AN INVESTIGATION OF THE VARIATION OF THE HEAT TRANSFER COEFFICIENT IN NATURAL CONVECTION DUE TO BOUNDARY LAYER INTERACTION IN ADJACENT VERTICAL PLATES

by

John H. Sununu

Submitted in Partial Fulfillment

of the Requirements for the

Degree of Bachelor of Science

at the

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May, 1961

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May 22, 1961

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Chairman, Departmental Committee of Theses

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An experimental investigation of natural convection for vertical, parallel plates is made. It is found that there exist three basic regions for the free convective phenomena for adjacent plates, each region with a different dependence on the plate spacing b.

The first region occurs for the case where the plates are at a large distance from each other, i.e. for large values of the dimensionless parameter Gr b/L. This is the situation for virtually no boundary layer interaction. For this situation the heat transfer coefficient is

$$\frac{\bar{h}\, b}{k} = .560\ (\text{Gr}\ b/L)^{1/4}$$

which turns out (as expected) to be independent of b.

For the region of very small Gr b/L the equation for the heat transfer coefficient was found to be

$$\frac{\bar{h}\, b}{k} = .0370\ (\text{Gr}\ b/L)$$

This is the region where the boundary layers interfere and suppress the convective process.

The third region is the region for intermediate values of Gr b/L. No exact solution to this region quite applies since the dependence is gradually changing in character. However an

approximate solution which matches the data reasonably well is

$$\frac{\bar{h}\,b}{k} = .170\ (Gr\ b/L)^{1/2}$$

This information is then used to obtain a condition for optimization of the free convection heat transfer in enclosed vertical spaces. It was found that the optimum value of Gr b/L was of the order of 100.

Thesis Supervisor: Dr. Steven I. Freedman

Position: Instructor

ACKNOWLEDGEMENT

To Dr. S. I. Freedman, my advisor, I am greatly indebted for his guidance, and for his constructive criticisms aimed toward developing in me a better understanding and approach to the methods and problems of research.

To Nancy, my wife, I wish to express my appreciation for her assistance and her limitless patience and understanding.

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NOMENCLATURE

A	Area of heat transfer, area of plates
b	Distance between adjacent plates
c_p	Specific heat of fluid
C_1	Proportionality constant
C_2	Proportionality constant
C_3	Proportionality constant
F	View Factor
$\tilde{F}$	Black-surface over-all interchange factor
$\mathcal{F}$	Over-all interchange factor
g	Acceleration due to gravity
Gr	Grashof Number $\frac{g \beta \Delta T b^3}{\nu^2}$
Gr_L	Grashof Number $\frac{g \beta \Delta T L^3}{\nu^2}$
h	Coefficient of heat transfer
$\bar{h}$	Mean value of the coefficient of heat transfer
k	Thermal conductivity of fluid
L	Height of plates
Nu	Nusselt Number $\bar{h}b/k$
Nu_L	Nusselt Number $\bar{h}L/k$
Nu_x	Local Nusselt Number hb/k
p	Pressure
Pr	Prandtl Number ν/α

q	Rate of heat transfer
T	Temperature
T_∞	Temperature of ambient fluid
T_m	Mean Temperature of Plate
ΔT	Temperature difference, $T_m - T_\infty$
v	Velocity of fluid
W	Dimension available for fins
x	Coordinate along plate normal to the driving force
y	Coordinate perpendicular to the driving force

Greek Letters

α	Thermal diffusivity, $k/c_p\rho$
β	Coefficient of volumetric expansion
δ	Boundary layer thickness
ϵ	Emissivity
ν	Kinematic viscosity $= \mu/\rho$
ρ	Density of fluid
θ	Temperature difference, $T - T_\infty$
θ_w	Temperature difference, $T_{WALL} - T_\infty$
μ	Viscosity of fluid
η_L	Dimensionless parameter from reference (6), $^{81}/Gr\ b/L$

INTRODUCTION

Fluid motion which is caused solely by the density gradient created by temperature differences is called natural or free convection. In such flow heat is transferred from the surface of the object to the fluid layers in its neighborhood.

Laminar free convection on a vertical surface has been a subject of study since Lorenz (1)* published his pioneer paper in 1881. Since then, however, a major part of the work on natural convection over vertical plates has been confined to single vertical plates under a variety of prescribed wall conditions. An exact solution of the boundary-layer differential equations for free convection on a vertical flat plate with uniform wall temperature exists (2), and some excellent approximate solutions are also available. (A solution from Eckert (3) is reproduced for reference in Appendix I)

It can be shown, from dimensional considerations, that the heat transfer in free convection is a function of the Grashof number and the Prandtl number. Therefore, the dimensionless heat transfer coefficient, the Nusselt number, is then a function of these same quantities:

$$Nu = f\,(Gr,\ Pr)$$

One of the applications which has been given considerably less attention is that concerning enclosed spaces, e.g. the case of a number of parallel vertical plates in close proximity. For this situation it can be shown that if the plates are close enough so that there is interaction in the convective phenomena of adjacent plates, the distance between the plates as well as the plate height must be included in the analysis and therefore

$$Nu = f\,(Gr,\ Pr,\ b/L)$$

The purpose of this paper is to investigate the functional dependence of the heat transfer coefficient of heated vertical plates to the spacing between the plates. In the experimental model to be used the enclosing fluid will be air.

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*Numbers in brackets pertain to numbered references in the bibliography.

COORDINATE SYSTEM

The coordinate system appropriate to this analysis is shown below.

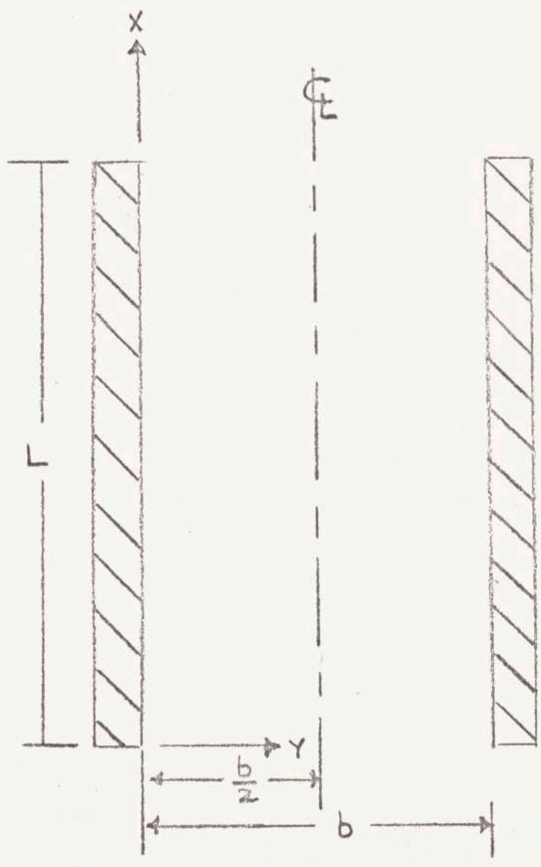


Figure I

THEORETICAL CONSIDERATIONS

For a single flat plate which is heated, the phenomena of laminar free convection can be explained as follows. As the plates are heated, the fluid in the immediate vicinity experiences an increase in temperature as a result of the heat transfer from the plate, and the fluid begins to rise vertically. In this manner a boundary layer is developed, with zero thickness at the lower edge and with increasing thickness in the upward direction.

If we have two adjacent plates, the effect is the same so long as the thickness of the boundary layer everywhere remains less than half the plate separation, i.e. so long as the two boundary layers never intersect.

An exact solution applicable to both the above situations can be found in reference (1). Also, an approximate solution using the relatively simpler integral technique is reproduced in Appendix I.

If instead of the case of non-interacting boundary layers we have the situation where they do meet, the problem becomes more involved. In fact an exact solution for this case is not available.

For the situation where the boundary layers do not interact we can derive the following expression for the boundary layer thickness.

$$\delta = 3.93 \left(\frac{\nu}{\alpha}\right)^{-1/2} \left(.952 + \nu/\alpha\right)^{1/4} \left(\frac{g\beta\theta_w}{\nu^2}\right)^{-1/4} x^{1/4}$$

We can make this expression dimensionless by dividing this by the plate spacing b, and noting that $\frac{\nu}{\alpha} = Pr$.

$$\frac{\delta}{b} = 3.93 \left(\frac{.952 + Pr}{Pr^2}\right)^{1/4} \left(\frac{g \beta \theta_w}{\nu^2}\right)^{-1/4} x^{1/4} \frac{1}{b}$$

By forming a Grashof number $\frac{g \beta \Delta T b^3}{\nu^2}$ and introducing L, the height of the plate we can obtain the following:

$$\frac{\delta}{b} = 3.93 \left(\frac{.952 + Pr}{Pr^2}\right)^{1/4} \left(Gr \frac{b}{L}\right)^{-1/4} \left(\frac{x}{L}\right)^{1/4}$$

It can be shown that

$h = 2k/\delta$ (see reference (3), pg. 315)

If we use this relationship and form a Nusselt number based on the plate spacing

$$Nu_x = \frac{hb}{k} = 2\, b/\delta$$

We can show that for the local Nusselt number

$$Nu_x = .508 \left(\frac{Pr^2}{.952 + Pr}\right)^{1/4} \cdot (Gr\ b/L)^{1/4} \left(\frac{L}{x}\right)^{1/4}$$

By integrating this expression over the length of the plate we can show that for non-interacting boundary layers the average value of the Nusselt number is 4/3 the local value at the top of the plates.

For air with a Prandtl number Pr = .714

$$Nu = .504 \left(Gr \frac{b}{L}\right)^{1/4}$$

This is valid for all situations where there is no boundary layer interaction, i.e, for values of Gr b/L which fulfill the condition.

$$b \geq 2\delta_{MAX}.$$

Using this criteria in the equation for the dimensionless boundary layer thickness we find that this expression for the average value of the Nusselt number is valid for

$$Gr\, b/L \geq 5000$$

It should be pointed out here that the parameter b shows up in the equations only because of the way we defined our functions. By writing these equations out

$$\bar{h}b/k = .504 \left(\frac{g \beta \Delta T b^3}{\nu^2} \frac{b}{L}\right)^{1/4}$$

and rearrange a bit

$$\bar{h}b/k = .504 \left(\frac{g \beta \Delta T}{\nu^2 L}\right)^{1/4} b$$

it is obvious that b can be cancelled out and that L is the important parameter. Note also that by multiplying both sides by L we obtain the standard form for the Nusselt number of a flat plate in free convection.

$$\frac{\bar{h}L}{k} = .504 \left(\frac{g \beta \Delta T L^3}{\nu^2} \right)^{1/4}$$

or

$$Nu_L = f\,(Gr_L)$$

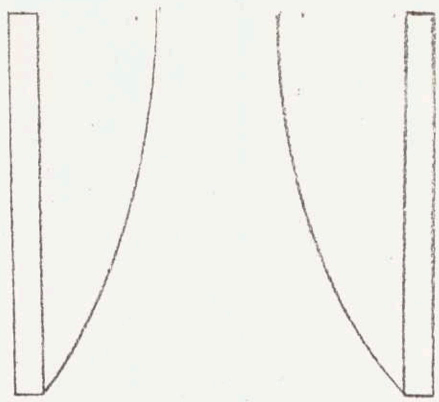
As noted previously, if there is interaction, the situation is much more complicated. The purpose of this paper is to analyze the situation emperically and arrive at a practical emperical equation which will describe the phenomena.

From a physical interpretation of the phenomena it is expected that in the region where there is interaction the Nusselt number will still depend on the same parameters with a particular dependence on the ratio b/L.......

$$Nu = f\,(Pr, Gr, b/L) = f\,(Pr, Gr\, b/L)$$

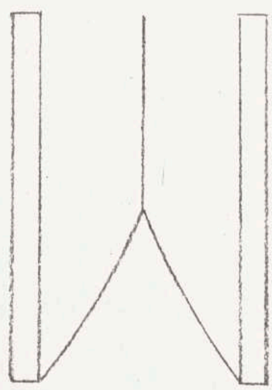
and for the case of constant Prandtl number

$$Nu = f\,(Gr\, b/L)$$



Non-interacting adjacent boundary layers

Adjacent boundary layers with complete interaction



Intermediate condition

Figure II

CONSIDERATION OF RADIATION EFFECTS

In order to properly investigate the phenomena of convection, it is important to correctly separate the effects of the convective process from the simultaneous process of radiation.

There are two choices presented for making this correction. One can either directly measure the radiation or one can calculate it, using the well known relationships of radiant heat transmission. Unfortunately the proper measuring techniques call for some relatively elaborate equipment which were not available for this investigation. Therefore, the method employed here was to calculate the radiation for each condition examined.

