THE USE OF DISCRIMINATORS IN THE LINEAR DETECTION OF F-M SIGNALS

By

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APPENDIX

I. INTRODUCTION

Although the discriminator circuits for the detection of F-M signals are familiar and widely used, little has been published that gives a thorough discussion of the linearity of detection as well as many design considerations. The purpose of the present work is:

A. to study the linearity of the ordinary discriminator circuit under idealized conditions.

B. to investigate the effects of the departures from the idealized case and the methods of minimizing them.

C. to formulate a definite design procedure in the light of the theoretical and experimental results.

D. to construct by following the design procedure an actual discriminator circuit that will work as the theory predicts.

II. GENERAL THEORETICAL INVESTIGATIONS

An ordinary discriminator circuit is shown in Figure 2-1. One chief part of the circuit is formed by the two resonant circuits shown separately from the remaining part in Figure 2-2, where

$$R_1 = \frac{R_p R_1'}{R_p' + R_1'} \qquad R_p \text{ being the plate resistance of the tube} \tag{2-1}$$

$$L_{e_1} = \frac{L_1 L_2 - M^2}{L_2 - M} \qquad L_{e_2} = \frac{L_1 L_2 - M^2}{L_1 - M} \qquad L_m = \frac{L_1 L_2 - M^2}{M} \tag{2-2}$$

The special arrangement of Figure 2-1 (a) where one end of the coupling condenser C_c is connected to the center of the primary inductance will be investigated at length. The voltages applied to the two diodes are:

$$\tfrac{1}{2}\,(E_1 - E_2) \text{ and } \tfrac{1}{2}\,(E_1 + E_2) \text{ respectively} \tag{2-3}$$

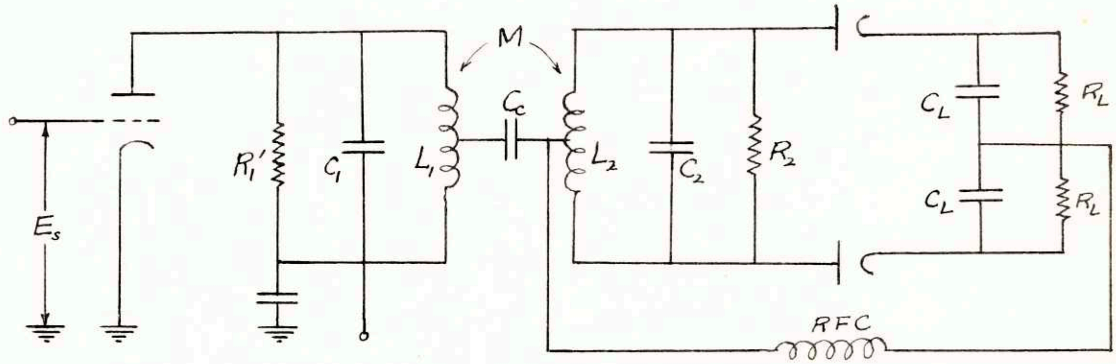
Where E_1, E_2 and I_1 are assumed as peak values. Assuming the diodes as perfect peak rectifiers, the detected voltage is therefore,

$$V = \tfrac{1}{2}\,|E_1 - E_2| - \tfrac{1}{2}\,|E_1 + E_2| \tag{2-4}$$

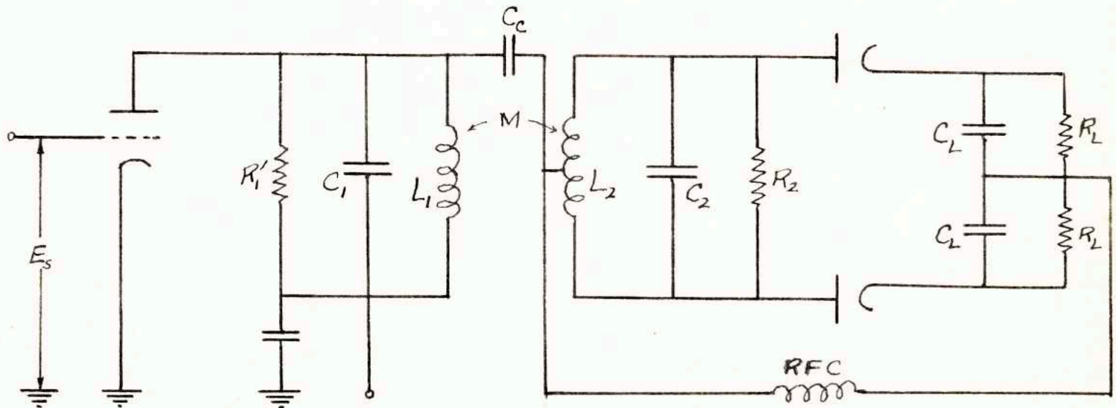
It has been pointed out by Professor Arguimbau that the treatment of resonant circuits can be simplified by normalizing the quantities[1]. It can easily be shown that the complex impedance of a parallel R, L_e, C network is given by[2]

$$Q = \frac{R}{1+jx}$$

1. L. B. Arguimbau "Notes on Tuned Circuits", M. I. T. Communications Laboratories Note, 1943
2. See Appendix I.

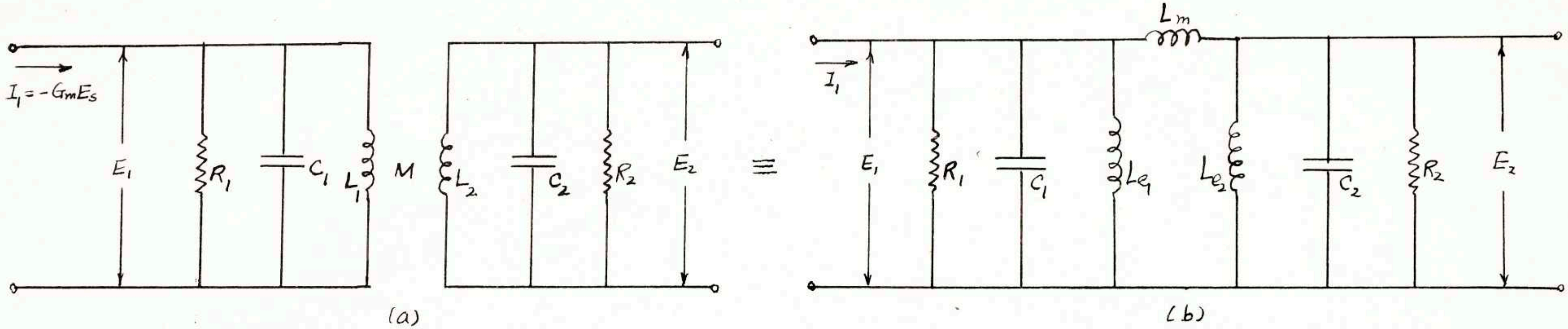

M
C_c
R_1'
C_1
L_1
L_2
C_2
R_2
C_L
R_L
C_L
R_L
E_s
RFC

(a)


C_c
M
R_1'
C_1
L_1
L_2
C_2
R_2
C_L
R_L
C_L
R_L
E_s
RFC

(b)

Fig. 2-1


$I_1 = -G_m E_s$
E_1
R_1
C_1
L_1
M
L_2
C_2
R_2
E_2
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L_m
I_1
E_1
R_1
C_1
L_{e_1}
L_{e_2}
C_2
R_2
E_2
(a)
(b)

Fig. 2-2

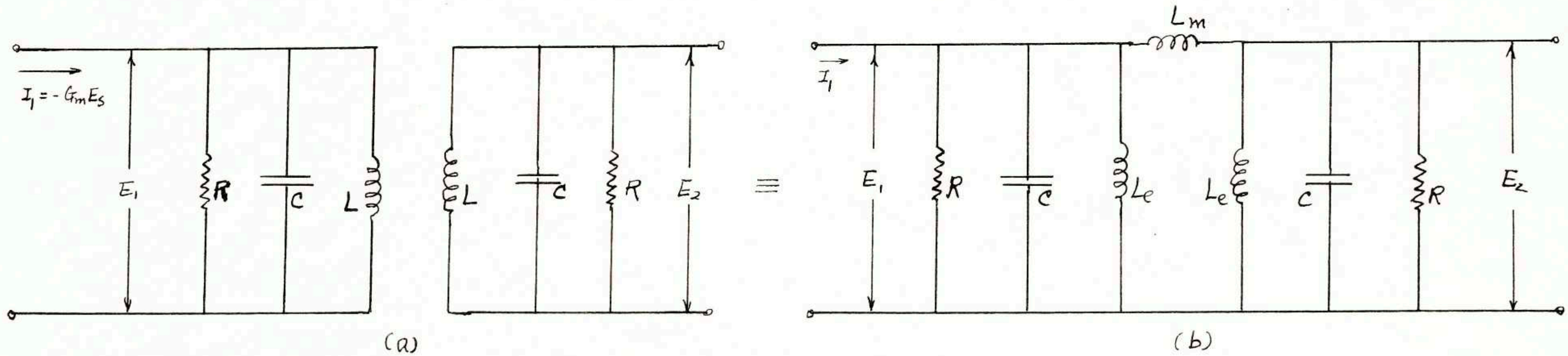

$I_1 = -G_m E_s$
E_1
R
C
L
L
C
R
E_2
≡
L_m
I_1
E_1
R
C
L_e
L_e
C
R
E_2
(a)
(b)

Fig. 2-3

$$\left.\begin{aligned} \text{Where} \qquad X &= Q_o\left(\frac{f}{f_o} - \frac{f_o}{f}\right) \qquad f_o = \frac{1}{2\pi\sqrt{L_e C}} \\ Q_o &= \frac{R}{2\pi f_o L_e} = 2\pi f_o CR = R\sqrt{\frac{C}{L_e}} \end{aligned}\right\} \tag{2-5}$$

The above relations will be used in discussing the discriminator circuit. The circuit of Figure 2-1 (a) with $L_1=L_2=L$, $C_1=C_2=C$, and $R_1=R_2=R$ will be first considered (Figure 2-3)

$$\left.\begin{aligned} \text{Then} \qquad M &= k\sqrt{L_1L_2} = kL \\ L_{e_1} = L_{e_2} = L_e &= \frac{L^2(1-k^2)}{L(1-k)} = (1+k)\,L \simeq L \\ L_m &= \frac{1-k^2}{k}\,L \simeq \frac{L}{k} \\ f_o^2 &= \frac{1}{4\pi^2(1+k)LC} \simeq \frac{1}{4\pi^2 LC} \end{aligned}\right\} \tag{2-6}$$

For this particular special case with $L_1=L_2=L$, $C_1=C_2=C$, and $R_1=R_2=R$, a simple expression easily accessible to mathematical interpretation can readily be derived, as is given in the M. I. T. Laboratories Note already cited[3]. The expression is:

$$\frac{V}{|I_1|} = \frac{R}{2}\,f(y) \quad \text{where} \quad f(y) = \frac{1}{\sqrt{1+(y-a)^2}} - \frac{1}{\sqrt{1+(y+a)^2}} \tag{2-7}$$

$$\left.\begin{aligned} \text{Where} \qquad y &= x-a \\ a &= Q_o\,\frac{k}{1-k^2} \simeq kQ_o \end{aligned}\right\} \tag{2-8}$$

The significance of the expression (2-7) has to be discussed. The expressions $\frac{1}{\sqrt{1+(y-a)^2}}$ and $\frac{1}{\sqrt{1+(y+a)^2}}$ are recognized as those representing resonance curves. The curve representing the expression (2-7) is obtained by combining the two resonance curves as indicated by the heavy line in

3. *See Appendix 2*

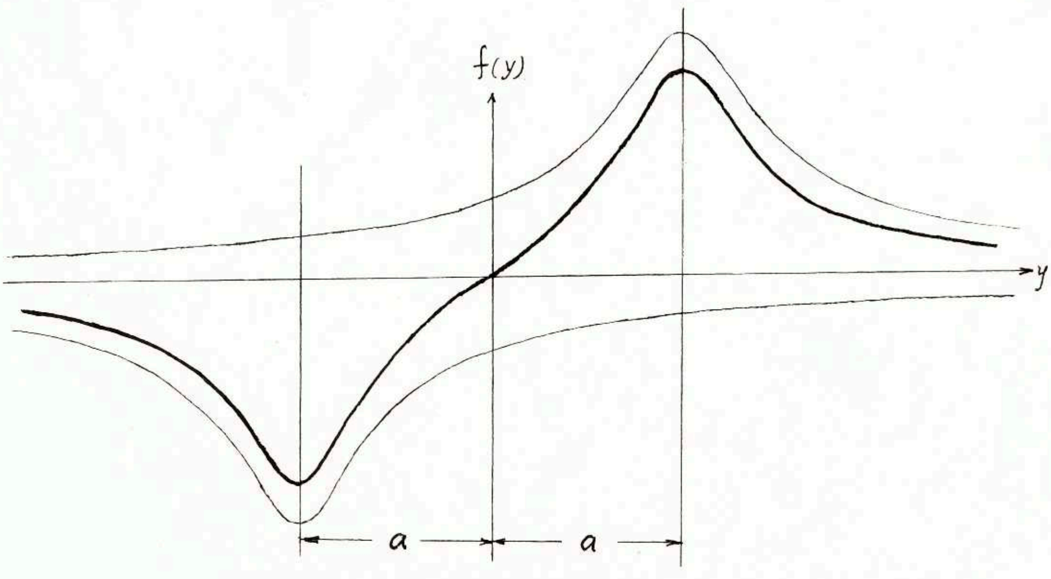


Fig. 2-4

Figure 2-4. The linearity of the resultant curve in the range $-a \leq y \leq a$ depends on the value of a.

