NUMERICAL SOLUTION OF THE CABLE DYNAMIC EQUATIONS USING THE LINEARIZED EQUIVALENT DAMPING FORCE

by

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Abstract

The nonlinear static and linear dynamic equations of a mooring line are studied. Through a linearization technique the nonlinear drag forces are also included by means of an equivalent damping coefficient. This results in substantial reduction of the computing cost relative to fully nonlinear codes, while providing reasonable accuracy. Also, mooring line terminal impedances are derived, which can be used for an efficient analysis of the dynamics of a mooring system. The investigation into the effect of the major parameters on the dynamic tension of a cable is given for both inextensible and extensible cables, horizontal and inclined.

Numerical examples to demonstrate the validity of the theoretical developments of this thesis show good agreement compared with existing results. The reduced computing time to obtain results of good accuracy, when using an equivalent damping coefficient, offers the designer the opportunity to check many different configurations of a mooring system operating in a wide variety of environmental conditions, and hence the possibility for improving the mooring system design process.

Thesis Supervisor: Michael S. Triantafyllou
Title: Associate Professor

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Nomenclature

A	Area of the cable in stretched condition
A_o	Area of the cable in unstretched condition
B	Buoyancy of the cable per unit length in stretched condition
C_{Dn}	Normal drag coefficient
C_{Dt}	Tangential (frictional) drag coefficient
D_o	Diameter of the cable in unstretched condition
d	Water depth
E	Young's modulus
EA	Extensional rigidity
H_0	Horizontal Tension
H_{0*}	Horizontal Tension projected on the chord ($H_0/\cos\phi_a$ in two dimensional case)
L	Length of the unstretched cable
Re	Reynolds number
S_{xx}, S_{yy}	Impedance transfer function
T_e	Effective tension
T_o	Effective static tension
T_1	Effective dynamic tension
U	Current velocity

e	Strain
e_o	Static strain
e_1	Dynamic strain
g	Gravity constant
$\vec{i}$	unit vector in x direction (fixed reference system)
$\vec{j}$	unit vector in y direction (fixed reference system)
k	Wavenumber
l	Length of the static span of the cable
m	Mass per unit unstretched length
m_a	Added mass per unit unstretched length
M	Mass plus the added mass per unit unstretched length
$\vec{n}$	Normal unit vector in the (x,y) system
$\vec{t}$	Tangential unit vector in the (x,y) system
$\vec{n}_0$	Normal unit vector in the (p,q) system
$\vec{t}_0$	Tangential unit vector in the (p,q) system
p	Dynamic tangential displacement in $\vec{t}_o$ direction
q	Dynamic normal displacement in $\vec{n}_o$ direction
s	Unstretched Lagrangian coordinate along the centerline of the cable
t	Time
v_n	Normal velocity of the cable in the (x,y) system
v_t	Tangential velocity of the cable in the (x,y) system

ϵ	Small perturbation quantity
λ	Ratio of the elastic stiffness and the catenary stiffness for horizontal cables
λ_*	Ratio of the elastic stiffness and the catenary stiffness for inclined cables
α	Ratio of the total weight to the horizontal tension (w_0L/H_0)
α_*	Ratio of the total weight to the projected horizontal tension (w_0L/H_{0*})
ρ	Density of the cable material
ρ_w	Density of the water
ϕ	Euler angle (see definitions). In two dimensions it is the angle between the cable and the horizontal
ϕ_o	Static component of the ϕ
ϕ_1	Dynamic component of the ϕ
ϕ_a	Angle of the chord of the cable and the horizontal in two dimensions
ω	Circular frequency
σ	Nondimensional unstretched length parameter

INTRODUCTION

The recent move towards deeper water in the offshore industry made mooring systems very attractive and created an interest in studying such systems. Semi-submersibles operating at greater depths are normally positioned with a multi-leg mooring system.

The main function of a mooring system is to provide a holding force for offshore structures. The first step in the design of a mooring system is, therefore, a static analysis. The statics of a mooring line is a well understood probrem. Analytic and numeric techniques, which provide a reliable estimate of the holding force achieved through an appropriate adjustment of the static configuration, are available in the design process.

Also the dynamic behaviour of a mooring line is of interest, since it affects the motions and loads, and therefore the performance of the overall system. Since the 18th century, many researchers have studied the cable dynamics. A good survey can be found in [Choo 73], [Berteaux 76] and [Irvine 81]. Available solution techniques for cable dynamics, such as finite elements, finite differences and perturbation analysis, were used to simulate the dynamic behaviour of cables. The effect of the principal parameter on the dynamic tension has been studied in [Irvine 81] and [Bliek 84].

In this thesis, great attention is paid to the linearization of the nonlinear drag forces, which is one of sources of cable nonlinearities. The use of the equivalent damping cofficients resulting from that linearization allows us to reduce the long computational time, required for computing the dynamic tension by nonlinear time domain analysis, while obtaining results of similar accuracy.

Chapter 1

STATIC EQUATIONS

1.1 Introduction

Static equations which include the hydrodynamic loading, can be obtained by applying the condition of static equilibrium and by studying the geometric configuration of an infinitesimal segment of the mooring line. A derivation of static equations from the three dimensional dynamic equations by retaining only the time independent terms has been done by Bliek [Bliek 84]. The catenary equations have been derived by Irvine [Irvine 81].

In this chapter, the nonlinear static equations of a mooring line whose static configuration lies in one plane are derived.

1.2 Nonlinear Static Equations

We obtain the following set of nonlinear static equations by using the condition of static equilibrium as shown in Figure 1-1.

$$\frac{dT_0}{ds} = w_0 \sin \phi_0 - F_{t0}$$

$$T_0 \frac{d\phi_0}{ds} = w_0 \cos \phi_0 + F_{n0} \tag{1.1}$$

with :

$$F_{t0} = 0.5 \pi \rho_w C_{Dt} D_0 U \cos \phi_0 \mid U \cos \phi_0 \mid (1+e_0/2)$$

$$F_{n0} = 0.5\, \rho_w\, C_{Dn}\, D_0\, U \sin \phi_0\, |\, U \sin \phi_0\, |\, (1+e_0/2)$$

where :

T_0 = Static effective tension

ϕ_0 = Static component of the angle ϕ (See Equation (2.2) and (2.8))

C_{Dt} = Tangential drag coefficient

C_{Dn} = Normal drag coefficient

w_0 = Net weight of the cable per unit unstretched length

U = Velocity of current

e_0 = Static strain

ρ_w = Density of water

The static equations are accompanied by the following two geometric relations.

$$\frac{dx}{ds} = (1+e_0) \cos \phi_0$$
$$\frac{dy}{ds} = (1+e_0) \sin \phi_0 \tag{1.2}$$

1.3 Formulation and Solution of the Mathematical Model for Statics

We can form a complete mathematical model for the static problem of a mooring line by combining equations (1.1) and relations (1.2) and the following boundary conditions.

at bottom : Fixed

at top : Given top tension

(or horizontal component of the top tension)

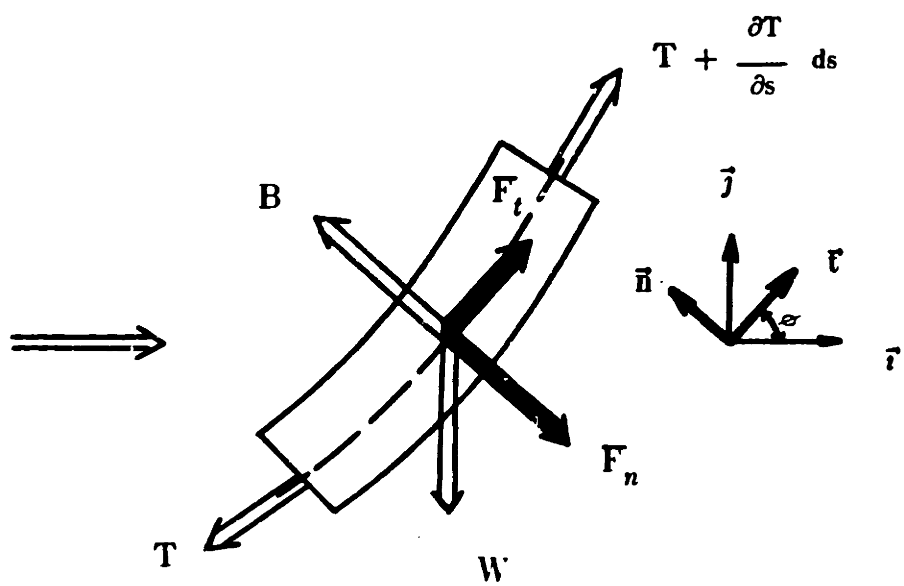

Figure 1-1: Static Forces

To determine the in-plane static configuration of the mooring line we can make use of the Runge-Kutta method to integrate the first order differential equations derived above. The fourth-order method is selected for small numerical errors [Hildebrand 49]. A shooting technique is used to solve the boundary value problem in terms of an initial value technique, combined with a stable iterative procedure. The Bisection method is selected.

Chapter 2

DYNAMIC EQUATIONS

2.1 Introduction

Many researchers have studied three-dimensional dynamics of a mooring line in various coordinate systems [Critescu 67], [Lenskii 78], and [Cannon 72]. Recently, Bliek has derived dynamic equations by considering kinematics and dynamics of a mooring line in three dimensions with compatibility relations [Bliek 84].

In this chapter we deal with the two-dimensional dynamics of a mooring line with coplanar static configuration. We pay particular attention to the nonlinear hydrodynamic forces. In the next chapter, the linearization of this nonlinear hydrodynamic force is fully investigated.

2.2 Decomposition of (Relative) Velocity and Drag Force

We have two different coordinate systems; one of which is based on the static configuration of a mooring line (static reference), while the other comes from the moving configuration (dynamic reference). The former is denoted by (p,q), the latter by (x,y). See Figur 2-1. We project the drag forces and velocities along both coordinate systems and relations between the corresponding components are derived.

Two Coordinate Systems

(p,q) : the fixed coordinate system

unit vectors $(\vec{t}_0,\vec{n}_0)$

(x,y) : the moving coordinate system

unit vectors $(\vec{t},\vec{n})$

Relations between Unit Vectors

$$\vec{t} = \vec{t}_0 \cos\phi_1 + \vec{n}_0 \sin\phi_1 \tag{2.1}$$

$$\vec{n} = -\vec{t}_0 \sin\phi_1 + \vec{n}_0 \cos\phi_1$$

Velocity

Direction	Cable	Current
$\vec{t}$	$v_t = \dfrac{\partial p}{\partial t} \cos \phi_1 + \dfrac{\partial q}{\partial t} \sin \phi_1$	$U \cos(\phi_1+\phi_0)$
$\vec{n}$	$v_n = -\dfrac{\partial p}{\partial t} \sin \phi_1 + \dfrac{\partial q}{\partial t} \cos \phi_1$	$-U \sin(\phi_1+\phi_0)$

(2.2)

where :

ϕ_0 = Static component of the angle ϕ

ϕ_1 = Dynamic component of the angle ϕ

v_t = Tangential component of the cable velocity

v_n = Nomal component of the cable velocity

Relative Velocity

$\vec{t}$

$$\frac{\partial p}{\partial t}\cos \phi_1 + \frac{\partial q}{\partial t} \sin \phi_1 - U \cos(\phi_1+\phi_0) \tag{2.3}$$

$\vec{n}$ $\qquad -\frac{\partial p}{\partial t}\sin\phi_1 + \frac{\partial q}{\partial t}\cos\phi_1 - [-U\sin(\phi_1+\phi_0)]$

Damping Force

$$F_T = F_{t1} + F_{n1}$$

$$= F_p + F_q \tag{2.4}$$

Note that Relative velocity means the relative velocity between the cable and the current.

