Design
for an
Iron Railway Bridge,
with a
Consideration of the Principles
determining the Design.

Geo. F. Swain.

Mass. Inst. Tech.

May, 1877.

Index.

Subject.	Page.
Introduction and General Statement	4
Limitation of subject.	6
Determination of span and number of piers.	7
(a.) General Considerations.	7
(b.) Theoretical solution.	14
(c.) application to the present case.	17
Type of Bridge.	18
(a.) General considerations, and difficulties in the way of a complete solution.	19
(b.) Comparison of the different forms of truss.	26
(c.) Table of results.	34
Materials.	39
Connections.	41
Dimensions assumed, etc.	41
Amount and Distribution of load.	42
Limits of Stress.	49
Determination of Stresses.	50
(a) Chords.	50

(b.) End post. 52

(c) Vertical ties 52

(d.) Web. 52

"Stress" Sheet. 56.

Determination of dimensions. 56.

(a) Ties. 57

(b.) Upper chord. 59

(c) End post. 62

(d) Web struts. 62

(e) Pins. 67

(f) Reinforcing plates 72

(g.) Rivets 74

(h.) Lateral bracing 78

(k.) Floor beams. 83.

(l.) Stringers. 87.

General Details. 88

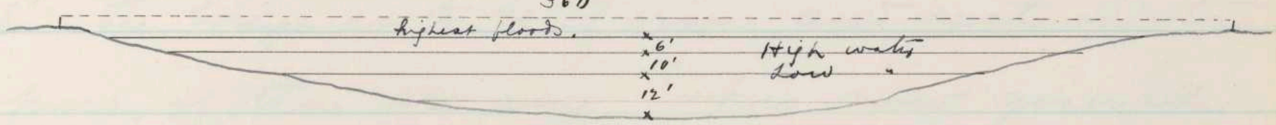
Introduction and General Statement.

It is attempted in this thesis to solve the following problem: it is required to design a truss bridge to carry a single track railway over a river, the length of span from abutment to abutment, nature of the soil, current, navigation, climate, etc., being assumed as will be presently described, and this must be done at the least cost consistent with thorough workmanship and ample strength and stability. In attempting a solution of this problem I shall discuss briefly the general principles determining my design, and shall endeavor to give a reason for every step. The following are the assumed conditions;—

1°. The bridge is to be straight, and the stream is crossed at right angles.

2°. The nature of the navigation in the river is not such as to prevent piers in the stream, but a water-way of twenty feet is required at all states of the water. The tides do not affect the river at the supposed location.

3°. The contour of the bed of the stream is as shown in the figure.



4°. The current is in general rather sluggish, but during floods it assumes a surface velocity in the centre of the stream of seven feet a second.

5°. The climate is quite cold, and in winter large quantities of ice are formed and brought down by the stream. The river does not close every year, and when it does, the duration of the close is uncertain. In still water the thickness of ice is from 12 to 15 inches; but in the river, on account of packing, the thickness is uncertain.

6° The soil is sand and gravel, mixed with clay.

and extends to a depth of from 10 to 20 feet below the bottom of the stream, at which depth a stratum of sandstone commences

7°. Plenty of timber is at hand, but rock can only be obtained with some difficulty.

8°. The bridge is to be of iron and rests on stone piers, if there are any. "It is now generally admitted that the original cheapness of timber structures does not compensate for their rapid decay, their frequent destruction by fire, and the constant repairs and watching which they demand."

I shall limit myself, for want of time, to the bridge proper, not considering the piers and abutments. These structures offer no particular difficulty, being calculated by the ordinary formulae for walls and buttresses. I originally intended to consider them, hence the above details, but want of time forbids.

Determination of Span and Number of Piers.

The first step in proceeding to design the required structure is to determine how many spans and piers there shall be. The object is, of course, to have the total cost of piers and superstructure a minimum, and a correct estimate in regard to the number of spans is a matter of great delicacy and importance. It may be doubted whether any very long bridge was ever built which absolutely fulfilled the above condition, and the cause of this fact is easily recognized on an examination of the variable elements entering into the problem, which are of such a nature that they can not be exactly estimated. The judgment of the engineer is thus the chief thing to be relied upon, and upon his skill and experience will depend the more or less accurate fulfilment of the given condition.

In a short bridge the determination becomes comparatively easy, and when the question is between one pier or none, the experienced engineer can scarcely hesitate, on a careful examination of the local circumstances.

As regards the structure itself a number of elements are constant, being the same whatever the number of spans; these include the track, flooring, abutments, etc. The variable elements are the piers, foundations, and main girders or trusses. The cost of a truss bridge is roughly proportional to the weight of material in it, and hence increases with the length of the spans and the consequent increase of the dimensions of the parts. If this were the only consideration, therefore, the smaller the span the better; but since numerous spans require numerous piers, and since the cost of piers in a long bridge is a considerable proportion of the total cost, we must endeavor to find at what point we can strike the best mean. It is often very difficult and expensive to build piers, for the river may be deep and rapid, the foundations bad, or the floods heavy. The cost of a pier will be nearly the same to support a long span as a short one, and will be roughly proportional to its bulk, as far as the masonry is concerned. The cost of foundations, however, can not be exactly estimated, and this element becomes therefore at once the most uncertain and important one to be considered. Let us consider the principal conditions affecting it. 1°. The most important point is the character of the river bed, and this should be carefully determined by soundings, dredgings, and trial borings. The best foundations are obtained on rock, hard gravel, or stiff clay. 2°. The greatest, mean, and least velocities of the stream should be found by one of the usual methods, almost the only one practicable in this case being that by current meters, in which the number of revolutions in a given time of a wheel driven by the current ~~are~~ is counted, and the velocity deduced therefrom by comparison with a table constructed experimentally by moving the meter through still water at known velocities. 3°. The additional velocity gained by the stream in consequence of the contraction of its water-way by piers, and the consequent scour of its bed, is a very important element in many cases in determining the depth to which the foundations should be sunk. This increased velocity continues until the scour of the bed increases the water way to its original area, and the depth of scour can be easily calculated approximately by dividing the deficiency to be made up by about three quarters of the breadth of the stream. To provide against this scour, two courses are open; the foundations may either be placed low enough to be below the reach of the scour, or they may be protected by riprap.

The nature of the navigation in the river is often of such a character as to determine to a great degree how many piers there shall be. When the navigation is large and important, it may be necessary to obstruct the water way as little as possible, and in that case few piers should be built. The piers should be so far apart and so constructed that no obstruction is offered to ice, driftwood, or rafts of timber. When great masses of ice or driftwood are liable to come against the piers, these must be strong enough to resist the pressure, and hence in such a case the piers should be few, large, and strong. When, on the contrary, there is little ice pressure, and foundations are easily obtained, it may be preferable to build a number of light piers at short distances apart. The limit in this direction is reached in crossing a shallow pond or marsh, when the bridge and piers may be replaced by trestlework.

The cost of suitable rock is a very important element in the problem we are considering. When it can not be obtained except at a good deal of expense, it may be advisable to build quite long spans, and as the cost of piers and foundations decreases, the number of spans should be increased. The minimum cost occurs when the cost of a single span of the main girder is ~~just~~ nearly equal to that of one pier. This statement may be proved as follows:

Let P = the cost of one pier,

n = number of spans,

L = total length,

l = length of each span, supposing them all equal. Then we have

Cost of piers $= P(n-1) = P(\frac{L}{l} - 1)$.

Cost of one span of girder $= al^2$, supposing that the cost varies as the square of the length, the depth being constant, an assumption that is approximately true, as will be seen hereafter, and a being a constant. Then total cost of trusses $= n a l^2 = \frac{L}{l} a l^2 = L a l$, and, total cost of piers and trusses $= y = Lal + P(\frac{L}{l} - 1)$

Differentiating with respect to l, we have

$\frac{dy}{dl} = La - \frac{PL}{l^2}$, and making this equal to zero,

$La = \frac{PL}{l^2}$; $l^2 La = PL$, $\therefore l^2 = \frac{P}{a}$.

also, $\frac{d^2y}{dl^2} = \frac{2PLl}{l^4} = \frac{2PL}{l^3}$, and this is positive for the value, $l = \sqrt{\frac{P}{a}}$, hence this value of l renders the total cost a minimum. But the cost of one pier is P, and that of one span is al^2, hence when $l^2 = \frac{P}{a}$ these two are equal.

The elements above enumerated are in general all that enter into the problem, and in examining the present case in the light they give, I should decide that on the whole it was best to have one pier in the centre of the stream, for the following reasons; 1° Rock suitable for piers is not to be obtained with great facility; 2°. The water is not very deep, but the foundations are on the whole moderately expensive, considering the cost of the rock and the height required; 3° The ice pressure is quite heavy, and the piers should have considerable strength.

Vose says " In ordinary cases, for large rivers, with piers of first class masonry, from 20 to 40 feet above the water, with common pile or caisson foundations, we may put the spans at from 150 to 200 feet as the most economical; being larger as the foundations are more expensive."

Theoretical solutions of the problem of economic span have been proposed, but as Vose remarks, such investigations are of little use in practice, since local circumstances exercise so important an influence. It may be well, however, to give one which is given by Unwin, and to apply the results to the present case, as a sort of check on our former conclusion. Let the following notation be assumed; W = total external distributed load in tons.

W' = wt. of truss itself in tons.

l = clear span in feet.

h = effective depth of truss in feet.

$r = \frac{l}{h}$.

s = average stress in tons per sq. in. on the gross section of the chords at the centre.

A = gross area in sq. in. of both chords at the centre.

Then we have, M_0 = moment at centre $= \frac{1}{8}(W+W')l$, and $\frac{M_0}{hs} = \frac{1}{2}A$. Now assuming that the wt. of the truss is proportional to Al, we have $\frac{\frac{1}{2}Al}{W'}$ = a constant = c, and

$$c' = 8c = \frac{4Al}{W'} = \frac{(W+W')l^2}{hsW'}.$$ This equation serves to deduce the value of the constant from girders of known wt., and from it we obtain $W' = \frac{Wl^2}{c'hs - l^2} = \frac{Wlr}{c's - lr}$, which serves to determine the wt. of a bridge approximately. Mr. Unwin makes the following remarks about the assumption involved in this equation :- " In all calculations with respect to bridges, it is constantly necessary to form an approximate estimate of the wt. of a beam, the load on which, exclusive of its own

weight, is already known. Various methods of proceeding have been proposed to compass this end, but no method appears more convenient than to express the wt. of a beam as a function of its load. To do this with perfect accuracy involves formulae of great complexity, but looking at the fact that from 2/3rds to 3/4ths of the whole wt. of metal in a girder is concentrated in the top and bottom booms, and that the volume of a well designed boom should be proportional to its length and to the area of its section at the centre, it appears probable that an approximate formula might be found to express the weight in terms of the load and the stress at the centre".

Admitting, then, this formula as approximately true, and substituting for W and W' the wts. per foot run in tons, w and w', we have

$$w' = \frac{wl^2}{c'ks - l^2} = \frac{wlr}{c's - lr}$$

Now let $L = nl$ = total length of bridge;

n = no. of spans; P = cost of one pier;

Q = cost of iron per ton. Then

total cost of piers = $(n-1)P$.

" " " trusses $= 2w'l\,n = \dfrac{2n\,wl^2r}{c's - lr}$

$$= \frac{2L^2wr}{c'sn - Lr}, \text{ and}$$

Total cost $= y = P(n-1) + \dfrac{2L^2wr}{c'sn - Lr}$, which is to be a minimum; hence

$$\frac{dy}{dn} = 0 = P - \frac{c's\,2L^2wr}{(c'sn - Lr)^2} \therefore$$

$$c'sn - Lr = \sqrt{\frac{c's\,2L^2wr}{P}}$$

$$n = \frac{L}{c's}\left[r + \sqrt{\frac{c's\,2wr}{P}}\right] \text{ and}$$

$$l = \frac{L}{n} = c's \div \left[r + \sqrt{\frac{c's\,2wr}{P}}\right].$$

Now suppose we assume in the present case

$$\begin{cases} r = 7\ ;\ P = \$20000.\ ;\ Q = \$140.\ ;\ s = 5.5\ ; \\ c' = 1200\ ;\ w = 1\tfrac{3}{4}\ ; \end{cases}$$ then substituting in the above formulae we get

$$\begin{cases} l = 6600 \div \left[7 + \sqrt{\dfrac{6600 \cdot 140 \cdot 7 \cdot 1\frac{3}{4}}{20000}}\right] = 214' \\ n = 1.65. \end{cases}$$

This agrees approximately with our former decision, and under the circumstances assumed it would probably be best to make two spans of 180' each.

