DEVELOPMENT OF A COMPARATIVE MARKOV MODEL FOR SHIP OVERHAUL POLICIES

by

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Submitted to the Department of Ocean Engineering and the Alfred P. Sloan School of Management on May 12, 1972, in partial fulfillment of the requirements for the degrees of Ocean Engineer and Master of Science in Management.

ABSTRACT

The object of the thesis is development of a method for describing the behavior of a ship's material condition over time so that the way in which various maintenance strategies affect this material condition may be examined. The thesis demonstrates the manner in which these strategies may then be evaluated based on total expected cost of a fleet of ships.

Ship failure and deterioration behavior is modeled as a transient Markov process, with a set of increasingly complex models. The most comprehensive of these allows such behavior to depend on the ship's present condition, its age, and the time since it was last overhauled. In the models a ship is treated as a single component, rather than an aggregation of several smaller components, and it is assumed that the ship's material condition can be categorized in a discrete manner.

Use of the models is demonstrated by the use of two example cases, one of which is based on actual maintenance data for Navy ships.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

I. INTRODUCTION

A. OBJECTIVES:

It is the intention of this thesis to develop an approach for examining the behavior of a ship's material condition over time and the manner in which that behavior is affected by various alternative maintenance policies. We will do this under the premise that a trade-off exists between the investment cost of a fleet of ships and maintenance expenditures for the fleet under various maintenance policies, given an operational requirement of the form that some number of ships x must be available for operations with probability p. For a specified x and p, our goal will be to determine that combination of ship acquisition and overhaul policies with the lowest expected cost, which obeys this constraint.

B. BACKGROUND:

In recent years nearly 20 percent, or approximately a billion dollars a year, of the Navy's entire operation and maintenance budget has been spent on the maintenance of the Navy's ships.[1] This does not include the cost of naval manpower, the cost of support ships and advance support bases, nor the capital cost of naval shipyards, all of which contribute significantly to the real cost of ship maintenance. Nor does it include any additional investment in ships that may result from choosing one maintenance policy over another.

A significant part of the cost of ship maintenance is incurred during the ship's regular overhaul. A regular overhaul is a scheduled period (under present Navy policy), occurring at specified intervals, during which a ship is assigned to a shipyard for work of a relatively major nature. It includes maintenance, both corrective and preventive, as well as (usually) some items of modernization.

Because a significant portion of the maintenance dollar is spent on regularly scheduled ship overhauls, a great deal of attention has been focused on improving the effectiveness of overhauls, i.e., attempting to get maximum benefit from the

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[1]See U.S. Office of Management and Budget, Budget of the United States Government and appendices thereto, 1969-1972, Government Printing Office, Washington, DC; and Department of the Navy, Naval Ship Systems Command, The Cost Estimate Fact Book, Volume I: Systems and Methods, Washington, DC, 1969.

work accomplished relative to the amount of time and money invested. This effort has generally proceeded along two lines: (1) By attempting to shorten overhauls while maintaining the same amount of work accomplished by improved planning, skill utilization, material procurement, etc., by such efforts as the Compressed Regular Overhaul (CROH) Program; (2) By examining the appropriateness of the length between regular overhauls and the manner in which a ship's material condition deteriorates between regular overhauls. It is in this latter area, perhaps, that the most effort has been exerted.

In this area, again, two basic approaches seem to have been taken: The first of these is to examine separately the failure behavior of the various components which make up a ship and then attempt to synthesize this into a description of the behavior of the material condition of the overall ship. To conduct such an analysis for just one component, assuming the data to support the analysis is available, is a rather extensive task. Frequently, sufficiently complete or accurate data to adequately support such an analysis is not available for the component. In considering an entire ship made up of several hundred subsystems the magnitude of the job becomes quite formidable. In the Navy, this is compounded by the lack of commonality between components of ships of the same type, or even of the same class of ship within a type. This has resulted in part from the Navy's policy (in the past, at least) of letting contracts for ships of a given class to several different building yards without requiring commonality of equipment.

A more promising path to take (and the second basic approach to the problem), at least in the short run, would seem to be to treat the whole ship as a single component. Indeed, using this approach, Farrar and Apple[2] were reasonably successful in developing useful formulas for repair requirements as well as information on the difference between scheduled and unscheduled repairs. This was done in spite of much of their data being incomplete and somewhat inaccurate. Their work tends to support the basic feasibility of this second approach, in that a great deal of useful, yet relatively inexpensive information can be derived by considering the ship as a single unit.

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[2] See Donald E. Farrar and Robert E. Apple, "Some Factors That Affect the Overhaul Cost of Ships: An Exercise in Statistical Cost Analysis," _Naval Research Logistics Quarterly_, vol. 10, no. 4, December 1963, pp. 335-368, and "Economic Considerations in Establishing an Overhaul Cycle for Ships: An Empirical Analysis," Institute of Naval Studies Research Contribution No. 7, April 1964, _Naval Engineer's Journal_, vol. 77, no. 1, February 1965, pp. 69-78.

C. THE APPROACH:

Here we pursue the "whole ship" approach, and develop an increasingly general set of stochastic models describing the behavior of a ship's overall material condition as the ship ages and as alternate overhaul strategies are employed. These models will be designed to be intuitively appealing to, and in fact to make use of, the informed judgment of Navy decision makers. A model which a decision maker does not understand, or which he finds at odds with his intuition, is unlikely to see much application.

Once a model has been developed through which a reasonable approximation of the behavior of a ship's material condition over its life can be made, a number of different investment and maintenance policies will be studied for their effect on the total cost of meeting a specified operational requirement. In particular, once the behavior of a single ship has been modeled, information can then be determined for an entire fleet of such ships. For example, the expected number of ships in overhaul at a given time for a given fleet size or, given the requirement for maintaining a specified number of operational ships with a specified probability, the required fleet size can be determined. We will show this in the thesis.

In this thesis we will consider cost as belonging to two categories: overhaul cost, and the ship acquisition and life cycle costs not including overhaul cost (reduced to present value). The latter will henceforth be referred to as "ship cost". In order to be valid for decision making, this must include the present value of the ship's entire life cycle costs less overhaul costs, including cost of crew, etc., as well as the actual acquisition cost. If we know or can reasonably estimate ship cost as well as overhaul cost, and if we can determine the expected number of overhauls the ship will experience under a given policy, then we can determine total expected lifetime cost.

The models developed in the thesis will include the capability to investigate the effect on total fleet cost of various overhaul policies and the required fleet size (number of ships) resulting from these policies.

We will model ship failure and deterioration behavior as a transient Markov process. Beginning with an extremely simple process we will elaborate it until it has the ability to reflect observed behavior in a reasonably complete manner. Once this has been done, data extracted from actual ship maintenance records will be introduced and use of the most comprehensive model as an aid to decision making will be demonstrated.

This approach to modeling a ship's change in material condition over time represents, it is felt, a somewhat unique conceptualization of the problem. Although Farrar and Apple treated the ship as a single component, their approach depended primarily on the use of regression analysis.

The model should be applicable to both naval and commercial ships, at least in basic concept. However, in the thesis, where specifics are required in either developing or demonstrating the model, they will be derived from and geared to destroyer-type ships of the U.S. Navy.

D. SUMMARY:

The remaining chapters of the thesis are summarized as follows: In Chapter II we first establish the underlying assumptions for the model. We then develop a simple model in which two categories of ship material condition are assumed (consider these as "failed" and "not failed") and in which the probability of failure is dependent solely on the ship's age. Chapter II concludes with a hypothetical example demonstrating use of the model as developed at that point.

In Chapter III we introduce into the model the assumption that not only will the probability of failure change with ship age, but with the length of time since the ship was last overhauled as well. We then expand our binary failure assumption (two categories of material condition) to one which allows for four categories of ship material condition, in which the worst of these, condition 4, corresponds to failure in the binary model. We conclude Chapter III with an example, using our four condition model, based on data extracted from the maintenance records of the Commander, Cruiser-Destroyer Force, Atlantic (COMCRUDESLANT).

Chapters II and III of necessity deal to a certain extent with the mechanics of computation for the model. It is not until we reach Chapter IV that we are actually able to apply the model to a comparison of alternative overhaul policies, utilizing the example of Chapter III as a basis. In Chapter

IV we examine the effect of overhaul policy on the number of ships necessary to meet a given operational requirement with a specified probability. Chapter IV compares overhaul policies of several specified intervals with the policy of overhauling a ship only upon failure, i.e. a "demand" dependent policy. Chapter IV further considers the effect of varying the specified operational requirement and the effect of imputing some penalty cost to an unscheduled overhaul, i.e., one which occurs on demand due to a ship failure.

In Chapter V, after discussing some possible directions for further work in this area, we arrive at the following conclusions resulting from the thesis:

(1) That the behavior of a ship's material condition over time can be successfully modeled by a transient Markov process.

(2) That such a model can be of use to Navy decision makers in comparing alternative overhaul policies.

(3) That actual implementation of the model will require initially a major data collection and reduction effort.

II. DEVELOPMENT OF THE BASIC MODEL

A. ASSUMPTIONS:

The basic assumptions underlying the models used in this report are: (1) that the change in a ship's material condition over time may be described by a Markov process, (2) that the ship can be classified into one of two or more discrete categories of ship material condition, and (3) that some reasonable approximation of the probabilities of remaining in the same or changing to a different material condition category as the ship ages can be determined.

A first order Markov process is one in which only the last state occupied by the process is relevant in determining its future behavior. Under the Markovian assumption the probability of making a transition to each state of the process depends only on the state presently occupied.[3]

As noted earlier, a ship, especially a warship, is sufficiently complex that its overall material condition is comprised of the net effect of the material condition of a large number of component systems and subsystems. Further, it can be significantly affected by a number of diverse factors, such as the intensity of operations, the funds available for repairs and spare parts, the qualification and motivation of the

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[3]See Ronald A. Howard, Dynamic Probabilistic Systems, Volume I: Markov Models, John Wiley and Sons, New York, 1971,p.3.

operating crew, etc. Consequently, a means of precisely determining a ship's overall material condition has thus far been somewhat elusive. Efforts to devise some sort of material condition index have been initiated and are continuing; however, no such usable index exists at this time.

It seems rather that the best means to try to determine a ship's overall material condition is to ask the operator/maintainer, in the present case the type commander. Such an approach is necessarily subjective and is more likely to be qualitative than quantitative in nature. The type commander, supported by his staff, is nonetheless likely to have a good "feel" for the condition of his ships since he is in frequent contact with them and has the benefit of experience with various ships. Based on this, the model assumes that a type commander, utilizing the collective knowledge and experience of his operating and maintenance staffs, can (and in practice does, although perhaps not formally) categorize his ships into two, three or four different levels of material condition. Some artificiality exists in that the model assumes discreteness where in fact a continuum exists. Nonetheless, experience has indicated that a type commander can fairly well tell which of his ships he considers "good", which are "bad", and which he would group in the middle as not belonging to either extreme category.

Initial estimates of the probability of transition from one of these categories to another, as the ship ages, must be made. Obviously the best means of doing this is to observe the behavior of a large number of similar ships over a period of time. Unfortunately, in the Navy to date there has been little or no formal categorization of total ship material condition as described above, so no past records are available from which estimators of transition probabilities may be determined directly, although a similar system is in effect in the Navy to denote the seriousness to the ship's operating capability of material casualties to individual components (part of the Casualty Reporting, or CASREPT, system). Consequently, the best approach would seem to be Bayesian in nature, where prior estimates of the behavior of a ship's material condition are established based on the best data and informed opinion available, and then are updated as better information becomes available. A discussion of how such prior estimates might be made will appear later.

B. THE TWO CONDITION, AGE DEPENDENT MODEL:

Given the foregoing assumptions, we will begin our analysis by considering a relatively simple case. In this simplest case, two possible conditions are assumed: these can be considered "good" and "bad", "operating" and "failed", etc. They will be designated as "1" and "2", respectively.

Intuitively, it seems reasonable to expect a higher probability of failure for an old ship than for a new one. While this may or may not be true, it would be desirable if the model were able to accomodate such a possibility. In order adequately to describe a state, therefore, both the ship's condition and age must be specified. Let the state indices be age t and condition k; the state then is described by the ordered pair:

(k,t) = ship is t units old and in condition k,
$k \varepsilon K$, $t \varepsilon H$,

where $K = \{1,2\}$ for a two condition model,

and $H = \{1,2,\ldots,T\}$, T being the life (i.e., maximum age the ship will reach) of the ship in terms of the basic time unit.

The assumption is made that the ship will occupy the state for all of a given time increment. This should be reasonably valid if the time increment is relatively short. In fact, it could correspond to a periodic appraisal of the condition of a fleet of ships, so that transition would take place only when the ships' condition was reappraised. In this simple model, it is assumed that the decision maker, upon finding a ship in condition 2, would immediately take corrective action consisting of an "overhaul". In other words, a "demand" (for overhaul) condition would be said to exist any time the ship reached condition 2. The objective of overhauling a ship presumably is to improve its material condition. It will therefore be assumed that the overhaul will with certainty return the ship to condition 1.

Since an overhaul, under the latest assumption, in effect represents a transition from condition 2 to condition 1, it is convenient to take as the basic time unit a period equal to the approximate length of overhaul of a ship, or a period of which length of overhaul is a multiple. A review of 59 recent overhauls of a variety of COMCRUDESLANT destroyer-type ships indicated an approximate four month overhaul length, on the average. The average time between overhauls was approximately 40 months. For this model therefore, unless otherwise stated, the basic unit of time is assumed to be four months, although the actual length of the increment should in general be of little importance to the applicability of the model. Four months should, however, provide sufficient time for the decision maker to perceive a change in condition. It provides, for a ship life of, say, 20 years, some 60 different ages.

Transition probabilities will be described by $p(k|i,t)$, which is the probability of the ship's being in condition k at age t+1 given that it was in condition i at age t. What results, then, is a transient Markov process lasting the duration of the ship's life (T time units) in which a state has zero probability of occupancy more than once.

With this model, multi-step transitional probabilities may be calculated. If some initial state is fixed, the multi-step transition probabilities will provide the probability of entering a given state (i.e., a given condition at a given age). Multi-step transition probabilities will be described by $P(k,t|k_o,t_o)$ which is interpreted as $Pr\{$entering state $(k,t)|$process began in state $(k_o,t_o)\}$. Since at each transition t must increase by exactly 1, the number of steps in the transition is given simply by $t-t_o$. For this model, multi-step transition probabilities are calculated by the recursion formula:

$$P(k,t|k_o,t_o) = \sum_{i=1}^{2} P(i,t-1|k_o,t_o)p(k|i,t-1),$$
$$k = 1,2.$$

Of particular interest in this model will be the probability of entering a given state, given that the ship is new as an initial condition. If "new" is assumed to mean that the ship is in condition 1 at time zero (in other words that the initial state is (1,0)), then this probability would be described by $P(k,t|1,0)$ for the probability of entering state (k,t). For ease of notation henceforth this will be written simply as $P(k,t)$.