Drag Force = - f (Relative Velocity)

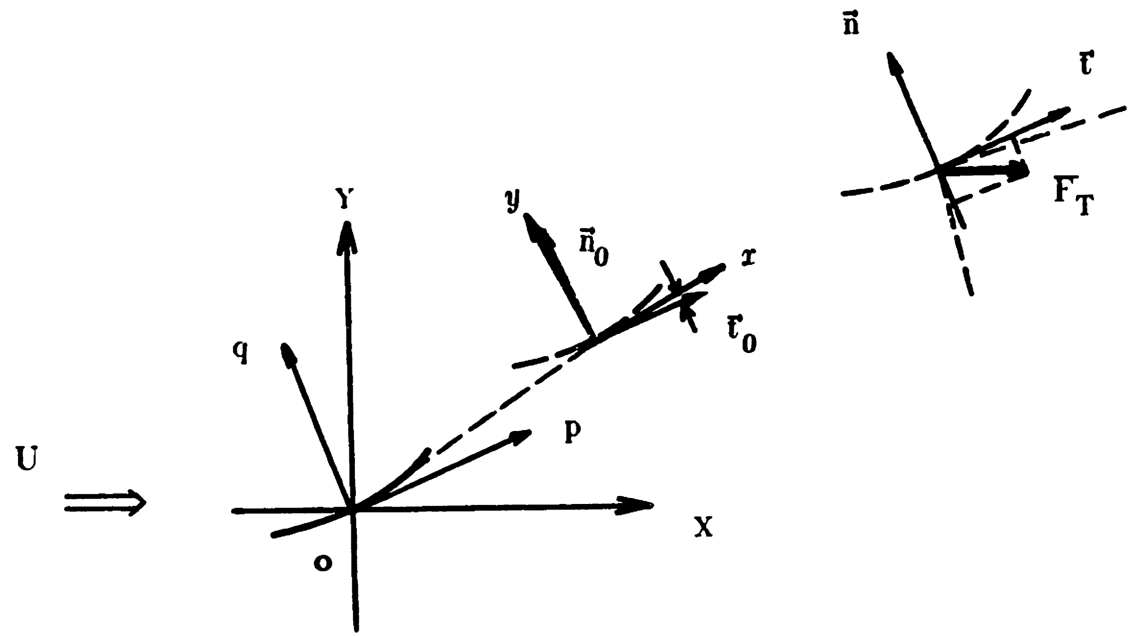

Figure 2-1: Coordinate Systems of Drag Forces and Velocities

2.3 Nonlinear Dynamic Equations

The dynamic equations, obtained from the dynamic equilibrium of forces acting on a mooring line in the moving coordinate system (x,y), including the nonlinear hydrodynamic forces are as follows :

$$m\left[\frac{\partial v_t}{\partial t} - v_n \frac{\partial \phi}{\partial t}\right] = \frac{\partial T_e}{\partial s} - w_0 \sin\phi + F_{t1}$$

$$m\left[\frac{\partial v_n}{\partial t} + v_t \frac{\partial \phi}{\partial t}\right] = T_e \frac{\partial \phi}{\partial s} - w_0 \cos\phi + F_{n1} - m_a \frac{\partial v_n}{\partial t} \tag{2.5}$$

with :

$$F_{t1} = 0.5\,\pi\,\rho_w\,C_{Dt}\,D_0\,(U\cos\phi - v_t)\,|U\cos\phi - v_t|\,(1+e/2)$$

$$F_{n1} = 0.5\,\rho_w\,C_{Dt}\,D_0\,(-U\sin\phi - v_n)\,|-U\sin\phi - v_n|\,(1+e/2)$$

$$v_t = \frac{\partial p}{\partial t}\cos\phi_1 + \frac{\partial q}{\partial t}\sin\phi_1$$

$$v_n = -\frac{\partial p}{\partial t}\sin\phi_1 + \frac{\partial q}{\partial t}\cos\phi_1$$

where :

m = Mass per unit unstretched length

T_e = Effective tension

e = Strain

The compatibility equations for the mooring lines must be satisfied. :

$$\frac{\partial v_t}{\partial s} - \frac{\partial \phi}{\partial s} v_n = \frac{\partial e}{\partial t}$$

$$\frac{\partial v_n}{\partial s} + \frac{\partial \varphi}{\partial s} v_t = (1+e) \frac{\partial \phi}{\partial t} \qquad (2.6)$$

Finally, a constitutive equation for the cable must be added :

$$T_e = f\left(e, \frac{\partial e}{\partial t}, s\right) \qquad (2.7)$$

Equations (2.5), (2.6) and (2.7) constitute the complete set of the nonlinear two-dimensional dynamic equations (5 equations with 5 unknowns).

An important simplification of the dynamic problem is obtained when the solution is assumed to consist of a quasi-static part, which is slowly varying in time and represents a change in the mean configuration of the mooring line, and a dynamic part, which constitutes small harmonic oscillations around the mean position. The governing equations can then be separated in nonlinear static and linearized dynamic equations. In this chapter, we investigate the linearization of the nonlinear dynamic equations

2.4 Linearization

We derive the equations governing the dynamics of small deviations (p,q) from the static configuration along the tangential and normal directions respectively, by expanding the set of nonlinear equations (2.5), (2.6) and (2.7) written in the moving coordinate system about the static configuration and neglecting higher order terms, i.e. we set :

$$T_e = T_0 + T_1 \qquad (2.8)$$

$$\phi = \varphi_0 + \varphi_1$$

Finally, we obtain the following set of equations in the (p,q) coordinate system :

$$m \frac{\partial^2 p}{\partial t^2} = \frac{\partial T_1}{\partial s} + F_{t1} - T_0 \frac{d\phi_0}{ds} \phi_1$$

$$M \frac{\partial^2 q}{\partial t^2} = \frac{d\phi_0}{ds} T_1 + \frac{\partial \phi_1}{\partial s} T_0 + F_{n1} + \frac{dT_0}{ds} \phi_1$$

$$\frac{\partial p}{\partial s} - q \frac{d\phi_0}{ds} = \frac{T_1}{EA} \tag{2.9}$$

$$\frac{\partial q}{\partial s} + p \frac{d\phi_0}{ds} = \phi_1 (1+e_0)$$

Also it can be shown that the equations for the in-plane dynamics (p,q) including the compatibility equations, are completely decoupled from the equation for the out-of-plane motion (r), for the case of a static two- dimensional configuration. For a proof, see [Bliek 84].

Chapter 3

LINEARIZATION OF THE NONLINEAR DRAG FORCES

3.1 Introduction

In this chapter we consider the possibility of using an equivalent linearization technique to model the drag forces. We will limit the analysis to cables with a sinusoidal excitation at the top. When an equivalent linearization technique is used, the motion equations can be solved in the frequency domain, reducing considerably the calculation time.

The cable experiences a significant nonlinear drag force when subjected to a top excitation. The drag force is a function of the relative velocity between the cable and the current. In this chapter we decompose this nonlinear force into a mean force plus an equivalent linear damping force, by minimizing the quadratic error between the equivalent representation and the nonlinear drag force, averaged over one period. The mean force can be included in the static analysis. Equivalent linearized cable dynamic equations can be formulated using the governing equations in chapter 2 and the damping coefficients derived in this chapter. These equivalent linearized dynamic equations can be easily solved in the frequency domain.

The equivalent linearization technique has been applied to the OTEC cold water pipe dynamics [Paulling 79] and riser dynamics [Krolikowski 80]. See also [Triantafyllou 82].

3.2 Nonlinear Drag forces in Two-Dimensions

The drag forces can be written in the (p,q) coordinate system according to the Morison's formula as follows :

$$\begin{aligned}
F_{ext,p}\,(1+e) = {} & 0.5\,\rho_w\, C_{Dn}(Re)\, D_o\, (U\sin\phi + v_n)\,|\,U\sin\phi + v_n\,| \\
& (1+e/2)\sin\phi_1 \\
& + 0.5\,\pi\,\rho_w\, C_{Dt}(Re)\, D_o\, (U\cos\phi - v_t)\,|\,U\cos\phi - v_t\,| \\
& (1+e/2)\cos\phi_1
\end{aligned} \tag{3.1}$$

$$\begin{aligned}
F_{ext,q}(1+e) = {} & -\,0.5\rho_w\, C_{Dn}(Re)\, D_o\, (U\sin\phi + v_n)\,|\,U\sin\phi + v_n\,| \\
& (1+e/2)\cos\phi_1 \\
& + 0.5\,\pi\,\rho_w\, C_{Dt}\,(Re)\, D_o\,(\,U\cos\phi - v_t)\,|\,U\cos\phi - v_t\,| \\
& (1+e/2)\sin\phi_1
\end{aligned}$$

where: $\phi = \phi_o + \phi_1$

$$v_t = \frac{\partial p}{\partial t}\cos\phi_1 + \frac{\partial q}{\partial t}\sin\phi_1$$

$$v_n = -\frac{\partial p}{\partial t}\sin\phi_1 + \frac{\partial q}{\partial t}\cos\phi_1$$

We can rewrite these equations in nondimensional form using the relative velocities between the cable and the current as;

$$\mathcal{F}_p = \frac{F_p}{0.5\,\rho_w\, D_o\, U|U|}$$

$$= C_{Dn}\, A\,|\,A\,|\,\sin\phi_1 + \pi\, C_{Dt}\, B\,|\,B\,|\,\cos\phi_1$$

(3.2)

$$\mathcal{F}_q = \frac{F_q}{0.5\,\rho_w\, D_o\, U|U|}$$

$$= -C_{Dn}\, A\,|\,A\,|\,\cos\phi_1 + \pi\, C_{Dt}\, B\,|\,B\,|\,\sin\phi_1$$

where:

$$A = (\sin\phi_o + \frac{1}{U}\frac{\partial q}{\partial t})\cos\phi_1 + (\cos\phi_o - \frac{1}{U}\frac{\partial p}{\partial t})\sin\phi_1$$

$$B = (\cos\phi_o - \frac{1}{U}\frac{\partial p}{\partial t})\cos\phi_1 - (\sin\phi_o + \frac{1}{U}\frac{\partial q}{\partial t})\sin\phi_1$$

The following new variables are introduced and substituted for the absolute signs.

$$\mathrm{sgn}(A) = \frac{|A|}{A} \qquad \mathrm{sgn}(B) = \frac{|B|}{B} \tag{3.3}$$

The nonlinear nondimensionalized drag forces can now be written as;

$$\mathcal{F}_p = C_{Dn}\, A^2\, \mathrm{sgn}(A)\, \sin\phi_1 + \pi\, C_{Dt}\, B^2\, \mathrm{sgn}(B)\, \cos\phi_1$$

(3.4)

$$\mathcal{F}_q = -\, C_{Dn}\, A^2\, \mathrm{sgn}(A)\, \cos\phi_1 + \pi\, C_{Dt}\, B^2\, \mathrm{sgn}(B)\, \sin\phi_1$$

3.3 Assumptions

Several assumptions are made in order to approximate the nonlinear drag force with an equivalent linearized force.

1. The linearized cable equations, except for the drag forces, are valid. The motions are small and the dynamic tension generated is small compared to the static tension.

2. The excitation is sinusoidal

3. The tangential drag forces are small compared to the normal drag forces ($\pi C_{Dt} = O(\epsilon)$).

Under the above conditions, the two dimensional sinusoidal motions can be written as:

$$
\begin{aligned}
p &= p_a \cos(\omega t+\epsilon_1) \\
q &= q_a \cos(\omega t+\epsilon_2) \qquad (3.5) \\
\phi_1 &= \phi_{1a} \cos(\omega t+\epsilon_3)
\end{aligned}
$$

where : p_a. q_a and ϕ_{1a} are small quantities of order ϵ

and ϵ_1, ϵ_2 and ϵ_3 represent the phase angles.

3.4 Approximation of the Nonlinear Drag Force for Small Motions

The expressions for the nonlinear drag forces can be simplified significantly if small motions and a small tangential drag coefficient are assumed. Generally we can distinguish three cases as a function of the ratio of the cable velocity to the current velocity.

$$\frac{\omega p_a}{U} \text{ and } \frac{\omega q_a}{U} \ll 1$$

$$\frac{\omega p_a}{U} \text{ and } \frac{\omega q_a}{U} \approx 1 \tag{3.6}$$

$$\frac{\omega p_a}{U} \text{ and } \frac{\omega q_a}{U} \gg 1$$

The two limiting cases, which represent the large current and the small current velocity (compared with the cable velocity) are studied here.

3.4.1 Large Current Velocity

For large current:

$$\frac{\omega p_a}{U} \text{ and } \frac{\omega q_a}{U} \approx \epsilon$$

The drag forces can be approximated to $O(\epsilon^2)$ as:

$$\begin{aligned} F_p(t) &\approx C_{Dn} \sin\phi_o \,|\sin\phi_o|\; \phi_{1a}\cos(\omega t+\epsilon_3) \\ &\quad + \pi\, C_{Dt} \cos\phi_o \,|\cos\phi_o\,| \\ F_q(t) &\approx -\, C_{Dn}\{ \sin\phi_o \,|\sin\phi_o\,| - 2\frac{\omega q_a}{U}\,|\sin\phi_o|\sin(\omega t+\epsilon_1) \\ &\quad + 2\,|\sin\phi_o\,|\cos\phi_o\; \phi_{1a}\cos(\omega t+\epsilon_3)\} \end{aligned} \tag{3.7}$$

3.4.2 Small Current Velocity

For small current:

$$\frac{U}{\omega p_a} \text{ and } \frac{U}{\omega q_a} \approx \epsilon$$

The drag forces can be approximated to O(1):

$$F_p(t) \approx - C_{Dn} \left|\frac{\omega q_a}{U} \sin(\omega t+\epsilon_2)\right|\frac{\omega q_a}{U} \sin(\omega t+\epsilon_2)\ \phi_{1a}\cos(\omega t+\epsilon_3)$$

$$+ \pi\, C_{Dt} \left|\frac{\omega p_a}{U} \sin(\omega t+\epsilon_1)\right| \frac{\omega p_a}{U} \sin(\omega t+\epsilon_1)$$

(3.8)

$$F_q(t) \approx C_{Dn} \left|\frac{\omega q_a}{U} \sin(\omega t+\epsilon_2)\right| \frac{\omega q_a}{U} \sin(\omega t+\epsilon_2)$$

