LONG-SPAN BRIDGE LIVE LOADS

by

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Submitted in partial fulfillment
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ABSTRACT

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A model of the renewal type is presented to characterize the truck arrival process on highway bridges. A unique feature of this model is that it implies the grouping or clustering of trucks as they cross a bridge. For long-span bridges, these groups of trucks are treated as the live loading unit to be considered in design and a method is outlined for predicting the load effect due to the presence of groups on a bridge. The model also serves to point out the underlying mechanisms for reduction of live load intensity with increase in bridge span length. Data is presented for truck headways and truck gross weights to substantiate the assumptions made in developing the model. Numerical examples are included to illustrate some possible uses of the model for highway bridge design.

Thesis Supervisor: Prof. C. Allin Cornell

Title: Associate Professor of Civil Engineering

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List of Symbols

(Note: Standard conventions are used throughout for probability-related notation. Any specially defined notation appears below.)

a length of an individual truck

k parameter of the gamma distribution of X, gross truck weight

λ parameter of the gamma distribution of X, gross truck weight

λ_G average arrival rate of groups following the departure of preceding groups

λ_W average arrival rate of trucks within groups

p parameter of the geometric distribution of N, the number of trucks in a group

r useful lifetime of bridge

s span length of bridge

u_{eq} equivalent uniform load

H_G intergroup headway

H_W intragroup truck headways

I moment influence variable

J number of intragroup headways in a group of N trucks

L sum of intragroup truck headways

L_G sum of M-1 intergroup headways

L_S sum of intragroup headways of M groups

L_T total group length

M number of groups present simultaneously on bridge

M_{MAX} maximum positive moment on simply supported bridge due to one group

M_P peak lifetime positive moment on simply supported span due to one group

N number of trucks in a group

T sum of L_S and L_G, or sum of all headways of M groups

W total group weight

X gross weight of an individual truck

Chapter 1: Highway Bridge Live Loads

1.1 Introduction

Presented in this chapter is a summary of the efforts made during the past sixty years to quantify and standardize the live loading for highway bridges. These efforts may be subdivided into two categories: the development of specifications for use in design of new structures, and the quest for probabilistic models to rationalize live load selection.

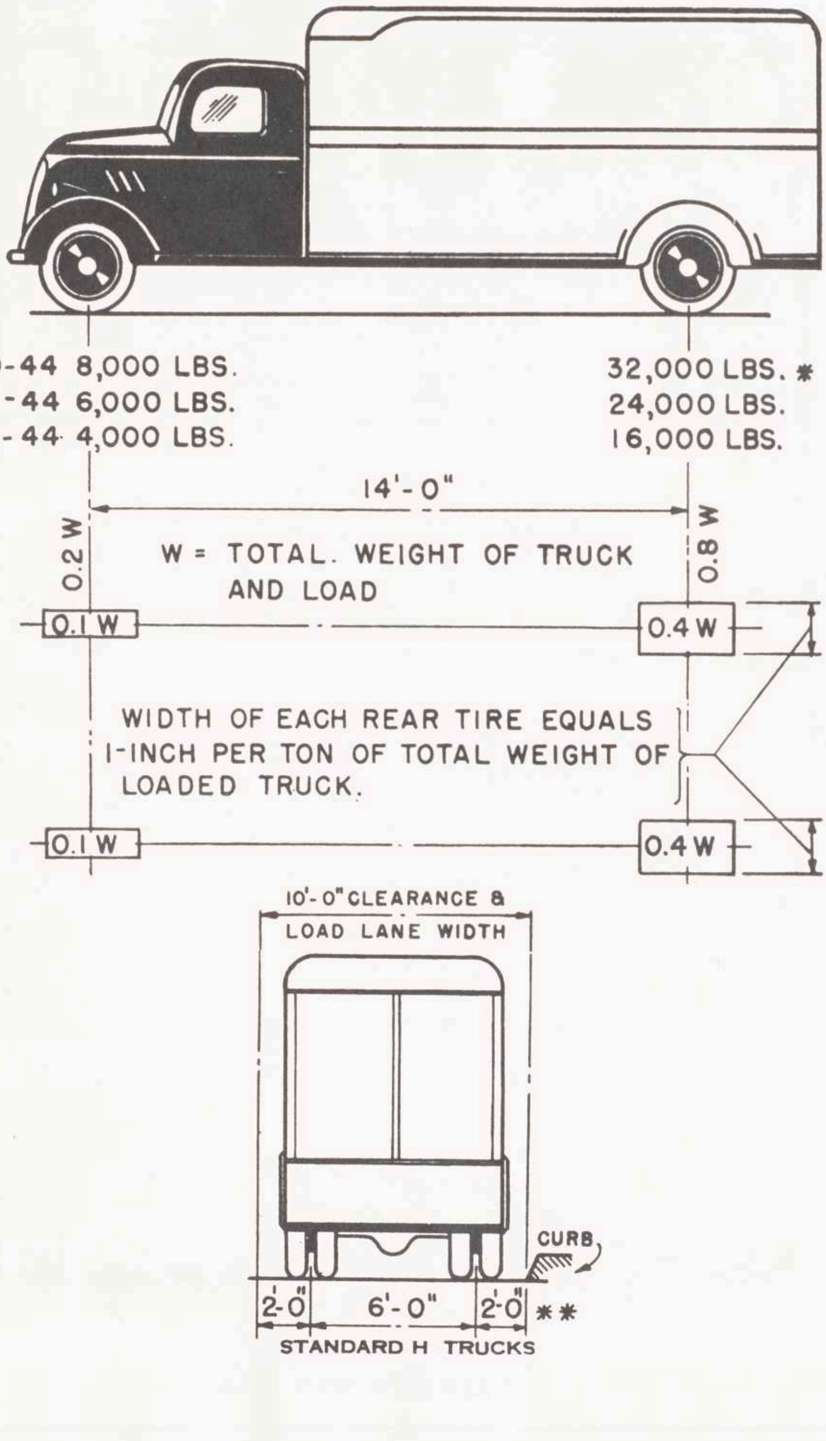
1.2 Development of Specifications

Before World War I, highway bridge live load design was based, for the most part, on horse-drawn vehicle characteristics, with consideration given to heavy loads produced by steam-driven traction engines. The earliest printed article dealing with the effects of motor trucks on bridges appeared in 1914[1-1], and it contained recommendations for the use of design loadings based on actual vehicles then in service rather than a uniform live load of 80-100 psf plus a concentrated load as used in design up to that point.

Standardization of highway bridge design throughout the United States began with the passage of the Federal Highway Act in 1916, and by 1919 the Federal Government was cooperating in a large proportion of highway projects. This participation by the Federal Government led to the standardization of loading requirements for highway bridges nationwide. The early load requirements were generally in terms of motor trucks with design loads of 10-20 tons, with 15 ton trucks taken as a minimum loading for main highway structures.

In 1921, the American Association of State Highway Officials (AASHO) formed a Committee on Bridges and Structures and in 1923 this Committee produced a tentative specification in which vehicular live loads were defined in terms of H loadings (two-axle standard trucks). In 1925, these loadings were published as part of the Standard Specifications for Highway Bridges and Incidental Structures, and a novel feature of the 1925 Specifications was that live loads were to be placed in definite traffic lanes. In 1941, the H-S loading was introduced, to reflect the growing appearance of combination vehicles (tractor and trailer) and the heavier weights of these vehicles. In 1944 further changes were made in the standard loads, by changing axle spacing and concentrated loads in the H-S loading. The live loadings as specified by AASHO have not changed since 1944, and are those in common use in U.S. bridge design at present.

Two loading patterns are presented in the current AASHO Specifications, the H loading and the H-S loading. The H loading is based on the 1935 truck train loading (Figure [1.1])



H20-44 8,000 LBS.
H15-44 6,000 LBS.
H10-44 4,000 LBS.
32,000 LBS. *
24,000 LBS.
16,000 LBS.
14'-0"
0.2 W
0.8 W
W = TOTAL. WEIGHT OF TRUCK AND LOAD
0.1 W
0.4 W
WIDTH OF EACH REAR TIRE EQUALS 1-INCH PER TON OF TOTAL WEIGHT OF LOADED TRUCK.
0.1 W
0.4 W
10'-0" CLEARANCE & LOAD LANE WIDTH
CURB
2'-0"
6'-0"
2'-0" **
STANDARD H TRUCKS

Figure [1.1a]

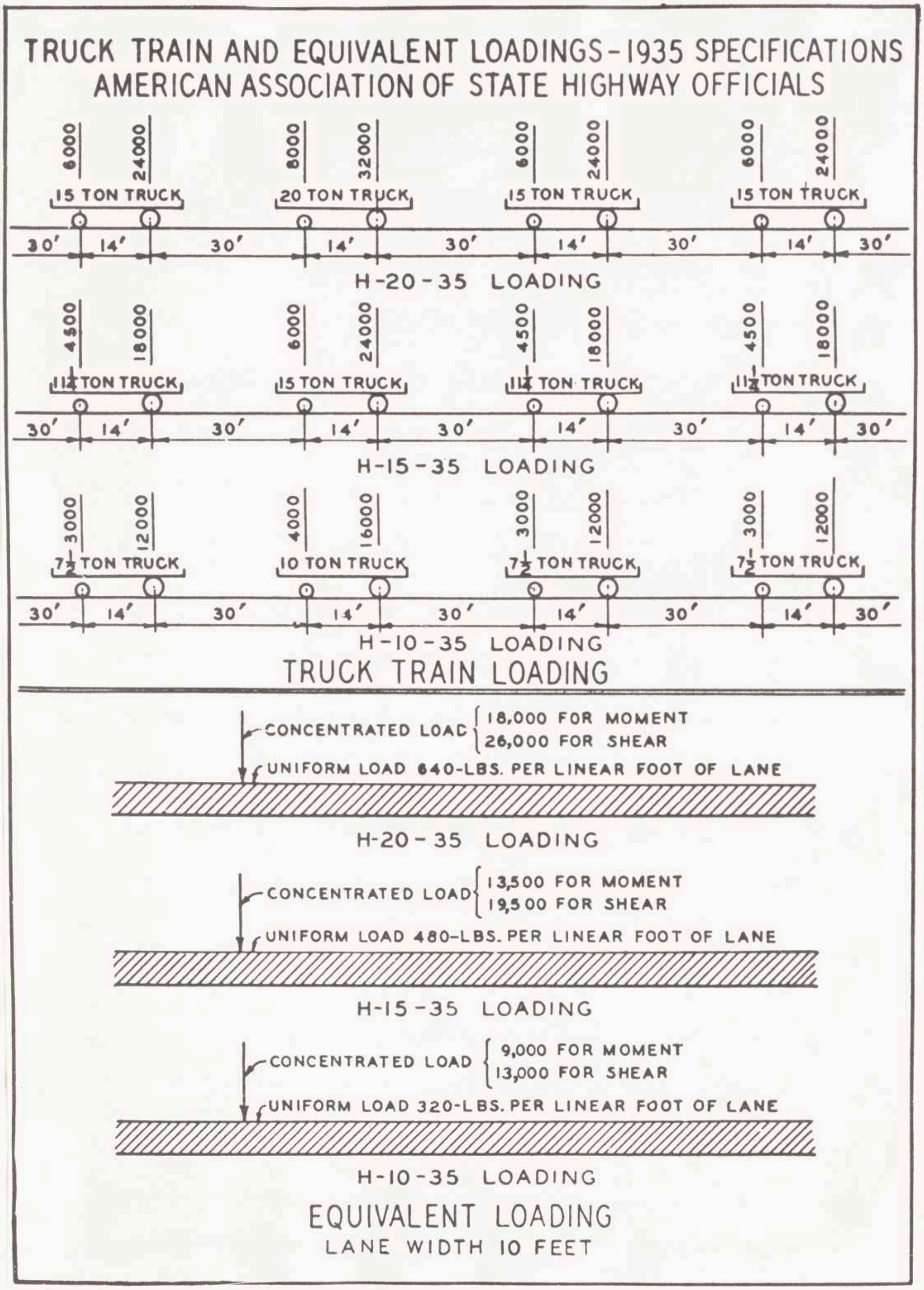
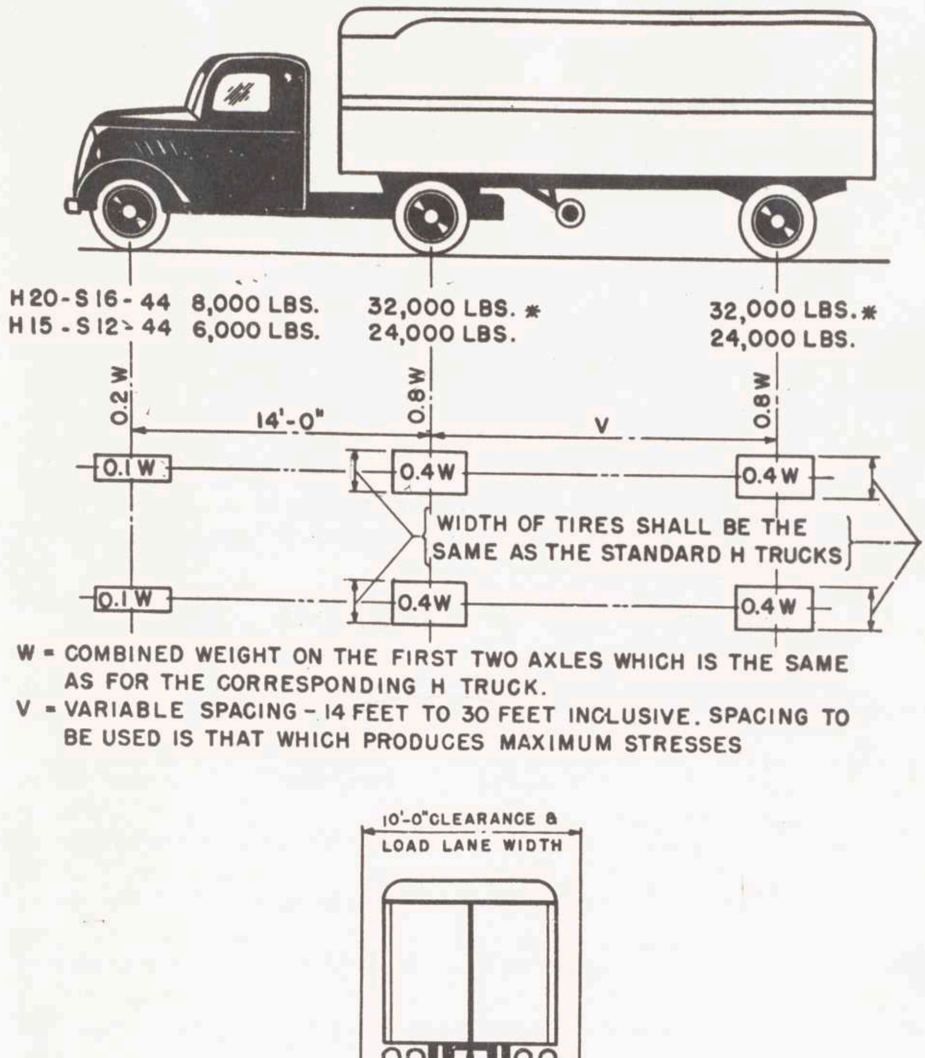

TRUCK TRAIN AND EQUIVALENT LOADINGS-1935 SPECIFICATIONS
AMERICAN ASSOCIATION OF STATE HIGHWAY OFFICIALS
6000
24000
15 TON TRUCK
8000
32000
20 TON TRUCK
30′
14′
H-20-35 LOADING
4500
18000
11½ TON TRUCK
H-15-35 LOADING
3000
12000
7½ TON TRUCK
4000
16000
10 TON TRUCK
H-10-35 LOADING
TRUCK TRAIN LOADING
CONCENTRATED LOAD
18,000 FOR MOMENT
26,000 FOR SHEAR
UNIFORM LOAD 640-LBS. PER LINEAR FOOT OF LANE
H-20-35 LOADING
CONCENTRATED LOAD
13,500 FOR MOMENT
19,500 FOR SHEAR
UNIFORM LOAD 480-LBS. PER LINEAR FOOT OF LANE
H-15-35 LOADING
CONCENTRATED LOAD
9,000 FOR MOMENT
13,000 FOR SHEAR
UNIFORM LOAD 320-LBS. PER LINEAR FOOT OF LANE
H-10-35 LOADING
EQUIVALENT LOADING
LANE WIDTH 10 FEET

Figure [1.1b]

which consists of a string of vehicles spaced to simulate truck traffic moving at 15 mph. A lane loading is also presented to be used on longer spans due to the inconvenience of applying the truck train loadings and carrying out the calculations that they entail. The H-S loadings presented in 1944 are designed to duplicate the truck train loading that was dropped at that time, with the advantage that in design only a single truck has to be applied to the structure rather than a whole string of trucks. Figure [1.2] is a graphical representation of the H-S loading. For design purposes, both the truck loading and the lane loading must be employed, with the governing loading being that which produces the maximum stress. Tabulated values of maximum moment and maximum shear for simply supported bridges of various loaded lengths less than or equal to 300 feet are available in the back of each of the editions of the AASHO Specifications, with notes explaining which loading governed in the determination of that maximum load effect.

With regard to spans longer than 300 feet, the Specifications make no direct recommendation, but suggest (in the Introduction to the 5th Edition, 1949) that the uniform live load be reduced for bridges longer than 300-400 feet. The reasoning behind the proposed reduction may best be summarized by a quote from Asplund[1-2]:

> There is a natural feeling that traffic loads per unit length of lane are smaller on long bridge spans



STANDARD H-S TRUCKS

Note: Lane Loading for H20-S16-44 is same as Lane Loading for H20-35, while Lane Loading for H15-S12-44 is same as Lane Loading for H15-35, both shown on the lower portion of Figure [1.1b]

Figure [1.2]

than on short. Observations on actual traffic lanes corroborate this feeling but no rational analysis of the proper magnitude of the reduction seems to exist.

In the absence of specifications for design loadings of bridges of span length in excess of 300-400 feet, the selection of the design live load magnitude falls into the province of the individual designers. Many long-span highway bridges have been designed for a variety of live loads arising from differences in loading conditions in different geographical areas and differences in expected service requirements of individual bridges.

One possible reason for the lack of specifications for live loading in the long span range of bridges is the infrequency of construction of such bridges. Table [1.1] contains a summary of the frequency distribution of bridge span lengths in the California State Highway System[1-3]. The number of spans over two hundred feet in length amount to less than one percent of the total number of spans. Additionally, as the span length increases, the proportion of dead load to design live load increases, to the point that on very long spans the live load is just a small portion of the total design load. The author has been told in a conversation with an executive of one of the design firms that has been responsible for the design of a number of the longest bridges in the world that live load can sometimes represent only 10% of the total load of a bridge, and the doubling of the design live load would

Frequency Distribution of Bridge Span Lengths
in the California State Highway System

Span Length (feet)	Number of Spans	Percent of Total No. of Spans	Cum. Percent
10 to 19	7,655	35.32	35.32
20 to 29	4,078	18.85	54.17
30 to 39	3,749	17.30	71.47
40 to 49	1,936	8.97	80.44
50 to 59	1,414	6.54	86.98
60 to 69	1,116	5.15	92.13
70 to 79	416	1.93	94.06
80 to 89	355	1.64	95.70
90 to 99	142	.70	96.40
100 to 109	214	.99	97.39
110 to 199	416	1.93	99.32
200 to 399	121	.56	99.88
400 to 999	14	.06	99.94
1000 to 4200	13	.06	100.00

Table [1.1]

have little effect upon the economics of the construction of such a bridge. Furthermore, this executive has submitted that the live load selection within his firm is based upon formulae derived from traffic observations made decades ago on some of the East River crossings in New York City. It is also common to read accounts of the addition of second levels or median strips to existing bridges not designed originally for these additions, the usual practice being to assume a more liberal live load magnitude than in the original design and to let the excess live load cover the additions to the bridge. Thus, it may be seen that the live load selection for long span bridges is open to various and individual interpretations, and may be based in some cases on antiquated data.

In the United States, minimum design loads for bridges designed to carry heavy truck traffic is specified as H15-S12 (AASHO classification). In designs for less than H20 or H20-S16 provision for overload must be made, while for multi-lane bridges reductions are specified for the live load due to the decreased probability of the simultaneous loading of all lanes, much the same as the reductions made for live load with increased span length in long span bridges. Current practice is to design for H20-S16-44 loadings on all bridges even though the specifications permit design for H15-S12, reflecting the probability that even secondary road bridges may be loaded with heavy industrial vehicles some time during their service life.

Foreign practice has generally been to specify several design loadings to be used depending upon the importance of the roads and bridges to be designed. In addition, some countries provide for the presence of abnormally heavy loads. Most foreign design loadings are based on traffic lanes and uniformly distributed loads in conjunction with a single vehicle or knife edge load. In the case of multiple traffic lanes, one or two fully loaded lanes are placed in the worst position transeversely and the remaining lanes are considered to contain only a fraction of the maximum lane loadings.

Incidental to state highway officials' enforcement of state regulations, much highway data has been collected in the form of loadometer surveys throughout the United States. The data collected from these surveys has found widespread usage as a planning tool in the design of new highways. Additionally, many investigators have used this data to verify the realism of the AASHO-specified live loads.

In the area of long-span bridge design, an attempt was made in 1953[1-4] to utilize survey data to establish a specification of live load magnitudes. Traffic studies of the lower deck of the San Francisco- Oakland Bay Bridge (which when surveyed was restricted to truck traffic) were reported, in addition to loadometer surveys of heavy rural traffic, and the weights and arrangements of military convoys. The loading pattern on the bridge was used as the basis of the recommended loading, because it approximated the convoy loading and more than adequately covered all conditions of loadings when automobiles were present in the traffic stream. Figure [1.3] shows the recommended loadings and compares them to the AASHO loading for the short-span ranges. As noted above, these recommendations were never accepted by the bridge design community as specifications for the live loading of long span bridges, and merely serve as a guide to bridge designers.

1.3 Probabilistic Models

A number of investigators have advanced probabilistic models over the years to represent the live loading of both short and long span highway bridges. Common to most of these models is an effort to point out the rationale behind the reductions of live load intensity with increase in span length and number of loaded lanes. Much of the work in the development of probabilistic models for highway bridge live loads was started in the early 1950's, and interest in this area seems to have waned since the mid 1950's. Only recently has renewed interest in this area been exhibited, especially with regard to long-span live loads and fatigue problems.

Asplund in 1955 published a paper[1-5] in which he developed a model to consider the loading due to stationary loads on a bridge. The unique features of his model are the assignment of slots of deterministic length to each vehicle

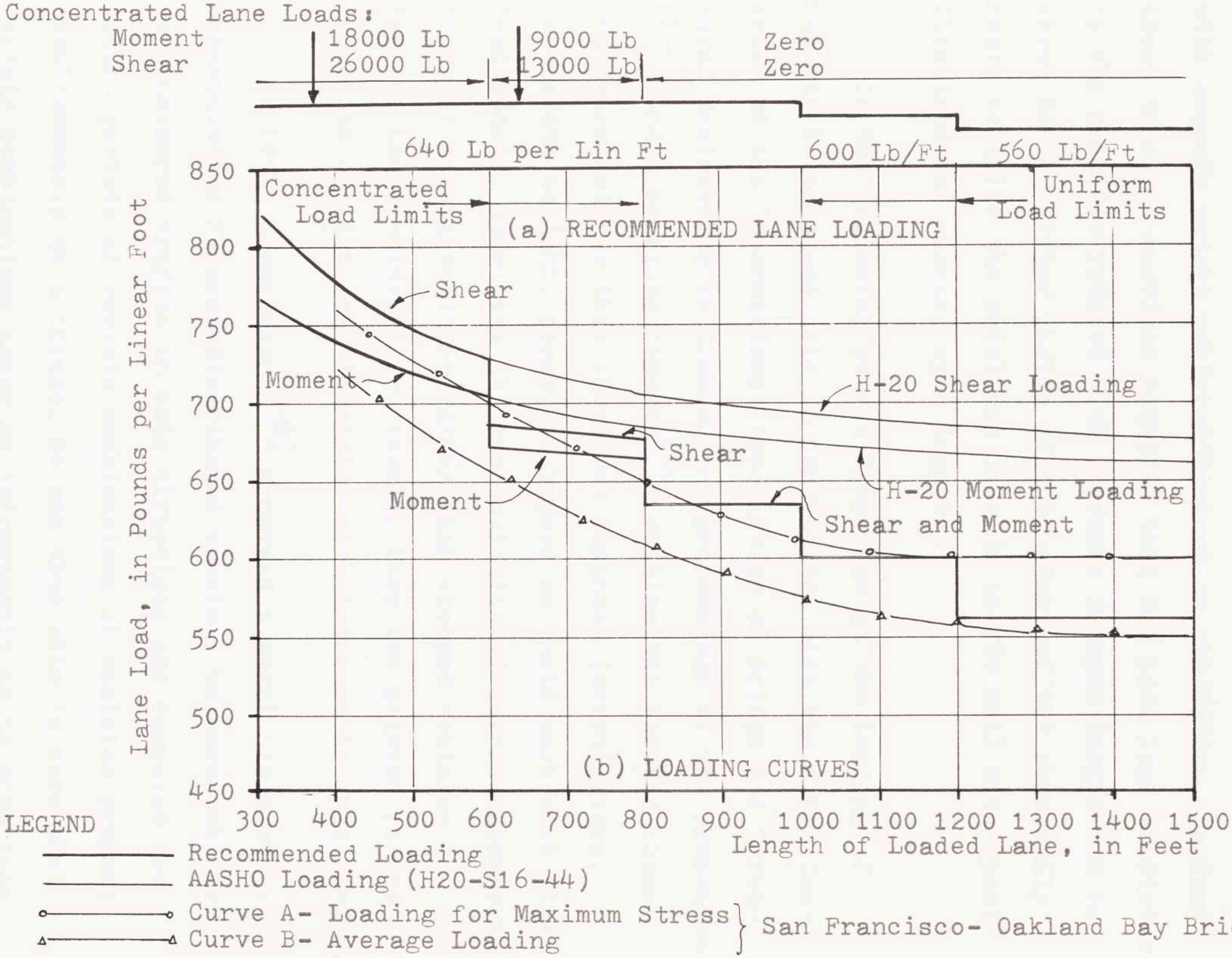

Concentrated Lane Loads:
Moment 18000 Lb 9000 Lb Zero
Shear 26000 Lb 13000 Lb Zero
640 Lb per Lin Ft
600 Lb/Ft
560 Lb/Ft
Concentrated Load Limits
Uniform Load Limits
(a) RECOMMENDED LANE LOADING
Lane Load, in Pounds per Linear Foot
850
800
750
700
650
600
550
500
450
Shear
Moment
H-20 Shear Loading
Shear
H-20 Moment Loading
Moment
Shear and Moment
(b) LOADING CURVES
300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500
Length of Loaded Lane, in Feet
LEGEND
Recommended Loading
AASHO Loading (H20-S16-44)
Curve A- Loading for Maximum Stress
Curve B- Average Loading
San Francisco- Oakland Bay Bridge

Figure [1.3]

and the consideration of abnormally heavy vehicles along with average weight vehicles present on the bridge. Asplund shows through numerical examples that the lane load predicted by his model is reduced with increase in span length and he shows how by proper choice of the weight of the abnormally heavy vehicles the model can be made to fit well with specifications for shorter span lengths.

In the following year a Symposium on the Loading of Highway Bridges was held in conjunction with the Fifth Congress of the International Association of Bridge and Structural Engineering in Lisbon. The proceedings of the Symposium [1-6] and a magazine report[1-7] outline the various views that were held at that time. Two Japanese investigators, Tahara and Konishi, presented papers on their work with live load models, examining the probabilities of severe concentrations of moving vehicles rather than stopped vehicles as Asplund had considered previously. They too pointed to the reduction of live load intensity with increase in span length.

In 1957, Stephenson[1-8] proposed a model based on the assumption of Poisson-distributed spacings between vehicles. He considered traffic in both directions and computed the return periods of certain combinations of vehicles present simultaneously on a bridge. He was thus able to show that certain combinations occur so infrequently as to drop them from consideration for design purposes.

In recent years, Garson, Goble, and Moses[1-9] have developed a probabilistic description of traffic loading. While their work has not been concentrated in developing a live load model but rather a fatigue prediction program, their description of traffic characteristics is useful for the development of components for a live loads model.

Chapter 2: Highway Bridge Live Load Model

2.1 Introduction

The model presented herein considers the full range of highway bridge spans and attempts to rationalize some of the traditional assumptions made in long-span bridge live load selection. The model serves to point out the underlying mechanisms leading to a reduction of live load intensities for longer spans, and may also be useful in quantifying the amount of reduction. Furthermore, the model presents a unified approach to live load considerations for all bridge spans, rather than singular approaches for the more traditional and arbitrary classifications: short spans and long spans.

Input variables for the model are parameters of vehicle behavioral characteristics as determined from data either currently widely available or collected recently by highway research groups. Verification of the model will depend upon the furtherance of the efforts of highway research groups in the collection of data, and the development of new mechanisms of data collection pointed to the verification of the specific assumptions made in the model.

Strictly speaking, the model is applicable only to multi-lane traffic streams with passing of vehicles allowed. Recognizing, however, that heavy vehicular traffic tends to bear to the right in multi-lane flow, either through convenience or through regulation, the great proportion of heavy vehicles will appear only in one lane, and hence applicability of this model may perhaps also be extended to use for the single right hand lane, e.g., in the study of the load effect on single girders.

2.2 Assumptions and Details of Model

2.2.1 Traffic Characteristics

Traffic on highway bridges is assumed to move at a uniform, constant speed. Thus, at least while crossing over a bridge, it is assumed that vehicles maintain constant relative spacings among themselves.

A certain degree of independence is assumed for vehicles in the traffic streams under consideration. In the case of trucks, which are the vehicles of interest in this model, it is assumed that the weights and driver characteristics of individual trucks are mutually independent of one another. For example, a traffic stream dominated by trucks dispatched in bunches from one location might well contradict the above assumption of independence.

