STUDIES IN PRODUCTION FUNCTIONS.

by

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ABSTRACT

Title : STUDIES IN PRODUCTION FUNCTIONS

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The first part of the thesis is concerned with the implications of temporally non-homothetic fixed coefficient technology for less developed agriculture. A disaggregated Leontief type production function - the 'Leading Input' production function-has been estimated using time series on paddy production in some Asian countries. Finally, an L.I. function for the agricultural sector has been plugged in a standard dual economy set up, and the qualitative solution and implications of it derived.

The final section of the thesis considers the implications of fixed coefficient technology at the firm level for a short run aggregate production function for the industry. The Houthakker model has been extended to cover analysis of disembodied technical change, movement of returns to scale when profits are reinvested, and some consequences of indepedent marginal cell distribution functions studied.

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A. THE LEADING INPUT PRODUCTION FUNCTION: AN ECONOMETRIC STUDY

I

One of the main lacunae in the aggregate models of economic development - both theoretical and empirical- is the absence of an adequate production function for the agriultural sector. There is a certain amount of descriptive knowledge of the state of peasant agriculture, for instance, in S.E. Asian countries, but few works in the area have attempted to incorporate the information in a production relation. Although there has been a lot of interest in the phenomenon of surplus labor, other facts of peasant agriculture have hardly received recognition from model-builders in the field. One would suspect that the conclusions of descriptive and optimal growth models will undergo qualitative change if one incorporates a production function, substantially different from the neo-classical, for the agricultural sector.

This paper puts forward a production function which is capable of explaining, over time, the major descriptive findings about peasant agriculture. The function has been tested against Indian, Japanese and Taiwanese data on rice production and has been found to work quite well in these contexts.

Pre-occuptation with neo-classical production functions in the context of peasant agriculture often gives rise to anomalies that cannot be consistently explained within the framework of traditional theory. The unhappy co-existence of surplus labor and neo-classical production structures in many dual economy models is a case in point. Such models can at best pay lip service to the structural characteristics of peasant agriculture at any point of time - let alone explaining the dynamics of growth.

In a recent book on Asian agriculture S. Ishikawa has analysed data that throw up interesting an structurally similar time characteristics of agricultural development in various Asian countries. Apart from discovering substantial complementarities in factor use, he also comes up against, what he calls, the phenomenon of 'leading inputs'. In one country after another, he has observed, that at different stages of development different factors tend to provide the impetus to growth, in the sense that output gets very sensitive even to small increases in that factor. In a Leontief type world - characterising complementarities in factor use - this would mean that the leading input will also have to be the factor constraining the growth of output for the time being. Analysing this phenomenon, Ishikawa has found that not merely can one factor or the other be identified as the 'leading input' for a country's agriculture at any particular point of time, but that over time, leading inputs might actually alternate. In the context of paddy production in S.E. Asian countries, he identifies two such leading inputs - irrigation water, and a composite input consisting of fertilizers, better seeds, etc. - as being the prime movers in the development process. He does this on the basis of simple correlation studies on the formidable amount of data that he has gathered, and some simple regressions. He does not claim that his correlation studies conclusively prove anything, but they do suggest a reasonable hypothesis about the pattern of growth in Asian agriculture.

As regards complementarity in factor use at any point of time, the hypothesis has often been raised in the context of peasant agriculture, but was never sufficiently backed by evidence. Eckaus had advanced it to explain the existence of surplus labor (2). Schultz had touched upon it in the context of explaining the relative fixity of techniques in peasant agriculture (Cf. 3). In recent years the publication of the Farm Management Studies (FMS) by the Government of India (10) has started a prolonged debate among Indian economists regarding the various findings on farm size and productivity. Among these, the issue that has perhaps attained maximum attention is the phenomenon of 'inefficiency' - as depicted by relatively low output per unit of land - of large farms. To us it seems, a lot of these findings can be explained by the hypothesis of complementary factors. The idea at the back of people's minds seems to have been that the growth in size is invariably accompanied by an increased in fixed capital and economies of scale, whereby the average cost schedule floats downwards and productivities rise. Findings to the contrary in the case of large Indian farms surprised people, for in this scenerio, a fall in productivity with a rise in size is a heresy and an 'anomaly' to be explained, may be, by the perverse socio-economic background of the people.

It seems that if something like a fixed coefficients production function is indeed in operation, a rise in size (as depicted by a rise in land endowment) would leave the capital/labor ratios unchanged and so long as we are in the early stages of development (where land is not a leading input), would, by itself, do nothing to raise output. Hence large and small farms alike would naturally have relatively similar capital labor ratios and there would be smaller output per unit of land as size increases. Of course, the deviations from the 'ideal' technique can be substantial in any cross-section sample to the extent fixed endowments (other than land) will differ from unit to unit for various reasons, even if the factor market is competitive, but in a long run time series study, one would expect such differences to be smoothed out. Also, one can raise the theoretical question as to whether any production function for that matter can be estimated using data involving 'inefficient' units, but this does not seem any more serious than the fact that there can be deviant units in competitive models too. The actual production of an inefficient unit cannot be predicted by any production function, but using a Leontief type function, one can attempt to estimate the extent of the deviation.

The three phenomena of (a) 'leading inputs', (b) surplus labor and (c) complementarity among inputs, suggested to us a fixed coefficient production function with productivities that vary over-time. In other words, we postulate a function of the form -

$$Q(t) = \min_i \left[\frac{X_1(t)}{a_1(t)}, \ldots\ldots, \frac{X_n(t)}{a_n(t)} \right] .$$

where Q(t) is output at time, $X_i(t)$'s are factors of production a(t)'s are the inverse of their respective average productivities. We will call this the L.I. function.

At any point of time, given the average productivities, the relative scarcity of one factor may generate unemployment among one or more of the other factors, and although techniques of production (or factor combinations) might change very slowly, one or the other factor might become the 'leading input' if their relative productivities change significantly over time.

As is well-known, such a function is also consistent with a wide range of factor prices. In particular, the wage rate can be quite low or fairly high depending upon demand and supply conditions and one does not have to make additional non-economic assumptions to explain why it is low. As a matter of fact, one can invoke any one of several reasons cited in the literature to explain whey agricultural wages should be non-zero. Apart from the ample empirical evidence (11) to the effect that labor is not in fact in absolute surplus in many of the so-called 'underdeveloped' or 'labor-surplus' economies, there is reason to believe that the actual level of wages may, in fact, be determined by such factors as subsistence needs, and the marginal disutility of labor. The latter factor is all the more important since with our production function, labor is a complementary input. Also the distribution of output will be strongly influenced by it through its impact on the labor-supply function. Finally, since demand for agricultural output is by and large coming out of wage income, a zero wage rate in agriculture is unlikely to be an interesting or a probable situation to occur. Similarly, so long as land is not in absolute excess supply over the economy, one can easily explain why the price of land is strictly positive despite the existence of land inefficient units. Besides, socio-institutional and expectational factors that have not all been taken into consideration in this study do have a significant role to play in explaining high land prices in such countries. Land-holding is a status-symbol, and one of the surest hedges against inflationary price rise.

On the face of it, the new function has little new about it excepting that the coefficients vary over time. But that apparently small change seems to go a long way in explaining movements of agricultural output. Testing this form of the function against the standard ones in use (e.g., Cobb-Douglas) on Indian, Japanese and Taiwanese data, we obtained much better results. On a priori grounds, it seems to be able to explain most of the observed phenomena in peasant agriculture. Making productivities functions of time is admittedly a sign of ignorance of the real forces at work, but it is no more or no less objectionable than the treatment of disembodied technical change, for instance. Specially, with the limited nature of data on peasant agriculture, this does not seem to be a bad first approximation.

In sum, the power of the L.I. function (leading Input, or Leontief-Ishikawa, for short) seems to be in the fact that it can explain the so-called anomalies of peasant agriculture within the framework of traditional economics. The phenomenon of labor or land surplus, as the case may be emerges directly from the technological realities of the situation. Also a whole range of factor prices is admissible with equilibrium - the exact level of such prices vary from one situation to another depending on demand and supply conditions.

There is a very useful feature about this production function, i.e., the case with which it can be estimated. By using the duality of cost and production functions, one can estimate some polynomial approximation of the functions $a_i(t)$'s by looking only at the cost function dual to the production function suggested above. Such a function happens to be linear in factor prices (8) with variable coefficients. Coefficients of the finite order polynomials used to approximate the $a_i(t)$ functions can be estimated directly by multiple regression. By tackling the problem from the cost function side, we automatically do away with the measurement of physical inputs and their aggregation, and problems of similar nature which have always bothered production function estimators.

Let us elaborate on the above a little bit. The duality properties of cost and production functions was first emphasized by Shephard (9) (Ist edition). In recent years the subject has been thoroughly examined in the light of conjugate functions. (Cf. McFadden, Rockafeller, Shephard). Given a production function,

$$y = f(x_1, ...x_n)$$

where y is output, x_i's inputs, and given factor prices w_i's, the cost function is defined as

$$c(y,w) = \min_{x}.\{w.\,x\} \text{ for } x \in V(y), W = (\omega_1, \cdots, \omega_n)$$

where $V = \{x \mid y \leq f(x)\}$, i.e., set of all x's that can produce y.

The derivation of the cost function nowhere depends on any assumption regarding the structure of the output market. However, it does usually involve the assumption of competition in the factor markets. But as Sehphard points out (Cf. (9), p.80), this is only a suf icient but not a necessary assumption. By redefining the units and factors one can get around the question of making any qualitative statement about the structure of the factor market. All that is necessary is to have the factor prices independent of the quantities used in the output market at hand. However, since the cost function is by definition minmum with respect ot input prices, it does involve an assumption about economic ef iciency in just the same way as a production function implies it. If in fact allocational inefficiency is rampant in peasant agriculture as some claim it is, - so that some of the observed points are strictly inside the production possibility from the it is indeed futile to attempt any kind of economic analysis of it.

Diewert's work mentioned above proves that the dual to a pure Leontief production function

$$Q = \min_i \left[\frac{X_i}{a_i} \right] ,$$

is given by

$$C\ (X\ ;\ Q) = \left\{ a_1 p_1 + \ \ldots. \ + a_n p_n \right\} Q.$$

where p_i's are prices of the factors x_i's respectively and Q is output. In our case the productivity coefficients are functions of time. Each such function $a_i(t)$ can be approximated by a polynomial

$$a_i(t) \approx \sum_j \lambda_{ij} . t^j \quad , \quad j = 0,1,2,\ldots.m$$

so that the cost function for unit level of ouput relevant for our case is

$$C\ (X\ ;\ 1) = \sum_{i=1}^{n} \sum_{j=1}^{m} p_i . \lambda_{ij} . t^j ,$$

This is a function linear in the coefficients λ_{ij}'s and is easily estimable by multiple regression given data on cost and factor prices. It is clear that this is tantamount to estimating the parameters $a_i(t)$'s of the original production function.

We shall first find out if the L.I. function works well with time series data. This has been done in the context of Japan, Taiwan and a district in Gujrat, India. Also, a case study for a district in S. Carolina, U.S.A.

is given as a study in constrast. The results are reported in section II. We then proceed to analytically formulate and statistically test the L.I. hypothesis with the estimated coefficients. This is done in section III. Section IV examines a dual economy model with an agricultural production function of the L.I. type. The model is explicitly solved for the initial labor surplus situation and consequences for growth analysed.

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II

This section presents the analysis of the case studies done in the context of paddy production in Japan, Taiwan, a district in Gujrat (India) and corn production in a country in South Carolina, U.S.A. The regression results and the data are presented in Appendices II-1 to II-4 respectively.

II-1 : Japan.

The best available long term economic data that we could get was on Japan. Also Japan seemed to be about the only Asian country to have gone through the whole spectrum of development. And although regional variances in soil-climatic patterns are by no means unsubstantial, they are small compared to those in case of a large country. In the absence of reliable micro data covering a reasonably long time period (Cf. sections II-3 and II-4), we tested the L.I. function against aggregate data on small homogeneous countries like Japan and Taiwan. Using some kind of a structural similarity argument as is done in cross-section studies, this exercise, togather with II-3 and II-4, is intended to give an indication of the robustness of the function against both micro/macro data.

The major source of reference for Japan is the ninth voluem (on Agriculture) in the series 'Estimates of Long Term Economic Statistics of Japan since 1968(LESJ) published by the Hitotsubishi University. We have had to supplement it by other sources of reference like S. Ishikawa's book, Rosovsky's 'Capital Formation in Japan'. and K. Okhawa's 'The Growth Rate of the Japanese Economy'. The period for which we did the estimation is 1878 to 1940, i.e., covering most of Meiji era and coming up to the second war. Although data after that data were available, possibility of fundamental structural changes in the post-war period prompted us to forego that information. Our choice of the initial data was mostly dictated by the availability of data.

Cost of Production data:

No data on cost of production of paddy as such was available for such a long period. What we did have, however, was a series in millions of yen (1934-36 prices) of the farm value of current input in all agriclture. We also had data on the farm value of rice produced and the farm value of total agricultural production. We assumed that for each year

$$\frac{\text{Farm value of current inputs in rice production(FVCIR)}}{\text{Farm value of current inputs in all agriculture(FVCI)}} = \frac{\text{Farm value of rice production (FVRP)}}{\text{Farm value of all agricultural production (FVTA)}}$$

so that $FVCIR = \frac{FVRP}{FVTA} \cdot FVCI$ for each time period.

Dividing this by total production of rice in Japan in millions koku (TPRJ) and deflating by the price index of 1904-6 (the choice of this particular index is explained below), we get the dependent variable series, cost of production of rice per koku (CPRK). Since the assumption implies: $\frac{FVCIR}{FVRP} = \frac{FVCI}{FVTA}$, it in effect boils down to some kind of a long-run equilibrium condition that the rate of profit earned in rice production is equal to the rate of profit earned in all agriculture. It also implies that the ratio of current inputs cost to total cost in rice production is roughly equal to the average ratio for all agriculture, since our cost of production series only includes current input costs. As regards the first of these assumptions, one would certainly expect to find deviations from it over cycles, but would expect it to average out over time. As regards the second, one can justify it under one of two situations: one if the ratio of current costs to fixed costs were equal for all commodities in the agricultural sector, indicating equal organic composition all around ; and two, if the ratio for paddy were somewhere near the average for all agricultural commodities with the share of the product of each to total agricultural production as respective weights. The first is much too unlikely a situation, whereas the second invokes much too complicated questions about index numbers and like. Yet since information on the degree of capital intensity of Japanese rice production vis-a-vis that of all agriculture is lacking, such an assumption is indispenable, and one would be inclined to defend it on account of the second reason rather than the first. For one can argue that, since the ratio of the farm value of rice to that of all agriculture in Japan has remained relatively high and stable over the sample period,* it is unlikely that the capital intensity in rice production will diverge significantly from the capital intensity in agriculture on average. A glance at Table (II.1) shows that the composition of agriculture has not changed very drastically either. Hence the assumption FVCIR = (FVRP/FVTA). FVCI on p. ( ) above is not very unwarranted in the absence of more specific information. What makes the estimation process intrinsically robust is the fact that the estimated coefficients are unique up to a scalar multiplication, so that even if FVCIR/FVRT were equal to constant times ($\frac{FVCI}{FVTA}$), the coefficients would exhibit the same pattern as before. It is only if the capital intensity in paddy production relative to the average capital intensity in all agriculture has changed significantly over the sample period, that our CP-series will be unrepresentative of the real state of affairs. Without any information to the effect that this is so, not to speak of the direction of relative change, if any, we decided to stick by the assumption FVCIR/FVRP = FVCI/FVTA on the ground that paddy being the most important crop in Japanese agriculture, its capital intensity cannot be very different from the average capital intensity for all agriculture.

<u>Fertilizer price data</u> : The LESJ (Long term Economic Statistics of Japan, cited above) contains a wealth of information on fertilizer prices. There is an index of fertilizer prices with 1934-6 as the base years, which we used by deflating it by 1904-6 prices. There are also price series on the unit values of nitrogenous, phosphate and potash content in fertilizers, as well as their respective uses over time in Japanese agriculture. For purposes of estimation, we tried both the price index cited above and the price series on nitrogenous fertilizers alone. However, we got better results by using a fertilizer price index that was computed from the price series of three kinds of fertilizers using their shares in total quantity as weights, and then deflating it by 1904-6 price index. This is our PF 3Z series.

<u>Substitute Water Rate</u> : As Ishikawa points out (p. 212) statistical surveys of irrigation facilities are very rare in Japan. About the only reliable survey of irrigation has been made is an ad hoc basic survey of agriculture in 1955, and even there data is sparse.

In Rosovsky's 'Capital Formation in Japan' (p. 17 Table 6) there is a division of investment in public works into 'traditional' and 'new' components - the first of which contains 'riparian' investment and the second contains 'water works'. The data covers the period 1890 to 1935 with gaps in between. While water works mainly involve construction of pipes and storages of water for consumption purposes, the estimates of riparian expenditure for central and local governments (Tables VII-1 to VIII-2, pages 164-174, Rosovsky) indicate expenditure mainly for irrigation and flood control. For the central Government, construction for harbors and riparian works data have been added together for the period 1868-1890. Judging from the trend in later years, we ascribed three-fourths of the total expenditure on riparian works alone. From 1891 to 1940, the value of riparian construction by central Japanese Government in ¥ 1000 is taken from Rosovsky. For local governments data on riparian expenses is available for the years 1875 to 1940 (Table VII-2, pp. 171-76). We added the total expenditure on riparian constructions by the Central (CGRW) and local governments (LGRW & LGAE), to get total expenses on such construction in the country as a whole. Then we made alternative assumptions about the life spans of such riparian works (10, 15 and 20 years) and calculated the cost streams that would result under three alternative hypotheses about the rate of interest (4, 8 and 12 % respectively). Assuming that revenue earned by imposing water rates on farmers covers such costs for each year, we divided each of the nine alternative revenue streams by acreage to get nine alternative series of estimated cost of irrigation water per acre over time. We use these as instrument variables for the price of irrigation water. Quite clearly, the proper unit to use is not the cost of irrigation water per acre, but the price per cubic feet of it per second. One might want to measure all inputs and outputs in terms of units per acre, but that would render it impossible to interpret the coefficients in the estimated cost function. If, however, one makes alternative assumptions regarding the trend increase in water use per acre, one might get alternative estimates of water rate. Suppose, for instance, one assumes that the rate of water use during the year is a constant, The total revenue earned from water charges in the year t is R_t, the acreage tilled is A_t and t, the total time during which water is released. With a constant time trend in water use α cubic feet per acre per second, one gets

$$\alpha \text{ cusee} \times \tau \times P = \frac{R_t}{A_t} = \text{Cost per acre.}$$

where P is the price of 1 cusee.

$$\therefore P = \frac{\text{Cost per acre}}{\alpha\tau \text{ cubic feet}} = \delta . \text{Cost per acre}$$

if the time trend and the time during which water was released are both constaint. Thus use of cost per acre as a substitute for price per cusee is justified in this case, since the two are equivaluent upto a scalar multiple.

In fact even if $\alpha + \tau$ (or, $\alpha\tau$) are allowed to vary within the year, so long as they vary in the same way for all the years, the same argument as above will hold and one can legitimately use cost of irrigation water per acre as a substitute variable for price of water per cubic.

In case the variability in water use is confined not within the year, but also spreads across years, cost per acre will not be a good substitute for price per cusec. The only way one can try to get from the one to the other in the absence of any information about the nature of such variability across years, would be to make alternative assumptions about it to get alternative estimates. If $\alpha\tau$ is a monotomically rising functions of time, the cost per acre series will overestimate the price per cusec for later years, If no clear trend is found the former would, in general, serve as a good enough instrument variable for the latter. It is seen that the general pattern of results is little affected by the choice of interest rate or the life span of riparian construction. For the purpose of comparison, we have also estimated the equations using a substitute variable for the water rate obtained by dividing each year's total expenses on riparin works by the year's estimated acreage. The results of these regression are reported in Table II-1-2. Equations II.1-1 to II.1-9 & Equations II.1-10 to II.1-15 respectively.

Other input prices: Some of the other inputs we have included in the various equations estimated are labor, land and tools and implements. Two different wage rates have been tried : the daily wage rate of male contract workers in agriculture (DWMA) and the yearly wage rate of male agricultural workers in yens per year (YWMA). The price for land has been represented by the rent of paddy field per tan (RPFT). As for tools and implements used in agriculture, a price index of tools and implements used in Japanese agriculture (TIPI) has been used. It is to be noted that neither land nor capital equipments can be included in current inputs as such. But as noted earlier, if we assume that the ratio of current costs to total costs is remaining approsimately constant over time current input cost can be made a proxy for total cost (Cf. discussion on regressions through the origin in the Indian example).

Choice of the base years : Most of the series were given terms of four different price indices : 1874-76, 1904-06, 1934-36 and current. We chose to express all our data in terms of the 1904-06 prices primarily because of the central location of the base years in reference to the period being examined.

In fitting the input price data on the per unit cost of production of rice in Japan, there are a few features that are present almost without exception, no matter what combination of input prices we consider- features that seem to corroborate the 'leading input' hypothesis of Ishikawa.

We can write

$$CP(t) = \sum_i a_i(t) P_i(t) + \sum_j b_j(t) P_j(t)$$

where the P_i's are the included prices and the P_j's are the excluded ones, and then assume either $\sum_j b_j P_j$ = constant, or $\sum_j b_j P_j = \lambda . CP$ in which case we shall be estimating $a_i(t)/(1-\lambda)$ instead of the $a_i(t)$'s. The estimated constant term might also include elements of fixed costs, and to that extent would be unidentifiable. However, the parameters we are interest in are the $a_i(t)$'s, and these can be approximated by polynomials in t of as high an order as one might choose. In our case, the estimated coefficients of t^j's ($j = 0, 1, 2, \cdots$) without exception fall quite drastically as j increases (roughly by the order of 10^2 or more). Hence we decided to approximate the $a_i(t)$ functions by quadratics in t, since beyond that, the additional loss of degrees of freedom - (one each for every higher power of t for each of the included factor prices) did not seem worthwhile. For results of the statistical tests to determine the power of the polynomials, Cf. section III below.

The interesting feature that emerges from all the regressions is the sign pattern of the $a_i(t)$

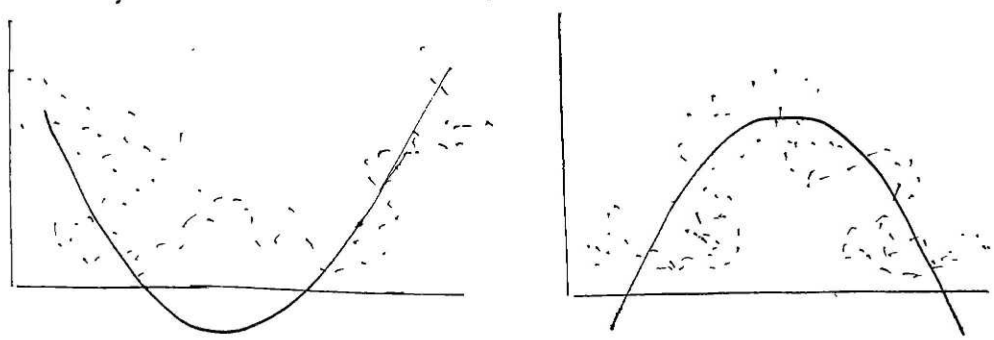
coefficients. If, for instance, $a_i(t)P_i(t)$ is approximated by

$$a_i^0 P_i(t) + a_i^1 \left\{t.\ P_i(t)\right\} + a_i^2 \left\{t^2 \cdot P_i\ (t)\right\},$$

then the estimated coefficients $(\hat{a}_i^0, \hat{a}_i^1, \hat{a}_i^2)$ almost invariably come out with alternating signs : (+, - , +) or (- , + , -) . If in an equation two input prices have been included with quadratic time functions as coefficients, and if one of them happen to be of the type (+, -, +), the other is almost invariably (-,+ , -), and vice versa. In case of three of more inputs, the alternating sign pattern is still very much there, only now the peaks and troughs of similar looking functions are set wide apart in time.