3.5 Method of Least Squares

We propose to approximate the above equations by forces proportional to the cable velocities as follows:

$$F_{ep} = F_{Fop} + C_p \frac{1}{U}\frac{\partial p}{\partial t}$$

(3.9)

$$F_{eq} = F_{Foq} + C_q \frac{1}{U}\frac{\partial q}{\partial t}$$

In order to minimize the integral of the square of the residual over one period, interval the Method of Least squares is used. The residuals are defined as:

$$E_p = F_p - F_{ep}$$

(3.10)

$$E_q = F_q - F_{eq}$$

The following conditions must be satisfied:

$$\frac{\partial \overline{E}_p^{\,2}}{\partial C_p} = 0 \qquad \frac{\partial \overline{E}_q^{\,2}}{\partial C_q} = 0 \tag{3.11}$$

3.6 Integral of the Square of the Residual over One Period

3.6.1 Large Current

$$E_p^{\,2} = 1/(2\pi)\int_0^{2\pi} \{\, C_{Dn} \sin\phi_o\, |\sin\phi_0|\, \phi_{1a} \cos(\omega\tau+\epsilon_3) + \pi\, C_{Dt} \cos\phi_o\, |\cos\phi_o| - F_{Fop} + C_q \frac{\omega p_a}{U} \sin(\omega\tau+\epsilon_1) \,\}^2\, d\tau \tag{3.12}$$

If the method of least squares is applied, we can find the equivalent mean static force and the linear force as follows:

$$F_{Fop} = \pi\, C_{Dt} \cos\phi_o\, |\cos\phi_o| \qquad C_p = -\, C_{Dn} \sin\phi_o\, |\sin\phi_o|\, \phi_{1a} \sin(\epsilon_1-\epsilon_3) \frac{U}{\omega p_a} \tag{3.13}$$

Similarly,

$$F_{Foq} = -\, C_{Dn} \sin\phi_o\, |\sin\phi_o| \qquad C_q = -\,2\, C_{Dn} |\sin\phi_o| + 2\, C_{Dn} |\sin\phi_o| \cos\phi_o\, \phi_{1a} \sin(\epsilon_2-\epsilon_3) \frac{U}{\omega q_a} \tag{3.14}$$

3.6.2 Small Current or No External Current

In this case we obtain:

$$F_{\mathrm{Fop}} = 0 \qquad\qquad F_{\mathrm{Foq}} = 0 \tag{3.15}$$

$$C_{\mathrm{p}} = -\ \pi\, C_{\mathrm{Dt}} \frac{8}{3\pi} \frac{\omega p_a}{U} \qquad\qquad C_{\mathrm{q}} = -\ C_{\mathrm{Dn}} \frac{8}{3\pi} \frac{\omega p_a}{U}$$

3.7 Linearized Equivalent Damping Coefficients

From the above relations we can obtain the equivalent damping coefficients written in dimensional form:

$$F_p = -\ b_p \frac{\partial p}{\partial t}$$

$$F_q = -\ b_q \frac{\partial q}{\partial t} \tag{3.16}$$

where b_p and b_q are the equivalent damping coefficients obtained from the equations (3.12), (3.13) and (3.14). The final forms for the equivalent damping coefficients can be obtained for each case as :

$$b_p = 0.5\, \rho_w\, C_{\mathrm{Dn}}\, D_0 \left(\frac{|U\sin\phi_o|\, U\sin\phi_o}{\omega p_a}\right) \phi_{1a} \sin(\epsilon_1 - \epsilon_3)$$

$$b_q = \rho_w\, C_{\mathrm{Dn}}\, D_0 \left\{ |U\sin\phi_o| - \left(\frac{|U\sin\phi_o|\, U\sin\phi_o}{\omega q_a}\right) \phi_{1a} \sin(\epsilon_2 - \epsilon_3) \right\} \tag{3.17}$$

For small current:

$$b_p = \rho_w \, \pi \, C_{Dt} \, D_0 \frac{4}{3\pi} \omega p_a$$

$$b_q = \rho_w \, C_{Dn} \, D_0 \frac{4}{3\pi} \omega q_a \tag{3.18}$$

The equivalent linearization of the nonlinear hydrodynamic drag forces can be used directly in the equations obtained in chapter 2. The matrix formulation is now in terms of complex quantities due to the damping forces and has been implemented numerically. To find the correct estimation of the linearized damping coefficient, an iteration procedure starting from an assumed initial amplitude is used. This numerical procedure has been implemented in a computer code.

Chapter 4

TERMINAL IMPEDANCES

4.1 Introduction

The concept of termination impedances has been widely used in the design of guyed masts (See for example [Davenport 65]). In mooring line design, the termination impedances can also be useful. In this chapter we will discuss some introductory concepts about terminal impedances. This will allow us to calculate the eigenfrequencies and the linear response for multi-leg systems. The concept discussed here is only valid for linear systems. In the case where equivalent drag damping is used, the superposition principle is not valid anymore, so that the combined response can only be obtained by adding the equivalent impedance coefficients with the correct motion for each leg of a multi-leg system. Further research in the domain of equivalent impedance coefficients for multi-leg systems is certainly required.

4.2 Terminal Impedances in Two-Dimensions

The upper end of the cable is excited by an externally imposed harmonic motion. The mooring line termination impedances are defined in the following way:

$$\begin{bmatrix} S_{xx}(\omega) & S_{xy}(\omega) \\ S_{yx}(\omega) & S_{yy}(\omega) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} \tag{4.1}$$

where: x: complex amplitude motion, horizontal direction

y: complex amplitude motion, vertical direction

F_x: complex amplitude force, horizontal direction

F_y: complex amplitude force, vertical direction

The resonance frequencies are the poles of the impedance transfer functions. The mooring line admittance matrix can be calculated as the inverse of the impedance matrix. In practical applications, the above transfer matrices can be used to find body motions and dynamic tensions.

At the top of the cable, sinusoidal motions in the x direction and the y direction are imposed. The dynamic forces at the top are obtained as:

$$S_{xx} = [\, T_{11} \cos\phi_o - T_o \sin\phi_o \, \phi_{11} \,]/A_x$$
$$S_{yx} = [\, T_{11} \sin\phi_o + T_o \cos\phi_o \, \phi_{11} \,]/A_x \qquad (4.2)$$

where: T_{11}, ϕ_{11} are the dynamic tension and the dynamic angle, respectively, caused by external motion in the x direction

$$S_{xy} = [\, T_{12} \cos\phi_o - T_o \sin\phi_o \, \phi_{12} \,]/A_y$$
$$S_{yy} = [\, T_{12} \sin\phi_o + T_o \cos\phi_o \, \phi_{12} \,]/A_y \qquad (4.3)$$

where: T_{12}, ϕ_{12} are the dynamic tension and the dynamic angle, respectively, caused by external motion in the y direction

The dynamic tension at the top due to unit motion in the x or the y direction is also a very important transfer function. It indicates how much dynamic tension is generated in the cable by a unit amplitude motion, and can be used directly in fatigue analysis. The dynamic tension transfer functions are defined as;

$$S_{Tx} = T_{11}/A_x \qquad (4.4)$$

$$S_{Ty} = T_{12}/A_y$$

The solutions of the above equations can be obtained by solving the linearized dynamic equations in the frequency domain. Solutions were obtained in this theory by using a finite difference scheme.

4.3 Application to a Two-leg System with a Co-planar Static Configuration

The impedance functions for the in-plane motions are completely uncoupled from the out-of-plane motions (assuming small motions). The total impedance function can be obtained by simple addition of the two single line impedance functions (see figure 4-1):

$$\begin{bmatrix} S^1_{xx} + S^2_{xx} & S^1_{xy} - S^2_{xy} \\ S^1_{yx} - S^2_{yx} & S^1_{yy} + S^2_{yy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} \tag{4.5}$$

Especially, if the two legs are symmetric, the above relation is reduced to the simple form:

$$\begin{bmatrix} 2\,S^1_{xx} & 0 \\ 0 & 2\,S^1_{yy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} \tag{4.6}$$

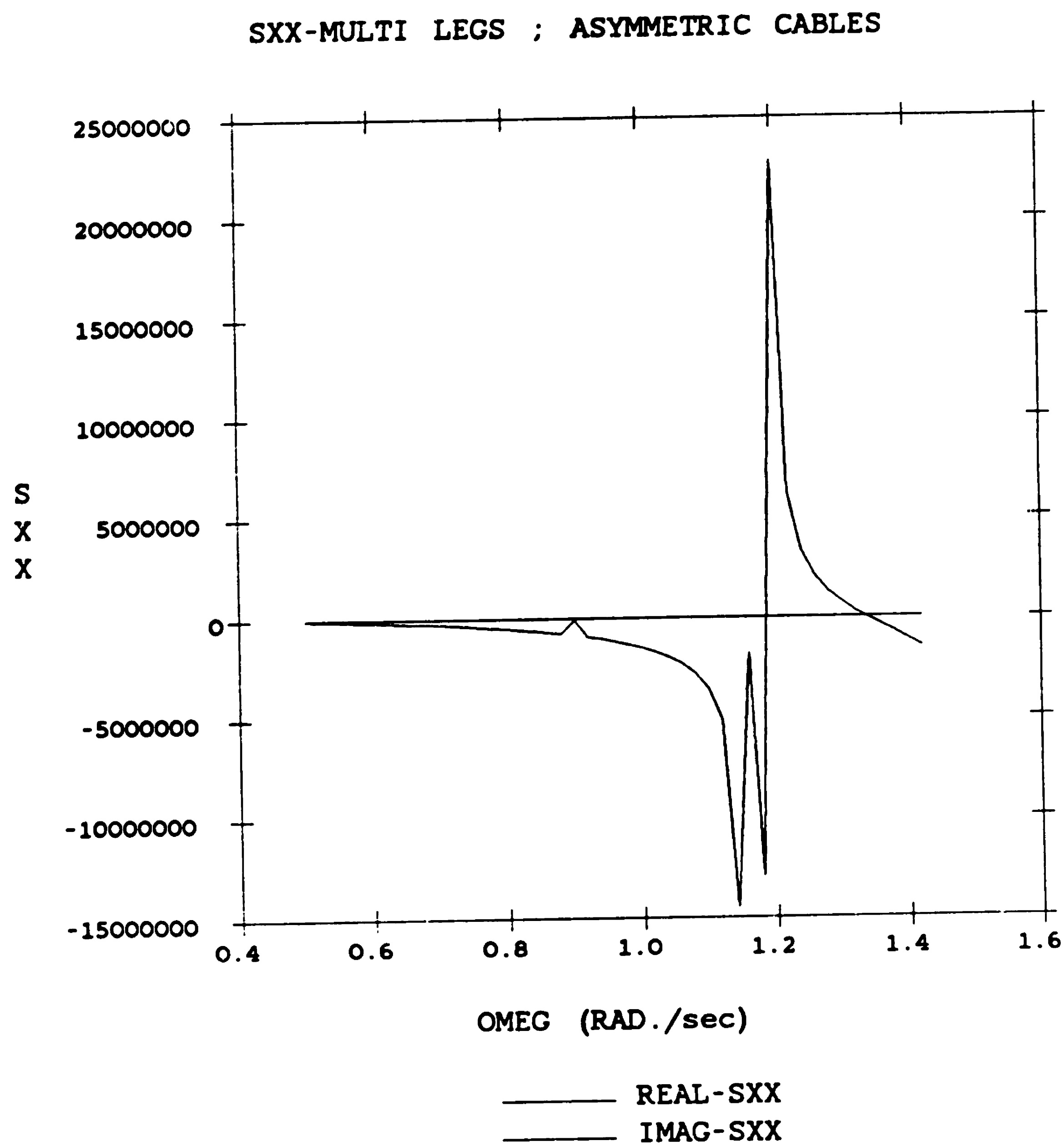

Figure 4-1: Impedance function for a Two-leg System

Chapter 5

NUMERICAL APPLICATIONS USING EQUIVALENT DAMPING COEFFICIENTS

5.1 Introduction

In this chapter, applications are provided, to demonstrate the practical implications of the theory presented in the preceding chapters.

5.2 Applications

For a comparison between the results of the nonlinear time domain analysis and those of the frequency domain analysis, a cable used for guyed towers is used. The principal dimensions are shown in Table 5-I.

Figures 5-1, 5-2 and 5-3 show normal displacements at each quarter point of the guy during one period of (a) the frequency domain analysis using the linearization technique and of (b) the nonlinear time domain analysis at the steady state condition, for different ratios of the amplitude of the top horizontal excitation to the diameter of the guy.

