Essays in Macroeconomics

by

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ABSTRACT

The thesis consists of three essays on macroeconomics. In the first essay, I study the price and wage-setting implications of monopsony models with nominal rigidities. I develop a New Keynesian model with wage posting and on-the-job search. I show how wage markdowns are related to the importance of hiring costs, and those are estimated to be an order of magnitude larger than previous calibrations. I show how at the individual level, both higher monopsony power and higher wage rigidity amplify the price response of idiosyncratic demand shocks. At the aggregate level, the main driver of inflation is not an increase in real wages but rather an increase in the cost of hiring workers. Given that firms have problems finding workers, they raise prices. In a calibrated model, I show how negative labor supply shocks reduce the real wage when the nominal wage increase is offset by the nominal price increase.

In the second essay (joint with Marc de la Barrera and Masao Fukui), we study how the interaction between China's productivity growth and currency peg to the US dollar affected the labor market and trade imbalance in the United States. Empirically, we document that in response to similar exposure to Chinese exports, countries pegging to the US dollar experienced larger unemployment and trade deficits compared to floating countries. Theoretically, we develop a dynamic model of trade featuring endogenous imbalances and nominal rigidity, and show that Foreign growth may hurt Home welfare and characterize optimal trade and monetary policy in this environment. Quantitatively, we find that China's currency peg is responsible for 447 thousand manufacturing jobs lost in the US over 2000-2012, one third of the total US trade deficit over the same period, and reduced US lifetime welfare gains from Chinese growth by 32% compared to an economy where an otherwise identically growing China had its currency floating. A short-run safeguard tariff may have effectively accommodated China's currency peg and ameliorated the labor market distortions.

In the third essay (joint with Tim de Silva), we explore a novel field that uses machine learning techniques to solve dynamic stochastic optimization problems. While most traditional approaches require the knowledge of a law of motion for exogenous states like income, we show a methodology that allows us to remain agnostic about the data-generating process of the state. Instead of calibrating a model mimic the dynamics of the state, we need to observe realizations of such state. Parametrizing the policy function with a neural network, we are able to solve the value function problem without ever knowing the law of motion of the state, which the neural network endogenously learns. We test our approach with the income fluctuations problem and show how our methodology is able to learn the income process when it is an AR(1), and is also able to solve the problem for an unspecified income process. We then compare the welfare loss of specifying a particular income process and evaluating the policy function without making any assumption on the income process, and we find that the miss optimization loss is negligible. A byproduct of this project is the publication of the python package `nndp` that is available for use and solve a wide array of finite horizon, dynamic stochastic optimization problems.

JEL Classification C63, E24, E32, F16, F31, G51, J63

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Chapter 1

Monopsony and Nominal Rigidities

1.1 Introduction

There is little debate that most firms set prices and wages, and that those decisions are subject to nominal rigidities. Yet the majority of workhorse macro models abstract away from it, either by assuming that firms take wages as given or they are bargained with workers. In this paper, we take a monopsonistic view of the labor market, where firms post wages in an environment with search frictions and nominal rigidities, and derive the macroeconomic implications of this natural assumption.

Monopsony, introduced by Robinson (1933) and popularized by Manning (2003), is a central topic in labor economics but it has not been developed in the wage rigidity literature. Given that most employment contracts are not bargained at all (Hall and Krueger (2012)), and wages are set infrequently (Grigsby, Hurst, and Yildirmaz, 2021), this is an important omission. Instead, the benchmark model for wage rigidity is the Erceg, Henderson, and Levin (2000) model, where unions set wages. Firms can hire in a frictionless labor market, and set prices subject to nominal rigiditites. While appealing for its tractability, in a country like the US, where only 10% of jobs are unionized it is unlikely to be the right framework to think about wage formation.

Given the documented prevalence of monopsony in the labor market (Azar et al., 2020, Lamadon, Mogstad, and Setzler, 2022, Berger, Herkenhoff, and Mongey, 2022), what is the effect it has on wage and price formation? To answer this question, we need a model where firms set prices and wages. Previous wage posting models are not well-suited to answer this question because firms lack price-setting power. Acclaimed New Keynesian models with search do not have the notion of monopsony.[1]

[1] Moscarini and Postel-Vinay (2016) or Coles and Mortensen (2016) solve the Burdett and Mortensen (1998) model in a real economy and Gertler, Sala, and Trigari (2008), Blanchard and Galí (2010) or Chris-

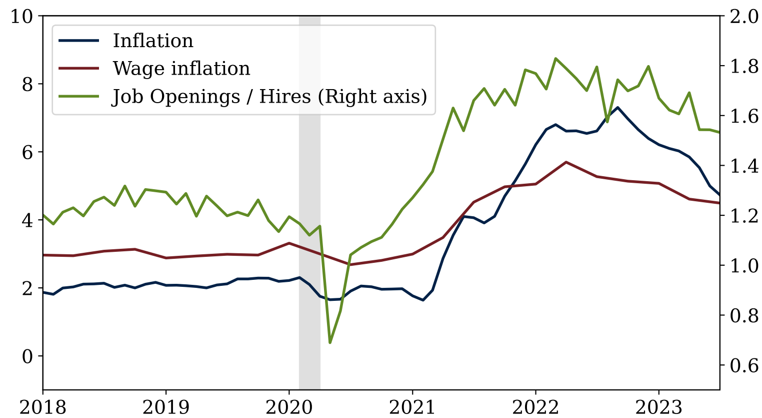

Figure 1-1: Price and wage inflation and time to hire a worker
Note: Dynamics of inflation (All-items less energy), wage inflation (Employment Cost Index) and job openings over hires prior and after the Covid pandemic. Left axis correspond to annual inflation rates, right axis the ratio of job openings divided by hires in a given month.

With search frictions, firms internalize in their pricing decisions that hiring workers is costly (Krause and Lubik, 2007). The first contribution of the paper is to show that monopsony increases the importance of such costs. Wages are set to trade off a higher wage bill versus lower turnover costs, and higher monopsony lowers wages and consequently raises turnover costs. Second, we derive the price and a novel wage Phillips curve. Inflation dynamics are determined by the evolution of the real wage and the cost of hiring a worker. Figure 1-1 shows the evolution price and wage inflation and the job openings over hires ratio, a proxy of how hard is to hire a worker, over the last inflationary period. The time to fill is an order of magnitude more volatile than the inflation rate. Monopsony, by increasing the importance of hiring frictions, *steepens* the price Phillips curve. The wage Phillips curve is governed by the poaching intensity of firms trying to poach workers from each other, and monopsony *flattens* it, because labor supply at the firm level is less responsive to it.

We start by defining the problem of a firm that jointly sets prices, wages and posts vacancies. It takes as given the acceptance rate and a turnover rate which depend on the wage posted. Proposition 1 states that the share devoted to hiring costs equals to the wage markdown, and provides a formula for it. To know how important are hiring costs, we need an estimate of the quit elasticity at the firm level. Section 1.2.1 describes how the firm responds to idiosyncratic demand shocks before considering aggregate shocks in general equilibrium. An increase in monopsony increases both the price and wage response to a positive demand shock. With higher labor market power, a desired increase in firm size

tiano, Eichenbaum, and Trabandt (2016), Benigno and Eggertsson (2023) are examples New Keynesian models with search without monopsony.

requires a larger increase of wages, which imply a larger price response. Next we consider the effect of wage rigidity. In models where firms take wages as given, wage rigidity dampens the marginal costs, but here is not the case. Firms *optimally* set wages, so any constraint on wage setting imply a larger price response.

Section 1.3 introduces the general equilibrium version of the monopsonistic model. It is a dynamic wage posting model with on-the-job search in the spirit of Burdett and Mortensen (1998) where firms also set prices. The consumption side of the model is standard, and worker behaviour is simplified. Unemployed workers accept any job regardless of the wage, and job-to-job decisions are made by comparing current wages plus an idiosyncratic taste shock. Taste shocks are introduced for two reasons. First, they rationalize that many job-to-job transitions are to lower-paying jobs (Sorkin, 2018) and can account for heterogeneous moving costs. Second, they reduce the labor supply elasticity at the individual firm level. If the variance of such shocks is sufficiently big, then Albrecht, Carrillo-Tudela, and Vroman (2018) show that a symmetric wage equilibrium exists, which will simplify the exposition. The model is closed by deriving the general equilibrium acceptance and turnover rate functions. While a single firm can raise wages to attract workers, if all firms do the same, the effect is offset and the only way to increase employment is by raising vacancies. This leads to a second round of effects, as more vacancies imply more quits, which further increases the incentive to increase wages and post more vacancies, and so on.

Having presented the main elements of the model, in Section 1.4 we derive the implications of monopsony for the price and wage Phillips curve. Proposition 3 expresses the dynamics of wages and prices as a function of labor market variables. In Erceg, Henderson, and Levin (2000), aggregate wages are driven by the marginal rate of substitution of unions. In search models like Gertler, Sala, and Trigari (2008) or Blanchard and Galí (2010), wages are determined by the worker outside option and its bargaining power. Here, like in Moscarini and Postel-Vinay (2023), they are determined by the competition across firms for currently employed workers. The time to fill a vacancy, employment growth, and the $\frac{EE}{UE}$ ratio are observable labor market variables that are indicative of nominal wage growth. Price inflation is determined by the evolution of the real wage and critically, the hiring costs. The weight put on hiring costs is increasing in monopsony power, and since those costs are more volatile than the real wage, the price Phillips curve becomes steeper. We end the section highlighting the importance of distinguishing vacancy costs from hiring costs. Most of the applied literature on monopsony estimates the elasticity of labor supply to be two times the quit elasticity (Manning, 2003). We show how this crucially depends on vacancies, not hires, being costly. If not, the relevant labor supply elasticity that the firm faces needs to be divided by two, which implies that current estimates of monopsony power are a lower bound and markdowns might be significantly higher than previously thought.

How important are marginal hiring costs? In Section 1.5 we quantify them using the sufficient statistics from Proposition 1. Hiring costs are equal to the wage markdown, and all we need is the quit elasticity at the firm level, which has been estimated by many studies, and the turnover rate which is observable. Sokolova and Sorensen (2021) propose using 3.5 as a best practice estimate, and the US turnover rate is 12% quarterly. Those values imply a wage markdown of 13%, which is equivalent to say that marginal hiring costs represent 13% of the total cost of employing a new worker. From here, we can obtain the value of a worker, which we estimate it to be 15 weeks of its wage. This value is in line with direct estimates of hiring costs provided by Muehlemann and Strupler (2018) or the actual pricing of staffing firms, who usually charge from 8 to 16 weekly wages. In contrast, standard calibrations of search models based on bargaining like Gertler, Sala, and Trigari (2008), Blanchard and Galí (2010) or Christiano, Eichenbaum, and Trabandt (2016) imply that these costs are below one week of wages, one order of magnitude smaller.

Finally, Section 1.6 calibrates the model to match several labor market characteristics and performs several exercises. An advantage of the wage posting model developed in this paper is that its calibration does not depend on unobserved and contested parameters like the bargaining power of workers or the value of unemployment.[2] We target the quit elasticity, which many papers estimate. When the economy is hit by a demand shock, individual firms post more vacancies, increase nominal wages, and prices. All other firms do the same, which offset the benefits of raising wages. More vacancies mean (i) more workers quit and (ii) the quit elasticity increases.

The resulting response is price and wage inflation, and, for the calibration used a drop in the real wage. Since the real wage falls, the sole driver of inflation is the increase in hiring costs, as in Krause and Lubik (2007). This is not the only paper that emphasises that demand shocks can reduce the real wage. Lorenzoni and Werning (2023) get the same result by imposing strong diminishing returns to labor, which increase marginal costs despite nominal wages not rising. We compare the model to an alternative model where wages are bargained with real wage rigidities as in Blanchard and Galí (2010). The bargained model features higher employment response and lower inflation response, with an increase in the real wage and a much higher volatility of market tightness.

We end the paper extending the model to account for labor supply shocks, supported by the evidence that not all unemployed accept all job offers Faberman et al. (2022). Instead

[2]For example, Hagedorn and Manovskii (2008) argues that worker's bargaining power is close to zero, Gertler, Sala, and Trigari (2008) calibrates it to be close to one, and Gagliardone, Gertler, et al. (2023) exogenously set to 0.5 to satisfy the Hosios condition.

they receive a flow value of being unemployed and are subject to the same idiosyncratic taste shocks than workers. We can then represent labor supply shocks by an increase in these unemployment benefits, or an increase in the disutility of work, as happened during the Covid recovery. A unique feature of the monopsonistic model is that labor supply shocks can *lower* the real wage. Other search models would predict the opposite, even those with nominal rigidities like Gertler, Sala, and Trigari (2008). When workers are unwilling to work, any individual firm has the incentive to increase wages to attract them, but also they pass the extra cost into prices. Whether they do most of the adjustment on prices or wages depends on the relative product and labor market elasticities and the rigidities they face. Firms do not internalize that by raising its own price, they are lowering the real wage of everyone.

Related Literature. This paper is mainly related to four strands of literature. It brings monopsony to a dynamic, general equilibrium model in a tractable way. It compares its mechanisms to standard New Keynesian models, with especial focus on those with search frictions. It emphasises the hiring costs, and finally it relates to papers discussing the recent inflation surge.

The monopsony literature distinguishes several sources of firm wage-setting power: search frictions, and preference heterogeneity Manning (2021). By having a wage posting model with idiosyncratic preference shocks, this paper contains elements of both, like D. Berger et al. (2023). The first view is pioneered by the Burdett and Mortensen (1998) wage posting model with on-the-job search, while the second got traction with Card et al. (2018) and bought into general equilibrium by Berger, Herkenhoff, and Mongey (2022). Albrecht, Carrillo-Tudela, and Vroman (2018) shows how idiosyncratic shocks in an otherwise standard wage posting model allows for the existence of a unique wage equilibrium, an assumption that we make here.

The notion that the labor supply at the firm level is not completely elastic dates back to Robinson (1933) and has gained popularity over the recent decades (Manning (2003)). Whether firms post wages or those are bargained is an empirical question, and the true answer is that it is a little bit of both. Hall and Krueger (2012) document that two-thirds of workers do not bargain at all their wage. The SCE Job Search Supplement (Faberman et al. (2022)) corroborate this finding and add that only 12% of job offers receive a counter-offer, giving empirical support for wage posting models over bargaining models. A critical measure of monopsony power is the elasticity of labor supply at the firm level, which many papers attempt to estimate. Sokolova and Sorensen (2021) provides a meta-study from which we will take the main estimate of the quit elasticity, and Bassier, Dube, and Naidu (2022) provide estimates by broad sectors.

While Burdett and Mortensen (1998) pioneered the idea that firms post higher wages to increase in firm size, the idea that firms increase wages of incumbents to save in turnover costs dates back to Salop (1979). Manning (2006) also acknowledges that firms can increase in size by posting higher wages or spending more in recruiting costs. The closest paper in terms of modeling the firm wage setting problem is Bloesch and Larsen (2023). The paper notes that when firms pay vacancy costs in terms of vacancy *rates*, then in steady state there is not a size-wage relationship at the firm level, like Coles and Mortensen (2016). Empirically, the importance of hiring costs has been documented in a series of papers by Blatter, Muehlemann, and Schenker (2012) and Muehlemann and Strupler (2018) using a rich dataset from Switzerland, which we use to compare to the estimate of hiring costs using our sufficient statistics result. In bargaining model, Silva and Toledo (2009) and Pissarides (2009) both emphasize the importance of post-matching hiring costs.

The goal of this paper is to bring monopsony and nominal wage rigidities to an otherwise standard New Keynesian model. Dennery (2020) and Alpanda and Zubairy (2022) have done it in a model with static monopsony without search frictions. Instead, the standard approach to wage rigidity is the one pioneered by Erceg, Henderson, and Levin (2000) in which unions set wages and firms determine quantities. While it might be a good model for some European countries, unions penetration is minimal in the United States. Yet, the simplicity of this model has made it very popular, used in most papers with wage rigidity Christiano, Eichenbaum, and Evans (2005), Huo and Ríos-Rull (2020), Lorenzoni and Werning (2023). In those models, nominal wages are set by unions which target some marginal rate of substitution. By definition there are not search frictions, but Galí (2011) reinterprets this model to be able to talk about unemployment.

Closer to this paper are New Keynesian models that feature search frictions. The general practice (Blanchard and Galí (2010), Gertler, Sala, and Trigari (2008),Christiano, Eichenbaum, and Trabandt (2016), Moscarini and Postel-Vinay (2023)) is to separate the product and the labor market trough a perfectly competitive intermediate layer of 'labor services' produced by firms subject to search frictions. This assumption is very convenient, as it allows one to disentangle forward-looking vacancy-posting and pricing decisions and thus simplify the analysis. Final good firms buy these labor services at the equilibrium price, produce goods and are subject to price rigidities. This separation makes the problem of the labor packer a real one, where productivity is the price of the intermediate good. Then several papers consider different wage determination protocols. The current paper does not use this two-layer economy and instead it is the same firm that posts prices and vacancies. Another that takes this approach is Thomas (2011). However, in that paper wages are negotiated among firms and workers, and it is the value that induces the right amount of hours required to supply the labor that is needed to satisfy demand.

Among those papers, Blanchard and Galí (2010) and Gertler, Sala, and Trigari (2008) explicitly model wage rigidities in a search environment. In a model with matching frictions, the bargaining set for wage determination is relatively wide, because the difficulty in locating matches creates match capital the moment a tentative match is made. Any wage within the bargaining set could be an outcome of the bargain. Wage rigidity helps pin down which of these points is selected as in R. Hall (2005). The main implication of this form of wage rigidity is that wages affect hires trough the vacancy creation incentives. Lower wages make matches more profitable from the firm's prespective which induces higher vacancy creation. In this paper, lower wages lowers the vacancy yield, which makes the hiring process costlier.

Finally, the recent inflation period has spurred several papers that try to explain it. Lorenzoni and Werning (2023) and Gagliardone, Gertler, et al. (2023) highlight the importance of the low substitutability of labor with other inputs like oil. Autor, Dube, and Mcgrew (2023) study the wage compression over this period, and in one of their analysis find that at the state level, labor market tightness spurred nominal wage growth *and* price growth, with a resulting null effect on the real wage. Cerrato and Gitti (2022) document a sharp steepening of the price Phillips curve during the Covid recovery.

1.2 A dynamic model of price and wage setting

We start by presenting the firm problem in partial equilibrium to understand the micro implications that monopsony has on wage and price setting. Traditional pricing models take the price of inputs, in this case the wage, as given, and most monopsonistic models are either static or do not consider the fact that the firm also sets prices.[3] The model presented encompasses several models of the labor and product market and thus generalizes and links both views. In this section, we will focus on the response of idiosyncratic demand shocks to an individual firm.

The firm problem Firms start the period with some employment level n_{t-1} and set prices p_t, wages w_t, and vacancies v_t to maximize profits. It faces a demand curve $z_t\mathcal{D}\left(\frac{p_t}{P_t}\right)$ with constant elasticity, where z_t is an idiosyncratic demand shock and P_t is the aggregate price level, and it produces using labor $y_t = f(n_t)$. Posting v_t vacancies costs $\phi(v_t, n_{t-1})$, and those are transformed into a hire at a rate $a(w_t/W_t)$, where W_t is a measure of the aggregate

[3]Some papers like Kline et al., 2019 or Bloesch and Larsen (2023) consider firms with price-setting power but their focus is on the wage setting.

wage. In each period, a fraction $\delta(w_t/W_t)$ of incumbent workers leave the firm. In Section 1.3 we will derive these acceptance and turnover functions. For the time being, the firm can set prices and wages in a flexible way, we will introduce nominal wage rigidities in Section 1.2.1. Let $J_t(n_{t-1})$ the value of starting the period with n_{t-1} workers. Then the Bellman equation is

$$J_t(n_{t-1}) = \max_{p_t, w_t, v_t, n_t} \frac{p_t}{P_t} z_t \mathcal{D}\left(\frac{p_t}{P_t}\right) - w_t \frac{n_t}{P_t} - \phi(v_t, n_{t-1}) + \beta E_t[J_{t+1}(n_t)] \qquad \text{s.t.}$$

$$n_t = \left(1 - \delta\left(\frac{w_t}{W_t}\right)\right) n_{t-1} + a\left(\frac{w_t}{W_t}\right) v_t. \tag{1.1}$$

$$z_t \mathcal{D}\left(\frac{p_t}{P_t}\right) = f(n_t). \tag{1.2}$$

Without defining $\delta(\cdot)$, $a(\cdot)$ and $\phi(v_t, n_{t-1})$ this problem encapsulates many models of the labor market. A perfectly competitive labor market is the case when $a(\cdot)$ and $\delta(\cdot)$ are step functions at the market wage and vacancies are costless. That is, if w_t is lower than W_t, no worker accepts a job and all workers quit. This is the traditional approach in most macro models (Galí, 2015, Erceg, Henderson, and Levin, 2000), where firms with price-setting power take wages as given. Any firm idiosyncratic shock will not have an effect on the wage setting. Static monopsony like Card et al. (2018) is a case where $\delta(\cdot) = 1$ and v_t is fixed.[4] In such case, we get a static relationship between the wage posted at t and employment size, $n_t = a(w_t/W_t) \equiv \mathcal{LS}(w_t/W_t)$. This formulation is taken into macro models by Berger, Herkenhoff, and Mongey (2022), Dennery (2020) and Alpanda and Zubairy (2022). Wage posting models in the spirit of Burdett and Mortensen (1998) are also represented here with two simplifications: acceptance and turnover decisions depend only on wages posted at time t, and the formulation assumes the existence of an aggregate wage W_t, instead of a distribution of wages. And finally, models of costly hiring (as opposed to costly vacancy posting) can be represented by setting $a(\cdot) = 1$, which implies that hiring h_t and vacancies are effectively the same. Silva and Toledo (2009) and Muehlemann and Strupler (2018) argue that vacancy costs are small compared to the cost of hiring a worker, which includes training and productivity losses.

In order to simplify the model and isolate the mechanism, consider the case where the turnover rate and the acceptance rate are constant elasticity and can be written as $\delta\left(w_t/W_t\right) = \bar{\delta}\left(\frac{w_t}{W_t}\right)^{-\epsilon_\delta}$ and $a(w_t/W_t) = \bar{a}\left(\frac{w_t}{W_t}\right)^{\epsilon_a}$. The constant $\bar{\delta}$ represents the turnover rate of a firm that sets its wage at the market wage. The constant $\bar{a}$ can't be separately

[4]We can get a fixed v_t by defining $\phi(v_t, n_{t-1}) = 0$ if $v_t \leq 1$ and infinite otherwise.

identified from the vacancy costs and therefore is normalized to 1.[5] For the main text, vacancy costs are assumed to be constant returns to scale $\phi(v_t, n_{t-1}) = \frac{\kappa_v}{1+\nu}\left(\frac{v_t}{n_{t-1}}\right)^{1+\nu} n_{t-1}$. We will denote $\phi'(x) \equiv \phi_v(x,1) = \kappa_v x^\nu$ the marginal cost of posting an extra vacancy. Coles and Mortensen (2016) and Bloesch and Larsen (2023) show how the assumption of constant returns to scale implies that wages in steady state are independent of firm size. Gouin-Bonenfant (2022) shows how wages are determined by firm *growth*. Production is linear $f(n_t) = n_t$. Decreasing returns to scale would have the standard implications in this model, as well as the effect of adding extra inputs in the production function as long as they can be bought in a competitive market.

Wage Setting First we consider the wage setting problem of the firm that targets an employment level n_t, when it starts the period with n_{t-1} workers. By taking a first order condition with respect to v_t, we can get the value of a worker, which we denote by μ_t as

$$\mu_t = \frac{\phi'\left(\frac{v_t}{n_{t-1}}\right)}{a(w_t/W_t)}.$$

This equation resembles the free entry condition of traditional search models like Pissarides (2017) where μ_t is the value of a job. Posting a vacancy costs $\phi'\left(\frac{v_t}{n_{t-1}}\right)$ and it successfully becomes a hire at a rate $a(w_t/W_t)$. Here, the definition of a vacancy and the definition of the acceptance probability are vague and hard to map to real and observable objects. A vacancy could be a job posting online, an interview with a prospective applicant, or a formal offer, we are agnostic about it. Consequently, the cost of a vacancy is very different in these three scenarios, as it is the definition of $a(\cdot)$. However, μ_t has a clear interpretation: is the marginal value of a worker. In other words, is the willignes to pay of a firm that wants to hire an extra worker, for having it immediately on the workforce. In Section 1.5 we will discuss reasonable values for μ_t. We will show that for traditional calibrations with bargaining as a wage-setting protocol, this value is abnormally low.

When setting wages, firms trade-off a paying a higher wage bill versus lowering turnover costs, both because less workers quit and vacancies are more effective in becoming hires. The first order condition is

$$\frac{1}{P_t} n_t = \frac{\phi'\left(\frac{v_t}{n_{t-1}}\right)}{a(w_t/W_t)} \left(-\delta'\left(\frac{w_t}{W_t}\right)\frac{1}{W_t} n_{t-1} + a'\left(\frac{w_t}{W_t}\right)\frac{1}{W_t} v_t\right). \tag{1.3}$$

The left hand side of Equation (1.3) represents the cost of raising the wage, namely paying

[5] Note that we do not constrain $a(w_t/W_t)$ to be a probability.

the entire workforce. The term inside the parenthesis on the right hand side is how many extra workers do not leave plus those that accept a vacancy now that the wage has increased. Each one of those workers is valued at $\frac{\phi'\left(\frac{v_t}{n_{t-1}}\right)}{a(w_t/W_t)}$. In steady state, quits and hires coincide, $\delta\left(\frac{w}{W}\right)n = a\left(\frac{w}{W}\right)v$. Then Equation (1.3) simplifies to

$$\frac{w}{P} = \frac{\phi'\left(\frac{\delta(w/W)}{a(w/W)}\right)}{a(w/W)}\delta\left(\frac{w}{W}\right)(\epsilon_\delta + \epsilon_a). \tag{1.4}$$

Equation (1.4) shows that in steady state, wages are independent of firm size, and therefore independent of the characteristics of the demand or production function. This is the consequence of assuming that vacancy costs are constant returns to scale Bloesch and Larsen, 2023 discusses how this assumption can reconcile a low firm size-wage relationship with significant monopsony power.

Price Setting Having pinned down wages, we turn our attention to the firm price setting. Unlike standard macro models where the flexible price is a static condition, here is dynamic. When firms want to have an extra worker, they have to pay the wage w_t plus the net cost of hiring her. This includes the cost of hiring the worker at t, minus the value of having her at $t+1$ in case she has not quit. Prices are set at a markup over marginal cost and the pricing condition is

$$\frac{p_t}{P_t} = \mathcal{M}_p \frac{w_t + \frac{\phi'\left(\frac{v_t}{n_{t-1}}\right)}{a(w_t/W_t)} - \beta E_t\left[\left(1-\delta\left(\frac{w_{t+1}}{W_{t+1}}\right)\right)\frac{\phi'\left(\frac{v_{t+1}}{n_t}\right)}{a(w_{t+1}/W_{t+1})} + \frac{\nu}{1+\nu}\phi'\left(\frac{v_{t+1}}{n_t}\right)\frac{v_{t+1}}{n_t}\right]}{f'(n_t)}, \tag{1.5}$$

where $\mathcal{M}_p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$ is the markup and $\epsilon_p \equiv -\frac{d\log\mathcal{D}}{d\log p_t}$. The last term of the equation comes from the fact that a hire at t reduces the hiring rate at $t+1$ given v_{t+1}. This equation is similar to Krause and Lubik (2007) with the difference that there, quits are exogenous and wages are not set by firms but rather bargained, so instead of $a(w_t/W_t)$ they have the rate at which firms meet workers, which is independent of the wage. An important feature of Equation (1.5) is that whether prices are increasing in wages is ambiguous. In models where wages are taken as given by firms or set by bargaining, marginal cost are increasing in wages. Under the monopsonistic view, these are set optimally to minimize the *total* cost of attaining a workforce n_t when previous employment is n_{t-1}. Raising wages increases the wage bill but it lowers the cost of turnover. For a reasonable calibration[6], the price is decreasing in wages.

[6] Parameters must be such that $\nu + \frac{\epsilon_a}{\epsilon_a + \epsilon_\delta} \geq \delta$. This is satisfied as long as (i) acceptance elasticity is not too small, and/or (ii) vacancy costs are convex. This assumption is not satisfied if firms pay per hire ($\epsilon_a = 0$) and hiring costs are linear ($\nu = 0$).

The intuition is clear for the static monopsony case. With a higher wage, firm employment increases and prices must decrease to sell all the produced output.

Linearizing (1.5) around steady state, after imposing that production is linear, we get

$$\hat{p}_t = \tau \hat{w}_t + (1-\tau)\hat{\gamma}_t. \tag{1.6}$$

γ_t is the net cost of hiring a worker, excluding the wage. τ is the share of wage costs relative to the total cost of a new worker, which will be a central parameter of our model. Without search or adjustment frictions, this value is one, and lower values of τ indicate that hiring costs are more relevant for firms when they set prices. The next proposition relates the importance of hiring costs, which is unobserved, to the wage markdown that firms charge and it can be estimated.

Proposition 1 (Monopsony and marginal hiring costs)**.** *In steady state, the share of the wage costs relative to the marginal cost of a new employee is equal to the monopsonistic wage markdown.*

$$\tau \equiv \frac{\omega}{\omega + (1-\tilde{\beta})\frac{\phi'}{a}} = \frac{\delta(\epsilon_\delta + \epsilon_a)}{\delta(\epsilon_\delta + \epsilon_a) + 1 - \tilde{\beta}} \equiv \mathcal{M}_w, \tag{1.7}$$

where $\tilde{\beta} \equiv \beta(1 - \frac{1}{1+\nu}\tilde{\delta})$ *is the effective discount rate.*

Proof. In steady state, wages are determined by (1.4). Use this condition to substitute $\frac{\phi'}{a}$ into the definition of τ to get that $\tau = \frac{\delta(\epsilon_\delta+\epsilon_a)}{\delta(\epsilon_\delta+\epsilon_a)+1-\tilde{\beta}}$. To see that it coincides with the wage markdown, plug it into the optimal price equation in steady state to obtain

$$w = \frac{\delta(\epsilon_\delta + \epsilon_a)}{\delta(\epsilon_\delta + \epsilon_a) + 1 - \tilde{\beta}} \frac{pf'(n)}{\mathcal{M}_p} = \mathcal{M}_w \frac{pf'(n)}{\mathcal{M}_p}.$$

The wage is a markdown $\mathcal{M}_w$ over the marginal revenue product of labor. □

The observation that the wage markdown coincides with the fraction devoted to hiring costs is general to other search models.[7] The advantage of Proposition 1 is that for monopsonistic search, we provide an expression for the wage markdown, which coincides with the one provided by Manning (2003), for which we have empirical estimates. For the case $\beta = 1$ and $\nu = 0$, we obtain the familiar formula $\mathcal{M}_w = \frac{\epsilon_w}{\epsilon_w+1}$, where $\epsilon_w = \epsilon_\delta + \epsilon_a$ is the labor supply elasticity.

[7]For example, for many models with wage bargaining, the free entry condition imposes

$$c(\theta_t) = \vartheta_t - w_t + \beta(1-\bar{\delta})c(\theta_{t+1}),$$

where $c(\theta_t)$ is the cost of a match, which depends on market tightness, but not wages since those are bargained ex-post. ϑ_t is the marginal product of labor. Then in steady state $\tau \equiv \frac{w}{w+(1-\beta(1-\delta)c(\theta)} = \frac{w}{\vartheta} \equiv \mathcal{M}_w$.

The importance of hiring costs will be central for the New Keynesian Phillips curve in Section 1.4. Inflation is determined by the dynamics of the real wage and turnover costs, but the latter is more volatile than the former. In Section 1.5 we will show how traditional bargaining models imply a calibrated τ very close to the unity, and so search costs play a minimal role in marginal costs. But before considering the aggregate implications of monopsony, first we study how firms react to idiosyncratic demand shocks.

Incumbent vs new hire wages We have imposed the standard monopsonistic assumption that the firm must set the same wage for currently employed workers and new hires. Kline et al. (2019) and Fukui (2020) consider the case where firms can discriminate between incumbents and new workers. The Appendix A.1.1 solves the firm problem allowing for it. It further generalizes the model by allowing for explicit vacancy *and* post-matching hiring costs. Firms set new hire wages to reduce vacancy costs and incumbent wages to prevent quits and reduce hiring costs. Whether incumbent hires are higher than new hire wages depends on the quit elasticity of incumbents, the acceptance elasticity of new hires, and the hiring costs unrelated to vacancy posting. When hiring costs are high, incumbent workers are more valuable than new hires and therefore are paid more. As in the benchmark case, wages are independent of firm productivity or demand.

1.2.1 Price and wage response to a demand shock

We now show the pass-through of prices and wages when the firm receives a transitory demand shock. Technology shocks, both transitory and permanent, are considered in Appendix A.1.3. Most of the pass-trough literature (Amiti, Itskhoki, and Konings, 2019, Ashenfelter and Jurajda, 2022), considers the case where input prices exogenously change at the firm level, and study how they change prices. Here, we acknowledge that both prices and wages are set by the same firm and study how prices and wage changes depend on the extent of product and labor market power. Kroft et al. (2023) also studies how price and wage setting power interact in a model of static monopsony to define the price and wage levels in steady state.

A benchmark case, widely used in macroeconomics, is a producer with constant returns to scale production facing a constant elasticity demand function, that can hire workers at the market wage. Under those assumptions, idiosyncratic demand shocks have no effect on prices, and by assumption, the wage is independent of the firm. We can get a response in prices if the firm operates under decreasing returns to scale, variable markups (Kimball, 1995, Atkeson and Burstein, 2008), or convex hiring costs. But none of those cases consider the wage-setting problem of the firm.

The price and wage setting problem of our firm is more nuanced and corresponds to the solution of the system (1.8),(1.2),(1.5) and (1.3). Before numerically solving it, we consider a first-order approximation of a simpler problem. A firm in steady state that desires to increase employment by $d \log n_t$. It can do it via two margins: raising wages or raising vacancies. Monopsony and hiring costs affect this trade-off, with two clear benchmarks. In a perfectly competitive labor market, wages do not increase and all the adjustment is trough vacancies. Static monopsony models like Card et al. (2018) only consider the wage as a margin of adjustment. Lemma 1 derives the optimal response for intermediate cases.

Lemma 1 (Wage and vacancy response to employment growth). *When firm employment increases by $d \log n_t$, the first order response of wages and vacancies is given by*

$$d \log w_t = \frac{1}{\delta} \frac{\nu + \frac{\epsilon_a}{\epsilon_w} - \delta}{(1+\nu)\epsilon_w + 1} d \log n_t$$
$$d \log v_t = \frac{1}{\delta} \frac{\delta \epsilon_w + \epsilon_\delta + 1}{(1+\nu)\epsilon_w + 1} d \log n_t,$$

where $\epsilon_w \equiv \epsilon_a + \epsilon_\delta$.

We can recover the benchmark cases by taking the limit of $\epsilon_w \to \infty$ and $\nu \to \infty$ respectively. While vacancies always increase, the sign of the change in wages is not determinate. The cost of raising wages increases with employment and the benefit comes from reducing the number of vacancies, either because fewer workers quit or more accept the job offers. While theoretically is possible that the optimal response of a firm that wants to grow is to reduce wages, Assumption 1 rules that out.

Assumption 1. *Parameters are such that in steady state, the following condition is satisfied*

$$\nu + \frac{\epsilon_a}{\epsilon_w} - \delta \geq 0$$

Given that the turnover rate δ is a small number, this assumption is likely to be satisfied as long as (i) vacancy costs are slightly convex and/or (ii) the acceptance elasticity is not too small. It does not hold in a particular situation: when the firm has linear costs by hire ($\epsilon_a = 0$, $\nu = 0$).

Having derived how wages and vacancies respond to an increase in labor demand, we turn the attention on how labor market power affects price setting and how product market power affects wage setting. To do so, make the problem static by assuming that $\beta = 0$, and further assume that the quit and acceptance elasticity coincide, $\epsilon_a = \epsilon_\delta$. The general equilibrium model presented in Section 1.3 will have this property, consistent with Manning

(2003). We will think of an increase in labor market power as a decrease in ϵ_w, keeping the turnover rate constant, and an increase in product market power as a decrease in ϵ_p.[8] Then, we can derive the next proposition.

Proposition 2 (Response to demand shocks)**.** *Under Assumption 1 and $\beta = 0$, in response to a positive demand shock, $d \log z_t$*

- *The price response $\frac{d \log p_t}{d \log z_t}$ is increasing in* labor *market power.*
- *The wage response $\frac{d \log w_t}{d \log z_t}$ is increasing in firm* product *market power.*

The proof is in Appendix A.1.2, where we also show that the wage response also increases with product market power. The intuition of Proposition 2 is as follows. For the first part, higher labor market power implies that the firm requires a larger wage increase to achieve a certain level of employment, which increases marginal costs and thus prices. For the second part, larger market power implies that any demand shock has a larger effect in quantities, and since wages increase with employment, wages also respond by more.

Numerical simulation We relax the assumption of $\beta = 0$ and numerically solve the firm problem. For the benchmark case, we set the quit elasticity and the acceptance elasticity to 3.5, as proposed by Sokolova and Sorensen (2021). We will target this elasticity in the general equilibrium model. We normalize the aggregate wage to 1 and set $\delta(w_t) = \bar{\delta} w_t^{-\epsilon_\delta}$, with $\bar{\delta} = 0.12$, which corresponds to the quarterly turnover rate when the firm sets the wage equal to the market wage. In each of the simulations, we calibrate κ_v so the steady state wage is equal to one, and vacancy costs are assumed to be quadratic ($\nu = 1$). The demand elasticity is set to 9, which implies a markup of 12.5% and the quarterly discount rate is $\beta = 0.995$.

Figure 1-2 shows the impulse response of a shock that raises demand by 10% on impact and decays exponentially at rate $\rho = 0.8$. In both panels, the solid blue line represents the benchmark case. When the firm faces an increase in demand, it raises prices by a little bit less than 1% and it raises wages by around 2% on impact. After the initial demand shock has passed, the firm has too many workers and lowers the wage to reduce its size. Firing workers is always dominated by reducing the wage so they voluntarily quit. [9] The green doted line represents the case when the firm has no price-setting power (in the upper panel) or has no wage-setting power (lower panel). In both cases, demand shocks have no effect on prices or wages, for unrelated reasons. When the product market is close to perfectly competitive,

[8] When changing labor market power, assume that turnover costs are recalibrated so both firms have the same turnover rate in steady state.

[9] A model where firing is optimal requires the notion of wage rigidity.

firms have little room to raise prices, since when they do, demand for their products falls a lot. Given that employment does not increase, neither do wages. With perfectly competitive labor markets, prices do not increase because marginal costs do not increase. The firm can hire more workers by marginally increasing wages, and thus they do not need to pass the cost into prices. For this result to hold, production must be constant returns to scale. The model comparison with the perfect competition case highlights the importance of taking into account the wage-setting power of firms when those are subject to idiosyncratic shocks.

Now we show how the results obtained in Proposition 2 also hold in the dynamic model. In the first panel of Figure 1-2, the elasticity of product demand is lowered to 4, which implies a markup of 33%. Here, an increase in product market power increases the response of prices and wages. The lower panel decreases the quit and acceptance elasticity, thereby giving more monopsony power to the firm. More wage-setting power increases the price response with an ambiguous effect on the wage. On one end, more monopsony power pushes for a higher wage response given employment growth, but the increase in prices reduces the labor demand of the firm.

Monopsony and wage rigidity In competitive models where firms take the wage as given, higher wages are always associated with higher marginal costs. This is not the case if we think of wages as being optimally set by firms. Under monopsony, a higher wage increases the wage bill but makes it easier to hire and retain workers. In Figure 1-2 we saw how the optimal response to a demand shock is to raise wages. If wages are rigid, the firm lacks one lever to adjust and therefore marginal costs are larger than in the scenario where wages were allowed to increase. The Appendix A.1.3 shows how, under Assumption 1 and $\beta = 0$, wage rigidity *increases* the price response of a demand shock, contrary to standard models.

This is what is shown in Figure 1-3 and the case for fixed wages. The price response on impact is around 20% higher and more persistent. In this case, firms would like to rise wages to hire more workers, and wether firms face upward wage rigidity is a debated issue. However, there is vast evidence that they dislike wage cuts Tobin (1972) and those are infrequent Grigsby, Hurst, and Yildirmaz (2021). The optimal response under flexible wages implies a big wage increase on impact and future wage reductions. A model with downward wage rigidity would also prevent the initial wage increase and have similar effects, since firms are reluctant to raise wages if they are unable to lower them in the future.

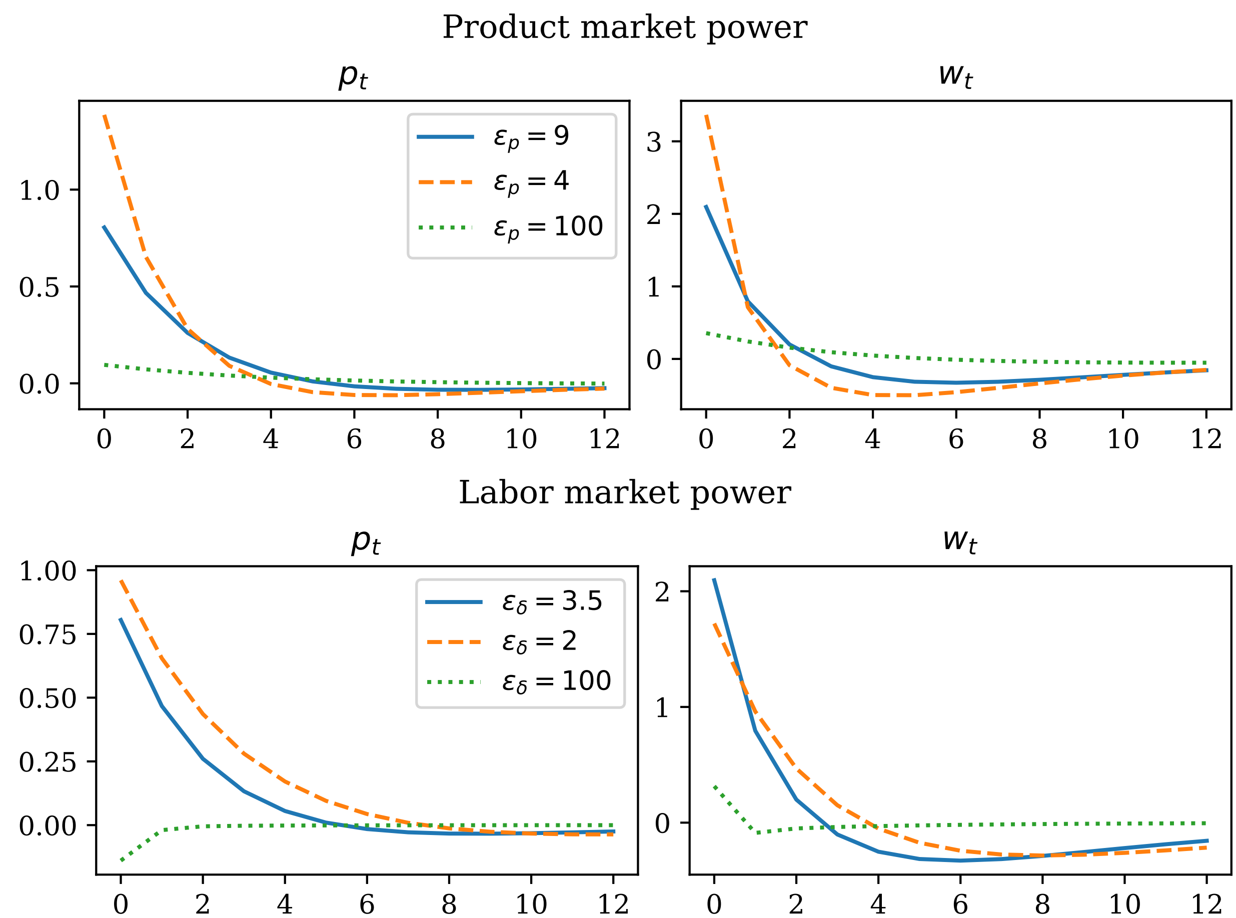

Figure 1-2: Price and wage response to a demand shock

Note: Price and wage response to a 10% demand shock that decays exponentially with persistence $\rho = 0.8$. Each period is a quarter, and units are in percentage points. The first panel shows the effect of product market power on the price and wage costs and the second panel shows the effect of labor market power.

1.3 A Monopolistic Model with Nominal Rigidities

The previous section introduced the firm problem and showed how it responds to idiosyncratic shocks. This section develops a general equilibrium model to think about the aggregate implications of monopsony. It endogenizes the acceptance, turnover and demand functions that before were taken as given. The starting point is a New Keynesian model with on-the-job search and wage posting. Firms are the same as before with the addition of nominal price and wage rigiditites. The household block is standard. It supplies one unit of labor inelastically and makes consumption-saving decisions. Unemployed workers always accept jobs and job-to-job transitions are subject to an idiosyncratic taste shock that allows for the existence of a unique equilibrium.

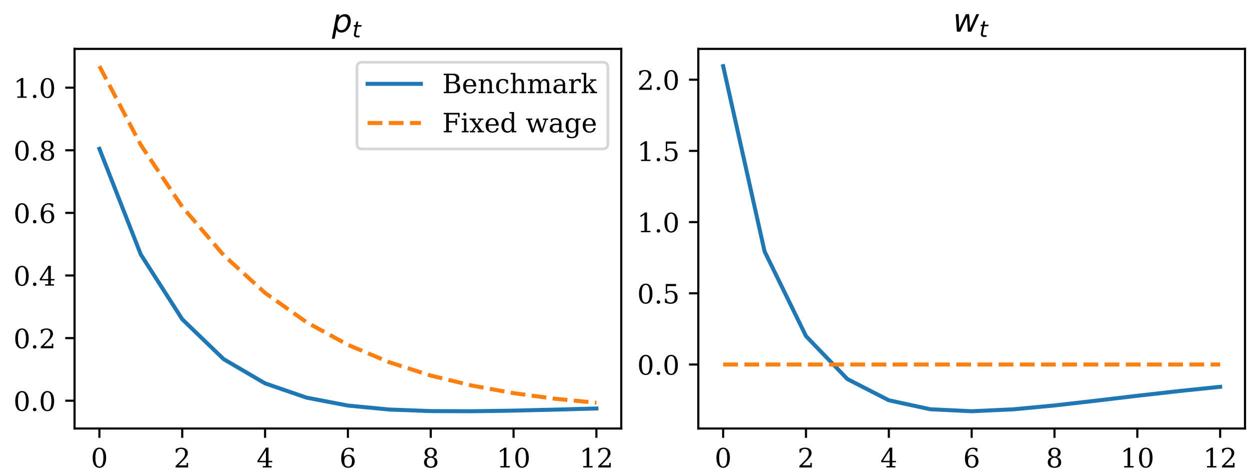

Figure 1-3: Price and wage response with wage rigidity

Note: Price and wage response to a 10% demand shock that decays exponentially with persistence $\rho = 0.8$. Each period is a quarter, and units are in percentage points. Comparison of the benchmark model and a model where wages are fixed.

Households. The household block is standard. There is a representative household with a continuum of members of measure unity that make consumption-saving decisions. They supply one unit of labor exogenously and take the total labor income $\int w_{it} n_{it} di$ as given. Employment is determined through a search and matching process that we describe below. The family provides perfect consumption insurance for its members, implying that consumption is the same for each person, regardless of whether he or she is currently employed. The preferences of the representative household are the equally weighted average of the preferences of its workers,

$$U_t = E_t \sum_{k=0}^{\infty} \beta^k u(C_{t+k}).$$

C_t is a CES aggregator of individual varieties c_{it} with elasticity ϵ_p across goods i, priced at p_{it}. Households can save on risk-free bonds B_t sold at a price Q_t set by the central bank that pays one nominal unit at $t+1$ and get profits from firms rebated Π_t. Their budget constraint is

$$\int_0^1 p_{it} c_{it} di + Q_t B_t = \int_0^1 w_{it} n_{it} + B_{t-1} + \Pi_t.$$

Demand for each variety is $c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\epsilon_p} C_t$ and the price level satisfies $P_t^{1-\epsilon_p} \equiv \int p_{it}^{1-\epsilon_p} di$. The Euler equation is

$$Q_t = \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \frac{1}{\Pi_{t+1}^p},$$

where $\Lambda_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$ is the discount factor and Π_t^p is price inflation.

Monopsonistic firms with nominal rigidities. Each variety i is sold by an infinitely lived firm as described in Section 1.2. In order to close the model in general equilibrium, we introduce a small timing variation. The firm ends the period $t-1$ with n_{it-1} employees but before period t starts, an exogenous fraction $\bar{\delta}$ quit. Workers become unemployed and ready to search for a job at t. These exogenous quits represent layoffs, retirements, or reallocations, which in the data represent around two-thirds of job separations and are not related to the wage that workers receive. Once wages have been posted, a fraction $\delta_t\left(\frac{w_t}{W_t}\right)$ of the remaining workforce is poached by other firms. The employment law of motion (1.8) is substituted by

$$n_{it} = \left(1 - (\bar{\delta} + (1-\bar{\delta})\delta_t\left(\frac{w_{it}}{W_t}\right)\right) n_{it-1} + a_t\left(\frac{w_{it}}{W_t}\right) v_{it}. \tag{1.8}$$

In a traditional wage posting models, the acceptance and turnover decision depend on the entire distribution of wages. The notation used here already presumes the existence of a symmetric wage equilibrium, which will be discussed later. We add the subscript t in the quit and acceptance rates because these depend on aggregate conditions. Let $\tilde{\delta}_t\left(\frac{w_{it}}{W_t}\right) \equiv \bar{\delta} + (1-\bar{\delta})\delta_t\left(\frac{w_{it}}{W_t}\right)$ be the total turnover at the firm level. Workers hired at time t are ready to produce.

The firm faces Rotemberg nominal rigidities in price and wage setting. Let $x_{it-1} \equiv (n_{it-1}, p_{it-1}, w_{it-1})$ be the relevant state variable t of a firm that ended the period $t-1$ with $n_{it-1}, p_{it-1}, w_{it-1}$. $J_t(x_{t-1})$ is the corresponding value function. Firms take as given the aggregate sequences $\{N_{t+k}, V_{t+k}, P_{t+k}, W_{t+k}, Y_{t+k}\}_{k=0}^{\infty}$, where Y_t is total output, which defines the function $\{\delta_{t+k}, a_{t+k}, \mathcal{D}_{t+k}\}_{k=0}^{\infty}$. They discount time using the stochastic discount factor of households. To simplify notation, we drop the subindex i from the Bellman equation, which is

$$\begin{aligned} J_t(x_{t-1}) = \max_{p_t, w_t, v_t, n_t} & \frac{p_t}{P_t}\mathcal{D}_t\left(\frac{p_t}{P_t}\right) - \frac{w_t}{P_t}n_t - \kappa_v v_t \\ & - \frac{\kappa_p}{2}\left(\frac{p_t}{p_{t-1}} - 1\right)^2 Y_t - \frac{\kappa_w}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 N_t + E_t[\Lambda_{t,t+1}J_{t+1}(x_t)] \end{aligned}$$

subject to (1.8) and $\mathcal{D}_t\left(\frac{p_t}{P_t}\right) = n_t$. To simplify the problem, it is assumed that vacancy costs are linear, posting a vacancy costs κ_v. Traditional wage posting models like Burdett and Mortensen (1998) assume linear production and then they require convexity in vacancy costs to determine firm size. Here, even with linear production and linear vacancy costs, firm size is determined by the shape of the demand function. κ_p and κ_w are price and wage rigidity parameters, respectively. The assumption of Rotemberg pricing simplifies the formulation by making the equilibrium symmetric. To pay for vacancy costs and the Rotemberg adjustment costs, firms buy a bundle of goods from all other firms as it is the case in a roundabout economy. They aggregate this bundle using the same elasticity as households do.

The problem formulation differs from standard New Keynesian models with search like Gertler, Sala, and Trigari (2008), Blanchard and Galí (2010) or Christiano, Eichenbaum, and Trabandt (2016) where the identity of the price setter and the wage setter is differentiated to keep both problems tractable. Krause and Lubik (2007) has a similar model where firms hire by posting vacancies workers and have price-setting power, but they take wages as given. In Bloesch and Larsen (2023), firms solve the same model in steady state and with flexible prices and wages.

Labor Markets. Labor markets are subject to search frictions and feature on-the-job search. There is a unit of workers willing to supply labor. Aggregate unemployment is measured at the end of the period and is given by $U_t = 1 - N_t$. As it has been noted, at the beginning of the period a fraction $\bar{\delta}$ of workers separate from their firms and search for another job. Thus, the pool of unemployed workers that search for a job at period t is $U_{t-1} + \bar{\delta}N_{t-1}$. On top of that, employed workers search with efficiency s relative to the unemployed ones. Then market tightness is

$$\theta_t = \frac{V_t}{U_{t-1} + \bar{\delta}N_{t-1} + s(1-\bar{\delta})N_{t-1}},$$

where $S_t \equiv U_{t-1} + \bar{\delta}N_{t-1} + s(1-\bar{\delta})N_{t-1}$ is total search effort. A constant returns matching function generates $m(V_t, S_t)$ matches at period t. The probability that a vacancy meets a potential worker is $q(\theta_t) = m(\theta^{-1}, 1) = \frac{m(S_t,V_t)}{V_t}$ with elasticity $\frac{d\log q}{d\log\theta} = -\eta$. Conditional on a successful match, the applicant is an employed worker with probability

$$p_t^E = \frac{s(1-\bar{\delta})N_{t-1}}{U_{t-1} + \bar{\delta}N_{t-1} + s(1-\bar{\delta})N_{t-1}}$$

and unemployed with $p_t^U = \frac{U_{t-1}+\bar{\delta}N_{t-1}}{U_{t-1}+\bar{\delta}N_{t-1}+s(1-\bar{\delta})N_{t-1}}$. This description of the labor market is standard in any model with on the job search like Faberman et al. (2022) or Moscarini and Postel-Vinay (2023). For a given level of vacancies, a higher employment level makes it costlier hire a worker because (i) it is less likely to match one of them and (ii) it is more likely that the matched worker is already employed, who is likely to reject the job offer, as we describe next.