I have deduced the values of c' for several of the bridges constructed by Clarke, Reeves and Co., and find that they vary between 900 and 1300, the average being about 1200. The formula for W' will not give accurately, with the same constant, the weights of girders of varying depth.

The above formulae show that the economic span, l, increases as r decreases; that is, as $\frac{h}{l}$ <u>increases</u>, hence a deep truss, besides economizing material in the chords, is economical as regards the number of piers.

<u>Type of Bridge.</u>

Having thus determined upon two spans of 180' each, the next step is to fix the type of bridge, the proportion of depth to span, and the number of panels and consequent inclination of the braces. These considerations are to a certain extent inter-dependent, for it is probable that a style of truss which is the most economical when combined with a certain length and depth of panel, may have to yield its place to some other type when those dimensions are changed. The exact determination, however, of the most economical type, depth, and panel length, even in a given case, is impossible, and still less is any general solution practicable, for the elements entering into the problem are so various and variable that they can not be taken account of. In the first place, a great deal depends upon the limit of stress, and the proper value of this will always be an open question. Again, our knowledge of the resistance of materials to crushing is very imperfect, and an accurate investigation, such as would allow for the failure of struts by bending, should rest on more exact formulae than either Hodgkinson's or Gordon's. The latter, when compared with Hodgkinson's experiments on round end pillars, gives results always less than the true breaking weight, while the former gives results sometimes greater and sometimes less than the true value. Then again, the forms of struts, being governed in great measure by convenience of connection, vary widely, and can not be allowed for.

It is possible, however, in a general way, to compare the different forms of trusses, and to determine approximately their relative economy, leaving out of consideration the connections. Assuming that there are no special circumstances of convenience or inconvenience attending the manufacture of any particular kind of truss, that is the best which consumes the least material in its construction, and all theoretical investigations are accordingly directed to the determination of the amounts of iron in the different forms. Theoretical results, however, must not be strictly relied upon, both for the reasons stated above, and on account of the fact that it is impossible to take account of all the material to be put into a

bridge. Unwin says,* regarding this question.
" It would appear at first sight extremely simple to determine, by exact calculation, which form of girder requires the least material; and if girders could be constructed with the sectional area of the parts exactly proportional to the theoretical strains, such a determination might easily be made. But in every girder material is introduced, according to the judgment of the engineer, to cover joints and for stiffening, whilst the sectional area of the booms towards the ends and of the bracing or web toward the centre cannot be reduced in practice to the dimensions which theory prescribes. The amount of this arbitrary excess of material far exceeds the difference in weight due to the form of the truss, and is not susceptible of any very exact estimation."

Mr. F. J. Bramwell, in a paper before the

---

* Iron Bridges and Roofs - pp. 80, 81.

Institution of Civil Engineers in England,[x] adopted a very simple method of comparison, which, neglecting difference of connections, extra material and also in a measure the tendency of struts to fail by bending, gives an accurate estimate of the value of the different trusses. This method admits of the maximum web stresses being taken account of, and is certainly rapid and very satisfactory. By an extension of it, which I shall employ, we may easily deduce the economical depth of a bridge of given type, span, and length of panel; and were it not that such a calculation would be quite complex and practically almost valueless, we might leave both the depth and length of panel undetermined, and find what values would reduce the total amount of iron to a minimum. Such a tedious calculation, however, would be unprofitable. The panel length of a bridge is determined by comparing the cost of

[x] Proceedings Inst. C.E. - 1862-63 - (Vol. XXII)

connections, platform, stringers, additional material necessary to resist the "panel bending moment" (if the floor beams are spaced closer than the length of the panel), and the additional material required in the strut, and is fixed by the best engineering precedent at from 10 to 20 feet, for spans of over 100 feet in length.

It is also a well known fact that the most economical angle for ties in a truss is 45°, as far as they alone, — independently of the chords — are concerned; and this would also be the most economical angle for struts, were it not for their tendency to fail by bending. As it is, deductions based on Hodgkinson's formulae for <u>cast iron</u> solid and hollow pillars show that their most economical angle is 39° 50' 31", nearly, (with the vertical).×

In view of all these considerations, I have <u>not considered it advisable, for our present</u>

× Merrill. — Iron truss Bridges for Railroads. Van Nostrand's Eclectic Eng. Mag. — March, 1869.

purpose, to discuss the results arrived at by Merill, Baker,* and others, but to make a comparison, following Mr. Bramwell's method, – which appears to me to be the best I have seen, – of the Warren, Pratt, Whipple-Murphy, Lattice, and Post trusses; – those being the principal ones now competing for favor. In Mr. Bramwell's original paper, the Fink, Bollman, and Lattice girders are compared (though without taking into account the maximum stresses in the latter), and the result stands (the above error not being corrected); Fink, 396 parts of iron

Bollman, 370 " " "

Lattice, 262 " " "

If we correct the above error, the result stands;

Fink, 396 parts of iron

Bollman, 370 " " "

Lattice, $268\frac{2}{3}$ " " ".

Supposing that in each case the live plus the dead

---

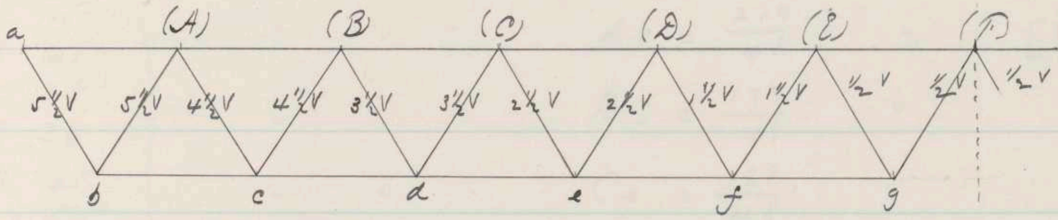
* { Long Span Bridges for Railroads, and
{ Short " " " "

load is <u>four</u> times the dead. This enormous difference practically excludes the first two trusses from the field, for even allowing all possible margin, it seems hardly probable that these trusses can be economical when compared with some of the others above mentioned; notwithstanding their extensive use in the South and West. The cause of this is evident: the stresses belonging to the centre of the bridge, and which might be got rid of in a short distance on each side of the centre, are carried to the very ends of the top chord, and this useless stress, so to speak, is increased in the ties by their great inclination.

The Jones or Howe truss, inasmuch as its ties are vertical, and its struts inclined and therefore longer, has its material evidently unfavorably disposed, and will not be considered. Iron bridges are seldom (I know of not one except the Ashtabula bridge) built on this type.

Assuming, then a panel length of 15 feet, and that the live + dead loads = 4 (and 3) · the dead load, I proceed to the comparison.

1° Triangular truss. (Deck.)



Assuming, – what is not exactly true. – that the wt. of the bridge is concentrated at the upper vertices, let the load at each vertex be one unit. Call the length of a panel unity, and the depth x, and assume one unit section of iron required to resist one unit of stress. We then have; in the top chord,

$$\begin{cases} \text{parts of iron in the half span} = 1 \cdot \frac{\frac{1}{2}}{x} 5\tfrac{1}{2} + \frac{\frac{1}{2}}{x}\left(5\tfrac{1}{2} + 5\tfrac{1}{2} + 4\tfrac{1}{2}\right) \\ + \frac{1}{2x}\left(5\tfrac{1}{2} + 5\tfrac{1}{2} + 4\tfrac{1}{2} + 4\tfrac{1}{2} + 3\tfrac{1}{2}\right) + \&c \end{cases}$$

$$= \frac{1}{2x}\left(5\tfrac{1}{2} + 15\tfrac{1}{2} + 23\tfrac{1}{2} + 29\tfrac{1}{2} + 33\tfrac{1}{2} + 35\tfrac{1}{2}\right) = \frac{143}{2x} \; \therefore$$

parts of iron in whole span (top chord) $= \frac{143}{x}$.

Similarly, in the bottom chord,

Half span $= \frac{1}{2x}\left(11 + 20 + 27 + 32 + 35 + (36 \times \tfrac{1}{2})\right) = \frac{143}{2x}$

Whole span $= \frac{143}{x}$, as in the top chord, which might have been seen by inspection. Again, Web ties;— max. stress on $ab = \left\{\frac{66}{12}(\text{live} + d.) - 0\right\} \frac{\sqrt{x^2+1/4}}{x}$ ∴

1	1
2	3
3	6
4	10
5	15
6	21
7	28
8	36
9	45
10	55
11	66

Since its length $= \sqrt{x^2+\frac{1}{4}}$,

parts of iron in $ab = \frac{66}{12} \cdot \frac{x^2+1/4}{x}$. Similarly

" " " " $Ac = \frac{219}{48} \cdot$ " $\left[\frac{55}{12} - \frac{1}{12 \cdot 4}\right]$

" " " " $Bd = \frac{177}{48}$ "

" " " " $Ce = \frac{138}{48}$ "

" " " " $Df = \frac{102}{48}$ "

" " " " $Eg = \frac{69}{48}$ "

Total for half span $= \frac{969}{48} \cdot \frac{x^2+\frac{1}{4}}{x}$

For the web struts we shall have the same result.

We can not allow the same section for compression as for tension, and though we can not take this difference accurately into account, we may do so approximately. Thus allowing for tension 5 tons per sq. in., suppose we allow only 3 tons for compⁿ; then to reduce these parts of iron to a common unit we may take off $\frac{2}{5}$ of that in tension (making the compⁿ area the unit), or add $\frac{2}{3}$ of that in compⁿ, (making the tension area the unit.) I shall do the former. To express it more plainly, this is equivalent to making the unit area that required to resist one unit of compressive stress. Hence we have

$$y = \text{total iron} = \frac{143}{x} + \frac{143}{x} - \frac{286}{5x} + \frac{969}{24}\cdot\frac{x^2+1/4}{x} + \frac{969}{24}\cdot\frac{x^2+1/4}{x}$$

$$- \frac{969}{60}\cdot\frac{x^2+1/4}{x} = \frac{1144}{5x} + \frac{646}{10}\cdot\frac{x^2+1/4}{x} \;\therefore$$

$$\frac{dy}{dx} = -\frac{1144}{5x^2} + \frac{646}{10}\left(\frac{2x^2-x^2-1/4}{x^2}\right) = -\frac{1144}{5x^2} + \frac{646}{10}\left(\frac{x^2-1/4}{x^2}\right),$$

which is to be ~~a minimum~~ zero, so that y shall be a minimum; hence

$$\frac{1144}{5x^2} = \frac{646}{10} - \frac{646}{40x^2}\;; \quad \frac{9798}{40x^2} = \frac{646}{10} = \frac{2584}{40} \;\therefore$$

$$x = \pm\sqrt{\frac{9798}{2584}} = \pm\frac{70}{36} = \pm 2, \text{ nearly.}$$

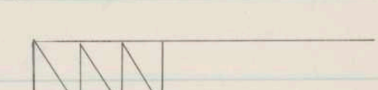
$$\frac{d^2y}{dx^2} = \frac{1144\cdot 2x}{5x^4} + \frac{646\cdot 2x}{40x^4} = \frac{2288}{5x^3} + \frac{1292}{40x^3},$$ which is positive for $x = 2$, hence this value makes y a minimum. Substituting this value of x, we have

$y = 251.7$, distributed as follows;
$$\left\{\begin{array}{l} \text{Chords} = 114.4 \\ \text{Web} = \underline{137.3} \\ \qquad\quad\;\; 251.7 \end{array}\right.$$

By applying this method to the other trusses mentioned above, I have obtained the following results. The details of the calculations it is not necessary to give.

2°. Triangular <u>through</u> bridge. The results are the same as above; $d_0 = 2$; $y = 251.7$

3°. Pratt. Single intersection. Through.



Economic depth = 2 ;

$$\text{total iron} = \left\{ \begin{array}{ll} \text{Chords}, & 118. \\ \text{Web}; & 146.2 \\ \hline \text{total}, & 264.2 \end{array} \right.$$

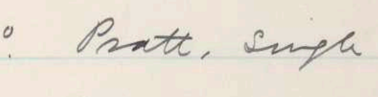
4°. Pratt, single intersection, <u>deck</u>.

