INVENTORY MODELS AND BACKLOG COSTS:

AN EMPIRICAL INVESTIGATION

by

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ABSTRACT

This dissertation contains three empirical essays on manufacturers' backlogs and inventories of finished goods.

The first essay (chapter II) examines the smoothness qualities inherent in the Holt, et al. (1961) linear-quadratic inventory model. It is shown that a central property of the model is that a certain weighted sum of variances and covariances of production, sales and inventories must be nonnegative. The weights are the basic structural parameters of the model. The model may be tested by seeing whether this sum in fact is nonnegative. This test has three advantages over the tests of cross-equation restrictions commonly used in recent studies of this model: it is computationally simpler, more robust, and, most important, economically more informative. When the test is applied to some non-durables data aggregated to the two-digit SIC code level, it almost always rejects the model, even thought the model does well by traditional criteria.

One possible reason for the rejection is that the model's formulation of backlog costs is inadequate. The second essay (chapter III) formulates and tests an inventory model for production to stock industries with special attention to backlog costs. Although backlogs (queues of orders yet to be filled) are small in these industries, the estimates from this chapter suggest that the backlogs are economically important. The estimates of the parameter reflecting the cost to the firm of putting a unit on the backlog are usually statistically significant, and significantly larger than the cost of putting an extra unit into inventory.

The third essay (chapter IV) formulates and tests an inventory model for production to order industries. It is hypothesized that backlogs have two effects. First, they shift demand in, since higher backlogs mean longer delivery lags. (Symmetrically, inventories are hypothesized to shift demand out.) Second, backlogs, since they facilitate grouping the production of similar items, allow production costs to be cut. Empirical results are generally supportive of both hypotheses.

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CHAPTER I:

INTRODUCTION

The linear-quadratic inventory model was first introduced by Holt et al. (1961) and has since been the source of much theoretical and empirical work on inventories (e.g., Belsley (1969), Blanchard(1982)). In its purest form (Blinder (1982), the model argues that finished goods inventories are held solely to cut (quadratic) production costs, subject to the constraint that randomly varying sales are met. Inventories should be built up when sales are low and drawn down when sales are high, with the exact pattern determined by the relative costs of production and of holding inventories. This intuitively plausible notion has received empirical support from regression estimates of equations derived from the model (Belsley (1969), Blanchard (1982)).

However, production smoothing cannot be the sole motive for holding inventories, if, as appears generally to be true (Blinder(1981b), chapter II of this thesis), production is more volatile than sales.[1] That such volatility contradicts the model was noted informally by Blinder (1981b) and Blanchard (1982). Chapter II develops the argument rigorously. The basic idea is simply this: if production is more volatile than sales, firms could cut their production costs simply by setting production equal to sales and setting inventories equal to zero each period. Formally, this basic idea can be expressed as an inequality stating that a certain weighted sum of variances and covariances of production, sales and inventories must be nonnegative. The weights are the basic structural parameters of the model. The model may be tested by seeing whether this sum in fact is nonnegative. This test has three advantages over the tests of cross-equation restrictions commonly used in recent studies of this model: it is computationally simpler, more robust, and, most important, economically more informative. Chapter II also tests statistically whether some non-durables data aggregated to the two digit SIC code level satisfy the inequality. In general, they do not, even though the model does well by traditional criteria.

One possible explanation for the excess volatility is that inventories can not only affect costs in the hypothesized fashion, but can as well affect revenue, in the following way. Firms that have a low stockpile of inventories may run out when demand is high, and may then be forced to place some orders on a backlog. These backlogs may impose costs on producers, perhaps in the from of lost future sales. Manufacturers will then tend to build up inventories when expected sales are high, but will find smaller stockpiles satisfactory when expected sales are low. As is explained in chapter III, this tendency of inventories to track sales can make production more volatile than sales.

The notion that backlog costs of this sort can be important forms the basis of the bulk of this thesis--chapters III, IV and part of chapter II. In chapter II, the production smoothing model is extended in a fashion suggested by its originators (Holt et al. (1961)) and others (e.g., Blanchard (1982)) to include a certain quadratic term intended to capture backlog costs. This extended model does not perform noticeably better than the basic model. The extra parameter reflecting backlog costs is rarely significant, and a variance inequality derived for the extended model is rarely satisfied by the data.

One possible explanation for the poor performance of the extended model is that the particular form of backlog costs chosen is inadequate. The next two chapters (chapters III and IV) therefore develop and estimate production smoothing models with more sophisticated (non-linear quadratic) formulations of backlog costs.

Chapter III does so for what are called production to stock industries. Firms in these industries ordinarily produce in advance of receipt of orders, store the output in a stock of finished goods inventories, and sell to their customers directly from this pre-existing stock. Examples are petroleum and rubber. Backlogs are small and transitory (Abramowitz (1951), Belsley (1969)), and this has led some investigators (e.g., Belsley (1969)) to conclude that backlog costs are unimportant in these industries. The estimates from chapter III suggest otherwise. The estimates of the parameter reflecting the cost to the firm of putting a unit on the backlog are usually statistically significant, and significantly larger than the cost of putting an extra unit into inventory.

Chapter IV formulates and estimates a production smoothing model with backlog costs for production to order industries. Firms in these industries tend to produce output that is more or less tailored to the individual customer and/or is costly to store. These firms ordinarily wait for customer orders to be placed before completing production, working off a backlog of customer orders already received. Examples are airplanes and mainframe computers. Both backlogs and inventory stocks are substantial, and some investigators studying production to order industries have indeed integrated backlogs into their inventory models (e.g., Belsley (1969)). Chapter IV also develops such an integrated model, departing from previous approaches in two significant ways. First, the hypothesized effect that inventories and backlogs have on revenue is captured directly in the demand curve, rather than indirectly as an opportunity cost to the firm. Second, the production cost effects of backlogs and inventories are sharply distinguished: a large backlog allows firms to cut production costs by grouping the production of similar products, a large inventory stock does not. Results suggest that both of these departures are warranted. Demand is shifted by backlogs and inventories in the hypothesized fashion (a higher backlog shifts demand out, a higher inventory stock shifts in in, *ceteris paribus*). And backlogs do appear to allow production costs to be cut.

The two remaining parts to the thesis are a final chapter (chapter V) containing summarizing conclusions and an Appendix discussing the use of constant dollar inventory data in regressions.

Before turning to chapter II a word on its approach to estimation is appropriate. That chapter, as well as chapters III and

IV, uses a maximizing model to derive estimable first order conditions. Instrumental variable methods are then used to estimate parameters in these first order conditions under the rational expectations assumption that agents use information to make forecasts efficiently. Thus the equations estimated are structural rather than reduced form, and the parameters retrieved characterize tastes and technology. They are therefore likely to be relatively stable across changes in the environment, at least as compared to parameters derived from reduced form regressions.[2]

FOOTNOTES

1.Volatility is measured by variance around trend, see chapter II.

2.Since the models are only approximations, the estimated parameters are not strictly structural and thus not totally invariant to changes in the environment. This has been emphasized to me by S. Fischer and pointed out in Blanchard (1982,p8).

CHAPTER II:

A VARIANCE BOUNDS TEST OF THE LINEAR QUADRATIC INVENTORY MODEL

The linear quadratic inventory model, originated by Holt et al. (1961), has been the source of some recent empirical work on inventories of finished goods (Blanchard (1982), Eichenbaum (1982)). These studies have been interpreted as being generally supportive of the model: while tests of overidentifying restrictions did tend to reject, estimates of the model's key parameters were almost always right-signed and significant. Results were considered reliable enough for use in answering questions on the dynamic interaction of inventories with sales and production that are central to understanding business cycles.

The approaches of both Blanchard (1982) and Eichenbaum (1982), however, while different in many respects, share three major shortcomings. First, estimation is computationally cumbersome. Both impose highly nonlinear cross-equation constraints that can make estimation numerically difficult (Blanchard (1982, pp33,43). Second, some strong assumptions about stochastic environment and market structure are required. Blanchard (1982,p38) assumed that inventories do not Granger-cause sales, an assumption apparently rejected by the data. Eichenbaum (1982,pp5-8) assumed a perfect competitor and that demand had a particularly simple structure. The third, and most important, the formulation and application of their test procedures contribute only a limited amount to our understanding of the model. For example, these procedures provide little guidance to the range of parameter estimates that are sensible, other than sign. This is illustrated in Blanchard (1982,p36). in which the estimates of a key parameter, one that determines a target inventory to sales ratio, vary over ten data sets from a low of about one to a high of about seventeen. All of these apparently are equally acceptable.

This paper suggests an alternative method of estimating and testing a linear quadratic inventory model, and then applies it to some non-durables data aggregated to the two digit SIC code level.[1] The method is computationally simpler, more robust, and economically more informative than the approaches of Blanchard (1982) and Eichenbaum (1982). The estimates are obtained in standard fashion from an Euler equation. They are then examined to see whether the firm could have expected to have been better off with the static policy of simply letting inventories increase from period to period at their trend rate of growth. This is done by comparing expected costs under the static policy and the policy that is optimal according to the model. The difference between the two, which should be nonnegative if the model is correct, may be expressed as a simple weighted sum of certain variances and covariances of inventories, sales and production. The weights are the basic structural parameters of the model, obtainable from the Euler equation. Even if all the estimates of parameters are right signed and significant, the estimate of this difference in principle may be insignificantly positive, or even negative. If it is, it seems unlikely that inventories truly are chosen in accordance with the supposedly optimal policy and therefore unlikely that the model is correct.

And in fact, for the non-durables data studied here, despite parameter estimates comparable to Eichenbaum (1982) and Blanchard (1982), it is found that the difference in expected costs is almost always negative--that is, the allegedly optimal policy could almost always have been expected to increase costs relative to the static, no-feedback one. The increase is statistically significant about half the time. Moreover, it is economically large, with expected deviations of costs from trend that are up to 16 percent higher than under the static policy. This would seem to provide strong evidence that the model is inconsistent with this data.

This inconsistency is especially evident since the test requires relatively few assumptions about stochastic environment and market structure. In particular, it is consistent with but does not require the assumptions of Blanchard (1982) and Eichenbaum (1982) mentioned above. Also, it is computationally straightforward, requiring only linear estimation. In fact, in some cases it could be concluded that the static inventory policy would be expected to cost less even without calculating any of the model's parameters. All that was required was the calculation of certain variances and covariances. Since the test easily extends to cover other linear quadratic models, and perhaps some non-linear models as well, it may be of general interest.

This is especially so since, in the present case at least, the difference in expected costs is theoretically important, quite apart from its usefulness for empirical work. Under the null hypothesis that the model is correct, the difference is zero only if there are no shocks to forcing variables. The difference thus summarizes the dynamic interactions between production, sales and inventories as these variables are optimally adjusted in response to shocks. And these interactions are precisely what the model is intended to explain.

The paper is organized as follows. Part II develops the test, part III contains empirical results, and part IV contains conclusions.

II.THE TEST

This section first describes the model and then derives an inequality that is central to the test.

A.The Model

The model under consideration is intended for finished goods inventories in so-called "production to stock" industries (Abramowitz (1951), Rowley and Trevedi (1975)). Its precise formulation varies from author to author, and this paper's empirical work tests two versions. Both may be derived from the following general model. Firms producing a single homogeneous good maximize expected discounted real profits:

$$(1)\quad \max E_0 \sum_{t=0}^{\infty} d_1^t \Big([p_t S_t] - d_2^t[a_0(\Delta Q_t)^2 + a_1 Q_t^2 + a_2(H_t - a_3 S_{t+1})^2] \Big)$$

$$\text{s.t. } Q_t = S_t + H_t - H_{t-1}$$

where

E_0 mathematical expectations, conditional on information available at time 0

d_1 fixed real discount rate, $0<d_1<1$

d_2 fixed rate of technological progress, $0<d_2<1$

p_t real price in period t

S_t units sold in period t

Q_t units produced in period t

H_t units of finished goods inventories at end of period t

a_i strictly positive parameters

Three general comments on (1) will be made, before the individual terms of the equation are briefly discussed. First, the firm's choice variables have intentionally been left unspecified. The estimation here is consistent with any of the standard ones: output only (Belsley (1969)) or inventories only (Blanchard (1982)) in models in which sales are exogenous; output, inventories and sales in models in which the firm is a perfect competitor (Blanchard and Melino (1981), Eichenbaum (1982))[2]; output, price and inventories in models in which the firm is a monopolist (Blinder (1980)). The firm's information set has been left unspecified for the same reason.

Second, cost shocks, present in recent studies using the model, have been suppressed for simplicity. Strictly speaking, the errors in the regressions are purely expectational. Parameter estimates were, however, made robust to the presence of these shocks.[3]

Third, for the present, all variables should be assumed to be deviations from trend (where trend should be understood to encompass all deterministic components, seasonal as well as secular). This assumption is purely for expositional convenience and will be relaxed shortly. What we wish to derive are some restrictions that are implied for arbitrary trend, and the algebra is less cluttered when trend terms are set to zero.

The first term in brackets in equation (1) is revenue, the second is costs. Although the revenue function will play no role in the bulk of this paper, it is worth pointing out some of the implications of its presence at this initial stage to emphasize the generality of the tests performed here. The market may be perfect (Eichenbaum (1982)) or imperfect (Blinder (1982)). Price speculation on the supply side (Eichenbaum (1982)) or perhaps even on the demand side may be present.

Pricing and production decisions may be made simultaneously (Eichenbaum (1982), Blinder (1982)) or separately (Holt, et al., (1961)). In short, Summers (1981) criticisms of inventory models that ignore interactions between firms and their customers are not relevant here.

The second term in brackets is costs. These are the focus of the model, and, here as elsewhere, are central. Total per period costs are the sum of three terms.

The first is the cost of changing production, which is quadratic in the period to period change in the number of units produced. This represents, for example, hiring and firing costs.

The second is the cost of production, which is quadratic in the number of units produced. This approximates an arbitrary concave cost function that results as usual from a decreasing returns to scale technology.

The third and final term embodies inventory and backlog costs, and is quadratic in how far inventories are from a target level. It is discussed at length in the next chapter; a brief explanation of its rationale will suffice here. Inventory holding costs (e.g., storage and handling charges) are reflected in a_2. The parameter a_3 is the inventory to expected sales ratio that would be set in the absence of both types of production costs ($a_0=a_1=0$). Not all authors agree that this ratio should be anything but zero, and the two major variations in (1) accommodated in the tests here turn on whether a_3 is allowed to be non-zero. Those who do so (Blanchard (1982), Eichenbaum (1982), Holt, et al., (1961)) argue that sales sometimes exceed inventories on hand, forcing firms to backlog orders. Firms face costs when such a backlog develops, perhaps because of loss of future sales. Thus, <u>ceteris paribus</u>, when expected sales are higher, inventories should be higher as well. The target level for inventories, $a_3E_tS_{t+1}$, trades off backlog and inventory costs. In this model <u>with a target level</u>, inventories can serve two functions. They can buffer production, allowing it to be smoothed in the presence of fluctuating demand. And they can cut backlog costs. Optimal inventories balance production, holding and backlog costs.

Some other authors, however, insist that in the absence of production costs, the target level for inventories would be zero (Auerbach and Green (1980), Belsley (1969), Blinder (1982)). They impose $a_3=0$. Inventories are then held purely to smooth production. In this model <u>without a target level</u>, optimal inventories balance savings in production costs against the costs of carrying inventories.

The tests performed here will thus accommodate equation (1) both with and without a target level for inventories.

<u>B.An Inequality</u>

We now derive an inequality by calculating the effect inventories have on expected costs.[4] (The algebra carries along a_3. The effect in models without a target level is obtained simply by setting $a_3=0$ in the manipulations that follow.) According to the model, firms solve (1), subject to transversality and market equilibrium conditions to select optimal H_t^* and/or Q_t^* (and, as noted above, possibly p_t^* and S_t^* as well) (Eichenbaum (1982), Hansen and Sargent (1980), Sargent (1981)). In this optimal closed loop policy, the endogenous control variables typically are set by a feedback rule, with their optimal period t values a function of their own past values and past and present values of forcing variables.

Let us assume that the sequences (H_t^*) (Q_t^*), and (S_t^*) are covariance stationary. Methods for calculating this stationary solution in particular cases may be found in Eichenbaum (1982), Holt, et al., (1961) and Blanchard (1982). Let $E_0V_0^*$ be the expectation at time t of the value of the objective function that results from this policy:

$$(2)\ E_0 \sum_{t=0}^{\infty} d_1^t \Big([p_t^* S_t^*] - d_2^t [a_0(\Delta Q_t^*)^2 + a_1(Q_t^*)^2 + a_2(H_t^* - a_3 S_{t+1}^*)^2] \Big)$$

Let $E_0V_0^A$ be the expectation at time t of the value of the objective function that would result from the alternative policy of setting $H_t^A=0$ in every period, $Q_t^A=S_t^A=S_t^*$. Price $p_t^A=p_t^*$ will in general still be consistent with buyers demanding $S_t^A=S_t^*$.[5] The value of the objective function under this alternative policy is then

$$(3)\ E_0 \sum_{t=0}^{\infty} d_1^t \Big([p_t^* S_t^*] - d_2^t [a_0(\Delta S_t^*)^2 + a_1(S_t^*)^2 + a_2(-a_3 S_{t+1}^*)^2] \Big)$$

This alternative decision rule clearly is feasible.[6] By assumption, then, since V_0^* is optimal, $E_0V_0^* \geq E_0V_0^A$. Now, $E_0V_0^*$ and $E_0V_0^A$ are random with respect to unconditional information and $E_0V_0^* - E_0V_0^A$ is a well-defined random variable with respect to this information set. Since it is nonnegative it has a nonnegative expectation. Thus $E(E_0V_0^* - E_0V_0^A) \geq 0$. By the law of iterated expectations, then

$$(4)\quad EV_0^* \geq EV_0^A$$

$$==> \quad E \sum_{t=0}^{\infty} d_1^t \Big([p_t^* S_t^*] - d_2^t [a_0(\Delta Q_t^*)^2 + a_1(Q_t^*)^2 + a_2(H_t^* - a_3 S_{t+1}^*)^2] \Big)$$

$$\geq$$

$$E \sum_{t=0}^{\infty} d_1^t \Big([p_t^* S_t^*] - d_2^t [a_0(\Delta S_t^*)^2 + a_1(S_t^*)^2 + a_2(-a_3 S_{t+1}^*)^2] \Big)$$

Let $var(Q^*)=E(Q_t^*)^2$ denote the variance of production and $cov(Q,Q_{-1})=E(Q_t^* Q_{t-1}^*)$ its first autocovariance, with analogous notation for other variables. (No time subscripts are necessary by the assumption of covariance stationarity.) Also define $d=d_1 d_2$. With this notation (4) becomes

$$(5) \quad \sum_{t=0}^{\infty} d_1^t \; E[p_t^* S_t^*] - \sum_{t=0}^{\infty} d^t \; [(a_0 var(\Delta Q^*) + a_1 var(Q^*) + a_2 var(H^* - a_3 S_{+1}^*)]$$

$$\geq$$

$$\sum_{t=0}^{\infty} d_1^t \; E[p_t^* S_t^*] - \sum_{t=0}^{\infty} d^t \; [(a_0 var(\Delta S^*) + a_1 var(S^*) + a_2 var(-a_3 S_{+1}^*)]$$

Using $Q_t=S_t+H_t-H_{t-1}$ where convenient, expanding $var(H^*-a_3S_{+1}^*)$ = $var(H^*)-2a_3cov(H^*,S_{+1}^*)+a_3^2var(S^*)$, moving all terms to the left hand side of the inequality, and then applying the standard formula for a geometric sum transforms (5) into

$$(6) \; 0 < (1-d)^{-1} \, [a_0(var(\Delta S^*) - var(\Delta Q^*)) + a_1(var(S^*) - var(Q^*)) - a_2 var(H^*) + 2a_2 a_3 cov(H^*, S_{+1}^*)]$$

It is the two versions of this inequality--with and without a target level--that will be tested:[7]

$$(7.1)\ 0 < (1-d)^{-1}\ [a_0(\text{var}(\Delta S)-\text{var}(\Delta Q)) + a_1(\text{var}(S)-\text{var}(Q)) - a_2\text{var}(H)]$$

$$(7.2)\ 0 < (1-d)^{-1}\ [a_0(\text{var}(\Delta S)-\text{var}(\Delta Q)) + a_1(\text{var}(S)-\text{var}(Q)) - a_2\text{var}(H) + 2\ a_2a_3\text{cov}(H,S_{+1})]$$

The "*" superscripts have been dropped in accordance with the null hypothesis that observed H, S and Q accord with the optimal solution to (1).

(7.1) and (7.2) have been derived assuming that all variables have zero unconditional expectations. These inequalities still hold even when such expectations are non-zero and firms account for them when maximizing expected discounted profits. For let the variables in (1) include deterministic components--constant, time trends, seasonal dummies, etc.--and add linear terms such as $a_{10}(\Delta Q_t)$ to the cost function in equation (1). It is then easily verified (see the Appendix to this chapter) that if the alternative policy is the no-feedback, open loop one that sets inventories equal to their unconditional expectation each period ($H_t^A=EH_t^*$, $p_t^A=p_t^*$, $S_t^A=S_t^*$, $Q_t^A=S_t^*+E(H_t^*-H_{t-1}^*)$), the inequalities in (7) still result.[8] For the remainder of the paper, (7.1) and (7.2) will be understood to apply to just such a model with deterministic terms. It should be noted again that for expositional convenience all such terms will be referred to as "trend," even though the word "trend" is perhaps somewhat misleading if deterministic seasonal fluctuations are present or if secular growth is not.

In this light, let us interpret (7.1) and (7.2). The right-hand sides of these two equations describe the cost savings that could be (unconditionally) expected to result from setting inventories optimally rather than without feedback. The first two terms express differences of production costs, the third that of inventory costs, and the fourth, in (7.2), that of costs of inventories that deviate from their target level. The expected difference in inventory holding costs, $-a_2 \text{var}(H_t)$, is always negative. Therefore, according to the model, these expected cost increases are more than offset by savings elsewhere (otherwise the optimal policy would not be optimal). Inequality (7.1), applicable when there is no target level, says that the firm must expect to save either on costs of changing production ($\text{var}(\Delta Q) < \text{var}(\Delta S)$), or on costs of production ($\text{var}(Q) < \text{var}(S)$), or both, and the expected savings must be large enough that overall expected costs are lower, i.e., (7.1) holds. Similarly, (7.2), applicable when there is a target level, says that the optimal policy must be expected to more than offset increases in expected inventory holding costs with expected savings in production and/or target level costs.