To perform these calculations it is necessary to know two fundamental parameters: the view factor F, and the emissivity ϵ Using these we can calculate the term

$$\mathcal{F} = f(\epsilon, F)$$

and use this in the radiation equation

$$q = A \mathcal{F} \sigma (T_1^4 - T_0^4)$$

The view factor of interest for our geometry of vertical plates is that which measures the fraction of the radiation leaving the reference plate which is intercepted by the cooler surroundings. This F_{10} is $1 - F_{12}$ where F_{12} is the radiation which is intercepted by the adjacent plates. If these adjacent plates were not held at the same temperature as the reference plate there would be some heat transfer q' where

$$q' = f(F_{12}, \epsilon, T_1, T_2)$$

Since however all plates were held at the same temperature the only radiant heat transmission from the reference plate, q, is given by the previously cited relationship

$$q = A \mathcal{F} \sigma (T_1^4 - T_0^4)$$

where

$$\mathcal{F} = \frac{1}{(\frac{1}{\epsilon} - 1) + \frac{1}{\tilde{F}_{10}}}$$

and

$$\tilde{F}_{10} = F_{10} + F_{12} F_{10}$$

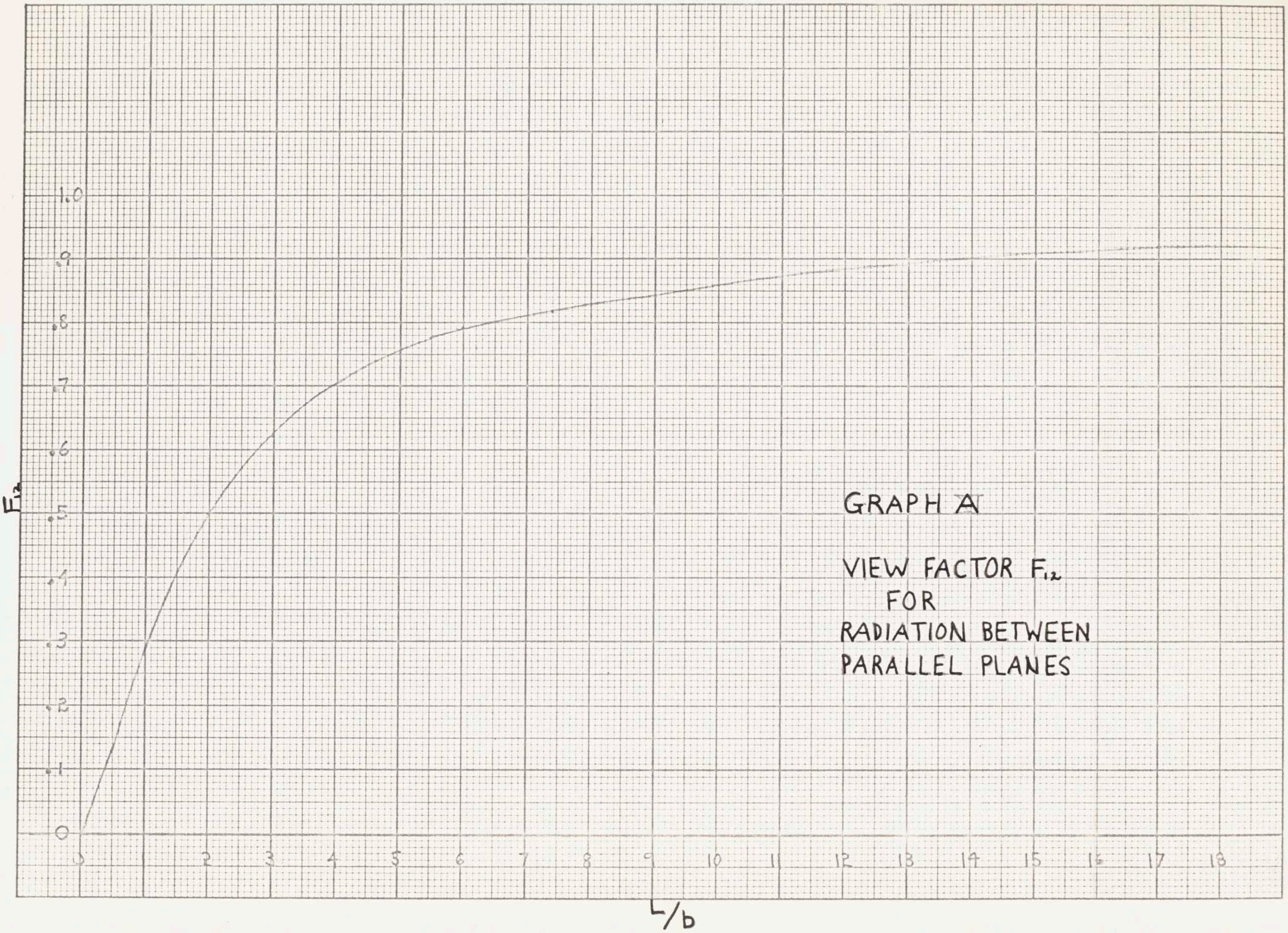
For any geometry employed the calculation of F_{12} involves a lengthy algebraic derivation. This calculation has been performed, however, for the particular geometry used, i.e., for 2:1 parallel rectangular planes, and has been tablulated in McAdams (4). Using these results a plot of F_{12} as a function of plate separation is plotted graphically on the following page. Also plotted is the curve $1-F_{12}$ as a function of plate separation.

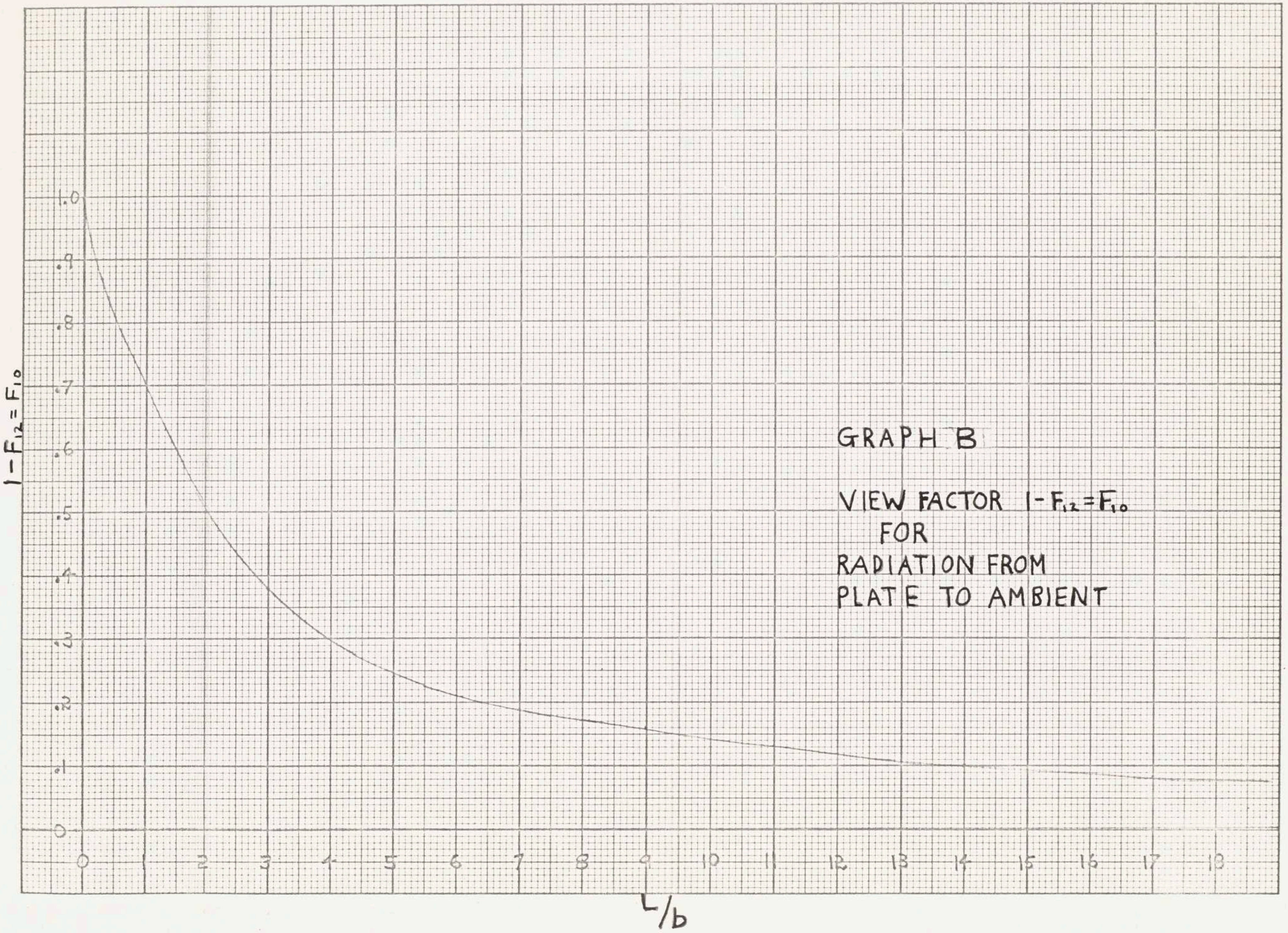
A more involved process was the calculation of the plate emissivity. In order to minimize the effect of any error resulting from the uncertainty of this parameter, different surface finishes with different emissivities were examined and the results of the separate convective evaluations were compared. A single vertical, heated plate of a given finish on the sides was allowed to stand until well past the time necessary to achieve steady state. The edges of the plate were painted with a heavy black lacquer of a known emissivity of .97.

The temperature and heat dissipated by the plate were noted and then, using the correlated data of previous investigations of free convection the heat given off in free convection was calculated.

Also the heat lost to radiation from the edges was calculated.

Both these terms, the heat dissipated in convection and by radiation from the edges were subtracted from the total dissipation. Then the remaining power was used to calculate the emissivity of the sides.


GRAPH A
VIEW FACTOR F_{12}
FOR
RADIATION BETWEEN
PARALLEL PLANES
F_{12}
L/b


GRAPH B
VIEW FACTOR 1-F12=F10
FOR
RADIATION FROM
PLATE TO AMBIENT
1-F12=F10
L/b
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18

All the tests run, at a number of power levels, compared very favorably within the limits of experimental error.

Once ϵ and F were known the calculation of heat lost due to radiation was relatively easy. It is of interest to know how important any error in the value of ϵ is in the final calculation. Some typi cal evaluations are shown in Appendix II.

TEST APPARATUS

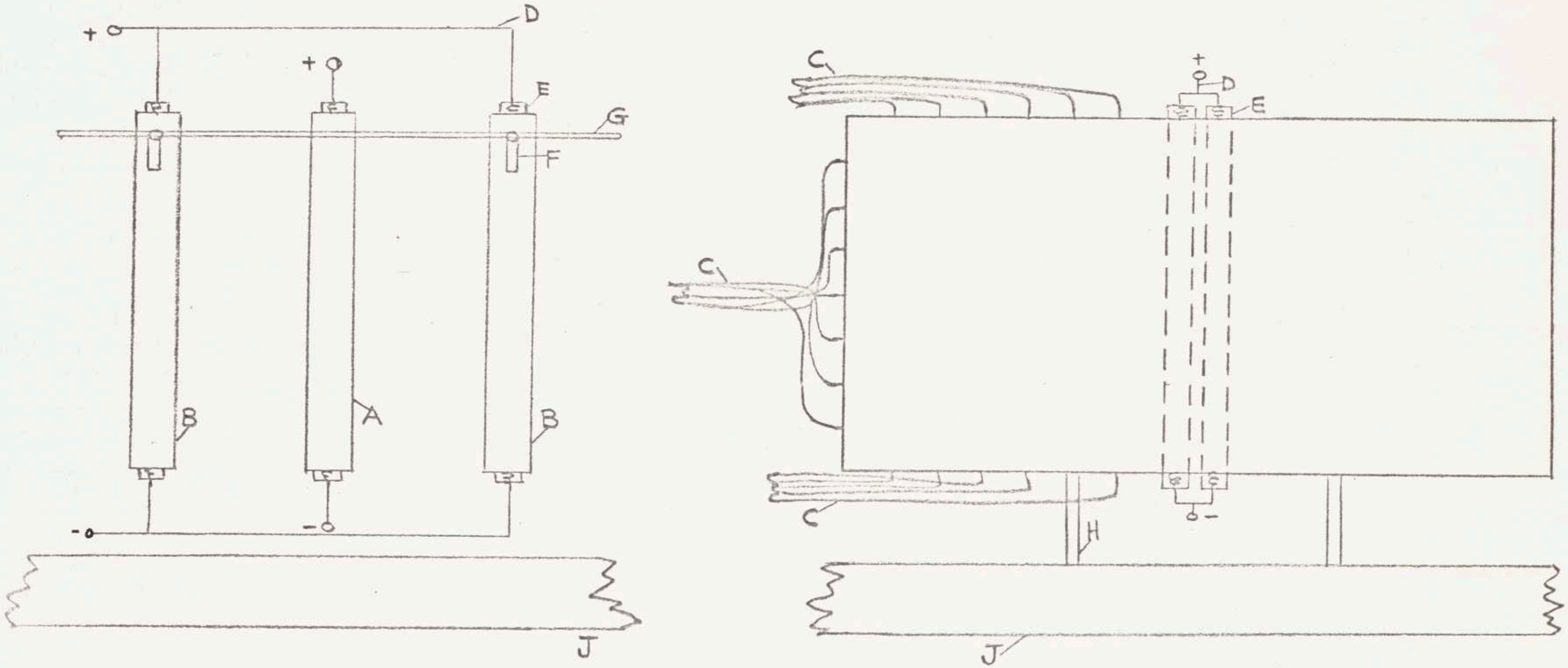
In the preceeding sections it has been noted that Nu = f (Gr, Pr, b/L). For the studies proposed here the Prandtl number of the enclosing fluid (air) may be considered to remain constant. Therefore we may say that Nu = f (Gr, b/L). The apparatus must be designed to permit the variation of these parameters and to measure the response of the system to these changes.

