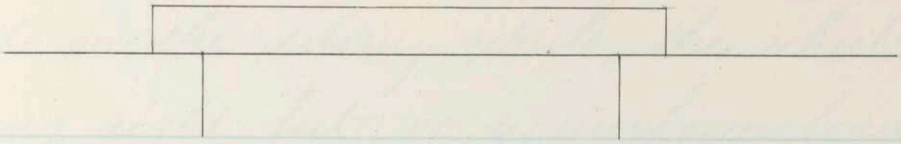
The Strains in Bridges.

It is the object of the following Thesis to give the practical method in use for calculating the strains in Railway Bridges of different lengths of span, and also the greatest weight that can be brought upon them by a certain class of engine.

The calculations are made on the principle of the composition and resolution of forces, and the work is proved by the theory of bending moments.

Written by S. M. Felton Jr. as his Graduating
Thesis at the Mass. Inst. of Technology
Class of 1873.

Case 1



For the first case, we will take a girder of twelve feet, clear span, such as we find in use for trestle works, or carrying a train from one panel to another in bridges where cross girders are placed at each panel only. Wood is very extensively used for this purpose, generally in the form of solid stringers.

In the first place we must find the greatest load we can have on the girder at one time, and the effect of that load when passing at high rates of speed. We will suppose, in all the following cases that the heaviest engine on the road weighs

70,000 pounds, with 45,000 concentrated on the driving wheels, the wheel base being eight feet, or a uniform load of 5625 lbs. per. ft. for eight feet.

The total wheel base of such an engine would be 22' 6" and the total load 70,000 lbs., making 3,111 lbs as the load per foot for a length of 22' 6".

On the span we are considering the greatest strain would be produced by the weight on one pair of driving-wheels at the centre of the span.

As both of the girders will be similar we will calculate one only; so, the centre weight on one driving wheel be weight on one driving wheel, or 11,250 lbs.

According to Rankine p. 306 Applied Mechanics, and the deductions of Fairbain, "a suddenly applied transverse load produces double the strain which a gradually applied load produces." Rankine makes allowance for this by using twice the factor of safety for the rolling load that he uses for the dead load, but, in the case of a short span like the one under consideration, the dead load is so small compared with the live load and the vibration is so great that it has been found much safer to design the girders for twice the maximum load, at the same time using large factors of safety.

From Stoney we find the formula for the centre breaking loaght of a rectangular beam supported at each end - $W = \frac{4ad^2S}{l}$

a = area of beam, d = depth, S = modulus of Rupture by cross-breaking. l = length of span.

We will now assume the depth of the beam as about 1/9 of the span, this being the best proportion for short spans. We have given W, d, l & S, So we shall have to transform the formula to find $\underline{b}$ = the breadth.

$$W = \frac{4bd^2S}{l} \text{ or } 16'' = \frac{Wl}{4d^2S}$$

$W = 22500$, $d = 16''$, $l = 144''$

$S = 1229$ (according to Stoney)

To prove this formula we take the well known theory of bending-moments.

According to Rankine, we take the greatest bending moment at a given cross section and equate it with the moment of resistance at that section; now the greatest bending moment is at the centre and the section being uniform, the moment of resistance is the same at any section.

The bending moment is given by the formula

$$M = \frac{Wl}{4}.$$

The moment of resistance for any case would be $M_0 = \frac{fI}{y}$; the beam being rectangular in section, I being equal to the moment of inertia, we can put $I = n'bh^3$, and $y = m'h$, let $\frac{n'}{m'} = n$, then $M_0 = nfbh^2$. Equating,

$M_0 = nfbh^2 = \frac{Wl}{4}$, $n = 1/6$, so,

$M_0 = \frac{fbh^2}{6} = \frac{Wl}{4}$, or $Wl = \frac{4fbh^2}{6}$, $W = \frac{2fbh^2}{3l}$

The modulus of rupture, S, given by Stoney is one sixth of that given by Rankine $= f$; so the formulae are practically the same both being derived from the same principle.

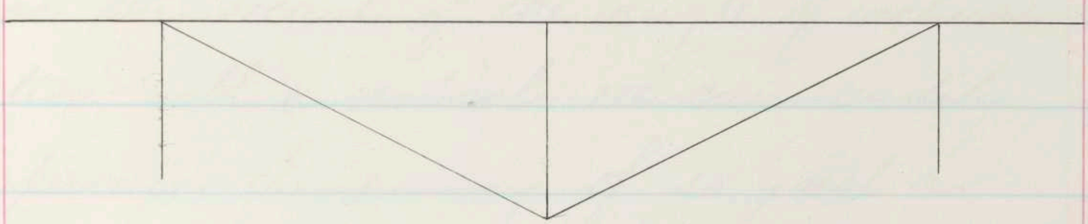
Solving this case:

$$b = \frac{5Wl}{4d^2S} = \frac{5 \times 22500 \times 144}{4 \times 2556 \times 1229} = 12.8.$$

S being Stoney's value for the modulus of rupture = 1229. the factor of safety is five, being the same as used by Prof. Vose for girders of wood.

We frequently find in practice wooden girders of twelve feet span under a heavy traffic with less dimensions than we have found, even as low as 12" x 12" but in such cases the deflections are quite apparent.

Case II



In this case, we will consider a trussed girder with a span of twenty four feet.

In this case and the following ones we will suppose wrought iron to be used in all points except connections, pedestals etc.

We will take a depth of six feet, one post dividing the truss into two panels of twelve feet each.

The depth at first glance, might seem to be too great, being equal to one fourth the span; but when we consider that the strain on the rods increases as the secant of the angle of inclination with a vertical we can readily perceive the economy of so great a depth for we can lengthen the post, to a certain extent, with much more economy than we can increase the area of the rods. The greatest weight on this girder will be the engine weight of 70,000, but we will have a concentrated weight at the centre of, say $\frac{70000}{2} = 35000$ pounds. This weight we will use to calculate the strains, first multiplying it by two to allow for the effects of high speeds and vibration. The compression on the post will be equal to the centre load = 35000 pounds. This weight is transmitted directly to the abutments by the truss rods.

