Determination of the Virtual Mass of the Akron due to the Potential Flow of Air about it.

By

Isabel C. Ebel

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for the Degree of
Bachelor of Science in Aeronautical Engineering
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Signature of Author ---- Signature redacted

Certification of the Staff --------------------

Professor in Charge ---- Signature redacted

Chairman of the Course Committee
on Undergraduate Students ---- Signature redacted

Head of the Course --------------------

Aero.
Thesis
1932

38

Cambridge, Massachusetts

May 25, 1932.

Professor A. L. Merrill
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:-

I herewith submit the following thesis entitled "Determination of the Virtual Mass of the Akron due to the Potential Flow of Air about it," in partial fulfillment for the requirements for the Bachelor of Science degree at the Massachusetts Institute of Technology.

Respectfully yours,

Signature redacted

Isabel C. Ebel

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ACKNOWLEDGEMENT

This thesis begun at the suggestion of Dr. Richard H. Smith could not have been carried to a satisfactory conclusion without his instructions, for which I wish to express my gratitude.

I am also indebted to Miss Hilda M. Lyon for her helpful criticisms and advice.

Determination of the Virtual Mass of the Airship Akron due to the Potential Flow of Air about it.

Deceleration tests used to determine the resistance of airships have shown that a discrepancy exists between the value of the actual mass of an airship and the virtual mass which occurs to give the forces that act on the airship, that is

$$R = ma$$

becomes

$$R = V_m a$$

where

R = resistance of the airship

a = acceleration

m = mass of the airship

V_m = virtual mass

In correcting for this variation in mass it has been assumed that the apparent addition of mass is due to the loss of a part of the ships propulsive energy into the surrounding medium. This loss is the sum of two effects:-

(I) Additional inertia due to the potential flow of a fluid about the body.

(II) Viscous drag within the boundary layer of a fluid at the surface of the body.

Consider the oblate spheroid to approximate the shape of all rigid airships. The oblate spheroid is a special type of ellipsoid, one in which the cross-sections perpendicular to the longitudinal axis are circular. The inertia coefficients of ellipsoids have been found by Horace Lamb and are listed in his text on hydrodynamics. The inertia coefficient of the ellipsoid of the same fineness ratio as the airship has been accepted as that fraction of additional mass, due to the potential flow. This then is the first of the effects mentioned above. The second is believed to have an equal value. So the total additional mass becomes twice the value indicated by the inertia coefficient of the ellipsoid.

Take the U.S.S. Los Angeles as an example. The longitudinal inertia coefficient of the corresponding ellipsoid is about .04. The additional mass of the airship is then taken as .08m or $V_m = 1.08m$.

The purpose of this thesis is to calculate a finite numerical value for that part of the virtual mass due to the potential flow about the airship Akron moving in purely translational motion. The exact shape of the Akron is used without approximation to more standard elliptical forms.

Treat the air as a fluid flowing about the body in question. The flow of the fluid is produced by a series of impulses. An impulse $P = \rho\varphi$

where P = impulse per unit area.

ρ = density of the fluid.

φ = velocity potential of the air flow.

The work done by an impulse, i.e., the change in kinetic energy, is proved in mechanics to be the product of the impulse and the average of the initial and final velocities in the direction of the impulse.

$\Delta T = \frac{v_n}{2} P$

ΔT = change in kinetic energy per unit area starting from rest.

v_n = final normal velocity of the unit area.

$= V \frac{dx}{dn} = \frac{\partial \varphi}{\partial n}$ (V = forward velocity of the body)

so $\Delta T = \frac{\rho}{2} \varphi \frac{d\varphi}{dn}$

and kinetic energy of the fluid becomes

$T = \frac{\rho}{2} \int \varphi \frac{d\varphi}{dn} da$

taken over the surface of the solid body and where da = unit area.

But from dynamics it is known that $\frac{1}{2} MV^2$ = kinetic energy. Therefore $T = \frac{1}{2} MV^2$ and $M = \frac{2T}{V^2} = \frac{\rho}{V^2} \int \varphi \frac{d\varphi}{dn} da$ which is the inertia mass we wish to find.

From this we see that that part of virtual mass due to the potential flow about any body may be found if the velocity potential is known.

