

THE MISSION FOR A MANNED EXPEDITION TO MARS

BY

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by

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ABSTRACT

It is the objective of this thesis to define the mission for a manned expedition to Mars with the major portion of the work being devoted to the selection of the best interplanetary transfer. The approach used is to develop simple approximate models in order to gain an understanding of the complicated orbital mechanics, make conclusions based on a study of the simple models, and verify the conclusions by accurate analysis with the electronic computer. For round-trip missions of about 400 days duration velocity savings over single-elliptical transfer of about 3000 feet per second are possible by making one leg of the trip a bi-elliptical transfer. During every third opposition period it is possible to increase the saving to about 7000 feet per second by making the bi-elliptical transfer during a close approach to Venus. The opportunity to perform this mission will exist for several months during late 1970. Conditions will not be as attractive again until

after the turn of the century. A method for analyzing all practical free-fall interplanetary mission combinations is presented. One specific mission for a manned expedition to Mars is recommended together with alternatives.

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CHAPTER 1

INTRODUCTION

The first attempts to place man into space outside the protecting cover of the Earth's atmosphere have met with such success that many manned space missions are now being planned. The current United States Apollo program has the objective of placing a man on the moon prior to 1970. The next logical mission in space after lunar exploration is the exploration of the planet Mars. The study of the mission for man on Mars is the subject of this thesis. The term "mission" includes the objectives of the expedition, the operations to be conducted by the crew during the flight and on Mars, the things to be brought back to Earth, the dates of departure and arrival and the flight paths to be followed.

1.1 The Reason for Going

The primary reasons for man to go to Mars initially are scientific research and exploration. The possibility that some form of life can or did at some time exist on Mars makes it a particularly noteworthy target. Of all the planets in the solar system, Mars is the only one suspected

capable of supporting life save Earth and possibly Venus. The exploration of Mars may be the only way open for man to establish the existence of extra-terrestrial life.

Mars and Venus are the two planets most easily reached from Earth. Of the two, Mars offers the more attractive setting for man's next mission in space after lunar exploration. The surface of Venus is covered by a dense yellow atmosphere which is opaque to visible light. Astronomers have never had a clear view of the planet's surface and consequently important parameters such as the rotational period are in doubt. Recent measurements taken aboard the Mariner spacecraft indicate that the surface temperature is several hundred degrees Fahrenheit. In contrast, the atmosphere of Mars is transparent to visible light and the surface has been studied by astronomers. An excellent summary of the history of this study has been made by de Vaucouleurs. (2)

Prior to the launch of a manned interplanetary expedition man will have placed optical telescopes in balloons, rockets, satellites or on the surface of the moon that will allow greater resolution than possible with present Earth-mounted telescopes that are limited by the distorting properties of the Earth's atmosphere. These viewing positions will allow the telescope to give more and better information about the Martian surface. The atmospheric pressure and density at the surface of Mars is similar to that of Earth

at an altitude of 56,000 feet. Surface conditions in general appear to make Mars quite adaptable to a manned mission.

The sciences having the most direct interest in the results of a manned expedition to Mars are biology, geophysics, astronomy and meteorology. The biologist is interested in the possibility of extra-terrestrial life on Mars. The geophysicist is interested in all the physical aspects of the planet. Having another planet besides Earth to study firsthand can help uncover the secrets of the origin of the solar system. The astronomer has studied Mars using telescope, spectrometer and theory for thousands of years. Firsthand information to verify or disprove the theories would be invaluable to the science of astronomy. Since the moon has no atmosphere Mars is the easiest place to study extra-terrestrial meteorology. Mars offers a model for the study of weather and climate that is expected to be less complex than for Earth because of the smaller size of the planet and the lesser amount of water in the atmosphere. An understanding of the meteorology of Mars would help in the understanding of our own more complex meteorological system. The volume of scientific information available from a manned expedition to Mars is immense.

Taylor and Blockley⁽³⁾ have considered in detail the reasons for man to go on space missions instead of sending

equipment alone. Even with the added difficulty of protecting the man from the space environment, man is the best piece of equipment that can be found to do the job. Despite the arguments pro and con whether man should go, history has proven that if he has the capability, he will go.

There are secondary reasons for going to Mars. The foremost of these is the fact that the results of scientific research have always lead to useful applications in society. In many cases the results could never have been predicted at the onset of the research. It is the research itself that uncovers the useful application. Just the possibility of what might be found on Mars offers a reason for going. It is speculative but not impossible that some millions of years ago Mars had a more abundant atmosphere and higher forms of life existed on the surface. If so, then evidence of their culture might be discovered and offer lessons for our own. If Mars is capable of supporting life then it is possible that Mars in time could be cultivated and even colonized by man. The important question initially is to what extent life can or did exist there, and the best way to establish the answer is for man to go.

In order that a manned expedition to Mars accomplish its objectives the crew must get to Mars, perform the planned scientific research and exploration, and return safely to Earth. The work to be done includes running the ship. This requires that the crew perform the functions of navi-

gation, power plant control, maintenance, communication and survival. The scientific research and exploration work involves making measurements, gathering samples, taking pictures and mapping. The details of this work will be determined by the "state of the art" at the time of the expedition in those sciences already mentioned as having the most interest in Mars' exploration. (It is expected that once a telescope is placed above most of the Earth's atmosphere there will be a major increase in the knowledge concerning Mars.) What will actually be accomplished on the surface of Mars will also depend upon what the crew finds there. Like any scientific research, the course of their work cannot be completely predicted before it is started. Probably the major question to be answered is whether Mars was, is, or could be, capable of supporting life at some time in the past, present or future.

1.2 The Work of Others

A number of proposals have been made suggesting specific mission profiles for a manned expedition to Mars. Notable is the early work of von Braun⁽⁷⁾⁽⁸⁾ who has shown with substantiating calculations that the voyage is feasible with present day propellants and technology. In Reference (7) von Braun describes in detail an expedition of ten vehicles manned by a crew of seventy, a very large undertaking. In Reference (8) Ley and von Braun describe in detail a more modest expedition of two vehicles manned by

a crew of twelve. Both calculations are based on coplanar circular orbits for Earth and Mars and both voyages go via a Hohman trajectory taking two hundred and sixty days each way with a wait time at Mars of four hundred and fifty days. Himmel et al⁽⁹⁾ have considered the controlling effect of the radiation shielding requirement in the mission planning and have recommended a four hundred day expedition using a nuclear rocket. The weight of the passive shielding alone takes up over seventy tons of the allowed payload weight.

Recently a large volume of work has been done on interplanetary missions under the EMPIRE (Early Manned Planetary-Interplanetary Roundtrip Expedition) program, initiated by the Future Projects Office of the National Aeronautics and Space Administration at the Marshall Space Flight Center, Huntsville, Alabama. NASA has contracted several companies to conduct studies of specific interplanetary missions. The Aeronutronic Division of Ford Motor Company⁽¹⁾ has studied dual planet flyby missions of the Crocco⁽⁵⁾ and symmetric types⁽¹⁾. These missions launch from an Earth orbit and free-fall past Venus and Mars returning to Earth in a direct atmospheric entry. The Missiles and Space Division of Lockheed Aircraft Corporation⁽⁴⁾ has made an extensive orbital parameter study of orbits to both Venus and Mars as well as single planet and dual planet flyby missions. The Astronautics Division of General Dynamics Corporation⁽⁶⁾ has studied initial landing missions and assumes the use of

thrust for velocity changes at each of the terminals of the interplanetary phase. Each of the Empire reports represents work by a complete study group consisting of several individuals.

Because of the large volume of work on different aspects of a mission to Mars it would be impossible to make reference to all of the literature on the subject. It should be noted, however, that the author has found no mention in the literature of the specific missions suggested in this work, namely trips to Mars via bi-elliptical transfer or via a Venus encounter that includes a significant velocity change near Venus.

1.3 Approach to the Problem

A manned expedition to Mars will be a large undertaking. The United States is presently planning to send a man to the surface of the Moon around 1967. The total cost of the lunar expedition has been estimated at approximately \$40 billion over roughly a five-year period. This averages a cost of about \$40 per year for every inhabitant of the country. The technological advances resulting from the lunar expedition can be applied directly to the Mars expedition, but it will still probably cost at least as much to go to Mars as it will to the Moon. If cost were the only factor a mission could be chosen by selecting that trip which minimized the cost. In the case of the lunar expedition the United States is choosing the mission profile

with the motivation of getting there as soon as possible. It is not realistic to assume that the mission will be based on optimization of cost or any other factor alone. It will depend on the unpredictable actions and decisions of many different people up to and including the heads of state of the major powers of the world. Looking at the United States Apollo program, similar decisions were based not on a scientific optimization but rather on the informed opinions of responsible persons who still publically disagreed after the decision was made⁽¹⁰⁾. In the case of the Mars trip it does seem universally agreed that the interplanetary phase of the trip, through one hundred million miles of space, is the predominant consideration in establishing the mission. Because of the long time of flight of the interplanetary transfer in comparison with the other phases, the major aspects of the mission will be determined by the interplanetary requirement. The problem is still complex due to the many engineering trade-offs involved in the determination of the interplanetary transfer paths. In order to obtain an understanding of the total problem the author has attempted to make gross simplifications of those aspects which are complex so that the major contributing factors are shown in clear perspective. An example of the application of this approach is the simplified approximate model described in Chapter 4. There the complexities of the orbital mechanics are reduced

to a crude linear model. Based on an understanding of the crude model conclusions may be drawn and subsequently checked by more rigorous analysis with the aid of the electronic computer. As verification of the value of this approach the savings associated with the bi-elliptical transfer including Venus encounter missions (described in Chapters 7, 11, 12) were predicted solely by use of the simple model and prior to the use of the electronic computer. The basic approach is to use simple models to gain an understanding of the problem, then make conclusions based on the simple model, and finally check the conclusions by more rigorous analysis using the electronic computer.

CHAPTER 2

THE ORBITAL GEOMETRY OF EARTH, VENUS, AND MARS2.1 Units

The unit for interplanetary distance will be the astronomical unit (a.u.) and for planetary distance the planet radius. Dates will be specified in Julian date. Time intervals specified in days refer to the terrestrial mean solar day. When time is expressed symbolically in an equation the unit is the terrestrial year (365.25 mean solar days). The unit for velocity is the Earth's Mean Orbital Speed (EMOS) which is equivalent to 2π a.u. per year or about 100,000 feet per second or 30 kilometers per second or 18.6 miles per second.

2.2 Major Aspects

The major aspects of the orbital geometry for Earth, Venus, and Mars are shown in Figures 1a and 1b. The opposition positions of Earth and Mars are given through 1978 and the conjunction positions of Earth and Venus are given for the period 1970 through 1977 in a sun-centered inertial frame. The orbit of Venus is quite circular ($\epsilon = .007$), that of Earth is approximately circular ($\epsilon = .017$), while

the eccentricity of Mars' orbit ($\epsilon = .093$) is sufficient to cause its radial distance from the Sun to vary as much as a quarter of an astronomical unit. Aphelion, α , and perihelion, π , for Earth, \oplus , Venus, \ominus , and Mars, $\♂$, are shown together with the first point of Aries, γ . The ascending node, Ω , of either Venus or Mars is shown as the point on the orbit where the planet crosses the ecliptic plane from south to north. The descending node, Υ , is displaced by 180 degrees from the ascending node. The orbital inclinations of $1^{\circ}51'$ for Mars and $3^{\circ}24'$ for Venus while small are important considerations in an orbital transfer. The polar obliquity of Mars ($24^{\circ} - 25^{\circ}$) is about the same as that of Earth giving rise to a similar cycle of seasons. To a first approximation the northern hemisphere of Mars celebrates summer in the part of the sky where Earth celebrates winter. An opposition of Mars which occurs near perihelion is a favorable opposition since at that time Mars is not only closest to the Sun but also closest to the Earth. The next favorable opposition can be seen to occur in 1971. The favorable oppositions due in 1986 and 1988 are not shown. If they were shown in Figure 1 they would straddle the 1971 opposition. The most favorable oppositions occur when Mars is near its maximum distance below the plane of the ecliptic. Similarly the least favorable oppositions occur when Mars is near its position of maximum distance above the plane of the ecliptic.

2.2 Synodic Periods

The relative positions of Earth, Venus, and Mars will be important in the work to follow. The easiest way to visualize these relative positions is to show the relative times of Mars' oppositions and Venus' conjunctions. The orbital period of Mars is 687 days (the term "day" will refer to the "terrestrial mean solar day"). The orbital period of Earth is 365 days. This causes the synodic period, the time between successive oppositions, to be about 780 days. The orbital period of Venus is 224 days causing the synodic period between successive conjunctions of Venus to be about 580 days. Considering the paths of the planets around the sun as a racetrack, Earth overtakes Mars at each opposition, and Venus overtakes Earth at each conjunction. These events are plotted on a time scale in Figure 2 for the years 1964-1980. Since Venus is the fastest of the three planets it overtakes both Earth and Mars. Since Mars is the slowest of the three, Mars is overtaken by Venus more often than Earth is overtaken by Venus. The period between successive alignments of Venus and Mars is about 340 days. These events are also plotted in Figure 2 and will be referred to later.

CHAPTER 3

THE ORBITAL PARAMETERS OF ALL POSSIBLE TRANSFER ORBITS3.1 The Planet-Centered Coordinate Frame

Define a planet-centered reference frame with x-axis pointing radially outward from the Sun through the planet in the plane of the planet's motion. The y-axis is in the circumferential direction also in the planet's orbital plane. The z-axis is perpendicular to the planet's orbital plane and completes a right-hand orthogonal set. For a planet in circular orbit the Sun is fixed on the negative x-axis and the velocity vector is along the positive y-axis. When velocity components are specified at a planet they will almost always be in this reference frame.

3.2 Loci of Constant Orbital Parameters

If a spaceship is to follow a free-fall trajectory between two planets then specification of the time of launch and the time of arrival is sufficient to uniquely determine the trajectory. The time of launch fixes the position of the launch planet at launch and hence determines the four coordinates in space and time of the launch

event. Similarly, the arrival date fixes the four coordinates of the arrival event. The difference between the times of arrival and launch determines the time of flight and this is sufficient to select a unique orbit. In a similar manner, the specification of any two independent parameters of the transfer orbit will determine a trajectory between two given planets. By choosing two of the initial velocity components of a spaceship as independent variables it is possible to express all the parameters of all possible orbits between two planets as a function of these two components of the initial launch velocity. Loci of constant orbital parameters can then be shown graphically.

The method described here is similar to a method suggested by Vertregt⁽¹¹⁾ and carried out by Dugan⁽¹²⁾. Vertregt's suggestion was to plot all the possible orbits on an eccentricity versus semi-latus rectum plane. Dugan constructed these plots assuming circular coplanar orbits. He then used the plots as "working charts" to develop curves for round-trip mission planning. The method described here is considered an improvement because it displays the information in a manner more useful for mission planning and can also take into account the orbital inclination and eccentricity in a simple manner. Furthermore, the present method can be directly applied to the problem of pairing two interplanetary orbits which

have a common terminus. This will be discussed in Chapter 11. In order to fix ideas the method will be applied to free-fall transfers between Earth and Mars.

3.3 Free-fall Orbits from Earth to Mars

This example assumes a circular orbit for Earth. The ellipticity and orbital inclination of Mars are handled by looking at trajectories that terminate at a few representative points on the true Martian orbit. All the orbital parameters except the out-of-plane velocity increment will be determined assuming coplanar orbits. The out-of-plane velocity increment will be added vectorially to the component in the Earth's orbital plane to obtain the total velocity increment. The V_x , V_y plane will be the x-y plane described in Section 3.1 with axes calibrated in velocity units. Plot the projection of the velocity of an interplanetary space vehicle relative to the Earth after escape as a point in the V_x , V_y plane. Any possible free-fall orbit is represented by one point on the plane. If the radial distance of Mars at arrival is specified, all orbits which do not reach Mars lie in one region of the plane. This region can be separated from the region containing all orbits which do reach Mars by the locus of those orbits which are just tangent to the orbit of Mars. See Figure 3a. Of the orbits which do reach Mars, the many orbital characteristics can be displayed by plotting the locus of orbits having the same value of the orbital parameter of interest.

A few of the more important parameters are now discussed.

3.4 Constant Launch Velocity, V

The launch velocity relative to Earth is indicative of the fuel required for the trip to Mars. These curves plot as circles about the origin. V is the projection of the total velocity increment on the Earth's orbital plane and does not yet include the out-of-plane component.

$$V_x^2 + V_y^2 = V^2 \quad (3.1)$$

3.5 Constant Orbital Period, T, or Semi-major Axis, a

These orbital parameters depend on the velocity relative to the Sun and plot as circles about the point $V_x = 0$, $V_y = -1$.

$$V_x^2 + (V_y + 1)^2 = \frac{2a - 1}{a} \quad (3.2)$$

$$T = (a)^{3/2} \quad (3.3)$$

3.6 Constant Angular Momentum, h, or Semi-latus Rectum, p

These orbital parameters depend only on the tangential velocity and plot as vertical straight lines.

$$1 + V_y = h \quad (3.4)$$

$$(1 + V_y)^2 = p = h^2 \quad (3.5)$$

3.7 Constant Eccentricity, ϵ

The eccentricity is given by Equation (3.6)

$$\epsilon^2 = 1 + [V_x^2 + (1 + V_y)^2 - 2] [1 + V_y]^2 \quad (3.6)$$

For V_x and V_y small the locus is a two by one ellipse

about the origin. The $\epsilon = 1$ locus is a circle about the $V_x = 0, V_y = -1$ point. The $\epsilon = 1$ locus separates the elliptical and hyperbolic orbits. If $V_y = 0$ then $\epsilon = V_x$.

3.8 Orbits which reach Mars

The criterion for reaching Mars is that the aphelion radial distance from the Sun be greater than the radius of the Martian orbit, r_M , at arrival. The locus of orbits which just reach Mars can be plotted from the ϵ and p loci that satisfy

$$r_M = \frac{p}{1 - \epsilon} \quad (3.7)$$

Because the radial distance of Mars varies considerably throughout its orbit the loci will be graphed for several values of r_M . The minimum value at perihelion, the average value near the line of nodes, and the maximum value at aphelion are sufficient to show the effect of eccentricity in a simple model. Each value of r_M corresponds to an arrival at Mars at a specific point in the Martian orbit.

3.9 Constant Time of Flight, t_f

The time of flight to reach Mars is one of the more important parameters. For elliptical orbits two loci will go through each point since there are two possible crossings of the Martian orbit. The expression for the time of flight depends in part on the type of orbit. These expressions are given in Reference (12). For the hyperbolic orbits, for example

$$t_f = \frac{a^{3/2}}{2\pi} [\gamma_1 - \gamma_2 - (\sinh \gamma_1 - \sinh \gamma_2)] \quad (3.8)$$

where

$$\gamma_1 = \text{arc cosh} \left(\frac{a+1}{a} \right)$$

$$\gamma_2 = \text{arc cosh} \left(\frac{a+r\delta}{a} \right)$$

3.10 Constant Heliocentric Trip Angle, θ

For elliptical orbits there will be two θ loci through each point since there are two possible crossings of the Martian orbit. For a hyperbolic orbit the locus is given by Equation (3.9). For the elliptical orbits there are sign changes to make Equation (3.9) applicable.

$$\theta = \left| \text{arc cos} \left(\frac{p-r\delta}{r\sigma\epsilon} \right) - \text{arc cos} \left(\frac{p-1}{\epsilon} \right) \right| \quad (3.9)$$

3.11 Constant Configuration at Launch, ϕ_L

The launch configuration angle is the heliocentric angle between Mars' radius vector and the Earth's radius vector at launch. See Figure 3b. This parameter sets the launch date relative to the current opposition date. It is important for determining the date of a launch opportunity or the duration of a launch window.

$$\phi_L = \theta_M - \theta \quad (3.10)$$

where θ_M = the heliocentric angle measured from Mars' radius vector at launch to Mars' radius vector at arrival.

3.12 Constant Configuration at Arrival, ϕ_A

This parameter gives the angular orientation of Earth and Mars at the completion of the trip to Mars. It is the heliocentric angle between Mars' radius vector and the Earth's radius vector at arrival. This parameter sets the arrival date relative to the current opposition. It is probably the most important parameter for round trip considerations because it represents the end conditions with which one must start to plan the return trip. It also gives the orientation of Earth and Mars at the start of the exploration phase which is important for communication requirements.

$$\phi_A = 2\pi t_f - \theta \quad (3.11)$$

3.13 The Out-of-Plane Velocity Increment, V_z

The out-of-plane velocity increment is given by

$$V_z = \frac{z(1 + V_y)}{r_{\sigma} \sin \theta} \quad (3.12)$$

The distance, z , of Mars above the ecliptic plane at arrival is determined by the corresponding value of r_{σ} . The relationship can be seen in Figure 1. The maximum positive value of z occurs near Martian aphelion and the maximum negative value of z occurs near perihelion. V_z should be added vectorially to V and a total velocity locus plotted to show the net effect of the orbital inclination on the

total velocity requirement.

3.14 Velocity Increment at Mars

Because of the possibility of using atmospheric braking at Mars the expression for the required velocity increment could be complex. A simple bound is given by the case using only thrust braking and the case using only atmospheric braking. The velocity increment at Mars can also be visualized by plotting all of the contours on the arrival planet's V_x , V_y plane.

There may be other important parameters to be considered but the point is that they can all be represented as loci on some V_x , V_y plane. To consider the round trip the same curves are constructed for orbits starting in the vicinity of Mars and proceeding to Earth. It is possible using a large scale with light colored lines to present all or several loci on one chart. The equations presented are based on a simplified two-body representation but the device described is adaptable to the presentation of the same information deduced from more accurate models. The advantage of the device is that all possible orbits are displayed and at a glance all properties of any proposed orbit may be determined. Further, the direction and magnitude of the velocity increment to accomplish the particular orbit is apparent. All combinations of flyby and stopover missions may be analyzed by pairing inbound and outbound velocity vectors at the destination planet.

CHAPTER 4

THE SIMPLIFIED APPROXIMATE MODEL4.1 The Need for Simplicity

Because the closed form analytic solutions of many problems in celestial mechanics are difficult or impossible to develop, recourse is made to a digital computer. Once the computer is used the basic understanding of what is going on becomes confused in graphs and tables. As an aid to mission planning it appears desirable to have a model that permits a simple visualization of complex orbital characteristics and gives at least approximate numerical results without requiring a computer solution. In Reference (13) the author developed such a model in connection with the problem of orbital rendezvous. The model expresses the position and velocity of a freely-falling spaceship as a function of time and initial conditions or as a function of time and the orbital parameters. The model shows the first order effects of ellipticity and orbital plane inclination. Transfer between sun-centered and planet-centered coordinate frames both inertial and rotating is simple and the effects of orbital maneuvering

can be visualized in the several frames even though the numerical results are approximate. The model is too crude for accurate trajectory selection but it is extremely useful for the demonstration of complicated orbital characteristics and optimizations under limiting conditions. Optimizations too complicated to be developed rigorously⁽¹⁴⁾ can be done relatively easily using the model. One such example is given in Appendix C. In those cases where optimizations have been done rigorously (by Lawden⁽¹⁴⁾ or Long⁽¹⁵⁾ for example) the conclusions using the model have been substantiated.

4.2 The Model

Define an Earth-centered frame (x, y, z) and a Sun-centered frame (ρ, ϕ, z) that rotate with respect to inertial space at the rotational velocity of the Earth around the Sun. The x, y, ρ and ϕ measurements are made in the Earth's orbital plane. See Figure 4. The z coordinate measures the distance above or below the Earth's orbital plane. For units use the astronomical unit (a.u.) for distance, the radian for angle, the year for time, and the Earth's Mean Orbital Speed for velocity.* At time $t = 0$ a vehicle leaves the Earth at E with a velocity relative to the Earth given by the components V_x, V_y and V_z . At a time t years later the vehicle is freely falling in the Sun's gravitational field at point P with coordinates $x, y,$ and z or ρ, ϕ and z . If P is close to E then the

*Note that one EMOS = 2π a.u. per year. The rationalized velocity unit is chosen to avoid the 2π factor.

results derived in Appendix A give the coordinates of P approximately by Equations (4.1).

$$x = sV_x + rV_y$$

$$y = -rV_x + qV_y$$

$$z = sV_z \tag{4.1}$$

where: $q = q(t) = 4\sin 2\pi t - 6\pi t$

$$r = r(t) = 2 - 2\cos 2\pi t$$

$$s = s(t) = \sin 2\pi t$$

Equivalently, if $\rho \approx 1$ a.u.

$$\rho = 1 + sV_x + rV_y$$

$$\phi = -rV_x + qV_y \tag{4.2}$$

$$z = sV_z$$

Because the distance from E to P is not always small compared with the distance from the Earth to the Sun the values of the coordinates are crude. The accuracy is discussed in Appendix B. The equations are, however, sufficiently representative of the orbital mechanics to be of great value in mission planning. The charts out-

lined in Chapter 3, for instance, can be sketched using only slide rule calculations. The numerical results are approximate but the orbital characteristics can be easily visualized.

4.3 Visualizations

The results of velocity components in the three directions are visualized simply. Referring to Figure 5, V_x alone gives rise to a 2 x 1 elliptical path. V_y alone gives rise to a cycloid-like motion when viewed in this rotating frame. A V_y velocity component is four times as effective as a V_x component in producing an x-displacement, but it carries with it a stronger y-displacement, and the times to realize the maximum displacements are different. V_z alone gives a simple harmonic motion above and below the plane of the ecliptic. In general the motion will consist of all three components. Since q , r , and s are functions of time only, the equations are a simple linear set when t is specified.

It should be noted that positive V_x and V_y velocity components cause the vehicle to fall behind Earth while negative V_x and V_y components cause the vehicle to run ahead of Earth. Since a round trip will require that the vehicle return to Earth, the total y displacement must sum to zero (or a multiple of 2π). Consequently trajectories cannot be chosen without concern for the y motion.

In this model a planet in a coplanar, circular orbit will move along the line:

$$x = \text{constant}; \quad z = 0 \quad (4.3)$$

and will have a velocity given by:

$$\begin{aligned} \dot{x} &= 0 \\ \dot{y} &= -\frac{3}{2}x \end{aligned} \quad (4.4)$$

This shows that Earth overtakes the outer planets such as Mars and in turn is overtaken by the inner planets such as Venus. The orbits of Venus and Mars are shown in Figure 4 as they would appear in this frame assuming circular orbits.

4.4 Relative Velocity at Arrival, V_a

To determine the velocity of the spaceship at arrival relative to a planet in circular orbit, differentiate^{*} Equations (4.1) and subtract (4.4)

$$\begin{bmatrix} V_{a_x} \\ V_{a_y} \end{bmatrix} = \begin{bmatrix} c & 2s \\ -\frac{2s}{c} & c \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} \quad (4.5)$$

$$\text{where } c = c(t) = \cos 2\pi t$$

A simple matrix transforms the launch velocity into the arrival velocity.

In order to study the orbits which return to Earth from space let V_x and V_y be the velocity components of

* Since Equations (4.1) were obtained in Appendix A by a direct integration there is no additional approximation here.

the vehicle relative to Earth at arrival instead of launch and let time run backwards, i.e. replace t by minus t in all equations. Equation (4.5) will then give \bar{V}_a as the velocity of the vehicle relative to the launch planet (in circular orbit) at launch. To visualize time running backwards in Figure 5 merely proceed in the direction opposite to the arrows.

4.5 Orbital Inclination and Eccentricity

Orbital inclination and eccentricity in the case of Mars cause the path of the destination planet to have a slight slope in this frame relative to the path of a planet in coplanar, circular orbit. Because of the orbital inclination the z of Mars can vary from $-.044$ to $+.054$ a.u. during its two-year orbit. Because of the orbital eccentricity the x of Mars can vary from $.38$ to $.67$ a.u. The actual values of x and z at arrival will be determined by the date of arrival. Because of round-trip considerations the arrival date is almost always within a few months of the current opposition date. This means that it is only necessary to substitute the x and z of Mars near the current opposition date to compare the effects of different opposition dates on the launch velocity.

4.6 Superposition

As a consequence of the linearity of the simple model, the result of two independent velocity increments applied to the vehicle at different times may be deduced by adding

the results of each increment applied separately. If velocity V_1 is applied at time t_1 and velocity, V_2 is applied at time t_2 , then the resulting motion for $t > t_2$ is given by

$$\begin{aligned} x &= s(t - t_1)V_{1_x} + r(t - t_1)V_{1_y} + s(t - t_2)V_{2_x} + r(t - t_2)V_{2_y} \\ y &= -r(t - t_1)V_{1_x} + q(t - t_1)V_{1_y} - r(t - t_2)V_{2_x} + q(t - t_2)V_{2_y} \\ z &= s(t - t_1)V_{1_z} + s(t - t_2)V_{2_z} \end{aligned} \quad (4.7)$$

The use of the superposition concept is more valuable as a visualization aid than it is to deduce Equation (4.7).

