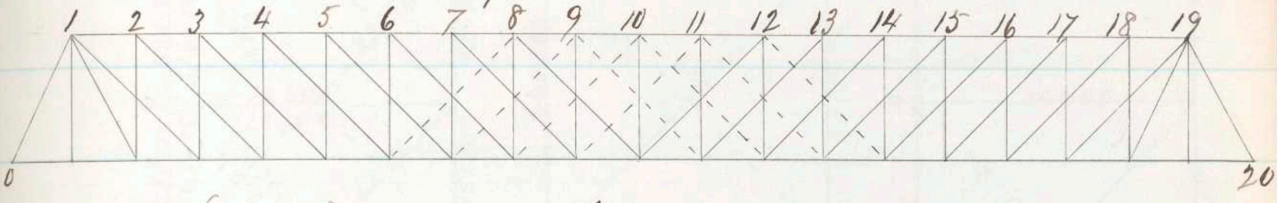
Thesis on the Linville Bridge.

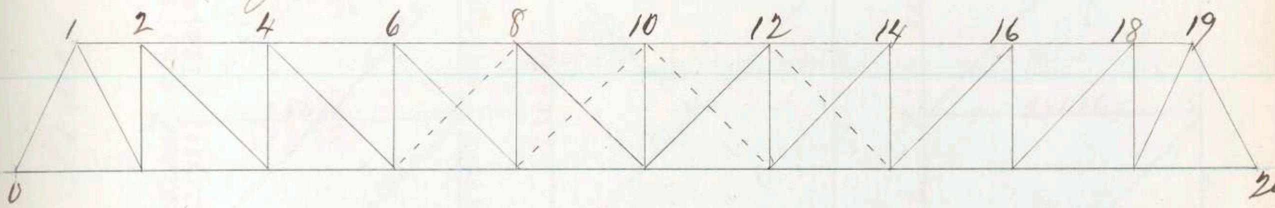
by
Joseph S. Emerson. Class of '74.

Linville Truss.

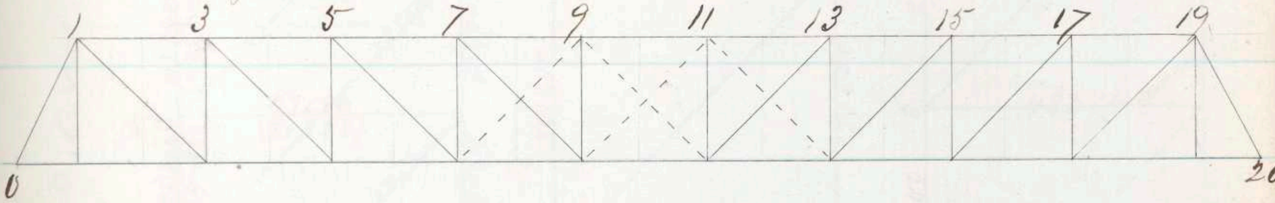
(Fig. 1) Compound Truss.



(Fig. 2.) Truss No. 1.



(Fig 3) Truss. No. 2.



Given, $l = 192'$ $k = 19'.2$ $N = 20$ $\frac{l}{N} = 9'.6$

fixed load = 800 lbs. per ft.

rolling " = 1200 " " "

Required to design a rail road bridge.

w = fixed load on one panel = $800 \times 9.6 = 7680$ lbs.

w' = rolling " " " " $= 1200 \times 9.6 = \underline{11520}$ "

$w + w'$ = total " " " " = 19200 "

Diagram of Forces.

Fig. (4)

