A COMPARISON OF THE EXISTING METHODS OF STUDYING THE STABILITY OF EARTH SLOPES

by

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ABSTRACT

COMPARISON OF EXISTING METHODS OF STUDYING THE STABILITY OF EARTH SLOPES

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Submitted in partial fulfillment of the requirements for the degree of Master of Science at the Massachusetts Institute of Technology, August 24, 1959.

The object of this thesis has been to compare the most recent developments in studying the stability of earth slopes. A theoretical study is presented of the general conditions of equilibrium that must be fulfilled for the problem to become statically determined. These considerations have led to a procedure having the above characteristics. From the application of different methods to practical examples, it is concluded that the Method of Bishop in its most simplified form is simple, reliable, and involves an error that is small when compared to a more rigorous method.

The results provided by a more exact procedure, such as the one presented here, seem to be a reason for further investigation.

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SECTION I

OBJECT OF THE THESIS AND CONCLUSIONS IN BRIEF

This thesis has been an attempt of comparison of different existing methods of studying the stability of earth slopes, with special emphasis on the procedure published by Bishop in 1954.

It has been the author's aim to give some thought to the mechanics of the analysis, as well as to the conditions of static equilibrium that the slopes must fulfill. It has been concluded that care must be taken in making only the necessary assumptions to overcome the indetermination of the problem since, otherwise, there exists the possibility of the problem to become overdetermined.

It is a general principle of engineering that any method of calculation will be accepted by practical engineers if it is simple, reliable in practice, and if the error involved is small as compared to a more rigorous procedure.

A method of the above characteristics is even more desirable in the analysis of stability of earth slopes by means of any of the slip circle procedures, in which a complicated calculation may lead to a number of trials unsafely small.

Since the accuracy of the Method of Bishop, when applied to homogeneous slopes, has been proved to be satisfactory in previous studies (Sevaldson, 1955; Bjerrum and Kjaerneli, 1956), more attention is given to a case on non-homogeneous slope.

The conclusion to be drawn from the examples presented herein is that the method of Bishop in its most simplified form is the procedure to be recommended since the error involved, in the safe side, as compared to a more rigorous one, seems to justify its use due to the fact that the time spent is much shorter.

SECTION II

GENERAL CONSIDERATIONS

The methods of analyzing stability of earth slopes may be classified in two groups: (1) methods in which the state of stresses are investigated in the whole earth mass, and (2) methods in which the state of stresses is only investigated along an assumed surface of failure.

The procedures of the first group, mostly based on the theory of Elasticity have been investigated and compared by Carrillo (Ref. 4), Terzaghi (Ref. 10), Bishop (Ref. 1), and Fröhlich (Ref. 5), whose general conclusions are that these methods are neither simple nor reliable. Furthermore, the theoretical assumptions and simplifications do not, in general, justify such painstaking and involved calculations.

The second group is based on the theory of limit design.

It is a well-known fact (Refs. 1 and 5) that when in a point of an earth mass the stresses reach its limit values, failure does not take place. On the contrary, a plastic behavior begins to develop. A further strain takes place without change in stress and a progressive phenomenon of relaxation is present in such a manner that only when the condition of flow has been reached along a continuous path across the earth mass does a real failure happen.

It appears from the above considerations that the limit design group of methods gives a better inside picture of the problem, originating simpler methods as well. Nevertheless, two shortcomings are apparent: (1) the use of an approximate surface of failure makes inexact any of the methods; and (2) the conditions of limit equilibrium are only valid when the slope is in the verge of failure that is precisely the situation that must be prevented.

It is at this second stage that the concept of factor of safety appears, being its definition a rather hard task.

First of all, a mathematical choice is open, being it usually defined as against shear strength (see Ref. 8, Chap. 16).

$$\tau = \frac{S}{F} = \frac{C_e}{F} + \bar{\sigma}_{ff} \frac{\tan\phi_e}{F}$$

But a more subtle point is here involved, which must be considered, referring to the mode of failure, which gives raise to two definitions: first, it can be assumed that the slope is on the verge of failure and the study of bare equilibrium will provide the shearing resistance required to maintain it; or second, the actual stable state is studied, thus obtaining the mobilized shear resistance required to this equilibrium. Both approaches involve a knowledge of the pore pressures set up in the slope and also depend upon the knowledge of shear strength. As the subject is not the scope of this thesis, no further discussion is given (see Ref. 11).

Furthermore, the problem of stability analysis is indetermined, as will be seen later, and an <u>exact</u> solution is not possible. Consequently, some assumptions must be done to obtain a feasible procedure and some conditions to be fulfilled are usually neglected, thus leading to different definitions of the factor of safety.

It is the author's feeling that any comparison of methods will be meaningless if one does not keep in mind how the factor of safety is actually defined.

SECTION III

MECHANICS OF THE ANALYSIS

Let it be assumed that a trial circle has been chosen, thus defining a free body whose equilibrium has to be studied, and let this body be divided into n slices by means of n - 1 vertical lines.

In Figure 1 the forces acting on a sample slice are shown as well as the corresponding polygon of force equilibrium.

The polygon of forces provides two of the three conditions necessary for the slice to be in equilibrium. These equations are obtained by projection on two non-parallel directions of the plane and are any two of Equations 1a, 1b, 1c, 1d shown in Figure 2. The third condition is obtained by taking moments about any point which is chosen to be the center of the circle O, and it is given by Equation 1e.

The number of equations will be three by slice, thus totaling 3n. The unknowns will be E_i, X_i, y_i $(i = 1, 2, \ldots . n - 1)$, that is, 3(n - 1); N_i $(i + 1,2, \ldots . n)$, that is, n more unknowns, plus the factor of safety F.

Therefore, the total number of unknowns is:

$$3(n - 1) + n + 1 = 4n - 2$$

and being 3n the number of equations the degree of indetermination is $(4n - 2) - 3n = n - 2$.

Two things will be noticed in the above considerations: (1) the reasoning is valid regardless of the shape of the assumed surface of failure; and (2) the moment equation is necessary for the slice equilibrium regardless of the existance of the actual center of rotation.

Some mathematical process is done to make the obtained equations of practical value. First, T_i and N_i are eliminated by means of Eqs. 3, and second, the set of equations (1d) may be substituted by the result of summing up these equations after giving to i the values 1, 2, 3, ..., i. No change in the compatibility conditions is caused by the last operation that is merely a linear substitution.

