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# THE DEVELOPMENT OF AN AUTOMATIC CURVE-FOLLOWING MECHANISM

by

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# I

## INTRODUCTION

One of the most important of the projects which are now being undertaken in the Electrical Research Laboratories of the Massachusetts Institute of Technology is the development of automatic computing machines. In connection with one of the details of this development, the present thesis was undertaken at the suggestion of Mr. S. H. Caldwell, in charge of the Laboratories. Mr. Caldwell also supervised the progress of the development, and the writer is much indebted to him for his interest and assistance. Acknowledgment is also made to Professor H. L. Hazen, who designed the servo-motor used in the lay-out and offered many valuable suggestions, and to Mr. G. S. Brown, who designed and built the d-c. amplifier.

Organization of Paper -- In the arrangement of this paper a procedure has been followed which it is hoped will make the desired information available, with a minimum of effort, to both those who want only a general idea of what was undertaken and what was accomplished and those who want a more detailed account of the procedure.

To this end the first section contains general information regarding the purpose of the development and the results obtained, together with a brief description of the operation of the apparatus. Following this are several sections which treat the design of the apparatus in detail, and finally, in order to facilitate future progress along this line, a section on "Future Development" has been written.

## II

## GENERAL REMARKS

Purpose -- Perhaps the most important tool which has been developed for the solution of ordinary differential equations with variable coefficients is the Differential Analyzer, developed in the Electrical Research Laboratories of the Massachusetts Institute of Technology by Dr. Bush and his associates. This machine* has proved to be of great practical importance and is now being used continuously by the students and faculty for the solution of a large variety of problems, many of great importance.

In the operation of this machine it is necessary in many cases for curves to be plotted on paper and placed on the input tables. There are four of these tables, the number used at any one time depending on the nature of the problem being solved. On each input table is a carriage which is driven parallel to the axis of abscissas by a lead screw, this lead screw being driven by the machine. A slider on the carriage is permitted to move parallel to the axis of ordinates, and this motion is controlled by a crank. In the process

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*For a description of the machine and its operation, see the Journal of the Franklin Institute, Vol. 212, No. 4, October, 1931.

of solving a problem, it is necessary for an operator to crank the slider so that as the carriage is moved across the paper, by the action of the machine, an index on the slider follows the curve.

Since this is done manually, it requires the presence of a number of men for the performance of a purely mechanical task and also brings the question of personal error into the solution of a problem. Although it is found that the error in following the curve balances out to a large extent, a greater consistency of results would be advantageous in many cases. For this reason some form of an automatic curve-following mechanism is desirable. In addition to the above there are indirect advantages offered by such a device which may be appreciated only by a thorough knowledge of the operation of the Analyzer.

Another point which may be mentioned is that in a future improved design of the Analyzer, which would be more compact, easier to use, and require much less attention than the present one, the elimination of cranking would help greatly.

For the foregoing reasons the development of an automatic curve-following device which would replace the hand cranking now employed would be a great step forward in further mechanizing the machine, and this is the subject of the present thesis. Of course, in addition to possessing the necessary ruggedness and reliability, the apparatus must have a degree of accuracy comparable to the rest of the Analyzer. It has been found that in individual units an accuracy of one part in a thousand may be obtained, although the over-all accuracy is somewhat less than this.

Description of Apparatus -- The apparatus* developed for this purpose is an application of the photoelectric cell. A picture of the apparatus is shown below:

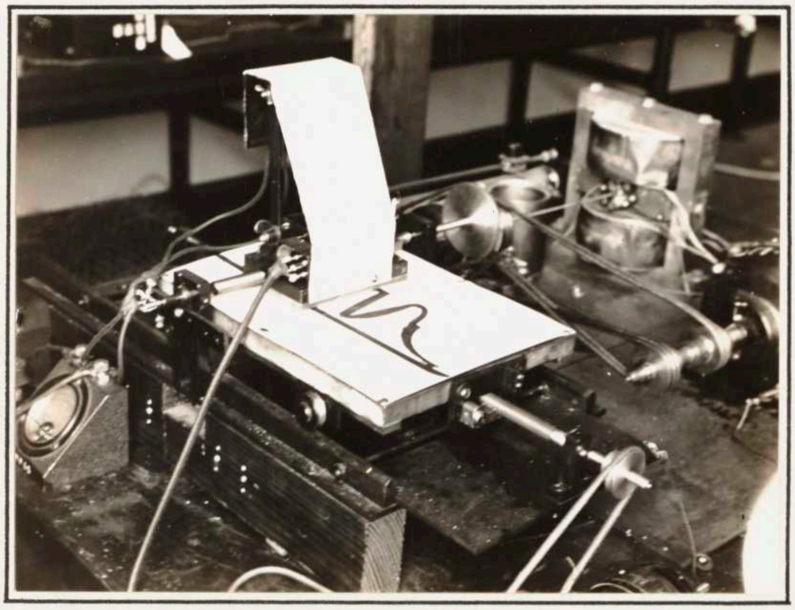

Figure 1.

This picture shows a small movable table upon which is fastened an inked curve. A lead screw driven by a small motor moves this table parallel to the axis of abscissas. A source of light and a photocell are mounted on a carriage in the bottom of which is a small

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*For details see p. 8 et. seq.

slot. The carriage may be driven parallel to the axis of ordinates by another lead screw. The light is focused on the slit, and a portion of the light reflected from the paper is caught by the photocell.

This reflected light causes a current to flow in the cell and through an external resistance. The voltage drop across this resistance is used as the input of a d-c. amplifier, and the output of this amplifier flows into the armature of a specially designed servo-motor. Since this servo-motor must operate on a rather small current (20 milliamperes produced 500 r.p.m.), its output is also small. In order to obtain a torque large enough to furnish the input to the Differential Analyzer, a torque amplifier is inserted between the servo-motor and the carriage.

Now it is seen that any change in the amount of light reflected into the photocell changes the grid bias of the first tube of the amplifier, thus changing the output and causing a current to flow in the motor. The motor then rotates and moves the carriage. If, in the beginning, the grid bias of the first tube of the amplifier is so adjusted that when the slit is placed over the curve and is half white and half black no current flows in the motor, then any deviation from this position will change the amount of reflected light and so cause the motor to move the carriage. If the direction of rotation is such as to restore equilibrium, then the essentials of an automatic curve-following mechanism are obtained.

This, briefly, is the way in which the apparatus works. A more detailed analysis of the operation is given later in the paper.

Results -- The accuracy with which the device follows a curve is dependent on several factors which will be discussed later, but, for a slope of 5 to 1 and for a table speed of about 5 in./min., the deviation did not exceed about one-fortieth of an inch. Other factors remaining the same, the deviation is proportional to the table speed so that by slowing down the driving motor any reasonable degree of accuracy may be obtained.

This slowing down of the speed of traverse, when steep slopes are encountered, is also necessary on the Analyzer as now operated. The automatic apparatus is capable of moving much faster than is possible with hand cranking.

The above degree of accuracy, together with the consistency with which curves are traced and retraced many times, is sufficient to show that a similar design, adapted to the Analyzer, will give the desired results.

# III

## DESIGN OF APPARATUS

<u>Choice of Method of Attack</u> -- Since, as far as the writer could ascertain, there had been no previous attempt made to develop such a curve-following device as was wanted for the present purpose, it was necessary to consider the various possible means by which the desired results could be obtained and to choose the most promising method for further investigation. Among the various possibilities considered were some form of electrostatic device, a single or double conducting line, a curve etched in copper and followed by a phonograph needle, and a photocell.

The electrostatic method of attack did not seem to offer as much promise as some of the others; so it was not given much attention. A way in which two parallel conducting lines could be used is shown by the following figure:

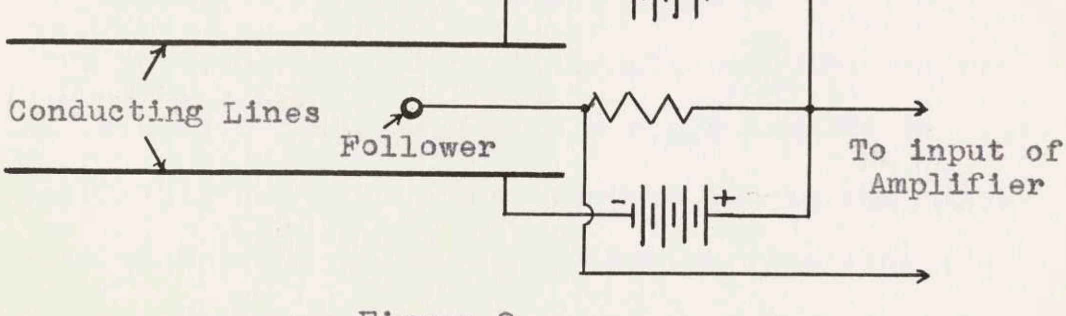


Figure 2.