Economic depth = d_0 = 2 ;

$$\text{total iron} = \left\{ \begin{array}{ll} \text{chords}, & 118. \\ \text{Web}, & 141.7 \\ \hline \text{total}, & 259.7 \end{array} \right\}$$

, the difference in the web being due to the posts, which in the previous case were 82.4, while in this they are 77.9

5°. Murphy-Whipple. Through. Double intersection.



d_0 = 3, nearly

$$\text{total iron} = \left\{ \begin{array}{ll} 80\tfrac{2}{3}, & \text{chords} \\ 131.4, & \text{web} \\ \hline 212.1 & \text{total} \end{array} \right\}.$$

With this great depth, however, the posts would require much extra material. Suppose we add 1/6 of the iron in the posts, making 7/6 in all, we get total iron = 224.2, which is probably nearer the true amt., as comparing this truss with the others.

If we make d = 2 in this bridge, we get

$$\left\{ \begin{array}{ll} \text{Chords}, & 121 \\ \text{Web}, & 100.8 \\ \hline \text{total}, & 221.8 \end{array} \right\}$$

This depth is preferable on many accounts, and would probably be more economical than the other, 3.

6°. Lattice. Double intersection; through.

$d_0 = 2$, nearly.

$$\text{iron} = \begin{cases} \text{Chords}, & 115.15 \\ \text{Web}, & 148.7 \\ \text{total}, & 263.85 \end{cases}$$

7°. Lattice. Triple system; through.

$d_0 = 2$, nearly

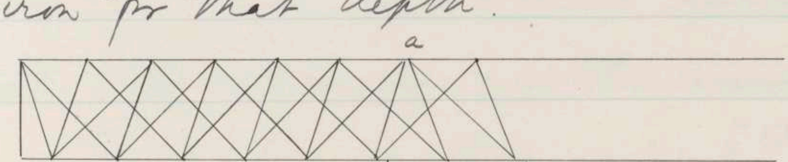
$$\text{iron} = \begin{cases} \text{Chords}, & 115.2 \\ \text{Web}, & 145.2 \\ \text{total}, & 260.4 \end{cases}$$

8°. Lattice. Quadruple system; through

$d_0 = 2$, nearly

$$\text{iron} = \begin{cases} \text{Chords}, & 115.35 \\ \text{Web}, & 152.3 \\ \text{total}, & 267.65 \end{cases}$$

9°. Post truss. Through. In this bridge the depth is $1\frac{1}{2}$ the panel length, hence I have simply found the iron for that depth.



$$\text{iron} = \begin{cases} \text{Chords} = 157.8 \\ \text{Web} = 90.75 \\ \text{total} = 248.55 \end{cases}$$ In calculating this truss I have supposed that owing to want of accuracy in construction any stress passing to the left abutment

through the tie ab (and similarly for the right abutment, of course) is likely to be taken wholly by either of the struts ad, ac. Sometimes the supposition is made that in this case half the stress goes to each strut, but it is practically impossible to construct a bridge with the lengths of the bars, tightness of the connections, etc., so proportioned as to insure such a distribution of the stress. The struts and ties should therefore be proportioned on the supposition mentioned above.

I have repeated the previous calculations supposing the live + dead loads = 3 · dead, — a supposition probably more nearly correct for a bridge as long as the one under consideration than the one on which the previous calculations are based. The following are the results:—

1°. $d_0 = 2$, and in that case $\begin{cases} \text{chords} = 114.4 \\ \text{web} = 135.6 \\ \text{total} \quad 250.0 \end{cases}$

2°. Same as 1°.

3°. $d_0 = 2$; $\quad \begin{aligned} \text{chords} &= 118.0 \\ \text{web} &= 143.1 \\ \hline \text{total} &= 261.1 \end{aligned}$

4°. d_0 = **2**, nearly.
Chords = 118.
Web = 139.1
total = 257.1

5°. d_0 = <u>2.8</u>, nearly, <u>and when $d = 3$</u>,
Chords = $80\frac{2}{3}$
Web = 129
total = $209\frac{2}{3}$

If **d** = 2, we have
Chords = 121
Web = 98.9
total = 219.9

6°. d_0 = about 1.8, and making it 2, we have
Chords = 115.15
Web = 146.9
total = 262.115

7°. d_0 = 2, nearly;
Chords = 115.2
Web = 142.8
total = 258.11

8°. d_{11} = 2. nearly;
Chords = 115.35
Web = 151.55
total = 265.911

9°.
Chords = 157.8
Web = 88.35
total = 246.15

The above calculations have been very carefully made, and although in such a large amount of arithmetical and algebraical work there is every liability to error, yet I feel quite sure that no mistake of importance has been committed.

Since the cost of connection will be nearly the same in trusses with the same number of joints, the main inaccuracies in these results are those in regard to the struts, and the error arising from supposing the dead load concentrated at the joints of the loaded chord only, instead of at all the joints, as it strictly should be. However, for trusses with the same number of joints, it seems to me that the above figures represent pretty correctly the relations between the total cost. For convenience, I group the results together in the tables on the next page, in the second of which the values for the Murphy-Whipple are given for two additional depths. It must be carefully noticed that the two tables above can not be compared with each other as they stand, since the unit of stress is different in the two cases, being the same weight of live and dead load. The live load being fixed, the unit of stress will

1°. Live + dead = 4 dead.

Name	d_{ii}	d corr. to figures given	Chords	Web	Total.	
Warren, {deck or through}	2	2	114.4	137.3	251.7	
Pratt, through	2	2	118.	146.2	264.2	
" , deck.	2	2	118.	141.7	259.7	
M.-W., through	3	3	80 2/3	131.4	212.1	
" " "	3	2	121	100.8	221.8	
Double lattice "	2	2	115.15	148.7	263.85	
Triple " "	2	2	115.2	145.2	260.4	
Quad. " "	2	2	115.35	152.3	267.65	
Post.- "	—	3/2	157.8	90.75	248.55	

2°. Live + dead = 3 dead

Warren {deck or through}	2	2	114.4	135.6	250	
Pratt, through.	2	2	118.	143.1	261.1	
" , deck.	2	2	118.	139.1	257.1	
M.W., through	2.8	3	80 2/3	129	209 2/3	
" " "	2.8	2	121	98.9	219.9	
Double lat., "	1.8	2	115.15	146.9	262.05	
Triple " "	2	2	115.2	142.8	258.	
Quad. " "	2	2	115.35	150.55	265.9	
Post — "	—	3/2	157.8	88.35	246.15	
M.W. — "	2.8	3/2	161.33	87.7	249.03	
" — — "	2.8	5/3	145.2	90.9	236.1	

1° Live + dead = 4 dead.

Name	d''	d corr. to figures given	Chords	Web	Total.
Warren {deck or through}	2	2	114.4	137.3	251.7
Pratt, through	2	2	118.	146.2	264.2
" , deck.	2	2	118.	141.7	259.7
M.-W., through.	3	3	80 2/3	131.4	212.1
" " "	3	2	121	100.8	221.8
Double Lattice "	2	2	115.15	148.7	263.85
Triple " "	2	2	115.2	145.2	260.4
Quad. " "	2	2	115.35	152.3	267.65
Post.- "	—	3/2	157.8	90.75	248.55

2° Live + dead = 3 dead

Warren {deck or through}	2	2	114.4	135.6	250
Pratt, through.	2	2	118.	143.1	261.1
" , deck.	2	2	118.	139.1	257.1
M.W., through	2.8	3	80 2/3	129	209 2/3
" " "	2.8	2	121	98.9	219.9
Double Lat., "	1.8	2	115.15	146.9	262.05
Triple " "	2	2	115.2	142.8	258.
Quad. " "	2	2	115.35	150.55	265.9
Post – "	— *	3/2	157.8	88.35	246.15
M.W. – "	2.8	3/2	161.33	87.7	249.03
" – – "	2.8	5/3	145.2	90.9	236.1

vary according to the assumption made regarding the dead.

These results show the singular fact that in every case but one the economical depth is very nearly twice the panel length. This I cannot explain, nor why the exception should occur. We also see that in all the trusses of the same depth, the chords contain very nearly the same amounts of iron, as we should naturally expect would be the case. From the example worked out in full we see also that in a given truss the amt. of iron in the chords varies inversely as the depth. A singular variation is to be noticed in the lattice girders, where the chord weights increase slightly as the systems become more numerous, while the web weight of the triple lattice is less than that of either of the others. We notice that the order of economy of the different trusses is precisely the same in both tables, the chord wts. remaining unchanged, while in the second table the web wts. are slightly less than in the first. As regards this indicated order of economy, the following remarks may be made:—

1°. The lattice girders, besides standing highest in the list, as it is, would require more material in the struts, on account of their length and the small stresses in them; and these two facts, added to the numerous joints required, render these bridges uneconomical in the present case. These bridges are very generally considered quite economical, and with more time at hand it would be interesting to calculate all the above bridges for a span of 91' (half of the present one), and thus to discover what effect the length of span has on the order of economy. If in these calculations the depth were kept the same as in the above cases tabulated, we could test the accuracy of the formula given on p. , viz. $W' = \frac{We^2}{c'hs - e^2}$, and thus gain some idea of the accuracy of the assumption involved in the equation for the economic span, given above. Lack of time prevent my making these calculations.

2°. We see that the most economical trusses are the Post and the Murphy-Whipple, and that in proportion to its depth the former takes the lead. One consideration, however, must modify the figures; its posts are longer than those in the Murphy-Whipple of the same depth, and this item would in all probability practically place the latter at the head, though we may reasonably conclude that these trusses are about equal. The Murphy-Whipple truss is a very common form, and has met with great favor on our railroads, being the standard truss built by the Phoenix Bridge Co., the Keystone Bridge Co., the Detroit Bridge & Iron Co., and other firms. Its practical advantages have thus been fully tested, and we may consider its efficiency and economy to be sanctioned thus by both theory and practice. I shall adopt this form for my design, and this once fixed, the next point to be considered is what depth to assume, or, in other words, at what point, as the depth is increased, will the diminution of chord weight cease to exceed the increase of web weight. This point is incapable of exact determination, and must be fixed by precedent. The economic depth indicated in the tables must (as before remarked) only be followed approximately. The best precedent fixes the general depth of girders as from $\frac{1}{6}$ to $\frac{1}{14}$ of the span, and on the whole it seems to me that a depth of 25 feet, with a panel length of 15 feet, would be best suited for this case. I shall therefore adopt these dimensions.

It is to be noticed that the tables above show conclusively that the economical angles for the web members, when the effect of their inclination on the chords is considered, is not what it is when those web members alone are considered.

In a few of the cases tabulated I have calculated the same truss as deck and as through, simply to show the small difference involved, which occurs in the posts. In most cases, other circumstances, - such as the water way required, the approaches, etc., - will determine whether a bridge shall be deck or through. (though a deck bridge is much more easily braced laterally than a through bridge), the small difference in the amounts of iron being insignificant.

Before proceeding to the calculations of stresses, a few words must be said about materials and connections.

Materials.

The metals used in the construction of bridges are cast-iron, wrought iron, and steel. Of these, the former has been used extensively for compression members, while the use of the latter has only begun in late years. The objections to the use of cast iron in a bridge are very weighty, and seem to me to overbalance its advantages. Its low specific gravity, its great resistance to crushing, its low cost, and the great ease with which it can be cast to different figures, would stamp it at once as beyond all comparison a better material than wrought iron for the compression members, were it not for its inability to resist sudden shocks, and its liability to hidden flaws and strains. Moreover, its smaller modulus of elasticity and coefficient of expansion cause extra stresses when it is used with wrought iron. On this account, a great annual range of temperature, such as occurs in our country, is not favorable to its use.

Although steel is gaining favor, I believe, as a material for railway bridges, its use is not at present sufficiently common to warrant its being used in the present case. Its advantages are very great in many respects, but it is somewhat liable to hidden flaws and strains, and its cost is high, so that I have decided to use it only ~~[illegible]~~ for the pins of this bridge, all the rest of the pieces being of wrought iron, except some castings for shoes and brackets.

Connections.

There has been a great deal of discussion regarding the relative advantages of rivetted and pin connections, and as yet no definite conclusion has been arrived at. European bridges are almost all rivetted, while pin connections are extensively used in this country. The question is, however, merely a matter of individual opinion as yet, and I see no reason why either system can not be economically used. I have decided to use pin connections in the present case.

