Thesis on a Murphy-Whipple Truss

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The following problem was given out, A Murphy-Whipple Truss is to be constructed for a single track railway. There are to be two trusses, each to be 18 feet high, 192 feet long divided into 16 panels each 12 feet long. Each truss is to bear a rolling load of 1200 pounds per foot run and we assume the fixed load at 800 pounds per foot for purposes of calculations.

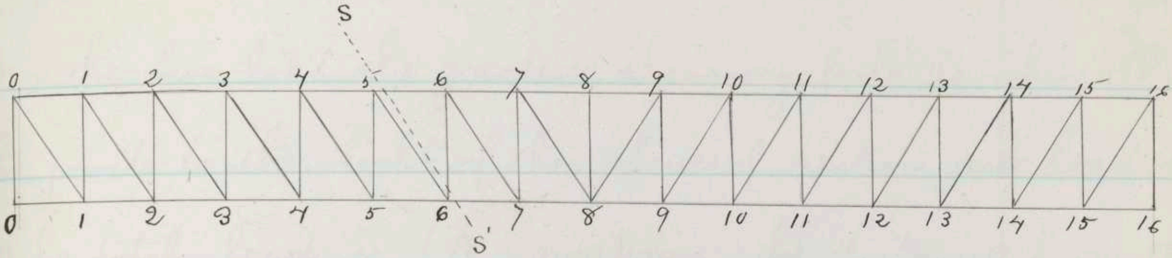
A Murphy-Whipple truss consists essentially of an upper chord to resist thrust, a lower chord to resist tension, a series of vertical posts to resist thrust, and system of diagonal tie bars sloping downward toward the centre. These diagonal ties extend only from the top of one post to the foot of the next. Certain other ties are introduced in the central portion of the bridge sloping in the opposite direction called counter-ties. This name Murphy-Whipple is only applied to ~~iron~~ bridges of this general structure.

The details which we have adopted can be best seen on consultation of the accompanying drawings. It is constructed mainly as follows. First the ~~series~~ of vertical posts whose cross-section is a hollow octagon abut above and below into ~~a~~ cast iron pieces which have tongues which fit the interior of the posts and hold them in place. The upper of these pieces is so arranged as to fit in an exactly similar manner the pieces of which the upper chord ~~chor~~ is composed which are of cross section similar to that of the posts. Through ~~this~~ each of these pieces a large steel pin runs from which the diagonal braces are suspended and to this casting also the lateral braces between the two trusses are attached.

The lower casting is hollow so as to admit the eye-bars of which the lower chord is made. These bars are held together by another steel pin and from this pin are hung the rods which suspend the cross girders which sustain the track. The diagonal ties are also eye bars and run between the pins in the two chords and are ~~a~~ both above and below entirely outside of the castings. The counter-ties are round rods and are put on outside of the main ties.

The general form of the bridge being understood ~~the~~ we now shall pursue the following order. First the deduction the proper formulae for calculation of the stresses; second computation of the stresses; third computation of proper dimensions for the pieces; fourth computation of those parts which relate to the bridge as a whole as cross-girders lateral braces &c.; fifth the weight; and finally recalculation (if necessary) of the stresses.

To deduce the formulae for the calculation of the stresses we assume all the weight to be concentrated at the joint at the lower end of each post. Commencing at the left hand point of support as 0 the joints on the two chords are to be numbered consequtively. Let n denote the number of any joint and N the total number



~~the~~ of divisions in the truss. The posts and diagonals are numbered by the number of the joint where their lower ends meet. The divisions of the upper chord are numbered by the greater number of the joints it lies between, those of the lower by the less. Let k denote the height of the girder measured from centre to centre of the pins in the upper and lower chords; s, the length of a diagonal $= \sqrt{k^2 + \frac{l^2}{N^2}}$

w, the uniform fixed load at each joint

w', the travelling load at each joint.