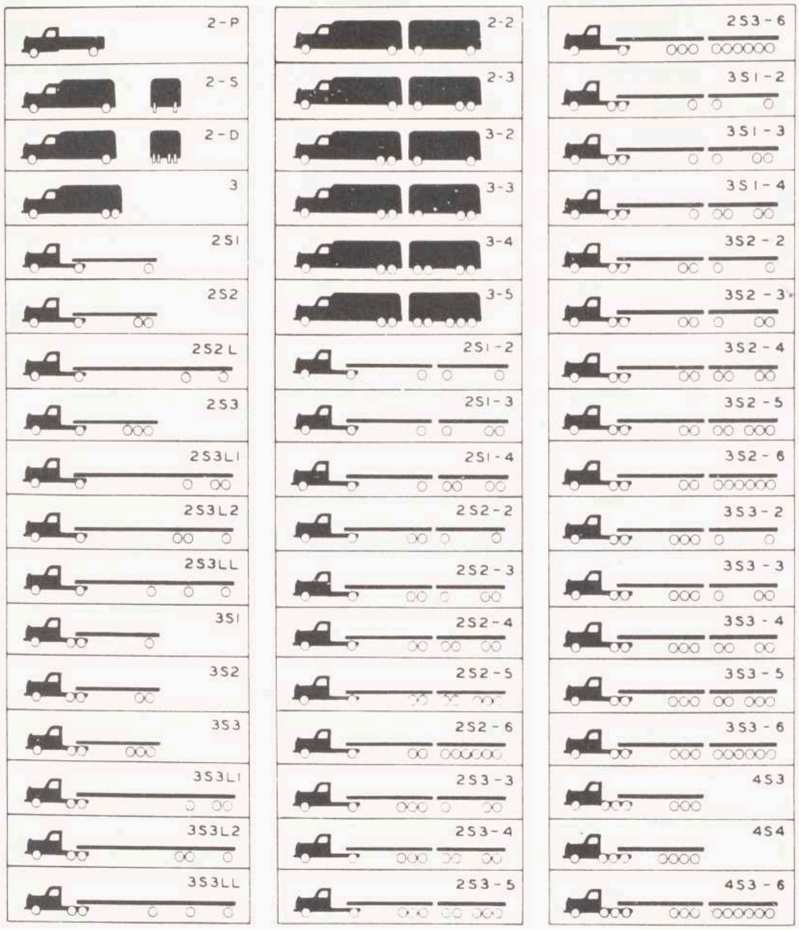
Traffic of interest may be characterized as consisting of <u>groups</u> of heavy vehicles, hereafter referred to as trucks, separated either by gaps in the traffic stream, in which no vehicles are present, or by lighter vehicles, hereafter referred to as cars. A group may be composed of one or more heavy vehicles, concentrated in spatial proximity to one another, and maintaining their relative positions while crossing over the bridge. It should be understood, however, that the groups hypothesized in this model may not be easily discernible to an observer on the highway. Rather, the groups serve as a convenient analytic tool; their existence can be identified after the processing of traffic headway data, as will be discussed in Chapter 3. They are of practical importance because they imply larger loads and load effects than non-grouped or random (Poisson) traffic streams.

The number of trucks in a group is taken to be the random number N (where N may take the following values: n= 1, 2, 3,...). A truck is understood to be a vehicle whose body type resembles one of the general classifications of truck body types as appears in Figure [2.1].

It is assumed that the distribution of N, the number of trucks in a group, is given by a geometric distribution (a more complete discussion concerning this choice of distribution appears in Chapter 3, Section 3.2), whose probability mass function (PMF) is defined as:

$$p_N(n) = P[N=n]$$

$$= \text{probability that a group is composed of } n \text{ vehicles}$$



Commercial Traffic Vehicle Types

Figure [2.1]

$$p_N(n) = (1-p)^{n-1} p \qquad n= 1,2,3,\ldots$$

which implies that a larger group is less likely than a smaller group.

The assumed PMF leads to the following:

$$F_N(n)= 1-(1-p)^n \qquad n= 1,2,3,\ldots$$

$$E[N]= m_N= \frac{1}{p}$$

$$Var[N]= \sigma_N^2= \frac{1-p}{p^2}$$

in which $F_N(n)$ denotes the cumulative mass function (CMF) of n, E[N] denotes the expected value (or mean) of N, and Var[N] denotes the variance (or square of standard deviation) of N.

2.2.2 Truck Characteristics

2.2.2.1 Gross Weight

The gross weight of an individual truck may be represented by the random variable X, and it is assumed that X is gamma distributed (for a full discussion of the appropriateness of the gamma distribution for truck gross weights the reader is referred to Chapter 3, Section 3.3). The probability density function of X may be expressed as:

$$f_X(x)= \frac{\lambda(\lambda x)^{k-1} e^{-\lambda x}}{\Gamma(k)} \qquad x \geq 0$$

where:

$$m_X = \frac{k}{\lambda}$$

$$X^2 = \frac{k}{\lambda^2}$$

The cumulative distribution function (CDF) of individual truck weights may be written as:

$$F_X(x) = \frac{\Gamma(k,\lambda x)}{\Gamma(k)}$$

where $\Gamma(k)$ is the gamma function, evaluated as:

$$\Gamma(k) = \int_0^\infty e^{-u} u^{k-1}\, du$$

and where $\Gamma(k,\lambda x)$ is the "incomplete gamma function", evaluated as

$$\Gamma(k,\lambda x) = \int_0^{\lambda x} e^{-u} u^{k-1}\, du$$

It may also be recalled that for integer k,

$$\Gamma(k) = (k-1)!$$

$\frac{\Gamma(k,\lambda x)}{\Gamma(k)}$ is also known as the "incomplete gamma ratio".

2.2.2.2 Truck Length

The variability of individual truck lengths plays only a minor role in this model, and therefore these lengths may be treated as deterministic quantities. The constant <u>a</u> will be taken to represent the length of a truck for use in the model. Individual truck lengths themselves play only a minor role in the model, especially in the long span range of bridges, where the span length is much greater than the length of an individual truck.

2.2.3 Group Characteristics

2.2.3.1 Group Length

Define L_T, the total length of a group of N trucks, as:

$$L_T = \sum_{i=0}^{N-1} H_{W_i} + a \qquad \text{(where } H_{W_0} = 0\text{)}$$

where H_W is defined as the headway between two consecutively arriving trucks within a group and is measured as the distance from the front axle of the preceding truck to the front axle of the following truck. A graphical representation of L_T is presented in Figure [2.2].

The headway between two groups is defined as H_G, and is measured as the distance between the front axle of the last car in a preceding group to the front axle of the first car of the following group.

For $N > 1$, define L, the sum of intragroup truck headways H_W as:

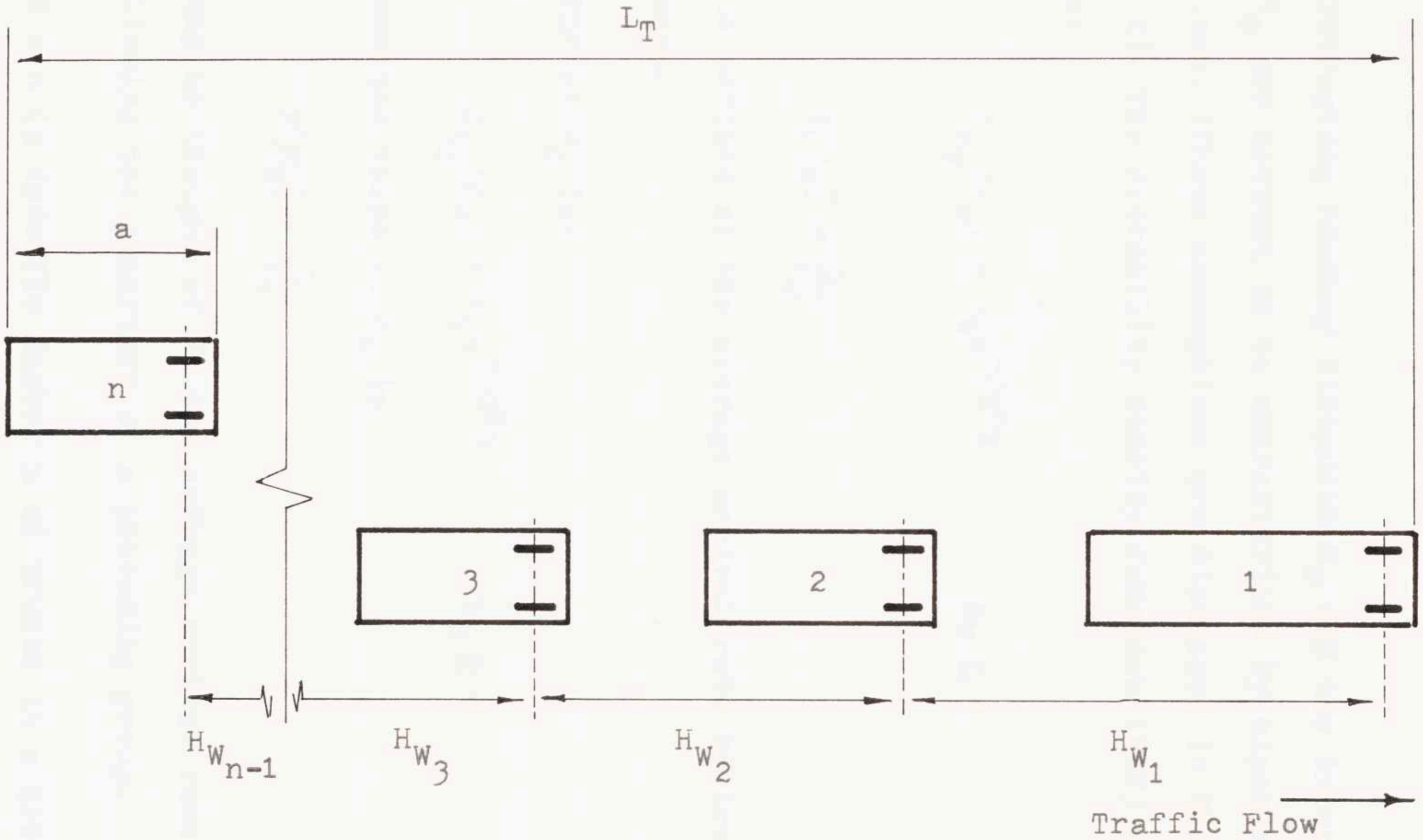


Illustration of Group Length of a Group of n Trucks
Figure [2.2]

$$L = \sum_{i=1}^{N-1} H_{W_i} \qquad (N > 1)$$

The intragroup headway distances H_W and the intergroup headways H_G are assumed to be characterized by exponential distributions. (These assumptions are discussed in Chapter 3, Section 3.2). The probability density function (PDF) of H_W is written as:

$$f_{H_W}(h_W) = \lambda_W e^{-\lambda_W h_W} \qquad h_W \geq 0$$

and

$$E[H_W] = \frac{1}{\lambda_W}$$

where λ_W is defined as the average arrival rate of trucks <u>within</u> groups.

The PDF of H_G is:

$$f_{H_G}(h_G) = \lambda_G e^{-\lambda_G h_G} \qquad h_G \geq 0$$

and the expected value of H_G is

$$E[H_G] = \frac{1}{\lambda_G}$$

where λ_G may be thought of as the average arrival rate of trucks following the departure of a preceding group.

For N = n (a specific number n of trucks in a group), the distribution of L, the sum of n independent, identically distributed exponential intragroup headways, is given by the gamma distribution[2-1]:

Define J, the number of intragroup headways in a group of N trucks:

$$J = N - 1$$

Then,

$$f_{L|J=j}(l) = \frac{\lambda_W(\lambda_W l)^{j-1} e^{-\lambda_W l}}{(j-1)!} \qquad l \geq 0, \quad j = 1,2,3,\ldots$$

or

$$f_{L|N=n}(l) = \frac{\lambda_W(\lambda_W l)^{n-2} e^{-\lambda_W l}}{(n-2)!} \qquad l \geq 0, \quad n = 2,3,4,\ldots$$

It should be noted that for a group of only one truck L is undefined since a group of one truck's length is composed only of its own individual length and no intragroup headway contribution.

The distribution of L for random N (provided that $N \geq 2$) may be expressed as:

$$f_{L|N\geq 2}(l) = \sum_{n=2}^{\infty} \left[f_{L|N=n}(l) \cdot p_{N_2}(n)\right] \qquad l \geq 0$$

where $p_{N_2}(n)$ is the geometric distribution of N renormalized for $N \geq 2$. This renormalized distribution of N is needed to provide a distribution for L that is truly an exhaustively defined distribution, since L is defined only for $N \geq 2$.

$$p_{N_2}(n) = p_{N|N\geq 2}(n)$$

$$= \frac{(1-p)^{n-1}p}{(1-p)} = (1-p)^{n-2}p \qquad n= 2,3,4,\ldots$$

Substituting in the above expression for the distribution of L:

$$f_{L|N\geq 2}(l) = \sum_{n=2}^{\infty}\left[\frac{\lambda_W(\lambda_W l)^{n-2}e^{-\lambda_W l}}{(n-2)!}\cdot(1-p)^{n-2}p\right]$$

$$= p\lambda_W e^{-\lambda_W l}\left[\sum_{i'=0}^{\infty}\frac{[\lambda_W l(1-p)]^{i'}}{(i')!}\right] \qquad l \geq 0$$

Introducing a term $e^{-\lambda_W l(1-p)}$ in the numerator of the bracketed summation, and dividing the term outside the brackets by the same quantity, the distribution may be rewritten as:

$$f_{L|N\geq 2}(l) = \frac{p\lambda_W e^{-\lambda_W l}}{e^{-\lambda_W l(1-p)}}\left[\sum_{i'=0}^{\infty}\frac{[\lambda_W l(1-p)]^{i'}e^{-\lambda_W l(1-p)}}{(i')!}\right]$$

Recalling the form of the Poisson distribution:

$$p_X(x) = \frac{\nu^x e^{-\nu}}{x!} \qquad x= 0,1,2,\ldots$$

it can be seen by inspection that the summation inside the brackets is the CDF of a Poisson distribution evaluated at i' equal to infinity, so that the bracketed term is equal to unity. Therefore, the distribution of L may be written simply as:

$$f_{L|N\geq 2}(l) = (p\lambda_W)e^{-(p\lambda_W)l} \qquad l \geq 0$$

which is recognized as an exponential distribution, with parameter $p\lambda_W$. The cumulative distribution function (CDF) is then:

$$F_{L|N\geq 2}(l) = 1 - e^{-(p\lambda_W)l} \qquad l \geq 0$$

Returning now to L_T, the total group length, it will be seen that the distribution of L_T is a mixed distribution, that is, a distribution composed of a discrete part and a continuous part.

For n=1, it has been stated above that L, the sum of intragroup headways, is undefined, and hence L_T, the total group length, is equal to the deterministic quantity a̲. The distribution of N gives the probability of n=1 as the finite quantity p, and hence there is a finite probability p that L_T=a̲.

For N≥2, the distribution of L_T is derived from the distribution of L, recognizing that L_T is simply a linear transformation of L, and the distribution of L is weighted by (1-p) to account for the proportion of groups with two or more trucks as compared to the whole family of truck groups.

Graphically, the mixed distribution of L_T appears in

Figure [2.3].

Recalling, that for arbitrary X,Y,a,b, where X and Y are random variables and a and b are constants:

If: $Y = a + bX$

then:

$$f_Y(y) = \left|\frac{1}{b}\right| f_X\left(\frac{y-a}{b}\right)$$

Therefore, for $L_T = L + a$,

$$f_{L_T|N\geq 2}(l_T) = f_{L|N\geq 2}(l_T - \underline{a})$$

$$= (p\lambda_W)e^{-(p\lambda_W)[l_T-\underline{a}]} \qquad l_T \geq a$$

The continuous portion of the distribution of L_T can then be expressed as:

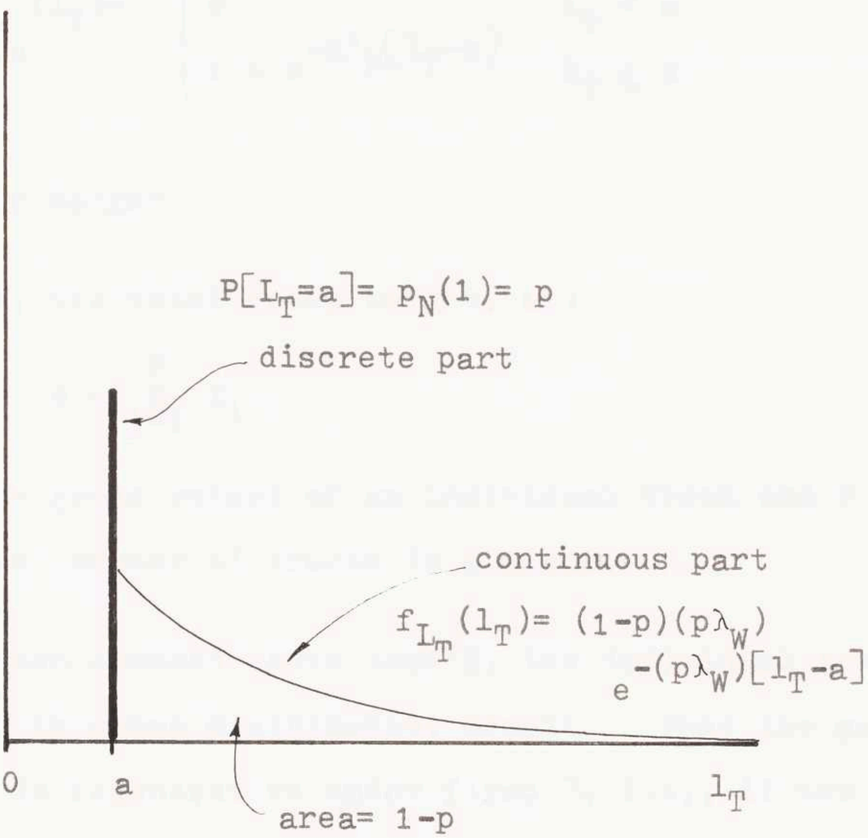
$$f_{L_T}(l_T) = (1-p)(p\lambda_W)e^{-(p\lambda_W)[l_T-a]}$$

where (1-p) represents the fraction of groups with two or more trucks, which is the fraction to which the continuous portion of the distribution of L_T applies.

In algebraic form the mixed distribution of L_T may be defined as follows:

$$F_{L_T}(l_T) = pF_{L_{T_d}}(l_T) + (1-p)F_{L_{T_c}}(l_T)$$

where F_{L_T} is a cumulative probability function of L_T, and



Graphical Representation of Mixed Distribution of L_T

Figure [2.3]

where $F_{L_{T_d}}$ is associated with the discrete values that L_T can assume and $F_{L_{T_c}}$ is associated with the continuous values that L_T can assume. These may be defined here as:

$$F_{L_{T_d}}(l_T) = \begin{cases} 0 & l_T < a \\ 1 & l_T \geq a \end{cases}$$

$$F_{L_{T_c}}(l_T) = \begin{cases} 0 & l_T < a \\ 1 - e^{-p\lambda_W[l_T - a]} & l_T \geq a \end{cases}$$

2.2.3.2 Group Weight

Define W, the total group weight, as:

$$W = \sum_{i=1}^{N} X_i$$

where X is the gross weight of an individual truck and N is the (random) number of trucks in a group.

It has been assumed above that X, the individual truck gross weight, is gamma distributed. Recall that the gamma distribution is regenerative under fixed λ, i.e., if two random variables are gamma distributed with identical parameter λ, and one is $G(k_1,\lambda)$, or gamma distributed with parameters k_1 and λ, and the other is $G(k_2,\lambda)$, then the distribution of the sum of the two gamma distributed variables is $G(k_1+k_2,\lambda)$, or gamma distributed with parameters (k_1,k_2) and λ; thus the distribution of W, total group weight, for a given N=n may be expressed as:

$$f_{W|N=n}(w) = \frac{\lambda(\lambda w)^{nk-1} e^{-\lambda w}}{\Gamma(nk)} \qquad w \geq 0$$

and

$$m_{W|N=n} = \frac{nk}{\lambda}$$

$$\sigma^2_{W|N=n} = \frac{nk}{\lambda^2}$$

Recognizing that N is a random variable, the distribution of W for random N may be written as:

$$f_{W_N}(w) = \sum_{n=1}^{\infty} [f_{W|N=n}(w) \cdot p_N(n)] \qquad w \geq 0$$

$$= \left(\frac{p}{1-p}\right) \frac{e^{-\lambda w}}{w} \sum_{n=1}^{\infty} \frac{(\lambda w)^{nk}(1-p)^n}{\Gamma(nk)}$$

and

$$m_{W_N} = m_N m_X$$

$$\mathrm{Var}[W_N] = m_N \sigma_X^2 + \sigma_N^2 m_X^2$$

2.3 Special Applications

With the basic definitions for the model given above, the model may be put to use answering certain questions concerning the design live loading of a given bridge of span length (clear distance between supports) s.

2.3.1 Simultaneous Occurrence of Groups on Bridge

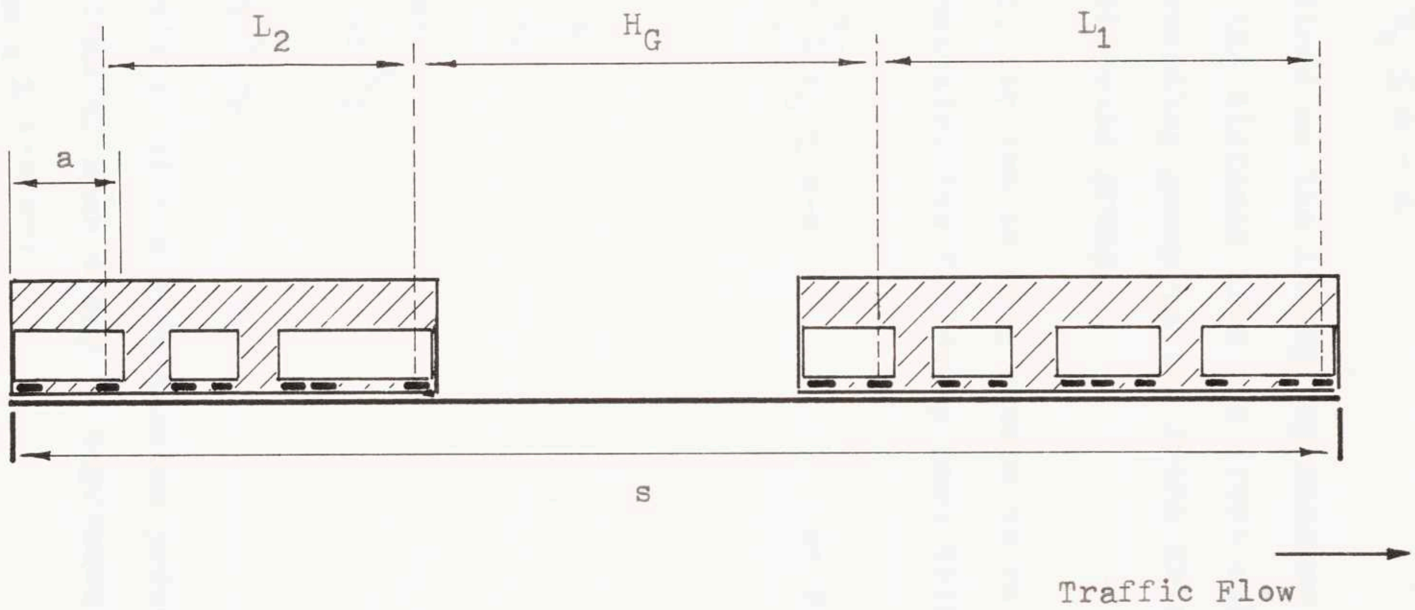
The number of groups simultaneously occurring on a bridge is defined here to be the set of full groups (i.e., pairs, triplets, etc.) which fits on a bridge of span s (i.e., whose total length is less than or equal to s). This section examines the following question: Given a bridge of span length s and assuming a certain probability level, what set of groups would fit on the bridge such that the fraction of those sets (pairs, triplets, etc.) is equal to or exceeds the preset probability level?

Define M to be the number of groups (members of a set) present simultaneously on a bridge.

For one group to be present fully on a bridge of span s, its group length must be less than or just equal to the span length. The probability of one group's length being less than or equal to span length s may be expressed as:

$$\begin{aligned} P[L_T \leq s] &= F_{L_T}(s) \\ &= 0 & s \leq a \\ &= p + (1-p)(1-e^{-p\lambda_W[s-a]}) & s \geq a \end{aligned}$$

For two entire groups to be present simultaneously on a bridge of span s, the sum of their individual headways plus the headway between groups must be less than s-a. This may be seen upon examination of Figure [2.4].



Bridge of Span s Loaded by the Simultaneous Presence of Two Groups

Figure [2.4]

Algebraically this can be expressed as:

$$\sum_{i=1}^{2} L_i + H_G \leq s - a$$

where H_G is defined as the intergroup headway or gap, which is measured as the distance from the front axle of the last truck in the preceding group to the front axle of the first truck in the following group.

In general, for two or more groups to be present on a bridge simultaneously, the following must hold:

$$\sum_{i=1}^{m} L + \sum_{i=1}^{m-1} H_G \leq s-a \qquad m= 2,3,4,\ldots$$

Define:

$$L_S= \sum_{i=1}^{m} L_i$$

$$L_G= \sum_{i=1}^{m-1} H_{G_i}$$

$$T= L_S + L_G$$

The probability of the simultaneous presence of M=m groups on a bridge of span s may be expressed as the probability that $T_{|M=m} \leq s-a$ or:

$$P[T_{|M=m} \leq s-a]= F_{T|M=m}(s-a)$$

What remains is to determine F_T.

Recalling that L, the sum of a random number ($N \geq 2$) of intragroup headways is exponentially distributed with parameter $p\lambda_W$, then the distribution of $L_{S|M=m}$, the sum of m independent, identically distributed exponential L's is given by the gamma distribution, with PDF:

$$f_{L_S|M=m}(l_S) = \frac{(p\lambda_W)[(p\lambda_W)l_S]^{m-1} e^{-p\lambda_W l_S}}{(m-1)!} \qquad l_S \geq 0$$
$$m=1,2,3,\ldots$$

The cumulative distribution function (CDF) of this gamma distribution for integer m may be expressed as:

$$F_{L_S|M=m}(l_S) = 1 - \sum_{i=0}^{m-1} \frac{e^{-p\lambda_W l_S}(p\lambda_W l_S)^i}{i!}$$

For example, for m=3,

$$F_{L_S|M=3}(l_S) = 1 - e^{-p\lambda_W l_S} - e^{-p\lambda_W l_S} p\lambda_W l_S - \frac{e^{-p\lambda_W l_S}(p\lambda_W l_S)^2}{2!}$$

The distribution of H_G, the intergroup headways, is assumed to be exponential (as explained in more detail in Chapter 3, Section 3.2) with probability density function (PDF):

$$f_{H_G}(h_G) = \lambda_G e^{-\lambda_G h_G} \qquad h_G \geq 0$$

where λ_G may be thought of as the average rate of arrival of groups following the departure of preceding groups, and where $\frac{1}{\lambda_G}$ is the average gap between groups.

For a specific number (m-1) of intergroup headways, the distribution of L_G, the sum of independent, identically distributed exponential intergroup headways, is given by the gamma distribution, with PDF:

$$f_{L_G|M=m}(l_G) = \frac{\lambda_G(\lambda_G l_G)^{m-2} e^{-\lambda_G l_G}}{(m-2)!} \qquad l_G \geq 0$$
$$m = 2,3,4,\ldots$$

and with CDF:

$$F_{L_G|M=m}(l_G) = 1 - \sum_{i=0}^{m-2} \frac{e^{-\lambda_G l_G}(\lambda_G l_G)^i}{i!}$$

Recalling that for two independent random variables X and Y:

If $Z = X + Y$

$$F_Z(z) = \int_{-\infty}^{\infty} F_X(x)\, f_Y(z-x)\, dx$$

then for $T_{|M=m} = L_{S|M=m} + L_{G|M=m}$:

$$F_{T\ M=m}(t) = \int_{-\infty}^{\infty} F_{L_S|M=m}(l_S)\, f_{L_G|M=m}(t-l_S)\, dl_S$$
$$m = 2,3,4,\ldots$$

Since $F_{L_S}(l_S)=0$ for $l_S<0$, the lower limit of the integral may be changed from $-\infty$ to 0; also, since $f_{LG}(l_G)=0$ for

$l_G<0$, the upper limit may be changed to t. Rewriting and substituting:

$$F_{T|M=m}(t)= \int_0^t (1- \sum_{i=0}^{m-1} \frac{e^{-p\lambda_W l_S}(p\lambda_W l_S)^i}{i!}$$

$$(\frac{\lambda_G(\lambda_G[t-l_S])^{m-2} e^{-\lambda_G[t-l_S]}}{(m-2)!}) dl_S$$

$$m= 2,3,4,\ldots$$

With the above, the probability of M (m= 1, 2, 3,...) groups simultaneously occupying the bridge can be evaluated for each value of m. This probability can then be matched to a preset probability level and a conclusion reached as to how many simultaneous group occurrences need be designed for. Typically, one would expect that the longer the span under consideration, the more groups it need be designed to carry simultaneously. For a single set of traffic parameters, bridges of different span lengths may then be classified as 1 or 2 or 3 . . . group bridges, the number of groups in their classification signifying the number of design groups that must be considered. This type of classification would supersede the rather arbitrary present day classification of bridges into long and short span categories, and would point the way to a more orderly consideration of bridge design requirements based on the traffic the bridges are to serve and based upon the proposed bridge spans.

2.3.2 Fully Loaded Bridges

A bridge will be termed "fully loaded" if it is occupied by one group of trucks whose group length, L_T, is equal to or greater than the bridge span length s. The probability that any one group will have length equal to or greater than s may be expressed as:

$$
\begin{aligned}
P[L_T \geq s] &= 1 - F_{L_T}(s) \\
&= 1 & s < a \\
&= 1 - [p + (1-p)(1 - e^{-p\lambda_W[s-a]})] \\
&= (1-p)e^{-p\lambda_W[s-a]} & s \geq a
\end{aligned}
$$

It is quite clear that for bridges whose span length is greater than or equal to the length of one truck, as the span length s increases, there is a decrease in the probability that the bridge will be fully loaded by heavy vehicles. All bridge specifications have recognized this fact, and have provided live load reductions for bridges in the so-called long-span category. The specifications have never provided evidence or any sound justification for these reductions and have appealed to the designer's intuitive feeling that reductions can be made. The above probability function points clearly to the origins of this reduction, and on at least a relative basis can quantify the amount of reduction for given traffic parameters and a range of span lengths.

2.3.3 Maximum Moment: Simply Supported Bridge

Another application of the model may be to derive a distribution for the maximum moment on a long, simply-supported bridge due to the loading of the bridge by one group of heavy vehicles. It is assumed that the group acts as a uniform load over its length L_T and that to produce the maximum moment effect the uniform load of the group is centered at the midpoint of the span length. (Any such group crossing the bridge would have to be so positioned at one point in its crossing.)

