Essays in International Trade and Macroeconomics

by

Bumsoo Kim

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Author: Bumsoo Kim
Department of Economics
May 15, 2024

Certified by: Arnaud Costinot
Professor of Economics; Associate Head
Thesis Supervisor

Certified by: Stephen Morris
Peter A. Diamond Professor of Economics
Thesis Supervisor

Accepted by: Isaiah Andrews
Professor of Economics
Chairman, Departmental Committee on Graduate Studies

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Abstract

The thesis consists of three essays on international trade, macroeconomics and finance. In the first essay (joint with Marc de la Barrera and Masao Fukui), we study how the interaction between China's productivity growth and currency peg to the US dollar affected the labor market and trade imbalance in the United States. Empirically, we document that in response to similar exposure to Chinese exports, countries pegging to the US dollar experienced larger unemployment and trade deficits compared to floating countries. Theoretically, we develop a dynamic model of trade featuring endogenous imbalances and nominal rigidity, and show that Foreign growth may hurt Home welfare and characterize optimal trade and monetary policy in this environment. Quantitatively, we find that China's currency peg is responsible for 447 thousand manufacturing jobs lost in the US over 2000-2012, one third of the total US trade deficit over the same period, and reduced US lifetime welfare gains from Chinese growth by 32% compared to an economy where an otherwise identically growing China had its currency floating. A short-run safeguard tariff may have effectively accommodated China's currency peg and ameliorated the labor market distortions.

In the second essay (joint with Ying Gao and Marc de la Barrera), we study the Federal Reserve's problem of disclosing the models it uses in supervisory stress tests of large banks. Banks argue that nondisclosure leads to inefficiencies from uncertainty, but regulators are concerned that full disclosure can lead to banks gaming the system. We formalize this trade-off in a stylized model where both the regulator and banks have private "models" about a risky asset, and the regulator uses its own model to 'stress test' the investment. We show that the regulator may benefit from hiding the model when the bank's model is more precise than the regulator's own model. The key idea is that hiding the regulator's model forces the bank to guess it using the bank's own models, effectively eliciting the bank's private information. We also show that if the regulator can fine-tune disclosure policies, the regulator can approximately enforce the first-best action as if the regulator fully knew all the private information held by banks. The intuition is an application of the Cremer and McLean (1988) information rent extraction result.

In the third essay, I investigate the rationale behind East Asian countries' adoption of currency pegging strategies during periods of accelerated growth and explores the optimal peg level in the context of nominal wage rigidity and financial frictions. By developing a policy framework under wage rigidity, imperfect substitution, and exchange rate determination, I find that while the optimal peg level significantly influences real outcomes, standard economic models struggle to rationalize undervalued currency pegs. Potential justifications for an undervalued peg, such as export-driven learning externalities, are explored through numerical simulations.

JEL Classification Codes: F16, F31, F41, F66, G28

Thesis Supervisor: Arnaud Costinot
Title: Professor of Economics; Associate Head

Thesis Supervisor: Stephen Morris
Title: Peter A. Diamond Professor of Economics

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Iván was extremely friendly and helpful, even though I reached out to him only very late in my PhD. Conversations with him often felt like the first time I wore glasses – I got an unparalleled sense of clarity as he navigated my model through his lens and took his time to write out equations, graphs and their implications on a whiteboard. This process always gave me new exciting questions and novel insights, enriching both this thesis and my future research trajectory.

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This thesis is dedicated to HaRyung, Kwonsoo, and Hyunsoo. Thank you for being such an integral part of my journey.

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Chapter 1

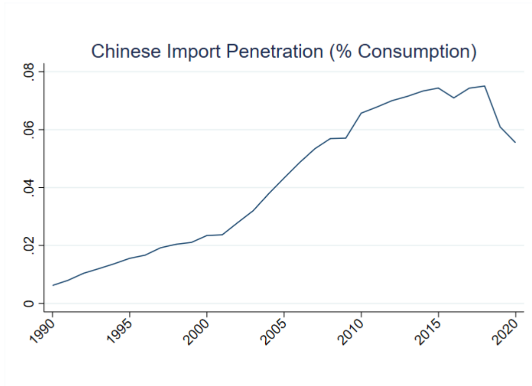
Currency Pegs, Trade Deficits and Unemployment: A Reevaluation of the China Shock

1.1 Introduction

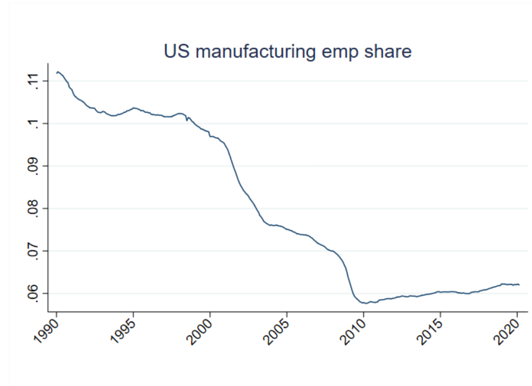
Four facts of the past two decades have drawn significant attention in both academic research and public discourse. First, China's exports to the US have grown significantly, driven by spectacular productivity growth and falling trade costs – henceforth the *China shock* (Figure 1.1a). Second, US manufacturing has undergone a significant decline, coupled with a rise in unemployment in manufacturing-heavy regions (Figure 1.1b). Third, the US has incurred a substantial trade deficit, while China ran a trade surplus (Figure 1.1c). Fourth, China has pegged its currency against the US dollar via an explicit peg (until 2004) or a managed band (after 2005) (Figure 1.1d).

An often-heard narrative in policy circles emphasizes how the last fact may have caused or magnified the first three. According to that narrative, *currency manipulation* by China might have been responsible for its sudden export surge to the US, large trade imbalances between the two countries, and, in turn, depressed the US labor market.[1] Although much has been said about the China shock in the trade and labor literature (Caliendo et al., 2019; Rodríguez-Clare et al., 2022; Dix-Carneiro et al., 2023), as well as the global savings glut in the international macro literature (Caballero et al., 2008; Mendoza et al., 2009; Kehoe et al., 2018), there has been no attempt at connecting the four facts collectively. This paper proposes to fill this gap by establishing a causal link between

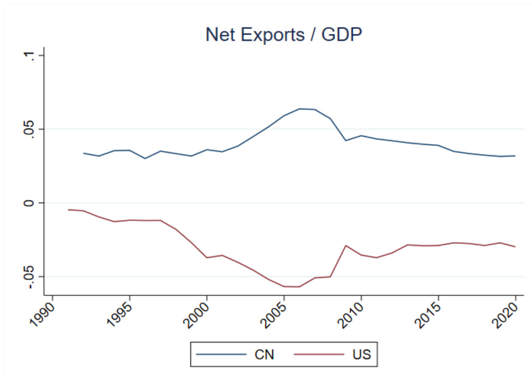
[1] Countries increase tariffs in response to unemployment (Bown and Crowley, 2013) and trade deficits (Delpeuch et al., 2021), consistent with this narrative and suggesting that it may have affected policy.



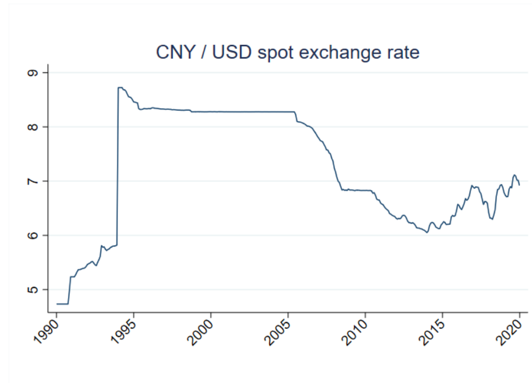
(a) Chinese import penetration



(b) US manufacturing employment



(c) Net exports (% GDP)



(d) CNY/USD exchange rate

Figure 1.1: Four stylized facts.

Sources: (a) Import of goods from China obtained from US Census Bureau and Bureau of Economic Analysis (BEA), US goods consumption obtained from BEA. (b) Bureau of Labor Statistics. (c) US Census and BEA. (d) Board of Governors of the Federal Reserve System (US). Retrieved from FRED.

the four facts, both empirically and theoretically, and reevaulate the consequences of the China shock and quantify the effect of China's peg in US outcomes.

Our contribution is threefold. First, we present an empirical finding that a country's exchange rate regime affects the incidence of the China shock on labor market outcomes and trade imbalances. We show that countries pegging to the US dollar – tying itself to Chinese currency – experienced a larger output decline, higher unemployment, and larger trade deficits in response to higher exposure to Chinese growth, unlike floating countries whose currency depreciated in response to China shock exposure. Second, we develop a model of trade with endogenous imbalances and wage rigidity that parsimoniously connects the four facts above by endogenizing the US trade deficit as a result of Chinese growth. We highlight the possibility that a country's welfare may decrease as a result of Foreign growth and study optimal policy responses. Third, we use a richer version of the same model to reevaluate the effects of the China shock and the role of China's exchange rate peg. We develop a highly efficient solution algorithm for solving dynamic macro-trade models with labor reallocation, and find that China's exchange rate peg contributed to a substantial part of the US trade deficit, decline in US manufacturing, unemployment, and reduced the welfare gains from the China shock.

In Section 1.2, we present evidence of the role of China's exchange rate peg in shaping labor market outcomes and trade imbalances in response to trade shocks. We use the joint fact that China's export growth post-2000 varied across sectors and that countries varied in their sectoral composition pre-2000 to construct a shift-share measure of country-specific exposure to the China shock, a cross-country analog of Autor et al. (2013, 2021). We then implement a triple-difference strategy that compares the *differential* impact of the *same* exposure between floating countries and countries pegged to the US dollar and, therefore, pegged to the Chinese currency. Our triple-difference strategy shows that a similar surge in exposure led to a lower manufacturing output, a temporary increase in unemployment, and larger trade deficits when the country's currency is pegged to the US dollar, relative to a country that floats.

In Section 1.3, we develop a dynamic model of trade with predictions consistent with the empirical findings and can jointly explain the four facts above. Our model is a two-period model with Armington trade in each period that allows consumption savings through an international bond market, and features short-run nominal wage rigidity. Under an exchange rate peg (Figure 1.1d), our model predicts that an increase in Foreign productivity (Figure 1.1a) causes a trade deficit at Home (Figure 1.1c) and Home workers face involuntary unemployment (Figure 1.1b). This holds provided that the trade elasticity σ is higher than the intertemporal elasticity γ, as documented empirically. The intuition is as follows: after Foreign growth, the Home relative wage should adjust through nominal wage or exchange rate. With both channels muted, the trade balance is determined by expenditure switching and relative inflation. When $\sigma > \gamma$, the expenditure switching channel dominates, Home runs a trade deficit, and shrinking global demand for Home goods causes unemployment at Home. This framework allows us to jointly explain the trade deficit and unemployment in manufacturing-heavy regions of the US as an endogenous outcome of Chinese growth under an exchange rate peg, parsimoniously explaining the stylized facts of the 2000s.[2]

[2]In related work, for which we explain in more detail below, Dix-Carneiro et al. (2023) study an environment with endogenous trade imbalances and unemployment due to search friction. As we show in the Appendix (Section 1.10), in such an environment with quantity friction, we get opposite predictions on the direction of trade imbalance, highlighting the role of nominal rigidity and exchange rate pegs in connecting these facts.

Turning to welfare and policy analysis, we show that Home welfare may even decrease as a result of Foreign growth when the trade elasticity is sufficiently high. Despite an improvement in terms-of-trade today, Foreign growth under a peg creates involuntary unemployment and future terms-of-trade deterioration due to required future trade surpluses. The higher the trade elasticity, the more expenditure is switched towards foreign goods, and the more severe the negative effects are. We show that the optimal short-run tariff in response to the shock is positive. Here, dynamic terms-of-trade considerations reinforce the standard motive for *safeguard* tariffs allowed by the WTO. We also highlight that Home's optimal monetary policy, barring constraints such as the Zero Lower Bound, would want to overshoot the output gap because it is borrowing and can set the global interest rate under a peg.

To explore the quantitative significance of the mechanism, Section 1.4 introduces a multi-country, multi-sector, infinite-horizon model consisting of two blocks. The first block is a workhorse trade model with input-output linkages and labor migration frictions (Caliendo et al., 2019), both of which shape how trade shocks affect the labor market. This trade block allows us to quantify the general equilibrium effects of the China shock using observed sector-level trade and worker reallocation data. The second block is a macroeconomic block comprising wage rigidity generating a New Keynesian Phillips Curve (Erceg et al., 2000), intertemporal balances from consumption-savings (Obstfeld and Rogoff, 2005), and exchange rate determination from financial flows (Itskhoki and Mukhin, 2021a). This macro block allows us to incorporate involuntary unemployment, endogenous trade imbalances, and compare exchange rate pegs with floating exchange rates.

We calibrate the model to exactly match the sectoral trade flow data from the World Input Output Database (WIOD) and labor adjustment data from the Current Population Survey (CPS). We develop a novel solution algorithm that allows us to quickly solve for the full sequence of wages, prices, labor allocation, and trade imbalances for any realized or counterfactual fundamentals and policies, including the exchange rate regime. We bring frontier computational methods from macroeconomics, leveraging the sequence-space Jacobian method introduced by Auclert et al. (2021a) and using advances in machine learning frameworks to efficiently solve for the equilibrium in minutes.

Section 1.5 conducts counterfactual and welfare analysis. We first quantify the effect of the China shock by comparing the realized economy with the counterfactual economy without Chinese productivity growth and trade liberalization. We find that the China shock can explain 2.25 percentage points of the US trade deficit between 2000 and 2012, 991 thousand manufacturing jobs lost, and may be responsible for a surge in unemployment of 3.04% over the same period, concentrated in the affected manufacturing sectors, estimates that are approximately double those in the previous literature. Turning to welfare analysis, we find that the China shock still increased the welfare of the US by 0.183%, an estimate lower than previous literature but still positive, showing that the surge in Chinese exports, even after accounting for involuntary unemployment and dynamic terms-of-trade effects due to the exchange rate peg, increases the welfare of the US.

We also consider an additional counterfactual economy without Chinese growth and trade liberalization, and also without China's *savings glut* – residual demand for savings by China, which we calibrate to match the trade imbalances of each country. We use this counterfactual to assess the contributions of China's savings glut to the outcomes of the US and find that the decline in manufacturing is nearly identical with or without China's savings glut. This reinforces the findings of Kehoe et al. (2018), which show that the global savings glut is responsible for only a small portion of the decline in US goods-sector employment (15.1%). We show that once we incorporate the exchange rate peg, China's residual savings glut had a negligible effect on the US manufacturing decline or the trade deficit. This finding underlines the centrality of the exchange rate peg in how the growth and savings of China affected the US.

Next, we isolate the effect of China's exchange rate peg on the same aggregate outcomes. The question we ask is: How different would the effects of the China shock have been without the peg? Comparing the realized economy with the counterfactual economy where an otherwise identically growing China floats its exchange rate, we find that China's peg to the US dollar is responsible for 1.3 percentage points of the US trade deficit (% GDP), 447 thousand manufacturing jobs lost. These equilibrium responses largely match those observed in the empirical findings (Section 1.2) and support the quantitative significance of the relevant channels in our theoretical model (Section 1.3). Balancing these factors, China's exchange rate peg lowered US lifetime welfare by 0.083% relative to an economy where the China shock occurred, but China floated its currency with respect to the US dollar.

Finally, we explore the consequences of counterfactual policies on labor market outcomes and US welfare. We ask the following questions: What would have been the impact on US welfare if different policy measures were implemented? What are the effects of a targeted tariff designed to reduce trade deficits? And finally, what is the role of monetary policy in shaping these outcomes? We find that a tariff of 15-20% on Chinese goods could have ameliorated the short-run labor market distortions, this positive effect remains even under retaliatory tariffs, and monetary policy could have been effective in reducing the distortion from the China shock, conditional on not being subject to the Zero Lower Bound.

The paper is accompanied by an Appendix containing a description of the data, proofs of the main propositions, and derivations of key equations, robustness tests, model extensions, further derivations, calibration details, and the solution algorithm.

Related Literature

Our paper contributes to a large trade and labor literature that studies the labor market consequences of globalization. On the empirical side, Autor et al. (2013, 2021), Acemoglu et al. (2016) have shown that US labor markets competing more with Chinese imports are hurt relatively more.[3] On the structural side, the seminal work by Caliendo et al. (2019) (henceforth CDP) quantifies the effect of the China shock across labor markets. We contribute to the structural trade literature by embedding a full New Keynesian macro block into CDP. This allows us to address involuntary unemployment, discuss the implications of endogenous imbalances, and study counterfactual policies.

Two closely related papers, Rodríguez-Clare et al. (2022) and Dix-Carneiro et al. (2023), also study unemployment in response to the China shock by augmenting CDP with labor market frictions. Rodríguez-Clare et al. (2022) (henceforth RUV) is most similar to ours in that they introduce wage rigidity. Our approach is different in two dimensions. First, we feature endogenous imbalances through consumption-savings and nominal rigidity generating a Phillips Curve. This complements their approach, which uses exogenous imbalances and demand anchors with a reduced-form downward nominal wage rigidity (DNWR). Second, our model underscores the central role of exchange rate pegs, allowing us to evaluate the welfare effect of China's USD peg on the United States. These differences allow our framework to highlight the effect of counterfactual monetary policies and exchange rate pegs.[4]

Dix-Carneiro et al. (2023) introduce endogenous consumption-savings to study the effect of the China shock and trade imbalances on the labor market and uses search

[3] Recent empirical papers that connect trade shocks with the labor market include Pierce and Schott (2016), Dix-Carneiro and Kovak (2017), Handley and Limão (2017), Carrère et al. (2020), Costinot et al. (2022). Autor et al. (2016) and Redding (2022) provide excellent review of the literature.

[4] In related work, Fadinger et al. (2023) study the effect of German growth on the Eurozone through a model of DNWR and consumption-savings, with an exogenous demand anchor. In such models, a floating exchange rate moves to clear all nominal frictions; on the other hand, a floating exchange rate in our model is financially driven and may not immediately adjust to clear the labor market across all sectors.

frictions à la Mortensen and Pissarides (1994) to generate unemployment.[5] However, the response to trade shocks qualitatively differs under nominal frictions (wage rigidity) and quantity friction (search) in two important ways. First, quantity friction amplifies terms-of-trade shocks and leads to a reduction in unemployment in response to Foreign trade shocks, in conflict with increased unemployment in regions more exposed to the China shock (Autor et al., 2013, 2021). Second, quantity friction generates a force for the US, not China, to run trade surpluses in response to Chinese productivity growth, necessitating an even larger exogenous *savings shock* to align with the observed trade imbalance. Under our model of wage rigidity, short-run unemployment and trade deficit in the US are endogenous outcomes of the Chinese productivity growth. Our framework can also investigate the effect of the exchange rate peg and study counterfactual tariffs or monetary policies, elements absent from their study.

We highlight how an exchange rate peg under nominal rigidity can generate trade imbalances. This contributes to the international finance literature that studies the "global savings glut" of the 2000s, a term first coined by Bernanke (2005). Recent work attributes the US current account deficit to financial frictions (e.g. Caballero et al. (2008, 2021), Mendoza et al. (2009)), business cycle dynamics (e.g. Backus et al. (2009), Jin (2012)) or demographics (e.g., Auclert et al. (2021b), Bárány et al. (2023)).[6] Our work highlights a goods-market explanation of the observed trade imbalances under exchange rate pegs that can exist concurrently with the financial origins. Through the lens of our quantitative model, we attribute 37.1% of the US deficit to China's exchange rate peg, with the remaining deficit attributable to other countries and potential financial mechanisms that we have abstracted from.

We contribute to the open economy macroeconomics literature by bridging it with structural trade models to study sector-level shocks, such as the China shock.[7] From Galí and Monacelli (2005, 2008) to more recent work such as Schmitt-Grohé and Uribe (2016) and Auclert et al. (2021c), the literature has studied the role of trade, exchange rates and monetary policy in the macroeconomy. We build on these papers along two dimensions. First, we consider the effects of the exchange rate peg for an economy facing a peg, necessitating a departure from the small open economy model, which a majority of the literature focuses on, and consider Home monetary policy that directly affects savings

[5]Kehoe et al. (2018) also study the effect of imbalances in the labor market, but do not study unemployment. Dix-Carneiro (2014), Kim and Vogel (2020, 2021), Galle et al. (2023) also embed search-and-matching into trade, without imbalances.

[6]See Gourinchas and Rey (2014) for a review of this literature.

[7]In doing so, we follow the recommendations of Rodríguez-Clare et al. (2022) by "*adding a Taylor Rule [..] allow agents to make savings and investment decisions, and incorporate international financial flows affecting exchange rates.*"

decisions abroad. Second, we incorporate a multisector trade model that allows us to investigate the macroeconomic effect of shocks such as the China shock that are very asymmetric across sectors.

Our work on tariffs and monetary policy in response to the China shock is closely related to the literature studying the macroeconomic consequences of trade policy and monetary policy in the open economy. The closest to our analysis are Jeanne (2020), Auray et al. (2023), and Bergin and Corsetti (2023), each of which studies the interaction of tariffs and monetary policy in an Open Economy New Keynesian model.[8] While our insights resonate well with theirs, these papers focus on steady-state and business-cycle optimal policy, whereas we study policies in a transition path in response to a permanent shock. As such, their government is focused on steady-state welfare maximization, while the government in our model seeks to affect dynamics, including endogenous imbalances.

We underscore the role of China's exchange rate peg in generating unemployment and a steeper decline for US manufacturing by worsening its competitiveness. This is closely related to the idea that flexible exchange rates are a shock absorber. Previous empirical evidence of such an absorber role has been documented in the goods market (Broda, 2001, 2004; Edwards and Levy Yeyati, 2005; Carrière-Swallow et al., 2021), labor market (Schmitt-Grohé and Uribe, 2016; Campbell, 2020; Ahn et al., 2022), and financial market (Ben Zeev, 2019). Our analysis in Section 1.2 provides additional support that flexible exchange rates operate as an adjustment margin for the China shock. Our model explicitly incorporates exchange rate regimes into a structural trade model, allowing us to quantify the welfare effects of a large emerging market economy's currency peg on the US.[9]

1.2 Empirics: Exchange Rate Regimes and the China Shock

This section presents motivating evidence for the relevance of China's exchange rate peg in how the China shock affected the US labor market and trade deficit. Public discourse puts trade deficits and the peg at the center of how China affected the US labor market: with Chinese productivity growth and a peg, cheap Chinese goods flood the US market,

[8] See also Barbiero et al. (2019); Lindé and Pescatori (2019); Barattieri et al. (2021); Auray et al. (2022) for tariffs, Ghironi (2000); Benigno and Benigno (2003); Devereux and Engel (2003); Faia and Monacelli (2008); Corsetti et al. (2010); Lombardo and Ravenna (2014) for monetary policy, and Erceg et al. (2018), Barattieri et al. (2021), Cacciatore and Ghironi (2021) for empirical analysis of tariffs, monetary policy and exchange rates.

[9] This also relates us to the exchange rate determination literature, such as Gabaix and Maggiori (2015), Itskhoki and Mukhin (2021a), Hagedorn (2021). Our model is a limit case of these setups.

shifting demand, exacerbating trade deficits, and harming US manufacturing. Would a floating exchange rate have functioned as a margin of adjustment? Establishing the sign and magnitude of the relationship between China's exchange rate peg and the labor market outcomes and trade balances is important in understanding the role the exchange rate plays in international trade.

To empirically answer this question, our focus must extend beyond the US and China, given the absence of a counterfactual scenario of Chinese export surge under a fully flexible exchange rate between the two countries. We overcome this challenge by comparing countries with different currency regimes vis-à-vis China's regime – peg to the US dollar – and similar exposure to Chinese exports. We construct a measure of each country's exposure to Chinese export growth, and conditional on the same exposure to the China shock, test (1) whether the nominal exchange rate responds to the China shock for floating countries, and if so, in which direction, and (2) whether countries pegged to the US dollar (including the US) experience a drop in output and employment, and larger trade deficits relative to countries that do not peg to the US dollar. Our findings are consistent with these two hypotheses and motivate our modeling framework and quantitative analysis in Sections 1.3 onwards.

1.2.1 Background: the China shock and exchange rate peg

A large literature investigates the role of Chinese productivity growth and decreased trade costs in disrupting the US labor market. Empirical evidence and quantitative estimations consistently find that the surge in Chinese exports is a key factor in the economic decline and potential welfare losses of regions and sectors with greater exposure. This *China shock* is primarily attributed to productivity growth (Hsieh and Ossa, 2016) and falling trade costs due to China's 2001 accession to the WTO (Handley and Limão, 2017), and plateaued after the early 2010s (Autor et al., 2021).

Concurrently to the export growth, China maintained an exchange rate peg to the US dollar. The renminbi (China's official currency) was pegged at a rate of 8.28:1 in June 1994 and sustained a hard peg until July 2005, which "contributed to the exploding exports and ballooning trade surpluses of the early 2000s" (Kroeber, 2014). Subsequently, the People's Bank of China (PBOC) implemented a managed *band*, allowing the currency to fluctuate within a narrow band. This band gradually widened from 0.3% in July 2005 to 1% in April 2012, with a hard peg during the Great Recession. The renminbi appreciated through a slow and controlled process, and Ilzetzki et al. (2019) classify China's exchange rate policy as a *de facto peg* from January 1994 to 2019.

1.2.2 Data and Measure of the China Shock

In this subsection, we outline the sources of our data and the construction of shocks. Additional details are provided in Appendix (Section 1.7).

Exposure to the China shock. To measure the exposure of a country i to the surge in Chinese exports, we follow Acemoglu et al. (2016) and Autor et al. (2021) to construct a shift-share measure of exposure that combines (1) a weight of each sector s for each country i and (2) global growth in Chinese exports for each sector s

$$S_i = \sum_s \underbrace{\lambda_i^s}_{\text{share}} \times \underbrace{g_C^s}_{\text{China shock in sector } s} \tag{1.1}$$

Here $g_C^s = \log(E_{CT}^s) - \log(E_{Ct}^s)$ is the global increase in Chinese export value for each sector s from the pre-shock period t to post-shock T ($t = 2000$ to $T = 2012$, following Autor et al. (2021)), and λ_i^s is a weight of each country i's exposure to Chinese export growth in sector s. Sectoral export data is obtained from the UN Comtrade database at the 4-digit SITC level, and we closely follow the cleaning procedures in Feenstra et al. (2005) and Atkin et al. (2022).

S_i is a shift-share measure (Bartik, 1991) of each country's exposure to the surge in Chinese exports and is akin to the local labor market exposure measure in Autor et al. (2013). From Equation 1.1, any variation in S_i across countries comes entirely from variations in sector share λ_i^s: countries with higher S_i face more competition from Chinese exports precisely because those countries had a larger share of sectors where Chinese exports increased. A sufficient condition for S_i identifying country i's exposure to the sectoral shocks is for the shocks g_C^s to be exogenous to demand-side confounders (Borusyak et al., 2022). We discuss this further in Section 1.2.5 find supporting evidence for shock exogeneity in the Appendix (Section 1.7).[10]

We define the weights λ_i^s of each sector s in country i. Gathering accurate data on 4-digit sector sizes across countries is difficult, and we proxy for the sector size using export value data, which is readily available. Thus, our baseline measure of each sector s's weight in each country i is given by

$$\lambda_i^s = \frac{E_{it}^s}{GDP_{it}}$$

[10] The assumption of exogenous shocks (or 'shifts') in the China shock context is standard and is used in Autor et al. (2013, 2021); Acemoglu et al. (2016).

where E_{it}^s is country i's total value of exports at the pre-period t; a higher share λ_i^s means country i is exporting relatively more to sector s. Thus, our measure of *exposure to China shock* for country i becomes

$$S_i = \sum_s \frac{E_{it}^s}{GDP_{it}} \Delta \log(E_C^s)$$

which has the following interpretation: a higher S_i means that country i is exporting more in sectors where Chinese exports globally increased. Thus, S_i measures how much country i's exports to third countries are substituted to China, which complements the China shock literature, which often studies domestic competition with imports from China. In the Appendix, we consider alternative weights λ_i and shocks g_C^s, showing that the results are robust to alternative choices.

Exchange rate regime. Because China's currency is pegged to the US dollar, we want to compare countries that use or peg to the US dollar to countries floating relative to the US dollar. We classify each country-year observation's de facto exchange rate regime using the Ilzetzki et al. (2019) (henceforth IRR) exchange rate classification. IRR categorizes every country's *de facto* exchange rate policy from 1946 to 2019 into a six-category classification, with the categories being: (1) peg; (2) a narrow band; (3) a broad band and managed float; (4) freely floating; (5) freely falling; (6) dual market with missing market data, with an anchor currency to each observation.[11]

We define the dummy variable Peg_{it} to be 1 if the country is the United States, or the country is classified as category 1 or 2 according to IRR and their anchor currency is the US dollar. We define Peg_{it} to be 0 if the country's currency is floating or is classified as category 1 or 2 and their anchor currency is not the US dollar. Observations in categories 3 (intermediate categories), 5 and 6 (freely falling or missing data) are dropped, and we also exclude countries whose Peg_{it} changes during our period of interest, as currency regime changes are highly endogenous and indicate turbulent economic conditions. In the remainder of this section, we say the country *pegs* if $\text{Peg}_{it} = 1$ and *floats* if $\text{Peg}_{it} = 0$, so that pegs and floats are with respect to the US dollar.

Outcome variables of interest. We consider the following outcome variables for each country: (1) nominal exchange rate; (2) real GDP; (3) manufacturing output; (4) unemployment; and (5) net exports. If the nominal exchange rate responds to higher S_i for floating countries but not for pegged countries, this is evidence that the exchange

[11] IRR also provides a fine 15-category classification. Details and the fine classification are given in the Appendix (Section 1.7).

rate is operating as an adjustment margin. Then, we investigate the effects of the margin through the dependent variables (2) to (5). Real GDP, manufacturing export, and trade balance are computed from the World Bank's World Development Indicators (WDI) database; the unemployment rate is from the International Labour Organization (ILO); the nominal exchange rate of a country is the effective exchange rate and obtained from Darvas (2012, 2021).

1.2.3 Empirical Design

Our goal is to test across different countries whether higher exposure to the China shock had differential effects depending on each country's exchange rate regime. Thus, we wish to test for countries i:

$$\mathbb{E}[\Delta Y_i | \Delta S_i, \text{Peg}_i = 1] \neq \mathbb{E}[\Delta Y_i | \Delta S_i, \text{Peg}_i = 0] \tag{1.2}$$

where Y_i denotes a dependent variable of interest (trade deficit, labor market, and goods market outcomes), S_i denotes exposure to the China shock, and Peg_i is a dummy variable for whether country i uses or pegs to the US dollar. This approach circumvents the heterogeneous exposure confounder – each country's differential exposure to the China shock – that may plague a simple binary test on the exchange rate regime.[12]

Triple-Difference Regression. Our novel analysis is to explore how the interaction between a country i's exposure to the China shock (S_i) and its currency regime (Peg_i) affects output, employment, and trade balances. We estimate first-difference models using successively longer time differences. For each year h, we implement Equation 1.2 through the following regression:

$$\Delta_h Y_{i,t+h} = \alpha_h + \beta_{1h} S_i + \beta_{2h} \text{Peg}_i + \beta_{3h} (S_i \times \text{Peg}_i) + X_i' \gamma + \epsilon_{ih}, \tag{1.3}$$

where $\Delta_h Y_{i,t+h} = Y_{i,t+h} - Y_{i,t}$ is the change in the outcome for country i between year $t + h$ and initial year t. X_i includes controls for country i's pre-period characteristics. This triple-difference design (over time, exposure, and exchange rate regime) compares how variations in outcomes between countries with similar exposure levels are influenced by the exchange rate regime. Rejecting the null $\beta_{3h} = 0$ supports the hypothesis in Equation 1.2: similar exposure to the China shock affects peggers and floaters differently.

[12] As such, confounders such as different industry composition or development levels should not affect our analysis, as they are captured by conditioning on S_i.

Following Autor et al. (2021), we focus on the period 2000 to 2019, comprising China's intense growth in the first decade and the plateauing in the second. Our definition of the China shock is growth in exports between $t = 2000$ and $t = 2012$. Hence, for $h < 12$, the estimate captures the effect of the partial shock from 2000 to $2000 + h$ on the outcome variables. For $h \geq 12$, the estimate is an event study of how the China shock impacts the outcome variable over a longer horizon.

Controls. The control vector X_i' includes country-specific characteristics that affect outcome variables of interest. We control for log population and log GDP per capita in each country at the starting period $t = 2000$. This is to control for the possibility that the effect of the China shock may interact with the size and development of this country. Since our construct of the shift-share exposure S_i implies $\sum_s \lambda_i^s \neq 1$ in general, we purge for the bias generated by incomplete shares, highlighted in (Borusyak et al., 2022) by including $\sum_s \lambda_i^s$ in our set of controls.[13] We control for the interaction of those controls with the Peg_i, to account for the possibility that the exchange rate peg is correlated with the shares, these variables, and affects the outcome variable differently. We also control for one lag of the outcome variable – if $Y_{i,t+h}$ is the outcome variable, we control for $Y_{i,t-1}$ for $h \geq 0$ and $Y_{i,t+h-1}$ for $h < 0$. The controls, with the exception of $\sum_s \lambda_i^s$, are obtained from the WDI.

Balanced Panel. Our empirical strategy rests on the identifying assumption that there are no omitted variables that are correlated with the exchange rate regime and affect the outcome variables differentially. Table 1.5 reports summary statistics in various observable characteristics between the countries pegging and floating with respect to the USD, and their differences. Pegging countries are smaller (Hassan et al., 2022), have a lower manufacturing share and moderately lower unemployment in 2000. However, peggers and floaters show broad similarity in other observable factors, including exposure to the China shock.

1.2.4 Results

Nominal exchange rate. We first ask whether the nominal exchange rate responds to the China shock. If exchange rates indeed serve as an adjustment margin, we would expect currencies of countries more exposed to the China shock to *depreciate more* under a floating

[13] We chose these weights because the alternative – divide by total exports – would mean that relatively closed countries are more exposed to the China shock, which is unrealistic. In the Appendix (Section 1.7), we conduct the same empirical specification with alternative weights λ_i^s that sum to 1.

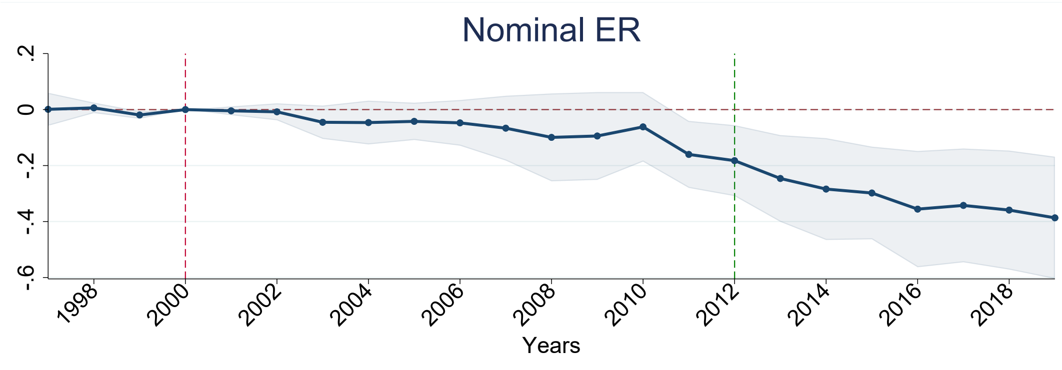


Figure 1.2: Exchange rate response to the China shock.

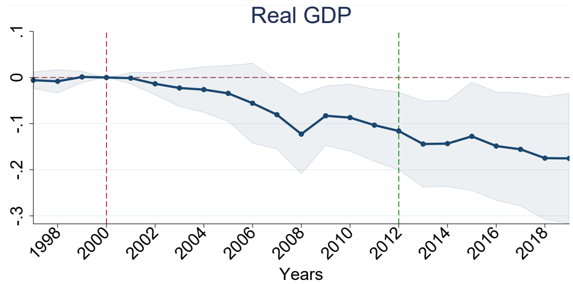
Note. The figure plots β_{3h} of the model 1.3 with the nominal exchange rate as the dependent variable across time. It shows the differential response of the nominal exchange rate among peggers and floaters to the China shock. In the Appendix, we plot the coefficient for the subset of countries where the currency is pegged versus floated against the US dollar respectively. A higher value of the nominal exchange rate implies depreciation of the currency. The shaded area is the 95% confidence band for each regression. The red dashed line indicates the beginning of the China shock (2000) and the green the end of the China shock (2012). The plotted coefficients have standard error of S_i normalized to 1.

regime. In contrast, we would not anticipate currency responses to the China shock for countries pegged to the US dollar. If true, this supports the hypothesis that competition with Chinese goods leads to depreciation in the currencies of floating economies, while the lack of such a response in pegged economies could lead to distortions.

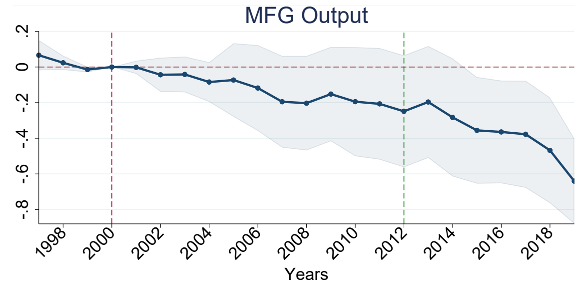
We report the estimated response of the nominal exchange rate to the interaction of the China shock and exchange rate regime using our triple difference strategy. Figure 1.2 displays the coefficients β_{3h} of the differential response between pegged and floating countries, together with the 95% confidence intervals. Conditional on similar China shock exposure S_i, floating countries have their currency *depreciate* compared to pegged countries.

The significance of this effect suggests that the exchange rate operates as an important margin of adjustment in global export competition. This perspective is often overlooked in the China shock literature, either empirically or structurally. We underscore that this role of the exchange rate may be relatively uncharted territory, and the absence of exchange rate adjustments may have real consequences, which we explore next.

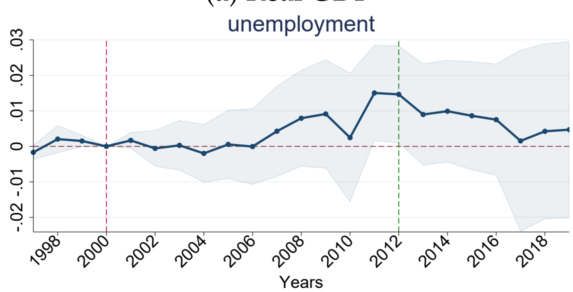
Output, Unemployment, and Net Exports. Next, we assess how the China shock affects pegged and floating economies *differently* for our variables of interest: real GDP, manufacturing output, unemployment rate, and net exports. If China's peg to the dollar influences the impact of the China shock on goods market outcomes and trade balances,



(a) Real GDP



(b) Manufacturing Output



(c) Unemployment rate

(d) Net Exports (% GDP)

Figure 1.3: Responses of peggers to USD vs floaters to USD to the China shock.

Note. The plotted coefficient β_{3h} is the differential response among peggers and floaters to the China shock. A positive coefficient implies that conditional on the same exposure to the China shock S_i, pegged countries' output variable response is higher than floating countries' response for the same variable. The shaded area is the 95% confidence band for each regression. The red dashed line indicates $t = 2000$, the start of the China shock, and the green line $t = 2012$, the end of the China shock. A comparison plot of the separate double-difference regressions for pegged and floating countries is provided in theAppendix (Section 1.7), in Figures 1.11 and 1.12 respectively. The plotted coefficients have standard error of S_i normalized to 1.

we should observe a non-zero β_{3t}, with the interpretation that countries more exposed to Chinese exports will experience a stronger decline in output, higher unemployment, and larger trade deficits if their currency is pegged to the US dollar.

Figure 1.3 plots our estimates of β_{3h} for those outcomes. For real GDP and manufacturing output, the left-hand side is $\log(Y_{i,t+h}) - \log(Y_{i,t-1})$ and is intended to measure percentage change. For the unemployment rate, we use the difference $Y_{i,t+h} - Y_{i,t-1}$, and for net exports, we use $\frac{NX_{i,t+h}}{Y_{i,t+h}} - \frac{NX_{i,t-1}}{Y_{i,t-1}}$. We report the double-difference results for the full sample and the pegged and floating countries separately in the Appendix (Section 1.7).

The top two panels of Figure 1.3 show that the real GDP and manufacturing output were more adversely affected by the China shock for pegging countries, even conditional on the same increase in exposure S_i. The negative effects on real GDP and manufacturing output for pegging countries build up during the trade exposure period and extend persistently for years after the shock.[14] Notably, the decline in manufacturing output attributable to the interaction of Chinese exports and currency peg is double the analogous effect on real GDP, suggesting that the manufacturing sectors are hurt more by higher exposure, in line with previous literature.

The bottom left panel (Figure 1.3c) shows that unemployment increases during the duration of the shock and reverts after the culmination of the shock. This finding suggests the existence of short-run friction in the labor market that is affected by higher exposure to the China shock when the currency is pegged, consistent with the notion that the friction in the labor market may be a *nominal* friction. The bottom right panel (Figure 1.3d) shows that the trade balances of pegged countries deteriorate more for pegged countries, and this decline persists.

In Figure 1.12, we show how peggers and floaters respond differently to higher S_i separately, by running regressions for each subsample and plotting β_1. We see that within the peggers, greater exposure to Chinese exports led to lower manufacturing output, a temporary increase in unemployment, and larger trade deficits. In sharp contrast, within the floaters, we find that nominal exchange rates adjust in a way that there is no material association between the exposure to Chinese exports and macroeconomic outcomes.

The difference of outcomes suggests that a country's peg to the US dollar – which pegs it to China – affects the incidence of the China shock on that country because the exchange rate cannot adjust to the China shock. These empirical findings provide additional support for the strand of literature that finds the costs of exchange rate pegs through the loss of a nominal adjustment margin (see e.g., Broda (2004) and Ahn et al. (2022)).

1.2.5 Discussion

Sensitivity analysis

Robustness. The results in Figures 1.2 and 1.3 are robust to alternative specifications. In the Appendix (Section 1.7), we progressively add and remove the controls, add additional controls, and change the time horizon of the China shock to be 2000-2010 and 2000-2007. In addition, we conduct a parallel analysis using an alternative shift-share instrument where the shares are now exports as a share of total exports from i (summing to 1) or

[14] Autor et al. (2021) suggest two reasons for why trade-exposed labor markets suffer long-lasting hardship; the first is that such regions are poorly positioned to recover because of a dearth of college-educated workers, and the second is that specialization in industries with Chinese competition left these regions exposed to industry-specific shocks that self-reinforce during decline (Dix-Carneiro and Kovak, 2017). We note that both are plausible.

where the shifts are increases in nominal export volumes. Our results are consistent across these alternative specifications.

Shift-share as leveraging shock exogeneity. As Borusyak et al. (2022) show, a sufficient condition for identification is for the industry-specific growth shocks g_C^s to be exogenous, clarifying the identifying assumptions in our analysis and the construction of the standard errors. In the Appendix (Section 1.7), we draw on recent literature (Borusyak et al., 2022; Borusyak and Hull, 2023) to test shock exogeneity and find supporting evidence for the shift-share measure S_i as leveraging quasi-random variation in the shocks g_C^s.[15]

Instruments and Bias. The shift-share analysis may be biased if Chinese exports and sectoral shares are both correlated with sectoral demand shocks. In studying US regions, Autor et al. (2013) overcome bias associated with US demand shocks by using exposure of other developed countries as instrument. As our concern is a global demand shock, we cannot construct an analogous instrument. However, such a shock would also violate the exogeneity of the aforementioned instrument in Autor et al. (2013). With lack of a superior alternative, we proceed with the OLS estimate.

Relation with exchange rate puzzles

Our empirical results raise the following question: how do we reconcile the fact that exchange rate regimes affect differential responses of macroeconomic aggregates to shocks to the fact that the unconditional correlation between exchange rates and output is close to zero? It is known that the exchange rate is disconnected from macroeconomic aggregates (Meese and Rogoff (1983), Itskhoki and Mukhin (2021a)), and while the nominal and real exchange rate volatility are highly correlated, (Mussa, 1986), such movements are orthogonal to behavior of other macro variables (Itskhoki and Mukhin, 2021b).

We argue that the *conditional* exchange rate response to exogenous shocks can be consistent with *unconditional* exchange rate disconnect.[16] Our empirical findings suggest that exchange rate movements *counteract* underlying shocks to fundamentals: a productivity growth leads to an increase in demand for that country's goods in partial

[15] Goldsmith-Pinkham et al. (2020) develop an alternative approach to identification of shift-share exposure based on the exogeneity of the initial-period shares λ_i^s; this is less suitable for our analysis.

[16] The conditional relation and unconditional disconnect can be microfounded through noisy expectation about future productivity (Chahrour et al., 2023) or through multiple financial shocks (Fukui et al., 2023).

equilibrium, and the general equilibrium response of the exchange rate moves in the opposite direction through an appreciation of that country's currency (Figure 1.2) – and the lack of this force has real consequences (Figure 1.3). This role of exchange rates as an insulator is documented in Broda (2004) using a VAR analysis of terms-of-trade shocks. Our analysis highlights that China's exchange rate peg to the US dollar can mute this insulator role for countries using the US dollar, leading to real consequences.

1.3 A two-period trade model with nominal rigidity

In this section, we develop a tractable model that rationalizes the unemployment in manufacturing and trade deficits as an outcome of Foreign productivity growth and an exchange rate peg, explaining concurrently the four facts (Figure 1.1) and corroborating the findings in Section 1.2. Our one-sector, two-period, two-country model highlights the role of exchange rate pegs and nominal wage rigidity. Using this model, we study the positive and normative implications of a trade shock and policy implications.[17] We keep the ingredients minimal for analytical tractability and extend the model in Section 1.4.

1.3.1 Model setup

Our environment has two countries, Home (H) and Foreign (F). In our application, Home will be the United States and Foreign will be China. There are two periods: $t = 0$ (short-run) and $t = 1$ (long-run). A representative household in each country consumes goods from both countries and supplies labor to firms that produce goods. Each country has its own nominal account; the price of country j's currency in units of country i's currency at time t is e_{jit}, with $e_{HHt} = e_{FFt} = 1$ and $e_{FHt} = \frac{1}{e_{HFt}}$. We denote $e_t = e_{FHt}$. Hence an increase in e_t is a depreciation of the Home currency.

Household preferences. In each country j, there is a representative agent who consumes goods C_{ijt} across origins i aggregated into a final good C_{jt}, supplies labor L_{jt}. The household has preferences represented by

$$\mathcal{U}_j = [u(C_{j0}) - v(L_{j0})] + \beta[u(C_{j1}) - v(L_{j1})], \tag{1.4}$$

[17] In the Appendix (Section 1.10), we analyze a two-sector tradable-nontradable model to study the decline in the share of manufacturing, and how trade shocks may propagate to nontradable sectors through aggregate demand. This section is intended to be minimal.

$$\text{where } u(C) = \frac{C^{1-\gamma^{-1}} - 1}{1-\gamma^{-1}}, \text{ and } C_{jt} = (C_{Hjt}^{\frac{\sigma-1}{\sigma}} + C_{Fjt}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}.$$

Here σ is the elasticity of substitution between domestic and foreign goods (the Armington elasticity), and γ is the elasticity of intertemporal substitution. We assume that the Armington elasticity is larger than unity, and the intertemporal elasticity is smaller: formally, $\sigma > 1$ and $\sigma > \gamma$.[18] $v(\cdot)$ is the disutility of supplying labor, which we assume is increasing and convex with $v(0) = 0$.

Technology. A representative firm in country i uses labor as input and has a constant returns to scale production function that requires $\frac{1}{A_{ij}}$ labor to supply a unit of good to market j. Thus for a firm in country i selling Y_{ij} goods to country j at time t using L_{ijt} labor, we have

$$Y_{ijt} = A_{ij} L_{ijt}.$$

A_{ij} implicitly incorporates trade frictions. Throughout we assume $A_{HF} \leq A_{HH}$ and $A_{FH} \leq A_{FF}$, implicitly assuming home bias in consumption.

Savings. Each country issues a domestic bond with zero net supply. In period 0, households in each country j have access to a claim of a unit of currency i in period 1, with the price of a claim being $\frac{1}{1+i_{i1}}$ in country i currency. We let B_{ij1} denote the amount of claims for i currency that households in country j own. We assume there is no risk, and bonds from Home and Foreign are perfect substitutes.

Labor Market and Nominal Rigidity. We consider the simplest form of short-run nominal wage rigidity. We assume that nominal wages in both countries are completely fixed in period $t = 0$ to an exogenous level $\{w_{j0}\}$, while wages $\{w_{j1}\}$ are flexible for $t = 1$. Since wages are rigid in period 0, we assume that the labor market is demand-determined in both countries, and workers supply whatever labor is demanded. In period 1, we assume that wages equalize labor supply and labor demand.[19]

Monetary policy and exchange rates. The monetary authority at Home sets the nominal

[18]Empirical estimates of σ range from 3-10 (Anderson and van Wincoop, 2003; Imbs and Mejean, 2017) to 1.5-3 (Boehm et al., 2023), but is consistently greater than 1. Estimates of γ are less than 1 and sometimes indistinguishable from 0. Section 1.3.5 draws on the literature to discuss this assumption. If we instead had $\sigma = \gamma = 1$, we are in the Cole and Obstfeld (1991) case, where the equilibrium always features trade balance. Thus this assumption is key to predicting the direction of trade imbalance.

[19]The assumption that wages are completely fixed is to highlight the intuition; any short-run friction in wage adjustment will yield qualitatively identical results.

interest rate according to a CPI-based Taylor rule with a coefficient of 1 on inflation:

$$\log(1+i_{H1}) = -\log(\beta) + \log(\frac{P_{H1}}{P_{H0}}) + \epsilon_{H0}, \tag{1.5}$$

where ϵ_{H0} is the discretionary monetary policy.[20] This rule implicitly sets the real rate $R_{H1} = (1+i_{H1})\frac{P_{H0}}{P_{H1}}$ at

$$R_{H1} = \frac{1}{\beta}\exp(\epsilon_{H0}).$$

We say a monetary policy *does not respond to shocks* if it sets $\epsilon_{H0} = 0$, or equivalently $R_{H1} = \frac{1}{\beta}$. In Sections 1.4 onwards, we consider a more standard Taylor rule, which delivers similar results.

Turning to Foreign monetary policy, we are interested in the equilibrium dynamics when Foreign pegs the nominal exchange rate to Home. We assume that Foreign monetary policy directly chooses the exchange rate

$$e_0 = e_1 = \bar{e}, \tag{1.6}$$

at an exogenous level $\bar{e}$.[21]

Trade taxes and subsidies. The government can also levy taxes on imports and subsidize exports. We assume that the Home government unilaterally chooses the short-run import tariff t_{FHt} and export subsidy s_{HFt}. If we denote the pre-tariff price of i goods to j at time t by P_{ijt}, Home government revenue is

$$T_{Ht} = t_{FHt}P_{FHt}C_{FHt} - s_{HFt}e_{FHt}P_{HFt}C_{HFt}. \tag{1.7}$$

We assume that the revenue T_{Ht} is rebated lump-sum to the representative household.

1.3.2 Competitive Equilibrium

In a competitive equilibrium, households maximize their utility, firms maximize their profit, and markets clear. We briefly derive each condition and relegate the details to the Appendix.

[20] This follows McKay et al. (2016), Auclert et al. (2021c), and allows our analysis to be orthogonal to the effects of monetary policy *rules*.

[21] An explicit monetary rule setting i_{Ft} that leads to the exchange rate peg can be found in Benigno et al. (2007).

Utility maximization. The household at country j chooses consumption $\{C_{ijt}\}$, $\{L_{it}\}_{t=1}$,$\{B_{ijt}\}$ to maximize utility $\mathcal{U}_H$ as described in Equation 1.4 subject to the sequential budget constraints,

$$\sum_i (1+t_{ij0})P_{ij0}C_{ij0} + \sum_i \frac{B_{ij1}}{1+i_{ijt}} e_{ij0} \leq W_{j0}L_{j0} + \Pi_{j0} + T_{j0}, \tag{1.8}$$

$$\sum_i (1+t_{ij1})P_{ij1}C_{ij1} \leq W_{j1}L_{jt} + \sum_i B_{ij1}e_{ij1} + \Pi_{j1} + T_{j1}, \tag{1.9}$$

where P_{ijt} is the (pre-tariff) prices for goods from country i to j in units of j currency, B_{j1} is a tradable claim to one nominal unit of account in period 1 with price $\frac{1}{1+i_{jt}}$, W_{jt} is the nominal wage, Π_{jt} is the profit of country j firms and T_{jt} is the government revenue rebated lump-sum.

The first-order conditions to this utility maximization problem are standard and given by:

$$P_{jt} = (\sum_i ((1+t_{ijt})P_{ijt})^{1-\sigma})^{1/(1-\sigma)}, \tag{1.10}$$

$$\lambda_{ijt} = \frac{((1+t_{ijt})P_{ijt})^{1-\sigma}}{\sum_l P_{ljt}^{1-\sigma}}, \tag{1.11}$$

$$v'(L_{j1}) = \frac{u'(C_{j1})w_{j1}}{P_{j1}}, \tag{1.12}$$

$$u'(C_{jt}) = \beta(1+i_{jt})\frac{P_{jt}}{P_{jt+1}}u'(C_{jt+1}) = \beta R_{jt} u'(C_{jt+1}), \tag{1.13}$$

$$\frac{1+i_{F1}}{1+i_{H1}} = \frac{e_1}{e_0}, \tag{1.14}$$

where P_{jt} denotes the consumer price index (CPI) in country j and λ_{ijt} the expenditure share. With the peg $e_1 = e_0 = \bar{e}$, the last condition becomes $i_{F1} = i_{H1}$ (trilemma).

Since wages $\{w_{j0}\}$ are rigid at $t = 0$ and the labor market is demand determined, we may have $v'(L_{j0}) \neq \frac{u'(C_{j0})w_{j0}}{P_{j0}}$. We define the *labor wedge* in period 0 as

$$\mu_{j0} = v'(L_{j0}) - \frac{u'(C_{j0})w_{j0}}{P_{j0}}, \tag{1.15}$$

how much the marginal value of working for households is away from the marginal return from working in utility terms. If $\mu_{j0} < 0$, households would like to supply more labor but cannot, so there is *involuntary unemployment*. If $\mu_{j0} > 0$, households are supplying more labor than they would want to, so the economy is *overheated*.

Firm optimization. The profits of a representative firm from j selling Y_{ijt} goods to market i is given by

$$\Pi_{it} = \sum_j \left[(1+s_{ijt}) \frac{1}{e_{ijt}} P_{ijt} - \frac{W_{it}}{A_{ij}} \right] Y_{ijt}$$

where s_{ijt} is an ad-valorem sales subsidy to i. Since firms are competitive, profits Π_{jt} are equal to 0, and the unit price is equal to marginal cost:

$$P_{ijt} = \frac{1}{1+s_{ijt}} e_{ijt} \frac{w_{it}}{A_{ij}}. \tag{1.16}$$

Market clearing. For each (i,t), the goods market clearing conditions are given by

$$L_{it} = \sum_j \frac{C_{ijt}}{A_{ij}}, \tag{1.17}$$

and the bonds market clearing condition is given by

$$B_{H1} + e_1 B_{F1} = 0. \tag{1.18}$$

Equilibrium. We are ready to define an equilibrium in the model as follows:

Definition 1.1. *Given fundamentals $\{A_{ij}\}$, rigid short-run wage w_{H0}, w_{F0}, policy $\{R_{H1}, t_{ijt}, s_{ijt}\}$ and pegged exchange rate $\bar{e} = e_0 = e_1$, a pegged equilibrium consists of prices $\{w_{it}, P_{it}, P_{ijt}\}$, household's choice variables $\{C_{ijt}\}$,$\{B_{it}\}$,$\{L_{it}\}_{t \geq 1}$ and demand-determined short-run labor $\{L_{i0}\}$ such that Equations 1.8 to 1.18 hold.*

1.3.3 Consequences of a trade shock

In this subsection, we highlight the equilibrium response to trade shocks in this model. As a benchmark, we consider the laissez-faire equilibrium where $t_{FHt} = s_{HFt} = 0$.

The timing of the model and the shock is as follows. Before the start of our setup ($t = -1$), productivities were at a level $\{A_{ij,-1}\}$, and nominal wages $w_{i,-1}$ and exchange rate e_{-1} were such that trade is balanced and labor wedge is zero. Right before $t = 0$, a shock permanently increases Foreign export productivity A_{FH}; we call this the *trade shock*. We assume that wages $\{w_{i0}\}$ are rigid at the pre-shock level $\{w_{i,-1}\}$, and the Foreign policymaker pegs the exchange rate $e_0 = e_1$ at the pre-shock level e_{-1}.

Equilibrium responses. To investigate the effects of the trade shock on trade balance and employment levels, we first observe how the terms-of-trade responds to a trade shock under a peg. We denote by $S_{HFt} = \frac{P_{HFt}\bar{e}}{P_{FHt}}$ the Home terms-of-trade at time t, where a higher terms-of-trade means a higher price of exports relative to imports. S_{HFt} is given by:

$$S_{HFt} = \frac{(\frac{w_{Ht}}{\bar{e}A_{HF}})\bar{e}}{\frac{w_{Ft}\bar{e}}{A_{FH}}} = \underbrace{(\frac{w_{Ht}}{w_{Ft}\bar{e}})}_{\text{relative wage}} \underbrace{(\frac{A_{FH}}{A_{HF}})}_{\text{productivity}} \tag{1.19}$$

If wages were flexible, an increase in A_{FH} affects S_{HF} in two ways. The *direct effect* increases S_{HF} by an equal proportion, improving Home terms-of-trade. The *general equilibrium (GE) effect* is that relative wage $\omega_t = \frac{w_{Ht}}{w_{Ft}\bar{e}}$ adjusts. Under the assumption that $\sigma > 1$, an increase in A_{FH} decreases Home's relative wage ω_t, so the GE effect reduces ω_t. If wages are flexible or the exchange rate is floating, the GE effect would take place immediately, and the equilibrium after the trade shock will be a new steady-state equilibrium with $\omega_0 = \omega_1$, without any dynamics between $t = 0$ and $t = 1$.[22]

However, when wages are rigid and the exchange rate is pegged, the GE effect is muted in the short-run. Then we have $\omega_0 > \omega_1$: Home's relative wage is higher in the short-run than the long-run. This results in the following comparative static:

Proposition 1.1. *In the pegged equilibrium, in response to a trade shock ($A_{FH} \uparrow$), Home runs a trade deficit ($B_{H1} < 0$). Moreover, if Home monetary policy does not respond ($R_{H1} = \frac{1}{\beta}$), then there is involuntary unemployment at Home ($\mu_{H0} < 0$).*

Proof. See Appendix 1.8. □

The logic behind the imbalances ($B_{H1} < 0$) is as follows. Home borrows if and only if:

$$\underbrace{\frac{\bar{e}\lambda_{HF0}P_{F0}C_{F0}}{\lambda_{FH0}P_{H0}C_{H0}}}_{t=0\text{ exports/imports}} < \underbrace{\frac{\bar{e}\lambda_{HF1}P_{F1}C_{F1}}{\lambda_{FH1}P_{H1}C_{H1}}}_{t=1\text{ exports/imports}} \Leftrightarrow \underbrace{\frac{\lambda_{HF0}/\lambda_{HF1}}{\lambda_{FH0}/\lambda_{FH1}}}_{\text{expenditure switching}} < \frac{\pi_F}{\pi_H}\frac{C_{H0}/C_{H1}}{C_{F0}/C_{F1}} = \underbrace{(\frac{\pi_F}{\pi_H})^{1-\gamma}}_{\text{relative inflation}} \tag{1.20}$$

Inequality 1.20 highlights the two forces that determine the sign of trade balance. The first force is *expenditure switching*. When $\sigma > 1$, we have $\omega_0 > \omega_1$, so both countries want to buy more Foreign goods today than tomorrow, implying $\lambda_{FH0} > \lambda_{FH1}$ and $\lambda_{HF1} < \lambda_{HF1}$, pushing towards Home deficit. The second force is *relative inflation*. With $\omega_0 > \omega_1$, Home's future prices increase *less* because of home bias in consumption. This pushes towards Home surplus if and only if $\gamma > 1$.[23] When $\sigma > \gamma$, expenditure switching

[22] The fact that a floating exchange rate can adjust for the GE effects under nominal rigidity is closely related to the Dornbusch (1976) overshooting model.

[23] In fact, estimates of γ are often 1 or less, whence relative inflation also leads to Home borrowing.

(governed by σ) outweighs relative inflation (governed by γ), resulting in Home trade deficit.[24]

Under a peg, Home's monetary policy cannot affect the sign of the trade imbalance. Home borrows regardless of R_{H1}, because R_{H1} affects the consumption-savings decision of both countries. In fact, when $\gamma = 1$, R_{H1} cannot even affect the magnitude of the deficit, as the effect of interest rates is exactly identical in both countries. We discuss this further in Section 1.3.4.