Thus it would seem to be a minimal economic requirement that (7.1) and (7.2) be satisfied by data that are to be explained by the model. The inequalities merely ask that the optimal policy be expected to cost less than the static one. The static policy is the one that would be optimal in the absence of all shocks to forcing variables. The inequalities therefore summarize how production, sales and inventories are expected to interact as they are dynamically adjusted in response to shocks. And this is precisely what the model purports to explain. It is perhaps reasonable, therefore, to ask that the data not only satisfy

(7.1) and (7.2), but do so to an extent that is significant in economic or statistical terms.

The next section sees how well some aggregate nondurables data satisfy these inequalities. Given that (7.1) and (7.2) have been derived for a single firm, however, it is appropriate to make a remark on aggregation before examining these empirical results. The inequalities do still hold at an aggregate level, provided that all the parameters representing technology (e.g., the a_i's) and the stochastic characteristics of forcing variables (i.e., their ARMA parameters) are the same for each individual firm. As is explained in detail in the Appendix, under these sufficient though perhaps not necessary conditions each firm's behavior is summarized by a set of linear regresions with identical coefficients on the regressors. As usual, therefore, the model aggregates exactly, and aggregate behavior is characterized by the same set of regressions. It is no surprise, then, that aggregate production, sales and inventories satisfy (7.1) and (7.2), for arbitrary correlation of production, sales and inventories across firms.

III.Empirical results

Data and estimation are decribed briefly before empirical results are presented.

A.Data

The data were real (1972 dollars) and monthly. Both seasonally adjusted and unadjusted data were used. Seasonally adjusted data were available for 1959 to 1980 for aggregate non-durables and four two-digit non-durables industries (chemicals (SIC 28), rubber (SIC 30), petroleum (SIC 29), and food (SIC 20)). Seasonally unadjusted data were available for aggregate non-durables and three two digit industries (chemicals, petroleum and rubber). (Again, durable goods industries were excluded because the model is intended to apply only to industries that produce to stock, and durable goods industries generally produce to order.)

Sales were obtained by using the appropriate wholesale price index (Citibank Economic Database files PWDMND, PWCH, PWRUB, PWFUEL, PWFOSA, PWPA) to deflate the Bureau of the Census nominal figures for sales (files MNS, MNSCH2, MNSCH5, MNSCH4, MNSFO, and MNSPR2 for the seasonally adjusted figures, and the Bureau of the Census (1978,1982) Manufacturer's Shipments, Inventories and Orders for the unadjusted figures). The seasonally adjusted inventory figures were obtained by converting the Bureau's recently calculated constant dollar seasonally adjusted inventory series (Hinrichs and Eckman (1981)) from "cost" to "market" so that one dollar of inventories represented the same physical units as one dollar of sales (see the Appendix "A Note on the

Econometric Use of Constant Dollar Inventory Series" for a definition of "cost" and "market" and an explanation of why a conversion was necessary). As in Reagan and Sheehan (1982) the seasonally unadjusted constant dollar inventory figures were obtained by multiplying the adjusted figures by the corresponding unadjusted to adjusted ratio for book value (nominal) inventories. (This procedure was adopted since no unadjusted constant dollar data appear to be available. It makes the plausible assumption that the "seasonal deflator" is the same for book value and constant dollar inventories.)[9] Production was obtained from the identity $Q_t=S_t+H_t-H_{t-1}$.

B.Estimation

The sample period covered 1959:5 to 1980:10, with 1980:11 and 1980:12 used for leads and 1959:2 to 1959:4 used for lags. All data were first regressed on a constant and time trend, and, for seasonally unadjusted data, on seasonal dummies as well. The resulting residuals were used in all subsequent regressions. Note that in the linear estimation performed here, parameter estimates resulting from the regressions using the detrended variables are identical to those that would have resulted from regressions using the original variables including trend. Also, it should be noted that in Reagan and Sheehan (1982) time series study of precisely the unadjusted data used here, it was found that seasonal dummies alone successfully accounted for the seasonal variation in inventories. There appeared to be no need to allow for indeterministic seasonal components.

Three specific aspects of estimation will be briefly discussed. These are estimation of the a_i, of the second moments of inventories, sales and production, and, finally, of the standard error of (7). (Throughout this section, references to "(7)" should be understood to be shorthand for "(7.1) and (7.2)"). Additional details will be found in the Appendix.

The a_i's in the model with a target level were obtained as follows. (The same procedure was applied to the model without a target level, except that $a_3=0$ was imposed.) A necessary first order condition to solve (1) at time $t \geq t_0$ is obtained by differentiating (1) with respect to H_t and setting the result equal to zero:[10]

$$
\begin{aligned}
(8)\ E_t\, 2[d^2a_0H_{t+2} &- (2d^2a_0+2da_0+da_1)H_{t+1} \\
&+ (d^2a_0+4da_0+a_0+da_1+a_1+a_2)H_t \\
&- (2a_0+2da_0+a_1)H_{t-1} + a_0H_{t-2} \\
&+ d^2a_0S_{t+2} - (d^2a_0+2da_0+da_1+a_2a_3)S_{t+1} \\
&+ (2da_0+a_0+a_1)S_t - a_0S_{t-1}\ ] = 0
\end{aligned}
$$

After defining lower case $q_t = dQ_t-Q_{t-1}$ and dividing this first order condition by two, the Euler equation (9) results:

$$(9)\ E_t[\ a_0dq_{t+2}-(a_1+a_0(1+d))q_{t+1}+a_0q_t+a_2H_t-a_2a_3S_{t+1}\ ]= 0$$

Now normalize $a_1+(1+d)a_0=1$ and write (9) as

$$(10)\ q_{t+1}= a_0(dq_{t+2}+q_t) + a_2H_t - a_2a_3S_{t+1} + u_{1t}$$

where u_{1t} is an MA(2) error. With monthly d=.995 imposed (corresponding annual discount rate is about six per cent) (10) can be estimated by instrumental variables.[11] The six instruments used were three lags each of inventories and sales. The estimation required two steps, as described in Hansen and Singleton (1982). The first step calculated the variance-covariance matrix of the u_{1t} and the second obtained the optimal instrumental variables estimator. See the Appendix to this chapter for further details. Since the equation is overidentified--the model without a target level has two right-hand side variables and that with has three--Hansen's (1982) test of overidentifying restrictions was calculated.

Variances and covariances were calculated from a bivariate (inventories,sales) autoregression of order three:[12]

$$(11)\quad H_t = \phi_{11}H_{t-1}+\phi_{12}H_{t-2}+\phi_{13}H_{t-3}+\phi_{14}S_{t-1}+\phi_{15}S_{t-2}+\phi_{16}S_{t-3}+u_{2t}$$
$$S_t = \phi_{21}H_{t-1}+\phi_{22}H_{t-2}+\phi_{23}H_{t-3}+\phi_{24}S_{t-1}+\phi_{25}S_{t-2}+\phi_{26}S_{t-3}+u_{3t}$$

The Yule-Walker equation using the estimated ϕ_{ij} was then used in the standard way (Anderson (1971,p 182)) to obtain the needed second moments of sales and inventories. The second moments of production were derived from the identity $Q_t=S_t+H_t-H_{t-1}$, e.g. $var(Q)=var(S)+2cov(S,H)-2cov(S,H_{-1})+2var(H)-2cov(H,H_{-1})$.

Finally, the standard error of the statistic (7) was derived as follows. Let θ be the (1 x 24) parameter vector needed to calculate (7). θ, defined precisely in the Appendix to this chapter, consists of the coefficients on the 21 right-hand side variables in three equation system consisting of (10) and (11) and the three elements of the covariance matrix of the error terms in (11). The estimated θ is asymptotically normal with a covariance matrix V defined in the Appendix. The statistic (7) is a function of θ, say, $g(\theta)$, and thus is asymptotically normal with covariance matrix $(dg/d\theta)V(dg/d\theta)'$. The standard error of (7) is the square root of $(dg/d\theta)V(dg/d\theta)'$. The derivatives $dg/d\theta$ were calculated numerically.

C.Results

We will shortly present estimates of the size and the standard errors of the right hand sides of (7.1) and (7.2) for the data described above. This will require estimates not only of the appropriate variances and covariances of inventories, sales and production, but of the a_i parameters as well. First, however, let us consider whether these data are qualitatively consistent with the inequalities, by examining the approriate second moments. Tables I and II have these, for seasonally adjusted and unadjusted data respectively.

It follows immediately from the trivial calculations underlying the entries in Tables I and II that for both seasonally adjusted and unadjusted data, the model without a target level violates (7.1) for almost all industries! (The only possible exception is chemicals.) Columns (5)-(7) indicate that for all but the chemical industry, $var(\Delta S_t)-var(\Delta Q_t)<0$, $var(S_t)-var(Q_t)<0$, and, of course, $var(H_t)>0$. Since the a_i are known a priori to be positive it follows that for all but chemicals, $0>a_0(var(\Delta S)-var(\Delta Q)) + a_1(var(S)-var(Q)) - a_2var(H)$. In other words, according to the model itself, the static, no-feedback policy of letting inventories grow at their trend rate would have been expected to be preferable to the optimal policy that the model claims actually was followed: lower costs of changing production, lower costs of production, and lower inventory costs. From these simple

TABLE I

BASIC VARIANCES AND COVARIANCES
(SEASONALLY ADJUSTED DATA)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	var(ΔS)	var(ΔQ)	var(S)	var(Q)	(1)-(2)	(3)-(4)	var(H)	$cov(H,S_{+1})$
Aggregate non-durables	124 428	170 406	1 288 760	1 311 910	-45 978.2	-23 240.0	1 326 370	715 325
Chemicals (SIC 28)	8 547	9 404	85 259	84 792	-856.9	967.7	71 597	16 129
Rubber (SIC 30)	2 474	3 687	22 188	23 186	-1 212.5	-998.5	26 960	14 312
Petroleum (SIC 29)	3 003	5 189	21 159	22 358	-2 185.9	-838.9	11 601	1 123
Food (SIC 20)	39 789	51 744	46 800	50 628	-11 954.7	-3827.5	97 778	21 828

Notes:
Units are millions of 1972 dollars squared. Data and calculation described in text.

TABLE II

BASIC VARIANCES AND COVARIANCES
(SEASONALLY UNADJUSTED DATA)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	var(ΔS)	var(ΔQ)	var(S)	var(Q)	(1)-(2)	(3)-(4)	var(H)	cov(H,S_{+1})
Aggregate non-durables	218 562	335 787	1 213 660	1 283 900	-117 225	-70 241.8	1 142 410	733 007
Chemicals	20 713	18 850	103 899	100 926	1 868	2 973.7	69 000	23 588
Rubber	4 249	5 291	19 088	20 331	-1 042	-1 243.2	16 745	7 801
Petroleum	3 480	5 956	21 188	22 502	-2 476	-1 314.3	11 315	-6 379

Notes:
Units are millions of 1972 dollars squared. Data and calculation described in text.

calculations we can conclude that with the possible exception of the chemical industry, the data studied here are inconsistent with the model without a target level. This suggests that backlog costs, whose existence are used to rationalize a non-zero target level, are of crucial importance to this model.

It also follows from Tables I and II that even the model with a target level is inconsistent with the seasonally unadjusted behavior of the petroleum industry, since inventories here covary negatively with next period's sales. Relative to the static policy, the optimal policy that supposedly was followed would have been expected to increase all the costs just noted, and the cost of being away from a target level as well. Thus, this data set is incompatible with the model, with or without a target level. For the remaining industries, (7.1) and (7.2) cannot be signed without the a_i's. Let us therefore turn to precise calculation of the inequalities.

In Tables III and IV are the a_i's for the models with and without a target level, respectively. Almost all of the parameter estimates are indeed positive. Consider the model without a target level first. With seasonally adjusted data 7 of 10 free signs on the a_i are correct, and with unadjusted the figure is 5 of 8. (The number of free signs is 10 and 8 rather than 15 and 12 because the normalization rule $a_1+(1+d)a_0=1$ constrains either a_0 or a_1 to be positive in each equation.) The comparable figures for the model with a target level are 13 of 15 and 9 of 12. Only two of the wrong-signed coefficients are significant at the .05 level (a_0 in the model with a target level, for both seasonally adjusted rubber and seasonally unadjusted aggregate non-durables). In

TABLE III

STRUCTURAL PARAMETERS, MODEL WITHOUT A TARGET LEVEL

	a_0	a_1	a_2	J
Seasonally adjusted				
Aggregate non-durables	.2430 (.0450)	.5150 (.0898)	.0130 (.0188)	12.74
Chemicals	.4054 (.0624)	.1913 (.1245)	.0160 (.0172)	12.92
Rubber	-.0499 (.1144)	1.0995 (.2282)	-.0083 (.0354)	7.91
Petroleum	.1392 (.0822)	.7222 (.1640)	.0418 (.0235)	7.42
Food	.3361 (.0582)	.3295 (.1161)	-.00002 (.0175)	6.61
Seasonally unadjusted				
Aggregate non-durables	-.1069 (.0929)	1.2132 (.1854)	-.0047 (.0276)	20.03
Chemicals	.3439 (.0845)	.3139 (.1686)	.0228 (.0202)	14.60
Rubber	.3873 (.1074)	.2274 (.2143)	-.0128 (.0270)	4.19
Petroleum	.3256 (.0554)	.3504 (.1105)	.0387 (.0127)	4.81

Notes:
1. Variables defined in text.
2. J distributed as chi-squared with four degrees of freedom, critical levels: 9.48 at .05, 13.28 at .01, 14.86 at .005.
3. Asymptotic standard errors in parentheses; standard error on $a_1=1-(1+d)a_0=1-1.995a_0$ calculated as 1.995 times the standard error on a_0.

TABLE IV

STRUCTURAL PARAMETERS, MODEL WITH A TARGET LEVEL

	a_0	a_1	a_2	a_3	J
Seasonally adjusted					
Aggregate non-durables	.1748 (.1110)	.6514 (.1326)	.0228 (.0232)	1.1240 (1.2216)	11.50
Chemicals	.3970 (.0667)	.2081 (.1331)	.0171 (.0177)	.3256 (.9832)	12.83
Rubber	-.2444 (.1184)	1.4878 (.2362)	.0199 (.0494)	4.5217 (10.6568)	1.79
Petroleum	.0772 (.0903)	.8461 (.1801)	.0367 (.0263)	1.1048 (1.0979)	3.50
Food	-.0782 (.2900)	1.1562 (.5786)	.0839 (.0868)	6.4670 (3.4104)	7.84
Seasonally unadjusted					
Aggregate non-durables	-.2427 (.1014)	1.4842 (.2023)	.0618 (.0470)	2.0831 (1.2243)	11.40
Chemicals	.1886 (.1389)	.6238 (.2771	.0391 (.0287)	.9111 (.8223)	13.43
Rubber	.2519 (.1711)	.4974 (.3413)	-.0098 (.0343)	-3.4508 (13.7254)	4.07
Petroleum	.2232 (.1018)	.5547 (.2031)	.0256 (.0206)	.8141 (1.2732)	3.58

Notes:
1.See Notes to Table III.
2.J distributed as chi-squared with three degrees of freedom, critical levels: 7.81 at .05, 11.34 at .01, 12.84 at .005.

most equations the production cost a_1 and the cost of changing production a_0 are significant. Somewhat puzzling is the imprecision of the estimates of the inventory holding cost a_2 and the target level parameter a_3, which are rarely significant at the .05 level. They are, however, almost always positive and stand here in about the same ratio to the other a_i and to each other as they did in Blanchard's (1982) estimates for the automobile industry.

However, these parameters, though positive and often significant, are not enough to make the model plausible. Results of the variance bounds test for the model without a target level are shown in Table V, and for the model with a target level in Table VI. It was noted above what would result for all data sets except possibly chemicals for the model without a target level, and for the seasonally unadjusted petroleum industry in the model with a target level. Thus it is no surprise that Tables V and VI indicate that (7.1) and (7.2) were violated for all of these. However, the inequality for the model without a target level was violated for chemicals as well, both seasonally adjusted and unadjusted, as was the inequality for the model with a target level for most of the data sets. Thus, the inequalities were violated in fourteen out of eighteen instances, and five of these were significant at the .05 level. The four data sets that did satisfy (7.2) did so insignificantly, with standard errors uniformly larger than the sizes of the inequality. Also, two of these four produced the only significantly wrong-signed parameter (a_0 for adjusted rubber and unadjusted aggregate non-durables). It therefore appears that the model does not well explain any of the data studied here.

Moreover, the increase in deviations of costs from trend

TABLE V

TEST STATISTICS, MODEL WITHOUT A TARGET LEVEL

	(1) Eq'n (7.1)	(2) Eq'n (12)	(3) 100 x (1)/(2)
Seasonally adjusted			
Aggregate non-durables	- 8 069 470 (6 667 880)	146 843 000	-5.50
Chemicals	-261 532 (291 782)	4 216 060	-6.20
Rubber	-162 262 (162 038)	5 016 600	-3.23
Petroleum	-279 009 (130 681)	3 470 090	-8.04
Food	-1 055 240 (492 792)	6 814 090	-15.49
Seasonally unadjusted			
Aggregate non-durables	-13 459 700 (7 388 180)	303 283 000	-4.44
Chemicals	-40 (291 782)	7 947 400	-0.00
Rubber	- 94 261 (99 273)	1 291 370	-7.30
Petroleum	-340 868 (115 259)	2 052 380	-16.61

Notes:
Units are billions of "normalized" 1972 dollars, obtained after normalizing one 1972 dollar to one unit of production and $a_0+a_1(1+d)$=one dollar.

TABLE VI

TEST STATISTICS, MODEL WITH A TARGET LEVEL

	(1) Eq'n (7.2)	(2) Eq'n (12)	(3) 100 x (1)/(2)
Seasonally adjusted			
Aggregate non-durables	-3 348 200 (6 922 430)	183 009 000	-1.83
Chemicals	- 237 244 (286 447)	4 494 560	-5.28
Rubber	169 912 (427 473)	8 113 910	2.09
Petroleum	-242 555 (125 596)	4 123 110	-5.88
Food	2 398 570 (3 028 000)	40 639 700	5.90
Seasonally unadjusted			
Aggregate non-durables	8 476 740 (17 958 000)	406 318 000	2.08
Chemicals	237 991 (704 357)	14 179 300	1.68
Rubber	- 37 183 (145 869)	1 702 580	-2.18
Petroleum	-367 652 (124 072)	2 945 460	-12.48

Notes:
See Notes in Table V.

attributable to the optimal policy would appear to be economically as well as statistically noticeable. Column (2) in Tables V and VI contain total deviations of costs from trend (again, in "normalized" dollars, $a_1+(1+d)a_0=1$):[13]

$$(12) \quad (1-d)^{-1}\left[a_0 var(\Delta Q)+a_1 var(Q)+a_2 var(H) - 2a_2a_3 cov(H,S_{+1})+a_2a_3^{"} var(S)\right]$$

When (7.1) or (7.2) is divided by (12) (possibly with $a_3=0$ imposed in (12)) the result is a dimensionless measure of the extent to which the optimal policy increases or decreases deviations of costs from trend relative to the static policy. This is shown in column 3 of Tables V and VI. The optimal policy <u>increases</u> expected cost deviations by up to 15 percent. If this increase were to be believed it would mean that deviations of profit margins from trend, and therefore presumably profit margins themselves, are substantially reduced.

It is of some interest to compare the results of the inequality tests with those of a common test of specification, the Hansen (1982) test of overidentifying restrictions that is reported in the columns labelled <u>J</u>. This was accepted at the .05 level for half the data sets (rubber, petroleum, food) and was rejected at the .05 but accepted at the .005 level for the three other data sets. This compares favorably with the tests of the overidentifying restrictions in the remaining chapters of this thesis, as well as in other recent studies (Blanchard (1982), Eichenbaum (1982)). Thus it is perhaps fair to say that this traditional test is supportive of the model. It would appear, then, that the variance bounds test was an essential element in assessing the reasonableness of this model for these data.

IV.CONCLUSIONS

This summarizes the basic conclusions of this chapter.

First, if the results of the previous section prove robust to, e.g., choice of sample period, it would seem that the linear quadratic model does a poor job of rationalizing these inventory data. In effect, a contradiction results when it is assumed that the actual inventory path chosen is the one that is optimal according to the model. The allegedly optimal path is dominated by a naive alternative path.

In the model without a target level for inventories, this follows simply because production is more variable than sales. Inventories therefore cannot be chosen simply to perform their putative function, smoothing production.[14] For the model with a target level, the matter is slightly more complicated. Inventories do usually track their target level (except in the petroleum industry). But this makes production and inventories so variable that inventories cannot be chosen as hypothesized, to minimize quadratic inventory, production and target-level costs.

This suggests two things. First inventories appear to serve some role other than production smoothing. Second, if this role results from backlog costs, it is inadequate to handle this by adding to a production smoothing model a simple cost of having inventories deviate from a target level.[15]

The remainder of this thesis attempts to model backlog costs in a more sophisticated fashion. The next chapter does so for the production to stock industries studied here, and chapter after that does so for some production to order industries.

FOOTNOTES

1.The four two digit industries were chemicals (SIC 28), rubber (SIC 30), and petroluem (SIC 29), both seasonally adjusted and unadjusted, and food (SIC 20) seasonally adjusted. These were the only non-durables industries for which data were readily available. The test was also applied to aggregate non-durables, both seasonally adjusted and unadjusted. Durables industries were excluded because of the "production to order" nature of their business, see chapter 4 and Abramowitz (1951) or Rowley and Trevedi (1975).

2.Eichenbaum's (1982) model does not fit precisely into this framework, even in its simplified version (1982, pp24-25). He includes the term "$a_4 w_{t+j} Q_{t+j}$" in the cost function, where w_{t+j} is the wage and a_4 another positive parameter. As will become apparent, the inequality to be derived here is apporximately correct if a_4(cov(w,Q)-cov(w,S)) is small compared to the other terms in the inequality.

