DEMAND ANALYSIS OF RAIL COMMUTER TRANSPORTATION

by

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ABSTRACT

During 1963 and 1964 the Boston and Maine Railroad participated in the mass transportation demonstration project conducted by the Mass Transportation Commission. The frequency of service and twenty-ride commutation and one-way fares were altered on all lines serving the city of Boston. A regression analysis has been conducted with the data generated during the project by four of these lines. The analysis revealed that peak hour demand is very inelastic with respect to the frequency of service and the twenty-ride fare and somewhat inelastic with respect to the one-way fare. Off-peak hour demand was found to be considerably elastic with respect to the one-way fare but somewhat inelastic with respect to the frequency of service and the twenty-ride fare.

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HISTORICAL BACKGROUND

Boston is one of five major cities in the United States having a rail commuter service. For Boston the service is supplied by both a public mass transit system and the passenger divisions of private railroads.

Mass Transit System

The public has long been concerned about the welfare of the mass transit system. Begun as speculative ventures by individual entrepreneurs the system soon became overextended and unprofitable. A public Control Act was needed in 1918 to assure the continuance of the much needed service supplied by the Boston Elevated Company to fourteen towns and cities. As a temporary measure each of the fourteen assumed a portion of the deficit in proportion to its share of the passenger volume. However, it became necessary to continue the public subsidy in 1930 and finally to establish public ownership in 1947. The then created Metropolitan Transit Authority has since been expanded into the Massachusetts Bay Transportation Authority.[1]

Private Railroads

Initially the private railroads covered their passenger service deficits by means of the "value of service" method of rate determination. However the transportation revolution of the thirties altered

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[1] Mass Transportation in Massachusetts, Mass Transportation Commission (Boston, 1964), pp. 23-24.

the situation. Passenger volumes decreased, increasing the deficit, as more and more commuters turned to the automobile and bus. The revenue available to cover the increasing deficits decreased as the industries most able to pay became those with the largest number of alternative means for moving their goods.[2] The railroads turned to fare increases and service reductions as a means of reducing the deficits. These remedies only aggravated the situation. Sixty percent service reductions were accompanied by seventy percent volume declines. As a last resort preparation was begun by the Boston and Maine on petitions to the Interstate Commerce Commission for the total discontinuance of all railroad commuter service into Boston.

Mass Transportation Commission

By then the need for coordination and cooperation in the planning of both public and private transportation facilities had become apparent. In 1959 the Governor and the General Court of Massachusetts established the Mass Transportation Commission. The Commission was charged with the responsibility of investigating the relationship of mass transportation facilities, land use and urban renewal to the economic needs and opportunities of the Commonwealth. A preliminary survey, "The Boston Region", revealed that inadequate information was available to decision makers. At the Commission's request a Joint Special Legislative Committee on Transportation was established in May, 1961. Together the two bodies recommended that the M.T.C.

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[2] John R. Meyer and others, Competition in the Transportation Industries (Cambridge, 1960), pp. 8-14.

conduct a mass transportation demonstration project. On October 5, 1962, the United States Housing and Home Finance Agency notified the Commission that its application for a $3.6 million demonstration grant had been approved. To this was added $1.8 million of the Commonwealth's funds.[3]

Demonstration Project

According to the Agency's administrator, Dr. Robert Weaver, the purpose of the demonstration was

> ... to provide data, not now available in any reliable form, upon which predictions can be based as to the effects of various service and fare changes, alone or in combination, on transit ridership
>
> . . .
>
> It is expected that the results of these experiments will enable the decision-makers of the Commonwealth of Massachusetts to make immediate as well as long-term decisions regarding transportation planning and financing which are based on reliable and acceptable facts.[4]

Of the $5.4 million approximately $4 million were allocated for railroad experiments, $500,000 for experiments with private bus companies and $900,000 for experiments with the M.T.A. The first of the thirty-five began on December 10, 1962, and the last ended on March 28, 1964.

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[3] Mass Transportation in Massachusetts, pp. 5-6.

[4] Ibid., p. 7.