The so-called Fundamental equations (4) are thus obtained.

SECTION IV

SLIP CIRCLE METHODS

In the previous section it has been seen that, being the problem indetermined, some distributional assumption must be done to overcome this situation (see Ref. 9, Art. 16.15), thus giving rise to a variety of methods.

(a) <u>Method of Fellenius</u>

The forces on the sides are neglected and $\Sigma N = 0$ is utilized to obtain the value of the direct stress on the base of the slice, thus leading to Equation 5 that defines the factor of safety. It is seen that there is no static equilibrium.

(b) <u>Method of Bishop</u>

By substituting Equation 3a in 4c the general definition of the factor of safety is obtained, Equation 6a. As a further condition of compatibility, $E_{i-1} - E_i$, is calculated in Equation 6b.

A general discussion of the procedures that may be followed is presented in Reference 2 although no detail is given. This refers to the kind of assumptions that must be done as well as to the successive approximations involved.

By making $X_{i-1} - X_i = 0$ Equation 6c is set up, defining the simplified method of Bishop.

(c) <u>Method of Janbu</u>

In Reference 6 it may be shown how Janbu did obtain its expression for the safety factor, derived from Equation 6b. To run the successive approximations a moment equation is utilized in which the forces on the sides are assumed to be applied at the third point.

(d) Modified Method of Slices

A general expression for the so-called modified methods (Ref. 12) may be obtained by introducing the angle β_i in Eq. 4b and Eq. 8 is obtained.

Different assumptions may be done:

(i) The forces on the sides are applied in a direction bisecting the angle formed by the slope direction and the tangent to the slip circle at the bottom of the corresponding vertical.
It can be seen that the problem becomes overdetermined since n - 1 new equations are introduced and only n - 2 were necessary for the equilibrium to be possible.
Anyway, this method is applied to the Example B, showing how the moment equation (6a) gives a different factor of safety in spite of the fact that Eq. 6b is fulfilled.

(ii) If the forces on the sides are assumed to be parallel, a solution is possible which establishes the static equilibrium since one more unknown is introduced, β, but the number of additional equations is n - 1. The general formula for this method is given by Eq. 9.
After a value of β is assumed, different values are given to F until it is obtained $\Sigma(E_{i-1} - E_i) = 0$. The obtained values of F and X_i are then introduced in Eq. 6a. If the Eq. 6a holds good, the problem is solved. Otherwise, more trials are necessary.

No special recommendation may be done about the choice of the angle β.
It must be kept in mind that it must be reasonable with regard to the physical properties of the soil involved.

SECTION V

EXAMPLES AND PROCEDURE

Two practical cases have been studied:

CASE A: Homogeneous Section

This case was thoroughly studied by Sutherland in Reference 8. Some additional analyses were made:

1. The parallel assumption of Method d-ii was repeated and Sutherland's results rechecked.
2. The simplified method of Bishop was utilized.
3. By utilizing the forces on the sides provided by Method d-ii, the line of thrust was obtained by means of Eq. 4a.

The physical and geometrical characteristics of the section as well as all calculations are presented in Appendix I, and Figure 3.

CASE B: Non-homogeneous Section

The section presented in Figure 4, pertaining to Hilfanli Dam in Turkey was obtained through the kindness of Mr. John Lowe.

The following methods were applied:

1. Method of Fellenius.
2. Simplified method of Bishop.
3. Method d-i with further use of Eq. 6a in order to determine the error involved in the procedure.
4. Method d-ii.
5. By utilizing the results obtained in Step No. 4,

the line of thrust was determined by means of

Eq. 4a.

The characteristics of the section as well as calculations are presented in Appendix II. For the sake of brevity the different trials done for the determination of the angle β are not presented and only the final solution is calculated here.

In both cases some graphical procedure would provide simpler study. It has not been done so because, this being an attempt of investigation, more accuracy was desired.

SECTION VI

CONCLUSIONS AND RECOMMENDATIONS

The following table summarizes the factors of safety as obtained by the various methods:

	Factor of Safety	Equilibrium
<u>CASE A</u>		
1. Fellenius	1.38	No
2. Method b-ii ($\beta = 14.4^o$)	1.53	Yes
3. Simplified Bishop	1.53	No
<u>CASE B</u>		
1. Fellenius	1.57	No
2. Simplified Bishop	1.77	No
3. Method b-i	1.91 (F_M=1.83)	No
4. Method b-ii ($\beta = 20^o$)	1.88	Yes

The conclusions to be drawn from the theoretical considerations and the practical examples may be summarixed as follows:

1. When the free body defined by trial circle is divided into <u>n</u> slices, the degree of indetermination is <u>n - 2</u>. This conclusion holds also good for any shape of the assumed surface of failure.

Therefore care must be taken that the number of assumptions does not succeed the degree of indetermination.

2. The method of Fellenius must be discarded as giving results too conservative.

3. The method b-i does not give a large error but it must be kept in mind the meaning of the safety factor which is provided by the equilibrium of forces, regardless of the moment equilibrium.

4. From the obtained lines of thrusts it is concluded that Method b-ii seems to be the most exact of all procedures utilized in this thesis. It not only establishes the static equilibrium but also gives a thrust reasonably distributed, thus confirming the method indicated by Janbu (Ref. 6).

5. The simplicity of the method as well as the errors involved as compared to more rigorous procedures makes the Simplified Bishop the method to be recommended.

6. It is the author's feeling that more investigation should be done in methods like b-ii in order to provide a more simple mathematical treatment to the problem. The reasonability of the results here obtained seems to make this study worthy of further research.