The test section as shown in the drawing on the following page, consisted of three parallel, vertical plates. The two outside plates were made of aluminum. The center plate, which may be referred to as the reference plate, was made of copper. All three plates had the same dimensions of 8" x 4" x 1/4".

At either end of each outside plate there was fastened a small, hooked metal "slider". These sliders were thermally insulated from the plates by means of a pair of phenolic fiber washers. These fixtures permitted the outside plates to slide on a pair of stainless steel rods and allowed the plate spacing to be adjusted to any value desired. This distance and the parallel alignment of the plates was measured by means of a vernier micrometer.

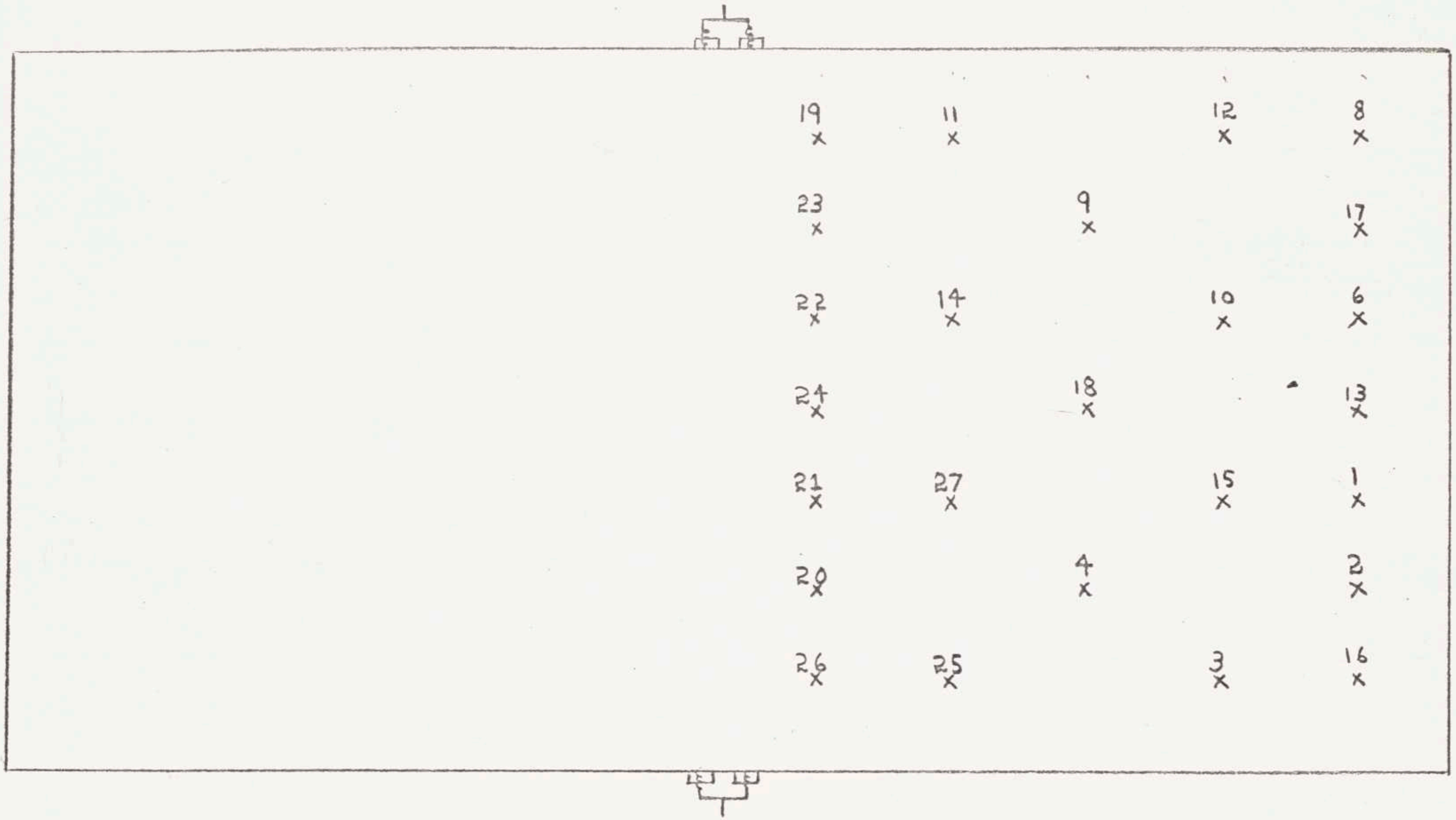
The center plate was monitored by means of twenty-five copper-constantan thermocouples imbedded in a network of shallow channels.

A Reference Plate
B Moveable Plate
C Thermocouples
D Heating Coils
E Glass Insulating Tubing
F Slider Hooks
G Stainless Steel Support Rods
H Reference Plate Support
J Plywood Stand



TEST SECTION

Figure III



THERMOCOUPLE LOCATIONS IN REFERENCE PLATE

Figure IX

These channels were then covered with a high conductivity, silver-aluminum epoxy and the surface then ground smooth. This eliminated as much as possible any variation of the temperature profile due to disturbance of the plate geometry. The outside plates were monitored by two thermocouples each. These thermocouples were inserted from the outer side of the plates to prevent distrubing the flow in the interior regions.

All three plates were heated by means of a pair of nichrome heating coils wound from #32 wire. These elements were encased in glass tubing and inserted in the center of the plates as shown in the schematic. Care was taken to wind the coils so that the electrical resistance of each was essentially the same. This was controlled by winding the coils longer than necessary and cutting them to the desired lengths after comparing their electrical characteristics at various power levels.

The reason copper was chosen for the center plate was to reduce the thermal gradient in the reference section of the system, i.e, to maintain as closely as possible an isothermal plate. It should be noted also that this minimization of the thermal gradient and the symmetry about the heating elements justify the monitoring of only half of the plate.

The two coils in the reference plate were connected in a parallel circuit and the power through them was regulated by one of the two power supplies available. The four coils in the outer plates were all connected in parallel and driven by the second power supply. By this arrangement the power or temperature of the reference plate could be controlled independently of the outer two.

The surface finish of the copper plate was varied. It consisted in turn of either the bare copper-epoxy surface, a black painted finish, and finally an aluminum surface.

This aluminum-on-copper surface was achieved by coating the surface of the plate with an extremely thin layer of the high conductivity epoxy and "ironing on" a piece of alloy aluminum foil which had been cut to size. When the epoxy set, the surface had adhered to the copper extraordinarily well. The thermal resistance of the epoxy layer was estimated to be of the order of 1/200 of a degree Farenheit per BTU/hr. or quite negligible relative to the parameters measured.

The thermocouple response was measured on a millivolt potentiometer. The responses were measured directly and at times, to make the procedure easier, through a rotary switch. There was no measurable variation between the two methods.

A tabulation of all the apparatus used can be found in Appendix III, List of Equipment.

EXPERIMENTAL PROCEDURE

The fundamental experimental procedure involved in this investigation was to monitor the temperature of a reference plate as a function of both the heat given off by the plate in free convection and its distance from two adjacent, parallel plates held at the same temperature as the reference plate.

A total of three plates (rather than two, as was the practice in some preceeding investigations (5))were used in order to make the reference plate more sensitive to the spacing and to be able to correlate the data with a minimum of corrections for extraneous effects. The temperature of the center plate, which will often be referred to as the reference plate, was monitored by means of twenty-five thermocouples. The outside plate temperature was monitored by means of four thermocouples. The ambient temperature was read on a mercury thermometer and was cross checked by means of a shielded thermocouple placed in the free air.

The process of taking the data was basically quite simple. It consisted of the following steps:

1. Adjusting the plate spacing on both sides of the reference plate to the desired value.
2. Heating the center plate and the outside plates.

3. Taking a preliminary set of temperature readings to check if the outside plates were at the same temperature as the reference plate.

4. Adjusting the power into the outside plates to correct any temperature difference found in the preceeding step.

5. Taking the ambient temperature and noting the total power into the reference plate.

6. Taking the complete set of temperature readings for the plates.

7. Calculating the corrections for the power lost to radiation, etc.

8. Reducing the data to a meaningful form.

In order to minimize any errors which may arise due to procedure, two basis cycling techniques were applied. The first consisted of making a set of runs at a constant spacing and allowing the power input to vary. And the second consisted of operating at a constant power level and allowing the spacing to vary.

By plotting the temperature of the reference plate as a function of time it was found that forty-five minutes were more than sufficient waiting time to allow the system to come to steady state.

The losses of the system through the insulation to supports was checked and found to be much less than any of the experimental errors.

We should also note here that the temperature gradient which existed in the copper plate was, at worst conditions, approximately 6% of the mean temperature difference from the copper plate to ambient. That is to say, the temperature of the plate was everywhere within approximately ±3% of the mean temperature.

The gradient in the aluminum plate was higher, but the temperatures were still within ±5% of the mean temperature. Due to this difference in the gradients it was obviously impossible to keep the plates at exactly the same temperature, point for point. It was, however, relatively easy to keep the mean temperatures the same and the point by point temperatures varied by no more than ±3%.

The corrections to the power dissipated consisted of subtracting the following terms from the net power in:

1. The heat given off by radiation from the sides of the plates.
2. The heat given off by radiation from the ends of the plates.
3. The heat given off by free convection from the ends of the plates.

All three of the above dissipations were extraneous to this investigation. The only power of interest was that which remained after these corrections, i.e. the heat dissipated by free convection from the sides facing adjacent plates. This is the power which is used in reducing the data in the following sections.

PRESENTATION AND DISCUSSION OF RESULTS

Since we have established that in free convection for adjacent parallel plates the Nusselt number is a function of the Grashof number and the ratio b/L, the most obvious and informative presentation of the data would be a curve showing the variation of Nu vs. Gr b/L. This has been done and is shown on Graph I. From this graph it should be possible to obtain a functional relationship between the parameters.

This curve is in very close agreement with that obtained by W. Elenbaas (5). Also the shape of the curve is similar to the relationship obtained by W. Hinkle (6) using an approximate integral technique. Hinkle's equation relating Nu to Gr b/L may be written

$$Nu = (A - B\, e^{-C\eta_L} + D\, e^{-E\eta_L})\ Gr\, b/L$$

where

$$\eta_L = 81 / Gr\, b/L$$

where A, B, C, D, and E are constants to be determined emperically. A relationship of this kind is very awkward to be used in any practical application. Therefore, rather than adjust the constants a simpler, approximate solution will be sought.

From Graph I it can be seen that for high values of Gr b/L the curve does approach the 1/4 power dependence predicted. There is disagreement in the value of the proportionality constant. From the graph

$$Nu = .560 \ (Gr\ b/L)^{1/4}$$

instead of

$$Nu = .504 \ (Gr\ b/L)^{1/4}$$

as predicted. Also the limit for the validity of this equation occurs at a lower value than expected. Emperically, the 1/4 power dependence extends to Grashof numbers as low as 150 rather than merely to 5×10^3.

These discrepancies are probably due to the fact that our constants were established by using results from the approximate solution. Although the form of the solution can be predicted reasonably well, such a variation in the constants must be expected when applying the integral technique.

In the region of the curve where Gr b/L is small the log-log relationship again assumes a straight line character and it can be seen that

$$Nu = \text{Constant} \ \ Gr\ b/L \ = .037 \ \ Gr\ b/L$$

This behavior applies for Gr b/L smaller than approximately 25.