This model will provide multistep transition probabilities (as has been described) and therefore the probability of the ship's being in overhaul at any given age. From this the expected number of overhauls over the ship's life may be determined. This is simply the probability of being in overhaul in each time unit of a ship's life, summed over the ship's life. Since we are assuming that a ship found to be in condition 2 will automatically undergo overhaul, we may write: E(number of overhauls over ship's life) $= \sum_{t=1}^{T-1} P(2,t)$. The upper limit of T-1 instead of T reflects the assumption that no ship would be overhauled during the last time unit of its life, just in time to be removed from service.

If we desire to determine the probability distribution of the number of overhauls the ship will experience during its life (or up to a given point in its life), we must do the following: Define $\pi(k,t,x)$ as the probability of the ship's being in state (k,t) and having experienced x overhauls (including the current one, if $k=2$) given that the ship was new at $t=0$. This probability may also be calculated by means of a recursion formula:

$$\pi(k,t,x) = \begin{cases} \sum_{i=1}^{2} \pi(i,t-1,x)p(k|i,t-1), k=1, \\ \sum_{i=1}^{2} \pi(i,t-1,x-1)p(k|i,t-1), k=2. \end{cases}$$

We may then find the probability distribution for the number of overhauls in a ship's lifetime as follows:

$$\Pr\left\{\begin{array}{c}\text{ship will experience} \\ \text{x overhauls in its lifetime}\end{array}\right\} = \sum_{k=1}^{2} \pi(k,T-1,x),$$

where T is the ship's life in terms of the basic time unit. We again use the value for t=T-1 instead of T.

Now consider a fleet of identical ships, all of the same age. This is obviously an oversimplification, but will serve to demonstrate the manner in which the model may be used in considering groups of ships. Given the above assumptions and independence between failures of different ships, the probability of having a certain number of ships in overhaul at any given time out of a fleet of a given size follows a binomial probability mass function. If a denotes the probability of a single ship being in overhaul at a given age, the probability of having y ships of that age in overhaul out of a fleet size of N ships is given by:

$$\binom{N}{y} a^y (1-a)^{N-y}$$

This is also, of course, the probability of having N-y ships available for operations out of a fleet of N ships (assuming that a ship not in overhaul is available for operations).

From this probability mass function can be determined the fleet size required to provide some number of operational ships (as a minimum) with a specified probability. Given such a requirement, a fleet size necessary to support it is assumed. The probability of having at least the minimum number of operational ships is computed using the binomial probability mass function. If this probability is greater than the specified probability, then the assumed fleet size is decreased until the requirement is just met. If this probability is less than that specified, the assumed fleet size is increased until the requirement is met.

Since the expected number of overhauls over a ship's life may be determined from the model, assume that both ship cost and overhaul cost are known. Then the expected total cost may be determined for a single ship by multiplying the overhaul cost by the expected number of overhauls and adding the result to the ship cost. Once this cost has been calculated for a single ship it is known for a fleet (of a given size) of such ships.

C. AN EXAMPLE:

Before proceeding with development of more comprehensive models it may be helpful to demonstrate application of the relatively simple model developed thus far in a hypothesized example.

Suppose, for the example, that the life of the sample ship is expected to be 10 years, or 30 time units; further, suppose that the probability of "failure" increases in stages as the ship ages, as follows:

$$p(2|1,t) = \begin{cases} .1 & t=0,\ldots,5 \\ .2 & t=6,\ldots,11 \\ .3 & t=12,\ldots,17 \\ .4 & t=18,\ldots,23 \\ .5 & t=24,\ldots,29 \end{cases}$$

It is assumed that the ship begins in state (1,0), and that if it should "fail", overhaul of the ship would begin immediately and would, with certainty, return the ship to condition 1 at the beginning of the next time unit.

The following have been computed for the example: Values of $P(k,t)$ for all k and t; values of $P(k,t|k_o,t_o)$ for t through ten time units; values of $\pi(k,t,x)$ for all possible k, t and x; and the distribution and expected number of overhauls the ship will experience in its life. All possible $P(k,t|k_o,t_o)$ were not computed as the result becomes somewhat unwieldy. For example, computing all values through 30 time units results in approximately nine times the output as for ten units. Because of this, and having demonstrated the manner in which multi-step transition probabilities may be calculated, henceforth only those values specifically required for some application will be computed, except for the special case of $P(k,t)$ which is in fact the family of state probabilities, given that the ship began in the "new" condition.

The computer program used for the calculations was developed from the recursion relationships of the model. The program, together with the results for the example, comprise Appendix A to the thesis. The program is written in the PL/1 programming language but the logic should present no problem to FORTRAN programmers.

The expected number of overhauls the sample ship will experience over its life is 6.38. The probability distribution of number of overhauls is shown in Table 1. These probabilities are determined by summing $\pi(1,29,x)$ and $\pi(2,29,x)$ for all values of x. A ship age of 29 instead of 30 is used since it is assumed that it would be a waste of resources to overhaul a ship during the final time unit of its life. The special family of transition probabilities $P(k,t)$ are given by Table 2. Since these give the probability of being in a specific state, given that the ship started "new", they will henceforth be referred to simply as "state probabilities" to

Number of Overhauls in Ship's Life	Probability
0	.00002
1	.00053
2	.00499
3	.02287
4	.08295
5	.17352
6	.24346
7	.23183
8	.14980
9	.06494
10	.01844
11	.00328
12	.00034
13	.00001
14	.00000+
15	.00000+

Note:

Because of the certainty of returning to condition 1 after overhaul, the ship cannot experience "failure" (and therefore overhaul) during two consecutive time units. Therefore the probability of more than 15 overhauls is zero.

TABLE 1

Probability Distribution of Number of Overhauls for the Two Condition, Age Dependent Model Example

k	t	P(k,t)
1	1	.900
2	1	.100
1	2	.910
2	2	.090
1	3	.909
2	3	.091
1	4	.909
2	4	.091
1	5	.909
2	5	.091
1	6	.909
2	6	.091
1	7	.818
2	7	.082
1	8	.836
2	8	.164
1	9	.833
2	9	.167
1	10	.833
2	10	.167
1	11	.833
2	11	.167
1	12	.833
2	12	.167
1	13	.750
2	13	.250
1	14	.775
2	14	.225

TABLE 2

State Probabilities for the Two Condition, Age Dependent Model Example

k	t	P(k,t)
1	15	.767
2	15	.233
1	16	.770
2	16	.230
1	17	.769
2	17	.231
1	18	.769
2	18	.231
1	19	.692
2	19	.308
1	20	.723
2	20	.277
1	21	.711
2	21	.289
1	22	.716
2	22	.284
1	23	.714
2	23	.286
1	24	.715
2	24	.285
1	25	.643
2	25	.357
1	26	.679
2	26	.321
1	27	.661
2	27	.339
1	28	.670
2	28	.330
1	29	.665
2	29	.335
1	30	.667
2	30	.333

TABLE 2 (continued)

differentiate them from other multi-step transition probabilities.

Under the assumption that a ship found to be in condition 2 will automatically undergo overhaul, the state probability P(2,t) is also the probability of the ship's being in overhaul at age t. For the present example, assume that the decision maker desires to maintain a fleet of ships identical to, and of the same age as, the sample ship. Assume further that he wishes to have a probability (say .95) of having a certain number of ships operational (that is, not in overhaul), say 30 ships. The problem is to determine the size of fleet required to give a probability of at least .95 of 30 or more operational ships.

With these assumptions, the required fleet size may be determined as follows: First, determine the highest probability of being in overhaul (that is, the maximum value of P(2,2) at any point in the ship's life. For the example this is P(2,25) (from Table 2) which has a value of .357. This represents the worst possible situation. Next, using the binomial probability mass function, obtain the probability distribution for the number of operational ships using the value .357 and an assumed fleet size. Increase or decrease the fleet size as required until the requirement of a .95 probability of 30 operational ships is just met. This is, of course, the smallest fleet size that will meet the specified requirement.

The minimum fleet size satisfactory to meet the requirements specified in the example is 56. The computer program developed to accomplish this is contained in Appendix B. The initial fleet size assumed was 50 ships. The probability density for the number of operational ships for a fleet size of 56 is provided in Table 3. As would be expected, the probability of 30 or more operational ships is .963 (slightly greater than .95 due to discreteness).

It should be emphasized that use of the binomial probability mass function is permitted only by the fact that the ships are identical and of the same age. The problem becomes more complex (in computation, although not in conceptualization) for a mixed fleet.

Obviously, with the expected number of overhauls determined, if a cost per overhaul can be estimated or is known, then overhaul costs over a ship's life can be calculated. Assume the following representative costs for the example: Ship cost, \$40 million; overhaul cost, \$1.5 million. Then the mean of the total cost for the example is:

$$\begin{aligned}&\$40.0 \text{ million} + 6.38 \times \$1.5 \text{ million}\\&= \$49.57 \text{ million.}\end{aligned}$$

This can be expanded to include the entire fleet.

Although the data is hypothetical and this model is relatively simple, the preceding example should serve to indicate the basic mechanics and some potential uses for such models.

Number of Operational Ships	Probability
0	.00000+
1	.00000+
2	.00000+
3	.00000+
4	.00000+
5	.00000+
6	.00000+
7	.00000+
8	.00000+
9	.00000+
10	.00000+
11	.00000+
12	.00000+
13	.00000+
14	.00000+
15	.00000+
16	.00000+
17	.00000+
18	.00000+
19	.00000+
20	.00001
21	.00003
22	.00008
23	.00022
24	.00053
25	.00123
26	.00264

TABLE 3

Probability Distribution of the Number of Operational Ships for a Fleet of 56 Ships (Two Condition, Age Dependent Example)

Number of Operational Ships	Probability
27	.00527
28	.00982
29	.01706
30	.02762
31	.04167
32	.05857
33	.07664
34	.09327
35	.10548
36	.11070
37	.10766
38	.09685
39	.08042
40	.06149
41	.04317
42	.02774
43	.01625
44	.00864
45	.00414
46	.00178
47	.00068
48	.00023
49	.00007
50	.00002
51	.00000+
52	.00000+
53	.00000+
54	.00000+
55	.00000+
56	.00000+

TABLE 3 (continued)

The following chapter will deal with development of more comprehensive models based on this one.

III. DEVELOPMENT OF MORE COMPREHENSIVE MODELS

A. THE EFFECT OF OVERHAUL ON TRANSITION PROBABILITIES:

As a first step in developing a more comprehensive model, consider the following: If one is willing to agree that some policy of overhauling ships on a regularly scheduled basis may be the most economical, one is implicity assuming that an overhaul may in some way "improve" succeeding transition probabilities. Otherwise, if no such improvement can be attained by an overhaul, there is no reason for overhauling. If a ship is just as likely to "fail" immediately following an overhaul, at some given age, as it is without a recent overhaul, at the same age, then there is obviously no justification for expending the resources to overhaul the ship until it actually fails.

It therefore appears that to describe adequately states and transition probabilities requires knowing the time (i.e., number of time units) since the last overhaul, as well as the ship's age and condition. Therefore, let the state indices now be age t, condition k, and time since last overhaul j; the state is described by the ordered triplet:

(k,t,j) = ship is t units old and has gone j units since its last overhaul, and is in condition k, $k\varepsilon K$, $t\varepsilon H$, $j\varepsilon J$,

where again, $K = \{1,2\}$ for a two condition model,

$H = \{1,2,...T\}$, T being the life of the ship in terms of the basic time unit,

and $J = \{1,2,...m\}$, m being the longest period a ship could possibly go without an overhaul; for a "demand" dependent (i.e., overhaul only upon ship's reaching condition 2) overhaul policy, m=T.

For a ship that has yet to undergo its first overhaul, j will be taken as equal to t.

Transition probabilities (one-step) will be described by $p(k|i,t,j)$, which is the probability of the ship's being in condition k at age t+1, j+1 units since its last overhaul, given that it was in condition i at age t, j units since last overhaul.

It will now be necessary to describe multi-step transition probabilities as follows:

$P(k,t,j|k_0,t_o,j_o)$,
Pr{entering state $(k,t,j)|$transition began in state (k_o,t_o,j_o)}.

Following the convention adopted earlier, the special family of transition probabilites $P(k,t,j|1,0,0)$, the "state" probabilities, will be designated by $P(k,t,j)$.

The expression $\pi(k,t,j,x)$ will be defined as the probability of the ship's being in state (k,t,j) having experienced x overhauls. Recursion formulas may be determined for multi-step transition probabilities and values of $\pi(k,t,j,x)$, given that the process started in state (1,0,0), for this two-condition, age and time-since-overhaul dependent model. These have been developed in a manner parallelling those developed earlier and are shown in Appendix C.

B. THE MULTI-CONDITION, AGE DEPENDENT MODEL:

While a structure which provides for only two categories of material condition serves sufficiently well to demonstrate the mechanics of the model, to say that a ship is either in good condition or is in bad condition and therefore in need of overhaul, with no middle ground, is intuitively unappealing. It seems more likely that the decision maker could group his ships into three, four, or even more approximately discrete groups based on his subjective (but informed) opinion.

At this point it will be assumed that a four condition model is reasonable in that it provides a wider range for categorizing a ship's condition without providing so many choices as to make such categorization overly difficult and arbitrary. There will be borderline cases, to be sure, regardless of the number of categories chosen.

First, we will consider a four condition model in which transition probabilities are unaffected by the time since the ship was last overhauled (although they still can vary with ship age). In this model, the beneficial effect of an overhaul can be represented by a change to a better condition, e.g., condition 1 with certainty or condition 1 and condition 2 with probabilities summing to unity, etc. Transition probabilities can be used to describe a gradually deteriorating material condition; all that is necessary is to structure the transition probabilities so that, over time, the ship's condition is more likely to get worse than it is to get better.

With this model, calculation of the entire set of (multistep) transition probabilities remains reasonable, although still cumbersome; further, we can still determine, within reasonable computational limits, the probability distribution for the number of overhauls which the ship will experience in its life, as well as the expected value of this number.

Computation of these probabilities is accomplished in a manner identical to that described in Chapter II, except that the set of possible conditions which the ship might reach, K, is {1,2,3,4}.