RAIL EXPERIMENTS

As the M.T.C. and the special legislative committee were preparing their application for funds, preliminary discussions were held with the three railroads supplying Boston's commuter service: the Boston and Maine, the New Haven, and the New York Central. All three indicated a willingness to participate in a demonstration project. Agreements at the policy and technical levels later proved inconclusive with the New York Central and, subsequently, it did not participate directly in the experiments.[5]

The New Haven

The New Haven's participation was divided into two phases. During the first phase, March 10, 1963, through September 7, 1963, off-peak hour service was increased on several lines while peak hour fares were decreased. For the second phase, September 8, 1963, through March 7, 1964, the experiment was essentially continued on only one line, that connecting Boston with Providence. Phase one revealed that where fares were reduced but service unchanged, the patronage increased little if at all. Where service had been increased, so had the patronage. During phase two patronage on all lines increased over that of phase one, even on the lines which had been dropped from the experiment.[6]

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[5] Ibid., p. 26.

[6] Ibid., pp. 40-43.

The Boston and Maine

The experiment involving the Boston and Maine was divided into three phases. During the first, January 1963 to July 1963, overall service was increased by 77 percent while fares were decreased by amounts varying from 12 to 72 percent. For phase two, August 1963 to December 1963, fares were essentially returned to their pre-experiment level with service being maintained at its phase one level. The third phase was unique. Between January 7, 1964, and March 21, 1964, an attempt was made to test the conclusions of the first two phases. Fares and service were adjusted sporadically. The results of the experiment are significant. Patronage increased by 27 percent during the first phase. Despite the return of fares to the pre-experiment level patronage increased even more during the second phase, being 37.5 percent over that of the similar period of 1962. It continued to increase during the third phase, being 44 percent above the similar 1962 period.[7]

Conclusions

The M.T.C. reported three chief operational findings of the rail experiments:

> 1. Additional passengers can be attracted to both peak and off-peak railroad suburban service.
>
> 2. Frequency of service is a more important factor than lower fares in both retaining present passengers and attracting additional passengers....

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[7] *Ibid.*, pp. 26-38.

3. Increases in commuter fares, when accompanied by a continuation of a high level of frequency of service, do not necessarily result in decreases in passenger volumes.[8]

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[8] Ibid., p. 44.

THE BOSTON AND MAINE RAILROAD

Route Structure

The Boston and Maine route structure fans out from Boston to the north and the west. Only the four lines of specific interest, the Eastern, Reading, Western, and Fitchburg, will be described here. To the northeast the Eastern Route serves Lynn, Salem, and Beverly. At Beverly the route splits with one line extending to Rockport, the other to Newburyport. The Reading Line extends for twelve miles to the north serving three suburban towns. Diverging from another at Wilmington the Western Route serves Lawrence and Haverhill, the latter being thirty three miles from Boston. The Fitchburg Line extends fifty miles to the industrial center of Fitchburg.[9]

Pre-experiment

During the twelve years prior to the experiments the Boston and Maine suffered a large decline in its passenger patronage. It carried approximately 5.3 million passengers to or from Boston during 1962, only 30.4 percent of its 1950 volume. The decline was partially the result of fare and service changes. Intrastate commutation fares had been increased by 25 percent in 1952 and by 35 percent in 1957. One-way and round trip fares had been increased by 5 percent in 1956, by 35 percent in 1957, and by 5 percent again in 1959. In 1960 and

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[9] *Ibid.*, p. 27.

1961 all fares were incremented by five cents per ride. Overall service had been reduced during this period by almost 60 percent. Service on several branches was completely discontinued, that on others gradually reduced.[10]

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[10] Rail Demonstration Project Technical Supplement, Mass Transportation Commission (Boston, 1964).

Table 1

BOSTON AND MAINE RAILROAD[11]

Patronage in and out of Boston

Year	Revenue Passengers (Millions)	Percent of 1950
1950	17.4	100.0
1951	16.5	94.8
1952	16.6	95.4
1953	14.7	84.5
1954	13.8	79.3
1955	13.0	74.7
1956	13.2	75.9
1957	11.2	64.4
1958	8.5	48.9
1959	7.1	40.8
1960	6.3	36.2
1961	5.7	32.8
1962	5.3	30.4

[11] Ibid.

THE ANALYSIS

The purpose of the experiments was to provide data "upon which predictions can be based as to the effects of various service and fare changes, alone or in combination, on transit ridership." In its "raw" form the data obtained from the experiments permits few conclusions and predictions of questionable reliability. The analysis which follows transforms the "raw" data into a form which not only permits more accurate predictions but also some measure of their reliability.

