Between the Lines

Encoding Relations Through Body, Tool, and Algorithm

by

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Abstract

The tools architects use orchestrate the discipline in seen and unseen ways. In recent decades, we have swapped early forms of mechanical drawing instruments for digital tools with unimaginable computing power. While this increased level of computational literacy allows us to script and code architectural forms more efficiently, it has also created incongruities between the computationally described object and material constructions. At times the digital tools we depend on today go as far as defining the aesthetic of our buildings. To complicate this further, the digital tools most often solicited by the architectural practice are non-native imports adapted for their visual potential and practical uses. Meaning embedded within the programming of tools that shape our buildings are residual values of other disciplines. For example, we can trace the origins of CAD software back to engineers and mathematicians at Boeing and here at MIT, who sought to mechanize the construction of splines and irregular curved surfaces for the production of slipstream automobiles, toothbrushes, and even letterforms. And much like the hidden algorithms in the background of our digital tools, there is an apparatus of choreography surrounding our physical tools that encode instructions on how the body engages with the object. In other words, the machines we use produce not only drawings but gestures as well, keying us into the always-present yet rarely discussed embodied dimensions of tools.

To expand upon the embodied dimensions of our tools today, we need to reconsider the machine as the site of intervention. Motion data and performance envelopes surrounding our tools extend beyond the projective reenactment of the machine and offer us a means to measure the derivative of what it takes to produce a drawing, a surface, or a construction. This thesis dislocates the spline from its formal geometry associated with slipstream construction and recasts it as a way to record the tumble-type inscriptions surrounding an object's performance — a tactic to mutually mark and negotiate the activity between humans and machines.

Thesis Supervisor: Brandon Clifford

Title: Associate Professor of Architecture

This thesis is a testament to the collective effort of many, and I am humbled by the experiences that have allowed me to develop this research.

To my committee:
Brandon Clifford, Jaffer Kolb, Curtis Roth, with Andrew Witt, and Axel Kilian

Your patience and encouragement have been essential to the success of this project. You have pushed me to sharpen the project's claims and ensured they had substance. With your guidance, this thesis was possible.

To my mentors:
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I can't thank you enough for your interminable support and advocacy over the years. Your impact will undoubtedly continue to shape my career as a designer and teacher. Thank you, you are the best.

To my family and friends:
My partner, Michael Graziano; my sisters, Brittany, Morgan, and Chelsea; my parents, Tammy, Mark, Steve, and Linda; and my friends near and far

Your encouragement has sustained me through the long hours and demanding work. Thank you for the love.

To my SMarchS cohort:
Il Hwan Kim, Kevin Malca, Demiçan Tas, Ganit Goldstein, and many others

I am truly lucky to have had the opportunity to study at MIT alongside your brilliant and diverse minds. You help me see the world in new and unexpected ways. I have loved my time here, thanks to you.

Between the Lines:

Encoding Relations Through Body, Tool, and Algorithm

0.01↙

https://zachschumacher.com/Between-the-Lines

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Between the Lines

Encoding Relations Through Body, Tool, and Algorithm

Collectively the tools architects use orchestrate the discipline. From the schematic phases of design to the act of construction, the tools architects depend upon to represent and realize projects are inseparable from how we think. Architects have swapped early forms of mechanical drawing instruments for digital tools with unimaginable computing in recent decades. They are expanding beyond Computer-aided design (CAD) software often associated with architectural production to tools offering a broader range of representation techniques. When soliciting new digital tools, architects often consider adjacent disciplines such as the automotive, industrial, or graphic industries. Smuggled within the exchange of imported digital tools, perhaps unknowingly, are the residual values of those disciplines embedded into the underwriting of their software. Consider the most stereotypical digital tool in architecture, CAD. Hidden behind the effects of the screen, the origins of CAD software trace back to engineers and mathematicians at Boeing, GM, British Aircraft Corporation, (and here at MIT). Following World War II, MIT assistant professor Steven Coons and his colleagues founded the CAD Project. This group sought to mechanize the construction of splines and irregular curved surfaces for the production of slipstream automobiles, airplanes, toothbrushes, and even letterforms. Fast forward several decades and the underwriting that produced the interpolation of an irregularly curved surface is still used faithfully in architecture today.

Most recently, there has been a shift in the dependency of our computational tools. Beyond the desire to script and code complex architectural forms more efficiently, the discipline seeks new ways of perceiving, recording, and drawing spaces. Video game engines, five-axis robots, physics simulators, photo-scanning, and typographic tools have become regular guests in architecture studio projects, straining the relationship between the computationally described object and material constructions. This exchange is not superficial but rather complex. It both enhances and threatens the seemingly stable values which define architecture -- at times, going as far as defining the aesthetic of our buildings.

While today's turbulent technical culture in architectural production may sometimes be overwhelming, it is also enriching. This constant exchange between architecture and other disciplines has allowed us to move into a broader range of representational techniques as a means to formalize ideas into a physical environment. Fortunately, however, when these extra-disciplinary channels are opened, it satisfies the desire for further complexity and performance. Digital representation and fabrication is not merely a way of doing the same old thing faster but an enormous shift in how architecture is conceived, represented, and understood.

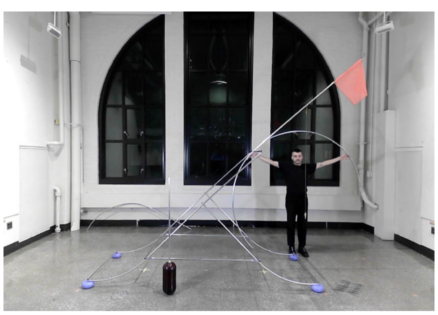
Helping us disentangle the discrepancies between our digital models, construction drawings, and building construction is the shared language of the algorithm. The algorithm offers an alternative to translating information by relying on a discrete list of executable steps that prioritizes proportion and relation over finite dimensions and form. A series of indefatigable instructions that speak to both the human and machine. As digital tools begin to suture the architectural discipline to the machine, we need to lean into the shared language of the algorithm that already exists in the background of our tools. What if all that matters is the communication between humans and machines?

One way to preserve this conduit between humans and machines is to look toward choreography. Much like the algorithms hidden in our tools, choreography is an apparatus of instruction that encodes information on how the body engages with an object. In other words, the machines we use produce not only drawings but gestures as well, keying us into the always-present yet rarely discussed embodied dimensions of tools. One way this temporal relationship is recorded is through motion data. A process of recording the movement of objects or people in space. Rather than recording the visual appearance of something, motion capture tracks the web of points and splines which trace the body's movement.

Between the Lines

→0.02

↘0.03



→0.01
An image taken in studio of in-progress tools and machines.

→0.02
A film still of myself and T. Deutch interjecting and redirecting the performance of die Schlange (the Snake).

→0.03
Film stills of myself instructing choreography with an apparatus.

Recording motion data and performance envelopes surrounding our tools extend beyond the projective reenactment of the machine and offer us a means to measure the derivative of what it takes to produce a drawing, a surface, or a construction. By revealing the mechanism present in the construction of something, we are able to exploit the variance and affordances of our tools. Therefore to expand upon the embodied dimensions of our tools today, we need to consider the transaction between the following:

1. Tool
The capacity of the digital to be literalized

2. Algorithm
Instruction as process and procedure

3. Body
The ability for the process to become content

This thesis is interested in the design tools that bridge the gap between the human body and the forms produced through its motions. One way to do this is to dislocate the spline from geometric specificity often associated with digital tools and can recast it as a method to reveal the implied volume of invisible gestures between the body and machine. We can trace the tumble-type inscriptions that spiral outward from an object's performance by recording motion data — a tactic to mutually mark and negotiate the activity between humans and machines. To expand upon the embodied dimensions of our tools today, we need to reconsider the machine and the gestures it produces as the site of intervention.

1. Tool

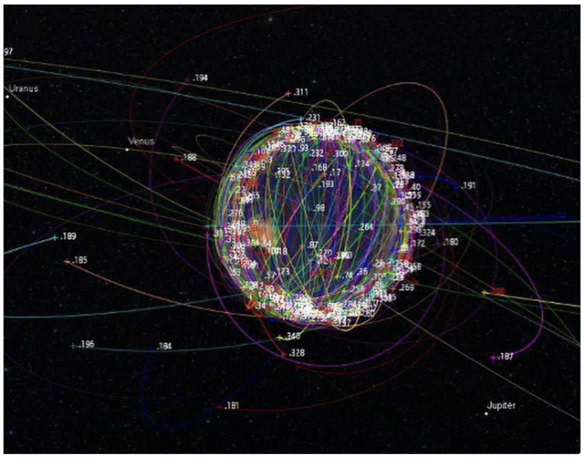
The capacity of the digital to be literalized

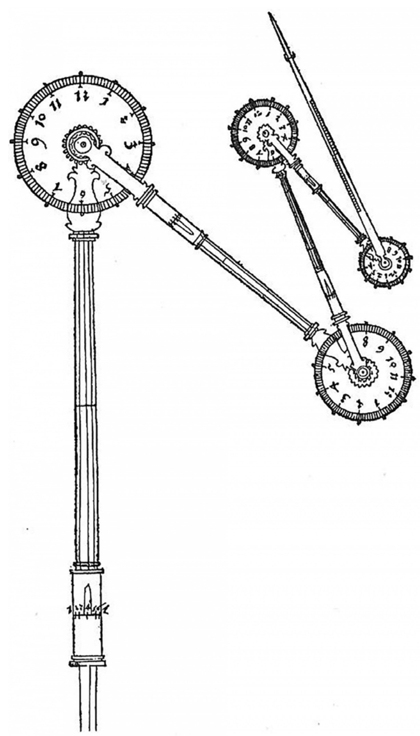
→1.01
The first prototype of die Schlange (the Snake) using mostly tripod parts, printed circuit boards, and stepper motors, 2022.

→1.01

1.02↖

1.03←





→1.04

How do we locate points in space? Rather, how do we locate the 3D position of points and the distances between them?

Throughout history, we have seen land surveyors use ropes, theodolites, scopes, and other tools to locate points with fair accuracy. Illustrated here is a knotted rope being stretched by the body to measure fields in ancient Egypt (Fig. 1.02). Instead of relying on bodies to hold the rope in tension, land surveyors today often rely on the many satellites floating in orbit to triangulate the location of a line on the ground with finite accuracy (Fig. 1.03). Analyzing the spline can help us sort out the difficulty of locating a 3D point in space.

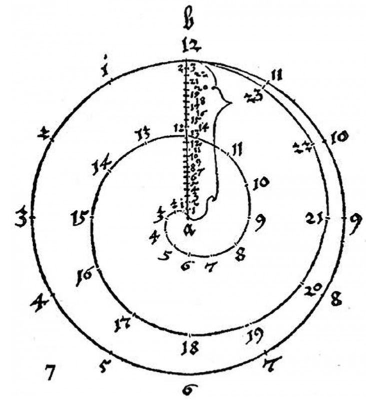
On a 15th-century piece of paper in black ink, Albrecht Dürer describes die Schlange (The Snake) (Fig. 1.04). Drawn by hand with enormous precision, it is not initially evident that this is a snake. It looks nothing like a snake. The Snake he refers to isn't only the drawing instrument, die Schlange, he has illustrated on the page, but the 3-dimensional serpentine curve absent from the diagram that die Schlange produces.[1] Although the illustration does not represent the curve, it does prescribe the kinematic arrangement of the five' rods' and four 'dials' of die Schlange, which would, hypothetically, be one of the earliest parametric drawing instruments intended to construct a 3-dimensional spline (or, as Dürer referred to it, "the serpentine line"). However, since no evidence suggests that this instrument was ever physically constructed, we can only speculate on its intended use. Some have hypothesized that the tool's

→1.02
In ancient Egypt, a rope stretcher (or harpedonaptai) was a surveyor who measured real property demarcations and foundations using knotted cords. The practice is depicted in tomb paintings of the Theban Necropolis.