This method would use practically the same apparatus as the photoelectric method which was finally adopted except that the change in grid bias on the first tube of the amplifier circuit would be produced by the follower touching either the upper or the lower of the conducting lines and so adding either a negative or a positive increment of voltage. This method has the disadvantage that a slight deviation from the desired path produces just as much current in the servo-motor, which drives the follower, as a large deviation. Thus the operation would be jerky and probably the follower would be in continuous oscillation between the two lines.

At this point it might be pointed out that in all the methods here mentioned the operation of the apparatus depends on there being a deviation from the curve, for until there is a deviation there is no torque established to move the apparatus. Although in many cases the necessary deviation may be made very small, still this remains as a fundamental difficulty.

Etching a curve on a metal plate or cutting a curve in some material such as celluloid and following it with a phonograph needle would seem to be quite practicable but rather inconvenient due to the difficulty of drawing the curve. A design involving the use of a photocell would be much more convenient in that it would follow a curve inked on ordinary paper. Consideration showed that such a design also promised to be sufficiently accurate and reliable for the purpose; so it was decided to investigate this method further before trying any other possibility.

There was available a specially designed d-c. motor which had been built for the cinema integraph (now being developed) and also a d-c. amplifier which had been constructed to furnish the necessary input to the motor. The contemplated design, which proved to be satisfactory and was adopted, utilized this apparatus and has already been described (pp. 5-7). The apparatus might be represented diagrammatically as follows:

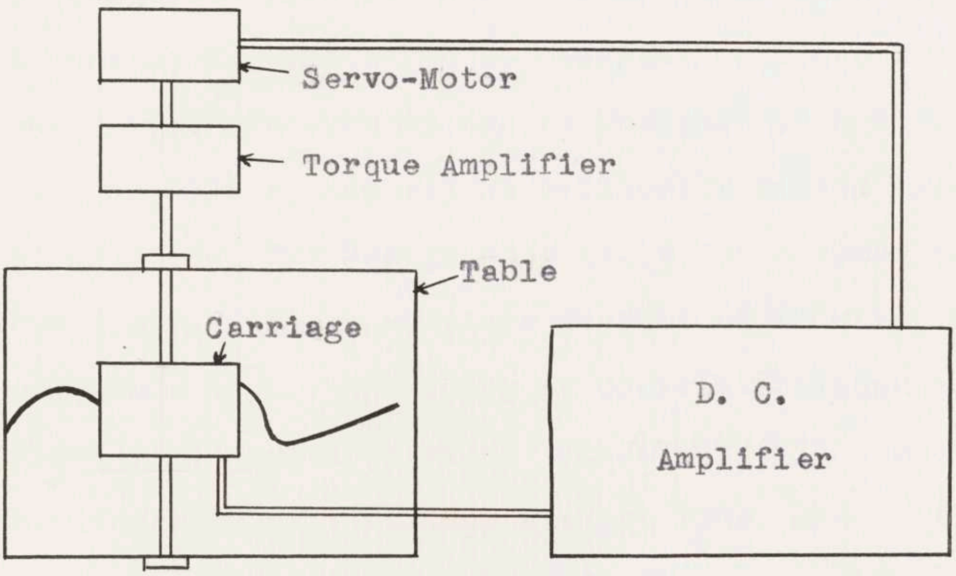


Figure 3.

The various elements of this design will now be considered in detail.

Motor Design -- As previously mentioned, the servo-motor which is used was designed for use on the cinema integraph. The complete design of this motor, as well as a detailed development of the theory of servo-mechanisms in general, is given in a paper soon to be published, probably in the Journal of the Franklin Institute*, so that a lengthy discussion of this subject is quite out of place in this paper. For this reason the salient features of this motor will be mentioned only, the reader being referred to Professor Hazen's paper for a justification of the statements.

The motor is a separately excited d-c. motor, and is characterized particularly by a high ratio of torque to inertia so that very large acceleration is obtained. The motor is designed to operate on small currents such as can be obtained from a vacuum-tube amplifier, and may be critically damped when so desired. The design data calls for a speed of 500 r.p.m. with an armature current of 20 mils, this magnitude of current being at present obtained with three-stage amplification. The motor will stand a considerable overload for a short time, and in the present application a speed of 800 r.p.m. with a current of 32 mils is sometimes used.

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*"Theory of Servo-Mechanisms, including the Design and Test of a High-Performance Servo-Mechanism," by Professor H. L. Hazen.

In order to obtain a large acceleration, the armature was made a very thin shell, and the iron core was made stationary and supported by the frame of the motor. This gave a much smaller moment of inertia than is ordinarily the case. Large field coils gave a high-flux density in the air gap, thus giving maximum torque for a given armature current. For the critically damped case, the time constant was found experimentally to be 0.024 seconds. This means that the motor will accelerate from zero to full speed in less than half a revolution. The damping in the motor is from three sources: frictional damping, the damping due to eddy-currents in the armature shell, and the damping due to the back e.m.f. in the armature conductors themselves. The proper proportioning of the damping in the last two sources is very important in the design.

The amount of damping may be adjusted by changing the amount of armature current which a revolution deflection of the motor shaft (with the rest of the apparatus connected) will cause to flow. The test data shows that a current of 54.8 milliamperes per revolution deflection with the motor at rest will give critical damping. This figure applies only when the motor is connected to the present amplifier and with a field current of 0.4 amperes. This is one of the requisites for the design of the photoelectric portion of the apparatus.

In order to obtain enough torque to drive a considerable load, it is necessary to employ a mechanical torque amplifier.* This piece of apparatus was developed for the Differential Analyzer, the basic idea being due to Mr. Neiman of the Bethlehem Steel Corporation, and several of them are being used continuously with very satisfactory results. The particular torque amplifier for use with which the servomotor was designed has an amplification of about 2,000. The output of the two units is about one foot-pound, and from experience this is known to be sufficient for the purpose. In any case, the amplification can be increased by suitable design.

D-C. Amplifier Design -- The circuit of the amplifier is shown on p. 14. It is seen that the currents in the last two stages flow through the motor in opposite directions. By properly setting the "C" biases on these stages, it is possible to obtain zero current in the motor, for a given "C" bias on the first stage, with all tubes working within their operating range. Furthermore, it is also possible to obtain a linear relationship between the motor current and the input grid voltage for a range of -20 to +20 mils, running-motor current. Beyond this there is a departure

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*See "The Differential Analyzer," by V. Bush, loc. cit.