To find the greatest shearing force at any ~~joint~~ ~~section~~ we take a section just to the right of that joint. (As SS' in the figure) For the dead load the shearing force is evidently equal the supporting force minus the number of weights on the left of the section or to express this algebraically

$F' = \frac{N-1}{2} w - nw = w\left[\frac{N-1}{2} - n\right]$

For the live load the greatest shearing force is when all the joints to the right of the plane of section are loaded. The total load is $(N - n - 1)\,w$, the distance from the nearest point of support is $\frac{(N-n)}{2}$ therefore the shearing force $F'' = (N - n - 1)\,\frac{(N-n)}{2}\,w'$

$$\therefore V_n = F' + F'' = w\left(\frac{N-1}{2} - n\right) + w'(N-n)\,\frac{N-n-1}{2N}$$

This formula evidently gives the compression in the posts. This stress in these posts is evidently due to the pull on it from the ties next nearest the centre therefore the ~~how~~ vertical components of these two stresses must be equal hence the stress in the direction of the diagonal $T_n = \frac{s}{k}\,V_{n-1}$

When the shorter segment is loaded with the live load the shearing force due to it is of the opposite kind of that due to the dead load. In this case

$$F_1'' = n w'\,\frac{n+1}{2N} \quad \text{and} \quad V_n' = w'\,\frac{n(n+1)}{2N} - w\left(\frac{N}{2} - n - 1\right)\cancel{\frac{(N-n)}{2}}$$

This resolved along the direction of the counter braces

$$t_n = \frac{s}{k}\left[w'\,\frac{n(n+1)}{2N} - w\left(\frac{N-n}{2} - 1\right)\right]$$ This only has to be taken into account when the results are positive.

The bending moment at any cross section
$M = \frac{(w+w')}{2} l \frac{(N-n)n}{2N}$ This is resisted by a couple having an arm k hence the tension in lower and compression in the upper chord is represented by the formula $H_n = \frac{w+w'}{2k} \cdot \frac{l(N-n)n}{2N}$

The ~~formulas becoms~~ reduce to the following when we introduce into them the given quantities

$V_n = 9600(7.5-n) + 450(15-n)(16-n)$ ~~with the factors of safety~~

$sV_n = 28800(7.5-n) + 2700(15-n)(16-n)$ with the factors of safety 3&6

$T_n = \frac{s}{k} V_{n-1} = 1.202 V_{n-1}$

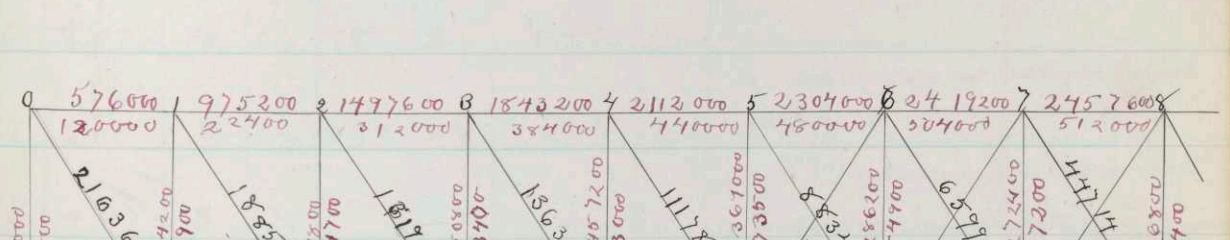
$st_n = 1.202[28800(7.5-n) + 2700\, n(n+1)]$ this includes factors of safety 3&6

$H_n = 8000\, n(16-n)$

$sH_n = 38400\, n(16-n)$ including factors of safety 3&6

The object in computing ~~these~~ some of the stress multiplied by their factors of safety is on account of the ~~we~~ formulae for the struts being in terms of the ultimate strength. ~~ease~~ The counter ties are calculated thus because by doubling the intensity of the dive load their number is increased and we ~~d~~wish to take the method which will produce the greatest ~~strains~~ stress on them.

	V	sV	T	st	H	sH
0	180000	864000	~~216360~~		0	
1	156900	754200	216360 ~~216360~~		120000	576000
2	134700	649800	188094 ~~161909~~		224000	975200
3	113400	550800	161909		312000	1497600
4	93000	457200	136307		384000	1843200
5	73500	369000	111786.	10818	440000	2112000
6	54900	286200	88347	84380	480000	2304000
7	37200	272400	65990	164434.	504000	2419200
8	20400	136800	44714		512000	2457600



Having found the stresses we now must find the size of the pieces which are to bear them.