→1.03
A representational image of satellites orbiting earth by Trevor Paglen and NASA, 2023.

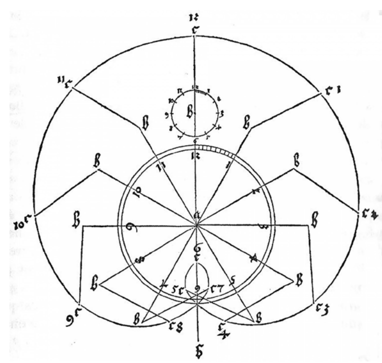
→1.04
Illustration of die Schlange (the snake) by Albrecht Dürer in Underweysung der Messung, 1525. Public Domain of The Metropolitan Museum of Art.

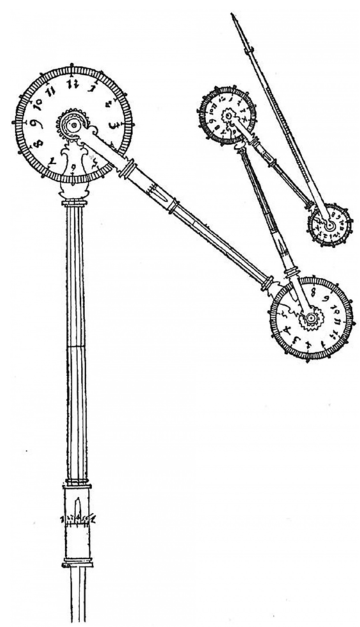
1.05←



→1.06

→1.07





→1.08

purpose was a sculptural jig or a mechanism to understand the movement of a body, while others speculate on the tool's association with witchcraft (Fig. 1.05). Regardless of the drawing instrument's applied use, it is clear that die Schlange has the potential to construct endless and varied geometric splines.

The discipline has adopted drawing tools for their visual potential and practical uses. However, unlike the orthographic drawings of military origin, or the construction of a single-point perspective drawing, the spline lacked the underwriting that allowed it to be a productive means of construction until recently. One example can be witnessed in the many drawing instruments constructed by Albrecht Dürer. Though Dürer is perhaps best known as a German Renaissance artist, he was also an early contributor to the development of visual perspective and mathematical scholarship through his many drawing instruments recorded in his seminal text, Underweysung der Messung, a treatise of geometry, c.1525. At the end of his text, past his mention of descriptive geometry and perspective, he pivots back toward his interest in human proportions and movement to illustrate three lesser-known but significant drawing machines, der Schnecke (the Snail), die Spinne (the Spider), and lastly die Schlange (the Snake) (Figs. 1.06, 1.07, and 1.08, respectively).

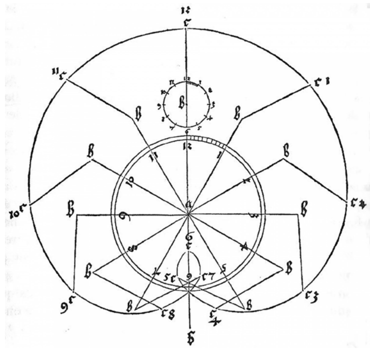
As we turn the pages from the Snail to the Spider to the Snake, the devices become increasingly more complex, adding additional hinge points which frustrate and reorder the

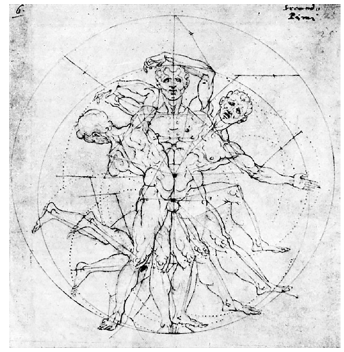
→1.05
Laocoön and His Sons excavated in Rome in 1506. It depicts the Trojan priest Laocoön and his sons Antiphantes and Thymbraeus being attacked by sea serpents. One of the unsubstantiated speculations of die Schlange's intended use has been suggested that it was a sculptural jig.

→1.06
Illustration of der Schnecke (the snail) by Albrecht Dürer in Underweysung der Messung, 1525. Public Domain of The Metropolitan Museum of Art.

→1.07
Illustration of die Spinne (the spider) by Albrecht Dürer in Underweysung der Messung, 1525. Public Domain of The Metropolitan Museum of Art.

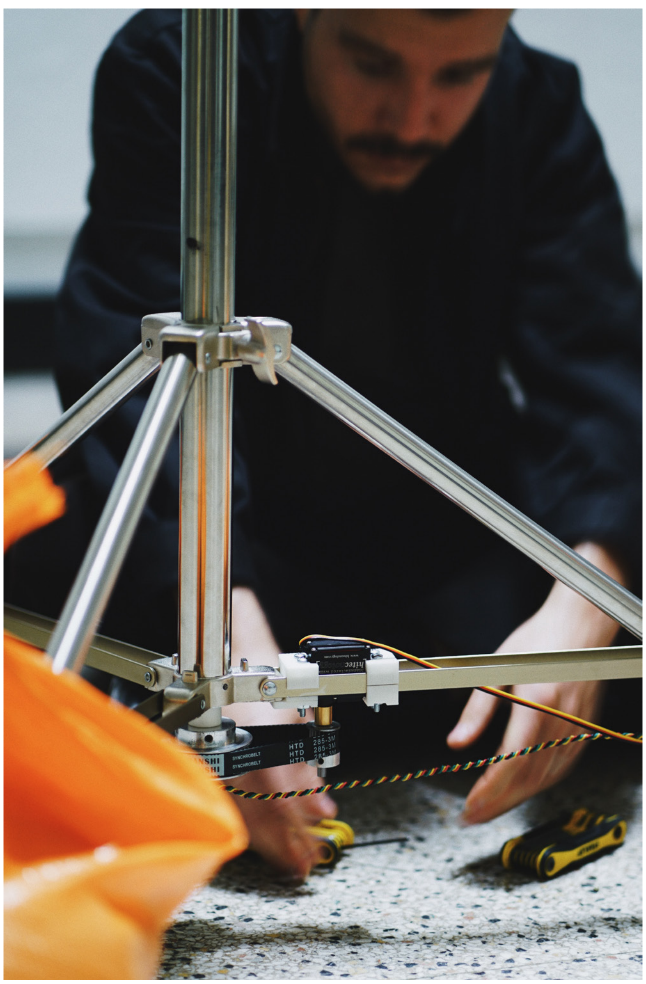
→1.08
Illustration of die Schlange (the snake) by Albrecht Dürer in Underweysung der Messung, 1525. Public Domain of The Metropolitan Museum of Art.





→1.09
↘1.10

1.11 ←



→1.09
Illustration of die Spinne (the spider) by Albrecht Dürer in Underweysung der Messung, 1525. Public Domain of The Metropolitan Museum of Art.

→1.10
Illustration of the human body's proportions by Carlo Urbino di Crema, Codex Huygens, c. 1570.

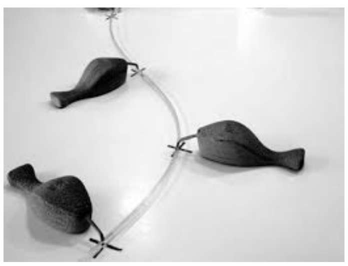
→1.11
A photo of me adjusting the servo motor at the base of the second die Schlange prototype.

curves. The Snail and the Spider are both described as 2-dimensional curves. One could easily imagine the similarities between the proportions of the Spider and da Vinci's Vitruvian Man or other figural studies drawn at the turn of the 15th century (Figs. 1.09, and 1.10, respectively). Whereas the Snake takes a significant transformation in the sequence to become a 3-dimensional curve, the spline. Unlike his more famous perspectival devices in which the eye of the observer plays a fundamental role, there is a fundamental shift in these three drawing instruments from image production to parametric agents that generate geometric figures. In Underweysung der Messung, we only find a diagram of the Snake, illustrating the "dials" and "rods" and this list of instructions, which has been translated here by Bernard Cache:[2]

> 1.) The instrument should be made with few or many dials and rods, according to the intended applications.
>
> 2.) The rods shall be arranged in a manner that they can be advanced by degrees and can be shortened or extended.
>
> 3.) The rods can be pulled apart or pushed together, also by degree, so that they become shorter or longer.

Orthographic projections, such as plan, section, and elevation, have an understandable trueness to them, allowing them to be reliable

1.12 ←



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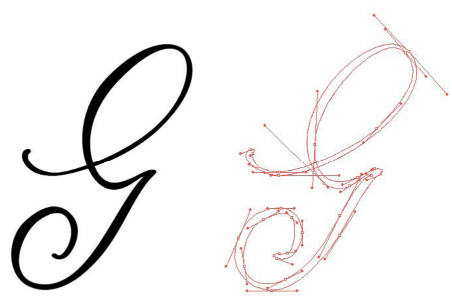
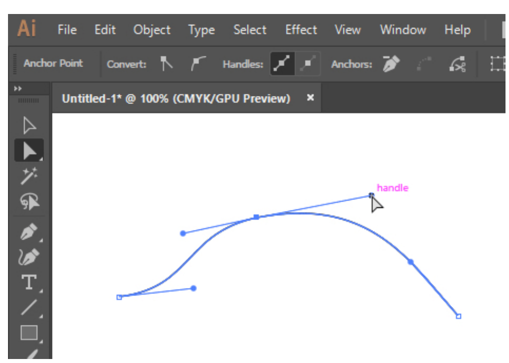


→1.14



→1.15

drawings for the mass production of buildings. In contrast, constructing curves, splines, and arcs has always required a series of relational objects to be reliable, a dependency in which a specific physical tool is required to construct a curved figure precise enough for construction. This unique closeness between the body and tool and the resulting curve is special to curves in a way that is unnecessary to construct other types of lines.

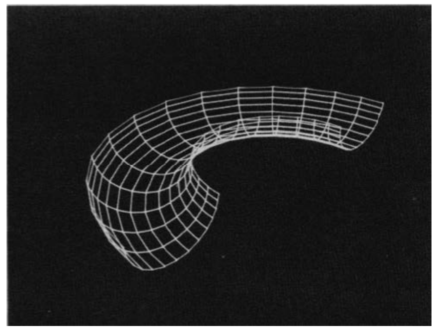
We have come to reflect our modern machines, literally, through our posture. A contemporary example of the codependent relationship between body and tool can be witnessed from the early days at Boeing when an engineer would anchor weights to flexible strips of metal and wood laths to construct precise curves (Fig. 1.12). When constructing splines for airplanes, engineers in the 1940s would crawl alongside the tool to build precise curves by continually repositioning whale-shaped weights, submitting to the contorted posture of the "draftsman's spline" (Fig. 1.13).[3] This same logic has been carried into our digital tools. In the mid-twentieth century, engineers were tasked with mechanizing the remarkably smooth and irregular curves for the construction of airplanes and cars. (Some of which were teaching here at MIT.) By using the fewest possible points to control the irregular curve, they were able to digitize the same logic recorded in the "draftsman's spline." Or in developing the Bezier curve, used to construct typefaces that mimic calligraphy (Fig. 1.14). Or even today, we continue to use this same logic when working in Adobe Illustrator and Rhino, a series of control points operating in the periphery of the curved line, acting as visual weights on our digital splines (Fig. 1.15).[4]

→1.12
Weighted spline whales (or ducks) used to pin the metal and wooden laths into the shape of an ideal curve.