Dimensions, Etc..

To recapitulate, then, this bridge is to be a single-track through Murphy-Whipple, with double intersections, leaning end posts, and pin connections. Steel is to be used for pins, and wrt. iron for all the other members. The dimensions are to be as follows:

Length, from c. to c. of end pins =	180'
Length of panel =	15'
Number of panels =	12

Depth, from c. to c. of pins = 25'
Width " " " " " = 16'

Amount and Distribution of load.

The great object to be attained in proportioning the parts of bridges, or, in fact, of any structure, is uniformity of strength. The strength of a bridge being measured by that of its weakest part, it follows that when this object is not attained, there is either waste of material in some parts of the structure, or insufficient strength in others. The greatest stress that can possibly occur in each piece should therefore be carefully calculated, and the area of the section made sufficient to resist it, allowing everywhere the same factor of safety. This factor must be determined by the engineer in any case, for no general rule can be laid down.

The amount of load to be provided for in designing a bridge will vary with the span and the nature of the traffic. A very heavy load may be concentrated in a short space, so that for short spans the load per running foot to be allowed for is much greater than for long spans, where a train of cars constitutes the load. Moreover, the dead load being small in short spans, a much greater proportion of the total load is live than is the case with long spans, and hence the proportional destructive effect is much greater.

It is the practice of some engineers to allow for the passage of a moderately heavy freight engine, weighing say 125000 lbs., and to depend on the factor of safety when an exceptionally heavy engine, such as only passes over the road occasionally, comes on. The weight to be provided for will vary according to the grade on which the bridge is situated. For it is evident that unless these exceptionally heavy engines are seldom run over the bridge, it would be better to calculate the bridge for them, and not to depend on the allowed margin of safety. Another practice is to allow for a rolling load per running foot corresponding to a moderate train, and to suppose, at the head of the train, one panel to have an additional uniform load. The method which I shall follow is different from either of these.

It has rarely been the practice to make an accurate or scientific allowance for the greater destructive effect of a live load over that of a dead load, and the general rule has been to "lump" together the dead and live loads and use a factor of safety of about five (5). In certain cases this does well enough, but in others it does not. The proper way to proceed is to allow about twice as large a factor of safety for live load as for dead, and in applying this rule we may proceed in three ways; 1° Double the live load and add it to the dead, treating the whole as dead load; 2° Multiply the live and dead loads by their respective factors of safety, and the sum is the ultimate dead load to be provided for; 3° Determine the proportion which the live load bears to the dead, and take the sum as a working load using a limit of stress which varies according to this ratio. Thus, calling $\underline{a}$ = the dead load, and $\underline{na}$ = the live load, and $\underline{s}$ = the limiting stress for dead loads, we shall have, supposing both loads applied to a bar,

area required for dead load $= \frac{a}{s}$.

" " " live " $= \frac{na}{\frac{s}{2}} = \frac{2na}{s}$

Total " " $= \frac{a}{s}(1+2n)$, and since

" load $= a + na = a(1+n)$, we have

limiting stress to be used $= \dfrac{a(1+n)}{\frac{a}{s}(1+2n)} = \dfrac{s(1+n)}{1+2n}$

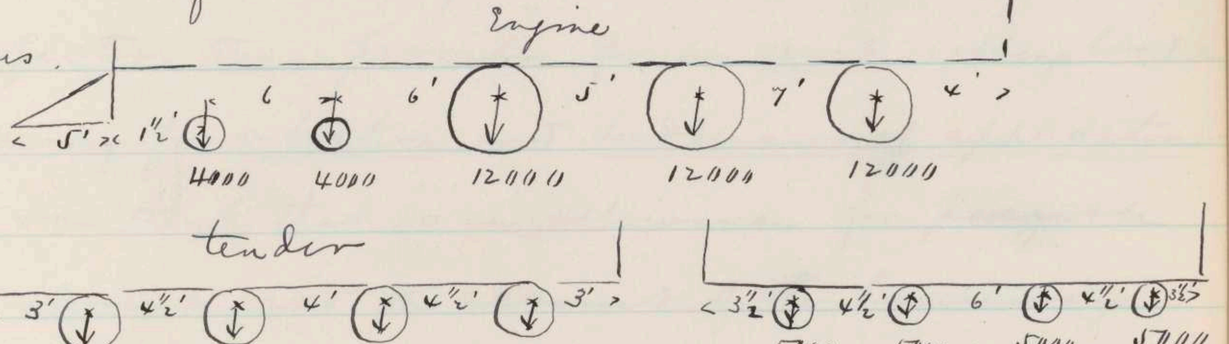
The following table is calculated by this formula.

(tons per sq. in.)

Live : dead.	Tension.	Compression
(All dead.) 0	7.	5.5
.25	5.83	4.6
.33	5.6	4.4
.571	5.25	4.12
.66	4.5.	3.93
1.00	4.66	3.66
2.00	4.2	3.3
(all live) ∞	3.5	2.75

The first method is the one I shall use, and instead of assuming the weight of the train as uniformly distributed, I shall place the train in different positions on the truss and resolve the load on each panel into components acting at the joints, finding the corresponding chord stress in the centre. Then I find the uniformly distributed load which will produce a central chord stress equal to the greatest found. The train that I shall assume will consist of two freight engines (with tenders) weighing 128000 lbs. each, in a length of 53½ feet, followed by a train of loaded coal cars (the heaviest in use).

Thus.



It will be very easy to see at once about what position of the train will give the greatest central chord stress, and generally not more than two trials will be necessary, except in a very long bridge. I find that the greatest chord stress in this case is about 94 tons, and to find the equivalent uniform load we have (calling x that load per panel)

$$94 = \left\{5\tfrac{1}{2}x \cdot 6 - x(5+4+3+2+1)\right\}\frac{1}{depth} = 18x \cdot \frac{3}{5} \therefore$$

$x = 8.7$ tons, nearly. Hence the load per ft. run is $8.7 \times 2000 \cdot \frac{1}{15} = 1160$ lbs., which is slightly greater than that given by the Phoenix Bridge Co. in their album (p. 23.) [This is all for one truss.]. The equivalent dead load $= 2 \cdot 1160 = 2320$ lbs. It is also considered by some that an additional fraction should be added, to provide for hammering of the train. The factor, two, provides for a quiet rolling load, allowing for vibration and suddenness of application — but some think that as an allowance for possible derailment and on account of the hammering caused by uneven tracks, etc., an additional fraction should be allowed. I shall therefore add 20% to the above, to obtain the equivalent dead load, especially as in the calculations I intend to consider

the dead load as concentrated at the lower joints.
Thus we have, live load reduced to dead = 2784 lbs. per ft.

In dealing with the dead load, "three courses may be pursued; 1°. The weight of the main girders ~~may~~ may be assumed from comparison with the weight of similar girders previously constructed, or from tables like those which Mr. Baker has calculated for the purpose"; 2°. The dimensions may be first calculated, neglecting the weight of the main girders, and then Prof. Rankine's method of allowance applied; 3°. "We may use the approximate formula given on page with constants derived from bridges of similar construction to that the wt. of which is required."
I have followed the first course, and by examining the table of bridges built by Clarke, Reeves & Co., it appears that 700 lbs. per ft. run for one truss will be about the weight. Hence, finally, the loads calculated for are, (for the chord system)

Live load, red. to dead = 2800 lbs. per ft. run (1 truss)
Dead load = 700 " – – – –

3500

The web of a bridge is usually calculated for a larger load than the chord system, on account of its being fully strained by the locomotive leading each train, and I shall assume for this purpose the following loads:

Live load, reduced to dead = 3600 lbs.

Dead "	= 700	"
Total.	4300	"

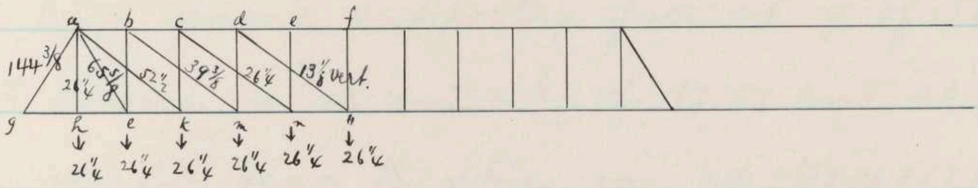
Now in regard to the limit of stress. As the above loads fully provide for vibration and concussion, and are considered quiet dead loads, a correspondingly large limit of stress will be allowable. All the materials being supposed of the best quality, the following limits of stress will be used.

Wrt. iron.	Tension	7 tons per. sq. in
	Bearing	7 " " "
	Compr.	5.5 " " " "
	Shear	6 " " " "
Steel.	Shearing	8 " " " "
	Cross breaking	10 " " " "
Oak.	Cross breaking.	2500 lbs. " " "

On comparing with the table just given we see that these values for wrought iron are ~~safer~~ lower than those usually adopted in the ratios 4.2 : 5 and 3.3 : 4, considering the live load $\frac{2800}{2} = 1400$ (= 2·700.)

Determination of Stresses.

1°. Chord Stresses.



$3500 \times 15 = 52500$ lbs $= 26\frac{1}{4}$ tons = total panel load.

$700 \times 15 = 10500$ " $= 5\frac{1}{4}$ " = dead " ".

The loads travel to the abutments in the manner shown, and by resolving the forces in the web we obtain the following chord stresses;–

$ab = (144\frac{3}{8} + 65\frac{5}{8})\frac{3}{5} + 52\frac{1}{2} \cdot \frac{6}{5} = 189$

$bc = 39\frac{3}{8} \cdot \frac{6}{5} + 189 = 236\frac{1}{4}$.

$cd = 236\frac{1}{4} + 26\frac{1}{4} \cdot \frac{6}{5} = 267\frac{3}{4}$

$de = ef = 267\frac{3}{4} + 13\frac{1}{8} \cdot \frac{6}{5} = 283\frac{1}{2}$.

$gh = he = 144\frac{3}{8} \cdot \frac{3}{5} = 86\frac{5}{8}$.

$ek = 86\frac{5}{8} + 65\frac{5}{8} \cdot \frac{3}{5} = 126$; $km = ab$; $mn = bc$; $no = cd$.

Even by this method of procedure it is possible that the stresses obtained in the end panels of the chords may be less than would be produced by the actual concentrated loads when in the position giving the maximum stresses in al and lk. In fact, in the present case, I find that one position of the actual load gives a supporting force at g of 50.14 tons, with stresses on al and ak of 17.77 and 20.97 tons respectively. Hence the stress in ab $= (50.14 + 17.77)^{3}/_{5} + (20.97)\frac{6}{5} = 65.91$ due to live load alone. Reducing to dead by multiplying by two and adding 20%, and then adding the stress due to the uniform load alone, the stress becomes 196 tons, dead. I also find the stresses in ah and hl to be 89.5, but in all the other chord members the stresses are less than those before obtained. This shows the difficulty of allowing accurately for concentrated loads by taking an "equivalent uniform load", although in this case the differences are so small that considering our margin of 20% they would do no harm. Having discovered them, however, I shall take account of them.

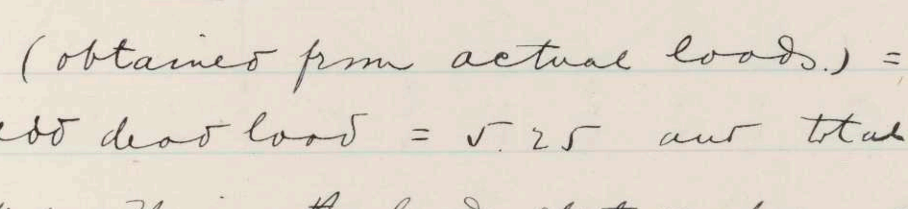
2°. End posts. Greatest supporting force is found to be 50.14 live. $(\underset{\text{(panels)}}{50.14} \times \underset{\text{(panel load)}}{2})\frac{120}{100} = 120.34$, and adding dead $= 28.9$ $(= 5\tfrac{1}{2} \times 5\tfrac{1}{4})$, the total is 149.24, which differs by only about 5 from that obtained from the uniform load. Resolving along ga, we find

stress in ga $= 149.24 \times \dfrac{\sqrt{1+(5/3)^2}}{5/3}$ = about 175 tons.

