PROBABILISTIC CHOICE SET GENERATION IN TRANSPORTATION DEMAND MODELS

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ABSTRACT

Motivated by practical and theoretical shortcomings of existing discrete choice models of travel behavior, especially in the application context of developing countries, this research proposes to investigate expansion of extant discrete choice models to include consideration of economic, social, and cultural constraints. A two-stage choice process, composed of the choice set formation and actual choice stages, is proposed and modelled. Various models of choice set formation and choice are proposed that specify, at differing levels of aggregation, the impact of constraints on individual choice.

The proposed methodology, deemed of special relevance to modelling travel behavior in developing countries, is empirically tested with data from two Brazilian cities, Maceio and São Paulo. Work mode choice is modelled in both cases. The models of choice set formation are compared against standard models of discrete choice in both statistical and predictive terms.

It is shown that in these two cities, for the work mode choice dimension, a standard logit choice model that accounts for alternative availability via the use of variables in the choice utilities functions is a robust, cheaper specification when compared to simple models of choice set formation. The São Paulo work, however, raises the possibility that explicit modelling of constraints will pay dividends in terms of improved models of travel behavior.

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CHAPTER 1

INTRODUCTION AND STATEMENT OF OBJECTIVES

1.1 The Role of Choice Set Formation Within the Classical Paradigm of Individual Choice

The theory of individual choice, developed in the field of psychology (see the review in Luce and Suppes, 1965) and later adapted to explain economic behavior of consumers with respect to discrete goods, has found a fertile field for application in transport demand estimation (see for example, Domencich and McFadden, 1975; Ben-Akiva, 1973; Ben-Akiva and Lerman, 1974; Adler and Ben-Akiva, 1976; Lave and Train, 1979; Westin and Gillen, 1978; Landau *et al.*, 1982; Train, 1980; Swait *et al.*, 1984).

The theory of disaggregate choice is based upon the axiom of a rational consumer, who, when faced with a choice among the discrete elements of some well-defined set, makes use of a pre-established decision rule to make his selection. Imbedded in this paradigm of choice are a series of assumptions (*e.g.* rationality, existence and type of decision rules) necessary to make the theory operational.

The assumption of special interest here concerns the set of discrete goods (to be called the <u>choice set</u>) from among which a final selection is made. In practical modelling applications it has traditionally been assumed that the analyst, who commonly has available only the revealed choice, is able to specify the choice set from which the observed choice was made. Specification of the choice set has almost invariably been dealt with in two ways:

- ignoring the issue and making all alternatives available to all decision-makers;
- using logical rules to impute the set, <u>e.g.</u> in mode choice models, no auto drive alternative for individuals without a license.

The effect of ignoring or misspecifying choice sets is apparent, after some thought, in modelling problems where it is expected that major changes in society, family structure, and economy will occur over time. For example, improvements in the transport infrastructure will not only result in a modification of the propensity to utilize one or another mode, it will also affect the availability of the modes to individuals. Hence, their choice sets and choices will both be potentially modified. It is to be expected that traditional discrete choice models, indeed demand models of any type that do not consider choice set generation, will display a lack of robustness (in terms of stability of parameters, hence forecasts) as the time horizon of the application increases.

Developing countries, characterized by high degrees of heterogeneity along social, cultural, and economic dimensions, are prime examples of application contexts in which flux is the norm. The scarcity of resources, juxtaposed to the many (and ever-increasing) needs of a burgeoning population, make it critical that planners have at their disposal the best possible information upon which to base their recommendations for resource allocation.

For some time, the transport demand and econometrics literature has also recognised the limitations of using the two approaches mentioned above to <u>a priori</u> define choice sets. Lerman(1975) was one to early on recognize the inappropriateness of allocating all alternatives to all decision-makers: "The underlying theory of the logit model requires that the choice set for each observation consist of feasible alternatives... in order to properly estimate a mode choice model, one must know which of the set of possible alternatives are actually available to the individual." (Lerman, 1975, p. 244) The scope of this observation is not, of course, limited to the logit specification or the mode choice context, but is applicable to any discrete model specification and choice dimension.

This type of reasoning has led to the widespread use of imputed choice sets in practical applications of discrete choice models. Ben-Akiva and Lerman(1974), for instance, in modelling the joint choice of auto ownership level and mode to work in Washington, DC, use rules such as (1) no auto ownership permissible to households without licensed drivers and (2) no transit option for households living in fringe suburban areas.

Train(1980) reports another study of this same pair of choice dimensions for the San Francisco area. As part of the calibration process, he estimates a model of mode choice conditional on auto ownership level; for this model, Train employs a number of rules to allocate to individuals the modal alternatives: (1) no auto drive for persons from households without an automobile; (2) any of the transit alternatives are eliminated if its use entails three or more transfers to or from work, or if the one-way travel time exceeds four hours.

These papers are only two examples (see also Quarmby, 1967; Watson, 1974) of the compromise that both practitioners and researchers have made to achieve reasonable model forms. Many researchers, however, have been aware of the problems that result from employing the approaches we've mentioned above. Williams and Ortuzar(1979), for example, study the effects (on both parameter estimation and forecasting) of giving all alternatives to all individuals when the true process of alternative allocation follows some known probabilistic process. They demonstrate the biasing of parameter estimates that occurs, as well as errors in forecasting, due to the misallocation of alternatives. McFadden and Reid(1975) and Westin(1974) show the same type of results when the procedure to define choice sets is to impute them by the use of logical rules. These researchers are not alone in pointing out problems with these methods of generating choice sets (see, for example, Stopher, 1980; Meyer, 1979); we shall investigate these references in more detail in Chapter 2.

Diagnosis has not been the only concern of researchers. Several investigators, trying to overcome the deficiencies of extant discrete choice model specifications, have formulated explicit models of choice set formation (notably McFadden, 1976a; Ben-Akiva, 1977; Meyer, 1979; Gaudry and Dagenais, 1979; and Pitschke, 1980). With the exception of Pitschke(1980) and Gaudry and Wills(1979), however, none of the researchers has actually attempted to calibrate models of discrete choice that incorporate choice set formation. And more importantly, only Meyer(1979) has given a behavioral interpretation to the underlying process of generation of an individual's choice set. The other authors have limited themselves to developing econometric specifications (and in two of the cases, actually calibrating models), but with little or no behavioral motivation for the choice set generation stage of their specifications.

1.2 Research Scope and Objectives

The summary of the state-of-the-art of choice set formation modelling that we have provided in the previous section demonstrates two omissions in the literature: first, there is a paucity of behavioral interpretation for the choice set formation process, which makes the development of specific models a more difficult matter since we are left with an unsound foundation upon which to stand and formulate behavioral hypotheses about that process; second, there is little empirical comparative work that tries to measure the costs and benefits of applying choice set formation modelling versus traditional discrete choice models. Accordingly, this research sets for itself three overall objectives:

(1) develop a behavioral framework that allows for incorporation of choice set formation in discrete choice models in general, and

in transport demand models in particular;

(2) formulate a number of hierarchically more complex and behaviorally sound models of choice set generation, evaluating their applicability to different choice contexts relevant to transport analysts, especially in developing countries; and

(3) empirically test the practicality of estimating several of the more promising models formulated under (2), to evaluate the benefits that are gained in terms of a greater accuracy and better understanding of individual behavior from the improved model realism, versus the cost, time and sophistication necessary to calibrate such models.

This study is not theoretical in nature: the theory it needs is extant, but is here aggregated from divers sources and cemented together in a manner suitable for analyzing individual transport-related behavior. Hence, the first objective given above.

Given its practical nature, this thesis will give equal emphasis to the second and third objectives in an effort to arrive at useable models incorporating the choice formation stage of the choice paradigm.

The third objective is carried out within the context of modelling work trip modal choice in Maceio and São Paulo, Brazil. The basic premise of the proposed approach is that choice set formation is a question of identifying relevant constraints on decision-making units, and a city in a developing country is a likely place to find constrained individuals.

The two cities form a sharply contrasting pair of case study areas. Maceio is a city of less than one-half million located in the Northeast of Brazil, a region historically plagued by droughts, almost completely reliant upon cash crops such as sugar cane, and held back from development by a semi-feudal social structure. Swait _et al._(1984) and Geltner and Barros(1984) report in some detail on the travel behavior of the residents of Maceio.

At the other extreme in Brazil we find São Paulo, a teeming metropolis of more than 11 million people. One of the fastest growing cities in the world, it is the heart of Brazil's industrial park. With a far more equitable income distribution than is found in Maceio, it is natural that we find in São Paulo a wider spectrum of economic and social strata than are to be found in the smaller, poorer city of the Northeast. The richness of behavioral variability in São Paulo adds significantly to the validity of the conclusions we make based upon the empirical work undertaken in this research.

As for the reasons for using modal choice as the vehicle for evaluating the practicality of calibrating choice set models, there are several.

Firstly, because of the degree of flexibility individuals associate with the modal dimension of transport behavior, it is very often the vehicle used by transport planners to implement policy objectives. In addition, there are many studies of this choice dimension that are not based on a detailed consideration of choice set formation (we have mentioned a few these studies before: Quarmby, 1967; Ben-Akiva and Lerman, 1974; Train, 1980; Westin and Gillen, 1978; others are Lerman, 1976; Liou et al., 1975; McFadden(1976b)), and can thus serve as a basis for comparison with our empirical conclusions.

Secondly, work modal choice can be changed in the short run by individuals that are not highly constrained. If the investigation demonstrates the importance and feasibility of modelling choice set formation for even this level of choice, how much more so will it not be for such choices as residential location, shopping destination, quantity and type of automobile, and so forth?

Lastly, the more general models of choice set formation that will be presented become infeasible to calibrate with a large number of alternatives; to permit an uniform comparison of model performance, modal choice was selected because it permitted calibrating even complex models within the available time and money budgets.

1.3 Outline Of the Thesis

Chapter 2 deals principally with two topics: first, we formulate a constraint-based approach to choice set formation that comprises the behavioral framework we desired in our first research objective (see Section 1.2); second, it investigates the practical and theoretical impacts of ignoring choice set formation by reviewing the research that has been presented in the literature, and by conducting a theoretical analysis of model misspecification for a specific model form and choice context.

Based on our knowledge of the deleterious impact of ignoring or misspecifying choice sets, we set out in Chapter 3 to formulate a number of choice models that include a probabilistic choice set formation stage, obeying the constraint-based approach of Chapter 2. This model development is not intended to be exhaustive, by any means, but is intended to exemplify our approach and prepare specific model forms for the empirical phase of the research.

In Chapters 4 and 5 we present the calibration results for standard discrete choice and choice set formation models for Maceio and São Paulo, respectively. In the first of these two chapters, the emphasis of the empirical work is upon comparing the costs and benefits of utilizing simple models of choice set formation as opposed to the logit model, for the work mode choice dimension.

The purpose of Chapter 5, which to some extent is to confirm the observations we make in Maceio, is more directed at exploring whether inclusion of ad hoc variables related to alternative availability in standard discrete choice models is a practical substitute for choice set formation modelling. The approach we take to investigate this question opens an interesting avenue for future research in the area of choice set formation modelling.

Recommendations for future investigation of this topic and others raised during the course of the empirical work are given in Chapter 6, which also presents an overview of the present research effort and summarizes its principal findings.

CHAPTER 2

THE ROLE OF CONSTRAINTS IN THE CHOICE SET FORMATION PROCESS

2.1 Introduction

This chapter presents the proposed approach to choice set modelling, outlining a typology of constraints generally applicable to individual urban travel contexts. Clearly, it is not possible to attempt the development of a set of constraints and/or models applicable to every conceivable situation of interest. This chapter presents a modelling _strategy_ or _methodology_ which must be tailored to fit a specific situation of interest. It is _not_ sufficient to characterize the attributes of the alternatives (_e.g._ time and cost in a modal choice context) related to a choice situation; it is also necessary to describe the relationships between the individual decision-maker and his environment (_i.e._ physical context, cultural traditions, family and group ties, situation within or without the market economy), which are important determinants of choice set formation.

Chapter 2 also presents a treatment of the manner in which use of the constraint typology, introduced herein, can be integrated into a modelling approach. This will serve as introduction to the subject matter of Chapter 3, in which a series of choice set formation models is formulated and discussed with respect to their applicability to several transport-related choice contexts.

To finalize the chapter, we investigate the impact of not modelling choice set formation upon both the estimation of choice model parameters and use of the models in forecasting. We do this in two stages: first, we review the literature dealing with this aspect of model misspecification; second, we conduct a detailed analysis of specification errors for a prototypical binary choice context, under the condition of incorrect choice set representation.

2.2 Towards a View of Choice Set Generation

2.2.1 Introduction

Classical economic choice theory postulates the following two-stage sequential process (Manski,1977):

(1) forces exogenous to the individual pose a choice problem (*i.e.* define the indi-

vidual's choice set C_n, where n represents an individual);

(2) with C_n well-defined, the individual selects an alternative from among those available according to some pre-established decision rule (e.g. utility maximization, elimination-by-aspects (Tversky, 1972), etc.).

In its turn, choice set formation can be viewed as the process of establishing the set of feasible alternatives available to an individual decision-maker. The factors that establish the infeasibility or not of specific alternatives can be identified by characterizing the relevant interconnections between the individual and his or her environment, as well as self-imposed restrictions (such as those of a psychological nature).

The two-stage exogenous approach above closely parallels the satisficing mode of rational behavior proposed by Simon(1955). The first stage, that of choice set generation, is exactly analogous to the process for definition of what Simon calls the "considered subset", that is, the subset of the objectively available alternatives that the decision-maker actually considers for choice. The process of constraint identification must not only consider constraints such as income and transport infrastructure, but must also account for informational, psychological, cultural, and social restrictions.

The next section is devoted to operationalizing the process of constraint identification by establishing a general typology of constraints operative on individuals undertaking spatial movement in urban areas. It is not intended to be exhaustive; gaps in its coverage of various choice contexts certainly exist. In addition, so as not to bog ourselves down in minutiae, the typology is of an abstract nature. Nonetheless, it fulfills the useful purpose of all prototypes: it establishes the practical feasibility of a concept. And by restricting itself to individual urban travel, it can be specific enough to be of direct use in the empirical phase of this thesis.

2.2.2 A Typology of Constraints on Individual Urban Travel

This section will outline a typology of constraints which should be generally useful to guide further thoughts on choice set modelling in urban transportation contexts. It is difficult to speak of constraints to individual decision-making without becoming aware of an implicit time frame for the discussion. What is deemed a constraint in one time frame is a decision variable over the longer run. However, in the interests of generality we have allowed ourselves the liberty of mixing time frames in the discussion that follows.

Any classification inevitably tends toward artificiality since interaction between categories becomes difficult to make explicit; nonetheless, the taxonomy can be helpful to the extent that the construct aids us in understanding the issues involved. In this spirit, and under the _caveat_ stated above, constraints are categorized here into three broad groupings:

(1) household/family constraints;

(2) societal constraints; and

(3) personal constraints.

2.2.2.1 Household/Family Constraints

It might seem natural to first consider constraints at the personal level, but is is argued here that constraints are primarily determined by one's household/family affiliations. The household of which an individual is a member exerts influences that alter the individual's perception of his needs and impose upon him responsibilities towards satisfaction of collective as well as personal necessities. Thus it seems more appropriate to focus first upon household level constraints.

The important point is that the individual decision-maker cannot be considered in isolation of this part of his social environment. Brog and Erl(1981), in developing their "situational" approach to the study of individual activity patterns in urban areas, report from their analysis efforts that "...it very quickly became obvious that the composition of pertinent activity patterns is only partially a result of a person's free choice; rather, it is frequently determined by members of his family." (Brog and Erl, 1981, p. 1)

Three major categories of household/family constraints will be discussed in the following paragraphs.

The first of these are physical constraints imposed by the household; examples of this category are residential location and resource availability (e.g. income sharing within the household, auto ownership, etc.), both of which affect an individual's choice set. Of the three household/family constraint categories treated here, this is the most straightforward to identify and integrate in models of choice set formation.

Perceptual constraints are the second category to be presented. The perception-forming impact that a household has upon its members can be very great, especially within cultures where the family receives primary emphasis over the individual. Within the household each member acts the role he or she has been assigned, and the resources available to him or her to carry out his or her responsibilities take into account his or her relative contribution to the collective good. Thus, a secondary worker may not actually have use of the automobile as an alternative in his work mode choice set because in his perception, molded by his familial environment, only the primary worker has a call upon the vehicle. Further, it may be that the primary worker is captive to this modal alternative due to his or her perception of the household's status within society.

The last category of household constraints is related to household <u>structure</u>, both in terms of its internal decision-making mode and the hierarchical positioning of its members. Modes of intra-household decision-making are an important consideration to have in mind because they result in differing patterns of activity and resource allocation to individual members. This is admittedly a difficult area to model explicitly, but perhaps differentiation along such dimensions as lifecycle and lifestyle might indirectly help to identify relevant constraints on individual behavior. Salomon(1980) shows that the latter is an interesting dimension along which to investigate individual travel behavior, so it is reasonable to assume that it can also aid in modelling choice set generation.

The hierarchical arrangement of household members is also of relevance. For instance, the individuals in a group of adults who merely share the same living space likely act as independent households with respect to their transportation needs. On the other hand, the actions and possibilities of members of a nuclear family are likely to be highly interrelated, so such individuals should not be considered in isolation of one another. This type of household constraint also seems most amenable to treatment via market segmentation schemes.

2.2.2.2 Societal Constraints

Societal constraints, the second of three major categories presented at the beginning of the section, will now be discussed. The first of its subcategories is the set of constraints resulting from the physical layout of the transport infrastructure in an urban area. These constraints are also straightforward to understand and incorporate in choice set models, and have often been applied. Ben-Akiva and Lerman(1974) is an example of the application of such constraints in the context of a simultaneous disaggregate model of auto ownership and mode to work. The authors mention restricting transit for households living in fringe suburban areas within their zone of study. Many constraints imposed by the transport infrastructure are related to public transportation modes, whose availability, frequency, reliability, and so forth, affect an individual's ability to participate in desired activities.

The second category of societal constraints is closely related to the previous class. The form and structure of the urban economy impose a variety of constraints on individuals. The spatial and temporal distributions of economic activities influence all aspects of travel decision-making, from trip frequency to mode choice. The location of shopping opportunities and store operating hours are specific examples of such constraints. Landau <u>et al.</u>(1982) present an example of utilizing travel time and store operating hours information to define workers' lunch-hour shopping destination choice sets.

<u>Information availability</u> is the third societal constraint category. It is important to distinguish between making information available to an individual and his ability to process and use that data. This latter aspect will be treated under the last major category, personal constraints. The degree of ubiquity of information concerning some product determines how high the cost of obtaining that data will be to the individual. Hence, information is relatively easy to acquire for an individual interested in purchasing an automobile in the U.S. since advertisements can be found in newspapers, magazines, and television, and further data can be obtained via word-of-mouth because auto ownership is so widespread. However, for certain types of automobiles little information is readily available and a higher search cost must be paid by the consumer. In fact, this cost may be so high as to remove the alternative from any consideration whatsoever.

2.2.2.3 Personal Constraints

Proceeding to the final constraint category, personal constraints are subdivided into two classifications, natural and perceptual. The term "natural" is used to depict conditions or circumstances external to the self cf the individual. For instance, the individual may be limited by such factors as age, sex, and specific handicaps. Role-related natural constraints account for the fact that the individual operates within and is influenced by his household, his social/economic class, and other groups to which he is affiliated. These, to a greater or lesser extent, determine his activity patterns (e.g. labor force participation) and dictate personal resources, hence helping the definition of his personal choice set.

Closely related to the latter constraints are the influences on perception exerted by one's role affiliations. The various groups an individual lives within operate to modify, condition, and direct the manner in which personal needs are met. In this context the groups are viewed as agents seeking some degree of conformity in individual behavior.

During the discussion on societal constraints, the subject of information availability was broached. At the personal level, an individual's ability to process and assimilate what information is provided may also help determine his choice set in a given situation. This does not refer only to the obvious restrictions such as literacy level, ownership of a radio or television, etc., but also to an individual's very attitude towards new information or new alternatives. Information availability alone does not guarantee that an alternative will become part of a choice set; all else being equal, it is necessary that an individual's "informational inertia" (or "habit") be overcome before an alternative becomes feasible and is incorporated into his choice set.

Meyer(1979) has attempted to deal with the dynamics of informational constraints and destination choice set formation by formulating a theory of how individuals learn about spatial opportunities when subjected to space and time constraints. Hall(1980), in the context of modelling residential location choice, also treats personal and societal information constraints by associating a search cost with each alternative incorporated into a household's choice set. When the cost of incorporating an additional alternative is unacceptably high, the search is terminated (i.e. the choice set is defined and, simultaneously, a choice is made).

Now, with a better idea of what constitutes a constraint, it is appropriate that we consider how to treat them in the modelling process.

2.3 The Treatment of Constraints In Choice Set Generation Models

The observer, external to the choice process under scrutiny, is assumed to have limited information about the sequence of events and the actors involved. In this context, a constraint can be thought of as deterministic or probabilistic depending upon the degree of confidence the observer places on information at hand. Hence, knowledge that an individual has no driver's license may lead one to *a priori* remove auto-drive from her set of possible mode choices, even though her household owns an auto. Another such example is omission of the bus alternative for an individual whose residence and work location are not both served by this mode.

Application of only deterministic constraints result in what Manski(1977) terms the "expected choice set". Many examples of this approach exist in the literature (*e.g.* Ben-Akiva and Lerman, 1974; Train, 1980). A most elaborate effort of applying deterministic constraints to choice set formation modelling is the work of Recker *et al*.(1983), concerning trip-chaining behavior of individuals in urban areas. Certainly, for most individuals, a rather large number of alternative activity patterns can be enumerated; however, only a smaller subset of these will actually be feasible, for any of a number of reasons. Recker and his associates utilize an intricate process for identifying the set of feasible activity patterns: alternatives are generated via a constrained combinatoric scheduling algorithm (which allows for restrictions posed by the individual's household and the transport infrastructure), and the alternatives are then processed via pattern recognition procedures to arrive at distinct alternatives; finally, all inferior distinct alternatives are eliminated, achieving Recker's representation of the individual's set of feasible activity patterns.

Ben-Akiva _et_ _al_.(1984) provide us with an unique application of deterministic choice set definition. They consider modelling auto drivers' route choice in a region of the Netherlands; clearly, this is a situation in which a very large number of physical alternative paths will exist, given the high population and road density in the area. While this presents a difficult practical problem to estimate a model of route choice, it also presents a conceptual difficulty since it is unlikely that an individual driver is really aware of the potentially large number of alternative routes.

Accordingly, Ben-Akiva _et_ _al_. propose to define a set of "labelled" paths as the choice set; these labelled paths are the _optimal physical_ paths, each with respect to some criterion function. These criteria are such measures as travel time, scenery, congestion, and so on. Thus, one labelled path corresponds to the shortest travel time physical path, another is the physical path of greatest scenic beauty, and so forth. By this hypothesis of behavior (i.e. that individuals classify actual routes in terms of the stated criteria before exercising choice, which is among the best route for each criterion), Ben-Akiva and his co-workers reduce the very large set of physical paths to one of more manageable proportions, and then go on to calibrate models of choice of route.

Another instance of deterministic choice set formation modelling, reported in O'Neill and Nelson(1981), concerns a transport study for the Baltimore area in which an explicit attempt was made to formulate work and school modal choice sets through the combined use of reasonableness (i.e. consistency) tests and user-supplied information on alternative availability. For a number of reasons detailed in the above reference, the Baltimore experience was not successful in terms of the choice set generation aspect of the effort.

Market segmentation, allied with specific choice model specifications, can be an useful tool for expressing the effects of constraints. For instance, in a mode choice context, our subdivision of the decision-makers into groupings reflecting different household structures (say single individual households, independent multi-individual households, single-worker nuclear households with children below 5 years of age,...) implicitly reflects constraints imposed by resource allocation mechanisms.

It is clear, however, that many constraints that are identifiable by an analyst do not lend themselves to this treatment with certainty. Consider, for example, the impact of access walk distance on modal availability: one might determine that bus is unavailable if this distance is greater than (some pre-determined figure), but it seems more reasonable to view the impact of this factor as probabilistic since different individuals may well have varying perceptions of what constitutes an acceptable walking distance.

A second rationale for considering a constraint probabilistic is our limited knowledge of the underlying choice set generation process. Kozel(1981) reports an extensive and in-depth analysis of the transport survey data for the Brazilian city of Maceio (this data is the same as that which we shall utilize in Chapter 4). The purpose of that study was to identify segments of the population whose activity patterns (e.g. shopping trip generation, lunch trip home from work, mode choice to work) and long-run mobility decisions (e.g. residential location, automobile ownership) are constrained by (1) economic, (2) time, (3) physical, (4) social/personal, or (5) household factors. The interesting conclusion of that effort was that Kozel was unable to isolate groups that are deterministically constrained from given choice alternatives. No matter how she segmented the available observations, some were always left whose actions were unexplainable in terms of the constraints she tested. This inability to detect deterministic contraints in the Maceio study, in fact, led to the current research effort into probabilistic choice set formation.

One may, of course, view all constraints as probabilistic. Wermuth(1978) is an example of this approach for a model of choice between auto drive and public transit. He contends that there are a number of constraints that must be dealt with since the "...individual modal choice process to a high degree is not based on rational comparison of characteristics of the competing modes." (Wermuth, 1978, p. 1) He categorizes a constraints hierarchy as shown below:

(1) sociodemographic factors;

(2) auto ownership;

(3) permanent car availability;

(4) trip specific car availability; and

(5) captivity to one or the other mode, or freedom to choose from both.

Each of the elements above is modelled stochastically to arrive at a full specification of the probability of an individual taking one or the other mode.

In theoretical terms, however, if we have information which is known certainly, its incorporation in the modelling and estimation processes results in more efficient parameter estimates. And in practical terms, use of this information can greatly decrease the effort needed to obtain the parameters. Thus, the approach to be adopted here for modelling the two-stage sequential choice process postulated previously is to allow for a hybrid of deterministic and probabilistic constraints.

Some of the constraints mentioned in the preceeding section, especially those of a psychological nature or having to do with the role affiliations of an individual, may be very difficult to account for explicitly. As indicated here, some of these may be susceptible to treatment by market segmentation schemes; while this may be the only practical solution, we must be aware of the more imprecise nature of this approach as compared to explicit modelling of a constraint.

Finally, some constraints must simply be ignored due to lack of information or a true understanding of the choice situation being analyzed.

2.4 Recognition of the Importance of Choice Set Formation

2.4.1 Previous Studies

Overwhelmingly, the discrete choice literature has concentrated its attention upon the second stage of the choice process defined in Section 2.2.1, both in theoretical and applied work. A result of this focus is signalled by the relatively wide variety of alternative structures and empirical applications of discrete choice models extant (see the reviews in Amemiya, 1975; Amemiya, 1981; Hensher and Johnson, 1981). The other result has been a paucity of theoretical development for the choice set generation stage.