Each rod carries ½ 35000 pounds = 17500 pounds. The strain is expressed by the following formula. W sec. θ. θ = the ~~strain made~~ angle made ~~by~~ by the rods with a vertical.

$$\text{Sec. } \theta = \frac{\text{length of rod}}{\text{depth of truss.}} = \frac{13.416}{6} = 2.236.$$

Strain on rods = W sec θ = 17500 × 2.236 = 39.130 lbs.

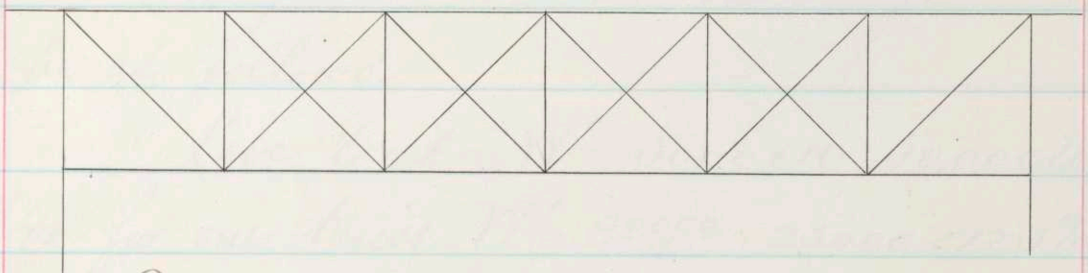
The strain on the upper member of this truss is found by the bending moment $M = \frac{Wl}{4} = \frac{35000 \times 24}{4} = \frac{840000}{4} = 210000.$

This bending moment is resisted by an equal and opposite couple with an area = depth of truss, so, $\frac{M}{d} = \frac{210000}{6} = 35000$ the horizontal strain at the centre of the truss—or the direct compression on the top member from the load uniformly distributed; besides this strain we have between the post and either end, a transverse strain from the rolling load. In this case as in Case 1. the greatest load would be the weight on one driving wheel = 11.250. this mult. by two for the effects of the live load; as before, we have $W = 22{,}500$.

$M = \frac{Wl}{4} = \frac{22500 \times 12}{4} = 67500$ lbs. Suppose we use two 12" I beams. Then we have a flange strain on the beams of 67500 lbs. and a direct compression of 35000 lbs. both of these strains must be provided for. The maximum strains are tabulated below.-

Ties	Wt	Strain	Posts	Chords	
		Tension		comp	Transverse
No. 1	17500	39,130		35000	67500 35000
" 2	17500	39,130	35000	TOTAL	102,500

Case III



We will suppose a bridge, in this case of 60' span, 6 panels, and 10' depth of truss - deck bridge.

The truss is known as the Pratt or Whipple and in the following cases, we shall adhere to the same style.

The greatest weight that can come 12
upon the whole bridge is one engine weighing with loaded tender 125000 lbs. or 2500 lbs per. ft. for its length; twice this load or 5000 lbs. per foot run will suffice for both the panel and chord systems allowing for vibration &c.

The panel Wts on the bridge will be as follows.

Live load = $W' = 5000 \times 10 = 50,000$ lbs or for one truss $W' = \frac{50000}{2} = 25000 = 12.5$ Tons.

Dead load for bridge track etc.

$W = 800 \times 10 = 8000$ lbs. $\frac{8000}{2} = 4000 = 2$ tons

Whenever ton is used. 2000 lbs is meant. The weights borne by the ties when the bridge is unloaded are as follows.

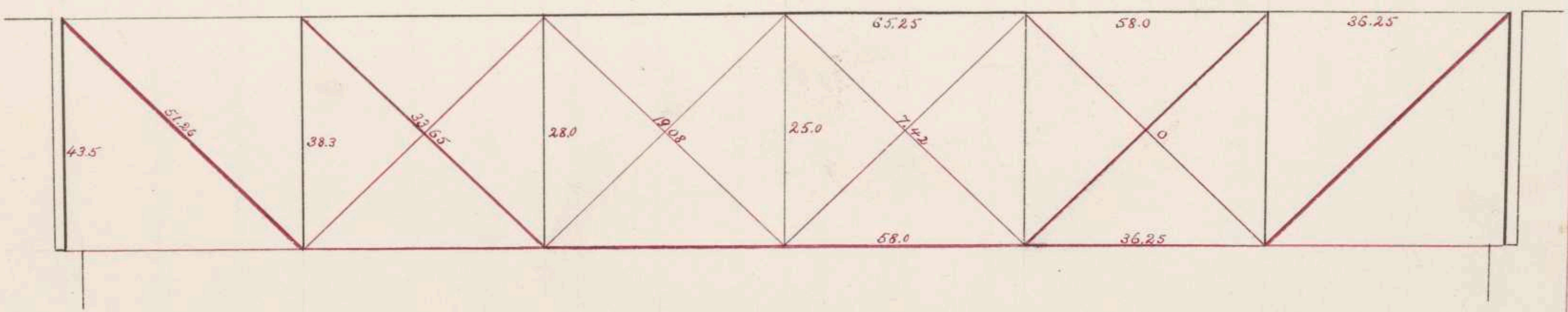
$T_3 = \frac{1}{2} W = 1$ Ton, $T_2 = 1\frac{1}{2} W = 3$ Tons

Diagram of Strains.

Span 60 Ft. Ht. 10 Ft.

W = 2 tons. W' = 12.5 tons.

CASE III.



Scale 8' = 1"

$T_1 = 2\tfrac{1}{2} W = 5$ Tons.

The weights borne by the ties as the live load comes upon the bridge are as follows, on the principal of the lever.

$C_1 = \tfrac{1}{8} W' = 2.08$ Tons. $C_2 = \tfrac{3}{6} W' = 6.25$ tons

$C_3 = \tfrac{6}{6}$ " $= 12.5$ " . $T_2 = \tfrac{10}{6}$ " $= 20.8$ "

$T_1 = \tfrac{15}{6}$ " $= 31.25$ " .