The intuition for Home unemployment is as follows. Short-run aggregate consumption C_{H0} is determined from the Euler equation. At C_{H0} and real wage $\frac{w_{H0}}{p_{H0}}$, Home workers would want to supply labor $L^S_{H0} = v'^{-1}(u'(C_{H0})\frac{w_{H0}}{P_{H0}})$. However, workers supply whatever is demanded, and the demand L_{H0} is pinned down by relative wage ω_0:

$$L_{H0} = \frac{1}{A_{HH}} \frac{\lambda_{HH0}(\omega_0) P_{H0}}{P_{HH0}} C_{H0} + \frac{1}{A_{HF}} \frac{\lambda_{HF0}(\omega_0) P_{F0}}{P_{HF0}} C_{F0}.$$

If ω_0 is higher, the desired supply L^S_{H0} increases but actual demand L_{H0} falls; this generates *involuntary unemployment*, with the unemployment rate given by $u_{H0} = 1 - \frac{L_{H0}}{L^S_{H0}}$.[25]

In contrast, under a floating exchange rate, we would observe neither deficits nor unemployment: as $\omega_0 = \omega_1$, the equilibrium is observationally equivalent to the new steady-state after the trade shock, with trade balance and full employment.

Proposition 1.1 parsimoniously connects the four facts in the introduction: the US trade deficit (Figure 1.1c) and surge in manufacturing unemployment (Figure 1.1b) can be endogenously explained by Chinese productivity growth (Figure 1.1a) and its exchange rate peg (Figure 1.1d). This contrasts with prior studies of the China shock, which typically perceive China's concurrent growth and savings as a puzzle. We show that China's peg under wage rigidity promotes a stronger short-term terms-of-trade during its growth, driving it to save.[26]

Proposition 1.1 supports the notion that nominal rigidity is the relevant friction in the China shock context, and allows us to differentiate from quantity friction such as search friction. This is because such frictions predict the opposite outcome – Home saves in

[24] An intuitive example is when $\sigma \to \infty$. Home wouldn't produce at all at $t = 0$, but it can compete against Foreign at $t = 1$. So Home wants to borrow to smooth consumption unless $\gamma = \infty$.

[25] In this economy, Foreign (China) is overheated and has employment rate greater than 1. We leave this open as a possibility and discuss potential microfoundations and implications in Section 1.6.

[26] Here we assumed that productivity A_{FH} increases from $t = -1$ but is the same between $t = 0$ and 1. If productivity were increasing between the two periods, there would be competition between our expenditure switching channel and the standard force for China to borrow. International finance papers such as Caballero et al. (2008) offer a financial solution.

response to Foreign growth. This is because relative wages across time is *reversed* under quantity friction: short-run Home relative wage is depressed, leading to Home saving and less unemployment. In the Appendix (Section 1.10), we formalize this by considering a quantity rigidity model, showing that indeed Home would save when Foreign grows.

Welfare effects. Next, we turn to the welfare implications of the trade shock. We first highlight that trade balances affect the future terms-of-trade: specifically, a deterioration in balances B_{H1} leads to a decrease in future relative wage ω_1. The intuition is closely related to the transfer problem: debt accumulated today becomes a future *transfer* for Foreign, which, combined with a home bias for demand, increases global demand for Foreign goods, improving their terms-of-trade and worsening Home's.

Using this fact, the next proposition highlights the possibility that Home's aggregate welfare may decrease as a result of Foreign growth:

Proposition 1.2. *In the pegged equilibrium where monetary policy does not respond ($R_{H1} = \frac{1}{\beta}$), a small increase in A_{FH} reduces Home welfare when σ is sufficiently high and improves Home welfare when σ is small (i.e. close to 1).*

Proof. See Appendix 1.8. □

An intuitive explanation is as follows. There are three channels through which productivity growth A_{FH} affects Home welfare:

$$\frac{d\mathcal{U}_H}{dA_{FH}} = -\underbrace{\frac{u'(C_{H0})}{P_{H0}} C_{FH0} \frac{dP_{FH0}}{dA_{FH}}}_{\text{terms-of-trade at } t=0} - \underbrace{\mu_0 \frac{dL_0}{dA_{FH}}}_{\text{labor wedge}} + \underbrace{\frac{\beta u'(C_{H1})}{P_{H1}} \left[C_{HF1} \frac{dP_{HF1}}{dA_{FH}} - C_{FH1} \frac{dP_{FH1}}{dA_{FH}} \right]}_{\text{terms of trade at } t=1} \tag{1.21}$$

The terms correspond to (1) the short-run effect of cheaper import goods, (2) labor market friction caused by wage rigidity, and (3) change in long-run terms-of-trade, including direct productivity effects and general equilibrium effects. If $\sigma \to 1$, preference becomes Cobb-Douglas, the pegged equilibrium coincides with the flexible-wage equilibrium, and trade is balanced as in Cole and Obstfeld (1991). Then the effects (2) and the general equilibrium component of (3) go to zero, leaving cheaper goods as the primary welfare benefit. In the opposite case, when $\sigma \to \infty$, short-run demand for Home goods becomes 0. Then, a small change in A_{FH} can cause a discrete loss of utility from the labor wedge and the trade deficit worsening future terms-of-trade, dwarfing welfare gains from cheaper goods.

The possibility of Foreign productivity growth harming Home welfare echoes immiserizing growth where Home's productivity growth worsens its terms-of-trade, negating gains from the expansion of the production frontier (Bhagwati, 1958). In our case, Foreign productivity growth improves Home terms-of-trade, and the peg magnifies this gain today, but unemployment moves Home production into the interior of the PPF and harms future terms-of-trade through trade deficit, offsetting the gains.

Proposition 1.2 cautions against using trade balance as a welfare indicator. Public discourse often views trade deficits as inherently undesirable. However, whenever σ exceeds 1 and surpasses γ, a trade deficit is the predicted outcome for Home under a trade shock under a peg. The shock may benefit Home welfare if σ is not excessively high. Conversely, a large γ with $\sigma \to 1$ results in Home's trade surplus and welfare gains, whereas with $\gamma > \sigma$ both large, Home faces welfare losses despite a trade surplus. In the next sections, we undertake a quantitative analysis of the substitution, rigidity, and productivity growth to assess whether the China shock improved or harmed aggregate US welfare.[27]

1.3.4 Policy response

In this subsection, we consider the unilateral problem of the Home government facing a growth in A_{FH} and an exchange rate peg. We assume the Home government can choose its short-run tariff level t_{FH0}, domestic subsidy s_{HF0} and monetary policy R_{H1}.[28] We assume the government cannot choose long-run tariff t_{FH1}, as the motivation for long-run tariffs as terms-of-trade manipulation is well understood since Graaff (1949).

Formally, the policy problem that the Home government faces is:

$$\max_{t_{FH0}, s_{HF0}, R_{H1}} \mathcal{U}_{\mathcal{H}} = \max_{t_{FH0}, s_{HF0}, R_{H1}} \sum_{t=0}^{1} \beta^t [u(C_{Ht}) - v(L_{Ht})] \tag{1.22}$$

subject to the same equilibrium conditions.

We first note that the planner can replicate the flexible price outcome. Indeed, if $\omega_{peg} = \frac{w_{H0}}{w_{F0}\bar{e}}$ is the short-run relative wage under peg, and $\omega_f = \frac{w^f_{H0}}{w^f_{F0}e^f}$ is the relative wage under flexible price (after the trade shock), the planner can set $R_{H1} = \frac{1}{\beta}$ and $t_{FH0} = s_{FH0} = \frac{\omega_f}{\omega_{peg}} - 1$. This tax and subsidy level sets the relative prices equal to the flexible price level, and the tax revenue and cost of subsidy cancel out exactly. Thus, we know the planner can undo the wedges and the potential welfare losses in Proposition 1.2.[29]

[27] Whether trade deficits are symptoms of welfare gains or losses is a different question to whether capital controls are beneficial. The next subsection shows that capital controls unambiguously hurt Home welfare.

[28] Since wages are rigid, we do not have Lerner symmetry, and subsidies and tariffs are independent.

[29] This connects with Farhi et al. (2014) that fiscal instruments can replicate currency devaluations.

However, this policy may not be optimal for the Home government. As an extreme example, if Foreign is offering goods for free, Home would be much better off taking those goods than setting high tariffs that distort consumption.

To solve for the optimal policy, we proceed in two steps. First, we solve for the optimal trade policy (t_{FH0}, s_{HF0}) given monetary policy R_{H1}, then we proceed to solve for the optimal R_{H1}. This approach makes the problem more tractable, and the inner problem may be a more reasonable benchmark of reality, where monetary policy is unable to fully respond to a sector-origin specific trade shock.[30] We give an executive summary of our results and discuss the details in the Appendix (Section 1.9).

Optimal trade policy

Given monetary policy R_{H1}, an indirect formula for the optimal trade policy can be obtained via a first-order variation argument. Starting from the optimal policy, the marginal effect of policy change in welfare must be zero, yielding the following formula:[31]

Lemma 1.1. *The optimal short-run tariff rate on imports t_{FH0} satisfies*

$$t_{FH0} = \frac{1}{P_{FH0}} \left[\underbrace{\frac{\mu_0}{\tilde{\lambda}} \frac{\partial L_{H0}}{\partial C_{FH0}}}_{\text{labor wedge}} - \frac{1}{(1+i_{H1})} \underbrace{\left(L_{HF1} \frac{\partial w_{H1}}{\partial C_{FH0}} - L_{FH1} \frac{\partial w_{F1}}{\partial C_{FH0}} \right)}_{\text{future terms-of-trade}} + \underbrace{s_{HF0} P_{HF0} \frac{\partial C_{HF0}}{\partial C_{FH0}}}_{\text{subsidy externality}} \right] \tag{1.23}$$

The optimal short-run subsidy rate on exports s_{HF0} satisfies

$$s_{HF0} = \frac{1}{P_{HF0}} \left[- \underbrace{\frac{\mu_0}{\tilde{\lambda}} \frac{\partial L_{H0}}{\partial C_{HF0}}}_{\text{labor wedge}} + \frac{1}{(1+i_{H1})} \underbrace{\left(L_{HF1} \frac{\partial w_{H1}}{\partial C_{HF0}} - L_{FH1} \frac{\partial w_{F1}}{\partial C_{HF0}} \right)}_{\text{future terms-of-trade}} - \underbrace{P_{HF0} C_{HF0} \frac{\partial s_{HF0}}{\partial C_{HF0}}}_{\text{terms-of-trade today}} \right] \tag{1.24}$$

where $\tilde{\lambda}$ is the Lagrange multiplier on the lifetime budget constraint.

Proof. See Appendix 1.8. □

[30] In the early 2000s, the government was tightening monetary policy in response to concerns over inflation and tightening of unused resources; loosening in response to the China shock was not the Federal Reserve Bank's goal (Federal Reserve Board, 2005). Following the Great Recession, the Federal Reserve Bank was subject to the Zero Lower Bound.

[31] A similar argument can be found in Costinot et al. (2022).

The first-order formula for tariffs succinctly captures the three *externalities* of imports that the Home government seeks to address via a tariff. First, tariffs and subsidies both reduce the labor wedge by stimulating demand for domestic labor. Second, tariffs and subsidies, by affecting relative prices of goods, improve current trade balance (Inequality 1.20), which improves the terms-of-trade in the future. Third, the fiscal externality (deadweight loss) of tariffs and subsidies interact in general equilibrium. In a competitive equilibrium, home households do not internalize any of these effects of an extra unit of import. Thus the tax level $t_{FH0}P_{FH0}$ and the subsidy level $s_{HF0}P_{HF0}$ can be considered a Pigouvian tax that corrects for the three externalities of consuming an extra unit of import or exporting an extra unit.

Using the formula, we can sign the optimal tariff and show that its magnitude *increases* with the Foreign shock A_{FH0}:

Proposition 1.3. *If there is unemployment at the zero-tariff economy ($\mu_{H0} < 0$ when $t_{FH0} = 0$), the optimal tariff t_{FH0} is positive and is increasing in the size of the trade shock A_{FH0}.*

Proof. See Appendix 1.8. □

The intuition that we can and should use tariffs as second-best instruments to fix distortions is well-known. The prediction obtained in Proposition 1.3 is sharper. We show that in an environment where trade shocks cause unemployment and trade deficits, the tariff should be positive and increase in the magnitude of the trade shock. In this context, the short-run tariff t_{FH0} is akin to *safeguard* tariffs allowed under the WTO Agreement on Safeguards.

But this is not the only role of tariffs in our model, as highlighted in the future terms-of-trade term in Equation 1.23. While tariffs do not affect today's terms-of-trade (due to wage rigidity and peg), a unilateral short-run tariff reduces Home's trade deficit, improving Home's future terms-of-trade. Hence, Home would want to set tariffs beyond the globally optimal "distortion-fixing" level, at the expense of Foreign welfare. As such, short-run tariffs are *safeguard* and *beggar-thy-neighbor* at the same time, even when the short-run terms-of-trade is rigid.[32]

Our model underscores that under an exchange rate peg, the optimal short-run tariff is increasing in the magnitude of the trade shock. This contrasts with the flexible exchange rate case, where the optimal tariff is pinned down primarily by the trade elasticity (Gros, 1987) and does not depend on the shock magnitude. Our framework focuses on tariffs

[32] By nature of being beggar-thy-neighbor, Foreign can retaliate with its own tariffs to undo the imbalance-adjusting channel of Home tariffs.

that correct a distortion caused by the peg and the trade shock, so the magnitude of the optimal tariff scales with the size of the distortion.

Proposition 1.3 assumes monetary policy does not clear unemployment. As aforementioned, the central bank may be unable to clear the output gap caused by sector-specific trade shocks because of multisector considerations, financial concerns, and liquidity constraints such as the Zero Lower Bound. Tariffs will be a useful tool in this second-best world.

Optimal monetary policy

What is the optimal monetary policy R_{H1}? An analogous first-order condition on monetary policy highlights the channels in which monetary policy affects welfare. We highlight a special case when the intertemporal elasticity is equal to 1 (consumption is log):

Proposition 1.4. *When $\gamma = 1$, optimal monetary policy R_{H1} satisfies the following equation:*

$$0 = \underbrace{-\mu_0 \frac{dL_0}{dR_{H1}}}_{\text{wedge}} + \tilde{\lambda}_r [\underbrace{R_{H1} t_{FH0} \frac{P_{FH0}}{P_{H0}} \frac{dC_{FH0}}{dR_{H1}}}_{\text{tariff fiscal externality}} + \underbrace{(NX_0)}_{\text{intertemporal TOT}}], \tag{1.25}$$

where $\tilde{\lambda}_r$ is the Lagrange multiplier on the Home lifetime real budget constraint normalized by P_{H0}.

As a special case, when $t_{FH0} = 0$, the optimal monetary policy R_{H1} is such that $\mu_0 > 0$: it is optimal to loosen monetary policy beyond clearing the output gap.

Proof. See Appendix 1.8. □

Proposition 1.4 highlights that when Foreign pegs, the optimal monetary policy for a borrowing Home will *overshoot* the output gap. This leverages Home's control of *global* monetary policy and manipulate intertemporal terms-of-trade to its favor. Particularly for the US, which influences global rates as the dominant currency (Gopinath et al., 2020) and runs current account deficits, the central bank may want to set a lower interest rate, with minimal risk of bond liquidation from pegging countries.

The proposition also clarifies that tariffs are second-best instruments when monetary policy cannot respond – whether due to the ZLB or multisectoral considerations. In fact, under a positive tariff, the additional losses from tariff fiscal externality compels Home to set a higher interest rate, reducing overall welfare.[33]

The assumption $\gamma = 1$ allows us to circumvent the effect of today's monetary policy on the magnitude of the trade deficit. When $\gamma = 1$, the effect of interest rate on consumption and output is proportionate in both countries: thus the real value of the deficit does not change, and monetary policy R_{H1} does not affect the intratemporal terms-of-trade in the future. On the other hand, when $\gamma \neq 1$, the optimal monetary policy equation (Equation 1.25) comes with an additional "future terms of trade" term: monetary policy may affect the magnitude of the deficit in real terms (but not the sign, as we discussed in Section 1.3.3), affecting the optimal policy.

Capital Controls

Lastly, we study the welfare effects of the endogenous deficits we highlighted in Proposition 1.1 by considering *capital controls* in addition to the tariffs and subsidies. We have established that deficits and unemployment can come from the same cause – trade shock and exchange rate peg – but are deficits inherently bad for Home welfare? While this is where some policy narratives go, the next proposition shows that this is not the case.

Proposition 1.5. *In the pegged equilibrium, removing international financial flows (forcing $B_{H1} = 0$) worsens Home unemployment (μ_{H0} decreases), and reduces Home welfare $\mathcal{U}_0$.*

Proof. See Appendix 1.8. □

Removing financial flows worsens Home unemployment because of home bias in consumption. Indeed, with trade costs, under the same price levels, Home borrowing to consume will increase demand for Home goods, while Foreign saving will decrease demand for Foreign goods. Since unemployment is determined by aggregate demand, Home's trade deficit in the short-run actually ameliorates unemployment, and capital controls will only worsen unemployment. As such, while deficits may be symptoms of a friction that may harm the economy, deficits themselves are not a friction to solve, and capital controls may harm Home welfare. The fact that financial transfers are welfare-improving under an exchange rate peg is closely related to the idea that fiscal unions are desirable under currency unions (Farhi and Werning, 2017); we highlight that the possibility of a dynamic budget-balanced (net current value zero) transfer is welfare-improving.

[33] In the Appendix (Section 1.9), we numerically solve for the joint optimal trade and monetary policy for various levels of the trade shock A_{FH0}. We find that the joint optimal policy involves no tariffs and a very loose monetary policy, highlighting the distortionary nature of tariffs. In a first-best one-sector world, Home would take advantage of the cheap goods and solve the labor wedge solely through monetary policy.

1.3.5 Discussion

Our framework shows that the consequences of trade shocks under a peg depend on labor market frictions, and tariffs and monetary policy can ameliorate welfare losses. Here we address potential questions, including the duration of nominal rigidity and the parameter values.

Duration of nominal rigidity. The prolonged impact of the China shock may raise questions on the role of nominal rigidity. Our answer is twofold. First, the China shock was a persistent event over the 2000s than a one-off event in 2000, aligning observed patterns with short-term mechanisms. Second, the relevant rigidity here is wage rigidity. Downward nominal wage rigidity (DNWR) is known to be persistent and can extend the effects of Foreign shocks well beyond the typical span of price rigidity, as discussed in Schmitt-Grohé and Uribe (2016).

The elasticities of substitution. Our findings rely on $\sigma > \gamma$: the consumption of goods across origins is more substitutable than across time. Trade elasticity (σ) estimates range from 1.5 to 10 but consistently above unity (Costinot and Rodríguez-Clare, 2014; Imbs and Mejean, 2017; Boehm et al., 2023), and recent literature (Teti, 2023) suggests that lower estimates might stem from tariff misreporting, indicating actual elasticity is closer to the higher estimates. The intertemporal elasticity (γ) is generally estimated to be below 1, with some studies finding it near zero (Hall, 1988; Best et al., 2020), supporting the assumption of $\sigma > \gamma$.[34] In Section 1.4, we introduce a multisector model of high substitution within sector but lower substitution across sectors, and confirm that high within-sector substitutability drives our results.

Multisector considerations. We used a one-sector model to highlight the main mechanism. In the Appendix (1.10), we introduce a two-sector model, separating tradables from nontradables in segmented labor markets. The expanded model predicts similar effects of Foreign growth under a peg: short-term trade deficits and tradable sector unemployment.

The extended model also highlights distributional effects. First, the output share of tradable declines even absent labor reallocation. Second, if monetary policy is

[34] The international macroeconomics literature uses a much lower macro-trade elasticity to rationalize International Real Business Cycle (IRBC) facts (Backus et al., 1994). Feenstra et al. (2018) estimate the macro- and micro-elasticities, and find that the macro-elasticity is "not as low as the value of unity sometimes found using macro time series methods," further supporting our notion that the trade elasticity is at least unity.

unresponsive, we have unemployment in both sectors: the recession *spills over* to the nontradable sector through aggregate demand. Third, monetary policy faces a trade-off between a recession in the tradable sector and overheating in the nontradable sector, explaining the US service sector boom in the 2000s. Further analysis is given in the Appendix, and the subsequent sections provide a quantification of the China shock through a general equilibrium multisector model.

1.4 Quantitative model

In this section, we extend the model in Section 1.3 so that it can be taken to sector-level trade data for a general equilibrium analysis of the effects of Chinese growth and the peg. We generalize the previous setup in two directions: (1) a multi-sector, multi-country model with Ricardian forces, input-output linkages and labor reallocation (Caliendo et al., 2019); (2) an infinite-period model with wage rigidity (Erceg et al., 2000), consumption-savings pinning down trade balances (Obstfeld and Rogoff, 1995) and exchange rate determination from financial channels (Itskhoki and Mukhin, 2021a). The first block allows us to investigate how the China shock, a sector-specific shock, affects other sectors, while the second block allows us to consider involuntary unemployment, endogenous trade imbalances, and the role of exchange rate pegs.

1.4.1 Model Setup and Equilibrium

In the model, time is discrete and indexed by $t = 0, 1, \cdots$. The economy consists of $i, j = 1, 2, \cdots, I$ countries, each with an exogenous labor endowment given by a continuum of workers with mass $\bar{L}_i$ (thus, we rule out migration across countries). There are $n, s = 1, 2, \cdots, S$ sectors. Unless otherwise stated, i is the producer/exporter, j is the importer/buyer, and we write exporters first in subscripts. Country 1 is the USA; country 2 is China; we are mainly interested in the interaction between these two countries. Each country has its nominal account, and nominal variables are denominated in the currency of the price-facing household. The exchange rate e_{jit} is the value of currency j with respect to currency i, so an increase in e_{jit} is a relative depreciation of i currency with respect to j currency. We present the main assumptions and relegate the derivations and details to Appendix 1.8.

Household preferences. In each country j, there is a representative household family that comprises atomistic *members* m of measure $\bar{L}_j$ and has preferences represented by

$$\mathcal{U}_j = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \delta_{jt} \int_0^{\bar{L}_j} \mathcal{U}_{jt}(m) dm, \tag{1.26}$$

where $\mathcal{U}_{jt}(m)$ is the member-specific utility, β is a discount factor common across all countries, and δ_{jt} is a country-specific intertemporal preference shifter which captures financial factors exogenous to our model. We implement our model at an annual frequency, so each period t corresponds to a year.

The utility of each member m depends on final goods consumption $C_{jt}(m)$, labor supply $\ell_{jt}(m)$, current sector $s_{jt}(m)$, future sector of choice $s_{jt+1}(m)$, and an idiosyncratic preference shifter $\epsilon_{jt}(m) = \{\epsilon^s_{jt}(m)\}_s$ across different future sectors. The preferences of member m is represented by

$$\mathcal{U}_{jt}(m) = u(C_{jt}(m)) + v(\ell_{jt}(m), s_{jt}(m), s_{jt+1}(m), \epsilon_{it}), \tag{1.27}$$

$$\text{where } u(C) = \frac{C^{1-\gamma^{-1}} - 1}{1 - \gamma^{-1}}, \text{ and } v(\ell, s, n, \epsilon_t) = -\theta^s_i \frac{1}{1+\varphi^{-1}} \ell_{it}^{1+\varphi^{-1}} + \eta^s_{it} - \chi^{sn}_{it} - \epsilon^n_{it}, \tag{1.28}$$

where γ is the elasticity of intertemporal substitution, φ is the Frisch elasticity of labor supply, and θ^s_i is the intensity of labor disutility in each sector s. η^s_{it} captures the non-pecuniary sector-specific benefits, and χ^{sn}_{it} captures the relocation costs of moving from sector s to sector n, measured in terms of utility. This formulation follows Artuç et al. (2010) with an additional endogenous labor supply term $\ell_{it}^{1+\frac{1}{\varphi}}$.[35]

We have perfect risk sharing across members of the family, so $C_{jt}(m) = C_{jt}$. Final goods C_{jt} is a Cobb-Douglas aggregate of consumption across each of the sectors $s = 1, 2, \cdots, S$ with shares α^s_{jt}. Consumption within each sector follows the Armington trade model, where consumption is a CES aggregate of goods from each of the I countries with an elasticity of substitution $\sigma_s > 1$ within each sector s. Consumption is given by

$$C_{jt} = \prod_s \left(\frac{C^s_{jt}}{\alpha^s_{jt}} \right)^{\alpha^s_{jt}}, \quad C^s_{jt} = \left[\sum_i (C^s_{ijt})^{\frac{\sigma_s - 1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s - 1}}$$

Savings. Analogously to Section 1.3, each country issues a nominal bond of price $\frac{1}{1+i_{it}}$. There is no aggregate risk, and bonds are perfect substitutes across origins.

[35]This can implicitly be interpreted as an intensive margin of labor supply; in Appendix 1.8, we microfound this as with an *extensive* margin interpretation, more suitable to study unemployment.

Firms and technology. Goods are distinguished by sector and origin. Sector s goods from country i are produced by competitive firms using Cobb-Douglas technology, with labor share ϕ_i^s and sector n input shares ϕ_i^{ns} satisfying $\phi_i^s + \sum_n \phi_i^{ns} = 1$. The total factor productivity of country i, sector s at time t is A_{it}^s, and exports from i to j face an iceberg cost τ_{ijt}^s with $\tau_{iit}^s = 1$ by normalization. Inputs from sector n across different goods are aggregated CES with elasticity σ_s, in the same way as consumption goods in sector n. Thus the production function F_{ijt}^s of a representative firm in country i, sector s at time t to destination j is

$$F_{ijt}^s(l_{ijt}^s, \{X_{ijt}^{ns}\}_n) = \frac{A_{it}^s}{\tau_{ijt}^s} \left(\frac{l_{ijt}^s}{\phi_i^s}\right)^{\phi_i^s} \prod_n \left(\frac{X_{ijt}^{ns}}{\phi_i^{ns}}\right)^{\phi_i^{ns}} \tag{1.29}$$

Unions and Wage Rigidity. We assume wage rigidity in each sector s through wage-setting unions facing nominal friction. A continuum of unions in sector s organizes the measure L_{it}^s of workers in sector s and employs them for an equal number of hours ℓ_{it}^s. Each union faces a labor demand curve and sets nominal wages W_{it}^s in each period to maximize the welfare of the sector s members with discount rate β.[36] We assume wage rigidity in the form of a Rotemberg friction $\Phi(W_t^s, W_{t-1}^s)$ and choose the union objective function so that the union's optimization problem leads to the wage Phillips curve,

$$\log(\pi_{it}^{sw} + 1) = \kappa_w(v'(\ell_{it}^s) - \frac{W_{it}^s}{P_{it}} u'(C_{it})) + \beta \log(\pi_{it+1}^{sw} + 1) \tag{1.30}$$

where $\pi_{it}^{sw} = \frac{W_{it}^s}{W_{it-1}^s} - 1$ denotes wage inflation at time t.[37]

Migration across sectors. We assume that each member m is forward-looking and faces a dynamic problem with discount factor β, labor reallocation costs χ_i^{sn} to move from sector s to n; these reallocation costs are time-invariant, additive, and measured in utility units. Each member m receives an idiosyncratic shock for each choice of sector, denoted by $\epsilon_{it} = \{\epsilon_{it}^n\}_n$. Since the per-worker labor supply ℓ_{it}^s is determined by the union, the member takes it as given. If we denote by $\mathcal{V}_{it}^s(\epsilon_{it})$ the lifetime utility of the worker in sector s with

[36]Here, we are implicitly assuming that the intertemporal preference shifters δ_{jt} are pure consumption shocks that affect consumption but not labor supply. We make this assumption for clarity of exposition, as the shifters are intended to match the realized trade imbalances and model financial shocks outside of the scope of our model.

[37]To a first order, the equation is identical to assuming Calvo rigidity, where the probability of keeping the wage fixed is θ_w, with $\kappa_w = \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}$.

preference shock ϵ_{it}, then we have the worker's Bellman equation,

$$\mathcal{V}^s_{it}(\epsilon_{it}) = \tilde{\lambda}_{it} W^s_{it} \ell^s_{it} - h(\ell^s_{it}) + \eta^s_{it} + \max_n [\beta \mathbb{E}[\mathcal{V}^n_{it+1}(\epsilon_{it+1})] + \epsilon^n_{it} - \chi^{sn}_{it}], \tag{1.31}$$

where $\tilde{\lambda}_{it} = \frac{u'(C_{it})}{P_{it}}$ is the Lagrange multiplier on the country i household family's period t budget constraint. Here $\tilde{\lambda}_{it} W^s_{it}$ is the marginal utility of labor by a worker in sector s. Workers internalize how their choice of sector affects the family budget. The solution to the Bellman equation above yields a transition matrix μ^{sn}_{it} and expected utility $V^s_{it} = \mathbb{E}[\mathcal{V}^s_{it}(\epsilon_{it})]$ given by

$$\mu^{sn}_{it} = \frac{\exp(\frac{1}{\nu}(\beta V^n_{it+1} - \chi^{sn}_{it}))}{\sum_{n'} \exp(\frac{1}{\nu}(\beta V^{n'}_{it+1} - \chi^{sn'}_{it}))}, \tag{1.32}$$

$$V^s_{it} = \tilde{\lambda}_{it} W^s_{it} \ell^s_{it} + \eta^s_{it} - v(\ell^s_{it}) + \nu \log \left(\sum_n \exp(\frac{1}{\nu}(\beta V^n_{it+1} - \chi^{sn}_{it})) \right). \tag{1.33}$$

Monetary policy. The monetary authority in each country i sets a nominal interest rate i_{it}. We assume that country 1 (USA) sets a Taylor rule on inflation

$$\log(1 + i_{1t}) = r_{1t} + \phi_\pi \log(1 + \pi_{1t}) + \epsilon^{MP}_{1t}, \tag{1.34}$$

where r_{1t} is the real interest rate, $\pi_{1t} = \frac{P_{it+1}}{P_{it}}$ is the CPI inflation, and interpret ϵ^{MP}_{1t} as any discretionary monetary policy the central bank of Country 1 may pursue.

The monetary policy of country 2 (China) may be a *peg* or a *float*. Under a peg, we assume that country 2 pegs the exchange rate to country 1, so i_{2t} is implicitly pinned down by $e_{12t} = \bar{e}$.[38] Under a float, country 2 pursues an independent Taylor rule of the form

$$\log(1 + i_{2t}) = r_{2t} + \phi_\pi \log(1 + \pi_{2t}) + \epsilon^{MP}_{2t}. \tag{1.35}$$

We assume that the rest of world ($i \geq 3$) floats its currency with respect to the US dollar, and assume that monetary policy in each of the countries is given by its own Taylor rule (Equation 1.34) responding to its CPI inflation.[39]

[38] Because bonds are perfect substitutes, we rule out pegging in the form of foreign exchange intervention. In fact, in a model with UIP deviations, the first-order linear consumption responses are identical whether China pegs the currency through moving interest rates, or fixing the interest rate and buying bonds (and financing this through lump-sum taxes), because the current account of the country (fiscal authority plus household) is identical in both cases. We formally explore this in a work in progress.

[39] Alternatively we may consider a middle ground, corresponding to a Taylor rule with an exchange rate target.

Exchange rate determination. Denote by $e_{it} = e_{i1t}$ the value of currency i with respect to the US dollar. We have $e_{ijt} = \frac{e_{it}}{e_{jt}}$. If country i pegs its currency, it sets e_{it} to an exogenous number $\bar{e}_i$. When country i floats its currency, the UIP condition pins down $\frac{e_{it+1}}{e_{it}}$. We assume that, if country i floats its currency, e_{i0} is the unique value such that

$$\lim_{t\to\infty} B_{it} = 0. \tag{1.36}$$

Equation 1.36 operationalizes the idea that there are financial forces that move exchange rates to clear long-run balance of payments, and can be microfounded as a limit case of financial frictions pinning down the exchange rate.[40]

Tariffs and fiscal policy. Each country j can choose a set of ad valorem import tariff rates $\{t^s_{ijt}\}$ on goods from country i to country j; the tariff revenues are rebated to households lump-sum, and the government balances its budget every period. Thus if we denote the pre-tariff price of sector s goods from i to j at time t by P^s_{ijt}, government j's revenue is

$$T_{jt} = \sum_{i,s} t^s_{ijt} P^s_{ijt} (C^s_{ijt} + X^s_{ijt}) \tag{1.37}$$

where C^s_{ijt} is consumption of (i,s) goods in country j, and X^s_{ijt} is total input use of (i,s) goods in country j. To focus on tariffs, we assume away export subsidies.

Equilibrium. We are now ready to define the equilibrium in the quantitative model.

Definition 1.2. *Given parameters $\{A^s_{it}, \tau^s_{ijt}, \delta^s_i, \chi^s_{it}, \eta^s_i\}$, previous period nominal wage $\{W^s_{i-1}\}$, initial bond holdings $\{B_{i0}\}$, labor allocation $\{L^s_{i0}\}$, and policy rules $\{i_{it}\}, \{t^s_{ijt}\}$, an equilibrium in this model consists of consumption $\{C_{jt}, C^s_{ijt}\}$, bond holdings $\{B^s_{it}\}$, labor supply $\{\ell^s_{it}\}$, labor allocation $\{L^s_{it}\}$, prices $\{P_{jt}, P^s_{jt}, P^s_{ijt}\}$, wage $\{W^s_{it}\}$ and exchange rates $\{e_{ijt}\}$ that satisfy the following:*

(a) Consumption and bond holdings solve the family optimization problem,

(b) Prices, labor, and input demand solve firm profit maximization,

(c) Labor supply and wages satisfy the Phillips curve,

(d) Labor reallocation and lifetime value solves the sector choice problem,

[40] This idea dates back to Meade (1951) and Friedman (1953). Equation 1.36 is a special case of the exchange rate determination literature with financial frictions (Kouri, 1976; Itskhoki and Mukhin, 2021a) where we take the limit of the magnitude of the friction to zero. We microfound this in Appendix 1.11.

(e) Monetary policy in the US is given by a Taylor rule,

(f) Monetary policy in other countries and exchange rates satisfy (a peg) or (zero long-run balances).

(g) Goods market, bond market clears, and the government balances its budget.

The formal equations and derivations are in Appendix 1.8.

1.4.2 Data and Calibration

We provide an overview of our data and calibration process and relegate the details to the Appendix (Section 1.8). Our quantitative model has six country aggregates: US, China, Europe (including UK), Asia, the Americas, and the rest of world. We consider 6 sectors: agriculture, low-, mid- and high-tech manufacturing, and low- and high-tech services, classified according to the North American Industry Classification System (NAICS).[41] The time of our data spans from $t = T_0 = 2000$ to $t = T_{data} = 2012$ annually.

Trade and production data. The primary dataset we use is the World Input-Output Database (WIOD) 2016 edition (Timmer et al., 2015). The WIOD compiles data from national accounts and bilateral trade data for 56 sectors and 44 countries. It has information on the value of trade flows X^s_{ijt} from country i to country j in sector s at year t for 56 sectors across 44 countries. It also contains data on purchases of inputs across sectors, value added of each sector in each country (which corresponds to the labor share in our model), consumption shares across sectors, and the net exports for each country. We obtain the price indices for each sector from the WIOD's Socioeconomic Accounts (WIOD SEA).

Labor and Sectoral Adjustments. We obtain the initial distribution of workers in the year 2000 by sectors using the WIOD SEA. We use data from the Current Population Survey (CPS) in the United States to construct the matrix of migration flows μ^{sn}_{it} across sectors in the US. We assume away migration flows between countries. For countries outside of the US and China, we assume that workers are immobile and fixed in that sector; for China, we assume that the cost of moving is fixed at the 2000 level.

Calibration. Table 1.1 provides a summary of the parameters, including the sources of parameters whose values we take from the literature or the moments that we target for

[41] This follows Dix-Carneiro et al. (2023).

Panel A. Fixed according to literature			
Parameter	Value	Description	Source
β	0.95	Discount factor	5% interest rate
ν	2.02	ϵ^n_{it} dispersion	Caliendo et al. (2019)
γ	1	Intertemporal Elasticity	Standard
φ	2	Frisch elasticity	Peterman (2016)
σ_s	5	Elasticity of substitution	Head and Mayer (2014)
κ	0.05	NKPC slope	Hazell et al. (2022)
ϕ_π	1.5	Taylor rule coefficient	Taylor (1993)
Panel B. Parameters we calibrate			
Parameter		Description	Target moments
α^s_{it}		Expenditure shares	WIOD consumption share
ϕ^s_{it}		Labor share	WIOD value added
ϕ^{sn}_{it}		Input-output matrix	WIOD input-output
θ^s_i		Intensity of labor disutility	$\ell^s_{i,2000} = 1$
η^s_i		Non-pecuniary utility	WIOD SEA labor distribution
χ^{sn}_{it}		Migration cost	CPS sector change
τ^s_{ijt}		Trade cost	WIOD trade flow
A^s_{it}		Productivity	WIOD trade flow and SEA price index
δ_{it}		Intertemporal preference shifter	WIOD net exports
r_{it}		US real interest rate	Full employment without China shock

Table 1.1: Calibrated parameters

the parameters we directly calibrate.

Values for parameters in Panel A of Table 1.1 are taken from the literature, as they are difficult to identify given available data, or our estimation strategy would be analogous to the literature. The time frequency is annual, and we use $\beta = 0.95$ to match the 5% annual interest rate. Estimating the dispersion ν of sectoral preference shocks ϵ^n_{it} requires panel data and instrumental variables; we impose this to be common across all countries and set them to be $\nu = 2.02$, following Caliendo et al. (2019). For the elasticity of intertemporal substitution, we follow standard practice in the macro and trade literature and set $\gamma = 1$, assuming log utility. The Frisch elasticity of labor supply is set to $\varphi = 2$, closer to macro estimates (Peterman, 2016). Measuring the elasticity of substitution of goods across origin often requires panel data on variation, so we set it to 5, which is standard in the literature (Head and Mayer, 2014; Rodríguez-Clare et al., 2022; Dix-Carneiro et al., 2023). We set the New Keynesian Phillips Curve slope to $\kappa = 0.05$ to match Hazell et al. (2022) which exploit variation across US states to obtain the response of inflation to the labor wedge.[42] The Taylor rule coefficient is set to 1.5, following the original paper by Taylor, as standard in the macro literature.

In Panel B of Table 1.1, we can directly compute the sectoral consumption expenditure share α_{it}^{s}, labor share ϕ_{it}^{s}, and input-output share ϕ_{it}^{sn} directly from the WIOD data. For the rest of the parameters, we rely on parts or all of the model to match the model-generated moments with the data. We divide our calibration into two steps: calibrating the initial period, and then calibrating how those parameters change in our model. We set the non-pecuniary utilities η_{i}^{s} such that the model-implied initial labor distribution $L_{i,2000}^{s}$ matches the realized labor distribution observed in the WIOD SEA, and the migration cost $\chi_{i,2000}^{sn}$ so that it matches the observed sector change flows in the CPS of the US; we assume that China faces the same sectoral migration costs, and countries besides US and China have an immobile labor market. We normalize θ_{i}^{s} so that the initial per-worker labor supply in our model is $\ell_{i}^{s}=1$. Turning to the trade side, we calibrate the trade costs τ_{ij0}^{s} and A_{i0}^{s} to match the trade flow in the initial period exactly up to normalization, following the exact hat algebra approach of Dekle et al. (2007) and Caliendo et al. (2019).

Next, we discuss the calibration of the *shocks* we extract. We extract three main sets of shocks from the WIOD data: changes in trade costs $\hat{\tau}_{ijt}^{s}=\frac{\tau_{ijt}^{s}}{\tau_{ij0}^{s}}$, changes in productivity $\hat{A}_{it}^{s}=\frac{A_{it}^{s}}{A_{i0}^{s}}$, and intertemporal preference shocks δ_{it}.[43] We calibrate these shocks to exactly match three realized 'shocks' in the WIOD data: changes in sectoral output price indices $\hat{P}_{it}^{s,dom}=\frac{P_{it}^{s,dom}}{P_{i0}^{s,dom}}$, changes in trade shares $\hat{\lambda}_{ijt}^{s}=\frac{\lambda_{ijt}^{s}}{\lambda_{ijt}^{0}}$, and net exports in each period as a share of GDP $NXGDP_{it}=\frac{NX_{it}}{GDP_{it}}$. We calibrate the trade cost shocks $\hat{\tau}_{ijt}^{s}$ to exactly match the gravity structure of trade flows up to normalization; we assume $\hat{\tau}_{iit}^{s}=1$. On the other hand, since prices are a function of wage and productivity, and the dynamics of wage (and its rigidity) are central to our channel, we cannot back out the productivity without solving for the full model. Thus, we employ a Simulated Method of Moments (SMM) approach, targeting the changes in output price and net exports as moments we exactly match. We also calibrate the sector change costs χ_{it}^{sn} in the US so that the model-implied migration μ_{it}^{sn} exactly match the sector reallocation data in the CPS. The details of this calibration procedure can be found in the Appendix (Section 1.11).

[42]Since their model is quarterly and the Phillips curve links price inflation with unemployment, we undergo a series of transformations to make our estimate consistent with their estimate of $\kappa'=0.0062$. Details are given in the Appendix (Section 1.11)

[43]We also assume that the preference and technology parameters $(\alpha_{it}^{s},\phi_{it}^{s},\phi_{it}^{sn})$ are time-varying, but we directly observe this as shares from the data.

1.4.3 Solution algorithm

We aim to study the employment, trade balance, and welfare effects of China's peg against the US dollar and revisit the effects of the China shock under this framework. We bring frontier computational methods from macroeconomics (Auclert et al., 2021a) and apply them to answer trade questions. We sketch our solution algorithm here and provide the details and discussions in the Appendix (Section 1.11).

Given the elasticities and parameters calibrated in Subsection 1.4.2 (Table 1.1), we directly solve for the equilibrium in the *sequence-space* of equilibrium objects

$$\{X_t\}_{t=T_0}^{T} = \{(B_{it}, P_{it}, C_{it}, e_{it}, W_{it}^s, \ell_{it}^s, L_{it}^s, V_{it}^s)\}_{t=T_0}^{T}$$

for $T \gg T_{data}$ such that the economy returns to a new steady-state by $t = T$. This requires solving a high-dimensional nonlinear equation.[44] The key idea is that the nonlinear system of equations that define $\{X_t\}$ is extremely sparse: each period t equilibrium condition only depends on variables of time $t, t-1, t+1$, and even those equations depend on a few parameters within each t. Then, the Jacobian of the equilibrium conditions can be efficiently constructed, and we employ nonlinear root-finding algorithms to solve for the full sequence of wages, consumptions, trade imbalances, and labor allocations. By leveraging the sequence-space Jacobian approach from Auclert et al. (2021a) and combining it with computational advances in machine learning, we can solve for the full nonlinear solution of our model in seconds to minutes depending on specification, allowing us to compute a wider dimension of counterfactual scenarios and explore policy implications.[45]

1.5 Effects of the China shock and the role of the peg

In this section, we use the model described in Section 1.4.1 and calibrated parameters from Section 1.4.2 to study the effect of the China shock and the China peg. In Section 1.5.1, we first define the "China shock", using the change in productivities, trade costs, and preference parameters observed over this period.

In Section 1.5.2, we revisit the effect of the China shock on the US labor market and trade deficit. We show how modeling wage rigidity, consumption-savings, and exchange

[44]With $I = S = 6$ and $T = 100$, the system of equations have over 20000 variables.

[45]The methods we use include parallelization, autodiff, just-in-time compiling, and Intel's PARADISO package for quickly solving large sparse systems, many of which are heavily used in machine learning contexts where the parameter space is even larger. The toolkits are available in the Python-based framework "JAX," which we use extensively. Details can be found in the Appendix (Section 1.11).

rate peg affects the predictions on the effect of the China shock, compared to estimates in the literature that ignore these channels. In Section 1.5.3, we quantify how the exchange rate peg *magnified* the effects of the China shock on the United States by comparing the realized economy with a counterfactual economy with otherwise identical evolution of parameters, but under a floating exchange rate.

1.5.1 The China shock

One goal of our quantitative model is to estimate the effect of the China shock under an exchange rate peg and nominal rigidity. In this subsection, we define what the China shock is in the context of our model.

In Section 1.4.2, we extract the realized evolution of parameters across time. This is the baseline, *realized* economy with the China shock. We consider two notions of the China shock. The main shock, which we call the *China trade shock* only considers the changes in China that are directly associated with increasing import penetration of Chinese goods: the productivity A_{it}^{s} and the trade costs τ_{ijt}^{s}. Thus the counterfactual economy without the *China trade shock* is the equilibrium where the calibrated parameters (Table 1.1) are identical to the realized equilibrium, with the exception of productivity A_{it}^{s} and the trade costs τ_{ijt}^{s} in China; for China, we fix the productivity A_{CN}^{s} and trade costs $\tau_{i\mathrm{CN}t}^{s}, \tau_{\mathrm{CN}it}^{s}$ to be fixed at their levels in $t = T_0$.[46]

Figure 1.4 plots the computed China shock on the productivities A_{CN}^{s} and the trade cost from China to US $\tau_{CN,US,t}^{s}$ as a ratio between the levels at time t versus the level at the initial period $t = T_0 = 2000$ for the six sectors. China's productivity increases in all sectors, but especially in the medium-tech and high-tech manufacturing sectors. China's trade costs also decrease for all sectors; while the decline seems to be most pronounced for the service sectors, this is driven by the fact that the service sectors are close to nontradable – the implied trade costs τ_{ijt}^{s} in 2000 are close to 70-80 that get reduced to 30 by 2012, but is still very high. Much of the effect on the US economy is driven by the shocks in the manufacturing sectors.

We also consider another set of shocks, which includes the intertemporal preference shock $\delta_{\mathrm{CN}t}$. While the changes in productivity A and trade cost τ capture the surge in Chinese exports, this is not the only structural change in China during this period. Rich financial dynamics outside the scope of our model will affect realized trade imbalances

[46] In the Appendix (Section 1.13), we discuss alternative notions of the *no China shock* counterfactual, such as (1) where China's global import penetration does not increase throughout the period (Caliendo et al., 2019; Rodríguez-Clare et al., 2022), or (2) Chinese productivity grows on par with the global average during this period (Dix-Carneiro et al., 2023). We find qualitatively similar results.

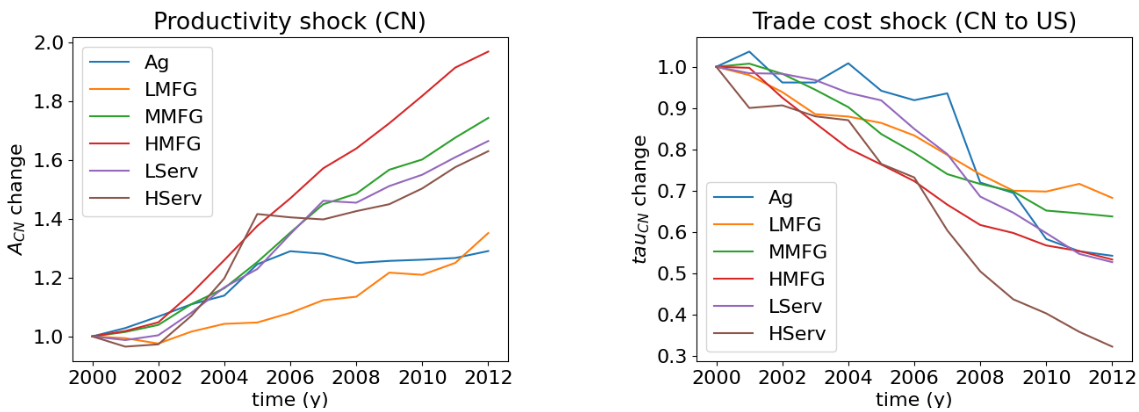


Figure 1.4: Calibrated values of the China trade shock.

and consumption-saving patterns. Those 'residuals' constitute the savings glut of China and are interpreted as part of the China shock in Dix-Carneiro et al. (2023). We call this shock the *China trade and savings shock*. Then, the counterfactual economy without the China trade and savings shock is the equilibrium with identical parameters as the realized equilibrium, with the exception of $A^s_{\mathrm{CN}}, \tau^s_{i\mathrm{CN}t}, \delta_{\mathrm{CN}t}$; we fix those values to be the values at $t = T_0$ in China.[47]

Comparing the realized economy with the economy without the *China trade shock* allows us to evaluate the effect of Chinese growth on US outcomes, such as the distribution of labor, trade balances, or unemployment. Comparing the realized economy to the economy without the *China trade and savings shock* gives us the effect of China's structural change, including the savings glut, on the same US outcomes. By looking at the difference between these two outcomes, we can evaluate the extent to which the realized US trade deficit and decline in manufacturing (Figure 1.1) can be causally attributed to Chinese growth.

For all our counterfactual scenarios, we assume in our baseline analysis that agents have no foresight of the shocks during this period for both the realized and counterfactual equilibrium, operationalizing the notion that "every year is a China shock" during the period of spectacular productivity growth in China. We discuss the details of our implementation, the rationalization for agents' foresight, and robustness exercises where we alternatively assume perfect foresight in the Appendix (Section 1.12).

[47] During this period, consumption shares α^s_{it} and input-output linkages, labor shares $\phi^s_{it}, \phi^{sn}_{it}$ vary over time. We match the varying shares in both the realized and counterfactual equilibrium.

1.5.2 Reevaluating the China shock

We start by revisiting the quantitative effects of the surge in China's imports – the *China shock* – on the US economy using our calibrated model. We are interested in asking the following question: what are the dynamic effects of the China shock on labor reallocation, unemployment, the trade balance of the US, and welfare consequences through the lens of our model? We revisit the effects of the China shock under wage rigidity and endogenous consumption-savings and compare how those ingredients lead to different implications of the China shock than three previous literature: Caliendo et al. (2019), which feature exogenous deficits and no involuntary unemployment, Rodríguez-Clare et al. (2022) which feature nominal rigidity but exogenous deficits, and Dix-Carneiro et al. (2023) which feature endogenous deficits but quantity rigidity instead.

To quantify our answer to this question, we first solve for the baseline economy with the actual evolution of fundamentals over 2000-2012. Then we solve the economy under both the *no China trade shock* counterfactual and the *no China trade and savings shock* counterfactual and treat the difference in outcomes such as the trade imbalance, labor market, and welfare outcomes between the realized and counterfactual outcomes as the effect of the shock.

Figure 1.5 shows the import penetration ratio of China to the US, the manufacturing share of US employment, the net exports of the US (as a percentage of contemporaneous GDP), and aggregate unemployment in the economy for the (1) realized economy, (2) the counterfactual economy without the China trade shock, and (3) the counterfactual economy without the China trade and savings shock. The first three figures replicate the four stylized facts we highlight in the introduction (Figure 1.1). Figure 1.5a clarifies that the growth in import penetration from China in this period is driven by productivity growth and trade liberalization of China. In fact, if China had not grown in this period, import penetration from China would have decreased, as other Asian countries growing in this period (most notably other parts of Asia) would have assumed the role of China.

Next, we study the decline in US manufacturing. Figure 1.5b investigates the impact of the China shock on the manufacturing share of employment. As we see, a sizable share of the exit of workers from manufacturing can be attributed to the China shock in our framework. In numbers, 991 thousand jobs lost in manufacturing could be attributed to the China trade shock. Most notably, the decline in manufacturing is almost identical in the *no China trade shock* case and the *no China trade and savings shock* case, suggesting that the residual savings glut of China plays a negligible role in the decline of US manufacturing. This goes further than the findings of Kehoe et al. (2018), which show that the savings glut is responsible for 15.1% of the decline in US manufacturing.

Our framework in Section 1.3 substantiates this viewpoint: Proposition 1.5 shows that US borrowing should mitigate the decline in manufacturing, as consuming more in the short-run would help a declining demand for Home goods.

Turning to trade deficits, Figure 1.5c shows that a significant proportion of realized US trade deficits can be explained by the China trade shock. In fact, taking the average from 2000 to 2012, 2.25 percentage points of the US annual deficit (% GDP) can be explained solely by the China trade shock, and if China had not grown, the US may have had balanced trade by 2012. The realized average annual trade deficit of the US during the same period was 3.4% of GDP, suggesting that two thirds of the US trade deficit over this period could be explained by the China shock. The residual savings glut δ_{it} plays little role in affecting the balances, suggesting that the theoretical channel we highlighted in Proposition 1.1 – permanent Foreign growth leading to Home deficits – is responsible for a majority of the US trade deficit of the 2000s.

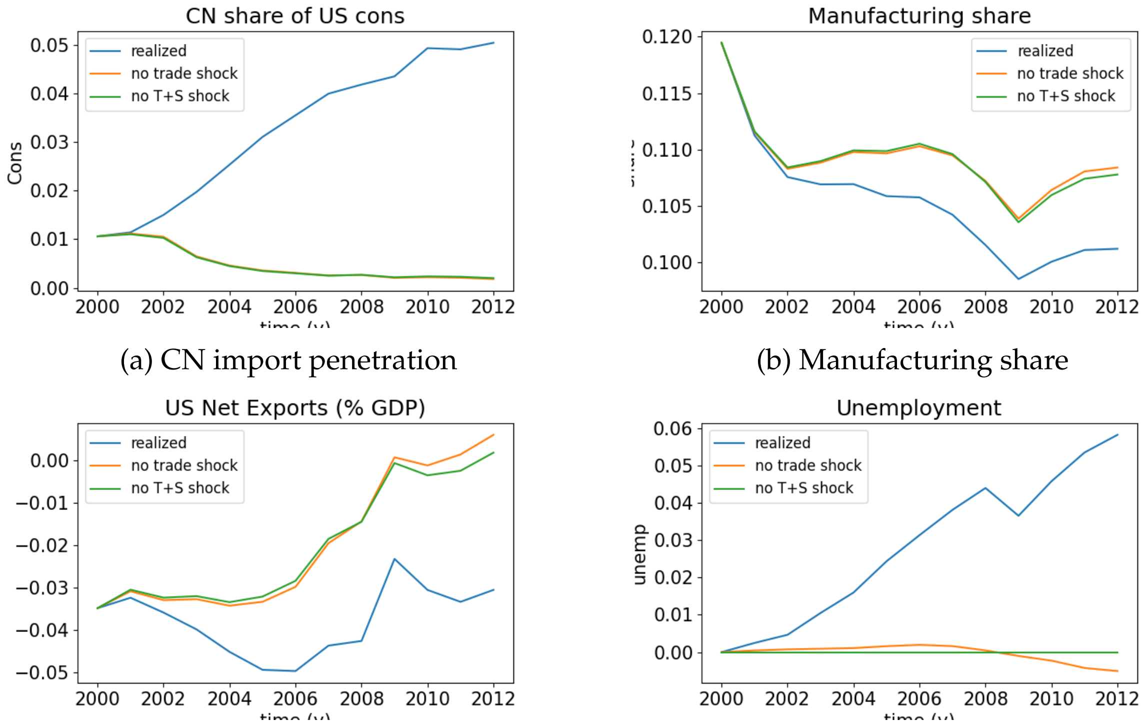
Next, we use our general equilibrium model to obtain the implied effects of the China shock on unemployment. Figure 1.5d plots the aggregate US unemployment response to the China shock according to our model. Unemployment increases through the span of the shock, and on average, the excess unemployment generated from the China shock from 2000 to 2012 is 3.04%; this unemployment is necessarily short-lived, and it reaches zero after the culmination of the China shock, as nominal wages adjust to the new equilibrium level.[48]

Finally, we measure the welfare implications of the China shock. The household family's utility comprises both consumption utility and the disutility of labor. In evaluating the welfare effects, we consider the aggregate discounted utility incorporating the full path of consumption and the disutility of labor. Thus we define the *welfare effect* of the shock as the lifetime compensating variation in consumption for the US; formally, the welfare effect of the China shock is the scalar ζ such that

$$\mathcal{U}_0(\{C_{CS}\}_t, \{\ell_{CS}\}_{s,t})) = \mathcal{U}_0(\{(1+\zeta)C_{\text{noCS}}\}_t, \{\ell_{\text{noCS}}\}_{s,t}), \tag{1.38}$$

or how much more lifetime consumption (in percentages) the household needs to be indifferent between the China shock case and the no China shock case. According to this metric, the China shock contributed to a 0.183% gain in lifetime welfare, a modest

[48]The unemployment level is high because the shock to manufacturing can spill over to the service sector through aggregate demand (highlighted in the two-sector model in the Appendix (Section 1.10)), and targeting CPI inflation is not an optimal monetary policy in this setup. We consider this result as a benchmark and consider alternative monetary policy rules in the Appendix (Section 1.13), and show that the decline in manufacturing share and trade deficits are robust.



(a) CN import penetration

(b) Manufacturing share

(c) Net exports (% GDP)

(d) Unemployment rate

Figure 1.5: Response of the economy to the China shock.

Note. The ‘realized’ graphs are the equilibrium outcome from the full sequence of parameters that were targeted to match realized moments. The ’no trade shock’ graphs are the equilibrium outcome from the sequence of parameters identical to the realized, except we remove the productivity growth and trade cost reduction in China. The ’no T+S shock’ graphs are the equilibrium outcome from the same sequence, except we remove the residual ’savings shocks’ in China. The similarities between the no trade shock and the no T+S shock suggest that the residual savings glut of China played close to zero role in the manufacturing decline or the trade deficits after we account for the effect of the exchange rate peg.

but significant gain, and the distortion margins we highlighted in Proposition 1.2 – unemployment and future terms-of-trade deterioration – did not flip the aggregate welfare implications of the China shock.

Table 1.2 compares the estimated effects of the China shock from our framework to three references in the literature. The first is Caliendo et al. (2019) (CDP19), which features no intra-sector labor market friction and models imbalances through systems of transfers. The second is Rodríguez-Clare et al. (2022), which features downward nominal wage rigidity but exogenous imbalances. The third is Dix-Carneiro et al. (2023), which models labor market friction through quantity friction (search and matching). Our model estimates close to double the number of manufacturing jobs lost through the China shock than the estimates of the previous literature, a much larger proportion of the realized US trade deficit than what Dix-Carneiro et al. (2023) attribute to the China shock and

Effect of China shock				
	Our model	CDP19	RUV22	DPRT23
MFG jobs lost	991k	550k	498k	530k
Deficit (% GDP)	2.25	N/A	N/A	0.8
Unemployment (%)	3.04	N/A	1.4	0
Welfare gains	0.183%	0.2%	0.229%	0.183%*
Wage rigidity	O	X	O	X
Search friction	X	X	X	O
Cons-savings	O	X	X	O
ER peg	O	X	X	X

Table 1.2: Effects of the China shock: comparison to existing literature.

Note. *: Dix-Carneiro et al. (2023) measure welfare using consumption only, without considering the labor market effects of welfare. We take into account the disutility of labor in measuring aggregate welfare.

more moderate welfare gains from the China shock. Our estimate of the number of manufacturing jobs lost is close to the estimates of Autor et al. (2013) – 982,000 jobs lost as a result of the China shock after 2000 – suggesting that the *missing intercept* may not be as large as previously thought. Interestingly, despite the manufacturing jobs lost that are about twice as large and a significant level of unemployment, the welfare consequences of the China shock are still positive and close to the literature's estimates.

In the following subsection, we show that the difference between our estimates and the literature's estimates can be almost entirely attributed to China's exchange rate peg.

1.5.3 The effect of the exchange rate peg

The second and most novel part of our quantitative analysis focuses on how much the peg interacted with the China shock to generate the realized effects of the China shock we saw in Section 1.5.2. If the empirical findings in Section 1.2 and the propositions in Section 1.3 hold, we should expect that the exchange rate peg is responsible for a sizable part of the trade deficit, the decline in manufacturing, and may affect the welfare implications of the China shock.

To quantify this, we compare the outcomes of the baseline economy to a counterfactual economy with identical fundamentals, except for one change: China's monetary policy no longer pegs to the US dollar. China's alternative monetary policy could be many things – a full-discretion policy, an interest rate with an exchange rate target – but to highlight the effect of the peg, we consider the simplest counterfactual by assuming that

China's monetary policy is symmetric to the US, an independent Taylor rule with the same coefficient on China's domestic CPI inflation. The difference in the outcomes of the economy with the peg and the economy without the peg, both with the China shock, is the causal effect of China's exchange rate peg on the US.

Figure 1.6 shows the same aggregate variables in the US – import penetration ratio of Chinese goods, manufacturing share of employment, net exports of US, and unemployment in the economy for the (1) realized economy, (2) the counterfactual economy without the China trade shock, and (3) the counterfactual economy with the same shocks as the realized economy, but China had a floating exchange rate.

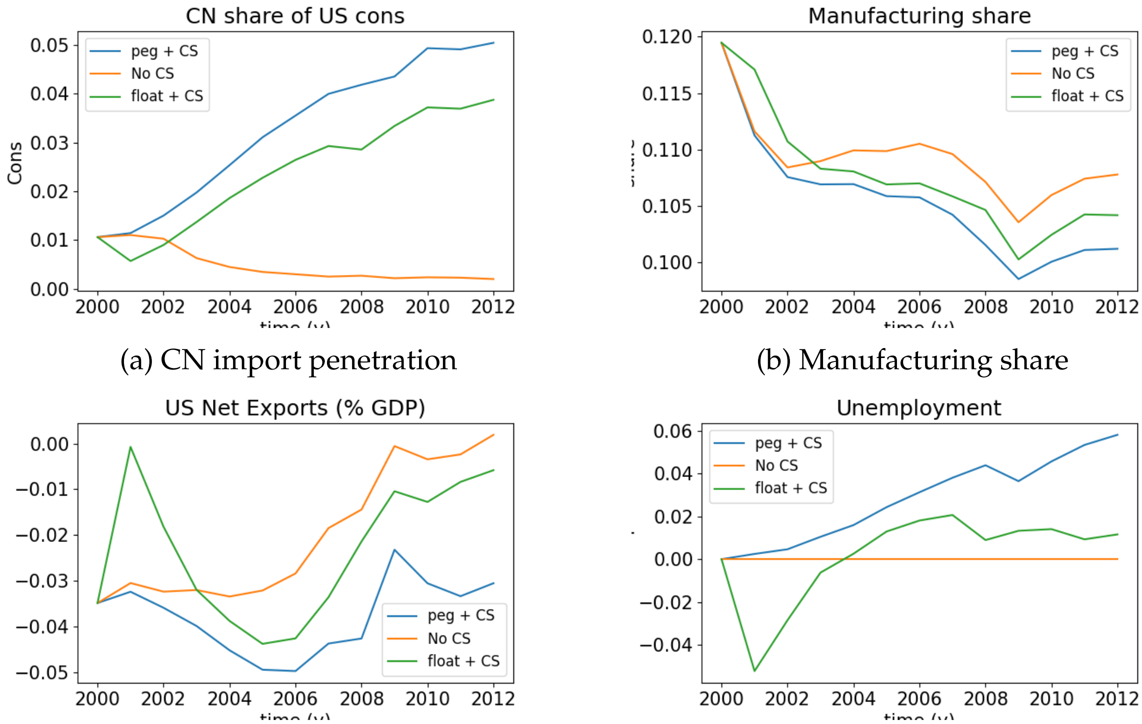
Figure 1.6a shows that the exchange rate peg played a role in Chinese import penetration to the US, and the actual penetration ratio would have been closer to 4% under a floating exchange rate. Under a float, Chinese currency would have appreciated during this period, and the increased price would have made Chinese goods less attractive to US consumers.