3°. Vertical ties, ah —. Greatest live load = 13.9 (obtained from actual loads.) = 33.36 dead.

add dead load = 5.25 and total = 38.61 tons

4° Web. Using the loads stated above, viz. 3600 lbs. per ft. live and 700 lbs. dead, we find stresses as follows:



$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \cdots \\ 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & \cdots \end{bmatrix}$$

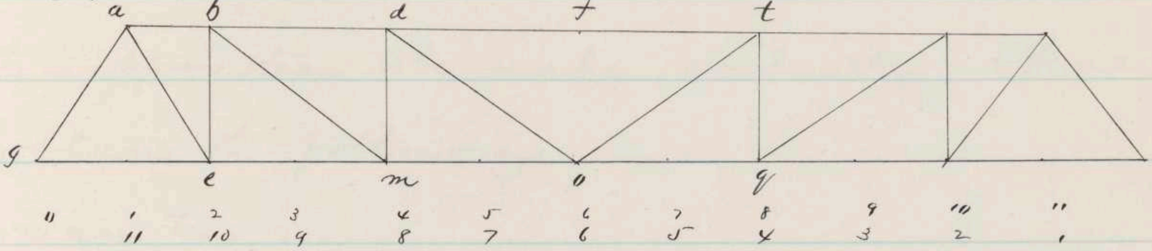
1 .. 1	2 .. 2
3 .. 4	4 .. 6
5 .. 9	6 .. 12
7 .. 16	8 .. 20
9 .. 25	10 .. 30
11 .. 36	↓
↓	

triangular numbers

live load per panel = 3600 × 15 = 54000 lbs.
= 27 tons
dead, as before = $5\tfrac{1}{4}$ "
total = $32\tfrac{1}{4}$ "

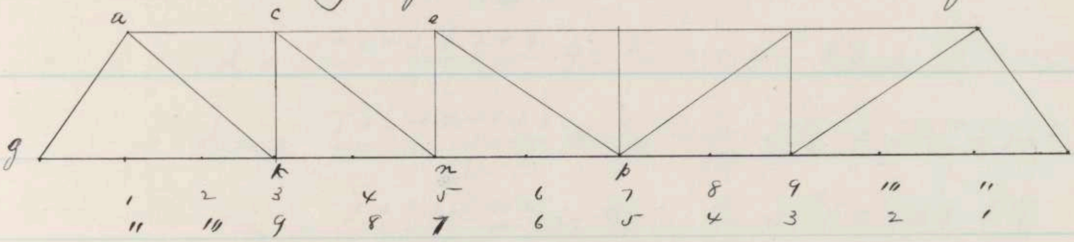
This truss is really two distinct trusses super-posed, one being $galbm\ldots$, shown in the following figure,

(1)



and the other being $gakcn\ldots$ shown in the following;

(2)



There is doubt as to the vertical ties ah - in this truss, for it is impossible to say whether ~~it is~~ they are to belong to one of the component trusses or to the other. Thus here, as in the Post truss, before alluded to, the worst case must be provided for. The difference is small and is shown by the following equations. If both vert. ties are considered as belonging to truss (2) we have stress in al, truss (1) $= \frac{2+4+6+8+10}{12} \cdot 32\tfrac{1}{4} - 0 \cdot 5\tfrac{1}{4}$, vertically.

If both are considered to belong to truss (1), stress in $al = \frac{1+2+4+6+8+10}{12} \cdot 32\tfrac{1}{4} - \tfrac{1}{2} \cdot 5\tfrac{1}{4}$, vertically; and if the right hand one is considered as belonging to truss (1), and the left hand one to truss (2), stress in al (vert.) $= \frac{1+2+4+6+8+10}{12} \cdot 32\tfrac{1}{4} - 11 \cdot 5\tfrac{1}{4}$. The latter, then, is the supposition to be made, evidently. Hence we obtain the following:

(a) main ties.

$\underline{\text{vert}}$. stress in $al = \frac{1+2+4+6+8+10}{12} \cdot 32\tfrac{1}{4} - 0 = 83.325$ tons

" " " $ak = \frac{1+3+5+7+9}{12} \cdot 32\tfrac{1}{4} - 0 = 67.2$ "

" " " $bm = \frac{1+2+4+6+8}{12} \cdot 32\tfrac{1}{4} - \frac{2}{12} \cdot 5\tfrac{1}{4} = 55.575$ "

" " " $cn = \frac{1+3+5+7}{12} \cdot 32\tfrac{1}{4} - \frac{3}{12} \cdot 5\tfrac{1}{4} = 41.69$ "

" " " $do = \frac{1+2+4+6}{12} \cdot 32\tfrac{1}{4} - \frac{2+4}{12} \cdot 5\tfrac{1}{4} = 32.325$ "

Resolving along the bars we have

Stress in $al = 97.213$ tons

" " $ak = 104.97$ "

" " $bm = 86.81$ "

" " $cn = 65.12$ "

" " $do = 50.49$ "

(b.) counters.

vert. stress in $ep = \frac{1+3+5}{12} \cdot 32\tfrac{1}{4} - \frac{3+5}{12} \cdot 5\tfrac{1}{4} = 20.69$ tons

" " " $fq = \frac{1+2+4}{12} \cdot 32\tfrac{1}{4} - \frac{2+4+6}{12} \cdot 5\tfrac{1}{4} = 13.575$ "

vert. stress in $sr = \frac{1+3}{12} \cdot 32\frac{1}{4} - \frac{7+5+3}{12} \cdot 5\frac{1}{4} = 4.19$ tons

" " " $tu = \frac{1+2}{12} \cdot 32\frac{1}{4} - \frac{8+6+4+2}{12} \cdot 5\frac{1}{4} = -.69$ ",

hence this last is not needed.

Resolving along the bars, we get,

stress in $ep = 32.32$ tons.

" " $fq = 21.21$ "

" " $sr = 6.55$ "

(C.) Posts. Referring to the vertical components of the stresses in the ties, we see at once,

stress in $bl. = 55.575$ tons.

" " $ck = 41.69$ "

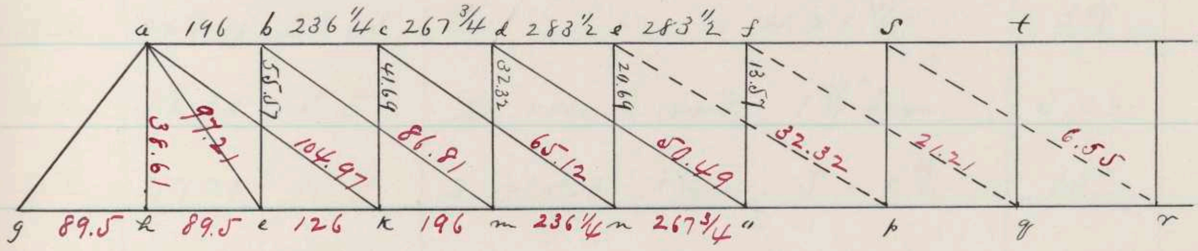
" " $dm = 32.325$ "

" " $en = 20.69$ "

" " $fo = 13.575$ "

Were this thesis not written with a desire to test and examine some of the methods in most common use, no account would be taken of the actual loads, for we see that a proper assumed uniform load provides with sufficient accuracy for all possible stresses; but having found in the examination some slight differences, I have thought I might as well use them. Thus, with the uniform load assumed we should find the stress on the vertical ties to be $32\frac{1}{4}$ tons, and on the end posts $177\frac{3}{8}$, but I use 38.6 and 175 instead.

The full "Stress" sheet stands as follows, according to the preceding calculations.



{ Red indicates tension;
{ black indicates compression

Determination of dimensions.

1° allowing 7 tons per sq. in. for tension, we can at once form the following table, which includes all the ties:

Ties.

Piece	Stress	Area req.	Satisfied by	Area
		□"		□"
gh	89.5	12.8	4 rect. bars, $3\frac{1}{2}'' \times 1''$	14
hl	89.5	12.8	" " " " "	14
ek	126	18	6 " " $3'' \times 1''$	18
km	196	28	" " " $4'' \times 1\frac{1}{4}''$	30
mn	236.25	33.75	" " " $4'' \times 1\frac{1}{2}''$	36
no	267.75	38.25	" " " $4'' \times 1\frac{5}{8}''$	39
ah	38.61	5.5	2 round rods, $1\frac{7}{8}''$ diam.	5.52
al	97.21	13.9	2 rect. bars, $5'' \times 1\frac{1}{2}''$	15.
ak	104.97	15.	" " " $5'' \times 1\frac{1}{2}''$	15.
bm	86.81	12.4	" " " $4'' \times 1\frac{5}{8}''$	13.
cn	65.12	9.3	" " " $4'' \times 1\frac{1}{4}''$	10.
do	50.49	7.21	" " " $3'' \times 1\frac{1}{4}''$	7.5
ep	32.32	4.62	" " " $3'' \times \frac{7}{8}''$	5.25
fq	21.21	3.3	1 round rod $2\frac{1}{8}''$ diam.	3.5
sr.	6.55	.94	" " " $1\frac{1}{8}''$ diam.	.99

2° Struts. (a) Upper chord. According to Rankine, (C. E. p. 562) the upper chord of a pin jointed bridge, is to be considered as made up of struts hinged at the ends. It seems to me, however, that this is inaccurate; and that although the divisions of the upper chord are not exactly fixed at the ends, yet ~~they~~ they are very nearly in this condition. If the upper chord could assume an undulating figure with points of inflexion at the joints, its condition would be nearly that of a strut with rounded ends; but since the chord receives its maximum stresses when the full load is on the truss, and since this load - nearly uniformly distributed, as a whole - acting through the ties on the upper chord, will effectually prevent that chord from assuming such a figure, it appears to me that it is nearer right to calculate it as a strut with fixed ends.

[In this bridge the track is to be supported at the joints only, so that the lower chord is exposed to no bending action.]

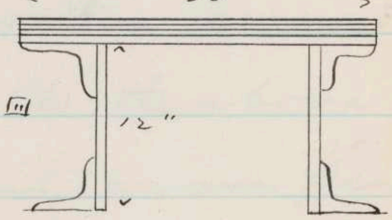
We have, then, the following calculations for the upper chord:

(de, ef, fs and st.) Stress = $283\frac{1}{2}$ tons.

Assume that the strut will bear 5 tons per sq. in.

Then area required $= \frac{283.5}{5} = 56.7$ sq. in.

Assume the following section,



4 upper horiz. plates, $20'' \times (\frac{1}{2}'' + \frac{3}{8}'' + \frac{1}{4}'' + \frac{1}{4}'') = 27.5$ □"

2 vert. plates, $12'' \times \frac{3}{4}'' =$ 18.

4 angle irons, $3'' \times 3'' \times \frac{1}{2} =$ 11.

56.5 sq. in.

If area of web $= B$; area of flanges $= A$; and depth of flanges + $\frac{1}{2}$ thickness of web $= h$, – then we have (see Rankine, p. 523, XI.), $r^2 = h^2 \left\{ \frac{A}{12(A+B)} + \frac{AB}{4(A+B)^2} \right\}$ or. substituting, $r^2 = 156 \left\{ \frac{18 + \frac{1}{2}(11)}{12(56\frac{1}{2})} + \frac{28\frac{1}{2} \cdot 33}{4 \cdot 56\frac{1}{2} \cdot 56\frac{1}{2}} \right\}$ nearly, or $r^2 = 14.88$, and substituting in Gordon's formula, viz. $\frac{P}{S} = \frac{f}{1 + \frac{e^2}{36000 r^2}}$, we have $\frac{P}{S} = \frac{5.5}{1 + \frac{(15 \cdot 12)^2}{36000 \cdot 14.88}}$

$= \frac{5.5}{1 + \frac{32400}{36000 \cdot 14.88}} = 5.2$ nearly, – more than was at just assumed. Hence the above section will answer, as the resistance to crushing by bending about an axis at right angles to the one assumed in the above calculations is much greater than the resistance as calculated. This point deserves notice. The values of r^2 given by Rankine are not, in all cases, the proper ones to be used, for in some cases the radius of gyration around another axis may be smaller. In the above case, however, we have, around an axis parallel to the web plate (vert.), $I = \frac{1}{12} \cdot B \cdot (20)^2 + A \cdot (6)^2$, nearly $= 1946.$, and $r'^2 = \frac{I}{A+B} = \frac{I}{56.5} = 34.+$, hence $\frac{P}{S}$ in this case is greater than in the other.