This is not to say, of course, that researchers are unaware of the ramifications of incorrect representation of choice sets. The work of Williams and Ortuzar(1979) and Stopher(1980), for example, give numerical and empirical verification of the problems that can arise when the first stage is ignored or misspecified.

Stopher(1980) studied empirically the impact of captives on the estimation of and forecasting with a binary mode choice model. Captivity is defined as the state of not having a choice prerogative (<u>i.e.</u> one's choice set has only one alternative), and it is immediately obvious that the choice of consumers in this category need not be modelled at all (insofar as there is no need to model captivity itself, of course; more on this topic in Chapter 3). However, if consumers who are truly captive to a given choice are assumed to have more than one alternative in their choice sets, and are included with non-captives in the data to estimate a choice model, Stopher states that the estimated coefficients for all attributes (both of alternatives and consumer-related) were smaller and less significant than in the "true" model (defined by him as the model estimated on data excluding the captives); and the alternative specific constants were larger and more significant than in the "true" model. Theoretical treatment of a binary choice model leads to similar conclusions concerning the existence of an estimation bias (see Section 2.4.2).

Among other issues concerning discrete choice models, Williams and Ortuzar(1979) chose to investigate the impact of the analyst assuming that the choice set generation process is such that consumers have all alternatives in their choice sets, when in reality the process follows some known probability law. Using randomly generated data from a known model, they show that if choice set generation is actually a function of variables in the choice model itself, biased predictions of behavior due to policy changes will result, even though the fit to the base case data is quite good.

Meyer(1979) argues that model parameters may be as much influenced by variations in choice sets among individuals (not fully accounted for in standard discrete choice model structures) as by variations in preferences (these are accounted for). Other authors with this same concern are Louviere(1979) and Pirie(1976). Meyer is of the opinion that the indeterminacy of these effects may result in a lack of stability of destination choice model parameters over time and space; it seems reasonable to extend the coverage of this opinion to contexts other than destination choice.

The valuation of travel time has long been a subject of interest to transportation researchers and practitioners alike (see Hensher, 1978, for a survey of that literature) due to the important role that travel time savings play in establishing the benefits of investing in transport infrastucture. A recent paper by Heggie(1983) deals with the empirical estimation of the value of non-work travel time, but he rightly points out the estimation sample must include only those individuals who are, in his terminology, the "choosers". In the terminology we employ here, it is necessary to isolate the captives and not permit them to bias the estimate of the value of time. Heggie utilizes information solicited from the user of the transport system to distinguish the choosers from the non-choosers (or captives).

Heggie's approach is inspired by the research of Brog and Erl(1981). The latter utilized a series of filters on a sample of 1200 urban households in West Germany to determine who is actually a chooser (i.e. explicitly compares alternatives) for the mode to work decision. Using objective criteria (availability of mode, type of employment, ...), information availability (is the individual aware of objectively available alternatives?), and subjective availability (does the person consider an alternative a possibility?), Brog and Erl estimate that only 10% of the workers actually make rational comparisons to determine their modal choice. One can debate the use of these subjective data in the analysis, but it is difficult to escape the conclusion that ignoring or misrepresenting the choice set generation stage in a discrete choice model can lead to serious impacts, as we have described above.

Several other studies have dealt with choice set generation, but they concentrate upon formulating choice set models rather than dealing with the consequences of model misspecification. Thus, they will be reviewed in Chapter 3.

2.4.2 An Analytical Investigation of Specification Errors

This section will present a specification error analysis for a simple binary choice context. Besides yielding information about this specific problem, the analysis will also provide insight into the general process of misspecifying choice sets.

In this analysis we consider a subject population which acts in accordance with a true model of behavior, known to us; our task will be to analyze the errors that result from assuming that the population behave according to an erroneously specified model.

The choice situation we shall study is identical to that of Stopher(1980): namely, the subject population, faced with a choice between discrete alternatives 1 and 2, is composed of two subgroups, I_1 and I_2. The first of these, I_1, has no restrictions placed upon it and is free to choose between the two alternatives according to the true model, which has a binary logit form with two parameters and one explanatory variable:

$$P(1|B^*,X_n) = 1/(1 + \exp(-b_1^* - b_2^* X_n)) \;, \quad \forall n \varepsilon I_1, \quad (2.1)$$

where $P(1|B^*,X_n)$ is the probability individual n chooses alternative 1;

B^* = (b_1^*,b_2^*) is the true vector of choice model parameters;

X_n is the single explanatory variable of the choice between alternatives 1 and 2, for individual n.

Subgroup I_2, however, is captive to alternative 1, that is,

$$P(1|B^*,X_n) = 1 \quad , \forall n \in I_2. \tag{2.2}$$

Of course,

$$P(2|B^*,X_n) = 1 - P(1|B^*,X_n), \quad \forall n. \tag{2.3}$$

This model can also be expressed jointly as follows:

$$P(1|B^*,X_n) = d_n + (1-d_n)[1/(1+\exp(-b_1^*-b_2^*X_n))], \ \forall n, \tag{2.4}$$

where $\quad d_n = \begin{cases} 1 & \text{if } n \in I_2 \\ 0 & \text{if } n \in I_1 \end{cases}$

Suppose that the state of captivity or not is unknown to us when we randomly draw an individual from the subject population. Thus, d_n is a random variable from our viewpoint. Assume that

$$Pr(d_n = 1) = k^* \quad , \tag{2.5}$$

where k^* is a constant parameter, $0 \le k^* \le 1$. This parameter is equal to the fraction of the population in subgroup I_2. (In Chapter 3 we will consider more complex captivity models in which the probability of captivity varies among individuals.) Thus, the true model for the probability that a random individual from the subject population chooses alternative 1 becomes

$$Q(1|B^*,k^*,X_n) = k^* + (1-k^*)[1/(1+\exp(-b_1^*-b_2^*X_n))], \ \forall n. \qquad (2.6)$$

This specification is a binary choice version of the logit captivity model of McFadden(1976) and Ben-Akiva(1977), which is identical to the so-called "Dogit" model of Gaudry and Dagenais(1979).

Suppose now that, as analysts, we have drawn at random from this population N individuals. Suppose further that we know the form of the true choice model (2.6), and wish only to estimate the values of the unknown parameters, k^* and B^*. Expressed as a maximum likelihood estimation problem, we are faced with

$$\max_{B,k} \ \Gamma_N(B,k,X) = \sum_{n=1}^{N} \sum_{i=1,2} \delta_{in} \ln Q(i|B,k,X_n) \quad (2.7)$$

where $\Gamma_N(B,k,X)$ is the log likelihood function;

B,k correspond, respectively, to B^* and k^*;

$$\delta_{in} = \begin{cases} 1 & \text{if person n observed choosing alternative i, i=1,2} \\ 0 & \text{otherwise} \end{cases}$$

Up to this point in the analysis no specification error has been committed. We assumed knowledge of the correct specification of the model, but not of the specific values of model parameters; for the purpose of estimating these values we have collected a sample of N individuals, and by solving the mathematical program (2.7) we will obtain the maximum likelihood estimates (MLEs) of the captivity and choice model parameters (i.e. k and B). With this level of knowledge, it is clear that these MLEs from (2.7) will be consistent and asymptotically unbiased estimates of k^* and B^*.

Now we will commit a single, but crucial error: namely, we will not use the knowledge that part of the population is captive to alternative 1. The estimation problem we are now faced with is a special case of (2.7) in which parameter k is forced to be zero, that is,

$$\max_{B} \quad \Gamma_N(B,0,X) = \sum_{n=1}^{N} \sum_{i=1,2} \delta_{in} \ln H(i|B,X_n) \quad , \quad (2.8)$$

where
$$H(1|B,X_n) = 1/(1+\exp(-b_1-b_2X_n)),$$
$$H(2|B,X_n) = 1 - H(1|B,X_n), \quad \forall n.$$

Clearly, ignoring the captivity phenomenon in the population must yield inconsistent estimates of the choice model parameters; a question of greater interest lies in whether we can obtain further insight about the resulting bias in terms of its direction, even of its magnitude.

Even before executing this asymptotic analysis of the estimator in (2.8), it is possible to note an unfortunate result of using (2.8). To find the maximum of (2.8) we first find its stationary points by differentiating with respect to the components of B to obtain the following first-order conditions (FOCs):

$$\sum_{n=1}^{N} [\, \delta_{1n} - H(1|B,X_n)] = 0 \tag{2.9}$$

$$\sum_{n=1}^{N} [\, \delta_{1n} - H(1|B,X_n)]\, X_n = 0 \tag{2.10}$$

The simultaneous solution of these two nonlinear equations will yield what are, in fact, the unique solutions for program (2.8). They are unique due to the concavity of (2.8) with respect to B, as shown in Domencich and McFadden(1975), p. 111. For analytical convenience, and using the fact that all elements in the sample from subgroup I_2 are captive to alternative 1 (hence $\delta_{1n} = 1 \;,\; \forall n \in I_2$), rewrite (2.9) as

$$\sum_{n \varepsilon I_1} \delta_{1n} = \sum_{n \varepsilon I_1} H(1|B,X_n) + \sum_{n \varepsilon I_2} H(1|B,X_n) - N_2 \quad , \qquad (2.11)$$

where N_2 is the number of individuals in the sample belonging to I_2 (i.e. the captives). This quantity is not, of course, known to us, but it is implicitly in (2.9) and hence can, for analytical purposes, be isolated in (2.11).

Since $0 < H(1|B,X_n) < 1$, we know that the term

$$\sum_{n \varepsilon I_2} H(1|B,X_n) - N_2$$

must be negative; thus,

$$\sum_{n \varepsilon I_1} \delta_{1n} < \sum_{n \varepsilon I_1} H(1|B,X_n) \quad , \qquad (2.12)$$

at the value of B which maximizes (2.8). The left-hand side of (2.12) is the actual number of individuals in the sample from group I_1 that choose alternative 1; the right-hand side is the predicted number of individuals in I_1 that choose 1.

That is to say, in any finite sample which contains elements captive to an alternative (in this case alternative 1), the predicted model parameters will overestimate the number of non-captive decision-makers that choose that alternative. A similar analysis for (2.10) results in

$$\sum_{n \varepsilon I_1} \delta_{1n} X_n \quad < \quad \sum_{n \varepsilon I_1} H(1|B,X_n)\, X_n \quad , \qquad (2.13)$$

which lead to similar conclusions as (2.12) regarding the predicted and observed average value of X_n in the subpopulation of I_1 choosing alternative 1.

Clearly, the estimate $\hat{B}$ of B^* resulting from the solution of (2.9-10) will not be correct; for (2.12) and (2.13) to hold, we intuitively expect $\hat{b}_1$ to be upwards biased ($\hat{b}_1 > b_1^*$) and $\hat{b}_2$ to be downwards biased ($\hat{b}_2 < b_2^*$), as claimed by Stopher(1980).

These results are in no wise a function of the specific choice model used here (<u>i.e.</u> the logit specification). Rather, they are a direct result of having applied the choice model to a group of the population which has no choice; any model specification which ignores this captivity should result in a similar outcome. The implications stemming from (2.12-13) corroborate the statements and empirical analyses of Stopher(1980) and Williams and Ortuzar(1979), but it is possible to go further in the analysis.

Let us now perform an analysis of the asymptotic properties of estimator (2.8), which will enable us to get a better grasp of the issue of its inconsistency and direction of bias.

Maximization of (2.8) is completely equivalent to the maximization of

$$D_N(B,0,X) = \Gamma_N(B,0,X)/N \qquad (2.14)$$

since division by the positive constant N does not change the form of the function. Now define D(B,0) as

$$D(B,0) = \operatorname*{plim}_{N\to\infty} D_N(B,0,X) \quad . \qquad (2.15)$$

Since $H(i|B,X_n)$ is continuous and everywhere positive, both $H(i|B,X_n)$ and $\ln H(i|B,X_n)$ are bounded. This guarantees that D(B,0) exists and is finite for all values of B. The function D(B,0) is the expectation of function (2.8) with respect to attribute X as the sample size grows without bound, <u>i.e.</u> over the population. Thus,

$$D(B,0) = \operatorname*{plim}_{N\to\infty} \left[(1/N) \sum_{n=1}^{N} \sum_{i=1,2} \delta_{in} \ln H(i|B,X_n) \right]$$

$$= \sum_{i=1,2} E_X[Q(i|B^*,k^*,X) \ln H(i|B,X)] \qquad (2.16)$$

where $Q(1|B^*,k^*,X)$ is given by (2.6) and $Q(2|B^*,k^*,X)=1-Q(1|B^*,k^*,X)$; $H(i|B,z)$, i=1,2, are defined following (2.8);

and $E_X[\,]$ represents the expectation operator with respect to the attribute X. Asymptotically, then, our estimation problem, originally stated as (2.8), has found the equivalent form below:

$$\max_{B} D(B,0) = \sum_{i=1,2} \int_X Q(i|B^*,k^*,z)\ \ln H(i|B,z)f(z)dz, \quad (2.17)$$

where f(X) is the probability density function for the attribute X. Substituting (2.6) into (2.17), we obtain

$$D(B,0) = (1-k^*)\ D_1(B,0) + k^*\ D_2(B,0)\ , \quad (2.18)$$

where

$$D_1(B,0) = \sum_{i=1,2} \int_X H(i|B^*,z)\ \ln H(i|B,z)f(z)dz\ , \quad (2.19)$$

$$D_2(B,0) = \int_X \ln H(1|B,z)f(z)dz \quad . \quad (2.20)$$

Consider now the following theorem:

<u>Theorem 2.1</u>: Let g(x) and h(x) be real-valued, concave functions on E_n, the n-dimensional argument space. Further, let x_g^* and x_h^* represent the

points where g and h attain their global maxima, respectively. Then $f(x)=u_1 g(x)+u_2 h(x)$, where $u_i \geq 0$, $i=1,2$, $u_1+u_2=1$, will attain its global maximum at $x^*=u_1 x_g^*+u_2 x_h^*$.

Pf. See Appendix 1.

According to Theorem 2.1, if $D_1(B,0)$ and $D_2(B,0)$ are both concave with respect to B, then the optimal solution to (2.18) can be found by obtaining the global maximum for function (2.19) and that of function (2.20) , and combining the two optima in the manner described in the theorem. As shown in Appendix 2, $D_1(B,0)$ and $D_2(B,0)$ are indeed both concave in B, so that Theorem 2.1 is applicable. Thus, if we let $\hat{B}$ be the optimal solution to (2.18) we can write

$$\hat{B} = (1-k^*)\,\tilde{B}_1 + k^*\tilde{B}_2 \tag{2.21}$$

where $\tilde{B}_1$ is the optimal solution for

$$\max_{B} \quad D_1(B,0) \tag{2.22}$$

and $\tilde{B}_2$ is the optimal solution for

$$\max_{B} \quad D_2(B,0) \quad . \tag{2.23}$$

To find the optimal of (2.22), we make use of a result from information theory, which is stated below:

Lemma 2.2: Let $g(s,x)$ be a real-valued function over a space $S \times X$ such that g is integrable with respect to a measure u over S and $g(s,x) \geq 0$, all $s \in S$, $x \in X$. Let x^* be an element of X such that $g(s,x^*) > 0$ for almost every $s \in S$ and $\int_S [g(s,x^*) - g(s,x)]du \geq 0$, all $x \in X$. Then

$$f(x) = \int_S g(s,x^*)[\ln g(s,x)]du$$

attains its maximum at $x = x^*$.

Pf. See Rao(1973), p.59.

The straightforward application of this lemma to function (2.19) leads us to conclude that the optimum of (2.22) is

$$\tilde{B}_1 = B^* \quad , \tag{2.24}$$

that is, the estimator of the choice model parameters in $D_1(B,0)$ is asymptotically equal to the true parameter values.

Now let us turn our attention to program (2.23), in which we must find the optimum of $D_2(B,0)$. By substituting the definition for $H(1|B,X)$ into (2.20), we obtain

$$D_2(B,0) = b_1 - \int_X \ln(\exp(b_1) + \exp(-b_2 z)) f(z)dz \quad . \quad (2.25)$$

To find the stationary points of (2.25), we differentiate with respect to the two parameters, which results in the following first-order conditions:

$$\int_X H(1|B,z)f(z)dz = 1 \qquad , \qquad (2.26a)$$

$$\int_X z\ H(1|B,z)f(z)dz = \int_X z\ f(z)dz \qquad . \qquad (2.26b)$$

Expression (2.26a) states that the expectation of $H(1|B,X)$, with respect to attribute X, must be equal to 1 at any stationary point. Since $H(1|B,X)$ is a proper cumulative probability density function, it is bounded by 0 and 1. Thus, to satisfy (2.26a) it is necessary that the following hold at the point : <u>$H(1|B,X)$ must be equal to 1 for all values of X</u>. This, in turn, implies two conclusions:

(1) for $H(1|B,X)$ to be equal to 1, it is necessary that b_1 grow arbitrarily large (*i.e.* $b_1 \to \infty$);

(2) for this result to hold for all values of X, b_2 must be equal to zero.

This solution also satisfies (2.26b), which indicates that a stationary point has been identified. Indeed, this degenerate solution is to be expected since $D_2(B,0)$ is monotonically increasing in b_1; since b_2 must be zero to satisfy conditions (2.26), we must simply increase b_1 towards infinity to maximize $D_2(B,0)$.

Therefore, we conclude that the maximum of (2.23) is attained at

$$\tilde{B}_2 = \begin{bmatrix} \infty \\ 0 \end{bmatrix} \tag{2.27}$$

This, then, is the result of using the improper estimator given in (2.8); by bringing the two solutions (2.24) and (2.27) together according to expression (2.21), we find that the erroneously estimated parameters for the choice model exhibit the following characteristics:

(1) $\hat{b}_1$, the estimate of b_1^* according to estimator (2.8), is a convex combination of b_1^* and an arbitrarily large number, whence we conclude that $\hat{b}_1 \to \infty$;

(2) $\hat{b}_2$, the estimate of b_2^* by (2.8), can be expressed as

$$\hat{b}_2 = (1-k^*)b_2^* .$$

Thus, the estimated alternative specific constant grows arbitrarily large, and the estimated coefficient of the attribute is smaller, in absolute terms, than the true value by a factor dependent upon the degree of captivity in the population (the greater the captivity, the smaller the estimated coefficient).

These conclusions are exactly those reached empirically by Stopher(1980), but we now have the backing of theoretical analysis to make the above assertions. It is also helpful to note that the asymptotic results above are also identically valid for a binary probit, rather than logit, choice model, assuming also a linear-in-parameters specification. In fact, it is correct for any binary choice model of the form

$$Pr(1|B,X_n) = F[\Delta V(B,X_n)] \quad ,$$

where F[] is a proper cumulative density function,

$$\Delta V(B,X_n) = b_1 + b_2X_n \ ,$$

and F[] is monotonically increasing in $\Delta V()$.

The specific form of the bias we've deduced for this binary choice context lends credence to Meyer(1979)'s spatial and temporal parameter instability argument, mentioned in Section 2.4.1. The bias of attribute related parameters is a function of the degree of captivity in the population; the mechanisms that result in captivity will likely vary from place to place, across cultures and in different time periods, so it becomes even more critical that we model these mechanisms before attempting spatial and temporal transfer of parameters.

In summary, this lengthy discussion of a relatively simple choice context in which the analyst does not account for the correct choice set structure present in the population, has served the useful purpose of pointing out the statistical and practical impacts of following such a course in real-world contexts.

Besides quantifying the direction, indeed, even the magnitude of the bias (albeit in terms of what is usually an unknown quantity, k^*), it is instructive to note that any function of the estimated parameters $\hat{B}$ will also, of course, be biased. In fact, consider the choice probability elasticity with respect to attribute X for the true model $P(1|B,X)$, but evaluated at the estimated parameter $\hat{B}$:

$$E_X^{P(1|B,X)}\Bigg|_{B=\hat{B}} = [1 - P(1|\hat{B},X)]\,\hat{b}_2\,X$$

$$= (1-k^*)[1 - P(1|\hat{B},X)]\,b_2^*\,X \quad (2.28)$$

Thus, the elasticity of the assumed choice probability model is also a function of the degree of captivity in the population; due to the inclusion of the captives in the estimation of the choice model parameters, the resulting elasticity will underpredict (by a factor of $1-k^*$) the elasticity of the non-captive population with respect to changes in X; the condition is worsened due to the fact that $P(1|\hat{B},X)$ overstates $P(1|B^*,X)$, the true probability of alternative 1 for the non-captive population.

2.5 Summary

Chapter 2 has undertaken two principal tasks within the framework of this research: first, it has attempted to motivate, from both practical and theoretical considerations, the need for explicitly studying the process of choice set formation when modelling individual choice; second, it has developed our view of implementing choice set formation models via the process of constraint identification. To accomplish this second task, a prototypical constraint typology for individual urban travel behavior has been presented and extensively discussed, and practical procedures for incorporating constraints in choice set models have also been mentioned.

In a natural flow of affairs, then, we now proceed to Chapter 3, wherein a number of specific choice set models are developed in light of the methodology presented here.

CHAPTER 3

MODELLING CHOICE SET FORMATION

3.1 Introduction

The function of this chapter is to implement the methodology for modelling choice set formation, previously outlined in Chapter 2, by formulating estimable specifications. The chapter is divided into three principal parts: the first will present the general mathematical framework reflecting the constraint-based methodology of Chapter 2; the second will present several specific model formulations, accompanied by a discussion of their usefulness and appropriate contexts for application; finally, the third part will discuss a number of practical and theoretical estimation issues that must be dealt with.

The model development presented in this chapter is intended as prototypical, in much the same spirit as the constraint typology of Chapter 2, Section 2.2.2. Because of the large number of models that can be formulated based on the framework of constraint modelling, it is undesirable to develop a general model that covers a wide spectrum of application contexts. Instead, our efforts have concentrated upon elucidating the general methodology for modelling choice set generation proposed herein, whilst formulating specific models that will subsequently be empirically tested in a real-world environment.

3.2 The Mathematical Framework for Modelling Choice Set Generation

3.2.1 Introduction

From the perspective of an analyst unknowing of the specific alternatives comprising an individual's choice set, the probability of observing the choice of some discrete alternative must be expressed as (Manski, 1977):

$$\Pr(j|B,D,X_n) = \sum_{C \in G_n(j)} \Pr(j|C,B,X_n) \Pr(C|G_n,D,X_n) \tag{3.1}$$

where j is a discrete alternative;

n is an individual in the population;

X_n is a matrix of socio-economic and alternative attributes;

B, D are vectors of parameters;

M_n is the set of all deterministically feasible alternatives for n ($M_n \subseteq M$);

M is the master choice set, comprising all possible alternatives available for the choice context and population in question;

C is a choice set;

G_n is the set of all non-empty subsets of M_n; and

$G_n(j)$ is the set of all non-empty subsets of M_n that contain alternative j ($G_n(j) \subset G_n$).

Expression (3.1) reflects a three-part model of the choice process:

(1) a model of choice given choice set, $Pr(j|C,B,X_n)$, a discrete choice model which by definition yields choice probabilities of zero for $j \notin C$;

(2) a deterministic choice set generation model which a priori defines the set M_n from the set M; and

(3) a stochastic choice set generation model, $Pr(C|G_n,D,X_n)$, expressing the probability that set $C \epsilon M_n$ is the individual's actual choice set.

Items (2) and (3) above are the mechanisms whereby the deterministic and probabilistic constraints to choice set formation, discussed in Chapter 2, Section 2.3, are incorporated in the choice set model.

A high degree of computational complexity is implied by (3.1). If $|X|$ is the number of elements in some set X, then $|G_n|$ is $(2^{|Mn|}-1)$, of which $(2^{|Mn|-1})$ choice sets actually contain any given alternative $j \varepsilon M_n$. Hence, $|G_n(j)|$ is $(2^{|Mn|-1})$, which is the number of terms involved in the summation in (3.1). To demonstrate how the number of possible choice sets can quickly become overwhelming, if M_n has 3 alternatives, then 4 terms must be summed; with 10 alternatives, the number of possibilities has increased to 512; with 30, an incredible 536,870,912 possibilities exist! This is true for the <u>estimation</u> of an unrestricted choice set model; for <u>prediction</u>, when we wish to forecast the probabilities of choice for all alternatives in M_n, we must deal with the much greater number of $(2^{|Mn|}-1)$.

Most choice contexts of interest are, unfortunately, characterized by many, rather than few, alternatives. A possible approach to reducing the dimensionality of the choice set generation problem is to place <u>a priori</u> restrictions on the members of G_n. That is, the analyst must be willing to assume that for all practical purposes, modelling the choice situation at hand requires only a subset of the $(2^{|Mn|}-1)$ possible sets. We add, thus, a further element in the proposed approach to choice set modelling. In fact, a rich and varied arsenal of choice set generation models is possible through manipulation of the structure of feasible choice sets; it is by this method that we shall find practical and useful models of choice set formation.

To our knowledge, McFadden(1976a) was the first to formulate a model of choice set generation that incorporates structural restrictions: in a little-known note, he considers a choice situation in which a decision-maker is either captive to an alternative j, $j \varepsilon M_n$, or is otherwise free to choose from the full set M_n according to a multinomial logit model. This logit captivity model was also independently developed by Ben-Akiva(1977) and Gaudry and Dagenais(1979). The latter named this the "Dogit" model. The name arises from the observation that the logit captivity model "dodges" the IIA (Independence of Irrelevant Alternatives) assumption of the logit model.

An important qualitative distinction should be made between the model development of McFadden(1976a) and Ben-Akiva(1977), on the one hand, and Gaudry and Dagenais(1979) on the other. The former have motivated their logit captivity formulation from the behavioral basis described above; the latter, however, develop the model (and later extensions of it, see Gaudry and Dagenais, 1981) purely from the viewpoint of circumventing the IIA property of the logit model. The objective of our work will be to develop model forms based on behavioral, rather than purely econometric, considerations, in the spirit of McFadden and Ben-Akiva.

A simple captivity model, such as the one above, represents a choice context in which the decision-maker is either captive to an alternative (i.e. $C_n=\{i\}$, for some $i \in M_n$) or has a full choice set (i.e. $C_n = M_n$). Thus, in the notation previously introduced, $G_n=\{(1),(2),...,(|M_n|),M_n\}$ for the captivity case. Ben-Akiva(1977) considers two other structures for G_n:

(1) either the decision-maker has available all alternatives ($C_n=M_n$), or otherwise, the full set less one alternative; the unavailability of this one alternative results in $|M_n|+1$ possible choice sets, that is, $G_n=\{(M_n-1),(M_n-2),...,(M_n-|M_n|), M_n\}$, where the notation (M_n-i), $i \in M_n$, denotes the set resulting from the removal of element i from M_n;

(2) a combination of the simple captivity case and (1) above, resulting in an universe of $(2|M_n|+1)$ possible choice sets, so that $G_n=\{(1),...,(|M_n|),(M_n-1),$

$\ldots,(M_n-|M_n|),M_n\}$.