Combining the effects of the dead and live loads. and multiplying by sec. θ. we have the maximum strains on the different rods, remembering, that the strains act in opposite directions beyond the centre post. the true strain being the difference between the live and dead loads.

θ 45°, sec. $\theta = 1.414$.

$T' = \left(\tfrac{15}{6} W' + 2\tfrac{1}{2} W\right)$ sec. $\theta = (31.25 + 5.)$ sec θ

$= 36.25 \times 1.414 = 51.26$ tons

$T_2 = \left(\frac{10}{6} W' + 1\tfrac{1}{2} W\right) \sec\theta = (20.8 + 3) \sec\theta$

$= 23.8 \times 1.414 = 33.65$ tons.

$T_3 = \left(\frac{6}{6} W' + \tfrac{1}{2} W\right) \sec\theta = (12.5 + 1) \sec\theta$

$= 13.5 \times 1.414 = 19.08$ tons.

$C_2 = \left(\frac{3}{6} W - \tfrac{1}{2} W\right) \sec\theta = (6.25 - 1) \sec\theta$

$= 5.25 \times 1.414 = 7.42$ tons.

$C_1 = \left(\frac{1}{6} W - 1\tfrac{1}{2} W\right) \sec\theta = (2.08 - 3) \sec\theta$

= negative quantity

To find the chord strains we must get first the weights borne by the main ties when the bridge is fully loaded, as the greatest strain on the chords occurs when the bridge is covered with the rolling load.

The total panel load is $W' + W =$ $12.5 + 2 = 14.5$ tons.

T_3' carries $\frac{1}{2}(W' + W) = 7.25$ tons.

T_2 carries $1\frac{1}{2}(W'+W) = 21.75$ tons. 15

T_1 " $2\frac{1}{2}(W'+W) = 36.25$ " .

This value of T_1 will be found to agree with the value of T_1 on the last page which should be equal to the vertical force at each end, one half panel being borne directly by each abutment. we have five panels or 72.5 tons borne by the ties $\frac{72.5}{2} = 36.25$ the vertical or shearing force at each end. This proves the vertical forces in the bridge to be right.

The general formulae for the horizontal strains is $H. = W \tan\theta$ in which $W =$ wt. borne by tie and $\theta =$ the angle made by tie with vertical $\theta = 45°$; $\tan\theta = 1.00$

$H_1 = W_{T_1} \times \tan\theta = 36.25 \times 1 = 36.25$

$H_2 = H_1 + W_{T_2} \times \tan\theta = 36.25 + 21.75 = 58.00$

$H_3 = H_2 + W_{T_3} \times \tan\theta = 58.00 + 7.25 = 65.25$

To prove this we use Rankins formula

$$H = \frac{(w' + w)l}{K} \cdot \frac{n(N-n)}{2N}$$; in which

$l = 60'$, $k = 10'$ $N = 6$, $n = 3$.

$$H = \frac{14.5 \times 60}{10} \times \frac{9}{12} = 87 \times \tfrac{3}{4} = 65.25.$$

Proving the horizontal strains to be correct. The bridge being a deck bridge the load is supported on the top chord; this form is the best for short spans, where the truss is not deep enough to allow lateral bracing over the track. The best method for supporting the track is to rest the cross beams or ties directly on the chord, and proportion it to stand the compression from the whole truss, and also the transverse

strain produced by the rolling load
between any two posts.

The greatest weight that can come upon one panel between the posts will be the weight on one driver. So we have $M = \frac{Wl}{4} = \frac{112.50 \times 10}{4} = 28,125$, or a bending moment of 28,125 pounds at the centre of a panel in addition to the direct compression from the bridge load.

The compression on the posts will be as follows:

$P_1 = W_{T_1} + 1/2(W' + W) = 43.5$ Tons

$P_2 = W_{T_2} + (W' + W) = 38.3$ "

$P_3 = W_{T_3} + (W' + W) = 28.0$

$P_4 = W_{T_4} + W_{C_2} + (W' + W) = 25.0$ Tons

The bridge being deck we have $W' + W$ brought directly upon the posts.

in addition to the weight of the adjacent ties, — the supposition being that the whole of the load is transmitted from the top.

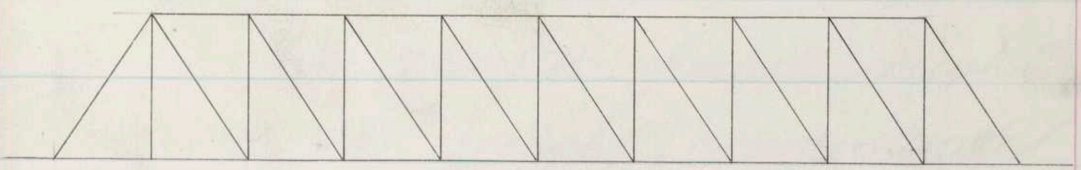
Below are the maximum strains on each point.

Post.	Load.	Strain	Pos.	Load	Position	Strains Low.	Up.
T_1	36.25	51.26	P_1	43.5	H_1	36.25	36.25
T_2	23.8	33.65	P_2	38.3	H_2	58.0	58.0
T_3	13.5	19.08	P_3	28.	H_3		65.25
C_2	5.25	7.42	P_4	25.			
C_1	Negative						

The strains are nothing in the lower chords, of the end panels, as the whole bridge is suspended by ties T.

It is customary in short spans to run the counter braces across the whole truss; even though there may be no strain on the end ones. they tend to prevent vibration.

Case IV



Span 150'; depth 22'; 10 panels.

