SYSTEM EFFECTIVENESS ANALYSIS
FOR
COMMAND AND CONTROL

by

VINCENT BOUTHONNIER

Ingenieur de l'Ecole Centrale des Arts et Manufactures

(1981)

SUBMITTED TO THE DEPARTMENT OF
MECHANICAL ENGINEERING
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE
IN
TECHNOLOGY AND POLICY

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 1982

Signature of Author ______________________________
Technology and Policy Program
August 18, 1982

Certified by ______________________________
Alexander H. Levis
Thesis Supervisor

Accepted by ______________________________
Richard de Neufville
Chairman, Technology and Policy Program

Accepted by ______________________________
Warren M. Rohsenow
Chairman, Departmental Graduate Committee,
Mechanical Engineering Department

SYSTEM EFFECTIVENESS ANALYSIS

FOR

COMMAND AND CONTROL

BY

VINCENT BOUTHONNIER

Submitted to the Department of Mechanical Engineering
on August 18, 1982
in partial fulfillment on the requirements for the degree of
Master of Science in Technology and Policy

ABSTRACT

A methodology for assessing the effectiveness of Command, Control and Communication (C^3) systems is developed. It is carried out by characterizing independently system and mission attributes. These attributes are determined as a function of the primitives that describe the system, the mission and the context within which both operate. Then the characteristics of the system and the mission are compared in a common attribute space. This comparison leads to the evaluation of measures of effectiveness which are in turn combined to yield a global measure of effectiveness. The assessment of a communication network is presented to illustrate the methodology.

Thesis Supervisor: Dr. Alexander H. Levis

Title: Senior Research Scientist

ACKNOWLEGMENT

I would like to express my sincere thanks to Dr. Alexander H. Levis for his guidance, support and contribution to this thesis. I have very much appreciated working with him.

Lisa Babine is gratefully acknowledged for the fine job she did at typing the manuscript.

Art Giordani of LIDS deserves also special thanks for drafting the figures.

Finally, I thank the LIDS members I have known and who have always been helpful and friendly to me.

This research was carried out at the MIT Laboratory for Information and Decision Systems with partial support provided by the Office of Naval Research under Contracts ONR/N00014-81-K-0495 (MIT OSP No. 90945) and ONR/N00014-77-C-0532. (MIT OSP No. 85552).

TABLE OF CONTENTS

Page

LIST OF FIGURES

SECTION 1
INTRODUCTION

1.1 INTRODUCTION

Over the past decades Command, Control, and Communication systems have become a major subject of concern in the Armed Forces. Colonel Ball (1980) illustrates this concern when he states: "we employ tactical aerospace forces, none more than 20 years old, including such equipment as the F-15, F-16, A-10, AWACS, etc., with command and control systems nearly all over 20 years old". Implicit in his statement is the need for improving C^3 systems. However, it is not clear how to understand the term "improving". Shanahan, Teates and Wise (1981) note that a commander is likely to describe his requirements for a C^3 systems in general, ambiguous terms as, for example, an "over-the-horizon detection or classification and targeting capability". On the other hand, the system developer, describes the capabilities of a C^3 system in terms of technical specifications (bit-error rates, bandwidth, etc.). This lack of clear communication between user and developer has often led toward more complex systems. As a result, for a long period of time,the rationale has been: The better the technical characteristics (i.e., more accurate, faster, more options), the more effective the system is likely to be. This perception is changing, with emphasis placed on defining the effectiveness of a system not only in terms of its performance specifications. General Welch (1980) gives as a definition for C^3 effectiveness "a specific measure of the C^3 system's capability to enhance the commander-battle staff decision process".

The work presented in this thesis is in the spirit of this definition. A methodology for assessing C^3 system is proposed which aims at measuring the effectiveness of a C^3 system by comparing its capabilities with the requirements of the mission in which it will be used. The advantage of this approach is that it relates the performance of a system to the tasks or mission it has to fulfill.

In the remainder of this section background information is provided and the system effectiveness methodology is introduced. This methodology is presented in detail in Section 2 where it is described how both aspects of system amd mission can be analyzed separately and then compared in a common attribute space. Furthermore, the derivation of a global measure of effectiveness from the results of utility theory is examined. In Section 3 a simple example of C^3 system, a communication network is introduced. This example is used to illustrate a practical application of the methodology. In Section 4 it is shown, in the case of the communication network, how attributes describing the system's capabilities can be derived. Similarly, Section 5 focuses on deriving attributes characterizing the requirements of the mission. These system and mission attributes are then compared in a common attribute space (Section 6). This section includes also a discussion on different global measures of effectiveness. Finally, conclusions and recommendations for further research on the subject are given in Section 7.

1.2 BACKGROUND

This section intends to introduce C^3 systems, to explain why they need to be assessed, to pinpoint the difficulties of any assessment process and, eventually, to present different techniques which can be used for evaluating C^3 systems.

D. B. Brick (1978) has given the following definition of C^3 systems: "C^3 systems designate a composite of equipment, skills and techniques which, while not instruments of combat, enable a commander to exercise continuous control of his forces and weapons in all situations by providing him with the information needed to make operational decisions and the means to disseminate then".

This definition emphasizes the complexity of C^3 systems. Typically, today's commander must be provided with warning, reconnaissance and damage assessment, and he must have extensive communications and data-processing capability. These complex interelationships have pushed toward increased sophistication of the C^3 components. It is not clear that this trend leads to increased overall effectiveness. For instance, it may not take into account human behavior; humans are integral parts of C^3 systems. Efforts to date have tended to treat the human components either as the "user" or an "input" to the C^3 systems rather than an integral component of the overall design. Recent work, Boettcher and Levis (1981), is trying to analyze the interrelationships between a C^3 system and the organization it supports.

Recently, an understanding has been evolving, however, that development of assessment procedures for C^3 systems is of utmost importance. H. L. Van Trees (1980) has written that "assessing the utility of C^3 systems is part of a larger problem, i.e., developing a basic understanding of command, control, and communications". T. P. Rona (1980) defines more precisely what is meant by C^3 system assessment "finding out how well a system can perform its intended task, i.e., to support the military in their assigned mission". Both of those statements indicates the need for a rationale for assessing, and choosing between two alternative systems. General Welch (1980) has provided clues to the answer by observing that both issues of knowing, on the one hand, what a system can provide, and on the other hand, what the system is intended to do, should be considered separately. But he also recognizes that they are difficulties inherent in the nature of C^3 systems that will make "C^3 analysis remain somewhat an art".

Among those difficulties is the fact that C^3 systems are not an end in themselves, but an integral part of the national defense structure. Thus, C^3 systems effectiveness depends largely on exogenous factors. These factors are of two types. The first is the interaction with the human component, the human organization which has been mentioned earlier on. The second is the environment or the context in which these systems have to operate. For instance, a system might have been very effective in a context (e.g., the Pacific) or set of contexts and not in another (e.g., the North Atlantic). Furthermore, the system could behave very poorly in unforeseen contexts. Thus, the method of analysis used would have to be flexible enough in order to account both for different and new contexts. Among the possible techniques that can be developed for evaluating C^3 systems, C.A. Zraket (1980) identifies four categories. These categories are shown on Figure 1.1 as a function of their costs.

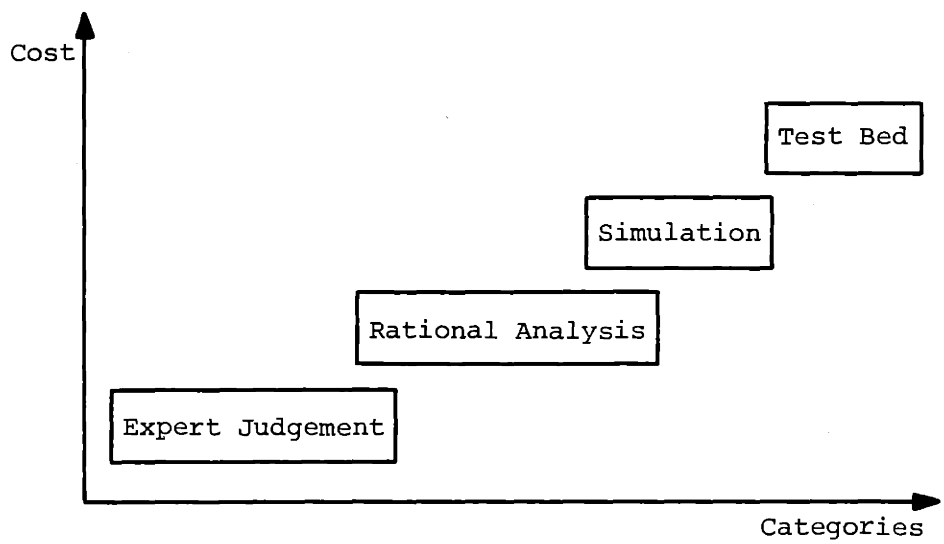


Figure 1.1 Relationship between cost and assessment category

Expert Judgement: denotes the assessment based on the expertise of military and technical persons. It is highly flexible, but the lack of analytical method may bias the assessment and make it subject to individual preferences or political pressures.

Rational Analysis: is a term which applies to that class of analysis which employs mathematical models to represent the system and its environment.

Simulations: represent the system by means of computer programs which model the process sequences and timing of the real system and of the external environment. They can give some insight on the functioning of the system, however, they provide only raw data (given the context, and the mission inputs) which need to be interpreted to yield a system assessment.

Test Beds: refer to the test performed on a real system but where the external environment is simulated. This technique gives certainly a more realistic idea of how a C^3 system would perform. However, it has the drawback of being expensive and highly inflexible.

Each of these techniques is not superior to one another; each of them has, however, to be used at one point or another in the assessment process. For instance, the use of *Rational Analysis* may point out that a system is not appropriate for its foreseen use and hence avoid further costly studies. In the next section the development of a technique based on *Rational Analysis* is discussed.

1.3 STATEMENT OF THE PROBLEM

In the spectrum of assessment techniques described in section 1.2, *Rational Analysis* is of special interest. It is a rather inexpensive technique that has the advantage of being flexible, e.g. various kinds of sensitivity analyses can be performed directly.

As general Welch has pointed out (Welch (1980)) the development of assessment techniques should aim at relating the performance of the system with the requirements of the mission. This is the basic idea of the methodology described throughout this thesis. It was first introduced by Dersin and Levis (1981, 1982) for the effectiveness analysis of power systems. Thus, this approach is rather new. Actually, for a long time the predominant idea was that technical improvements were increasing the effectiveness of C^3 systems. Now, this assumption is questioned, namely, because of the difference in rates of advancements between the human performance and the technical performance.

The following (hypothetical) policy issue illustrates this point. If a new kind of aircraft has to be developed with the possibility, on the one hand, to build a very sophisticated aircraft with a wide range of missions, and on the other hand to build two or three different aircraft with a more limited range of missions for each, which of the two alternatives should be chosen? Proponents of technical improvement would opt for the first alternative. However, the integration in the same aircraft of many different functions might decrease the effectiveness of each individual function. Hence the aircraft with a large range of missions can be less effective, each mission taken one by one, than the aircraft with a limited range of missions. It might also be more difficult for the pilot to cope with the sophistication of the aircraft. Hence the overall effectiveness would be diminished because of the limitation of the human component. It might, finally, not be necessary to have all functions available in the same aircraft,if any mission requires only part of the functions.

This streches the importance of the mission and of the context in which the system has to operate. Unfortunately, the mission or set of missions a system has to perform is often defined in too broad terms. it might also be possible that the analysis is misleading in that it concentrates on only one aspect of the mission. For instance, let us take the example of a communication device that should improve the reliability of a network. Thus, one might be led to believe that the effectiveness of the network has been increased but, at the same time, the introduction of this device might also increase the vulnerability of the system (i.e., possibility of detection by enemy forces). This shows that a trade-off exists between higher reliability and higher vulnerability and this trade-off should appear in the effectiveness analysis.

Once the effectiveness analysis has been carried out it should then be able to help the decisionmaker in choosing which improvement should be pursued or which system should be implemented. But how can such an effectiveness analysis be made? The answer is by defining a methodology relating the performance of the system to the requirements of the mission. This methodology would be based on analytical measures assessing the effectiveness of the system given the mission and the context in which it has to operate. In Section 2, the different steps of such a methodology are detailed.

SECTION 2

METHOD OF ANALYSIS

2.1 INTRODUCTION

This section describes the proposed methodology for assessing the effectiveness of C^3 systems. A comprehensive analysis of the questions that should be raised is presented and a description is given of the successive steps that should be taken in the assessment process. Section 2.2 contains an overview of the proposed methodology with emphasis on the close interrelationships between the concepts that characterize the system, the mission, and their context. In section 2.3, the procedure is described for characterizing separately both the system (or, more precisely, subsets of the system) and the mission in terms of common attributes, i.e., aggregate variables that describe what the system can provide and what the mission requires the system to provide. It will be also shown how those attributes characterizing either the system or the mission can be compared for assessing the effectiveness of the subset of the system that is being analyzed. Finally, in section 2.4, the issue of deriving a global measure of effectiveness is raised. The discussion will focus on how - and to what extent - utility theory can be used for aggregating the partial measures into a global measure.

2.2 DESCRIPTION OF THE METHODOLOGY

This section contains an overview of the methodology which is developed in this thesis. First, the notions of system, mission and context are presented. Then, the framework for the assessment is set up. The issues of defining subsets of interest for the system and selecting criteria of assessment are discussed.

2.2.1 <u>System, Mission and Context</u>

Before developing the principles of the methodology, it is necessary to define the notions of system, mission and context that will be used throughout the thesis.

System: This term has already been defined in Section 1. It refers to a C^3 system, i.e., a composite of skills and equipment at the disposal of the military commander(s).

Mission: This term designates the task the military commanders have been assigned to accomplish. It should not be interpreted broadly, but instead as the specific description of a particular task. For instance, a mission described as "the defense of the national territory against enemy attacks" is too broad a mission. We are more concerned with specific tasks that support the overall goal. What is sought is a methodology for assessing how the C^3 system, intended to assist the commander, contributes to this task.

Context: This term designates the environment, the "milieu" in which the mission takes place. Typically, this would include such information as the geographical location of the mission, the expected enemy forces, the weather conditions or, in the case of communications networks, the enemy's capability for jamming.

The basic idea in this methodology is to compare what the mission requires from the system with the capabilities of the system. Therefore, assessment criteria are needed. These criteria or attributes are aggregate variables of the system or functions of the mission characteristics. However, the system's capabilities cannot be assessed, if the context in which it has to operate is not defined. For instance, a helicopter operates differently in mountainous terrain and in deserts with sandstorms, i.e., the probability of failure of the components are different and the causes of failure vary a lot. Hence the system needs to be placed in the context of the mission in order to be assessed correctly. The distinction between the context and the mission should be emphasized. The context describes a preexisting situation that is known in advance possibly with some degree of uncertainty. The mission involves a certain number of tactical choices based on the overall objectives and also on the circumstances defined by the context. Hence the context influences both system and mission. This distinction needed to be made before proceeding further with the analysis. It is the first and necessary step in the assessment process of the system. Now the next steps of the analysis are presented.

2.2.2 Methodological Framework

After having defined the system that is to be assessed, the next question is how to decompose the system into subsystems which are easier to assess and to which corresponds a certain subset of the mission. Often, the decomposition is likely to be obvious. Parts of the system which have similar uses or missions can be grouped together to form a subset of the system. The second advantage of such a decomposition is to select those parts of the system that are likely to operate in the given context. Then the mission itself can be decomposed into submissions corresponding to the subsystems just identified.

The next issue is the selection of the criteria for assessing the effectiveness of each subsystem with respect to the mission it has to accomplish. These criteria have to be specified in relation to the characteristics of the system. For instance, for a sensor system, reliability, accuracy and speed of response are important attributes.

The next step is to specify the system and mission attributes. System attributes are measures of system (or subsystem) properties; mission attributes express the requirements of the mission. The determination of those attributes is carried out separately for the system and for the mission. The rationale for such a separate analysis is that: (i) System and mission attributes do not necessarily depend on the same basic variables or primitives. (ii) The basic characteristics of the system (resp. of the mission) are often related to each other and as a result need to be aggregated into a new set of variables before being compared. (iii) This approach imposes a discipline in carrying out the analyses of both the system and the mission.

These aggregation steps in the methodology are shown in Figure 2.1.

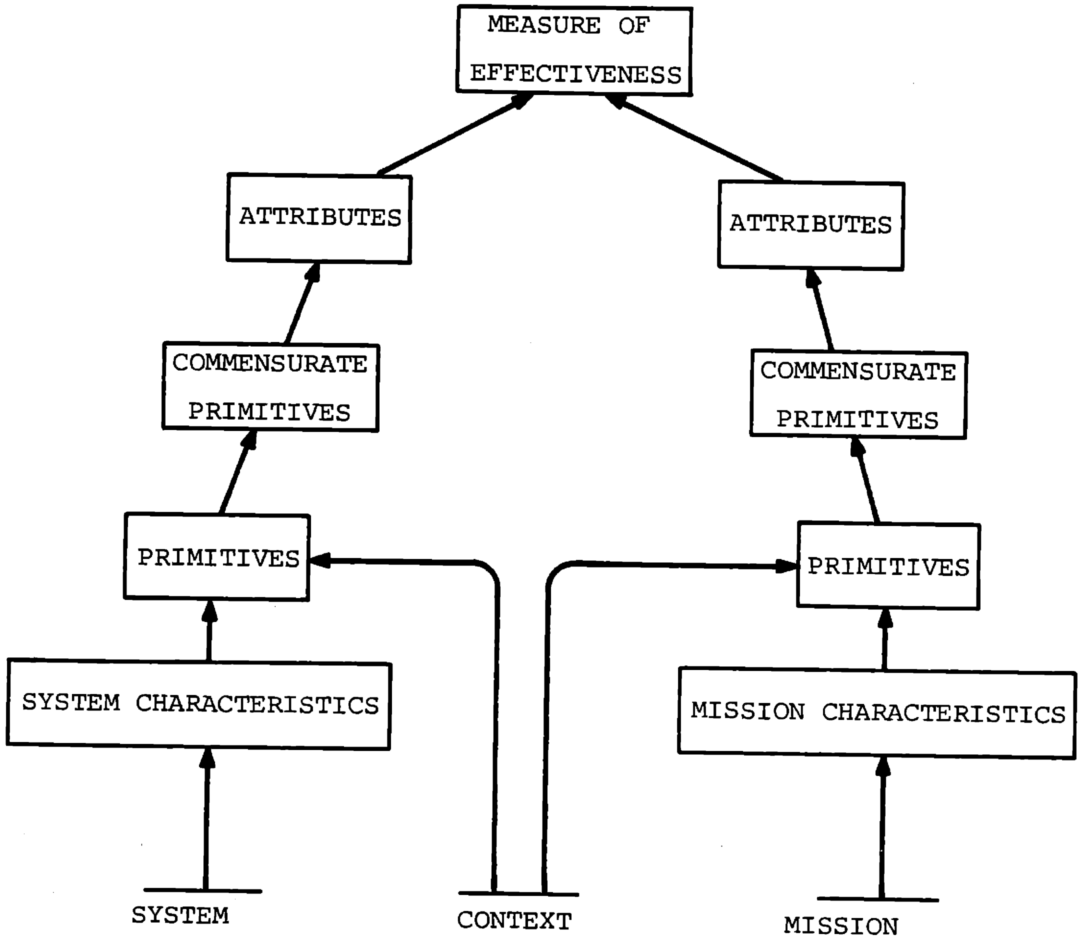


Figure 2.1 Assessment Procedure

The close interrelationship between system and context and mission and context is recalled. The aggregation steps are as follows:

Step 1: Define from the system characteristics and from the context the variables (or primitives) that are likely to influence the system attributes.