For each of these cases he formulates models for the full choice process.

At the current state-of-the-art of transport demand modelling and computational capability, a fully general choice set generation model is simply prohibitive for most practical choice contexts. Accordingly, most of our efforts will henceforth be directed towards choice set models with a priori restrictions on G_n.

First, however, some necessary definitions are in order.

3.2.2 Probabilistic Constraints and Availability

This section concerns itself with the introduction of two basic concepts used throughout this chapter: the first of these is the idea of random constraints and the second that of probabilistic alternative availability.

As we have noted earlier, the methodology that is proposed here to model choice set formation is based upon the description of the interconnections existing between an individual decision-maker and his/her environment, which results in the identification of operative constraints. As analysts, however, we are limited by a series of factors (lack of knowledge of human behavior, insufficient and/or poor quality data, lack of money and time, ...) that result in an inaccurate (to varying degrees) understanding of the phenomenon of interest.

Our inability to correctly describe all the restrictions active on the population of decision-makers leads us to formalize the concept of probabilistic constraints in the following manner: let a set of constraints be denoted as

$$H_k(D,X_{in}) \geq 0, \ \forall i \in M_n, \ \forall k \in K, \tag{3.2}$$

where K is the constraint index set; and

X_{in} is the vector of characteristics of individual n and attributes of alternative i.

The analyst's lack of knowledge, etc., is introduced by allowing $H_k()$, $k \in K$, to be a random variable, decomposible additively as

$$H_k(D,X_{in}) = h_k(D,X_{in}) - v_{ikn}, \ \forall i \in M_n, \ \forall k \in K, \tag{3.3}$$

where $h_k()$ is the deterministic, specifiable portion of the k^{th} constraint; and

v_{ikn} is the random, unknown component, which is unobservable, hence a random variable.

Given an assumption on the joint distribution function of the v's it is possible to derive a model of choice set formation, as will be shown subsequently.

Let availability of alternative i for decision-maker n be defined as the event described by the binary random variable

$$A_{in} = \begin{cases} 1 & \text{if } H_k(D,X_{in}) \geq 0, \ \forall k \in K \\ 0 & \text{otherwise} \end{cases} \qquad (3.4)$$

That is, alternative i is available if all relevant constraints are satisfied, and unavailable otherwise.

From (3.4) one can readily write the probability that $i \in M_n$ is available to decision-maker n as

$$Pr(A_{in}=1) = Pr[H_k(D,X_{in}) \geq 0, \ \forall k \in K], \ i \in M_n. \qquad (3.5)$$

From (3.5) we formulate the probability that some collection of alternatives in M_n is individual n's choice set. If we call C this choice set, then

$$Pr[C \text{ is n's choice set} | \text{not all } A_{ln}=0, \ l \in M_n]$$

$$= \frac{Pr[A_{in}=1, \ \forall i \in C \text{ and } A_{jn}=0, \ \forall j \in M_n - C]}{Pr[\text{not all } A_{ln}=0, \ l \in M_n]}, \qquad (3.6)$$

$$C \in G_n$$

The notation M_n-C denotes the complement of $M_n \cap C$.

Substituting (3.4) into (3.6) yields for $C \varepsilon G_n$,

$$\Pr(C|M_n,D,X_n)$$

$$= \frac{\Pr[\{H_k(D,X_{in})>0, \forall k \varepsilon K, \forall i \varepsilon C\} \text{ and } \{H_k(D,X_{jn})<0, \text{ at least one } k \varepsilon K, \forall j \varepsilon M_n - C\}]}{1 - \Pr[H_k(D,X_{ln})<0, \text{ at least one } k \varepsilon K, \forall l \varepsilon M_n]}$$

$$= \frac{\Pr[\{h_k(D,X_{in})>v_{ikn}, \forall k \varepsilon K, \forall i \varepsilon C\} \text{ and } \{h_k(D,X_{jn})<v_{jkn}, \text{ at least one } k \varepsilon K, \forall j \varepsilon M_n - C\}]}{1 - \Pr[h_k(D,X_{ln})<0, \text{ at least one } k \varepsilon K, \forall l \varepsilon M_n]}$$

(3.7)

With differing assumptions concerning the joint distribution function of the v's and the structure of G_n, we can use (3.7) to develop different models of choice set generation.

Suppose that the following constraints are operative upon individuals considering mode choice for work:

(1) $(c_{in}/Y_n) \leq \tilde{D}_{1n}$, $i=1,2,3,$

where c_{in} is the travel cost per trip by mode i for person n;

Y_n is the personal income of n;

$\tilde{D}_{1n}$ is an unknown, critical value for person n.

If the travel cost to income ratio for mode i is less than or equal to $\tilde{D}_{1n}$, alternative i is feasible for n.

(2) $_1W_{in} \leq \tilde{D}_{2n}$, i=1,2,3,

where $_1W_{in}$ is the total walk access distance for individual n if mode i is utilized;

$\tilde{D}_{2n}$ is a second unknown, critical value.

Again, if the total walk access distance is within the acceptable range, i is feasible; otherwise, not.

(3) $Y_n \geq \tilde{D}_{3n}$, i = 3 (auto drive).

The auto drive mode will be feasible for n only if his or her income is greater than some (unknown) threshold value $\tilde{D}_{3n}$.

For ease of exposition, let us assume that the unknown critical values $\tilde{D}_{mn}$, m=1,2,3, are independently distributed normal variates with mean D_m and variance s_m^2, for all m. Based on these assumptions, we define the probability of availability of each alternative as

$$Pr(A_{1n}=1) = \left[1-\Phi\left(\frac{r_{1n} - D_1}{s_1}\right)\right] \cdot \left[1-\Phi\left(\frac{_1W_{1n} - D_2}{s_2}\right)\right]$$

$$Pr(A_{2n}=1) = \left[1-\Phi\left(\frac{r_{2n} - D_1}{s_1}\right)\right] \cdot \left[1-\Phi\left(\frac{_1W_{2n} - D_2}{s_2}\right)\right]$$

$$\Pr(A_{3n}=1) = \left[1 - \Phi\left(\frac{r_{3n} - D_1}{s_1}\right)\right] \cdot \left[1 - \Phi\left(\frac{l^{W}_{3n} - D_2}{s_2}\right)\right] \cdot$$

$$\Phi\left(\frac{Y_n - D_3}{s_3}\right) \quad , \tag{3.8}$$

where $r_{in} = \dfrac{c_{in}}{y_n}$,$i=1,2,3$; and

$\Phi()$ is the standard normal cumulative distribution function.

Based upon expressions (3.8), it is straightforward to express the probabilities of specific choice sets, once we make certain assumptions about the structure of choice sets and the interaction of availability of various alternatives.

3.3 Some Models of Choice Set Formation

3.3.1 G_n Unrestricted - Independent Availability

One of the simplest assumptions that can be made about the joint distribution of the v's is that they are independently distributed across alternatives, but not within. Such an assumption, though analytically and computationally more convenient than some others, does imply certain characteristics about the choice set formation process which may or may not be desirable in a given situation (much as the IID assumption made in random utility formulations). Specifically, in certain choice contexts it is counterintuitive to assume that the availability of one or more alternatives is either statistically or behaviorally unlinked to the availability or not of some other alternative(s).

For example, the choice of shopping destination is dependent upon an individual's knowledge of the available shopping opportunities. This mental map, built up over a length of time, is unlikely to include the entire area under analysis since the individual's knowledge will be incomplete. Therefore, one would expect that all destinations outside the boundaries of the mental map are infeasible for the person. Thus, availability of alternatives is conditioned on their relationship to one another and to the mental map.

Similarly, in an automobile type choice context, the infeasibility of one compact car due to the decision-maker's internal space requirements should imply a like result for similar sized vehicles.

Despite its limitations, the independent availability assumption still provides a rich specification of the choice set generation process, and so is worth a closer look. Further, in the interests of obtaining operational models it is necessary to achieve compromises between behavioral realism, computational feasibility, and data requirements.

Assuming that the v's are independently distributed across alternatives, though not within an alternative, we rewrite (3.6) as

$$\Pr(C|M_n,D,X_n) = \frac{\prod_{i \in C} \Pr(A_{in}=1) \prod_{j \in M_n-C} \Pr(A_{jn}=0)}{1 - \prod_{j \in M_n} \Pr(A_{jn}=0)}, \quad C \in G_n. \tag{3.9}$$

If we do not further disaggregate the constraints themselves (as in (3.7)), we can define

$$D_i = \Pr(A_{in}=1), \ \forall i \in M, \ \forall n, \tag{3.10}$$

where M is, again, the master choice set. Now rewrite (3.9) as

$$\Pr(C|M_n,D,X_n) = \frac{\prod_{i \in C} D_i \prod_{j \in M_n-C} (1-D_j)}{1 - \prod_{j \in M_n} (1-D_j)}, \quad C \in G_n. \tag{3.11}$$

If we join to (3.11) the assumption that the choice given choice set model is of a multinomial logit form, we will obtain what may be termed the <u>independent availability</u>

logit model. This specification will be tested empirically in Chapter 4.

Rather than assuming a constant parameter across all decision-makers, it is perhaps more appropriate to describe the aggregate impact of constraints with a parametrized function, such as the one suggested by Ben-Akiva(1977),

$$\Pr(A_{in}=1) \propto \exp(D\ X_{in}),\ \forall i \in M; \tag{3.12}$$

this will result in a generalization of (3.11) which allows the aggregate impact of constraints to vary by person and alternative. The calibration of a model using (3.12) and a logit choice given choice set model must be performed with care: the same socio-economic variables present in both the choice set and choice models may cause identification problems. It is possible to identify $|M|$ constants in the functions (3.12), and $|M|-1$ in the choice given choice set specification, as long as the specification is dependent upon at least one of the variables.

We can further attempt to explicitly formulate constraints and use (3.7) to express choice set formation models. However, the limited practical applicability of choice set formation models without choice set structural restrictions (see Section 3.2.1 for a discussion of this topic) encourages us to transfer attention to other model specifications. Before terminating this section, we will review the work of Pitschke(1980), who presents the only known example, thus far, of development and estimation of an unrestricted choice set generation model.

It is clear from (3.7) that the likelihood of some set C being an individual's choice set (logically) depends not only upon all the alternatives in C being feasible, but in addition, all alternatives in $(M_n\text{-}C)$ being infeasible. This important point eluded Pitschke(1980), a pioneering effort in modelling choice set generation. He assumes that there is some measure of utility accruing to an individual from a specific choice set C, which can be described by a stochastic measure

$$\tilde{W}_n(C,\theta,X_n) = \tilde{w}_n(C,\theta,X_n) + \tilde{e}_n(C) \quad ,$$

in strict analogy to random utility formulations in standard choice models. Assuming that the error terms $\tilde{e}_n$ are IID Gumbel, Pitschke employs a multinomial logit choice set formation model, which he empirically tested in a mode choice context with data from the Minneapolis-St. Paul area in the United States.

From our development of (3.7), however, it would seem more reasonable to have defined the stochastic choice set "benefit" as

$$W_n(C,\theta,X_n) = w_{1,n}(C,\theta,X_n) - w_{2,n}(M_n\text{-}C,\theta,X_n) + e_n(C) \quad ,$$

where $w_{1,n}()$ describes the "benefit" accruing to n from the availability of the alternatives in C, and

$w_{2,n}()$ is the "benefit" associated with the unavailability of the alternatives in M_n-C.

Making an identical distributional assumption as Pitschke leads us to the model

$$Pr(C|M_n,\Theta,X_n)$$

$$= \frac{\exp[w_{1,n}(C,\Theta,X_n) - w_{2,n}(M_n-C,\Theta,X_n)]}{\sum_{C' \varepsilon G_n} \exp[w_{1,n}(C',\Theta,X_n) - w_{2,n}(M_n-C',\Theta,X_n))]} \tag{3.13}$$

from which we deduce that the only route to achieve Pitschke's specification is to assume that

$$w_{2,n}(M_n-C,\Theta,X_n) = w_{2,n}(M_n-C',\Theta,X_n) \tag{3.14}$$

for all C, $C' \varepsilon G_n$, $C \neq C'$. That is to say, the "disbenefit" resulting from not having available <u>any</u> group of alternatives must be a constant. This seems an undesirable assumption to make. Consider, for example, the case in which one dominant alternative in M_n has a probability of availability close to one ; then, if choice set C does not include this alternative, one would expect its likelihood to be much smaller than that of some other set C', which does include the dominant alternative. Pitschke's formulation, however, constrains the "disbenefit" of not having available (M_n-C) and (M_n-C') to be equal, which is clearly a strong assumption, especially if it is not necessary to assume this restriction to arrive at tenable models.

The following section will direct our interest towards models with *a priori* restrictions on choice set structure.

3.3.2 *A Priori* Restrictions on Choice Set Structure

In this section we will develop several models of choice set formation that will exemplify the rich variety of assumptions that are possible with regard to choice set structure. There is no intention for the development to be exhaustive; the models presented are appropriate in certain contexts and inappropriate in others.

3.3.2.1 G_n Restricted - Captivity To a Group of Alternatives

Lack of choice is the way of life for countless millions in less developed countries. Kozel(1981), discussed in Section 2.3, and Bajpai(1984) provide interesting descriptive analyses of this phenomenon in an attempt to characterize the sources and degree of choice captivity in a medium-sized Brazilian city and in several Indian cities, respectively.

Although the more general choice set formulation given in (3.7) can address the issue of captivity, the computational difficulties of calibrating such a model motivate us to postulate a choice framework more directly applicable to this phenomenon. This, again, is an example of trading off generality for tractability.

As before, let M_n be the set of deterministically feasible alternatives for n, a member of a population divided into two groups, captives and non-captives. The overall set M_n is subdivided into R non-empty, mutually exclusive, and collectively exhaustive subsets, denoted by M_{nr}, $r=1,\ldots,R$. Either an individual is free to choose from the full choice set M_n, or he is captive to one of its subsets. For this structure, we express the set of non-empty choice sets as

$$\tilde{G}_n = \{M_{n1},\ldots,M_{nR},M_n\} \ , \tag{3.15}$$

where

$$M_n = \bigcup_{r=1}^{R} M_{nr} \qquad ,$$

$$M_{nr} \quad M_{nr'} = \emptyset, \ \forall r' \neq r,$$

Since any alternative j M_n can be in only one other set, say $M_{nr(j)}$, the probability that alternative $j \varepsilon M_n$ is actually chosen by individual n is given by

$$Pr(j|B,D,X_n,\tilde{G}_n) = \frac{1}{K(\tilde{G}_n,D,X_n)}$$

$$[Pr(j|M_{nr(j)},B,X_n)\ Pr(M_{nr(j)}|G_n,D,X_n) +$$

$$Pr(j|M_n,B,X_n)\ Pr(M_n|G_n,D,X_n)] ,$$

$$j \varepsilon M_n, \qquad (3.16)$$

where the term $K(\tilde{G}_n,D,X_n)$ is a normalization constant to account for the restrictions imposed on choice set structure to arrive at $\tilde{G}_n$ (i.e. all the sets in G_n that are being ignored). To allow the restricted choice set probabilities to sum to one, we let

$$K(\tilde{G}_n,D,X_n) = \sum_{C \varepsilon \tilde{G}_n} Pr(C|G_n,D,X_n) \quad . \qquad (3.17)$$

Note that in (3.16) we have implicitly defined

$$r(C|\tilde{G}_n,D,X_n) = \frac{Pr(C|M_n,D,X_n)}{K(\tilde{G}_n,D,X_n)}, \quad C \varepsilon \tilde{G}_n. \qquad (3.18)$$

where $Pr(C|M_n,D,X_n)$ is the probability of choice set C with unrestricted choice set structure. Expression (3.18) states that the probability of some set $C \varepsilon \tilde{G}_n$ being individual n's choice set, given the restricted choice set structure of (3.15), is given by the probability of C being n's set when all possible sets in G_n are considered, normalized by the total probability of the elements of $\tilde{G}_n$ being n's choice set, given all possible sets.

We may now rewrite (3.16) as

$$\Pr(j|B,D,X_n,\tilde{G}_n) = [\Pr(j|M_{nr(j)},B,X_n)\ \Pr(M_{nr(j)}|\tilde{G}_n,D,X_n) + \Pr(j|M_n,B,X_n)\ \Pr(M_n|G_n,D,X_n)], \qquad (3.19)$$

where $M_{nr(j)}$ continues to be the one set, besides M_n, of which alternative j is an element. This, then, constitutes our general captivity model. Let us now study some specific developments.

If we were to assume, for instance, that $|M_{nr}|=1$, $r=1,\ldots,R$, so that each subset contains only one alternative, we have a choice context identical to the classical captivity model studied by McFadden(1976a), Ben-Akiva(1977), and Gaudry and Dagenais(1979) (see Section 3.2.1 for a discussion of these references). In addition, assume that the error terms of the constraints are independent across alternatives, so that we may then use (3.9) to express the probability of $M_{nr(j)}$ or M_n being the decision maker's choice set. For convenience in development, define

$$c_j = \Pr(A_{jn}=1), \ \forall j \in M, \qquad (3.20)$$
$$a_j = 1-c_j = \Pr(A_{jn}=0), \ \forall j \in M.$$

Then we can write the <u>unrestricted</u> probabilities of M_{nj}, $j=1,\ldots,R$, and M_n as

$$\Pr(M_{nj}\,|\,G_n,c,X_n) = \frac{c_j}{a_j}\, Q(M_n), \qquad j=1,\ldots,R, \qquad (3.21a)$$

$$\Pr(M_n\,|\,G_n,c,X_n) = \prod_{j\in M_n} \frac{c_j}{a_j}\, Q(M_n), \qquad (3.21b)$$

where

$$Q(M_n) = \frac{\prod_{j\in M_n} a_j}{1 - \prod_{j\in M_n} a_j} \qquad (3.22)$$

From expressions (3.21a,b) we form the normalization constant $K(\tilde{G}_n,c,X_n)$:

$$K(\tilde{G}_n,c,X_n) = Q(M_n)\left[\sum_{l\in M_n} \frac{c_l}{a_l} + \prod_{j\in M_n} \frac{c_j}{a_j}\right]. \qquad (3.23)$$

Further, define

$$D_i = \frac{c_i}{a_i}, \ \forall i \in M, \qquad (3.24a)$$

$$\tilde{D}_n = \prod_{i \varepsilon M_n} D_i , \quad \forall n. \qquad (3.24b)$$

Note that it will not be possible to identify the $(n+|M|)$ parameters above. In fact, it is possible to identify at most $|M|$ of them, so we will set $\tilde{D}_n=1$, $\forall n$. We can then rewrite (3.19), for the present model specification, as

$$\Pr(j|B,D,X_n,\tilde{G}_n) = \frac{1}{1+\sum_{i\varepsilon M_n} D_i} [D_j+\Pr(j|M_n,B,X_n)]. \qquad (3.25)$$

If we further assume that the specification for the choice given choice set model is multinomial logit, (3.25) will be exactly the logit captivity model mentioned before.

As in the case of the independent availability model discussed in Section 3.3.1, it is possible to generalize (3.25) by assuming an appropriate functional form, such as (3.12), for the parameters D_i, $i \varepsilon M$.

The model portrayed in (3.19) is, however, more general than the specification given in (3.25). Suppose, for example, that we have a work modal choice context in which we assume that all individuals have one of three types of choice sets:

(1) they may exercise choice between only bus (indexed as mode 1) and ride-sharing

(indexed as 2), which are available to everyone;

(2) they may be captive to auto drive (indexed as mode 3), if the alternative is available; or

(3) they may have the full choice set of bus, ride-sharing, and auto drive (if available) from which to choose.

For individuals without the auto drive alternative, $\tilde{G}_n$ is composed only of the set

$$M_n = \{1,2\}, \tag{3.26a}$$

whereas for individuals with all three alternatives $\tilde{G}_n$ is composed of three sets:

$$M_{n,1} = \{1,2\},\ M_{n,2} = \{3\},\ M_n = \{1,2,3\}. \tag{3.26b}$$

For purposes of estimating the parameters of a choice set formation model, individuals without the auto drive alternative are irrelevant (they of course contribute to the determination of the choice given choice set model parameters). Thus, using the notation developed before and making the same assumptions concerning the independence of constraints across alternatives, we may write the unrestricted choice set probabilites for decision-makers with $\tilde{G}_n$ given by (3.26b) as

$$\Pr(M_{nr}|G_n,c,X_n) = Q(M_n) \prod_{i \in M_{nr}} \frac{c_i}{a_i},\ r=1,2, \tag{3.27a}$$

$$\Pr(M_n|G_n,c,X_n) = Q(M_n) \prod_{i\varepsilon M_n} \frac{c_i}{a_i} \quad , \tag{3.27b}$$

$Q(M_n)$ being given by (3.22). Define

$$D_r = \prod_{i\varepsilon M_{nr}} \frac{c_i}{a_i} \quad , \ r=1,2, \tag{3.28}$$

$$\tilde{D}_n = \prod_{i\varepsilon M_n} \frac{c_i}{a_i} \, . \tag{3.29}$$

As occurred in the development of (3.25), it is not possible to identify $\tilde{D}_n$, so we normalize it to 1 for all n. Therefore,

$$K(\tilde{G}_n,D,X_n) = Q(M_n)\ [1+D_1+D_2], \tag{3.30}$$

and we can finally express the choice probabilities for individuals with $M_n = \{1,2,3\}$ as

$$\Pr(1|B,D,X_n,\tilde{G}_n) = \frac{1}{1+D_1+D_2} [D_1 \Pr(1|M_{n1},B,X_n) + \Pr(1|M_n,B,X_n)] \quad ,$$

$$\Pr(2|B,D,X_n,\tilde{G}_n) = \frac{1}{1+D_1+D_2} [D_1 \Pr(2|M_{n1},B,X_n) + \Pr(2|M_n,B,X_n)] \quad ,$$

$$Pr(3|B,D,X_n,\tilde{G}_n) = \frac{1}{1+D_1+D_2} [D_2 + Pr(3|M_n,B,X_n)] \quad . \tag{3.31}$$

For individuals with $M_n=\{1,2\}$,

$$\begin{aligned} Pr(1|B,D,X_n,\tilde{G}_n) &= Pr(1|M_n,B,X_n), \\ Pr(2|B,D,X_n,\tilde{G}_n) &= 1-Pr(1|M_n,B,X_n). \end{aligned} \tag{3.32}$$

In practice, issues such as data availability, behavioral realism, and available monetary, computational and time resources will dictate the choice between a richer specification, such as embodied in (3.8) or (3.12), and a simpler one, such as (3.31,32).

The specific choice set structure restriction that we have imposed in this section (which results in the set of possible choice sets given in (3.15)) is applicable to other choice contexts than the modal one considered in the previous examples. Consider an urban area that has an active downtown shopping area and, in addition, a number of suburban shopping malls. In a model of shopping destination for which we decide to adopt choice set structure (3.15), we can divide the full set of possible destinations into a set containing the various downtown destinations, plus a set for each of the suburban shopping malls. The appropriateness of such a structure will depend largely upon the analyst's intuition about the magnitude of the impact of constraints that might lead to the phenomenon of individuals being captive to the downtown or a single suburban shopping mall.

The next section will introduce a second possible restriction to the choice set structure.

3.3.2.2 G_n Restricted - Maximum Choice Set Size

The limited ability of human beings as information processors is a well-established fact, both from one's own experience as well as from psychological and marketing experiments. Simply put, individuals are unable to handle large amounts of information; in situations in which they are faced with the prospect of dealing with much data for the purposes of reaching some decision (such as the choice of some alternative), most individuals will adopt screening and decision criteria that spare them the tedium and effort of viewing each separate datum. This aspect of decision-making processes has long been known, and has led to choice models that attempt to incorporate this screening mechanism, as in the "satisficing" model of Simon(1955) or the elimination-by-aspects model of Tversky(1972).

In Chapter 2, in discussing the constraint typology on individual urban travel, we mentioned information-related restrictions to which individuals are subject. These restrictions are of two types: those imposed by the society and reflecting the extent to which information is made available to individuals, and those from within decision-makers, reflecting the degree to which people can assimilate information they receive or collect. One can, certainly, adopt the complementary view of the impact of information on choice set formation as a dynamic learning process, wherein an individual receives or collects data from various sources (experience, media reports, and word-of-mouth) upon which to base a decision. Naturally, this information collection and processing involves costs which the individual must meet if he or she is to use the data. Offsetting these costs are the aspirations of the individuals: if aspiration levels are higher than the satisfaction levels afforded by currently known alternatives, individuals embark upon a search for new alternatives to satisfy their perceived needs. This type of decision process is reflected in the works of Hall(1980), Richardson(1982) and Meyer(1979).

To this point, then, we have described a two-tiered choice set formation process. Firstly, there exist effects accruing from objective constraints, which dictate the feasibility of alternatives on the basis of factors external to the self of the individual. For example, the transport infrastructure, household/family activity requirements, and group affiliations (all discussed in Chapter 2, Section 2.2.2) are typical of these constraints. In our models, some of these constraints are treated deterministically, others probabilistically, as in the models of Sections 3.3.1 and 3.3.2.1. Secondly, there are the information-related constraints we have just discussed; or as may seem more convenient, but completely analogous, there is a search process within the set of objectively feasible alternatives to actually define the elements of the choice set. Following all of this, actual choice is exercised.

Up to now we have only reformulated the choice set formation paradigm that leads to expression (3.1). Any models we develop from this approach will suffer the same practical limitations that beset (3.1). Thus, we will make the plausible simplifying assumption that individuals are unable (or unwilling) to consider simultaneously more than some known number of discrete alternatives. The marketing research literature (which we discuss in the following paragraphs), especially that dealing with the pre-test-marketing of new products, gives us empirical evidence of the plausibility of this assumption.

Market researchers have for some time adopted the working concept of an "evoked set", one definition of which is a collection of some number of items, all representing some specific product, which the consumer (1) has used in the past, (2) is currently using, (3) will seriously consider using, and (4) will not outright reject using (see Allaire, 1973, Silk and Urban, 1978, and Urban and Hauser, 1980). Based on this concept, which parallels the constraint-based choice set generation process we have previously described, several marketing researchers report statistics concerning the size of individuals' evoked sets. For example, in an experiment involving choice of deodorants (of which quite a large number are on the market), it was found that 93% of the individuals in a sample of 299 U.S. residents had sets of five or fewer elements; in fact, all the individuals had seven or less elements. These finding are entirely consistent with those of Urban(1975), Wierenga(1974), and Massy et al.(1968), for different consumer goods.

Clearly, the small investment in time, effort and money needed to gather information concerning deodorants is hardly typical of the choice contexts of interest to transport planners. However, these results clearly suggest the plausibility of a modelling approach that incorporates a choice set size restriction. The size limitation may be viewed as a proxy for the inherent information-processing limitations of individuals. In the following pages we will view this size limitation as part of the choice set generation process, and will develop a specific model for this aspect of the choice paradigm.