Also, the value of the constant is found to be .037.

In the region between the two extremes of the curve, the character changes gradually from the linear dependence to the 1/4 power dependence. As a first approximation we can assume a 1/2 power dependence in this region. The resulting approximate curve for this region is shown as a dotted line on Graph I. This fits the data well within 10%.

The table below summarizes these results.

Gr b/L	Nu
$Gr\ b/L < 25$	$Nu = .037\ Gr\ b/L$
$25 < Gr\ b/L < 150$	$Nu = .170\ (Gr\ b/L)^{1/2}$
$150 < Gr\ b/L$	$Nu = .560\ (Gr\ b/L)^{1/4}$

The physical phenomena behind the different behavior of these regions may be reasonably explained in the following manner.

At high values of Gr b/L, where the dependence is the 1/4 power equation, we have the situation of non-interacting boundary layers. This was discussed at length in the chapter on Theoretical Considerations. In this region the effective boundary layer thickness is everywhere less than 1/2 the plate spacing and the heat transfer is independent of the spacing.

$$\frac{\bar{h}b}{k} = C_3 \left(\frac{g \beta \Delta T\, b^3}{\nu^2} \frac{b}{L}\right)^{1/4}$$

$$\bar{h} = C_3 \left(\frac{g \beta \Delta T}{\nu^2 L}\right)^{1/4} k$$

In the region of linear dependence, at low values of the Grashof number, the free convection is suppressed. This region represents the situation when the boundary layers interact almost immediately. This means that δ , the boundary layer thickness becomes greater than 1/2 the plate spacing just above the leading edge and therefore meets the boundary layer of the adjacent plate. The combined layer then virtually fills the enclosed space and for a major part of the distance along the plates the center line temperature may be well above the actual ambient temperature of the fluid.

Looking at the equations for the heat transfer coefficient we find a strong dependence on the plate spacing.

$$\frac{\bar{h}\, b}{k} = C_1 \left(\frac{g \beta \Delta T\, b^4}{\nu^2 L}\right)$$

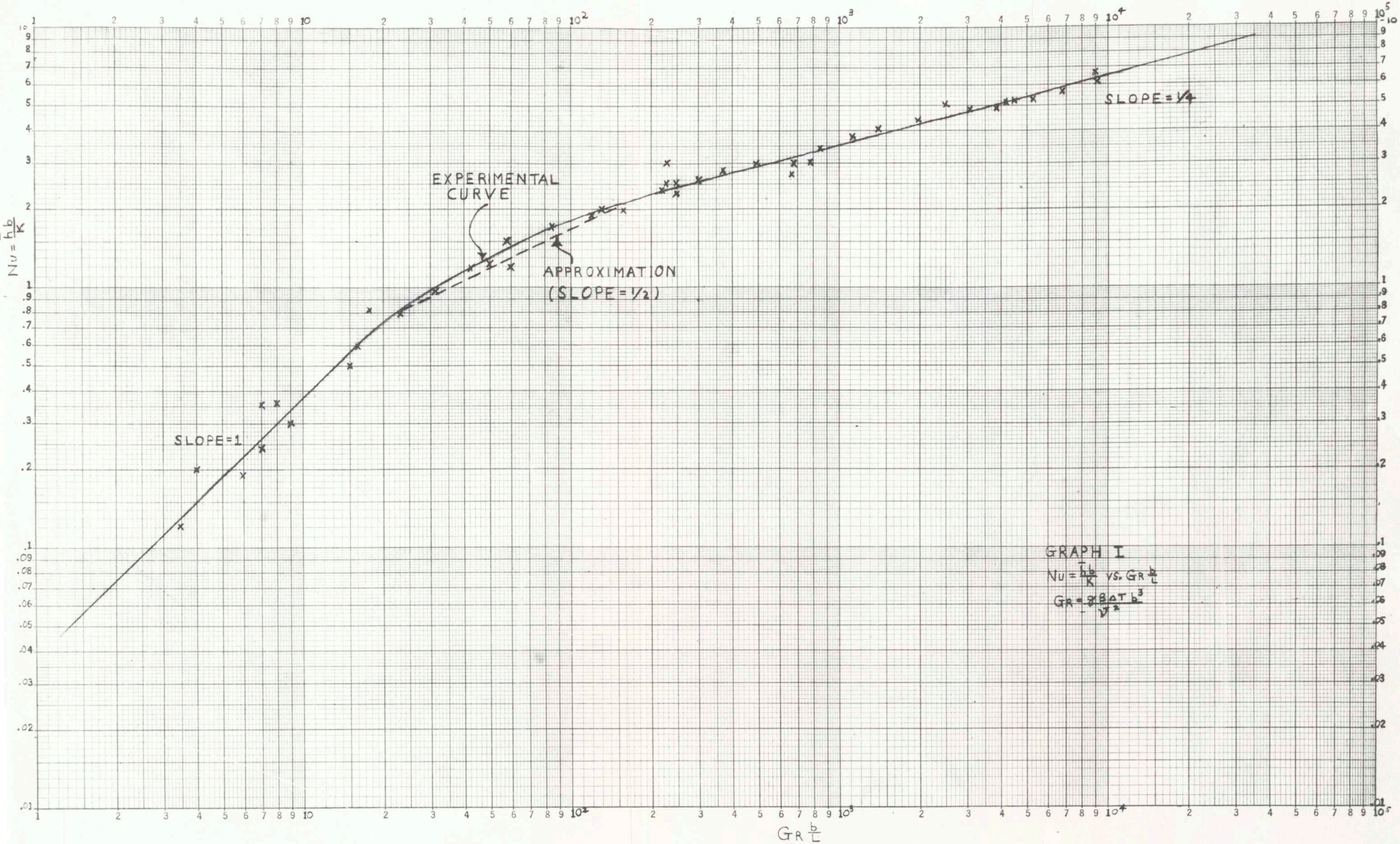
$$\bar{h} = C_1\, k \left(\frac{g \beta \Delta T}{\nu^2 L}\right) b^3$$

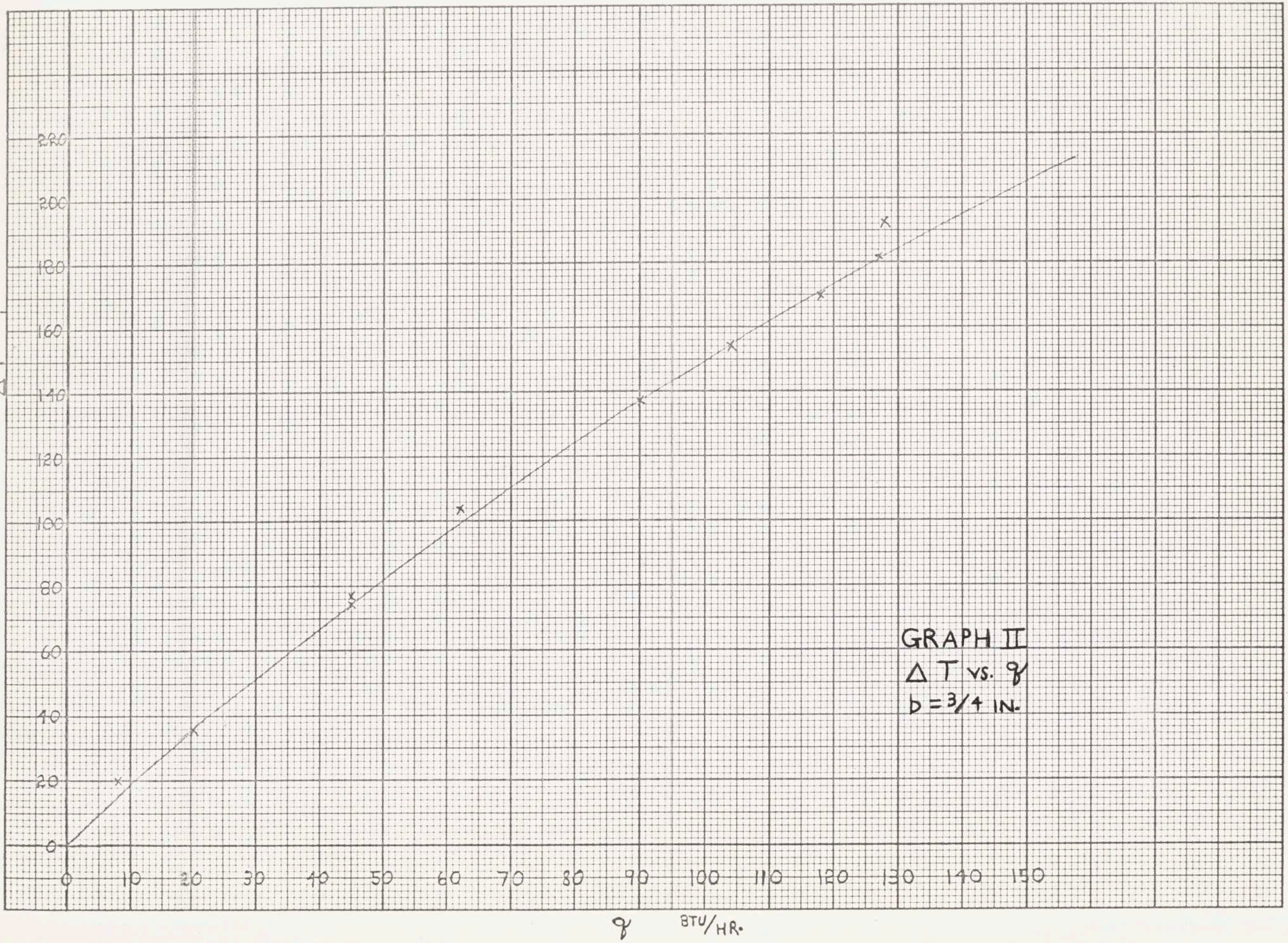
In fact the heat transfer coefficient is proportional to the spacing cubed.

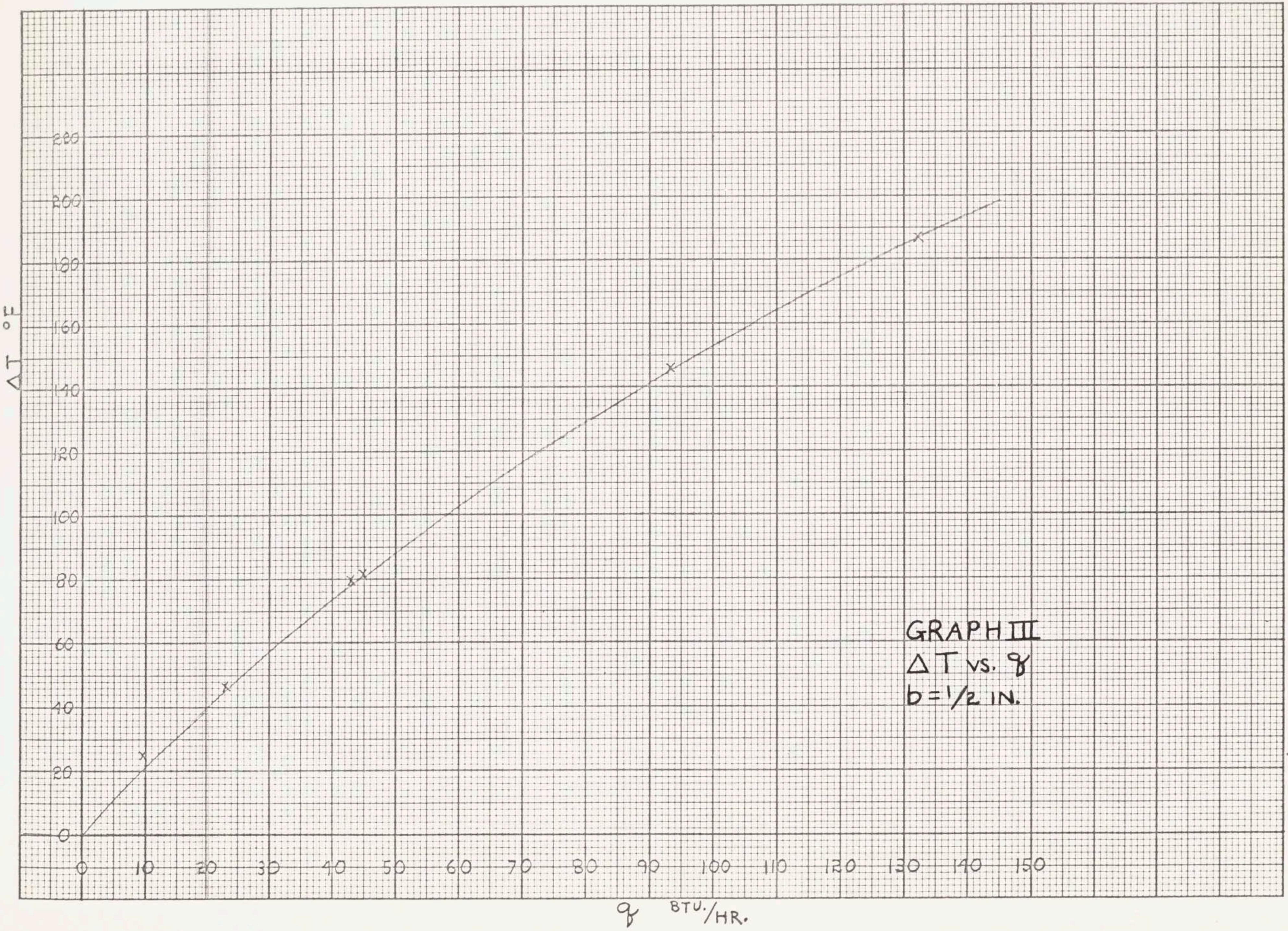
The region in between these extremes is the region between the situation of complete interaction and no interaction. That is, the transition region is the case where the boundary layers interact part of the way up the channel. This corresponds to the gradual change in slope of the curve, which indicates that the length of the region of interference is gradually changing.