The results of the first velocity increment can be considered as the motion of a fictitious vehicle which only made one velocity change. The results of the second velocity increment can be interpreted as the motion of the true vehicle relative to the fictitious vehicle. The sum of the two results gives the motion of the true vehicle relative to the Earth. Each individual result can be visualized by reference to Figure 5 as before. This concept will be useful in the discussion of bi-elliptical transfer in Chapter 7.

4.7 Other Uses of the Model

Besides its application to mission planning the model can be useful wherever visualization of orbital characteristics is desirable. As mentioned earlier, it was originally

adapted to the problem of orbital rendezvous⁽¹³⁾. The model is also useful in the study of navigational corrections. Stern⁽¹⁹⁾ has developed an analytical expression for any mid-course velocity correction. The model described here allows a visualization of the effect of those corrections. It also offers a quick approximate check of any results derived from more accurate models.

CHAPTER 5

THE ROUND TRIP5.1 Conditions Necessary to Go

In order that man can go to Mars he must have a vehicle capable of taking the necessary payload to the destination and back. If the vehicle is an impulse-type rocket then the requirement can be expressed by two curves. One is the velocity increment required for the mission as a function of the total mission time and the other is the weight of the required payload as a function of the total mission time. For some total mission time the vehicle must be capable of imparting the velocity increment to the corresponding payload weight.

The required velocity increment curve is obtained in this manner. First, represent all the possible orbits to Mars and back in some convenient manner (such as that outlined in Chapter 3, for example). Second, select a total time for the mission and choose from all those orbit pairs that give the selected total time the one pair that gives the minimum required velocity increment. With this required velocity increment and total mission time plot

* For this analysis the velocity increment required is the sum of the velocity increments at all four terminals of the interplanetary transfers.

one point of the curve. This procedure has been carried out by Dugan⁽¹²⁾ and Johnson and Smith⁽¹⁶⁾ for orbits which start and terminate in close circular orbits about Earth and Mars. Both works assumed circular, coplanar, orbits about the Sun for Earth and Mars. Dugan's results are shown in Figure 6. The characteristic shape can be seen to have relative minima near 450 days and 1,000 days. Decreasing the total trip time below 450 days requires an ever increasing velocity increment.

The payload weight curve is obtained by considering the weight of the crew and their equipment including all the items necessary for life support outside the protecting cover of the Earth's atmosphere. The major contribution of the weight of the payload appears to be the radiation shielding for the crew. The weight requirements for passive shielding have been studied by Wallner and Kaufman⁽¹⁷⁾ and the best figures available are only estimates because of the basic lack of knowledge about both the existing solar radiation and man's tolerable dosage. Kash and Tooper⁽¹⁸⁾ have indicated the possibility of major weight reductions through the use of active shielding but the techniques are far from developed. Because of the unknown

information about the major contribution to the weight of the payload the curve is estimated by bounding lines shown in Figure 6. The maximum slope is near one month. The increase is due to the increased weight of shielding necessary to provide protection from a major solar flare. The probability of encountering such a flare at some time during the trip increases rapidly if the trip time is greater than one month. The question of how to shield against the possibility of a giant solar flare is largely a conjecture at this time. Also, little is known about the shielding requirements for equipment such as the electronic components. The weight of food, oxygen and water is estimated at about 15 pounds per man per day. In general the slope will be greater than this value because the weight of the required shielding also increases with time. It is possible to have a negative slope as the time of flight gets extremely short if the power plant is of the nuclear type. The increased thrust requirement for very short missions increases the radiation from the engine. If the engine does not offer a radiation hazard then extremely small payloads are required for very short duration missions.

Because the interplanetary vehicle discussed above will have to first be boosted into an Earth orbit, its initial weight is a controlling factor. The easiest and consequently the first capability for man to go to Mars will be with the lightest vehicle that meets the requirements of

the two curves.

For an impulse type rocket assuming infinite staging:

$$\Delta v = c \ln \frac{W_i}{W_f} \quad (5.1)$$

where:

Δv = velocity increment given the payload

c = exhaust velocity of the propellant

W_i = initial weight of the rocket

W_f = final weight of the rocket.

Rewriting Equation (5.1)

$$c \ln W_i = \Delta v + c \ln W_f \quad (5.2)$$

Assume that W_i and W_f are continuous functions of the mission time, t_m . For a relative minimum:

$$\frac{\partial \Delta v}{\partial t_m} + \frac{c}{W_f} \frac{\partial W_f}{\partial t_m} = 0 \quad (5.3)$$

For the comparison of any two relative minima (or any two missions) designated 1 and 2, the first will be the true minimum if

$$\Delta V_1 - \Delta V_2 < c \ln \frac{W_{f2}}{W_{f1}} \quad (5.4)$$

Set c at 30,000 ft./sec. which is a nominal value for a nuclear rocket. The first rocket capable of taking man

to Mars will probably be a nuclear rocket. The conditions of Equation (5.3) and (5.4) can now be used with the curves of Figures 6 and 7 to select the mission that gives the minimum vehicle weight. This occurs with a mission time of around 400 days. Therefore the easiest and hence the first capability to go at all will involve a mission time of around 400 days. Faster trips are more desirable from almost all viewpoints except the velocity required, so it is concluded that the actual mission will be 400 days or less. The figures to reach this conclusion are obviously approximate but in any case it appears that they justify the use of the branch of the required velocity curve in Figure 6 to the left of the 450 day minimum.

For all missions taking less than 450 days an increase in the stay time at Mars causes an increase in the required total velocity increment. For the 400-day mission Ref. (12) shows this is about 300 feet per second per day at Mars. For the minimum velocity trip there is relatively little choice. Stay time on Mars will have to be near 450 days or there is no advantage in resorting to this type of trip for the manned mission. In terms of a "scram" capability, the short missions can depart Mars prior to the planned time with less fuel than anticipated for the mission. The longer missions must remain at Mars 450 days or else expend more fuel than was planned for the mission.

Several types of continuous-thrust engines are now

appearing as possible competitors to the impulsive-type rocket⁽²⁰⁾. A curve analogous to the velocity-increment-required curve could be developed for a continuous-thrust engine. However, since the continuous-thrust engines are in an early stage of development the impulsive-type rocket will almost certainly be the first to offer a manned Mars capability. If a continuous-thrust engine is to compete it will have to offer the capability of a 400-day mission or less.

The conclusion drawn here is that man cannot go to Mars until the capability exists. Once the capability exists man will probably go to Mars with as short a mission time as possible. That mission time will be about 400 days' duration or less.

5.2 Discussion of the Conclusion

The analysis of Section 5.1 is crude. The fact that the payload weight curve is not well bounded is not too serious because the criteria for a minimum depends more on the slope, and the slope can be estimated within a tighter bound than the curve itself. The assumption that all velocity changes will be accomplished with thrust is a poor one because the use of atmospheric braking at either terminal can afford weight savings. Atmospheric braking will be discussed further in Chapter 9. The assumption of a nuclear rocket is critical. The optimum mission time depends directly on the exhaust velocity of

the rocket propellant. If the nuclear rocket is not available and chemical rockets ($c \approx 10,000$ ft./sec.) must be used, then the optimum mission time by the analysis of Section 5.1 will be longer unless there is some way to reduce the required velocity.

There are other reasons why shorter missions are desirable besides the decreased payload weight. One major factor is the effect of the long mission duration on the crew. A year's flight in a space capsule could be compared to a year in prison. The history of aviation has shown that man's inherent impatience has continually caused him to develop faster and faster aircraft. The traffic on airlines indicates that man is willing to pay a higher fare in order to reach his destination in less time. A faster trip to Mars would mean lower durability and reliability requirements on the equipment. From practically all viewpoints, except the velocity increment required, shorter trips are to be desired. Since favorable launch opportunities come about every 780 days the results of a longer mission would not be available for application to a new trip for over four years. The results of a shorter mission would be available for use during the next "Mars season".

The discussion of this Chapter should introduce the nature of the many trade-offs involved in this study. While an analytical method is presented for choosing an optimum

mission time there are subjective arguments for deviating from the optimum. In summary, it is concluded that if the nuclear rocket is available, the easiest capability to go to Mars will be with a mission time of 400 days or less.

The analyses in this Chapter and Chapter 6 are based on single-elliptical transfers between Earth and Mars. The conclusions which are drawn are needed to introduce the newer concepts of bi-elliptical transfer and transfer via Venus. Consequently the statements made in these two chapters do not necessarily apply to missions of the latter type.

CHAPTER 6

PATHS IN SPACE6.1 An Earth-centered Rotating Frame

The orbital characteristics of round-trip missions to Mars are visualized most easily in an Earth-centered or a Mars-centered coordinate frame. Figure 8 shows the motion of Mars relative to the Earth-centered frame described in Chapter 3. Mars appears to rotate clockwise with a period of about 780 days. The position of Mars is shown at monthly intervals before and after opposition. Each pair of trajectories that represents a point on the curve of Figure 6 can be plotted in this frame. The opportunity for executing the mission represented by any one point occurs only once every 780 day period. These missions fall into two basic types. The minimum velocity mission is shown by a dotted line in Figure 8 and represents one type. It is seen that the major portion of the mission is spent on Mars while Mars is distant from the Earth. The other type of mission arrives at Mars close to the opposition position. This type includes all the missions of less than 400 days' duration. All these missions arrive at Mars between the two quadrature

positions. A typical mission of this type is indicated by a solid line in Figure 8. For a circular Mars' orbit and a given total mission time the total velocity increment is a minimum when the arrival at Mars is displaced either side of the opposition position. The total velocity increment does not change much when the arrival position is varied either side of the minimum. Because of this insensitivity to time of arrival the arrival date can vary either side of the opposition date by as much as one quarter of the round-trip travel time with less than 15% velocity penalty. This allows the arrival point at Mars to be shifted away from the opposition position towards perihelion and thus take advantage of the decreased radial distance from the Sun. This result can also be used to pair orbits so that the out-of-plane component does not constitute a penalty. This means that a typical mission will probably arrive at Mars on the perihelion side of the current opposition but in any case not further from opposition than the quadrature position.

6.2 An Earth-centered Inertial Frame

Figure 9 shows the position of Mars relative to an Earth-centered frame that is non-rotating with respect to inertial space. Position is indicated at monthly intervals before or after opposition. The minimum velocity mission is indicated in the Figure by a dotted line. Again, this mission requires the crew to remain at Mars for 450

days. The conclusion concerning the manned mission to Mars appears obvious when the motion is observed in this frame. For fast missions to Mars it is easiest to visit near the opposition position. In this frame all the missions of less than 400 days arrive at Mars inside a flat 15° cone about the position of Mars at opposition.

6.3 A Mars-centered Rotating Frame

Figure 10 shows the position of Earth relative to a Mars-centered frame which rotates so that the Sun has constant direction. The position of Earth is indicated at monthly intervals before or after opposition and the minimum velocity mission is shown by a dotted line. The launch date from Earth and the return date to Earth are indicated by the position of Earth at either time relative to the opposition. While Figure 8 shows the major effect of changing the arrival date at Mars, Figure 10 shows the major effect of changing the launch or return date at Earth. For a given total mission time a change in the date of Earth launch must be accomplished by a change in the date of Mars arrival or there will be an increase in the required velocity increment. The many trade-offs are best evaluated using the charts suggested in Chapter 3 where all the parameters of all the possible orbits to Mars are displayed together.

Figures 8-10 are drawn assuming circular orbits for Earth and Mars. The true orbits will bring the planets

closer or further apart than shown depending upon the opposition date.

6.4 The 400-Day Trips

If the mission time is constrained to be about 400 days, the mission profile that uses the minimum total velocity increment looks very similar to the one shown by the solid line in Figure 8. The going trajectory launches with a velocity relative to Earth that is pointed almost directly toward the Sun. As shown in Figure 5, this causes the vehicle to go towards the Sun and ahead of Earth before going out to intercept the Mars' orbit. The time of flight to Mars is about 240 days. In a Sun-centered, inertial frame the heliocentric trip angle would be about 270 degrees. As shown in Chapter 4, this is an uneconomical way to get to Mars, but it is chosen because one of the two halves of the round trip must get ahead of Earth if the other half gets behind. This aspect of round-trip mission planning is what makes it so complex in comparison with interplanetary trips which only go one way.

Other observations from Figure 8 are that the spaceship never gets further than one astronomical unit from the Earth and during favorable oppositions could be kept within a half astronomical unit. The point of closest approach to the Sun occurs at a distance which is approximately equal to Venus' orbital radius.

Similar observations can be made for the reciprocal

mission which uses the shorter transfer for the outbound leg and the longer transfer for the inbound leg.

CHAPTER 7

BI-ELLIPTICAL TRANSFER7.1 Possible Savings through Bi-elliptical Transfer

The following observations are made based on the simple models already presented. Low energy transfers to the outer planets are characterized by the fact that the spaceship falls behind Earth while low energy transfers to the inner planets cause the spaceship to run ahead of Earth. The manned mission to Mars will probably be accomplished with a mission duration of about 400 days or less. Missions of this duration which use two single-elliptical transfers require the use of one uneconomical transfer in order to get ahead of Earth. By starting a trip to Mars with a low energy transfer toward Venus, the spaceship will run ahead of Earth and be in a position for two low energy transfers, first to Mars and then back to Earth. The reciprocal mission is also possible by making a low energy transfer to Mars with a bi-elliptical transfer on return.

The above argument in favor of a bi-elliptical transfer may be justified on more rigorous grounds by considering

Figure 11. Suppose that $E'\pi M$ is the optimum single-ellipse transfer to Mars for some round-trip mission. Barrar⁽²¹⁾ has given analytical proof that transfer by two impulses, one at E and one at π , is always more economical than a single impulse transfer at E' to achieve the orbit πM . In addition the $E\pi M$ transfer will place the spaceship farther ahead of Earth at arrival than the $E'\pi M$ transfer. It is true that the $E\pi M$ transfer will have a longer flight time than the $E'\pi M$ transfer but this can be partially made up on the return trip by virtue of the more favorable distance ahead of Earth on arriving at Mars.

7.2 Analysis

The simple model is already equipped to handle bi-elliptical transfer by virtue of the superposition principle explained in Section 4.6. The approximate coordinates of the spaceship are given by Equations (4.7). The effect of each individual velocity increment can be visualized using Figure 5. It is possible to minimize the total velocity increment under any set of constraint conditions by using the model together with the Calculus of Variations as in Appendix A. The method is straightforward but it requires the solution of a simultaneous set of equations of high order for most realistic constraint conditions.

In the analysis of a single-elliptical transfer between Earth and Mars it was pointed out that the specification of a launch date and an arrival date was sufficient to uniquely

determine a trajectory between the two planets. In the case of a bi-elliptical transfer the number of possible pairs of ellipses for a given launch and arrival date is infinite. Furthermore, the choice of the ellipses involves several trade-offs. To make the required launch velocity increment a minimum the angle $ES\pi$ (see Figure 11) should be near 180 degrees. To make the time of flight a minimum the angle $ES\pi$ should be as small as possible. Similarly, the angle πSM should be near 180 degrees for minimum velocity relative to Mars at arrival but as small as possible for a short time of flight. In order to make comparisons between the two types of transfer it appears desirable to fix the configuration of the bi-elliptical transfer so that the specification of a launch and an arrival date will uniquely determine a trajectory. To this end, assume throughout the analysis that the two ellipses $E\pi$ and πM have a common periapse at π . Now if the angle πSM is specified, the choice of a launch date and an arrival date is sufficient to determine a unique bi-elliptical transfer. The velocity relative to Mars at arrival is directly related to the size of the angle πSM . If this velocity is to be dissipated by means of atmospheric braking then the choice of the angle πSM will depend directly on the limitations imposed by drag braking in the Martian atmosphere. In addition to offering fuel savings at launch bi-elliptical transfer gives the mission planner control over the arrival

velocity at Mars making it possible to fully exploit the use of atmospheric braking at Mars. Atmospheric braking will be discussed in greater detail in Chapter 9. Once the desired arrival velocity at Mars is selected the angle π_{SM} is chosen to give approximately that arrival velocity. Specification of the launch and arrival date will now give a unique bi-elliptical transfer which can be compared with the conventional, single-ellipse transfer corresponding to the two dates.

7.3 Advantages of Bi-elliptical Transfer

The fuel saving afforded through bi-elliptical transfer is very great if drag brakes can dissipate high approach velocity at Mars without heavy weight penalty. It makes possible the full utilization of the soft Martian atmosphere to terminate a fast economical transfer to Mars. Even with thrust-braking at both Mars and Earth the technique still offers savings over the best pair of single-elliptical transfers of equivalent mission duration. The saving is the largest for mission durations of 400 to 500 days. Bi-elliptical transfer will always provide a trajectory at least as good as single-elliptical transfer because the latter is the limiting case of the former when the mid-course velocity increment approaches zero. The fact that the bi-elliptical transfer allows independent adjustment of the terminal velocities in both magnitude and direction is important. Many of the best single-elliptical missions

in 1971, for instance, launch almost directly into the Sun. This factor could be a distinct disadvantage during early tracking from Earth-fixed observatories. An additional fuel saving is possible in that the first navigational correction and the second orbital transfer can both be made in one thrust maneuver.

7.4 Planetary Environment

One factor that limits the gains anticipated with bi-elliptical transfer is that the second orbital transfer is accomplished while beyond the influence of any planetary gravity. As shown in Chapter 8, the planetary environment allows a velocity increment measured relative to a planet to produce a greater than one to one ratio velocity change when evaluated relative to the Sun after planetary escape. In crude terms, it "costs" more fuel to generate a velocity change when distant from the planetary environment. This means that the bi-elliptical transfer would be even more effective if the second orbital transfer could be accomplished during an encounter with Venus while en route to Mars. By making a small velocity change close to Venus at high velocity and low potential energy, a large orbital energy change will be generated to place the spaceship on a trajectory to Mars. The disadvantage of this type of mission is that it requires a specific orientation of Earth, Venus and Mars, the synodic period of their relative positions being about six years. Because of the timing requirement the bi-

elliptical transfer to Mars via Venus encounter will be treated in Chapter 11 as a three-legged mission of single-elliptical transfers. The Venus encounter mission is mentioned here primarily because it was conceived as an extension of the bi-elliptical transfer.

CHAPTER 8

PLANETARY ENCOUNTER8.1 Analytic Model

The analytic model for the planetary encounter assumes a planet-centered, inertial, reference frame. The only force assumed to be acting is due to the planet's gravitational field. All orbital transfers are accomplished by impulsive velocity changes. This is consistent with conventional patched-conic trajectory analysis.

There are three basic maneuvers that will be of interest. The first maneuver is the direct descent from a solar orbit at effectively an infinite distance from the planet down to the top of the planet's atmosphere via hyperbolic transfer. The direct ascent from the top of the planet's atmosphere to escape from the planet's gravitational field via hyperbolic transfer is considered as the same basic maneuver. The second maneuver is the transfer into or out of a circular parking orbit from an infinite distance via hyperbolic transfer. The third maneuver is a flyby of the planet with a velocity impulse applied at the point of closest approach to the planet. The inbound trajectory

and the outbound trajectory are both hyperbolas with a common periapse.

All three maneuvers involve hyperbolic trajectories and in each case the velocity increment will be accomplished by a thrust impulse applied at the periapse of the hyperbolic path. In practice the thrust increment cannot be applied in an exact impulse at the exact periapse position. Zee⁽²²⁾ has reported an introductory investigation into the errors associated with the assumption of impulsive thrust application. He indicates that the assumption of impulsive thrust will give results sufficiently accurate for mission planning.

The unit for distance in this analysis will be the planet radius. Velocity will continue to be expressed in EMOS units. The following quantities are defined for the analysis of the three basic planetary-encounter maneuvers.

r_{π} = the radial distance of the pericenter of the hyperbola from the center of the planet.

V_{π} = the velocity of the vehicle relative to the planet at the pericenter of the hyperbola.

V_H = the hyperbolic approach or departure velocity of the vehicle relative to the planet after escape from the planet's gravitational field.

V_C = circular satellite velocity at the planet's surface.

V_S = circular satellite velocity at the radial distance, r_π .

V_E = escape velocity at the radial distance, r_π .

δ = the angle between the vectors \bar{V}_π and \bar{V}_H .

ΔV = the velocity increment applied at the periapse.

ϵ = the eccentricity of the hyperbola.

a = the semi-major axis of the hyperbola.

p = the semi-latus rectum of the hyperbola.

In the case of the flyby maneuver the subscript I or O will be added to refer the parameter to the inbound or outbound hyperbola, respectively.

8.2 General Hyperbolic Motion

The main parameters of the hyperbolic motion which are needed in each of the basic planetary-encounter maneuvers are V_H , V_π , r_π and δ . The parameter, r_π , can be conveniently expressed in terms of the circular satellite velocity, V_S , at that radial distance by

$$r_\pi = \left(\frac{V_C}{V_S} \right)^2 \quad (8.1)$$

V_C , the circular satellite velocity at the planet's surface, is a constant for each planet and is given below for Earth, Venus and Mars.

$$V_{C\oplus} = .266 \text{ EMOS}$$

$$V_{CQ} = .243 \text{ EMOS}$$

$$V_{CO} = .121 \text{ EMOS}$$

The escape velocity, V_E , is simply related to the circular satellite velocity, V_S .

$$V_E^2 = 2V_S^2 \quad (8.2)$$

By using V_S and V_E instead of r_π all the main characteristics of the general hyperbolic motion are described by two equations.

$$V_\pi^2 = V_E^2 + V_H^2 \quad (8.3)$$

$$\sin \delta = \frac{1}{1 + \left(\frac{V_H}{V_S}\right)^2} \quad (8.4)$$

If the conventional parameters are required they can be obtained from the velocity parameters using Equations (8.5) - (8.7).

$$\epsilon = 1 + \left(\frac{V_H}{V_S}\right)^2 \quad (8.5)$$

$$a = \left(\frac{V_C}{V_H} \right)^2 \quad (8.6)$$

$$p = a(\epsilon^2 - 1) \quad (8.7)$$

8.3 Direct Ascent or Descent

The problem here is usually to find the velocity at which a vehicle enters the planetary atmosphere knowing the hyperbolic approach velocity, V_H , from the interplanetary trajectory analysis. Assuming that the vehicle enters the atmosphere near the periapse, the atmospheric entry velocity is given by V_π in Equation (8.3) where V_E is determined by knowing the radial distance of the top of the atmosphere. Little error is introduced by assuming the top of the atmosphere to be at one planet radius. Equation (8.3) is plotted in Figure 12. The figure also includes a memory aid for Equation (8.3) plus approximate values of V_E for Earth, Venus and Mars.

8.4 Parking Orbit

A parking orbit is a circular orbit used as an intermediate transfer point when arriving or departing a planet. The plane of the parking orbit for Earth departure must contain the hyperbolic departure velocity vector, \bar{V}_H . The plane of the parking orbit at Mars must contain both the hyperbolic departure and arrival vectors and is therefore

determined by the choice of interplanetary trajectories. The plane of both parking orbits will need to be adjusted slightly for orbital precession due to planet oblateness. The velocity increment required, ΔV , to arrive or depart the parking orbit is just the difference between V_{π} and V_S .

$$\Delta V = V_{\pi} - V_S \quad (8.8)$$

A plot of $\frac{\Delta V}{V_S}$ vs. $\frac{V_H}{V_S}$ is given in Figure 13 together with the values of V_S for Earth, Venus and Mars at nominal parking orbit radii of 1.1, 1.1, and 1.3 respectively. In working with interplanetary trajectories the velocities being considered are the hyperbolic approach and departure velocities at the planet. The velocity which a rocket would actually be required to deliver is the ΔV velocity. In order to keep in mind what a V_H "costs" in terms of a ΔV it is a useful guide to note that the ratio

$$\frac{V_H}{\Delta V} = 1 \quad \text{when} \quad V_H = \frac{V_S}{2} .$$

For hyperbolic velocities less than half the circular satellite velocity the "cost" is greater than one for one. For larger hyperbolic velocities the "cost" is less than one for one.

8.5 Flyby Maneuver

The flyby maneuver consists of an inbound and an outbound hyperbola that are joined by a common periapse. Transfer from the inbound to the outbound trajectory is accomplished by a velocity impulse, ΔV , applied at the periapse. The plane of the motion is determined by the plane of the inbound and outbound hyperbolic velocity vectors. The advantage of the maneuver is that in addition to the rotation of the relative velocity vector a large hyperbolic velocity change can be generated with a small velocity change at the periapse. The maneuver is shown schematically in Figure 14.

The general problem is to find the two hyperbolas and the associated ΔV , given the two hyperbolic velocity vectors \bar{V}_{HI} and \bar{V}_{HO} from the interplanetary analysis. Let A be the angle between the vectors \bar{V}_{HI} and \bar{V}_{HO} .

$$A = \arccos \frac{\bar{V}_{HI} \cdot \bar{V}_{HO}}{V_{HI} V_{HO}} \quad (8.9)$$

The two hyperbolas can be determined by the simultaneous solution of Equations (8.10) - (8.12) for V_S .

$$\sin \delta_I = \frac{1}{1 + \left(\frac{V_{HI}}{V_S}\right)^2} \quad (8.10)$$

$$\sin \delta_0 = \frac{1}{1 + \left(\frac{V_{HO}}{V_S}\right)^2} \quad (8.11)$$

$$\delta_I + \delta_0 = A \quad (8.12)$$

Knowing V_S , $V_{\pi I}$ and $V_{\pi O}$ can be found using Equation (8.3).

The velocity increment, ΔV , is given by

$$\Delta V = V_{\pi O} - V_{\pi I} \quad (8.13)$$

Equations (8.10) - (8.12) are a transcendental set and require an iteration for an accurate solution. A very good approximation for V_S is given by Equation (8.4) if V_H is taken as the average of V_{HO} and V_{HI} and δ is taken as $\frac{A}{2}$.

A special case of the general flyby maneuver occurs when the velocity increment, ΔV , is zero, so that the planetary encounter imparts only a directional change to the hyperbolic velocity. Most of the agencies studying "flyby missions" have restricted their analysis to this special case⁽¹⁾⁽⁴⁾.

As noted in Section 8.4, it is convenient to know the "cost" of a hyperbolic velocity change in terms of the actual velocity increment applied, i.e. the ratio $\frac{\Delta V_H}{\Delta V}$. For this type of maneuver the "cost" is always less than one for one.

The cost approaches zero as V_H approaches zero. In general, the smaller r_π , the smaller the "cost". To a first approximation r_π is determined by the size of the angle, A . The larger the angle, A , the closer the vehicle must pass the planet. Since r_π must be greater than one to avoid a collision with the planet, there is an upper limit on the angle through which the hyperbolic velocity vector may be turned.

When $|\bar{V}_{HI}| = |\bar{V}_{HO}| = V_S$ then $A = 60$ degrees.

This gives a rule of thumb that the turn angle, A , cannot be greater than 60 degrees if the hyperbolic approach velocity is greater than the circular satellite velocity, V_C , at the planet's surface.

CHAPTER 9

ATMOSPHERIC BRAKING9.1 Nature of the Problem

When a vehicle arrives at a planet after an interplanetary voyage it must dissipate part of its high approach velocity in order to enter a parking orbit or to descend for a landing. This velocity change could be accomplished with rocket thrust but in most cases it is more economical to enter the planet's atmosphere and dissipate the energy in the form of heat through atmospheric braking. Once the vehicle has been slowed to circular satellite speed it may either exit the atmosphere into a parking orbit or continue deceleration to a direct landing on the surface. For the first manned expedition to Mars it will be necessary to establish a parking orbit before descending to the surface in order to map the terrain, select a landing site and verify that there is no unforeseen, hostile environment which might preclude a landing on the surface. Upon return to Earth a direct descent to the surface is feasible.

The primary reason for using aerodynamic braking instead of thrust-braking is that the mass ratio of the initial

mass to the final mass is smaller using aerodynamic braking. Therefore the major objective of the analysis for mission planning purposes is to obtain some expression of the required mass ratio for aerodynamic braking as a function of the approach velocity. The secondary objective of the analysis is to obtain a general understanding of the problems of aerodynamic braking. The author intends no contribution to this area of technology, but instead is reporting on a study of the subject for application to mission planning.