→1.13
Weighted spline whales (or ducks) used to pin the metal and wooden laths into the shape of an ideal curve.

→1.14
In the production of typefaces and fonts, we still rely on the underwriting of the Bézier curve, created by Pierre Bézier in the early days of CAD computer graphics.

→1.15
A screenshot of a spline tool used in Adobe Illustrator indicates the anchors and handles on the curve's periphery.



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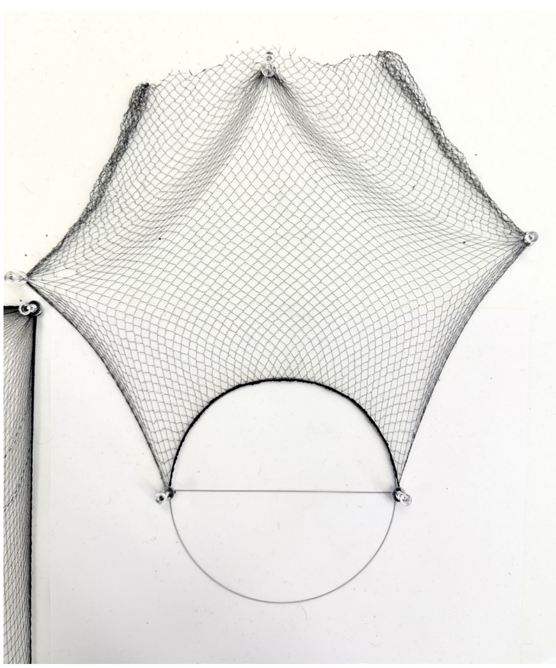
→1.17



1.18←

Smuggled within the translation of physical tools to digital tools, perhaps unknowingly, are the values of slipstream construction and the realization that we both have a shared interest in surface articulation (Fig. 1.16). While the smoothness of surface modeling is highly beneficial for the production of cars and airplanes, it is less suited for a discipline that continues to be produced with sheet materials (Fig 1.17).In other words, our irregular surfaces carry with them the embedded priorities associated with the aerodynamic construction of the mid-twentieth century that is defined by the boundary curve of a surface and the blending functions which interpolate the space between them. Or here, when simulated with a hairnet stretched across a surface (Fig. 1.18). We can see the deformation between the edge that is pinned in place and the outlining curves that negotiate the space between the apparatus and the intended target, shifting the priority toward a multiplicity of vectors rather than a single coordinate.

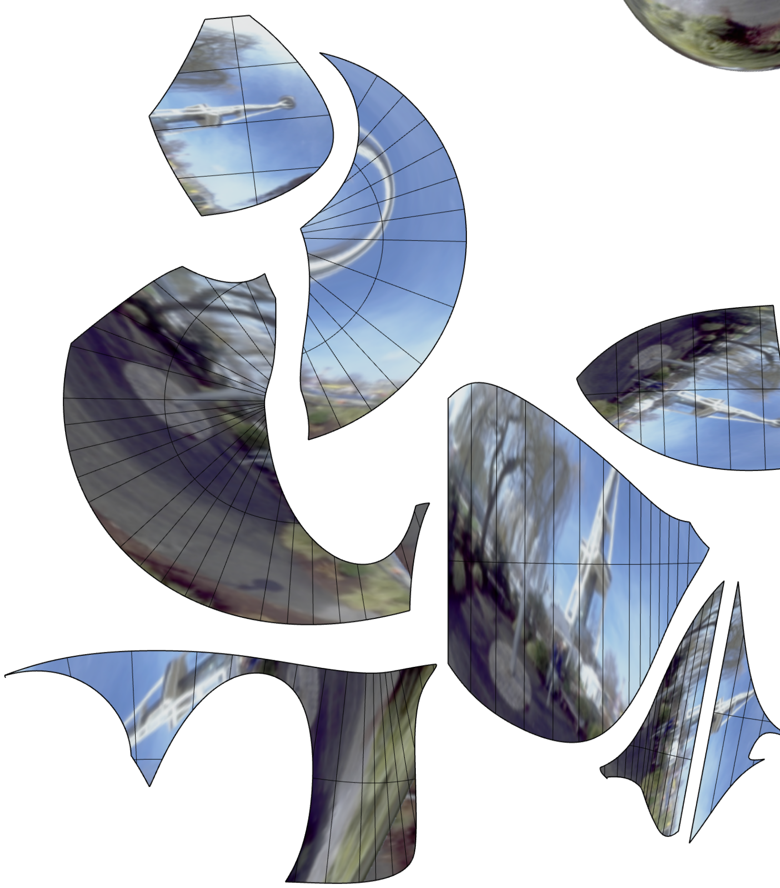
Historically, drawing machines have shifted drawing from a purely visual exercise to an indexed calculation and rapid production process. Re-animating the kinematic arrangement of “die Schlange” validates the contribution of Albrecht Dürer’s deep inquiry into the parametrically designed spline. Although we are increasingly reliant on contemporary tools such as 3D scanning for visual and practical uses, drawing instruments demonstrate their instructive potential to record and survey their environment. Therefore by revealing the mechanism present in the construction of something, we are able to exploit the variance and affordances of our tools.

→1.16
A polygon mesh approximating a curved surface.

→1.17
Ivan Sutherland’s students mark and measure Marsha Sutherland’s VW Beetle for digitization, 1972.

→1.18
A hairnet used to produce the interpolation of a surface connecting the boundary curve of the circle with the pins holding it in place.

→1.19



1.20 ←

→1.19
Irregular surfaces carry with them the embedded priorities associated with the aerodynamic construction of the mid-twentieth century that is defined by the boundary curve of a surface and the blending functions which interpolate the space between them. A digital object demonstrating the priorities of smooth continuous surfaces embedded within our digital tools today. Pictured here is an object reflecting the landscape around the Space Needle.

→1.20
Pictured here is an unrolled surface drawing of the smooth reflective surface of the spline geometry.

End notes:

1.
Dürer, Albrecht. Underweysung der Messung mit dem zirckel un richt scheyt (A Treatise of Geometry) was published in 1525. This publication is available for viewing at the Metropolitan Museum of Art.

2.
I was originally introduced to die Schlange through Bernard Cache's essay, Instruments of Thought: Another Classical Tradition. In The Cornell Journal of Architecture. 9, Mathematics: from the Ideal to the Uncertain. Cache also has a lecture online titled "Dürer - Vitruvius - Plato. Instruments of Thought".

3.
Several texts reference the "draftsman spline," but one of the most thorough texts is from Robin Forrest, An Introduction to Splines for Use in Computer Graphics and Geometric Modeling, 1995.

4.
Recently Jacob Gaboury published a book based on his dissertation at NYU titled, Image Objects: an Archaeology of Computer Graphics. It is an excellent cross-section through the contemporary digital tools and history of computer graphics commonly used today.

5.
Iman Fayyad has authored a clever essay on the translation from complex curve geometry to built and virtual architecture in Log 51 titled, On Flatness: The Virtual Turn.

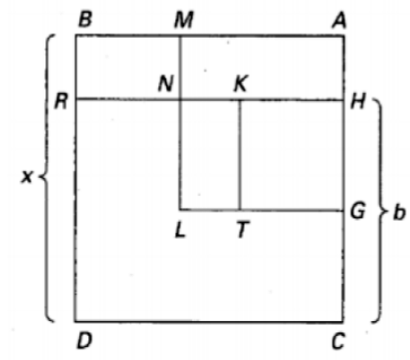
All images are by the author unless otherwise credited.

2. Algorithm

Instruction as process and procedure

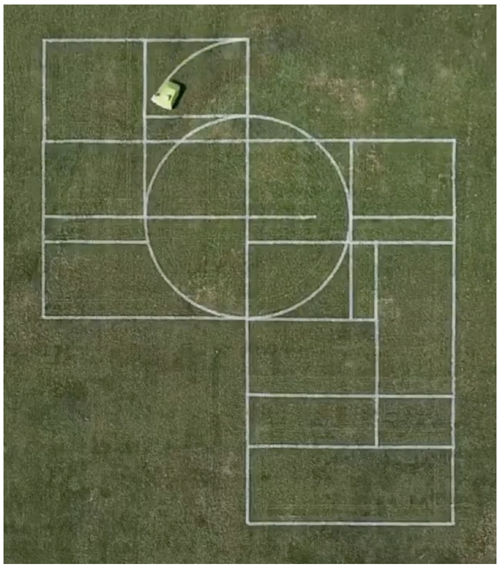
→2.01
Video stills of the fully reconstructed die Schlange, standing nearly 12ft tall and made mostly of aluminum tripod parts, printed circuit boards, and servo motors, recording each rod and dial in motion (photo by Daisy Ziyan Zhang).

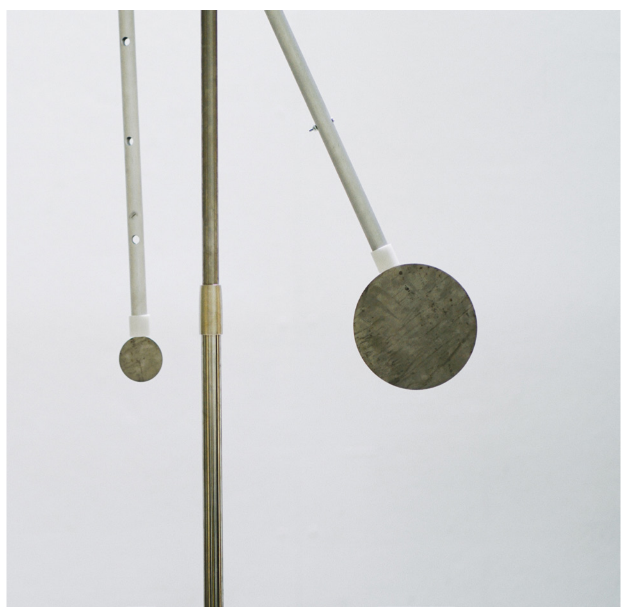
→2.01



→2.02

2.03 ←





→2.04

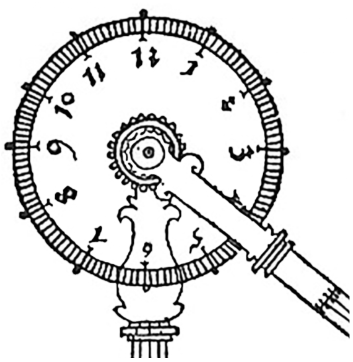
→2.02
Muhammad ibn Mūsā al-Khwārizmī, algorithmic proof from The Compendious Book on Calculation by Completion and Balancing, c. 825

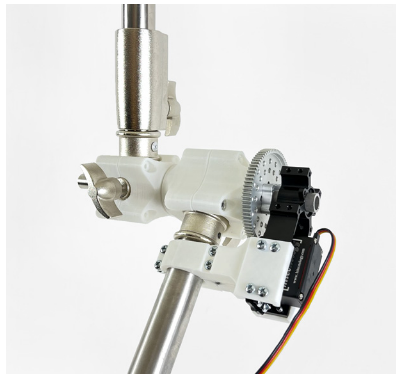
→2.03
Zach Schumacher with Outpost Office (photo by Erik Herrmann), Spake Scapes, 2021

→2.04
Photo of the reconstructed die Schlange (photo by Daisy Ziyan Zhang).