For this section, an unemployed worker that receives an offer accepts it regardless of the wage, which will be relaxed in Section 1.6.4. This is a simplifying assumption that makes aggregate labor supply fixed and allows firms to set wages only considering the behavior of the employed workers. In particular, a worker earning w_{it} accepts an offer that pays w_{jt} if $\varepsilon_{jt} w_{jt} \geq w_{it}$, where $\varepsilon_{jt} \sim F_{\varepsilon}$ is a multiplicative taste shock. Therefore, the probability that a worker accepts the offer from firm j is $1 - F_{\varepsilon}\left(\frac{w_{it}}{w_{jt}}\right)$. We can think of $F_{\varepsilon}(1) > 0.5$ as moving costs, in the sense that the probability of accepting a job offer that pays the same as the current job is less than $1/2$. This behavior corresponds to a myopic worker that fully discounts the future and makes job decisions according to the time t best option. At a steady state equilibrium, workers expect $w_{it} = w_{it+1}$ so the assumption is innocuous. But it greatly simplifies the problem of the firm out of the steady state, that otherwise would be untractable. The implications of the effect of monopsony would be the same at the expense of losing analytical tractability.

It is well known that in a wage posting model a la Burdett and Mortensen (1998), a symmetric equilibrium where all firms post the same wage does not exist. If the wage distribution had any mass point, it would be profitable for firms in that point to deviate and offer a slightly higher wage. This argument misses the fact that jobs are heterogeneous and people do not move from job to job solely based on the wage paid. Preference heterogeneity is also regarded as one of the main sources of monopsony power by firms. Albrecht, Carrillo-Tudela, and Vroman (2018) show that if workers have idiosyncratic taste shocks for different firms with $f_{\varepsilon}(1)$ high enough, then a symmetric equilibrium where all firms post the same wage can be sustained. The reason is that preference heterogeneity reduces the elasticity of the turnover and acceptance rates, which makes deviating unprofitable because the wage increase required to the entire workforce does not compensate the reduced turnover and increased acceptance rate. $f_{\varepsilon}(1)$ is the mass of workers indifferent between two jobs that pay the same. The traditional search model is the case where $f_{\varepsilon}(1) \to \infty$, and workers always move to the better paying job. Trought the paper, we assume that the taste dispersion is big enough such that the symmetric equilibrium exists and let W_t be the symmetric market wage.

With the labor markets defined, the acceptance rate of vacancies is given by

$$a_t\left(\frac{w_t}{W_t}\right) = q(\theta_t)\left(1 - p_t^E F_{\varepsilon}\left(\frac{W_t}{w_t}\right)\right).$$

With probability $q(\theta_t)$, the vacancy meets a worker. Conditional on the match, with probability p_t^E this worker is already employed and rejects the job offer with probability $F_{\varepsilon}\left(\frac{W_t}{w_t}\right)$.

Similarly, the endogenous job-to-job turnover rate is

$$\delta_t\left(\frac{w_t}{W_t}\right) = \frac{V_t q(\theta_t) p_t^E \left(1 - F_\varepsilon\left(\frac{w_t}{W_t}\right)\right)}{(1-\bar{\delta})N_{t-1}}.$$

When there are V_t vacancies, a fraction $q(\theta_t)$ matches with a worker, and with probability p_t^E this worker is employed. $1 - F_\varepsilon\left(\frac{w_t}{W_t}\right)$ is the probability that the worker accepts the outside offer and leaves the firm. All these matches are divided by the current mass of workers $(1-\bar{\delta})N_{t-1}$ under the assumption that no worker receives more than one offer at any given period.

Finally, since there is on the job search, the aggregate law of motion of employment differs from the firm law of motion because job-to-job transitions do not add new workers into the workforce. Only vacancies that match unemployed workers add to employment. The law of motion for aggregate employment is

$$N_t = (1-\bar{\delta})N_{t-1} + V_t q(\theta_t) p_t^U. \tag{1.9}$$

Given that the unemployed workers accept all job offers, this law of motion is independent on the wage.

Market clearing and monetary policy To close the model, a central bank sets interest rates according to a Taylor rule that targets current inflation. It sets the bond prices to

$$Q_t = e^{m_t}\beta\left(\Pi_t^p\right)^{-\phi_\pi},$$

where Π_t^p is the inflation rate and ϕ_π is its Taylor coefficient. m_t is a monetary policy shock that is interpreted as a demand shock.

Total output Y_t is devoted to consumption and to pay for the vacancy and Rotemberg costs. The market clearing condition is

$$Y_t = C_t + \kappa_v V_t + \frac{\kappa_p}{2}\left(\Pi_t^p - 1\right)^2 Y_t - \frac{\kappa_w}{2}\left(\Pi_t^w - 1\right)^2 N_t.$$

Demand for each firm is $\mathcal{D}_t\left(\frac{p_t}{P_t}\right) = \left(\frac{p_t}{P_t}\right)^{-\epsilon_p} Y_t$.

Equilibrium The equilibrium definition is standard. Firms take as given aggregate variables as given and maximize profits by choosing p_{it}, w_{it}, v_{it} and n_{it} given their initial state $(n_{it-1}, p_{it-1}, w_{it-1})$. In the symmetric equilibrium, $P_t = p_{it}, W_t = w_{it}, V_t = v_{it}$ and $N_t = n_{it}$

for all i and no firm finds it profitable to deviate.

1.4 Monopsony and the Phillips Curves

The Phillips curve is a central topic in monetary economics that help us understand the relationship between economic activity and inflation. The conventional NKPC literature uses unemployment or the output gap as measure of economic activity (Hazell et al., 2022), but for this formulation to be correct, several assumptions have to be made. In particular, wages are rigid. As emphasized by both Galí and Gertler (1999) and Sbordone (2002), the primitive form of the curve features real marginal costs as the forcing variable for price inflation. This section derives the marginal cost Phillips curve of the monopsonistic model and a novel wage Phillips curve that can be expressed as a function of labor market variables.

We start by solving for the real wage of an economy with flexible nominal wages. We can use the wage setting equation (1.3) and the definitions of $a_t(\cdot)$ and $\delta_t(\cdot)$ to get its general equilibrium counterpart

$$\frac{1}{P_t}n_t = \frac{\kappa_v}{q(\theta_t)\left(1 - p_t^E F_\varepsilon\left(\frac{W_t}{w_t}\right)\right)}\left(\frac{V_t q(\theta_t) p_t^E f_\varepsilon\left(\frac{w_t}{W_t}\right)\frac{1}{W_t}}{(1-\bar{\delta})N_{t-1}}(1-\bar{\delta})n_{t-1} + q(\theta_t)p_t^E f_\varepsilon\left(\frac{W_t}{w_t}\right)\frac{W_t}{w_t^2}v_t\right).$$

Applying symmetry greatly simplifies the problem. In this economy, the flexible real wage is given by

$$\omega_t^{flex} = \kappa_v 2\epsilon_{a,t}\frac{V_t}{N_t}. \tag{1.10}$$

$\epsilon_{a,t} \equiv \frac{d\log a_t}{d\log w} = \frac{p_t^E f_\varepsilon(1)}{1-p_t^E F_\varepsilon(1)}$ is the individual firm acceptance elasticity, which coincides with the quit elasticity $\epsilon_{\tilde{\delta},t} \equiv -\frac{d\log\tilde{\delta}_t}{d\log w}$ in steady state. Manning (2003) shows that, under certain conditions satisfied here, both elasticities coincide and hence the 2 multiplying in the right-hand side. This is because every endogenous quit is related to a hire. The acceptance elasticity varies over the business cycle because p_t^E is increasing in the employment level, because it becomes more likely that firms have to hire already employed workers. Autor, Dube, and Mcgrew (2023) documents that empirically. An increase in worker search effort, which is assumed to be constant here, would also increase the quit elasticity (Faccini and Melosi, 2023).

Equation (1.10) substitutes the wage setting equation of other models. Wages in general equilibrium are pinned down by competition among firms for workers, as is the case in Burdett and Mortensen (1998). Under bargaining, the real wage is a weighted average between the labor productivity and the unemployment benefits plus the outside option of the worker. In models with classical labor supply, the real wage is equal to the marginal rate of substitution of households. Here, real wages raise when the quit elasticity and vacancies are high, which imply that more firms poach workers from each other. It decreases with employment, because the cost of raising wages is proportional to the workforce. Note that we do not need a concept of unemployment here; all workers are willing to accept the first job they find. The appendix shows how we can express (1.10) in log deviations from the steady state as a function of labor market observables

$$\hat{\omega}_t^{flex} = \hat{V}_t - \hat{H}_t + \frac{1-\bar{\delta}}{\bar{\delta}} \Delta N_t + \hat{EE}_t - \hat{UE}_t. \tag{1.11}$$

$\hat{V}_t - \hat{H}_t = \hat{a}_t$ is the acceptance rate, where H_t is total hiring. It can be understood as the deviation on the time to fill a vacancy. When it takes time to fill a vacancy it means that the market is tight, either because labor supply is low or labor demand is high, which increases the real wage. The next element is employment growth, which also raises competition for workers. And the last term is the ratio of employment to employment vs unemployment to employment transitions.

Adding nominal wage rigidities in prices and wages, the next proposition defines the price and wage Phillips curve, and the implications of monopsony on the transmission of demand shocks into prices and wages. We consider that monopsony increases when the wage markdown increases, which by Proposition 1, it coincides with the relevance of hiring costs. Next we see how both product market power and labor market power interact in the price and wage formation process.

Proposition 3 (Price and Wage Phillips curves)**.** *In response to a demand shock, the dynamics of price and wage inflation of the monopsonistic economy satisfy the following system of equations:*

$$\pi_t^p = \frac{\epsilon_p}{\kappa_p} \left(\tau \hat{\omega}_t + \frac{1-\tau}{1-\tilde{\beta}} \left(\hat{V}_t - \hat{H}_t - \beta \left[(1-\tilde{\delta})(\hat{V}_{t+1} - \hat{H}_{t+1}) - \tilde{\delta} \hat{\tilde{\delta}}_{t+1} \right] E_t \right) \right) + \beta E_t[\pi_{t+1}^p] \tag{1.12}$$

$$\pi_t^w = \frac{1}{\kappa_w} \frac{\tau}{\mathcal{M}_p} \left(\hat{V}_t - \hat{H}_t + \frac{1-\bar{\delta}}{\bar{\delta}} \Delta \hat{N}_t + \hat{EE}_t - \hat{UE}_t - \hat{\omega}_t \right) + \beta E_t[\pi_{t+1}^w] \tag{1.13}$$

$$\Delta \hat{\omega}_t = \pi_t^w - \pi_t^p. \tag{1.14}$$

A higher degree of monopsony (lower τ) raises the importance of hiring costs in the price Phillips curve and flattens the wage Phillips curve.

In models without labor market frictions and constant marginal product of labor, the only driver of inflation is the real wage. Search frictions, regardless of the wage protocol used, acknowledge the cost of hiring workers, which raises when the labor market becomes tighter, as measured by $\hat{V}_t - \hat{H}_t$. When vacancies are high relative to the total hires in a period, it means that many of those vacancies are either unable to match a worker or they are turned down. If workers are more likely to leave the firm at $t + 1$, then the net cost of hiring it at t also increases because it reduces the net present value of a hire. This price Phillips curve is similar to Krause, Lopez-Salido, and Lubik (2008) with two differences. On the job search makes the separation rate is endogenous and depends on market conditions, and (nominal) wages are posted by firms instead of having the real wage bargained.

Like prices, nominal wages also increase when workers are harder to find, when firms want to grow, and when the market is tight in the sense that more hires come from other employees, consistent with evidence provided by Autor, Dube, and Mcgrew (2023). Unemployed acceptance decision is independent of the wage offered so firms have low incentives to increase wages when they are mostly hiring from the unemployment pool. Moscarini and Postel-Vinay (2023) argue for the oposite sign in the EE/UE ratio which they call Acceptance Ratio AC. Theirs is a model of a job ladder, and a high AC ratio means that workers are missmatched, containing wage pressure.

A novel feature of this model is that both labor market power and product market power affect both Phillips curves. Product market power (lower ϵ_p, higher $\mathcal{M}_p$) flattens both curves. This result comes from the assumption that nominal rigidities have a menu cost component and firms set wages. When firms face Rotemberg costs, they trade off the explicit costs of increasing prices versus the benefits of having a price closer to the optimum. These benefits depend on the curvature of the profit function with respect to the price, which decreases with product market power. If the elasticity of demand is low, firms can afford being misspriced so the incentive to change prices is reduced. If demand is very elastic, then Rotemberg pricing converges to flexible prices. A similar thing happens in the wage Phillips curve. Higher monopsony power implies that having an optimal wage is less important, and wages move by less.

The results on Proposition 3 are not enough to conclude whether monopsonistic economies are more or less inflationary. It increases the importance of hiring costs relative to the real wage. However, over the business cycle, the volatility of hiring costs is around one order of magnitude larger than the volatility of the real wage. Therefore, higher monopsony power steepens the wage Phillips curve.

We have assumed that the marginal product of labor is constant for simplicity, which implies that the only driving force of inflation is related to the labor market. In the quantitative section, we allow for production to have decreasing returns to scale. Lorenzoni and Werning (2023) using a stylized unions model and Gagliardone and Gertler (2023) in a bigger DSGE model with Nash bargaining, present a model with two inputs, labor and oil, with very low elasticity of substitution. Then demand shocks or oil supply shocks are inflationary because they sharply reduce the marginal product of labor which raises marginal costs.

Costly hire We have assumed that firms pay vacancy costs, but an equally valid assumption would have been that firms pay per hire. Pissarides (2009), Silva and Toledo (2009), and Christiano, Eichenbaum, and Trabandt (2016) emphasize the importance of post-match hiring cost. These costs are independent of the labor market condition and mute the wage response to aggregate business cycle. Bargaining models have a particular way of adding these costs: they are paid *after* the match has been created but *before* the bargaining takes place, which means they are sunk once the bargaining starts. In Christiano, Eichenbaum, and Trabandt (2016), the calibration implies that these costs represent 94% of the total cost of hiring a worker. While these costs mute the wage response to aggregate market conditions, it is the variation in the matching costs that ultimately defines the wage. The monopsonistic model instead is well behaved even if vacancies are totally free. The key is on-the-job search. Firms set wages to avoid workers from leaving to other firms, because replacing them is costly. The Appendix A.2.4 develops this case, here we discuss the two main implications.

The first is that the relevant elasticity of labor supply that every firm faces is doubled with respect to the case when posting vacancies is costly: firms do not care about the acceptance elasticity. The traditional approach to estimate the degree of market power is to estimate quit elasticities and multiply them by two as Manning (2003) proposes. This methodology crucially consider that vacancies, not hiring, is costly. Therefore, they provide a lower bound on monopsony. For example, the benchmark 3.5 quit elasticity that we use in this paper implies a markdown of 13% if we consider that vacancies are costly, but it increases to 23% in case hiring is costly.

The second implication is that as expected, the wage and price response to market tightness is muted. Hiring costs do not increase because workers are harder to find, but rather because they quit more often.

1.5 The Importance of Hiring Costs

Hiring workers is costly, and firms internalize that when making price and wage-setting conditions. The previous section has emphasized the importance of such costs and how they relate to monopsony power, but it remains to assess whether they are indeed a significant driver of marginal costs. First, we note that previous search models already have the notion that hiring costs matter for inflation dynamics, but their calibration implies that their effect is negligible. Then using the result in Proposition 1, that in the monopsonistic model presented here, a reasonable calibrated model implies that search frictions are an important driver of the inflation.

The traditional approach. The general approach of modeling New Keynesian models with search frictions is to assume a two-layer economy. Christiano, Eichenbaum, and Trabandt (2016), Gertler, Sala, and Trigari (2008), Moscarini and Postel-Vinay (2023), Blanchard and Galí (2010), to put some examples, share this structure. This assumption allows to separate the identity of the price setter and the wage setter, simplifying both problems. Firms subject to search frictions hire workers, and sell 'labor services' at a perfectly competitive price ϑ_t, using Christiano, Eichenbaum, and Trabandt (2016) notation, and retailers buy these services, differentiate them, and sell them subject to nominal rigidities and a downward sloping demand curve.

Let's consider the labor market, where firms can post vacancies paying some cost to be defined. Letting J_t be the value of a single job, the bellman equation that governs it

$$J_t = \vartheta_t - w_t + \beta(1 - \bar{\delta})J_{t+1}. \tag{1.15}$$

When there is a match, the job generates a flow surplus of $\vartheta_t - w_t$ at t and with probability $(1 - \bar{\delta})$ it survives another period. The papers mentioned, to the exception of Moscarini and Postel-Vinay (2023), do not feature on-the-job search so separations are exogenous. We do not need to specify how the wage is defined, different papers have different wage-setting protocols. The model is closed by imposing a free entry condition, equating the expected benefit of a vacancy to its cost. In reduced form, it pins down the value of a job given some function $c(\cdot)$ that potentially depends on market tightness,

$$J_t = c(\theta_t).$$

The standard case, as in Pissarides (2017), is when vacancies have a cost to post κ and are matched to a worker with probability $q(\theta_t)$. Then the value of a job is $J_t = \frac{\kappa}{q(\theta_t)}$. Other models consider convex vacancy costs, or as in Pissarides (2009) or Christiano, Eichenbaum, and Trabandt (2016), fixed costs that are paid after the match has been realized. With the free-entry condition, we can rewrite (1.15) as

$$\vartheta_t = w_t + \frac{\kappa}{q(\theta_t)} - \beta(1 - \bar{\delta})\frac{\kappa}{q(\theta_{t+1})}. \tag{1.16}$$

The second layer in the economy are retailers that buy the labor services at price ϑ_t, and

Paper	τ	Method/Target
Christiano, Eichenbaum, and Trabandt (2016)	0.994	Free entry
Gertler, Sala, and Trigari (2008)	0.996	Free entry
Blanchard and Galí (2010)	0.989	Hiring costs is 1% of GDP
Krause, Lopez-Salido, and Lubik (2008)	0.95	Labor share and output elasticity of labor
This paper	0.87	Quit elasticity and turnover

Table 1.1: Implied τ by popular models with search frictions.

sell their own variety subject to a downard sloping demand curve and nominal rigidities. If we assume that production is linear $y_t = n_t$, then the marginal cost is the prive of the labor service, so $\lambda_t = \vartheta_t$.[10] Linearized, we obtain Equation 1.6, which we rewrite here to ease of exposition:

$$\hat{\lambda}_t = \tau \hat{w}_t + (1 - \tau)\hat{\gamma}_t$$

with $\hat{\gamma}_t$ being the net cost of hiring a worker. The value of τ is rarely reported in the aforementioned papers. In most cases, the cost of posting a vacancy is calibrated to target the employment level, instead of being a parameter that can be observed in the data.

What are the costs implied by standard calibrations of models of the labor market? In Christiano, Eichenbaum, and Trabandt (2016) case, the total cost associated with hiring a new worker is roughly 7 percent of their quarterly wage rate. This means that in steady state, $c(\theta) = 0.07w$, but $\tau = \frac{w}{w+(1-\tilde{\beta})c(\theta)}$ takes into the account that a worker hired at t will most likely stay at the firm at $t+1$. Therefore, it would be wrong that hiring costs represent 7% of the cost of hiring a worker because those costs are only paid once. Taking that into account, we get $\tau = 0.994$. The real wage represents virtually all the cost of hiring a worker, the labor market tightness has little effect on marginal costs besides it's effect it has on the wage. Christiano, Eichenbaum, and Trabandt (2016) is not an exotic calibration but rather the norm. In Gertler, Sala, and Trigari (2008), the marginal cost of hiring a worker represents a 3.3% of its quarterly wage, which implies $\tau = 0.996$. As a third example, Blanchard and Galí (2010) calibration implies $\tau = 0.989$. As a general rule, models where the cost of hiring a worker is around 0-10% its quarterly wage will have a hard time getting a τ significantly below one. Taken to the real world, a value of 5% implies that a firm values a worker that earns \$40.000 a year by \$500. It would rather look for another worker than face a one-time-off cost of \$501 to keep the incumbent.

[10] If that was not the case, marginal costs would be the price of the labor services over their marginal product, but any conclusion with respect to the importance of hiring costs would remain unchanged

The monopsony view. Proposition 1 presented a way to back up τ. It coincides with the wage markdown, and it provides a formula for it. All we need to know is the turnover rate $\tilde{\delta}$, which is an observable variable, the quit elasticity $\varepsilon_{\tilde{\delta}}$, which has been estimated by many papers, and β, the discount rate, which we assume to be 0.995, 2% annual.[11]

Sokolova and Sorensen (2021) provide a meta-study of 1,320 estimates the elasticity of labor supply from 53 studies. Out of those that compute a separation elasticity, the authors conclude that the best practice estimate for $\epsilon_{\tilde{\delta}}$ is 3.5, although they report a lot of variance across estimates. Together with a quarterly turnover rate of $\tilde{\delta} = 0.12$, it implies that $\tau = 0.87$, which is equivalent to say that workers get 87% of their marginal product, or that the wage represents 87% of the cost of hiring a worker, and the remaining 13% is the net cost of hiring him. This fraction is an order of magnitude larger than the one indirectly calibrated by previous papers.

Is this order of magnitude reasonable? The wage setting condition in steady state is $\omega = \frac{\kappa}{a}\tilde{\delta}2\epsilon_{\tilde{\delta}}$. As in many search models, the definition a vacancy, it's cost and the acceptance rate are objects hard to define and measure. One could interpret a vacancy as a cheap 'now hiring' add, with very low cost and also very low probability that anybody looking at this add ends up being employed. Another interpretation is that a vacancy is a formal job offer after several rounds of interviews and after discarding other candidates. The cost of this job offer is much higher but the likelihood that the candidate accepts the job is also high. Without taking a stance on what v, κ and a are, the term $\frac{\kappa}{a}$ has a clear interpretation: is the willingness to pay for a fully productive new worker. This value equals to 1.2 times the quarterly wage of this worker, or 15 weeks of wage, versus the less than 1 week of wage when we take standard calibration values.

Quantifying hiring costs is challenging. Empirical evidence on how firms recruit employees is still scarce, largely as a result of data limitations. In two papers using the same rich dataset of Swiss workers, Blatter, Muehlemann, and Schenker (2012) and Muehlemann and Strupler (2018) estimate that hiring costs of skilled workers range from 10 to 17 weeks, consistent with the findings reported here. Their dataset consists of a questionnaire of 4032 firms for the 2012 paper and 8874 firms for the 2018 paper administrated by the Swiss Federal Statistical Office and the Centre for Research in Economics of Education at the University of Bern. There, the human resources department filled out a questionnaire answering questions precisely related to the cost of hiring a worker. In particular, they were asked about average advertising costs, time spend in recruiting activities, time for the worker to become fully

[11] In case vacancy costs are not linear but can be written as $\frac{\kappa}{1+\nu}\left(\frac{v_t}{(1-\bar{\delta})n_{t-1}}\right)^{1+\nu}(1-\bar{\delta})n_{t-1}$, then the effective discount rate is $\tilde{\beta} = \beta\left(1 - \frac{1}{1+\nu}\tilde{\delta}\right)$, so we should also know the convexity of the vacancy function. However, $\nu > 0$ has a small effect into τ

Sector	$\epsilon_{\tilde{\delta}}$	$\tilde{\delta}$	τ
Art, accomodation & food	1.20	0.19	0.70
Wholesale, trade & transport	1.39	0.11	0.73
Education and healt	2.15	0.07	0.80
Manufacturing	2.29	0.07	0.81
Prof. business & financial services	3.91	0.12	0.88

Table 1.2: Implied hiring costs by sector

Note: Separation elasticities from Bassier, Dube, and Naidu (2022), turnover is from the LEHD J2J dataset and implied τ by sector

productive, and training costs spent per hire.[12]

An alternative way to assess how costly is to hire an employee is to observe its market price. Many firms opt for externalizing its hiring process to staffing firms, whose job is to find ideal candidates for every job opening. Indeed, one of the largest employment websites in the world, reports that external recruiters charge a commission of 15% to 30% of the hired employee's first-year salary, or equivalently from 8 to 16 weeks. Consistent with that the Staffing Industry Analysis survey of 300 North America staffing firms report that their median fee is 20%, or 10.4 weeks. A consideration with these values is that (i) they include a price markup which overestimates the cost of hiring an employee but (ii) they do not include the adaptation and training costs, which underestimates the cost of a new hire. Importantly, these three pieces of direct evidence point to the same order of magnitude as the value obtained using the model.

The model presented here only has one sector, but we can compute the importance of hiring costs by sector if we have estimates of the quit elasticity. Bassier, Dube, and Naidu (2022) provide such estimates for workers in the state of Oregon using LEHD data for 5 aggregated sectors. They estimate separation elasticities by comparing workers with similar work histories that moved to high vs low wage firms, and then compare the quit rate of both workers in the new firms. Using their preferred specification, they find a quit elasticity of 2.1, lower than the 3.5 best estimate proposed by Sokolova and Sorensen (2021). Table (1.2) shows their estimates by sector, together with the average turnover rates from 2000:2022 taken from the LEHD J2J data set and the implied value of the wage markdown and τ. There is substantial heterogeneity across sectors, and monopsony power is more prevalent in low-wage industries as expected. Despite high turnover rates, the elasticity with respect to the wage in such sectors is low. Professional business, on the other side, gets closer to a perfectly competitive market where workers are elastic to the wage and move often.

While a wage markdown of 70% in the Art, accommodation & food sector is big, but a

[12] See Blatter, Muehlemann, and Schenker (2012) Appendix A for the wording of the questionnaire.

credible estimate, it is hard to argue that firms devote 30% of their labor costs in to hiring workers. For ease the exposition, we have set up the problem such that vacancy costs are linear in vacancies, which equates marginal costs with average costs. If vacancy costs are convex in the vacancy rate, then the the 30% refers to the marginal cost, while the average costs can be significantly lower. If instead of κv_t we had $\frac{\kappa}{1+\nu}\left(\frac{v_t}{n_{t-1}}\right)^{1+\nu} n_{t-1}$, where ν is a measure of the convexity of the vacancy cost, assuming quadratic costs ($\nu = 1$) would implies that the wage bill represents 83% of the total cost of the workforce, in the most adverse scenario where the wage markdown is 0.7.[13]

1.6 Quantitative evaluation

Having derived the properties of the model, in this last section we calibrate it and evaluate it numerically. We performing several exercices and extensions, and compare it with a standard search model where wages are bargained and the real wage is rigid as in Blanchard and Galí (2010).

1.6.1 Calibration

A time period is a quarter. First, we discuss standard parameters. The discount factor is set to $\beta = 0.995$ to target a steady state real rate of two percent. Utility is log and the product market power is set at $\epsilon_p = 6$, which implies a markup of twenty percent over marginal cost. In the previous section, production was linear. Here we allow for decreasing returns to scale with $f(n_t) = An_t^\alpha$, normalizing $A = 1$ and we set $\alpha = 0.7$.

For the labor market, we obtain employment data from FRED and LEHD, using data from 2000:2020. The unemployment rate target over this period is 5.8% and the separations rate is 10.6%. The ratio of employment-to-employment quits relative to total quits is 0.32, which coincides with the ratio of employment-to-employment hires relative to total hires. This means that in steady state, the exogenous separation rate is $\bar{\delta} = 7.2\%$ and the endogenous one is $\delta = 3.7\%$. We set the elasticity of the matching function to 0.5. The relative search effort of the employed worker is obtained from the Job Search Supplement of the Survey of Consumer Expectations by Faberman et al. (2022). The survey allows to directly observe the incidence of offer arrivals by employment status, and employed workers search efficiency is $s = 0.23$. While they document that conditional on searching for a job, employed

[13]That is,

$$\frac{wn}{wn + \frac{\kappa}{2}\left(\frac{v}{n}\right)^2 n} = 0.83.$$

Parameter	Definition	Value
β	Discount factor	0.995
ϵ_p	Demand elasticity	6
$\bar{\delta}$	Exogenous turnover	0.072
s	Search intensity	0.23
α	Decreasing returns	0.7
η	Elasticity of the matching function	0.5

Table 1.3: Parameters used in the model

workers are more likely to receive offers, not all employed workers actively seek for jobs. In this benchmark model, we are assuming that unemployed accept any job offer, which will be relaxed later. Knowing the ratio EE/UE, the search efficiency of employed workers, and the unemployed level, we can infer the probability that employed workers accept a job offer that pays the same as the current one. That is, $1 - F_\varepsilon(1) = 0.31$. While this model does not feature heterogeneity, this value lines up with Sorkin (2018) evidence that documents that 37% of employment-to-employment transitions see earning declines. Finally, we use the quit elasticity of labor supply to calibrate $f_\varepsilon(1)$, the density of the taste shocks at 1. We use $\varepsilon_{\bar{\delta}} = 3.5$ as we just argued in the previous section, which implies that the contribution of hiring costs to marginal costs is 13%. Note that despite being a search model, we do not need to specify nor calibrate the unemployment benefits. In principle, a firm deviate by offering very low wages targeting the unemployed workers only. That firm knows that no employed worker will accept its offer and all workers will quit as soon as they receive another job offer fom another firm. If that was the case, the turnover rate of such firm would be 23% . We rule out this case by assuming that there is a minimum wage that prevents this deviation from being profitable, which implies that the derived wage Phillips curve corresponds to a global optimum.

For the price rigidity parameter, we match the slope of the price Phillips curve estimated by Gagliardone, Gertler, et al. (2023), who provide a mapping from marginal costs to inflation. They estimate this pass-through to be 0.05 using a rich administrative data-set of Belgian firms. This slope implies a coefficient κ_p of 120, which corresponds to a frequency of price adjustment of 5 quarters. This is in the upper end of value for price rigidity, and Gagliardone, Gertler, et al. (2023) get a frequencey of 3.3 quarters because they take into consideration the strategic price-seting behavior obtained from departing from the monopolistic CES case. For the wage rigidity parameter, we set it so for the firm is equally costly to change prices by 1% than it is to raise wages by 1%, which implies $\kappa_w = 122$.

Target	Description	Value
u	Unemployment	0.058
$\tilde{\delta}$	Separations Rate	0.10
$\frac{EE}{UE}$	EE-UE ratio	0.32
$\epsilon_{\tilde{\delta}}$	Quit elasticity	3.5
$\frac{\epsilon_p}{\kappa_p}$	Price NKPC slope	0.05

Table 1.4: Targets

1.6.2 Alternative model: bargaining with real wage rigidity

The model presented here does not have a clear benchmark to compare, since there is not an established model of search and wage determination. In Section 2 we briefly mention how the Phillips wage curve is different from the one that assumes that unions set wages as in Erceg, Henderson, and Levin (2000). A fairer comparison of the monopsonistic model is a model that features search frictions but Nash bargaining. We provide a standard model with real wages are rigid as in Blanchard and Galí (2010) or Krause and Lubik (2007), which has been recently used by Gagliardone and Gertler (2023). Since the model is standard, the details are left to the appendix.

Workers receive a flow value from being unemployed b and bargaining wage ς. Firms hire them by posting a vacancy at a cost κ, which meets a worker with probability $q(\theta)$. There is no job search, and workers quit exogenously with probability $\bar{\delta}$. Each worker produces one unit of 'labor services', sold at price ϑ_t. These services are used by a final output firm that uses the same production function $y_t = f(n_t)$ and set prices subject to Rotemberg rigidities.

Wages are Nash-bargained and subject to real rigidities. Let $J_t(\omega_t)$ be the value of a match for a firm and $H_t(\omega_t)$ the worker surplus when the negotiated real wage is ω_t. The wage that would arise in a flexible environment would be

$$\omega_t^{Nash} = \arg\max_{\omega} H_t(\omega)^{\varsigma} J_t(\omega)^{1-\varsigma}.$$

Real wage rigidities are introduced by assuming that the real wage does not fully adjust to the Nash negotiated wage, but instead the real wage is

$$\omega_t = \left(\omega_t^{Nash}\right)^{1-\gamma} \omega^{\gamma}.$$

where ω is the real wage in steady state. Under reasonable parametrizations, this behavior is consistent with rational behavior as it lies within the bargaining set, i.e. it is never above firm's reservation wage (the value to the firm of a worker) nor it is ever below worker's reservation wage (the flow value of unemployment). One way to interpret this wage setting protocol is as the firm providing some insurance to workers by offering a smoother real wage than would be the case under period-by-period Nash bargaining. [14]

This model introduces three new parameters, (b, ς, γ). We target a replacement rate of $b/\omega = 0.7$ as proposed by R. E. Hall and Milgrom (2008). which implies $b = 0.36$. This includes not ony the unemployment benefits but also the leisure utility of not having to work. The literature on Nash bargaining has not converged on the right value for ς, and we set it to 0.5 as Gagliardone and Gertler (2023) to satisfy the Hosios condition. There is no concensus on what this parameter should be, and there is not empirical evidence to guide its calibration. Hagedorn and Manovskii (2008) argues it should be close to zero (0.05), while Gertler, Sala, and Trigari (2008) calibrate it to be close to one (0.9). A clear advantage of the monopsonistic model is that its calibration does not rely on parameters like this bargaining weight with is usually either set exogenously or calibrated but with a difficult interpretation. The wage rigidity parameter is set to 0.9.

The calibration of the bargaining model implies that the value of an employee, $\frac{\kappa}{q(\theta)}$ in steady state represents 11% of the worker wage, or 1.5 weeks of wage. This implies that the contribution of the wage into the cost of hiring a worker is $\tau = 0.993$, in line with the values obtained in Table 1.1.

1.6.3 Impulse Response

Having presented both models we compare its response to a demand shock. Assume that the central bank announces a drop of 0.5% on the interest rate on impact that reverts back to the steady state with persistence $\rho_m = 0.8$. This shock can also be interpreted as a drop in the household discount factor, making them more impatient. Figure 1-4 shows the impulse response of selected macroeconomic variables. The response of employment and inflation in both cases is similar, with the monopsonistic model being slightly more inflationary. The dynamics and drivers of the real wage and wage inflation are significantly different, which we comment below.

In the monopsonistic model, when firms face a demand shock they need to hire workers to satisfy it. They post more vacancies and raise wages to increase the vacancy yield and reduce quits. In partial equilibrium, prices increase by two channels. With higher employment, the marginal product of labor decreases if there are decreasing returns, and marginal vacancy costs increase if vacancy costs are convex. Contrary to standard models, it is not direct that in partial equilibrium raising wages increases marginal costs, because they reduce hiring costs. But that's not the case in general equilibrium, where all firms raising wages nullifies

[14] See Gertler and Trigari (2009) and Christiano et al. (2016) for formal models of real wage rigidity in a search and matching setting.

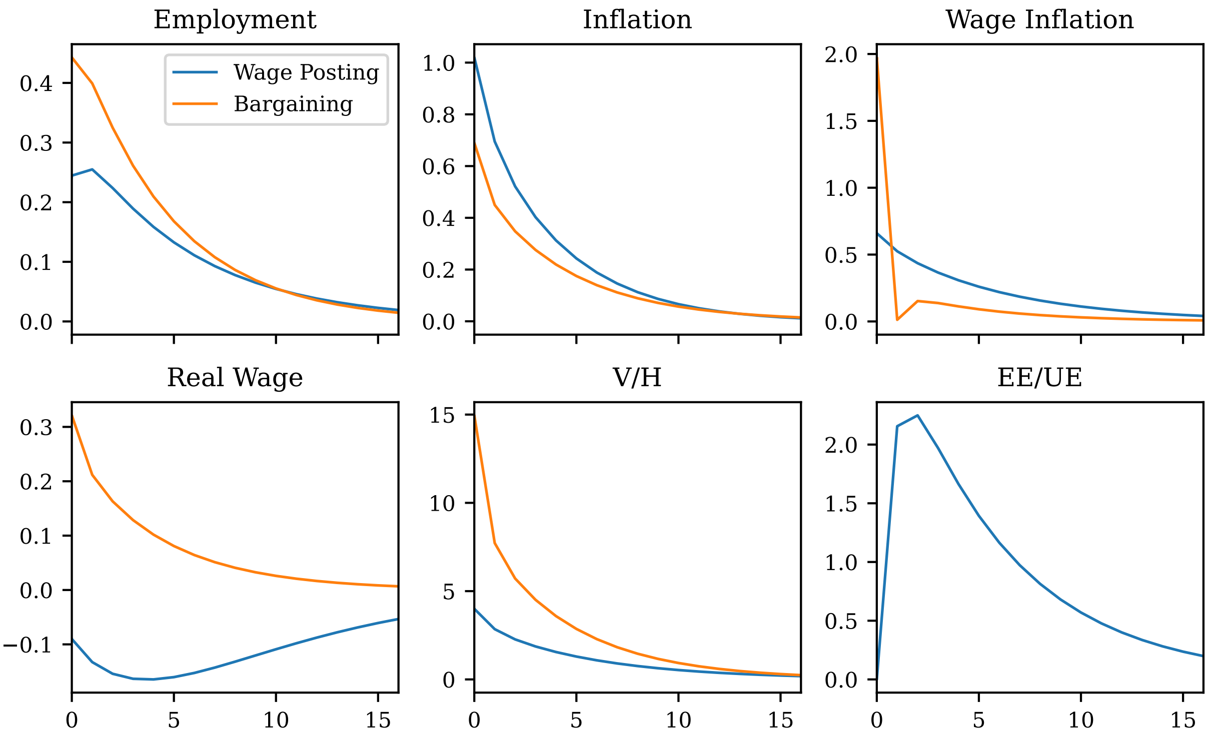

Figure 1-4: Response to an aggregate demand shock.

Note: Response to a demand shock. All variables are presented as log deviations from steady state. Price and wage inflation are expressed in annual terms.

the effect of any individual firm. Moreover, all firms posting more vacancies mean that (i) each vacancy is less likely to meet with an applicant and (ii) more vacancies meet currently employed workers that may quit, which induces further vacancy creation. The net effect is nominal wage inflation, more quits and hiring difficulties. Both effects are inflationary and pass it into prices. The effect on the real wage is ambiguous and depends on the relative slopes of the price and wage Phillips curves.

In the bargaining model with a two-layer economy, the mechanism is different. When final good demand rises, demand for labor services also increases. In order to induce vacancy creation, the real price of these services increases. This is equivalent to a productivity shock for the firm that provides labor services. As is general in those cases, the real wage increases, but given the real wage rigidity assumption, not as much as the Nash bargained solution.

In the monopsonistic model, the real wage decreases with the demand shock. Traditional models of nominal wage rigidity like Erceg, Henderson, and Levin (2000) can also share this feature, as exemplified in Lorenzoni and Werning (2023). But the mechanism is different. In models where firms take wages as given, if production has decreasing returns then increasing production raises nominal marginal costs, even if nominal wages do not rise. In Lorenzoni

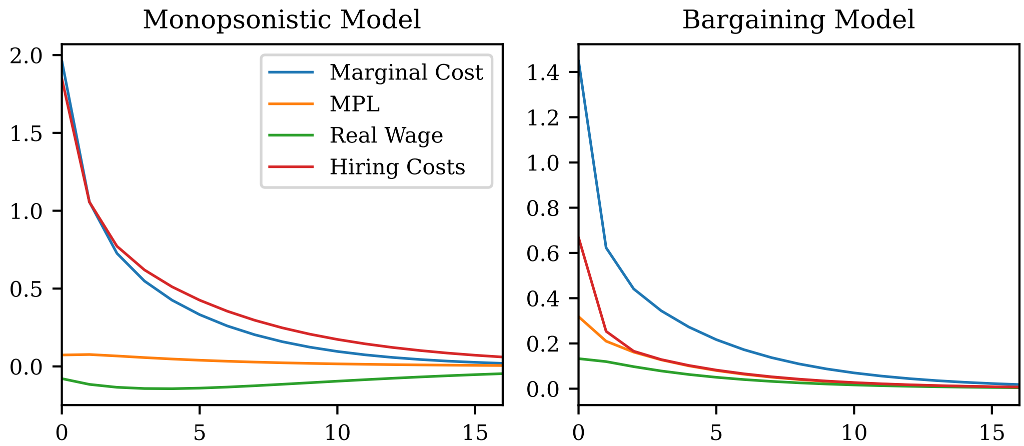

Figure 1-5: Split of marginal costs.
Note: Deviation of marginal costs relative to their steady state value. In the monopsonistic model, the driver of inflation is the cost of hiring a worker, not the real wage, which falls.

and Werning (2023) or Gagliardone and Gertler (2023), marginal cost raise sharply given the low substitutability between labor and oil. In the model presented here, this channel is also present, but its not the only, nor the main reason, why the real wage decreases. Figure 1-5 shows the evolution of marginal costs and its main components: the real wage, the marginal product of labor, and the hiring costs, for both the monopsonistic model and the bargaining model. In the monopsonistic model, hiring costs are the driver of marginal costs, with a small effect of the decreasing returns to labor offset by a decrease in the real wage. Firms rise prices because they do not find workers, not because they have to pay them more (in real terms). In the bargaining model, the three effects are positive and add up, to have an almost identical response of marginal costs.

Costly Hire

Now we consider the case when vacancies are free to post, but hiring is costly. While at the firm level, vacancies are irrelevant for firms, this is not the case at the aggregate level, because aggregate vacancies determine poaching. The calibration is exactly the same as the previous case but now the relevance of hiring costs is amplified because the effective labor supply elasticity is divided by two. Figure 1-6 shows the response of a demand shock and compares both models.

The model with costly hire significantly mutes the price and wage inflation response to the shock, which implies a larger employment response. This is expected because the cost of hiring a worker is independent of the state of the labor market. Pissarides (2009) and Christiano, Eichenbaum, and Trabandt (2016) emphasize the role of fixed costs of hiring to

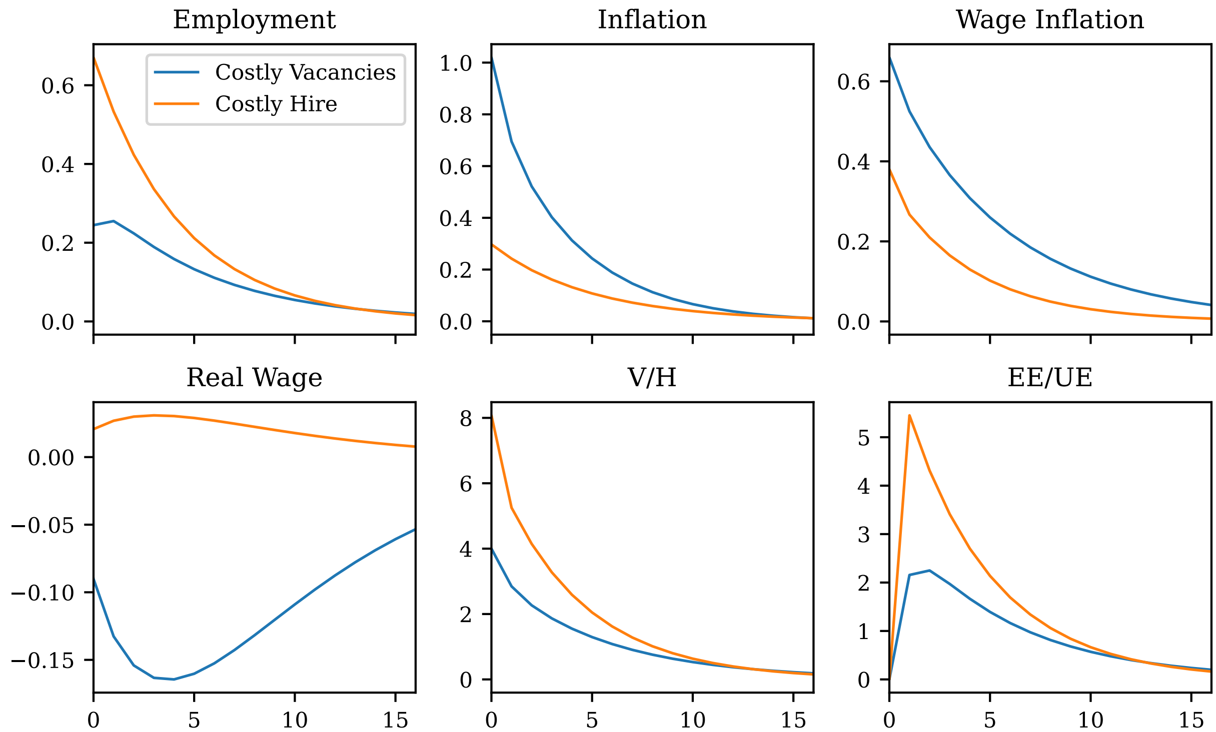

Figure 1-6: Costly hire versus costly vacancies
Note: In blue, impulse response of the model with costly vacancies, and in orange, response of costly hire (vacancies are free)

generate wage inertia. However, these two papers have a particular formalization of such costs. They are paid *after* the match has been created but *before* the wage negotiation starts. If those costs were paid after the worker has bargained the wage, they would not affect the wage bargaining problem. The model presented here is not subject to that critique.

If the cost of hiring workers is independent of the labor market, what drives the dynamics of prices and wages? Firms aim to reduce hiring needs by retaining more workers by paying them more. As we see in the bottom right panel, employment-to-employment transitions spike as poaching intensity increases. The increase in nominal wages also implies an increase in prices. The cost of hiring a worker is independent of labor market conditions, but the *net* cost of hiring does depend on it via the quit rate. This adds further pressure in to marginal costs, but is small compared with the effect of the increase in market tightness that operates in the model with costly vacancies. Hiring costs are not enough to overturn the positive real wage response.

1.6.4 Endogenous Labor supply

For simplicity, it has been assumed that unemployed workers accepted all job offers regardless of the wage. This implied that the aggregate supply of labor is independent of the wage as we see in equation (1.9). This is the norm in most search models except those that have an endogenous labor participation equation like Graves, Huckfeldt, and Swanson (2023). However, Faberman et al. (2022) document that the unemployed reject 50% of the best offer received over the last month. This could be the result of and heterogeneous distribution of outside options as in Burdett and Mortensen (1998), or as this paper assumes, idiosyncratic taste shocks.

Unemployed workers receive a flow b_t from being unemployed. When offered to work for a wage ω_t, they accept if $\omega_t \varepsilon_t \geq b_t$, the same way workers compare job offers. In the previous section, the symmetry assumption implied that only information about $f_\varepsilon(1)$ and $F_\varepsilon(1)$ was required to define the evolution of the economy, but now we need to make an assumption on the distribution of ε_t. We assume that follows a lognormal distribution with mean μ_ε and standard deviation σ_ε which we calibrate, as well as the value from being unemployed. The aggregate low of motion becomes

$$N_t = (1-\bar{\delta})N_{t-1} + V_t q(\theta_t) p_t^U \left(1 - F_\varepsilon\left(\frac{b_t}{\omega_t}\right)\right).$$

The inclusion of endogenous labor supply adds a new mechanism that was muted in the simplified model. We just saw that a demand shock lowers the real wage. Despite nominal wages being higher, unemployed workers are less willing to accept jobs, which exacerbates labor supply shortages. The appendix shows that the effect is not significative for demand shocks.

Labor supply shocks. The inclusion of an acceptance decision by the unemployed allows us to think about the effect of labor supply shocks, and compare it with the standard bargaining model. Figure 1-7 shows the response of such shock, that increases the value of unemployment by 5%, and compares it to the model with bargaining and real wage rigidities. While for demand shocks the reaction of employment and inflation were pretty similar, here they are starkly different.

In the monopsonistic model, a drop in the willingness to work by unemployed workers makes hiring more costly. Each vacancy sent is more likely to be turned off which pushes price and nominal wage inflation up, with a negative effect on the real wage. Vacancies take longer to fill and having trouble hiring unemployed workers, firms start competing among them for the employed ones.

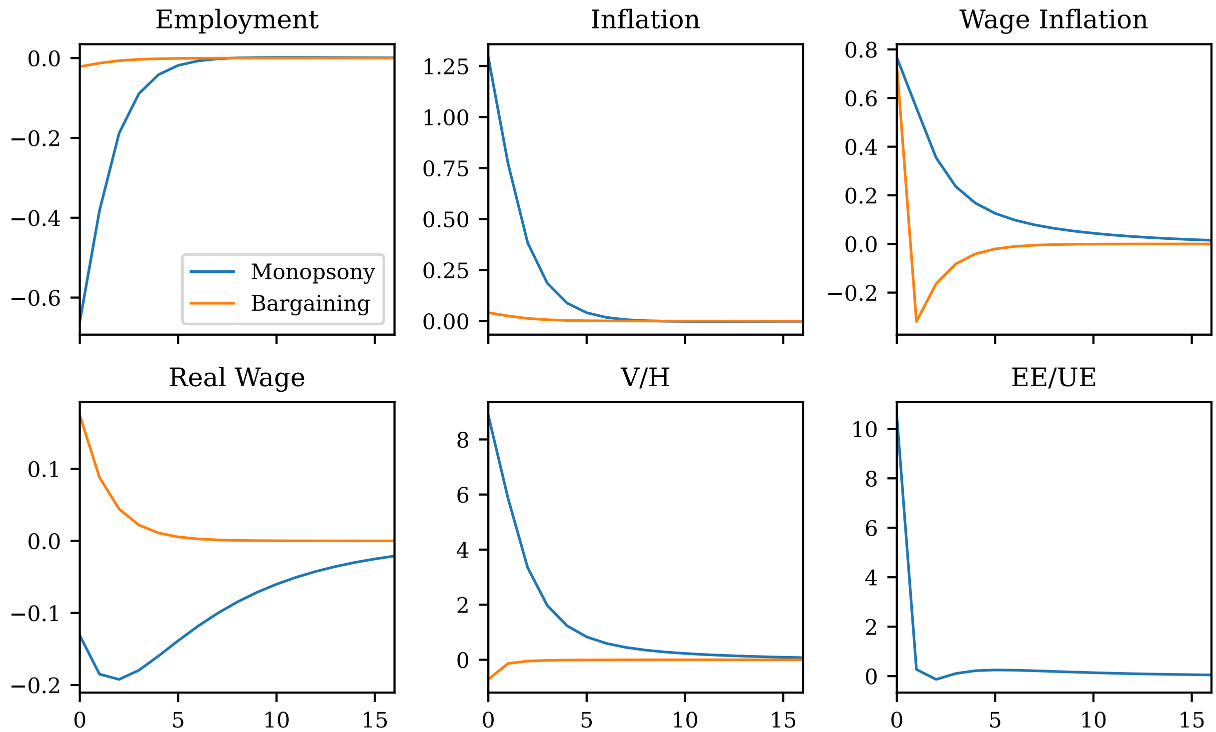

Figure 1-7: Response to a 1% labor supply shock

In contrast, in a bargaining model with real wage rigidities a labor supply shock has very little effect on employment and inflation. The same would happen if we were considering a model with nominal rigidities like Gertler, Sala, and Trigari (2008) or the Erceg, Henderson, and Levin (2000) unions model. In all these models, the rate at which firms can hire workers is independent of the wage, as long as it is inside the bargaining bands (R. Hall (2005)). The disutility of work only affects the economy trough the wage bargaining condition, and if wages are rigid, then it does not effect the economy at all. While demand shocks behave similarly in the bargaining vs monopsonistic model, the implications for labor supply shocks are very different and help explain the post-covid inflation.

1.7 Concluding remarks

This paper has presented a model of monopsony in the New Keynesian framework. Compared with alternative models of the labor market, it offers several advantages. It is more realistic and easier to interpret, since firms are the ones setting prices and wages. It does not rely on wage-setting protocols for which we hardly have evidence of. To calibrate the model all we need is the quit elasticity of labor supply, which has been widely studied in the labor economics literature. The implied cost of hiring a worker is more in line with empirical evidence than other models for which it is negligible.

These advantages come at a cost. Solving wage posting models out of steady state is notoriously difficult and we have made some simplifying assumptions to be able to do it analytically. Firms are symmetric, workers are myopic and nominal frictions are à la Rotemberg. Generalizing these simplifying assumptions would not change the main results and message of the model but it would be valuable. It is left for future research. In a steady state environment, de la Barrera (2023) solves a wage posting model with heterogeneous firms and forward-looking workers subject to idiosyncratic taste shocks.

We have highlighted the importance of hiring frictions in determining not only wages but also prices, an element that is not present by construction in models without search frictions and neglected by the calibrations in those that have them. Monopsony increases the importance of those marginal hiring costs and we provided a sufficient statistics for it; the wage markdown. An average worker costs around 15 weeks of its salary, consistent with market estimates.

The theoretical model presented here can inspire several empirical questions that would corroborate the model implications. While the monopsonistic model and alternative models behave similarly when shocked with demand shocks, the price and wage dynamics of supply shocks are significantly different. A challenge is to identify labor supply shocks. In this line, Autor, Dube, and Mcgrew (2023) finds that since the onset of the pandemic, regions were the market were tighter saw a bigger wage increase but also a price increase of the same magnitude.