Figures 5-4, 5-5 and 5-6 show normal displacements of the guy at intervals of 1/20th of the period at the steady state condition, by using the nonlinear time domain analysis, for different ratios of the amplitude of the top horizontal excitation to the diameter of the guy. The amplitude to diameter ratio takes the values 0.01, 1, 10 and 100

Cable used in Figures 5-1 through 5-6

and Figure 6-6

T_0	=	1,332,000 N
m	=	48.7 kg/m
M	=	55.0 kg/m
w	=	414.98 N/m
EA	=	1.30×10^9N (nominal)
L	=	1,036m
d	=	426.7m
C_{Dn}	=	1.20
C_{Dt}	=	0.05
ϕ_a	=	24.4^0

Table 5-I: Cable used in Guyed Tower

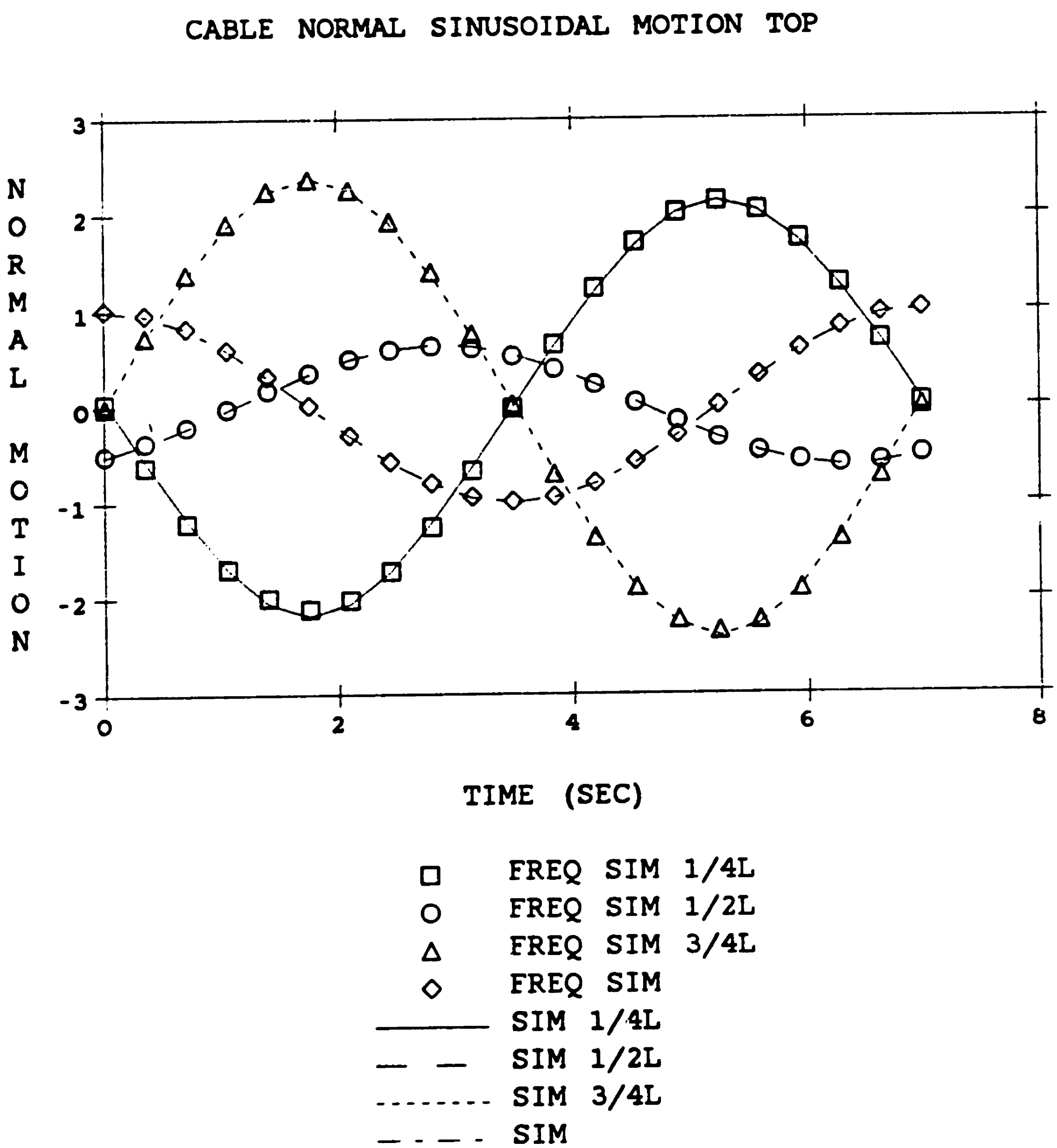

Figure 5-1: Normal Displacement at Each Quarter Point. 1D

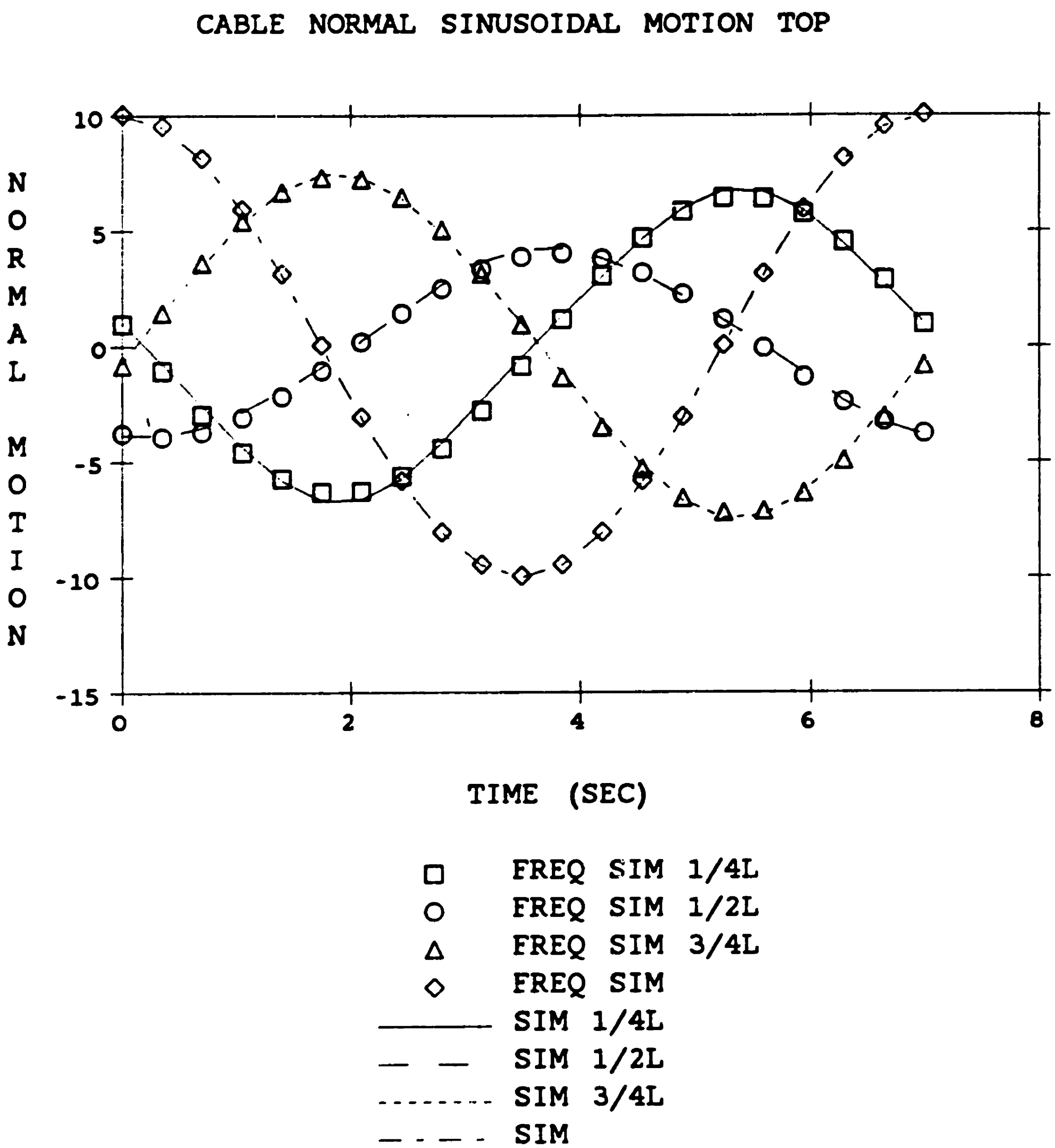

Figure 5-2: Normal Displacement at Each Quarter Point. 10D

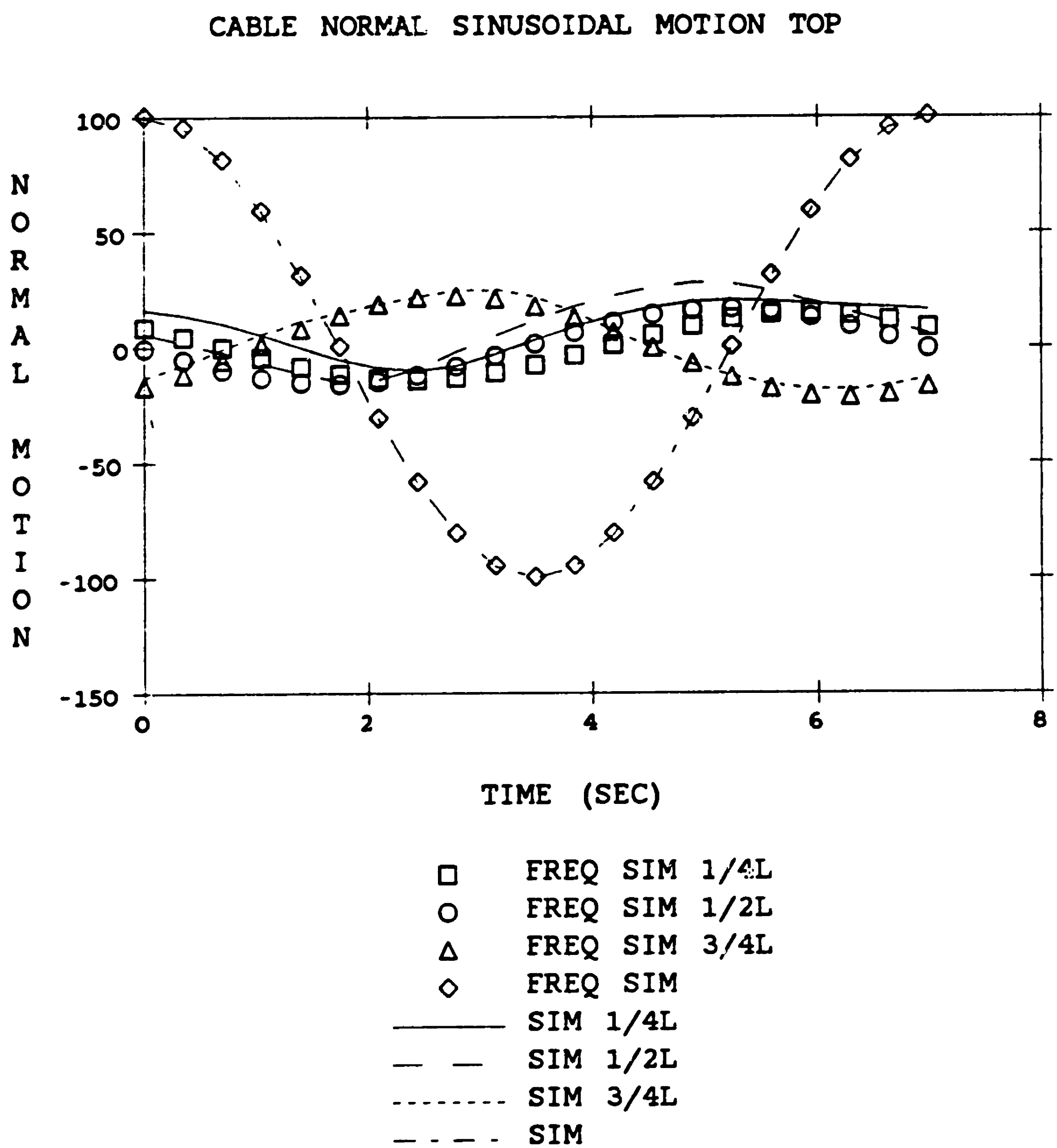

Figure 5-3: Normal Displacement at Each Quarter Point. 100D

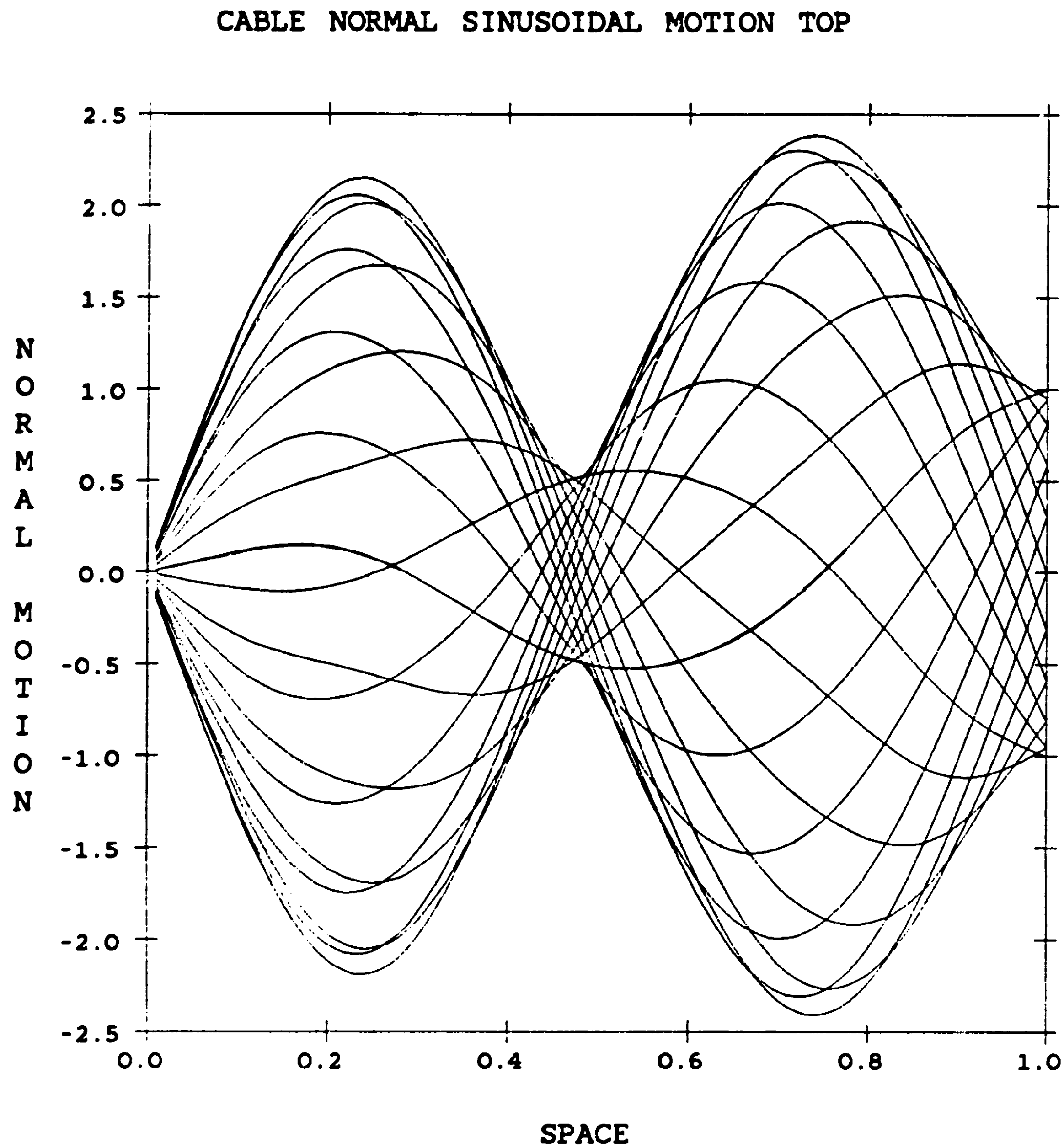

Figure 5-4: Normal Displacement at Each 1/20th of one Period. 1D

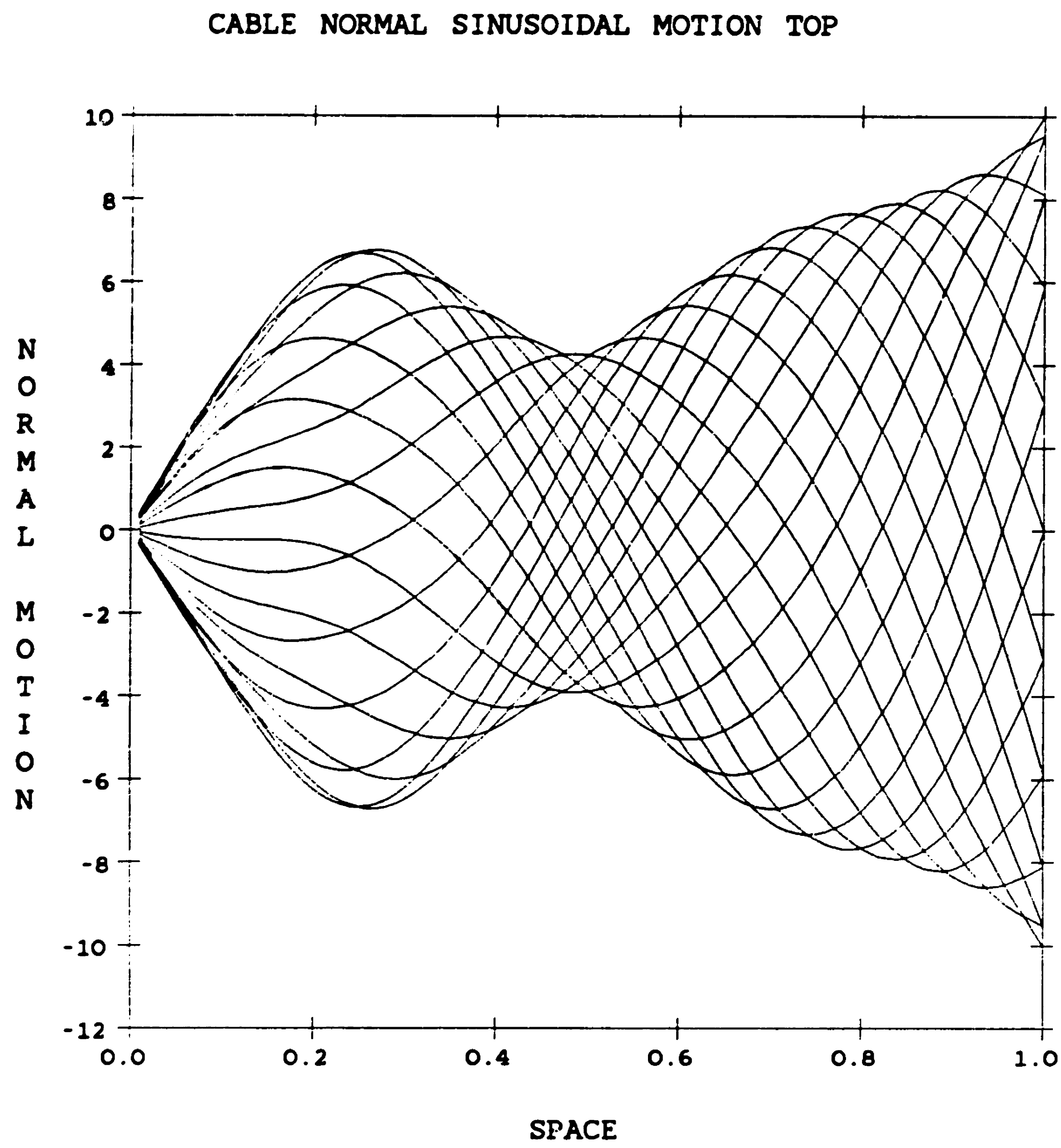

Figure 5-5: Normal Displacement at Each 1/20th of one Period. 10D

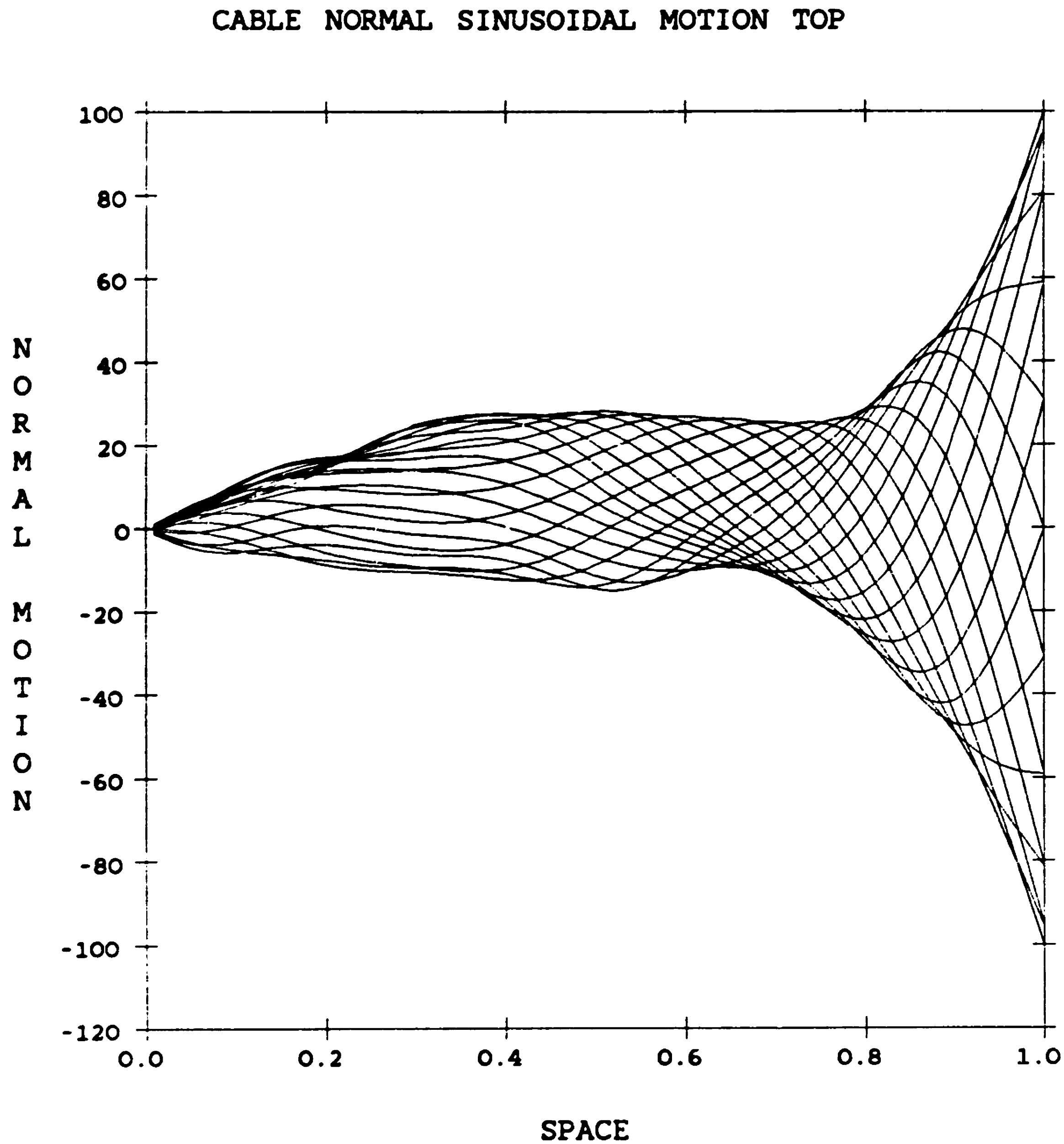

Figure 5-6: Normal Displacement at Each 1/20th of one Period. 100D

Chapter 6

DYNAMIC PROPERIES OF A MOORING LINE WITH RESPECT TO LAMBDA

6.1 Introduction

A cable always vibrates both transversely and axially, thus differing basically from a taut string. Due to the curvature it can accomodate an imposed motion in the axial direction by readjusting its configuration or by stretching, or by combining the two. As a result, the dynamic tension depends strongly on the ratio of the elastic stiffness (stretching) to catenary stiffness (change of configuration). Therefore the major independent parameter λ^2 ,which accounts for geometric and elastic effects, is of fundamental importance in the dynamic (and also static) responce of a cable.