Let us now rewrite the unconditional probability of choice of some alternative $j \in M_n$ as

$$\Pr(j|B,\Theta,\gamma,X_n,S_n) = \frac{1}{K(M_n,S_n,\Theta,\gamma)}$$

$$\sum_{l=1}^{S_n} \sum_{C \in G_n(j,l)} \Pr(j|C,B,X_n) \Pr[C|\Theta,X_n] \Pr[L_n=l|\gamma,X_n] \tag{3.33}$$

where

B,Θ,γ	are unknown parameters;
S_n	is an analyst-determined maximum possible choice set size for individual n, $S_n \leq M_n$;
L_n	is the true, unknown size of n's choice set, but it is known to be less than or equal to S_n;
$G_n(j,l)$	is the set of all non-empty subsets of M_n containing j and having <u>exactly</u> l alternatives, $l=1,\ldots,S_n$;
	other quantities as previously defined.

The normalization factor $K(M_n,S_n,\Theta,\gamma)$ is given by

$$K(M_n,S_n,\Theta,\gamma) = \sum_{l=1}^{S_n} \sum_{C \in G_n(j,l)} \Pr(C|\Theta,X_n) \Pr(L_n=l|\gamma,X_n) \tag{3.34}$$

Expression (3.33) reflects the various hypotheses discussed above: $Pr(C|\Theta,X_n)$ is the model of objective feasibility of the alternatives in C, and $Pr(L_n=l|\gamma,X_n)$ will be the model in which we describe the individual's access to and processing of information about alternatives. Let us specify in turm each of these models.

We shall assume that the objective feasibility of a set of alternatives is the joint feasibility of all its individual alternatives. If we further assume that the error terms of the constraints are independently distributed across alternatives (though not within alternatives), we can write

$$Pr(C|\Theta,X_n) = \frac{\prod_{i\varepsilon C} Pr(A_{in}=1)}{1 - \prod_{k\varepsilon M_n} Pr(A_{kn}=0)}, \quad C\varepsilon G_n(j,l), \quad l=1,\ldots,S_n \qquad (3.35)$$

We are now left to deal with the final component of (3.33), namely the choice set size model $Pr(L_n=l|\gamma,X_n)$, $l=1,\ldots,S_n$. There is, unfortunately, no theory (and little empirical evidence, for that matter) to direct our formulation of this model. About the only generalization possible is that the size of choice sets, especially in contexts with many alternatives, is probably significantly smaller than the universe of possibilities. One possible approach that is consistent with the present development is to assume the existence of a latent variable $Z(\rho, X_n)$, which describes the individual's ability to process data as a function of his or her socio-economic attributes. The vector ρ is a component of the γ vector of parameters. The choice set size variable L_n is related to $Z(\rho, X_n)$ as follows:

$$L_n = \begin{cases} 1 & \text{if } Z(\rho, X_n) \varepsilon (-\infty, \mu_1] \\ l & \text{if } Z(\rho, X_n) \varepsilon (\mu_{l-1}, \mu_l],\ l=2,\ldots,S_n-1 \\ S_n & \text{if } Z(\rho, X_n) \varepsilon (\mu_{S_n-1}, \infty) \end{cases} \tag{3.36}$$

where μ_m is the m^{th} component of the parameter vector μ (also, as with ρ, a component of γ), $m=1,\ldots,(S_n-1)$.

We can proceed to make distributional assumptions concerning the stochastic component of $Z(\rho, X_n)$ and thus derive the desired distribution for L_n.

For modelling contexts in which the number of alternatives is not very great, the formulation in (3.36) is possible to execute. However, the number of parameters is a direct function of the maximum possible size of choice sets, max $\{S_n, \forall n\}$. This makes the approach computationally infeasible for most problems when we realize that other parameters (vectors Θ and B) exist in the full formulation of the choice model.

Therefore, we abandon (3.36) and resort to making an arbitrary assumption concerning the distribution of choice set sizes in the population. For the present derivation, we will assume that the choice set size distribution is given by a truncated Poisson distribution function, as follows:

$$\Pr[L_n=l|\gamma,X_n] = \frac{1}{\eta(\gamma,X_n,S_n)} \frac{[Z(\rho,X_n)]^l \exp[-Z(\rho,X_n)]}{l!},$$

$$l=1,\ldots,S_n,\; Z(\rho,X_n)>0. \tag{3.37}$$

where $Z(\rho,X_n)$ is a non-stochastic function related to the information processing capability of individual n, required to be strictly positive;

$\eta(\gamma,X_n,S_n)$ is a normalization factor to account for the range of L_n, <u>i.e.</u>

$$\eta(\gamma,X_n,s_n) = \sum_{m=1}^{S_n} \frac{[Z(\rho,X_n)]^m \exp[-Z(\rho,X_n)]}{m!} \tag{3.38}$$

There remains the task of bringing together the three component models, (3.35) and (3.37), into expression (3.33). It is first necessary, however, to be more specific about the formulation of (3.35), the alternative feasibility model. For expositional purposes, we will make the simplifying definition that

$$\Theta_i = \Pr(A_{in}=1),\ \forall i \in M,\ \forall n. \tag{3.39}$$

We can now write (3.33) as

$$\Pr(j|B,\Theta,\gamma,X_n,S_n)$$

$$= \frac{1}{K(M_n,S_n,\Theta,\gamma)} \sum_{l=1}^{S_n} \sum_{C \in G_n(j,l)} \Pr(j|C,B,X_n)$$

$$\frac{\prod_{i \in C} \Theta_i}{1-\prod_{k \in M_n}(1-\Theta_i)} \qquad \frac{[Z(\rho,X_n)]^l \exp[-Z(\rho,X_n)]}{l!\ \eta(\gamma,X_n,S_n)} \tag{3.40}$$

all quantities as previously defined.

Expression (3.40) finishes the development of a class of choice set models that incorporate a maximum choice set size limitation. However, this size limitation is arbitrary only to the extent that an upper limit is specified by the model analyst. The universe of possible choice sets, in contrast to conventional choice model specifications, is quite large; at the same time, judicious choice of S_n, the maximum choice set size, results in models that need not suffer from the computational limitations of a model such as

(3.9), when applied to choice contexts with a large number of alternatives.

It is impossible to set general guidelines for choosing an appropriate value of S_n: this parameter is case-specific and must be decided upon for each choice context being modelled. Intuitively, it is better to err on the side of too many, rather than too few, alternatives; offsetting this we have the concommitant computational burden that must be borne. However, if one measures computational effort in terms of the number of choice sets involved, the marginal effort due to increasing the maximum choice set size by one element decreases as S_n approaches $|M_n|$. For example, if one has established S_n as 4 when $|M_n|=10$, then to increase S_n to 5 results in 65% more choice sets to handle; if S_n is 7, again $|M_n|=10$, increasing S_n to 8 results in about 5% more possible choice sets. Clearly, there is a significant trade-off between small and large values of S_n, but far less difference between proximate, and large, maximum choice set sizes.

3.4 Summary

This chapter formulates, based upon the approach of constraint identification, a series of models of the choice set generation process. The models range from those with no prior knowledge concerning possible choice set structures to those imposing structural restrictions on the space of possible choice sets. These latter models result from the need to achieve a workable compromise between model realism and computational constraints, and are especially useful in choice contexts involving a large number of alternatives.

The high degree of nonlinearity and absence of desirable properties, such as concavity, of the functions embodying the estimators that may be used to calibrate the models we've proposed here, present a number of numerical and algorithmic difficulties that must be addressed by the solution method adopted to solve the estimation problem. These difficulties are discussed in Appendix 3, and one solution strategy that involves use of the Berndt-Hall-Hall-Hausman(1974) algorithm is implemented for the empirical stage of this research.

The next two chapters address the empirical estimation of choice set formation models.

CHAPTER 4

AN EMPIRICAL APPLICATION OF CHOICE SET FORMATION MODELLING - MACEIO, BRAZIL

4.1 Introduction

This chapter presents and discusses a number of calibration results for the city of Maceio, Brazil, for which we have estimated several models of choice and choice set formation. Specifically, we provide a spectrum of models of work mode choice, ranging from a standard logit choice model, to a logit captivity formulation, to an independent availability logit specification. Augmented by the use of market segmentation, these results indicate that the models of choice set formation are statistically significant with respect to the standard of comparison, the logit model.

To permit a fuller evaluation of the merit of modelling choice set formation, we also provide comparisons of aggregate measures of response based upon the various models. Since the difficulties of calibrating the choice set formation models can be significant when compared to traditional models of choice with deterministic choice set allocation rules, it is necessary that we go beyond statistical significance as a criterion of evaluation to consider the impact of the more complex specification on measures of behavior useful to policy-makers.

We begin this presentation by briefly describing the city of Maceio.

4.2 The City of Maceio

Maceio, the capital of the state of Alagoas, is the export outlet for what is traditionally the state's principal product, cane sugar. The city is characteristic of the main urban areas of poverty-stricken northeastern Brazil:

(1) it has experienced a tremendous population growth in the last 20 years, averaging an annual growth rate of 4.6% (from a 1960 population of 170,000, the city had a population of about 365,000 in 1978, and is expected to reach one-half million by the mid-80's);

(2) the personal income distribution is highly skewed: in 1970, 87.3% of the population earned one federally mandated minimum wage or

less, and only 1.5% earned over 3 minimum wages; over time there has been a marked tendency towards further concentration;

(3) the unemployment level is high, and the employment that does exist is dominated by the service sector (public and private services constitute 93% of the employment), and little industrialization is evident.

These same factors are common concerns to all planners and transportation analysts in developing countries (see, for example, Bajpai, 1984, for discussion of the Indian context, and Geltner and Barros, 1984, for a similar analysis of Maceio, Brazil).

Due to geography, historical development, and economic forces Maceio displays a higher residential density (about 40 persons/hectare) than comparably sized cities in more developed regions (Geltner and Barros, 1984). In addition, as would be expected in a city in a developing country, the geographical distribution of high and low income households is heterogeneous, with elements of many economic strata being found in close proximity one to the other. There emerges, however, a generally radial sectoral pattern with predominantly low income sectors alternating with high income areas.

Given the small overall dimensions of the urban area, it is not surprising that employment is concentrated in the CBD (Central Business District) and the nearby port; in fact, the high proportion of 60% of jobs are to be found in these two areas. Interestingly, despite the high employment concentration the CBD remains a residential area of moderate density.

With respect to the physical layout of the transport infrastructure, Maceio's street and transit networks are basically radial in nature, molding themselves, in part, to the geographical features of the area and to the distribution of economic and social activity patterns in the city. The high concentration of employment in the CBD and the layout of the transport system contribute directly to traffic congestion, fuel consumption, and accessibility levels of the urban poor.

The data we utilize in our study was collected in 1977 by the Empresa Brasileira de Planejamento de Transportes - GEIPOT (see GEIPOT, 1982). The household survey recorded the travel patterns, over a 24-hour period, of members over 5 years of age for more than 3,200 randomly selected households. In addition, a number of household and person-specific socio-economic attributes were collected and are utilized in our model specifications. The travel impedance measures were also provided us by GEIPOT. They are derived from calibrated automobile and transit networks constructed for the thirty-five traffic zones defined for the study area.

The city and the travel patterns of its residents have been studied extensively elsewhere (see Swait *et al.*, 1984; Barros and Geltner, 1984), and the interested reader is urged to consult these references.

4.3 Model Calibration Results

4.3.1 The Choice Context

The particular choice dimension we shall investigate here is that of home-based work mode choice for full-time workers. The unit of observation is the modal choice pattern for a working day because of the wide-spread habit of returning home for lunch and important policy implications of manipulating this type of behavior. An investigation of the observed modal choice patterns of Maceio workers, captured in a 1977 household survey, reveals that less than 5% of the workers chose travel patterns involving more than one mode. Hence, our universe of alternatives is reduced to round trips to work by

(1) bus,
(2) taxi,
(3) auto drive, and
(4) auto passenger.

Thus, when we speak of a modal alternative we are actually referring to the use of that mode by the worker for all the trips taken that day that are home-based work.

The following deterministic constraints were applied to the above alternatives to allocate them to individuals:

(1) the auto drive alternative is available only to individuals from auto-owning households that are 18 years or older (no information was available in the survey on driver's license); and

(2) if the one-way network travel time for the mode is greater than 2 hours, it is unavailable.

Thus, bus, taxi and auto passenger are ubiquitous; auto drive is limited to individuals <u>eligible</u> for a driver's license and whose households own a vehicle. The travel time limitation is a further imposition.

To provide a basis for comparison, we first estimated a standard logit model on a random sample of 1,477 workers. Next, in accordance with the discussion in Section 2.3 of Chapter 2, we utilize market segmentation as a first attempt to account for the impact of constraints on choice. Following that, we present in turn the logit captivity and independent availability logit models that have been calibrated for Maceio.

4.3.2 The Standard Logit Model

Table 4-1 presents the calibration results for the standard logit model of home-based full-time worker mode choice. The 19 parameter model includes time, cost, income, family size, car competition, and role-related variables, which with one exception evince high levels of significance and correct signs. Though no extensive efforts were expended to obtain an improved specification, it is felt that the model as it now stands represents a reasonable standard for comparison.

Inclusion of variables such as auto competition, income and family size in the utility functions of alternatives can be interpreted as an _ad hoc_ model of alternative availability. To see this, suppose there exists a function $A_i(SE)$, where i is a discrete alternative, and SE is a vector of socio-economic attributes. Then we can write the probability of choice of i in set C as

$$\Pr(i|C) = \frac{\exp[V_i + \ln A_i(SE)]}{\sum_{j \varepsilon C} \exp[V_j + \ln A_j(SE)]} \qquad (4.1)$$

where V_i is the deterministic choice utility of alternative i. We can think of the functions $A_i(SE)$ as measures of the

Table 4-1 - Maceió Home-Based Work Tour Mode Choice Model - Logit Specification - Full Data Set

	Estimated Parameters (Asymptotic t-statistics in parentheses)
1. Alternative Specific Constants	
- Bus	-0-
- Taxi	-1.287 (-2.5)
- Auto Passenger	-2.883 (-7.5)
- Auto Drive	0.015 (0.0)
2. Total Travel Time (min/day)	-0.0119(-3.5)
3. Total Travel Cost (Cr$ 1977/day) divided by ℓn (Household Income, KwH/month)	-0.2960(-5.9)
4. Household Income, KwH/month	
- Bus	-0-
- Taxi	0.0050 (2.7)
- Auto Passenger	0.0066 (5.1)
- Auto Drive	0.0062 (4.8)
5. Number of Household Members	
- Bus	-0-
- Taxi	-0.1102(-2.1)
- Auto Passenger	-0.1464(-3.0)
- Auto Drive	-0.2065(-3.9)
6. Auto Competition (#cars/#workers)	
- Bus	-0-
- Taxi	1.8144 (4.3)
- Auto Passenger	3.1017(11.3)
- Auto Drive	2.8752 (9.6)
7. CBD work location (zones 4 & 5) and lunch trip home	
- Bus	-0-
- Taxi	0.660 (2.1)
- Auto Passenger and Drive	-0.183 (-0.7)
8. Female Worker	
- Bus, Taxi, Auto Passenger	-0-
- Auto Drive	-1.654 (-7.0)

Table 4-1 - Continued

	Estimated Paremeters (Asymptotic t-statistics in parentheses)
9. Professional worker (codes 5 & 7) and lunch trip home	
- Bus	-0-
- Taxi	0.792 (2.3)
- Auto Passenger and Drive	1.006 (4.2)
Summary Statistics	
log likelihood at equal probability	-1779.0
log likelihood at convergence	-720.9
rho-squared	0.5948
adjusted rho-squared[1]	0.5894
# parameters	19
Sample Description	
Choosing - Bus	931
- Taxi	55
- Auto Passenger	120
- Auto Drive	371
Total Observations	1477

[1]See Horowitz (1982) for the definition of this measure.

effect of the constraints on the availability of i, in much the same way as size variables are utilized to correct for aggreagtion of elemental alternatives in logit models of destination choice (see Lerman, 1975).

To maintain uniformity during the comparisons, we have used this same specification for the choice model throughout our study of Maceio, opening exceptions only in cases of unidentifiability. We thus hope to more clearly pinpoint the effect of modelling choice sets.

4.3.3 Market Segmentation Models

Market segmentation is an useful technique to account for taste variation in a population, but as we have pointed out before, it can also be used to bring out the impact of constraints on choice. Accordingly, we have calibrated logit choice models for two market segmentation schemes that should reflect the existence of constraints, in addition to taste variations. One of these schemes is of an economic nature, and the other is both economic and role-related.

First, we have separated the observations along the household income dimension, in Maceio proxied by monthly household electrical energy consumption (see Swait _et al._, 1984, for more discussion of this measure). The three income groupings are (1) less than 80 KwH/month (LOW), (2) 80 to

130 KwH/month (MEDIUM) and (3) greater than 130 KwH/month (HIGH). The sample contains 528, 446, and 503 workers in each category, respectively. Located in an economically depressed area of Brazil, it is to be expected that income should play a significant role in determining mode choice in Maceio; and indeed, it is the most significant of the three segmentations we present. Table 4-2 shows the three income segment logit models. Again, with a a very high level of significance (a chi-squared statistic of 71.2, compared to a critical value of 63.7 for a 99% significance level and 40 degrees of freedom), we reject the hypothesis of choice model parameter equality across the income segments. The apparent parameter differences seem to mainly be concentrated in the socio-economic attributes, such as income (whose effect is quite diminished in the income segment models, indicating that the segmentation has significantly reduced within group variation with respect to this variable), household size, and auto competition. The travel impedance parameters are not very different across segments.

The second segmentation is according to the gender of the worker. This scheme is inspired by the magnitude and significance of the female worker dummy in the models presented thus far. The parameter of this variable indicates the significantly lesser occurrence of female auto drivers versus male auto drivers in households with cars. Table 4-3

Table 4-2 - Maceió Home-Based Work Tour Mode Choice Model - Logit Specification - Income Segmentation

	Estimated Parameters (Asymptotic t-statistics in parentheses)		
	LOW	MEDIUM	HIGH
1. Alternative Specific Constants			
- Bus	-0-	-0-	-0-
- Taxi	0.047	-1.299	0.784
	(0.0)	(-0.5)	(0.7)
- Auto Passenger	-3.472	-3.301	-2.166
	(-2.0)	(-1.8)	(-2.3)
- Auto Drive	1.137	0.497	0.122
	(0.6)	(0.3)	(0.1)
2. Total Travel Time (min/day)	-0.0078	-0.0142	-0.0109
	(-1.0)	(-2.3)	(-1.8)
3. Total Travel Cost (Cr$ 1977/day) divided by ℓn (Household Income, KwH/month)	-0.2448	-0.3179	-0.3417
	(-2.1)	(-3.8)	(-2.6)
4. Household Income, KwH/month			
- Bus	-0-	-0-	-0-
- Taxi	0.0010	-0.0102	0.0008
	(0.1)	(-0.4)	(0.3)
- Auto Passenger	0.0203	0.0156	0.0027
	(1.0)	(0.9)	(1.2)
- Auto Drive	0.0076	0.0255	0.0020
	(0.4)	(1.5)	(0.9)
5. Number of Household Members			
- Bus	-0-	-0-	-0-
- Taxi	-0.4431	0.1798	-0.3162
	(-2.5)	(1.7)	(-2.9)
- Auto Passenger	-0.2566	-0.1624	-0.1659
	(-1.9)	(-1.7)	(-2.1)
- Auto Drive	-0.2589	-0.5647	-0.1726
	(1.1)	(-4.1)	(-2.2)

Table 4-2 - Continued

	Eastimated Parameters (Asymptotic t-statistics in parentheses)		
	LOW	MEDIUM	HIGH
6. Auto Competition (#cars/#workers)			
- Bus	-0-	-0-	-0-
- Taxi	3.9430	1.3676	2.0277
	(2.8)	(1.4)	(3.3)
- Auto Passenger	3.6191	2.4660	3.8396
	(2.8)	(6.2)	(8.3)
- Auto Drive	2.0442	1.8923	3.9182
	(2.1)	(3.3)	(8.2)
7. CBD work location (zones 4 & 5) <u>and</u> lunch trip home			
- Bus	-0-	-0-	-0-
- Taxi	0.299	0.281	1.299
	(0.3)	(0.5)	(2.4)
- Auto Passenger and Drive	-0.296	-0.489	0.134
	(-0.4)	(-1.2)	(0.3)
8. Female Worker			
- Bus, Taxi, Auto Passenger	-0-	-0-	-0-
- Auto Drive	-1.663	-2.001	-1.552
	(-1.8)	(-4.2)	(-5.0)
9. Professional Worker <u>and</u> lunch trip home			
- Bus	-0-	-0-	-0-
- Taxi	-0.527	0.390	1.441
	(-0.4)	(0.5)	(2.8)
- Auto Passenger and Drive	0.934	0.596	1.480
	(1.2)	(1.4)	(3.6)

Table 4- 2 - Continued

	Estimated Parameters (Asymptotic t-statistics in parentheses)		
	LOW	MEDIUM	HIGH
Summary Statistics			
log likelihood at equal probability	-596.5	-532.8	-649.8
log likelihood at convergence	-129.0	-237.3	-319.0
rho-squared	0.7837	0.5546	0.5090
adjusted rho-squared	0.7677	0.5368	0.4944
# parameters	19	19	19
Sample Description			
Choosing - Bus	467	301	163
- Taxi	12	18	25
- Auto Passenger	14	39	67
- Auto Drive	35	88	248
Total Observations	528	446	503

presents the two gender segment models, and again we reject parameter equality at about the 90% significance level (the chi-squared statistic is 28.0, compared to a critical value of 26.0 with 18 degrees of freedom). As with the other segmentation scheme, parameter differences seem to derive from different distributions of socio-economic attributes between the two groups of workers.

Although the market segmentation results are encouraging, especially the income group models, it is impossible to attribute any part of the improvement to a better choice model specification because of accounting for taste variations, or to improved "modelling" of constraints to choice (see expression (4.1)). It would be possible to test more elaborate market segmentation schemes (such as those suggested in Section 2.3); it is likely, however, that the indeterminacy of the source of improved explanatory power of the resulting models would continue. The next section will explore the separation of these two effects by presenting estimation results for the logit captivity specification.

4.3.4 The Logit Captivity Models

The simple logit captivity model, formulated in Chapter 3 as expression (3.25), represents a choice context in which

Table 4-3 - Maceió Home-Based Work Tour Mode Choice Model - Logit Specification - Segmentation by Gender

	Estimated Parameters (Asymptotic t-statistics in parentheses)			
	Female		Male	
1. Alternative Specific Constants				
- Bus	-0-		-0-	
- Taxi	-1.396	(-1.4)	-1.254	(-2.0)
- Auto Passenger	-3.235	(-3.5)	-2.835	(-6.5)
- Auto Drive	-3.709	(-3.9)	0.574	(1.2)
2. Total Travel Time (min/day)	-0.0133	(-1.8)	-0.0114	(-2.9)
3. Total Travel Cost (Cr$ 1977/day) divided by ℓn (Household Income, KwH/month)	-0.2234	(-2.0)	-0.3272	(-5.5)
4. Household Income, KwH/month				
- Bus	-0-		-0-	
- Taxi	0.0021	(0.3)	0.0062	(3.2)
- Auto Passenger	0.0070	(2.5)	0.0063	(4.2)
- Auto Drive	0.0070	(2.3)	0.0060	(4.1)
5. Number of Household Members				
- Bus	-0-		-0-	
- Taxi	-0.2464	(-2.3)	-0.0647	(-1.0)
- Auto Passenger	-0.2144	(-1.8)	-0.1129	(-2.1)
- Auto Drive	-0.1733	(-1.4)	-0.2161	(-3.6)
6. Auto Competition (#cars/#workers)				
- Bus	-0-		-0-	
- Taxi	2.7255	(3.2)	1.3822	(2.6)
- Auto Passenger	3.9416	(6.5)	2.7975	(8.3)
- Auto Drive	4.9656	(7.9)	2.2857	(6.2)
7. CBD work location (zones 4 & 5) and lunch trip home				
- Bus	-0-		-0-	
- Taxi	1.074	(1.6)	0.5563	(1.5)
- Auto Passenger and Drive	-0.142	(-0.3)	-0.1488	(-0.5)

Table 4- 3- Continued

	Estimated Parameters (Asymptotic t-statistics in parentheses)			
	Female		Male	
8. Professional worker and lunch trip home				
- Bus	-0-		-0-	
- Taxi	1.505	(2.1)	0.3275	(0.7)
- Auto Passenger and Drive	1.669	(3.5)	0.7330	(2.5)

Summary Statistics

	Female	Male
log likelihood at equal probability	-454.4	-1324.6
log likelihood at convergence	-192.7	-514.2
rho-squared	0.5759	0.6118
adjusted rho-squared	0.5561	0.6050
# parameters	18	18

Sample Description

	Female	Male
Choosing - Bus	267	664
- Taxi	18	37
- Auto Passenger	45	75
- Auto Drive	51	320
Total Observations	381	1096

the decision-maker is either captive to one of the alternatives thought to be available to him, or is otherwise free to choose from the full set of alternatives according to a multinomial logit model. Table 4-4 shows the calibration results for this specification on the full sample of 1,477 workers; the choice model parameters (i.e. those for the logit component of the captivity model) are directly comparable to the parameters in tables 4-1 through 4-3. Note that the model in Table 4-4 maintains the hypothesis of no captivity to the auto drive mode for workers who have this alternative. While this restriction seems plausible for the city of Maceio, it is not an arbitrary one: it is the result of the parameter being driven to zero during the process of optimization of the log likelihood function for the Maceio sample.

With a chi-squared statistic of 3.8 with 3 degrees of freedom, we cannot reject the hypothesis, at a 90% significance level, that the captivity parameters are jointly zero. Thus, there seems to be little evidence of captivity for the workers as a whole. This is not, of course, a surprising result: the radical choice set structure (i.e. captivity or complete freedom of choice) of this model is unlikely to be generally applicable to the entire population of workers.