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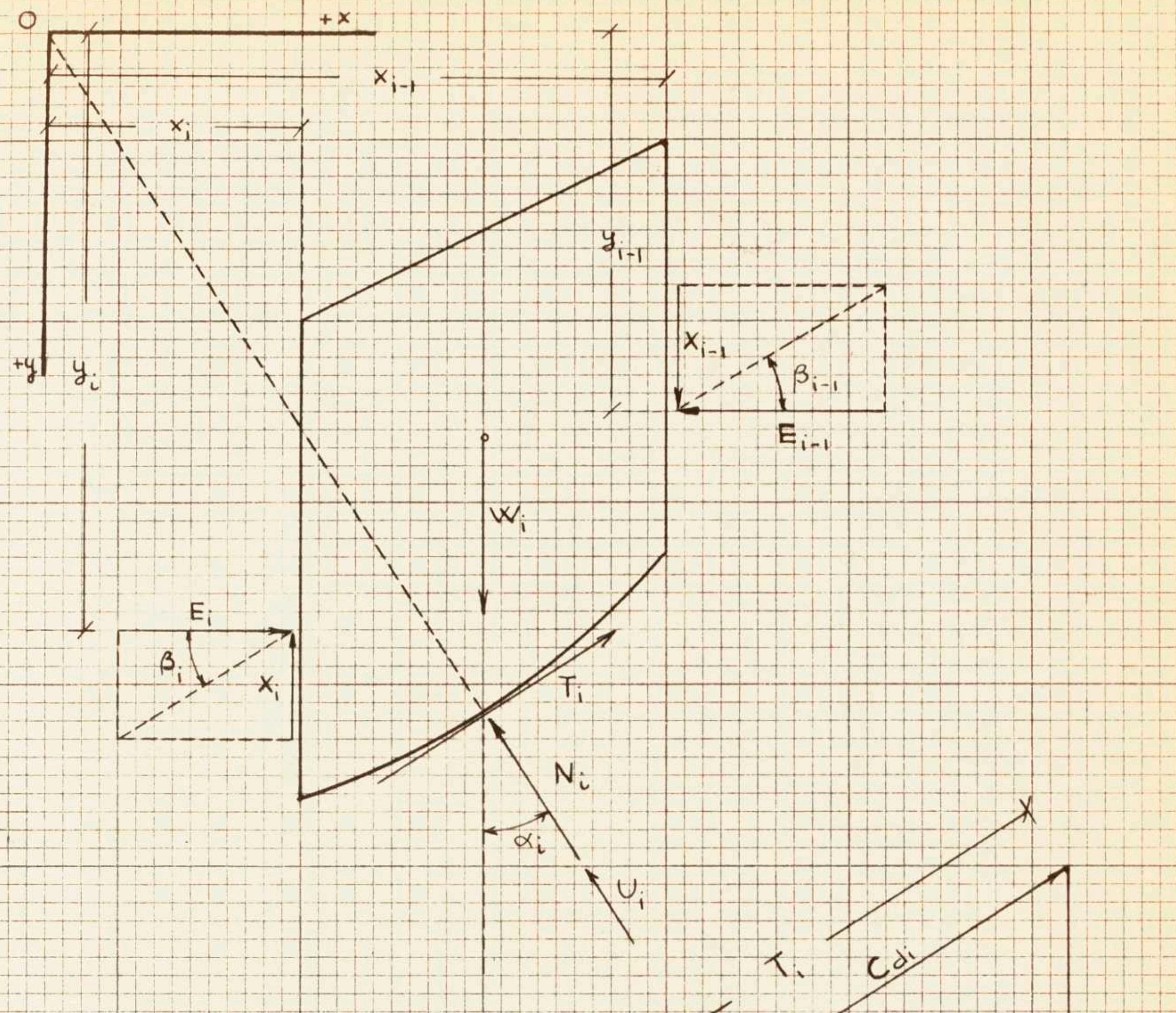


SYMBOLS

$$N_i = \bar{\sigma}_i \cdot \widehat{L}_i$$

$$C_{di} = \frac{C_{ei}}{F} = \frac{c_{ei} \cdot \widehat{L}_i}{F}$$

$$\tan\phi_{di} = \frac{1}{F} \tan\phi_{ei}$$

$$U_i = u_{ave.i} \widehat{L}_i$$

FIGURE 1: KEY

EQUATIONS OF EQUILIBRIUM

$$\Sigma V = 0 \qquad W_i + X_{i-1} - X_i = (N_i + U_i)\cos\alpha_i + T_i \sin\alpha_i \qquad (1a)$$

$$\Sigma H = 0 \qquad E_{i-1} - E_i = -(N_i + U_i)\sin\alpha_i + T_i \cos\alpha_i \qquad (1b)$$

$$\Sigma T = 0 \qquad T_i = (W_i + X_{i-1} - X_i)\sin\alpha_i + (E_{i-1} - E_i)\cos\alpha_i \qquad (1c)$$

$$\Sigma N = 0 \qquad N_i + U_i = (W_i + X_{i-1} - X_i)\cos\alpha_i - (E_{i-1} - E_i)\sin\alpha_i \qquad (1d)$$

$$\Sigma M = 0 \qquad x_{i-1}X_{i-1} + y_{i-1}E_{i-1} - (x_i X_i + y_i E_i) = W_i d_i - RT_i \qquad (1e)$$

AUXILIARY COEFFICIENTS

$$M_i = \cos\alpha_i + \sin\alpha_i \tan\phi_{di} \qquad (2a)$$

$$M_i' = \sin\alpha_i - \cos\alpha_i \tan\phi_{di} \qquad (2b)$$

FUNDAMENTAL EQUATIONS

From (1a) $\quad N_i M_i = W_i + X_{i-1} - X_i - U_i\cos\alpha_i - C_{di}\sin\alpha_i \qquad (3a)$

From (1b) $\quad -N_i M_i' = E_{i-1} - E_i + U_i \sin\alpha_i - C_{di}\cos\alpha_i \qquad (3b)$

From (3a) $\quad T_i = \frac{1}{M_i}\left[C_{di}\cos\alpha_i + (W_i + X_i - X_{i-1} - U_i\cos\alpha_i)\tan\phi_{di}\right] \qquad (3c)$

Therefore

$$\boxed{M_i'(X_{i-1} - X_i) + M_i(E_{i-1} - E_i) = C_{di} - W_i M_i' - U_i \tan\phi_{di}} \qquad (4a)$$

In (1e) $i = 1, 2, 3, \ldots i$ and summing up

$$\boxed{x_i X_i + y_i E_i = R\left[\sum_{i=1}^{i} T_i - \Sigma W_i \frac{d_i}{R}\right]} \qquad (4b)$$

FIG. 2A DERIVATIONS ~

METHOD OF FELLENIUS.

$X_{i-1} - X_i = 0$, $E_{i-1} - E_i = 0$

From (1d) $N_i = W_i \cos\alpha_i - U_i$

In (4b) $i = n$, and N_i subtituted:

$$F = \frac{\sum \left[C_{ei} + (W_i \cos\alpha_i - U_i) \tan\phi_{ei} \right]}{\sum W_i \sin\alpha_i} \tag{5}$$

METHOD OF BISHOP.