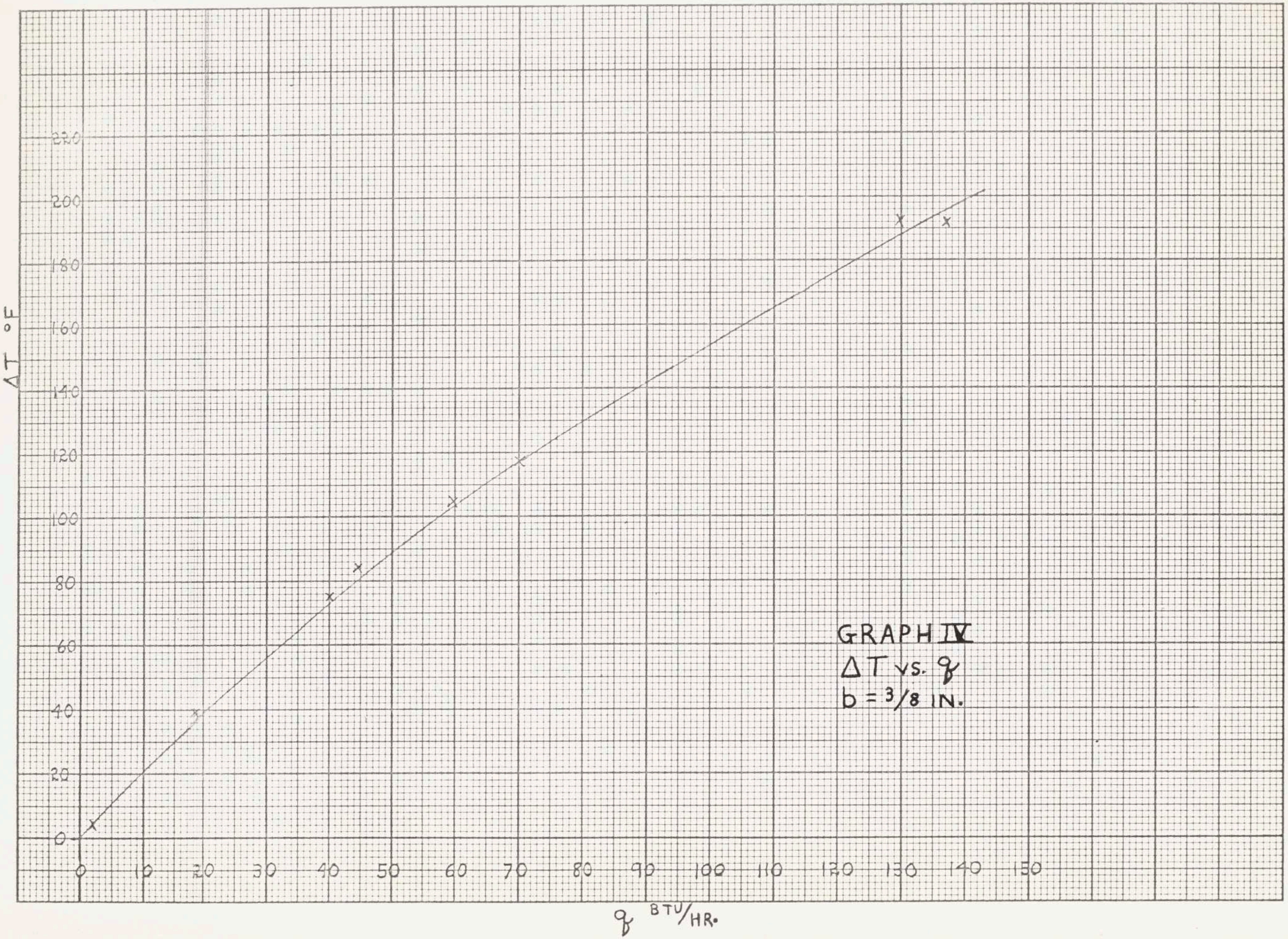
Taking this into consideration a better solution for the heat transfer coefficient in this region (relative to the first approximate equation of $\frac{hb}{k} = C_2 \left(\left(\frac{g\beta\Delta T b^4}{\nu^2 L} \right)^{1/2} \right)$ can be obtained by doing this analytical computation in stages. The thickness of the boundary layer along the plates can be calculated and the points of intersection found. Then for the region below the point of intersection, the equation for no interaction may be used and for the region above, the equation for complete interaction. The two results then may be suitably averaged to give the net heat transfer coefficient. Although this would still be an approximation it should give a result slightly closer to the actual curve than the initial 1/2 power approximation.

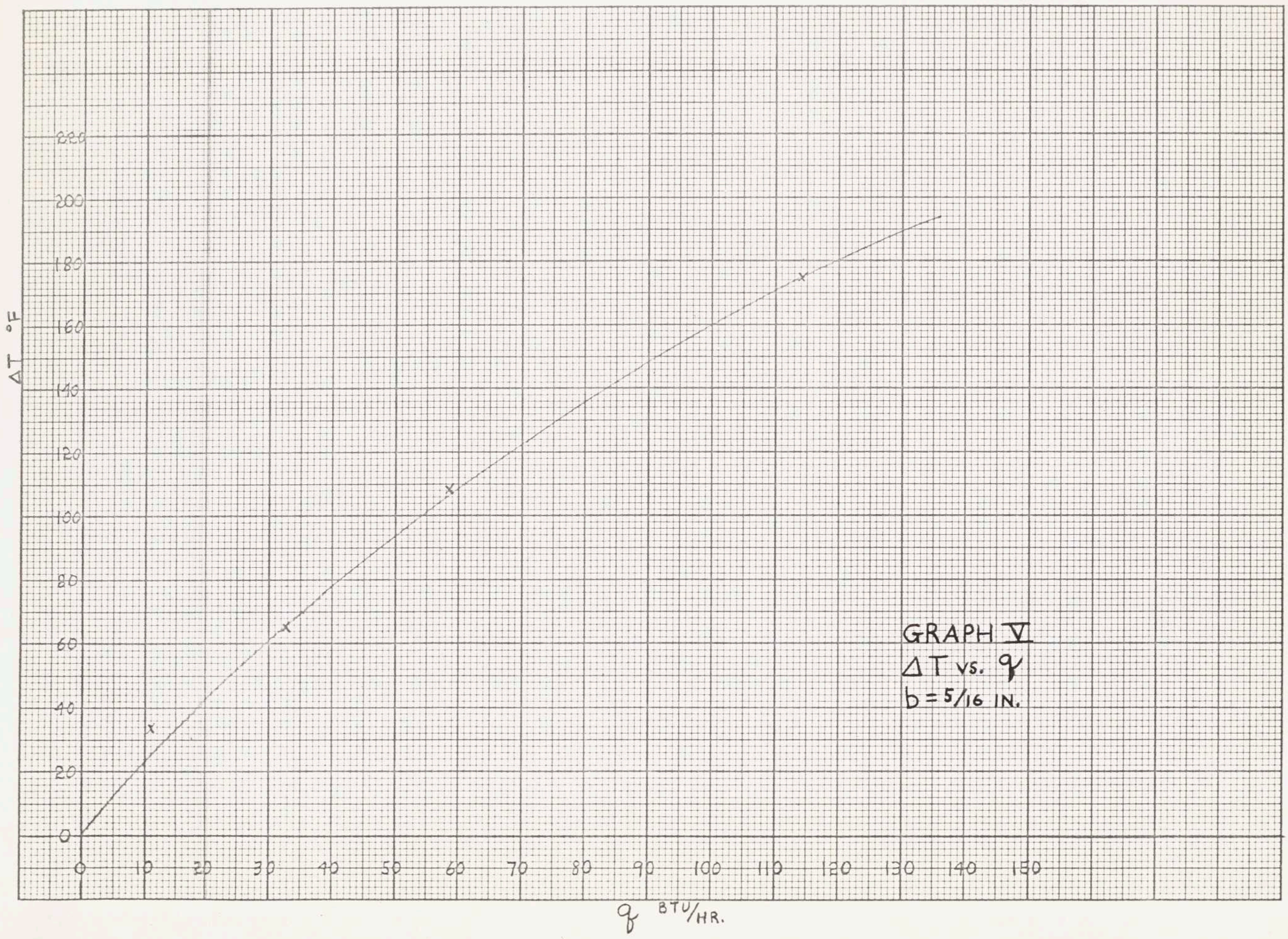
In order to give an overall picture of the net effect of varying the plate spacing, a more direct presentation of the data is often meaningful. This direct effect can be shown by plotting the mean temperature rise of the plate versus the power input for various values of the spacing. This has been done for some of the data and is shown in Graphs II-VIII and a composite of these curves in Graph I. From these curves it is immediately obvious that as the spacing is decreased from 3/4", the effectiveness of the free convective phenomena decreases noticeably. This is also emphasized in Graph I where mean heat transfer coefficient for a typical ΔT is plotted against the ratio b/L.

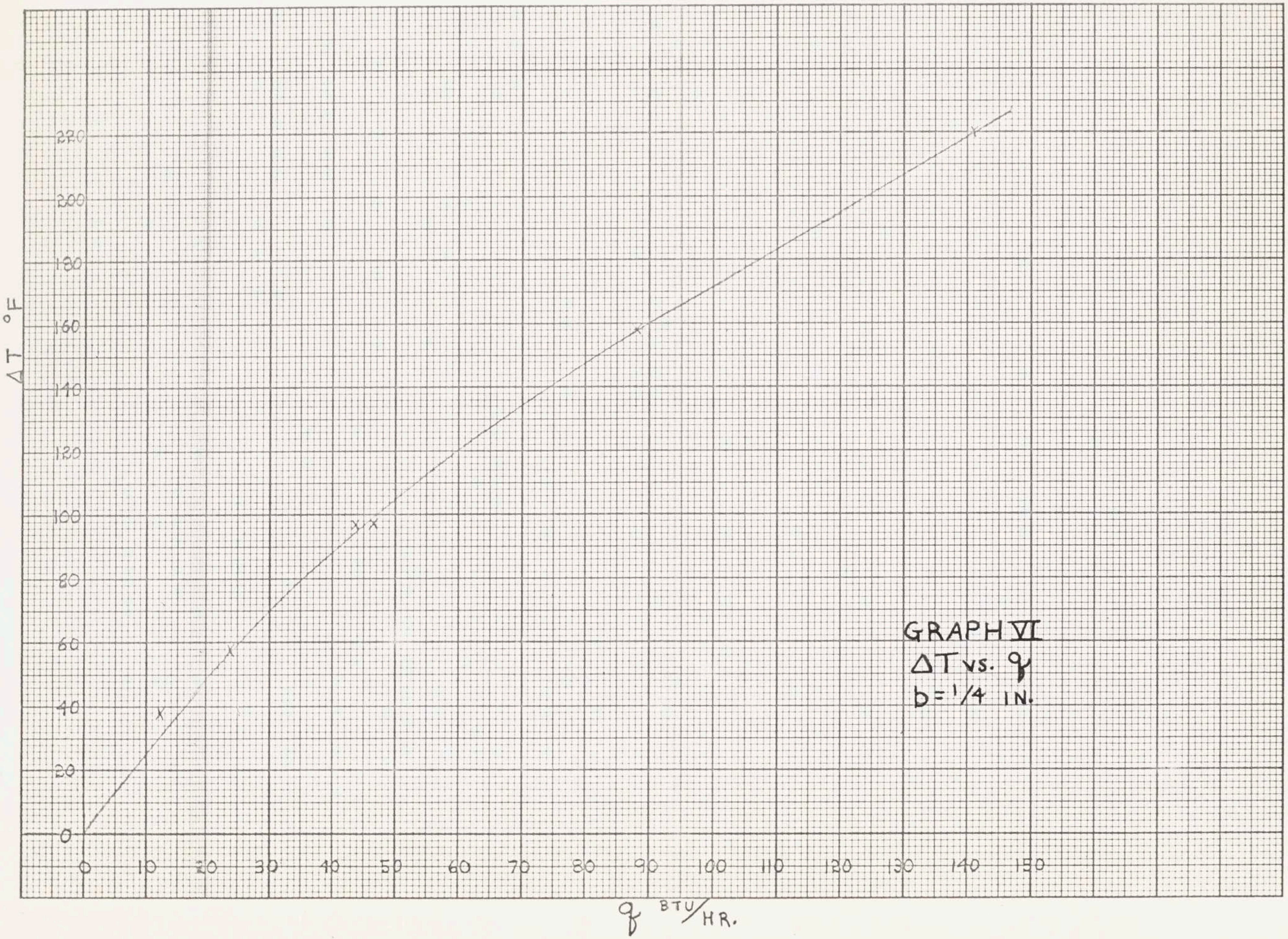

EXPERIMENTAL CURVE
APPROXIMATION (SLOPE = 1/2)
SLOPE = 1/4
SLOPE = 1
GRAPH I
Nu = hb/K vs. Gr b/L
Gr = gβΔT b³/ν²
Gr b/L
Nu = hb/K