This lack of significant improvement over the fit of the logit specification and the significant improvement

Table 4-4 - Maceió Home-Based Work Tour Mode Choice Model - Logit Captivity Specification - Full Data Set

Choice Model Parameters	Estimated Parameters (Asymptotic t-statistics in parentheses)
1. Alternative Specific Constants	
- Bus	-0-
- Taxi	-1.128 (-2.0)
- Auto Passenger	-3.020 (-6.3)
- Auto Drive	-0.069 (-0.2)
2. Total Travel Time (min/day)	-0.0136 (-3.4)
3. Total Travel Cost (Cr$ 1977/day) divided by ℓn (Household Income, KwH/month)	-0.3561 (-4.7)
4. Household Income, KwH/month	
- Bus	-0-
- Taxi	0.0055 (2.7)
- Auto Passenger	0.0067 (4.2)
- Auto Drive	0.0063 (3.9)
5. Number of Household Members	
- Bus	-0-
- Taxi	-0.1116 (-1.9)
- Auto Passenger	-0.1649 (-2.7)
- Auto Drive	-0.2172 (-3.6)
6. Auto Competition (#cars/#workers)	
- Bus	-0-
- Taxi	1.8700 (3.1)
- Auto Passenger	3.6323 (8.1)
- Auto Drive	3.4434 (7.6)
7. CBD work location (zones 4 & 5) <u>and</u> lunch trip home	
- Bus	-0-
- Taxi	0.759 (2.2)
- Auto Passenger and Drive	-0.138 (-0.5)
8. Female Worker	
- Bus, Taxi, Auto Passenger	-0-
- Auto Drive	-1.7494 (-6.6)

Table 4-4 - Continued

	Estimated Parameters (Asymptotic t-statistics in parentheses)	
9. Professional worker <u>and</u> lunch trip home		
- Bus	-0-	
- Taxi	0.956	(2.4)
- Auto Passenger and Drive	1.106	(3.8)
Captivity Parameters		
1. Bus	1.3451 E-2	(1.2)
2. Taxi	0.3663 E-2	(0.9)
3. Auto Passenger	0.7893 E-2	(0.8)
4. Auto Drive	-0-	
Summary Statistics		
log likelihood at equal probability	-1779.0	
log likelihood at convergence	-719.0	
rho-squared	0.5958	
adjusted rho-squared	0.5896	
# parameters	22	
Sample Description		
Choosing - Bus	931	
- Taxi	55	
- Auto Passenger	120	
- Auto Drive	371	
Total Observations	1477	

obtained by the income segmentation (see Table 4-2) as compared to the pooled logit specification of Table 4-1, led us to hypothesize that we could perhaps uncover evidence of captivity by calibrating logit captivity models by income group.

These results are presented in Table 4-5. The income segmentation indeed brings to light significant captivity to the bus mode in the low income group, and to the bus and auto passenger modes in the medium income group. There is indicated a small degree of captivity to auto passenger in the high income group, but this is not statistically significant because of the large variance of the respective captivity parameter. It is clear that the income segment captivity models are a statistically significant improvement over the pooled logit captivity of Table 4-4.

The income segment logit captivity models are also jointly a statistically significant improvement over the income segment logit models of Table 4-2. We test the hypothesis that the captivity parameters are all jointly zero by using a chi-squared statistic of 24.8, which can be compared to a critical value of 23.2 at the 99% significance level with a conservative 10 degrees of freedom. Therefore we reject this hypothesis; the data indicates that besides the taste variation that are captured by the income segmentation in Table 4-2, there is in addition a variation

Table 4-5 - Maceió Home-Based Work Tour Mode Choice Model - Logit Captivity Specification - Income Segmentation

Choice Model Parameters	Estimated Parameters (Asymptotic t-statistics in parentheses)		
	LOW	MEDIUM	HIGH
1. Alternative Specific Constants			
- Bus	-0-	-0-	-0-
- Taxi	2.136	0.368	1.347
	(0.6)	(0.1)	(1.0)
- Auto Passenger	-2.160	-5.723	-2.320
	(-0.8)	(-1.3)	(-2.0)
- Auto Drive	2.789	2.151	0.165
	(0.7)	(0.6)	(0.2)
2. Total Travel Time (min/day)	-0.0121	-0.0412	-0.0130
	(-0.4)	(-2.4)	(-1.8)
3. Total Travel Cost (Cr$ 1977/day) divided by ℓn (Household income, KwH/month)	-0.5429	-0.8256	-0.4512
	(-1.4)	(-2.8)	(-2.8)
4. Household Income, KwH/month			
- Bus	-0-	-0-	-0-
- Taxi	0.1117	-0.0136	0.0010
	(0.9)	(-0.4)	(0.3)
- Auto Passenger	0.0343	0.0294	0.0023
	(1.0)	(0.7)	(0.8)
- Auto Drive	0.0230	0.0409	0.0011
	(0.6)	(1.3)	(0.4)
5. Number of Household Members			
- Bus	-0-	-0-	-0-
- Taxi	-3.1245	0.2429	-0.3648
	(-1.2)	(1.7)	(-2.8)
- Auto Passenger	-0.7120	-0.3614	-0.2515
	(-1.6)	(-1.4)	(-2.0)
- Auto Drive	-0.5786	-1.0581	-0.2203
	(-1.1)	(-2.9)	(-2.2)

Table 4-5 - Continued

Choice Model Parameters	Estimated Parameters (Asympotic t-statistics in parentheses) LOW	MEDIUM	HIGH
6. Auto Competition (#cars/#workers)			
- Bus	-0-	-0-	-0-
- Taxi	11.6667	2.4800	2.1121
	(1.7)	(1.7)	(1.5)
- Auto Passenger	10.7902	4.6122	5.1644
	(2.1)	(3.1)	(5.0)
- Auto Drive	10.4267	2.8327	5.5720
	(2.2)	(2.0)	(5.3)
7. CBD work location (zones 4 & 5) and lunch trip home			
- Bus	-0-	-0-	-0-
- Taxi	-0.198	1.775	1.6648
	(-0.1)	(1.8)	(2.5)
- Auto Passenger and Drive	-0.3420	-0.397	0.1862
	(-0.2)	(-0.5)	(0.4)
8. Female Worker			
- Bus, Taxi, Auto Passenger	-0-	-0-	-0-
- Auto Drive	-4.351	3.180	-1.936
	(-3.0)	(-3.1)	(-4.3)
9. Professional worker and lunch trip home			
- Bus	-0-	-0-	-0-
- Taxi	NI	0.234	1.826
		(0.2)	(2.9)
- Auto Passenger and Drive	NI	1.247	1.856
		(1.5)	(2.9)

Table 4-5 - Continued

	Estimated Parameters (Asympotic t-statistics in parentheses)		
	LOW	MEDIUM	HIGH
Captivity Parameters			
1. Bus	16.7199E-2	9.8778E-2	1.1472E-2
	(2.1)	(2.0)	(1.1)
2. Taxi	1.5882E-2	0.9947E-2	0.6714E-2
	(2.4)	(1.3)	(1.1)
3. Auto Passenger	0.6859E-2	5.8262E-2	4.3642E-2
	(1.0)	(2.7)	(1.4)
4. Auto Drive	-0-	-0-	-0-
Summary Statistics			
log likelihood at equal probability	-596.5	-532.8	-649.8
log likelihood at convergence	-124.9	-234.5	-313.5
rho-squared	0.7907	0.5599	0.5175
adjusted rho-squared	0.7739	0.5393	0.5006
# parameters	20	22	22
Sample Description			
Choosing - Bus	467	301	163
- Taxi	12	18	25
- Auto Passenger	14	39	67
- Auto Drive	35	88	248
Total Observations	528	446	503

in the choice set structure of individuals that must be accounted for in the choice model specification.

We also note that the logit captivity models provide statistically better fit across all three income segments when compared to their standard logit counterparts of Table 4-2. It is even more interesting to note some of the significant changes that have occurred in certain individual parameters.

Consider, for example, the coefficients of the travel time and cost variables. Those in the logit captivity model are uniformly larger than the corresponding parameters in Table 4-2. Conceptually, the removal of captives from consideration in the calibration of the choice model removes their diluting effect upon i's parameters; only the true choosers affect the choice model parameters. In fact, all of the travel impedance and socio-economic parameters grow in magnitude, some of them quite drastically (e.g. auto competition in the low income group).

These changes have occurred in the presence of what is not a large captivity effect. After all, the most significant degree of captivity is that to bus in the low income segment, for which there is an estimated probability of 0.14 of captivity to that mode.

We can also note, in Table 4-5, the captivity parameters decrease in magnitude with increasing income for all modes (except for auto drive, which has no captivity parameters), doing so most sharply for the bus alternative. This suggests that the captivity effect is income-related, and that the explanatory power of these choice set formation models could be greatly enhanced by specifying captivity functions for each alternative, rather than single parameters. We shall explore this approach in Chapter 5.

4.3.5 The Independent Availability Logit Models

Once more, we shall begin with a pooled model that is directly comparable to the restricted logit model of Table 4-1 and the logit captivity specification of Table 4-4. Table 4-6 displays this model. Contrary to the logit captivity model, the independent availability model provides a significantly better fit to the observed choices than the standard logit model: we reject the hypothesis that the availability parameters are all jointly one (indicating deterministic availability of all alternatives allocated by the analyst to the individuals) at the 95% level. This improvement is explained by the independent availability model's improved representation of the choice set structure, as opposed to the extreme assumption underlying the logit captivity model.

Again, as in the case of the logit captivity models, we note a general increase in the magnitude of the choice

Table 4-6 - Maceió Home-Based Work Tour Mode Choice Model - Independent Availability Logit - Full Data Set

Choice Model Parameters	Estimated Parameters (Asymptotic t-statistics in parentheses)
1. Alternative Specific Constants	
- Bus	-0-
- Taxi	-0.439 (-0.3)
- Auto Passenger	-2.905 (-6.6)
- Auto Drive	0.823 (1.1)
2. Total Travel Time (min/day)	-0.0151 (-3.5)
3. Total Travel Cost (Cr$ 1977/day) divided by ℓn (Household Income, KwH/month)	-0.3248 (-5.2)
4. Household Income, KwH/month	
- Bus	-0-
- Taxi	0.0051 (2.3)
- Auto Passenger	0.0070 (4.3)
- Auto Drive	0.0067 (3.5)
5. Number of Household Members	
- Bus	-0-
- Taxi	-0.1103 (-1.8)
- Auto Passenger	-0.1504 (-2.9)
- Auto Drive	-0.3115 (-3.3)
6. Auto Competition (#cars/#workers)	
- Bus	-0-
- Taxi	2.0623 (3.4)
- Auto Passenger	3.5875 (8.2)
- Auto Drive	5.1082 (5.5)
7. CBD work location (zones 4 & 5) and lunch trip home	
- Bus	-0-
- Taxi	0.735 (2.0)
- Auto Passenger and Drive	-0.349 (-1.2)
8. Female Worker	
- Bus, Taxi, Auto Passenger	-0-
- Auto Drive	-3.108 (-5.1)

Table 4-6 - Continued

	Estimated Parameters (Asymptotic t-statistics in parentheses)
9. Professional worker <u>and</u> lunch trip home	
- Bus	-0-
- Taxi	0.872 (2.1)
- Auto Passenger and Drive	1.101 (3.8)
<u>Availability Parameters</u>	
1. Bus	1.0
2. Taxi	0.50386 (1.0)
3. Auto Passenger	0.87517 (7.9)
4. Auto Drive	0.83105(28.6)
<u>Summary Statistics</u>	
log likelihood at equal probability	-1779.0
log likelihood at convergence	-715.0
rho-squared	0.5981
adjusted rho-squared	0.5919
# parameters	22
<u>Sample Description</u>	
Choosing - Bus	931
- Taxi	55
- Auto Passenger	120
- Auto Drive	371
Total Observations	1477

model parameters. In the model of Table 4-6, this increase is attributable not only to inclusion of captivity but to the consideration of all the trade-off situations that each specific decision-maker can face. For example, if an individual has available bus (B), taxi (T), and auto passenger (AP), not only is there a probability that his choice of bus is from the set (B,T,AP), but there is now a probability that the choice is from (B,T) and (B,AP).

The improvement in fit provided by the independent availability logit model, albeit statistically significant, is certainly not dramatic. Once again, this has led us to segment the sample of workers along the income dimension, and calibrate separate models for each. These results are given in Table 4-7. The hypothesis that all the availability parameters are equal across income segments can be rejected at the 95% significance level, so it has been a definite improvement to segment the sample.

Now let us compare the independent availability logit income segment models with the logit captivity models of Table 4-5. Although it is possible to perform a formal statistical hypothesis test (remember, however, that the logit captivity specification is not nested within the independent availability model), it will be sufficient to compare the two specifications for each segment utilizing the Akaike Information Criterion (see expression 3 in Appendix 3), AIC. Together, the three logit captivity models

Table 4-7 - Maceió Home-Based Work Tour Mode Choice Model - Independent Availability Logit - Income Segmentation

Choice Model Parameters	Estimated Parameters (Asymptotic t-statistics in parentheses)		
	LOW	MEDIUM	HIGH
1. Altenative Specific Constants			
- Bus	-0-	-0-	-0-
- Taxi	1.349	-1.276	1.754
	(0.6)	(-0.5)	(1.1)
- Auto Passenger	-1.797	-3.410	-2.281
	(-0.7)	(-1.9)	(-1.9)
- Auto Drive	1.745	1.202	0.457
	(0.8)	(0.5)	(0.4)
2. Total Travel Time (min/day)	0.0046	-0.0170	-0.0154
	(0.4)	(-2.5)	(-2.1)
3. Total Travel Cost (Cr$ 1977/day) divided by ℓn (Household income, KwH/month)	-0.2381	-0.3363	-0.4503
	(-1.2)	(-3.8)	(-2.9)
4. Household Income, KwH/month			
- Bus	-0-	-0-	-0-
- Taxi	0.0224	-0.0095	0.0011
	(0.6)	(-0.4)	(0.4)
- Auto Passenger	0.0367	0.0165	0.0030
	(1.3)	(0.9)	(1.0)
- Auto Drive	0.0178	0.0399	0.0006
	(0.9)	(1.5)	(0.2)
5. Number of Household Members			
- Bus	-0-	-0-	-0-
- Taxi	-1.3512	0.1773	-0.3387
	(-1.6)	(1.6)	(-2.4)
- Auto Passenger	-0.5662	-0.1683	-0.1742
	(-2.0)	(-1.7)	(-1.8)
- Auto Drive	-0.3588	-0.8299	-0.2298
	(-1.4)	(2.9)	(-1.9)

Table 4-7 - Continued

Choice Model Parameters	Estimated Parameters (Asymptotic t-statistics in parentheses)		
	LOW	MEDIUM	HIGH
6. Auto Competition (#cars/#workers)			
- Bus	-0-	-0-	-0-
- Taxi	4.6992	1.3107	2.1947
	(2.6)	(1.3)	(2.2)
- Auto Passenger	4.5498	2.3939	5.4002
	(2.6)	(5.6)	(5.6)
- Auto Drive	2.2039	2.0340	7.2517
	(2.0)	(2.4)	(5.0)
7. CBD work location (zones 4 & 5) and lunch trip home			
- Bus	-0-	-0-	-0-
- Taxi	0.019	0.346	1.528
	(0.0)	(0.6)	(2.3)
- Auto Passenger and Drive	-0.596	-0.702	-0.121
	(-0.6)	(-1.6)	(-0.2)
8. Female Worker			
- Bus, Taxi, Auto Passenger	-0-	-0-	-0-
- Auto Drive	-2.103	-2.914	-2.729
	(-1.9)	(-3.3)	(-4.1)
9. Professional worker and lunch trip home			
- Bus	-0-	-0-	-0-
- Taxi	-0.681	0.423	1.817
	(-0.4)	(0.6)	(2.7)
- Auto Passenger and Drive	1.273	0.709	1.841
	(1.3)	(1.6)	(3.5)

Table 4-7 - Continued

	Estimated Parameters (Asymptotic t-statistics in parentheses)		
	LOW	MEDIUM	HIGH
Availability Parameters			
1. Bus	0.98487 (129.8)	1.0	1.0
2. Taxi	1.0	1.0	0.68756 (2.4)
3. Auto Passenger	0.3562 (1.4)	1.0	0.81269 (7.3)
4. Auto Drive	1.0	0.86821 (12.1)	0.86881 (24.5)
Summary Statistics			
log likelihhod at equal probability	-596.5	-532.8	-649.8
log likelihood at convergence	-126.3	-236.3	-311.7
rho-squared	0.7882	0.5565	0.5203
adjusted rho-squared	0.7707	0.5377	0.5034
# parameters	21	20	22
Sample Description			
Choosing - Bus	467	301	163
- Taxi	12	18	25
- Auto Passenger	14	39	67
- Auto Drive	35	88	248
Total Observations	538	446	503

have an AIC of 736.9, compared to 737.3 for the independent availability logit, so that the former specification seems, in the aggregate, to be somewhat superior to the latter (as a basis for comparison, the income logit models have an AIC of 742.3). By comparing the individual pairs of AIC values by income group, we note that the source of the captivity models' improved performance comes from the low income segment; in the high income group, the independent availability assumption seems better than the captivity choice set structure.

This result highlights an important practical conclusion. It makes clear that the restrictions imposed upon the choice set structure cannot be arbitrary; rather, they must reflect the population in question and the source of the constraints upon them. Hence, in the present context, it would seem reasonable to adopt the logit captivity model for the low income group, and the independent availability specification for the most unconstrained group, the high income segment of the workers. The choice of model for the medium income segment is somewhat arbitrary since the AIC's are so similar.

In Table 4-8 we show the predicted choice set probabilities according to the logit captivity and independent availability logit models (both for the full data set and the three income segments). The table has two

Table 4-8 - Predicted Choice Set Probabilities

(a) Available Alternatives - B, T, AP, AD[3]

	Logit Captivity				Independent Availability Logit			
Choice Set	FULL	LOW	MEDIUM	HIGH	FULL	LOW	MEDIUM	HIGH
B	0.013	0.141	0.085	0.011	0.010	0	0	0.008
T	0.004	0.013	0.009	0.006	0	0	0	0
AP	0.008	0.006	0.050	0.041	0	0	0	0
AD	0	0	0	0	0	0	0	0
B,T	0	0	0	0	0.011	0	0	0.017
B,AP	0	0	0	0	0.073	0	0	0.033
B,AD	0	0	0	0	0.051	0	0	0.051
T,AP	0	0	0	0	0	0	0	0
T,AD	0	0	0	0	0	0.01	0	0
AP,AD	0	0	0	0	0	0	0	0
B,T,AP	0	0	0	0	0.075	0	0.132	0.073
B,T,AD	0	0	0	0	0.052	0.634	0	0.112
B,AP,AD	0	0	0	0	0.361	0	0	0.221
T,AP,AD	0	0	0	0	0	0.005	0	0
B,T,AP,AD	0.976	0.840	0.857	0.942	0.366	0.351	0.868	0.485

[3] B=bus, T=taxi, AP=auto passenger, AD=auto drive

Table 4-8 - Continued

(b) Available Alternatives - B, T, AP

	Logit Captivity				Independent Availability Logit			
Choice Set	FULL	LOW	MEDIUM	HIGH	FULL	LOW	MEDIUM	HIGH
B	0.013	0.141	0.085	0.011	0.062	0	0	0.059
T	0.004	0.013	0.009	0.006	0	0.010	0	0
AP	0.008	0.006	0.050	0.041	0	0	0	0
B,T	0	0	0	0	0.063	0.634	0	0.129
B,AP	0	0	0	0	0.434	0	0	0.254
T,AP	0	0	0	0	0	0.005	0	0
B,T,AP	0.976	0.840	0.857	0.942	0.441	0.351	1.000	0.559

parts, (a) and (b), the first corresponding to a worker with all four modal alternatives in his or her deterministic choice set, and the second to an individual without the auto drive alternative.

While many useful inferences can be drawn from the table, an interesting one comes from part (a) for the independent availability model and the high income segment. This group is naturally the one that displays the higher rate of auto ownership, and hence is the one in which workers will most often have auto drive allocated to them by our choice set construction rules (see Section 4.3.1). Yet for these individuals there is estimated a less than 50% chance that they will actually be selecting from the <u>full</u> choice set that includes auto driver. Medium income workers who have auto drive available, on the other hand, have an estimated probability of 87% to be choosing from the full choice set of four alternatives. A third observation can be made concerning low income workers who have bus, taxi, and auto passenger available (see the independent availability logit results for this group in Table 4-8(b)). The choice set construction rules we adopted allowed auto passenger to all workers; there is only a 35% chance, however, that a low income worker with this three alternative deterministic set actually chooses from it. It is nearly twice as likely that he will choose between bus and taxi instead.

4.4 Model Prediction Results

As we stated at the beginning of this Chapter, it is necessary that we go beyond measures of model fit to evaluate the gains provided by the choice set formation models of Section 4.3. There we showed that the captivity and independent availability choice set formation models, combined with market segmentation of the worker population, result in choice models statistically superior to the standard logit models forming the datum for comparison. This result holds in spite of apparent weaknesses in the choice set formation models (i.e. the strong assumption of independence of alternative availability, or the extreme scenario of captivity or full choice).

On the other hand, the additional difficulty of calibrating a choice model with choice set formation stage can be significant when compared to estimating the parameters of a standard discrete choice model, such as the logit specification. The loss of certain convenient properties of the log likelihood function (see Appendix 3) of the sample creates serious obstacles for the analyst, and jeopardizes the practical usefulness of choice set models, in general.

However, if the additional benefits gained by calibrating the choice set formation models exceed the added practical difficulties (extra time, cost, effort), it seems worthwhile to undertake the necessary effort. Accordingly, we have chosen to define the additional benefits provided by a specification as being its predicted aggregate response to certain measures of interest to policy-makers. More specifically, if aggregate measures of behavioral response differ significantly between the simpler standard of comparison and the model in question, the extra effort seems justified for the policy-maker will have more accurate information upon which to base a decision.

The approach we take is to perform the comparison between models using three different sets of measures. We compare, in each case, the income segment logit models (Table 4-2) with the logit captivity models of Table 4-5 for the low and medium income groups and the independent availability logit specification of Table 4-7 for the high income group. We utilize the two sets of models to predict ridership changes due to

(1) uniform changes (two levels, low and high) across the population for travel time and cost;

(2) implementation of a specific policy alternative;

(3) and shifts in the distributions of a socio-

economic variable, specifically income.

It should be noted that the choice set models we have chosen for these comparisons comprise the specifications that are statistically best, in terms of model fit, for each income group.

4.4.1 Uniform Changes in Travel Time and Cost

Two levels of change in the time and cost variables are implemented herein, 10% (low) and 100% (high). The reason for this two-level test is that the benefits of choice set modelling may be nonlinear, and only appear under conditions of extreme change in these variables.

Tables 4-9 and 4-10 show the predicted ridership changes under a 10% travel time increase, for the income segment logit and the income segment choice set specifications, respectively. In those tables, the results are presented both by income segment and over the entire sample of workers.

If we study the low income predictions in the two tables (4-9a and 4-10a), we note that the changes in ridership from the logit captivity formulation are less than or equal to the corresponding prediction from the standard logit model for the group; this is to be expected, of course, given that the predicted degree of captivity to bus is highest in this segment of the workers, for which this

Table 4- 9 - Predicted Impact (% Change in Ridership) of 10% Travel Time Increase- Income Segment Logit Specifications

(a) LOW INCOME (<80 KwH/month)

	Predicted Response in Mode			
	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-0.3	3.3	4.0	1.4
Taxi	0.1	-3.3	0	0.3
Auto	0.1	0.8	-1.3	-0.6

(b) MEDIUM INCOME (80-130 KwH/month)

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-1.8	6.7	6.6	1.9
Taxi	0.3	-6.7	0.3	0.1
Auto	0.5	0.6	-1.8	-0.9

(c) HIGH INCOME (>130 KwH/month)

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-2.3	3.2	1.9	0.6
Taxi	0.4	-6.0	0.4	0.2
Auto	0.6	1.2	0.7	-0.8

(d) OVERALL

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-1.1	4.2	3.7	1.0
Taxi	0.2	-5.6	0.3	0.2
Auto	0.3	0.9	-0.3	-0.8

Table 4-10 - Predicted Impact (% Change in Ridership) of Uniform 10% Travel Time Increase - Income Segment Choice set Specifications

(a) LOW INCOME (<80 KwH/month)

	Predicted Response in Mode			
	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-0.2	1.7	3.4	0.8
Taxi	0	-1.7	0	0.3
Auto	0.1	0.8	-1.4	0
(b) MEDIUM INCOME (80-130 KwH/month)				
Change Bus	-2.6	12.8	7.3	2.8
Taxi	0.6	-12.3	0.3	0.2
Auto	0.6	0.6	-1.5	-1.3
(c) HIGH INCOME (>130 KwH/month)				
Change Bus	-2.3	4.4	2.7	0.4
Taxi	0.6	-6.4	0.6	0.1
Auto	0.6	1.2	-0.3	-0.4
(d) OVERALL				
Change Bus	-1.3	6.6	4.3	1.0
Taxi	0.3	-7.3	0.4	0.2
Auto	0.3	0.9	-0.8	-0.6

mode is also the most frequently chosen. The estimation sample of 528 workers in the low income segment has an observed frequency of choice of bus of 88%, so it is understandable that the dampening effect mentioned is present. The predicted ridership changes in Tables 4-9 and 4-10 are average changes: we have not provided error bounds for these measures because the differences are small between models (with few exceptions), and it is our a priori opinion that none of the differences are statistically significant at any reasonable level of significance. Even in the case of the taxi mode, for which there is a 100% difference between the predictions of the two models, it is unlikely that they are statistically different since this mode is the least well explained by any of the models we have presented.

For the medium income segment of the Maceio workers (Tables 4-9b, 4-10b) we have the opposite result, as compared to the low income group: namely, the choice set specification in general states that the medium income workers are more sensitive to the 10% travel time increase than predicted by the standard logit specification. This segment, like the low income group, has a high incidence of choice of bus (67%), but the choice set specification predicts a smaller degree of captivity in this group as compared to the low income segment. At the same time, we see that the travel time coefficients in the income segment logit and logit captivity specifications differ by a magnitude of almost 3 times. However, it again seems that the predicted differences are not significant.

The high income segment is also predicted by its independent availability model to be more sensitive to the 10% travel time increase than predicted by the standard logit specification (see Tables 4-9c, 4-10c).

In aggregate, Tables 4-9d and 4-10d show a tendency of the standard logit specification to underpredict the effect of the travel time increases on the worker population as compared to the choice set model system. The source of this disparity between the models are the medium and high income groups, which the choice set models predict to be more sensitive to the change than stated by the logit formulation. Because of the aggregation, the overall changes in ridership predicted to occur by each set of models are even more uniform than if viewed by income segment, as we have just done.

The predicted impacts due to the low (10%) level cost increase are shown in Tables 4-11 and 4-12. An examination of these two tables leads us to reach the same conclusions of differences by income segment and overall, so no new information is provided by these data.

