THE EFFECT OF TWO SETS OF JOINTS ON ROCK SLOPE RELIABILITY

by

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ABSTRACT

A probabilistic model is developed that simulates both the variation of persistence of each of two distinct joint sets within a rock mass and their combined effect on rock slope reliability.

Each of the sets of joints is pseudo-randomly simulated to form a joint pattern, and a stability analysis is performed on each simulated realization. Thus in a typical realization, a number of joint planes will intersect the slope face to form exit points of potential failure paths which are generally non planar and pass through joints and intact rock. Consequently, from a number of realizations, reliability may be determined for part or the entire slope. Reliability is measured in the probability of a joint plane exiting on the slope face given that this exit point belongs to a failure path.

An extensive sensitivity study is made with respect to the main parameters that influence slope stability. Results obtained aided in establishing a set of recommendations for design and stability analyses.

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Special thanks and particular credit to Kevin O'Reilly who has be responsible for previously establishing the general guidelines and computer methods to be used for carrying out this work; and to Joan McCusker who was endlessly patient and accurate in typing and retyping this manuscript.

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CHAPTER 1

INTRODUCTION

1.1 The Problem

Rock slope stability crtically depends on uncertain geometric and strength parameters. The reason for uncertainty is that is is practically impssible to measure the values of all governing parameters in a typical rock mass. As a result, present deterministic design methods incorporate high safety margins. A method of analysis which takes the stochastic character of rock masses into account is needed. More precisely, what is needed at the present time is:

(1) An understanding of the reliability distribution of the various parameters influencing the stability of a rock mass;

(2) Development of analytical methods that consider the stochastic character of parameters in making stability predictions.

The purpose of this thesis is to study the effect of two joint sets on the reliability of a rock slope. Reliability of rock slopes was first studied by Glynn (1978) and later by O'Reilly (1980). Both took into consideration the two criteria mentioned above. A model is developed and used to study the dependence of rock slope safety on various parameters.

This introductory chapter gives a brief description of the parameters on which rock slope reliability depends. A brief discussion on the merit of the deterministic approach and the possible merit of a deterministic-stochastic approach to be used in rock engineering will follow.

1.2 Slope Stability Parameters

The major parameters affecting slope stability are:

(1) In-Situ state of stress

(2) Intact rock shearing resistance

(3) First joint set orientation

(4) Second joint set orientation

(5) Shearing resistance of the joints

(6) Length of joints

The relative importance and variability of each parameter is discussed in detail next.

State of Stress

The in-situ state of stress can only be determined at a few points within a rock mass. Shearing resistance of intact rock and discontinuities depends both on rock strength and on the in-situ state of stress. Accurate methods are complex and generally impractical for design use (i.e., finite element), and still in the development stages. Consequently, design approaches must use simplifying assumptions.

Intact Rock Shearing Resistance

Intact rock shearing resistance is uncertain because of:

(1) Natural spatial variability of intact rock strength parameters.

(2) Measurement errors.