The main goal of this paper is to rethink the labor market in macroeconomic models. Firms that post wages and prices is a more realistic assumption which comes at its costs. More research on this topic should be done to overcome those and better understand what drives the dynamics of nominal prices, wages and the real wage.

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Chapter 2

Currency Pegs, Trade Deficits and Unemployment: A Reevaluation of the China Shock

2.1 Introduction

Four facts of the past two decades have drawn significant attention in both academic research and public discourse. First, China's exports to the US have grown significantly, driven by spectacular productivity growth and falling trade costs – henceforth the *China shock* (Figure 2-1a). Second, US manufacturing has undergone a significant decline, coupled with a rise in unemployment in manufacturing-heavy regions (Figure 2-1b). Third, the US has incurred a substantial trade deficit, while China ran a trade surplus (Figure 2-1c). Fourth, China has pegged its currency against the US dollar via an explicit peg (until 2004) or a managed band (after 2005) (Figure 2-1d).

An often-heard narrative in policy circles emphasizes how the last fact may have caused or magnified the first three. According to that narrative, *currency manipulation* by China might have been responsible for its sudden export surge to the US, large trade imbalances between the two countries, and, in turn, depressed the US labor market.[1] Although much has been said about the China shock in the trade and labor literature (Caliendo, Dvorkin, and Parro, 2019; Andrès Rodríguez-Clare, Ulate, and Vásquez, 2022; Dix-Carneiro, Pessoa, et al., 2023), as well as the global savings glut in the international macro literature (Ricardo J. Caballero, Farhi, and Gourinchas, 2008; Mendoza, Quadrini, and Ríos-Rull, 2009; T. J.

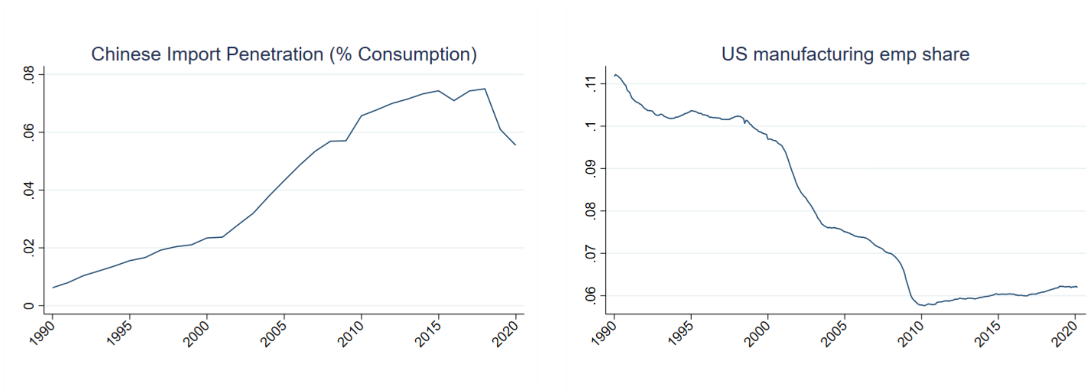
[1]Countries increase tariffs in response to unemployment (Bown and Crowley, 2013) and trade deficits (Delpeuch, Fize, and Martin, 2021), consistent with this narrative and suggesting that it may have affected policy.

Kehoe, Ruhl, and Steinberg, 2018), there has been no attempt at connecting the four facts collectively. This paper proposes to fill this gap by establishing a causal link between the four facts, both empirically and theoretically, and reevaulate the consequences of the China shock and quantify the effect of China's peg in US outcomes.

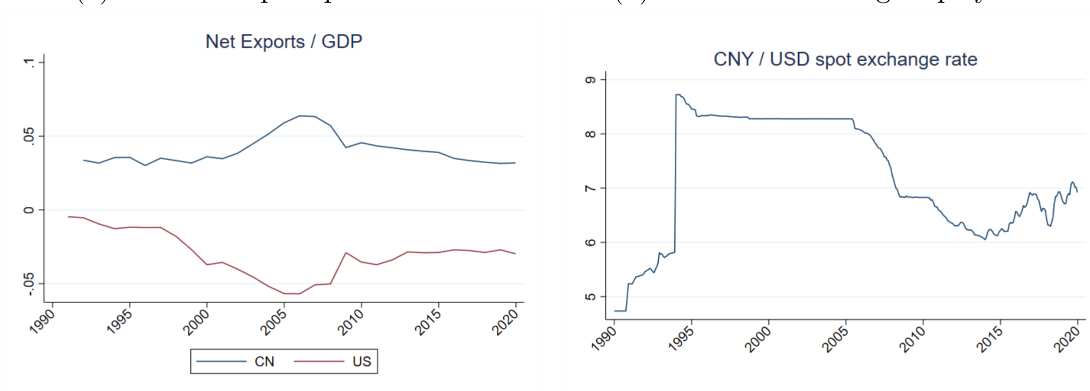
Our contribution is threefold. First, we present an empirical finding that a country's exchange rate regime affects the incidence of the China shock on labor market outcomes and trade imbalances. We show that countries pegging to the US dollar – tying itself to Chinese currency – experienced a larger output decline, higher unemployment, and larger trade deficits in response to higher exposure to Chinese growth, unlike floating countries whose currency depreciated in response to China shock exposure. Second, we develop a model of trade with endogenous imbalances and wage rigidity that parsimoniously connects the four facts above by endogenizing the US trade deficit as a result of Chinese growth. We highlight the possibility that a country's welfare may decrease as a result of Foreign growth and study optimal policy responses. Third, we use a richer version of the same model to reevaluate the effects of the China shock and the role of China's exchange rate peg. We develop a highly efficient solution algorithm for solving dynamic macro-trade models with labor reallocation, and find that China's exchange rate peg contributed to a substantial part of the US trade deficit, decline in US manufacturing, unemployment, and reduced the welfare gains from the China shock.

In Section 2.2, we present evidence of the role of China's exchange rate peg in shaping labor market outcomes and trade imbalances in response to trade shocks. We use the joint fact that China's export growth post-2000 varied across sectors and that countries varied in their sectoral composition pre-2000 to construct a shift-share measure of country-specific exposure to the China shock, a cross-country analog of Autor, Dorn, and Hanson (2013) and Autor, Dorn, and Hanson (2021). We then implement a triple-difference strategy that compares the *differential* impact of the *same* exposure between floating countries and countries pegged to the US dollar and, therefore, pegged to the Chinese currency. Our triple-difference strategy shows that a similar surge in exposure led to a lower manufacturing output, a temporary increase in unemployment, and larger trade deficits when the country's currency is pegged to the US dollar, relative to a country that floats.

In Section 2.3, we develop a dynamic model of trade with predictions consistent with the empirical findings and can jointly explain the four facts above. Our model is a two-period model with Armington trade in each period that allows consumption savings through an international bond market, and features short-run nominal wage rigidity. Under an exchange rate peg (Figure 2-1d), our model predicts that an increase in Foreign productivity (Figure 2-1a) causes a trade deficit at Home (Figure 2-1c) and Home workers face involuntary



(a) Chinese import penetration (b) US manufacturing employment



(c) Net exports (% GDP) (d) CNY/USD exchange rate

Figure 2-1: Four stylized facts.

Sources: (a) Import of goods from China obtained from US Census Bureau and Bureau of Economic Analysis (BEA), US goods consumption obtained from BEA. (b) Bureau of Labor Statistics. (c) US Census and BEA. (d) Board of Governors of the Federal Reserve System (US). Retrieved from FRED.

unemployment (Figure 2-1b). This holds provided that the trade elasticity σ is higher than the intertemporal elasticity γ, as documented empirically. The intuition is as follows: after Foreign growth, the Home relative wage should adjust through nominal wage or exchange rate. With both channels muted, the trade balance is determined by expenditure switching and relative inflation. When $\sigma > \gamma$, the expenditure switching channel dominates, Home runs a trade deficit, and shrinking global demand for Home goods causes unemployment at Home. This framework allows us to jointly explain the trade deficit and unemployment in manufacturing-heavy regions of the US as an endogenous outcome of Chinese growth under an exchange rate peg, parsimoniously explaining the stylized facts of the 2000s.[2]

[2] In related work, for which we explain in more detail below, Dix-Carneiro, Pessoa, et al. (2023) study an environment with endogenous trade imbalances and unemployment due to search friction. As we show in the Online Supplement, in such an environment with quantity friction, we get opposite predictions on the direction of trade imbalance, highlighting the role of nominal rigidity and exchange rate pegs in connecting these facts.

Turning to welfare and policy analysis, we show that Home welfare may even decrease as a result of Foreign growth when the trade elasticity is sufficiently high. Despite an improvement in terms-of-trade today, Foreign growth under a peg creates involuntary unemployment and future terms-of-trade deterioration due to required future trade surpluses. The higher the trade elasticity, the more expenditure is switched towards foreign goods, and the more severe the negative effects are. We show that the optimal short-run tariff in response to the shock is positive. Here, dynamic terms-of-trade considerations reinforce the standard motive for *safeguard* tariffs allowed by the WTO. We also highlight that Home's optimal monetary policy, barring constraints such as the Zero Lower Bound, would want to overshoot the output gap because it is borrowing and can set the global interest rate under a peg.

To explore the quantitative significance of the mechanism, Section 2.4 introduces a multi-country, multi-sector, infinite-horizon model consisting of two blocks. The first block is a workhorse trade model with input-output linkages and labor migration frictions (Caliendo, Dvorkin, and Parro, 2019), both of which shape how trade shocks affect the labor market. This trade block allows us to quantify the general equilibrium effects of the China shock using observed sector-level trade and worker reallocation data. The second block is a macroeconomic block comprising wage rigidity generating a New Keynesian Phillips Curve (C. J. Erceg, Henderson, and Levin, 2000), intertemporal balances from consumption-savings (Obstfeld and Kenneth S. Rogoff, 2005), and exchange rate determination from financial flows (Itskhoki and Mukhin, 2021a). This macro block allows us to incorporate involuntary unemployment, endogenous trade imbalances, and compare exchange rate pegs with floating exchange rates.

We calibrate the model to exactly match the sectoral trade flow data from the World Input Output Database (WIOD) and labor adjustment data from the Current Population Survey (CPS). We develop a novel solution algorithm that allows us to quickly solve for the full sequence of wages, prices, labor allocation, and trade imbalances for any realized or counterfactual fundamentals and policies, including the exchange rate regime. We bring frontier computational methods from macroeconomics, leveraging the sequence-space Jacobian method introduced by Auclert, Bardóczy, et al. (2021) and using advances in machine learning frameworks to efficiently solve for the equilibrium in minutes.

Section 2.5 conducts counterfactual and welfare analysis. We first quantify the effect of the China shock by comparing the realized economy with the counterfactual economy without Chinese productivity growth and trade liberalization. We find that the China shock can explain 2.25 percentage points of the US trade deficit between 2000 and 2012, 991 thousand manufacturing jobs lost, and may be responsible for a surge in unemployment of 3.04% over the same period, concentrated in the affected manufacturing sectors, estimates that are approximately double those in the previous literature. Turning to welfare analysis, we find that the China shock still increased the welfare of the US by 0.183%, an estimate lower than previous literature but still positive, showing that the surge in Chinese exports, even after accounting for involuntary unemployment and dynamic terms-of-trade effects due to the exchange rate peg, increases the welfare of the US.

We also consider an additional counterfactual economy without Chinese growth and trade liberalization, and also without China's *savings glut* – residual demand for savings by China, which we calibrate to match the trade imbalances of each country. We use this counterfactual to assess the contributions of China's savings glut to the outcomes of the US and find that the decline in manufacturing is nearly identical with or without China's savings glut. This reinforces the findings of T. J. Kehoe, Ruhl, and Steinberg (2018), which show that the global savings glut is responsible for only a small portion of the decline in US goods-sector employment (15.1%). We show that once we incorporate the exchange rate peg, China's residual savings glut had a negligible effect on the US manufacturing decline or the trade deficit. This finding underlines the centrality of the exchange rate peg in how the growth and savings of China affected the US.

Next, we isolate the effect of China's exchange rate peg on the same aggregate outcomes. The question we ask is: How different would the effects of the China shock have been without the peg? Comparing the realized economy with the counterfactual economy where an otherwise identically growing China floats its exchange rate, we find that China's peg to the US dollar is responsible for 1.3 percentage points of the US trade deficit (% GDP), 447 thousand manufacturing jobs lost. These equilibrium responses largely match those observed in the empirical findings (Section 2.2) and support the quantitative significance of the relevant channels in our theoretical model (Section 2.3). Balancing these factors, China's exchange rate peg lowered US lifetime welfare by 0.083% relative to an economy where the China shock occurred, but China floated its currency with respect to the US dollar.

Finally, we explore the consequences of counterfactual policies on labor market outcomes and US welfare. We ask the following questions: What would have been the impact on US welfare if different policy measures were implemented? What are the effects of a targeted tariff designed to reduce trade deficits? And finally, what is the role of monetary policy in shaping these outcomes? We find that a tariff of 15-20% on Chinese goods could have ameliorated the short-run labor market distortions, this positive effect remains even under retaliatory tariffs, and monetary policy could have been effective in reducing the distortion from the China shock, conditional on not being subject to the Zero Lower Bound.

The paper is accompanied by an Appendix containing a description of the data, proofs of the main propositions, and derivations of key equations, and a longer Online Supplement, that contains robustness tests, model extensions, further derivations, calibration details, and the solution algorithm.

Related Literature

Our paper contributes to a large trade and labor literature that studies the labor market consequences of globalization. On the empirical side, Autor, Dorn, and Hanson (2013) and Autor, Dorn, and Hanson (2021), Acemoglu et al. (2016) have shown that US labor markets competing more with Chinese imports are hurt relatively more.[3] On the structural side, the seminal work by Caliendo, Dvorkin, and Parro (2019) (henceforth CDP) quantifies the effect of the China shock across labor markets. We contribute to the structural trade literature by embedding a full New Keynesian macro block into CDP. This allows us to address involuntary unemployment, discuss the implications of endogenous imbalances, and study counterfactual policies.

Two closely related papers, Andrès Rodríguez-Clare, Ulate, and Vásquez (2022) and Dix-Carneiro, Pessoa, et al. (2023), also study unemployment in response to the China shock by augmenting CDP with labor market frictions. Andrès Rodríguez-Clare, Ulate, and Vásquez (2022) (henceforth RUV) is most similar to ours in that they introduce wage rigidity. Our approach is different in two dimensions. First, we feature endogenous imbalances through consumption-savings and nominal rigidity generating a Phillips Curve. This complements their approach, which uses exogenous imbalances and demand anchors with a reduced-form downward nominal wage rigidity (DNWR). Second, our model underscores the central role of exchange rate pegs, allowing us to evaluate the welfare effect of China's USD peg on the United States. These differences allow our framework to highlight the effect of counterfactual monetary policies and exchange rate pegs.[4]

Dix-Carneiro, Pessoa, et al. (2023) introduce endogenous consumption-savings to study the effect of the China shock and trade imbalances on the labor market and uses search frictions à la Mortensen and Pissarides (1994) to generate unemployment.[5] However, the

[3]Recent empirical papers that connect trade shocks with the labor market include Pierce and Schott (2016), Dix-Carneiro and Kovak (2017), Handley and Limão (2017), Carrère, Grujovic, and Robert-Nicoud (2020), Costinot, Sarvimäki, and Vogel (2022). Autor, Dorn, and Hanson (2016) and Redding (2022) provide excellent review of the literature.

[4]In related work, Fadinger, Herkenhoff, and Schymik (2023) study the effect of German growth on the Eurozone through a model of DNWR and consumption-savings, with an exogenous demand anchor. In such models, a floating exchange rate moves to clear all nominal frictions; on the other hand, a floating exchange rate in our model is financially driven and may not immediately adjust to clear the labor market across all sectors.

[5]T. J. Kehoe, Ruhl, and Steinberg (2018) also study the effect of imbalances in the labor market, but do not study unemployment. Dix-Carneiro (2014), Kim and Vogel (2020) and Kim and Vogel (2021), Galle, Andrés Rodríguez-Clare, and Yi (2023) also embed search-and-matching into trade, without imbalances.

response to trade shocks qualitatively differs under nominal frictions (wage rigidity) and quantity friction (search) in two important ways. First, quantity friction amplifies terms-of-trade shocks and leads to a reduction in unemployment in response to Foreign trade shocks, in conflict with increased unemployment in regions more exposed to the China shock (Autor, Dorn, and Hanson, 2013; Autor, Dorn, and Hanson, 2021). Second, quantity friction generates a force for the US, not China, to run trade surpluses in response to Chinese productivity growth, necessitating an even larger exogenous *savings shock* to align with the observed trade imbalance. Under our model of wage rigidity, short-run unemployment and trade deficit in the US are endogenous outcomes of the Chinese productivity growth. Our framework can also investigate the effect of the exchange rate peg and study counterfactual tariffs or monetary policies, elements absent from their study.

We highlight how an exchange rate peg under nominal rigidity can generate trade imbalances. This contributes to the international finance literature that studies the "global savings glut" of the 2000s, a term first coined by Bernanke (2005). Recent work attributes the US current account deficit to financial frictions (e.g. Ricardo J. Caballero, Farhi, and Gourinchas (2008) and Ricardo J Caballero, Farhi, and Gourinchas (2021), Mendoza, Quadrini, and Ríos-Rull (2009)), business cycle dynamics (e.g. D. Backus et al. (2009), Jin (2012)) or demographics (e.g., Auclert, Malmberg, et al. (2021), Bárány, Coeurdacier, and Guibaud (2023)).[6] Our work highlights a goods-market explanation of the observed trade imbalances under exchange rate pegs that can exist concurrently with the financial origins. Through the lens of our quantitative model, we attribute 37.1% of the US deficit to China's exchange rate peg, with the remaining deficit attributable to other countries and potential financial mechanisms that we have abstracted from.

We contribute to the open economy macroeconomics literature by bridging it with structural trade models to study sector-level shocks, such as the China shock.[7] From Galí and Monacelli (2005) and Galí and Monacelli (2008) to more recent work such as Schmitt-Grohé and Uribe (2016) and Auclert, Rognlie, et al. (2021), the literature has studied the role of trade, exchange rates and monetary policy in the macroeconomy. We build on these papers along two dimensions. First, we consider the effects of the exchange rate peg for an economy facing a peg, necessitating a departure from the small open economy model, which a majority of the literature focuses on, and consider Home monetary policy that directly affects savings decisions abroad. Second, we incorporate a multisector trade model that allows us to investigate the macroeconomic effect of shocks such as the China shock that are very

[6] See Gourinchas and Rey (2014) for a review of this literature.

[7] In doing so, we follow the recommendations of Andrès Rodríguez-Clare, Ulate, and Vásquez (2022) by "*adding a Taylor Rule [..] allow agents to make savings and investment decisions, and incorporate international financial flows affecting exchange rates.*"

asymmetric across sectors.

Our work on tariffs and monetary policy in response to the China shock is closely related to the literature studying the macroeconomic consequences of trade policy and monetary policy in the open economy. The closest to our analysis are Jeanne (2020), Auray, Devereux, and Eyquem (2023), and Bergin and Corsetti (2023), each of which studies the interaction of tariffs and monetary policy in an Open Economy New Keynesian model.[8] While our insights resonate well with theirs, these papers focus on steady-state and business-cycle optimal policy, whereas we study policies in a transition path in response to a permanent shock. As such, their government is focused on steady-state welfare maximization, while the government in our model seeks to affect dynamics, including endogenous imbalances.

We underscore the role of China's exchange rate peg in generating unemployment and a steeper decline for US manufacturing by worsening its competitiveness. This is closely related to the idea that flexible exchange rates are a shock absorber. Previous empirical evidence of such an absorber role has been documented in the goods market (Broda, 2001; Broda, 2004; Edwards and Levy Yeyati, 2005; Carrière-Swallow, Magud, and Yépez, 2021), labor market (Schmitt-Grohé and Uribe, 2016; Campbell, 2020; Ahn, Choi, and Rivadeneyra, 2022), and financial market (Ben Zeev, 2019). Our analysis in Section 2.2 provides additional support that flexible exchange rates operate as an adjustment margin for the China shock. Our model explicitly incorporates exchange rate regimes into a structural trade model, allowing us to quantify the welfare effects of a large emerging market economy's currency peg on the US.[9]

2.2 Empirics: Exchange Rate Regimes and the China Shock

This section presents motivating evidence for the relevance of China's exchange rate peg in how the China shock affected the US labor market and trade deficit. Public discourse puts trade deficits and the peg at the center of how China affected the US labor market: with Chinese productivity growth and a peg, cheap Chinese goods flood the US market, shifting demand, exacerbating trade deficits, and harming US manufacturing. Would a

[8]See also Barbiero et al. (2019), Lindé and Pescatori (2019), Barattieri, Cacciatore, and F. Ghironi (2021), and Auray, Devereux, and Eyquem (2022) for tariffs, F. P. Ghironi (2000), G. Benigno and P. Benigno (2003), Devereux and Engel (2003), Faia and Monacelli (2008), Corsetti, Dedola, and Leduc (2010), and Lombardo and Ravenna (2014) for monetary policy, and C. Erceg, Prestipino, and Raffo (2018), Barattieri, Cacciatore, and F. Ghironi (2021), Cacciatore and F. Ghironi (2021) for empirical analysis of tariffs, monetary policy and exchange rates.

[9]This also relates us to the exchange rate determination literature, such as Gabaix and Maggiori (2015), Itskhoki and Mukhin (2021a), Hagedorn (2021). Our model is a limit case of these setups.

floating exchange rate have functioned as a margin of adjustment? Establishing the sign and magnitude of the relationship between China's exchange rate peg and the labor market outcomes and trade balances is important in understanding the role the exchange rate plays in international trade.

To empirically answer this question, our focus must extend beyond the US and China, given the absence of a counterfactual scenario of Chinese export surge under a fully flexible exchange rate between the two countries. We overcome this challenge by comparing countries with different currency regimes vis-à-vis China's regime – peg to the US dollar – and similar exposure to Chinese exports. We construct a measure of each country's exposure to Chinese export growth, and conditional on the same exposure to the China shock, test (1) whether the nominal exchange rate responds to the China shock for floating countries, and if so, in which direction, and (2) whether countries pegged to the US dollar (including the US) experience a drop in output and employment, and larger trade deficits relative to countries that do not peg to the US dollar. Our findings are consistent with these two hypotheses and motivate our modeling framework and quantitative analysis in Sections 2.3 onwards.

2.2.1 Background: the China shock and exchange rate peg

A large literature investigates the role of Chinese productivity growth and decreased trade costs in disrupting the US labor market. Empirical evidence and quantitative estimations consistently find that the surge in Chinese exports is a key factor in the economic decline and potential welfare losses of regions and sectors with greater exposure. This *China shock* is primarily attributed to productivity growth (Hsieh and Ossa, 2016) and falling trade costs due to China's 2001 accession to the WTO (Handley and Limão, 2017), and plateaued after the early 2010s (Autor, Dorn, and Hanson, 2021).

Concurrently to the export growth, China maintained an exchange rate peg to the US dollar. The renminbi (China's official currency) was pegged at a rate of 8.28:1 in June 1994 and sustained a hard peg until July 2005, which "contributed to the exploding exports and ballooning trade surpluses of the early 2000s" (Kroeber, 2014). Subsequently, the People's Bank of China (PBOC) implemented a managed *band*, allowing the currency to fluctuate within a narrow band. This band gradually widened from 0.3% in July 2005 to 1% in April 2012, with a hard peg during the Great Recession. The renminbi appreciated through a slow and controlled process, and Ilzetzki, Reinhart, and Kenneth S Rogoff (2019) classify China's exchange rate policy as a *de facto peg* from January 1994 to 2019.

2.2.2 Data and Measure of the China Shock

In this subsection, we outline the sources of our data and the construction of shocks. Additional details are provided in Appendix B.1.

Exposure to the China shock. To measure the exposure of a country i to the surge in Chinese exports, we follow Acemoglu et al. (2016) and Autor, Dorn, and Hanson (2021) to construct a shift-share measure of exposure that combines (1) a weight of each sector s for each country i and (2) global growth in Chinese exports for each sector s

$$S_i = \sum_s \underbrace{\lambda_i^s}_{\text{share}} \times \underbrace{g_C^s}_{\text{China shock in sector } s} \tag{2.1}$$

Here $g_C^s = \log(E_{CT}^s) - \log(E_{Ct}^s)$ is the global increase in Chinese export value for each sector s from the pre-shock period t to post-shock T ($t = 2000$ to $T = 2012$, following Autor, Dorn, and Hanson (2021)), and λ_i^s is a weight of each country i's exposure to Chinese export growth in sector s. Sectoral export data is obtained from the UN Comtrade database at the 4-digit SITC level, and we closely follow the cleaning procedures in Robert C Feenstra et al. (2005) and Atkin, Costinot, and Fukui (2022).

S_i is a shift-share measure (Bartik, 1991) of each country's exposure to the surge in Chinese exports and is akin to the local labor market exposure measure in Autor, Dorn, and Hanson (2013). From Equation 2.1, any variation in S_i across countries comes entirely from variations in sector share λ_i^s: countries with higher S_i face more competition from Chinese exports precisely because those countries had a larger share of sectors where Chinese exports increased. A sufficient condition for S_i identifying country i's exposure to the sectoral shocks is for the shocks g_C^s to be exogenous to demand-side confounders (Borusyak, Hull, and Jaravel, 2022). We discuss this further in Section 2.2.5 find supporting evidence for shock exogeneity in Appendix B.1.[10]

We define the weights λ_i^s of each sector s in country i. Gathering accurate data on 4-digit sector sizes across countries is difficult, and we proxy for the sector size using export value data, which is readily available. Thus, our baseline measure of each sector s's weight in each country i is given by

$$\lambda_i^s = \frac{E_{it}^s}{GDP_{it}}$$

where E_{it}^s is country i's total value of exports at the pre-period t; a higher share λ_i^s means country i is exporting relatively more to sector s. Thus, our measure of *exposure to China*

[10]The assumption of exogenous shocks (or 'shifts') in the China shock context is standard and is used in Autor, Dorn, and Hanson (2013), Autor, Dorn, and Hanson (2021), and Acemoglu et al. (2016).

shock for country i becomes

$$S_i = \sum_s \frac{E^s_{it}}{GDP_{it}} \Delta \log(E^s_C)$$

which has the following interpretation: a higher S_i means that country i is exporting more in sectors where Chinese exports globally increased. Thus, S_i measures how much country i's exports to third countries are substituted to China, which complements the China shock literature, which often studies domestic competition with imports from China. In the Online Supplement, we consider alternative weights λ_i and shocks g^s_C, showing that the results are robust to alternative choices.

Exchange rate regime. Because China's currency is pegged to the US dollar, we want to compare countries that use or peg to the US dollar to countries floating relative to the US dollar. We classify each country-year observation's de facto exchange rate regime using the Ilzetzki, Reinhart, and Kenneth S Rogoff (2019) (henceforth IRR) exchange rate classification. IRR categorizes every country's *de facto* exchange rate policy from 1946 to 2019 into a six-category classification, with the categories being: (1) peg; (2) a narrow band; (3) a broad band and managed float; (4) freely floating; (5) freely falling; (6) dual market with missing market data, with an anchor currency to each observation.[11]

We define the dummy variable Peg_{it} to be 1 if the country is the United States, or the country is classified as category 1 or 2 according to IRR and their anchor currency is the US dollar. We define Peg_{it} to be 0 if the country's currency is floating or is classified as category 1 or 2 and their anchor currency is not the US dollar. Observations in categories 3 (intermediate categories), 5 and 6 (freely falling or missing data) are dropped, and we also exclude countries whose Peg_{it} changes during our period of interest, as currency regime changes are highly endogenous and indicate turbulent economic conditions. In the remainder of this section, we say the country *pegs* if $\text{Peg}_{it} = 1$ and *floats* if $\text{Peg}_{it} = 0$, so that pegs and floats are with respect to the US dollar.

Outcome variables of interest. We consider the following outcome variables for each country: (1) nominal exchange rate; (2) real GDP; (3) manufacturing output; (4) unemployment; and (5) net exports. If the nominal exchange rate responds to higher S_i for floating countries but not for pegged countries, this is evidence that the exchange rate is operating as an adjustment margin. Then, we investigate the effects of the margin through the dependent variables (2) to (5). Real GDP, manufacturing export, and trade balance are computed from

[11]IRR also provides a fine 15-category classification. Details and the fine classification are given in Appendix B.1.

the World Bank's World Development Indicators (WDI) database; the unemployment rate is from the International Labour Organization (ILO); the nominal exchange rate of a country is the effective exchange rate and obtained from Darvas (2012) and Darvas (2021).

2.2.3 Empirical Design

Our goal is to test across different countries whether higher exposure to the China shock had differential effects depending on each country's exchange rate regime. Thus, we wish to test for countries i:

$$\mathbb{E}[\Delta Y_i | \Delta S_i, \text{Peg}_i = 1] \neq \mathbb{E}[\Delta Y_i | \Delta S_i, \text{Peg}_i = 0] \tag{2.2}$$

where Y_i denotes a dependent variable of interest (trade deficit, labor market, and goods market outcomes), S_i denotes exposure to the China shock, and Peg_i is a dummy variable for whether country i uses or pegs to the US dollar. This approach circumvents the heterogeneous exposure confounder – each country's differential exposure to the China shock – that may plague a simple binary test on the exchange rate regime.[12]

Triple-Difference Regression. Our novel analysis is to explore how the interaction between a country i's exposure to the China shock (S_i) and its currency regime (Peg_i) affects output, employment, and trade balances. We estimate first-difference models using successively longer time differences. For each year h, we implement Equation 2.2 through the following regression:

$$\Delta_h Y_{i,t+h} = \alpha_h + \beta_{1h} S_i + \beta_{2h} \text{Peg}_i + \beta_{3h}(S_i \times \text{Peg}_i) + X_i' \gamma + \epsilon_{ih}, \tag{2.3}$$

where $\Delta_h Y_{i,t+h} = Y_{i,t+h} - Y_{i,t}$ is the change in the outcome for country i between year $t+h$ and initial year t. X_i includes controls for country i's pre-period characteristics. This triple-difference design (over time, exposure, and exchange rate regime) compares how variations in outcomes between countries with similar exposure levels are influenced by the exchange rate regime. Rejecting the null $\beta_{3h} = 0$ supports the hypothesis in Equation 2.2: similar exposure to the China shock affects peggers and floaters differently.

Following Autor, Dorn, and Hanson (2021), we focus on the period 2000 to 2019, comprising China's intense growth in the first decade and the plateauing in the second. Our definition of the China shock is growth in exports between $t = 2000$ and $t = 2012$. Hence,

[12] As such, confounders such as different industry composition or development levels should not affect our analysis, as they are captured by conditioning on S_i.

for $h < 12$, the estimate captures the effect of the partial shock from 2000 to $2000 + h$ on the outcome variables. For $h \geq 12$, the estimate is an event study of how the China shock impacts the outcome variable over a longer horizon.

Controls. The control vector X_i' includes country-specific characteristics that affect outcome variables of interest. We control for log population and log GDP per capita in each country at the starting period $t = 2000$. This is to control for the possibility that the effect of the China shock may interact with the size and development of this country. Since our construct of the shift-share exposure S_i implies $\sum_s \lambda_i^s \neq 1$ in general, we purge for the bias generated by incomplete shares, highlighted in (Borusyak, Hull, and Jaravel, 2022) by including $\sum_s \lambda_i^s$ in our set of controls.[13] We control for the interaction of those controls with the Peg_i, to account for the possibility that the exchange rate peg is correlated with the shares, these variables, and affects the outcome variable differently. We also control for one lag of the outcome variable – if $Y_{i,t+h}$ is the outcome variable, we control for $Y_{i,t-1}$ for $h \geq 0$ and $Y_{i,t+h-1}$ for $h < 0$. The controls, with the exception of $\sum_s \lambda_i^s$, are obtained from the WDI.

Balanced Panel. Our empirical strategy rests on the identifying assumption that there are no omitted variables that are correlated with the exchange rate regime and affect the outcome variables differentially. Table B.2 reports summary statistics in various observable characteristics between the countries pegging and floating with respect to the USD, and their differences. Pegging countries are smaller (Hassan, Mertens, and Zhang, 2022), have a lower manufacturing share and moderately lower unemployment in 2000. However, peggers and floaters show broad similarity in other observable factors, including exposure to the China shock.

2.2.4 Results

Nominal exchange rate. We first ask whether the nominal exchange rate responds to the China shock. If exchange rates indeed serve as an adjustment margin, we would expect currencies of countries more exposed to the China shock to *depreciate more* under a floating regime. In contrast, we would not anticipate currency responses to the China shock for countries pegged to the US dollar. If true, this supports the hypothesis that competition with Chinese goods leads to depreciation in the currencies of floating economies, while the lack of such a response in pegged economies could lead to distortions.

[13] We chose these weights because the alternative – divide by total exports – would mean that relatively closed countries are more exposed to the China shock, which is unrealistic. In the Online Supplement, we conduct the same empirical specification with alternative weights λ_i^s that sum to 1.

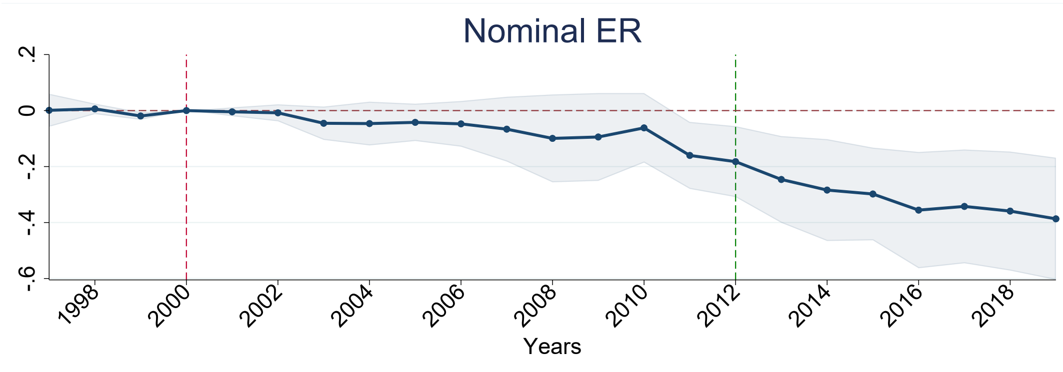


Figure 2-2: Exchange rate response to the China shock.

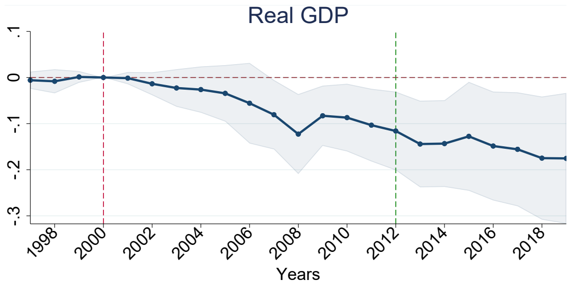
Note. The figure plots β_{3h} of the model 2.3 with the nominal exchange rate as the dependent variable across time. It shows the differential response of the nominal exchange rate among peggers and floaters to the China shock. In the Appendix, we plot the coefficient for the subset of countries where the currency is pegged versus floated against the US dollar respectively. A higher value of the nominal exchange rate implies depreciation of the currency. The shaded area is the 95% confidence band for each regression. The red dashed line indicates the beginning of the China shock (2000) and the green the end of the China shock (2012). The plotted coefficients have standard error of S_i normalized to 1.

We report the estimated response of the nominal exchange rate to the interaction of the China shock and exchange rate regime using our triple difference strategy. Figure 2-2 displays the coefficients β_{3h} of the differential response between pegged and floating countries, together with the 95% confidence intervals. Conditional on similar China shock exposure S_i, floating countries have their currency *depreciate* compared to pegged countries.

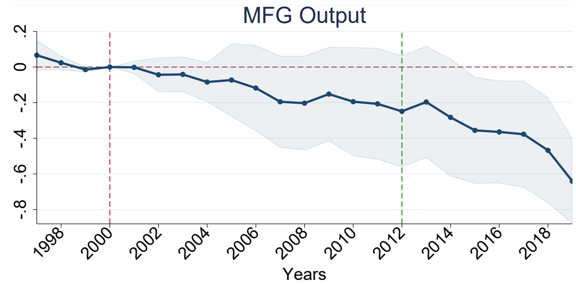
The significance of this effect suggests that the exchange rate operates as an important margin of adjustment in global export competition. This perspective is often overlooked in the China shock literature, either empirically or structurally. We underscore that this role of the exchange rate may be relatively uncharted territory, and the absence of exchange rate adjustments may have real consequences, which we explore next.

Output, Unemployment, and Net Exports. Next, we assess how the China shock affects pegged and floating economies *differently* for our variables of interest: real GDP, manufacturing output, unemployment rate, and net exports. If China's peg to the dollar influences the impact of the China shock on goods market outcomes and trade balances, we should observe a non-zero β_{3t}, with the interpretation that countries more exposed to Chinese exports will experience a stronger decline in output, higher unemployment, and larger trade deficits if their currency is pegged to the US dollar.

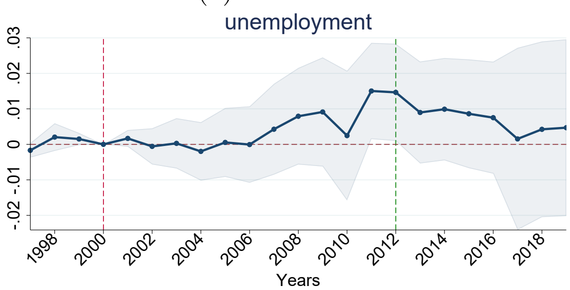
Figure 2-3 plots our estimates of β_{3h} for those outcomes. For real GDP and manufacturing output, the left-hand side is $\log(Y_{i,t+h}) - \log(Y_{i,t-1})$ and is intended to measure percentage



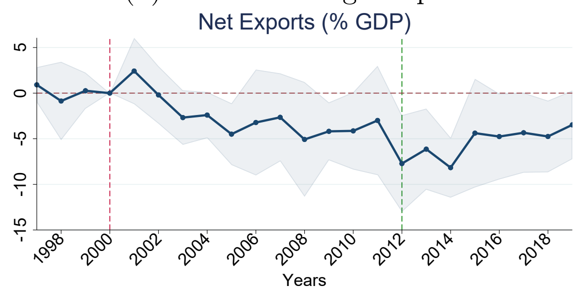
(a) Real GDP



(b) Manufacturing Output



(c) Unemployment rate



(d) Net Exports (% GDP)

Figure 2-3: Responses of peggers to USD vs floaters to USD to the China shock.

Note. The plotted coefficient β_{3h} is the differential response among peggers and floaters to the China shock. A positive coefficient implies that conditional on the same exposure to the China shock S_i, pegged countries' output variable response is higher than floating countries' response for the same variable. The shaded area is the 95% confidence band for each regression. The red dashed line indicates $t = 2000$, the start of the China shock, and the green line $t = 2012$, the end of the China shock. A comparison plot of the separate double-difference regressions for pegged and floating countries is provided in Appendix B.1, in Figures B-3 and B-4 respectively. The plotted coefficients have standard error of S_i normalized to 1.

change. For the unemployment rate, we use the difference $Y_{i,t+h} - Y_{i,t-1}$, and for net exports, we use $\frac{NX_{i,t+h}}{Y_{i,t+h}} - \frac{NX_{i,t-1}}{Y_{i,t-1}}$. We report the double-difference results for the full sample and the pegged and floating countries separately in Appendix B.1.

The top two panels of Figure 2-3 show that the real GDP and manufacturing output were more adversely affected by the China shock for pegging countries, even conditional on the same increase in exposure S_i. The negative effects on real GDP and manufacturing output for pegging countries build up during the trade exposure period and extend persistently for years after the shock.[14] Notably, the decline in manufacturing output attributable to the interaction of Chinese exports and currency peg is double the analogous effect on real GDP, suggesting that the manufacturing sectors are hurt more by higher exposure, in line with

[14] Autor, Dorn, and Hanson (2021) suggest two reasons for why trade-exposed labor markets suffer long-lasting hardship; the first is that such regions are poorly positioned to recover because of a dearth of college-educated workers, and the second is that specialization in industries with Chinese competition left these regions exposed to industry-specific shocks that self-reinforce during decline (Dix-Carneiro and Kovak, 2017). We note that both are plausible.

previous literature.

The bottom left panel (Figure 2-3c) shows that unemployment increases during the duration of the shock and reverts after the culmination of the shock. This finding suggests the existence of short-run friction in the labor market that is affected by higher exposure to the China shock when the currency is pegged, consistent with the notion that the friction in the labor market may be a *nominal* friction. The bottom right panel (Figure 2-3d) shows that the trade balances of pegged countries deteriorate more for pegged countries, and this decline persists.

In Figure B-4, we show how peggers and floaters respond differently to higher S_i separately, by running regressions for each subsample and plotting β_1. We see that within the peggers, greater exposure to Chinese exports led to lower manufacturing output, a temporary increase in unemployment, and larger trade deficits. In sharp contrast, within the floaters, we find that nominal exchange rates adjust in a way that there is no material association between the exposure to Chinese exports and macroeconomic outcomes.

The difference of outcomes suggests that a country's peg to the US dollar – which pegs it to China – affects the incidence of the China shock on that country because the exchange rate cannot adjust to the China shock. These empirical findings provide additional support for the strand of literature that finds the costs of exchange rate pegs through the loss of a nominal adjustment margin (see e.g., Broda (2004) and Ahn, Choi, and Rivadeneyra (2022)).

2.2.5 Discussion

Sensitivity analysis

Robustness. The results in Figures 2-2 and 2-3 are robust to alternative specifications. In the Online Supplement, we progressively add and remove the controls, add additional controls, and change the time horizon of the China shock to be 2000-2010 and 2000-2007. In addition, we conduct a parallel analysis using an alternative shift-share instrument where the shares are now exports as a share of total exports from i (summing to 1) or where the shifts are increases in nominal export volumes. Our results are consistent across these alternative specifications.

Shift-share as leveraging shock exogeneity. As Borusyak, Hull, and Jaravel (2022) show, a sufficient condition for identification is for the industry-specific growth shocks g_C^s to be exogenous, clarifying the identifying assumptions in our analysis and the construction of the standard errors. In Appendix B.1, we draw on recent literature (Borusyak, Hull, and Jaravel, 2022; Borusyak and Hull, 2023) to test shock exogeneity and find supporting evidence for the shift-share measure S_i as leveraging quasi-random variation in the shocks g_C^s.[15]

Instruments and Bias. The shift-share analysis may be biased if Chinese exports and sectoral shares are both correlated with sectoral demand shocks. In studying US regions, Autor, Dorn, and Hanson (2013) overcome bias associated with US demand shocks by using exposure of other developed countries as instrument. As our concern is a global demand shock, we cannot construct an analogous instrument. However, such a shock would also violate the exogeneity of the aforementioned instrument in Autor, Dorn, and Hanson (2013). With lack of a superior alternative, we proceed with the OLS estimate.

Relation with exchange rate puzzles

Our empirical results raise the following question: how do we reconcile the fact that exchange rate regimes affect differential responses of macroeconomic aggregates to shocks to the fact that the unconditional correlation between exchange rates and output is close to zero? It is known that the exchange rate is disconnected from macroeconomic aggregates (Meese and K. Rogoff (1983), Itskhoki and Mukhin (2021a)), and while the nominal and real exchange rate volatility are highly correlated, (Mussa, 1986), such movements are orthogonal to behavior of other macro variables (Itskhoki and Mukhin, 2021b).

We argue that the *conditional* exchange rate response to exogenous shocks can be consistent with *unconditional* exchange rate disconnect.[16] Our empirical findings suggest that exchange rate movements *counteract* underlying shocks to fundamentals: a productivity growth leads to an increase in demand for that country's goods in partial equilibrium, and the general equilibrium response of the exchange rate moves in the opposite direction through an appreciation of that country's currency (Figure 2-2) – and the lack of this force has real consequences (Figure 2-3). This role of exchange rates as an insulator is documented in Broda (2004) using a VAR analysis of terms-of-trade shocks. Our analysis highlights that China's exchange rate peg to the US dollar can mute this insulator role for countries using the US dollar, leading to real consequences.

[15] Goldsmith-Pinkham, Sorkin, and Swift (2020) develop an alternative approach to identification of shift-share exposure based on the exogeneity of the initial-period shares λ_i^s; this is less suitable for our analysis.

[16] The conditional relation and unconditional disconnect can be microfounded through noisy expectation about future productivity (Chahrour et al., 2023) or through multiple financial shocks (Fukui, Nakamura, and Steinsson, 2023).

2.3 A two-period trade model with nominal rigidity

In this section, we develop a tractable model that rationalizes the unemployment in manufacturing and trade deficits as an outcome of Foreign productivity growth and an exchange rate peg, explaining concurrently the four facts (Figure 2-1) and corroborating the findings in Section 2.2. Our one-sector, two-period, two-country model highlights the role of exchange rate pegs and nominal wage rigidity. Using this model, we study the positive and normative implications of a trade shock and policy implications.[17] We keep the ingredients minimal for analytical tractability and extend the model in Section 2.4.

2.3.1 Model setup

Our environment has two countries, Home (H) and Foreign (F). In our application, Home will be the United States and Foreign will be China. There are two periods: $t = 0$ (short-run) and $t = 1$ (long-run). A representative household in each country consumes goods from both countries and supplies labor to firms that produce goods. Each country has its own nominal account; the price of country j's currency in units of country i's currency at time t is e_{jit}, with $e_{HHt} = e_{FFt} = 1$ and $e_{FHt} = \frac{1}{e_{HFt}}$. We denote $e_t = e_{FHt}$. Hence an increase in e_t is a depreciation of the Home currency.

Household preferences. In each country j, there is a representative agent who consumes goods C_{ijt} across origins i aggregated into a final good C_{jt}, supplies labor L_{jt}. The household has preferences represented by

$$\mathcal{U}_j = [u(C_{j0}) - v(L_{j0})] + \beta[u(C_{j1}) - v(L_{j1})], \tag{2.4}$$

$$\text{where } u(C) = \frac{C^{1-\gamma^{-1}} - 1}{1 - \gamma^{-1}}, \text{ and } C_{jt} = (C_{Hjt}^{\frac{\sigma-1}{\sigma}} + C_{Fjt}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}.$$

Here σ is the elasticity of substitution between domestic and foreign goods (the Armington elasticity), and γ is the elasticity of intertemporal substitution. We assume that the Armington elasticity is larger than unity, and the intertemporal elasticity is smaller: formally, $\sigma > 1$ and $\sigma > \gamma$.[18] $v(\cdot)$ is the disutility of supplying labor, which we assume is increasing

[17] In the Online Supplement, we analyze a two-sector tradable-nontradable model to study the decline in the share of manufacturing, and how trade shocks may propagate to nontradable sectors through aggregate demand. This section is intended to be minimal.

[18] Empirical estimates of σ range from 3-10 (Anderson and Wincoop, 2003; Imbs and Mejean, 2017) to 1.5-3 (Boehm, Levchenko, and Pandalai-Nayar, 2023), but is consistently greater than 1. Estimates of γ are less than 1 and sometimes indistinguishable from 0. Section 2.3.5 draws on the literature to discuss this assumption. If we instead had $\sigma = \gamma = 1$, we are in the Cole and Obstfeld (1991) case, where the equilibrium

and convex with $v(0) = 0$.

Technology. A representative firm in country i uses labor as input and has a constant returns to scale production function that requires $\frac{1}{A_{ij}}$ labor to supply a unit of good to market j. Thus for a firm in country i selling Y_{ij} goods to country j at time t using L_{ijt} labor, we have

$$Y_{ijt} = A_{ij} L_{ijt}.$$

A_{ij} implicitly incorporates trade frictions. Throughout we assume $A_{HF} \leq A_{HH}$ and $A_{FH} \leq A_{FF}$, implicitly assuming home bias in consumption.

Savings. Each country issues a domestic bond with zero net supply. In period 0, households in each country j have access to a claim of a unit of currency i in period 1, with the price of a claim being $\frac{1}{1+i_{i1}}$ in country i currency. We let B_{ij1} denote the amount of claims for i currency that households in country j own. We assume there is no risk, and bonds from Home and Foreign are perfect substitutes.

Labor Market and Nominal Rigidity. We consider the simplest form of short-run nominal wage rigidity. We assume that nominal wages in both countries are completely fixed in period $t = 0$ to an exogenous level $\{w_{j0}\}$, while wages $\{w_{j1}\}$ are flexible for $t = 1$. Since wages are rigid in period 0, we assume that the labor market is demand-determined in both countries, and workers supply whatever labor is demanded. In period 1, we assume that wages equalize labor supply and labor demand.[19]

Monetary policy and exchange rates. The monetary authority at Home sets the nominal interest rate according to a CPI-based Taylor rule with a coefficient of 1 on inflation:

$$\log(1 + i_{H1}) = -\log(\beta) + \log(\frac{P_{H1}}{P_{H0}}) + \epsilon_{H0}, \tag{2.5}$$

where ϵ_{H0} is the discretionary monetary policy.[20] This rule implicitly sets the real rate $R_{H1} = (1 + i_{H1})\frac{P_{H0}}{P_{H1}}$ at

$$R_{H1} = \frac{1}{\beta} \exp(\epsilon_{H0}).$$

always features trade balance. Thus this assumption is key to predicting the direction of trade imbalance.

[19]The assumption that wages are completely fixed is to highlight the intuition; any short-run friction in wage adjustment will yield qualitatively identical results.

[20]This follows McKay, Nakamura, and Steinsson (2016), Auclert, Rognlie, et al. (2021), and allows our analysis to be orthogonal to the effects of monetary policy *rules*.

We say a monetary policy *does not respond to shocks* if it sets $\epsilon_{H0} = 0$, or equivalently $R_{H1} = \frac{1}{\beta}$. In Sections 2.4 onwards, we consider a more standard Taylor rule, which delivers similar results.

Turning to Foreign monetary policy, we are interested in the equilibrium dynamics when Foreign pegs the nominal exchange rate to Home. We assume that Foreign monetary policy directly chooses the exchange rate

$$e_0 = e_1 = \bar{e}, \tag{2.6}$$

at an exogenous level $\bar{e}$.[21]

Trade taxes and subsidies. The government can also levy taxes on imports and subsidize exports. We assume that the Home government unilaterally chooses the short-run import tariff t_{FHt} and export subsidy s_{HFt}. If we denote the pre-tariff price of i goods to j at time t by P_{ijt}, Home government revenue is

$$T_{Ht} = t_{FHt} P_{FHt} C_{FHt} - s_{HFt} e_{FHt} P_{HFt} C_{HFt}. \tag{2.7}$$

We assume that the revenue T_{Ht} is rebated lump-sum to the representative household.

2.3.2 Competitive Equilibrium

In a competitive equilibrium, households maximize their utility, firms maximize their profit, and markets clear. We briefly derive each condition and relegate the details to the Online Supplement.

Utility maximization. The household at country j chooses consumption $\{C_{ijt}\}$, $\{L_{it}\}_{t=1}$,$\{B_{ijt}\}$ to maximize utility $\mathcal{U}_H$ as described in Equation 2.4 subject to the sequential budget constraints,

$$\sum_i (1 + t_{ij0}) P_{ij0} C_{ij0} + \sum_i \frac{B_{ij1}}{1 + i_{ijt}} e_{ij0} \leq W_{j0} L_{j0} + \Pi_{j0} + T_{j0}, \tag{2.8}$$

$$\sum_i (1 + t_{ij1}) P_{ij1} C_{ij1} \leq W_{j1} L_{jt} + \sum_i B_{ij1} e_{ij1} + \Pi_{j1} + T_{j1}, \tag{2.9}$$

where P_{ijt} is the (pre-tariff) prices for goods from country i to j in units of j currency, B_{j1} is a tradable claim to one nominal unit of account in period 1 with price $\frac{1}{1+i_{jt}}$, W_{jt} is the

[21] An explicit monetary rule setting i_{Ft} that leads to the exchange rate peg can be found in G. Benigno, P. Benigno, and F. Ghironi (2007).

nominal wage, Π_{jt} is the profit of country j firms and T_{jt} is the government revenue rebated lump-sum.

The first-order conditions to this utility maximization problem are standard and given by:

$$P_{jt} = (\sum_i ((1+t_{ijt})P_{ijt})^{1-\sigma})^{1/(1-\sigma)}, \tag{2.10}$$

$$\lambda_{ijt} = \frac{((1+t_{ijt})P_{ijt})^{1-\sigma}}{\sum_l P_{ljt}^{1-\sigma}}, \tag{2.11}$$

$$v'(L_{j1}) = \frac{u'(C_{j1})w_{j1}}{P_{j1}}, \tag{2.12}$$

$$u'(C_{jt}) = \beta(1+i_{jt})\frac{P_{jt}}{P_{jt+1}}u'(C_{jt+1}) = \beta R_{jt} u'(C_{jt+1}), \tag{2.13}$$

$$\frac{1+i_{F1}}{1+i_{H1}} = \frac{e_1}{e_0}, \tag{2.14}$$

where P_{jt} denotes the consumer price index (CPI) in country j and λ_{ijt} the expenditure share. With the peg $e_1 = e_0 = \bar{e}$, the last condition becomes $i_{F1} = i_{H1}$ (trilemma).