Least-Squares Estimator

What is desired is the best estimate of the portion of the relative change of the passenger volume which can be "explained" by the relative change of each of the independent variables, the frequency of service and the fares. If the portions are considered to be linearly additive, the problem is to determine the best estimate of the coefficients of the regression equation

$$X_1 = \beta_0 + \beta_2 X_2 + \beta_3 X_3 + \epsilon \quad ,$$

where ϵ may be regarded as the portion of X_1 "not explained" by X_2 and X_3.

To approximate the values of β_0, β_2 and β_3 an estimator,

$$X_1 = b_0 + b_2 X_2 + b_3 X_3 \quad ,$$

is introduced. A "working model" is then constructed,

$$X_1' = b_0 + b_2 X_2 + b_3 X_3 + e \quad ,$$

where X_1' is the observed value of the dependent variable and e the portion of X_1' "not explained" by the estimator. The best estimator is one for which the sum of the squares of the differences between the observed values of X_1 and those predicted by the estimator,

$$\sum e^2 = \sum (X_1' - b_0 - b_2X_2 - b_3X_3)^2 \quad ,$$

is a minimum.

This minimum occurs where the partial derivatives of $\sum e^2$ with respect to b_0, b_2, and b_3 are zero. The resulting equations can readily be solved for the best estimated values of b_0, b_2, and b_3.

REGR II

The above regression analysis was conducted on the IBM 1620 II computer at the Sloan School of Management of the Massachusetts Institute of Technology. The regression program, referred to as REGR II, is in storage and readily accessible. In addition to calculating the regression coefficients the program supplies several means of testing the significance of the coefficients and the estimator.

Multiple Correlation Coefficient

The multiple correlation coefficient represents the maximum correlation to be expected between X_1, the dependent variable, and a linearly additive combination of X_2 and X_3. The percentage of the variance of the dependent variable which has been explained by the independent variables is indicated by the square of the multiple correlation coefficient.

Fisher's F and Student's t

The significance of the multiple correlation coefficient is established by means of Fisher's F. A null hypothesis is made that the coefficient results from random sampling errors. It is implied that if enough observations were available the correlation would be zero. The values of F have been calculated according to the null hypothesis for various levels of significance and degrees of freedom. The calculated value of F for the correlation in question is compared to those in the table for the same number of degrees of freedom. The level of significance for the coefficient is then that for which the calculated F is greater than the null hypothesis F. A .001 level of significance infers that the probability of the correlation having resulted from random sampling errors is .001. Student's t performs essentially the same function for the estimated coefficients of the regression equation.[12]

The Analysis

The analysis consisted of a series of steps with three regression equations being considered at each step. Peak and off-peak data were considered separately to separate the commuter from the shopper or pleasure rider.[13] As the peak inbound and peak outbound data represent two totally distinct time periods it was considered best to analyze

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[12] Sir Ronald A. Fisher, Statistical Methods for Research Workers (New York, 1958).

Harold Freeman, Introduction to Statistical Inference (Reading, Massachusetts, 1963).

J. Johnston, Econometric Methods (New York, 1963).

[13] Peak hour service is defined as the weekday trains which enter the city between 7:30 and 9:30 a.m. and leave the city between 4:30 and 6:30 p.m. All other service is designated off-peak.

them separately. The off-peak inbound and outbound data do represent approximately the same period and were considered separate observations for the same analysis.

Dependent Variable

Some measure of the variation of the passenger volumes was needed for the dependent variable. Two choices were available. Audits recording the number of revenue passengers according to station and type of ticket had been taken during one week of each month of the experiment. However, only two weeks had been audited the previous year. The alternative was the headcounts taken daily by trainmen entering and leaving Boston. While these counts are less accurate and do include non-revenue as well as revenue passengers, they are considered to be "reasonably reliable for trends and comparisons".[14] Headcounts were also available for the entire previous year as well as for the experiment. If similar periods of the year could be compared seasonal fluctuations in passenger volumes would be removed. Thus the headcounts were chosen. It was believed that the greater number of potential observations would more than compensate for the loss in accuracy.[15]

Independent Variables

The three important independent variables are the fractional changes in the frequency of service and in twenty-ride commutation and one-way fares. The service figures were calculated in terms of

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[14] "Supplement No. 3," Mass Transportation in Massachusetts, Mass Transportation Commission (Boston, 1964), p. 13.