For inextensible horizontal cables of small sag-to-span ratio the solution by Routh [Routh 55] and Rohrs [Rohrs 51] applies. For extensible horizontal cables of small sag-to-span ratio, the solution by Irvine and Caughey [Irvine 74] and Simpson [Simpson 66] applies.

6.2 Statics

If no current exists (the weight is the only external force), the following simple static relations are satisfied :

$$T_0 \frac{d\phi_0}{d\sigma} = w_0 L \cos \phi_0 \tag{6.1}$$

$$\frac{dT_0}{d\sigma} = w_0 L \sin \phi_0$$

where : σ is a nondimensionalized Largrangian coordinate. ($s = L\sigma$)

Also, the basic relations are obtained from Equation (6.1) :

$$T_0 = \frac{H_0}{\cos \phi_0} \tag{6.2}$$

$$\frac{d\phi_0}{d\sigma} = \alpha \cos^2 \phi_0$$

Assuming that the ratio of sag-to-span is less than 1/8, the following approximations are valid :

$$T_0 = \frac{H_0}{\cos \phi_0} \simeq H_0 \tag{6.3}$$

$$\frac{d\phi_0}{d\sigma} = \alpha \cos^2 \phi_0 \simeq \alpha$$

with : $\alpha = \dfrac{w_0 L}{H_0}$

where : H_0 = Horizontal static tension

α = Ratio of the net weight of the cable in the fluid and the horizontal static tension.

6.3 Free Vibrations of a Horizontal, Small Sag-to-Span Cable

6.3.1 Horizontal, Small Sag, Inextensible Cable

The study of the eigenfrequencies and modes of an inextensible chain hanging between two points at the same level, was initiated by Rohrs in 1851 [Rohrs 51].

When small amplitude harmonic motions are assumed, the set of governing equations for the dynamics of inextensible cables is :

$$-mL^2\omega^2\xi_a = \frac{dT_{1a}}{d\sigma} - T_0\frac{d\phi_0}{d\sigma}\phi_{1a}$$

$$-ML^2\omega^2\eta_a = \frac{d\phi_0}{d\sigma}T_{1a} + \frac{d\phi_{1a}}{d\sigma}T_0 + \frac{dT_0}{d\sigma}\phi_{1a} \tag{6.4}$$

$$\frac{d\xi_a}{d\sigma} - \eta_a\frac{d\phi_0}{d\sigma} = 0$$

$$\frac{d\eta_a}{d\sigma} + \xi_a\frac{d\phi_0}{d\sigma} = \phi_{1a}$$

where : ω is the frequency of given excitation.