We take 10 000 pounds as the safe tension for wrought-iron. The following table gives the required cross-section and dimensions of all main and counter-ties and chord-links. All dimensions are in inches

	2 Bars Main Ties			4 Bars 6" wide Chord-links		2 Rods Counter-Ties	
n	Required cross sec	depth	thickness	Required cross sec	thickness	Required Sectional area	Diameter
0				0	1/2*		
1	21.64	5	2 3/16	12			
2	18.86	5	2	22.4	1		
3	16.17	5	1 5/8	31.2	1 3/8		
4	13.63	5	1 3/8	38.4	1 5/8	0	1**
5	11.18	5	1 3/16	44.	1 7/8	.1082	1
6	8.84	4	1 1/8	48.	2	.844	1 1/8
7	6.60	3	1 1/8	50.4	2 1/8	1.644	1 1/2
8.	4.47	3	3/4				

* 2 Bars 6" × 1/2" are put here to give lateral stiffness

** This extra counter brace is put in to allow greater intensity which may be on the shorter segment.

(The dimensions of the Main Ties and Chord Links are afterwards changed)

To calculate the compression members we use a formula derived from one on page 523 of Rankine's Civil Engineering and which is a slightly modified form of what is know as Gordon's Formula.

$S = P\left[\frac{1}{36000} + \left(\frac{l}{36000a}\right)^2\right]$ where S = required sectional area

P = ~~b~~breaking weight of the strut

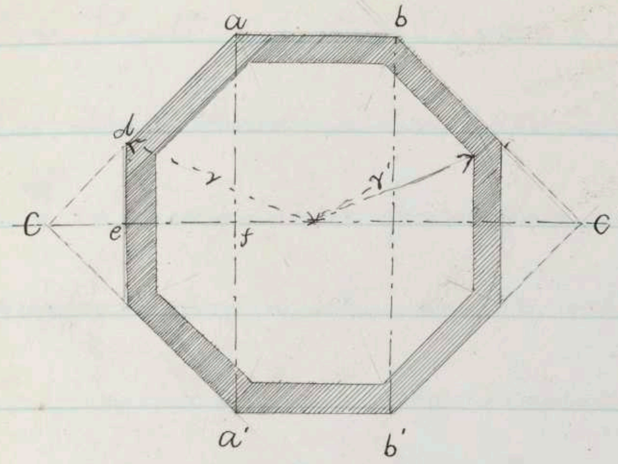
l = length

a = "radius of gyration" for the cross sec

The square of the radius of gyration is the moment of inertia I divided by the area of the cross-section, A. $a^2 = \frac{I}{A}$.

The moment of intertia of the octagon (see figure on page 10 8) is found to be the sum of the moments of inertia ~~of~~ the rectangle a b b' a' and the ~~m~~ four triangles a c f &c less ~~the~~ that of the four triangles ~~about~~ d e c &c all taken about c c' the neutral axis of the whole figure. Calling these three I_1, I_2 and I_3 and that of whole octagon I'

$I' = I_1 + I_2 - I_3$



$$I_1 = \tfrac{1}{12}\left[2r\sqrt{\tfrac{1}{2}-\tfrac{1}{4}\sqrt{2}} \times 8r^3\left(\tfrac{1}{2}+\tfrac{1}{4}\sqrt{2}\right)^{\frac{3}{2}}\right] = \frac{8}{6}r^4\left(\tfrac{1}{2}+\tfrac{1}{4}\sqrt{2}\right)\left(\tfrac{1}{4}-\tfrac{1}{8}\right)^{\frac{1}{2}} = \frac{1}{6}r^4(1+\sqrt{2})$$

$$I_2 = 4\int_0^{y_1}(z_1 - z)y^2\,dy = \tfrac{4}{12}y_1^4 = \tfrac{4}{12}\left(\tfrac{1}{4}+\tfrac{1}{8}+\tfrac{1}{4}\sqrt{2}\right)r^4 = \tfrac{4}{12}\left(\tfrac{3}{8}+\tfrac{1}{4}\sqrt{2}\right)r^4$$