All this seem to indicate that over the course of time, one or the other factor has played the role of the limitational factor - or the 'leading input' - in the production of rice in Japan, and also that the role of the 'leading input' has been played by different factors at different points of time. Although to determine the exact location of such switch points in time would require more sophisticated estimating techniques and better quality data than we have at our disposal, the general pattern of alternating leading inputs seems to fairly clear (Cf. Section III).

To illustrate the point, consider Eq. (II·1-) In this equation, yearly wage of contractual agricultural labor has come out with statistically insignificant coefficients for all powers of t. (Incidentally, this result emerges quite systematically from the equations we have estimated - suggestions, perhaps, that most agricultural laborers are self-employed). As for the fertilizer price and the rent of rice-producing land, the $a_i(t)$ curves, when plotted against time, have the following shapes:



Clearly the negative values in either case for certain ranges of t are inadmissible, and have most probably come about due to the forced approximation of the functions by quadratics in t. Scatters shown in the diagram, for instance, when approximated by quadratic functions would indeed generate some negative values. The point to note is that if over time the actual combinations of factors measured in natural units (i.e., techniques of production) do not change very radically, this pattern of input productivity curves (or their inverses, to be exact) would necessitate the alternating emergence of different factors as leading inputs. To elaborate, let

$$Q(t) = \min_i \left[\frac{X_i(t)}{a_i(t)} \right], \quad i = 1 \ldots\ldots n$$

be the production function and let $(x_1(t), \ldots . x_i(t), \ldots x_n(t))$ be the technique of production at any t. If $\frac{x_i(t)}{x_j(t)}$

$i, j = 1, \ldots n$; $i \neq j$ are relatively smooth functions of time, the alternating patterns of behaviour of the $a_i(t)$ coefficients will ensure the sequential emergence of different factors as 'leading inputs' in different stages of development.

For eqs. II-1-1 to II-1-9, using the nine alternative substitute water rates, the general pattern of results can be described as follows: estimating the cost functions with price of fertilizer, rent of paddy field and any of nine alternative water rates as dependent variables, the three coefficients of the quadratic productivity functions of the respective factors are of the sign (+, - , +), (- , +, -) and (+, -, +) respectively. The trough of the inverse fertilizer productivity function oocurs between the periods 32 to 40, and that of any of the nine irrigation water productivity functions towards the end of the sample period and the peak of the land input productivity also occurs towards the end of the sample period; i.e., independent of the life span of riparian works and the prevailing rate of interest, here (as in the Indian and the Taiwanese cases), the results seem to indicate that irrigation water is the first constraining factor. The similar looking profiles of water and fertilizer productivity functions in all the three cases cited might indicate a basic structural similarity between them, i.e., the sequential complementarity of irrigation water and fertilizer input (although the troughs are widely separated in time). The peak obtained by the rent coefficient function towards the end of the sample period indicates the approach to limits of intensive cultivation under prevailing techniques.

II.2 Production of rice in Taiwan, 1901-1939

Our second example of the worability of the L.I. hypothesis is drawn on the basis of 39 years of rice production data from pre-War Taiwan. As for Japan, macro data were used, with the tacit assumption that production techniques did not vary widely from region to region --not a bad assumption a priori for a country the size of Taiwan. Our main sources of reference in this case are (i) Agricultural Development of Taiwan, 1913-1960 by Yhi-Min Ho (Vanderbilt University Press, 1966) and (ii) the volume on 'Prices' in the series 'Long Term Economic Statistics of Japan' (Vol.8, published by the Hitotsubashi University).

Taiwan's growth experience in the early twentieth country parallels, or surpasses, that of Japan in the Meiji era. Pre-war Taiwan had the advantage of borrowing the technical know-how of Imperial Japan. In the first twenty odd years of the present century, emphasis was on developing irrigational and drainage facilities and in the next two decades, leading up to Second World War, Taiwan experienced a very high rate of growth in agriculture, which coincided with utilization of new and improved seeds and much more intensive use of fertilizers than before -- mostly imported from Japan. The War affected Taiwan's economy quite severely and in the post-war years emphasis on non- traditional inputs and mechanisation indicates a structural break from the past.

This is one reason for selecting 1939 as the terminal date for our sample.

Apart from importing large quantities of fertilizers and production know-how from Japan, from our point of view, Taiwan is similar to Japan in more than one ways. Rice is the most important crop for both the countries. Both are small countries with relatively homogeneous production conditions (Taiwan more so than Japan), both countries are characterized by numerous farms with small average holdings. It is no wonder that the results obtained are similar for the two cases.

Here as in the case of Japan and India, the L.I. function seems to operate much better than the Cobb-Douglas. The included input prices are again those of fertilizer and irrigation water, and as in the other two cases they emerge as complementary inputs, with similar looking time profiles of productivity functions. Labour and land prices were excluded for - apart from the usual problem of non-existence of reliable data - neither of these inputs could be deemed binding on expirical grounds. Since, as for Japan, we did not have data on water rates, but only on investment on irrigation projects, we calculated the average cost of irrigation water per hectare of land and used this as an instrument variable for the price of irrigation water. As for Japan again, we did this for different assumed life-spans of projects (10, 15 and 20 years) and alternative rates of interest (4, 8 and 12 %), and the general pattern of results is very robust with respect to the choice of either.

Availability of data in the precise form in which we want them was, as usual, a problem. One has to make the best use of what is obtainable and manipulate it according to need, all the while being conscious of the assumptions made in the process. In what follows we discuss the data as were obtained, and the use to which we put them under specific assumptions.

Cost of production data :

No data on cost of production were available. However, Ho had calculated an aggregate input index for agriculture which takes into account the main items of cost including working and fixed capital. Also an aggregate output index for all agriculture has been calculated by Ho for a total of 74 different products. Since no consistent price index exist for Taiwan covering the whole period concerned, whereas data on physical inputs and outputs were available, Ho had calculated both the indices in terms of the 1952-56 average prices. This procedure is admissible to the extent the relative price structure of the period chosen as base is representative of the entire period. It is very hard to test that hypothesis when the data on relative prices for the period are lacking. However, there is a similar aggregate output index for agriculture constructed by S.C. Hsieh and T.H. Lee of the Chinese-American Joint Commission on Rural Development, Taiwan (JCRR), in 1935-37 prices. Comparison of the H_0 index and the Hsieh-Lee index shows very little discrepancy.

To the extent the base period relative price structure is representative of that for the whole period, an index of productivity, or an index constructed by dividing the input index by the output index, should be a fair approximation of the behaviour of unit cost of production over time. As in Japan, since paddy is by far the most important product in Taiwan's agriculture, we assume that such an index for rice production, were it available, would not be very different from the index of per unit cost that has been computed for all agriculture. Hence the latter can be taken as a proxy for the index of per unit cost of production of rice in Taiwan.

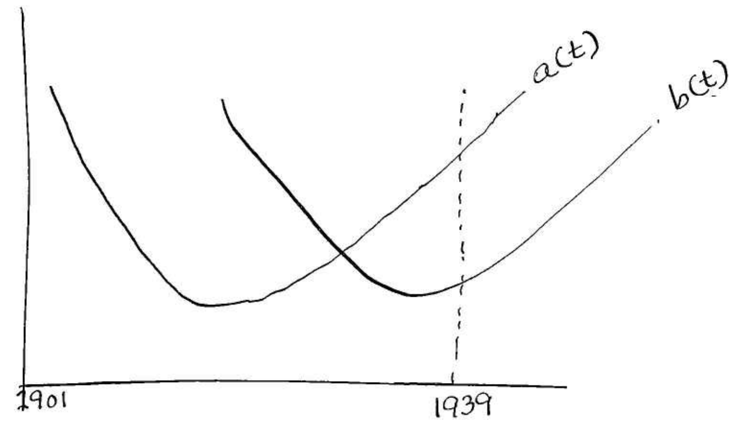
Price of fertilizers:

Most of Taiwan's fertilizers for the period concerned was not domestically produced, but was imported from Japan. Before 1912, domestic production was negligible. Even in the late twenties and the thirties, when use of commercial fertilizers went up at a high rate, the major source of supply of fertilizers was Japan. Thus the relevant variable to use in this case would be the import price of Japanese fertilizers. Such data were not available for the whole of the period concerned. So we looked in Japanese sources to get in idea of the export price of fertilizers over the sample period. Volume 8 (on Prices) of the Hitotsubashi University's series on Estimates of Long Term Statistics of Japan contains an index of the export price of Japanese 'drugs and chemicals' of which various kinds of fertilizers and insecticides constitute a sizeable fraction. We have used this series to represent the movement of fertilizer prices in Taiwan for the period covered. As with all indices, the use of this one too entails some assumptions on the behaviour of relative prices and shares of commodities over time.

<u>Water rate</u>:

Agricultural Development of Taiwan contains data on investment expenditure on irrigation by private and government concerns. Assuming that cost of a particular irrigation project is paid up in 10,15 or 20 years, we calculated alternate revenue streams with three rates of interest (4,8 and 12%). Each of the nine resulting revenue streams were then divided by total crop area to give nine alternative series of average cost of irrigation water per hactare. These were used as instrument variables for the price of irrigation water.

The equations estimated with the above data come out with a consistent pattern of the coefficient functions: the functions reach their minima within an average span of five years of each other in the late twenties and the early thirties.



a (t) : Inverse of fertilizer productivity

b (t) : Inverse of water productivity

If irrigation had been the first leading input in Taiwan's agriculture as Ishikawa claims it was, then the result we have obtained here helps us to establish the range of years within which the switch to fertilizers and better seeds as the next leading input is most likely to come about, i.e., the range of years that spans the two successive minima. It is within this period, that the requirement of fertilizer per unit of output began to rise sharply relative to that of irrigation water and this corroborates the empirical finding that new seeds and fertilizers did become the prime movers in Taiwanese agriculture around that period.

II.3 Kakrapar Weir and Canal Project : India.

In order to verify the hypothesis of the L.I. production function, first we take up a micro level study of paddy production under the Kakrapar Project in the Gujrat region of India. The L.I. function is supposed to operate well for agricultural units under traditional agriculture. Provided this is so, by Houthakker's logic (1), it sould be fairly representative of the aggregative production structure too if the units are not too different from one another in the techniques they use. Hence for our purposes, micro data are more appropriate than macro data. However, reliable micro data are very hard to come by. The best we could hope for is data on a small, reasonably homogeneous region. This is one reason for selecting this specific case. Also, we figured that the only way we could hope to get anything remotely close to a series on price of irrigation water would be to use the rates charged by government under the various irrigation projects. This prompted us to look into government reports on such projects. The Government of India publication titled 'Evaluation of Major Irrigation Projects - Some Case Studies' (2), or EMIP, for short, covers eight major projects in India. Among these the

1. Houthakker, : The Pareto Distribution and the Cobb-Douglas Production Function, Review of Economic Studies, 1955.

Kakrapar Weir and Canal Project is the one most exhaustively analysed and has the maximum amount of reported data. In view of the doctoring it needed, one can well imagine the scarcity of information on the other projects.

Because of the paucity of the data and numerous gaps in it that had to be filled, it may be worthwile to describe in some detail the way we constructed the various time series needed from whatever data was available.

Data on Water rates:

As expected a series for water rates charged for paddy production under the project proved to be the one hardest to come by. Given the available data, a lot of estimation and computation was in order. In this section we shall try to explain the what, how and why of it all.

We have a matrix of actual crop patterns under the project for the years 1958-59 to 1963-64 for all the major crops (excepting that for the year 1963-64, the data on the Rabi crops were missing). From this we projected forward the acreage under different crops, assuming the distribution of the crop pattern to remain constant at the average % - distribution over the years 1958-59 to 1963-64. This runs upto the year 1967-68 when our projection of total irrigated area calculated on the basis of Table

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2. Programe Evaluation Organization:Planning Commission, Govt.of India-1965.P.E.O.Publication No. 50.

hits the series 'of planned' irrigated area as estimated by the project planners. So for period 1968-69 to 1971-72, we allow our estimate of total irrigated area to run along this ceiling and get the acreage under various cross for this period by assuming the same distribution pattern of crops specified in II·3-3. Having gotten the A_{it} matrix, we multiply each row of it by the only series of water rates we have, the vector r_{it*}, listed in Table 6. The resulting vector $\tilde{R}_t$ is the vector of revenues that would have accrued if water rates had remained constant over the entire period. The ratio $R_t/\tilde{R}_t$ (3) is the correction factor that is applied at each t, to the water rate specified for paddy at t=t* (i.e., r_{jt*}) in order to get the 'competitive' water rate vector WR(t) used in the equations to represent the price of irrigation water for paddy production. Two assumptions are implicit. One, in using $R_t/\tilde{R}_t$ as the correction factor, we are assuming that the ratio of the water rates charged for any two crops is the same for all the years, i.e., the relative importance of irrigation water to the different crops remains unchanged over the years. Two, since the very formulation of the cost function implies, or necessitates, competitive factor markets (just as a production function implies efficiency), and since the R_t series seems to have

(3) R_t is total revenue expected from the project: it was estimated by project planners on the basis of net benefits to farmers, their ability to pay, as also to cover 'interest costs' (EMIP, p.74).

been computed on the basis of the net benefit to the farmers due to the use of irrigation water and their ability to pay, the computed water rate series WR(t) is the closest we can get to competitive price for irrigation water for paddy.

Note that the computed series is not a water rate series per se but rather a series of average cost of irrigation water per acre of paddy land. The reason this can be a fairly good instrument variable for irrigation water rate is a specific condition prevailing in most of these irrigation projects. Water is usually rationed out to users according to their estimated needs. Assuming that this is so, cost of water per unit of land can be taken as a substitute of the price of water.

Since our water rate series depends vitally upon the assumed acreage distribution of crops, we decided to do some sensitivity analysis by altering our assumption in this respect. To get an alternative series of water rates and to test its effect on the results of the regressions, we extrapolated the acreage under different crops separately. The projected acreage is given in Table II.3.3 along with the R* series that emerges from the pattern (Table II.3 -4) Note that the new series of projected acreage lags far behind the 'planned' acreage of the project planners. Hence the resulting water rate series is likely to be higher than the original WR series, especially towards the end of the sample period.

Cost of production:

The cost of production series is obtained by solving a first order difference equation. Something of this nature is inevitable since the marginal cost of production of rice is stated directly in the project report for only one of the 14 sample years.(1) We are, however, told that an Indian government study projection of 3.39% annual rate of growth of paddy in the region has been used in the calculations of project planners,(2) i.e., $\dot{X}/X = .0339$. In the absence of any other information, we had assumed dC/dX to be constant over the sample period. The exact figure for dC/dt was then derived as follows:

$$\text{Given} \quad \frac{\dot{X}}{X} = .0339$$

$$\text{or,} \quad X(t) = X(0)e^{.0339t},$$

we have

$$\dot{X}/X$$

$$dX/dt = X(0)\ .0339\ e^{.0339t}$$

As we are concerned with cost per pound, X(0) is assumed to be 1.

(1) Cf.Tables and data of page 27. The first entry in the 6th column of Table on p.245(EMIP), is derived by dividing the cost of additional yield of paddy per acre by the additional yield per acre per Ib.and X is output, dC/dX= .07.

(2) 'Growth Rates in Agriculture'(Directorate of Economics and Statistics) EMIP, p.243/5)

$\therefore$ $dX/dt = .0339(1+.0339t+(.0339t)^2/2! + \ldots)$

$\approx .0339$

$\therefore$ $dC/dt = dC/dX . dX/dt = .07 . (.0339) = .0025$

One way to generalize the above result would be to include some more terms in the expansion for dX/dt, but we decided against it since the resulting quantitative difference for the given sample did not seem to be worth the effort.

The alternative series in cost of production- CP1 & CP 2 are derived in the following way: Unable to make any a priori alternative assumption regarding the behavior of marginal cost from given information, we decided to tackle the problem from an altogether different angle, which would, avoid the constancy of marginal cost. Note that

$$\frac{\dot{C}'}{C} = \frac{1}{C}\,\frac{dC}{dt} = \left(\frac{X}{C} \cdot \frac{dC}{dX}\right)\left(\frac{1}{X} \cdot \frac{dX}{dt}\right)$$

$$= \frac{d\log C}{d\log X}\ , \quad .0339$$

For the year 1961-62, the cost/revenue ratio of per unit output of rice is 7/17 (Cf. Table attached), Assuming this ratio to persist over the sample period, we have

$$\frac{d\log C}{d\log X} = \frac{7}{17} \cdot \frac{d\log R}{d\log X}$$

Since $R = p.X$, where p is price per unit of output

$$\frac{d\log R}{d\log X} = 1 + \frac{d\log p}{d\log x}$$

where $\frac{d\log p}{d\log x} = \frac{1}{\mathcal{E}}$ is the inverse of the price elasticity of demand for food which, in all probability, is less than one. We have calculated the expression (a) for two alternative values of the parameter $\mathcal{E}$,=0.5 and 0.25, so that the corresponding figures for dlog R/dlog X are respectively 1 + 1/.5 = 3, and 1 = 1/.25 = 5. Therefore, two alternative cost of production series emerge:

$$\text{(i)}\ \dot{C}/C = .0339 \times 3 \times 7/17$$

$$\approx .04.$$

so that

$$C(t) = C(0)\ e^{.04t}$$

$$\text{(ii)}\ \dot{C}/C = .0339 \times 5 \times 7/17$$

$$\approx .07.$$

so that

$$C(t) = C(0)e^{.07t}.$$

These two series are called CP1 and CP2 respectively. The initial C(0) is obtained in both the cases from C(4) =.07.

An alternative approach might have been to assume long run competitive conditions to exist, so that C=R. We calculated the cost of production series that emerges from this assumption and tested our equations against them. The results were less satisfactory than if we allowed a cost/revenue gap to persist over the years. In fact this latter assumption is better on a priori grounds too, since the cost quoted here is cost at source of production whereas the revenue per unit would include such things as a transportation costs, storage costs, dealer charges and so on. It would have been better if we could get the actual C/R ratio for each year separately for the sample period, but as usual, data were lacking.

The exercise performed with the Kakrapar Project data is purely a demonstrative one. For one thing, the degrees of freedom are much too few to make any kind of conclusive statement. Given such limitations, however, the L.I. cost function seems to fare reasonably well vis-a-vis, say, the Cobb-Douglas cost function (Eq. II.3-6 to 8). which is the dual to a Cobb-Douglas production function.

We start from Ishikawa's identification of the two leading inputs in S.E. Agr., as being irrigation water and a composite input represented by fertilizers. The only two input prices included in the cost of production are the price of fertilizer and the water rate. Apart from trying to examine the explanatory power of the L.I. hypothesis, we wish to examine the possibility of switches in the role of these two inputs. As for the remaining factors, we can make either of two assumptions:

(i) that the sum total of their contribution to cost remains constant over the period examined, or,

(ii) that the sum total of their contribution as a % of total cost remains constant over the period.

For the first case, we should include a constant in the regression equation ; for the second, we sould not include any. In this case the estimated coefficients would pick up a factor of proportionality.

If $CP(t) = \sum_i a_i(t)\ P_i(t) + \sum_j b_j(t) P_j(t)$,

where P_i's are the included prices and P_j's are the excluded ones, and if

$$\frac{\sum_j b_j(t) P_j(t)}{CP(t)} = \lambda \text{ , a constant for all t,}$$

then the equation above can be rewritten as

$$CP(t) = a_i(t)\ P_i(t) + \lambda .\ CP(t)$$

$$\text{or, } CP(t) = \sum_i \left\{ \frac{a_i(t)}{1 - \lambda} \right\} . P_i(t) = \sum_i d_i(t)\ P_i(t)$$

To the extent λ is small, the estimated coefficients d_i's will be close to the a_i's . But the thing to note is that the identifiability of the $a_i(t)$ coefficients is not of primary importance, since what we are primarily interested in is the time profile of the productivity coefficients, which do remain identifiable up to a scalar multiplication. However, there are two corrections to be made in the printed equations without a constrant term. First is the value of R^2. Printed R^2 is calculated on the basis of the formula

$$R^2 = 1 - \frac{\sum e_t^2}{\sum (y_t - \bar{y})^2}$$

This formula is correct if there is a constant term in the equation, for then $\bar{e} \equiv 0$. It is not so ff the equation contains no constant term as an explanatory variable. The proper formula for such cases is

$$R^2 = 1 - \frac{\Sigma_t e_t^2}{\Sigma_t y_t^2}.$$

Secondly, the printed D-W statistics for equations without a constant have no bearing on the actual degree of auto-correlation present in these equations, since the D-W is computed on the basis of the assumption that the estimated equation contains a constant term.

The results obtained are quite robust with respect to the alternative cost of production series, but not so with the alternative water rates. This is because the divergence between planned acreage (used to calculate the WR series) and acreage projected from past trend (used to calculate WR 1) is very high -- almost of the order of 3 to 1 for the later years.

Equations II.3-1 to 3 are regressions run against the three alternative cost of production series with WR 1 as the price of water. The t-values of the coefficients are almost always significant. Also, all three equations indicate similar time profiles of productivity coefficients. similar to one another and to the broad pattern that emerges from the other two cases (Japan and Taiwan) studied here. The sign profile of quadratic productivity coefficients for both the inputs are also identical -- possibly indicating the complementarity of irrigation water and chemical fertilizers, although the points of extreme are separated in time inside the sample period. The equations with WR instead of WR1 as an explanatory variable do not come out with such clear cut features (Cf. eqs.4 and 5). As indicated earlier, this could be caused by the unnaturally low WR values, especially in the later years, generated by an unrealistically high 'planned' irrigated acreage series.

The same data tested against Cobb-Douglas cost functions (eqs.6 -8) come out with results clearly worse than L. I. functions-- often with highly significant negative coefficients. The C-D functions with CP1 & CP2 as dependent variables have multiplicative technical change function A (t) of the form t where is a constant, rather than ofthe usual exponential form $A(t) = e^{rt}$, where 'r' is the constant rate of technical progress. This modification of the usual form was necessitated by the nature ofthe dependent variable series CPI and CP2, which are themselves solutions of exponential equations.

II.4 Production of corn in Dillon County, South Carolina: A study in contrast:

This case-study is motivated by the desire to find out how badly, or well, the L.I. function fares when the presumed conditions for its applicability are absent, i.e. where ex-ante substitution possibilities are indeed varied, and the choice of techniques of production at any particular point of time is presumably dependent on relative factor prices. The obvious choice was the United States, both because of the sophistication of its agricultural production techniques, and the easy availability of data. We decided to perform the experiment on micro data because in estimating a production function via its dual cost function, we are treating factor prices as parameters --and that is a more defensible position to take if the data are of the micro type.

The choice of Dillon County in South Carolina is motivated by no particular fact but, again, availability of some data over and above what is obtained in the Agricultural Census volumes and other standard U.S. Department of Agricultural publications.(Cf. section on data sources), & the history of this part of the country as a stable corn (and other crops) producing region in recent years. Going through the agricultural history of S. Carolina, and the old issues of the Census of Agriculture, it was clear that up until the early part of the 20th century, the northwest section of S. Carolina, where Dillon County is located, had cotton as its major agricultural produce. But gradually around the twenties corn replaced cotton as the main production items, and since then it has remained that way. We figured that if the L.I. function depends for its applicability on the prevailing conditions of under-developed agriculture, one would expect it to give not-too-good results in the context of developed agriculture : indeed,standard neo-classical production functions like the Cobb-Douglas ought to explain facts better than the L.I. in such contexts. We tested corn production data in Dillon County for the years 1945 to 1964 - the year for which the latest census data were available --against both the L.I. and Cobb-Douglas cost functions, and true to our hunch, the Cobb-Douglas fared better than the L.I. in explaining the present data, and fared relatively much better than in the previous cases. On the other hand, the L.I. function fares much worse than before, --sometimes giving out coefficients that stay negative for the whole range of the sample period and beyond, even with a quadratic time function, and more often than not failing to show up the alternating parabolic time profiles as before.