3.This was done by by suitable adjustment for moving average errors and choice of instruments. This also allowed for the possibility that production is decided before sales are known, with sales expectational errors resulting in unintended inventory investment Thus all individual elements in the inequalities to be derived are consistently estimated if these shocks are present or if sales information arrives with a lag. But the inequalities omit any direct contribution to expected costs of either of these.

4.I thank both R. Shiller and L. Summers for (independently) suggesting to me the basic argument of this section.

5. Except if the firm has some market power and demand depends on actual or expected production or inventories. As far as I know, this assumption has never been made in this class of models.

6. Strictly speaking the alternative policy is not feasible if production takes place with a lag and inventories absorb sales expectational errors as in Blinder (1982). But even here the inequality about to be developed may be considered approximately correct if these errors are small relative to the size of the inventory stock, as seems plausible. See footnote 3.

7.This shows that if we ignore unintended investment (see footnote 3), Blinder (1981a) is incorrect in his hunch that var(Q)>var(S) is possible in his (1982) model under certain circumstances. This model was as in (1) except that $a_0=a_3=0$. Thus from (7.1) var(Q)<var(S) unambiguously.

8.To prevent confusion, it should be stressed that this alternative policy entails varying inventories from period to period if inventories display a time trend and/or deterministic seasonal variation. See Bertsekas (1976, pp191-2) for a definition of an "open loop" policy. Strictly speaking, setting $H_t^A=H_t^*$ is the open loop policy only if inventories are the only control.

9.An alternative method for calculating unadjusted constant dollar inventories would be to deflate book value inventories by the appropriate wholesale price index. Given the massive switch from FIFO to LIFO accounting in the 1970s, and cyclical differences in output price versus input cost (see Foss, et al. (n.d.)), this is likely to lead to estimates substantially inferior than those derived as described in the text.

10.This assumes $dp_{t+j}S_{t+j}/dH_t=0$. This is consistent with any linear quadratic inventory model that I am aware of, including not only those in which sales are exogenous (e.g. (Belsley (1969)) but also those in which they are jointly endogenous with inventories (Eichenbaum (1981), Blinder (1982)).

11.u_{1t} is MA(1) if it comes from purely expectational errors and production and sales decisions are made simultaneously. But if unobserved shocks are present, or if production is decided before sales are known, the errors are MA(2). It thus seemed desirable to adopt a procedure that was consistent even in the presence of these errors. In addition, it should be noted that monthly d=.999 and d=.995 were also tried, with results virtually identical to those reported in the Tables for d=.995 (e.g., the estimated a_i's were identical to the first two decimal places).

12.This is not to say that the model (1) implies that inventories and sales follow such an autoregression. In general, however, it does imply that they follow a bivariate ARMA process of some order (Hansen and Sargent (1981)). The order of the ARMA process cannot be tied down without making auxiliary assumptions that we have been at pains to avoid making. The AR process assumed in the text should be considered an approximation to this ARMA process.

13.S. Fischer has pointed out to me that if another first order condition which contained p_t were available, the dollar value of the a_i could be determined. But this would probably require that an assumption be made about market structure, so that (1) could be differentiated with respect to p_t and/or S_t. This seems undesireable.

14.This has been conjectured by Blinder (1981b) and Blanchard (1982).

15.For a criticism of the use of this simple cost in a completely different inventory model, see Carlson and Wehrs (1974) and Feldstein and Auerbach (1976).

APPENDIX

This discusses:

1.The derivation of inequalities (7.1) and (7.2) when trend is present.

2.Verification that under suitable conditions (7.1) and (7.2) hold at the aggregate industry level if they hold for individual firms.

3.Estimation of the variances and covariances from the Yule-Walker equation.

4.The structure of the variance-covariance matrix of the three-equation system consisting of (10) and (11).

1.Trend

This will derive (7.2), the inequality when a target level is present. Inequality (7.1) follows by setting $a_3=0$ below.

When trend is present, expected costs are:

$$
\begin{aligned}
\text{(A1)}\ \max E_0\sum_{t=0}^{\infty} d_1^t]([p_tS_t \\
& - d_2^t[a_0(\Delta Q_t)^2+a_1Q_t^2+a_2(H_t-a_3S_{t+1})^2 \\
& - [\ d_3^t(a_{100}+\sum_1^{11}a_{10j}M_j)\ Q_t \\
& \quad + d_4^t(a_{110}+\sum_1^{11}a_{11j}M_j)Q_t \\
& \quad + d_5^t(a_{120}+\sum_1^{11}a_{12j}M_j)(H_t-a_3S_{t+1})\ ] \\
& - [a_{20}t\Delta Q_t + a_{21}tQ_t + a_{22}t(H_t-a_3S_{t+1})] \\
& - [F_{0t} + F_{1t} + F_{2t}\ )
\end{aligned}
$$

The a_{ij}, i=10,11,12, j=0,....11, represent traditional linear costs and the M_j's are seasonal dummies (M_j=1 in month j for seasonally unadjusted data, M_j=0 in all months for seasonally adjusted data); the d_i,

$i=3,4,5$, technological progress ($0<d_i\leq 1$), the a_i, $i=20,21,22$, trends induced by technological progress and/or secular growth in exogenous variables, and the F_{it}, fixed costs associated with each of the three types of costs.

We wish to establish that $EV_0^*-EV_0^A$ is as on the right hand side of (7.2), where EV_0^* is (A1) evaluated at the optimum and EV_0^A is (A1) evaluated at the alternative policy $H_t^A=EH_t^*$, $p_t^A=p_t^*$, $S_t^A=S_t^*$, $Q_t^A=S_t^*+E\Delta H_t^*$. It immediately follows that $EH_t^A=EH_t^*$, $Ep_t^AS_t^A=Ep_t^*S_t^*$, $EQ_t^A=ES_t^*+E\Delta H_t^*=EQ_t^*$. Thus, for both policies, revenue and the elements of costs contained in the last five lines of (A1) are the same, so

(A2) $EV_0^*-EV_0^A =$

$$\sum_{t=0}^{\infty} d^t \Big[a_0(E(\Delta Q_t^A)^2-E(\Delta Q_t^*)^2)+a_1(E(Q_t^A)^2-E(Q_t^*)^2) + a_2(E(H_t^A-a_3S_{t+1}^A)^2-E(H_t^*-a_3S_{t+1}^*)^2) \Big]$$

(Recall that $d=d_1d_2$.) By definition, for any random variable X_t, $EX_t^2=(EX_t)^2+var(X_t)$, $var(X_t)=E(X_t-EX_t)^2$. Thus, since the unconditional expectations of production, sales and inventories are the same for both policies, we have

(A3) $EV_0^*-EV_0^A =$

$$\sum_{t=0}^{\infty} d^t\Big[a_0(var(\Delta Q_t^A)-var(\Delta Q_t^*))+a_1(var(Q_t^A)-var(Q_t^*)) + a_2((var(H_t^A-a_3S_{t+1}^A)-var(H_t^*-a_3S_{t+1}^*)) \Big]$$

Finally, $var(\Delta Q_t^A) = E[\Delta S_t^*+E\Delta^2H_t^* - E(\Delta S_t^*+E\Delta^2H_t^*)]^2 = E[\Delta S_t^*-E\Delta S_t^*]^2 = var(\ S_t^*)$, and, similarly, $var(Q_t^A)=var(S_t^*)$, $var(H_t^A-a_3S_{t+1}^A)=var(-a_3S_{t+1}^*)$. Inequality (7.2) then follows directly.

2. Aggregation over firms

This section presents sufficient conditions for and (7.2) to hold for aggregate production, sales and inventories in an industry, given that it holds for each of the firms in the industry. (The argument for (7.1) follows by setting $a_3=0$ below.) These conditions are that the structural parameters representing the linear stochastic characteristics of forcing variables and technology are the same for all firms. As will become apparent, this will not restrict interfirm correlations of production, sales and inventories. These are arbitrary.

Let there be K firms, indexed by k, k=1,...,K. Thus H_{kt} is inventories of the k^{th} firm in period t, S_{kt} its sales, etc. Let X_{kt} be the (n x 1) vector of forcing variables facing the firm, with Wold representation $X_{kt}=B(L)e_{kt}$; $B(L) = \sum_{j=0}^{\infty} B_j L^j$, each B_j a (n x n) matrix and L the lag operator. (Example: n=1 and $X_{kt}=S_{kt}$ (Holt, et al. (1961)). Even in this case, n>1 is perhaps appropriate if variables other than past sales help predict future sales.) The e_{kt} may be arbitrarily correlated across firms ($Ee_{kt}e_{jt}$ arbitrary), but are uncorrelated across time periods ($Ee_{kt}e_{js}=0$ for $t\neq s$, all j,k). It should be pointed out that this allows perfect correlation ($e_{kt}=e_{jt}$) as will happen if, say, X_{kt} consists exclusively of common prices facing all firms. Note that the structural parameters B(L) are assumed to be the same for all firms.

Each firm chooses its endogenous variables by solving a set of first order conditions, subject to the law of motion of its forcing variables, $X_{kt}=B(L)e_{kt}$. One of these is the Euler equation (9):

(9) $E_t[\ a_0 d q_{kt+2} - (a_1 + a_0(1+d)) q_{kt+1} + a_0 q_{kt} + a_2 H_{kt} - a_2 a_3 S_{kt+1}\ ] = 0$

Note that the structural parameters d and the a_i's are assumed to be the same across all firms. Similarly, we assume that parameters comparable to d and the a_i's are the same for all firms in other first order conditions needed to solve (1) (e.g., in a first order condition derived by differentiating (1) with respect to sales S_{kt}).

The firm solves these first-order conditions to get its optimal feedback rules

$$(A2.1)\quad \begin{aligned} &H_{kt}=A_1(L)X_{kt}=A_1(L)B(L)e_{kt}=B_1(L)e_{kt} \\ &S_{kt}=B_2(L)e_{kt} \\ &Q_{kt}=(B_2(L)+(1-L)B_1(L))e_{kt}=B_3(L)e_{kt} \\ &\Delta S_{kt}=(1-L)B_2(L)e_{kt}=B_4(L)e_{kt} \\ &\Delta Q_{kt}=(1-L)B_3(L)e_{kt}=B_5(L)e_{kt} \end{aligned}$$

where, e.g., $B_1(L)=\sum_{i=0}^{\infty}B_{1i}L^i$, and degenerate cases such as $X_{kt} = S_{kt}$ are covered by allowing $B_2(L)=B(L)$. Note that the $B_i(L)$ are not subscripted by k. Given that the structural parameters of the problem are the same for all firms, this fact easily follows by examining the constructive solution techniques described in Hansen and Sargent (1980, 1981), Eichenbaum (1982), or a standard dynamic programming text such as Bertsekas (1976, ch. 3.1).

Now, it was shown in the paper that

$$(A2.2)\ 0 < (1-d)^{-1} \times [a_0(\mathrm{var}(S_k)-\mathrm{var}(\Delta Q_k)) + a_1(\mathrm{var}(S_k)-\mathrm{var}(Q_k)) - a_2\mathrm{var}(H_k) + 2\, a_2a_3\mathrm{cov}(H_{kt},S_{kt+1})]$$

Let $V_k=Ee_{tk}e_{tk}'$. Equation (A2.2) may be written in terms of the B_{ij}, i=1,...,5 as

$$(A2.3)\ 0 < (1-d)^{-1} \times [a_0(\sum_j B_{4j}V_kB_{4j}'-\sum_j B_{5j}V_kB_{5j}') + a_1(\sum_j B_{2j}V_kB_{2j}'-\sum_j B_{3j}V_kB_{3j}') - a_2(\sum_j B_{1j}V_kB_{1j}') + 2a_2a_3(\sum_j B_{1j}V_kB_{2j+1}')]$$

Note that the argument in the text has shown that (A2.2) holds for arbitrary positive definite V_k. That is, nowhere did that argument constrain the elements of V_k relative to one another or to any of the structural parameters.

Finally, let variables without subscripts denote market aggregates, e.g., $e_t=\sum_{k=1}^{K} e_{kt}$. Also let $V=Ee_te_t'$. Then we have from (A2.1) that

$$
\begin{aligned}
H_t &= B_1(L)e_t\\
S_t &= B_2(L)e_t\\
Q_t &= B_3(L)e_t\\
\Delta S_t &= B_4(L)e_t\\
\Delta Q_t &= B_5(L)e_t
\end{aligned}
$$

==>

$$
(1-d)^{-1}\,[a_0(var(\Delta S)-var(\Delta Q)) + a_1(var(S)-var(Q)) - a_2 var(H) + 2\,a_2a_3cov(H_t,S_{t+1})]
$$

$$
= (1-d)^{-1}\,[a_0(\sum_j B_{4j}VB_{4j}'-\sum_j B_{5j}VB_{5j}') + a_1(\sum_j B_{2j}VB_{2j}'-\sum_j B_{3j}VB_{3j}') - a_2(\sum_j B_{1j}VB_{1j}') + 2a_2a_3(\sum_j B_{1j}VB_{2j+1}')\,]
$$

$$
> 0
$$

The last inequality holds because, as noted, we have proved that (A2.3) holds for an arbitrary variance-covariance matrix V_k. In particular, it holds for $V_k=V$.

3.Variances and Covariances

Variances and covariances were calculated by solving the Yule-Walker equations in the iterative manner suggested by Anderson (1971, p177). The pair of equations (11) may be written in quasi-first order form as $X_t=BX_{t-1}+U_t$, where X_t and U_t are (6 x 1), $X_t=(H_t,H_{t-1},H_{t-2},S_t,S_{t-1},S_{t-2})'$, $U_t=(u_{2t},0,0,u_{3t},0,0)'$, and the (6 x 6) matrix B is

$$
\begin{matrix}
\phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16}\\
1 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0\\
\phi_{21} & \phi_{22} & \phi_{23} & \phi_{23} & \phi_{25} & \phi_{26}\\
0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0
\end{matrix}
$$

Let $C=EX_tX_t'$, $D=EU_tU_t'$. It follows then (Anderson, (1971), p182) that $C=BCB'+D$. After the parameters in B were obtained as described in the next section, the elements of C were found numerically. An initial guess of C=D was made and C was updated iteratively by $C_j = BC_{j-1}B' + D$ until the change in each of the elements of C was acceptably small. All of the second moments needed for (7) may be calculated from the elements of C, e.g., $var(Q)=var(S)+2cov(S,H)-2cov(S,H_{-1})+2var(H)-2cov(H,H_{-1}) = C(3,4)+2C(1,4)-2C(2,4)+2C(1,1)-2C(1,2)$.

4. Variance-covariance matrix of parameter estimates

The reader is warned that the notation here does not precisely match that in the text.

We have a trivariate system

$$\underset{(1\times 1)}{\tilde{y}_{1t+1}} = \underset{(1\times n)}{\tilde{X}'_{1t+1}} \underset{(n\times 1)}{b_1} + \underset{(1\times 1)}{\tilde{u}_{1t+1}}$$

$$\underset{(1\times 1)}{y_{2t}} = \underset{(1\times k)}{Z'_t} \underset{(k\times 1)}{b_2} + \underset{(1\times 1)}{u_{2t}}$$

$$\underset{(1\times 1)}{y_{3t}} = \underset{(1\times k)}{Z'_t} \underset{(k\times 1)}{b_3} + \underset{(1\times 1)}{u_{3t}}$$

The first equation is the Euler equation (10), and n=4 or 5 depending on whether or not a target level is allowed. The second and third are the bivariate (H,S) autoregression (11), and k=8. $\tilde{u}_{1t+1}$ is MA(2), $\tilde{u}_{1t+1} = (q_{t+1}-E_t q_{t+1}) - a_0 d(q_{t+2}-E_t q_{t+2}) + a_2 a_3(S_{t+1}-E_t S_{t+1}) - a_2(H_t - E_t H_t) +$ period t cost shocks. (The last two terms may be present as described in on page 5 and footnote 3.) $\tilde{u}_{1t+1}$ thus contains expectational errors and/or cost shocks occuring in periods t+2, t+1 and t, and so is correlated with all the right hand side variables in (10). u_{2t} and u_{3t} are iid, but are correlated with $\tilde{u}_{1t+1}$, $\tilde{u}_{1t}$ and $\tilde{u}_{1t-1}$.

The Euler equation is led one period relative to the autoregression equations to make the right-hand side variables in the latter legitimate instruments in the former. That is, H_{t-1} and S_{t-1} are elements of Z_t, and are correlated with $\tilde{u}_{1t}$ since part of u_{1t} reflects period t-1 expectational errors and/or cost shocks; they are not, however, correlated with $\tilde{u}_{1t+1}$.

Let $y_{1t}=\tilde{y}_{1t+1}$, $X_{1t}=\tilde{X}_{1t+1}$, and $u_{1t}=\tilde{u}_{1t+1}$. Stack the three equations as

$$\underset{(T\times 1)}{y_1} = \underset{(T\times n)}{X}\underset{(n\times 1)}{b_1} + \underset{(T\times 1)}{u_1}$$

$$\underset{(T\times 1)}{y_2} = \underset{(T\times k)}{Z}\underset{(k\times 1)}{b_2} + \underset{(T\times 1)}{u_2}$$

$$\underset{(T\times 1)}{y_3} = \underset{(T\times k)}{X}\underset{(k\times 1)}{b_3} + \underset{(T\times 1)}{u_3}$$

(Strictly speaking these matrices should be subscripted by the number of observations T. This additional subscript is omitted here and in subsequent definitions to keep the notation uncluttered.) Let $V_{1j}=Eu_1u_j'$, $j=1,2,3$, with estimate $\hat{V}_{1j}$ calculated from residuals from 2SLS. The V_{1j} are (T x T) matrices with zeroes everywhere except on the diagonal and first two off-diagonal bands. Let $A=\text{plim}\,(T^{-1}X'Z)(T^{-1}Z'V_{11}Z)^{-1}$ = Hansen's (1982) (n x k) optimal weighting matrix for 2SLS solution of the Euler equation, with estimate $\hat{A}=X'Z(Z'\hat{V}_{11}Z)^{-1}$; let $K=2k+n$ = number of regression coefficients; and let $\sigma_{22}=Eu_{2t}^2$, $\sigma_{23}=Eu_{2t}u_{3t}$, $\sigma_{33}=Eu_{3t}^2$. The estimate of the ((K+3) x 1) parameter vector $\hat{\theta} = (\hat{b}_1',\hat{b}_2',\hat{b}_3',\hat{\sigma}_{22},\hat{\sigma}_{23},\hat{\sigma}_{33})$ was calculated by solving the orthogonality conditions

(A4)

$$0 = T^{-1}\Sigma h_t(\hat{\theta}) = \begin{bmatrix} T^{-1}\Sigma \hat{A} Z_t(y_{1t}-X_t'\hat{b}_1) \\ T^{-1}\Sigma Z_t(y_{2t}-Z_t'\hat{b}_2) \\ T^{-1}\Sigma Z_t(y_{3t}-Z_t'\hat{b}_3) \\ \hat{\sigma}_{22}-T^{-1}\Sigma(y_{2t}-Z_t'\hat{b}_2)^2 \\ \hat{\sigma}_{23}-T^{-1}\Sigma(y_{2t}-Z_t'\hat{b}_2)(y_{3t}-Z\hat{b}_3) \\ \hat{\sigma}_{33}-T^{-1}\Sigma(y_{3t}-Z_t'\hat{b}_3)^2 \end{bmatrix}$$

$$= \begin{bmatrix} T^{-1}\hat{A}\, Z'(y_1-X\hat{b}_1) \\ T^{-1}Z'(y_2-Z\hat{b}_2) \\ T^{-1}Z'(y_3-Z\hat{b}_3) \\ \hat{\sigma}_{22}-T^{-1}\Sigma(y_{2t}-Z_t'\hat{b}_2)^2 \\ \hat{\sigma}_{23}-T^{-1}\Sigma(y_{2t}-Z_t'\hat{b}_2)(y_{3t}-Z_t'\hat{b}_3) \\ \hat{\sigma}_{33}-T^{-1}\Sigma(y_{3t}-Z_t'\hat{b}_3)^2 \end{bmatrix},$$

(In this section all summations run from 1 to T unless otherwise noted.) Thus $\hat{b}_1=(\hat{A}\, Z'X)^{-1}\hat{A}\, Z'y_1$ as in Hansen and Singleton (1982), $\hat{b}_2$ and $\hat{b}_3$ are OLS estimates, and the $\hat{\sigma}_{ij}$ are solved from sample moments. These orthogonality conditions were chosen in part because they force sample moments to asymptotically equal population moments. That is, for each t, $Eh_t(\theta)|_{\theta^*} = 0$, where θ^* is the true but unknown θ. Hansen (1982,pp1038ff) establishes that $T^{1/2}(\theta - \theta^*)$ therefore is asymptotically normal with covariance matrix $V = (\text{plim } T^{-1}\, \Sigma h_{t\theta})^{-1}S(\text{plim } T^{-1}\, \Sigma h_{t\theta}')^{-1}$, where $h_{t\theta}$ is the ((K+3) by (K+3)) matrix of derivatives of h_t with respect to θ and $S=\sum_{j=-\infty}^{\infty} Eh_th_{t-j}' = \sum_{j=-2}^{2} Eh_th_{t-j}'$ (the summation stops at 2 because $Eh_th_{t-j}'=0$ for $|j|>2$). We will first derive $h_{t\theta}$ and then S.