In (4b) $i = n$, and (3c) is subtituted:

$$F = \frac{\sum \left[C_{ei} \cos\alpha_i + (W_i + X_{i-1} - X_i - U_i \cos\alpha_i) \tan\phi_{ei} \right] \frac{1}{M_i}}{\sum W_i \sin\alpha_i} \tag{6a}$$

If $X_{i-1} - X_i = 0$, or $\sum (X_{i-1} - X_i) \frac{\tan\phi_{ei}}{M_i} = 0$,

SIMPLIFIED BISHOP:

$$F = \frac{\sum \left[C_{ei} \cos\alpha_i + (W_i - U_i \cos\alpha_i) \tan\phi_{ei} \right] \frac{1}{M_i}}{\sum W_i \sin\alpha_i} \tag{6b}$$

From (1c):

$$E_{i-1} - E_i = T_i \sec\alpha_i - (W_i + X_{i-1} - X_i) \tan\alpha_i$$

Summing up:

$$\sum (E_{i-1} - E_i) = 0 = \sum T_i \sec\alpha_i - \sum (W_i + X_{i-1} - X_i) \tan\alpha_i \tag{6c}$$

METHOD OF JANBU.

By introducing (3c) in (6c) F is obtained:

$$F = \frac{\sum \left[C_{ei} \cos\alpha_i + (W_i + X_{i-1} - X_i - U_i \cos\alpha_i) \tan\phi_{ei} \right] \frac{\sec\alpha_i}{M_i}}{\sum (W_i + X_{i-1} - X_i) \tan\alpha_i} \tag{7}$$

Some moment equation is utilized.

FIG 2B: METHODS

GENERAL SOLUTION

By doing $X_i = E_i \tan\beta_i$, and introducing the auxiliary coefficients:

$$P_i = M_i + M_i' \tan\beta_i$$

$$P_i' = M_i + M_i' \tan\beta_{i-1}$$

Eq. 4a becomes:

$$P_i' E_{i-1} - P_i E_i = C_{di} - W_i M_i' - U_i \tan\phi_{di} \qquad (8)$$

This equation is utilized in Method d-i

PARALLEL SOLUTION:

By making $\beta_i = \beta$ = constant

$$P_i = P_i' = M_i + M_i' \tan\beta$$

and (8) becomes:

$$E_{i-1} - E_i = \frac{1}{P_i}\left[C_{di} - W_i M_i' - U_i \tan\phi_{di}\right] \qquad (9)$$

The so-called Method d-ii consists of:

1. Reasonable β assumed
2. F is obtained by condition $\Sigma(E_{i-1} - E_i) = 0$ & Eq. 9
3. F is introduced in Eq. (6a)
4. If Eq. 6a not fulfiled, another β is assumed.

FIG. 2B (CONT.)

APPENDIX I

HOMOGENEOUS SECTION

TABLE A-I: CHARACTERISTICS OF THE BODY

Slice	W_i (Kips)	α_i	$\sin\alpha_i$	$\cos\alpha_i$	$W_i \sin\alpha_i$ (Kips)	U_i (Kips)	C_{ei}
1	73.00	-15.20°	-0.2630	0.9644	-19.20	0.00	13.20
2	182.00	-3.80°	-0.0659	0.9973	-12.00	19.90	12.75
3	252.00	$+7.10^\circ$	0.1238	0.9889	+31.20	54.00	13.00
4	275.00	$+18.40^\circ$	0.3142	0.9484	+86.40	70.00	13.30
5	247.00	$+30^\circ$	0.5020	0.8640	+124.00	65.00	14.60
6	130.00	$+45^\circ$	0.7015	0.7015	+91.20	29.00	18.30

TABLE A-II: AUXILIARY COEFFICIENTS

$\beta = 14.4^\circ$

$F = 1.53$

$\tan \phi_d = 0.2614$

Slice	$\sin\alpha_i$	$\cos\alpha_i$	$\sin\alpha_i \tan\phi_d$	$\cos\alpha_i \tan\phi_d$	M_i	M'_i	$M'_i \tan\beta$	P_i
1	-0.2630	0.9644	-0.0688	0.2521	0.8956	-0.5151	-0.1324	0.7633
2	-0.0659	0.9973	-0.0172	0.2607	0.9800	-0.3267	-0.0840	0.8961
3	0.1238	0.9889	0.0324	0.2585	1.0213	-0.1347	-0.0346	0.9866
4	0.3142	0.9484	0.0821	0.2479	1.0305	0.0662	0.0170	1.0475
5	0.5020	0.8640	0.1313	0.2259	0.9952	0.2762	0.0710	1.0662
6	0.7015	0.7015	0.1834	0.1834	0.8850	0.5181	0.1332	1.0181

TABLE A-III: FORCES ON THE SIDES OF SLICES

($\beta = 14.4^{\circ}$, F = 1.53)

Slice	W_i	U_i	(a) $W_i M'_i$	(b) $U_i \tan\phi_{di}$	(c) $C_{di} = \frac{C_{ei}}{F}$	(a)+(b)	(c)-(a)-(b)	P_i	$E_{i-1} E_i$	$X_{i-1} X_i$	E_i	X_i
1	73.00	0.00	-37.61	0.00	8.63	-37.61	46.24	0.7633	60.60	15.57	-60.60	-15.57
2	182.00	19.90	-59.45	5.20	8.33	-54.25	62.58	0.8961	69.87	17.96	-130.47	-33.53
3	252.00	54.00	-33.95	14.12	8.50	-19.83	28.33	0.9866	28.84	7.41	-159.31	-40.94
4	275.00	70.00	+18.22	18.30	8.69	+36.52	-27.83	1.0475	-26.54	-6.82	-132.77	-34.12
5	247.00	65.00	+68.21	16.99	9.54	+85.20	-75.66	1.0662	-70.94	-18.23	-61.83	-15.89
6	130.00	29.00	+67.36	7.58	11.96	+79.94	-62.98	1.0181	-61.83	-15.89		

TABLE A-IV: MOMENT EQUILIBRIUM

(GENERAL BISHOP)