One ought to take these results with a grain of salt, however,For one thing, here as in the Indian case, the degrees of freedom were very few. The reason is, of course, scarcity of data. It is virtually impossible to get published data useful for our purpose on a county by county basis. The

The best we could do was to take census data published once every five years, and interpolate for the intervening years --a procedure quite unsatisfactory from the point of view of econometric estimation, and yet the best we could think of under the circumstances. The best we could hope for was to get a verfication of the hypothesis that the L.I.function fares relatively worse, and the Cobb-Douglas fares relatively better, in this case as compared to the previous ones discussed, and the results clearly seem to indicate that.

Data and data sources:

Cost of production of corn per bushel in Dillon County:

No data on cost of production of corn in Dillon County was available. We had to construct the series on the basis of various assumptions. A manuscript entitled 'Selected South Carolina Economic Data' by J.D. Conklin and R.A. Quesinberry (published by the Bureau of Business and Economic Research, University of S. Carolina, December, 1969) contains a series on S. Carolina Farm Production Expenses (Table 5, pp.15-16) for the years 1949 to 1968. The series on the farm value of corn production in Dillon County and in all of South Carolina were available at intervals of five years from the various issues of Census of Agriculture. The farm value of total agricultural production for the two places was also available from the same source. For calculating the cost of production of corn in Dillon Country, we made the following assumptions:

$$\text{(i)}\ \frac{\text{Production expense of corn in Dillon County}}{\text{Farm value of corn production in Dillon Co.}} = \frac{\text{Production expense of corn in South Carolina}}{\text{Farm value of corn production in South Carolina}}$$

$$\text{(ii)}\ \frac{\text{Production expense of corn in South Carolina}}{\text{Farm value of corn production in South Carolina}} = \frac{\text{Production expenses in all agriculture in South C.}}{\text{Farm value of all agricultural production in S.C.}}$$

Assumption (i) implies that the rate of profit earned in corn production in Dillon County is equal to average rate or profit earned in corn production in all of South Carolina, and assumption (ii) implies that the rate of profit earned in corn production in South Carolina is equal to the average rate of profit earned in all agricultural production in the State. The first is a pretty defensible assumption in the absence of specific facts to the contrary, specially since Dillon County appears to be pretty much in the middle of the range of S.C. countries so far as earnings per acre go. The second assumption implies that corn is neither in the most profitable nor in the least profitable zones of all agricultural production in S. Carolina. Note that unlike in the Japanese case, this assumption does not imply anything about the relative organic compositions of the various agricultural sectors, nor does it depend on corn being the most important agricultural produce of the state, since in this case, unlike in the Japanese, production expenses include depreciation and replacement allowance for fixed capital equipment, rent of

Land and such expenses as interest payments on mortgages.

Putting these two assumptions together, we have

Production expenses of corn in Dillon County

= Farm value of corn production in Dillon County, times the average rate of profit for all agricultural production in South Carolina, where this latter is defined as (being equal to

$$\frac{\text{Production expenses for all agriculture in S.C.}}{\text{Farm value of all agricultural production in S.C.}}$$

The production expenses of corn in Dillon County has then been divided by the total number of bushels of corn harvested in D.C. in Corresponding years and converted to 1957-59 prices to give the cost of production of corn in Dillon County per bushel (CCDC) -- the dependent variable in our equations. The figures for the years 1945 to 1948 are estimates.

Farm Wage rate:

The series of wage rate for farm laborers is taken from the book 'Selected South Carolina Economic Data' cited above. The specific series chosen is the annual average for South Carolina of wage rate with room and board in dollars per day. This is given in current prices, and has been converted to 1957-59 prices to give our wage rate series WRDLZ.

Price of farm machinery:

No series of farm machinery prices was available for South Carolina. Hence we decided to use the series on the index of prices of arm machinery paid by farmers in all United States. This series is taken from various issues of 'Agricultural Statistics' published by the U.S. Department of Agriculture. The original series was given in 1910-14 prices. We recomputed it in 1957-59 prices to get the farm machinery price index series (FMPIZ) used in our equations.

<u>Price of fertilizers</u>:

Of the two fertilizer prices ued in these equations, one is the price per ton of sulphate of ammonia taken from the various issues of the Dept. of Agriculture's annual publication 'Agricultural Statistics' cited above. The other is a price index of fertilizers paid by the farmers in all U.S., also taken from the same source. Both have been redone in 1957-59 prices. These are our two series PSAFZ and FPOBZ respectively.

Equations 1 to 3 are applications of the L.I. function on the data at hand, whereas equations 4 and 5 are of the Cobb-Douglas form. In general the L.I. functions have come out with better R^2, but then this could be attributed to the relative meagreness of the degrees of freedom as compared to the other case. The interesting thing to note is that the L.I. function shows consistently worse performance than in the previous cases. The t-values are in general worse than in the Cobb-Douglas functions, The coefficients are often negative for all or most of the sample period, and the alternating pattern of time profiles of coefficients is not as evident as before. On the other hand, the Cobb-Douglas functions seems to be in much better shape than before. The t-values are good, except for the wage rate, which is insignificant in both the cases. The sign pattern of coefficients in eq. II 4.4 are just what one would expect if the true cost function were Cobb-Douglas : positive for the factor price coefficients, and negative for the constant term and the coefficient of time -- indicating a 0.7% rate of technical progress. In equation II 4-5 we had the logarithm of total output as an explanatory variable to take care of the possibility of non-constant returns to scale, and the coefficient of the term comes out to be highly significant. The coefficient of time in this equation is positive, contrary to usual expectations, but this can be explained, perhaps, by the introduction of the phenomenon of increasing returns to scales via the total output variable.

Not all is well with the Cobb-Douglas representation though. The D-W is quite unsatisfactory in both the cases. The D-W is also pretty bad for equations II.4-1 & 2. The thing to remember in emperical investigations of this nature, specially where data are so scarce, and often so unrepresentative, and autocorrelated, is that the results can best be indicative of a pattern, and nothing more. With all such reservations in mind, it seems to be reasonably clear that the relative positions of the L.I. and the Cobb-Douglas hypotheses as descriptions of the production structures have indeed reversed in this case as compared to the two cases analysed earlier.

III

Statistical analysis of the L.I. function:

Section III is concerned with statistical analysis of the L.I. function. The reasons behind the choice of a Leontief type function with time-varying coefficients and the specific statistical hypotheses tested are described below.

Choice of a Leontief type function

By the Diamond-McFadden - Rodiguez theorem('Identification of the Elasticity of Substitution and Bias of Technical Change' in 'An Econometric approach to Production Theory' edited by D.L. McFadden), it is not possible to simultaneously identify the elasticity of substitution and biases in technical change from price-quantity data. In a recent paper L. Lau & S. Tamura (JPE, 1972) have used and estimated a non-homothetic Leontief-type production function.
On the basis of the above mentioned theorem, they have a priori assumed the production function in Japanese petrochemical industry to be 'limitational' (Leontief type) and proceeded to study other effects. In our case, too, the presumption of complementarity in production was based on uniformly superior performance of the L.I. function vis-a-vis the Cobb-Douglas. For Lau & Tamura, the assumption of a 'limitational' function was presumably prompted by similar earlier studies for Japanese chemical

industry where it seemed to have performed well. In our case the hypothesis of a Leontief function was suggested by Ishikawa's findings about strong complementarity in some input uses, together with the usual experience in developing countries of the malperformance of 'traditional' factors after a certain stage of development is reached . (e.g., Schultz:Crisis in World agriculture). The empirical evidence he cites neatly fits into our scheme of things.

Choice of the input requirement functions:

The input demand function for any input X_i implied in our model is $X_i(t) = a_i(t) \cdot Y(t), \quad a_i \neq a_j$ ..... (1)

where Y(t) is output at time t.

The input demand functions estimated by Lau & Tamura are

$$X_i(t) = a_i(t)\, h_i[Y(t)] \quad \cdots\cdots\cdots \quad (2)$$

Here total input demand is subjected to two kinds of effects (i) scale effect, coming through changes in Y, and (ii) technical progress effect a_i represented by an exponential function of time. Although (2) is apparently more general looking than (1) the appearance is deceptive. For under no circumstances does their technical progress parameter (γ_i) come out to be significantly different from zero; i.e., in effect for $\forall i$, (2) can be written as

$$x_i(t) = a_0\, h_i[Y(t)] \quad - - - - - \quad (3)$$

This is so because, some kind of impossibility theorem operates for the simultaneous identification of scale effect and technical progress effect also. As they conclude : 'Any technical progress ...must be embodied in scale' (p.1184). To the extent that this si so, the two effects are not separately identifiable. This unpossibility is all the more prominent if all the data we have are time series data, as in our case.

Thus although in form (1) is a special case of (2) with $h_i(Y) = Y \ \forall i$, the generality is hard to capture in practice. The choice boils down to one between 1 & 3, & a priori there is not much reason to prefer one over another. However, in our case, choice of (1) is prompted by two reasons :(i) all the data we have are time series data for which (1) seems to be the more appropriate alternative since the one thing that will vary in a time series study is time, - even if output stays constant and (ii) since we wanted to estimate cost functions rather than individual input demand functions, Choice of (3) would have led to non-linearities in exogenous variables: time being a non-stochastic variable, the problem of non-linearity in estimation is avoided by selecting (1).

Keeping these observations in mind, the $a_i(t)$ functtions should be interpreted not as technical progress functions but as unit input requirement functions.

Movement of Unit input requirement under (2) :

The input demand functions that Lau and Tamura estimates are of the form:

$$X_i(t) = a_i(t)\ h_i(Y(t)).$$

$\therefore$ Unit input requirement

$$x_i = \frac{X_i}{Y} = a_i \cdot \frac{h_i(Y)}{Y}$$

$$\therefore \frac{\dot{x}_i}{x_i} = \frac{\dot{a}_i}{a_i} + \frac{\dot{Y}}{Y}\left[\frac{h_i'(Y).Y}{h_i(Y)} - 1\right] \quad .....(4)$$

where $\frac{h_i'(Y).Y}{h_i(Y)}$ is the elasticity of the function $h_i(Y)$ with respect to Y. Since a_i is the technical progress function, $\frac{\dot{a}_i}{a_i}$ is < 0. [Note that the $a_i(t)$ functions are the inverse of the usual technical progress functions]. But in general we cannot say anything regarding the sign of the other term. If there is constant returns to an input X_i, $\frac{h_i'Y}{h_i} = 1$, and $\frac{\dot{x}_i}{x_i}$ is definitely < 0. If not, $\frac{h'Y}{h(Y)}$ may have increasing returns $\left(\frac{h'Y}{h(Y)} < 1\right)$, but $\frac{\dot{Y}}{Y}$ may be < 0, so that $\frac{\dot{x}}{x}$ may be > 0. Or, for positive $\frac{\dot{Y}}{Y}$, the returns to an input may be decreasing -- as in the case of the capital input for Ozaki (Economies of scale and Input-output Coefficients : Essays in honour of Wassily Leontief, ed. by Carter & Brody)--so that $\frac{\dot{Y}}{Y}\left(\frac{h'Y}{h} - 1\right)$ will be

> 0, and for small $\frac{\dot{a}_i}{a_i}$ (as for Lau & Tamura), $\frac{\dot{\mathfrak{X}}}{\mathfrak{X}}$ may well be > 0. In general, the model imposes no a priori restrictions on the movement of unit input requirement over time. This is consistent with our model where also no restrictions are placed on the movement of unit input requirements so long as the temporal dominance condition is largely satisfied; i.e., so long as a later period technique is not dominated by one, or a convex combination of more than one, previous per od techniques (Cf. appendix II-1 on dominance condition).

The L.I. function :

Having stated above the reasons for the general formulation, we shall now try to define more sharply what we mean by the L.I. function. It is a function which should satisfy the following three properties:

(i) It belongs to the class of Leontief functions with parameters varying over time.

(ii) Given exogenous factor supplies, the parameters change in such a way that different factors become binding constaints over disinct predictable time intervals.

(iii) Over a period in which an input X_i is binding, unit input requirements of other factors will fall as the supply of X_i rises. The input requirement of the leading input X_i itself may or may not rise over time.

or may not rise over time.

Reasons for the choice of a Leontief function have been given above. We have tested for the order of the time polynomial representations of the parameters, which also tests for the hypothesis of pure homotheticity over time (pure Leontief). It is to be noted that the parameter functions $a_i(t)$'s are estimated using a cost function, so their estimates are linear combinations of the input price vectors and the cost of production (effective at the corner points of L - shaped iso quants) and so they are independent of factor supplies. Sine the $a_i(t)$'s have been estimated from the dual cost side, in principle they are independent of factor supplies. Drastic changes in factor supply composition over time would indeed change the <u>level of output</u> drastically, but in our set-up, they should not affect the unit input requirements. This reflects the lack of responsiveness in the model of choice of techniques to short run composition of factor supplies. In long time series studies, one can presumably ignore such considerations.

We have constructed the following tests on one representative equation each of Japan and Taiwan. They are:

$$(CPRK)=C+a_1(t)(PFS3Z)+a_2(t)(SWR5Z)+a_3(t)RPFTZ$$ – for Japan

$$ICP = C + a_1(t)(PIDC)+a_2(t)(SWR5)$$ – For Taiwan.

Here, PFS3Z, PIDC are fertilizer prices & SWR5, SWR5Z are water rates. RPFTZ is rent on paddy field.

In testing for the order of the time polynomials approximating the true $a_i(t)$ functions, we found that if we write

$$a_i(t) = a_{i0} + a_{i1}t + a_{i2}t^2 + a_{i3}t^3$$

the total explanatory power of the cubic coefficients a_{i3} (i = 1,2 for Taiwan & i = 1,2,3 for Japan) is not significantly different from Zero at 99% confidence level-whereas the quadratic parameters $a_{i2}'s$ come out with highly significant explanatory power. Testing for the hypothesis that the functions are quadratics in time as agains that they are cubic, linear or pure Leontief, the latter are always rejected at 1% critical level. However, if H_0 is that the true functions are linear in time one cannot reject the hypothesis that they are pure Leontief for Taiwan. The relevant F-values are given in the following table. Polynomials of order higher than the cubic have not been tested.

Let the cost functions be given by:

$$C(t) = \sum_{i=1}^{m} \left[\sum_{j=0}^{3} a_{ij} t^j \right] P_i(t).$$

where m = 2 for Taiwan & = 3 for Japan.

Tests	JAPAN Computed F	JAPAN Critical F(at1%)	JAPAN Results	TAIWAN Computed F	TAIWAN Critical F(at 1%)	TAIWAN Result
Test 1						
H_0 : a_{ij} =0 i,& j=3	F^3_{53}= 1.24	F^3_{53}= 4.20	Reject H_A	F^2_{30}= 2.01	F^2_{30}= 5.39	Accept H_o
H_A : a_{ij} 0 i,& j=3			Accept H_o			
Test 2						
H_o : a_{ij}=0 i, & j=2,3	F^3_{56} = 12.73	F^3_{56} = 4.16	Reject H_o	F^2_{32} =15.97	F^2_{32} = 5.34	Reject H_o
H_A = a_{ij} 0 i, j=2						
and a_{ij} = i, j =3						
Test 3						
H_o : a_{ij}=0, i,& j=1,2,3	F^3_{59}=10.47	F^3_{59}=4.13	Reject H_o	F^2_{34} = 2.47	F^2_{34}=5.29	Accept H_o
H_A : a_{ij} = 0 i,j = 1,&						
a_{ij} = 0 i & j=2,3						
Test 4						
H_o : a_{ij} = 0 i,j=1,2,3	F^6_{56} =8.27	F^6_{56}= 3.16	Reject H_o	F^4_{32} =10.32	F^4_{32}=3.97	Reject H_o
H_A a_{ij} 0 i,& j=1,2						
& a_{ij} = 0 i, & j = 3						

(ii) Given the form of the estimated functions,

$$C\ (t) = \sum_{i=1}^{m} \sum_{j=o}^{2} \left(a_{ij}\ t^{j} \right).\ P_i\ (t),$$

We estimated the following equations:

$$\frac{\Delta\left\{\hat{a}_1\ (t)\ /\ \hat{a}_2\ (t)\right\}}{\left\{\hat{a}_1\ (t)\ /\ \hat{a}_2\ (t)\right\}} = \alpha_1 + \beta_1 \cdot t \qquad \ldots(i)$$

and

$$\frac{\left\{X_1(t)\ /\ X_2(t)\right\}}{\left\{X_1(t)\ /\ X_2.(t)\right\}} = \alpha_2 + \beta_2 \cdot t$$

where $\hat{a}_1(t)$ and $\hat{a}_2(t)$ are estimates for $a_1(t) + a_2(t)$ --coefficients for fertilizer and irrigation water prices respectively, and X_1, X_2 are supplies of the respective inputs. The objective is to use the estimated $\hat{a}_1(t)$ functions to test for the possibility of a swithc in the leading inputs inise the sample period. The $a_i(t)$ functions are estimated at the corner of the L - shaped isoquants - by virtue of the fact that a cost function is by definition, efficient. But actual output will depend on factor supplies as well. It is impossible with the available data to tackle the problem in its full generality.What we have done here is to test for a rejection of the hypothesis that a swithc in the binding constraints has occured within the sample period. Conser the situation where

$$\frac{X_1(o)}{a_1(o)} > \frac{X_2\ (o)}{a_2\ (o)}\ ,$$

i.e, that irrigation water is the first binding constraint. Then a switch cannot occur if X_2/X_1, rises at a rate lower than that of a_2/a_1. Therefore, if we approximate the time rates of change of these two ratios by linear functions, the hypothesis $H_o: \beta_1 > \beta_2$ being rejected would imply the rejection of the hypothesis of no swtich, where β_i's are the coefficients of time in (i) above.

$$\therefore \quad H_o : \beta_1 - \beta_2 \leq 0 .$$
$$H_A : \beta_1 - \beta_2 > 0 .$$

Japan:

Computed $t = 3.34$

Critical at 1% $t = 2.61$

Reject H_o

Taiwan

Computed $t = 9.89$

Critical t at 1%=2.63

Reject H_o

Thus hypothesis of no swtich is rejected at 99% for both Japan & Taiwan.

(iii) Knowing that there is likelihood of a swtich, we tried to locate the approximate time for both Japan & Taiwan. From a comparison of exogenous factor supplies with the estimated $\hat{a}_i(t)$ functions the swtich for Japan seems to have occured between 30-35 years (where $X_i/X_j \approx \hat{a}_i / \hat{a}_j$). Prior to this, irrigation would be the leading input and after this fertilizer.

Test (ii) above reveals that irrigation being the first constraint is consistent with the data. We tested for property (iii) by taking estimated input requirement for fertilizer for the first 30 years for Japan and regressing it on supply of irrigation water and land. As expected, coefficient for irrigation is negative and significant,that for land is not. If the time horizon is expanded to the first 35 years and then to 40 years, the explanatory power of irrigation as the leading input goes doen as manifested by gradual fall in the t-values. If in these same intervals, we regress estimated coefficient of irrigation water rate on fertilizer supply, the resulting coefficient is positive. Hence we reject the hypothesis that fertilizer is the first leading input.

We next take up the second time periods (31-63,36-63 and 41-63), where the hypothesis is that fertilizer is the L.I. and repeat the same experiements. Once again the sign pattern of coefficients turn out to be the way we want them to be :(a) input requirement of irrigation is negatively and significantly related to supply of fertilizer but not conversely and (b) this relationship is best for the time interval which corresponds to the best interval for water being L.I. in the first period.

For Taiwan we tried the same set of regressions with the crucial interval placed around 20-25 years. The role of irrigation as L.I. in the first interval is less clear than in Japan, but one can say for sure that fertilizer input in the second interval satisfies the needed condition (iii) and irrigation water does not.

From these results, though not very sophisticated, we thought that the first tentative conclusions might be derived. It is quite possible for all the data to have been generated by some involved shifts in production function of a completely different nature : we have not proved that to be impossible. However, we do not know whether such a proof is at all possible in empirical work, and secondly, we do not really know if it is necessary. We tried to formalize the oft-mentioned phenomenon of complementarity in agricultural economies in a way amenable to simple analytical examination & incorporate it into a qualitative model of development, and study the consequences. We showed that as we defined the L.I. function, it can explain the role played by different inputs at different times in history. We did not get a full explanation of <u>what moves</u> those $a_i(t)$functions,but then it turns out that may not be possible anyway.And even though we did not, for the purpose of predicting the behaviour of input demands and output response,it is possible that we do not need that information.We are aware of the possibility that this kind of analysis may,in fact,be suited best for micro studies.And that L.I.as a macro phenomenon may be less supportable on a priori grounds. But availability of data is the major constraint in this. All we are claiming is that this is a better way than labelling everything as capital and labor and this may be a useful way of trying to find out how some key inputs affect the system.

Japan

I : Irrigation

F : Fertilizer

T : Land

x : Input requirement of I

y : Input requirement of F'

z : Input requirement of T.

	Dependent variable Y		Dependent variable X	
Interval: Years	Coeff. of I	Coeff. of T	Coeff. of F	Coeff. of T.
(1-30)	-2.10 (-3.06)	17.87 (5.57)	0.005 (5.65)	-0.0007 (-1.68)
(1-35)	-1.22 (-2.04)	11.92 (5.56)	0.005 (5.65)	-0.006 (-1.39)
(1-40)	-0.48 (-0.95)	8.14 (5.61)	0.001 (4.08)	-0.0004 (-1.51)
(31-63)	0.54 (0.39)	-1.14 (-3.76)	-0.96 (-1.65)	49.57 (5.51)
(36-63)	0.98 (0.32)	-127.8 (-1.39)	-1.31 (-1.42)	46.26 (3.72)
(41-63)	-1.72 (-.57)	13.28 (12.96)	-0.0001 (-8.37)	-0.001 (-22.92)

- Figures in brackets are the t - values.

Taiwan

I : Irrigation x : Input requirement of I

F : Fertilizer y : Input requirement of F

	Dependent Variable : y	Dependent Variable : x
Interval: Years	Coefficient of I	Coefficient of F
(1-20)	-0.34 (-6.62)	-0.46 (-11.97)
(1-25)	-0.25 (-2.16)	-0.60 (-11.09)
(1-30)	-0.05 (-0.41)	-0.77 (-11.62)
(21-39)	+0.001 (+3.23)	-2.59 (-16.43)
(26-39)	0.01 (2.23)	-4.13 (-3.01)
(31-39)	0.002 (1.38)	-3.28 (-21.24)

IV.

The econometric study of time series data on paddy production of various Asian countries above has suggested that a somewhat disaggregated production function of the form

$$Q(t) = \min_i \left[\frac{X_i(t)}{a_i(t)} \right]$$

worksbetter than standard neo-classical functions.The idea is similar to S. Ishikawa's 'Leading Input' hypothesis, which states that at various stages of development of such economies different inputs seem to generate the major thrust in agricultural activities, although (a) at any point of time, the coefficients a's are constant and (b) over deteminate time intervals different inputs play the leading role. It has been shown that estimation of the cost function dual to the above equarion for Q leads to the a (t) functions. In the cases studied, irrigation water and fertilizer/better seeds play the role of the first and second leading inputs. Generally labor is one of the non-binding constraints.