For the airship Akron moving head-on through a uniform stream of air the velocity potential is a single valued function and

$$P = \rho(\varphi_1 - \varphi_2)$$

$\varphi_1 = \int_{s_1}^{s_2} q\,ds$, for the body fixed in a moving stream.

$\varphi_2 = Ux$, for the moving stream.

where q = resultant tangential velocity at the surface of the Akron.

U = velocity of the uniform stream of air, and is taken equal to unity.

ds = element of the surface of the Akron.

x = distance along the longitudinal axis of the Akron.

then $P = \rho\left[\int_0^s q\,ds - Ux\right] = \rho\varphi_A$

and $T = \frac{\rho}{2}\int \varphi_A \frac{d\varphi_A}{dn}\,da$

Summarizing:-

$$\varphi_A = \int_0^s q\,ds - Ux$$

$$\frac{d\varphi_A}{dn} = U\sin\theta$$

$$da = 2\pi r\,ds$$

where r = cross-sectional radius of the airship.

θ = angle which the tangent to the surface makes with the horizontal.

The tangential velocities I have used here are those found by Mr. Eaton of this Institute. They were obtained by means of Dr. Van Karman's method of finding theoretical pressure distributions of airship forms.

The angle, θ, and radius, r, were found graphically from a .0085 scale drawing of the Akron, the ordinates of which are here included.

Procedure:-

(1) Find φ_A:-

a) Plot q vs. s.

b) Integrate this curve.

c) Find $\int q \, ds - Ux$

for each section.

(2) Find $\frac{2T}{V^2}$:-

a) Multiply $\varphi \times \frac{d\varphi}{dn} \times da$

for each section.

b) Plot $\varphi \frac{d\varphi}{dn} da$ vs. s.

c) Integrate this curve to find $\frac{2T}{\rho V^2}$

(3) Find volume of Akron:-

a) Plot πr^2 vs. x.

b) Integrate this curve to find the volume.

(4) Plot, K, longitudinal coefficients of ellipsoids vs. c/a fineness ratio of ellipsoids. Locate K for c/a = 5.9 fineness ratio of the Akron.

where c = longitudinal axis.

a = maximum cross-sectional diameter.