## Circuit Diagram of D-C. Amplifier

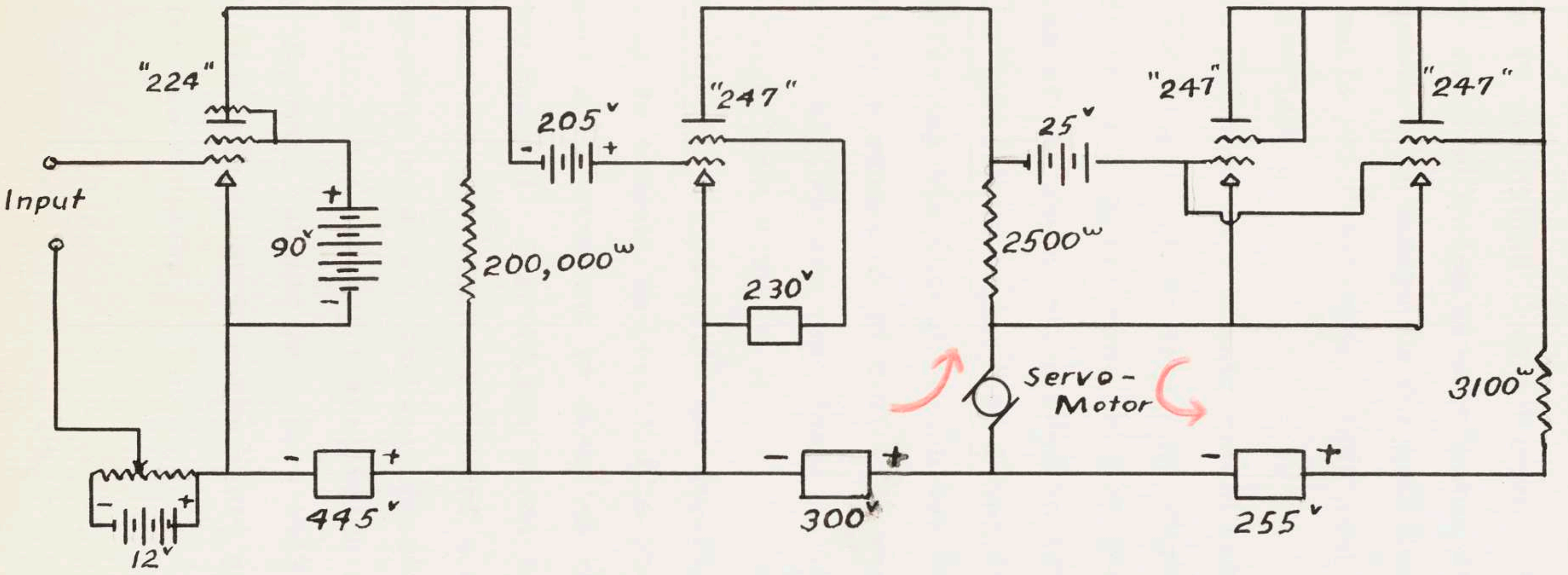


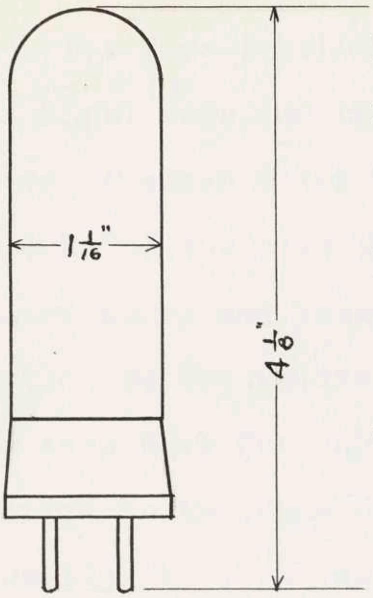

Plate Voltages supplied from an A-C. power pack

Figure 4.

from the linear relationship, and the maximum current obtainable is about 32 mils. In the preceding section it was stated that a current of 54.8 milliamperes per revolution gives critical damping. This corresponds to a change in the grid input of about 0.205 volts per revolution. This test is with the motor stalled.

In addition to a linear relationship between input and output, it is seen that since the amplifier is resistance coupled there is also practically no time lag at the frequencies used in this application. In order to prevent high frequencies in the input from affecting the output, which was found to be undesirable, a condenser of 0.01 mfd. was connected across the 200,000 ohm resistance in the plate circuit of the first tube.

<u>Photocell</u> -- Since it was desired to make the apparatus as compact as possible, a photocell which was small and convenient in shape as well as sensitive was desired. A PJ 23 was found to fulfill these requirements best. This cell is of the gas-filled, caesium-oxide type, and the specifications are given on page 16. Preliminary tests, which are described in the appendix, showed that this cell when operated at a plate voltage of 90 volts would give ample current for the purpose.




Specifications

| | |
|---|---|
| Number of Electrodes | 2 |
| Ratio Anode Current at 90 volts to Anode Current at 25 volts | 10 (max.) |
| Average Characteristics at 90 volts | |
| Sensitivity (μa./lumen) | 50 |
| Interelectrode Capacitance (μμf.) | 4 |
| Window Area (sq. in.) | 0.9 |

Figure 5.

<u>Scanning</u> <u>Slit</u> -- The exact proportions of the scanning slit are very important in determining the accuracy of following. As described before, the beam of light goes through the slit and is reflected from the paper. This reflected light goes into the photocell as shown below:

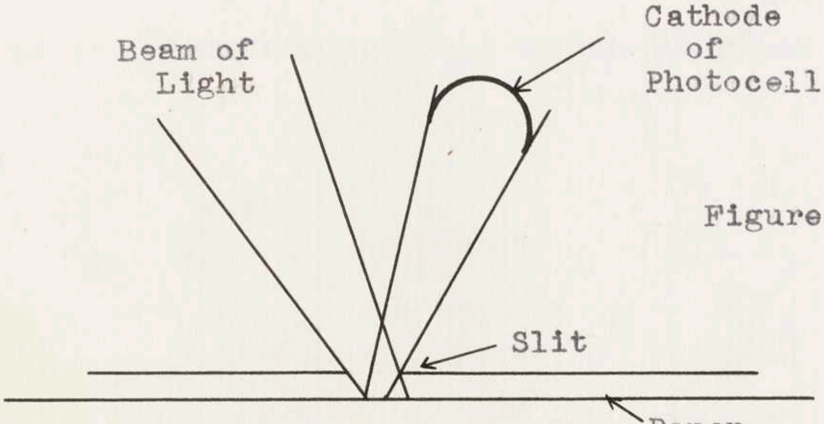



Figure 6.

The figure shows that the photocell does not see the same portion of the paper that the source of light sees and that the portion of the illuminated area of paper which the cell does see depends on the height of the slit above the paper. If the slit had sharp edges and rested flat on the paper, the cell would see the entire illuminated area. From this it is seen that for accurate following the slit must always be the same distance from the paper and, preferably, as close as possible.

The size and shape of the area seen by the cell is also important. For the best operation it is desirable to have the motor current proportional to the deviation from the curve, for then the speed with which the error is corrected is proportional to the amount of error. For this condition to obtain, the area must be a rectangle, and the proportion of the rectangle depends on the steepness of the slope which the apparatus is designed to follow. The way in which this is determined is shown by the diagrams in Fig. 7.

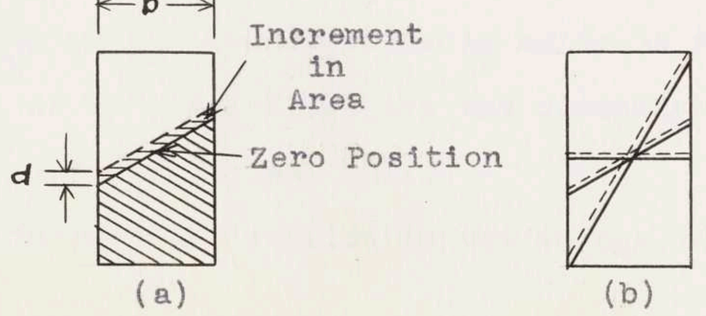



Figure 7.

From diagram (a) a deviation of amount d changes the relative areas of black to white by an increment of area, d x b, since the area is a parallelogram. Diagram (b) shows that this increment, for small deviations, remains a parallelogram of area d x b for all slopes up to that corresponding to the diagonal of the rectangle. For slopes greater than this, the relationship does not hold, and the increment in area for unit deviation becomes a function of the slope.

Assuming uniform illumination over the rectangle, the change in reflected light is proportional to the change in relative areas. Thus for a rectangular slit, for all slopes up to that corresponding to the diagonal (very nearly), the change in reflected light per unit deviation is a constant. This causes the change in photocell current per unit deviation, and, therefore, the change in servo-motor current per unit deviation to be a constant; so the motor current is a linear function of the deviation. As a consequence, the restoring torque of the motor will be proportional to the deviation, which is the desired relationship.

If the scanning slit is 1/4" x 1/32", the maximum slope for which the relationship holds is a slope of 8 to 1, or $83^{\circ}$, and these are the dimensions used in the apparatus which was built.

A third consideration is the change in amount of illumination per unit deviation. This factor must be such that the necessary value of 54.8 mils/revolution deviation be obtained in the armature of the motor. It is evident that this factor depends on the intensity of illumination, the reflection coefficient of the paper, the width of the slit, and other factors which will be mentioned later.

A fourth consideration is the degree of curvature of the line. If, in the initial position, the apparatus is adjusted so that half the slit is white and the other half is black, then correct following means that the apparatus moves so as to keep the slit half white and half black at all times. Now the apparatus should follow a curve point by point, and consideration shows that this is only possible when the segment of the line seen by the photocell is straight, and when this is true the apparatus will always follow the mid-point of the segment which it sees. In order to secure this condition, the slit should be as narrow as is consistent with the preceding consideration, for the narrower the slit, the less is the effect of the curvature.

In the present apparatus the slit has been made of manila paper. This is easily done by taking a piece of paper and cutting a slot 1/4" wide as shown on the following page:

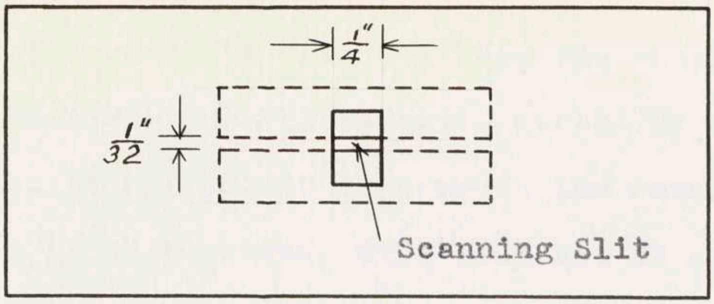



Figure 8.