Slice	W_i	$X_{i-1}-X_i$	(a) $W_i+X_{i-1}-X_i$	U_i	(b) $U_i\cos\alpha_i$	(c) (a)-(b)	(d) (c)$\tan\phi_{ei}$	C_i	(e) $C_i\cos\alpha_i$	(f) (d)+(e)	(g) (f)/M_i	T_i (g)/F
1	73.00	15.57	88.57	- -	- -	88.57	35.428	13.20	12.730	48.158	53.769	34.97
2	182.00	17.96	199.96	19.90	19.85	180.11	72.044	12.75	12.715	84.759	86.489	56.36
3	252.00	7.41	259.41	54.00	53.40	206.01	82.404	13.00	12.856	95.260	93.277	60.79
4	275.00	- 6.82	268.18	70.00	66.39	201.79	80.716	13.30	12.613	93.329	90.567	59.02
5	247.00	-18.23	228.77	65.00	56.16	172.61	69.044	14.60	12.614	81.658	82.050	53.46
6	130.00	-15.89	114.11	29.00	20.34	39.77	37.508	18.30	12.838	50.346	56.891	37.00
											Σ = 463.043	

$$F_M = \frac{463.043}{301.600} = 1.53$$

TABLE A-V: POINTS OF APPLICATION OF FORCES ON THE SIDES

Slice	E_i	X_i	T_i	x_i	$W_i \sin\alpha_i$	$W_i \sin\alpha_i - T_i$	$\Sigma(W_i \sin\alpha_i - T_i)$	$R\Sigma$	$x_i X_i$	$R\Sigma - x_i X_i$	$\frac{R\Sigma - x_i X_i}{E_i}$
1-2	-60.60	-15.57	34.97	-36.33	-19.20	-54.17	-54.17	-11,890.32	565.71	-12,456.03	205.5
2-3	-130.47	-33.53	56.36	+6	-12.00	-68.36	-122.53	-26,895.34	-201.18	-26,694.16	204.6
3-4	-159.31	-40.94	60.79	+48.33	+31.20	-29.59	-152.12	-33.390.34	-1978.63	-31,411.71	197.2
4-5	-132.77	-34.12	59.02	+90.66	+86.40	+27.38	-124.74	-27,380.43	-3093.32	-24,287.11	182.9
5-6	- 61.83	-15.89	53.46	+132.99	+124.00	+70.54	-54.20	-11,896.90	-2113.21	-9,783.78	158.2
			37.00		+ 91.20	+54.20	- -	- -	- -	- -	

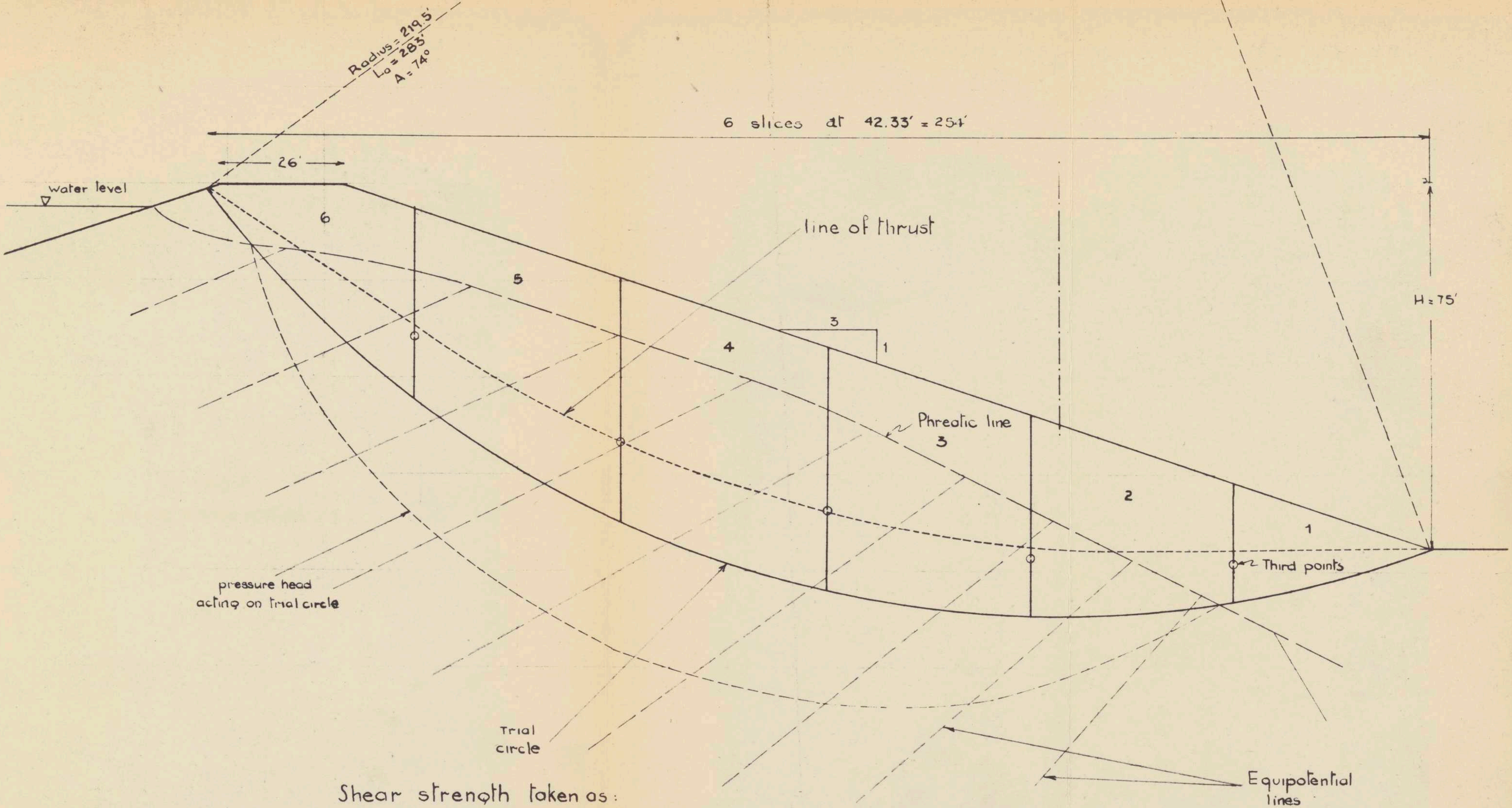
TABLE A-VI: ORDINARY BISHOP

$(X_{i-1} - X_i = 0)$

Slice	W_i	$U_i \cos\alpha_i$	(a) $W_i - U_i \cos\alpha_i$	(b) (a)$\tan\phi_e$	(c) $C_i \cos\alpha_i$	(d) (b)+(c)	F = 1.53 M_i	(d)/M_i
1	73.00	- -	73.00	29.200	12.730	41.930	0.8956	46.816
2	182.00	19.85	162.15	64.860	12.715	77.575	0.9800	79.157
3	252.00	43.40	198.60	74.440	12.856	92.296	1.0213	90.375
4	275.00	66.39	208.61	83.444	12.613	96.057	1.0305	93.214
5	247.00	56.16	190.84	76.336	12.614	88.950	0.9952	89.377
6	130.00	20.34	109.66	43.864	188.38	567.02	0.8850	64.074