Let the unmodulated i-f frequency be f_i. At f_i, f(y) should be 0. Hence, y = 0 at f_i. Remembering

$$y = Q_o \left(\frac{f}{f_o} - \frac{f_o}{f}\right) - a$$

hence $$\left(\frac{f_i}{f_o}\right)^2 - \frac{a}{Q_o}\frac{f_i}{f_o} - 1 = 0.$$

and $$\frac{f_i}{f_o} = \frac{a}{2Q_o} + \sqrt{\frac{a^2}{4Q_o^2} + 1} \simeq 1 + \frac{a}{2Q_o} + \frac{a^2}{8Q_o{}^2} \simeq 1 + \frac{a}{2Q_o} \qquad (2\text{-}9)$$

This expression reveals an interesting fact that f_i should not be equal to, though very near, the resonant frequency f_o of L_e and C.

Let the instantaneous frequency f be

$$f = f_i + \delta f \qquad (2\text{-}10)$$

then $$\frac{f}{f_i} = 1 + \frac{\delta f}{f_i} \qquad \frac{f_i}{f} \simeq 1 - \frac{\delta f}{f_i}$$

$$y = Q_o\left(\frac{f}{f_i}\frac{f_i}{f_o} - \frac{f_i}{f}\frac{f_o}{f_i}\right) - a$$

$$\simeq Q_o\left[\left(1 + \frac{\delta f}{f_i}\right)\left(\sqrt{1 + \frac{a^2}{4Q_o^2}} + \frac{a}{2Q_o}\right) - \left(1 - \frac{\delta f}{f_i}\right)\left(\sqrt{1 + \frac{a^2}{4Q_o^2}} - \frac{a}{2Q_o}\right)\right] - a$$

hence $$y \simeq 2Q_o\sqrt{1 + \frac{a^2}{4Q_o^2}}\,\frac{\delta f}{f_i} \simeq 2Q_o\frac{\delta f}{f_i} \qquad (2\text{-}11)$$

The expression (2-11) shows the important fact that if the resultant curve of Figure 2-4 is essentially linear, distortionless detection is achieved.

Suppose that $\frac{\delta f}{f_i}$ has a maximum value $\frac{\Delta f}{f_i} = m$ where Δf is usually equal to 75 kc. Denote the value of y for $\frac{\delta f}{f_i} = m$ by Y and express Y in terms of a as $Y = \alpha a$ where $\alpha \leqq 1$ usually; then

$$\alpha a \simeq 2Q_o \sqrt{1+\frac{a^2}{4Q_o^2}} \; m$$

$$Q_o \simeq \frac{\alpha a}{2m} \sqrt{1-\frac{m^2}{\alpha^2}} \simeq \frac{\alpha a}{2m}\left(1-\frac{1}{2}\frac{m^2}{\alpha^2}\right) \simeq \frac{\alpha a}{2m} \tag{2-12}$$

By (2-8) $$k \simeq \frac{a}{Q_o} = \frac{2m}{\alpha} \frac{1}{\sqrt{1-\frac{m^2}{\alpha^2}}} = \frac{2m}{\alpha}\left(1+\frac{m^2}{2\alpha^2}\right) \simeq \frac{2m}{\alpha} \tag{2-13}$$

The expression (2-7) will be investigated more closely. Since, by the well-known property of Legendres polynomials,

$$\left.\begin{aligned}\left[1+(y-a)^2\right]^{-\frac{1}{2}} &= \frac{1}{\sqrt{1+a^2}}\left[1-2\frac{a}{\sqrt{1+a^2}}\frac{y}{\sqrt{1+a^2}}+\left(\frac{y}{\sqrt{1+a^2}}\right)^2\right]^{-\frac{1}{2}}\\ &= \frac{1}{\sqrt{1+a^2}}\left[P_o\left(\frac{a}{\sqrt{1+a^2}}\right)+P_1\left(\frac{a}{\sqrt{1+a^2}}\right)\left(\frac{y}{\sqrt{1+a^2}}\right)+P_2\left(\frac{a}{\sqrt{1+a^2}}\right)\left(\frac{y}{\sqrt{1+a^2}}\right)^2+\cdots\right]\\ \text{and similarly } \left[1+(y+a)^2\right]^{-\frac{1}{2}} &= \frac{1}{\sqrt{1+a^2}}\left[P_o\left(\frac{a}{\sqrt{1+a^2}}\right)-P_1\left(\frac{a}{\sqrt{1+a^2}}\right)\left(\frac{y}{\sqrt{1+a^2}}\right)+P_2\left(\frac{a}{\sqrt{1+a^2}}\right)\left(\frac{y}{\sqrt{1+a^2}}\right)^2\cdots\right]\end{aligned}\right\} \tag{2-14}$$

where Pn is the Legendre's polynominal of the nth order, therefore

$$f(y) = \frac{2}{\sqrt{1+a^2}}\sum_{n=o}^{\infty} P_{2n+1}\left(\frac{a}{\sqrt{1+a^2}}\right)\left(\frac{y}{\sqrt{1+a^2}}\right)^{2n+1} \tag{2-15}$$

If the frequency modulation is sinusoidal, then

$$y = Y\sin 2\pi f_a t = \alpha a \sin 2\pi f_a t \tag{2-16}$$

where f_a is the audio frequency, then

$$f(y) = \frac{2}{\sqrt{1+a^2}}\sum_{n=o}^{\infty} P_{2n+1}\left(\frac{a}{\sqrt{1+a^2}}\right)\left(\frac{\alpha a}{\sqrt{1+a^2}}\right)^{2n+1}\sin^{2n+1} 2\pi f_a t \tag{2-17}$$

By the principle of Fourier Analysis, assume

$$f(y) = \sum_{m=0}^{\infty} A_{2m+1}\sin\left[2\pi(2m+1) f_a t\right] \tag{2-18}$$

$$A_1 = \frac{2}{\sqrt{1+a^2}}\sum_{n=o}^{\infty} P_{2n+1}\left(\frac{a}{\sqrt{1+a^2}}\right)\left(\frac{\alpha a}{\sqrt{1+a^2}}\right)^{2n+1}\frac{2}{\pi}\int_0^{\pi}\sin^{2n+2}\theta\, d\theta$$

Since $\displaystyle\int_0^{\pi} \sin^{2n+2}\theta \, d\theta = \frac{1.3.5\cdots(2n+1)}{2.4.6\cdots(2n+2)}\pi = \frac{\pi}{2^{2n+1}} \frac{(2n+1)!}{(n+1)!\,n!}$ hence

$$A_1 = \frac{4}{\sqrt{1+a^2}} \sum_{n=0}^{\infty} P_{2n+1}\left(\frac{a}{\sqrt{1+a^2}}\right)\left(\frac{\alpha a}{\sqrt{1+a^2}}\right)^{2n+1} \frac{1.3.5\cdots(2n+1)}{2.4.6\cdots(2n+2)} \tag{2-19}$$

$$A_{2m+1} = \frac{2}{\sqrt{1+a^2}} \sum_{n=0}^{\infty} P_{2n+1}\left(\frac{a}{\sqrt{1+a^2}}\right)\left(\frac{\alpha a}{\sqrt{1+a^2}}\right)^{2n+1} \frac{2}{\pi} \int_0^{\pi} \sin^{2n+1}\theta \sin(2m+1)\theta \, d\theta \tag{2-20}$$

(2-21)

Now

$$\left.\begin{aligned} \int_0^{\pi} \sin^{2n+1}\theta \sin(2m+1)\theta\, d\theta &= (-1)^m \pi \, 2n(2n-2)\cdots(2n-2m+2) \frac{1.3.5\cdots(2n+1)}{2.4.6\cdots(2n+2m+2)} \\ &= (-1)^m \frac{\pi}{2^{2n+1}} \frac{(2n+1)!}{(n+m+1)!(n-m)!} \quad \text{if } n \geqq m \text{ and } m \geqq 1 \\ \int_0^{\pi} \sin^{2n+1}\theta \sin(2m+1)\theta\, d\theta &= 0 \qquad \text{if } n < m \end{aligned}\right\}$$

hence,
$$A_{2m+1} = (-1)^m \frac{4}{\sqrt{1+a^2}} \sum_{n=m}^{\infty} P_{2n+1}\left(\frac{a}{\sqrt{1+a^2}}\right)\left(\frac{\alpha a}{\sqrt{1+a^2}}\right)^{2n+1} (2n)(2n-2)\cdots(2n-2m+2)$$

$$\frac{1.3.5\cdots(2n+1)}{2.4.6\cdots(2n+2m+2)} \tag{2-22}$$

Where $m \geqq 1$. The magnitudes of the various harmonics relative to that of the fundamental can therefore be computed if a and α are given.

Define sensitivity S as

$$S = \left|\frac{\text{Fundamental of } V}{I_1}\right| \frac{1}{\frac{\delta f}{f_i}} \tag{2-23}$$

$$S = \frac{\frac{1}{2}RA_1}{m} = \frac{2R}{\sqrt{1+a^2}} \frac{1}{m} \sum_{n=0}^{\infty} P_{2n+1}\left(\frac{a}{\sqrt{1+a^2}}\right)\left(\frac{\alpha a}{\sqrt{1+a^2}}\right)^{2n+1} \frac{1.3.5\cdots(2n+1)}{2.4.6\cdots(2n+2)} \tag{2-24}$$

Now $R \simeq \dfrac{Q_0}{2\pi f_i C} \simeq \dfrac{\alpha a}{4\pi f_i C m}$

$$\therefore S \simeq \frac{\alpha a}{2\pi f_i C} \frac{1}{m^2} \frac{1}{\sqrt{1+a^2}} \sum_{n=0}^{\infty} P_{2n+1}\left(\frac{a}{\sqrt{1+a^2}}\right)\left(\frac{\alpha a}{\sqrt{1+a^2}}\right)^{2n+1} \frac{1.3.5\cdots(2n+1)}{2.4.6\cdots(2n+2)}$$

The design procedure is based on choosing a and α as two parameters. It would be desirable to compute S and the first few $\dfrac{|A_{2m+1}|}{A_1}$ for different values of a and α. Unfortunately, the factor $(\frac{\alpha a}{\sqrt{1+a^2}})^{2n+1}$ in the expressions (2-19), (2-22) and (2-24) decreases slowly unless $\frac{\alpha a}{\sqrt{1+a^2}} < 0.5$ (say). These expressions, therefore, converge slowly and the numerical computation will be tedious. When $P_3\,(\frac{a}{\sqrt{1+a^2}}) = o$, i. e., when $a = \sqrt{\frac{3}{2}}$, the first term contributing the third harmonic vanishes. This means that if $Y = \alpha a$ is very small, there is practically no third harmonic when $a = \sqrt{\frac{3}{2}}$. If Y is not small, $a = \sqrt{\frac{3}{2}}$ is not the optimum condition since the remaining terms contributing the third harmonic and the terms contributing higher harmonics become important.