It should be emphasized that to assume that the total weight of the group is uniform over its length and that this uniform load acting on the bridge produces a moment equivalent to the moment produced by the single units of the group acting through their axle loads is also to assume that the group length is small in comparison to the span length of the bridge under consideration. It can be seen from the mixed distribution of group length presented above that the shorter group lengths are more common. It is only in the range of shorter group lengths and long bridge spans that the following analysis provides near accurate results. As the group length (and consequently N, the number of trucks in a group) increases or as the span length decreases, the results become progressively more inaccurate, as the assumptions made above fail to hold true.

The maximum moment produced by a uniform load of length L_T and total weight W centered over the midpoint of a simply supported bridge of span s is expressed as:

$$M_{MAX} = W\left(\frac{s}{4} - \frac{L_T}{8}\right) \qquad L_T \leq s$$

Defining a new variable, I, (which is basically a moment influence variable) as:

$$I = \begin{cases} \dfrac{s}{4} - \dfrac{L_T}{8} & L_T \leq s \\ \dfrac{s}{8} & L_T > s \end{cases}$$

the expression for M_{MAX} may be rewritten as:

$$M_{MAX} = W \cdot I$$

where W and I are random variables and M_{MAX} is the random variable product of W and I.

It should be noted here that in the case of $L_T > s$, or length of group greater than span length of the bridge, only a portion of the total group weight W is effective in producing the maximum moment, i.e., only that portion of W which is on the bridge. However, as the analysis presented here assumes that the moment producing effect of the group can be approximated by a uniform load over its length, this approximation will only be valid when the group length is shorter than the bridge span. Certainly, this is not the case when $L_T > s$, and therefore it should be clearly understood that the analysis presented here is only valid when $P[L_T > s]$, or the probability that the total group length is greater than the bridge span, is very small. To account accurately for the $L_T > s$ case would also require modifying the assumption below of conditional independence of the load on the bridge and I.

Recalling that the distribution of the product of two independent random variables, as for example, Z= XY, is defined for X and Y continuous as:

$$f_Z(z) = \int_{-\infty}^{\infty} \left|\frac{1}{y}\right| f_X\left(\frac{z}{y}\right) f_Y(y)\, dy$$

then the distribution of $M_{MAX} = W \cdot I$, provided that W and I are independent, is given by:

$$f_{M_{MAX}}(m_{max}) = \int_{-\infty}^{\infty} \left|\frac{1}{i}\right| f_W\left(\frac{m_{max}}{i}\right) f_I(i)\, di$$

Since this integral is valid only for W and I independent, it must be recognized in formulating the derivation of this distribution that W and L_T (and hence I) are both related to n, the number of trucks in a group, and are independent only for a given value of N. Proceeding as in other cases of random variables independent conditional on n, the distribution of M_{MAX} for a specific n may be expressed as:

$$f_{M_{MAX}|N=n}(m_{max}) = \int_{-\infty}^{\infty} \left|\frac{1}{i}\right| f_{W|N=n}\left(\frac{m_{max}}{i}\right) f_{I|N=n}(i)\, di$$

However, for n=1 this distribution is not strictly valid, as it is the distribution of the product of two random variables, and when n=1, I is no longer a random variable but rather a deterministic variable with value $(\frac{s}{4} - \frac{a}{8})$. (This will be seen to be true by recalling that for n=1, l_T=a, the length of one truck.)

In the special case of n=1, $M_{MAX} = W(\frac{s}{4} - \frac{a}{8})$ and the distribution of M_{MAX} is a linearly transformed distribution of W, where

$$f_{M_{MAX}|N=1}(m_{max}) = \frac{1}{(\frac{s}{4} - \frac{a}{8})} f_{W|N=1}\left(\frac{m_{max}}{\frac{s}{4} - \frac{a}{8}}\right)$$

$$= \frac{1}{(\frac{s}{4} - \frac{a}{8})} \left(\frac{\lambda[\lambda(\frac{m_{max}}{\frac{s}{4} - \frac{a}{8}})]^{k-1} e^{-\lambda(\frac{m_{max}}{\frac{s}{4} - \frac{a}{8}})}}{\Gamma(k)} \right)$$

$$m_{max} \geq 0$$

Returning to the general distribution of M_{MAX} for $n \neq 1$, the distribution of I for N=n may be expressed as a linearly transformed distribution of L_T, (assuming $L_T \leq s$):

$$f_{I|N=n}(i) = 8f_{L_T|N=n}\left(\frac{i - \frac{s}{4}}{-\frac{1}{8}}\right) = 8f_{L_T|N=n}(2s - 8i)$$

The distribution of L_T for a given value of n may be expressed as a linearly transformed distribution of L, the sum of intragroup headways, since $L_T = L + a$. Thus:

$$f_{L_T|N=n}(l_T) = f_{L|N=n}(l_T-a) = \frac{\lambda_W(\lambda_W[l_T-a])^{n-2} e^{-\lambda_W[l_T-a]}}{(n-2)!}$$

$$n=2,3,4,\ldots$$

The distribution of I for a given n (n=2,3,4,...) may then be written as:

$$f_{I|N=n}(i) = 8f_{L|N=n}(2s-8i-a) =$$

$$\frac{\lambda_W[\lambda_W(2s-8i-a)]^{n-2} e^{-\lambda_W(2s-8i-a)}}{(n-2)!}$$

The distribution of W given N=n may be recalled to be:

$$f_{W|N=n}(w) = \frac{\lambda(\lambda w)^{nk-1} e^{-\lambda w}}{\Gamma(nk)} \qquad w \geq 0$$

Recalling that $f_{L_T|N=n}(l_T)$ is non-zero only for $l_T \geq a$, for $f_{I|N=n}(i)$ to be non-zero:

$$(2s - 8i) \geq a$$

or

$$i \leq \frac{s}{4} - \frac{a}{8}$$

It may also be recalled that for this analysis to be valid it is assumed that $P[L_T > s]$ is very small, and values of I greater than $\frac{s}{8}$ are the only ones to be considered.

Therefore, in the expression for $f_{M_{MAX}|N=n}(m_{max})$ (which must now be restricted to values of $N \geq 2$), the lower limit of the integral may be changed from $-\infty$ to $\frac{s}{8}$, while the upper limit may be changed from ∞ to $(\frac{s}{4} - \frac{a}{8})$.

Substituting and replacing the integral limits leads to the following expression:

$$f_{M_{MAX}|N=n}(m_{max}) = \int_{\frac{s}{8}}^{\frac{s}{4}-\frac{a}{8}} [(\frac{1}{i})(\frac{\lambda[\lambda(\frac{m_{max}}{i})]^{nk-1} e^{-\lambda(\frac{m_{max}}{i})}}{\Gamma(nk)})$$

$$(\frac{8\lambda_W[\lambda_W(2s-8i-a)]^{n-2} e^{-\lambda_W(2s-8i-a)}}{(n-2)!})]\, di$$

$$n = 2,3,4,\ldots$$

The expression for the distribution of M_{MAX} for random N is:

$$f_{M_{MAX_N}}(m_{max}) = \sum_{n=1}^{\infty} f_{M_{MAX}|N=n}(m_{max}) \cdot p_N(n) \qquad m_{max} \geq 0$$

$$= f_{M_{MAX}|N=1}(m_{max}) \cdot p_N(1)$$

$$+ \sum_{n=2}^{\infty} f_{M_{MAX}|N=n}(m_{max}) \cdot p_N(n)$$

$$f_{M_{MAX_N}}(m_{max}) = \frac{1}{\frac{s}{4}-\frac{a}{8}} \left(\frac{\lambda[\lambda(\frac{m_{max}}{\frac{s}{4}-\frac{a}{8}})]^{k-1} e^{-\lambda(\frac{m_{max}}{\frac{s}{4}-\frac{a}{8}})}}{\Gamma(k)} \right) \cdot p$$

$$+\sum_{n=2}^{\infty}\left(\int_{\frac{s}{8}}^{\frac{s}{4}-\frac{a}{8}}\left[\left(\frac{1}{i}\right)\left(\frac{\lambda\left[\lambda\left(\frac{m_{max}}{i}\right)\right]^{nk-1}e^{-\lambda\left(\frac{m_{max}}{i}\right)}}{\Gamma(nk)}\right)\left(\frac{8\lambda_w\left[\lambda_w(2s-8i-a)\right]^{n-2}e^{-\lambda_w(2s-8i-a)}}{(n-2)!}\right)\right]di\cdot p(1-p)^{n-1}\right)$$

$$f_{M_{MAX_N}}(m_{max}) = \frac{p}{\frac{s}{4}-\frac{a}{8}}\left(\frac{\lambda\left[\lambda\left(\frac{m_{max}}{\frac{s}{4}-\frac{a}{8}}\right)\right]^{k-1}e^{-\lambda\left(\frac{m_{max}}{\frac{s}{4}-\frac{a}{8}}\right)}}{\Gamma(k)}\right)$$

$$+\left(\frac{p}{p-1}\right)\frac{8e^{-\lambda_w(2s-a)}}{m_{max}\,\lambda_w}\sum_{n=2}^{\infty}\left(\frac{[\lambda_w(1-p)]^{n}[\lambda m_{max}]^{nk}}{(n-2)!\,\Gamma(nk)}\int_{\frac{s}{8}}^{\frac{s}{4}-\frac{a}{8}}\left[\left(\frac{1}{i}\right)^{nk}e^{-\lambda\left(\frac{m_{max}}{i}\right)+8\lambda_w i}(2s-8i-a)^{n-2}\right]di\right)$$

The probability that M_{MAX} takes on a value of m_{max} $\pm\Delta m_{max}$ may be expressed as:

$$P[m_{max}-\Delta m_{max} \le M_{MAX} \le m_{max}+\Delta m_{max}] = \int_{m_{max}-\Delta m_{max}}^{m_{max}+\Delta m_{max}} f_{M_{MAX_N}}(m_{max})\,dm_{max}$$

For example, the probability that the maximum positive moment on a simply-supported bridge due to the passage of an arbitrary group takes on a value of 5000 ft-kips ± 0.5 ft-kips may be evaluated as:

$$\int_{4999.5}^{5000.5} f_{M_{MAX_N}}(m_{max})\,dm_{max}$$

As an approximation:

$$P[4999.5^{'k} \le M_{MAX} \le 5000.5^{'k}] \approx f_{M_{MAX_N}}(5000^{'k})$$

The term corresponding to n=1 in the distribution of M_{MAX} may be interpreted as the probability of the attainment of a certain range of values of M_{MAX} (where $m_{max}-\Delta m_{max} \leq M_{MAX} \leq m_{max}+\Delta m_{max}$) due to the presence of only one heavy vehicle (truck) on the bridge, while each of the succeeding terms gives the additional probability of attaining that range of values of M_{MAX} for each additional truck present on the bridge. Thus, for existing bridges of span length s for which the critical loading is due to the presence of only one group on the bridge, examination of each of the terms of this distribution could point to the need for regulating the number of trucks allowed on the bridge at one time to maintain a tolerable level of maximum positive moment.

For any specific value m_{max}, the uniform load covering the entire bridge span under investigation that would produce the same value of m_{max} will be termed u_{eq} (equivalent uniform load). Thus,

$$u_{eq} = \frac{8m_{max}}{s^2}$$

u_{eq} will be useful for comparison purposes between different loadings and between the loads specified in different specifications, such as the AASHO specifications for highway bridge loadings.

2.3.4 Maximum Lifetime Moment: Simply Supported Bridge

The results of the previous section are for the maximum moment caused by one group during the passage of an arbitrary or randomly selected group from the many which pass over the bridge during its useful lifetime, r. The distribution of the peak lifetime moment is:

$$1 - F_{M_p}(m_p) = P[M_p \geq m_p] =$$

$$\sum_{i=1}^{\infty} P[\text{Maximum moment of } i \text{ groups} \geq m_p] \cdot P[i \text{ groups occur in lifetime of bridge}]$$

For independent group effects:

$$1 - F_{M_p}(m_p) = \sum_{i=1}^{\infty} [1 - (P[\text{Moment of any one group} < m_p])^i] \cdot P[i \text{ groups occur}]$$

$$= \sum_{i=1}^{\infty} [1 - (1 - P[\text{Moment of any one group} > m_p])^i] \cdot P[i \text{ groups occur}]$$

$$\sum_{i=1}^{\infty} (1 - 1 + iP[\text{Moment of any one group} > m_p] - \frac{i(i-1)}{2} P[\ ]^2 + \ldots) \cdot P[i \text{ groups occur}])$$

For small risks of interest:

$$1 - F_{M_p}(m_p) \approx \sum_{i=1}^{\infty} (iP[\text{Moment of any one group} > m_p]) \cdot P[i \text{ groups occur}]$$

$$1 - F_{M_p}(m_p) \approx P[\text{Moment of any one group} > m_p] \cdot \sum_{i=1}^{\infty} iP[i \text{ groups occur}]$$

$$\approx (1 - F_{M_{MAX}}(m_p))\, E[\text{number of groups in life of bridge}]$$

The expected number of groups is the expected number of trucks divided by the expected number of trucks per group ($\frac{1}{p}=m_N$), i.e., it is p times the expected number of trucks in the bridge lifetime.

Therefore, to find the 0.90 fractile of M_p one must find the value which is the (1 - 0.10/E[number of groups in life of bridge]) fractile of M_{MAX}. For example, if the expected number of trucks is 10^5 and $p= 0.7$, then the 0.9 (or 1-0.1) fractile of M_p corresponds to the $1 - 0.10/0.7\text{x}10^5$ or $1 - 0.14 \times 10^{-5}$ fractile of M_{MAX}. The difficulty in developing a highly reliable distribution of M_p is readily apparent, as the distribution of M_{MAX} depends both on limited statistical data and on a hypothetical model of truck group behavior.

2.3.5 Further Applications

In the future, areas of development for the model presented herein include the derivation of distributions for other load effects, such as shear, the generalization of the moment distribution for any influence surface to handle individual members rather than gross bridge moment, and the handling of cases of loading by two or more groups present simultaneously on the bridge.

The model presented is valid for a range of spans, including short. However, the moment analysis outlined here for one group is only valid for longer spans. The same model should be used with an extension of the analysis presented to consider the case of shorter spans, excluding those spans of single truck length (or less) where the model is too gross.

Additionally, consideration has been given here only to one direction of traffic flow. While a good portion of highway bridges have separate superstructures for each direction of travel, the majority of bridges carry two-way traffic, and to be general enough the model should be extended to consider such cases.

Chapter 3: Estimation of Model Parameters

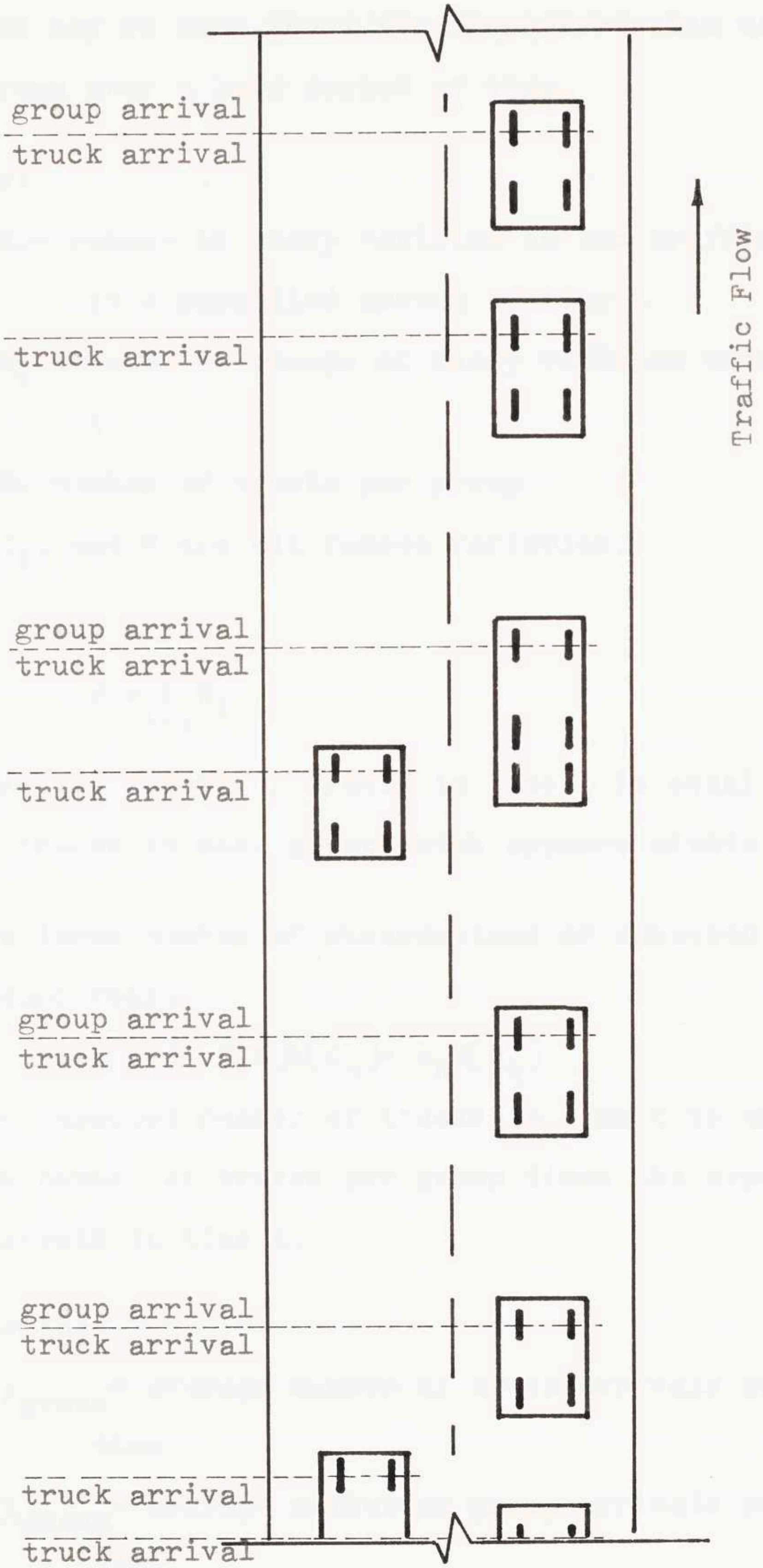
3.1 Introduction

Presented in this chapter are explanations for the various assumptions made in Chapter 2, where the details of a live load model for highway bridges were presented. These explanations involve a review of currently available data in the areas pertinant to the model presented, the rationale behind the assumptions made based on the data, and the techniques for estimating model parameter values from the raw data.

3.2 Group Behavior of Trucks

In the model advanced in this thesis, the concept of groups, or bunches of trucks acting within the traffic stream has been introduced as the live loading unit to be considered for a range of bridge span lengths, in place of the conventional approach of considering the load effect of individual truck units or axle groupings of individual trucks. While this hypothesis of group behavior is purely a conceptual one, and not easily verifiable by the casual observation of bridge traffic, some evidence supporting this type of behavior may be advanced.

Imagining a traffic stream with groups of trucks bunched together within the stream, as depicted in Figure [3.1], some



Idealized Traffic Stream With Bunching of Trucks

Figure [3.1]

observations may be made about the characteristics of the traffic stream over a long period of time.

Define:

N_t= number of heavy vehicles in the traffic stream in a specified amount of time t

G_t= number of groups of heavy vehicles within time t

N= number of trucks per group

where N_t, G_t, and N are all random variables.

Then:

$$N_t = \sum_{i=1}^{G_t} N_i$$

or in words: the number of trucks in time t is equal to the sum of the trucks in each group which appears within time t.

Over a large number of observations of duration t it would be found that:

$$E[N_t] = E[N]E[G_t] = m_N E[G_t]$$

or that the expected number of trucks in time t is equal to the average number of trucks per group times the expected number of groups in time t.

Now define:

λ_{gross}= average number of truck arrivals per unit time

λ_{group}= average number of group arrivals per unit time

λ_W= average number of truck arrivals within groups per unit time

Over a large number of observations the following relation would hold:

$$\lambda_{gross} = m_N \lambda_{group}$$

or that the average number of truck arrivals per unit time is equal to the average number of trucks per group times the average number of group arrivals per unit time.

Furthermore,

$$\lambda_W = b\lambda_{gross}$$

or the average number of truck arrivals within groups per unit time is some multiple b of the average number of truck arrivals per unit time. By definition, b is the ratio of within group arrival rates to the arrival rate of trucks in the traffic stream as a whole. For grouping to occur, the value of this ratio would typically be greater than unity.

Turning attention now to the headways (times or distances between arrivals) of trucks and groups, and defining:

H_{group}= headway between groups measured as the distance between front axle of lead truck in preceding group to front axle of lead truck in following group

H_W= headway between trucks travelling in groups measured as the distance between front axles of preceding and following trucks within groups

H_G= intergroup gap or headway, measured as the distance between the front axle of the last truck in a group to the front axle of the first truck in the following group

Figure [3.2] is a graphical representation of these defined headways.

The following average values would follow from the above definitions:

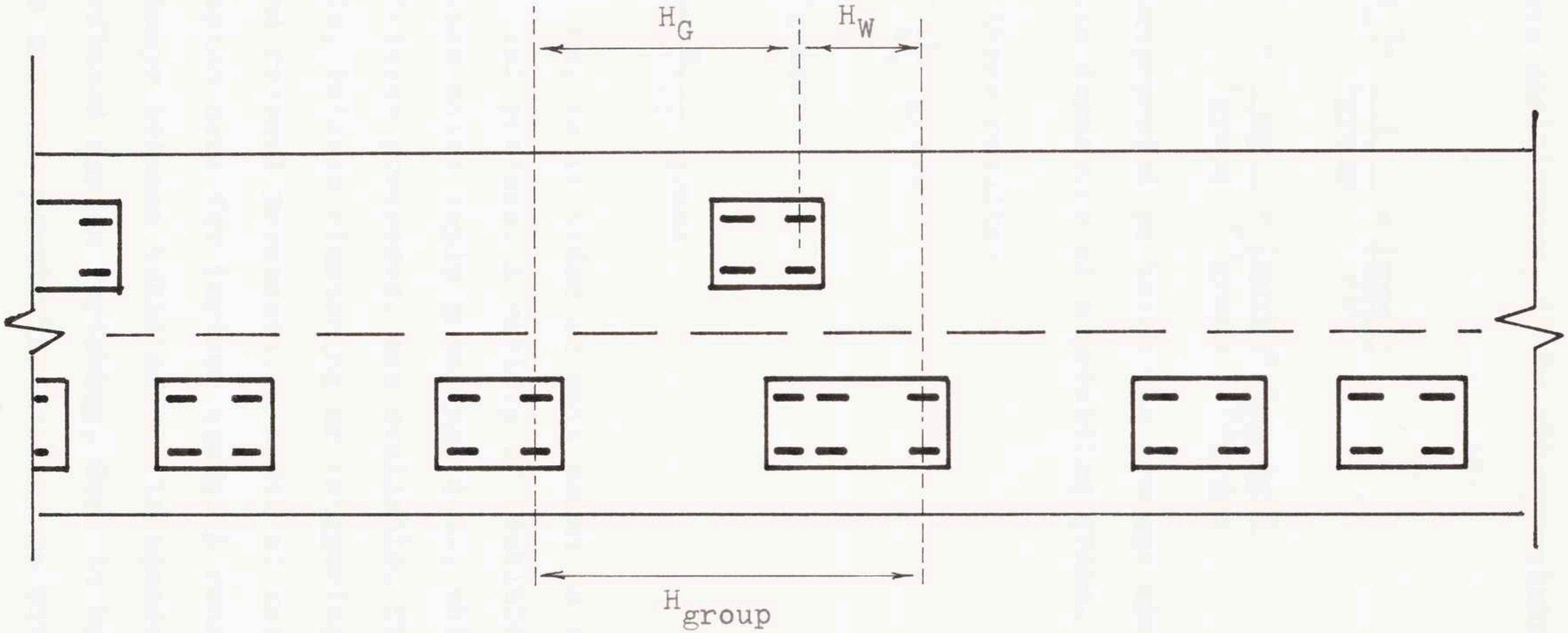
$E[H_{group}] = \frac{1}{\lambda_{group}}$ = average time between arrival of groups

$E[H_W] = \frac{1}{\lambda_W}$ = average time between arrivals of trucks within groups

$E[H_G] = \frac{1}{\lambda_{group}} - \frac{(m_N-1)}{\lambda_W}$ = average time between arrival of lead truck of group and departure of last truck of preceding group

where $\frac{m_N-1}{\lambda_W}$ is equal to the average of the sum of the intergroup headways of a group.

For the headways above to represent average distances, the average time between arrivals may be multiplied by the average speed of travel.



Definition of Truck Headways Within Traffic Stream

Figure [3.2]

From the above definitions, it is also possible to define $\frac{1}{\lambda_G}$ as:

$$\frac{1}{\lambda_G} = E[H_G] = \frac{1}{\lambda_{group}} - \frac{(m_N-1)}{\lambda_W}$$

$$= \frac{m_N}{\lambda_{gross}} - \frac{(m_N-1)}{b\lambda_{gross}} = \frac{bm_N-m_N+1}{b\lambda_{gross}}$$

where λ_G may be interpreted as being the average arrival rate of trucks after the departure of a preceding group.

Summarizing these results:

$$\lambda_{group} = \frac{1}{m_N} \lambda_{gross}$$

$$\lambda_W = b\ \lambda_{gross}$$

$$\lambda_G = \frac{b}{m_N(b-1)+1} \lambda_{gross}$$

A digression may be in order at this point to characterize the truck arrival process. A variety of feasible models of arrival processes which imply grouping, i.e., which are not independent Poisson processes, are available. These include, for example, Poisson clustering or triggering models, Markov models, and renewal processes. A model of the renewal type has been adopted here for further study. A renewal model implies that headways between vehicles are independent, identically distributed random variables. Here it has in effect been assumed that any particular headway is with probability p an intergroup headway (H_G) and with probability (1 -p) an intragroup headway (H_W). Intergroup headways are assumed to be exponentially distributed with parameter λ_G; i.e., its cumulative distribution function (CDF) may be written as:

$$F_{H_G}(h_G) = 1 - e^{-\lambda_G h_G} \qquad h_G \geq 0$$

Intragroup headways (H_W) are also assumed to be exponentially distributed but with parameter λ_W and the CDF of H_W may be expressed as:

$$F_{H_W}(h_W) = 1 - e^{-\lambda_W h_W} \qquad h_W \geq 0$$

Presumably, for grouping to occur the mean intergroup headway must be greater than the mean intragroup headway or $m_{H_G} > m_{H_W}$, where:

$$m_{H_G} = \frac{1}{\lambda_G}$$

$$m_{H_W} = \frac{1}{\lambda_W}$$

thus: $\lambda_W > \lambda_G$ or (from the expressions above for λ_W and λ_G):

$$b > \frac{b}{m_N(b-1)+1}$$

(which can be seen by algebraic manipulation to yield the result that for grouping to occur, b> 1).

The implication of the above assumptions is that the distribution of headways in general is a mixture of two exponential distributions:

$$F_H(h) = (1-p)\, F_{H_W}(h) + (p)\, F_{H_G}(h)$$

For any traffic stream, headways between trucks are assumed to be independent random variables, with a constant portion p of headways of the intergroup type, and the complementary portion of the intragroup type. Thus, the headways represent a sequence of Bernoulli trials. Therefore, the probability that the first (n-1) trials yield an intragroup headway and the n^{th} trial yields an intergroup headway may be expressed as:

$$(1-p)^{n-1}p \qquad\qquad n = 1,2,3,\ldots$$

n-1 intragroup headways would result from the arrival of a group of n trucks, and thus the above probability, which may be recalled to be of the form of the geometric distribution, may be viewed as the probability that a group consists of exactly n member trucks. This, then, explains the assumption of the geometric distribution for N, the group size, in the model presented in Chapter 2, which is a natural consequence of the assumptions made regarding headways.

It is useful to look at the limiting case. If there were no grouping, there would be no headways of the intragroup type, or (1-p) = 0. This would mean that p, the proportion of intergroup headways, would be equal to one, or that all headways would be of the intergroup type. Since N, the number of trucks in a group, is seen to be geometrically distributed, E[N], the expected (average) value of N is expressed as:

$$E[N] = m_N = \frac{1}{p}$$

and if p = 1, m_N, or the average number of trucks in a group, is equal to one. In fact, the distribution on N reduces to a single probability mass of one on the value n = 1, i.e., all "groups" are of size one. Also, in the case of no grouping, the distribution of headways may be expressed as:

$$F_H(h) = (1)\, F_{H_G}(h) + (1-1)\, F_{H_W}(h)$$

$$= 1 - e^{-\lambda_G h} \qquad h \geq 0$$

$$= 1 - e^{-\frac{1}{m_N}\lambda_{gross} h}$$

$$= 1 - e^{-\lambda_{gross} h}$$

This may be recognized as the exponential distribution of headways which also results from an independent Poisson arrival process, with truck arrival rate equal to λ_{gross}. Thus, the headway model assumed here may be viewed as being a "collapsible" model, one that in general considers the grouping of trucks but which in the extreme case collapses to a simple Poisson arrival model with no grouping.

Headway data is commonly plotted as complementary CDF $(1-F_H(h))$, or percentage of headways exceeding a given value of headway versus headway, on a semilogarithmic scale. Headway data plotted on semilog paper would appear as a straight line for headways resulting from independent Poisson arrivals, as illustrated in Figure [3.3]. The slope of this straight line is proportional to λ_{gross}, or the average number of arrival events per unit time. Given the number of trucks counted in a specified length of time, the average number of truck arrivals per unit time may easily be calculated, and a theoretical line plotted for the complementary cumulative distribution function of truck headways. For any set of observations, comparison of plotted data to such a line would give an indication of the validity of the Poisson assumption.