$\Sigma = 463.013$

$$F = \frac{463.013}{301.600} = 1.53$$



Shear strength taken as:

$s = 300 + 0.4\,\bar{\sigma}$ lb psf

Unit weight constant, equal to 125 pcf

FIGURE 3 : CASE A (after Sutherland)

SCALE : 1" = 20'

APPENDIX II

NON-HOMOGENEOUS SECTION

CASE B: CHARACTERISTICS

	(MT/M^3)				C_e
Material	$\gamma_{sat.}$	$\gamma_{subm.}$	γ_{dry}	φ_e	(MT/M^2)
Sound rock	2.19	1.19	1.86	45°	0
Gabbro	2.19	1.19	1.86	40°	0
Filter material	2.19	1.19	1.86	35°	0
Impervious core	2.09	1.09	1.74	20°	2.7

TABLE B-I: CHARACTERISTICS OF THE FREE BODY

Slice	Gabbro rock and Filter		Impervious Core		Total Weight (MT)	α_i (Degr.)	Slope Angle	$\hat{L}_i$ (M)	x_i
	Drained	Subm.	Saturated	Subm.					
1	88.50	204.00	6.40	44.60	343.50	54°	37.6°	34.6	45.25
2	--	223.00	--	68.00	291.00	40.5°	37.6°	13.0	35.25
3	--	223.00	--	60.00	283.00	29°	37.6°	11.9	25.25
4	--	246.00	--	24.50	270.59	19°	22.6°	11.8	15.25
5	--	143.00	--	--	143.00	11°	22.6°	5.9	9.75
6	--	231.00	--	--	231.00	4°	22.6°	9.7	-0.25
7	--	179.00	--	--	179.00	-5°	22.6°	10.3	-10.25
8	--	130.00	--	--	130.00	-15°	22.6°	17.3	

TABLE B- II: METHOD OF FELLENIUS

Slice	W_i	$\sin\alpha_i$	$\cos\alpha_i$	$W_i\sin\alpha_i$	$N_i = W_i\cos\alpha_i$	$\tan\phi_{ei}$	$N_i\tan\phi_{ei}$	c_{ei}	$\widehat{L}_i$	$C_{ei}=c_{ei}\widehat{L}_i$
1	343.5	0.8092	0.5878	277.90	201.90	0.3640	73.49	2.70	34.6	93.42
2	291.0	0.6494	0.7604	188.99	221.28	0.3640	80.54	2.70	13.0	35.10
3	283.0	0.4848	0.8746	137.20	247.52	0.3640	90.09	2.70	11.9	32.13
4	270.5	0.3256	0.9455	88.07	255.76	0.3640	93.07	2.70	11.8	31.86
5	143.0	0.1908	0.9816	27.29	140.37	0.7002	98.29	-	5.9	-
6	231.0	0.0698	0.9976	16.11	230.44	0.8391	193.36	-	9.7	-
7	179.0	-0.0872	0.9962	-15.60	178.32	0.8391	149.63	-	10.3	-
8	130.0	-0.2588	0.9659	-33.65	125.57	0.8391	105.37	-	17.3	-

$\Sigma W_i\sin\alpha_i = 686.31$ $\quad\quad$ $\Sigma N_i\tan\phi_{ei} = 883.84$ $\quad\quad$ $\Sigma C_{ei} = 192.51$

$$F = \frac{883.84 + 192.51}{686.31} = 1.57$$

TABLE B- III: METHOD d-i

AUXILIARY COEFFICIENTS

(F = 1.91)

	$\sin\alpha_i$	$\cos\alpha_i$	$\tan\phi_d$	$\sin\alpha_i \tan\phi_d$	$\cos\alpha_i \tan\phi_d$	M_i	M_i'	β_i	$\tan\beta_i$	$\tan\beta_i M_i'$	$\tan\beta_{i-1} M_i'$	P_i	P_i'
1	0.8090	0.5878	0.1906	0.1542	0.1120	0.7420	0.6970	42.3°	0.9099	0.6342	-	1.3762	0.7420
2	0.6494	0.7604	0.1906	0.1238	0.1449	0.8842	0.5045	36.3°	0.7346	0.3706	0.4591	1.2548	1.3433
3	0.4848	0.8746	0.1906	0.0924	0.1667	0.9670	0.3181	30.8°	0.5961	0.1897	0.2337	1.1866	1.2307
4	0.3256	0.9455	0.1906	0.0620	0.1802	1.0075	0.1454	18.3°	0.3307	0.0481	0.0867	1.0556	1.0942
5	0.1908	0.9816	0.3666	0.0699	0.3599	1.0515	-0.1691	15.6°	0.2792	-0.0472	-0.0559	1.0043	0.9956
6	0.0698	0.9976	0.4393	0.0306	0.4382	1.0282	-0.3685	10.8°	0.1908	-0.0703	-0.1029	0.9579	0.9253
7	-0.0872	0.9962	0.4393	-0.0383	0.4376	0.9579	-0.5248	6.3°	0.1104	-0.0579	-0.1001	0.9000	0.8578
8	-0.2588	0.9659	0.4393	-0.1137	0.4244	0.8522	-0.6832	-	-	-	-0.0754	0.8522	0.7768

TABLE B- IV: METHOD d-i

FORCES ON THE SIDES (F = 1.91)