9.2 Thermodynamic Considerations

When the vehicle enters a planetary atmosphere at supersonic speed a shock wave is formed ahead of the vehicle. The stagnation temperature is sufficiently high to ionize the gas behind the shock wave, directly in front of the vehicle. Heat is transferred to the vehicle by convection through the boundary layer and also by radiation directly from the hot, ionized gas. At entry speeds below about 0.45 EMOS convective heating predominates and it is minimized by a blunt vehicle which creates a strong detached shock wave well ahead of the vehicle. The capsules designed for the Mercury, Gemini and Apollo programs all have this configuration. At speeds above 0.45 EMOS radiative heating predominates and it is minimized by a sharp-nosed vehicle which creates a weak shock. The heating rate due to radiative heating goes up extremely rapidly with increased

velocity and becomes a big problem if not the limiting factor at high speeds.

For radiative heating the stagnation temperature increases with increasing velocity and increasing air density. The actual vehicle temperature at the stagnation point may differ greatly from the stagnation temperature due to configuration, real gas effects, materials' properties, and exposure time, but the trend of increasing temperatures for higher velocities and air density is the same. A typical temperature as given by Lockheed⁽³⁹⁾ is about 1400 degrees Fahrenheit at an altitude of 50 nautical miles in the Earth's atmosphere at a velocity of 0.45 EMOS. It rises to about 2100 degrees Fahrenheit at 45 nautical miles altitude at the same velocity. A temperature of 1400 degrees Fahrenheit is about the service limit for most common metal alloys. A temperature of 2100 degrees Fahrenheit is above the service limit for the best high temperature metal alloys and molybdenum or ceramics are required. The vehicle may be protected from the heat that is absorbed by insulation with an ablative, fibre-glass material that chars, melts and vaporizes. The ablative weight is usually less than 15 per cent of the total vehicle weight. If the vehicle temperature can be kept low enough the absorbed heat can be dissipated by direct reradiation from the vehicle surface. Normally, ablation would be accomplished at a lower altitude, say 40 miles in the Earth's atmosphere. The heating rate would be high but the deceleration period would be short so the total heat absorbed would be small. Reradiation

would be accomplished at a higher altitude, say 50 miles in the Earth's atmosphere. The heating rate would be lower, but the deceleration period would be longer so that the total heat absorbed is larger. In general, the flight path with the minimum total heat input is the one with the highest allowable deceleration ("G" load).

9.3 Aerodynamic Considerations

In order to arrive at the proper altitude for atmospheric braking the periapse of the hyperbolic approach trajectory must lie close to the deceleration zone. The acceptable limits of the periapse position determine the atmospheric braking corridor which is shown in Figure 15. Vehicles which arrive below the undershoot boundary will exceed the acceptable deceleration limit (nominally 10 "G") or the designed heating rate. Vehicles which arrive above the overshoot boundary will not be decelerated sufficiently to effect a proper capture by the planet's gravitational field. Vehicles arriving within the corridor but near the undershoot boundary need to direct the lift vector radially outward to arrive in the proper deceleration zone. The entry is characterized by high deceleration and high heating rate, but low total heat is absorbed. Vehicles arriving within the corridor but near the overshoot boundary need to direct the lift vector radially inward to arrive in the proper deceleration zone. This entry is characterized by low deceleration and low heating rate, but high total

heat is absorbed.

Following Chapman⁽²⁵⁾ the corridor width is defined as the difference in the radial distance to the periapse of each of the two boundary trajectories. In general, the use of aerodynamic lift increases the corridor width. The size depends on the aerodynamic technique used, i.e. constant altitude, constant angle of attack, constant deceleration, etc. Corridor width also increases with slower approaches, higher allowable decelerations, and lower atmospheric density gradients. Small corridor widths require greater navigational accuracy.

If the vehicle is to descend for a landing the use of aerodynamic lift gives more control over the landing point. The lift vector is maneuvered in a manner similar to the operation of a glider. The footprint for a vehicle with $L/D = 1$ is about as large as the continental United States.

9.4 Work That Has Been Done

The references used in the atmospheric braking study will be mentioned in this section. They are representative of work that has been done but they by no means exhaust the field.

Eggers et al⁽²³⁾ have developed equations for ballistic-type vehicles which enter the atmosphere at sufficiently steep angles that the gravitational and centrifugal forces can be disregarded. This assumption is not valid for

atmospheric braking although some of the analysis is helpful. The most useful analytical contribution has come from Chapman⁽²⁴⁾⁽²⁵⁾. His basic assumption is that the percentage change in radial distance is small compared to the percentage change in velocity. The resulting analytical expressions are quite valid at speeds above circular satellite velocity and apply readily to the problem of atmospheric braking. By using dimensionless parameters he makes the results applicable to any planet. He points out that the small planet size and the low density gradient make Mars one of the easiest planetary atmospheres to enter. Several plots included in Reference (25) allow quick determination of corridor widths. Luidens has investigated the corridor widths associated with specific aerodynamic techniques⁽²⁶⁾ and specific vehicle geometry⁽²⁷⁾. The latter reference considered a delta-wing vehicle with high L/D ratio of about one or two. He concluded that the constant-G path with total deceleration at the vehicle limit provided the minimum total heat absorption. Working with Himmel et al⁽⁹⁾ on the study of a manned nuclear-rocket mission to Mars he recommended the use of a delta-wing vehicle for Earth re-entry. For a delta-wing configuration, he concluded that the low vehicle density at Mars, due to large amounts of stored hydrogen propellant, caused high structural weights and offered no saving over thrust-braking. He did not consider other configurations for

atmospheric braking at Mars. Ford Aeronutronic has made a comparative study of three re-entry vehicles, a drag brake, a high L/D vehicle, and the Apollo capsule for Earth re-entry from specific interplanetary flyby missions⁽¹⁾. They concluded that the three vehicle types are competitive for their mission in that no single one varied more than 20 per cent in weight from the others.

Two interesting and promising drag brake configurations have been proposed although less work has been done on the drag brake than the other configurations. The first of these is the Avco drag brake which varies its frontal area like an umbrella. Hayes and Vander Velde⁽²⁸⁾ have presented a landing control system which modulates the drag on the basis of measured deceleration. Their work has also suggested the use of modulated drag in conjunction with a lifting vehicle in order to take advantage of the good features of both lift and drag control.

The second drag brake configuration has been proposed by Lockheed. It uses rocket thrust to produce a component of lift directed radially inward toward the planet. The artificial lift created by the rocket allows the drag brake to enter the atmosphere at high velocity near the overshoot boundary. Ragsac and Titus⁽²⁹⁾ of Lockheed in reporting a technique for the local optimization of interplanetary missions have assumed the use of a drag brake with a propulsion system to augment vehicle lift for atmospheric braking at

both Earth and Mars. Plots are given of the vehicle weights as a function of approach velocity. They limit the atmospheric entry velocity at Earth to 50,000 feet per second and at Mars to 40,000 feet per second.

9.5 Specific Vehicle Types

The three representative vehicle types for atmospheric braking and re-entry are the drag brake, the Apollo-type vehicle, and the hypersonic glider with a high lift to drag ratio. A comparison of their representative characteristics for an Earth re-entry is given in Table 9.1. Any specific vehicle type will probably be one of these three or a combination of these with rocket thrust. All three vehicles can make a direct descent to a landing at Earth without rocket augmentation at atmospheric entry speeds of up to 0.45 EMOS with competitive vehicle weights.

The technique of multiple braking passes is a possible method of atmospheric braking which allows the vehicle to cool between passes. It is not considered for the manned Mars mission for the following reasons. Multiple passes at Earth cause repeated exposure of the crew to the Van Allen radiation belts. In addition, at Mars, the velocity increment required for circular satellite velocity is not that much larger than the velocity increment required for capture to offer much saving.

The most attractive vehicle for atmospheric braking

Table 9.1 Comparison of Re-entry Vehicles

	Drag Brake	Apollo Vehicle	Hypersonic Glider
Max. Lift to Drag Ratio L/D	.2	.5	2
Altitude	50 naut. mi.	40 naut. mi.	40 naut. mi.
G Level	6	8	4
Heat Protection	Reradiation	Ablative Shield	Ablative Nose
Ballistic Coef. $\frac{W}{C_D A}$	5 psf	50 psf	500 psf

All figures are approximate.

to enter a parking orbit at Mars is the Lockheed drag brake which has a propulsion system to augment vehicle lift. The mass ratio of the vehicle is given as an analytical function of the entry velocity in Reference (30). When the atmospheric entry velocity, V_{π} , is expressed in EMOS units, this function becomes

$$\frac{M_1}{M_F} = 1.9 (V_{\pi} - .04)^{1/4} \quad (9.1)$$

where M_1 = the initial mass before atmospheric braking

M_F = the final useful mass in parking orbit

The mass ratio can also be expressed in terms of the hyperbolic approach velocity, V_H , by using the analysis of Chapter 8. This has been done and the result is plotted in Figure 16. The required mass ratios for thrust-braking with both a nuclear rocket and a chemical rocket are shown for comparison.

9.6 Conclusions for Mission Planning

It is concluded that atmospheric braking should be used at Mars for deceleration prior to entering a parking orbit. Atmospheric braking should be used at Earth in a direct re-entry to a landing. For mission planning, the mass ratio given by Equation (9.1) or Figure 16 will

be used for atmospheric braking at Mars for atmospheric entry speeds, V_{π} , up to 0.34 EMOS. This is about three times circular satellite velocity for Mars and is equivalent to a hyperbolic approach velocity, V_H , of 0.3 EMOS. This limit is more conservative than that given in Reference (29). If approach velocities in excess of this limit are considered, the deceleration down to the aerodynamic braking speed will have to be done with rocket thrust-braking. At Earth the limiting atmospheric entry speed, V_{π} , will be set at 0.45 EMOS. This is about 1.8 times circular satellite velocity for Earth and is equivalent to a hyperbolic approach velocity, V_H , of 0.25 EMOS. It will be assumed that the required vehicle weight is constant up to entry speeds of 0.45 EMOS. Above that limit thrust-braking by a rocket must be used to slow the vehicle to 0.45 EMOS.

Conservative estimates have been made here of aerodynamic braking capabilities in order not to overestimate the savings associated with bi-elliptical transfer. The savings from bi-elliptical transfer increase as the limiting velocity for atmospheric braking at Mars increases. This fact should encourage more study of the problem of atmospheric braking at Mars with the aim of extending the approach velocity limit.

CHAPTER 10

USE OF THE COMPUTER10.1 The Role of the Computer

The basic approach used in this thesis toward the problem of interplanetary transfer has been to develop a simple model to gain an understanding of the problem, to make conclusions based on the simple model, and finally check the conclusions by more rigorous analysis using the electronic computer. Accurate hand calculation of the large number of trajectories needed for meaningful mission planning would be out of the question. Fortunately the electronic computer is ideally suited for this type of computation. The approach often taken by others toward the same problem has been to compute a large number of trajectories, plot certain parameters of the trajectories and attempt to interpret the plots. With the author's approach an understanding of the orbital mechanics is accomplished with the simple model and it is only necessary to read the accurate data required from the computer print-out sheets. Using computed results before the problem is fully understood can be a very dangerous practice.

10.2 General Trajectory Program

The General Trajectory Program was written to compute single-elliptical trajectories between any two given planets as a function of the launch date and arrival date. The computation is based on the known two-body solution for the motion of a particle falling freely in the solar gravitational field. No perturbations due to disturbing gravitational fields are included. Within this framework the computation is rigorous, that is, none of the approximations made in the simple models discussed earlier are used in the computer program.

The input to the program consists of the launch planet, the Julian date of launch, the arrival planet, and the Julian date of arrival. This information is sufficient to determine a unique conical trajectory between the two planets. If that conic is an ellipse then the program will compute all its orbital elements. The data read out includes the longitude at launch, the longitude at arrival, all components of the hyperbolic velocity, V_H , of the spaceship relative to the planet after escape from the planetary gravitation at both launch and arrival, the semi-major axis, the eccentricity, the radial distance at perihelion and the longitude of perihelion. The program will compute trajectories for a matrix of launch and arrival dates at any requested interval.

The Ephemeris data consisted of the mean elements of

Equations (10.1) - (10.3) determine planet position from the date and the orbital elements. The problem that remains is to determine the one elliptical trajectory which has the launch planet's position at the launch date and the arrival planet's position at the arrival date. Battin⁽³⁴⁾ has treated this problem at length and his computational procedure was followed closely. With Dr. Battin's kind permission the iteration routine to obtain the semi-major axis of the transfer ellipse from the time of flight was taken directly from a computer program which he wrote. After the transfer ellipse is determined it is a straightforward vector subtraction to obtain the velocity of the spaceship relative to the planet as the difference between the spaceship's velocity and the planet's velocity. The components of the relative velocity vector are expressed in the frame defined in Chapter 3, that is, in the radial direction, the circumferencial direction, and in a direction perpendicular to the planet's orbital plane.

10.3 Bi-elliptical Transfer Program

The advantages of bi-elliptical transfer were discussed in Chapter 7. In order to compare computed bi-elliptical transfers with single-elliptical transfers it is desirable that the specification of a launch date and an arrival date uniquely determine a trajectory. As explained in Section 7.2, this is possible if certain constraints are placed on the choice of the two ellipses.

Referring to Figure 11, it is necessary that the two ellipses have a common perihelion point at π and that the angle πSM be specified. With this done the launch date and the arrival date fix the positions of E and M respectively. It remains to determine the distance $S\pi$. The total time of flight can be expressed as a function of the unknown distance $S\pi$. A Newton iteration is then performed to give the value of $S\pi$ which satisfies the time of flight expression. The rest of the Bi-elliptical Transfer Program is the same as the General Trajectory Program. In addition, the read-out includes the velocity increment required at perihelion. The program was written for three values of the angle πSM , 120 degrees, 135 degrees, and 150 degrees. The size of πSM tends to control the arrival velocity at Mars.

10.4 Computer Results

The General Trajectory Program and the Bi-elliptical Transfer Program were written by the author for the IBM 7090 Computer using FORTRAN⁽³⁵⁾. The programs were compiled and executed using the facilities of the M.I.T. Computation Center in Cambridge, Massachusetts. Copies of the programs can be made available to interested persons. On the 7090 over 1000 trajectories are computed in less than one minute of machine time. To date all practical trips between planets have been computed at ten-day intervals through 1980. It is a simple matter to make

additional runs for dates not already computed. The print-out sheets read very much like a train schedule. Find the desired departure date and arrival date and read across the line to find the required hyperbolic velocity, V_H , at launch and arrival, the direction of launch and arrival, and the point of closest approach to the Sun.

The computed results can be applied to any type of mission. One, two, three, or four-legged trips can be analyzed by picking the individual legs from the computed data and pairing the legs at each terminal with the type of planetary encounter desired using the analysis of Chapter 8. This procedure will be used in the next chapter to select a mission to Mars that makes a flyby encounter with Venus en route.

CHAPTER 11

TRANSFER VIA VENUS11.1 Possible Savings

In Chapter 7 it was observed that the savings associated with a bi-elliptical transfer were limited because the mid-course velocity increment was not applied in a planetary environment. It was also observed that if Venus were in approximately the same position as the intended perihelion point it would be possible to make large savings over the best single-elliptical mission to Mars. Because of the relative orientation of Earth, Venus and Mars which is required, the transfer is treated as two single-elliptical transfers. The first transfer is from Earth to Venus and the second transfer is from Venus to Mars. The problem still remains to determine when the relative orientation of the planets will be right.

The relative orientation of the planets through 1980 is indicated by the plot of the dates of planetary alignment in Figure 2. Using the simple model described in Chapter 4 it can be shown that low energy transfers between planets are characterized by the fact that alignment

of the launch and arrival planet occurs about midway through the transfer. Consequently, in order that a low energy transfer to Venus can be followed by a low energy transfer to Mars and a low energy transfer back to Earth, the Earth-Venus, Venus-Mars, and Mars-Earth alignment dates must occur in that order with a spacing of about 140 days between them. This orientation is seen to occur about every six years or prior to every third opposition of Mars. When the orientation is right for an Earth-Venus-Mars transfer before the opposition it is also right for a Earth-Mars-Venus transfer after the opposition. Examples of these favorable orientations are shown in Figure 2 around the oppositions of 1965, 1971, and 1978. The opposition of 1971 is also favorable because it occurs near Mars' perihelion, closest to both the Earth and the Sun. An orientation which allows an Earth-Venus-Mars transfer will not occur in so favorable a position again until after the turn of the century.

11.2 Trip Selection

The selection of an Earth-Venus-Mars trip is performed in the following manner. First, a favorable launch period, such as the one prior to the 1971 opposition, is selected by observation of Figure 2. Trips are then computed between the planets at ten-day intervals during the favorable period using the computer as explained

in Chapter 10. Unfavorable trips are eliminated by scanning the print-out sheets and marking those trips which depart Earth with V_H greater than 0.2, which arrive at Mars with V_H greater than 0.3, or which go closer to the Sun than 0.7 a.u. Trips arriving at Venus on dates for which there are no acceptable trips to Mars are also eliminated as are trips departing Venus on dates for which there are no acceptable trips from Earth. The trips which remain will define a launch window of acceptable transfers to Mars via Venus providing the two legs of the trip can be paired at Venus using the planetary encounter analysis of Section 8.5.

All the acceptable transfers can be plotted on the $V_x - V_y$ plane at Venus as explained in Chapter 3. The loci of constant arrival date at Venus and constant departure date from Venus show the hyperbolic velocity vectors which can be paired. Knowledge of the mechanics of the planetary encounter allows the mission planner to choose an economical pair of trips by scanning the plot. With a little experience he can make the selection without plotting. The typical trip described in the next section was chosen in this manner. For the 1971 period the launch window described by the above constraints exists at Earth for 120 days. The arrival date at Venus can vary over 50 days and the arrival date at Mars can vary over 120 days.

The Mars-Venus-Earth trip, or any multi-legged mission, is handled in the same manner as described here. Use of the simple model and the time plot in Figure 2 show the mission planner where to look with the computer. The planetary encounter analysis of Chapter 8 shows him how to pair any possible inbound and outbound legs.

For the special case of a planetary flyby with no velocity change, the magnitude of the inbound and outbound hyperbolic velocity must be identical. In this case it will almost certainly be necessary to plot all acceptable pairs to find two with exactly the same velocity magnitude. This special case has been the type of mission reported by the agencies studying dual-planet "flyby" missions. (1)(4) The major disadvantage of this restrictive type of mission is that the launch window extends for only a day or two. Furthermore, it does not fully exploit the advantages associated with bi-elliptical transfer.

11.3 Typical Trips

An Earth-Venus-Mars trip that arrives at Mars 84 days prior to the 1971 opposition and a Mars-Venus-Earth trip which departs Mars 56 days after the 1971 opposition are described in this section as typical of the type of trip which makes a transfer via Venus. The specific examples were chosen because they represent the most attractive mission opportunities for the manned expedition to Mars.

For the first trip the spaceship launches from an Earth orbit on 1 September 1970 at a longitude of 338 degrees east of Aries. The propulsive velocity requirement is 0.126 EMOS units and the vehicle will have a hyperbolic velocity of 0.126 EMOS units relative to the Earth after escape from the Earth's gravitational field. On 31 December 1970 the spaceship approaches Venus with a hyperbolic velocity of 0.171. The velocity components are $V_x = -0.027$; $V_y = 0.084$; $V_z = -0.146$. Venus is passed on the southern, sunlit side at a distance of 1.74 planet radii. At the point of closest approach to Venus a velocity increment of 0.024 is added which gives a hyperbolic velocity after escape of 0.211. This relative velocity vector has been rotated 57 degrees from the inbound velocity vector so that it now has components $V_x = 0.027$; $V_y = 0.208$; $V_z = -0.020$. Mars is reached on 19 May 1971 with a hyperbolic approach velocity of 0.284. The spaceship decelerates by means of a drag brake in the Martian atmosphere and enters a parking orbit about Mars. The mass ratio required for the atmospheric braking operation is 1.42. A summary of this itinerary is given in Table 11.1 and the path through the solar system is shown in Figure 17.

For the second trip the spaceship launches from a parking orbit at Mars on 6 October 1971 at a longitude of 353 degrees east of Aries. The propulsive velocity

requirement is 0.141 and the hyperbolic velocity after escape is 0.196. On 3 April 1972 Venus is approached with a hyperbolic velocity of 0.265. The components are $V_x = -0.196$; $V_y = 0.154$; $V_z = -0.089$. By subtracting a velocity increment of 0.045 at a distance of 1.2 planet radii on the southern, sunlit side of Venus the hyperbolic departure velocity is reduced to 0.186. The components are $V_x = -0.095$; $V_y = 0.106$; $V_z = 0.121$. The spaceship arrives at Earth 22 July 1972 with a hyperbolic approach velocity of 0.226. It makes an entry into the atmosphere with a direct descent to the Earth's surface. This trip is also summarized in Table 11.1 and the path through the solar system is shown in Figure 18. It is conceivable (but not recommended) that the velocity reduction at Venus could be made by using a drag brake in the Venusian atmosphere. The braking control accuracy required and the consequences of a miscalculation caution against such an attempt on early missions.

11.4 Mission Advantages

The trips of the type just described offer several advantages over single-elliptical transfers for round-trip missions to Mars. The Earth-Venus-Mars transfer arrives at Mars far earlier for less required velocity. The minimum hyperbolic velocity at Earth in order to arrive at Mars on 19 May 1971 by single-elliptical transfer is 0.366 EMOS. This requires a propulsive velocity

Table 11.1 Transfers Via Venus

Julian Date	Calendar Date	Planet	Longitude	Hyperbolic Velocity, V_H	Propulsive Requirement, V
2440830	1 Sept. 1970	♁	338°	.126	.126
2440950	31 Dec. 1970	♀	127°	.171 - .211	.024
2441090	19 May 1971	♂	267°	.284	-
2441230	6 Oct. 1971	♂	353°	.196	.141
2441410	3 Apr. 1972	♀	144°	.265 - .186	.045
2441520	22 July 1972	♁	299°	.226	-

increment of 0.26 which is almost twice the requirement for the transfer via Venus.

The chief advantage of the Mars-Venus-Earth transfer is that it allows a late departure from Mars. In order to depart Mars on 6 October 1971 with a hyperbolic velocity less than 0.2 by single-elliptical transfer, it would be necessary to accept a hyperbolic arrival speed at Earth of 0.3. Although the transfer via Venus on return is more economical than single-elliptical, the saving is not as spectacular as it is going via Venus.

The only disadvantage of transfer via Venus is the velocity requirement for planetary homing. This is far less than the major velocity savings.

Economical transfers via Venus tend to be of slightly longer duration than the best single-elliptical transfers for round-trip missions. Despite this characteristic, the typical saving for missions of the same duration is about 0.07 EMOS in favor of transfer via Venus. This velocity when transformed into a weight saving in an Earth orbit is about 25 per cent for a nuclear propulsion system. For chemical propulsion it cuts the initial weight in half. In fact, for almost any attractive single-elliptical mission which arrives at Mars prior to the 1971 opposition there will be a transfer via Venus which leaves Earth and arrives at Mars on the same days but does so with less total velocity required.

There are other advantages besides the velocity saving. Man is afforded the opportunity of a close approach to Venus without any additional cost, two planets for less than the price of one. It also offers a psychological advantage by breaking up the trip for the crew. It will be three and a half months to Venus and four and a half months to Mars instead of eight months to Mars. The advantages of transfer via Venus are so attractive that a maximum effort should be made to develop a manned Mars capability prior to the 1970 launch opportunity.

CHAPTER 12

RECOMMENDED MISSION TO MARS12.1 The Program

During the remainder of this century the most favorable conditions for a manned expedition to Mars will exist in the 1970-72 period associated with the 1971 opposition of Mars. During the 1971 opposition Mars will not only be closer to the Earth than during other oppositions but Venus will be in a position to allow a transfer via Venus. It is not unreasonable to anticipate a manned Mars capability by 1970 if effort is expended toward reaching that goal. The advantages are sufficiently attractive to warrant the effort.

The recommended mission will launch in 1970 and go to Mars via Venus on the trajectory described in Section 11.3. This brings the expedition into a parking orbit at Mars 84 days prior to the 1971 opposition. The early arrival date allows several options. The most optimistic of these is for the expedition to remain on Mars for 140 days and return to Earth via Venus using the second trajectory described in Section 11.3. If the expedition

were equipped for this long mission then an expeditious return to Earth could be made at any earlier date using less fuel and life-support provisions than required for the longer stay. Examples of the requirements for typical early-return trips after zero days, 10 days, 40 days and 90 days are given in Table 12.1. The propulsive requirement assumes that the hyperbolic departure velocity, V_H , lies in the plane of the parking orbit. This might not be the case exactly if the parking orbit had been established for a different departure. The low satellite velocity at Mars and the fact that most of the departure velocity vectors lie close to a common plane make the associated penalty small. The early-return trips do not make a Venus encounter on return. They are, to some extent, emergency procedures to be exercised in the event that fuel or provisions run low or some unforeseen danger dictates an early departure. The total trip for departure after 10 days is shown in Figure 17.

Back at Earth there is another option of launching a second vehicle which can economically depart Earth 30 days after the arrival of the first at Mars and arrive at Mars prior to the planned departure of the first vehicle from Mars. The second vehicle could replenish the first, if necessary, or bring any needed equipment based on the report of the first launch. There would

Table 12.1 Typical Early-Return Trips From Mars

Julian Date	Calendar Date	Planet	Longitude	Hyperbolic Velocity, V_H	Propulsive Requirement, V
2441090	19 May 1971	\odot	267°	.170	.121
2441190	27 Aug. 1971	\oplus	333°	.235	-
0 days on Mars		Total mission = 360 days			
2441100	29 May 1971	\odot	272°	.171	.122
2441200	6 Sept. 1971	\oplus	342°	.198	-
10 days on Mars		Total mission = 370 days			
2441130	28 June 1971	\odot	290°	.180	.128
2441280	25 Nov. 1971	\oplus	062°	.177	-
40 days on Mars		Total mission = 450 days			
2441180	17 Aug. 1971	\odot	321°	.185	.132
2441400	24 Mar. 1972	\oplus	183°	.276	.012
90 days on Mars		Total mission = 570 days			

be 290 days in which to assess the knowledge gained from the first Earth launch, 170 days to assess the Venus encounter and 30 days to assess the findings at Mars before making the second launch. The second launch would plan to return in company with the first making an encounter with Venus en route to Earth. The itinerary for the second launch is given in Table 12.2 and the total trip is shown in Figure 18.

Table 12.2 Second Launch Itinerary

Julian Date	Calendar Date	Planet	Longitude	Hyperbolic Velocity, V_H	Propulsive Requirement, V
2441120	18 June 1971	♁	266°	.167	.142
2441220	26 Sept. 1971	♂	346°	.236	-
2441230	6 Oct. 1971	♂	353°	.196	.141
2441410	3 Apr. 1972	♀	144°	.265 - .186	.045
2441520	22 July 1972	♁	299°	.226	-

10 days on Mars

Total mission = 400 days

The total mission duration for the first launch appears to contradict the conclusions reached in Chapter 5. It should be pointed out that those conclusions were based on the use of single-elliptical transfers. The mission for the first launch transfers via Venus both going and returning. Furthermore, the largest contribution to the payload weight in the analysis of Chapter 5 was the radiation shielding. During the 140 days on Mars the shielding requirement should be drastically reduced because of the natural shielding from Mars' magnetic field and atmosphere. If the long mission does not appear feasible as 1970 approaches any of the shorter missions given in Table 12.1 could be substituted. The mission shown in Figure 17, for instance, is more attractive than any single-elliptical mission. The second launch could still be made taking advantage of the findings of the first launch. The second launch could use things left in orbit or on the surface by the first launch, but there would be no overlap of personnel.

12.2 The Advantages

The main advantage of the recommended program is that it fully exploits the favorable location of the planets during the 1970-72 period. In addition to economically providing for 140 days of Mars' exploration it also offers two or three close approaches to Venus without any extra cost. The mission can safely change its planned itinerary to adjust for unpredictable events which occur subsequent

to launch. The knowledge gained during one opposition period is available for use during the same period. Launch windows are wide and velocity requirements are low. The reduced velocity requirements and the proposed use of atmospheric braking make it conceivable that the mission can be accomplished with chemical rockets in the event the nuclear rocket is unavailable by 1970.

By comparison, it would be completely impractical to attempt the proposed itinerary using single-elliptical transfers. The initial weight in Earth orbit would be increased by 45 per cent for nuclear propulsion and by a factor of 3 for chemical propulsion. The best 350-day mission which goes via single-elliptical transfer, arrives at Mars prior to the 1971 opposition, and stays 10 days, would require about 25 per cent greater initial weight for nuclear propulsion and about twice as much for chemical propulsion over the equivalent mission going via Venus.

The main reason for manned planetary exploration is scientific research. The cost of the program is expected to run near forty billions of dollars. The amount of scientific data which can be obtained during three close approaches to Venus and 140 days on Mars is probably in excess of any mission proposed to date. Because of the savings associated with transfer via Venus the total cost will probably be less than that anticipated for any previously proposed Mars stopover mission. The overall

gain in the amount of scientific information per dollar is so large that it should certainly justify the effort to accomplish this program in time for the first launch in September of 1970.