Such a model will permit us to make comparisons between a demand dependent overhaul policy and one in which the ship is overhauled at regularly scheduled intervals (i.e., at certain regularly spaced points in its life). Such a scheduled overhaul policy will have the advantage of permitting at least part of the overhauls in a ship's life to be scheduled in advance rather than resulting from the ship' reaching a demand condition (i.e., unscheduled). However, with this particular model, we cannot, of course, base a ship's entry into a scheduled overhaul on the time since it was last overhauled.

C. THE MULTI-CONDITION, AGE AND TIME-SINCE-OVERHAUL DEPENDENT MODEL:

The most comprehensive model we will consider combines both the dependence on ship age and time since overhaul of the model of section A with the multi-condition capability of section B. This allows ship failure behavior to depend in an arbitrary manner on ship age, time since overhaul, and present material condition. This gain in generality is not without its sacrifice. As we will see, by requiring three indices to describe a state we have lost the ability to obtain some information about the process due to computational limitations. The recursion formulas for this multi-condition, ship age and time-since-overhaul dependent model are shown in Appendix C.

D. AN EXAMPLE FOR THE MOST COMPREHENSIVE MODEL:

It follows from the description of this model that there are T x m sets of transition probabilities, since probability of transition is dependent on both ship age and time since last overhaul. If one considers a ship's life, T, to be about 25 years, in the case where a ship isn't overhauled until the "demand" condition is reached, m is also 25 years. This represents 75 time units in the model, which implies 75 x 75 = 5625 possible sets of transition probabilities. This is clearly unmanageable. The answer, therefore, is to attempt to group ships which seem to follow similar "failure" rates into categories based upon age and time since overhaul.

Following this approach, use of the multi-condition, age and time-since-overhaul dependent model will be demonstrated utilizing transition probabilites estimated from the RAV (restricted availability) history of 33 CRUDESLANT destroyer-type ships. The manner in which these probabilities were estimated is described in Appendix D. The ships were grouped into three categories by age: Category I, 1 to 3 years; category II, 4 to 13 years, and category III, 14 to 25 years, inclusive. Three categories of time since overhaul were decided on: one (four month) time unit, two to three time units, and four or more time units. This results in nine sets of transition probability matrices. These are shown in Table 4.

Age Category I

j = 1

$p(1|1,t,1) = 0.505$

$p(2|1,t,1) = 0.253$

0)3|1,t,1) = 0.121

$p(4|1,t,1) = 0.121$

$p(i|k,t,1)$ are of no concern for $i = 2,3,4$, since ship can only be in condition 1 at $j = 1$.

j = 2,3

$p(1\|1,t,j) = 0.663$	$p(1\|2,t,j) = 0.263$	$p(1\|3,t,j) = 0.263$
$p(2\|1,t,j) = 0.263$	$p(2\|2,t,j) = 0.663$	$p(2\|3,t,j) = 0.263$
$p(3\|1,t,j) = 0.037$	$p(3\|2,t,j) = 0.037$	$p(3\|3,t,j) = 0.437$
$p(4\|1,t,j) = 0.037$	$p(4\|2,t,j) = 0.037$	$p(4\|3,t,j) = 0.037$

j = 4 or greater

$p(1\|1,t,j) = 0.682$	$p(1\|2,t,j) = 0.282$	$p(1\|3,t,j) = 0.282$
$p(2\|1,t,j) = 0.282$	$p(2\|2,t,j) = 0.682$	$p(2\|3,t,j) = 0.282$
$p(3\|1,t,j) = 0.018$	$p(3\|2,t,j) = 0.018$	$p(3\|3,t,j) = 0.418$
$p(4\|1,t,j) = 0.018$	$p(4\|2,t,j) = 0.018$	$p(4\|3,t,j) = 0.018$

TABLE 4

One-Step Transition Probabilities for the Multi-Condition, Age and Time-Since-Overhaul Dependent Example

Age Category II

$j = 1$

$p(1\mid 1,t,1) = 0.333$

$p(2\mid 1,t,1) = 0.167$

$p(3\mid 1,t,1) = 0.250$

$p(4\ 1,t,1) = 0.250$

$j = 2,3$

$p(1\mid 1,t,j) = 0.662$	$p(1\mid 2,t,j) = 0.262$	$p(1\mid 3,t,j) = 0.262$
$p(2\mid 1,t,j) = 0.262$	$p(2\mid 2,t,j) = 0.662$	$p(2\mid 3,t,j) = 0.262$
$p(3\mid 1,t,j) = 0.038$	$p(3\mid 2,t,j) = 0.038$	$p(3\mid 3,t,j) = 0.438$
$p(4\mid 1,t,j) = 0.038$	$p(4\mid 2,t,j) = 0.038$	$p(4\mid 3,t,j) = 0.038$

$j = 4$ or greater

$p(1\mid 1,t,j) = 0.594$	$p(1\mid 2,t,j) = 0.194$	$p(1\mid 3,t,j) = 0.194$
$p(2\mid 1,t,j) = 0.194$	$p(2\mid 2,t,j) = 0.594$	$p(2\mid 3,t,j) = 0.194$
$p(3\mid 1,t,j) = 0{,}106$	$p(3\mid 2,t,j) = 0.106$	$p(3\mid 3,t,j) = 0.506$
$p(4\mid 1,t,j) = 0.106$	$p(4\mid 2,t,j) = 0.106$	$p(4\mid 3,t,j) = 0.106$

TABLE 4 (continued)

<u>Age Category III</u>

<u>$j = 1$</u>

$p(1|1,t,1) = 0.571$

$p(2|1,t,1) = 0.285$

$p(3|1,t,1) = 0.072$

$p(4|1,t,1) = 0.250$

<u>$j = 2,3$</u>

$p(1\|1,t,j) = 0.664$	$p(1\|2,t,j) = 0.264$	$p(1\|3,t,j) = 0.264$
$p(2\|1,t,j) = 0.264$	$p(2\|2,t,j) = 0.664$	$p(2\|3,t,j) = 0.264$
$p(3\|1,t,j) = 0.036$	$p(3\|2,t,j) = 0.036$	$p(3\|3,t,j) = 0.436$
$p(4\|1,t,j) = 0.036$	$p(4\|2,t,j) = 0.036$	$p(4\|3,t,j) = 0.036$

<u>$j = 4$ or greater</u>

$p(1\|1,t,j) = 0.627$	$p(1\|2,t,j) = 0.227$	$p(1\|3,t,j) = 0.227$
$p(2\|1,t,j) = 0.227$	$p(2\|2,t,j) = 0.627$	$p(2\|3,t,j) = 0.227$
$p(3\|1,t,j) = 0.073$	$p(3\|2,t,j) = 0.073$	$p(3\|3,t,j) = 0.473$
$p(4\|1,t,j) = 0.073$	$p(4\|2,t,j) = 0.073$	$p(4\|3,t,j) = 0.073$

TABLE 4 (continued)

The computer model used for the calculations is shown in Appendix E. This model omits two features of the model used in earlier calculations (Appendix A). These are: (1) The computation of multi-step transition probabilities, for reasons discussed earlier, and (2) The computation of $\pi(k,t,j,x)$, i.e., the probability of being in a given state and having experienced a given number of overhauls. The computation of $\pi(k,t,j,x)$ was omitted due to the computer space required to accomodate this four dimensional array. For the demand dependent policy, where j may take on any value up to t, this requires 4 x 75 x 75 x 38 = 755,000 computer words, beyond the core storage capacity of all but the largest computers. Since determination of the probability distribution of the number of overhauls is the only advantage to computing $\pi(k,t,j,x)$, and since, as we have seen, we can compute the expected value of this random variable by other methods, under the circumstances omission of this calculation seems a reasonable trade-off, provided we assume that decisions will be risk neutral (i.e., based on expected cost). Given a computer with sufficient capacity, calculation of $\pi(k,t,j,x)$ would be no more difficult conceptually than it was in the earlier model.

The state probabilities ($P(k,t,j)$) resulting from the model are calculated utilizing the transition probabilities of Table 4 for a demand dependent overhaul policy. Because of the large number of states, these probabilites have been aggregated over all j for a given pair, k,t. This results in the probability of the ship's being in condition k at age t without consideration of the time since last overhaul. This probability will be denoted by $\phi(k,t)$. Values of $\phi(k,t)$ for this example are displayed in Appendix F.

The expected number of overhauls the ship will experience over its life is 5.546. This value was obtained in the same manner as that for the example in Chapter II: The probability of a ship's being in the "demand" condition (condition 4, in this case) at any given ship age is the probability of being in overhaul at that age. Since only one time unit is involved, this probability is also the expected number of overhauls that the ship will undergo at that age. The expected number of overhauls throughout the ship's life is obtained by summing these values over the ship's life, i.e., all possible ages.

In the following chapter, a comparison of the demand dependent overhaul policy reflected here will be made with a policy of some fixed overhaul cycle length.

E. RELAXATION OF CERTAIN ASSUMPTIONS:

Throughout the development of the various models, we have retained certain assumptions: That, under a demand dependent policy, a ship would not be overhauled until it reached the worst condition; that, when it reached this condition, it would be overhauled with probability 1; and, that this overhaul would return the ship to condition 1 with certainty. While we will continue to make these assumptions, the approach can accomodate relaxing them with little difficulty. Since being in the worst condition would no longer be tantamount to being in overhaul, some additional computation would be required to determine the probability of being in overhaul, given the ship's age and the length of time since its last overhaul. Further, since overhaul would no longer ensure the ship's returning to condition 1, P(1,t,1) would no longer be equal to unity (for all t), and one or more of P(k,t,1) would have values greater than zero for k = 2,3,4. Again, the change would be more computational then conceptual.

IV. A COMPARISON OF OVERHAUL POLICIES

A. A SCHEDULED OVERHAUL POLICY:

We will now use the most comprehensive model to consider a policy by which a ship is overhauled at some specified interval, rather than waiting for some demand condition to be met before overhauling. This is the policy currently in effect in the Navy. To do this requires very little modification to this model. For an overhaul cycle length (i.e., end-of-overhaul to end-of-overhaul) of j time units, we modify $p(k|i,t,j-1)$ as follows:

$$p(k|4,t,j-1) = 1, \quad k = 1,2,3;$$

$$p(k|i,t,j-1) = 0, \quad k = 1,2,3; \quad i = 1,2,3.$$

In this manner we artificially "force" a demand condition on the ship. The mechanics of the model do not require any change at all. This modification does reflect the assumption that, if the ship should reach condition 4 prior to the end of the cycle, overhaul would take place ahead of schedule with the next cycle beginning at that time.

We may use the multi-condition, age and time-since-overhaul dependent model, then, to compare policies of overhauling at some fixed interval with the demand dependent polciy described earlier. We will utilize our previous example (Section D of Chapter III) to demonstrate this.

With the model modified to reflect a fixed overhaul policy as described above, the results shown in Table 5 were obtained. In the example of Chapter II we used the highest probability of being in overhaul over the ship's life as a basis for determining the necessary fleet size to meet specified requirements. This highest probability is shown in Table 5, along with the corresponding value of j, as well as the cycle length (in years) and the expected number of overhauls the ship will experience under that policy. The symbol "∞" implies a strictly demand dependent policy. We observe that the highest probability of being in overhaul occurs at the end of the first cycle in each case (i.e., at j = 3 for a one year cycle length, j = 6 for a two year cycle length, etc.). To use these values to determine a required fleet size would imply a whole fleet of exactly the same age. Although we accepted this assumption in the example of Chapter II, such an assumption is obviously unrealistic and would distort the decision making process in this case. Even if all the ships in a fleet were new at the same time, we must realize that in actuality the decision maker would stagger the point at which ships' overhaul cycles were begun. Consequently, we must find some way to overcome this problem.

Overhaul Cycle Length (Years)	Expected Number of Overhauls (Lifetime)	Highest Probability of Being in Overhaul	Ship Age at which Highest Probability Occurs
1	25.67	.879	j = 3
2	13.96	.811	j = 6
3	10.24	.765	j = 9
4	8.43	.641	j = 12
5	7.42	.480	j = 15
6	6.82	.371	j = 18
7	6.42	.294	j = 21
8	6.16	.238	j = 24
9	5.98	.199	j = 27
10	5.85	.171	j = 30
∞	5.55	.121	j = 2

where j is the ship's age in terms of the basic time unit.

TABLE 5

Expected Number of Overhauls and Highest Probabilities of Being in Overhaul for Various Overhaul Policies

B. THE MIXED FLEET:

It is much more likely that in reality only a few new ships would be acquired at a time. We will assume that our ship acquisition policy calls for purchasing some number of new ships every two years. If we assume that this takes place over, say, ten years, then we essentially have five different types of ship to consider, since the probability of being in overhaul varies with ship age, even though we still assume that the ships are otherwise identical.

In actual practice in the Navy a fleet of ships is continuously experiencing replacement of old ships with new ones. Consequently we should examine our "mixed" fleet over some period where we have essentially reached a "steady state" in number of ships (rather than some point where we are either still acquiring ships on net or have begun decreasing the total fleet size), say a five year period beginning when our ships' ages vary from 12 to 20 years. We will wish to examine the mixed fleet at perhaps five points in time during this period, say a year apart, in order again to determine the total fleet size required to provide us with a given number of operational ships with a specified probability. Unfortunately, since all our ships do not simultaneously have the same probability of being in overhaul we cannot apply the binomial probability mass function as we did in Chapter II.

There is, however, another method of approaching the problem. First, assume that we have a relatively large number of ships. Secondly, suppose that the probability of one ship's being in overhaul is independent of the probability of any other ship's being in overhaul. This should in general be true except for combatant situations, natural disasters, etc., or unless the capacity of the overhauling shipyards becomes a constraint.