To simplify the numerical computations, the problem can be approached in a somewhat different, though a little artificial, manner. The resultant curve of Fig. 2-4 is redrawn in Fig. 2-5. The range actually utilized lies between $y = -Y = -\alpha a$ and $y = Y = \alpha a$ where $\alpha \leq 1$ usually. In this range the curve can be approximated by a straight line RS whose equation is

$$f_s\,(y) = Ay = \frac{2}{R}\,\frac{V_s}{|I_1|} \qquad (2\text{-}25)$$

Where V_s is the fictitious value of the detected voltage if the curve were the straight line RS. Let

$$\varphi(y) = \frac{f(y)-f_s(y)}{f_s(y)} = \frac{1}{Ay}\left[\frac{1}{\sqrt{1+(y-a)^2}} - \frac{1}{\sqrt{1+(y+a)^2}}\right] - 1 \qquad (2\text{-}26)$$

The expression (2-26) is a measure of the percentage distortion. The instantaneous "distortion" D(y) in db is defined as

$$D(y) = 20\,\log_{10}\varphi(y) = 20\,\log_{10}\frac{1}{Ay}\left[\frac{1}{\sqrt{1+(y-a)^2}} - \frac{1}{\sqrt{1+(y+a)^2}}\right] \qquad (2\text{-}27)$$

The value of A is so chosen that within the range $-\alpha a \leq y \leq \alpha a$

$$\text{Max. } D(y) = \text{Max}\left[-D(y)\right] = -\text{Min. } D(y) = D_o \qquad (2\text{-}28)$$

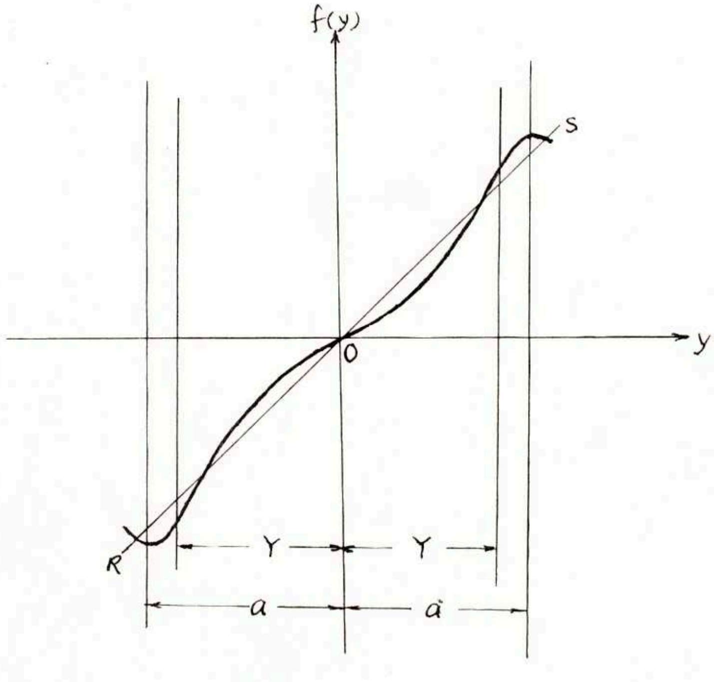


Fig. 2-5

D_o is a measure of the overall distortion within the given range.

$$\text{Max. } D(y) = 20 \text{ Max. } \log_{10} \frac{1}{y}\left[\frac{1}{\sqrt{1+(y-a)^2}} - \frac{1}{\sqrt{1+(y+a)^2}}\right] - 20 \log_{10} A$$

$$\text{Mim. } D(y) = 20 \text{ Min. } \log_{10} \frac{1}{y}\left[\frac{1}{\sqrt{1+(y-a)^2}} - \frac{1}{\sqrt{1+(y+a)^2}}\right] - 20 \log_{10} A$$

$$\log_{10} A = \frac{1}{2}\left\{\text{Max. } \text{Log}_{10} \frac{1}{y}\left[\frac{1}{\sqrt{1+(y-a)^2}} - \frac{1}{\sqrt{1+(y+a)^2}}\right] + \text{Min. } \log_{10} \frac{1}{y} \left[\frac{1}{\sqrt{1+(y-a)^2}} - \frac{1}{\sqrt{1+(y+a)^2}}\right]\right\} \tag{2-29}$$

and $$D_o = 10\left\{\text{Max. } \log_{10} \frac{1}{y}\left[\frac{1}{\sqrt{1+(y-a)^2}} - \frac{1}{\sqrt{1+(y+a)^2}}\right] - \text{Min. } \log_{10} \frac{1}{y} \left[\frac{1}{\sqrt{1+(y-a)^2}} - \frac{1}{\sqrt{1+(y+a)^2}}\right]\right\} \tag{2-30}$$

By the change of variable $\qquad z = \frac{y}{a}$

$$\log_{10} A = \frac{1}{2}\left\{\text{Max. } \log_{10} \frac{1}{z}\left[\frac{1}{\sqrt{1+a^2(z-1)^2}} - \frac{1}{\sqrt{1+a^2(z+1)^2}}\right] + \text{Min. } \log_{10} \frac{1}{z} \left[\frac{1}{\sqrt{1+a^2(z-1)^2}} - \frac{1}{\sqrt{1+a^2(z+1)^2}}\right]\right\} - \log_{10} A \tag{2-29a}$$

$$D_o = 10\left\{\text{Max. } \log_{10} \frac{1}{z}\left[\frac{1}{\sqrt{1+a^2(z-1)^2}} - \frac{1}{\sqrt{1+a^2(z+1)^2}}\right] - \text{Min. } \log_{10} \frac{1}{z} \left[\frac{1}{\sqrt{1+a^2(z-1)^2}} - \frac{1}{\sqrt{1+a^2(z+1)^2}}\right]\right\} \tag{2-30a}$$

Re-define sensitivity S as

$$S = \left|\frac{V_s}{I_1}\right| \frac{1}{\frac{\delta f}{f_i}}$$

Then $$S = \frac{R}{2} f_s(y) \frac{1}{\frac{\delta f}{f_i}} = \frac{1}{2} RA \frac{y}{\frac{\delta f}{f_i}} \tag{2-31}$$

In virtue of (2-11)

$$S \simeq RAQ_o = \frac{AQ_o^2}{2\pi f_i C} \simeq \frac{A(\alpha a)^2}{8\pi f_i C m^2} \propto A(\alpha a)^2 \qquad (2-32)$$

In order to get good sensitivity C is preferably small, but limited by the consideration that C should be much larger than the stray capacitances between the coils. In actual designs f_i and m are chosen in conformity with many considerations not concerned in the present investigations. The permissible distortion D_o is also a given quantity. Hence, $A(\alpha a)^2$ is a measure of sensitivity. The magnitudes of D_o and $A(\alpha a)^2$ for different values of a and α can be found once and for all by carrying out the actual computations. The optimum design corresponds to the choice of a and α such that $A(\alpha a)^2$ is a maximum, while D_o is not larger than the allowable limit.

For the arrangement of Figure 2-1 (b) with $L_1 = L_2 = L$, $C_1 = C_2 = C$, $R_1 = R_2 = R$, it can easily be proved that the detected voltage V is given by the relation

$$\frac{V}{|I_1|} = \left|\frac{E_1 - \frac{E_2}{2}}{I_1}\right| - \left|\frac{E_1 + \frac{E_2}{2}}{I_1}\right| = \frac{R}{\sqrt{1+(y+a)^2}\sqrt{1+(y-a)^2}}\left[\sqrt{1+\left(y+\frac{a}{2}\right)^2} - \sqrt{1+\left(y-\frac{a}{2}\right)^2}\right] \qquad (2-33)$$

Where $y = x - a$, $\quad a = \frac{k\,Q_o}{1-k^2} \simeq kQ_o$ as before.

In this paper, if no particular mention is made, the circuit of Figure 2-1 (a) is understood.

III. OTHER CONSIDERATIONS

In the foregoing discussions idealized conditions have been assumed. Departures therefrom have to be studied.

A. Clipping of High Modulating Audio Frequency.

The time constant $R_L C_L$ of the diode circuit (Figure 2-1) should be much larger than the period of the i-f frequency, but small enough to avoid slipping of modulation. The voltage across either diode is an amplitude-modulated wave in the sense that the amplitude of the wave bears a definite relation to the frequency deviation, though evidently the envelope has no resemblance to the original wave form of the audio modulating signal. Assume

$$y = \alpha a \sin 2\pi f_a t \qquad (3\text{-}1)$$

The peak value of the i-f voltage applied to either diode at any instant is

$$E_o = \left| \frac{E_1+E_2}{2} \right| = \frac{\frac{1}{2}R|I_1|}{\sqrt{1+a^2\,(1+\alpha\sin\theta)^2}} \quad \text{where } \theta = 2\pi f_a t$$

Near a positive peak of the i-f swing, the load condenser C_L is charged to approximately E_o and then discharges until the next peak swing occurs. To insure no clipping of the modulation, let the magnitude of the initial time rate of discharge of C_L through R_L larger than the magnitude of the time variation of the envelope of the i-f wave. Hence

$$\left| E_o \frac{d}{d\tau} e^{-\frac{1}{R_L C_L}\tau} \right|_{\tau=o} \geqq \left| \frac{dE_o}{dt} \right| \qquad \text{for any value of } \theta$$

or $$\frac{1}{R_L C_L} \frac{\frac{1}{2}R\,|I_1|}{\sqrt{1+a^2\,(1+\alpha\sin\theta)^2}} \geqq \left| \frac{\frac{1}{2}RI_1\; a^2\,(1+\alpha\sin\theta)}{\left[1+a^2\,(1+\alpha\sin\theta)^2\right]^{\frac{3}{2}}}\,\alpha\, 2\pi f_a\,\cos\theta \right|$$

Therefore $$\frac{1}{R_L C_L} \geqq \left| \frac{2\pi f_a\,\alpha\, a^2\,(1+\alpha\sin\theta)\,\cos\theta}{1+a^2\,(1+\alpha\sin\theta)^2} \right| \qquad (3\text{-}2)$$

This expression cannot be further simplified unless some approximation is made. It is on the safe side to let

$$\frac{1}{R_L C_L} \geqq \left| \frac{2\pi f_a\,\alpha\; a^2\,(1+\alpha\sin\theta)}{1+a^2\,(1+\alpha\sin\theta)^2} \right|$$

The right-hand side is of the form $\frac{x}{1+x^2}$ which is a maximum when $x=1$. Equating $\frac{1}{R_L C_L}$ to the maximum value of the right-hand quantity gives

$$\frac{1}{R_L C_L} = \alpha\, a\, \pi f_a \qquad (3\text{-}3)$$

R_L should be large to increase the efficiency of detection. Hence C_L should be small so long as it remains much larger than the stray capacitances across the diodes. By (3-3), C_L having been chosen, it is possible to compute the maximum value of R_L if f_a is known. Strictly speaking, the above derivation holds only for a pure sinusoidal frequency modulation, but it can serve as a guide for complex-wave modulation. In speech and music very high audio frequencies are not present as fundamentals, but as harmonics necessary for the exact reproduction of the signal. In (3-3) if α is the actual value used, f_a need not be the highest useful harmonic frequency. It is sufficient to choose a value reasonably higher than the highest useful fundamental frequency.

B. <u>Diode Loading Effect</u>.

It has been so far assumed that the diode circuit takes no appreciable

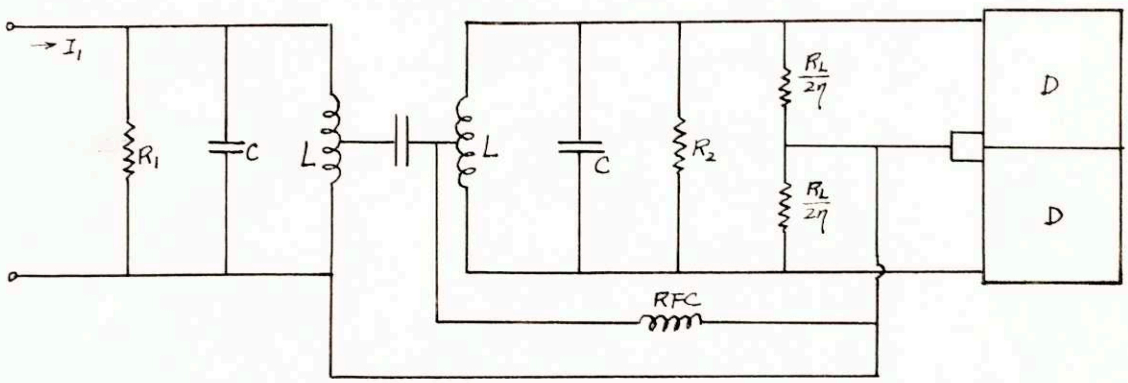


Fig. 3-1

power. As the value of R_L cannot be made large without limit in order to avoid the clipping of modulation, the diode loading effect is actually important. It can be shown that the loading effect of a diode is equal to a resistance of value $\frac{R_L}{2\eta}$ where η is the efficiency of detection, thus resulting in the equivalent circuit of Figure 3-1, where D denotes an ideal detector with an infinite input impedance and the efficiency of detection η.