12.3 The Alternatives

If advantage is not taken of the 1971 opposition it will be six years before transfer via Venus is possible again and after the turn of the century before such an attractive opportunity reoccurs. It is possible to go to Mars without transfer via Venus during each opposition period every two years. If Venus is not in position then the trip should be accomplished with a bi-elliptical transfer for one leg. The saving over single-elliptical transfer is modest, typically about 0.03 EMOS or 3000 feet per second. After 1971 the distance of Mars from Earth at opposition gets increasingly greater until 1980 when it again starts to decrease. The increased distance requires longer trips and consequently larger launch weights to accomplish the mission.

Waiting for the launch windows associated with the oppositions of 1973 or 1975 could double or triple the required launch weight in an Earth parking orbit by comparison with the 1970 launch. Failure to take advantage of the 1970 opportunity will mean that all during the decade the cost of a manned Mars expedition will be increasing. The distance of Mars at opposition will decrease again

during the 1980's and there will be favorable oppositions in 1986 and 1988. Unfortunately the planet orientation will not be right for a transfer via Venus during either of these two favorable oppositions. During the 1990's the distance of Mars at opposition again increases so that it will be after the year 2000 before conditions are as attractive as in the 1970-72 period.

Manned flyby missions are possible alternatives to missions which land at Mars. A flyby at Venus is a reasonable objective since there is practically no hope of a manned landing. (Barrett⁽³⁶⁾ has set the surface temperature on Venus near 800 degrees Fahrenheit on the basis of microwave measurements made aboard the Mariner-2 spacecraft.) It does not seem reasonable, however, to have men travel several hundred million miles spending over a year of their lives in a space capsule to enjoy only a few short hours close to a planet where the real objective is to land on the surface. If man is going to all the expense necessary to get within a few hundred miles of Mars he should at least stop in a parking orbit and make enough scientific observations to justify the expense. The methods developed in this thesis are completely applicable to missions which make one, two, or three flybys at Venus or Mars, but the author has spent little time investigating these missions because of the limited reward in terms of scientific data per dollar by

comparison with missions which stop at Mars. A sample analysis of a mission which makes a single flyby of Mars is given in Appendix D to show how the simple model can be used in trajectory selection.

CHAPTER 13

SUMMARY AND CONCLUSIONS13.1 Summary of the Work Accomplished

The purpose of this thesis was to define the mission for a manned expedition to Mars. The approach was to use simple models in order to gain an understanding of the problem, to make conclusions on the basis of the simple models, and verify the conclusions by accurate analysis with the electronic computer. The first step was to consider all possible single-elliptical transfers between planets. The orbital mechanics was reduced to a simple model which showed all the characteristics of interplanetary transfers while giving approximate numerical parameters. Round-trip missions to Mars were studied by pairing all possible single-elliptical transfers. The best round-trip mission was selected as the one which required the minimum initial launch weight. Analysis of the two single-elliptical transfers making up the best round trip showed that one transfer was extremely uneconomical. This led to the development of the bi-elliptical transfer which produced a round trip of the same total duration but with

a lower velocity requirement. The savings associated with the bi-elliptical transfer were limited because the mid-course velocity increment had to be applied beyond the influence of any planetary environment. By use of the simple models it was shown that the planets would be in a position once every six years to allow the bi-elliptical trajectory change to be made during a flyby encounter with Venus. Having reached these conclusions with the simple models accurate two-body single-elliptical and bi-elliptical trajectories were generated with the electronic computer for all practical launch dates through 1980. The computer results confirmed the savings predicted for both bi-elliptical transfer and transfer via Venus.

The contributions made by the thesis are itemized below.

1. The complicated orbital mechanics are reduced to a simple model extremely valuable for mission planning.
2. A useful display of the characteristics of all practical single-elliptical transfers between planets is presented.
3. The addition of a velocity increment during a fly-by of Venus is shown to widen the launch window and reduce the velocity requirement for a trajectory which passes both Venus and Mars.
4. A calculation of the hyperbolic launch and arrival velocity vectors for attractive planet to planet

trajectories through 1980 has been accomplished.

A method is outlined for connecting these trajectories to analyze multi-legged mission combinations.

5. A computer program for the computation of bi-elliptical trajectories to Mars has been written and used to compute attractive trajectories of this type through 1980.
6. A mission is recommended for a manned expedition to Mars in 1970. By making a velocity change near Venus the known advantages associated with a two-planet fly-by and a perhelion opposition are adapted to a stopover mission.

13.2 Conclusions

The most economical round trips to Mars are accomplished with an interplanetary transfer which makes a trajectory change during a close approach to Venus. The orientation of the planets is right for this maneuver every six years or during every third opposition period of Mars. During opposition periods when the planet orientation does not allow a transfer via Venus the round trip should be accomplished using one bi-elliptical transfer. Stopping at Mars is to be accomplished with atmospheric braking using a drag brake. Return to Earth can terminate with a direct re-entry to a landing. Vehicle types for Earth re-entry are competitive. A 1970 mission to Mars via Venus is recommended to take advantage of the economical

launch opportunities which will not be as attractive again until the turn of the century. Failure to take advantage of the 1970 launch windows will mean an increasing cost for a manned Mars expedition throughout the decade.

13.3 Recommendations for Further Study

The concepts of bi-elliptical transfer and transfer via Venus require an extension of our present interplanetary navigational techniques. The current approach⁽¹⁹⁾⁽³⁴⁾ is to apply linear perturbation theory about a reference trajectory which is precomputed. For bi-elliptical transfer some modification must be made since the first navigational maneuver is to be accomplished at a discontinuity in the reference trajectory. For transfer via Venus it is necessary to be able to relate the effect of maneuvering in the planetary environment during the fly-by maneuver to the arrival conditions at the next planet. An extension of Stern's work⁽¹⁹⁾ to include navigation along a hyperbolic path could lead to a complete analytical formulation of the navigational problem along a reference trajectory. Again a method for handling the navigational correction at the discontinuity is required. The recommended mission calls for the capability of departing Mars any time within a 140-day period. Either many reference trajectories must be stored on board or some sort of adaptive navigational system needs to be developed. Any proposed system must consider the possibility of a failure of the onboard computer.

Ragsac and others⁽²⁹⁾⁽³⁰⁾ working at Lockheed have developed computer techniques for selecting interplanetary missions which provide minimum initial launch weight. They have considered only single-elliptical transfers, but their method should be adaptable to the trajectories suggested here. Because the bi-elliptical transfer allows some measure of control over the approach velocity at Mars more consideration should be given to raising the approach speed capability of a drag brake at Mars. This factor will become increasingly important if transfers are anticipated during the unfavorable oppositions from 1975 through 1982. The techniques for selecting the best mission require knowledge of the drag brake weight as a function of approach speed.

Transfer via Venus requires a velocity change to be accomplished during a close approach to the planet. The development here assumed impulsive thrusting exactly at the periapse. An error analysis should be made to determine the effects of finite burning time and failure to apply the thrust exactly at the periapse. The introductory work of Zee⁽²²⁾ only considered entry into a Hohman transfer. Ross⁽³⁷⁾ has suggested that correctional thrust during a flyby maneuver should not be applied exactly at the point of closest approach because it would disturb observational experiments in progress. A complete study of the trade-offs associated with the flyby maneuver should be conducted.

A large amount of study has recently been initiated by NASA under the EMPIRE program anticipating manned interplanetary expeditions in the early 1970's. This entire study should be accelerated in order to develop a manned Mars capability prior to the attractive launch opportunities in 1970.

APPENDIX A

MATHEMATICAL MODELS OF THE INTERPLANETARY ORBITAL MECHANICS

Postulate the existence of an inertial frame, i , centered at any arbitrary inertial point, I . An inertial frame is defined as a frame in which Newton's laws of motion are valid and is considered to be non-rotating with respect to the distant galaxies. Denote the position of a particle at some point, P , relative to I as the position vector, \bar{R}_{IP} . Denote the second derivative of the position vector relative to the inertial frame, i , as $p_i^2 \bar{R}_{IP}$. By Newton's second law

$$p_i^2 \bar{R}_{IP} = \bar{G}_{SP} + \sum_B \bar{G}_{BP} \quad (\text{A.1})$$

where \bar{G}_{SP} is the gravitational force per unit mass on a particle at P due to the Sun and \bar{G}_{BP} is the gravitational force per unit mass on a particle at P due to any other body, B , other than the Sun. By applying Newton's second law to the Sun, centered at S ,

$$p_i^2 \bar{R}_{IS} = \sum_B \bar{G}_{BS} \quad (\text{A.2})$$

Subtracting (A.2) from (A.1)

$$p_i^2 \bar{R}_{SP} = \bar{G}_{SP} + \sum_B (\bar{G}_{BP} - \bar{G}_{BS}) \quad (\text{A.3})$$

Equation (A.3) says that Newton's laws of motion are valid for a particle in a Sun-centered inertial frame provided the particle is considered to be acted upon by the solar gravity plus the gravitational "gradient" force per unit mass given by the summation term in (A.3). This is the general n -body problem.

Define the accurate two-body model as the solution of Equation (A.3) with the "gradient" forces neglected. The resulting motion is well known with the particle describing a conic section in an inertial frame. If the motion is expressed relative to the Earth in this model then the elliptic character of the Earth's orbit will be preserved.

Define the standard model as the solution of Equation (A.3) with the "gradient" forces neglected but with the motion expressed relative to the Earth which is assumed to be in circular orbit about the Sun.

Define the simple model as the approximate solution of Equation (A.3) which follows, based primarily on the assumption of a linearized solar gravitational field. The xyz frame is centered at the Earth which is assumed to be in circular orbit about the Sun. The frame rotates with respect to inertial space at a constant rate of 2π radians per year. Figure 4 shows the orientation of the x and y axes in the Earth's orbital plane. The z axis is perpendicular to the Earth's orbital plane. By application of the theorem of Coriolis the inertial acceleration, $p_i^2 \bar{R}_{SP}$, of a point, P , with coordinates, (x, y, z) , is given by

$$\begin{aligned} p_i^2 \bar{R}_{SP} = & [\ddot{x} - 4\pi\dot{y} - 4\pi^2(1+x)] \bar{i} \\ & + [\ddot{y} + 4\pi\dot{x} - 4\pi^2 y] \bar{j} \\ & + \ddot{z} \bar{k} \end{aligned} \quad (A.4)$$

where the unit of time is the year and the unit of distance is the astronomical unit. By virtue of the choice of units the product of the gravitational constant and the mass of the Sun is $4\pi^2$ a.u. per year². The gravitational force per unit mass at P due to the Sun, \bar{G}_{SP} , is

$$\bar{G}_{SP} = \frac{-4\pi^2}{[(1+x)^2 + y^2 + z^2]^{3/2}} [(1+x)\bar{i} + y\bar{j} + z\bar{k}] \quad (A.5)$$

Assume that the gravitational force of the Sun is the only force acting on a particle at P. Then the components of (A.4) can be equated to the components of (A.5) to give the equations of motion for the particle.

$$\begin{aligned}\ddot{x} &= \frac{-4\pi^2(1+x)}{[(1+x)^2 + y^2 + z^2]^{3/2}} + 4\pi\dot{y} + 4\pi^2(1+x) \\ \ddot{y} &= \frac{-4\pi^2 y}{[(1+x)^2 + y^2 + z^2]^{3/2}} - 4\pi\dot{x} + 4\pi^2 y \\ \ddot{z} &= \frac{-4\pi^2 z}{[(1+x)^2 + y^2 + z^2]^{3/2}}\end{aligned}\quad (\text{A.6})$$

Assuming that x , y , and z are all small compared to the astronomical unit so that a power series expansion will converge

$$[(1+x)^2 + y^2 + z^2]^{-3/2} \cong 1 - 3x + 6x^2 - \frac{3}{2}y^2 - \frac{3}{2}z^2 + \dots \quad (\text{A.7})$$

$$\begin{aligned}\bar{G}_{SP} &\cong -4\pi^2(1 - 2x + 3x^2 - \frac{3}{2}y^2 - \frac{3}{2}z^2 + \dots)\bar{i} \\ &\quad - 4\pi^2(y - 3xy + \dots)\bar{j} \\ &\quad - 4\pi^2(z - 3xz + \dots)\bar{k}\end{aligned}\quad (\text{A.8})$$

Neglecting powers or products of second order or higher in x , y , and z , the linearized equations of motion in the rotating frame become

$$\begin{aligned}\ddot{x} &= 4\pi y + 12\pi^2 x \\ \ddot{y} &= -4\pi\dot{x} \\ \ddot{z} &= -4\pi^2 z\end{aligned}\quad (\text{A.9})$$

At $t = 0$ let $x = y = z = 0$

$$\dot{x} = 2\pi V_x \quad \dot{y} = 2\pi V_y \quad \dot{z} = 2\pi V_z$$

The 2π factor puts velocity in units of Earth Mean Orbital Speed (EMOS). The solution gives the equations defining the simple model.

$$x = sV_x + rV_y$$

$$y = -rV_x + qV_y$$

$$z = sV_z$$

where $q = q(t) = 4 \sin 2\pi t - 6\pi t$

$$r = r(t) = 2 - 2 \cos 2\pi t \tag{A.10}$$

$$s = s(t) = \sin 2\pi t$$

To the same order of accuracy, x may be replaced by $\rho - 1$, where ρ is the radial distance of the particle from the Sun, and y may be replaced by ϕ , where ϕ is the heliocentric angle by which the particle is ahead of Earth.

$$\rho = 1 + sV_x + rV_y$$

$$\phi = -rV_x + qV_y \tag{A.11}$$

$$z = sV_z$$

In Equations (A.10) and (A.11) velocity is in the EMOS unit, distance is in the astronomical unit, and time is measured in years. The fact that distance and velocity appear to be dimensionally the same is because the choice of units has suppressed the numerical factor of Earth's heliocentric angular rate of 2π radians per year. Note also that the velocity of one astronomical unit per year differs from the velocity of one EMOS unit by the factor of 2π .

The magnitude of the acceleration error made in the simple model is given by the terms neglected in Equation (A.8). The general accuracy of each of the models is discussed in the next appendix.

APPENDIX B

ACCURACY OF THE MATHEMATICAL MODELS

The accurate two-body model assumes that a vehicle acts like a particle and the gravitational "gradient" forces due to all bodies other than the Sun are neglected. The effects of perturbations due to the departure or arrival planet are taken into consideration automatically by defining the initial and terminal velocities for the interplanetary phase as the hyperbolic velocity after escape from the departure planet's gravity or prior to capture by the arrival planet's gravity. For flights to the near planets the neglect of Jupiter causes the greatest error. Jupiter has a mass 318 times that of Earth at an orbital radius of about 5 a.u. Lawden⁽⁴⁰⁾ has calculated the error due to neglecting Jupiter during a Mars trip to be equivalent to a launch velocity error of about five feet per second. He argues that there is no need for greater accuracy because the launching rocket will not be able to deliver it. Following Lawden's suggestion, define the acceptable accuracy for mission planning to be equal to the expected initial launch accuracy, nominally about one hundred feet per second. The accurate two-body model therefore provides acceptable accuracy for mission planning.

The standard model assumes the Earth to be in a circular orbit about the Sun. The actual position of a body in elliptic orbit is given by

$$r = a(1 - \epsilon \cos E) \quad (\text{B.1})$$

where:

- a = the semi-major axis
- ϵ = the eccentricity
- E = the eccentric anomaly

Since $\epsilon = 0.017$ for Earth, the maximum error in r due to assuming $\epsilon = 0$ is 0.017 a.u.. The inertial velocity, V , of a body in elliptical orbit is given in EMOS units by

$$V^2 = \frac{1 + \epsilon^2 + 2\epsilon \cos f}{a(1 - \epsilon^2)} \quad (\text{B.2})$$

where:

f = the true anomaly

Expanded in powers of ϵ

$$V = \frac{1}{\sqrt{a}}(1 + \epsilon \cos f + \epsilon^2 \dots) \quad (\text{B.3})$$

The maximum error in velocity due to assuming $\epsilon = 0$ is about 0.017 EMOS. The standard model does not provide acceptable accuracy for mission planning and provides only minor simplification of the orbital mechanics.

The simple model produces even less accurate positions and velocities but is valuable because it provides gross simplification of the orbital mechanics. Equations (A.10) or (A.11) represent the mathematical model. Rigorous equations for the same coordinates cannot be written in closed form because the position of a body in an elliptic orbit is only expressed as a function of the time through the medium of the eccentric anomaly which requires a transcendental solution of Kepler's equation. The coordinates can be written to any degree of approximation by using MacMillan's⁽³³⁾ expansion of Kepler's equation, which is a power series in the eccentricity. By definition of ρ and ϕ , and assuming Earth to be in a circular orbit

$$\rho = \frac{p}{1 + \epsilon \cos f} \quad (\text{B.4})$$

$$\phi = f - f_0 - 2\pi t \quad (\text{B.5})$$

where:

p = the semi-latus rectum

f_0 = the true anomaly at launch

From MacMillan's expansion

$$f = M + 2\epsilon \sin M + \frac{5}{4} \epsilon^2 \sin 2M + \frac{\epsilon^3}{12} (13 \sin 3M - 3 \sin M) + \dots \quad (\text{B.6})$$

where M is the mean anomaly defined by

$$M = \frac{2\pi(t-\tau)}{a^{3/2}} \quad (\text{B.7})$$

Here τ is the time of perihelion passage deduced from Kepler's equation

$$\tau = \frac{a^{3/2}}{2\pi} (E_0 - \epsilon \sin E_0) \quad (\text{B.8})$$

E_0 is the eccentric anomaly at launch and is given, from its definition, by

$$E_0 = 2 \tan^{-1} \left[\sqrt{\frac{1-\epsilon}{1+\epsilon}} \tan \frac{f_0}{2} \right] \quad (\text{B.9})$$

The true anomaly at launch, f_0 , is found from Equation (B.4) when

$t = 0$ and $\rho = 1$ a.u.

$$f_0 = \cos^{-1} \left(\frac{p-1}{\epsilon} \right) \quad (\text{B.10})$$

The orbital parameters a , p , and ϵ are expressed in terms of the initial velocity components

$$a = \frac{2}{1 - V_x^2 - (V_y + 1)^2} \quad (\text{B.11})$$

$$p = (1 + V_y)^2 \quad (\text{B.12})$$

$$\epsilon = \sqrt{1 + [V_x^2 + (1 + V_y)^2 - 2] (1 + V_y)^2} \quad (\text{B.13})$$

Direct substitution into (B.4) and (B.5) gives ρ and ϕ as functions of the time and initial velocity with an error less than ϵ^4 which for typical Mars' trips is about 0.01, the same order of accuracy expected by assuming Earth's orbit to be circular. To compare the coordinates given by these equations with those given by (A.11) two special transfers are considered. In the first case let the initial velocity be $V_x = 0$ and $V_y = 0.1$ EMOS unit. The coordinates ρ and ϕ as deduced by both models are shown in Figure 20a as a function of the time. A crossplot of ρ vs. ϕ is shown in Figure 20b for both models. The positional accuracy is seen to deteriorate sharply after about one half year. For the second case let the magnitude of the initial velocity be 0.4 EMOS unit but let the direction be such that the orbital period of exactly one year is produced by each model. This requires that $V_x = -0.392$, $V_y = -0.08$ for the standard model and $V_x = -0.4$, $V_y = 0$ for the simple model. The coordinates are plotted for both models in Figure 21a, b and c. The coordinates do not deteriorate with time in this case but the direction of the initial velocity vector is off by 11.5 degrees for the simple model.

APPENDIX C

EXAMPLE OPTIMIZATION USING THE SIMPLE MODEL

In Appendix A the coordinates x , y , z , of a freely-falling spaceship were expressed in terms of the initial velocity by Equations (A.10) which are rewritten below.

$$\begin{aligned}x &= sV_x + rV_y \\y &= -rV_x + qV_y \\z &= sV_z\end{aligned}\tag{A.10}$$

where

$$\begin{aligned}q &= q(t) = 4\sin 2\pi t - 6\pi t \\r &= r(t) = 2 - 2\cos 2\pi t \\s &= s(t) = \sin 2\pi t\end{aligned}$$

The problem is to minimize the initial velocity for a given value of x and z to be achieved at any time, t , after launch. By trigonometric identity

$$\sin^2 2\pi t + \cos^2 2\pi t = 1\tag{C.1}$$

Combining (C.1) with the definitions

$$4s^2 + r^2 - 4r = 0\tag{C.2}$$

The problem is reduced to an exercise in the Calculus of Variations. Minimize

$$v^2 = v_x^2 + v_y^2 + v_z^2\tag{C.3}$$

Subject to constraints:

$$A: \quad x - sV_x - rV_y = 0 \quad (C.4)$$

$$B: \quad z - sV_z = 0 \quad (C.5)$$

$$C: \quad 4s^2 + r^2 - 4r = 0 \quad (C.6)$$

Use the technique of Lagrange multipliers⁽³⁸⁾ to minimize

$$F = V^2 + aA + bB + cC \quad (C.7)$$

$$\frac{\partial F}{\partial V_x} : 2V_x - as = 0 \quad (C.8)$$

$$\frac{\partial F}{\partial V_y} : 2V_y - ar = 0 \quad (C.9)$$

$$\frac{\partial F}{\partial V_z} : 2V_z - bs = 0 \quad (C.10)$$

$$\frac{\partial F}{\partial r} : -aV_y + 2c(r - 2) = 0 \quad (C.11)$$

$$\frac{\partial F}{\partial s} : -aV_x - bV_z + 8cs = 0 \quad (C.12)$$

The simultaneous solution of Equations (C.7) - (C.12) together with the three constraint equations provides the solution to the problem. From (C.8) and (C.9)

$$\frac{V_x}{V_y} = \frac{s}{r} \quad (C.13)$$

From (C.9) and (C.11)

$$c = \frac{V_y^2}{r(r - 2)} \quad (C.14)$$

Substituting for a, b, and c in (C.12)

$$-\frac{V_x^2}{s} - \frac{V_z^2}{s} + \frac{4V_y^2 s}{r(r - 2)} = 0 \quad (C.15)$$

From (C.13) and (C.15)

$$V_z^2 = V_x^2 \frac{3r + 2}{r - 2} \quad (C.16)$$

Dividing (C.5) by (C.4)

$$\frac{z}{x} = \frac{V_z}{V_x} \frac{s^2}{r^2 + s^2} \quad (C.17)$$

From (C.16) and (C.17)

$$\frac{z}{x} = \frac{s^2}{r^2 + s^2} \left(\frac{3r + 2}{r - 2} \right)^{1/2} \quad (C.18)$$

Since s and r are both functions of the time, t , Equation (C.18) gives $\frac{z}{x}$ as a function of the time. Since Equation (C.18) is transcendental a plot or an iteration is required to obtain the time corresponding to minimum V when z and x are given. From (C.4) and (C.13)

$$V_x = \frac{sx}{r^2 + s^2} \quad (C.19)$$

$$V_y = \frac{rx}{r^2 + s^2} \quad (C.20)$$

Substituting in (C.3)

$$V_{\min} = \left[\frac{x^2}{r^2 + s^2} + \frac{z^2}{s^2} \right]^{1/2} \quad (C.21)$$

Equation (C.21) gives the value of the minimum velocity,

V_{\min} , when r and s are the values found from (C.18).

When z is zero the optimum transfer is through 180 degrees

with $V_{\min} = \left| \frac{x}{L} \right|$. This is the model's representation of a

Hohman transfer. When x is zero the optimum transfer is purely a plane change, the spaceship travels through only 90 degrees, and $V_{\min} = |z|$. With both x and z non-zero the transfer angle for minimum initial velocity will vary from 90 to 180 degrees depending upon the ratio, $\frac{z}{x}$.

It is sometimes suggested that when both x and z are non-zero a Hohman transfer be made at 180 degrees and a plane change at 90 degrees. The total velocity, V^* , required for this maneuver is

$$V^* = \left| \frac{x}{4} \right| + |z| \quad (\text{C.22})$$

In order to compare these two transfers, $\frac{V_{\min}}{|x|}$ and $\frac{V^*}{|x|}$ are both plotted against the ratio, $\frac{z}{x}$, in Figure 19. Since V_{\min} is always less than V^* there is no advantage to performing the transfer in two steps.

APPENDIX D

SAMPLE MISSION ANALYSIS USING THE SIMPLE MODEL

To show the utilization of the simple model a sample mission will be analyzed. Consider the problem of selecting the best trajectory which can launch from Earth, make a close approach to Mars, and return to Earth without any additional velocity change enroute. The trip is to start in 1970 and take less than 1.5 years. The trip will be selected using only the simple model neglecting the effect of the small perturbation due to the close approach at Mars, and the results will be compared with the best trajectory selected from data computed with the accurate two-body model which includes the slight help obtained from the rotation of the relative velocity vector at Mars. (The smaller mass of Mars makes the perturbation small in comparison with a close approach to Venus.)

The problem is to find a trajectory which will reach Mars and return to Earth within 1.5 years. From the standpoint of Equations (A.11) it is necessary to find the values of t , less than 1.5, which allow $\rho - 1$ and ϕ to vanish simultaneously. The requirement becomes

$$\frac{V_x}{V_y} = -\frac{r}{s} = \frac{q}{r} \quad (D.1)$$

which reduces to

$$4(1 - \cos 2\pi t) = 3\pi t \sin 2\pi t \quad (D.2)$$

There are solutions of (D.2) at $t = 0$, $t = 1.0$, and $t = 1.41$ years. The $t = 0$ solution is trivial. The $t = 1.0$ solution corresponds to a one year orbit and is the same trajectory discussed in Appendix B requiring a

minimum launch velocity of 0.4 EMOS to reach Mars. The solution of (D.1) when $\bar{t} = 1.41$ gives $q = -24.35$, $r = 3.67$, $s = 0.55$ and $V_x = -6.67 V_y$. The maximum value of ρ occurs when $t = 0.705$, half way through the trip. At that time $\phi = 0$ which means that if Mars is passed at the date of the current opposition ($\phi = 0$) then the launch velocity will be a minimum. The approximate coordinates of Mars on Julian Date 244 1175, the date of the 1971 opposition, are $\rho = 1.4$ and $z = -0.04$. Substituting these values back into Equations (A.11) when $t = 0.705$ gives the launch velocity components as $V_x = -0.298$, $V_y = 0.045$, $V_z = 0.047$. Subtracting the time to Mars in days from the opposition date gives the Julian Date of launch as 244 0920. The velocity components upon return are the same as at launch except for two sign changes.

At this point the simple model has predicted all the parameters of the proposed mission. It is necessary only to compute accurately a relatively few trajectories from Earth to Mars and from Mars to Earth at dates near those predicted by the simple model. In this case the computed data indicate the best trip is near the same dates but requires slightly less velocity than predicted by the simple model. The parameters of the mission found using data from the accurate two-body model are compared with the parameters predicted solely by using the simple model in Table D.1.

In Reference (4) trips of this class are called "symmetric" round trips and are analyzed using the standard model. The author states, "In this case, however, Lambert's Equation must be inverted for the solution, and the use of a digital computer is dictated." This means that if the standard model is used then the computer is required not only to compute

Table D.1 Parameters for the 1.5 year trip.

<u>Parameter</u>	<u>Simple Model</u>	<u>Accurate Two-body Model</u>
Launch date	244 0920	244 0930
Launch Velocity	0.304	0.238
V_x	-0.298	-0.234
V_y	0.045	0.021
V_z	0.047	0.037
Mars Arrival Date	244 1175	244 1180
Earth Return Date	244 1430	244 1440
Earth Arrival Velocity	0.304	0.295
V_x	0.298	0.294
V_y	0.045	0.005
V_z	-0.047	-0.033

the trajectory but also to find it originally. The value of the simple model over the standard model should be evident.