[15] Headcount statistics employed for the analysis are those reported in Supplement No. 3 of the M.T.C.'s final report, pp. 39-52.

the number of trains operating per unit of time. The fare reductions represent a weighted average of the reductions effective at each line's major stations.

Weather

A need was felt to include a means to measure the extent to which weather affected the surburban rail patronage. Two variables were constructed, one for peak, the other for off-peak hours. The equations used to construct them may seem somewhat arbitrary. In the belief that the effect of a given amount of precipitation becomes greater if it falls on several days rather than one, the total precipitation that fell during an observation period was multiplied by the number of days on which it fell. For the peak variable, only that which fell between six and nine a.m. was included, to which was added the amount of snow on the ground at the earlier hour divided by ten.

$$W_{PEAK} = \text{(DAYS)(TOTAL PRECIPITATION)} + \text{(SNOW ON GROUND)}/10$$

For the off-peak variable total daily precipitation figures were used, to which was added the snowfall for the same period divided by ten.

$$W_{OFF\text{-}PEAK} = \text{(DAYS)(TOTAL PRECIPITATION)} + \text{(SNOWFALL)}/10$$

All measurements were recorded at Boston's Logan Airport. The fractional change in the weather variables could not be calculated due to the possibility of zero denominators. Thus the weather variables in the regression equations represent the absolute change of weather.

Observations

The period chosen for each observation includes five non-holiday weekdays. Where a holiday did occur during an observation period the headcount and weather statistics for the remaining days were extrapolated to correspond to a five day period.

Because the third phase was atypical only weeks during the first and second phases were selected as observations. Scatter diagrams revealed the presence of an initial adjustment period of approximately five weeks. Therefore, the first five weeks of each phase were omitted. An additional week during the first phase was deleted for three of the lines. Unknown adjustments had been made on the raw headcount data to bring holiday weeks together for comparison. The entire first phase had to be omitted for the other line. A station which had been a stop on the Western Line during 1962 had been shifted to another line for the initial months of 1963. With these omissions the number of observations available for the analysis was forty for three of the lines and seventeen for the fourth.

LINEAR REGRESSIONS

Three linear regressions comprised the initial step of the analysis. The regression equations were of the form

$$\frac{\Delta Q}{Q} = \beta_0 + \beta_2 \frac{\Delta S}{S} + \beta_3 \frac{\Delta P_{ow}}{P_{ow}} + \beta_4 \frac{\Delta P_{tr}}{P_{tr}} + \beta_5 \Delta W \quad ,$$

where Q = passenger volume

S = frequency of service

P_{ow} = one-way fare

P_{tr} = twenty-ride commutation fare

and W = weather.

Here their best estimators will be discussed individually with regard to their statistical significance. Their economic significance will be considered later.

Peak Inbound

$$\frac{\Delta Q}{Q} = .3732 - .1769 \frac{\Delta S}{S} - .0697 \frac{\Delta P_{ow}}{P_{ow}} - .0518 \frac{\Delta P_{tr}}{P_{tr}} + .0653 \Delta W$$

The linear regression of the peak inbound sample yielded a best estimator with a multiple correlation coefficient of .5052 which is significant at the .001 level. However, only two of the beta-coefficients, those of the constant and the weather variable, are significant at the .05 level, none at the .01 level. Two of the independent variables are significantly correlated with the dependent variable, two pairs of independent variables with each other. Thus while approximately one quarter of the variance of $\frac{\Delta Q}{Q}$ has been "explained", the portion contributed by each of the independent variables has not been accurately determined.

Peak Outbound

$$\frac{\Delta Q}{Q} = .4829 - .3373 \frac{\Delta S}{S} - .0055 \frac{\Delta P_{ow}}{P_{ow}} - .1068 \frac{\Delta P_{tr}}{P_{tr}} + .0433 \Delta W$$

The multiple correlation coefficient for the peak outbound sample, .5409, is also significant at the .001 level. Here only the beta-coefficient of the service variable is significant at the .05 level, none are significant at the .01 level. Again two independent variables are correlated significantly with the dependent variable while three pairs are with each other. These results are essentially the same as those of the peak inbound sample.