If these conditions are satisfied then we may apply the central limit theorem of probability. Essentially, the central limit theorem states (for our application) that the sum of a sequence of N independently distributed random variables converges in distribution as $N \to \infty$ to a random variable that is normally distributed with mean equal to the sum of the component means and with variance equal to the sum of the component variances, provided that Liapunov's condition is met. Cramér[4] states Liapunov's condition as follows: Let z_1, $z_2, \ldots$ be independent random variables, and denote by m_i and σ_i the mean and the standard deviation of z_i. Suppose that the third absolute moment of z_i about its mean

$$\rho_i^3 = E(|z_i - m_i|^3)$$

is finite for every i, and write

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[4]See Harald Cramér, Mathematical Methods of Statistics, Princeton University Press, Princeton, NJ, 1946, pp. 215-218.

$$\rho^3 = \rho_1^3 + \rho_2^3 + \rho_3^3 + \ldots + \rho_k^3.$$

If the condition

$$\lim_{k \to \infty} \frac{\rho}{\sigma} = 0$$

is satisfied, then the sum $z = \sum_{i=1}^{k} z_i$ is asymptotically normal (m,σ) where m and σ are given by:

$$m = m_1 + m_2 + \ldots + m_k,$$
$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_k^2.$$

For a set of Bernoulli trials

$$\rho_i^3 = E(|z_i - p_i|^3) = p_i q_i (p_i^2 + q_i^2)$$
$$\leq p_i q_i$$

where

$$q_i = 1 - p_i.$$

Therefore

$$\rho^3 \leq \sum_{i=1}^{k} p_i (1 - p_i).$$

Now:

$$\sigma^2 = \sum_{i=1}^{k} p_i (1 - p_i)$$

Therefore

$$\frac{\rho}{\sigma} \leq \left(\sum_{i=1}^{k} p_i (1 - p_i) \right)^{-1/6}$$

If the series $\sum_{i=1}^{\infty} p_i(1 - p_i)$ is <u>divergent</u>, Liapunov's condition is satisfied and thus the variable z is asymptotically normal

$$\left(\sum_{i=1}^{k} p_i, \sqrt{\sum_{i=1}^{k} p_i(1 - p_i)}\right).$$

The series $\sum_{i=1}^{\infty} p_i(1 - p_i)$, where p_i is the probability of the ith ship's being in overhaul, will be divergent if for all i p_i equals neither 0 nor 1. This can be reasonably assumed for our application.

The approximating normal distribution will then have a mean of $\sum_{i=1}^{N} p_i$ and a variance of $\sum_{i=1}^{N} p_i(1 - p_i)$.

Since we have our ships in five groups and are assuming that all ships within a group are identical, we may simplify the summation process somewhat as follows: If we have n_i ships with probability p_i of being in overhaul at some specified time, $i = 1,\ldots,5$, then we may write: $\mu = \sum_{i=1}^{5} n_i p_i$, and $s^2 = \sum_{i=1}^{5} n_i p_i (1 - p_i)$, in order to obtain the mean, μ, and the variance, s^2, of our approximating normal distribution.

If we take $N = \sum_{i=1}^{5} n_i$, then, using the normal distribution, we may approximate the probability of having k out of N ships in overhaul, where k is equal to or less than a specified value b, as follows:

$$\Pr\{k \leq b\} \cong \Phi\left[\frac{b + 1/2 - \mu}{s}\right]$$

$\Phi(y_o)$ is defined as the cumulative distribution function for the unit normal probability density function, and

$$\Phi(y_o) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y_o} e^{-y^2/2}\, dy.$$

The +1/2 term is added to the argument of Φ in order to reflect the discreteness of the distribution we are approximating. The reasoning here is similar to that in the DeMoivre-LaPlace limit theorem.

Using this approach, then, we may determine the size of fleet necessary to meet our requirement. Obviously, given that we have initially assumed a fleet size as before, any increase or decrease may be implemented among our five different groups of ships in various ways. Where we decide to make such an increase or decrease may have some effect on our outcome.

Now that we have developed the approach to the problem using the normal approximation, there is no reason why we must restrict ourselves to ships which are identical except for age. We are equipped to consider ships with entirely different sets of material condition transition probabilities. For each type we must, of course, go through the procedure demonstrated in the example of Chapter III for determining the state probabilities.

C. AN EXAMPLE:

Returning to the results of the example of Chapter III: Suppose we wish to consider a fleet of five different groups of such ships, initially varying in age from twelve to twenty years, over a five year period. Define τ as the epoch in time under consideration. For convenience let τ be equal to the age of the oldest group of ships. Table 6 shows values for τ and the ages of the different groups, in terms of the basic (four month) time unit, at each of our five observations.

For the example we will assume that the decision maker requires a probability of .95 for the event {20 or more ships not in overhaul}. We will initially assume a required fleet size of 25 ships: five in each group. If it is necessary to increase the fleet size to meet the requirement, we will begin by adding a ship to Group 5, then one to Group 4, etc. If, on the other hand, we wish to decrease the fleet size because the assumed size is more than sufficient to meet the requirement, we will remove one ship from Group 1, then one from Group 2, etc.

In the example we will consider overhaul cycle lengths of four, six, eight, and ten years as well as the demand dependent case. For each of these we will take the observation with the largest fleet size necessary to support the decision maker's requirement and compare the policies on a mean value basis, economizing total cost. Here we will assume, as we

Observation	τ	Age of Group 1 (oldest)	Age of Group 2	Age of Group 3	Age of Group 4	Age of Group 5
1	60	60	54	48	42	36
2	63	63	57	51	45	39
3	66	66	60	54	48	42
4	69	69	63	57	51	45
5	72	72	66	60	54	48

Age is given in terms of the basic time unit.

TABLE 6

Ship Ages for the Mixed Fleet Example

have throughout the thesis, that our decision maker is not a risk averter and will therefore base his decision on expected cost.

Table 7 shows the values of p_i (the probability of being in overhaul for the ith group) for each of the five observations under each of the five overhaul policies. Using these values we now compute the number of ships in each group (under the decision rules previously established) needed to meet our requirement. The computer model used to do this is shown as Appendix G. This model, in the form shown, actually computes the probabilities for the event {20 or more ships not in overhaul} for all combinations of four, five, and six ships in each of our five groups, so the decision maker could examine various rules for increasing or decreasing the initially assumed fleet size.

The requirements resulting from the computation are tabulated in Table 8 for each of the five overhaul policies we have chosen to consider. The n_i are the requirement for the number of ships in each of the five groups with N being total fleet size.

For the various policies, the maximum N (upon which we will base our decision) is summarized as follows:

Four Year Overhaul Cycle

Observation	p_1	p_2	p_3	p_4	p_5
1	.11683	.10314	.12567	.09749	.17841
2	.10860	.10721	.11084	.10418	.11817
3	.10600	.11683	.10314	.12567	.09749
4	.10830	.10860	.10721	.11084	.10418
5	.11260	.10600	.11683	.10314	.12567

Six Year Overhaul Cycle

Observation	p_1	p_2	p_3	p_4	p_5
1	.08313	.09742	.08469	.07609	.15261
2	.08511	.09094	.08631	.08047	.10574
3	.08619	.08313	.09742	.08469	.07609
4	.08656	.08511	.09094	.08631	.08047
5	.08980	.08619	.08313	.09742	.08469

TABLE 7

Probabilities of Being in Overhaul at Each Observation for the Mixed Fleet Example

Eight Year Overhaul Cycle

Observation	p_1	p_2	p_3	p_4	p_5
1	.07856	.07533	.08715	.06887	.10810
2	.08183	.07668	.07215	.07217	.10670
3	.07494	.07856	.07533	.08715	.06887
4	.07598	.08183	.07668	.07215	.07217
5	.07875	.07494	.07856	.07533	.08715

Ten Year Overhaul Cycle

Observation	p_1	p_2	p_3	p_4	p_5
1	.07678	.06968	.06854	.06591	.10064
2	.07114	.07034	.06909	.06818	.10061
3	.07252	.07678	.06968	.06854	.06591
4	.07466	.07114	.07034	.06909	.06818
5	.07064	.07252	.07678	.06968	.06854

TABLE 7 (continued)

Demand Dependent Policy

Observation	p_1	p_2	p_3	p_4	p_5
1	.06395	.06395	.06394	.06165	.09980
2	.06395	.06395	.06395	.06407	.09980
3	.06395	.06395	.06395	.06394	.06165
4	.06395	.06395	.06395	.06395	.06407
5	.06395	.06395	.06395	.06395	.06394

TABLE 7 (continued)

Four Year Overhaul Cycle

Observation	n_1	n_2	n_3	n_4	n_5	N
1	5	5	5	5	6	26
2	5	5	5	5	5	25
3	5	5	5	5	5	25
4	5	5	5	5	5	25
5	5	5	5	5	5	25

Six Year Overhaul Cycle

Observation	n_1	n_2	n_3	n_4	n_5	N
1	5	5	5	5	5	25
2	4	5	5	5	5	24
3	4	5	5	5	5	24
4	4	5	5	5	5	24
5	4	5	5	5	5	24

TABLE 8

Fleet Size Requirements for the
Mixed Fleet Example

Eight Year Overhaul Cycle

Observation	n_1	n_2	n_3	n_4	n_5	N
1	4	5	5	5	5	24
2	4	5	5	5	5	24
3	4	5	5	5	5	24
4	4	5	5	5	5	24
5	4	5	5	5	5	24

Ten Year Overhaul Cycle

Observation	n_1	n_2	n_3	n_4	n_5	N
1	4	5	5	5	5	24
2	4	5	5	5	5	24
3	4	5	5	5	5	24
4	4	5	5	5	5	24
5	4	5	5	5	5	24

TABLE 8 (continued)

Demand Dependent Policy

Observation	n_1	n_2	n_3	n_4	n_5	N
1	4	5	5	5	5	24
2	4	5	5	5	5	24
3	4	4	5	5	5	23
4	4	4	5	5	5	23
5	4	4	5	5	5	23

TABLE 8 (continued)

POLICY	N_{max}
Four Year Overhaul Cycle	26
Six Year Overhaul Cycle	25
Eight Year Overhaul Cycle	24
Ten Year Overhaul Cycle	24
Demand Dependent Policy	24

Recalling the expected number of overhauls from Table 5, we may compute total expected cost for the fleet, under each policy, as follows:

Total expected cost = N [Ship cost + Overhaul cost (Expected number of overhauls)]

As before, we will assume a ship cost of \$40 million and an overhaul cost of \$1.5 million. Thus for the four year overhaul cycle policy, total expected cost would be computed as follows:

$$\text{Total expected cost} = 26[\$40 \times 10^6 + \$1.5 \times 10^6 (8.43)]$$
$$= \$1.37 \text{ billion.}$$

For the remaining policies considered, total expected cost is:

POLICY	TOTAL EXPECTED COST
Six Year Overhaul Cycle	\$1.26 billion
Eight Year Overhaul Cycle	\$1.18 billion
Ten Year Overhaul Cycle	\$1.17 billion
Demand Dependent Policy	\$1.16 billion

The decision maker would then choose the demand dependent policy as that which would be most economical (based on expected costs). As one would expect, and as we see, due to the fact that ship cost is relatively large compared to overhaul cost, the total expected cost is more sensitive to required fleet size than to expected number of overhauls.

Suppose now that the decision maker is willing to relax his requirement to a probability of .90 for the event {20 or more ships not in overhaul}. In the same manner as before, we obtain the following results:

POLICY	N	TOTAL EXPECTED COST
Four Year Overhaul Cycle	25	$1.32 billion
Six Year Overhaul Cycle	24	$1.21 billion
Eight Year Overhaul Cycle	24	$1.18 billion
Ten Year Overhaul Cycle	23	$1.12 billion
Demand Dependent Policy	23	$1.11 billion

On the other hand, if he requires a probability of, say, .98 for the event {20 or more ships not in overhaul}, we obtain the following:

POLICY	N	TOTAL EXPECTED COST
Four Year Overhaul Cycle	27	$1.42 billion
Six Year Overhaul Cycle	26	$1.31 billion
Eight Year Overhaul Cycle	25	$1.23 billion
Ten Year Overhaul Cycle	24	$1.17 billion
Demand Dependent Policy	24	$1.16 billion

We observe that, while the decision is relatively insensitive to such a change in requirements, the required size of fleet is not.

Up to this point we have assumed that the cost of a scheduled overhaul (under a policy calling for an overhaul cycle of some specified length) is the same as that of an unscheduled overhaul (i.e., one resulting from the ship's reaching the demand condition prior to the end of the overhaul cycle). For a strictly demand dependent policy, of course, all overhauls are unscheduled. An unscheduled overhaul should be expected to cost more than a scheduled one, due to uncertainty as to the number of ships in overhaul at any given time, the inability to plan specific work ahead of time based on an individual ship, etc.

Using our multi-condition, age and time-since-overhaul dependent model, the expected number of both scheduled and unscheduled overhauls which a ship will experience, under a policy calling for specified cycle lengths, can be computed as follows:

$$E(\text{number of scheduled overhauls}) = \sum_{t=1}^{T-1} P(4,t,L_c);$$

$$E(\text{number of unscheduled overhauls} = \sum_{t=1}^{T-1} \sum_{j=1}^{L_c-1} P(4,t,j)$$

where L_c is the specified overhaul cycle length. These values have been computed for our example and are shown in Table 9

Overhaul Cycle Length (Years)	Expected Number of Scheduled Overhauls	Expected Number of Unscheduled Overhauls	Total
1	21.70	3.97	25.67
2	9.95	4.01	13.96
3	5.59	4.65	10.24
4	3.48	4.95	8.43
5	2.27	5.15	7.42
6	1.54	5.28	6.82
7	1.06	5.36	6.42
8	0.74	5.42	6.16
9	0.52	5.46	5.98
10	0.37	5.48	5.85
∞	0.00	5.55	5.55

TABLE 9

Expected Number of Scheduled and Unscheduled Overhauls for the Multi-Condition, Age and Time-Since-Overhaul Dependent Model Example for overhaul cycle lengths of one to ten years and for the demand dependent policy.

Suppose we assume that the cost per overhaul of an unscheduled overhaul is \$2 million, while that of a scheduled overhaul is \$1.5 million. We will again examine the total expected fleet cost, taking this assumption into account. If we require a probability of .90 for the event {20 or more ships not in overhaul} (denote this as $p_{req}(20)$), we obtain the following results:

POLICY	N	TOTAL EXPECTED COST
Four Year Overhaul Cycle	25	\$1.38 billion
Six Year Overhaul Cycle	24	\$1.27 billion
Eight Year Overhaul Cycle	24	\$1.25 billion
Ten Year Overhaul Cycle	23	\$1.18+ billion
Demand Dependent Policy	23	\$1.18- billion

For a $p_{req}(20)$ of .95 we obtain:

POLICY	N	TOTAL EXPECTED COST
Four Year Overhaul Cycle	26	\$1.43 billion
Six Year Overhaul Cycle	25	\$1.32 billion
Eight Year Overhaul Cycle	24	\$1.25 billion
Ten Year Overhaul Cycle	24	\$1.24 billion
Demand Dependent Policy	24	\$1.23 billion

And, for $p_{req}(20) = .98$:

POLICY	N	TOTAL EXPECTED COST
Four Year Overhaul Cycle	27	\$1.49 billion
Six Year Overhaul Cycle	26	\$1.37 billion
Eight Year Overhaul Cycle	25	\$1.30 billion
Ten Year Overhaul Cycle	24	\$1.24 billion
Demand Dependent Policy	24	\$1.23 billion

As we can see, the decision as to the most economical policy would remain unchanged with the assumption that an unscheduled overhaul costs \$2.0 million (\$0.5 million more than a scheduled one).