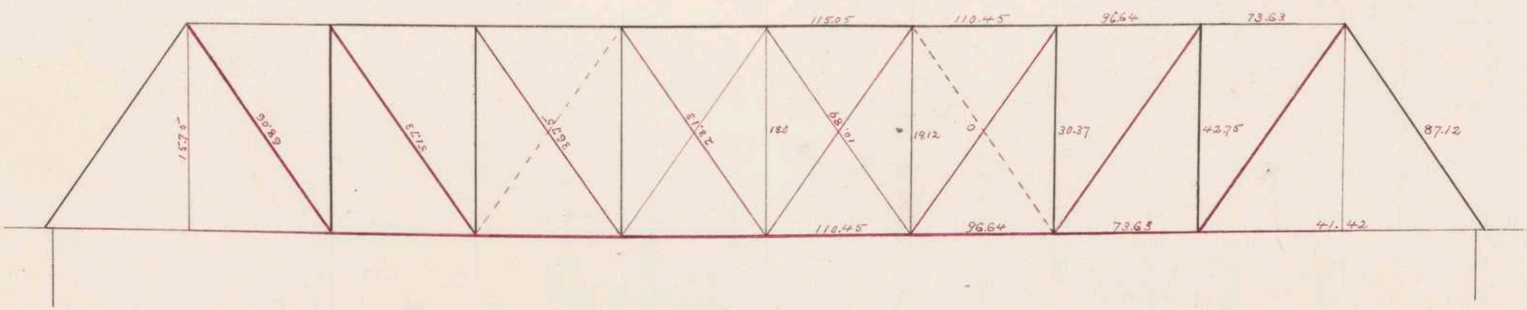
This bridge is known as a single intersection through bridge; for the length of span it is considered the most economical.

In this case we will consider the greatest load that can be brought upon the whole bridge at one time, as our data for the

Diagram of Strains.

Span 150 Ft. Ht. 22 Ft.

CASE IV



W = 11.25 tons. W″ = 9 tons. W = 4.5 tons.

SCALE 20 Ft. = 1 In.

calculation of the chord strains and the greatest panel load for the calculations of the vertical strains. In this truss as in the larger ones, this matter is important but for short spans there is very little difference.

The greatest panel weight will be the weight concentrated on the driving wheels, or 45000 lbs this would give $\frac{45000}{15} = 3000$ pounds per linear ft. for the panel system.

The greatest load that can come upon the whole length of the bridge would be a string of engines with tenders weighing 125000 pounds, each, including tenders and measuring 52 ft. from out to out.

or say 2400 lbs per lin. foot as the greatest uniform load that can come upon the bridge. Let

$W' = \frac{3000}{2} = 1500$ lbs panel load on one truss or $1500 \times 15 = 22,500 = 11.25$ Tons on one panel. $W'' =$ uniform live load or 2400 lbs per ft $= \frac{2400}{2} = 1200$ on one truss, $1200 \times 15 = 18000$ lbs per panel $= 9$ tons. We make no allowance for the effects of a fast train in this or in the following cases as the bridge weight becomes sufficient to counteract any great vibration.

The dead weight of this bridge would be 1200 lbs per ft; let

$W = \frac{1200}{6} = 600$ lbs on one truss or $600 \times 15 = 9000$ lbs $= 4.5$ Tons per panel.

The weights borne by the ties will be

as follows for the dead load

$T_5 = \frac{1}{2} W = 2.25$ Tons

$T_4 = 1\frac{1}{2} W = 6.75$ "

$T_3 = 2\frac{1}{2} W = 11.25$ "

$T_2 = 3\frac{1}{2} W = 15.75$ "

$T_1 = W = 4.5$ "

The weights borne by the ties from the moving load would be as follows, on the principal of the lever.

$c_2 = \frac{1}{10} W' = 1.12$ Tons

$c_3 = \frac{3}{10} W' = 3.37$ "

$c_4 = \frac{6}{10} W' = 6.75$ "

$c_5 = \frac{10}{10} W' = 11.25$ "

$T_5' = \frac{15}{10} W' = 16.87$ "

$T_4' = \frac{21}{10} W' = 23.62$ "

$T_3' = \frac{28}{10} W' = 31.5$ "

$T_2' = \frac{36}{10} W' = 40.5$ "

$T_1' = \frac{9}{10} W' = 10.12$ "

Now. by combining the effects of both dead and live loads, remembering that beyond the centre of the truss the actions of the loads are in opposite directions, we have the maximum load on any tie by multiplying this load by the secant of the angle the tie makes with a vertical we get the strain.

Secant $\theta = \frac{\text{length of tie}}{\text{depth of truss}} = \frac{26.627}{22} = 1.21$

$T_1 = (W' + W) = (11.25 + 4.5) = 15.75$ Tons

$T_2 = (\frac{36}{10}W' + 3\frac{1}{2}W)\sec\theta = (40.5 + 15.75)1.21$

$= 56.25 \times 1.21 = 68.06$

$T_3 = (\frac{28}{10}W' + 2\frac{1}{2}W)\sec\theta = (31.5 + 11.25)$

$= 42.75 \times 1.21 = 51.73$

$T_4 = (\frac{21}{10}W' + 1\frac{1}{2}W)\sec\theta = (23.62 + 6.75)$

$= 30.37 \times 1.21 = 36.75$

$T_5 = (\frac{15}{10}W' + \frac{1}{2}W)\sec\theta = (16.87 + 2.25)$

$= 19.12 \times 1.21 = 23.13.$

$C_5 = (\tfrac{10}{10} W' - \tfrac{1}{2} W) \sec\theta = (11.25 - 2.25$

$= 9.0 \times 1.21 = 10.89$

$C_4 = (\tfrac{6}{10} W' - 1\tfrac{1}{2} W) \sec\theta = (6.75 - 6.75)$

$= 0$ - negative strains.

We see from this that counter bracing is required only one panel beyond the centre.

In order to determine the chord strains, we must get the reaction of the uniform live load from the centre toward each end for "the greatest bending moment occurs when the travelling load extends over the whole length of the beam." Rankine P.p. 248. The horizontal strains in flanges attain their greatest value when the

load covers the whole girder"
Stoney Pp 41.