This circuit can be solved by setting up and solving a set of simultaneous equations. The result thus obtained, however, is too complicated to be accessible to the simple interpretation as permitted when the diode loading effect is negligible. Unfortunately, the theory based on the expression (2-7) holds only for the special case when the assumption that there is no diode loading effect is justified. The general case remains to be solved.

C. Circuit Unbalance.

The first type of unbalance is due to the stray capacitive couplings. Their effects must be made negligible.

The second type of unbalance is due to the distributed nature of the self-inductance and the mutual inductance. A simple example will make this point clear. Consider two coupled circuits as shown in Figure 3-2. If B is the geometrical center of the secondary winding AC, then evidently $L_2' = L_2''$. Let M' and M'' be the mutual inductances between the primary winding and the sections of the secondary AB and BC respectively. Then obviously $M' > M''$ and $E_2' > E_2''$. If the diode loading effect is negligible the ideal condition is $E_2' = E_2'' = \frac{E_2}{2}$ while L_2'

and L_2'' do not necessarily have to be equal. Hence, under this assumption B should not be chosen as the geometrical center, but rather the point such that $E_2' = E_2'' = \frac{E_2}{2}$. When the diode loading effect is important it can be seen that the following conditions must be satisfied in order to achieve perfect balance.

1. The four half-sections of the two windings must have the same self-inductance.

2. The mutual inductance between either half-section of the primary winding and either half-section of the secondary winding must be the same.

3. The two half-sections of both the primary and the secondary must be closely coupled.

D. <u>Change of Effective Coupling between the Two Resonant Circuits Due to Stray Capacitive Couplings</u>.

Consider two coils magnetically coupled together as shown in Figure 3-4 Due to the magnetic coupling E_1 and E_2 is related by $\frac{E_2}{E_1} = k\sqrt{\frac{L_2}{L_1}}$ where $k = \frac{M}{\sqrt{L_1 L_2}}$. This relation is no longer true if the stray capacitive couplings cannot be neglected. Nevertheless, their presence does not impair the validity of the expression (2-7) provided that the circuit balance is still preserved, except that the value of a is now not the same as with the case of pure magnetic coupling. This is true because the frequency deviation Δf is a very small fraction of f_i and it does not matter what kind of coupling actually exists between the two coils. If in Figure 3-4 the effective coefficient of coupling keff is defined as

$$K_{eff} = \frac{E_2}{E_1} \sqrt{\frac{L_1}{L_2}}$$

where E_1 and E_2 are the actual values, then the expression (2-7) is always true if $a = Q_o \quad \frac{k_{eff}}{1-k_{eff}^2} \simeq k_{eff}\, Q_o$

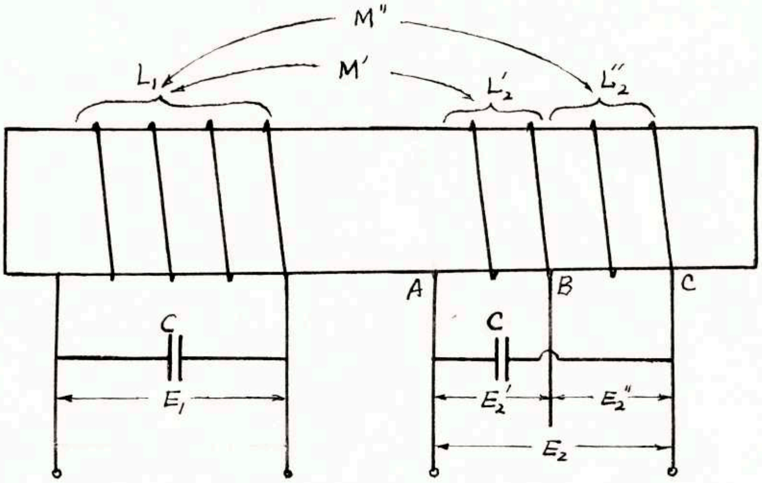


Fig. 3-2

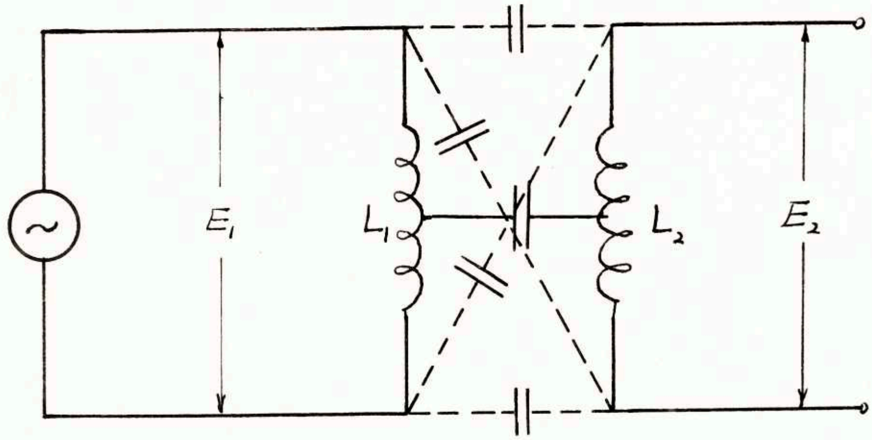


Fig. 3-3

IV. OUTLINE OF DESIGN PROCEDURE

As already pointed out the computations needed for the design of the discriminator circuit can be worked out once and for all by using expressions (2-29a) and (2-30a). For various values of a and α, D_o and $A(\alpha a)^2$ are computed and listed in a table. The optimum design is accomplished by choosing a and α such that $A(\alpha a)^2$ is a maximum while D_o remains less than the prescribed limit. Having determined a and α the data for the coil design are complete. The useful expressions are grouped together for convenient reference.

$$Q_o \simeq \frac{\alpha a}{2m}\sqrt{1-\frac{m^2}{\alpha^2}} \simeq \frac{\alpha a}{2m}\left(1-\frac{m^2}{2\alpha^2}\right) \simeq \frac{\alpha a}{2m} \tag{4-1}$$

$$k \simeq \frac{2m}{\alpha}\frac{1}{\sqrt{1-\frac{m^2}{\alpha^2}}} \simeq \frac{2m}{\alpha}\left(1+\frac{m^2}{2\alpha^2}\right) \simeq \frac{2m}{\alpha} \tag{4-2}$$

$$L = \frac{1}{4\pi^2 f_o^2\ C\ (1+k)} \simeq \frac{\left(1+\frac{a}{2Q_o}\right)^2}{4\pi^2 f_i^2\ C\ (1+k)} \simeq \frac{1}{4\pi^2 f_i^2\ C} \tag{4-3}$$

$$R = \frac{Q_o}{2\pi f_o C} \simeq \frac{Q_o\left(1+\frac{a}{2Q_o}\right)}{2\pi f_i\ C} \simeq \frac{Q_o}{2\pi f_i\ C} \tag{4-4}$$

C is chosen to be small but still much larger than the stray capacitances between the coils. R includes the effective shunt resistances due to the losses in the coil and the diode loading effect.

By taking into consideration the non-unity efficiency of detection η, the sensitivity S is given by

$$S = \left|\frac{V_s}{I_1}\right|\frac{1}{\frac{\delta f}{f_i}} \simeq \eta\frac{A(\alpha a)^2}{8\pi f_i\ C\ m^2} \tag{4-5}$$

Another useful expression is

$$\frac{1}{R_L C_L} = \alpha\, a \pi f_a \tag{4-6}$$

C_L is chosen to be small but still much larger than the stray capacitances across the diode.

To construct the table first compute for various values of a and α the values of

$$F(z) = \log_{10} \frac{1}{z} \left[\frac{1}{\sqrt{1+a^2\ (z-1)^2}} - \frac{1}{\sqrt{1+a^2\ (z+1)^2}} \right]. \tag{4-7}$$

$$F(o) = \log_{10} \frac{2a^2}{(1+a^2)^{\frac{3}{2}}} \tag{4-8}$$

TABLE I.

(a) $a = \sqrt{\frac{3}{2}}$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
F(z)	0.880-1	0.880-1	0.880-1	0.880-1	0.878-1	0.875-1	0.872-1	0.859-1	0.843-1	0.822-1	0.794-1

(b) $a = \sqrt{2}$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
F(z)	0.887-1	0.887-1	0.888-1	0.890-1	0.894-1	0.892-1	0.890-1	0.884-1	0.872-1	0.852-1	0.824-1

(c) $a = \sqrt{3}$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	1.0
F(z)	0.875-1	0.875-1	0.881-1	0.886-1	0.891-1	0.898-1	0.904-1	0.907-1	0.903-1	0.887-1	0.873-1	0.859-1

(d) $a = 2$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
F(z)	0.855-1	0.857-1	0.862-1	0.870-1	0.880-1	0.893-1	0.905-1	0.914-1	0.917-1	0.907-1	0.880-1	0.832-1

(e) $a = 3$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
F(z)	0.755-1	0.759-1	0.768-1	0.781-1	0.802-1	0.830-1	0.861-1	0.896-1	0.926-1	0.940-1	0.922-1	0.862-1	0.771-1

Table II

$\alpha \backslash a$	$\sqrt{\frac{3}{2}}$	$\sqrt{2}$	$\sqrt{3}$	2	3	
0.5	0.05 0.231	0.07 0.275	0.23 0.334	0.38 0.374	0.75 0.465	D_o $A(\alpha a)^2$
0.6	0.08 0.331	0.07 0.396	0.29 0.484	0.50 0.546	1.06 0.695	D_o $A(\alpha a)^2$
0.7	0.21 0.445	0.10 0.537	0.32 0.660	0.59 0.751	1.41 0.985	D_o $A(\alpha a)^2$
0.8	0.37 0.570	0.22 0.692	0.32 0.863	0.62 0.985	1.71 1.33	D_o $A(\alpha a)^2$
0.9	0.58 0.705	0.42 0.856	0.32 1.091	0.62 1.247	1.85 1.71	D_o $A(\alpha a)^2$
1.0	0.86 0.842	0.70 1.022	0.48 1.323	0.62 1.54	1.85 2.11	D_o $A(\alpha a)^2$
1.1				0.85 1.813	1.85 2.56	D_o $A(\alpha a)^2$
1.2					1.85 3.04	D_o $A(\alpha a)^2$
0.95			0.35 1.212			D_o $A(\alpha a)^2$

From Table II it is seen that the practical values of a lie between $\sqrt{3}$ and 2. $a = \sqrt{2}$ and $\alpha = 0.8$ gives excellent linearity. $a = \sqrt{3}$ and $\alpha = 1$ gives greater sensitivity and yet still very good linearity. If higher values of a are used, there is no reason to choose α less than 1. The sensitivity increases with the value of a.

Example 1.

Given data: $f_i = 5 \times 10^6$ cycles, $\Delta f = 75 \times 10^3$ cycles.

$D_o \leqq 0.7$ db $\qquad C = 50\ \mu\mu f$

From Table II, choose $a = 2$, $\alpha = 1$, then $D_o = 0.62$, $A(\alpha a)^2 = 1.54$

From (4-1) $\quad m = \frac{\Delta f}{f_i} = \frac{75 \times 10^3}{5 \times 10^6} = 15 \times 10^{-3}$

$$Q_o \simeq \frac{2}{2 \times 15 \times 10^{-3}} = 66.7$$

From (4-2) $\quad k \simeq 2 \times 15 \times 10^{-3} = 0.03$

From (4-3) $\quad L \simeq \frac{\left(1 + \frac{2}{2 \times 66.7}\right)^2}{4\pi^2 \times 25 \times 10^{12} \times 50 \times 10^{-12} \times 1.030} = 20.3 \times 10^{-6}$ henry

From (4-4) $\quad R \simeq 66.7 \times \frac{1 + \frac{2}{2 \times 66.7}}{2\pi \times 5 \times 10^6 \times 50 \times 10^{-12}} = 43100$ ohms

Assume $\eta = 0.8$, from (4-5)

$$S \simeq 0.8 \frac{1.54}{8\pi \times 5 \times 10^6 \times 50 \times 10^{-12} \times 225 \times 10^{-6}} = 0.872 \times 10^6$$

Assume $G_m = 2000 \times 10^{-6}$ mho, $|E_s| = 1$ peak value, then

$$|I_1| = G_m |E_s| = 2000 \times 10^{-6}$$

For $\frac{\delta f}{f_i} = \frac{1}{100}$

$$|V_s| \simeq 0.872 \times 10^4 \times 2000 \times 10^{-6} = 17.4 \text{ volts.}$$

From (4-6) $$\frac{1}{R_L C_L} = 2\pi f_a.$$

Choose $C_L = 50\ \mu\mu f$, $f_a = 10000 \sim$, then

$$R_L = \frac{1}{2\pi \times 10000 \times 50 \times 10^{-12}} = 0.318 \times 10^6 \text{ ohms.}$$

Use $R_L = 0.3 \times 10^6$ ohms

Example 2.