Investigating the decline in manufacturing (Figure 1.6b) and the US trade deficit (Figure 1.6c), we see that the exchange rate peg played a significant role in both. Even if China were identically growing, if China had a floating currency, close to 50% of the manufacturing decline attributable to the China shock and a significant proportion of the US trade deficit would disappear. Likewise, the level of unemployment is much closer to the 'no China shock' case (Figure 1.6d).[49]

Finally, we study the change in welfare. While the above results – the effect of the peg on the trade balance and the labor market – suggest that the peg may have adverse effects on the US economy, the peg comes with a clear benefit: the terms-of-trade improves, as China is selling goods at a price cheaper than in a flexible-price equilibrium. This force lowers the price index and increases consumption given the same budget. At the same time, unemployment moves the budget inwards, and this is a force that leads to a decline in consumption. Using the same compensating variations formula, we see that the China peg contributes to a welfare loss of 0.083% compared to the counterfactual economy with an identically growing but floating China.

Table 1.3 summarizes the quantitative effects of the interaction of the peg and the China shock. The first column summarizes the realized effects of the China shock under a peg, while the second column summarizes the counterfactual effect of the China shock when China is floating; the third and fourth columns compare the differences in relative

[49] The 'jump' in 2001 comes from the fact that our analysis takes the realized wages and distribution of labor in 2000 as fixed initial conditions, and these values were under a peg. When we report the average trade deficit and unemployment below, we take the average from 2003 to 2012 to trim this discontinuity.



(a) CN import penetration

(b) Manufacturing share

(c) Net exports (% GDP)

(d) Unemployment rate

Figure 1.6: Response of economy to China's peg.

Note. The 'peg + CS' graphs are the equilibrium outcome from the full sequence of parameters targeted to match realized moments. The 'no CS' graphs are the equilibrium outcome from the *no China trade shock* assumption. The 'float + CS' graphs are the equilibrium outcome from the full sequence of parameters identical to the 'peg + CS' case (realized equilibrium), but under the counterfactual assumption that China did not peg its exchange rate and had its own independent Taylor rule.

and absolute terms. As we see, the China shock interacted with the peg significantly. In absolute terms (Column 3), we see that China's currency peg is responsible for 447 thousand manufacturing jobs lost, 1.34% (as a fraction of GDP) US trade deficit, and 1.84% (in percentage points) unemployment in the US, and the welfare gains are reduced by 0.083 percentage points, compared to a counterfactual economy where an otherwise identical China floats. In relative terms (Column 4), China's currency peg *magnifies* the manufacturing jobs lost from the China shock by 82%, the trade deficits caused by the China shock by 161%, unemployment by 176%, and reduces the welfare gains by 32%.

The last column takes the literature's estimates from the three papers we discussed in the previous subsection (Caliendo et al., 2019; Rodríguez-Clare et al., 2022; Dix-Carneiro et al., 2023). The effect of the China shock under a counterfactual 'floating' economy (second column) is strikingly similar to the structural estimates of the effects of the China shock in the literature. The manufacturing jobs lost are close to 550 thousand in all of the

Decomposing China shock vs China peg					
	CS + peg	CS + float	$Y_p - Y_f$	$Y_p/Y_f - 1$	Lit estimate
MFG jobs lost	991k	543k	447k	+82%	550k
Deficit (% GDP)	2.25	0.86	1.34	+161%	0.8%
Unemployment (%)	3.04	1.10	1.84	+176%	1.4%
Welfare gains	0.183%	0.268%	-0.083p.p	−32%	0.2%

Table 1.3: Effects of the China peg

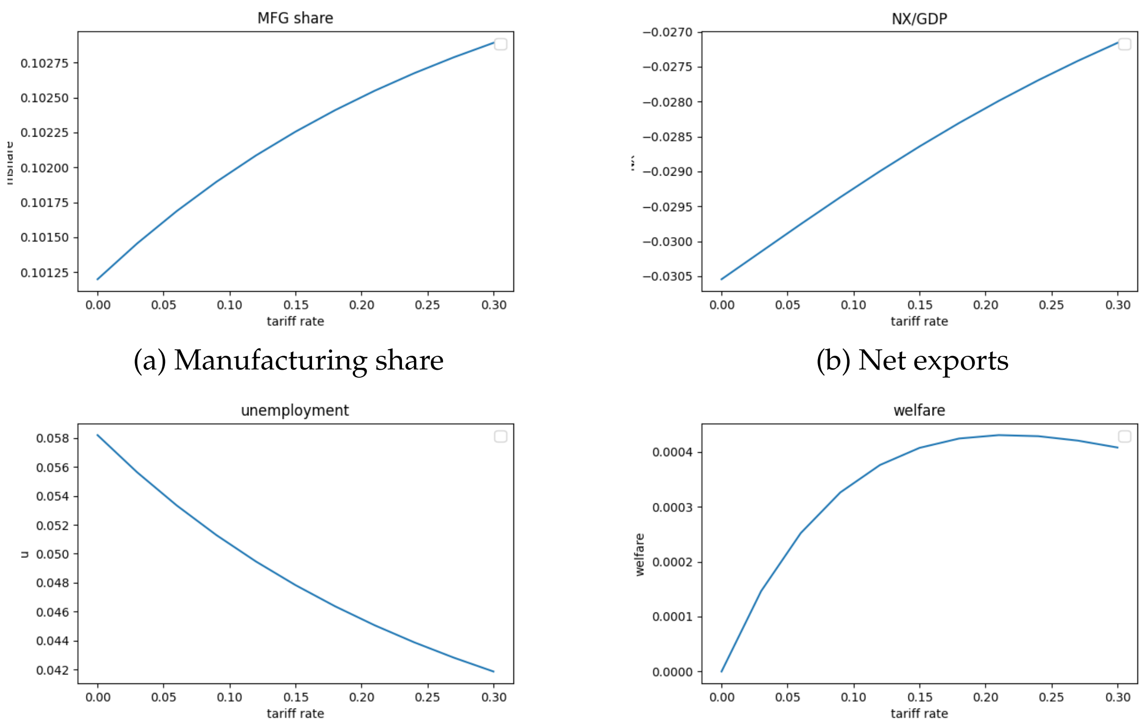
Note. The first column shows the realized effect of the China shock when the exchange rate is pegged. The second column shows the counterfactual effect of the identical China shock when China floats its currency. The third and fourth columns show the difference and ratio of the two, respectively. The fifth column shows the literature's estimates from Table 1.2.

three aforementioned papers, while we estimate 543 thousand under float. The US trade deficit caused by the China shock is estimated to be 0.8% of GDP in Dix-Carneiro et al. (2023); the US trade deficit attributed to the China shock under a (counterfactual) floating economy is 0.86% of GDP. The unemployment effect estimated by Rodríguez-Clare et al. (2022) is 1.4%; under our modeling framework, the counterfactual effect of the China shock under a float is 1.10%. These results suggest that explicitly modeling the exchange rate peg is essential in a general equilibrium analysis of the effects of China shock on the US.

1.5.4 Counterfactual policies

We conclude by studying how policies such as tariffs and monetary policy may have altered the effects of the China shock. Suppose we wanted a quantitative answer to policy questions such as: (1) Could the US have mitigated the negative consequences of the China shock with a tariff on Chinese goods in the early 2000s? (2) Does the answer to this question depend on whether China retaliates? (3) Should the US have pursued a different monetary policy to counter the effects of the exchange rate peg? Our quantitative framework is especially suitable for studying the effects of alternative policies, as we can quickly compute the counterfactual equilibrium under any set of policies. We can answer such questions by comparing the realized equilibrium with a counterfactual equilibrium with different tariff rates t_{ijt}^s, or alternative monetary policies, expressed either through a discretionary monetary policy response given by ϵ_{1t}^{MP} in the US monetary policy Taylor rule (Equation 1.34), or alternative rules of monetary policy.

The first counterfactual exercise we consider is a unilateral tariff that the US imposes on Chinese goods. Could protective tariffs have helped ameliorate the short-run losses

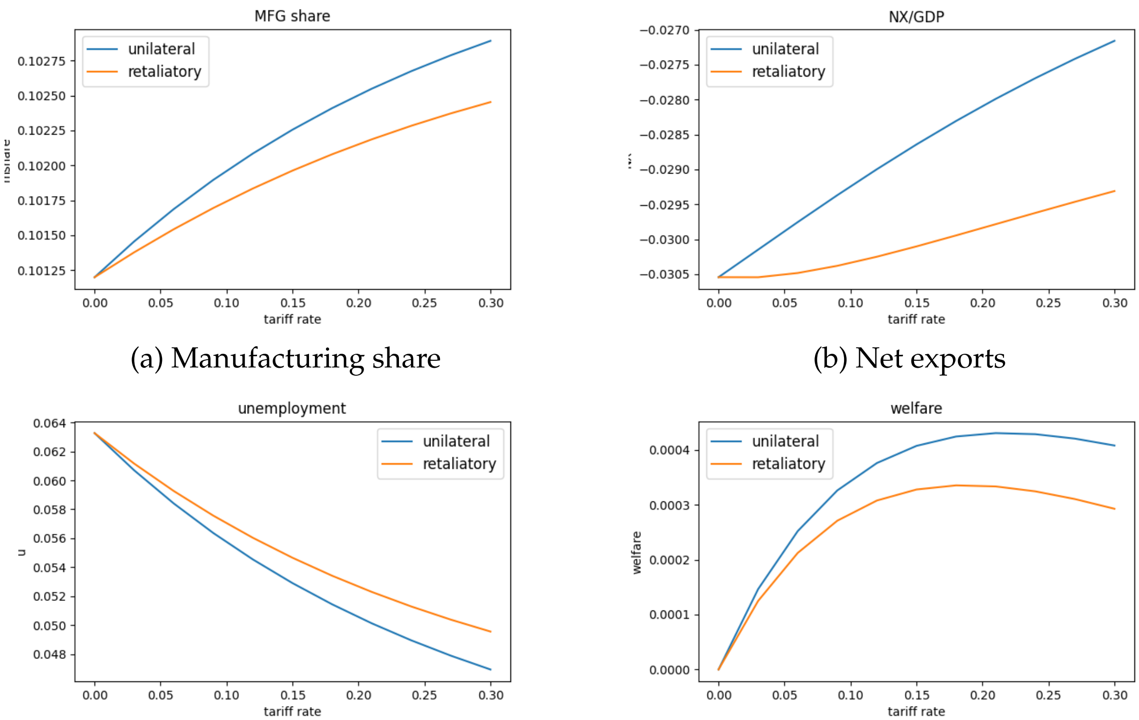


(a) Manufacturing share
(b) Net exports
(c) Unemployment
(d) Home welfare

Figure 1.7: Effect of unilateral tariffs.

from China's growth and exchange rate peg? The specific policy experiment we analyze is a uniform tariff rate of $x\%$ for $x \in [0, 0.3]$ imposed by the United States on Chinese goods from 2000 to 2012. In Figure 1.7, we highlight the effects of the tariffs on four key variables affected by the China shock: the share of manufacturing employment, US trade deficit as a percentage of GDP, unemployment rate, and aggregate welfare in the United States. The first three indicators are measured as their level in 2012, whereas aggregate welfare is computed using compensating variations relative to the realized equilibrium.

Figure 1.7 shows that a unilateral tariff reduces the decline in the share of manufacturing in the short-run, reduces the deficits, and reduces the unemployment rate. The welfare-maximizing tax rate is close to 20%, and this rate is much lower than the rate that restores full employment or restores the balance of trade. The tariff reduces 25% of the unemployment associated with the China shock and 10% of the realized trade deficit. The welfare gains from the tariff are modest, about 0.04% of lifetime welfare. This is about half of the welfare costs of the China peg (0.083%), suggesting that tariffs may help alleviate some of the welfare costs of the exchange rate peg. In this context, while a *safeguard* tariff helps alleviate the welfare losses from labor market frictions,



(a) Manufacturing share
(b) Net exports
(c) Unemployment
(d) Home welfare

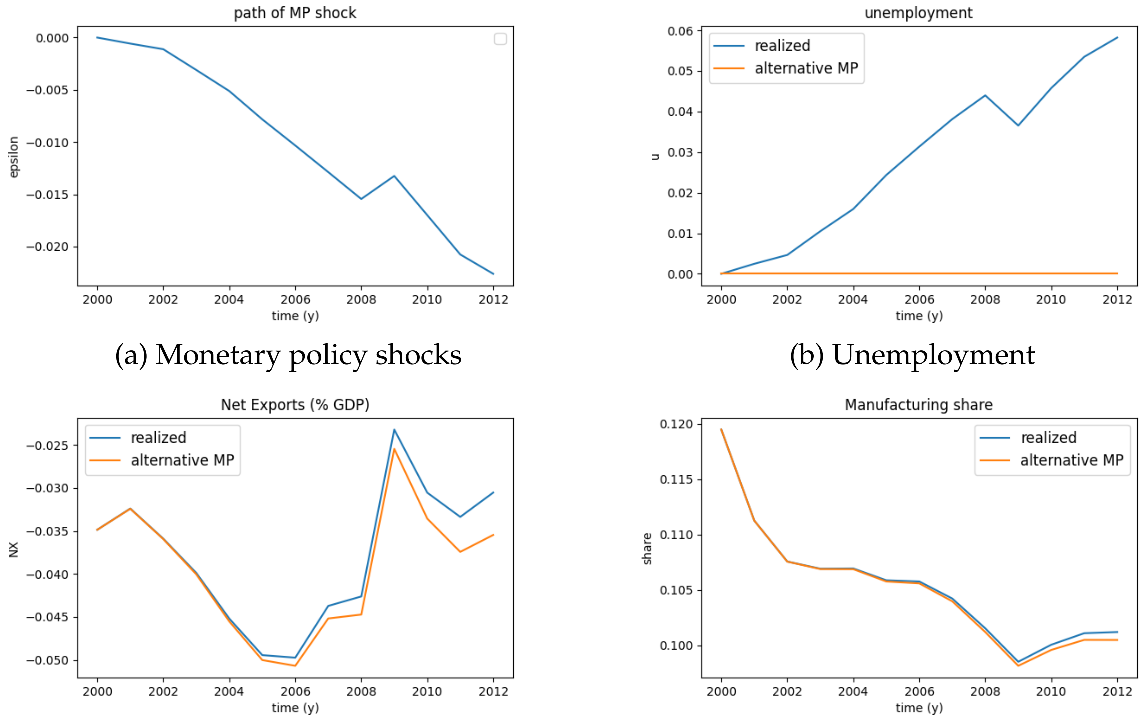
Figure 1.8: Effect of tariffs with retaliatory tariffs of equal magnitude

the distortionary impact of tariffs on consumption is substantial enough so that the US government will not fully undo the distortions using tariffs. This analysis clarifies the quantitative relevance of the different welfare channels in the optimal tariff formula (Equation 1.23).

In the second counterfactual exercise, we consider the same tariffs on Chinese exports to the US but assume that China retaliates with a tariff of equal magnitude. The possibility of retaliatory tariffs undoing any gains from tariffs is well understood in the trade context without nominal rigidity and is often used as an argument for free trade agreements. How do the welfare effects of safeguard tariffs change when such tariffs are faced with retaliatory tariffs?

Figure 1.8 shows the response of the same aggregate variables for different tariff rates set by the US, with a retaliatory tariff from China of the same magnitude. Retaliatory tariffs weaken the effectiveness of tariffs on the manufacturing share, net exports, and unemployment. Still, the safeguard nature remains even with retaliatory tariffs: short-run unemployment in the US is lowered.

In the next experiment, we assess the effects of monetary policy loosening in this



(a) Monetary policy shocks

(b) Unemployment

(c) Net exports

(d) Manufacturing share

Figure 1.9: Effect of alternative monetary policy

economy. In the baseline equilibrium (Figure 1.5), we saw that aggregate unemployment increased due to the China shock when the monetary policy was a Taylor rule targeting CPI inflation. How much looser should monetary policy be to undo the unemployment effects, and what are the effects of this additional discretionary monetary policy by the US? We simulate the model with different Home monetary policy shocks ϵ_{1t}^{MP} over 2000-2012 to find $\hat{\epsilon}_{1t}^{MP}$ that sets aggregate unemployment to zero from 2000 to 2012, and plot the economy's response to this monetary policy shock.

As Figure 1.9 shows, to clear unemployment, the nominal interest rate needs to be lower in 2000-2012 than the rate implied by the Taylor rule by up to 2%. This restores full aggregate employment but does not change the trade deficit or the decline in manufacturing share, confirming the role of monetary policy as an aggregate, not a distributional tool. Monetary policy loosening does not affect the trade deficit much because of the Chinese peg – if the US loosens monetary policy, the effective interest rate in China declines, too.[50]

[50] In the Appendix, we study alternative monetary policy rules that are better suited to target unemployment under permanent trade shocks. In a work in progress, we study optimal monetary policy rules in this environment.

In summary, we have found that a modest short-run tariff on Chinese goods in the early 2000s may help alleviate some of the labor market distortion caused by Chinese growth combined with the exchange rate peg.

1.6 Concluding remarks

What is the role of the exchange rate regime in shaping short-to-medium-run responses to trade shocks? The conventional trade literature sidesteps this question by focusing on flexible price equilibrium. We use the three different angles – empirical, theoretical, and quantitative – to revisit the effects of the China shock consistently suggest that China's currency peg against the US dollar is qualitatively and quantitatively pivotal in determining the labor market, trade balance, and welfare response.

We have empirically documented that countries using or pegging to to the US dollar exhibit lower real GDP, a larger decline in manufacturing, and deteriorating trade balances in response to the China shock, compared to countries with similar China shock exposure that float to the US dollar. Notably, the floating countries have their currency appreciate in response to a larger exposure to the China shock, suggesting that the exchange rate operates as an adjustment margin. We develop a simple model of wage rigidity that can explain these findings, where we analytically characterize how exchange rate pegs interact with Foreign productivity growth to generate trade deficits and unemployment at Home. When we calibrate the multi-sector trade model to match the trade and sectoral reallocation data, we find that China's peg against the US dollar is quantitatively significant in shaping the effects of the China shock in the US trade deficit, unemployment, and decline in manufacturing.

While we intentionally focused our analysis on the China shock and the US dollar, the intuition of the direction of trade imbalances and labor market adjustments under exchange rate pegs apply more broadly. The post-WWII East Asian growth stories, most notably Japan and South Korea, involve having the currency follow the US dollar and running large trade surpluses in the growth path. Our framework can also give a better understanding of trade balances within the Eurozone, such as the persistent trade surplus of Germany and Ireland, and the deficit of Greece in the Eurozone.

One aspect of the model we intentionally abstracted from is China's policy goal. Why does China peg the exchange rate to the US dollar by effectively overheating its economy to supply cheap goods to the world? Potential explanations missing in our model include financial stability and an increase in investment coming from exchange rate stability, a myopic government seeking to maximize short-run output, learning-by-doing models (where more exports lead to productivity growth), and an increase in trade leading to technology diffusion (Perla et al., 2021). These are all mechanisms outside the scope of our model that can rationalize an exchange rate peg for a growing country, which we do not take a stance on.

One final direction forward is to consider heterogeneous agents in our model. In our model, since the consumption-savings decision is made at a family level, and unemployment is only at the intensive margin, our estimates of the losses from the exchange rate peg are underestimates. With a concave utility, involuntary unemployment in the extensive margin will aggravate losses for the unemployed and may have precautionary saving implications for manufacturing workers in the US. A model of heterogeneous agents and savings in incomplete markets may better highlight the distributional consequences of the China shock and the China peg. Probing this direction would further enrich our understanding of the China shock, and the role of the exchange rate as a shock absorber.

1.7 Empirical Appendix

1.7.1 Description of Data

Table 1.4: Ilzetzki et al. (2019)'s Exchange Rate Classification

Fine	Coarse	Description	Example
1	1	No separate legal tender	Eurozone, Cameroon
2	1	Pre-announced peg	Argentina, Malaysia
3	1	Pre-announced horizontal band $< \pm 2\%$	N/A
4	1	De facto peg	China, Egypt, Saudi Arabia
5	2	Pre-announced crawling peg; band $< \pm 1\%$	Nicaragua
6	2	Pre-announced crawling band $< \pm 2\%$	Sweden, Venezuela
7	2	De facto crawling peg	Russia, Vietnam
8	2	De facto crawling band $< \pm 2\%$	Iceland, Canada
9	3	Pre-announced crawling band $> \pm 2\%$	Hungary, Sri Lanka
10	3	De facto crawling band $< \pm 5\%$	Paraguay, Turkey
11	3	Moving band $< \pm 2\%$	Korea, Thailand
12	3	Managed floating	Brazil, Mexico, United Kingdom
13	4	Freely floating	Japan, United Stats
14	5	Freely falling	Congo, Zimbabwe
15	6	Dual market with missing data	Afghanistan, Myanmar

Note: The table lists the fine and coarse exchange rate regime classification of Ilzetzki et al. (2019). $<$ stands for 'narrower than', and $>$ stands for 'wider than', and denotes the size of the (horizontal, crawling, moving) band. The last column lists some example countries that was classified as that regime as of June 2000.

Table 1.5: Summary statistics for pegs and floats

Variable	Pegs	Floats	Diff
log(population)	1.512	1.677	-0.689*
	(2.341)	(1.512)	(0.372)
log(GDP per capita)	8.421	8.562	-0.141
	(1.374)	(1.628)	(0.283)
MFG share (%)	11.414	14.213	-2.798**
	(6.428)	(7.692)	(1.394)
export (% GDP)	27.977	29.419	-1.442
	(26.995)	(22.065)	(4.561)
import (% GDP)	39.598	34.523	5.075
	(24.433)	(18.492)	(4.001)
NFA / GDP	-0.336	-0.106	-0.230
	(1.097)	(1.262)	(0.221)
CPI inflation	0.0437	0.0346	0.00910
	(0.0562)	(0.0315)	(0.00903)
unemployment rate	0.0870	0.1016	-0.0285**
	(0.0504)	(0.0871)	(0.0135)
S_i (china shock)	0.03493	0.04115	-0.00621
	(0.03022)	(0.03885)	(0.00643)
No. of obs	56	63	

Note: The first two columns report summary statistics for pegging countries and floating countries, with standard deviation in parentheses. The third column reports regression coefficients for regressions of the characteristics on a dummy variable for whether the country's currency is pegged to the US dollar, with the dependent variables on the left, with standard errors for the coefficients in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

1.7.2 Additional results

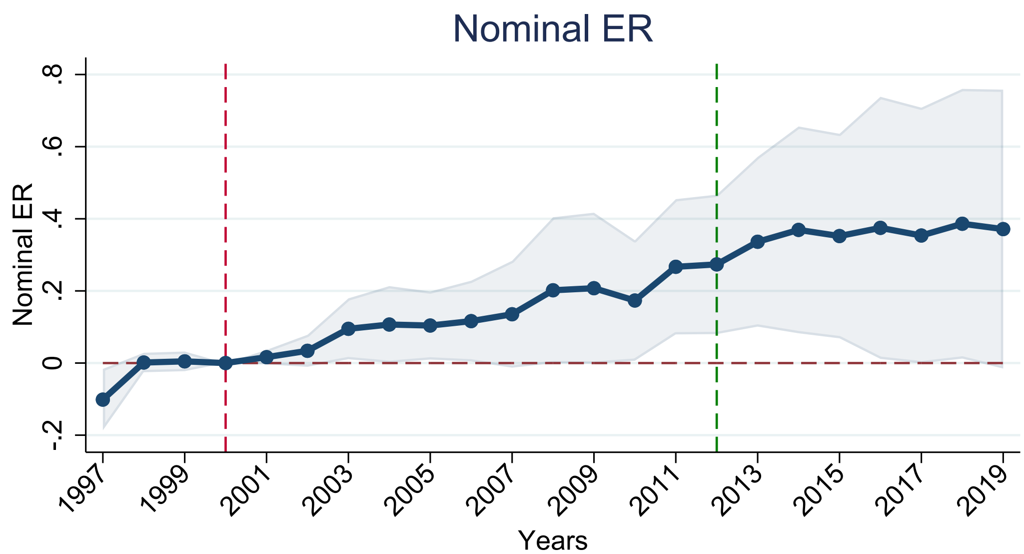
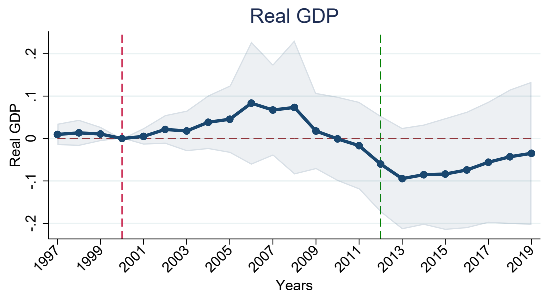
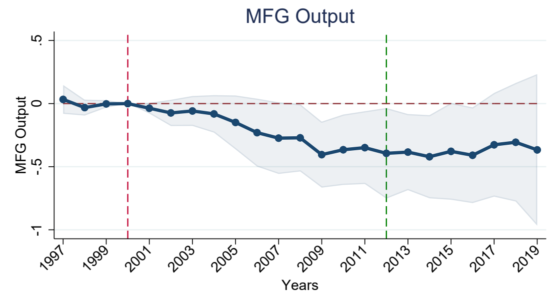


Figure 1.10: Average responses to the China shock across countries.

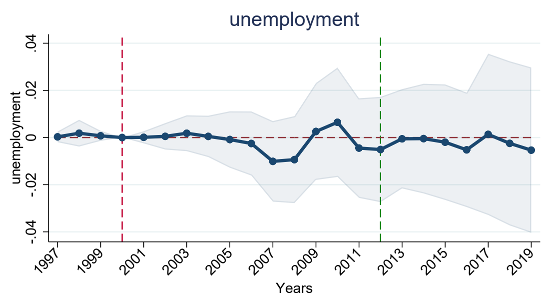
Note. The figure plots the double-difference regression result of the exchange rate against the China shock across all countries. The shaded area is the 95% confidence band for each local projection regression. The red dashed line indicates the beginning of the China shock (2000) and the green the end of the China shock (2012). On average countries' currencies depreciate in response to higher exposure to the China shock; the latter figure shows that the effect is completely driven by floaters.



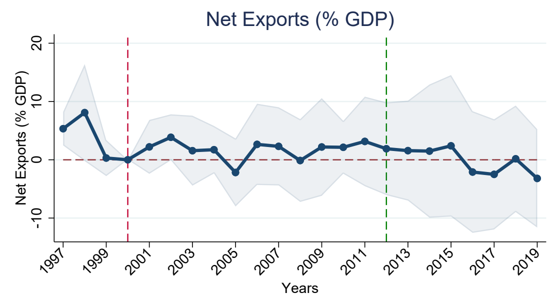
(a) Real GDP



(b) Manufacturing Output



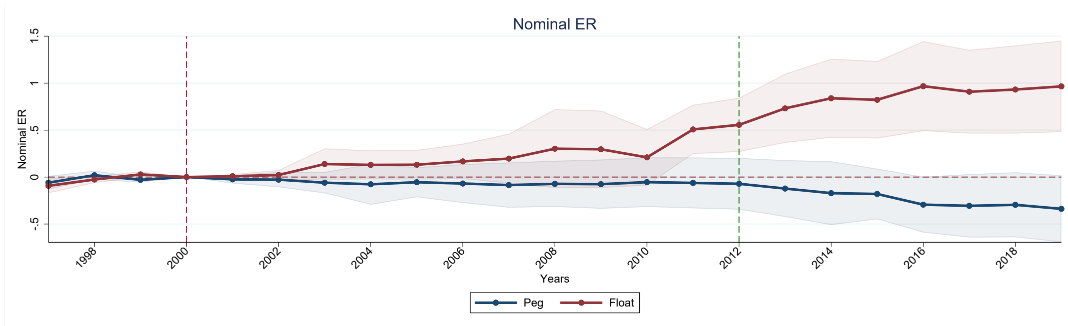
(c) Unemployment

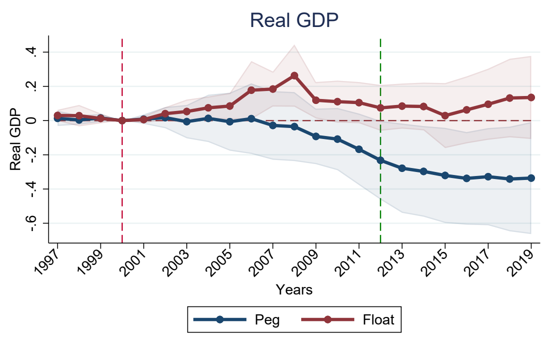


(d) Net Exports (% GDP)

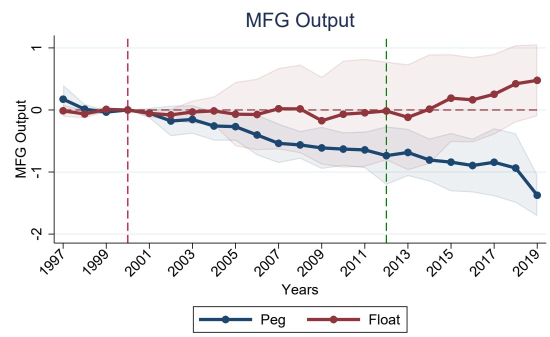
Figure 1.11: Average responses to the China shock across countries.

Note. The plotted coefficient β_{1h} is the average response to the China shock, without taking into account the heterogeneity in exchange rates: this is the 'double-difference' equivalent of Figure 1.3. As we see, the heterogeneity in exchange rate regime masks the true effect of the China shock. The shaded area is the 95% confidence band for each local projection. The red dashed line indicates $t = 2000$, the start of the China shock and the green line $t = 2012$, the end of the China shock.

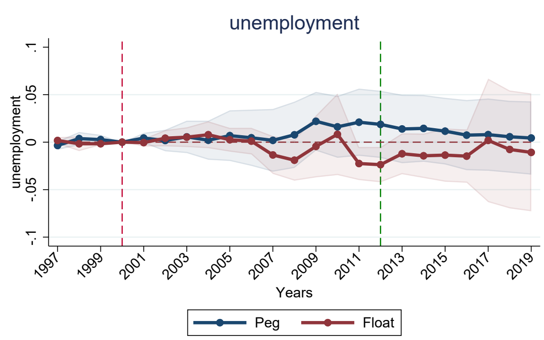




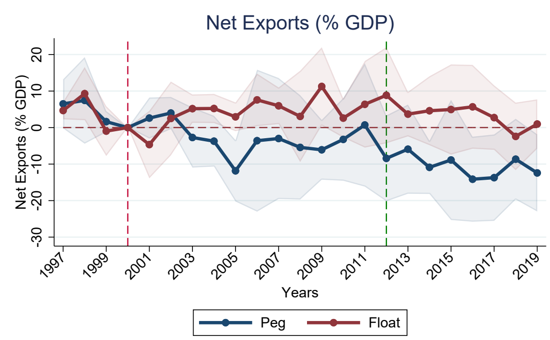
(a) Real GDP



(b) Manufacturing Output



(c) Unemployment



(d) Net Exports (% GDP)

Figure 1.12: Differential response of the China shock.

Note. This regression plots the coefficient for the subset of countries where currency is pegged versus floated against the US dollar respectively. The shaded area is the 95% confidence band for each local projection regression. The red dashed line indicates $t = 2000$, the start of the China shock and the green line $t = 2012$, the end of the China shock. The figures show that the nominal exchange rate for floaters appreciated, and for floaters, higher exposure to the China hsock did not affect manufacturing output, unemployment, or net exports (red lines); in sharp contrast, greater exposure to Chinese export led to lower manufacturing output, a temporary increase in unemployment, and larger trade deficits for pegging countries (blue lines).

1.7.3 Causal identification and inference

In this subsection, we discuss the identification and inference properties of our shift-share instrument, in relation to recent literature on such instruments (Borusyak et al., 2022; Borusyak and Hull, 2023).

Borusyak et al. (2022) (henceforth BHJ) derive sufficient conditions for causal identification in empirical setups that measure the exposure of a shock through a 'shift-share', or an average of a set of shocks with exposure share weights. Their sufficient condition is in terms of a quasi-random assignment of the shocks: in our context, the 'shock', or the growth in global Chinese exports $\Delta \log E_C^s$ is as good as random conditional on the exposure shares s_i. This holds if the shares are exogenous (Goldsmith-Pinkham et al., 2020), or if the large-sample covariance between the export shocks g_C^s and the unobserved shocks ϵ_{ih} in the regression equation (Equation 1.3) is zero. Our preferred interpretation is the latter, following the China shock literature Autor et al. (2013, 2021), henceforth ADH); as highlighted in BHJ, it is *a priori* implausible that the 2000 industry shares λ_i^s are uncorrelated with the errors ϵ_{ih}, as the latter will capture unobserved industry-level shocks. As such, we interpret our empirical strategy as assuming shift exogeneity, rather than share exogeneity.

ADH studies variation within US across commuting zones, and uses Chinese export surge into other developed countries as instruments to purge US-specific demand shocks that may bias their results, adding support to their a priori justification of shift exogeneity. This is unavailable for us, as we study global surge in Chinese exports. However, if there is an unobserved global demand shock towards Chinese goods, either (1) one may interpret this as a part of the 'China shock', or (2) this demand shock violates the exogeneity condition of the ADH instrument. As such, while our analysis is reduced-form, we believe that there is a priori justification for 'global surge in Chinese exports' in each sector being as-good-as-random.

With this in mind, we follow the framework of BHJ to test for the validity and robustness of our exposure measure.

Industry shocks and exposure measures

For the shift-share exposure measure to be valid under the shock exogeneity assumption, it is sufficient to have that g_C^s is as good as random conditional on the shares λ_i^s (Assumption 1 of BHJ). Moreover, for the measured coefficient to be consistent, we need the effective sample size $1/E[\sum_s (\lambda_i^s)^2]$ to be large enough (Assumption 2 of BHJ). Following BHJ, we summarize the distribution of the shocks g_C^s and the industry-level

Table 1.6: Shock and share summary statistics

Mean	1.757
Standard deviation	1.525
Interquartile range	1.596
Effective sample size (1/HHI)	24.38
Largest λ^s weight	0.189
2nd largest λ^s weight	0.022
Effective sample size, SITC3	18.44
Largest λ^s, SITC3	0.214
2nd largest λ^s, SITC3	0.027
No. of shocks (SITC4 industries)	782
No. of SITC3 groups	237

Note: The table summarizes the global China export shock g_C^s across sectors s.

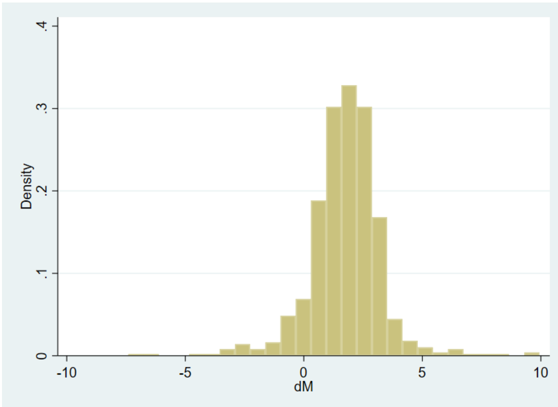
weights $\lambda^s \propto \sum_i \lambda_i^s$ (normalized to add up to one).

Table 1.6 reports summary statistics for the shocks and the shares.[51]. The distribution of the shock is quite regular, with the average of 1.757, a standard deviation of 1.525, and an interquartile range of 1.596. Figure 1.13 shows the histogram of the shocks g_C^s and a Q-Q plot of the realized distribution against the quantile of the normal distribution (using the `qnorm` command of Stata) shows that the distribution is close to normal, which adds support to the shock exogeneity assumption. The inverse HHI – the "effective sample size" according to BHJ – is 24.38. This is smaller than the sample size in BHJ (191.6, 58.4 when acoross SIC3 groups), and the main cause is that some countries in our sample have high concentration in petroleum and crude oil products (code 3330, share 18.9%). Thus we have suggestive evidence that the shocks are as good as random, and the effective sample size is reasonable for causal inference.

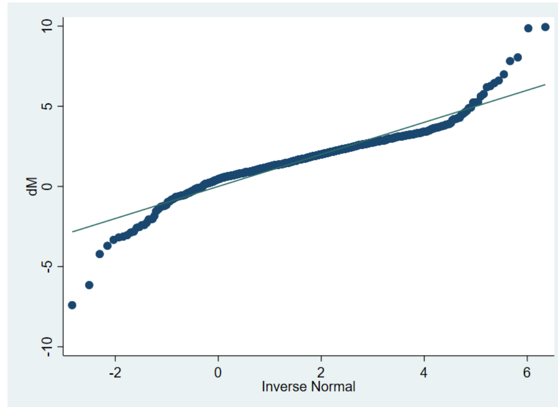
Besides these conditions, Assumption 2 of BHJ require the shocks to be sufficiently mutually uncorrelated. BHJ recommend analyzing the correlation patterns of shocks across the industries using available industry classifications. Following their methodology, we compute intra-class correlation coefficients (ICCs) of shocks within different industry groups. We use a random effects model with nested random effects:

$$g_C^s = \mu + a_{sitc1(s)} + b_{sitc2(s)} + c_{sitc3(s)} + \epsilon_s \tag{1.39}$$

[51] This table is the analogue of Table 1 in BHJ.



(a) Histogram



(b) Q-Q wrt normal distribution

Figure 1.13: Distribution of global China export shock g_C^s

	Estimate	SE
SITC 1-digit	0.225	(0.142)
SITC 2-digit	0.193	(0.087)
SITC 3-digit	0.281	(0.089)
4-digit (residual)	1.594	(0.096)
No. of SITC1 groups	10	
No. of SITC2 groups	69	
No. of SITC3 groups	237	
No. of shocks (SITC4 industries)	782	

Table 1.7: China export shock intra-class correlations

Note: This table reports intra-class correlation coefficients for the g_C^s China export shocks in Section 1.2, estimated from the hierarchical model (Equation 1.39).

where $a_{sitc1(s)}, b_{sitc2(s)}, c_{sitc3(s)}$ respecitvely denote random effects generated by the SITC 1-digit sectors, 2-digit sectors, and 3-digit sectors respectively. We estimate Equation 1.39 as a hierarchical linear model with maximum likelihood assuming Gaussian residuals. Table 1.7 reports the results from this mixed linear model; there is moderate clustering of shock residuals at each level of the SITC (0.225, 0.193, 0.281), but the residual component at the 4-digit level is largest. This supports the assumption that shocks are sufficiently mutually uncorrelated.

Non-random exposure

Next, we purge bias coming from non-random exposure to shocks, following Borusyak and Hull (2023). If some countries structurally have higher exposure to the quasi-random

China shock because they have higher shares λ_i^s, this will create a bias in the regression coefficient; notably, in our example, if pegged countries structurally have higher (lower) shares, the estimated effect of the interaction term will be biased upwards (downwards). This is econometrically equivalent to the 'incomplete shares' issue raised in BHJ; even if the DGP for the shocks $\Delta \log E_C^s$ is truly random, if some countries have structurally high exposure shares λ_i^s, the regression coefficients will be biased.

In this subsection, we briefly explain our implied DGP, and how using $\sum_s \lambda_i^s$ is equivalent to the re-centering instrument. We assume that the shocks $g = g_C^s$ come from a distribution G with mean $E[g] = \sum_s \frac{g_C^s}{S}$. In this case, countries with higher $\sum_s \lambda_i^s$ is going to have a higher *expected* exposure $E[\lambda_i^s g_C^s]$ conditional on the DGP, and this is going to bias our regression which seeks to evaluate the effect of causal higher g_C^s on outcomes. Borusyak and Hull (2023) show that 're-centering' the exposure $S_i = \sum_s \lambda_i^s g_C^s$ by instrumenting S_i with

$$\hat{S}_i = \sum \lambda_i^s g_C^s - E[\sum_s \lambda_i^s g^s | g \in G],$$

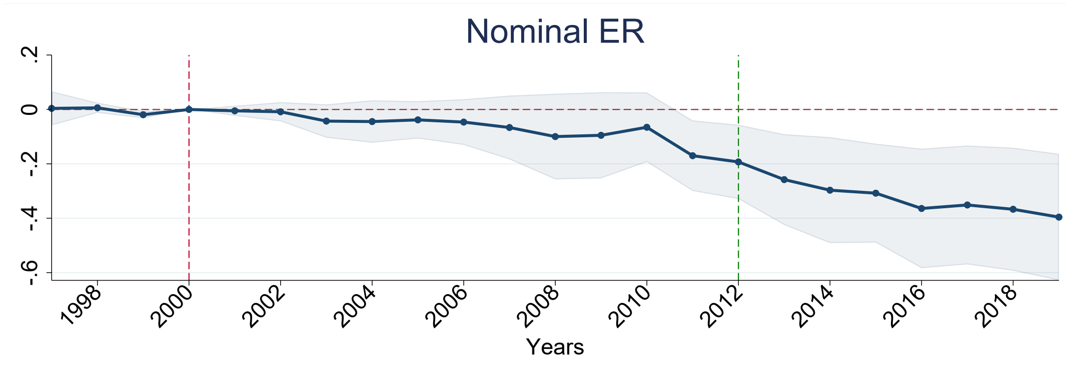
or alternatively controlling for $E[\sum_s \lambda_i^s g^s | g^s \in G]$ in the regressions is sufficient to purge this bias. But in linear shift-share settings such as ours under conditional exogeneity of the shock, we have

$$E[\sum_s \lambda_i^s g^s | g^s \in G] = \sum_s \lambda_i^s E[g^s],$$

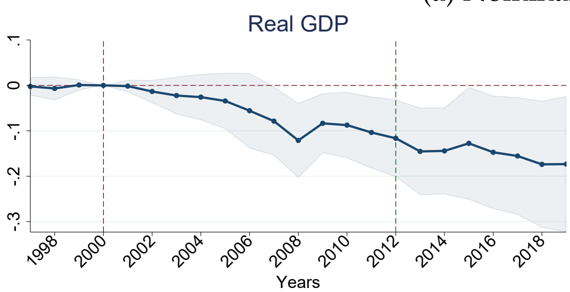
so this is equivalent to controlling for $\sum_s \lambda_i^s$ in the regression; this is exactly the solution for the 'incomplete shares' problem in Borusyak et al. (2022). Since we control for $\sum_s \lambda_i^s$ in our regressions, this is sufficient to purge the bias coming from non-random exposure.

In this section, we perform several robustness exercises, including: progressively adding and removing controls including lagged variables, running the regression for an alternative meausre of the *share*, and running the regression for an alternative measure of the *shock*. We find that our results are robust across these broad specifications.

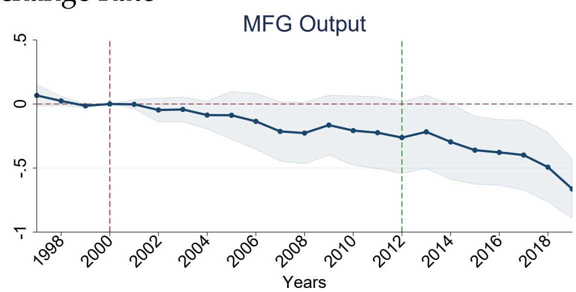
1.7.4 Adding and removing controls



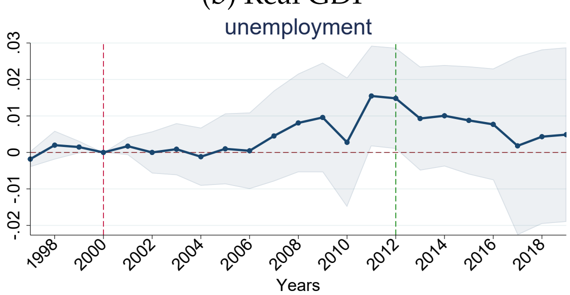
(a) Nominal Exchange Rate



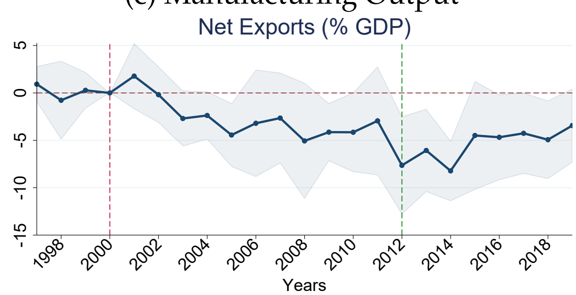
(b) Real GDP



(c) Manufacturing Output

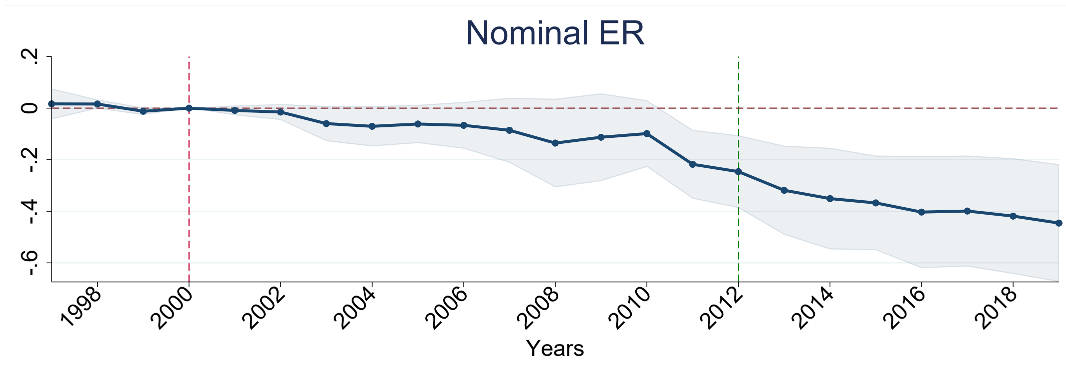


(d) Unemployment rate

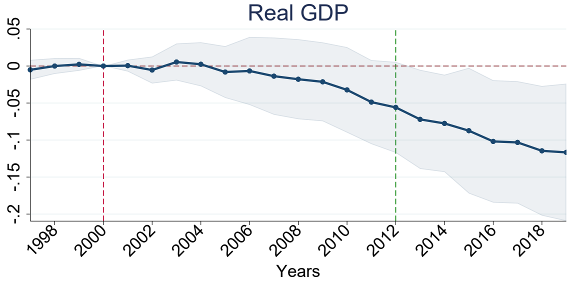


(e) Net Exports

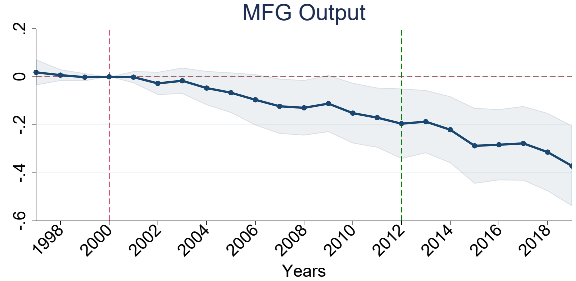
Figure 1.14: Plot of β_{3h} with the following controls: log(GDP), log(pop), $\sum_s \lambda_i^s$, and $(\sum_s \lambda_i^s) \times \text{Peg}_i$. Does not include the interaction of the other controls and the peg.



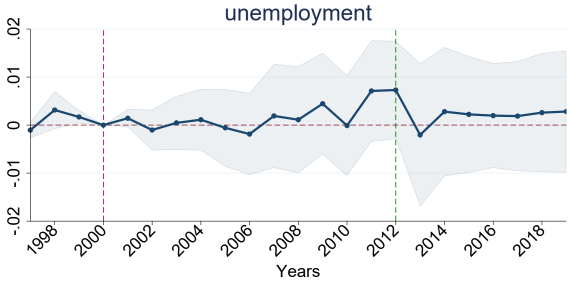
(a) Nominal Exchange Rate



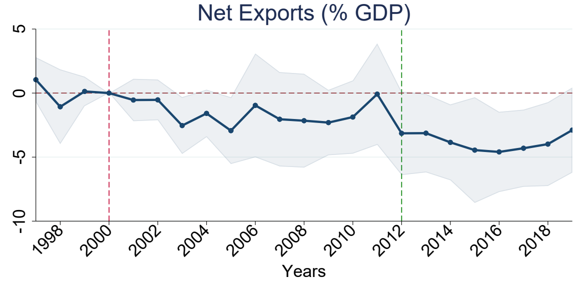
(b) Real GDP



(c) Manufacturing Output

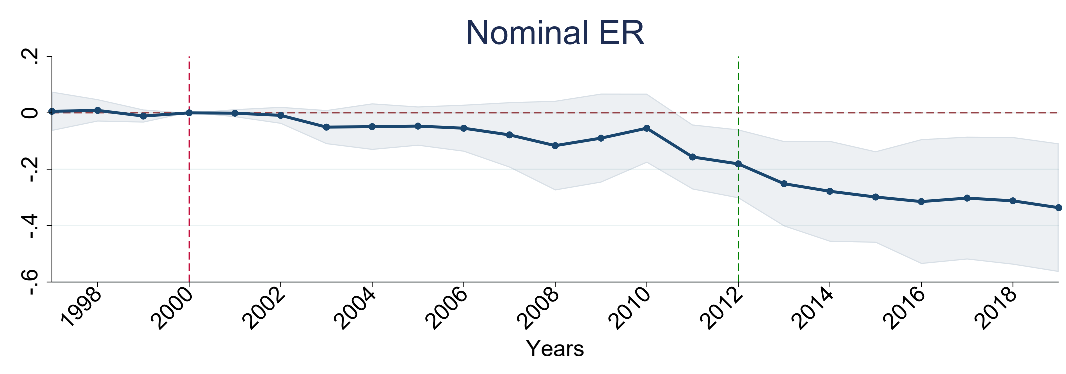


(d) Unemployment rate

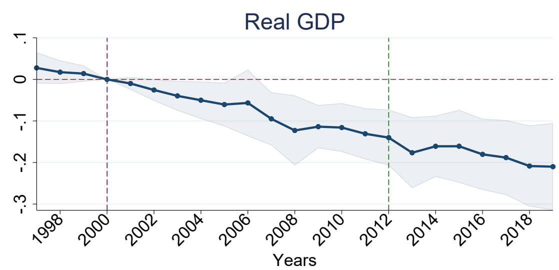


(e) Net exports

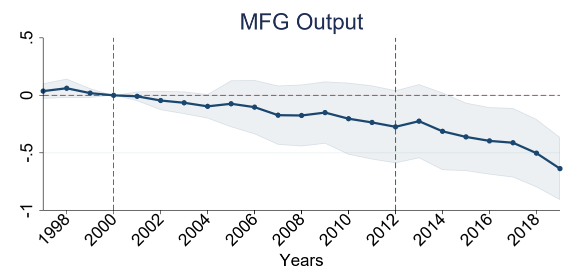
Figure 1.15: Plot of β_{3h} with the following controls: log(GDP), log(pop), $\sum_s \lambda_i^s$. No interactions.



(a) Nominal Exchange Rate

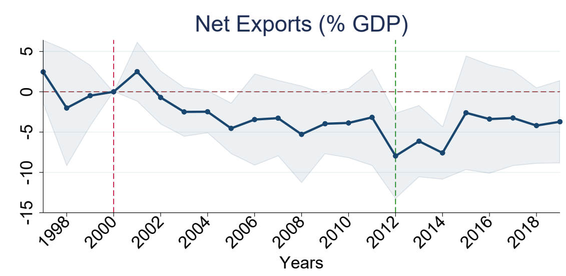


(b) Real GDP



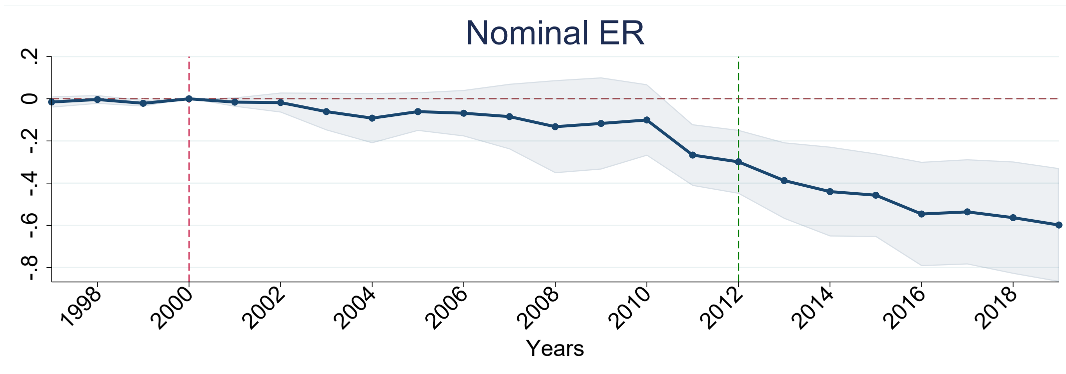
(c) Manufacturing Output

(d) Unemployment rate

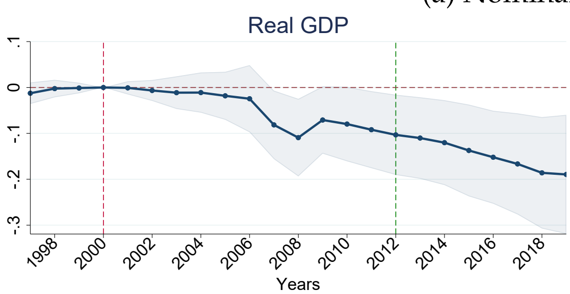


(e) Net exports

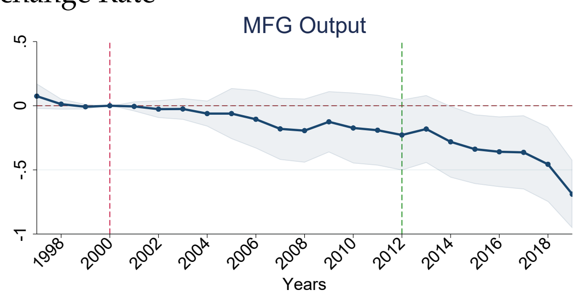
Figure 1.16: Plot of β_{3h} with the same regression, except no lagged DV.



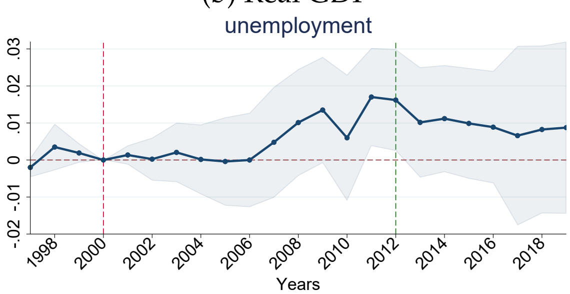
(a) Nominal Exchange Rate



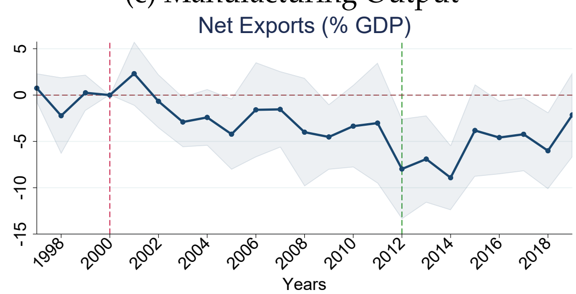
(b) Real GDP



(c) Manufacturing Output

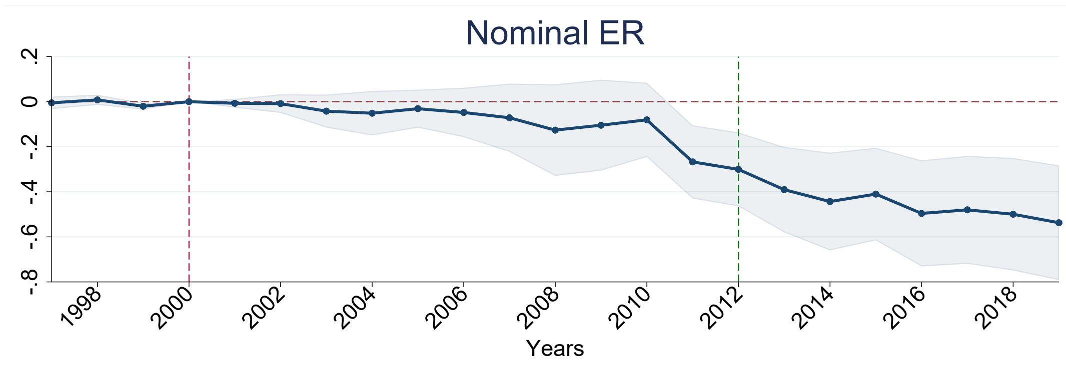


(d) Unemployment rate

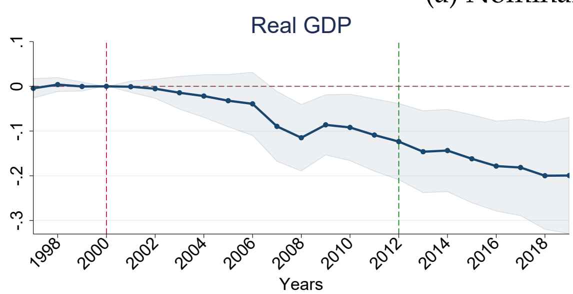


(e) Net exports

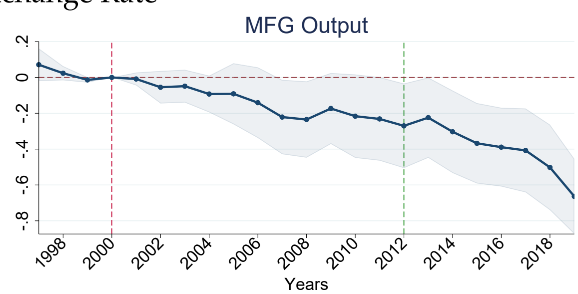
Figure 1.17: Plot of β_{3h} with 2000 Unemployment rate, and its interaction with peg, as additional cnotrol.



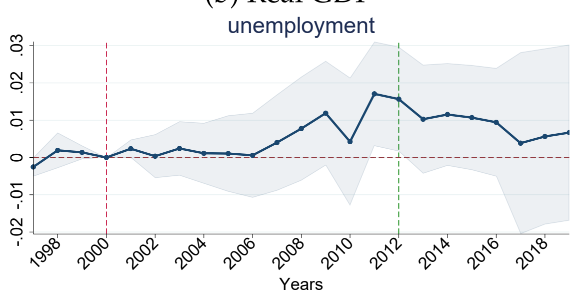
(a) Nominal Exchange Rate



(b) Real GDP



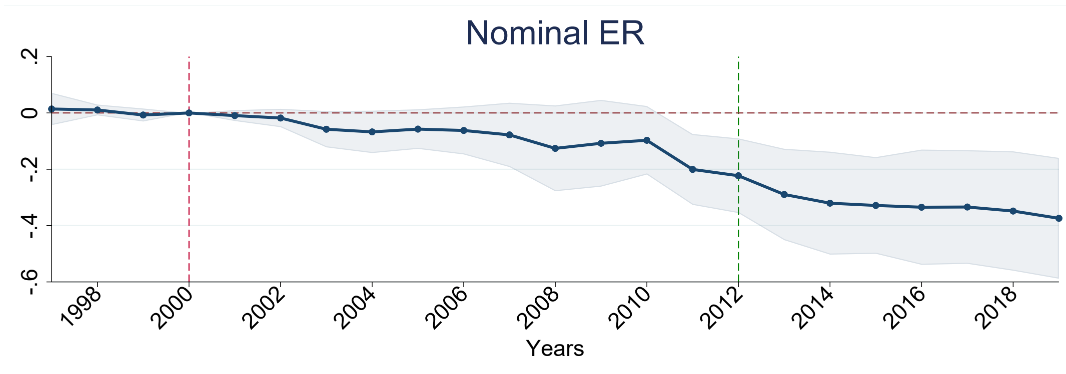
(c) Manufacturing Output



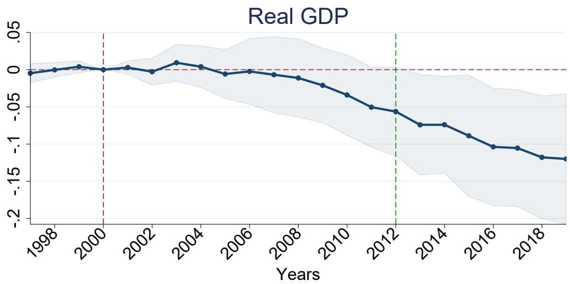
(d) Unemployment rate

(e) Net exports

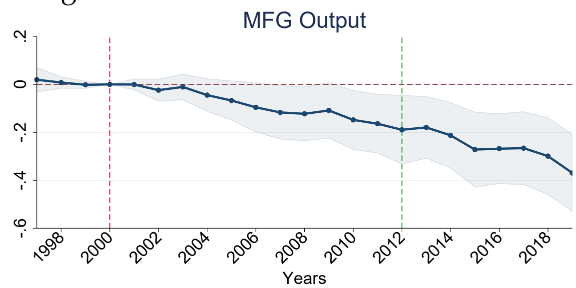
Figure 1.18: Plot of β_{3h} with 2000 Unemployment rate as additional control, without its interaction with the peg.



