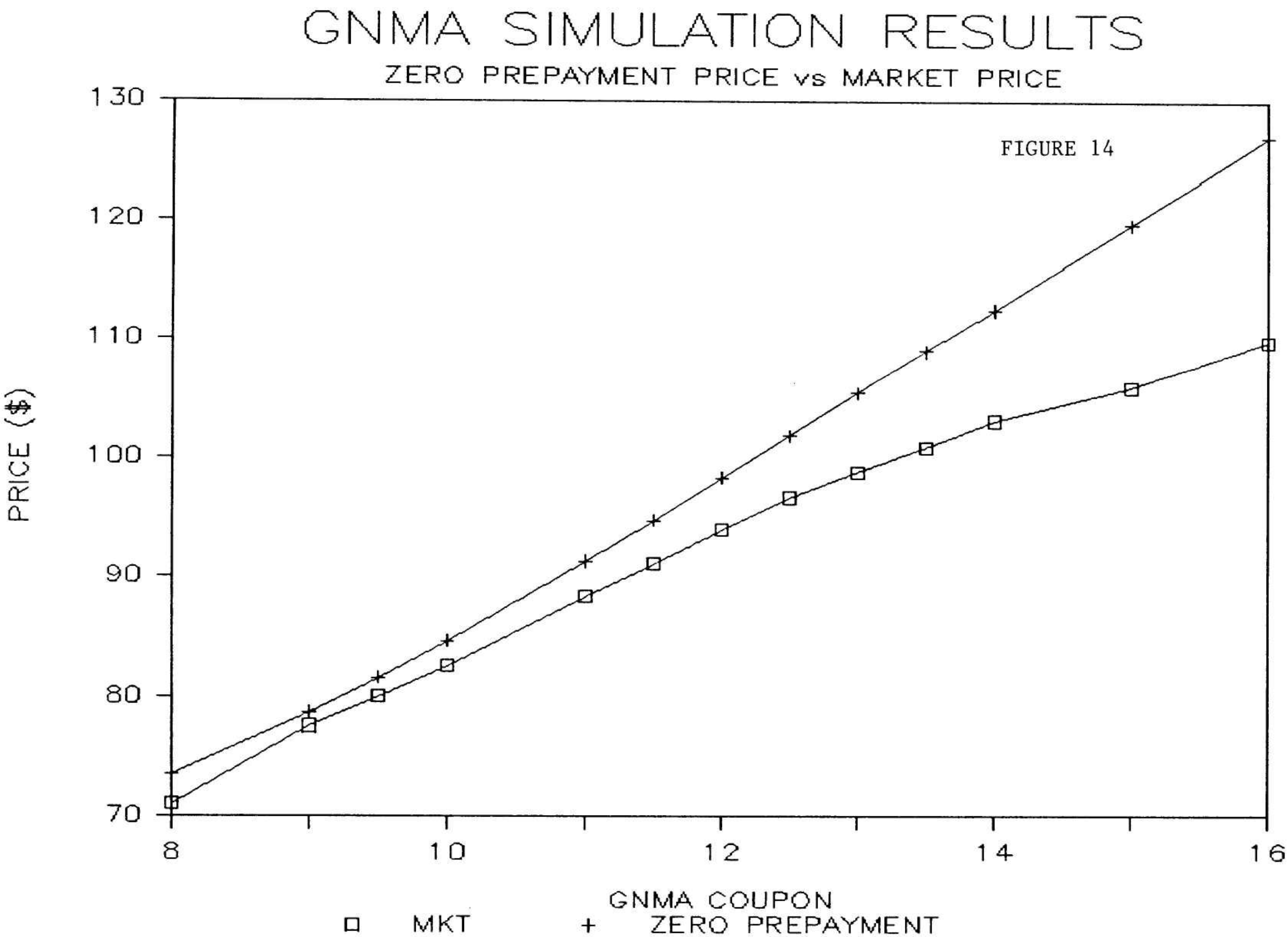
To address the question of the value of the compound option, Table 10 compares the market price with the present value of a cash flow stream if the mortgage had no call provisions. Figure 14 shows graphically that the difference between the non-callable mortgage and the market price increases as the coupon rate increases. This reflects the market's assessment that the premium GNMAs will be called early. It is interesting to note that there is a positive difference between the non-callable mortgage and the discount GNMAs. This implies that an investor could buy a discount GNMA and sell zero coupon treasuries matching the cash flows received from the GNMA to the cash outflows to serve the debt. By doing this, the investor can collect a positive profit from this transaction. The investor is exposed to a reinvestment risk if the GNMA prepays early since the investor has issued securities at the same rate as the GNMA coupon. If the GNMA prepays because it is optimal to refince the mortgage, then it is unlikely that the investor can receive a sufficient return on the prepaid principal to meet his cash outflow requirements in the future. This is why the difference between the non-callable mortgage and the GNMA increases with an increased coupon. This concept of using mortgages to fund the cash flow of bonds is the driver behind the issuance of collateralized mortgage obligations (CMO). A CMO has the added