When the load covers the whole girder we have no counter strains, so we resolve the direct action of the load borne by the ties by the formula: $H = W \tan\theta$
W being the wt. borne by the tie

$\tan\theta = \frac{\text{length of Panel}}{\text{Ht. of Truss}} = \frac{15}{22} = .6818$

$T_5 = \frac{1}{2}(W + W'') = 2.25 + 4.5) = 6.75$ Tons

$T_4 = 1\frac{1}{2}(W + W'') = 20.25$

$T_3 = 2\frac{1}{2}(W + W'') = 33.75$

$T_2 = 3\frac{1}{2}(W + W'') = 47.25$

$P_1 = T_1 + T_2 = (W + W') + 3\frac{1}{2}(W + W'') = 60.75$

Resolving into horizontal components we have.

$H_1 = W P_1 \tan\theta = 60.75 \times 68.18 = 41.42$

$H_2 = H_1 + (W_{T_2} \tan\theta) = 41.42 + 47.25 \times .6818$

$= 41.42 + 32.21 = 73.63$

$\underline{H}_3 = \underline{H}_2 + (W_{T_3} \tan\theta) = 73.63 + 33.75 \times .6818 =$

$= 73.63 + 23.01 = 96.64$

$\underline{H}_4 = \underline{H}_3 + (W_{T_4} \tan\theta) = 96.64 + 20.25 \times .6818 =$

$= 96.64 + 13.81 = 110.45$

$\underline{H}_5 = \underline{H}_4 + (W_{T_5} \tan\theta) = 110.45 + 6.75 \times .6818$

$= 110.45 + 4.60 = 115.05$

To prove the accuracy of these calculations we take Rankines formula:

$$\underline{H} = \frac{(n+w')l}{\underline{k}} \cdot \frac{n(\underline{N}-n)}{2\underline{N}}$$

$l = 150 \quad \underline{K} = 22 \quad \underline{N} = 10 \quad n = 5.$

$$\underline{H} = \frac{(4.5+9)150}{22} \times \frac{5(10-5)}{2 \times 10}$$

$$\therefore = 92.04 \times \frac{25}{20} = 115.05 \text{ Tons}$$

This gives the chord strains at the centre and coincides exactly with the value given above – proving the work to be correct.

The compression on the posts will be as follows.

$P_1 = (W_{T_1} + W_{T_2}) \sec\theta = (15.73 + 56.25)1.21 = 87.12$

$P_2 = W_{T_3} = 42.75$

$P_3 = W_{T_4} = 30.37$

$P_4 = W_{T_5} = 19.12$

$P_5 = (W_{T_5} + W_{C_5}) = 9. + 9. = 18.0$

The first post being inclined the strain is equal to the weight multiplied by the secant of θ. The other posts being vertical, there is no strain except that brought by ties connected with the top of the posts.

In this bridge the floor beams are supported at every panel and the track is carried by longitudinal stringers. The cross girders or floor beams are subjected to a strain

9

given by Rankines formula Pg
46 by (3) in which. k = gauge of rails from centre to centre in inches
W = wt. on two pair of driving wheels in this case; l = span of girders, or width between trusses.
l = 16 ft. = 192 inches k = 58 inches
W = 45000 pounds.

$$\underline{M} = \frac{W(l-k)}{4} = \frac{45000(192-58)}{4}$$

$$= \frac{6030000}{4} = 1,507,500$$

Then using two 15" I beams for the girders, we would have a flange strain of $\frac{1507500}{15}$ = 100,500 lbs
The weight might be reduced theoretically, as the stringers would transmit a portion of it, when the drivers were not over the girders, but this consideration is left out

as the effects of a suddenly ap-
plied load are felt sometimes severely by the floor system of bridges.-

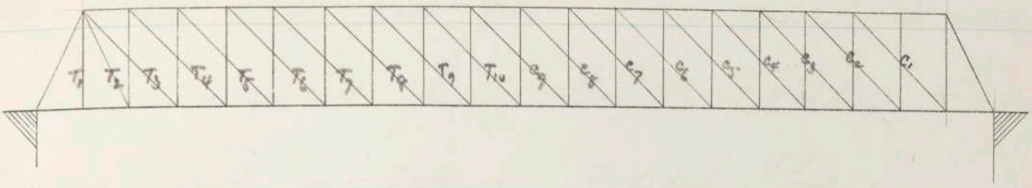
The following table gives the maximum strain that can be brought upon any member by the assumed loads.

Ties Position	Ties Load	Ties Strain	Posts Position	Posts Load	Chords Position	Chords Strains Lower	Chords Strains Upper
T_1	15.75	15.75	P_1	87.12			
T_2	56.25	68.06	P_2	42.75	H_1	41.42	
T_3	42.75	51.73	P_3	30.37	H_2	73.63	73.63
T_4	30.37	36.75	P_4	19.12	H_3	96.64	96.64
T_5	19.12	23.13	P_5	18.0	H_4	110.45	110.45
C_4	9.0	10.89			H_5		115.05
C_3	.0	0					

The compression on any post is equal to the weight borne by the tie or ties.

meeting at its top - except the end 30
ones as explained before.

Case V.



In this we will consider what is known as a double intersection through bridge
Span 250', depth of truss 25' length of panels 12.5' number of panels 20. The weight that can come on the panel system will be the weight on the drivers or $\frac{45000}{12.5}$ = 3600 lbs. per. foot.

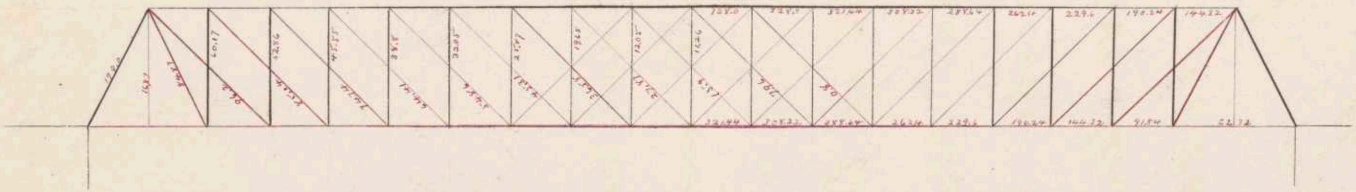
We have seen in the last case

Diagram of Strains.