Since wages $\{w_{j0}\}$ are rigid at $t = 0$ and the labor market is demand determined, we may have $v'(L_{j0}) \neq \frac{u'(C_{j0})w_{j0}}{P_{j0}}$. We define the *labor wedge* in period 0 as

$$\mu_{j0} = v'(L_{j0}) - \frac{u'(C_{j0})w_{j0}}{P_{j0}}, \tag{2.15}$$

how much the marginal value of working for households is away from the marginal return from working in utility terms. If $\mu_{j0} < 0$, households would like to supply more labor but cannot, so there is *involuntary unemployment*. If $\mu_{j0} > 0$, households are supplying more labor than they would want to, so the economy is *overheated*.

Firm optimization. The profits of a representative firm from j selling Y_{ijt} goods to market i is given by

$$\Pi_{it} = \sum_j \left[(1+s_{ijt})\frac{1}{e_{ijt}}P_{ijt} - \frac{W_{it}}{A_{ij}}\right] Y_{ijt}$$

where s_{ijt} is an ad-valorem sales subsidy to i. Since firms are competitive, profits Π_{jt} are equal to 0, and the unit price is equal to marginal cost:

$$P_{ijt} = \frac{1}{1+s_{ijt}} e_{ijt} \frac{w_{it}}{A_{ij}}. \tag{2.16}$$

Market clearing. For each (i,t), the goods market clearing conditions are given by

$$L_{it} = \sum_j \frac{C_{ijt}}{A_{ij}}, \tag{2.17}$$

and the bonds market clearing condition is given by

$$B_{H1} + e_1 B_{F1} = 0. \tag{2.18}$$

Equilibrium. We are ready to define an equilibrium in the model as follows:

Definition 1. *Given fundamentals $\{A_{ij}\}$, rigid short-run wage w_{H0}, w_{F0}, policy $\{R_{H1}, t_{ijt}, s_{ijt}\}$ and pegged exchange rate $\bar{e} = e_0 = e_1$, a pegged equilibrium consists of prices $\{w_{it}, P_{it}, P_{ijt}\}$, household's choice variables $\{C_{ijt}\}$,$\{B_{it}\}$,$\{L_{it}\}_{t\geq 1}$ and demand-determined short-run labor $\{L_{i0}\}$ such that Equations 2.8 to 2.18 hold.*

2.3.3 Consequences of a trade shock

In this subsection, we highlight the equilibrium response to trade shocks in this model. As a benchmark, we consider the laissez-faire equilibrium where $t_{FHt} = s_{HFt} = 0$.

The timing of the model and the shock is as follows. Before the start of our setup $(t = -1)$, productivities were at a level $\{A_{ij,-1}\}$, and nominal wages $w_{i,-1}$ and exchange rate e_{-1} were such that trade is balanced and labor wedge is zero. Right before $t = 0$, a shock permanently increases Foreign export productivity A_{FH}; we call this the *trade shock*. We assume that wages $\{w_{i0}\}$ are rigid at the pre-shock level $\{w_{i,-1}\}$, and the Foreign policymaker pegs the exchange rate $e_0 = e_1$ at the pre-shock level e_{-1}.

Equilibrium responses. To investigate the effects of the trade shock on trade balance and employment levels, we first observe how the terms-of-trade responds to a trade shock under a peg. We denote by $S_{HFt} = \frac{P_{HFt}\bar{e}}{P_{FHt}}$ the Home terms-of-trade at time t, where a higher terms-of-trade means a higher price of exports relative to imports. S_{HFt} is given by:

$$S_{HFt} = \frac{(\frac{w_{Ht}}{\bar{e}A_{HF}})\bar{e}}{\frac{w_{Ft}\bar{e}}{A_{FH}}} = \underbrace{(\frac{w_{Ht}}{w_{Ft}\bar{e}})}_{\text{relative wage}} \underbrace{(\frac{A_{FH}}{A_{HF}})}_{\text{productivity}} \tag{2.19}$$

If wages were flexible, an increase in A_{FH} affects S_{HF} in two ways. The *direct effect* increases S_{HF} by an equal proportion, improving Home terms-of-trade. The *general equilibrium (GE) effect* is that relative wage $\omega_t = \frac{w_{Ht}}{w_{Ft}\bar{e}}$ adjusts. Under the assumption that $\sigma > 1$, an increase in A_{FH} decreases Home's relative wage ω_t, so the GE effect reduces ω_t. If wages are flexible or the exchange rate is floating, the GE effect would take place immediately, and the equilibrium after the trade shock will be a new steady-state equilibrium with $\omega_0 = \omega_1$, without any dynamics between $t = 0$ and $t = 1$.[22]

However, when wages are rigid and the exchange rate is pegged, the GE effect is muted in the short-run. Then we have $\omega_0 > \omega_1$: Home's relative wage is higher in the short-run than the long-run. This results in the following comparative static:

Proposition 4. *In the pegged equilibrium, in response to a trade shock ($A_{FH} \uparrow$), Home runs a trade deficit ($B_{H1} < 0$). Moreover, if Home monetary policy does not respond ($R_{H1} = \frac{1}{\beta}$), then there is involuntary unemployment at Home ($\mu_{H0} < 0$).*

Proof. See Appendix B.2. □

The logic behind the imbalances ($B_{H1} < 0$) is as follows. Home borrows if and only if:

$$\underbrace{\frac{\bar{e}\lambda_{HF0}P_{F0}C_{F0}}{\lambda_{FH0}P_{H0}C_{H0}}}_{t=0 \text{ exports/imports}} < \underbrace{\frac{\bar{e}\lambda_{HF1}P_{F1}C_{F1}}{\lambda_{FH1}P_{H1}C_{H1}}}_{t=1 \text{ exports/imports}} \Leftrightarrow \underbrace{\frac{\lambda_{HF0}/\lambda_{HF1}}{\lambda_{FH0}/\lambda_{FH1}}}_{\text{expenditure switching}} < \frac{\pi_F}{\pi_H}\frac{C_{H0}/C_{H1}}{C_{F0}/C_{F1}} = \underbrace{(\frac{\pi_F}{\pi_H})^{1-\gamma}}_{\text{relative inflation}} \tag{2.20}$$

Inequality 2.20 highlights the two forces that determine the sign of trade balance. The first force is *expenditure switching.* When $\sigma > 1$, we have $\omega_0 > \omega_1$, so both countries want to buy more Foreign goods today than tomorrow, implying $\lambda_{FH0} > \lambda_{FH1}$ and $\lambda_{HF1} < \lambda_{HF1}$, pushing towards Home deficit. The second force is *relative inflation.* With $\omega_0 > \omega_1$, Home's future prices increase *less* because of home bias in consumption. This pushes towards Home surplus if and only if $\gamma > 1$.[23] When $\sigma > \gamma$, expenditure switching (governed by σ) outweighs relative inflation (governed by γ), resulting in Home trade deficit.[24]

Under a peg, Home's monetary policy cannot affect the sign of the trade imbalance. Home borrows regardless of R_{H1}, because R_{H1} affects the consumption-savings decision of both countries. In fact, when $\gamma = 1$, R_{H1} cannot even affect the magnitude of the deficit, as the effect of interest rates is exactly identical in both countries. We discuss this further in Section 2.3.4.

The intuition for Home unemployment is as follows. Short-run aggregate consumption C_{H0} is determined from the Euler equation. At C_{H0} and real wage $\frac{w_{H0}}{p_{H0}}$, Home workers

[22]The fact that a floating exchange rate can adjust for the GE effects under nominal rigidity is closely related to the Dornbusch (1976) overshooting model.

[23]In fact, estimates of γ are often 1 or less, whence relative inflation also leads to Home borrowing.

[24]An intuitive example is when $\sigma \to \infty$. Home wouldn't produce at all at $t = 0$, but it can compete against Foreign at $t = 1$. So Home wants to borrow to smooth consumption unless $\gamma = \infty$.

would want to supply labor $L^S_{H0} = v'^{-1}(u'(C_{H0})\frac{w_{H0}}{P_{H0}})$. However, workers supply whatever is demanded, and the demand L_{H0} is pinned down by relative wage ω_0:

$$L_{H0} = \frac{1}{A_{HH}} \frac{\lambda_{HH0}(\omega_0) P_{H0}}{P_{HH0}} C_{H0} + \frac{1}{A_{HF}} \frac{\lambda_{HF0}(\omega_0) P_{F0}}{P_{HF0}} C_{F0}.$$

If ω_0 is higher, the desired supply L^S_{H0} increases but actual demand L_{H0} falls; this generates *involuntary unemployment*, with the unemployment rate given by $u_{H0} = 1 - \frac{L_{H0}}{L^s_{H0}}$.[25]

In contrast, under a floating exchange rate, we would observe neither deficits nor unemployment: as $\omega_0 = \omega_1$, the equilibrium is observationally equivalent to the new steady-state after the trade shock, with trade balance and full employment.

Proposition 4 parsimoniously connects the four facts in the introduction: the US trade deficit (Figure 2-1c) and surge in manufacturing unemployment (Figure 2-1b) can be endogenously explained by Chinese productivity growth (Figure 2-1a) and its exchange rate peg (Figure 2-1d). This contrasts with prior studies of the China shock, which typically perceive China's concurrent growth and savings as a puzzle. We show that China's peg under wage rigidity promotes a stronger short-term terms-of-trade during its growth, driving it to save.[26]

Proposition 4 supports the notion that nominal rigidity is the relevant friction in the China shock context, and allows us to differentiate from quantity friction such as search friction. This is because such frictions predict the opposite outcome – Home saves in response to Foreign growth. This is because relative wages across time is *reversed* under quantity friction: short-run Home relative wage is depressed, leading to Home saving and less unemployment. In the Online Supplement, we formalize this by considering a quantity rigidity model, showing that Home would save when Foreign grows.

Welfare effects. Next, we turn to the welfare implications of the trade shock. We first highlight that trade balances affect the future terms-of-trade: specifically, a deterioration in balances B_{H1} leads to a decrease in future relative wage ω_1. The intuition is closely related to the transfer problem: debt accumulated today becomes a future *transfer* for Foreign, which, combined with a home bias for demand, increases global demand for Foreign goods, improving their terms-of-trade and worsening Home's.

Using this fact, the next proposition highlights the possibility that Home's aggregate welfare may decrease as a result of Foreign growth:

[25] In this economy, Foreign (China) is overheated and has employment rate greater than 1. We leave this open as a possibility and discuss potential microfoundations and implications in Section 2.6.

[26] Here we assumed that productivity A_{FH} increases from $t = -1$ but is the same between $t = 0$ and 1. If productivity were increasing between the two periods, there would be competition between our expenditure switching channel and the standard force for China to borrow. International finance papers such as Ricardo J. Caballero, Farhi, and Gourinchas (2008) offer a financial solution.

Proposition 5. *In the pegged equilibrium where monetary policy does not respond ($R_{H1} = \frac{1}{\beta}$), a small increase in A_{FH} reduces Home welfare when σ is sufficiently high and improves Home welfare when σ is small (i.e. close to 1).*

Proof. See Appendix B.2. □

An intuitive explanation is as follows. There are three channels through which productivity growth A_{FH} affects Home welfare:

$$\frac{d\mathcal{U}_H}{dA_{FH}} = \underbrace{-\frac{u'(C_{H0})}{P_{H0}} C_{FH0} \frac{dP_{FH0}}{dA_{FH}}}_{\text{terms-of-trade at } t=0} - \underbrace{\mu_0 \frac{dL_0}{dA_{FH}}}_{\text{labor wedge}} + \underbrace{\frac{\beta u'(C_{H1})}{P_{H1}} \left[C_{HF1} \frac{dP_{HF1}}{dA_{FH}} - C_{FH1} \frac{dP_{FH1}}{dA_{FH}} \right]}_{\text{terms of trade at } t=1} \tag{2.21}$$

The terms correspond to (1) the short-run effect of cheaper import goods, (2) labor market friction caused by wage rigidity, and (3) change in long-run terms-of-trade, including direct productivity effects and general equilibrium effects. If $\sigma \to 1$, preference becomes Cobb-Douglas, the pegged equilibrium coincides with the flexible-wage equilibrium, and trade is balanced as in Cole and Obstfeld (1991). Then the effects (2) and the general equilibrium component of (3) go to zero, leaving cheaper goods as the primary welfare benefit. In the opposite case, when $\sigma \to \infty$, short-run demand for Home goods becomes 0. Then, a small change in A_{FH} can cause a discrete loss of utility from the labor wedge and the trade deficit worsening future terms-of-trade, dwarfing welfare gains from cheaper goods.

The possibility of Foreign productivity growth harming Home welfare echoes immiserizing growth where Home's productivity growth worsens its terms-of-trade, negating gains from the expansion of the production frontier (Bhagwati, 1958). In our case, Foreign productivity growth improves Home terms-of-trade, and the peg magnifies this gain today, but unemployment moves Home production into the interior of the PPF and harms future terms-of-trade through trade deficit, offsetting the gains.

Proposition 5 cautions against using trade balance as a welfare indicator. Public discourse often views trade deficits as inherently undesirable. However, whenever σ exceeds 1 and surpasses γ, a trade deficit is the predicted outcome for Home under a trade shock under a peg. The shock may benefit Home welfare if σ is not excessively high. Conversely, a large γ with $\sigma \to 1$ results in Home's trade surplus and welfare gains, whereas with $\gamma > \sigma$ both large, Home faces welfare losses despite a trade surplus. In the next sections, we undertake a quantitative analysis of the substitution, rigidity, and productivity growth to assess whether the China shock improved or harmed aggregate US welfare.[27]

[27] Whether trade deficits are symptoms of welfare gains or losses is a different question to whether capital controls are beneficial. The next subsection shows that capital controls unambiguously hurt Home welfare.

2.3.4 Policy response

In this subsection, we consider the unilateral problem of the Home government facing a growth in A_{FH} and an exchange rate peg. We assume the Home government can choose its short-run tariff level t_{FH0}, domestic subsidy s_{HF0} and monetary policy R_{H1}.[28] We assume the government cannot choose long-run tariff t_{FH1}, as the motivation for long-run tariffs as terms-of-trade manipulation is well understood since Graaff (1949).

Formally, the policy problem that the Home government faces is:

$$\max_{t_{FH0},s_{HF0},R_{H1}} \mathcal{U}_{\mathcal{H}} = \max_{t_{FH0},s_{HF0},R_{H1}} \sum_{t=0}^{1} \beta^t [u(C_{Ht}) - v(L_{Ht})] \tag{2.22}$$

subject to the same equilibrium conditions.

We first note that the planner can replicate the flexible price outcome. Indeed, if $\omega_{peg} = \frac{w_{H0}}{w_{F0}\bar{e}}$ is the short-run relative wage under peg, and $\omega_f = \frac{w^f_{H0}}{w^f_{F0}e^f}$ is the relative wage under flexible price (after the trade shock), the planner can set $R_{H1} = \frac{1}{\beta}$ and $t_{FH0} = s_{FH0} = \frac{\omega_f}{\omega_{peg}} - 1$. This tax and subsidy level sets the relative prices equal to the flexible price level, and the tax revenue and cost of subsidy cancel out exactly. Thus, we know the planner can undo the wedges and the potential welfare losses in Proposition 5.[29]

However, this policy may not be optimal for the Home government. As an extreme example, if Foreign is offering goods for free, Home would be much better off taking those goods than setting high tariffs that distort consumption.

To solve for the optimal policy, we proceed in two steps. First, we solve for the optimal trade policy (t_{FH0}, s_{HF0}) given monetary policy R_{H1}, then we proceed to solve for the optimal R_{H1}. This approach makes the problem more tractable, and the inner problem may be a more reasonable benchmark of reality, where monetary policy is unable to fully respond to a sector-origin specific trade shock.[30] We give an executive summary of our results and discuss the details in the Online Supplement.

Optimal trade policy

Given monetary policy R_{H1}, an indirect formula for the optimal trade policy can be obtained via a first-order variation argument. Starting from the optimal policy, the marginal effect of

[28]Since wages are rigid, we do not have Lerner symmetry, and subsidies and tariffs are independent.

[29]This connects with Farhi, Gopinath, and Itskhoki (2014) that fiscal instruments can replicate currency devaluations.

[30]In the early 2000s, the government was tightening monetary policy in response to concerns over inflation and tightening of unused resources; loosening in response to the China shock was not the Federal Reserve Bank's goal (Federal Reserve Board, 2005). Following the Great Recession, the Federal Reserve Bank was subject to the Zero Lower Bound.

policy change in welfare must be zero, yielding the following formula:[31]

Lemma 2. *The optimal short-run tariff rate on imports t_{FH0} satisfies*

$$t_{FH0} = \frac{1}{P_{FH0}} \left[\underbrace{\frac{\mu_0}{\tilde{\lambda}} \frac{\partial L_{H0}}{\partial C_{FH0}}}_{\text{labor wedge}} - \frac{1}{(1+i_{H1})} \underbrace{(L_{HF1} \frac{\partial w_{H1}}{\partial C_{FH0}} - L_{FH1} \frac{\partial w_{F1}}{\partial C_{FH0}})}_{\text{future terms-of-trade}} + \underbrace{s_{HF0} P_{HF0} \frac{\partial C_{HF0}}{\partial C_{FH0}}}_{\text{subsidy externality}} \right] \tag{2.23}$$

The optimal short-run subsidy rate on exports s_{HF0} satisfies

$$s_{HF0} = \frac{1}{P_{HF0}} \left[- \underbrace{\frac{\mu_0}{\tilde{\lambda}} \frac{\partial L_{H0}}{\partial C_{HF0}}}_{\text{labor wedge}} + \frac{1}{(1+i_{H1})} \underbrace{(L_{HF1} \frac{\partial w_{H1}}{\partial C_{HF0}} - L_{FH1} \frac{\partial w_{F1}}{\partial C_{HF0}})}_{\text{future terms-of-trade}} - \underbrace{P_{HF0} C_{HF0} \frac{\partial s_{HF0}}{\partial C_{HF0}}}_{\text{terms-of-trade today}} \right] \tag{2.24}$$

where $\tilde{\lambda}$ is the Lagrange multiplier on the lifetime budget constraint.

Proof. See Appendix B.2. □

The first-order formula for tariffs succinctly captures the three *externalities* of imports that the Home government seeks to address via a tariff. First, tariffs and subsidies both reduce the labor wedge by stimulating demand for domestic labor. Second, tariffs and subsidies, by affecting relative prices of goods, improve current trade balance (Inequality 2.20), which improves the terms-of-trade in the future. Third, the fiscal externality (deadweight loss) of tariffs and subsidies interact in general equilibrium. In a competitive equilibrium, home households do not internalize any of these effects of an extra unit of import. Thus the tax level $t_{FH0}P_{FH0}$ and the subsidy level $s_{HF0}P_{HF0}$ can be considered a Pigouvian tax that corrects for the three externalities of consuming an extra unit of import or exporting an extra unit.

Using the formula, we can sign the optimal tariff and show that its magnitude *increases* with the Foreign shock A_{FH0}:

Proposition 6. *If there is unemployment at the zero-tariff economy ($\mu_{H0} < 0$ when $t_{FH0} = 0$), the optimal tariff t_{FH0} is positive and is increasing in the size of the trade shock A_{FH0}.*

Proof. See Appendix B.2. □

The intuition that we can and should use tariffs as second-best instruments to fix distortions is well-known. The prediction obtained in Proposition 6 is sharper. We show that in

[31] A similar argument can be found in Costinot, Sarvimäki, and Vogel (2022).

an environment where trade shocks cause unemployment and trade deficits, the tariff should be positive and increase in the magnitude of the trade shock. In this context, the short-run tariff t_{FH0} is akin to *safeguard* tariffs allowed under the WTO Agreement on Safeguards.

But this is not the only role of tariffs in our model, as highlighted in the future terms-of-trade term in Equation 2.23. While tariffs do not affect today's terms-of-trade (due to wage rigidity and peg), a unilateral short-run tariff reduces Home's trade deficit, improving Home's future terms-of-trade. Hence, Home would want to set tariffs beyond the globally optimal "distortion-fixing" level, at the expense of Foreign welfare. As such, short-run tariffs are *safeguard* and *beggar-thy-neighbor* at the same time, even when the short-run terms-of-trade is rigid.[32]

Our model underscores that under an exchange rate peg, the optimal short-run tariff is increasing in the magnitude of the trade shock. This contrasts with the flexible exchange rate case, where the optimal tariff is pinned down primarily by the trade elasticity (Gros, 1987) and does not depend on the shock magnitude. Our framework focuses on tariffs that correct a distortion caused by the peg and the trade shock, so the magnitude of the optimal tariff scales with the size of the distortion. We discuss this in more detail in the Online Supplement.

Proposition 6 assumes monetary policy does not clear unemployment. As aforementioned, the central bank may be unable to clear the output gap caused by sector-specific trade shocks because of multisector considerations, financial concerns, and liquidity constraints such as the Zero Lower Bound. Tariffs will be a useful tool in this second-best world.

Optimal monetary policy

What is the optimal monetary policy R_{H1}? An analogous first-order condition on monetary policy highlights the channels in which monetary policy affects welfare. We highlight a special case when the intertemporal elasticity is equal to 1 (consumption is log):

Proposition 7. *When $\gamma = 1$, optimal monetary policy R_{H1} satisfies the following equation:*

$$0 = \underbrace{-\mu_0 \frac{dL_0}{dR_{H1}}}_{wedge} + \tilde{\lambda}_r [\underbrace{R_{H1} t_{FH0} \frac{P_{FH0}}{P_{H0}} \frac{dC_{FH0}}{dR_{H1}}}_{tariff\ fiscal\ externality} + \underbrace{(NX_0)}_{intertemporal\ TOT}], \tag{2.25}$$

where $\tilde{\lambda}_r$ is the Lagrange multiplier on the Home lifetime real budget constraint normalized by P_{H0}.

[32] By nature of being beggar-thy-neighbor, Foreign can retaliate with its own tariffs to undo the imbalance-adjusting channel of Home tariffs.

As a special case, when $t_{FH0} = 0$, the optimal monetary policy R_{H1} is such that $\mu_0 > 0$: it is optimal to loosen monetary policy beyond clearing the output gap.

Proof. See Appendix B.2. □

Proposition 7 highlights that when Foreign pegs, the optimal monetary policy for a borrowing Home will *overshoot* the output gap. This leverages Home's control of *global* monetary policy and manipulate intertemporal terms-of-trade to its favor. Particularly for the US, which influences global rates as the dominant currency (Gopinath et al., 2020) and runs current account deficits, the central bank may want to set a lower interest rate, with minimal risk of bond liquidation from pegging countries.

The proposition also clarifies that tariffs are second-best instruments when monetary policy cannot respond – whether due to the ZLB or multisectoral considerations. In fact, under a positive tariff, the additional losses from tariff fiscal externality compels Home to set a higher interest rate, reducing overall welfare.[33]

The assumption $\gamma = 1$ allows us to circumvent the effect of today's monetary policy on the magnitude of the trade deficit. When $\gamma = 1$, the effect of interest rate on consumption and output is proportionate in both countries: thus the real value of the deficit does not change, and monetary policy R_{H1} does not affect the intratemporal terms-of-trade in the future. On the other hand, when $\gamma \neq 1$, the optimal monetary policy equation (Equation 2.25) comes with an additional "future terms of trade" term: monetary policy may affect the magnitude of the deficit in real terms (but not the sign, as we discussed in Section 2.3.3), affecting the optimal policy.

Capital Controls

Lastly, we study the welfare effects of the endogenous deficits we highlighted in Proposition 4 by considering *capital controls* in addition to the tariffs and subsidies. We have established that deficits and unemployment can come from the same cause – trade shock and exchange rate peg – but are deficits inherently bad for Home welfare? While this is where some policy narratives go, the next proposition shows that this is not the case.

Proposition 8. *In the pegged equilibrium, removing international financial flows (forcing $B_{H1} = 0$) worsens Home unemployment (μ_{H0} decreases), and reduces Home welfare $\mathcal{U}_0$.*

Proof. See Appendix B.2. □

[33] In the Online Supplement, we numerically solve for the joint optimal trade and monetary policy for various levels of the trade shock A_{FH0}. We find that the joint optimal policy involves no tariffs and a very loose monetary policy, highlighting the distortionary nature of tariffs. In a first-best one-sector world, Home would take advantage of the cheap goods and solve the labor wedge solely through monetary policy.

Removing financial flows worsens Home unemployment because of home bias in consumption. Indeed, with trade costs, under the same price levels, Home borrowing to consume will increase demand for Home goods, while Foreign saving will decrease demand for Foreign goods. Since unemployment is determined by aggregate demand, Home's trade deficit in the short-run actually ameliorates unemployment, and capital controls will only worsen unemployment. As such, while deficits may be symptoms of a friction that may harm the economy, deficits themselves are not a friction to solve, and capital controls may harm Home welfare. The fact that financial transfers are welfare-improving under an exchange rate peg is closely related to the idea that fiscal unions are desirable under currency unions (Farhi and Werning, 2017); we highlight that the possibility of a dynamic budget-balanced (net current value zero) transfer is welfare-improving.

2.3.5 Discussion

Our framework shows that the consequences of trade shocks under a peg depend on labor market frictions, and tariffs and monetary policy can ameliorate welfare losses. Here we address potential questions, including the duration of nominal rigidity and the parameter values.

Duration of nominal rigidity. The prolonged impact of the China shock may raise questions on the role of nominal rigidity. Our answer is twofold. First, the China shock was a persistent event over the 2000s than a one-off event in 2000, aligning observed patterns with short-term mechanisms. Second, the relevant rigidity here is wage rigidity. Downward nominal wage rigidity (DNWR) is known to be persistent and can extend the effects of Foreign shocks well beyond the typical span of price rigidity, as discussed in Schmitt-Grohé and Uribe (2016).

The elasticities of substitution. Our findings rely on $\sigma > \gamma$: the consumption of goods across origins is more substitutable than across time. Trade elasticity (σ) estimates range from 1.5 to 10 but consistently above unity (Costinot and Andrés Rodríguez-Clare, 2014; Imbs and Mejean, 2017; Boehm, Levchenko, and Pandalai-Nayar, 2023), and recent literature (Teti, 2023) suggests that lower estimates might stem from tariff misreporting, indicating actual elasticity is closer to the higher estimates. The intertemporal elasticity (γ) is generally estimated to be below 1, with some studies finding it near zero (Hall, 1988; Best et al., 2020), supporting the assumption of $\sigma > \gamma$.[34] In Section 2.4, we introduce a multisector model of

[34] The international macroeconomics literature uses a much lower macro-trade elasticity to rationalize International Real Business Cycle (IRBC) facts (D. K. Backus, P. J. Kehoe, and Kydland, 1994). Robert C.

high substitution within sector but lower substitution across sectors, and confirm that high within-sector substitutability drives our results.

Multisector considerations. We used a one-sector model to highlight the main mechanism. In the Online Supplement, we introduce a two-sector model, separating tradables from nontradables in segmented labor markets. The expanded model predicts similar effects of Foreign growth under a peg: short-term trade deficits and tradable sector unemployment.

The extended model also highlights distributional effects. First, the output share of tradable declines even absent labor reallocation. Second, if monetary policy is unresponsive, we have unemployment in both sectors: the recession *spills over* to the nontradable sector through aggregate demand. Third, monetary policy faces a trade-off between a recession in the tradable sector and overheating in the nontradable sector, explaining the US service sector boom in the 2000s. Further analysis is given in the Online Supplement, and the subsequent sections provide a quantification of the China shock through a general equilibrium multisector model.

2.4 Quantitative model

In this section, we extend the model in Section 2.3 so that it can be taken to sector-level trade data for a general equilibrium analysis of the effects of Chinese growth and the peg. We generalize the previous setup in two directions: (1) a multi-sector, multi-country model with Ricardian forces, input-output linkages and labor reallocation (Caliendo, Dvorkin, and Parro, 2019); (2) an infinite-period model with wage rigidity (C. J. Erceg, Henderson, and Levin, 2000), consumption-savings pinning down trade balances (Obstfeld and K. Rogoff, 1995) and exchange rate determination from financial channels (Itskhoki and Mukhin, 2021a). The first block allows us to investigate how the China shock, a sector-specific shock, affects other sectors, while the second block allows us to consider involuntary unemployment, endogenous trade imbalances, and the role of exchange rate pegs.

2.4.1 Model Setup and Equilibrium

In the model, time is discrete and indexed by $t = 0, 1, \cdots$. The economy consists of $i, j = 1, 2, \cdots, I$ countries, each with an exogenous labor endowment given by a continuum of workers with mass $\bar{L}_i$ (thus, we rule out migration across countries). There are

Feenstra et al. (2018) estimate the macro- and micro-elasticities, and find that the macro-elasticity is "not as low as the value of unity sometimes found using macro time series methods," further supporting our notion that the trade elasticity is at least unity.

$n, s = 1, 2, \cdots, S$ sectors. Unless otherwise stated, i is the producer/exporter, j is the importer/buyer, and we write exporters first in subscripts. Country 1 is the USA; country 2 is China; we are mainly interested in the interaction between these two countries. Each country has its nominal account, and nominal variables are denominated in the currency of the price-facing household. The exchange rate e_{jit} is the value of currency j with respect to currency i, so an increase in e_{jit} is a relative depreciation of i currency with respect to j currency. We present the main assumptions and relegate the derivations and details to Appendix B.3.

Household preferences. In each country j, there is a representative household family that comprises atomistic *members* m of measure $\bar{L}_j$ and has preferences represented by

$$\mathcal{U}_j = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \delta_{jt} \int_0^{\bar{L}_j} \mathcal{U}_{jt}(m) dm, \tag{2.26}$$

where $\mathcal{U}_{jt}(m)$ is the member-specific utility, β is a discount factor common across all countries, and δ_{jt} is a country-specific intertemporal preference shifter which captures financial factors exogenous to our model. We implement our model at an annual frequency, so each period t corresponds to a year.

The utility of each member m depends on final goods consumption $C_{jt}(m)$, labor supply $\ell_{jt}(m)$, current sector $s_{jt}(m)$, future sector of choice $s_{jt+1}(m)$, and an idiosyncratic preference shifter $\epsilon_{jt}(m) = \{\epsilon^s_{jt}(m)\}_s$ across different future sectors. The preferences of member m is represented by

$$\mathcal{U}_{jt}(m) = u(C_{jt}(m)) + v(\ell_{jt}(m), s_{jt}(m), s_{jt+1}(m), \epsilon_{it}), \tag{2.27}$$

$$\text{where } u(C) = \frac{C^{1-\gamma^{-1}} - 1}{1-\gamma^{-1}}, \text{ and } v(\ell, s, n, \epsilon_t) = -\theta^s_i \frac{1}{1+\varphi^{-1}} \ell_{it}^{1+\varphi^{-1}} + \eta^s_{it} - \chi^{sn}_{it} - \epsilon^n_{it}, \tag{2.28}$$

where γ is the elasticity of intertemporal substitution, φ is the Frisch elasticity of labor supply, and θ^s_i is the intensity of labor disutility in each sector s. η^s_{it} captures the non-pecuniary sector-specific benefits, and χ^{sn}_{it} captures the relocation costs of moving from sector s to sector n, measured in terms of utility. This formulation follows Artuç, Chaudhuri, and McLaren (2010) with an additional endogenous labor supply term $\ell_{it}^{1+\frac{1}{\varphi}}$.[35]

We have perfect risk sharing across members of the family, so $C_{jt}(m) = C_{jt}$. Final goods C_{jt} is a Cobb-Douglas aggregate of consumption across each of the sectors $s = 1, 2, \cdots, S$ with shares α^s_{jt}. Consumption within each sector follows the Armington trade model, where

[35]This can implicitly be interpreted as an intensive margin of labor supply; in Appendix B.3, we microfound this as with an *extensive* margin interpretation, more suitable to study unemployment.

consumption is a CES aggregate of goods from each of the I countries with an elasticity of substitution $\sigma_s > 1$ within each sector s. Consumption is given by

$$C_{jt} = \prod_s \left(\frac{C^s_{jt}}{\alpha^s_{jt}}\right)^{\alpha^s_{jt}}, \quad C^s_{jt} = \left[\sum_i (C^s_{ijt})^{\frac{\sigma_s - 1}{\sigma_s}}\right]^{\frac{\sigma_s}{\sigma_s - 1}}$$

Savings. Analogously to Section 2.3, each country issues a nominal bond of price $\frac{1}{1+i_{it}}$. There is no aggregate risk, and bonds are perfect substitutes across origins.

Firms and technology. Goods are distinguished by sector and origin. Sector s goods from country i are produced by competitive firms using Cobb-Douglas technology, with labor share ϕ^s_i and sector n input shares ϕ^{ns}_i satisfying $\phi^s_i + \sum_n \phi^{ns}_i = 1$. The total factor productivity of country i, sector s at time t is A^s_{it}, and exports from i to j face an iceberg cost τ^s_{ijt} with $\tau^s_{iit} = 1$ by normalization. Inputs from sector n across different goods are aggregated CES with elasticity σ_s, in the same way as consumption goods in sector n. Thus the production function F^s_{ijt} of a representative firm in country i, sector s at time t to destination j is

$$F^s_{ijt}(l^s_{ijt}, \{X^{ns}_{ijt}\}_n) = \frac{A^s_{it}}{\tau^s_{ijt}} \left(\frac{l^s_{ijt}}{\phi^s_i}\right)^{\phi^s_i} \prod_n \left(\frac{X^{ns}_{ijt}}{\phi^{ns}_i}\right)^{\phi^{ns}_i} \tag{2.29}$$

Unions and Wage Rigidity. We assume wage rigidity in each sector s through wage-setting unions facing nominal friction. A continuum of unions in sector s organizes the measure L^s_{it} of workers in sector s and employs them for an equal number of hours ℓ^s_{it}. Each union faces a labor demand curve and sets nominal wages W^s_{it} in each period to maximize the welfare of the sector s members with discount rate β.[36] We assume wage rigidity in the form of a Rotemberg friction $\Phi(W^s_t, W^s_{t-1})$ and choose the union objective function so that the union's optimization problem leads to the wage Phillips curve,

$$\log(\pi^{sw}_{it} + 1) = \kappa_w(v'(\ell^s_{it}) - \frac{W^s_{it}}{P_{it}} u'(C_{it})) + \beta \log(\pi^{sw}_{it+1} + 1) \tag{2.30}$$

where $\pi^{sw}_{it} = \frac{W^s_{it}}{W^s_{it-1}} - 1$ denotes wage inflation at time t.[37]

[36] Here, we are implicitly assuming that the intertemporal preference shifters δ_{jt} are pure consumption shocks that affect consumption but not labor supply. We make this assumption for clarity of exposition, as the shifters are intended to match the realized trade imbalances and model financial shocks outside of the scope of our model.

[37] To a first order, the equation is identical to assuming Calvo rigidity, where the probability of keeping

Migration across sectors. We assume that each member m is forward-looking and faces a dynamic problem with discount factor β, labor reallocation costs χ_i^{sn} to move from sector s to n; these reallocation costs are time-invariant, additive, and measured in utility units. Each member m receives an idiosyncratic shock for each choice of sector, denoted by $\epsilon_{it} = \{\epsilon_{it}^n\}_n$. Since the per-worker labor supply ℓ_{it}^s is determined by the union, the member takes it as given. If we denote by $\mathcal{V}_{it}^s(\epsilon_{it})$ the lifetime utility of the worker in sector s with preference shock ϵ_{it}, then we have the worker's Bellman equation,

$$\mathcal{V}_{it}^s(\epsilon_{it}) = \tilde{\lambda}_{it} W_{it}^s \ell_{it}^s - h(\ell_{it}^s) + \eta_{it}^s + \max_n [\beta \mathbb{E}[\mathcal{V}_{it+1}^n(\epsilon_{it+1})] + \epsilon_{it}^n - \chi_{it}^{sn}], \tag{2.31}$$

where $\tilde{\lambda}_{it} = \frac{u'(C_{it})}{P_{it}}$ is the Lagrange multiplier on the country i household family's period t budget constraint. Here $\tilde{\lambda}_{it} W_{it}^s$ is the marginal utility of labor by a worker in sector s. Workers internalize how their choice of sector affects the family budget. The solution to the Bellman equation above yields a transition matrix μ_{it}^{sn} and expected utility $V_{it}^s = \mathbb{E}[\mathcal{V}_{it}^s(\epsilon_{it})]$ given by

$$\mu_{it}^{sn} = \frac{\exp(\frac{1}{\nu}(\beta V_{it+1}^n - \chi_{it}^{sn}))}{\sum_{n'} \exp(\frac{1}{\nu}(\beta V_{it+1}^{n'} - \chi_{it}^{sn'}))}, \tag{2.32}$$

$$V_{it}^s = \tilde{\lambda}_{it} W_{it}^s \ell_{it}^s + \eta_{it}^s - v(\ell_{it}^s) + \nu \log \left(\sum_n \exp(\frac{1}{\nu}(\beta V_{it+1}^n - \chi_{it}^{sn})) \right). \tag{2.33}$$

Monetary policy. The monetary authority in each country i sets a nominal interest rate i_{it}. We assume that country 1 (USA) sets a Taylor rule on inflation

$$\log(1 + i_{1t}) = r_{1t} + \phi_\pi \log(1 + \pi_{1t}) + \epsilon_{1t}^{MP}, \tag{2.34}$$

where r_{1t} is the real interest rate, $\pi_{1t} = \frac{P_{it+1}}{P_{it}}$ is the CPI inflation, and interpret ϵ_{1t}^{MP} as any discretionary monetary policy the central bank of Country 1 may pursue.

The monetary policy of country 2 (China) may be a *peg* or a *float*. Under a peg, we assume that country 2 pegs the exchange rate to country 1, so i_{2t} is implicitly pinned down by $e_{12t} = \bar{e}$.[38] Under a float, country 2 pursues an independent Taylor rule of the form

$$\log(1 + i_{2t}) = r_{2t} + \phi_\pi \log(1 + \pi_{2t}) + \epsilon_{2t}^{MP}. \tag{2.35}$$

the wage fixed is θ_w, with $\kappa_w = \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}$.

[38]Because bonds are perfect substitutes, we rule out pegging in the form of foreign exchange intervention. In fact, in a model with UIP deviations, the first-order linear consumption responses are identical whether China pegs the currency through moving interest rates, or fixing the interest rate and buying bonds (and financing this through lump-sum taxes), because the current account of the country (fiscal authority plus household) is identical in both cases. We formally explore this in a work in progress.

We assume that the rest of world ($i \geq 3$) floats its currency with respect to the US dollar, and assume that monetary policy in each of the countries is given by its own Taylor rule (Equation 2.34) responding to its CPI inflation.[39]

Exchange rate determination. Denote by $e_{it} = e_{i1t}$ the value of currency i with respect to the US dollar. We have $e_{ijt} = \frac{e_{it}}{e_{jt}}$. If country i pegs its currency, it sets e_{it} to an exogenous number $\bar{e}_i$. When country i floats its currency, the UIP condition pins down $\frac{e_{it+1}}{e_{it}}$. We assume that, if country i floats its currency, e_{i0} is the unique value such that

$$\lim_{t \to \infty} B_{it} = 0. \tag{2.36}$$

Equation 2.36 operationalizes the idea that there are financial forces that move exchange rates to clear long-run balance of payments, and can be microfounded as a limit case of financial frictions pinning down the exchange rate.[40]

Tariffs and fiscal policy. Each country j can choose a set of ad valorem import tariff rates $\{t^s_{ijt}\}$ on goods from country i to country j; the tariff revenues are rebated to households lump-sum, and the government balances its budget every period. Thus if we denote the pre-tariff price of sector s goods from i to j at time t by P^s_{ijt}, government j's revenue is

$$T_{jt} = \sum_{i,s} t^s_{ijt} P^s_{ijt} (C^s_{ijt} + X^s_{ijt}) \tag{2.37}$$

where C^s_{ijt} is consumption of (i, s) goods in country j, and X^s_{ijt} is total input use of (i, s) goods in country j. To focus on tariffs, we assume away export subsidies.

Equilibrium. We are now ready to define the equilibrium in the quantitative model.

Definition 2. *Given parameters $\{A^s_{it}, \tau^s_{ijt}, \delta^s_i, \chi^s_{it}, \eta^s_i\}$, previous period nominal wage $\{W^s_{i-1}\}$, initial bond holdings $\{B_{i0}\}$, labor allocation $\{L^s_{i0}\}$, and policy rules $\{i_{it}\}, \{t^s_{ijt}\}$, an equilibrium in this model consists of consumption $\{C_{jt}, C^s_{ijt}\}$, bond holdings $\{B^s_{it}\}$, labor supply $\{\ell^s_{it}\}$, labor allocation $\{L^s_{it}\}$, prices $\{P_{jt}, P^s_{jt}, P^s_{ijt}\}$, wage $\{W^s_{it}\}$ and exchange rates $\{e_{ijt}\}$ that satisfy the following:*

1. *Consumption and bond holdings solve the family optimization problem,*

[39] Alternatively we may consider a middle ground, corresponding to a Taylor rule with an exchange rate target.

[40] This idea dates back to Meade (1951) and Friedman (1953). Equation 2.36 is a special case of the exchange rate determination literature with financial frictions (Kouri, 1976; Itskhoki and Mukhin, 2021a) where we take the limit of the magnitude of the friction to zero. We microfound this in Appendix B.3.

2. *Prices, labor, and input demand solve firm profit maximization,*

3. *Labor supply and wages satisfy the Phillips curve,*

4. *Labor reallocation and lifetime value solves the sector choice problem,*

5. *Monetary policy in the US is given by a Taylor rule,*

6. *Monetary policy in other countries and exchange rates satisfy (a peg) or (zero long-run balances).*

7. *Goods market, bond market clears, and the government balances its budget.*

The formal equations and derivations are in Appendix B.3.1.

2.4.2 Data and Calibration

We provide an overview of our data and calibration process and relegate the details to the Online Supplement. Our quantitative model has six country aggregates: US, China, Europe (including UK), Asia, the Americas, and the rest of world. We consider 6 sectors: agriculture, low-, mid- and high-tech manufacturing, and low- and high-tech services, classified according to the North American Industry Classification System (NAICS).[41] The time of our data spans from $t = T_0 = 2000$ to $t = T_{data} = 2012$ annually.

Trade and production data. The primary dataset we use is the World Input-Output Database (WIOD) 2016 edition (Timmer et al., 2015). The WIOD compiles data from national accounts and bilateral trade data for 56 sectors and 44 countries. It has information on the value of trade flows X^s_{ijt} from country i to country j in sector s at year t for 56 sectors across 44 countries. It also contains data on purchases of inputs across sectors, value added of each sector in each country (which corresponds to the labor share in our model), consumption shares across sectors, and the net exports for each country. We obtain the price indices for each sector from the WIOD's Socioeconomic Accounts (WIOD SEA).

Labor and Sectoral Adjustments. We obtain the initial distribution of workers in the year 2000 by sectors using the WIOD SEA. We use data from the Current Population Survey (CPS) in the United States to construct the matrix of migration flows μ^{sn}_{it} across sectors in the US. We assume away migration flows between countries. For countries outside of the

[41] This follows Dix-Carneiro, Pessoa, et al. (2023).

Panel A. Fixed according to literature			
Parameter	Value	Description	Source
β	0.95	Discount factor	5% interest rate
ν	2.02	ϵ_{it}^{n} dispersion	Caliendo, Dvorkin, and Parro (2019)
γ	1	Intertemporal Elasticity	Standard
φ	2	Frisch elasticity	Peterman (2016)
σ_s	5	Elasticity of substitution	Head and Mayer (2014)
κ	0.05	NKPC slope	Hazell et al. (2022)
ϕ_π	1.5	Taylor rule coefficient	Taylor (1993)
Panel B. Parameters we calibrate			
Parameter		Description	Target moments
α_{it}^{s}		Expenditure shares	WIOD consumption share
ϕ_{it}^{s}		Labor share	WIOD value added
ϕ_{it}^{sn}		Input-output matrix	WIOD input-output
θ_{i}^{s}		Intensity of labor disutility	$\ell_{i,2000}^{s} = 1$
η_{i}^{s}		Non-pecuniary utility	WIOD SEA labor distribution
χ_{it}^{sn}		Migration cost	CPS sector change
τ_{ijt}^{s}		Trade cost	WIOD trade flow
A_{it}^{s}		Productivity	WIOD trade flow and SEA price index
δ_{it}		Intertemporal preference shifter	WIOD net exports
r_{it}		US real interest rate	Full employment without China shock

Table 2.1: Calibrated parameters

US and China, we assume that workers are immobile and fixed in that sector; for China, we assume that the cost of moving is fixed at the 2000 level.

Calibration. Table 2.1 provides a summary of the parameters, including the sources of parameters whose values we take from the literature or the moments that we target for the parameters we directly calibrate.

Values for parameters in Panel A of Table 2.1 are taken from the literature, as they are difficult to identify given available data, or our estimation strategy would be analogous to the literature. The time frequency is annual, and we use $\beta = 0.95$ to match the 5% annual interest rate. Estimating the dispersion ν of sectoral preference shocks ϵ_{it}^{n} requires panel data and instrumental variables; we impose this to be common across all countries and set them to be $\nu = 2.02$, following Caliendo, Dvorkin, and Parro (2019). For the elasticity of intertemporal substitution, we follow standard practice in the macro and trade literature and set $\gamma = 1$, assuming log utility. The Frisch elasticity of labor supply is set to $\varphi = 2$, closer to macro estimates (Peterman, 2016). Measuring the elasticity of substitution of goods across origin often requires panel data on variation, so we set it to 5, which is standard in the literature (Head and Mayer, 2014; Andrès Rodríguez-Clare, Ulate, and Vásquez, 2022; Dix-Carneiro, Pessoa, et al., 2023). We set the New Keynesian Phillips Curve slope to $\kappa = 0.05$ to match Hazell et al. (2022) which exploit variation across US states to obtain the response of inflation to the labor wedge.[42] The Taylor rule coefficient is set to 1.5, following the original paper by Taylor, as standard in the macro literature.

In Panel B of Table 2.1, we can directly compute the sectoral consumption expenditure share α_{it}^s, labor share ϕ_{it}^s, and input-output share ϕ_{it}^{sn} directly from the WIOD data. For the rest of the parameters, we rely on parts or all of the model to match the model-generated moments with the data. We divide our calibration into two steps: calibrating the initial period, and then calibrating how those parameters change in our model. We set the non-pecuniary utilities η_i^s such that the model-implied initial labor distribution $L_{i,2000}^s$ matches the realized labor distribution observed in the WIOD SEA, and the migration cost $\chi_{i,2000}^{sn}$ so that it matches the observed sector change flows in the CPS of the US; we assume that China faces the same sectoral migration costs, and countries besides US and China have an immobile labor market. We normalize θ_i^s so that the initial per-worker labor supply in our model is $\ell_i^s = 1$. Turning to the trade side, we calibrate the trade costs τ_{ij0}^s and A_{i0}^s to match the trade flow in the initial period exactly up to normalization, following the exact hat algebra approach of Dekle, Eaton, and Kortum (2007) and Caliendo, Dvorkin, and Parro (2019).

Next, we discuss the calibration of the *shocks* we extract. We extract three main sets of shocks from the WIOD data: changes in trade costs $\hat{\tau}_{ijt}^s = \frac{\tau_{ijt}^s}{\tau_{ij0}^s}$, changes in productivity $\hat{A}_{it}^s = \frac{A_{it}^s}{A_{i0}^s}$, and intertemporal preference shocks δ_{it}.[43] We calibrate these shocks to exactly match three realized 'shocks' in the WIOD data: changes in sectoral output price indices $\hat{P}_{it}^{s,dom} = \frac{P_{it}^{s,dom}}{P_{i0}^{s,dom}}$, changes in trade shares $\hat{\lambda}_{ijt}^s = \frac{\lambda_{ijt}^s}{\lambda_{ijt}^0}$, and net exports in each period as a share of GDP $NXGDP_{it} = \frac{NX_{it}}{GDP_{it}}$. We calibrate the trade cost shocks $\hat{\tau}_{ijt}^s$ to exactly match the gravity structure of trade flows up to normalization; we assume $\hat{\tau}_{iit}^s = 1$. On the other hand, since prices are a function of wage and productivity, and the dynamics of wage (and its rigidity) are central to our channel, we cannot back out the productivity without solving for the full model. Thus, we employ a Simulated Method of Moments (SMM) approach, targeting the changes in output price and net exports as moments we exactly match. We

[42] Since their model is quarterly and the Phillips curve links price inflation with unemployment, we undergo a series of transformations to make our estimate consistent with their estimate of $\kappa' = 0.0062$. Details are given in the Online Supplement.

[43] We also assume that the preference and technology parameters $(\alpha_{it}^s, \phi_{it}^s, \phi_{it}^{sn})$ are time-varying, but we directly observe this as shares from the data.

also calibrate the sector change costs χ_{it}^{sn} in the US so that the model-implied migration μ_{it}^{sn} exactly match the sector reallocation data in the CPS. The details of this calibration procedure can be found in the Online Supplement.

2.4.3 Solution algorithm

We aim to study the employment, trade balance, and welfare effects of China's peg against the US dollar and revisit the effects of the China shock under this framework. We bring frontier computational methods from macroeconomics (Auclert, Bardóczy, et al., 2021) and apply them to answer trade questions. We sketch our solution algorithm here and provide the details and discussions in the Online Supplement.

Given the elasticities and parameters calibrated in Subsection 2.4.2 (Table 2.1), we directly solve for the equilibrium in the *sequence-space* of equilibrium objects

$$\{X_t\}_{t=T_0}^{T} = \{(B_{it}, P_{it}, C_{it}, e_{it}, W_{it}^s, \ell_{it}^s, L_{it}^s, V_{it}^s)\}_{t=T_0}^{T}$$

for $T \gg T_{data}$ such that the economy returns to a new steady-state by $t = T$. This requires solving a high-dimensional nonlinear equation.[44] The key idea is that the nonlinear system of equations that define $\{X_t\}$ is extremely sparse: each period t equilibrium condition only depends on variables of time $t, t-1, t+1$, and even those equations depend on a few parameters within each t. Then, the Jacobian of the equilibrium conditions can be efficiently constructed, and we employ nonlinear root-finding algorithms to solve for the full sequence of wages, consumptions, trade imbalances, and labor allocations. By leveraging the sequence-space Jacobian approach from Auclert, Bardóczy, et al. (2021) and combining it with computational advances in machine learning, we can solve for the full nonlinear solution of our model in seconds to minutes depending on specification, allowing us to compute a wider dimension of counterfactual scenarios and explore policy implications.[45]

2.5 Effects of the China shock and the role of the peg

In this section, we use the model described in Section 2.4.1 and calibrated parameters from Section 2.4.2 to study the effect of the China shock and the China peg. In Section 2.5.1, we

[44] With $I = S = 6$ and $T = 100$, the system of equations have over 20000 variables.

[45] The methods we use include parallelization, autodiff, just-in-time compiling, and Intel's PARADISO package for quickly solving large sparse systems, many of which are heavily used in machine learning contexts where the parameter space is even larger. The toolkits are available in the Python-based framework "JAX," which we use extensively. Details can be found in the Online Supplement.

first define the “China shock", using the change in productivities, trade costs, and preference parameters observed over this period.

In Section 2.5.2, we revisit the effect of the China shock on the US labor market and trade deficit. We show how modeling wage rigidity, consumption-savings, and exchange rate peg affects the predictions on the effect of the China shock, compared to estimates in the literature that ignore these channels. In Section 2.5.3, we quantify how the exchange rate peg *magnified* the effects of the China shock on the United States by comparing the realized economy with a counterfactual economy with otherwise identical evolution of parameters, but under a floating exchange rate.

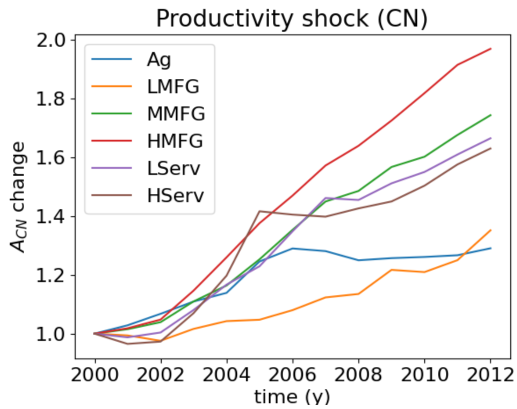
2.5.1 The China shock

One goal of our quantitative model is to estimate the effect of the China shock under an exchange rate peg and nominal rigidity. In this subsection, we define what the China shock is in the context of our model.

In Section 2.4.2, we extract the realized evolution of parameters across time. This is the baseline, *realized* economy with the China shock. We consider two notions of the China shock. The main shock, which we call the *China trade shock* only considers the changes in China that are directly associated with increasing import penetration of Chinese goods: the productivity A_{it}^s and the trade costs τ_{ijt}^s. Thus the counterfactual economy without the *China trade shock* is the equilibrium where the calibrated parameters (Table 2.1) are identical to the realized equilibrium, with the exception of productivity A_{it}^s and the trade costs τ_{ijt}^s in China; for China, we fix the productivity A_{CN}^s and trade costs $\tau_{i\text{CN}t}^s, \tau_{\text{CN}it}^s$ to be fixed at their levels in $t = T_0$.[46]

Figure 2-4 plots the computed China shock on the productivities A_{CN}^s and the trade cost from China to US $\tau_{CN,US,t}^s$ as a ratio between the levels at time t versus the level at the initial period $t = T_0 = 2000$ for the six sectors. China's productivity increases in all sectors, but especially in the medium-tech and high-tech manufacturing sectors. China's trade costs also decrease for all sectors; while the decline seems to be most pronounced for the service sectors, this is driven by the fact that the service sectors are close to nontradable – the implied trade costs τ_{ijt}^s in 2000 are close to 70-80 that get reduced to 30 by 2012, but is still very high. Much of the effect on the US economy is driven by the shocks in the manufacturing sectors.