APPENDIX E

THE FORTRAN PROGRAM FOR BI-ELLIPTICAL TRANSFERS

The contents of this appendix consists of a copy of the FORTRAN program used to compute bi-elliptical transfers and a sample of the output data which the program produces. The meaning of the major FORTRAN variables is as follows:

THB	=	the heliocentric angle between perihelion and the destination planet at arrival (this angle is referred to as π_{SM} in Chapter 7)
TJL	=	the time of launch in Julian date - 244 0000 days
GF1L	=	the longitude in degrees of the launch planet at launch
TJA	=	the time of arrival in Julian date - 244 0000 days
GF2A	=	the longitude in degrees of the arrival planet at arrival
V13RL	}	the radial, circumferential, out-of-plane, and total components respectively in EMOS units of the hyperbolic velocity after escape of the vehicle relative to the launch planet in the frame defined by the launch planet's orbit at launch
V13GL		
V13ZL		
V13TL		
V23RA	}	the radial, circumferential, out-of-plane, and total components respectively in EMOS units of the hyperbolic velocity prior to capture of the vehicle relative to the arrival planet in the frame defined by the arrival planet's orbit at arrival
V23GA		
V23ZA		
V23TA		
DV	=	the velocity increment in EMOS units required at perihelion

RP3 = the radial distance of the vehicle from the Sun in
a.u. at perihelion

GFP3 = the longitude in degrees of the vehicle at perihelion

BIELLIPTIC TRANSFER PROGRAM (THB = 150 DEGREES)

```

PI=3.1415927
1 READ 9
9 FORMAT (50H
PRINT 9
REAC 10, ETL, ETA, DT, PLA, A1, E1, TJP1, GFP1
10 FORMAT (XF10.5)
THE 221 H31 F31 ORG31 H331 HCOL H+OR
READ 11, A2, E2, TJP2, GFP2, GEN, AINC
11 FORMAT (6F10.5)
C ETL AND ETA ARE THE EARLIEST LAUNCH AND ARRIVAL DATES COMPUTED
C DT = ITERATION INTERVAL
C PLA = NO. OF PLANET THAT HAS TIME ITERATED FIRST
C 1. = LAUNCH PLANET 2. = ARRIVAL PLANET
C A = SEMIMAJOR AXIS
C E = ECCENTRICITY
C TJP = JULIAN DATE OF PERIHELION PASSAGE
C GFP = LONG. OF PERIHELION (RELATIVE TO F = FIRST POINT OF ARIES)
C GEN = LONG. OF THE ASCENDING NODE OF ARRIVAL PLANET REL. TO LAUNCH
C PLANET PLANE
C AINC = INCLINATION ANGLE BETWEEN THE TWO PLANET ORBITS
C I, J, K, M ARE ITERATION COUNTERS
C K = 0
C M = 0
2 I = 0
C J = 0
PRINT 44
44 FORMAT (130H0 TJL GF1L TJA GF2A V13RL V13GL V13ZL
1 V13TL V23RA V23GA V23ZA V23TA DV AT PERIHELION
2RP3 GFF3 )
TJL = ETL
TJA = ETA
3 TLA = TJA - TJL
TFY = TLA/365.25
C TFY = TIME OF FLIGHT IN YEARS
ANCM1 = (TJL-TJP1)*2.*PI/(SQRT((A1**3)*365.25)
F1 = ANCM1 + 2.*E1*SINE(ANCM1) + 1.25*E1*E1*SINE(2.*ANCM1)
R1 = (A1*(1.-E1*E1))/(1.+E1*COSE(F1))
G1 = GFP1*PI/180. + F1
VS1L = SQRT(2./R1 - 1./A1)
GG1 = ATANF(F1*SINE(F1)*R1/(A1 - A1*E1*E1))
VS1G = VS1L*COSE(GG1)
VS1R = VS1L*SINE(GG1)
ANCM2 = (TJA - TJP2)*2.*PI/(SQRT((A2**3)*365.25)
F2 = ANCM2 + 2.*E2*SINE(ANCM2) + 1.25*E2*E2*SINE(2.*ANCM2)
R2 = (A2*(1. - E2*E2))/(1. + E2*COSE(F2))
G2 = GFP2*PI/180. + F2
VS2A = SQRT(2./R2 - 1./A2)
GG2 = ATANF(E2*SINE(F2)*R2/(A2 - A2*E2*E2))
VS2G = VS2A*COSE(GG2)
VS2R = VS2A*SINE(GG2)
SINTH = SINE(G2 - G1)
COSTH = COSE(G2 - G1)
IF (SINTH) 14, 15, 14
15 PRINT 10
16 FORMAT (50H TRANSFER ANGLE = 150 DEGREES
GC TC 100

```

BIELLIPTIC TRANSFER PROGRAM (THB = 150 DEGREES)

```

C   ROUTINE TO DETERMINE RP3 FROM GIVEN TIME OF FLIGHT
14  THB = 5.*PI/6.
    COSTHB = CCSF(THB)
    COSTHA = CCSF(G2-G1-THB)
    THA = ACOSF(COSTHA)
    RP3 = .7
    IT = 0
89  IF(RP3-R1*CCSTHA) 90,90,92
90  PRINT 91
91  FORMAT (50H      HYPERBOLIC VELOCITY REQUIRED
    GO TO 100
92  EA = (R1-RP3)/(RP3-R1*COSTHA)
    EB = (R2-RP3)/(RP3-R2*COSTHB)
    IF (1.-EA) 90,90,93
93  IF (1.-EB) 90,90,94
94  PERA = SQRTF((RP3/(1.-EA))**3)
    PERB = SQRTF((RP3/(1.-EB))**3)
    ECA = 2.*ATANF(TANF(THA/2.)*SQRTF((1.-EA)/(1.+EA)))
    ECB = 2.*ATANF(TANF(THB/2.)*SQRTF((1.-EB)/(1.+EB)))
    TCFR = (ECA-EA*SINF(ECA))*PERA/(2.*PI) +
    1 (ECB-EB*SINF(ECB))*PERB/(2.*PI)
    IF (ABSF(TCFR-TFY)-.0001) 400,95,95
95  IT = IT + 1
    IF (IT - 30) 299,98,98
98  PRINT 99
99  FORMAT (50H      ITERATION FOR RP3 DOES NOT CONVERGE
    GO TO 100
299 IF (IT-2) 300,301,302
300 TX = TCFR
    RX = .7
    RP3 = .8
    GO TO 89
301 TY = TCFR
    RY = .8
    GO TO 303
302 RX = RY
    RY = RP3
    TX = TY
    TY = TCFR
303 RP3 = RY+(TFY-TY)*(RY-RX)/(TY-TX)
    GO TO 89
400 SQA = SQRTF(RP3*(1.+EA))
    SQB = SQRTF(RP3*(1.+EB))
    VS3GL = SQA/R1
    VS3GA = SQB/R2
    VS3RL = EA*SINF(G1-G2+THB)/SQA
    VS3RA = EB*SINF(THB)/SQB
    DV = SQB/RP3 - SQA/RP3
    V13RL = VS3RL - VS1RL
    V23RA = VS3RA - VS2RA
    V13ZL = SINF(AINC*PI/180.)*SINF(G2-GFN*PI/180.)*VS3GL/SINTH
    DOG1 = VS3GL*VS3GL - V13ZL*V13ZL
    IF (DOG1) 15,522,522
522 V13GL = SQRTF(DOG1) - VS1GL
    V23ZA = SINF(AINC*PI/180.)*SINF(G1-GFN*PI/180.)*VS3GA/SINTH
    DOG2 = VS3GA*VS3GA - V23ZA*V23ZA

```


BIELLIPTIC TRANSFER PROGRAM (THR = 150 DEGREES)

```

IF (DOG2) 15, 533, 533
533 V23GA = SQRTF(DOG2) - VS2GA
V13TL = SQRTF(V13RL*V13RL + V13GL*V13GL + V13ZL*V13ZL)
V23TA = SQRTF(V23RA*V23RA + V23GA*V23GA + V23ZA*V23ZA)
GFP3 = (G2-THR)*180./PI
C GET LONGITUDE BETWEEN ZERO AND 360 DEGREES
GFP3 = GFP3 - 1080.
IF (GFP3) 69, 70, 70
69 GFP3 = GFP3 + 360.
IF (GFP3) 69, 70, 70
70 GF1L = 180.*G1/PI - 1080.
IF (GF1L) 71, 72, 72
71 GF1L = GF1L + 360.
IF (GF1L) 71, 72, 72
72 GF2A = 180.*G2/PI - 1080.
IF (GF2A) 73, 74, 74
73 GF2A = GF2A + 360.
IF (GF2A) 73, 74, 74
74 PRINT 50C, TJJ, GF1L, TJA, GF2A, V13RL, V13GL, V13ZL, V13TL, V23RA,
1V23GA, V23ZA, V23TA, DV, RP3, GFP3
500 FORMAT(F6.C, F7.1, F8.0, F7.1, 8F8.3, F17.3, F14.3, F7.1)
100 IF (PLA - 1.5) 4, 5, 5
4 TJJ = TJJ + DT
I = I + 1
IF (I - 16) 3, 6, 6
6 ETA = ETA + DT
ETL = ETL + DT
K = K + 1
IF (K - 16) 2, 1, 1
5 TJA = TJA + DT
J = J + 1
IF (J - 16) 3, 7, 7
7 ETL = ETL + DT
ETA = ETA + DT
M = M + 1
IF (M - 16) 2, 1, 1
END(1, 1, 0, 0, 0, C, 1, C, C, 0, 0, 0, 0, 0, 0)

```

1970 BIELLIPTIC TRIPS FROM EARTH TO MARS

TJL	GFIL	TJA	GF2A	V13KL	V13GL	V13ZL	V13TL	V23RA	V23GA	V23ZA	V23TA	DV AT PERIHELION	RP3	GFP3
72C	232.3	1040.	239.3	0.024	-C.020	-C.045	0.055	C.119	-0.104	-0.010	0.158	0.106	0.907	89.3
73C	242.0	1040.	239.3	0.037	-C.046	0.116	0.130	C.143	-0.128	0.106	0.219	0.108	0.838	89.3
74C	251.6	1040.	239.3	0.038	-C.063	0.024	0.077	C.170	-0.139	0.039	0.223	0.113	0.769	89.3
75C	261.1	1040.	239.3	0.073	-C.089	0.014	0.093	C.202	-0.160	0.030	0.259	0.118	0.697	89.3
76C	270.7	1040.	239.3	-0.010	-C.113	0.009	0.115	C.235	-0.180	0.026	0.297	0.122	0.634	89.3
77C	282.2	1040.	239.3	-0.050	-C.110	0.007	0.123	C.236	-0.179	0.024	0.297	0.120	0.636	89.3
78C	289.8	1040.	239.3	-0.092	-C.110	0.006	0.144	C.238	-0.179	0.023	0.299	0.116	0.636	89.3
79C	299.3	1040.	239.3	-0.137	-C.107	0.006	0.174	C.241	-0.180	0.022	0.302	0.110	0.634	89.3
80C	308.9	1040.	239.3	-0.186	-C.103	0.005	0.213	C.245	-0.181	0.021	0.306	0.101	0.630	89.3
81C	318.4	1040.	239.3	-0.242	-C.096	0.005	0.261	C.250	-0.182	0.021	0.310	0.087	0.626	89.3
82C	328.1	1040.	239.3	-0.308	-C.087	0.005	0.320	C.254	-0.184	0.020	0.315	0.067	0.620	89.3
83C	337.7	1040.	239.3	-0.389	-C.072	0.005	0.395	C.259	-0.186	0.019	0.320	0.037	0.614	89.3
84C	347.4	1040.	239.3	-0.489	-C.048	0.006	0.492	C.264	-0.189	0.019	0.325	-0.007	0.607	89.3
85C	357.1	1040.	239.3	-0.622	-C.010	0.006	0.622	C.268	-0.191	0.018	0.330	-0.078	0.600	89.3
86C	6.9	1040.	239.3	-0.809	C.057	0.007	0.811	C.272	-0.193	0.017	0.334	-0.198	0.594	89.3

HYPERBOLIC VELOCITY REQUIRED

TJL	GFIL	TJA	GF2A	V13RL	V13GL	V13ZL	V13TL	V23RA	V23GA	V23ZA	V23TA	DV AT PERIHELION	RP3	GFP3
74C	242.0	1050.	244.6	0.026	-0.039	-C.080	0.186	C.122	-0.117	-0.109	0.201	0.104	0.895	94.6
75C	251.6	1050.	244.6	0.035	-C.045	0.066	0.087	C.147	-0.129	0.069	0.207	0.107	0.826	94.6
76C	261.1	1050.	244.6	0.031	-C.067	0.027	0.079	C.175	-0.145	0.040	0.231	0.112	0.756	94.6
77C	270.7	1050.	244.6	0.011	-C.094	0.017	0.096	C.208	-0.166	0.032	0.269	0.116	0.684	94.6
78C	280.2	1050.	244.6	-0.024	-0.105	0.013	0.109	C.225	-0.175	0.028	0.286	0.118	0.655	94.6
79C	289.8	1050.	244.6	-0.062	-C.104	0.013	0.121	C.226	-0.175	0.025	0.287	0.116	0.656	94.6
80C	299.3	1050.	244.6	-0.102	-C.101	0.009	0.144	C.229	-0.175	0.024	0.289	0.111	0.655	94.6
81C	308.9	1050.	244.6	-0.144	-0.098	0.008	0.175	C.232	-0.176	0.023	0.292	0.104	0.652	94.6
82C	318.4	1050.	244.6	-0.192	-0.093	0.008	0.214	C.236	-0.177	0.021	0.296	0.094	0.649	94.6
83C	328.1	1050.	244.6	-0.248	-C.086	0.008	0.262	C.240	-0.179	0.020	0.300	0.080	0.644	94.6
84C	337.7	1050.	244.6	-0.313	-C.075	0.008	0.322	C.245	-0.180	0.020	0.305	0.058	0.638	94.6
85C	347.4	1050.	244.6	-0.393	-C.058	0.008	0.397	C.249	-0.182	0.019	0.309	0.027	0.632	94.6
86C	357.1	1050.	244.6	-0.495	-0.031	0.009	0.496	C.253	-0.185	0.017	0.313	-0.022	0.626	94.6
87C	6.9	1050.	244.6	-0.632	C.014	0.010	0.632	C.256	-0.186	0.016	0.317	-0.099	0.620	94.6
88C	16.8	1050.	244.6	-0.824	C.095	0.013	0.834	C.259	-0.188	0.015	0.320	-0.235	0.616	94.6

HYPERBOLIC VELOCITY REQUIRED

TJL	GFIL	TJA	GF2A	V13RL	V13GL	V13ZL	V13TL	V23RA	V23GA	V23ZA	V23TA	DV AT PERIHELION	RP3	GFP3
74C	251.6	1060.	249.9	0.027	-C.106	C.384	0.399	C.125	-0.182	0.305	0.377	0.102	0.884	99.9
75C	261.1	1060.	249.9	0.032	-C.048	0.055	0.080	C.151	-0.133	0.061	0.210	0.106	0.815	99.9
76C	270.7	1060.	249.9	0.024	-C.072	0.029	0.081	C.180	-0.151	0.041	0.239	0.110	0.744	99.9
77C	280.2	1060.	249.9	-0.002	-0.099	0.020	0.101	C.214	-0.172	0.032	0.277	0.115	0.673	99.9
78C	289.8	1060.	249.9	-0.036	-C.097	0.016	0.105	C.215	-0.171	0.029	0.277	0.114	0.675	99.9
79C	299.3	1060.	249.9	-0.072	-C.095	0.013	0.120	C.217	-0.171	0.026	0.278	0.111	0.675	99.9
80C	308.9	1060.	249.9	-0.109	-C.093	0.012	0.144	C.220	-0.171	0.024	0.280	0.106	0.674	99.9
81C	318.4	1060.	249.9	-0.150	-C.089	0.011	0.175	C.224	-0.172	0.023	0.283	0.099	0.671	99.9
82C	328.1	1060.	249.9	-0.197	-C.083	0.011	0.214	C.227	-0.173	0.021	0.287	0.088	0.667	99.9
83C	337.7	1060.	249.9	-0.251	-C.075	0.010	0.262	C.231	-0.175	0.020	0.290	0.073	0.662	99.9
84C	347.4	1060.	249.9	-0.315	-C.063	0.011	0.322	C.235	-0.177	0.019	0.294	0.050	0.657	99.9
85C	357.1	1060.	249.9	-0.396	-C.044	0.011	0.398	C.238	-0.178	0.017	0.298	0.016	0.651	99.9
86C	6.9	1060.	249.9	-0.499	-C.013	0.013	0.499	C.241	-0.180	0.016	0.302	-0.036	0.646	99.9

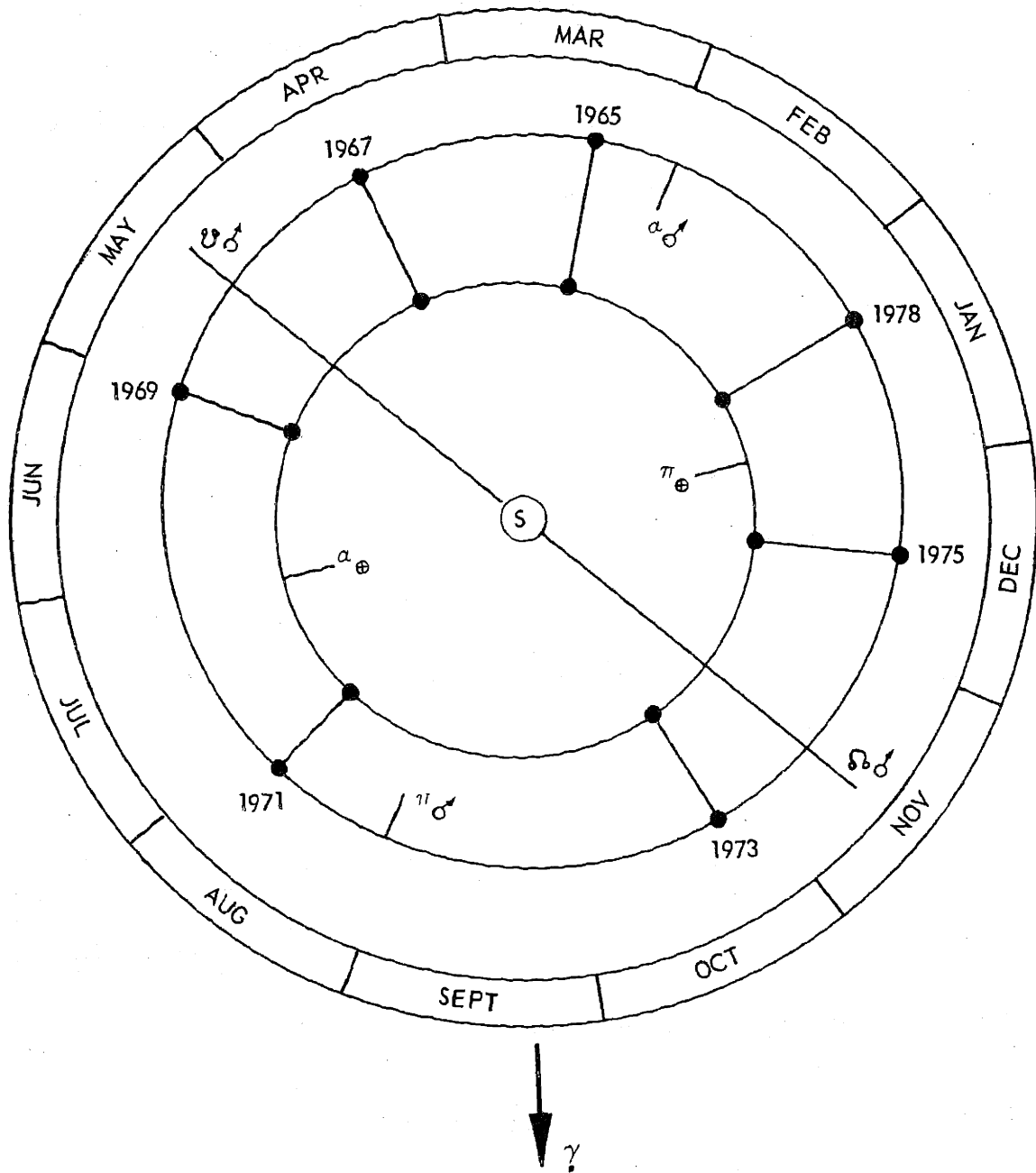


Fig. 1a. Oppositions of Mars 1965-1978.

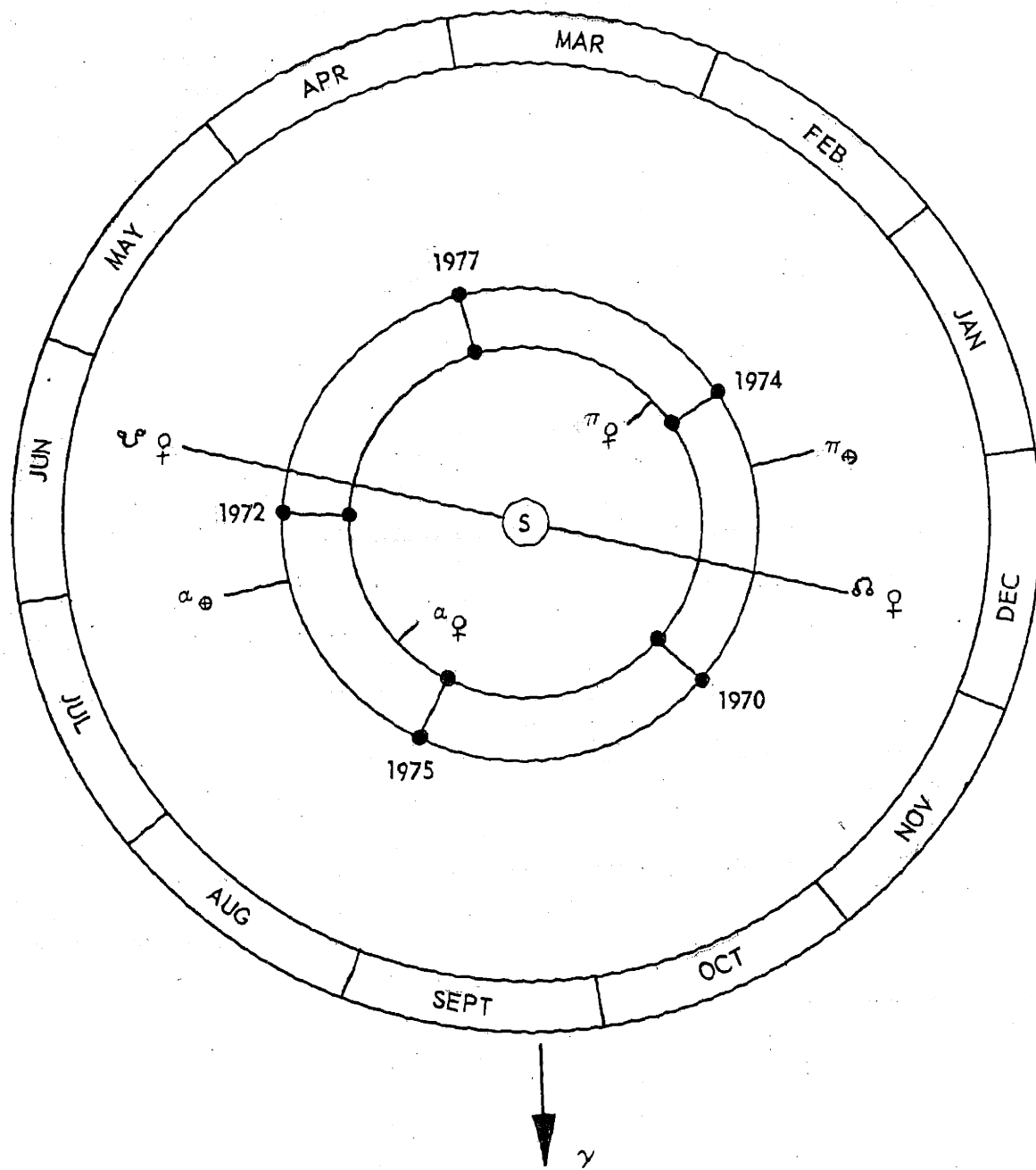


Fig. 1b. Conjunctions of Venus, 1970-1977.

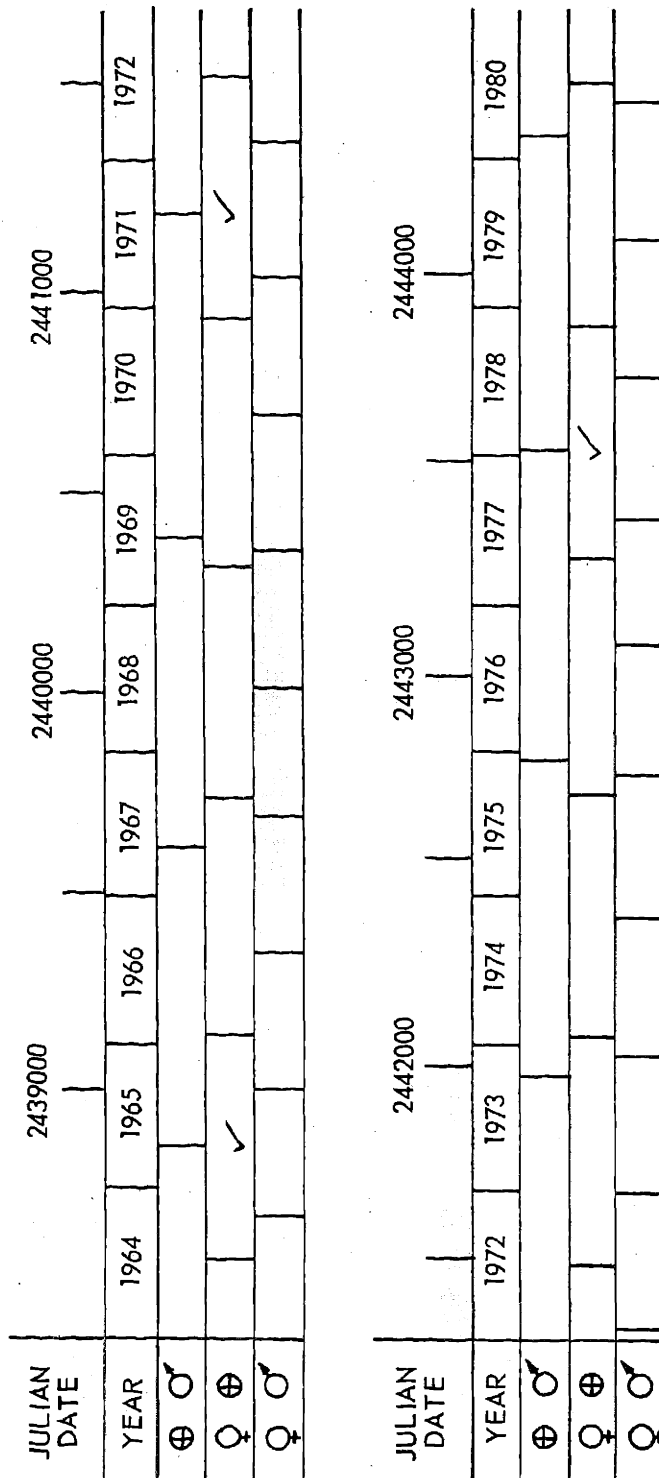


Fig. 2. Dates of planetary alignment, 1964-1980.

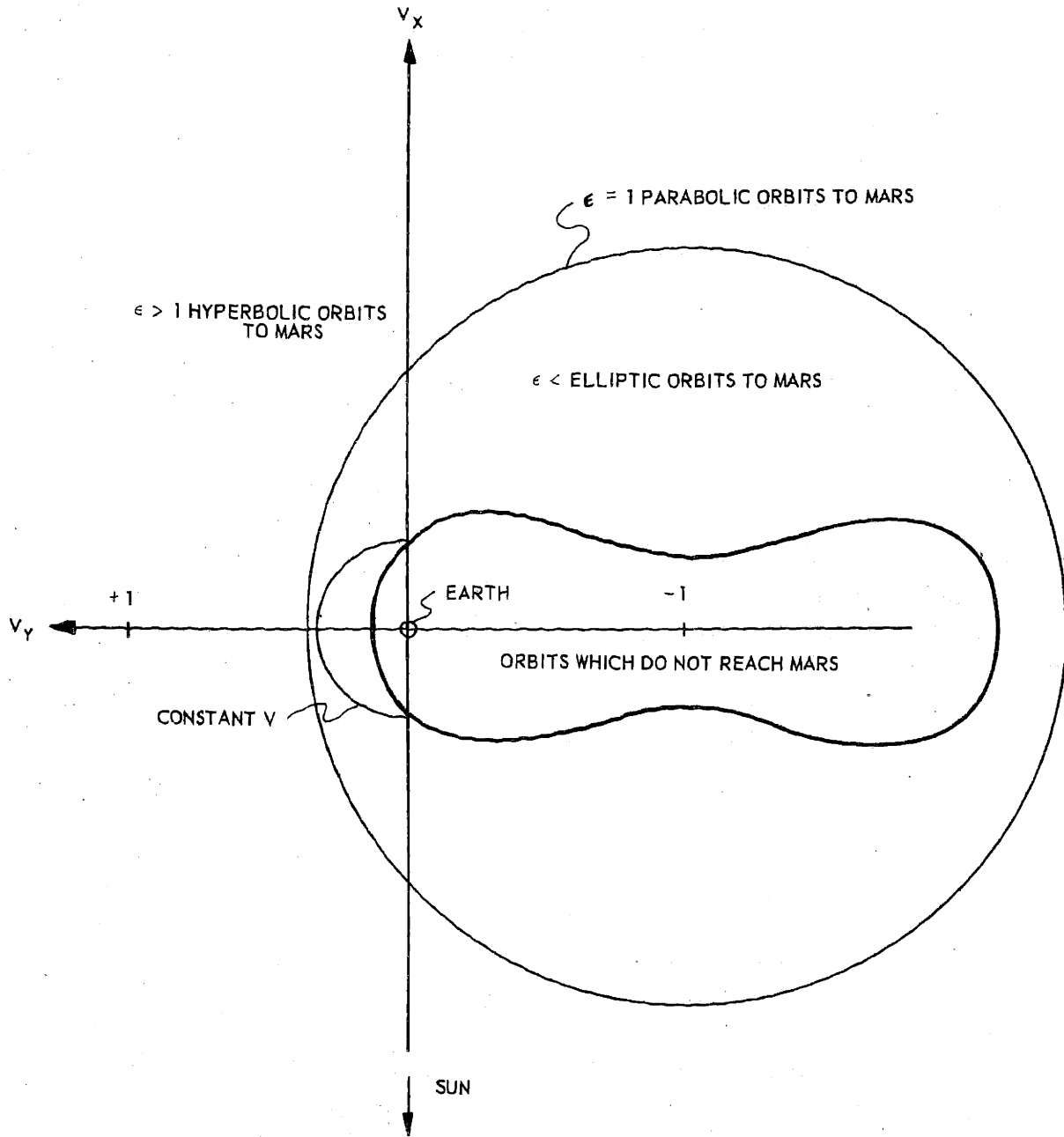


Fig. 3a. The $V_x - V_y$ plane.