Given Data: $f_i = 5 \times 10^6 \sim$, $\Delta f = 75 \times 10^3$

$D_o \leqq 0.4$ db $\qquad C = 50\ \mu\mu f$

By Table II, choose $a = \sqrt{3}$, $\alpha = 0.95$

Then $D_o = 0.35$ $\qquad A(\alpha a)^2 = 1.212$

$$m = 15 \times 10^{-3}$$

$$Q_o \simeq \frac{\sqrt{3} \times 0.95}{2 \times 15 \times 10^{-3}} = 54.9$$

$$k \simeq \frac{2 \times 15 \times 10^{-3}}{0.95} = 0.0316$$

$$L \simeq \frac{\left(1 + \frac{\sqrt{3}}{2 \times 54.9}\right)^2}{4\pi^2 \times 25 \times 10^{12} \times 50 \times 10^{-12} \times 1.032} = 20.3 \times 10^{-6} \text{ henry}$$

$$R \simeq \frac{54.9\ \left(1 + \frac{\sqrt{3}}{2 \times 54.9}\right)}{2\pi\ 5 \times 10^6 \times 50 \times 10^{-12}} = 35500 \text{ ohms}$$

Assume $\eta = 0.8$

$$S \simeq 0.8 \frac{1.212}{8\pi \times 5 \times 10^6 \times 50 \times 10^{-12} \times 225 \times 10^{-6}} = 0.686 \times 10^6$$

Assume $G_m = 2000 \times 10^{-6}$ mhos, $|E_s| = 1$ peak value.

For $\frac{\delta f}{f_i} = \frac{1}{100}$

$$|V_s| \simeq 0.686 \times 10^4 \times 2000 \times 10^{-6} = 13.7 \text{ volts}$$

$$\frac{1}{R_L C_L} = 0.95 \times \sqrt{3}\,\pi f_a = 5.17 f_a$$

$$z = \frac{y}{a} \qquad f(az) = \frac{1}{\sqrt{1 + a^2(z-1)^2}} - \frac{1}{\sqrt{1 + a^2(z+1)^2}} \qquad (4\text{-}9)$$

The two useful values of a are $a = 2$ and $a = \sqrt{3}$.

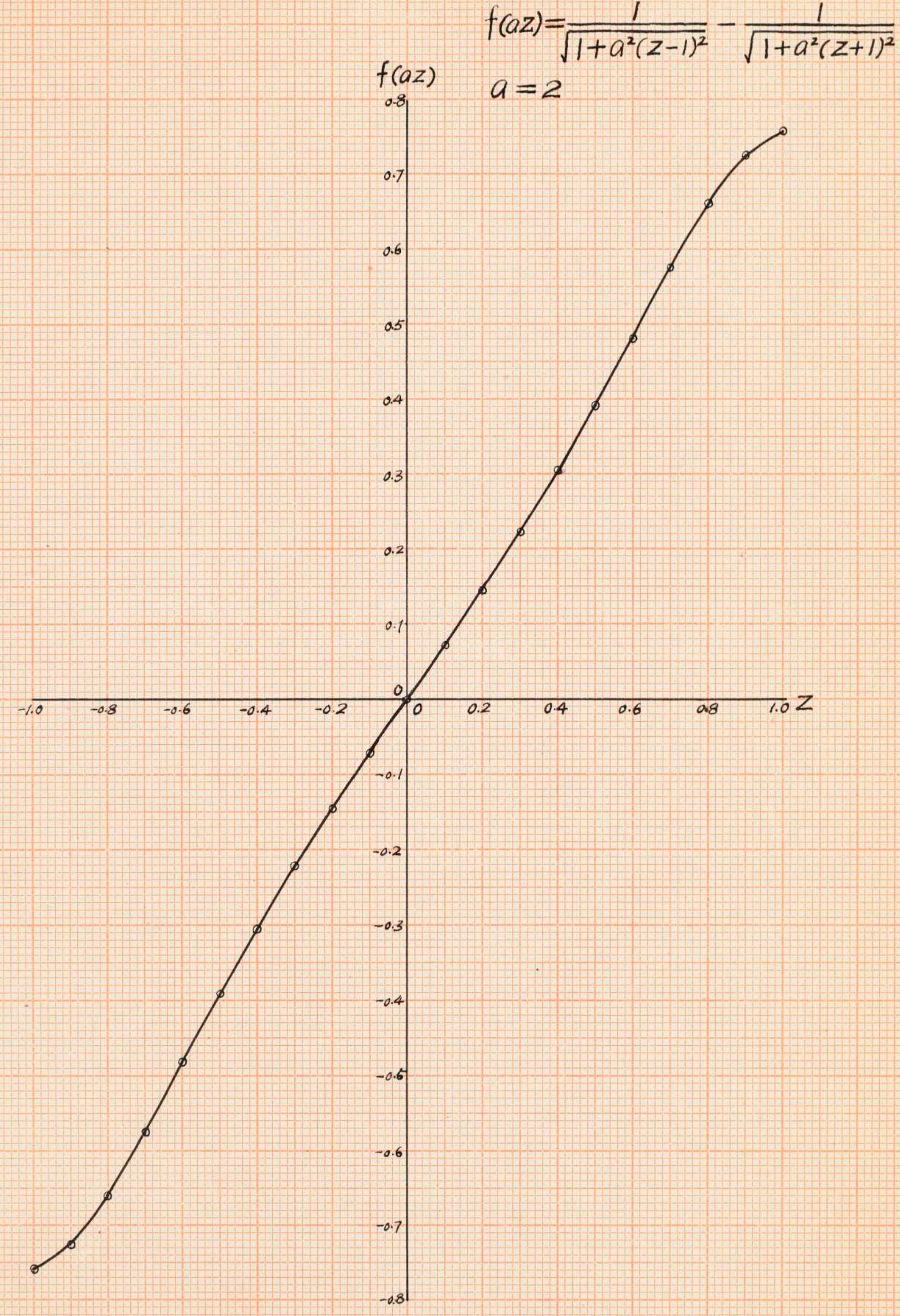
1. $a = 2$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
f(az)	0	0.0716	0.146	0.222	0.304	0.391	0.482	0.576	0.660	0.726	0.758

2. $a = \sqrt{3}$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
f(az)	0	0.0752	0.152	0.230	0.312	0.396	0.4825	0.565	0.639	0.694	0.7225

These curves are plotted in Figure 4-1 and Figure 4-2.



$$f(az) = \frac{1}{\sqrt{1+a^2(z-1)^2}} - \frac{1}{\sqrt{1+a^2(z+1)^2}}$$
$a = 2$
f(az)
Z
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-1.0
-0.8
-0.6
-0.4
-0.2
0.2
0.4
0.6
0.8
1.0

Fig. 4-1

$$f(az)=\frac{1}{\sqrt{1+a^2(z-1)^2}}-\frac{1}{\sqrt{1+a^2(z+1)^2}}$$

$a=\sqrt{3}$

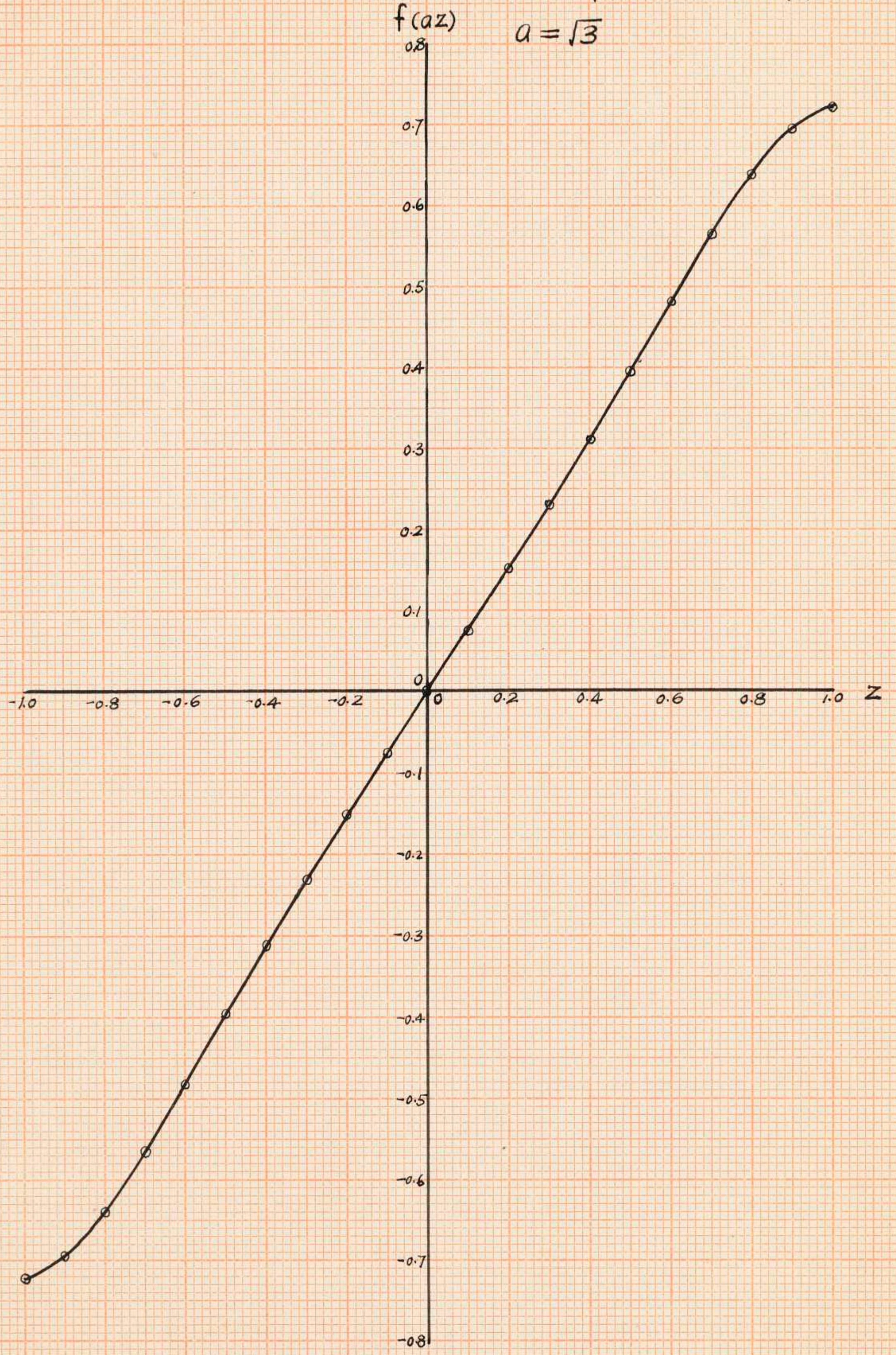


Fig. 4-2

V. EXPERIMENTAL SETUP AND RESULTS

The complete experimental discriminator circuit is shown in Figure 5-1. A list of parts is given below:

T: commercially made discriminator transformer

$$\left.\begin{array}{ll} L_1 = 46.8 \times 10^{-6}\ h & Q_1 = 53 \\ L_2 = 42.4 \times 10^{-6}\ h & Q_2 = 55 \end{array}\right\} \text{unmounted and without shield.}$$

The more balanced winding is chosen as the secondary.

Unfortunately L_1 and L_2 are not exactly equal.

$$\left.\begin{array}{l} C_1 \\ C_2 \end{array}\right\} \text{adjusted to } f_o \simeq 3.8 \text{ mc.}$$

R_1, R_2: subjected to choice.

R_3, R_4: 2MΩ carbon resistors.

R_5: 250Ω carbon resistor.

R_6: 62500Ω carbon resistor.

R_7: 1MΩ carbon resistor.

R_8: 0.5 MΩ carbon resistor.

R_9: 50000Ω carbon resistor

R_{10}: 0.5MΩ carbon resistor.

C_3: 100 $\mu\mu f$ mica condenser.

C_4, C_5, C_6: 0.001 μf mica condensers.

C_7: 100 $\mu\mu f$ mica condenser.

C_8: 50 μf electrolytic condenser.

C_9: Electrolytic condenser contained in the a-c voltage power supply rectifier circuit.