Results:-

Inertia mass of the Akron due to potential

flow = 7211.916 ρ (meters)^3

Actual mass of the Akron

= 208,947.000 ρ (meters)^3

Inertia coefficient of the Akron

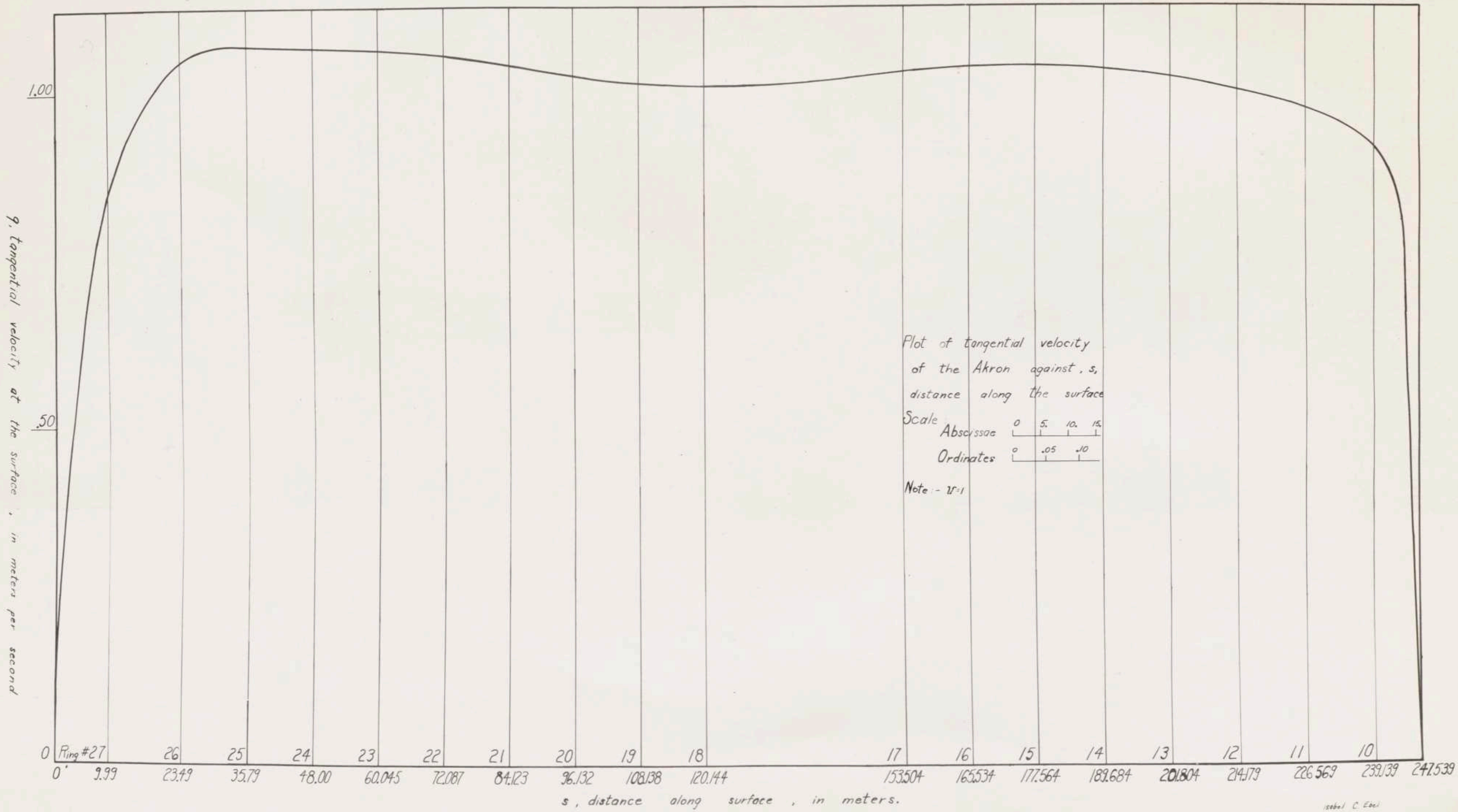
= .0345.

Inertia coefficient of the corresponding ellipsoid

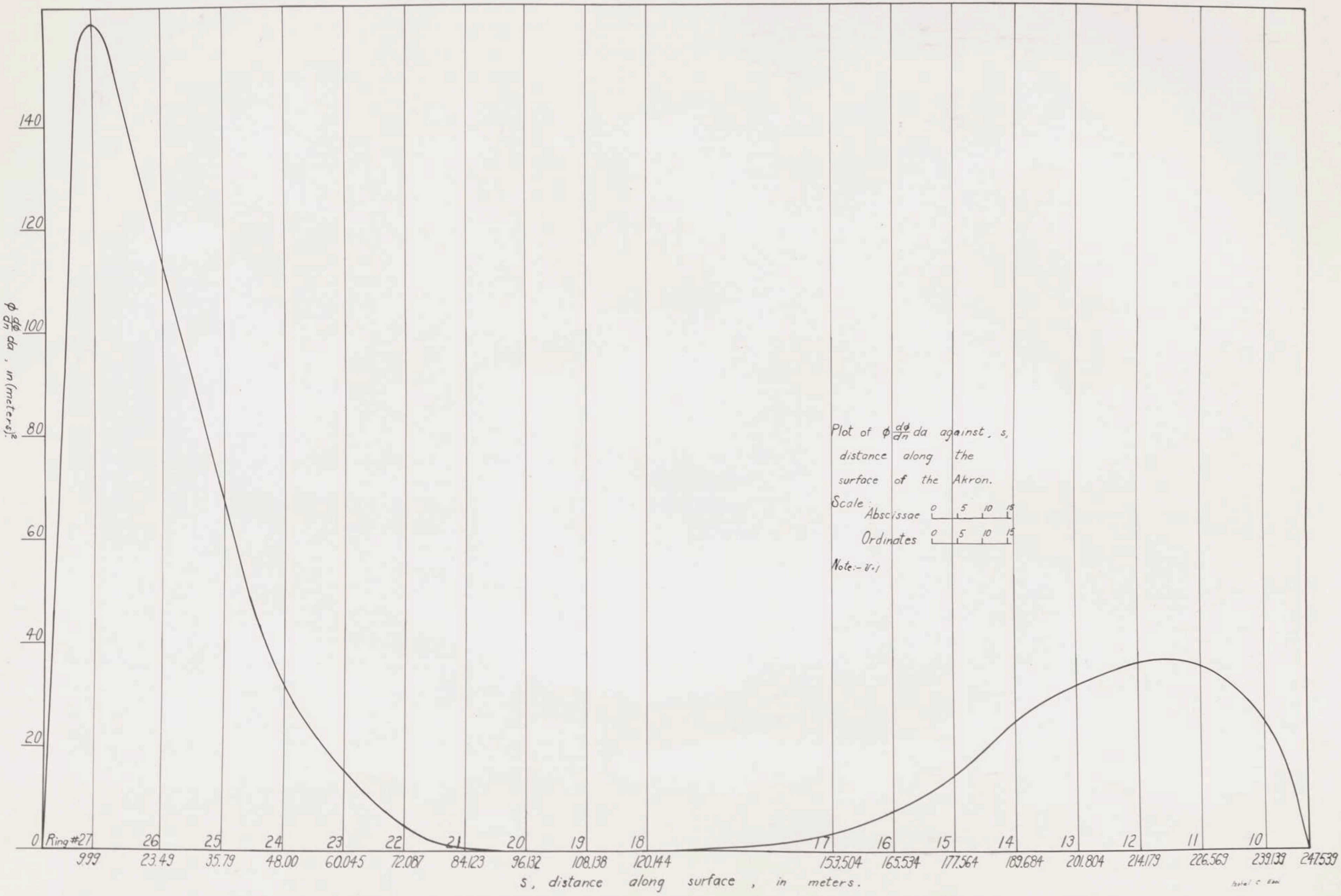
= .0460.