TABLE 10

COMPARISON OF MARKET PRICE WITH

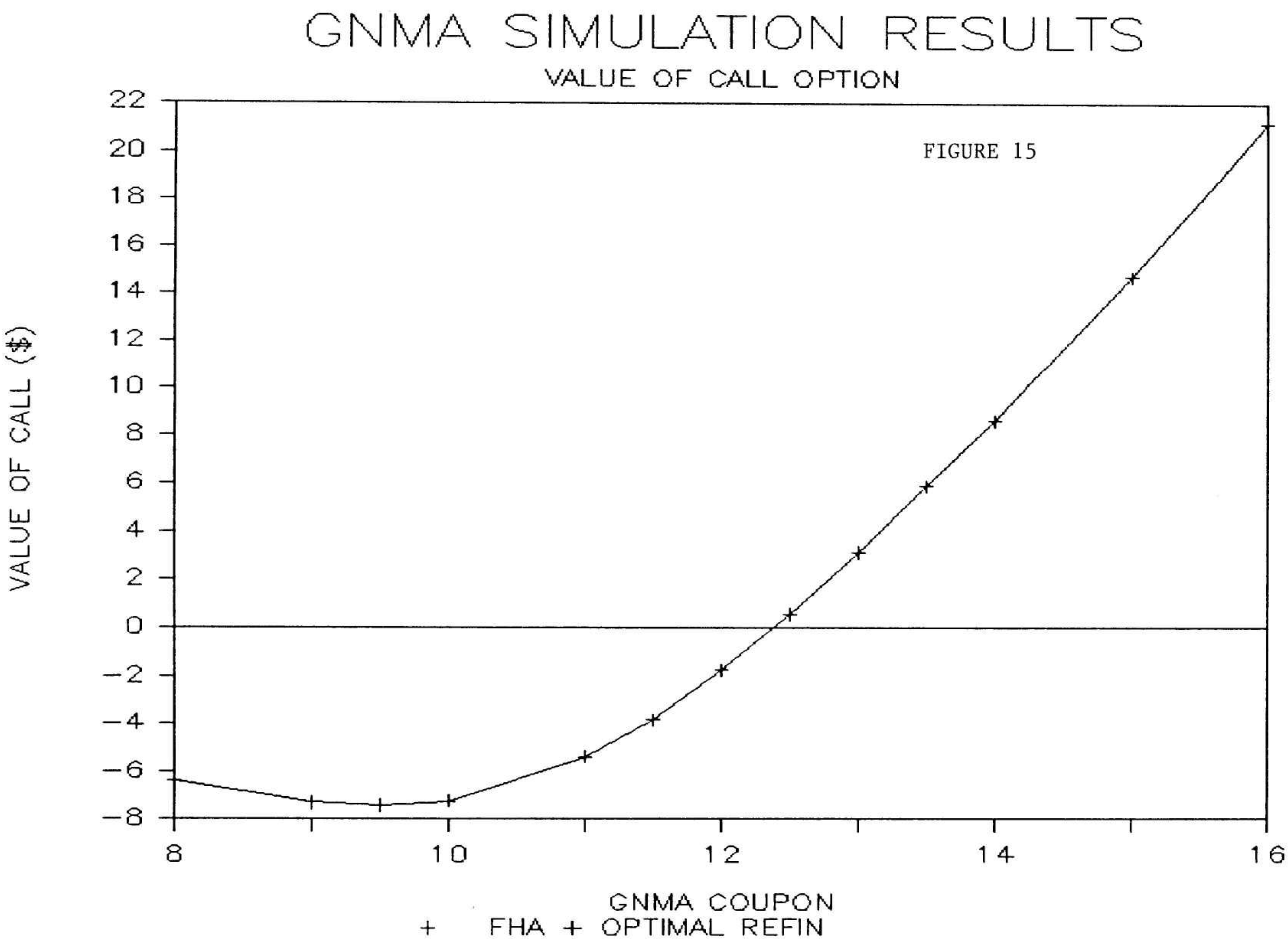
PRICE ASSUMING NO PREPAYMENT

GNMA COUPON	MARKET PRICE	PRICE WITH NO PREPAYMENT
8	71.09	73.50
9	77.59	78.68
9.5	80.00	81.55
10	82.53	84.60
11	88.31	91.23
11.5	91.03	94.67
12	93.91	98.24
12.5	96.59	101.78
13	98.75	105.38
13.5	100.81	109.01
14	103.03	112.45
15	105.84	119.72
16	109.78	127.08


GNMA SIMULATION RESULTS
ZERO PREPAYMENT PRICE vs MARKET PRICE
FIGURE 14
PRICE ($)
130
120
110
100
90
80
70
8
10
12
14
16
GNMA COUPON
MKT
ZERO PREPAYMENT

benefit of no reinvestment risk since all prepayments on the mortgage collateral are passed through to the holders of the shortest traunche in the CMO. [4] This transaction known as "CMO arbitrage" has been a major reason for the tremendous volume of CMOs that have been issued in the past year.

The value of the call option feature of the GNMA can be determined by looking at the difference between the price of the non-callable mortgage and the price derived from the simulation. This difference reflects the effect of the ability to exercise the option and prepay the principal early. This option is a compound option because it consists of a series of 29 european call options with each one expiring at the end of each year. The exercise of one option precludes the ability to expire another option at a later date. The value of this compound option is shown in Figure 15. For discount GNMAs, the exercise of the option is not optimal causing the price of the mortgage to increase. Therefore, the value is negative. For GNMAs selling above the current coupon, the option has a positive value reflecting the fact that it is possible to refinance the mortgage as a positive net present value