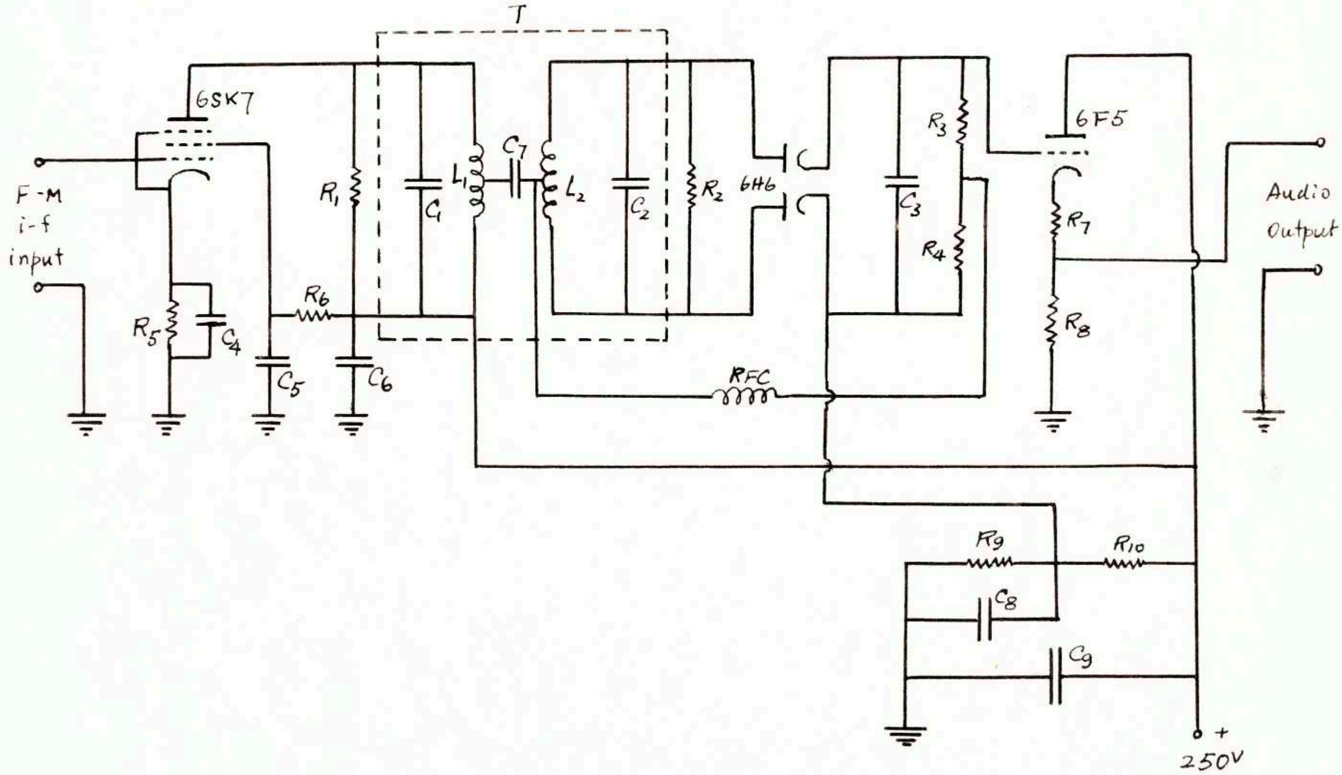

T
6SK7
F-M
i-f
input
R5
C4
C5
R6
C6
R1
C1
L1
C7
L2
C2
R2
6H6
R3
C3
R4
6F5
R7
R8
Audio
Output
RFC
R9
C8
R10
C9
+
250V

Fig. 5-1

The time constant of $R_3 + R_4$ and C_3 is seen to be too large to avoid the clipping of high audio modulating frequencies. As the present purpose is to check the results derived from the expression (2-7), it is advisable in the experiment to use a low modulating frequency and high values for R_3 and R_4 so that the diode loading effect is minimized. The 6F5 stage is a cathode follower introduced to eliminate the effect of the measuring instrument on the diode load. This stage introduces distortion less than 0.1%.

The entire experimental setup is shown in Figure 5-2. The modulating frequency is chosen as 100. The filter used is a simple band-pass filter formed by two coupled resonant circuits.

Measurements were first made on the coils of the discriminator transformer. The better balanced winding is chosen as the secondary. As measured with a Q-meter, the primary and the secondary inductances, unmounted and without the shield, are

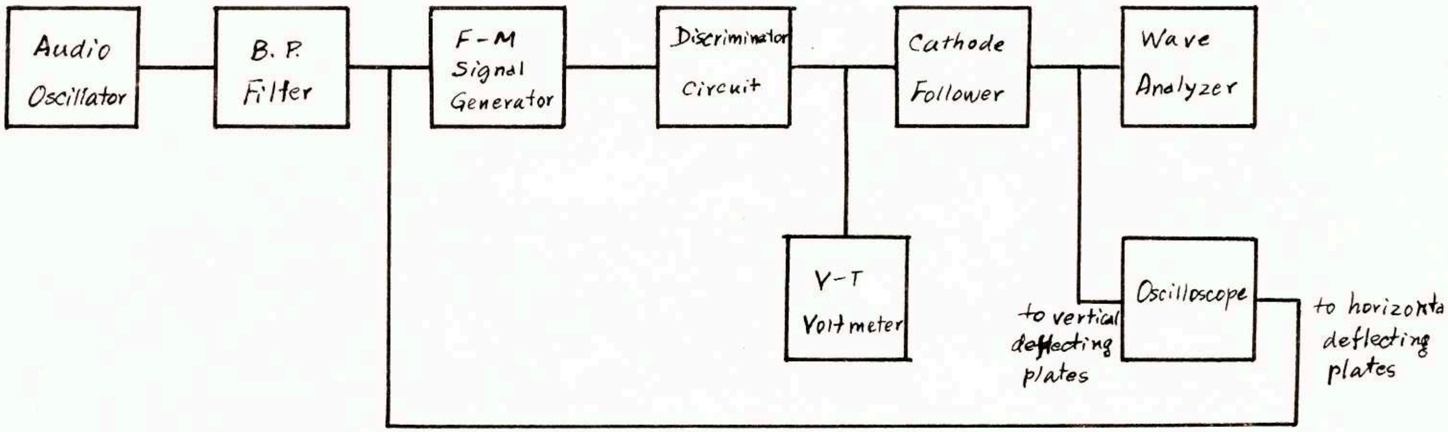
$$L_1 = \frac{1}{4\pi^2\ (2.6)^2 80} = 46.8 \times 10^{-6} \text{ heavy}$$

$$L_2 = \frac{1}{4\pi^2\ (2.6)^2 88.4} = 42.4 \times 10^{-6} \text{ heavy}$$

$$Q_1 = 53 \text{ at } \ f = 3.7 \text{ mc.}$$

$$Q_2 = 55 \text{ at } \ f = 3.7 \text{ mc.}$$

The condensers associated with the coils are variable air condensers ranging approximately from 7 $\mu\mu f$ to 40 $\mu\mu f$. The mutual inductive coupling is measured with the arrangement of Figure 5-3.
The condenser C is adjusted to give *null* output in the earphone. At balance, since $C \ll C_1$, and $C \ll C_2$



Audio Oscillator
B. P. Filter
F-M Signal Generator
Discriminator Circuit
Cathode Follower
Wave Analyzer
V-T Voltmeter
Oscilloscope
to vertical deflecting plates
to horizontal deflecting plates

Fig. 5-2

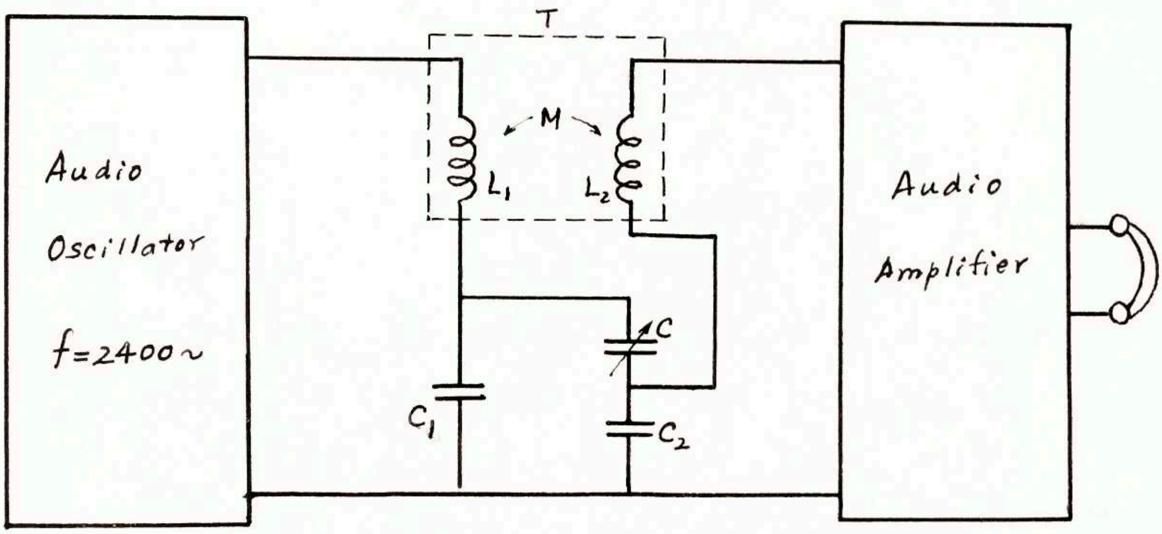


Fig. 5-3

$$2\pi f \; k \sqrt{L_1 L_2} = \frac{1}{2\pi f C_1} \; \frac{C}{C_2}$$

or $$k = \frac{C}{4\pi^2 f^2 \sqrt{L_1 L_2} \; C_1 C_2} \qquad (5\text{-}1)$$

Now $C = 517\mu\mu f$, $C_1 = C_2 = 1\mu f$ gives the balance point, hence

$$k = \frac{517}{4\pi^2 \times 24^2 \times 10^6 \times \sqrt{42.4 \times 46.8 \times 10^6}} = 0.051$$

To check with the theory, the value of a is made small for two reasons: (1) when a is small, the Q's of the resonants are low and consequently the diode loading effect is less important, (2) when a is small the expressions (2-19) and (2-22) converge more quickly, resulting in easier numerical computations.

Let $f_i = 3.8$ mc. After mounting it was found $Q_1 = 45$, $Q_2 = 47$. Neglecting the diode loading effect and taking into consideration the effect of the plate resistance of the 6SK7 tube on the primary it can be shown that for $Q = 30$, $R_1 = 114000$, $R_2 = 84200$ (Fig. 5-1)

$$a \simeq kQ_o = 0.051 \times 30 = 1.53 \simeq \sqrt{2.5}$$

The value of a is further checked by utilizing the relation (4-1). In the arrangement of Figure 5-2, the frequency deviation is increased until the peak detected voltage corresponding to $\alpha = 1$ is just reached, as indicated by the oscillograph. It was found that at $f_i = 3.8$ mc $\alpha = 1$ occurs at $\Delta f = 100$ KC. Hence,

$$a = 2 \times \frac{100 \times 10^3}{3.8 \times 10^6} \times 30 = 1.58 = \sqrt{2.5}$$

It is to be pointed out that the assumption that the diode loading effect is negligible may not be quite justified in the present case.