For the assumed group behavior, the expression for the complementary CDF of headways may be written as:

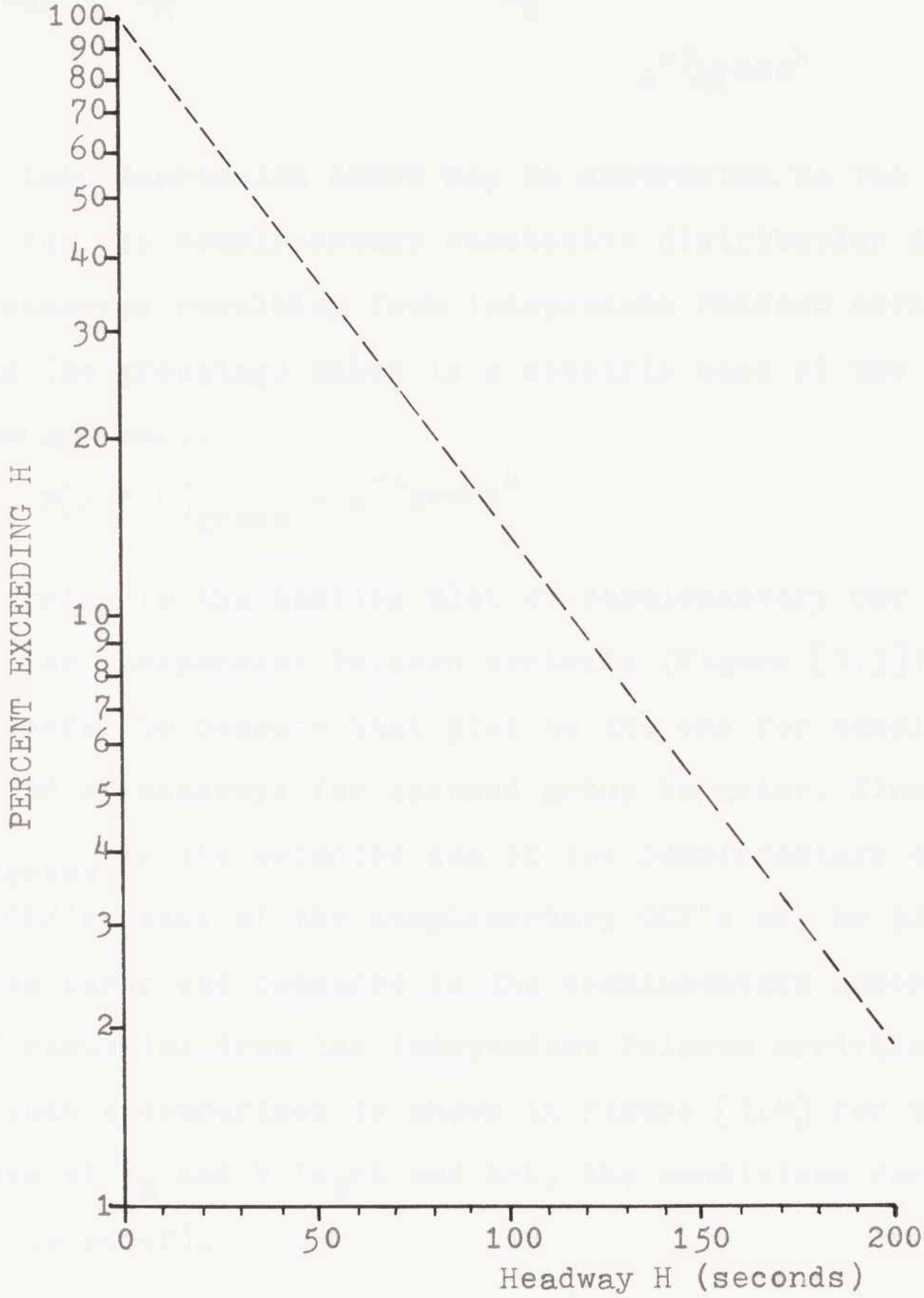
$$P[H > h]_{group} = P[H>h \text{ within a group}] \cdot P[\text{headway is within a group}] + P[H>h \text{ group to group}] \cdot P[\text{headway is group to group gap}]$$

Recalling that $p = P[\text{headway is within a group}]$, that $m_N = \frac{1}{p}$ (where m_N is equal to the mean number of trucks in a group), and that both the intragroup and intergroup headways are assumed to be exponentially distributed, the former with parameter λ_W, and the latter with parameter λ_G, $P[H > h]_{group}$ may be expressed as:

$$P[H > h]_{group} = \frac{(m_N - 1)}{m_N} e^{-\lambda_W h} + \frac{1}{m_N} e^{-\lambda_G h}$$

or:

$$P[H>h]_{group} = \left(\frac{m_N - 1}{m_N}\right) e^{-b\lambda_{gross} h} + \left(\frac{1}{m_N}\right) e^{-\left(\frac{b}{m_N(b-1)+1}\right)\lambda_{gross} h}$$



Complementary CDF for Headways Resulting From Independent Poisson Arrivals

Figure [3.3]

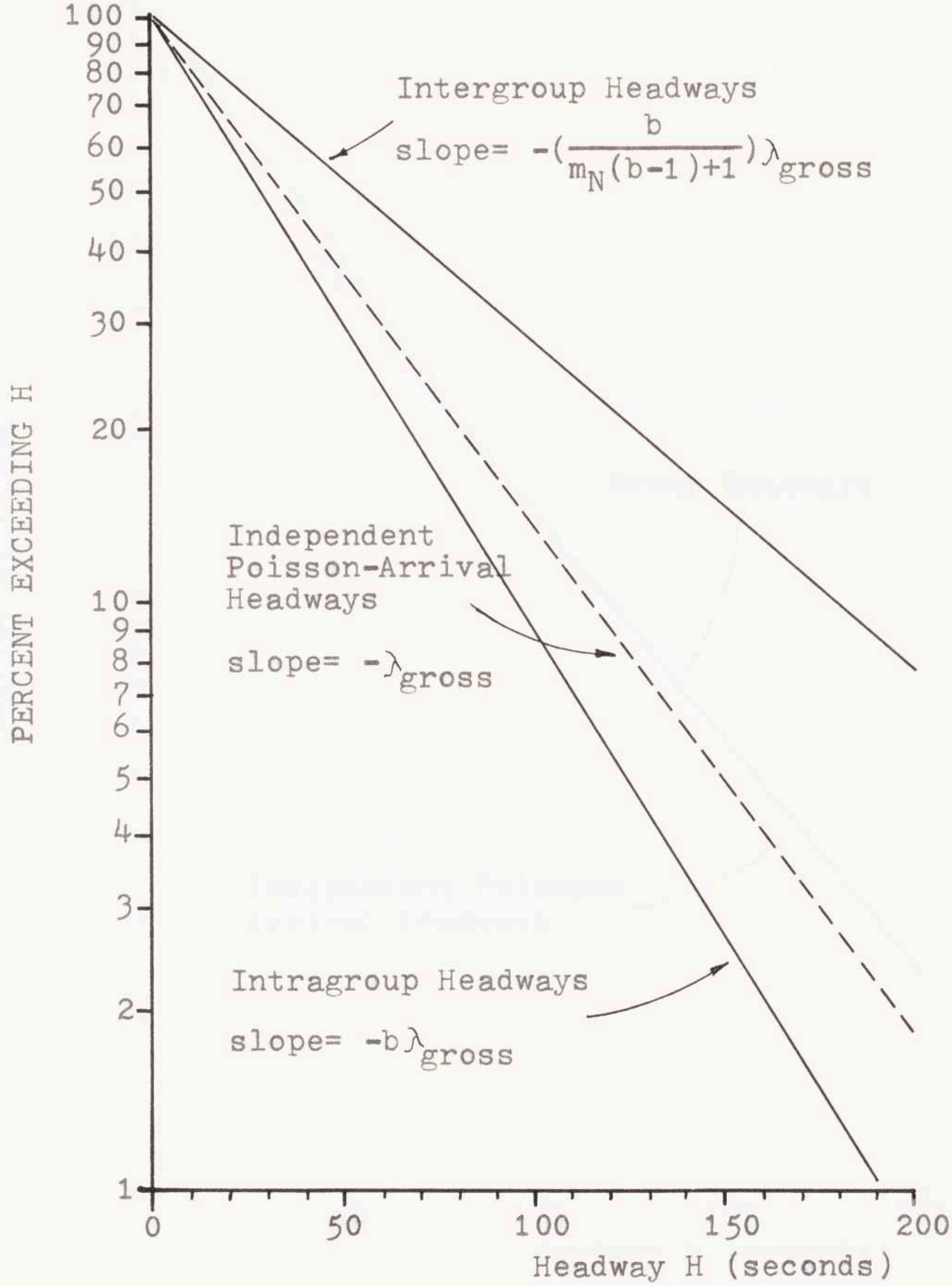
$$P[H>h]_{group}=\left[\left(\frac{m_N-1}{m_N}\right)e^{(1-b)\lambda_{gross}h}+\left(\frac{1}{m_N}\right)e^{\left(1-\frac{b}{m_N(b-1)+1}\right)\lambda_{gross}h}\right]e^{-\lambda_{gross}h}$$

The last expression above may be contrasted to the expression for the complementary cumulative distribution function of headways resulting from independent Poisson arrivals of trucks (no grouping) which is a specific case of the above (with m_N=1 and b=1):

$$P[H > h]_{gross} = e^{-\lambda_{gross}h}$$

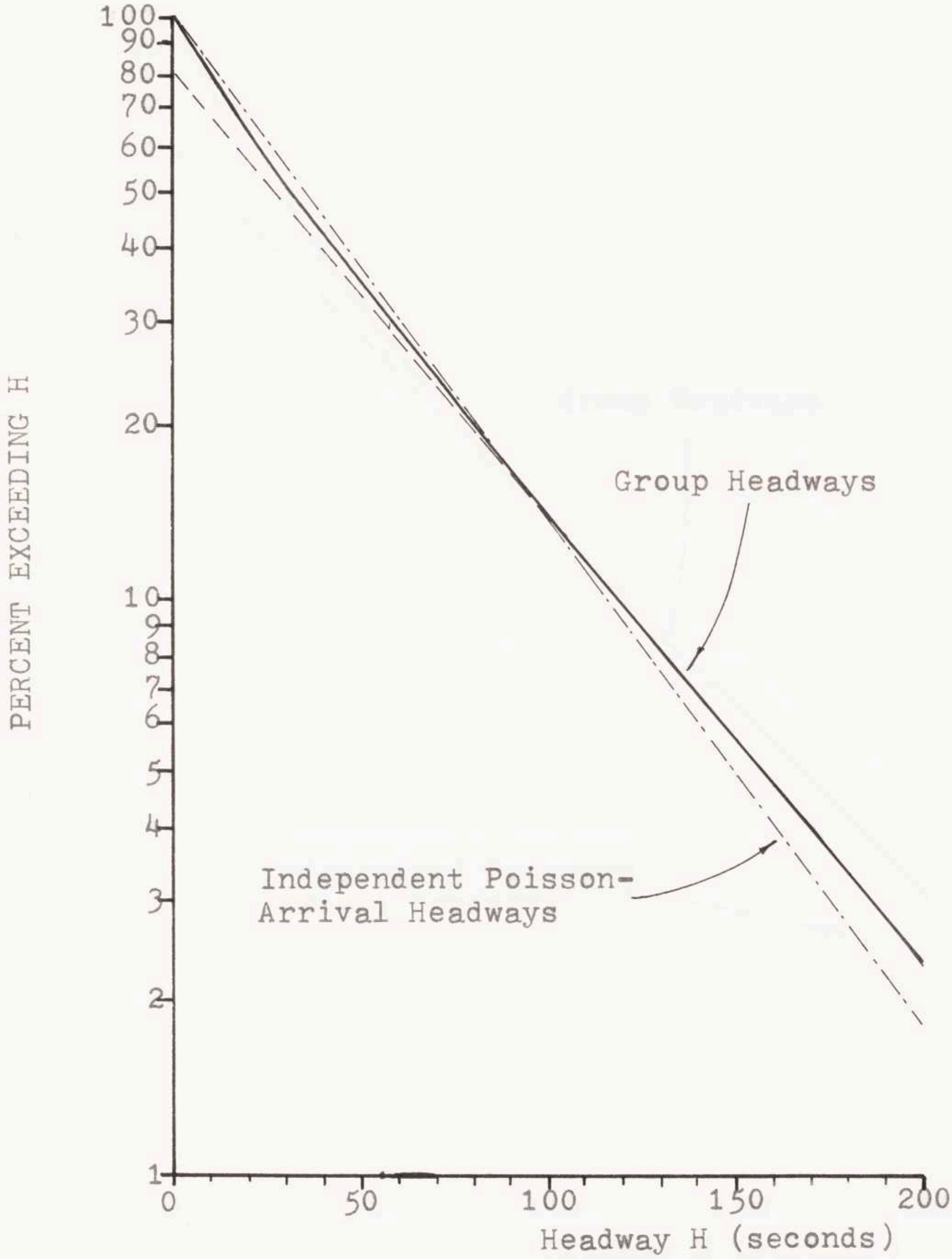
Returning to the semilog plot of complementary CDF of headways for independent Poisson arrivals (Figure [3.3]), it will be useful to compare that plot to the one for complementary CDF of headways for assumed group behavior. Since $P[H > h]_{group}$ is the weighted sum of two complementary exponential CDF's, each of the complementary CDF's may be plotted on semilog paper and compared to the complementary exponential CDF resulting from the independent Poisson arrivals of trucks. Such a comparison is shown in Figure [3.4] for typical values of m_N and b (m_N>1 and b>1, the conditions for grouping to occur).

$P[H > h]_{group}$, the weighted sum of the complementary CDF's of intra- and intergroup headways, would appear typically on a semilog plot as shown in Figure [3.5] for a range of values of parameter p.



Typical Headway Plots for Intragroup, Intergroup, and Independent Poisson Arrival Headways

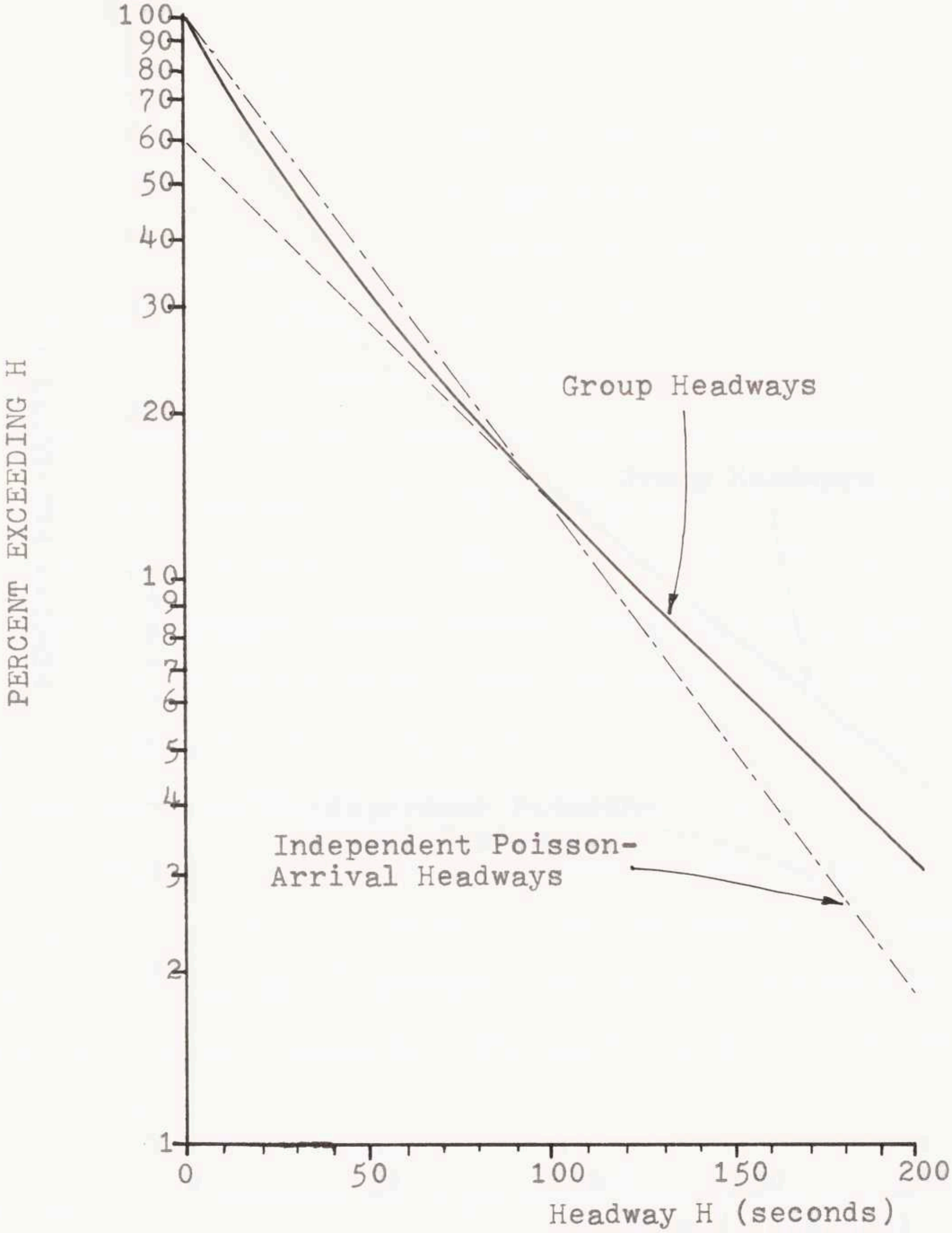
Figure [3.4]



Headway Plot for Assumed Group Behavior

p= 0.81

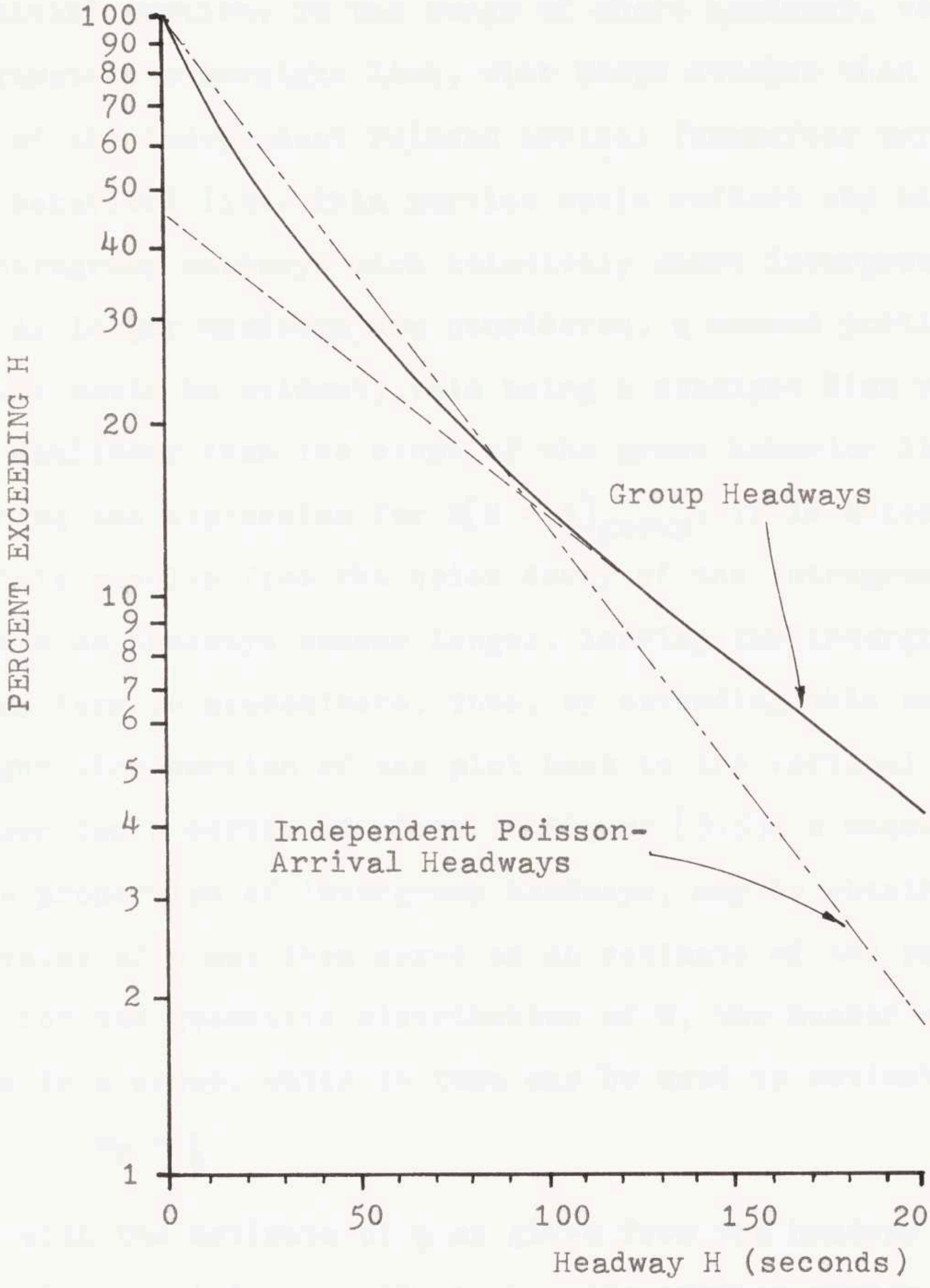
Figure [3.5a]



Headway Plot for Assumed Group Behavior

p= 0.60

Figure [3.5b]



Headway Plot for Assumed Group Behavior

p= 0.45

Figure [3.5c]

Typically, such a plot would be composed of two portions: The initial portion, in the range of short headways, would be approximately a straight line, with slope steeper than the slope of the independent Poisson arrival (hereafter termed gross behavior) line. This portion would reflect the mix of the intragroup headways with relatively short intergroup headways. As longer headways are considered, a second portion of the plot would be evident, this being a straight line with slope shallower than the slope of the gross behavior line. Examining the expression for $P[H > h]_{group}$, it is evident that this results from the quick decay of the intragroup headway term as headways become longer, leaving the intergroup headway term to predominate. Thus, by extending this second straight line portion of the plot back to the vertical axis, as shown for a series of plots in Figure [3.5], a measure of p, the proportion of intergroup headways, may be obtained. This value of p may then serve as an estimate of the parameter for the geometric distribution of N, the number of trucks in a group, which in turn may be used to estimate m_N:

$$m_N = \frac{1}{p}$$

Thus, with the estimate of p as given from the headway plot, it is also possible to estimate m_N, the average number of trucks per group.

The other important headway parameter, b, the ratio of within group arrival rate to gross arrival rate may be estimated by the knowledge of the value of m_N and consideration of the straight line portion of the $P[H > h]_{group}$ plot in the range of short headways. This may be demonstrated as follows:

It will be useful at this point to consider the Taylor Series expansion for e^x, which is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

and the expansion for e^{-x}, which is

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$$

For small values of x, e^x may be approximated as

$$e^x = 1 + x$$

and e^{-x} may be approximated as

$$e^{-x} = 1 - x$$

If small values of headway h are considered, then the expression for complementary CDF of group behavior headways may be approximated as:

$$P[H>h]_{group}=[(\frac{m_N-1}{m_N})(1+[1-b]\lambda_{gross}h)+(\frac{1}{m_N})(1+[1-\frac{b}{m_N(b-1)+1}]\lambda_{gro}h)]$$

$$(1-\lambda_{gross}h)$$

$$=[1+\lambda_{gross}h(1+\frac{[m_N(-b^2+b)+b(b-2)]}{m_N(b-1)+1})]\ (1-\lambda_{gross}h)$$

$$P[H>h]=1-\lambda_{gross}h+[1+\frac{[m_N(-b^2+b)+b(b-2)]}{m_N(b-1)+1}]\lambda_{gross}h$$

$$-[1+\frac{[m_N(-b^2+b)+b(b-2)]}{m_N(b-1)+1}]\lambda^2_{gross}h^2$$

Since only small values of h are being considered, the last term in the latter expression (involving h^2) may be neglected, and $P[H>h]_{group}$ may be expressed approximately as:

$$P[H>h]_{group} = 1-[-(\frac{m_N[-b^2+b]+b[b-2]}{m_N(b-1)+1})]\lambda_{gross}h$$

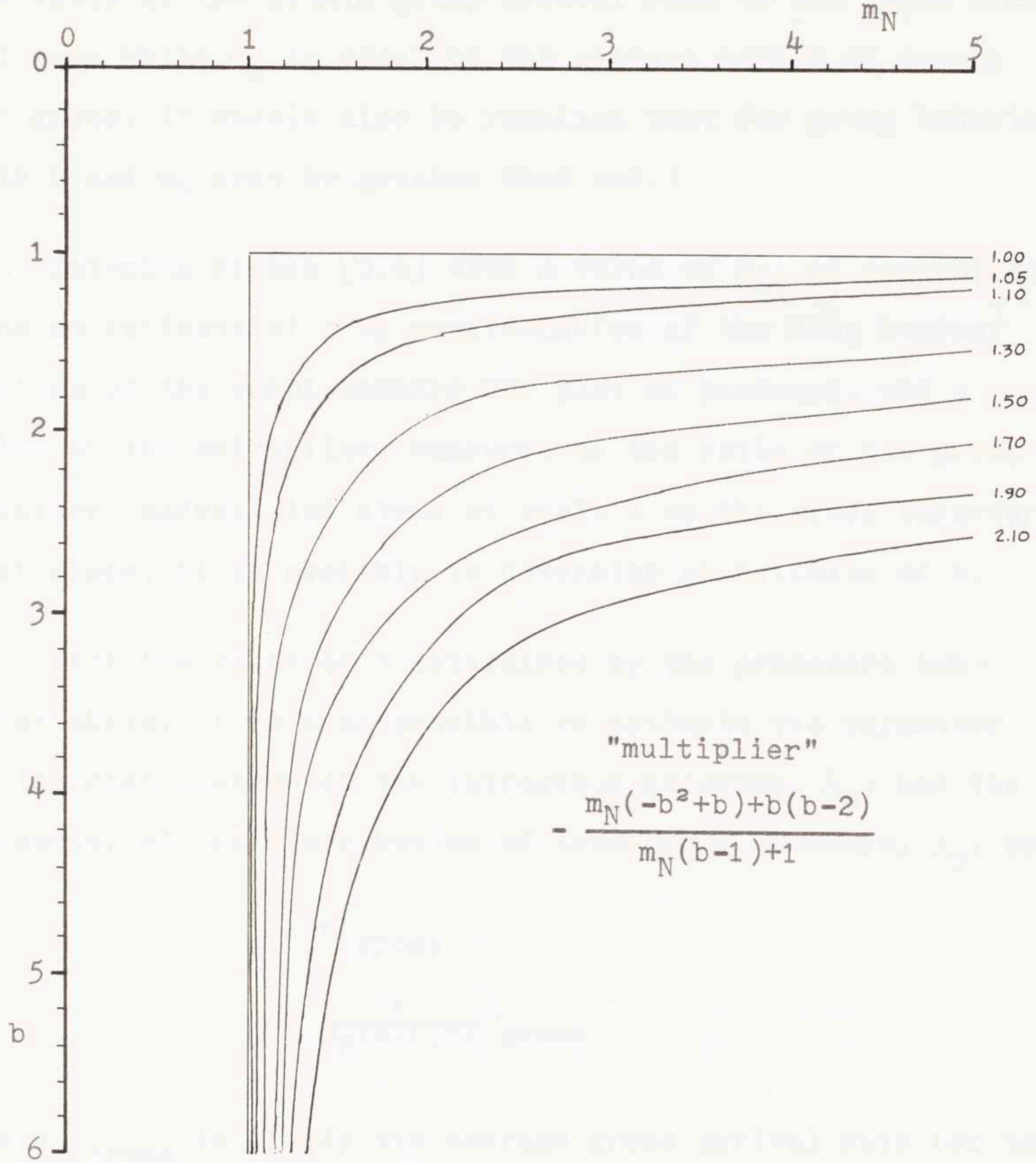
For pure Poisson behavior and small h, $P[H>h]_{gross}$ may be approximated as:

$$P[H>h]_{gross}=1-\lambda_{gross}h$$

Comparison of the two approximations above for group and for pure Poisson behavior show that they differ only in that the $\lambda_{gross}h$ term for the group case is modified by a multiplier term whose value is:

$$-(\frac{m_N[-b^2+b]+b[b-2]}{m\ (b-1)+1})$$

Figure [3.6] shows plotted contour lines of the value of this multiplier term for various combinations of b and m_N of interest. (The reader is reminded that $b = \frac{\lambda_W}{\lambda_{gross}}$ or



m_N
0
1
2
3
4
5
0
1
2
3
4
5
6
b
1.00
1.05
1.10
1.30
1.50
1.70
1.90
2.10
"multiplier"
$-\frac{m_N(-b^2+b)+b(b-2)}{m_N(b-1)+1}$

Figure [3.6]

the ratio of the within group arrival rate to the gross arrival rate while m_N is equal to the average number of trucks per group. It should also be recalled that for group behavior both b and m_N must be greater than one.)

Entering Figure [3.6] with a value of m_N, as determined from an estimate of p by consideration of the long headway portion of the complementary CDF plot of headways, and a value of the multiplier, measured as the ratio of the group behavior headway plot slope at small h to the gross behavior plot slope, it is possible to determine an estimate of b.

With the value of b determined by the procedure outlined above, it is then possible to estimate the parameter of the distribution of the intragroup headways, λ_W, and the parameter of the distribution of intergroup headways, λ_G, as:

$$\lambda_W = b\lambda_{gross}$$

$$\lambda_G = \frac{b}{m_N(b-1)+1}\lambda_{gross}$$

where λ_{gross} is simply the average gross arrival rate (or the number of trucks arriving during the observation period during which a set of headway data is collected divided by the total observation time). Figure [3.7] shows plotted contour lines of $\frac{b}{m_N(b-1)+1}$, the ratio of $\frac{\lambda_G}{\lambda_{gross}}$.