We next consider the differences in model predictions under a high (100%) uniform perturbation in time and cost. First, Tables 4-13 and 4-14 present the model predictions

Table 4-11 - Predicted Impact (% Change in Ridership) of Uniform 10% Travel Cost Increase - Income Segment Logit Specifications

Predicted Response in Mode

(a) LOW INCOME (<80 KwH/month)

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-0.1	1.7	2.0	0.6
Taxi	0.3	-15.8	0.7	0.9
Auto	0.2	0.8	-4.0	-1.1

(b) MEDIUM INCOME (80-130 KwH/month)

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-0.5	2.2	1.8	0.6
Taxi	0.8	-17.3	0.8	0.3
Auto	0.8	0.6	-3.3	-1.3

(c) HIGH INCOME (>130 KwH/month)

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-0.8	1.2	0.6	0.2
Taxi	1.4	-19.1	1.3	0.6
Auto	1.2	2.0	0.7	-1.2

(d) OVERALL

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-0.4	1.6	1.2	0.3
Taxi	0.7	-17.8	1.1	0.5
Auto	0.6	1.3	-1.2	-1.2

Table 4-12 - Predicted Impact (% Change in Ridership) of Uniform 10% Travel Cost Increase - Income Segment Choice Set Specifications

(a) LOW INCOME (<80 KwH/month)

	Predicted Response in Mode			
	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-0.1	0.8	2.0	0.6
Taxi	0.2	-8.3	0.7	0.6
Auto	0.2	0.8	-4.8	-0.6

(b) MEDIUM INCOME (80-130 KwH/month)

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-0.7	3.9	1.5	0.7
Taxi	1.3	-25.1	0.5	0.4
Auto	0.8	0.6	-2.5	-1.6

(c) HIGH INCOME (>130 KwH/month)

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-0.8	2.0	0.9	0.1
Taxi	1.9	-19.2	1.6	0.3
Auto	1.0	1.6	-1.0	-0.5

(d) OVERALL

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-0.4	2.4	1.2	0.3
Taxi	0.8	-18.8	1.2	0.3
Auto	0.5	1.1	-2.0	-0.8

due to a large change in travel time for the income segment logit and choice set specifications, respectively. As in the case of the low level changes shown previously, we note that while there are certain outstanding differences between the predictions (mainly in the taxi mode, once again), there is no obvious apparent trend in the values shown, at least at first sight.

There are a few individual differences, however, worthy of note. For example, in Table 4-13b we note that the income segment logit specification predicts that an uniform doubling of the auto travel time results in a more than 11% decrease in ridership for both the auto passenger and drive modes. The corresponding prediction in Table 4-14b, however, shows a 20% loss of ridership in the auto drive mode and about a 5% increase in ridership for the passenger mode. Thus, the standard logit specification for the medium income group suggests that a 100% increase in auto travel time causes a shift away from the mode entirely; the captivity specification, however, suggests that there will instead be a shift within the auto mode via the mechanism of increased ride-sharing. The predictions of the latter model violate our intuition, and we are unable to explain why they occur.

Careful study of 4-13 and 4-14 does bring to light one interesting pattern of differences between the two sets of models. Note that in the aggregate results of each table (4-13d and 4-14d) that the income segment logit specification

Table 4-13 - Predicted Impact (% Change in Ridership) of Uniform 100% Travel Time Increase - Income Segment Logit Specification

(a) LOW INCOME (<80 KwH/month)

		Predicted Response in Mode			
		Bus	Taxi	Auto Pass.	Auto Drive
Change	Bus	-3.7	40.8	54.7	12.3
	Taxi	0.6	-30.0	1.3	1.7
	Auto	0.7	3.3	-11.3	-5.7

(b) MEDIUM INCOME (80-130 KwH/month)

		Bus	Taxi	Auto Pass.	Auto Drive
Change	Bus	-20.7	84.9	88.2	14.2
	Taxi	2.4	-49.7	2.0	0
	Auto	4.6	4.5	-12.0	-11.4

(c) HIGH INCOME (>130 KwH/month)

		Bus	Taxi	Auto Pass.	Auto Drive
Change	Bus	-21.8	34.3	22.3	4.9
	Taxi	3.4	-45.4	3.3	1.4
	Auto	6.4	14.3	14.3	-9.6

(d) OVERALL

		Bus	Taxi	Auto Pass.	Auto Drive
Change	Bus	-12.4	52.4	47.6	7.8
	Taxi	1.7	-43.5	2.6	1.3
	Auto	3.0	8.9	2.6	-9.7

Table 4-14 - Predicted Impact (% Change in Ridership) of Uniform 100% Travel Time Increase - Income Segment Choice Set Specification

(a) LOW INCOME (<80 KwH/month)

		Predicted Response in Mode			
		Bus	Taxi	Auto Pass.	Auto Drive
Change	Bus	-2.5	13.3	54.4	6.4
	Taxi	0.2	-12.5	1.4	0.8
	Auto	0.5	2.5	-10.9	-2.5

(b) MEDIUM INCOME (80-130 KwH/month)

		Bus	Taxi	Auto Pass.	Auto Drive
Change	Bus	-36.5	219.0	144.8	14.5
	Taxi	3.3	-63.1	1.3	1.1
	Auto	5.1	5.6	4.8	-20.2

(c) HIGH INCOME (>130 KwH/month)

		Bus	Taxi	Auto Pass.	Auto Drive
Change	Bus	-24.1	49.2	28.6	3.2
	Taxi	4.8	-48.4	4.0	0.7
	Auto	5.6	9.6	2.7	-5.3

(d) OVERALL

		Bus	Taxi	Auto Pass.	Auto Drive
Change	Bus	-17.3	96.7	69.7	6.2
	Taxi	2.0	-45.4	2.8	0.8
	Auto	2.9	6.7	1.7	-8.6

generally predicts a <u>smaller</u> response to a doubling of bus travel time than predicted by the choice set specifications; conversely, a 100% increase in auto travel time is said to result in <u>greater</u> changes than predicted by the choice set models. Further study of the two tables indicates that the source of these aggregate level differences between the two model systems stem from identical patterns in the individual income groups of workers, though both the effects mentioned above are not necessarily present in each worker segment. What we observe here is perhaps the result of a two-fold effect:

(1) the choice set specifications predict a greater response to a change in bus travel time because of an increased sensitivity these models display to travel impedance as compared to the standard logit specifications (compare the travel time coefficients of Table 4-2 to those in Tables 4-5 and 4-7, noting that the former are uniformly less in absolute value than the latter); and

(2) the choice set models predict a smaller impact of changing auto travel time because of their fuller consideration of alternative availability (<u>i.e.</u> an individual's captivity

to the auto passenger mode makes him or her insensitive to changes in the mode's travel time).

Tables 4-15 and 4-16 correspond to the predictions due to a doubling of the travel cost. For both sets of models we first note that Maceio workers are predicted to be less sensitive to changes in travel cost than time; this difference in sensitivity to the two measures is less for the choice set specifications than for the standard logit models. The reason for this relative insensitivity may be that costs in the base case are low, so that changing them is not as onerous as changing travel times, which are originally at a high level.

The same pattern of smaller response to a doubling of bus travel time and greater response to a doubling of auto travel time predicted by the standard logit specifications as compared to the choice set models is apparent for travel cost. The reasons for this pattern are hypothesized to be the same given above for the travel time measure, i.e. greater sensitivity to cost in the choice set models and improved representation of the choice sets themselves, which has different effects on the various modes.

With this last comparison we conclude this section. The results presented thus far are not supportive of any strong superiority of the choice set specifications over the

Table 4-15 - Predicted Impact (% Change in Ridership) of Uniform 100% Travel Cost Increase - Income Segment Logit Specification

(a) LOW INCOME (<80 KwH/month)

	Predicted Response in Mode			
	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-1.5	18.3	19.3	5.4
Taxi	1.6	-77.5	3.3	4.0
Auto	1.7	8.3	-23.3	-14.9

(b) MEDIUM INCOME (80-130 KwH/month)

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-5.2	22.3	18.9	4.8
Taxi	4.1	-82.1	3.3	1.5
Auto	6.5	5.6	-17.6	-16.2

(c) HIGH INCOME (>130 KwH/month)

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-7.8	13.5	6.9	1.9
Taxi	6.7	-85.7	6.4	2.5
Auto	10.9	22.3	24.5	-16.2

(d) OVERALL

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-3.8	17.5	12.3	2.9
Taxi	3.3	-82.9	5.0	2.4
Auto	4.9	13.8	4.9	-15.9

Table 4-16 - Predicted Impact (% Change in Ridership) of Uniform 100% Travel Cost Increase - Income Segment Choice Set Specifications

(a) LOW INCOME (<80 KwH/month)

	Predicted Response in Mode			
	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-1.5	10.0	29.9	3.6
Taxi	0.7	-35.8	4.1	2.0
Auto	1.3	7.5	-18.4	-11.8

(b) MEDIUM INCOME (80-130 KwH/month)

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-7.3	49.2	19.7	6.1
Taxi	3.9	-76.0	1.5	1.2
Auto	6.2	5.0	8.6	-25.9

(c) HIGH INCOME (>130 KwH/month)

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-7.9	17.6	8.4	1.2
Taxi	8.7	-85.6	7.0	1.1
Auto	9.2	14.4	5.2	-8.9

(d) OVERALL

	Bus	Taxi	Auto Pass.	Auto Drive
Change Bus	-4.5	26.2	14.7	2.6
Taxi	3.1	-71.6	4.9	1.2
Auto	4.3	9.8	4.2	-13.2

standard logit formulation for the choice dimension in examination. Certain significant differences between the predictions of the two model systems have been pointed out, but none of them seem to alone be worth the extra effort necessary to calibrate the choice set models. On the other hand, neither is the uniform change scenario reflected in the previous predictions necessarily a realistic one for application of these models. This has led us to test for differences in predictions when the two model systems are applied in the context of evaluation of a specific policy scenario.

4.4.2 Evaluation of a Specific Policy

The policy scenario to be used in this section is inspired by an actual policy evaluation previously reported by Geltner and Swait(1981) for Maceio. The specific policy considered envisions extensive traffic engineering improvements in the CBD (Central Business District) of the city, including the implementation of "bus only" streets and improved loading and unloading spaces and procedures, and prohibition of parking of private automobiles in certain areas of the CBD. The impact of such changes is assumed to affect trips to and through the CBD in the following manner:

(1) bus trips - decrease of 10 minutes per leg of the trip, due to improved flow of

traffic;

(2) auto trips - increase of 5 minutes per leg, due to increased walking distances in the CBD; and

(3) taxi trips - no effect.

Table 4-17 shows the predicted average impacts of implementing this policy. In part (a) of the table we show the predictions of the logit models, and in (b) those of the choice set models. The income segment logit specification <u>understates</u> the impact of the policy on the bus and taxi modes for the medium and high income groups, and conversely <u>overstates</u> the impacts on the low income group, when compared to the corresponding choice set model predictions. This result can be explained by the sensitivity of the medium and high income groups to travel time in the choice set models being greater than the alternative availability effect; the opposite holds in the low income group. For the private modes there is no such clear-cut pattern. In either case, however, it is unclear that any of the observed differences in predictions between model systems is actually statistically significant.

This result is not unexpected given the homogeneity of predictions presented in Section 4.4.1. It had been hoped that by targeting a specific group of the workers' population, namely those working in the CBD or travelling

Table 4-17 - Predicted Impact (% Change in Ridership with Respect to Base Case) of CBD Improvement Policy Alternatives

(a) INCOME SEGMENT LOGIT SPECIFICATION

Income Group		Predicted Response in Mode			
		Bus	Taxi	Auto Pass.	Auto Drive
	LOW	0.8	-5.0	-10.0	-4.3
	MEDIUM	5.4	-15.6	-16.9	-8.0
	HIGH	8.3	-4.4	-6.9	-3.2
	OVERALL	3.6	-8.4	-10.5	-4.4

(b) INCOME SEGMENT CHOICE SET SPECIFICATIONS

Income Group		Bus	Taxi	Auto Pass.	Auto Drive
	LOW	0.5	-1.7	-12.0	0.0
	MEDIUM	7.2	-30.2	-12.3	-12.8
	HIGH	8.5	-10.0	-8.7	-2.3
	OVERALL	4.0	-14.7	-10.3	-4.6

through it to reach their workplace, significant differences between the model systems could be detected. Though at the time Table 4-17 was constructed this thought had not occurred to us, it is possible that differences may indeed exist if we examine the impacts only on CBD workers or those travelling through that part of the city.

4.4.3 Shifts in a Socioeconomic Characteristic

The previous two sections have evaluated differences in predictions between the two model systems under consideration in contexts that could best be labelled as short-range. In both cases, while certain trends are apparent, it remains unclear if one of the model systems is undoubtedly superior to the other. The purpose of this section is to evaluate the differences when the scenario simulated corresponds to long-range shifts in the composition of the worker population in Maceio. Specifically, we shall simulate two different shifts in the income distribution.

Table 4-18 displays the observed worker household income distribution, and the two postulated shifts in the distribution, labelled as Scenarios A and B. Both hypotheses represent a significant worsening of the income distribution as compared to the observed case, with Scenario B being the most extreme of the two. We simulate the shift in the income

Table 4-18 - Observed and Postulated Income Distributions for Maceió Workers

Income Category (KwH/month)	Observed Distribution	Scenario A	Scenario B
0-40	15.1%	16.9%	20.3%
41-60	9.0	16.9	20.3
61-80	12.4	13.5	16.9
81-100	12.5	13.5	16.9
101-120	13.5	10.2	10.2
121-150	12.5	10.2	10.2
151-200	10.2	9.3	3.2
201-250	5.6	3.4	0.7
251-300	2.7	2.7	0.7
> 300	6.5	3.4	0.7
TOTAL	100.0	100.0	100.0

distribution by assigning a weight to each observation corresponding to the ratio of the postulated to the observed frequency for its income group (e.g. 16.9/15.1 for the lowest income category in Scenario A). Note that the actual income value of an observation is not changed, merely the weight given to the predictions for the observation. This methodology assumes that all other characteristics remain constant within the sample (e.g. there are no accompanying shifts in the auto ownership distribution).

Tables 4-19 and 4-20 present the predictions for Scenarios A and B, respectively, for each of the model systems. Comparison of parts (a) and (b) of each table shows little or no difference in the predictions of the standard logit versus choice set specifications.

Though not presented here, another simulation of a shift in the auto ownership distribution has been carried out with similar results across the two model systems.

4.5 Conclusions

Section 4.3 presented an extensive modelling exercise aimed at evaluating the statistical validity of modelling probabilistic choice set formation. The calibrated models presented therein attest to the need for modelling choice set formation, particularly in environments such as Maceio, where the travelling public is subject to significant

Table 4-19 - Predicted Impact (% Change of Ridership with Respect to Base Case) of Income Distribution Shift - Scenario A

(a) INCOME SEGMENT LOGIT SPECIFICATION

Income Group		Predicted Response in Mode			
		Bus	Taxi	Auto Pass.	Auto Drive
	LOW	30.8	29.2	25.3	24.9
	MEDIUM	-9.0	-8.4	-11.2	-10.6
	HIGH	-18.3	-20.7	-26.5	-26.2
	OVERALL	9.3	-6.0	-15.1	-17.7

(b) INCOME SEGMENT CHOICE SET SPECIFICATIONS

Income Group		Bus	Taxi	Auto Pass.	Auto Drive
	LOW	30.7	26.7	23.3	28.6
	MEDIUM	-9.6	-8.4	-10.0	-9.3
	HIGH	-18.3	-21.9	-27.1	-25.9
	OVERALL	9.1	-6.9	-15.3	-16.8

Table 4-20 - Predicted Impact (% Change of Ridership with Respect to Base Case) of Income Distribution Shift - Scenario B

(a) INCOME SEGMENT LOGIT SPECIFICATION

		Predicted Response in Mode			
		Bus	Taxi	Auto Pass.	Auto Drive
Income Group	LOW	58.7	57.5	52.0	52.3
	MEDIUM	3.3	4.5	-1.0	0.2
	HIGH	-50.8	-55.3	-68.0	-71.0
	OVERALL	21.6	-11.2	-31.5	-42.4

(b) INCOME SEGMENT CHOICE SET SPECIFICATION

		Bus	Taxi	Auto Pass.	Auto Drive
Income Group	LOW	58.5	54.2	49.3	56.6
	MEDIUM	2.7	5.0	0.5	1.7
	HIGH	-50.6	-80.2	-68.9	-70.7
	OVERALL	21.3	-12.4	-31.8	-41.4

constraints of many types that cannot be observed. The choice set formation stage should be of even greater importance in the more discretionary types of behavior, such as shopping destination/mode choice and trip generation.

Market segmentation, while an indispensable technique to improve the explanatory power of the choice models for a population with taste variation, is too crude a tool to, alone, substitute for explicit models of choice set formation. Allied to the latter, however, market segmentation is of great value.

Another result of the empirical work in Maceio is the confirmation of the important effect of the assumption of choice set structure on the explanatory power of the full choice model. A strategy that combines market segmentation and appropriate choice set restrictions seems most likely to work well, as in the Maceio case, where the logit captivity model seems best for the low income group, while the independent availability logit specification seems superior for the high income group.

This very factor may indeed be the reason for the inability of the choice set specifications to present clearly different predictions from the standard logit specifications, under the various scenarios considered in Sections 4.4.1 to 4.4.3. Despite the statistical superiority of the choice set models, when compared to the standard logit models, it is felt that the homogeneity of the predictions across the two model systems is due in part to limitations of the choice set structure representation inherent in the captivity and independent availability models. Perhaps the assumptions made by each of these choice set models, while somewhat better than the deterministic choice set representation of traditional discrete choice models, are nonetheless inappropriate (even simplistic) for the populations in question.

A drawback of the choice set formation models is the greatly increased difficulty of calibrating them. The departure from the standard logit linear-in-parameters formulation can be a costly one since we lose the convenient property of concavity of the log likelihood function, which guarantees the uniqueness of the parameters at the point of convergence. Hence, a greater degree of care and sophistication on the part of the analyst is necessary, not to mention specialized estimation software.

One aspect of the general formulation (3.7) has been left untouched at this point: namely, there is yet need to explore the formulation and calibration of the individual constraint equations. In the work we have reported upon here, we have simply aggregated the effect of all relevant constraints into a small number of parameters. Future research should explore the issue of explicit constraint specification.

However, while this approach is more general and flexible than the simpler specifications we have calibrated in Maceio, and hence liable to result in improvements in explanatory power, it is not necessarily the case that the much greater extra effort needed to calibrate such models is in practice worthwhile. In Maceio, use of market segmentation and the standard logit specification (and even the simpler choice set formation models) is shown to be an effective set of tools.

CHAPTER 5

A SECOND APPLICATION OF CHOICE SET FORMATION MODELLING - SÃO PAULO, BRAZIL

5.1 Introduction

São Paulo contains the core and principal part of Brazil's industrial park, and this alone distinguishes it significantly from Maceio, our first application area (see Chapter 4). In addition, its physical size, population and history of development further distance it from the smaller Northeastern city.

This chapter will report upon several models of work mode choice that include the choice set formation stage. As in the case of Maceio, these models are compared to a standard discrete choice model with deterministic choice sets. The modelling exercise for São Paulo is not nearly as extensive as that reported for Maceio in Chapter 4; nonetheless, the results for São Paulo are significant because (1) they represent a second application environment and (2) São Paulo is very different from Maceio in terms of economic, social, and cultural criteria.

5.2 The City of São Paulo

As with any large metropolis of today, the process of urban development of São Paulo began from a single city center, which over the years dominated the economic and cultural life of the area. The very size of the city, however, has reversed this monopolizing effect of the traditional city center. Today we find in the city definite evidence of decentralization of city life, especially in the revitalization of the traditional centers of the smaller towns engulfed by the growth of the metropolis.

This decentralization of activity poses one of the most challenging transport problems that the city planners of São Paulo must face. The existing transport system is still heavily oriented towards providing radial service from the outskirts to the old center of São Paulo, but a 1977 origin-destination survey (EMPLASA, 1978a) shows that a very large number of the trips are actually circumferential in nature. The trip-makers are often forced to travel to the old downtown, then back out to the periphery to their final destination.

Remedying this deficiency of the transport system is one of the main objectives of current planning strategies for the city (EMPLASA, 1983). These plans include expansion to the existing metro, renewal and modernization of the suburban train network, and implementation of a priority bus system (with extensive use to be made of trolley buses, thus taking advantage of one of Brazil's great natural resources, hydroelectric power). Besides these construction options, physical and fiscal integration of public transport modes are short- and medium-range policies high on the planners' agenda.

São Paulo is one of the fastest growing cities of the world. From a little over half a million population in 1920, it grew to 1.3 million in 1940, and to approximately 11 million in 1977 (Abril, 1980; EMPLASA, 1978a). This growth is due not only to high birth rates but also to great numbers of migrants from other regions of Brazil, principally the Northeast. In addition, since the late nineteenth century significant numbers of Italian and Japanese immigrants have contributed to the high population growth. In addition, the growth process of São Paulo has resulted in a diverse population in cultural, social, ethnic, and economic terms.

As described in EMPLASA(1978b), the data we utilize in the following sections were collected during the 1977 home interview origin-destination survey. The sampling frame of buildings spanned the geographical area of Greater São Paulo (26 municipalities are included). The sampling of units was random within strata defined by architectural standards (size, type of finish, state of maintenance). All households in a selected building would be interviewed to determine socio-economic attributes and transportation-related behavior on the previous day. A total of 31,380 households were thus sampled, which provided a coverage of 108,000 persons who made a total of 182,000 trips (EMPLASA, 1978b).

Whereas the 1977 O/D survey data relates trip-making behavior to a detailed 633 traffic zone system, the impedance measures we have available are from a more coarse 125 zone network that was utilized in a posterior aggregate policy analysis (CET, 1979). Impedance measures were made available from separate walk, auto, bus, and rail networks. The latter is a combined network for the physical metro and suburban rail modes; since the two are non-overlapping, they are represented in one network.

5.3 Model Calibration Results

5.3.1 The Choice Context

The choice we shall examine in our empirical application is again mode choice for work trips. More specifically, we shall use home-based, morning peak period (arrival time before 10:30 AM) trips. Of the 17,088 such one way trips contained in the 1977 São Paulo Origin/Destination Survey (EMPLASA, 1978a), 1,746 were randomly selected for inclusion in the estimation data set.

The O/D survey provides a rich source of information on modal choice, including access, egress, and principal modes. Unfortunately, the currently available impedance measures (which represent the shortest path over all access, egress, and main mode combinations) have forced us to perform an aggregation of the elemental choices into the following alternatives:

(1) Bus,

(2) Auto Drive,

(3) Auto Passenger,

(4) Train,

(5) Metro, and

(6) Walk.

The first three modes are the same as in Maceio. Note that in contrast to the smaller city, taxis account for a negligible fraction of home-based work trips (less than 2%, EMPLASA, 1978a) in São Paulo, and so have been omitted from the analysis. No two zones are served by the two rail modes since the physical networks are non-overlapping. For our study, these two modes have been combined into a single rail alternative. Finally, the availability of walk trips for São Paulo adds a whole new dimension to the applicability of our modelling results. In fact, the unavailability of walk trips in Maceio is one of the deficiencies noted by Swait <u>et</u>

al.(1984) concerning the data usually collected in transport studies in developing countries.

As with Maceio, the deterministic allocation of alternatives to individual trip makers followed certain rules:

(1) the network connection for the observed and generated modes had to exist;

(2) the maximum allowed one-way travel time was 3 hours for bus, auto, and walk, and 4 hours for the rail mode (clearly the latter restriction applies, in practice, to the suburban train mode, not the metro alternative).

Contrary to Maceio, we are unable to restrict the auto drive alternative to individuals from auto-owning households since a significant number of observed drivers declared an automobile ownership level of zero.

The application of the above rules eliminated 20 observed trips, so that the final estimation data set consists of 1,726 trips. For informational purposes only, the distribution of deterministic choice sets in the estimation data set is given below.

Choice Set	Observed Trips
Bus, Auto Drive, Auto Pass.	67
Bus, Auto Drive, Auto Pass., Rail	231
Bus, Auto Drive, Auto Pass., Walk	942
Bus, Auto Drive, Auto Pass., Rail, Walk	486
	1726

The actual estimation data set, however, has one less observation. Following initial calibration results, an outlier analysis was performed, and an observation with a miscoded income value was detected and removed from the data. Therefore, the calibration results are for a random sample of 1,725 workers.

5.3.2 Model Estimation Results

The São Paulo modelling effort is more limited in scope than the detailed work executed for Maceio; in fact, we approach this second application with the aim of further delving into the separation of the effects of taste variation upon choice from those of choice set generation.

The rich specification of socio-economic characteristics in the Maceio standard logit choice models, allied to income group segmentation, result in few differences of note when they are compared to the choice set formation models used in Chapter 4. The latter, though statistically somewhat superior to the standard logit models, predict behavioral responses little different from the former. The choice set formation models calibrated in Chapter 4, however, represent the impacts of constraints in a particularly simple manner: the aggregate impact for each alternative is a single parameter, invariant across the population. We found significant improvements in fit when we separated the Maceio workers into income groups, thus permitting the choice set availability or captivity parameters to vary, albeit in a limited manner, with the workers' characteristics. At the same time, parameters in the utility functions of the choice model were also permitted to vary, so it is not possible to distinguish the improvement in model fit in terms of taste variation versus choice set generation.

As a first step in exploring this distinction, consider the two models shown in Table 5-1. The first of these is a linear-in-parameters logit specification with no socioeconomic variables (other than through an interaction term in the cost variable, in which income is introduced). This formulation stands in marked contrast to the Maceio specifications, where liberal use of socio-economic variables was made in an ad hoc model of alternative availability (see Section 4.3.2, expression (4.1)). To compare with this logit choice given choice set model we present in the same table a logit captivity model in which the aggregate captivity effect for each alternative is represented by one parameter.

Comparing the two models of Table 5-1 with a formal statistical test, we conclude that the captivity formulation is highly significant with respect to the standard logit model. As in the case of the Maceio choice set

Table 5-1 - São Paulo Home-Based AM Peak Work Mode Choice - Simple Logit and Logit Captivity Specifications

Choice Model Parameters	Logit		Logit Captivity	
	Estimated Parameters (Asymptotic t-statistics in parentheses)			
1. Alternative Specific Constants				
- Walk	-0-		-0-	
- Bus	-1.487	(-8.4)	-2.751	(-4.9)
- Auto Drive	-1.960	(-12.8)	-3.475	(-6.9)
- Auto Passenger	-3.550	(-20.0)	-5.647	(-9.5)
- Rail	-3.142	(-13.0)	-5.953	(-7.4)
2. - Walk Total Travel Time, minutes, one-way	-0.0674	(-10.7)	-0.1214	(-6.7)
- Walk (Total Travel Time)^2	2.2222E-4	(4.8)	3.8182E-4	(2.4)
3. Motorized Modes In-Vehicle Travel Time, minutes, one-way				
- Bus	-0.0158	(-3.8)	-0.0428	(-4.6)
- Auto	0.0146	(1.4)	0.0902	(3.2)
- Rail	-0.0023	(-0.8)	-0.0067	(-1.4)
4. (Cost, Cr$1977)/(Income, Cr$1977/month)				
Income = { Personal income if boarder or visitor; Household income, otherwise }				
- Walk	-0-		-0-	
- Bus	-241.98	(-1.7)	-386.45	(-1.3)
- Auto	-629.02	(-16.1)	-3098.98	(-6.5)
- Rail	-362.32	(-2.1)	-808.44	(-2.0)
Captivity Parameters				
- Walk	-0-		0.152565	(3.8)
- Bus	-0-		0.148355	(5.5)
- Auto Drive	-0-		0.051704	(4.7)
- Auto Passenger	-0-		0.088347	(2.5)
- Rail	-0-		0.054214	(2.5)
Summary Statistics				
log likelihood for equal probability	-2480.5		-2480.5	
log likelihood at convergence	-1888.3		-1825.0	
rho-squared	0.2388		0.2643	
adjusted rho-squared	0.2363		0.2608	
# parameters	12		17	

Table 5-1 - Continued

Sample Description

Choosing	- Walk	409
	- Bus	626
	- Auto Drive	466
	- Auto Passenger	95
	- Rail	129
		1725

specifications, there is a general increase in the scale of the choice utility parameters, and since the captivity effect here is significant, the shift is quite large in some cases. The captivity model predicts, for example, that an individual who has available all five alternatives has only a 67% chance of choosing from his or her full choice set.