[46]In the Online Supplement, we discuss alternative notions of the *no China shock* counterfactual, such as (1) where China's global import penetration does not increase throughout the period (Caliendo, Dvorkin, and Parro, 2019; Andrès Rodríguez-Clare, Ulate, and Vásquez, 2022), or (2) Chinese productivity grows on par with the global average during this period (Dix-Carneiro, Pessoa, et al., 2023). We find qualitatively similar results.



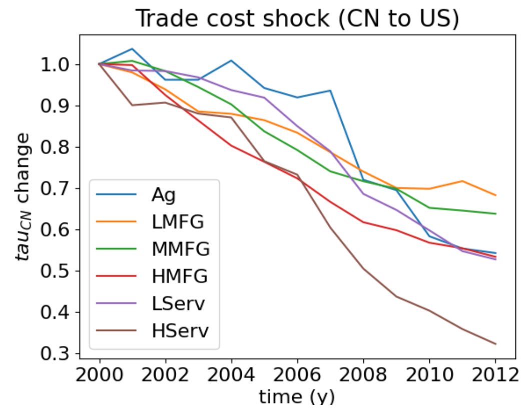


Figure 2-4: Calibrated values of the China trade shock.

We also consider another set of shocks, which includes the intertemporal preference shock $\delta_{\mathrm{CN}t}$. While the changes in productivity A and trade cost τ capture the surge in Chinese exports, this is not the only structural change in China during this period. Rich financial dynamics outside the scope of our model will affect realized trade imbalances and consumption-saving patterns. Those 'residuals' constitute the savings glut of China and are interpreted as part of the China shock in Dix-Carneiro, Pessoa, et al. (2023). We call this shock the *China trade and savings shock*. Then, the counterfactual economy without the China trade and savings shock is the equilibrium with identical parameters as the realized equilibrium, with the exception of $A^s_{\mathrm{CN}}, \tau^s_{i\mathrm{CN}t}, \delta_{\mathrm{CN}t}$; we fix those values to be the values at $t = T_0$ in China.[47]

Comparing the realized economy with the economy without the *China trade shock* allows us to evaluate the effect of Chinese growth on US outcomes, such as the distribution of labor, trade balances, or unemployment. Comparing the realized economy to the economy without the *China trade and savings shock* gives us the effect of China's structural change, including the savings glut, on the same US outcomes. By looking at the difference between these two outcomes, we can evaluate the extent to which the realized US trade deficit and decline in manufacturing (Figure 2-1) can be causally attributed to Chinese growth.

For all our counterfactual scenarios, we assume in our baseline analysis that agents have no foresight of the shocks during this period for both the realized and counterfactual equilibrium, operationalizing the notion that "every year is a China shock" during the period of spectacular productivity growth in China. We discuss the details of our implementation, the rationalization for agents' foresight, and robustness exercises where we alternatively assume perfect foresight in the Online Supplement.

[47]During this period, consumption shares α^s_{it} and input-output linkages, labor shares $\phi^s_{it}, \phi^{sn}_{it}$ vary over time. We match the varying shares in both the realized and counterfactual equilibrium.

2.5.2 Reevaluating the China shock

We start by revisiting the quantitative effects of the surge in China's imports – the *China shock* – on the US economy using our calibrated model. We are interested in asking the following question: what are the dynamic effects of the China shock on labor reallocation, unemployment, the trade balance of the US, and welfare consequences through the lens of our model? We revisit the effects of the China shock under wage rigidity and endogenous consumption-savings and compare how those ingredients lead to different implications of the China shock than three previous literature: Caliendo, Dvorkin, and Parro (2019), which feature exogenous deficits and no involuntary unemployment, Andrès Rodríguez-Clare, Ulate, and Vásquez (2022) which feature nominal rigidity but exogenous deficits, and Dix-Carneiro, Pessoa, et al. (2023) which feature endogenous deficits but quantity rigidity instead.

To quantify our answer to this question, we first solve for the baseline economy with the actual evolution of fundamentals over 2000-2012. Then we solve the economy under both the *no China trade shock* counterfactual and the *no China trade and savings shock* counterfactual and treat the difference in outcomes such as the trade imbalance, labor market, and welfare outcomes between the realized and counterfactual outcomes as the effect of the shock.

Figure 2-5 shows the import penetration ratio of China to the US, the manufacturing share of US employment, the net exports of the US (as a percentage of contemporaneous GDP), and aggregate unemployment in the economy for the (1) realized economy, (2) the counterfactual economy without the China trade shock, and (3) the counterfactual economy without the China trade and savings shock. The first three figures replicate the four stylized facts we highlight in the introduction (Figure 2-1). Figure 2-5a clarifies that the growth in import penetration from China in this period is driven by productivity growth and trade liberalization of China. In fact, if China had not grown in this period, import penetration from China would have decreased, as other Asian countries growing in this period (most notably other parts of Asia) would have assumed the role of China.

Next, we study the decline in US manufacturing. Figure 2-5b investigates the impact of the China shock on the manufacturing share of employment. As we see, a sizable share of the exit of workers from manufacturing can be attributed to the China shock in our framework. In numbers, 991 thousand jobs lost in manufacturing could be attributed to the China trade shock. Most notably, the decline in manufacturing is almost identical in the *no China trade shock* case and the *no China trade and savings shock* case, suggesting that the residual savings glut of China plays a negligible role in the decline of US manufacturing. This goes further than the findings of T. J. Kehoe, Ruhl, and Steinberg (2018), which show that the savings glut is responsible for 15.1% of the decline in US manufacturing. Our framework in Section 2.3 substantiates this viewpoint: Proposition 8 shows that US borrowing should mitigate the decline in manufacturing, as consuming more in the short-run would help a declining demand for Home goods.

Turning to trade deficits, Figure 2-5c shows that a significant proportion of realized US trade deficits can be explained by the China trade shock. In fact, taking the average from 2000 to 2012, 2.25 percentage points of the US annual deficit (% GDP) can be explained solely by the China trade shock, and if China had not grown, the US may have had balanced trade by 2012. The realized average annual trade deficit of the US during the same period was 3.4% of GDP, suggesting that two thirds of the US trade deficit over this period could be explained by the China shock. The residual savings glut δ_{it} plays little role in affecting the balances, suggesting that the theoretical channel we highlighted in Proposition 4 – permanent Foreign growth leading to Home deficits – is responsible for a majority of the US trade deficit of the 2000s.

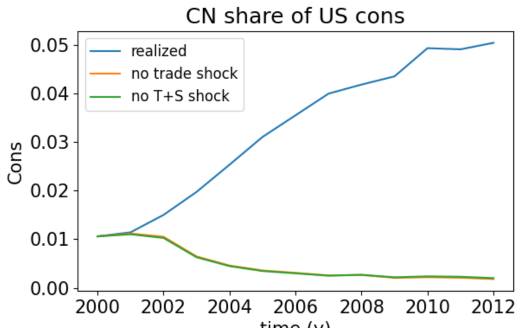
Next, we use our general equilibrium model to obtain the implied effects of the China shock on unemployment. Figure 2-5d plots the aggregate US unemployment response to the China shock according to our model. Unemployment increases through the span of the shock, and on average, the excess unemployment generated from the China shock from 2000 to 2012 is 3.04%; this unemployment is necessarily short-lived, and it reaches zero after the culmination of the China shock, as nominal wages adjust to the new equilibrium level.[48]

Finally, we measure the welfare implications of the China shock. The household family's utility comprises both consumption utility and the disutility of labor. In evaluating the welfare effects, we consider the aggregate discounted utility incorporating the full path of consumption and the disutility of labor. Thus we define the *welfare effect* of the shock as the lifetime compensating variation in consumption for the US; formally, the welfare effect of the China shock is the scalar ζ such that

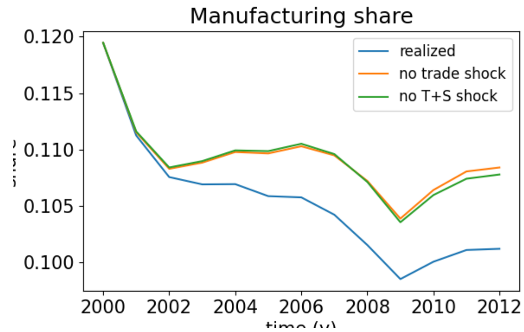
$$\mathcal{U}_0(\{C_{CS}\}_t, \{\ell_{CS}\}_{s,t})) = \mathcal{U}_0(\{(1+\zeta)C_{\text{noCS}}\}_t, \{\ell_{\text{noCS}}\}_{s,t}), \tag{2.38}$$

or how much more lifetime consumption (in percentages) the household needs to be indifferent between the China shock case and the no China shock case. According to this metric, the China shock contributed to a 0.183% gain in lifetime welfare, a modest but significant gain, and the distortion margins we highlighted in Proposition 5 – unemployment and future terms-of-trade deterioration – did not flip the aggregate welfare implications of the China shock.

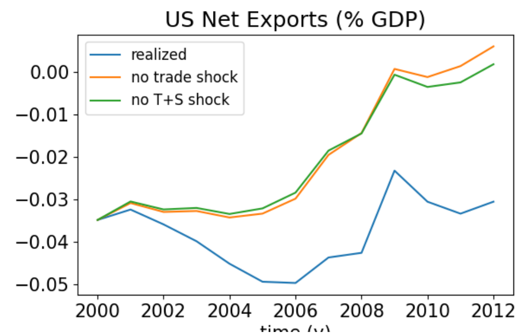
[48] The unemployment level is high because the shock to manufacturing can spill over to the service sector through aggregate demand (highlighted in the two-sector model in the Online Supplement), and targeting CPI inflation is not an optimal monetary policy in this setup. We consider this result as a benchmark and consider alternative monetary policy rules in the Online Supplement, and show that the decline in manufacturing share and trade deficits are robust.



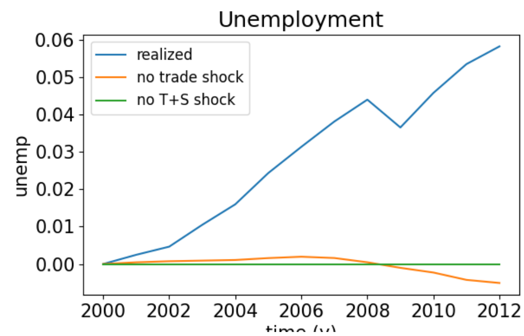
(a) CN import penetration



(b) Manufacturing share



(c) Net exports (% GDP)



(d) Unemployment rate

Figure 2-5: Response of the economy to the China shock.

Note. The 'realized' graphs are the equilibrium outcome from the full sequence of parameters that were targeted to match realized moments. The 'no trade shock' graphs are the equilibrium outcome from the sequence of parameters identical to the realized, except we remove the productivity growth and trade cost reduction in China. The 'no T+S shock' graphs are the equilibrium outcome from the same sequence, except we remove the residual 'savings shocks' in China. The similarities between the no trade shock and the no T+S shock suggest that the residual savings glut of China played close to zero role in the manufacturing decline or the trade deficits after we account for the effect of the exchange rate peg.

Table 2.2 compares the estimated effects of the China shock from our framework to three references in the literature. The first is Caliendo, Dvorkin, and Parro (2019) (CDP19), which features no intra-sector labor market friction and models imbalances through systems of transfers. The second is Andrès Rodríguez-Clare, Ulate, and Vásquez (2022), which features downward nominal wage rigidity but exogenous imbalances. The third is Dix-Carneiro, Pessoa, et al. (2023), which models labor market friction through quantity friction (search and matching). Our model estimates close to double the number of manufacturing jobs lost through the China shock than the estimates of the previous literature, a much larger proportion of the realized US trade deficit than what Dix-Carneiro, Pessoa, et al. (2023) attribute to the China shock and more moderate welfare gains from the China shock. Our estimate of the number of manufacturing jobs lost is close to the estimates of Autor, Dorn, and Hanson (2013) – 982,000 jobs lost as a result of the China shock after 2000 –

Effect of China shock				
	Our model	CDP19	RUV22	DPRT23
MFG jobs lost	991k	550k	498k	530k
Deficit (% GDP)	2.25	N/A	N/A	0.8
Unemployment (%)	3.04	N/A	1.4	0
Welfare gains	0.183%	0.2%	0.229%	0.183%*
Wage rigidity	O	X	O	X
Search friction	X	X	X	O
Cons-savings	O	X	X	O
ER peg	O	X	X	X

Table 2.2: Effects of the China shock: comparison to existing literature.

Note. *: Dix-Carneiro, Pessoa, et al. (2023) measure welfare using consumption only, without considering the labor market effects of welfare. We take into account the disutility of labor in measuring aggregate welfare.

suggesting that the *missing intercept* may not be as large as previously thought. Interestingly, despite the manufacturing jobs lost that are about twice as large and a significant level of unemployment, the welfare consequences of the China shock are still positive and close to the literature's estimates.

In the following subsection, we show that the difference between our estimates and the literature's estimates can be almost entirely attributed to China's exchange rate peg.

2.5.3 The effect of the exchange rate peg

The second and most novel part of our quantitative analysis focuses on how much the peg interacted with the China shock to generate the realized effects of the China shock we saw in Section 2.5.2. If the empirical findings in Section 2.2 and the propositions in Section 2.3 hold, we should expect that the exchange rate peg is responsible for a sizable part of the trade deficit, the decline in manufacturing, and may affect the welfare implications of the China shock.

To quantify this, we compare the outcomes of the baseline economy to a counterfactual economy with identical fundamentals, except for one change: China's monetary policy no longer pegs to the US dollar. China's alternative monetary policy could be many things – a full-discretion policy, an interest rate with an exchange rate target – but to highlight the effect of the peg, we consider the simplest counterfactual by assuming that China's monetary policy is symmetric to the US, an independent Taylor rule with the same coefficient on China's domestic CPI inflation. The difference in the outcomes of the economy with the peg and the economy without the peg, both with the China shock, is the causal effect of China's exchange rate peg on the US.

Figure 2-6 shows the same aggregate variables in the US – import penetration ratio of Chinese goods, manufacturing share of employment, net exports of US, and unemployment in the economy for the (1) realized economy, (2) the counterfactual economy without the China trade shock, and (3) the counterfactual economy with the same shocks as the realized economy, but China had a floating exchange rate.

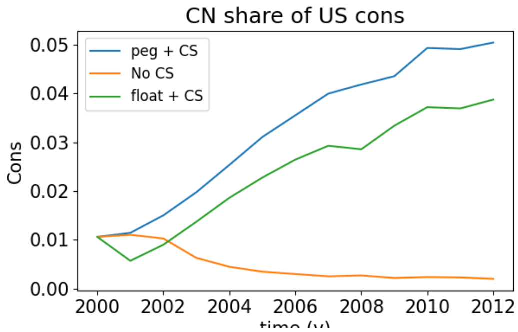
Figure 2-6a shows that the exchange rate peg played a role in Chinese import penetration to the US, and the actual penetration ratio would have been closer to 4% under a floating exchange rate. Under a float, Chinese currency would have appreciated during this period, and the increased price would have made Chinese goods less attractive to US consumers.

Investigating the decline in manufacturing (Figure 2-6b) and the US trade deficit (Figure 2-6c), we see that the exchange rate peg played a significant role in both. Even if China were identically growing, if China had a floating currency, close to 50% of the manufacturing decline attributable to the China shock and a significant proportion of the US trade deficit would disappear. Likewise, the level of unemployment is much closer to the 'no China shock' case (Figure 2-6d).[49]

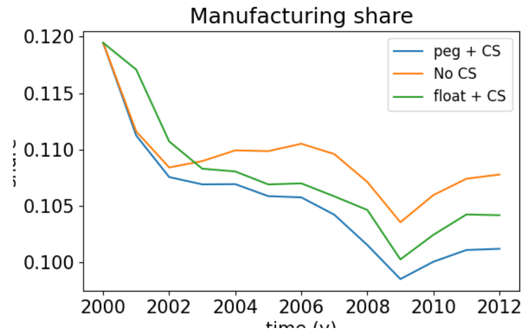
Finally, we study the change in welfare. While the above results – the effect of the peg on the trade balance and the labor market – suggest that the peg may have adverse effects on the US economy, the peg comes with a clear benefit: the terms-of-trade improves, as China is selling goods at a price cheaper than in a flexible-price equilibrium. This force lowers the price index and increases consumption given the same budget. At the same time, unemployment moves the budget inwards, and this is a force that leads to a decline in consumption. Using the same compensating variations formula, we see that the China peg contributes to a welfare loss of 0.083% compared to the counterfactual economy with an identically growing but floating China.

Table 2.3 summarizes the quantitative effects of the interaction of the peg and the China shock. The first column summarizes the realized effects of the China shock under a peg, while the second column summarizes the counterfactual effect of the China shock when China is floating; the third and fourth columns compare the differences in relative and absolute terms. As we see, the China shock interacted with the peg significantly. In absolute terms (Column 3), we see that China's currency peg is responsible for 447 thousand manufacturing jobs lost, 1.34% (as a fraction of GDP) US trade deficit, and 1.84% (in percentage points)

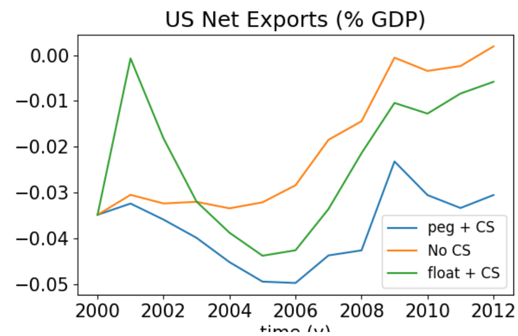
[49]The 'jump' in 2001 comes from the fact that our analysis takes the realized wages and distribution of labor in 2000 as fixed initial conditions, and these values were under a peg. When we report the average trade deficit and unemployment below, we take the average from 2003 to 2012 to trim this discontinuity.



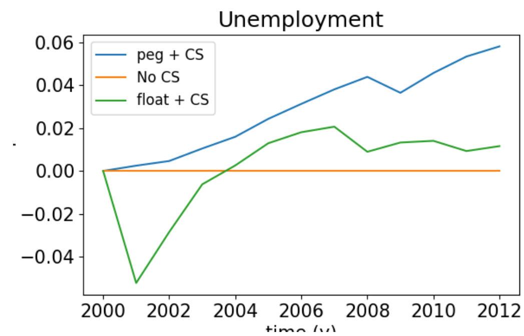
(a) CN import penetration



(b) Manufacturing share



(c) Net exports (% GDP)



(d) Unemployment rate

Figure 2-6: Response of economy to China's peg.

Note. The 'peg + CS' graphs are the equilibrium outcome from the full sequence of parameters targeted to match realized moments. The 'no CS' graphs are the equilibrium outcome from the *no China trade shock* assumption. The 'float + CS' graphs are the equilibrium outcome from the full sequence of parameters identical to the 'peg + CS' case (realized equilibrium), but under the counterfactual assumption that China did not peg its exchange rate and had its own independent Taylor rule.

unemployment in the US, and the welfare gains are reduced by 0.083 percentage points, compared to a counterfactual economy where an otherwise identical China floats. In relative terms (Column 4), China's currency peg *magnifies* the manufacturing jobs lost from the China shock by 82%, the trade deficits caused by the China shock by 161%, unemployment by 176%, and reduces the welfare gains by 32%.

The last column takes the literature's estimates from the three papers we discussed in the previous subsection (Caliendo, Dvorkin, and Parro, 2019; Andrès Rodríguez-Clare, Ulate, and Vásquez, 2022; Dix-Carneiro, Pessoa, et al., 2023). The effect of the China shock under a counterfactual 'floating' economy (second column) is strikingly similar to the structural estimates of the effects of the China shock in the literature. The manufacturing jobs lost are close to 550 thousand in all of the three aforementioned papers, while we estimate 543 thousand under float. The US trade deficit caused by the China shock is estimated to be 0.8% of GDP in Dix-Carneiro, Pessoa, et al. (2023); the US trade deficit attributed to the

Decomposing China shock vs China peg					
	CS + peg	CS + float	$Y_p - Y_f$	$Y_p/Y_f - 1$	Lit estimate
MFG jobs lost	991k	543k	447k	+82%	550k
Deficit (% GDP)	2.25	0.86	1.34	+161%	0.8%
Unemployment (%)	3.04	1.10	1.84	+176%	1.4%
Welfare gains	0.183%	0.268%	-0.083p.p	-32%	0.2%

Table 2.3: Effects of the China peg

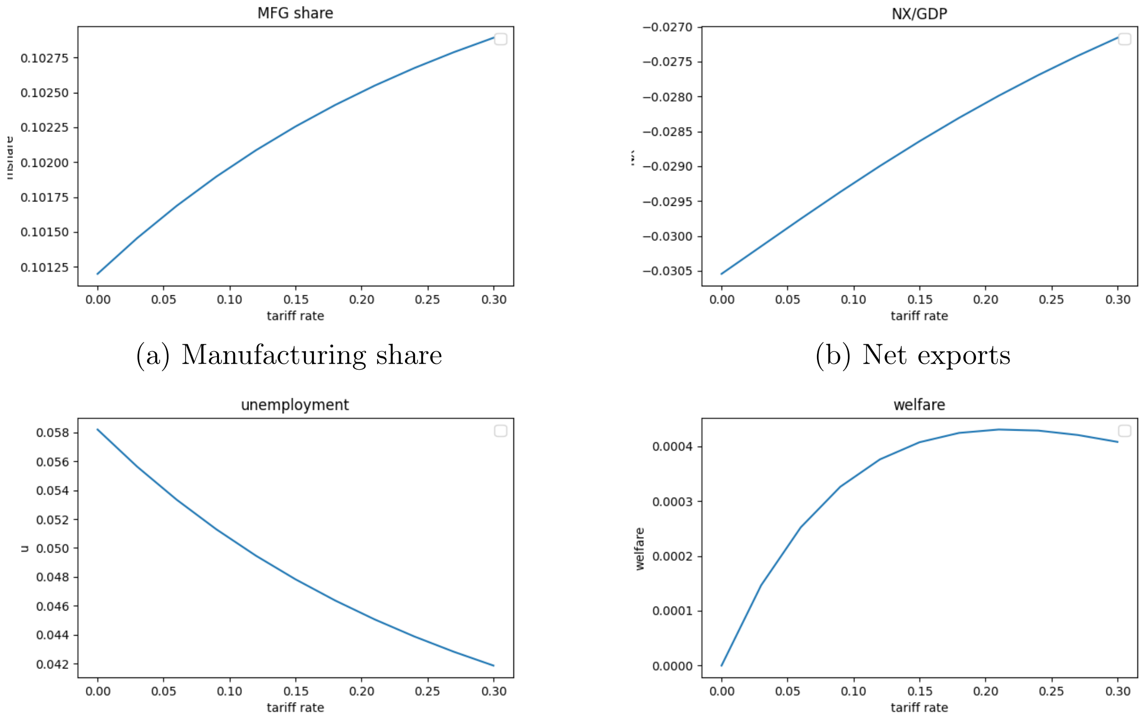
Note. The first column shows the realized effect of the China shock when the exchange rate is pegged. The second column shows the counterfactual effect of the identical China shock when China floats its currency. The third and fourth columns show the difference and ratio of the two, respectively. The fifth column shows the literature's estimates from Table 2.2.

China shock under a (counterfactual) floating economy is 0.86% of GDP. The unemployment effect estimated by Andrès Rodríguez-Clare, Ulate, and Vásquez (2022) is 1.4%; under our modeling framework, the counterfactual effect of the China shock under a float is 1.10%. These results suggest that explicitly modeling the exchange rate peg is essential in a general equilibrium analysis of the effects of China shock on the US.

2.5.4 Counterfactual policies

We conclude by studying how policies such as tariffs and monetary policy may have altered the effects of the China shock. Suppose we wanted a quantitative answer to policy questions such as: (1) Could the US have mitigated the negative consequences of the China shock with a tariff on Chinese goods in the early 2000s? (2) Does the answer to this question depend on whether China retaliates? (3) Should the US have pursued a different monetary policy to counter the effects of the exchange rate peg? Our quantitative framework is especially suitable for studying the effects of alternative policies, as we can quickly compute the counterfactual equilibrium under any set of policies. We can answer such questions by comparing the realized equilibrium with a counterfactual equilibrium with different tariff rates t^s_{ijt}, or alternative monetary policies, expressed either through a discretionary monetary policy response given by ϵ^{MP}_{1t} in the US monetary policy Taylor rule (Equation 2.34), or alternative rules of monetary policy.

The first counterfactual exercise we consider is a unilateral tariff that the US imposes on Chinese goods. Could protective tariffs have helped ameliorate the short-run losses from China's growth and exchange rate peg? The specific policy experiment we analyze is a uniform tariff rate of $x\%$ for $x \in [0, 0.3]$ imposed by the United States on Chinese goods from 2000 to 2012. In Figure 2-7, we highlight the effects of the tariffs on four key variables



(a) Manufacturing share

(b) Net exports

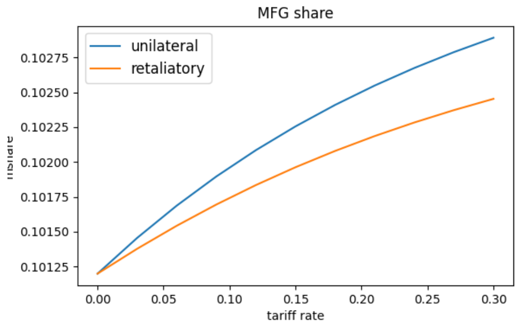
(c) Unemployment

(d) Home welfare

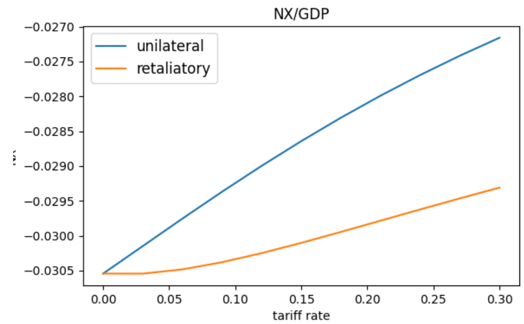
Figure 2-7: Effect of unilateral tariffs.

affected by the China shock: the share of manufacturing employment, US trade deficit as a percentage of GDP, unemployment rate, and aggregate welfare in the United States. The first three indicators are measured as their level in 2012, whereas aggregate welfare is computed using compensating variations relative to the realized equilibrium.

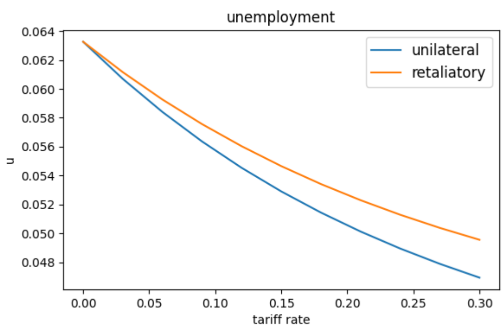
Figure 2-7 shows that a unilateral tariff reduces the decline in the share of manufacturing in the short-run, reduces the deficits, and reduces the unemployment rate. The welfare-maximizing tax rate is close to 20%, and this rate is much lower than the rate that restores full employment or restores the balance of trade. The tariff reduces 25% of the unemployment associated with the China shock and 10% of the realized trade deficit. The welfare gains from the tariff are modest, about 0.04% of lifetime welfare. This is about half of the welfare costs of the China peg (0.083%), suggesting that tariffs may help alleviate some of the welfare costs of the exchange rate peg. In this context, while a *safeguard* tariff helps alleviate the welfare losses from labor market frictions, the distortionary impact of tariffs on consumption is substantial enough so that the US government will not fully undo the distortions using tariffs. This analysis clarifies the quantitative relevance of the different welfare channels in the optimal tariff formula (Equation 2.23).



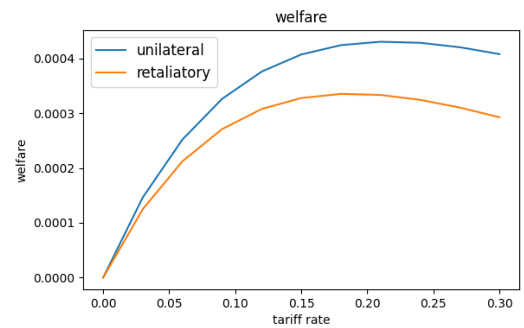
(a) Manufacturing share



(b) Net exports



(c) Unemployment



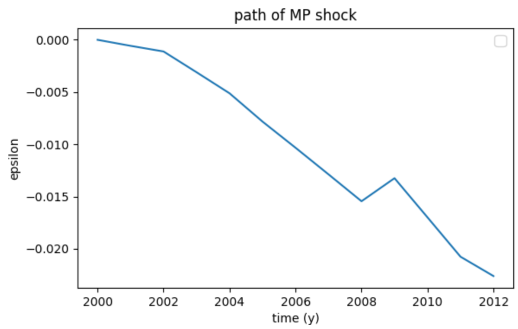
(d) Home welfare

Figure 2-8: Effect of tariffs with retaliatory tariffs of equal magnitude

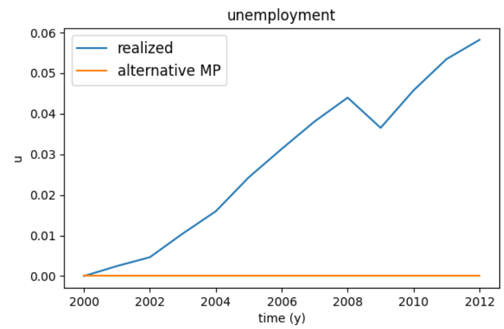
In the second counterfactual exercise, we consider the same tariffs on Chinese exports to the US but assume that China retaliates with a tariff of equal magnitude. The possibility of retaliatory tariffs undoing any gains from tariffs is well understood in the trade context without nominal rigidity and is often used as an argument for free trade agreements. How do the welfare effects of safeguard tariffs change when such tariffs are faced with retaliatory tariffs?

Figure 2-8 shows the response of the same aggregate variables for different tariff rates set by the US, with a retaliatory tariff from China of the same magnitude. Retaliatory tariffs weaken the effectiveness of tariffs on the manufacturing share, net exports, and unemployment. Still, the safeguard nature remains even with retaliatory tariffs: short-run unemployment in the US is lowered.

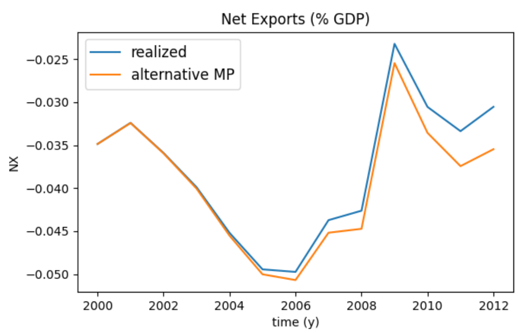
In the next experiment, we assess the effects of monetary policy loosening in this economy. In the baseline equilibrium (Figure 2-5), we saw that aggregate unemployment increased due to the China shock when the monetary policy was a Taylor rule targeting CPI inflation. How much looser should monetary policy be to undo the unemployment effects, and what are the effects of this additional discretionary monetary policy by the US? We simulate the



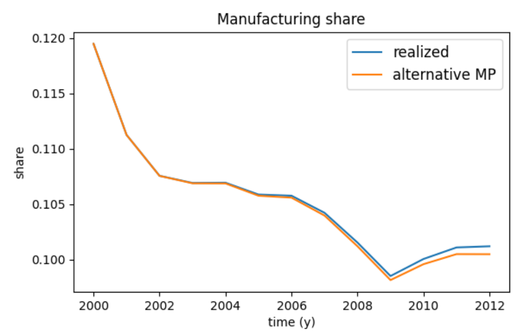
(a) Monetary policy shocks



(b) Unemployment



(c) Net exports



(d) Manufacturing share

Figure 2-9: Effect of alternative monetary policy

model with different Home monetary policy shocks ϵ_{1t}^{MP} over 2000-2012 to find $\hat{\epsilon}_{1t}^{MP}$ that sets aggregate unemployment to zero from 2000 to 2012, and plot the economy's response to this monetary policy shock.

As Figure 2-9 shows, to clear unemployment, the nominal interest rate needs to be lower in 2000-2012 than the rate implied by the Taylor rule by up to 2%. This restores full aggregate employment but does not change the trade deficit or the decline in manufacturing share, confirming the role of monetary policy as an aggregate, not a distributional tool. Monetary policy loosening does not affect the trade deficit much because of the Chinese peg – if the US loosens monetary policy, the effective interest rate in China declines, too.[50]

In summary, we have found that a modest short-run tariff on Chinese goods in the early 2000s may help alleviate some of the labor market distortion caused by Chinese growth combined with the exchange rate peg.

[50]In the Appendix, we study alternative monetary policy rules that are better suited to target unemployment under permanent trade shocks. In a work in progress, we study optimal monetary policy rules in this environment.

2.6 Concluding remarks

What is the role of the exchange rate regime in shaping short-to-medium-run responses to trade shocks? The conventional trade literature sidesteps this question by focusing on flexible price equilibrium. We use the three different angles – empirical, theoretical, and quantitative – to revisit the effects of the China shock consistently suggest that China's currency peg against the US dollar is qualitatively and quantitatively pivotal in determining the labor market, trade balance, and welfare response.

We have empirically documented that countries using or pegging to to the US dollar exhibit lower real GDP, a larger decline in manufacturing, and deteriorating trade balances in response to the China shock, compared to countries with similar China shock exposure that float to the US dollar. Notably, the floating countries have their currency appreciate in response to a larger exposure to the China shock, suggesting that the exchange rate operates as an adjustment margin. We develop a simple model of wage rigidity that can explain these findings, where we analytically characterize how exchange rate pegs interact with Foreign productivity growth to generate trade deficits and unemployment at Home. When we calibrate the multi-sector trade model to match the trade and sectoral reallocation data, we find that China's peg against the US dollar is quantitatively significant in shaping the effects of the China shock in the US trade deficit, unemployment, and decline in manufacturing.

While we intentionally focused our analysis on the China shock and the US dollar, the intuition of the direction of trade imbalances and labor market adjustments under exchange rate pegs apply more broadly. The post-WWII East Asian growth stories, most notably Japan and South Korea, involve having the currency follow the US dollar and running large trade surpluses in the growth path. Our framework can also give a better understanding of trade balances within the Eurozone, such as the persistent trade surplus of Germany and Ireland, and the deficit of Greece in the Eurozone.

One aspect of the model we intentionally abstracted from is China's policy goal. Why does China peg the exchange rate to the US dollar by effectively overheating its economy to supply cheap goods to the world? Potential explanations missing in our model include financial stability and an increase in investment coming from exchange rate stability, a myopic government seeking to maximize short-run output, learning-by-doing models (where more exports lead to productivity growth), and an increase in trade leading to technology diffusion (Perla, Tonetti, and Waugh, 2021). These are all mechanisms outside the scope of our model that can rationalize an exchange rate peg for a growing country, which we do not take a stance on.

One final direction forward is to consider heterogeneous agents in our model. In our model, since the consumption-savings decision is made at a family level, and unemployment is only at the intensive margin, our estimates of the losses from the exchange rate peg are underestimates. With a concave utility, involuntary unemployment in the extensive margin will aggravate losses for the unemployed and may have precautionary saving implications for manufacturing workers in the US. A model of heterogeneous agents and savings in incomplete markets may better highlight the distributional consequences of the China shock and the China peg. Probing this direction would further enrich our understanding of the China shock, and the role of the exchange rate as a shock absorber.

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Chapter 3

Model-Agnostic Dynamic Programming

3.1 Introduction

Many economic models formulate the decision problems of economic agents as dynamic optimization problems (see Stokey, Lucas, and Prescott 1989 for numerous examples). A dynamic optimization problem is characterized by the following ingredients: (i) a state vector that describes an agent's decision environment in the current period; (ii) a set of actions that an agent takes; (iii) a utility function that specifies the agent's payoff associated with choosing a given set of actions in a given state; and (iv) a law of motion for the state vector (i.e., a transition process), which specifies the probability distribution over states in the next period given the state and actions chosen in the current period.

The traditional approach to solving dynamic optimization problems starts with specifying the law of motion for the exogenous components of the state vector and estimating its parameters. For example, in the canonical life cycle consumption-saving problem of Gourinchas and Parker, 2002, there are three states: age, assets, and income. The law of motion for these states is straightforward: age in the next period equals age in the current period plus one; assets in the next period are determined by the consumption-saving choice and the budget constraint; and income in the next period evolves exogenously according to a Markov process. The first step in the traditional approach to solving this model would be to specify a process for income (i.e., the only exogenous state), which is usually a combination of a deterministic component and a stochastic component that follows a normal-AR1 process, and estimate the parameters of this process using income data. After doing so, the traditional approach would then solve this model using standard techniques, such as value or policy function iteration.

This paper proposes a new methodology for solving dynamic programming problems that sidesteps the need to specify a law of motion for exogenous states, which we call Model-

Agnostic Dynamic Programming (Agnostic DP, for short). Our motivation for developing this approach comes from the growing literature on labor income risk, which shows that administrative income data exhibit properties (e.g., excess skewness and kurtosis) that cannot be captured by the traditional income processes (Guvenen, Ozkan, and Song, 2014; Guvenen, Karahan, et al., 2021; Braxton et al., 2021). This evidence thus raises the question of what model of income processes should be used when solving dynamic consumption-saving problems (Guvenen, McKay, and Ryan, 2022).

Motivated by this evidence, the Agnostic DP approach that we develop in this paper does not require specifying the data-generating process (DGP) for exogenous states. Instead, we sample actual realizations of exogenous states (e.g., income in the consumption-savings example) directly and then use reinforcement learning techniques to find the optimal policy function given the unknown DGP for exogenous states. In particular, we follow V. Duarte, D. Duarte, and Silva, 2023 and parameterize the policy function using a deep neural network. We then use a stochastic gradient descent algorithm to solve for the network parameters that maximize the agent's expected lifetime utility. Unlike traditional approaches to solving dynamic programming problems, at no point in this solution process do we need to specify the law of motion for exogenous states, which allows us to remain "agnostic" about the data-generating process. The only requirement is that we have enough realizations of exogenous states from the data in order to train the network (i.e., perform stochastic gradient descent).

In Section 3.2, we pose a generic (finite-horizon) dynamic programming problem and describe our solution method in full generality. An agent in state s_t takes an action a_t that delivers the agent utility $u(s_t, a_t)$. We partition the state space into exogenous states k_t (e.g., time, income, or productivity) and endogenous states x_t (e.g., assets). We assume throughout that we have sample paths of exogenous states and a distribution for the initial value of the endogenous state. Given these preliminaries, we can then define a transition function of the state, $s_{t+1} = \hat{m}(s_t, a_t, k_{t+1})$, which, given a state, an action, and a future realization of exogenous states, returns the state at $t + 1$. We then show how to parameterize the policy function as a neural network, as in V. Duarte, Fonseca, et al., 2022, and describe the algorithm to solve the model while simultaneously sampling k_{t+1} from data realizations. The innovation of this approach is that it does not require specifying a parametric model for how k_{t+1} relates to s_t.

Section 3.3 applies our method to a canonical life cycle consumption-saving problem. In this model, an agent lives for T periods and receives an exogenous stream of income. In each period, the agent makes a consumption-savings decision subject to a borrowing limit and an exogenous interest rate. To solve the model, we parametrize the policy function using a deep neural network that the current states as inputs (time, assets, and income) and outputs consumption. Since the agent (like the modeler) does not know the law of motion for income (i.e., the only exogenous state in the model), we solve the model by optimizing the over 1 million parameters of the neural network to maximize expected lifetime utility, given income realizations that are sampled directly from the Current Population Survey (CPS) based on individuals' current income and age.

Having obtained the optimal policy function without specifying the DGP for income, we then ask the following question: what is the welfare loss from imposing a parametric model of the income process? To answer this question, we solve the model using the traditional approach by specifying and estimating a standard parametric income process. We then compare the expected lifetime utility of an agent that uses this "Classic" policy function, which has an incorrect representation of the DGP for income, with that of an agent that uses the Agnostic DP policy function from our method. Surprisingly, we find that these two values are virtually identical, implying that the cost of assuming a functional form for income is negligible.

One possibility for this negligible difference is that Agnostic DP does not achieve the global optimum. We rule out this possibility by comparing our method with the classical approach in a model in which the DGP is known. This is a useful exercise because we know that the classical approach delivers the optimal policy function when the DGP is known. However, in this case, we find that Agnostic DP is able to replicate the optimal policy function and achieve the same value function as Classic, which illustrates the reliability of our solution method.

Although the benefits of being "agnostic" about the data-generating process are small in our specific application, there are two notable benefits of Agnostic DP relative to the Classic approach. First, Agnostic DP is much simpler to code. Second, and more importantly, the Agnostic DP algorithm does not change as the number of states and actions increases, while the "curse of dimensionality" limits the Classic approach from solving more complex models. In principle, our Agnostic DP algorithm can handle as many states and actions as required, but more attention needs to be put into the architecture of the neural network as the model becomes more complex.

We conclude by discussing plans for future work. One reason why the welfare loss from assuming a functional form for income would be low is because we are using survey rather than administrative data; in future work, we plan to implement our approach using administrative earnings data. Two other directions for future work include generalizing the method to allow for infinite horizon problems and general equilibrium.

Related literature This paper adds to a nascent literature that develops methods to solve dynamic stochastic economic models using machine learning techniques (see, e.g., V. Duarte, Fonseca, et al. 2022; V. Duarte, D. Duarte, and Silva 2023; Scheidegger and Bilionis 2019; Fernández-Villaverde, Hurtado, and Nuño 2023; Azinovic, Gaegauf, and Scheidegger 2022). Closest to our paper is V. Duarte, Fonseca, et al., 2022, who introduce the deep reinforcement learning method for solving finite-horizon dynamic stochastic programming problems. They use this method to solve a rich model of life cycle portfolio choice, which includes many ingredients only modeled in isolation in prior work. However, their solution method is also "classical" in the sense that they need to specify the DGP for exogenous states in the model. In contrast, the contribution of our paper is to show how to leverage their solution technique to be agnostic about this DGP.

This paper is also part of an extensive literature in macroeconomics and household finance that studies household consumption, savings, and portfolio choices using life cycle models. The standard approach in this literature is to formulate a dynamic stochastic model of household behavior over the life cycle and then estimate it in two stages (e.g., Gourinchas and Parker, 2002; Catherine, 2022). In the first stage, the parameters that govern the DGP for exogenous states (e.g., income) are carefully estimated; in the second stage, any remaining parameters are estimated using indirect inference. In contrast to the first stage of this approach, our solution approach does not take a stand on the DGP for exogenous states. Our current application of this approach works with a simpler model than most of this literature, but in future work, we plan to explore its use in more complex models.

3.2 Model-Agnostic Approach

3.2.1 Framework

Setup

We are interested in solving a canonical dynamic programming as in Stokey, Lucas, and Prescott, 1989 of the form

$$\begin{aligned} V(s_0) = \max_{\{a_t \in \Gamma(s_t)\}_{t=0}^{T}} \mathbb{E}_0 \left[\sum_{t=0}^{T} u(s_t, a_t) \middle| s_0 \right] \qquad \text{subject to} \\ s_{t+1} = m(s_t, a_t, \epsilon_t). \end{aligned} \tag{3.1}$$

Over T periods, the agent takes *actions* a_t to maximize utility $u(s_t, a_t)$, which depends on the action taken and the *state* s_t. By convention, time t is the first element of s_t, which implies that $u(s_t, a_t)$ can depend on t in any general form. We partition the state into two elements, $s_t = (k_t, x_t)$, where k_t are *exogenous* states and x_t are *endogenous* states. The action space is constrained by $\Gamma(s_t)$, which defines the set of possible values that a_t can take, given s_t. The law of motion of the state is governed by the function $m(s_t, a_t, \epsilon_t)$, where ϵ_t is a random variable and makes the problem stochastic.

The goal is to find a policy function $a_t = \pi(s_t)$, which specifies which action to take given a current state s_t. Unless this policy can be solved analytically, it is common to parametrize this policy with some vector parameter θ and approximate π by $\hat{\pi}(s_t, \theta)$. θ can be the parameters of an interpolator or the weights in a neural network. We will consider this later case since neural networks are universal approximators and can handle many states easily. We now can define $\tilde{V}(s, \theta; \hat{\pi})$ as the expected value of using the policy function $\hat{\pi}(s, \theta)$ as

$$\begin{aligned} \tilde{V}(s_0, \theta; \hat{\pi}) =& \mathbb{E}_0 \left[\sum_{t=0}^{T} u(s_t, \hat{\pi}(s_t, \theta)) \bigg| s_0 \right] \qquad \text{subject to} \\ s_{t+1} =& m(s_t, \hat{\pi}(s_t, \theta), \epsilon_t). \end{aligned} \tag{3.2}$$

$\tilde{V}(s_0, \theta; \hat{\pi})$ is the expected value the agent gets when it starts at s_0, and follows the policy $\hat{\pi}$ parametrized by θ. Through the paper, we will take $\hat{\pi}$ as given and will be looking for θ. The optimal choice for $\hat{\pi}$ is left for further research, and we will use a multi-layer neural network with a fixed number of layers. In order to get rid of the expectation term, we define $\tilde{V}(s_0, \theta; \hat{\pi}, \{\epsilon_t\}_{t=0}^T)$ as the value of the policy $\hat{\pi}(s_t, \theta)$ when the initial state is s_0 for a particular shock realization $\{\epsilon_t\}_{t=0}^T$,

$$\tilde{V}(s_0, \theta; \hat{\pi}, \{\epsilon_t\}_{t=0}^T) = \sum_{t=0}^{T} u(s_t, \hat{\pi}(s_t, \theta)) \quad \text{subject to} \quad s_{t+1} = m(s_t, \hat{\pi}(s_t, \theta), \epsilon_t)$$

Traditional dynamic programming like (3.1) solve for a function $V(s_0)$ that gives the value *for every* s_0. However, once we are solving a problem for a given policy function, defined by $(\hat{\pi}, \theta)$, we need a way to compare different policy functions that do not depend on the initial state s_0. Thus, we assume the initial state s_0 comes from a distribution $F(\cdot)$, and the problem we will be solving is to find the parameters θ that maximize

$$\bar{V}(\hat{\pi}) = \max_{\theta} \mathbb{E}[\tilde{V}(s_0, \theta; \hat{\pi})], \tag{3.3}$$

where the expectation is taken with respect to s_0.

In order to solve (3.3), we will leverage advances in computer science and machine learning that allow taking the gradient of $\tilde{V}(s_0, \theta; \hat{\pi})$ with respect to θ, which we denote by $\nabla_\theta \tilde{V}(s_0, \theta; \hat{\pi})$. We will replace expectations by simulations of s_0 and paths for ϵ_t. The central contribution of this paper is how we think about the law of motion m.

Constraining the Action Space

The policy function will be defined by a neural network, which takes as input the state s_t and returns some actions a_t. These actions are constrained by $\Gamma(s_t)$, so we need to make sure that the neural network outputs a feasible action. For this, we have to determine the last activation layer according to the constraints of the problem and do a transformation that depends on the state. Suppose the action space is a scalar, and let $\tilde{a}(s_t)$ be the output of the neural network before we make sure it satisfies $a_t \in \Gamma(s_t)$. Then we can be in on of the following cases.

When the action space is bounded below and above, so $a_t \in [\underline{b}(s_t), \bar{b}(s_t)]$, then the neural network should output a probability $\tilde{a}(s_t) \in [0, 1]$ and then transform the action to $a(s_t) = \underline{b}(s_t) + (\bar{b}(s_t) = \underline{b}(s_t))\tilde{a}$. For that, using a sigmoid/logistic activation function is a good choice. An example of this constraint is consumption, which has to be positive and is bounded above by some value determined by assets, income, and borrowing constraints. The neural network then outputs the share of the total available consumption that is actually consumed.

Another possibility is that the action space is one-sided bounded like $a_t \geq \underline{b}(s_t)$. In such case, using a ReLU activation function, which is bounded below by zero, and setting $a(s_t) = \underline{b}(s_t) + \tilde{a}(s_t)$ would achieve the desired result. Similarly if $a_t \leq \bar{b}(s_t)$, set $a(s_t) = \bar{b}(s_t) - \tilde{a}(s_t)$. This constraint is natural in many models of labor supply where labor is not bounded above but required to be positive.

Finally, if the action is unconstrained and $a_t \in \mathbb{R}$, then any unbounded activation function suffices to satisfy $a_t \in \Gamma(s_t)$. In particular, the linear activation function is a good choice for that case.

The previous three cases considered a scalar action, but in most models, the action space is a vector, with components a_{it}. In such case, it is not always the case that we can constrain a_{ti} using the information of the state only because it depends on other actions a_{tj}. A clear example is a life cycle model with endogenous labor supply: how much an agent is allowed to consume depends on how much labor the agent supplies. We can still ensure that $a_t \in \Gamma(s_t)$, noting that labor constraints are independent of consumption, and given labor, we can constrain consumption.

The Law of Motion

The focus of this paper is on the law of motion of the state m. Traditional solution methods of the problem (3.1) assume a functional form for m and use it to take first-order conditions and compute expectations. The proposed methodology does not require such tractability and instead treats m like a *black-box*: given a state s_t and an action a_t, returns a new state. ϵ_t shocks make this mapping stochastic. The agent does not need to *know* m, but rather it *learns* it. All we need to provide is a function that samples s_{t+1} given s_t and a_t.

If the modeler knows m, it is an input of the model. This is how most economic models start: by defining a law of motion of the state that will be estimated/calibrated. We have developed a `Python` package `nndp`, that solves problem (3.1) given m, u and Γ. The advantage of using neural networks is that handling more states is straightforward, given that neural networks are meant to work with many inputs and provide many outputs. We use automatic differentiation and just-in-time compilation to speed up the convergence process by leveraging `jax`.

The most interesting case is when the modeler does not know m or wants to remain agnostic about the data-generating process. The next section discusses this case.

3.2.2 Solution Algorithm

The standard way of solving economic models is to (i) use data to estimate a process for the law of motion of the state and (ii) solve a traditional dynamic problem model given the estimated law of motion. Our methodology proposes to bypass step (i) and instead solve the dynamic problem while feeding real data realizations. This way, the modeler remains agnostic about the data-generating process and does not constrain it at all.

We have partitioned the state s_t into exogenous states k_t (time, income, interest rates...) and endogenous states x_t (assets). Given s_0, we can solve (3.1) if one is able to get enough path realizations. We define the value function of following a policy $\hat{\pi}$, parametrized by θ, given a future process for exogenous states $\{k_t\}_{t=0}^T$ and initial endogenous state x_0 as

$$\begin{aligned} \hat{V}(x_0, \theta, \{k_t\}_{t=0}^T; \hat{\pi}) = \sum_{t=0}^{T} u(s_t, \hat{\pi}(s_t, \theta)) \quad \text{subject to} \\ s_{t+1} = \hat{m}(s_t, \hat{\pi}(s_t, \theta), k_{t+1}) \\ s_0 = (k_0, x_0). \end{aligned} \tag{3.4}$$

We have not included the constraint $a_t \in \Gamma(s_t)$ because we assume that the policy function

$\hat{\pi}(s_t, \theta)$ already outputs feasible actions, as discussed in the previous section. Note that (3.4) is fully deterministic and differentiable with respect to θ. Algorithm 1 shows how we can find the parameter θ that maximizes (3.4) using stochastic gradient descent techniques.

Algorithm 1: Agnostic DP

Primitives: reward function $u(s_t, a_t)$, transition function $\hat{m}(s_t, a_t, k_{t+1})$, constraints $a_t \in \Gamma(s_t)$, and T.

Data: M paths with x_{i0} and $\{k_{it}\}_{t=0}^T$ indexed by i

Define a neural network architecture $\hat{\pi}$ that satisfies $a_t \in \Gamma(s_t)$ parameterized by θ;

Initialize the policy function parameters θ^0;

for $n < N$ **do**

  Sample $m < M$ paths, and let $\mathcal{I}^n$ be the set of paths selected. Compute the gradient $g(\theta^n) = \nabla_\theta \frac{1}{m} \sum_{i \in \mathcal{I}^n} \hat{V}(s_{i0}, \theta^n, \{k_{it}\}_{t=0}^T; \hat{\pi})$;

  Update $\theta^{n+1} = update(\theta^n, g(\theta^n))$

end

3.3 Application to Life Cycle Consumption-Saving Problem

In this section, we show an application of our method to a standard consumption-savings problem widely used in economics and household finance. It consists of a household with some wealth w_t that lives for T periods, receives an exogenous stream of income y_t, and can save at a constant interest rate R. The only decision that the household makes is how much to consume, so $a_t = c_t$. The value function of an agent at state $s_t = (t, y_t, w_t)$ is

$$
\begin{aligned}
V(t, y_t, w_t) = \max_{\{c_{t+k}\}_{k=t}^{T-1}} \mathbb{E}_t &\left[\sum_{k=t}^{T-1} \beta^t \frac{c_{t+k}^{1-\gamma}}{1-\gamma} + \beta^T b(w_T) \middle| y_t \right] \\
w_{t+1} &= R\left(w_t + y_t - c_t\right) \\
w_t &\geq 0 \quad c_t \geq 0.
\end{aligned}
\tag{3.5}
$$

We have added a bequest motive at the time of death T. We adopt De Nardi, French, and Jones, 2016 functional form $b(w_T) = b_0 \frac{(w_T + b_1)^{1-\gamma}}{1-\gamma}$, in part to showcase how utility can be time-invariant once t is part of the state space. b_0 is the intensity of the bequest motive, while b_1 determines the curvature of the bequest function and, hence, the extent to which bequests are luxury goods. In the notation of (3.1), the reward function is given by

$$u(s_t, a_t) = \begin{cases} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} & \text{if } t < T \\ \beta^T b_0 \frac{(w_t+b_1)^{1-\gamma}}{1-\gamma} & \text{if } t = T \end{cases}.$$

For simplicity, we assume that wealth has to be positive, allowing for positive borrowing is straightforward. We partition the state into exogenous states $k_t = (t, y_t)$ and endogenous states $x_t = w_t$. Problem (3.5) is missing the law of motion of y_t, which we have not specified. Suppose instead that we have a panel where we observe the income realizations of individuals over their lifetime. In the example provided, we have assumed that each household lives exactly for T periods, the appendix describes how the model could be enriched with (agnostic) random death. With that dataset, we could use Algorithm 1 to solve for $V(t, y_t, w_t)$ without making any assumption on the income process y_t. Next, we propose a method to generate that panel drawing from real income realizations.