Although today's construction methods continue to rely on the faithful reproduction of relationships recorded in static images and symbols, the algorithm suggests potent forms of transmission across space and time that are not image-based but process-based. Consider the method of constructing geometry by polymath Al-Khwārizmī. In the 8th century, polymath Al-Khwārizmī developed the algorithmic method for surveying land by departing from the Greek tradition. Working in a verbal custom through discrete steps allowed landowners to measure and subdivide their land by keying into the shared harmonics of 2D geometry (Fig. 2.02). For the contemporary architect, Al-Khwārizmī's instructions read like pseudocode, an artificial and informal language that helps programmers develop complex algorithms and software.[1] However, the result of these discrete and executable steps is, in fact, spatial. One example can be seen here, by using the algorithmic process to survey land, Al-Khwārizmī relies on geometry's relational harmonics to allow those who didn't understand mathematical equations to be still able to measure their land.[2] In a collaboration between myself and Outpost Office, we have plotted some of Al-Kwarizmi's proofs by painting lines with a GPS-guided robot, a tool commonly used to stripe sports fields (Fig. 2.03).

Drawing instruments often limit the number of possible inputs to expand upon the range of possible outputs.[3] Die Schlange's inputs are determined by its degrees of freedom (Fig. 2.04). For example, a compass has one degree of freedom, a center pin, and a rod that specifies the





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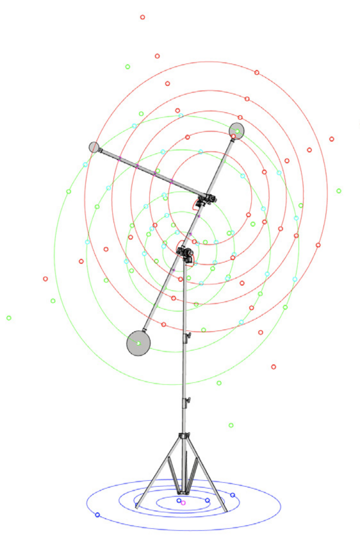
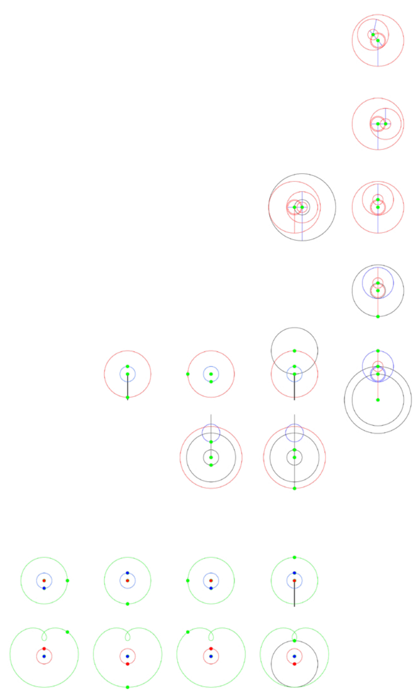
2.06 ↙





→2.07

↘2.08

radius. Die Schlange, however, has five degrees of freedom; four upper dials that rotate in the XY-direction and a fifth degree of freedom at the base rotating the central shaft in the Z-direction, implementing a limited set of inputs along with a list of executable steps. For example, in die Schlange, the instrument's dials are positioned to align with numerical values that match the face of a clock (Fig. 2.05). Whereas, in the reconstructed machine, the position is controlled by a stepper motor which is scripted to rotate by any given degree to allow the machine to construct a specific arc determined by its code (Fig. 2.06). In doing so, the parent-child relationship of the machine's kinematic rig synchronizes the machine's physical performance to the digital algorithm.

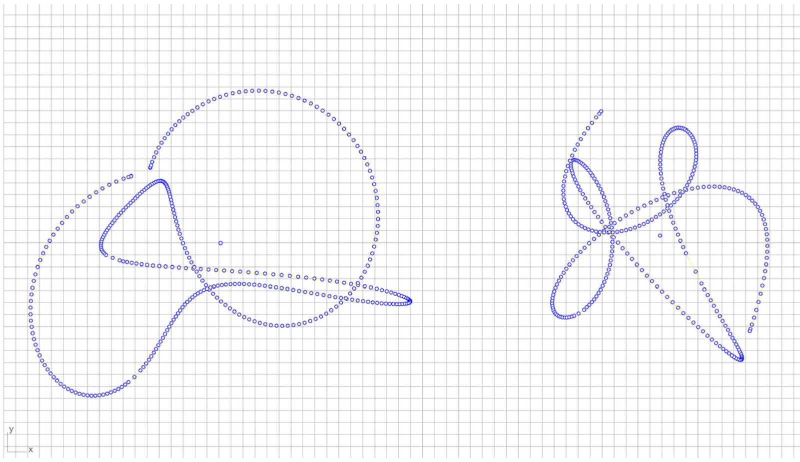
Matching today's mechanical components and technology to a 15th-century drawing instrument with 5 degrees of freedom has contributions on multiple levels. First, this framework is designed specifically for the mixed physical/virtual case to understand better and exploit the possibilities of kinematics within the architectural discipline. Kinematics is a branch of mechanics concerned with the motion or transformation of objects, systems, and bodies without considering the forces that cause them to move (Fig. 2.07). In animation, there are two types of kinematics, Forward Kinematics, in which you rotate each rod individually, and Inverse Kinematics, in which the movement of the parent-child chain is determined by its intended target by positioning the end of the chain.[4] To evaluate the kinematic performance

→2.05
Illustration of die Schlange's clock-like dials by Albrecht Dürer in Underweysung der Messung, 1525.

→2.06
A detailed photo of the servo motor at the connection between two rods on the reconstructed die Schlange. Reinterpreting the original clock-like dial as a servo motor allows for the careful control of each dial.

→2.07
A diagram of die Schlange's range of motion as indicated with the various concentric rings where each rod can be repositioned and the network of outlying points for each curve.

→2.08
Following the range of motion afforded by each dial, these epicycles demonstrate the relationship between each ring relative to the central point.



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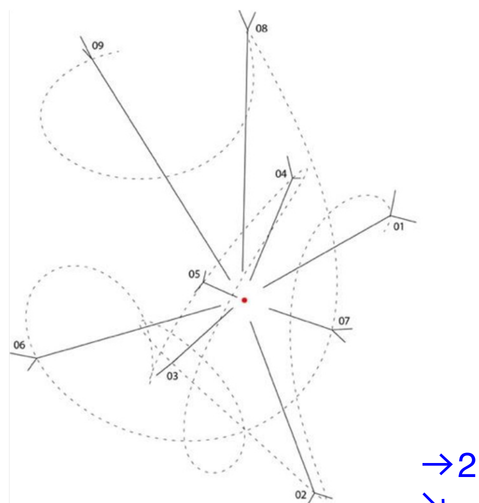

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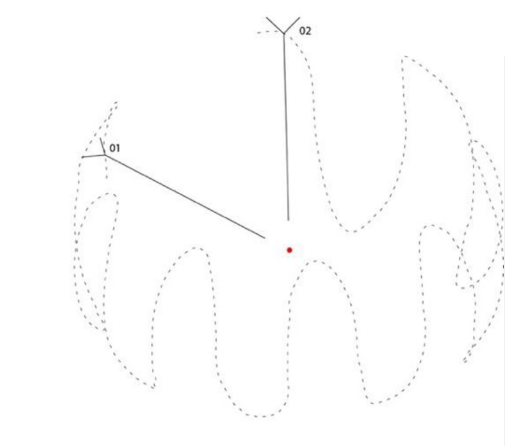


→2.11

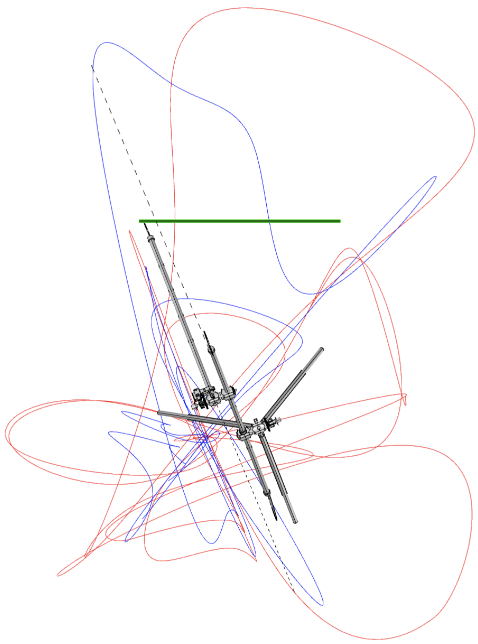
of the robotic instrument, I have recorded the device with a motion tracking software, OptiTrack, a technique that records the XYZ coordinate of the end-effector's implied volume at two frames per second. This can be seen here in the motion capture data of the instrument's kinematic performance recorded at MIT.nano's Immersion Lab to study its movement through plotted points and simulated digital models (Fig. 2.09). I am not reconstructing drawing instruments as a historian but as a designer. The motion data extracted from MIT.nano's Immersion Lab goes beyond the serial reproduction of drawings or the projective reenactment of the original machine. It is a litmus test for evaluating its performance directly informing each sequential prototype's kinematic arrangement. In other words, it is perhaps through the execution of the instrument's mechanical reconstruction that we can grasp its visual potential and practical uses. The numeric output is a shared language between the physical machine and computing.

Pictured left is an elevation of the result of plotting the motion tracking data of the end effector when the machine is demonstrating a Forward Kinematic movement. Here you see the red dot of the machine's central leader and the direction of the spline indicated by the annotated sequential numbers (Fig. 2.10). Similarly, we can use the OptiTrack software to record the machine's Inverse Kinematic movement when generating a sine curve (Fig. 2.11). The plotted motion tracking data demonstrates two of the endless variations of the splines performance envelopes drawing instruments can illustrate.

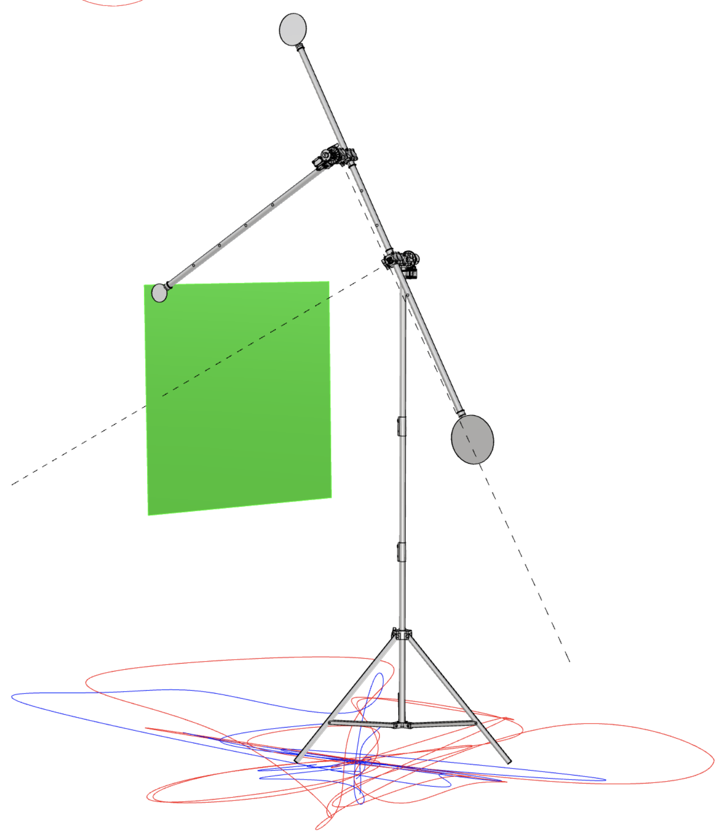
→2.09
Motion capture data of the instrument's kinematic performance recorded at MIT.nano's Immersion Lab with OptiTrack, a technique that records the XYZ coordinate of the end-effector's implied volume at two frames per second.

