MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HAYSTACK OBSERVATORY

WESTFORD, MASSACHUSETTS 01886

September 12, 2011

Telephone: 781-981-5400
Fax: 781-981-0590

To: EDGES Group

From: Alan E.E. Rogers

Subject: Absolute calibration of sky noise spectrum using a wideband antenna.

The EDGES uses a 3-position switch between the LNA and the antenna. The point at which the antenna connects to the switch is chosen as the reference plane. At this point we measure the antenna impedance by connecting a VNA or impedance analyzer to the antenna and then measure the LNA impedance by connecting the VNA to the EDGES input. In addition to measurements of antenna and LNA impedance the LNA noise waves need to be measured. This is accomplished by measuring the spectrum of an open (or shorted) low loss cable. The noise waves are then measured by fitting the spectrum to a model.

1] Noise wave model

The LNA Model is given in memo 76 which give the power from the LNA

$$\text{Power} = \text{wave}\left(\Gamma_a, \Gamma_\ell, T_{sky}, T_{amb}, L, t_u, t_c, t_s\right) + t_{\text{lna}}$$

$$= T_{sky}\left(1-|\Gamma_a|^2\right)L|F|^2 + T_{amb}\left[\left(1-L\right)L|\Gamma_a|^2 + \left(1-L\right)\right]|F|^2 + T_u|\Gamma_a|^2 L^2|F|^2$$
$$+|\Gamma_a|L|F|\left(t_c\cos\phi + t_s\sin\phi\right) + t_{\text{lna}}$$

Where $F = \left(\left(1+|\Gamma_\ell|^2\right)^{1/2} \Big/ \left(1-\Gamma_a\Gamma_\ell L\right)\right)$

ϕ = phase of $\Gamma_a F$

Γ_a = antenna reflection coefficient

Γ_ℓ = LNA reflection coefficient

T_{sky} = sky temperature

T_{amb} = ambient sky temperature

L = attenuation loss factor of an attenuator placed between antenna and EDGES antenna input.

t_u = uncorrelated noise

t_c, t_s = cosine and sine components of correlated noise

From this model we can derive a model for the output of the 3-position switch:

$$w3p\left(\Gamma_a, \Gamma_\ell, T_{sky}, T_{amb}, L, t_u, t_c, t_s\right)$$

$$= t_{cal}\left(p_{ant} - p_{load}\right) / \left(p_{cal} - p_{load}\right) + T_{amb}$$

Where $p_{ant} = wave\left(\Gamma_a, \Gamma_\ell, T_{sky}, T_{amb}, L, t_u, t_c, t_s\right)$

$$p_{load} = wave\left(0, \Gamma_\ell, T_{amb}, T_{amb}, 1, t_u, t_c, t_s\right)$$

$$p_{cal} = wave\left(0, \Gamma_\ell, T_{cal} + T_{amb}, T_{amb}, 1, t_u, t_c, t_s\right)$$

Note that the constant noise, t_{lna}, is not needed as it is eliminated in the 3 position switching.

2] Solving for LNA noise waves

When an open cable is connected to the EDGES input

$$\Gamma_a = L_c e^{-i\omega\tau}$$

L_c = cable loss factor

τ = cable delay

ω = frequency (radians/s)

The LNA noise wave parameters, t_u, t_c, t_s can be found by fitting the data to the model

$$w3p\left(\Gamma_a, \Gamma_\ell, T_{amb}, T_{amb}, 1, t_u, t_c, t_s\right)$$

Since the noise wave parameters change with frequency a polynomial is used for each parameter and their coefficients found by weighted least squares fitting to the appropriate function provided by the model.

3] Obtaining the calibrated sky noise

The sky noise spectrum for the 3-position switch is corrected for the LNA noise waves by adding the model for a zero Kelvin cold 50 Ω load

$$w3p\left(0, \Gamma_\ell, 0, T_{amb}, L, t_u, t_c, t_s\right)$$

And subtracting the model for an antenna looking at a cold sky

$$w3p\left(\Gamma_a, \Gamma_\ell, 0, T_{amb}, L, t_u, t_c, t_s\right)$$

Next the result is then divided by $\left(1 + \left|\Gamma_a\right|^2\right)\left|G\right|^2 L$

Where $G = 1/\left(1 - \Gamma_a \Gamma_\ell L\right)$

to obtain the calibrated spectrum. Further correction may be needed for ground loss and antenna resistive loss as given in memo 78.