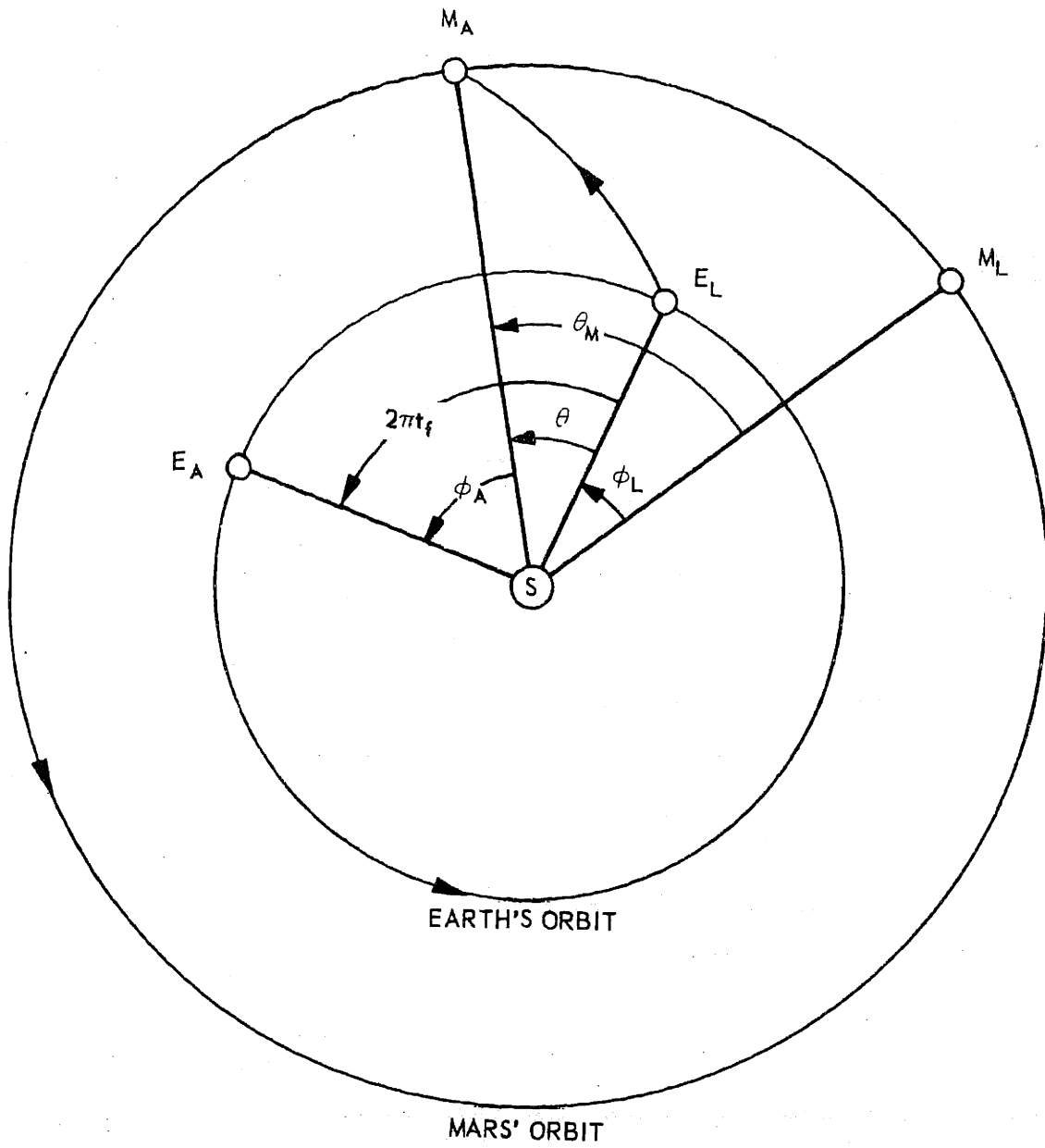


Fig. 3b. The orbital parameters of all possible orbits to Mars.

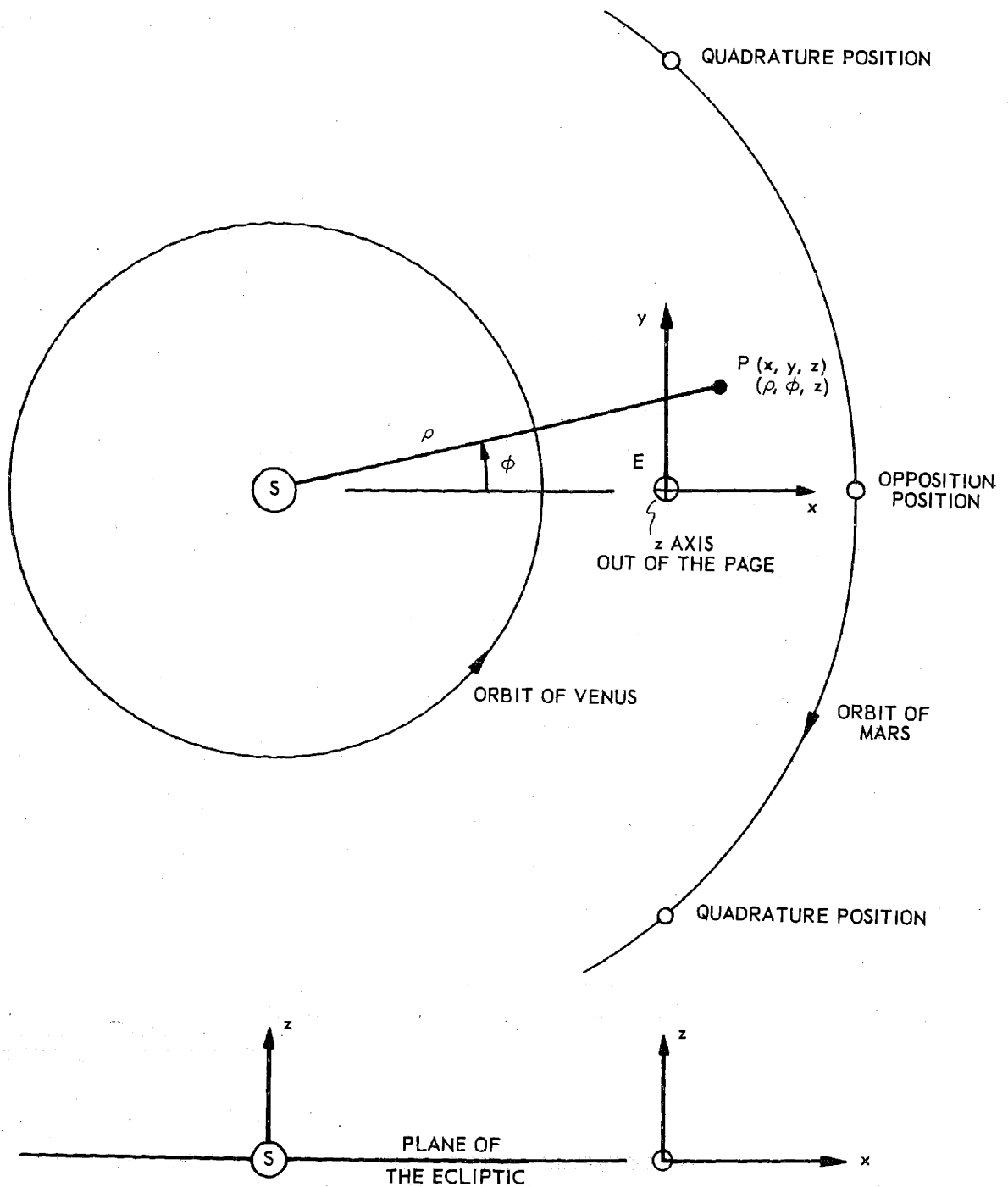


Fig. 4. The simplified approximate model.

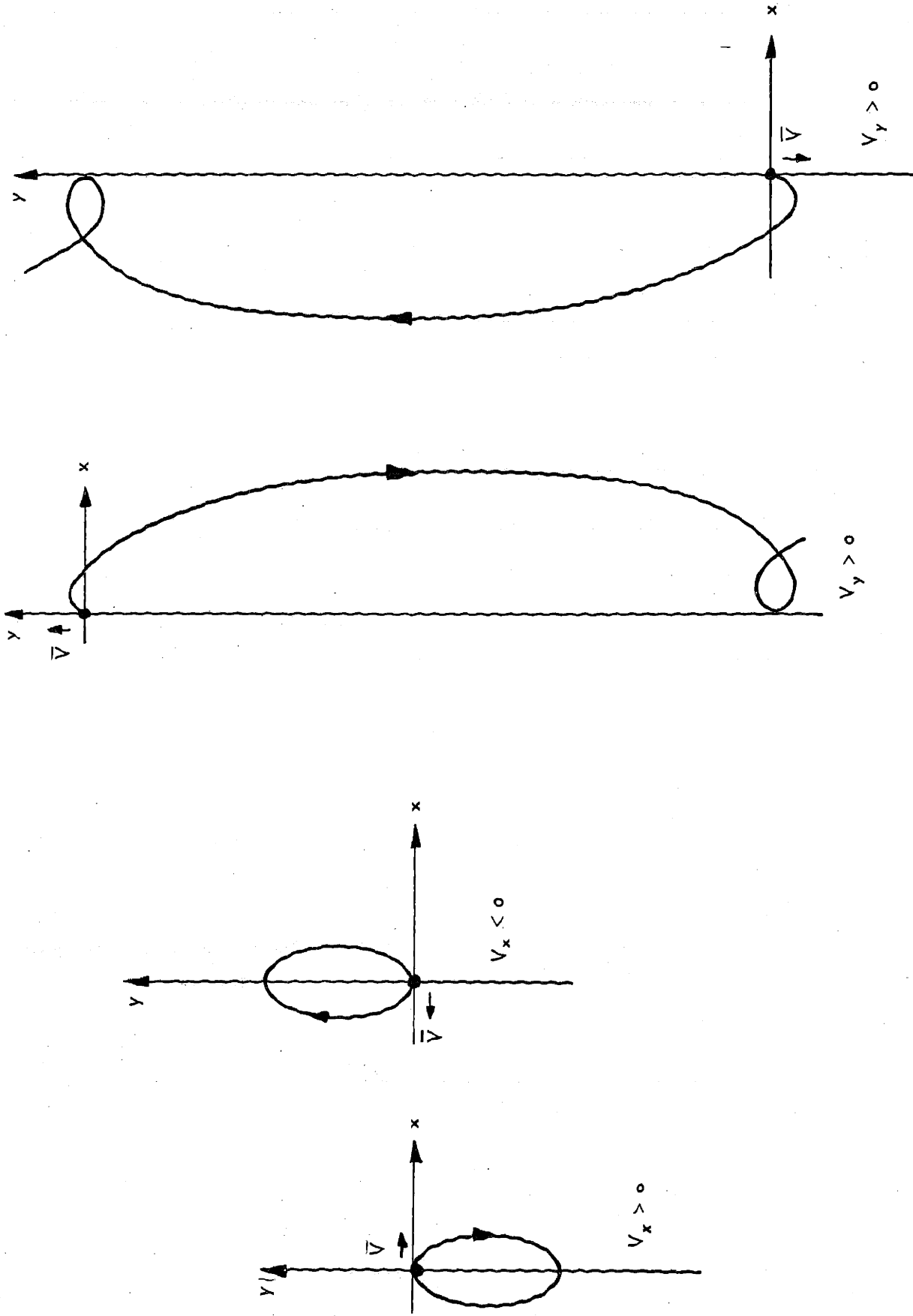


Fig. 5. Resulting orbits for initial radial velocity, V_x , and initial circumferential velocity, V_y .

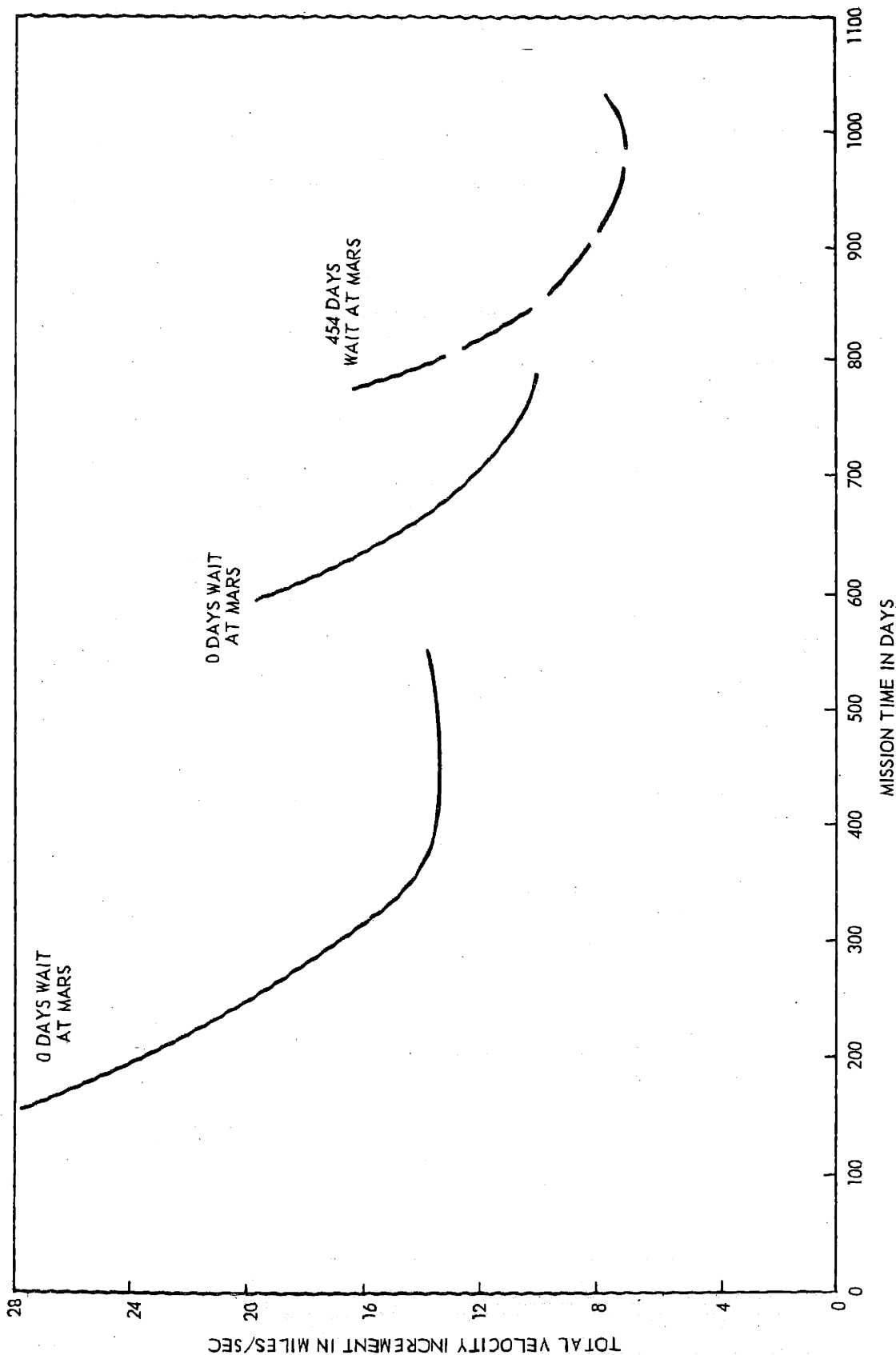


Fig. 6. Required velocity increment vs. total mission time ref. (12).

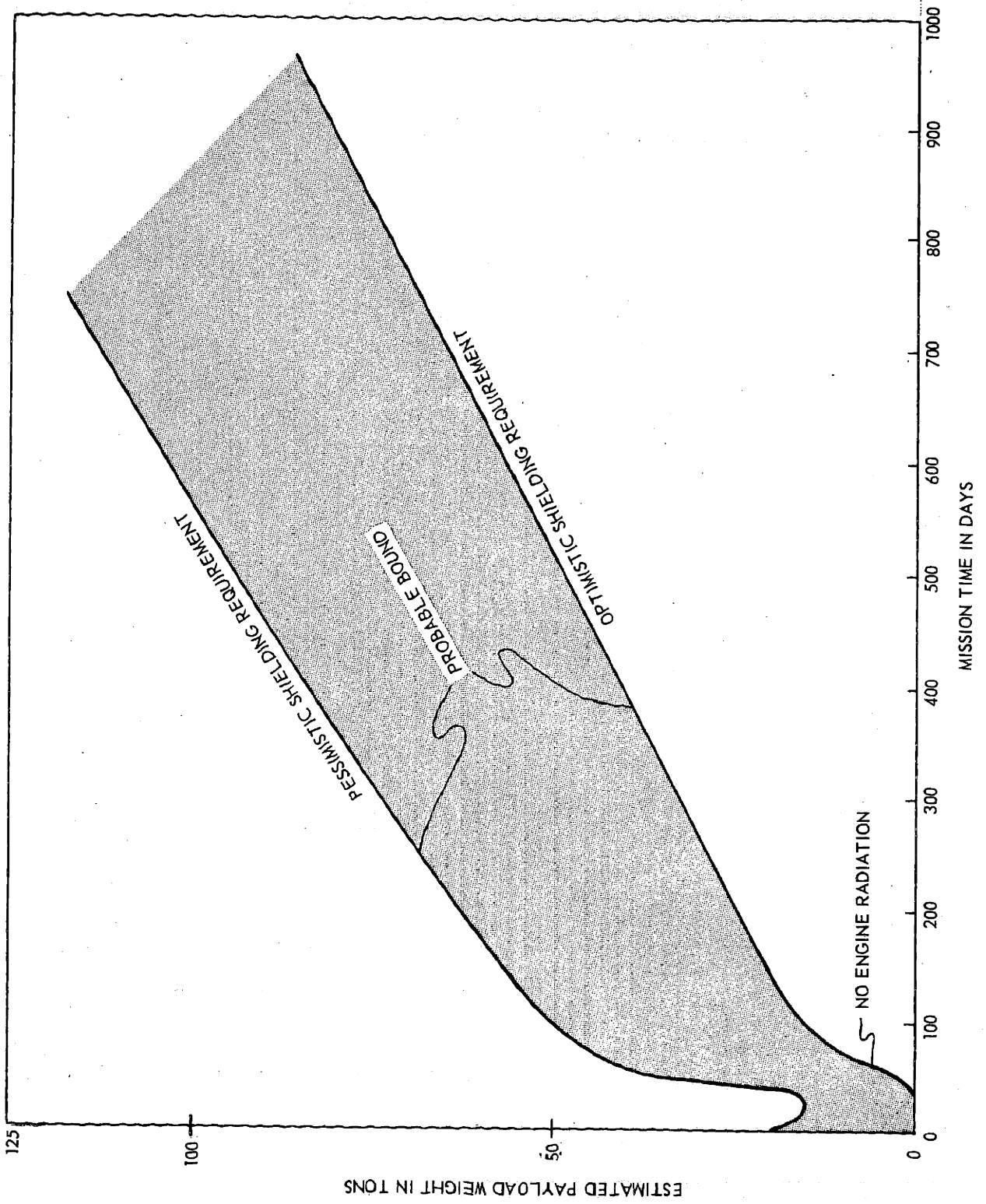


Fig. 7. Estimated payload weight vs. total mission time.

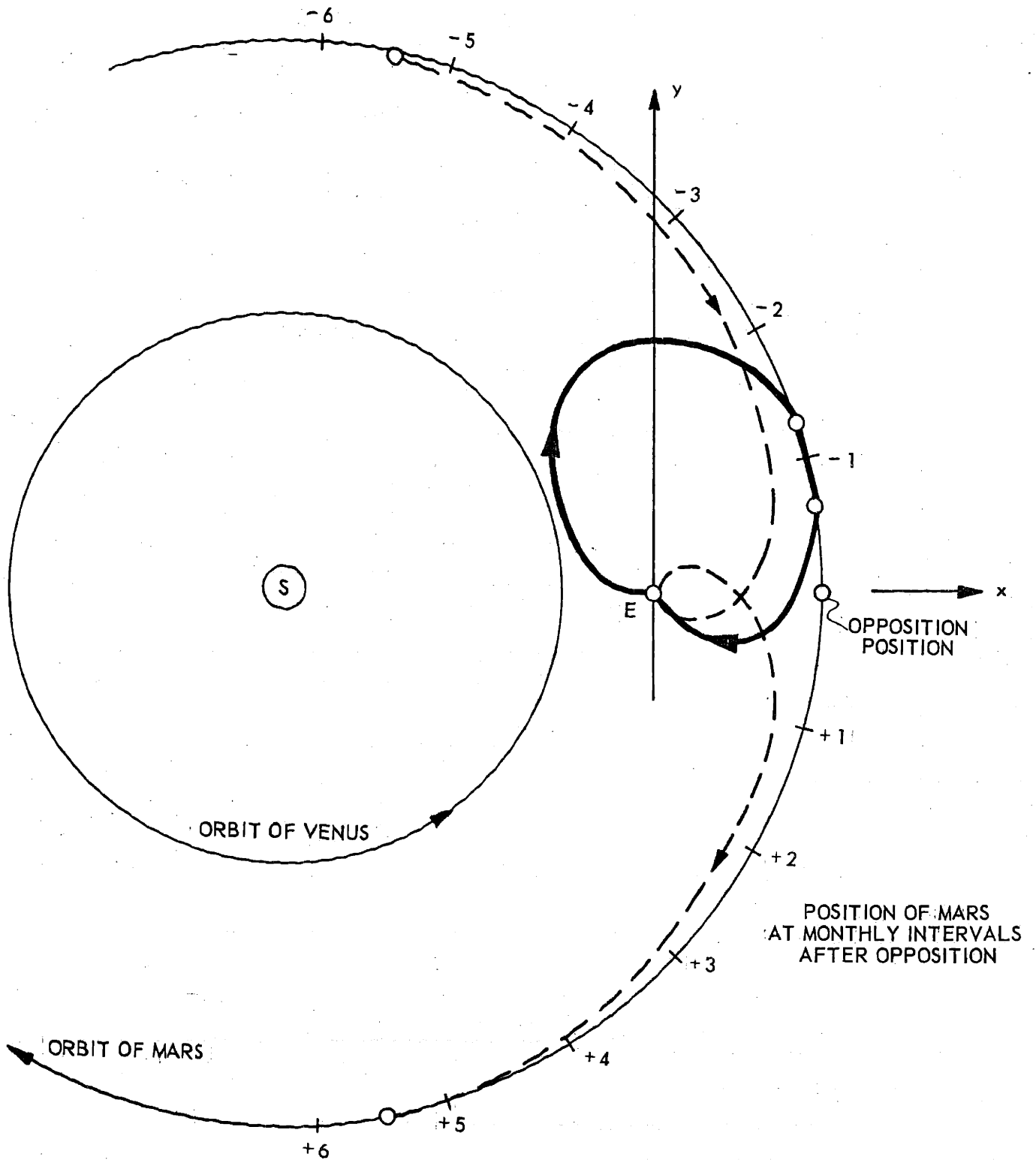


Fig. 8. An Earth-centered rotating frame.

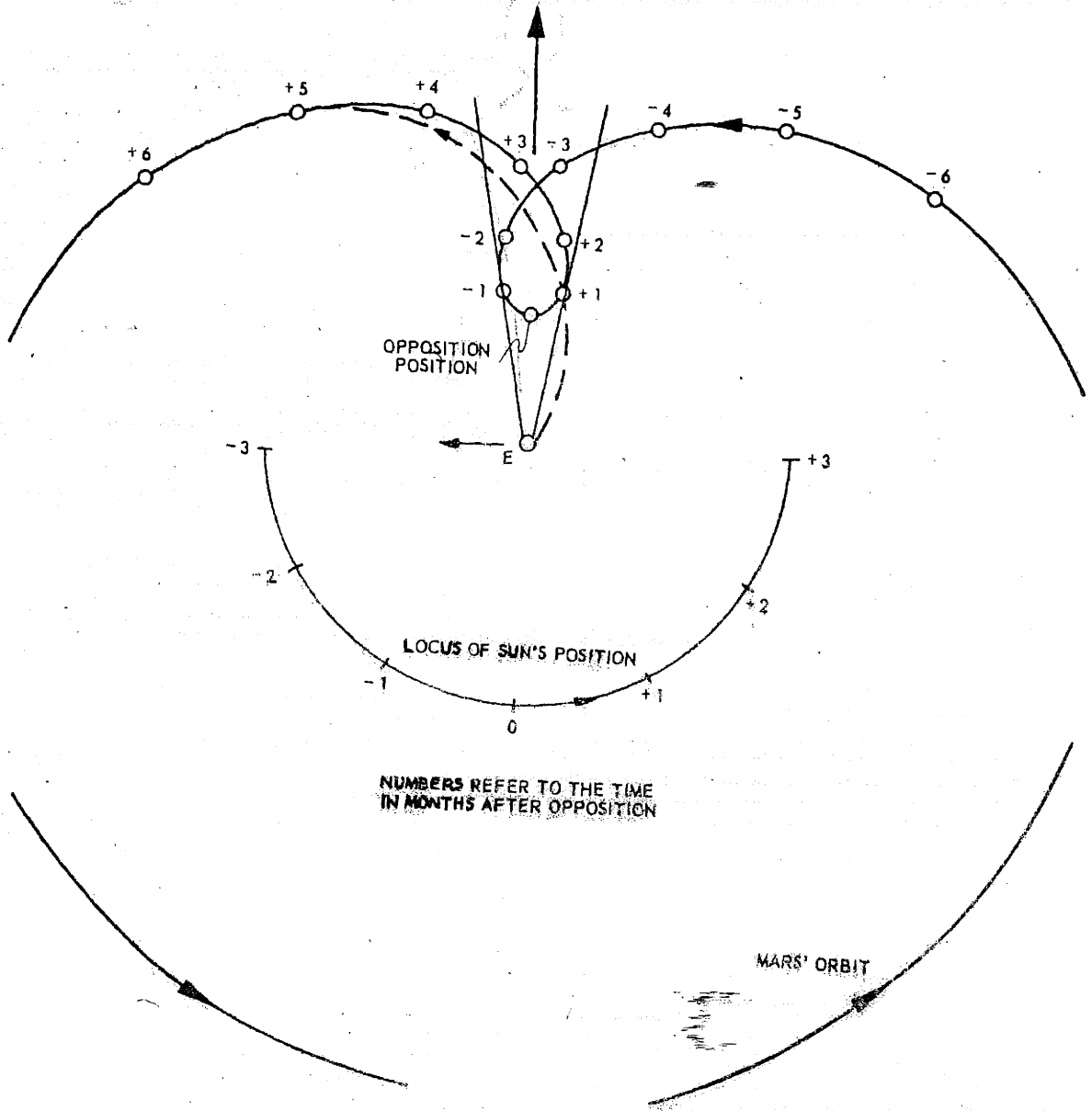


Fig. 9. An Earth-centered inertial frame.