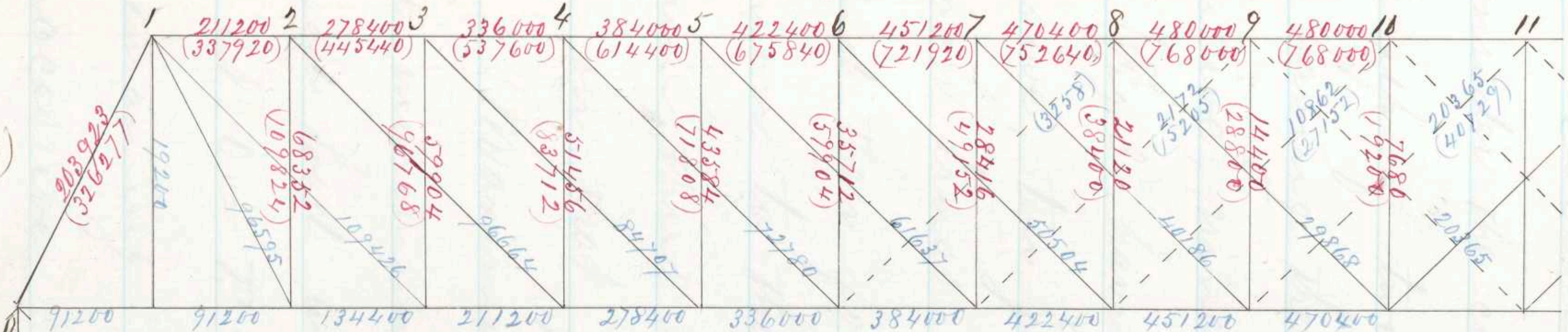


Fig. (5)

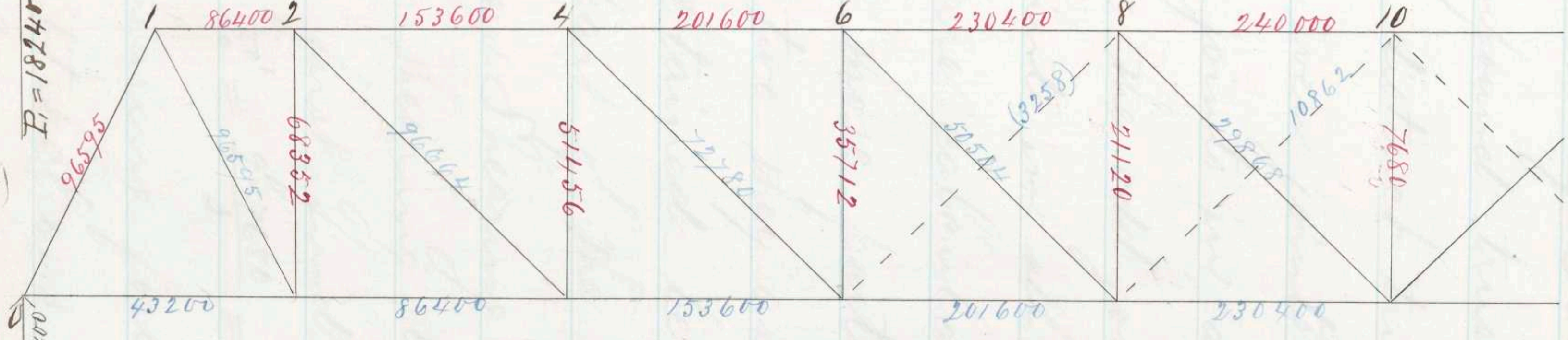
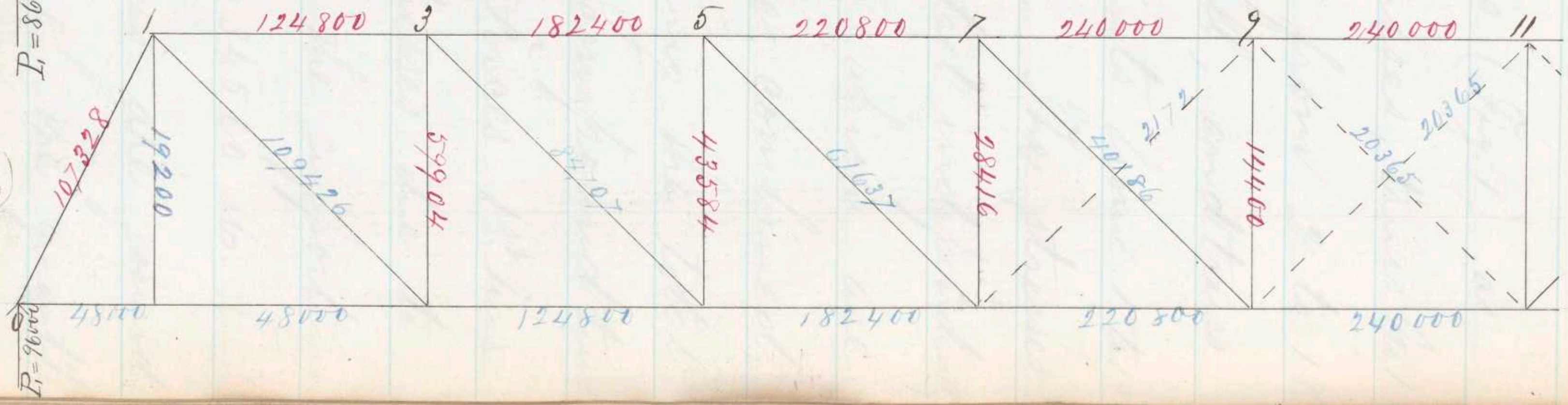


Fig. (6)



Calculation of Stresses.