These simplistic specifications clearly do not perform well in terms of overall fit. In addition, the anomalous positive travel time coefficient in the logit model for the auto modes is further exacerbated in the captivity specification. In the logit model, the positive coefficient occurs because the model must explain the use of the auto for trips longer than the average; the limited specification is forced to show a positive utility for increased impedance, which is counterintuitive. This condition is worsened in the captivity specification: by removing the captives to auto drive and passenger, the counterintuitive time coefficient is forced to become more positive and significant.

In keeping with these observations , we now include socio-economic characteristics in these specifications to obtain the calibration results of Table 5-2. (The models of Table 5-1 are nested within those of 5-2.) First, note that either of the specifications of Table 5-2 is by far superior

Table 5-2 - São Paulo Home-Based AM Peak Work Mode Choice - Logit and Logit Captivity Specifications With Socioeconomic Characteristics

Choice Model Parameters	Logit		Logit Captivity	
	Estimated Parameters (Asymptotic t-statistics in parentheses)			
1. Alternative Specific Constants				
- Walk	-0-		-0-	
- Bus	-0.701	(-2.7)	-0.731	(-2.7)
- Auto Drive	-4.530	(-10.7)	-5.357	(-5.3)
- Auto Passenger	-4.154	(-10.4)	-4.491	(-8.2)
- Rail	-2.038	(-4.7)	-2.184	(-4.8)
2. - Walk Total Travel Time, minutes, one-way	-0.0638	(-9.3)	-0.0679	(-8.3)
- Walk (Total Travel Time)^2	1.9961E-4	(4.1)	1.9862E-4	(3.1)
3. Motorized Modes In-Vehicle Travel Time, minutes, one-way				
- Bus	-0.0122	(-2.6)	-0.0148	(-2.9)
- Auto	-0.0322	(-2.5)	-0.0381	(-2.8)
- Rail	-0.0005	(-0.2)	-0.0012	(-0.4)
4. (Cost, Cr$1977)/(Income, Cr$1977/month)				
Income = { Personal income if boarder or visitor; Household income, otherwise				
- Walk	-0-		-0-	
- Bus	-655.29	(-4.2)	-638.48	(-3.8)
- Auto	-76.75	(-1.6)	-64.59	(-1.1)
- Rail	-713.02	(-3.9)	-710.51	(-3.8)
5. Income (see above for definition)				
- Walk	-0-		-0-	
- Bus	-2.11E-6	(-0.2)	4.06E-6	(0.3)
- Auto Drive	23.01E-6	(2.2)	33.28E-6	(2.6)
- Auto Passenger	9.59E-6	(0.7)	14.29E-6	(0.8)
- Rail	-9.73E-6	(-0.5)	-2.49E-6	(-0.1)

Table 5-2 - Continued

6. Number of Household Members				
Members = { 0 if boarder; # persons otherwise				
- Walk	-0-		-0-	
- Bus	-0.0870	(-2.4)	-0.0977	(-2.5)
- Auto Drive	-0.1338	(-2.8)	-0.1354	(-2.5)
- Auto Passenger	-0.0973	(-1.7)	-0.1366	(-1.8)
- Rail	-0.1459	(-2.7)	-0.1586	(-2.8)
7. Competition for Auto				
Competition = { 0 if boarder; # cars/# workers otherwise				
- Walk	-0-		-0-	
- Bus	-0.9289	(-3.5)	-0.8548	(-3.0)
- Auto Drive	1.2071	(6.1)	1.3597	(6.1)
- Auto Passenger	-0.3078	(-0.8)	-0.6906	(-1.1)
- Rail	-0.3651	(-0.9)	-0.2771	(-0.7)
8. Auto Drive Mode - dummy variables				
a) 1 if auto-owning household	2.660	(6.8)	3.225	(3.4)
b) 1 if auto-owning household <u>and</u> head of household	0.647	(3.2)	0.700	(3.3)
c) 1 if female worker	-1.207	(-4.9)	-1.271	(-4.7)
9. Auto Passenger Mode - dummy variable				
1 if auto-owning household	1.857	(5.4)	2.405	(4.1)
10. Auto Drive/Passenger Modes - dummy variable				
1 if high school or university education	0.377	(2.3)	0.396	(2.2)
<u>Captivity Parameters</u>				
- Walk	-0-		-0-	
- Bus	-0-		0.006383	(1.1)
- Auto Drive	-0-		0.010515	(1.5)
- Auto Passenger	-0-		-0-	
- Rail	-0-		0.015387	(1.1)

Table 5-2 - Continued

Summary Statistics

log likelihood for equal probability	-2480.5	-2480.5
log likelihood for convergence	-1437.1	-1435.3
rho-squared	0.4206	0.4214
adjusted rho-squared	0.4148	0.4149
# parameters	29	32

Sample Description

Choosing - Walk	409
- Bus	626
- Auto Drive	466
- Auto Passenger	95
- Rail	129
	1,725

to the simple models without socio-economic characteristics we saw before. Second, the inclusion of the socio-economic variables has corrected the anomalous travel time coefficient. Third, and most worthy of note, the logit and logit captivity specifications of Table 5-2 are not statistically different from one another. The captivity parameters are nearly zero and have a large variance.

A comparison of the logit model of Table 5-2, with the additive _ad hoc_ model of alternative availability, and the logit captivity specification in Table 5-1 indicate that the _ad hoc_ model is superior to the simple choice set generation without the socio-economic attributes in the choice utility function. Indeed, the inclusion of the additive socio-economic variables in the logit captivity model (see Table 5-2) eliminates the captivity effect detected in the corresponding simple model of Table 5-1.

These results, which repeat the observations we made for Maceio in Chapter 4, raise the possibility that a choice set generation model with a richer alternative availability specification than a single parameter, but a choice utility without the _ad hoc_ socio-economic attributes (such as in Table 5-1), may fit the observed choices at least as well as the logit model with _ad hoc_ availability variables. We report in Table 5-3 a different choice set generation model than those we've calibrated thus far: in it, the impact of constraints on an alternative will be given by a _function_ rather than a single parameter. The choice set structure of the model will be that of captivity, so that we write the full probability of choice as

$$\Pr(j|B,D,X_n,\tilde{G}_n) = \frac{1}{1+\sum_{i \varepsilon M_n} \exp(DX_{in})}[\exp(DX_{jn})+\Pr(j|M_n,B,X_n)], \tag{5.1}$$

where the notation we employ is identical to that defined in Chapter 3. The model (5.1) can be directly compared to expression (3.25), which states the equivalent choice probability when captivity to an alternative is represented by a single parameter. The model of choice we will adopt continues to be the logit formulation, hence the name parametrized logit captivity model for (5.1).

Let us now turn our attention to the actual calibration results for the parametrized logit captivity model, as shown in Table 5-3. Note that we have adopted a very simple specification for the logit choice given choice set model, identical to that of Table 5-1. We have omitted all socio-economic effects from the choice given choice set model and included them in the choice set formation specification. To compare the models of Table 5-2 to that of 5-3 we must employ the Akaike Information Criterion (AIC, see expression 3 of Appendix 3) since the models are non-nested:

Table 5-3 - São Paulo Home-Based AM Peak Work Mode Choice - Parametrized Logit Captivity Specification

	Estimated Parameters (Asymptotic t-statistics in parentheses)	
Choice Model Parameters		
1. Alternative Specific Constants		
- Walk	-0-	
- Bus	-2.450	(-2.40)
- Auto Drive	$-\infty$	
- Auto Passenger	-6.128	(-5.1)
- Rail	-5.196	(-4.6)
2. - Walk Total Travel Time, minutes, one-way	-0.1118	(-4.1)
- Walk (Total Travel Time)^2	3.6846E-4	(2.5)
3. Motorized Modes In-Vehicle Travel Time, minutes, one-way		
- Bus	-0.0404	(-4.3)
- Auto Passenger	-0.0783	(-2.0)
- Rail	-0.0081	(-1.7)
4. (Cost, CR\$1977)/(Income, Cr\$1977/month) Income = { Personal income if boarder; Household income otherwise		
- Walk	-0-	
- Bus	-497.63	(-1.8)
- Auto Passenger	-132.28	(-1.0)
- Rail	-1006.26	(-2.6)
Captivity Function Parameters		
1. Walk		
- Constant	-4.628	(-4.6)
- Income (see above)	-24.79E-6	(-0.8)
- Members = { 0 if boarder or visitor; # persons otherwise	0.2004	(1.9)
- Competition = { 0 if boarder; # cars/# persons otherwise	2.8487	(6.1)

Table 5-3 - Continued

2. Bus		
- Constant	-1.470	(-2.8)
- Income	-4.80E-6	(-0.1)
- Members	-0.0612	(-0.7)
- Competition	-0.3338	(-0.4)
3. Auto Drive		
- Constant	-4.422	(-7.0)
- Income	23.68E-6	(2.3)
- Members	-0.0441	(-0.9)
- Competition	2.3449	(6.5)
- 1 if auto-owning household	2.713	(4.2)
- 1 if head of household	0.853	(1.2)
- 1 if head of household and auto-owning household	-0.252	(-0.4)
- 1 if female worker	-1.137	(-4.6)
- 1 if high school or university education	0.401	(2.2)
4. Auto Passenger		
- Constant	-5.467	(-4.5)
- Income	17.62E-6	(1.2)
- Members	0.0182	(0.2)
- Competition	0.7813	(1.6)
- 1 if auto-owning household	3.235	(3.0)
- 1 if high school or university education	0.004	(0)
5. Rail		
- Constant	-1.049	(-1.4)
- Income	-3.27E-6	(-0.1)
- Members	-0.5104	(-1.9)
- Competition	1.1336	(1.3)

Summary Statistics

log likelihood at equal probability	-2480.5
log likelihood at convergence	-1441.6
rho-squared	0.4188
adjusted rho-squared	0.4111
# parameters	38

Sample Description

Choosing - Walk	409
- Bus	626
- Auto Drive	466
- Auto Passenger	95
- Rail	129
	1,725

model	AIC
logit	1466.1
logit captivity	1467.3
parametrized logit captivity	1479.6

Clearly, in terms of overall model fit, the logit specification with socio-economic variables for taste variation is superior to the parametrized logit captivity model, as is the logit captivity formulation.

However, let us consider the above result more carefully in terms of its implications: on the one hand we have a model which accounts in a make-shift manner for alternative availability (at best a simplistic representation of constraints and choice set generation), and on the other a model with a much richer specification of the constraints on choice set formation. The result is that the former model, albeit statistically superior to the latter, is not that much better. In fact, the removal of a number of insignificant parameters from the captivity functions of Table 5-3 would already improve matters quite a bit with respect to the AIC measure without even exploring the option of adding further important variables to those functions.

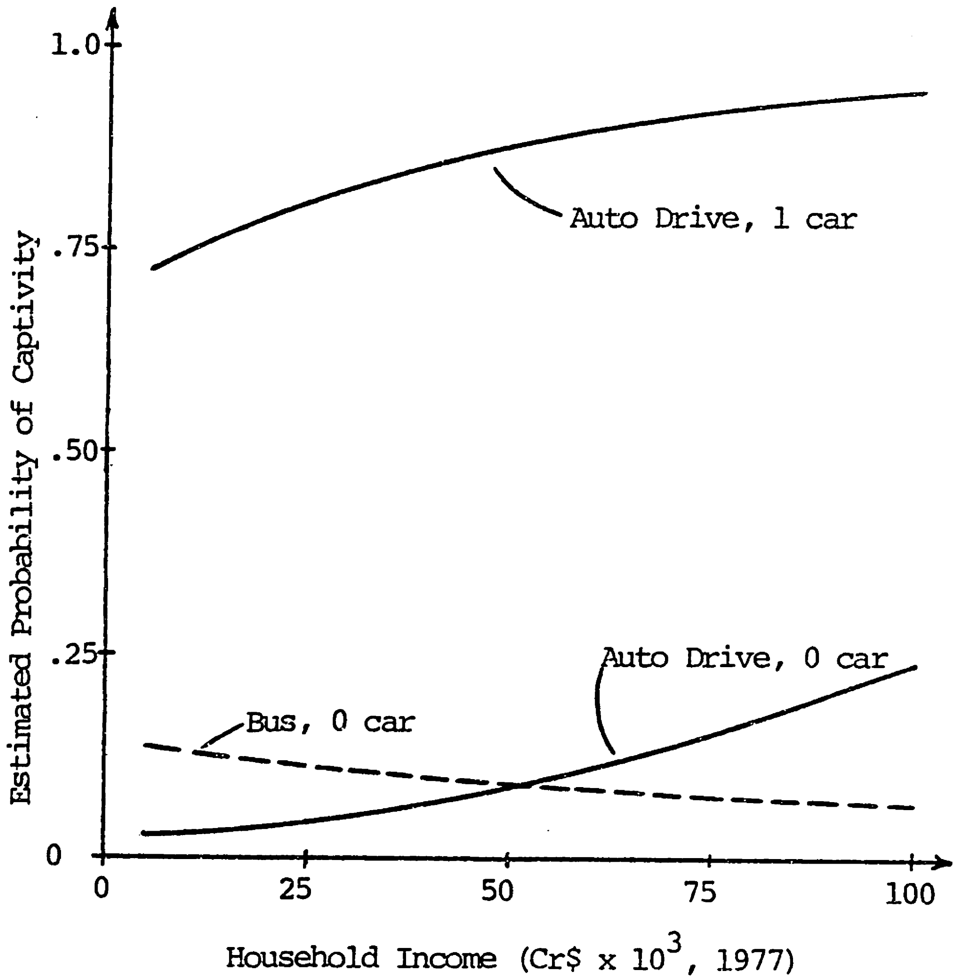
It is straightforward to imagine variables to add to the captivity functions that would doubtless improve the explanatory power of the parametrized logit captivity model.

For example, addition of gender and agre-related dummy variables to the walk captivity function may improve model fit; travel distance is another possible variable for the walk function. The auto drive captivity function could possibly benefit from the inclusion of one or more occupation category dummies that explain workers who need automobiles for their employment (e.g. travelling sales personnel), which would increase the likelihood of their being captive to the mode.

In addition, we must consider the fact that the choice set structure in (5.1) is quite extreme: captivity or full choice set. It is quite plausible that a less extreme choice set model that includes some intermediate choice sets, and is also parametrized in the manner of (5.1), could be superior to a standard choice model with a rich ad hoc specification of alternative availability. In our opinion, the calibration results in Table 5-3 argue strongly for the need to explore the trade-offs between ad hoc models of alternative availability and alternate parametrized choice set models.

There are several points of interest in the calibration results of Table 5-3. Firstly, note the alternative specific constant for the auto drive alternative: during the optimization process, this parameter was driven to a very large negative number and held there. This result states that individuals in the sample who chose auto drive are best replicated by the captivity portion of the full choice model rather than the choice given choice set portion. Selection of the auto drive mode is indicated to be independent of its own level-of-service, as well as those of the other modes. Secondly, note that the auto travel time coefficient is now negative and significant, in contrast to the models of Table 5-1, which have the same parsimonious specification (in terms of socio-economic attributes) as the parametrized logit captivity model of Table 5-3.

Now let us concentrate upon the captivity functions themselves. In general, the signs of the coefficients are as expected; in the few cases where counterintuitive results are encountered, the parameters are not statistically different from zero. To exemplify the rich range of inferences about the choice set formation stage that can be drawn from the parametrized logit captivity model, we provide Figure 5-1. There we show the estimated probabilities of captivity to auto drive and bus as a function of income and auto ownership level. The worker in question is the male head of a four-member household (and he's the only worker), and in addition has a high school education or better. The worker has available all five modal alternatives. If the household does not own an automobile, we see that the probability of captivity to the auto drive mode is relatively small, even for large income; on the



Household Characteristics: 4 members, 1 worker

Worker Characteristics: head of household, male, high school or university education, all alternatives available

Figure 5-1. Estimated Probabilities of Captivity by Income and Auto Ownership Level - Parametrized Logit Captivity Model other hand, if the household owns a vehicle, the probability is quite high (and grows with income) that the worker is captive to auto drive. This is consistent with the alternative specific constant of the auto drive alternative being driven to minus infinity in the logit choice given choice set model. This indicates the importance of the availability of the automobile to its actual use for the work trip. For the bus captivity, we see that it decreases with income for non-auto owning households, and up to about Cr$55,000 is larger than the probability of auto captivity. If the worker is from an auto-owning household, the probability of captivity to bus is very low, being less than 2% at all income levels.

5.4 Model Prediction Results

Here we shall compare the predictions of two of the models presented in Section 5.3: the logit model of Table 5-2 and the parametrized logit captivity model of Table 5-3. We shall compare the predictions for two levels of uniform change of time and cost across the worker population, and in addition we shall simulate the impact of shifts in the distributions of two socio-economic characteristics, income and auto ownership level.

5.4.1 Uniform Changes in Travel Time and Cost

Table 5-4 presents the predicted changes in ridership due to a 10% travel time increase for all workers. Part (a) of the table gives the results for the logit choice (L) model, and (b) for the parametrized logit captivity (LC) formulation. One difference we note immediately is to be expected: the LC model predicts the auto drive mode to be completely insensitive not only to its own travel time, but also to that of the other modes. The logit model, however, predicts the drive own- and cross-elasticities to be significantly higher. In contrast, the logit model understates the impacts of increasing bus and train/metro travel times when compared to the LC specification. The walk mode is deemed less sensitive by the LC than the logit model to this low level uniform change in travel time.

In Table 5-5 we have corresponding predictions for a doubling in travel time, again for both model specifications. The same patterns present in Table 5-4 are repeated here for the motorized modes. For the walk mode, both the LC and L models predict counterintuitive results (not shown here); these are due to the quadratic travel time specification for the walk mode (see Tables 5-2 and 5-3), which attributes positive utility to long travel times (>150 minutes). Within the estimation sample and under low levels

Table 5-4 - Predicted Impact (% Change in Ridership) of Uniform 10% Travel Time Increase

(a) Logit Choice Model

		Predicted Response in Mode				
		Bus	Auto Drive	Auto Pass.	Rail	Walk
	Bus	-2.5	0.9	2.3	4.3	1.0
Change	Auto	0.6	-1.6	1.2	0.9	0.3
	Rail	0	0	0	-0.3	0
	Walk	1.7	0.8	1.5	0.5	-4.1

(b) Parametrized Logit Captivity Model

	Bus	-4.6	0	5.6	12.8	1.9
Change	Auto	0	0	-0.3	0	0
	Rail	0.5	0	0.2	-2.9	0
	Walk	1.8	0	0.7	0.5	-3.3

Table 5-5 - Predicted Impact (% Change in Ridership) of Uniform 100% Travel Time Increase

(a) Logit Choice Model

		Predicted Response in Mode				
		Bus	Auto Drive	Auto Pass.	Rail	Walk
	Bus	-24.3	7.8	23.7	42.6	9.4
Change	Auto	5.6	-15.6	13.7	9.5	3.1
	Rail	0.4	0.1	0.2	-2.6	0

(b) Parametrized Logit Captivity Model

		Bus	Auto Drive	Auto Pass.	Rail	Walk
	Bus	-43.8	0	75.9	98.6	20.1
Change	Auto	0.2	0	-1.6	0.2	0
	Rail	4.6	0	2.0	-25.2	0.2

of change in travel time (such as in Table 5-4) the specification performs well, but with high levels of change (such as a doubling of everyone's walk travel time) the counterintuitive forecast we see results. This problem is a shortcoming of the polynomial specification for the variable. Due to time and monetary constraints, however, we are unable to repeat the calibration of the models to correct this variable.

If we return to the motorized modes in Table 5-5, we see that the differences between the LC and L models are very large for the auto modes. That is, doubling automobile travel time is forecast by the LC model to not affect the drive alternative and decrease by 1.6% the ridership of the passenger mode; the L formulation, however, shows large shifts occurring from the drive mode to auto passenger, bus train/metro, and even walk. It can certainly be argued that the LC model probably understates the true sensitivity of the population to such a large change in travel time, but it may well paint a more realistic picture of the impact of this variable on traveller behavior than is done by the standard logit model.

Tables 5-6 and 5-7 show the predictions by the two model systems for a 10% and a 100% increase in travel cost by each mode, respectively. In contrast to the travel time increases, both models predict similar results at both levels of cost change. The same patterns we noted above are

Table 5-6 - Predicted Impact (% Change in Ridership) of Uniform 10% Travel Cost Increase

(a) Logit Choice Model

		Predicted Response in Mode				
		Bus	Auto Drive	Auto Pass.	Rail	Walk
	Bus	-1.5	0.3	1.2	2.5	0.8
Change	Auto	0.1	-0.3	0.1	0.2	0
	Rail	0.5	0.1	0.2	-3.1	0.5

(b) Parametrized Logit Captivity Model

		Bus	Auto Drive	Auto Pass.	Rail	Walk
	Bus	-0.6	0	0.7	1.5	0.3
Change	Auto	0	0	-0.1	0	0
	Rail	0.5	0	0.2	-2.7	0

Table 5-7 - Predicted Impact (% Change in Ridership) of Uniform 100% Travel Cost Increase

(a) Logit Choice Model

		Predicted Response in Mode				
		Bus	Auto Drive	Auto Pass.	Rail	Walk
	Bus	-14.0	3.0	12.3	24.0	7.6
Change	Auto	1.0	-2.5	1.6	1.9	0.3
	Rail	4.0	0.9	2.1	-25.4	0.4

(b) Parametrized Logit Captivity Model

		Bus	Auto Drive	Auto Pass.	Rail	Walk
	Bus	-6.2	0	7.8	15.0	3.2
Change	Auto	0.1	0	-0.5	0.1	0
	Rail	4.1	0	1.8	-22.1	0.3

again present in these tables.

In summary, the LC model predicts a lower response level than the logit model to uniform changes in both travel time and cost (for small and large increases in these variables). The former model, however, is quite different from the logit for the travel time dimension; the two models produce much smaller discrepancies for the cost variable.

5.4.2 Shifts in Socioeconomic Characteristics

Now we shall simulate shifts in the income and auto ownership distributions for São Paulo workers. Table 5-8 presents the observed and postulated income distributions, with the latter representing a further concentration into the lower levels of income. The predicted changes in ridership under this worsened income distribution are given in Table 5-9 for both models. Note that the forecasts are given by three income classes as well as over the whole sample. The income class of the individual observation is not changed to make the predictions. Instead, the prediction under constant income is weighted differently than in the base case (where an observation is given a weight of 1.0).

As with the Maceio results (see Section 4.4.3), at the aggregate level the two models systems produce similar forecasts. This result is also valid in the high and medium income segments of the workers, though differences do appear

Table 5-8 - Observed and Postulated Income Distributions for São Paulo Workers

Income Category (#min. salaries)[1,2]	Observed Distributions	Postulated Distribution
0-1	0.6%	5.8%
1-3	15.1	23.2
3-5	19.9	20.3
5-7	16.1	17.4
7-9	10.1	11.6
9-11	8.4	8.7
11-13	4.8	2.9
13-15	6.4	2.9
15-17	2.6	2.9
17-19	2.4	2.9
19+	13.8	1.5
Total	100.0	100.0

[1] 1 min. salary ≈ Cr$1,000 in May, 1977.

[2] Upper limit is exclusive.

Table 5-9 - Predicted Impact (% Change in Ridership) of Income Distribution Shift

(a) Logit Choice Model[3]

Income Group		Predicted Response in Mode				
		Bus	Auto Drive	Auto Pass.	Rail	Walk
Income Group	LOW	48.1	30.0	58.4	69.2	43.9
	MEDIUM	8.0	5.0	7.1	5.9	22.8
	HIGH	-12.8	-44.0	-31.8	-21.4	-1.1
	OVERALL	10.0	-30.6	-10.7	16.2	14.9

(b) Parametrized Logit Captivity Model[3]

		Bus	Auto Drive	Auto Pass.	Rail	Walk
Income Group	LOW	53.2	21.8	47.9	59.1	43.2
	MEDIUM	8.8	4.0	5.2	6.0	22.5
	HIGH	-14.4	-44.0	-32.4	-22.9	2.1
	OVERALL	11.1	-31.4	-11.9	10.6	16.4

[3] LOW - [0,4) minimum salaries (1 min. sal. ≈Cr$1,000 1977)
MEDIUM - [4,8] minimum salaries
HIGH - (8,∞) minimum salaries

in the low income segment, notably in the auto modes and the rail mode. In general, model LC predicts a greater degree of sensitivity of modal split to the income shift than predicted by the logit model. The rail mode, in the low income bracket, displays the opposite behavior between the two modes.

Figure 5-1 suggests that auto ownership is a very important determinant of the use of the auto drive mode. Accordingly, we present in Table 5-10 a proposed shift in the auto ownership distribution (reflecting an 18% decrease in auto ownership) that is the basis for the forecasts of Table 5-11. Comparison of the predictions by each model leads us to the same observations we made above for the income shift simulation, though we note an even greater degree of similarity in the predictions than for the income case.

5.5 Conclusions

In general, the São Paulo work travel mode choice model calibration results tend to reinforce the conclusions we reached for Maceio (see Section 4.5). In practical terms, the present work reinforces the view that a well-specified (*i.e.* rich specification of *ad hoc* availability variables) standard logit choice model is robust with respect to the simpler forms of choice set models, such as the simple logit

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Table 5-10 - Observed and Postulated Auto Ownership Distributions for São Paulo Workers

Auto Ownership Level	Observed Distributions	Postulated Distributions
0	49.9%	58.0%
1+	51.1	42.0
Total	100.0	100.0

Table 5-11 - Predicted Impact (% Change in Ridership) of Auto Ownership Distribution Shift

(a) Logit Choice Model

		Predicted Response in Mode				
		Bus	Auto Drive	Auto Pass.	Rail	Walk
Income Group	LOW	14.2	-11.4	6.2	14.6	8.8
	MEDIUM	11.3	-14.4	-4.0	10.2	6.8
	HIGH	0.8	-15.8	-13.0	-4.1	1.1
	OVERALL	8.2	-15.3	-8.3	7.2	4.3

(b) Parametrized Logit Captivity Model

		Bus	Auto Drive	Auto Pass.	Rail	Walk
Income Group	LOW	14.1	-12.2	5.0	14.6	11.1
	MEDIUM	10.5	-14.3	-3.0	9.9	8.0
	HIGH	-0.6	-15.8	-12.9	-4.1	2.8
	OVERALL	7.3	-15.3	-7.8	6.7	6.1

captivity or independent availability logit, where the impact of constraints on an alternative is represented by a single parameter.