Differentiating (A4) to obtain $h_{t\theta}$ gives $T^{-1}\Sigma h_{t\theta}$ =

$$\left[\begin{array}{ccc|ccc} -T^{-1}AZ'X & 0 & 0 & 0 & 0 & 0 \\ 0 & -T^{-1}Z'Z & 0 & 0 & 0 & 0 \\ 0 & 0 & -T^{-1}Z'Z & 0 & 0 & 0 \\ \hline 0 & 2T^{-1}\Sigma Z_t(y_{2t}-Z_tb_2) & 0 & 1 & 0 & 0 \\ 0 & -T^{-1}\Sigma Z_t(y_{3t}-Z_tb_3) & -T^{-1}\Sigma Z_t(y_{2t}-Z_tb_2) & 0 & 1 & 0 \\ 0 & 0 & 2T^{-1}\Sigma Z_t(y_{3t}-Z_tb_3) & 0 & 0 & 1 \end{array}\right]$$

Since the Z_t are instruments, the (3 x K) lower left hand block plims to zero and thus $(\text{plim } T^{-1}\Sigma h_{t\theta})^{-1}$ =

$$\text{plim} \left[\begin{array}{ccc|c} -(T^{-1}AZ'X)^{-1} & 0 & 0 & \\ 0 & -(T^{-1}Z'Z)^{-1} & 0 & 0 \\ 0 & 0 & -(T^{-1}Z'Z)^{-1} & \\ \hline & 0 & & I_3 \end{array}\right]$$

Partition S as

$$\left[\begin{array}{cc} \underset{(K \times K)}{S_{11}} & \underset{(K \times 3)}{S_{12}} \\ \underset{(3 \times K)}{S_{12}'} & \underset{(3 \times 3)}{S_{22}} \end{array}\right]$$

Consider first S_{11}. From (A4) and the definition of S, the (n x n) block in the upper left hand corner of S_{11} is $\sum_{j=-2}^{2} EAZ_tu_{1t}u_{1t-j}Z_{t-j}'A'$ =

$\sum_{j=-2}^{2} EAZ_tZ'_{t-j}A'Eu_{1t}u_{1t-j} = \text{plim } T^{-1}AZ'V_{11}ZA' = \text{plim } T^{-1}AZ'X$ (the last equality follows from the definition of A). Similar algebra applied to the other elements of S_{11} yields

$$S_{11} = \text{plim } T^{-1} \begin{bmatrix} AZ'X & AZ'V_{12}Z & AZ'V_{13}Z \\ & \sigma_{22}Z'Z & \sigma_{23}Z'Z \\ & & \sigma_{33}Z'Z \end{bmatrix}$$

To evaluate S_{12}, assume that random variables assumed to be uncorrelated are independent as well. Specifically, assume that the Z_t are independent of u_{it-j} for $j\leq 0$, i=1,2,3, and that u_{2t-j} and u_{3t-j} are independent of u_{1t} for j>0. (Recall that since $u_{1t}=\tilde{u}_{1t+1}$, by assumptions made above u_{2t-j} and u_{3t-j} are correlated with u_{1t} only for j=0,-1,-2.) We will show that it then follows that $S_{12}=0$. Now, S_{12} is

$$\sum_{j=-2}^{2} E \begin{bmatrix} AZ_tu_{1t}(\sigma_{22}-u_{2t-j}^2) & AZ_tu_{1t}(\sigma_{23}-u_{2t-j}u_{3t-j}) & AZ_tu_{1t}(\sigma_{33}-u_{3t-j}^2) \\ Z_tu_{2t}(\sigma_{22}-u_{2t-j}^2) & Z_tu_{2t}(\sigma_{23}-u_{2t-j}u_{3t-j}) & Z_tu_{2t}(\sigma_{33}-u_{3t-j}^2) \\ Z_tu_{3t}(\sigma_{22}-u_{2t-j}^2) & Z_tu_{3t}(\sigma_{23}-u_{2t-j}u_{3t-j}) & Z_tu_{3t}(\sigma_{33}-u_{3t-j}^2) \end{bmatrix}$$

Consider the matrix in the upper left-hand corner. $AEZ_tu_{1t}(\sigma_{22}-u_{2t-j}^2) = A\sigma_{22}EZ_tEu_{1t} - AEZ_tu_{1t}u_{2t-j}^2 = -AEZ_tu_{1t}u_{2t-j}^2$. For $j\leq 0$ Z_t is independent of $u_{1t}u_{2t-j}^2$, so $EZ_tu_{1t}u_{2t-j}^2 = EZ_tEu_{1t}u_{2t-j}^2 = 0 \times Eu_{1t}u_{2t-j}^2 = 0$. (Recall that the Z_t have zero mean by construction.) For j>0, u_{1t} is independent of u_{2t-j} and Z_t and so $EZ_tu_{1t}u_{2t-j}^2 = Eu_{1t}EZ_tu_{2t-j}^2 = 0 \times EZ_tu_{2t-j}^2 = 0$. A similar argument shows that the remaining matrices in S_{12} are zero.

Finally, let us consider S_{22}. It is the sum from j=-2 to 2 of the expectation of the following matrix:

$$\begin{bmatrix} u_{2t}^2 u_{2t-j}^2 - \sigma_{22}^2 & u_{2t}^2 u_{2t-j} u_{3t-j} - \sigma_{22}\sigma_{23} & u_{2t}^2 u_{3t-j}^2 - \sigma_{22}\sigma_{33} \\ u_{2t}u_{3t}u_{2t-j}^2 - \sigma_{22}\sigma_{23} & u_{2t}u_{3t}u_{2t-j}u_{3t-j} - \sigma_{23}^2 & u_{2t}u_{3t}u_{3t-j}^2 - \sigma_{33}\sigma_{23} \\ u_{3t}^2 u_{2t-j}^2 - \sigma_{22}\sigma_{23} & u_{3t}^2 u_{2t-j}u_{3t-j} - \sigma_{23}\sigma_{33} & u_{3t}^2 u_{3t-j}^2 - \sigma_{33}^2 \end{bmatrix}$$

These expectations were calculated from sample moments from the residuals, with the fact that they are zero for j≠0 imposed. Thus, the element in the upper left corner, for example, was calculated as $T^{-1}\Sigma u_{2t}^4 - (T^{-1}\Sigma u_{2t}^2)^2$. The other elements were calculated analagously.

Thus, the asymptotic variance of $\hat{\theta}$ is

$$V = \begin{bmatrix} V_{11} & 0 \\ 0 & S_{22} \end{bmatrix}$$

where S_{22} is defined above and the upper triangle of the symmetric matrix V_{11} is

$$\begin{bmatrix} V_{111} & V_{112} & V_{113} \\ & V_{114} & V_{115} \\ & & V_{116} \end{bmatrix}$$

where

$$V_{111} = \text{plim } (T^{-1}AZ'X)^{-1}$$

$$V_{112} = \text{plim } [(T^{-1}AZ'X)^{-1}(T^{-1}AZ'V_{12}Z')(T^{-1}Z'Z)]$$

$$V_{113} = \text{plim}\ (T^{-1}AZ'X)^{-1}(T^{-1}AZ'V_{13}Z')(T^{-1}Z'Z)$$

$$V_{114} = \sigma_{22}(\text{plim}\ T^{-1}Z'Z)^{-1}$$

$$V_{115} = \sigma_{23}(\text{plim}\ T^{-1}Z'Z)^{-1}$$

$$V_{116} = \sigma_{33}(\text{plim}\ T^{-1}Z'Z)^{-1}$$

CHAPTER III:

BACKLOG COSTS IN PRODUCTION TO STOCK INDUSTRIES: SOME EMPIRICAL ESTIMATES

The distinction between "production to stock" and "production to order" industries was first noted by Abramowitz (1951), and has been central to much theoretical and empirical work on manufacturing behavior (e.g., Belsley (1969), Maccini (1973)). Production to stock industries tend to be those whose output is homogeneous and easily stored. Firms in these industries ordinarily produce in advance of receipt of orders, store the output in a stock of finished goods inventories, and sell to their customers directly from this pre-existing stock. Examples are petroleum and rubber. Production to order industries, by contrast, tend to be those whose output is more or less tailored to the individual customer and/or is costly to store. Firms in these industries ordinarily wait for customer orders to be placed before completing production, working off a backlog of customer orders already received. Examples are airplanes and mainframe computers.[1]

Backlogs (i.e., unfilled orders, or orders received for items yet to be produced or shipped) therefore presumably tend to be small in production to stock industries, although data apparently are not available. The Department of Commerce, for example, does not even publish data on backlogs in industries that produce primarily to stock.[2] But the conventional wisdom, presumably correct, that backlogs are empirically small, has led many investigators to conclude that these have no significant role in explaining production and inventory behavior.

Some recent theoretical and empirical work, generalizing and extending the production smoothing model originated by Holt et al. (1961), provides an example (Belsley (1969), Blinder (1982), Eichenbaum (1982)).[3] In one recent theoretical exposition of the model (Blinder (1982)), it is argued that finished goods inventories are held solely to cut the costs of adjusting production to meet randomly varying sales. They should be built up in periods when sales are low and drawn down when sales are high, with the exact pattern determined by the relative costs of production and of holding inventories. Backlogs are mentioned in passing as negative inventories, and basic questions such as why inventories in production to stock industries are positive on average are not answered. Moreover, production smoothing appears not to be the sole motive for holding inventories, since, as shown in the previous chapter, production is more volatile than sales.

The question, then, is what is making production more volatile than sales. Using the identity production = sales + change in finished goods inventories, the variance of production can be broken down as var (production) = var (sales) + var (change in inventories) + 2 x cov(sales, change in inventories). A model that allows a positive covariance between sales and inventory investment thus is compatible with production being more volatile than sales. As noted in the previous chapter, the production smoothing model has been extended to allow just such a covariance. Some authors have replaced the inventory cost, $a_2H_t^2$, with a cost of having inventories deviate from a target level, $a_2(H_t-a_3S_{t+1})^2$, where H_t is period t finished goods inventories,

S_{t+1} is period t+1 sales, $a_3>0$, and a_3S_{t+1} is the target level (Holt et al. (1961), Blanchard (1982), Eichenbaum (1982)).[4] This quite directly makes the model predict that inventories and sales will move together, but its underlying economic rationale is unclear. Eichenbaum (1982,p8), for example, says he includes this term to "capture...the view that inventories are held for reasons... which are related to expected sales" but otherwise leaves the term unexplained. Often the term is assumed to reflect both inventory carrying costs on the one hand and stockout or backlog costs on the other (e.g., Blanchard (1982,p21)).

The only thoroughly worked out rationale for the notion that the cost of deviating from a target level captures these two costs appears to be in Holt et al. (1961, ch. 11). They argue that the cost of deviating is an approximation to the sum of two distinct underlying costs, a backlog cost and an inventory cost, and formally derive the relationship between the cost of being away from a target level and the two underlying costs. The former cost, however, in general is time varying, even if the underlying costs are fixed, and is a complicated function of, among other things, the cumulative distribution function of sales. The parameters appearing in the simple time invariant cost of being away from a target level should therefore not be considered structural, as has been noted by Blanchard (1982,p21).

In addition, since the relationship between the underlying inventory and backlog costs on the one hand and the time invariant cost of being away from a target level on the other are unspecified, it is difficult to decide what are reasonable values of estimated parameters. For example, as noted in the previous chapter, in Blanchard (1982, p41), the key parameter a_3 determining the target level varies from a low of about one to a high of about seventeen over his ten data sets. Still worse, it appears that the estimated parameters cannot be used to deduce even approximately the underlying inventory and backlog costs; this would at least indirectly put a bound on what values of the target level are reasonable. This extended production smoothing model thus seems consistent with any explanation for why inventories in production to stock industries track sales, not just backlog costs.[5]

It is clear, however, that backlog costs themselves have the potential to explain the tendency of these inventories to track sales (although the work of Abel (1982) points out that this tendency follows only if the backlog or stockout costs are properly specified). Backlogs in production to stock industries indicate a temporary inability of the supplying firm to meet existing demand (Belsley (1969), Holt et al. (1961)). Insofar as it is standard industry practice to sell out of stock, without forcing customers to wait for delivery, an individual firm's backlog may cause it bad customer relations or lost future sales, and hence impose costs. Manufacturers will then tend to build up inventories when expected sales are high and will find smaller stockpiles satisfactory when expected sales are low, at least if production is decided before sales are known. This tendency of inventories to track sales may cause production to be more volatile than sales.

The present paper explores the idea that backlog costs may be important in production to stock industries. It formulates a rational expectations version of a production smoothing model that includes not only production and inventory costs, but an estimable backlog cost as well. It estimates and tests the model using data aggregated to the two digit SIC code level, as in the previous chapter. The method of estimation makes possible both recovery of structural parameters and testing of the model's overidentifying restriction. It is to be emphasized that a structural backlog cost is estimable, in contrast to the backlog or stockout cost sometimes argued to underlie the target level for inventories. Thus, it may be checked for plausiblity, allowing confirmation of the notion that it is the backlog costs and not some forces that are causing production to be volatile.

And the results are in fact encouraging. The implied values of structural parameters are plausible. They indicate that a given number of units backlogged costs the representative firm about one-tenth to one-half of what the same number of units produced costs and about two to eight times what the same number held in inventory for a month costs. Coefficients are usually significant. In particular, the backlog cost is almost always significantly greater than the inventory cost. This suggests that even in production to stock industries, in which backlogs are empirically small, backlog costs may be an important determinant of production and inventory behavior. Estimates derived from models that ignore these costs (e.g., Belsley (1969)) therefore may be seriously deficient.

However, tests of this paper's overidentifying restrictions almost always reject at decisive significance levels. For this reason, the estimates here should be interpreted cautiously.

The paper is organized as follows. Part II sets out the model, part III discusses estimation and empirical results, and part IV contains conclusions.

II. The Model

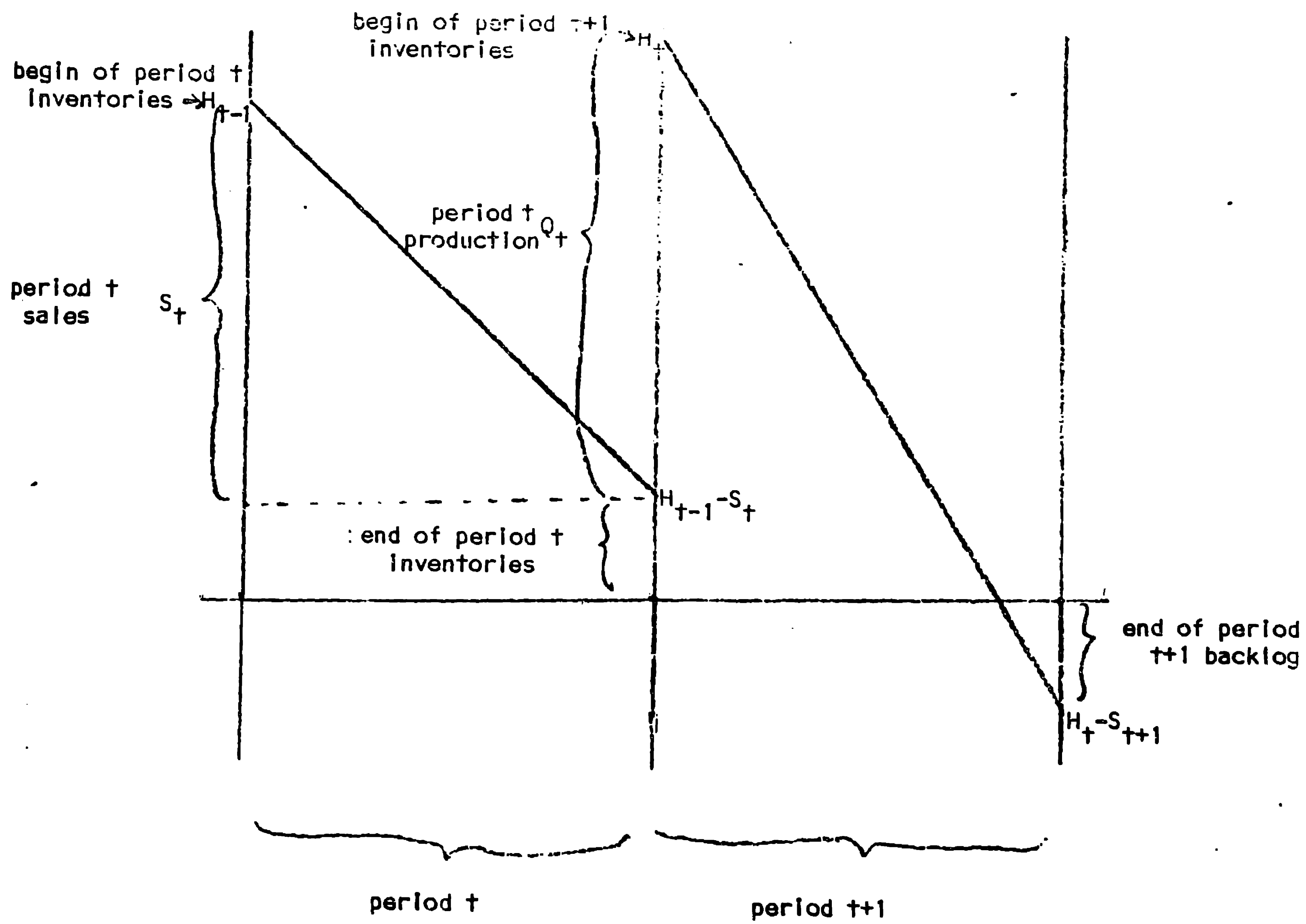
This section begins by explaining how production and sales are coordinated and what determines whether an order is backlogged or not. The formal model then follows. This section owes most to Holt et al. (1961).

The general environment is assumed to be as follows. N identical firms produce a single homogeneous good with a lag. At the beginning of each period, each firm initiates a production process that will not yield output Q_t until the end of the period. Throughout the period sales S_t arrive randomly. These may be exogenous in toto, as in Holt et al. (1961) or Blanchard (1982), or they may be beyond the firms control only with respect to a random shock, as in Blinder (1982).

The sales S_t streaming in are satisfied if possible from the beginning of period t stock of inventories H_{t-1}, and are backlogged if not. At the end of the period the firm gets its output Q_t and satisfies its backlog S_t-H_{t-1}, if in fact sales were larger than inventories and the firm backlogged some orders. The residual output $H_t=Q_t+H_{t-1}-S_t$, assumed positive, constitutes the stock of inventories at the beginning of the next period. See Figure I for a two period illustration of the case in which the firm has a backlog in period t+1 but not in period t.

Several clarifying comments are in order, before we turn to the model. First, the dating of variables may be confusing and deserves a word. The stock of inventories that exists on the borderline between period t and period t+1 is usually (e.g., in Chapter II) arbitrarily dated period t. To conform to the existing literature , this convention is followed here. In the present model, however, H_t is best understood as beginning of period t+1 inventories and thus is described as such.

FIGURE 1


begin of period t inventories → H_{t-1}
begin of period t+1 inventories → H_t
period t production Q_t
period t sales S_t
$H_{t-1}-S_t$
end of period t inventories
end of period t+1 backlog
H_t-S_{t+1}
period t
period t+1

This is to be contrasted with the end of period t+1 inventories H_t-S_{t+1}, the stock that remains after sales S_{t+1} but before production Q_{t+1} have arrives. Of course if $H_t-S_{t+1}<0$ and the firm backlogged some orders, end of period t+1 inventories are zero. Also, to conform to standard notation, when expectations are introduced below, "information available at time t" will be defined as information available when Q_t is selected. However, since the firm does not at this time know S_t, and therefore does not know H_t, this information set does not include S_t and H_t. Thus $E_tS_t \neq S_t$, $E_tH_t \neq H_t$ (E_t = expectations conditional on information available at time t).

Second, production is a choice variable, inventories are not. The firm can choose an expected but not an exact beginning or end of period stock, since sales are not known when production is decided. Actual inventories incorporate the negative of expectational errors in sales, increasing by one unit for each unit of an overestimate of sales. This is done to be consistent with the stylized fact that inventory accumulation or decumulation is often involuntary.

Third, the assumption that period t sales stream in continuously in advance of period t production is made to accord with the apparent fact that for many goods output and orders arrive asynchronously, with the output resulting from a given period's production decision indeed not available until after that period's orders have arrived (Holt et al. (1961, ch 11)).

Finally, it is assumed that the backlog is never carried from period to period, and the firm never simultaneously maintains a backlog and a stock of inventories. With considerable complication, these two could probably be weakened, and comparable statements about the firm's environment derived rather than asserted. There appears to be little doubt that they are accurate for production to stock industries: backlogs do appear, but do not persist for long (Abramowitz (1951, ch11), Belsley (1969, ch. 2), Holt et al. (1961, ch11)). It thus seems advisable for simplicity to just make these assumptions. Note that the implied sales-new order equality makes the recorded sales figures we see the only ones relevant to the firm's decision process.

Formally, the firm's problem is as follows. The firm maximizes the expected present discounted value of its real cash flow:

(1) $$\max E_0 \sum_{t=0}^{\infty} d_1^t \, (p_t S_t - d_2^t \, C_t)$$

$$\text{s.t. } Q_t = S_t + H_t - H_{t-1}$$

where

E_0	mathematical expectations, conditional on information available at time 0
d_1	fixed real discount rate, $0<d_1<1$
d_2	fixed rate of technological progress, $0<d_2<1$
p_t	real price in period t
S_t	units sold in period t
C_t	real costs in period t (detailed below)
Q_t	units produced in period t
H_t	units in inventory in period t

Revenue $p_t S_t$ will play no role in the bulk of this paper. It is present at this initial stage to emphasize the robustness of the model with respect to the revenue side of cash flow. The only restriction placed on revenue is that there be some forcing variables, beyond the firm's control, that make period t sales uncertain when period t production is decided. As noted above, sales may be exogenous in toto, or they may be dependent up to a shock on the firm's price (or perhaps it price relative to the average market price or to its competitors' prices). Beyond this, as in the previous chapter the model is consistent with a variety of market structures, choice variables and demand structures, so that as in the previous chapter, Summers (1981) criticisms of inventory models that ignore interactions between firms and their customers are not relevant here.

Costs C_t are the focus of the model. Per period costs have three quadratic components:

1)An inventory holding cost, representing, e.g., storage and handling charges. The period t+1 cost is assumed for simplicity to be a function of just the beginning of period t+1 stock, which is H_t, and the end of period t+1 stock, which is H_t-S_{t+1} if $H_t>S_{t+1}$, 0 if $H_t\leq S_{t+1}$. (These two sample points are used to give a measure of average inventory costs over the period.) Let ex-post inventory costs be c_{10} times the inventory level plus $c_0/2$ times its square. Period t+1 inventory costs C_{1t+1} expected as of time t are thus

$$(2)\ E_tC_{1t+1} = c_{10}E_tH_t + c_{10}\int_0^{Q_t+H_{t-1}} (Q_t+H_{t-1}-S_t-S_{t+1})dF_t(S_t+S_{t+1})$$

$$+ (c_0/2)E_tH_t^2$$

$$+ (c_0/2)\int_0^{Q_t+H_{t-1}} (Q_t+H_{t-1}-S_t-S_{t+1})^2dF_t(S_t+S_{t+1})$$

where $dF_t(S_t+S_{t+1})$ is the density of S_t+S_{t+1} conditional on information known at time t. (The expression for $E_tC(1t+i)$ for arbitrary $i\geq1$ generalizes (1) in the obvious fashion. Since it is messy, and is not needed to derive a first order condition for the firm, it is omitted. For the same reason, a general expression for E_tC_{2t+i} will be omitted below. Note that due to the production lag, production in period t first affects inventory costs in period t+1, not period t.)