Appendix A contains a set of plots of P[H>h] for unpub-

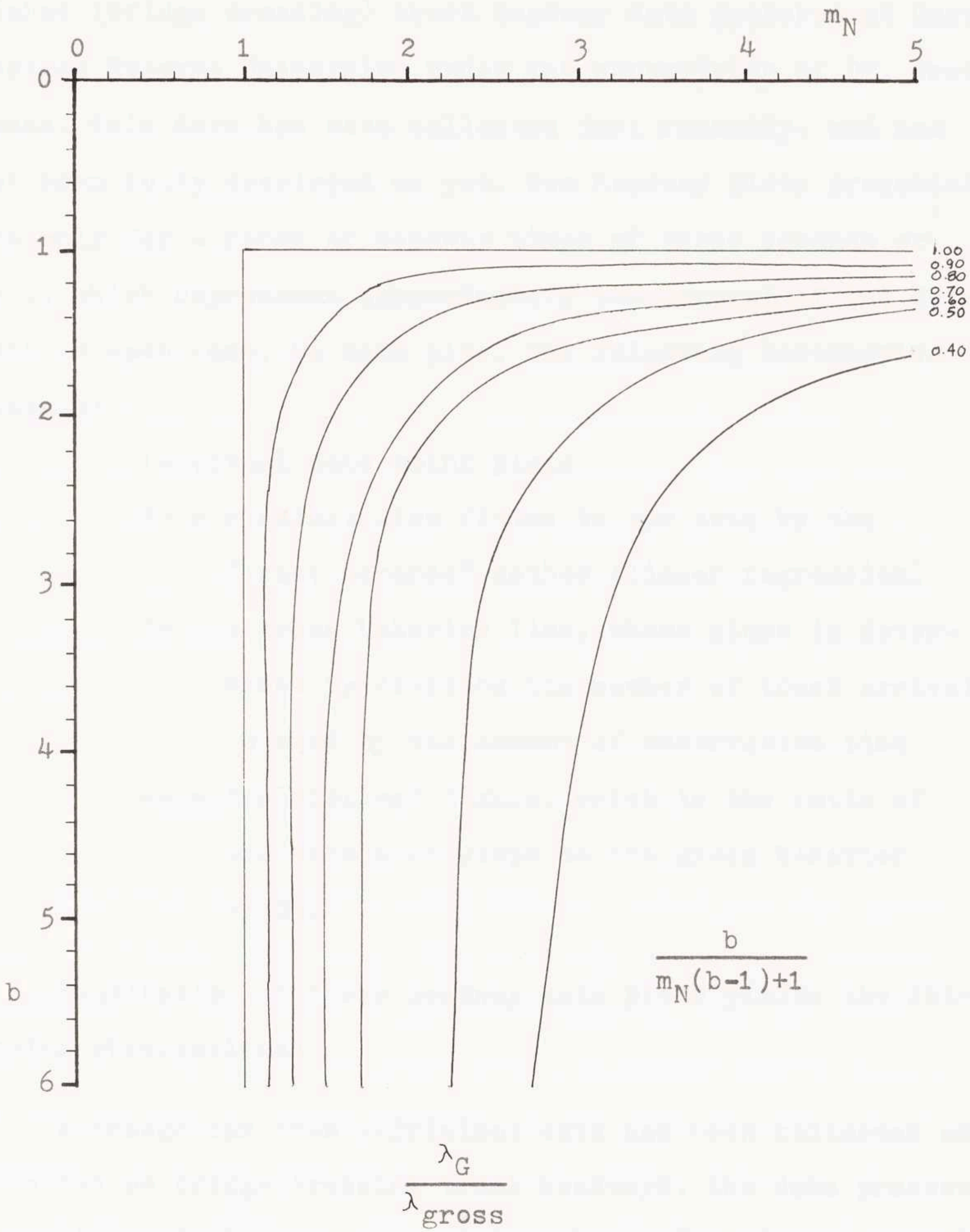

m_N
0
1
2
3
4
5
0
1
2
3
4
5
6
b
1.00
0.90
0.80
0.70
0.60
0.50
0.40
$\frac{b}{m_N(b-1)+1}$
$\frac{\lambda_G}{\lambda_{gross}}$

Figure [3.7]

lished (bridge crossing) truck headway data gathered at Case Western Reserve University under the supervision of Dr. Fred Moses. This data has been collected just recently, and has not been fully developed as yet. Ten headway plots presented are only for a range of headway times of three seconds or less, which represents approximately ten percent of the data sets in each case. On each plot, the following information appears:

1- actual data point plots

2- a straight line fitted to the data by the "least squares" method (linear regression)

3- the gross behavior line, whose slope is determined by dividing the number of truck arrivals counted by the amount of observation time

4- a "multiplier" figure, which is the ratio of the data plot slope to the gross behavior slope.

Examination of these headway data plots yields the following observations:

Although far from sufficient data has been collected and reported on bridge crossing truck headways, the data presented in Appendix A shows a trend for the headway data to result from some type of grouping or bunching phenomenon, rather than from an independent Poisson arriving process. The assumptions that have been set forth in this thesis lead to one possible explanation of the data plots as presented in Appendix A in terms of a renewal process, with independent, identically exponentially distributed headways between vehicles. It should be emphasized that although there is a correspondence between the data and the predicted behavior as given by the model, this correspondence may not be construed as "proof" of the validity of the hypotheses. Rather, what is intended here is to show that the hypotheses advanced are consistent with available data, and intuitively such hypotheses seem rational and plausible models of real-life behavior. What is necessary at this point is more data on truck headways and new mechanisms to verify the assumed group behavior.

3.3 Truck Gross Weight

Appendix B contains gross weight histograms for trucks based on weight data gathering studies (loadometer studies) conducted in the State of Massachusetts in 1967 and 1968[3-1] (which are the last two years that individual states published the results of their studies. Since 1968 the Federal Government has taken charge of data collection from all of the states and distributes this data to state and federal agencies for use in planning and design purposes). These histogram shapes are in agreement with the histograms presented and discussed by Garson, Goble, and Moses[3-2]. They have found that a composite normal distribution (the weighted sum of two normal distributions) represents well the gross weight data for trucks of the single unit type and for trucks of the tractor-trailer type (combination vehicles) for a variety of states. Although they state that the composite normal distribution needs further verification with the examination of more weight data from more states, a casual examination of the weight histograms presented for single unit and combinatorial vehicles reveals the suitability of this composite normal distribution for each vehicle type group. (A composite normal distribution is characterized generally by two humps. Garson, Goble, and Moses explain that each of the normal distributions combined in the composite normal distribution characterize the loaded and unloaded vehicles in each group type, respectively.) Garson, Goble, and Moses do not, however, advance a distribution for all truck gross weight, single and combinatorial vehicles combined. It is this distribution that is required to characterize X, the gross weight of an individual truck, in the model presented in Chapter 2. Presumably, on the basis of the observations made by Garson, Goble, and Moses, the distribution of truck gross weights for all truck types would be the result of the weighting of two composite normal distributions. (The resulting composite composite normal distribution would be characterized by four individual humps. Examination of the truck weight histograms for all vehicles reveals the presence of four distinct humps over the range of gross weights.) This procedure, however, would be too unwieldy for use in conjunction with the model, and was therefore rejected. For the purposes of the model presented here, a simple gamma distribution has been assumed to characterize truck gross weights for all truck body types. While not verified for a variety of truck weight histograms from different sources, it is assumed that due to its nature, the gamma distribution should provide a good fit for a variety of different histograms from different locations. Figures [3.8] and [3.9] show a gamma distribution superimposed upon truck gross weight data for all Massachusetts loadometer locations in 1967 and 1968, respectively. It should be pointed out that these histograms represent data from a particular mix of several stations, not all of which permit certain types of trucks. Each application may therefore warrant the development of different distributions. Should another distribution prove better than the gamma, the model presented here has to be modified, i.e., wherever the distribution of X, individual truck gross weights, or W, group gross weights appears, new distributions would have to be substituted in their place.

The parameters for the distributions shown in Figures [3.8] and [3.9], λ and k, were estimated from:

$$\hat{m}_X = \bar{x} = \frac{k}{\lambda}$$

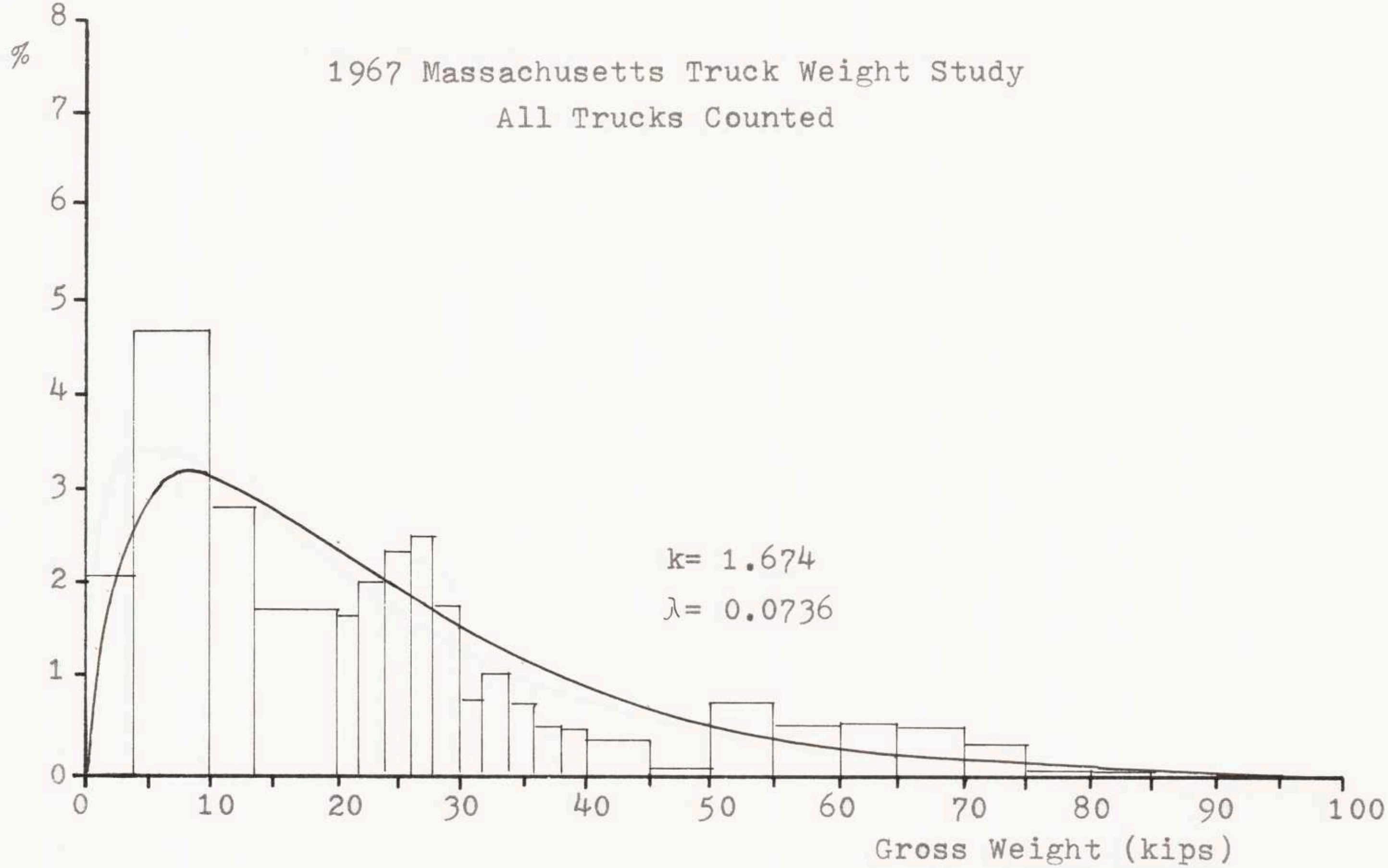

1967 Massachusetts Truck Weight Study
All Trucks Counted
k= 1.674
λ= 0.0736
%
0
1
2
3
4
5
6
7
8
0
10
20
30
40
50
60
70
80
90
100
Gross Weight (kips)

Figure [3.8]

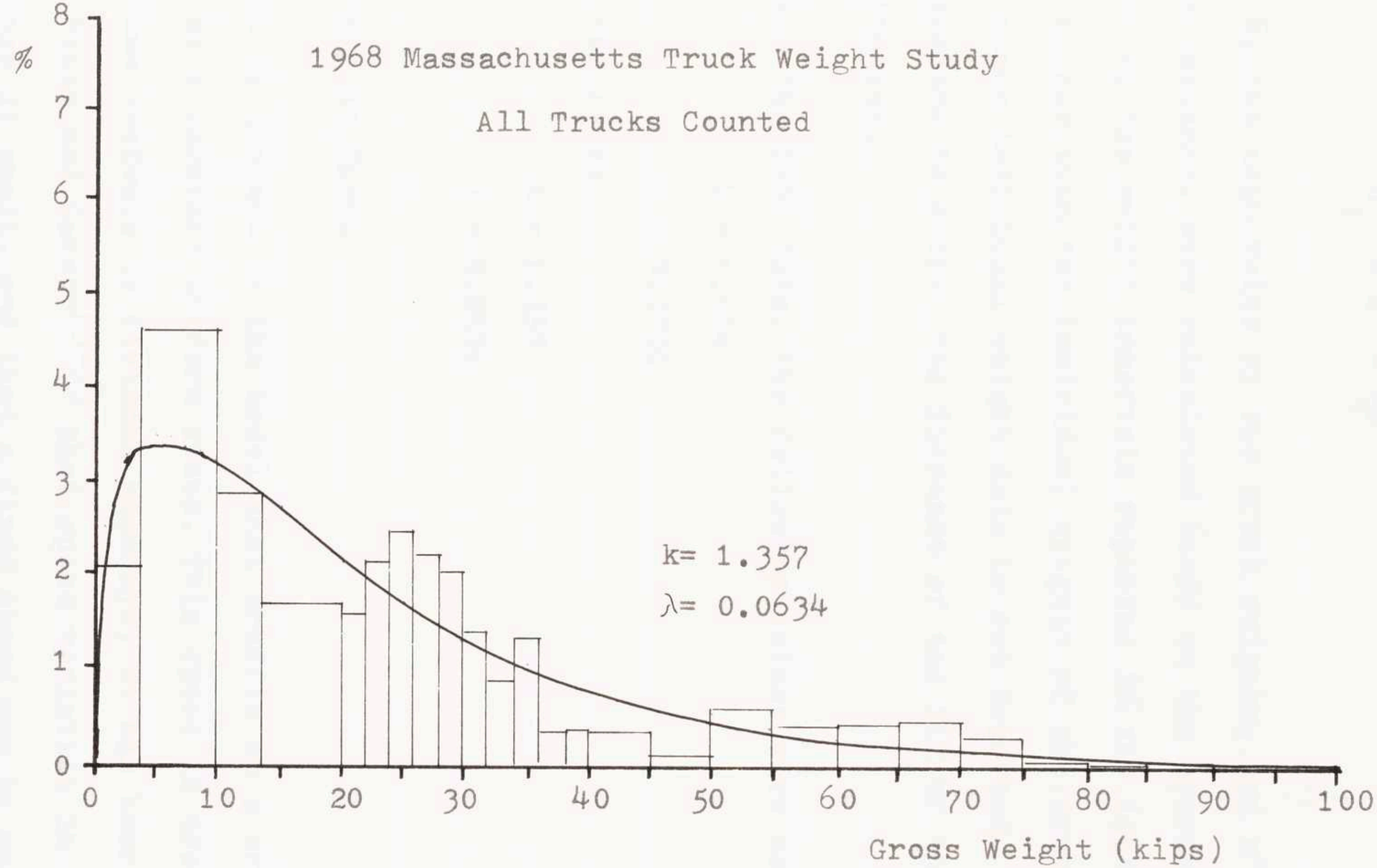

1968 Massachusetts Truck Weight Study
All Trucks Counted
%
8
7
6
5
4
3
2
1
0
0
10
20
30
40
50
60
70
80
90
100
k= 1.357
λ= 0.0634
Gross Weight (kips)

Figure [3.9]

$$\hat{\sigma}_X^2 = s^2 = \frac{k}{\lambda^2}$$

where $\bar{x}$, the mean value of the truck weights, and s^2, the sample variance, were calculated based on the average weight of each of the weight intervals reported in the loadometer study rather than the individual weights of the trucks surveyed since individual weight data is not reported. This is not expected to affect the fineness of the fit of the gamma distribution.

For the 1967 data, the following values were calculated:

$$k = 1.674$$

$$\lambda = 0.0736$$

while for 1968:

$$k = 1.357$$

$$\lambda = 0.0634$$

3.4 Traffic Speed

It is assumed in the model that traffic on a bridge moves at a constant uniform speed. This speed is used to convert time headways to distance headways. It has been suggested by Moses and Garson[3-3] that speed variation in bridge crossings is small, and that a fixed speed may be used. They suggest the use of the mean speed when available, or five miles per hour less than the posted speed when mean speed is not available.

3.5 Lane of Travel in Multi-lane Situations

It has been stated in the introduction to the model that truck traffic bears to the right in multi-lane flow. Evidence to this effect has been reported[3-4]. Investigators found that in the case of a three lane bridge on Route I-95 in Virginia more than three-fourths of the trucks crossed the structure in the right lane, while less than one percent used the left-most lane in a 98 hour period, and in the case of a two lane bridge on Route I-81 in Virginia, 98 percent of the trucks crossed the bridge in the right lane over approximately 70 hours.

While implicit to the assumptions about headways made in the model is the ability of trucks to pass each other or run alongside each other, the evidence presented above and common experience suggests that trucks do not usually avail themselves of the passing lanes on multi-lane highway bridges. This then suggests that the model may be applicable to single lane situations, even though such situations would contradict some of the model assumptions (e.g., the model permits two arrivals in a distance less than one truck length).

In general, when gathering data for use in estimation of the model parameters, an effort should be made to take measurements in areas and situations similar to those which the model is to be applied to, e.g., if the model is to be used in conjunction to the design of a three lane bridge (in one direction), then headway data and truck weight data should be sought from existing three lane bridges or highways either in the vicinity or in an area with traffic characteristics similar to those expected.

Chapter 4: Numerical Examples

4.1 Statement of Problem

The examples included in this chapter are intended to illustrate the usage of the model developed in Chapters 2 and 3 and to point to the shortcomings of the model in its present state of development and the areas in which the model need be expanded.

For the purposes of illustration, it is assumed that the governing body in charge of bridge design in a certain geographical area is contemplating the construction of bridges of span length varying from 100 to 4000 feet in length. The bridges are to carry two lanes of traffic in one direction only, and the characteristics of the bridge traffic are assumed to be similar in all cases.

A policy must be adopted for the live load intensity to be used in conjunction with the design of each bridge. Thus, in the framework of the model developed in the previous chapters, the following activities must be performed for the model to be useful in the development of live load criteria:

First, the input parameters for the model must be estimated from data from conditions similar to those expected on the bridges. With these estimates of parametric values the probabilities of occurrence of certain conditions on the various bridges may be evaluated: the simultaneous occurrence of groups, fully-loaded bridges, and the maximum moment on simply supported bridges due to the presence of one group.

4.2 Estimates of Parametric Values

It is assumed for purposes of illustration that the traffic expected on the various proposed bridges will be similar in composition, characteristics, and volume to the traffic measured for the headway data plots that appear in Figures [A.11] and [A.12]. Both figures were developed from the same set of headway data, with Figure [A.11] a plot of the very short headways (less than 4 seconds) while Figure [A.12] is a full plot of headway values up to 200 seconds. As per the discussion in Chapter 3, Section 3.2, the following values may be estimated from these figures: First, from Figure [A.11] the value of the multiplier, or ratio of the slope of the straight line fit to the data points to the slope of the gross behavior line, is 1.538 (where the gross behavior slope is proportional to the gross arrival rate of trucks and is equal to 0.0255 trucks/sec.). From Figure [A.12] the value of p, or the proportion of intergroup headways may be estimated as 0.60 by extending back a straight line through the long headway data points to the "percent exceeding H" axis. It may be recalled that m_N, the mean number of trucks per group, is the reciprocal of p, or 1/0.60 or 1.667.

Entering Figure [3.6] with a value for m_N of 1.667 and a value of the multiplier of 1.538, b, the ratio of within group arrival rate to gross arrival rate, may be estimated as 2.77. Recalling the assumptions made in Chapter 3, Section 3.4, it may further be assumed that the bridges are to be designed for a posted speed of 45 miles per hour, and thus 40 mph will be used here as the average speed on the bridges. With the above estimates, the following parameters may be evaluated as:

$$\lambda_{gross} = 0.0255 \frac{\text{trucks}}{\text{sec.}} \times \frac{1 \text{ hour}}{40 \text{ miles}} \times \frac{1 \text{ mile}}{5280 \text{ ft.}} \times \frac{3600 \text{ sec.}}{1 \text{ hour}} =$$

$$0.0004346 \text{ trucks/ft.}$$

$$\lambda_W = b\lambda_{gross} = 2.77 \times 0.0004346 \text{ trucks/ft.} = 0.0012 \text{ trucks/ft.}$$

$$\lambda_G = \frac{b}{m_N(b-1)+1} \lambda_{gross} = \frac{2.77}{(1.67)(1.77)+1} 0.0004346 =$$

$$0.0003 \text{ trucks/ft.}$$

It may further be assumed that the traffic conditions expected are similar to those measured in the 1968 Massachusetts Truck Weight Study. Thus, from Figure [3.9], the following values may be taken as estimates of the gamma distribution of individual truck weights:

$$k = 1.357$$

$$\lambda = 0.0634$$

Additionally, it will be assumed that an individual truck length a will be equal to 30 feet.

Summarizing the estimates of the parameters needed for input to the model:

$p = 0.60$

$\lambda_W = 0.0012$

$\lambda_G = 0.0003$

$k = 1.357$

$\lambda = 0.0634$

$a = 30.0$

4.3 Simultaneous Occurrence of Groups

The expression for the probability of m groups simultaneously loading the bridge was developed in Chapter 2, Section 2.3.1 as:

$$m = 1: \quad P[L_T \leq s] = p + (1-p)(1 - e^{-p\lambda_W[s-a]}) \qquad s \geq a$$

$$m > 1: \quad F_{T|M=m}(s-a) = \int_0^{s-a} \left(1 - \sum_{i=0}^{m-1} \frac{e^{-p\lambda_W l_S}(p\lambda_W l_S)^i}{i!}\right) \left(\frac{\lambda_G(\lambda_G[s-a-l_S])^{m-2} e^{-\lambda_G[s-a-l_S]}}{(m-2)!}\right) dl_S$$

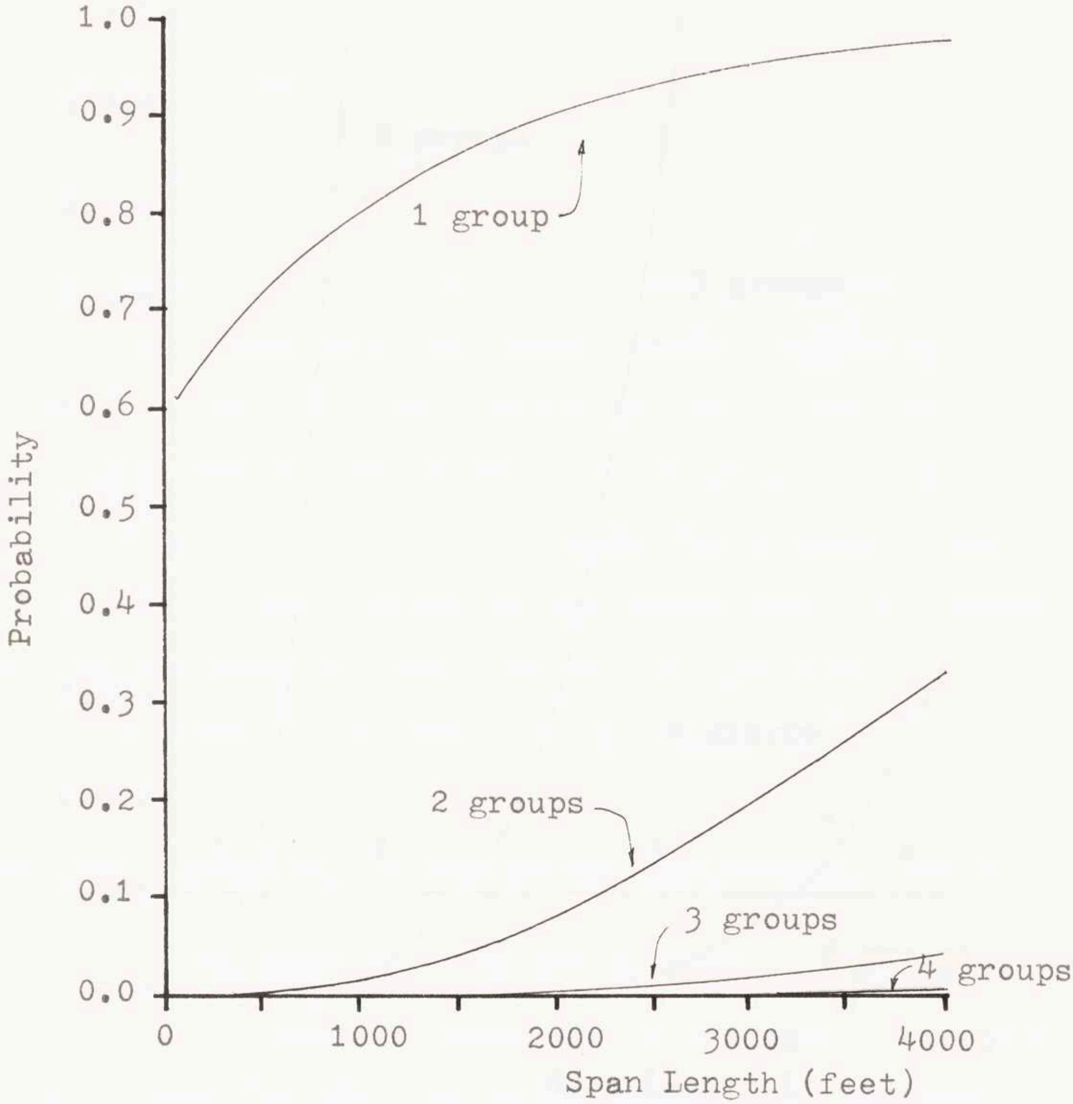
where s is the span length of the bridge in feet.

For the parametric values of this example, plots of the above probabilities are shown in Figures [4.1] and [4.2]. Figure [4.2] is an exaggerated plot of the lower portion of Figure [4.1] with probabilities ranging from 0.0 to 0.01. With such a plot it is possible to set a probability level, i.e., a probability of simultaneous occurrence of arbitrary groups over a range of bridge span lengths. Suppose for illustrative purposes that the probability level is set at p'= 0.001. Examination of Figure [4.2] yields the following interpretation: For a probability level of 0.001, bridges designed on the basis of the data considered need be designed for only one group up to a span length of 400 feet; 400 to 1500 foot spans need be designed for the simultaneous occurrence of two groups; 1500 to 3150 foot spans for threee groups simultaneously, and 3150 to 4000 foot spans for four groups.

Different sets of design data and different probability levels would result in different ranges for the 1,2,3,... group categories of bridges. It is clear, however, that this method provides for a consideration of the probabilities of simultaneous occurrence of live loading units on bridges of varying spans, and for a preset probability level this method points to a new classification scheme for live load considerations.

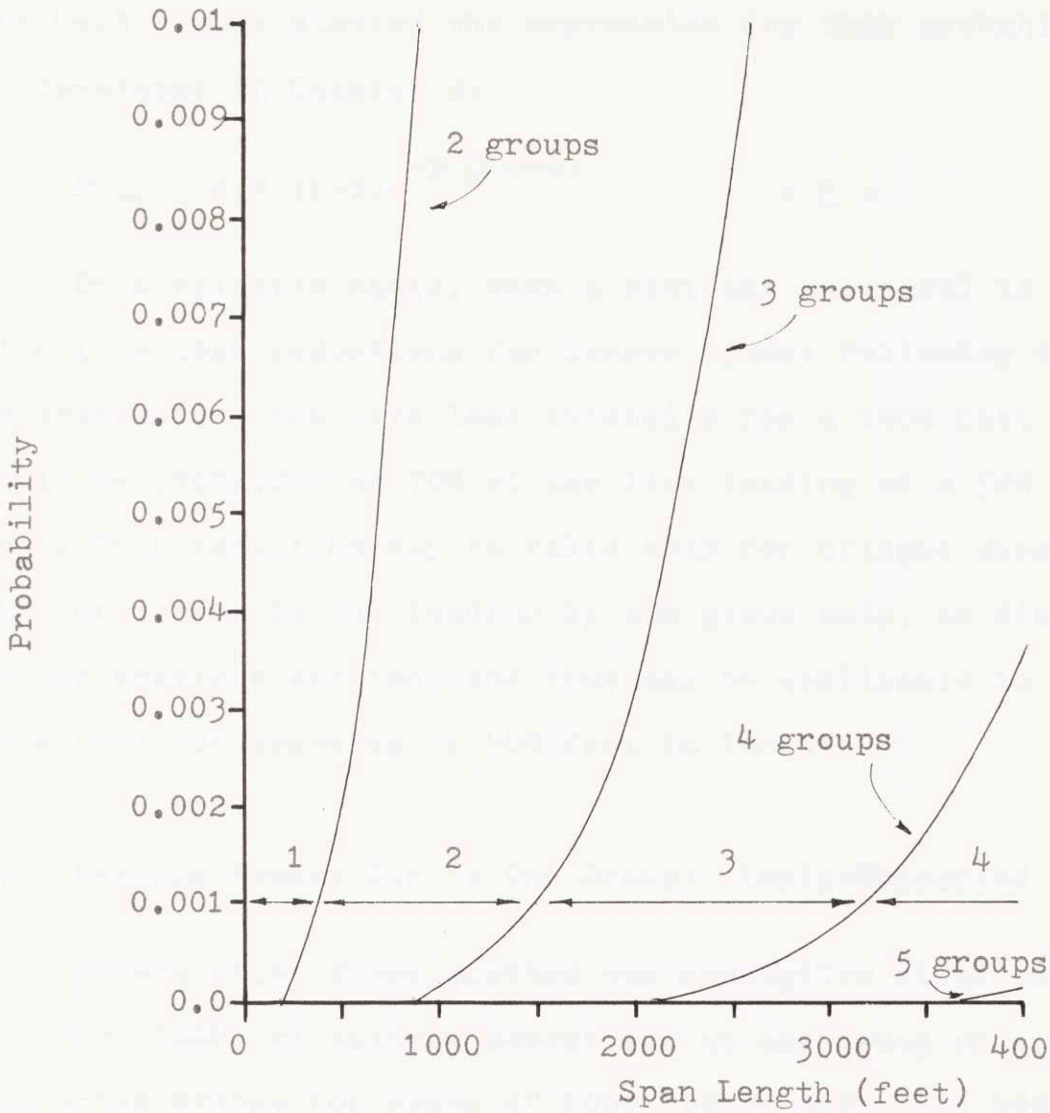
4.4 Fully-loaded Bridge

The probability that a bridge will be fully-loaded has



Simultaneous Occurrence of Groups

Figure [4.1]



Simultaneous Occurrence of Groups

Figure [4.2]

been defined in Chapter 2, Section 2.3.2 as the probability that a bridge will be occupied by one group whose length is equal to or greater than the span length of the bridge. Figure [4.3] shows plotted the expression for this probability as developed in Chapter 2:

$$P[L_T \geq s] = (1-p)e^{-p\lambda_W[s-a]} \qquad s \geq a$$

On a relative basis, such a plot may be useful to define live load reductions for longer spans; Following this interpretation the live load intensity for a 1000 foot span would be .200/.285 or 70% of the live loading of a 500 foot span. This reduction may be valid only for bridges whose design criterion is the loading by one group only, as discussed in the previous section, and thus may be applicable in this case only for spans up to 400 feet in length.