project. This option value is a lower bound on the true value of the option because the simulation assumes the option is exercised immediately upon achieving a positive value. If a home owner expects interest rates to continue to fall in the future, he may not refinance immediately, but instead prefer to wait to lock in a more


GNMA SIMULATION RESULTS
VALUE OF CALL OPTION
FIGURE 15
VALUE OF CALL ($)
22
20
18
16
14
12
10
8
6
4
2
0
−2
−4
−6
−8
8
10
12
14
16
GNMA COUPON
+ FHA + OPTIMAL REFIN

advantageous rate. While it is true that it is not optimal to exercise an option and prepay a discount mortgage when a home owner moves, usually the ability to assume a low rate mortgage makes a house more attractive. This causes the selling price to increase allowing the home owner to collect the option premium as he sells the house and transfers the existing mortgage to a new owner.

Chapter Five

Conclusions

There have been many models proposed in the past for valuing a GNMA security. Most of these models have made assumptions about the future prepayment profiles to determine the future cash flows. Once knowing the cash flows, calculating yield and duration is straight forward. However, most of these models were developed in a period of stable interest rates where the opportunity to refinance an existing mortgage due to a drop in interest rates was very rare. Most prepayments were due to moving, disaster, or default. In today's market, with dramatic interest rate shifts, models for GNMA valuation have to address the call option feature imbedded in the mortgage. A home owner can exercise this option and prepay the existing outstanding principal balance when it is optimal to refinance his existing mortgage.