Span 250 Ft. Ht. 25 Ft.

$W' = 11.25 \quad W'' = 7.5 \quad W = 5.62$ tons.

CASE V.



SCALE 40 Ft. = 1 Inch.

that a string of engines would weigh 2400 lbs per foot, so we will take this as the uniform live load. The dead load of bridge proper track &c, would be 1800 lbs per ft. We will have the following panel weights for one truss. – ton = 2000 lbs.

W = dead load = 11,250 lbs = 5.62 Tons
W' = variable live load = 22,500 lbs = 11.25 "
secant θ = 1.414 for all but end panels
tan θ = 1.000 for all but end panels.
secant θ_1 = 1.118 for end panels.
tan. θ_1 = .5 for end panells.

The ties and counter ties, on the principle of the lever, will bear the following proportions of the variable live load.

$C_1 = 1/20\, W' = .56$ Tons.

$C_2 = 2/20\, W' = 1.12$ "

$C_3 = 4/20\, W' = 1.68$ "

$C_4 = 6/20\, W' = 3.37$ "

$C_5 = 9/20\, W' = 5.05$ "

$C_6 = 12/20\, W' = 6.75$ "

$C_7 = 16/20\, W' = 9.00$ "

$C_8 = 20/20\, W' = 11.25$ "

$C_9 = 25/20\, W' = 14.06$ "

$T_{10} = 30/20\, W' = 16.87$ "

$T_9 = 36/20\, W' = 20.25$ "

$T_8 = 42/20\, W' = 23.62$ "

$T_7 = 49/20\, W' = 27.56$ "

$T_6 = 56/20\, W' = 31.5$ "

$T_5 = 64/20\, W'\ 36.0$ "

$T_4 = 72/20\, W' = 40.5$ "

$T_3 = 81/20\, W' = 45.56$ "

$T_2 = 90/20\, W' = 50.62$

$T_1 = {}^{19}/_{20} W' = 10.69$ Tons

The true strains on the ties are as follows.

$T'_1 = (W' + W) = 16.87$ Tons.

$T'_2 = ({}^{19}/_{20} W' + 4\frac{1}{2} W) \sec \theta_1 = (50.62 + 25.29) 1.118$

$= 75.91 \times 1.118 = 84.87$ Tons.

$T'_3 = ({}^{81}/_{20} W' + 4 W) \sec \theta = (45.56 + 22.48) 1.414$

$= 68.04 \times 1.414 = 96.2$

$T'_4 = ({}^{72}/_{20} W' + 3\frac{1}{2} W) \sec \theta = (40.5 + 19.67) 1.414$

$= 60.17 \times 1.414 = 85.08$

$T'_5 = ({}^{64}/_{20} W' + 3 W) \sec \theta = (36 + 16.86) 1.414$

$= 52.86 \times 1.414 = 74.74$

$T'_6 = ({}^{56}/_{20} W' + 2\frac{1}{2} W) \sec \theta = (31.5 + 14.05) 1.414$

$= 45.55 \times 1.414 = 64.41$

$T'_7 = ({}^{49}/_{20} W' + 2 W) \sec \theta = (27.56 + 11.24) 1.414$

$= 38.8 \times 1.414 = 54.86$

$T'_8 = ({}^{42}/_{20} W' + 1\frac{1}{2} W) \sec \theta = (23.62 + 8.43) 1.414$

$= 32.05 \times 1.414 = 45.31$

$T_9 = \left(\frac{36}{20}\overline{W}' + \overline{W}\right)\sec\theta = (21.25 + 5.62)\,1.414$

$= 25.87 \times 1.414 = 36.58$

$T_0 = \left(\frac{30}{20}\overline{W}' + \frac{1}{2}\overline{W}\right)\sec\theta = (16.87 + 2.81)\,1.414$

$= 19.68 \times 1.414 = 27.82$

$C_9 = \left(\frac{25}{20}\overline{W} - \frac{1}{2}\overline{W}\right)\sec\theta = (14.06 - 2.81)\,1.414$

$= 11.25 \times 1.414 = 15.9$

$C_8 = \left(\frac{20}{20}\overline{W}' - \overline{W}\right)\sec\theta = (11.25 - 5.62)\,1.414$

$= 5.63 \times 1.414 = 7.96$

$C_7 = \left(\frac{16}{20}\overline{W}' - 1\frac{1}{2}\overline{W}\right)\sec\theta = (9.0 - 8.43)\,1.414$

$= .57 \times 1.414 = 0.80$

$C_6 = \left(\frac{12}{20}\overline{W}' - 2\overline{W}\right)\sec\theta = (6.75 - 11.24)\,1.414$

$=$ negative.

The compression on the posts will be as follows:

$P_1 = \left(\overline{W}_{T_1} + \overline{W}_{T_2} + \overline{W}_{T_3}\right)\sec\theta_1 = 160.82 \times 1.118 = 179.8$

$P_2 = \overline{W}_{T_4} = 60.17$

$P_3 = \overline{W}_{T_5} = 52.87$

$P_4 = \overline{W}_{T_6} = 45.55$

$P_5 = W_{T_7} = 38.8$

$P_6 = W_{T_8} = 32.05$

$P_7 = W_{T_9} = 25.87$

$P_8 = W_{T_{10}} = 19.68$

$P_9 = W_{C_7} + W_{C_9} = 12.05$

$P_{10} = W_{C_8} + W_{C_8} = 5.63 \times 2 = 11.26$

To find the chord strains we must find the reactions of the uniform live load from the centre toward either end,

$T_{10} = \frac{1}{2}(W'' + W) = 6.56$

$T_9 = W'' + W = 13.12$

$T_8 = 1\frac{1}{2}(W'' + W) = 19.68$

$T_7 = 2(W'' + W) = 26.24$

$T_6 = 2\frac{1}{2}(W'' + W) = 32.8$

$T_5 = 3(W'' + W) = 39.36$

$T_4 = 3\frac{1}{2}(W'' + W) = 45.92$

$T_3 = 4(W'' + W) = 52.48$

$T_2 = 4\tfrac{1}{2}\,(W'' + W) = 59.04$

$T_1 = W'' + W = 13.02$

$P_1 = T_1' + T_2' + T_3' = 9\tfrac{1}{2}\,(W'' + W) = 124.64$

By resolving these forces into horizontal components, we have the following.