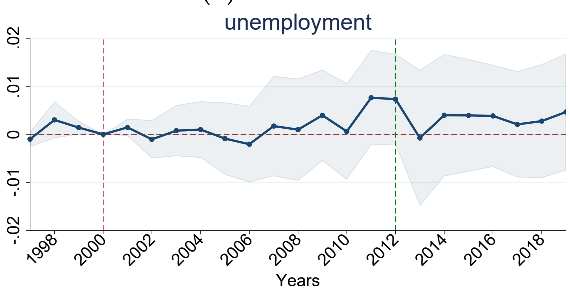
(a) Nominal Exchange Rate



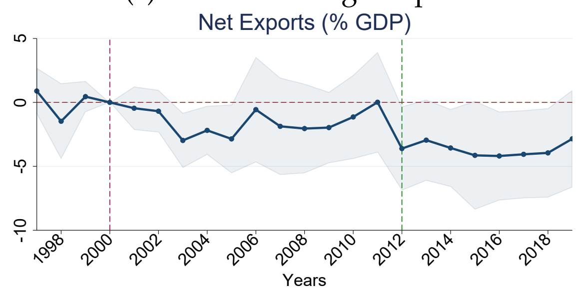
(b) Real GDP



(c) Manufacturing Output

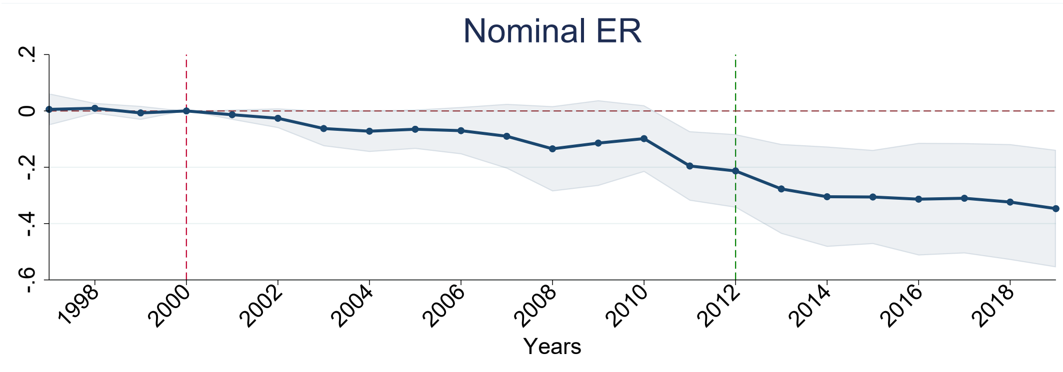


(d) Unemployment rate

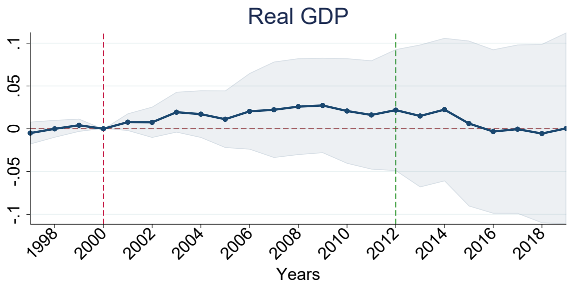


(e) Net exports

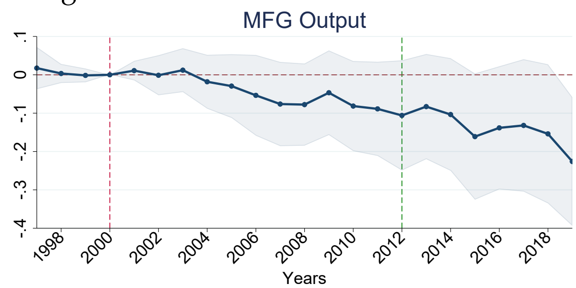
Figure 1.19: Plot of β_{3h} with GDP per capita and $\sum_s \lambda_i^s$ as controls.



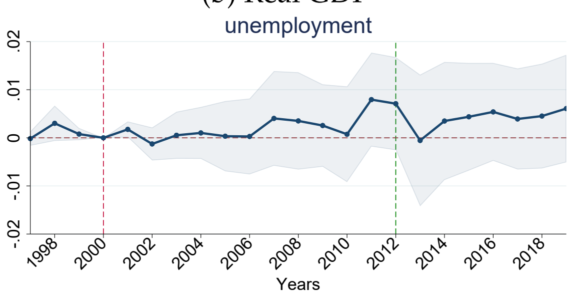
(a) Nominal Exchange Rate



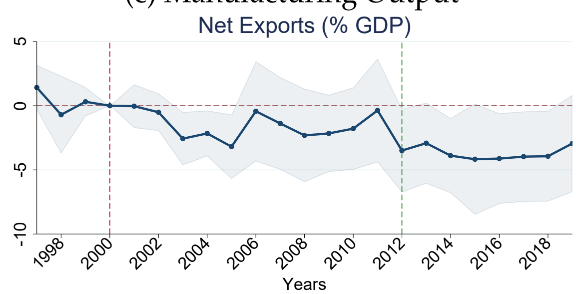
(b) Real GDP



(c) Manufacturing Output



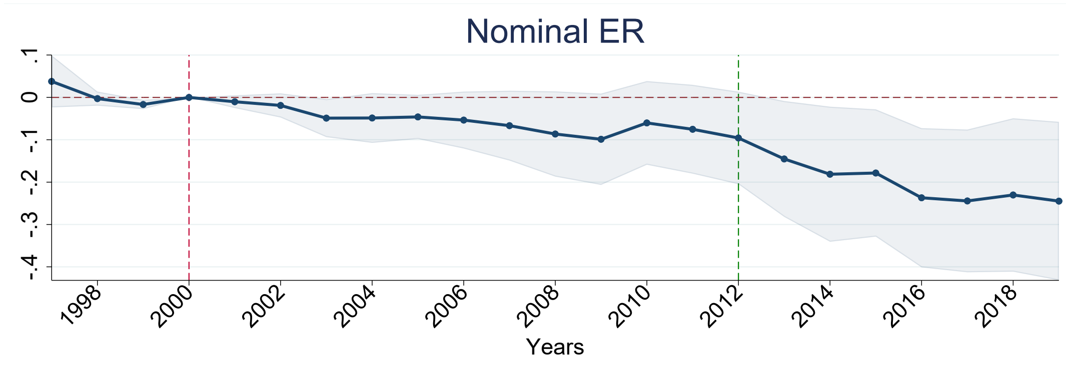
(d) Unemployment rate



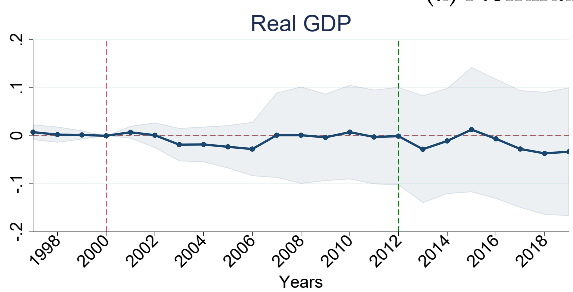
(e) Net exports

Figure 1.20: Plot of β_{3h} with $\sum_s \lambda_i^s$ as controls.

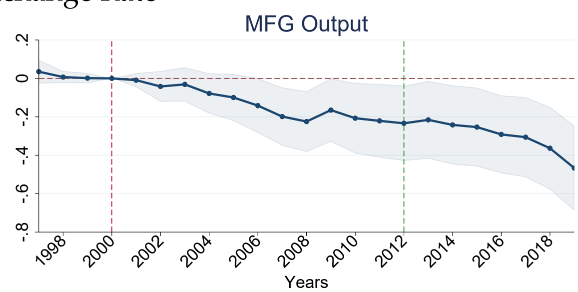
1.7.5 Different measures of the exposure



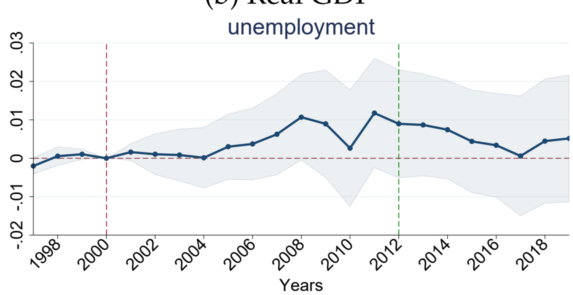
(a) Nominal Exchange Rate



(b) Real GDP



(c) Manufacturing Output

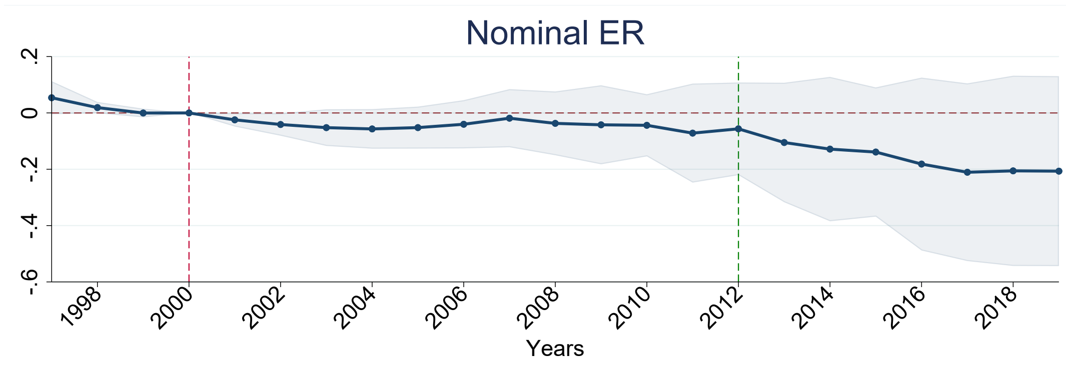


(d) Unemployment rate

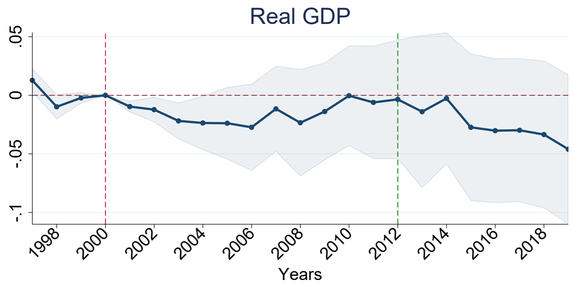
(e) Net exports

Figure 1.21: Regression with alternative shares

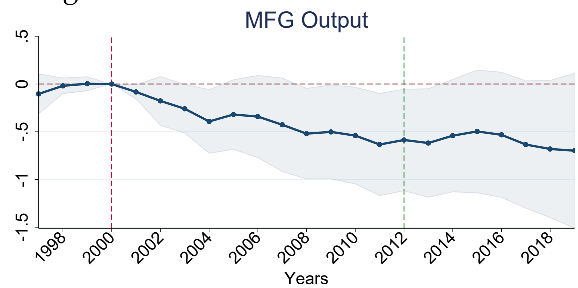
Note. This is the result of the main text regression with the same controls, but with an alternative definition of the China shock S_i, given by $S_i = \sum_s \frac{E^s_{it}}{EXP_{it}} \Delta \log(E^s_C)$. Here the shares are exports to a sector s divided by total *exports* of the country i, so that the weights sum to 1.



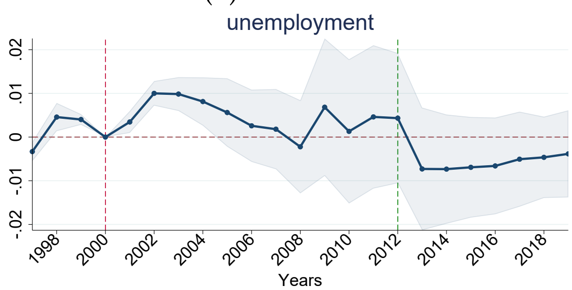
(a) Nominal Exchange Rate



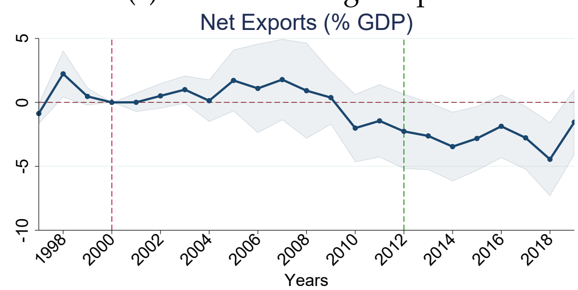
(b) Real GDP



(c) Manufacturing Output



(d) Unemployment rate

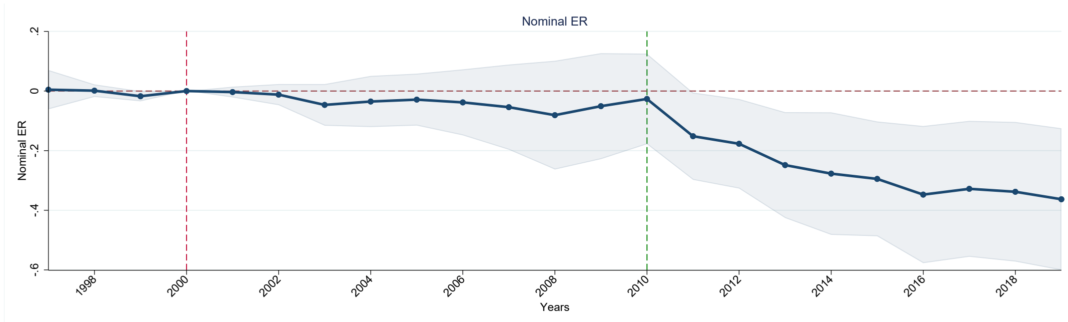


(e) Net exports

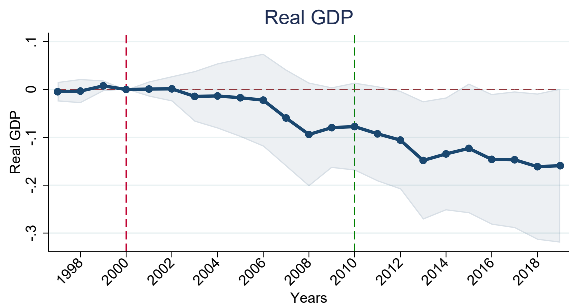
Figure 1.22: Regression with alternative shocks

Note. This is the result of the main text regression with the same controls, but with an alternative definition of the China shock S_i, given by $S_i = \sum_s \frac{E^s_{it}}{GDP_{it}} \frac{\Delta E^s_C}{L_{it}}$. Here the *shifts* of the shift-share is the difference in total export volume of China in sector s, divided by the total employment in country i.

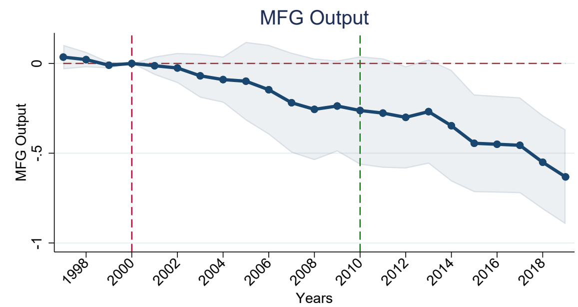
1.7.6 Different post-shock periods



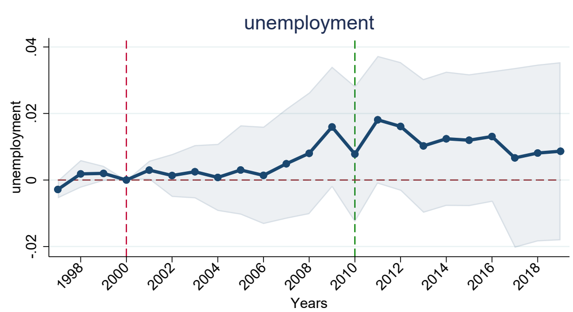
(a) Nominal Exchange Rate



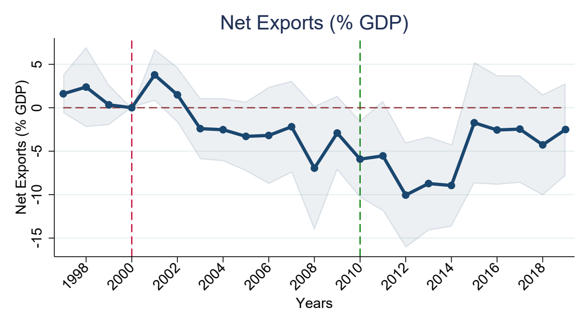
(b) Real GDP



(c) Manufacturing Output

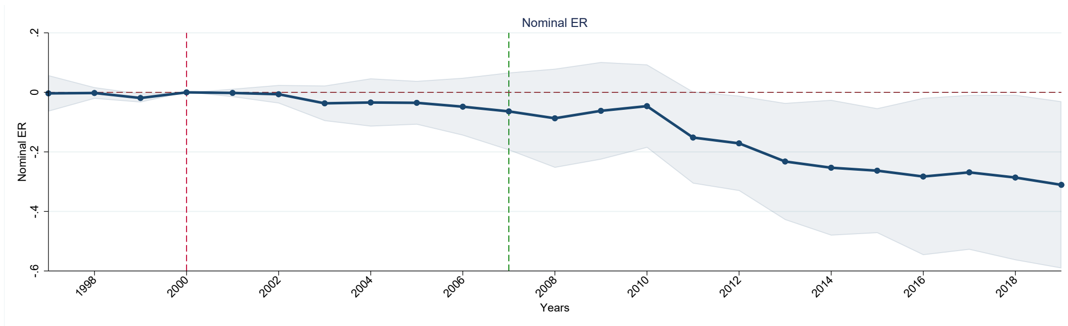


(d) Unemployment rate

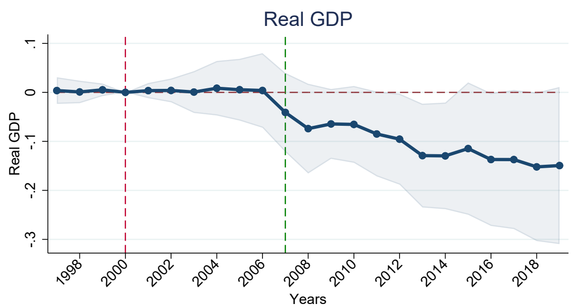


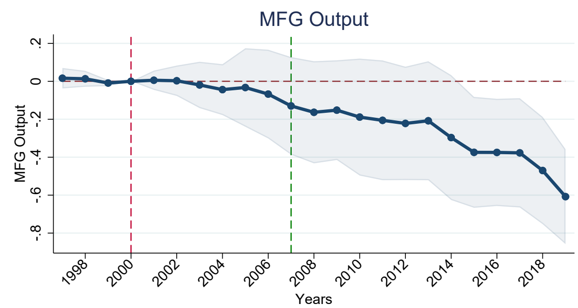
(e) Net exports

Figure 1.23: Regression with China shock constructed to end at 2010.
Note. This is the result of the main text regression with the same specification as the main text, except that the shock component g^s_C of the China shock is constructed as growth between 2000 and 2010.



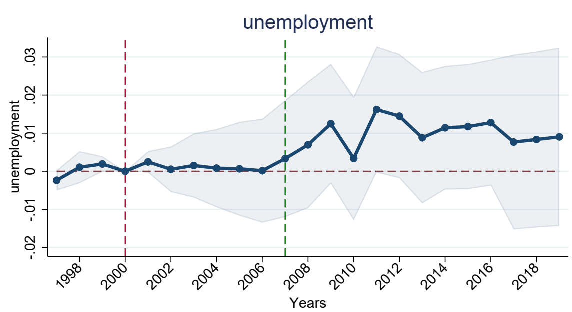
(a) Nominal Exchange Rate

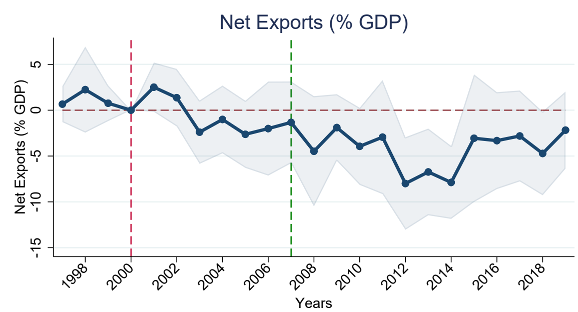




(b) Real GDP

(c) Manufacturing Output





(d) Unemployment rate

(e) Net exports

Figure 1.24: Regression with China shock constructed to end at 2007.

Note. This is the result of the main text regression with the same specification as the main text, except that the shock component g_C^s of the China shock is constructed as growth between 2000 and 2007.

1.8 Proofs of propositions

1.8.1 Proofs for Subsection 1.3.3

In this section I prove the Propositions in Section 1.3.3. In the equilibrium under the exchange rate peg, I assume without loss of generality that $\bar{e} = 1$. I first highlight a number of properties of the laissez-faire equilibrium that I extensively use in the proof.

Lemma 1.2. *Denote by $\omega_t = \frac{w_{Ht}}{w_{Ft}}$ the relative wage of Home at period $t \in \{0, 1\}$. The following properties hold:*

(a) The real wage $\frac{w_{jt}}{P_{jt}}$ and expenditure share λ_{ijt} depend on $\{w_{Ht}, w_{Ft}\}$ only through ω_t.

(b) Home real wage $\frac{w_{Ht}}{P_{Ht}}$ increases in ω_t, while Foreign real wage decreases in ω_t.

(c) Expenditure share for Home goods λ_{Hjt} is a decreasing function of ω_t; $\lambda_{Fjt} = 1 - \lambda_{Hjt}$ is an increasing function of ω_t

(d) Home relative wage is higher in period 0: $\omega_0 > \omega_1$.

(e) The real wage of Home is higher in period 0: $\frac{w_{H0}}{P_{H0}} > \frac{w_{H1}}{w_{P1}}$.

(f) Relative inflation is higher at Foreign. If we define $\pi_j = \frac{P_{j1}}{P_{j0}}$, we have $\pi_F > \pi_H$.

Proof. (a) We have

$$\frac{w_{Ht}}{P_{Ht}} = \frac{w_{Ht}}{(P_{HHt}^{1-\sigma} + P_{FHt}^{1-\sigma})^{1/(1-\sigma)}} = \frac{w_{Ht}}{((w_{Ht}/A_{HH})^{1-\sigma} + (w_{Ft}/A_{FH})^{1-\sigma})^{1/(1-\sigma)}}$$
$$= \frac{1}{((1/A_{HH})^{1-\sigma} + (\omega_t/A_{FH})^{1-\sigma})^{1/(1-\sigma)}}$$

and analogously for w_{Ft}/P_{Ft}. Likewise, we have

$$\lambda_{Hjt} = \frac{P_{Hjt}^{1-\sigma}}{P_{Hjt}^{1-\sigma} + P_{Fjt}^{1-\sigma}} = \frac{1}{1 + (\frac{w_{Ft}/A_{Fj}}{w_{Ht}/A_{Hj}})^{1-\sigma}} = \frac{1}{1 + (\omega_t)^{\sigma-1}(\frac{A_{Hj}}{A_{Fj}})^{1-\sigma}}$$

and $\lambda_{Fjt} = 1 - \lambda_{Hjt}$. In general, the real wage and expenditure share are functions of ω_t for *any* homothetic aggregator of Home and Foreign goods $C_j = C_j(C_{Hjt}, C_{Fjt})$.

(b) By inspection of the previous formula, we see that when $\sigma > 1$, $\frac{w_{Ht}}{w_{Ft}}$ is increasing in ω_t.

(c) Likewise, when $\sigma > 1$, λ_{Hjt} is decreasing in ω_t.

(d) Denote by $\omega^*(\{A_{ij}\})$ the Home relative wage under a *static, flexible-price* economy under productivity $\{A_{ij}\}_{i,j\in\{H,F\}}$, which can be solved by the trade balance equation:

$$\lambda_{FH} w_H L_H = \lambda_{HF} w_F L_F \;\; \Rightarrow \;\; \omega^* \frac{L_H}{L_F} = \frac{\lambda_{HF}(\omega^*)}{\lambda_{FH}(\omega^*)}$$

Now since L_j is increasing in $\frac{w_j}{P_j}$, the left-hand side is increasing in ω^* while the right-hand side is decreasing in ω^*. Thus there is a unique ω^*.

Consider the trade shock that increases A_F. Since λ_{FH} is increasing in A_F, λ_{FH} is decreasing in A_F, we have that a higher A_F decreases the right-hand side. Thus to satisfy equality, an increase in A_F must be accompanied by a *decrease* in ω^*.

We assumed that Home relative wage ω_0 is rigid at $\omega_0 = \omega^*(\{A_{ij,-1}\})$. Given an increase in A_F, $\omega_0 = \omega^*(\{A_{ij,-1}\}) > \omega^*(\{A_{ij0}\})$. Now, if we assumed for sake of contradiction that $\omega_1 \geq \omega_0 > \omega^*(\{A_{ij0}\}) = \omega^f$, we would have

$$\omega_t \frac{L_H(\omega_t)}{L_F(\omega_t)} > \frac{\lambda_{HF}(\omega_t)}{\lambda_{FH}(\omega_t)} \text{ for } t = 0, 1$$

but this would break the lifetime trade balance condition – Home's relative wage is too high in both periods, so Home cannot balance the lifetime budget. Thus we have $\omega_0 > \omega_1$.

(e) This follows from 2 and 5.

(f) We have

$$\begin{aligned}
\left(\frac{P_{Ht}}{P_{Ft}}\right)^{1-\sigma} &= \frac{P_{HHt}^{1-\sigma} + P_{FHt}^{1-\sigma}}{P_{HFt}^{1-\sigma} + P_{FFt}^{1-\sigma}} = \frac{(\omega_t \frac{A_{FF}}{A_{HH}})^{1-\sigma} + (\frac{A_{FF}}{A_{FH}})^{1-\sigma}}{(\omega_t \frac{A_{FF}}{A_{HF}})^{1-\sigma} + 1} \\
&= (\frac{A_{HF}}{A_{HH}})^{1-\sigma} (1 + \frac{(\frac{A_{HH}A_{FF}}{A_{HF}A_{FH}})^{1-\sigma} - 1}{(\omega_t \frac{A_{FF}}{A_{HF}})^{1-\sigma} + 1})
\end{aligned}$$

Since $\sigma > 1$ and $\frac{A_{HH}A_{FF}}{A_{HF}A_{FH}} > 1$ (Home bias, equivalently $\tau_{FH}\tau_{HF} \geq 1$), the last expression is decreasing in ω_t. Then since $\omega_0 > \omega_1$ and again $\sigma > 1$, we have $\frac{P_{H0}}{P_{F0}} > \frac{P_{H1}}{P_{F1}}$. Rearranging, we get $\pi_F > \pi_H$.

□

Using these properties, we prove the propositions.

Proposition 1.1. *In the pegged equilibrium, in response to a trade shock ($A_{FH} \uparrow$), Home runs a trade deficit ($B_{H1} < 0$). Moreover, if Home monetary policy does not respond ($R_{H1} = \frac{1}{\beta}$), then there is involuntary unemployment at Home ($\mu_{H0} < 0$).*

Proof. For the first part ($B_{H1} < 0$), note that Home borrows in the short-run if the following inequalities hold:

$$\underbrace{\lambda_{HF0} P_{F0} C_{F0}}_{t=0 \text{ Home exports}} < \underbrace{\lambda_{FH0} P_{H0} C_{H0}}_{t=0 \text{ Home imports}} \quad \text{and} \quad \underbrace{\lambda_{HF1} P_{F1} C_{F1}}_{t=1 \text{ Home exports}} > \underbrace{\lambda_{FH1} P_{H1} C_{H1}}_{t=1 \text{ Home imports}} \tag{1.40}$$

Invert the second inequality and multiply with the first to have

$$\frac{\lambda_{HF0}}{\lambda_{HF1}} \frac{P_{F0} C_{F0}}{P_{F1} C_{F1}} < \frac{\lambda_{FH0}}{\lambda_{FH1}} \frac{P_{H0} C_{H0}}{P_{H1} C_{H1}}$$

Rearrange to have:

$$\frac{\lambda_{HF0}/\lambda_{HF1}}{\lambda_{FH0}/\lambda_{FH1}} < \frac{\pi_F}{\pi_H} \frac{C_{H0}/C_{H1}}{C_{F0}/C_{F1}} \tag{1.41}$$

where $\pi_j = \frac{P_{j1}}{P_{j0}}$ denote inflation in country j. Note that if $B_1 > 0$, both inequalities are flipped in Inequality 1.40, so we have the exact opposite inequality, so Inequality 1.41 is a necessary and sufficient condition for Home borrowing. Since both countries face the same nominal interest rate under a peg, we have

$$C_{j0}^{-1/\gamma} = \beta(1+i) \frac{1}{\pi_j} C_{j1}^{-1/\gamma} \quad \Rightarrow \quad \frac{C_{j0}}{C_{j1}} = [\beta(1+i)\pi_j^{-1}]^{-\gamma}$$

Use this to rewrite Inequality 1.41 as

$$\frac{\lambda_{HF0}/\lambda_{HF1}}{\lambda_{FH0}/\lambda_{FH1}} < [\frac{\pi_F}{\pi_H}]^{1-\gamma} \Leftrightarrow B_{H1} < 0$$

(Note that the left-hand-side is the first 'variation in terms-of-trade across time' governed by σ, while the right-hand-side is the second 'home bias and relative prices' governed by γ, as described in the main text.)

With the CES parametric assumption, we may rewrite the expenditure shares λ_{ij} as

$$\frac{\lambda_{HF0}}{\lambda_{HF1}} = \frac{(P_{HF0}^{1-\sigma}/P_{F0}^{1-\sigma})}{(P_{HF1}^{1-\sigma}/P_{F1}^{1-\sigma})} = \pi_F^{1-\sigma} (\frac{w_{H0}}{w_{H1}})^{1-\sigma}$$
$$\frac{\lambda_{FH0}}{\lambda_{FH1}} = \frac{(P_{FH0}^{1-\sigma}/P_{H0}^{1-\sigma})}{(P_{FH1}^{1-\sigma}/P_{H1}^{1-\sigma})} = \pi_H^{1-\sigma} (\frac{w_{F0}}{w_{F1}})^{1-\sigma}$$

Hence,

$$\frac{\lambda_{HF0}/\lambda_{HF1}}{\lambda_{FH0}/\lambda_{FH1}} = (\frac{\pi_F}{\pi_H})^{1-\sigma}(\frac{w_{H0}/w_{H1}}{w_{F0}/w_{F1}})^{1-\sigma}$$

This is smaller than $[\frac{\pi_F}{\pi_H}]^{1-\gamma}$ if and only if

$$\begin{aligned}
(\frac{\pi_F}{\pi_H})^{1-\sigma}(\frac{w_{H0}/w_{H1}}{w_{F0}/w_{F1}})^{1-\sigma} &< (\frac{\pi_F}{\pi_H})^{1-\gamma} \\
\Leftrightarrow (\frac{w_{H0}/w_{H1}}{w_{F0}/w_{F1}})^{1-\sigma} &< (\frac{\pi_F}{\pi_H})^{\sigma-\gamma}
\end{aligned}$$

We have that the left-hand side is less than 1 by $\sigma > 1$ and part (d) of Lemma 1.2. We have that the right-hand side is greater than 1 by $\sigma > \gamma$ and part (f) of Lemma 1.2. Thus we have $RHS > 1 > LHS$.

For the second part ($\mu_{H0} < 0$ when $R_{H0} = 1/\beta$), we first have

$$v'(L_{H1}) = u'(C_{H1})\frac{w_{H1}}{P_{H1}}$$

From part (e) of Lemma 1.2, we have $\frac{w_{H0}}{w_{P0}} > \frac{w_{H1}}{w_{P1}}$. At the same time, we have $u'(C_{H1}) = u'(C_{H0})$ with $R_H = \frac{1}{\beta}$. Thus, if we can show $L_{H1} > L_{H0}$, we have

$$\mu_{H0} = v'(L_{H0}) - u'(C_{H0})\frac{w_{H0}}{P_{H0}} < v'(L_{H1}) - u'(C_{H1})\frac{w_{H1}}{P_{H1}} = 0$$

We proceed to show $L_{H1} > L_{H0}$. Goods market clearing condition is $L_{Ht} = \tau_{HH}C_{HHt} + \tau_{HF}C_{HFt}$, and since $C_{H1} = C_{H0}$ and $\lambda_{HH0} < \lambda_{HH1}$ by $\frac{w_{H0}}{w_{F0}} > \frac{w_{H1}}{w_{F1}}$, we have $C_{HH0} < C_{HH1}$. Moreover, with $\sigma > 1$ and $\sigma > \gamma$, we have

$$\begin{aligned}
\frac{C_{HF0}}{C_{HF1}} = \frac{(\frac{P_{HF0}}{P_{F0}})^{-\sigma}C_{F0}}{(\frac{P_{HF1}}{P_{F1}})^{-\sigma}C_{F1}} &= \frac{(\frac{P_{HF0}}{P_{F0}})^{-\sigma}}{(\frac{P_{HF1}}{P_{F1}})^{-\sigma}} \cdot (\beta(1+i)\frac{P_{F0}}{P_{F1}})^{-\gamma} \\
&< \frac{(\frac{P_{HF0}}{P_{F0}})^{-\gamma}}{(\frac{P_{HF1}}{P_{F1}})^{-\gamma}} \cdot (\frac{P_{H1}}{P_{H0}}\frac{P_{F0}}{P_{F1}})^{-\gamma} \\
&= (\frac{P_{HF0}}{P_{HF1}}\frac{P_{H1}}{P_{H0}})^{-\gamma} = (\frac{w_{H0}}{w_{H1}}\frac{P_{H1}}{P_{H0}})^{-\gamma} < 1
\end{aligned}$$

where we have the intermediate inequality because $(\frac{P_{HF0}}{P_{F0}}/\frac{P_{HF1}}{P_{F1}}) > 1$ (which follow from $\omega_0 > \omega_1$) and $\sigma \geq \gamma$, and the last inequality from part (e) of Lemma 1.2. Thus we have $C_{HH0} < C_{HH1}$ and $C_{HF0} < C_{HF1}$, so $L_{H0} < L_{H1}$, and we obtain $\mu_{H0} < 0$. □

For the next proposition, we first prove that deficits hurt future terms-of-trade.

Lemma 1.3. *Suppose Home borrows more in real terms, so that $\frac{B_{H1}}{w_{H1}}$ decreases. Then $\frac{w_{H1}\bar{e}}{w_{F1}}$ falls: Home future relative wage worsens as a result of Home borrowing.*

Proof. The goods market clearing condition for Home goods at $t = 1$ can be rewritten as

$$w_{H1}L_{H1} = \lambda_{HH1}(w_{H1}L_{H1} + B_{H1}) + \lambda_{HF1}(w_{F1}L_{F1} - B_{H1})$$

Rearranging this equation and writing everything in terms of $S_{H1} = \frac{w_{H1}}{w_{F1}}$ and $b = \frac{B_{H1}}{w_{H1}}$, we may write

$$1 = \lambda_{HH1}(1 + \frac{b}{L_{H1}}) + \lambda_{HF}(\frac{1}{S}\frac{L_{F1}}{L_{H1}} - \frac{b}{L_{H1}})$$
$$b[\frac{\lambda_{HH} - \lambda_{HF}}{L_H}] = 1 - \lambda_{HH} - \lambda_{HF}(\frac{1}{S}\frac{L_F}{L_H})$$

We have $\frac{\partial \lambda_{HH1}}{\partial S}, \frac{\partial \lambda_{HF1}}{\partial S} < 0$ (Home better terms-of-trade $\iff$ Home goods more expensive), $\frac{\partial L_H}{\partial S} > 0, \frac{\partial L_F}{\partial S} < 0$ (Home better TOT $\iff$ Home workers have better real wage, want to work more). Then the RHS is increasing in S. Moreover, from home bias we have $\lambda_{HH} + \lambda_{FF} > 1 \rightarrow \lambda_{HH} > \lambda_{HF}$, so the coefficient on b is positive. Thus $\frac{\partial b}{\partial S} > 0$; then $\frac{\partial S}{\partial b} = \frac{1}{\frac{\partial b}{\partial S}} > 0$ so running more debt ($b \downarrow$) will lead to worsening terms of trade $S \downarrow$. □

Proposition 1.2. *In the equilibrium where policy does not respond ($R_{H1} = \frac{1}{\beta}$), the effect of a small increase of A_{FH} on Home welfare $\mathcal{U}_H$ is ambiguous, and depends on σ. For small changes in $\epsilon_A = A_{FH0} - A_{FH-1}$, we have that:*

- *When $\sigma \rightarrow 1$, we have Home welfare increases as a result of the Foreign shock: $\frac{d\mathcal{U}_H}{dA_{FH}} > 0$.*
- *When $\sigma \rightarrow \infty$, we have Home welfare decreases as a result of the Foreign shock: $\frac{d\mathcal{U}_H}{dA_{FH}} < 0$*

Proof. We first derive the first-order welfare equation 1.21:

$$\frac{d\mathcal{U}_H}{dA_{FH}} = \underbrace{-\frac{u'(C_{H0})}{P_{H0}}C_{FH0}\frac{dP_{FH0}}{dA_{FH}}}_{\text{cheap goods}} + \underbrace{\mu_0\frac{dL_0}{dA_{FH}}}_{\text{labor wedge}} + \underbrace{\frac{\beta u'(C_{H1})}{P_{H1}}[C_{HF1}\frac{dP_{HF1}}{dA_{FH}} - C_{FH1}\frac{dP_{FH1}}{dA_{FH}}]}_{\text{terms of trade at } t=1}$$

Home agent's lifetime utility is

$$\mathcal{U}_H = U(C_{HH0}, C_{FH0}, C_{HH1}, C_{FH1}, L_{H0}, L_{H1})$$

and is subject to the lifetime budget constraint

$$P_{HH0}C_{HH0} + P_{FH0}C_{FH0} + \frac{1}{1+i_{Ht}}(P_{HH1}C_{HH1} + P_{FH1}C_{FH1}) = w_{H0}L_{H0} + \frac{1}{1+i_{H1}}w_{H1}L_{H1}$$

Invoking the Envelope theorem, the first-order effect of A_F on $\mathcal{U}_H$ can be written as

$$\frac{d\mathcal{U}_H}{dA_{FH}} = \sum_{t=0}^{1}\sum_{i\in\{H,F\}}\frac{dU}{dC_{iHt}}\frac{dC_{iHt}}{dA_{FH}} + \sum_{t=0}^{1}\frac{dU}{dL_{Ht}}\frac{dL_{Ht}}{dA_{FH}} \tag{1.42}$$

If we denote by $\tilde{\lambda}$ the Lagrange multiplier on the lifetime budget constraint, we have:

$$\frac{dU}{dC_{iH0}} = \tilde{\lambda}P_{iH0}, \quad \frac{dU}{dC_{iH1}} = \frac{\tilde{\lambda}}{1+i_{H1}}P_{iH1}, \quad \frac{dU}{dL_{H1}} = -\frac{\tilde{\lambda}}{1+i_{H1}}w_{H1}$$

while we may have $\frac{dU}{dL_{H0}} \neq -\tilde{\lambda}w_{H0}$ because households do not choose L_{H0}: in fact, we have

$$\frac{dU}{dL_{H0}} + \tilde{\lambda}w_{H0} = -v'(L_{H0}) + \frac{u'(C_{H0})}{P_{H0}}w_{H0} = -\mu_0.$$

Plugging these into Equation 1.42, we get

$$\frac{d\mathcal{U}_H}{dA_{FH}} = \tilde{\lambda}\left[\sum_{i\in\{H,F\}}(P_{iH0}\frac{dC_{iH1}}{dA_F} + \frac{P_{iH1}}{1+i_{H1}}\frac{dC_{iH0}}{dA_F}) - w_{H0}\frac{dL_{H0}}{dA_{FH}} - \frac{w_{H1}}{1+i_{H1}}\frac{dL_{H1}}{dA_{FH}}\right] - \mu_0\frac{dL_0}{dA_{FH}} \tag{1.43}$$

Now, if we take the derivative of the budget constraint, we have

$$\begin{aligned}
&\sum_{i\in\{H,F\}}\left(P_{iH0}\frac{dC_{iH0}}{dA_F} + \frac{P_{iH1}}{1+i_{H1}}\frac{dC_{iH1}}{dA_F}\right) - w_{H0}\frac{dL_{H0}}{dA_{FH}} - \frac{1}{1+i_{H1}}w_{H1}\frac{dL_{H1}}{dA_{FH}} \\
= &-\sum_{i\in\{H,F\}}\left(C_{iH0}\frac{dP_{iH0}}{dA_F} + \frac{C_{iH1}}{1+i_{H1}}\frac{dP_{iH1}}{dA_F}\right) + L_{H0}\frac{dw_{H0}}{dA_{FH}} + \frac{L_{H1}}{1+i_{H1}}\frac{dw_{H1}}{dA_{FH}} \\
= &-C_{FH0}\frac{dP_{FH0}}{dA_{FH}} - \sum_{i\in\{H,F\}}\frac{C_{iH1}}{1+i_{H1}}\frac{dP_{iH1}}{dA_F} + \frac{L_{H1}}{1+i_{H1}}\frac{dw_{H1}}{dA_{FH}}
\end{aligned}$$

where the last expression follows from the fact that w_{H0} is fixed, so we have $\frac{dw_{H0}}{dA_{FH}} = \frac{dP_{HH0}}{dA_{FH}} = 0$. Now to further simplify the last term $-\sum_{i\in\{H,F\}}\frac{C_{iH1}}{1+i_{H1}}\frac{dP_{iH1}}{dA_F} + \frac{L_{H1}}{1+i_{H1}}\frac{dw_{H1}}{dA_{FH}}$, we note that the Home goods market clearing condition in period 1 is

$$L_{H1} = \frac{1}{A_H}C_{HH1} + \frac{\tau_{HF1}}{A_H}C_{HF1}$$

and $P_{HH1} = w_{H1}/A_H$ so $dP_{HH1} = \frac{1}{A_H} dw_{H1}$. From this, we can rewrite

$$
\begin{aligned}
-\sum_{i \in \{H,F\}} C_{iH1} \frac{dP_{iH1}}{dA_F} + L_{H1} \frac{dw_{H1}}{dA_{FH}} &= -C_{HH1} \frac{dP_{HH1}}{dA_F} + C_{FH1} \frac{dP_{FH1}}{dA_{FH}} + (\frac{1}{A_H} C_{HH1} + \frac{\tau_{HF1}}{A_H} C_{HF1}) \frac{dw_{H1}}{dA_{FH}} \\
&= -C_{FH1} \frac{dP_{FH1}}{dA_{FH}} + \frac{\tau_{HF1}}{A_H} C_{HF1} \frac{dw_{H1}}{dA_{FH}} \\
&= -C_{FH1} \frac{dP_{FH1}}{dA_{FH}} + C_{HF1} \frac{dP_{HF1}}{dA_{FH}}
\end{aligned}
$$

Substitute everything into Equation 1.43 to obtain

$$
\frac{d\mathcal{U}_H}{dA_{FH}} = -\tilde{\lambda} C_{FH0} \frac{dP_{FH0}}{dA_{FH}} - \mu_0 \frac{dL_0}{dA_{FH}} + \frac{\tilde{\lambda}}{1 + i_{H1}} (C_{HF1} \frac{dP_{HF1}}{dA_{FH}} - C_{FH1} \frac{dP_{FH1}}{dA_{FH}}) \tag{1.44}
$$

and we substitute in $\tilde{\lambda} = \frac{u'(C_{H0})}{P_{H0}} = \frac{\beta(1+i_{H1})u'(C_{H1})}{P_{H1}}$ to obatin Equation 1.21.

The terms have natural interpretations:

- The first term, $-\tilde{\lambda} C_{FH0} \frac{dP_{FH0}}{dA_{FH}}$ correspond to utility gains from cheaper consumption at $t = 0$. As A_F increases, $\frac{dP_{FH0}}{dA_{FH}}$ takes on a negative value, so the utility increases.

- The second term $-\mu_0 \frac{dL_0}{dA_{FH}}$ is the *labor wedge* at $t = 0$. Labor is away from where the consumer wants to supply it. As a result of a higher A_F we have $\mu_0 < 0$ (from Proposition 1.1) and $dL_0 < 0$, so there is a loss in welfare.

- The third term $C_{HF1} \frac{dP_{HF1}}{dA_{FH}} - C_{FH1} \frac{dP_{FH1}}{dA_{FH}}$ can be interpreted as the terms-of-trade in $t = 1$; it pins down how much total revenue changes from an additional import versus an additional export, multiplied by the marginal utility of a dollar at $t = 1$. This is affected by both the permanent increase in A_F and the trade imbalance that is incurred that affects future terms-of-trade (Lemma 1.3).

Now we can prove the proposition. Consider a small shock that increases $A_F \to A_F + \epsilon$.

When $\sigma \to 1$, we know that $\mu_0 \to 0$, and $B_{H1} \to 0$. (This is known from Cole and Obstfeld (1991), but we can directly inspect the proof of Proposition 1.1 and see that all the inequalities become equalities at $\sigma = 1$). So the *first-order relevant* welfare changes are the decrease in prices resulting from the productivity gains (term (1) and the productivity component of term (3)). Thus there is a welfare gain when $\sigma \to 1$.

On the other hand, as $\sigma \to \infty$, the welfare losses from term (2) are discrete. Specifically, consider the following formulation:

$$d\mathcal{U}_H = -\tilde{\lambda} C_{FH0} dP_{FH0} - \mu_0 dL_0 + \frac{\tilde{\lambda}}{1+i_{H1}}(C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1})$$

When $0 < dA_{FH} < \epsilon$, the first and third terms are bounded by the price changes, which are also at most epsilon: so we have

$$\| -\tilde{\lambda} C_{FH0} dP_{FH0} + \frac{\tilde{\lambda}}{1+i_{H1}}(C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1}) \| < \epsilon_M$$

On the other hand, as $\sigma \to \infty$, we have $L_0 \to 0$, and $\mu_0 \to \mu < 0$; there is a *discrete* loss of welfare associated with an *infinitesimal* change in A_F. As such, we have that for small ϵ and large σ, $\frac{d\mathcal{U}_H}{dA_{FH}} < 0$: there is a welfare loss associated with trade.

Remark. We conjecture that $\frac{d\mathcal{U}_H}{dA_{FH}}$ is *monotonic* in σ, so that there exists a σ^* such that there are welfare gains when $\sigma < \sigma^*$ and losses when $\sigma > \sigma^*$. This seems intuitive, as all three effects (gains from cheaper goods, labor wedge, and future terms-of-trade) should naturally be monotonic in σ. However, we are unable to prove this, and leave this as a possibility. □

1.8.2 Proofs for Subsection 1.3.4

Here we prove the propositions for the optimal policy subsection. For this, we prove the following Lemma, also defined in the next Appendix (Section 1.9).

Lemma 1.4. *The first-order effect of a tariff and subsidy on Home welfare can be written as:*

$$\begin{aligned} d\mathcal{U}_H = &- \underbrace{\mu_0 dL_0}_{\textit{labor wedge}} + \frac{u'(C_{H0})}{P_{H0}}[\underbrace{t_{FH0}P_{FH0}dC_{FH0}}_{C_{H0}\ \textit{distortion}} - \underbrace{d(s_{HF0}P_{HF0}C_{HF0})}_{\textit{cost of subsidy}}] \\ &+ \frac{\beta u'(C_{H1})}{P_{H1}}(\underbrace{C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1}}_{\textit{future terms-of-trade}}) \end{aligned}$$

Proof. Re-normalize the tariffs $t_{FH0} \to t_{FH0}/P_{FH0}$, and subsidies $s_{HF0} \to s_{HF0}/P_{HF0}$ so that they have the interpretation of a 'flat addition in price', and we can renormalize them back later.

The rest of the argument is similar to the proof of Proposition 1.2 above. Home agent's lifetime utility is

$$\mathcal{U}_H = U(C_{HH0}, C_{FH0}, C_{HH1}, C_{FH1}, L_{H0}, L_{H1})$$

and is subject to the lifetime budget constraint

$$\begin{aligned}&P_{HH0}C_{HH0} + (P_{FH0} + t_{FH0})C_{FH0} + \frac{1}{1+i_{Ht}}(P_{HH1}C_{HH1} + P_{FH1}C_{FH1})\\&= w_{H0}L_{H0} + \frac{1}{1+i_{H1}}w_{H1}L_{H1} + T_{H0}\end{aligned}$$

with $T_{H0} = t_{FH0}C_{FH0} - s_{HF0}C_{HF0}$.

Analogously to the proof of Proposition 1.2, the first-order effect of any policy on welfare can be written as

$$d\mathcal{U}_H = \sum_{t=0}^{1}\sum_{i\in\{H,F\}}\frac{dU}{dC_{iHt}}dC_{iHt} + \sum_{t=0}^{1}\frac{dU}{dL_{Ht}}dL_{Ht} \tag{1.45}$$

If we denote by $\tilde{\lambda}$ the Lagrange multiplier on the lifetime budget constraint, we have:

$$\begin{aligned}\frac{dU}{dC_{HH0}} &= \tilde{\lambda}P_{HH0}, & \frac{dU}{dC_{FH0}} &= \tilde{\lambda}(P_{FH0} + t_{FH0})\\ \frac{dU}{dC_{HH1}} &= \frac{\tilde{\lambda}}{1+i_{H1}}P_{HH1}, & \frac{dU}{dC_{FH1}} &= \frac{\tilde{\lambda}}{1+i_{H1}}P_{FH1}\\ \frac{dU}{dL_{H0}} &= -\mu_0 - \tilde{\lambda}w_{H0}, & \frac{dU}{dL_{H1}} &= -\frac{\tilde{\lambda}}{1+i_{H1}}w_{H1}\end{aligned}$$

Plugging these into Equation 1.45, we get

$$\begin{aligned}d\mathcal{U}_H =& \tilde{\lambda}\left[\sum_{i\in\{H,F\}}\left(P_{iH0}dC_{iH0} + \frac{P_{iH1}}{1+i_{H1}}dC_{iH1}\right) - w_{H0}dL_{H0} - \frac{w_{H1}}{1+i_{H1}}dL_{H1}\right]\\&+ \tilde{\lambda}t_{FH0}dC_{FH0} - \mu_0 dL_0\end{aligned}$$

Now the household lifetime budget constraint, with the tax revenue plugged in, is

$$\begin{aligned}&P_{HH0}C_{HH0} + P_{FH0}C_{FH0} + \frac{1}{1+i_{Ht}}(P_{HH1}C_{HH1} + P_{FH1}C_{FH1})\\&= w_{H0}L_{H0} + \frac{1}{1+i_{H1}}w_{H1}L_{H1} - s_{HF0}C_{HF0}\end{aligned}$$

Take the derivative of this, and rearrange to obtain

$$\begin{aligned}&\sum_{i\in\{H,F\}}\left(P_{iH0}dC_{iH0}+\frac{P_{iH1}}{1+i_{H1}}dC_{iH1}\right)-w_{H0}dL_{H0}-\frac{1}{1+i_{H1}}w_{H1}dL_{H1}\\&=\frac{1}{1+i_{H1}}(C_{HF1}dP_{HF1}-C_{FH1}dP_{FH1})-d(s_{HF0}C_{HF0})\end{aligned}$$

where we use the fact that $dP_{HH0}=dP_{FH0}=dw_{H0}=0$ by rigidity, and then further simplify using the Home labor market clearing condition. Then the first-order welfare effects are given by

$$\begin{aligned}d\mathcal{U}_H&=-\mu_0 dL_0+\tilde{\lambda}t_{FH0}dC_{FH0}-\tilde{\lambda}d(s_{HF0}C_{HF0})+\frac{\tilde{\lambda}}{1+i_{H1}}(C_{HF1}dP_{HF1}-C_{FH1}dP_{FH1})\\&=-\mu_0 dL_0+\frac{u'(C_{H0})}{P_{H0}}[t_{FH0}dC_{FH0}-d(s_{HF0}C_{HF0})]+\frac{\beta u'(C_{H1})}{P_{H1}}(C_{HF1}dP_{HF1}-C_{FH1}dP_{FH1})\end{aligned}$$

□

Lemma 1.1. *The optimal short-run tariff rate on imports t_{FH0} satisfies*

$$t_{FH0}=\frac{1}{P_{FH0}}\left[\underbrace{\frac{\mu_0}{\tilde{\lambda}}\frac{\partial L_{H0}}{\partial C_{FH0}}}_{\text{labor wedge}}-\frac{1}{(1+i_{H1})}\underbrace{(L_{HF1}\frac{\partial w_{H1}}{\partial C_{FH0}}-L_{FH1}\frac{\partial w_{F1}}{\partial C_{FH0}})}_{\text{future terms-of-trade}}+\underbrace{s_{HF0}P_{HF0}\frac{\partial C_{HF0}}{\partial C_{FH0}}}_{\text{subsidy externality}}\right] \tag{1.46}$$

The optimal short-run subsidy rate on exports s_{HF0} satisfies

$$s_{HF0}=\frac{1}{P_{HF0}}\left[-\underbrace{\frac{\mu_0}{\tilde{\lambda}}\frac{\partial L_{H0}}{\partial C_{HF0}}}_{\text{labor wedge}}+\frac{1}{(1+i_{H1})}\underbrace{(L_{HF1}\frac{\partial w_{H1}}{\partial C_{HF0}}-L_{FH1}\frac{\partial w_{F1}}{\partial C_{HF0}})}_{\text{future terms-of-trade}}-\underbrace{P_{HF0}C_{HF0}\frac{\partial s_{HF0}}{\partial C_{HF0}}}_{\text{terms-of-trade today}}\right] \tag{1.47}$$

where $\tilde{\lambda}$ is the Lagrange multiplier on the lifetime budget constraint.

Proof. Under variation in tariffs, the optimal tariff rate with $d\mathcal{U}_H=0$ will satisfy

$$t_{FH0}=\frac{1}{P_{FH0}\frac{dC_{FH0}}{dt_{FH0}}}\left[\frac{\mu_0}{\tilde{\lambda}}\frac{dL_{H0}}{dt_{FH0}}+\frac{d(s_{HF0}P_{HF0}C_{HF0})}{dt_{HF0}}-\frac{1}{(1+i_{H1})}(L_{HF1}\frac{dw_{H1}}{dt_{FH0}}-L_{FH1}\frac{dw_{F1}}{dt_{FH0}})\right]$$

The multiplier $\frac{1}{P_{FH0}\frac{dC_{FH0}}{dt_{FH0}}}<0$ corresponds to the inverse elasticity of domestic demand with respect to tariffs; a lower elasticity implies a higher tariff rate. The first term is the effect of tariff on the labor wedge. Since $\frac{dL_{H0}}{dt_{FH0}} > 0$, when there is unemployment ($\mu_0 < 0$), we want a higher tariff. The second term is the effect of tariffs on subsidy revenue; a higher tariff will decrease real wage in Foreign, leading them to work/consume less, decreasing subsidy revenue. The third term is how much future terms-of-trade moves, in terms of how much marginal revenue from exports vs expenditure from imports move. A higher tariff will lead to less borrowing, leading to improving terms-of-trade, increasing the term.

In summary, when there is unemployment ($\mu_0 < 0$), the three terms inside the bracket are all negative; thus the optimal tariff t_{FH0} is *positive*.

A special case is when the Home economy is small; here today's tariffs cannot affect (1) tomorrow's terms-of-trade and (2) the subsidy revenue, so the optimal tariff is simply

$$t_{FH0} = \frac{1}{P_{FH0}\frac{dC_{FH0}}{dt_{FH0}}} \frac{\mu_0}{\tilde{\lambda}} \frac{dL_{H0}}{dt_{FH0}}$$

and this immediately shows that (1) the tariff is positive and (2) the tariff leaves some unemployment ($\mu_0 < 0$; otherwise, we have a contradiction.)

Now, considering variation in subsidies, we have

$$\begin{aligned} s_{HF0} = &\frac{1}{P_{HF0}\frac{dC_{HF0}}{ds_{HF0}}}[-P_{HF0}C_{HF0} + t_{FH0}P_{FH0}\frac{dC_{FH0}}{ds_{HF0}} \\ &- \frac{\mu_0}{\tilde{\lambda}}\frac{dL_{H0}}{ds_{HF0}} + \frac{1}{(1+i_{H1})}(L_{HF1}\frac{dw_{H1}}{ds_{HF0}} - L_{FH1}\frac{dw_{F1}}{ds_{HF0}})] \end{aligned}$$

The multiplier $\frac{1}{P_{HF0}\frac{dC_{HF0}}{s_{HF0}}} > 0$ corresponds to the inverse elasticity of foreign demand with respect to exports, and is positive. The first term is the resource cost of the subsidy; it costs to sell cheap goods. The second term is how much consumption distortion by tariffs is affected by subsidies; with a positive tariff, domestic subsidies will be a resource cost that reduces spending overall. The last two terms deliver similar intuition to the tariff case, with both forces implying a *positive* subsidy. □

Proposition 1.3. *If there is unemployment at the zero-tariff economy ($\mu_{H0} < 0$ when $t_{FH0} = 0$), the optimal tariff t_{FH0} is positive and is increasing in the size of the trade shock A_{FH0}.*

Proof. When $\mu_{H0} < 0$, all three terms in the optimal tariff formula (Equation 1.23) are positive:

- The first term is positive since an increase in imports C_{FH0} reduce demand for Home labor.

- the second is positive since an increase in C_{FH0} decrease w_{H1} relative to w_{F1} tomorrow (transfer affecting future terms-of-trade effect).

- The third term is positive since an increase in C_{FH0} is associated with an increase in exports C_{HF0}.

Likewise, all three forces increase when the magnitude of A_{FH0} increases. □

Proposition 1.4. *When $\gamma = 1$, optimal monetary policy R_{H1} satisfies the following equation:*

$$0 = \underbrace{-\mu_0 \frac{dL_0}{dR_{H1}}}_{\text{wedge}} + \tilde{\lambda}_r [\underbrace{R_{H1} t_{FH0} \frac{P_{FH0}}{P_{H0}} \frac{dC_{FH0}}{dR_{H1}}}_{\text{tariff fiscal externality}} + \underbrace{(NX_0)}_{\text{intertemporal TOT}}], \tag{1.48}$$

where $\tilde{\lambda}_r$ is the Lagrange multiplier on the Home lifetime real budget constraint normalized by P_{H0}.

As a special case, when $t_{FH0} = 0$, the optimal monetary policy R_{H1} is such that $\mu_0 > 0$: it is optimal to loosen monetary policy beyond clearing the output gap.

Proof. Since the central bank is choosing the real rate R_{H1}, we rewrite the budget constraint to incorporate R_{H1}:

$$R_{H1} \frac{1}{P_{H0}} (P_{HH0} C_{HH0} + (P_{FH0} + t_{FH0}) C_{FH0}) + \frac{1}{P_{H1}} (P_{HH1} C_{HH1} + P_{FH1} C_{FH1})$$
$$= R_{H1} \frac{1}{P_{H0}} (w_{H0} L_{H0} + T_{H0}) + \frac{w_{H1}}{P_{H1}} L_{H1}$$

Then the Lagrange multiplier on this *real* budget constraint is $\tilde{\lambda}_r = \frac{u'(C_{H0})}{R_{H1}} = \beta u'(C_{H1})$

Recall that the central bank's monetary policy rule sets interest rate according to Equation 1.5:

$$\log(1 + i_{H1}) = -\log(\beta) + \log(\frac{P_{H1}}{P_{H0}}) + \epsilon_{H0} \Leftrightarrow R_{H1} = \frac{1}{\beta} \exp(\epsilon_{H0})$$

We consider variations in $\exp(\epsilon_{H0})$ that leave inflation constant; notably, P_{H1} does not move in this variation.