Two smaller pieces, indicated by dotted lines, are then glued to the larger piece with a 1/32" space between them. The result is a slit of the desired dimensions, 1/4" x 1/32", with very clean edges. The manila paper is covered with india ink or an optical black paint to cut down the reflection. A fine hair may be glued crosswise of the slit for a reference point by which to determine just how accurately the apparatus is following the curve.

<u>Light</u> <u>Source</u> -- The characteristics which the source of light should have may be determined from the preceding section. It has been shown that the narrower the slit, the better the following, but it has also been stated that for proper operation of the amplifier and motor the input to the amplifier should be 0.205 volts per revolution displacement of the motor shaft. Since the motor is coupled to the carriage which contains the photoelectric cell, slit, and source of light, this displacement causes a corresponding displacement of the slit. Fulfillment

of the requirement is obtained when the change in current in the photocell circuit, caused by this displacement, is sufficient to produce the required change in input voltage. Evidently the amount of change in light which will produce this required voltage depends on the photoelectric circuit, but for a given circuit the intensity of illumination must increase as the slit is made narrower.

From preliminary tests it was found that an ordinary two-cell flashlight, when the beam of light was condensed by a spherical lens, would give sufficient intensity of illumination for the purpose required. The details of the tests are given in the appendix.

A second requirement is that the illumination must be uniform over the area of the slit. As shown previously, the operation of the apparatus is such as always to maintain equal areas of white and black. Since the action of the photocell depends on the total amount of reflected light, this condition may be maintained for all slopes only when the amount of light reflected from each portion of the white and from each portion of the black is the same.

As regards this condition, a flashlight is not particularly suitable. It was found, however, that by throwing the light slightly out of focus and carefully adjusting the position, the illumination could be made fairly uniform over a slit 1/4" x 1/32", while maintaining the necessary degree of intensity. There is no doubt but that this condition can be improved, and should be in a future design, but for the purpose of demonstrating the feasibility of the design, the flashlight was quite sufficient and had the advantage of being easy to obtain and use.

Photoelectric Circuit -- It is now possible to design a circuit which will satisfy the necessary requirements as outlined above. Only a very simple circuit, as shown below, is necessary.

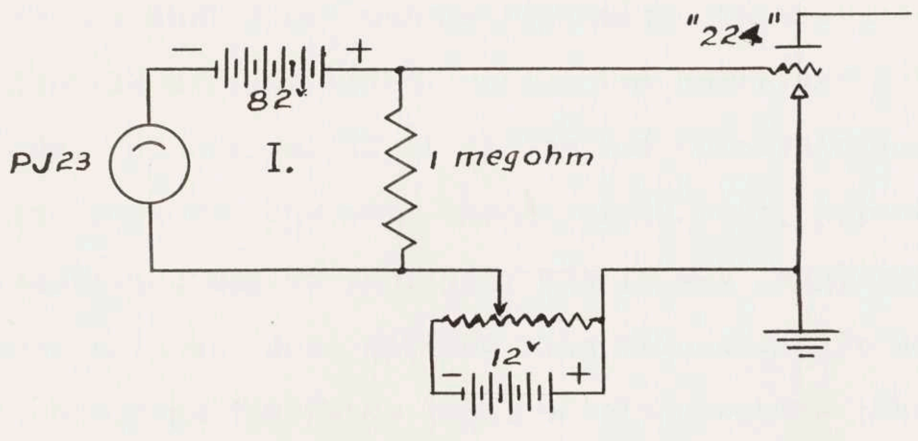



Figure 9.

Light reflected into the photocell causes current to flow in circuit I. This current produces a voltage drop in the megohm resistance, and this voltage forms part of the "C" bias of the first tube of the amplifier. The current in a photocell is proportional to the amount of light shining into it (there is a slight deviation for gas-filled cells, but this is negligible for the present purpose) so that the the voltage drop across the resistance will be proportional to the amount of light reflected into the cell. As stated before, the current output of the amplifier is linear with respect to the input voltage; so the current in the armature of the motor is proportional to the change in light in the cell. Therefore, the motor current is proportional to the deviation of the slit from its equilibrium position, and this is the desired relationship.

It was found from preliminary tests and calculations that a one megohm resistance in the photoelectric circuit would provide the necessary grid bias. Of course, if it is desired to obtain a greater grid bias for the same amount of current, a larger resistance may be employed, but it was found that when this was done difficulties were encountered due to the high impedance level of the circuit. More will be said concerning this point in the section on future development.

Another way of increasing the grid bias for a given illumination is to increase the plate voltage on the photocell. The maximum voltage for a PJ 23 is 90 volts, and the amplification within the cell (due to ionization of the gas) is quite sensitive to voltage variations near this point. In this connection, an effect might be mentioned although for the present purpose it is quite negligible. This effect is that a change in current in the cell also causes a change in the applied voltage because of the drop in the resistance. This causes a slight departure from linearity of the current-illumination relationship. Of course, there is a slight departure from linearity in any case at high anode voltages, but both of these have a negligible effect in the present application.

In the interest of economy, the leads to the resistance might be reversed from the position shown on page 22 so that the grid bias produced by stray light (that portion reflected from the interior of the apparatus and constant in value) would aid the bias given by the battery instead of bucking it as in the circuit shown.

Carriage and Table -- The actual design of the apparatus by which the curve-following is accomplished will now be discussed. This apparatus consists of a table which can be moved in one direction and a carriage which can be moved perpendicularly to this direction. The picture on page 5 shows the construction. The table on which the curve is placed is driven by a small series motor through a worm gear drive. Any speed up to about 300 r.p.m. can be obtained by means of a potentiometer for varying the applied voltage.

The lead screw has twenty threads to the inch. When applied to the Differential Analyzer, this drive would be furnished by the machine itself.

The metal cover on the carriage is to prevent the room lights from affecting the operation of the device. One side and the back were left open so that an observer could see the slit and check the accuracy of following. In order to obtain a clearer understanding of the construction and arrangement, this cover was removed, and the pictures shown below were taken.

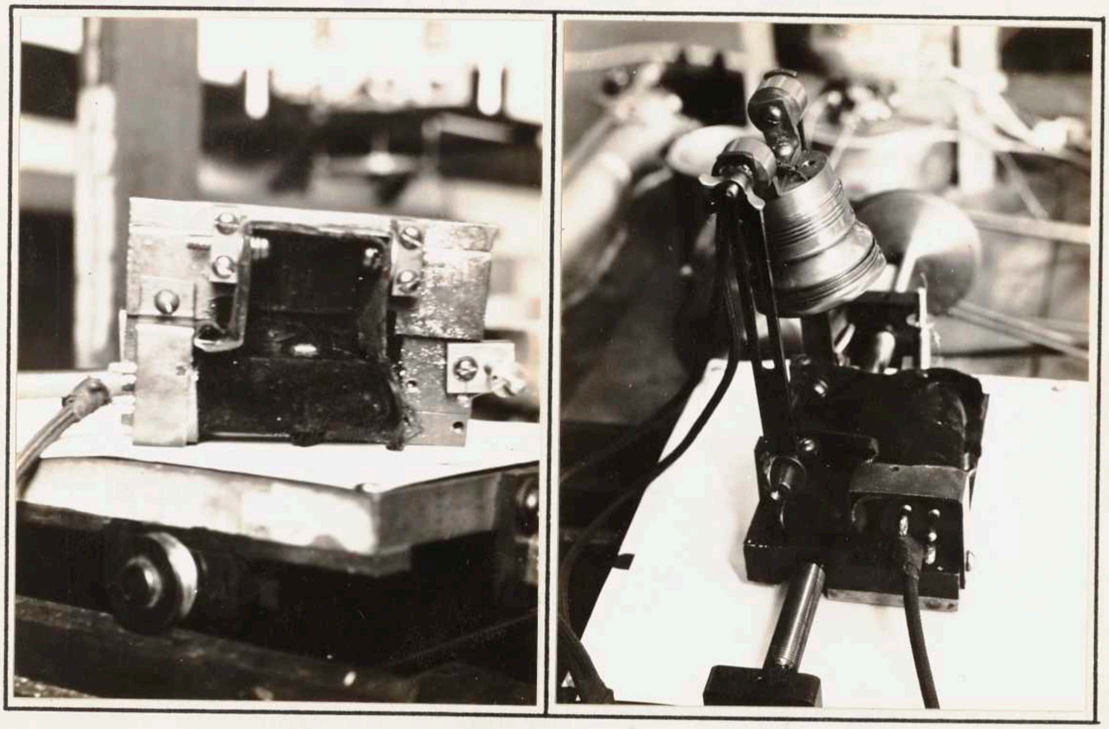


Figure 10.