→2.10
Motion tracking data of the end effector when the machine is demonstrating a Forward Kinematic movement. Here you see the red dot of the machine's central leader and the direction of the spline indicated by the annotated sequential numbers.

→2.11
Motion tracking data of the end effector when the machine is tasked with generating a sine curve with an Inverse Kinematic movement.



2.13 ←



→2.14

In contrast, the photographs collected from the end effector show the drawing machine's ability to survey and record our environments. The recursive act of reconstructing this elusive snake has made me realize that new interpretations of drawing instruments have the instructive potential to confront the incongruities between the computationally described object and the constructed surface.

Unlike the draftsperson, the machine has no concept of the shape it is drawing. While the code only defines the robot's relationship with the world, it translates to motion. For the architect, the uniquely expansive symbolic language of the algorithm can converge effectively with the machine to become one continuously interactive space. Similarly, in die Schlange's performance, the serpentine line reveals itself as each arm swings, carving an envelope of motion when simulated digitally with a light trail. The light trail, a technique that records the end effector's implied volume at two frames per second, helps to disentangle the kinematic performance by visualizing die Schlange's movements and its numeric output — expanding the dialogue between the robotic armature and the body beyond the typical smooth transaction from CAD user to the programmable machine (Figs. 2.13 and 2.14). Today's architectural reality exists in manyspaces, both real and virtual. There is a parallelism between these two domains and the operations between the various interactive spaces. While no movements of die Schlange are precisely identical, the rigor of instructions assures fidelity in each "version" while leaving

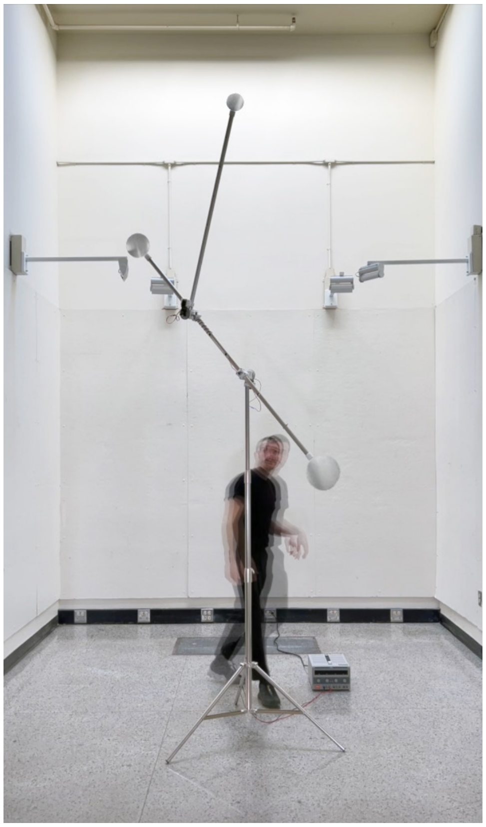
→2.13
A simulation of die Schlange controlled by inverse kinematics to scribe the boundary of a green square. The spline curves on the ground plane record the movements of the rods as they navigate to match the coordinates of the green square.

→2.14
A perspective view of the above

→2.15

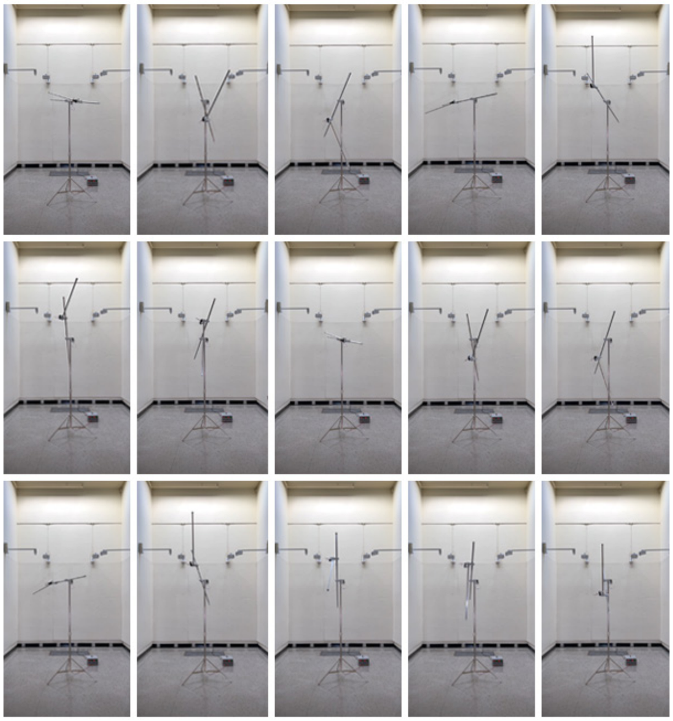
→2.15
Video stills record the movement of die Schlange's performance using a GoPro as the end-effector.

suitable space for chance and happenstance, unlike digital fabrication, where the whole point is determinacy, to gain control over matter. Die Schlange and the body don't subscribe to the same degree of predictability that we embed into the programming of our contemporary tools, such as a Kuka robot or CNC machine. The movements of die Schlange and the body are more chaotic. The uniquely expansive language of the algorithm brings the polarities of geometry and perception into creative contact, which goes beyond the faithful reproduction of relationships recorded in static images and symbols to suggest forms of transmission across space and time that are not image-based but process-based (Figs. 2.15). The algorithm acts as an indispensable interface between humans and machines by relying on the data flows to situate the drawing instrument in a larger technosocial complex. The shared lineage between tool and algorithm allows us to record our spaces and performance envelopes in a real sense of embodiment.



→2.16

2.17←



→2.16
An image of the fully reconstructed die Schlange, standing nearly 12ft tall and made mostly of aluminum tripod parts, printed circuit boards, and servo motors.

→2.17
Video stills recording the initial test of each rod and dial in motion.

End notes:

1.
At the 59th International Art Exhibition, La Biennale di Venezia, The Art and Culture Development Foundation of the Republic of Uzbekistan (ACDF) presented “Dixit Algorizmi — The Garden of Knowledge.” Included in this show was a short essay by Outpost Office and myself about Al-Khwārizmī’s proofs titled, “Spake Scapes.”

2.
Muhammad ibn Mūsā al-Khwārizmī, algorithmic proof from The Compendious Book on Calculation by Completion and Balancing, c. 825.

3.
The closest thing to a textbook on the tools used during the Renaissance is Martin Kemp’s The Science of Art, 1990.

4.
The subject of inverse and forward kinematics is a deep inquiry, and I’ve only scratched the surface. Serdar Kucuk and Zafer Bingul have compressed this information in the text Robot Kinematics: Forward and Inverse Kinematics, Industrial Robotics: Theory, Modelling, and Control. INTECH Open Access Publisher, 2006.

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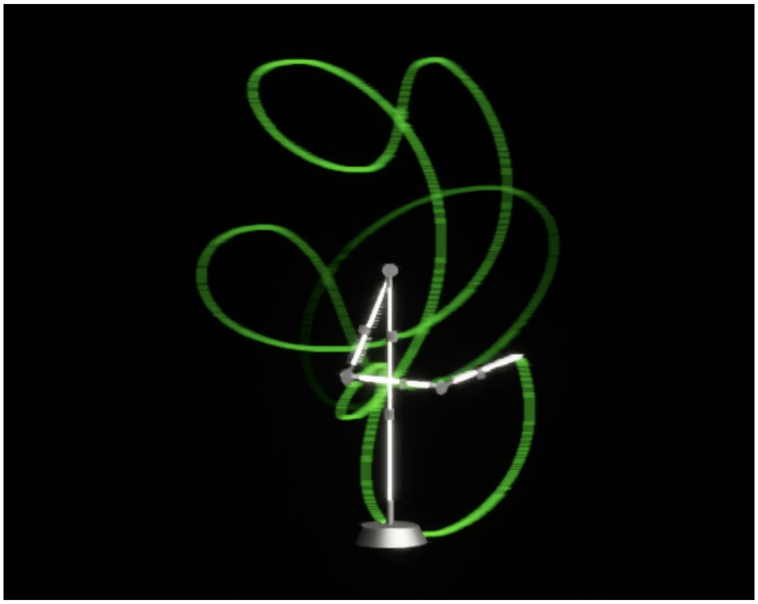
3. Body

The ability for process to become content

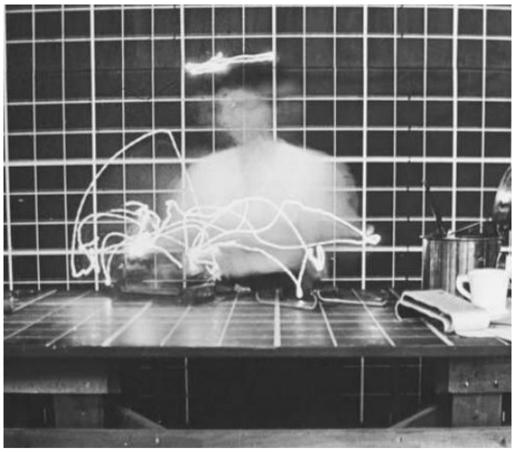
→3.01
The Four Elements is a painting by Louis Finson in 1611. The bodies in the painting represent (clockwise from the top left) Air, Fire, Earth, and Water. Each element entangled with the other, contorting their posture to push and pull the other elements with clasping hands, fists of hair, and extended legs.

→3.01



→3.02

3.03 ←



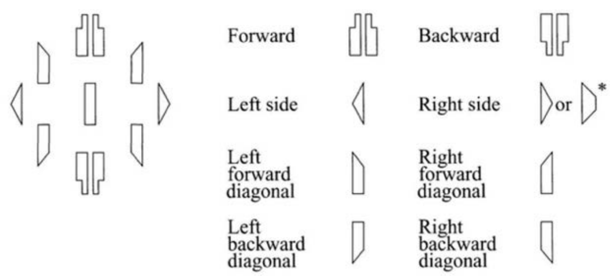
One way to preserve the conduit between humans and machines is to look toward choreography. Much like the algorithms hidden in the background of our tools, choreography is an apparatus of instruction surrounding our physical tools that encode information on how the body engages with an object. In other words, the machines we use produce not only drawings but gestures as well, keying us into the always-present yet rarely discussed embodied dimensions of tools.