The goal of this thesis was to develop and test a model for improved estimation of prepayment profiles. The GNMA security can be viewed as a non-callable mortgage with a compound call option. The compound option consists of a series of twenty nine european options which allow the homeowner to call his loan at the end of each year in the mortgage term. The exercise of one of these options preclude the exercise of any option with a longer term to maturity. The complexity of the compound call option makes simulation the only practical way to approach the question of its valuation.

The model developed in this work allows for two kinds of prepayment. One class of prepayment is characterized as sub-optimal where the mortgage is called even though prevailing interest rates are higher than the interest rate of the existing mortgage. These sub-optimal prepayments are usually generated by moving, disaster, default, etc. In addition to sub-optimal prepayment, my model allows for optimal prepayment where interest rates have fallen sufficiently to make refinancing a positive net present value project. Interest rate dynamics are modeled using the Salomon Brother's implied forward rate model developed for valuing adjustable rate mortgages.

The results of the simulation show that discount GNMAs are priced using FHA prepayment statistics with a 110 basis point spread off of the treasury curve. However, in the current and premium coupon GNMAs, the value of the compound option becomes significant and static models for prepayment, such as FHA or twelve year average life, do not give good results. The model described herein assumes that a mortgage will be refinanced at the first instant that it becomes optimal. Transaction costs are included by assuming the new mortgagor pays a number of origination points upfront at the initiation of a new mortgage. In the premium GNMA sector, it became necessary to include a probability of optimal refinancing to reflect the fact that not all home owners refinance their mortgages at the first possible instant that it becomes optimal. This could be due to the perception on the home owners part that rates will drop further, thereby allowing him to lock in even a better rate when he refinances. Another possibility is that the home owner is not aware of the optimal refinancing opportunity, so he does not prepay his mortgage. While consumers are becoming more financially savvy, it may still take time for the home owner to become aware of optimal refinancing possibilities.

The results of this thesis demonstrate that the concept of my model for estimating prepayment is feasible. However, since GNMAs are priced at a positive spread to treasuries, there are some empirical constants which need to be tracked over time. If these constants are fairly stable, then this model can be successfully used for the valuation of GNMA securities.

Appendix A

Government National Mortgage Association

This Appendix is reproduced from Dexter Senft's paper published in "The Handbook of Fixed Income Securities." [1] The Government National Mortgage Association (GNMA) is a wholly owned U. S. government corporation within the Department of Housing and Urban Development. GNMA is authorized to guarantee the timely payment of principal and interest on securities issued by approved institutions and backed by pools of FHA-insured or VA-guaranteed mortgages. GNMA was created in 1968 as a spin-off from FNMA.

Nickname: GNMA I or "Ginnie Mae"

Originators: Mortgage bankers, savings and loan associations, savings banks, and commercial banks.

Servicer: The originator is responsible for servicing and otherwise administering all component mortgages in a pool.

Types of Mortgage Securities Issued: GNMA Single Family securities (SF), GNMA Graduated-Payment Mortgage securities (GPM), GNMA Mobile Home securities (MH), GNMA Project Loan securities (PL), GNMA Buydown securities (BD), GNMA Construction Loan securities (CL), GNMA Serial Note securities (SN).

Component Mortgages in General: All mortgages in GNMA pools must be insured under the National Housing Act, guaranteed under Title V of the Housing Act of 1949 or insured or guaranteed under the Servicemen's Readjustment Act of 1944 or Chapter 37 of Title 38, United States Code.

Payments

Frequency: Monthly

Made by: Originator direct to holder(s) for each pool

Record Date: 30th day of each month

Cutoff Date: 25th day of each month for prepayments

Prepayment Penalties: None

Interest on Prepayments: The holder is due a full 30 days interest at the coupon rate on the unpaid principal balance at the beginning of the month, regardless of any prepayments up to the cutoff date.