3.3.1 Generating a Sampler Dataset

Not making any assumption about the data-generating process of the exogenous state comes at a cost: we need to observe many paths of income to learn its process to avoid overfitting. The universe of US tax returns (e.g., Guvenen, Ozkan, and Song, 2014) would be ideal for our solution method. Unfortunately, we don't have access to these data, so we use an alternative method to generate sample paths from real data realizations.

Income Paths

The CPS is a monthly U.S. labor force survey covering the period 1962 to the present and gathers information on over 65,000 households. It is the main survey to compute unemployment statistics, and it also contains information on education, labor force status, demographics, and other aspects of the U.S. population. The CPS interviews sample members eight times. Respondents are interviewed for four consecutive months, are rotated out for eight months, and then are included in the sample for another four months. In March, the Annual Social and Economic (ASEC) Supplement provides the usual monthly labor force data, but in addition, provides supplemental data on work experience, income, noncash benefits, and migration.

In particular, the ASEC Supplement has information about total income at the individual level and has a panel structure. In a given March, around 50,000 households are surveyed, and around 45,000 individuals are followed from year t to $t + 1$. We consider individuals aged from 25 to 65 to define an income path of $T = 40$ years. We deflate income using the cpi deflator and round it to the nearest multiple of $\underline{y} = \$2,500$, and call this variable y_{it}. We define t as the number of years since the person turned 25.

In order to obtain a complete path of length T, we sample from realized transitions from k_t to k_{t+1}. We start by sampling an individual aged 25 and we observe its income y_{i0} and y_{i1}. Then we sample one observation from all individuals aged 26 with income $y_{i'1} = y_{i1}$, for whom we also observe $y_{i'2}$. Any sample is done using ASEC weights. Rounding income to the nearest multiple of $\underline{y}$ allows for a higher likelihood to find at least one observation with $y_{i'1} = y_{i1}$. With that, we generate a path of length 3 consisting of $y_{i0}, y_{i1} = y_{i'1}, y_{i'2}$. We iterate further by sampling individuals aged 27 and with income $y_{i''2} = y_{i'2}$, to generate an observation for $y_{i''3}$. Once we reach $T = 40$, we consider we have generated a new income path. It might be the case that given $k_t = (t, y_t)$, we are unable to find a match to iterate forward. If that happens, we start the path from the beginning. This algorithm allows us to generate as many balanced paths as required to train and evaluate the model. Figure 3-2 shows 5 paths of income sampled using this procedure. We see some persistence and, at the same time, big jumps. Our methodology ensures that every pair y_{it}, y_{it+1} has been observed in the CPS.

The algorithm generates paths where the transitions y_t, y_{t+1} are data realizations. We are assuming that only age and income are determinants of future income, a simplification that is also shared by many other models, like the one we will use in Section 3.3.4. We can represent any income process of the form $y_{t+1} = f(t, y_t, \epsilon_t)$, with ϵ_t being an idiosyncratic shock with any distribution. The sampling mechanism can be generalized if we condition the matching on more observable states. We could add gender in the state space k_t or industry group. The obvious caveat of expanding the state space is that it becomes harder to find matches given the CPS sample size. There are many individuals aged 35 with \$50,000 income from whom we observe income at 36 (and thus we can sample one realization) than women aged 35 with \$50,000 income and working in accounting.

Distribution of Initial Assets

In addition to income paths, we require knowledge of the distribution of initial assets of individuals at age 25. We obtain those from the Survey of Consumer Finances, which is a triennial statistical survey of the balance sheet, pension, income, and other demographic characteristics of families in the United States. We pool all years from 1989 to 2022 and consider the net worth of observations aged 25-30. After filtering, 21,689 observations remain. Since we do not consider borrowing in our model, we set any observation with negative net worth to 0, which is 21% of the sample, and winsorize at 99%. Median assets are \$100,308, and mean assets are \$500,286, consistent with a fat tail distribution. Figure 3-1 shows the distribution of wealth.

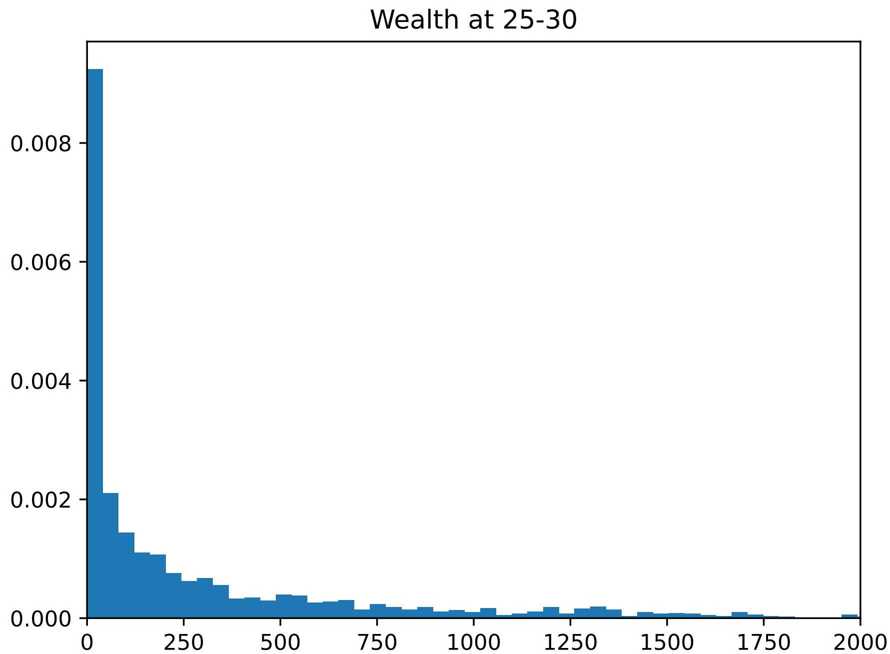

Figure 3-1: Wealth distribution at $t = 0$

Note: Wealth distribution at age 25-30 from the Survey of Consumer Finances.

Sampling Paths of States

With data on income and initial assets, we can sample initial states by sampling one income realization of somebody aged 25 from the CPS and the initial assets of somebody aged 25-30. A caveat of this approach is that we can't jointly sample income and assets because they come from different datasets. It is worth noting, though, that the distribution of initial states s_0 is important insofar as we want the neural network to optimize the policy around states that are more likely to be realized. In this sense, not having a correlation between income and assets helps to sample from a more dispersed distribution. We visit more states at the cost of visiting less frequently states that are more representative of an individual at 25.

What is relevant is that now we can generate as many realizations $(x_{i0}, \{k_{it}\}_{t=0}^{T})$ as we want, which are needed to solve our problem. Given an agent at state $s_{it} = (t_{it}, y_{it}, b_{it})$, an action $a_t = c_t$, we can define the function $m(s_t, a_t, k_{t+1})$, which returns s_{t+1}. The modeler specifies the law of motion for wealth but is agnostic about the process of y_{t+1}, which comes from k_{t+1}.

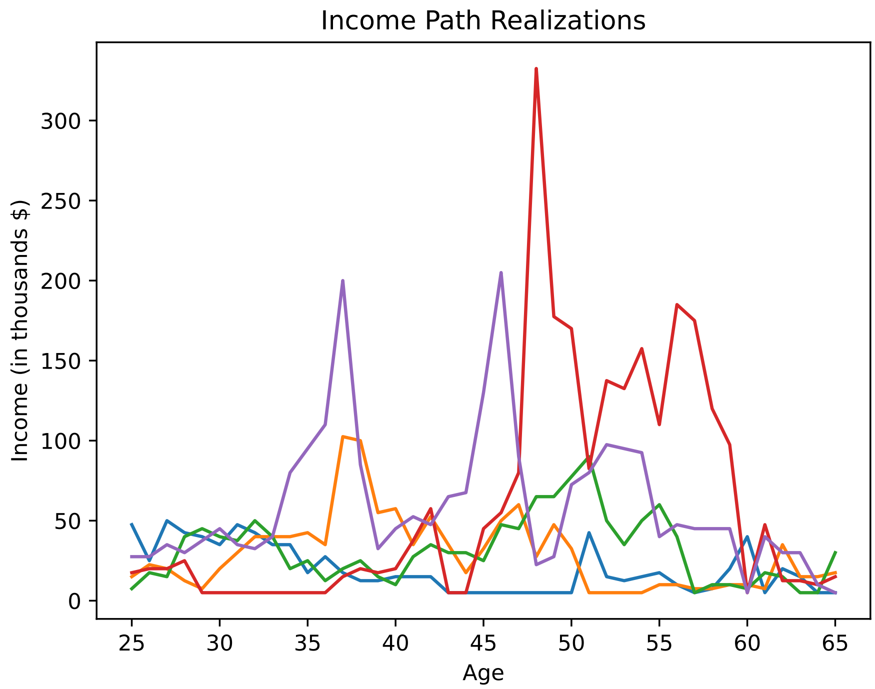

Figure 3-2: Synthetic Income realizations from the CPS.

3.3.2 Solution Method

We parametrize the policy function with a deep neural network $\hat{\pi}(s_t, \theta)$ that takes a 3-dimensional input and the parameters we optimize over and returns a 1-dimensional output bounded from 0 to 1. The output represents the share of cash-on-hand that the household consumes.[1] We build a network with $n_{layers} = 5$ hidden layers and each layer has $n_{nodes} = 500$ nodes, totalling 1,255,001 parameters. Each internal layer has a *tanh* activation function, which generates an output from -1 to 1, and the output activation function is a sigmoid, which generates a value from 0 to 1, which then is converted to total consumption. Figure 3-3 shows a graphical representation of the network.

We train the network with $N = 750$ epochs, and in each epoch we sample $m = 200$ paths $(x_{i0}, \{k_{it}\}_{t=0}^{T})$ as described in Section 3.3.1. Sampling more paths allows for a better calculation of expected values. Gradients are computed using `optax`, which is a gradient processing and optimization library for JAX (Bradbury et al., 2018), and we use Adam

[1]With a positive borrowing limit, it would represent the share of available cash-on-hand.

optimization routine (Kingma and Ba, 2015) with a learning rate of 10^{-4} . Optimization of hyperparameters and neural network structure is out of the scope of this paper, but there is room for making the network lighter and, therefore, faster.

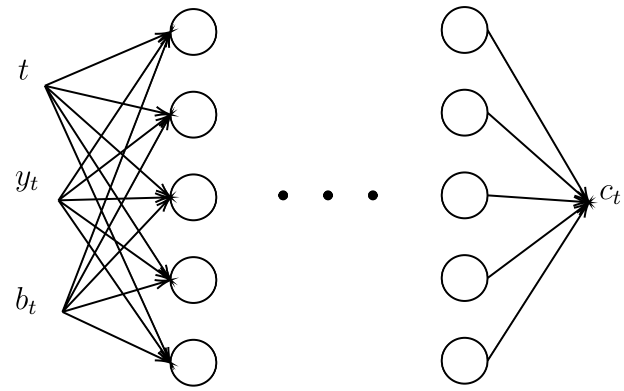

Figure 3-3: Neural network representation

Note: Neural network used to train the model, with 3 inputs, 1 output and 5 hidden layers, with 500 nodes each. Hidden layers are activated with *tanh*, and the output layer is activated using *sigmoid*

3.3.3 Results

Figure 3-4 shows the convergence of the value function as the neural network is trained, and the value function at the test dataset, that is, $E[\hat{V}(s_0, \theta, \{k_t\}_{t=0}^{T}; \hat{\pi}]$ across realizations of s_0 and $\{k_t\}_{t=0}^{T}$. Most of the training gains are achieved after 200 epochs, and each epoch has 200 paths, which means that the neural networks need to observe 40,000 paths of income to learn its process. Figure 3-4 shows the value function of an agent at age 25 that starts with 0 assets as a function of income. As expected, is increasing and convex. The function is not completely smooth because expected values are computed by sampling from income realizations. Since in the CPS, there are not many observations aged 25 earning more than \$100,000, we suffer from small sample problems at this high-income realizations.

In order to understand the policy function generated by the neural network, Figure 3-5 shows it of an agent with 0 assets as a function of income and time. The first panel shows the policy at age 25 ($t = 0$), and the second panel shows it at age 65 ($t = T = 40$). The figure also adds the income realization distribution at those ages since the neural network optimizes better states that are visited often. As expected, a young agent with low income behaves as a hand-to-mouth, consuming the majority of it. At age 65, the agent consumes a larger fraction of their income, although those with income higher than \$40,000 save some as a bequest motive.

Another way to evaluate the policy function is to plot, for an agent aged 25 with median income $\bar{y}$, which is \$25,000, as assets increase. Cash on hand is $a_0 + \bar{y}$ and the green line

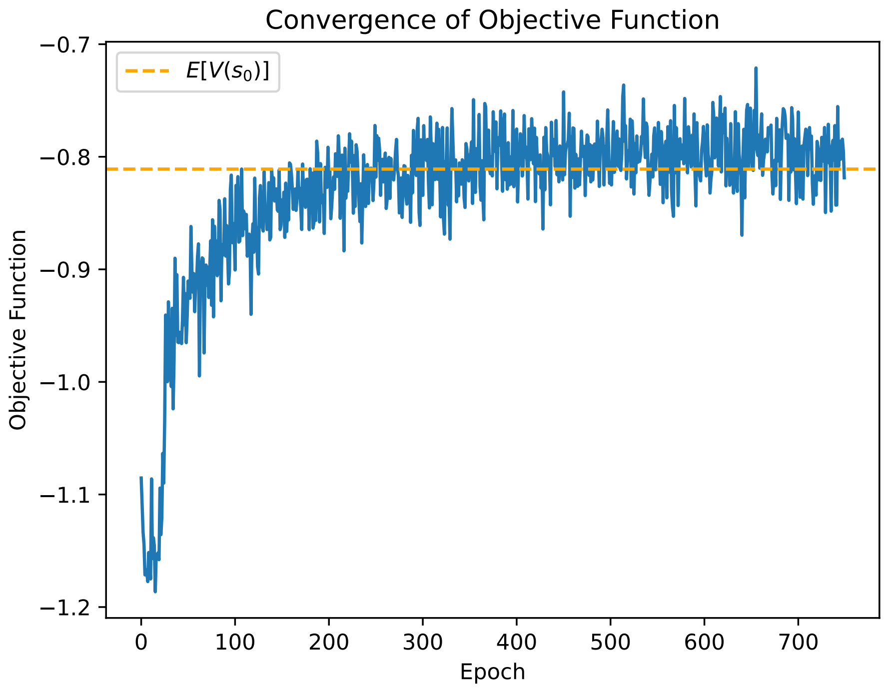

Figure 3-4: Convergence of the training algorithm.
Note: In each epoch, we use 200 sample paths (blue line). The orange dashed line denotes the value of the test data.

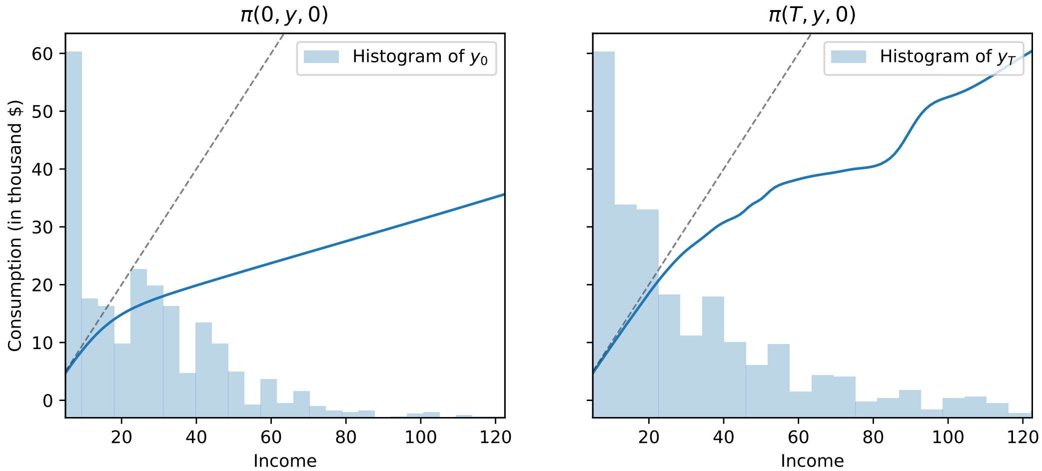

Figure 3-5: Policy functions

Note: Policy function at t=0 (25 year old) and $t = 40$ (65 year old). The histogram represents the distribution of income at that age.

represents consumption. The point where the solid blue line a_1 crosses the 45-degree line represents the dissaving threshold. Agents with less than $50,000 are closer to the borrowing limit and therefore save for the future, while wealthier agents dissave to increase present consumption.

3.3.4 The Cost of a Parameter Model for Income

In this section, we solve the model with standard techniques, which we call it "classical" method, to assess the cost of imposing a functional form for the income process. The exercise is to (i) fit an income process for y_t, (ii) solve the income-fluctuations process with classical methods, and (iii) evaluate the policy function with real data realizations. That is, we solve a model assuming a particular data-generating process for y_t but then we evaluate it with observational data.

To this end, we assume that $\log y_t$ is the sum of a deterministic component $f(t)$ that is a function of age and a disturbance η_{it} that follows an AR(1) process with persistence ρ.

$$\log y_{it} = f(t) + \eta_{it} \tag{3.6}$$

$$\eta_{it} = \rho \eta_{it-1} + \epsilon_{it}, \qquad \epsilon_{it} \sim N(0, \sigma) \tag{3.7}$$

As is standard, we fit the deterministic component of income to a polynomial of second degree

$$f(t) = \delta_0 + \delta_1 t + \delta_2 t^2. \tag{3.8}$$

The model is solved using backward iteration, which is robust and ensures that we reach a global optimum given the convexity of u. We discretize a grid of assets and income using 200 points for each dimension and approximate the normal shocks using Gauss-Hermite quadrature using 11 points to take expectations.

Table 3.1 shows the estimated parameter values, and Figure 3-6 shows the age profile, which is an inverted U, as expected. We obtain a standard deviation of the income shocks of 0.71, which is higher than the standard estimation of income processes. This is primarily due to the inclusion of zeros in the dataset to estimate the AR1. To explain big drops in income, a high volatility of the idiosyncratic shocks is required.

Parameter	Value
ρ	0.71
σ	0.71
δ_0	3.025
δ_1	0.035
δ_2	-0.000797

Table 3.1: Calibration of the income process.
Note: The income process is assumed to follow $\log y_{it} = \delta_0 + \delta_1 t + \delta_2^2 + \eta_{it}$ and $\eta_{it} = \rho\eta_{it-1} + \epsilon_{it}$

By solving the model, we obtain $\pi^{cl}_{AR1}(t, y_t, a_t)$, which is the policy function computed using the classic method when the data generating process for income is $AR1$. We can compare it with $\pi^{ag}_{AG}(t, y_t, a_t)$ which is the solution derived in Section 3.3.3, which is the policy function of the neural network when it has been trained in agnostic (model-free) data.

Figure 3-7 compares these two policy functions for randomly generated states at $t = 0$. Policies are fairly similar except for high values of consumption. At above \$30,000, the agnostic policy consumes more than the policy policy that assumes a functional process for income. Note that there is no reason to expect both policies to coincide because expectations of future income differ in both cases.

We want to quantify how much we are missing by specifying an income process versus letting the neural network learn it. Figure 3-8 shows the value of following each policy when they are evaluated using data realizations. Both policies derive fairly the same value, with insignificant differences between one and the other policy. This means that the cost of assuming a fairly simple income process is negligible.

There are two potential explanations for this unexpected result. One option is that the income process is indeed represented well by a quadratic polynomial and an AR1 residual. This is unlikely to be the case given that the deterministic trend regression has an adjusted R^2 of 0.012, while the adjusted R^2 of the regression (3.7) is 0.5. An alternative is that the income process is mostly driven by the idiosyncratic shocks. In this case, the benefit of

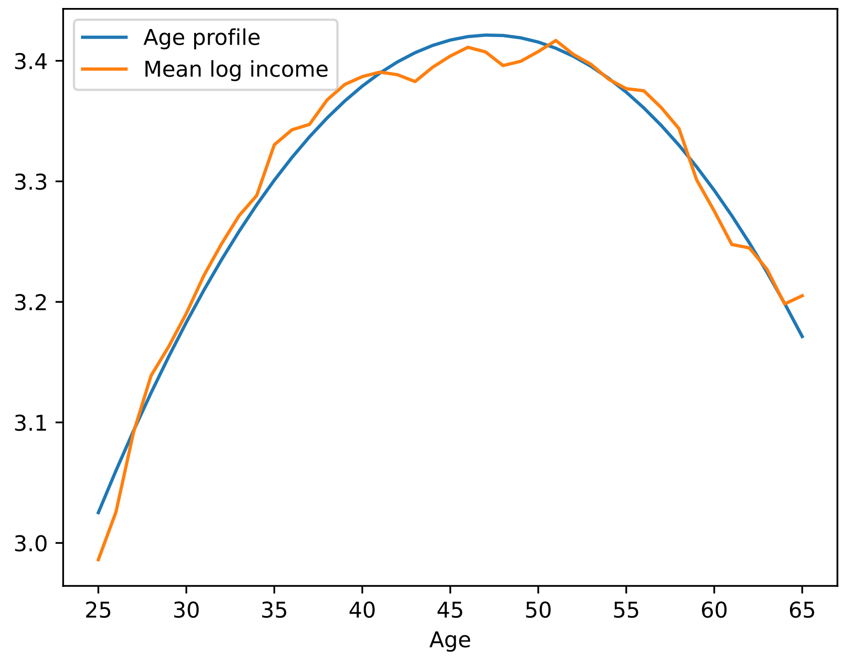

Figure 3-6: Age profile for the process of $\log y_t$

Note: Age profile obtained from the regression $\log y_{it} = b_0 + b_1 \text{Age}_i + b_2 \text{Age}_i^2 + \epsilon_{it}$

learning the true DGP of income is low, given the high noise-to-info ratio of the process.

3.3.5 Validation of Agnostic DP Solution Algorithm

In Section 3.3.3, we were agnostic about the true DGP of the process and, therefore, unsure if our methodology resulted in an optimal policy. In this section, we test whether Agnostic DP can achieve the same results as classical methods when trained with data for which we know its DGP.

To this end, we simulate data from the income process that we fitted in the previous section and use it as the input for Algorithm 1. In this case, we do know the optimal value and policy function up to the interpolation error induced in the classic solution method. Figure 3-9 shows how the agnostic method virtually achieves the global optimum in every state except those with bad realizations. This is normal since these are states that are less frequently visited by the neural network.

Overall, we can conclude that, at least for this simple model, training a model on agnostic data is as effective as solving a model with traditional methods that assume a functional form for the exogenous states.

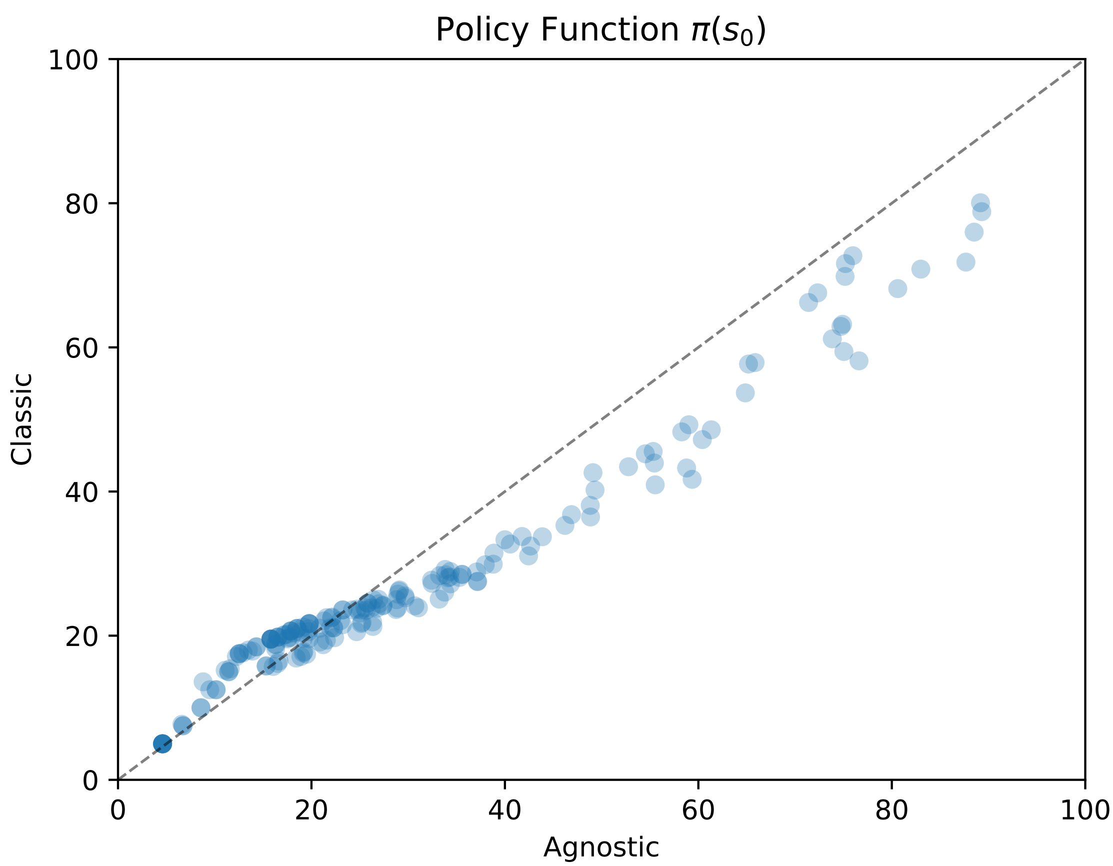

Figure 3-7: Policy function of the agnostic model and the classic model.
Note: Each dot represents a random initial state at age 25. That the agnostic policy function has been trained on agnostic data and the classic has been trained in fitted data, so they do not have to coincide.

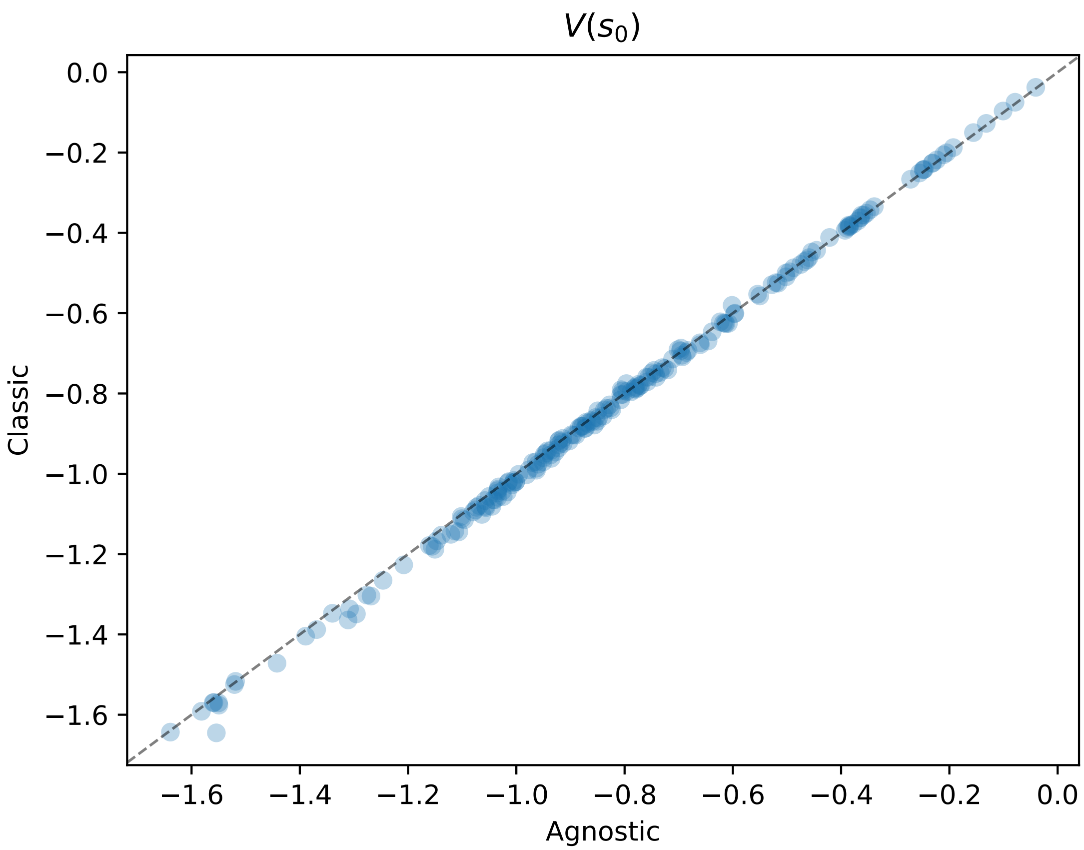

Figure 3-8: Value function comparison on real realizations.

Note: Each dot represents $E[V(s_0)]$ for a given s_0, computed with the agnostic policy function and the classic policy function. Income paths are sampled from data realizations.

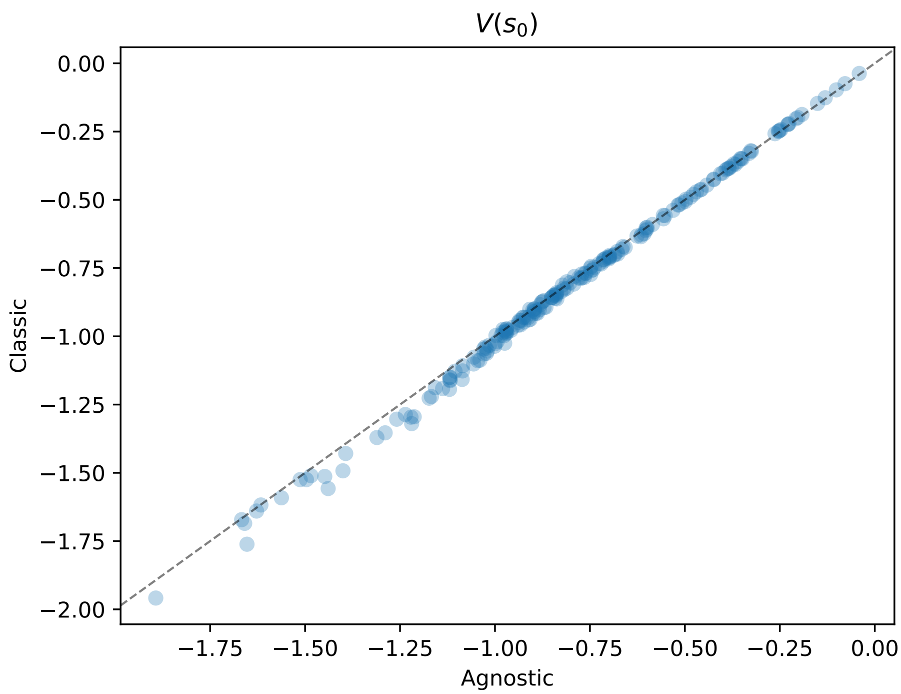

Figure 3-9: Value function comparison on specified income process.
Note: Each dot represents $E[V(s_0)]$ for a given s_0, computed with the agnostic policy function and the classic policy function. Income realizations are sampled from the income process defined in Section 3.3.4

3.4 Conclusion

In this paper, we propose a new approach for solving dynamic programming problems that does not require specifying the data-generating process for exogenous states, which we call Agnostic DP. Instead of using data to estimate a process and then solving the model in the standard way, we directly feed data realizations into the solution method, sidestepping the estimation part. Our method is more general because it allows for *any* underlying data generating process, at the cost of requiring more observations to train the model and avoid overfitting.

A clear advantage of using neural networks and gradient-based algorithms is that those naturally handle many states. We provide a `Python` package where the solution method is independent of the problem that we aim to solve. The researcher must specify a reward function $u(s_t, a_t)$, a law of motion $m(s_t, a_t, k_{t+1})$, and a neural network that takes states s_t as an input and outputs a_t, and the code does not change with the dimensionality of the states or actions.

We apply our methodology to a standard life cycle consumption-saving model. We start by proposing a method of sampling income paths using CPS data. For every observation of a given age and income bin, we sample a future realization of all individuals with the same age and income bin. While the method is not perfect, we use it for illustrational purposes. Our contribution is orthogonal to the input data or how it was created. For future work, we aim to either train the model with real income paths obtained from administrative earnings data or train a neural network to sample income realizations while being agnostic about the DGP that generates it.

Within this application, we show how to solve a dynamic programming problem while being completely agnostic about the data-generating process for exogenous states. In this application, the model generates reasonable policy functions that are more optimized in the states that are more likely to be realized. We then discuss the cost of assuming a functional form for the exogenous state and find that this cost is low for the dataset that we have generated. And finally, we confirm that our methodology does reach a global optimum by comparing the solution we obtain with Agnostic DP when we know the DGP, and the solution obtained with traditional methods, which we know is globally optimum.

This paper is a proof of concept of the power of Agnostic DP, and opens several avenues of research. A caveat of the solution method proposed is that it is defined for finite horizon problems. The problem becomes harder to optimize as we increase the horizon, partly due to discounting. Early in life, mistakes in the policy function when old have very little effect on total utility. The methodology works for solving partial equilibrium problems. A natural extension would be to close the model in general equilibrium. It is straightforward to have aggregate variables as a state variable, but satisfying market clearing conditions is not.

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Appendix A

Appendix of Monopsony and Nominal Rigidities

A.1 The firm problem in partial equilibrium

This appendix develops further the problem of an individual firm that is subject to idiosyncratic shocks. Appendix A.1.1 generalizes the model, allowing for vacancy *and* hiring costs, and drops the fairness constraints so firms can post different wages for incumbent and new hires. Appendix A.1.2 linearizes the problem of the firm around the steady state and determines the condition under which wages increase when wages increase. In the main text we discussed the effect of demand shocks, Appendix A.1.3 examines the effect of a transitory and a permanent technology shock.

A.1.1 Generalization of the monopsistic model

This section considers the problem of a firm that can set wages differentially for new workers and incumbent ones. This is the case in Kline et al. (2019) or Fukui (2020). We maintain the assumption that workers are not forward-looking, so acceptance and turnover decisions only depend on wages posted at t. A hired worker at t earns w_t^h and at $t+1$ it becomes an incumbent worker earning w_{t+1}^i. We further add explicit hiring costs that are independent of the vacancies posted $\psi(h_t, n_{t-1})$, with $\psi'(x_t) \equiv \psi_h(x, 1) = \kappa_h x_t^{\nu_h}$. The Bellman equation is

$$
\begin{aligned}
J_t(n_{t-1}) = \max_{p_t, w_t^h, w_t^i, v_t, n_t} & \frac{p_t}{P_t} z_t \mathcal{D}\left(\frac{p_t}{P_t}\right) - \frac{w_t^i}{P_t}\left(1 - \delta\left(\frac{w_t^i}{W_t}\right)\right) n_{t-1} \\
& - \frac{w_t^h}{P_t} h_t - \phi(v_t, n_{t-1}) - \psi(h_t, n_{t-1}) + \beta E_t[J_{t+1}(n_t)]
\end{aligned}
$$

$$
\begin{aligned}
n_t &= \left(1 - \delta\left(\frac{w_t^i}{W_t}\right)\right) n_{t-1} + h_t \\
z_t \mathcal{D}\left(\frac{p_t}{P_t}\right) &= f(n_t) \\
h_t &= a\left(\frac{w_t^h}{W_t}\right) v_t
\end{aligned}
$$

The first order condition with respect to vacancies is.

$$
\mu_t = \frac{w_t^h}{P_t} + \frac{\phi_v(v_t, n_{t-1})}{a\left(\frac{w_t^h}{W_t}\right)} + \psi_h(h_t, n_{t-1}).
$$

μ_t is the Lagrange multiplier of the employment law of motion. A new hire costs (i) its wage, (ii) the cost of finding him and (iii) the cost of training him. The first order condition for new hire wage gives:

$$
\frac{1}{P_t} h_t = \frac{\phi_v(v_t, n_{t-1})}{a\left(w_t^h / W_t\right)} a'\left(\frac{w_t^h}{W_t}\right) \frac{1}{W_t} v_t
$$

This condition is similar to (1.3). Raising the new hire wage implies paying all the new hires dw_t^h, but it attracts $a'\left(\frac{w_t^h}{W_t}\right) \frac{1}{W_t} v_t dw_t^h$ workers. Importantly, post-match hiring costs do not affect the first order condition for new hire wages, because they are the same regardless of the wage. If the firm wants to reduce training costs, it has to do it through reducing quits. Multiply by w_t^h to get a simpler expression

$$
\frac{w_t^h}{P_t} = \frac{\phi_v(v_t, n_{t-1})}{a\left(\frac{w_t^h}{W_t}\right)} \epsilon_a.
$$

This allows to write the value of a worker as

$$
\mu_t = \frac{\epsilon_a + 1}{\epsilon_a} \frac{w_t^h}{P_t} + \psi_h(h_t, n_{t-1}).
$$

Hiring an extra worker implies paying all the new hires a little more and training the worker.

The first-order condition for the incumbent wage is

$$\frac{1}{P_t}\left(1-\delta\left(\frac{w_t^i}{W_t}\right)\right)n_{t-1}+\frac{w_t^i}{P_t}\delta'\left(\frac{w_t^i}{W_t}\right)\frac{1}{W_t}n_{t-1}=\mu_t\left(-\delta'\left(\frac{w_t^i}{W_t}\right)\frac{1}{W_t}n_{t-1}\right)$$

Increasing the incumbent wage requires paying incumbents this amount plus paying those who would have otherwise left. The firm does not need to hire that many new workers in exchange. We can rearrange to get an expression that relates incumbent wages and new hire wages

$$\frac{w_t^i}{P_t}=\frac{\epsilon_\delta\delta\left(\frac{w_t^i}{W_t}\right)}{1-\delta\left(\frac{w_t^i}{W_t}\right)+\epsilon_\delta\delta\left(\frac{w_t^i}{W_t}\right)}\left(\frac{\epsilon_a+1}{\epsilon_a}\frac{w_t^h}{P_t}+\psi_h(h_t,n_{t-1})\right)$$

Whether incumbent workers are paid more than new hires is unclear, since the term $\frac{\epsilon_\delta}{\epsilon_\delta+\frac{1-\delta(w_t^i/W_t)}{\delta(w_t^i/W_t)}}$ is less than one and the term $\frac{\epsilon_a+1}{\epsilon_a}$ is more than one. They will be paid more if training costs are relevant (high $\phi_h(h_t,n_{t-1})$.

The pricing condition is the same as in the case with fairness constraints, but now the relevant wage is the new hire wage.

$$\frac{p_t}{P_t}=\mathcal{M}_p\frac{\frac{w_t^h}{P_t}+\frac{\phi_v(v_t,n_{t-1})}{a(w_t^h/W_t)}+\psi_h(h_t,n_{t-1})-\beta J'_{t+1}(n_t)}{f'(n_t)}.$$

Applying the envelope condition, we have that

$$\begin{aligned}J'_t(n_{t-1})=&\left(\frac{w_t^h}{P_t}-\frac{w_t^i}{P_t}+\frac{\phi_v(v_t,n_{t-1})}{a(w_t^h/W_t)}+\psi_h(h_t,n_{t-1})\right)\left(1-\delta\left(\frac{w_t^i}{W_t}\right)\right)\\&+\frac{\nu}{1+\nu}\phi'\left(\frac{v_t}{n_{t-1}}\right)\frac{v_t}{n_{t-1}}+\frac{\nu_h}{1+\nu_h}\phi'\left(\frac{h_t}{n_{t-1}}\right)\frac{h_t}{n_{t-1}}.\end{aligned}$$

This equation is similar to (1.5), the only new term is $\frac{w_{t+1}^h}{P_t}-\frac{w_{t+1}^i}{P_t}$. The firm understands that a hired worker at t becomes an incumbent worker at $t+1$, and is paid differently.

In steady state, we have that $\delta\left(\frac{w_t^i}{W_t}\right)n_t=a\left(\frac{w_t^h}{W_t}\right)v_t$. Incumbent and new hire wages solve the following system of equations:

$$\frac{w^h}{P}=\frac{\phi'\left(\frac{\delta(w^i/W)}{a(w^h/W)}\right)}{a(w^h/W)}\epsilon_a$$

$$\frac{\epsilon_\delta+\frac{1-\delta\left(w^i/W\right)}{\delta(w^i/W)}}{\epsilon_\delta}\frac{w^i}{P}=\frac{\epsilon_a+1}{\epsilon_a}\frac{w^h}{P}+\psi'(\delta(w^i/W))$$

As in the main text, wages in steady state are independent of firm productivity or firm demand.

A.1.2 Linearization of the firm problem with idiosyncratic shocks

In this section, we linearize the firm problem, taking as constant the demand function $\mathcal{D}(\cdot)$, the production function $f(\cdot)$ and the quit and acceptance probabilities $\delta(\cdot)$ and $a(\cdot)$. Abusing notation denote by δ and a without an argument the value of the turnover and acceptance rate in steady state. The market clearing $z_t\mathcal{D}(p_t) = f(n_t)$ condition becomes

$$\hat{z}_t - \epsilon_p \hat{p}_t = \alpha \hat{n}_t.$$

$\alpha \equiv \frac{d \log f}{d \log n}$ is the output elasticity with respect to labor. The law of motion of employment $n_t = (1 - \delta_t(w_t))\, n_{t-1} + a_t(w_t)v_t$, once linearized becomes

$$\hat{n}_t = (1-\delta)\hat{n}_{t-1} + \delta((\epsilon_\delta + \epsilon_a)\hat{w}_t + \hat{v}_t).$$

The wage first order condition $n_t = \frac{\phi'\left(\frac{v_t}{n_{t-1}}\right)}{a_t(w_t)} \left(-\delta'(w_t)n_{t-1} + a_t'(w_t)v_t\right)$ is less strightforward to linearize.

$$\hat{n}_t = \nu(\hat{v}_t - \hat{n}_{t-1}) - \epsilon_a \hat{w}_t + \frac{\epsilon_\delta}{\epsilon_a + \epsilon_\delta}\left(\frac{\delta''}{\delta'} w\hat{w} + \hat{n}_{t-1}\right) + \frac{\epsilon_a}{\epsilon_a + \epsilon_\delta}\left(\frac{a''}{a'} w\hat{w}_t + \hat{v}_t\right)$$

were we have used that in steady state $\frac{-\delta' n}{-\delta' n + a' v} = \frac{-\delta' n}{n\frac{a}{\phi'}} = \frac{-\delta'}{\delta(\epsilon_a + \epsilon_\delta)\frac{1}{w}} = \frac{\epsilon_\delta}{\epsilon_a + \epsilon_\delta}$ and a similar argument for the term $\frac{\epsilon_a}{\epsilon_a + \epsilon_\delta}$. Define the superelasticitites $\epsilon_{\delta\delta}$ and ϵ_{aa} as

$$\epsilon_{\delta\delta} \equiv \frac{\delta''(w/P)}{\delta'(w/P)}\frac{w}{P} + 1 + \epsilon_\delta, \qquad \epsilon_{aa} \equiv \frac{a''(w/P)}{a'(w/P)}\frac{w}{P} + 1 - \epsilon_a,$$

and substitute it into the linearized condition to get

$$\hat{n}_t = \nu(\hat{v}_t - \hat{n}_{t-1}) - \epsilon_a \hat{w}_t + \frac{\epsilon_\delta}{\epsilon_a + \epsilon_\delta}\left((\epsilon_{\delta\delta} - 1 - \epsilon_\delta)\hat{w} + \hat{n}_{t-1}\right) + \frac{\epsilon_a}{\epsilon_a + \epsilon_\delta}\left((\epsilon_{aa} - 1 + \epsilon_a)\hat{w}_t + \hat{v}_t\right).$$

For the CES case, $\epsilon_{\delta\delta}$ and ϵ_{aa} are equal to zero. Constant elasticity implies that $\delta(w)$ is convex. This implies that as wages increase, the benefit of doing so in terms of reducing quits diminishes. For $a(w)$, convexity or concavity depends on ϵ_a being larger or smaller than one. A convex a implies that the benefit of raising wages increases with the wage itself. If we want to consider the case for $a(w)$ with elasticity larger than 1, but concave, then ϵ_{aa}

must be negative and satisfy $\epsilon_{aa} \leq -(\epsilon_a - 1)$. We can rearrange the equation and obtain

$$\hat{n}_t = \left(\nu + \frac{\epsilon_a}{\epsilon_a + \epsilon_\delta}\right)\hat{v}_t - (1+\epsilon_\delta)\hat{w}_t + \left(\frac{\epsilon_\delta}{\epsilon_a + \epsilon_\delta} - \nu\right)\hat{n}_t$$

where we have used that $\frac{-\epsilon_a(\epsilon_a+\epsilon_\delta)-\epsilon_\delta(1+\epsilon_\delta)+\epsilon_a(\epsilon_a-1)}{\epsilon_a+\epsilon_\delta} = -(1+\epsilon_\delta)$. It might come at a surprise that ϵ_a does not interact with $\hat{w}_t$. This is because as the wage increases, the value of a new worker is reduced because now it is easier to hire, but at the same time the vacancy yield is increased by the same amount. Both effects cancel out.

Finally we linearize the price setting equation

$$p_t = \mathcal{M}_p \frac{w_t + \frac{\phi'\left(\frac{v_t}{n_{t-1}}\right)}{a_t(w_t/W_t)} - \beta E_t\left[\left(1 - \delta_{t+1}\left(\frac{w_{t+1}}{W_{t+1}}\right)\right)\frac{\phi'\left(\frac{v_{t+1}}{n_t}\right)}{a_{t+1}(w_{t+1}/W_{t+1})} + \frac{\nu}{1+\nu}\phi'\left(\frac{v_{t+1}}{n_t}\right)\frac{v_{t+1}}{n_t}\right]}{f_t'(n_t)},$$

We can divide the problem into two. The price is a weighted average of the wage costs and the net hiring costs

$$\hat{p}_t = \tau\hat{w}_t + (1-\tau)\hat{\gamma}_t - (1-\alpha)\hat{n}_t.$$

The linearized hiring costs are

$$\hat{\gamma}_t = \frac{1}{1-\tilde{\beta}}\left(\hat{\mu}_t - \beta(1-\delta)(\hat{\mu}_{t+1} + \frac{\delta}{1-\delta}\epsilon_\delta\hat{w}_{t+1}) + \frac{\beta\nu\delta}{1+\nu}((1+\nu)(\hat{v}_{t+1} - \hat{n}_t)\right),$$

where $\hat{\mu}_t = \nu(v_t - \hat{n}_{t-1}) - \epsilon_a\hat{w}_t$ is the value of a hire. The expression can be rearranged to

$$\hat{\gamma}_t = \frac{1}{1-\tilde{\beta}}\left(\hat{\mu}_t - \beta\left((1-\delta)\hat{\mu}_{t+1} + \delta\epsilon_\delta\hat{w}_{t+1} + \nu\delta(\hat{v}_{t+1} - \hat{n}_t)\right)\right).$$

The solution of the firm problem is the following system of dynamic equations

$$\hat{z}_t - \epsilon_p\hat{p}_t = \alpha\hat{n}_t$$

$$\hat{n}_t = (1-\delta)\hat{n}_{t-1} + \delta((\epsilon_\delta + \epsilon_a)\hat{w}_t + \hat{v}_t)$$

$$\hat{n}_t = \left(\nu + \frac{\epsilon_a}{\epsilon_a + \epsilon_\delta}\right)\hat{v}_t - (1+\epsilon_\delta)\hat{w}_t + \left(\frac{\epsilon_\delta}{\epsilon_a + \epsilon_\delta} - \nu\right)\hat{n}_{t-1}$$

$$\hat{p}_t = \tau\hat{w}_t + \frac{1-\tau}{1-\tilde{\beta}}\left(\hat{\mu}_t - \beta\left((1-\delta)\hat{\mu}_{t+1} + \delta\epsilon_\delta\hat{w}_{t+1} + \nu\delta(\hat{v}_{t+1} - \hat{n}_t)\right)\right) - (1-\alpha)\hat{n}_t$$

We can derive Lemma 1 using the law of motion of employment and the optimal wage setting to derive wages and vacancies when the firm grows Δn_t

$$\hat{w}_t = \frac{\nu - \delta + \frac{\epsilon_a}{\epsilon_a + \epsilon_\delta}}{\delta((1+\nu)(\epsilon_a + \epsilon_\delta) + 1)} \Delta \hat{n}_t \tag{A.1}$$

$$\hat{v}_t = \frac{\delta(\epsilon_a + \epsilon_\delta) + (\epsilon_\delta + 1)}{\delta((1+\nu)(\epsilon_a + \epsilon_\delta) + 1)} \Delta \hat{n}_t. \tag{A.2}$$

From here we see that Assumption 1 imposes that wages share the same sign as firm growth. Dividing both equations, we get an expression of $\hat{v}_t$ as a function of $\hat{w}_t$,

$$\hat{v}_t = \frac{\delta \epsilon_w + \epsilon_\delta + 1}{\nu + \frac{\epsilon_a}{\epsilon_w} - \delta} \hat{w}_t.$$

When vacancies and wages are optimally set, they move together. Moreover, if $\epsilon_a = \epsilon_\delta$, then $\frac{\epsilon_a}{\epsilon_w} = \frac{1}{2}$ and an increase in monosony power implies that a given increase in employment requires a larger wage change and it has an ambigous effect on vacancies. On one side, a higher wage change attracts more workers, but on the other, more monopsony implies that the effect of the wage change is muted.

Proof of Proposition 2

Proposition 2 has two parts. We start by showing that in response to a demand shock, the wage response is increasing in *product* market power.

By setting $\beta = 0$, and $\epsilon_w = \epsilon_a + \epsilon_\delta$, the price setting condition becomes

$$\hat{p}_t = \frac{\delta \epsilon_w}{\delta \epsilon_w + 1} \hat{w}_t + \frac{1}{\delta \epsilon_w + 1} \left(\nu \hat{v}_t - \epsilon_a \hat{w}_t \right),$$

where we have already substituted τ. Substituting $\hat{v}_t$ we get an expression for the price that only depends on the wage

$$\hat{p}_t = \frac{1}{\delta \epsilon_w + 1} \left(\delta \epsilon_w + \nu \frac{\delta \epsilon_w + \epsilon_\delta + 1}{\nu + \frac{\epsilon_a}{\epsilon_w} - \delta} - \epsilon_a \right) \hat{w}_t,$$

Similarly, we can write the market clearing condition plus the expression of the wage change given $\hat{n}_t$ to have:

$$\hat{z}_t - \epsilon_p \hat{p}_t = \alpha \delta \frac{(1+\nu)\epsilon_w + 1}{\nu + \frac{\epsilon_a}{\epsilon_w} - \delta} \hat{w}_t$$

We have a system of two equations and two unknowns, $\hat{p}_t, \hat{w}_t$, given the demand shock $\hat{z}_t$. We can solve this system graphically, for two values of product market power ϵ_p and $\epsilon_p' < \epsilon_p$, as Figure A-1 shows. Market power only affects the market clearing equation. The resulting

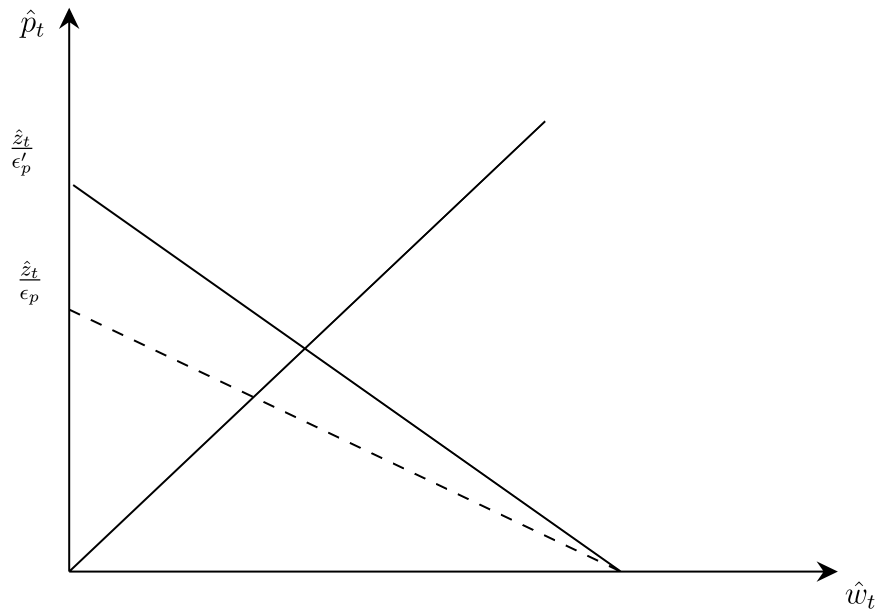

Figure A-1: Effect of product market power.