In actually laying out the apparatus, the following points had to be considered: First, in order for the photocell to receive as much of the reflected light as possible, it should be as close to the slit as possible, thus subtending the maximum solid angle. However, since the beam of light shines on the slit from above, the photocell must not interfere with the passage of the light. Furthermore, for semi-matte surfaces, such as manila paper, the directional distribution of the reflected light is as shown in the following diagram:

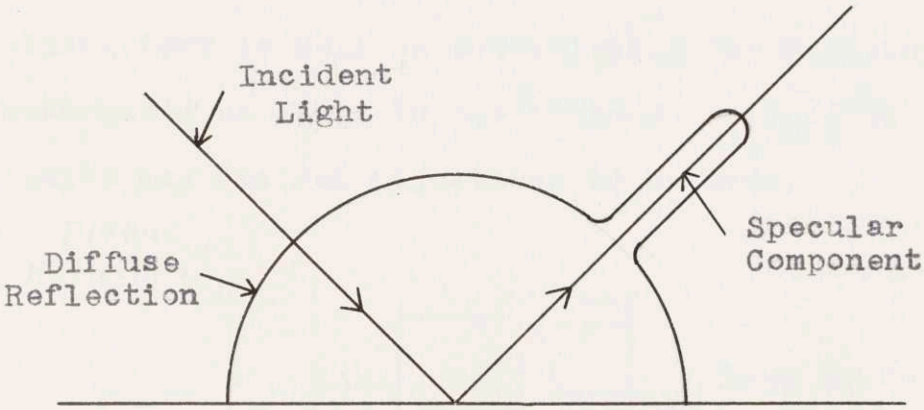



Figure 11.

In order to avoid the influence of the specular reflection, the positions of photocell and light source were so chosen that the hump fell outside of the space occupied by the photocell. Of course, if the ratio of specular reflection to diffuse reflection is the same for both the white and the black portions of the paper, then the specular reflection would do no harm. For certain positions of the light source on the present apparatus specular reflection is not avoided, but the effect of this is not known.

For the light source, the head was removed from a focusing flashlight. A base, as shown in figure 12, was made to screw into the head, and a small socket was mounted inside this base. A 7 cm. focal length spherical lens was inserted between the reflector of the flashlight and the plain glass front. The focus of the resulting system could be adjusted by screwing the head in or out slightly. This proved to be quite satisfactory as well as convenient. The mounting was constructed as shown in the picture on page 25, and permits any desired adjustment to be made.

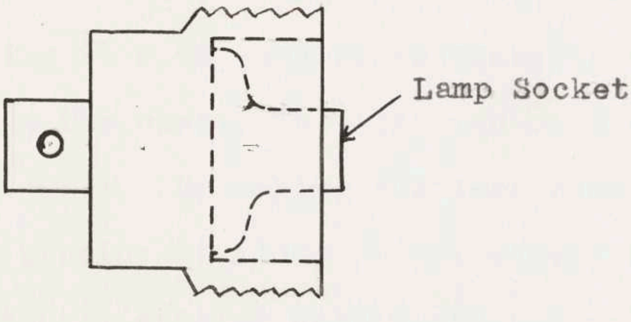



Figure 12.

The base of the carriage itself has a half nut and rides on a lead screw which is driven through the torque amplifier (the lead screw has 20 threads to the inch). It is further supported by a pivot which is shown in figure 13.

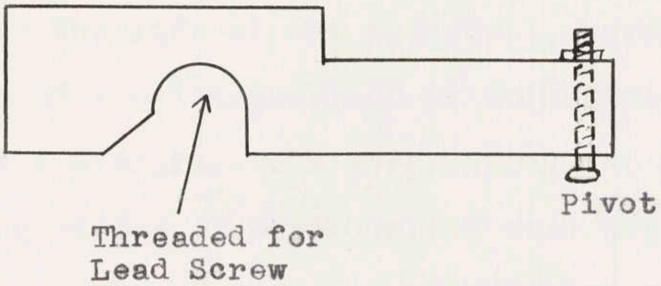



Figure 13.

By adjusting this pivot the position of the slit with respect to the paper may be determined. This type of support insures the slit's always being at the same distance from the paper provided the surface of the supporting platen is not too irregular.

The slit, made according to directions previously given, is glued to the bottom of the apparatus. This makes it possible to readily change the size of the slit, and by spacing it with several thicknesses of paper and adjusting the pivot, the slit can be placed very close to the paper. By making the base sufficiently large and placing it close to the paper, room light is prevented from getting underneath the carriage and reaching the area of the slit, thus affecting the intensity of illumination.

Since the beam of light is not confined to the area of the slit, there is quite a bit of stray light reflected into the photocell from the interior of the carriage. This stray light, since it is constant, only results in providing part of the constant grid bias of the amplifier and so does not affect the operation. The life of the photocell, however, is a function of the intensity of illumination of the cathode; therefore, this stray light was cut down by gluing pieces of black velvet over the interior of the apparatus. Those portions which could not be covered in this manner were painted with an optical black paint.

It was found that under certain conditions of operation the carriage was slightly unsteady; so a lead weight was placed in the carriage to make the movement more stable. In a future design this defect should be eliminated.

A small pencil was placed at one end of the carriage so that a permanent record of the following could be secured. The pencil is a nicely machined steel rod with a hole and set-screw at one end for holding a piece of lead. A holder was made of brass and a hole was drilled so that the rod could slide very smoothly in it. A small spring keeps the pencil always in contact with the paper.

Since the entire system is to be so adjusted that one revolution displacement of the motor shaft will cause a current of 54.8 mils to flow in the armature, it is evident that it is necessary to determine the correct ratio of drive from the motor to the carriage. On the input tables of the present Differential Analyzer, hand cranking at the rate of 200 r.p.m. (very fast cranking) will move the slider at the rate of 20 inches per minute on the highest gear ratio. For this same speed of following, the lead screw of the present apparatus should turn 400 r.p.m. If the ratio of drive between the motor and lead screw be one to one, a maximum speed of 800 r.p.m. could be obtained, and this ratio was chosen for trial.

<u>Complete</u> <u>Set-up</u> -- A diagram showing the relative positions of the various elements of the design is shown on page 10. The carriage lead screw is belted to the torque amplifier, and the tension is adjusted by moving the base which supports the carriage and table. The amount of torque required to move the carriage is very small, so there is no belt slippage. The table lead screw is also belted to its driving motor.

For the plate voltage supply of the d-c. amplifier, a power pack is used. Since the carriage and table are grounded to the cathode of the first amplifier tube, there is a potential of 750 volts between the carriage and the servo-motor (see p. 14). This potential exists directly across the driving

modified in later designs G.S. Brown

pulleys for the carriage and so requires due care in operating the apparatus.

The bias for the first amplifier tube is supplied by a potentiometer arrangement using a low-resistance rheostat as a vernier. It was found helpful to place these in a shielded box, with the shield grounded, and also to place the megohm resistance (wire-wound) of the photocell circuit with them.

# IV

## TECHNIQUE OF OPERATION

Method of Drawing Curve -- Since in normal opertion the area of the scanning slit is half white and half black, it is clear that the curve must have a width equal to at least half the length of the slit. However, for the sake of safety in case the zero setting shifts, the curve should have a width slightly greater than the length of the slit. The apparatus, however, follows the upper edge of the curve (or, by reversing the motor connections, the lower edge if desired). An easy way to construct the curve is to draw the desired curve with a ruling pen, as usual, and then fill in below this line with a small brush and ink to the necessary width. Ordinary india ink proves to be quite satisfactory and is easy to apply. Since the pivot supporting the carriage is not directly at the slit, some difficulty was experienced by the warping of the paper due to the india ink. The difficulty is easily removed by dry-mounting the curve on a piece of cardboard or sheet metal.

To prevent damage to the apparatus in case it should run off the curve, a band of ink may be drawn horizontally across the paper as shown in figure 14.

This figure is a reproduction of a curve actually followed by the apparatus.

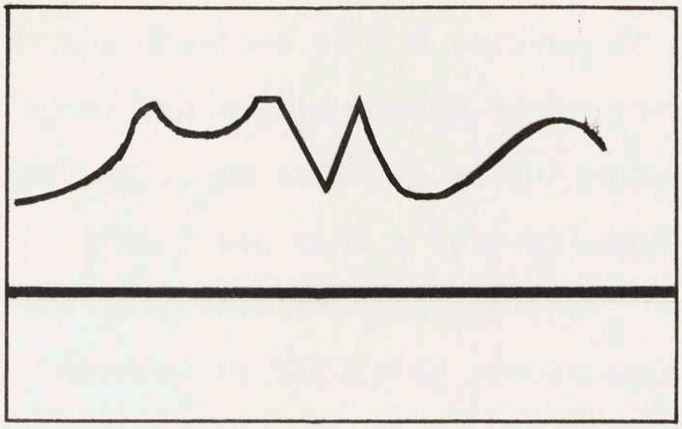


Figure 14.