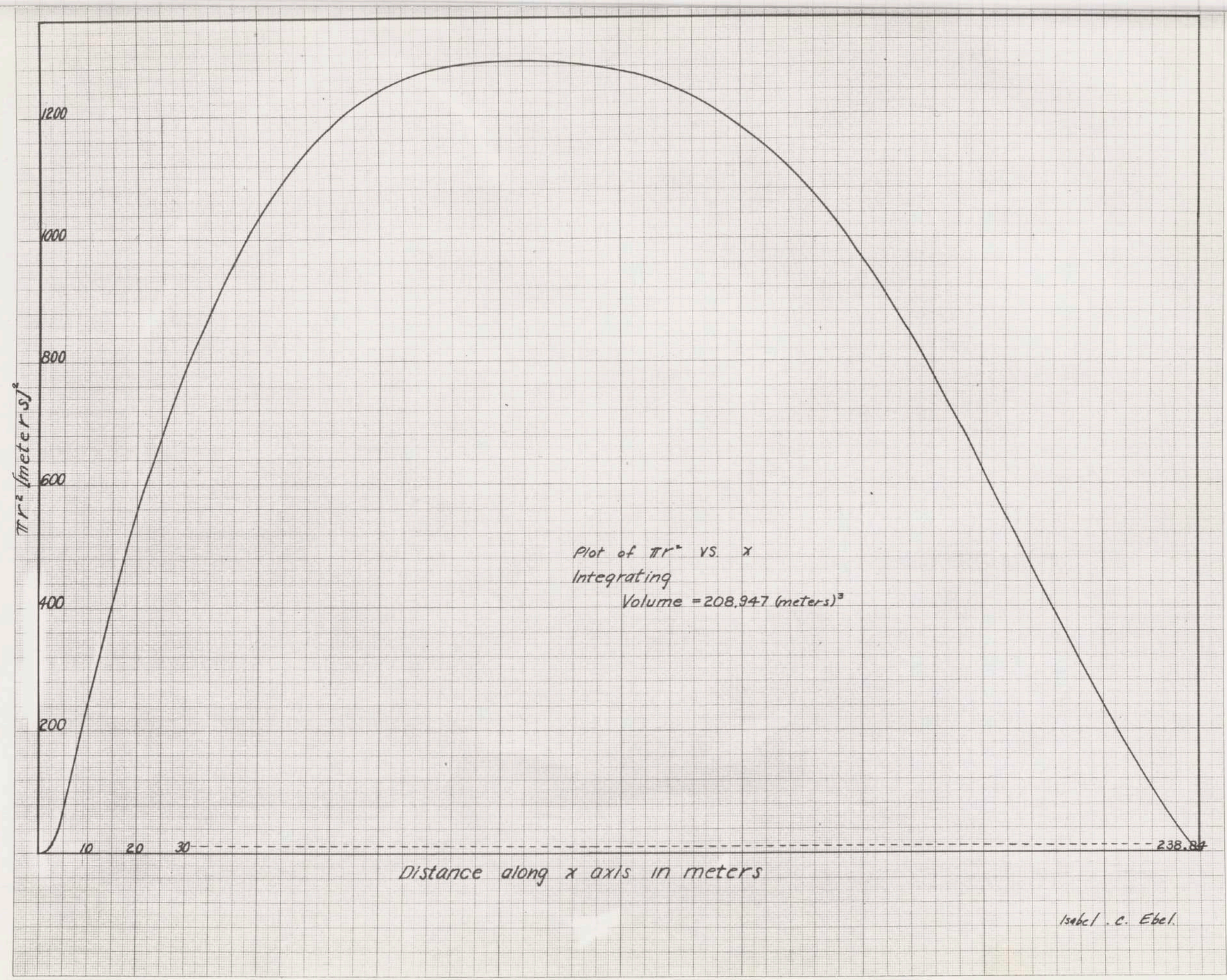
Virtual mass of the Akron becomes 1.0345m. due to the potential flow of air about it.