Off-Peak Inbound and Outbound

$$\frac{\Delta Q}{Q} = 1.7027 + .9992 \frac{\Delta S}{S} - 3.3314 \frac{\Delta P_{ow}}{P_{ow}} - .8849 \frac{\Delta P_{tr}}{P_{tr}} - .0011 \Delta W$$

The off-peak sample yielded better results. The multiple correlation coefficient is .6059, significant at the .001 level. With the exception of that of the weather variable, all beta-coefficients are significant at the .01 level. While the correlations among the variables are generally lower, several are significant at the .01 level. Thus, while being the best results obtained from the linear regression step, they are of questionable statistical reliability.

LOGARITHMIC REGRESSIONS

The major problem revealed by the first step of the analysis was the existence of significant correlations among the independent variables. In an attempt to remove or reduce their significance logarithmic regressions were conducted.[16] The regression equation was of the form

$$\log \frac{\Delta Q}{Q} = \beta_0 + \beta_2 \log \frac{\Delta S}{S} + \beta_3 \log \left|\frac{\Delta P_{ow}}{P_{ow}}\right| + \beta_4 \log \left|\frac{\Delta P_{tr}}{P_{tr}}\right| .$$

A subroutine was added to the regression program to transform the data into logarithms. Twenty-ride commutation fares had remained at the pre-experiment level on the Reading Line during the second phase of the experiment. As the logarithm of zero is undefined these observations could not be included in the logarithmic regressions. All other fare changes were negative, permitting their absolute values to be transformed into logarithms. The existence of both positive and negative changes in the weather variables prevented their inclusion in this step of the analysis.

Peak Inbound

$$\log \frac{\Delta Q}{Q} = 8.9734 + 5.3138 \log \frac{\Delta S}{S} + 9.2463 \log \left|\frac{\Delta P_{ow}}{P_{ow}}\right| - .3647 \log \left|\frac{\Delta P_{tr}}{P_{tr}}\right|$$

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[16] The variable pair, X_1 and X_2, was considered and five pairs of values were chosen from a table of random digits. The correlation between the numbers themselves was found to be about .2 higher than the correlation between their logarithms.

The multiple correlation coefficient for the logarithmic regression of the peak inbound sample, .5142, is significant at the .001 level and greater than that for the linear regression involving the same variables. All of the beta-coefficients are now significant at the .01 level, another improvement over the linear regression. However the correlation situation remains essentially the same for the variables. The omission of the Western Line reduced the number of changes of the independent variables for the regression, removing the advantage gained by the transformation into logarithms.

Peak Outbound

$$\log \frac{\Delta Q}{Q} = 5.1644 + 2.2298 \log \frac{\Delta S}{S} + 6.0583 \log \left|\frac{\Delta P_{ow}}{P_{ow}}\right| - .0835 \log \left|\frac{\Delta P_{tr}}{P_{tr}}\right|$$

The logarithmic regression for the peak outbound sample had a multiple correlation coefficient of .4732, significant at the .001 level but less than that of the corresponding linear regression. None of the beta-coefficients are significant at the .1 level, only two are at the .5 level. Again the correlation situation of the variables remained unimproved.

Off-Peak Inbound and Outbound

$$\log \frac{\Delta Q}{Q} = 3.0466 + .9629 \log \frac{\Delta S}{S} + 2.8818 \log \left|\frac{\Delta P_{ow}}{P_{ow}}\right| + .3229 \log \left|\frac{\Delta P_{tr}}{P_{tr}}\right|$$

The multiple correlation coefficient for the logarithmic regression for the off-peak sample was .7958, considerably greater than that of the corresponding linear regression and significant at the .001 level. All of the beta-coefficients are also significant at that level. For the corresponding linear regression they were significant at only the .01 level. However, the variables again remained significantly correlated among themselves.

RESIDUAL REGRESSIONS

The attempt to reduce or remove the significance of the correlations proved unsuccessful. The reduction in the number of observations necessitated by the transformation into logarithms appears to have removed any advantage gained by the transformation itself. The last step of the analysis is an attempt to avoid the inaccuracies caused by the correlations if the correlations themselves cannot be avoided.