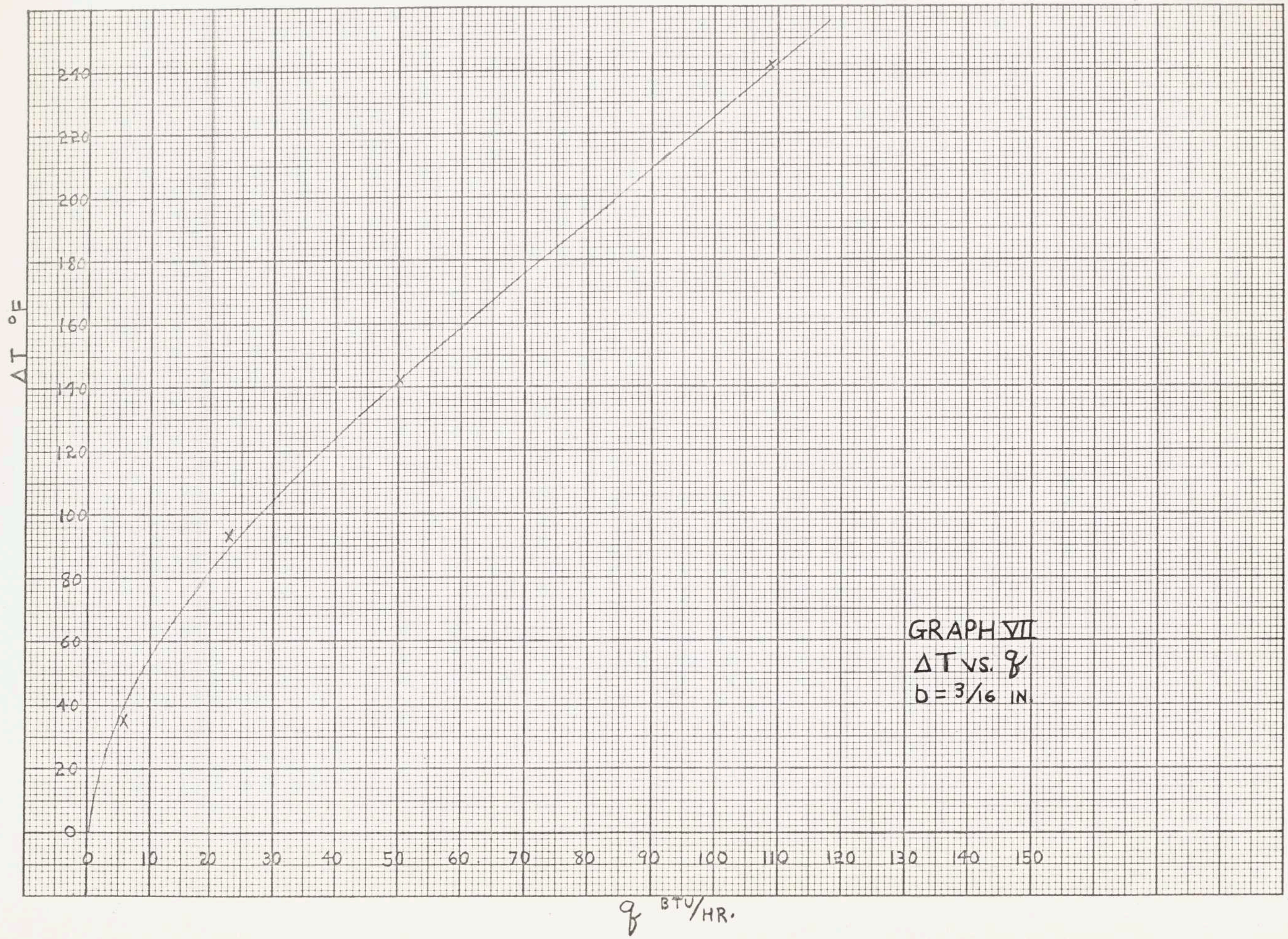

GRAPH II
ΔT vs. q
b = 3/4 IN.
ΔT °F
q BTU/HR.
0
20
40
60
80
100
120
140
160
180
200
220
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150


GRAPH III
ΔT vs. q
b = 1/2 IN.
ΔT °F
q BTU./HR.
0
20
40
60
80
100
120
140
160
180
200
220
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

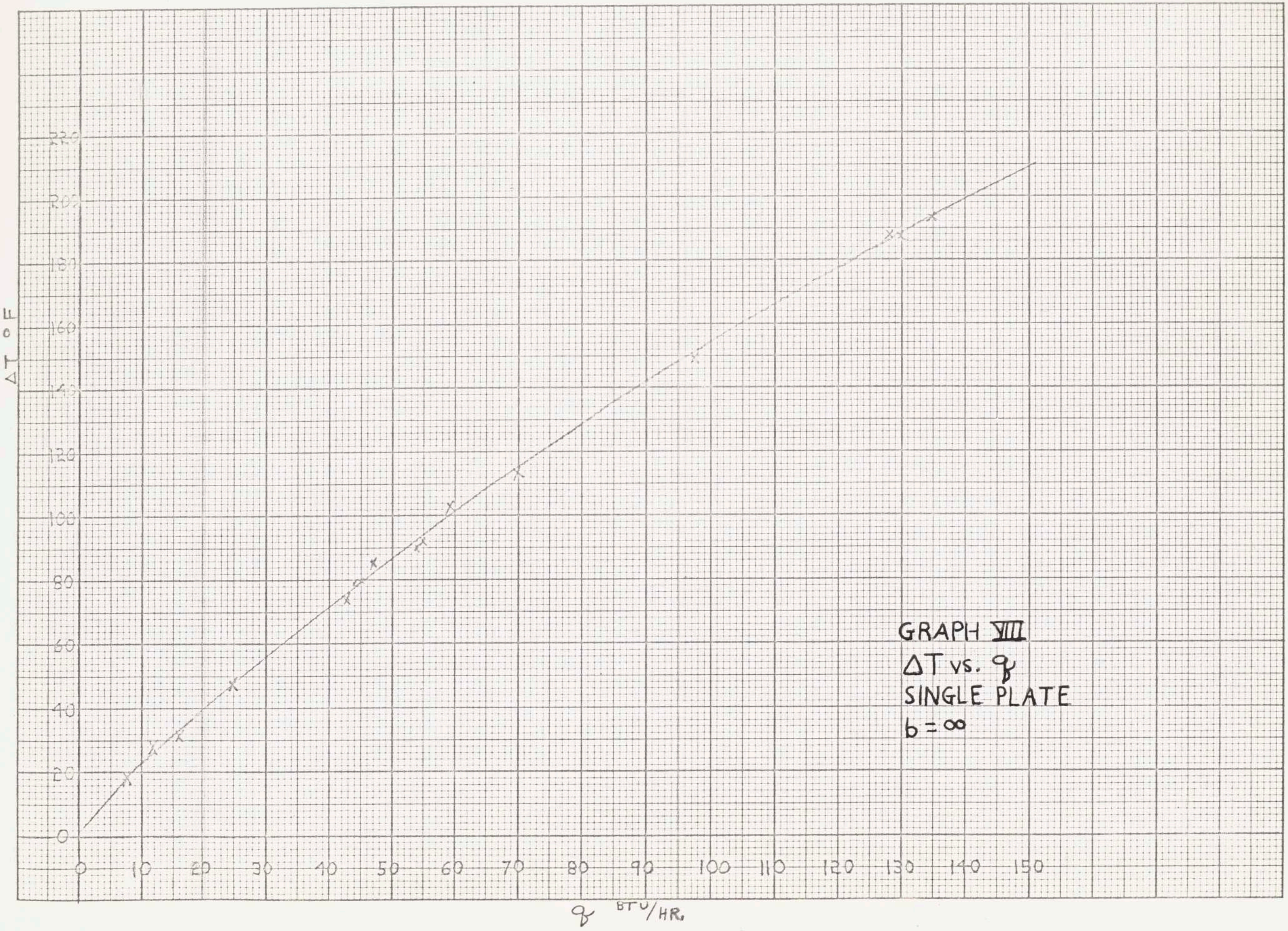

GRAPH IV
ΔT vs. q
b = 3/8 IN.
ΔT °F
q BTU/HR.
0
20
40
60
80
100
120
140
160
180
200
220
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150


GRAPH V
ΔT vs. q
b = 5/16 IN.
ΔT °F
q BTU/HR.
0
20
40
60
80
100
120
140
160
180
200
220
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150

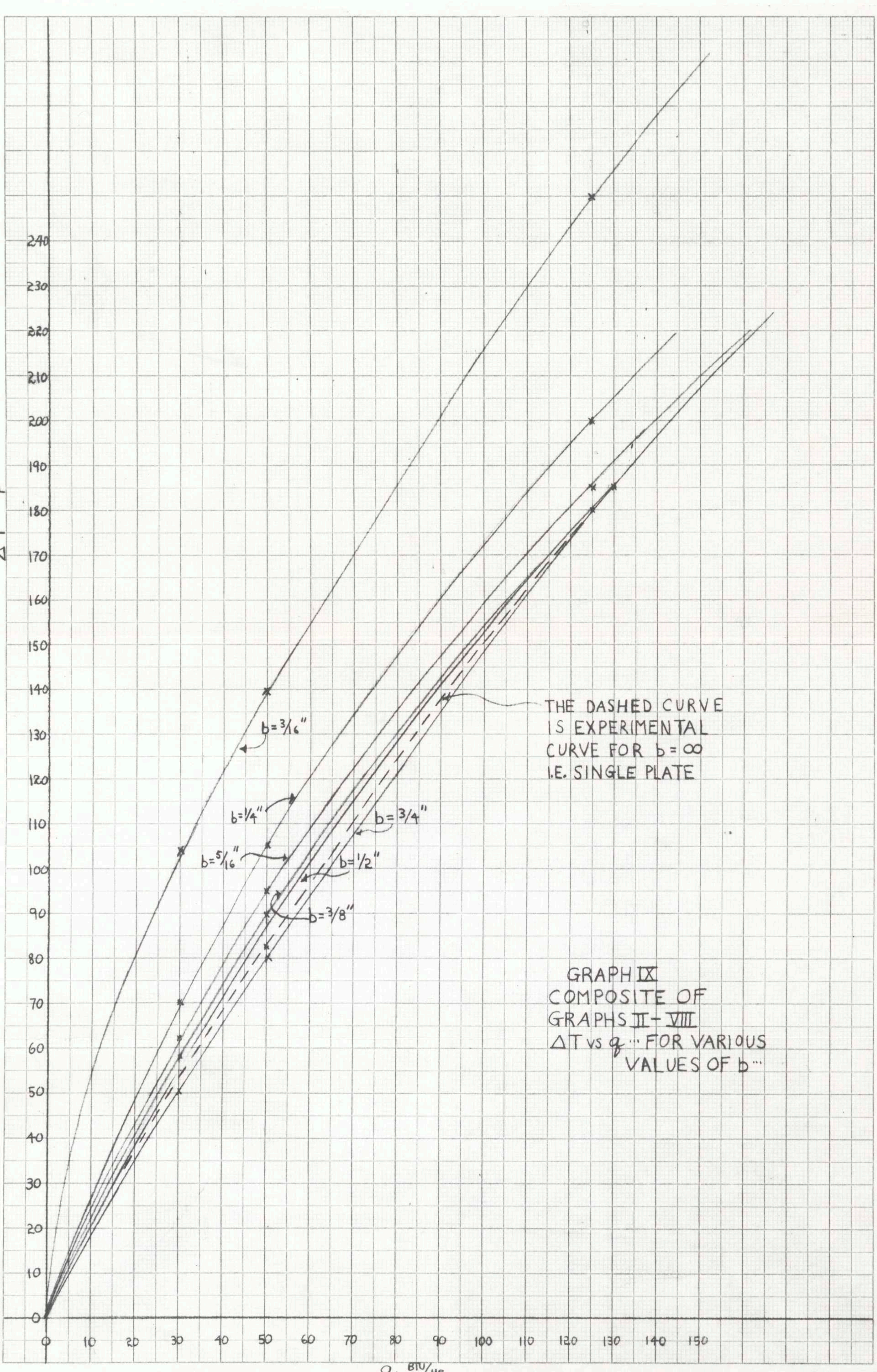

GRAPH VI
ΔT vs. q
b = 1/4 IN.
ΔT °F
q BTU/HR.
0
20
40
60
80
100
120
140
160
180
200
220
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150