Sequence of Operation -- The actual operation of the apparatus as at present developed will now be given. The filaments of the amplifier tubes and the light source are lighted, and the carriage is moved to that the slit is half white and half black. The plate voltage is then applied, and the grid bias of the first tube is adjusted to give zero current in the motor armature. When this is done the motor which furnishes the power for the torque amplifier is started, and the field switch of the servo-motor is closed. If this has been done correctly, the apparatus will have no tendency to move except for slight movements which are quite negligible.

Now the apparatus should be tested for proper damping. This is done by rotating the servo-motor shaft a quarter-turn, by hand, and reading the resulting motor current. This current should be about 13 or 14 mils, although it was found that successful operation was obtained with a current of about 16 mils. The action was found not to be very critical in this respect, and indeed a slight amount of underdamping (slightly too high a current) may be beneficial due to an increased acceleration. Too much underdamping, however, is likely to result in undesired oscillations. If the motor current is found to be too small, the intensity of illumination on the slit is increased, at the same time adjusting the grid bias on the amplifier so as to maintain the zero setting of the slit. This must be done very cautiously for the apparatus is very sensitive to slight changes in each of these variables.

When the above adjustments have been made, the apparatus is ready to follow the curve. The motor which drives the table is started, and the applied voltage is adjusted to give the desired speed of traverse. For a given speed of traverse, it is evident that the speed of the carriage depends on the steepness of the curve. Since a higher carriage speed means a larger motor current, this requires a greater deviation from the curve. Thus the faster the carriage has to move in order to remain on the curve, the greater the error in following. It is obvious that, with other factors constant, merely slowing down the speed of traverse will result in greater accuracy of following.

If, while the apparatus is running, the light on the carriage should burn out, the carriage will immediately be driven toward the upper edge of the paper. In order to prevent damage to the apparatus, in case this should happen, a switch should be provided so that, if the carriage should get too near the upper edge, the motor circuit would be opened.

At the present time the motor which drives the table is not reversible; so in order to be able to move the table in either direction, two belts are provided, one slightly longer than the other. The shorter belt is used for the direct drive, and the longer belt is crossed to give the reverse drive. The thread of the lead screw is turned off at the ends of the traverse to prevent over-travel.

# V

## RESULTS OBTAINED

After the complete apparatus had been set up, the necessary adjustments made, and satisfactory operation secured, a few observations were made to gain a more accurate knowledge of the performance. These tests were not at all rigorous or elaborate for such a procedure would be quite out of place in a preliminary design such as this one. However, it was felt desirable to obtain sufficient information to show the feasibility of the design, to give an idea of the accuracy involved, and to point the way toward further improvement.

Preliminary Observations -- The apparatus was started, as described in the preceding section, and let run for a few minutes to give the parts time to warm up. Then, with the table at rest, the following points were tested:

1. The shaft of the servo-motor was very quiet with scarcely noticeable vibration. This is important since there is sometimes quite a bit of a-c. pick-up which has a bad effect on the motor operation. Previous experience had shown that grounding the cathode of the first amplifier tube greatly reduces

this vibration.

2. Touching and moving the various leads in the photocell circuit and in the amplifier circuit did not appear to have any effect except when the grid terminal of the first amplifier tube was touched and when the battery in the photocell circuit was kicked. Touching the grid terminal, thus grounding it, would naturally be expected to change the bias and upset the equilibrium of the system, while kicking the battery is much harsher treatment than would ordinarily be employed. This test showed the circuit to be quite free from body capacity effects.

3. Most of the tests were made with the room nearly dark in order to eliminate any error due to people moving between the apparatus and the lights. It was hoped that the metal shield (lined with velvet) which had been placed over the photocell and light source would remove this necessity. With the current in the servo-motor initially at zero and the torque amplifier motor shut off (to keep the carriage from moving), the room lights were turned on. The change in motor current was about 5 mils. The grid bias was readjusted to give zero motor current, and the torque amplifier motor was started. It was found that moving near the apparatus and holding the hands near the carriage had no effect on the apparatus. This showed

that the shield was sufficiently effective to permit operation with the room lights on and also that, by completely enclosing the carriage, the effect of changes in the general level of illumination in the room could be eliminated.

4. It was next found that a half-revolution displacement of the carriage caused a current of about 25 mils to flow in the servo-motor. The motor, when displaced either side of zero, came back quickly and accurately to its initial position. When the displacement was maintained for several seconds, the return to zero was equally accurate.

5. Next, the permanence of the zero setting over a length of time was tested. This test was very important for it is easy to see that, no matter how accurately the servo-motor may maintain the apparatus in its equilibrium position, the following will not be accurate unless the equilibrium position itself, that is, with the slit half white and half black, be constant. A ten minute test showed that the zero shifted not more than a quarter turn in either direction, ending at a quarter turn too far into the black. This quarter turn corresponded to a movement of the carriage of 1/80". Resistances having a very small temperature coefficient are being obtained for the amplifier, and this may decrease the zero shift slightly.

It was later found that there was a loose part in the torque amplifier, and this may have caused some of the error. In any case, the error may be made as small as necessary by a proper choice of pulley ratios, although at the expense of speed of following.

6. Running Tests -- It was of interest to know the maximum speed at which the carriage could be moved. To determine this the carriage was moved onto the white portion of the paper (by holding a finger in the beam of light), the distance measured, and the time necessary for the carriage to return to the line noted. A maximum speed of 800 r.p.m. was obtained with 32 mils motor current. This corresponded to a linear speed of 40 inches per minute. The speed was limited by the fact that this was the maximum output of the amplifier regardless of the change in grid bias. If the linear relationship of photocell illumination and amplifier output still held, the current would be far in excess of 32 mils. It was noticed that, after this had been done several times, the zero had shifted into the black, possibly due to heating in the amplifier resistances.

7. The table was now set in motion, and the apparatus followed a curve such as is shown in figure 15. In order to determine the accuracy of following, the armature current of the servo-motor was observed. Since

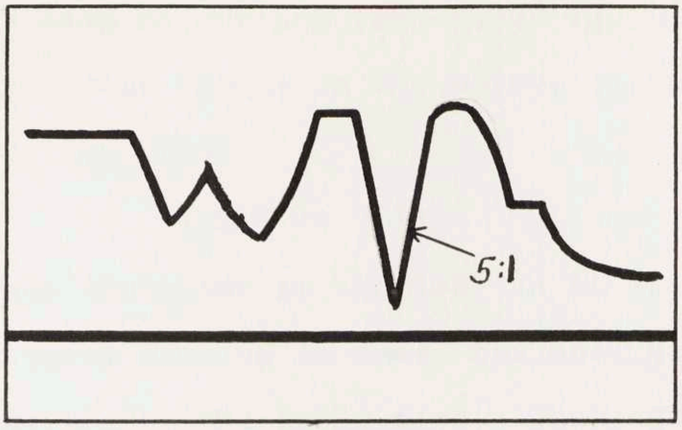



Figure 15.

this current is directly proportional to the deviation from the zero setting (in this test a current of 25 mils, stalled current, corresponded to half a revolution displacement or 1/40"), it was easy to calculate the exact amount of deviation. Of course, this is strictly correct only so long as the system remains linear, that is, for motor currents not exceeding 20 mils.

It was found that on the 5 to 1 slope, with a table speed of approximately 100 r.p.m., the motor current was about 20 mils. At this point it is necessary to make a distinction between stalled current and running current. On page 12 it has been stated that part of the damping of the motor is due to the back e.m.f. produced. This back e.m.f. causes a current to flow in the circuit composed of the armature conductors and the output circuit of the amplifier. If this damping current is denoted $i_d$, the current which produces torque in the motor, $i_t$, and the input current, $i_{in}$, then

$$i_{in} = i_t - i_d$$

When the motor is stalled, as it is when the test for critical damping is made, the back e.m.f. is zero and, therefore, the $i_d$ is zero. Thus, for this case the input current is also the torque-producing current.