ξ_a and η_a are nondimensionalized quantities. ($p_a = L\xi_a$ and $q_a = L\eta_a$)

Note that $\xi_a.\eta_a.T_{1a}$ and ϕ_{1a} denote small amplitudes of the corresponding quantities, and they are function of σ only (not time t).

From the tangential dynamics, using (6.4) we obtain the following dynamic tension :

$$T_{1a}(\sigma) = T_{10} + \int_0^\sigma [-mL^2\omega^2\xi_a + T_0\frac{d\phi_0}{d\sigma}\phi_{1a}]\, d\sigma \tag{6.5}$$

where : T_{10} is an integration constant.

Using the statics (6.3) for a horizontal, shallow sag cable, we derive the following equation for the normal dynamics. :

$$-ML^2\omega^2\eta_a = \alpha\, [\, T_{10} + \int_0^\sigma \{-mL^2\omega^2\xi_a + T_0\frac{d\phi_0}{d\sigma}\phi_{1a}\}\, d\sigma\,]$$

$$+ H_0 \frac{d\phi_{1a}}{d\sigma} + \frac{dT_0}{ds} \phi_{1a} \tag{6.6}$$

Neglecting higher order terms, the equation and boundary conditions are :

$$H_0 \frac{d^2\eta_a}{d\sigma^2} + M\omega^2L^2\eta_a = -\alpha\, T_{10}$$

with : $\eta_a(-1/2) = \eta_a(1/2) = 0$ (6.7)

The solution is :

$$\eta_a(\sigma) = C_1 \sin [kL\sigma] + C_2 \cos [kL\sigma] - \frac{\alpha\, T_{10}}{M\omega^2L^2} \tag{6.8}$$

where : $k = \omega\sqrt{M/H_0}$

In order to derive the antisymmetric modes, the following relations must be satisfied. :

$C_2 = 0$

$T_{10} = 0$ (that is, no additional tension)

Therefore, the antisymmetric eigenmode is : (6.9)

$$\sin \frac{kL}{2} = 0$$

For symmetric modes, we apply the boundary conditions (6.7) to the solution (6.8) to find the following expression for the normal motion :

$$\eta_a(\sigma) = \frac{\alpha\, T_{10}}{M\omega^2L^2} \left[\frac{\cos kL\sigma}{\cos kL/2} - 1 \right] \tag{6.10}$$

From Equation (6.3) and the third relation of (6.4), the tangential motion is obtained as :

$$\xi_a(\sigma) = \frac{\alpha T_{10}}{M\omega^2 L^2} \left[\frac{1}{kL} \frac{\sin kL\sigma}{\cos kL/2} - \sigma \right] + C \tag{6.11}$$

with : $\xi_a(1/2) = \xi_a(-1/2) = 0$

where : C is a integration constant.

We derive, therefore, the following transcendental equation from Equation (6.11) together with the boundary conditions :

$$\tan(kL/2) = kL/2 \tag{6.12}$$

Note that the first symmetric eigenfrequency is located at 2.86π (see Figure 6-1) compared with the value of π for the first eigenfrequency of a string, and dynamic tension is generated. For further discussion, refer to [Bliek 84].

6.3.2 Horizontal, Small Sag, Extensible Cables

The derivation for extensible cables is completely similar with the one for inextensible cables. The major differences come from the compatibility equations in (6.4).

$$-mL^2\omega^2\xi_a = \frac{dT_{1a}}{d\sigma} - T_0 \frac{d\phi_0}{d\sigma}\phi_{1a}$$

$$-ML^2\omega^2\eta_a = \frac{d\phi_0}{d\sigma} T_{1a} + \frac{d\phi_{1a}}{d\sigma} T_0 + \frac{dT_0}{ds}\phi_{1a}$$

(6.13)

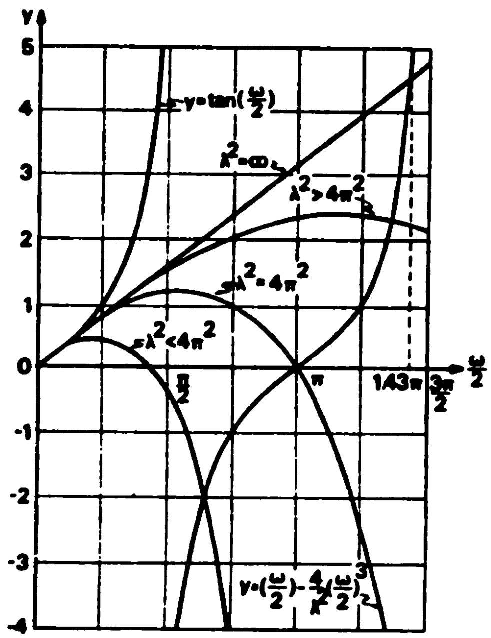

Figure 6-1: Graphs of eigenfrequency vs λ from [Irvine 81]

$$\frac{d\xi_a}{d\sigma} - \eta_a \frac{d\phi_0}{d\sigma} = \frac{T_{1a}}{EA}$$

$$\frac{d\eta_a}{d\sigma} + \xi_a \frac{d\phi_0}{d\sigma} = \phi_{1a}\,(1+e_0)$$

where : EA is the sectional rigidity, and allowance is made for stretching.

In the case of an extensible cable, the previous relations (6.6), (6.7), (6.8)and (6.9) from the inextensible cable are still valid except for $H_0/(1+e_0)$ instead of H_0.

$$\frac{H_0}{1+e_0}\frac{d^2\eta_a}{d\sigma^2} + M\omega^2L^2\eta_a = -\alpha\, T_{10} \tag{6.14}$$

with : $\eta_a(-1/2) = \eta_a(1/2) = 0$

The solution is :

$$\eta_a(\sigma) = C_1 \sin[\mathrm{kL}\sigma] + C_2 \cos[\mathrm{kL}\sigma] - \frac{\alpha T_{10}}{M\omega^2 L^2} \tag{6.15}$$

with : $k = \omega\sqrt{M(1+e_0)/H_0}$

For the solution to provide antisymmetric modes, the following relations must be satisfied :

$$C_2 = 0$$

$$T_{10} = 0 \qquad \text{(that is, no additional tension)} \tag{6.16}$$

$$\sin\frac{kL}{2} = 0$$

From the boundary conditions (6.14) and Equation (6.15), the two arbitray constants are determined in (6.17). :

$$C_1 = 0$$

$$C_2 = [\frac{\alpha T_{10}}{M\omega^2 L^2}]/\cos(kL/2) \tag{6.17}$$

$$\eta_a(\sigma) = \frac{\alpha T_{10}}{M\omega^2 L^2}[\frac{\cos kL\sigma}{\cos kL/2} - 1] \tag{6.18}$$

For the symmetric modes, however, attention should be paid to the tangential compatibility relation in (6.13) :

$$\frac{d\xi_a}{d\sigma} = \eta_a \frac{d\phi_0}{d\sigma} + \frac{T_{1a}}{E\,A} \tag{6.19}$$

Integrating the equation (6.19) along the cables with (6.3) and (6.5) gives the following relation for the dynamic tension of symmetric modes :

$$\xi_a(1/2) - \xi_a(-1/2) = \int_{-0.5}^{0.5} \eta_a(\sigma)\, \frac{d\phi_0}{d\sigma}\, d\sigma + \int_{-0.5}^{0.5} \frac{T_{1a}}{E\,A}\, d\sigma \tag{6.20}$$

$$0 = \alpha \int_{-0.5}^{0.5} \eta_a(\sigma)\, d\sigma + \frac{T_{10}}{E\,A} + O\,(\epsilon^2)$$

By substituting (6.18) in (6.20) we can find a simple relation for the eigenfrequencies, by following the steps shown below :

$$\frac{T_{10}}{E\,A} = -\,\alpha \int_{-0.5}^{0.5} \frac{\alpha\, T_{10}}{M\omega^2 L^2}\, [\, \frac{\cos kL\sigma}{\cos kL/2} - 1\,]\, d\sigma \tag{6.21}$$

$$\frac{M\omega^2 L^2}{E\,A\alpha^2} = [\,\sigma - \frac{1}{kL}\frac{\sin kL\sigma}{\cos kL/2}\,]_{\sigma=0.5} - [\,\sigma - \frac{1}{kL}\frac{\sin kL\sigma}{\cos kL/2}\,]_{\sigma=-0.5}$$

Finally, $$\frac{M\omega^2 L^2}{E\,A\alpha^2} = 1 - \frac{2}{kL}\tan\,(kL/2)$$

Using (6.3) and (6.15) we can derive the following transcendental equation. :

$$\tan\,(\frac{kL}{2}) = \frac{kL}{2} - \frac{4}{\lambda^2}(\frac{kL}{2})^3 \tag{6.22}$$

where : $\lambda^2 = (\frac{w_0 L}{H_0})^2 \frac{E A_0}{H_0}$

$A_0 = (1+e_0)A$

Equation (6.22) is of fundamental importance in the theory of cable vibrations.

6.3.3 Dynamic Properties of Extensible Cables with respect to Lambda

The equation (6.22) allows us to find the symmetric eigenfrequencies for a wide range of values of the parameter λ^2 involving cable elasticity and geometry.

If $\lambda^2 = 4\pi^2$, the frequencies of the antisymmetric mode and the symmetric mode are equal and the normal displacement is tangential to the profile at the ends. This value of λ is called the first modal cross-over. The modal cross-over phenomena occur at the value of $\lambda = 2n\pi$ (See Figures 6-2 and 6-3). When λ^2 is very large, the previous relation (6.12) for inextensible cables is recovered.

6.3.4 Extension to an Inclined Cable

In the case of an inclined cable, Irvine extended his theory by using a transformation of the coordinate systems [Irvine 78] and Bliek treats better the problem with a more general analysis [Bliek 84]. When the small sag approximation can be used to predict the symmetric modes, we derive the same relation as before for the symmetric eigenfrequencies of an inclined cable, except for the use of equivalent λ and H, identified by the subscript *.

$$\tan (\frac{k_* L}{2}) = \frac{k_* L}{2} - \frac{4}{\lambda_*^2} (\frac{k_* L}{2})^3$$

$$\text{with : } k_* = \omega\sqrt{M(1+e_0)/H_{0*}} \tag{6.23}$$

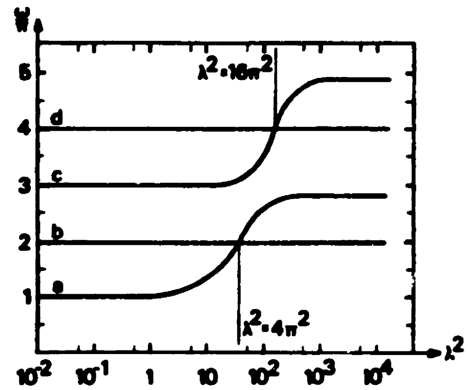

Figure 6-2: Natural Frequencies
of a cable as a function of λ from [Irvine 81]

where : $\lambda_*^2 = \left(\frac{w_0 L}{H_{0*}}\right)^2 \frac{E A_0}{H_{0*}} \cos^2\phi_a$

$$H_{0*} = \frac{H_0}{\cos\phi_a}$$

$\phi_a =$ an Inclination angle between the cable chord and the horizontal line .

For inclined cables, however, the analysis by the perturbation and numerical methods shows no cross-over phenomena. (See Figure 6-4)

6.3.5 Numerical Applications

To demonstrate the effect of the parameter λ^2 on the dynamic tension of a mooring line, a cable used for mooring semi-submersibles is employed. The

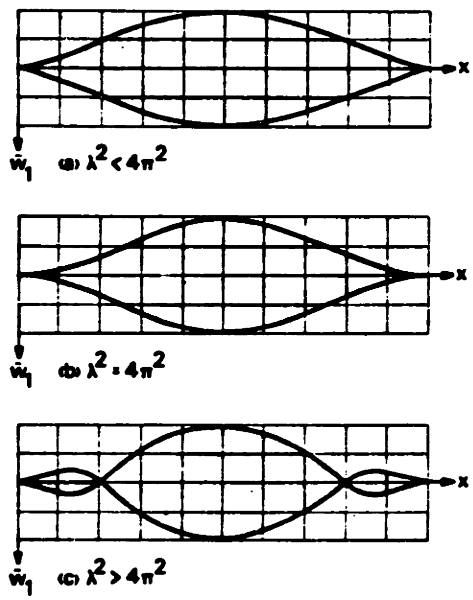

Figure 6-3: the First Symmetric In-plane Mode from [Irvine 81]

principal dimensions are shown in Table (6-I). Figure 6-5 shows the dynamic tension per unit amplitude in the horizontal direction (RAO) (a) by using the equivalent linearized damping and (b) by using the nonlinear time simulation.

Figure 6-6 shows the terminal impedance Sxx of the cable in Table 5-I for 10 D amplitude of excitation.

Figures 6-7, 6-8 and 6-9 show the effect of the independent parameter λ^2 on the dynamic tension at the first two eigenfrequencies without current : The ratio of the amplitude of top excitation to the diameter of the cable is 0.01 D, 1 D and 10 D respectively.