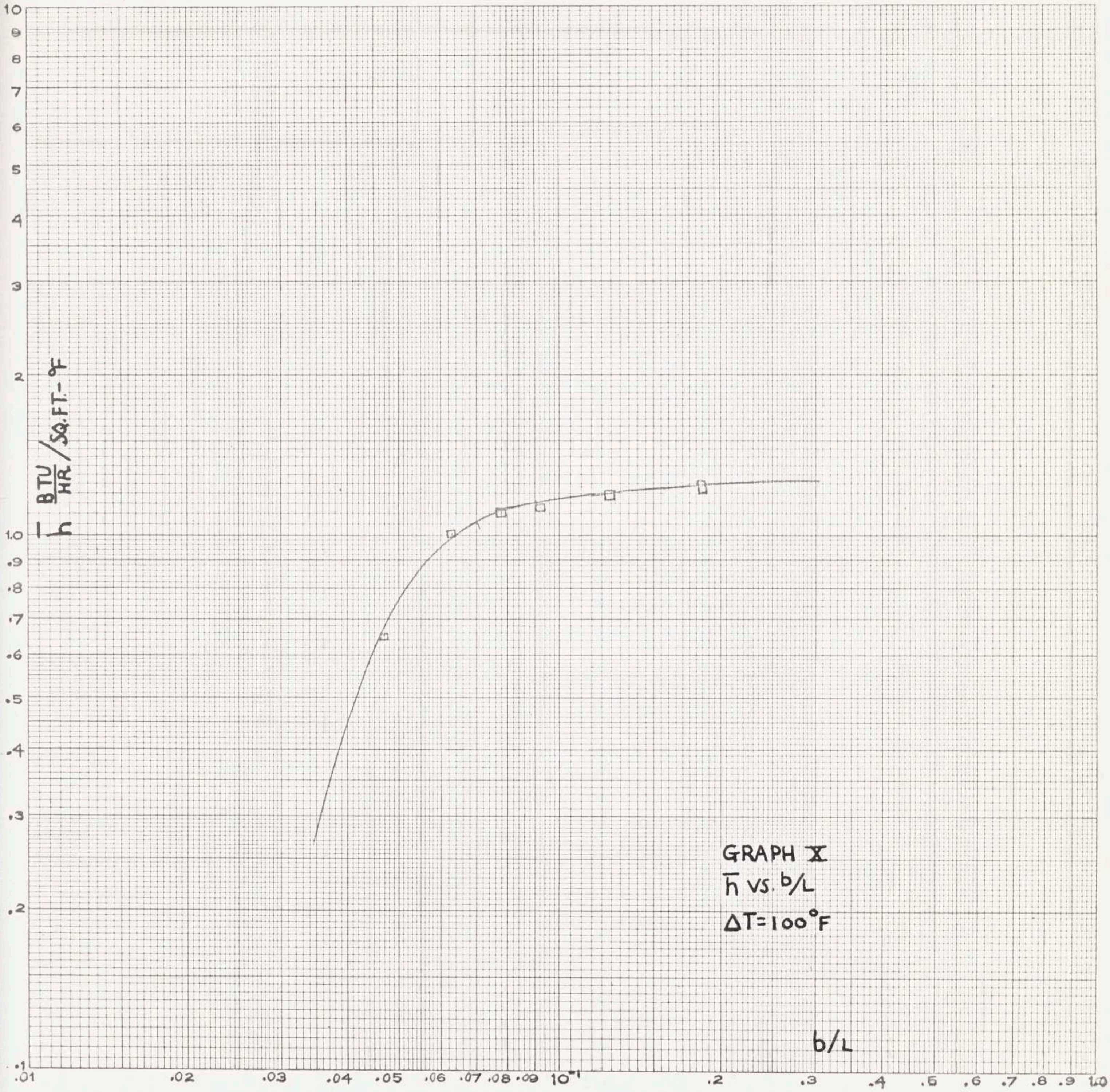


ΔT °F
240
220
200
180
160
140
120
100
80
60
40
20
0
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
q BTU/HR.
GRAPH VII
ΔT VS. q
b = 3/16 IN.



GRAPH VIII
ΔT vs. q
SINGLE PLATE
b = ∞
ΔT °F
q BTU/HR.
0
20
40
60
80
100
120
140
160
180
200
220
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150


ΔT °F
q BTU/HR
b = 3/16"
b = 1/4"
b = 5/16"
b = 3/8"
b = 1/2"
b = 3/4"
THE DASHED CURVE IS EXPERIMENTAL CURVE FOR b = ∞ I.E. SINGLE PLATE
GRAPH IX
COMPOSITE OF GRAPHS II - VIII
ΔT vs q ... FOR VARIOUS VALUES OF b ...


GRAPH X
$\bar{h}$ VS. b/L
ΔT=100°F
$\bar{h}$ BTU/HR./SQ.FT.-°F
b/L
10
9
8
7
6
5
4
3
2
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.01
.02
.03
.04
.05
.06
.07
.08
.09
10^{-1}
.2
.3
.4
.5
.6
.7
.8
.9
1.0

There is also another fact that should be noted. The curve for b = 3/4" indicates that at this spacing the plate was cooler than the plate for b = ∞, rather than warmer as would be expected from all that has been presented thus far.

There is no conclusive explanation for this. There is, however, a physical argument which can be given to make this behavior plausable.

The boundary layer in natural convection is an "envelope" for two fluid parameters: the temperature and the velocity. Although the two are coupled, the correlation between the two is not a one to one dependence. In the mode of interactions discussed previously the principle behind the effects was essentially the fact that the mid-plane temperature for the interacting boundary layers was effectively higher than the actual ambient temperature.

Furthermore it has been noted that the region for "destructive" interference occurred at values of b/L much smaller than the actual boundary layer thickness.

Perhaps then, in addition to the destructive interference found for boundary layers whose mutual penetration is larger, there is a secondary beneficial effect for those which just overlap.

Analytically, this argument may be justified by noting that, for boundary layers which just overlap, there no longer exists the condition that the fluid velocity go to zero at δ . Therefore, the velocity of the flow may be greater for slightly interacting boundary layers without a corresponding increase of the mid-plane temperature above the actual ambient. If this effect does exist then an improvement of the nature observed might be expected.

On the other hand, there is another potential explanation. There is the possibility that though the radiation corrections discussed were mutually consistant for the finite spacings investigated, the jump to an infinite spacing (F=1) might have introduced a small error which would show up as the effect observed. As plausable as this possibility seems, the care taken to avoid this would tend to favor the former argument of a beneficial interaction.

PRACTICAL CONSIDERATIONS

There are a great number of engineering applications for free convection cooling (and heating) which make use of vertical parallel plates or fins. In such applications it is quite important to optimize the efficiency of the device. Using the relationships found in the preceeding sections, some criteria for maximizing the efficiency of a free convection system will be derived.

Since our curve for the Nusselt number as a function of the Grashof number and b/L is continuous, and increases continually with increasing Gr b/L, our three regions can be investigated individually and the results matched at the boundary where the regions meet.

In region I, for constant ΔT and L, it can be shown that

$$\bar{h} = .037 \left(\frac{g\beta\Delta T b^4}{\nu^2 L}\right) \frac{k}{b} \sim b^3$$

In region II

$$\bar{h} = .170 \left(\frac{g\beta\Delta T b^4}{\nu^2 L}\right)^{1/2} \frac{k}{b} \sim b$$

In region III

$$\bar{h} = .560 \left(\frac{g\beta\Delta T b^4}{\nu^2 L}\right)^{1/4} \frac{K}{b} = \text{constant}$$

The heat dissipated by a given system in free convection is proportional to the total surface area. The total area in a finned system is equal to the area of a single fin times the number of fins.

For a given available dimension W, the number of fins (if we neglect the fin thickness) is equal to $\frac{W}{b}$. Therefore the area is proportional to $\frac{W}{b}$.

The heat transfer for a constant ΔT is proportional to $\bar{h}A$. Combining this with the above equations we obtain the following dependence in the three regions.

Region I

$$q \sim b^3 \cdot W/b \sim b^2$$

Region II

$$q \sim b \cdot W/b = \text{constant}$$

Region III

$$q \sim \text{constant} \cdot W/b \sim 1/b$$

Therefore if the product of Gr b/L for our system is in region I, the net heat transfer by natural convection will be increased considerably by using fewer fins at a wider spacing. The opposite is true of region III. Here more fins can be added even though we reduce the spacing. If Gr b/L is within region II, there is no change in the heat transfer for a slight change in spacing.

Matching these three criteria at the points where the regions merge we find that the optimum condition is for the system is that the product Gr b/L should be in region II. This is indicated in the brief sketch below.

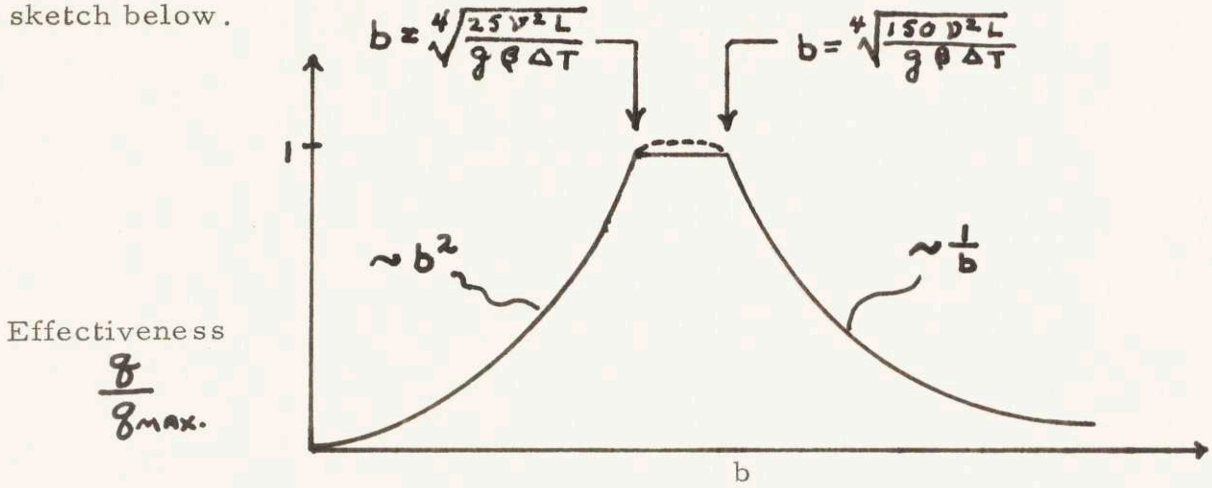


FIGURE V

It is worth emphasizing here that this analysis is valid only if the boundary layers are essentially two dimensional. This condition is satisfied if the dimension perpendicular to both b and L is large compared to δ. If these results are to be applied to three dimensional systems, e.g. vertical tubes, an equivalent diameter must be used in place of the spacing b.

Also it should be pointed out that a more exact analysis of the optimum criteria could be carried out using a more accurate representation for region II than the 1/2 power approximation. This would then result in more sharply defined conditions for optimum operation.

This would replace the constant effectiveness of region II as sketched above by a curved condition. However, after referring to the shape of Graph I, it does not seem likely that the curve would vary from the present horizontal line by very much.

CONCLUSION

From the data obtained in this investigation the following conclusions may be drawn:

There are effectively three regions of flow in natural convection heat transfer. First there is the region in which the boundary layer forms naturally and remains unmolested. This corresponds to the mode of free convection which is most familiar and the case usually assumed in the design of systems. This occurs when the free convection surfaces are far enough apart that there is no mutual interference between the flow patterns.