The above method of calculating struts, by using Gordon's formula with Rankine's modification, is stated not to give very accurate results when compared with experiment, but for struts of figures regarding which we have no experimental data, it is the best method I know of. I shall therefore use it on all the struts.

If we were to consider the upper chord as hinged, we should have, $\frac{P}{S} = \frac{5.5}{1 + \frac{32400}{9000 \cdot 14.88}} = 4.43$, instead of 5.2, as before. This difference is not large, and as it seems to me that the divisions of the upper chord approach much more nearly to the condition of struts with ~~[illegible]~~ fixed ends, than to that of those with rounded ends, I think the above section is amply large ~~enough~~.

(cd.) Stress = $267\frac{3}{4}$ tons. Assume 5 tons per sq. in.

Area req. = 53.55 sq. in. If the top plate is removed, the area left is 51.5, and

$$r^2 = 156\left\{\frac{23\frac{1}{2}}{12 \cdot 51.5} + \frac{23\frac{1}{2} \cdot 28}{4 \cdot 51\frac{1}{2} \cdot 51\frac{1}{2}}\right\} = 15.6$$

$$\frac{P}{S} = \frac{5.5}{1 + \frac{324''}{36''' \cdot 15.6}} = 5.2, \text{ and } \frac{267\frac{3}{4}}{5.2} = 51.5.$$

I will therefore stop the top plate at d.

(bc). Stress = $236\frac{1}{4}$. Assume 5 tons. Area req. = 47.25

If the next plate is removed, the area left = 46.5

$$r^2 = 156\left\{\frac{23.5}{12 \cdot 46.5} + \frac{23.5 \cdot 23}{4 \cdot 46.5 \cdot 46.5}\right\} = 16.3$$

$$\frac{P}{S} = \frac{5.5}{1 + \frac{9}{163}} = 5.2 \text{ and } \frac{236\frac{1}{4}}{5.2} = 45.4.$$

I will therefore stop this plate at c.

(ab.) Stress = 196. r^2 increases as the top plates are removed, hence 5 tons can be assumed at once and the calculations not repeated. $\frac{196}{5} = 39.2$. If we remove the 3/8" plate on top the area left = 39, which will be safe enough.

This chord is to be latticed on the under side, as shown on the detail plate, and the plates on top, before being stopped, are to be carried one foot beyond the joint, to act as reinforcing plates on the top.

(b.) End post. This rests on a cast iron shoe, and may be regarded as fixed at the ends. Hence, assuming 5 tons at first, area required $= \frac{175}{5} = 35$ sq. in. Take section same as ab. Then we find $r^2 = 16.5$, and $\frac{P}{S} = 4.5$, and $\frac{175}{4.5} = 39$, so that this is near enough. This post, like the top chord, is to be latticed on its under side.

(c) Struts in the web. In proportioning these struts, regard must be had to the fact that they are to be considered as hinged at the ends as regards flexure in the plane of the truss, and fixed at the ends as regards flexure at right angles to that plane. I shall use for strut two very shallow channel bars, rolled for the purpose, and an I beam between them. The width of ~~#~~ any strut must then not be greater than $12\frac{1}{4}''$, the distance between the vertical plates of the upper chord. The sections of the posts will be considered as H sections, and for this section we have (See Rankine, p. 523, X.), $r^2 = \frac{b^2}{12} \cdot \frac{A}{A+B}$, where b = breadth of flanges, A = their joint area, B = area of web.

(bl.) Assume 3.5 tons per sq. in. Then required area $= \frac{55.57}{3.5} = 15.9$ sq. in. Take section as follows:

$$\left\{\begin{array}{ll} \text{1 I beam No. 8 (See Phoenix Price list), area} = & 4.^{\square''} \\ \text{2 channels } 9''\ [\ \tfrac{5}{8},\ \tfrac{1}{2}\text{, area} = 2(6.25) = & \underline{12.5} \\ & 16.5 \end{array}\right.$$

Then we have, $r^2 = \frac{81}{12} \cdot \frac{12.5}{16.5} = 5.1$

$$\frac{P}{S} = \frac{5.5}{1 + \frac{(12.25)^2}{9000 \cdot 5.1}} = 1.8$$, much less than was assumed. The section, then, will not do, and a new one must be tried. Assume now 2.5 tons per sq. in. Then area required = 22.23 sq. in. Take section as follows.

1 I beam No. 7 (Phenix Price list)	=	5.5
2 channels $12''$ [$\frac{5}{8}$, $\frac{3}{4}$	$= 2(12 \cdot \frac{5}{8} + 2 \cdot \frac{5}{8} \cdot \frac{3}{4}) =$	$\underline{17.}$
		22.5

Then $r^2 = \frac{12^2}{12}\cdot\frac{17}{22.5} = 9$, nearly. $\frac{P}{S} = \frac{5.5}{1+\frac{40000}{9000\cdot 4}} = \frac{5.5}{2.1} = 2.6$,
hence this section will be sufficient.

(ck.) Assume $2\frac{1}{4}$ tons. Area req. $= \frac{41.69}{2.25} = 18.5$

Take section as follows:

I beam No. 7, = - - - - - - - 5.5

2 channels 12" $\frac{1}{2}$ $\frac{1}{2}$ $2(6+\frac{1}{2}) =$ 13

18.5

$r^2 = 12\cdot\frac{13}{18.5} = 8.4$; $\frac{P}{S} = \frac{5.5}{2.2} = 2.5$, hence the section will do.

(dm.) One of the counters (corresponding to sr) passes through this post.

Assume 1.8 tons. Area required $= \frac{32.32}{1.8} = 18$ sq. in

(+ thickness of web of I beam × diam of counter sr.)

Take section as follows;

I beam No. 8 (thickness of web = $\frac{1}{4}$") = - - - - - - - 4.

2 channels 9" $\frac{3}{4}$ $\frac{1}{2}$ $= 2\left(9\cdot\frac{3}{4} + \frac{3}{2}\cdot\frac{1}{2}\right) =$ 15.

19.

$r^2 = \frac{81}{12}\cdot\frac{15}{19} = 5.3$

$\frac{P}{S} = \frac{5.5}{1+\frac{11}{5.3}} = 1.9$, which is near enough.

(En.) The counter corresponding to fq passes through this post. Assume 1.5 tons. Area req. $= \frac{21.69}{1.5} = 13.8$

(+ thickness of web of I beam × diam of counter fq.)

Take section as follows:

I beam no. 106 (thickness of web = .3) = 3.6

2 Channels 9" $^{5/8}_{1/2}$ $= 2(9 \cdot \frac{5}{8} + \frac{5}{4} \cdot \frac{1}{2}) =$ $\underline{12.5}$

16.1

$$r^2 = \frac{81}{12} \cdot \frac{12.5}{16.1} = 5.2$$

$\frac{P}{S} = \frac{5.5}{2.92} = 1.9$, which is close enough, as in these long and slender struts it is best to be cautious.

(f0.) Assume 1 ton per sq. in $\therefore$ area req. = 13.57 □"

Take section as follows:

I beam No. 106 . - - - - - = 3.6

2 Channels 9" $^{1/2}_{1/2}$ $= 2(9 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2}) =$ $\underline{10.}$

13.6

$$r^2 = \frac{81}{12} \cdot \frac{10}{13.6} = 5.$$

$\frac{P}{S} = \frac{5.5}{3} = 1.8$, so that we can use, instead of I beam No. 106, - No. 105. This is the only change I shall make.

The above sections should be tested for bending sideways, in which case the following formulae are necessary.

$$\left.\begin{aligned} r^2 &= \left(\frac{Ac^2}{4} + \frac{Bc^2}{12}\right) \div (A+B) \\ \frac{P}{S} &= \frac{5.5}{1 + \frac{90000}{36000\, r^2}} . \end{aligned}\right\}$$ c being the depth of the web.

In applying these to the preceding cases, we have

(bl) $r^2 = 10.2$, greater even than the previous one, so that even if the strut were hinged at the ends as regards flexure in this direction, the safe stress would be greater than before.

(ck.) $r^2 = 4.9$. $\therefore$ safe.

(dm.). $r^2 = 7 +$ $\therefore$ safe.

(en.) $r^2 = 5.2 +$ $\therefore$ safe

(fo.) $r^2 = 5 +$ $\therefore$ safe.

For the upper chord we have $r^2 = \left(\frac{A(12)^2}{4} + \frac{B\cdot(20)^2}{12}\right) \div A+B$

$= (36\,A + 33\tfrac{1}{3}\,B) \div A+B$. $\therefore$

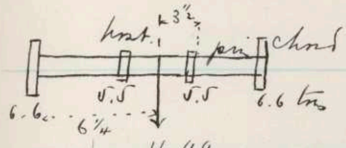
(ef) $r^2 = 34 +$, greater than before. . . . . . .

(ap.) $r^2 = 35 +$ " " ". Hence all are safe.

The great variations in the limiting stress in the above cases show how very general and approximate are the results arrived at in comparing the different girders, and confirm the previous conclusion regarding the lattice.

3." Pins. It is difficult to calculate these accurately, for the reason that it is hard to tell just how the stresses are to act. The first point to consider is whether the ties of the web and lower chord are to run in planes parallel to the truss. If they do not, they must either be made slightly bent, like a ⌠, so that the pins at the ends may fit tightly and the bearing stress be uniform — or if not, where they swing more than a few inches in a panel length, the pins can not fit accurately, and there is danger of the stress being concentrated at and near one edge of the bar. On the other hand, if the ties run only in planes parallel to the truss, washers must be extensively used, and the pins become subjected, in some instances, to quite large bending moments. I have preferred to run the ties crooked, but to make them swing as little as possible. The method of calculating the pins, which I have used, will be seen from the following examples.

(at f) The connections at f are shown on the detail sheet of drawings, and it is evident that the greatest bending moment will occur when fm is acting, in which case we have, horiz. comp. of fm = 13.2 (6.6 at each end.)

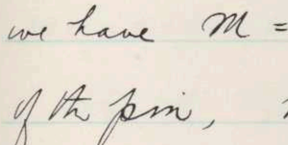


vert. " " " = 10.9 (5.5 " " ")

∴ <u>single</u> shear of chord (all the rest being double shears) = 6.6 tons ∴ area req. = $\frac{6.6}{8}$ = .83 sq. in;
and greatest shear = $\frac{16.49}{2}$ ∴ area req. = $\frac{16.49}{2 \times 8}$ = 1.116 sq. in.
But the bending moments are, horiz = 6.6 × 6¼ = 41.25 inch tons, and vert. = 5.5 × 3½ ∴ max = 41.25. Now we have $M = nfbh^2$, or in this case, d being the diameter of the pin, $M = nfd^3 = d^3 \cdot 10 \cdot .0982 = .982\,d^3$ ∴

$41.25 = .982\,d^3$ ∴ $d^3 = 42$ ∴ $d = 3\tfrac{1}{2}''$.

(at e.) The details of arranging the tie rods it is unne-



cessary to give. Suffice it to say, that they must be arranged so as not to interfere with each other, and if they can be attached to the chords without being run crooked, and so as not to require a very large pin, they should be run straight. At e the tie rod ep is attached as shown above.

Greatest shearing area required = $\frac{12.4}{8}$ = 1.5 sq. in.

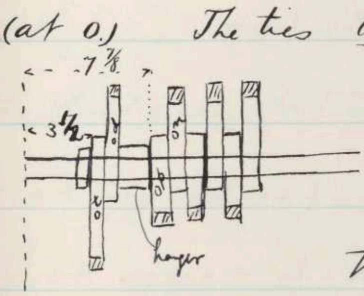
Horiz. moment = 12.4 × 1 = 12.4. Ek is left out of account, since ek and ep do not both pull their greatest stress at the same time. Vert. moment = 10.15 × 2 (about) = 20.3 ∴

$$d^3 = \frac{20.3}{.982} = 20 \therefore d = 3\tfrac{1}{4}''.$$

These examples are sufficient to show the method for the upper chord, and by applying it to the other joints, I find the following diameters for the pins: at d, 3¼; at c, 3½; at b, 3½; at a, 5½". Hence I make all the pins of the top chord 3½" in diameter, except that at a, which is 5½".