Step 2: Repeat step 1 for the mission primitives.

Step 3: Group the related primitives together and normalize them, so that all primitives are commensurate.

Step 4: Repeat step 3 for the mission.

Step 5: Relate the commensurate primitives to those attributes they are influencing.

Step 6: Repeat step 5 for the mission primitives and attributes.

Step 7: Compare, in a common attribute space, the system and mission attributes. Define measures of effectiveness which account for the way the requirements of the mission are met by the system.

Step 8: Combine the partial measures of effectiveness obtained for each subsystem into a global measure of effectiveness for the system as a whole.

These steps will be reviewed in more detail in the next sections.

2.3 EFFECTIVENESS ANALYSIS OF A SUBSYSTEM

In this section, the aggregation procedures for the system and for the mission are presented in more detail. Then it is shown how measures of effectiveness can be derived by comparing system and mission attributes in an attribute space.

2.3.1 Aggregation into System Attributes

The procedure for relating the system attributes A_i^s, where i varies between 1 and n to the system primitives p_j^s, where j varies between 1 and r, is completely system dependent so that it is difficult to describe explicitly in a general framework; in Section 4, such a procedure is illustrated for communication networks. Nevertheless, the assessment relies essentially on the physical properties of the system studied. In that respect, it is possible to model the operation of the system in order to relate the component characteristics to the system attributes. This can be expressed as:

$$A_i^s = f_i^s(p_1^s, p_2^s, \ldots, p_r^s) \qquad \text{for all } i \tag{2.1}$$

Either all the attributes have distinct primitives and then the A_i^s's vary independently from each other, or some A_i^s's have certain primitives in common and then their variation is linked. In some cases, it might even be possible to obtain for some of the attributes an implicit relationship of the type:

$$g^s(A_x^s, \ldots, A_y^s, A_z^s) \geq 0 \tag{2.2}$$

In turn, the primitives define a primitive set of dimension r. In general, each primitive takes value on a certain interval. Assuming a uniform probability distribution for each primitive, then the locus of all the possible primitive is a hyper-cube in the r-dimensional space. When the p_j^s's vary within this hyper-cube, relation (2.2) defines a locus in the attribute space which represent the feasibility set, i.e., the combinations of system attribute values that are feasible. This feasibility set is to be compared with the locus defined by the mission attributes which represent what the system should be able to provide.

2.3.2 Aggregation into Mission Attributes

As noted earlier, the mission primitives may have nothing in common with the system primitives. They arise from tactical data that may be expressed in terms of the objectives of the mission. As there is no way to know the end result of the mission before it actually takes place, the engagement needs to be modelled to derive conditions for the primitives. These conditions will translate into conditions for the mission attributes since the value of the latter is a function of the degree of achievement required from the mission. In Section 5, such a model is developed. It might be possible to write, then, the mission attributes A_i^m as a function of the mission primitives p_j^s

$$A_i^m = f_i^m \, (p_1^m, p_2^m, \ldots, p_r^m) \quad \text{for all } i \tag{2.3}$$

and a relationship among the attributes might be derived if they have common primitives.

$$g^m(A_x^m, \ldots, A_y^m, A_z^m) \geq 0 \tag{2.4}$$

Similarly to what has been said for the system attributes, the mission attributes define a locus in the attribute space. This locus corresponds to the set of values that satisfy the requirements of the mission. The next step consists of comparing both of these loci in the n-dimensional attribute space. Therefore, a measure of effectiveness has to be developed for assessing the performance of the subsystem studied.

2.3.3 Effectiveness Analysis in the Attribute Space

In order to compare the system and mission attributes these need to take values in the same interval. The rationale for normalizing the attributes is that the scale in which they are measured should not bias the measure in favor of one of the attributes. In the next sections the interval [0,1] has been chosen, however, any other interval could have been chosen. The normalized attributes take now values in the unity hyper-cube of the n-dimensional space. Similarly, both mission and system loci are confined to this hypercube.

Two cases may arise:

(1) Both loci do not intersect, i.e., there is no set of system attributes $(A_1^s, \ldots, A_n^s)$ which belongs to the mission locus. In this case the system does not satisfy at all the requirements of the mission.

(2) The two loci intersect. Two subcases of interest occur when the system locus is completely included in the mission locus and, conversely, when the mission locus is included in the system locus.

In the first case, the effectiveness measure associated could be simply null. However, this would not account for the fact that the system locus might be close to the mission locus or remote from it. Hence, an improvement would be to introduce the shortest distance between both loci. This issue is discussed in more detail in Section 6.

When both loci intersect, then a natural effectiveness rating is given by the ratio of a measure of the intersection of system and mission loci to a measure of the system loci. For instance, if the system locus is defined by a set of functions f_k:

$$f_k^s\ (A_i^s) \geq 0 \quad ; \quad k \in K \tag{2.5}$$

then the measure to be chosen is the volume of the system locus, S_L,

$$V_s = \iint \cdots \int_{S_L} dA_1^s \ldots dA_n^s \tag{2.6}$$

If the system locus is no longer a volume in the n-dimensional space but a manifold then the appropriate measure is the surface of the manifold.

$$V_s = \iint \cdots \int_{\Omega \varepsilon S_L} d\Omega \tag{2.7}$$

Similarly, the misssion volume (or surface) can be computed:

$$V_m = \iint \cdots \int_{\Omega \varepsilon S_m} d\Omega \tag{2.8}$$

where S_m designates the mission locus. Let the volume of the intersection of system and mission loci be called the truncated volume, V_t. It denotes the part of the system volume (or surface) which satisfies the requirements of the mission. V_t can be expressed as:

$$V_t = \int\int \cdots \int_{\substack{\Omega \varepsilon S_L \\ \text{and} \\ \Omega \varepsilon S_m}} d\Omega \tag{2.9}$$

The measure of effectiveness derived from the values of V_s and V_t will be called E^i, where the superscript i denotes the i-th subsystem that is being assessed:

$$E^i = q(V_s, V_t) \tag{2.10}$$

For example, function q in equation (2.10) may be the ratio of V_t over V_s. Such a measure is defined in Section 6.

The interest of this effectiveness measure is that, whenever the system locus is included in the mission locus, the measure is equal to its maximum value, one. This is consistent with an intuitive notion of effectiveness: if the system meets completely the requirements of the mission then it has a high effectiveness. Inversely when the mission volume is included in the system volume the system meets more than it is required. In this case the effectiveness measure is a function of the ratio of the volume (or of the surface) of the mission locus to the volume of the system locus. The lower this ratio will be the more ineffective (or mismatched) the system will be. Section 6 also shows that it is preferrable to work in the n-dimensional space than to work on projections of the loci in subspaces. By projecting much of the information is lost in a way that leads to overestimation of the effectiveness measure.

In this section it has been assumed that the system amd mission loci as well as their volumes (or surfaces) could have an analytical expression. Although the example developed in Section 6 exhibits such a property it may not always be the case. Then numerical methods should be used to compute the proposed measure of effectiveness.

The measure E^i defined previously, eq. (2.10), will vary between zero and one. Also, in order to apply the results of utility theory it will be necessary to map those E^i's from the interval [0,1) into R^+. The mapped variables or $\tilde{E}^i$ will always denote the effectiveness of the i-th subsystem, but will now take values in R^+.

Section 2.4 shows how utility theory can be used to derive a measure of effectiveness for the system as a whole. This measure will be associated with the particular function q chosen in (2.10). When other functions q are chosen, then corresponding partial measures E are obtained. A global measure $\hat{E}$ will be derived by combining all these partial measures.

2.4 GLOBAL EFFECTIVENESS ANALYSIS

The last section has shown how a measure of effectiveness could be derived for a particular subsystem. Once all subsystems have been assessed, the next step is to derive a global effectiveness measure. In section 2.4, it will be explained why utility theory can now be used. The limitations of utility theory will also be pointed out. Finally, in section 2.4.2, the necessary steps for applying utility theory to the assessment of system effectiveness will be described.

2.4.1 On Utility Theory

This section aims at justifying the usefulness of utility theory for aggregating the partial effectiveness measures into a global measure. First some elements on multi-attribute utility theory are recalled.

Background on Utility Theory. Utility theory relies on a certain number of assumptions and axioms necessary to define a utility function on the attribute set.

(1) It is assumed that there is a finite number of attributes (or commodities), say n, that can be represented by a vector $\underline{\tilde{E}}$ in $\mathcal{R}^n$.

$$\underline{\tilde{E}} = (\tilde{E}^1, \ldots, \tilde{E}^n) \qquad (2.11)$$

The set, $\mathcal{E}$, of all possible vectors $\underline{\tilde{E}}$ will be called the commodity set and it will be further assumed that it is defined on the non-negative orthant of $\mathcal{R}^n$.

(2) It is possible for the decisionmaker to choose the vector he prefers between any two vectors of $\mathcal{E}$. This choice should be consistent with the property of transitivity.

(3) For any two vectors $\underline{\tilde{E}}$ and $\underline{\tilde{E}}'$ of $\mathcal{E}$ which have exactly the same values except for the j-th component where $\tilde{E}^j$ is greater than $\tilde{E}'^j$, then $\underline{\tilde{E}}$ is preferred to $\underline{\tilde{E}}'$. This axiom is called the dominance axiom.

(4) For any vector $\underline{\tilde{E}}$ in $\mathcal{E}$, the set of vectors preferred to $\underline{\tilde{E}}$ and the set of vectors over which $\underline{\tilde{E}}$ is preferred are closed in $\mathcal{E}$. This is the axiom of continuity.

Debreu (1958) has shown that assumptions and axioms (1) to (4) are sufficient conditions for the existence of a real-valued function which is a continuous function of the commodities $\tilde{E}^1$ through $\tilde{E}^n$.

If the attribute or commodity $\tilde{E}^i$ describes the effectiveness of the i-th subsystem then all the preceding axioms apply, especially the dominance axiom. This implies that the effectiveness function is increasing with its arguments. This property is likely to be desired by the decisionmaker. It shows also why utility theory could not have been considered as a useful tool earlier in the analysis. We recall from the analysis in the attribute space that a larger system locus does not yield automatically higher effectiveness. Actually, if the mission locus is included in the system locus the effectiveness may even diminish.

This illustrates why utility theory could not have been used appropriately at this level: the dominance axiom is not satisfied.

Other aspects of utility theory are, however, of less interest, e.g., maximization of the utility function under a budget constraint in order to find the optimal quantity $\tilde{E}^i$. This analysis could not be used in the assessment of C^3 systems because (i) the emphasis is not so much at finding optimum values for the $\tilde{E}^i$'s as at determining which of two alternative systems is best suited for the assigned mission. (ii) the determination of a budget constraint in terms of the $\tilde{E}^i$'s is difficult to conceive: it would mean introduing a price system associated with the $\tilde{E}^i$'s. An attempt in this area, Valavani (1981), has not shown promising results.

So, further research should be limited at defining interesting functions which can reflect the preference ordering of the decisionmaker. This issue is addressed in more detail in Section 6. Section 2.4.2 lays out, next, the basis for the definition of effectiveness measures.

2.4.2 Utility Theory Applied to System Effectiveness Analysis

The elements of utility theory recalled in section 2.4.1 show that the arguments of the utility function belongs to the positive orthant of $\mathcal{R}^n$, i.e., they are unbounded from above. The partial effectiveness measures, E^i, derived from the analysis of section 2.3 do not exhibit such a property since unity is their upper bounds. A mapping from $[0,1)$ into $[0, +\infty)$ has then to be found in order to have the E^i's transformed into unbounded variables $\tilde{E}^i$.

Then, utility theory can be applied to derive an effectiveness measure for the system as a whole. If the function chosen in (2.10) is q_r, where r varies between 1 and s, then the effectiveness measure for the system will be denoted by E_r. One can compute s of such effectiveness measures, one for each function q. These partial effectiveness measures can finally be aggregated to yield a global effectiveness $\hat{E}$:

$$\hat{E} = f(E_1, \ldots, E_s) \tag{2.12}$$

Table 2.1 summarizes all the notation used until now.

TABLE 2.1

NOTATION	MEANING OF NOTATION
E^i	Partial effectiveness for subsystem i. Varies between 0 and 1.
$\tilde{E}^i$	Mapped effectiveness for subsystem i. Varies between 0 and infinity.
E	System effectiveness with partial measure q.
E_r	System effectiveness with partial measure q_r.

The global effectiveness measure $\hat{E}$ can also be derived by applying the elements of utility theory developed earlier. The partial measures replace the subsystem measures $\tilde{E}^i$; the commodity set is defined now as the set of all effectiveness measures E_1 through E_r.

2.4.3 Conclusion

The framework of the methodology for assessing C^3 systems has been presented throughout Section 2. The analysis has remained voluntarily general bringing only the broad outline of the method. Now these methods will be detailed by applying them to a particular example.

SECTION 3

SYSTEM EFFECTIVENESS ANALYSIS: THE CASE OF A COMMUNICATION NETWORK

3.1 INTRODUCTION

This section presents a simple example of a C^3 system: a communication network. This example will serve to illustrate the application of the methodology described in Section 2. First the system characteristics are defined. Then the notions of mission and context are introduced and their interaction with the network is described. This leads to the specification of origin-destination pairs, which in the description of the methodology have been called subsystems. Finally, assessment criteria or attributes are defined. These attributes will be used in Sections 4 and 5 to define the system and mission loci.

3.2 PRESENTATION OF THE NETWORK

In this section the network's topology and characteristics are presented first. Then the notion of mission and context are introduced and their interactions are analyzed.

3.2.1 Network Topology and Characteristics

The communication network we consider is represented by links and nodes. The chosen topology is shown in Figure 3.1. It consists of 7 nodes and 13 links (each link having a two-way communication capability). Each node corresponds to a decision center. The networks allows these centers to communicate with each other. It is assumed that only links are subject to destruction. Each link has a probability of failure. It also has a certain capacity, i.e., it can handle a limited number of messages per unit of time. A routing algorithm is specified according to the load assigned to the network: the paths used for communication between two nodes are determined as a function of the incoming loads. There are C_7^2 or 21 com-

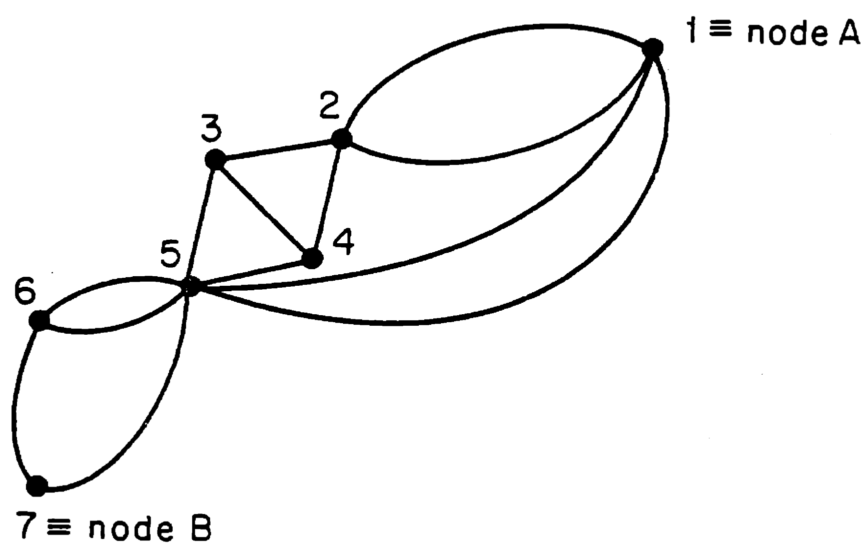


Figure 3.1 Simple communication network.

munication pairs that can exist in the network of Figure 3.1. Clearly, not all of them will be of interest. The next section, which introduces the context and the mission associated with this network, addresses this issue.

3.2.2 Context and Mission

The context for the communication network presented in the preceding section determines (1) where the system will operate, i.e., geographical location, (2) what the weather conditions will be, (3) the enemy's capability for jamming. This information will be part of the primitives for the system.

As noted earlier, each node is a command and control center. However, the centers cannot act independently from each other. They need to communicate and exchange information: this is precisely the task of the network. For instance, if node 7 is a sensor platform and node 5 is provided with weapon systems, then the network has to transmit the information obtained by node 7 to node 5 so that node 5 be able to aim at the targets.

The terms of the mission are defined in the tactical plan, i.e., the tasks and objectives assigned to each command center. These tasks and objectives have been defined according to (1) the overall goal of the mission (2) the specific information provided by the context (location of enemy forces, expected number of enemy forces, etc.). These are primitives for the mission. They will help define the mission attributes, i.e., what the mission requires from the system to fulfill its goals. But before defining attributes, we first need to speficy which part of the system to assess. In Section 2, this stage of the methodology was called *defining subsystems of interest*. In the present case those subsystems are origin-destination communication pairs. Out of the 21 possible subsystems only a limited number may be interesting given the tasks each node can fulfill, the context and the mission. For instance, the communication pair composed of nodes 5 and 7 described, reduces to the subsystem shown in Figure 3.2. All links and nodes which are not involved in this communication pair have been deleted.

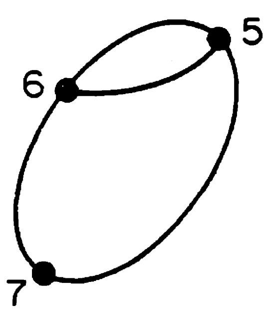


Figure 3.2 Subsystem (5,7) of the communication network.

Let other subsystems of interest be the communication pair (2,5) shown in Figure 3.3 and the pair (1,7) shown in Figure 3.1. In this last case the subsystem is equal to the system as a whole. Nodes 1 and 7 will be denoted as node A and node B in what follows. This particular subsystem will serve as an illustration for the development of the methodology in Sections 4 and 5. Next assessment criteria are defined for the subsystems.

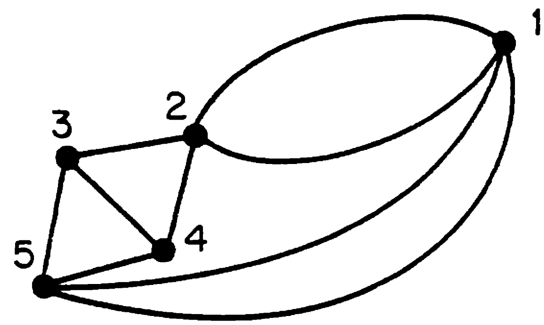


Figure 3.3 Subsystem (2,5) of the communication network.