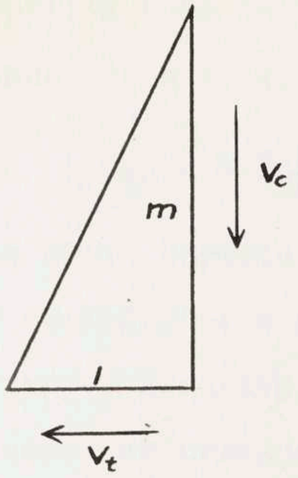
However, when the motor is running, there is a back e.m.f. and an $i_d$, and so the input current is not equal to the torque-producing current but is less than it by an amount equal to the damping current. For the case of critical damping and with the present amplifier, it has been found that

$$i_d = \frac{1}{2} i_{in}$$

and, therefore,

$$i_t = \frac{3}{2} i_{in}$$

Returning to the consideration of the test data, 25 mils of torque-producing current corresponds to a deviation of 1/40" or, from the preceding formula, 16-2/3 mils input current under running conditions corresponds to 1/40 of an inch deviation. A simple formula may be easily derived to obtain the error, given the slope of the line and the speed of traverse.




$v_t$ = speed of traverse

$v_c$ = speed of carriage

m = slope of line

Figure 16.

From this figure:

$$v_c = m v_t$$

Also, since 20 mils motor current gives a speed of 500 r.p.m. and 500 r.p.m. gives a carriage speed of 25" per minute, 20/25 mils gives a speed of 1" per minute and $v_c$ may be written:

$$v_c = \frac{25}{20}\ i_{in} = 1.25\ i_{in}$$

$i_{in}$ = motor current in mils

In the present test a running motor current of 16-2/3 mils corresponded to an error of 1/40". Therefore, since 40 x 16-2/3 mils gives an error of 1":

$$\text{Error in inches} = \frac{i_{in}}{40 \times 16\text{-}2/3} = \frac{3 i_{in}}{2000}$$

$$= \frac{3\ v_c}{1.25 \times 2000} = \frac{3 v_c}{2500}$$

$$= \frac{3\ m\ v_t}{2500}$$

Applying this to the given case, m = 5, $v_t = \frac{100}{20}$ = 5 inches per minute, and the error becomes:

$$\text{Error} = \frac{3 \times 5 \times 5}{2500} = 0.03 \text{ inches}$$

The above formula clearly shows that, as previously stated, for a given set-up accuracy of following varies directly as the slope of the curve and as the speed of traverse.

It might be mentioned at this point that in the above trials whenever the apparatus came to the end of the table where the curve, for a short distance, was very steep the carriage and torque amplifier went into a state of continuous oscillation. It was thought at the time that this was due to the carriage being slightly unstable because of the way in which it was supported. This proved partially correct, but, in addition, it was later found that the input bar of the torque amplifier had become loose, and this doubtlessly contributed to the instability. After this had been fixed, subsequent trials showed a much smoother operation.

A rather severe test was given the apparatus on May 6, which was Open House at the Institute. The apparatus was run steadily for three hours tracing and retracing a curve, and even after thirty runs the lines drawn by the pencil for each run showed a scarcely perceptible deviation from each other. The consistency of the results was very encouraging.

# VI

# FUTURE DEVELOPMENT

Since the apparatus so far developed is only an experimental design, further investigation along certain lines will be necessary and a design made which is directly applicable to the Differential Analyzer. For this reason the present section is devoted to a collecting together of the information which the writer feels is of value for future development. The paragraph headings are sufficient to indicate the scope of the sections.

Fundamentals of Carriage Design -- The mechanical design of the carriage must be such that the movement is very positive, smooth, and steady, with no possibility of its producing or sustaining oscillations. Any back lash or end play must be completely eliminated. This is evident from a consideration of the high amplification which is involved. A suitable design might be that which is used on the sliders of the present Analyzer, in which the weight of the slider is supported by two rods, and the function of the lead screw is to move the slider only.

Furthermore, the mechanical design must be such that the slit is always kept at a fixed distance from the paper, preferably as close to the paper as practicable. A possible design which would satisfy this condition would be to have the slit, photocell, and light source supported on a separate framework permitted to slide up and down on some nicely fitted supports, the supports being attached to the body of the carriage. A socket containing a single ball bearing (such as that on a ball-bearing top) would be located as near the slit as possible, and thus the entire apparatus would ride on the surface of the paper, the ball bearing insuring freedom of motion in any direction.

As stated before, the slit should have sharp, thin edges and be as narrow as possible. The principal factors determining the width are the intensity of illumination, the resistance in the photocell circuit, the amount of amplification of the d-c. amplifier, and the ratio of pulleys between the torque amplifier and carriage, for the resulting combination must be such as to give the required amount of damping in the servo-motor. The length of the slit is determined by the width and the maximum slope which it is desired to follow as well as by the ability of the light to provide uniform illumination over the entire area of the slit.

The photocell should be placed as close to the slit as possible in order to subtend as large a solid angle of light as possible, but it must not interfere with the beam of incident light. It should be as sensitive as possible, and the current should be linear with respect to the amount of illumination or very nearly so.

In placing the source of light, due regard should be had for the above. Specular reflection should be avoided (see p. 26). The light should be uniform and intense, the degree of intensity depending on the design of the rest of the system. Compactness is desirable, and stray light should be eliminated. A suggestion in this connection is a light having a straight line filament and a cylindrical mirror with an elliptical cross-section. If the filament is placed at one of the foci and the slit at the other, the arrangement should be satisfactory.

It has been stated that for critical damping the motor should have a stalled current of 54.8 mils per revolution deflection. Now if the motor is operated somewhat under-damped, the speed of response will be faster and the steady-state error less (see Professor Hazen's paper, loc. cit.). This condition is obtained by increasing the current per revolution deflection. In this case, however, there is a slight oscillation in the transient before the steady-state is reached, and when this was tried on the present apparatus, continuous oscillations and very unstable action resulted. This was probably due in some degree to a slight play and unsteadiness in the carriage, but it was later found that the torque amplifier was in very bad adjustment at the time of testing, and this probably was the major cause. Thus the effect of underdamping has not been determined, and it is very likely that a small amount will prove very beneficial.

It is evident that the deviation necessary to give the above damping current is dependent not only on the design of the photocell circuit, amplifier, etc., but also on the ratio of pulleys between the torque amplifier and the carriage.

A further point in the carriage design is the exclusion of room light. This is easily done by putting a cover on the apparatus to exclude light from above and by making the base large enough and close enough to the paper to exclude it in that direction. If the intensity of illumination on the slit is sufficiently great, and the inside of the apparatus is lined with velvet, this effect is of minor importance even with the present cover which is open on two sides.

Accuracy versus Speed of Traverse -- It has been shown that greater accuracy of following may be obtained by slowing down the table speed since the error for a given slope is directly proportional to the speed of traverse. A second method of accomplishing the same results also seems available, this method being to change the pulley ratios so that the servo-motor rotates faster than the lead screw. In this way the maximum carriage speed would be cut down, and this loss would be compensated for by a gain in accuracy as in the case of the table speed.

Careful analysis, however, shows that this is not the case, and the reasoning involved may be of interest. Assume that in case (1) the pulley ratio is one to one, the required damping current is 50 mils per revolution of the motor, and a running current of 20 mils gives 500 r.p.m. In case (2) assume the pulley ratio to be a step down in speed of two to one from motor to carriage, the rest of the factors being the same as in case (1).

Now for a given slope and a given table speed, the carriage speed must be the same in both cases. Assume this speed to be 250 r.p.m. of the lead screw. In case (1) this requires a motor speed of 250 r.p.m. and, therefore, a motor current of 10 mils. This current is produced by a certain deviation, d, the amount of the deviation being determined by the necessary condition that one revolution of the motor and, therefore, of the carriage lead screw give 50 mils stalled motor current.

The same reasoning applied to case (2) shows that a carriage speed of 250 r.p.m. requires a servo-motor speed of 500 r.p.m. and, therefore, a current of 20 mils. Investigating the condition for critical damping shows that in this case one-half revolution of the lead screw, corresponding to one revolution of the servo-motor, must give a current of 50 mils, and a unit deviation of the carriage in case (2) must give twice the current that a unit deviation produces in case (1). Therefore, the same deviation which will give 10 mils in case (1) will give 20 mils in case (2), and these currents produce the same carriage speed for the respective cases.

From this it is concluded that for a given slope and a given speed of traverse the error in following the curve will be the same regardless of the ratio of driving pulleys. This implies, of course, that the required speed of following is within the ability of the servo-motor to supply. The question of what the change in ratio does do comes to mind, and consideration shows that a change in ratio for a given carriage speed changes the torque output of the motor. Of course, the torque amplifier may be adjusted to compensate for this change in input torque.

From the above discussion it is evident that the only way to increase the accuracy of following after the slit has been made narrow enough to eliminate the error due to curvature is to decrease the speed of traverse or to slightly under-damp the motor, this last method possibly creating difficulties due to oscillations.