Payment Delay: Payments due on the 1st of the month are passed through on the 15th day of that month. Note that an investor who buys a GNMA on the 1st of a month does not receive any payments until the 15th of the following month. This is referred to as a 45 day payment delay.

Guarantees: Timely payment of interest and amortized principal is unconditionally guaranteed by GNMA. A decision of a U. S. assistant attorney general on 12/9/69 states that such guarantees constitute general obligations of the U. S. government and are backed by its full faith and credit.

Insurance: No pool or hazard insurance is used.

Minimum Pool Size: $1 million for SF and GPM securities, $.5 million for other types.

Form and Denomination: Fully registered, $25,000 minimum, $5,000 increments thereafter. For pools with odd principal amounts, one certificate per pool may reflect such odd amount, but this certificate must be for at least $25,000.

Transfer: Certificates are freely and fully transferable. Transfer agent is Chemical Bank in New York.

How Issued: Originator obtains commitment from GNMA to guarantee a pool of mortgages. Upon completion of such pool, a GNMA certificate is issued, and the originator is free to sell it.

Geographic Distribution: Narrow. Most pools contain 50 mortgages or fewer from one originator, so pools tend to be highly regionalized.

Comment: GNMA was the first pass-through issuer, is certainly the largest issuer, and offers the only mortgage securities that are government guaranteed.

GNMA Single-Family (SF) Pass-Throughs

Component Mortgages: Single-family, level payment loans insured by FHA or guaranteed by VA or the Farmers Home Administration (FmHA). VA guarantees must cover at least 25 percent of the property value. FmHA-guaranteed loans must be made under the Section 502 (Guaranteed Single-Family Rural Housing) Program, and only the fully guaranteed loan portions may be included. Each pool must contain at least 12 loans, and no loan may represent more than 10 percent of the original principal amount.

Maturity: Mortgages may have any maturity allowable under the various FHA, VA, and FmHA programs eligible for inclusion, but at least 90 percent of the loans must have a minimum of 20 years to maturity originally. Most GNMA SF pools contain 30-year mortgages. All loans must be pooled within one year of origination. Maturity of the pool is stated to be that of the longest component mortgage.

Interest Rate: Mortgages all carry interest rates 0.5 percent higher than the stated coupon rate on the security. From this .5 percent, 44 basis points go to the servicer as a servicing fee, and 6 basis points go to GNMA as a guarantee fee.

Yield Calculation: Assumes 30 year maturity, 12 year prepayment, 0.5 percent servicing fee, and 45 day delay.

Comment: GNMA I Single-families, often called regular GNMAs, are the premier mortgage security and constitute 80 percent of the pass-through market. In 1983, the GNMA II program was introduced. [19] This program takes advantage of many technological improvements that have emerged since the first GNMA I securities were issued. The key improvements include a central paying agent, Chemical Bank, and larger more geographically dispersed multiple issuer pools. The central paying agent provides investors with a single monthly check for all GNMA II holdings. The larger more diverse pools give investors improved prepayment consistency. GNMAs are highly liquid and are the only mortgage securities with an established futures market. Exchange traded options do exist, however, volume is low. Most options on GNMAs are written over the counter.

Appendix B

Simulation Results

GNMA SIMULATION RESULTS

GNMA COUPON: 8%
VOLATILITY: 12%

SIMULATION PRICE: $79.89
AVERAGE LIFE: 12.47 YEARS
YEARS SEASONED: 8

YEAR	TOTAL NUMBER TERMINATED	OPTIMAL REFINANCINGS
1	782	93
2	539	30
3	445	55
4	422	49
5	403	54
6	410	68
7	384	50
8	352	39
9	369	50
10	355	74
11	293	54
12	268	37
13	249	25
14	269	22
15	289	9
16	305	13
17	322	13
18	312	14
19	290	17
20	309	21
21	407	16
22	2226	0
23	0	0
24	0	0
25	0	0
26	0	0
27	0	0
28	0	0
29	0	0
30	0	0