4.5 Maximum Moment Due to One Group: Simply-Supported Bridge

Figure [4.4] shows plotted the cumulative distribution function (CDF) of maximum moment due to one group on a simply-supported bridge for spans of 1000, 2000, 3000, and 4000 feet. As explained in Chapter 2, Section 2.3.3, the development of the expression for the probability density function (PDF) of maximum moment as presented here is valid only in the case where $P[L_T \geq s]$ is very small, i.e., only in those cases when the total length of one group exceeds the span length with

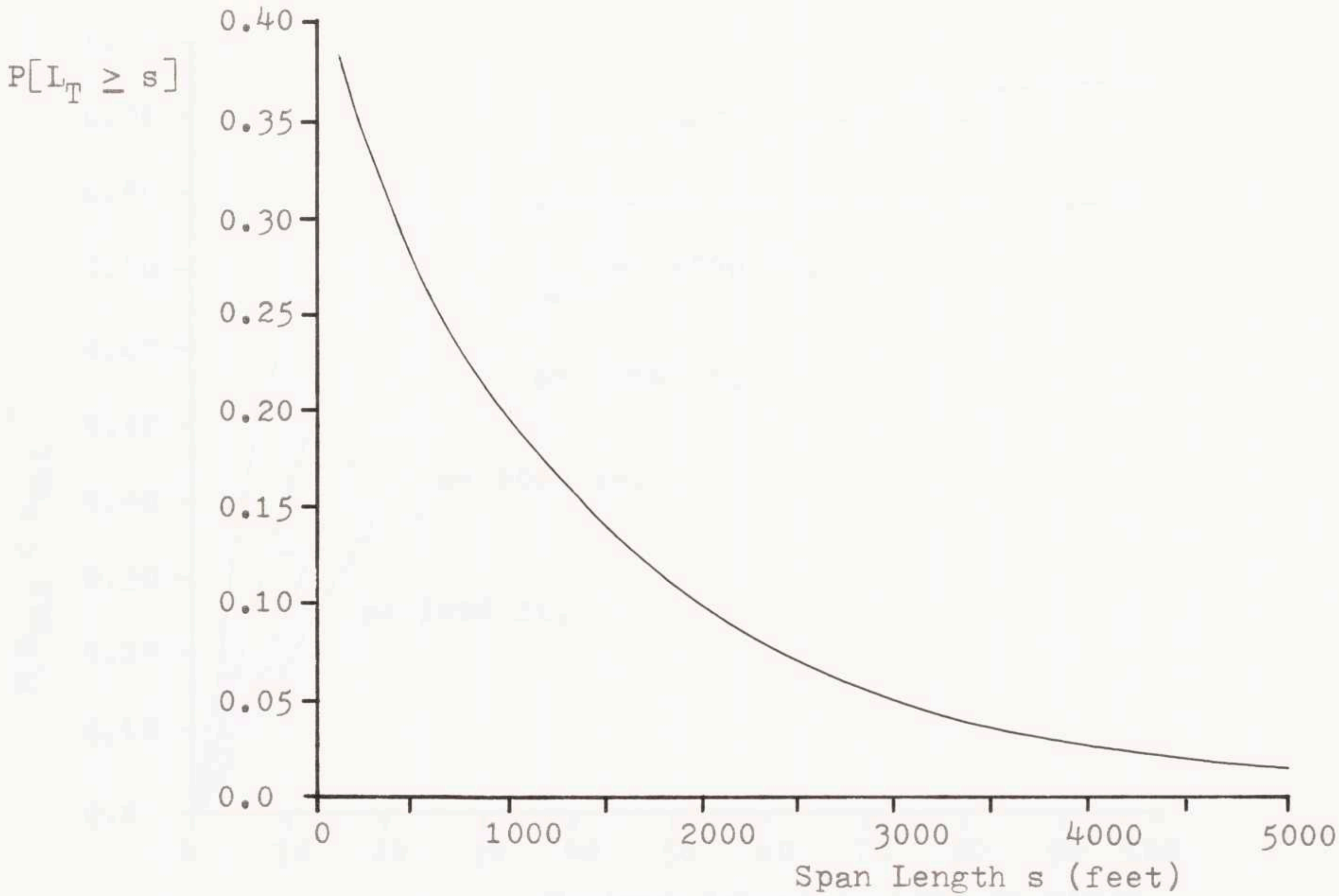

0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.0
P[L_T ≥ s]
0
1000
2000
3000
4000
5000
Span Length s (feet)

Figure [4.3]

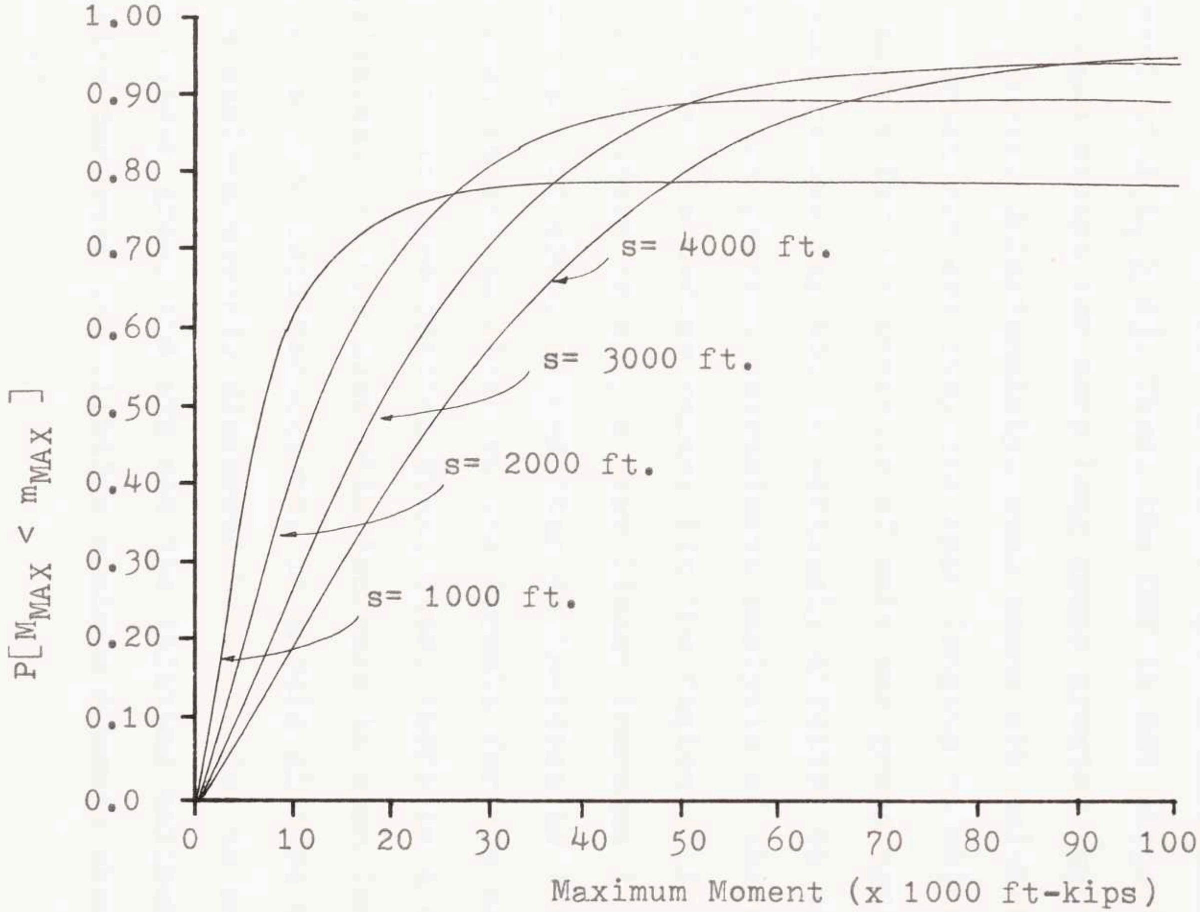

1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.0
$P[M_{MAX} < m_{MAX}]$
s= 4000 ft.
s= 3000 ft.
s= 2000 ft.
s= 1000 ft.
0
10
20
30
40
50
60
70
80
90
100
Maximum Moment (x 1000 ft-kips)

Figure [4.4]

only a small probability. As can be seen by reference to Figure [4.3], in the case of this example, the probability that $L_T > s$ is still greater than 2% even at a span length of 4000 feet. Examination of Figure [4.4] reveals that the maximum value of the CDF for any span length is precisely the complement of $P[L_T \geq s]$. Thus, the CDF is not properly defined in this case except for very long spans greater than 4000 feet in length. Unfortunately, such spans are neither of practical interest nor are they the span lengths to which a moment analysis for the presence of only one group applies. The median values may not be seriously affected by this inadequacy in the present approximate analysis of the model. Note that for the median values (in the region of P= 50%, for example) there is only a near linear increase in moment with increase in span, rather than an increase as the square of the span length as given by the formula for the maximum moment on a simply-supported span. Thus, there is a decay in the equivalent uniform load with increase in span length, pointing to live load reductions. It should also be recalled that the maximum moments discussed here are due to an arbitrarily chosen group and are not the lifetime maximum moments. For design purposes the lifetime maximum moments should be considered.

The difficulties outlined above point to the need for extending the model in two areas: First, in the area of short spans to consider cases of loading by one group where the group length is greater than the span length and hence the total group weight is not on the bridge all at the same time. This also entails a different approach to the loading by one group, i.e., the load may no longer be considered uniformly distributed over the group length. The second area is the development of procedures for the handling of multiple group occurrences for longer spans.

Footnotes

[1-1] L.R. Manville and R.W. Gastmeyer, "Motor-Truck Loading on Highway Bridges," Engineering News, LXXII (September 3, 1914), 492-4.

[1-2] Sven Olaf Asplund, "Probabilities of Traffic Loads on Bridges," Proceedings of ASCE, LXXXI, Separate no. 585 (January, 1955), 1.

[1-3] Stewart Mitchell and Gerald F. Borrman, "Vehicle Loads and Highway Bridge Design," Journal of the Structural Division; Proceedings of ASCE, LXXXIII, ST4, 1302 (July, 1957), 13.

[1-4] R.J. Ivy, et.al., "Live Load for Long Span Highway Bridges," Transactions; ASCE, CIX (1954), 981-1004.

[1-5] Asplund, op.cit., 1-12.

[1-6] International Association for Bridge and Structural Engineering, Symposium on Loading of Highway Bridges; Oporto 1956 (Stockholm: IABSE, 1956).

[1-7] "Highway Bridges: A Rational Approach to Heavy Loads," Engineering, CLXXXII (August 24, 1956), 235-6.

[1-8] Henson K. Stephenson, "Highway Bridge Live Loads Based on Laws of Chance," Journal of the Structural Division; Proceedings of ASCE, LXXXIII, ST4, 1314 (July, 1957).

[1-9] Robert C. Garson, George G. Goble, and Fred Moses, "Traffic Loading and its Analytical and Measured Description," in Specialty Conference on Safety and Reliability of Structures, sponsored by ASCE, Pittsburgh, Penn., November 2-3, 1972 (New York: ASCE) pp. 27-54.

[2-1] Jack R. Benjamin and C. Allin Cornell, Probability, Statistics and Decision for Civil Engineers (New York: McGraw-Hill, 1970).

[3-1]Mass. Dept. of Public Works, Bureau of Transportation Planning and Development, Report on the 1968 Truck Weight Study.

[3-2]Garson, Goble, and Moses, op.cit.

[3-3]Fred Moses and Robert Garson, Probability Theory for Highway Bridge Fatigue Stresses, Final Report (Cleveland, Ohio: Case Western Reserve Univ., 1972).

[3-4]Wallace T. McKeel, Jr., et.al., "Loading History Study of Two Highway Bridges in Virginia," Highway Research Record, 382, 27-37.

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"Highway Bridges: A Rational Approach to Heavy Loads." Engineering, CLXXXII (August 24, 1956), 235-6.

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Ivy, R.J., et.al. "Live Load for Long Span Highway Bridges." Transactions; ASCE, CIX (1954), 981-1004.

McGuire, William. Steel Structures. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1968.

McKeel, Wallace T., Jr., et.al. "Loading History Study of Two Highway Bridges in Virginia." Highway Research Record, 382, 27-37.

Manville, L.R., and Gastmeyer, R.W. "Motor-Truck Loading on Highway Bridges." Engineering News, LXXII (September 3, 1914), 492-4.

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Moses, Fred, and Garson, Robert. Probability Theory for Highway Bridge Fatigue Stresses. Final Report. Cleveland, Ohio: Case Western Reserve University, 1972.

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Appendix A: Headway Data

Included in this section are plots of complementary CDF of truck headways (P[H > h]) on a semilogarithmic scale. The plots were developed from data furnished by Dr. Fred Moses, Professor of Civil Engineering, Case Western Reserve University, Cleveland, Ohio. The data was collected under a contract from the Ohio Department of Highways and is not to be published without their consent. Each of the plots is labeled with the original tape designation for reference purposes.

The first ten plots, Figures [A.1-A.10] are for very short headways only, and cover only an average of ten percent of the data of any of the sets. Each of the data sets is for a bridge with two lanes of travel in one direction, with the exception of A37,39, which is for three lanes in one direction, and is so noted. Unfortunately, this data is just being collected and processed at the time of this writing, and the rest of the data has not been developed for the full range of headways. This data is useful, however, for getting an indication of the trend of short headways. On each of the plots the following information is recorded:

1- the total number of trucks counted

2- the total observation time

3- the "least squares" straight line fitted to the data points

4- the gross behavior line, i.e., a straight line with slope $_{gross}$, the total number of trucks counted divided by the total observation time

Examination of these plots leads to the observation that the straight line fitted to the data does not run through the origin (H=0, P=100%). Rather, the straight line intercepts the line P=100% at H > 0. For a distribution that would truly be the sum of two exponential distributions as hypothesized in the model herein, this straight line would pass through the origin. However, since the data lines cross the P=100% line very close to H=0, i.e., at very small values of headway, this shift from the assumed weighted sum of exponentials model is not considered significant, and may be explained simply as an indication that the trucks on the bridges measured did not take advantage of the opportunity present on the bridges with respect to passing other trucks.

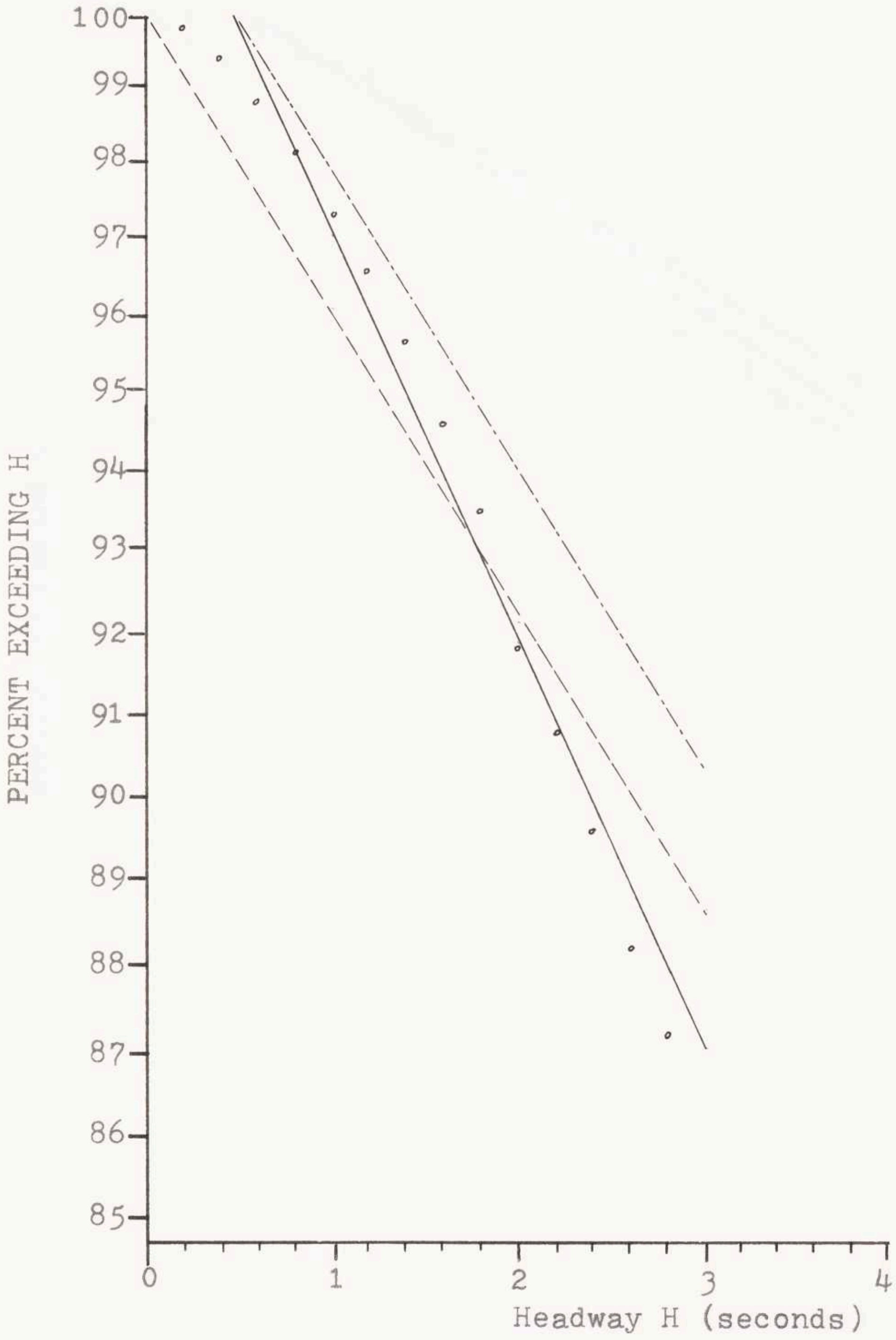
In each of the plots the gross behavior line is offset parallel to itself to cross the P=100% line at the same point as the "least squares" data line for purposes of comparison of their slopes. Table [A.1] contains a summary of this data, with the slopes of the gross behavior line and of the least squares data line and the multiplier values listed. It should be noted that in 8 out of 10 cases the multiplier is greater than unity, while in two cases it is less than one. As discussed in Chapter 3, for group behavior the multiplier value should be greater than one, and the data seems to bear this out. In the two cases where the multiplier is less than one, it may be noted that it is not much less than one, and it is quite possible that with a larger sample of data, i.e., headways up to 10 seconds, the value of the multiplier might be found to be closer to unity.

It is entirely possible that the slopes presented here for the data lines as fitted by the least squares method would change with a larger data set. This points to the problem of how much data in the short headway range is needed to establish the value of the multiplier, or in other words, how much data should be considered in a least squares analysis. In the case of the first ten data sets, this was an elementary question, in that only a very limited amount of data was available and least squares analysis was applied to all available data points. Figures [A.11] and [A.12], however, were developed from a full set of headway values up to 400 seconds. In the case of Figure [A.11], the first four seconds of data were used to establish the multiplier value. More work should be done in this area with the availability of more complete data sets to establish the sensitivity of the slope of the least squares lines in the short headway range to the differing number of data points considered.

Figure [A.12] shows a plot of P[H > h] for headways up to 200 seconds. Extension of the long headway range data line to the P axis leads to an estimate of p, the fraction of intergroup headways, as discussed in Chapter 3. Again, the problem which presents itself is how to select the data points in the long headway range for fitting a straight line to estimate p. In this case too some sensitivity studies would have to be made to determine how sensitive the estimate of p is to the number of data points considered in the long headway range.

Key to Figures [A.1-A.11]:

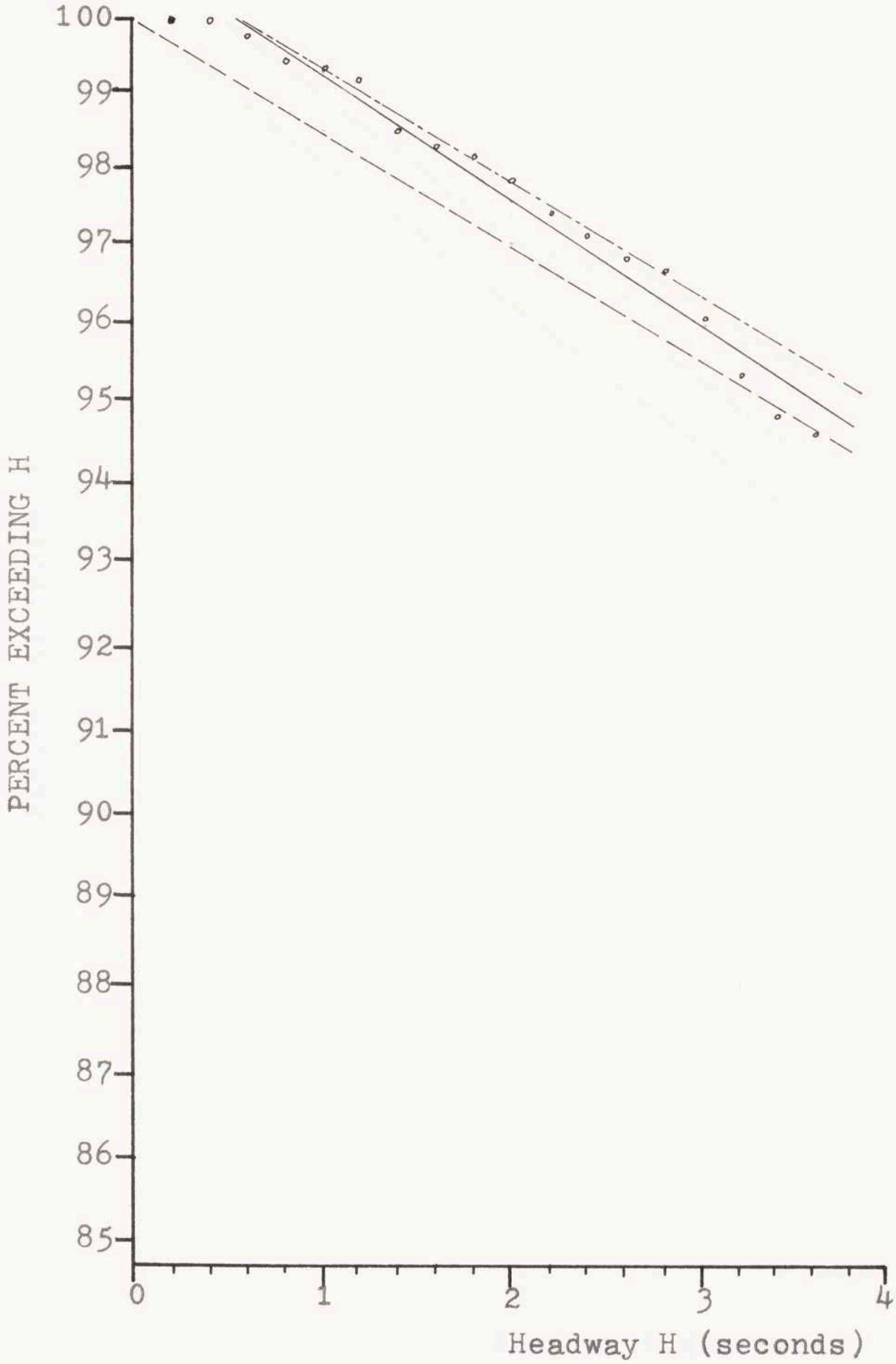
————————	Data Line fitted by Least Squares Method
- - - - - - -	Gross Behavior Line
—·—·—·—	Offset Gross Behavior Line



Headway Histogram H2,4
2012 trucks
14 hours

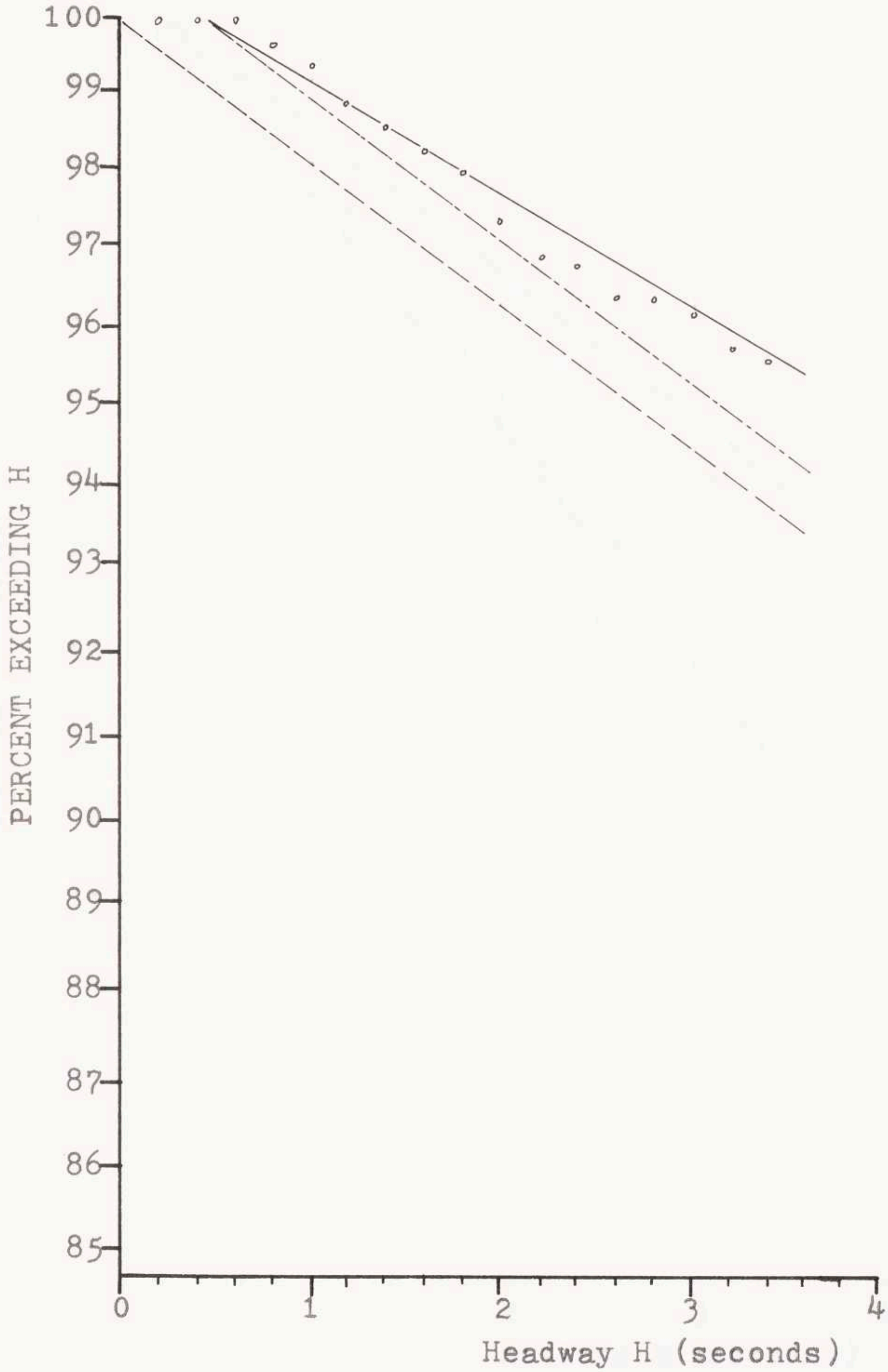
multiplier= 1.358

Figure [A.1]



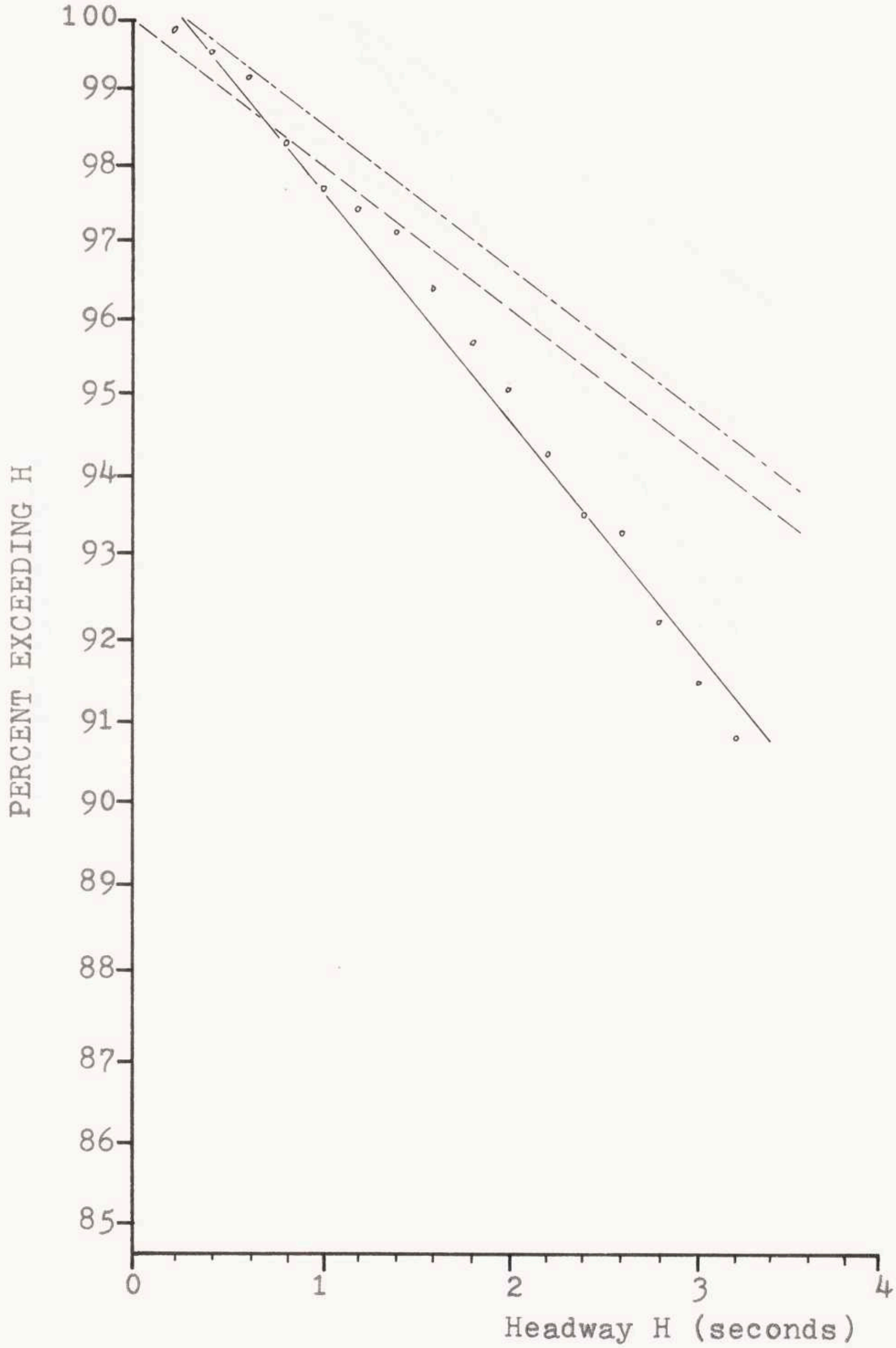
Headway Histogram W1,3
581 trucks
10.75 hours
multiplier= 1.127