Fundamental Limitation -- This leads directly to the fundamental limitation of this design which lies in the servo-motor. To see more clearly just what this limitation is, take the case in which the motor is critically damped. It has been shown that with the present motor and amplifier increased accuracy may be obtained only by sacrificing speed of following. What is desirable is to obtain increased accuracy without sacrificing speed (still maintaining critical damping). To see how this may be done, the following equations are taken from Professor Hazen's paper, already referred to:

$f^2 = 4\ J\ k$

$f$ = damping torque per unit speed

$J$ = moment of inertia of system

$k$ = torque per unit angle

For a given speed the steady-state error is proportional to

$$\text{Error} \sim \frac{f}{k}$$

$$\sim \frac{4\ J}{f}$$

Also, rearranging the above equation,

$$k = \frac{f^2}{4J}$$

Now suppose that f, the damping torque, is doubled. From the preceding equation (assuming J constant), the torque per unit angle k is quadrupled. At the same time, the error for a given speed is halved, the desired result. It is evident, however, that if all the damping is considered to be in the damping shell of the motor (this means infinite amplifier output impedance so that there is no damping due to back e.m.f.) then at any given speed the damping torque is twice that which it was before, and so the heating is twice as much. Since the permissible amount of heating is the factor which limits the motor speed, it is seen that for these new conditions the maximum permissible speed is half that of the previous design.

This last result would seem to counteract the benefits gained from the decreased error, since the motor cannot go as fast as before, but a simple numerical case will show that this is not true. Suppose that in the original motor

500 r.p.m. $\Leftrightarrow$ 20 mils $\Leftrightarrow$ 0.1 in. deflection

corresponds to the maximum permissible speed. From this, 250 r.p.m. $\Leftrightarrow$ 10 mils $\Leftrightarrow$ 0.05 in. deflection.

Take the new motor in which f has been doubled, thus quadrupling k while all other factors have remained the same. For constant speed the driving torque equals the damping torque (neglecting mechanical torques); therefore, at any given speed the driving torque in this case must be twice that in the preceding case, and the current must be twice as great. But since the maximum permissible speed is now half the previous maximum, the following will hold:

250 r.p.m. $\approx$ 20 mils $\approx$ 0.025 in. deflection (half the previous deflection).

The pulley ratio may be changed to step-up this speed, resulting as follows:

500 r.p.m. $\approx$ 20 mils $\approx$ 0.05 in. deflection.

This shows that by doubling the damping coefficient the error in following may be halved while keeping the speed of following constant. Of course, the photoelectric system and amplifier must now be designed so that a given motor current may be obtained with half the previous deflection.

In obtaining better performance as indicated above, a new motor is necessary. But, in order to more easily determine the effect of changing f, it was assumed that all the electromagnetic damping was furnished by the damping shell in the motor. This is not the case in a practical application for the output impedance of the amplifier is not infinite. A consideration of this aspect shows a means of improving the performance of the present motor without re-designing it.

With the same amount of internal damping, the external damping may be increased, thereby increasing f without increasing the motor heating. In this way the same motor speed may be maintained, and the increased accuracy due to a higher f may also be obtained. Another, and perhaps more satisfactory, way of showing this is to divide the input current into two parts: torque-producing current and damping current.

$$i_{in} = i_t - i_d$$

Motor heating is determined by $i_{in}$ so that increasing $i_t$ and $i_d$ by the same amount will not affect this limitation. The $i_t$ is the current which is proportional to the deflection. The way to increase the $i_d$ is by decreasing the output impedance of the amplifier.

The preceding paragraphs have outlined methods by which better performance could be secured with the motor as at present designed and by re-designing the motor. The study has by no means been complete, and a more thorough analysis will probably show several other ways by which the desired results may be obtained. Experience gained in the construction of the present motor has shown that there is little doubt but that the performance may be greatly improved.

Necessary Precautions -- In the present apparatus a great deal of trouble was experienced due to insufficient shielding, particularly in the photocell circuit. It was found necessary to use shielded cable throughout the circuit and to ground the shield. The potentiometers, by which the bias on the first amplifier tube was obtained, and the megohm resistance in the photocell circuit were placed in a box lined with copper and ~~was~~ grounded. It was also necessary to ground the carriage, table, and supporting stand on which the table ran. These precautions, when the cathode of the first amplifier tube was also grounded, proved very effective, and practically no a-c. pickup could be felt in the servo-motor shaft, this being quite a sensitive indicator. It was also important to have very good connections in the photocell circuit, trouble being experienced with joints in wires which had to be free to move.

Another precaution, the importance of which was not determined, is in drawing the curve. It is evident that different coefficients of reflection from different points on the curve will affect the results, but the degree of care which is necessary was not investigated.

Improvements Along Present Line -- In addition to the points which have been mentioned thus far in this section, there are several other interesting problems which might be mentioned. The necessity for a uniform light has been pointed out previously, as well as the need for high intensity of light. Now it is also equally important, for good following, for this intensity not to change while a run is being made. It was found that the apparatus is extremely sensitive to very small variations in the voltage applied to the lamp; therefore, it is desirable to arrange some device which will automatically compensate for these changes. An idea which promises success is to use a second photocell which looks directly at the source of light. The current produced by this direct illumination would pass through a resistance, and the voltage across this resistance would be used as part of the grid bias of the first amplifier tube, as in the case of the other photocell. By adjusting the value of this resistance and the amount of light entering the photocell (or the anode voltage), it could be arranged so that the changes in grid bias caused by each photocell when the light intensity varied would exactly offset each other, and the zero setting of the slit would not be changed.

A second point in this same connection is the automatic setting of the zero for the initial position. That is, when a given curve is placed on the table, how is it possible to make the apparatus automatically place itself on the curve and then begin following it? This problem would be interesting although perhaps not of such importance as many others. The uniformity of the reflection coefficients of curves drawn on separate sheets of paper would probably need to be investigated.

Another desirable device is something to regulate the speed of traverse of the curve. It has been shown that for a given accuracy the speed of traverse should be inversely proportional to the slope of the curve. Some form of regulator which would be actuated by the servo-motor current would give the desired result. Then, if the current exceeded a certain value corresponding to the maximum allowable deviation, the table speed would be automatically reduced.

A further improvement, and perhaps the most important one of all, is a way to avoid the difficulty experienced due to the high impedance level of the photocell circuit and the resulting necessity for careful shielding. A suggested method is to mount the first amplifier tube on the carriage with the photocell, and then carefully shield the carriage. If this were done, the leads from the carriage to the amplifier would be at a low impedance level and so cause no trouble.

Another way to avoid the difficulty of high impedance levels is to use a selenium cell. The objection which immediately comes to mind in this connection is the time lag in the response of these cells, but the writer recently heard of the development by a sound-recording company of a selenium cell whose response characteristic compares favorably with that of the photocell. If this proves to be the case, it might be possible to use it to advantage.

Other Possible Designs -- Two more possibilities will be mentioned which should be investigated before a final decision is made. The first of these is the possibility of breaking up the light beam into a series of flashes, perhaps by a rotating disc with slits cut in it. A periodic voltage instead of a d-c. voltage would then be obtained and could be fed into a transformer-coupled amplifier, with resulting simplification of the amplifier design.

The second possibility is the use of some kind of a thyratron circuit to replace the d-c. amplifier.

# APPENDIX

Preliminary Tests on Lights -- These preliminary tests were rough tests for the purpose of finding out the order of magnitude of currents which could be produced by sources of light which were readily obtainable. The photocell was placed in a cardboard box which had an opening about one-quarter inch square. A piece of paper was held before the opening so that the beam of light could be reflected into the photocell.

Various sources of light were tried including an ordinary desk lamp, a 200-watt lamp and a large condensing lens, and an ordinary flashlight with a small, seven-centimeter focal length, condensing lens. The photocell was a PJ 23 with 90 volts on the anode. The first two systems gave currents of the order of tenths of a microampere while the flashlight gave currents of the order of one or two microamperes, the light being reflected from white paper. The current rating of a PJ 23 for d-c. linear operation is 2 microamperes; therefore, the flashlight was amply sufficient for the purpose. With a resistance of one megohm in the circuit, this amount of current would give one to two volts, which was thought to be all that would be necessary.

Preliminary Tests on Reflection -- Rough tests were also made on the relative reflection coefficients of manila paper covered with an optical black paint, manila paper covered with india ink, and black velvet cloth. Tests showed that the india ink reflected slightly more than the optical black paint, but the difference was not great compared with the difference between either of the two and the plain manila paper; therefore, india ink was used for the curves because of its greater convenience. The tests also showed that the black velvet cloth reflected far less light than the optical black paint, about half as much. For this reason the cloth was used for lining the carriage to eliminate stray light.