Transform the marginal change in utility in a way analogous to Lemma 1.4 to write

$$
\begin{aligned}
d\mathcal{U}_H =&\tilde{\lambda}_r\left[\sum_{i\in\{H,F\}}\left(R_{H1}\frac{P_{iH0}}{P_{H0}}dC_{iH0}+\frac{P_{iH1}}{P_{H1}}dC_{iH1}\right)-R_{H1}\frac{w_{H0}}{P_{H0}}dL_{H0}-\frac{w_{H1}}{P_{H1}}dL_{H1}\right]\\
&+\tilde{\lambda}_r R_{H1}\frac{t_{FH0}}{P_{H0}}dC_{FH0}-\mu_0 dL_0
\end{aligned}
$$

Taking the derivative of the budget constraint, we get:

$$
\begin{aligned}
&\sum_{i\in\{H,F\}}\left(R_{H1}\frac{P_{iH0}}{P_{H0}}dC_{iH0}+\frac{P_{iH1}}{P_{H1}}dC_{iH1}\right)-R_{H1}\frac{w_{H0}}{P_{H0}}dL_{H0}-\frac{w_{H1}}{P_{H1}}dL_{H1}\\
=&\frac{1}{P_{H1}}(C_{HF1}dP_{HF1}-C_{FH1}dP_{FH1})+dR_{H1}(\frac{1}{P_{H0}}NX_{H0})
\end{aligned}
$$

where $NX_{H0}=(w_{H0}L_{H0}+T_{H0})-P_{HH0}C_{HH0}-(P_{FH0}+t_{FH0})C_{FH0}=\frac{B_{H1}}{R_{H1}}$ is the net export in period 0. Plugging this in and replacing $t_{FH0}\to t_{FH0}P_{FH0}$, we get

$$
\begin{aligned}
d\mathcal{U}_H=&-\mu_0 dL_0+\tilde{\lambda}_r[R_{H1}\frac{t_{FH0}P_{FH0}}{P_{H0}}dC_{FH0}\\
&+\frac{1}{P_{H1}}(C_{HF1}dP_{HF1}-C_{FH1}dP_{FH1})+dR_{H1}(\frac{1}{P_{H0}}NX_{H0})]
\end{aligned}
$$

Now we note that when $\gamma=1$, the equilibrium level of real balances $\frac{B_{H1}}{P_{H1}}$ do not depend on R_{H1}. This is because after any change in $R_{H1}\to\zeta R_{H1}$ for some constant ζ, the equilibrium conditions exactly hold if we replace C_{ij1}, C_{i1}, L_{i1} with $\zeta C_{ij1}, \zeta C_{i1}, \zeta L_{i1}$; monetary policy affects period 0 without affecting any real variables in period 1. (We can verify by inspecting the equilibrium conditions)

Thus, the period 1 variables do not depend on R_{H1}, and under the optimal monetary policy, the above equation becomes

$$
0-\mu_0 dL_0+\tilde{\lambda}_r[R_{H1}\frac{t_{FH0}P_{FH0}}{P_{H0}}dC_{FH0}+dR_{H1}(\frac{1}{P_{H0}}NX_{H0})] \tag{1.49}
$$

which is exactly the equation in the proposition. □

1.8.3 Equations in the quantitative model

In this subsection, I derive the equations for the quantitative model. The equations characterizing the equilibrium (Definition 1.2) in the case when China pegs is given by the following conditions:

(a) Family optimization:

$$P_{jt} = \prod_s (P^s_{jt})^{\alpha^s_j} \tag{1.50}$$

$$P^s_{jt} = [\sum_i ((1+t^s_{ijt})P^s_{ijt})^{1-\sigma_s}]^{\frac{1}{1-\sigma_s}} \tag{1.51}$$

$$\lambda^s_{ijt} = \frac{((1+t^s_{ijt})P^s_{ijt})^{1-\sigma_s}}{\sum_k ((1+t^s_{kjt})P^s_{kjt})^{1-\sigma_s}} \tag{1.52}$$

$$\tilde{\lambda}_{it} = \frac{u'(C_{it})}{P_{it}} \tag{1.53}$$

$$u'(C_{jt}) = \beta\hat{\delta}_{jt}(1+i_{jt})\frac{P_{jt}}{P_{jt+1}}u'(C_{jt+1}) \tag{1.54}$$

$$1+i_{it} = (1+i_{jt})\frac{e_{ijt+1}}{e_{ijt}} \tag{1.55}$$

$$P_{jt}C_{jt}\bar{L}_j + \frac{1}{1+i_{jt}}B_{jt+1} \leq B_{jt} + \sum_s W^s_{jt}\ell^s_{jt}L^s_{jt} + \Pi_{jt} + T_{jt} \tag{1.56}$$

(b) Firm optimization: if R^s_{jt} is total revenue of sector s in country j at time t, we have

$$P^s_{ijt} = e_{ijt}\tau^s_{ijt}\frac{1}{A^s_{it}}(W^s_{it})^{\phi^s_i}\prod_n (P^n_{it})^{\phi^{ns}_i} \tag{1.57}$$

$$W^s_{it}\ell^s_{it}L^s_{it} = \phi^s_i R^s_{it} \tag{1.58}$$

(c) Labor supply: given by New Keynesian Phillips curve

$$\log(\pi^{sw}_{it}+1) = \kappa_w(v'(\ell^s_{it}) - \frac{W^s_{it}}{P_{it}}u'(C_{it})) + \beta\log(\pi^{sw}_{it+1}+1) \tag{1.59}$$

(d) Labor reallocation and worker's value function:

$$\mu^{sn}_{it} = \frac{\exp(\frac{1}{\nu}(\beta V^n_{it+1} - \chi^{sn}_{it}))}{\sum_{n'}\exp(\frac{1}{\nu}(\beta V^{n'}_{it+1} - \chi^{sn'}_{it}))} \tag{1.60}$$

$$V^s_{it} = \tilde{\lambda}_{it}W^s_{it}\ell^s_{it} + \eta^s_{it} - v(\ell^s_{it}) + \nu\log\left(\sum_n \exp(\frac{1}{\nu}(\beta V^n_{it+1} - \chi^{sn}_{it}))\right) \tag{1.61}$$

$$L^n_{it+1} = \sum_s \mu^{sn}_{it}L^s_{it} \tag{1.62}$$

(e) Monetary policy and exchange rates:

$$\log(1+i_{1t}) = r_{1t} + \phi_\pi \log(1+\pi_{1t}) + \epsilon_{1t} \tag{1.63}$$

$$e_{2t} = \bar{e} \tag{1.64}$$

$$\log(1+i_{jt}) = r_{it} + \phi_\pi \log(1+\pi_{jt}) + \epsilon_{jt} \quad (j \geq 3) \tag{1.65}$$

$$\lim_{T\to\infty} B_{jT} = 0 \quad (j \geq 3) \tag{1.66}$$

(f) Market clearing conditions:

$$R^s_{it} = \sum_j e_{jit}\lambda^s_{ijt}\left[\alpha^s_j P_{jt}C_{jt} + \sum_n \phi^{sn}_j R^n_{jt}\right] \tag{1.67}$$

$$0 = \sum_i B_{it}e_{i1t} \tag{1.68}$$

The equilibrium is: given calibrated parameters and initial conditions $w^s_{j,-1}$, B_{j0}, L^s_{j0}, a sequence of variables $\{X_t\}_{t=0}^\infty$ where

$$X_t = (B_{jt}, C_{jt}, P_{jt}, e_{jt}, W^s_{jt}, P^s_{jt}, L^s_{jt}, \ell^s_{jt}, V^s_{jt})$$

that satisfy Equations (1.50) to (1.68). In the case where China floats its exchange rate, we replace $e_{2t} = \bar{e}$ with an analogous Taylor rule for China along with $\lim_{T\to\infty} B_{2T} = 0$.

In the next subsections, we derive each of the equations, especially the ones that are new in the quantitative setup.

New Keynesian Phillips curve

Suppress the country and sector index (i, s). In each labor market, the maximization problem of the labor packer ι at time t facing a labor demand curve with elasticity ϵ_w is

$$\max_{w_t(l)} \sum_{t\geq t'} \beta^{t'-t}[\tilde{\lambda}_{t'}w_{t'}(\iota)l_{t'}(\iota) - \int v(l_{t'}(\iota))d\iota - \Phi(\frac{w_{t'}(\iota)}{w_{t'-1}(\iota)})L_{t'}]$$

where $l_{t'}(\iota) = (\frac{w_{t'}(\iota)}{w_{t'}})^{-\epsilon_w}L_t$. The FOC wrt $w_t(\iota)$ is:

$$\begin{aligned} 0 = {} & \tilde{\lambda}_t(1-\epsilon_w)(\frac{w_t(\iota)}{w_t})^{-\epsilon}L_t + v'(l_t(\iota))\epsilon_w(\frac{w_t(\iota)}{w_t})^{-\epsilon_w-1}\frac{L_t}{w_t} \\ & - \Phi'(\frac{w_t(\iota)}{w_{t-1}(\iota)})\frac{1}{w_{t-1}(\iota)}L_t + \beta\Phi'(\frac{w_{t+1}(\iota)}{w_t(\iota)})\frac{w_{t+1}(\iota)}{w_t(\iota)^2}L_{t+1} \end{aligned}$$

Impose symmetry $w_t(\iota) = w_t$ and $l_t(\iota) = \ell_t$, if we let wage inflation $1 + \pi_t^w = \frac{w_t}{w_{t-1}} - 1$, the above equation becomes

$$0 = \tilde{\lambda}_t(1 - \epsilon_w)L_t w_t + v'(\ell_t)\epsilon_w L_t - \Phi'(1 + \pi_t^w)(1 + \pi_t^w)L_t + \beta\Phi'(1 + \pi_{t+1}^w)(1 + \pi_{t+1}^w)L_{t+1}$$

If we let $\Phi(x) = \epsilon_w \frac{1}{2\kappa_w}(\log x)^2$, then $\Phi'(\pi) = \frac{\epsilon_w}{\kappa_w}\frac{1}{x}\log x$. Moreover, $\tilde{\lambda}_t = \frac{u'(C_t)}{P_t}$, and letting $\mu_w = \frac{\epsilon_w}{\epsilon_w - 1}$ be markup, we have

$$\log(1 + \pi_t^w) = \kappa_w \underbrace{\left(v'(\ell_t) - w_t \frac{u'(C_t)}{P_t}\mu_w\right)}_{\text{output gap}} + \beta \log(1 + \pi_{t+1}^w)\frac{L_{t+1}}{L_t}$$

Note that when cost of adjustment is zero, $\kappa_w \to \infty$ so output gap becomes zero. Since we are not interested in the markup that unions charge, we assume that every period we tax w_t so that wage markup is undone and any tax revenue is rebated to the household lump-sum, we have the desired New Keynesian Phillips Curve:

$$\log(1 + \pi_t^w) = \kappa_w(v'(L_t) - w_t\frac{u'(C_t)}{P_t}) + \beta \log(1 + \pi_{t+1}^w)\frac{L_{t+1}}{L_t}$$

Exchange rate determination

In Section 1.4, for each floating country i, we defined the exchange rate in period e_{i0} to be the unique value such that

$$\lim_{t\to\infty} B_{it} = 0. \tag{1.36}$$

Here we microfound this condition through the *segmented financial market* model, a reduced-form version of Itskhoki and Mukhin (2021a). We assume that the household family in country i cannot directly trade any assets with one another, and the international asset positions are intermediated by the financial sector. As in the main text, households in each country i demand a quantity B_{it+1} of home-currency bonds in time t, giving identical optimization conditions, minus the UIP condition (since we do not have free bond markets).

The financial sector features two additional types of agents that trade bonds internationally: arbitraguers and noise traders. We assume countries $i \geq 2$ have each type of them, and they trade domestic bonds and US dollars only.[52] Each period, arbitraguers of mass m_i in country i choose a zero-capital portfolio (d_{it+1}, d_{it+1}^U) such that $\frac{d_{it+1}}{R_{it}} + \frac{1}{e_{it}}\frac{d_{it+1}^U}{R_{1t}} = 0$, where $R_{it} = 1 + i_{it}$ is the gross return, or the inverse price of bonds

[52] This can be relaxed, and is mainly for clarity of exposition.

of country i at time t, and $e_{it} = e_{i1t}$ is the value of currency i with respect to the US dollar. Their profits are rebated lump-sum to the household in i, and seek to maximize the CARA utility of the real return in units of country i goods:

$$\max_{d_{it}} \mathbb{E}_t \left[-\frac{1}{\omega} \exp\left(-\omega \frac{(R_{it} - R_{1t}\frac{e_{it+1}}{e_{it}})d_{it+1}}{P_{it+1}} R_{it} \right) \right] \tag{1.69}$$

where ω is the risk aversion parameter.

In addition, the financial market features a liquidity demand from a measure n_i of symmetric noise traders in each country $i \geq 2$. The total positions in US dollar bonds invested by noise trader in country i is modeled as an exogenous process

$$\frac{N^U_{it+1}}{1+i_{it}} = n(e^{\psi_t} - 1) \quad \text{with} \quad \psi_t = \rho_\psi \psi_{t-1} + \sigma_\psi \epsilon^{\psi_t}_t. \tag{1.70}$$

and they invest in country i bonds equivalent to this.

Denoting the total position of arbitraguers as $D_{it+1} = m_i d_{it+1}$, we have the portfolio balance condition for each i:

$$B_{it+1} + N_{it+1} + D_{it+1} = 0 \quad \text{and} \quad B_{1t+1} + \sum_{i\geq 2}(N^U_{it+1} + D^U_{it+1}) = 0 \tag{1.71}$$

The fact that intermediaries are risk-averse ($\omega > 0$) require them to take some compensation, and yields the *modified* UIP condition for each country with respect to the US dollar:

Lemma 1.5. *(Lemma 1 of Itskhoki and Mukhin (2021a).) The equilibrium condition in the finnacial market, log-linearized around a symmetric steady-state with $\bar{B}_i = 0, \bar{R} = \frac{1}{\beta}$, is given by*

$$i_{it} - i_{1t} = \mathbb{E}_t \Delta e_{t+1} + \chi_1 \psi_t - \chi_2 b_{t+1} \tag{1.72}$$

where $\chi_1 = \frac{n}{\beta}\frac{\omega\sigma_e^2}{m}$ and $\chi_2 = \bar{Y}\frac{\omega\sigma_e^2}{m}$.

Consider the limit of this economy, first where $n \to 0$, sending the magnitude of the noise trader to zero, while fixing $\frac{\omega}{\sigma_e^2}m$ (with an appropriate adjusting financial shock volatility). The UIP deviation then becomes

$$i_{it} - i_{1t} = \mathbb{E}_t \Delta e_{t+1} - \chi_2 b_{t+1}. \tag{1.73}$$

Note that this condition can alternatively be microfounded through convex portfolio adjustment costs (Kouri, 1976) or debt-elastic interest rate premiums (Schmitt-Grohé and Uribe, 2003); the business-cycle level equivalence of these models are explored in (Schmitt-Grohé and Uribe, 2003).

We highlight that under Equation 1.73, the model is stationary, and when e_{it} is pursuing an independent monetary policy, we must have

$$\lim_{t\to\infty} b_{t+1} = 0, \tag{1.74}$$

in any steady-state. If we take the limit $\chi_2 \to 0$, the condition converges to

$$i_{it} - i_{1t} = \mathbb{E}_t \Delta e_{t+1} \tag{1.75}$$

which is the UIP condition, and a terminal condition given by Equation 1.74.

Discussion on relevance. Why do we need an extra 'terminal' condition under UIP? This is closely related to the indeterminacy result by Kareken and Wallace (1981). Under frictionless bond markets with pure interest rate targets, the exchange rate at $t = 0$ after a shock is indeterminate. While this fact is a pure nominal result without real consequences in Kareken and Wallace (1981), in our model, each *level* of the nominal exchange has real implications on output and labor supply, as it connects with the *nominal wage anchor* from $t = -1$: different exchange rates correspond to different levels of output and demand in each country. The fact that the indeterminacy result could have real implications in setups of nominal rigidity and independent interest rates is also explored in Caballero et al. (2021), and the nonstationarity of a pure UIP model is also discussed in (Schmitt-Grohé and Uribe, 2003).

Labor and unemployment as extensive margin

In our current formulation, all supply of labor is at the intensive margin. We provide a microfoundation of the labor supply problem in terms of the extensive margin, following Gali (2008). We assume that each member m draws idiosyncratic productivity shocks $\{\epsilon^n_{it}(m)\}$ distributed Type 1 EV, and moving fromm sector s to n involves moving costs of χ^{sn}_{it}:

$$v(\{\epsilon^n_{it}(m)\}_n, s_{it}(m), s_{it-1}(m)) = \sum_{n,k} [\epsilon^n_{it}(m) - \chi^{sn}_{it}]\, \mathbb{I}(s_{it}(m) = n, s_{it-1}(m) = s),$$

Then, given sectoral choice $n = s_{it}(m)$, we pin down optimal work decisions at that sector (under full employment). Each member m has a disutility from wage inflation and work according to

$$\Phi\left(\iota_{it}(m), \{\pi_{it}^{w,s}\}\right) = -\iota_{it}(m) - \Phi_{it}^{s}(\pi_{it}^{w,s})$$

where $\iota_{it}(m)$ is the disutility from working. Once a member m is in sector n, we assume that the households draw idiosyncratic disutility from work after choosing a sector n:

$$\iota_{it}(m) = \tilde{\iota}^{\nu}, \quad \tilde{\iota} \sim_{iid} U[0,1].$$

Households decide to work if

$$\bar{v}\tilde{\iota}^{\nu} \leq \tilde{\lambda}_{it} w_{it}^{n},$$

where $\tilde{\lambda}_{it}$ is the Lagrangian multiplier on the budget constraint, and w_{it}^{n} is the wage. Then, conditional on choosing sector n, fraction $\ell \in [0,1]$ member will want to work where

$$\ell_{it}^{n} \in \arg\max_{\ell \in [0,1]} w_{it}^{n} \lambda_{it} - v(\ell)$$

with

$$v(\ell) = \bar{v} \int^{\ell} \tilde{\iota}^{\nu} d\tilde{\iota} = \bar{v} \frac{\ell^{1+\nu}}{1+\nu}.$$

1.9 Policy in the theoretical model

1.9.1 Optimal tariffs and subsidies

Given monetary policy R_{H1}, we solve for the optimal tariff t_{FH0} and the optimal export subsidy s_{HF0}. We note that under the optimum policy, any first-order variation on policy should have zero effect on welfare. To utilize this, we write the first-order effect of policy as a sum of first-order policy goals of the Home government.

Lemma 1.6. *For any variation in tariffs and subsidies (dt_{FH0}, ds_{HF0}), the first-order effect on Home welfare $\mathcal{U}_H$ is given by*

$$d\mathcal{U}_H = -\underbrace{\mu_0 dL_0}_{wedge} + \tilde{\lambda}[\underbrace{t_{FH0}P_{FH0}dC_{FH0}}_{t\ fiscal\ externality} - \underbrace{s_{HF0}P_{HF0}dC_{HF0}}_{s\ fiscal\ externality} - \underbrace{P_{HF0}C_{HF0}ds_{HF0}}_{terms\text{-}of\text{-}trade\ today}]$$
$$+ \frac{\tilde{\lambda}}{1+i_{H1}}(\underbrace{C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1}}_{future\ terms\text{-}of\text{-}trade})$$

where $\tilde{\lambda}$ is the Lagrange multiplier on the Home lifetime budget constraint.

Proof. The formal proof is based on a variations argument following Costinot and Werning (2022). See Appendix 1.8 of the main text. □

Lemma 1.6 captures the trade-off between different objectives of the Home planner. The first two terms capture the standard second-best use of tariffs to counter existing distortions at the cost of distorting consumption. The first term is the labor wedge – if there is unemployment ($\mu_0 < 0$), both an increase in t_{FH0} and s_{HF0} lead to welfare gains. The second term represents the fiscal externality, or 'Harberger triangle' of tariffs and subsidies. The third term is the terms-of-trade effect of subsidies. Since wages are rigid and firms are competitive, a subsidy on exports translates one-to-one to lower the price of exports that Foreign faces, worsening terms-of-trade.

The last term concerns the future terms-of-trade, which are worsened by trade deficits today. Tariffs and subsidies today will increase demand for Home goods, mitigating trade deficits and improving future terms-of-trade. This suggests that short-run tariffs can improve future terms-of-trade by affecting trade imbalances, corroborating observed policy interventions to improve trade balances (Delpeuch et al., 2021). Importantly, this mechanism is effective solely for unilateral tariffs, as retaliatory measures will undo the balance effects.

The optimal tariff and subsidy level balances between these objectives. To characterize the optimum level attained at $d\mathcal{U}_H = 0$, we apply a change of variables to treat each equilibrium variable X as a function $X(C_{HF0}, C_{FH0})$ of export quantity C_{HF0} and import quantity C_{FH0}. This is equivalent to defining $X(C_{HF0}, C_{HF0}) = X(s^*_{HF0}, t^*_{FH0})$ where s^*, t^* are tax and subsidy levels such that exports are C_{HF0} and imports are C_{FH0}. Under this notation, $\frac{\partial X}{\partial C_{FH0}}$ meausres the change of any equilibrium variable associated with a change in taxes and subsidies that induces a marginal change in imports C_{FH0}, and likewise for C_{HF0}.[53]

We explore two variations in (dC_{FH0}, dC_{HF0}), each leading to an optimal tariff formula and an optimal subsidy formula. The first variation we consider is a variation that keeps subsidies s_{HF0} constant, while the second is a variation that keep imports C_{FH0} constant.

Proposition 1.6. $(ds_{HF0} = 0)$ *The optimal short-run tariff on imports t_{FH0} satisfies*

$$t_{FH0} = \frac{1}{P_{FH0}} \left[\underbrace{\frac{\mu_0}{\tilde{\lambda}} \frac{\partial L_{H0}}{\partial C_{FH0}}}_{\text{labor wedge}} - \frac{1}{(1+i_{H1})} \underbrace{\left(L_{HF1} \frac{\partial w_{H1}}{\partial C_{FH0}} - L_{FH1} \frac{\partial w_{F1}}{\partial C_{FH0}}\right)}_{\text{future terms-of-trade}} + \underbrace{s_{HF0} P_{HF0} \frac{\partial C_{HF0}}{\partial C_{FH0}}}_{\text{subsidy externality}} \right] \tag{1.76}$$

$(dC_{FH0} = 0)$ *The optimal short-run subsidy on exports s_{HF0} satisfies*

$$s_{HF0} = \frac{1}{P_{HF0}} \left[- \underbrace{\frac{\mu_0}{\tilde{\lambda}} \frac{\partial L_{H0}}{\partial C_{HF0}}}_{\text{labor wedge}} + \frac{1}{(1+i_{H1})} \underbrace{\left(L_{HF1} \frac{\partial w_{H1}}{\partial C_{HF0}} - L_{FH1} \frac{\partial w_{F1}}{\partial C_{HF0}}\right)}_{\text{future terms-of-trade}} - \underbrace{P_{HF0} C_{HF0} \frac{\partial s_{HF0}}{\partial C_{HF0}}}_{\text{terms-of-trade today}} \right] \tag{1.77}$$

Proof. See Appendix 1.8 of the main text. □

While both formulas follow mechanically from Lemma 1.6, they highlight the different welfare channels of tariffs and subsidies. First, tariffs and subsidies both reduce the labor wedge by stimulating demand for domestic labor. Second, tariffs and subsides improve current trade balance, which improves the terms-of-trade in the future. In an economy with unemployment and trade deficits caused by trade shocks and an exchange rate peg, these are forces for *positive* tariffs and subsidies. The last term on the optimal tariff concern the general equilibrium interaction of subsidy and tariff externality, while the last term on the subsidies concern the terms-of-trade effect of subsidies: the subsidy is a transfer from Home to Foreign that directly affects the relative border price of goods.

In a competitive equilibrium, home households do not internalize any of these effects. Thus, the tax level $t_{FH0}P_{FH0}$ and the subsidy level $s_{HF0}P_{HF0}$ can be considered

[53] This notation follows Adão et al. (2023).

a Pigouvian tax that corrects for the three externalities of consuming an extra unit of import, or exporting an extra unit.

In the tariff formula (Equation 1.23), the three forces all suggest a positive tariff when there is unemployment ($\mu_0 < 0$). Tariffs alleviate labor market distortions, improve future terms-of-trade, and mitigate the fiscal externality of subsidies by reducing exports in general equilibrium. Moreover, in response to a larger trade shock (higher A_{FH}), unemployment and deficit effects are larger, justifying a larger tariff. Thus we have the following corollary:

Corollary 1. *If there is unemployment at the zero-tariff economy ($\mu_{H0} < 0$ when $t_{FH0} = 0$), the optimal tariff t_{FH0} is positive and is increasing in the size of the trade shock A_{FH0}.*

The intuition that we can and should use tariffs as second-best instruments to fix distortions is already well-known. We show that in an environment where trade shocks cause unemployment and trade deficits, we can sign the short-run tariff: it should be positive, and increasing in the magnitude of the trade shock.

The sign of the tariff depends on monetary policy not already clearing existing unemployment. The central bank may be unable to clear the output gap caused by sector-specific trade shocks, because of multisector considerations, financial concerns, and liquidity constraint such as the Zero Lower Bound. Sector-specific tariffs will be a useful tool in this case.[54]

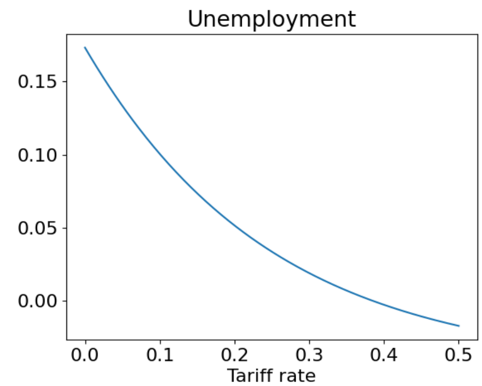
Our model underscores that under an exchange rate peg, the optimal tariff is increasing in the magnitude of the trade shock. In a floating exchange rate environment, the optimal tariff is pinned down primarily by the trade elasticity (Gros, 1987). In contrast, our framework focuses on tariffs that correct a distortion caused by the peg and the trade shock, so the magnitude of the optimal tariff scales with the size of the distortion.

Numerical simulation. To clarify the intuition, we conduct a simplified numerical exercise to investigate the effect of policy responses to the trade shocks. The exercise assumes that Home and Foreign are symmetric countries at $t = -1$ and introduce an unanticipated trade shock to Foreign productivity A_F.

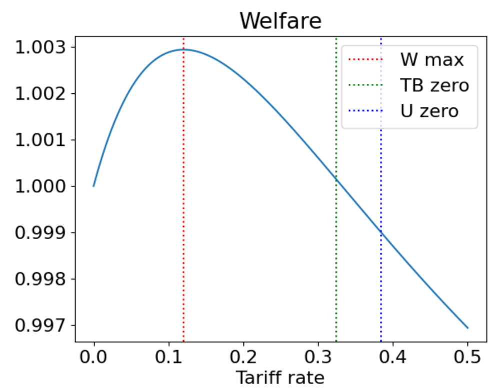
Figure 1.25 plots the trade balance and unemployment effects of varying tariff levels, as well as welfare – in compensating variations vis-à-vis the laissez-faire baseline –

[54] Indeed, in the early 2000s, the government was tightening monetary policy in response to concerns over inflation and tightening of unused resources; loosening in response to the China shock was not the Federal Reserve Bank's goal (Federal Reserve Board, 2005). Following the Great Recession, the Federal Reserve Bank was subject to the Zero Lower Bound.

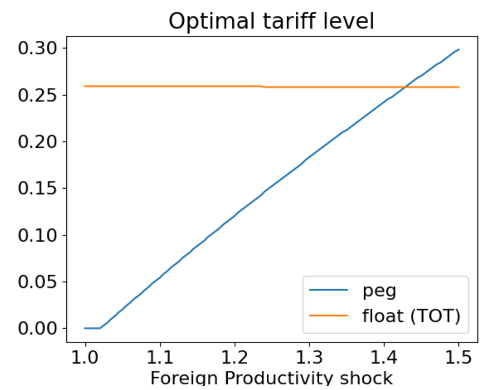
(a) Trade Balance / GDP



(b) Unemployment



(c) Welfare



(d) Optimal tariff vs shock

Figure 1.25: Effect of tariff on aggregates and welfare

Note. The top two figures plot the trade balance and short-run unemployment under various levels of tariffs, in response to a 20% shock in Foreign productivity, assuming monetary policy does not respond ($R_{H1} = \frac{1}{\beta}$). The bottom left figure plots the welfare response of tariffs; the dotted line represents the tariff level that maximizes welfare, that clears trade balances, and that clears unemployment respectively. The bottom right figure plots the optimal short-run tariff for varying levels of productivity shocks, under our *peg* model, and a flexible-price model a la Gros (1987).

under the trade shock. The top two panels show that tariffs attenuate trade deficits and unemployment, but with diminishing returns as tariffs increase. From Figure 1.25c, the welfare-optimal tariff is below the level erasing trade deficits or achieving full employment, due to the convexity of tariff-induced fiscal externalities (Harberger triangle). Figure 1.25d shows that under an exchange rate peg, the optimal tariff level increases with the magnitude of the trade shock, while under an economy with floating rates, tariffs target terms-of-trade manipulation, targeting the trade elasticity $\frac{1}{\sigma-1}$ as in the literature.

1.9.2 Optimal monetary policy

What is the optimal monetary policy given a tariff schedule? Analogously to the tariff section, we may use first-order variations to characterize the optimal monetary policy:

Lemma 1.7. *The optimal monetary policy satisfies the following equation:*

$$0 = \underbrace{-\mu_0 \frac{dL_0}{dR_{H1}}}_{wedge} + \tilde{\lambda}_r [\underbrace{R_{H1} t_{FH0} \frac{P_{FH0}}{P_{H0}} \frac{dC_{FH0}}{dR_{H1}}}_{t\ fiscal\ externality} + \underbrace{\frac{1}{P_{H1}} (C_{HF1} \frac{dP_{HF1}}{dR_{H1}} - C_{FH1} \frac{dP_{FH1}}{dR_{H1}})}_{future\ terms\text{-}of\text{-}trade} + \underbrace{(NX_0)}_{intertemporal\ TOT}]$$

where $NX_0 = \frac{w_{H0}}{P_{H0}} L_{H0} - C_{H0} = \frac{B_{H1}}{P_{H0} R_{H1}}$ *is Home's real net exports at period 0, and* $\tilde{\lambda}_r = \frac{u'(C_{H0})}{R_{H1}} = \beta u'(C_{H1})$ *is the Lagrange multiplier on the real budget constraint.*

The first three terms have already been discussed in Lemma 1.6. The last term captures how monetary policy controls the trade-off between current and future goods. Notably, due to the peg, there is an absence of distortionary costs of monetary policy. Home effectively determines the global savings rate rather than distorting its rate to a global benchmark.

When the government loosens monetary policy R_{H1}, each term affects welfare as follows. First, welfare improves by alleviating the labor wedge. Second, when $t_{FH0} > 0$, the tariff fiscal externality worsens as households consume more. Third, future terms-of-trade may be affected due to changes in trade imbalances. Finally, the intertemporal terms-of-trade effect enhances welfare because $NX_0 < 0$ (Proposition 1 of the main text), as looser policy allows Home to get more imports today for future exports. The latter two terms clarify that the optimal monetary policy may deviate from targeting the output gap in an open economy where Foreign pegs to Home.

Whether or not the central bank wants to loosen or tighten monetary policy in response to a trade shock depends on the relative strengths of these forces. However, when the intertemporal elasticity of substitution γ is equal to 1, the following lemma shows that *future terms-of-trade* is independent of monetary policy:

Lemma 1.8. *When* $\gamma = 1$*, monetary policy* R_{H1} *does not affect the trade balances* B_{H1}*. When* $\gamma < 1$*, a looser monetary policy (lower* R_{H1}*) increases the balances* R_{H1}*; the opposite holds when* $\gamma > 1$*.*

Proof. See Appendix 1.8 of the main text. □

Lemma 1.8 may seem counterintuitive at first – if Home loosens it's monetary policy, Home agents should want to borrow more. However, Home's monetary policy not only affects Home's borrowing decisions, but also Foreign's decision because the peg moves the interest rate equally in both countries. When $\gamma = 1$, Home's desire to borrow moves exactly as much as Foreign's, and while short-run aggregate demand may be affected, the resulting future transfers are unaffected. This allows us to simplify Lemma 1.7 to the following proposition:

Proposition 1.7. *When $\gamma = 1$, optimal monetary policy satisfies the following equation.*

$$0 = \underbrace{-\mu_0 \frac{dL_0}{dR_{H1}}}_{wedge} + \tilde{\lambda}_r [\underbrace{R_{H1} t_{FH0} \frac{P_{FH0}}{P_{H0}} \frac{dC_{FH0}}{dR_{H1}}}_{tariff\ fiscal\ externality} + \underbrace{(NX_0)}_{intertemporal\ TOT}] \tag{1.78}$$

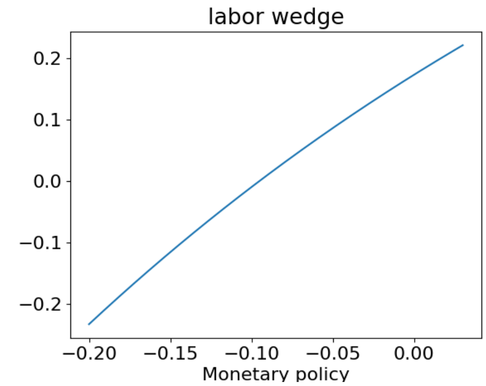
Notably, when $t_{FH0} = 0$, optimal monetary policy R_{H1} is such that $\mu_0 > 0$: it is optimal to loosen the monetary policy beyond the output gap.

Proof. See Appendix 1.8 of the main text. □

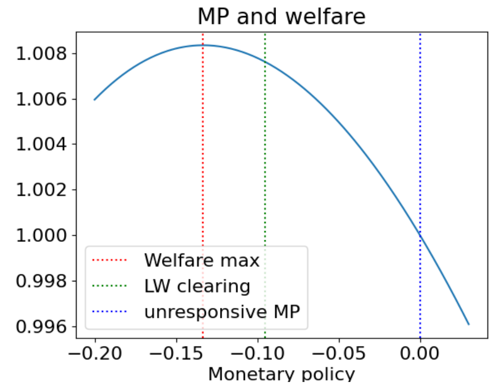
Proposition 1.7 highlights that in an open economy where Home is subject to a Foreign peg, optimal monetary policy may want to *overshoot* the output gap when Home is borrowing from Foreign. This is because Home has power in setting *global* monetary policy. This is especially relevant to the US, which effectively sets the interest rate for many countries being the dominant currency (Gopinath et al., 2020), and runs current account deficits; the central bank may want to set a looser interest rate, with minimal risk of bond liquidation from dollar-pegging countries. The proposition also clarifies again that tariffs are primarily second-best instruments to be used when monetary policy is unable to respond – whether due to the ZLB or multisectoral considerations. In fact, under a strictly positive tariff, the additional losses from tariff fiscal externality compels Home to adopt a more contractionary monetary stance, reducing overall welfare.

Numerical simulation. Figure 1.26 plots the economy's response to Home monetary policy. The top two panels consider optimal monetary policy absent tariffs ($t_{FH0} = 0$). The left panel (Figure 1.26a) depicts short-run unemployment response to monetary policy; as expected, a looser monetary policy reduces unemployment. The top right panel (Figure 1.26b) highlight that monetary policy may *overshoot* beyond the output-gap clearing level (Proposition 1.7).

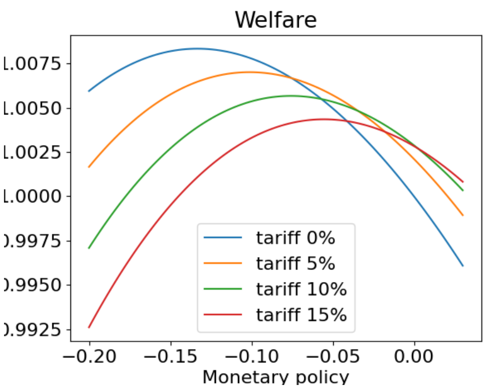
The bottom two panels consider the joint optimal tariffs and monetary policy. Figure 1.26c plots the welfare response of monetary policy under varying levels of tariffs. Two things stand out: (1) the optimal monetary policy is *tighter* with higher tariffs, because the unemployment is lower, and tariff distortions are worsened with looser monetary policy;



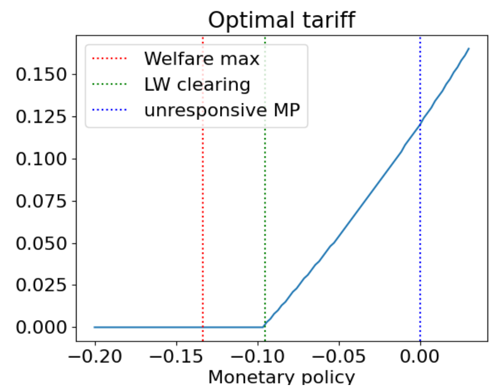
(a) Unemployment



(b) Welfare



(c) Welfare under varying tariffs



(d) Optimal tariff under MP

Figure 1.26: Effect of monetary policy.

Note. The top two figures plot the unemployment and welfare responses to varying levels of Home monetary policy shocks away from the unresponsive case $R = \frac{1}{\beta}$, in response to a 20% shock in Foreign productivity, assuming no tariffs $t_{FH0} = 0$. The bottom left figure plots the welfare under varying levels of tariffs. The bottom right figure plots the optimal short-run tariff for varying levels of monetary policy.

(2) the joint optimal policy seems to involve no tariffs and a very loose monetary policy. Figure 1.26d shows the optimal tariff level for each monetary policy level; it clarifies that the tariffs are positive only when monetary policy does not fully close the output gap, as in Corollary 1.

1.10 Alternative Theoretical Models

1.10.1 Two-sector model: tradables and nontradables

In this section, we develop a tractable model that rationalizes the manufacturing decline and trade deficits as an outcome of Foreign productivity growth and an exchange rate peg, explaining concurrently the four facts (Figure 1.1) in Section 1.1 and corroborating the findings in Section 1.2. Our model is a two-sector, two-period, two-country model that highlights the role of exchange rate pegs and nominal wage rigidity. Using this model, we study the positive and normative implications of a trade shock, including welfare and policy implications. We keep the ingredients minimal for analytical tractability, and extend the model into a multi-sector infinite horizon in the quantitative analysis in Section 1.4.

Model setup

Our environment has two countries, Home (H) and Foreign (F). On application to the China shock, Home will be the United States and Foreign will be China. There are two periods: $t = 0$ (short-run) and $t = 1$ (long-run). Each country is populated by a representative household that consumes goods from both countries, and supplies labor to firms that produce goods. Each country has its own nominal account (the US dollar for Home, and the renminbi for Foreign); the price of country j's currency in units of country i's currency at time t is e_{jit}, with $e_{HHt} = e_{FFt} = 1$ and $e_{FHt} = \frac{1}{e_{HFt}}$. We denote $e_t = e_{FHt}$. Hence an increase in e_t is a depreciation of Home currency. There are two sectors: tradables T and nontradables N. By definition, only tradables are tradable.

Household preferences. In each country j, there is a representative agent who consumes final good $\mathcal{C}_{jt}$, supplies labor L_{jt} for tradable goods and N_{jt} for nontradable goods. The household has preferences represented by

$$\mathcal{U}_j = [u(\mathcal{C}_{j0}) - v(L_{j0}) - v(N_{j0})] + \beta[u(\mathcal{C}_{j1}) - v(L_{j1}) - v(N_{j1})], \tag{1.79}$$

$$\text{where } u(C) = \frac{C^{1-\gamma^{-1}} - 1}{1-\gamma^{-1}},\ \mathcal{C}_{jt} = (C_{jt})^{\alpha}(C_{jt}^{N})^{1-\alpha},\text{ and } C_{jt}^{T} = ((C_{Hjt})^{\frac{\sigma-1}{\sigma}} + (C_{Fjt})^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}.$$

Here α is the share of tradable goods in consumption, σ is the elasticity of substitution between domestic and foreign tradable goods, and γ is the elasticity of intertemporal substitution. We assume that $\sigma > 1$ and $\sigma > \gamma$.[55] $v(\cdot)$ is the disutility of supplying labor in each sector, which we assume is increasing and convex with $v(0) = 0$.[56]

Technology. For tradables, a representative firm in country i uses labor as input and has a constant returns to scale production function that requires $\frac{1}{A_{ij}}$ labor to supply a unit of good to market j. Thus for a firm in country i selling Y_{ij} goods to country j at time t using L_{ijt} labor, we have

$$Y_{ijt} = A_{ij} L_{ijt}.$$

We assume there is home bias in consumption, which is equivalent to $A_{HF} \leq A_{HH}$ and $A_{FH} \leq A_{FF}$.[57]

For nontradables, we assume that a representative firm uses nontradable labor as input and supplies nontradable goods with productivity 1, hence $C^N_{jt} = N_{jt}$. We use N_{jt} to denote both labor supply and consumption of nontradables.

Savings. Each country issues a domestic bond with zero net supply. In period 0, households in each country j have access to a claim of a unit of currency i in period 1, with the price of claim being $\frac{1}{1+i_{i1}}$ in country i currency. We let B_{ij1} denote the amount of claims for i currency that households in country j own. We assume there is no risk and bonds from Home and Foreign are perfect substitutes.

Labor Market and Nominal Rigidity. We consider the simplest form of short-run nominal wage rigidity. We assume that nominal wages in both countries for both sectors are completely fixed in period $t = 0$ to an exogenous level $\{w^T_{j0}, w^N_{j0}\}$, while wages $\{w^T_{j1}, w^N_{j1}\}$ are flexible for $t = 1$. Since wages are rigid in period 0, we assume that the labor market is demand-determined in both countries, and workers supply whatever labor is demanded. In period 1, we assume that wages equalize labor supply and labor demand.[58]

[55] Empirical estimates of σ range from 3-10 (Anderson and van Wincoop, 2003; Imbs and Mejean, 2017) to 1.5-3 (Boehm et al., 2023), but is consistently greater than 1. Estimates of γ vary, but from Hall (1988) to Best et al. (2020), most estimates are less than 1 and sometimes indistinguishable from 0. It is reasonable to assume that substitution across origin of goods is easier than substitution across time.

[56] A natural interpretation of the utility is that there are two agents in each country, supplying tradable and nontradable labor, and perfectly insure themselves against risk so consumption is equalized. We abstract from within-country consumption heterogeneity, as we are interested in aggregate dynamics.

[57] We can equivalently formulate this in terms of productivity A_i and trade cost $\tau_{ij} \geq 1$.

[58] The assumption that wages are completely fixed is to highlight the intuition; any form of adjustment with friction will yield qualitatively identical results.

Monetary policy and exchange rates. The monetary authority at Home sets the nominal interest rate according to a CPI-based Taylor rule with a coefficient 1 on inflation:[59]

$$\log(1+i_{H1}) = -\log(\beta) + \log(\frac{P_{H1}}{P_{H0}}) + \epsilon_{H0}, \tag{1.80}$$

where ϵ_{H0} is the discretionary monetary policy, which sets the real rate $R_{H1} = (1+i_{H1})\frac{P_{H0}}{P_{H1}}$ at

$$R_{H1} = \frac{1}{\beta}\exp(\epsilon_{H0}).$$

We say a monetary policy *does not respond to shocks* if it sets $\epsilon_{H0} = 0$, or equivalently $R_{H1} = \frac{1}{\beta}$. In Sections 1.4 onwards, we consider a more standard Taylor rule, which deliver similar results.

Turning to Foreign monetary policy, we are interested in the equilibrium dynamics when Foreign pegs the nominal exchange rate to Home. We assume that Foreign monetary policy directly chooses the exchange rate $e_0 = e_1 = \bar{e}$ for an exogenous level $\bar{e}$.[60]

Trade taxes and subsidies. Besides monetary policy, the government can also levy taxes on imports, and subsidize exports. We assume that the Home government unilaterally chooses the short-run import tariff t_{FH0} and export subsidy s_{HF0}, and cannot choose long-run policies t_{FH1}, s_{HF1}. If we denote the pre-tariff price of i goods to j at time 0 by P_{ij0}, Home government revenue is

$$T_{H0} = t_{FH0}P_{FH0}C_{FH0} - s_{HF0}e_{FH0}P_{HF0}C_{HF0}. \tag{1.81}$$

We assume the revenue T_{H0} is rebated lump-sum to the representative household.

Competitive Equilibrium

In a competitive equilibrium, households maximize their utility, firms maximize their profit, and markets clear. We derive each condition.

Utility maximization. The household at country j chooses consumption $\{C_{ijt}\}$, $\{L_{it}\}$,$\{B_{ijt}\}$ to maximize utility $\mathcal{U}_H$ as described in Equation 1.79 subject to the sequential budget

[59] This follows McKay et al. (2016), Auclert et al. (2021c), and allows our analysis to be orthogonal to the effects of monetary policy *rules*.

[60] An explicit monetary rule setting i_{Ft} that leads to the exchange rate peg can be found in Benigno et al. (2007).

constraints,

$$\sum_i (1+t_{ij0})P_{ij0}C_{ij0} + P^N_{j0}N_{j0} + \sum_i \frac{B_{ij1}}{1+i_{ijt}} e_{ij0} \le w^T_{j0}L_{j0} + w^N_{j0}N_{j0} + \Pi_{j0} + T_{j0}, \tag{1.82}$$

$$\sum_i (1+t_{ij1})P_{ij1}C_{ij1} + P^N_{j1}N_{j1} \le w^T_{j1}L_{jt} + w^N_{j1}N_{j1} + \sum_i B_{ij1}e_{ij1} + \Pi_{j1} + T_{j1}, \tag{1.83}$$

where P_{ijt} is the (pre-tariff) prices for tradable goods from country i to j in units of j currency, B_{j1} is a tradable claim to one nominal unit of account in period 1 with price $\frac{1}{1+i_{jt}}$, W^s_{jt} is the nominal wage, Π_{jt} is the profit of country j firms and T_{jt} is the government revenue rebated lump-sum.

The first-order conditions to the previous utility maximization problem imply:

$$\mathcal{P}_{jt} = (P_{jt})^{\alpha}(P^N_{jt})^{1-\alpha}, \tag{1.84}$$

$$P_{jt} = (\sum_i ((1+t_{ijt})P_{ijt})^{1-\sigma})^{1/(1-\sigma)}, \tag{1.85}$$

$$\lambda_{ijt} = \frac{((1+t_{ijt})P_{ijt})^{1-\sigma}}{\sum_l P^{1-\sigma}_{ljt}}, \tag{1.86}$$

$$u'(C_{jt}) = \beta(1+i_{jt})\frac{\mathcal{P}_{jt}}{\mathcal{P}_{jt+1}}u'(C_{jt+1}) = \beta R_{jt}u'(C_{jt+1}), \tag{1.87}$$

$$\frac{1+i_{F1}}{1+i_{H1}} = \frac{e_1}{e_0}, \tag{1.88}$$

$$v'(L_{j1}) = \frac{u'(C_{j1})w_{j1}}{P_{j1}} \tag{1.89}$$

The first two conditions equate intratemporal substitution and gives the CPI price index $\mathcal{P}_{jt}$, tradable price index P_{jt} and expenditure share λ_{ijt}; the third equates the intertemporal substitution; the fourth is the uncovered interest rate parity (UIP) condition, which follows from Home and Foreign bonds being perfect substitutes. With an exchange rate peg $e_1 = e_0 = \bar{e}$, the condition becomes $i_{F1} = i_{H1}$ (trilemma). The last condition equates marginal utility with marginal disutility from labor in period 1.

Since wages $\{w^s_{j0}\}$ are rigid at $t = 0$ and the labor market is demand determined, we may have $v'(L_{j0}) \neq \frac{u'(C_{j0})w_{j0}}{P_{j0}}$. We define the *labor wedge* in period 0 as

$$\mu^T_{j0} = v'(L_{j0}) - \frac{u'(\mathcal{C}_{j0})w^T_{j0}}{\mathcal{P}_{j0}}, \;\; \mu^N_{j0} = v'(N_{j0}) - \frac{u'(\mathcal{C}_{j0})w^N_{j0}}{\mathcal{P}_{j0}} \tag{1.90}$$

how much the marginal value of working for households is away from the marginal return from working in utility terms. If $\mu^s_{j0} < 0$, households would like to supply more labor but cannot, so there is *involuntary unemployment*. If $\mu^s_{j0} > 0$, households are supplying more labor than they would want to, so the economy is *overheated*.

Firm optimization. The profits of a representative firm from j selling Y_{ijt} tradable goods to market i is given by

$$\Pi_{it} = \sum_j \left[(1+s_{ijt}) \frac{1}{e_{ijt}} P_{ijt} - \frac{w^T_{it}}{A_{ij}} \right] Y_{ijt}$$

where s_{ijt} is an ad-valorem sales subsidy to i. Since firms are competitive, profits Π_{jt} are equal to 0, and the unit price is equal to marginal cost:

$$P_{ijt} = \frac{1}{1+s_{ijt}} e_{ijt} \frac{w^T_{it}}{A_{ij}}. \tag{1.91}$$

For nontradables, the firm optimization problem gives

$$P^N_{jt} = w^N_{jt}. \tag{1.92}$$

Market clearing. For each (i,t), the tradable goods market clearing conditions are given by

$$L_{it} = \sum_j \frac{C_{ijt}}{A_{ij}} \tag{1.93}$$

and the bonds market clearing condition is given by

$$B_{H1} + e_1 B_{F1} = 0 \tag{1.94}$$

Equilibrium. We can define an equilibrium in the model as follows:

Definition 1.3. *Given fundamentals $\{A_{ij}\}$, rigid short-run wage $\{w^s_{H0}, w^s_{F0}\}$, policy $\{R_{H1}, t_{ijt}, s_{ijt}\}$ and pegged exchange rate $\bar{e}$, a pegged equilibrium consists of prices $\{w_{it}, P_{it}, P_{ijt}, P^N_{jt}\}$, household's choice variables $\{C_{ijt}\}$,$\{B_{it}\}$,$\{L_{it}\}_{t\geq 1}$, demand-determined short-run labor $\{L_{i0}\}$ and labor wedge $\{\mu_{j0}\}$such that Equations 1.82 to 1.94 hold.*

Consequences of a trade shock

In this subsection, we highlight the equilibrium response to trade shocks in this model. As a benchmark, we consider the laissez-faire equilibrium where $t_{FHt} = s_{HFt} = 0$.

The timing of the model and shock is as follows.

(a) Before the start of our setup ($t = -1$), assume that tradable goods productivities were at a level $\{A_{ij,-1}\}$, and nominal wages $w_{i,-1}$ and exchange rate e_{-1} were at a level where trade is balanced and labor wedge is zero.

(b) Right before $t = 0$, a trade shock occurs, where Foreign export productivity A_{FH} permanently increases.

We assume that Wages $\{w_{i0}\}$ are rigid at the pre-shock level $\{w_{i,-1}\}$, and the Foreign policymaker pegs the exchange rate $e_0 = e_1$ at the pre-shock level e_{-1}.

Equilibrium responses. To investigate the effects of the trade shock on trade balance and employment levels, we first observe how the terms-of-trade responds to a trade shock under a peg. We denote by $S_{HFt} = \frac{P_{HFt}\bar{e}}{P_{FHt}}$ the Home terms-of-trade at time t, where a higher terms-of-trade means getting more imports per unit of export. S_{HFt} is given by:

$$S_{HFt} = \frac{(\frac{w^T_{Ht}}{\bar{e}A_{HF}})\bar{e}}{\frac{w^T_{Ft}\bar{e}}{A_{FH}}} = \underbrace{(\frac{w^T_{Ht}}{w^T_{Ft}\bar{e}})}_{\text{relative wage}} \underbrace{(\frac{A_{FH}}{A_{HF}})}_{\text{productivity}} \tag{1.95}$$

In a model where wages are flexible, there are two effects of an increase of A_{FH} on S_{HF}. The *direct effect* increases S_{HF} by an equal proportion, improving Home terms-of-trade. The *general equilibrium effect* is that relative wage $\omega_t = \frac{w^T_{Ht}}{w^T_{Ft}\bar{e}}$ adjusts. When $\sigma > 1$, an increase in A_{FH} decreases Home's relative wage ω_t, so the general equilibrium effect reduces ω_t.

However, when wages are rigid, the exchange rate peg mutes the general equilibrium channel in the short-run. As such, we have $\omega_0 > \omega_1$ and $S_{HF0} > S_{HF1}$: Home tradable relative wage is higher in the short-run than the long-run. From the dynamics of relative wage, we can first prove the following proposition:

Proposition 1.8. *In the pegged equilibrium, in response to a trade shock ($A_{FH} \uparrow$), the following hold:*

(a) Home runs a trade deficit: $B_{H1} < 0$.

(b) *The share of labor used in tradables (or equivalently output) declines as a result of the shock:* $\frac{L_{H0}}{L_{H0}+N_{H0}}$ *is below the pre-shock level.*

(c) *Monetary policy faces a trade-off between tradable unemployment* ($\mu^T_{H0} < 0$) *and nontradable overheating* ($\mu^N_{H0} > 0$)

(d) *When monetary policy is unresponsive* ($R_{H1} = \frac{1}{\beta}$)*, there is unemployment in both tradables and nontradables:* $\mu^T_{H0}, \mu^N_{H0} < 0$.

Proof. See Appendix 1.8. □

The intuition for the first part ($B_{H1} < 0$) is as follows. Home borrows if and only if:

$$\underbrace{\frac{\bar{e}\lambda_{HF0}\alpha\mathcal{P}_{F0}\mathcal{C}_{F0}}{\lambda_{FH0}\alpha\mathcal{P}_{H0}\mathcal{C}_{H0}}}_{t=0\text{ exports/imports}} < \underbrace{\frac{\bar{e}\lambda_{HF1}\alpha\mathcal{P}_{F1}\mathcal{C}_{F1}}{\lambda_{FH1}\alpha\mathcal{P}_{H1}\mathcal{C}_{H1}}}_{t=1\text{ exports/imports}} \Leftrightarrow \underbrace{\frac{\lambda_{HF0}/\lambda_{HF1}}{\lambda_{FH0}/\lambda_{FH1}}}_{\text{expenditure switching}} < \frac{\pi_F}{\pi_H}\frac{\mathcal{C}_{H0}/\mathcal{C}_{H1}}{\mathcal{C}_{F0}/\mathcal{C}_{F1}} = \underbrace{(\frac{\pi_F}{\pi_H})^{1-\gamma}}_{\text{relative inflation}} \tag{1.96}$$

Inequality 1.96 highlights the two forces that determine the sign of trade balance. The first force on the left-hand side is *expenditure switching*. When $\sigma > 1$, we have $\omega_0 > \omega_1$, both countries want to buy more Foreign goods today, leading to higher import expenditure λ_{FH0} at Home and lower import expenditure λ_{HF0} at Foreign at $t = 0$ than $t = 1$. This is a force towards Home running a trade deficit. The second force on the right-hand side is *relative inflation*. With $\omega_0 > \omega_1$, Home's future prices increase *less* because of home bias in tradables consumption, while nontradable prices are unaffected. This becomes a force towards Home surplus or deficit, depending on whether $\gamma > 1$. When $\sigma > \gamma$, expenditure switching (governed by σ) outweighs the relative inflation (governed by γ), resulting in Home borrowing.[61]

The second part is a natural conclusion of the fact that tradables are the sector *directly* affected by the trade shock, while the nontradable sector is *indirectly* affected through aggregate demand channels. After the shock, labor supply is demand-determined, and in the short-run, global demand for Home tradables decline more than the nontradables.

The latter two parts highlight the trade-off that monetary policy faces. At the onset of the shock, if monetary policy does not respond, there is going to be unemployment in tradables. The intuition is as follows: the short-run Home consumption $\mathcal{C}_{H0}$ is determined from the Euler equation. At $\mathcal{C}_{H0}$ and real wage $\frac{w^T_{H0}}{p_{H0}}$, Home tradable workers would want to supply labor $L^S_{H0} = v'^{-1}(u'(\mathcal{C}_{H0})\frac{w^T_{H0}}{\mathcal{P}_{H0}})$. However, workers supply whatever is

[61] In fact, standard estimates of γ are often 1 or less, whence relative inflation also leads to Home borrowing.

demanded, and the demand L_{H0} is pinned down by relative wage ω_0:

$$L_{H0} = \frac{1}{A_{HH}} \frac{\lambda_{HH0}(\omega_0) P_{H0}}{P_{HH0}} C_{H0} + \frac{1}{A_{HF}} \frac{\lambda_{HF0}(\omega_0) P_{F0}}{P_{HF0}} C_{F0}.$$

If ω_0 is higher, the desired supply L^S_{H0} increases but actual demand L_{H0} falls; this generates *involuntary unemployment*, with the unemployment rate given by $u_{H0} = 1 - \frac{L_{H0}}{L^s_{H0}}$.

Now, with unemployment in the tradable sector, aggregate demand for Home nontradables is *also* going to decline; while the nontradable sector is not directly hit by the shock, the recession in aggregate Home tradable demand spills over to the nontradable sector through the household first-order condition. As such, if monetary policy does not react, there will be unemployment in both tradables and nontradables. And monetary policy cannot resolve both labor wedges: if it chooses tradable

Proposition 1.8 parsimoniously connects the four facts in the introduction: the US trade deficit and the decline in manufacturing in the 2000s can be endogenously explained by Chinese productivity growth and its exchange rate peg. This contrasts with prior studies of the China shock which typically perceive China's concurrent saving and growth as a puzzle. We show that China's exchange rate peg with wage rigidity promotes a stronger short-term comparative advantage in tradables during its growth, driving its endogenous decision to save.[62] Moreover, we can also explain the rise in unemployment in manufacturing-heavy regions as documented in Autor et al. (2013), which find that a $1,000 per worker increase in import exposure to China increases the unemployment to population rate by 0.22 percentage points.

Proposition 1.8 substantiates the role of nominal rigidity as an important cause of the labor market's sluggish response to trade shocks, contrasting with alternative frameworks using quantity friction such as search models (Dix-Carneiro et al., 2023; Galle et al., 2023). In a quantity friction framework, trade balance response to trade shocks would invert, with US saving and China borrowing. This is because relative wages across time is *reversed* in a search model: quantity friction induces a short-run labor surplus, depressing Home relative wage, prompting Home to save, and reducing Home unemployment.[63] In Appendix 1.10, we present an otherwise identical model incorporating quantity rigidity, confirming that Home indeed runs a short-run trade surplus and experiences declining unemployment in response to a trade shock.

[62] This is a force independent of balances being affected by Foreign's forex market intervention to maintain the peg. In a model with these features, these two forces will complement each other.

[63] *"The large trade surplus that China has been running since the early 2000s is a puzzle for models in which the main driving forces are productivity shocks."* (Dix-Carneiro et al., 2023)

Welfare effects. Next, we turn to welfare implications of the trade shock. We first highlight that trade balances affect the future terms-of-trade: specifically, a deterioration in balances B_{H1} leads to a decrease in future relative wage ω_1. The intuition is closely related to the transfer problem: debt accumulated today becomes a future *transfer* for Foreign, which, combined with home bias for demand, increases global demand for Foreign goods, improving their terms-of-trade and worsening Home's.

We study the welfare effects of the endogenous deficits we highlighted in Proposition 1.8. We have established that deficits and unemployment can come from the same cause – trade shock and exchange rate peg – but are deficits inherently bad for Home welfare? While this is where some policy narratives go, the next proposition shows that this is not the case.

Proposition 1.9. *In the pegged equilibrium, removing international financial flows (forcing $B_{H1} = 0$) worsens Home unemployment (μ_{H0} decreases), and reduces Home welfare W_0.*

Proof. See Appendix 1.8. □

Removing financial flows worsens Home unemployment because of Home bias in consumption. Indeed, with trade costs, under the same price levels, Home borrowing to consume will increase demand for Home goods, while Foreign saving will decrease demand for Foreign goods. Since unemployment is determined by aggregate demand, the fact that Home runs a trade deficit in the short-run actually *ameliorates* unemployment. As such, while deficits may be symptoms of a friction that may harm the economy, deficits themselves are not *frictions* to solve, and policies that restrict financial flows to curb deficits will not improve Home welfare.

Next, we study aggregate welfare implications of the trade shock. The next proposition highlights the possibility that Home welfare may decrease as a result of Foreign productivity growth:

Proposition 1.10. *In the pegged equilibrium where monetary policy does not respond ($R_{H1} = \frac{1}{\beta}$), an increase in A_{FH} reduces Home welfare when σ is sufficiently high and improves Home welfare when σ is small (i.e. close to 1).*

Proof. See Appendix 1.8. □

An intuitive explanation is as follows. As derived in Appendix 1.8, there are three channels through which productivity growth A_{FH} affects Home welfare:

$$\frac{d\mathcal{U}_H}{dA_{FH}} = -\underbrace{\frac{u'(C_{H0})}{P_{H0}} C_{FH0} \frac{dP_{FH0}}{dA_{FH}}}_{\text{terms-of-trade at } t=0} - \sum_{s\in\{T,N\}} \mu^s_{H0} \underbrace{\frac{dL^s_0}{dA_{FH}}}_{\text{labor wedge}} + \underbrace{\frac{\beta u'(C_{H1})}{P_{H1}} \left[C_{HF1} \frac{dP_{HF1}}{dA_{FH}} - C_{FH1} \frac{dP_{FH1}}{dA_{FH}} \right]}_{\text{terms of trade at } t=1} \tag{1.97}$$

The terms correspond to (1) the short-run effect of cheaper import goods (2) labor market friction caused by wage rigidity (3) change in long-run terms-of-trade, including direct productivity effects and trade balance effects on future terms-of-trade. If σ approaches 1, preference becomes Cobb-Douglas, the pegged equilibrium approaches the flexible-wage equilibrium, and trade is balanced (Cole and Obstfeld, 1991). Thus the effects (2) and the general equilibrium component of (3) go to zero, leaving cheaper goods as the primary welfare benefit. In the opposite case when $\sigma \to \infty$, Home and Foreign goods become perfect substitutes, and short-run demand for Home goods become 0. Then even an infinitesimal change in A_{FH} can lead to a discrete loss of utility from the labor wedge and the future transfer worsening terms-of-trade; this dwarfs any welfare gains from cheaper goods.

The possibility of Foreign productivity growth harming Home welfare is reminiscent of immiserizing growth, characterized by Home productivity growth worsening Home terms-of-trade, which can potentially outweigh the gains from expansion of the production possibilities frontier (PPF) (Bhagwati, 1958). In our context, Foreign productivity growth improves Home terms-of-trade. The exchange rate peg further improves the short-run terms-of-trade, but it moves Home production into the interior of the PPF because of unemployment, and hamper future terms-of-trade through current trade deficit, offsetting the gains from better terms-of-trade.