Procedure

The correlation problem can be avoided if one of the independent variables is either absent or of insignificant size for a portion of the observations of the dependent variable. Consider the regression equation,

$$X_1 = \beta_0 + \beta_2 X_2 + \beta_3 X_3 \quad ,$$

where X_2 and X_3 are correlated and X_3 is absent from a portion of the observations. From these observations the best estimator

$$X_1 = b_0 + b_2 X_2$$

is determined. A second regression is then conducted with the remaining observations. The residual, that portion of the behavior of X_1 not explained by the best estimator, is the dependent variable and X_3 the independent variable.

$$X_1 - b_0 - b_2 X_2 = \beta_0 + \beta_3 X_3$$

The portion of the behavior of X_1 explained by both X_2 and X_3 is then given by

$$X_1 = \beta_0 + \beta_2 X_2 + \beta_3 X_3 \quad ,$$

where β_0 is the sum of the constants of the two regressions.[17]

For each line of the Boston and Maine being considered here the frequency of service remained constant over the first and second phases of the experiment. One-way fares had been altered between phases.

The peak hour fare reductions were slightly less for phase two than they were for phase one. Thus the service variable was considered to be more important during the second phase. Linear regressions were conducted with it and the phase two observations. The resulting best estimators were then included in residual regressions with the one-way fare variable and the observations from phase one.

The off-peak hour one-way fare reductions were considerably greater during the second phase. Thus, the fare variable was considered to be insignificant during the first phase. The service variable and the phase one observations provided the estimator for inclusion in a residual regression with the one-way fare variable and the phase two observations.

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[17] J. Johnston, p. 201.

Peak Inbound

The best estimator of the one-way fare regression for the second phase,

$$\frac{\Delta Q}{Q} = .1406 - .4070 \frac{\Delta P_{ow}}{P_{ow}} ,$$

had a correlation coefficient of .3610, significant at the .05 level but not the .01 level. The beta-coefficients are significant at the .01 level. The residual regression,

$$R = .1415 - .1958 \frac{\Delta S}{S} ,$$

had a correlation coefficient of .6443. It and the beta-coefficients are significant at the .001 level. The resulting estimator is

$$\frac{\Delta Q}{Q} = .2821 - .1958 \frac{\Delta S}{S} - .4070 \frac{\Delta P_{ow}}{P_{ow}} .$$

Peak Outbound

The best estimator of the linear regression with the one-way fare variable,

$$\frac{\Delta Q}{Q} = -.0023 - .7528 \frac{\Delta P_{ow}}{P_{ow}} ,$$

had a correlation coefficient significant at the .01 level and a beta-coefficient for the fare variable significant at the .001 level. For the residual regression,

$$R = .1740 - .2462 \frac{\Delta S}{S} ,$$

all coefficients are significant at the .001 level. The resulting estimator is

$$\frac{\Delta Q}{Q} = .1717 - .2462 \frac{\Delta S}{S} - .7528 \frac{\Delta P_{ow}}{P_{ow}} .$$

Off-Peak Inbound and Outbound

The linear regression of the service variable for the first phase,

$$\frac{\Delta Q}{Q} = -.3379 + .9880 \frac{\Delta S}{S} ,$$

had a correlation coefficient of .2828, significant at the .05 level. The beta-coefficient of the service variable is significant at the .1 level. The residual regression,

$$R = 1.0566 - 2.7849 \frac{\Delta P_{ow}}{P_{ow}} ,$$

had a correlation coefficient of .4993, significant at the .001 level. Both beta-coefficients are significant at the same level. The resulting regression estimator is

$$\frac{\Delta Q}{Q} = .7187 + .9880 \frac{\Delta S}{S} - 2.7849 \frac{\Delta P_{ow}}{P_{ow}} .$$

BEHAVIOR MODELS

The analysis has essentially provided two possible models of the behavior of the suburban rail commuter.

$$\frac{\Delta Q}{Q} = \beta_0 + \beta_2 \frac{\Delta S}{S} + \beta_3 \frac{\Delta P_{ow}}{P_{ow}} + \beta_4 \frac{\Delta P_{tr}}{P_{tr}} + \beta_5 \Delta W$$

$$\frac{\Delta Q}{Q} = \beta_0 \left(\frac{\Delta S}{S}\right)^{\beta_2} \left(\frac{\Delta P_{ow}}{P_{ow}}\right)^{\beta_3} \left(\frac{\Delta P_{tr}}{P_{tr}}\right)^{\beta_4}$$

Of equal importance, it has provided the means for determining the reliability and the accuracy of each model, a measure of the reliability and the accuracy of conclusions and predictions based on the model.