3.3 DEFINITION OF ATTRIBUTES

In this section attributes for the communication network described in section 3.2 are defined. They are *Survivability*, *Reliability*, *Time Delay*, and *Input Flow*. These attributes will be related to the corresponding primitives.

3.3.1 Survivability

The attribute *Survivability* denotes the capability of the system to resist enemy jamming. System survivability denotes the system's ability to continue functioning in the presence of jamming, while mission survivability designates the survivability required by the mission. This attribute has been chosen because of the growing importance of electronic warfare in designing communication systems. The success of a mission will depend more and more on (1) the accuracy of the information transmitted and (2) the inability of the enemy to decode the information. The attribute *Survivability* intends to account for these factors. The system survivability can be specified more precisely: it will be assumed, in a simple model, that it depends on the probability of the enemy's jamming of each link (this information, as seen before, is context related) as well as the probability of failure of each link in the event of an enemy's jamming.

3.3.2 Reliability

The attribute *Reliability* denotes the capability of the system to provide the command center with working and reliable communication devices. Also the system reliability takes into consideration only the intrinsic characteristics of the components; hence the system primitives considered are the failure probabilities of the links. On the mission side, the primitives are more diffuclt to specify since they are mission dependent. For instance, in the example developed in Section 5, *Reliability* (and also *Survivability*) are related to the measurement radius in target acquisition. The smaller this radius is, the higher the mission reliability should be.

It is necessary to emphasize the distinction between *Reliability* and *Survivability: Reliability* is concerned with the interaction of the network with its natural environment (aging, time, weather, etc.) while *Survivability* is concerned with the interaction of the network with the enemy's electronic warfare capability.

3.3.3 Time Delay

The attribute *Time Delay* introduces the notion of duration of information transmission. This criterion is considered to be critical because in many instances the target acquisition by a weapon system depends on the rapidity with which the information is transmitted from the sensor platform. For each communication pair the system time delay is defined as the sum of the delay in each of the links of the path used. The time delay will be related, in each link, to the capacity of the link (a system primitive). Section 4 describes a model for defining the system time delay. On the other hand, the mission time delay is related to the mission primitives. An example is developed in Section 5.

3.3.4 Input Flow

The attribute *Input Flow* denotes the amount of information transmitted from one node of the network to another. The underlying assumption is that the more input flow is transmitted between two nodes, the better the target acquisition is (in the case of a sensor platform and a weapon system).

The system input flow defines what the system can provide. It will be seen in Section 4 that the links' capacities limit the input flow. Furthermore, it will be shown that a trade-off exists between *Time Delay* and *Input Flow*. On the mission side, *Input Flow* is related to the target acquisition.

3.3.5 Conclusion

In Section 3 a simple communication network has been presented that will be used in illustrating the methodology outlined in Section 2. Subsystems and assessment criteria have been defined. These elements will be used now for defining in more detail the system attributes in Section 4 and the mission attributes in Section 5.

SECTION 4

METHODOLOGY FOR DEFINING SYSTEM ATTRIBUTES

4.1 INTRODUCTION

This section aims at defining system attributes that illustrate the capabilities of the communication network introduced in Section 3. The system attributes Survivability, Reliability, Input Flow and Time Delay are related to the system primitives and analytical expressions are derived. Results from network analysis are presented in section 4.2. They are helpful in carrying out the computation of Reliability and Survivability for single communication pairs that is developed in Section 4.3. In section 4.4 some results of queueing theory are recalled. These results are applied then to the analysis of delay in communication networks. This leads to the definition of the attributes Time Delay and Input Flow in the case of single origin-destination pairs. Furthermore, it is shown how both of these attributes relate to each other and how they can be rescaled or normalized so that all attributes have common range.

4.2 BACKGROUND ON NETWORK ANALYSIS

In this section, the structural relationship between the network and its components (nodes, links) is analyzed. In this approach, a link is either functioning or has failed. Then the results of this first part are used to determine analytical expressions for the reliability or the survivability of the network (at this stage the analysis is valid for both attributes). In the second part, the analysis is probabilistic, i.e., each link has a certain probability of failure (the cause of the failure remaining unspecified in this section).

4.2.1 Structural Analysis

Two nodes of the network, node A and node B, are considered. Let us assign to each link belonging to a path going from node A to node B a binary variable x_i such that:

$$x_i = \begin{cases} 1 & \text{if link } i \text{ is functioning} \\ 0 & \text{if link } i \text{ is failed} \end{cases}$$

Similarly, the binary variable ϕ indicates the state of the communication between A and B.

$$\phi = \begin{cases} 1 & \text{if node A and Node B are able to communicate} \\ 0 & \text{if Node A and Node B are unable to communicate} \end{cases}$$

If we assume that the state of the communication is determined completely by the state of the links, then we may write

$$\phi = \phi(\underline{x})$$

$$\text{where } \underline{x} = (x_1, x_2, .., x_i, ..., x_n) \tag{4.1}$$

The function $\phi(\underline{x})$ is called the structure function of the communication pair (A,B). For example, in the single series structure shown in Figure 4.1, the structure function is given by:

$$\phi(\underline{x}) = \prod_{i=1}^{n} x_i = \min_i (x_i) \tag{4.2}$$

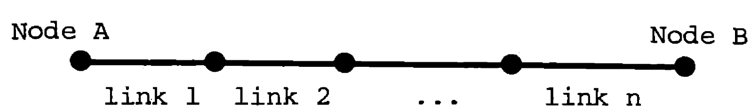


Fig. 4.1 Series structure.

Similarly, for a parallel structure, as shown in Figure 4.2, the structure function is given by

$$\phi(\underline{x}) = \coprod_{i=1}^{n} x_i = \max_i \; (x_i)$$

$$\text{where} \quad \coprod_{i=1}^{n} x_i = 1 - \prod_{i=1}^{n} (1-x_i) \qquad (4.3)$$

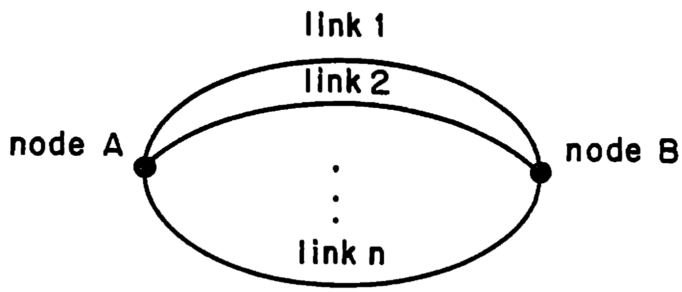


Figure 4.2 Parallel structure.

It can be easily shown, see Barlow and Proschan (1974), that for any type of subnetwork between node A and node B,

$$\prod_{i=1}^{n} x_i \leq \phi(\underline{x}) \leq \coprod_{i=1}^{n} x_i \qquad (4.4)$$

However, if one wants to determine the exact form of the structure function for the communication pair (A,B) it is necessary to use the notion of path-set or cut-set.

Let's assume that the subset of the network between node A and node B consist of n links. Let the n-dimensional binary vector $\underline{x}$ indicate the state of those links: a zero in the i-th coordinate signifying that the i-th link has failed, and a one signifying that the i-th link is operational.

Then let us denote by a path between node A and node B a vector $\underline{x}$ such that $\phi(\underline{x})$, its structure function, is equal to one. The path is said to be a minimal path when the removal of one of its links causes the communication to fail.

Let us denote by a cut between node A and node B a vector $\underline{x}$ such that $\phi(\underline{x})$ is equal to zero. The cut is said to be minimal when substitution of an operational link for any one link in the cut reestablishes communication between A and B.

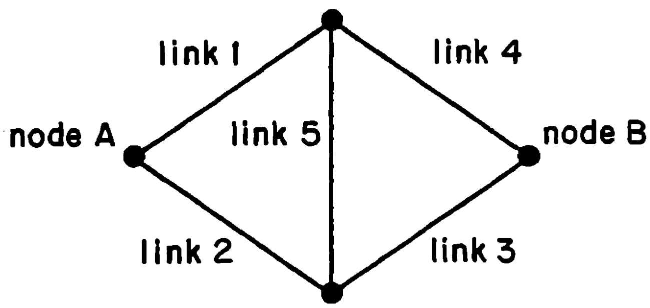


Figure 4.3 Bridge structure.

For instance if we consider the bridge structure shown in Figure 4.3, the minimal path set is:

$P_1 = (1,0,0,1,0)$, i.e., link 1 and 4
$P_2 = (0,1,1,0,0)$, i.e., link 2 and 3
$P_3 = (1,0,1,0,1)$, i.e., link 1,3 and 5
$P_4 = (0,1,0,1,1)$, i.e., link 2, 4 and 5

Similarly the minimal cut set is:

$C_1 = (0,0,1,1,1)$, i.e., link 1 and 2
$C_2 = (1,1,0,0,1)$, i.e., link 3 and 4
$C_3 = (0,1,0,1,0)$, i.e., link 1, 3 and 5
$C_4 = (1,0,1,0,0)$, i.e., link 2, 4 and 5

With each path P_j of the path set it is possible to associate a binary function ρ_j with arguments x_i, $i \in P_j$:

$$\rho_j(\underline{x}) = \prod_{i \in P_j} x_i \qquad (4.5)$$

If each of the links belonging to path j are functioning, then ρ_j takes the value one, otherwise the value is zero.

Since only one path needs to be working for node A and node B to be able to communicate, the structure function can be expressed as:

$$\phi(\underline{x}) = \coprod_{j=1}^{P} \rho_j = 1 - \prod_{j=1}^{P} [1-\rho_j(\underline{x})] \qquad (4.6)$$

where P is the total number of paths between A and B. Similarly, with each cut C_j of the cut set it is possible to associate a binary function η_j with argument x_i, $i \in C_j$.

$$\eta_j = \coprod_{i \in C_j} x_i \qquad (4.7)$$

It follows that η_j takes the value zero if all the links of the cut fail and the value 1 otherwise.

Since only one cut is necessary to have the communication fail, i.e., one of the η_j be equal to zero, the structure function can be expressed as:

$$\phi(\underline{x}) = \prod_{j=1}^{r} \eta_j = \prod_{j=1}^{r} \coprod_{i \in C_j} x_i \qquad (4.8)$$

where r is the total number of cuts between A and B. Thus, with equation (4.6) we have found two different expressions for the same quantity. This result will be used to advantage in section 4.2.2. It should be noted that algorithms exist to find all paths or all cuts. For example Barlow and Proschan (1974, pp. 256-266) have described an algorithm based on event trees for finding the cut set of a given structure.

4.2.2 Probabilistic Analysis

In this section, the variable x_i defined in the previous section is now a random variable, with p_i defined as:

$$\text{prob } (x_i=1) = p_i = E(x_i) \quad \text{for } i=1,\ldots,n \tag{4.9}$$

where E(x) denotes the expected value of the random variable x.

Hence, in what follows, p_i will denote the probability that link i is functioning; obviously $1-p_i$ will denote the probability that link i has failed (the reason of the failure is not stated here).

Similarly, for the origin-destination pair as a whole, the probability that both A and B are able to communicate is given by:

$$\text{prob } (\phi(\underline{x}) = 1) = h = E[\phi(\underline{x})] \tag{4.10}$$

where $E[\phi(\underline{x})]$ is the expected value of the random variable $\phi(\underline{x})$.

In section 4.3, distinction will be made between Reliability and Survivability. At this level of the analysis it is, however, not necessary. Thus a generic function h will be used.

If it is assumed that the links are statistically independent, i.e., the failure of one link is not influenced by the failure of another link, then the generic function h can be expressed as a function of the individual p_i:

$$h = h(\underline{p}) \tag{4.11}$$

where $\underline{p} = (p_1,\ldots,p_n)$ and n is the number of links. By (4.6) and (4.8) we have that:

$$\phi(\underline{x}) = \coprod_{j=1}^{P} \prod_{i\varepsilon P_j} x_i$$

and

$$\phi(\underline{x}) = \prod_{j=1}^{r} \coprod_{i \epsilon C_j} x_i$$

Expanding the right-hand side into multinomial expressions in the x_i's and using the idenpotency of x_i (i.e., $x_i^2 = x_i$), we can compute the function by taking the expectation:

$$h(\underline{p}) = E\left[\coprod_{j=1}^{p} \prod_{i \epsilon P_j} x_i\right] \quad (4.12)$$

$$h(\underline{p}) = E\left[\prod_{j=1}^{r} \coprod_{i \epsilon C_j} x_i\right] \quad (4.13)$$

For instance, in the case of the bridge structure shown in Figure 4.3, assuming that $p_1 = \ldots = p_5 = p$, then:

$$h(p) = 2p^2 + 2p^3 - 5p^4 + 2p^5 \quad (4.14)$$

This computation is already complex for a rather simple network. Thus it is desirable to compute approximate expressions.

Actually the different paths between node A and node B cannot be assumed in general to be independent (since it is likely that some of them have components in common). Barlow and Proschan (1974) have shown that, when at least two paths (resp. two cuts) overlap, the following inequality holds:

$$b(\underline{p}) < h(\underline{p}) < B(\underline{p}) \quad (4.14)$$

where

$$b(\underline{p}) = \prod_{j=1}^{r} \coprod_{i \epsilon C_j} p_i$$

and

$$B(\underline{p}) = \coprod_{j=1}^{p} \prod_{i \varepsilon P_j} p_i$$

In the case where no cuts or no paths overlap than either:

$$h(\underline{p}) = B(\underline{p})$$

or

$$h(\underline{p}) = b(\underline{p})$$

Application of this result to the bridge structure with all p_i equal to p yields:

$$b(p) = p^4 (2-p^2)^2 (3-3p+p^2)^2 \tag{4.15}$$

$$B(p) = 1 - (1-p^2)^2 (1-p^3)^2 \tag{4.16}$$

Figure 4.4 presents a graphical comparison of b(p), h(p) and B(p) for the bridge structure.

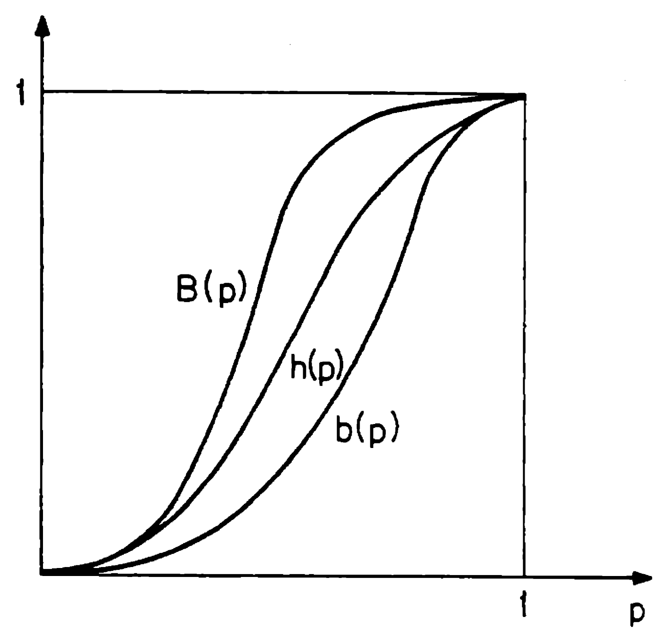


Figure 4.4 Bounds on the h function for the bridge structure.

An expression of b(p) when p is close to one (i.e., low probability of failure) shows that:

$$h(p) - b(p) = 5\,\varepsilon^4 + O(\varepsilon^4) \tag{4.17}$$

with $1-p = \varepsilon$

whereas the difference between B(p) and h(p) is only of the order of ε^2. So clearly, for high p (the case for communication networks) b(p) is the best approximation of the true value h(p). For instance, when p is set equal to 0.8 the error between the approximated value obtained with eq. (4.15) and the true value obtained with (4.14) is less than 0.7 percent. This result has been shown in a specific example but it remains true for most of the usual configurations. In any case, the true value is in the range [b(p), B(p)].

In this section it has been shown how to compute the true value of the function h, and also an approximate value for it. These results are used next in defining the notion of Survivability and Reliability.

4.3 SURVIVABILITY AND RELIABILITY

This section introduces to the computation of Survivability and Reliability for a single origin-destination pair. First, based on the example of a network given in Section 3, it is shown how to develop a function h for the chosen pair. Then Survivability and Reliability are introduced and related to the system primitives.

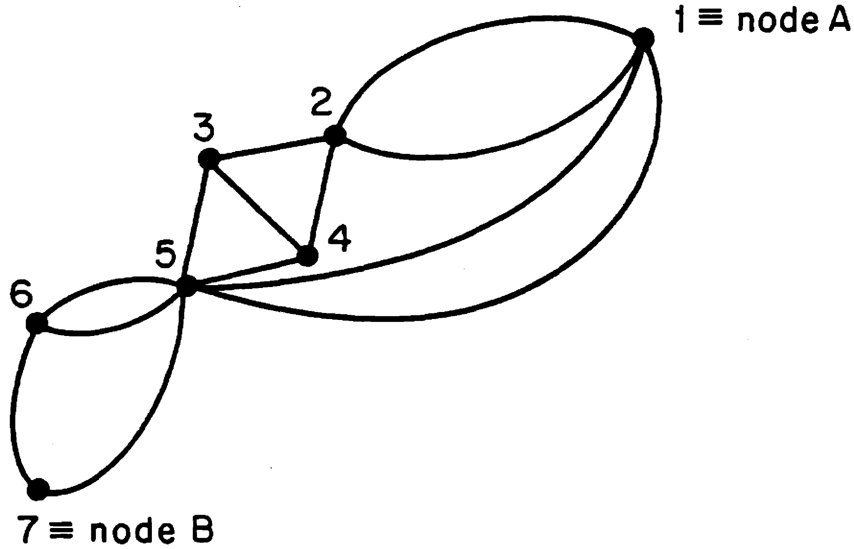
4.3.1 Determination of the Function h

In determining $h(\underline{p})$ two cases must be considered:

(a) the subset of the network between node A and node B has no identifiable disjoint subsystems. In this case the methods of section 4.2.2 have to be applied.

(b) The subset of the network between A and B can be decomposed in disjoint subsystems or modules. These modules, in turn, can be replaced by equivalent links with probabilities equal to the function h derived for the module. By doing so, the network topology is considerably simplified and the procedure can then be repeated until the network reduces to one link.

The following example derived from the network shown in Section 3 and reproduced in Figure 4.5 illustrates this method.



Figire 4.5 Simple communication network

The nodes of interest are 1 and 7. It is impractical to find all paths (or cuts) between node A and node B and apply the results of section 4.2.2. More simply stated, the network between A and B can be decomposed into modules. In the example of Figure 4.5, four modules can be identified:

- the two-link connection between 1 and 2 denoted by m_1
- the two-link connection between 1 and 5 denoted by m_2
- the bridge-connection between 2 and 5 denoted by m_3
- the four-link connection between 5 and 7 denoted by m_4.