Proposition 1.10 underscores the need to be cautious in using trade balance as a welfare indicator in trade. Public discourse often consider trade deficits as inherently undesirable. However, whenever σ exceeds 1 and surpasses γ, a trade deficit is a likely outcome for the home country in response to a Foreign productivity growth under currency pegging. This may be beneficial for Home welfare if σ is not excessively high, but suggest welfare losses if Home and Foreign goods are highly substitutable. Conversely, if γ is very large and $\sigma \to 1$, Home runs a trade surplus and has welfare gains, whereas if both σ and γ are sufficiently large with $\gamma > \sigma$, Home runs a trade surplus but incur welfare losses. Therefore, it is crucial to undertake a quantitative analysis of the specific degree of substitution, rigidity, and productivity growth in order to assess whether a trade shock accompanied by a deficit is detrimental or beneficial to aggregate welfare.

Optimal policy

This subsection analyzes the effect of policy on welfare under the exchange rate peg. We consider the short-run unilateral problem of the Home government a growth in A_{FH} and an exchange rate peg, which can choose its short-run tariff level t_{FH0}, domestic subsidy s_{HF0} and monetary policy R_{H1}.[64] These policies are akin to using second-best policies to fix an existing distortion, aligning with *safeguard* tariffs allowed by the WTO. We assume the government cannot choose long-run tariff t_{FH1}, as the only motivation for long-run tariffs is terms-of-trade manipulation, which is well understood in the literature (Gros, 1987), and is forbidden by the WTO.

Formally, the *policy problem* that the Home government faces is:

$$\max_{t_{FH0}, s_{HF0}, R_{H1}} \mathcal{U}_{\mathcal{H}} \tag{1.98}$$

subject to Equations 1.82 to 1.94.

We first note that the planner can replicate the flexible price outcome. Indeed, if $\omega_{peg} = \frac{w_{H0}}{w_{F0}\bar{e}}$ is the relative wage under peg at $t = 0$, and $\omega_f = \frac{w^f_{H0}}{w^f_{F0}e^f}$ is the relative wage under flexible price (both computed after the trade shock), the planner can set $R_{H1} = \frac{1}{\beta}$ and $t_{FH0} = s_{FH0} = \frac{\omega_f}{\omega_{peg}} - 1$. This tax and subsidy level sets the relative prices equal to the flexible price level, and the tax revenue and cost of subsidy cancel out exactly. Thus we know the planner can undo the wedge present in the model, thereby undoing the welfare losses which could potentially be significant as in Proposition 1.10.[65]

However, this policy may not be optimal for Home government. If Foreign exchange rate policy is offering cheap goods for Home, policies that undo this could potentially be suboptimal – in the extreme case, if Foreign is offering goods for free, Home would be much better off taking those goods than setting high tariffs that distort consumption.

We proceed in two steps: first, we solve the inner problem of optimal trade policy (t_{FH0}, s_{HF0}) given monetary policy R_{H1}, then we proceed to solve for the optimal R_{H1}. This approach makes the problem more tractable, and may be a more reasonable benchmark for reality, where monetary policy is unable to fully respond to a sector-origin specific trade shock.

[64] Since wages are rigid, we no longer have Lerner symmetry, and subsidies and tariffs are independent instruments.

[65] More generally, this connects with Farhi et al. (2014) that fiscal instruments can replicate currency devaluations.

Optimal trade policy

Given monetary policy, an indirect formula for the optimal trade policy can be obtained via a first-order variations argument: marginal effect of policy change in welfare must be zero.[66]

Lemma 1.9. *The optimal short-run tariff rate on imports t_{FH0} satisfies*

$$t_{FH0} = \frac{1}{P_{FH0}} \left[\underbrace{\frac{\mu_0}{\tilde{\lambda}} \frac{\partial L_{H0}}{\partial C_{FH0}}}_{\text{labor wedge}} - \frac{1}{(1+i_{H1})} \underbrace{(L_{HF1} \frac{\partial w_{H1}}{\partial C_{FH0}} - L_{FH1} \frac{\partial w_{F1}}{\partial C_{FH0}})}_{\text{future terms-of-trade}} + \underbrace{s_{HF0} P_{HF0} \frac{\partial C_{HF0}}{\partial C_{FH0}}}_{\text{subsidy externality}} \right] \tag{1.99}$$

The optimal short-run subsidy rate on exports s_{HF0} satisfies

$$s_{HF0} = \frac{1}{P_{HF0}} \left[- \underbrace{\frac{\mu_0}{\tilde{\lambda}} \frac{\partial L_{H0}}{\partial C_{HF0}}}_{\text{labor wedge}} + \frac{1}{(1+i_{H1})} \underbrace{(L_{HF1} \frac{\partial w_{H1}}{\partial C_{HF0}} - L_{FH1} \frac{\partial w_{F1}}{\partial C_{HF0}})}_{\text{future terms-of-trade}} - \underbrace{P_{HF0} C_{HF0} \frac{\partial s_{HF0}}{\partial C_{HF0}}}_{\text{terms-of-trade today}} \right] \tag{1.100}$$

Proof. See Appendix 1.9 for a detailed intuition, and Appendix 1.8 for a formal proof. □

The first-order formula for tariffs succinctly captures the *externalities* of imports that the Home government seeks to address via a tariff. First, tariffs and subsidies both reduce the labor wedge by stimulating demand for domestic labor. Second, tariffs and subsides improve current trade balance, which improves the terms-of-trade in the future. Third, the fiscal externality (deadweight loss) of tariffs and subsidies interact in general equilibrium. In a competitive equilibrium, home households do not internalize any of these "effects" of an extra unit of import; thus the tax level $t_{FH0}P_{FH0}$ and the subsidy level $s_{HF0}P_{HF0}$ can be considered a Pigouvian tax that corrects for the three externalities of consuming an extra unit of import, or exporting an extra unit.

Using the formula, we can sign the optimal tariff, and show that its magnitude *increases* with the Foreign shock A_{FH0}:

Proposition 1.11. *Given R_{H1}, if there is tradables unemployment at the zero-tariff economy ($\mu^T_{H0} < 0$ when $t_{FH0} = 0$), the optimal tariff t_{FH0} is positive and is increasing in the size of the trade shock A_{FH0}.*

Proof. See Appendix 1.8. □

[66] This method follows Costinot et al. (2022).

The intuition that we can and should use tariffs as second-best instruments to fix distortions is already well-known. We show that in an environment where trade shocks cause unemployment and trade deficits, we can sign the short-run tariff: it should be positive, and increasing in the magnitude of the trade shock, when monetary policy fails to clear existing unemployment. The central bank may be unable to clear the output gap caused by sector-specific trade shocks, because of multisector considerations, financial concerns, and liquidity constraint such as the Zero Lower Bound. Tariffs will be a useful tool in this second-best world.[67]

Our model underscores that under an exchange rate peg, the optimal tariff is increasing in the magnitude of the trade shock. In a floating exchange rate environment, the optimal tariff is pinned down primarily by the trade elasticity (Gros, 1987) and does not depend on the shock magnitude. In contrast, our framework focuses on tariffs that correct a distortion caused by the peg and the trade shock, so the magnitude of the optimal tariff scales with the size of the distortion.

Optimal monetary policy

What is the optimal monetary policy given a tariff schedule? An analogous first-order condition on monetary policy highlights the channels in which monetary policy affects welfare. We highlight a special case, when the intertemporal elasticity is equal to 1 (consumption is log):

Proposition 1.12. *When $\gamma = 1$, optimal monetary policy satisfies the following equation.*

$$0 = \underbrace{-\sum_{s\in\{T,N\}} \mu_0 \frac{dL_0}{dR_{H1}}}_{wedge} + \tilde{\lambda}_r [\underbrace{R_{H1} t_{FH0} \frac{P_{FH0}}{P_{H0}} \frac{dC_{FH0}}{dR_{H1}}}_{\text{tariff fiscal externality}} + \underbrace{(NX_0)}_{\text{intertemporal TOT}}] \tag{1.101}$$

Notably, when $t_{FH0} = 0$, optimal monetary policy R_{H1} is such that $\mu_0 > 0$: it is optimal to loosen the monetary policy beyond the output gap.

Proof. See Appendix 1.9 for a detailed intuition, and Appendix 1.8 for a formal proof. □

Proposition 1.12 highlights that in an open economy where Home is subject to a Foreign peg, optimal monetary policy may want to *overshoot* the output gap when Home is borrowing from Foreign. This is because Home has power in setting *global* monetary

[67]Indeed, in the early 2000s, the government was tightening monetary policy in response to concerns over inflation and tightening of unused resources; loosening in response to the China shock was not the Federal Reserve Bank's goal (Federal Reserve Board, 2005). Following the Great Recession, the Federal Reserve Bank was subject to the Zero Lower Bound.

policy. This is especially relevant to the US, which effectively sets the interest rate for many countries being the dominant currency (Gopinath et al., 2020), and runs current account deficits; the central bank may want to set a looser interest rate, with minimal risk of bond liquidation from dollar-pegging countries. The proposition also clarifies again that tariffs are primarily second-best instruments to be used when monetary policy is unable to respond – whether due to the ZLB or multisectoral considerations. In fact, under a strictly positive tariff, the additional losses from tariff fiscal externality compels Home to adopt a more contractionary monetary stance, reducing overall welfare.

1.10.2 Quantity-side friction

In this section, I explore a model analogous to the stylized model in Section 1.3, except with one difference: *nominal* rigidity is replaced with *quantity* rigidity. We explore the model implications, most notably how the propositions change.

Setup and equilibrium. The model setup is identical to Section 1.3, so I skip the details. The main difference is that we replace wage rigidity ($\{w_{i0}\}$ is exogenously given) with quantity rigidity, such that $\{L_{i0}\}$ is exogenous and given by the pre-shock steady-state equilibrium value. Since there is no nominal rigidity, the *absolute level* of nominal variables and the nominal exchange rate play no role in the equilibrium. Thus we normalize the exchange rate to $e_t = 1$.

The laissez-faire (no tariffs) equilibrium conditions are given by

$$P_{jt} = (\sum_i (P_{ijt})^{1-\sigma})^{1/(1-\sigma)}, \tag{1.102}$$

$$\lambda_{ijt} = \frac{(P_{ijt})^{1-\sigma}}{\sum_l P_{ljt}^{1-\sigma}}, \tag{1.103}$$

$$u'(C_{jt}) = \beta(1+i_{jt})\frac{P_{jt}}{P_{jt+1}}u'(C_{jt+1}) = \beta R_{jt} u'(C_{jt+1}), \tag{1.104}$$

$$\frac{1+i_{F1}}{1+i_{H1}} = \frac{e_1}{e_0}, \tag{1.105}$$

$$v'(L_{j1}) = \frac{u'(C_{j1})w_{j1}}{P_{j1}} \tag{1.106}$$

$$P_{ijt} = \frac{w_{it}}{A_{ij}} \tag{1.107}$$

$$L_{it} = \sum_j \frac{C_{ijt}}{A_{ij}} \tag{1.108}$$

$$w_{i0}L_{i0} + \frac{w_{i1}L_{i1}}{1+i_i} = P_{i0}C_{i0} + \frac{P_{i1}C_{i1}}{1+i_i} \tag{1.109}$$

with an exogenous $\{L_{i0}\}$ set at the steady-state values under the pre-shock $\{A_{ij}\}$ at $t = -1$.

First, since there is no nominal rigidity, we have monetary neutrality; this is a *real* economy where nominal values only play a unit of account role. Thus we may normalize Home relative wage to 1 in both periods, and the exchange rate $e_0 = e_1 = 1$ in both periods. Thus what matters is the relative wage of Foreign in each period.

We first make the following assumption about the *flexible-quantity* equilibrium:

Assumption 1.1. *If we denote $L_i^*(\{A_{ij}\})$ the labor supply under a static, flexible-quantity economy under productivity $\{A_{ij}\}$, we have*

$$L_H^*(\{A_{ij,0}\}) < L_H^*(\{A_{ij,-1}\}) \quad \textit{and } L_F^*(\{A_{ij,0}\}) < L_F^*(\{A_{ij,-1}\})$$

The assumption states that in response to Foreign productivity growth, Home workers would want to supply less labor, whereas Foreign workers would want to supply more labor. At Home, this would hold because import prices decline, real wage goes up, so both the income effect and substitution effect work towards less labor supply. At Foreign, a productivity growth would imply more labor supply iff the Frisch elasticity is large (so labor supply responds more to higher income). Alternatively, any multi-sector model with a sectoral shock (labor moves into this sector in China, and moves out in the US) would generate this direction.

Thus, the *quantity rigidity* on short-run labor $\{L_{H0}, L_{F0}\}$ (which could be motivated by search friction a la Mortensen and Pissarides (1994), or sectoral reallocation a la Artuç et al. (2010)) is such that Home wants to supply less labor but cannot, and Foreign wants to supply more labor but cannot.[68] Under this framework, we have the following properties:

Proposition 1.13. *Under the above quantity rigidity framework, we have:*

(a) $\omega_0 < \omega_1$*: Home relative wage is lower in the short-run than the long-run.*

(b) $B_{H1} > 0$*: Home saves in the short-run.*

(c) $\mu_{H0} < 0$*: there is overheating at Home in the short-run.*

Proof. (Sketch of proof)

The first part follows from our assumption: since L_H under flexible quantity would have been lower, we have $L_{H0} > L_{H1}$ and $L_{F0} < L_{F1}$. Since this pins down total goods supply, for the goods market to clear in each period we must have $\omega_0 < \omega_1$.

The second part's proof is analogous to the proof of Proposition 1.1 (in the Appendix of the main text). Given $\omega_0 < \omega_1$, we can rearrange the terms to get a sufficient condition based on expenditure switching and relative inflation; with $\sigma > \gamma$, we get $B_{H1} > 0$.

The proof of the third part is a combination of two facts:

- Short-run Home relative wage is lower in the short-run than the flexible-quantity level ($\omega_0 < \omega^{flex}$, follows from the fixed labor supply)
- Short-run Home labor is higher than in the flexible-quantity level ($L_0 > L^{flex}$)
- Home's consumption C_{H0} is pinned down by relative wage ω_0 and the labor supply L_{H0}, L_{F0} (by solving the system of equations governing labor supply).

Analogously to Proposition 1.1 of the main text, we can verify that the desired labor supply is larger than the flexible-quantity labor supply, which in turn is greater than the labor demanded at $t = 0$. □

The above proposition highlights the differential predictions of quantity rigidity models and nominal rigidity models. In response to a permanent Foreign growth, if the source of labor friction is on nominal wages, Proposition 1.1 of the main text shows that Home's relative wage is *higher* in the short-run, Home runs a trade *deficit*, and Home

[68] This is the dynamics in Dix-Carneiro et al. (2023).

faces *unemployment*. On the other hand, if the source of labor friction is on quantity, Proposition 1.13 shows that Home's relative wage is *lower* in the short-run, Home runs a trade *surplus*, and Home faces *overheating*. (Indeed, in the quantity rigidity model of Dix-Carneiro et al. (2023), we find that the US borrows in response to Chinese growth.)

The stylized facts of the 2000s (Figure 1.1 of the main text) are consistent with the wage rigidity model, and we have verified the predictions of the wage rigidity framework through our empirical findings in Section 1.2. This provides supporting evidence that, in analyzing the labor market response to the China shock, an important channel is nominal rigidity that generates involuntary unemployment in the US.

1.11 Data, Calibration, and Solution Algorithm

This Appendix builds on Section 1.4.2 and describes the construction of our data and our calibration strategy.

1.11.1 WIOD data

Our main source of trade data is the World Input-Output Database (WIOD) 2016 release Timmer et al. (2015). The World Input-Output Table in the WIOD cover 44 countries and a rest-of-world aggregate, and the data spans from 2000 to 2014.

List of country aggregates and sectors. We follow Dix-Carneiro et al. (2023) and divide the world into six country aggregates and six sectors, focusing on US (country 1) and China (Country 2). Table 1 shows our country aggregates, and Table 2 shows how the 56 sectors in the WIOD are mapped to the six broad sectors considered in our model.

	Group	Countries in group
1	USA	USA
2	China	China
3	Europe	Austria (AUT), Belgium (BEL), Bulgaria (BGR), Switzerland (CHE), Cyprus (CYP), Czech Republic (CZE), Germany (DEU), Denmark (DNK), Spain (ESP), Estonia (EST), Finland (FIN), France (FRA), United Kingdom (GBR), Greece (GRC), Croatia (HRV), Hungary (HUN), Ireland (IRL), Italy (ITA), Lithuania (LTU), Luxembourg (LUX), Latvia (LVA), Malta (MLT), Netherlands (NLD), Norway (NOR), Poland (POL), Portugal (PRT), Romania (ROU), Slovakia (SVK), Slovenia (SVN), Sweden (SWE)
4	Asia/Oceania	Australia (AUS), Japan (JPN), Korea (KOR), Taiwan (TWN)
5	Americas	Brazil (BRA), Canada (CAN), Mexico (MEX)
6	Rest of World	Indonesia (IDN), India (IND), Russia (RUS), Turkey (TUR), ROW

Table 1.8: Country definitions

	Sector aggregate	WIOD sector
1	Agriculture and Mining	Agriculture (1-3), Mining (4)
2	LT Manufacturing	Wood (7), Paper and Printing (8-9), Coke and Petroleum (10), Basic and Fabricated Metals (15-16), other mfg (22)
3	MT Manufacturing	Food (5), Textiles (6), Rubber (13), Mineral (14)
4	HT Manufacturing	Chemical and Pharmaceutical (11-12), Machinery, Computers and Motor Vehicles (17-23)
5	LT Services	Utilities (24-26), Construction (27), Wholesale and Retail (28-30), Transportation (31-35), Accommodation (36), Other Service (54), Household (55), Miscellaneous (56)
6	HT Services	Media and Telecommunications (37-39), IT (40), Finance (41-43), Real Estate (44), Legal (45), Architecture (46), Science (47), Advertising (48), Other Professional (49), Government and Education (50-52), Health (53)

Table 1.9: Sector definitions

Note: The numbers inside parentheses denote the WIOD sectors, which follow the International Standard Industrial Classification revision 4 (ISIC Rev. 4). The classification of the six broad sectors follow Dix-Carneiro et al. (2023). In the sector aggregate classifications, (L,M,H) stand for Low-, Medium-, High- and T stands for Technology.

Constructed variables

The World Input-Output Table of WIOD contains the following raw data:

- M_{ijt}^{sn}, goods produced in sector s at country i that is used as inputs for goods in sector n at country j.

- F_{ijt}^{s}, goods produced in sector s at country i that is used as final expenditure in country j. (There are five expenditure categories; three consumption and two investment. We aggregate them.)

- GO_{it}^{s}, VA_{it}^{s}, ITM_{it}^{s} denote gross output, value added and international transport margins in country, sector (i, s) respectively.

Since the data comprises 44 countries and 56 sectors, we map this into our 6-sector, 6-country model by a direct sum.

From M_{ijt}^{sn} and F_{ijt}^{s}, we obtain the following:

- X^s_{ijt}, the total exports from i to j in sector s, given by

$$X^s_{ijt} = F^s_{ijt} + \sum_n M^{sn}_{ijt}$$

- λ^s_{ijt}, the share of sector s expenditure in j that originates from i, given by

$$\lambda^s_{ijt} = \frac{X^s_{ijt}}{\sum_{i'} X^s_{i'jt}}$$

- IO^{sn}_{it}, the input-output table of country i, given by

$$IO^{sn}_{it} = \sum_{i'} M^{sn}_{i'it}$$

- E^s_{it}, expenditure of country i in sector s, by

$$E^s_{it} = \sum_s F^s_{i'it}$$

We also obtain the net exports of country i by

$$NX_{it} = \sum_s VA^s_{it} + \sum_s ITM^s_{it} - \sum_s C^s_{it}$$

To ensure that net exports sum to zero, we assign any error to the rest-of-world.

From the WIOD Socio-Economic Accounts (SEA), we obtain the following:

- Industry-level employment $L^s_{i,2000}$ at period $t = 0$: we use the 2000 values as the initial condition for our model.

- Sectoral prices. We obtain $P^{s,dom}_{it}$, the domestic output price (price deflator) of country i in WIOD sector j expressed in millions of dollars. We closely follow the procedure in Dix-Carneiro et al. (2023) to construct $P^{s,dom}_{it}$ for our 6 country aggregates i and 6 sectors j.

We use the constructed $\{X^s_{ijt}, \lambda^s_{ijt}, IO^{sn}_{it}, E^s_{it}, NX_{it}, VA_{it}, GO_{it}, L^s_{i,2000}, P^{s,dom}_{it}\}$ in our calibration.

1.11.2 CPS data

To construct labor transition across sectors, we use the Current Population Survey (CPS). We rely on the annual retrospective questions from the Annual Social and Economics Supplement (ASEC) of the CPS. We map the 1990 Census industry codes in the CPS to the WIOD sector codes (based on ISIC Rev. 4) then into our 6 sectors, and obtain the transition ratio of employment from sector s to sector n at time t:

$$\mu_t^{sn} = \frac{1_{s,t-1} 1_{nt} wt_{it}}{\sum_{s'} 1_{s,t-1} 1_{s't} wt_{it}}$$

1.11.3 Calibration of parameters outside of the model

The parameters in Panel A of Table 1.1 are calibrated outside the model. We make note of two parameters important in our model, which are σ (Armington elasticity) and κ (slope of the New Keynesian Phillips Curve with respect to the output gap).

Calibration of σ. We use $\sigma = 5$ as the elasticity of within-sector goods substitution across different origins. This is identical to the elasticity used in Rodríguez-Clare et al. (2022), and generates the same gravity trade equation as in Dix-Carneiro et al. (2023)[69].

Calibration of κ. Hazell et al. (2022) estimate the slope of the following equation for unemployment:

$$\pi_t = -\kappa' \hat{u}_t + \beta E_t \pi_{t+1} + \nu_t$$

where $\hat{u}_t = \bar{u}_t - u_t$ is the gap from full employment, and using inter-state panel data, at a quarterly frequency, and find $\kappa' = 0.0062$. In our context, our time is annual, so the equivalent form is

$$\pi_t = -\kappa'(1 + \beta^{1/4} + \beta^{2/4} + \beta^{3/4}) \hat{u}_t + \beta E_t \pi_{t+1}.$$

Moreover, their measure of unemployment is $u_t = 1 - N_{Ht}$. In our context, our wage NKPC is given by

$$\log(1 + \pi_t^w) = \kappa(v'(\ell_t) - \frac{w_t}{P_t} u'(C_t)) + \beta \log(1 + \pi_{t+1}^w)$$

[69]The formulation is different, because Dix-Carneiro et al. (2023) use a Eaton-Kortum model of perfect competition with a continuum of goods. In our model, the gravity equation is governed by a scale of $(1 - \sigma)$, whereas in their model it is governed by $-\lambda$ where λ is the Frechet scale parameter. Dix-Carneiro et al. (2023) use $\lambda = 4$, generating the same gravity equation.

The output gap can be rewritten as $v'(\ell_t) - \frac{w_t}{P_t}u'(C_t) = v'(\ell_t) - v'(\ell_t^D)$ where ℓ_t^D is the desired labor supply at this level. Linearizing v near the full-employment level $\ell_t = 1$, we have

$$\pi_t^w = \kappa\frac{\theta}{\varphi}(\ell_t - 1) + \beta\pi_{t+1}^w$$

Lastly, if wages increase by $X\%$ everywhere, the price index would also increase proportionately because production technology has constant returns to scale. Thus, the κ value consistent with Hazell et al. (2022) is given by

$$\kappa = \varphi\kappa'\frac{1}{\theta}(1 + \beta^{1/4} + \beta^{2/4} + \beta^{3/4}) = 0.05$$

using our values of $\varphi = 1.0, \beta = 0.95$, and the population average of θ given by 0.966.

1.11.4 Calibration of parameters in our model

The next paragraphs detail the calibration of parameters in Panel B of Table 1.1, using the WIOD and CPS data above. In this section, a variable with a bar above $(\overline{X})$ denotes variables directly observable in the data, and all other variables denote equilibirum objects.

We first note that the preference shares and production function parameters are directly measurable from the data:

$$\alpha_{it}^s = \frac{\overline{E}_{it}^s}{\sum_n \overline{E}_{it}^n} \tag{1.110}$$

$$\phi_{it}^{sn} = \frac{\overline{IO}_{it}^{sn}}{\sum_{s'} \overline{IO}_{it}^{s'n}} \tag{1.111}$$

$$\phi_{it}^s = \frac{\overline{VA}_{it}^s}{\overline{GO}_{it}^s} \tag{1.112}$$

The calibration of the remaining parameters $\tau_{ijt}^s, A_{it}^s, \delta_{it}^s, \eta_{it}^s, \chi_{it}^{sn}$ requires use of our model. We first calibrate the 2000 values, and then calibrate the 'shocks' to these variables.

Calibration of the initial period

Since δ_{it}^s govern intertemporal preference shocks, we need not calibrate it for the year 2000. We assume that the model is in steady-state in the year 2000, which implies two assumptions: the first is that the labor market is in full employment (no output gap), and that the labor distribution L_i^s in 2000 is the steady-state distribution of labor under the assumption that parameters $\{\tau_{ijt}^s, A_{it}^s, \eta_{it}^s, \chi_{it}^{sn}, \theta_{it}^s\}$ do not change from 2000 values. This simplifies our analysis without hurting the 'effect of the China shock.'

Suppressing the time subscripts t, we calibrate the 2000 values of $\{\tau_{ij}^s, A_i^s, \eta_i^s, \chi_i^{sn}, \theta_i^s\}$ to match the following observed data:

- Trade cost τ_{ij}^s and productivity A_i^s matches the sector-level expenditure share $\overline{\lambda}_{ij}^s$
- Intensity of labor disutility θ_i^s is such that $\ell_i^s = 1$ in the initial period;
- Nonpecuniary utility η_i^s matches the 2000 distribution of labor L_i^s:
- Migration costs χ_{it}^{sn} match the migration flow from 1999 to 2000 $\mu_{i,-1}^{sn}$.

Productivity A_i^s and trade costs τ_{ij}^s. We first construct the equation identifying trade costs τ_{ij}^s using the gravity equation and firm pricing equation, following Head and Ries (2001) and Eaton et al. (2016). The firm pricing equation is given by

$$P_{ij}^s = e_{ij}\tau_{ij}^s \frac{1}{A_i^s}(W_i^s)^{\phi_i^s} \prod_n (P_i^n)^{\phi_i^{ns}} = e_{ij}\tau_{ij}^s P_i^{s,dom}$$

and we normalize $\tau_{ii}^s = 1$ (so A_i^s fully captures the productivity). The gravity equation of trade shares is

$$\lambda_{ij}^s = \frac{(P_{ij}^s)^{1-\sigma_s}}{(P_j^s)^{1-\sigma_s}}$$

so we have

$$\frac{P_{ij}^s}{P_{jj}^s} = (\frac{\lambda_{ij}^s}{\lambda_{jj}^s})^{\frac{1}{1-\sigma}} \tag{1.113}$$

Combining the two equations above, we get

$$\tau_{ij}^s = \frac{e_j P_j^{s,dom}}{e_i P_i^{s,dom}} (\frac{\lambda_{ij}^s}{\lambda_{jj}^s})^{\frac{1}{1-\sigma_s}} \tag{1.114}$$

We normalize the productivities $A_i^s = 1$ and set the wages so that $W_i^s L_i^{s,data}$ fits the data on value added in each sector (so we can match the net export flows). Then, we calibrate the "trade block" of $\{A_i^s, \tau_{ij}^s\}$ under the assumption that labor supply is fixed at $L_i^{s,data}$ and households find it optimal to supply $\ell_i^s = 1$, and calibrate the rest of the parameters so that our model endogenously generates $L_i^s = L_i^{s,data}$ and $\ell_i^s = 1$. This solves the following system of equations:

$$P_i = \prod (P_i^s)^{\alpha_i^s} \tag{pindex}$$

$$(P_j^s)^{1-\sigma} = \sum_i (P_{ij}^s)^{1-\sigma_s} \tag{pindex}$$

$$\sum_s W_i^s L_i^{s,data} = P_i C_i + NX_i \tag{budget}$$

$$R_i^s = \sum_j \lambda_{ij}^s (\alpha_j^s P_j C_j + \sum_n \phi_j^{sn} R_j^n) \tag{goods market}$$

with the auxiliary variables

$$P_{ij}^s = \frac{1}{A_i^s} \tau_{ij}^s (W_i^s)^{\phi_i^s} \prod_n (P_i^n)^{\phi_i^{ns}} \tag{unit cost}$$

$$P_{i,dom}^s = \frac{1}{A_i^s} (W_i^s)^{\phi_i^s} \prod_n (P_i^n)^{\phi_i^{ns}} \tag{domestic}$$

$$\lambda_{ij}^s = \frac{(P_{ij}^s)^{1-\sigma}}{(P_j^s)^{1-\sigma}} \tag{trade share}$$

$$\phi_i^s R_i^s = W_i^s L_i^s \tag{labor share}$$

The solution to this model gives the trade block parameters $\{A_i^s, \tau_{ij}^s\}$.

Disutility of labor. We calibrate θ_i^s such that $\ell_i^s = 1$ in equilibrium. In our calibration of $\{A_i^s, \tau_{ij}^s\}$, we obtain C_i, W_i^s, P_i. Then the labor supply equation is

$$\theta_i^s (\ell_i^s)^{\varphi^{-1}} = v'(\ell_i^s) = \frac{W_i^s}{P_i} u'(\frac{C_i}{L_i})$$

so the calibrated value of θ_i^s that satisfy $\ell_i^s = 1$ is $\theta_i^s = \frac{W_i^s}{P_i} u'(\frac{C_i}{L_i})$.

Migration costs. We follow the literature and recover the migration frictions χ_i^{sn} from the observed migration flow μ_i^{sn} form the CPS. Assuming that bilateral migration frictions are symmetric in the initial period only ($\chi_{it}^{sn} = \chi_{it}^{ns}$) and normalizing own migration frictions to zero ($\chi_{it}^{ss} = 0$), the model's gravity equations for migration in Equation 1.32 imply

$$\log \left(\frac{\mu_i^{sn} \mu_i^{ns}}{\mu_i^{ss} \mu_i^{nn}} \right) = -\frac{2}{\nu} \chi_i^{sn} \tag{1.115}$$

so we can back out χ_i^{sn} for the US. For other countries, we assume the same migration costs.[70]

Nonpecuniary utility. Given the above calibrated parameters, we invert the realized labor supply $\{L_i^s\}$ to obtain the nonpecuniary utilities $\{\eta_i^s\}$ such that the model-implied L_i^s exactly match the data.

Calibration of the shocks

We calibrate the productivity, trade shocks, preference shocks, and the time-varying migration costs $\{A_{it}^s, \tau_{ijt}^s, \delta_{it}, \chi_{it}^{sn}\}_{t=2000}^{T_{data}}$. We use data from 2000-2012, so we calibrate 12 years of shocks, and assume these parameters are constant at the levels of $T = 2012$.

Since we have the initial values of productivity and trade costs, we match the *shocks* to these variables: $\hat{\tau}_{ijt}^s = \frac{\tau_{ijt}^s}{\tau_{ij0}^s}, \hat{A}_{it}^s = \frac{A_{it}^s}{A_{i0}^s}$. We calibrate $\{\hat{\tau}_{ijt}^s, \hat{A}_{it}^s, \delta_{it}, \chi_{it}^{sn}\}$ to match the following observed data:

- Changes in expenditure shares: $\hat{\lambda}_{ijt}^s = \frac{\lambda_{ijt}^s}{\lambda_{ij0}^s}$
- Changes in output prices in USD terms, observed from WIOD SEA.
- Net exports as a fraction of GDP: $NXGDP_{it} = \frac{NX_{it}}{VA_{it}}$
- Migration flows: μ_{it}^{sn}

First, we can back out $\hat{\tau}_{ijt}^s$ directly from the gravity equation without solving for the full model. Indeed, From Equation 1.114, we have

$$\hat{\tau}_{ijt}^s = \frac{\hat{e}_{jt}\hat{P}_{jt}^{s,dom}}{\hat{e}_{it}\hat{P}_{it}^{s,dom}}\left(\frac{\hat{\lambda}_{ijt}^s}{\hat{\lambda}_{jjt}^s}\right)^{\frac{1}{1-\sigma_S}} \tag{1.116}$$

But $\hat{e}_{jt}\hat{P}_{jt}^{s,dom}$ is precisely the changes in output prices in USD terms; hence the right-hand side is directly observable from data, and thus we obtain the left-hand side from the gravity equation.

Now we turn to productivity shocks $\hat{A}_{it}^s$, savings shocks δ_{it}, and the reallocation costs χ_{it}^{sn}. We use the full structure of the model by using the method of simulated moments: we solve the model given any sequence of shocks $\{\hat{A}_{it}^s, \delta_{it}\}$, and match the constructed

[70] In practice, we can break symmetry and calibrate χ_i^{sn} by time-differencing migration flow across time – under this method, migration costs as the residual term of the Artuç et al. (2010) regression used to compute ν, used also in Caliendo et al. (2019); Dix-Carneiro et al. (2023). However, the calibrated migration costs have implausibly large noise, mainly because the migration flow values themselves are highly noisy and we're taking log differences of them; notably, the migration costs are highly heterogeneous across time, while time-differencing requires assumption of constant migration cost. Thus we consider this a more 'stable' approach, as we match the initial labor distribution exactly.

output price data $\hat{P}_{it}^{s,dom} = \frac{P_{it}^{s,dom}}{P_{it}^{0,dom}}$, net exports as a share of GDP ($NXGDP_{it}$) and migration flows μ_{it}^{sn} respectively to exactly match the realized data.

This section presents the algorithms we use to estimate the model, calibrate the shocks, and perform counterfactual simulations. We assume convergence to the steady-state in T periods for a large enough T. In our baseline specification, we assume $T = 100$, so the economy converges to the new steady-state in 100 years. In our robustness tests, we compare the results with the results for $T = 200$ and verify that the results are quantitatively similar.

1.11.5 Variables and equations

As outlined in Section 1.4.3, we solve the economy in the *sequence-space*. Thus we consider a sequence of variables $\{X_t\}_{t=0}^{T}$, and each period's variables X_t comprise

$$X = (B_i, P_i, C_i, e_i, W_i^s, P_i^s, \ell_i^s, L_i^s, V_i^s).$$

Table 1.10 lists the definitions of the variables of interest, and auxiliary variables we use in our solution algorithm.

Panel A. Variables of interest	
Variable	Description
B_i	NFA in USD
P_i	Final goods price
C_i	Final goods consumption
e_i	Exchange rate
W_i^s	Sectoral wage
P_i^s	Sectoral goods price
ℓ_i^s	Per-worker labor supply
L_i^s	Distribution of labor
V_i^s	Worker value function

Panel B. Auxiliary variables	
Variable	Description
R_i^s	Revenue of i in s
E_i^s	Expenditure of i in s
$\mu_i^{ss'}$	Worker transition matrix
P_{ij}^s	Unit price of good
λ_{ij}^s	Trade shares
i_{it}	Nominal interest rate

Table 1.10: Variables to solve for

We denote the *auxiliary* variables as such because they can be directly computed from the variables in X:

$$R_i^s = \frac{W_i^s L_i^s}{\phi_i^s} \tag{Labor share}$$

$$E_i^s = \alpha_i^s P_i C_i + \sum_n \phi_i^{sn} R_i^n \tag{Expenditure}$$

$$\mu_i^{ss'} = \frac{\exp(\beta V_i^{s'} - \chi_i^{ss'})^{1/\nu}}{\sum_n \exp(\beta V_i^n - \chi_i^{sn})^{1/\nu}} \tag{Worker transition}$$

$$P_{ij}^s = \frac{1}{e_{ij}} \tau_{ij}^s \frac{1}{A_i^s} (W_i^s)^{\phi_i^s} \prod_{s'} (P_i^{s'})^{\phi_i^{s's}} \tag{Unit cost}$$

$$\lambda_{ij}^s = \frac{(P_{ij}^s)^{1-\sigma}}{\sum_l (P_{lj}^s)^{1-\sigma}} \tag{Trade share}$$

$$\log(1 + i_{it}) = r_{it} + \phi_\pi \log(P_{it+1}/P_{it}) + \epsilon_{it}^{MP} \tag{Taylor rule}$$

with China's interest rate i_{2t} identical to that of the US (peg and UIP).

We take the logs of the positive variables C, P, W, e, L, ℓ to ensure stability of our algorithm. Given the variables X_t, the equations of the quantitative model (in Section 1.8.3) can be written as:

$$F_1(X_t) = p_i - \sum_s \alpha_i^s p_i^s \tag{price index}$$

$$F_2(X_t) = \exp((1-\sigma) p_j^s - \sum_i \exp((1-\sigma) p_{ij}^s) \tag{sector price}$$

$$F_3(X_t) = R_i^s - \sum_j e_{ji} \lambda_{ij}^s E_{js} \tag{goods market}$$

$$F_4(X_t, X_{t+1}) = \exp(p_i + c_i) + \frac{1}{1+i} B_{i,t+1} - B_i - \sum_s \exp(w_i^s + \ell_i^s) \tag{HH budget}$$

$$F_5(X_t, X_{t+1}) = (-\frac{1}{\gamma} c_i - p_i) - (-\frac{1}{\gamma} c_{i,t+1} - p_{i,t+1}) - \log(\beta(1 + i_i)) - \log(\delta) \tag{Euler}$$

$$F_6(X_t, X_{t+1}) = e_{t+1} - e_t + \log(1 + i_i) - \log(1 + i_1) \tag{UIP}$$

$$F_7(X_t, X_{t+1}) = \exp(l_{i,t+1}^s) - \sum_n \mu_i^{ns} \exp(l_i^n) \tag{mig}$$

$$F_8(X_t, X_{t+1}) = v_{is} - \frac{\exp(w_{is} + n_{is})}{\exp(l_{is} + p_{is})} u'(c_i) - \nu \log[\sum \exp(\frac{1}{\nu}(\beta v_{is}^{t+1} - \chi_{iss'}))] \tag{Value}$$

$$\begin{aligned} F_9(\{w_{i,t-1}^s\}, X_t, X_{t+1}) = &(w_{is} - w_{is}^{t-1}) - \kappa_w [v'(\ell_{is}) - u'(c_{is}) \exp(w_{is} - p_i)] \\ &- \beta_w (w_s^{t+1} - w_s) \exp(l_s^{t+1} - l_s) \end{aligned} \tag{NKPC}$$

This set of equations is the main set of equations we use to solve for the equilibrium. Note that the period t equilibrium conditions only depend on t, $t+1$ variables and the previous period wage.

1.11.6 Solving for the steady-state

We first solve for the long-run steady-state: an equilibrium with persistent net foreign asset positions (in USD) and relative wages. Per our assumptions, for China, which pegs to the US, we may have $B_i \neq 0$, and for countries other than China and the US, we have $B_i = 0$.[71] Given any values of the fundamentals and parameters in Table 1.1, and the terminal real NFA $\{B_i\}_i$, the steady-state comprises $2I + 5IS$ variables $X_T = (P_i, C_i), (W_i^s, P_i^s, L_i^s, \ell_i^s, V_i^s)$ that solve the following system of equations, written using the form in Section 1.11.5:

$$G_{ss}(X_T) = \begin{pmatrix} F_1(X_T) \\ F_2(X_T) \\ F_3(X_T) \\ F_4(X_T, X_T) \\ F_7(X_T, X_T) \\ F_8(X_T, X_T) \\ F_9(X_T, X_T, X_T) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{1.117}$$

taking advantage of the fact that in steady-state, $X_{T-1} = X_T = X_{T+1}$. The algorithm for solving for this steady-state is as follows: it robustly converges for any given parameters.

Step 1. Make an initial guess for the solution $X_T^{(1)}$.

Step 2. Update the initial guess of the solution $X_T^{(1)} \to X_T^{(2)}$ using the price index equation F_2, the value function and labor transition equations F_7, F_8: each of them are contraction mappings, so given the initial guess, we can iterate the function a finite number of times.

Step 3. Use **gradient descent** to update the guess $X_T^{(2)} \to X_T^{(3)}$ (we use 20 iterations with learning rate 10^{-12}).

Step 4. Use **Newton's method** on $G_{ss}(X_T)$ to update the guess $X_T^{(3)} \to X_T^{(4)}$, until the error tolerance $\|G_{ss}(X_T)\|$ is below a certain threshold (we use 10^{-20}).

[71] More generally, if we consider a model with more countries, for currency unions, we may have $B_i \neq 0$ for members of a union in steady-state, but $\sum_{i\in\mathcal{I}} B_i = 0$ for any currency union $\mathcal{I}$.

The resulting set of variables $X_T^{(4)}$ is the set that solves the system G_{ss} given B_T. See Section 1.11.9 for the bolded nonlinear solvers.

1.11.7 Estimation algorithm for pegged economy

Given any set of dynamic parameters and fundamentals in Table 1.1 and the initial conditions $\{w^s_{i,-1}, L^s_{i0}, B_i\}$, China's pegged exchange rate $e_2 = \bar{e}$, and any policy $\{T^s_{ijt}\}, \{\epsilon^{MP}_{it}\}$, the economy is defined in the sequence-space as the set of variables

$$X = \{X_t\}_{t=0}^{T} = \{(B_{it}, P_{it}, C_{it}, e_{it}, W^s_{it}, P^s_{it}, \ell^s_{it}, L^s_{it}, V^s_{it})\}_{t=0}^{T}$$

that satisfy the equilibrium conditions. The period-t equilibrium conditions are given by

$$G_t(X_t, \{w^s_{it-1}\}, X_{t+1}) = \begin{pmatrix} F_1(X_t) \\ F_2(X_t) \\ F_3(X_t) \\ F_4(X_t, X_{t+1}) \\ F_5(X_t, X_{t+1}) \\ F_6(X_t, X_{t+1}) \\ F_7(X_t, X_{t+1}) \\ F_8(X_t, X_{t+1}) \\ F_9(\{w^s_{it-1}\}, X_t, X_{t+1}) \end{pmatrix} \tag{1.118}$$

The set of equations for the *path* $\{X_t\}_{t=0}^{T-1}$, given a terminal steady-state X_T, is

$$\mathcal{G}(\{X_t\}_{t=0}^{T-1}, X_T) = \begin{pmatrix} G_0(X_0, \{w^s_{i,-1}\}, X_1) \\ G_1(X_1, \{w^s_{i,0}\}, X_2) \\ \cdots \\ G_{T-2}(X_{T-2}, \{w^s_{i,T-3}\}, X_{T-1}) \\ G_{ss-1}(X_{T-1}, \{w^s_{i,T-2}\}, X_T) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 0 \end{pmatrix} \tag{1.119}$$

where G_{ss-1} is the period $T-1$ condition that links the sequence-space to the terminal steady-state, and is given by:

$$G_{ss-1}(X_{T-1}, \{w^s_{i,T-2}\}, X_T) = \begin{pmatrix} F_1(X_{T-1}) \\ F_2(X_{T-1}) \\ F_3(X_{T-1}) \\ \hat{F}_4(X_{T-1}) \\ C_{T-1} - C_T \\ F_7(X_{T-1}, X_T) \\ \hat{F}_9(\{w^s_{i,T-2}\}, X_{T-1}, X_T) \end{pmatrix} \tag{1.120}$$

The following are the differences between the last condition G_{ss-1} and a generic G_t:

- We replace the Euler equation with $C_{T-1} = C_T$, signifying that we have reached a terminal state.
- We replace the Household budget clearing $F_4(X_{T-1}, X_T)$ with

  $$\hat{F}_4 = \begin{pmatrix} e_{iT-1} = \bar{e} & \text{if } i = 2 \\ \exp(p_i + c_i) - B_i - \sum_s \exp(w^s_i + \ell^s_i) & \text{if } i > 2 \end{pmatrix}$$

  We encode the fact that floating countries have bond zero, and that pegging China has its exchange rate 1. Note that we have not used the household budget constraint of US and China (by Walras, this is one condition). We get back to this.
- We remove the UIP condition F_6 and the labor migration equations F_8, which are forward-looking equations.
- In the NKPC F_9, we impose $w^T_s = w^{T-1}_s$, again signifying that we are in staedy-state by period T.

Technical note: all of this is necessary because our model is nonstationary and the exchange rate features a unit root.

Given our construction of $\mathcal{G}$, we implement our solution algorithm in two steps: inner loop and outer loop.

Inner loop. Solve for the *path* $X_{path} = \{X_t\}_{t=0}^{T-1}$ that solves $\mathcal{G}(X_{path}, X_T)$ given a terminal state X_T. In an abuse of notation, we remove the dependency of $\mathcal{G}$ X_T.

Step 1. Make an initial guess for $X^{(1)}_{path}$. Here it is important that the sequence $\{X_t\}$ *converges* to the terminal state X_T for the algorithm to be stable.

Step 2. Use **gradient descent** on $\mathcal{G}(X_{path})$ to improve the initial guess $X^{(1)}_{path} \to X^{(2)}_{path}$.

Step 3. Use **quasi-Newton's method** on $\mathcal{G}(X_{path})$ to update the guess $X^{(2)}_{path} \to X^{(3)}_{path}$. In practice we repeat until $\|\mathcal{G}(X_{path})\| < 10^{-8}$.

Step 4. Use **Levenberg-Marquardt algorithm** on $\mathcal{G}(X_{path})$ to fine-tune the guess $X^{(3)}_{path} \to X^{(4)}_{path}$. In practice we repeat until $\|\mathcal{G}(X_{path})\| < 10^{-10}$.

Step 3 requires quick construction and inversion of the Jacobian of $\mathcal{G}(X_{path})$, which is a large matrix (in our main specification, with $I = S = 6$ and $T = 100$, the Jacobian has dimension 20000×20000). We have knowledge of the structure of $\mathcal{G}$: each time t equation G_t depends only on X_t, X_{t+1} and $\{w^s_{i,t-1}\}$. Thus we know the sparsity structure of the Jacobian (i.e. where all the nonzero elements are). so we use automatic differentiation (autodiff) to speed up this process, and construct the Jacobian $J_{\mathcal{G}}$ as a sparse matrix. Then we use Intel's PARDISO package[72] to quickly invert $J_{\mathcal{G}}$.

Outer loop. Solve for the X_T that is consistent with the path X_{path}.

Step 1. Start from an initial guess of $B^{(1)}_T$. We only need to keep track of China's (or pegged countries') bond position, as the floaters will have $B_T = 0$.

Step 2. Given $B^{(i)}_T$, solve for $X^{(i)}_T$ using the steady-state solution (Section 1.11.6).

Step 3. Given $X^{(i)}_T$, solve for $X^{(i)}_{path}$ (inner loop).

Step 4. Using $X^{(i)}_{T-1}$ and China's Household budget constraint, find $B^{(i)}_{T,implied}$.

Step 5. The map $B^{(i)}_T \to B^{(i)}_{T,implied}$ is a monotonically decreasing map. Find the unique fixed point B_T by iterative search, using **secant-bisection**.

Once the outer loop converges, we have a solution in the sequence-space $\{X_t\}$. When $I = S = 6$ and $T = 100$, with our current code, the solution is usually found between 1-3 minutes on a Dell PowerEdge R940xa server (208 cores, 3TB RAM) with a NVIDIA Tesla V100 (32GB) GPU accelerator.

[72]See https://www.intel.com/content/www/us/en/resources-documentation/developer.html

1.11.8 Estimation algorithm for floating economy

For the floating economy, we first replace the auxiliary condition on China's monetary policy with an independent Taylor rule:

$$\log(1 + i_{2t}) = r_{2t} + \phi_\pi \log(P_{it+1}/P_{it}) + \epsilon_{2t}^{MP}$$

And we replace the 'linking' condition to the terminal steady-state as

$$G_{ss-1}^{float}(X_{T-1}, \{w_{i,T-2}^s\}, X_T) = \begin{pmatrix} F_1(X_{T-1}) \\ F_2(X_{T-1}) \\ F_3(X_{T-1}) \\ F_4(X_{T-1}) \\ C_{T-1} - C_T \\ F_7(X_{T-1}, X_T) \\ \hat{F}_9(\{w_{i,T-2}^s\}, X_{T-1}, X_T) \end{pmatrix} \tag{1.121}$$

where the difference between this and the pegged case is that we impose $B_T = 0$ for all countries. With this in mind, the solution algorithm is as follows:

Step 1. Solve for the long-run steady-state X_T consistent with $B_T = 0$.

Step 2. Make an initial guess for $X_{path}^{(1)}$. Here it is important that the sequence $\{X_t\}$ *converges* to the terminal state X_T for the algorithm to be stable.

Step 3. Use **gradient descent** on $\mathcal{G}^{float}(X_{path})$ to improve the initial guess $X_{path}^{(1)} \rightarrow X_{path}^{(2)}$.

Step 4. Use **quasi-Newton's method** on $\mathcal{G}^{float}(X_{path})$ to update the guess $X_{path}^{(2)} \rightarrow X_{path}^{(3)}$. In practice we repeat until $\|\mathcal{G}^{float}(X_{path})\| < 10^{-8}$.

Step 5. Use **Levenberg-Marquardt method** on $\mathcal{G}(X_{path})$ to fine-tune the guess $X_{path}^{(3)} \rightarrow X_{path}^{(4)}$. In practice we repeat until $\|\mathcal{G}^{float}(X_{path})\| < 10^{-10}$.

The solution in Step 5 corresponds to the solution of the floating economy, and there is no need for an outer loop. This is because under a floating economy, the model is stationary, and we *know* which steady-state we converge to.

1.11.9 Nonlinear solver algorithms

This subsection describes the generic nonlinear solvers we use in our solution algorithms.

Gradient descent. Given a function $f : \mathbb{R}^n \to \mathbb{R}^n$, we approximate the root of f by applying gradient descent on $g = \|f\|_2^2 = \sum_i f_i^2$.

Input: function $g = \|f\|_2^2$; gradient ∇g of g; learning rate λ; number of iterations m; tolerance tol.

Algorithm:

Step 1. Start from an initial guess $x^{(0)}$.

Step 2. Evaluate ∇g, the gradient of g, at $x^{(i)}$.

Step 3. Update the guess $x^{(i+1)} = x^{(i)} - \lambda \cdot \nabla g(x^{(i)})$ for sufficiently small λ.

Step 4. Repeat 2-3 for m iterations, terminate if $g(x^{(i+1)}) < tol$.

Note. In practice, this is too slow to converge to the root. We use this to *update* the initial guess, to feed in to the next solvers.

Newton's method. Given a function $f : \mathbb{R}^n \to \mathbb{R}^n$, we approximate the root of f by Newton's method on f.

Input: f, the function; J, the Jacobian J_f of f; $g = \|f\|_2^2$; number of iterations m; tolerance tol.

Algorithm:

Step 1. Start from an initial guess $x^{(0)}$.

Step 2. Use autodiff to compute J_f at $x^{(i)}$

Step 3. Use PARDISO to evaluate $J_f(x^{(i)})^{-1} f(x^{(i)})$.

Step 4. Update $x^{(i+1)} = x^{(i)} - J_f(x^{(i)})^{-1} f(x^{(i)})$; u

Step 5. Repeat 2-4 for m iterations, terminate if $g(x^{(i+1)}) < tol$

Note. Newton's algorithm requires a good initial guess. In static problems (solving for the terminal state), we use parts of the equation (which are contraction mappings) to construct the initial guess close to the solution, In dynamic problems, our initial guess is close to the terminal steady-state: this 'anchors' the problem and allows for convergence. But for efficiency reasons, we use the quasi-Newton method below for the high-dimensional dynamic problem.

Quasi-Newton's method. Given a function $f : \mathbb{R}^n \to \mathbb{R}^n$, we approximate the root of f by quasi-Newton's method on f.

Input: f, the function; J, the Jacobian J_f of f; $g = \|f\|_2^2$; grid s; number of iterations m; tolerance tol.

Algorithm:

Step 1. Start from an initial guess $x^{(0)}$.

Step 2. Use autodiff to compute J_f at $x^{(i)}$. Here it is essential that our autodiff procedure is sparse-aware, that is, aware of the nonzero elements of J_f.

Step 3. Use PARDISO to evaluate $dx = J_f(x^{(i)})^{-1} f(x^{(i)})$.

Step 4. Construct *candidate* updates $x(s) = x^{(i)} - s \cdot dx$ for a grid s. In practice we use a linear grid from 0.1 to 5.

Step 5. Compute $g(x(s))$ for each s and update $x^{(i+1)}$ to be $x(s)$ with the minimal $g(x)$.

Step 6. Repeat 2-4 for m iterations, terminate if $g(x^{(i+1)}) < tol$.

Note. The advantage of this approach is as follows: the bottleneck in Newton's method is computing and inverting the Jacobian $J_f(x^{(i)})$. By searching over a full grid after each computation of $J_f(x)^{-1} f(x)$, we can effectively search for more candidates with minimal time cost. In reality, Newton's method can overshoot in the first few steps so it's better to have small s, whereas closer to the root, the optimal s seems to be $2-4$. This may be due to the fact that the Jacobian is singular near the solution – smallest norm eigenvalue reaches zero – and it is known that a coefficient s with the multiplicity of the root gets us to quadratic convergence.

Levenberg-Marquardt Method. Given a function $f : \mathbb{R}^n \to \mathbb{R}^n$, we approximate the root of f by Levenberg-Marquardt algorithm.

Input: f, the function; J, the Jacobian J_f of f; $g = \|f\|_2^2$; dampening parameter λ; number of iterations m; tolerance tol.

Algorithm:

Step 1. Start from an initial guess $x^{(0)}$, and

Step 2. Use autodiff to compute J_f at $x^{(i)}$. Here it is essential that our autodiff procedure is sparse-aware, that is, aware of the nonzero elements of J_f.

Step 3. Compute $A = J^T J + \lambda \cdot (diag(J^T J))$ where $J^T J$ is the transpose of J multiplied by J, and $diag(M)$ is the matrix of diagonal entries of M.

Step 4. Use PARDISO to construct candidate update $x_n = x^{(i)} - A^{-1} J^T f(x^{(i)})$.

5-1. If $g(x_n) > g(x^{(i)})$, multiply λ by λ_{up}, and return to Step 3.

5-2. If $g(x_n) < g(x^{(i)})$, accept $x^{(i+1)} = x_n$, and divide λ by λ_{down}.

Step 6. Terminate if $g(x^{(i+1)}) < tol$, or $i = M$. Otherwise return to Step 2.

Heuristically, this allows us to get *closer* to the solution faster than quasi-Newton. In practice we use $\lambda_{up} = 2$ and $\lambda_{down} = 5$ (this is called *delayed gratification*.)

Secant-Bisection. Given a decreasing function $f : \mathbb{R} \to \mathbb{R}$, find x^* such that $f(x^*) = x^*$. Useful when the evaluation of f involves solving a high-dimensional nonlinear system in the background (see above).

Algorithm:

Step 1. Start from an initial guess $x^{(0)}$.

Step 2. Evaluate f at $x^{(0)}$. If $f(x^{(0)}) > x^{(0)}$, then we know $x^* > x^{(0)}$. Set $x^{(1)} = x^{(0)} + s(f(x^{(0)}))$ for small s, and iterate until we find $x^{(i)}$ such that $f(x^{(i)}) < x^{(i)}$, therefore $x^{(i)} > x^*$.

Step 3. Given $x_{lb} \leq x^{(i)} \leq x_{ub}$ with knowledge that $x^* \in [x_{lb}, x_{ub}]$, evaluate f at $x^{(i)}$. If $f(x^{(i)}) > x^{(i)}$, then replace x_{lb} with $x^(i)$; otherwise replace x_{ub} with $x^{(i)}$.

Step 4. Bisection method would update $x_b^{(i+1)} = \frac{x_{ub}+x_{lb}}{2}$. Secant method would update $x_s^{(i+1)}$ as the intersection of the line connecting $(x_{ub}, f(x_{ub}))$ and $(x_{lb}, f(x_{lb}))$ and the x-axis. We update $x^{(i+1)} = sx_s^{(i+1)} + (1-s)x_b^{(i+1)}$ for some step size $s \in (0, 1)$. (In practice we use $s = 0.9$ by heuristics.)

Step 5. Repeat until convergence.

This method is useful because bisection is too slow (if evaluation takes 1 minute, and we want error margin 10^{-5}, we need 16 evaluations; whereas secant method is much faster for 'regular' functions, it may get stuck in corner. The hybrid method converges quickly – within 3-5 attempts maximum – to the fixed point within desired tolerance.

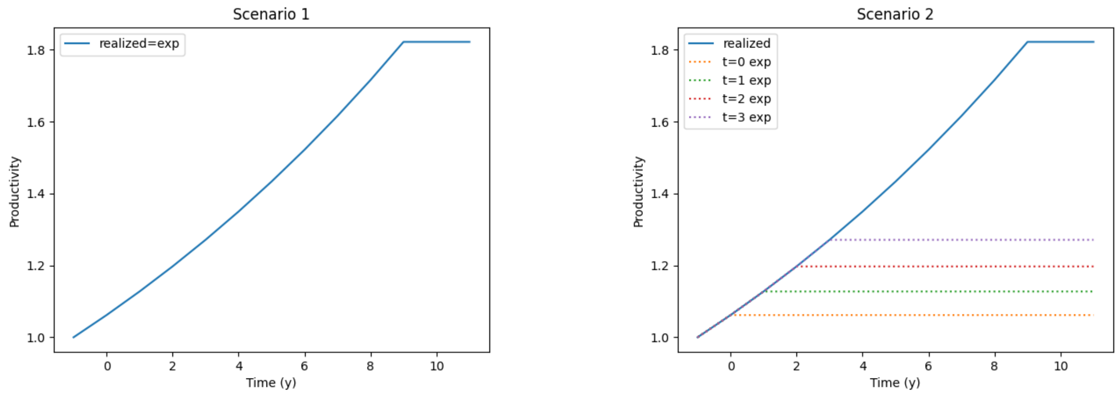
1.12 Foresight of the China Shock

We discuss anticipation of the shock by the households of the model, as agents' foresight of the China shock is important in determining the economy's response to the shock. The literature on structurally estimating the effect of the China shock (Caliendo et al., 2019; Rodríguez-Clare et al., 2022; Dix-Carneiro et al., 2023) all implicitly assume that every agent in the economy at $t = T_0$ have perfect foresight of the full sequence of productivities for $t \in [T_0, T_{data}]$ including the China shock and makes forward looking choices, including sectoral reallocation and consumption-savings, anticipating the development of the full path of the China shock at the start of the model (usually 2000). If the China shock was truly a shock, this is equivalent to assuming that nobody knew of the productivity growth in 1999, but everyone woke up at 2000 and learned the full sequence of the China shock, including that it will plateau at around 2010 (Autor et al., 2021).[73] The problem with this approach is that the model implies a lot of frontloading in transition – wages will adjust incorporating not only the immediate shock but all future shocks, manufacturing workers in 2000 would have a higher desire to leave, and Chinese households will borrow large amounts if they foresaw the full extent of Chinese growth – and the calibrated parameters have to take extreme values to reconcile this with the observed migration and net exports.

We consider an alternative assumption – that agents face a *series* of unanticipated shocks for each t between T_0 and T_{data}. Specifically, in the baseline equilibrium with the realized China shock, at every year t between T_0 and T_{data}, agents learn the new fundamentals at time t $\Theta_t = \{\tilde{\tau}^s_{ijt}, \tilde{\delta}_{it}, \tilde{A}^s_i\}$, and agents (incorrectly) assume that the fundamentals are constant for $t' > t$. In this sense, every year between T_0 and T_{data} is a *China shock*.

To test the validity of this assumption, we estimate the response of our economy to a gradual productivity shock in the low-tech manufacturing sector of China over T_c years, but using two polar opposite assumptions about agents' foresight. In the first exercise, we assume that agents *do not foresee* the shocks in full: for T_c years, the agents face an unanticipated productivity shock every year, and makes decisions assuming that there are no more shocks onwards. In the second exercise, we assume instead, analogously to the literature, that agents in the model have perfect foresight of the full sequence of productivity shocks in $t = T_0 = 2000$. All remaining fundamentals are fixed at calibrated values in $t = T_0$, so the only deviation is the productivity shocks, and to highlight the role productivity shocks play in our model, we assume, for this thought exercise only, that the

[73] One of the reasons why the literature assumes this strong form of perfect foresight is computational tractability. Our modeling framework and solution algorithm (Section 1.4.3) allows us to bypass these challenges.



(a) Perfect foresight

(b) Compare peg and float

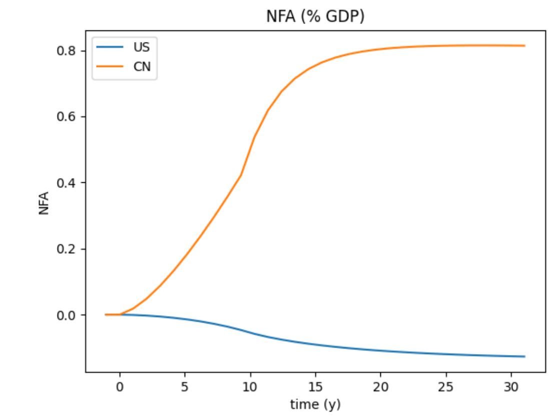
Figure 1.27: Productivity growth in low-tech manufacturing. 6% per year for 10 years.

economy is in steady-state under the initial parameters at $T_0 = 2000$, so any transition dynamics can be fully attributed to the productivity shock.