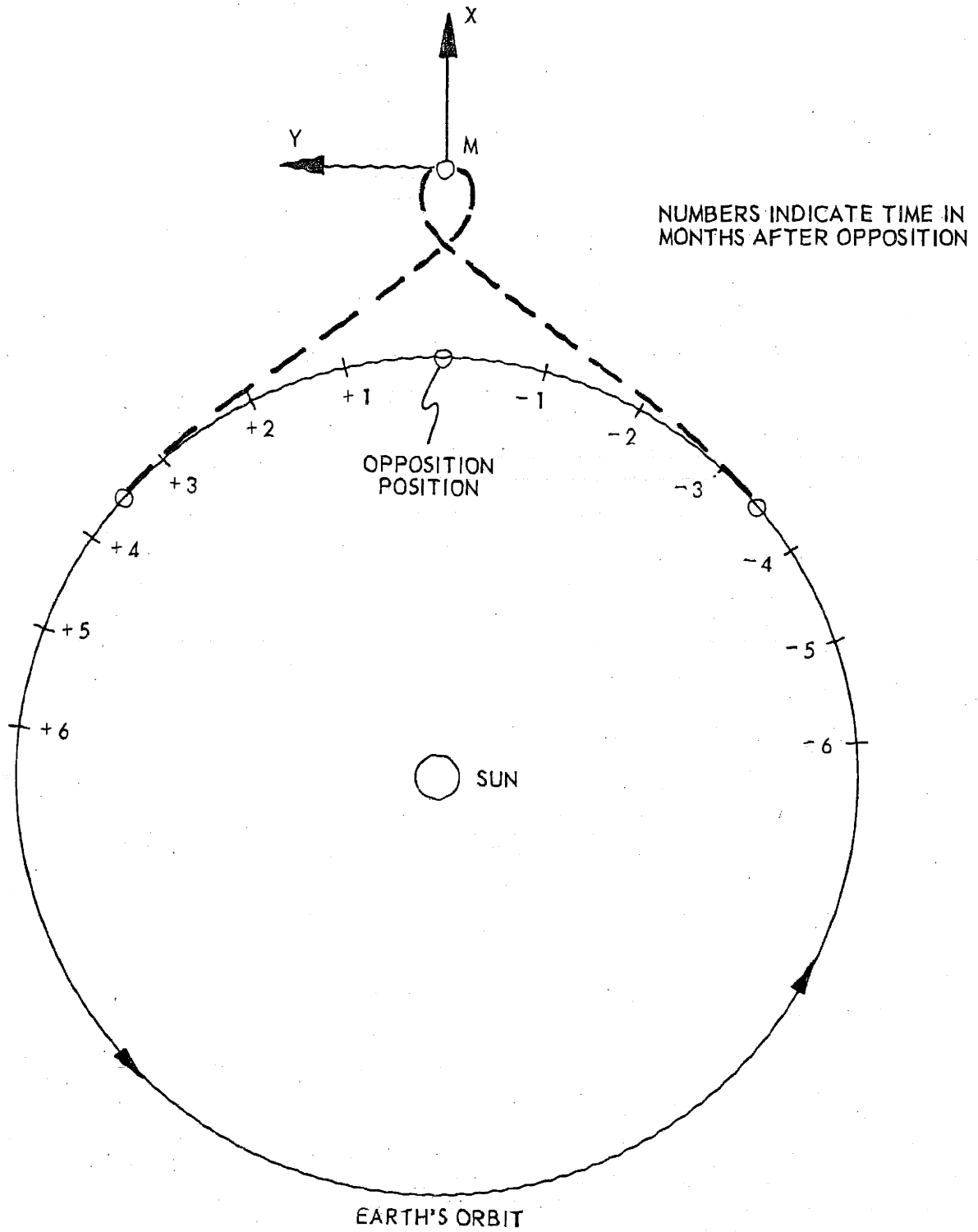
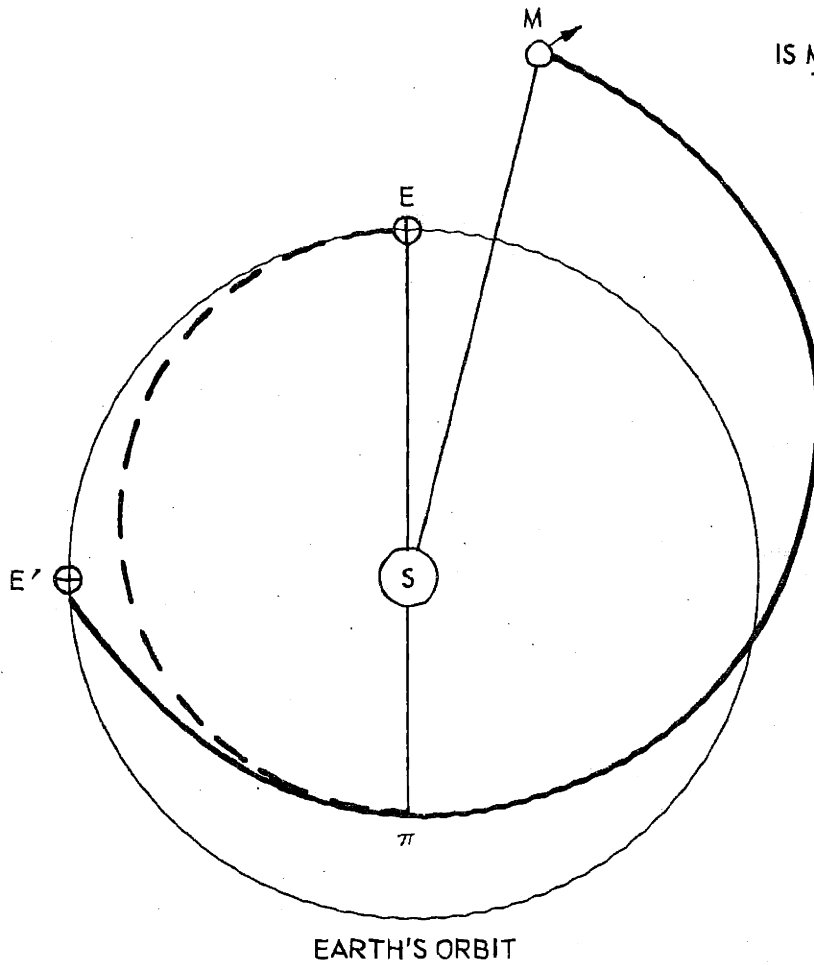


Fig. 10. A Mars-centered rotating frame.



THE TRANSFER $E \pi M$
IS MORE ECONOMICAL THAN
THE TRANSFER $E' \pi M$

Fig. 11. Bi-elliptical transfer to Mars.

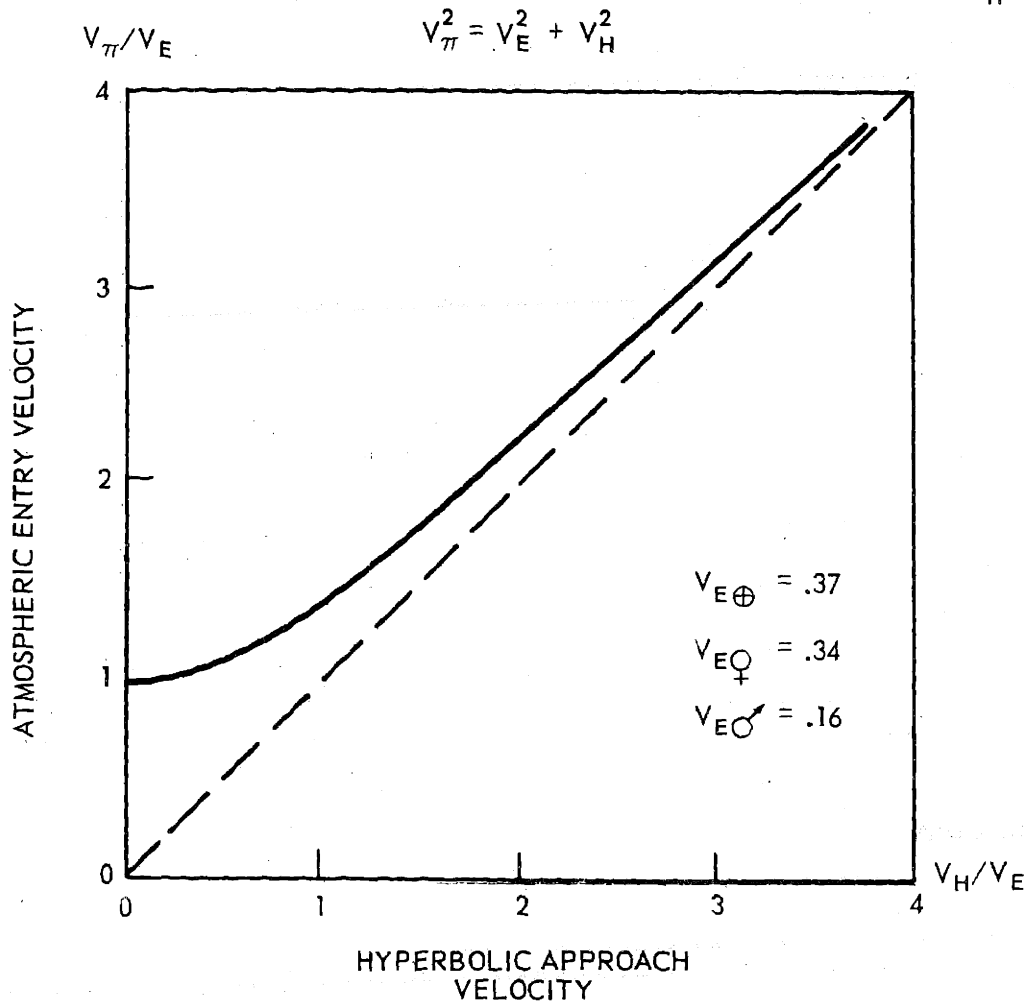
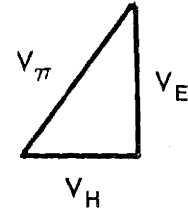


Fig. 12. Atmospheric entry velocity vs. hyperbolic approach velocity.

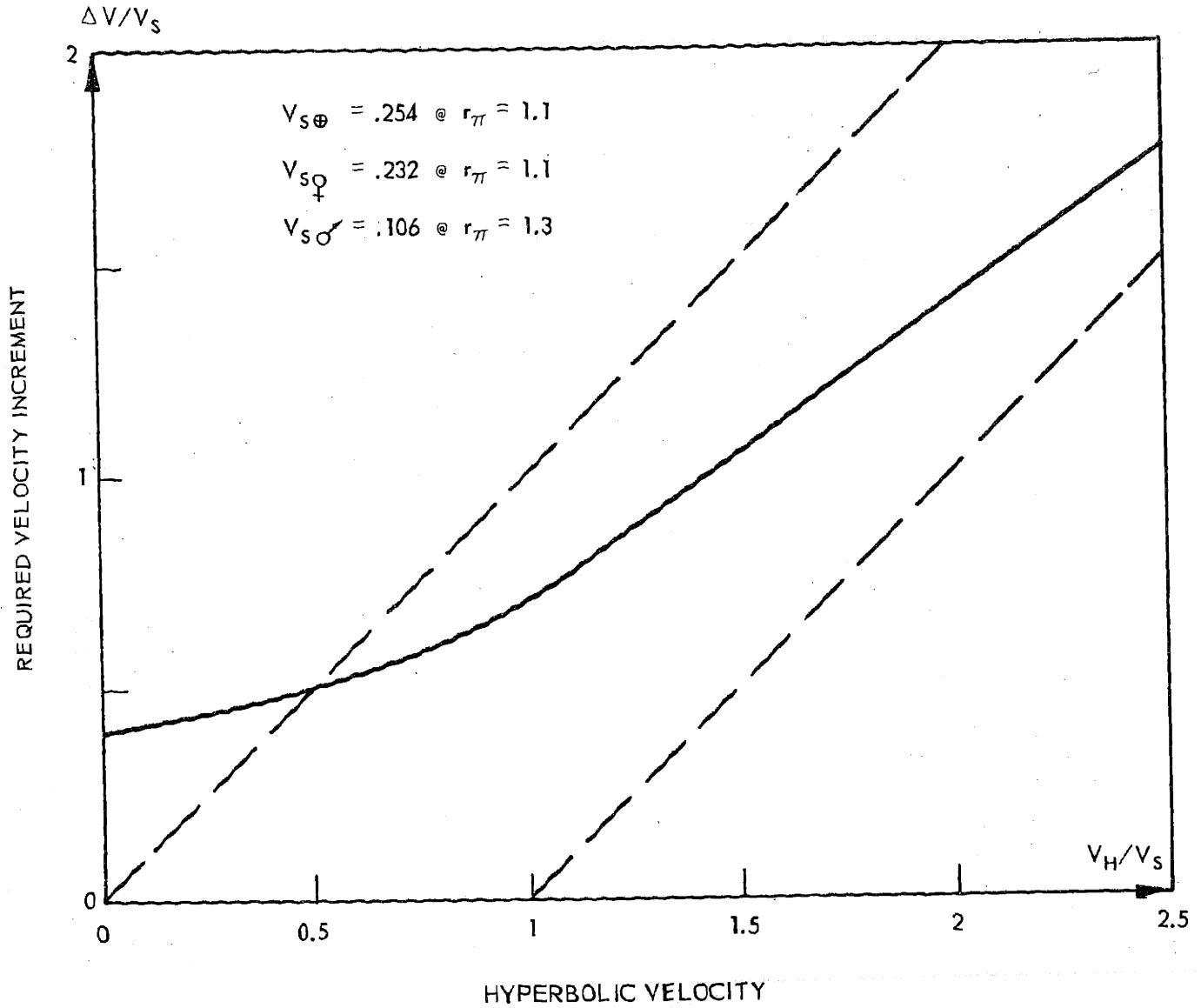


Fig. 13. Required velocity increment entering or departing a parking orbit vs. hyperbolic velocity.

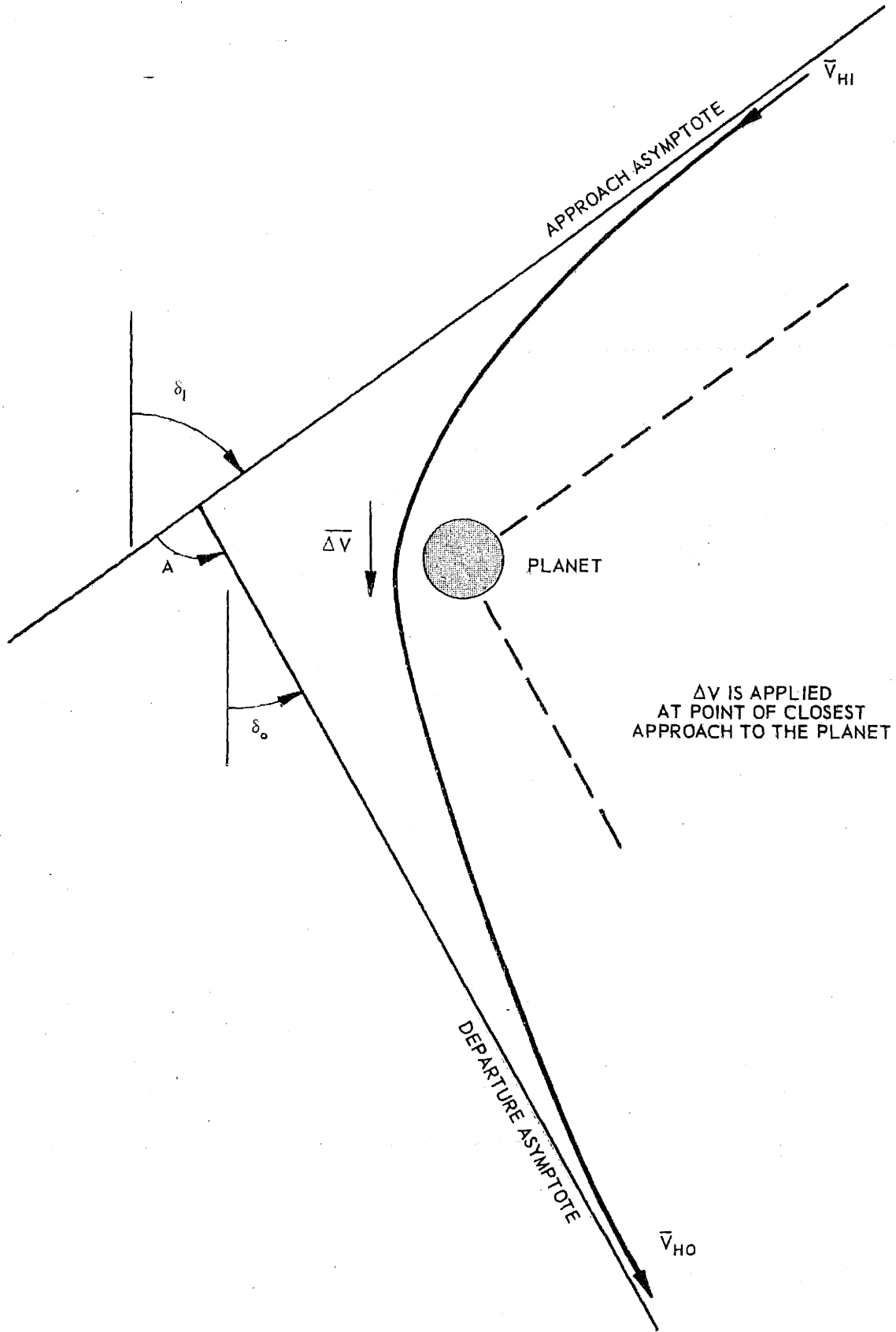
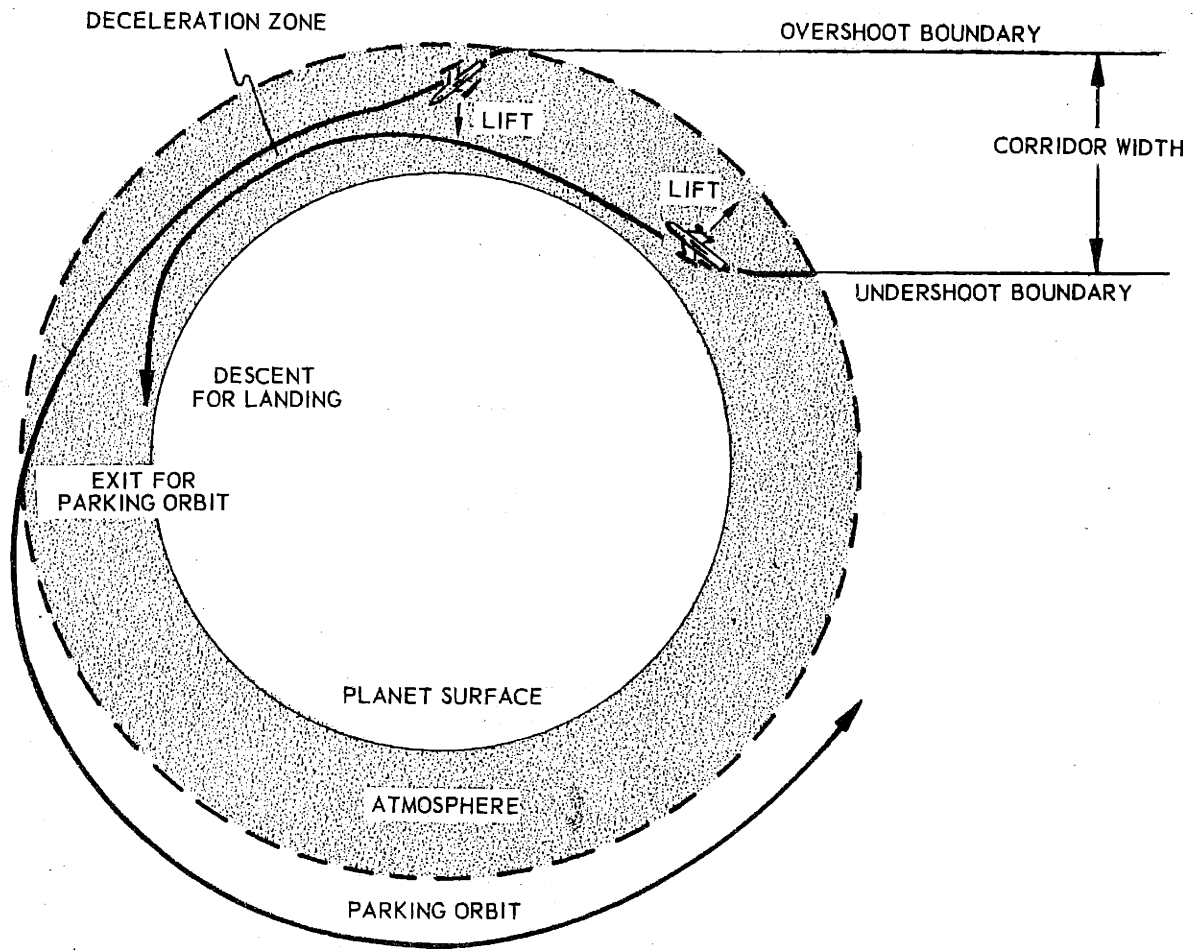


Fig. 14. The fly by maneuver.



NOTE: NOT DRAWN TO SCALE

Fig. 15. Atmospheric braking corridor.

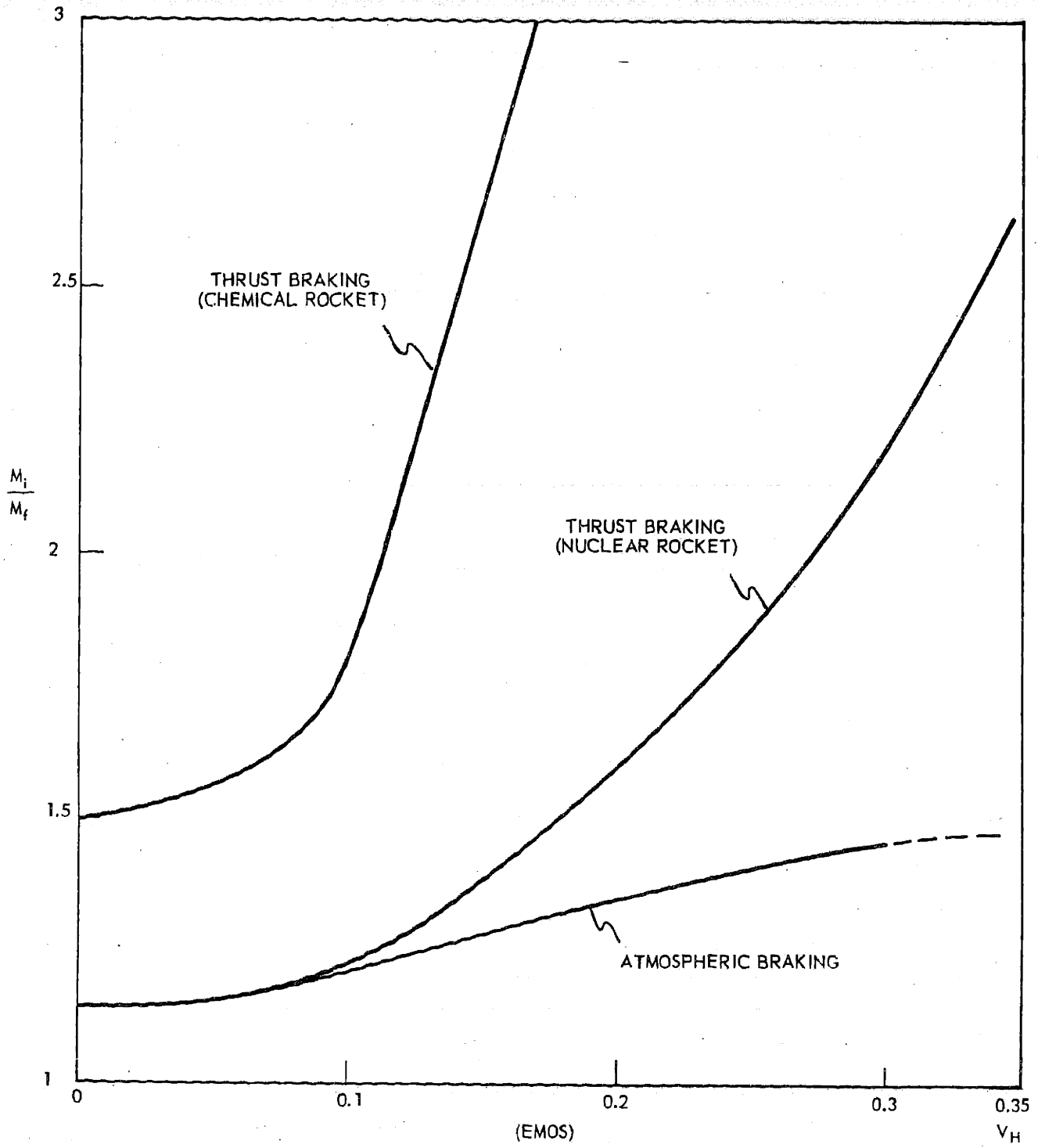


Fig. 16. Mass ratio vs. hyperbolic approach velocity to establish parking orbit at Mars.

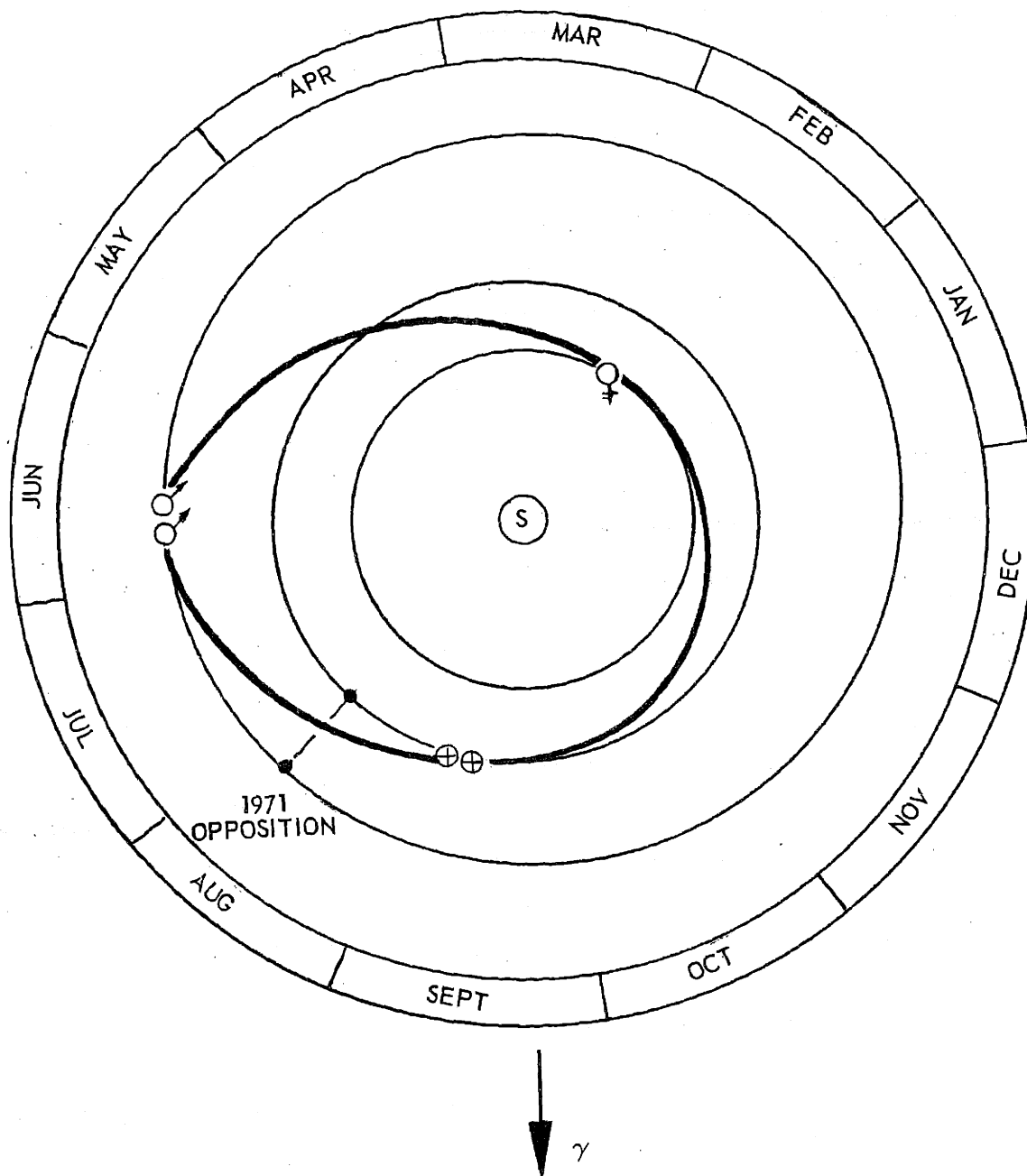


Fig. 17. Venus encounter enroute to Mars, 1970-71, 370 day mission - 10 days on Mars.