For the bottom chord, the following examples will suffice;

(at o,) The ties on transmit a certain stress to the ties op, and the first question is, — how large a pin is needed for this transmission alone. The stress on each



bar $\underline{on} = \frac{267\tfrac{3}{4}}{6} = 44.63$, hence, since one of the bars acts with single shear, area req. $= \frac{44.63}{8} = 5.58$ sq. in. If the pin fits accurately, there will be no moment in this transmission, but a moment will arise when od or ot acts, and the next thing to do is to take account of this. The max. stresses in each bar of ad and ot are equal, and = $\frac{51.49}{2}$ = 25.25 tons, but these max. stresses do not act together. We have now two moments to consider, a horizontal and a vertical. The vertical force is half the same wt. = ½(38.61), and its arm half the diameter of the hanger, since the latter is a round rod (1⅞" diam., as will be shown farther on.) (the load from the post, if the bars od and ot are close to it, produces but a small moment); hence $M = 19.3 \cdot \frac{15}{16} = 18$ in. tons. The horizontal moment is more difficult to estimate. Its force = difference of the horiz. components of od and ot, or since one only acts at a time ~~we may consider it~~ the horiz. comp. of one = $(\frac{6}{5} \cdot 32.325) \cdot \frac{1}{2}$ = $\frac{38.8}{2}$, and its arm may probably be taken as the distance between the bar ot (supposing that bar to be acting alone) and the innermost bar on. The first effect of the action of ot will be to relieve the first bar on of its share of the stress – or part of it – which is transmitted through op, as well as to partly relieve the innermost bar op; – and to allow the innermost bar on to take up its own stress (ot). It is difficult to say just what the effect is, and how far the innermost bar *op* is relieved. If it were relieved to any great extent, it is evident that the other bars *op* would be overstrained, but the deflection of the pin being so very small, and the fact that *od* is pulling in the opposite direction to resist distortion, make it seem probable that it will be safe to take the arm = width of *od* + w. of hanger + w. of *op*. = $4\frac{3}{4}''$. At any point except o, there are not two bars as *ot* and *od*, whose pulls in opposite directions partly counterbalance each other, and hence the above remarks would have still greater importance. We have, then, $M = 4\frac{3}{4} \times 19.4 = 92.1 \therefore d^3 = \frac{92.1}{.982} = 94 \therefore d = 4\frac{1}{2}''$. Similarly I find pins as follows: at n, $4''$; at m, $4''$; at k, $4''$; at l, $4\frac{1}{2}''$; at h, $3\frac{1}{2}''$; at g, $3\frac{1}{2}''$.

It is not usual, I believe, to go into long calculations regarding the pins, for the subject is indefinite, and after all, experience is here the best guide. As to the present case, I have no doubt that ~~if~~ all the pins of the lower chord could be made $4''$ in diam. with perfect safety, though I let them remain as above.

4". Reinforcing plates and bearing areas.

(a) Upper chord. Bearing areas.

(at A) all the ties at A, or at any other point, will not pull their maximum stresses at the same time, but by considering that they do we save trouble and are safe. Thus at A we have, resolving the stresses horizontally, ~~and~~ along ag, and vertically.

total vert. stress = 189 ; total horiz = 244 ; total along ag = 220.6

max. = 244, or 122 at each end of the pin. Hence, since we allow 7 tons per sq. in for bearing, we must have area req. = $\frac{122}{7}$ = 17.5 sq. in, and since pin is $5\frac{1}{2}$" in diam. the thickness of plates = $\frac{17.5}{5.5}$ = 3.2". The vert. plate of the chord is 3/4" thick, hence we require 2.45" more. I put a 3/4" plate outside, and 2 3/4" plates inside, making in all 4 · 3/4 = 3", which will be ample.

I also put a 3/4" cover plate at A, extending 2' over chord and over end post.

(at B.) Horiz. stress = 66.69 or 33.35 on each end of pin. Vert. stress = 55.575 ∴ less. ∴ area required = $\frac{33.35}{7}$ = 4.77 sq. in. ∴ $\frac{4.77}{3.5}$ = 1.4" = thickness required. I put a $\frac{3}{4}$" plate on the outside.

Similarly, I find the plates at the other points as follows; at c, $\frac{3}{8}$"–; at d & other pts. none. But plates are required at all these points, for the vertical plates of the chord break at the joints, hence I put at all points but a, one plate on the outside, – a $\frac{3}{4}$" at b, and a $\frac{1}{2}$" at all other points. The length of these plates will be considered under the head of "rivets".

(b.) Posts. The bearing area on the posts will be calculated supposing that the web or I beam does not bear on the pin at all, although it does so in all cases but two. We have, then, the following;

(bl) $\frac{55.57}{2} = 27.79$ at each end. $\frac{27.79}{7} = 3.97$ sq. in. req.

$\frac{3.97}{3.5} = 1.1$" thickness req. at top, hence by referring to the section of bl we see that when the trough is filled up by a plate there is ample area. Since the pins at the bottom chord joints are larger than those at the top, the required thickness at these pts. is diminished, hence there is no need of calculating that thickness, for the trough is to be filled up both at top and bottom.

(ck) $\frac{41.69}{2.7 \cdot 3.5} = .9$, and referring to the section, we see that we have enough, and similarly for all the other post.

5°. Hangers for floor beams. These consist of two round rods, hence area of each $= \frac{38.61}{2.7} = 2.8$ sq. in. $\therefore$ diam. $= 1\frac{7}{8}$".

6°. Rivets. (a) Post. The distance apart at which the rivets in the post are placed is dictated by experience, but where the post meets the chord the length of the reinforcing plate and the size and spacing of the rivets is determined by the fact that the share of the total stress borne by the I beam must be transmitted to that beam through rivets within the limit of the reinforcing plate, since we suppose the I beam not to bear on the pin. Thus for bl we have the prop. of the stress borne by the I beam is $\frac{5.5}{22.5} \cdot 55.57 = 13.6$ tons, hence $\frac{13.6}{2} = 6.8$ tons must pass through each side. Using ½" rivets for the post, their area being .2$\square$" nearly, and allowing 6 tons for shear, we have, area req. = 1.1 sq. in., and number of rivets is 6, or 3 in a row. I put in 8, spacing them 2½", from centre to centre, and carrying the reinforcing plate 1' above (or below) the edge of the pin, and I do the same for all the other posts.

(b.) Top Chord. (at a) At each end of the pin we have $4 \cdot \frac{3}{4}$" plates, and 196 tons is transmitted to them, or 98 to each end, each plate bearing $\frac{98}{4} = 24.5$ tons; hence $3 \cdot 24.5 = 73.5$ tons must be transmitted by rivets to the vert. plate of the chord inside the length of the 3 reinforcing plates. Using $\frac{7}{8}$" rivets for the Chords, whose area = .6$\square$", each rivet will bear $6 \times .6 = 3.6$ tons. But since one plate is on the outside and two on the inside, and the rivets extend through all, it follows that if we have rivets sufficient to transmit the stress of the two inside plates only, those same rivets, acting in double shear, will serve for the outside plate. Hence $\frac{2 \cdot 24.5}{3.6} = 14$ rivets, which I use. These being spaced $2\frac{1}{2}$" from c. to c. a 2' plate will be long enough as concerns them. But we must also consider the transmission to the upper plates of their share of the stress, which must take place inside the limit of the reinforcing plates.

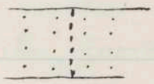
Each vertical plate of the chord is capable of bearing $12 \cdot 3/4 \cdot 5 = 45$ tons, hence all that must be insured is the transmission of $\frac{196}{2} - 45 = 53$ tons to the upper and lower angle irons on each side. The lower angle iron can bear $2 3/4 \cdot 5 = 13 3/4$, hence about 40 tons goes to the upper one $\therefore \frac{40}{3.6} = 11$ rivets required. Spacing them 2" from c. to c. the 2' plate gives length enough for 10, and this will be sufficient. During the remainder of the panel length the 196 tons distributes itself uniformly among the various plates and angles, the rivets being spaced 6", c. to c.

At 6, 40 tons more is added to the chord stress. At this point the vertical plates of the chord are bearing $\frac{18^{\square''}}{39^{\square''}} \cdot 196 = 90$ tons, just what they are capable of, and so are the lower angle irons hence in all the remaining plates, all the additional chord stress coming in must be transmitted to the upper plates, and within the length of the reinforcing plate.

$40 \div 3.6 = 11$ rivets, or ~~6~~ 5.5 on a side. If the reinforcing plate be carried 1' beyond the pin, 6 rivets can be placed in it, and 5 in the upper angle iron, which will be enough.

At all other points 6 rivets are placed in the reinforcing plate itself, and 4 in the upper angle iron.

But the vertical chord plates are not continuous, but break just to the side of every pin (see detail sheet.) Hence we must consider the transmission of their stress, which is to be done through the same plates just calculated, but on the other side of the pin. 90 tons must be transmitted, and although a large part might be transmitted by direct bearing of the plates themselves, it is best to consider that it all – or a large portion of it; – goes through the rivets. If we make this supposition, it follows that the reinforcing plates must be as large as the vertical chord plates themselves, but since the supposition is not true I believe the size of the reinforcing plates as previously fixed, will be sufficient, provided the connection of the chord plates be accurately made. $\frac{90}{2.3.6} = 12.5$ rivets would be needed, did they transmit all the stress, and I have put in from 5 to 8.

Where the horizontal chord plate breaks, reinforcing plates should be placed, and the maximum stress to be transmitted is $5 \cdot \frac{1}{2} \cdot 20 = 50$ tons, which requires $\frac{50}{3.6} = 16$ rivets, under the above supposition. I put in 8, thus . I do not know whether it is the practice to allow for sufficient rivets to transmit all the stress or not.

7°. Lateral bracing and width of bridge.

I assumed a width of 16' as agreeing with precedent. This dimension will of course depend on the size of the cars and engines, but it is also important to notice that during gales the entire weight of the train is liable to be thrown on one ~~track~~ rail, and that provision should be made for this event by properly adapting to each other the strength of the main trusses, the width of the bridge, and the speed of the train. Some deck bridges are built so narrow that in such a case a very large proportion of the whole weight of the train would fall upon one truss. Even in such a case, however, if the assumed loads had been determined by the method which I have used, there would be no danger, provided that trains were obliged to run slowly during such gales, for in that case a large proportion of the destructive action of the live load would be got rid of. As regards the floor system, of course, the narrower the bridge the greater the economy. I do not know the exact weights & dimensions of the cars now in use on our roads, but assuming the weight of a loaded Pullman car, which gives the greatest wind surface, to be 71600 lbs., and its length 75' and height (of box) 12', + total ht. = 15', we have wind surface = 75 × 12 = 900 sq. ft., and if x = wind pressure in lbs. per sq. ft. sufficient to throw all the wt. on one rail,

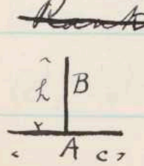
$x \cdot 900 \cdot 9 = 2\tfrac{1}{3} \cdot 71600 \therefore x = 21-$ lbs., a pressure which is by no means uncommon in this climate. But the heaviest cars do not give the greatest wind surface, and it would probably take a much greater pressure to throw the whole wt. of a train of coal cars on one rail, although the heaviest gales would probably do it. We see, however, that the wind pres.

sure on the cars does not affect the lateral bracing, or at least does so only very slightly.