For each of the modules m_i, either the function $h_{m_i}(\underline{p})$ is already known or it can be computed according to the method of section 4.2.2.

In the case of the previous example, this yields, with all p_i's equal to p,

$$h_{m_1}(p) = 1 - (1-p)^2$$

$$h_{m_2}(p) = 1 - (1-p)^2$$

$$h_{m_3}(p) = 2p^2 + 2p^3 - 5p^4 + 2p^5 \qquad (4.18)$$

$$h_{m_4}(p) = 1 - (1-p^2(2-p)^2)(1-p)$$

It is, then, possible to replace the network of Figure 4.5 by the equivalent network shown in Figure 4.6.

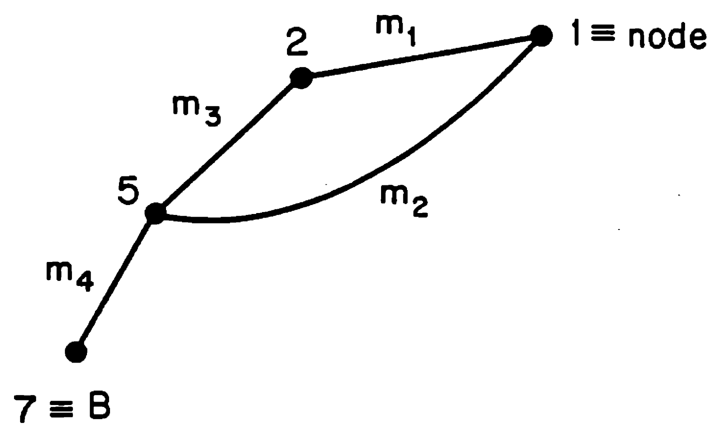


Figure 4.6 Equivalent network.

where each module m_i is replaced by a link with probability of failure $1-h_{mi(p)}$.

Finally this reduces to a single link between node A and node B with function $h_{A,B}(p)$ equal to:

$$h_{A,B}(p) = h_{m_4}(p) \bullet [1-(1-h_{m_2}(p)) \bullet (1-h_{m_1}(p) \bullet h_{m_3}(p))] \qquad (4.19)$$

The final expression for the function h is obtained by substituting into (4.19) the expressions for the modules shown in (4.18).

This method is likely to be preferred to the path-cut set method especially when the network consists of simple and well known modules.

4.3.2 System Reliability

Until now, it has not been necessary to specify the meaning of the p_i's introduced in section 4.2.2. It was only stated that they represented the probability of a link functioning. However, the complementary probability $(1-p_i)$, i.e., the probability of failure is likely to have different causes. In this section on Reliability the probability of failure of a link is assumed to be due to the intrinsic characteristics of the links. It depends on such characteristics as the reliability of the components of the links, the time of the year, or the weather conditions (magnetic thunderstorms may disturb radio communication). Reliability denotes the capability of the network to deliver a message from node A to node B when only the technical properties of the link are taken into account. Hence Reliability can be expressed as a function of the individual p_i's (see section 4.3.1):

$$R = h_{A,B}(\underline{p}) \qquad (4.20)$$

where $p = (p_1,\ldots,p_n)$.

Generally the p_i's will take different values since (i) link reliability is likely to vary with time, (ii) it can be assumed that link components are replaced after a certain number of hours of service so that ageing does not occur. Hence each p_i is likely to take values in an interval of [0,1]. It is possible for the p_i's not to have uniform probability distribution; this case has not been studied. Thus, in the n-dimensional primitive space $(p_1,\ldots,p_n)$ the feasibility set is a hypercube, in which the reliability function R takes its value. On this hypercube R takes minimum values and maximum values (because R is a continuous function defined on a closed set of $\mathbb{R}^n$). Furthermore, the minimum value for R is going to be obtained for the lowest vertex of the hypercube and the maximum value for the highest vertex; the lowest and highest vertices defined respectively, as the vertex with the smallest sum of its coordinates, and the vertex with the largest sum of its cooridnates. The reason, therefore, is that the Reliability, $R=h_{A,B}(p)$, is an increasing function of its arguments.

Once the bounds for R, R_{min} and R_{max}, have been found, the system's capability in terms of the attribute Reliability for the origin-destination pair considered is:

$$R_{min} \leq R \leq R_{max} \tag{4.21}$$

A similar analysis will be carried out, next, for the attribute Survivability.

4.3.3 System's Survivability

The attribute Survivability, in contrast to Reliability, does not depend on the links' physical deterioration, but on the links' capabilities to resist an enemy attack. For a communication network, this capability reflects the jamming resistance of the links. Hence, Survivability does not account for technical failures; when the enemy does not jam one particular link then the Survivability p_i of the link is unity.

This shows that two factors have to be taken into account when computing the Survivability of a communication pair:

(a) The probability that the enemy is going to attack link i. This information is not a system characteristic, but comes from the context in which the mission is going to take place. Military experts are expected to have some information on the enemy's jamming capability in different environments.

(b) The probability of a link being jammed in the event of an attack. This information is system dependent, since it is related to the properties of the link.

So, if the probability of an attack on link i is denoted by e_i and the probability of jamming link i under the event of an attack by f_i then the Survivability p_i of link i is given by:

$$p_i = 1 - e_i f_i \tag{4.22}$$

The method described in section 4.2.1 and 4.2.2 can be applied to this situation. Survivability, S, can be expressed as:

$$S = h_{A,B}(\underline{p}) \tag{4.23}$$

where $p = (p_1,\ldots,p_n)$ and p_i is the survivability of link i.

For the same reasons as described in section 4.2.2 for Reliability, the system's capability in terms of the attribute Survivability for the origin-destination pair considered is defined by:

$$S_{min} \leq S \leq S_{max} \tag{4.24}$$

The results of (4.21) and (4.24) will be used in Section 6 to define the system attribute space which is going to be compared, there, with the mission attribute space.

4.4 TIME DELAY AND INPUT FLOW

In this section some elements of queueing theory are reviewed and then applied to communication networks. The Time Delay and Input Flow attributes for a single origin-destination pair are defined next. Eventually, both of these attributes are rescaled so that they take values between zero and one.

4.4.1 Elements of Queueing Theory

Queueing theory is used to model the processes in which customers arrive, wait their turn for service, and then leave. Five components characterize queueing systems: (i) the interarrival time probability density function, (ii) the service time probability density function, (iii) the number of servers, (iv) the queueing discipline, (v) the amount of buffer space in the queue.

The case of infinite buffer, single server system using the first come first serve discipline will be used. Moreover, the interarrival probability and the service time probability will be assumed to be exponential. This is a, so called, M/M/1 model (see Figure 4.7) which has been studied extensively and is easy to use.

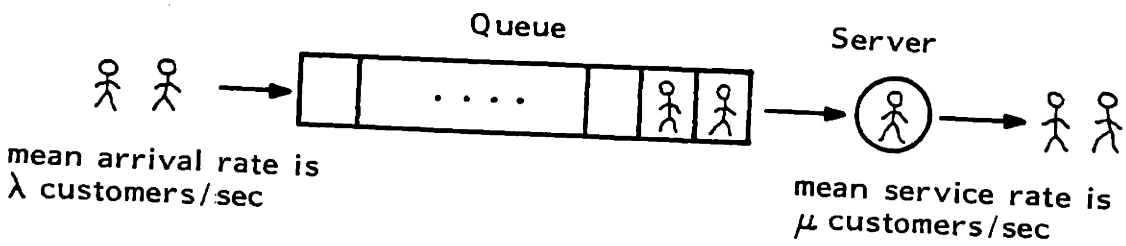


Figure 4.7 A single server queueing system.

Let f(t) be the density function for the interarrival time,

$$f(t) = \lambda e^{-\lambda t}$$

then we know that the mean average time between two arrivals is $1/\lambda$ sec, i.e., λ customers per second.

Similarly, let g(t) be the service time probability density function,

$$g(t) = \mu e^{-\mu t}$$

then this yields a mean service time of $1/\mu$ sec/customer.

Intuitively it can be seen that a stable system, i.e., a system whose queue does not go to infinity has to have a mean service rate μ greater than the mean arrival rate λ. This condition appears in the expression for the total number N of customers in the system (see Tanenbaum (1981)):

$$N = \frac{\lambda}{\mu - \lambda} \tag{4.25}$$

The total mean waiting time T is then

$$N = \lambda\, T \tag{4.26}$$

Substituting for N in (4.25) by its expression from (4.26) yields:

$$T = \frac{N}{\lambda} = \frac{1}{\mu - \lambda} \tag{4.27}$$

This value is used, next, to compute the delay at one node of a communication network.

4.4.2 Application of M/M/1 Models to Communication Networks

The M/M/1 model can be applied to the problem of finding the queueing delay for packets (grouping of coded information) at a node. Let us take the example of two nodes, say i, and j, connected with a link (see Figure 4.8). Let the probability density function for packet size in bits be $\mu e^{-\mu x}$ with mean of $1/\mu$ bits/packet.

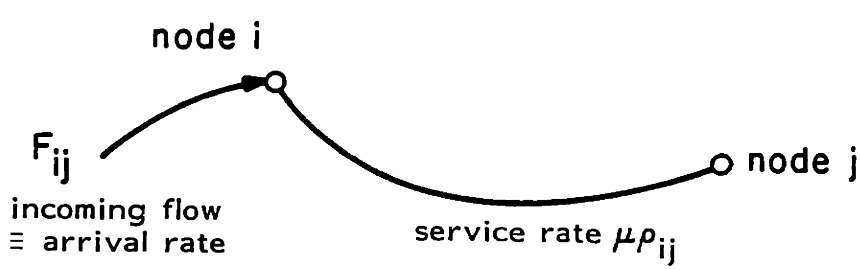


Figure 4.8 M/M/1 applied to a communication link.

Now introduce the capacity C_{ij} of the link i-j measured in bits/sec. The product μC_{ij} is then the service rate in packets/sec. The arrival rate, or input flow, for the link i-j is F_{ij} packets/sec. Equation (4.27) can now be rewritten as:

$$T_{ij} = \frac{1}{\mu C_{ij} - F_{ij}} \tag{4.28}$$

T_{ij} is the mean delay of a packet arriving at node i before being transmitted to node j (it includes both queueing and transmission time). One can see that delay never goes to zero even when traffic is light ($F_{ij} \to 0$): the minimum value of the delay is then equal to $1/\mu C_i$ which represents the transmission time.

However, one major problem when applying M/M/1 models to networks is that the output of one link becomes the input to another. Burke (1956) has solved the problem by showing that if the outputs of several M/M/1 servers feed into the input queue of another server, the resulting input process is also a Poisson process, with mean equal to the means of the feeding processes. A further difficulty arises from the fact that when a packet moves around the network, it maintains it size. This property introduces non random correlation into the system. The "Independence Assumption" allows us to get around this problem by assuming that every time a packet arrives at a node it looses its idendity and a new length is chosen at random. Tanenbaum (1981) and Schwartz (1977) have developed this issue in more detail.

4.4.3 Computation of the Time Delay and Input Flow

The expression for the time delay, (4.28), will be used in this section for computing the total delay for a given origin-destination pair. Furthermore, it will allow us to relate total delay to input flow.

For illustrative purposes, we return to the network introduced in Section 3 and shown in Figure 4.5. We want to find the transmission delay for a packet sent from node A to node B. Each link of the network between

A and B has its own capacity, C_k, with $k \varepsilon \{1,2,...,13\}$. They are 30 different paths that can be chosen to send a message from A to B. Also 30 time delays can be computed, one for each path. For instance, if the path P_j is chosen, then the total delay T_{P_j} can be expressed as:

$$T_{P_j} = \sum_{C_k \varepsilon P_j} \frac{1}{\mu C_k - F} \tag{4.29}$$

where F is the input flow that can be transmitted from node A to node B and $1/\mu$ the mean number of bits/packet.

Thus, it is possible to define over the path set P:

$$T_{min} = \underset{P_j \varepsilon \underline{P}}{\text{Min}} \; (T_{P_j}) \tag{4.30}$$

$$T_{max} = \underset{P_j \varepsilon \underline{P}}{\text{Max}} \; (T_{P_j}) \tag{4.31}$$

So depending on the routing algorithm chosen T may vary between T_{min} and T_{max}:

$$T_{min} \leq T_{A,B} \leq T_{max} \tag{4.32}$$

It has beem assumed, however, that the capacities, C_k, are fixed while they may also vary in a certain range. This assumption does not change much the result obtained in (4.32). Simply T_{min} is obtained when all capacities are set to their highest values, and T_{max} when all capacities are set to their smallest values.

For instance, we may further assume that for any K;

$$C_k = C \qquad \text{for all } k$$

and that C may vary between C_{min} and C_{max}. Then it is easy to see for the network of Figure 4.5 that the total delay between A and B satisfies:

$$\frac{2}{\mu C_{max}-F} \leq T \leq \frac{6}{\mu C_{min}-F} \qquad (4.33)$$

Relations (4.32) and (4.33), hence, show that it is possible to relate the Time Delay and Input Flow attributes. These conditions define the network's capabilities; relation (4.33) defines in the plane (F,T) the locus of possible values for both attributes.

4.4.4 Scaling of Input Flow and Time Delay

The expression found in (4.33) shows that T may vary between $2/\mu C_{max}$ and infinity. Similarly F may vary between zero and μC_{min} (excluded). However, both other attributes, Reliability and Survivability vary between 0 and 1. So, in order to have the same range for all attributes it is necessary to normalize T and F so that they take values between 0 and 1.

An obvious scaling factor for T is T* defined as the maximum duration of the mission. Let t be the scaled value of the Time Delay T then,

$$t = \frac{T}{T^*} \quad \text{and } 0 \leq t \leq 1 \qquad (4.34)$$

For the Input Flow F the most appropriate scaling factor F* is defined by:

$$F^* = \mu \underset{i}{\text{Max}}\, C_i \qquad (4.35)$$

where i is the link index.

Let K then be scaled input flow:

$$K = \frac{F}{F^*} \qquad (4.36)$$

These values of t and K will be used later in Section 6 where the system's capabilities are compared with the requirements of the mission.

SECTION 5

METHODOLOGY FOR RELATING THE MISSION ATTRIBUTES

5.1 INTRODUCTION

This section focuses on the relationships between the mission attributes for the communication network presented in section 3.2. The mission requirements and their interactions are described in section 5.2. A simple engagement model based on Lanchester equations is introduced in section 5.3. From this model, certain conditions are derived in section 5.4. These conditions have to be met by the mission's attributes in order that the mission's requirements be satisfied.

5.2 MISSION REQUIREMENTS

We recall from Section 3 that in any particular situation only certain nodes need communicate and exchange information. These origin-destination pairs are intended to support the overall goals of the mission as efficiently as possible. The following analysis looks at the effectiveness of each of those communication pairs in terms of the attributes *Survivability*, *Reliability*, *Delay* and *Input Flow*. For simplicity, only one origin destination pair will be considered. Within the tactical plan specified by the mission, each of the nodes, say A and B, has been assigned a specific task.

Node A, for instance, can be a sensor platform whose mission is to locate and identify enemy forces. However, this information is without any value, if it cannot be communicated to node B, whose mission is to defend against the forces located by node A. For instance, node B may be an aircraft carrier which needs information on the location of the enemy's fleet in order to direct its aircraft toward their targets.

The task assigned to B may be, for example, to destroy as much as m percent of the enemy forces while its own forces (aircraft) should suffer no more than n percent of losses.

Let x(t) denote the number of our forces (blue) and y(t) the number of the enemy's units (orange). Then the two conditions can be written as:

$$\frac{x(t)}{x(0)} \geq n \tag{5.1}$$

$$\frac{y(t)}{y(0)} \leq m \tag{5.2}$$

The quantity x(0) denotes the number of blue forces before the engagement. This information is clearly a primitive for the mission, since the number of aircraft allocated to the mission depends on the tactical plan (and is restricted by the number of units available). Similarly the quantity y(0) denotes the number of orange units before the engagement. This primitive depends both on the context and on the mission; on the context because the geographical location specifies the size and type of expected enemy forces; on the mission because the tactical plan and the resources available may restrict the blue forces to attack only part of the orange forces. Criteria for deciding which orange forces to attack may be, for example, the concentration of their forces or their capabilities to be replaced during the engagement.

The success of blue's mission also requires that the engagement never result in a situation where the blue forces have no unit left while the orange forces have some units still available. This can be expressed simply by requiring that:

$$x(t) = 0 \quad \text{and} \quad y(t) > 0$$

should *not hold* for any time t.

5.3 LANCHESTER-TYPE COMBAT MODEL

Lanchester models aim at describing modern warfare where dispersed forces can focus their fire power on a single target. The type of Lanchester model considered is the "salvo fire" engagement. In this model each blue unit (resp. each orange unit) fires every t_x (resp. t_y) units of time at random at any orange unit (resp. at any blue unit). By introducing p_x, the single shot probability of kill of an orange unit by a blue unit, and p_y, the single shot probability of kill of a blue unit by an orange unit, (Mangulis (1980)) has shown that the solution to this problem verifies the set of equations:

$$\frac{dx}{dt} = \frac{x}{t_y}\left[1 - \left(1 - \frac{p_y}{x}\right)^y\right]$$

$$\frac{dy}{dt} = \frac{y}{t_x}\left[1 - \left(1 - \frac{p_x}{y}\right)^x\right] \qquad (5.3)$$

If the single shot probabilities of kill are small then (5.4) can be approximated by:

$$\frac{dx}{dt} = -\frac{p_y}{t_y}\, y$$

$$\frac{dy}{dt} = -\frac{p_x}{t_x}\, x \qquad (5.4)$$

This simplified version of the "salvo fire" engagement is a particular case of the "square law" attrition process:

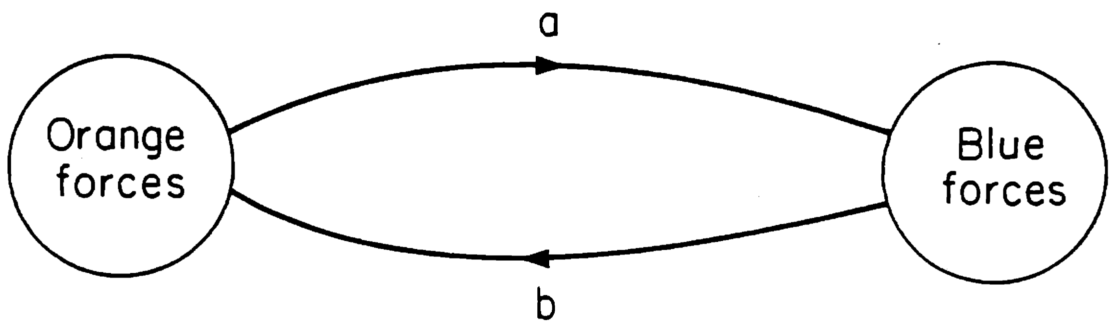
$$\frac{dx}{dt} = -\, ay$$

$$\frac{dy}{dt} = -\, bx \qquad (5.5)$$

where a and b denote the attrition rate coefficients (Figure 5.1 gives a representation of the process). Clearly, in (5.4) the attrition rate of the blue forces is $a = p_y/t_y$ and the attrition rate of the orange forces is $b = p_x/t_x$.