The theoretical computations using the expression (4-9) are tabulated below:

$$a = \sqrt{2.5} = 1.58$$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
f(az)	0	0.0765	0.1539	0.2325	0.313	0.3955	0.4775	0.554	0.6225	0.671	0.6985

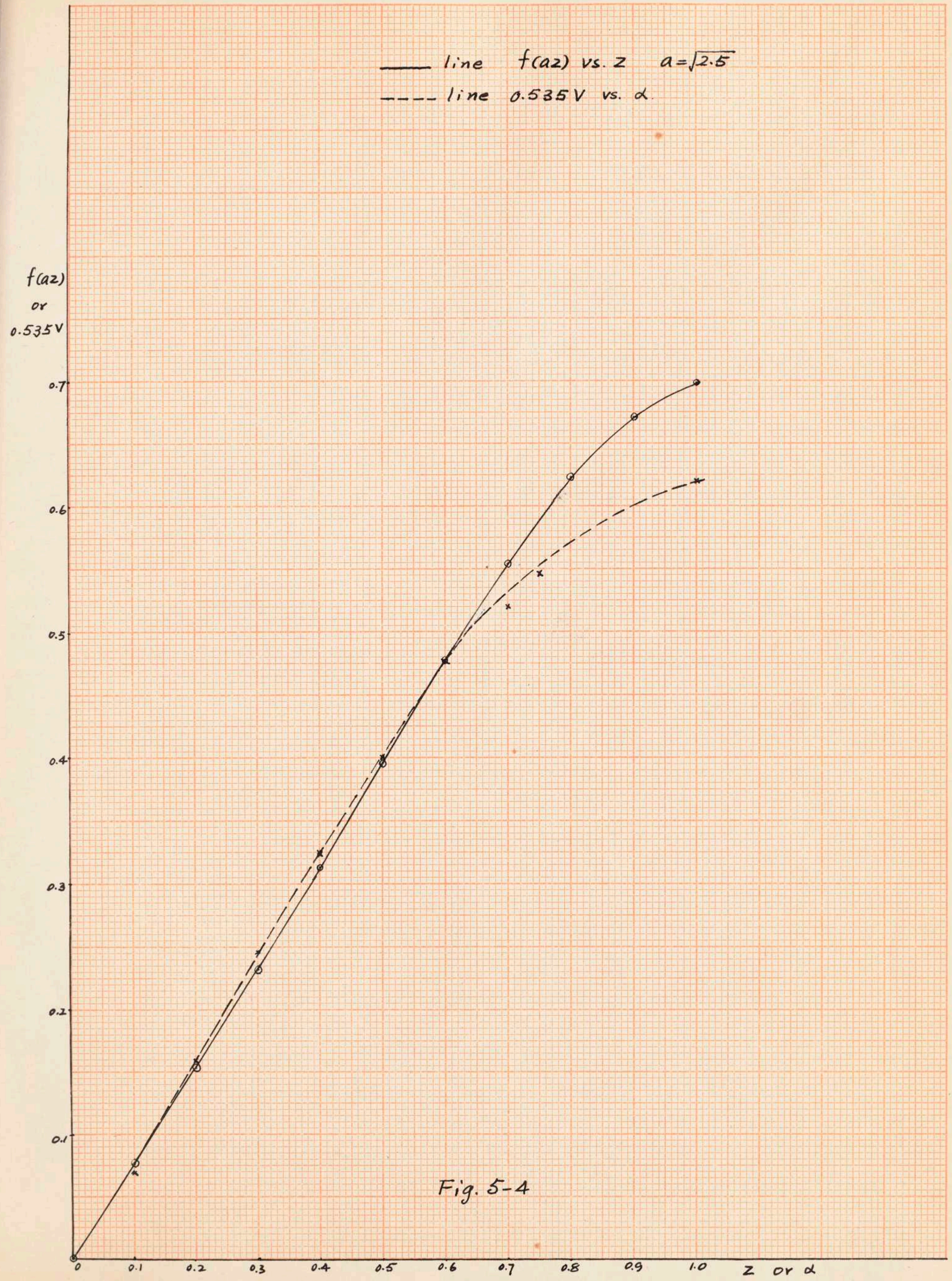
To compare with the computed values the detected voltage at different frequency deviations was measured with the i-f input voltage kept constant. Since R_L is very high, η is almost unity and essentially constant. The experimental results are:

f(kc)	100	75	70	60	50	40	30	20	10
α	1.0	0.75	0.70	0.60	0.50	0.40	0.30	0.20	0.10
V(volts)	1.16	1.02	0.98	0.89	0.75	0.61	0.46	0.3	0.13
0.535V	0.62	0.546	0.525	0.476	0.401	0.326	0.246	0.16	0.0695

0.535V is plotted against α in Figure 5-4 and compared with the curve of f(az) vs. z plotted in the same figure. The discrepency will be discussed later.

The harmonic contents for $a = \sqrt{2.5}$ are computed using the formulas (2-19) and (2-22). The experimental values measured with a wave analyzer do not check with computed ones. This is not surprising because the instruments used are not adequate when small distortions are to be measured. Actually, it is possible that the linearity of the F-M signal generator used is worse than the linearity of the discriminator for $a = \sqrt{2.5}$. The following quotation is taken from an article by the designers of the signal generator[4]: "At swings of plus and minus 75kc, the departure from linearity does not exceed 1%. This is for the modulator only. In order that the overall distortion of the complete signal generator shall not greatly exceed this figure, it has been found

4. A. W. Barber, C. J. Franks, and Richardson, "A Signal Generator for Frequency Modulation," Electronics, vol. 14, pp. 36-38, 92-95, April 1941



line f(az) vs. z a=√2.5
line 0.535V vs. α.
f(az) or 0.535V
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
z or α

Fig. 5-4

necessary to drive the reactance tube modulator from a low-impedance source." In measuring the harmonic contents the audio output voltage was kept constant instead of the i-f input voltage.

Computed harmonic contents:

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7
A_1	0.0765	0.153	0.232	0.312	0.393	0.473	0.552
A_3	-36.9×10^{-6}	-0.000295	-0.000861	-0.00184	-0.00289	-0.00329	-0.00204
A_5	-0.0891×10^{-6}	-2.9×10^{-6}	-25.9×10^{-6}	-0.000114	-0.000300	-0.00106	-0.0025
$100\frac{\lvert A_3\rvert}{A_1}$	0.0482	0.193	0.371	0.590	0.735	0.697	0.37
$100\frac{\lvert A_5\rvert}{A_1}$	0	0	0.0112	0.0365	0.0764	0.224	0.453

In this table although three significant figures are given, there is no claim to accuracy to three significant figures.

Measured harmonic contents

f (kc)	100	75	70	60	50	40	30
α	1	0.75	0.70	0.60	0.50	0.40	0.30
2nd(%)	0.28	0.4	0.22	0.3	0.5	0.57	0.54
3rd (%)	8.4	4.4	4.0	3.2	2.3	1.85	1.7
4th(%)	0.5	0.45	0.26	0.28	0.1	0.06	0.07
5th(%)	0.53	0.25	0.21	0.25	0.35	0.38	0.46

2. Let $Q = 11$, then $R_1 = 16600\,\Omega$, $R_2 = 14500\Omega$ and $a \simeq 0.577 = \frac{1}{\sqrt{3}}$

The computed third harmonic contents are listed in the following table.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$\frac{\sqrt{1+a^2}}{4} A_1$	0.0125	0.025	0.0369	0.0487	0.06	0.0706	0.0806
$\frac{\sqrt{1+a^2}}{4} A_3$	6.84×10^{-6}	5.47×10^{-5}	18.4×10^{-5}	43.3×10^{-5}	81.9×10^{-5}	0.001435	0.00222
$\frac{\lvert A_3 \rvert}{A_1} 100$	0.0547	0.219	0.50	0.889	1.37	2.05	2.76

It is observed that the distortion of the signal generator is still far from negligible in comparison. The measured harmonic contents are as given below:

Δf(kc)	100	75	70	60	50	40
2nd(%)	0.73	0.54	0.61	0.54	0.45	0.36
3rd(%)	6.7	3.90	3.6	2.8	2.0	1.65
5th(%)	0.35	0.25	0.28	0.3	0.33	0.36

The fourth harmonic was found very small.

The following point might be pointed out. The two resonant circuits must be tuned to the same frequency, together with the shunt stray capacitances. It was found not very practical to tune the circuits separately with a vacuum-tube voltmeter as an indicator of resonance without affecting the circuit. Hence, the circuit was adjusted by the criterion that the even harmonics should be a minimum, theoretically zero. As the signal generator introduces a large amount of second harmonics, the fact that the second harmonic is a minimum in the detected output voltage does not indicate the right tuning adjustment. In fact, the adjustment to give the least second harmonic in the detected output voltage was found to vary with the frequency deviation, as might be expected in the presence of actual imperfections. But due to the lack of other good methods, this method of adjustment was used.

The measured distortion is the overall magnitude contributed both by the signal generator and the discriminator. Although of no theoretical interest, it might be mentioned that with some cut-and-try adjustments, small overall distortion can be achieved. For example, if $R_1 = 0.5\ M\Omega$, $R_2 = 0.25\ M\Omega$ (Figure 5-1), the measured harmonic contents are as listed below.

Δf (Kc)	75	70	60	50	40	30
α	0.75	0.70	0.60	0.50	0.40	0.30
2nd(%)	0.51	0.35	0.34	0.41	0.48	0.58
3rd(%)	1.20	1.1	0.66	0.64	0.88	1.20
4th(%)	0.16	0.09	0.09	0.1	0.1	0.11
5th(%)	0.55	0.38	0.7	0.3	0.37	0.45

The pictures of the patterns shown by the oscilloscope for different values of a are given in Figures 5-5, 5-6 and 5-7. The picture shown in Figure 5-8 illustrates the clipping of high audio modulating frequencies.