By reinterpreting die Schlange with aluminum parts and servo motors, I can synchronize the tool's physical construction with the algorithm. In the instrument's performance, the Snake reveals itself as each arm swings, carving an envelope of motion (Fig. 3.02). The implied volume recorded from the die Schlange's movement may be reminiscent of the early 20th-centrury light drawings by Frank and Lillian Gilbreth. The Gilbreth's "motion models," are recordings of the virtual volumes of human action and labor. The long exposure photographs of a light trail, recorded the efficiency of a factory worker performing a repeatable task, which was then studied to optimize workflow (Fig. 3.03). Even in these images, used to optimize workflow in a factory, the motion data that records a body's movement goes beyond the serial reproduction of a task. These images expose the invisible motion envelopes around the projective reenactment of machines. Through Die Schlange's mechanical reconstruction, we can grasp its visual potential and practical uses. In the reconstruction, Dürer's interest in the shared harmonics between the movement of the body and machine is revealed, allowing us to encode rules of engagement

→3.02
Die Schlange's performance was simulated with the video game software Unreal Engine.
As the arms swing, the 'snake' reveals itself with a light trail.

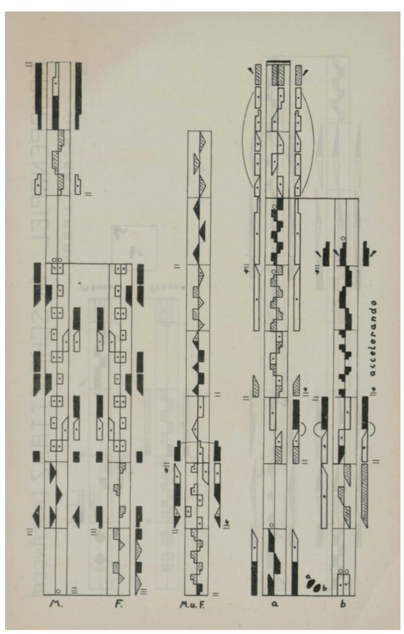
→3.03
In the 1950s, Frank and Lilleth Gilbreth used long-exposure photography to study the efficiency of a factory worker completing a task.

3.04←


Forward
Backward
Left side
Right side
or
*
Left forward diagonal
Right forward diagonal
Left backward diagonal
Right backward diagonal

→3.05

→3.06


M.
F.
M.u.F.
a
b
accelerando

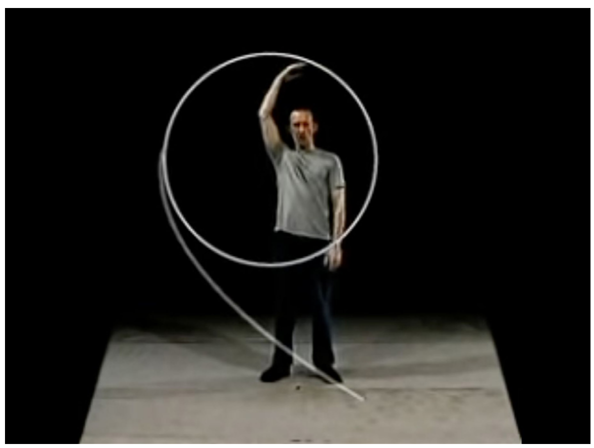
between human and machine. In other words, we may consider the illuminating role that early drawing automation could play as a conceptual system to collapse the relationship between body and tool.

Many devices and illustrations have recorded a body or tool's reach or performance envelope throughout history. For example, consider NASA's Multiple Axis Space Test Motion Inertia Facility (MASTIF), 1960. In the mid-twentieth century, NASA constructed a multi-axis rig known as the 'gimbal rig' to motion-test the tumble-type maneuvers that astronauts may encounter in space (Fig. 3.04).[1] In this study of the proposed space habitat, the body's performance is recorded with a light drawing by NASA through a series of hinged vectors that graze corresponding arcs, reaching the full extent of the pilot in motion to describe the figure. The motion data extracted goes beyond the serial reproduction of drawings or the projective reenactment of the original machine.[2] It is a litmus test for evaluating its performance, which directly informs the kinematic arrangement of the reconstructed apparatus. When NASA records the corresponding arcs of the 'gimbal rig,' the energetic lines of choreography begin to imply something else. Suddenly the spline is dislocated from a formal material geometry and relocated as a way to map embodied energy, an invisible form that records the implications of what it takes to produce something in space. Light drawing mutually marks and negotiates the activity between the human and the machine, a tactic to represent unseen realities.

→3.04
Time-lapsed photo of NASA's gimbal rig used for Mercury astronaut training in 1960.

→3.05
Pictured are the eight main directions used as the foundational instruction for Labanotation, illustrated by Ann Hutchinson. Labanotation is a structured system for analyzing and recording movement with symbols.

→3.06
Rudolf von Laban, dance artist, choreographer, and dance theorist, is best known for his annotated instructions for dance, Labanotation (Gruppenspiel für Schrifttanz.) Pictured here are dance instructions from 1930–1931 by his daughter Azra von Laban.

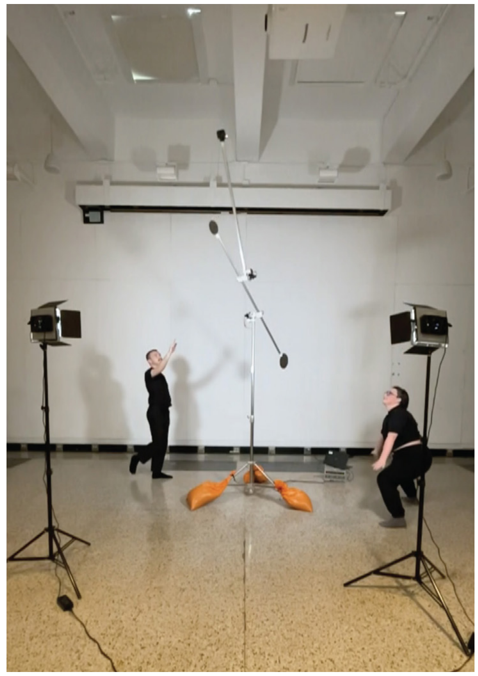


→3.07

→3.07
William Forsythe's "Three Dropping Curves" is one of many instructional videos recorded by the choreographer that illustrates the relations between the body and surrounding space.

Collapsing the distinction between machine and body is choreography (Figs. 3.05 and 3.06).[3] Much like the algorithm, choreographers encode instructions about the performance to expand upon the body's capabilities and definitions. One who helps situate the body alongside the algorithm is William Forsythe. Forsythe is challenging the sequential act of ballet and choreography, working at the intersection of the body, the algorithm, and the curve. Choreography as it relates to algorithms and its attempt to reveal the structural relationship between the body and the mechanism. Or, as the Heideggerian definition of technology suggests, "Technology is a human activity."[4] Because of this, his performances have been criticized for bordering on randomness for having the appearance that he has severed the traditional underlying structure of poses associated with ballet. By limiting the number of possible inputs to expand the number of possible outputs, Forsythe demonstrates the body's relationship with itself and the surrounding space to construct a serial list of movements that relies on the lines of the body to scribe a spline that is both nonstandard and repeatable (Fig. 3.07). In his instructional video "Three Dropping Curves," Foresythe is able to collapse the distinction between choreography, as it relates to algorithms, to reveal the structural apparatus.

Embodiment denotes a form of participant status, and when the body inserts itself into the machine's performance, the machine becomes flexible and subjective. The embedded indeterminacy of die Schlange and its reliance on



↗3.08
→3.09

→3.08
A video still of myself and T. Deutch interjecting and redirecting die Schlange's performance.

→3.09
Video stills record the movement of die Schlange's performance using a GoPro as the end-effector.

the body makes the performance unpredictable in a way that Rhino and other CAD software don't allow by moving together in inhuman and unknowable ways. By relinquishing the determinism of the tool, the tool is no longer bound to generate geometric results based on an intended target by following the end-to-end control of inverse or forward kinematics. Like anchors and other control points, the body redirects the machine's deterministic motion in chaotic ways, exposing the affordances of the machine by allowing the body to intervene and disrupt a predetermined set of instructions. Disentangling methods of design that are often fixated on dimension and form to pivot toward proportion, relationship, and tempo.

With each new technology, our codependent relationship with the tools we use is only heightened. The postures of our bodies and our machines reflect one another in reciprocal relations, conforming to one another (Figs. 3.08 and 3.09). Motion data and performance envelopes surrounding our tools extend beyond the projective reenactment of the machine and offer us a means to measure the derivative of what it takes to produce a drawing, a surface, or a construction. One way to do this is to dislocate the spline from its formal geometry associated with slipstream construction and recast it as a way to record the tumble-type inscriptions surrounding an object's performance. Therefore to expand upon the embodied dimensions of our tools today, we need to reconsider the machine as the site of intervention.

→3.10

→3.10
This photo joiner records the panoramic movement of die Schlange's performance by using a GoPro as the end-effector.

End notes:

1.
The Multiple Axis Space Test Motion Inertia Facility, fondly reffered to as the "the gimbal rig," simulated maneuvers that might be encountered in space flight. The term "tumble-type" refers to the movement of the gimal rig.

2.
No text has been more influential in this thesis than Andrew Witt's Formulations: Architecture, Mathematics, Culture. Andrew and I have had many conversations on this subject, and I can't thank him enough for his feedback throughout this thesis. Thanks, Andrew :)

3.
Rudolf von Laban, dance artist, choreographer, and dance theorist, is best known for his annotated instructions for dance, Labanotation (Gruppenspiel für Schrifttanz.) You can read more about his and others' dance theory in the article by Flora L. Brandl titled, On a Curious Chance Resemblance: Rudolf von Laban's Kinetography and the Geometric Abstractions of Sophie Taeuber-Arp.

4.
I don't think it's possible to write about the embodied dimension of tools without citing Martin Heidegger's The Question Concerning Technology and Other Essays.

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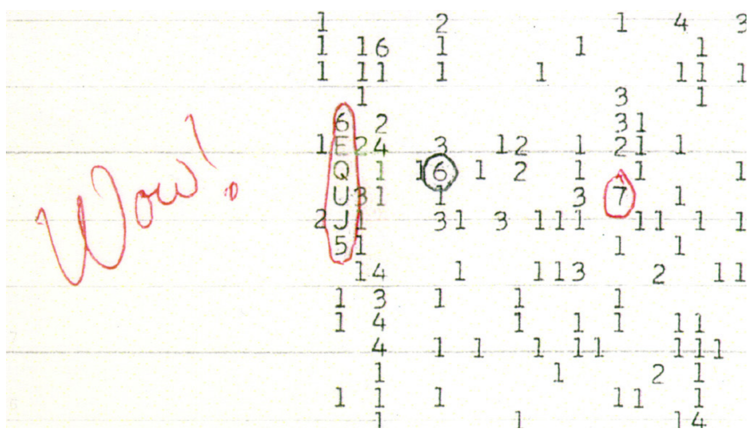
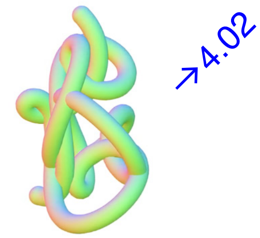
4. Outro

→4.01
The “Wow! signal” was recorded in 1977 by Ohio State University’s Big Ear radio telescope. The signal “6EQUJ5” was marked by astronomer Jerry E. Ehman as an exclamation to indicate the 72-second window during which Big Ear could detect a response from the direction of the Sagittarius constellation, suggesting a sign of extraterrestrial intelligence. While I am unsure about the claims of this radio signal, this printout indicates a potent form of communication between humans and machines across space and time.