An interesting property of such a model is revealed by integrating (5.3) to yield:

$$a\, y^2(t) - b\, x^2(t) = a\, y^2(0) - b\, x^2(0) \qquad (5.6)$$



x(t): number of units in the blue forces.

y(t): number of units in the orange forces

a: attrition rate on the blue forces

b: attrition rate on the orange forces

Figure 5.1 Lanchester combat model.

This result will be used to advantage in the next section where the attrition rate on the orange forces, b, is related to the mission attributes. It should be clear, however, that, if this model gives a good basis for describing the requirements of the mission, it remains rather simplistic. Wohl (1981) and Ekchian (1982) notice that the square law attrition process assumes aimed fire which is not always a realistic assumption. It also assumes homogeneity of forces, i.e., forces are composed of identical units, which is most often not the case (two vessels at random are unlikely to be identical). The replacement or withdrawal of force is disregarded and the assumption of small single-shot probabilities of kill is debatable. Despite these shortcomings it is interesting to explore the applicability of such a model for deriving conditions on the mission attributes.

5.4 DERIVATION OF CONDITIONS FOR THE MISSION ATTRIBUTES

Our intent, in this section, is to relate the mission attributes, Survivability, denoted by S, Reliability , denoted by R, Input Flow, denoted by F, and Delay denoted by T to each other using the Lanchester warfare model described in the previous section. In what follows the attrition coefficient, a, from equation (5.5) will be considered a constant characterizing the enemy's fighting capability. On the other hand the attrition rate, b, will be expressed as a function of the weapons characteristics of the blue forces as well as of the mission attributes.

5.4.1 Computation of the Attrition Rate b

The attrition rate b is defined in terms of the kill probability of a salvo, p_x, and of the time between salvos, t_x. If T is the delay in transmitting information from node A (sensor) to node B (aircraft carrier) then the time interval between salvos becomes $t_x + T$ where t_x is an intrinsic characteristic of the weapon system.

Let r_x denote the kill radius of the blue weapon system. Let R_x denote the radius of uncertainty in locating the orange targets. Then, assuming a perfect network, i.e., all the information provided by the sensor is transmitted to the platform instantaneously without failure, p_x can be expressed as:

$$p_x = \frac{\pi r_x^2}{\pi R_x^2} \tag{5.7}$$

However, the network is never so perfect as to provide the "ideal" value R_x. Also, in reality, the actual value of the measurement radius, R_x' , is going to be a function of the attributes S, R, F, T as well of the velocity V of the orange targets. Hence R_x' can be written as

$$R_x' = f(S,R)\ g(F,T,V) \tag{5.8}$$

Clearly, since R_x is the smallest radius of uncertainty that can be obtained, R_x should satisfy (5.8) when S is set to 1, R to 1, F to maximum value F_{max} and T to 0.

The function g can be specified more precisely:

$$g(F,T,V) = h(F) + VT \tag{5.9}$$

where h(F) accounts for the fact that the accuracy in the measurement radius depends on the flow of information from node A to node B. The terms VT accounts for the fact that the target has moved during the transmission time T. The velocity V is assumed to be constant.

Combining equations (5.7), (5.8) and (5.9) yields:

$$p_x = \frac{r_x^2}{f^2(S,R)\ (h(F) + VT)^2} \tag{5.10}$$

and for the corresponding attrition rate

$$b = \frac{r_x^2}{f^2(S,R)\ (h(F) + VT)^2\ (t_x + T)} \tag{5.11}$$

5.4.2 <u>Relationships Between the Attributes</u>

In this section a condition for the mission attributes is going to be derived from the requirements of the mission.

The solutions of the square law attrition model shown in (5.5) can be expressed as:

$$x(t) = \frac{1}{2}\left(x(0) + \sqrt{\frac{a}{b}}\,y(0)\right) e^{-\sqrt{ab}\,t} + \frac{1}{2}\left(x(0) - \sqrt{\frac{a}{b}}\,y(0)\right) e^{\sqrt{ab}\,t}$$

$$y(t) = \frac{1}{2}\left(x(0)\sqrt{\frac{b}{a}} + y(0)\right) e^{-\sqrt{ab}\,t} + \frac{1}{2}\left(-x(0)\sqrt{\frac{b}{a}} + y(0)\right) e^{\sqrt{ab}\,t}$$

(5.12)

These solutions are valid for all positive t. However, when either x(t) or y(t) becomes null the combat ends. As noticed in section 5.2 the mission requires that:

$$x(t) = 0 \quad \text{and} \quad y(t) > 0 \tag{5.13}$$

never holds. From (5.12) this implies that

$$x(0) - \sqrt{\frac{a}{b}}\, y(0) > 0$$

or

$$\frac{b}{a} > \frac{y^2(0)}{x^2(0)} \tag{5.14}$$

Hence, whenever condition (5.14) holds, the orange forces are defeated.

However, other conditions for the success of the mission have been stated in (5.1) and (5.2). Substituting these conditions into (5.6) yields:

$$b\, x^2(0) - a\, y^2(0) > b\, n^2\, x(0) - a\, m^2\, y(0)$$

which reduces finally to:

$$\frac{b}{a} > \frac{y^2(0)\ (1-m^2)}{x^2(0)\ (1-n^2)} \tag{5.15}$$

Defining s as the force ratio, $s=x(0)/y(0)$, it follows from (5.14) that the ratio of the attrition rates, b/a, has to be greater than the inverse of the square of s, whereas in (5.15) a new factor, $1-m^2/1-n^2$, modifies the condition. Clearly (5.15) yields a less restrictive condition than (5.14) when m is greater than n. However, we can expect this condition not be verified since the terms of the mission certainly try to minimize blue losses while maximizing orange losses. Thus inequality (5.15) is the relation to be used for defining the constraint on the mission attributes. Substituting for b in (5.11) yields:

$$b = \frac{r_x^2}{f^2(S,R)\ (h(F) + VT)^2 (t_x+T)} \quad > \quad a\ \frac{1 - m^2}{s\ (1-n^2)} \qquad (5.16)$$

Inequality (5.16) might not be satisfied for any values of S, R, F and T. This happens when b_{max} does not satisfy (5.15), where b_{max} is defined as the maximum value of b obtained when the radius of uncertainty R_x' takes its smallest value, R_x. This occurs when the network is working perfectly. Simplification of equation (5.11) yields for b_{max}:

$$b_{max} = \frac{r_x^2}{R_x^2\ t_x}$$

Obviously, for any S, R, F and T, the attrition rate b as defined in (5.11), satisfies

$$b = P(S,R,F,T,) \leq b_{max}$$

Clearly, if b_{max} does not satisfy (5.15)

$$b_{max} < \frac{a\ (1-m^2)}{s^2(1-n^2)}$$

inequality (5.16) will never hold. Thus, in order to have a solvable problem one has to make sure that condition (5.15) is satisfied by b_{max}.

5.4.3 Scaling of the Input Flow and Delay Attributes

The scaling proposed for F and T is the same as in Section 4. The rationale for rescaling is that all attributes have the same range, the interval [0,1].

An appropriate scaling factor for the input flow attribute may be F*, defined as the upper bound on the flow in the link with the highest capacity among all paths between node A and node B. Then, as in equation (4.36) K is defined as:

$$K = \frac{F}{F^*} \quad \text{and } 0 \leq K \leq 1 \tag{5.17}$$

In the same way the delay attributes T can be rescaled into t by using as scaling factor T*, defined as the maximum duration of the mission as seen in equation (4.34). Then T becomes:

$$t = \frac{T}{T^*} \quad \text{and } 0 \leq T \leq 1 \tag{5.18}$$

Consequently, condition (5.16) can be rewritten now using the normalized attributes S,R,K and t as:

$$b = \frac{r_x^2}{f^2(S,R)\ (h(F^*K) + VT^*t)^2 (t_x + T^*t)} > \frac{a(1-m^2)}{s^2(1-n^2)} \tag{5.19}$$

Inequality (5.19) defines a certain locus in the four-dimensional space (S,R,K,t). In the next section this locus is compared with the locus derived from the system attributes.

SECTION 6

MEASURES OF EFFECTIVENESS

6.1 INTRODUCTION

This section addresses the definition of measures of effectiveness. This is achieved by defining first measures of effectiveness for a single origin-destination pair and then, by combining those measures for all the origin destination pairs, forming an overall or global measure of the system's effectiveness. Section 6.2 focuses on comparing the analytical expressions characterizing the system and the mission. Then, in section 6.3, it is shown how the partial measures of effectiveness derived in Section 6.2 can be combined to specify the criteria that may allow the decisionmaker to define a global effectiveness measure.

6.2 EFFECTIVENESS ASSESSMENT OF SINGLE ORIGIN-DESTINATION PAIRS

This section develops a measure of effectiveness for a single origin-destination pair. The proposed method is first described and then the conditions derived from the analysis of the system and of the mission are presented. Eventually, the requirements of the system and of the mission are compared in a multidimensional attribute space.

6.2.1 Comparison Between System Capabilities and Mission Requirements

In Sections 4 and 5, it was shown how to derive conditions for the system attributes and for the mission attributes. As noted earlier, these attributes are the same for the mission and for the system. On the one hand, they determine what is required from the system to meet the specifications of the mission; on the other hand, they determine what the system can actually provide given the context in which the mission takes place. The four attributes being considered are:

S = Survivability

R = Reliability

K = Scaled Input Flow

t = Scaled Time Delay

Clearly, they can be represented in a four dimensional space (S,R,K,t) or, more precisely, in the unit hypercube with the origin as one of its vertices. Hence, the best way to compare the system and the mission would be to work in this four-dimensional space and determine if the locus defined by the system and the locus defined by the mission within the hypercube intersect. A possible simplification would be to look at the projections of the four-dimensional conditions into three-dimensional spaces. This idea appears appealing at first, however, it will be shown that relying on such projections might be highly misleading. In particular, the projection methods tends to over estimate the actual effectiveness measure. This does not mean that the projections should not be used. They can, actually, be very helpful in bringing more insight on the actual shape of the system and mission loci. They should, however, be used very cautiously as the following analysis will show.

The measure of effectiveness is based on the comparison of the volumes defined by the system and mission loci. The following analysis is valid in a n-dimensional space (if n attributes are considered). It will be applied later to the four-dimensional example discussed earlier.

Typically, three configurations may be encountered in comparing the volumes of the system and mission loci:

(1) The system does not satisfy at all the requirements of the mission. In this case, the locus of points that characterize the capabilities of the system will not intersect with the locus specified by the mission requirements.

(2) The system satifies only partly the requirements of the mission. In this case, the two loci will intersect.

(3) The system satisfies entirely the mission requirements. In this case, the locus for the system is included in the locus specified by the mission.

The problem that arises is how to assess in a quantitative way the performance of the system. A first answer would be to look at the ratio of the "volume" of that part of the system locus which satisfies the mission to the volume of the total system locus. However, such a measure, which uses volume as a metric, does not account for the possibility that the system may not meet at all, or exceed the requirements of the mission. For instance, whether the system fails by little or by much to have a common intersection with the mission is not reflected in the previous measure: in both cases the contribution to the effectiveness rating is null.

A possible approach would be to introduce as a measure, in the case of no common intersection, the shortest distance between the boundary of the system locus and the boundary of the mission locus. In the case of a common intersection between both loci, then the measure would be the ratio of the volume of the intersection of the system amd mission loci to the volume of the system locus. The measure will be defined so that it varies continuously between 0 and 1.

For instance, such a measure could be defined as follows. Let V_s denote the volume defined by the system locus in the unit-cube (since all attributes vary between 0 and 1), V_m denote the volume defined by the mission in the unit-cube, d*denote the shortest distance between the system locus and the mission locus when both loci do not intersect and E denote the measure of effectiveness derived from the comparison of system and mission attributes; with $E \varepsilon [0,1]$.

- If $V_s \cap V_m = \emptyset$

Then $$E = \frac{\sqrt{n}-d^*}{\sqrt{n}} \times (0.1) \qquad (6.1)$$

- If $V_s \cap V_m \neq \emptyset$

$$\text{Then } E = 0.1 + \frac{V_s \cap V_m}{V_s} \times (0.9)$$

where $\sqrt{n}$ denotes the longest diagonal in the unit cube.

Figure 6.1 through 6.4 illustrate and justify this choice in the case of a three-dimensional space. Shown in Figure 6.1 is the case when V_m and V_s do not intersect, i.e., the system does not meet any requirement of the mission. It follows from (6.1) that the effectiveness E may vary between 0 and 0.1. In figure 6.2, the system meets partially the requirements of the mission so that the effectiveness E is somewhere between 0.1 and 1. In Figure 6.3, the system meets completely the requirements of the mission and then the effectiveness is 1. Eventually in Figure 6.4, the system exceeds the requirements of the mission and then clearly the ratio $V_s \cap V_m / V_s$ is equal to V_M/V_S. One sees easily that the smaller the ratio is, the less effective the system is: the rationale being that the system is not well matched to the mission.

The next sections present the projection method in the case of the four dimensional example (S,R,K,t). Four particular projections have been chosen by taking all combinations of 3 out of 4 attributes. The volumes of the projections of system and mission loci are compared using the effectiveness measure defined in (6.1). A global measure for the communication pair is then derived by averaging those three-dimensional measures. This approximate measure is compared with the actual measure computed in the four-dimensional space. This emphasizes the shortcomings of the projection method and shows the necessity to work in the four-dimensional space (or more generally in the n-dimensional space) to derive an accurate effectiveness measure.

6.2.2. Presentation of the System Locus

In Section 4, the procedure for deriving expressions for Survivability, Reliability, Delay and Input Flow, was shown. The results of that analysis

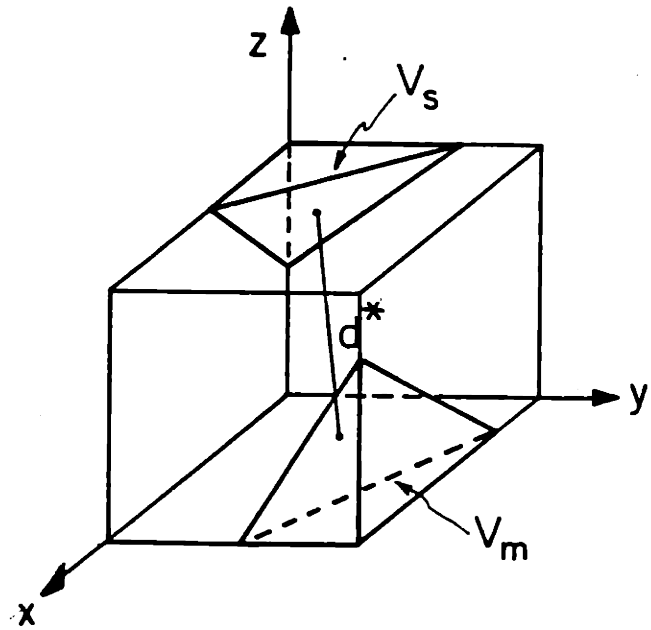


Figure 6.1 Representation in the attribute space (x,y,z) of $V_m \cap V_s = \phi$

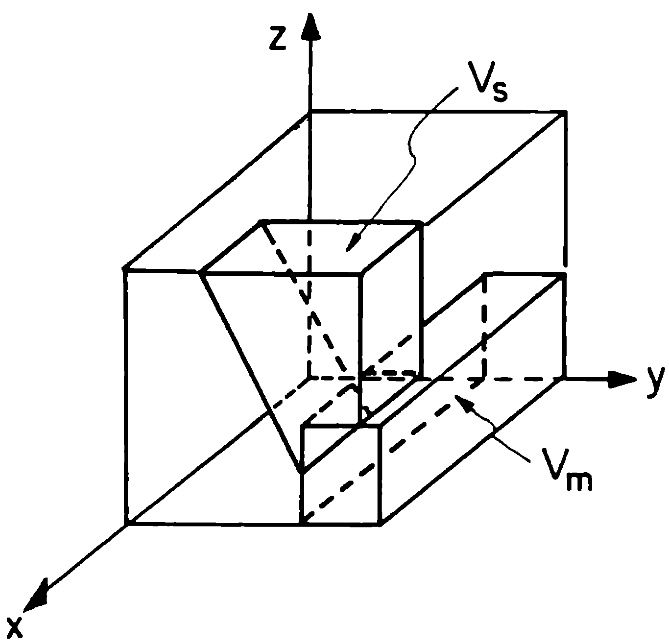


Figure 6.2 Representation in the attribute space (x,y,z) of $V_m \cap V_s \neq \phi$

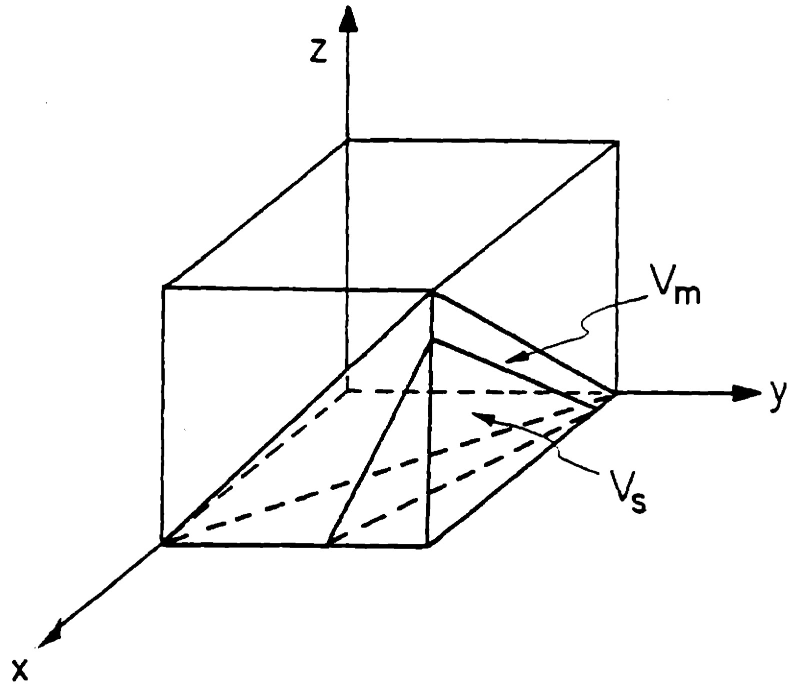


Figure 6.3 Representation in the attribute space (x,y,z) of $V_s \cap V_m = V_s$

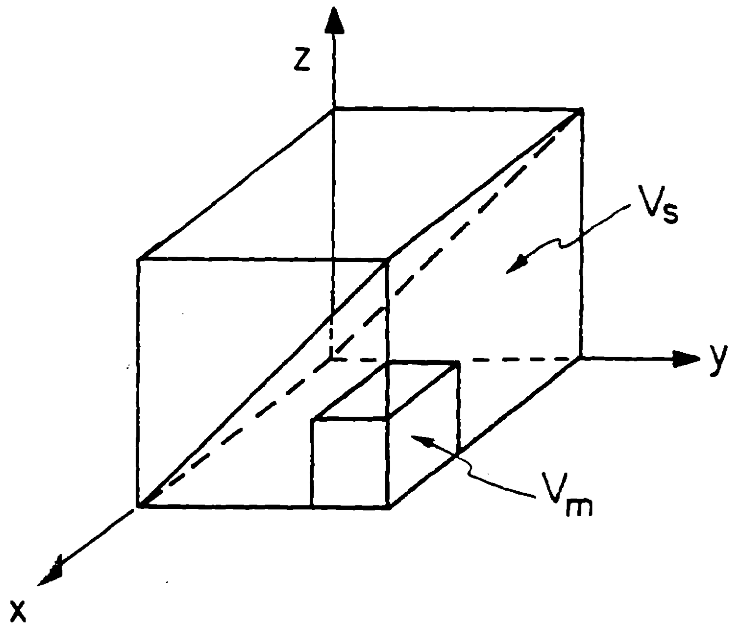


Figure 6.4 Representation in the attribute space (x,y,z) of $V_s \cap V_m = V_m$

are reviewed and then the representation of the locus described.