Quite clearly, when we switch to something like a two-factor two-sector economy, some of the most important features of the analysis will be lost.Still to get simple, qualitative results which can contrasted with models of the pure neo-classical type, one falls back upon the old capital jelly and hopes that some of the flavour would still be maintained. We will construct a dual economy model where the only important innovation is in the agricultural production function which we call the L.I. function (for Leading Inputs, or

Leontief-Ishikawa) with parameter constraints so as to make capital (K) perform the role of the first leading input. Since at any point of time the productivity coefficients are fixed, one can now postulate a wide range of agricultural wage rates consistent with competitive equilibrium. We will solve the general model where the saving propensities of wage-earners and capitalists are different, and each lies between zero and one. A more general version would distinguish between s_{ω}^{A} and s_{ω}^{I} (propensities to save out of wage income in agriculture and industry respectively) on the one hand, and s_{p}^{A} and s_{p}^{I} (same for profit income) on the other. While having one s_{ω} for all wage income and one s_{p} for all profit income are simplifying, putting $s_{w} = 0$ turns out to be critical. As we shall show, the Kaldorian case $s_{\omega} = 0$, $s_{p} = 1$ most often used in the literature (cf. Dixit (1)) imposes serious restrictions on distributive shares and other economic variables once the agricultural production function is admitted of the L.I. type. In particular, one is forced to choose a Cobb-Douglas production function for industry. In this context, one might refer to the work of Little and Mirrlees (3) citing evidence that savings out of agricultural wage income are strictly positive. On the other hand, the assumption of a Cobb-Douglas production function for industry has also serious analytical and statistical implications (Cf. Marglin, (4)). The ganeral model we have solved will bring up these issues clearly.

Once we introduce a Leontief type production function in agriculture, the question arises, how are the factor prises determined ? We will assume that the wage rate in terms of corn in agriculture is $w = \bar{w}$, given. The assumption is an old one in such models; however, with our production function as opposed to a nwo-classical one, (i) it arises as a technological necessity rather than a bio-social one, and - a related consequence - (ii) it allows us to study the behavior of solutions under parametric variations of $\bar{w}$ between 0 and $1/b(t)$, where $b(t)$ is the imverse of the average productivity of labor at time t. Having fixed $\bar{w}$, and knowing the $b(t)$ and $a(t)$ (inverse of the average productivity of capital), the assumption of L.I. is now consistent with initial agricultural labor surplus $(L_e^A(0) \quad L^A(0))$. Also $\bar{w}L^A$ gives the wage bill in agriculture, so that the average return to capital in land is known $r(t) = (1 - \bar{w}b(t))/a(t)$. Hence under competitive conditions with shiftable capital, one can find the equilibrium $r(t)/p(t)$ in industry for various terms of trade $p(t)$.

Variables.

$Q^A(t)$: Agricultural output, corn, at time t.

$L^A(t)$: Labor gainfully employed in agriculture.

$K^A(t)$: Capital employed in agriculture.

C^A, C^I : Consumption of corn in agriculture and industry respectively.

$p(t)$: Price of the industrial good in terms of corn.

$r(t)$: Rental rate in terms of corn.

$Q^I(t)$: Industrial output.

$K^I(t)$: Industrial capital.

$L^I(t)$: Industrial labor.

$K(t)$: Total capital stock at time t.

$w^I(t)$: Industrial wage rate in terms of corn.

Parameters.

$\bar{w}$: Institutionally fixed wage rate in terms of corn in agriculture.

s_w^A, s_p^A : Savings propensities out of agricultural wages and profits respectively, assumed constant.

s_w^I, s_p^I : Same for industry.

$a(t), b(t)$: Unit input requirements of agricultural capital and labor respectively, at time t.

The model.

The following equations describe the model for the general case where $0 \leq s_w^A \neq s_w^I \leq 1$ and $0 \leq s_p^A \neq s_p^I \leq 1$.

$$Q(t) = \min\left[\frac{K^A(t)}{a(t)}, \frac{L^A(t)}{b(t)}\right]$$: Agricultural production function.

$$\text{or,} \quad Q(t) = K^A(t)/a(t) \quad \text{........(1)}$$

$$= L_e^A(t)/b(t) \quad \text{........(2)}$$

where $L_e^A(0) < L^A(0)$, and K^A is the bindibg constraint.

$$Q^I(t) = F(K^I(t), L^I(t)) \quad \ldots\ldots (3)$$

the industrial production function. We shall assume it to have constant returns to scale and also that $F_i > 0$, $F_{ii} < 0$.

$$\frac{\partial F}{\partial K^I(t)} = \frac{r(t)}{p(t)} \quad \ldots\ldots (4)$$

competitive capital demand in industry.

$$\frac{\partial F}{\partial L^I(t)} = \frac{\omega^I(t)}{p(t)} \quad \ldots\ldots (5)$$

competitive labor demand in industry.

Consumption in the agricultural sector is given by

$$C^A = (1-s_\omega^A)\,\bar{\omega}\,L_e^A + (1-s_p^A)\,rK^A \quad \ldots\ldots\ldots (6)'$$

Consumption in the industrial sector is

$$C^I = (1-s_\omega^I)\,\omega^I L^I + (1-s_p^I)\,rK^I \quad \ldots\ldots\ldots (7)'$$

Since all agricultural output is consumed,

$$Q^A = C^A + C^I$$

$$= (1-s_\omega^A)\,\bar{\omega}\,L_e^A + (1-s_\omega^I)\,\omega^I L^I + (1-s_p^A)\,rK^A + (1-s_p^I)\,rK^I \quad ----(8)'$$

Rental rate on capital, r(t), as fixed in agriculture, is

$$r(t) = \frac{1-\bar{\omega}\,b(t)}{a(t)} \quad \ldots\ldots\ldots (9)$$

Labor supply function to industry is taken to be

$$L^{I} = f\left(\frac{w^{I}-\bar{w}}{p}\right), \quad f'>0, \quad f(0)=0 \quad .....(10)$$

Here we make the implicit assumption that wages are paid in kind in both the sectors, so that industrial wage earners receive their wages in terms of machines, which they exchange for corn. Hence the terms of trade p apperas in the labor supply equation.

Assuming full employment of capital at each t,

$$K(t) = K^{A}(t) + K^{I}(t) \quad .... (11)$$

Lastly, there is the identity equating total investment ($\dot{K}$) to the output of the industrial sector, Q^{I}. Assuming savings-investment equality for the whole economy, we have

$$\dot{K} \equiv Q^{I} = \dot{K}^{A} + \dot{K}^{I}$$

$$= \frac{1}{p}\left[s_{w}^{A}\bar{w}L_{e}^{A} + s_{w}^{I}w^{I}L^{I} + s_{p}^{A}rK^{A} + s_{p}^{I}rK^{I} \right] \quad ...(12).$$

Solution procedure

The static part of the model consists of the first eleven equations in thetwelve unknowns : Q^{A}, K^{A}, L_{e}^{A}, Q^{I}, K^{I}, L^{I}, r, p, w^{I}, C^{A}, C^{I}, and K. Hence, in principle, one can solve for the first eleven variables in terms of the remaining one, K. Substituting in the R.H.S. of (12), we get the differential equation for K describing the law of motion of the system.

Closed form solution of the model as specified above may not be possible and will certainly be very cumbrous. Hence we will assume $s_{\omega}^{A} = s_{\omega}^{I} = s_{\omega}$ and $s_{p}^{A} = s_{p}^{I} = s_{p}$. Then equations (6)′, (7)′, (8)′ and (12)′ become

$$C^{A} = (1-s_{\omega})\bar{\omega}L_{e}^{A} + (1-s_{p})rK^{A} \quad .. (6)$$

$$C^{I} = (1-s_{\omega})\omega^{I}L^{I} + (1-s_{p})rK^{I} \quad ..(7)$$

$$Q^{A} = (1-s_{\omega})(\bar{\omega}L_{e}^{A}+\omega^{I}L^{I}) + (1-s_{p})rK \quad .. (8)$$

and $$\dot{K} \equiv Q^{I} = \frac{1}{p}\left[s_{\omega}(\bar{\omega}L_{e}^{A}+\omega^{I}L^{I}) + s_{p}rK\right] \quad ..(12).$$

Since in the model (equations (1) - (12)), factor shares in industry can rise, fall or stay constant, assumption of a Cobb-Douglas production function for this sector is a purely simplifying one (Cf. section on distributive shares below).

Solution

Assumption of Cobb-Douglas function for the industrial sector replaces (3) above by

$$Q^{I} = (K^{I})^{\alpha}(L^{I})^{1-\alpha}, \qquad 0 < \alpha < 1 \quad ... (13)$$

Then $$\frac{\partial Q^{I}}{\partial L^{I}} = (1-\alpha)\left(K^{I}/L^{I}\right)^{\alpha} \quad ... (14)$$

The industrial labor supply function is given by

$$L_S^I = f\left(\frac{\omega^I - \bar{\omega}}{p}\right), \qquad f' > 0$$

Ideally for $w^I = \bar{w}$, L_S^I should be 0, i.e., unless industrial wage rate is higher than the agricultural wage rate, no supply of labor will be forthcoming to industry, so that $f(x) = 0$ for $x = w^I - \bar{w} \leq 0$. One can postulate an Arthur Lewisian type infinitely elastic L_S schedule fo industry, making $f' = \infty$. This is unsatisfactory for more than the usual reason fo positive transfer costs. For one thing, as labor moves from agriculture to industry, cet. par., per capita consumption in agriculture rises, and one would need to offer higher and higher wages in the industrial sector to induce surplus labor to move. Using the Implicit Function Theorem, we can then write

$$\frac{\omega^I}{p} = \phi(L^I; \bar{\omega}) \qquad \text{where } \phi' > 0 \text{ and } \phi(0) = \frac{\bar{\omega}}{p} \ldots (15)$$

Equating (14) and (15) we have for equilibrium L

$$\phi(L^I) = (1-\alpha)\,(K^I/L^I)^{\alpha}$$

$$\text{Or, } \left\{\alpha + \left(\frac{\phi' L^I}{\phi}\right)\right\} \frac{\dot{L}^I}{L^I} = \alpha \cdot \frac{\dot{K}^I}{K^I}$$

$$\text{or, } (\alpha + \xi) \cdot \frac{\dot{L}_I}{L^I} = \alpha \cdot \frac{\dot{K}^I}{K^I}$$

where $\xi = \frac{\phi' L^I}{\phi}$, is the inverse of the elasticity of the labor supply curve. For a rising L_S , with a positive intercept, ξ is generally a variable. But for large L^I and small $\bar{w}$, we can approximate it by a constant. To see this, let

$$\frac{w^I}{p} = \phi(L^I) = (L^I)^\eta + \bar{w} , \qquad \eta \text{ constant.}$$

Then $$\xi = \frac{\phi' L^I}{\phi} = \eta \left[\frac{1}{1 + \frac{\bar{w}}{(L^I)^\eta}}\right] \rightarrow \eta \qquad \text{as } \frac{\bar{w}}{L^I} \rightarrow 0.$$

Taking η as an approximation for ξ ,the labor market equilibrium situation for industry can be written as

$$(L^I)^{\alpha+\eta} = (1-\alpha)(K^I)^\alpha$$

or, $$L^I = (1-\alpha)^{\frac{1}{\alpha+\eta}} (K^I)^{\frac{\alpha}{\alpha+\eta}} \qquad \ldots (16).$$

From (4),

$$\frac{r}{p} = \frac{\partial F}{\partial K^I} = \alpha (K^I)^{\alpha-1} (L^I)^{1-\alpha}$$

$$= \alpha (K^I)^{\alpha-1} \left\{ (1-\alpha)^{\frac{1}{\alpha+\eta}} (K^I)^{\frac{\alpha}{\alpha+\eta}} \right\}^{1-\alpha}$$

$$= \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha+\eta}} (K^I)^{\frac{\eta(\alpha-1)}{\alpha+\eta}}.$$

Or, $$p = \frac{r}{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha+\eta}}} \cdot (K^I)^{\frac{\eta(1-\alpha)}{\alpha+\eta}} \qquad \ldots (17).$$

Hence

$$w^I \approx p \cdot (L^I)^\eta = \frac{r}{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha+\eta}}} \cdot (K^I)^{\frac{\eta(1-\alpha)}{\alpha+\eta}} \left[(1-\alpha)(K^I)^\alpha\right]^{\frac{\eta}{\alpha+\eta}}.$$

$$= \frac{r}{\alpha (1-\alpha)^{\frac{1-\alpha}{\alpha+\eta}}} \cdot (K^I)^{\frac{\eta}{\alpha+\eta}} \qquad \ldots \quad (18)$$

Putting (16) in (13),

$$Q^I = (K^I)^{\alpha} [(1-\alpha)(K^I)^{\alpha}]^{\frac{1-\alpha}{\alpha+\eta}}$$

$$= (1-\alpha)^{\frac{1-\alpha}{\alpha+\eta}} \cdot (K^I)^{\frac{\alpha(1+\eta)}{\alpha+\eta}} \qquad \ldots \quad (19)$$

From (8), using (1), (2) and (9) we get

$$K^A \left[s_\omega \cdot \frac{\bar{\omega} b}{a} + r \cdot s_p \right] = (1 - s_\omega)\, \omega^I L^I + (1 - s_p) r K^I$$

Substituting for $w^I L^I$ in terms of K^I(equations (16) and (18)) one gets

$$K^A = \left\{ \frac{\frac{(1-s_\omega)(1-\alpha)}{\alpha} + (1-s_p)}{s_\omega \cdot \frac{\bar{\omega} b}{a} + s_p \cdot r} \right\} r K^I \qquad \ldots \quad (20)$$

From eqn. (12) we have

$$\dot{K} = s_\omega \cdot \frac{\omega^I L^I}{p} + s_\omega \cdot \frac{\bar{\omega} b}{a p} K^A + s_p \cdot \frac{r}{p} K \qquad \ldots \quad (21)$$

Since L^I, p, w^L and K^A have all been solved in terms of K^I ((16), (17), (18) and (20)), we can write the R.H.S. of (21) in terms of K^I and K. In (20), replace K^I by $(K - K^A)$ from (12). Then solve for K^A in terms of K. Substituting back in (20), we get K^I in terms of K and all other variables in terms of K, as was required. (21) is now a differential equation in K :

$$\dot{K} = K^{\frac{\alpha(1+\eta)}{\alpha+\eta}}\left[\frac{s_\omega(1-\alpha)^{1+\frac{1-\alpha}{\alpha+\eta}}}{(1+rC)^{\frac{\alpha(1+\eta)}{\alpha+\eta}}} + s_\omega\cdot\frac{\bar{\omega}b}{a}\cdot\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha+\eta}}\cdot C\,(1+rC)^{-\frac{\alpha(1+\eta)}{\alpha+\eta}} + s_p\cdot\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha+\eta}}(1+rC)^{\frac{\eta(1-\alpha)}{\alpha+\eta}}\right].$$

where $C = \left\{\dfrac{\dfrac{(1-s_\omega)(1-\alpha)}{\alpha} + (1-s_p)}{s_\omega\cdot\dfrac{\bar{\omega}b}{a} + s_p\cdot r}\right\}$

r being a known function of time.

Simplifying , we have

$$\dot{K} = K^{\frac{\alpha(1+\eta)}{\alpha+\eta}}\cdot\frac{(1-\alpha)^{\frac{1-\alpha}{\alpha+\eta}}}{(1+rC)^{-\frac{\eta(1-\alpha)}{\alpha+\eta}}}\cdot$$

$$\cdot\left\{\frac{s_\omega(1-\alpha)}{1+rC} + \frac{s_\omega\cdot\frac{\bar{\omega}b}{a}\cdot\alpha\cdot C}{1+rC} + \alpha s_p\right\} \qquad \ldots (22)$$

We want to evaluate the expression inside the bracket, E.

$$E = \frac{s_\omega(1-\alpha)}{1+rC} + \frac{s_\omega\cdot\frac{\bar{\omega}b}{a}\cdot\alpha\cdot C}{1+rC} + \alpha s_p$$

$$= \frac{1}{1+rC}\left[s_\omega(1-\alpha) + s_\omega\cdot\frac{\bar{\omega}b}{a}\cdot\alpha\cdot C + \alpha s_p(1+rC)\right].$$

$$= \frac{1}{1+rC}\left[\{s_\omega(1-\alpha) + s_p\alpha\} + C\left\{\frac{\alpha}{a}s_\omega\cdot\bar{\omega}b + \alpha\cdot r\cdot s_p\right\}\right]$$

$$= \frac{1}{1+rC}\left[\{(1-\alpha)s_\omega + \alpha s_p\} + \left\{\frac{(1-s_\omega)(1-\alpha) + \alpha(1-s_p)}{s_\omega\bar{\omega}b + s_p a r}\right\}\cdot\{s_\omega\bar{\omega}b + s_p\cdot ar\}\right]$$

(substituting for C).

Cancelling terms in the R.H.S. we get

$$E = \frac{1}{1+rC}\left[(1-\alpha)s_\omega + \alpha s_p + (1-\alpha)(1-s_\omega) + \alpha(1-s_p)\right].$$

$$= \frac{(1-\alpha)+\alpha}{1+rC} = \frac{1}{1+rC} \qquad \ldots(23)$$

To simplify further

$$1 + rC = 1 + \frac{ar}{\alpha} \cdot \frac{(1-s_\omega)(1-\alpha) + \alpha(1-s_p)}{s_\omega \cdot \bar{\omega} b + s_p ar}$$

$$= \frac{\alpha(s_\omega \bar{\omega} b + s_p . ar) + ar\{(1-s_\omega)(1-\alpha) + \alpha(1-s_p)\}}{\alpha(s_\omega . \bar{\omega} b + s_p . ar)}$$

$$= \frac{\alpha(s_\omega . \bar{\omega} b + s_p . ar) + ar(1 - s_\omega - \alpha + \alpha s_\omega + \alpha - \alpha s_p)}{\alpha(s_\omega . \bar{\omega} b + s_p ar)}$$

$$= \frac{\alpha s_\omega . \bar{\omega} b + ar - ar s_\omega + \alpha ar s_\omega}{\alpha(s_\omega \bar{\omega} b + s_p ar)}$$

Using $\bar{w}b = 1 - ar$, this gives

$$1 + rC = \frac{\alpha s_\omega (1-ar) + ar - ar s_\omega + \alpha ar s_\omega}{\alpha(s_\omega \bar{\omega} b + s_p ar)}$$

$$= \frac{\alpha s_\omega + ar(1-s_\omega)}{\alpha\{s_\omega(1-ar) + s_p ar\}} \qquad \ldots (24)$$

Substituting (23) and (24) into (22) we get

$$\dot{K} = K^{\frac{\alpha(1+\eta)}{\alpha+\eta}} . \alpha^{\frac{\alpha(1+\eta)}{\alpha+\eta}} . (1-\alpha)^{\frac{1-\alpha}{\alpha+\eta}} . \left\{\frac{s_p . ar + s_\omega(1-ar)}{\alpha s_\omega + ar(1-s_\omega)}\right\}^{\frac{\alpha(1+\eta)}{\alpha+\eta}}$$

$$\ldots (25)$$

This gives the equation for the growth of total capital K over time as function of the saving propensities of the two classes (s_ω and s_p), the technological parameters of the two sectors (a(t), b(t) and α) and the institutionally given agricultural wage rate ($\bar{w}$, since

$r = \frac{1-\bar{\omega}b}{a}$). Note that the exponent of K is positive but less than one ($\frac{\alpha(1+\eta)}{\alpha+\eta} = \frac{1+\eta}{1+\frac{\eta}{\alpha}} < 1$, for $\eta > 0$ and $0 < \alpha < 1$) so that the rate of growth of capital is damped (falls with rising K) unless $\eta = 0$ (infinitely elastic labor supply), where growth is exponential.

The rate of growth of capital as given by (25) varies directly with changes in the savings propensities of both wage earners and capitalists. The expression within brackets on the R.H.S. of (25) can be rewritten as

$$A = \frac{s_p ar + s_\omega(1-ar)}{ar + s_\omega(\alpha - ar)} \qquad \dots (26)$$

It is clear, since s_p enters the expression only in the numerator, a rise in s_p will raise the value of A and hence, the rate of growth of capital. To study the effect of a rise in s_ω on A we note that

$$\frac{dA}{ds_\omega} = \frac{1}{D^2}\left[\{ar + s_\omega(\alpha - ar)\}(1-ar) - \{s_p ar + s_\omega(1-ar)\}(\alpha - ar)\right]$$

(where $D = ar + s_\omega(\alpha - ar)$)

$$= \frac{1}{D^2}\left[ar(1-ar) + s_\omega(1-ar)(\alpha-ar) - s_p ar(\alpha - ar) - s_\omega(1-ar)(\alpha-ar)\right]$$

$$= \frac{ar}{D^2}\left[(1-ar) - (\alpha - ar)s_p\right]$$

which is always positive for $\alpha, s_p, ar < 1$. Hence rise in workers' savings propensity also raises the rate of growth but at a different rate.

To see the effect on $\dot{K}$ of an exogenous rise in r, note that

$$\frac{\alpha\{(s_p - s_\omega)ar + s_\omega\}}{\alpha s_\omega + ar(1-s_\omega)} = \frac{1}{1+rC} \qquad \text{from (24)}$$

Also from (22), $K^A/K^I = rC$, so that

$$\frac{1}{1+rC} = \frac{1}{1+K^A/K^I} = \frac{K^I}{K^A+K^I} = \frac{K^I}{K}$$

Hence

$$\frac{\alpha\{(s_p - s_\omega)ar + s_\omega\}}{\alpha s_\omega + ar(1-s_\omega)} = \frac{K^I}{K}$$ =Share of industrial capital in total capital.

If r rises (given a and b , $\bar{w}$ falls) exogenously,

$$\frac{dA}{dr} = \frac{d}{dr}\left\{\frac{1}{\alpha(1+rC)}\right\} = \frac{-\alpha C}{\{\alpha(1+rC)\}^2}$$, which is < 0.

Computing the expression for $\frac{dA}{dr}$, this becomes

$$\frac{dA}{dr} = \frac{1}{D^2}\left[\alpha(s_p - s_\omega) - (1-s_\omega)\right]$$

where D stands for the denominator. It is clear that the sign of this expression has to be negative for s_p , s_ω , and $\alpha \in (0,1)$ and $s_p > s_\omega$.

Thus any exogenous rise in r induces a fall in K^I/K and in the overall ~~gr~~ rate of growth of capital. This is understandable, since a fall in the wage-rental ratio in the industrial sector would induce a shift towards less capital intensive techniques, whereas the production function in the agricultural sector being of the L.I. type , there is no substitution from capital there. This might help explain the relative paucity of private capital in industry and why despite a presumably high and rising rate of return on capital, private investment seems to stay away from industry.

In conclusion, one result supposed to be rather strange comes out easily from the model. It concerns the response of food consumption in industry C^I, to the price of food, 1/p (Dixit, op. cit.). Depending on the sign of the elasticity of industrial L_S^I, it can go either way. For

$$C^I = \left[\frac{(1-s_\omega)(1-\alpha)}{\alpha} + (1-s_p) \right] r K^I$$

Hence $$\frac{dC^I}{dp} = \left[\frac{(1-s_\omega)(1-\alpha)}{\alpha} + (1-s_p) \right] r \cdot \frac{dK^I}{dp}$$

Since $$p = \frac{r}{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha+\eta}}} \cdot (K^I)^{\frac{\eta(1-\alpha)}{\alpha+\eta}}.$$

Or, $$1 = \left\{ \frac{r}{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha+\eta}}} \right\} \cdot \frac{\eta(1-\alpha)}{\alpha+\eta} \cdot (K^I)^{-\frac{\alpha(1+\eta)}{\alpha+\eta}} \cdot \frac{dK^I}{dp}.$$

so that $\text{sgn}\left(\frac{dK^I}{dp}\right) = f(\text{sgn}(\alpha+\eta))$.

For a backward sloping labor supply curve, η is less than 0. For a very perversely behaved L_S schedule, η can be less than $-1/\alpha$, where α is the elasticity of industrial output with respect to capital. In such cases we shall have the peculiar situation where industrial consumption falls if price of machinery p goes up or if price of food, 1/p, falls.

Distributive shares, savings propensities and the Cobb-Douglas production function.