We may approach the trade-off between scheduled and unscheduled overhauls from another viewpoint, however. Suppose we wish to find some coefficient c, where

$$c = \frac{\text{cost of unscheduled overhaul}}{\text{cost of scheduled overhaul}},$$

such that the decision maker would be indifferent between two alternative overhaul policies. To illustrate the manner in which we can do this, consider the ten year overhaul cycle policy and the demand dependent policy for $p_{req}(20) = .95$. Since the required fleet size is the same, we need only consider single ship expected overhaul cost in order to determine a value for c. We do this as follows:

E(number of scheduled overhauls for 10 yr. policy)

+ c x E(number of unscheduled overhauls for 10 yr. policy)

= E(number of scheduled overhauls for demand dependent policy) + c x E(number of unscheduled overhauls for demand dependent policy).

Substituting the values from Table 9 we get:

$$.37 + 5.48c = .00 + 5.55c$$

$$c = \frac{.37}{.07} = 5.3$$

This implies that for the particular set of transition probabilities assumed for the example, the cost of an unscheduled overhaul would have to be of the order of five times that of a scheduled one in order for the decision maker to change his decision to adopt the demand dependent overhaul policy.

In our example the demand dependent overhaul policy has consistently been the choice, economically. This will always be true unless we impute a penalty to the cost of an unscheduled overhaul. Given such a penalty, and different behavior of transition probabilities from our example (especially if probability of "failure" continues to increase as ship age and time since overhaul increase), we could easily have reached a different decision.

We have shown, then, how our model can be used to compare alternative overhaul policies by considering total expected cost of a fleet of ships in addition to the cost differential between scheduled and unscheduled overhauls.

V. RECOMMENDATIONS, SUMMARY, AND CONCLUSIONS

A. RECOMMENDATIONS FOR FURTHER WORK:

The next step, and the most important toward actual implementation of the models, and in particular the most comprehensive (multi-condition, age and time-since-overhaul dependent) model, is determination of reasonable estimators for transition probabilities based on the best data available. Once these have been determined, the model implemented, and transition probabilities refined so that the results of the model are relatively reliable, the model may be used to examine a large number of possible maintenance strategies. For example, the decision maker might want to consider the alternative of assigning his ships to shipyards upon their reaching condition 3 for limited maintenance not actually amounting to overhaul (as we have used the term here). There are no doubt other maintenance policies of this nature that can be evaluated using this multi-condition, time-since-overhaul dependent model as a basis.

If it were found to be reasonable to describe transition probabilities analytically in terms of ship age and time since last overhaul, the model certainly could be modified to accommodate such a change. As a matter of fact, transition probabilities could be computed internally by the computer model. This would eliminate the necessity of reading in (in some manner) long lists of transition matrices, as we have done here.

Efforts to develop criteria for an acceptable material condition index for ships should continue, although the complexity and variety of configurations encountered in warships does not make this an easy assignment. An index of some kind is essential if one is to expect valid results from this model. In the interim, however, the model should be implemented utilizing whatever rough index of material condition that can be developed in the short run.

It must be emphasized that results obtained from the models are only as good as their input. The models are geared to categorization of material condition either by subjective judgment or by some quantitative scheme for determining a material condition index. Critical to the model's usefulness as an aid to decision making is accurate determination of transition probabilities, as indicated above. As pointed out in Appendix D, these should be estimated initially from the best historical data available, using criteria which, once decided upon, should be applied consistently to all ships. Then, using the Bayesian approach,[5] transition probabilities should be updated as actual ship behavior is observed.

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[5]For one example of how this approach can be used, see Chapter 5 of John W. Devanney III, Marine Decisions Under Uncertainty, National Sea Grant Program, Sea Grant Project GH-88, M.I.T., Cornell Maritime Press, Cambridge, Maryland, November 1971.

We have demonstrated the model using only one set of transition probability matrices. It would be useful to hypothesize transition probabilities based on different assumed forms of behavior of the ship's material condition (e.g., deterioration more or less linear over time, etc.) and compare the results with those obtained here. In this manner some "feel" may be obtained for the dynamics of the model.

An approach to modeling the change in a ship's condition over time that deserves some consideration involves the use of a semi-Markov process model. In a semi-Markov process, the states occupied on successive transitions are governed by the transition probabilities (as in our model) of a Markov process (called the imbedded Markov process). The time the process stays in a particular state, however, is a random variable (either discrete or continuous) described by an exponential density (Poisson process). This random variable is referred to as the "holding" time and is dependent on the state presently occupied and on the state to which the next transition will be made. Howard, Volume II,[6] provides a rather thorough discussion of semi-Markov processes.

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[6] Ronald A. Howard, Dynamic Probabilistic Systems, Volume II: Semi-Markov and Decision Processes, John Wiley and Sons, New York, 1971, Chapters 10 and 11.

B. SUMMARY AND CONCLUSIONS:

Beginning with a very simple description of the behavior of a ship's material condition, we have developed a set of models which, while still conceptually simple, is capable of handling a rather wide range of ship deterioration behavior. The approach used in developing the models considers the ship as a single component, rather than a set of many smaller components. Modeling the behavior of the ship's material condition over time by a transient Markov process represents an unusual if not unique approach to this area of no small concern to the Navy.

With the central limit theorem, as we have seen, there is no problem in going from considerations of a fleet of identical ships, all of the same age, to a "mixed" fleet, where not only are ships not all of the same age but are dissimilar in other respects as well.

We have seen how the models can be used to determine the effect of various overhaul policies on ship acquisition requirements as well as on total expected fleet cost. Further, we have seen the importance of considering the real cost differential between scheduled and unscheduled overhauls.

The significant conclusions to be drawn from this thesis are threefold:

(1) That the use of transient Markov processes to model ship deterioration and failure behavior is reasonable.

(2) That such models are potentially of significant use to Navy (and other) decision makers.

(3) That a great deal of initial effort is required in the area of data analysis to develop material condition indices and reasonably accurate estimates of transition probabilities.

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APPENDIX A

Computer Program and Results of Example for the Two-Condition, Age Dependent Model

```
OWHLS: PROC OPTIONS(MAIN);
    DCL P(2,2,0:30),PCAP(2,0:30,2,0:30),PI(2,C:30,-1:30),
    (I,K,KO,T,TO,X) FIXED,SHIP_LIFE FIXED INITIAL(30);
/*                                                                    */
/*                                                                    */
/*  SET ALL ELEMENTS OF ARFAYS TO ZERC.                               */
/*                                                                    */
    P=0.0;
    PCAP=C.0;
    PI=0.0;
/*                                                                    */
/*                                                                    */
/*  P(K,I,T) IS THE PROBABILITY OF BEING IN CONDITION K AT AGE T+1    */
/*  GIVEN CONDITICN I AT AGE T.                                       */
/*                                                                    */
/*  PCAP(K,T,KO,TO) IS THE MULTI-STEP TRANSITION PROBABILITY CF       */
/*  STATE (K,T) GIVEN STATE (KO,TO).                                  */
/*                                                                    */
/*  PI(K,T,X) IS THE PROBAEILITY OF BEING IN STATE (K,T) AND HAVING   */
/*  EXPERIENCED X OVERHAULS.                                          */
/*                                                                    */
/*                                                                    */
/*  ENTER INITIAL CONDITICNS.                                         */
/*                                                                    */
    PCAP(1,0,1,0)=1.0;
    PI(1,0,0)=1.0;
/*                                                                    */
/*                                                                    */
/*  ENTER TRANSITION PROBAEILITIES.                                   */
/*                                                                    */
    DC T=0 TO SHIP_LIFE-1;
         P(1,2,T)=1.0;
         END;
    DC T=C TO 5;
         P(1,1,T)=0.9;
         P(2,1,T)=0.1;
```

```
        END;
    DO T=6 TO 11;
        P(1,1,T)=0.8;
        P(2,1,T)=0.2;
        END;
    DO T=12 TO 17;
        P(1,1,T)=0.7;
        P(2,1,T)=0.3;
        END;
    DO T=18 TO 23;
        P(1,1,T)=0.6;
        P(2,1,T)=0.4;
        END;
    DO T=24 TO 29;
        P(1,1,T)=0.5;
        P(2,1,T)=0.5;
        END;
/*                                                                        */
    DO T=1 TO SHIP_LIFE;
        DO K=1 TO 2;
            PCAP(K,T,K,T)=1.0;
            END;
        END;
    DO T=1 TO SHIP_LIFE;
        DO K=1 TO 2;
            DO I=1 TO 2;
                PCAP(K,T,1,0)=PCAP(K,T,1,0)+PCAP(I,T-1,1,0)*
                P(K,I,T-1);
                END;
            END;
        END;
    DO T=2 TO 10;
        DO K=1 TO 2;
            DO T0=1 TO T-1;
                DO K0=1 TO 2;
                    DO I=1 TO 2;
```

```
                              PCAP(K,T,KO,TO)=PCAP(K,T,KO,TO)+
                              PCAP(I,T-1,KO,TO)*P(K,I,T-1);
                              END;
                         END;
                    END;
               END;
          END;
     DO T=1 TO SHIP_LIFE;
          DO X=0 TO T;
               DO I=1 TO 2;
                    PI(1,T,X)=PI(1,T,X)+PI(I,T-1,X)*P(1,I,T-1);
                    PI(2,T,X)=PI(2,T,X)+PI(I,T-1,X-1)*P(2,I,T-1);
                    END;
               END;
          END;
/*                                                                        */
/*                                                                        */
/*   THE FOLLOWING DO LOOP REFLECTS THE ASSUMPTION THAT NO SHIP           */
/*   WOULD BE OVERHAULED DURING THE FINAL TIME UNIT OF ITS LIFE, I.E.     */
/*   WITH NO FURTHER USEFUL LIFE REMAINING.                               */
/*                                                                        */
     DO X=1 TO SHIP_LIFE;
          PI(2,SHIP_LIFE,X-1)=PI(2,SHIP_LIFE,X);
          END;
     PI(2,SHIP_LIFE,SHIP_LIFE)=0.0;
/*                                                                        */
/*   PRINT DESIRED OUTPUT.                                                */
/*                                                                        */
     PUT SKIP(5);
     DO T=1 TO SHIP_LIFE;
          DO K=1 TO 2;
               PUT SKIP EDIT (K,T,PCAP(K,T,1,0)) (X(18),F(1),X(15),F(2),
               X(15),F(8,6));
               END;
          END;
     PUT PAGE;
```

```
    PUT SKIP(5);
    DO T=2 TO 10;
         DO K=1 TO 2;
              DO TO=1 TO T-1;
                   DO KO=1 TO 2;
                        PUT SKIP EDIT (K,T,KO,TO,PCAP(K,T,KO,TO))
                        (X(15),F(1),X(8),F(2),X(8),F(1),X(8),F(2),
                        X(8),F(8,6));
                        END;
                   END;
              END;
         END;
    PUT PAGE;
    PUT SKIP(5);
    DO T=1 TO SHIP_LIFE;
         DO X=0 TO T;
              DO K=1 TO 2;
                   PUT SKIP EDIT (K,T,X,PI(K,T,X)) (X(15),F(1),X(10),
                   F(2),X(10),F(2),X(10),F(8,6));
                   END;
              END;
         END;
/*                                                                       */
/*                                                                       */
/*  CALCULATE EXPECTED NUMBER OF OVERHAULS, EXP_NOH.                     */
/*                                                                       */
    EXP_NOH=0.0;
    DO T=1 TO SHIP_LIFE-1;
         EXP_NOH=EXP_NOH+PCAP(2,T,1,0);
         END;
    PUT PAGE;
    PUT SKIP(15);
    PUT EDIT (EXP_NOH) (X(30),F(8,5));
END OVHLS;
```

k	t	P(k,t|1,0)
1	1	0.900000
2	1	0.100000
1	2	0.910000
2	2	0.090000
1	3	0.909000
2	3	0.091000
1	4	0.909100
2	4	0.090900
1	5	0.909090
2	5	0.090910
1	6	0.909091
2	6	0.090909
1	7	0.818181
2	7	0.181818
1	8	0.836363
2	8	0.163636
1	9	0.832726
2	9	0.167273
1	10	0.833454
2	10	0.166545
1	11	0.833308
2	11	0.166691
1	12	0.833337
2	12	0.166662
1	13	0.749997
2	13	0.250001
1	14	0.774999
2	14	0.224999
1	15	0.767499
2	15	0.232500
1	16	0.769749
2	16	0.230250
1	17	0.769074
2	17	0.230925
1	18	0.769276
2	18	0.230722
1	19	0.692288
2	19	0.307710
1	20	0.723083
2	20	0.276915
1	21	0.710765
2	21	0.289233
1	22	0.715692
2	22	0.284306
1	23	0.713721
2	23	0.286277
1	24	0.714509
2	24	0.285488
1	25	0.642743
2	25	0.357255
1	26	0.678626
2	26	0.321371
1	27	0.660684
2	27	0.339313

1	28	0.669655
2	28	0.330342
1	29	0.665170
2	29	0.334828
1	30	0.667412
2	30	0.332585

k	t	k_o	t_o	$P(k,t\|k_o,t_o)$
1	2	1	1	0.900000
1	2	2	1	1.000000
2	2	1	1	0.100000
2	2	2	1	0.000000
1	3	1	1	0.910000
1	3	2	1	0.900000
1	3	1	2	0.900000
1	3	2	2	1.000000
2	3	1	1	0.090000
2	3	2	1	0.100000
2	3	1	2	0.100000
2	3	2	2	0.000000
1	4	1	1	0.909000
1	4	2	1	0.910000
1	4	1	2	0.910000
1	4	2	2	0.900000
1	4	1	3	0.900000
1	4	2	3	1.000000
2	4	1	1	0.091000
2	4	2	1	0.090000
2	4	1	2	0.090000
2	4	2	2	0.100000
2	4	1	3	0.100000
2	4	2	3	0.000000
1	5	1	1	0.909100
1	5	2	1	0.909000
1	5	1	2	0.909000
1	5	2	2	0.910000
1	5	1	3	0.910000
1	5	2	3	0.900000
1	5	1	4	0.900000
1	5	2	4	1.000000
2	5	1	1	0.090900
2	5	2	1	0.091000
2	5	1	2	0.091000
2	5	2	2	0.090000
2	5	1	3	0.090000
2	5	2	3	0.100000
2	5	1	4	0.100000
2	5	2	4	0.000000
1	6	1	1	0.909090
1	6	2	1	0.909100
1	6	1	2	0.909100
1	6	2	2	0.909000
1	6	1	3	0.909000
1	6	2	3	0.910000
1	6	1	4	0.910000
1	6	2	4	0.900000
1	6	1	5	0.900000
1	6	2	5	1.000000
2	6	1	1	0.090910
2	6	2	1	0.090900
2	6	1	2	0.090900
2	6	2	2	0.091000