- *Survivability*, S. This system attributes depends, as seen in Section 4, on two primitives: (a) the probability that a link is being jammed in an attack and (b), the probability that a link is being attacked. It is assumed that S varies independently from any other attribute. Furthermore, assuming that the probability p can vary between 0.368 and 0.393, equation (4.19) leads to a variation in the attribute S:

$$0.4 \leq S \leq 0.45 \tag{6.2}$$

- *Reliability*, R. This attribute depends on the probability that a link fails because of component failures due to internal causes. It is assumed that R varies independently from any other attributes and, furthermore, with p varying between 0.368 and 0.393 equation (4.19) yields:

$$0.4 \leq R \leq 0.45 \tag{6.3}$$

- *Delay*, t. The scaled or normalized delay t is related to the normalized input flow K, but is independent of R and S. It has been shown, eq. (4.33), that for the communication pair (A,B) of Figure 4.5:

$$\frac{2}{\mu C_2 - F} \leq T \leq \frac{6}{\mu C_1 - F} \tag{6.4}$$

with $C_1 \leq C \leq C_2$. For illustrative purposes, it is assumed that (6.4) reduces to

$$\frac{0.1}{1-K} \leq t \leq \frac{0.1}{0.7-K} \tag{6.5}$$

where the normalized attributes K and t have been substituted.

This inequality is shown in Figure 6.5. It defines a surface in the plane (t,K). It also indicates that

$$0.1 \leq t \leq 1 \tag{6.6}$$

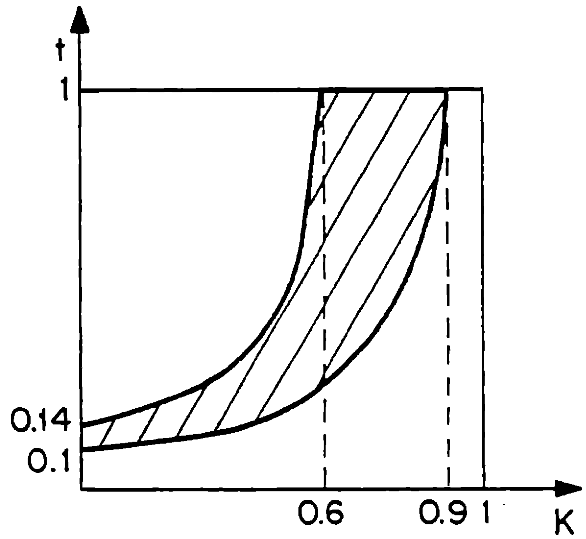


Figure 6.5 Normalized delay as a function of normalized input.

● *Input Flow*, K. The scaled normalized input flow is related to the normalized delay as shown previously. It can also be derived from Figure 6.5 that,

$$0 \leq K \leq 0.9 \tag{6.7}$$

Conditions (6.2) to (6.7) will be represented now in the three-dimensional spaces, as described in section 6.1.1. Figures 6.6 through 6.8 show the projections of the above conditions into the three-dimensional spaces. In Figure 6.8 the unspecified axis is either S or R. The next step is to compare the volumes of the projected system loci with the volumes of the projected mission loci.

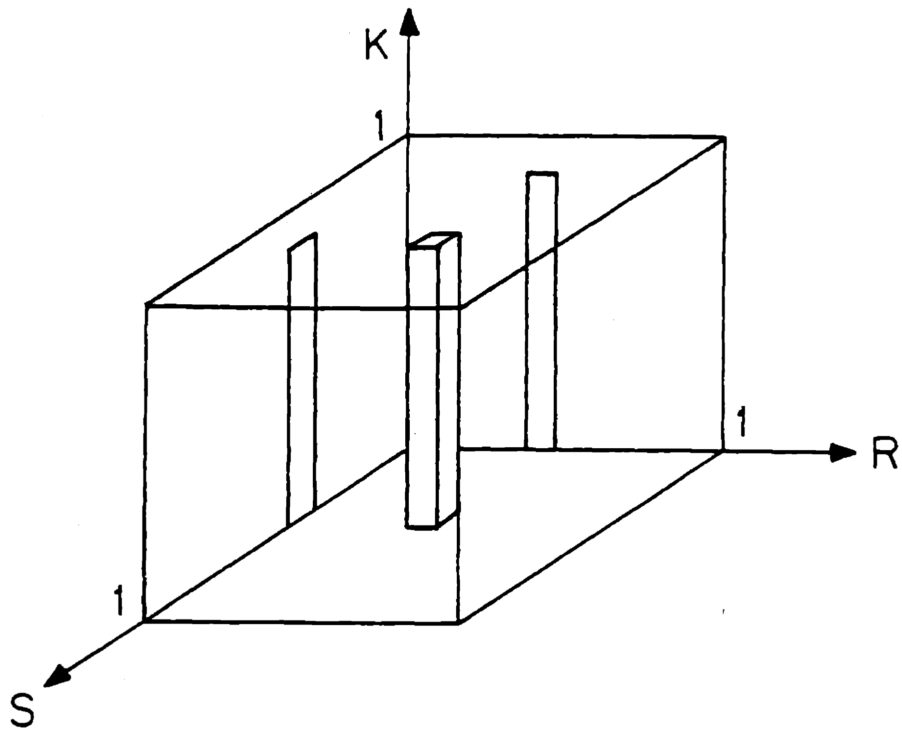


Figure 6.6 Projection of the system locus in the space (S, R, K)

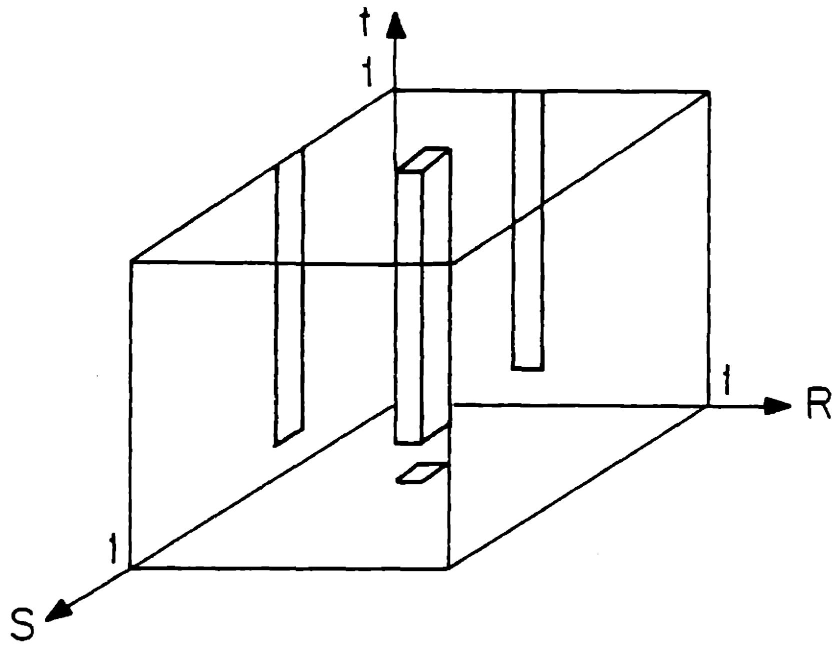


Figure 6.7 Projection of the system locus in the space (S, R, t)

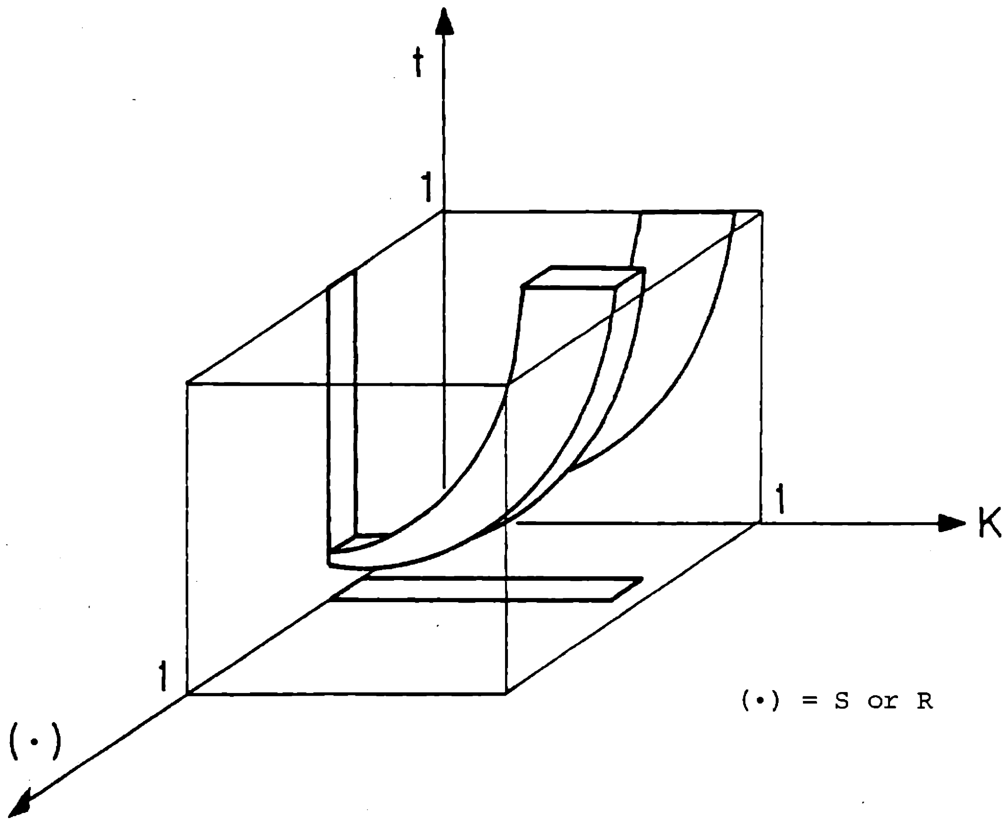


Figure 6.8 Projection of the system locus in the space (S, K, t) or (R,K,t)

6.2.3 Presentation of the Mission Locus

In Section 5, a condition for the mission attributes (S,R,K,t) has been derived, eq. (5.19). However, the funtions f and h had not been specified. Let us assume that:

$$f(S,R) = 2(S + R)^{-1}$$

and

$$h(F*K) = 10\ R_x\ (1 - \frac{9}{10} F*K)$$

If it is also assumed that

$$t_x >> T*t,$$

then equation (5.19) takes the form:

$$S + R + cK - dt > e \tag{6.8}$$

where c, d, e are coefficients depending on a, m, n, R_x, t_x, T*, x(0) and y(0).

For illustrative purposes, it will be assume in what follows that

$$c = d = e = 1$$

so that (6.8) can be expressed as:

$$\begin{aligned} &S + R + K - t > 1 \\ &0 \leq S \leq 1 \\ &0 \leq R \leq 1 \\ &0 \leq K \leq 1 \\ &0 \leq t \leq 1 \end{aligned} \tag{6.9}$$

Equation (6.9) defines a volume in the four dimensional space (S,R,K,t). For the reasons given in section 6.1, it will be projected into four three-dimensional spaces.

- *First projection:* (S,R,K) All possible cuts of the volume defined in (6.9) by a constant t plane are contained between the two planes:

$$S + R + K > 1 \qquad \text{for } t = 0$$

and

$$S + R + K > 2 \qquad \text{for } t = 1$$

Hence the projection in (S,R,K) is given by:

$$S + R + K > 1$$

and is shown in Figure 6.9.

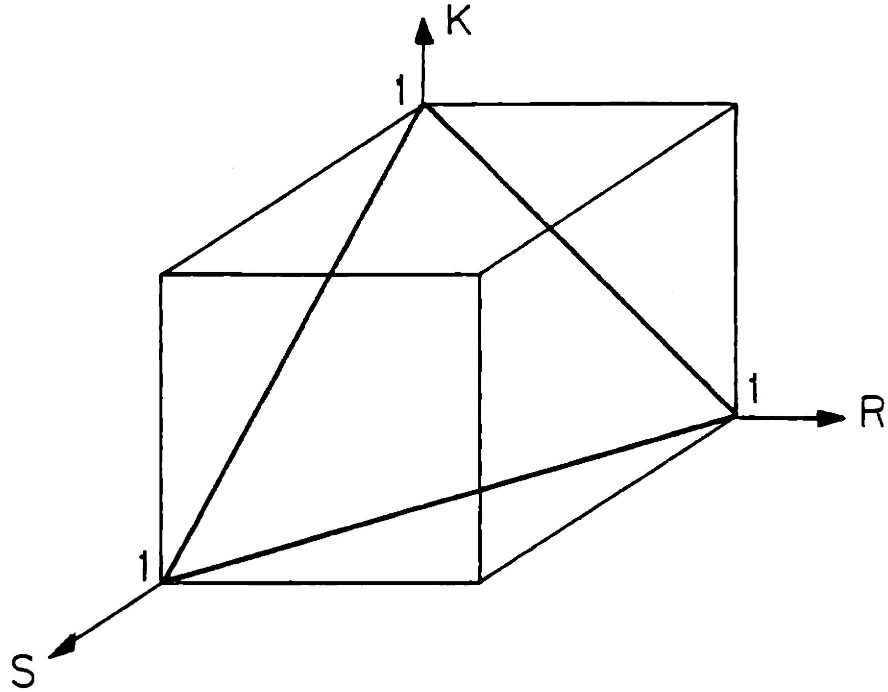


Figure 6.9 Projection of the mission locus in the space (S, R, K).

- *Second projection:* (S,R,t) All possible cuts of the mission volume by a constant K plane are contained between the two planes:

$$S + R - t > 1 \qquad \text{for } K = 0$$

and

$$S + R - t > 0 \qquad \text{for } K = 1$$

The projection in (S,R,t) is then given by:

S + R - t > 0

and it is shown in Figure 6.10. The axes are not specified because the same figure is also valid for the two remaining projections. In this case, the axes are S and R.

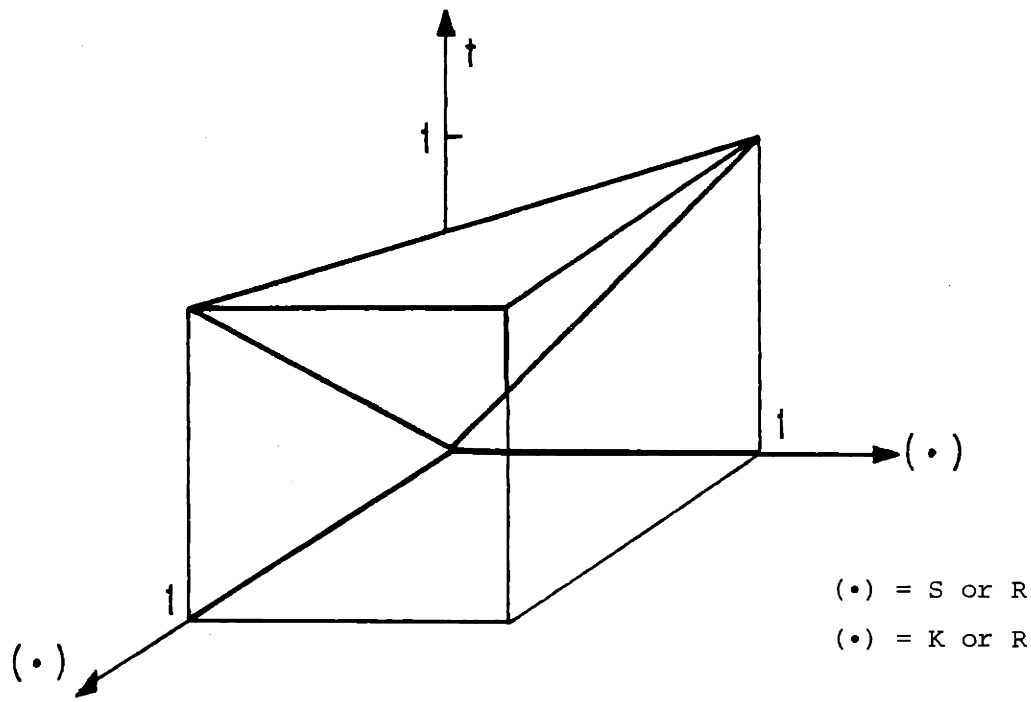


Figure 6.10 Projection of the mission locus in the space (S,R,t), (S,K,t) or (R,K,t).

- *Third projection:* (R,K,t) The same remarks as for the second projection with S in place of R can be made. The projection in (R,K,t) is then given by:

R + K - t > 0

It is shown in Figure 6.10. The unspecified axes are R and K.

- *Fourth projection:* (S,K,t) This is given by

S + K - t > 0

and it is shown in Figure 6.10. The axes are S and K.

6.2.4 Comparison of the Mission and System Attributes

Figures 6.11 to 6.13 show the relationship between the projections of volumes defined by the system and those defined by the mission. In each case, the intersection is shown by the shaded volumes. The remaining problem is to identify in which of the two cases described previously (see eq. (6.1)) each of the four figures can be classified.

The boundary surface defining the projected mission volume is either of the form:

$$x + y + z = 1 \tag{6.10}$$

or

$$x + y - z = 0 \tag{6.11}$$

where x, y, and z stand for the attributes. Then, when (6.10) has to be used, let x', y', z' denote a point in the attribute space, V_s. Then conditions (6.12) and (6.13) are evaluated.

$$x'_{min} + y'_{min} + z'_{min} \lesseqgtr 1 \tag{6.12}$$

$$x'_{max} + y'_{max} + z'_{max} \lesseqgtr 1 \tag{6.13}$$

If the inequality signs in both (6.12) and (6.13) are "$\leq$", then the system does not meet at all the requirements of the mission. If a "$\leq$" sign is obtained in (6.12) and a ">" sign in (6.13), then the system and mission volumes intersect. Finally, if both signs are ">", then the system meets totally the requirements of the mission.