Discussion of standard two sector models usually do not include those on distributive shares in the two sectors. In a dual economy model, this can be a tricky problem, especially since agricultural wages are assumed to be institutionally given. A model with (a) neo-classical production functions in both the sectors, (b) perfect capital market with shiftable capital between the two sectors and (c) unlimited labor supply to industry at a given wage rate, can generate non-zero labor share only because of a forced positive w. In fact Jorgenson (2) avoids the problem of distributive shares in agriculture because his model has no capital in the agricultural production function. The question comes into sharp focus when one uses a Leontief type production function for the agricultural sector. It can be shown that in the general model we have solved above, if we impose the standard assumption regarding savings behavior of the two economic classes ($s_{\omega}^{A} = s_{\omega}^{I} = 0$, and $s_{p}^{A} = s_{p}^{I} = 1$), or if the share of capital in the two sectors are equal (see below), distributive shares in the industrial sector get frozen, so that the only production function for this sector that is consistent with the rest of the model is Cobb-Douglas. However, no constraint is imposed on the form of theindustrial production function in the model presented above when savings out of wage income are non-zero and capital share in the two sectors are apriori unconstrained. In a model like this, agricultural wage earners would include tenant farmers, as well as self-employed cultivators, and in general s will be strict ly positive. Empirical evidence to this effect can be found in (3) cited above.

To see this, and the conditions for resource transfer between sectors, lwt us considerthe behavior of agricultural savings.

$$S^A = s_\omega \cdot \bar{\omega} L_e^A + s_p r K^A$$

$$= \left(s_\omega \cdot \frac{\bar{\omega} b_e^A}{a} + s_p r \right) K^A \qquad \text{(i)}$$

From p. above,

$$K^A = K - K^I = \frac{rC}{1+rC} \cdot K$$

Substituting in (i),

$$S^A = \left(s_\omega \cdot \frac{\bar{\omega} b}{a} + s_p \cdot r \right) \left(\frac{rC}{1+rC} \right) K \qquad \text{(ii)}$$

Agricultural investment

$$I^A = p\dot{K}^A = p \left(\frac{rC}{1+rC} \right) \dot{K}$$

$$= K \left(\frac{rC}{1+rC} \right) \left[\frac{r}{\alpha} \cdot \frac{1}{1+rC} \right] \qquad \text{(iii)}$$

- substituting from (19), (24) and (25).

Hence surplus savings in agriculture is

$$S^A - I^A = K. \ \frac{rC}{(1+rC)^2} \cdot \frac{1}{a\alpha} \cdot s_\omega (\alpha - ar) \qquad \text{(iv)}$$

Therefore,

$$S^A \gtrless I^A \text{ according as } \alpha \gtrless ar = 1 - \bar{w}b \qquad \ldots(a)$$

In addition,

$$S^A = I^A \text{ if } s^W = 0 \qquad \ldots(b)$$

From (a), if share of capital in industry is greater than that in agriculture, there is surplus for transfer to industry from agriculture. The lower the institutionally fixed $\bar{w}$, the higher is the share of agricultural capital, and the smaller is the surplus savings transferred to industry, given b. For low enough $\bar{w}$, there may even be a drai drain of savings from industry to agriculture. One policy implication of this might be a stiff rate of taxation on rental income in agriculture. If, however, either ar = α and/or $s_w = 0$, then savings equal investments in the two sectors <u>separately</u>. Hence the following equations hold as well :

$$\left.\begin{aligned} p\dot{K}^A &= s_w \cdot \bar{w} L_e^A + s_p r K^A \\ \& \quad p\dot{K}^I &= s_w \; w^I L^I + s_p r K^I \end{aligned}\right\} \text{ for } s_w \neq 0. \qquad \ldots(v)$$

$$\left.\begin{aligned} \text{and} \quad p\dot{K}^A &= s_p r K^A \\ \& \quad p\dot{K}^I &= s_p r K^I \end{aligned}\right\} \text{ for } s_w = 0. \qquad \ldots(vi)$$

From (12) above we have,

$$\frac{rK^I}{pQ^I} = \frac{rK^I}{s_w \cdot \frac{\bar{w}b}{a} K^A + s_w \cdot w^I L^I + s_p r K^A + s_p r K^I}$$

$$\text{or,} \quad \frac{rK^I}{pQ^I} = \frac{1 - s_w}{(s_p - s_w) + \left(s_w \cdot \frac{\bar{w}b}{a} + s_p\right) K^A/K^I} \text{, for } s_w \neq 0 \ldots(vii)$$

and,

$$\frac{rK^I}{pQ^I} = \frac{1}{s_p(1+K^A/K^I)} \quad \text{for } s_w = 0 \text{ ...(viii)}$$

If $s_w = 0$, $\dot{K}^A/K^A = \dot{K}^I/K^I = \dot{K}/K = \frac{r}{p} \cdot s_p$ (from (vi)), so that K^A/K^I is constant, and hence, from (viii), rK^I/pQ^I is so too.

If $s_w \neq 0$, but respective factor shares are equal in the two sectors, then $\bar{w}b/ar = w^I L^I / rK^I$, and from(v)

$$\frac{\dot{K}^A}{K^A} = \left(\frac{r}{p}\right)\left(s_w \cdot \frac{\bar{w}b}{a} + s_p\right)$$

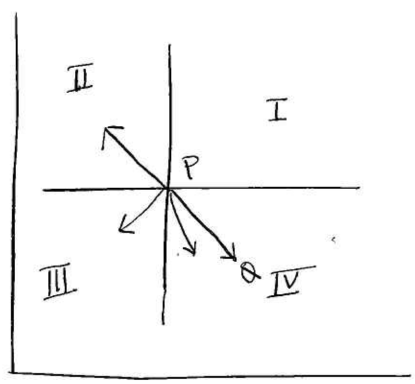
and $$\frac{\dot{K}^I}{K^I} = \left(\frac{r}{p}\right) \cdot \left(s_w \cdot \frac{w^I L^I}{rK^I} + s_p\right)$$

and the two are again equal, so that K^A/K^I is a constant, and from (vii) rK^I/pQ^I si again a constant. Note that this is not true if $s_w^A \neq s_w^I$). In either of these two cases, the only production function for the industrial sector that is consistent with the rest of the model is the Cobb-Douglas. The von Neumann balanced growth path for dual economies needs assumptions we may not be prepared to make.

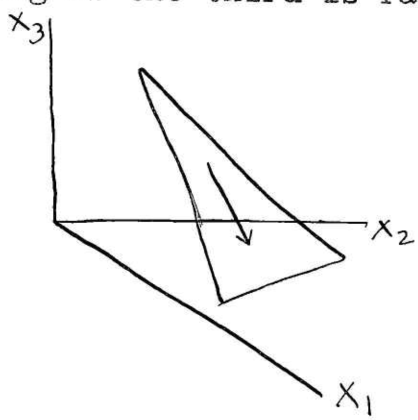
One last point should be mentioned. If indeed the industrial production function is Cobb-Douglas and the agricultural one has fixed coefficients, <u>and</u> $s_w \neq 0$, then one should not , at the same time, postulate an exogenously fixed wage (and hence profit) rate in agriculture along with competitive and shiftable capital between sectors. For then the total ~~savings~~ shares of of labor and capital in G.N.P. get frozen and workers' savings enter into the supply of savings. Then the equilibrium rate of interest on capital will be determined by the joint supply of savings of workers and capitalists matched against the demand for capital and hence we can no longer <u>assume</u> a rate of interest fixed from agriculture. In short, for each choice of K^I/L^I ratio in industry, the <u>total labor income</u> will change since labor transferred from agriculture earns a different wage rate. Hence, unless the factor shares are <u>equal</u> in the two sectors (the second case we studied) with $s_w \neq 0$, changing K/L in industry changes total savings and hence equilibrium investment and profits. This is what does not come out when s_w is assumed to be zero.

Appendix - I.

This appendix is concerned with some regularity conditions regarding productivity movements over time. Since we are estimating time functions a(t) and b(t) which equal the two input requirements per unit of output, what is the guarantee that we shall not encounter a situation which would seem to be ruled out a priori, viz., both rising ? In other words, what if the function predicts both productivities to fall over time ? It would imply that in period 1 , the farmer combines two inputs in a way inferior to that of period 0, and that he does not stay put with the better technique. If he is at P in one period, he can move to quadrants II, III and IV, but not to I (including the boundaries). Any movement to II and IV would seem to indicate improvement in the use of one of the two inputs, and that is admissible. Also if he gets to a point like Q, then he should stay below the line PQ in subsequent periods.



Before turning to an examination of the proposition, first we tried to find out whether such a situation did actually happen. The answer is almost nowhere. It would have been nice to plot the points, but it would have been a cumbrous and laborous job for 50 observations and 2 variables for India and Taiwan combined with about 5 equations for each, and ghastly for Japan, with 63 observations and 3 variables. So we first decided to check what kind of input/output ratios were implied in the regressions. All the tables are attached herewith. In the case of Japan, the variables are moving on a plane and two input coefficients can rise as long as the third is falling. One can slide along the face



of the ABC plane towards the base line AB on the (X_1, X_2) plane and still satisfy the condition of not doing worse than previous periods For a guaranteed 'better' performance, however, one has to move along a surface that is concave to the origin.

However, this is not all the defense of the production function and we should sound a caution against applying restrictions which seem 'evident', but only so under assumptions which may not be applicable here. Indeed if a rise in input requirements of both the factors were a disaster for the model, one way out would be to devise an algorithm by which the input combination chosen in a particular period cannot lie above the envelop of linear combinations of techniques chosen in all previous periods. Of course, if both input requirements are falling, new techniques will always lie below that envelop, and the constraint will never be operative. But then here we are concerned with primitive agriculture over anywhere between 15 to 60 years, and the only significant changes that occurred over such vast time spans are the introduction of the two leading inputs - irrigation and fertilizers. If one specifies a neo-classical timeless production function with positive first and negative second partial derivatives, very little would be asked. There is no provision in traditional neo-classical theory for increasing returns to inputs. But in our model, we submit, we have the only case where one could probably talk about historical diminishing returns and for a valid reason - the static nature of the economic system involved until machinery appears in any large scale (note the bad performance of the L.I. function for Dillon County). We would argue that until that happens, a(t) and b(t) functions are picking up, apart from technical progress, whatever that means, this phenomenon of historical variations of returns to inputs. Inso doing, we wolld at least have the support of Ricardo and other classical economists who were concerned with such phenomena. We submit that it would be worthwhile to take note of this basis institutional difference between modern and traditional agriculture rather than introduce additional constraints in the model which might make it conform to a different economic system, in time or place. Also, traditional agriculture, unlike modern agriculture or industry, is largely conditioned by exogenous factors. Imposing any kind of monotonicity of production techniques would automatically rule out the effect of such factors.

Lastly, falling productivity for even one factor in isolation need not necessarily be interpreted as forgetting of technology. There may be factors not recorded in the model that render an old technique inaccessible - like decline in the fertility of land for instance. Once again, this suggests something akin to the negativity of second partial derivative of neo-classical production functions.

Table I-1 : Japan : Inverse Productivity Coefficients

Equation II 1-3			Equation II 1-10		
a	b	c	a	b	c
2.497	0.367	-0.003	0.003	1.288	-0.061
2.369	0.380	-0.002	0.003	1.244	-0.059
2.245	0.392	-0.002	0.003	1.199	-0.057
2.123	0.405	-0.002	0.003	1.156	-0.055
2.004	0.417	-0.002	0.002	1.113	-0.053
1.888	0.428	-0.002	0.002	1.071	-0.051
1.775	0.440	-0.001	0.002	1.030	-0.049
1.665	0.451	-0.001	0.002	0.989	-0.047
1.557	0.462	-0.001	0.002	0.949	-0.045
1.453	0.473	-0.001	0.001	0.910	-0.043
1.551	0.434	-0.001	0.001	0.871	-0.041
1.253	0.494	-0.001	0.001	0.833	-0.039
1.157	0.584	-0.000	0.001	0.795	-0.038
1.064	0.514	-0.000	0.001	0.760	-0.036
0.974	0.523	-0.000	0.000	0.724	-0.034
0.887	0.532	-0.000	0.000	0.889	-0.032
0.803	0.541	-0.000	0.000	0.654	-0.031
0.722	0.550	0.000	0.000	0.621	-0.029
0.643	0.558	0.000	0.000	0.588	-0.027
0.568	0.566	0.000	0.000	0.588	-0.026
0.495	0.574	0.000	0.000	0.524	-0.024
0.425	0.582	0.000	0.000	0.493	-0.023
0.358	0.589	0.000	0.000	0.463	-0.021

Table I-1 Japan Contd.

0.294	0.596	0.000	-0.000	0.433	-0.020
0.233	0.603	0.001	-0.000	0.404	-0.018
0.175	0.610	0.001	-0.000	0.376	-0.017
0.120	0.616	0.001	-0.000	0.349	-0.015
0.067	0.622	0.001	-0.000	0.322	-0.014
0.018	0.628	0.001	-0.000	0.296	-0.013
-0.028	0.634	0.001	-0.000	0.270	-0.011
-0.072	0.639	0.002	-0.000	0.246	-0.010
-0.112	0.644	0.002	-0.000	0.222	-0.009
-0.150	0.649	0.002	-0.000	0.198	-0.007
-0.186	0.653	0.002	-0.000	0.176	-0.006
-0.218	-.658	0.002	-0.000	0.154	-0.005
-0.247	0.662	0.002	-0.000	0.133	-0.004
-0.273	0.665	0.002	-0.000	0.112	-0.003
-0.297	0.669	0.002	-0.000	0.092	-0.002
-0.318	0.672	0.003	-0.000	0.073	-0.001
-0.335	0.675	0.003	-0.000	0.055	-0.000
-0.350	0.678	0.000	-0.000	0.037	0.000
-0.362	0.680	0.003	-0.000	0.020	0.001
-0.371	0.682	0.003	-0.000	0.003	0.002
-0.377	0.684	0.003	-0.000	-0.011	0.003
-0.381	0.686	0.003	-0.000	-0.026	0.004
-0.381	0.687	0.003	0.000	-0.041	0.005
-0.379	0.689	0.004	0.000	-0.054	0.006

Table I.2: Inverse Productivity Coefficients: Taiwan

Equation II.2-2		Equation II.2-5	
0.459	0.835	0.499	1.128
0.427	0.789	0.462	1.067
0.396	0.745	0.425	1.009
0.367	0.702	0.392	0.952
0.339	0.660	0.357	0.897
0.312	0.619	0.326	0.844
0.287	0.580	0.295	0.792
0.263	0.542	0.266	0.742
0.241	0.506	0.238	0.693
0.220	0.470	0.212	0.646
0.200	0.436	0.187	0.601
0.182	0.403	0.164	0.557
0.165	0.372	0.141	0.516
0.150	0.342	0.121	0.475
0.136	0.313	0.101	0.437
0.123	0.285	0.084	0.400
0.112	0.259	0.067	0.365
0.102	0.234	0.052	0.331
0.094	0.211	0.038	0.299
0.087	0.188	0.026	0.269
0.081	0.167	0.015	0.240
0.077	0.147	0.006	0.213
0.074	0.129	-0.002	0.188
0.073	0.112	-0.008	0.165

Table 1-2 : Taiwan - contd.

Equation II.2-2		Equation II 2-5	
0.073	0.096	-0.014	0.143
0.074	0.081	-0.018	0.122
0.077	0.068	-0.020	0.104
0.081	0.056	-0.021	0.087
0.087	0.046	-0.021	0.071
0.094	0.036	-0.019	0.058
0.102	0.028	-0.016	0.046
0.112	0.021	-0.012	0.035
0.123	0.016	-0.006	0.027
0.136	0.012	0.001	0.020
0.150	0.009	0.009	0.014
0.165	0.007	0.020	0.011
0.182	0.007	0.031	0.009
0.200	0.008	0.044	0.008
0.220	0.011	0.058	0.010

Table I.3: Inverse Productivity Coefficients:Kakrapar Project

Equation II.3-1		Equation II.3-3	
0.31	2.08	0.27	2.48
0.28	1.90	0.25	3.25
0.25	1.75	0.22	2.06
0.23	1.63	0.20	1.93
0.21	1.55	0.18	1.85
0.19	1.51	0.17	1.82
0.17	1.51	0.16	1.83
0.15	1.54	0.16	1.90
0.14	1.61	0.16	2.01
0.13	1.72	0.16	2.17
0.12	1.86	0.16	2.38
0.11	2.04	0.17	2.65
0.11	2.26	0.19	2.96
0.10	2.51	0.20	3.32

Appendix II-1

Variable names : Japan.

(Unless otherwise stated, the source of reference is 'Estimate of Long-Term Economic Statistics of Japan Since 1868'(LESJ) Vol.IX, published by the Hitotsubashi University).

FVRP : Farm value of rice production , in millions of yens (Table 3, col. 1, pp.150-1).

FVTA : Farm value of total agricultural production, millions of yen. Table 3, col.14, pp.150-1.

FVCI : Farm value of current inputs, millions yen. Table 16, col.13, pp.186-7.

TPRJ : Total production of rice in Japan,millions of koku. (Paddy and upland rice combined. 1.2 koku = 2.3 metric tons). Table 12, col. 1 ,pp.166-9.

IAP1 : Index of agricultural prices (all commodities) - base 1904-6. Table 7, col.13, pp.158-9.

IAP2 : Index of agricultural prices (all commodities) - base : 1934-6. Table 8, col. 13, pp.160-1.

IFP1 : Index of fertiliser prices - base : 1934-6. Table 18, col. 6, pp.192-3.

UVNF : Unit value of nitrogen content in fertilizers in thousand yens per metric ton. Table 23, col. 1, pp.202-3. (Reported series starts from the year 1883. Figures for the years 1878-82 are estimates based on later trend).

UVPF : Unit value of potash content in fertilizers in thousands of yen per metric ton. Table 25, col.1, pp.206-7. Data for the years 1878 to 1882 are estimates.

UVPH : Unit value of phosphate content in fertilizers in thosand yen per metric ton. Table 24, col.1. pp. 204-5. Data for the years 1878 to 1882 are estimatøs.

INNF : Input of nitrogen content in fertilizers in thousands of metric tons of N. Table 20, col. 1 , pp, 196-7.

INPH : Input of phosphate content in fertilizers, in thou sands of metric tons of P_2O_5. Table 21, co. 1 , pp. 198-99.

INPF : Input of potash content in fertilizers, in thousands of metric tons of K_2O. Table 22, col. 1, pp. 200-1.

TIPI : Price index of tools and implements , base : 1934-36 (agriculture). Table 31, col.3, p.215.

PFAJ : Paddy field area for all Japan, in hundreds of chō Table 32, col. 10, pp.216-7.

CGRW : Central government riparian works - a part of 'Public Works' - in thousands of yen (Rosovsky, VII-1,IIC). Data from the year 1878 to 1889 are projected on the basis of total expenditure on harbor and riparian construction ; two-thirds of this total is taken to be on riparian work alone. Data for the years 1890-1940 are taken from Rosovsky. All figures are in current prices.

LGRW : Local government riparian works - a part of 'Public Works' - in thousands of current yen.Rosovsky - Table VII-2, col. IIC, pp.171,174.

LGAE : Local government agricultural expenditure (mainly irrigation), in thousands of current yen.Rosovsky - Table VII-2, col. III, pp.172,175.

DWMA : Daily wage rate of male contract workers in agriculture in yens per day. Data for the years 1878 to 1885 and 1889-91 are estimates. Table 34, col. 3, pp.220-1.

YWMA : Yearly wage rate of male agricultural workers in yens. Table 34, col. 1, pp.220-1.

RPFT : Rent of paddy field per 'tan' in yens. Table 34, col. 12, pp.220-1. Data for the years 1878-84, 1886-88, 1890-97 and 1899-1902 are estimates.

SW1Z : Substitute water rate obtained by assuming that the average life span of riparian works (M) is 10 years and the rate of interest is 4% per yr.

SW2Z : Same as above, with M = 15, r/i = 4%.

SW3Z : " , with M = 20, r/i = 4%.

SW4Z : " , with M = 10, r/i = 8%.

SW5Z : " , with M = 15, r/i = 8%.

SW6Z : " , with M = 20, r/i = 8%.

SW7Z : " , with M = 10, r/i = 12%.

SW8Z : " , with M = 15, r/i = 12%.

SW9Z : " , with M = 20, r/i = 12%.

PFS3Z : Price of fertilizer, series 3, as explained in the text.

RPFTZ : Rent of paddy field per 'tan' in yens. The letter Z at the end of a variable name indicates that the variable has been expressed in terms of 1904-6 prices.

PFS3ZT : PFS3Z x TIME. A variable name ending in 'T' indicates that it has been multiplied by time.

PFS3ZQ : PFS3Z x $(TIME)^2$. A variable name ending with a 'Q' indicates that it has been multiplied by the square of the corresponding time period.

LPFS3Z : Log(PFS3Z). A variable starting with 'L' is the logarithmic transformation of the variable defined by the following characters.

a(t) : Coefficient of PFS3Z, quadratic in time.

b(t) : Coefficient of SWiZ (i=1,..,9), quadratic in time.

c(t) : Coefficient of RPFTZ : quadratic in time.

Table II 1.1a : Regression Results for Japan[(1)]

Eqn. No.	Subst. Water Rate Index i.	Dependent Variable	Explanatory Variables: Constant	PFS3Z	PFS3ZT	PFS3ZQ	SWiZ	SWiZt	SWiZQ	RPFTZ	RPFTZT	RPFTZQ	R^2	F-statistics (3)	D-W	t* for min a(t) =a(t*) (4)	t* for min b(t) = b(t*)	t* for max c(t) = c(t*).
II.1-1	1	CPRK[(2)]	2.3969*[(5)] (23.5)	0.0048* (2.53)	-0.0002* (-4.17)	0.00001* (3.35)	1.3060 (1.11)	-0.0427 (-1.09)	0.0008 (1.02)	-0.0751* (-1.78)	0.0029 (1.48)	-0.0001 (-1.11)	.63	10.16	2.34	37	63	52
II.1-2	2	"	2.4314* (22.58)	0.0041* (2.13)	-0.0003* (-3.94)	0.00001* (3.58)	2.1393 (1.54)	-0.0710 (-1.53)	0.0005 (1.46)	-0.0611 (-1.45)	0.0019 (-0.96)	-0.00001 (-0.46)	.64	10.63	2.39	36	71	80
II.1-3	3	"	2.3592* (22.95)	0.0052* (2.74)	-0.0003* (-4.97)	0.000004* (3.63)	2.4086 (1.10)	-0.0806 (-1.12)	0.0006 (1.13)	-0.0741 (-1.54)	0.0028 (1.16)	-0.00002 (0.79)	.64	10.24	2.39	40	61	58
II.1-4	4	"	2.3991* (23.23)	0.0047* (2.51)	-.00029* (-4.18)	0.000004* (3.37)	1.2984* (1.04)	-0.0424 (-1.02)	0.0003 (.096)	-0.0743* (-1.71)	0.0029 (1.47)	-0.00003 (-1.10)	,63	10.12	2.37	36	63	55
II.1-5	5	"	2.3465* (21.79)	0.0042* (2.20)	-0.0003* (-3.74)	0.000004* (3.42)	1.3343 (1.19)	-0.0458 (-1.23)	0.0003 (1.13)	-0.063 (-1.54)	0.0022 (1.08)	-0.00002 (-0.82)	.64	10.44	2.36	34	64	76
II.1-6	6	"	2.4325* (21.85)	0.0054* (2.94)	-0.0003* (-5.16)	0.000004* (3.47)	2.4932 (0.94)	-0.0834 (-0.93)	0.0006 (0.95)	-0.0767 (-1.54)	0.0029 (1.17)	-0.00002 (-0.82)	.63	10.20	2.39	41	61	56
II.1-7	7	"	2.4123* (22.68)	0.0047* (2.50)	-0.0004* (-4.21)	0.00003* (3.38)	1.3881 (1.08)	-0.0456 (-1.07)	0.0004 (1.01)	-0.0756* (-1.82)	0.0029 (1.52)	-0.00003 (-1.13)	.63	10.15	2.37	36	63	56
II.1-8	8	"	2.4462* (20.98)	0.0039* (2.07)	-0.0003* (-3.76)	0.00001* (3.47)	2.0699 (1.63)	-0.0711* (-1.65)	-0.0005 (1.57)	-0.0577 (-1.42)	0.0017 (0.88)	-0.00001 (-0.31)	.65	10.76	2.39	34	64	88
II.1-9	9	"	2.4091* (21.56)	0.0052* (3.02)	-0.0003* (-4.84)	0.000003 (3.08)	0.8523 (0.27)	-0.3587 (-0.33)	0.0005 (0.39)	-0.0858* (-1.66)	0.0036 (1.36)	-0.00004 (-1.05)	.63	10.15	2.29	39	52	51

(1) Number of observations = 63

(2) For explanation of all variable names, cf. p.