→4.01

Architects have a codependent relationship with their tools. As the desire to script and code complex architectural forms more efficiently increases, the discipline has shifted its gaze toward computational tools with unimaginable processing power. While today's turbulent technical culture in architectural production may sometimes be overwhelming, it is also enriching. This constant exchange of digital tools between architecture and other disciplines has allowed us to move into a broader range of representational techniques as a means to formalize ideas in a physical environment. Collectively these newly minted tools offer a unique language to unify perception and performance across disciplines and space. Digital representation and fabrication is not merely a way of doing the same old thing faster but an enormous shift in how architecture is conceived, represented, and understood.



As digital tools begin to suture the architectural discipline of the machine, we need to lean into the shared languages that already exist in the background of our tools. Helping us disentangle the discrepancies between our digital models and on-site construction is the shared language of the algorithm. The algorithm, and an overall increased level of computational literacy, have allowed us to participate in the constant exchange with other tools and disciplines. In addition to making our relationship with software more transient, the value of the algorithm extends beyond the smooth transaction between tools. At its core, the algorithm is a series of indefatigable instructions that speak to both the human and machine.

→4.02
A polygon mesh approximating a curved spline recorded from die Schlange with an applied normal map.

Much like the hidden algorithms hidden behind the effects of the screen, there is an apparatus of choreography surrounding our physical tools that encode instructions on how the body engages with an object. In other words, the machines we use produce not only drawings but gestures as well, keying us into the always-present yet rarely discussed embodied dimensions of tools. Leaning into these protocols related to encoded instruction may be a critical way to move the discipline forward. The space of intervention is the machine and the traces of the implied virtual volume of the machine's kinetic movement. One way this temporal relationship is recorded is through motion data, a process of recording the activity of objects or people in space. The architectural relevance of these techniques is made visible by a temporal light trail that demarcates the threshold between human agents and the space they occupy around the machine, carving an envelope of motion. Recording motion data and performance envelopes surrounding our tools extend beyond the projective reenactment of the device and offers us a means to measure the derivative of what it takes to produce a drawing, a surface, or a construction. The performance envelopes recorded with motion data are the truest constraints of human habitation.

This thesis dislocates the spline from its formal geometry associated with slipstream construction and recasts it as a way to record the tumble-type inscriptions surrounding an object's performance — a tactic to mutually mark and negotiate the activity between humans and machines.

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An image taken in studio of in-progress tools and machines.

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A film still of myself and T. Deutch interjecting and redirecting the performance of die Schlange (the Snake).

0.03
A film still of myself instructing choreography with an apparatus.

1. Tool

1.01
My first prototype of die Schlange using mostly tripod parts, printed circuit boards, and stepper motors, 2022.

1.02
In ancient Egypt, a rope stretcher (or harpedonaptai) was a surveyor who measured real property demarcations and foundations using knotted cords. The practice is depicted in tomb paintings of the Theban Necropolis.

1.03
A representational image of satellites orbiting earth by Trevor Paglen and NASA, 2023.

1.04
Illustration of die Schlange (the snake) by Albrecht Dürer in Underweysung der Messung, 1525. Public Domain of The Metropolitan Museum of Art.

1.05
Laocoön and His Sons excavated in Rome in 1506. It depicts the Trojan priest Laocoön and his sons Antiphantes and Thymbraeus being attacked by sea serpents. One of the unsubstantiated speculations of die Schlange's intended use has been suggested that it was a sculptural jig.

1.06
Illustration of der Schnecke (the snail) by Albrecht Dürer in Underweysung der Messung, 1525. Public Domain of The Metropolitan Museum of Art.

1.07
Illustration of die Spinne (the spider) by Albrecht Dürer in Underweysung der Messung, 1525. Public Domain of The Metropolitan Museum of Art.

1.08
Illustration of die Schlange (the snake)by Albrecht Dürer in Underweysung der Messung, 1525. Public Domain of The Metropolitan Museum of Art.

1.09
Illustration of die Spinne (the spider) by Albrecht Dürer in Underweysung der Messung, 1525. Public Domain of The Metropolitan Museum of Art.

1.10
Illustration of the human body's proportions by Carlo Urbino di Crema, Codex Huygens, c. 1570.

1.11
A photo of me adjusting the servo motor at the base of the second die Schlange prototype.

1.12
Weighted spline whales (or ducks) used to pin the metal and wooden laths into the shape of an ideal curve.

1.13
Weighted spline whales (or ducks) used to pin the metal and wooden laths into the shape of an ideal curve.

1.14
In the production of typefaces and fonts, we still rely on the underwriting of the Bézier curve, created by Pierre Bézier in the early days of CAD computer graphics.

1.15
A screenshot of a spline tool used in Adobe Illustrator indicates the anchors and handles on the curve's periphery.

1.16
A hairnet used to produce the interpolation of a surface connecting the boundary curve of the circle with the pins holding it in place.

1.17
A polygon mesh approximating a curved surface.

1.18
Ivan Sutherland's students mark and measure Marsha Sutherland's VW Beetle for digitization, 1972.

1.19
Irregular surfaces carry with them the embedded priorities associated with the aerodynamic construction of the mid-twentieth century that is defined by the boundary curve of a surface and the blending functions which interpolate the space between them. A digital object demonstrating the priorities of smooth continuous surfaces embedded within our digital tools today. Pictured here is an object reflecting the landscape around the Space Needle.

1.20
Pictured here is an unrolled surface drawing of the smooth reflective surface of the spline geometry.

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2. Algorithm

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Video stills of the fully reconstructed die Schlange, standing nearly 12ft tall and made mostly of aluminum tripod parts, printed circuit boards, and servo motors, recording each rod and dial in motion (photo by Daisy Ziyan Zhang).

2.02
Muhammad ibn Mūsā al-Khwārizmī, algorithmic proof from The Compendious Book on Calculation by Completion and Balancing, c. 825

2.03
Zach Schumacher with Outpost Office (photo by Erik Herrmann), Thus Spake Al-Khwārizmī, 2021

2.04
Photo of the reconstructed die Schlange (photo by Daisy Ziyan Zhang).

2.05
Illustration of die Schlange's clock-like dials by Albrecht Dürer in Underweysung der Messung, 1525.

2.06
A detailed photo of the servo motor at the connection between two rods on the reconstructed die Schlange. Reinterpreting the original clock-like dial as a servo motor allows for the careful control of each dial.

2.07
A diagram of die Schlange's range of motion as indicated with the various concentric rings where each rod can be repositioned and the network of outlying points for each curve.

2.08
Following the range of motion afforded by each dial, these epicycles demonstrate the relationship between each ring relative to the central point.

2.09
Motion capture data of the instrument's kinematic performance recorded at MIT.nano's Immersion Lab with OptiTrack, a technique that records the XYZ coordinate of the end-effector's implied volume at two frames per second.

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Motion tracking data of the end effector when the machine is demonstrating a Forward Kinematic movement. Here you see the red dot of the machine's central leader and the direction of the spline indicated by the annotated sequential numbers.

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Motion tracking data of the end effector when the machine is tasked with generating a sine curve with an Inverse Kinematic movement.

2.13
A simulation of die Schlange controlled by inverse kinematics to scribe the boundary of a green square. The spline curves on the ground plane record the movements of the rods as they navigate to match the coordinates of the green square.

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A perspective view of the above

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Video stills record the movement of die Schlange's performance using a GoPro as the end-effector.

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An image of the fully reconstructed die Schlange, standing nearly 12ft tall and made mostly of aluminum tripod parts, printed circuit boards, and servo motors.

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Video stills recording the initial test of each rod and dial in motion.

3. Body

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The Four Elements is a painting by Louis Finson in 1611. The bodies in the painting represent (clockwise from the top left) Air, Fire, Earth, and Water. Each element entangled with the other, contorting their posture to push and pull the other elements with clasping hands, fists of hair, and extended legs.

3.02
Die Schlange's performance was simulated with the video game software Unreal Engine.
As the arms swing, the 'snake' reveals itself with a light trail.

3.03
In the 1950s, Frank and Lilleth Gilbreth used long-exposure photography to study the efficiency of a factory worker completing a task.

3.04
Time-lapsed photo of NASA's gimbal rig used for Mercury astronaut training in 1960.

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Pictured are the eight main directions used as the foundational instruction for Labanotation, illustrated by Ann Hutchinson. Labanotation is a structured system for analyzing and recording

movement with symbols.
3.06
Rudolf von Laban, dance artist, choreographer, and dance theorist, is best known for his annotated instructions for dance, Labanotation (Gruppenspiel für Schrifttanz.) Pictured here are dance instructions from 1930–1931 by his daughter Azra von Laban.

3.07
William Forsythe's "Three Dropping Curves" is one of many instructional videos recorded by the choreographer that illustrates the relations between the body and surrounding space.

3.08
A video still of myself and T. Deutch interjecting and redirecting die Schlange's performance.

3.09
Video stills record the movement of die Schlange's performance using a GoPro as the end-effector.

3.10
This photo joiner records the panoramic movement of die Schlange's performance by using a GoPro as the end-effector.

4. Outro

4.01
The "Wow! signal" was recorded in 1977 by Ohio State University's Big Ear radio telescope. The signal "6EQUJ5" was marked by astronomer Jerry E. Ehman as an exclamation to indicate the 72-second window during which Big Ear could detect a response from the direction of the Sagittarius constellation, suggesting a sign of extraterrestrial intelligence. While I am unsure about the claims of this radio signal, this printout indicates a potent form of communication between humans and machines across space and time.

4.02
A polygon mesh approximating a curved spline recorded from die Schlange with an applied normal map.

Bibliography

Astengo, Gregorio. De mechanisering van het parallelle teken (Mechanising Parallel Lines). Tijdschrift voor architectuur (Journal for Architecture). OASE #105 Practices of drawing Tekenpraktijken. OASE foundation, Rotterdam, The Netherlands, 2020.

Benjamin, Walter. “The Work of Art in the Age of Mechanical Reproduction.” In A Museum Studies Approach to Heritage, 1:226–243. 1st ed. Routledge, 2019.

Brandl, Flora L. “On a Curious Chance Resemblance: Rudolf von Laban’s Kinetography and the Geometric Abstractions of Sophie Taeuber-Arp.” Arts 9, no. 1 (2020).

Cache, Bernard. Instruments of Thought: Another Classical Tradition. Essay. In The Cornell Journal of Architecture. 9, Mathematics : from the Ideal to the Uncertain. Edited by O’Donnell, Caroline. Ithaca, NY: Cornell University, College of Architecture, Art and Planning, 2012.

Cardozo Llach,Daniel. Builders of the Vision: Software and the Imagination of Design (New York: Rooutledge, 2015), 49-72.

Coons, Steven A. (Steven Anson). Surfaces for Computer-Aided Design of Space Forms. Cambridge, Mass: M.I.T. Project MAC, 1967.

Davis, Paul. Worcester Polytechnic Institute. B-Splines and Geometric Design. SIAM News 29(5), June 1996.

De Boor, Carl. “On Calculating with B-Splines.” Journal of approximation theory 6, no. 1 (1972): 50–62.

Dürer, Albrecht. Underweysung der Messung mit dem zirckel un richt scheyt. Nuremberg, Germany: Hieronymus Andreae, German, Bad Mergentheim, 1525.

Fayyad, Iman. “On Flatness: The Virtual Turn.” Log, Anyone Corporation, issue 51, 2021, pp. 119-130.