We consider the compound truss (Fig. 1) as made up of two distinct trusses; truss No. 1. (Fig. 2) having the even joints from 2 to 18 inclusive, nine joints in all, and truss No. 2 (Fig. 3) having the odd joints from 1 to 19 inclusive, ten joints in all. The stresses on each truss are calculated independently and then for those parts which are common to the two they are combined, the sum thus obtained being the total stress on that part of the compound truss.

(Maximum Shearing Forces 1st Truss.

For the maximum shearing stresses due to the fixed load we first find the supporting force at $0 = P_1 = F_0' = \frac{9 \times 7680}{2} = 34560$ lbs.

The remaining shearing forces are found by successive subtractions of the quantity $w = 7680$ lbs. General formula $F_n' = F_{n-1}' - w$

For the maximum shearing stress due to the travelling load at any point, we consider the longer segment of the truss as loaded, and the shorter as unloaded. The supporting force at the nearest end gives the shearing force required. For the point n

no. of panels loaded $= \frac{18-n}{2}$

dist. of C.G. from 20 $= \frac{20-n}{2}$

$\therefore F_n'' = \frac{(18-n)(20-n)\,11520}{4\times 20} = 144(18-n)(20-n)$

Total shearing force $F_n = F_n' + F_n''$

The diagonal thrust T_0 is found by multiplying the shearing force F_0 by the secant of the angle which the diagonal makes with the vertical. This secant $= \frac{\sqrt{(19.2)^2+9.6^2}}{19.2}$ $= 1.118$ $\therefore T_0 = 1.118\,F_0$. The pull on the diagonal tie (1) is found in the same way. The pulls on the remaining diagonal ties are found by the formula $T_n = F_n \sec 45°$

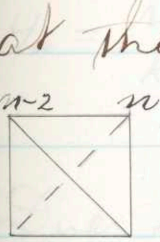
$T_n = 1.4142\,F_n$

Table of Shearing and Diagonal Stresses

1st Truss.

n	F_n' lbs.	F_n'' lbs.	F_n lbs.	T_n lbs.
0	34560	51840	86400	96595
2	26880	41472	68352	96664
4	19200	32256	51456	72780
6	11520	24192	35712	50504
8	3840	17280	21120	29868
10	−3840	11520	7680	10862

The thrust on the vertical strut at any point n is equal to the shearing force at that point, or $V_n = F_n$.



For maximum pull on counter tie at n, we find stress on counter tie n when the shorter segment is loaded with travelling load, and subtract the stress on the tie $(n-2)$ caused by the fixed load. This difference which is the actual maximum pull on the tie at $n - t_n$

$= (F''_{20-n} - F'_{n-2})$ sec. $45° = 1.4142\,(F''_{20-n} - F'_{n-2})$

n	F''_{20-n} lbs.	F'_{n-2} lbs.	$F''_{20-n} - F'_{n-2}$ lbs.	$1.4142(F''_{20-n} - F'_{n-2})$ lbs.
8	6912	11520	–	–
10	11520	3840	7680	10862

Horizontal Stresses 1st Truss.

$$H_1 = \frac{M_1}{k} = \frac{86400 \times 9.6}{19.2} = 43200 \text{ lbs}$$

$$H_2 = \frac{M_2}{k} = \frac{86400 \times 9.6 \times 2}{19.2} = 86400 \text{ "}$$

$$H_4 = \frac{M_4}{k} = \frac{86400 \times 9.6 \times 4 - 19200 \times 9.6 \times 2}{19.2} = 153600 \text{ "}$$

$$H_6 = \frac{M_6}{k} = \frac{86400 \times 9.6 \times 6 - 19200 \times 9.6 \times 3}{19.2} = 201600 \text{ "}$$

$$H_8 = \frac{M_8}{k} = \frac{86400 \times 9.6 \times 8 - 19200 \times 9.6 \times 4}{19.2} = 230400 \text{ "}$$

$$H_{10} = \frac{M_{10}}{k} = \frac{86400 \times 9.6 \times 10 - 19200 \times 9.6 \times 5}{19.2} = 240000 \text{ "}$$

Maximum Shearing Forces 2nd Truss.