When (6.11) has to be used, the previous analysis holds, but instead $(x'_{min}, y'_{min}, z'_{max})$ and $(x'_{max}, y'_{max}, z'_{min})$ should be used in conditions (6.12) and (6.13).

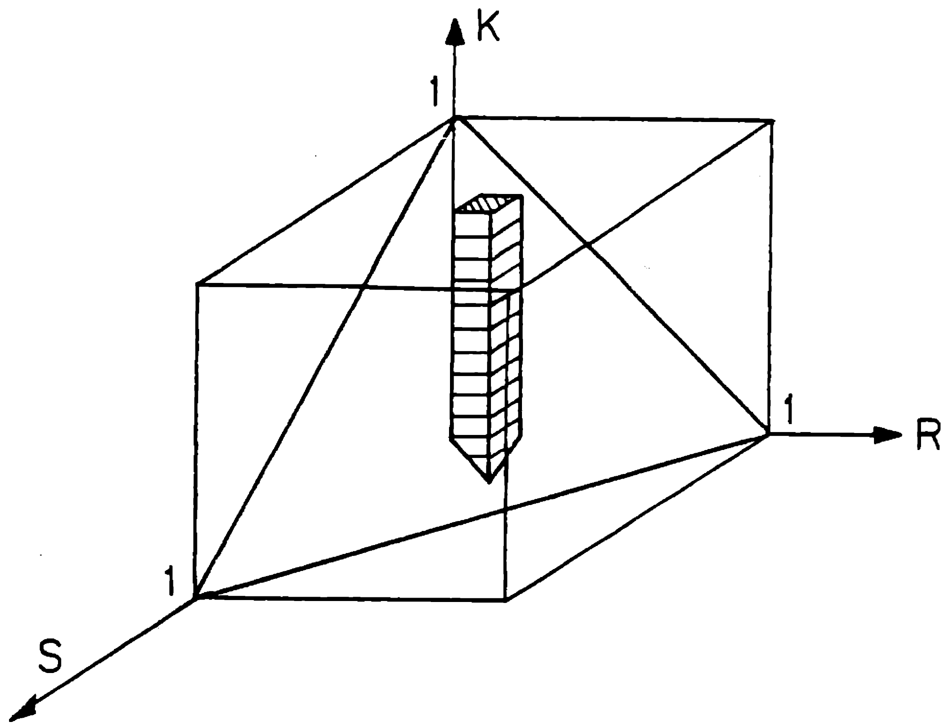


Figure 6.11 Effectiveness analysis in the space (S,K,R)

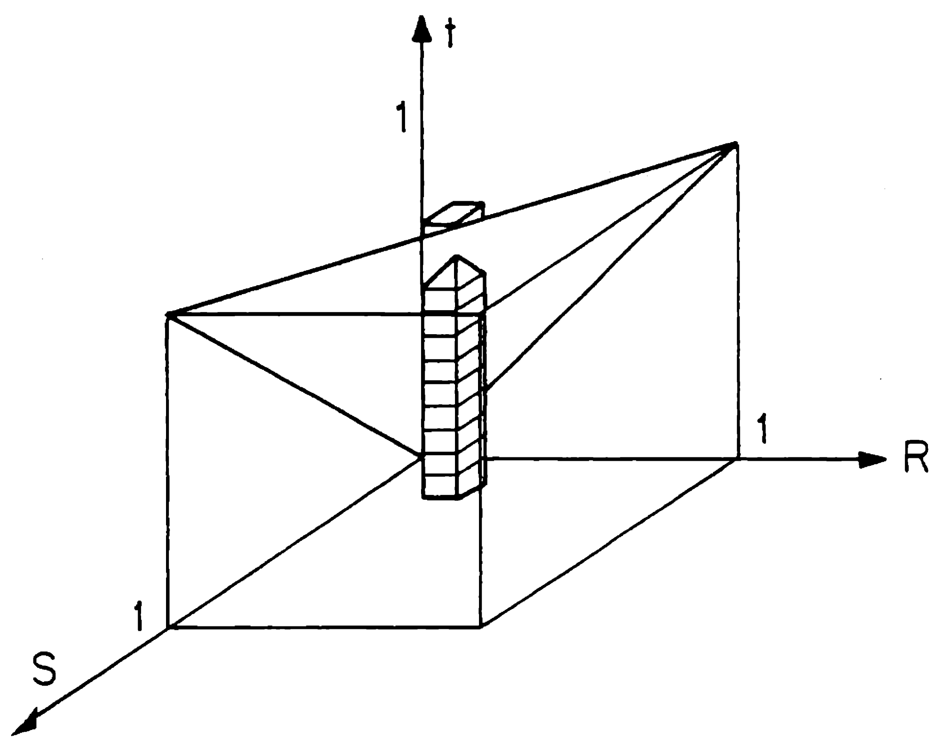


Figure 6.12 Effectiveness analysis in the space (S,R,t)

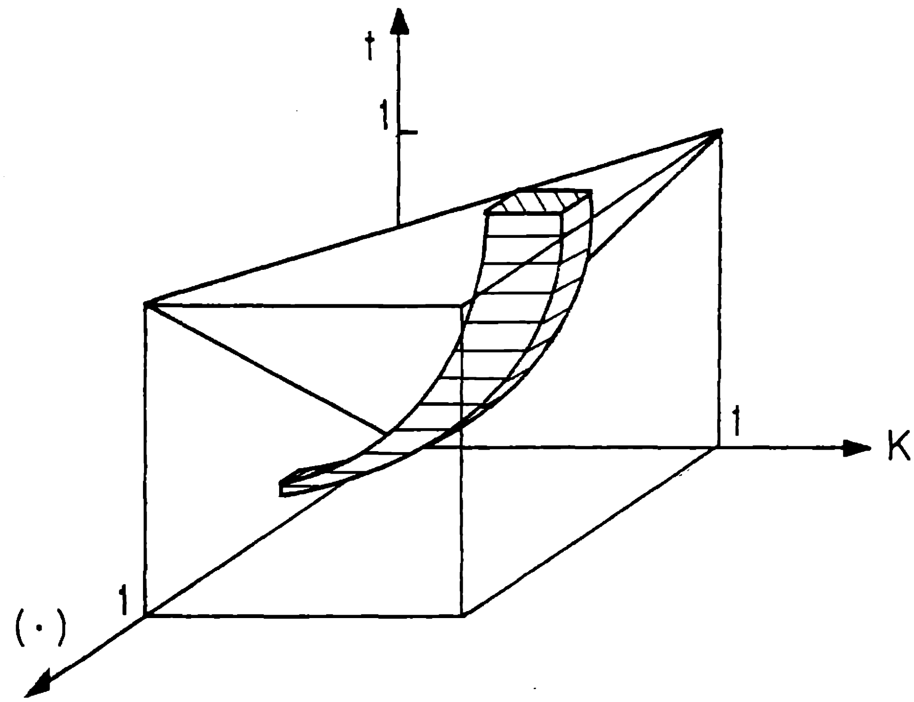


Figure 6.13 Effectiveness analysis in the space (S,K,t) or (R,K,t)

If both volumes have no intersection, then the next step is to compute the minimum distance d* defined as

$$d^* = \min_{\substack{P \in V_m \\ Q \in V_s}} d(P,Q) \qquad (6.14)$$

Typically, both P and Q will belong to the surface of V_m and V_s.

In the case where both volumes intersect then it is necessary to compute the system volume first and the volume of the intersection $V_m \cap V_s$. Except for the trivial cases where $V_m \cap V_s$ is equal to V_s or V_m, a truncated volume V_t has to be computed.

$$V_t = \iiint_{V_s} dx\,dy\,dz$$

$$\text{and } x + y \pm z \; > {}^{1}_{0} \qquad (6.15)$$

Each of the four cases, Figures 6.11 to 6.13 are analyzed now in more detail.

Effectiveness Analysis in (S,R,K). The system volume is defined by:

$$0.4 \le S \le 0.45$$

$$0.4 \le R \le 0.45$$

$$0 \le K \le 0.9$$

and the mission volume by:

$$S + R + K > 1.$$

Applying the method previously described shows that both volumes intersect (see Figure 6.11). The total volume for the system is:

$$V_s = (0.05)^2 \times 0.9$$

The truncated volume V_t, is given by:

$$V_t = \int_{0.4}^{0.45} \int_{0.4}^{0.45} \int_{1-(S+R)}^{0.9} dS\,dR\,dK$$

which yields

$$V_t = (0.05)^2\, 0.75$$

Hence the effectiveness as given by (6.1), is

$$E_1 = 0.1 + \frac{0.75}{0.9} \times 0.9 = 0.85 \qquad (6.16)$$

Effectiveness Analysis in (S,R,t). The system volume is defined by:

$$0.4 \leq S \leq 0.45$$

$$0.4 \leq R \leq 0.45$$

$$0.1 \leq t \leq 1$$

and the mission volume by:

$$S + R - t > 0$$

The two volumes intersect as shown in Figure 6.13. The total volume for the system is:

$$V_s = (0.05)^2 \times 0.9$$

The truncated volume, V_t, is given by:

$$V_t = \int_{0.4}^{0.45} \int_{0.4}^{0.45} \int_{0.1}^{S+R} dR\, dS\, dt$$

which yields

$$V_t = (0.05)^2 \times 0.75$$

The effectiveness measure, as given by (6.1), is

$$E_2 = 0.1 + \frac{0.75}{0.9} \times 0.9 = 0.85 \qquad (6.17)$$

Effectiveness Analysis in (S,K,t). The system volume is defined by:

$$\frac{0.1}{1-K} \leq t \leq \frac{0.1}{0.7-K}$$

$$0.4 \leq S \leq 0.45$$

and the mission volume by:

$$S + K - t > 0$$

It can be seen on Figure 6.14 (see also Figure 6.13) that the mission volume is included in the system volume. Hence the effectiveness measure as given by (6.1) is:

$$E_3 = 0.1 + 0.9 = 1 \qquad (6.18)$$

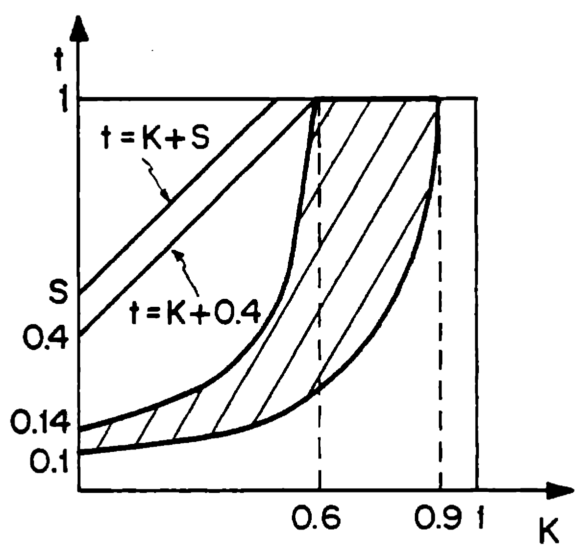


Figure 6.14 Cut of the system and mission volumes by a constant S or R plane.

Effectiveness Analysis in (R,K,t). The system volume is defined by:

$$0.4 \leq R \leq 0.45$$

$$\frac{0.1}{1-K} \leq t \leq \frac{0.1}{0.7-K}$$

and the mission volume by:

$$R + K - t > 0$$

As for the analysis in (S,K,t) the mission volume is included in the system volume. Hence the measure of effectiveness is (see Figures 6.13 and 6.14):

$$E_4 = 0.1 + 0.9 = 1 \qquad (6.19)$$

A global measure of effectiveness will be derived in the next section for the communication pair (A,B).

6.2.5 Effectiveness of the Communication Pair (A,B)

The measures of effectiveness have been computed for each of the four three-dimensional projections. As noted earlier, an overall measure of effectiveness for the communication pair (A,B) could be derived by averaging the values E_1 through E_4. All of them are fairly high; E_3 and E_4 are equal to unity. This yields, by attributing equal weights to the E_i's, a global measure E.

$$E = \frac{1}{4} \sum_{i=1}^{4} E_i = 0.925 \qquad (6.20)$$

The actual effectiveness measure can be computed by carrying out the analysis in the four-dimensional space. As pointed out before this will illuminate the shortcomings of the projection method.

In the attribute space (S,R,K,t) the system locus is defined by:

$$
\begin{aligned}
0.4 \leq S \leq 0.45 \\
0.4 \leq R \leq 0.45 \\
\frac{0.1}{1-K} \leq t \leq \frac{0.1}{0.7-K}
\end{aligned}
\qquad (6.21)
$$

and the mission locus by:

$$S + R + K - t > 1 \qquad (6.22)$$

A cut by a plane (S+R=constant) shows that both system and mission loci intersect. The computations of the system volume and of the truncated volume, i.e., $V_s \cap V_m$, are carried out in the Appendix. The result for the system volume V_s is

$$V_s = 6.6 \times 10^{-4}$$

and for the truncated volume V_t,

$$V_t = 2.01 \times 10^{-4}$$

Those values of V_s and V_t yield an effectiveness measure, eq. (6.1), of

$$E = 0.1 + \frac{2.01}{6.61} \times 0.9 = 0.37 \qquad (6.23)$$

This result illustrates that the approximated value can be much higher (in this case) than the actual value. Also, the projection method does not appear to be reliable. The result obtained depends a lot on the choice of the projection space. For instance, a projection in (S = R,K,t) would have yielded an effectiveness measure of 0.47, much lower then those averaged in (6.20). Thus, it is preferable to work directly in the original space using projections or even cuts for defining bounds on the real effectiveness measure.

6.3 GLOBAL EFFECTIVENESS

6.3.1 Introduction

This section is concerned with the derivation of a global measure of effectiveness for the system. In section 6.1 it was shown how a measure of effectiveness could be defined for each of the single communication pairs in the network. The problem arising now is to find a way to combine those partial measures of effectiveness (partial because they just describe the effectiveness of a subset of the network) into a global measure assessing the effectiveness of the system as a whole. Therefore, section 6.3.2 explores the issue of mapping the measures obtained for single communication pairs from $[0,1)$, i.e., their range of variation as defined in (6.1), to $[0, +\infty)$ in order to be able to apply some of the results of utility theory. In Section 6.3.3 some analytical expressions are proposed as measures for global effectiveness. These expressions are functions of the new measures derived in section 6.3.2. While no definite expression is given, it is shown upon which criteria the choice of a particular expression could be based.

6.3.2 Mapping of the Partial Measures of Effectiveness

The measure of effectiveness for single communication pairs as defined in (6.1) may vary between 0 and 1. However, Debreu (1958) and Phlips (1974) -among others- have shown that, in order to define a measure of effectiveness on a set of variables $(\tilde{E}^1,\ldots\tilde{E}^n)$ it is necessary for each $\tilde{E}^i$ to belong to the positive orthant of R^n, i.e, it is unbounded from above. The partial measures derived from (6.1), the E^i's, do not meet this condition (since unity is the upper bound); thus they need to be mapped from $[0,1)$ into $[0, +\infty)$. The upper bound, 1, has to be excluded because the image of a closed bounded set of R^n through a continuous function is also a closed bounded set of R^n, and hence the mapping into $[0, +\infty)$ would not have been possible.

Figures 6.15 through 6.22 show some of the possible mappings. For instance, the functions in Figures 6.15 to 6.18 have a positive slope at the origin and are convex. Such mappings may be used when the values of E are broadly distributed over the interval [0,1). The functions in Figures 6.19 and 6.20 have a zero slope at the origin and are convex. The slope remains small until E reaches a certain threshold. These mappings may be used when the E's take values within a subinterval of the interval [0,1) (for example when the E's are all above the threshold level).

The two remaining mappings shown in Figures 6.21 and 6.22 have an infinite slope at the origin and have an inflection point. For small and high values of E, i.e., close to zero and close to unity the slope of the mapping is high so that such a mapping could be used to discriminate among the E's when they are small, or when the E's take both small and high values. In the case, where a mapping would have to be used for E's with diverse distributions, a neutral mapping of the type presented in Figure 6.15 might be preferred.