(3) F-statistic is given for (9,53) d.f.. F for all equations is significant at 99%.

(4) For explanation of a(t), b(t) and c(t), cf. p.

(5) The figures within brackets are respective t-values; * indicates significance at 95%, and , that at 90% levels respectively.

Table II. 1.1b : Regression Results for Japan (contd.).[1]

Equation Number	Dependent Variable	Constant	PFS3Z	PFS3ZT	PFS3ZQ	SWRIZ	SWRIZT	SWRIZQ	RPFTZ	RPFTZT	RPFTZQ	TIPIZ	TIPIZT	TIPIZQ	YWMAZ	YWMAZT	YWMAZQ	R^2	F-Statistic.	D-W
II.1-10	CPRK	2.2602* (13.39)	-0.0018 (-0.76)	-0.0001 (-1.06)	0.00003* (1.96)	0.9521 (1.22)	-.0351 (-.94)	.0003 (0.74)				.0003* (1.70)	.00006 (.10)	-.000005 (.0.95)				.63	10.18	2.33
II.1-11	"	2.2818* (21.04)	0.0046* (2.67)	-0.0003* (-5.35)	0.000004 (3.03)				-0.2036* (-1.85)	.0093* (1.91)	-.0001* (-1.75)				0.0162 (0.84)	-.0005 (-.60)	.000005 (.44)	.62	9.78	2.51
II.1-12	"	2.6589* (18.17)				2.6788* (3.90)	-.1319* (-4.45)	0.0014* (4.35)	-0.3542* (-3.47)	.0132* (2.61)	-.0013* (-2.02)				-.0033	0.0002	.000001	.58	8.24	2.13
II.1-13	"	2.4829* (17.20)				1.1620 (1.51)	-.0441 (-1.19)	0.0004 (0.88)	-.2807* (2.89)	.0125* (3.71)	-.00013* (-4.53)							.65	11.33	2.43

(1) Cf. notes to Table II.1.1a above.

Table II 1c : Regression Results for Japan (contd)[1].

Equation Number	Dependent Variable	Constant	LPFS3Z	LSWRIZ	LYWMAZ	LTIPRJ	TIME	R^2	F - Statistic	D-W
II.1-14	LCPRK	1.7651* (6.84)	-0.2593* (-3.63)	-0.0720 (-2.07)	0.1208* (1.84)		0.0005 (0.21)	.26	5.17 (4.58)	1.36
II.1-15	"	3.4550* (8.45)	-0.2306* (-3.80)	-0.0757* (-2.58)	0.1358* (2.44)	-0.5363* (-4.89)	0.0058* (2.51)	.48	10.56 (5,57)	.86

(1) Cf. notes to Table II.1.1a

Table II.1-2 : JAPAN.

YEAR	FVRP	FVTA	FVCI	TPRJ	IAP1	IAP2	UVNF
1878	424	720	251.5	32.8	46.5	28.6	...
1879	455	773	266.2	35.2	60.2	35.8	...
1880	447	792	255.9	34.6	69.8	39.9	...
1881	433	771	272.1	33.5	75.1	44.4	...
1882	433	791	274.9	33.5	62.7	37.0	...
1883	438	787	274.0	33.9	48.1	29.1	473
1884	423	792	273.3	32.7	42.2	25.5	384
1885	473	848	264.6	36.6	48.0	28.0	421
1886	501	896	262.2	38.8	46.0	28.5	438
1887	534	943	262.9	41.3	42.8	26.9	536
1888	510	923	265.1	39.5	40.4	24.7	593
1889	429	839	264.5	33.2	45.9	27.6	586
1890	557	978	267.0	43.1	59.8	34.4	668
1891	493	941	275.4	38.2	53.8	32.0	632
1892	535	972	275.0	41.4	55.4	33.1	641
1893	481	935	283.6	37.3	56.7	33.9	659
1894	541	1025	285.7	41.9	63.6	37.4	636
1895	516	1025	284.8	40.0	64.0	37.9	605
1896	468	937	288.6	36.2	68.4	40.4	717
1897	427	924	296.3	33.0	80.9	46.7	724
1898	612	1133	297.1	47.4	95.4	53.5	845
1899	513	1044	303.6	39.7	81.9	47.8	812
1900	536	1102	310.9	41.5	85.8	49.8	765
1901	606	1174	317.8	46.9	81.1	47.1	680
1902	477	1016	324.0	36.9	88.3	50.8	650

YEAR	FVRP	FVTA	FVCI	TPRJ	IAP1	IAP2	UVNF
1903	600	1166	330.4	46.5	100.2	57.2	727
1904	664	1250	322.5	51.4	97.1	54.8	885
1905	493	1063	318.7	38.2	98.5	56.9	855
1906	598	1202	321.6	46.3	104.4	60.6	791
1907	634	1291	343.5	49.1	115.6	67.2	818
1908	671	1328	364.2	51.9	110.3	62.7	706
1909	677	1337	385.8	52.4	98.4	56.9	627
1910	602	1282	394.5	46.6	100.1	57.4	705
1911	668	1391	422.2	51.7	123.2	68.5	728
1912	649	1393	421.1	50.2	143.6	78.2	791
1913	649	1422	451.2	50.3	142.6	78.4	778
1914	737	1513	438.4	57.0	102.3	59.5	738
1915	723	1526	431.9	55.9	96.5	55.8	677
1916	755	1620	441.9	58.5	109.6	64.0	789
1917	707	1584	463.4	54.6	154.9	88.8	1035
1918	707	1573	476.3	54.7	240.3	133.0	1303
1919	787	1696	531.7	60.8	331.2	182.0	1544
1920	817	1695	496.5	63.2	262.6	145.6	1540
1921	713	1580	491.7	55.2	253.3	143.3	982
1922	784	1640	496.8	60.7	213.7	127.8	1036
1923	716	1578	537.9	55.4	236.4	138.5	1054
1924	739	1628	539.2	57.2	265.9	148.5	1076
1925	771	1762	549.5	59.7	260.3	148.8	1180

YEAR	FVRP	FVTA	FVCI	TPRJ	IAP1	IAP2	UVNF
1926	718	1696	596.2	55.6	236.0	134.1	971
1927	802	1807	592.2	62.1	207.6	117.3	824
1928	779	1801	612.5	60.3	204.2	115.6	817
1929	769	1833	638.1	59.6	201.4	124.3	768
1930	864	1970	616.4	66.9	133.0	75.3	542
1931	713	1763	656.1	55.2	123.5	69.3	402
1932	780	1838	612.7	60.4	141.9	77.8	459
1933	915	2072	608.3	70.8	151.6	85.6	534
1934	670	1752	649.1	51.8	174.5	92.2	530
1935	742	1833	664.1	57.5	187.6	101.0	599
1936	870	1985	729.1	67.2	196.5	106.8	565
1937	857	2019	716.1	66.3	218.9	118.0	602
1938	851	1946	767.1	65.9	235.1	126.8	603
1939	892	2096	768.9	69.0	311.5	171.8	772
1940	786	1964	704.6	60.9	340.4	190.2	693.

YEAR	UVPH	UVPF	INNF	INPH	INPF	IFP1	TIPI
1878	...	...	23.9	15.9	18.8	72.8	43.9
1879	...	...	28.2	19.9	18.7	94.7	44.8
1880	...	...	27.5	19.1	19.1	125.1	42.1
1881	...	...	27.0	18.8	18.9	122.8	42.4
1882	...	...	23.4	16.5	18.9	95.2	40.8
1883	135	137	23.4	16.6	18.8	71.2	40.1
1884	110	113	29.1	18.7	19.0	57.9	34.1
1885	120	123	24.7	15.8	18.7	63.4	32.1
1886	125	128	25.4	16.7	19.1	66.0	29.9
1887	152	151	23.0	17.1	19.1	80.4	36.1
1888	167	164	24.1	16.6	19.3	88.8	41.0
1889	166	162	23.1	16.1	19.2	87.8	38.4
1890	188	182	23.9	15.8	19.1	99.9	39.5
1891	179	177	26.5	18.0	19.7	94.8	38.0
1892	182	179	24.7	17.3	19.6	96.2	36.6
1893	186	186	28.8	20.6	19.6	98.8	37.2
1894	181	187	29.8	18.7	19.8	95.8	36.9
1895	173	183	30.0	19.3	20.0	91.4	40.7
1896	205	213	29.6	19.5	19.8	108.2	43.5
1897	207	218	30.4	21.4	19.9	109.3	46.5
1898	238	252	27.8	22.5	19.6	127.2	46.1
1899	228	239	30.8	26.6	20.2	122.3	52.5
1900	225	220	37.9	25.4	21.7	115.7	45.1
1901	201	192	43.7	30.4	23.0	102.8	46.8
1902	176	187	49.9	26.8	24.8	97.0	47.5

YEAR	UVPH	UVPF	INNF	INPH	INPF	IFP1	TIPI
1903	197	207	51.0	44.5	24.1	108.4	50.3
1904	225	258	38.4	44.2	21.9	130.9	53.1
1905	243	232	50.9	44.8	24.9	128.0	57.0
1906	239	276	54.7	51.4	25.6	122.0	58.7
1907	227	247	69.2	68.2	27.6	123.0	63.3
1908	192	223	80.5	65.2	30.7	106.2	60.8
1909	176	196	96.5	82.9	40.3	94.7	58.0
1910	188	225	92.5	84.3	38.9	105.8	58.7
1911	179	208	105.1	96.8	44.7	107.0	60.8
1912	168	200	107.4	99.7	44.9	112.9	64,7
1913	164	193	135.4	120.9	52.3	110.8	64.7
1914	187	228	125.1	112.4	49.2	119.6	61.9
1915	163	175	121.1	91.0	47.2	98.5	62.6
1916	176	172	124.4	95.5	47.3	112.3	75.6
1917	203	202	147.9	104.9	52.2	143.9	94.9
1918	258	256	167.7	115.6	59.0	181.5	124.4
1919	410	456	218.8	152.1	69.9	230.1	152.6
1920	678	859	187.6	128.3	58.2	268.6	167.7
1921	312	392	177.1	130.5	60.8	154.8	129.7
1922	300	445	177.1	129.5	61.1	161.9	126.6
1923	306	358	212.8	137.5	66.4	161.2	129.0
1924	343	396	200.4	147.7	69.3	168.4	133.6
1925	340	364	204.2	157.7	75.3	178.8	130.5

YEAR	UVPH	UVPF	INNF	INPH	INPF	IFP1	TIPI
1926	327	384	259.1	183.7	82.3	154.5	115.7
1927	299	368	250.5	207.0	85.5	134.6	110.1
1928	286	357	262.9	207.9	91.3	132.2	110.7
1929	276	352	281.1	218.3	100.4	125.6	107.6
1930	253	349	271.9	210.8	101.5	97.6	88.6
1931	208	320	300.7	197.4	90.5	76.5	74.9
1932	237	420	263.8	210.2	70.1	89.4	83.0
1933	214	442	252.3	214.7	74.7	96.8	94.9
1934	223	362	271.9	213.4	93.6	94.1	97.0
1935	242	357	290.6	233.1	108.0	103.4	99.5
1936	253	408	363.0	262.3	107.3	102.5	103.7
1937	365	441	305,8	270.0	140.8	117.8	125.7
1938	472	478	382.5	230.9	153.9	128.7	132.5
1939	503	629	361.1	265.8	141.3	156.6	146.6
1940	508	864	359.3	252.1	90.2	157.0	164.2

YEAR	PFAJ	DWMA	YWMA	RPFT	CGRW	LGRW	LGAE
1878	26984				681.5	1895	
1879	27206				889.	1989	
1880	27398				319	3087	
1881	27428				355	2510	
1882	27416				685	3681	
1883	27541				771	2980	
1884	27555				840	3132	
1885	27620			4.86	1083	3730	
1886	27663	0.15			503	3058	
1887	27766	0.15			977	3276	
1888	27857	0.14	22		901	3166	
1889	27895			7.52	965	5725	
1890	27950	8			874	5845	
1891	28006				675	6213	
1892	28024	0.20	29		1253	5455	
1893	28060	0.25			779	6817	
1894	28074	0.25	34		540	7168	
1895	28137	0.28	37		826	5753	
1896	28144	0.34	41		1230	11067	
1897	28171	0.40	52		3954	14960	
1898	28263	0.34	59		2847	11970	
1899	28336	0.37	53	9.92	2367	12163	
1900	28411	0.36	56		2220	9757	1725
1901	28489	0.36	53		2921	7064	1648
1902	28500	0.40	57		3318	7474	1552

YEAR	PFAJ	DWMA	YWMA	RPFT	CGRW	LGBW	LGAE
1903	28576	0.42	63	13.54	1798	10680	1833
1904	28643	0.38	60	12.56	1048	7267	1432
1905	28711	0.38	60	11.47	1076	7088	1816
1906	28823	0.41	66	13.41	1673	8124	1898
1907	28911	0.44	71	15.32	3349	12605	2408
1908	29071	0.45	76	14.63	4640	15704	2484
1909	29278	0.41	69	11.96	3986	11677	2525
1910	29409	0.42	75	12.40	6671	16676	2819
1911	29548	0.50	82	16.85	9207	26024	3197
1912	29702	0.56	94	20.23	18798	12590	3719
1913	29817	0.59	91	20.36	10530	14517	4119
1914	29917	0.51	77	13.27	10667	18435	4582
1915	30045	0.49	75	12.98	7319	18052	3398
1916	30180	0.52	84	14.65	6790	11843	3000
1917	30338	0.65	102	20.94	6938	11240	3967
1918	30421	1.01	141	34.94	13718	18753	4769
1919	30494	1.63	222	50.11	19551	22765	6284
1920	30665	1,64	221	39.38	36888	35720	8826
1921	30318	1,57	224	38.48	31543	41020	8702
1922	30914	1.51	223	27.81	33989	36441	13218
1923	31052	1.47	227	33.35	31321	36325	17833
1924	31154	1.42	230	38.96	21676	37705	17040
1925	31285	1.44	232	35.59	16754	34217	20570

YEAR	PFAJ	DWMA	YWMA	RPFT	CGRW	LGRW	LGAE
1926	31390	1.36	230	32.60	21672	40560	24099
1927	31575	1.43	228	27.22	22496	43276	31498
1928	31749	1.39	193	25.94	24405	41280	30884
1929	31900	1.31	205	25.49	17434	35125	23137
1930	32019	1.12	174	16.01	22092	26526	22329
1931	32095	0.89	142	16.01	14247	30880	20359
1932	32173	0.78	129	19.20	34935	38640	21758
1933	32233	0.81	132	19.19	44547	57102	23179
1934	32200	0.81	138	25.58	32771	40122	23478
1935	32176	0.86	145	26.33	27288	51163	
1936	32156	0.90	153	26.29	24510	54034	
1937	32157	1.01	180	30.24	31382	46215	
1938	32076	1.21	198	31.94	24507	48453	
1939	32072	1.60	230	40.72	32616	49924	
1940	32043	1.90	213	41.78	28175	63763	

Appendix II.2 : Taiwan.

List of variable names :

ICP : Index of cost of productioon obtained by dividing the Ho input index by his aggregate output index, both in 1952-56 prices.

PIDC : Price index of drugs and chemicals, the export price index of Japan being used as substitute for fertilizer price in Taiwan.

SWRi ; i= 1,...,9 ; Substitute water wate calculated on the basis of alternative life span and alternative rates of interest.

LICP : Log(ICP). Any variable starting with 'L' is the natural logarithm of the variable defined by the following characters.

SWR5T : SWR5 x TIME. Any variable ending with 'T' indicates that the variable has been multiplied by time.

PIDCQ : PIDC x $(TIME)^2$. Any variable ending with a 'Q' has been multiplied by the square of the corresponding time value.

a(t) : Inverse of fertilizer productivity.

b(t) : Inverse of irrigation water productivity.

TABLE II. 2.2a : Regression Results for Taiwan

Equation Number	Subst. Water Rate Index i	Dependent Variable	Explanatory Variables: Constant	FIDC	PIDCT	PIDCQ	SWRi	SWRiT	SWRiQ	R^2	F-statistic (6,32)	D-W
II.2-1	1	ICP	0.5876	.0033	-.0002	.000003	303.022	-17.179	0.2269*	.86	34.63	1.39
			(9.33)	(3.15)	(-3.20)	(2.63)	(2.30)	(-2.19)	(1.69)			
II.2-2	2	"	0.4931	.0049	-.0003	.000006	882.767	-47.701	0.6512	.88	42.10	1.55
			(7.05)	(4.46)	(-4.29)	(3.64)	(4.06)	(-4.34)	(4.02)			
II.2-3	3	"	0.4931	.0048	-.0003	.000005	1302.497	-73.293	1.0631	.88	41.05	1.51
			(7.05)	(4.14)	(-4.10)	(3.18)	(3.89)	(-4.25)	(4.09)			
II.2-4	4	"	0.5822	.0034	-.0002	.000004	339.504	-19.366	0.2614	.86	35.36	1.43
			(9.68)	(3.36)	(-3.38)	(2.76)	(2.59)	(-2.53)	(2.03)			
II.2-5	5	"	0.4697	.0053	-.0004	.000007	1189.89	-62.622	0.8326	.90	48.31	1.63
			(8.03)	(5.20)	(-4.90)	(4.18)	(4.86)	(-[illegible].28)	(5.12)			
II.2-6	6	"	0.4722	.0052	-.0003	.000005	917.782	-105.975	1.5263	.89	47.11	1.56
			(7.29)	(4.76)	(-4.60)	(3.34)	(4.58)	(-5.05)	(5.11)			
II.2-7	7	"	0.5880	.0033	-.0002	.000004	347.972	-20.022	0.2723	.87	36.40	1.47
			(10.56)	(3.51)	(-3.51)	(2.84)	(2.78)	(-2.75)	(2.23)			
II.2-8	8	"	0.4786	.0053	-.0004	.000008	1453.121	-75.924	0.9966	.91	54.41	1.67
			(9.56)	(5.83)	(-5.28)	(4.55)	(5.46)	(-5.98)	(6.00)			
II.2-9	9	"	0.4967	.0049	-.0003	.000005	2443.600	135.623	1.9588	.90	49.98	1.51
			(8.87)	(5.02)	(-4.72)	(3.01)	(4.89)	(-5.40)	(5.66)			

(1) Number of observations = 39.

(2) For explanation of all variable names, cf. p.

(3) The figures within brackets are the respective t-values.
All t-values are significant at 99% level, except the ones marked*
All F-values are significant at 99% level.

Table II.2.2b : Regression Results for Taiwan (contd).[1]

Equation Number	Subst. Water Rate Index i	Dependent Variable	Explanatory Variables: Constant	LPIDC	LSWRi	TIME	R^2	F	D-W
II.2-10	1	LICP	-0.9172* (-3.09)	0.0052 (0.10)	-0.0658* (-6.03)		.55	22.55 (2,36)	0.53
II.2-11	5	"	-1.1281* (-3.69)	0.0482 (0.87)	-0.0639* (-6.41)		.58	25.21 (2,36)	0.59
II.2-12	1	"	-0.5557* (-2.60)	0.0896* (2.19)	-0.0009 (-0.07)	-0.0129* (-0.0129)	.79	44.64 (3,35)	1.09

(1) Cf. notes to Table II.2-2a above.

(2) * indicates significance at 95% level.

Table II.2-2 : Taiwan.

Year	Output Index (a)	Input Index (b)	Cost Index (c)	Irrigation Investment (m.T$)(d)	PIDC (e)
1901	100.0	85.8	85.8	0.3	70.2
1902	101.1	90.4	89.4	1.7	78.1
1903	130.0	100.0	76.9	2.4	71.3
1904	145.6	108.8	74.7	3.7	75.1
1905	153.9	109.9	71.4	0.7	76.8
1906	148.1	111.2	75.1	6.8	86.1
1907	160.9	115.1	71.6	8.8	67.6
1908	169.1	118.2	69.9	8.6	58.8
1909	172.2	121.2	70.4	56.2	63.0
1910	168.1	124.6	74.1	34.9	73.3
1911	177.6	140.8	79.3	24.1	70.2
1912	159.8	131.0	82.0	17.5	88.0
1913	181.8	133.7	73.6	23.9	70.6
1914	174.1	134.8	77.4	-	71.9
1915	188.5	140.7	74.6	4.7	85.8
1916	197.9	149.3	75.5	14.3	109.8
1917	215.2	153.3	71.2	21.3	116.4
1918	203.3	145.4	71.5	13.7	167.5
1919	206.0	152.7	74.1	18.5	223.0
1920	191.4	146.9	76.8	36.2	136.2
1921	201.2	145.4	72.2	65,0	162.0
1922	225.4	142.7	63.3	70.7	181.0
1923	219.3	146.3	66.7	78.8	193.5
1924	257.3	155.3	60.3	84.0	190.5

Table II.2-2 (contd).

1925	271.8	161.8	59.5	93.2	141.4
1926	266.8	164.5	61.7	120.0	108.4
1927	281.7	169.1	60.0	145.2	118.0
1928	293.9	175.0	59.6	369.8	108.8
1929	295.3	172.1	58.3	159.0	99.3
1930	316.3	177.0	56.0	134.5	84.2
1931	323.4	184.2	57.0	115.4	93.2
1932	370.7	182.6	49.3	42.0	113.9
1933	333.9	190.2	57.0	48.5	107.7
1934	363.9	200.7	55.1	64.9	94.4
1935	390.5	213.2	54.6	66.8	93.9
1936	405.8	218.5	53.9	7.5	103.2
1937	411.4	222.9	54.2	5.8	124.3
1938	430.6	224.2	52.1	7.2	145.9
1939	434.9	223.5	51.3	32.5	153.3

(a) Source : Index of aggregate output, 1901-1960. 'Agricultural Development of Taiwan' by Yhi-Min Ho. Table 1, p.17.

(b) Source : Same as above. Table 23, pp.64-5.

(c) Col. (a) divided by col. (b).

(d) In millions of Taiwan dollars. Includes both private and government irrigation investment. The deflator used is the 'derived price index of farm products' Ho , op. cit., Table E-1, pp.153-4.

(e) Source : LESJ, Vol. 8, Table 20, p.215, col.5.2.

Appendix II.3

List of variable names : Kakrapar Project.

CP : Original cost of production series calculated on the basis of constant marginal cost assumption.

CP1 : New cost of production series based on the assumption that cost/revenue ratio is constant over the sample years and that the price elasticity of demand for rice is 0.5.

CP2 : Same as above, except that the price elasticity of demand for rice is taken to be 0.25.

WR : The original water rate series computed under the assumption that the distribution of acreage under different crops remains constant at the sample average, and total irrigated area approaches the planned figure in finite time (within the sample period).

WR1 : The new water rate series obtained by projecting the acreage under different crops separately.

PF : Price of fertilizer.

LCP : Log(CP). Same for any variable starting with 'L'.

TOPF : TIME x PF. Same for any variable starting with 'T'.

TTPF : $TIME^2$ x PF. " " " " 'TT'.

a(t) : Coefficient of PF, linear or quadratic in time.

b(t) : Coefficient of water, old or new, linear or quadratic in time.

Table II. 3 -1. :

Regression Results for Kakrapar Weir and Canal Project, India.