Figure [A.2]



Headway Histogram W6,8
750 trucks
11 hours
multiplier= 0.815

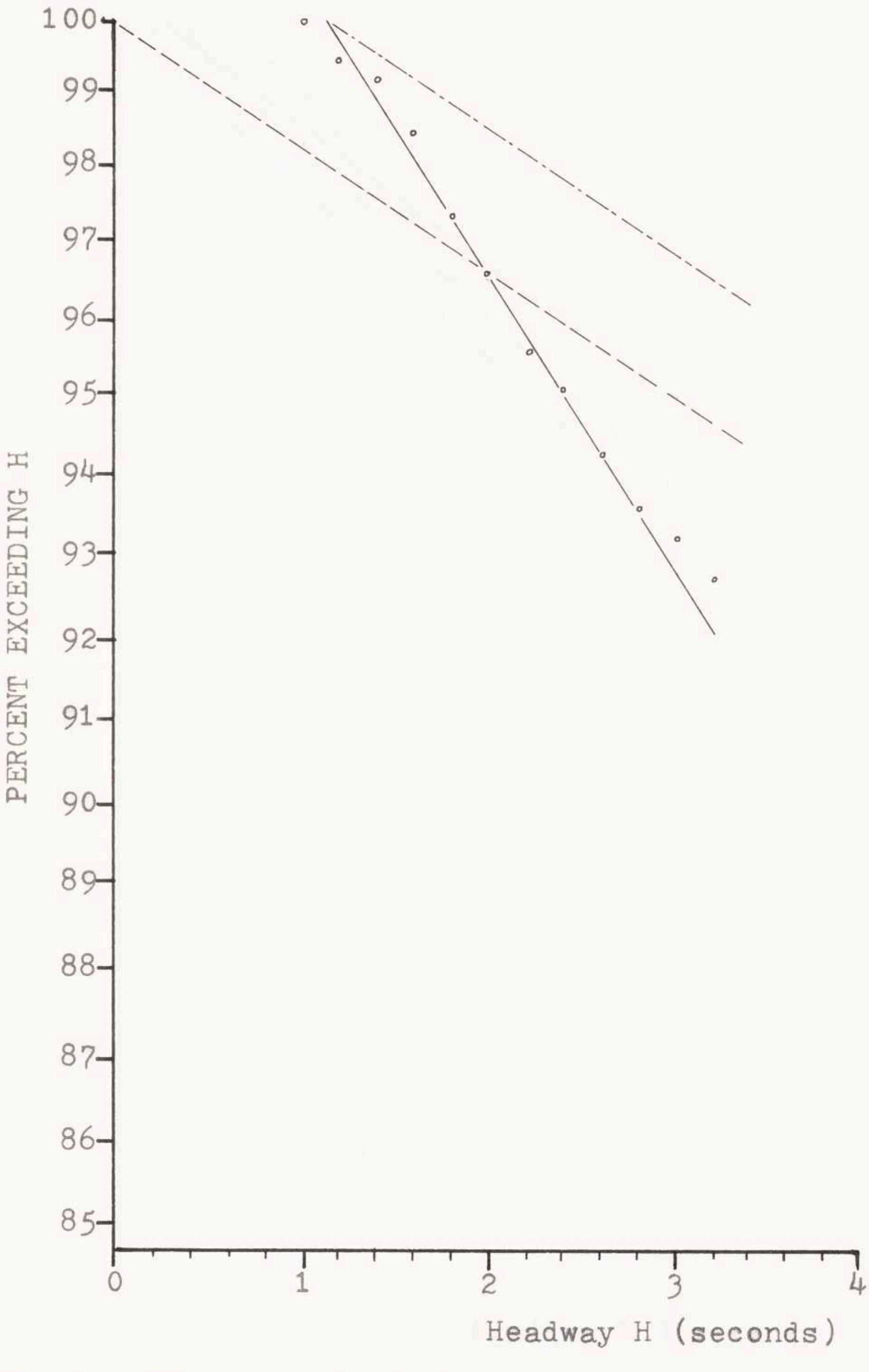
Figure [A.3]



Headway Histogram Ash5,7
865 trucks
12 hours

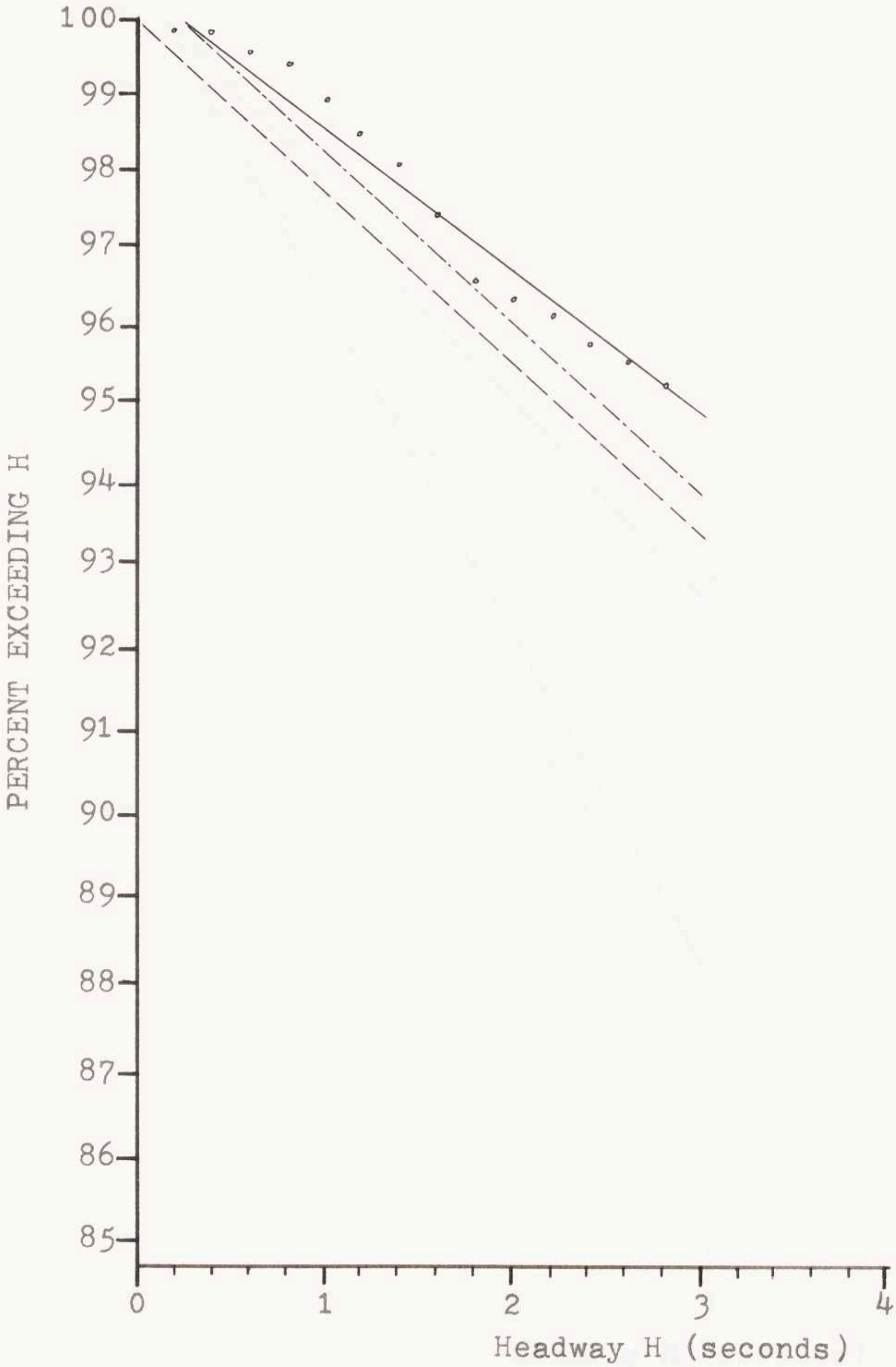
multiplier= 1.550

Figure [A.4]



Headway Histogram Ash1,3
730 trucks
12 hours
multiplier= 2.450

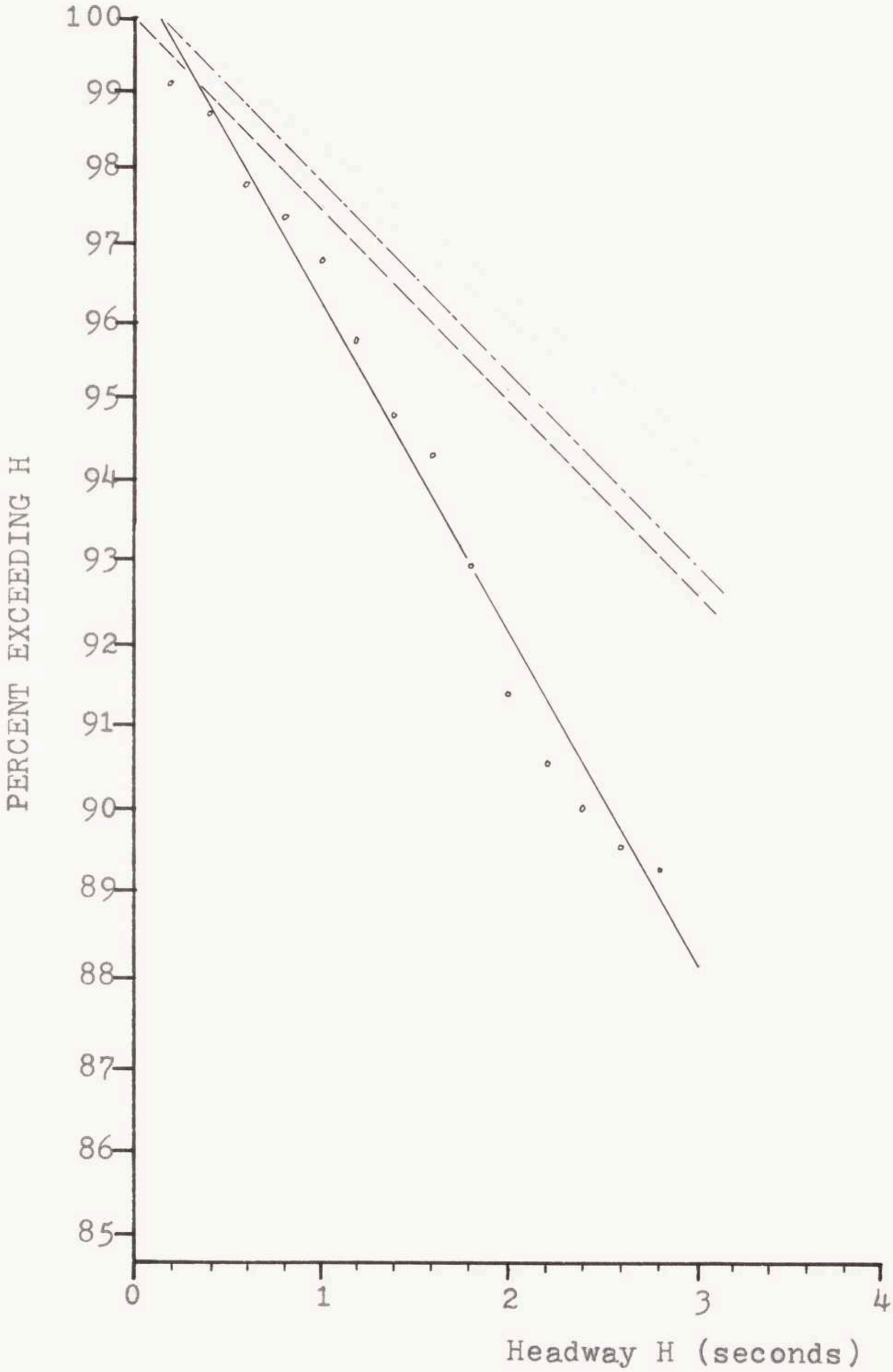
Figure [A.5]



Headway Histogram A37,39
1013 trucks
12 hours

multiplier= 0.857

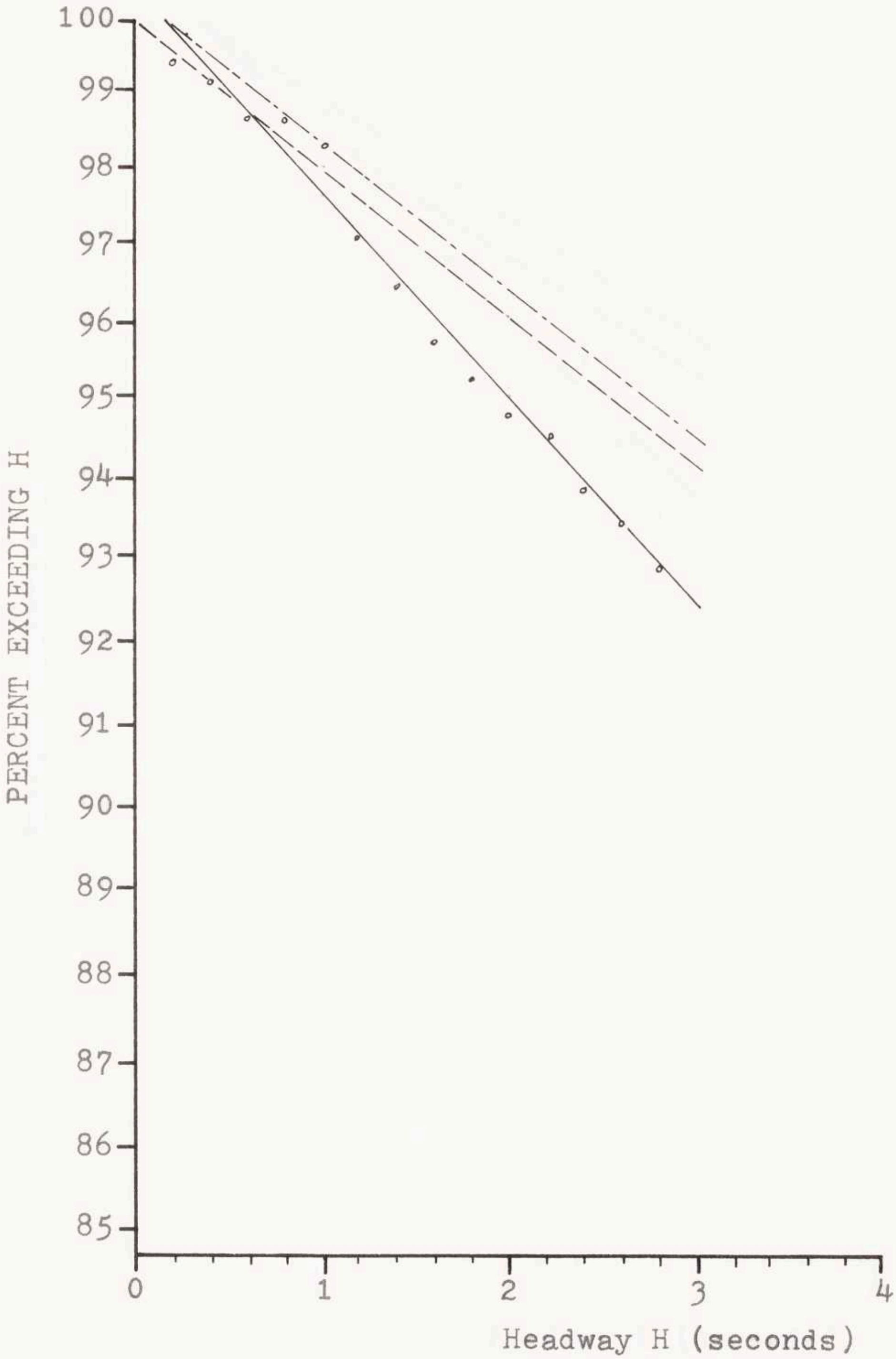
Figure [A.6]



Headway Histogram A41,43
982 trucks
11 hours

multiplier= 1.774

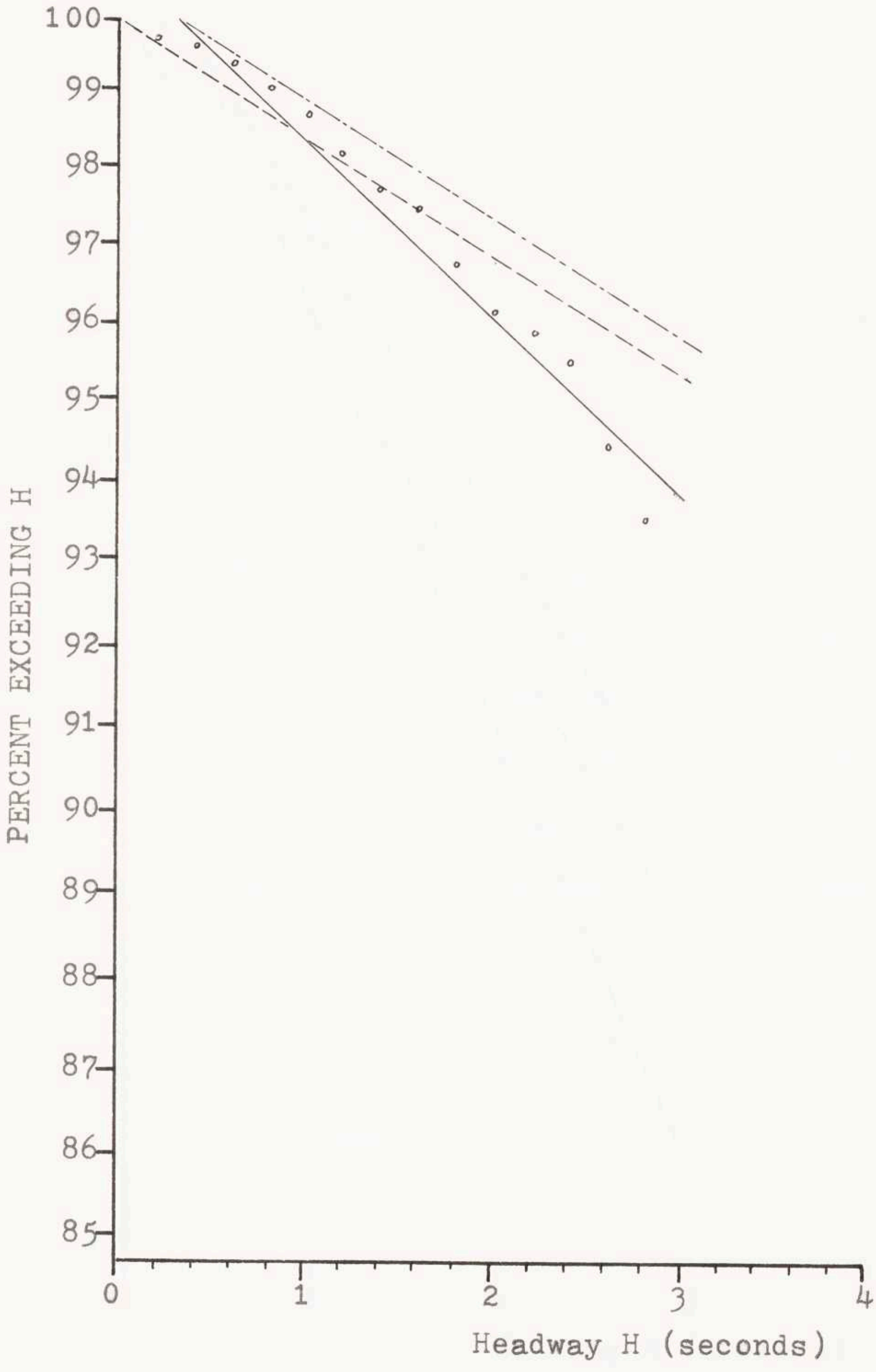
Figure [A.7]



Headway Histogram D29,31
789 trucks
11 hours

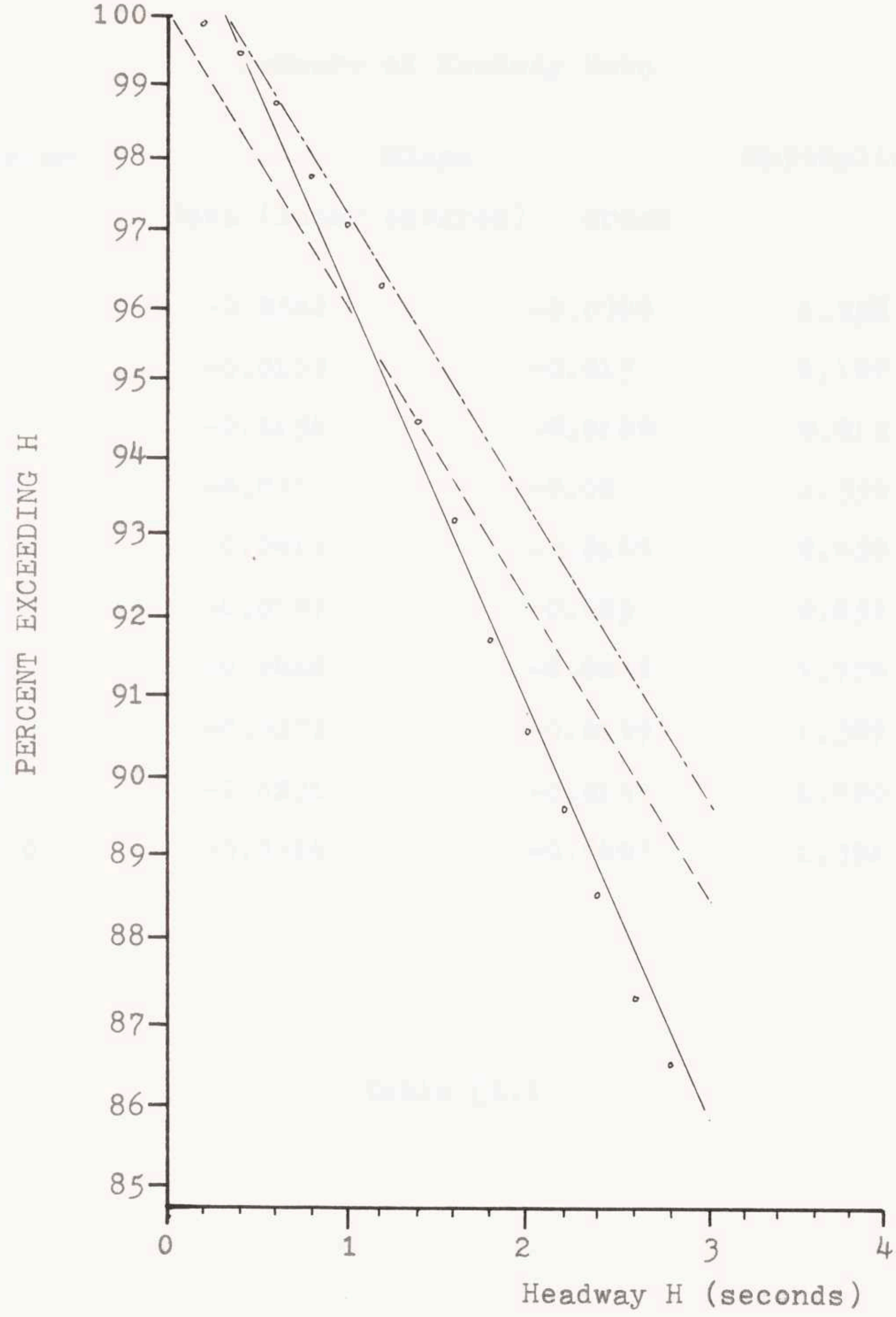
multiplier= 1.367

Figure [A.8]



Headway Histogram D34,36
612 trucks
11 hours
multiplier= 1.490

Figure [A.9]



Headway Histogram H6,8,10
1759 trucks
12 hours
3 lanes

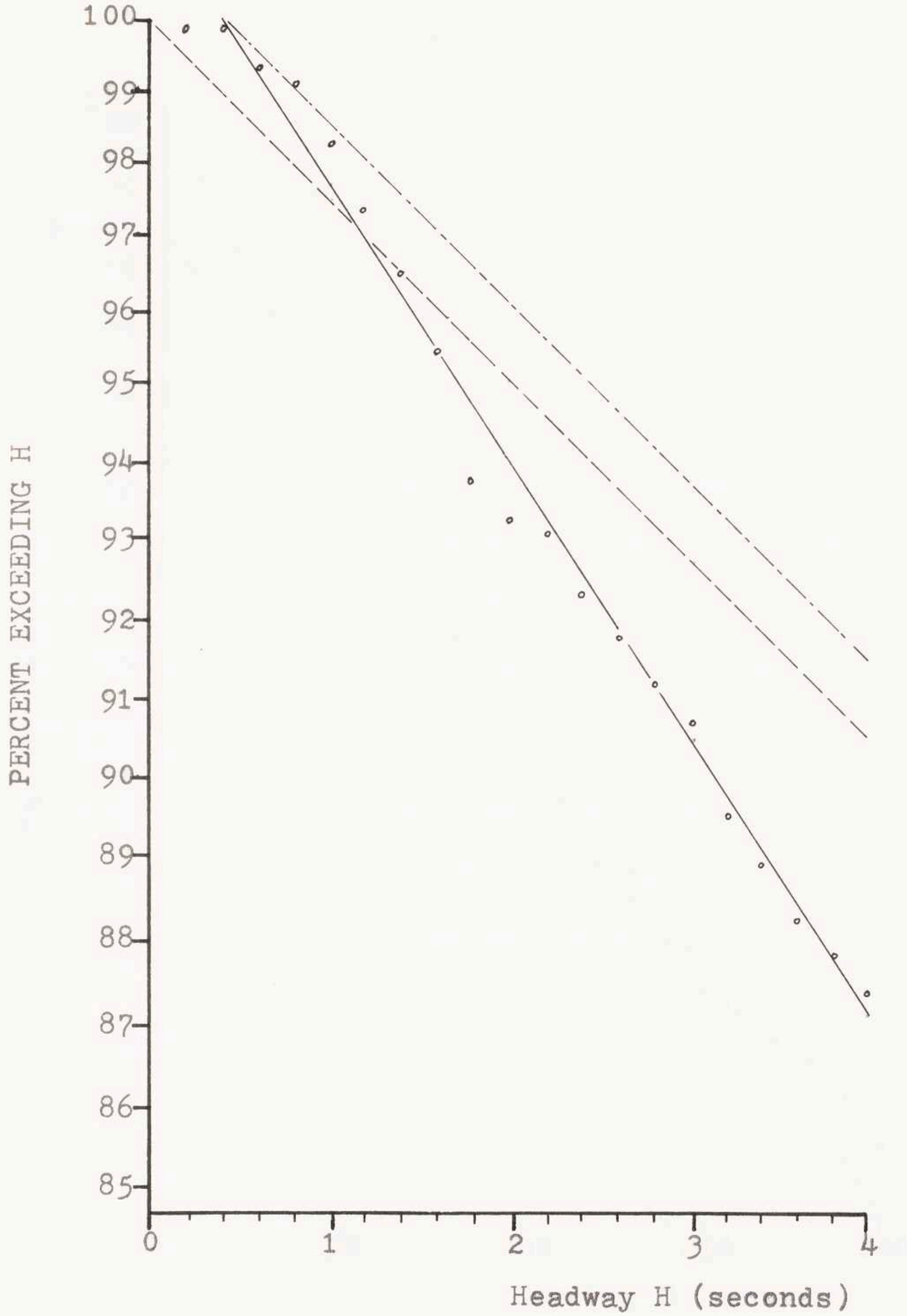
multiplier= 1.391

Figure [A.10]

Summary of Headway Data

Histogram	Slope		Multiplier
	Data (least squares)	Gross	
H2,4	-0.0542	-0.0399	1.358
W1,3	-0.0169	-0.015	1.127
W6,8	-0.0154	-0.0189	0.815
Ash5,7	-0.031	-0.02	1.550
Ash1,3	-0.0414	-0.0169	2.450
A37,39	-0.0197	-0.023	0.857
A41,43	-0.0440	-0.0248	1.774
D29,31	-0.0272	-0.0199	1.367
D34,36	-0.0231	-0.0155	1.490
H6,8,10	-0.0566	-0.0407	1.391

Table [A.1]



1077 trucks
11.75 hours multiplier= 1.538

Figure [A.11]

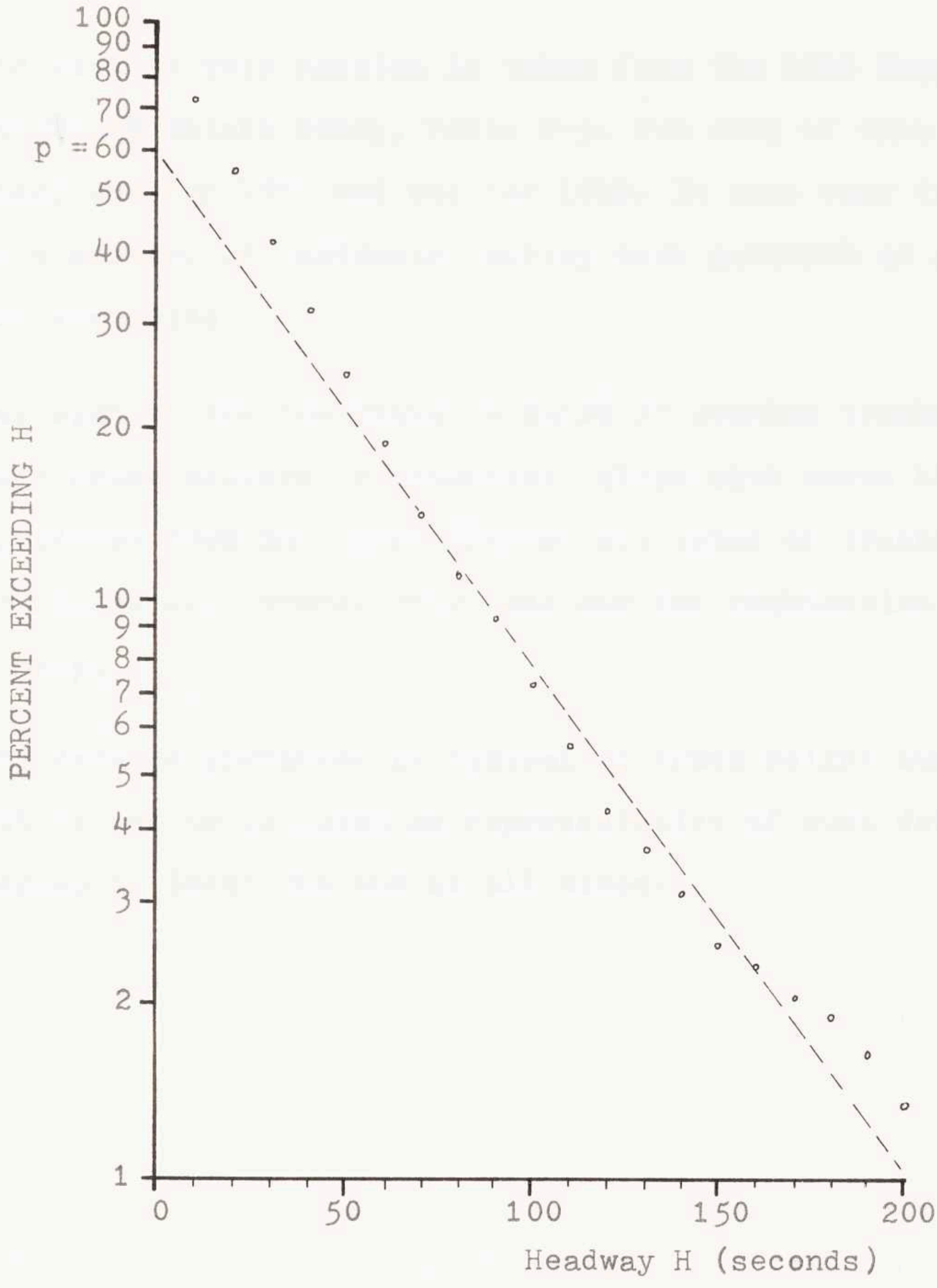

100
90
80
70
p = 60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
PERCENT EXCEEDING H
0
50
100
150
200
Headway H (seconds)

Figure [A.12]

Appendix B: Truck Weight Data

The data in this section is taken from the 1968 Massachusetts Truck Weight Study, Table W-5. Two sets of data are presented, one for 1967 and one for 1968. In each case the data is a summary of loadometer survey data gathered at all stations statewide.