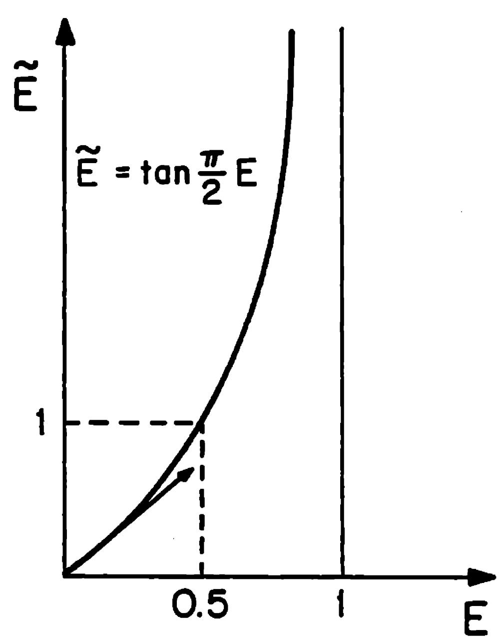


Figure 6.15 Mapping of the effectiveness by $\tan\left(x \frac{\pi}{2}\right)$

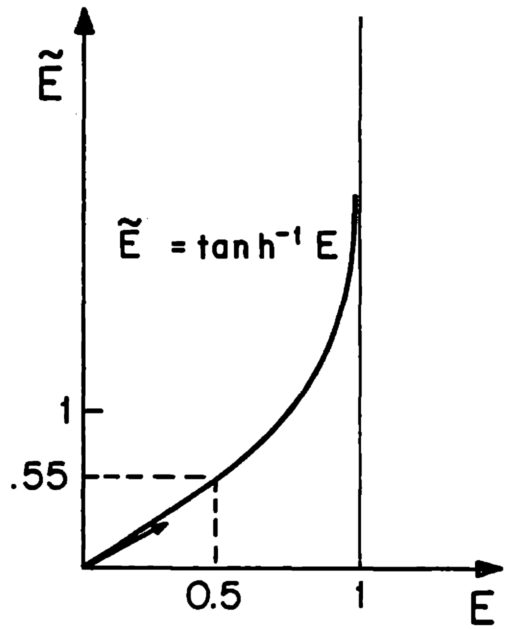


Figure 6.16 Mapping of the effectiveness by $\tanh^{-1}x$

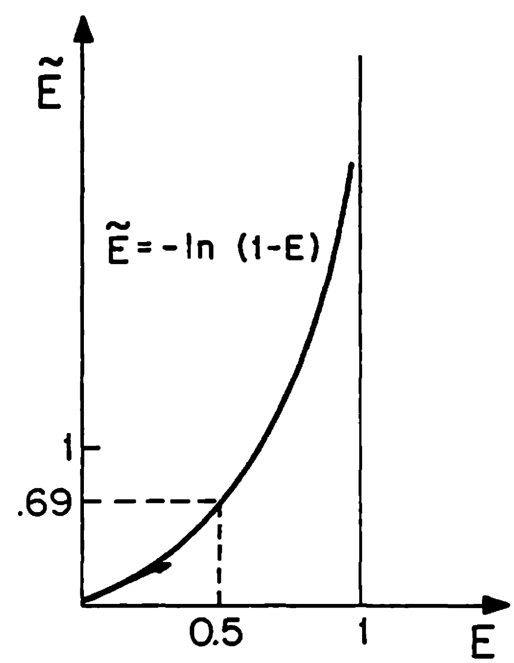


Figure 6.17 Mapping of the effectiveness by $-\ln(1-x)$

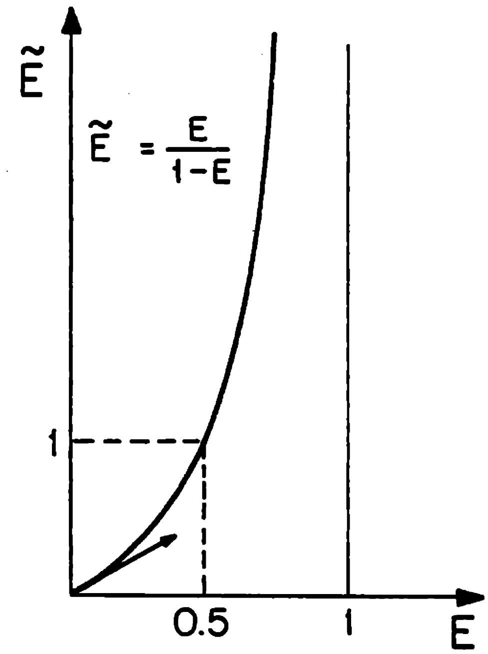


Figure 6.18 Mapping of the effectiveness by x/1-x

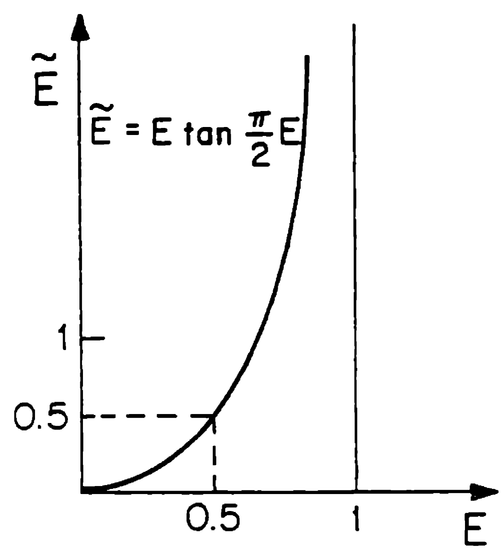


Figure 6.19 Mapping of the effectiveness by $x \tan\left(x \frac{\pi}{2}\right)$

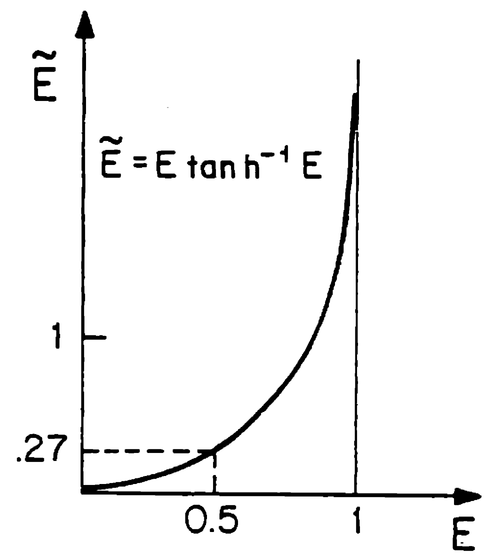


Figure 6.20 Mapping of the effectiveness by $x \tanh^{-1} x$

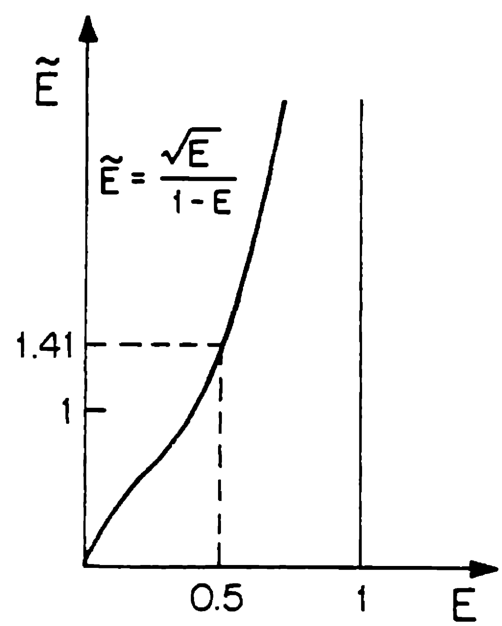


Figure 6.21 Mapping of the effectiveness by $\sqrt{x}/1-x$

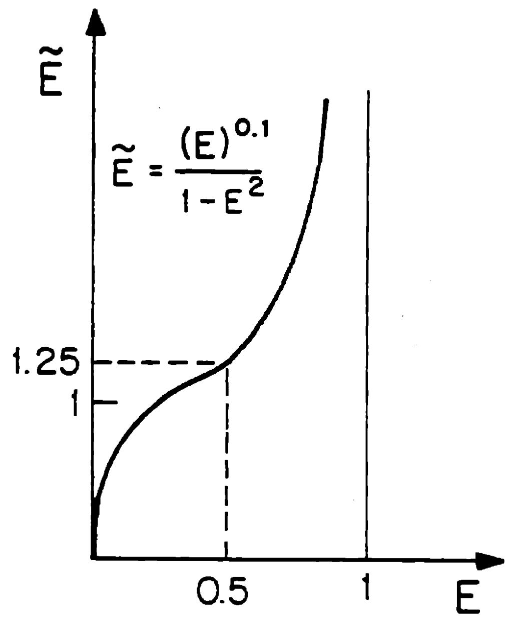


Figure 6.22 Mapping of the effectiveness by $x^{0.1}/1-x^2$

6.3.3 Global Effectiveness Measures

The first characteristic of the function that should transform the mapped measures of effectiveness into a global measure is that it should be an increasing function of all $\tilde{E}$'s. Let this function be u; then

$$\frac{\partial u}{\partial \tilde{E}^j} \geq 0 \qquad (6.24)$$

However, there are many ways to choose an increasing function. The simplest way is certaintly to choose an additive function:

$$E = u(\tilde{E}^1,\ldots,\tilde{E}^n) = u_1(\tilde{E}^1) + u_2(\tilde{E}^2) + \ldots + u_n(\tilde{E}^n) \qquad (6.25)$$

Such a function satisfies the relationship:

$$\frac{\partial^2 u}{\partial \tilde{E}^i \partial \tilde{E}^j} = 0 \qquad i \neq j \tag{6.26}$$

This means that the marginal effectiveness of the i-th communication pair is not influenced by the variation in the effectiveness of any other pair. An effectiveness function defined as a weighted average of the values of the $\tilde{E}^i$'s is a typical example. However, the decisionmaker might not want to consider that all the variables (the $\tilde{E}^i$'s) are unrelated. Hence, it might be interesting to group together those that appear to be close substitutes. For instance, the global effectiveness might be perceived as unchanged when two of the $\tilde{E}$'s, say $\tilde{E}^i$ and $\tilde{E}^j$, vary in a prescribed manner. A typical illustration might be the case of two alternative paths between some origin-destination pair of the network. Clearly, the global effectiveness depends on some aggregate measure of $\tilde{E}^i$ and $\tilde{E}^j$. In such a case, the effectiveness function is said to be separable and is written as:

$$u(\tilde{E}^1,\ldots,\tilde{E}^n) = t(\tilde{E}^1,\ldots,v(\tilde{E}^i,\tilde{E}^j),\ldots,\tilde{E}^n) \tag{6.27}$$

and if the marginal effectiveness of $\tilde{E}^j$ and of $\tilde{E}^i$, i.e., $\partial t/\partial \tilde{E}^i$ and $\partial t/\partial \tilde{E}^j$ is independent of any other variable, then the effectiveness function u is said to be strongly separable and can be written as:

$$u(\tilde{E}^1,\ldots,\tilde{E}^n) = u_1(z_1) + \ldots + u_r(z_r)$$

where

$$z_i = v_i(\tilde{E}^j,\ldots,\tilde{E}^k)$$

with each $\tilde{E}^j$ appearing in only one of the function v_i.

It might be interesting to choose for v_i Cobb-Douglas production functions of the type:

$$V(\tilde{E}^1,\ldots,\tilde{E}^r) = A(\tilde{E}^1)^{\alpha_1}(\tilde{E}^2)^{\alpha_2}\ldots(\tilde{E}^r)^{\alpha_r} \qquad (6.28)$$

where A, and the α_i's are constant.

Such a function exhibits a constant elasticity with respect to each of its variables. For the variable $\tilde{E}^i$ this constant elasticity is just the constant α_i. Three cases may be encountered:

$$\sum_{i=1}^{r} \alpha_i < 1 \qquad (6.29)$$

$$\sum_{i=1}^{r} \alpha_i = 1 \qquad (6.30)$$

$$\sum_{i=1}^{r} \alpha_i > 1 \qquad (6.31)$$

If, respectively, eq. (6.29), (6.30), or (6.31) holds, then function v exhibits, respectively, decreasing, constant, or increasing returns to scale.

The case of constant returns to scale might be of interest for the decisionmaker since then, the function v is homogeneous of degree one, i.e., multiplying all arguments by the same constant C will multiply also the effectiveness function v by the same constant C.

When only two arguments, $\tilde{E}^i$ and $\tilde{E}^j$, are considered, other things being equal, then the isoquants of function v are hyperbolic. Along an isoquant the rate of substitution is equal, ceteris paribus, to $\alpha_i\tilde{E}^j/\alpha_j\tilde{E}^i$. Thus, the ratio α_i/α_j measures the difficulty of substituting $\tilde{E}^i$ by $\tilde{E}^j$. Also the decisionmaker may want to make it difficult to substitute $\tilde{E}^i$ by $\tilde{E}^j$ by choosing a high value for α_i/α_j.

When all the arguments are considered, then with condition (6.30), the α_i's can be chosen in classifying the communication pairs by their strategic importance attributing to the more important the greatest value for α, and so on, to the lowest ranking pair. Then, once the effectiveness measures E_r have been defined, the global effectiveness $\hat{E}$ can be derived using the set $(E_1,\ldots,E_s)$ as a commodity set. The preceding analysis can be applied again to the E's to yield the global measure $\hat{E}$.

6.3.4 Conclusion

It has been shown in section 6.3.2 how to map the partial measures of effectiveness E^i into a new measure $\tilde{E}^i$ which takes values in $[0, +\infty)$. Section 6.3.3 has presented a framework for defining global effectiveness measures. The issues of additivity and separability have been explored. The example of the separable function v, given in (6.28), has shown how the value of the coefficients α_i could be related to the way the decision-maker assesses the contribution of the different subsets of the network to the global effectiveness.

The next section contains some overall conclusions and recommendations for further research and development of this methodology for C^3 systems effectiveness analysis.

SECTION 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 INTRODUCTION

In this section final conclusions will be drawn from the research presented in this thesis. The originality of the approach will be emphasized and other areas of implementation of the methodology will be addressed. Then recommendations for the improvement of the methodology and further research will be given.

7.2 CONCLUSIONS

In this thesis a new approach for assessing the effectiveness of C^3 systems has been developed. The key idea of this methodology is to relate the capabilities of a system to the requirements of the mission(s) it has been assigned to fulfil. This idea departs from the assessment methods based on the pure performance of systems. More precisely, each stage of the methodology (specification of system and mission characteristics, context related primitives, attributes, subsystems of interest) gives a good idea of *what* the system is intended to do, *where* it is intended to be used and *how* it is intended to be used. Answering these questions is essential in order to assess correctly a system. And this is not only true for C^3 systems,but it can be also applied to any large scale systems, especially those which provide or deliver a service. Work carried out on the effectiveness analysis of power systems, see Dersin and Levis (1981, 1982), has shown that the methodology developed throughout this thesis could be applied with advantage. Clearly, the methods used for deriving system and mission attributes will differ from one system to the other, but the procedure will remain the same. However, the methodology in this thesis needs to be improved; the next section deals with this issue.

7.3 RECOMMENDATIONS

Recommendations for further research address each stage of the methodology: derivation of system and mission attributes, comparison of the system and mission loci in a common attribute space and derivation of a global measure of effectiveness. Each of these points will now be developed.

System and Mission Attributes. Assessing the effectiveness of C^3 systems makes further research necessary for deriving system and mission attributes. For system attributes, this can be based on the technical characteristics of the system. Numerous research efforts concerned with system performances have been carried out. However, techniques used (for instance simulations) provide raw data. These data need to be aggregated into attributes. Also, common attributes should be defined for similar systems (i.e., systems which can accomplish the same array of missions) so that they can be compared. For mission attributes, simulations can be helpful in modeling engagements. However, the data provided should also be related and aggregated into attributes.

Partial Measure of Effectiveness. The comparison of the system and mission loci in a common attribute space can be further improved by (i) developing methods for computing the surfaces or volumes of the loci (a method using projections has been shown not to be very accurate) (ii) developing alternatives measures to those proposed in Section 6 (for instance the case where the system does not meet any of the requirements of the mission can be treated separately). This can be carried out by studying manifolds and volumes in an n-dimensional space.

Global Measure of Effectiveness. Further research for deriving a global measure of effectiveness may benefit from other work carried out on multi-attribute assessment. Moreover, the use of utility theory can be developed by relating the possible needs of the decisionmaker with the shape of the functions chosen.

APPENDIX

COMPUTATION OF THE ACTUAL VOLUMES V_s AND V_t IN THE FOUR-DIMENSIONAL SPACE.

System Volume. The volume of the system locus in the space (S,R,K,t) is given by:

$$V_s = (0.05) \left[(0.1) \int_0^{0.6} \frac{dk}{0.7-K} - (0.1) \int_0^{0.9} \frac{dK}{1-K} + 0.3 \right] \quad (A.1)$$

which,by integration,yields:

$$V_s = (0.05)^2 \; [0.3 - 0.1 \text{ Log } 0.7]$$

$$V_s = 6.608 \; 10^{-4}$$

Truncated Volume. The volume of the intersection of the mission and system loci is given by:

$$V_t = \int_{0.8}^{0.85} (u-0.8) \; A(u)du + \int_{0.85}^{0.9} (0.9-u) \; A(u)du \quad (A.2)$$

and

$$A(u) = \frac{1}{2} u \sqrt{u^2-0.4} + 0.1 \; \ln \frac{1}{2} \left[u-\sqrt{u^2-0.4}\right] - 0.1 \; \ln \frac{1}{2} \left[u+\sqrt{u^2+0.4}\right]$$

This has been obtained by making a change of variable

$$u = S + R$$

and by computing at constant u the surface defined by

$$\frac{0.1}{1-K} \leq t \leq \frac{0.1}{0.7-K}$$

$$t \leq K + u-1$$

By introducing the change of variable:

$$u = \sqrt{0.4} \cosh \rho$$

this yields:

$$\int A(u)du = \frac{1}{6}\, 0.4 \sqrt{0.4} + \sinh^3 \rho + 0.2 \sqrt{0.4}\ (\sinh \rho - \rho \cosh \rho) + C$$

$$\int uA(u)du = 0.02\ (\frac{1}{8} \sinh 4\rho - \frac{\rho}{2}) + 0.01\ (\sinh 2\rho - 2\rho \cosh 2\rho) + D$$

Then substituting in (A.2), we obtain:

$$V_t = 2.007 \times 10^{-4}$$

Hence the measure of effectiveness, (6.1), is:

$$E = 0.1 + \frac{2.007}{6.608} \times 0.9 = 0.373 \qquad \text{(A.3)}$$

This measure of effectiveness is the actual measure we were looking for.

REFERENCES

Ball, W. H., (1980). Is Command and Control Really Necessary? Why not Buy More Airplanes? Information Processing and Decision making for Battle management, MITRE Corporation, Bedford, MA.

Boettcher, K. L., and A. H. Levis (1982). Modeling the Interacting Decision-maker with Bounded Rationality. IEEE Trans. on Systems, Man and Cybernetics, Vol. SM-12, No. 3, pp. 334-348.

Brick, D. B. (1978). Airforce Tactical C^3I System. Working paper. Electronic System Division, Hanscom Air Force Base, MA.

Burke, P. J., (Dec. 1956). The Output of a Queueing System. Oper. Res., Vol. 4, pp. 699-704.

Debreu, G. (1958). Theory of Value: An Axiomatric Analysis of Economic Equilibrium. John Wiley and Sons, New York

Dersin, P., and A. H. Levis (1981). Large Scale System Effectiveness Analysis. Report LIDS-FR-1072, Laboratory for Information and Decision Systems, MIT, Cambridge, MA.

Dersin, P., and A. H. Levis (Nov. 1982). Characterization of Electricity Demand in a Power System for Service Sufficiency Evaluation. European Journal of Operational Research.

Ekchian, L. K. (1982). An Overview of Lanchester-Type Combat Models for Warfare Scenarios. LIDS-P- 1193, Laboratory for Information and Decision Systems, MIT, Cambridge, MA.

Mangulis, V. (1980). Lanchester-Type Equations of Combat which Account for Multiple Killing Hits. Oper. Res., 28, pp. 560-659.

Phlips, L. (1974). Applied Consumption Analysis. North-Holland/American Elsevier, Amsterdam, Oxford, New York.

Rona, T. P. (1980). Survey of C^3 Assessment. Concepts and Issues. Proceedings for Quantitative Assessment of Utility of Command and Control Systems. National Defense University, Washington D.C.

Schwartz, M. (1977). Computer-Communication Network Design and Analysis. Prentice-Hall, Englewoods Cliff, N.J.

Shanahan, E. J., H. B. Teates and B. B. Wise (1980). Defining and Measuring C^2. Proceedings of the Third MIT/ONR Workshop on Distributed Information and Decision Systems. Laboratory for Information and Decision Systems, MIT, Cambridge, MA.

Tanenbaum, A. S. (1981) Computer Networks. Prentice-Hall, Englewood Cliffs, New Jersey.

Valavani, L. (1981). Effectiveness Assessment of Power Systems. Working paper. Systems Control, Inc., Burlington, MA.

Van Trees, H. L. (1980). Keynote Address. Proceedings for Quantitative Assessment of Utility of Command and Control Systems. National Defense University, Washington D.C.

Welch, J.A. (1980). State of the Art of C^2 Assessment. Proceedings for Quantitative Assessment of Utility of Command and Control Systems. National Defense University, Washington DC.

Wohl, J. G., D. Gootkind and H. D'Angelo (1981). Measures of Merit for Command and Control. MITRE Technical Report, MITRE Corp., Bedford, MA.

Zraket, C. A. (1980). Issues in Command and Control Evaluation. Proccedings for Quantitative Assessment of Utility of Command and Control Systems. National Defense University, Washington DC.