Eqn. No.	Dependent Variable	Explantory Variables														R^2	F	D-W
		Constant	PF	TOPF	TTPF	WR1	TOWR1	TTWR1	WR	TOWR	TTWR	LPF	LWR	LWR1	TIME			
II.3-1	CP		.345* (20.6)	-.032* (-2.7)	.001 (1.29)	2.31* (4.1)	-.242 (-2.05)	.018 (1.68)								.98	80.42 (5,8)	3.07
II.3-2	CP1		.342* (20.01)	-.033* (-2.76)	.001 (1.5)	2.38* (4.06)	-.254 (-2.1)	.019 (1.75)								.98	129.94 (5,8)	3.08
II.2-3	CP2		.310* (15.86)	-.034* (-2.42)	.002 (1.9)	2.76* (4.13)	-.304 (-2.2)	.025 (1.9)								.99	400.03 (5,8)	3.11
II.2-4	CP		.319* ([illegible].39)	.008 (.061)	-.003 (-.23)				.464 (.34)	-.021 (-.05)	.004 (.11)					.97	54.43 (5,8)	2.46
II.2-5	"	83.66 (36.2)	-.126* (-9.7)	.015* (13.9)	-.002 (-1.9)				-.167 (-1.5)	.024 (.72)	-.001 (-.3)					.99	7673. (6,7)	3.08
II.2-6	LCP1	2.75 (1.47)										.276 (.81)	-.062 (-1.1)		.192* (5.0)	.96	116.6 (3,10)	.78
II.2-7	LC 2	[illegible].93* (3.15)										.239 (1.4)		-.253* (-6.0)	.588* (10.1)	.97	493.8 (3,10)	2.28
II.2-8	LCP	4.22* (35.73)										-.024 (1.07)		.011* (4.8)	-.031* (-41.3)	.99	7272. (3,10)	1.21

(1) Number of observations = 14.

(2) For explanation of variable names, cf. p.

(3) * indicates significance of t at 95% level.

Table II.3-2

Year	CP (1)	CP1 (1)	CP2 (1)	PF	Irrigated Area : Planned (3)	Irrigated Area : Actual (4)
1958-9	62.50	62.04	56.74	184	85700	21633
195960	65.0	64.62	60.86	182	108000	49806
1960-1	67.50	67.26	65.27	182	157800	63025
1961-2	70.00	70.00	70.00	183	214050	81784
1962-3	72.50	72.86	75.08	175	277800	93577
1963-4	75.00	75.86	80.52	175	335050	131008
1964-5	77.50	78.94	86.36	171	389100	183411*
1965-6	80.00	82.15	92.62	180	435188	266755*
1966-7	82.50	85.50	99.33	203	474413	373485*
1967-8	85.00	88.99	106.54	210	505988	482126*
1968-9	87.50	92.62	124.26	215	528713	528713
69-70	90.00	96.40	122.55	207	543688	543688
1970-1	92.50	100.33	131.43	224	553913	553913
1971-2	95.00	104.43	140.96	227	559625	559625

(1) Cost of production per 1000lb of rice .Cf. text.

(2) Price paid by peasants per 100kg of ammonium sulphate- UN/FAO Yearbooks, relevant years.

(3) As planned by project planners. EMIP, p.32.

(4) Actual area irrigated. EMIP, p.34.

* Estimate based on growth rate of irrigated area - coincides with the 'planned' series 1968-69 onwards.

Table II.3-3 : Given Acreage Under Crops and Respective Water Rates, ('60-1).

	Kharif Rice	Kharif Perennials	Kharif: Others	Rabi Wheat	Rabi: Perens.	Rabi: Others	Hot Weather Perens.	Hot Weather Others	Total
Water Rate(Rs) '60-1	12	12	5	10	20	10	28	20	
'58-59	10856	1397	5251	479	1515	3240	...	...	22738
'59-60	4069	1059	3247	1311	4675	5506	988	21	20876
'60-61	14886	5781	7800	2004	7694	5159	4604	49	47977
'61-62	13709	7808	9475	3202	9585	9402	6960	4	60145
'62-63	26849	9351	12315	4204	10659	5306	8128	44	76856
'63-64	20980	11361	11062	4163*	12686*	10563*	9449	4	93557*
Average: '58-59 to 63-4.	15225	6126	8192	2240	6826	5683	6026	20	50338
%	30.25	12.17	16.27	4.45	13.56	11.29	11.97	0.39	-

Source : EMIP . * indicates estimate.

Table II.3-4.

Year	(a) R_t	(b) R_t	(c) R^*_t
1958-59	37000	252541	...
1959-60	183000	267525	...
1960-61	566000	642406	...
1961-62	1121000	818199	...
1962-63	1818000	1031839	...
1963-64	2537000	1109034	...
1964-65	3325000	1274519	1276196
1965-66	4061000	2498058	1480102
1966-67	4773000	3633476	1692621
1967-68	5260000	5086866	1865160
1968-69	5657000	6891557	2057699
1969-70	5910000	7201071	2250381
1970-71	6078000	7408095	2442777
1971-72	6192000	7544295	2635396

(a) Net revenue series as estimated by project planners,EMIP.

(b) Revenue series that would have emerged if 1960-61 rates were charged at all periods.

(c) Alternate revenue series (at 1960-61 rates) that emerges if 'planned' acreage is replaced by projected acreage.

All values are in Rupees.

Appendix II.4

Variable names : Dillon County, South Carolina, U.S.A.

CCDC : Cost of producing one bushel of corn.

WRDL : Wage rate of agricultural day labor(S.Carolina).

PSAF : Price of ammonium sulphate, $/ton.

PINB : Price index, all agriculture, base : 1957-9.

PIOB : Price index, all agriculture, base : 1910-14.

FPOB : Price index, all fertilizers, as paid by American farmers.Base: 1910-14.

FMPP : Price index of farm machinery used by American farmers, base : 1910-14.

TAPESC : Total agricultural production costs (S.Carolina) - current $. Includes depreciation, replacementof farm machinery, interest on mortgage payments.

VAASC : Value of all agricultural production(S.C.),current $.

VCDC : Farm value of corn production, Dillon County, in current $.

CHDC : Bushels of corn harvested in Dillon County.

LCCDC : Log(CCDC).Same for any variable name starting with L.

FMPIZ : FMPI converted to '57-59 base. Same for any variable name ending with 'Z'.

FMPIZT : FMPIZ x TIME. Same for any variable name ending with 'T'.

FMPIZQ : FMPIZ x $TIME^2$. Same for any variable name ending with 'Q'.

Table II. 4 - 1 : Regression Results for Dillon County, South Carolina (1).

Equation	Dependent Variable	Explanatory Variables																		R^2	F	D-W
		Constant	FMPIZ	FMPIZT	FMPIZQ	PSFAZ	PSFAZT	PSFAZQ	WRDLZ	WRDLZT	WRDLZQ	FPOBZ	FPOBZT	FPOBZQ	LWRDLZ	LPSAFZ	LFMPIZ	LCHDC	TIME			
II.4-1	CCDC	1.085 (1.48)	-.008 (-.93)	-.001 (-.57)	.000008 (.38)	.002 (.26)	-0024* (2.35)	.000001 (.01)												-70	5.27 (6,13)	1.1
II.4-2	"	.561 (-.55)	.009 (.97)	-.002 (-1.48)	.00004 (.59)				.245 (1.60)	.069* (2.35)	-.002 (-.94)									.76	.711 (6,13)	1.62
II.4-3	"	.254 (2.8)	-.002 (-.07)	-.006 (-1.7)	.003* (2.66)				.258* (2.36)	.0100 (.23)	-.002 (-.73)	-.018 (-.37)	.017* (2.13)	-.001 (1.68)						.90	10.26 (9,10)	2.21
II.4-4	LCCDC	-7.80*													.5323* (1.65)	1.123* (3.41)	0.586* (2.02)		-.008 (-.32)	.56	4.88 (4,15)	1.07
II.4-5	"	-.5004 (-.52)													.0566 (.51)	0.987* (5.83)		-2.82* (-7.6)	.0331* (6.77)	.88	29.3 (4,15)	1.03

(1) Number of observations = 20.

(2) For explanation of variable names, cf. p.

(3) Figures within brackets are respective t-values.

* indicates significance at 95% level.

Table II.4-2 : Data : Dillon County, S. Carolina.

YEAR	PIOB [1]	PINB [1]	FPOB [1]	PSAF [1]	WRDL [2]	FMPI [1]
1945	207	57.4	120	46.80	2.86*	176
1946	236	65.7	121	48.60	2.91*	182
1947	276	63.8	134	60.00	2.97*	206
1948	287	87.2	146	69.60	3.02*	240
1949	250	83.5	150	74.40	3.05	270
1950	258	86.8	144	66.30	3.05	277
1951	302	96.7	152	66.20	3.40	298
1952	288	94.0	156	68.00	3.70	308
1953	255	92.7	157	69.80	3.70	311
1954	246	92.9	158	68.40	3.60	312
1955	232	93.2	155	66.00	3.75	312
1956	230	96.2	152	61.80	3.90	326
1957	235	99.0	153	58.90	3.95	342
1958	253	100.4	153	60.00	3.92	357
1959	240	100.6	152	58.80	4.05	372
1960	238	100.7	152	57.80	4.10	382
1961	240	100.3	154	58.40	4.20	391
1962	244	100.6	153	56.90	4.45	398
1963	242	100.3	152	52.60	4.70	405
1964	236	100.5	152	52.60	4.85	414

[1] Source: Agricultural Statistics : U.S. Dept. of Agr..Annual No.

[2] Source :Selected South Carolina Economic Data - J.D.Conklin and R.A.Quesinberry. Dec., 1969

* Estimates.

Table II.4-2 (contd).

YEAR	VCDC [1]	VAASC [2]	TAPESC [3]	CHDC [4]
1945*	565553*	280.9*	195.9*	585229*
1946	635695	273.8	201.1	616269
1947	708831	266.7	236.7	647309
1948	781967	259.4	243.6	678349
1949*	928240	252.2	239.8	740429
1950	830707	251.3	231.0	643575
1951	733177	249.9	242.7	546729
1952	635647	248.7	252.3	449865
1953	538117	247.4	251.1	353010
1954*	440587	246.2	255.5	256155
1955	504135	249.3	270.1	329240
1956	567683	252.7	266.6	402325
1957	631231	254.4	273.5	475410
1958	694779	257.9	279.3	548495
1959*	758326	261.5	282.7	621579
1960	818068	266.5	281.9	665047
1961	877810	271.5	298.2	708515
1962	937552	276.4	300.5	751983
1963	997294	281.7	320.8	795451
1964*	1057035	286.6	330.4	838917

(1) In dollars; (2),(3) : In millions of dollars.

(4) Number of bushels harvested.

* Reported figures.Rest estimated. Census of U.S.Agiculture: 1945,'49,'54,'59,'64. S.Carolina and Counties.

B. Analysis of Growth, Returns to scale and Technical Progress in an Aggregate Production Function.

Houthakker had suggested (1) an interesting device to generate aggregate production functions from Leontief type firm production functions, depending on the form of the cell distribution functions of the fixed factor. If the cell distribution function is Pareto, the aggregate function turns out to be Cobb- Douglas. Solow had drawn attention to this contribution in a survey article on capital theory (2). In recent years more research has been done in the field. Levhari extended Houthakker's result by establishing a one-to-one correspondence between cell distribution functions and aggregate production functions (3). A significant contribution in the field has been made by Prof. Johansen in a recently published book on Production Functions (4).

Several extensions of the original Houthakker proposed structure are still possible. Johansen concerns himself with embodied technical progress in the model, and as one might expect, comes up with very complicated expressions. In macro functions, however, analysis of disembodied technical progress still seems to be a fruitful endeavour. Similarly, one might want to study the growth and decay of an industry emanating from usual production calculus of the constituent units. Also, distribution functions other than Pareto can be tried to find out if they yield aggregate production functions in explicit forms. This paper seeks to deal with these aspects of the problem.

The Model:

Prof. Houthakker's model consists of numerous small firms (cells) each producing a homogeneous commodity with different variable factor combinations, but each with a Leontief type production function. Each cell is endowed with a certain amount of the 'fixed factor' which does not show up in the short-run aggregate production function. Whether a firm will be producing the commodity or not will be determined by whether or not its variable costs per unit in the period are being covered by price per unit of output. If one assumes constant returns to scale in all the factors -- variable and fixed -- so that one firm producing ten units of output is equivalent to ten firms each producing one unit, then corresponding to each point in the positive orthant of the variable input space one can associate the quantity of output that is being produced by that particular technique. In other words, one can postulate a distribution function of output defined over the space, which is equivalent to the underlying distribution function of the fixed factor up to a scalar multiplication. Under these circumstances, if $f(x_1,x_2)$ is the distribution of output over the variable input space (x_1,x_2), and if X_0, X_1 and X_2 are the aggregate quantities of output and the two variable inputs respectively, and if $G = \{(x_1,x_2) \mid p_1x_1+p_2x_2 \leq p_0 = 1\}$ be the feasibility (i.e., non-negative short-run profit) region, then one can write

$$X_0 = \iint_G f(x_1, x_2)\, dx_1 dx_2 \quad \ldots\ldots \quad (1)$$

$$X_1 = \iint_G x_1 f(x_1, x_2)\, dx_1 dx_2 \quad \ldots\ldots \quad (2)$$

$$X_2 = \iint_G x_2 f(x_1, x_2)\, dx_1 dx_2 \quad \ldots\ldots \quad (3)$$

Given these relations, one can solve out the two price variables from the three equations (1) - (3) so that we get an expression connecting X_0, X_1 & X_2, which is the aggregate production function we looking for. If the region over which $f(x_1, x_2)$ is $>$ 0 is wide enough the aggregate function exhibits neo-classical properties of substitutiblity (Cf. Houthakker (1), Johansen (4)). If however this so-called 'region of substitution' (Johansen) is the degree of substitutiblity in the aggregate function is reduced until in the extreme case, where the region is a line (or curve) through the origin, the aggregate function, like the cell functions, becomes a pure Leontief type. This happens since the marginal variances and covariances of the cell distribution function along the budget line $p_1 x_1 + p_2 x_2 = 1$ all disappear, leading to an indeterminary in the prices --- a characteristic of the Leontief function.

The aggregate production function thus generated will, in general, be neo-classical, and it exhibits all properties of competitive profit maximization Cf.Johansen

Also it always has diminishing returns to scale in the variable factors. This is easy to see. If $X_0 = X_0(X_1, X_2)$ is the macro function, the returns to scale in it can be expressed by the elasticity of output to factor changes, defined for proportional factor variations.

$$\mathcal{E} = \frac{d\log X_0}{d\log X_1} = \frac{d\log X_0}{d\log X_2} \qquad \ldots.(4)$$

$$\text{i.e.,} \quad \mathcal{E} = \frac{dX_0}{dX_1} \cdot \frac{X_1}{X_0} = \frac{dX_0}{dX_2} \cdot \frac{X_2}{X_0}$$

$$\text{where} \quad \frac{dX_1}{X_1} = \frac{dX_2}{X_2} \quad \text{by definition} \qquad \ldots(5)$$

It can be seen that $\mathcal{E}$ is always < 1.

$$\because \quad \mathcal{E} = \frac{X_1}{X_0} \cdot \frac{dX_0}{dX_1}$$

$$\text{or,} \qquad \varepsilon X_0 = dX_0 \cdot X_1 / dX_1 \quad \ldots\ldots. \qquad (6)$$

The production function is

$$X_0 = X_0(X_1, X_2)$$

we have

$$dX_0 = \frac{\partial X_0}{\partial X_1} \cdot dX_1 + \frac{\partial X_0}{\partial X_2} \cdot .dX_2 \quad \ldots\ldots\ldots.(7)$$

the aggregate production function satisfies competitive profit maximization properties (Cf. (4)). Substituting (7) in (6)

$$\varepsilon X_0 = (p_1 dX_1 + p_2 dX_2)\frac{X_1}{dX_1}$$

$$= p_1 X_1 + p_2 \cdot X_1 \frac{dX_2}{dX_1}$$

$$= p_1 X_1 + p_2 X_2 \quad \ldots\ \text{from (5)}$$

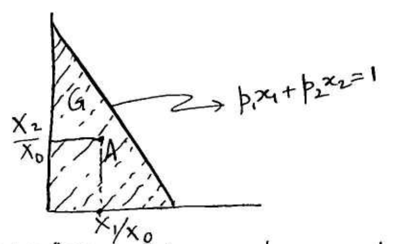
$$\therefore \varepsilon = \frac{p_1 X_1 + p_2 X_2}{X_0} \qquad (8)$$

Substituting in (8). The definitional expressions for the aggregate categories, we have

$$\varepsilon = p_1 \left[\frac{\iint_G x_1 f(x_1,x_2)\,dx_1 dx_2}{\iint_G f(x_1,x_2)\,dx_1 dx_2}\right] + p_2 \left[\frac{\iint_G x_2 f(x_1,x_2)\,dx_1 dx_2}{\iint_G f(x_1,x_2)\,dx_1 dx_2}\right].$$

where G is the region of integration

It is clear that $\frac{X_i}{X_0}$ $(i = 1,2)$ is the average value of the variable $x_i (i=1,2)$ over the region G. Hence,



unless the distribution obtains positive values only on the border line $p_1 x_1 + p_2 x_2 = 1$ and is zero everywhere inside the triangle G, this average value will be located somewhere inside the triangle for both i's. If A is the point with co-ordinates $\left(\frac{X_1}{X_0}, \frac{X_2}{X_0}\right)$ for given $f(x_1, x_2)$ and (p_1, p_2), then of necessaty

$$p_1\left(\frac{X_1}{X_0}\right) + p_2\left(\frac{X_2}{X_0}\right) < 1$$

$$\text{i.e,} \quad \varepsilon = \frac{p_1 X_1 + p_2 X_2}{X_0} < 1 \qquad - - - - \qquad (9)$$

II

Technical change.

Let us consider the question of technical change in this model. Johansen has indicated an approach for analyzing embodied technical change in the model by incorporating new components to the cell distribution function in an additive manner. The only kind of technical change we are considered with here, however, is of the disembodied type.

Disembodied technical change in the context of the Houthakker model can be classified into two broad categories:

(i) Rise in the productivity of the fixed factor for all the cells, and

(ii) Rise in the productivity of the variable factors for all the cells. One can study the impact of such changes for cell functions on the aggregate production function.

(i) Rise in the productivity of the fixed factor: the case of uniform multiplicative shift:-

Suppose all cells experience an α % rise in the productivity of the fixed factor while the productivity of the variable factors remains unchanged. This means that all firms that could previously produce 1 unit of output with 1 unit of the fixed factor can now produce 'α' units of output ($\alpha > 1$) with the same 1 unit of the fixed factor. However, since variable factor productivity is unchanged, output/v. factor ratio is unchanged, although v. factor/fixed factor ratio actually rises. Since we are assuming Leontief type production functions, the physical amount of variable factors will change in proportion to output, leaving the feasibility zone, or the region of integration, same as before. In other words, the technique of production changes with a bias against the fixed factor.

In terms of the Houthakker model, if the post-change cell distribution function is designated $f^1(x_1, x_2)$ and the pre-change one is called $f^0(x_1, x_2)$ then

$$f^1(x_1, x_2) = \alpha f^0(x_1, x_2) \qquad \forall\ (x_1, x_2).$$

Since, as explained before, neither the productivity nor the prices of the variable factors have undergone any change, the region of integration is still defined by $G(x) = \{(x_1, x_2) \mid p_1 x_1 + p_2 x_2 \leq 1\}$.

writing the expressions for the macro-variables in the post-change situation, we have.

$$X_0(1) = \alpha \iint_G f^0(x_1, x_2)\, dx_1\, dx_2 = \alpha X_0(0) \quad \text{.... (10)}$$

$$X_1(1) = \alpha \iint_G x_1 f^0(x_1, x_2)\, dx_1\, dx_2 = \alpha X_1(0) \quad \text{---- (11)}$$

$$\text{\&} \quad X_2(1) = \alpha \iint_G x_2 f^0(x_1, x_2)\, dx_1\, dx_2 = \alpha X_2(0) \quad \text{--- (12)}$$

If

$$X_0(1) = H\left(X_1(1), X_2(1); F\right)$$

$$\text{\&} \quad X_0(0) = \Phi\left(X_1(0), X_2(0), F\right)$$

define the production functions in the post and pre-change situations respectively, we have

$$H(x_1(1), x_2(1); F) = x_0(1) = \alpha x_0(0) = \alpha \phi(x_1(0), x_2(0); F)$$
$$= \phi(\alpha x_1(0), \alpha x_2(0); \alpha F) \quad \text{by C.R.S.}$$
$$= \phi(x_1(1), x_2(1); \alpha F) \quad \cdots \quad (13)$$

Hence the effect of uniform multiplicative shift in the cell distribution functions resulting from a rise in the productivity of the fixed factor for firms is reflected in a fixed factor augmenting technical change of the same magnitude in the macro function.

(ii) Rise in the productivity of variable factors : variable input saving technical change:-

The other type of disembodied technical change one can think of is of the variable -input-saving type, with productivity of the fixed factor remaining unchanged.

Suppose a cell in the pre-change situation uses (x_1, x_2) units of the variable inputs to produce 1 unit of the output, In the post-change situation we assume that the same firm needs only $(x_1/a_1, x_2/a_2)$ units of varible inputs to produce 1 unit of final product, where $a_1, a_2 \geq 1$, with strict inequality holding in at least one case. We further assume that all the cells in the industry experience identical rates of technical change.

The phenomenon of input-saving change indicates that old firms that were already producing with profit, can now earn a higher rate of quasi - rent, that previously marginal firms noe start earning positive profits, and that some new firms, which found it unprofitable to produce before, can now start producing profitably since variable input requirements all around have fallen.

Thus there are two things that apparently seem to change from the old situation to the new: one, the cell distribution function, and two, the budget constraint:

	Pre-change situation	Post-change situation
Cell dist. function	$f^0(x_1, x_2)$	$f^1(x_1, x_2) = f^0(a_1x_1, a_2x_2)$, $a_1, a_2 \geq 1$.
Feasible region	$G(x) = \{(x_1, x_2) \mid p_1x_1 + p_2x_2 \leq 1\}$	$R(x) = \{(x_1, x_2) \mid p_1\frac{x_1}{a_1} + p_2\frac{x_2}{a_2} \leq 1\}$.

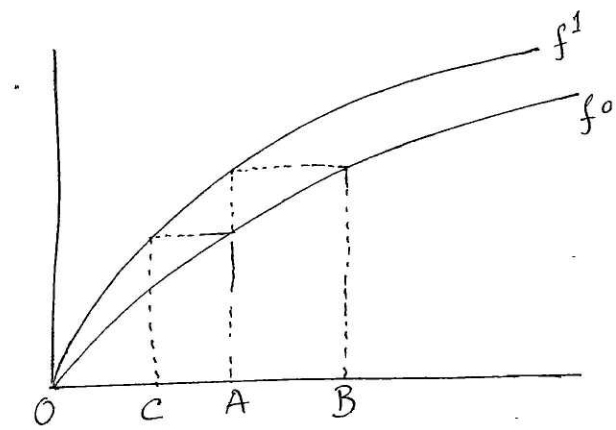
What one has to be careful about is that a change from the old situation to the new can be brought about by incorporating <u>either one</u> of the changes listed above and that enforcement of both would be incorrect.

This is so because as between the old situation and the new, productivity of the fixed factor has not changed, nor has its endowment to individual cells. Hence the output that the old firms can produce now is exactly the amount they could produce before -- only they are making more profits now -- and the increase in total output in the new situation is explained solely by the entry of new firms made possible by a fall/in variable input requirements. We can look at the new situation in either of the two ways:

(a) The same distribution function with a new budget line, or,

(b) A new distribution function with the old budget line, or, alternately, the old function defined in efficiency units, plus the old budget line,

The two situations are depicted in the attached figure for the case of one variable input:



If OA is the feasible region initially, then we either keep the distribution function unchanged at f^o and raise the feasible region to OB (situation (a)), or we keep the feasible region fixed at OA and raise the distribution function to f^1 (situation (b)) -- where OC/OA = OA/OB. In situation (a) the identification of cells to points on the (x_1, x_2) plain is maintained, whereas in situation (b) that identity is lost and variable inputs are now defined in efficiency units.