Due to fixed load $F'_0 = \dfrac{10 \times 7680}{2} = 38400$ lbs.

General formula $F'_n = F_{n-1} - w$.

Due to rolling load $F''_0 = \dfrac{10 \times 11520}{2} = 57600$ lbs.

For any other point n

no. of panels loaded $= \frac{20-(n+1)}{2} = \frac{19-n}{2}$

dist. of C.G. from 20 $= \frac{20-(n+1)}{2} = \frac{19-n}{2}$

$\therefore F_n'' = \frac{19-n}{2} \times \frac{(19-n)11520}{2\times 20} = 144\,(19-n)^2$

Total shearing force $F_n = F_n' + F_n''$

The diagonal thrust $T_0 = 1.118\,F_0$ and the pulls on the diagonal ties are found by the formula $T_n = 1.4142\,F_n$ as in the 1st truss.

Table of Shearing and Diagonal Stresses

2nd Truss.

n	F_n'	F_n''	F_n	T_n
0	38400	57600	96000	107328
1	30720	46656	77376	109426
3	23040	36864	59904	84707
5	15360	28224	43584	61637
7	7680	20736	28416	40186
9	0	14400	14400	20365

The vertical tie rod at 1 has to support the wt. on one panel length. Its stress is therefore given by the formula $V_1 = w + w' = 19200$ lbs. The remaining vertical stresses are thrusts on the vertical struts whose stresses are given, as in the 1st truss, by the formula $V_n = F_n$. For pulls on counter ties we have the formula, already explained,

$$t_n = 1.4142\left(F''_{20-n} - F'_{n-2}\right)$$

n	F''_{20-n}	F'_{n-2}	$F''_{20-n} - F'_{n-2}$	$t_n = 1.4142\left(F''_{20-n} - F'_{n-2}\right)$
7	5184	15360	–	–
9	9216	7680	1536	2172
11	14400	0	14400	20365

Horizontal Stresses 2nd Truss

$$H_1 = \frac{M_1}{k} = \frac{96000 \times 9.6}{19.2} = 48000 \text{ lbs.}$$

$$H_3 = \frac{M_3}{k} = \frac{96000 \times 9.6 \times 3 - 19200 \times 9.6 \times 2}{19.2} = 124800 \text{ ''}$$

$$H_5 = \frac{M_5}{k} = \frac{96000 \times 9.6 \times 5 - 19200 \times 9.6 \times 3}{19.2} = 182400 \text{ ''}$$

$$H_7 = \frac{M_7}{k} = \frac{96000 \times 9.6 \times 7 - 192000 \times 9.6 \times 4}{19.2} = 220800 \text{ ''}$$

$$H_9 = \frac{M_9}{k} = \frac{96000 \times 9.6 \times 9 - 192000 \times 9.6 \times 5}{19.2} = 240000 \text{ ''}$$

$$H_{11} = \frac{M_{11}}{k} = \frac{96000 \times 9.6 \times 11 - 192000 \times 9.6 \times 6}{19.2} = 240000 \text{ ''}$$

Combined Stresses due to 1st & 2nd Trusses.

Thrust on Strut 0.

	1st truss lbs.	2nd truss lbs.	compound truss lbs.
T_0	96595	107328	203923

Horizontal Stresses.

n	H_n 1st truss lbs.	H_n 2nd truss lbs.	H_n compound truss. lbs.
1	43200	48000	91200
1½	86400	48000	134400
2	86400	124800	211200
3	153600	124800	278400
4	153600	182400	336000
5	201600	182400	384000
6	201600	220800	422400
7	230400	220800	451200
8	230400	240000	470400
9	240000	240000	480000
10	240000	240000	480000

Factors of Safety.

In applying Gordon's Formula (see Rankine's C.E. pg. 523) for the strengths of wrought iron struts, it becomes necessary to introduce the following factors of safety, viz.

for travelling load 6

" fixed " 3.

In order to do this the more readily, we calculate the thrusts for the load $(w+2w')$ on each panel length and then use the factor of safety, 3, once for all.