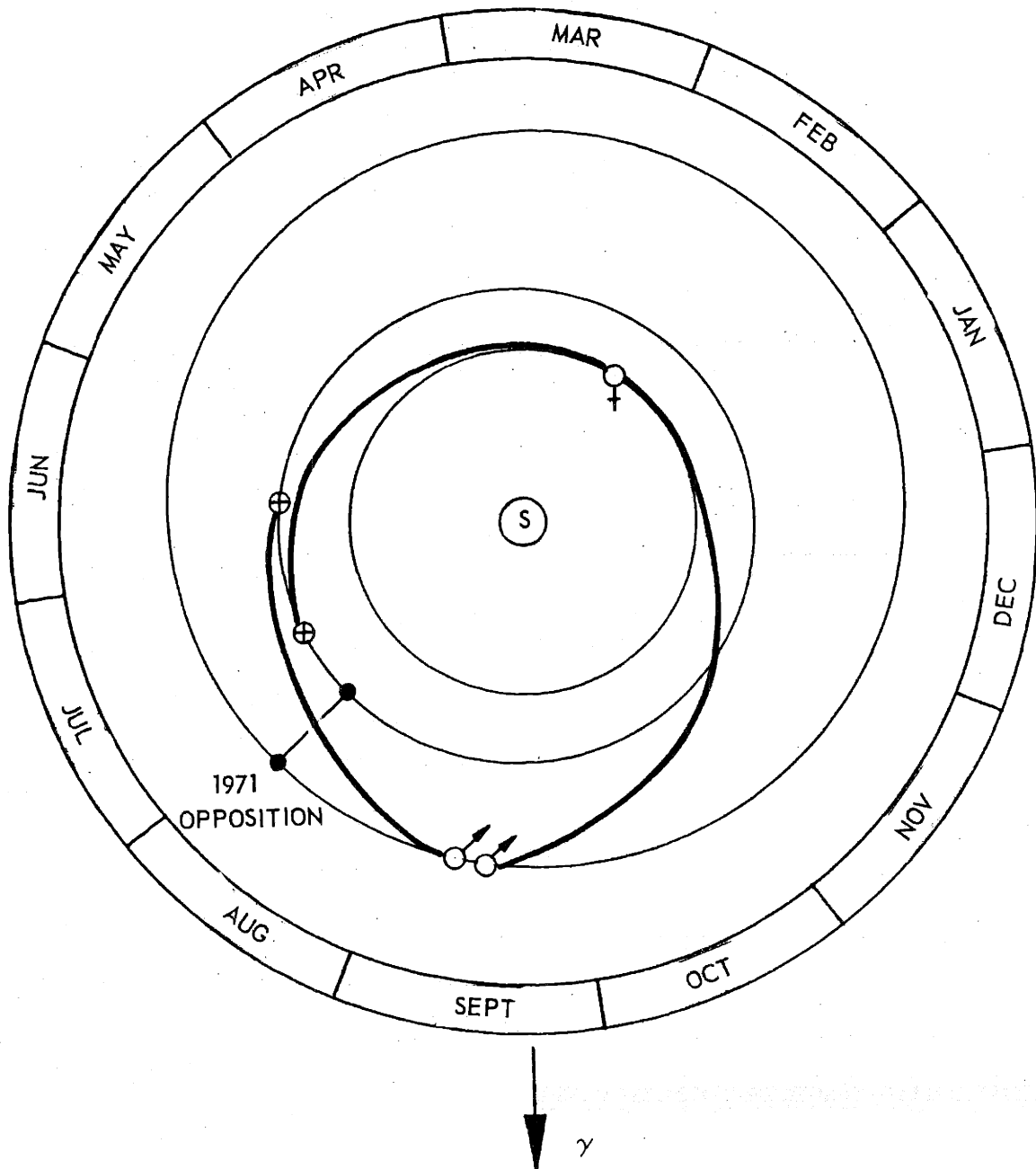


Fig. 18. Venus encounter upon return from Mars, 1971-72, 400 day mission - 10 days on Mars.

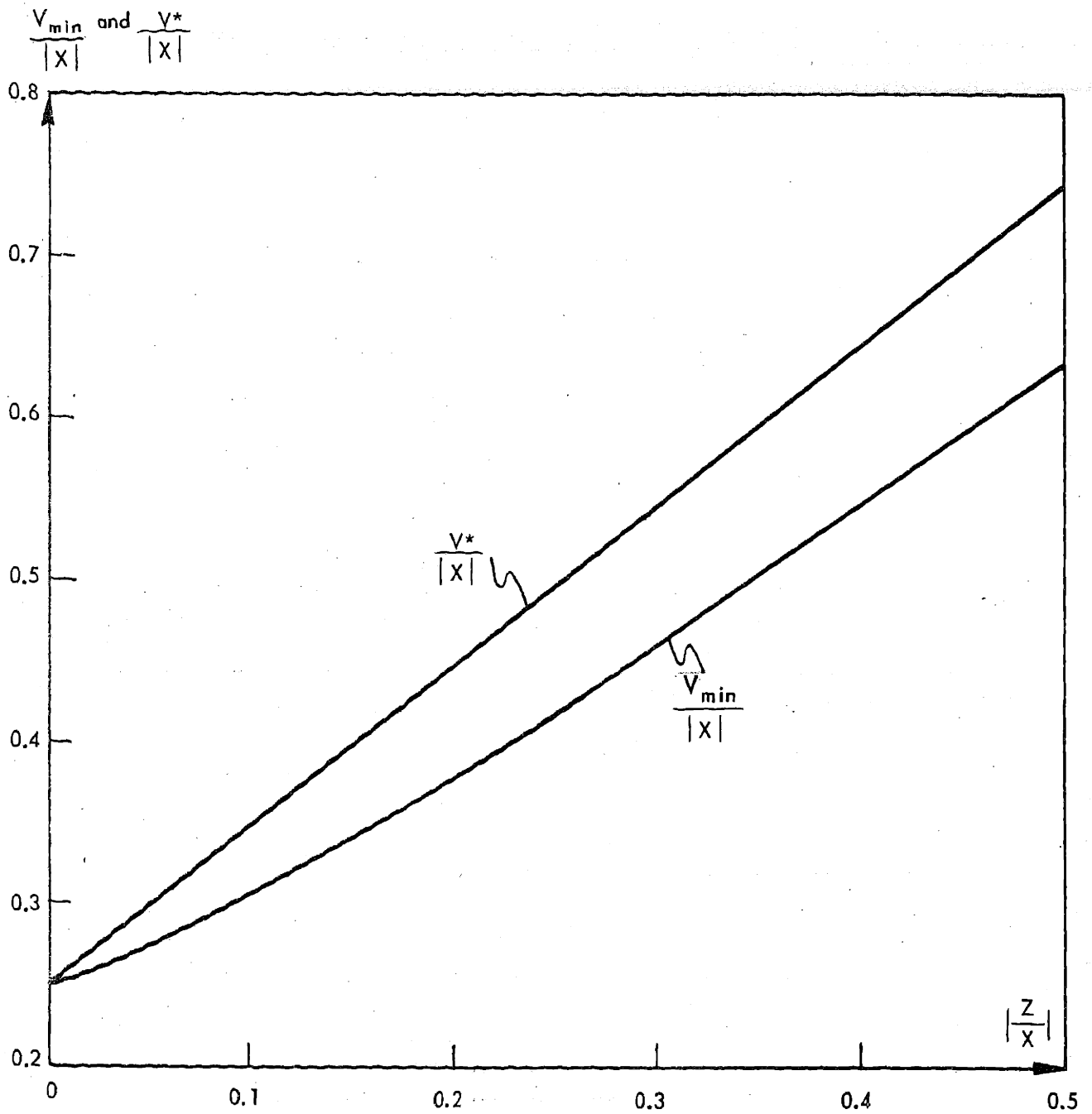


Fig. 19. Comparison of V_{\min} and V^* (Appendix C).

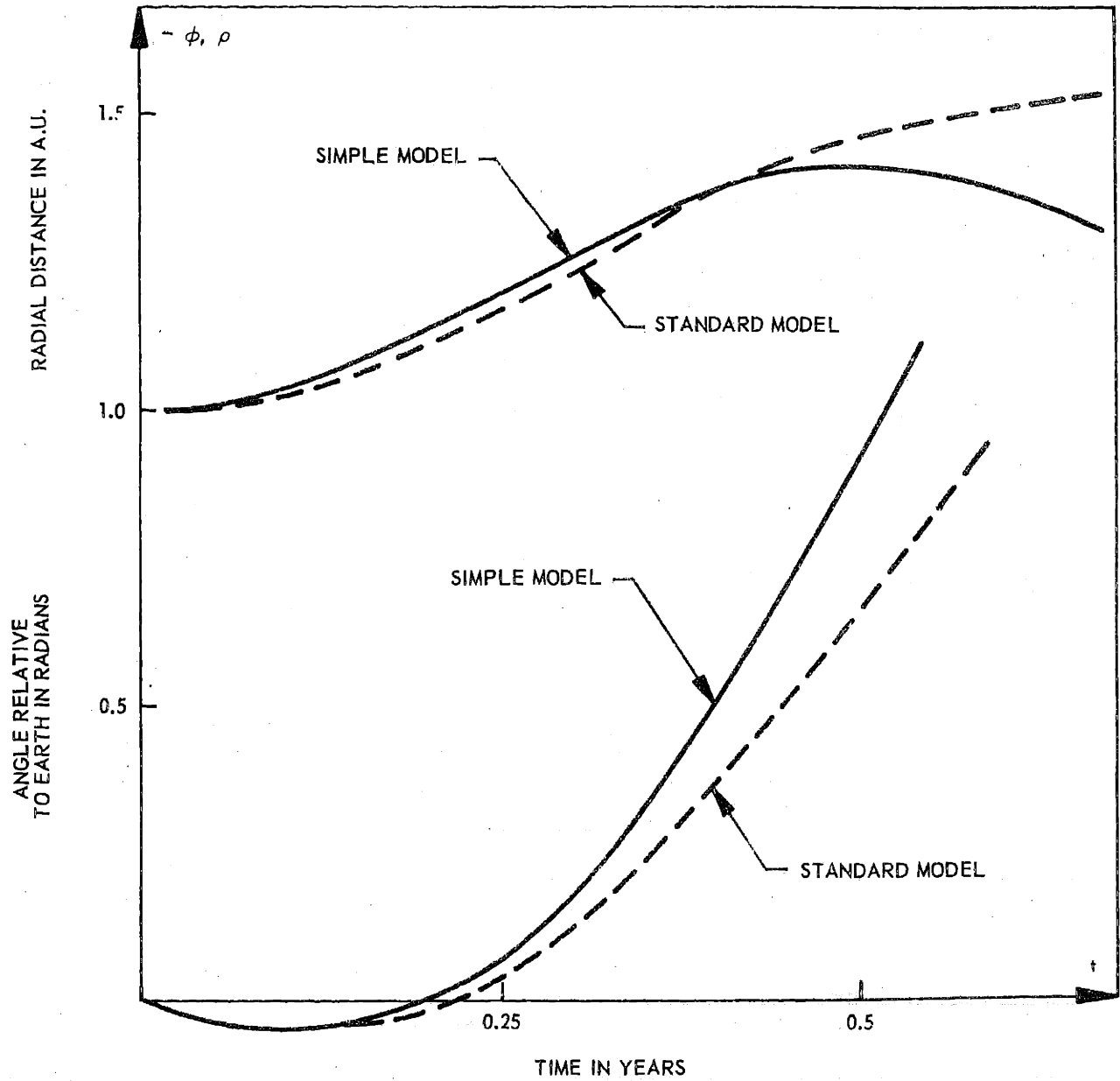


Fig. 20a. Comparison of simple model with standard model ρ and ϕ vs. t for $V_y = 0.1$.

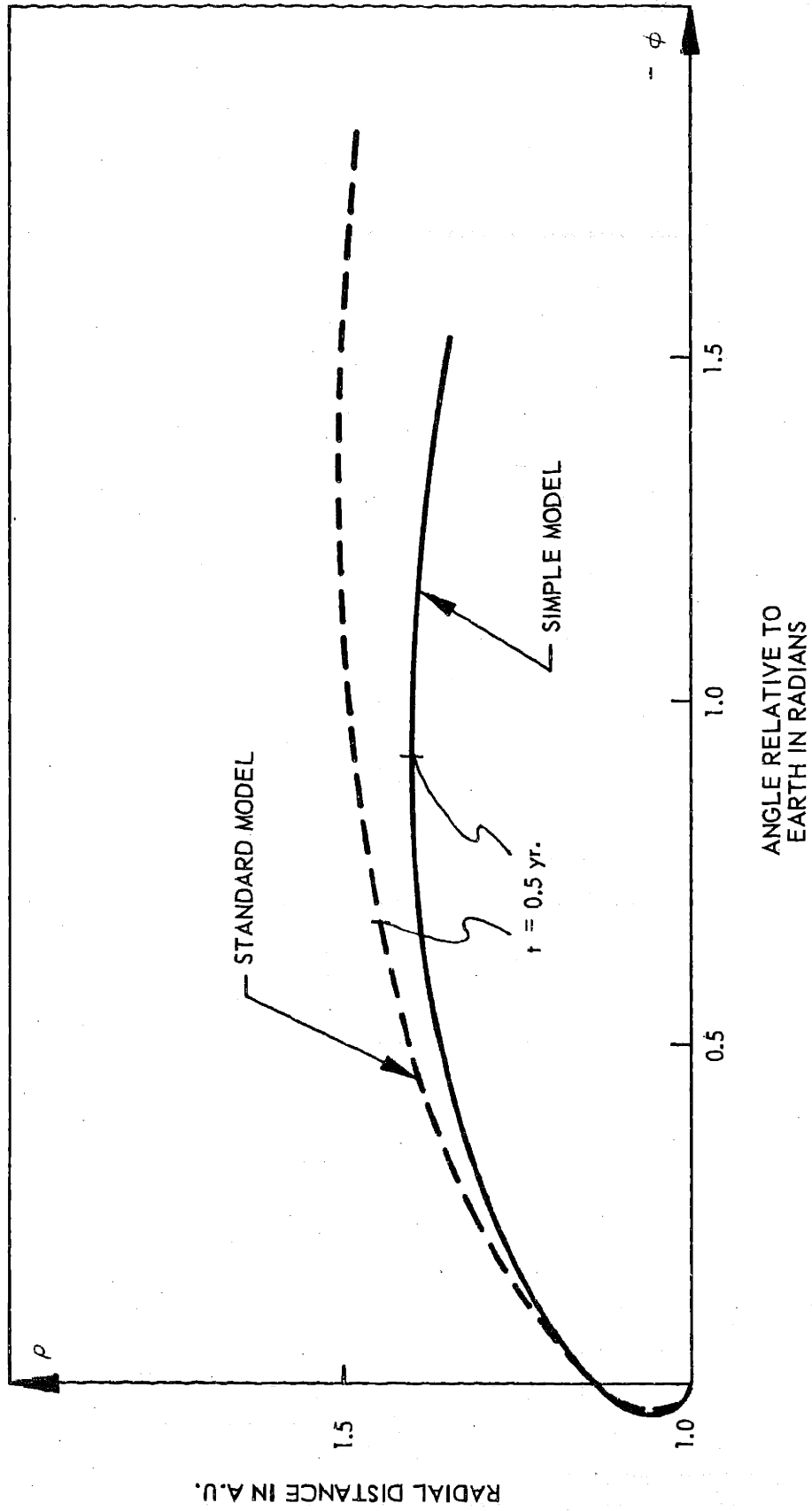


Fig. 20b. Comparison of simple model with standard model ρ vs. ϕ for $V_y = 0.1$.

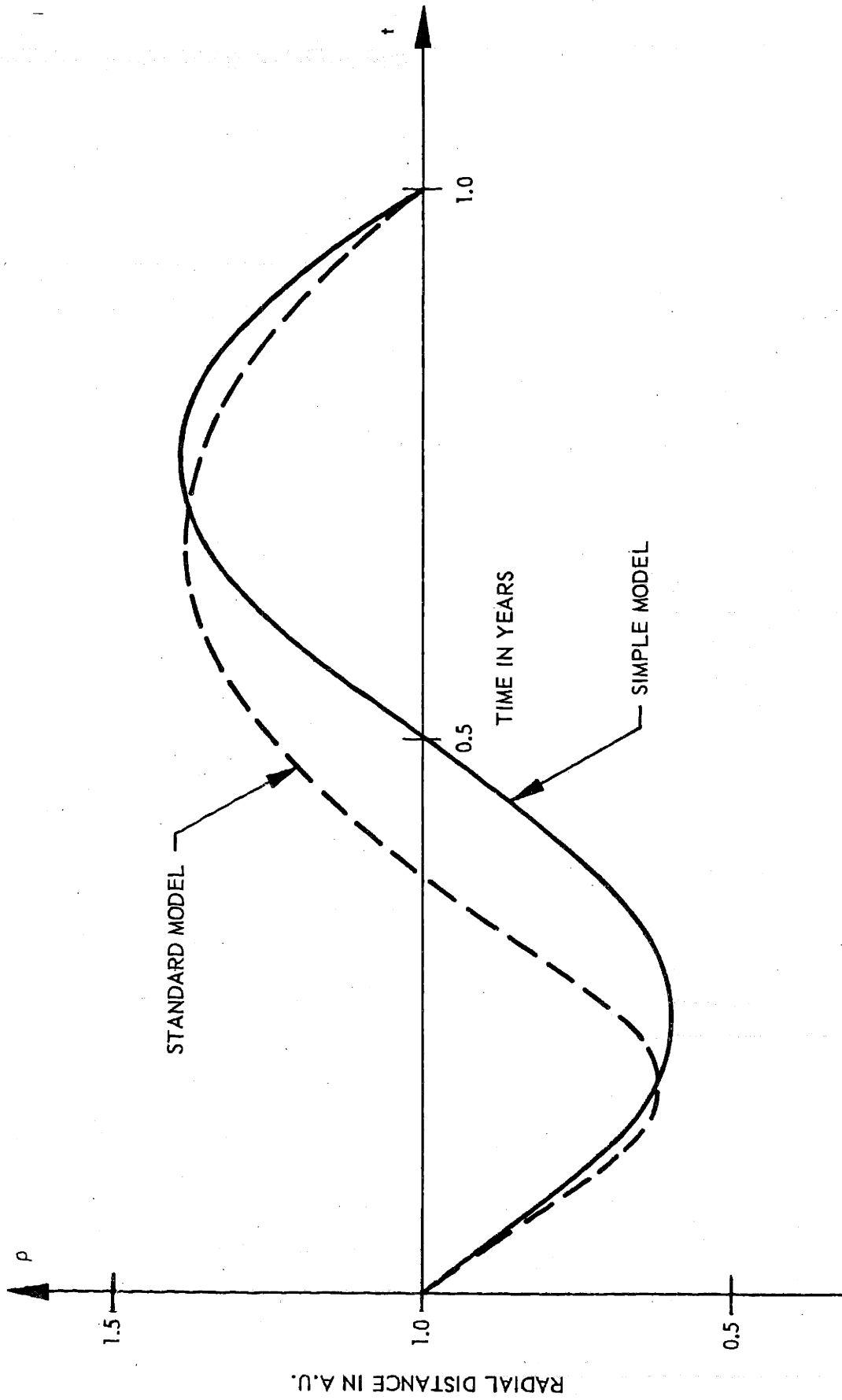


Fig. 21a. Comparison of simple model with standard model ρ vs. t for one year orbit.

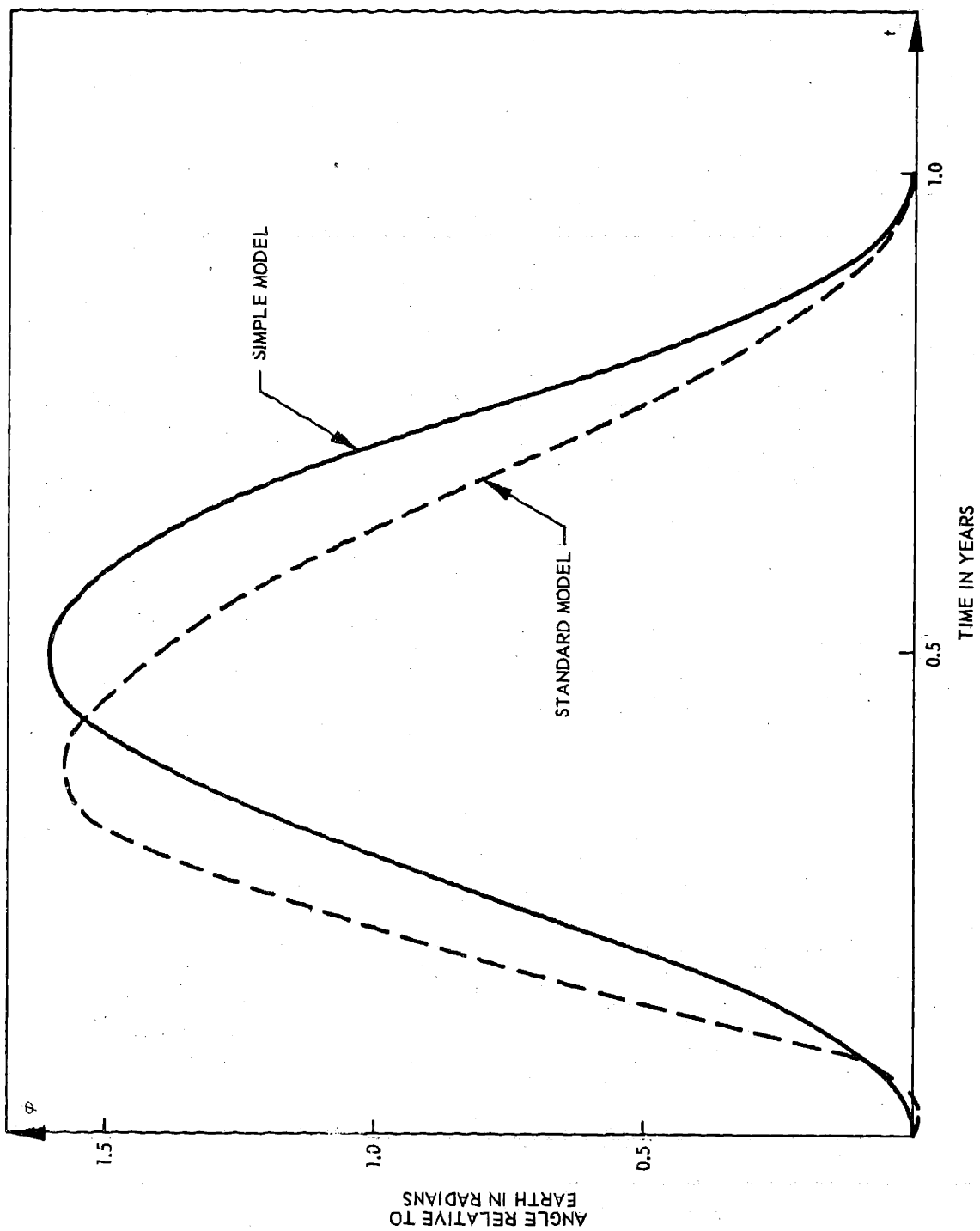


Fig. 21b. Comparison of simple model with standard model ϕ vs. t for one year orbit.

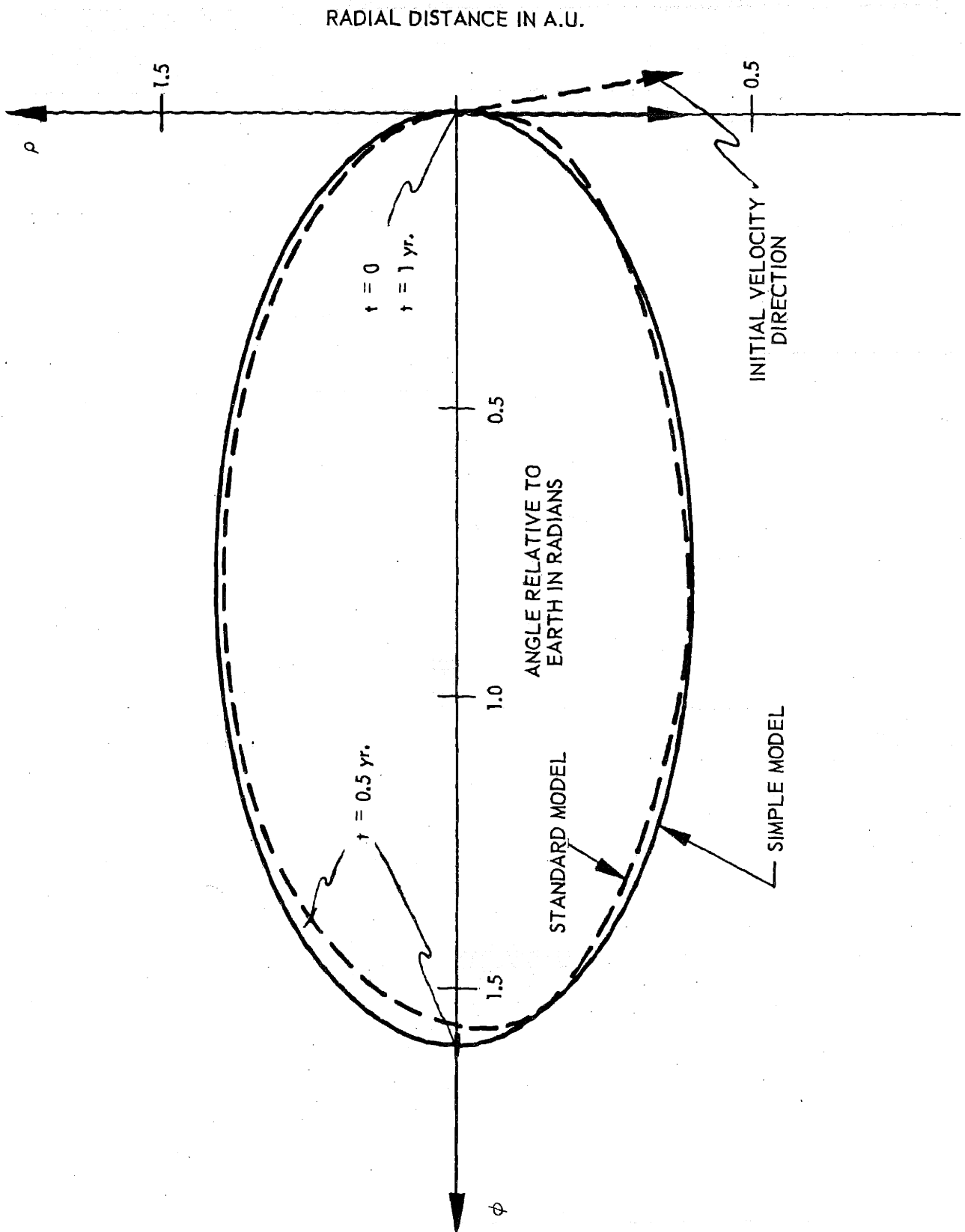


Fig. 21c. Comparison of simple model with standard model ρ vs. ϕ for one year orbit.

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BIOGRAPHICAL SKETCH

Walter Mark Hollister was born on 22 November 1930 in St. Johnsbury, Vermont. He graduated from the Rye High School as valedictorian in 1948. He received both the degree of Bachelor of Arts from Middlebury College and the degree of Bachelor of Science in Electrical Engineering from the Massachusetts Institute of Technology in 1953 under a combined program. As an undergraduate he was elected to Phi Beta Kappa, Tau Beta Pi, Eta Kappa Nu, and Sigma Xi.

After graduation he became a Field Engineer for the Sperry Gyroscope Company, working with the autopilot installed in the B-47. In 1954 he was commissioned in the United States Navy. Lt. Hollister served as a Naval Aviator, Electronics Officer, Ordnance Officer, and Aviation Safety Officer. Since his release from full-time active duty in 1958 he has remained active in the Naval Reserve and was recalled for one year during the Berlin crisis in 1961-62. Through participation in the reserve program he has maintained his pilot proficiency in both jet and conventional models of service aircraft.

Upon release from active duty in 1958 he returned to M.I.T. where he received the degree of Master of Science in 1959. Since 1958 he has been, in consecutive years, Sperry Gyroscope Company Fellow, Convair Fellow, Teaching Assistant, and Instructor in the Department of Aeronautics and Astronautics. His teaching experience includes courses in gyroscopic instrument theory, fire control, and inertial guidance. In addition to his S.M. thesis, Reference (13), he is currently writing a textbook in collaboration with Dr. Walter Wrigley on gyroscopic instrument theory.