2)A backlog cost, representing, e.g., charges incurred by bad customer relations. The period t+1 cost is assumed for simplicity to be a function of the end of period t+1 backlog, which is $S_{t+1}-H_t$ if $H_t\leq S_{t+1}$, 0 if $H_t<S_{t+1}$ Let ex-post backlog costs be c_{11} times the size of the backlog and $c_1/2$ times its square. Period t+1 backlog costs C_{2t+1} expected as of period t are thus

$$(3)\ E_tC_{2t+1} = c_{11} \int_{Q_t+H_{t-1}}^{\infty}(S_t+S_{t+1}-Q_t-H_{t-1})dF_t(S_t+S_{t+1})$$

$$+ \ (c_1/2) \int_{Q_t+H_{t-1}}^{\infty}(S_t+S_{t+1}-Q_t-H_{t-1})^2dF_t(S_t+S_{t+1})$$

3)A standard period t production cost C_{3t}:

$$(4)\ E_tC_{3t} = (a_1/2)(Q_t+f_1(t))^2 + f_{11}(t)Q_t$$

where $f_1(t)$ and $f_{11}(t)$ are linear deterministic functions. In the empirical work, they are assumed to depend on a constant, linear time trend, and, in the case of seasonally unadjusted data, on seasonal dummies as well.

Total expected per period costs are $E_tC_t = E_tC_{1t}+E_tC_{2t}+E_tC_{3t}$. The firm chooses Q_t and appropriate variables on the revenue side (see above) to maximize (1). It is assumed that in this model as in the linear one, a transversality condition and stationarity of forcing variables around a time trend ensure that the equilibrium vales of Q_t, S_t and H_t are also stationary around a time trend (see Hansen and Sargent (1980,1981)). This complicated equilibrium solution, however, is not needed here. A compact first order condition sufficient for understanding inventory behavior may be derived by noting that at the optimal production level Q_t^*, a unit increase in production this period offset by a unit decrease in production next period cannot decrease expected costs.[7] This marginal change in the production plan affects expected costs only via expected production costs in periods t and t+1, inventory costs in period t+1, and backlog costs in period t+1. Period t inventory and backlog costs are already foregone when Q_t is selected, and this change leaves all other expected costs unchanged as well.

More specifically, this implies

$$
\begin{aligned}
(5)\quad 0 = E_t \{ \quad & c_{10} + c_{10} \int_0^{Q_t+H_{t-1}} dF_t(S_t+S_{t+1}) \\
& + c_0 H_t + (c_0/2) \int_0^{Q_t+H_{t-1}} (Q_t+H_{t-1}-S_t-S_{t+1})\, dF_t(S_t+S_{t+1}) \\
& - c_{11} \int_{Q_t+H_{t-1}}^{\infty} dF_t(S_t+S_{t+1}) \\
& - (c_1/2) \int_{Q_t+H_{t-1}}^{\infty} (S_t+S_{t+1}-Q_t-H_{t-1})\, dF_t(S_t+S_{t+1}) \\
& - d a_1 Q_{t+1} + a_1 Q_t + (1-d) a_1 f_1(t) + (1-d) f_{11}(t) \quad \}
\end{aligned}
$$

The first two lines represent the expected change in inventory costs, the third, backlog costs, and the fourth, production costs. Each is obtained by differentiating the appropriate terms in the cost function. It has been implicitly assumed that it is possible to parameterize the problem so that the density of sales $dF_t(S_t+S_{t+1})$ does not change with production.

Define $q_t = dQ_t-Q_{t-1}$, $f(t) = (1-d)a_1f_1(t) + (1-d)f_{11}(t) + 2c_{10}$. Note that

$$c_{10}\int_0^{Q_t+H_{t-1}} dF_t(S_t+S_{t+1}) = c_{10} - c_{10}\int_{Q_t+H_{t-1}}^{\infty} dF_t(S_t+S_{t+1})$$

and

$$c_0\int_0^{Q_t+H_{t-1}} (Q_t+H_{t-1}-S_t-S_{t+1})dF_t(S_t+S_{t+1}) =$$

$$c_0E_t(Q_t+H_{t-1}-S_t-S_{t+1})$$

$$- c_0\int_{Q_t+H_{t-1}}^{\infty}(Q_t+H_{t-1}-S_t-S_{t+1})dF_t(S_t+S_{t+1})$$

$$= c_0E_t(H_t-S_{t+1})$$

$$+ c_0\int_{Q_t+H_{t-1}}^{\infty}(S_t+S_{t+1}-Q_t-H_{t-1})dF_t(S_t+S_{t+1})$$

Using the equalities just noted, (5) may be written

$$(6)\ 0 = E_t\ \{ -(c_{10}+c_{11})\int_{Q_t+H_{t-1}}^{\infty} dF_t(S_t+S_{t+1}) + c_0(2H_t-S_{t+1})$$

$$+(c_0-c_1)\int_{Q_t+H_{t-1}}^{\infty}(S_t+S_{t+1}-Q_t-H_{t-1})dF_t(S_t+S_{t+1})$$

$$-da_1q_t + f(t)\ \}$$

This is the equation that will be estimated. To understand it, it will be helpful to consider some special cases in which some of the parameters are zero.

Case 1 Consider first the case of no production costs, $a_1=f(t)=0$. and only linear inventory and backlog costs, $c_0=c_1=0$. Then $\int_{Q_t+H_{t-1}}^{\infty} dF_t(S_t+S_{t+1}) = 2c_{10}/(c_{10}+c_{11})$. Q_t is chosen so that the probability that $S_{t+1}+S_t>Q_t+H_{t-1}$ = probability that $S_{t+1} > H_t$ = probability of a backlog = (2 x inventory cost) / (inventory + backlog cost). This condition makes sense only if $c_{11}\geq c_{10}$-- i.e., only if the backlog cost is at least as large as the inventory cost. If $c_{11}<c_{10}$, a corner and not an interior solution to the maximization problem occurs and the optimal policy is to set inventories to zero to guarantee a backlog. In line with the assumption that backlogs are not carried from period to period (see above), it is henceforth assumed that $c_{11}\geq c_{10}$. It may then be seen from the above equation that the larger the backlog cost c_{11} relative to the inventory cost c_{10}, the lower the probability that the firm will backlog an order, and conversely. (The factor of two enters because at the margin an extra unit of inventories held for the entire period costs the firm twice: once because it is there at the beginning and once because it is there at the end. Backlogs, on the other hand, are assumed to never be carried through an entire period, and only cost the firm at the end. Again, this is intended to capture the fact that in production to stock industries, an item may stay in inventory for a considerable period of time, whereas backlogs usually are quickly cleared up.)

<u>Case 2</u> Now consider the case of no production costs, $a_1=f(t)=0$, and only quadratic inventory and backlog costs, $c_{10}=c_{11}=0$. Then

$$(7)\quad 0 = E_t$$

$$\{ 2c_0H_t - c_0S_{t+1} + (c_0-c_1) \int_{Q_t+H_{t-1}}^{\infty}(S_t+S_{t+1}-Q_t-H_{t-1})dF_t(S_t+S_{t+1})] \}$$

If the inventory cost c_0 equals the backlog cost c_1, expected inventories equal half of expected sales. If backlog costs c_1 are larger than inventory costs c_0, then expected inventories are set greater than half of next period's sales, and conversely if the backlog cost is lower.[8]

Finally, consider the case of no inventory or backlog costs, $c_0=c_1=c_{10}=c_{11}=0$, and only quadratic production costs, $f_{10}(t)=f_{11}(t)=0$. Then using $Q_{t+1} = d(H_{t+1}+S_{t+1}-H_t) - (H_t+S_t-H_{t-1})$, (6) becomes

$$(8)\quad 0 = a_1E_t\{ (1-L)(d-L)H_{t+1} + (d-L)S_{t+1} \}$$

where L is the lag operator, $LX_{t+1}=X_t$. Canceling the common lag factors and the non-zero scalar a_1 gives $E_t [(1-L)H_{t+1} + S_{t+1}] = 0$. Thus, on average, $H_{t+1}-H_t = -S_{t+1}$. Unsurprisingly, with no backlog or inventory costs, the optimal policy is to allow inventories to passively absorb sales, driving inventories monotonically downwards.

In general, of course, inventories evolve according to a complicated interaction of these three special cases. It is to be emphasized that the full model, with its attention to backlog costs and its assumption that production is decided before sales are known, does indeed suggest a tendency for inventories and sales to move together, and thus for production to be more volatile than sales. This follows immediately from the first two special cases above, in which inventories vary directly with sales. Thus, this model, in contrast to the pure production smoothing model (Blinder (1982)) is compatible with production being more volatile than sales.

We now turn to empirical results.

III.Empirical results

Estimation is described briefly before empirical results are presented. The data are described in the "Data" subsection of the previous chapter. The sample period is also the same as in the previous chapter, covering 1959:5 to 1980:10, with 1980:11 used for leads and 1959:2 to 1959:4 used for lags.

A.Estimation

After normalizing $a_1=1$ and imposing a monthly discount rate of .995 (corresponding annual figure is about 6 per cent), (6) may be written in regression format

$$(9) \quad q_t = f(t) - (c_{10}+c_{11}) \int_{Q_t+H_{t-1}}^{\infty} dF_t(S_t+S_{t+1}) + c_0 E_t(2H_t-S_{t+1}) + (c_0-c_1) \int_{Q_t+H_{t-1}}^{\infty} (S_t+S_{t+1}-Q_t-H_{t-1}) dF_t(S_t+S_{t+1}) + u_t$$

where $u_t = Q_{t+1} - E_tQ_{t+1}$ is an MA (1) error. This subsection discusses estimation of the coefficients on the integrals, a non-trivial issue since the integrals are unobserved variables. Two different methods were used. The first requires that the industry act as one firm, or, equivalently, that sales across all firms are perfectly correlated. The second does not, and is likely to be less sensitive to aggregation over firms and over time. Two variants of the second were tried, resulting in three sets of estimates for each data set. An explanation of the two methods is introduced with a brief discussion of the related and familiar technique used to estimate c_0, the coefficient on the unobserved variable $E_t(2H_t-S_{t+1})$. The reader uninterested in details may skip to the "Results" section below.

Estimation of c_0 is easily handled by McCallum's (1976) and Hansen and Singleton's (1982) instrumental variables procedures. The expected value was replaced by the actual ex-post value, $2H_t-S_{t+1}$, and then instrumented by variables known at time t or earlier and correlated with the ex-post value (instruments used included in particular suitably lagged values of H and S). Using the ex-post values as regressors add to the regression error the expectational error $-c_0[2H_t-S_{t+1} - E_t(2H_t-S_{t+1})]$. However, under rational expectations, this expectational error is orthogonal to anything known in period t or earlier, has unconditional expectation zero, is correlated with at most one of its own lags, and under suitable conditions is homoscedastic. (The error is correlated with itself lagged once since $S_{t+1}-E_tS_{t+1}$ is a function of sales innovations in both period t $(S_t-E_tS_t)$ and period t+1 $(S_{t+1}-E_{t+1}S_{t+1})$.) An instrumental variables procedure, with the appropriate adjustment to the covariance matrix to account for the MA(1) nature of the errors, delivers consistent parameter and covariance estimates.

The first method of handling the coefficients on the integrals is similar. This will be explained in detail for $\int_{Q_t+H_{t-1}}^{\infty} dF_t(S_t+S_{t+1})$; the argument for the other integral is analogous. Define the dichotomous random variable B_{t+1}, $B_{t+1}=0$ if $Q_t+H_{t-1}>S_{t+1}+S_t$, $B_{t+1}=1$ if $Q_t+H_{t-1}\leq S_{t+1}+S_t$. Thus B_{t+1} is one if the firm backlogged some orders in period t+1, zero if it did not. Note that

E_tB_{t+1} =

E_tB_{t+1} =

$1 \times$ (probability using period t information that

$Q_t+H_{t-1} \leq S_t+S_{t+1}$)

$+ 0 \times$ (probability using period t information that

$Q_t+H_{t-1}>S_{t+1}+S_t$)

$$= 1 \times \int_{Q_t+H_{t-1}}^{\infty} dF_t(S_t+S_{t+1})$$

Equation (6) thus is equivalent to a regression in which the regressor multiplied by c_0 is E_tB_{t+1}. Again we can replace E_tB_{t+1} by the actual ex-post value of B_t and instrument it (suitably lagged values of B_t are legitimate here). And again this adds an expectational error $+(c_{10}+c_{11})(B_{t+1}-E_tB_{t+1})$ to the regression error which again is orthogonal to anything known in period t or earlier, has unconditional expectation zero, and is correlated with only one lag of itself. This time, however, the expectational errors in general are conditionally heteroscedastic:

(11) $E_t(B_{t+1}-E_tB_{t+1})^2$ =

$(1-E_tB_{t+1})^2 \times$ (prob using period t information that $B_{t+1}=1$)

$+ (0-E_tB_{t+1})^2 \times$ (prob using period t information that $B_{t+1}=0$)

$= (1-E_tB_{t+1})^2 \times E_tB_{t+1} + (-E_tB_{t+1})^2 \times (1-E_tB_{t+1})$

$= (1-E_tB_{t+1})E_tB_{t+1}$,

the usual formula for the variance of a binomial random variable. Conditional heteroscedasticity results if E_tB_{t+1} is time varying. Empirically, it does appear to be time varying, since it is (positively) serially correlated. And theoretically, it perhaps can be expected to be so.[9] The conditional heteroscedasticity is handled with the heteroscedasticity consistent covariance matrix of White (1982) and Hansen and Singleton (1982).

The equation to be estimated by this first method thus is

$$\text{(9a)} \quad q_t = f(t) - (c_{10}+c_{11})B_{t+1} + c_0E_t(2H_t-S_{t+1}) + (c_0-c_1)(S_t+S_{t+1}-Q_t-H_{t-1})B_{t+1} + v_t$$

$v_t = u_t$ + expectational errrors, $v_t \sim MA(1)$, B_{t+1} defined in (10).

Two further problems with this estimation technique should be noted. First, these expectational errors may be unconditionally heteroscedastic as well, and thus there is a question as to the correctness of the standard formulas for the covariance matrix of the estimated parameters.[10]

Second, if $B_t=1$ or $B_t=0$ over the entire sample period, the matrix of right hand side variable in (9a) will not be of full rank and estimation will not be viable. (And even if B_t does vary, but only a little, so that there are relatively few ones or zeros, for numerical reasons multicollinearity may still be a problem.) According to the model, $B_t=1$ (i.e., $S_t+S_{t+1} > Q_t+H_{t-1}$) in all time periods implies that the endogenous probability of a backlog is so high that we need more time periods than we have yet observed to obtain $B_t=0$ in the sample even once (and conversely if $B_t=0$ in all time periods). Alternatively, it may mean that the decision period on production is less than the observation period (e.g., decisions are made more than once a month); if this were the case period t+1 production may be used to satisfy period t+1 sales and $S_t+S_{t+1} >$ Q_t+H_{t-1} need not imply backlogs. This suggests that it is desirable to use an alternative measure of demand pressure, not as sensitive to discrepancies between the decision and observation periods. In any case, an alternative meaasure was needed to obtain estimates from the food industry, since $B_t=1$ in all time periods for it.

In this second estimator, $\int_{Q_t+H_{t-1}}^{\infty} dF_t(S_t+S_{t+1})$ is proxied by E_tR_{1t+1} or E_tR_{2t+1}, and $\int_{Q_t+H_{t-1}}^{\infty}(S_t+S_{t+1}-Q_t-H_{t-1})dF_t(S_t+S_{t+1})$ by $(Q_t+H_{t-1})(1-E_tR_{1t+1})$ or $(Q_t+H_{t-1})(1-E_tR_{2t+1})$, where

(12) $R_{1t+1} = (S_t+S_{t+1}) \ / \ (S_t+S_{t+1}+Q_t+H_{t-1})$

$R_{2t+1} = (S_t+S_{t+1}) \ / \ (S_t+S_{t+1}+Q_t+H_{t-1}+Q_{t+1})$.

R_{2t+1} allows for the possibility that period t+1 production is used in part to satisfy sales within period t+1. Aggregate monthly production may then in part be used to satisfy aggregate monthly sales.

The basic justifications for using E_tR_{1t+1} and E_tR_{2t+1} to proxy $\int_{Q_t+H_{t-1}}^{\infty} dF_t(S_t+S_{t+1})$ are that they move in the same direction as the ratio of sales to the stock of goods available to satisfy sales, and thus seem well suited as a proxy for the type of demand pressure that leads to backlogs. In addition, they fall between zero and one and are based in information known at period t (as probabilities should).

The other regressor, $(Q_t+H_{t-1})(1-E_tR_{it+1}), i=1,2$, was suggested by the following reasoning. $\int_{Q_t+H_{t-1}}^{\infty}(S_t+S_{t+1}-Q_t-H_{t-1})dF_t(S_t+S_{t+1}) = E_t(S_t+S_{t+1} | S_t+S_{t+1}>Q_t+H_{t-1}) - (Q_t+H_{t-1}) \times \text{prob}(S_t+S_{t+1}>Q_t+H_{t-1}) \geq Q_t+H_{t-1} \times [1-\text{prob}(S_t+S_{t+1}>Q_t+H_{t-1})]$, approximately $(Q_t+H_{t-1}) \times$

$(1-E_tR_{it+1})$. The inequality in the preceding sentence suggests that the regressor is likely to be too low when E_tR_{1t+1} is used. This bias is perhaps offset when $E_tR_{2t+1} \geq E_tR_{1t+1}$ is used.

The coefficients on E_tR_{it+1} and $Q_t+H_{t-1}(1-E_tR_{it+1})$ again were estimated by replacing the regressors with their ex-post values and instrumenting. The equations estimated by this second method then, were:

(9b) $$q_t = f(t) - (c_{10}+c_{11})R_{1t+1} + c_0E_t(2H_t-S_{t+1}) + (c_0-c_1)(Q_t+H_{t-1})(1-R_{1t+1}) + v_t$$

(9c) $$q_t = f(t) - (c_{10}+c_{11})R_{2t+1} + c_0E_t(2H_t-S_{t+1}) + (c_0-c_1)(Q_t+H_{t-1})(1-R_{2t+1}) + v_t$$

$v_t = u_t$ + expectational errors, $v_t \sim MA(1)$, R_{1t+1} and R_{2t+1} defined in (12).

Let $X_t = B_t$, R_{1t+1} or R_{2t+1}. For all data sets but food, the ten instruments used were three lags apiece of inventories and sales, one lag of X_{t+1}, one lag of the regressor multiplied by c_0-c_1, and deterministic terms (constant, time trend, and, for seasonally unadjusted data, seasonal dummies). Numerical problems were encountered when using this instrument list to estimate parameters from the food industry. (The optimal weighting matrix, defined in Hansen (1982), was not positive definite.) This problem disappeared when estimates from the food industry were obtained with a six element instrument list, consisting of two lags each of sales and inventories, and deterministic terms.

B.Results

TABLE I

PARAMETER ESTIMATES, EQUATION 9.A

	(1) $c_{10}+c_{11}$	(2) c_0	(3) c_1	(4) c_0-c_1	(5) J
Seasonally adjusted					
Aggregate non-durables	-128.6	-.0559	.0761	-.1321	8.71
	(261.6)	(.0534)	(.0634)	(.0487)	
Chemicals	-3.0	-.0214	.1256	-.1470	6.85
	(56.9)	(.0235)	(.1844)	(.1887)	
Rubber	41.1	-.0444	.1634	.1189	9.65
	(71.1)	(.0534)	(.4617)	(.4435)	
Petroleum	-75.0	-.1049	.0883	-.1932	5.72
	(108.0)	(.0996)	(.0558)	(.0558)	
Seasonally unadjusted					
Aggregate non-durables	-490.7	-.0757	.2500	-.3257	2.46
	(1556.6)	(.0889)	(.3744)	(.4334)	
Chemicals	-51.7	-.0052	.2683	-.2735	4.12
	(147.3)	(.0480)	(.3432)	(.3407)	
Rubber	64.5	-.0965	.2402	-.3367	.86
	(140.5)	(.0971)	(.8215)	(.8330)	
Petroleum	-326.3	-.1718	.0759	-.2478	3.42
	(490.0)	(.1713)	(.1009)	(.2571)	

Notes:
1. Variables defined in text.
2. J distributed as chi-squared with five degrees of freedom, critical levels: 11.07 at .05, 15.08 at .01, 16.75 at .005.
3. Asymptotic standard errors in parentheses; standard error on column (3) = column (2) - column (4) calculated as $[\mathrm{var}(c_0)+\mathrm{var}(c_0-c_1)-2\mathrm{cov}(c_0,c_1))^{1/2}$.
4. Estimates for food not available, see text.

Table I contains the results for the first method of estimation, Tables IIA and IIB those for each of the two variants of the second method of estimation. Coefficients on deterministic terms are not reported, although all regressions run included constant and trend terms, and, for seasonally unadjusted data, seasonal dummies as well. Also some preliminary experimentation was done to see if the results were sensitive to the sample period, and they appeared not to be. Thus only the results over the entire sample are reported. The column labelled "J" contains Hansen's (1982) test of overidentifying restrictions. For all data sets except food, it is distributed as chi-squared with five degrees of freedom. (Five equals the number of instruments minus number of regressors, ten minus five for seasonally adjusted data and twenty one minus sixteen for unadjusted data). For food, it is distributed as chi-squared with one degree of freedom.

As may be seen from colmns (2) and (3) in Table I, in the first method only 7 out of 16 signs on the quadratic inventory and backlog cost parameters c_0 and c_1 are correct.[11] The seven all occurred on the quadratic backlog cost parameters c_1, while all eight of the quadratic inventory cost parameters c_0 were negative. None of these sixteen were significantly different from zero at the five per cent level. The backlog cost, however, was always larger than the inventory cost, though rarely signficantly so. The sum of the linear inventory and backlog cost, $c_{10}+c_{11}$, is generally wrong-signed and insignificant.