Note:- The determination of that part of the virtual mass due to viscous drag within the boundary layer is not found in this work. It may or may not be equal to that due to the potential flow.



Isobel C Ebel


Plot of φ dφ/dn da against, s, distance along the surface of the Akron.
Scale:
Abscissae 0 5 10 15
Ordinates 0 5 10 15
Note:- V=1
φ dφ/dn da, in (meters)²
140
120
100
80
60
40
20
0
Ring #27
26
25
24
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9.99
23.49
35.79
48.00
60.045
72.087
84.123
96.132
108.138
120.144
153.504
165.534
177.564
189.684
201.804
214.179
226.569
239.139
247.539
s, distance along surface, in meters.


Plot of πr^2 vs. x
Integrating
Volume = 208,947 (meters)^3
πr^2 (meters)^2
1200
1000
800
600
400
200
10
20
30
238.84
Distance along x axis in meters
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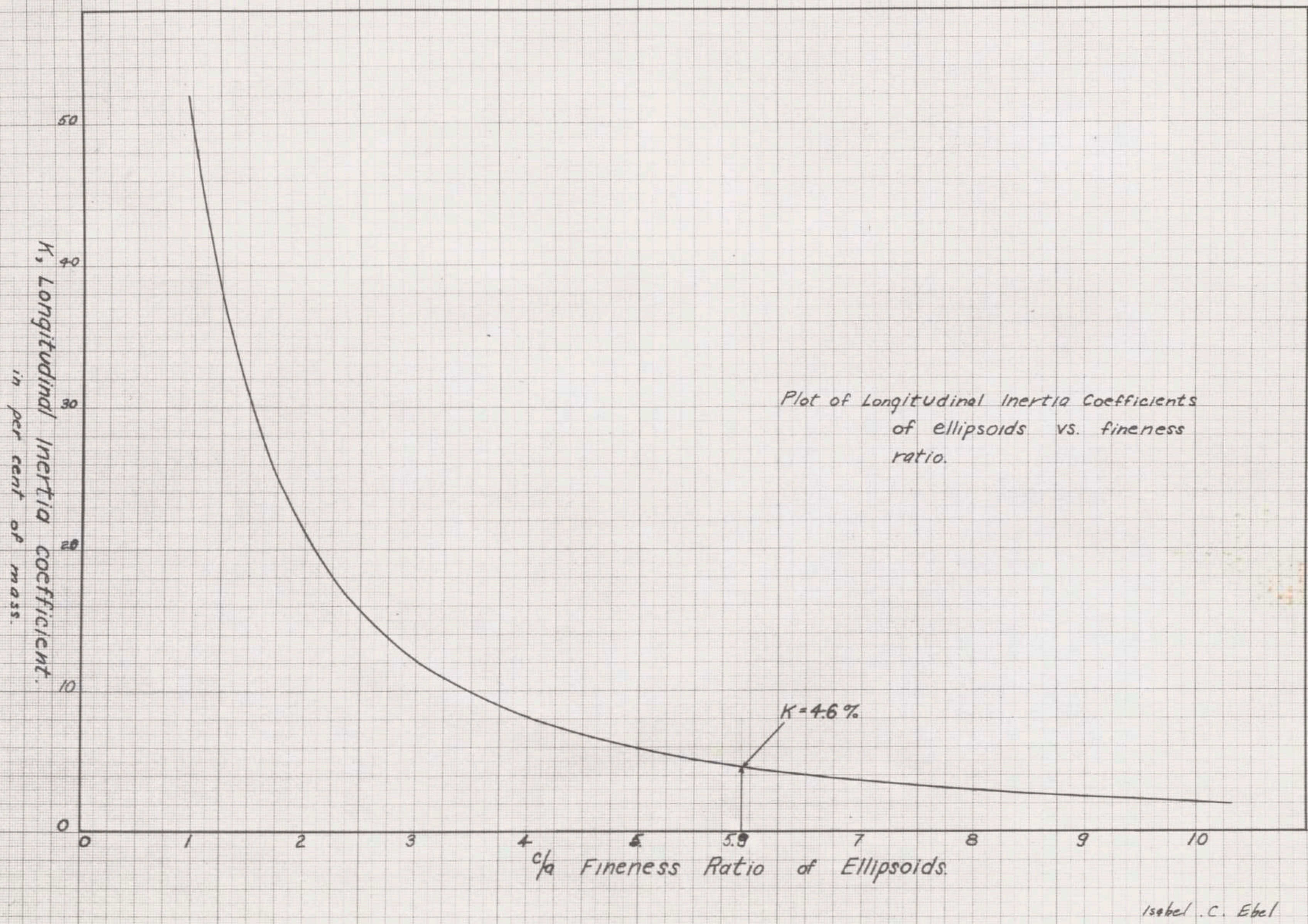
K, Longitudinal Inertia coefficient.
in per cent of mass.
50
40
30
20
10
0
0
1
2
3
4
5
5.9
7
8
9
10
c/a Fineness Ratio of Ellipsoids.
Plot of Longitudinal Inertia Coefficients of ellipsoids vs. fineness ratio.
K=4.6 %
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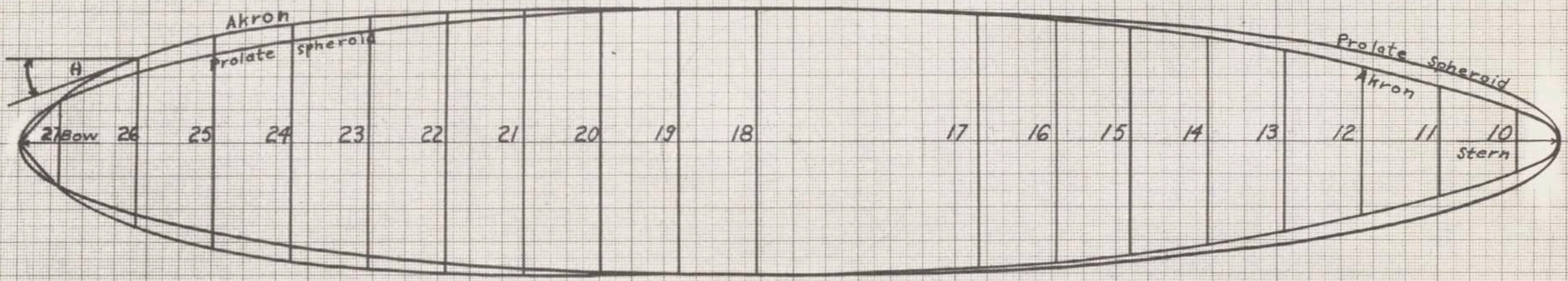

Scale drawing of the Akron with ellipsoid, of same fineness ratio, superimposed.

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Tabulation of Values Used.