Peak Hour Models

The residual regressions have provided the best estimators of passenger behavior during the peak hours. They accounted for approximately one-half of the observed variation in the passenger volumes with only two of the independent variables. The logarithmic regressions accounted for only one-quarter of the variation with three independent variables. In addition the beta-coefficients for the logarithmic regressions are unreasonable. The peak outbound model predicts that a fifty percent reduction in the one-way fare accompanied by a one hundred percent decrease in the twenty-ride fare and a one hundred percent increase in service would cause passenger volumes to increase by almost one billion percent. The peak inbound model yields an even more ridiculous prediction. The linear models do produce reasonable predictions but their insignificant beta-coefficients cause the predictions to be inaccurate. The residual regressions do provide significant beta-coefficients for the frequency of service and one-way fare variables. Thus accurate predictions of the effects of these two variables on peak hour patronage are possible.

Off-Peak Hour Models

The linear and residual regressions provided very similar models of off-peak hour passenger behavior. The logarithmic regression again had a tendency to produce unreasonable predictions. According to it a fifty percent one-way fare reduction accompanied by a two hundred percent service would cause a one million percent increase in patronage.

The beta-coefficients of the residual model were again more significant. However all were significant at the .01 level and quite consistent. Because of this significance and consistency very reliable predictions regarding off-peak hour patronage are possible.

Summary

The best models or estimators of the behavior of the rail commutation patronage are:

for the peak inbound hours,

$$\frac{\Delta Q}{Q} = .2821 - .1958 \frac{\Delta S}{S} - .4070 \frac{\Delta P_{ow}}{P_{ow}} \quad ;$$

for the peak outbound hours,

$$\frac{\Delta Q}{Q} = .1717 - .2462 \frac{\Delta S}{S} - .7528 \frac{\Delta P_{ow}}{P_{ow}} \quad ;$$

and for the off-peak hours,

$$\frac{\triangle Q}{Q} = .7187 + .9880 \frac{\triangle S}{S} - 2.7849 \frac{\triangle P_{ow}}{P_{ow}} \quad .$$

ECONOMIC ANALYSIS

The models of the behavior of the suburban rail commuter are of the form

$$\frac{\Delta Q}{Q} = \beta_0 + \beta_2 \frac{\Delta S}{S} + \beta_3 \frac{\Delta P}{P} \quad .$$

If β_0 and $\frac{\Delta S}{S}$ are equal to zero,

$$\beta_3 = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\Delta Q}{\Delta P} \frac{P}{Q} \quad .$$

β_3 is then the elasticity of demand with respect to the price of the one-way ticket. It indicates the degree of responsiveness of the public to changes in the one-way fare. Similarly β_2 indicates the responsiveness of patronage to changes in the frequency of service.

For a continuous demand function the elasticity is the same regardless of the direction of the movement along the function. However, this cannot be assumed to be true for the elasticities provided by the analysis. Fares were always reduced, frequency of service always increased. Nothing is known of the elasticity with respect to a fare increase or a service decrease.

Frequency of Service

One would expect an increase in the frequency of service to cause an increase in the passenger volume, that the beta-coefficients of the service variables would be positive. The opposite was true for the peak hours of the experiment. For the inbound and outbound, linear and residual regressions these beta-coefficients were negative and varied in size from -.1769 to -.3373. The beta-coefficient for the off-peak hours was positive and approximately unity.

While the peak and off-peak service variables have different values they are significantly correlated. Because of this the responsiveness of the peak hour patronage to the frequency of off-peak service has been included in the peak service beta-coefficients. They then represent not the elasticity of demand with respect to the peak hour frequency of service but the "net" elasticity with respect to the peak and the off-peak service variables. The negative values indicate a net transfer of patronage to the off-peak hours. They also suggest that demand is relatively inelastic for the peak service variable.