$H_1 = W_{P_1} \times \tan\theta_1 = 124.64 \times 5 = 62.32$

$H_2 = (W_{T_2} \tan\theta_1) + H_1 = 29.52 + 62.32 = 91.84$

$H_3 = (W_{T_3} \tan\theta) + H_2 = 52.48 + 91.84 = 144.32$

$H_4 = W_{T_4} \tan\theta) + H_3 = 45.92 + 144.32 = 190.24$

$H_5 = (W_{T_5} \tan\theta) + H_4 = 39.36 + 190.24 = 229.6$

$H_6 = (W_{T_6} \tan\theta) + H_5 = 32.8 + 229.6 = 262.4$

$H_7 = (W_{T_7} \tan\theta) + H_6 = 26.24 + 262.4 = 288.64$

$H_8 = (W_{T_8} \tan\theta) + H_7 = 19.68 + 288.84 = 308.32$

$H_9 = (W_{T_9} \tan\theta) + H_8 = 13.12 + 308.32 = 321.44$

$H_{10} = W_{T_{10}} \tan\theta) + H_9 = 6.56 + 321.44 = 328.0$

To prove this we have $H = \frac{(w + w')l}{K} \cdot \frac{n(N-n)}{2N}$

$n = 10,\ N = 20,\ K = 25$

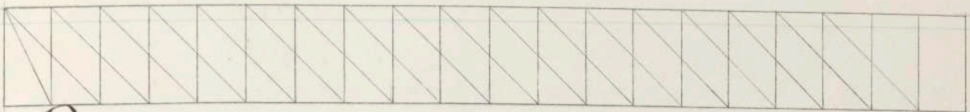
$H_{10} = \frac{13.12 \times 250}{25} \times \frac{10 \times 10}{40} = 131.2 \times \frac{100}{40} = 328.00$

This proves the work.

The width of truss is 16', so the cross girders would be the same as in the preceding case. Tabulating the results we have the following maximum strains.

T_1 =	16.87	16.87	P_1	179.0	H_1	62.32	
T_2 =	75.91	84.87	P_2	60.17	H_2	91.84	
T_3	68.08	96.2	P_3	52.86	H_3	144.32	144.32
T_4	60.17	85.08	P_4	45.55	H_4	190.24	190.24
T_5	52.86	74.74	P_5	38.8	H_5	229.6	229.6
T_6	45.55	64.41	P_6	32.05	H_6	262.4	262.4
T_7	38.8	54.86	P_7	25.87	H_7	288.64	288.64
T_8	32.05	45.31	P_8	19.68	H_8	308.32	308.32
T_9	25.87	36.58	P_9	12.05	H_9	321.44	321.44
T_{10}	19.68	27.87	P_{10}	11.26	H_{10}		328.00
C_9	11.25	15.9					
C_8	5.63	7.96					
C_7	0.57	0.8					

Case VI



For this case we will take a span such as the one to be built across the Hudson River at Pough-keepsie N.Y. This, probably will be a good illustration of long span bridges

Span 520 ft. Ht. = 52 ft. No. of panels = 20. panel length 26 ft. Top chord trussed midway between posts. Deck bridge.

The maximum panel weights will be as follows.

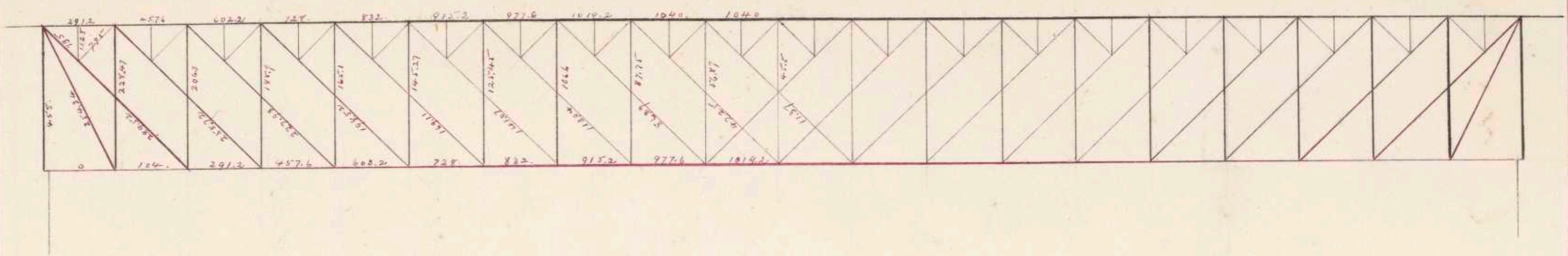
W' = variable live load = 3000 lbs per ft. this being about the wt. per foot of the standard engine

Diagram of Strains.

Span 520 Ft. Ht. 52 Ft.

W' = 19.5 tons. W'' = 15.6 tons. W = 26 tons.

CASE VI



SCALE = 65 Ft. = 1 In.

without the tender. 3000 × 26 = 78000 lbs per panel

The uniform live load = W'' will be the same as in the preceding case = 2400 lbs per ft. 2400 × 26 = 62400 lbs per panel.

The dead load will be 4000 lbs per ft = 104000 lbs per panel.

Reducing to tons of 2000 lbs we have for one truss.

$W' = 19.5$ Tons, $W'' = 15.6$ Tons, $W = 26.0$

θ = angle of ties with a vertical =

secant θ_1 = 1.118 for end panels.

tan θ_1 = .5000 " " "

sec θ = 1.414 for middle, tan θ = 1.00

The principals of calculation in this truss will be the same as the preceding one with the exception of the

difference between a dead load and
through bridge.