Forrest, A. Robin. An Introduction to Splines for Use in Computer Graphics and Geometric Modeling, ed. Richard Bartels, Joh Beatty, and Brian Barsky. Burlington, MA: Morgan Kaufmann Publishers, 1995.

Gaboury, Jacob. Image Objects : an Archaeology of Computer Graphics. Cambridge, Massachusetts: The MIT Press, 2021.

Gilbreth, Frank Bunker, and Lillian Moller Gilbreth. Essay, Applied Motion Study: A Collection of Papers on the Efficient Method to Industrial Preparedness. New York City, NY: Macmillan, 1919.

Grandine, Thomas A. The Extensive Use of Splines at Boeing. SIAM News, Volume 38, Number 4, May 2005.

Donna J. Haraway, “A Cyborg Manifesto: Science, technology, and Socialist-Feminism in the Late Twentieth Century,” in Simians, Cyborgs, and Women: The Reinvention of Nature (New York: Routledge, 1991), 149-181.

Heidegger, Martin. The Question Concerning Technology and Other Essays. Translated by William Lovitt. (New York: Harper & Row, 1977), pp. 3-35. The German text appears in Martin Heidegger, Vorträge und Aufsätze (Pfullingen: Günther Neske Verlag, 1954), pg.13-44.

Imperiale, Alicia. New Flatness : Surface Tension in Digital Architecture. Basel, Birkhäuser, 2000.

Kemp, Martin. The Science of Art. Yale University Press. New Haven, Conn, 1990.

Knuth, Donald E. “Mathematical Typography,”Bulletin (New Series) of the American Mathematical Society 1, no. 2 March 1979): 337-372.

Kucuk, Serdar and Bingul, Zafer. Robot Kinematics: Forward and Inverse Kinematics, Industrial Robotics: Theory, Modelling and Control. INTECH Open Access Publisher, 2006.

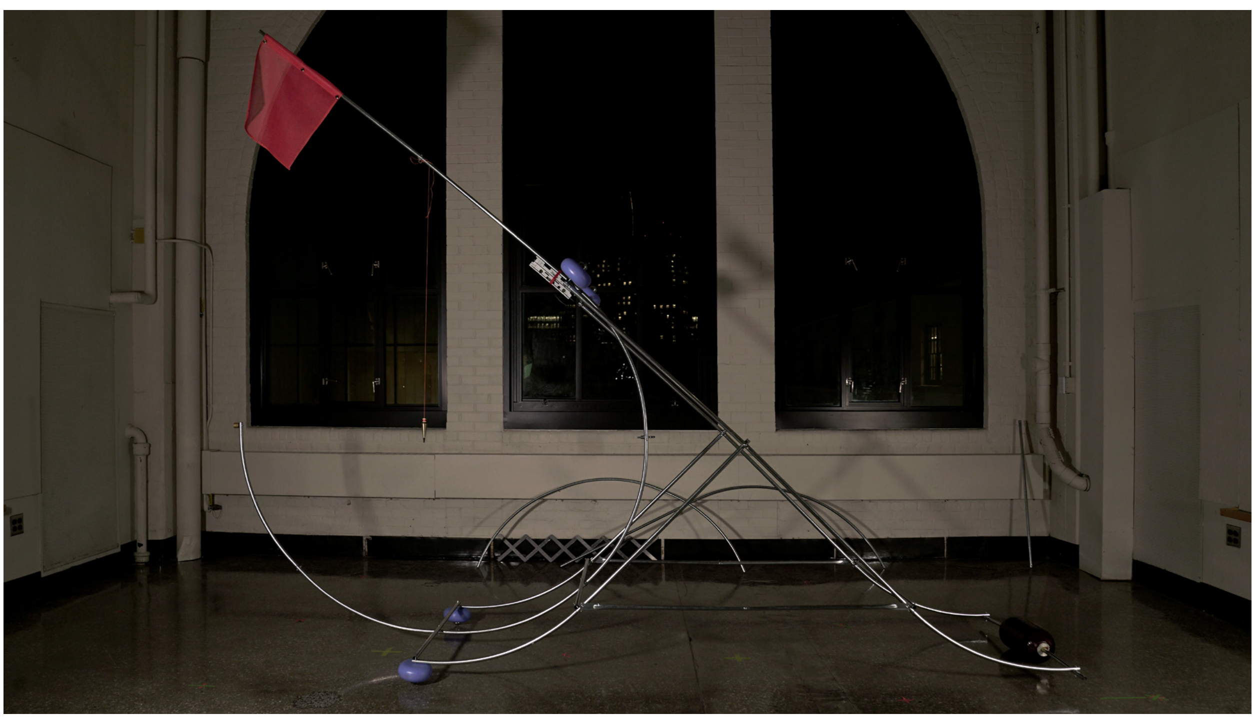
Linder, Mark. Nothing Less Than Literal : Architecture after Minimalism. Cambridge, Mass: MIT Press, 2004.

Lynn, Greg. Animate Form. New York: Princeton Architectural Press, 1999.

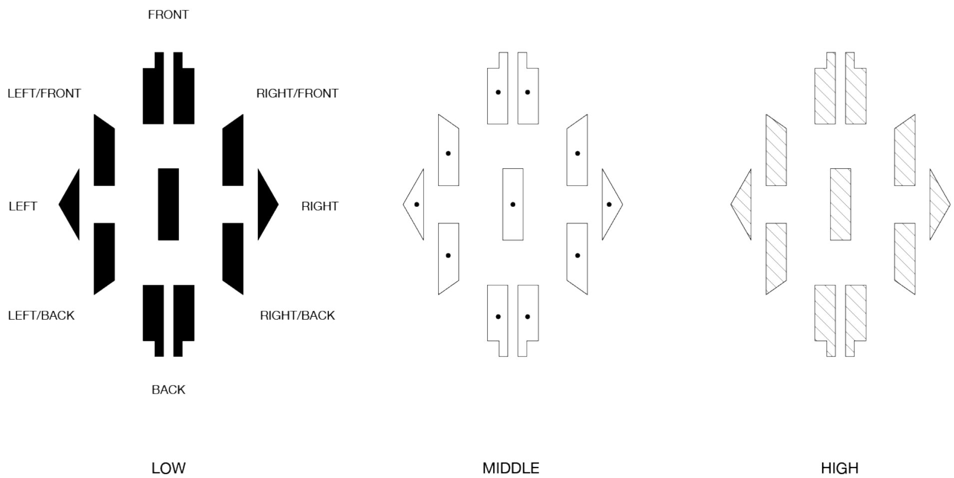
Schoenberg, Isaac Jacob. “Contributions to the Problem of Approximation of Equidistant Data by Analytic Functions,” Quarterly of Applied Mathematics 4 (1946): 45-99, 112-141.

Witt, Andrew. Formulations: Architecture, Mathematics, Culture. Cambridge: MIT Press, 2022.

Appendix



Appendix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RIGHT
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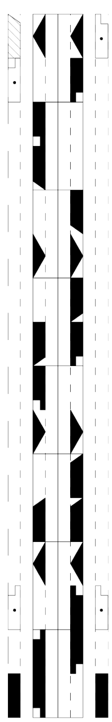
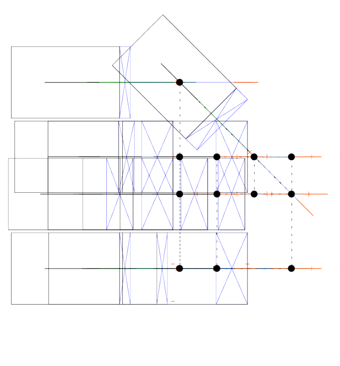
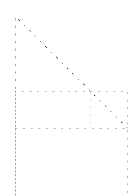
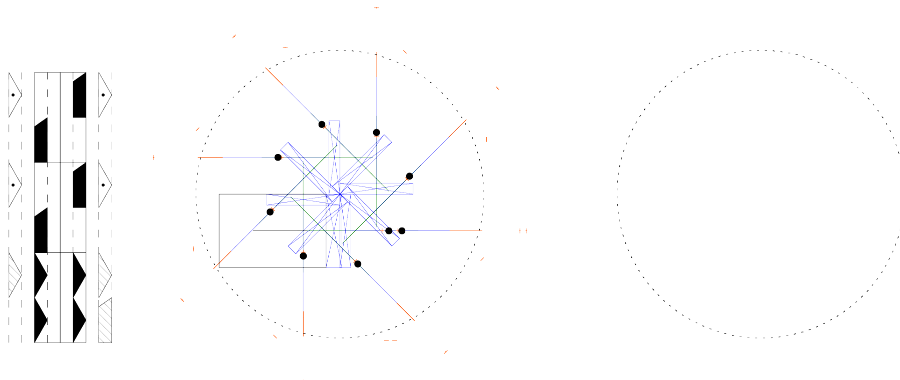
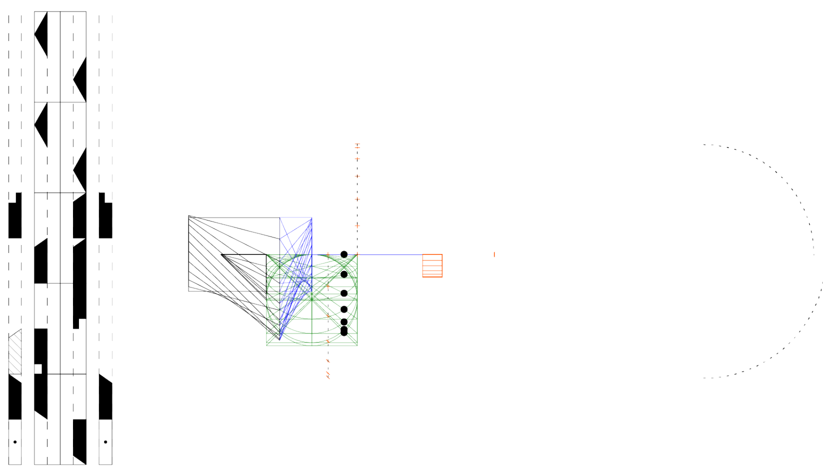
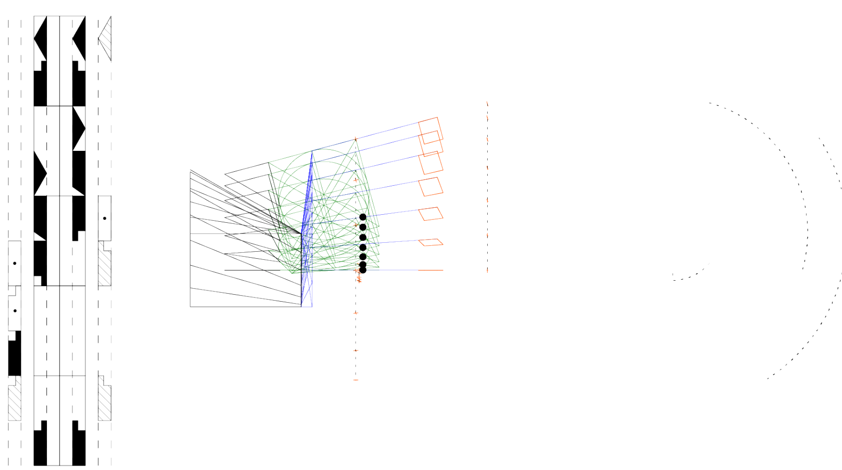
LOW
MIDDLE
HIGH







Appendix