Note: The dashed line corresponds to a firm with low market power ($\epsilon_p > \epsilon'_p$). In response to a demand shock, both prices and wages increase by more in the case of a firm with high market power.

effect is an increase in both prices and wages. It remains to ensure that the pricing equation is upward sloping, which is the case for ν sufficiently large.

Now we show the polar case. The price response to a demand shock is increasing in labor market power. For that, we write the price setting condition as a function of the employment growth $\hat{n}_t$, using the inverse of equations (A.1) and (A.2). We get an expression of the form:

$$\hat{p}_t = \frac{1}{\delta\epsilon_w + 1}\frac{1}{\delta}\frac{1}{(1+\nu)\epsilon_w + 1}\left((\delta\epsilon_w - \epsilon_a)\left(\nu + \frac{\epsilon_a}{\epsilon_w} - \delta\right) + \nu\left(\delta\epsilon_w + \epsilon_\delta + 1\right)\right)\hat{n}_t$$

Together with the market clearing condition, Figure A-2 shows the graphical solution of the system. The condition that makes prices and employment comove is the same as the condition that makes wages and prices comove. Now, an increase in monopsony power shifts the curve counter-clock wise. The resulting equilibrium features a higher price response and lower employment. However, the wage response is ambiguous because even though employment responds by less, by (A.1), wages respond by more given an employment growth level.

A.1.3 Response to an idiosyncratic technology shock

This section compares the monopsonistic response to a technology shock. Figure A-3 shows the evolution of wages, prices and employment, and compares it with a competitive wage

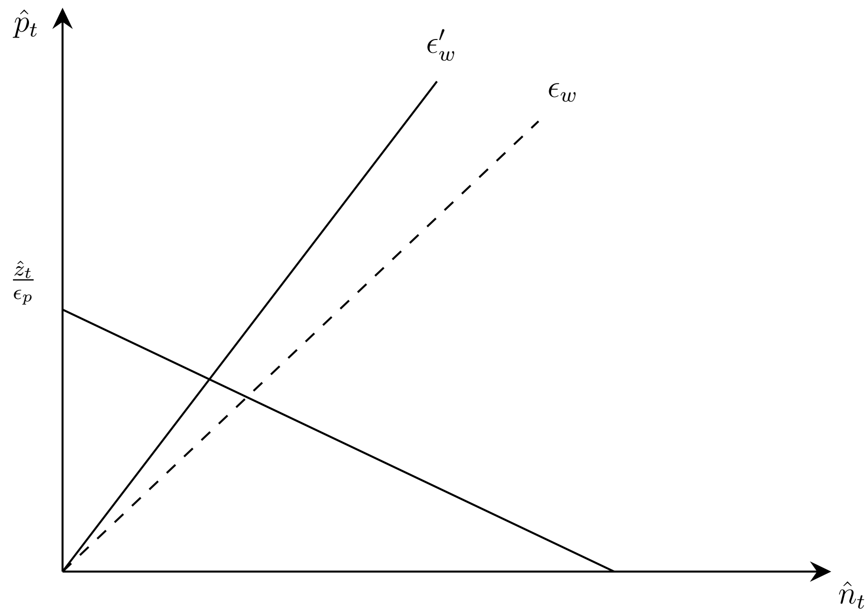

Figure A-2: Effect of labor market power.
Note: The dashed line corresponds to a firm with low market power ($\epsilon_w > \epsilon'_w$). In response to a demand shock, prices increase and employment decreases.

benchmark. Then the firm becomes more productive, marginal cost decreases and the firm lowers its price. This induces more demand that needs to be satisfied. Given Assumption 1, employment growth calls for higher wages. Both wages and higher hiring costs increase marginal costs, which raises prices.

In comparison, if the input market is perfectly competitive, all these effects are absent. Wages are constant and marginal costs do not increase due to the assumption of constant returns to scale. Adding decreasing returns to scale would have the same effect in both models, it would increase marginal costs. We don't add them here for a cleaner comparison. Figure A-4 plots the response to a permanent technology shock. When the firm needs to grow it does so by raising wages, but as employment growth slows down, wages go back to the steady state, since those are independent of firm characteristics. This contrasts with other models of monopsony where a permanent technology shock leads to a permanent increase in wages.

Wage Rigidity

Here we show how any constraint on wage updating implies a larger price response when the firm is hit by a demand shock. To do so, we solve the problem of the firm (1.1),(1.2) and (1.3), dropping (1.5) and specifying exogenously the wage process. We assume that $\beta = 0$ and Assumption 1 is satisfied. Then we can get vacancies required to reach an employment

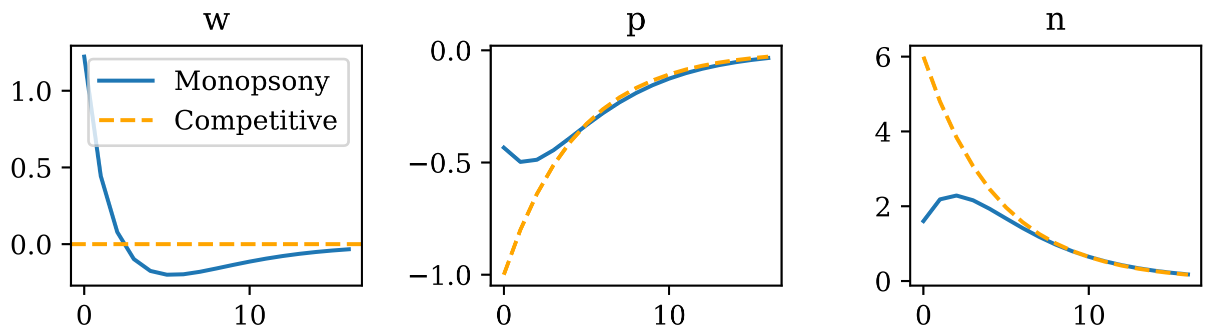

Figure A-3: Response to a temporary technology shock

Note: Transitory productivity shock that increases productivity by 1%. The blue solid line represent the monopsony case, and the yellow dashed line represents the competitive benchmark.

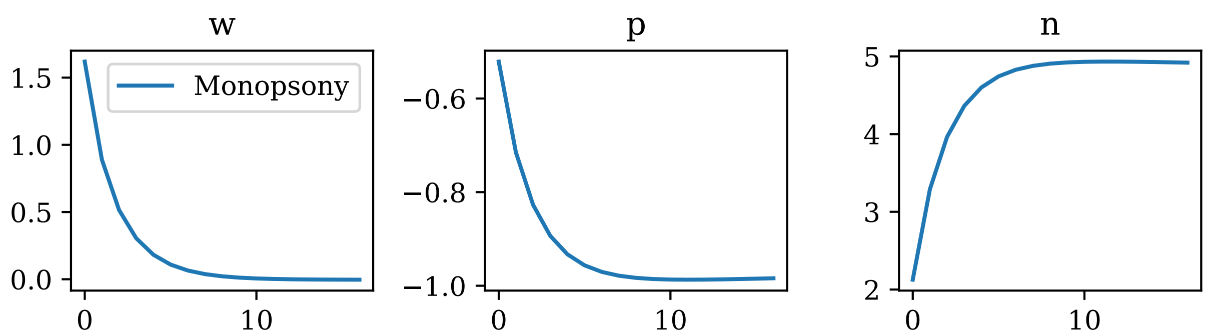

Figure A-4: Response to a permanent productivity shock level $\hat{n}_t$ (when $\hat{n}_{t-1} = 0$ as

$$\hat{v}_t = \frac{1}{\delta}\hat{n}_t - \epsilon_w \hat{w}_t = \frac{1}{\delta}(\hat{z}_t - \epsilon_p \hat{p}_t) - \epsilon_w \hat{w}_t.$$

In the second equality, we have imposed the market clearing condition. Then we can get the pricing condition as a function of the demand shock and the posted wage,

$$\left(\delta\epsilon_w + 1 + \frac{\nu\epsilon_p}{\delta}\right)\hat{p}_t = \frac{\nu}{\delta}\hat{z}_t + \left(\delta - \nu - \frac{\epsilon_a}{\epsilon_w}\right)\hat{w}_t. \tag{A.3}$$

From (A.3), we see that the sign of the coefficient on the wage is positive since it coincides with Assumption 1. Thus, an increase in the wage reduces the price response of a demand shock.

A.2 Derivations for the general equilibrium model

This appendix derives the firm problem and the linearized Phillips curves of Proposition 3. Appendix A.2.1 solves the problem of the firm in general equilibrium and derives the non-linear Phillips curves. Appendix A.2.2 discusses the steady state and Appendix A.2.3 linearizes the Phillips curves and expresses them in observable labor market variables. Finally A.2.4 considers the case when hiring, and not posting vacancies, is costly.

A.2.1 Non-linear problem

We start with the firm problem, allowing for a general production function $y_t = A_t f(n_t)$. The Bellman equation at t is

$$\begin{aligned} J_t(x_{t-1}) = \max_{p_t, w_t, v_t, n_t} & \frac{p_t}{P_t}\mathcal{D}_t\left(\frac{p_t}{P_t}\right) - \frac{w_t}{P_t}n_t - \kappa_v v_t \\ & - \frac{\kappa_p}{2}\left(\frac{p_t}{p_{t-1}} - 1\right)^2 Y_t - \frac{\kappa_w}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 N_t + E_t[\Lambda_{t,t+1}J_{t+1}(x_t)] \end{aligned}$$

subject to:

$$n_t = \left(1 - \tilde{\delta}_t\left(\frac{w_t}{W_t}\right)\right)n_{t-1} + a_t\left(\frac{w_t}{W_t}\right)v_t$$

$$\mathcal{D}_t\left(\frac{p_t}{P_t}\right) = A_t f(n_t)$$

Wage Setting Let μ_t be the Lagrange multiplier of the law of motion for employment and λ_t be the Lagrange multiplier of the constraint on the production function. The former is the value of a worker and the latter the real marginal cost. Taking a first-order condition with respect to v_t we get

$$\mu_t = \frac{\kappa_v}{a_t\left(\frac{w_t}{W_t}\right)}.$$

The first order condition with respect to the wage is

$$\begin{aligned}\frac{1}{P_t}n_t + \kappa_w\left(\frac{w_t}{w_{t-1}} - 1\right)N_t\frac{1}{w_{t-1}} - \kappa_w E_t\left[\Lambda_{t,t+1}\left(\frac{w_{t+1}}{w_t} - 1\right)\frac{w_{t+1}}{w_t^2}N_{t+1}\right] \\ = \mu_t\left(-\tilde{\delta}_t'\left(\frac{w_t}{W_t}\right)\frac{1}{W_t}n_{t-1} + a_t'\left(\frac{w_t}{W_t}\right)\frac{1}{W_t}v_t\right).\end{aligned}$$

Multiply the equation by W_t, divide it by N_t, and apply symmetry so $w_t = W_t$ and $n_t = N_t$ to obtain

$$\omega_t + \kappa_w\left(\Pi_t^w - 1\right)\Pi_t^w - \kappa_w E_t\left[\Lambda_{t,t+1}\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w\frac{N_{t+1}}{N_t}\right] = \frac{\kappa_v}{a_t(1)}\left(-\tilde{\delta}_t'(1)\frac{N_{t-1}}{N_t} + a_t'(1)\frac{V_t}{N_t}\right).$$

Plug $\delta_t'(1)$ and $a_t'(1)$

$$\omega_t + \kappa_w\left(\Pi_t^w - 1\right)\Pi_t^w - \kappa_w E_t\left[\Lambda_{t,t+1}\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w\frac{N_{t+1}}{N_t}\right] = \frac{\kappa_v}{a_t(1)}\left(\frac{V_t}{N_t}q(\theta_t)p_t^E f_\varepsilon(1) + q(\theta_t)f_\varepsilon(1)\frac{V_t}{N_t}\right)$$

Rearrange to get the non-linear wage Phillips curve:

$$(\Pi_t^w - 1)\Pi_t^w = \frac{1}{\kappa_w}\left(2\frac{\kappa_v}{a_t(1)}\frac{V_t}{N_t}q(\theta_t)p_t^E f_\varepsilon(1) - \omega_t\right) + E_t\left[\Lambda_{t,t+1}\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w\frac{N_{t+1}}{N_t}\right]$$

Now, we can get the acceptance elasticity by noting that

$$\frac{q(\theta_t)p_t^E f_\varepsilon(1)}{a_t(1)} = \frac{a_t'(1)\frac{1}{W}}{a_t(1)}W = \epsilon_{a,t}.$$

To relate it to the quit elasticity, we can use the fact that in steady state, $a(1)V = \tilde{\delta}(1)N$

$$\frac{q(\theta)p^E f_\varepsilon(1)}{a(1)} = \frac{\frac{q(\theta)p^E f_\varepsilon(1)V}{N}}{\tilde{\delta}(1)} = \frac{\tilde{\delta}'(1)\frac{1}{W}}{\tilde{\delta}}W = \epsilon_{\tilde{\delta}}$$

Out of the steady state, both elasticities do not coincide. Instead, we have that

$$\epsilon_{\tilde{\delta},t} \equiv \frac{\frac{V_t q(\theta_t) p_t^E f_\varepsilon(1)}{N_{t-1}}}{\tilde{\delta}_t(1)} = \frac{V_t q(\theta_t) p_t^E f_\varepsilon(1)}{a_t(1) V_t \left(1 - \frac{N_t - N_{t-1}}{N_t - (1-\tilde{\delta}_t(1)) N_{t-1}}\right)} = \left(1 + \frac{1}{\tilde{\delta}_t(1)} \frac{N_t - N_{t-1}}{N_{t-1}}\right) \epsilon_{t,a}.$$

With that, we get the non-linear wage Phillips curve

$$(\Pi_t^w - 1)\Pi_t^w = \frac{1}{\kappa_w}\left(2\kappa_v \epsilon_{a,t} \frac{V_t}{N_t} - \omega_t\right) + E_t\left[\Lambda_{t,t+1}\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w \frac{N_{t+1}}{N_t}\right].$$

Price Setting The first order condition for prices is standard for a model with Rotemberg rigidities is

$$\begin{aligned} \frac{1}{P_t}\mathcal{D}_t\left(\frac{p_t}{P_t}\right) + &\frac{p_t}{P_t}\mathcal{D}_t'\left(\frac{p_t}{P_t}\right)\frac{1}{P_t} - \kappa_p\left(\frac{p_t}{p_{t-1}} - 1\right)\frac{1}{p_{t-1}} Y_t + \\ &E_t\left[\Lambda_{t,t+1}\kappa_p\left(\frac{p_{t+1}}{p_t} - 1\right)\frac{p_{t+1}}{p_t^2} Y_{t+1}\right] - \lambda_t \mathcal{D}_t'\left(\frac{p_t}{P_t}\right)\frac{1}{P_t} = 0. \end{aligned}$$

λ_t is the Lagrange multiplier of the market clearing constraint and represent marginal costs. Multiply by $\frac{P_t}{Y_t}$ and apply symmetry:

$$1 - \epsilon_p - \kappa_p\left(\Pi_t^p - 1\right)\Pi_t^p + E_t\left[\Lambda_{t,t+1}\kappa_p\left(\Pi_{t+1}^p - 1\right)\Pi_{t+1}^p \frac{Y_{t+1}}{Y_t}\right] + \epsilon_p \lambda_t = 0$$

And rearrange to get the non-linear price Phillips curve

$$(\Pi_t^p - 1)\Pi_t^p = \frac{1}{\kappa_p}\left(1 - \epsilon_p + \epsilon_p \lambda_t\right) + E_t\left[\Lambda_{t+1}(\Pi_{t+1}^p - 1)\Pi_{t+1}^p \frac{Y_{t+1}}{Y_t}\right].$$

Marginal Costs Finally λ_t, marginal costs, are obtained from the first order condition with respect to n_t,

$$-\frac{w_t}{P_t} - \mu_t + \lambda_t A_t f'(n_t) + E_t\left[\Lambda_{t,t+1}\left(1 - \tilde{\delta}_{t+1}\left(\frac{w_{t+1}}{W_{t+1}}\right)\right)\mu_{t+1}\right] = 0,$$

and rearranging we get that marginal costs are

$$\lambda_t = \frac{\omega_t + \frac{\kappa_v}{a_t(1)} - E_t\left[\Lambda_{t,t+1}(1 - \tilde{\delta}_t(1))\frac{\kappa_v}{a_{t+1}(1)}\right]}{A_t f'(n_t)}.$$

A.2.2 The steady state

We drop the t subindex to denote steady state variables. The pricing equation in steady state is

$$1 = \mathcal{M}_p \frac{\omega + (1-\beta(1-\tilde{\delta}))\frac{\kappa_v}{a}}{Af'(n)}.$$

The wage setting condition, using the fact that in steady state $\epsilon_a = \epsilon_{\tilde{\delta}}$, and $\frac{V}{N} = \frac{\tilde{\delta}}{a}$ we get

$$\omega = \frac{\kappa_v}{a} 2\tilde{\delta}\epsilon_{\tilde{\delta}}.$$

Combining both equations we get the real wage in steady state

$$\omega = \frac{1}{\mathcal{M}_p} \frac{2\tilde{\delta}\epsilon_{\tilde{\delta}}}{2\tilde{\delta}\epsilon_{\tilde{\delta}} + 1 - \tilde{\beta}} Af'(n).$$

This equation contrasts with the first-order condition for an individual firm that sets wages (Equation 1.4). The real wage is below the marginal product of labor for two reasons. Market power adds a markup over marginal costs, and search frictions add a wage markdown $\mathcal{M}_w = \frac{2\tilde{\delta}\epsilon_{\tilde{\delta}}}{2\tilde{\delta}\epsilon_{\tilde{\delta}}+1-\tilde{\beta}}$. As in Proposition 1 shows, this markdown also coincides with the wage share over the total cost of hiring a worker

$$\tau \equiv \frac{\omega}{\omega + (1-\tilde{\beta})\frac{\kappa_v}{a}} = \frac{2\tilde{\delta}\epsilon_{\tilde{\delta}}}{2\tilde{\delta}\epsilon_{\tilde{\delta}} + 1 - \tilde{\beta}}.$$

A.2.3 Linearizing the Phillips curves

Wage Phillips curve Here we derive Proposition 3. We start with the wage Phillips curve. First we start noting that the term inside the parenthesis in equation A.2.1 can be written as:

$$\frac{\kappa_v}{a_t(1)} \frac{V_t}{N_t} q(\theta_t) 2p_t^E f_\varepsilon(1) = \frac{\kappa_v}{a_t(1)} \frac{N_t - (1-\bar{\delta})N_{t-1}}{N_t} 2\frac{p_t^E}{p_t^U} f_\varepsilon(1),$$

after using the law of motion of aggregate employment $N_t = (1-\bar{\delta})N_{t-1} + V_t q(\theta_t) p_t^U$ and substituting $V_t q(\theta_t)$. Also, we have that

$$\frac{EE_t}{UE_t} = \frac{V_t q(\theta_t) p_t^E (1 - F_\varepsilon(1))}{V_t q_t p_t^U} = \frac{p_t^E}{p_t^U}(1 - F_\varepsilon(1)), \tag{A.4}$$

where EE_t and UE_t are the employment-to-employment flows and the unemployment-to-employment transitions. This means that $\hat{p}_t^E - \hat{p}_t^U = \hat{EE}_t - \hat{UE}_t$ and $\hat{a}_t(1) = \hat{H}_t - \hat{V}_t$. With that, we get the wage Phillips curve linearized and expressed as a function of labor market variables. The time to fill $\hat{V}_t - \hat{H}_t$, employment growth, the $\frac{EE}{UE}$ ratio, and the deviation from the real wage.

$$\pi_t^w = \frac{\omega}{\kappa_w}\left(\hat{V}_t - \hat{H}_t + \frac{1-\bar{\delta}}{\bar{\delta}}\Delta\hat{N}_t + \hat{EE}_t - \hat{UE}_t - \hat{\omega}_t\right) + \beta E_t \pi_{t+1}^w$$

Using that $\omega = \frac{\tau}{\mathcal{M}_p}$ (here we assume that production is linear) we can express the slope of the curve as a function of product and labor market power.

$$\pi_t^w = \frac{1}{\kappa_w}\frac{\tau}{\mathcal{M}_p}\left(\hat{V}_t - \hat{H}_t + \frac{1-\bar{\delta}}{\bar{\delta}}\Delta\hat{N}_t + \hat{EE}_t - \hat{UE}_t - \hat{\omega}_t\right) + \beta E_t \pi_{t+1}^w$$

Price Phillips Curve The price Phillips linearized is standard for any model with Rotemberg costs,

$$\pi_t^p = \frac{\epsilon_p}{\kappa_p}\hat{\lambda}_t + \beta E_t[\pi_{t+1}^p].$$

And marginal costs are

$$\hat{\lambda}_t = \tau\hat{\omega}_t + (1-\tau)\left(\frac{1}{1-\tilde{\beta}}(\hat{V}_t - \hat{H}_t) - \frac{\tilde{\beta}}{1-\tilde{\beta}}\left(\hat{V}_{t+1} - \hat{H}_{t+1} - \frac{\tilde{\delta}}{1-\tilde{\delta}}\hat{\tilde{\delta}}_{t+1}\right)\right) - \hat{MPL}_t$$

$\hat{MPL}_t$ is the log deviation of the marginal product of labor, which can be expressed as

$$\hat{MPL}_t = \hat{A}_t - (1-\alpha)\hat{N}_t.$$

$\alpha \equiv \frac{d\log f(n_t)}{d\log n_t}$ is a measure of decreasing returns to scale. And we get the Equation (1.12)

$$\pi_t^p = \frac{\epsilon_p}{\kappa_p}\left(\tau\hat{\omega}_t + (1-\tau)\left(\frac{1}{1-\tilde{\beta}}(\hat{V}_t - \hat{H}_t) - \frac{\tilde{\beta}}{1-\tilde{\beta}}\left(\hat{V}_{t+1} - \hat{H}_{t+1} - \frac{\tilde{\delta}}{1-\tilde{\delta}}\hat{\tilde{\delta}}_{t+1}\right)\right) - \hat{MPL}_t\right) + \beta E_t[\pi_{t+1}^p]$$

A.2.4 Alternative formulation: free vacancies, costly hire

The model previously presented assumes that vacancies are costly, as is standard in the search literature. Pissarides (2009) and Christiano, Eichenbaum, and Trabandt (2016) emphasize the importance of fixed costs of hiring that are independent of the labor market conditions to reduce wage volatility. Empirically, Muehlemann and Strupler (2018) find that pre-matching hiring costs (those related to search) account for just 21% of a firm's hiring costs. In this section we take the extreme assumption that firms face hiring costs and vacancies are virtually free.

The objective of considering this case is twofold. First, it makes it clear how the wage-setting problem differs from standard search models. Witouth search costs, bargaining models collapse to a competitive model, because there is no surplus to be shared. The dynamics of the wage are driven by the cost of matching workers. Here, while firms can costlessly post vacancies, hiring is costly and wages are set to reduce quits, more in line with a model where there are abundant applicants as in Salop (1979). Second, it challenges the monopsonistic theory that the relevant elasticity at the firm level is the sum of the quit elasticity and the acceptance elasticity.

The structure of the model is exactly the same with the exception of the firm Bellman equation, which now becomes

$$\begin{aligned} J_t(x_{t-1}) = \max_{p_t, w_t, v_t} & \frac{p_t}{P_t}\mathcal{D}_t\left(\frac{p_t}{P_t}\right) - \frac{w_t}{P_t}n_t - \kappa_h h_t \\ & - \frac{\kappa_p}{2}\left(\frac{p_t}{p_{t-1}} - 1\right)^2 Y_t - \frac{\kappa_w}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 N_t + \beta E_t[J_{t+1}(x_t)] \end{aligned}$$

where $h_t = a(w_t)v_t$ are hires at period t. This formulation makes vacancies payoff-irrelevant, which means they are free to post. What allows for a positive wage in equilibrium are fairness constraints, the fact that firms have to pay the same to new and incumbent workers. Wages are not set to attract new workers since firms can post as many vacancies as needed to achieve this goal, but rather to keep incumbents from being poached by other firms. Having costless vacancies is not the same as not having search frictions. In this case, while firms can find a worker without incurring any cost, workers can only wait unemployed or at a firm until a job offer is handled to them. The the real wage is given by

$$\omega = \tilde{\delta}\kappa_h \epsilon_{\tilde{\delta}}.$$

The next proposition adapts Proposition 2 to the case when posting vacancies is free:

Proposition 3 (Costly hire) *If firms face costs to hire workers instead of posting vacancies, the price and wage Phillips curves become:*

$$\pi_t^p = \frac{\epsilon_p}{\kappa_p}\left(\tau\hat{\omega}_t + \frac{1-\tau}{1-\tilde{\beta}}\beta(1-\bar{\delta})\delta\hat{\delta}_{t+1}\right) + \beta E_t[\pi_{t+1}^p]$$

$$\pi_t^w = \frac{1}{\kappa_w}\frac{\tau}{\mathcal{M}_p}\left(\frac{1-\bar{\delta}}{\bar{\delta}}\hat{\Delta}N_t + \hat{EE}_t - \hat{UE}_t - \hat{\omega}_t\right) + \beta E_t[\pi_{t+1}^w]$$

and (1.14), where $\tau = \frac{\tilde{\delta}\epsilon_{\tilde{\delta}}}{\tilde{\delta}\epsilon_{\tilde{\delta}}+1-\tilde{\beta}}$. *Paying per hire instead of per vacancy reduces* τ *and flattens* *both the price and the wage Phillips curves.*

There are 2 differences between Proposition 2 and Proposition 3. First, the wage markdown increases as the effective elasticity of labor supply is reduced by a half. This observation is important for the literature that estimates wage markdowns and monopsony power. It is conventional to assume that the labor supply elasticity is $\epsilon_a + \epsilon_{\tilde{\delta}}$ and by lack of estimation of ϵ_a and using general equilibrium arguments, set $\epsilon_a = \epsilon_{\tilde{\delta}}$. This is the case in this model, but when hiring is costly ϵ_a does not matter in the wage determination. Firms do not set wages to increase the acceptance rate but rather to reduce the turnover rate. Most certainly the reality is in between, with some costs being related to finding workers (job ads, interviews...) and those can be reduced by increasing the wage, and some others are independent of the wage. This means that the effective labor supply that firms face lie on the interval $[\epsilon_{\tilde{\delta}}, 2\epsilon_{\tilde{\delta}}]$, depending on how important are costs related to vacancies versus costs related to hiring.

The second difference is that the terms related to the acceptance rate $V_t - H_t$ are not present in Proposition 3. This reduces the price and wage reaction to labor market conditions, because the cost of getting an extra employee no longer depends on market tightness. However, market tightness is still what drives the dynamics of wages because when the market is hot, it is more likely that workers recieve outside offers, which pushes firms to increase wages to prevent them from quitting. On the marginal cost side, an increased turnover rate reduces the net cost of hiring a worker since it is more likely that he will quit at $t + 1$.

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Appendix B

Appendix of Trade Rigidities

B.1 Empirical Appendix

B.1.1 Description of Data

Table B.1: Ilzetzki, Reinhart, and Rogoff (2019)'s Exchange Rate Classification

Fine	Coarse	Description	Example
1	1	No separate legal tender	Eurozone, Cameroon
2	1	Pre-announced peg	Argentina, Malaysia
3	1	Pre-announced horizontal band $< \pm2\%$	N/A
4	1	De facto peg	China, Egypt, Saudi Arabia
5	2	Pre-announced crawling peg; band $< \pm1\%$	Nicaragua
6	2	Pre-announced crawling band $< \pm2\%$	Sweden, Venezuela
7	2	De facto crawling peg	Russia, Vietnam
8	2	De facto crawling band $< \pm2\%$	Iceland, Canada
9	3	Pre-announced crawling band $> \pm2\%$	Hungary, Sri Lanka
10	3	De facto crawling band $< \pm5\%$	Paraguay, Turkey
11	3	Moving band $< \pm2\%$	Korea, Thailand
12	3	Managed floating	Brazil, Mexico, United Kingdom
13	4	Freely floating	Japan, United Stats
14	5	Freely falling	Congo, Zimbabwe
15	6	Dual market with missing data	Afghanistan, Myanmar

Note: The table lists the fine and coarse exchange rate regime classification of Ilzetzki, Reinhart, and Rogoff (2019). $<$ stands for 'narrower than', and $>$ stands for 'wider than', and denotes the size of the (horizontal, crawling, moving) band. The last column lists some example countries that was classified as that regime as of June 2000.

Table B.2: Summary statistics for pegs and floats

Variable	Pegs	Floats	Diff
log(population)	1.512	1.677	-0.689*
	(2.341)	(1.512)	(0.372)
log(GDP per capita)	8.421	8.562	-0.141
	(1.374)	(1.628)	(0.283)
MFG share (%)	11.414	14.213	-2.798**
	(6.428)	(7.692)	(1.394)
export (% GDP)	27.977	29.419	-1.442
	(26.995)	(22.065)	(4.561)
import (% GDP)	39.598	34.523	5.075
	(24.433)	(18.492)	(4.001)
NFA / GDP	-0.336	-0.106	-0.230
	(1.097)	(1.262)	(0.221)
CPI inflation	0.0437	0.0346	0.00910
	(0.0562)	(0.0315)	(0.00903)
unemployment rate	0.0870	0.1016	-0.0285**
	(0.0504)	(0.0871)	(0.0135)
S_i (china shock)	0.03493	0.04115	-0.00621
	(0.03022)	(0.03885)	(0.00643)
No. of obs	56	63	

Note: The first two columns report summary statistics for pegging countries and floating countries, with standard deviation in parentheses. The third column reports regression coefficients for regressions of the characteristics on a dummy variable for whether the country's currency is pegged to the US dollar, with the dependent variables on the left, with standard errors for the coefficients in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

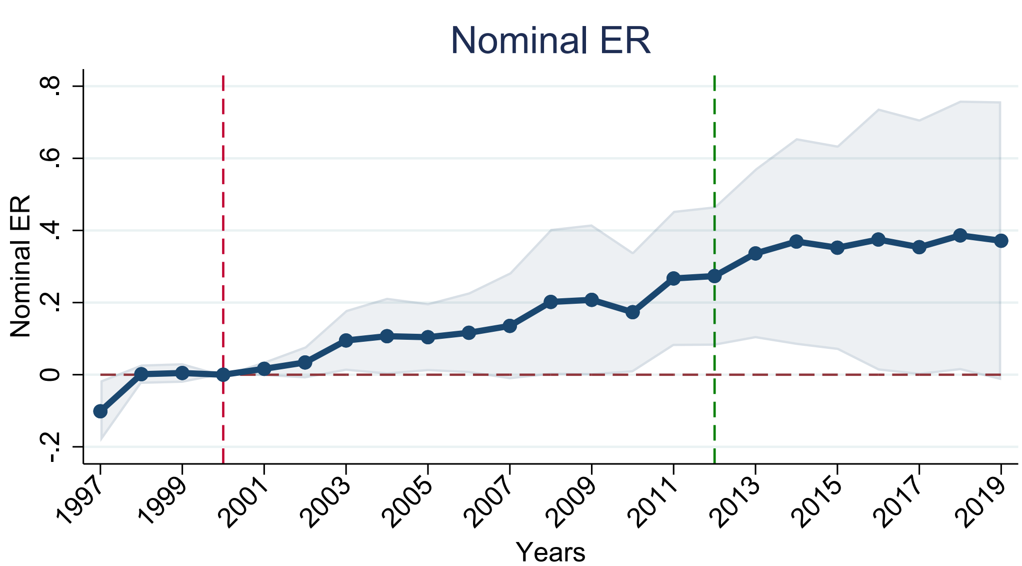


Figure B-1: Average responses to the China shock across countries.

B.1.2 Additional results

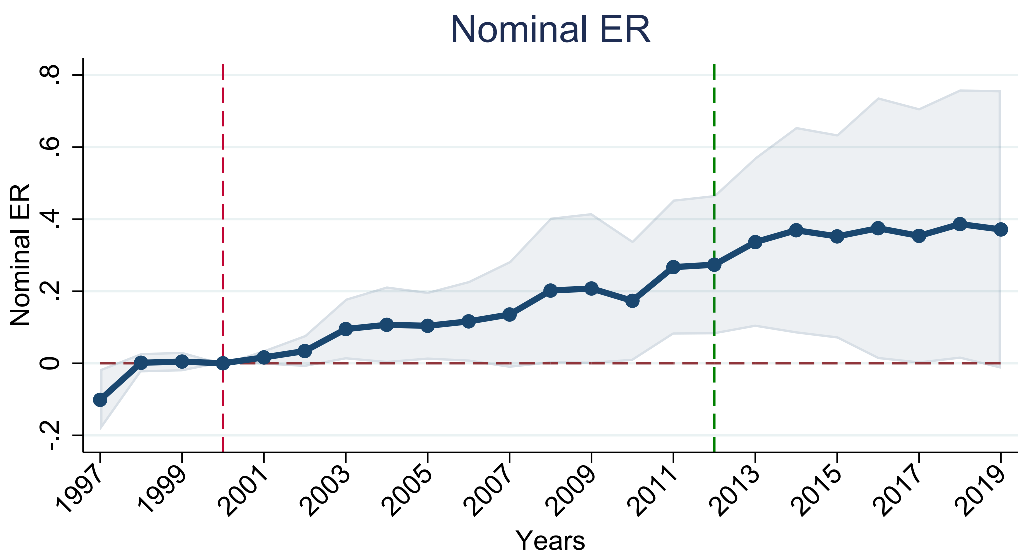
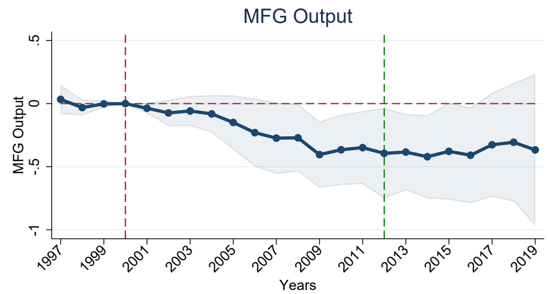


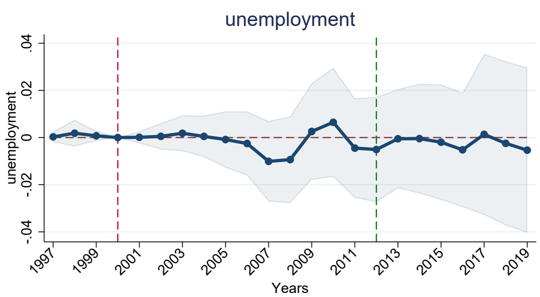
Figure B-2: Average responses to the China shock across countries.

Note. The figure plots the double-difference regression result of the exchange rate against the China shock across all countries. The shaded area is the 95% confidence band for each local projection regression. The red dashed line indicates the beginning of the China shock (2000) and the green the end of the China shock (2012). On average countries' currencies depreciate in response to higher exposure to the China shock; the latter figure shows that the effect is completely driven by floaters.

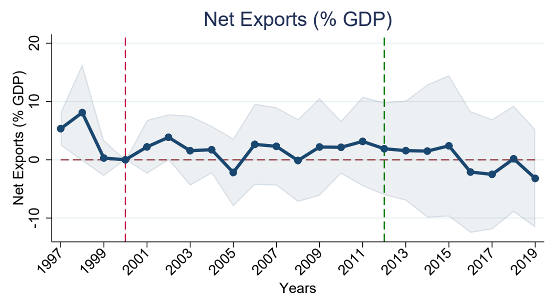
(a) Real GDP



(b) Manufacturing Output



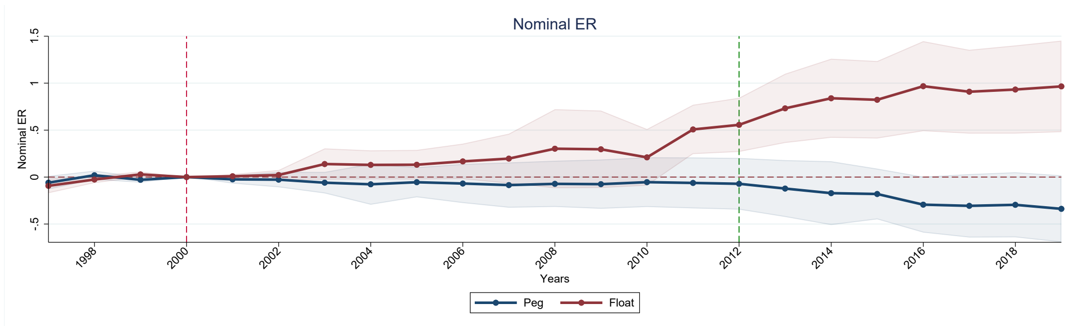
(c) Unemployment

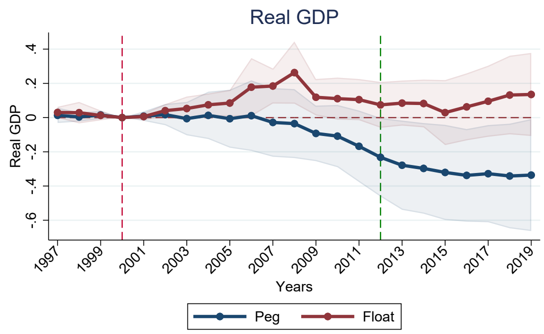


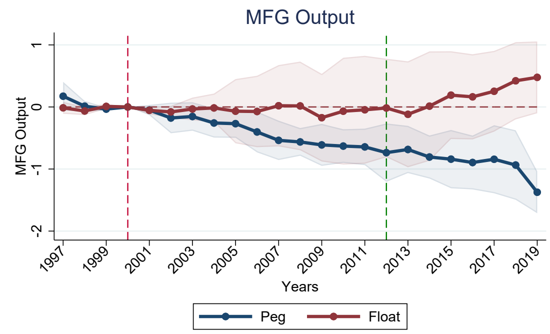
(d) Net Exports (% GDP)

Figure B-3: Average responses to the China shock across countries.

Note. The plotted coefficient β_{1h} is the average response to the China shock, without taking into account the heterogeneity in exchange rates: this is the 'double-difference' equivalent of Figure 2-3. As we see, the heterogeneity in exchange rate regime masks the true effect of the China shock. The shaded area is the 95% confidence band for each local projection. The red dashed line indicates $t = 2000$, the start of the China shock and the green line $t = 2012$, the end of the China shock.

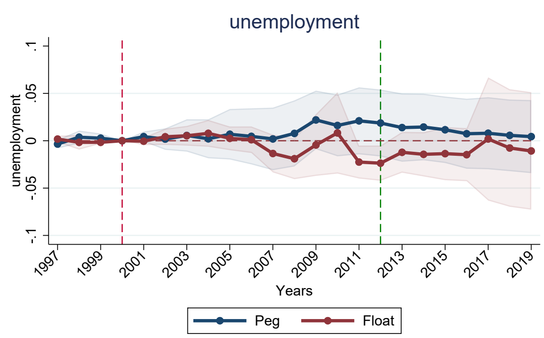


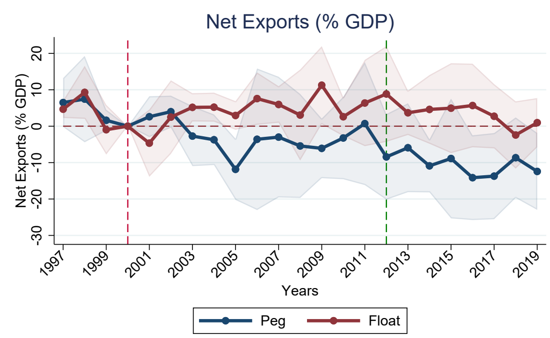




(a) Real GDP

(b) Manufacturing Output





(c) Unemployment

(d) Net Exports (% GDP)

Figure B-4: Differential response of the China shock.

Note. This regression plots the coefficient for the subset of countries where currency is pegged versus floated against the US dollar respectively. The shaded area is the 95% confidence band for each local projection regression. The red dashed line indicates $t = 2000$, the start of the China shock and the green line $t = 2012$, the end of the China shock. The figures show that the nominal exchange rate for floaters appreciated, and for floaters, higher exposure to the China hsock did not affect manufacturing output, unemployment, or net exports (red lines); in sharp contrast, greater exposure to Chinese export led to lower manufacturing output, a temporary increase in unemployment, and larger trade deficits for pegging countries (blue lines).

B.1.3 Causal identification and inference

In this subsection, we discuss the identification and inference properties of our shift-share instrument, in relation to recent literature on such instruments (Borusyak, Hull, and Jaravel, 2022; Borusyak and Hull, 2023).

Borusyak, Hull, and Jaravel (2022) (henceforth BHJ) derive sufficient conditions for causal identification in empirical setups that measure the exposure of a shock through a 'shift-share', or an average of a set of shocks with exposure share weights. Their sufficient condition is in terms of a quasi-random assignment of the shocks: in our context, the 'shock', or the growth in global Chinese exports $\Delta \log E_C^s$ is as good as random conditional on the exposure shares s_i. This holds if the shares are exogenous (Goldsmith-Pinkham, Sorkin, and Swift, 2020), or if the large-sample covariance between the export shocks g_C^s and the unobserved shocks ϵ_{ih} in the regression equation (Equation 2.3) is zero. Our preferred interpretation is the latter, following the China shock literature Autor, Dorn, and Hanson (2013) and Autor, Dorn, and Hanson (2021), henceforth ADH); as highlighted in BHJ, it is *a priori* implausible that the 2000 industry shares λ_i^s are uncorrelated with the errors ϵ_{ih}, as the latter will capture unobserved industry-level shocks. As such, we interpret our empirical strategy as assuming shift exogeneity, rather than share exogeneity.

ADH studies variation within US across commuting zones, and uses Chinese export surge into other developed countries as instruments to purge US-specific demand shocks that may bias their results, adding support to their a priori justification of shift exogeneity. This is unavailable for us, as we study global surge in Chinese exports. However, if there is an unobserved global demand shock towards Chinese goods, either (1) one may interpret this as a part of the 'China shock', or (2) this demand shock violates the exogeneity condition of the ADH instrument. As such, while our analysis is reduced-form, we believe that there is a priori justification for 'global surge in Chinese exports' in each sector being as-good-as-random.

With this in mind, we follow the framework of BHJ to test for the validity and robustness of our exposure measure.

Industry shocks and exposure measures

For the shift-share exposure measure to be valid under the shock exogeneity assumption, it is sufficient to have that g_C^s is as good as random conditional on the shares λ_i^s (Assumption 1 of BHJ). Moreover, for the measured coefficient to be consistent, we need the effective sample size $1/E[\sum_s (\lambda_i^s)^2]$ to be large enough (Assumption 2 of BHJ). Following BHJ, we summarize the distribution of the shocks g_C^s and the industry-level weights $\lambda^s \propto \sum_i \lambda_i^s$ (normalized to add up to one).

Table B.3: Shock and share summary statistics

Mean	1.757
Standard deviation	1.525
Interquartile range	1.596
Effective sample size (1/HHI)	24.38
Largest λ^s weight	0.189
2nd largest λ^s weight	0.022
Effective sample size, SITC3	18.44
Largest λ^s, SITC3	0.214
2nd largest λ^s, SITC3	0.027
No. of shocks (SITC4 industries)	782
No. of SITC3 groups	237

Note: The table summarizes the global China export shock g_C^s across sectors s.

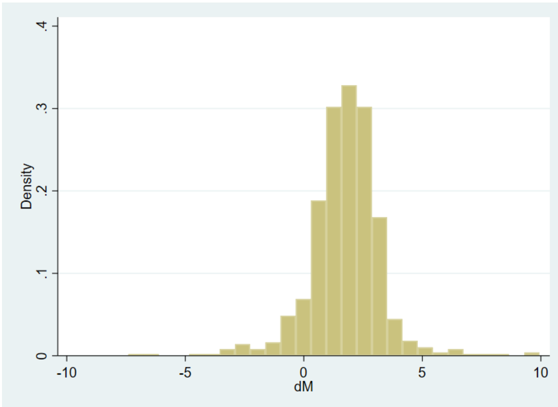
Table B.3 reports summary statistics for the shocks and the shares.[1]. The distribution of the shock is quite regular, with the average of 1.757, a standard deviation of 1.525, and an interquartile range of 1.596. Figure B-5 shows the histogram of the shocks g_C^s and a Q-Q plot of the realized distribution against the quantile of the normal distribution (using the `qnorm` command of Stata) shows that the distribution is close to normal, which adds support to the shock exogeneity assumption. The inverse HHI – the "effective sample size" according to BHJ – is 24.38. This is smaller than the sample size in BHJ (191.6, 58.4 when acoross SIC3 groups), and the main cause is that some countries in our sample have high concentration in petroleum and crude oil products (code 3330, share 18.9%). Thus we have suggestive evidence that the shocks are as good as random, and the effective sample size is reasonable for causal inference.

Besides these conditions, Assumption 2 of BHJ require the shocks to be sufficiently mutually uncorrelated. BHJ recommend analyzing the correlation patterns of shocks across the industries using available industry classifications. Following their methodology, we compute intra-class correlation coefficients (ICCs) of shocks within different industry groups. We use a random effects model with nested random effects:

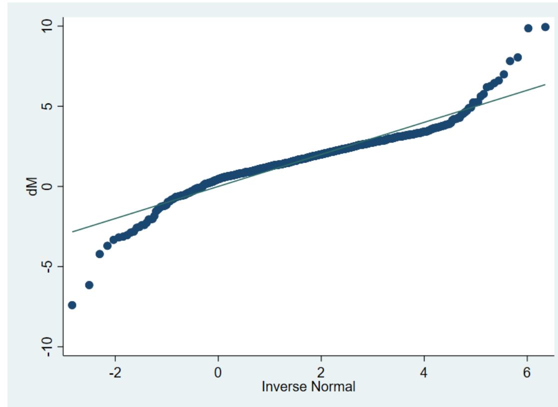
$$g_C^s = \mu + a_{sitc1(s)} + b_{sitc2(s)} + c_{sitc3(s)} + \epsilon_s \tag{B.1}$$

where $a_{sitc1(s)}, b_{sitc2(s)}, c_{sitc3(s)}$ respecitvely denote random effects generated by the SITC 1-

[1]This table is the analogue of Table 1 in BHJ.



(a) Histogram



(b) Q-Q wrt normal distribution

Figure B-5: Distribution of global China export shock g_C^s

	Estimate	SE
SITC 1-digit	0.225	(0.142)
SITC 2-digit	0.193	(0.087)
SITC 3-digit	0.281	(0.089)
4-digit (residual)	1.594	(0.096)
No. of SITC1 groups	10	
No. of SITC2 groups	69	
No. of SITC3 groups	237	
No. of shocks (SITC4 industries)	782	

Table B.4: China export shock intra-class correlations

Note: This table reports intra-class correlation coefficients for the g_C^s China exprot shocks in Section 2.2, estimated from the hierarchical model (Equation B.1).

digit sectors, 2-digit sectors, and 3-digit sectors respectively. We estimate Equation B.1 as a hierarchical linear model with maximum likelihood assuming Gaussian residuals. Table B.4 reports the results from this mixed linear model; there is moderate clustering of shock residuals at each level of the SITC (0.225, 0.193, 0.281), but the residual component at the 4-digit level is largest. This supports the assumption that shocks are sufficiently mutually uncorrelated.

Non-random exposure

Next, we purge bias coming from non-random exposure to shocks, following Borusyak and Hull (2023). If some countries structurally have higher exposure to the quasi-random China shock because they have higher shares λ_i^s, this will create a bias in the regression coefficient; notably, in our example, if pegged countries structurally have higher (lower) shares, the estimated effect of the interaction term will be biased upwards (downwards). This is econometrically equivalent to the 'incomplete shares' issue raised in BHJ; even if the DGP for the shocks $\Delta \log E_C^s$ is truly random, if some countries have structurally high exposure shares λ_i^s, the regression coefficients will be biased.

In this subsection, we briefly explain our implied DGP, and how using $\sum_s \lambda_i^s$ is equivalent to the re-centering instrument. We assume that the shocks $g = g_C^s$ come from a distribution G with mean $E[g] \ = \sum_s \frac{g_C^s}{S}$. In this case, countries with higher $\sum_s \lambda_i^s$ is going to have a higher *expected* exposure $E[\lambda_i^s g_C^s]$ conditional on the DGP, and this is going to bias our regression which seeks to evaluate the effect of causal higher g_C^s on outcomes. Borusyak and Hull (2023) show that 're-centering' the exposure $S_i = \sum_s \lambda_i^s g_C^s$ by instrumenting S_i with

$$\hat{S}_i = \sum \lambda_i^s g_C^s - E[\sum_s \lambda_i^s g^s | g \in G],$$

or alternatively controlling for $E[\sum_s \lambda_i^s g^s | g^s \in G]$ in the regressions is sufficient to purge this bias. But in linear shift-share settings such as ours under conditional exogeneity of the shock, we have

$$E[\sum_s \lambda_i^s g^s | g^s \in G] = \sum_s \lambda_i^s E[g^s],$$

so this is equivalent to controlling for $\sum_s \lambda_i^s$ in the regression; this is exactly the solution for the 'incomplete shares' problem in Borusyak, Hull, and Jaravel (2022). Since we control for $\sum_s \lambda_i^s$ in our regressions, this is sufficient to purge the bias coming from non-random exposure.

B.2 Proofs of propositions

B.2.1 Proofs for Subsection 2.3.3

In this section I prove the Propositions in Section 2.3.3. In the equilibrium under the exchange rate peg, I assume without loss of generality that $\bar{e} = 1$. I first highlight a number of properties of the laissez-faire equilibrium that I extensively use in the proof.

Lemma 3. *Denote by* $\omega_t = \frac{w_{Ht}}{w_{Ft}}$ *the relative wage of Home at period* $t \in \{0, 1\}$*. The following properties hold:*

1. *The real wage* $\frac{w_{jt}}{P_{jt}}$ *and expenditure share* λ_{ijt} *depend on* $\{w_{Ht}, w_{Ft}\}$ *only through* ω_t*.*

2. *Home real wage* $\frac{w_{Ht}}{P_{Ht}}$ *increases in* ω_t*, while Foreign real wage decreases in* ω_t*.*

3. *Expenditure share for Home goods* λ_{Hjt} *is a decreasing function of* ω_t*;* $\lambda_{Fjt} = 1 - \lambda_{Hjt}$ *is an increasing function of* ω_t

4. *Home relative wage is higher in period 0:* $\omega_0 > \omega_1$*.*

5. *The real wage of Home is higher in period 0:* $\frac{w_{H0}}{P_{H0}} > \frac{w_{H1}}{w_{P1}}$*.*

6. *Relative inflation is higher at Foreign. If we define* $\pi_j = \frac{P_{j1}}{P_{j0}}$*, we have* $\pi_F > \pi_H$*.*

Proof. 1. We have

$$\frac{w_{Ht}}{P_{Ht}} = \frac{w_{Ht}}{(P_{HHt}^{1-\sigma} + P_{FHt}^{1-\sigma})^{1/(1-\sigma)}} = \frac{w_{Ht}}{((w_{Ht}/A_{HH})^{1-\sigma} + (w_{Ft}/A_{FH})^{1-\sigma})^{1/(1-\sigma)}}$$
$$= \frac{1}{((1/A_{HH})^{1-\sigma} + (\omega_t/A_{FH})^{1-\sigma})^{1/(1-\sigma)}}$$

and analogously for w_{Ft}/P_{Ft}. Likewise, we have

$$\lambda_{Hjt} = \frac{P_{Hjt}^{1-\sigma}}{P_{Hjt}^{1-\sigma} + P_{Fjt}^{1-\sigma}} = \frac{1}{1 + (\frac{w_{Ft}/A_{Fj}}{w_{Ht}/A_{Hj}})^{1-\sigma}} = \frac{1}{1 + (\omega_t)^{\sigma-1}(\frac{A_{Hj}}{A_{Fj}})^{1-\sigma}}$$

and $\lambda_{Fjt} = 1 - \lambda_{Hjt}$. In general, the real wage and expenditure share are functions of ω_t for *any* homothetic aggregator of Home and Foreign goods $\mathcal{C}_j = \mathcal{C}_j(C_{Hjt}, C_{Fjt})$.

2. By inspection of the previous formula, we see that when $\sigma > 1$, $\frac{w_{Ht}}{w_{Ft}}$ is increasing in ω_t.

3. Likewise, when $\sigma > 1$, λ_{Hjt} is decreasing in ω_t.

4. Denote by $\omega^*(\{A_{ij}\})$ the Home relative wage under a *static, flexible-price* economy under productivity $\{A_{ij}\}_{i,j\in\{H,F\}}$, which can be solved by the trade balance equation:

$$\lambda_{FH} w_H L_H = \lambda_{HF} w_F L_F \quad \Rightarrow \quad \omega^* \frac{L_H}{L_F} = \frac{\lambda_{HF}(\omega^*)}{\lambda_{FH}(\omega^*)}$$

Now since L_j is increasing in $\frac{w_j}{P_j}$, the left-hand side is increasing in ω^* while the right-hand side is decreasing in ω^*. Thus there is a unique ω^*.