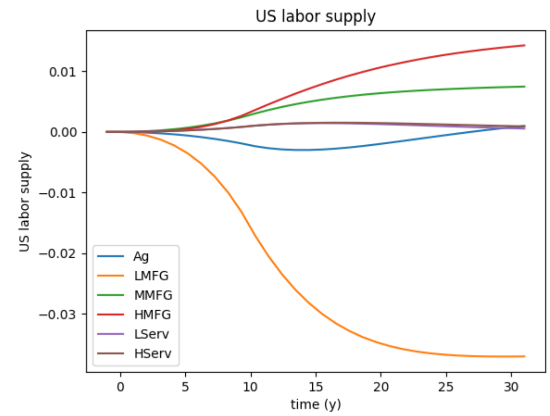
Exercise 1. Gradual shock, no foresight. First we study the no-foresight assumption, as represented by the right panel of Figure 1.27. In this case, the economy started at the 2000 levels, then Chinese productivity in low-tech manufacturing grows by 6% for 10 years, but every year, agents are surprised by the new productivity level; in this sense, every year is a China shock for 10 years.

Figure 1.28 plots the net foreign asset position, wage, labor reallocation, and unemployment response of the US in response to this shock. From the top left panel, we see that the net foreign asset B_{it} for the US is negative, while the net foreign asset for China is positive; so China saves while US borrows, in line with the observed data. In this sense, our channel – exchnage rate peg interacting with a productivity shock – can *endogenously generate the savings glut*, as seen in Proposition 1.1 in Section 1.3. The top right plot, which shows labor reallocation, is analogous to the perfect foresight case, where workers slowly move out of the affected sector, and move into and out of other sectors depending on the input-output linkage.

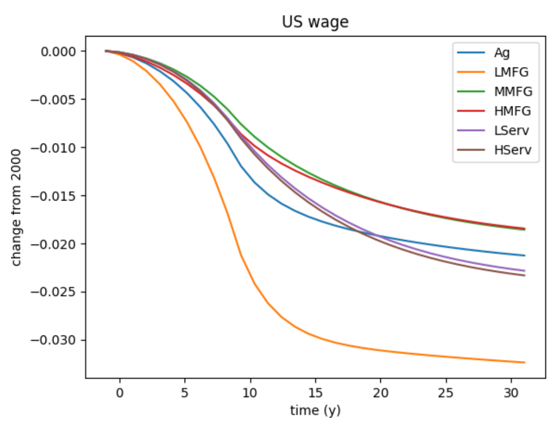
The bottom two figures show the labor market's response in terms of wages and unemployment. Both plots match the observations in Section 1.2, theoretical prediction in Section 1.3, and matches evidence found in literature (Autor et al., 2013, 2021). Wages in the most affected sector fall, but wages in other sectors fall too because of the shock propagating to other sectors through input-output linkage. Lastly, the China shock induces unemployment in the US that grows over time as Chinese productivity



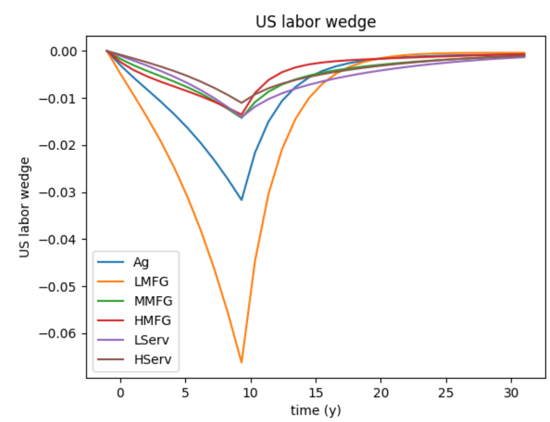
(a) NFA (% GDP)



(b) US labor reallocation



(c) US wage



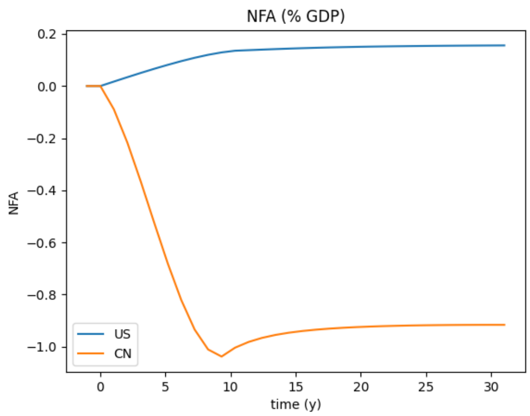
(d) US labor wedge

Figure 1.28: Response of the US economy to the gradual shock with no foresight.

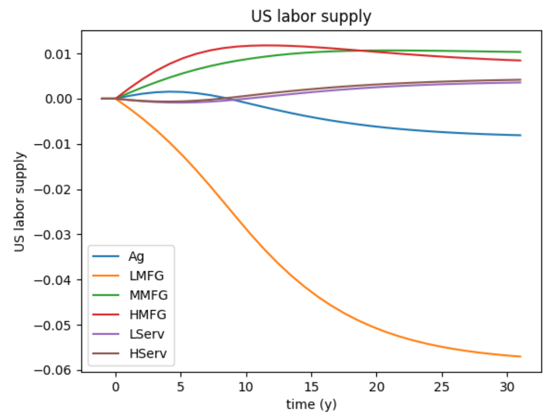
grows over time, and reverts to zero as Chinese growth plateaus and the economy slowly adjusts to the new steady-state. Notably, while the directly exposed sector is most harmed, unemployment increases for workers in other sectors as well, because of input-output linkages.

Exercise 2. Gradual shock, perfect foresight. Next we consider the perfect foresight model, as represented in the left panel of Figure 1.27. In this case, the economy started at the 2000 levels, then Chinese productivity in low-tech manufacturing grows by 6% for 10 years, and all agents in the model expect the full path of Chinese productivity growth.

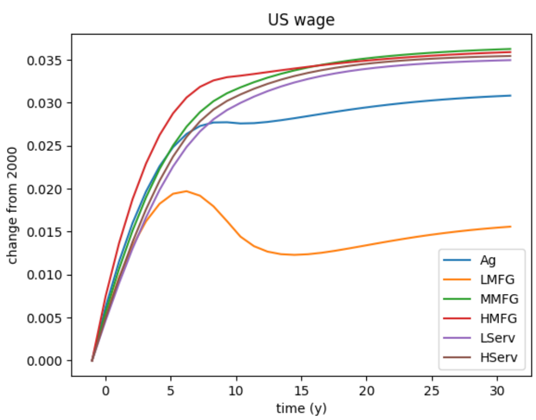
Figure 1.29 plots the net foreign asset position, wage, labor reallocation, and unemployment response of the US in response to this shock. As the top left panel shows, if everyone in the model has perfect foresight of the China shock, Chinese agents



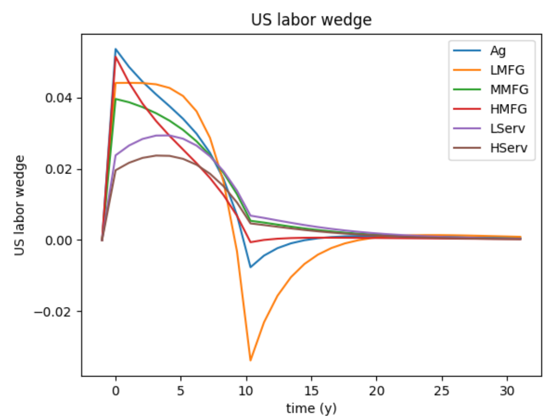
(a) NFA (% GDP)



(b) US labor reallocation



(c) US wage



(d) US labor wedge

Figure 1.29: Response of the US economy to the gradual shock with perfect foresight.

have incentive to borrow because they foresee that their productivity in 10 years will be double their productivity today; likewise, US anticipates that Chinese goods will be much cheaper in the future, so it saves. The top right panel shows the labor reallocation response of the China shock, which is in line with what we would expect; since low-tech manufacturing in China grows, workers move out into other sectors. At the same time, some sectors grow more than others because of input-output linkages.

The bottom two panels of Figure 1.29 show the wage and unemployment responses of the China shock. From the left panel, we see that wages *increase* in response to a Chinese productivity growth across all sectors. This is because of the combination of the fact that US borrows to consume more today, and home bias in the model. The most interesting response is the labor wedge, as observed in Figure 1.29d. Since the economy faces a sudden surge in US goods demand (due to US saving and home bias), and both wages and labor supply are slow to adjust, there is *excess demand* for domestic goods – the US economy is overheated because of the expectation of future growth in China. As we see, neither the consumption-savings, nor the unemployment responses match those of the China shock.

We note that reality is somewhere in between these polar opposite assumptions (no foresight vs perfect foresight). Because the consumption-savings and labor market responses of the no foresight assumption are more consistent with the empirical evidence (Section 1.2 and Autor et al. (2021)), in our main text, we calibrate and solve for the baseline and counterfactual economies under the assumption that households did not foresee the China shock. In Section 1.13 of this Supplement, we answer the same counterfactuals as in Section 1.5, but under the perfect foresight assumption.

1.13 Robustness: Quantitatives

1.13.1 Alternative monetary policy

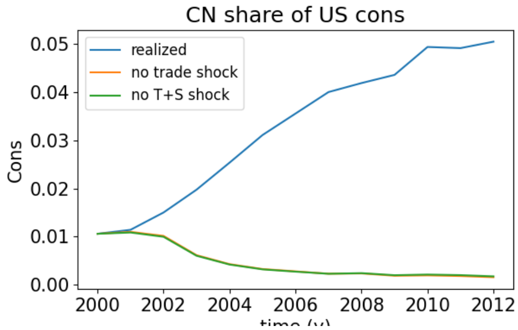
In our main text, we assumed that the floating countries (US and the world except China) used a Taylor rule targeting CPI inflation. This is not an 'optimal' monetary policy rule, as we do not have divine coincidence: targeting price inflation does not adequately target the nominal friction – labor wedge – in the economy. As we highlighted in the discussion in Section 1.3 and in Section 1.10 of this supplement, in this case, the recession may spill over to the nontraded sectors, resulting in potentially high aggregate unemployment, not just manufacturing.

In this subsection, we redo the exercises in Section 1.5.2 (Reevaluating the China shock), except we replace the monetary policy rules, both for the realized economy and the counterfactual economy without the China shock or the peg, with a Taylor rule that targets the employment-weighted labor wedge:

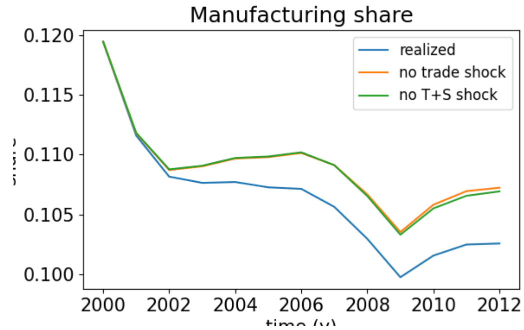
$$\log(1+i_{it}) = r_{it} + \phi_{og}(\sum_s \frac{L_{it}^s}{\bar{L}_i}(v'(\ell_{it}^s) - \frac{W_{it}^s}{P_{it}u'(C_{it})})). \tag{1.122}$$

We note that a change in monetary policy changes the price indices and the relative savings of each country. As such, we need to recalibrate our shocks to the economy $\{A_{it}^s, \delta_{it}^s\}$ to ensure that the resulting equilibrium under our realized shocks correspond to the realized economy in the targeted moments. Thus we recalibrate the China shock for the purpose of this exercise. We also note that the choice of employment-weighted labor wedge is a 'rule of thumb' choice. We study the optimal weights, and optimal monetary policy, under such environments, in a companion work in progress.

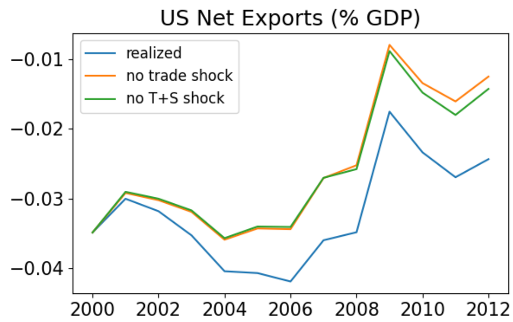
The results under the alternative monetary policy are shown in Figure 1.30 and Table 1.11. The decline in manufacturing as a result of the China shock is still larger than estimated in the literature, with the estimate being 700 thousand jobs. The deficit explained by the China shock is smaller (0.82% of GDP each year, compared to 2.25% in the baseline model) but still significant. The aggregate unemployment is close to zero, suggesting that the aggregate level of unemployment is primarily caused by the CPI-inflation targeting Taylor rule. The US economy balances between unemployment in manufacturing and overheating in services.



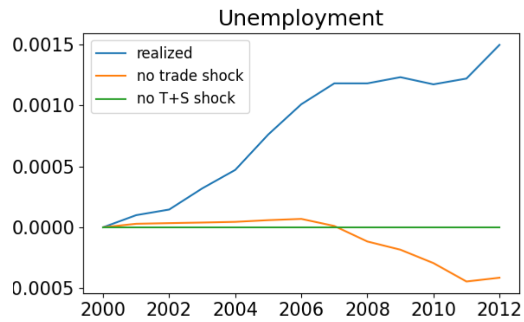
(a) CN import penetration



(b) Manufacturing share



(c) Net exports (% GDP)



(d) Unemployment rate

Figure 1.30: Response of the economy to the China shock, alternative monetary policy.

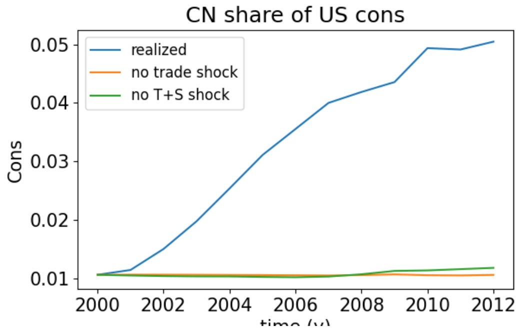
Effect of China shock					
	Alt MP	main text	CDP19	RUV22	DPRT23
MFG jobs lost	700k	991k	550k	498k	530k
Deficit (% GDP)	0.82	2.25	N/A	N/A	0.8
Unemployment (%)	0.095	3.04	N/A	1.4	0
Welfare gains	0.215%	0.183%	0.2%	0.229%	0.183%
Wage rigidity	O	O	X	O	X
Search friction	X	X	X	X	O
Cons-savings	O	O	X	X	O
ER peg	O	O	X	X	X

Table 1.11: Effects of the China shock, alternative definition

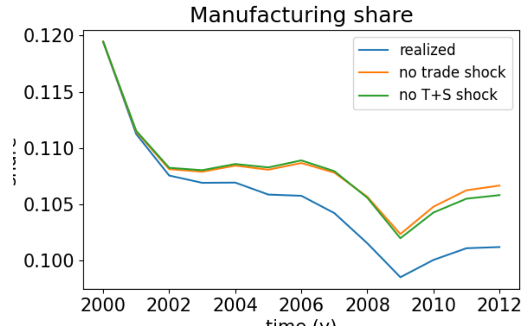
1.13.2 Alternative China Shocks

In our main text, our baseline assumption on the counterfactual 'no China shock' economy was an economy where productivity A^s_{it} and trade costs τ^s_{ijt} for China are fixed at the 2000 level. In this subsection, we redo the exercises in Section 1.5.2 (Reevaluating the China shock) with an alternative definition by defining the 'no China shock' economy as an economy where productivity A^s_{it} and export trade costs τ^s_{ijt} are calibrated to values such that λ^s_{ijt} for China is fixed at the 2000 values. This would be closer to specifications that calibrate the China shock to match regression coefficients on observed growth in export shares, used in Caliendo et al. (2019); Rodríguez-Clare et al. (2022).

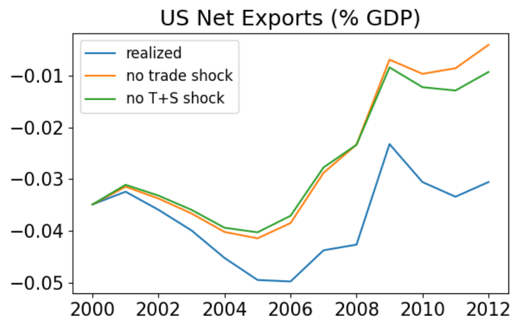
The results under this alternative China shock are similar to our results in the main text. As we see from the first subfigure of Figure 1.31, the counterfactual economy without the China shock has Chinese share of US consumption flat. Even in this case, the manufacturing jobs lost, trade deficit, and unemployment numbers are smaller than our baseline results but still significantly larger than the literature's estimates, highlighting the relevance of the exchange rate peg. Moreover, since China's growth is smaller, under this specification, we get a *smaller* welfare gain from the China shock.



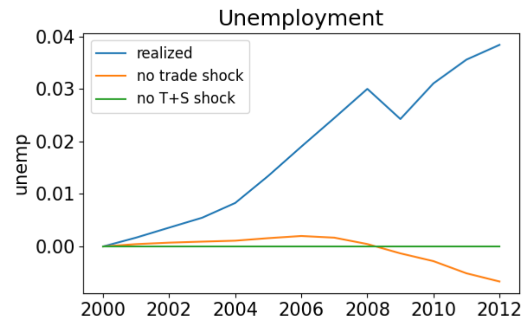
(a) CN import penetration



(b) Manufacturing share



(c) Net exports (% GDP)



(d) Unemployment rate

Figure 1.31: Response of the economy to the China shock, alternative measure.

Effect of China shock					
	Alt shock	baseline	CDP19	RUV22	DPRT23
MFG jobs lost	822k	991k	550k	498k	530k
Deficit (% GDP)	1.50	2.25	N/A	N/A	0.8
Unemployment (%)	2.02	3.04	N/A	1.4	0
Welfare gains	0.145%	0.183%	0.2%	0.229%	0.183%*
Wage rigidity	O	O	X	O	X
Search friction	X	X	X	X	O
Cons-savings	O	O	X	X	O
ER peg	O	O	X	X	X

Table 1.12: Effects of the China shock, alternative definition

Chapter 2

Model (non-)disclosure in supervisory stress tests

2.1 Introduction

Every year, the Federal Reserve Bank performs a stress test on major banks and financial institutions in the US to assess the financial stability of the system under the Dodd-Frank Act. These stress tests are loss projections computed by the Fed given theoretical scenarios and the banks' balance sheet. The Federal Reserve Bank reveals parts, but not all, of the mapping ('model') from scenarios and balance sheets to loss projections, and this creates a tension in *model disclosure*, which this paper seeks to investigate.

Risk managers claim that not knowing the model parameters the Fed is using adds further risk to their decisions. On top of the underlying risk any financial action has, they must project how this will affect the stress test results. Under that view, banks do not know the regulatory cost of risk, and this regulatory uncertainty induces banks to make inefficient decisions, such as excessively reducing lending (Gissler et al. (2016)).

On the other hand, the regulators are concerned of potential dangers of revealing the stress test models. According to the Federal Reserve's memo,[1] there are three reasons to not keep the test model parameters and assumptions transparent:

(a) *Gaming the system*: "Firms could [..] make modifications to their businesses that change the results of the stress test without changing the risks they face."

(b) *Correlation*: "[Full disclosure] could increase correlation in asset holdings [..] making the financial system more vulnerable."

[1] https://www.govinfo.gov/content/pkg/FR-2017-12-15/pdf/2017-26856.pdf. Retrieved May 13, 2020.

(c) *Model Monoculture*: “Full disclosure could incentivize banks to simply use models similar to the Federal Reserve’s, rather than build their own capacity to identify, measure, and manage risk."

The Federal Reserve’s concern on *gaming the system* and *model monoculture* motivate our stylized model. In our framework, both the Fed and banks have imperfect information (‘models’) about the underlying state regarding a risky asset, and banks take actions (‘investments’) which affect the Fed’s payoffs. The Fed cares more about systemic risk or the possibility of a financial crisis than individual banks; we highlight this by assuming that the bank would always like to invest in a risky asset, while the regulator’s utility (‘social welfare’) is such that the regulator wants the bank to invest only in good states. The Fed’s private information corresponds to the Fed’s model and information about the economy, which the Fed uses to test the bank’s investments under stress, and debates whether to release more or less information about.

In this setup, the trade-off is as follows. The Federal Reserve can give banks more information about the models they use for the test, which would help the banks make more efficient decisions. At the same time, giving more information about these models could incentivize banks to simply use those models to assess their actions, instead of using their own models. Under this trade-off, what is the optimal disclosure policy for the Fed?

In our stylized model, there is a regulator and a bank, each with private, imperfect information about the riskiness of an asset. The regulator can ‘punish’ banks for making investments in ‘bad’ assets determined by its own model, but can choose to disclose additional information about the punishment to affect the beliefs and the action of the bank. In a theoretical sense, our setup is a mixture of both optimal mechanism design and information design (à la Bergemann and Morris (2016a)): the principal can design punishments, as well as disclose information.

The model we propose concisely captures the trade-off in disclosing more information. Disclosing more information reduces the variance in the bank’s decision problem, allowing a more informed decision; at the same time, disclosing more information allows the bank to optimize their action with respect to the regulator’s punishment without using their own information. If the regulator fully discloses its model, banks already know everything about the punishment, and can expose themselves to (socially) excessive risk as long as they can pass the regulator’s stress test. Facing such a trade-off, what is the optimal disclosure policy for the regulator?

Our answer is as follows. We first show that the regulator fully disclosing everything is not first-best, because the bank will game the test, investing in the risky asset if they can pass the test, even if their private information indicate significant risk. Then we show that even when the regulator is restricted to a binary choice between full disclosure to no disclosure, not disclosing any information can outperform fully disclosing if the bank has more precise information than the regulator (Proposition 2.1). Next, if the regulator can choose arbitrary partial disclosure policies, the regulator can fine-tune a disclosure policy to approximate the first-best - make the bank choose the socially optimal action, incorporating both the regulator and the bank's private information, which is impossible in both full and no disclosure settings (Proposition 2.2). We discuss the implications of this proposition.

2.1.1 Relevant Literature

This paper contributes to a strand of literature in stress test design, especially on the question of *disclosure* in stress tests. Recent developments in the question of stress test disclosure include Goldstein and Leitner (2018) which study the disclosure problem of stress tests to agents with heterogeneous beliefs, Parlatore and Philippon (2018) which study the design of stress scenarios, Inostroza and Pavan (2020) which cast the coordination problem of receivers into a global game with private information, and Parlasca (2019) studying the time inconsistency problem in stress testing. The Handbook of Financial Stress Testing by Farmer et al. (2022) include several chapters dedicated to stress test design and disclosure, most notably Goldstein and Leitner (2022) which overview the literature of stress test disclosure.

Most of the papers in the literature of stress test disclosure concern the disclosure of stress test *results to investors* and its implications, after the stress test takes place. On the other hand, the disclosure problem and the trade-offs we study are regarding the disclosure of stress test *model to banks*. The paper that explicitly studies the question of *model disclosure* is Leitner and Williams (2022), which is the first paper to provide a unified framework to study disclosure of regulators' stress test models, which we build on. The main departure we have from their framework is that their paper implicitly assumes that the banks have an asymmetric informational advantage about the state: everything that the regulator knows is an obfuscation, or a *garbling* of what the banks know.

Instead, our framework considers the possibility that the banks may not be fully informed, or that the regulator may have some informational advantage over banks. We believe this is a more reasonable assumption that leads to different conclusions. Indeed, the Federal reserve does seem to have some informational advantages over individual banks: (1) macro-variables that the Fed would pay attention to, more than individual banks[2] (2) systemic risk that aggregates information reported from all banks; each bank knows its own position, but it's only the Fed that knows and takes into account each bank's position in the market. This is relevant to supervisory stress tests since any such informational advantage the Fed has will be used in conducting the stress tests.

If the banks are not fully informed, by releasing information about the underlying state, the regulator can provide *guidance* to help agents make a more informed decision, which can be beneficial for everyone. By excluding this effect, Leitner and Williams (2022) conclude the regulator should never fully disclose their information. We clarify in Proposition 2.1 that the regulator does not want to disclose if and only if it believes that the banks are more informed. In both papers, the regulator can do better by partially disclosing its information. However, we expand the disclosure policies in Section 2.4, show how the regulator can achieve the first best outcome with sufficient flexibility on the information structure: notably, we show that approximating the first-best outcome is possible by a disclosure structure that almost fully discloses the regulator's information.

From a theoretical standpoint, our modeling framework relates to information disclosure in mechanism design. Our setup can be considered as a partially informed principal having the ability to design incentives and choose the amount of private information it wants to disclose, to a partially informed agent. Supervisory stress tests have this characteristic and we use it as an application to contribute to this literature at large.

The question of whether a principal with private information should fully disclose any information she has is of considerable interest, yet the literature gives mixed answers. In auction environments, Milgrom and Weber (1982) and Ottaviani and Prat (2001) discover the linkage principle and argue that the seller has incentive to fully reveal any information related with the buyer's valuation. On the other hand, it is well known in the persuasion literature (Kamenica and Gentzkow (2011), Bergemann and Morris (2016a), Bergemann and Morris (2016b)) that an information designer can benefit from not fully disclosing, and instead randomizing over information structures.

In standard Bayesian persuasion literature, the payoff structures are exogenously given. On the other hand, standard literature on principal-agent problems feature a principal given a fixed information structure and choosing the optimal contract satisfying incentive compatibility constraints. The question of our paper involves endogenizing both the payoffs and the information by allowing the principal to design mechanisms as well as a disclosure policy to induce a certain action. Thus this paper most closely

[2] Former Boston Fed President Eric Rosengren: "We asked BofA to tell us their exposure to subprime mortgages.. they had no idea"

relates to the literature on information disclosure in mechanism design where the principal controls the private information agents learn beyond their initial private information. Developments in this literature include Eso and Szentes (2007), Bergemann and Pesendorfer (2007), Li and Shi (2017), Yamashita (2018) and Krähmer (2020).

There are two main differences between our paper and the aforementioned literature on disclosure and mechanism design. First, the previous papers analyze the case where either the agent or the principal with private information, while our paper concerns the case where both the principal and the agent have private, imperfect information. This matters for two reasons: first, in our setup, the principal discloses her private, imperfect information about the underlying state, whereas previous work assumes that the principal discloses information about the underlying state itself: thus the meaning of full disclosure differs in their setting and ours. This distinction is critical, since our setup has a trivial solution once we assume the principal has perfect information. The second difference is that previous papers concern mechanism design environments with transfers from the agent to the principal, while we consider principal-agent problems where the principal can punish the agent, but the disutility to the agent from punishment does not translate into gains for the principal. We believe our assumptions are closer to reality in studying financial stress tests, where a regulator has imperfect information and wants to influence an agent's action using punishments based on this imperfect information.

Our paper highlights the principal's gains from garbling information by keeping the agent guessing using his own private information, which is correlated with the underlying state and the principal's information. This intuition is closely related to the literature on surplus extraction through correlation, building on the seminal result of Crémer and McLean (1988) which shows that for any (quasi-linear) utility function and a finite type space, if the agents' signals are correlated, there exists a mechanism that extracts all the surplus. McAfee and Reny (1992) prove this result in a continuum; Rahman (2012) generalizes this result to arbitrary type spaces. The main argument in Cremer-McLean is to use other bidders' bids that are correlated with each bidder's valuation to construct payoffs, to make each bidder effectively report their own value. In a similar vein, the main strategy of a principal with the power to arbitrarily disclose her information and design arbitrary punishments is to use her own information, which is correlated with the agent's information: by making the agent guess the principal's information, the principal can effectively elicit the agent's private information.

A contribution we make to the surplus extraction literature is providing a real-life application to the Cremer-McLean surplus extraction result. While a celebrated result in theory, applying Cremer-McLean's main insight of "taking advantage of correlations to extract information rent" in practice is difficult: Milgrom (2004) has described the full surplus extraction result as "nothing that is found in practice and reminds us of how important it is to check the practical reasonableness of solutions suggested by a model before implementing any practical policy based on the model." Our setup and argument gives a practical application of a principal taking advantage of correlation structures in information, by "keeping the agent guessing" about the principal's private information. Proposition 2.2 extends to suggest the optimality of partial disclosure and the advantages in using the correlation in information to the principal's benefit.

Our result is also relevant to the distinction between "omniscient persuasion" and "public persuasion" (Bergemann and Morris (2016b)). We show that with carefully designed information structures and incentives, public persuasion can allow the sender to achieve utility arbitrarily close to that from omniscient persuasion. In public persuasion setups, incentive compatibility constraints restrict the set of attainable payoffs for the principal. Our paper analyzes the case where the principal cannot elicit the agent's private information through standard contracts because the principal is tied to only making binary 'punishment' decisions. We show that in such circumstances, randomizing on the principal's own information allows her to elicit the agent's private information.

2.2 Model Setup

There is a bank and a regulator. The bank can choose between two actions: invest in a risky asset or a safe asset. While both the bank and the regulator know the payoff of the safe asset, both the bank and the regulator only have private, imperfect information about the payoff of the risky asset. The regulator can choose two objects of interest: a *stress test*, which is an assessment of the expected performance of the risky asset in a given scenario, conducted using what the regulator knows; and a *disclosure policy* about what the regulator knows. The formal model specifications are laid out below.

Agents and Fundamentals. The economy consists of a bank (B) and a regulator (Fed, F)[3]. The bank can choose between two actions: invest in a risky asset or a safe asset. The payoff from investing in the safe asset is known; we normalize it to zero for both the bank and the regulator. The payoff of the risky asset depends on the realization of

[3]The model naturally extends to multiple banks; we keep it to one bank for clarity. We discuss this extension in Section 2.5.

a random variable $\omega \in \Omega \subseteq \mathbb{R}$. We call ω the **underlying state** of the risky asset. The bank's payoff from the risky asset is $u^B(\omega)$, which are the private gains of investing in the risky asset (relative to the safe asset). Assume $u^B(\omega)$ is bounded over Ω, so that utility obtainable in the best possible state is $\bar{u}^B < \infty$. The regulator's payoff is $u^F(\omega)$, which can be considered as the social welfare associated with the risky asset, which the regulator, as a constrained social planner, seeks to maximize.

There is a conflict of interest: the bank always prefers the risky asset to the safe asset, while the regulator regulator prefers the risky asset only if the underlying state is good (safe). Formally,

Assumption 2.1. *u^F is an increasing function of ω, and there is a ω_0 such that $u^F(\omega) \geq 0$ iff $\omega \geq \omega_0$.*

Assumption 2.2. *u^B is an increasing function of ω and $u^B(\omega) \geq 0$ for all ω.*

Assumptions 2.1 and 2.2 highlight the main conflict of interest: the bank wants to invest even in risky states, whereas the Fed wants the bank to invest only in good states. This is the only assumption we make – we don't make any structural or parametric assumptions about the payoff function.[4] The following example previews the types of situation we are interested in:

Example. Assume there is a financial crisis with probability p. Let R_g be the (known) gross return of a risky asset when there is no crisis, and ω be the bank's return of the asset when the financial crisis occurs. Let $u^B(\omega) = (1-p)R_g + p\omega - 1$, $u^F(\omega) = (1-p)R_g + p\omega - pL - 1$. The assumption $u^B(\omega) \geq 0$ highlights the fact that the banks want to invest even when the return in the bad state is low, indicating a higher willingness to take risk and/or moral hazard associated with the Fed put; whereas the regulator is more concerned about the macroeconomic costs, including having to bail out such banks, and externalities from financial crisis (Caballero and Simsek, 2013).

This example generalizes to any adequate interpretation to an uncertainty regarding a risky asset: ω could denote Value-at-Risk, beta, or quality outstanding bonds. Depending on the level of risk aversion, we can construct an appropriate utility function for both the regulator and the bank.

Information (Model). Both the regulator and the bank have their own private 'models' of ω. Formally, we assume that both the regulator and the bank have a common prior $\omega \sim f$

[4] The assumption that $u^B(\omega) \geq 0$ for all ω highlight the bank's stronger desire to invest, or some form of moral hazard in the banks' belief that they will be rescued by a bailout when the underlying state turns out to be bad (such as a financial crisis). It also helps simplify our analysis.

on the underlying state, and their private model of ω is an imperfect signal on ω.[5] The regulator's model of ω is represented by a signal $s^F \in \mathcal{S}^F \subset \mathbb{R}$ drawn from a distribution with CDF $F_{s^F}(\cdot|\omega)$ and density $f_{s^F}(\cdot|\omega)$, and the bank's model of ω is represented by a signal $s^B \in \mathcal{S}^B \subset \mathbb{R}$, drawn from a distribution with CDF $F_{s^B}(\cdot|\omega)$ and density $f_{s^B}(\cdot|\omega)$. A natural interpretation of s^F, s^B is model-implied values of ω.

Here the prior f captures any common knowledge between the regulator and the bank, while s^F, s^B capture what the regulator and the bank respectively know about the underlying state, represented in their models. We assume that the bank observes s^B and the regulator observes s^F before the bank makes its investment decision (hence a "model" of the risky asset before the returns are realized). We call s^F (resp. s^B) the regulator (resp. bank)'s information or signal. In practice, the FED model to asses losses from bank's positions is comprised of several regression equations per loan proftolio with unknown coefficients, estimated with previous data.

We do not impose parametric forms on the distributions, but we assume that both the regulator and bank are more likely to observe higher signals when the state ω is higher:

Assumption 2.3. *$s^F|\omega$ and $s^B|\omega$ satisfy monotone likelihood ratio property (MLRP): for any $\omega_1 > \omega_2$, both $f_{s^F}(s|\omega_1)/f_{s^F}(s|\omega_2)$ and $f_{s^B}(s|\omega_1)/f_{s^B}(s|\omega_2)$ are strictly increasing in s.*

This ensures that both the regulator and the bank is more likely to observe higher signals when the underlying state is larger; moreover, if the bank sees a higher signal, the regulator's signal is likely to be higher too (and vice versa).

As we pointed out in the introduction, this formulation implicitly assumes that the Fed's model of ω contains some information *of intrinsic value* that the bank does not know (captured by s^F). This departs from previous work such as Leitner and Williams (2020) where the bank's private information is clearly superior to the regulator's[6]. We believe our assumption that the regulator and banks independently have some nonoverlapping private information is realistic, and delivers crisp implications based on the relative 'quality' of information and models that the regulator and banks may have.

Stress test. After the bank makes its investment decision, the regulator conducts a supervisory stress test to determine whether to pass or fail the bank. Investments in safe assets always pass the test. If the bank invests in the risky asset, the regulator uses its private information to 'test' the investment. Specifically, the bank's investment passes the regulator's stress test if and only if the regulator's private information s^F is above

[5] The common prior assumption is not essential; the regulator and bank can agree to disagree on a different prior. As long as the heterogeneous prior is common knowledge, our analysis follows through.

[6] In Leitner and Williams (2020), the regulator's information is a strict garbling of the bank's information.

some threshold s^*. In practice, this threshold could represent model-implied threshold Value-at-Risk in adverse scenarios, or minimum capital requirements. The regulator chooses and announces this threshold s^* *before* the bank and Fed observe s^F, s^B.

Assigning a threshold rule on s^F for supervisory stress tests highlight the idea that supervisory stress tests are the regulator testing whether the risky investment is safe enough under an adverse scenario, using its own models and parameters, as opposed to using the bank's own assessments of their investments. Indeed, s^F encapsulates all of the regulator's private information that it uses to estimate or evaluate ω. Thus it is natural to interpret s^F as an output of the regulator's model it uses to test the bank's risky investment, or a key coefficient/parameter in the regulator's model.[7]

We can interpret s^* as the **strictness** of the test – with a higher s^*, the regulator is less likely to give a pass to a risky investment, so the test is stricter.

Failure. If the bank invests in the risky asset and fails the test, a private cost c is incurred to the bank. We call (s^*, c) the *stress test structure*.

Multiple interpretations for this cost exist in this context – one is a cost to the bank from "failure" being publicly announced to the market; this would lead to a decline in the market's confidence to the bank, incurring costs such as stock price declines (Flannery, Hirtle, Kovner). Another is direct transaction costs associated with having to liquidate assets. Yet another can be a direct "punishment" imposed by the Fed, such as restrictions on share repurchases or dividends, cease-and-desist orders to correct practices in risk management, or even penalty fees. All of these punishments have been used by the Federal Reserve in conducting the Dodd-Frank Act Stress Test (DFAST).

Note that the aforementioned interpretations are ambiguous as to whether the regulator may have control of c. It seems that under some interpretations (direct punishments/regulations or intensity of signalling to the market), the Fed may have some control over the magnitude of the cost, whereas it seems too strong to assume that the Fed can fine-tune c. In this paper, We explore the case when c is exogenous, and discuss the case where the Fed can set c optimally.

We can interpret c as the **harshness** of the test: a higher c implies that the costs of failing the test are harsher on banks.

Final payoffs. After the stress test and punishments, the payoffs are as follows. If the bank invests in the safe asset, the regulator and the bank's payoffs are 0. If the bank

[7]The debate around disclosure of the Federal Reserve's Dodd-Frank Act Stress Tests is centered on revealing the coefficients of regressions that the Fed uses to estimate losses.

invests in the risky asset and passes the stress test, it receives payoff $u^B(\omega)$ and the regulator receives payoff $u^F(\omega)$. If the bank invests in the risky asset and fails the stress test, it receives payoff $u^B(\omega) - c$ and the regulator receives payoff $u^F(\omega)$.

Disclosure. The main question we seek to answer is whether the regulator should reveal its "model" s^F to the bank. We analyze this in a Bayesian persuasion framework as in Kamenica and Gentzkow (2011): the regulator can *partially disclose* information about s^F by committing to an information structure that provides the bank with signals about s^F. In the supervisory stress test framework, this corresponds to giving some, but not all, of the parameters and model specifications that the Fed uses in estimating ω.

Formally, before observing s^F, the regulator discloses information about s^F by committing to a *disclosure policy* (M, π) that consists of a set M of messages and conditional distributions $\pi : \mathcal{S}^F \to \Delta(M)$, where $\pi_{s^F} \in \Delta(M)$ denotes the distribution of messages conditional on s^F. An equivalent way to state the definition of π is as follows: after observing s^F, the regulator generates a *message* m about s^F according to $m|s^F \sim \pi_{s^F}$ and reveal m to the bank. This message m captures (partial) information about the regulator's model that it discloses to the banks.

Two benchmark information structures are *full disclosure* corresponding to $\pi_{s^F}^{full} = \delta_{s^F}$, the point mass on s^F, and *no disclosure* corresponding to $\pi_{s^F}^{no} = \delta_m$, the point mass on some m independent of s^F: the regulator sends the same message m for any s^F, disclosing no information about s^F.

Timing. The timing of the model is as follows:

(a) (Ex-ante stage) The regulator chooses a disclosure policy π about s^F and a punishment threshold s^*, and publicly announces it.
(b) (Interim stage) The regulator observes s^F, and the bank observes s^B.
(c) (Disclosure stage) The regulator discloses $m \sim \pi_{s^F}$ to the bank, and the bank updates its posterior on ω and s^F.
(d) Knowing s^B and m, the bank decides whether or not to invest in the risky asset.
(e) If the bank decides to invest and $s^F < s^*$, the bank fails the test.

Problem. The regulator and the bank's problem is given as follows.

- **Bank's problem.** The bank decides whether or not to invest in the risky asset, knowing the bank's own signal s^B and the regulator's message m. Given s^B, m, the bank believes investing in the risky asset will *pass* the stress test with probability $q = P[s^F > s^* | s^B, m]$. Then the bank invests if and only if

$$E[u^B(\omega)|s^B, m] - (1-q)c \geq 0 \tag{2.1}$$

Note that q is increasing in s^B, and $u^B(\omega)$ is monotonic in ω, so the expectation is increasing in s^B. Thus given m, the bank invests if and only if $s^B \geq s^{B*}(m, c)$ for some s^{B*}.

- **Regulator's problem.** The regulator chooses s^* (stress test threshold), pi (disclosure policy) in the ex-ante stage (before observing s^F) to maximize

$$\mathbb{E}_{\omega, s^F, s^B, m}[u^F(\omega)], \tag{2.2}$$

where the expectation is over all possible realizations of ω, s^F, s^B and m.

The actual realization of ω is irrelevant to the regulator or the bank's decision problem, as the decision happens before the realization. This is natural in our context: any decision related to investment in risky assets, including supervisory stress tests and punishments, cannot depend on the actual payoff of the risky asset. We define the equilibrium as:

Definition 2.1. *(Equilibrium) An equilibrium is a disclosure policy $\pi : \mathcal{S}^F \to \Delta(M)$, stress test threshold s^*, and action chosen by the bank (invest in safe or risky asset) such that the regulator and bank each solve their optimization problem.*

First-best action for the regulator. Given information μ about ω, the regulator wants the bank to invest in the risky asset if and only if $E[u^F(\omega)|\mu] \geq 0$. There are two sets of information available in the market: s^F known by the regulator and s^B known by the bank. Thus the first-best action is to invest iff

$$E[u^F(\omega)|s^F, s^B] \geq 0.$$

This takes into account both the information that the regulator and bank knows. Note that the expectation is increasing in both s^F and s^B, by Assumption 2.3.

Call the set of signals $S^{FB} = \{(s^F, s^B) | E[u^F(\omega)|s^F, s^B] \geq 0\}$; this is the set of *realizations of information* for which the regulator wants the bank to invest. Also denote by $U^F(s^F, s^B) = E[u^F(R)|s^F, s^B]$ the expected utility of the regulator when the bank invests in the risky asset, when the Fed knows s^F and the bank knows s^B. We seek to answer the following questions:

Research Question. Should the regulator reveal s^F to the bank prior to the investment? Can hiding, or garbling and partially revealing the regulator's model/information be welfare enhancing than fully disclosing it? How strict should the stress test be, and how does this depend on the fundamentals?

The next sections seek to address these questions.

2.3 Full vs. No Disclosure when banks want to pass

In this section, we compare between two disclosure policies: full disclosure where the regulator reveals s^F fully to the bank, and no disclosure where the regulator reveals no information about s^F to the bank. Restricting the regulator to this binary choice allows us to concisely deliver the intuition that disclosure may hurt the regulator because the banks would rely less on their own private information in investment decisions and just follow the supervisory stress tests. Moreover, this is not a gross simplification – if the regulator cannot credibly commit to more complicated disclosure policies, the only choices would be to reveal or not reveal.

For simplicity, we assume that banks always want to pass the stress test:

Assumption 2.4. *There is some $c_0 > \bar{u}_B$ that lower bounds the minimum possible punishment that the Fed can carry out conditional on failure.*

Since punishment outweighs any possible benefit to the bank from investing, banks will never invest in an asset if they are certain it will result in stress test failure.

2.3.1 Full Disclosure

When the Fed fully discloses s^F, the bank knows exactly when it will pass or fail the test. Thus the expected utility from investing in the risky asset is

$$U^{B,FD}(s^B, s^F) = \begin{cases} E[u^B(\omega)|s^B, s^F] > 0 & \text{if } s^F \geq s^* \\ E[u^B(\omega)|s^B, s^F] - c < 0 & \text{if } s^F < s^* \end{cases}$$

As mentioned before, the cost of failing the stress test is assumed to be high, i.e. $c \geq E[u^B(\omega)|s^B]$ for any s^B. The bank will invest if and only if $s^F \geq s^*$, i.e. if it knows the

Fed's signal is high enough that it passes the test. The regulator's ex-ante utility from full disclosure with threshold s^* is

$$U^{F,FD}(s^*) = E[U^F(s^F, s^B)|s^F > s^*]Pr(s^F > s^*) = \int_{s^F \geq s^*} U^F(s^F, s^B)dF(s^F)$$

The optimal threshold s^{*FD} is the unique solution to $E[U^F(s^F, s^B)|s^F = s^{*FD}] = 0$. The reason why full disclosure cannot achieve the first-best for the regulator is clear; the bank would always invest if they knew they would pass the test, even if their information s^B suggests that the social welfare resulting from the investment is low. This is to say, the bank "games the system" when the Fed's signal is high enough to allow them to comfortably pass.

2.3.2 No disclosure

On the other hand, assume that the regulator discloses nothing about s^F. The bank's utility from investing in the risky asset is

$$U^{B,ND}(s^B) = E[u^B(\omega)|s^B] - (1-q)c$$

where $q = P[s^F \geq s^*|s^B]$ is now the bank's estimate of the probability of passing the stress test: the bank needs to "guess" the test results using its information s^B. Since q and $E[u^B(\omega)|s^B]$ are both increasing in s^B, $U^{B,ND}(s^B)$ is increasing in s^B, there is $s^{B*}(s^*, c)$ such that bank will invest iff $s^B \geq s^{B*}(s^*, c)$.

Thus the set of signals where the bank invests in the risky asset and passes the test is given by

$$S^{ND}(s^*) = \{(s^F, s^B)|s^B \geq s^{B*}(s^*, c)\}.$$

The mapping $s^* \to s^{B*}(s^*, c)$ is increasing in s^* and c; if the test is stricter or harsher, the bank would need a higher signal to invest. Thus $s^{B*}(s^*, c)$ is invertible with respect to s^*, and for a fixed c, we can define $s^*(s^{B*})$ be the value of s^* that implements s^{B*}.

We let

$$U^{F,ND}(s^*) = E[U^F(s^F, s^B)|s^B \geq s^{B*}(s^*, c)]$$

be the regulator's payoff when it chooses the bank's threshold s^{B*}. Let $U^{ND*} = \max_{s^{B*}} U^{F,ND}(s^{B*})$ be the highest payoff to the regulator. The optimal policy satisfies $E[U^F(s^F, s^B)|s^B = s^{B*}(s^*, c)] = 0$.

The reason why no disclosure cannot achieve the first-best for the regulator is different now; as the bank does not know s^F, the bank is not fully informed. This clarifies the trade-off with disclosure: full disclosure makes the banks more informed, but at the same time, opens the door for the banks to *game the system*, investing even when the bank's assessment is bad (s^F high, s^B low).

Comparative statics

The payoff to the Fed under the optimal full-disclosure threshold, $U^{*FD} = U^{F,FD}(s^{*FD})$, is unaffected by the distribution of the bank's signal, and increases with the informativeness of the Fed's signal. To see the former, observe that the bank's decision is independent of its own signal. Note, for the latter, that if a signal $s^{F'}$ is Blackwell more informative than s^F, then $s^F = s^{F'} + \epsilon$ where ϵ is a noise term distributed $F_\epsilon(\epsilon|s^{F'})$.[8] If the optimal threshold under signal structure s^F is s^*, then under $s^{F'}$ the regulator could mimic expected outcomes under s^F by randomizing whether to allow banks to invest, and using an interior probability

$$p^F(s^{F'}) = 1 - F_\epsilon(s^* - s^{F'}|s^{F'})$$

of announcing to banks that they will be allowed to pass the stress test even if they invest in the risky asset. However, because there is a uniquely optimal cutoff under $s^{F'}$, and elsewhere it is either strictly optimal to invest or strictly optimal not to invest given s^F, randomization is suboptimal. The Fed therefore does strictly better by passing banks above the cutoff and failing them below, which it can enact when playing optimally with the more-informative signal $s^{F'}$, but not under the less-informative s^F.

On the other hand, the Fed's payoff under the no-disclosure threshold depends only on the informativeness of the bank's signal. Here, a more informative signal for the bank is better. The argument is similar to the previous case: since s^* determines the outcome through s^{B*}, the implied cutoff in the bank's private signal that determines whether they invest or not, and $U^{F,ND}$ depends only on s^{B*} and the joint distribution of s^B and ω. If $s^B = s^{B'} + \epsilon$ with $\epsilon \sim F(\epsilon|s^{B'})$, then the outcome of the optimal policy s^{B^*} under s^B is equivalent to having the bank that observes $s^{B'}$ randomize, and invest with probability

$$p^B(s^{B'}) = 1 - F_\epsilon(s^{B*} - s^{B'}|s^{B'}).$$

Again, because the Fed's expected payoff from investment is also strictly increasing in the bank's signal, randomization is suboptimal everywhere except the optimal cutoff, and it is better for the Fed if the bank plays their best response to $s^{*'}$ using their more informative signal $s^{B'}$.

[8] We assume that the garbling preserves monotonicity of returns to investing in s^F; if not, then a cutoff rule may no longer be optimal.

2.3.3 Full disclosure vs no disclosure

Which of the two regimes is preferred? The benefit of full disclosure is that the bank is more informed in its decision about ω, but it comes at a cost – namely, the bank *overinvests* in states where it knows it'll pass the test, and may *underinvest* when it knows it'll fail the test. On the other hand, without disclosure, the bank's investment decision is made only using its information, so some information is lost.

The comparative statics in the previous section show that full disclosure improves when the Fed has a precise signal, while no disclosure improves when the bank's signal is more informative. It follows that which regime is better depends on "whose model is better", i.e., on the relative precision of s^F and s^B.

Proposition 2.1. *Suppose the regulator chooses between full disclosure (FD) and no disclosure (ND), and c is exogenously given and fixed.*

(a) Fix s_F. If $U^{FD*} > U^{ND*}$ under s^B, then the same is true for all $s^{B'}$ that are garblings of s^B. In other words, full disclosure becomes more attractive to the Fed as the bank's model worsens.

(b) Fix s_B. If $U^{FD*} > U^{ND*}$ under s^F, then the same is true for all $s^{F'}$ that are Blackwell more informative than s^B – when the Fed's own model improves, full disclosure becomes more attractive.

(c) If s_F is a garbling of s_B, then no disclosure is better than full disclosure. If s_B is a garbling of s^F, then full disclosure outperforms no disclosure.

Essentially, the Fed would like to disclose its model ONLY if its information is precise enough so that the Fed benefits from dictating the investment. Otherwise, it should hide its model, in order to prevent banks, which have more valuable information about tail-end risks, from discarding their own predictions, and to make banks leverage their own information in the Fed's favor by guessing the Fed's model.[9]

Proposition 2.1 clarifies to us that if there's any motivation for the regulator to disclose information, it is not because the regulator believes the banks are better equipped to make investment decisions and wants to delegate the decision to the bank. Rather, the regulator would like to fully disclose only if it believes that its own models are more accurate than the banks, to *provide guidance*, or make binding recommendations on investment decisions based on the regulator's model. This may have been true in the first few years following

[9]The intuition resembles that of Cremer and McLean (1985): as long as the Fed's signal is not fully known to the bank, it is correlated with the bank's own model, and this correlation can be exploited to make the bank use its private information in a way that best benefits the Fed.

the Great Recession, when there was good reason to believe that banks themselves did not have sufficient capacity to identify systemic risk.

On the other hand, in more recent times, where it is generally believed that banks employ more sophisticated models and may have some proprietary information that allows them to have a more precise estimate of risk, the fact that banks have better models may actually incentivize the regulator to *hide* information, precisely because the regulator does not want banks to overinvest in risky assets that pass the regulator's test. In our framework, it is precisely because the regulator believes banks may have better private information that they hide the regulator's information, as the regulator wants to align the bank's interests with social welfare, while ensuring that the bank uses its private information.

2.3.4 Comparative statics on strictness and harshness

We investigate some comparative statics on fundamentals of interest – such as the passing threshold s^* ('strictness'), cost of failure c ('harshness'), and how they relate to the disclosure policy. We test predictions such as the following:

(a) If the regulator discloses information, the regulator would have to make the tests stricter ("minimum required capital levels would need to be materially increased") to counteract gaming. This was suggested by former Fed governor Tarullo (2017).[10]

(b) If the cost of punishment is higher to the banks, the test should be less strict.

We first show that the first prediction need not necessarily be true. Formally, if s^{*FD} is the regulator's optimal threshold under full disclosure, and $s^{*ND}(c)$ is the regulator's optimal threshold under no disclosure, the statement is equivalent to $s^{FD*} > s^{*ND}(c)$. However, this is not necessarily true: it depends on the specific information structure, and what investment threshold the regulator wants to induce the bank to use in the case of no disclosure.

On the other hand, we have seen from the analysis of the full and no disclosure case that the second proposition is true for no disclosure, but not for full disclosure. Indeed, under no disclosure, a higher c is accompanied by a lower s^*. That is, if the cost of failure is *harsher*, the regulator responds by making the test *less strict*; if the cost of failure is lower, the regulator makes the test harder to pass. On the other hand, under full disclosure, as long as the cost of failing the stress test is high enough so that no bank would voluntarily fail the test with probability 1 (Assumption 2.4), the optimal threshold s^{*FD} is simply

[10]See: https://www.federalreserve.gov/newsevents/speech/tarullo20170404a.htm

given by $E[U^F(s^F, s^B)|s^F = s^{*FD}] = 0$. As such, the trade-off between strictness and harshness exists only when the regulator hides information about the test.

2.3.5 Punishment flexibility

In Section 2.2, we discussed the possibility that the regulator may be able to *choose* the harshness of punishment c, through either sending stronger messages about the investment's riskiness, or directly choosing which punishment to levy (freezing / coercing to sell assets, fining for failure etc.) What happens if the regulator chooses c optimally, in either full disclosure or no disclosure? We briefly discuss the intuition here.

Under full disclosure, as we have seen in the above section, as long as the punishment c is large enough, the bank's investment decision solely depends on s^*, the threshold set by the regulator; banks will invest if and only if they know they will pass the test, as no bank will *intentionally* fail the test. Thus under Assumption 2.4, there is no gains from punishment flexibility. Instead, if we allow the regulator to set punishment c low so that *some banks may intentionally fail the test and invest in risky assets*, this may strictly benefit the regulator, because the banks who would be willing to take the risk are selected to be the ones whose models implied a *better state*; so the regulator's utility can improve if the regulator allows some banks to fail. However, allowing some bavnks to intentionally fail is unrealistic – most notably, market participants learning failure is a strong enough disincentive sets a lower bouind for c, so it is likely that many banks to not want to intentionally fail the test.

Under no disclosure, choosing c and choosing s^* are *dual* in the sense that both are tools to affect the bank's investment threshold $s^{B*}(c, s^*)$; we have seen above that a higher c (harsher) is associated with a lower s^* (less strict). As such, the ability to choose c does not affect the optimal policy. Thus, under both full disclosure and no disclosure, the regulator's capacity to choose c does not significantly affect our analysis.

2.4 Partial Disclosure

We now assess the optimality of *partial disclosure* policies, where the regulator can commit to providing some information about its private information about the state of the world. In this setup, the regulator, upon observing its private signal s^F, can choose to disclose partial information about s^F, without fully disclosing it. As mentioned in Section 2.2, we model this as the regulator choosing a disclosure policy (M, π) *before observing* s^F such that, upon observing s^F, the bank discloses $m \sim \pi_{s^F}$. The bank updates its posterior of the state ω and the regulator's private signal s^F using m and makes its investment decision.

Under this 'middle ground' between full disclosure and no disclosure, how much better can the regulator perform? We argue here that fine-tuning the disclosure policy is sufficient to be arbitrarily close to the first-best, as long as the harshness c is sufficiently large.

2.4.1 The bank and the regulator's problem

Suppose regulator chooses the strictness s^* and the disclosure policy (M, π). Upon seeing s^F, the Fed discloses $m \sim \pi_{s^F}$. The bank, upon observing m, will form posteriors on ω and s^F given s^B and m: $\hat{\omega} \sim (\omega | s^B, m)$ and $\hat{s}^F \sim (s^F | s^B, m)$. Given this information, the bank will invest if and only if

$$U^B(s^B, m) = E[u^B(\omega)|s^B, m] - (1 - P[s^F > s^*|s^B, m])c \geq 0$$

This expected utility is increasing in s^B. As such, the bank invests if and only if $s^B \geq s^{B*}(m, c)$ for some mapping s^{B*}. Since the utility is decreasing in c, this mapping s^{B*} is clearly increasing in c; the banks need a higher signal to invest in a risky asset when the punishment is harsher. The dependence on m is trickier because we have no guarantee that m is monotonic - for example, the regulator may want to pool extremely good signals with extremely bad signals. But if we rule out such disclosure policies, we have a monotonicity result:

Lemma 2.1. *Suppose that the regulator's disclosure policy $m|s^F$ satisfies MLRP: for $s_1^F > s_2^F$, $f(m|s_1^F)/f(m|s_2^F)$ is increasing in m. Then the bank's expected utility $U^B(s^B, m)$ is increasing in m, so $s^{B*}(m, c)$ is decreasing in m.*

The regulator needs to construct a disclosure policy (M, π) and s^* to maximize ex-ante utility

$$\mathbb{E}_{\omega, s^F, s^B, m}[u^F(\omega)]$$

Recall that the first-best action that the regulator would like the bank to take is to invest in the risky asset iff

$$\mathbb{E}[u^F(\omega)|s^F, s^B] \geq 0$$

Since $s^F|\omega$ and $s^B|\omega$ both satisfy MLRP, the left-hand side is increasing in s^F, s^B. So there exists some decreasing $s^{B,FB}(s^F)$ such that the first-best action is for the bank to invest if and only if $s^B \geq s^{B,FB}(s^F)$.

From this formulation we clearly see the trade-offs of disclosure. Under full disclosure, the bank's decision rule $s^{B*} \geq s^{B*}(m,c)$ will depend exclusively on $m = s^F$, and not on s^B. With less disclosure, however, the bank's decision rule $s^{B*} \geq s^{B*}(m,c)$ will be noisy compared to the first-best $s^{B,FB}(s^F)$ since m is not s^F, and when the noise is sufficiently large, the banks will just rely on s^B to make the decision.

2.4.2 Approximating the first-best

The natural question is: can the regulator choose disclosure policy (M,π) and stress test s^* such that $s^B \geq s^{B,FB}(s^F)$ and $s^B \geq s^{B*}(m,c)$ are identical, or close enough? In this subsection, we show that if c is sufficiently large (or if the regulator can choose c), the regulator can design a stress test such that the regulator can induce the banks to approximate the first-best action that incorporates the signals of both the regulator and the bank. Thus this is clearly a strict improvement over full disclosure – when the bank can game the test – and over no disclosure – when the bank's action depends only on its signal.

Proposition 2.2. *Suppose the regulator can choose any information structure to partially disclose its signal. Then the regulator can design a disclosure policy with infinitesimal noise to approximate the "efficient investment" decision that incorporates both the information known by the regulator and the bank. Formally, for any $\epsilon > 0$, the Fed can choose the stress test (s^*, c) and a disclosure policy π such that the bank, after observing m will invest in the risky asset if and only if $s^B \geq s^{B*}(m,c)$, and this action satisfies*

$$E[u^F(\omega)1_{s^B \geq s^{B*}}] > U^{FB} - \varepsilon$$

where U^{FB} is the first-best utility of the regulator, and the expectation on the left-hand side is taken over all signal and message realizations.

An intuition for the proof of the proposition is as follows. For each bank with a signal s^B, the regulator sets up a disclosure policy such that the regulator almost fully discloses their model s^F, and almost all of the banks pass the test, but there is always a small probability of failing the test, and the conditional probability of failure is larger when the bank sees a lower message ('higher probability that the Fed believes the investment is risky'). The regulator fine-tunes the disclosure policy (M,π) such that the bank's 'threshold signal' $s^{B*}(m)$ after seeing the regulator's message m closely approximates the welfare-maximizing investment threshold.

Specifically, the regulator chooses the disclosure policy (M, π), cost of failure c, and passing threshold s^* such that:

- $M = \mathcal{S}^F$: the set of messages is simply the set of private signals of the regulator.
- $\pi_{s^F} = (1 - \epsilon(s^F))\delta_{s^F} + \epsilon_{s^F} U(\mathcal{S}^F)$: the disclosure policy fully reveals s^F with probability $1 - \epsilon(s^F)$, but the message may be garbled with some noise with full support, so that the banks can never be 100% sure they will pass the test.
- Construct the 'noise probabilities' $\epsilon(s^F)$ to be small enough so that the disclosed message m is almost always equal to s^F, but upon seeing any message m, there is a probability $p(m, s^B)$ such that the bank fails the test.
- Set c high, and fine-tune the probabilities $p(m, s^B)$ (using $\epsilon(s^F)$) such that $s^{B*}(m, c)$ is sufficiently close to $s^{B,FB}(m)$, which is sufficiently close to $s^{B,FB}(s^F)$ as long as $\epsilon(s^F)$ is small enough.

A rigorous construction and proof of approximate optimality is given in the Appendix (Section 2.7).

This result highlights the main benefit of partially disclosing. The regulator wants the bank to 'guess' the regulator's signal s^F using s^B, because guessing elicits the bank's private information; however, hiding information is inherently costly as it creates additional noise, when the regulator's goal is to match the bank's actions with the information that both the regulator and banks know. But by disclosing almost everything but leaving a small noise, the regulator can keep the benefits of 'not disclosing' – eliciting the bank's private information and preventing gaming the system – while keeping almost all of the benefits from disclosing – allowing banks to make a more informed decision in investments.

This result too, is in line with the Federal Reserve's actual policy regarding disclosure in the Dodd-Frank Act Stress Test (DFAST). The Fed is increasingly disclosing more information because it "helps financial institutions [..] understand the capital implications of changes to their business activities, such as acquiring or selling a portfolio of assets," at the same time cautioning against full disclosure because "doing so could permit firms to reverse-engineer the stress test." Disclosing most, but not all, of the relevant models of the stress test allows the regulator to reap most of the benefits associated with disclosing, while keeping enough uncertainty to shut down the perils associated with disclosure.

In the limit when our disclosure policy approximates the first-best ($\varepsilon \to 0$), we have that: banks almost fully know the regulator's information ($P[m = s^F] \to 1$), the banks are punished harshly ($c \to \infty$), but the test is not strict ($P[s^F \leq s^*] \to 0$), and the bank's investment decision, conditional on the message, is the first-best investment decision. Our result is suggestive that the regulator disclosing more information is associated with a harsher test (Section 2.3.4), and if the regulator can choose between strictness and harshness (Section 2.3.5), it is better to make the test harsh (punish severely) but not strict (punish only a few banks); however, since our result shows one way to approximate first-best but doesn't show that it is the *only* way, other disclosure policies and punishments may be able to approximate the first-best as well.