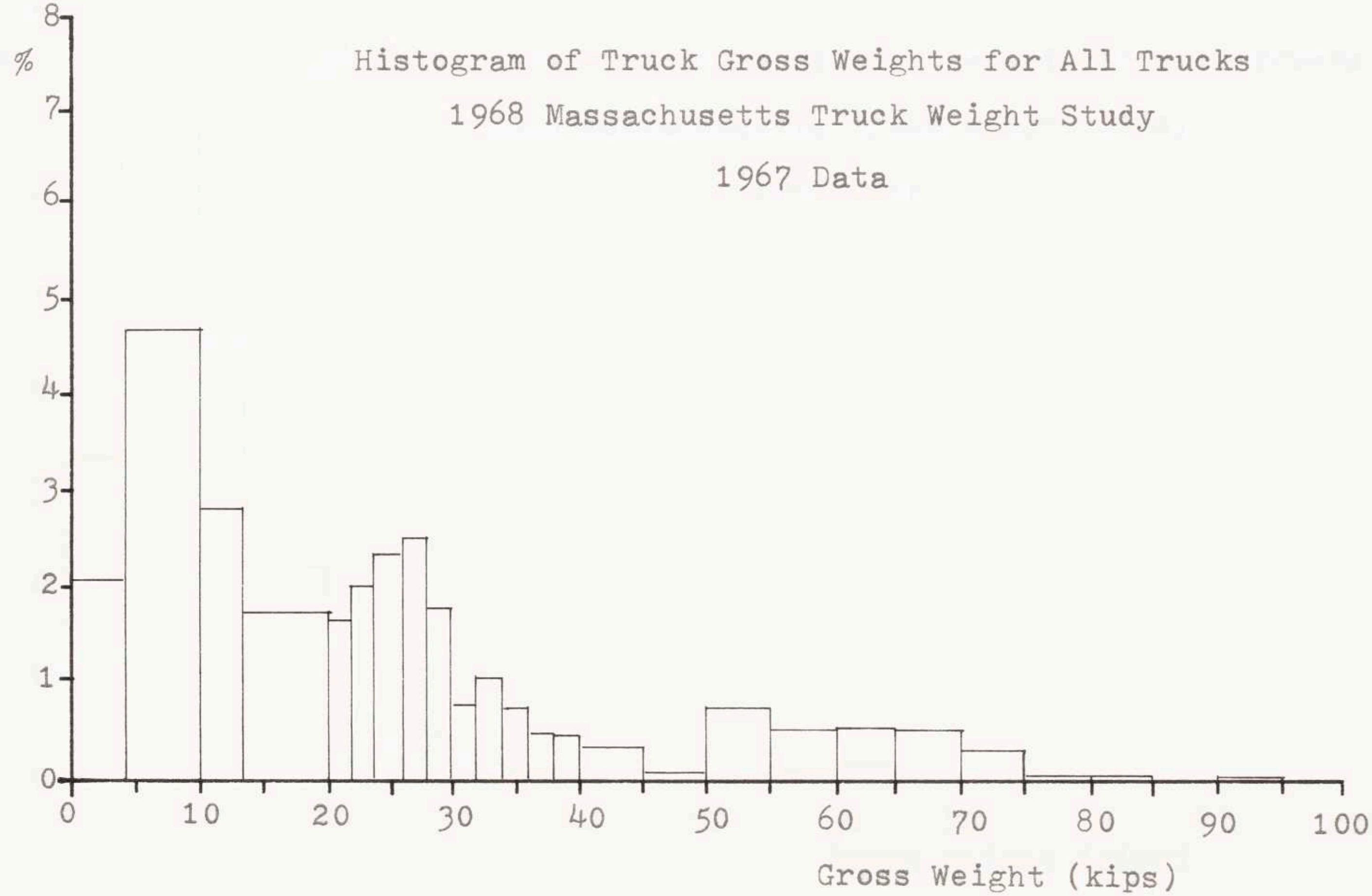
For each of the two years, a table of counted trucks and their gross weights is presented, along with three histograms produced from the data, one for all types of trucks, one for single unit trucks only, and one for combination trucks only.

The data is presented as typical of truck weight survey data but is not to be taken as representative of such data in all geographic locations and at all times.

1967 Data From 1968 Massachusetts Truck Weight Study
(Table W-5, Sheet 7, Number Counted)

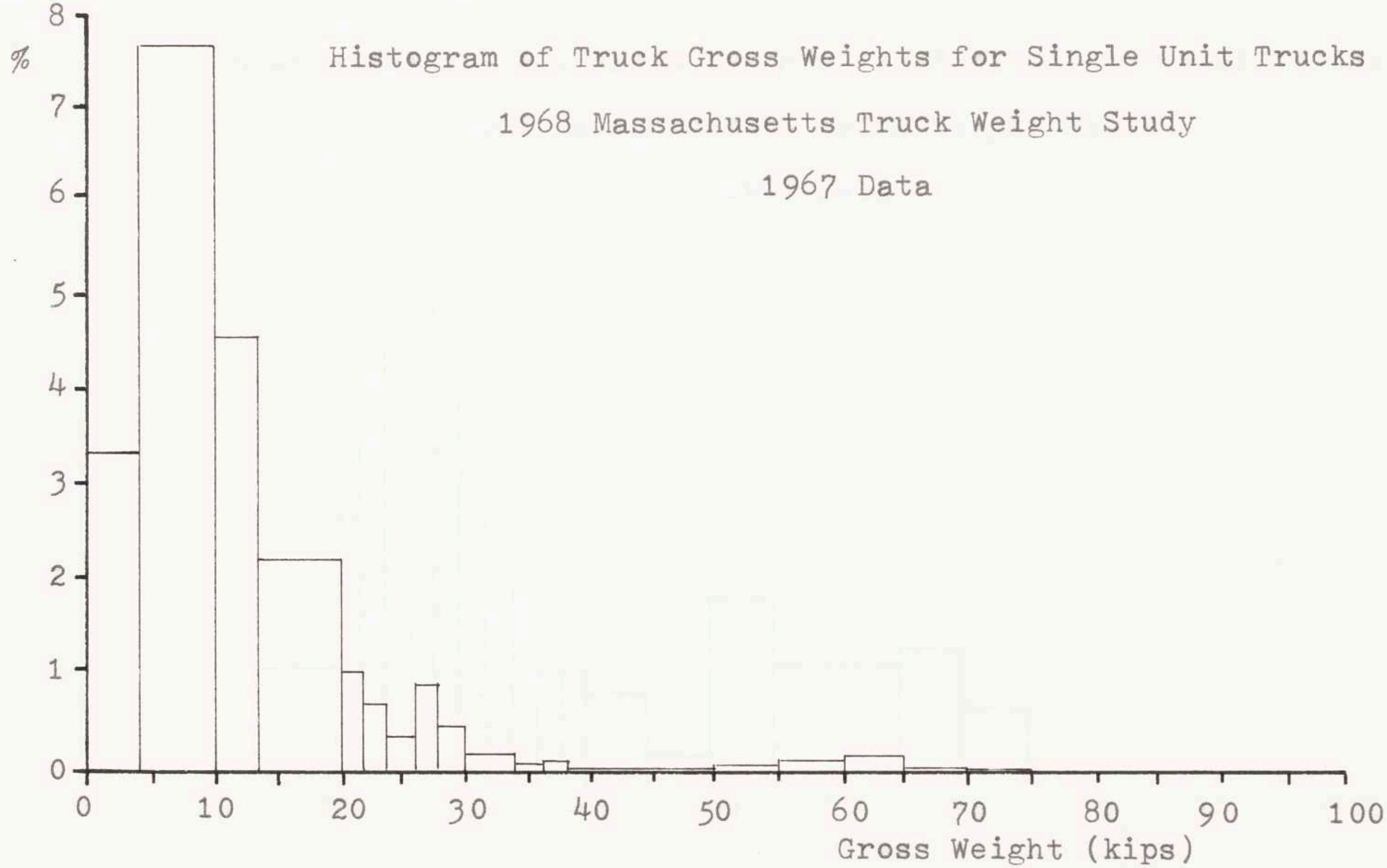
Gross Weight	Panel & Pickup	2 Axle 4 Tire	2 Axle 6 Tire	3 Axle	Total Single Unit	3 Axle	4 Axle	5 Axle	Total Semi-Tr Comb.	Total
<4	166	17			183					183
4-10	231	180	214		625					625
10-13.5	1	4	210	2	217					217
13.5-20		1	186	7	194	45	12		57	251
20-22		1	24	2	27	28	16	1	45	72
22-24		1	12	5	18	22	46	3	71	89
24-26			7	2	9	21	48	26	95	104
26-28			11	12	23	12	37	38	87	110
28-30			4	7	11	9	33	26	68	79
30-32			2	3	5	5	12	11	28	33
32-34			1	4	5	10	22	8	40	45
34-36	1		1	2	2	6	16	6	28	30
36-38			1	1	3	5	10	2	17	20
38-40				2	1	4	10	4	18	19
40-45				2	2	3	20	11	34	36
45-50				4	2		5	2	7	9
50-55				7	4	3	56	18	77	81
55-60				11	7		30	18	48	55
60-65				2	11		15	33	48	59
65-70				1	2		8	45	53	55
70-75					1		6	23	29	30
75-80							2	2	4	4
80-85							1	3	4	4
85-90										
90-95							1	1	2	2
95-100										
100-105										
105-155							1	1	2	2
					1352				862	2214

Table [B-1]



Histogram of Truck Gross Weights for All Trucks
1968 Massachusetts Truck Weight Study
1967 Data
%
8
7
6
5
4
3
2
1
0
0
10
20
30
40
50
60
70
80
90
100
Gross Weight (kips)

Figure [B.1]



%
8
7
6
5
4
3
2
1
0
Histogram of Truck Gross Weights for Single Unit Trucks
1968 Massachusetts Truck Weight Study
1967 Data
0
10
20
30
40
50
60
70
80
90
100
Gross Weight (kips)

Figure [B.2]

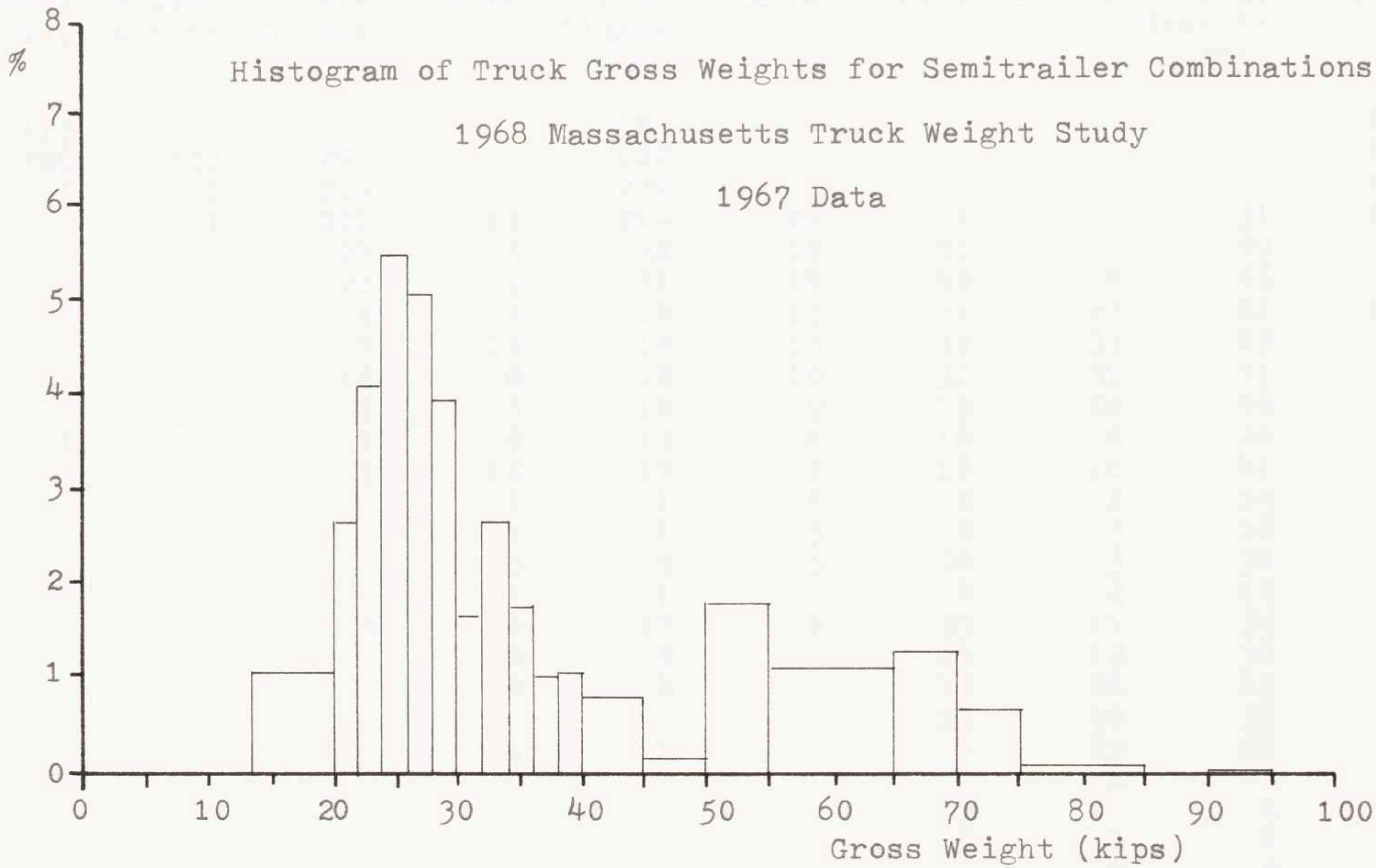

Histogram of Truck Gross Weights for Semitrailer Combinations
1968 Massachusetts Truck Weight Study
1967 Data
%
0
1
2
3
4
5
6
7
8
0
10
20
30
40
50
60
70
80
90
100
Gross Weight (kips)

Figure [B.3]

1968 Data From 1968 Massachusetts Truck Weight Study
(Table W-5, Sheet 7, Number Counted)

Gross Weight	Panel& Pickup	2 Axle 4 Tire	2 Axle 6 Tire	3 Axle	Total Single Unit	3 Axle	4 Axle	5 Axle	Total Semi-Tr Comb.	Total
<4	173	14			187					187
4-10	226	153	241		620					620
10-13.5	1	1	219	1	222					222
13.5-20		1	201	12	214	29	2		31	245
20-22			27	5	32	19	20		39	71
22-24			22	9	31	18	40	8	66	97
24-26			9	9	18	15	51	25	91	109
26-28			7	12	19	13	32	35	80	99
28-30			10	8	18	10	31	31	72	90
30-32			8	6	14	9	19	20	48	62
32-34	1		4	8	13	4	14	6	24	37
34-36			5	12	17	7	18	16	41	58
36-38				1	1	4	8	2	14	15
38-40				1	1	5	8	3	16	17
40-45				5	5	5	24	5	34	39
45-50				1	1		8	2	10	11
50-55			3	14	17	6	33	11	50	67
55-60				9	9		24	12	36	45
60-65				4	4		17	26	43	47
65-70							12	40	52	52
70-75				1	1		1	25	26	27
75-80								6	6	6
80-85							2		2	2
85-90							2	4	6	6
90-95								2	2	2
95-100								2	2	2
100-105								1	1	1
					1444				792	2236

Table [B.2]

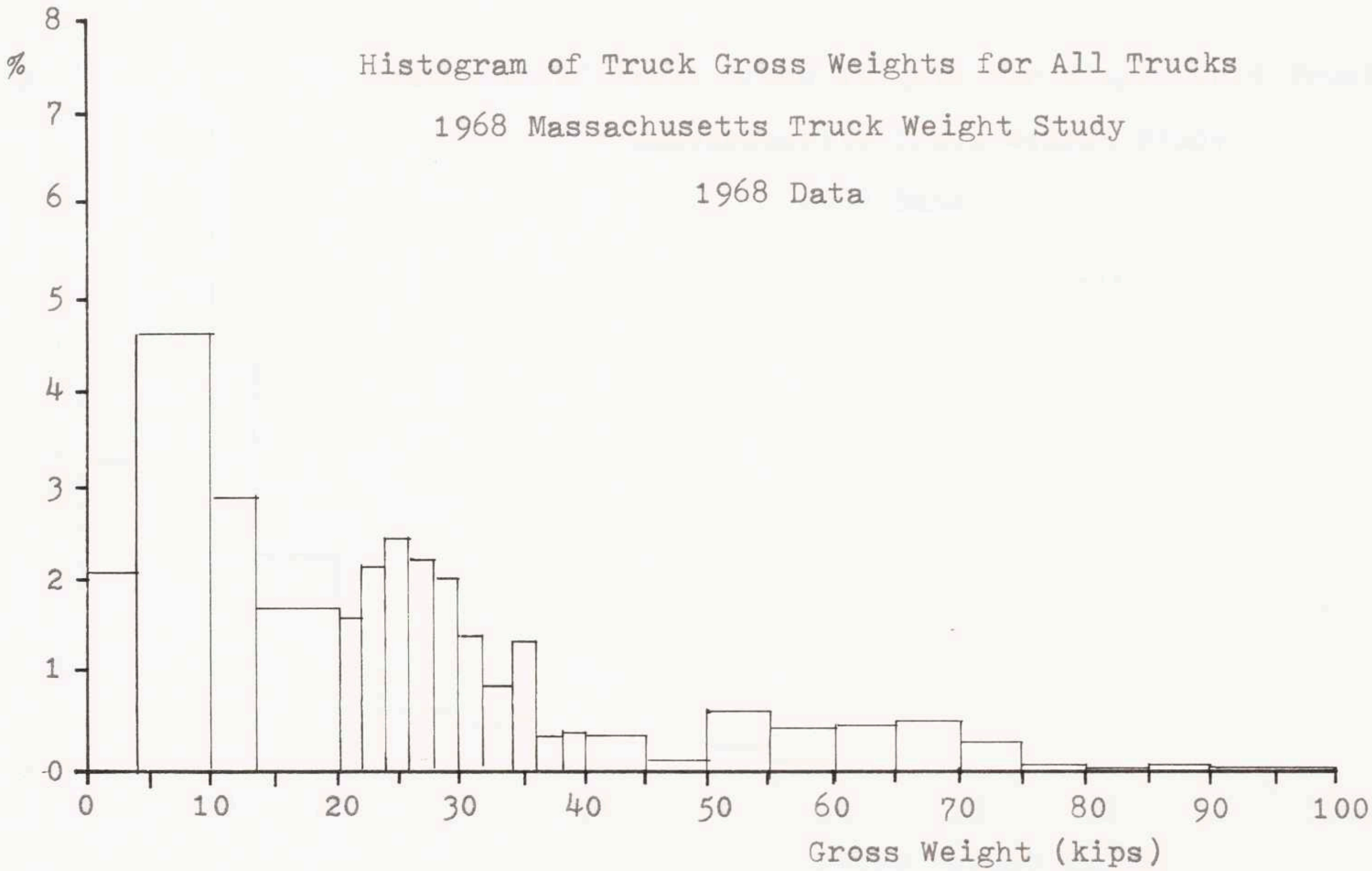

Histogram of Truck Gross Weights for All Trucks
1968 Massachusetts Truck Weight Study
1968 Data
%
8
7
6
5
4
3
2
1
0
0
10
20
30
40
50
60
70
80
90
100
Gross Weight (kips)

Figure [B.4]

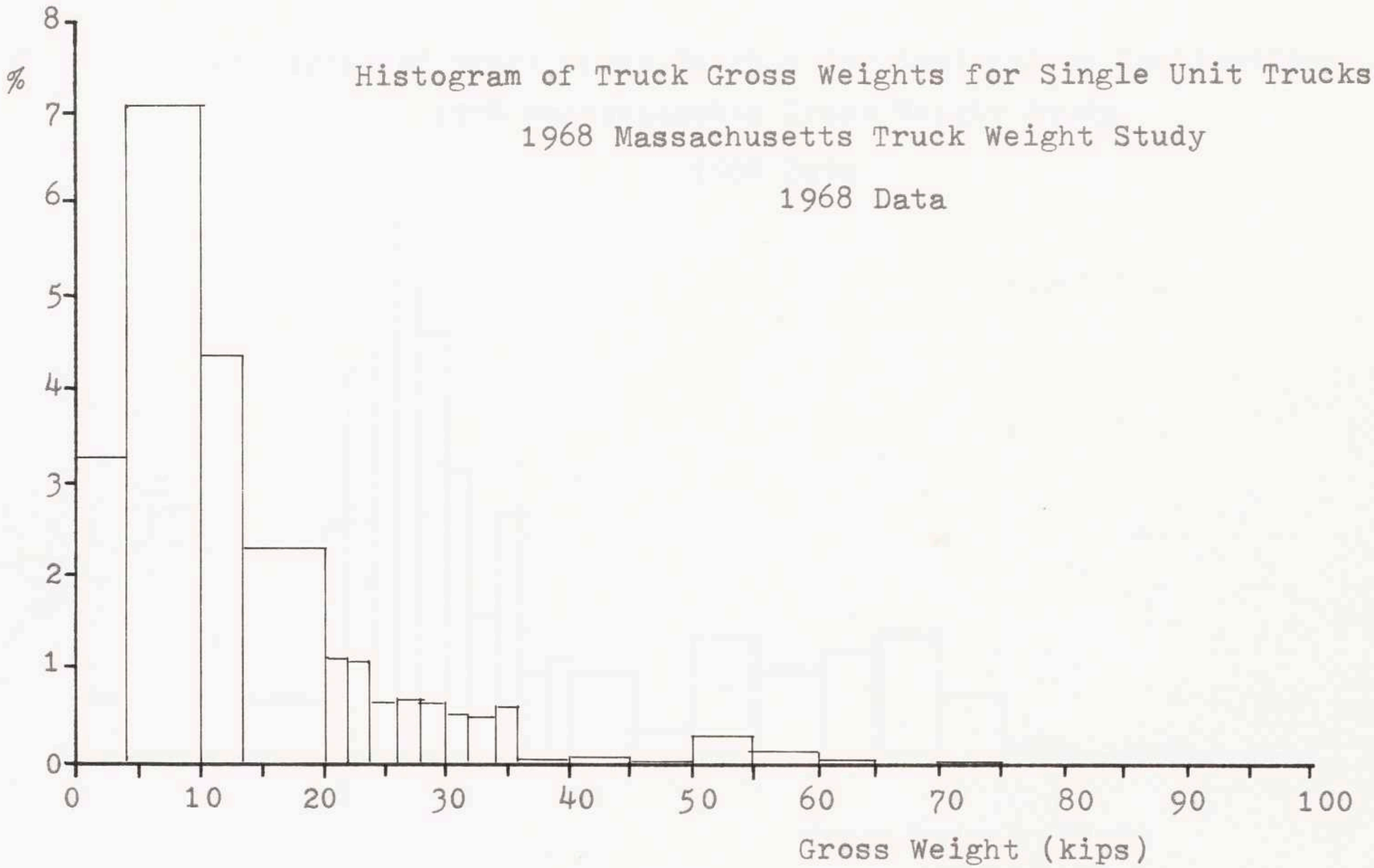
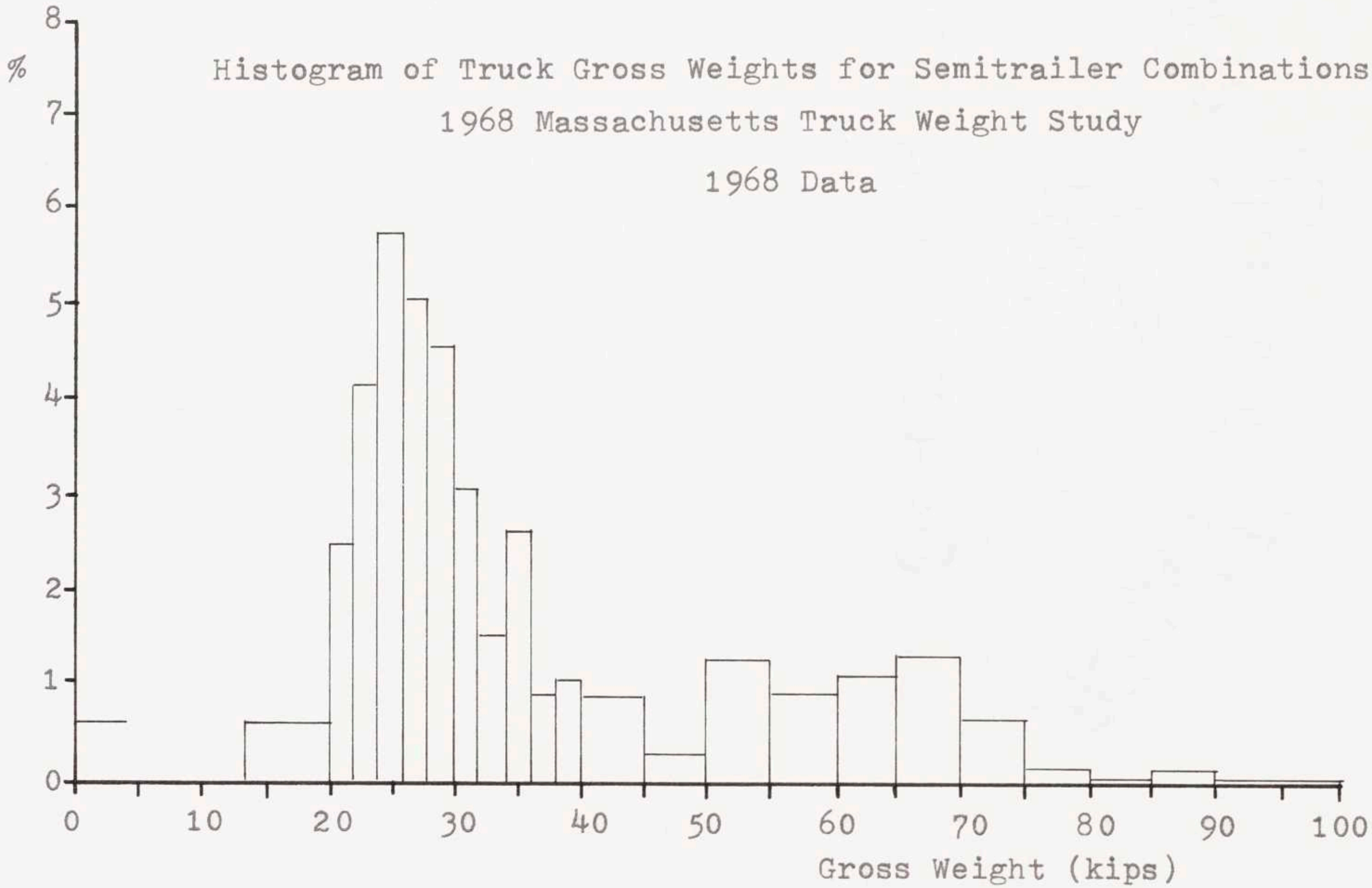

Histogram of Truck Gross Weights for Single Unit Trucks
1968 Massachusetts Truck Weight Study
1968 Data
%
0
1
2
3
4
5
6
7
8
0
10
20
30
40
50
60
70
80
90
100
Gross Weight (kips)

Figure [B.5]



%
Histogram of Truck Gross Weights for Semitrailer Combinations
1968 Massachusetts Truck Weight Study
1968 Data
0
1
2
3
4
5
6
7
8
0
10
20
30
40
50
60
70
80
90
100
Gross Weight (kips)

Figure [B.6]