As noted, this first estimation method seems likely to be sensitive to aggregation. Let us therefore turn to the second method, which will be the focus of the rest of the discussion. Here,

TABLE IIA

PARAMETER ESTIMATES, EQUATION 9.B

	(1) $c_{10}+c_{11}$	(2) c_0	(3) c_1	(4) c_0-c_1	(5) J
Seasonally adjusted					
Aggregate non-durables	-23 637 (49 502)	.1081 (.1689)	.2510 (.3053)	-.1430 (.1381)	55.1
Chemicals	-3 343 (2 623)	.0828 (.0593)	.2240 (.1280)	-.1410 (.0722)	87.5
Rubber	-104 (763)	.0370 (.0490)	.1622 (.1006)	-.1252 (.0577)	38.6
Petroleum	-2 896 (1 425)	.0991 (.0420)	.3034 (.0796)	-.2043 (.0493)	115.1
Food	135 352 (125 311)	-.8254 (1.0585)	-.2739 (1.3301)	-.5514 (.3047)	6.8
Seasonally unadjusted					
Aggregate non-durables	93 223 (66 243)	-.2833 (.2314)	-.4213 (.4819)	.1380 (.1891)	58.1
Chemicals	-3 651 (2 840)	.1252 (.0633)	.3202 (.1350)	-.1950 (.0754)	46.2
Rubber	-2 276 (8 891)	.1962 (.0578)	.5126 (.1172)	-.3163 (.0658)	12.7
Petroleum	-1 138 (1 585)	.0481 (.0405)	.3610 (.0705)	-.3129 (.0559)	92.2

1.See Notes to Table I.
2."J" for food distributed as chi-squared with one degree of freedom, critical levels: 3.84 at .05, 6.63 at .01, 7.88 at .005. For all others J distributed as chi-squared with five degrees of freedom, critical levels as listed in Table I. See text.

TABLE IIB

PARAMETER ESTIMATES, EQUATION 9.C

	(1) $c_{10}+c_{11}$	(2) c_0	(3) c_1	(4) c_0-c_1	(5) J
Seasonally adjusted					
Aggregate non-durables	386 527 (228 442)	-.7650 (.4671)	.4783 (.3215)	-1.2433 (.7881)	95.8
Chemicals	-5 436 (4 551)	.0747 (.0605)	.1866 (.1202)	-.1120 (.0630)	80.8
Rubber	-850 (1 178)	.0514 (.0459)	.0514 (.0459)	-.1148 (.0458)	35.6
Petroleum	-3 276 (2 329)	.0609 (.0404)	.2077 (.0728)	-.1469 (.0421)	98.6
Food	-259 048 (150 991)	2.0830 (1.0518)	4.1825 (1.8791)	-2.0995 (.8359)	.2
Seasonally unadjusted					
Aggregate non-durables	104 163 (55 756)	-.1833 (.1154)	.2391 (.1984)	.0558 (.0847)	97.3
Chemicals	-16 001 (6 318)	.2420 (.0842)	.4976 (.1394)	-.2854 (.0868)	39.0
Rubber	-4 913 (1 735)	.2241 (.0688)	.5154 (.1272)	-.2913 (.0651)	8.8
Petroleum	2 807 (2 500)	.0400 (.0391)	.2954 (.0487)	-.2554 (.0487)	77.2

See Notes to Tables I and IIA.

by contrast, 30 of 36 signs on the quadratic inventory and backlog costs were correct, as indicated by columns (2) and (3) in Tables IIA and IIB. (Five of the six wrong signs were for aggregate non-durables, suggesting that aggregation is still a problem for this method). About half of these were significant at the five percent level. The backlog cost is significantly larger than the inventory cost fully three-fourths of the time (see column (4) in Tables IIA and IIB). This strongly suggests that the two costs should not be used interchangeably, with backlogs considered negative inventories, as they occasionally are (Belsley (1969), Blinder (1982)).

There were, however, some puzzling aspects to the second method's results. The sum of the linear backlog and inventory costs $c_{10}+c_{11}$ is wrong-signed about two-thirds of the time, although never significantly so (column (1) in Tables IIA and IIB). Also the overidentifying restrictions almost always reject at decisive significance levels (column (5) in Tables IIA and IIB), as seems common in rational expectations inventory models (Blanchard (1982), Eichenbaum (1982)).

The results on balance, however, do suggest an important role for backlog costs as a determinant of inventory behavior. The quadratic backlog cost in Tables IIA and IIB ranged from about .2 to .5, with the comparable figures for the inventory cost about .05 to .25. These are to be compared with the production cost, normalized at one. Since these parameters are the quadratic terms in the cost function, they dominate total costs when the units backlogged, held in inventory, or produced are large enough. Thus, for a large enough number of units, total costs of a given number of units backlogged are about one-fifth to one-half that of the same number of units produced, and about two to eight times that of the same number of units held in inventory.

This seems consistent with the role for backlogs hypothesized here, although I have been unable to locate any studies by economists that contain estimates that may usefully be compared to these. However, the examples in Holt et al. (1961) and Killeen (1969) use values for similar parameters that are intended to be realistic though apparently not calculated from any real world firm data. Holt et al. (1961,p235) have a backlog cost to inventory cost ratio of twelve to one and a backlog cost to production cost ratio of one to two. Killeen (1969, p53) has a backlog to inventory cost ratio of about twenty to one and a backlog to production cost ratio of 5 to 3. The values estimated here thus are comparable.

IV.CONCLUSIONS

This suggests steps to be taken in future research.

The estimates here indicate that backlog costs are indeed significant in production to stock industries. Substantiation of these results on different data and with different models is of course highly desirable. In this connection, two directions are especially worth mentioning. First, the use of raw individual firm data appears to be very appropriate. The model did markedly better on the two digit SIC codes than on the aggregate, and slightly better on seasonally unadjusted than on adjusted data. This implies that data that are disaggregated and unprocessed are more appropriate to the model than those that are not.

Second, it would seem very useful to obtain estimates of backlog costs from production to stock firms or industries in which data on both backlogs and finished goods inventories are available. (The model used in this paper would of course not be directly applicable.) This once again suggests using individual firm data, since, as noted above, Department of Commerce data on backlogs in production to stock industries are not available.

They are available, however, in production to order industries. While production to order industries differ from production to stock industries in some fundamental ways, it appears plausible that for both industries the larger the backlog and the smaller the inventory stock (both suitably normalized) the higher are the opportunity costs of lost sales. With direct backlog data available, it is possible to put parameterize these costs naturally and to allow demand to be shifted by backlogs and inventories.

This is studied in the following chapter.

FOOTNOTES

1.Detailed evidence on what differentiates production to order from production to stock industries may be found in Zarnowitz (1973).

2.The two digit SIC codes for which they do publish data are industries that produce mainly to order, see the next chapter or Belsley (1969).

3.The original work on the model (Holt et al. (1961)), however, does not: the authors were careful to point out the importance of backlogs in production to stock industries.

4.This term is perhaps most familiar from work based on Lovell's flexible accelerator inventory model (Lovell (1964).

5.See Rowley and Trevedi (1975, p52) for justifications other than backlog costs for the target level.

6.A cost of changing production, $a_0(Q_t-Q_{t-1})^2$, sometimes present in a production smoothing model (see the previous chapter) is suppressed because it did not perform well in the regressions.

7.I thank Julio Rotemberg for suggesting this argument.

8.This may be verified by using the implicit function theorem to show that $[dH_t/d(c_1/c_0)]>0$ along (7). This comparative statics argument of course requires that c_0 and c_1 not appear in other first order conditions (if any). It is thus not valid if they appear in the first order condition for price, so that expected sales shifts along with c_1/c_0.

9.Or at least it can be if one special case ($c_{10}=c_{11}=0$, $c_0=c_1$) is representative, so that the model becomes linear quadratic and the techniques of Hansen and Sargent (1980) can be used to solve the model explicitly. In this case it may be shown that E_tB_{t+1} is time varying.

10.An implausible but simple example in which the errors are unconditionally heteroscedastic is: $Q_t+H_{t-1}-S_t-S_{t+1} = t + e_t$, e_t white noise. In this cases the conditional and unconditional distributions of B_t are the same, with the variance time varying as in (10).

11.Again, food was not estimated by this method since for this data set, $B_t=1$ in all time periods.

CHAPTER IV:

BACKLOG COSTS IN PRODUCTION TO ORDER INDUSTRIES:

SOME EMPIRICAL ESTIMATES

Production to order industries are those that ordinarily wait for customer orders to come in before completing production. Backlogs, or queues of orders yet to be filled, therefore tend to be substantial. In particular, in these industries the value of orders in the backlog is several times larger than the value of finished goods inventories. For the data studied in this chapter, this may be seen in lines one and two of Table I. The magnitudes of the backlogs alone suggest that insofar as they are economically linked to inventories, it may be no small omission to ignore them altogether when studying durable goods inventories, as is often done (e.g., Feldstein and Auerbach (1976)).

This last point has been noted before, and some authors have integrated backlogs into inventory models. Perhaps the most prominent example of this type of model is that first introduced by Holt et al. (1961) and extended by, among others, Belsley (1969) and Childs (1967), cited in Rowley and Trevedi (1975). It is a straightforward adaptation of the production smoothing model in chapter 2. Let N_t be new orders net of cancellations, B_t the number of units on the backlog at the end of period t, and, as before S_t = period t sales, Q_t = period t production, and H_t = finished goods inventories at the end of period t. By definition, then, $N_t = S_t+B_t-B_{t-1}$ and $Q_t = S_t+H_t-H_{t-1}$, so $Q_t = N_t+(H_t-B_t)-(H_{t-1}-B_{t-1})$. If backlogs are treated simply as negative

TABLE I

MEANS / STANDARD DEVIATIONS OF BASIC VARIABLES
(1959-1980)

	Aggregate Durables	Electrical Machinery	Metals
(1)Backlogs	81257 / 11300	16045 / 1678	22068 / 3601
(2)Finished goods inventories	22431 / 686	3050 / 159	6223 / 314
(3)Shipments	25826 / 1952	4010 / 311	8429 / 809
(4)Production	25922 / 1972	4028 / 321	8448 / 805
(5)New orders	26000 / 2568	4068 / 421	8475 / 1017

Notes:
1.Data are monthly, seasonally adjusted, 1959:2 to 1980:12. Sources described in section III. Note that the aggregate durables figure is net of non-electrical machinery (SIC 35).
2.The mean is a simple sample mean. The standard deviation is the standard error of a regression of the relevant variable on a constant and a time trend, i.e., the standard deviation of the variable around a time trend.
3.Units are millions of 1972 dollars.

inventories (see Holt et al. (1961) and Belsley (1969) for justifications), "net" inventories H_t-B_t may be considered to be a choice variable for the firm. Some authors have done this (Belsley(1969), Childs(1967) cited in Rowley and Trevedi (1975)). By analogy to the production smoothing model described in chapter 2, one could assume that firms minimize costs, with per period costs

$$(1) \qquad a_0(Q_t-Q_{t-1})^2 + a_1Q_t^2 + a_4[-(H_t-B_t) - a_5Q_t]^2$$

Clearly the model has implications for the variance of production, as did the model in chapter 2. It is worth digressing for a moment to comment on this. It may be shown by an argument similar to that of Chapter 2[1] that equation (1) implies that

$$\begin{aligned} &a_0\mathrm{var}(Q_t-Q_{t-1}) + (a_1+a_4a_5^2)\mathrm{var}(Q_t) \\ &\quad + 2a_4a_5\mathrm{cov}(H_t-B_t,Q_t) + a_4\mathrm{var}(H_t-B_t) < \\ &a_0\mathrm{var}(N_t-N_{t-1}) + (a_1+a_4a_5^2)\mathrm{var}(N_t) \end{aligned}$$

From lines (4) and (5) in Table I, var(Q) < var(N) for all three data sets. It is also true (although not reported in Table I) that for all three $\mathrm{var}(Q_t-Q_{t-1}) < \mathrm{var}(N_t-N_{t-1})$ and $2a_2a_3\mathrm{cov}(H_t-B_t,Q_t) < 0$. Thus, in contrast to the production to stock industries studied in chapter 2, the variance inequality for this Holt et al. (1961) model holds.[2]

Nevertheless, the model seems inadequate for other reasons, and its shortcomings turn on the term multiplied by a_4. This is intended to capture two distinct effects of backlogs: on demand and on production costs.[3] The first effect, on demand, is familiar from the previous chapter: large backlogs or long delivery lags discourage customers and hence impose costs, perhaps in the form of lost sales.

The second effect of backlogs, on production costs, may be found only in order and not in stock industries. Since production to order firms tend to produce a heterogeneous product, often tailored differently for different customers, a large backlog makes it likelier that the firm will be able to group production of similar items. This batch processing (discussed in slightly more detail in the next section) will cut machine set-up time and hence costs.

Backlogs undoubtedly do have both these aspects, and this paper's model allows for both. But the $a_4(.)$ term in equation (1) seems unsatisfactory for either. Insofar as a large net backlog B_t-H_t adversely affects demand, it would seem desirable to parameterize these effects directly in the demand curve if possible.[4] And proxies for delivery lags can easily be constructed using backlog, inventory and other data (e.g., as in Zarnowitz (1973), Trevedi (1970)). This paper also constructs such a proxy, and then puts it directly into the demand curve to be estimated.[5] In contrast to previous chapters, then, this one will specify a demand curve and make an assumption about market structure. This would seem to be the price that has to be paid to model in this natural way the demand-side effects of backlogs and inventories.

The $a_4(.)$ term also seems inadequate to capture the second effect of backlogs, on production costs. The validity of the term depends crucially on backlogs being considered negative inventories. In certain respects they are--perhaps in their effects on demand (as indicated by the empirical results below) or in the qualitative sense that both allow production smoothing (i.e., both allow firms to produce at a relatively low level when demand is relatively high and conversely). But backlogs manifestly are not just negative inventories in production to order industries, in that inventories cannot permit the batch processing described above. In addition, empirically, as seen in lines (1) and (2) of Table I, inventories and backlogs do not behave as "equal and opposites" even superficially. Backlogs are not only larger, but also much more volatile. Their coefficient of variation (i.e, standard deviation divided by mean, not reported in Table I) is several times larger than that of inventories, for each of the three data sets. For both theoretical and empirical reasons, then, backlogs and inventories should not be considered equal and opposite.[6] And equally incorrect is the argument that since inventories are so much smaller than backlogs the "net backlog" B_t-H_t is "almost" like the backlog B_t itself (Belsley (1969, p54)). For even if the average values of inventories are small relative to those of backlogs the values on the margin of B_t-H_t and B_t might be quite different.

This paper, then, estimates a version of the production smoothing model that attempts to capture the revenue effects of the net backlog directly in the demand curve, and sharply distinguishes between the cost effects of backlogs and inventories. It is indeed found that the net backlog shifts demand, and that the backlog has the hypothesized effects on costs. A one percent increase in the net backlog/shipment ratio causes demand to fall by about one percent; the backlog causes the extra increase in marginal cost (i.e., second derivative of cost function with respect to production) to fall by about eighty per cent.[7] The parameters used to calculate these elasticities usually are statistically significant. However, these results should be interpreted with caution, since tests of overidentifying restrictions generally rejected, and a certain cost parameter was always wrong-signed.

The paper is organized as follows. Part II sets out the model, part III discusses the data, estimation and empirical results, and part IV contains conclusions.

II.The Model.

The general environment is as follows. A monopolist in a production to order industry supplies customers whose order quantity depends not only on the monopolist's price but on their estimates of its delivery lag as well. That is, when customers decide on the quantity of new orders they observe an explicitly quoted price, and estimate the delivery lag from past industry levels of backorders, inventories, production and shipments. The monopolist unilaterally decides not only price, but production and shipments as well. Inventories are decided according to production = shipments + change in inventories, and backorders according to new orders = shipments + change in unfilled orders.

Several comments on this are appropriate, before turning to the formal model. First, there is a single prevailing market price and delivery lag for all new orders in a given period. This rules out, for example, simultaneous existence of a higher priced production to stock and lower priced production to order market. These do occasionally exist (e.g., in steel (de Vany and Frey (1982)). But such spot markets apparently are of secondary importance, at least if Zarnowitz (1973) was correct in virtually ignoring them in his exhaustive treatment of production to order industries.

Second, the new orders are considered irrevocable commitments, i.e., cancellations are ruled out. This appears to be a relatively harmless assumption, since cancellations per period typically are less than five percent of gross new orders (Zarnowitz (1973, pp26-27)). In any case it is necessary since the only new orders data available are net of cancellations.

Third, in contrast to the production to stock firm described in the previous chapter, the firm does simultaneously maintain a stock of finished goods inventories and backorders. This again matches the fact that both stocks are substantial in production to order industries. Formally, this follows since inventories and backorders affect costs differently (see below).

The model is as follows. As in previous chapters, the firm maximizes the expected present discounted value of its real cash flow:

$$\text{(2)} \quad \max E_0 \sum_{t=0}^{\infty} d_1^t (R_t - d_2^t C_t)$$

$$\text{s.t. } Q_t = S_t + H_t - H_{t-1}, \quad N_t = S_t + B_t - B_{t-1}$$

where

E_0	mathematical expectations, conditional on information available at time 0
d_1	fixed real discount rate, $0<d_1<1$
d_2	fixed rate of technological progress, $0<d_2<1$
R_t	real revenue in period t (detailed below)
C_t	real costs in period t (detailed below)
Q_t	units produced in period t
S_t	units shipped in period t
H_t	units of finished goods inventory at end of period t
N_t	units newly ordered in period t
B_t	units on backorder at end of period t

As in previous chapters, the information set is left unspecified since estimation here is consistent as long as the firm's set includes at a minimum previous levels of backorders, inventories and shipments, and certain functions of these defined below.

In contrast to previous chapters, not only costs but demand and revenue will be specified precisely. Following are discussions of revenue, demand, costs, and of the first order conditions of the monopolist. Constant and linear trend terms analagous to those listed in the Appendix to chapter 2 should be assumed to be present in the equations to follow, although in general they have been suppressed for notational simplicity.

Demand

It is assumed that demand (new orders) may be written

$$(3)\quad N_t = (1/e_1)\,[\,-p_t - e_2(B_{t-1}-H_{t-1})/S_{t-1} + e_3 YD_t + u_t\,]$$

The e_i are positive parameters. A term $e_{00}+e_{01}t$ should be understood to be present inside the brackets. The variables in the demand curve are explained in turn.

The first is real price p_t, that is, the price of the firm's good relative to the prevailing price level.

The second variable, $(B_{t-1}-H_{t-1})/S_{t-1}$, is assumed to capture the effects of delivery lags on demand. The net backlog $B_{t-1}-H_{t-1}$ is the number of orders received but not yet produced. The higher is this relative to the shipment rate S_{t-1}, the longer the buyer is likely to have to wait for delivery and, thus, the lower is demand. Here, as elsewhere (Trevedi (1970), Zarnowitz (1973, pp278ff)) a simple ratio is used to roughly approximate the number of periods the buyer will have to wait to receive shipment of his order.[8]

The assumption that a single previous period's net backlog to shipments ratio alone suffices to capture the effects of delivery lags on demand is a simplifying one. More complicated dynamic behavior is certainly possible, but it is hoped that this simple ratio provides a useful first approximation.[9]

The next element, YD_t, is a variable that shifts demand. Ideally this would be income of the buyers, or, insofar as the purchasers are manufacturers themselves, the demand for the product that the purchasers produce. Such an ideal variable did not appear to be available. It was proxied in the empirical work with real disposable income.

The final variable in (3), u_t, is a white noise shock.

Revenue

In a production to order market, revenue may be derived from demand in two different ways. Both variants were used in the empirical work. In one, the price paid for a good is the price prevailing at the time of shipment, so that revenue is R_t equals p_tS_t. This is appropriate when escalator clauses are built into orders contracts. This has been becoming increasingly more common since 1973, and even been standard in at least one (steel) long before that (Foss et al., (n.d., pp155-156)).

However, for most industries over most of the sample period, the price paid for the good was the price prevailing at the time the order was placed (Zarnowitz (1973, p25)), Foss et al. (n.d, pp155-56). Revenue from this period's new orders is thus p_tN_t. However, with delivery lags, the present discounted value of the revenue is less than p_tN_t, since payment presumably usually is not made until the shipment is received by the customer.[10] For simplicity, however, this discounting is ignored in the empirical work and the second method of handling revenue set $R_t = p_t N_t$.

For estimation it was convenient to write the demand curve in inverse form. The two versions of revenue estimated, then, were

$$(4)\ [\ -e_1 N_t - e_2(B_{t-1}-H_{t-1})/S_{t-1} + e_3 YD_t + u_t\ ]S_t$$
$$(5)\ [\ -e_1 N_t - e_2(B_{t-1}-H_{t-1})/S_{t-1} + e_3 YD_t + u_t\ ]N_t$$

Costs Costs have two components.[11]

1)Costs relating to production and backlogs:

$$(6)\quad (a_1/2)Q_t^2[1-a_4 B_t(B_t+Q_t)^{-1}] + a_{10}Q_t[1-a_{40}B_t(B_t+Q_t)^{-1}]$$

The a_i are all positive.

The term is intended to capture the idea that in the range of production and backlogs typically observed, an increase in the size of the backlog lessens production costs for a given rate of production. But it does so at a decreasing rate with the marginal benefit of an increase in the backlog approaches zero for any fixed rate of production.

Thus, the ratio $B_t/(B_t+Q_t)$ is concave in B_t. With $B_t=0$, (6) degenerates to two of the terms included in the previous two chapters' cost functions, $(a_1/2)Q_t^2 + a_{10}Q_t$. With no backlog, therefore, this term reflects increasing costs associated with decreasing returns to scale technology, as in previous chapters. And as B_t grows, costs fall, but do so at a decreasing rate.

The economic forces underlying this term are in part similar to those identified by Holt et al. (1961) and others, see the introduction to this chapter. For typical rates of capacity utilization, if the firm increases its backlog while maintaining its existing rate of production, it increases the possibility that similar orders may be grouped and then produced. This production bunching can cut costs by allowing longer run times and fewer setups. Thus the larger backlog both increases the time that machines are productively available and decreases labor costs associated with setups. Such a backlog also lessens the probability that specialized machines will have to sit idle. For a more extended discussion, see Belsley (1969, p48) and Holt et al. (1961, pp316-317).