However, there are two caveats in applying the insights from this proposition in reality:

(a) The optimal disclosure policy may be difficult to implement in practice. To approximate the first-best, the regulator has to add an infinitesimally small probability of 'failure,' and fine-tune the probabilities so that the bank with signal s^B finds it exactly indifferent between investing and not investing at $s^B = s^{B*}(m, c)$. While the Federal Reserve definitely has some signaling capacity, it may be unrealistic to assume that the Federal Reserve can fine-tune the message to this extent.

(b) The possibility of misspecification makes our result less robust. Our first-best approximation result assumes that the regulator is fully aware of the information acquisition structure of itself and the bank, and the banks fully know and believe the complicated disclosure policy and updates accurately in a Bayesian manner. If the regulator is misspecified in any of these steps, combining a small noise with large punishments could lead to very inefficient outcomes.

As such, Proposition 2.2 should not be taken literally, and should be considered as a benchmark that highlights the substance – hiding *some* information is always better than revealing all information, whereas hiding too much information is going to hurt social welfare. At the same time, the intuition the proposition suggests – as long as the regulator does not disclose its private information fully, more disclosure could be socially beneficial if it is accommodated by an appropriate punishment – is quite relevant in the stress test context.

2.5 Discussion and extensions

In this section, we discuss the implications of our main propositions and their intuition, and discuss possible extensions of our framework.

(a) What if there is a social cost of banks failing the stress test that the regulator must account for? We assumed for simplicity that the regulator does not internalize the cost of punishment that is incurred to bank as a result of failing the stress test. If such social welfare costs of punishment exist, no disclosure will be relatively worse than full disclosure compared to our initial analysis, as under no disclosure some banks may still invest and get punished, while under full disclosure no bank will ever voluntarily invest knowing it will be punished. However, if the social cost is not as large as the private costs to banks (which include management changes, ban on additional transactions, etc.), it may still be the case that no disclosure outperforms full disclosure. Moreover, partial disclosure strictly dominates both full and no disclosure.

(b) A natural extension would be to add multiple banks which make independent investment decisions. If the regulator has multiple banks, each with their own 'models' s_i^B, then the disclosure problem becomes whether or not to disclose a common s^F to multiple banks with their own s_i^B. In such a scenario, we immediately see that there is stronger incentive for the regulator to not fully disclose, as each bank using their own information would likely lead to a better outcome than every bank blindly investing the maximum according to the stress test rule. This can be formalized if the regulator has a specific aversion to banks' actions being correlated – either investment is multidimensional and there is some correlation disutility, or the regulator's social welfare function dislikes banks' investment thresholds being too close to one another. This highlights the concern expressed by the Federal Reserve against disclosing: "[Full disclosure] could increase correlation in asset holdings [..] making the financial system more vulnerable."

(c) In a similar vein, to make our stylized model more realistic, we may treat investment and 'models' as multidimensional objects. This would clarify the 'information' in the disclosure problem as parameters in question as specific coefficients into the regulator's 'stress test' model, and make explicit some of the goals of the regulator (decrease the correlation across banks – low beta) versus the bank (increase correlation with market – high beta). In this case, there's an additional layer of optimal disclosure that can be discussed – the optimal number of dimensions that the regulator may disclose. This can be naturally interpreted as the number of coefficients to disclose in actual stress tests conducted by the Federal Reserve.

(d) Endogenizing information acquisition by the banks. Another existing concern by the Federal Reserve argument against model disclosure is that "full disclosure could incentivize banks to simply use models similar to the Federal Reserve's, rather than build their own capacity to identify, measure, and manage risk." What if banks' draw s^B is not drawn from an exogenous distribution, but banks could invest in knowing a more precise s^B, through some information acquisition that is more costly in the precision of the draw? If the regulator fully discloses, then banks will never invest in knowing a better s^B. Hiding the regulator's model s^F would incentivize banks to "do their own research" to pass the stress test, hence investing in their own, independent models, and therefore also improves the outcomes of less relative to more disclosure. This allows room for investigating the optimal level of disclosure, if banks investing in their capacity comes with a social cost.

(e) The paper focuses its analysis on supervisory stress tests in the financial sector, but the intuition and logic naturally extends to any environment where a regulator has imperfect information about an underlying state and wants to influence an agent's action by assessing and punishing agents' actions through her imperfect information. Applying our analysis to scenarios such as traffic cameras, law enforcement, or firm-employee relationships can explain why intentional obfuscations/garblings exist in many principal-agent relationships with private information.[11]

2.6 Conclusion

Should the Federal Reserve disclose its stress test models to banks? To shed light on this question, we propose a stylized model where a regulator and bank has private information about the state of a risky asset, and the regulator uses its own information to test the investment. Our framework incorporates the main trade-off of disclosure: disclosure allows banks to make a more informed decision, but at the same time, allows banks to *game the test*.

Our main contribution is highlighting the possibility of the regulator to elicit the banks' private information by hiding the regulator's private information used for the stress tests, and keeping the banks guessing about this information. Our first result comparing full disclosure to no disclosure highlights this intuition: we show that no disclosure can be better than full disclosure if banks hold better information.

[11] A previous version of this paper titled 'Optimal Disclosure in Principal-Agent Problems with Imperfect Information' discusses this in more detail.

Our second result shows that if the regulator can commit to arbitrarily complex disclosure policies, the regulator can choose a disclosure policy that reveals almost everything, but leaves a small noise, and fine-tune this so that the bank's investment action is arbitrarily close to the socially optimal investment decision, and combining this with a sufficiently harsh punishment.

The second result may shed some light on the Federal Reserve's strategies in recent years, where they release more information on the crucial parameters and modeling structures, but keep some parts opaque, and punishment for failing the test is severe enough such that no major bank voluntarily fails the test. However, at the same time, the second result relies on fine-tuning information disclosure and committing to them; it should be thought of as a benchmark, and not too literally. When the regulator lacks commitment power, it may make more sense to think of the regulator's question as choosing between full disclosure and no disclosure, whence we are back to our initial framework in which the signals' relative informativeness matters.

While we lay out our intuition for the main trade-off of gaming the system, our model is stylized in nature, and cannot answer quantitative policy questions on stress test disclosure by itself. A natural next step will be to build a close-to-reality model of stress test design by following the literature, such as Parlatore and Philippon (2018) or Parlasca (2019) with multiple banks, each endowed with their own private information and making investment decisions, and the regulator deciding 'how much' information to disclose. Such a quantitative model will be able to guide policymakers on the 'optimal stress test model disclosure' debate.

2.7 Proofs of propositions

Lemma. *Assume two random variables v, w are given such that $v|w$ satisfies MLRP; that is, if $w_1 > w_2$, $f(v|w_1)/f(v|w_2)$ is increasing in v. Then $w|v$ satisfies MLRP too.*

(In words: if I get to see v first, upon seeing higher v, I expect a higher w. Then, if I get to see w first, upon seeing higher w, I expect a higher w.)

Proof. For any $v_1 > v_2$ and $w_1 > w_2$, by $v|w$ MLRP, we have $f(v_1|w_1)/f(v_1|w_2) > f(v_2|w_1)/f(v_2|w_2)$. Rearrange to obtain $f(v_1|w_1)/f(v_2|w_1) > f(v_1|w_2)/f(v_2|w_2)$

Then by Bayes rule (we use p to denote unconditional distributions), we have

$$\begin{aligned}\frac{f(w_1|v_1)}{f(w_1|v_2)} &= \frac{f(v_1|w_1)p(w_1)/p(v_1)}{f(v_2|w_1)p(w_1)/p(v_2)} \\ &= \frac{f(v_1|w_1)}{f(v_2|w_1)}\frac{p(v_2)}{p(v_1)} \\ &> \frac{f(v_1|w_2)}{f(v_2|w_2)}\frac{p(v_2)}{p(v_1)} \\ &= \frac{f(v_1|w_2)p(w_2)/p(v_1)}{f(v_2|w_2)p(w_2)/p(v_2)} = \frac{f(w_2|v_1)}{f(w_2|v_2)}\end{aligned}$$

Since this holds for any $v_1 > v_2$ and $w_1 > w_2$, $w|v$ satisfies MLRP. □

Proposition 2.2. *Suppose the regulator can choose any information structure to partially disclose its signal. Then the regulator can design a disclosure policy with infinitesimal noise to approximate the "efficient investment" decision that incorporates both the information known by the regulator and the bank. Formally, for any $\epsilon > 0$, the Fed can choose the stress test (s^*, c) and a disclosure policy π such that the bank, after observing m will invest in the risky asset if and only if $s^B \geq s^{B*}(m, c)$, and this action satisfies*

$$U^{PD} = \mathbb{E}[u^F(\omega)1_{s^B \geq s^{B*}}] > U^{FB} - \epsilon$$

where U^{FB} is the first-best utility of the regulator, and the expectation on the left-hand side is the expected regulator's utility (social welfare) under the partial disclosure policy, taken over all signal and message realizations.

Proof. Fix an $\epsilon > 0$; we claim that there exists a partial disclosure policy achieving utility $> U^{FB} - \epsilon$. We proceed in the following steps.

Step 1. We approximate the signal space into a discrete space; with a close enough approximation with large number of elements N, the distributions should converge to the distribution in continuum, and the difference in ex-ante utility from the discrete signal space and continuous signal space should differ by an infinitesimal amount.

Specifically, we assume that discrete signals $d^F, d^B|\omega$ are drawn from $\{s_1, s_2, \cdots, s_{2N}\} \subset S$, such that $P(d^F = s_i|\omega) = P(s^F \leq s_i|\omega) - P(s^F \leq s_{i-1}|\omega)$, and the endpoints are $P(d^F = s_1|\omega) = P(s^F \leq s_1)$, $P(d^F = s_i|\omega) = 1 - P(s^F|\omega)$, and analogously for s^B. This should naturally define the posterior distributions $d^F|d^B$ and $\omega|d^B$. Define $s_0 = -\infty$ and $s_{2N+1} = +\infty$.

Moreover, we choose $s_1, \cdots, s_n$ such that the ex-ante probabilities $\max(P(d^F \leq s_n), P(d^B \leq s_n) < \delta$ for some δ sufficiently small: the first n signals in the discrete space happen with small probability. These correspond to 'fail' signals. We can choose $s_{N+1}, \cdots, s_{2N}$ arbitrarily, as long as they converge to the continuous distribution as $N \to \infty$. Let them be distributed of equal probability.

Step 2. Define the "optimal bank threshold" under $d^F = s_{N+i}$ for each i.

Specifically, for each $i \in \{1, 2, \cdots, n\}$, given $d^F = s_{N+i}$, choose the smallest j for which $E[u^F(\omega)|d^F = s_i, d^B = s_j] \geq 0$. Then by monotonicity, this s_j is the smallest bank's signal for which the regulator would like the bank to invest. Denote this $s_j = s^{B*}(s_{N+i})$: then by definition, bank investing iff $d^B \geq s^{B*}(d^F)$ is the first-best investment rule.

Step 3. We define the disclosure structure $\pi : m|d^F$ and the punishment threshold s^* and c as follows:

(a) Define the threshold s^* to be s_n: banks fail the test and are punished iff $d^F \leq s_N$. This would guarantee ex-ante that only δ of the banks are punished.

(b) Define the disclosure policy π as follows:

- Upon observing $d^F = s_i$ ($i \geq N+1$), send message $m = s_i$: reveal d^F fully.
- Upon observing $d^F = s_i$ ($i < N$), send message $m = s_i$ with probability $1 - \epsilon_i$ and send $m = s_{N+i}$ with probability ϵ_i, where $\max[\epsilon_i] < \epsilon$.

From this we immediately see that $m = d^F$ with ex-ante probability $\geq 1 - \epsilon$.

(c) For each i, choose c sufficiently large and $\epsilon_i < \epsilon$ such that bank with signal $d^B = s^{B*}(s_{N+i})$ and seeing message $m = s_{N+i}$ gets expected utility ϵ' from investing in risky asset; banks will never invest if seeing signal $m \leq s_N$.

The equation defining c, ϵ_i is

$$\begin{aligned}\epsilon' =& \mathbb{E}[u^B(\omega)|d^B = s^{B*}(s_{N+i}), m = s_{N+i}] - cP[d^F \leq s_N|d^B = s^{B*}(s_{N+i}), m = s_{N+i}] \\ =& P[d^F = s_{N+i}|d^B, m = s_{N+i}]\mathbb{E}[u^B(\omega)|d^B = s^{B*}(s_{N+i}), d^F = s_{N+i}] \\ &+ P[d^F = s_i|d^B = s^{B*}(s_{N+i}), m = s_{N+i}]\mathbb{E}[u^B(\omega)|d^B = s^{B*}(s_{N+i}), d^F = s_i] \\ &- cP[d^F = s_i|d^B = s^{B*}(s_{N+i}), m = s_{N+i}]\end{aligned}$$

Write $q(\epsilon_i) = \mathbb{P}[d^F = s_i|d^B = s^{B*}(s_{N+i}), m = s_{N+i}]$. This is the posterior probability that a bank fails the test when it receives message $m = s_{N+i}$ and has signal $s^{B*}(s_{N+i})$. Also write $u_{i1} = \mathbb{E}[u^B(\omega)|d^B = s^{B*}(s_{N+i}), d^F = s_{N+i}]$ and $u_{i2} = \mathbb{E}[u^B(\omega)|d^B = s^{B*}(s_{N+i}), d^F = s_i]$. Then the above equation is simplified to:

$$\epsilon' = (1 - q(\epsilon_i))u_{i1} + q(\epsilon_i)u_{i2} - q(\epsilon_i)c$$

Now $q(\epsilon_i)$ is a continuous function on ϵ_i and $q(\epsilon_i) \to 0$ as $\epsilon_i \to 0$. So for sufficiently large c, there is some $q(\epsilon_i) \ll 1$ that satisfy this equation, and we can set $\max \epsilon_i < \epsilon$ if c is large enough.

Send $\epsilon' \to 0$ first, the bank's investment rule is: upon seeing $m = s_{N+i}$, invest if and only if $d^B \geq s^{B*}(m) = s^{B*}(s_{N+i})$. Banks will be punished with probability δ; and $\bar{d}$ and m are identical with probability at least $1 - \epsilon$.

As $\epsilon \to 0$ (and $c \to \infty$), $m = d^F$ with probability almost 1, and the bank's investment rule $d^B \geq s^{B*}(m)$ is identical to the first-best rule.

$\square$

Chapter 3

Exchange Rate Policy for a Growing Economy

3.1 Introduction

Why did East Asian countries, such as Japan, South Korea, and China, employ currency pegging strategies at an undervalued level during periods of accelerated growth? The exchange rate plays a pivotal role in shaping the economic landscape of developing nations, particularly in the context of rapid economic growth. This paper studies the rationale behind these policy choices and their implications for reserve accumulation, and seek to understand whether the level of the peg matters in a *mercantilist* way. I seek to contribute to the exchange rate policy literature by studying the relationship between exchange rates and economic growth, and how monetary policy and foreign exchange intervention intermediates both.

I address these questions by developing a policy framework that emphasizes nominal wage rigidity and financial frictions. The model integrates the exchange rate determination and optimal policy framework by Itskhoki and Mukhin (2023) with the nominal wage rigidity and imperfect substitution framework of Kim et al. (2024) to study the implications of exchange rate policies for a growing economy. As in the canonical framework, the model features noise traders and arbitraguers, and monetary policy and forex interventions have a one-to-one correspondence to output gaps and exchange rates. The model diverges from the conventional analysis by emphasizing wage rigidity, as opposed to price rigidity, which generate different and realistic terms-of-trade and trade imbalance effects. I use this framework to examine the policy dynamics of rapidly growing economies than those in steady-state.

A central focus of this paper is determining the optimal level of peg. In an environment with nominal rigidities and imperfect substitution, the level at which the peg is set, in conjunction with the underlying path of productivity, can significantly influence real outcomes through its impact on the substitution of tradable goods. This line of inquiry allows us to ask whether a "mercantilist" peg that systematically undervalues the domestic currency is feasible, and whether it is welfare-improving, within a canonical framework of exchange rate determination.

After building the framework in Section 3.2, I analyze the positive and normative implications of exchange rate policies. I first ask the optimal level of peg problem: conditional on a peg, what is the optimal level? I demonstrate that a lower pegged exchange rate would reduce unemployment, increase the trade surplus and net foreign asset, but simultaneously deteriorate the domestic terms-of-trade. This analysis uncovers the implausibility of systematic devaluation: the Home government might prefer an overvalued currency relative to the laissez-faire exchange rate. Furthermore, the anticipation of future economic growth provides a stronger incentive for the government to favor a higher pegged rate, as current market rates do not fully account for prospective growth.

Turning to optimal policy, I first solve for the constrained optimum consumption for the Home planner. I show that the optimal consumption given resource constraints eliminate the international risk sharing wedge, and the optimal consumption balances the labor wedge with the optimal terms-of-trade. Based on this analysis, I hypothesize that a forex intervention that negates noise trader liquidity demand, combined with a monetary policy that sets the exchange rate slightly above the laissez-faire level constitutes an optimal strategy. Based on these results, standard economic models of exchange rates encounter difficulties in rationalizing policies that favor an undervalued currency peg.

In Section 3.4, I explore a potential justification for an undervalued peg: the productivity-enhancing effects of leraning-by-exporting externalities. This channel is realistic in the context of aforementioned East Asian developing economies, and can rationalize selling goods abroad at cheaper prices. I analytically characterize the first-order effects of a lower exchange rate, and numerically simulate to test the welfare implications of an undervalued peg. My numerical simulations suggest that the learning-by-doing externality needs to be sufficiently strong, and even then the terms-of-trade motive and the labor wedge may make the optimal exchange rate policy unclear.

Related Literature

This paper contributes to a strand of literature that studies the role of exchange rates in both goods and financial markets. My model heavily draws on the exchange rate determination and optimal policy framework of Itskhoki and Mukhin (2021a) and Itskhoki and Mukhin (2023). This builds on a long-standing literature on UIP deviations from financial frictions by Kouri (1976), Devereux and Engel (2003), Schmitt-Grohé and Uribe (2003), Alvarez et al. (2009), Gabaix and Maggiori (2015). This paper contributes by considering the conditional response of exchange rates in response to a large productivity growth, the optimal level of peg under such growth, and whether a systematic undervaluation can be rationalized.

An important channel of exchange rate policy affecting real outcomes is expenditure switching, and the role of exchange rates as a shock absorber. This idea dates back to Meade (1951) and Friedman (1953), and have been studied by (Broda, 2001, 2004; Edwards and Levy Yeyati, 2005; Carrière-Swallow et al., 2021) in the goods market and (Schmitt-Grohé and Uribe, 2016; Campbell, 2020; Ahn et al., 2022) in the labor market. Our positive predictions suggest that even under financial frictions and volatility determining exchange rate, the underlying fundamental productivity moves the nominal exchange rate on average, and the optimal exchange rate policy closely follws this 'natural' exchange rate, and the differences caused by terms-of-trade externalities.

A mechanism studied in Section 4 of the paper is how a currency undervaluation can be rationalized by learning-by-exporting externalities. Ito et al. (1999) and Rodrik (2008) suggest that an undervaulation of the real exchange rate is correlated with economic growth. Aizenman and Lee (2010) and Korinek and Servén (2016) are similar to our study in that it links the learning-by-doing externality with net exports and real exchange rates, but I go further and connect this with an exchange rate peg. Lucas and Moll (2014), Bloom et al. (2016), Sampson (2016), Perla et al. (2021) offer evidences and different theoretical mechanisms of learning-by-doing externality. The results in this paper suggest that this externality is necessary for rationalizing currency undervaluation, and conducts a numerical analysis to test the validity of this channel. A concurrent paper, Ottonello et al. (2024), study the implications of exchange rates as an industrial policy in the presence of externalities; this paper explicitly considers financial fluctuations of the exchange rate, and consider the efficiency of an explicit currency peg.

This paper proceeds as follows. Section 3.2 presents the model framework and the policy problem. Section 3.3 derives the positive predictions of the economy under a laissez-faire policy, a peg at a specific value, and the optimal policy. Section 3.4 introduces the wedges and learning-by-exporting externality and simulates the economy. Section 3.5

concludes.

3.2 Model Setup

In this section, I develop a tractable model of exchange rate determination, nominal wage rigidity and trade imbalances. I impose a number of strong assumptions for analytical tractability. Future work seeks to extend this model into a multi-sector model, conducting a quantitative analysis akin to Kim et al. (2024).

3.2.1 Model Setup

The environment has two countries, Home (H) and Foreign (F). Since our context is the pegging problem of a developing country to a dominant currency, the Foreign can be considered the United States, or the rest of world. The model is a infinite-period model. Home is populated by a representative household that consumes goods from both Home and Foreign, and supplies labor to firms that produce goods. Each country has its own nominal account: the Foreign nominal account is the dollar, and I will use this term flexibly. The exchange rate e_t at time t is defined to be the price of dollars in units of Home currency. Hence an increase in e_t is a depreciation of Home currency.

To simplify the problem, I assume that Home is a small open economy; this implies that import prices (in dollars) and the interest rates can be taken as exogenous, and I additionally assume that the demand curve for Home goods is taken as exogenous. There is one tradable sector in the model, and goods are distinguished by origin; the model can be naturally extended to a tradable-nontradable model.

Household preferences. At Home, there is a representative agent of mass L who consumes domestic goods C_{Ht} and imports C_{Ft} aggregated into a final good C_t, and supplies labor L_t. The household has preferences represented by a log-linear utility

$$U_{j0} = \sum \mathbb{E}_0[\beta^t(\log C_t - \gamma L_t)] \quad \text{where} \quad C_t = C_{Ht}^{\gamma} C_{Ft}^{1-\gamma} \tag{3.1}$$

where γ is the share of expenditure on Home goods, potentially capturing the degree of home bias or iceberg trade costs.[1] Here as domestic goods and imports are imperfect

[1]The Cobb-Douglas assumption is for simplifying the exposition. Using a CES utility $C_t = (\gamma^{\frac{1}{\sigma}} C_{Ht}^{\frac{\sigma-1}{\sigma}} + (1-\gamma)^{\frac{1}{\sigma}} C_{Ft}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$ with $\sigma > 1$ the Armington elasticity would lead to analogous results.

substitutes, the law of one price fails. The household can borrow or lend using a one-period risk-free home-currency bond B_t, leading to the budget constraint:

$$P_{Ht}C_{Ht} + P_{Ft}C_{Ft} + \frac{B_t}{R_t} = B_{t-1} + W_t L_t + T_t \tag{3.2}$$

where T_t is a lump-sum transfer from the government and financial agents, and R_t is the interest rate set by the Home government.

Foreign demand. Since the Home economy is a small open economy, we need a notion of demand for Foreign goods. I assume that the global residual demand for Home exports is isoelastic with elasticity $\sigma > 1$:

$$C^*_{Ht} = C^* \cdot (P^*_{Ht})^{-\sigma} \tag{3.3}$$

where P^*_{Ht} is the unit export price in dollars and C^* is a constant that Home takes as exogenous. Net exports in dollars is given by

$$NX_t = \underbrace{P^*_{Ht} \cdot C^*_{Ht}}_{\text{exports}} - \underbrace{C_{Ft}}_{\text{imports in dollars}}$$

Here the assumption $\sigma > 1$ is important, as it guarantees that domestic goods are gross substitutes to Foreign-produced goods, and expenditure shifts to Home following productivity growth.

Technology. A representative, competitive firm at Home uses labor as an input and has a constant returns to scale production function that requires $\frac{1}{A_t}$ labor to supply a unit of good to Home and the global market. Thus for the Home firm, output is given by

$$Y_t = A_t L_t$$

Since Home is a small open economy, Home faces an exogenous export price P_{Ft}. I assume a stable price level of imports in dollars: $P^*_{Ft} = 1$, or $P_{Ft} = e_t P_{Ft} = e_t$. This essentially assumes that Foreign productivity is fixed, and thus has the implication that Home productivity growth is a **relative** productivity growth, connecting with (JMP) that relative productivity plays a part in imbalances and inflation. I assume that Home productivity is a geometric random walk, given by

$$\log A_t = \log A_{t-1} + \epsilon^A_t \tag{3.4}$$

where ϵ_t^A is a productivity shock.[2] In the baseline model, I assume that the shock ϵ_t^A is exogenous and distributed $N(0, \sigma_{At}^2)$. I endogenize the productivity shock ϵ_t^A in Section 3.4 to incorporate learning-by-doing externalities.

Labor market and nominal rigidity. I assume that there is friction in nominal wages, leading to different terms-of-trade and labor market dynamics compared to the frictional price case. I assume that nominal wage resetting is subject to Rotemberg friction that generates a wage Phillips curve:

$$\log(\frac{W_t}{W_{t-1}}) = \kappa \mu_t^L + \beta \log(\frac{W_{t+1}}{W_t}). \tag{3.5}$$

where $\mu_t^L = 1 - \frac{W_t}{P_{Ht}C_{Ht}}$ is the labor wedge that measures the gap between the Home household's optimal labor supply and the actual labor demanded, such that $\mu_t^L > 0$ implies unemployment, and $\mu_t^L < 0$ implies the economy is overheated. Since wages are not in equilibrium, we assume that the labor market is demand-determined through domestic and export demand, and workers supply whatever labor is demanded at the goods price.

Financial Market and Exchange Rates. The setting here follows Itskhoki and Mukhin (2023). There are two bonds: domestic bonds with rate R_t set by the Home government, and a global bond with an exogenous interest rate R_t^*. The household at Home can only invest in its domestic bonds, and three types of agents intermediate home and foreign currency bonds in the international financial market. Those are noise traders, arbitraguers and the government: the first two are the source of friction in the financial market that determine the exchange rate, while the third participates in the market to manage, and possibly peg the exchange rate.

There is a continuum of one-period noise traders of mass n that represent the idiosyncratic demand for liquidity in foreign currency. They take a zero-capital portfolio (N_t, N_t^*) such that $N_t/R_t + e_t N_t^*/R_t^* = 0$. I assume that this demand for foreign currency is an exogenous process, in that

$$\frac{N_t^*}{R_t^*} = n(e^{\psi_t} - 1) \text{ with } \psi_t = \rho_t \psi_{t-1} + \sigma_\psi \epsilon_t^\psi$$

[2]This is a limit case of the Itskhoki and Mukhin (2023) setup where productivity follows an AR(1) process, with the limit $\rho \to 1$. This paper is interested in the context of a permanent growth in developing countries as opposed to a temporary mean-reverting growth, so this assumption is more realistic.

A positive $\frac{N_t^*}{R_t^*}$ implies that there is a positive demand for Foreign bonds, and vice versa.

The arbitraguers are also one-period agents that also hold a zero-capital portfolio (D_t, D_t^*) such that $D_t/R_t + e_t D_t^*/R_t^* = 0$, with the carry trade income is

$$\pi_{t+1}^{D^*} = D_t^* - D_t/e_t = \tilde{R}_{t+1}^* \cdot \frac{D_t^*}{R_t^*}$$

in dollars, where $\tilde{R}_{t+1}^* = R_t^* - R_t \frac{e_t}{e_{t+1}}$ is the UIP deviation in dollar terms. The arbitraguer chooses their portfolio to maximize min-variance preferences over profits:

$$V_t(\Pi_{t+1}^D) = E_t[\frac{e_t}{e_{t+1}} \Theta_{t+1} \Pi_{t+1}^D] - \frac{\omega}{2} var(\Pi_{t+1}^D) \tag{3.6}$$

where $\Theta_{t+1} = \beta \frac{C_{Ft}}{C_{Ft+1}}$ is the stochastic discount factor (SDF) of the Home household and the second term reflects the risk aversion, with ω being the risk aversion parameter. Both the noise traders and arbitraguers are agents at Home, and their income and losses are transferred to the household through a lump-sum transfer T_t.

The Home government can also invest in bonds from both countries. Specifically, the Home government buys Home and Foreign bonds (F_t, F_t^*) that have total value $\frac{F_t}{R_t} + \mathcal{E}_t \frac{F_t^*}{R_t^*}$ in domestic currency. Any holdings are rebated to (or taken from) the Home household through a lump-sum transfer (or tax). The Foreign government sets an interest rate R_t^* on Foreign bonds, which the Home government takes as exogenous.

Government and policy objective. The government has two policy tools: monetary policy R_t and foreign exchange reserves (F_t, F_t^*) to maximize the utility of the household – with one difference: the government puts less weight on the labor disutility. Specifically, Home government chooses R_t and (F_t, F_t^*) to maximize

$$U_g = \sum \beta^t (\log C_t - \gamma L_t),$$

subject to the equilibrium constraints. We also assume that instead of choosing R_t, the government directly chooses aggregate expenditure $P_t C_t$, because there is a one-to-one map between R_t and $P_t C_t$ given wage rigidity and the domestic bond market clearing condition.

3.2.2 Competitive equilibrium

In a competitive equilibrium, households maximize their utility, firms and arbitraguers maximize their profit, and markets clear. In this subsection, we derive each condition.

Utility maximization. The household at country j chooses consumption and savings to maximize discounted utility as described in Equation 3.1 subject to the budget constraint (Equation 3.2). The first-order conditions to the household utility maximization problem are given by:

$$\frac{\gamma}{1-\gamma} = \frac{P_{Ht}C_{Ht}}{P_{Ft}C_{Ft}} \tag{3.7}$$

$$1 = \beta R_t \mathbb{E}_t \frac{P_{Ft}}{P_{Ft+1}} \frac{C_{Ft}}{C_{Ft+1}} \tag{3.8}$$

where P_t denotes the consumer price index. If wages were flexible, we would additionally have the labor supply constraint $C_{Ht} = \frac{W_t}{P_{Ht}}$, but wages are sticky and this condition is replaced by the wage Phillips curve (Equation 3.5).

Firm optimization. Since firms are competitive, domestic firm profits are zero, and the unit price is equal to marginal cost. Import prices are stable in dollar terms, giving us the following equations for prices:

$$P_{Ht} = \frac{W_t}{A_t}, \;\; P^*_{Ht} = \frac{1}{e_t}\frac{W_t}{A_t}, \;\; P_{Ft} = e_t. \tag{3.9}$$

Arbitraguers. The arbitraguers choose their capital portfolio (D_t, D^*_t) to maximize their mean-variance utility (Equation 3.6). The optimal portfolio choice is given by

$$\frac{D^*_t}{R^*_t} = \frac{1}{\omega\sigma^2_t}\mathbb{E}_t(\Theta_{t+1}\tilde{R}_{t+1}) \tag{3.10}$$

where $\sigma^2_t = var_t(\tilde{R}^*_{t+1}) = R^2_t \cdot var_t(\frac{e_t}{e_{t+1}})$ measures the volatility of the nominal exchange rate, coming from both Home productivity volatility and noise trader's currency demand volatility. From this, we see that the UIP deviation is the risk premium that the arbitraguers are charging, measured by the volatility of the exchange rate; it would be zero if the exchange rate is fully pegged.

Market clearing. The goods market clearing condition is given by

$$Y_t = A_t L_t = C_{Ht} + C^*_{Ht} \tag{3.11}$$

The financial market clearing condition requires that the home-currency bonds sum to zero:

$$B_t + N_t + D_t + F_t = 0 \tag{3.12}$$

and Foreign bonds is supplied with an exogenous interest rate R^*_t in dollar terms. The government's net income from the financial market is given by

$$T^g_t = F_{t-1} + e_t F^*_{t-1} - (\frac{F_t}{R_t} + \frac{e_t F^*_t}{R^*_t}),$$

and the profits of noise traders and arbitraguers is given by

$$\Pi^*_t = e_t \tilde{R}^*_t \frac{N^*_{t-1} + D^*_{t-1}}{R^*_{t-1}},$$

and so the transfer to the household is

$$T_t = T^g_t + \Pi^*_t = F_{t-1} + e_t F^*_{t-1} - (\frac{F_t}{R_t} + \frac{e_t F^*_t}{R^*_t}) + e_t \tilde{R}^*_t \frac{N^*_{t-1} + D^*_{t-1}}{R^*_{t-1}} \tag{3.13}$$

The market clearing condition allows us to transform the household budget constraint and the arbitraguer's optimality constraint in simpler forms. First, the household budget constraint can be simplified using the net foreign asset (NFA) position B^*_t in dollars, as in Itskhoki and Mukhin (2023). The net foreign asset position is given by

$$B^*_t := \frac{R^*_t}{\epsilon_t}(\frac{B_t + F_t}{R_t}) + F^*_t = F^*_t + N^*_t + D^*_t$$

where the second equality follows from the fact that noise traders and arbitraguers have zero-capital. Then plugging in the household transfers (Equation 3.13), the household budget constraint can be rewritten in NFA terms as

$$\frac{B^*_t}{R^*_t} - B^*_{t-1} = NX_t \tag{3.14}$$

which show that the net foreign asset in dollars evolve through the dollar value of net exports. Here we assumed that Home owns the financial sector, so there is no international transfers.

Moreover, we can rearrange the arbitraguer's optimality condition (Equation 3.10) to remove the dependency on D_t^* and get the international risk sharing condition:

$$\beta R_t^* \mathbb{E}_t \frac{C_{Ft}}{C_{Ft+1}} = 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*} \tag{3.15}$$

where the last term corresponds to UIP deviations.

Initial conditions. The last ingredient of an equilibrium in this economy is an initial condition. There are three initial condition variables: $A_{-1}, B_{-1}^*, W_{-1}, e_{-1}$. We assume that the economy was in a trade-balance steady-state at $t = -1$ (this pins down C^*, and gives a relation between W_{-1} and e_{-1}). This is to naturally interpret the results as deviations from the steady-state.

Equilibrium. The equilibrium is as follows. Given initial conditions $\{A_{-1}, B_{-1}^*, W_{-1}\}$, exogenous $\{R_t^*, C^*\}$, stochastic processes $\{A_t, N_t^*\}$, and Home government policy $\{C_t, F_t, F_t^*\}$, an equilibrium comprises $\{C_{Ht}, C_{Ft}, C_{Ft}^*, B_t^*, e_t, L_t, W_t, D_t^*\}$ and an implied volatility σ_t^2 such that households maximize their utility, arbitraguers maximize their profits, and goods and bond market clears. These correspond to Equations 3.7 to 3.15.[3]

3.3 Response to growth and optimal policy

In this section, we study the positive and normative predictions in a growing economy without externalities. First, we study the positive effect of a positive permanent productivity shock ($\epsilon_0^A > 0$) given specific policies at Home. Next, we turn to normatives, and study the optimal policy (R_t, F_t^*) for the Home government that faces $\epsilon_0^A > 0$.

3.3.1 Positive predictions

How would the exchange rate, trade imbalances, and macroeconomic aggregates for the Home country eolve when its productivity permanently grows? The answer to this question critically depends on the conduct of domestic policy. In this subsection, we analyze two polar opposite cases as benchmark: when the Home government's policy is

[3]Given $P_t C_t$, W_t, and e_t, the household optimization problem gives C_t, C_{Ht}, C_{Ft}; the Foreign optimization problem gives C_{Ht}^* and NX_t, hence B_t^*, D_t^*, L_t are determined.

fully inward-looking (“Laissez-faire forex policy"), and the opposite case, where Home government pegs the currency at a rate $e_t = \bar{e}$.

Laissez-faire policy. Let's first consider the baseline case where the government is fully inward-looking: $F_t = F_t^* = 0$, and monetary policy is such that Home achieves full employment ($\mu_t^L = 0$). What happens to the exchange rate and trade balance of Home under this economy? The next proposition gives the answer.

Proposition 3.1. *At the limit $\sigma_A \to 0, \sigma_\psi \to 0$, in response to a positive productivity shock ($\epsilon_0^A > 0$), we have (1) Home currency appreciates ($e_0 \downarrow$), and (2) trade is balanced in period 0 ($NX_0 = 0$). With a larger σ_t^ψ, this is true in expectation, and the following comparative statics hold: with a higher N_t^*, Home currency depreciates ($e_0 \downarrow$), and Home runs a trade surplus.*

The intuition behind the first part is as follows. Following a positive productivity shock, Foreign demand for Home goods rise, and Home monetary policy raises wages until full employment is restored. If the magnitude of future shocks go to zero, the equilibrium exchange rate at $t = 0$ will be at the new trade-balance rate.

On the other hand, if $\sigma_\psi > 0$, then the financial shocks N_0^* create a demand shock for Foreign currency that the arbitraguers will absorb, requiring carry trade profits. This carry trade creates deviations from UIP, a wedge in the international risk-sharing condition; this moves exchange rate e_0 away from the frictionless level. The proposition clarifies that following productivity growth, absent forex interventions, exchange rate appreciates on average, but the sign of the net exports is ambiguous, and the liquidity demand shock N_t^* can drive both directions if the sign is large enough.

Exchange rate peg. Next, let's study the effects of a currency peg at an exogenous level $\bar{e}$. A question we ask is what are the implications of pegging at different levels? At what levels would a peg be sustainable, and what are the NFA and welfare implications? We first prove two auxiliary statements about the currency peg.

Lemma 3.1. *Under a full currency peg $e_t = \bar{e}$, the Home interest rate is fixed at $R_t = R_t^*$, and Home government cannot choose monetary policy R_t independently.*

This is because under a currency peg, the volatility of the exchange rate is zero, so arbitraguers are taking riskless investments: they could substitute across both bonds freely, and this undoes the segmented capital markets.

Proposition 3.2. *The marginal effect of a currency undervaluation (higher $\bar{e}$) is given by the following:*

- *NX_t increases: Home runs a trade surplus.*
- *μ_0 increases; unemployment improves, overheats economy if already hot.*
- *Terms-of-trade worsens, and gradually recovers.*

With these effects, the marginal change in welfare is given by

$$\frac{dU}{d\bar{e}} = \underbrace{\mu_{H0}\frac{dC_{H0}}{d\bar{e}} + \mu^*_{H0}\frac{dC^*_{H0}}{d\bar{e}}}_{\text{labor wedge}} + \underbrace{\lambda_0 C^*_{H0}\frac{dP^*_{H0}}{d\bar{e}}}_{\text{today TOT}} + \underbrace{\frac{dU_1}{dW_0}\frac{dW_0}{d\bar{e}}}_{\text{future TOT}} \tag{3.16}$$

where λ_0 is the marginal utility of a dollar to the Home household, and U_1 is the expectation of future utility.

Proof. Derivation in Appendix (Section 3.6). □

Pegging at a higher exchange rate has three effects: Home would run a trade surplus as exports rise and imports fall. Because wages are rigid, the change in Home wages cannot fully undo the exchange rate effect. At the same time, the increase in export demand would increase aggregate demand for Home labor, so if there was unemployment, it would be lessened, and possibly overheat the economy. Third effect is terms-of-trade in each period that is worsened by the exchange rate devaluation – this effect dissipates over time as Home's nominal wage adjusts. Since the short-run terms-of-trade manipulation motive is not internalized by the Household, we conjecture the following:

Conjecture 1. *If the laissez-faire exchange rate is given by e_0^{fl}, the optimal level of currency peg satisfies $\bar{e} < e_0^{fl}$: Home government should set the pegged exchange rate at an overvalued level.*

I conjecture this to be true because e_0^{fl} is at the equilibrium value, the changes in welfare resulting from the labor wedge should be second-order, but the gains in terms-of-trade from setting $\bar{e}$ at an appreciated value would be of first-order. As such, the Home government in a growing economy would want to set an exchange rate that is *overvalued* compared to the laissez-faire exchange rate.

A caveat is that we only treated the case when there was one growth shock $\epsilon_0^A > 0$. What happens if there is a sequence of growth shocks $\epsilon_t^A > 0$ and the Home government anticipates it? The result should be that the pegged exchange rate $\bar{e}$ should be even more appreciated, as the level pegged needs to account for not just A_0, but future productivity as well. These results suggest that, in a standard model, a peg that *undervalues* its currency is difficult to rationalize, and if anything, the pegged currency should be *overvalued* when considering terms-of-trade effects.

3.3.2 Optimal policy

In this subsection, we study optimal policy of the Home government in this economy. Would the optimal policy involve completely pegging the currency, losing monetary independence? Would the optimal policy allow some floating, and target full employment? We assume that the Home government has full commitment power, and chooses monetary policy $\{R_t\}$ and forex intervention $\{F_t, F_t^*\}$ in this economy to maximize Home welfare. To simplify our exposition, we replace the wage Phillips curve (Equation 3.5) with the condition that wages are completely rigid ($W_t = \bar{W}$); this is equivalent to sending $\kappa \to 0$.

Policy problem. Given the constraints and equilibrium definition, the policy problem in the baseline economy can be written as:

$$\max \mathbb{E} \sum_t \beta^t [\gamma \log C_{Ht} + (1-\gamma) \log C_{Ft} - \gamma L_t]$$

subject to the following constraints:

$$\begin{aligned}
L_t &= \frac{C_{Ht} + C_{Ht}^*}{A_t} \\
C_{Ht}^* &= C^* \cdot (P_{Ht}^*)^{-\sigma} \\
P_{Ht}^* &= \frac{W_t}{A_t e_t} \\
\beta R_t \mathbb{E}_t \frac{P_t}{P_{t+1}} \frac{C_t}{C_{t+1}} &= 1 \\
\frac{B_t^*}{R_t^*} - B_{t-1}^* &= P_{Ht}^* C_{Ht}^* - C_{Ft} \\
\beta R_t^* \mathbb{E}_t \frac{C_t}{C_{t+1}} &= 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*} \\
\sigma_t^2 &= R_t^2 var_t(\frac{e_t}{e_{t+1}})
\end{aligned}$$

To solve this policy problem, let's instead solve for the constrained optimization problem, leaving F_t^* and R_t as a free variable. Plugging in the market clearing condition, the constrained optimization problem can be written solely in terms of the dollar budget constraint:

$$\max_{C_{Ht}, C_{Ft}, C_{Ht}^*, B_t^*} \mathbb{E} \sum_t \beta^t \left[\gamma \log C_{Ht} + (1-\gamma) \log C_{Ft} - \frac{\gamma}{A_t} (C_{Ht} + C_{Ht}^*) \right] \tag{3.17}$$

such that

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = P_{Ht}^*(C_{Ht}^*)C_{Ht}^* - C_{Ft}$$

The first-order optimality conditions of this problem describe what the optimal policy would target. The three first-order conditions are:

$$\frac{1}{C_{Ht}} = \frac{1}{A_t} \tag{C_{Ht}}$$

$$\beta R_t^* \mathbb{E}_t \frac{C_{Ft}}{C_{Ft+1}} = 1 \tag{C_{Ft}}$$

$$\lambda_t \left(\frac{dP_{Ht}^*}{dC_{Ht}^*} C_{Ht}^* + P_{Ht}^*\right) = \frac{\gamma}{A_t} \tag{C_{Ht}^*}$$

The first condition states that the labor wedge must be zero: since $P_{Ht} = \frac{W_t}{A_t}$, the right-hand-side is the real wage, and this equates marginal utility of consumption with marginal disutility of labor. The second condition states that the constrained optimum satisfies international risk sharing without wedge: the government should play the role of the arbitraguer, taking on the liquidity demand from noise traders and households without charging the arbitraguers' carry trade premium. The third condition highlights the terms-of-trade manipulation motive – Home has market power and can charge markup, but the representative, competitive firm does not internalize this, and the constrained optimum would take advantage.

However, we show that the government cannot target all three objectives:

Lemma 3.2. *With uncertainty in underlying productivity ($\sigma_A > 0$), achieving all three objectives through R_t and F_t^* is impossible.*

The reason is as follows: the only way to undo the risk sharing wedge is to guarantee $\sigma_t^2 = 0$ or $F_t^* = B_t^* + N_t^*$. Under the former, future exchange rate e_{t+1} has to be determined before realization of A_t, so terms-of-trade cannot be targeted; under the latter, we only have monetary policy R_t^* to target both labor wedge and terms-of-trade. At the same time, we see that if we have an additional instrument (such as an export tax), we can effectively target all three objectives, reaching the constrained optimum.

Given two instruments, we conjecture that the optimum exchange rate *follows* the productivity level A_t, at a slightly higher level to take advantage of terms-of-trade manipulation motive. This would not be a peg, but a floating exchange rate that is heavily managed through forex manipulation F_t, F_t^*, with an intuition similar to Fanelli and Straub (2021) and Itskhoki and Mukhin (2023).

3.4 Learning-by-doing Externalities

In this section, we explore a potential channel in which a mercantilist peg could be rationalized: through a learning-by-doing externality.

3.4.1 Learning-by-doing: setup and analytical results

The setup is identical to the model setup in Section 3.2, with one difference: I replace the exogenous productivity innovation ϵ_t^A with an endogenous function of exports:

$$\log A_t = \log A_{t-1} + f(X_{t-1}, A_{t-1})$$

where f_A is the learning-by-doing innovation that depends on both the previous period export X_{t-1} and productivity A_{t-1}. We use exports instead of total output, because the specific channel of learning we are interested in, and has more support in the literature, is learning through trade. The dependency on previous-period productivity is also natural, as learning-by-doing is likely to have decreasing returns as the country grows. We assume the following regularity conditions for $f(\cdot)$:

Assumption 3.1. *The learning-by-doing externality $f(\cdot, \cdot)$ satisfies the following:*

- *$f(X, A)$ is an increasing function of X and a decreasing function of A.*
- $\log A + f(X, A)$ *is bounded.*
- *$f(X, A) < 0$ for small X, $f(X, A) > 0$ for large X.*

The second condition ensures that Home productivity cannot explode through learning-by-doing. Under these conditions, I analytically and numerically investigate the merits of an undervalued peg.

Effects of a peg. I first study the effects of a currency peg at an exogenous level $\bar{e}$, when there is a learning-by-doing externality. What happens under a full peg? Replicating the logic in Proposition 3.2 gives us the following result:

Lemma 3.3. *The marginal effect of a currency undervaluation (higher $\bar{e}$) is:*

- *NX increases; higher surplus*
- *μ_0 increases; unemployment improves, overheats economy if already hot.*
- *Terms-of-trade worsens, recovers over time*

- *Future productivity A_{t+1} is higher.*

In welfare terms, if we denote

$$\mathcal{U}_t = \mathbb{E}_t[\sum_{s=t}^{\infty} \beta^{s-t}(\log C_s - \gamma L_s)|A_{t-1}, B_{t-1}, W_{t-1}]$$

the conditional utility at time t given initial conditions, we have the following first-order effect of change in $\bar{e}$:

$$\frac{dU_t}{d\bar{e}} = \underbrace{\mu_{Ht}\frac{dC_{Ht}}{d\bar{e}} + \mu^*_{Ht}\frac{dC^*_{Ht}}{d\bar{e}}}_{\text{labor wedge}} + \underbrace{\lambda_t C^*_{Ht}\frac{dP^*_{H0}}{d\bar{e}}}_{\text{TOT at time t}} + \underbrace{\frac{dU_{t+1}}{dW_t}\frac{dW_t}{d\bar{e}}}_{\text{future TOT}} + \underbrace{\frac{dU_{t+1}}{dA_{t+1}}\frac{dA_{t+1}}{d\bar{e}}}_{\text{LBD externality}} \tag{3.18}$$

Notably, $\frac{dA_{t+1}}{d\bar{e}} = A_t \frac{df(Y_t, A_t)}{dY_t}\frac{dY_t}{d\bar{e}} > 0$, so Home would want to undervalue its currency relative to the no-externality benchmark.

The proof is analogous to the proof of Proposition 3.2, with the last term correpsonding to the learning-by-doing externality. The last term in Equation 3.18 highlights the learning-by-doing externality: undervaluing the exchange rate (higher $\bar{e}$) increases the output (Y_t), which increases future productivity A_{t+1} which increases lifetime welfare. If this LBD externality is strong enough, it can potentially outdo the terms-of-trade channel (which calls for a higher exchange rate) and rationalize a peg at a lower value.

But this isn't saying that an undervalued peg is optimal; there may be alternative policies that may be better. We compare the results of alternative policies numerically in the next section.

3.4.2 Numerical illustrations

To complement the analysis, I conduct a numerical simulation of the model, and compare the welfare implications of different policies. This gives us a sense of whether pegging at a lower value can be rationalized, and a rough quantification if it can be. We compare the outcome and welfare implications of different policies.

Specifically, we assume that $f(Y, A) = \alpha(1 - A)(\log X - \log X_{-1})$, where X_{-1} is the pre-period export level; there is no externality in the baseline. With this functional form, we have that A_{t+1} grows faster when A_t is lower (further away from the global frontier of $A = 1$), when X_t is higher, and productivity *decreases* once we reach the global frontier $A = 1$.[4]

[4] Numerically we never reach $A = 1$; exports decline as wage increases, undoing the externalities.

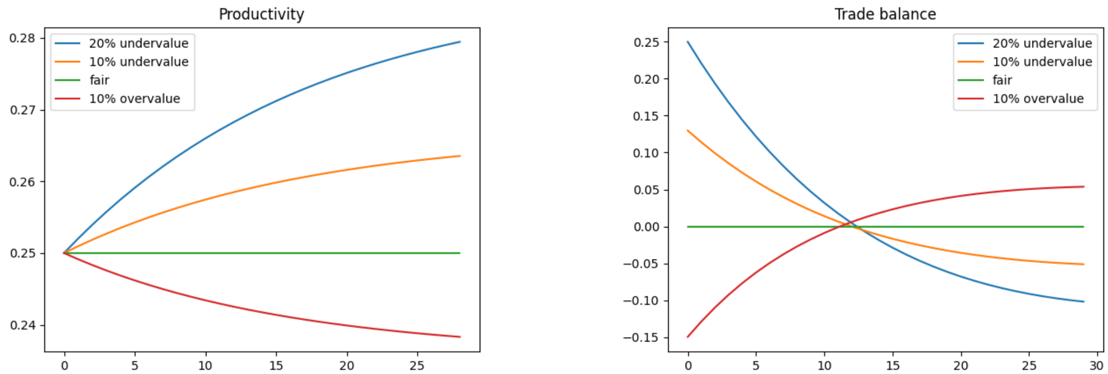


Figure 3.1: Weak learning by doing $\alpha = 0.01$

Parameter values. We also assume the following values for parameters. Time frequency is annual, and I set $\beta = 0.96$ and $R_t^* = \frac{1}{\beta}$. We set $\sigma = 5$, and $\kappa = 0.05$ in line with the literature. We set $\gamma = 0.8$, roughly matching imports as 20% of total expenditure. We set $W_{-1} = 1$, $A_0 = \frac{1}{4}$ (indicating lower productivity), and C^* is such that the economy was in full employment trade balance. α is a somewhat arbitrary parameter that we cannot match, so we test two different cases, one with a weak LBD externality with $\alpha = 0.01$, and one with a strong LBD externality with $\alpha = 0.05$.

Simulated policies. We simulate the economy under a perpetual peg at level $\bar{e}$ for different levels of $\bar{e}$, and compare the learning-by-doing productivity, trade balance, and welfare results.

Figure 3.1 shows the evolution of productivity and trade balances under weak learning by doing ($\alpha = 0.01$). Here, the productivity gain is slow with an undervaluing peg, and Home runs a moderate trade surplus over this period.

Figure 3.2 shows the evolution of productivity and trade balances under strong learning by doing ($\alpha = 0.05$). Here, the productivity gain is much faster, and Home runs a larger trade surplus over a shorter time period due to undervaluing, but Home reaches trade balance quickly and ends up running trade deficits to repay the debt.

What are the welfare implications of a currency peg? In our setup, we actually find that a currency undervaluation *decreases* Home welfare significantly despite the learning-by-doing externality: a 10% undervaluing peg causes 8% welfare losses in compensating variations in the weak LBD case, and 9% in the strong LBD case.

Why is it that there are welfare losses despite learning-by-doing externalities? This can be primarily attributed to two reasons: (1) the losses are large in the short-run;

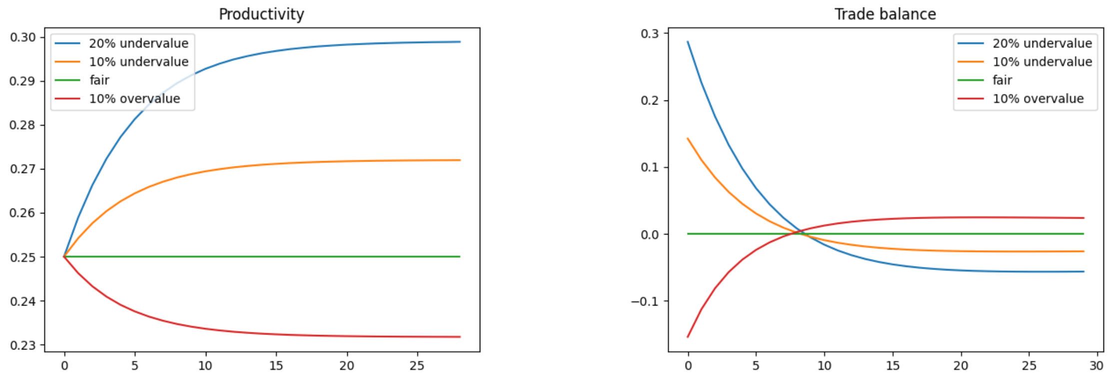


Figure 3.2: Strong learning by doing $\alpha = 0.05$

Home needs to work more, beyond the market clearing level and export more while consuming less (trade surpluses), all while worsened terms-of-trade (2) the benefits of higher productivity come in the long-run, but this is after welfare is significantly discounted for.[5] Using a proper microfoundation for the learning-by-doing externality, with higher growth levels seem necessary to rationalize an undervalued peg.

3.5 Concluding remarks

This paper studies the exchange rate policy for a growing economy, in an attempt to rationalize the common strategy of pegging to the US dollar at a fixed, pre-growth rate, for many developing countries. Through the lens of a workhorse model with productivity shocks and exchange rates determined from financial flows, we study the optimal monetary policy and FX interventions of a small open economy. We show that while a currency peg may be justifiable, the standard model cannot rationalize the mercantilist policy of keeping the exchange rate pegged at a lower level, and show that a learning-by-exporting externality can rationalize the level of peg. As such, we highlight that while financial flows are important in determining the exchange rates, the specific conduct of exchange rate policy depends heavily on the underlying real economy; and the optimal policy for a growing economy will be different from the optimal policy for an economy in steady-state, and the optimal policy would necessarily involve building extensive foreign reserves. In a future version of this paper, I hope to empirically test the validity of this channel, and test a variant of the optimal policy that involves *de-pegging* once the learning-by-doing externality decreases.

[5] Moreover, the productivity gains are still modest in the “strong" LBD case.

One channel we abstract from is investments. A commonly cited reason for a developing country for pegging its currency is that hosting foreign capital allows investing in infrastructure, and having a stable currency makes it easier to attract the former. Since the trade surplus resulting from a cheaper currency is stored as bonds without real investment value, a model where those savings lead to additional productivity spillovers may be a channel worth investigating theoretically and empirically.

3.6 Proof and derivations

Proof of Proposition 3.2. In each period, the initial condition that defines the system is B^*_{t-1} and W_{t-1}. As such, we can rewrite the utility function in recursive form:

$$U_t(B^*_{t-1}, W_t) = \gamma \log C_{Ht} + (1-\gamma)\log C_{Ft} - \gamma \log L_t + \mathbb{E}_t U_{t+1}(B^*_t, W_t)$$

$$dU = \frac{dU}{dC_{H0}}dC_{H0} + \frac{dU}{dC_{F0}}dC_{F0} + \frac{dU}{dL_0}dL_0 + \frac{d\mathbb{E}_t U_1}{dB^*_0}dB^*_0 + \frac{d\mathbb{E}_t U_1}{dw_0}dw_0$$

Subject to two constraints: the resource constraint and the dollar budget constraint.

$$L_0 = \frac{1}{A_0}(C_{H0} + C^*_{H0})$$
$$\frac{B^*_0}{R_0} - B^*_{-1} = P^*_{H0}C^*_{H0} - C_{F0}$$

Denote the Lagrange multiplier on the dollar budget constraint be λ_0. Then we have

$$dL_0 = \frac{1}{A_0}(dC_{H0} + dC^*_{H0})$$

plug this in: we have

$$\begin{aligned} dU =& \left(\frac{dU}{dC_{H0}} - \frac{1}{A_0}\frac{dU}{dL_0}\right)dC_{H0} + \left(P^*_{H0}\lambda_0 - \frac{1}{A_0}\frac{dU}{dL_0}\right)dC^*_{H0} \\ &+ \frac{dU}{dC_{F0}}dC_{F0} - P^*_{H0}\lambda_0 dC^*_{H0} + \frac{d\mathbb{E}_t U_1}{dB_0}dB^*_0 + \frac{d\mathbb{E}_t U_1}{dw_0}dw_0 \end{aligned}$$

where the first line is the labor wedge in domestic good production and export production. The derivative of the dollar BC:

$$-\frac{1}{R_0}dB_0^* = dP_{H0}^* C_{H0}^* + P_{H0}^* dC_{H0}^* - dC_{F0}$$

From the household maximization problem, we have

$$\begin{aligned}\frac{d\mathbb{E}_t U_1}{dB_0} &= -\frac{1}{R_0}\lambda_0 \\ \frac{dU_0}{dC_{F0}} &= \lambda_0\end{aligned}$$

Thus, we can plug in the above equations to get

$$\begin{aligned}dU =&(\text{labor wedge}) \\ &+ \lambda_0(dC_{F0} - P_{H0}^* dC_{H0}^* - \frac{1}{R_0}dB_0) + \frac{d\mathbb{E}_t U_1}{dW_0}dW_0 \\ =&(\text{laborwedge}) + \lambda_0 C_{H0}^* dP_{H0}^* + \frac{d\mathbb{E}_t U_1}{dW_0}dW_0\end{aligned}$$

Take the derivative with respect to the exchange rate, and we are complete.

□

3.6.1 Equations for numerical part

This is the set of equations that I put in a numerical solver to solve for the system. In every part, we implicitly assume that the government policy removes risk premia – so international risk sharing holds.

The set of equations defining an equilibrium are given by

$$\begin{aligned}\frac{P_{Ht}C_{Ht}}{P_{Ft}C_{Ft}} &= \frac{\gamma}{1-\gamma} \\ \beta R_t \mathbb{E}_t \frac{P_{Ht}C_{Ht}}{P_{Ht+1}C_{Ht+1}} &= 1 \\ \log\frac{W_t}{W_{t-1}} &= \kappa(1 - \frac{W_t}{P_{Ht}C_{Ht}}) + \log\frac{W_{t+1}}{W_t} \\ A_t L_t &= C_{Ht} + C_{Ht}^* = C_{Ht} + C^*(P_{Ht}^*)^{-\sigma} \\ \beta B_{t+1}^* - B_t^* &= P_{Ht}^* C^*(P_{Ht}^*)^{-\sigma} - C_{Ft} \\ \log A_{t+1} &= \log A_t + F(Y, A)\end{aligned}$$

where $P_{Ht} = \frac{W_t}{A_t}$, $P_{Ft} = e_t$, $P_{Ht}^* = \frac{W_t}{A_t e_t}$ is the unit cost of each good. Plugging these in to remove P dependencies, we have

$$\begin{aligned}
C_{Ft} &= \frac{1-\gamma}{\gamma} C_{Ht} \frac{W_t}{A_t e_t} \\
\beta R_t (\frac{W_t}{A_t}) C_{Ht} &= \frac{W_{t+1}}{A_{t+1}} C_{Ht+1} \\
\log \frac{W_t}{W_{t-1}} &= \kappa (1 - \frac{A_t}{C_{Ht}}) + \log \frac{W_{t+1}}{W_t} \\
A_t L_t &= C_{Ht} + C^* (\frac{W_t}{A_t e_t})^{-\sigma} \\
\beta B^*_{t+1} - B^*_t &= C^* (\frac{W_t}{A_t e_t})^{1-\sigma} - C_{Ft} \\
\log A_{t+1} &= \log A_t + F(A_t L_t, A_t)
\end{aligned}$$

This is the dynamic system of equations that we seek to numerically solve for.

Initial condition. We assumed that the economy was in full employment and trade balance at $t = -1$. The set of equations in this case is:

$$\begin{aligned}
C^* (\frac{W}{Ae})^{1-\sigma} &= -\frac{1-\gamma}{\gamma} C_H \frac{W}{Ae} \\
C_H &= A \\
AL &= C_H + C^* (\frac{W}{Ae})^{-\sigma}
\end{aligned}$$

Plug in $A = \frac{1}{4}, W = 1$ to solve for e and C^*. We use this value of C^* as the "global aggregate consumption" that Home takes as exogenous.

Currency peg. Under a peg $e_t = \bar{e}$, we have

$$\begin{aligned}
\frac{W_t}{A_t} C_{Ht} &= \frac{W_{t+1}}{A_{t+1}} C_{Ht+1} \\
\log \frac{W_t}{W_{t-1}} &= \kappa (1 - \frac{A_t}{C_{Ht}}) + \log \frac{W_{t+1}}{W_t} \\
A_t L_t &= C_{Ht} + C^* (\frac{W_t}{A_t \bar{e}})^{-\sigma} \\
\beta B^*_{t+1} - B^*_t &= C^* (\frac{W_t}{A_t e_t})^{1-\sigma} - \frac{1-\gamma}{\gamma} C_{Ht} \frac{W_t}{A_t \bar{e}} \\
\log A_{t+1} &= \log A_t + F(C^* (\frac{W_t}{A_t \bar{e}})^{-\sigma}, A_t)
\end{aligned}$$

as the equations defining the path of equilibrium.

There are four variables in each period: A_t, C_{Ht}, W_t, B_t^* with four equations (L_t is whatever clears the market).

The terminal condition (as $t \to \infty$, under a peg) must have productivity A_t converge to a constant (the LBD innovation stops), and $C_t = A_t$ by full employment. The budget balance condition as $t \to \infty$ becomes

$$C^*(\frac{W}{A\bar{e}})^{1-\sigma} - \frac{1-\gamma}{\gamma}\frac{W}{\bar{e}} = (1-\beta)B^*$$

We can use this condition as a *terminal* condition to solve for the equilibrium.

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