$$I_3 = \tfrac{1}{3}\left(\tfrac{3}{8}-\tfrac{1}{4}\sqrt{2}\right)r^4$$

$$I' = \frac{r^4}{6}(1+\sqrt{2}) + \frac{r^4}{3}\left(\tfrac{3}{8}+\tfrac{1}{4}\sqrt{2}\right) - \frac{r^4}{3}\left(\tfrac{3}{8}+\tfrac{1}{4}\sqrt{2}\right) = \frac{r^4}{6}(1+2\sqrt{2})$$

Hence the moment of inertia of the hollow octagon

$$I = \frac{1+2\sqrt{2}}{6}(r^4 - r'^4)$$

$$A = 2\sqrt{2}(r^2 - r'^2)$$

$$a^2 = \frac{\frac{1+2\sqrt{2}}{6}(r^4 - r_1^4)}{2\sqrt{2}(r^2 - r_1^2)} = \frac{1+2\sqrt{2}}{12\sqrt{2}}(r^2 + r_1^2)$$ calling $r = r_1$ we get the

approximate formula $a^2 = \frac{1+2\sqrt{2}}{6\sqrt{2}}r^2 = .451\,r^2$

This introduced into the prost formula for struts making $l = 216$ inch for the posts; and $l = 144$ inches, $r = 7$ inches for the upper chord give the formulae

For Posts, $S = P(.0000278 + \frac{.0000798}{r^2})$

For Upper Chord $S = .0000285 \cdot P$

In using these formulae we have used r as the radius corresponding to the inner octagon.

To find the neccessary thickness we divide the sectional area by the perimeter of this same octagon which of course makes ~~all them th~~ all a little thicker than necessary but the error is very slight and errs on the side of greater strength. The formula used is $t = S \div 6.123r$

	Posts			Upper chords	
n	S	r	t	S	t
0	26.27	6	3/4		
1	23.30	5.5	11/16	16.78	7/16
2	20.53	5	11/16	27.04	5/8
3	17.84	4.5	11/16	42.83	1
4	14.81	4.5	9/16	52.71	1 1/4
5	12.43	4	1/2	60.40	1 7/16
6	9.64	4	3/8	65.90	1 1/2
7	9.67	3.5	1/2	69.19	1 5/8
8	6.39	3.5	5/16	70.28	1 5/8

Dimensions in inches.

For the steel pins we take 12 000 pounds for the safe shearing force per square inch of section.

	Upper Chord		Lower Chord	
n	Required Cross.sect	diam	Req Cross-sec	Diam
0	9.015	3 7/16	0 ~~9.015~~	3
1	7.8	3 1/4	9.015 ~~7.8~~	3 7/16
2	6.7	3	78 ~~6.7~~	3 1/4
3	5.68	3	67 ~~5.68~~	3
4	4.66	2 1/2	568 ~~4.66~~	3
5	3.68	2 1/2	466 ~~5.00~~	3
6	2.75	2 1/2	5.00	3
7	1.86	2 1/2	525	3
8	1.35	2 1/2	525	3

It will be seen from the above table that economy of material has been sacrified to uniformity. This extra diameter also serves to allow for unforeseen contingences which would be very likely to affect the stresses on these pins to considerable extent.

We now pass to the consideration of those parts which relate to the whole bridge. The first thing is the cross-girders. These have the following dead load to sustain

2 Stringers ea. 12' x 14" x 9"		1134 lbs
9 Ties	each 10' x 8" x 8"	2160 ..
Rails	8 yards	520 ..
		3814 lbs = w_t

These girders are in the same condition with a beam acted upon by two equal and opposite couples as is shown by the figure

The greatest bending moment $M_0 = 65(6w' + \frac{3w_t}{2})$

including the proper factors of safety 3 & 6 $=$ 5987865 in. lbs.