	W_i	C_{di}	$W_iM'_i$	$W_iM'_i-C_i$	P_i	P'_i	$E_{i-1}P'_i$	$W_iM'_i-C_i+E_{i-1}P'_i$	E_i	$X_i=E_i\tan\beta_i$	$X_{i-1}-X_i$
1	343.5	48.91	239.42	190.51	1.3762	0.7420	-	190.51	138.57	126.09	-126.09
2	291.0	18.38	146.82	128.44	1.2548	1.3433	186.14	314.59	250.71	184.16	- 58.07
3	283.0	16.82	90.15	73.33	1.1866	1.2307	308.79	382.12	322.02	191.20	- 7.04
4	270.5	16.68	39.33	22.65	1.0556	1.0942	352.36	375.01	355.24	117.49	73.71
5	143.0	-	-24.17	-24.17	1.0043	0.9956	353.69	329.51	328.09	91.60	15.89
6	231.0	-	-85.12	-85.12	0.9579	0.9253	303.59	218.47	228.07	43.51	48.09
7	179.0	-	-93.94	-93.94	0.9000	0.8578	195.63	101.7	113.00	12.48	31.03
8	130.0	-	-88.81	-88.81	0.8522	0.7768	87.78	- 0.1	-	-	12.48

TABLE B- V: METHOD d-i

- CHECK OF MOMENT EQUILIBRIUM -

	W_i	$X_{i-1}-X_i$	(a) $W_i+X_{i-1}-X_i$	(b) $\tan\phi_{ei}$	(c) (a)x(b)	C_{ei}	$\cos\alpha_i$	$C_{ei}\cos\alpha_i$	(c)+(d)	M_i	$\frac{(c)+(d)}{M_i}$
1	343.5	-126.09	217.41	0.3640	79.14	93.42	0.5878	54.91	134.05	0.7420	180.66
2	291.0	- 58.07	232.93	0.3640	84.79	35.10	0.7604	26.69	111.48	0.8842	126.08
3	283.0	- 7.04	275.96	0.3640	100.50	32.13	0.8746	28.10	128.60	0.9670	132.99
4	270.5	73.71	344.21	0.3640	125.29	31.86	0.9455	30.12	155.41	1.0075	154.25
5	143.0	15.89	168.89	0.7002	118.26	-		-	118.26	1.0515	112.47
6	231.0	48.09	279.09	0.8391	234.18	-		-	234.18	1.0282	227.76
7	179.0	31.03	210.03	0.8391	176.24	-		-	176.24	0.9579	183.99
8	130.0	12.48	142.48	0.8391	119.55	-		-	119.55	0.8522	140.28
											Σ = 1,258.48

$$F_M = \frac{\Sigma\left[C_{ei}\cos\alpha_i + (W_i+X_{i-1}-X_i)\tan\phi_{ei}\right]\frac{1}{M_i}}{\Sigma W_i \sin\alpha_i} = \frac{1,258.48}{686.30} = 1.83$$

TABLE B- VI: METHOD d-i_i

FORCE EQUILIBRIUM (β = 20° F = 1.88)

	$\sin\alpha_i$	$\cos\alpha_i$	$\tan\phi_{di}$	$\sin\alpha_i\tan\phi_{di}$	$\cos\alpha_i\tan\phi_{di}$	M_i	M'_i	$M'_i\tan\beta_i$	P_i	Cd_i	$W_iM'_i$	$Cd_i-W_iM'_i$	$E_{i-1}E_i$
1	0.8090	0.5878	0.1936	0.1566	0.1138	0.7444	0.6952	0.2531	0.9975	49.69	238.80	-189.11	-189.69
2	0.6494	0.7604	0.1936	0.1257	0.1472	0.8861	0.5022	0.1828	1.0689	18.67	146.14	-127.47	-119.36
3	0.4848	0.8746	0.1936	0.0939	0.1693	0.9686	0.3155	0.1148	1.0833	17.09	89.29	- 72.20	- 66.76
4	0.3256	0.9455	0.1936	0.0630	0.1830	1.0085	0.1426	0.0519	1.0604	16.95	38.57	- 21.62	- 20.50
5	0.1908	0.9816	0.3724	0.0711	0.3655	1.0527	-0.1747	-0.0635	0.9892	-	-24.98	24.98	25.14
6	0.0698	0.9976	0.4463	0.0312	0.4452	1.0288	-0.3754	-0.1366	0.8922	-	-86.72	86.72	97.09
7	-0.0872	0.9962	0.4463	-0.0389	0.4446	0.9573	-0.5318	-0.1936	0.7637	-	-95.19	95.19	124.53
8	-0.2588	0.9659	0.4463	-0.1155	0.4311	0.8504	-0.6899	-0.2511	0.5993	-	-89.69	89.69	149.55

$\Sigma(E_{i-1}-E_i) = 0$

TABLE B- VII: METHOD d-ii

- MOMENT EQUILIBRIUM -
($\beta = 20^{\circ}$ F = 1.88)

	$X_{i-1}-X_i$	(a) $W_i+X_{i-1}-X_i$	(b) (a)x$\tan\phi_{ei}$	(c) $C_{ei}\cos\alpha_i$	(b)+(c)	(d) $\frac{(b)+(c)}{M_i}$	(e) $T_i = \frac{(d)}{F}$	(f) $W_i \sin\alpha_i$	(e)-(f)	$\Sigma[(e)-(f)]$	R x Σ
1	-69.05	-274.45	99.90	54.91	154.81	207.97	110.62	277.90	167.13	167.13	10,362.06
2	-43.45	247.55	90.11	26,69	116.80	131.81	70.11	188.99	118.72	285.85	17,722.70
3	-24.30	258.70	94.17	28.10	122.27	126.25	67.15	137.20	69.90	355.75	22,056.50
4	- 7.46	263.04	95.75	30.12	125.87	124.81	66.39	88.07	21.52	377.27	23,386.40
5	9.15	152.15	106.54	-	106.54	101.21	53.84	27.28	-26.71	350.56	21,734.72
6	35.34	266.34	223.49	-	223.49	217.23	115.55	16.11	-99.60	250.96	15,559.52
7	45.33	224.33	188.24	-	188.24	196.64	104.60	-15.60	-120.35	130.61	8,097.82
8	54.44	184.44	154.76	-	154.76	181.98	96.80	-33.65	-130.61	0.00	-

$$\Sigma \frac{(b) + (c)}{M_i} = 1,287.90$$

$$F_M = \frac{1,287.90}{286.30} = 1.88$$

TABLE B- VIII: METHOD d-ii

POINT OF APPLICATION OF FORCES ON THE SIDES

slice	(a) R x Σ	$X_{i-1}-X$	X_i	x_i	(b) x_iX_i	(c) (a)-(b)	$E_{i-1}-E_i$	E_i	$y_i=\frac{(c)}{E_i}$
1	10,362.06	-69.05	69.05	45.25	3,124.51	7,237.55	-189.69	189.69	38.15
2	17,722.70	-43.45	112.50	35.25	3,965.63	13,757.07	-119.36	309.05	44.51
3	22,056.50	-24.30	136.80	25.25	3,454.20	18,602.30	- 66.76	375.81	49.50
4	23,386.40	- 7.46	144.26	15.25	2,199.97	21,186.43	- 20.50	396.31	53.46
5	21,734.72	9.15	135.11	9.75	1,317.32	20,417.40	+ 25.14	371.17	55.01
6	15,559.52	35.34	99.77	- 0.25	- 24.94	15,584.46	- 97.09	274.08	56.86
7	8,097.82	45.33	54.44	-10.25	- 558.01	8,653.83	+124.53	149.55	57.88
8	-	54.44	0.00	-	-	-	+149.55	-	