Appendix C

Lotus Spreadsheet Results

GNMA PRICING ANALYSIS ASSUMING FHA PREPAYMENT ONLY

COUPON RATE	8.0%YRS SEASONED		8
EFFECTIVE MARGIN	152 BASIS POINTS		
MARKET PRICE	71.09 MARGIN PRICE		71.10
MODEL PRICE	77.07		
YIELD TO MATURITY	13.80%MOD DURATION		5.86

YEAR	TREASURY YIELD	MORTGAGE PAYMENT	MORTGAGE INTEREST	MORTGAGE PRINCIPAL	PREPAY % ORIG POOL	PRINCIPAL PREPAY	MORTGAGE CASH FLOW	DISCOUNTED CASH FLOW	EFF MARGIN DISC CF
0					0		-71.09		
1	11.04%	9.80	8.00	1.80	0.04101	4.03	13.83	12.46	12.29
2	12.49%	9.40	7.53	1.87	0.03070	2.95	12.36	9.89	9.63
3	12.44%	9.10	7.15	1.95	0.02574	2.42	11.52	8.20	7.88
4	12.95%	8.85	6.80	2.05	0.02364	2.17	11.02	6.95	6.58
5	12.58%	8.62	6.46	2.16	0.02293	2.05	10.67	5.97	5.58
6	12.58%	8.39	6.12	2.27	0.02247	1.95	10.34	5.14	4.74
7	12.95%	8.17	5.79	2.39	0.02173	1.82	9.99	4.40	4.01
8	12.95%	7.96	5.45	2.51	0.02102	1.70	9.66	3.76	3.38
9	12.95%	7.75	5.11	2.64	0.02014	1.56	9.31	3.21	2.85
10	12.00%	7.55	4.78	2.78	0.01915	1.41	8.97	2.76	2.42
11	12.00%	7.37	4.44	2.93	0.01809	1.27	8.63	2.38	2.05
12	12.00%	7.19	4.11	3.08	0.01699	1.12	8.31	2.04	1.74
13	12.00%	7.02	3.77	3.25	0.01590	0.97	8.00	1.75	1.47
14	12.00%	6.87	3.43	3.44	0.01756	0.99	7.86	1.54	1.27
15	12.00%	6.69	3.08	3.62	0.01889	0.96	7.66	1.34	1.09
16	12.00%	6.51	2.71	3.80	0.01987	0.90	7.41	1.16	0.93
17	12.00%	6.31	2.34	3.98	0.02050	0.80	7.12	0.99	0.79
18	12.00%	6.11	1.95	4.16	0.02078	0.67	6.79	0.84	0.66
19	12.00%	5.91	1.57	4.34	0.02072	0.52	6.43	0.71	0.55
20	11.00%	5.71	1.18	4.53	0.02036	0.36	6.06	0.61	0.46
21	11.00%	5.51	0.79	4.72	0.02617	0.24	5.75	0.52	0.39
22	11.00%	5.25	0.39	4.86	0.02638	.00	5.25	0.43	0.32
23	11.00%	0.00	.00	.00	0	0.00	0.00	0.00	0.00
24	11.00%	0.00	.00	.00	0	0.00	0.00	0.00	0.00
25	11.00%	0.00	.00	.00	0	0.00	0.00	0.00	0.00
26	11.00%	0.00	.00	.00	0	0.00	0.00	0.00	0.00
27	11.00%	0.00	.00	.00	0	0.00	0.00	0.00	0.00
28	11.00%	0.00	.00	.00	0	0.00	0.00	0.00	0.00
29	11.00%	0.00	.00	.00	0	0.00	0.00	0.00	0.00
30	12.00%	0.00	.00	.00	0	0.00	0.00	0.00	0.00