The beta-coefficient of the off-peak service variable then over-estimates the responsiveness of the public to off-peak service increases. It includes the patronage lost by the peak hour period. If the peak hour demand is very inelastic, which it does appear to be, the net transfer indicated by the peak beta-coefficients is approximately equal to the total patronage transferred. The change in off-peak passenger volumes resulting from the attraction of "new" riders can be approximated by

$$\frac{\Delta Q}{Q} = \beta_{PI}\left[\frac{\Delta S}{S}\right]_{P} + \beta_{PO}\left[\frac{\Delta S}{S}\right]_{P} + \beta_{OP}\left[\frac{\Delta S}{S}\right]_{OP} .$$

If the peak and off-peak service variables are equal

$$\frac{\Delta Q}{Q} = \left[\beta_{PI} + \beta_{PO} + \beta_{OP}\right]\left[\frac{\Delta S}{S}\right] .$$

With the beta-coefficients of the residual models this becomes

$$\frac{\Delta Q}{Q} = .5460 \frac{\Delta S}{S} \quad .$$

Thus, off-peak demand is also inelastic with respect to the frequency of service.

One-Way Fares

The beta-coefficients indicate that peak hour demand is inelastic with respect to the one-way fare variable, the peak inbound coefficient being -.4070, the peak outbound -.7528. Off-peak hour demand is quite elastic, the most reliable beta-coefficient being -2.7849.

Twenty-ride Fares

No accurate measure of the elasticity of demand with respect to the twenty-ride commutation fare was provided by the analysis for the peak hours. The logarithmic regressions indicate that the fare reductions actually caused a decrease in peak hour patronage. The more reliable linear models indicate that demand is quite inelastic, the reductions having caused slight increases in patronage.

The "raw" data, once the demand elasticities with respect to the other variables are known, also indicates that demand is inelastic with respect to the twenty-ride fare. Passenger volumes increased during the second phase even more than they had during the first. Since the demand is almost completely inelastic with respect to the frequency of service, the fact that it had remained at its phase one level does not explain the continuing high passenger volumes. The twenty-ride fares had been returned to their pre-experiment level during the second phase, having been reduced considerably for the first. Patronage then could only have remained at or above its phase one level if the demand for the peak hours is almost completely inelastic with respect to the twenty-ride fare.

Reliable beta-coefficients were available for the off-peak hours. The linear and the logarithmic regressions both indicate that off-peak demand is inelastic with respect to the twenty-ride fare.

Weather

Two general conclusions can be made regarding the weather. Precipitation will cause peak hour patronage to increase and off-peak hour patronage to decrease. In addition a given amount of precipitation will have a greater effect on peak hour than off-peak hour passenger volumes.

Summary

The analysis reveals that peak hour demand is very inelastic with respect to the frequency of service and the twenty-ride commutation fare. It is somewhat inelastic with respect to the one-way fare. While the off-peak hour demand is somewhat inelastic with respect to the frequency of service and the twenty-ride fare it is considerably elastic with respect to the one-way fare.

APPENDIX I

Table 2[18]

BOSTON AND MAINE RAILROAD

Service Changes

	Peak Hour	Off-Peak Hour
Eastern	.81	1.43
Reading	1.17	1.09
Western	.58	.44
Fitchburg	.68	1.12

[18] Rail Demonstration Project Technical Supplement.

Table 3[19]

BOSTON AND MAINE RAILROAD

Fare Reductions

Phase One

	Twenty-ride	One-way
Eastern	.29	.36
Reading	.22	.27
Western	.32	.37
Fitchburg	.30	.39

Phase Two

	Twenty-ride	One-way	
		Peak	Off-Peak
Eastern	.02	.31	.54
Reading	.00	.14	.27
Western	.01	.37	.59
Fitchburg	.03	.36	.52

[19] Ibid.

BIBLIOGRAPHY

Fisher, Sir Ronald A. Statistical Methods for Research Workers. New York: Hafne Publishing Company, Inc., 1958.

Freeman, Harold. Introduction to Statistical Inference. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1963.

Johnston, J. Econometric Methods. New York: McGraw-Hill Book Company, Inc., 1963.

Mass Transportation in Massachusetts. Boston: Mass Transportation Commission, 1964.

Meyer, John R., and others. Competition in the Transportation Industries. Cambridge: Harvard University Press, 1960.

Rail Demonstration Project Technical Supplement. Boston: Mass Transportation Commission, 1964.

"Supplement No. 3." Mass Transportation in Massachusetts. Boston: Mass Transportation Commission, 1964.