Consider the trade shock that increases A_F. Since λ_{FH} is increasing in A_F, λ_{FH} is decreasing in A_F, we have that a higher A_F decreases the right-hand side. Thus to satisfy equality, an increase in A_F must be accompanied by a *decrease* in ω^*.

We assumed that Home relative wage ω_0 is rigid at $\omega_0 = \omega^*(\{A_{ij,-1}\})$. Given an increase in A_F, $\omega_0 = \omega^*(\{A_{ij,-1}\}) > \omega^*(\{A_{ij0}\})$. Now, if we assumed for sake of contradiction that $\omega_1 \geq \omega_0 > \omega^*(\{A_{ij0}\}) = \omega^f$, we would have

$$\omega_t \frac{L_H(\omega_t)}{L_F(\omega_t)} > \frac{\lambda_{HF}(\omega_t)}{\lambda_{FH}(\omega_t)} \text{ for } t = 0, 1$$

but this would break the lifetime trade balance condition – Home's relative wage is too high in both periods, so Home cannot balance the lifetime budget. Thus we have $\omega_0 > \omega_1$.

5. This follows from 2 and 5.

6. We have

$$\left(\frac{P_{Ht}}{P_{Ft}}\right)^{1-\sigma} = \frac{P_{HHt}^{1-\sigma} + P_{FHt}^{1-\sigma}}{P_{HFt}^{1-\sigma} + P_{FFt}^{1-\sigma}} = \frac{(\omega_t \frac{A_{FF}}{A_{HH}})^{1-\sigma} + (\frac{A_{FF}}{A_{FH}})^{1-\sigma}}{(\omega_t \frac{A_{FF}}{A_{HF}})^{1-\sigma} + 1}$$
$$= (\frac{A_{HF}}{A_{HH}})^{1-\sigma}(1 + \frac{(\frac{A_{HH}A_{FF}}{A_{HF}A_{FH}})^{1-\sigma} - 1}{(\omega_t \frac{A_{FF}}{A_{HF}})^{1-\sigma} + 1})$$

Since $\sigma > 1$ and $\frac{A_{HH}A_{FF}}{A_{HF}A_{FH}} > 1$ (Home bias, equivalently $\tau_{FH}\tau_{HF} \geq 1$), the last expression is decreasing in ω_t. Then since $\omega_0 > \omega_1$ and again $\sigma > 1$, we have $\frac{P_{H0}}{P_{F0}} > \frac{P_{H1}}{P_{F1}}$. Rearranging, we get $\pi_F > \pi_H$.

□

Using these properties, we prove the propositions.

Proposition 4. *In the pegged equilibrium, in response to a trade shock ($A_{FH} \uparrow$), Home runs a trade deficit ($B_{H1} < 0$). Moreover, if Home monetary policy does not respond ($R_{H1} = \frac{1}{\beta}$), then there is involuntary unemployment at Home ($\mu_{H0} < 0$).*

Proof. For the first part ($B_{H1} < 0$), note that Home borrows in the short-run if the following inequalities hold:

$$\underbrace{\lambda_{HF0}P_{F0}C_{F0}}_{t=0 \text{ Home exports}} < \underbrace{\lambda_{FH0}P_{H0}C_{H0}}_{t=0 \text{ Home imports}} \text{ and } \underbrace{\lambda_{HF1}P_{F1}C_{F1}}_{t=1 \text{ Home exports}} > \underbrace{\lambda_{FH1}P_{H1}C_{H1}}_{t=1 \text{ Home imports}} \tag{B.2}$$

Invert the second inequality and multiply with the first to have

$$\frac{\lambda_{HF0}}{\lambda_{HF1}} \frac{P_{F0}C_{F0}}{P_{F1}C_{F1}} < \frac{\lambda_{FH0}}{\lambda_{FH1}} \frac{P_{H0}C_{H0}}{P_{H1}C_{H1}}$$

Rearrange to have:

$$\frac{\lambda_{HF0}/\lambda_{HF1}}{\lambda_{FH0}/\lambda_{FH1}} < \frac{\pi_F}{\pi_H}\frac{C_{H0}/C_{H1}}{C_{F0}/C_{F1}} \tag{B.3}$$

where $\pi_j = \frac{P_{j1}}{P_{j0}}$ denote inflation in country j. Note that if $B_1 > 0$, both inequalities are flipped in Inequality B.2, so we have the exact opposite inequality, so Inequality B.3 is a necessary and sufficient condition for Home borrowing. Since both countries face the same nominal interest rate under a peg, we have

$$C_{j0}^{-1/\gamma} = \beta(1+i)\frac{1}{\pi_j}C_{j1}^{-1/\gamma} \quad \Rightarrow \quad \frac{C_{j0}}{C_{j1}} = [\beta(1+i)\pi_j^{-1}]^{-\gamma}$$

Use this to rewrite Inequality B.3 as

$$\frac{\lambda_{HF0}/\lambda_{HF1}}{\lambda_{FH0}/\lambda_{FH1}} < [\frac{\pi_F}{\pi_H}]^{1-\gamma} \Leftrightarrow B_{H1} < 0$$

(Note that the left-hand-side is the first 'variation in terms-of-trade across time' governed by σ, while the right-hand-side is the second 'home bias and relative prices' governed by γ, as described in the main text.)

With the CES parametric assumption, we may rewrite the expenditure shares λ_{ij} as

$$\begin{aligned}
\frac{\lambda_{HF0}}{\lambda_{HF1}} &= \frac{(P_{HF0}^{1-\sigma}/P_{F0}^{1-\sigma})}{(P_{HF1}^{1-\sigma}/P_{F1}^{1-\sigma})} = \pi_F^{1-\sigma}(\frac{w_{H0}}{w_{H1}})^{1-\sigma} \\
\frac{\lambda_{FH0}}{\lambda_{FH1}} &= \frac{(P_{FH0}^{1-\sigma}/P_{H0}^{1-\sigma})}{(P_{FH1}^{1-\sigma}/P_{H1}^{1-\sigma})} = \pi_H^{1-\sigma}(\frac{w_{F0}}{w_{F1}})^{1-\sigma}
\end{aligned}$$

Hence,

$$\frac{\lambda_{HF0}/\lambda_{HF1}}{\lambda_{FH0}/\lambda_{FH1}} = (\frac{\pi_F}{\pi_H})^{1-\sigma}(\frac{w_{H0}/w_{H1}}{w_{F0}/w_{F1}})^{1-\sigma}$$

This is smaller than $[\frac{\pi_F}{\pi_H}]^{1-\gamma}$ if and only if

$$\begin{aligned}
(\frac{\pi_F}{\pi_H})^{1-\sigma}(\frac{w_{H0}/w_{H1}}{w_{F0}/w_{F1}})^{1-\sigma} &< (\frac{\pi_F}{\pi_H})^{1-\gamma} \\
\Leftrightarrow (\frac{w_{H0}/w_{H1}}{w_{F0}/w_{F1}})^{1-\sigma} &< (\frac{\pi_F}{\pi_H})^{\sigma-\gamma}
\end{aligned}$$

We have that the left-hand side is less than 1 by $\sigma > 1$ and part (d) of Lemma 3. We have that the right-hand side is greater than 1 by $\sigma > \gamma$ and part (f) of Lemma 3. Thus we have $RHS > 1 > LHS$.

For the second part ($\mu_{H0} < 0$ when $R_{H0} = 1/\beta$), we first have

$$v'(L_{H1}) = u'(C_{H1})\frac{w_{H1}}{P_{H1}}$$

From part (e) of Lemma 3, we have $\frac{w_{H0}}{w_{P0}} > \frac{w_{H1}}{w_{P1}}$. At the same time, we have $u'(C_{H1}) = u'(C_{H0})$ with $R_H = \frac{1}{\beta}$. Thus, if we can show $L_{H1} > L_{H0}$, we have

$$\mu_{H0} = v'(L_{H0}) - u'(C_{H0})\frac{w_{H0}}{P_{H0}} < v'(L_{H1}) - u'(C_{H1})\frac{w_{H1}}{P_{H1}} = 0$$

We proceed to show $L_{H1} > L_{H0}$. Goods market clearing condition is $L_{Ht} = \tau_{HH}C_{HHt} + \tau_{HF}C_{HFt}$, and since $C_{H1} = C_{H0}$ and $\lambda_{HH0} < \lambda_{HH1}$ by $\frac{w_{H0}}{w_{F0}} > \frac{w_{H1}}{w_{F1}}$, we have $C_{HH0} < C_{HH1}$. Moreover, with $\sigma > 1$ and $\sigma > \gamma$, we have

$$\begin{aligned}
\frac{C_{HF0}}{C_{HF1}} &= \frac{(\frac{P_{HF0}}{P_{F0}})^{-\sigma}C_{F0}}{(\frac{P_{HF1}}{P_{F1}})^{-\sigma}C_{F1}} = \frac{(\frac{P_{HF0}}{P_{F0}})^{-\sigma}}{(\frac{P_{HF1}}{P_{F1}})^{-\sigma}} \cdot (\beta(1+i)\frac{P_{F0}}{P_{F1}})^{-\gamma} \\
&< \frac{(\frac{P_{HF0}}{P_{F0}})^{-\gamma}}{(\frac{P_{HF1}}{P_{F1}})^{-\gamma}} \cdot (\frac{P_{H1}}{P_{H0}}\frac{P_{F0}}{P_{F1}})^{-\gamma} \\
&= (\frac{P_{HF0}}{P_{HF1}}\frac{P_{H1}}{P_{H0}})^{-\gamma} = (\frac{w_{H0}}{w_{H1}}\frac{P_{H1}}{P_{H0}})^{-\gamma} < 1
\end{aligned}$$

where we have the intermediate inequality because $(\frac{P_{HF0}}{P_{F0}}/\frac{P_{HF1}}{P_{F1}}) > 1$ (which follow from $\omega_0 > \omega_1$) and $\sigma \geq \gamma$, and the last inequality from part (e) of Lemma 3. Thus we have $C_{HH0} < C_{HH1}$ and $C_{HF0} < C_{HF1}$, so $L_{H0} < L_{H1}$, and we obtain $\mu_{H0} < 0$. □

For the next proposition, we first prove that deficits hurt future terms-of-trade.

Lemma 4. *Suppose Home borrows more in real terms, so that $\frac{B_{H1}}{w_{H1}}$ decreases. Then $\frac{w_{H1}\bar{e}}{w_{F1}}$ falls: Home future relative wage worsens as a result of Home borrowing.*

Proof. The goods market clearing condition for Home goods at $t = 1$ can be rewritten as

$$w_{H1}L_{H1} = \lambda_{HH1}(w_{H1}L_{H1} + B_{H1}) + \lambda_{HF1}(w_{F1}L_{F1} - B_{H1})$$

Rearranging this equation and writing everything in terms of $S_{H1} = \frac{w_{H1}}{w_{F1}}$ and $b = \frac{B_{H1}}{w_{H1}}$, we may write

$$\begin{aligned}
1 &= \lambda_{HH1}(1 + \frac{b}{L_{H1}}) + \lambda_{HF}(\frac{1}{S}\frac{L_{F1}}{L_{H1}} - \frac{b}{L_{H1}}) \\
b[\frac{\lambda_{HH} - \lambda_{HF}}{L_H}] &= 1 - \lambda_{HH} - \lambda_{HF}(\frac{1}{S}\frac{L_F}{L_H})
\end{aligned}$$

We have $\frac{\partial \lambda_{HH1}}{\partial S}, \frac{\partial \lambda_{HF1}}{\partial S} < 0$ (Home better terms-of-trade $\iff$ Home goods more expensive), $\frac{\partial L_H}{\partial S} > 0, \frac{\partial L_F}{\partial S} < 0$ (Home better TOT $\iff$ Home workers have better real wage, want to work more). Then the RHS is increasing in S. Moreover, from home bias we have $\lambda_{HH} + \lambda_{FF} > 1 \rightarrow \lambda_{HH} > \lambda_{HF}$, so the coefficient on b is positive. Thus $\frac{\partial b}{\partial S} > 0$; then $\frac{\partial S}{\partial b} = \frac{1}{\frac{\partial b}{\partial S}} > 0$ so running more debt ($b \downarrow$) will lead to worsening terms of trade $S \downarrow$. $\square$

Proposition 5. *In the equilibrium where policy does not respond ($R_{H1} = \frac{1}{\beta}$), the effect of a small increase of A_{FH} on Home welfare $\mathcal{U}_H$ is ambiguous, and depends on σ. For small changes in $\epsilon_A = A_{FH0} - A_{FH-1}$, we have that:*

- *When $\sigma \rightarrow 1$, we have Home welfare increases as a result of the Foreign shock: $\frac{d\mathcal{U}_H}{dA_{FH}} > 0$.*

- *When $\sigma \rightarrow \infty$, we have Home welfare decreases as a result of the Foreign shock: $\frac{d\mathcal{U}_H}{dA_{FH}} < 0$*

Proof. We first derive the first-order welfare equation 2.21:

$$\frac{d\mathcal{U}_H}{dA_{FH}} = \underbrace{-\frac{u'(C_{H0})}{P_{H0}} C_{FH0} \frac{dP_{FH0}}{dA_{FH}}}_{\text{cheap goods}} + \underbrace{\mu_0 \frac{dL_0}{dA_{FH}}}_{\text{labor wedge}} + \underbrace{\frac{\beta u'(C_{H1})}{P_{H1}} [C_{HF1} \frac{dP_{HF1}}{dA_{FH}} - C_{FH1} \frac{dP_{FH1}}{dA_{FH}}]}_{\text{terms of trade at } t=1}$$

Home agent's lifetime utility is

$$\mathcal{U}_H = U(C_{HH0}, C_{FH0}, C_{HH1}, C_{FH1}, L_{H0}, L_{H1})$$

and is subject to the lifetime budget constraint

$$P_{HH0}C_{HH0} + P_{FH0}C_{FH0} + \frac{1}{1+i_{Ht}}(P_{HH1}C_{HH1} + P_{FH1}C_{FH1}) = w_{H0}L_{H0} + \frac{1}{1+i_{H1}} w_{H1}L_{H1}$$

Invoking the Envelope theorem, the first-order effect of A_F on $\mathcal{U}_H$ can be written as

$$\frac{d\mathcal{U}_H}{dA_{FH}} = \sum_{t=0}^{1} \sum_{i \in \{H,F\}} \frac{dU}{dC_{iHt}} \frac{dC_{iHt}}{dA_{FH}} + \sum_{t=0}^{1} \frac{dU}{dL_{Ht}} \frac{dL_{Ht}}{dA_{FH}} \tag{B.4}$$

If we denote by $\tilde{\lambda}$ the Lagrange multiplier on the lifetime budget constraint, we have:

$$\frac{dU}{dC_{iH0}} = \tilde{\lambda} P_{iH0}, \quad \frac{dU}{dC_{iH1}} = \frac{\tilde{\lambda}}{1+i_{H1}} P_{iH1}, \quad \frac{dU}{dL_{H1}} = -\frac{\tilde{\lambda}}{1+i_{H1}} w_{H1}$$

while we may have $\frac{dU}{dL_{H0}} \neq -\tilde{\lambda} w_{H0}$ because households do not choose L_{H0}: in fact, we have

$$\frac{dU}{dL_{H0}} + \tilde{\lambda} w_{H0} = -v'(L_{H0}) + \frac{u'(C_{H0})}{P_{H0}} w_{H0} = -\mu_0.$$

Plugging these into Equation B.4, we get

$$\frac{d\mathcal{U}_H}{dA_{FH}} = \tilde{\lambda} \left[\sum_{i \in \{H,F\}} (P_{iH0} \frac{dC_{iH1}}{dA_F} + \frac{P_{iH1}}{1+i_{H1}} \frac{dC_{iH0}}{dA_F}) - w_{H0} \frac{dL_{H0}}{dA_{FH}} - \frac{w_{H1}}{1+i_{H1}} \frac{dL_{H1}}{dA_{FH}} \right] - \mu_0 \frac{dL_0}{dA_{FH}} \tag{B.5}$$

Now, if we take the derivative of the budget constraint, we have

$$\begin{aligned}
& \sum_{i \in \{H,F\}} \left(P_{iH0} \frac{dC_{iH0}}{dA_F} + \frac{P_{iH1}}{1+i_{H1}} \frac{dC_{iH1}}{dA_F} \right) - w_{H0} \frac{dL_{H0}}{dA_{FH}} - \frac{1}{1+i_{H1}} w_{H1} \frac{dL_{H1}}{dA_{FH}} \\
= & -\sum_{i \in \{H,F\}} \left(C_{iH0} \frac{dP_{iH0}}{dA_F} + \frac{C_{iH1}}{1+i_{H1}} \frac{dP_{iH1}}{dA_F} \right) + L_{H0} \frac{dw_{H0}}{dA_{FH}} + \frac{L_{H1}}{1+i_{H1}} \frac{dw_{H1}}{dA_{FH}} \\
= & -C_{FH0} \frac{dP_{FH0}}{dA_{FH}} - \sum_{i \in \{H,F\}} \frac{C_{iH1}}{1+i_{H1}} \frac{dP_{iH1}}{dA_F} + \frac{L_{H1}}{1+i_{H1}} \frac{dw_{H1}}{dA_{FH}}
\end{aligned}$$

where the last expression follows from the fact that w_{H0} is fixed, so we have $\frac{dw_{H0}}{dA_{FH}} = \frac{dP_{HH0}}{dA_{FH}} = 0$. Now to further simplify the last term $-\sum_{i \in \{H,F\}} \frac{C_{iH1}}{1+i_{H1}} \frac{dP_{iH1}}{dA_F} + \frac{L_{H1}}{1+i_{H1}} \frac{dw_{H1}}{dA_{FH}}$, we note that the Home goods market clearing condition in period 1 is

$$L_{H1} = \frac{1}{A_H} C_{HH1} + \frac{\tau_{HF1}}{A_H} C_{HF1}$$

and $P_{HH1} = w_{H1}/A_H$ so $dP_{HH1} = \frac{1}{A_H} dw_{H1}$. From this, we can rewrite

$$\begin{aligned}
-\sum_{i \in \{H,F\}} C_{iH1} \frac{dP_{iH1}}{dA_F} + L_{H1} \frac{dw_{H1}}{dA_{FH}} &= -C_{HH1} \frac{dP_{HH1}}{dA_F} + C_{FH1} \frac{dP_{FH1}}{dA_{FH}} + (\frac{1}{A_H} C_{HH1} + \frac{\tau_{HF1}}{A_H} C_{HF1}) \frac{dw_{H1}}{dA_{FH}} \\
&= -C_{FH1} \frac{dP_{FH1}}{dA_{FH}} + \frac{\tau_{HF1}}{A_H} C_{HF1} \frac{dw_{H1}}{dA_{FH}} \\
&= -C_{FH1} \frac{dP_{FH1}}{dA_{FH}} + C_{HF1} \frac{dP_{HF1}}{dA_{FH}}
\end{aligned}$$

Substitute everything into Equation B.5 to obtain

$$\frac{d\mathcal{U}_H}{dA_{FH}} = -\tilde{\lambda} C_{FH0} \frac{dP_{FH0}}{dA_{FH}} - \mu_0 \frac{dL_0}{dA_{FH}} + \frac{\tilde{\lambda}}{1+i_{H1}} (C_{HF1} \frac{dP_{HF1}}{dA_{FH}} - C_{FH1} \frac{dP_{FH1}}{dA_{FH}}) \tag{B.6}$$

and we substitute in $\tilde{\lambda} = \frac{u'(C_{H0})}{P_{H0}} = \frac{\beta(1+i_{H1})u'(C_{H1})}{P_{H1}}$ to obatin Equation 2.21.

The terms have natural interpretations:

- The first term, $-\tilde{\lambda}C_{FH0}\frac{dP_{FH0}}{dA_{FH}}$ correspond to utility gains from cheaper consumption at $t = 0$. As A_F increases, $\frac{dP_{FH0}}{dA_{FH}}$ takes on a negative value, so the utility increases.

- The second term $-\mu_0\frac{dL_0}{dA_{FH}}$ is the *labor wedge* at $t = 0$. Labor is away from where the consumer wants to supply it. As a result of a higher A_F we have $\mu_0 < 0$ (from Proposition 4) and $dL_0 < 0$, so there is a loss in welfare.

- The third term $C_{HF1}\frac{dP_{HF1}}{dA_{FH}} - C_{FH1}\frac{dP_{FH1}}{dA_{FH}}$ can be interpreted as the terms-of-trade in $t = 1$; it pins down how much total revenue changes from an additional import versus an additional export, multiplied by the marginal utility of a dollar at $t = 1$. This is affected by both the permanent increase in A_F and the trade imbalance that is incurred that affects future terms-of-trade (Lemma 4).

Now we can prove the proposition. Consider a small shock that increases $A_F \to A_F + \epsilon$.

When $\sigma \to 1$, we know that $\mu_0 \to 0$, and $B_{H1} \to 0$. (This is known from Cole and Obstfeld (1991), but we can directly inspect the proof of Proposition 4 and see that all the inequalities become equalities at $\sigma = 1$). So the *first-order relevant* welfare changes are the decrease in prices resulting from the productivity gains (term (1) and the productivity component of term (3)). Thus there is a welfare gain when $\sigma \to 1$.

On the other hand, as $\sigma \to \infty$, the welfare losses from term (2) are discrete. Specifically, consider the following formulation:

$$d\mathcal{U}_H = -\tilde{\lambda}C_{FH0}dP_{FH0} - \mu_0 dL_0 + \frac{\tilde{\lambda}}{1 + i_{H1}}(C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1})$$

When $0 < dA_{FH} < \epsilon$, the first and third terms are bounded by the price changes, which are also at most epsilon: so we have

$$\| - \tilde{\lambda}C_{FH0}dP_{FH0} + \frac{\tilde{\lambda}}{1 + i_{H1}}(C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1})\| < \epsilon_M$$

On the other hand, as $\sigma \to \infty$, we have $L_0 \to 0$, and $\mu_0 \to \mu < 0$; there is a *discrete* loss of welfare associated with an *infinitesimal* change in A_F. As such, we have that for small ϵ and large σ, $\frac{d\mathcal{U}_H}{dA_{FH}} < 0$: there is a welfare loss associated with trade.

Remark. We conjecture that $\frac{d\mathcal{U}_H}{dA_{FH}}$ is *monotonic* in σ, so that there exists a σ^* such that there are welfare gains when $\sigma < \sigma^*$ and losses when $\sigma > \sigma^*$. This seems intuitive, as all three effects (gains from cheaper goods, labor wedge, and future terms-of-trade) should naturally be monotonic in σ. However, we are unable to prove this, and leave this as a possibility. $\square$

B.2.2 Proofs for Subsection 2.3.4

Here we prove the propositions for the optimal policy subsection. For this, we prove the following Lemma.

Lemma 5. *The first-order effect of a tariff and subsidy on Home welfare can be written as:*

$$\begin{aligned} d\mathcal{U}_H = &- \underbrace{\mu_0 dL_0}_{\textit{labor wedge}} + \frac{u'(C_{H0})}{P_{H0}}[\underbrace{t_{FH0}P_{FH0}dC_{FH0}}_{C_{H0}\ \textit{distortion}} - \underbrace{d(s_{HF0}P_{HF0}C_{HF0})}_{\textit{cost of subsidy}}] \\ &+ \frac{\beta u'(C_{H1})}{P_{H1}}(\underbrace{C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1}}_{\textit{future terms-of-trade}}) \end{aligned}$$

Proof. Re-normalize the tariffs $t_{FH0} \to t_{FH0}/P_{FH0}$, and subsidies $s_{HF0} \to s_{HF0}/P_{HF0}$ so that they have the interpretation of a 'flat addition in price', and we can renormalize them back later.

The rest of the argument is similar to the proof of Proposition 5 above. Home agent's lifetime utility is

$$\mathcal{U}_H = U(C_{HH0}, C_{FH0}, C_{HH1}, C_{FH1}, L_{H0}, L_{H1})$$

and is subject to the lifetime budget constraint

$$\begin{aligned} &P_{HH0}C_{HH0} + (P_{FH0} + t_{FH0})C_{FH0} + \frac{1}{1+i_{Ht}}(P_{HH1}C_{HH1} + P_{FH1}C_{FH1}) \\ &= w_{H0}L_{H0} + \frac{1}{1+i_{H1}}w_{H1}L_{H1} + T_{H0} \end{aligned}$$

with $T_{H0} = t_{FH0}C_{FH0} - s_{HF0}C_{HF0}$.

Analogously to the proof of Proposition 5, the first-order effect of any policy on welfare can be written as

$$d\mathcal{U}_H = \sum_{t=0}^{1} \sum_{i\in\{H,F\}} \frac{dU}{dC_{iHt}} dC_{iHt} + \sum_{t=0}^{1} \frac{dU}{dL_{Ht}} dL_{Ht} \tag{B.7}$$

If we denote by $\tilde{\lambda}$ the Lagrange multiplier on the lifetime budget constraint, we have:

$$\frac{dU}{dC_{HH0}} = \tilde{\lambda} P_{HH0}, \quad \frac{dU}{dC_{FH0}} = \tilde{\lambda}(P_{FH0} + t_{FH0})$$
$$\frac{dU}{dC_{HH1}} = \frac{\tilde{\lambda}}{1+i_{H1}} P_{HH1}, \quad \frac{dU}{dC_{FH1}} = \frac{\tilde{\lambda}}{1+i_{H1}} P_{FH1}$$
$$\frac{dU}{dL_{H0}} = -\mu_0 - \tilde{\lambda} w_{H0}, \quad \frac{dU}{dL_{H1}} = -\frac{\tilde{\lambda}}{1+i_{H1}} w_{H1}$$

Plugging these into Equation B.7, we get

$$\begin{aligned} d\mathcal{U}_H =& \tilde{\lambda}\left[\sum_{i\in\{H,F\}}\left(P_{iH0}dC_{iH0} + \frac{P_{iH1}}{1+i_{H1}}dC_{iH1}\right) - w_{H0}dL_{H0} - \frac{w_{H1}}{1+i_{H1}}dL_{H1}\right] \\ &+ \tilde{\lambda} t_{FH0} dC_{FH0} - \mu_0 dL_0 \end{aligned}$$

Now the household lifetime budget constraint, with the tax revenue plugged in, is

$$\begin{aligned} & P_{HH0}C_{HH0} + P_{FH0}C_{FH0} + \frac{1}{1+i_{Ht}}(P_{HH1}C_{HH1} + P_{FH1}C_{FH1}) \\ &= w_{H0}L_{H0} + \frac{1}{1+i_{H1}} w_{H1}L_{H1} - s_{HF0}C_{HF0} \end{aligned}$$

Take the derivative of this, and rearrange to obtain

$$\begin{aligned} & \sum_{i\in\{H,F\}}\left(P_{iH0}dC_{iH0} + \frac{P_{iH1}}{1+i_{H1}}dC_{iH1}\right) - w_{H0}dL_{H0} - \frac{1}{1+i_{H1}}w_{H1}dL_{H1} \\ =& \frac{1}{1+i_{H1}}(C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1}) - d(s_{HF0}C_{HF0}) \end{aligned}$$

where we use the fact that $dP_{HH0} = dP_{FH0} = dw_{H0} = 0$ by rigidity, and then further simplify using the Home labor market clearing condition. Then the first-order welfare effects are given by

$$\begin{aligned} d\mathcal{U}_H &= -\mu_0 dL_0 + \tilde{\lambda} t_{FH0} dC_{FH0} - \tilde{\lambda} d(s_{HF0}C_{HF0}) + \frac{\tilde{\lambda}}{1+i_{H1}}(C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1}) \\ &= -\mu_0 dL_0 + \frac{u'(C_{H0})}{P_{H0}}[t_{FH0}dC_{FH0} - d(s_{HF0}C_{HF0})] + \frac{\beta u'(C_{H1})}{P_{H1}}(C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1}) \end{aligned}$$

$\square$

Lemma 2. *The optimal short-run tariff rate on imports t_{FH0} satisfies*

$$t_{FH0} = \frac{1}{P_{FH0}} \left[\underbrace{\frac{\mu_0}{\tilde{\lambda}} \frac{\partial L_{H0}}{\partial C_{FH0}}}_{\text{labor wedge}} - \frac{1}{(1+i_{H1})} \underbrace{(L_{HF1} \frac{\partial w_{H1}}{\partial C_{FH0}} - L_{FH1} \frac{\partial w_{F1}}{\partial C_{FH0}})}_{\text{future terms-of-trade}} + \underbrace{s_{HF0} P_{HF0} \frac{\partial C_{HF0}}{\partial C_{FH0}}}_{\text{subsidy externality}} \right] \tag{B.8}$$

The optimal short-run subsidy rate on exports s_{HF0} satisfies

$$s_{HF0} = \frac{1}{P_{HF0}} \left[\underbrace{-\frac{\mu_0}{\tilde{\lambda}} \frac{\partial L_{H0}}{\partial C_{HF0}}}_{\text{labor wedge}} + \frac{1}{(1+i_{H1})} \underbrace{(L_{HF1} \frac{\partial w_{H1}}{\partial C_{HF0}} - L_{FH1} \frac{\partial w_{F1}}{\partial C_{HF0}})}_{\text{future terms-of-trade}} - \underbrace{P_{HF0} C_{HF0} \frac{\partial s_{HF0}}{\partial C_{HF0}}}_{\text{terms-of-trade today}} \right] \tag{B.9}$$

where $\tilde{\lambda}$ is the Lagrange multiplier on the lifetime budget constraint.

Proof. Under variation in tariffs, the optimal tariff rate with $d\mathcal{U}_H = 0$ will satisfy

$$t_{FH0} = \frac{1}{P_{FH0} \frac{dC_{FH0}}{dt_{FH0}}} \left[\frac{\mu_0}{\tilde{\lambda}} \frac{dL_{H0}}{dt_{FH0}} + \frac{d(s_{HF0} P_{HF0} C_{HF0})}{dt_{HF0}} - \frac{1}{(1+i_{H1})} (L_{HF1} \frac{dw_{H1}}{dt_{FH0}} - L_{FH1} \frac{dw_{F1}}{dt_{FH0}}) \right]$$

The multiplier $\frac{1}{P_{FH0} \frac{dC_{FH0}}{dt_{FH0}}} < 0$ corresponds to the inverse elasticity of domestic demand with respect to tariffs; a lower elasticity implies a higher tariff rate. The first term is the effect of tariff on the labor wedge. Since $\frac{dL_{H0}}{dt_{FH0}} > 0$, when there is unemployment ($\mu_0 < 0$), we want a higher tariff. The second term is the effect of tariffs on subsidy revenue; a higher tariff will decrease real wage in Foreign, leading them to work/consume less, decreasing subsidy revenue. The third term is how much future terms-of-trade moves, in terms of how much marginal revenue from exports vs expenditure from imports move. A higher tariff will lead to less borrowing, leading to improving terms-of-trade, increasing the term.

In summary, when there is unemployment ($\mu_0 < 0$), the three terms inside the bracket are all negative; thus the optimal tariff t_{FH0} is *positive.*

A special case is when the Home economy is small; here today's tariffs cannot affect (1) tomorrow's terms-of-trade and (2) the subsidy revenue, so the optimal tariff is simply

$$t_{FH0} = \frac{1}{P_{FH0} \frac{dC_{FH0}}{dt_{FH0}}} \frac{\mu_0}{\tilde{\lambda}} \frac{dL_{H0}}{dt_{FH0}}$$

and this immediately shows that (1) the tariff is positive and (2) the tariff leaves some unemployment ($\mu_0 < 0$; otherwise, we have a contradiction.)

Now, considering variation in subsidies, we have

$$\begin{aligned} s_{HF0} =& \frac{1}{P_{HF0}\frac{dC_{HF0}}{ds_{HF0}}}[-P_{HF0}C_{HF0} + t_{FH0}P_{FH0}\frac{dC_{FH0}}{ds_{HF0}} \\ & - \frac{\mu_0}{\tilde{\lambda}}\frac{dL_{H0}}{ds_{HF0}} + \frac{1}{(1+i_{H1})}(L_{HF1}\frac{dw_{H1}}{ds_{HF0}} - L_{FH1}\frac{dw_{F1}}{ds_{HF0}})] \end{aligned}$$

The multiplier $\frac{1}{P_{HF0}\frac{dC_{HF0}}{s_{HF0}}} > 0$ corresponds to the inverse elasticity of foreign demand with respect to exports, and is positive. The first term is the resource cost of the subsidy; it costs to sell cheap goods. The second term is how much consumption distortion by tariffs is affected by subsidies; with a positive tariff, domestic subsidies will be a resource cost that reduces spending overall. The last two terms deliver similar intuition to the tariff case, with both forces implying a *positive* subsidy. □

Proposition 6. *If there is unemployment at the zero-tariff economy ($\mu_{H0} < 0$ when $t_{FH0} = 0$), the optimal tariff t_{FH0} is positive and is increasing in the size of the trade shock A_{FH0}.*

Proof. When $\mu_{H0} < 0$, all three terms in the optimal tariff formula (Equation 2.23) are positive:

- The first term is positive since an increase in imports C_{FH0} reduce demand for Home labor.
- the second is positive since an increase in C_{FH0} decrease w_{H1} relative to w_{F1} tomorrow (transfer affecting future terms-of-trade effect).
- The third term is positive since an increase in C_{FH0} is associated with an increase in exports C_{HF0}.

Likewise, all three forces increase when the magnitude of A_{FH0} increases. □

Proposition 7. *When $\gamma = 1$, optimal monetary policy R_{H1} satisfies the following equation:*

$$0 = \underbrace{-\mu_0\frac{dL_0}{dR_{H1}}}_{\text{wedge}} + \tilde{\lambda}_r[\underbrace{R_{H1}t_{FH0}\frac{P_{FH0}}{P_{H0}}\frac{dC_{FH0}}{dR_{H1}}}_{\text{tariff fiscal externality}} + \underbrace{(NX_0)}_{\text{intertemporal TOT}}], \tag{B.10}$$

where $\tilde{\lambda}_r$ is the Lagrange multiplier on the Home lifetime real budget constraint normalized by P_{H0}.

As a special case, when $t_{FH0} = 0$, the optimal monetary policy R_{H1} is such that $\mu_0 > 0$: it is optimal to loosen monetary policy beyond clearing the output gap.

Proof. Since the central bank is choosing the real rate R_{H1}, we rewrite the budget constraint to incorporate R_{H1}:

$$\begin{aligned} & R_{H1}\frac{1}{P_{H0}}(P_{HH0}C_{HH0} + (P_{FH0} + t_{FH0})C_{FH0}) + \frac{1}{P_{H1}}(P_{HH1}C_{HH1} + P_{FH1}C_{FH1}) \\ & = R_{H1}\frac{1}{P_{H0}}(w_{H0}L_{H0} + T_{H0}) + \frac{w_{H1}}{P_{H1}}L_{H1} \end{aligned}$$

Then the Lagrange multiplier on this *real* budget constraint is $\tilde{\lambda}_r = \frac{u'(C_{H0})}{R_{H1}} = \beta u'(C_{H1})$

Recall that the central bank's monetary policy rule sets interest rate according to Equation 2.5:

$$\log(1 + i_{H1}) = -\log(\beta) + \log(\frac{P_{H1}}{P_{H0}}) + \epsilon_{H0} \;\Leftrightarrow\; R_{H1} = \frac{1}{\beta}\exp(\epsilon_{H0})$$

We consider variations in $\exp(\epsilon_{H0})$ that leave inflation constant; notably, P_{H1} does not move in this variation.

Transform the marginal change in utility in a way analogous to Lemma 5 to write

$$\begin{aligned} d\mathcal{U}_H =& \tilde{\lambda}_r \left[\sum_{i\in\{H,F\}} \left(R_{H1}\frac{P_{iH0}}{P_{H0}}dC_{iH0} + \frac{P_{iH1}}{P_{H1}}dC_{iH1} \right) - R_{H1}\frac{w_{H0}}{P_{H0}}dL_{H0} - \frac{w_{H1}}{P_{H1}}dL_{H1} \right] \\ & + \tilde{\lambda}_r R_{H1}\frac{t_{FH0}}{P_{H0}}dC_{FH0} - \mu_0 dL_0 \end{aligned}$$

Taking the derivative of the budget constraint, we get:

$$\begin{aligned} & \sum_{i\in\{H,F\}} \left(R_{H1}\frac{P_{iH0}}{P_{H0}}dC_{iH0} + \frac{P_{iH1}}{P_{H1}}dC_{iH1} \right) - R_{H1}\frac{w_{H0}}{P_{H0}}dL_{H0} - \frac{w_{H1}}{P_{H1}}dL_{H1} \\ & = \frac{1}{P_{H1}}(C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1}) + dR_{H1}(\frac{1}{P_{H0}}NX_{H0}) \end{aligned}$$

where $NX_{H0} = (w_{H0}L_{H0} + T_{H0}) - P_{HH0}C_{HH0} - (P_{FH0} + t_{FH0})C_{FH0} = \frac{B_{H1}}{R_{H1}}$ is the net export in period 0. Plugging this in and replacing $t_{FH0} \to t_{FH0}P_{FH0}$, we get

$$\begin{aligned} d\mathcal{U}_H =& -\mu_0 dL_0 + \tilde{\lambda}_r[R_{H1}\frac{t_{FH0}P_{FH0}}{P_{H0}}dC_{FH0} \\ & + \frac{1}{P_{H1}}(C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1}) + dR_{H1}(\frac{1}{P_{H0}}NX_{H0})] \end{aligned}$$

Now we note that when $\gamma = 1$, the equilibrium level of real balances $\frac{B_{H1}}{P_{H1}}$ do not depend on R_{H1}. This is because after any change in $R_{H1} \to \zeta R_{H1}$ for some constant ζ, the equilibrium conditions exactly hold if we replace C_{ij1}, C_{i1}, L_{i1} with $\zeta C_{ij1}, \zeta C_{i1}, \zeta L_{i1}$; monetary policy affects period 0 without affecting any real variables in period 1. (We can verify by inspecting the equilibrium conditions)

Thus, the period 1 variables do not depend on R_{H1}, and under the optimal monetary policy, the above equation becomes

$$0 - \mu_0 dL_0 + \tilde{\lambda}_r[R_{H1}\frac{t_{FH0}P_{FH0}}{P_{H0}}dC_{FH0} + dR_{H1}(\frac{1}{P_{H0}}NX_{H0})] \tag{B.11}$$

which is exactly the equation in the proposition. □

B.3 Derivations and microfoundations

In this section, we derive the equations in the main text in 2.4.

B.3.1 Equilibrium in the quantitative model

The equations characterizing the equilibrium (Definition 2) in the case when China pegs is given by the following conditions:

1. Family optimization:

$$P_{jt} = \prod_s (P_{jt}^s)^{\alpha_j^s} \tag{B.12}$$

$$P_{jt}^s = [\sum_i ((1+t_{ijt}^s)P_{ijt}^s)^{1-\sigma_s}]^{\frac{1}{1-\sigma_s}} \tag{B.13}$$

$$\lambda_{ijt}^s = \frac{((1+t_{ijt}^s)P_{ijt}^s)^{1-\sigma_s}}{\sum_k ((1+t_{kjt}^s)P_{kjt}^s)^{1-\sigma_s}} \tag{B.14}$$

$$\tilde{\lambda}_{it} = \frac{u'(C_{it})}{P_{it}} \tag{B.15}$$

$$u'(C_{jt}) = \beta\hat{\delta}_{jt}(1+i_{jt})\frac{P_{jt}}{P_{jt+1}}u'(C_{jt+1}) \tag{B.16}$$

$$1 + i_{it} = (1+i_{jt})\frac{e_{ijt+1}}{e_{ijt}} \tag{B.17}$$

$$P_{jt}C_{jt}\bar{L}_j + \frac{1}{1+i_{jt}}B_{jt+1} \leq B_{jt} + \sum_s W_{jt}^s \ell_{jt}^s L_{jt}^s + \Pi_{jt} + T_{jt} \tag{B.18}$$

2. Firm optimization: if R_{jt}^s is total revenue of sector s in country j at time t, we have

$$P_{ijt}^s = e_{ijt}\tau_{ijt}^s \frac{1}{A_{it}^s}(W_{it}^s)^{\phi_i^s}\prod_n (P_{it}^n)^{\phi_i^{ns}} \tag{B.19}$$

$$W_{it}^s \ell_{it}^s L_{it}^s = \phi_i^s R_{it}^s \tag{B.20}$$

3. Labor supply: given by New Keynesian Phillips curve

$$\log(\pi_{it}^{sw}+1)=\kappa_w(v'(\ell_{it}^s)-\frac{W_{it}^s}{P_{it}}u'(C_{it}))+\beta\log(\pi_{it+1}^{sw}+1) \tag{B.21}$$

4. Labor reallocation and worker's value function:

$$\mu_{it}^{sn}=\frac{\exp(\frac{1}{\nu}(\beta V_{it+1}^n-\chi_{it}^{sn}))}{\sum_{n'}\exp(\frac{1}{\nu}(\beta V_{it+1}^{n'}-\chi_{it}^{sn'}))} \tag{B.22}$$

$$V_{it}^s=\tilde{\lambda}_{it}W_{it}^s\ell_{it}^s+\eta_{it}^s-v(\ell_{it}^s)+\nu\log\left(\sum_n\exp(\frac{1}{\nu}(\beta V_{it+1}^n-\chi_{it}^{sn}))\right) \tag{B.23}$$

$$L_{it+1}^n=\sum_s\mu_{it}^{sn}L_{it}^s \tag{B.24}$$

5. Monetary policy and exchange rates:

$$\log(1+i_{1t})=r_{1t}+\phi_\pi\log(1+\pi_{1t})+\epsilon_{1t} \tag{B.25}$$

$$e_{2t}=\bar{e} \tag{B.26}$$

$$\log(1+i_{jt})=r_{it}+\phi_\pi\log(1+\pi_{jt})+\epsilon_{jt}\quad(j\geq 3) \tag{B.27}$$

$$\lim_{T\to\infty}B_{jT}=0\quad(j\geq 3) \tag{B.28}$$

6. Market clearing conditions:

$$R_{it}^s=\sum_j e_{jit}\lambda_{ijt}^s\left[\alpha_j^sP_{jt}C_{jt}+\sum_n\phi_j^{sn}R_{jt}^n\right] \tag{B.29}$$

$$0=\sum_i B_{it}e_{i1t} \tag{B.30}$$

The equilibrium is: given calibrated parameters and initial conditions $w_{j,-1}^s$, B_{j0}, L_{j0}^s, a sequence of variables $\{X_t\}_{t=0}^{\infty}$ where

$$X_t=(B_{jt},C_{jt},P_{jt},e_{jt},W_{jt}^s,P_{jt}^s,L_{jt}^s,\ell_{jt}^s,V_{jt}^s)$$

that satisfy Equations (B.12) to (B.30). In the case where China floats its exchange rate, we replace $e_{2t}=\bar{e}$ with an analogous Taylor rule for China along with $\lim_{T\to\infty}B_{2T}=0$.

In the next subsections, we derive each of the equations, especially the ones that are new in the quantitative setup.

New Keynesian Phillips curve

Suppress the country and sector index (i, s). In each labor market, the maximization problem of the labor packer ι at time t facing a labor demand curve with elasticity ϵ_w is

$$\max_{w_t(l)} \sum_{t \geq t'} \beta^{t'-t} [\tilde{\lambda}_{t'} w_{t'}(\iota) l_{t'}(\iota) - \int v(l_{t'}(\iota)) d\iota - \Phi(\frac{w_{t'}(\iota)}{w_{t'-1}(\iota)}) L_{t'}]$$

where $l_{t'}(\iota) = (\frac{w_{t'}(\iota)}{w_{t'}})^{-\epsilon_w} L_t$. The FOC wrt $w_t(\iota)$ is:

$$\begin{aligned} 0 = &\tilde{\lambda}_t (1 - \epsilon_w)(\frac{w_t(\iota)}{w_t})^{-\epsilon} L_t + v'(l_t(\iota)) \epsilon_w (\frac{w_t(\iota)}{w_t})^{-\epsilon_w - 1} \frac{L_t}{w_t} \\ &- \Phi'(\frac{w_t(\iota)}{w_{t-1}(\iota)}) \frac{1}{w_{t-1}(\iota)} L_t + \beta \Phi'(\frac{w_{t+1}(\iota)}{w_t(\iota)}) \frac{w_{t+1}(\iota)}{w_t(\iota)^2} L_{t+1} \end{aligned}$$

Impose symmetry $w_t(\iota) = w_t$ and $l_t(\iota) = \ell_t$, if we let wage inflation $1 + \pi_t^w = \frac{w_t}{w_{t-1}} - 1$, the above equation becomes

$$0 = \tilde{\lambda}_t (1 - \epsilon_w) L_t w_t + v'(\ell_t) \epsilon_w L_t - \Phi'(1 + \pi_t^w)(1 + \pi_t^w) L_t + \beta \Phi'(1 + \pi_{t+1}^w)(1 + \pi_{t+1}^w) L_{t+1}$$

If we let $\Phi(x) = \epsilon_w \frac{1}{2\kappa_w} (\log x)^2$, then $\Phi'(\pi) = \frac{\epsilon_w}{\kappa_w} \frac{1}{x} \log x$. Moreover, $\tilde{\lambda}_t = \frac{u'(C_t)}{P_t}$, and letting $\mu_w = \frac{\epsilon_w}{\epsilon_w - 1}$ be markup, we have

$$\log(1 + \pi_t^w) = \kappa_w \underbrace{(v'(\ell_t) - w_t \frac{u'(C_t)}{P_t} \mu_w)}_{\text{output gap}} + \beta \log(1 + \pi_{t+1}^w) \frac{L_{t+1}}{L_t}$$

Note that when cost of adjustment is zero, $\kappa_w \to \infty$ so output gap becomes zero. Since we are not interested in the markup that unions charge, we assume that every period we tax w_t so that wage markup is undone and any tax revenue is rebated to the household lump-sum, we have the desired New Keynesian Phillips Curve:

$$\log(1 + \pi_t^w) = \kappa_w (v'(L_t) - w_t \frac{u'(C_t)}{P_t}) + \beta \log(1 + \pi_{t+1}^w) \frac{L_{t+1}}{L_t}$$

Exchange rate determination

In Section 2.4, for each floating country i, we defined the exchange rate in period e_{i0} to be the unique value such that

$$\lim_{t \to \infty} B_{it} = 0. \tag{2.36}$$

Here we microfound this condition through the *segmented financial market* model, a reduced-form version of Itskhoki and Mukhin (2021). We assume that the household family in country i cannot directly trade any assets with one another, and the international asset positions are intermediated by the financial sector. As in the main text, households in each country i demand a quantity B_{it+1} of home-currency bonds in time t, giving identical optimization conditions, minus the UIP condition (since we do not have free bond markets).

The financial sector features two additional types of agents that trade bonds internationally: arbitraguers and noise traders. We assume countries $i \geq 2$ have each type of them, and they trade domestic bonds and US dollars only.[2] Each period, arbitraguers of mass m_i in country i choose a zero-capital portfolio (d_{it+1}, d^U_{it+1}) such that $\frac{d_{it+1}}{R_{it}} + \frac{1}{e_{it}}\frac{d^U_{it+1}}{R_{1t}} = 0$, where $R_{it} = 1 + i_{it}$ is the gross return, or the inverse price of bonds of country i at time t, and $e_{it} = e_{i1t}$ is the value of currency i with respect to the US dollar. Their profits are rebated lump-sum to the household in i, and seek to maximize the CARA utility of the real return in units of country i goods:

$$\max_{d_{it}} \mathbb{E}_t \left[-\frac{1}{\omega} \exp\left(-\omega \frac{(R_{it} - R_{1t}\frac{e_{it+1}}{e_{it}})d_{it+1}}{P_{it+1}} R_{it} \right) \right] \tag{B.31}$$

where ω is the risk aversion parameter.

In addition, the financial market features a liquidity demand from a measure n_i of symmetric noise traders in each country $i \geq 2$. The total positions in US dollar bonds invested by noise trader in country i is modeled as an exogenous process

$$\frac{N^U_{it+1}}{1 + i_{it}} = n(e^{\psi_t} - 1) \quad \text{with} \quad \psi_t = \rho_\psi \psi_{t-1} + \sigma_\psi \epsilon^{\psi_t}_t. \tag{B.32}$$

and they invest in country i bonds equivalent to this.

Denoting the total position of arbitraguers as $D_{it+1} = m_i d_{it+1}$, we have the portfolio balance condition for each i:

$$B_{it+1} + N_{it+1} + D_{it+1} = 0 \quad \text{and} \quad B_{1t+1} + \sum_{i \geq 2} (N^U_{it+1} + D^U_{it+1}) = 0 \tag{B.33}$$

The fact that intermediaries are risk-averse ($\omega > 0$) require them to take some compensation, and yields the *modified* UIP condition for each country with respect to the US dollar:

Lemma 6. *(Lemma 1 of Itskhoki and Mukhin (2021).) The equilibrium condition in the finnacial market, log-linearized around a symmetric steady-state with* $\bar{B}_i = 0, \bar{R} = \frac{1}{\beta}$, *is*

[2] This can be relaxed, and is mainly for clarity of exposition.

given by

$$i_{it} - i_{1t} = \mathbb{E}_t \Delta e_{t+1} + \chi_1 \psi_t - \chi_2 b_{t+1} \tag{B.34}$$

where $\chi_1 = \frac{n}{\beta} \frac{\omega \sigma_e^2}{m}$ *and* $\chi_2 = \bar{Y} \frac{\omega \sigma_e^2}{m}$.

Consider the limit of this economy, first where $n \to 0$, sending the magnitude of the noise trader to zero, while fixing $\frac{\omega}{\sigma_e^2} m$ (with an appropriate adjusting financial shock volatility). The UIP deviation then becomes

$$i_{it} - i_{1t} = \mathbb{E}_t \Delta e_{t+1} - \chi_2 b_{t+1}. \tag{B.35}$$

Note that this condition can alternatively be microfounded through convex portfolio adjustment costs (Kouri, 1976) or debt-elastic interest rate premiums (Schmitt-Grohé and Uribe, 2003); the business-cycle level equivalence of these models are explored in (Schmitt-Grohé and Uribe, 2003).

We highlight that under Equation B.35, the model is stationary, and when e_{it} is pursuing an independent monetary policy, we must have

$$\lim_{t \to \infty} b_{t+1} = 0, \tag{B.36}$$

in any steady-state. If we take the limit $\chi_2 \to 0$, the condition converges to

$$i_{it} - i_{1t} = \mathbb{E}_t \Delta e_{t+1} \tag{B.37}$$

which is the UIP condition, and a terminal condition given by Equation B.36.

Discussion on relevance. Why do we need an extra 'terminal' condition under UIP? This is closely related to the indeterminacy result by Kareken and Wallace (1981). Under frictionless bond markets with pure interest rate targets, the exchange rate at $t = 0$ after a shock is indeterminate. While this fact is a pure nominal result without real consequences in Kareken and Wallace (1981), in our model, each *level* of the nominal exchange has real implications on output and labor supply, as it connects with the *nominal wage anchor* from $t = -1$: different exchange rates correspond to different levels of output and demand in each country. The fact that the indeterminacy result could have real implications in setups of nominal rigidity and independent interest rates is also explored in Caballero, Farhi, and Gourinchas (2021), and the nonstationarity of a pure UIP model is also discussed in (Schmitt-Grohé and Uribe, 2003).

Labor and unemployment as extensive margin

In our current formulation, all supply of labor is at the intensive margin. We provide a microfoundation of the labor supply problem in terms of the extensive margin, following Gali (2008). We assume that each member m draws idiosyncratic productivity shocks $\{\epsilon_{it}^n(m)\}$ distributed Type 1 EV, and moving fromm sector s to n involves moving costs of χ_{it}^{sn}:

$$v(\{\epsilon_{it}^n(m)\}_n, s_{it}(m), s_{it-1}(m)) = \sum_{n,k} [\epsilon_{it}^n(m) - \chi_{it}^{sn}]\, \mathbb{I}(s_{it}(m) = n, s_{it-1}(m) = s),$$

Then, given sectoral choice $n = s_{it}(m)$, we pin down optimal work decisions at that sector (under full employment). Each member m has a disutility from wage inflation and work according to

$$\Phi\left(\iota_{it}(m), \{\pi_{it}^{w,s}\}\right) = -\iota_{it}(m) - \Phi_{it}^s(\pi_{it}^{w,s})$$

where $\iota_{it}(m)$ is the disutility from working. Once a member m is in sector n, we assume that the households draw idiosyncratic disutility from work after choosing a sector n:

$$\iota_{it}(m) = \tilde{\iota}^\nu, \quad \tilde{\iota} \sim_{iid} U[0,1].$$

Households decide to work if

$$\bar{v}\tilde{\iota}^\nu \leq \tilde{\lambda}_{it} w_{it}^n,$$

where $\tilde{\lambda}_{it}$ is the Lagrangian multiplier on the budget constraint, and w_{it}^n is the wage. Then, conditional on choosing sector n, fraction $\ell \in [0,1]$ member will want to work where

$$\ell_{it}^n \in \arg\max_{\ell \in [0,1]} w_{it}^n \lambda_{it} - v(\ell)$$

with

$$v(\ell) = \bar{v} \int^{\ell} \tilde{\iota}^\nu d\tilde{\iota} = \bar{v} \frac{\ell^{1+\nu}}{1+\nu}.$$