The strains on the ties will be:

$T_1 = \left(\frac{100}{20}W' + 4\frac{1}{2}W\right)\sec\theta = 227.5 \times 1.118 = 254.34$

$T_2 = \left(\frac{90}{20}W' + 4\frac{1}{2}W\right)\sec\theta = 204.75 \times 1.414 = 289.52$

$T_3 = \left(\frac{81}{20}W' + 4W\right)\sec\theta = 182.97 \times 1.414 \; 258.72$

$T_4 \left(\frac{72}{20}W' + 3\frac{1}{2}W\right)\sec\theta = 161.2 \times 1.414 = 227.93$

$T_5 = \left(\frac{64}{20}W' + 3W\right)\sec\theta = 140.4 \times 1.414 = 198.52$

$T_6 = \left(\frac{56}{20}W' + 2\frac{1}{2}W\right)\sec\theta = 119.6 \times 1.414 = 169.11$

$T_7 = \left(\frac{49}{20}W' + 2W\right)\sec\theta = 99.77 \times 1.414 = 141.07$

$T_8 = \frac{42}{20}W' + 1\frac{1}{2}W)\sec\theta = 79.95 \times 1.414 = 113.04$

$T_9 = \left(\frac{36}{20}W' + W\right)\sec\theta = 61.1 \times 1.414 = 86.39$

$T_{10} = \left(\frac{30}{20}W' + \frac{1}{2}W\right)\sec\theta = 42.25 \times 1.414 \; 59.74$

$C_9 = \left(\frac{25}{20}W - \frac{1}{2}W\right)\sec\theta = 11.37 \times 1.414 = 16.07$

We can see from the calculations that the counter strains would only extend one panel beyond the centre for the next rod would give – 6.5 Tons. The reasons for this, are the great difference between the dead, and uniform live load.

The compression on the posts will be as follows, On the supposition that the whole weight of the bridge comes upon the top chord.

$P_1 = W_{T_1} + W_{T_2} + \frac{1}{2}(W' + W) = 432.25 + 22.75 = 455$

$P_2 = W_{T_3} + (W' + W) = 182.97 + 45.5 = 228.47$

$P_3 = W_{T_4} + (W' + W) = 161.2 + 45.5) = 206.7$

$P_4 = W_{T_5} + (W' + W) = 140.4 + 45.5 = 185.9$

$P_5 = W_{T_6} + (W' + W) = 119.6 + 45.5 = 165.1$

$P_6 = W_{T_7} + (W' + W) = 99.77 + 45.5 = 145.27$

$P_7 = W_{T_8} + (W' + W) = 79.95 + 45.5 - 125.45$

$P_8 = W_{T_9} + (W' + W) = 61.1 + 45.5 = 106.6$

$P_9 = W_{T_{10}} + (W' + W) = 42.25 + 45.5 = 87.75$

$P_{10} = W_{C_9} + (W' + W) = 11.37 + 45.5 = 56.87$

$P_{11} = W' + W = 45.5.$

Post P_1 should bear one half of the weight of the truss and load or $45.5 \times 20 = 910$ Tons; $\frac{910}{2} = 455.$ the compression on post P_1. This proves the vertical effect of the load.

The chord strains will be calculated as before from $W'' + W$.

$H_1 = 5(W'' + W) \tan\theta_1 = 208 \times .5 = 104$

$H_2 = 4\frac{1}{2}(W' + W) \tan\theta + H_1 = 291.2$

$H_3 = 4 (W'' + W) \tan\theta + H_2 = 457.6$

$H_4 = 3\frac{1}{2}(W'' + W) \tan\theta + H_3 = 603.2$

$H_5 = 3 (W'' + W) \tan\theta + H_4 = 728.$

$H_6 = 2\frac{1}{2}(W'' + W) \tan\theta + H_5 = 832$

$H_7 = 2 (W'' + W) \tan\theta + H_6 = 915.2$

$H_8 = 1\frac{1}{2}(W'' + W) \tan\theta + H_7 = 977.6$

$H_9 = (W'' + W) \tan\theta + H_8 = 1019.2$

$H_{10} = (W'' + W)\tan\theta + H_9 = 104.0.0$

To prove the chord strains, we have $H_{10} = \frac{(W'' + w)l}{K} \cdot \frac{n(N-n)}{2N}$

$n = 10,\ N = 20\ K = 52\ \ l = 520$

$H_{10} = \frac{41.6 \times 520}{52} \times \frac{10.10}{40} = 41.6 \times \frac{100}{40} = 104.0$ Tons

proving the horizontal strains to be correct. The trussed chord would be designed the same as the truss in case II. The post would come down to the intersection of the lines of a main, and counter tie, now $\theta = 45^\circ$, so the post would be 13 feet deep; the span would be equal to the panel length, or 26 ft. The greatest load would be the weight on a pair of drivers at the centre; this would be for our standard engine

45000 lbs; on one truss 22500 lbs = 11.25 T. 44
Compression on Post = 11.25 Tons.
Tension on ties = $\frac{1}{2}W \times \sec\theta$ = 5.62 × 1.414 = 7.95
Compression on Top chord $\frac{Wl}{4d}$ =
= $\frac{11.25 \times 26}{4 \times 13}$ = 5.62 Tons.
This will require an extra area in the chords and extra ties.

The whole arrangement being probably the most economical for the length of span.

Cross girders will be 13 ft. apart

Table in following page will give the maximum strain which can be brought on any member by the assumed loads.

The object of this Thesis has been 46
to avoid the use of complicated
formulae in all cases yet to make
the as accurate and thorough
as any practical considerations
should require. The methods
of calculating the strains on the
ties and counter-ties is the same
in principle as Rankine's given
in the case of a Howe truss, but
much more simple to under-
stand.

The engine weight assumed
might not be large enough for
some coal roads but the general
principle has been illustrated, so
that it could be applied to any
similar structure.

In the diagrams of strains, the red lines denote tension and the black lines compression, the shading of the lines, showing the varying strains.