Ring #	r (meters)	x distance from bow (meters)	ds (meters)	s (meters)
27	7.22	6.95	9.99	9.990
26	13.10	18.95	13.50	23.490
25	16.10	30.95	12.30	35.790
24	18.02	42.95	12.21	48.000
23	19.11	54.95	12.045	60.045
22	19.78	66.95	12.042	72.087
21	20.09	78.95	12.036	84.123
20	20.24	90.95	12.009	96.132
19	20.25	102.95	12.006	108.138
18	20.20	114.95	12.006	120.144
17	19.21	148.16	33.360	153.504
16	18.40	160.16	12.030	165.534
15	17.23	172.16	12.030	177.564
14	15.70	184.16	12.120	189.684
13	13.80	196.16	12.120	201.804
12	11.40	208.16	12.375	214.179
11	8.50	120.16	12.390	226.569
10	4.70	232.16	12.570	239.139
Stern tip.		238.84	8.400	247.539

ring	θ degrees	sin θ	q^2	q
27	- 39.20	.6320	.7199	.8485
26	- 17.40	.2990	1.0934	1.0457
25	- 10.90	.1891	1.1355	1.0656
24	- 6.40	.1115	1.1249	1.0606
23	- 4.20	.0732	1.1337	1.0649
22	- 2.00	.0349	1.1185	1.0576
21	- .60	.0105	1.0864	1.0423
20	- .30	.0052	1.0442	1.0219
19	.00	.000		
18	+ .80	.0140		
17	+ 3.25	.0567	1.0608	1.0295
16	+ 4.60	.0802	1.0537	1.0265
15	+ 6.30	.1097	1.0742	1.0365
14	+ 8.60	.1495	1.0405	1.0201
13	+ 10.30	.1788	1.0406	1.0201
12	+ 11.80	.2045	.9818	.9916
11	+ 14.90	.2571	.9350	.9669
10	+ 20.075	.3433	.8239	.9076

Note:-Angles measured below the horizontal are negative.

Velocities, q^2 and q are for q^2/U^2 and q/U where $U = 1$

Ring	q ds	$\int$ q ds	dx	Ux
Bow tip.	- 5.895	- 107.47	- 6.95	- 102.95
27	-13.000	- 101.575	-12.00	- 96.00
26	-13.050	- 88.575	-12.00	- 84.00
25	-13.000	- 75.525	-12.00	- 72.00
24	-12.765	- 62.525	-12.00	- 60.00
23	-12.685	- 49.760	-12.00	- 48.00
22	-12.570	- 37.075	-12.00	- 36.00
21	-12.340	- 24.505	-12.00	- 24.00
20	-12.165	- 12.165	-12.00	- 12.00
19				
18	+12.065	+ 12.065	+12.00	+ 12.00
17	+33.650	+ 45.715	+33.21	+ 45.21
16	+12.340	+ 58.055	+12.00	+ 57.21
15	+12.390	+ 70.445	+12.00	+ 69.21
14	+12.450	+ 82.895	+12.00	+ 81.21
13	+12.355	+ 95.250	+12.00	+ 93.21
12	+12.405	+ 107.655	+12.00	+ 105.21
11	+12.140	+ 119.795	+12.00	+ 117.21
10	+11.855	+ 131.650	+12.00	+ 129.21
tip.	+ 5.300	+ 136.950	+ 6.68	+ 135.89

Integration of velocity curve from maximum diameter of the Akron, where $\varphi = 0$.

i.e., from Ring 19-to the left to bow tip, negative.
-to the right to stern tip, positive.

Ring	$q\,ds^{\varphi} - Ux$	$\frac{d\varphi}{dn}$ U sin θ	da $2\pi r ds$	$\varphi \frac{d\phi}{dn} da$
Bow tip.	- 4.520	- 1.0000	2π 0 rds	0
27	- 5.575	- .6320	" 17.22 "	1029.0
26	- 4.575	- .2990	" 13.10 "	1894.0
25	- 3.525	- .1891	" 16.10 "	1107.0
24	- 2.525	- .1115	" 18.02 "	1571.4
23	- 1.760	- .0732	" 19.11 "	269.8
22	- 1.075	- .0349	" 19.78 "	110.8
21	- .505	- .0105	" 20.09 "	24.072
20	- .165	- .0052	" 20.24 "	4.203
19	0	0	" 20.25 "	.6003
18	+ .065	+ .0140	" 20.20 "	.72036
17	+ .505	+ .0567	" 19.21 "	36.68000
16	+ .845	+ .0802	" 18.40 "	61.34
15	+ 1.235	+ .1097	" 17.23 "	132.40
14	+ 1.680	+ .1495	" 15.70 "	240.00
13	+ 2.040	+ .1788	" 13.80 "	347.90
12	+ 2.445	+ .2045	" 11.40 "	420.90
11	+ 2.585	+ .2571	" 8.50 "	447.00
10	+ 2.440	+ .3433	" 4.70 "	392.30
Stern tip.	+ 1.060	+ 1.0000	" 0 "	121.80
				7211.91566