To see that input-saving technical change give us identical results, note that by def.

(a), total output in the post-change situation is given by

$$X_o^{(a)}(1) = \iint_{R(x)} f^o(x_1,x_2)\,dx_1\,dx_2\,, \text{ where } R(x) = \left\{(x_1,x_2) \middle| p_1\frac{x_1}{a_1} + p_2\frac{x_2}{a_2} \le 1\right\} \quad \ldots\ldots (14)$$

By def. (b), post-change output is given by

$$X_o^{(b)}(1) = \iint_{G(x)} f^o(a_1x_1, a_2x_2)\,d(a_1x_1)\,d(a_2x_2)\,, \text{ where } G(x) = \left\{(x_1,x_2) \middle| p_1x_1 + p_2x_2 \le 1\right\}. \quad \ldots\ldots (15)$$

Setting $a_1x_1 = y_1$ and $a_2x_2 = y_2$, we have,

$$X_o^{(b)}(1) = \iint_{R(y)} f^o(y_1,y_2)\,dy_1\,dy_2\,, \quad \text{where } R(y) = \left\{(y_1,y_2) \middle| p_1\frac{y_1}{a_1} + p_2\frac{y_2}{a_2} \le 1\right\}.$$

which is identical to $X_o^{(a)}(1)$.

Using def. (a) for new total output, and omitting the superscript, we have

$$\begin{aligned} X_o(1) &= \iint_{R(x)} f^o(x_1,x_2)\,dx_1\,dx_2 \\ &= \iint_{G(x)} f^o(x_1,x_2)\,dx_1\,dx_2 + \iint_{R(x)-G(x)} f(x_1,x_2)\,dx_1\,dx_2 \\ &= \text{Output of old firms} + \text{Output of new firms.} \end{aligned}$$

Thus, using G and R as superscripts to denote the old and new regions of integration respec., we can write

$$X_o(1) = X_o^G(0) + X_o^{R-G}(0) \quad \ldots\ldots (16)$$

Also, $X_1(1) = \iint_{R(x)} \frac{x_1}{a_1} f^o(x_1,x_2)\,dx_1\,dx_2 = \frac{1}{a_1} X_1^R(0) \quad \ldots (17)$

& $X_2(1) = \frac{1}{a_2} X_2^R(0)$. $\quad \ldots\ldots (18)$

Let the old production function be given by

$$X_0^G(0) = \phi(X_1^G(0), X_2^G(0); F(G)) \quad \cdots\cdots \quad (19)$$

Since a change in the region of integration should not change the form of the production function, which depends only on the form of the cell distribution function, when the feasible zone changes from G to R, we should still have

$$X_0^R(0) = \phi(X_1^R(0), X_2^R(0); F(R))$$

The only difference between the two being the change in the amount of the fixed factor used brought about by the entry of some new firms.

Let the production function in the post-change situation be given by

$$X_1(0) = H[X_1(1), X_2(1); F(R)].$$

Substituting values of $X_1(1)$ and $X_2(1)$ we get

$$X_1(0) = H[a_1 X_1^R(0), a_2 X_2^R(0); F(R)] \quad \cdots \quad (20)$$

Comparing with (1) this shows that the technical change is of the variable input augmenting type, the extent of input augumentation being exactly the same as that of the cells. If productivity of the fixed factor. F. rises, measure F in efficiency units F^*, where, $F^*(t) = \alpha(t) F(t)$, $\alpha > 1$. same for the variable factors. If the fixed factor, (or the variable factors, all measured in efficiency units) to output ratios remain constant (thus redefining the concept of fixed coefficients), these ratios where all categories are measured in natural units will be higher. So wherever a factor's productivity is changing over time, one might measure it in efficiency units and it is clear why the micro results should carry over exactly to the macro level.

In the above cases we have considered situations where technical change is proportionate for all firms. An interesting extension of this would be when it occurs at different rates for different firms. One such case is discussed in the following section.

III

Reinvestment and aggregate returns to scale

Suppose that the extent of 'factor-saving' that a firm can enjoy is not totally exogenous, but a function of its quasi-rents, assuming that it invests all or part of it in improving the quality of its fixed factor endowment. Since quasi-rents earned by the different firms are different, it is clear that the extent of this endogenously propelled factor saving will also vary from firm to firm.

Alternatively, one might postulate that the quasi-rents earned in one period are reinvested in procuring more fixed factors in the next. This will generate a pattern of growth of individual cells which will get reflected on the aggregate level.

Let us suppose that each firm reinvests a function $\alpha(r)$ of its quasi-rent r, where $\alpha' > 0$, $\alpha(0) = 0$

If $f^0(x_1, x_2)$ be the original

distribution function, the new distribution function will then be given by

$$f^1(x_1, x_2) = (1 + \alpha(r))\, f^0(x_1, x_2) \qquad \cdots (21)$$

$$\text{where} \quad r = 1 - p_1 x_1 - p_2 x_2$$

making the simplest assumption about $\alpha(r)$, let us suppose all firms reinvest <u>all</u> their profits, so that

$$\alpha(r) = r$$

substituting this in (21) we have

$$\begin{aligned} f^1(x_1, x_2) &= (1 + r)\, f^0(x_1, x_2) \\ &= (2 - p_1 x_1 - p_2 x_2)\, f^0(x_1, x_2) \\ &= 2 f^0(x_1, x_2) - p_1 x_1 f^0(x_1, x_2) - p_2 x_2 f^0(x_1, x_2) \end{aligned}$$

Integrating over the region G $(p_1 x_1 + p_2 x_2 \leq 1)$, we have, as before,

$$X_0(1) = 2 X_0(0) - p_1 X_1(0) - p_2 X_2(0)$$

$$\text{or,} \quad \frac{X_0(1)}{X_0(0)} = 2 - \frac{p_1 X_1(0) + p_2 X_2(0)}{X_0(0)} = 2 - \varepsilon(0) \quad \ldots (22)$$

where $\varepsilon(0)$ is the returns to scale in the initial period

We have

$$\varepsilon(0) = \frac{p_1 X_1(0) + p_2 X_2(0)}{X_0(0)}$$

$$\begin{aligned} \varepsilon(1) &= \frac{p_1 X_1(1) + p_2 X_2(1)}{X_0(1)} \\ &= \frac{2[p_1 X_1(0) + p_2 X_2(0)] - \iint_G (p_1 x_1 + p_2 x_2)^2 f^0(x_1, x_2)\, dx_1 dx_2}{2 X_0(0) - \iint_G (p_1 x_1 + p_2 x_2) f^0(x_1, x_2)\, dx_1 dx_2} \end{aligned}$$

$\therefore\ \varepsilon(0) \gtreqless \varepsilon(1)$ according as

$$\frac{p_1 X_1(0) + p_2 X_2(0)}{X_0(0)} \gtreqless \frac{2[p_1 X_1(0) + p_2 X_2(0)] - \iint_G (p_1x_1 + p_2x_2)^2 f^0(x_1,x_2)\,dx_1 dx_2}{2X_0(0) - \iint_G (p_1x_1+p_2x_2) f^0(x_1,x_2)\,dx_1 dx_2}$$

i.e, acc. as $\left[\iint_G (p_1x_1+p_2x_2) f^0(x_1,x_2)\,dx_1 dx_2\right]^2$

$$\gtreqless \left[\iint_G f^0(x_1,x_2)\,dx_1 dx_2\right] \cdot \left[\iint_G (p_1x_1+p_2x_2)^2 f^0(x_1,x_2)\,dx_1 dx_2\right].$$

i.e, acc. as $\iint_G (p_1x_1+p_2x_2) f^0(x_1,x_2)\,dx_1 dx_2$

$$\gtreqless \left[\iint_G f^0(x_1,x_2)\,dx_1 dx_2\right]^{1/2} \left[\iint_G (p_1x_1+p_2x_2)^2 f^0(x_1,x_2)\,dx_1 dx_2\right]^{1/2} \quad \text{---- (23a)}$$

By using Holder's inequality, we have

$$\int h(x)\,g(x)\,dx \le \left[\int \{h(x)\}^p dx\right]^{1/p} \cdot \left[\int \{g(x)\}^{p'} dx\right]^{1/p'}$$

for $p' = \frac{p}{1-p}$

Let $p = p' = 2$

$h(x) = [f^0(x_1,x_2)]^{1/2}$

& $g(x) = (p_1x_1+p_2x_2)[f^0(x_1,x_2)]^{1/2}$

Substituting these values in the expression for Holder's inequality and integrating over the region G, we have:

$$\iint_G \{f^{\circ}(x_1,x_2)\}^{1/2}\left[(p_1x_1+p_2x_2)\{f^{\circ}(x_1,x_2)\}^{1/2}\right]dx_1dx_2$$
$$\leq \left[\iint_G \{(f^{\circ}(x_1,x_2))^{1/2}\}^2 dx_1dx_2\right]\cdot\left[\iint_G \{(p_1x_1+p_2x_2)(f^{\circ})^{1/2}\}^2 dx_1dx_2\right].$$

$$\text{Or,}\quad \iint_G (p_1x_1+p_2x_2)f^{\circ}(x_1,x_2)\,dx_1dx_2 \leq$$
$$\left[\iint_G f^{\circ}(x_1,x_2)\,dx_1dx_2\right]\left[\iint_G (p_1x_1+p_2x_2)^2 f^{\circ}(x_1,x_2)\,dx_1dx_2\right] \quad ---(23b)$$

Comparing this with (23 a) we have

$$\varepsilon(0) \not< \varepsilon(1) \qquad \text{for any } (p_1,p_2).$$

i.e., whenever profits are reinvested b, all the firms in a certain fixed percentage, aggregate returns to scale for the industry as a whole falls, or at best remains constant, but can never go up.

There is another anle from which all this can be looked at. A reduction in the returns to scale is associated with shift in the point of means$\left(\frac{X_1}{X_0},\frac{X_2}{X_0}\right)$further towards the orign (Cf. p.~~268~~ above). All this is very natural since a fixed % reinvestment of profits by each firm will, pari passu, make the cell distribution more and more skewed in favour of the more efficient firms, thus pulling the point of means towards the orign. It is interesting to note that this tendency towards increased concentration of percentage output in the hands of the more efficient firms exists even without assuming indivisibility of the fixed factor - one of the often quoted reasons for tendency towards concentration. This tendency is accentuated if one clamps a positive rate of depreciation on the model. Assumption of a certain amount of lumpivers in the fixed factor together with a positve rate of depreciation might explain why some marginal firms might eventually be driven out of business. Thus initial efficiency of some firms, vis-a-vis others, however obtained, has a tendency to accentuate the gap between firms even in an otherwise competitive situation. The choice of the particular reinvestment function $\alpha(r) = 1 + r$, was made solely for the purpose of convenience. The basic result of increased skewness of the Cell distribution holds with any reinvestment function $\alpha(r)$, satisfying the property $\alpha(0) = 0$ & $\alpha'(r) > 0 \ \forall r$.

IV

All of the previous analysis has been based on the assumption that cell production functions are fixed coefficient. What happens if they are not? Can one still associate a cell distribution function unequivocally with an aggregate production function? Our answer is no, and this is primarily because with neo-classical cell production functions, relative prices lose their parametric nature, and are no longer washed out as before at the end of the calculations.

Suppose there are an firms in an industry each with a neo-classical (convex) production possibility set. A produnction possibility set for any $y \geq 0$ 0 is defined as

$$L(y) = \{ x \mid f(x) \geq y, \; x \geq 0 \}, \quad \text{where } f(x)$$

is a quasi-concaue function defined on the positive quadrant of R^n . By a well known theorem, any ~~convex~~ vector ~~survey~~ sum of such sets will be convex, and hence a production possibility set in its own right. The weights defined by the convex combination act as probability numbers. Hence with each set of a production possibility sets and any convex combination of them, is associated an aggregate production possibility set or an aggregate production function.

The probability distribution defined here is quite different from the type Houthakker has in mind; for the ones Houthakker talks about are truncated and independent of the prices. One mgith say that this difference can be at least formally removed by normalizing Houthakker's distributions by their comulative totals for any given. set of relative prices - and thereby by making them functions of the trucation point (i.e., of relative prices. But the difference between the two cases runs deeper than that. In the case where the cell production functions are neo-classical, even if one postulates a cell distribution function specifying the allocation of the fixed factor among cells, unlike in the Leontief case, the distribution of output and variable inputs are no longer determinate unless one also knows the factor prices. For now, unlike in the previous case, the extent of utilization of capacity for each firm becomes a variable -- itself depending on relative prices of factors.

It seems, therefore, that if we want to think in terms of some kind of cell distribution function over the variable input space where cell production functions are neo-classical, such a function will have to defend not merely on $x_1 + x_2$, but also on $p_1 + p_2$. In other words, with neo-classical cell production functions, thinking in terms of Houthakker type cell distribution functions does not seem to be very paying.

V

Coming back to the case of fixed coefficient cell production functions, one might experiment with different known forms of distribution and production function to find out if either generates a reasonable looking form of the other function associated with it. The association of the Pareto distribution with Cobb-Douglas production function in the aggregate, as proved by Houthakker, is very striking, but unfortunately seems to be somewhat of an exception, in that both functions are well-known, compact and manipulable. The distribution function that Levhari has derived from a one-variable factor C.E.S. production function in the aggregate (3), is indeed a legitimate distribution function, but much too cumbrons in form and certainly not very recognizable in appearance. Explicit solutions for aggregate production functions from known cell distribution functions are indeed obtainable, but mostly in principle. This is so because although in theory it is possible to drive out m relative factor prices out of (m + 1) equations --- one each for the m factors and one for the output -- more often than not the equations are non-linear in relative prices and simplifying manipulations cannot be done. Johansen has worked out explicit solutions for a few manouvorable cases like that of the rectangular distribution in two variable factor situations. But in general, explicit solutions are unlikely to exist. Local approximations are one way to deal with this. But without going into that, it is still possible to glean some interesting information about the aggregate function when the cell distribution function have specific characteristics.

Let us assume that the cell distribution function has independent marginal distributions,

i.e., $f(x_1, x_2) = g(x_1)h(x_2)$

Then

$$X_0(p_1, p_2) = \int_0^{1/p_1} \int_0^{\frac{1-p_1x_1}{p_2}} g(x_1)\, h(x_2)\, dx_1\, dx_2$$

$$= \int_0^{1/p_1} g(x_1) \left[\int_0^{y} h(x_2)\, dx_2 \right] dx_1 , \qquad y = \frac{1-p_1x_1}{p_2}$$

$$= \int_0^{\frac{1}{p_1}} g(x_1)\, H(y)\, dx_1 , \qquad H(y) = \int_0^y h(x)\, dx$$

$$\therefore \; X_0(p_1, p_2) = \int_0^{1/p_1} F(x_1)\, dx_1 , \qquad F(x_1) = g(x_1) H(y), \qquad ---(24)$$

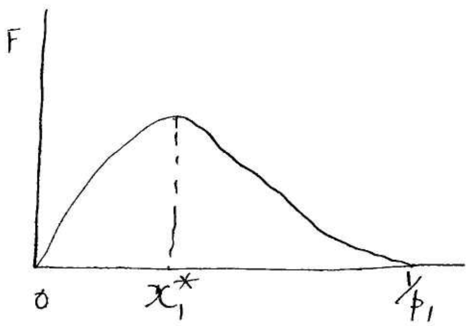
The function $F(x_1)$ is defined over $\left[0, \frac{1}{p_1}\right]$

At $x_1 = 0$, $F(x_1)\Big|_{x_1=0} = g(0)\, H\left(\frac{1}{p_2}\right) = 0$

At $x_1 = \frac{1}{p_1}$, $F(x_1)\Big|_{x_1=\frac{1}{p_1}} = g\left(\frac{1}{p_1}\right) H(0) = 0.$

$\therefore F(x_1) = 0$ at both the end points $x_1 = 0$ & $1/p_1$

since it is positive for all intermediate values of x_1 by Rolle's Theorem there exists at least one such x_1^* that



$F(x_1^*) = \max F(x_1)$, $F'(x_1^*) = 0$
Thus the function F can look something like the figure attached. It need not be unimodel through.

We can also express X_1 & X_2 in terms of F. To see this:

$$X_1(p_1, p_2) = \int_0^{1/p_1} x_1 g(x_1) \int_0^{y} h(x_2)\, dx_2\, dx_1$$

$$= \int_0^{1/p_1} x_1 \left[g(x_1) H(y) \right] dx_1$$

$$= \int_0^{1/p_1} x_1 F(x_1)\, dx_1 \quad \text{------} \quad (25)$$

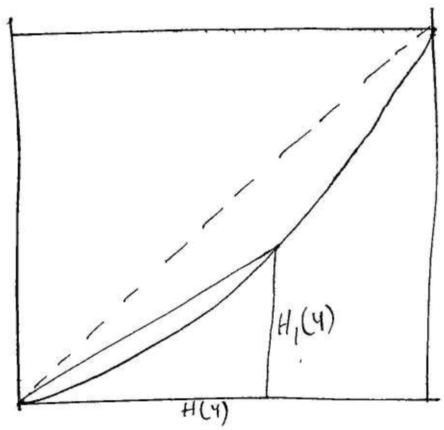
$$X_2(p_1, p_2) = \int_0^{1/p_1} g(x_1) \int_0^{y} x_2 h(x_2)\, dx_2\, dx_1$$

$$= \int_0^{1/p_1} g(x_1) H(y) \left\{ \frac{\int_0^{y} x_2 h(x_2)\, dx_2}{\int_0^{y} h(x_2)\, dx_2} \right\} dx_1$$

$$= \int_0^{1/p_1} F(x_1) \left[\frac{H_1(y)}{H(y)} \right] dx_1$$

$$= \int_0^{1/p_1} F(x_1)\, \Phi(x_1)\, dx_1 \quad \text{------} \quad (26)$$

It is clear that the ratio $\frac{H_1(y)}{H(y)} = L(h;y)$ is the Lorentz angle with respect to the function h. By the property of Lorentz curves,



$$L_y > 0 \;\&\; L_{yy} \gg 0$$

$$\therefore \phi'(x_1) = \phi_y \cdot \frac{dy}{dx_1}$$

$$= \phi_y\left(-\frac{p_1}{p_2}\right) < 0$$

And $\phi''(x_1) = \phi_{yy}\left(\frac{dy}{dx_1}\right)^2 + \phi_y\left(\frac{d^2y}{dx_1^2}\right) \geq 0 \quad \because \phi_{yy} \geq 0 \;\&\; \frac{d^2y}{dx_1^2} = 0$

$\therefore$ Any function $\phi(x_1)$ with $\phi'(x) < 0$ and $\phi''(x) \geq 0$ is a potential candidate for this case

$\therefore$ Writing (24) - (26) in one place,

we have

$$X_0 = \int_0^{1/p_1} F(x_1)\, dx_1$$

$$X_1 = \int_0^{1/p_1} x_1 F(x_1)\, dx_1$$

$$X_2 = \int_0^{1/p_1} \phi(x_1) F(x_1)\, dx_1 ,$$

$$\phi' < 0, \quad \phi'' \geq 0 .$$

One simple from of $\phi(x_1)$ satisfying the restrictions will be $\phi(x_1) = c - \alpha x_1$

Substituting this in the expression for X_2 we note

$$\alpha X_1 + X_2 = \int_0^{1/p_1} \alpha x_1 F(x_1)\, dx_1 + \int_0^{1/p_1} (c - \alpha x_1) F(x_1)\, dx_1$$

$$= \int_0^{1/p_1} \{\alpha x_1 + (c - \alpha x_1)\} F(x_1)\, dx_1$$

$\therefore \alpha X_1 + X_2$

$$= c \int_0^{1/p_1} F(x_1)\, dx_1 = c X_0$$

Thus $\phi(x_1) = c - \alpha x_1$ entails a linear aggregate production function

Another form $\phi(x_1)$ can take is $\phi(x_1) = \frac{1}{x_1}$ satisfying the restrictions $\phi' < 0$, $\phi'' \geq 0$

$$X_1 = \int_0^{1/p_1} x_1 F(x_1)\, dx_1 = \int_0^{1/p_1} F(x_1)\, d\left[\frac{x_1^2}{2}\right].$$

$\because$ F is continuous and $(x_1^2/2)$ is monotonic function in $[0, 1/p_1]$, one can apply the First Mean Value Theoreum of integral calculus to get

$$X_1 = F(x_1^{**}) \left[\frac{1}{2p_1^2} - 0\right] = C_1 F(x_1^{**}) \quad \text{---- (i)}$$

where $x_1^{**} \in [0, 1/p_1]$

Similarly, $X_0 = F(x_1^*) \cdot 1/p_1 = C_0 F(x_1^*)$ ---- (ii)

and $X_2 = F(x_1^{***}) \log\left(\frac{1}{p_1}\right) = C_2 F(x_1^{***})$ ---- (iii)

Since x_1^*, x_1^{**} + x_1^{***} are fixed points in $[0, 1/p_1]$ for constant p_1, one can expand (i) + (iii) around x_1^*, and keeping the linear terms alone as a first approximation, get

$$X_1 = \hat{c}_1 F(x_1^*) \quad \& \quad X_2 = \hat{c}_2 F(x_1^*),$$

so that $X_0 = A X_1^{1/2} X_2^{1/2} \longrightarrow$ Cobb-Douglas

Although this result should be extended to cover the relation of x_1^* to x_1^{**} & x_1^{***} through out the interval, this seems to be a useful first approximation. It implies that a long-linear cell distribution function and the Lorentz coefficient of one of the marginal distribution functions being an inversely falling function of the other variable, together imply an aggregate production function which is Cobb-Douglas as a first approximation.

In conclusion, one might add a few words regarding the relationship between this aggregation problem à la Houthakker vis-a-vis the standard aggregation problem in economics dealt with by G. Fisher, Green, Nataf et. al..

We have seen that under ordinary circumstances, Houthakker's method of aggregation fails when the firm production functions are neo-classical. In Houthakker type set-up, the way this came about is via the dependence of output of a producing firm on the prevailing relative prices of variable inputs. If it were possible to know exactly how a firm with a given technology will behave given a set of relative prices, in that case alone an effort at aggregation would perhaps been fruitful.

The classical problem of aggregation, though different in the type of questions it predominently deals with, is similar to ours at least in its negative content. It is similar to ours also in so far as both are concerned with the problem of consistency of micro and macro results. Apart from this fundamental similarity, the two approaches are quite different.

(i) A major part of the classical discussion on the possibility of aggregation deals with aggregation over factors. Heterogeneity of a factor like capital was only one of the problems to be tackled. The most general condition for aggregation over factors has been recognized to be that of functional separability or what is commonly known as the 'Leontief condition' : i.e., the MRS between any two inputs that are to be aggregated should be independent of the remaining inputs in the production function. Our problem, on the other hand, abstracts completely from the problem of aggregation among factors and deals exclusively with aggregation over firms. Nevertheless, as Prof. Fisher has shown in his various papers on the subject, this condition does impose some very strong restrictions on the condition for the existence of a capital aggregate boils down to the condition that all the firm production functions be exactly identical save for a capital augmenting technical difference : i.e., if $f(k_1, l_1)$ be the production function of the first firm, then that of the second has to be of the form $f(ak_1, l_2)$, where a is a constant. Similar conditions exist for labor aggregation. Although this does not relate directly to our problem, it does show that the problems of aggregation over factors and over firms are related, and to that extent might have some bearing on our problem.

(ii) There has been some discussion on the problem of aggregation over firms directly rather then via the functional separability conditions. Nataf has shown that a necessary and sufficient condition for micro production functions to be aggregable is that each such function be additively separable in the factors. This seems to be a relevant result for our purpose too, since with additively separable (or linear) production functions, relative prices of inputs no longer determine the extent of capacity utilization, and to that extent both the approaches seem to indicate the existence of aggregate production functions.

(iii) Most of the discussion on aggregation in recent years has been done on the basis of prior optimal allocation of at least the 'moveable' reasources like labor between firms so as to maximize output given total employment. Our approach does not do that.

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