2	6	1	3	C.091000
2	6	2	3	C.090000
2	6	1	4	0.090000
2	6	2	4	C.100000
2	6	1	5	0.100000
2	6	2	5	0.000000
1	7	1	1	C.818182
1	7	2	1	0.818180
1	7	1	2	0.818180
1	7	2	2	0.818200
1	7	1	3	C.818200
1	7	2	3	C.818000
1	7	1	4	0.818000
1	7	2	4	C.820000
1	7	1	5	C.820000
1	7	2	5	0.800000
1	7	1	6	C.800000
1	7	2	6	1.000000
2	7	1	1	0.181818
2	7	2	1	C.181820
2	7	1	2	0.181820
2	7	2	2	C.181800
2	7	1	3	C.181800
2	7	2	3	0.182000
2	7	1	4	C.182000
2	7	2	4	0.180000
2	7	1	5	0.180000
2	7	2	5	C.200000
2	7	1	6	0.200000
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2	10	1	8	C.160000
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1	3	0	0.729000
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2	3	2	0.010000
1	3	3	0.000000
2	3	3	0.000000
1	4	0	0.656100
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1	4	1	0.243000
2	4	1	0.072900
1	4	2	0.010000
2	4	2	0.018000
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1	4	4	0.000000
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2	5	2	0.024300
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2	5	3	0.001000
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1	5	5	0.000000
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2	23	14	0.000000
1	23	15	0.000000
2	23	15	0.000000
1	23	16	0.000000
2	23	16	0.000000
1	23	17	0.000000
2	23	17	0.000000
1	23	18	0.000000
2	23	18	0.000000
1	23	19	0.000000
2	23	19	0.000000
1	23	20	0.000000
2	23	20	0.000000
1	23	21	0.000000
2	23	21	0.000000

1	23	22	0.000000
2	23	22	0.000000
1	23	23	0.000000
2	23	23	0.000000
1	24	0	0.000765
2	24	0	0.000000
1	24	1	0.009182
2	24	1	0.000510
1	24	2	0.045069
2	24	2	0.005555
1	24	3	0.119082
2	24	3	0.024504
1	24	4	0.187417
2	24	4	0.057621
1	24	5	0.183585
2	24	5	0.079819
1	24	6	0.113536
2	24	6	0.067858
1	24	7	0.044006
2	24	7	0.035742
1	24	8	0.010372
2	24	8	0.011492
1	24	9	0.001396
2	24	9	0.002161
1	24	10	0.000095
2	24	10	0.000217
1	24	11	0.000003
2	24	11	0.000010
1	24	12	0.000000
2	24	12	0.000000
1	24	13	0.000000
2	24	13	0.000000
1	24	14	0.000000
2	24	14	0.000000
1	24	15	0.000000
2	24	15	0.000000
1	24	16	0.000000
2	24	16	0.000000
1	24	17	0.000000
2	24	17	0.000000
1	24	18	0.000000
2	24	18	0.000000
1	24	19	0.000000
2	24	19	0.000000
1	24	20	0.000000
2	24	20	0.000000
1	24	21	0.000000
2	24	21	0.000000
1	24	22	0.000000
2	24	22	0.000000
1	24	23	0.000000
2	24	23	0.000000
1	24	24	0.000000
2	24	24	0.000000
1	25	0	0.000382
2	25	0	0.000000
1	25	1	0.005101
2	25	1	0.000382
1	25	2	0.028089
2	25	2	0.004591

1	25	3	0.084045
2	25	3	0.022535
1	25	4	0.151329
2	25	4	0.059541
1	25	5	0.171611
2	25	5	0.093708
1	25	6	0.124626
2	25	6	0.091792
1	25	7	0.057745
2	25	7	0.056768
1	25	8	0.016678
2	25	8	0.022003
1	25	9	0.002859
2	25	9	0.005186
1	25	10	0.000265
2	25	10	0.000698
1	25	11	0.000011
2	25	11	0.000048
1	25	12	0.000000
2	25	12	0.000001
1	25	13	0.000000
2	25	13	0.000000
1	25	14	0.000000
2	25	14	0.000000
1	25	15	0.000000
2	25	15	0.000000
1	25	16	0.000000
2	25	16	0.000000
1	25	17	0.000000
2	25	17	0.000000
1	25	18	0.000000
2	25	18	0.000000
1	25	19	0.000000
2	25	19	0.000000
1	25	20	0.000000
2	25	20	0.000000
1	25	21	0.000000
2	25	21	0.000000
1	25	22	0.000000
2	25	22	0.000000
1	25	23	0.000000
2	25	23	0.000000
1	25	24	0.000000
2	25	24	0.000000
1	25	25	0.000000
2	25	25	0.000000
1	26	0	0.000191
2	26	0	0.000000
1	26	1	0.002933
2	26	1	0.000191
1	26	2	0.018635
2	26	2	0.002550
1	26	3	0.064557
2	26	3	0.014045
1	26	4	0.135206
2	26	4	0.042022
1	26	5	0.179514
2	26	5	0.075665
1	26	6	0.154105
2	26	6	0.085806

1	26	7	0.085641
2	26	7	0.062313
1	26	8	0.030342
2	26	8	0.028872
1	26	9	0.006615
2	26	9	0.008339
1	26	10	0.000830
2	26	10	0.001429
1	26	11	0.000053
2	26	11	0.000132
1	26	12	0.000001
2	26	12	0.000006
1	26	13	0.000000
2	26	13	0.000000
1	26	14	0.000000
2	26	14	0.000000
1	26	15	0.000000
2	26	15	0.000000
1	26	16	0.000000
2	26	16	0.000000
1	26	17	0.000000
2	26	17	0.000000
1	26	18	0.000000
2	26	18	0.000000
1	26	19	0.000000
2	26	19	0.000000
1	26	20	0.000000
2	26	20	0.000000
1	26	21	0.000000
2	26	21	0.000000
1	26	22	0.000000
2	26	22	0.000000
1	26	23	0.000000
2	26	23	0.000000
1	26	24	0.000000
2	26	24	0.000000
1	26	25	0.000000
2	26	25	0.000000
1	26	26	0.000000
2	26	26	0.000000
1	27	0	0.000096
2	27	0	0.000000
1	27	1	0.001657
2	27	1	0.000096
1	27	2	0.011868
2	27	2	0.001466
1	27	3	0.046323
2	27	3	0.009318
1	27	4	0.109625
2	27	4	0.032278
1	27	5	0.165422
2	27	5	0.067603
1	27	6	0.162858
2	27	6	0.089757
1	27	7	0.105133
2	27	7	0.077053
1	27	8	0.044044
2	27	8	0.042820
1	27	9	0.011647
2	27	9	0.015171

1	27	10	0.001845
2	27	10	0.003308
1	27	11	0.000159
2	27	11	0.000415
1	27	12	0.000006
2	27	12	0.000027
1	27	13	0.000000
2	27	13	0.000001
1	27	14	0.000000
2	27	14	0.000000
1	27	15	0.000000
2	27	15	0.000000
1	27	16	0.000000
2	27	16	0.000000
1	27	17	0.000000
2	27	17	0.000000
1	27	18	0.000000
2	27	18	0.000000
1	27	19	0.000000
2	27	19	0.000000
1	27	20	0.000000
2	27	20	0.000000
1	27	21	0.000000
2	27	21	0.000000
1	27	22	0.000000
2	27	22	0.000000
1	27	23	0.000000
2	27	23	0.000000
1	27	24	0.000000
2	27	24	0.000000
1	27	25	0.000000
2	27	25	0.000000
1	27	26	0.000000
2	27	26	0.000000
1	27	27	0.000000
2	27	27	0.000000
1	28	0	0.000048
2	28	0	0.000000
1	28	1	0.000924
2	28	1	0.000048
1	28	2	0.007400
2	28	2	0.000829
1	28	3	0.032479
2	28	3	0.005934
1	28	4	0.087091
2	28	4	0.023161
1	28	5	0.150314
2	28	5	0.054813
1	28	6	0.171186
2	28	6	0.082711
1	28	7	0.129619
2	28	7	0.081429
1	28	8	0.064842
2	28	8	0.052567
1	28	9	0.020995
2	28	9	0.022022
1	28	10	0.004230
2	28	10	0.005823
1	28	11	0.000495
2	28	11	0.000922

1	28	12	0.000030
2	28	12	0.000080
1	28	13	0.000001
2	28	13	0.000003
1	28	14	0.000000
2	28	14	0.000000
1	28	15	0.000000
2	28	15	0.000000
1	28	16	0.000000
2	28	16	0.000000
1	28	17	0.000000
2	28	17	0.000000
1	28	18	0.000000
2	28	18	0.000000
1	28	19	0.000000
2	28	19	0.000000
1	28	20	0.000000
2	28	20	0.000000
1	28	21	0.000000
2	28	21	0.000000
1	28	22	0.000000
2	28	22	0.000000
1	28	23	0.000000
2	28	23	0.000000
1	28	24	0.000000
2	28	24	0.000000
1	28	25	0.000000
2	28	25	0.000000
1	28	26	0.000000
2	28	26	0.000000
1	28	27	0.000000
2	28	27	0.000000
1	28	28	0.000000
2	28	28	0.000000
1	29	0	0.000024
2	29	0	0.000000
1	29	1	0.000510
2	29	1	0.000024
1	29	2	0.004529
2	29	2	0.000462
1	29	3	0.022174
2	29	3	0.003700
1	29	4	0.066707
2	29	4	0.016240
1	29	5	0.129969
2	29	5	0.043545
1	29	6	0.168304
2	29	6	0.075157
1	29	7	0.146239
2	29	7	0.085593
1	29	8	0.084988
2	29	8	0.064810
1	29	9	0.032519
2	29	9	0.032421
1	29	10	0.007938
2	29	10	0.010497
1	29	11	0.001170
2	29	11	0.002115
1	29	12	0.000094
2	29	12	0.000247

1	29	13	0.000003
2	29	13	0.000015
1	29	14	0.000000
2	29	14	0.000000
1	29	15	0.000000
2	29	15	0.000000
1	29	16	0.000000
2	29	16	0.000000
1	29	17	0.000000
2	29	17	0.000000
1	29	18	0.000000
2	29	18	0.000000
1	29	19	0.000000
2	29	19	0.000000
1	29	20	0.000000
2	29	20	0.000000
1	29	21	0.000000
2	29	21	0.000000
1	29	22	0.000000
2	29	22	0.000000
1	29	23	0.000000
2	29	23	0.000000
1	29	24	0.000000
2	29	24	0.000000
1	29	25	0.000000
2	29	25	0.000000
1	29	26	0.000000
2	29	26	0.000000
1	29	27	0.000000
2	29	27	0.000000
1	29	28	0.000000
2	29	28	0.000000
1	29	29	0.000000
2	29	29	0.000000
1	30	0	0.000012
2	30	0	0.000012
1	30	1	0.000279
2	30	1	0.000255
1	30	2	0.002727
2	30	2	0.002264
1	30	3	0.014787
2	30	3	0.011087
1	30	4	0.049593
2	30	4	0.033353
1	30	5	0.108530
2	30	5	0.064985
1	30	6	0.159309
2	30	6	0.084152
1	30	7	0.158712
2	30	7	0.073119
1	30	8	0.107303
2	30	8	0.042494
1	30	9	0.048681
2	30	9	0.016260
1	30	10	0.014467
2	30	10	0.003969
1	30	11	0.002700
2	30	11	0.000585
1	30	12	0.000295
2	30	12	0.000047

1	30	13	0.000017
2	30	13	0.000002
1	30	14	0.000000
2	30	14	0.000000
1	30	15	0.000000
2	30	15	0.000000
1	30	16	0.000000
2	30	16	0.000000
1	30	17	0.000000
2	30	17	0.000000
1	30	18	0.000000
2	30	18	0.000000
1	30	19	0.000000
2	30	19	0.000000
1	30	20	0.000000
2	30	20	0.000000
1	30	21	0.000000
2	30	21	0.000000
1	30	22	0.000000
2	30	22	0.000000
1	30	23	0.000000
2	30	23	0.000000
1	30	24	0.000000
2	30	24	0.000000
1	30	25	0.000000
2	30	25	0.000000
1	30	26	0.000000
2	30	26	0.000000
1	30	27	0.000000
2	30	27	0.000000
1	30	28	0.000000
2	30	28	0.000000
1	30	29	0.000000
2	30	29	0.000000
1	30	30	0.000000
2	30	30	0.000000

The expected number of overhauls
the ship will experience is:

6.37877

APPENDIX B

Computer Program for Determining Required Fleet Size, Homogeneous Fleet

```
FLEET: PROC OPTIONS(MAIN);
/*                                                                         */
/*  THIS PROCEDURE COMPUTES THE FLEET SIZE REQUIRED TO PROVIDE AT         */
/*  LEAST NREQ OPERATIONAL SHIPS WITH A PROBABILITY OF PMIN, GIVEN        */
/*  THE PROBABILITY OF A SINGLE SHIP'S BEING IN OVERHAUL, POHMAX.         */
/*                                                                         */
    DCL ALGAM ENTRY (FIXED DEC (5,0)),(I,NOP,NDOWN) FIXED,PNOP(0:100),
    PMIN INITIAL(0.95),FLEET_SIZE FIXED INITIAL(50),NREQ FIXED
    INITIAL(30),COUNTER FIXED INITIAL(0);
    POHMAX=3.57254E-01;
LABEL1: DO NOP=0 TO FLEET_SIZE;
          NDOWN=FLEET_SIZE-NOP;
          IF NOP=0 THEN DO;
               PNOP(NOP)=POHMAX**FLEET_SIZE;
               GO TO LABEL2;
               END;
          IF NOP=FLEET_SIZE THEN DO;
               PNOP(NOP)=(1.0-POHMAX)**NOP;
               GO TO LABEL2;
               END;
          A=ALGAM(FLEET_SIZE+1)-ALGAM(NOP+1)-ALGAM(NDOWN+1)+NOP*
          LOG(1.0-POHMAX)+NDOWN*LOG(POHMAX);
          PNOP(NOP)=EXP(A);
LABEL2:   PUT LIST ('THE PROBABILITY OF'||NOP||
          ' SHIPS OPERATING AND '||NDOWN||' SHIPS IN OVERHAUL IS '
          ||PNOP(NOP));
          PUT SKIP;
          END;
         PUT SKIP(3);
    PCUM=0.0;
    DO I=NREQ TO FLEET_SIZE;
          PCUM=PCUM+PNOP(I);
          END;
    IF PCUM>PMIN THEN IF COUNTER=1
          THEN GO TO LABEL3;
          ELSE DO;
```

```
              FLEET_SIZE=FLEET_SIZE-1;
              GO TO LABEL1;
              END;
     IF PCUM<PMIN THEN DO;
          FLEET_SIZE=FLEET_SIZE+1;
          COUNTER=1;
          GO TO LABEL1;
          END;
LABEL3: PUT LIST ('FOR A PROBABILITY OF '||PMIN||
     ' OF HAVING AT LEAST'||NREQ||' OPERATIONAL SHIPS, A FLEET SIZE OF'
     ||FLEET_SIZE||' IS REQUIRED.');
/*                                                                        */
/*   THE INTERNAL SUBROUTINE ALGAM PROVIDES THE NATURAL LOGARITHM OF      */
/*   THE GAMMA FUNCTION OF THE GIVEN ARGUMENT.                            */
/*                                                                        */
ALGAM:PROC(X);
     DCL X FIXED;
     Z=X;
     IF X<=1.E10 THEN IF X<=1.E-09 THEN RETURN(-1.E75);
                      ELSE DO;
                           TERM=1.E0;
AGAIN:                     IF Z<=18.E0 THEN DO;
                                            TERM=TERM*Z;
                                            Z=Z+1.E0;
                                            GO TO AGAIN;
                                            END;
                                       ELSE DO;
                                            RZ2=1.E0/Z**2;
DLNG=(Z-0.5E0)*LOG(Z)-Z+0.9189385E0-LOG(TERM)+(1.E0/Z)*
      (.8333333E-01-(RZ2*(.9277777E-02+(RZ2*(.7936508E-03-
       (RZ2*(.5952381E-03)))))));
                                            RETURN(DLNG);
                                            END;
                               END;
                 ELSE IF X<1.E70 THEN RETURN(Z*(LOG(Z)-1.E0));
                                 ELSE RETURN(1.E75);
```

```
END ALGAM;
/*                                                                                    */
    END FLEET;
```

APPENDIX C

Recursion Relationships for Age and Time-Since-Overhaul Dependent Models

APPENDIX C

RECURSION RELATIONSHIPS FOR AGE AND TIME-SINCE-OVERHAUL DEPENDENT MODELS

The following are the recursion formulas developed for the age and time-since-overhaul models for both 2 and 4 conditions:

1. The 2 Condition Model:

$$P(k,t,j|k_o,t_o,j_o) = P(1,t-1,j-1|k_o,t_o,j_o)p(k|1,t-1,j-1)$$
$$j = 2,\ldots,t$$
$$k = 1,2$$

Since being in condition 2 is tantamount to undergoing an overhaul which will return the ship to condition 1 with certainty, and since j takes a value of 1 only upon the ship's coming out of overhaul,

$$P(k,t,j|k_o,t_o,j_o) = \begin{cases} \sum_{\ell=1}^{t-1} P(2,t-1,\ell|k_o,t_o,j_o) & j=1,\ k=1 \\ 0 & j=1,\ k=2. \end{cases}$$

Also,

$$\pi(k,t,j,x) = \pi(1,t-1,j-1,x)p(k|1,t-1,j-1)$$
$$j=2,\ldots,t; \quad k=1$$

$$\pi(k,t,j,x) = \pi(1,t-1,j-1)p(k|1,t-1,j-1),$$
$$j=2,\ldots,t;\ k=2.$$

Again,

$$\pi(k,t,j,x) = \begin{cases} \sum_{\ell=1}^{t-1} \pi(2,t-1,\ell,x) & j=1,\ k=1 \\ 0 & j=1,\ k=2. \end{cases}$$

2. The multi-condition (4) model, following the same approach:

$$P(k,t,j|k_o,t_o,j_o) = \begin{cases} \sum_{i=1}^{3} P(i,t-1,j-1|k_o,t_o,j_o)p(k|i,t-1,j-1) \\ \qquad j=2,\ldots,t;\quad k=1,\ldots,4 \\ \sum_{\ell=1}^{t-1} P(4,t-1,\ell|k_o,t_o,j_o) & j=1,\ k=1 \\ 0 & j=1;\ k=2,3,4. \end{cases}$$

$$\pi(k,t,j,x) = \begin{cases} \sum_{i=1}^{3} \pi(i,t-1,j-1,x)p(k|i,t-1,j-1) & j=2,\ldots,t;\ k=1,2,3 \\ \sum_{i=1}^{3} \pi(i,t-1,j-1,x-1)p(k|i,t-1,j-1) & j=2,\ldots,t;\ k=4 \\ \sum_{\ell=1}^{t-1} \pi(4,t-1,\ell,x) & j=1,\ k=1 \\ 0 & j=1;\ k=2,3,4. \end{cases}$$

APPENDIX D

Determination of Transition Probabilities for the Example for the Multi-Condition, Age and Time-Since-Overhaul Dependent Model

APPENDIX D

DETERMINATION OF TRANSITION PROBABILITIES FOR THE MULTI-CONDITION, AGE AND TIME-SINCE-OVERHAUL DEPENDENT MODEL

The available data provided the dates of overhauls and restricted availabilities for CRUDESLANT ships from 1963 to 1971 (approximately). Thirty-three ships were selected for which such data was available for a complete cycle (i.e., overhaul to overhaul). These ships were grouped into the three age categories described in Chapter III. For each category a histogram (Figure D-1) was developed showing the total number of restricted availabilities occurring during each time unit after overhaul (up to three years for category I and four years for categories II and III - since COMCRUDESLANT has never operated with an overhaul policy with cycles in excess of four years, little data was available for ships out of overhaul for longer than that period).

The results of this were then examined for some logical grouping of ships based on time since last overhaul. It appears that in each case the number of RAV's during the first unit of time is markedly higher than that for subsequent time periods. Further, there also appears to be a relatively well defined difference between the second and third periods and later periods. Therefore, the first category of time out of overhaul consists simply of j = 1, the second category only of

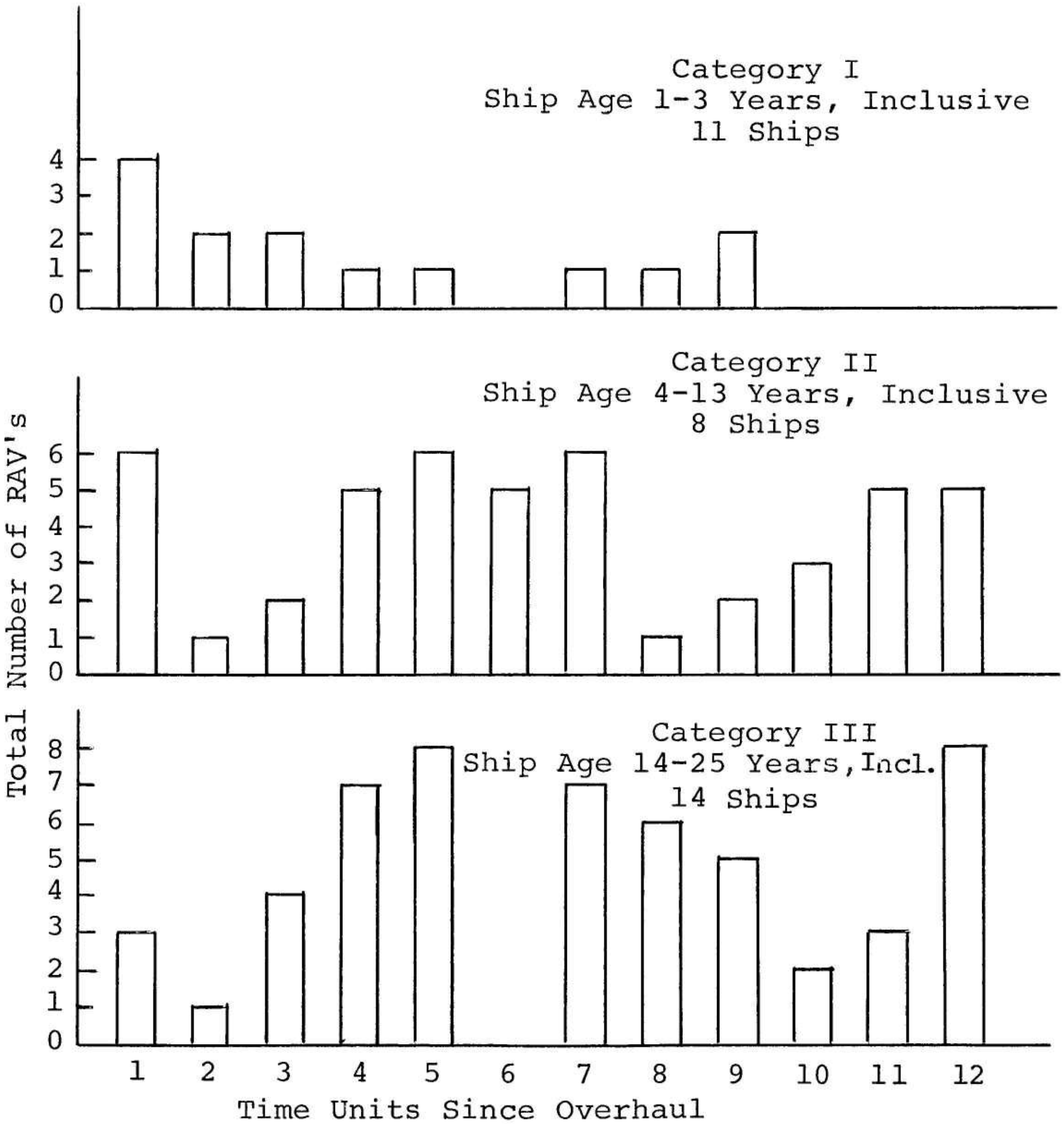

FIGURE D-1

Histogram of RAV Frequency

$j = 2,3$, while the third therefore consists of $j = 4,\ldots,t$ (unless j is otherwise restricted by some overhaul policy).

With time since last overhaul categorized thusly, some estimate of state probabilities was required. This was done by first making the arbitrary assumption that one out of every three RAV's (which, in general, represent unexpected equipment failures of a relatively critical nature) was due to a ship's being in condition 4. Therefore, one-third the number of RAV's averaged over time and number of ships (for each category - there now being 3 x 3, or 9, categories), provided an estimator of the probability of being in condition 4. It was then arbitrarily assumed that the state probabilities for condition 3 were equal to those for condition 4, and that those for condition 1 equaled those for condition 2. Since these must sum to unity, these assumptions define the state probabilities. Working "backwards" from the estimates of state probabilities, estimators for transition probabilities were developed, with the primary assumption being that a ship would be more likely to remain in the same condition over a transition than it would be to change to any other given condition. The matrices of these transition probability estimators comprise Table (4) in Chapter III.

The high "failure rate" immediately following overhaul is somewhat counterintuitive. It is felt by the author that this is due primarily to two factors. First, when a ship is overhauled, disruption to major mechanical or electrical/

electronic systems is frequently involved. Such disruptions may in some cases result in casualties to these systems once operation is resumed. Secondly, what is administratively described as a restricted availability, immediately following overhaul, may in fact be a continuation of the overhaul to complete one or more unfinished jobs. This could result from official disfavor toward failing to complete an overhaul on schedule.

Certain of the assumptions made in developing the transition probability matrices would not be tenable in actually applying the model. However, for the results of the model to be reasonably valid, it is important that the initial values estimated for transition probabilities be based on complete, accurate data insofar as possible, and the criteria for estimating a ship's material condition be applied in a consistent manner to all ships. As noted in Chapter II, while efforts continue to develop a material condition index which will permit a quantitative description of a ship's material condition, no such usable index currently exists. Some reasonable approximation of such an index is required, then. Of significant value in developing such an approximation might be the results of type commanders' material inspections, or inspections by the Board of Inspection and Survey (INSURV Board). Other possible parameters that could be used in estimating a ship's material condition include RAV frequency (as in the example), CASREPT data (frequency, severity, etc.), and the number of man-hours expended in maintenance or the volume of deferred maintenance (under the "3-M" system) reported by a ship. Here one should be careful, however, as inconsistencies occur from ship to ship in the accuracy and thoroughness of the reporting of maintenance data.

Whatever combination of parameters is decided upon, the best approach to estimating transition probabilites is probably to choose several representative ships and use past data to "track" them over time, observing how they changed in material condition and noting age and time since overhaul at the time changes took place.

It should be obvious that once transition probability estimators have been determined and a model such as this implemented, the estimators (as well as the criteria for grouping ships into categories) should be updated as behavior of ship material condition over time is actually observed.