Note:- U equals unity.

= 7211.916

To find Volume.

x	dx	r	πr^2	Volume
from bow	0	0		
.09	.09	.23	.145	.005
1.34	1.25	1.53	7.354	3.60
4.34	3.00	4.93	76.355	96.00
8.34	4.00	8.33	218.000	592.00
14.34	6.00	11.43	410.900	1896.00
24.34	10.00	14.68	676.000	5560.00
34.34	10.00	16.77	882.500	7880.00
44.34	10.00	18.17	1037.000	9640.00
54.34	10.00	19.10	1145.000	10940.00
64.34	10.00	19.68	1215.000	11880.00
74.34	10.00	20.02	1258.000	12400.00
84.34	10.00	20.19	1279.000	12710.00
94.34	10.00	20.24	1286.500	12840.00
104.34	10.00	20.25	1287.500	12860.00
114.34	10.00	20.20	1280.500	12845.00
124.34	10.00	20.06	1262.000	12720.00
134.34	10.00	19.81	1231.000	12490.00
144.34	10.00	19.41	1181.500	12080.00
154.34	10.00	18.83	1112.500	11490.00
164.34	10.00	18.04	1021.000	10690.00
174.34	10.00	17.00	906.500	9680.00
184.34	10.00	15.70	774.000	8440.00

Continued.

x	dx	r	πr^2	Volume
194.34	10.00	14.11	624.500	6880.00
204.34	10.00	12.21	467.500	5470.00
214.34	10.00	9.97	312.050	3880.00
224.34	10.00	7.32	168.250	2420.00
233.34	9.00	4.24	56.400	1008.00
235.84	2.50	3.07	29.580	107.50
237.84	2.00	1.74	9.505	40.00
238.84	1.00	.0	0	5.00
				209,543.105

From Graphical integration

Volume = 209,543 $(\text{meters})^3$

Using planimeter

Volume = 208,350 $(\text{meters})^3$

Average = 208,947

Inertia mass = $\frac{7211.916}{208,947}$ = .0345 m.

To find inertia coefficient of ellipsoid with fineness ratio of 5.895.

c/a = fineness ratio.

K = inertia coefficient. (longitudinal).

c/a	K
1.	.5
1.50	.305
2.00	.209
2.51	.156
2.99	.122
3.99	.082
4.99	.059
6.01	.045
6.97	.036
8.01	.029
9.02	.024
9.97	.021
∞	0

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Ordinates of the Airship

XRS-4 Akron.

Station	Distance from tail (meters)	Radius (meters)
0	.0	.0
1	1.00	1.74
2	3.00	3.07
3	5.50	4.24
4	14.50	7.32
5	24.50	9.97
6	34.50	12.21
7	44.50	14.11
8	54.50	15.70
9	64.50	17.00
10	74.50	18.04
11	84.50	18.83
12	94.50	19.41
13	104.50	19.81
14	114.50	20.06
15	124.50	20.20
16	134.50	20.25
17	144.50	20.24
18	154.50	20.19
19	164.50	20.02
20	174.50	19.68
21	184.50	19.10

Ordinates of the Airship

XRS-4 Akron.

Station	Distance from tail (meters)	Radius (meters)
22	194.50	18.17
23	204.50	16.77
24	214.50	14.68
25	224.50	11.43
26	230.50	8.33
27	234.50	4.93
28	237.50	1.53
29	238.75	.23

$$\text{Fineness ratio} = \frac{238.84}{2 \times 20.25} = 5.895$$

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