At the other extreme is the case which is the situation where complete mutual interference has occurred. This is the case corresponding to placing free convection surfaces too close to each other so that the boundary layers overlap considerably and the convection phenomena is suppressed.

The third case is the region of transition. This is the region which defines the gradual change from no interaction to complete interaction. As would be expected the convection in this region is effectively an average of the two extremes.

For each of these regions a practical emperical equation was derived relating the effective heat transfer coefficient to the parameters of the system. The limits of the three regions and the equations governing the heat transfer are summarized below.

Gr b/L	Nu
$Gr\, b/L < 25$	$Nu = .037\ Gr\, b/L$
$25 < Gr\, b/L < 150$	$Nu = .170\ (Gr\, b/L)^{1/2}$
$150 < Gr\, b/L$	$Nu = .560\ (Gr\, b/L)^{1/4}$

In addition to the pronounced suppression of convection at low values of Gr b/L, another effect was noted to occur for slightly overlapping boundary layers. It appears that for such a case there is the possibility of beneficial interaction, that is, a possible increase in the effectiveness of the heat transfer. This occurred for values of Gr b/L around 10^3. It is expected that this effect is due to mutual reinforcement of the velocities in the boundary layer.

Using the basic data on the effectiveness of heat transfer for the three regions a further analysis was carried out to discover if there existed a condition which maximizes the hA product in free convection. This would provide criteria for obtaining maximum heat dissipation by free convection per unit volume. The condition for optimum operation is that the product of Gr b/L should be equal to 100. Any variation of plate spacing which increases or decreases this value decreases the amount of heat that can be dissipated at a given ΔT. Since this product Gr b/L is proportional to b^4, it is obvious that the proper spacing is quite critical for optimum performance.

RECOMMENDATIONS

Although the results obtained in this investigation appear quite inclusive concerning the natural convection process for adjacent, parallel plates there remains much work which would be quite informative.

In particular the possibility of increased heat transfer due to the beneficial interaction in slightly overlapping boundary layers should be investigated further.

Also a correlation of the results obtained here with a similar investigation which might be carried out with b fixed and L as the variable parameter would be of value.

In addition it is recommended that an investigation of free convection in three dimensional systems such as vertical tubes be undertaken. In such a system the interference is no longer along a single surface, but rather occurs in more than one direction. It would be of interest to know if there exists an effective parameter similar to b which depends on the tube geometry and can be used in equations of the type obtained here for two dimensional boundary layers.

APPENDIX I

An Approximate Solution of the Natural Convection Boundary Layer Equations for a Single Vertical Plate

The coordinate system appropriate for this analysis is shown in the figure on the following page.

This derivation makes the following assumptions: The flow is steady, two-dimensional, incompressible, viscous, and has constant properties. Furthermore, as in most boundary layer investigations, the gradient of a quantity along the surface is assumed negligible in comparison to the gradient in the direction normal to the surface. With these approximations we can reduce our analysis to a solution of the following equations:

Conservation of mass

$$\frac{\partial u}{\partial X} + \frac{\partial v}{\partial Y} = 0$$

Conservation of momentum

$$u\frac{\partial u}{\partial X} + v\frac{\partial u}{\partial Y} = -g - \frac{1}{\rho}\frac{\partial p}{\partial X} + \nu\frac{\partial^2 u}{\partial Y^2}$$

Conservation of energy

$$u\frac{\partial T}{\partial X} + v\frac{\partial T}{\partial Y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial Y^2}$$

COORDINATE SYSTEM

The coordinate system appropriate to this analysis is shown below.

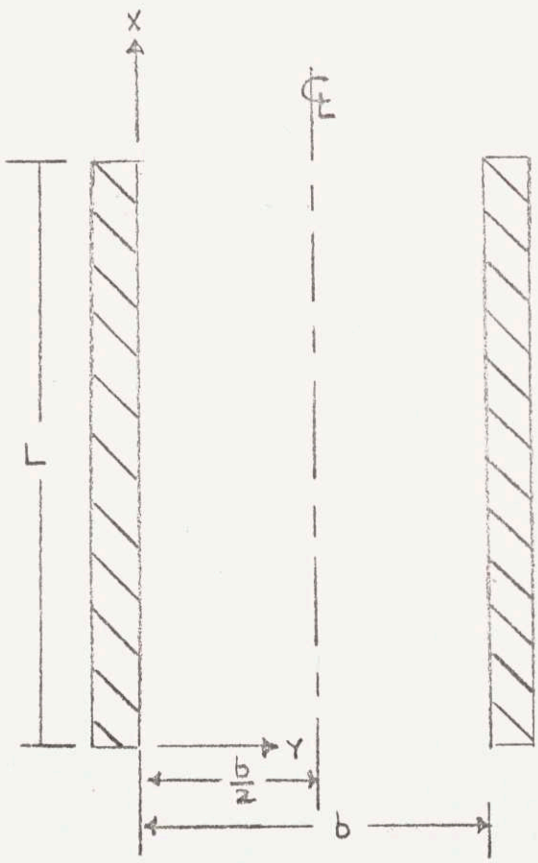


Figure I

If we integrate these equations over the thickness of the boundary layer and rearrange the terms, and include the buoyancy force $dx\, g\rho\beta\int_0^\delta \theta\, dy$ we obtain the following integral equations:

$$\frac{d}{dx}\int_0^\delta u^2\, dy = g\beta\int_0^\delta \theta\, dy - \nu\left(\frac{du}{dy}\right)_{WALL}$$

and

$$\frac{d}{dx}\int_0^\delta u\theta\, dy = -\alpha\left(\frac{d\theta}{dy}\right)_{WALL}$$

We now approximate the temperature profile by a parabola $\theta = \theta_w\left(1-\frac{y}{\delta}\right)^2$ (which fulfills the boundary conditions $\theta = \theta_w$ at y = o and θ = o at y = δ. We also make an approximation for the velocity profile of the form $u = u_o \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)^2$.

Noting that

$$\int_0^\delta u^2 dy = \frac{u_o^2\delta}{105}\ ;\quad \int_0^\delta \theta\, dy = \frac{\delta\theta_w}{3}\ ;\quad \int_0^\delta \theta u\, dy = \frac{\delta u_o\theta_w}{30}$$

we can reduce our boundary layer equations to the form

$$\frac{1}{105}\frac{d}{dx}\left(u_o^2\delta\right) = \frac{1}{3}g\beta\delta\theta_w - \frac{\nu u_o}{\delta}$$

$$\frac{1}{30}\theta_w\frac{d}{dx}\left(u_o\delta\right) = 2\alpha\frac{\theta_w}{\delta}$$

We solve these equations by assuming u_0 and δ to be exponential functions of x:

$$u_0 = H_1 x^m$$

$$\delta = H_2 x^n$$

Using these relations we obtain

$$\frac{2m+n}{105} H_1^2 H_2 x^{2m+n-1} = g\beta\theta_w \frac{H_2}{3} x^n - \frac{H_1}{H_2} \nu x^{m-n}$$

$$\frac{m+n}{30} H_1 H_2 x^{m+n-1} = \frac{2\alpha}{H_2} x^{-n}$$

$\therefore$

$$2m+n-1 = n = m-n$$

$$m+n-1 = n$$

By solving for the exponents we find

$$m = 1/2$$

$$n = 1/4$$

Using these values

$$\frac{H_1^2 H_2}{84} = g\beta\theta_w \frac{H_2}{3} - \frac{H_1}{H_2}\nu$$

$$\frac{H_1 H_2}{40} = \frac{2\alpha}{H_2}$$

$\therefore$

$$H_1 = 5.17\nu\left(\frac{20}{21}+\frac{\nu}{\alpha}\right)^{-1/2}\left(\frac{g\beta\theta_w}{\nu^2}\right)^{1/2}$$

$$H_2 = 3.93\left(\frac{20}{21}+\frac{\nu}{\alpha}\right)^{1/4}\left(\frac{g\beta\theta_w}{\nu^2}\right)^{-1/4}\left(\frac{\nu}{\alpha}\right)^{-1/2}$$

We now have an expression for the maximum velocity

$$u_{MAX} = \frac{4}{27}u_0 = .766\,\nu\left(.952+\frac{\nu}{\alpha}\right)^{-1/2}\left(\frac{g\beta\theta_w}{\nu^2}\right)^{1/2} x^{1/2}$$

and the boundary layer thickness

$$\delta = 3.93\left(\frac{\nu}{\alpha}\right)^{-1/2}\left(.952+\frac{\nu}{\alpha}\right)^{1/4}\left(\frac{g\beta\theta_w}{\nu^2}\right)^{-1/4} x^{1/4}$$

Using the above we can derive an expression for the dimensionless heat transfer coefficient

$$Nu_x' = 0.508\,Pr^{1/2}\,(0.952+Pr)^{-1/4}\,(Gr_x^{1/4}) = \frac{hx}{k}$$

For air with a Prandtl number of 0.714 we can obtain

$$Nu'_x = 0.378 \ (Gr_x)^{1/4}$$

This compares excellently with the exact solution obtained by E. Pohlhausen which resulted in a numerical constant of 0.360 instead of 0.378 as in the above equations.

The solution presented here is an approximate solution, due to Von Karman[1], and discussed in Eckert and Drake (3), pages 312-315.

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1 T. Von Karman, Z. angew, Math. Mech, 1, 235 (1921)

APPENDIX II

An analysis of the Error Introduced in the Calculation of the Heat Dissipated by Radiation from the Sides Due to an Error in the Surface Emissivity.

As was shown in the chapter on Radiation the heat dissipated from the sides due to radiation is proportional to $\mathcal{F}$ which can be obtained from the following equations.

$$\mathcal{F} = \frac{\tilde{F}_{10}\epsilon}{\tilde{F}_{10} - \tilde{F}_{10}\epsilon + \epsilon}$$

$$\tilde{F}_{10} = F_{10} + F_{12} F_{10}$$

Using these relations the value of $\mathcal{F}$ for some of the spacings used will be compared for various values of the emissivity.

The value for $F_{10} \rightarrow \tilde{F}_{10}$ the black surface over-all interchange factor, is taken from the graphs reproduced from McAdams (4) in the Chapter on Radiation.

I $b = 3/4''$

ϵ	$\tilde{F}_{10}$	$\mathcal{F}$
.18	.421	.144
.26	.421	.195
.30	.421	.210
.39	.421	.253
.52	.421	.299
1.00	.421	.421

II $b = 1/2''$

ϵ	$\tilde{F}_{10}$	$\mathcal{F}$
.18	.312	.129
.26	.312	.165
.30	.312	.180
.39	.312	.215
.52	.312	.244
1.00	.312	.312

III $b = 1/4''$

ϵ	$\tilde{F}_{10}$	$\mathcal{F}$
.18	.135	.079
.26	.135	.099
.30	.135	.104
.39	.135	.110
.52	.135	.123
1.00	.135	.135

From the above it can be seen that an error in ϵ does not give an equivalent magnitude of error in the correction for radiation from the sides. For instance at 3/4" a 50% error in ϵ would result in only a 30% difference in the radiation term and a 16% error in ϵ would mean only a difference of 8% in the radiation. At 1/4" this is even more pronounced. A 50% error results in a 22% difference and a 16% error in only a 5% discrepancy.

Therefore, it may be concluded that any error which may have occurred in the values of emissivity derived would not show up as quite so large an error in the final calculation of the radiation correction.

It should be noted here that the values of emissivity were consistent to within ± 8% so that the corrections can be considered fairly accurate.

Appendix III

List of Equipment

2 Transistorized D.C. Power Supplies, Consolidated Avionics Corporation, Serial Z50-15

1 Millivolt Potentiometer, Leeds and Northrup Co., Model No. 8686

1 Rotary Switch, Leeds and Northrup Co.

1 Mercury Thermometer, Marsh Instrument Company

1 Vernier Micrometer

APPENDIX IV

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3. Eckert, E., and Drake, R., "Heat and Mass Transfer", McGraw-Hill, New York, (1959)

4. McAdams, W.H., "Heat Transmission", 3rd Ed., McGraw-Hill, New York, (1954)

5. Elenbaas, W., "Heat Dissipation of Parallel Plates by Free Convection", Vo. IX, No. 1 (Jan. 1942)

6. Hinkle, W., "An Application of the Von Karman Integral Technique to Natural Convection in an Enclosed Space", M. I. T. Thesis, (May 1960)

7. Carlson, W.O., "Interferometric Studies of Convective Flow Phenomena in Vertical, Plane Enclosed Air Layers", University of Minnesota Thesis, (1956)