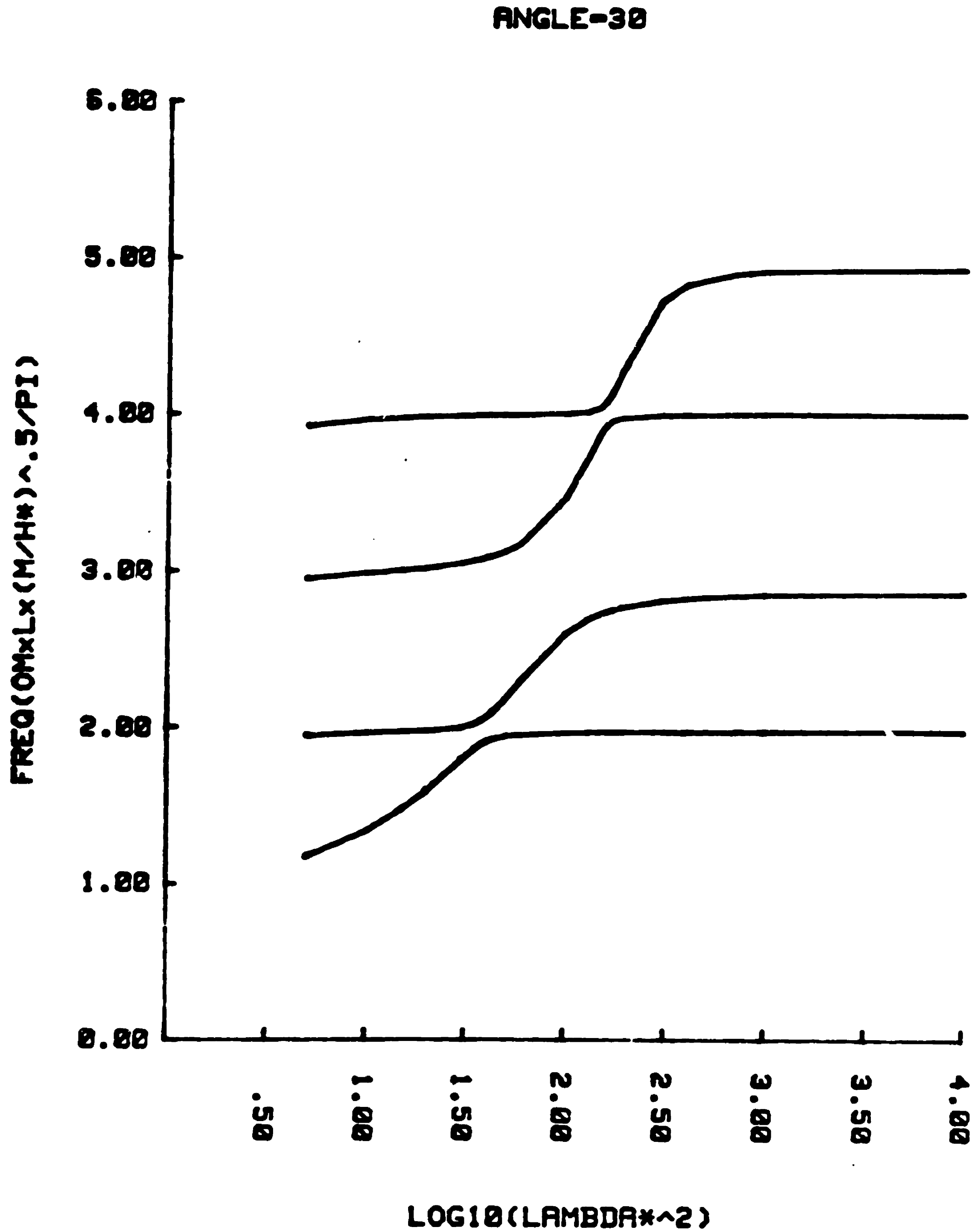

Figure 6-4: Natural Frequencies of a cable as a function of λ from [Bliek 84]

Cable used in Figures 6-6 through 6-8

$T_0 \;=\; 1{,}400{,}000\ N^*$

$m \;=\; 33.51\ kg/m$

$M \;=\; 39.81\ kg/m$

$w \;=\; 290\ N/m$

$EA \;=\; 0.424 \times 10^9 N$ (nominal)

$L \;=\; 1{,}700m$

$d \;=\; 700m$

$C_{Dt} \;=\; 1.20$

$C_{Dn} \;=\; 0.05$

$\phi_a \;=\; 24.37^0$ (varying slighyly with EA)*

* For Figure 6-4 $\quad T_0 = 700{,}000\ N$

$\phi_a = 24.97^0$

Table 6-I: Cable used in Semi-submersible

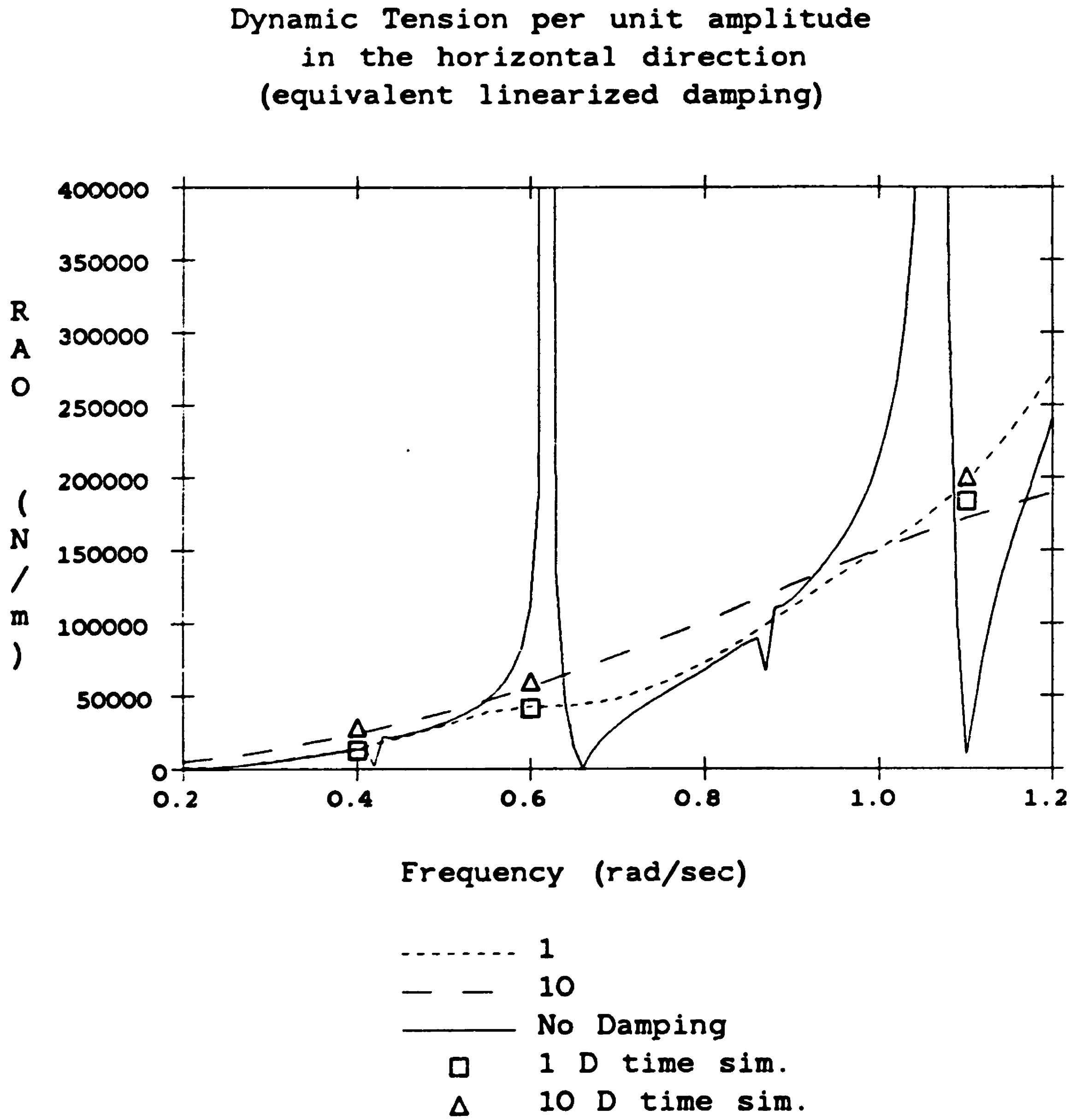

Figure 6-5: Dynamic Tension per unit amplitude in the horizontal direction using equivalent damping coefficient

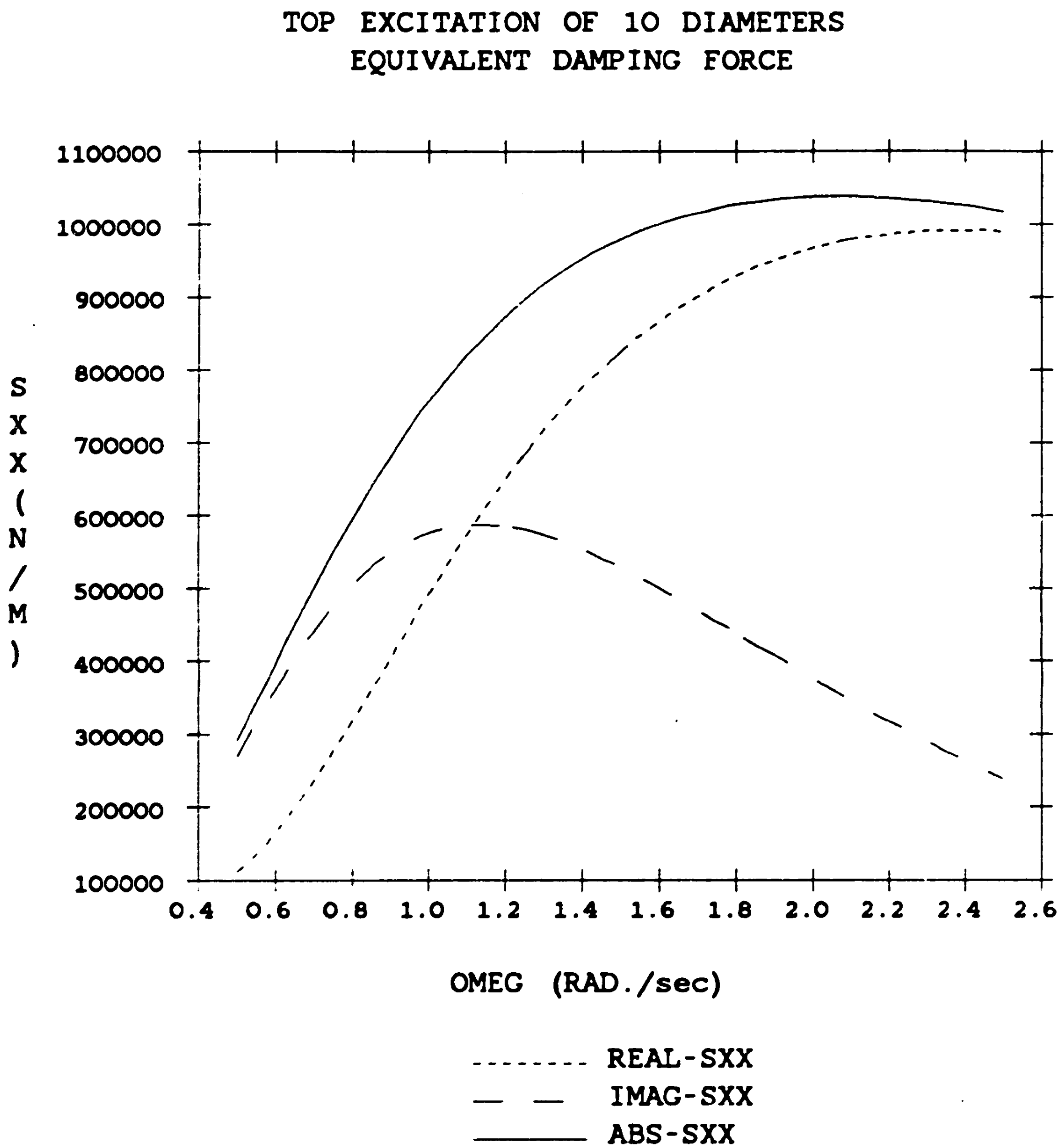

Figure 6-6: Sxx - Terminal Impedance
for Top Excitation Amplitude 10 diameters : Table 5-I