TABLE B- IX:

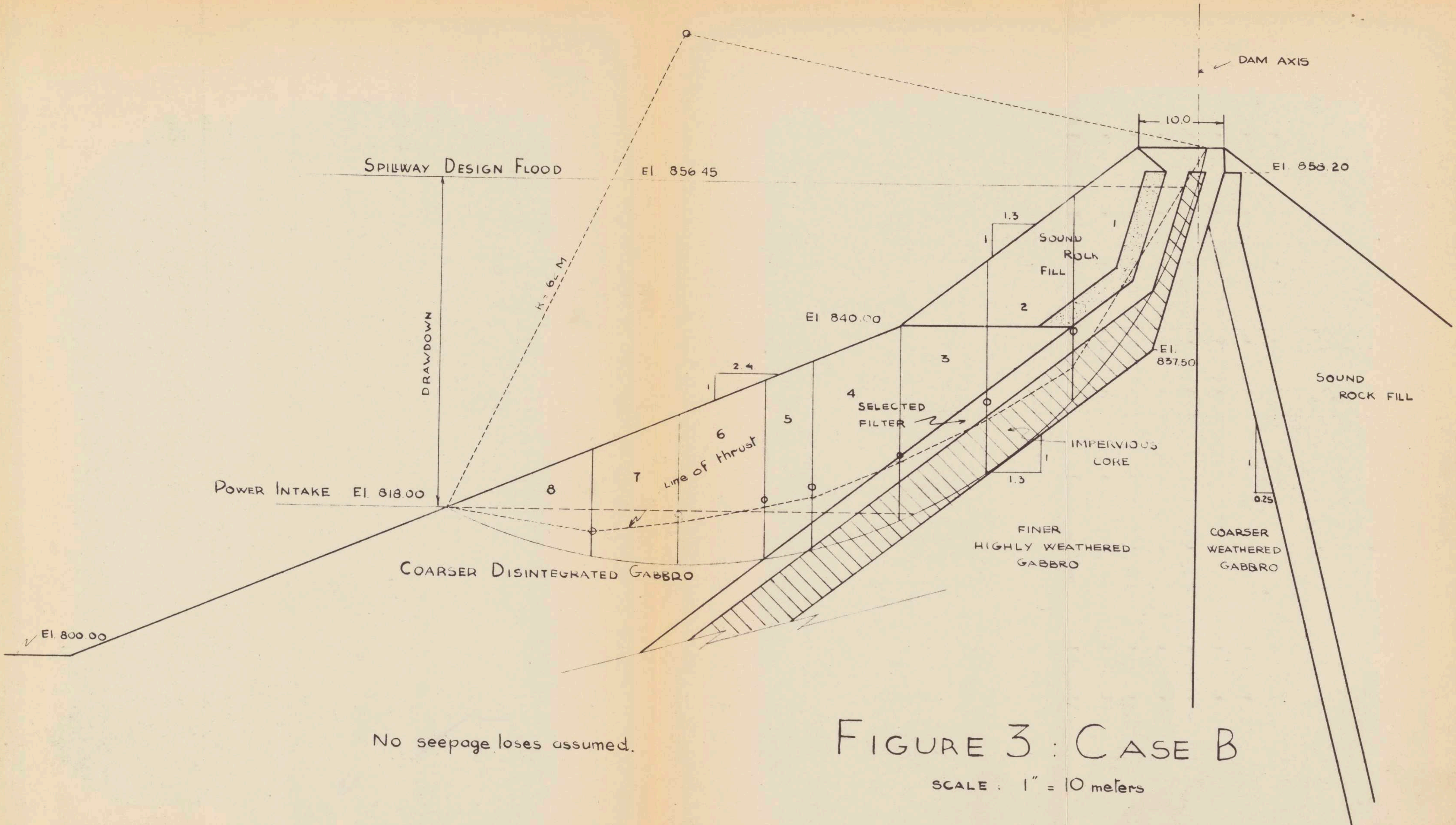
Method Simplified of Bishop

	W_i	$\tan\phi_{ei}$	(a) $W_i \tan\phi_{ei}$	(b) $C_{ei} \cos\alpha_i$	(a)+(b)	F = 1.80: M_i	F = 1.80: $\frac{(a)+(b)}{M_i}$	F = 1.75: M_i	F = 1.75: $\frac{(a)+(b)}{M_i}$
1	343.5	0.3640	125.02	54.91	179.93	0.7514	239.47	0.7560	237.99
2	291.0	0.3640	105.92	26.69	132.61	0.8917	148.71	0.8955	148.08
3	283.0	0.3640	103.00	28.10	131.10	0.9727	134.79	0.9747	134.51
4	270.5	0.3640	98.45	30.12	128.57	1.0114	127.14	1.0132	126.90
5	143.0	0.7002	100.13	-	100.13	1.0558	94.83	1.0580	94.64
6	231.0	0.8391	193.83	-	193.83	1.0301	188.17	1.0310	188.00
7	179.0	0.8391	150.20	-	150.20	0.9556	157.18	0.9544	157.34
8	130.0	0.8391	109.08	-	109.08	0.8453	129.05	0.8418	129.58
							Σ=1219.34		Σ=1217.04

$F_1 = 1.80$ $\qquad F = \frac{1219.34}{686.30} = 1.777$

$F_2 = 1.75$ $\qquad F = \frac{1217.04}{386.30} = 1.773$

$F = 1.77$



No seepage loses assumed.

FIGURE 3 : CASE B

SCALE : 1" = 10 meters