Our São Paulo work has, however, raised a number of interesting research questions that should be dealt with in the future. Namely, the statistical performance of the parametrized logit captivity model opens up the possibility that future research should concentrate upon the formulation of alternate functional forms for the constraint functions as well as explore less restrictive choice set structures than the captivity assumption. Comparison of predictions of the parametrized logit captivity and the standard logit choice models shows, in contrast to Maceio, some areas of significant difference. It would seem to us that future concentration of research effort into the parametrization of constraints and more flexible choice set structures could pay off in terms of an improved understanding of constrained travel behaviour.

CHAPTER 6

SUMMARY AND RECOMMENDATIONS FOR FUTURE RESEARCH

6.1 An Overview of the Research

This work begins by placing in theoretical and empirical perspective the process of definition of a discrete set of alternatives (called the choice set) prior to a decision-maker's exercise of his or her prerogative of choice.

In Chapter 2 we investigate the theoretical and practical impacts of omitting the specification of choice set formation from models of discrete choice; it is shown that biases in parameter estimates and forecasts result from such an omission. This observation is supported by numerical and empirical work by other researchers (e.g. Williams and Ortuzar, 1979; Stopher, 1980; Meyer, 1979; Louviere, 1979) as well as our own theoretical analysis of a simple binary choice model. At that stage we propose the hypothesis that choice set formation is the resultant of a series of forces (i.e. constraints) acting on the individual. In the context of travel behavior in developing countries, this proposition seems of great interest because of the highly constrained economic and physical environment in which people must act. Towards integrating this view of the choice set formation stage in models of discrete choice of travel behavior, we propose a constraint typology for individual urban travel that encompasses (1) household/family, (2) societal, and (3) personal constraints.

Besides discussing the types of constraints to which individuals are subject, we discuss several approaches to incorporating restrictions to behavior in models of choice. The approach we adopt is to allow for a hybrid of deterministic (i.e. those which the analyst is certain about) and probabilistic (i.e. those of which we are unable to be 100% positive about their effect, but which we can model in some fashion) constraints.

Within the hybrid framework of constraints proposed in Chapter 2, a number of models of choice and choice set formation are developed in Chapter 3. Due to the unobservability of the choice set in the real world, one of the greatest problems to present themselves in the modelling of choice set formation is the dimensionality of the choice set space: as the number of discrete alternatives grows, the size of the choice set space increases exponentially. This forces us to concentrate our model development efforts on a compromise solution: we formulate specific restrictions to the choice set space itself.

Based upon the straightforward concept of probabilistic availability of alternatives (thus incorporating the hybrid approach described above), we formulate a model of choice set formation that assumes no restrictions on the choice set space, but does assume that the constraints defining availability are independently distributed between discrete alternatives, though not within alternatives. Following that, we explore the more practical approach of restricting the choice set space: we propose a general captivity model, and a second specification in which the choice set size (i.e. number of elements) is modelled as a proxy for information-processing limitations that human beings suffer. This model development is by no means exhaustive; rather, its main purpose is to buttress the empirical applications that follow in Chapters 4 and 5.

Although standard statistical hypothesis tests are applicable to the specifications we develop, the actual task of obtaining parameter estimates for the choice and choice set formation models can be a difficult one. The estimator functions lose the convenient property of concavity, which guarantees the uniqueness of the optimum of the function. In addition, any of a number of numerical problems (e.g. bad scaling of the data, inappropriate optimization algorithm) can plague the estimation procedure (see Appendix 3 for a more detailed discussion).

To test the practicality of overcoming such difficulties and obtaining models of choice set formation, we carry out empirical testing against the standard logit model in two Brazilian cities, Maceio and São Paulo. The first city is a small state capital in poverty-stricken Northeastern Brazil, the second the sprawling, bustling metropolis containing the heart of the nation's industrial park. These two cities were chosen due to the availability of data, but they also represent economic, social and cultural extremes that lend to cur empirical results a wider applicability than might otherwise be the case.

In Maceio we model home-based mode choice to work. We calibrate standard logit choice models (both pooled and by socio-economic segments) that serve as the datum. The choice set models we calibrate are the logit captivity and independent availability specifications presented in Chapter 3. In both choice set models, we maintain the hypothesis that the choice model is also a logit model. The principal results of the Maceio exercise are that

(1) the standard logit choice model, when well-specified by using socio-economic variables in the utility functions and market segmentation to form, in an <u>ad hoc</u> fashion,

a model of alternative availability, performs well in statistical and predictive terms when compared to models of choice set formation where the effects of constraints are modelled by a single parameter per alternative;

(2) it is important that the choice set restrictions applied to derive a model of choice set formation be realistic for the population in question;

(3) with simple models of choice set formation, it becomes important to calibrate choice models by market segment;

(4) the calibration of choice set formation models is a costly and time-consuming affair compared to obtaining parameter estimates for a linear-in-parameters logit choice model.

All of these items point to the general conclusion that a well thought out standard choice model is a robust, cheaper alternative for modelling the obligatory type of travel behavior, such as work mode choice, that we analyzed in Maceio, if the competing specifications are choice set models in which the impacts of constraints are aggregated into a single parameter per alternative, or in which the choice set restrictions are of a simple nature, such as in the captivity model. This conclusion may be extensible to other dimensions of choice with the same characteristics, such as school travel; however, for discretionary travel (e.g. shopping destination/mode choice) this may not hold.

The São Paulo work, which also deals with home-based work mode choice, is aimed at exploring the robustness (in terms of forecasting) of standard logit models with ad hoc availability variables when compared to choice set generation models with complex availability specifications. In São Paulo we see that when compared to a logit model with no ad hoc availability variables, even a simple captivity model with the impact of constraints aggregated into single parameters is statistically more significant. When we compare these same models, but both with improved availability variables, we can detect no statistically significant difference between the model forms. This suggests to us that an improved description of the alternative availability process may result in better models of choice behavior.

To explore this possibility, we have calibrated for São Paulo a logit captivity specification in which the captivity is no longer embodied in a single parameter, but is given by a function of socio-economic attributes (i.e. income, auto ownership, level of education, gender, etc.). In addition, all the alternative availability variables are removed from the logit choice model itself. We find that this specification has almost as much explanatory power as the standard logit model with taste variation variables, and in addition gives some rather different forecasting results under certain. It is felt that further work on the captivity functions, or even exploration of alternate choice set assumptions allied to specification of availability functions (rather than single parameters), will yield models that significantly surpass the standard logit model in explanatory power.

The parametrized logit captivity model, as we have called the specification described above, opens a new avenue of research by shifting the emphasis of explaining behavior from the choice given choice set model to the choice set model itself.

6.2 Contributions of This Research

At the theoretical level, we have given a behavioral interpretation to choice set generation which is general and well-suited to incorporation of this stage of choice in transport demand models. The constraint-based approach, wherein the decision-maker's environment is modelled to show its effects on the availability of discrete alternatives, is of special interest in applications of discrete choice models in developing countries. The power of such a theoretical construct is that specific econometric models can be evaluated not only in terms of their statistical and predictive performance, but also in terms of their behavioral plausibility.

Previous studies in choice set generation modelling have been limited in scope. They have either proposed theoretical models of choice set formation but performed no empirical analysis of the validity of their specification (e.g. McFadden, 1976a; Ben-Akiva, 1977; Gaudry and Dagenais, 1979; Meyer, 1979; Richardson, 1982), or they have performed empirical work with little or no behavioral interpretation (e.g. Pitschke, 1980; O'Neill and Nelson, 1981; Gaudry and Wills, 1979; Recker et al., 1983), or they have been diagnostic in nature (e.g. Stopher, 1980; Williams and Ortuzar, 1979; Louviere, 1979).

With the theoretical underpinning we provide, we believe that the empirical comparison of standard discrete choice versus choice set formation models for the two Brazilian cities of Maceio and São Paulo represent an unique contribution on two planes: firstly, this study has succeeded in linking its theoretical and empirical phases to achieve a better understanding of the process of choice set generation in transport demand models, and thus represents an incremental increase in our empirical understanding of the processes of transport demand; secondly, the empirical work with Brazilian data from two such different urban areas has resulted in an improved understanding of urban transport demand in that country (and to some extent, for other developing countries, also).

The empirical work we have undertaken constitutes the first systematic investigation of several models of choice set formation along a variety of dimensions, with an emphasis upon evaluating the incremental benefit they provide planners when compared to the more traditional discrete choice models of demand. Just as importantly, the empirical results for São Paulo are especially useful in pointing out new research directions (see Section 6.3) that emphasize the parametrization of constraint or availability functions as opposed to the choice utility functions in choice model specifications.

6.3 Recommendations For Future Research

This thesis has only begun to explore the practical and theoretical costs and benefits of incorporating choice set formation models in models of discrete choice. As we look back upon the research, it is our opinion that for practical modelling of obligatory travel behavior in Brazil, it is fairly safe to argue that standard discrete choice models with well-specified *ad hoc* models of alternative availability (see Section 4.3.2) are quite satisfactory when compared to simple choice set formation models. We feel that further exploration of choice set restrictions (other than the captivity and independent availability) combined with simple availability representation (i.e. with single parameters per alternative) should be explored.

More importantly, for practical transport demand estimation, it is necessary that we analyze differences in statistical and predictive performance of standard versus choice set models for other dimensions of choice, such as shopping and recreational travel. Perhaps in these more discretionary choice contexts even the simple choice set formation models will yield significantly different results than given by standard models. This type of work would help to better establish the types of travel behavior and economic and social contexts where choice set formation models are most applicable.

Finally, we feel that theoretical research into modelling choice set generation should explore explicit modelling of availability and constraint functions, as opposed to the aggregate constraint modelling we have presented for Maceio. The São Paulo work mode choice parametrized logit captivity model indicates that there is great potential in this approach. Hence, we should proceed to the parametrization of constraint functions, which allied to more general choice set restrictions than the captivity assumption tested at this point, may result in greatly improved models of travel behavior in constrained environments.

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Appendix 1 - Proof of Theorem 2.1

The notation we employ here is defined in the statement of the theorem.

To prove Theorem 2.1, we must first show that

$$f(x^*) \geq f(x), \text{ any } x \neq x^*.$$

By definition,

$$f(x^*) = u_1 g(x^*) + u_2 h(x^*). \tag{1}$$

The concavity of g and h result in

$$\begin{aligned} g(x^*) &= g(u_1 x_g^* + u_2 x_h^*) \geq u_1 g(x^*) + u_2 g(x_h^*), \\ h(x^*) &= h(u_1 x_g^* + u_2 x_h^*) \geq u_1 h(x_g^*) + u_2 h(x_h^*), \end{aligned} \tag{2}$$

which results in the inequality

$$\begin{aligned} f(x^*) \geq\ & u_1[u_1 g(x_g^*) + u_2 g(x_h^*)] + \\ & u_2[u_1 h(x_g^*) + u_2 h(x_h^*)]. \end{aligned} \tag{3}$$

Now take any point $\tilde{x} \neq x^*$. Then

$$f(\tilde{x}) = u_1 g(\tilde{x}) + u_2 h(\tilde{x}), \tag{4}$$

by definition.

The optimality of x_g^* and x_h^* implies that $g(x_g^*) \geq g(\tilde{x})$ and $h(x_h^*) \geq h(\tilde{x})$. Since u_1 and u_2 are positive, (4) becomes

$$f(\tilde{x}) \leq u_1 g(x_g^*) + u_2 h(x_h^*). \tag{5}$$

If we add the negative of (5) to (3) we obtain

$$\begin{aligned} f(x^*) - f(\tilde{x}) \geq\ & u_1[u_1 g(x_g^*) + u_2 g(x_h^*) - g(x_g^*)] + \\ & u_2[u_1 h(x_g^*) + u_2 h(x_h^*) - h(x_h^*)] \end{aligned} \tag{6}$$

Since $u_2 = 1 - u_1$, (6) becomes

$$\begin{aligned} f(x^*) - f(\tilde{x}) &\geq u_1[(1-u_1)g(x_h^*) - (1- u_1)g(x_g^*)] \\ &\quad + (1-u_1)[\ u_1 h(x_g^*) - u_1 h(x_h^*)] \\ f(x^*) - f(\tilde{x}) &\geq u_1(1-u_1)[g(x_h^*) - g(x_g^*)] \\ &\quad + u_1(1-u_1)[h(x_g^*) - h(x_h^*)] \end{aligned} \tag{7}$$

Again, the optimality of x_g^* and x_h^* implies that

$$g(x_h^*) - g(x_g^*) \leq 0, \text{ and}$$

$$h(x_g^*) - h(x_h^*) \leq 0.$$

Hence,

$$f(x^*) \geq f(\tilde{x}), \text{ any } \tilde{x} \neq x^*, \tag{8}$$

which proves the optimality of x^*.

To prove that x^* is a global maximum is straightforward. It is well-known that any non-negative linear combination of concave functions, over the same convex set, is itself concave (see, for example, Martos, 1975, p.60, Corollary 42); hence, $f(x)$ is concave. A second well-known result is that any local maximum of a concave function is also a global maximum (see Martos, 1975, p.89, Theorem 2); hence x^* is globally optimal for f().

Appendix 2 - Proof of Concavity of (2.19) and (2.20) With Respect to B

Before proving the desired results, it is necessary to prove several intermediate results that will be of later use.

Lemma A2.1 - If x is an element of the real numbers, then

$$\ln\left[\frac{1}{1+e^x}\right] \quad \text{and} \quad \ln\left[\frac{e^x}{1+e^x}\right]$$

are both concave with respect to x.

Proof. To prove that either function is concave, it is necessary that we show that its second-order derivative is everywhere non-positive. As it happens,

$$\frac{d^2}{dx^2}\ln\left[\frac{1}{1+e^x}\right] = \frac{d^2}{dx^2}\ln\left[\frac{e^x}{1+e^x}\right] = -\frac{e^x}{(1+e^x)^2},$$

which is everywhere non-positive. Q.E.D.

Using Lemma A2.1, it is possible to prove the following result:

Lemma A2.2 - Let $V(R,s)=-R^Ts$, where R is a vector of real numbers of dimension L, and s is another vector of real numbers of the same dimension, so that V(R,s) is a real-valued scalar function, then

$$\ln\left[\frac{1}{1+\exp(V(R,s))}\right] \quad \text{and} \quad \ln\left[\frac{\exp(V(R,s))}{1+\exp(V(R,s))}\right]$$

are both concave in R.

Proof. By Lemma A2.1, both functions in the present lemma are concave with respect to V(R,s). By Daganzo(1979), Appendix D, Property 4.3, a concave function of a linear function (which V(R,s) certainly satisfies) is itself concave, which concludes the proof. Q.E.D.

We shall also need the lemma stated below to prove the desired results.

Lemma A2.3 - Let g(x) and f(x,y) be real-valued functions defined over the spaces X and X x Y, respectively. The function g is defined as

$$g(x) = \int_Y f(x,u)du$$

Further, f(x,y) is concave with respect to x. Then, g is also concave with respect to x.

Proof. One way of proving concavity is to show that

$$g(a_1x_1 + a_2x_2) \geq a_1g(x_1) + a_2g(x_2),$$

where a_1, a_2 are two positive scalars, x_1 and x_2 are elements of X. Thus,

$$g(a_1x_1 + a_2x_2) = \int_Y f(a_1x_1 + a_2x_2,u)du.$$

The concavity of f implies that

$$f(a_1x_1 + a_2x_2,y) \geq a_1f(x_1,y) + a_2f(x_2,y)$$

for all values y Y. Therefore,

$$g(a_1x_1 + a_2x_2) \geq \int_Y [a_1f(x_1,u) + a_2f(x_2,u)]du$$

$$g(a_1x_1 + a_2x_2) \geq a_1 \int_Y f(x_1,u)du + a_2 \int_Y f(x_2,u)du$$

$$g(a_1x_1 + a_2x_2) \geq a_1g(x_1) + a_2g(x_2)$$

Q.E.D.

With these several results, we can now prove the concavity of functions $D_1(B,0)$ and $D_2(B,0)$.

Theorem A2.4 - The functions $D_1(B,0)$ and $D_2(B,0)$, given in expressions (2.19) and (2.20), respectively, are concave with respect to the elements of the vector B.

Proof. The notation employed below is first defined in Chapter 2.

First let us prove the concavity of (2.19). Consider the terms

$$r(i,B,B^*,z)=W(i,B^*,z)\ln H(i|B,z),\ i=1,2,$$

where $W(i,B^*,z) = Q(i|B^*,z)f(z),\ i=1,2,$

so that we rewrite

$$D_1(B,0) = \sum_{i=1,2} \int_X r(i,B,B^*,z)dz.$$

By Lemmas A2.1 and A2.2, $\ln H(i|B,X)$, $i=1,2$, is concave with respect to the B vector. Since $W(i,B^*,z)$, $i=1,2$, is independent of B, it can be treated as a constant (which is positive) in this analysis. Thus, $r(i,B,B^*,z)$, $i=1,2$, is concave with respect to B.

Use of Lemma A2.3 insures us that

$$\int_X r(i,B,B^*,z)dz, \quad i=1,2,$$

is concave with respect to B. If to this we add the well-known result that the sum of concave functions is also concave (see Martos, 1975, p.60, Corollary 42), we finally have the desired result that $D_1(B,0)$ is concave with respect to B.

Now for the case of $D_2(B,0)$. Rewrite (2.20) as

$$D_2(B,0) = \int_X q(B,z)dz,$$

where $q(B,z)=\ln H(1|B,z)f(z)$. By Lemma A2.2, $\ln H(1|B,z)$ is concave in B; since $f(x)$ is independent of B and it is positive, we conclude that $q(B,z)$ is also concave in B. A straightforward application of Lemma A2.3 leads us to the result that $D_2(B,0)$ is concave with respect to B.

Q.E.D.

Appendix 3 - Hypothesis Tests and Estimation of Choice Set Formation Models

No statistical difficulties are forthcoming either with respect to estimation or inference, when we deal with the choice models presented in Chapter 3. Assuming that our presumed knowledge of the causal structure for the processes of choice set generation and choice is indeed correct, we will obtain consistent and asymptotically efficient parameter estimates by maximizing the likelihood of observing our sample of decision-makers.

We are able to benefit from the great amount of research that has been performed in recent years concerning alternative sample designs and estimators for discrete choice models (see Manski and Lerman, 1977; Manski and McFadden, 1981; Cosslett, 1978, 1981; and Hsieh et al., 1983). Thus, for example, we may still obtain consistent and asymptotically normal, though not efficient, parameter estimates by working with a purely choice-based sample and utilizing the Weighted Exogenous Sample Maximum Likelihood, or WESML, estimator (see Manski and Lerman, 1977). This estimator requires no more effort to implement than that required for the ML estimator, and we can benefit from all the advantages available from a choice-based sample. Other estimators for endogenously stratified samples, such as the

Conditional Maximum Likelihood (CML) or Full Information Concentrated Likelihood Equations (FICLE), both discussed in Cosslett(1981) and Hsieh et al.(1983), may also be adopted. Both of these are consistent and asymptotically normal. The former, the CML estimator, is computationally of the same order of difficulty as the WESML estimator, but the FICLE estimator, though it presents the additional advantage of being efficient (which the other estimators mentioned here are not), is somewhat more difficult to implement.

Whichever of the above estimators is actually adopted, conventional asymptotic theory hypothesis testing can be applied to parameter vectors to test restrictions on the parameters of the choice or choice set generation models. If our estimation sample is purely random or exogenously stratified, we may use the likelihood ratio statistic

$$-2[\Lambda_N(\Gamma_R) - \Lambda_N(\Gamma_u)] \overset{\text{a.d.}}{\sim} \chi^2(k), \qquad (1)$$

where Γ_R, Γ_U are the restricted and unrestricted parameter vectors, respectively,

$\Lambda_N(\Gamma)$ is the likelihood of a sample the size N evaluated at the specific parameter vector Γ,

a.d. means asymptotically distributed as,

$\chi^2(k)$ is the chi-squared distribution with k degrees of freedom, and

k is the number of restrictions imposed upon Γ_U to obtain Γ_R,

to test the hypothesis that some linear set of restrictions of the form $R\Gamma_U=\Gamma_R$, where R is a matrix embodying the restrictions, holds in the observed sample.

In the case of an endogenously stratified sample, such as a purely choice-based one, for which we use the WESML or CML estimators, we can test the same hypothesis R $_U$= $_R$ by using the Wald statistic

$$(R\Gamma_U-\Gamma_R)^T[RV_n(\Gamma_U)R^T]^{-1}(R\Gamma_U-\Gamma_R) \overset{a.d.}{\sim} \chi^2(k) \qquad (2)$$

where $V_N(\Gamma_U)$ is the estimated variance-covariance matrix of the respective estimator evaluated at Γ_U, other quantities as previously defined. Consistent estimates of the variance matrices are given in Hsieh <u>et al</u>.(1983) for the two estimators. These estimates are especially convenient in that they do not require knowledge of the second-order derivatives of the appropriate objective functions.

As for statistical comparisons of non-nested hypotheses, whether they be of the choice given choice set model subject to the same choice sets, or between two different choice set generation models, we may use the measure known as the Akaike Information Criterion (AIC) (see

Akaike, 1973, or the more understandable presentation given by Amemiya, 1980) to compare non-nested hypotheses. This measure is defined as

$$\mathrm{AIC}(\Gamma) = -\Lambda(\Gamma) + k; \tag{3}$$

we prefer the model with the smaller AIC.

Though no new statistical issues are raised in the calibration of the combined choice and choice set generation models proposed here, potentially serious computational issues do arise. Firstly, there is the issue of combinatoric growth of choice set dimensionality with the number of possible alternatives. This problem so determines the practicality of modelling choice set formation that we have dealt with it extensively by formulating choice set models with a priori restrictions on choice set structure (see Section 3.3.2). To the extent that such structural restrictions are realistic and acceptable to the analyst, this compromise solution circumvents the dimensionality issue.

Secondly, given any of the full choice models formulated in this chapter, there still remains the problem of actually obtaining parameter estimates. None of the models presented yield objective functions (i.e. maximum likelihood, WESML or CML estimators) with desirable properties, such as the concavity of the log likelihood function for a random sample with a linear-in-parameters logit model (which guarantees the uniqueness of parameter estimates as long as no correlation exists between independent variables). Thus, for the choice set generation models we have proposed, it is possible for numerical optimization algorithms to converge to stationary points in the parameter space that are not actually the global maximum of the estimator function. We can partially circumvent this shortcoming by initiating the algorithm from a number of widely spaced points in the parameter space, and perhaps identify with greater assurance the global maximum.

Given the highly nonlinear nature of the estimation problem, a host of numerical problems beset any attempt to obtain parameter estimates. As explained by Gill *et al.*(1981), these problems range from inappropriate scaling of the data (*i.e.* the constraint and utility functions) and algorithmic termination criteria, to poor choice of optimization algorithm.

It is necessary to carefully assess the appropriateness of a given model structure to the population to which it is being applied. For instance, if a logit captivity model is to be applied to a market segment which, in reality, does not display any evidence whatsoever of captivity, it is not possible to identify both the captivity constants and the intercepts of the choice given choice set utility functions. Apart from the question of misspecification of the choice model for this group, it is clear that grave numerical problems arise for any optimization algorithm due to the inappropriateness of the model.

The Newton Method, which has often been applied to maximum likelihood estimation problems, requires knowledge of second-order derivatives. Given the degree of complexity of the model formulations presented in Chapter 3 and the investigative nature of our work, we have chosen to utilize instead a quasi-Newton optimization method, which utilizes an estimate of the Hessian (i.e. the matrix of second derivatives) of the objective function. Different methods of estimating the Hessian result in different quasi-Newton algorithms (see, for example, Gill et al., 1981, Himmelblau, 1972).

Berndt-Hall-Hall-Hausman(1974) suggest, for maximum likelihood estimation of parameters of nonlinear models from simple or exogenously stratified random samples, that we utilize the identity (see Theil, 1971, pp.387-388)

$$-E\left[\frac{\partial^2 \Lambda(X,\Gamma)}{\partial\Gamma\ \partial\Gamma^T}\right]_{\Gamma=\Gamma^*} = V\left[\frac{\partial\Lambda(X,\Gamma)}{\partial\Gamma}\right]_{\Gamma=\Gamma^*} \quad (4)$$

where $\Lambda(X,\Gamma)$ is the log likelihood function for a random sample;

X is a matrix of attributes;

Γ^* is the true vector of parameters; and

E[ ],V[ ] are the expectation an variance, respectively, of their arguments;

to formulate

$$Q_N(X,\Gamma) = - \sum_{n=1}^{N} \frac{\partial \ln Pr(i_n|X_n,\Gamma)}{\partial \Gamma} \cdot \frac{\partial \ln Pr(i_n|X_n,\Gamma)}{\partial \Gamma^T} \qquad (5)$$

as the sample estimate of the Hessian for the log likelihood function (at Γ) of any of the choice and choice set generation models that have been proposed in Chapter 3. Result (4) insures that $Q_N(X,\Gamma)$ converges to the Hessian of the log likelihood funtion as $N \to \infty$. Expression (5) neatly avoids the need to compute an analytic Hessian, which significantly reduces the computational effort involved to find the MLEs. The quasi-Newton algorithm for ML estimation, using (5) as the estimate of the Hessian, has become known in the econometrics literature as the BHHH algorithm, after its proponents.

This same approach may be used for the other estimators that have been mentioned. Consider, for example, the WESML objective function for a purely choice-based sample:

$$W_N(X,\Gamma) = \sum_{n=1}^{N} w(i_n) \ln Pr(i|X,\Gamma) \qquad (6)$$

where $w(i_n)$ is a weight given to observation n, equal to the ratio of the proportion of individuals in the population choosing i_n to the proportion in the sample choosing i_n, $i_n \varepsilon M$.

As shown in Hsieh et al.(1983), we may use the straightforward extension of (5):

$$Q_N^W(X,\Gamma) = \sum_{n=1}^{N} w(i_n) \frac{\partial \ln Pr(i_n|X_n,\Gamma)}{\partial \Gamma} \cdot \frac{\partial \ln Pr(i_n|X_n,\Gamma)}{\partial \Gamma^T} \qquad (7)$$

as the estimate of the Hessian of (6). Similar results may be had for the CML estimator.