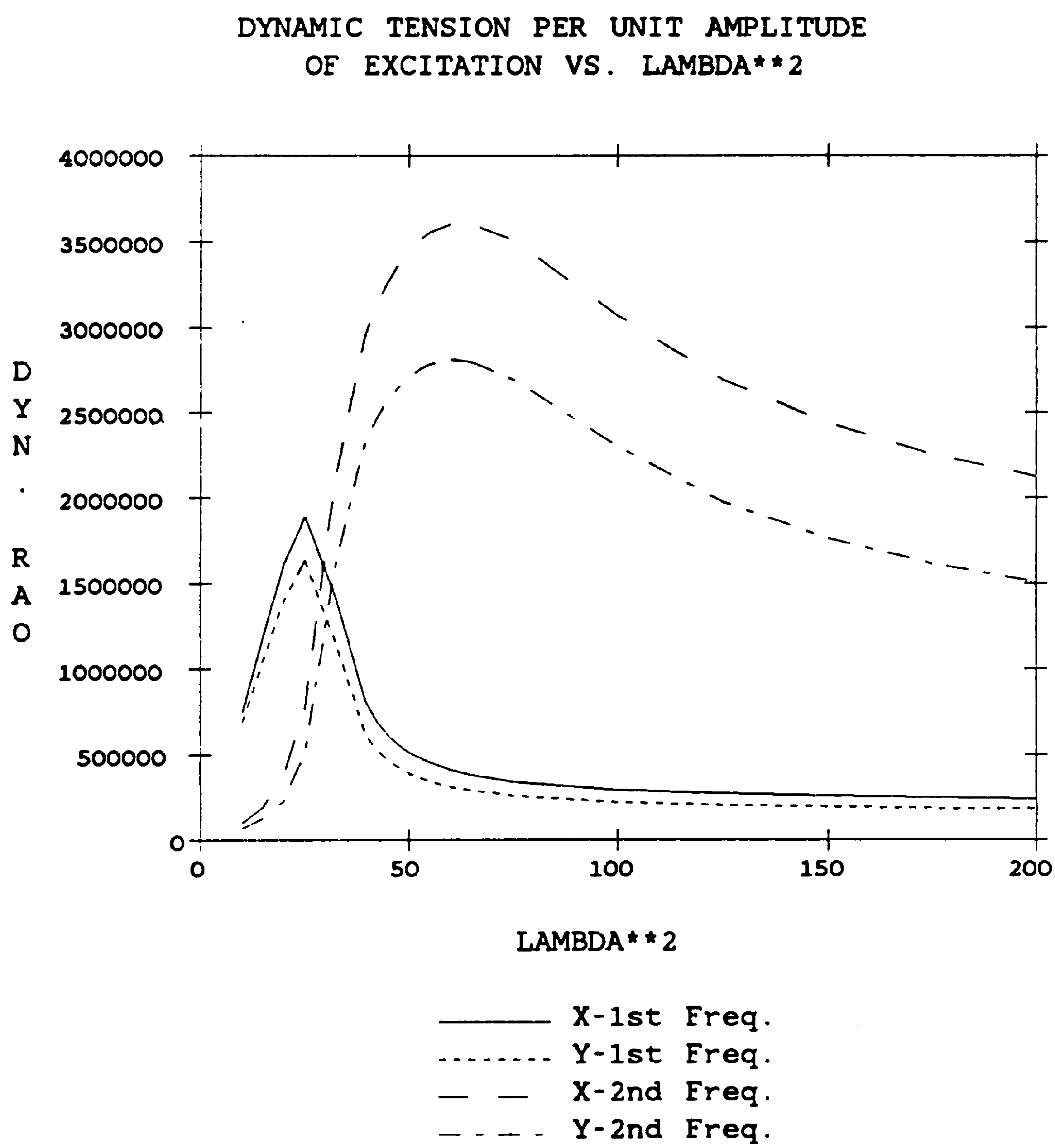

Figure 6-7: Dynamic Tension per unit Amplitude for Top Excitation Amplitude 0.01 D vs λ^2

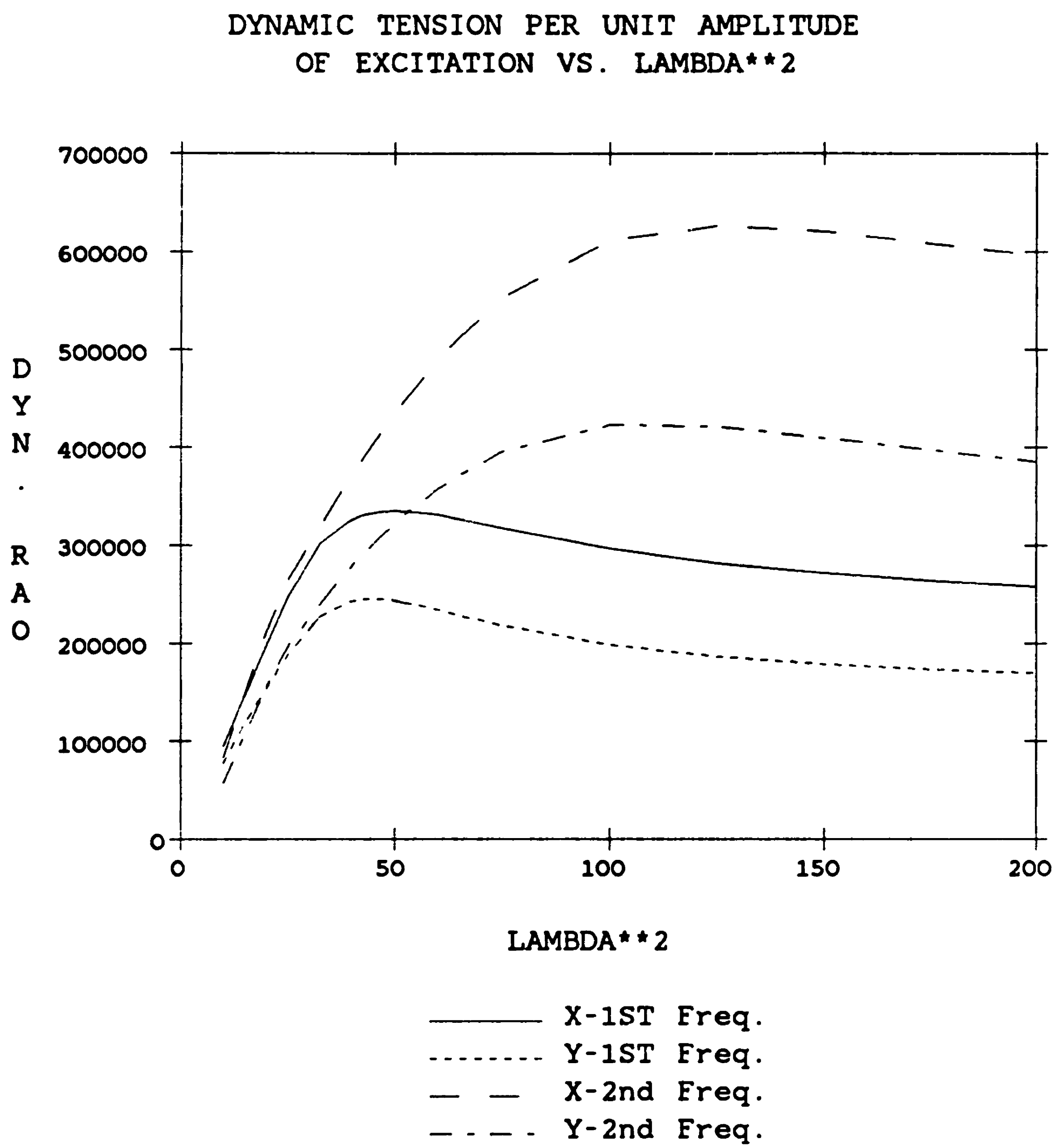

Figure 6-8: Dynamic Tension per unit Amplitude for Top Excitation Amplitude 1 D vs λ^2

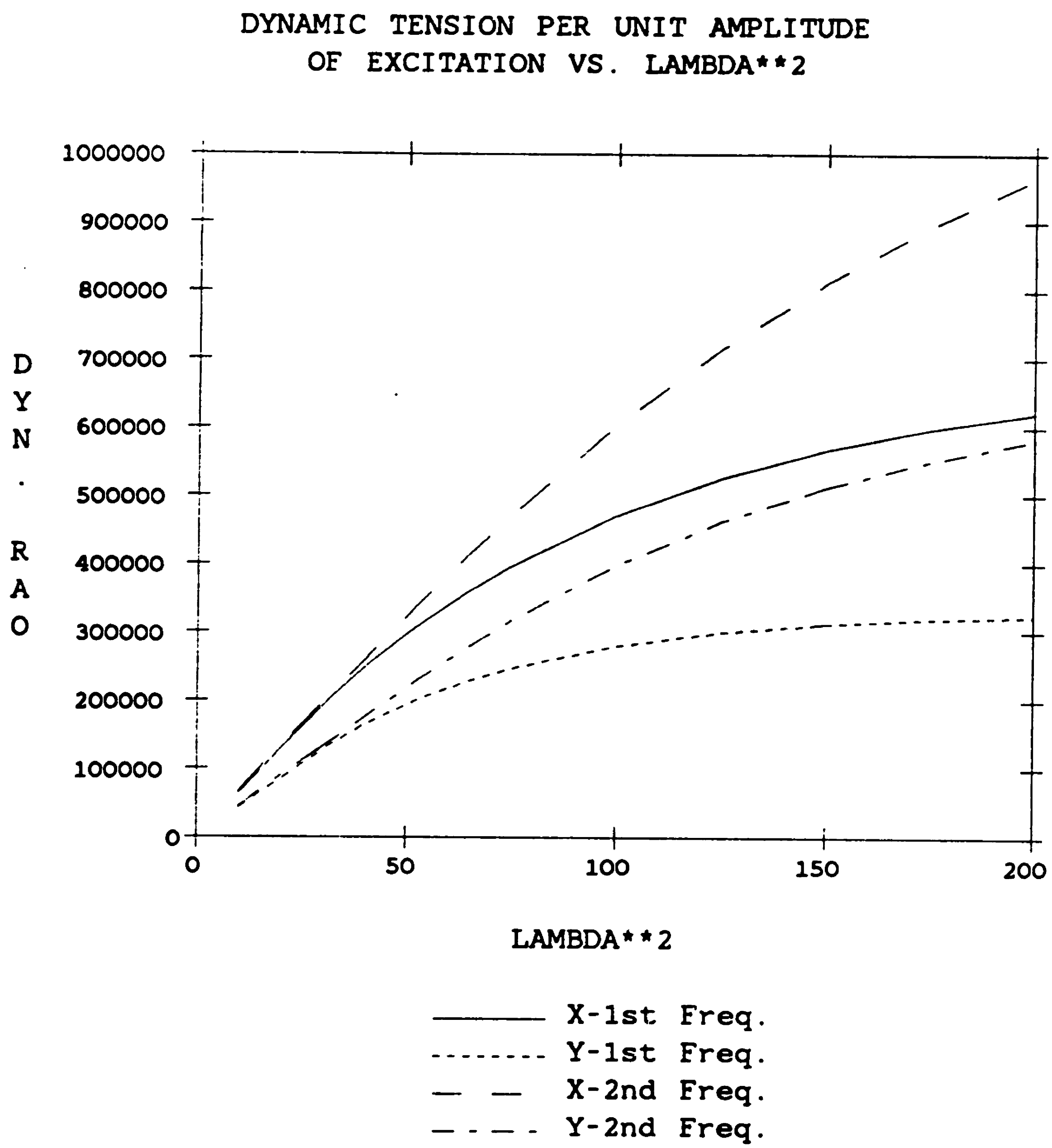

Figure 6-9: Dynamic Tension per unit Amplitude for Top Excitation Amplitude 10 D vs λ^2

CONCLUSIONS

The results obtained by using the equivalent damping coefficient show good agreement with those derived by the nonlinear time domain simulation in Figures 5-1, 5-2 and 5-3. High computing costs required for the nonlinear time simulations in the preliminary design phase of a mooring system, prevent the designer from checking the dynamic behaviour in a wide variety of different environmental conditions.

Further research into the intermediate case of having the ratio of the cable velocity to current velocity near one, is required [when $\frac{w\xi_a}{U}$ and $\frac{w\eta_a}{U} = O(1)$].

The principal parameter λ^2, which is the ratio of the elastic stiffness-to-catenary stiffness, alters the natural frequencies and dynamic tension substantially. Dynamic tension of a mooring line in a region adjacent to $\lambda=2n\pi$ should be examined more carefully including extensible and inextensible inclined cables for application to heavy sea states.

In Figure 6-6 the dynamic tension per unit amplitude of excitation is shown as a function of λ^2. The amplitude at the top is small (0.01 D) so the drag forces are insignificant. As seen in Figure 6-6 the dynamic tension is important in the first natural mode for $\lambda < 2\pi$, and in the second natural mode for $\lambda > 2\pi$. This is in agreement with the linear theory which predicts that the first natural mode is symmetric for $\lambda < 2\pi$, while for $\lambda > 2\pi$ the first mode is antisymmetric and the second mode becomes symmetric. It should be remembered that the dynamic tension is large for a symmetric mode only.

With an increase of the excitation amplitude (Figure 6-5), the fluid drag force becomes predominant over the other forces, leading to a substantial increase of the amplitude of the dynamic tension over a whole range of frequencies. Due to the substantial resistance of the drag forces against lateral motion, the catenary stiffness of the cable appears to be substantially increased. As a result, the dynamic tension from the first natural mode extends over to the region $\lambda > 2\pi$ (see Figures 6-8 and 6-9), while the cable accomodates the motion primarily by stretching, thus providing substantially larger dynamic tension.

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