The ratio between the horizontal thrusts on the upper boom due to the panel load $(w+2w')$ and the corresponding thrusts due to the panel load $(w+w')$, already found, will be simply $\frac{w+2w'}{w+w'}$ $= \frac{800+2400}{800+1200} = \frac{32}{20} = 1.6$ This ratio also applies to the thrusts on strut O.

The thrust on any vertical strut is due to the panel load $(w+2w')$ is given by the formula $T'_n = F'_n + 2F''_n$, F'_n and F''_n having already been calculated.

Table of Thrusts due to Panel Load $w+2w'$.

	due to $w+w'$	due to $w+2w'$
On Strut 0	$T_0 =$ 203923 lbs	$T'_0 = 1.6\,T_0 =$ 326277 lbs

On Upper Boom.

n	H_n	$H'_n = 1.6\,H_n$
2	211200 lbs	337920 lbs
3	278400 "	445440 "
4	336000 "	537600 "
5	384000 "	614400 "
6	422400 "	775840 "
7	451200 "	721920 "
8	470400 "	752640 "
9	480000 "	768000 "
10	480000 "	768000 "

Tension in Counter Ties due to Panel Load $w+2w'$.

If we calculate the strains in the counter ties, taking into account the fact that the factor of safety for the rolling load is double that for the fixed load, we shall find that more ties are required than our former calculations called for. On this account it seems safer to adopt the following formula for calculating their tensions, viz. $t'_n = 1.4142\,(2F''_{20-n} - F'_{n-2})$. The following table gives the results.

1st Truss

n	$2F''_{20-n}$ lbs.	F'_{n-2} lbs.	$2F''_{20-n} - F'_{n-2}$ lbs.	$t'_n = 1.4142(2F''_{20-n} - F'_{n-2})$ lbs.
6	6912	19200	—	—
8	13824	11520	2304	3258
10	23040	3840	19200	27152

2nd Truss.

n	$2F''_{20-n}$ lbs.	F'_{n-2} lbs.	$2F''_{20-n} - F'_{n-2}$ lbs.	$t'_n = 1.4142(2F''_{20-n} - F'_{n-2})$ lbs.
7	10368	15360	—	—
9	18432	7680	10752	15205
11	28800	0	28800	40729

On Pg. 2. is a diagram of forces, giving the results of all the preceeding calculations, in which thrusts are given in red and pulls in blue. Stresses due to the panel load $w+2w'$ are enclosed in a parenthesis.

Design of the Parts.

The Struts — These I make of wrought iron Phoenixville columns and in applying Gordon's formula, for the working strength, I use the stresses in parenthesis with a factor of safety, 3. The formula thus becomes $\frac{P}{S} = \frac{12000}{1+\frac{l^2}{36000 r^2}}$

Substituting for r^2 its value $\frac{h^2}{8}$ in a thin cylindrical cell, and solving with respect to S, we get $S = \frac{\left(1+\frac{l^2}{4500h^2}\right)P}{12000}$

For the end strut it is near enough to call $l = 20'$ and $h = 15''$, as the inside diameter is to be $14\frac{3}{8}''$. Putting in these values the formula reduces to $S = \frac{P}{11354}$ square inches. Using the same internal diameter for the struts of the upper boom and calling $l = 9'$ and $h = 15''$, the formula reduces to $S = \frac{P}{11863}$ sq. inches. From these data and the tables of the Phoenixville Manufacturing Company I obtain the following results

End Strut.

area in sq inches	thickness in inches	lbs. wt. per ft.
29	$\frac{5}{16}$	96.7

Horizontal Struts, Upper Boom.

No.	area in sq. inches	thickness in inches	lbs. wt. per ft.
2	29	$\frac{5}{16}$	96.7
3	38	$\frac{1}{2}$	126.7
4	46	$\frac{5}{8}$	153.3
5	52	$\frac{3}{4}$	173.3
6	57	$\frac{13}{16}$	190.0
7	62	$\frac{7}{8}$	206.7
8	64	$\frac{15}{16}$	213.3
9	65	$\frac{15}{16}$	216.7
10	65	$\frac{15}{16}$	216.7

For the vertical struts we will use columns whose internal diameter is $7\frac{2}{16}''$. Calling $l = 18'$ and $h = 8''$ our formula becomes $S = \frac{P}{10327}$ sq. inches, which gives the following results.

Vertical Struts.

No.	area in sq inches	thickness in inches	lbs. wt. per. ft.
2	10.5	$\frac{5}{8}$	35.0
3	9.5	$\frac{9}{16}$	31.7
4	8.	$\frac{1}{2}$	26.7

We will make all the rest the same as the 4th

The Ties.