APPENDIX E

Computer Program for the Multi-
Condition, Age and Time-Since
Overhaul Dependent Model

```
SHIPS: PROC OPTIONS (MAIN);
    DCL P(4,4,0:75,0:75), PCAP(4,0:75,0:75), PHI(4,0:75),
    (T,J,K,L,M) FIXED,
    SHIP_LIFE FIXED INITIAL(75);
/*                                                                           */
/*                                                                           */
/*  SET ALL ELEMENTS OF ARRAYS TO ZERO.                                      */
/*                                                                           */
    P=0.0;
    PCAP=0.0;
    PHI=0.0;
/*                                                                           */
/*                                                                           */
/*  P(K,I,T,J) IS THE PROBABILITY OF BEING IN CONDITION K AT AGE             */
/*  T+1, J+1 UNITS SINCE LAST OVERHAUL, GIVEN CONDITION I AT AGE T,          */
/*  J UNITS SINCE LAST OVERHAUL.                                             */
/*                                                                           */
/*  PCAP(K,T,J) IS THE PROBABILITY OF BEING IN STATE (K,T,J) GIVEN           */
/*  THAT THE SHIP STARTED 'NEW'.                                             */
/*                                                                           */
/*  PHI(K,T) IS THE PROBABILITY OF BEING IN CONDITION K AT AGE T.            */
/*                                                                           */
/*                                                                           */
/*  ENTER INITIAL CONDITION.                                                 */
/*                                                                           */
    PCAP(1,0,0)=1.0;
/*                                                                           */
/*                                                                           */
/* THE FOLLOWING THREE NESTS OF DO LOOPS ENTER NINE TRANSITION               */
/*  PROBABILITY MATRICES, THE NINE MATRICES RESULTING FROM THREE             */
/*  CATEGORIES OF SHIP AGE TIMES THREE CATEGORIES OF TIME OUT OF             */
/*  OVERHAUL.  THESE TRANSITION PROBABILITIES ARE ESTIMATED FROM             */
/*  MAINTENANCE DATA (RAV FREQUENCY) FOR 33 CRUDESLANT SHIPS DURING          */
/*  THE PERIOD 1963-1971.                                                    */
/*                                                                           */
    DO T=0 TO 8;
```

```
P(1,1,T,1)=0.505;
P(2,1,T,1)=0.253;
P(3,1,T,1)=0.121;
P(4,1,T,1)=0.121;
DO J=2 TC 3;
     P(1,1,T,J)=C.663;
     P(2,1,T,J)=0.263;
     P(3,1,T,J)=C.037;
     P(4,1,T,J)=0.037;
     P(1,2,T,J)=0.263;
     P(2,2,T,J)=C.663;
     P(3,2,T,J)=0.037;
     P(4,2,T,J)=C.037;
     P(1,3,T,J)=C.263;
     P(2,3,T,J)=0.263;
     P(3,3,T,J)=C.437;
     P(4,3,T,J)=C.037;
     END;
DO J=4 TC 8;
     P(1,1,T,J)=C.682;
     P(2,1,T,J)=C.282;
     P(3,1,T,J)=C.018;
     P(4,1,T,J)=0.018;
     P(1,2,T,J)=C.282;
     P(2,2,T,J)=C.682;
     P(3,2,T,J)=0.018;
     P(4,2,T,J)=C.018;
     P(1,3,T,J)=0.282;
     P(2,3,T,J)=C.282;
     P(3,3,T,J)=C.418;
     P(4,3,T,J)=0.018;
     END;
END;
```

```
DC T=9 TO 38;
     P(1,1,T,1)=0.333;
     P(2,1,T,1)=0.167;
```

```
        P(3,1,T,1)=0.250;
        P(4,1,T,1)=0.250;
        DO J=2 TO 3;
             P(1,1,T,J)=0.662;
             P(2,1,T,J)=0.262;
             P(3,1,T,J)=0.038;
             P(4,1,T,J)=0.038;
             P(1,2,T,J)=0.262;
             P(2,2,T,J)=0.662;
             P(3,2,T,J)=0.038;
             P(4,2,T,J)=0.038;
             P(1,3,T,J)=0.262;
             P(2,3,T,J)=0.262;
             P(3,3,T,J)=0.438;
             P(4,3,T,J)=0.038;
             END;
        DO J=4 TO T;
             P(1,1,T,J)=0.594;
             P(2,1,T,J)=0.194;
             P(3,1,T,J)=0.106;
             P(4,1,T,J)=0.106;
             P(1,2,T,J)=0.194;
             P(2,2,T,J)=0.594;
             P(3,2,T,J)=0.106;
             P(4,2,T,J)=0.106;
             P(1,3,T,J)=0.194;
             P(2,3,T,J)=0.194;
             P(3,3,T,J)=0.506;
             P(4,3,T,J)=0.106;
             END;
        END;
   DO T=39 TO 74;
        P(1,1,T,1)=0.571;
        P(2,1,T,1)=0.285;
        P(3,1,T,1)=0.072;
        P(4,1,T,1)=0.072;
```

```
        DO J=2 TO 3;
             P(1,1,T,J)=0.664;
             P(2,1,T,J)=0.264;
             P(3,1,T,J)=0.036;
             P(4,1,T,J)=0.036;
             P(1,2,T,J)=0.264;
             P(2,2,T,J)=0.664;
             P(3,2,T,J)=0.036;
             P(4,2,T,J)=0.036;
             P(1,3,T,J)=0.264;
             P(2,3,T,J)=0.264;
             P(3,3,T,J)=0.436;
             P(4,3,T,J)=0.036;
             END;
        DO J=4 TO T;
             P(1,1,T,J)=0.627;
             P(2,1,T,J)=0.227;
             P(3,1,T,J)=0.073;
             P(4,1,T,J)=0.073;
             P(1,2,T,J)=0.227;
             P(2,2,T,J)=0.627;
             P(3,2,T,J)=0.073;
             P(4,2,T,J)=0.073;
             P(1,3,T,J)=0.227;
             P(2,3,T,J)=0.227;
             P(3,3,T,J)=0.473;
             P(4,3,T,J)=0.073;
             END;
        END;
    P(1,1,0,0)=1.0;
/*                                                                    */
/*                                                                    */
    DO T=1 TO SHIP_LIFE;
         DO J=1 TO T;
              DO L=1 TO 3;
                   DO K=1 TO 4;
```

```
                        PCAP(K,T,J)=PCAP(K,T,J)+PCAP(L,T-1,J-1)*
                        P(K,L,T-1,J-1);
                        END;
                    END;
                END;
            DO M=0 TO T-1;
                PCAP(1,T,1)=PCAP(1,T,1)+PCAP(4,T-1,M);
                END;
            DO J=1 TO T;
                DO K=1 TO 4;
                    PHI(K,T)=PHI(K,T)+PCAP(K,T,J);
                    END;
                END;
            END;
/*                                                                    */
/*                                                                    */
/*  PRINT DESIRED OUTPUT.                                             */
/*                                                                    */
    PUT SKIP(5);
    DO T=1 TO SHIP_LIFE;
        DO L=1 TO 4;
            PUT SKIP EDIT (L,T,PHI(L,T)) (X(22),F(1),X(11),F(2),
            X(12),F(8,6));
            END;
        END;
    PUT PAGE;
/*                                                                    */
/*                                                                    */
/*  CALCULATE EXPECTED NUMBER OF OVERHAULS, EXP_NOH.                  */
/*                                                                    */
    EXP_NOH=0.0;
    DO T=1 TO SHIP_LIFE-1;
        EXP_NOH=EXP_NOH+PHI(4,T);
        END;
    PUT SKIP(15) EDIT (EXP_NOH) (X(30),F(8,5));
    END SHIPS;
```

APPENDIX F

Aggregated State Probabilities ($\phi(k,t)$) for the Multi-Condition, Age and Time-Since-Overhaul Dependent Model Example

k	t	$\phi(k,t)$
1	1	1.000000
2	1	0.000000
3	1	0.000000
4	1	0.000000
1	2	0.505000
2	2	0.253000
3	2	0.121000
4	2	0.121000
1	3	0.554177
2	3	0.332377
3	3	0.080923
4	3	0.032523
1	4	0.489522
2	4	0.386187
3	4	0.078330
4	4	0.045961
1	5	0.503031
2	5	0.420550
3	5	0.053875
4	5	0.022543
1	6	0.488774
2	6	0.440040
3	6	0.046367
4	6	0.024817
1	7	0.490047
2	7	0.449073
3	7	0.039713
4	7	0.021166
1	8	0.487707
2	8	0.453824
3	8	0.037176
4	8	0.021291
1	9	0.487845
2	9	0.456134
3	9	0.035444
4	9	0.020574
1	10	0.402857
2	10	0.374581
3	10	0.118369
4	10	0.104191
1	11	0.436053
2	11	0.325366
3	11	0.142963
4	11	0.095615
1	12	0.420386
2	12	0.304877
3	12	0.165959
4	12	0.108774
1	13	0.431193
2	13	0.298589
3	13	0.168298
4	13	0.101915
1	14	0.430217
2	14	0.300714

3	14	0.168191
4	14	0.100872
1	15	0.431027
2	15	0.302202
3	15	0.167020
4	15	0.099744
1	16	0.431010
2	16	0.303340
3	16	0.166226
4	16	0.099418
1	17	0.430644
2	17	0.303499
3	17	0.166170
4	17	0.099680
1	18	0.430686
2	18	0.303412
3	18	0.166181
4	18	0.099713
1	19	0.430589
2	19	0.303291
3	19	0.166292
4	19	0.099819
1	20	0.430624
2	20	0.303219
3	20	0.166332
4	20	0.099815
1	21	0.430622
2	21	0.303202
3	21	0.166349
4	21	0.099816
1	22	0.430629
2	22	0.303202
3	22	0.166348
4	22	0.099809
1	23	0.430631
2	23	0.303208
3	23	0.166344
4	23	0.099804
1	24	0.430630
2	24	0.303212
3	24	0.166341
4	24	0.099804
1	25	0.430630
2	25	0.303213
3	25	0.166340
4	25	0.099803
1	26	0.430628
2	26	0.303212
3	26	0.166340
4	26	0.099804
1	27	0.430628
2	27	0.303212
3	27	0.166340
4	27	0.099804
1	28	0.430627
2	28	0.303211
3	28	0.166340
4	28	0.099804
1	29	0.430627
2	29	0.303211

3	29	C.166340
4	29	C.099804
1	3C	C.430627
2	30	0.303211
3	3C	C.166340
4	30	C.099804
1	31	C.430626
2	31	0.303210
3	31	C.166340
4	31	C.099803
1	32	C.430625
2	32	0.303210
3	32	C.166339
4	32	C.C99803
1	33	C.430625
2	33	0.303210
3	33	0.166339
4	33	0.099803
1	34	0.430624
2	34	C.303209
3	34	C.166339
4	34	C.099803
1	35	0.430624
2	35	0.303209
3	35	0.166339
4	35	C.C99803
1	36	C.430623
2	36	0.303208
3	36	0.166338
4	36	0.099803
1	37	0.430623
2	37	C.303208
3	37	C.166338
4	37	C.099802
1	38	C.430622
2	38	C.303208
3	38	C.166338
4	38	C.099802
1	39	C.430621
2	39	C.303207
3	39	C.166338
4	39	C.099802
1	4C	0.476233
2	4C	C.336843
3	4C	C.126713
4	4C	0.060179
1	41	C.464512
2	41	C.359954
3	41	C.113C93
4	41	0.062408
1	42	C.464399
2	42	C.367C28
3	42	0.106888
4	42	C.061651
1	43	C.462283
2	43	0.368798
3	43	C.105820
4	43	C.063065
1	44	C.461337
2	44	C.367906

3	44	0.106525
4	44	0.064197
1	45	0.461803
2	45	0.367422
3	45	0.106675
4	45	0.064065
1	46	0.461846
2	46	0.367347
3	46	0.106720
4	46	0.064050
1	47	0.461945
2	47	0.367399
3	47	0.106654
4	47	0.063966
1	48	0.461953
2	48	0.367470
3	48	0.106600
4	48	0.063939
1	49	0.461935
2	49	0.367495
3	49	0.106585
4	49	0.063946
1	50	0.461931
2	50	0.367499
3	50	0.106583
4	50	0.063949
1	51	0.461927
2	51	0.367496
3	51	0.106585
4	51	0.063952
1	52	0.461927
2	52	0.367493
3	52	0.106586
4	52	0.063952
1	53	0.461928
2	53	0.367492
3	53	0.106586
4	53	0.063952
1	54	0.461927
2	54	0.367492
3	54	0.106586
4	54	0.063952
1	55	0.461927
2	55	0.367492
3	55	0.106586
4	55	0.063952
1	56	0.461927
2	56	0.367492
3	56	0.106586
4	56	0.063951
1	57	0.461926
2	57	0.367491
3	57	0.106585
4	57	0.063951
1	58	0.461925
2	58	0.367490
3	58	0.106585
4	58	0.063951
1	59	0.461925
2	59	0.367491

3	59	0.106585
4	59	0.063951
1	60	0.461925
2	60	0.367490
3	60	0.106585
4	60	0.063951
1	61	0.461924
2	61	0.367490
3	61	0.106585
4	61	0.063951
1	62	0.461924
2	62	0.367489
3	62	0.106584
4	62	0.063951
1	63	0.461923
2	63	0.367489
3	63	0.106585
4	63	0.063951
1	64	0.461922
2	64	0.367488
3	64	0.106584
4	64	0.063951
1	65	0.461922
2	65	0.367488
3	65	0.106584
4	65	0.063951
1	66	0.461921
2	66	0.367487
3	66	0.106584
4	66	0.063951
1	67	0.461921
2	67	0.367487
3	67	0.106584
4	67	0.063951
1	68	0.461920
2	68	0.367486
3	68	0.106584
4	68	0.063951
1	69	0.461919
2	69	0.367486
3	69	0.106584
4	69	0.063950
1	70	0.461919
2	70	0.367486
3	70	0.106583
4	70	0.063950
1	71	0.461918
2	71	0.367485
3	71	0.106583
4	71	0.063950
1	72	0.461918
2	72	0.367485
3	72	0.106583
4	72	0.063950
1	73	0.461917
2	73	0.367484
3	73	0.106583
4	73	0.063950
1	74	0.461917
2	74	0.367484

3	74	0.106583
4	74	0.063950
1	75	0.461916
2	75	0.367483
3	75	0.106582
4	75	0.063950

APPENDIX G

Computer Program for Determining Required Fleet Size, Mixed Fleet

```
MXFLT: PROC OPTIONS(MAIN);
/*                                                                        */
/*  THIS PROCEDURE COMPUTES PROBABILITIES OF VARIOUS COMBINATIONS OF */
/*  OPERATIONAL SHIPS FOR THE 'MIXED' FLEET, USING THE NORMAL         */
/*  APPROXIMATION BASED ON THE CENTRAL LIMIT THEOREM, AS DESCRIBED    */
/*  IN SECTION C OF CHAPTER IV.  REQUIRED AS INPUT ARE THE            */
/*  PROBABILITIES OF BEING IN OVERHAUL FOR INDIVIDUAL SHIPS.          */
/*                                                                        */
    DCL PR(125), (I,J,N1,N2,N3,N4,N5) FIXED, MU FLOAT, (X,P,D) FLOAT
    BIN,N FIXED;
    GET LIST ((PR(N) DO N=1 TO 125));
    DO I=0 TO 100 BY 25;
    DO J=0 TO 20 BY 5;
    PR1=PR(I+J+1);
    PR2=PR(I+J+2);
    PR3=PR(I+J+3);
    PR4=PR(I+J+4);
    PR5=PR(I+J+5);
    PUT DATA (PR1,PR2,PR3,PR4,PR5);
    PUT SKIP;
    DO N1=4 TO 6;
    DO N2=4 TO 6;
    DO N3=4 TO 6;
    DO N4=4 TO 6;
    DO N5=4 TO 6;
    MU=N1*PR1+N2*PR2+N3*PR3+N4*PR4+N5*PR5;
    VAR=N1*PR1*(1.0-PR1)+N2*PR2*(1.0-PR2)+N3*PR3*(1.0-PR3)+N4*PR4*
    (1.0-PR4)+N5*PR5*(1.0-PR5);
    B=N1+N2+N3+N4+N5-20;
    X=(B+0.5-MU)/SQRT(VAR);
    CALL NDTR(X,P,D);
    A=P;
    PUT DATA (N1,N2,N3,N4,N5);
    PUT DATA (A);
    PUT SKIP;
    END; END; END; END; END;
```

```
PUT SKIP(5);
END;
PUT SKIP(10);
END;
END MXFLT;
```