PRINCIPAL AMORTIZATION	69.120
PRINCIPAL PREPAYMENT	30.880
DISCOUNTED VALUE	77.0681955

COUPON RATE	8.0%	YRS SEASONED	8
EFFECTIVE MARGIN	258 BASIS POINTS		
MARKET PRICE	71.09	MARGIN PRICE	71.09
MODEL PRICE	79.89		
YIELD TO MATURITY	14.83%	MOD DURATION	4.96

YEAR	TREASURY YIELD	MORTGAGE PAYMENT	MORTGAGE INTEREST	MORTGAGE PRINCIPAL	PREPAY % ORIG POOL	PRINCIPAL PREPAY	MORTGAGE CASH FLOW	DISCOUNTED CASH FLOW	EFF MARGIN DISC CF
0					0		-71.09		
1	11.04%	9.80	8.00	1.80	0.07820	7.68	17.48	15.74	15.39
2	12.49%	9.04	7.24	1.80	0.05390	5.19	14.22	11.39	10.88
3	12.44%	8.51	6.68	1.83	0.04450	4.19	12.70	9.04	8.44
4	12.95%	8.07	6.20	1.87	0.04220	3.88	11.95	7.53	6.88
5	12.58%	7.66	5.74	1.92	0.04030	3.60	11.26	6.31	5.63
6	12.58%	7.26	5.30	1.96	0.04100	3.56	10.82	5.38	4.70
7	12.95%	6.86	4.86	2.00	0.03840	3.22	10.08	4.44	3.79
8	12.95%	6.48	4.44	2.04	0.03520	2.84	9.33	3.64	3.03
9	12.95%	6.14	4.05	2.09	0.03690	2.86	9.00	3.11	2.53
10	12.00%	5.78	3.65	2.12	0.03550	2.62	8.40	2.59	2.06
11	12.00%	5.43	3.27	2.16	0.02930	2.05	7.48	2.06	1.60
12	12.00%	5.14	2.94	2.21	0.02680	1.76	6.91	1.70	1.29
13	12.00%	4.88	2.62	2.26	0.02490	1.52	6.40	1.40	1.05
14	12.00%	4.64	2.32	2.32	0.02690	1.52	6.15	1.20	0.88
15	12.00%	4.37	2.01	2.36	0.02890	1.48	5.85	1.02	0.73
16	12.00%	4.09	1.70	2.39	0.03050	1.38	5.47	0.85	0.59
17	12.00%	3.79	1.40	2.39	0.03220	1.26	5.05	0.70	0.48
18	12.00%	3.47	1.11	2.36	0.03120	1.01	4.49	0.56	0.37
19	12.00%	3.17	0.84	2.33	0.02900	0.73	3.90	0.43	0.28
20	11.00%	2.88	0.59	2.29	0.03090	0.54	3.42	0.34	0.22
21	11.00%	2.58	0.37	2.21	0.04070	0.37	2.95	0.27	0.17
22	11.00%	2.18	0.16	2.02	0.22260	.00	2.18	0.18	0.11
23	11.00%	0.00	0.00	0.00	0.00000	0.00	0.00	0.00	0.00
24	11.00%	0.00	0.00	0.00	0.00000	0.00	0.00	0.00	0.00
25	11.00%	0.00	0.00	0.00	0.00000	0.00	0.00	0.00	0.00
26	11.00%	0.00	0.00	0.00	0.00000	0.00	0.00	0.00	0.00
27	11.00%	0.00	0.00	0.00	0.00000	0.00	0.00	0.00	0.00
28	11.00%	0.00	0.00	0.00	0.00000	0.00	0.00	0.00	0.00
29	11.00%	0.00	0.00	0.00	0.00000	0.00	0.00	0.00	0.00
30	12.00%	0.00	0.00	0.00	0.00000	0.00	0.00	0.00	0.00

PRINCIPAL AMORTIZATION	46.730
PRINCIPAL PREPAYMENT	53.270
DISCOUNTED VALUE	79.8890974

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