If we take the height of these girders as 15 inches the greatest chord stress = 399191 lbs requiring a sectional area of 11.08 square inches. This is about double that of the Trenton "15 inch heavy beam" and therefore two of these beams will be just sufficient to sustain the load

These cross girders are suspended at each end by two rods. These rods have to bear for dead load

$\frac{w_c}{4}$ = 858 953

Quarter of the weight of the girders 567 1520 lbs

The greatest live load that can come up on them is when the wheels of a locomotive come directly over a girder. Call this weight qE and assume the greatest weight of an engine as 40 tons. Then $qE = .75 \times 40 \times 2000 = 60\,000$ lbs

This divided among four rods for each 15000 lbs

Taking 3 for dead and 8 for live (on account of the greater liability to shocks) as factors of safety we get as the necessary ultimate strength for these rods = 124560 lbs, requiring a sectional area of 2.49. We shall therefore make the diameter = $1\frac{3}{4}$

In order to brace the two girders together laterally we adopt a system precisely similar to that of which the trusses themselves are composed except that we have to provide a system of main diagonals running each way since the thrust due to the wind may be on either side of the bridge.

On the lower chord the greatest ~~R~~ shearing force ~~wt~~ will be when a train of cars extends across the whole bridge. Assume the height of the train as ten feet and the thrust occasioned by the wind as 40 pound per square foot the ~~the~~ greatest shearing force will be $\frac{10 \times 192 \times 40}{2} = 38400$ lbs. This resolved along the direction of the diagonal gives a section area of 4.70 requiring a diameter of $2\frac{7}{16}$ in. which for simplicity we will adopt for all these ties. The cross girders supply the place of strutts.

On the upper chord the great thrust will be due to the thrust on the upper half of the bridge. The greatest shearing force $547 \times 20 = 10940$ lbs which requires ~~a~~ an area for the diagonals of 1.34 in or diameter of $1\frac{5}{16}$. For strutts we use hollow cast iron cylinders of a diameter $4\frac{1}{2}$ inches. From a formula in Rankine's Civil Engineering we derive the ultimate strength per square inch of section. $P = 80000 \div 1 + \frac{32400}{800 \times 20.25} = 26666.6$

Divided this by the proper factor of safety. ~~and~~ equals 5333.3
The thrust occasioned by the wind requires a a cross-section of 2.06 sq. in and we will make them 1/4 inch thick.

The weight of the bridge is some what uncertain on account of the casting which are made as hollow as possible and the irregularity of the figure thus formed makes it impossible to calculate the solidity. The precise weight of the strutts is ~~nott~~ not to be obtained unless we could obtain tables of the ~~resultant~~ weights as given by the Keystone Company by which this form is built. The following table gives the results ~~per~~ per panel

Lower Chord	1430
Main Ties	975
Upper Chord	1819
Posts	1217.3
Counter Ties	41.
Cross Girders	1234.4
Rails Stringers &c	1907.
Steel Pins	124.6
Castings	1300
Lateral Braces	487.1
	10535.4

Assuming 10600 per panel for the dead load we recalculate the bridge with the results given below. These stress are somewhat greater than before and will ~~necess~~ require that the ties be increased in size as below. The struts already have considerable greater sectional area (in the flanges &c) than was necessary so they ~~did~~ need not be increased.

n	V	T	H	Main Ties (2 Bars) Required section	width	thick.	Chord Links (4 Bars 6" wide) Req. sec.	thick.
0	187500						~~12.50~~	1/2
1	163400	225375	125000	22.54	5	2 1/4	12.50	1/2
2	140250	196406.8	233333	19.64	5	2	23.33	1
3	117900	168520.4	325000	16.85	5	1 11/16	32.50	1 3/8
4	96500	141715.8	400000	14.17	5	1 7/16	40.00	1 11/16
5	76000	115993.	458333	11.60	5	1 3/16	45.83	1 7/8
6	56400	91352	500000	9.13	4	1 1/8	50.00	2 1/8
7	37700	67928	525000	6.78	3	1 3/16	52.50	2 1/8
8	25200	45315.4	533333	4.53	3	3/4		

Although this bridge turns out to be rather heavy I am inclined to think that is more the consequence of the details we have taken than of the general form of the bridge which might be built in many other ways.

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