The heaviest wind pressure of which I have any knowledge (in this climate) is 72 lbs. per sq. ~~in.~~ ft., which occurred in Boston last Winter? The surface exposed per ft. run is about 6 sq. ft. for each truss or 12 in all, exclusive of cars. Gales like that above mentioned being exceedingly rare, it will be safe to assume 60 lbs. per sq. ft. pressure. Hence panel load $= 15 \cdot 60 \cdot 12 = 5.4$ tons (10800 lbs.) There is no need of assuming partial loads, for the wind pressure is nearly uniform. Hence each supporting force $= 5.4 \times 5.5 = 29.7$; call it 30. Hence the shears are: 30; 24.6; 19.2; 13.8; 8.4; 3. But these are for two systems, the upper and the lower, hence for each we have the shears, 15; 12.3; 9.6; 6.9; 4.2; 1.5. Resolving along the bars, the stresses become: $20\frac{5}{8}$; 16.9; 13.2; 9.5; 5.8; 2.1. The wind pressure, though not so destructive in its action as a train of cars is far from being a dead load, but we can

safely use 7 tons per sq. in as the limiting stress for tension, for we have assumed a very large
ft.
wind pressure. Rankine gives 55 lbs. per sq. in. as the maximum. Using 7 tons, then, we have the following areas; 2.93 ; 2.4 ; 1.9 ; 1.4 ; .8 ; .3 , and I use the following rods and areas:
$1\frac{7}{8}''$, 2.76 □ ; $1\frac{3}{4}''$, 2.4 □ ; $1\frac{5}{8}''$, 2.1 □ ; $1\frac{3}{8}''$. 1.48 □ ; $1\frac{1}{4}''$, 1.23 □ ; $1''$, .78 □. These dimensions approach very nearly the common practice. The chord stresses due to the wind pressure are so small in comparison with those due to the loads that the margins in the areas of the chords are sufficient to allow for them. There only remain, therefore, the struts. Those of the upper system consist of two angle irons rivetted together, and to the chords on which they rest. The stresses on them are;
15 ; 12.3 ; 9.6 ; 6.9 ; 4.2 ; 3 . (a) Use a limiting stress of 5 tons. Assume ~~4~~ 3 first. struts being fixed at ends by being rivetted to the chords. Area req = $\frac{15}{3\cancel{4}} = \cancel{3\frac{3}{4}}\ 5$

The ~~Rankine's~~ formulae for a T iron are $r^2 = \dfrac{Ac^2}{12(A+B)}$ and



$$r'^2 = h^2\left\{\frac{B}{12(A+B)} + \frac{AB}{4(A+B)^2}\right\};\quad \frac{p}{S} = \frac{5}{1+\dfrac{l^2}{36000\,r_1^2}}.$$

Assume 2 angle irons, $3 \times 3 \times \frac{1}{2}$, area $= 2(2\frac{3}{4})$

$r^2 = \frac{11}{4} \cdot 36 \cdot \frac{1}{12} \cdot \frac{1}{5\frac{1}{2}} = \frac{3}{2}$: $r'^2 = 9\left\{\frac{1}{24} + \frac{1}{16}\right\} = 1-$ ∴

$r_1^2 = (\text{min. } r)^2 = 1$ ∴ $\frac{p}{s} = \frac{5}{1 + 1.02} = 2.4$, hence we must assume less than 3. Take 2.5; then area req. $= \frac{15}{2.5} = 6$ sq. in. Take $2(4 \times 4 \times \frac{1}{2})$; area $= 2(3\frac{3}{4})$

$r^2 = \frac{64}{24} = 2\frac{2}{3}$; $r'^2 = 16\left\{\frac{1}{24} + \frac{1}{16}\right\} = 1\frac{2}{3}$:

$\frac{p}{s} = \frac{5}{1 + \frac{36864}{36000 \cdot \frac{5}{3}}} = 3.4$, hence this would do, but for convenience of connection, since the tie is quite large and a large nut is required at its end, I use $2 \cdot 6 \times 4 \times \frac{1}{2}$ angle irons.

(b.) Stress $= 12.3$ tons. Assume 2.5; area req. $= 5^{\square}$.

Take 2 angle irons, $5 \times 3 \times \frac{1}{2}$, area $= 2(3\frac{3}{4})$

$r_1^2 = 1.2$ ∴ $\frac{p}{s} = 2.5$ nearly, hence the section will do.

(c.) Stress $= 9.6$ tons. Assume 1.8 tons: area req $= 5.3$

Take 2 angle irons $4 \times 4 \times \frac{1}{2}$. area $= 2(3\frac{3}{4})$

$r_1^2 = \frac{5}{3}$ ∴ $\frac{p}{s} = 3.4$, - far larger than assumed.

(f.) and (e) The sizes being thus greatly governed by convenience of connection, I assume the last two $(4 \times 3 \times \frac{1}{2}) \cdot 2$ and the others as given above.

(d) I assume this $4 \times 4 \times \frac{1}{2}$. Thus they are as follow:

(a) 2 angle irons, $6 \times 4 \times \frac{1}{2}$

(b) " " ", $5 \times 3 \times \frac{1}{2}$.

(c) " " ", $4 \times 4 \times \frac{1}{2}$

(d) " " ", $4 \times 4 \times \frac{1}{2}$

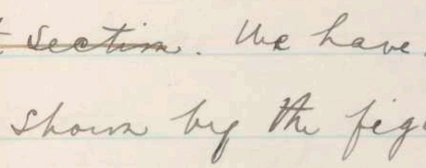
(e) " " ", $4 \times 3 \times \frac{1}{2}$

(f) " " ", $4 \times 3 \times \frac{1}{2}$

The lower lateral system has the floor beams for its struts, and these will be considered farther on.

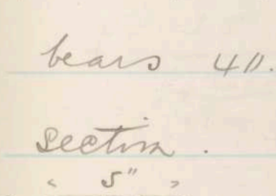
The next thing to consider is the attachment of the transverse posts to the chords, which is done by rivets. The greatest stress to be transmitted is $\frac{15}{16} \cdot 15 = 14$ tons, and allowing 6 tons for shear, and using $7/8$" rivets, we have the number of rivets needed $= \frac{14}{6 \times .6} = 4$. I put in from 8 to 10, for perfect rigidity.

8°. Floor beams. These are rolled I beams, two at each joint. It will probably be unnecessary to take account of ~~the~~ the fact that the whole weight of a train is liable to be thrown on one rail, for, as before remarked, if the gale is so heavy as to throw the whole weight of the <u>heaviest</u> cars on one rail (?), the train should be run slowly, and this will protect the trusses and floor beams sufficiently to make up for this concentration of load, and also for the stress in the floor beams due to their position as the struts of the lateral system. ~~This latter, however, will be allowed for by a slight addition to the section~~. We have, then, a beam 16' long, loaded as shown by the figure, since each driver of the



engine bears 6 tons, and 220% of this = ~~about~~ 14.4 tons

$M = 14.4 \cdot 5\frac{2}{3} = 81.6$ ft. tons. Hence each I beam bears 40.8 ft. tons moment. Take the following section. Area of web, top & bottom $= 15 \times .6 = 9.0$ sq. in

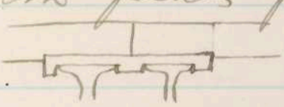


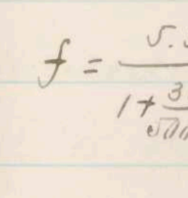
" " each flange $= 4.4 \times 1$ ∴ total $= \underline{8.8}$

17.8 sq. in.

$I = \frac{1}{12} \cdot 9 \cdot 225 + 8.8 \cdot 49$

$= 600$. Also, from Rankine, p. 526, we have $f = \dfrac{5.5}{1 + \dfrac{36864}{5000 \cdot 25}} = \dfrac{5.5}{1.3} = 4.2$ tons per sq. in.

Hence $\frac{fI}{y} = \frac{4.2 \times 600}{7.5} = 336$ inch tons. But 40.8 ft. tons = 489.6 inch tons. Hence this I beam will not do. Take now the following case. Partially stiffen the I beams by having the stringers rest on cast iron plate projecting over the sides, in section thus . The I beam can not bend now without deranging all the stringers, and although we can not consider it fixed at all these points, yet it will be amply safe to consider its length as 5', hence $f = \frac{5.5}{1+\frac{3600}{5000\, b^2}}$. Take now the following section



Area of web $= 15 \cdot .65 = \quad 9.75$

" " both flanges $= \quad \underline{10.25}$

$20.$

$$I = \frac{1}{12} \cdot \frac{39}{4} \cdot 225 + 10.25 \cdot 49$$
$$= 685.25.$$

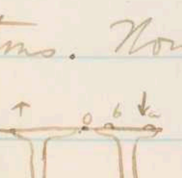
$$f = \frac{5.5}{1+\frac{3600}{5000 \cdot \frac{441}{16}}} = 5.36 \quad \therefore \quad \frac{fI}{y} = \frac{5.36 \cdot 685.25}{7.5} = 489.7,$$

and I think this will be sufficient. To allow for the lateral bracing, I think no addition will be needed, for I believe that I beams of the dimensions given above have been found to be sufficient in practice. However, since the Phoenix

Iron Co. roll a beam exactly similar to the above, except that the width of flange is $5\frac{5}{16}$", I shall use that, in which case t would be made slightly greater, and the effect of wind pressure just about allowed for.

The connection of the lateral bracing is shown sufficiently well by the detail drawings accompanying this thesis. Connection is made with the chords by rivetting the posts to the I beams. The calculations for the rivets are the same as those previously given for the upper system.

At h, 2 plates are put in especially to connect the lateral system with the chord. As these plates can turn round the pin, it may be necessary to calculate the rivets here by moment. There is $\frac{15}{16} \cdot 15$ tons $= 14$ tons to be transmitted to the chord, and it acts at a distance of 1' below the centre of the pin (about). Hence its moment $= 14 \cdot 12 = 168$ inch tons. Now there are two rows of rivets in each I beam.



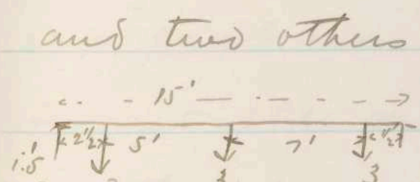
and the required moment is given by

the stress in them round an axis through O. Hence if Oa = 5" and Ob = 2½", we have, considering the stress to be uniformly varying;

$168 = 2x \cdot 5 \cdot 7 + 2x \cdot 2\tfrac{1}{2} \cdot 7$, x being the rivet area in each row – a – b, and 7 the stress per sq. in. $\therefore 168 = 70x + 35x = 105x \quad \therefore x = 1.6$ sq. in.

Hence if 7/8" rivets are used 3 will be needed in each row, which I have 6.

9° Stringers. These are of oak, and the sleepers are notched into them. They rest on castings on the floor beams as before described, and as shown on the first plate of drawings, one stringer bears half the load on a rail (not exactly, since the tie is a continuous girder). The greatest bending moment occurs when one driving wheel is in the centre, and two others near the ends, thus. In this



case $M = \frac{4.1}{\cancel{15}_{2}} \cdot \frac{\cancel{15}^{3}}{2} - 3 \cdot 5 = 15\tfrac{3}{4}$ foot tons. If two drivers are equally dist. from the centre, $M = 15$ ft. tons

$15\tfrac{3}{4} \times 2 + 20\%_0 = 37\tfrac{1}{2}$ nearly. $= 450$ inch tons.

$\frac{1}{6} \cdot 2500 \cdot b h^2 = 450 \cdot 2000.$ assume $h = 16''$.

Then $\frac{1}{6} \cdot 2500 \cdot b \cdot 256 = 900000$ $\therefore$

$b = 9''$ (about.)

General Details.

The ties are of hard pine, 8"x8", spaced 8" in clear, an arrangement quite common, and perfectly safe.

Where the stringers run on the abutment, they rest on plates of cast iron, notched into the stone, as shown on Pl. I.

There are two guard rails of hard pine, 8"x 8", bolted down at every third tie.

The end posts rest on castings which extend 2½" into the posts. This casting, at the movable end of the span, rests on rollers, below which is a cast iron wall plate notched down, and secured still more by four bolts with screws at their lower ends which screw into two wedge shaped pieces, the whole acting like a lewis. To this casting, the lower system of lateral bracing is attached by a bolt and nut, countersunk underneath, so as not to interfere with the rollers. The fixed end of the span is the same as the movable, except that the rollers are omitted, and the casting and wall plate are combined, making only one piece.

The lattice bars for the top chord and end posts are 2" x 1/4" and have a run of 2'.

In order to prevent the weight of the top chord from being transmitted to the post through the pin, and thus causing a permanent stress on the upper parts of the reinforcing plates at the joint, the chord rests on cast iron brackets, which are bolted to the posts lower down, thus transferring the weight directly.

The portal bracing consists of two pair of curved angle irons, 3 x 3 x 1/2, securely rivetted to the end posts and to a second pair of angle irons, 6 x 4 x 1/2, extending straight across. These three members, - the curved angle irons, the straight ses, and the end posts, - are also tied together by rectangular flat bars, 2" × 1/4". The arrangement is clearly shown on Pl. 2.

The details of bridge construction can be best learned from experience, and although some of the above may be faulty, I hope that the general methods of design and calculation are correct.

Geo. F. Swain.

(Note.) Lack of time prevents me from making a calculation of the weight of the bridge. Should such a calculation show the weight to be very much greater than that assumed, the dimensions of the pieces would have to be changed, but I should not consider it worth while to notice a moderate variation, say up to 50 lbs. per ft. run.