These are made of wrought iron bars, the sectional area required at any point being found by dividing the stress at that point, as given in the diagram of forces, by 10000, the working tensile strength of iron bars.

Chord Links. – These have 3" eyes, and are thickened around the eyes to once and a half the thickness of the rest of the link to give bearing area and strength.

The following table gives the number, size etc. of the chord links.

panel.	area cross section sq. inches	no of links.	size in inches	total lbs. wt. per. ft.
1	10.5	2	$6'' \times \frac{7}{8}''$	35
2	10.5	2	$6 \times \frac{7}{8}$	35
3	18.0	4	$6 \times \frac{3}{4}$	60
4	24.0	4	6×1	80
5	31.5	6	$6 \times \frac{7}{8}$	105
6	40.5	6	$6 \times 1\frac{1}{8}$	135
7	42.0	8	$6 \times \frac{7}{8}$	140
8	42.0	8	$6 \times \frac{7}{8}$	140
9	48.0	8	6×1	160
10	48.0	8	6×1	160

Vertical and Diagonal Tie Rods.

These do not require to be thickened around the eyes to give bearing area, but are made 2" wide all around to insure sufficient tensile strength.

The following table gives the number size etc. of the vertical and diagonal tie rods.

	area cross section sq inches	no. of ties	size in inches	total lbs. wt. per ft.
Vert. tie 1	3	2	$3 \times \frac{1}{2}$	10

Diagonal Ties numbered from above.

No				
1 – 2	10.125	2	$3 \times 1\frac{11}{16}$	33.75
1 – 3	11.25	2	$3 \times 1\frac{7}{8}$	37.5
2	10.125	2	$3 \times 1\frac{11}{16}$	33.75
3	8.625	2	$3 \times 1\frac{7}{16}$	28.75
4	7.5	2	$3 \times 1\frac{1}{4}$	25.
5	6.75	2	$3 \times 1\frac{1}{8}$	22.5
6	5.25	2	$3 \times \frac{7}{8}$	17.5
7	4.125	2	$3 \times \frac{11}{16}$	13.75
8	3.	2	$3 \times \frac{1}{2}$	10

Counter Ties.

These are made of round bars with a lengthening bar ~~swivel band screw~~ in the middle by which they are to be kept tight.

Their number and dimensions are given in the following table.

No.	area cross section in sq. inches	no of ties	diameter in inches	total lbs. wt. per ft
6	.884	2	$\frac{3}{4}$	2.95
7	.884	2	$\frac{3}{4}$	2.95
8	.884	2	$\frac{3}{4}$	2.95
9	1.571	2	1	5.24
10	2.455	2	$1\frac{1}{4}$	8.15
11	3.535	2	$1\frac{1}{2}$	11.78

The Pins

These are made of steel. For the two, at either extremities of the upper boom, the diameter is $3\frac{1}{2}''$, and for all the rest the diameter is $3''$. Taking the shearing strength of steel at 12000 lbs. per. sq. inch. I get the following results for the shearing strength etc. of the pins.

diameter in inches	area crossection in sq inches	shearing stress in lbs.	lbs. wt per ft
3	7.071	84852	23.57
3.5	9.624	115488	32.08

The arrangement of the links and ties is such that in no case does any pin have to bear more than the above shearing forces.

The Castings.

Figs. 11 and 14 show the construction of the casting at the bottom of the end strut. Figs. 9 and 12 show that at the top of the same strut, while in Figs. 15 and 18 are shown the castings that connect the sections of the upper boom. It is only necessary to add that the holes for the pins are drilled and the faces against which the struts abut are carefully planed.

In Figs. 17 and 20 are shown the castings to which the lower horizontal cross bracing is secured.

The Platform.

This is shown in Figs 7 & 8 and is made up of 4 stringers 8"×14", with cross ties 8"×8" placed 16" from centre to centre at the ends of which are the two guard rails 8"×12".

The I Iron Cross Girders.

These have to support the weight of the train, platform and their own weight.
The wt of the platform and I irons is, assuming the I irons to weigh 200 lbs. per yard, 5786 lbs. The greatest rolling load ~~weight~~ per panel that can come on the I irons is what rests on 4 driving wheels or 40000 lbs, making in all 45786 lbs on 2

I irons. We consider this whole load as concentrated at two points, one under each rail 66" from the point of support. This gives a bending moment of

$\frac{45786 \times 66}{4} = 754469$ inch lbs. for each of the 2 I irons which requires a Phoenix ville I iron 200 lbs to the yard, ht.=15" sectional area 20 sq. inches.