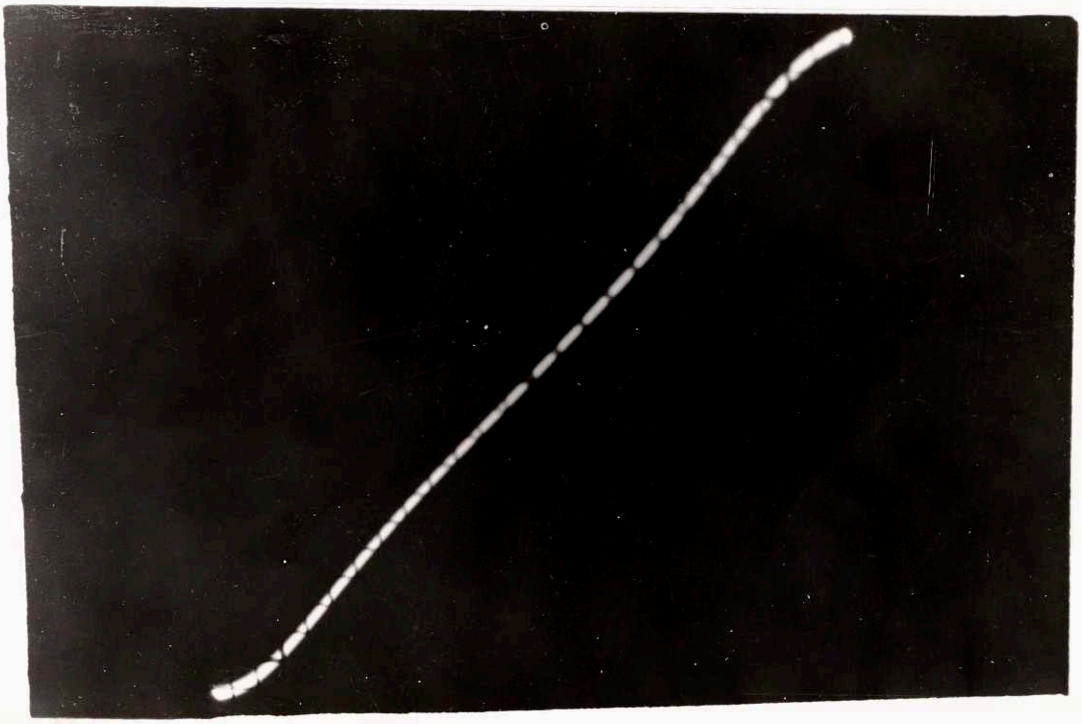


Fig. 5-5 $a = 2.3, \quad \alpha = 1$

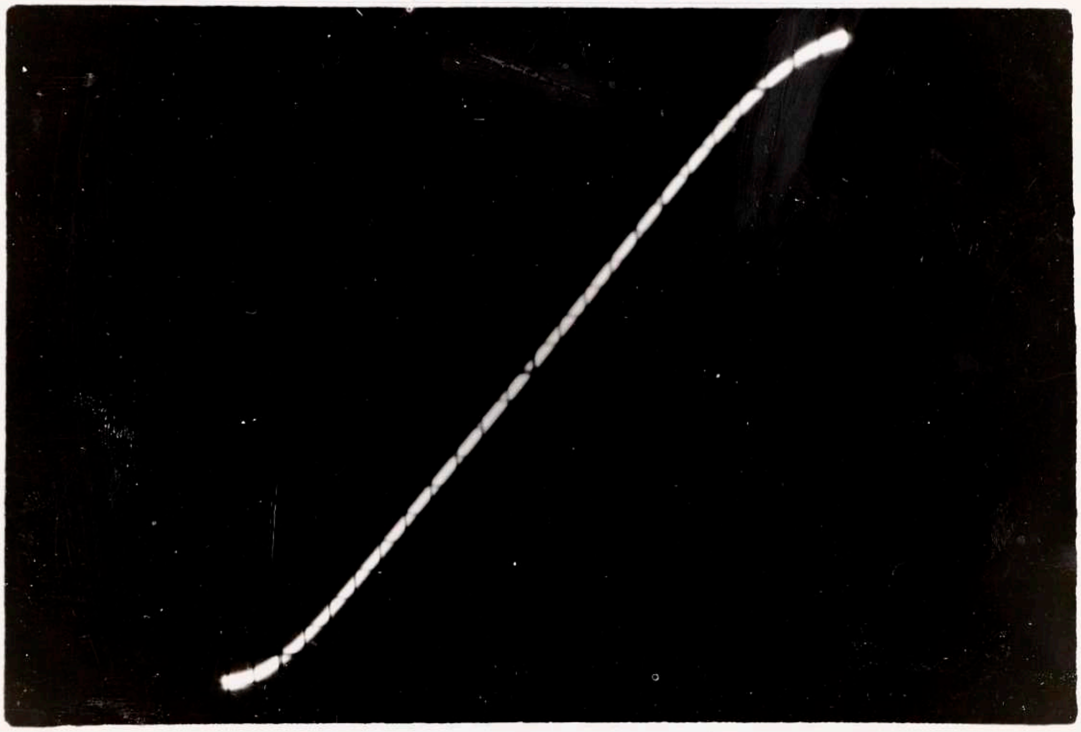


Fig. 5-6 $a = \sqrt{2 \cdot 5} = 1.58$ $\alpha = 1$

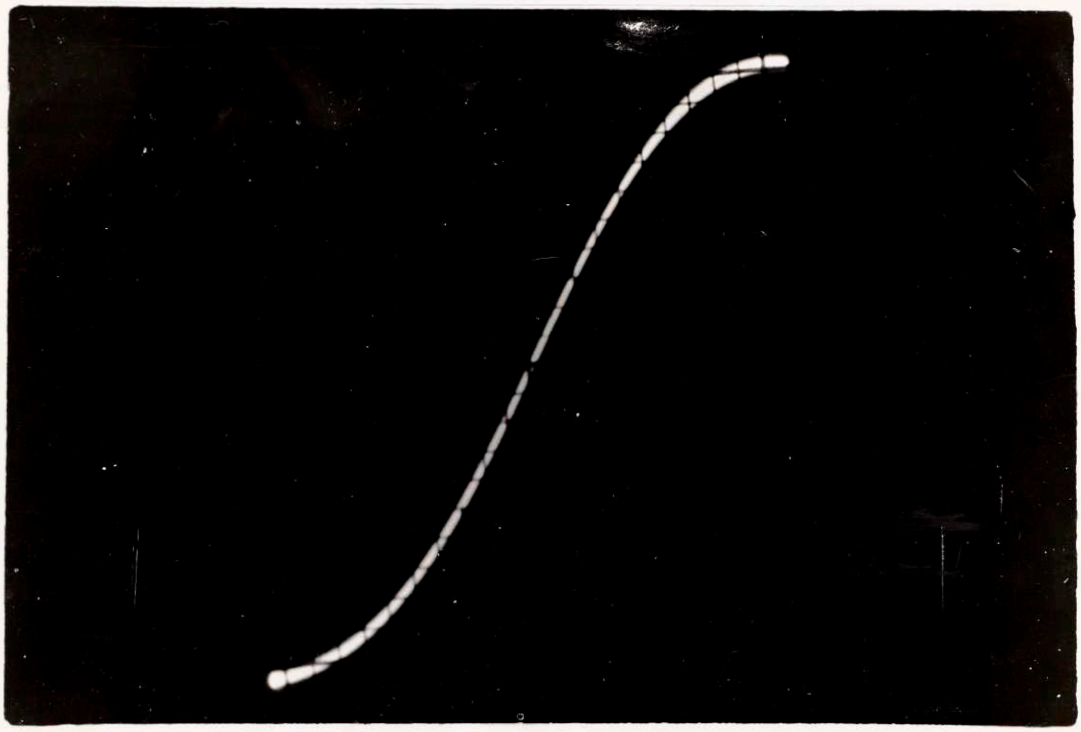


Fig. 5-7 $\quad a = \frac{1}{\sqrt{3}} = 0.577 \quad \alpha = 1$

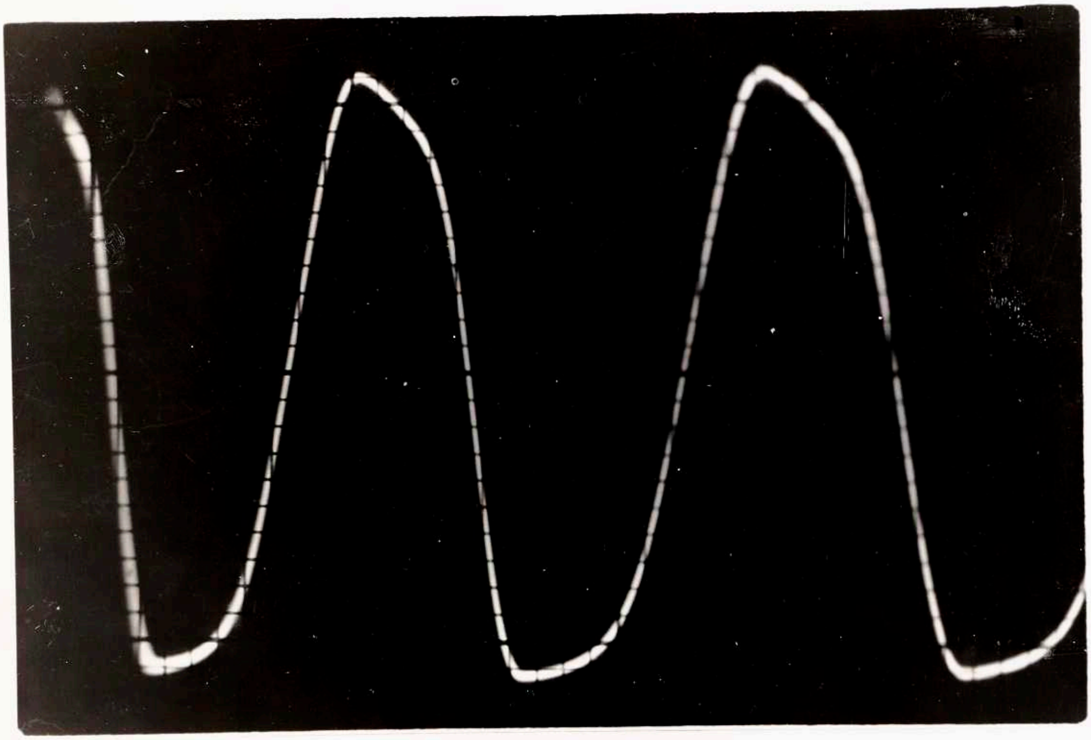


Fig. 5-8

VI. CONCLUSION

The whole theoretical discussion involves many approximations based on the assumption that $m = \frac{\Delta f}{f_i}$ is very small, which is usually true. Furthermore, it is assumed that the diode loading effect is negligible. Unfortunately, the latter assumption is probably not quite justified in most cases, and this problem remains to be solved. Experimentally, it was found that with the diode load resistance as low as 0.1 M the Q's of the resonant circuits can be adjusted to give practically as low distortions as when the diode load resistance is high, although no numerical check with the idealized theory is to be expected. Hence, while not presenting any serious difficulty in achieving low-distortion detection of F-M signals, the diode loading effect does impair the simplicity and the elegance of the theoretical treatment of the idealized case.

In the experiment many imperfections are present. The most serious of all is the error of the instruments as discussed in the preceding section. They prevent accurate measurements with reliable results. Other minor imperfections are due to the inaccuracy of the circuit constants such as L, C, Q etc. and due to stray capacitive couplings.

In spite of the idealized assumption that there is no diode loading effect in the theoretical discussions, the work is useful in that it is the first approach to the problem. With the idealized assumption the whole treatment is simple and nice and the design procedure can be constructed on a rational basis. A good check with the idealized theory can be expected if a distortionless F-M signal generator is available and if great care is taken in winding the coils and measuring the circuit constants and, of course, if the diode load resistances are high enough.

APPENDIX 1.

The Derivation of Expressions (2-5)

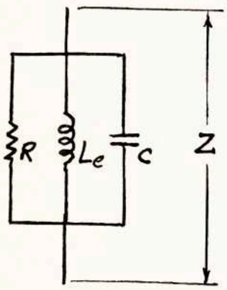


Fig. A1-1

$$Z = \frac{RZ'}{R+Z'} \qquad \text{(A1-1)}$$

$$\text{where} \quad Z' = \frac{j\omega L_e \frac{1}{j\omega c}}{j\omega L_e + \frac{1}{j\omega c}} = \frac{\frac{L_e}{C}}{j\omega L_e \left(1 - \frac{f_o^2}{f^2}\right)}$$

$$= \frac{\frac{R^2}{Q_o^2}}{j \frac{R f}{Q_o f_o} \left(1 - \frac{f_o^2}{f^2}\right)} = \frac{R}{j Q_o \left(\frac{f}{f_o} - \frac{f_o}{f}\right)}$$

Let $x = Q_o \left(\frac{f}{f_o} - \frac{f_o}{f}\right)$ then

$$Z' = \frac{R}{jx} \qquad \text{(A1-2)}$$

$$Z = \frac{\frac{R^2}{jx}}{R + \frac{R}{jx}} = \frac{R}{1+jx} \qquad \text{(A1-3)}$$

APPENDIX 2.

The Derivation of Expressions (2-7) and (2-8)

Consider the circuit of Figure 3(b), which can be redrawn as in Figure A2-1,

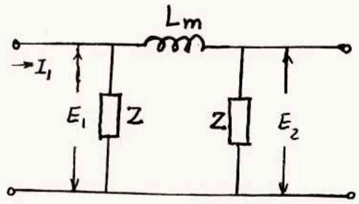


Figure A2-1

where $Z = \frac{R}{1+jx}$

$$E_1 = I_1 \frac{Z(Z + j\omega L_m)}{2Z + j\omega L_m} \tag{A2-1}$$

$$E_2 = I_1 \frac{Z(Z + j\omega L_m)}{2Z + j\omega L_m} \frac{Z}{Z + j\omega L_m} = I_1 \frac{Z^2}{2Z + j\omega L_m} \tag{A2-2}$$

$$\frac{E_1 + E_2}{I_1} = \frac{Z(Z + j\omega L_m)}{2Z + j\omega L_m} + \frac{Z^2}{2Z + j\omega L_m} = Z = \frac{R}{1+jx} \tag{A2-3}$$

$$\frac{E_1 - E_2}{I_1} = \frac{Z(Z + j\omega L_m)}{2Z + j\omega L_m} - \frac{Z^2}{2Z + j\omega L_m} = \frac{j\omega L_m Z}{2Z + j\omega L_m}$$

Let $L_m = k_e L_e \qquad k_e = \frac{1-k^2}{k} \simeq \frac{1}{k}$ (A2-4)

$$j\omega L_m = j \frac{f}{f_o} k_e \omega_o L_e = j \frac{f}{f_o} k_e \frac{R}{Q_o} \simeq jk_e \frac{R}{Q_o} \tag{A2-5}$$

$$\frac{E_1 - E_2}{I_1} \simeq \frac{j k_e \frac{R}{Q_o} \frac{R}{1+jx}}{\frac{2R}{1+jx} + jk_e \frac{R}{Q_o}} = \frac{j \frac{k_e}{Q_o} R}{2 + j \frac{k_e}{Q_o}(1+jx)}$$

or $$\frac{E_1 - E_2}{I_1} = \frac{R}{1 + j\left(x - \frac{2Q_o}{k_e}\right)}$$

Let $a = \frac{Q_o}{k_e} = Q_o \frac{k}{1-k^2} \simeq kQ_o$ (A2-6)

and $y = x - a$ (A2-7)

Then
$$\left.\begin{aligned} \frac{E_1 + E_2}{I_1} &= \frac{R}{1 + j(y+a)} \\ \frac{E_1 - E_2}{I_1} &= \frac{R}{1 + j(y-a)} \end{aligned}\right\} \tag{A2-8}$$

$$\therefore \quad f(y) = 2\frac{V}{|I_1|} = \left|\frac{E_1 - E_2}{I_1}\right| - \left|\frac{E_1 + E_2}{I_1}\right| = \frac{R}{\sqrt{1+(y-a)^2}} - \frac{R}{\sqrt{1+(y+a)^2}} \qquad \text{(A2-9)}$$

APPENDIX 3.

List of Instruments Used.

1. F-M signal generator (modulated by an external audio oscillator)

   Type 150-A, Boonton Radio Corporation

2. Audio oscillator

   Type 205-A, Boonton Radio Corporation

3. V-T voltmeter

   Type 726-A, General Radio Company

4. Wave analyzer

   Type 736-A, General Radio Company

5. Oscilloscope

   Type 208, Dumont Laboratories Inc.

6. Precision condenser

   Type 722-D, General Radio Company

7. Audio amplifier

APPENDIX 4.

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