(3) A standard inventory cost:

$$(7) \quad (a_2/2)(H_t)^2$$

This reflects storage and carrying costs.

First order conditions

It will be convenient here to write these without substituting out the constraints $Q_t=S_t+H_t-H_{t-1}$ and $N_t=S_t+B_t-B_{t-1}$. (When the system was actually estimated, as discussed in the next section, the constraints were of course substituted out.) Define the product of the discount rate and the rate of technological progress as $d=d_1d_2$. The first order conditions will be written out for the case revenue $R_t=p_tS_t$; the case $R_t=p_tN_t$ is similar.

The problem to be solved is (2) with

$$(8)\qquad R_t - C_t = [\, -e_1N_t - e_2(B_{t-1}-H_{t-1})/S_{t-1} + e_3YD_t + u_t\,]S_t$$
$$- (a_1/2)Q_t^2[1-a_4B_t(B_t+Q_t)^{-1}]$$
$$- a_{10}Q_t[1-a_{40}B_t(B_t+Q_t)^{-1}] - (a_2/2)(H_t)^2$$

Let m_{1t} be the Lagrangian associated with $Q_t=S_t+H_t-H_{t-1}$, m_{2t} that with $N_t=S_t+B_t-B_{t-1}$. The first order conditions (again, with deterministic terms suppressed) then are

$$(9a)\ H:\quad E_t\{\, m_{1t} + a_2H_t - d_1e_2(S_{t+1}/S_t) = dm_{1t+1} \,\}$$
$$(9b)\ B:\quad E_t\{\, m_{2t} - (a_1a_4/2)Q_t^3(Q_t+B_t)^{-2} - (a_{10}a_{40})Q_t^2(Q_t+B_t)^{-2}$$
$$+ d_1e_2(S_{t+1}/S_t) = dm_{2t+1} \,\}$$
$$(9c)\ Q:\quad E_t\{\, a_1Q_t - (a_1a_4/2)(2Q_tB_t^2 + Q_t^2B_t)(Q_t+B_t)^{-2}$$
$$- a_{10}a_{40}B_t^2(B_t+Q_t)^{-2} \,\} = m_{1t}$$
$$(9d)\ S:\quad E_t\{\, p_t + d_1e_2[(B_t-H_t)S_{t+1})]/(S_t^2)$$
$$= m_{1t} + m_{2t} \,\}$$
$$(9e)\ N:\quad E_t\{\, e_1S_t = m_{2t} \,\}$$

(9a) says that the value of an extra unit of inventories this period equals its expected shadow value next period. The marginal value of an extra unit of inventories this period has a rising component due to carrying costs as in Blinder (1982) and, here, a falling one due to its expected effects on next period's demand (see Blanchard and Melino (1982,p8)). Thus, this extra unit increases both costs and expected revenues.

Similarly, (9b) says that the marginal value of an extra unit on the backlog this period equals its expected shadow value next period. An extra unit increases the customers' estimate of the delivery lag and thus decreases expected revenue. This is offset in that the unit also decreases production costs. (9c) says as in Blinder (1982) to produce until the marginal cost of production equals the shadow value of inventories; the marginal cost falls with the level of the backlog. (9d) says ship until the marginal value of a shipment equals the sum of the shadow values of backorders and inventories. The value of a shipment depends not only on its direct effect on marginal revenue p_t but also on its effects on delivery lags. Finally (9e) says accept new orders until the marginal value of one equals the shadow value of an extra unit on the backlog.[12]

Empirical Results

Data and estimation are described briefly before empirical results are presented.

A. Data

The data were real (1972 dollars), monthly and seasonally adjusted, 1959-1980. Data were available for aggregate durables exclusive of non-electrical machinery (i.e., net of SIC 35), electrical machinery (SIC 36) and metals (sum of primary metals SIC 33 and fabricated metals SIC 34). Nominal backlog and shipment data were obtained from Citibank Economic Database (files MDU, MDS, MDSMAC, MDUMAC, MDSMA4, MDSMU4, MDSM2, MDUM2, MSDM4, MDUM4). These were deflated by the appropriate wholesale price index (files PWDMD, PWME, PW117, PWMET). The constant dollar inventory figures were obtained as in previous chapters by converting the Bureau of Economic Analysis's recently calculated constant dollar inventory series (Hinrichs and Eckman (1981)) from cost to market so that a dollar of inventories represented the same physical units as a dollar of sales. (See the Appendix "A Note on the Econometric Use of Constant Dollar Inventory Series" for a definition of "cost" and "market" and an explanation of why the conversion was necessary.) Real disposable income was obtained from Citibank file GMYD72. Production was obtained from the identity $Q_t=S_t+H_t-H_{t-1}$, new orders from $N_t=S_t+B_t-B_{t-1}$.

Estimation

The sample period covered 1959:5 to 1980:10, with 1980:11 used for leads, and 1959:2 to 1959:4 for lags. All regressions included constants and time trends, although these coefficients are suppressed from the equations that follow. The trivariate system associated with revenue $R_t=p_tS_t$ is discussed first in some detail, and then the trivariate system associated with p_tN_t is presented.

After substituting out the Lagrangians, using the demand curve to substitute out for price p_t, and again letting $d=d_1d_2$ denote the product of the discount rate and the rate of technological progress, the system (9) becomes

$$(10)\quad E_t \{ (a_1a_4/2)(dQB2_{t+1}-QB2_t) + a_{10}a_{40}(dQB1_{t+1}-QB1_t) - a_1(dQ_{t+1}-Q_t) + a_2H_t - d_1e_2(S_{t+1}/S_t) \} = 0$$

$$E_t \{ (a_1a_4/2)QB4_t + a_{10}a_{40}QB3_t + e_1(dS_{t+1}-S_t) - d_1c_2(S_{t+1}/S_t) \} = 0$$

$$E_t \{ (a_1a_4/2)QB2_t + a_{10}a_{40}QB1_t - a_1Q_t - e_1(S_t+N_t) - - e_2(BSS_{t+1}-HSS_{t+1}) + e_3YD_t + u_t \} = 0$$

where $QB1_t$, $QB2_t$, $QB3_t$, $QB4_t$, BSS_{t+1} and HSS_{t+1} are

$$(11)\quad QB1_t = B_t^2(Q_t+B_t)^{-2}$$

$$QB2_t = (2Q_tB_t^2+B_tQ_t^2)(Q_t+B_t)^{-2}$$

$$QB3_t = Q_t^2(Q_t+B_t)^{-2}$$

$$QB4_t = Q_t^3(Q_t+B_t)^{-2}$$

$$BSS_{t+1} = (B_tS_{t+1})/(S_t^2) - (B_{t-1}/S_{t-1})$$

$$HSS_{t+1} = (H_tS_{t+1})/(S_t^2) - (H_{t-1}/S_{t-1})$$

and, again, YD_t is disposable income.

As in previous chapters, a normalization is needed to identify parameters. Since a_1a_4 appears in every equation, it is a natural choice for normalization: with $a_1a_4=2$ in each equation, all parameters are identified up to the same scale factor and thus can be compared across equations. Let lower case $x_t = dX_t - X_{t-1}$, $X_t = B_t$, Q_t, $QB2_t$ or $QB1_t$. Also replace expected values of regressors with their observed ex-post values. Set the monthly rate of technological progress $d_2 = .997$ (corresponding annual rate is about 3.5 per cent) and the discount rate $d_1 = .998$ (annual rate 2.5 percent); their product d is then .995 (annual rate 6 per cent). With these conventions, (10) becomes

$$
\begin{aligned}
(12.s)\quad qb2_{t+1} &= a_1 q_{t+1} - a_2 H_t + d_1 e_2(S_{t+1}/S_t) - a_{10}a_{40} qb1_{t+1} + v_{1t} \\
QB4_t &= -e_1(d_1 S_{t+1} - S_t) + d_1 e_2(S_{t+1}/S_t) - a_{10}a_{40} QB3_t + v_{2t} \\
QB2_t &= a_1 Q_t + e_1(S_t + N_t) - e_2(BSS_{t+1} - HSS_{t+1}) - \\
&\qquad - e_3 YD_t - a_{10}a_{40} QB1_t + v_{3t}
\end{aligned}
$$

The estimation allowed the residuals v_{1t} and v_{2t} to be MA(1) and v_{3t} to be MA(2). This permits the period t demand shock u_t and period t disposable income YD_t to be unobserved when the firm makes its period t decisions. For example, $v_{1t} = -a_1(q_{t+1} - E_t q_{t+1}) - de_2[(S_{t+1}/S_t) - E_t(S_{t+1}/S_t)] + a_{10}a_{40}(qb1_{t+1} - E_t qb1_{t+1}) + (qb2_{t+1} - E_t qb2_{t+1})$, and is MA(1) if not only u_{t+1} and YD_{t+1} but u_t or YD_t as well are not part of the period t information set. A similar argument applies to v_{2t} and v_{3t}.[13]

An unconstrained version of the system (12.s) was estimated by two-step, three stage least squares, as described in Hansen (1982) and Hansen and Singleton (1982). That is, the equations in (12.s) were stacked in the usual way (e.g., Theil (1972,p523)) to get a 3T x 1 vector of residuals $y - Xb_1$; X is (3T x 19), b_1 is (19 x 1). (The seven parameters above and beyond the twelve written out on the right hand sides of (12.s) are the constant and trend terms in the three equations, and separate coefficients on BSS_{t+1} and HSS_{t+1} in the third equation in (12.s). These last regressors were entered separately to facilitate testing of the hypothesis that backlogs and inventories have equal and opposite effects on demand. See below.) Let Q be the (3T x 3T) weighting matrix which given the instruments optimally accounts for the MA nature of the errors, as defined in Hansen (1982) or Hansen and Singleton (1982). The thirteen instruments used were: constant, time trend, two lags each of sales, inventories, backlogs and disposable income, and one lag each of S_{t+1}/S_t, BSS_{t+1} and HSS_{t+1}. The parameter vector b_1 was chosen to minimize $(y-Xb_1)'Q(y-Xb_1) \Longrightarrow b_1 = (X'QX)^{-1}X'Qy$, with $V_1 = (X'QX)^{-1}$ the variance of b_1

A parameter vector b_2 that satisfies the cross-equation constraints implicit in (12.s) was obtained from b_1 as follows. Write the constraints that connect the parameters across the equations in (12.s) (e.g., that the coefficient on q_{t+1} in the first equation equals that on Q_t in the last equation) as $Rb_1=0$. R is a 7 x 19 matrix. Finally, let b_2 be an estimate of the (19 x 1) parameter vector that minimizes the same objective function $(y-Xb_2)'Q(y-Xb_2)$ subject to the constraint $Rb_2 = 0$. Let V_2 be its covariance matrix. Only twelve of the elements of b_2 are distinct (a_1, a_2, $a_{10}a_{40}$, e_1, e_2, e_3, and the six deterministic terms) and V_2 is singular. Then the constrained system may be calculated from the unconstrained one by (see Theil (1971, p 288))[14] by

$$(13)\quad b_2 = [I - V_1R'(RV_1R')^{-1}R]b_1$$
$$V_2 = [I - V_1R'(RV_1R')^{-1}R]V_1$$

The increase in the objective function, $(y-Xb_2)'Q(y-Xb_2)$ -

$(y-Xb_1)'Q(y-Xb_1)$ may be used to test the null hypothesis that the cross-equation constraints are valid. It is asymptotically distributed as chi-squared with seven degrees of freedom under the null hypothesis.[15]

To test whether inventories and backlogs had equal and opposite effects on demand, the constraint that BSS_{t+1} and HSS_{t+1} have equal and opposite signs was dropped. (The constraint that the coefficient on S_{t+1}/S_t in the first equation be .998 tmes that of HSS_{t+1} in the third equation was still imposed, and similarly for S_{t+1}/S_t in the second equation and BSS_{t+1} in the third.) The system was then estimated subject to the remaining six constraints, by the two-step procedure just described. Let the resulting coefficient vector be b_3. Under the null hypothesis that backlogs and inventories have equal and opposite effects on demand, the difference in distance functions $(y-Xb_2)'Q(y-Xb_2)$ - $(y-Xb_3)'Q(y-Xb_3)$ is asymptotically distributed as chi-squared with one degree of freedom.

With revenue p_tN_t, the system that was estimated was

$$
\begin{aligned}
(12.n)\quad qb2_{t+1} &= a_1q_{t+1} - a_2H_t + d_1e_2(N_{t+1}/S_t) \\
&\quad - a_{10}a_{40}qb1_{t+1} + v_{1t} \\
QB4_t &= a_4B_t - 2e_1(d_1N_{t+1}-N_t) - e_2(BNS1_{t+1} - HNS1_{t+1}) \\
&\quad + e_3(d_1YD_{t+1}-YD_t) - a_{10}a_{40}QB3_t + v_{2t} \\
QB2_t &= a_1Q_t + 2e_1N_t - e_2(BNS2_{t+1} - HNS2_{t+1}) \\
&\quad - e_3YD_t - a_{10}a_{40}QB1_t + v_{3t}
\end{aligned}
$$

where

$$BNS1_t = (d_1B_{t-1}-N_t)/S_{t-1} - (B_{t-2}/S_{t-2})$$

$$HNS1_t = d_1 H_{t-1}/S_{t-1} - (H_{t-2}/S_{t-2})$$

$$BNS2_t = d_1 B_{t-1} N_t/(S_{t-1}^2) - (B_{t-2}/S_{t-2})$$

$$HNS2_t = d_1 H_{t-1} N_t/(S_{t-1}^2) - (H_{t-2}/S_{t-2})$$

The thirteen instruments used were: constant, time trend, two lags each of sales, inventories, backlogs and disposable income, and one lag each of N_{t+1}/S_t, $BNS1_{t+1}$ and $HNS1_{t+1}$. The constrained system and the statistic testing whether inventories and backlogs have equal and opposite effects on demand were estimated analagously to those for revenue p_tS_t. The number of constraints this time was nine and the number of parameters in the unconstrained system 21. The two extra constraints and parameters come from the presence of $(d_1 YD_{t+1} - YD_t)$ and and $HNS1_t$ in the second equation of (12.n).

Results

Table II has the results for revenue $R_t = p_tS_t$, Table III for $R_t = p_tN_t$. Coefficients for deterministic terms are not reported, since they are considered to be of secondary interest.

Of the 21 parameters in Table II, 14 have the correct sign. The mediocre performance basically reflects the production-backlog cost $a_{10}a_{40}<0$ in all three data sets and the overall poor performance of the electrical machinery equation. The parameter estimates for the latter, in column (2), suggest that demand slopes up ($e_1<0$ in row (5)). The coefficient e_2 on the net backlog-shipment ratio is also wrong-signed (row(6)). (Note, however, that the incorrect signs on e_2 and e_1 combine to put the right sign on the coefficient e_2/e_1 in the demand equation. See equation (3) above.) The other two data sets (aggregate ex

TABLE II

PARAMETER ESTIMATES, EQUATION 12.S

	(1) Aggregate Durables	(2) Electrical Machinery	(3) Metals
(1) a_1	1.3347 (.0377)	1.4722 (.0283)	1.2159 (.0518)
(2) a_2	.0097 (.0065)	-.0096 (.0043)	.0101 (.0078)
(3) a_4	1.4984 (.0424)	1.3584 (.0262)	.6448 (.0700)
(4) $a_{10}a_{40}$	-41644 (823)	-6193 (144)	-10820 (864)
(5) e_1	.0090 (.0212)	-.0044 (.0151)	.0133 (.0282)
(6) e_2	841.8 (658.4)	-46.70 (67.23)	188.6 (301.9)
(7) e_3	2.5718 (2.3326)	1.0594 (.2514)	-2.2481 (1.4340)
(8) J_1	63.9	71.1	44.7
(9) J_2	123.3	46.1	88.7
(10) J_3	5.3	10.0	3.3
(11) J_4	.3	.01	1.1

Notes:
1. Variables defined in text; a_4 calculated as $2/a_1$.
2. Asymptotic standard errors in parentheses.
3. The J_i are chi-squared distributed with the indicated degrees of freedom. Critical levels:
   - J_1 20 d.f., critical levels: 31.4 at .95, 40.0 at .995
   - J_2 7 d.f., critical levels: 14.1 at .95, 20.3 at .995
   - J_3 5 d.f., critical levels: 11.1 at .95, 16.7 at .995
   - J_4 1 d.f., critical levels: 3.8 at .95, 7.9 at .995

non-electrical machinery and metals) have attractive parameter estimates, save for the significantly negative linear production-backlog cost $a_{10}a_{40}$. For both of these data sets, both inventory costs a_2 and the net-backlog shipment ratio parameter e_2 are positive, although disappointingly insignificant. In addition, the inventory cost a_2 is perhaps smaller than might have been expected, given results from previous chapters.[16] But for all three data sets, the quadratic production-backlog cost parameters a_1 and a_4 were highly significant.

Table III has results for revenue $R_t = p_t N_t$. This performs somewhat better, with 15 of 21 parameters correctly signed. The exceptions were again the estimates of the linear production-backlog cost $a_{10}a_{40}$ (significantly wrong-signed in all three equations) along with two of the three estimates of the inventory cost a_2, and one estimate of the parameter e_3 capturing the effects of disposable income on demand. Once again, the quadratic production-backlog cost a_1 and a_4 were highly significant. In addition, the net backlog-shipment ratio strongly affects demand (e_2 significant at the 1 per cent level of electrical machinery and metals, significant at the five per cent level for aggregate). Finally, the demand curve coefficient on price, e_1, is significant at the five per cent level for both electrical machinery and backlogs.

The last four rows of Tables II and III report tests of several hypotheses. Row (8), labeled J_1, is Hansen's (1982) test of overidentifying restrictions, and row (9) is the test of cross-equation restrictions described in the previous subsection. Both rejected at the .5 per cent level for all data sets. One experiment was performed in an

TABLE III

PARAMETER ESTIMATES, EQUATION 12.N

	(1) Aggregate Durables	(2) Electrical Machinery	(3) Metals
(1) a_1	1.3267 (.0377)	1.4019 (.0283)	1.1983 (.0518)
(2) a_2	-.0026 (.0065)	-.0069 (.0034)	.0024 (.0049)
(3) a_4	1.5075 (.0258)	1.4260 (.0170)	1.6690 (.0204)
(4) $a_{10}a_{40}$	-42294 (866)	-5641 (165)	-10125 (372)
(5) e_1	.0103 (.0111)	.0270 (.0076)	.0515 (.0126)
(6) e_2	385.2 (179.2)	67.17 (17.09)	197.2 (48.3)
(7) e_3	.0414 (2.0197)	.5371 (.2346)	-.4279 (1.2253)
(8) J_1	53.0	87.0	78.8
(9) J_2	104.3	77.8	110.5
(10) J_3	11.1	40.3	25.0
(11) J_4	3.6	.01	.9

Notes:
1.See notes to Table II.
2The J_i are chi-squared distributed with the indicated degrees of freedom. Critical levels:

J_1: 18 d.f., critical levels: 28.9 at .95, 37.2 at .995
J_2: 9 d.f., critical levels: 16.9 at .95, 23.6 at .995
J_3: 7 d.f., critical levels: 14.1 at .95, 20.3 at .995
J_4: 1 d.f., critical levels: 3.8 at .95, 7.9 at .995

attempt to narrow down the source of the latter (row (9)) rejection. The significantly wrong-sign on $a_{10}a_{40}$ suggested that it might be a source of trouble. Thus the cross equation restrictions that it be the same in all equations was relaxed, and the resulting chi-square statistic with five (system 12.s) or seven (system 12.n) degrees of freedom was calculated. This is reported on line (10).[17] It accepted at the five per cent significance level for all three data sets using (12.s), and for aggregate durables using (12.n) as well. It appears, then, that the linear term $a_{10}Q_t[1-a_{40}B_t(B_t+Q_t)^{-1}]$ is a major source of unsatisfactory aspects of the results of the estimation.

The last row of Tables II and III, row (11), contains the results of the test that inventories and backlogs have equal and opposite effects on demand. This test was accepted quite comfortably for all three data sets for both (12.s) and (12.n). This lends support to the Holt et al. (1961) hypothesis that backlogs may be considered negative inventories with respect to their effect on demand.[18]

The mixed results on tests of overidentifying restrictions and the significantly negative estimates of the linear production-backlog cost $a_{10}a_{40}$ suggest problems with the specification of the model. A likely source of trouble is equation (6), specifying production-backlog costs. The question of how backlogs affect production costs in production to order industries clearly is one that needs further study.

The results do, however, suggest that the net backlog strongly affects demand, and that backlogs strongly affect production costs. The implied demand elasticities suggest that the net backlog-shipment ratio is economically as well as statistically significant. As indicated in

Table IV, a one per cent increase in the net backlog-shipment ratio causes new orders to fall by roughly one per cent.

The economic effect of backlogs on costs is indicated in Table V. As in previous chapters, the second derivative of the cost function is identifiable. It is:

$$(14)\quad a_1 - a_1 a_4 B_t^3 (B_t + Q_t)^{-3} + a_{10} a_{40} B_t^2 (B_t + Q_t)^{-3}$$

This is contained in row (2) of Table V, evaluated at the mean levels of backlogs B_t and production Q_t. Without backlogs, the second derivative would of course just be a_1. In row (3) of Table V is the percentage fall in this second derivative attributable to backlogs: 100 x $[1-(14)/a_1]$. Thus, at the sample means, the increase in the marginal cost of production was twenty percent of what it would have been without backlogs. While this figure perhaps overstates the impact of backlogs on production costs in that it uses the significantly wrong-signed estimates of $a_{10}a_{40}$, it does suggest that backlogs strongly affect production costs.

TABLE IV

DEMAND ELASTICITIES WITH RESPECT TO THE NET BACKLOG-SHIPMENT RATIO

	Aggregate	Electrical	Metals
Eq'n (12.s)	8.19	.85	.31
Eq'n (12.n)	3.28	.20	.08

Notes:
1.All elasticities calculated at sample means.
2.Elasticities calculated as $(e_2/e_1)/\{N/[(B-H)/S]\}$, where new orders N, backlogs B, inventories H and shipments S were all evaluated at their sample means.