The platform is suspended by 4 pins, each holding $\frac{45786}{4} = 11446$ lbs.

This calls for a pin of 1.2" sectional area, but as these bolts are peculiarly exposed to shocks, I make them 1½" diameter or 1.8" cross section.

The large I iron girders rest at each end upon 3 small ones, as shown in Figs. 17 and 20. These latter are of the Phoenixville make. 4" in height.

10 lbs. to the ft. And these, in turn, rest on a wrought iron plate $17.5'' \times 8'' \times \frac{1}{2}''$, supported by the bolts already described.

Top Cross Bracing.

The struts I make of Phoenixville columns of the smallest size.

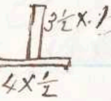
Thickness in inches	$\frac{1}{8}$
Area in sq. inches	2.8
Lbs. wt per ft.	9.3
Inside diameter in inches	$3.\frac{5}{8}$

These are fitted into the castings as shown in Figs. 9, 12, 13 and 18.
The end tie rods are $1\frac{1}{4}''$ in diameter. As we go towards the centre they diminish ~~by $\frac{1}{8}''$~~ successively by $\frac{1}{8}''$ until the size of $\frac{3}{4}''$ is reached, when all the rest are made of that uniform size.

Lower Cross Bracing.

Here the T irons act as struts.
Beginning at the ends with ties $2\frac{1}{2}''$ in diameter, they successively decrease by $\frac{1}{8}''$ in diameter as we proceed to the centre.

End Brace.

This is a wrought iron semicircular arch made of an inverted T shaped piece, 

with a cross section of $5\frac{1}{2}$ sq. inches.
It is firmly bolted to the end posts and to the 1st strut of the upper cross bracing.
The above ~~dimen~~ dimensions I have adopted as sufficient to resist the lateral force of the wind when blowing with a force of 50 lbs per. sq. ft. when the bridge was

unloaded, and 25 lbs per sq. ft. when a train was passing over it.

In the following tables I have given the results of my calculations of the weight of each panel. The weight of the wood work has been taken as 36 lbs. per cubic ft. throughout, this being an average weight of spruce pine. By adding all the panels together I make the total weight of one half of the bridge 148510 lbs or an average of 14851 lbs. per panel. The panel load on each girder is thus 7426 lbs. and the weight of the whole bridge is 297020 lbs. or 148½ tons.

Table of Weights in pounds.

Panel	1 lbs.	2 lbs.	3 lbs.	4 lbs.	5 lbs.
4 Stringers	1075	1075	1075	1075	1075
Cross ties	1498	1498	1498	1498	1498
Guard Rails	461	461	461	461	461
Rails	384	384	384	384	384
Large & small I irons	1204	2408	2408	2408	2408
Castings	2300	500	400	400	400
Pins, nuts, bolts, etc.	272	544	544	544	544
Top cross struts	76	151	151	151	151
Posts	—	660	1257	1099	1005
Top chord	1708	1708	2238	2708	3061
Chord links	700	700	1200	1600	2100
Main ties	195	3780	1890	1610	1400
Counter ties	—	—	—	—	—
Upper cross ties	—	163	132	104	80
Lower " "	557	497	440	387	336
End arch	418				
Total	10848	14529	14078	14429	14903

Table of weights in pounds.

Panel	6 lbs.	7 lbs.	8 lbs.	9 lbs.	10 lbs.
4 Stringers	1075	1075	1075	1075	1075
timber Cross ties	1498	1498	1498	1498	1498
Guard Rails	461	461	461	461	461
Rails	384	384	384	384	384
Large & small Irons	2408	2408	2408	2408	2408
Castings	400	400	400	400	400
Pins, bolts, nuts, etc	544	544	544	544	544
Top cross struts	151	151	151	151	151
Posts	1005	1005	1005	1005	1005
Top chord	3490	3652	3768	3828	3828
Chord links	2700	2800	2800	3200	3200
Main ties	1260	980	770	560	—
Counter ties	165	165	165	294	1116
Upper cross ties	59	59	59	59	59
Lower " "	291	247	208	172	139
Total	15891	15829	15696	16039	16268