TABLE V
EFFECTS OF BACKLOGS ON SECOND DERIVATIVE OF PRODUCTION COSTS

	Aggregate Durables	Electrical Machinery	Metals
(12.s):			
a_1	1.33	1.47	1.22
Eq'n (14)	.24	.25	.27
$[1-(14)/a_1]*100$	82	83	78
(12.n):			
a_1	1.33	1.40	1.15
Eq'n (14)	.22	.20	.22
$[1-(14)/a_1]*100$	83	86	81

IV.CONCLUSIONS

This suggests steps for future research. Generalization and substantiation of two aspects of the model seem particularly important.

This first concerns the interaction between backlogs and production costs. While results were generally favorable to the hypothesis that backlogs help cut production costs , they were by no means unambiguously so. Thus, substantiation using other models or data or both appears especially desirable . This seem especially so since to my knowledge this chapter contains the first estimates of structural parameters summarizing this production-backlog interaction, even though the interaction is often said to be important (e.g., Belsley (1969), Holt et al. (1961), Maccini (1973)).[19]

The second aspect requiring generalization is more central to the basic argument of this thesis, that backlogs and inventories affect demand as well as costs. The assumption that the industry acts as a monopolist appeared to be required by the aggregate data. Such data seems to make it impossible to identify such plausible effects as a given firm's demand depending on its net backlog-shipment ratio relative to its competitors' ratios. Thus the use of individual firm data appears appropriate.

FOOTNOTES

1.Specifically, compare expected costs under the optimal policy with these under an alternative policy that sets $Q_t=N_t$, $H_t-B_t=0$. The left hand side of the inequality is expected per period costs under the optimal policy, the right hand side expected per period costs under the alternative policy.

2.Equation (1) has no implications for the variance of production relative to the variance of shipments, since the model does not determine inventories H_t. Thus the fact that production is more volatile than shipments for aggregate durables and electrical equipment, as indicated by lines (3) and (4) in Table 1, neither contradicts nor supports the model.

3.See Belsley (1969,pp46-53), or Holt et al. (1961, pp314-317) for details of the discussion to follow.

4.This has has been noted by Maccini(1973,1976).

5.With adequate backlog data this approach, of putting backlogs in the demand curve, is possibly applicable to production to stock industries as well, see footnote 8.

6.For a detailed exposition on the empirical aspect of this point, see Reagan and Sheehan (1982).

7. All figures were calculated using sample means. Details are in part III.

8.It can sensibly be interpreted as such an approximation only if the backlog B_{t-1} is larger than the inventory stock H_{t-1}, as is the case in all periods for all the data studied here. Since this is presumably not the case for production to stock industries, these ratios perhaps are not appropriate for use in stock industries, even if backlog data were available. Also, not all authors would enter these two ratios separately to capture the effects of delivery lags. For example Zarnowitz (1973,pp279ff) assumes the backlog-shipment ratio alone captures these effects.

9.One simple generalization is to enter B_{t-1}/S_{t-1} and H_{t-1}/S_{t-1} separately, to allow the effects of the two ratios to be different. This was tried, but the coefficients on the two ratios were insignificantly different. See section III.

10.An exception to this rule is certain products manufactured for the government, see Foss et al. (n.d., ch. 11).

11.Linear terms suppressed for notational convenience from the cost function as written in the text should be understood to include some terms in addition to those corresponding to the quadratic terms For generality, linear and trend terms coresponding to all basic variables in the model should be understood to be present. For example, $c_1 B_t$ or $c_2 N_t$, with c_1 and c_2 positive parameters, might be present to reflect overhead associated with backlogs and new orders. Note that individual deterministic terms are not identified.

12.For a more thorough analysis of a set of first order conditions for a production to order firm in a non stochastic but otherwise more general environment, see Maccini (1973).

13.The higher order MA component may also result from cost shocks as in Blanchard (1982), although these have been suppressed for simplicity.

14.Theil actually shows this only for GLS on seemingly unrelated regressions. But his proof readily extends to 3SLS with moving average errors.

15.See Gallant and Jorgenson (1981) for a proof that this difference may be used in the case of iid errors. That it may be used as well when there are MA errors, and the model and the constraints are linear, is easily shown. The proof mimics that of the standard linear model that the Wald test is identical to the difference between the constrained and unconstrained objective functions (Theil, (1972), pp143-44)).

16.The fact that estimates of the inventory-production cost ratio a_2/a_1 are small relative to previous previous chapters' estimates is particularly troublesome in that production to order industries supposedly are charaterized by especially large inventory carrying costs. See the introduction to the previous chapter.

17.Most of the estimates of (12.s) and (12.n) subject to this subset of the cross-equation constraints were similar to those obtained by imposing the entire set of constraints, and thus are not reported. The only exceptions were the estimates of $a_{10}a_{40}$. These were all significantly negative, but sometimes had values markedly different fro those reported in the tables. For example, for aggregate durables, system (12.s), the values of $a_{10}a_{40}$ from the three eqautions (asymptotic standard errors in parentheses): were: -35517 (3120), -24276 (2187), -43123 (915).

18.This is not to suggest that backlogs and inventories are equal and opposite in all respects, as argued in various places in this chapter.

19.Belsley's (1969) model appears to allow him to use his regression coefficients to obtain such structural parameters, but he does not attempt to do so.

CHAPTER V:

CONCLUSIONS

This concluding section summarizes the results of chapters III and IV, and suggests directions for future research.

The results of these two chapters have in general been supportive of the hypothesis that inventories and backlogs affect revenues as well as costs. Chapter III studied production to stock industries, in which backlogs are sometimes thought to be unimportant since they are numerically small (e.g., Auerbach and Green (1980,pp1-2)). The revenue effects of inventories and backlogs were formulated as an opportunity cost to holding a backlog. The backlog cost in general was statistically significant, significantly greater than the inventory cost, and, since it was estimated to be as much as half the production cost, economically significant as well. Thus it appears that backlog costs are important in these industries.

Chapter IV studied production to order industries, in which, although backlogs are sometimes (but not always) thought to be important, they are rarely allowed to affect demand directly, or to affect production costs differently than do inventories (e.g., Belsley (1969)). Chapter IV allowed for both of these affects. The revenue effects of inventories and backlogs were modeled by allowing these two variables (suitably normalized) to shift demand. Empirical results suggest that the two do shift demand, with implied elasticities of about one. Also, backlogs do turn out to significantly affect production costs, in a fashion that inventories do not. Typical levels of backlogs cause the increase in the marginal cost of the typical level of production to fall by about eighty per cent.

These results are of course not definitive, and some suggestions for generalizing and extending the results seem appropriate. These seem especially important since the results were not uniformly favorable: tests of overidentifying restrictions almost always rejected, and not all certain parameters were persistently wrong-signed.

One possible extension has been suggested in the concluding sections to chapters III and IV. It is possibly true that individual firm data are more appropriate than the aggregate data used here. It would therefore be desirable to test the models in these two chapters on disaggregated data, perhaps modifying them in the fashion suggested in those concluding sections.

At least two more substantial extensions seem particularly worth investigating. The first would allow for a richer specification of demand. In chapter III demand was left unspecified (although some assumptions are implicit in the specific form of opportunity costs used to model the revenue effects of inventories). Leaving demand unspecified undoubtedly leads to more robust estimates than those resulting when demand is precisely specified as in chapter IV, but also leads to estimates that are less efficient than those resulting when demand is correctly specified. The extra efficiency that results from specifying demand as well as supply could be marked, since the demand curve is the natural place to model the revenue effects of inventories and backlogs. Chapter IV does indeed specify a demand curve, but the curve was basically static and backward looking. Thus there appears to be a wide range of plausible demand side behavior yet to be explored.

The second extension would specify production costs more richly. In this thesis real production cost were allowed to vary only in certain limited ways. Now, production costs are of course of central importance, as indicated by the statistical significance of estimates of production cost parameters. Since these costs plausibly vary with real wages, material prices and interest rates, it would seem desirable to model them as so varying. Thus there appears to be much unexplored scope for modeling production costs.

In sum, the basic hypothesis that inventories and backlogs affect revenues as well as costs appears to have received sufficient support in this thesis to warrant further investigation using different models and different data.

APPENDIX:

A NOTE ON THE ECONOMETRIC USE OF CONSTANT DOLLAR INVENTORY SERIES

Economic models of production, sales and finished goods inventories are usually formulated in terms of physical units, with costs and revenues assumed to be functions of the number of physical units produced or sold (e.g., Belsley (1969), Blinder (1982), Eichenbaum (1982)). In empirical estimation of these models, it is assumed that in the data one dollar of finished goods inventories represents the same amount of goods as one dollar of sales. Unfortunately, this is not the case when the standard macroeconomic data sources are used, Department of Commerce constant dollar inventory series on the one hand and constant dollar shipments (calculated by deflating Department of Commerce nominal shipments by the appropriate wholesale price index) on the other. The problem is that the inventory figures are evaluated at what accountants call "cost," while the sales figures are evaluated at what accountants call "market."[1]

A simple example will illustrate the problem. In the first line of Table I is a two period sequence of production of physical units Q_t, sales S_t and end of period inventories H_t; the variables of course obey the identity $Q_t=S_t+H_t-H_{t-1}$. Sales in current dollars in the second line of the table is calculated simply by multiplying units sold by the market price P. The current dollar (book) value of

TABLE I

	H_0	Q_1	PI_1	UC_1	P_1	S_1	H_1	CGS_1	Q_2	PI_2	UC_2	P_2	S_2	H_2	CGS_2
physical units	100					90	110		100				120	90	
current dollars	90		100	1.0	2.0	180	109	81		110	1.1	2.1	252	99	120
constant (period one) dollars	100					180	110						240	90	

Q=production, PI=purchases of inventoriable goods, labor and overhead,
UC=unit cost=PI/Q, P=market price, S=sales, H=inventories,
CGS=cost of goods sold = PI-ΔH

inventories on this line is derived as follows. First, the firm's purchases of inventoriable goods, labor and overhead are summed to get the variable called PI in the table. Exactly what expenditures constitute PI depend on the firm's accounting policy[2] and is not important in the present context; but it is worth noting that for a firm with positive accounting profits, PI in general is less than current dollar sales. PI may be divided by the total number of units produced to get a unit cost UC. Book value of inventories equals the sum of the unit costs of all goods in inventory. If the unit cost of all 100 units of period zero inventories is \$.90, and the firm uses FIFO accounting, then the book value of inventories is as indicated on line 2 of the table (e.g., \$109 = (10 x \$.90) + (100 x \$1)). Cost of goods sold CGS is defined as PI-ΔH.

As may be seen, the ratio of nominal sales to book value of inventories overstates the ratio of units sold to units in inventory. This is because market prices will in general be above unit cost when the firm is making an accounting profit. And this overstatement remains even if sales and inventory figures are deflated to period one dollars, as may be seen in line 3 of the table. The Department of Commerce computes constant dollar inventory series in effect by calculating what the firm would have evaluated its inventories at if there were no inflation in the prices paid for purchases of inventoriable goods, labor and overhead. Since market price will in general be higher than unit cost, an overstatement remains.

It is obvious that if a dollar of inventories is to represent the same amount of goods as a dollar of sales, the constant dollar inventory figures should be multiplied in all periods by the base period ratio of market price to unit cost (equivalently, the constant dollar sales figure should be divided by this ratio). This ratio may be approximated from an individual firm's annual report as the ratio of S/CGS. (In the example, $S_1/CGS_1=2.2$ and thus overstates the correct conversion factor of 2.0. The calculated figure is off because the firm uses book value and not constant dollar inventories in its computation of CGS. The example is misleading in that in real data from manufacturing the bias this induces is likely to be very small.[3]) In aggregate data this same ratio may be computed from the Internal Revenue Service data on revenues and expenses which is available annually at the two digit SIC code level in its publication Statistics of Income-- Corporate Income Tax Returns. As suggested in Foss et al. (n.d. , p47n) the ratio of (business receipts) / (cost of sales and operations + rent + repairs + depreciation + taxes) approximates the ratio of (shipments)/(cost of goods sold).[4] Table II contains these ratios for aggregate manufacturing at the two digit SIC code level for 1972 (1972 was chosen because it is the base year for the latest Department of Commerce constant dollar inventory series). As may be seen, it is substantial, implying that in general a dollar's worth of finished goods inventories represents about twenty five per cent more physical goods than a dollar's worth of sales.

We close by noting the effect on regression estimates in linear models of adjusting the constant dollar figures. Suppose we are studying aggregate manufacturing finished goods inventories with a

TABLE II

Estimates of ratio of shipments/cost of goods sold
(SIC codes in parentheses)

(20) Food	1.1973	(24) Lumber	1.2243
(21) Tobacco	1.3997	(25) Furniture	1.2899
(22) Textiles	1.1779	(32) Stone	1.2842
(23) Apparel	1.2407	(33) Primary Metals	1.1632
(26) Paper	1.2548	(34) Fabr. Metals	1.2636
(27) Printing	1.4149	(35) Machinery	1.3646
(28) Chemicals	1.4064	(36) Electrical	1.3218
(29) Petroleum	1.2081	(37) Motor vehicles	1.2057
(30) Rubber	1.3079	(37) Transportation	1.1819
(31) Leather	1.2325	(38) Instruments	1.4823
		(39) Other	1.3615
All non-durables	1.2582	All durables	1.2629

All manufacturing 1.2605

Source: calculated as described in the text from Table 2 in IRS (1976) regression equation that has inventories on the left hand side and lagged inventories and lagged and current sales on the right hand side. The coefficient estimates on the sales variables that result when these are divided by the 1.2605 ratio from Table II will be 1.2605 times as large as the ones that result if sales are not properly adjusted. By not adjusting, then, we would underestimate how repsonsive the level of inventories is to sales by 26 per cent. This suggests, for example, that inventories are not quite as implausibly unresponsive to sales as some economists (Carlson and Wehrs (1974), Feldstein and Auerbach (1976)) have claimed, since their arguments have rested on comparison of the values plausibly predicted by their models with estimates produced by precisely this regression.

FOOTNOTES

1.Foss et al. (n.d., pp10,48) and the IRS (1981, p106) state that manufacturing inventories are generally evaluated by the firm at cost or the lower of cost or market. For finished goods inventories this in effect means inventories are evaluated at cost. That Department of Commerce constant dollar inventory series still evaluate inventories at cost is implicit in the discussion of how it deflates inventori in Foss et al. ((n.d)., chapters 5 and 13), Herman et al. (19 pp20-22), Hinrichs and Eckman (1981) and was stated explicitly by Mr. John Hinrichs of the Bureau of Economics Analysis in telephone conversations with me. The problem noted here is more serious for finished goods than for other stages of inventories, because (1) the spread between cost and market is likely to be largest here, and (2) it is here that economic models generally insist that inventories and sales be expressed in equivalent physical units. However, since for inventories at all stages of fabrication the Department of Commerce constant dollar inventory series evaluate inventories at cost, the series for other stages may be unsatisfactory for the same reason.

2.At a minimum PI must include all variable costs such as wages of production workers, purchases of materials used in production, expenditures on heat and light and rent; taxes and the amortized value of fixed costs such as depreciation may be included at the option of the firm, provided IRS regulations are met (Foss et al. (n.d.), chapters 2 and 10)).

3.This is because for manufacturing firms the mean absolute value of $H_t - H_{t-1}$ is very small compared to sales or production (see Feldstein and Auerbach (1976)). Thus the bias from using the book value of $H_t - H_{t-1}$ is likely to be small.

4.Foss et al. presumably suggest adding rent and repairs to compensate for direct costs not accounted for in the IRS figure for cost of sales and operations (see IRS (1976, p159)), depreciation and taxes to account for "full cost absorption" (Foss et al. (n.d.), chapter 10)). The IRS data unsuprisingly does not yield the ratio desired for other reasons as well: it covers tax returns filed over the course of a twelve month period centered on December of 1972 and thus reflects neither all nor only economic activity in 1972; its SIC code definitions apparently do not precisely match the Department of Commerce's (IRS (1976, p1)).

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REFERENCES

Abel, Andrew (1982). "Inventories, Stockouts and Production Smoothing," manuscript.

Abramowitz, Moses (1950). Inventories and Business Cycles, National Bureau of Economic Research: New York.

Anderson, T.W. (1971). The Statistical Analysis of Time Series, John Wiley and Sons: New York.

Auerbach, Alan J. and Jerry Green (1980). "Components of Manufacturing Inventories, NBER Working Paper no. 491

Belsley, David (1969). Industry Production Behavior: The Order-Stock Distinction, North-Holland: Amsterdam.

Bertsekas, Dimitri (1976). Dynamic Programming and Stochastic Control, Academic Press: New York.

Blanchard, Olivier (1982). "The Production and Inventory Behavior of the American Automovile Industry," NBER Working Paper no. 891

Blanchard, Olivier and Angelo Melino (1981). "Cyclical Behavior of Prices and Quantities in the Automobile Market," manuscript.

Blinder, Alan (1981a). "Reappraising the Production Smoothing Model of Inventory Behaivor," manuscript.

Blinder, Alan (1981b). "Retail Inventory Behavior and Business Fluctuations," Brookings Papers on Economic Activity, pp443-520.

Blinder, Alan (1982). "Inventories and Sticky Prices," American Economic Review, 72 (3) (June), pp334-348.

Bureau of the Census (1978). Manufacturers Shipments, Inventories and Orders: 1958-1977. Washington: U. S. Government Printing Office.

Bureau of the Census (1982). Manufactruers Shipments, Inventories and Orders: 1977-1982. Washington: U. S. Government Printing Office.

Carlson, J.A. and W.E. Wehrs (1974). "Aggregate Inventory Behavior," pp 311-332 in George Horwich and Paul Samuelson (eds.), Trade, Stability and Growth: Essays in Honor of L. A. Metzler,

Childs, Gerald D. (1967) Inventories and Unfilled Orders. North Holladn: Amsterdam.

Eichenbaum, Martin (1982). "Some Evidence on the Smoothing Properties of Finished Goods Inventories," manuscript.

Feldstein, Martin and Alan Auerbach (1976). "Inventory Behavior in

Durable Goods Manufacturing: the Target Adjustment Model," Brookings Papers on Economic Activity, pp351-401.

Foss, Murray F., and Gary Fromm and Irving Rottenberg (n.d.). Measurement of Business Inventories. Washington: U.S. Government Printing Office.

Gallant, A. Ronald and Dale W. Jorgenson (1981). "Statistical Inference in a System of Simultaneous, Non-linear, Implicit Equations in the Context of Instrumental Variables Estimation", Journal of Econometrics, 11 (2/3) (Oct.-Dec.), pp275-302.

Hansen, Lars Peter (1982). "Large Sample Properties of Generalized Method of Moments Estimators," Econometrica, 50(4) (July) pp1029-1054.

Hansen, Lars Peter and Thomas Sargent (1980). "Formulating and Estimating Dynamic Linear Rational Expectations Models," Journal of Economic Dynamics and Control, 2, pp7-46.

Hansen, Lars Peter and Thomas Sargent (1981). "Formualting and Estimating Linear Rational Expectations for Dynamically Interrelated Variables," pp127-156 in Lucas, Robert E. and Thomas Sargent (eds.) Rational Expectations and Econometric Practice, Minneapolis: University of Minnesota Press.

Hansen, Lars Peter and Kenneth Singleton (1982). "Generalized Instrumental Variable Estimation of Nonlinear Rational Expectations Models," Econometrica, 50(5) (September), pp1269-1286.

Hinrichs, John C. and Anthony D. Eckman (1981). "Constant Dollar Manufacturers' Inventories," Survey of Current Business, 61 (November), pp16-23.

Holt, Charles, and Franco Modigliani, John Muth and Herbert Simon (1961). Planning, Production, Inventories and Work Force, Prentice Hall: Englewood Cliffs.

Internal Revenue Service (various years). Statistics of Income--Corporations' Income Tax Returns, U.S. Government Printing Office: Washingont, D.C.

Irvine, F. Owen, Jr. (1981). Merchant Wholesales Inventory Investment and the Cost of Capital," American Economic Review, 71(1), (May). pp23-29.

Killeen, Louis M. (1969). Techniques of Inventory Management, American Management Association.

Lovell, Michael C. (1961). "Manufacturers' Inventories, Sales Expectations, and the Accelerator Principle," Econometrica, 29(3), (July), pp293-314.

Lovell, Michael C. (1964). "Determinants of Inventory Investment," pp197-224 in Models of Income Distribution, Princeton University Press: Princeton.

Maccini, Louis (1973) "On Optimal Delivery Lags," Journal of Economic Theory, 6 (April) pp107-125.

Maccini, Louis (1977) "An Empirical Model of Price and Output Behavior," Economic Inquiry, XV (October), pp493-510.

Maccini, Louis M. and Robert Rossana (1981). "Investment in Finished Goods Inventories: An Analysis of Adjustment Speeds," American Economic Review, 71(2), (May), pp42-47.

McCallum, Bennet (1976). "Rational Expectations and the Natural Rate Hypothesis: Some Consistent Estimates," Econometrica, 54/1, (January), pp43-52.

Mood, Alexander and Franklin Greybill and Duane Boes (1974). Introduction to the Theory of Statistics, New York: McGraw Hill.

Reagan, Patricia B. and Dennis P. Sheehan (1982). "The Stylized Facts about the Behavior of Manufacturers' Inventories and Backorders over the Business Cycle, 1959-1980," manuscript.

Rowley, J.R. and P.K. Trevedi (1975). Econometrics of Investment, John Wiley and Sons: New York.

Sargent, Thomas (1981). "VII: Linear Dynamic Equilibrium Models," manuscript.

Summers, Lawrence (1981). "Comment," Brookings Papers on Economic Activity, pp513-517.

Theil, Henri (1971). Principles of Econometrics, Amsterdam: North Holland.

Trevedi, P.K. (1970) "The Relation Between the Order-Delivery Lag and The Rate of Capaity Utilization in the Engineering Industry in the U.K., 1958-1967", Economica, XXXVIII, pp54-67.

deVany, Arthur and Frey, Gail (1982). "Backlogs and the Value of Excess Capacity in the Steel Indusry," American Economic Review, 72/3 (June), pp 441-45

White, Halbert(1982). "Instrumental Variables Regression with Independent Observations," Econometrica, 52/2 (March) pp483-500.