Joint Orientation

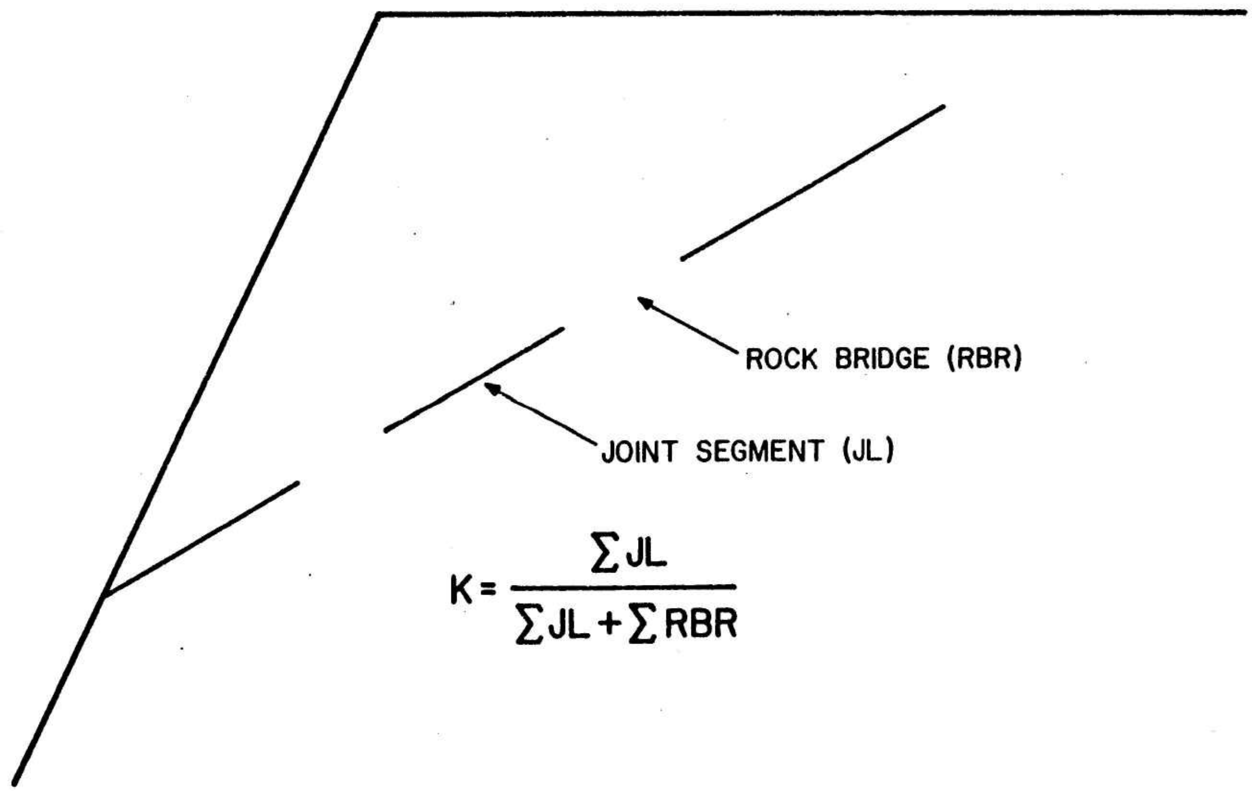
The effect of joint orientation on stability is an established fact included in current rock slope stability analysis.

Joint Shearing Resistance

Failure in a jointed rock slope occurs when intact "wedges" or "blocks", bounded by joints, move in the direction of one of the joint planes. Since sliding is commonly assumed to take place along joints, a reliable prediction of joints shear strength is critical. Some degree of spatial variablity in joint resistance will always be found. Variation in measured values of joint friction angle Φ and joint cohesion C_j will be either due to random measurement errors or in-situ variability.

Joint Persistence

Joint persistence is a measure of joint continuity. Quantitatively, it can be considered to be the percentage of a "joint plane" which is actaully discontinuous. For a block bounded by non-completely persistent joints to fail, intact "rock bridges" must fail. (Refer to Figure 1.1.)



$$K = \frac{\sum JL}{\sum JL + \sum RBR}$$

FIGURE 1.1 2 - DIMENSIONAL JOINT PERSISTENCE (K)

Slope reliability is, in general, highly sensitive to variation in persistence. Due to this high sensitivity, no current design method satisfactorily treats joint resistance. Commonly the "persistence Problem" is conservatively ignored by assuming joints with 100% persistence (O'Reilly, 1980).

Predictions of slope performance are made even more complex by the fact that each of these parameters is, to at least some extent, variable within the slope.

The effects of the cleft water pressure are ignored since they are not well known at the present time.

1.3 Design Approaches

There are currently two basic approaches to evaluate stability of a jointed rock slope. The first approach uses limit equilbrium analysis with single values of the parameters. It yields a factor of safety against failure for a single 'potential failure body'. Uncertainty associated with each of the parameters is controlled by an appropriate selection of a factor of safety.

The second approach yields a probability of failure for 'potential failure bodies' rather than a safety factor. In such an approach, varability in parameters affecting stability as well as the expected number of 'potential failure bodies' are considered. The final goal of a probabilistic design is to evaluate the probability of failure of an entire rock slope.

Recently, probabilistic design methods have been developed that are capable of calculating the probability of failure of blocks or wedges within a rock slope rather than a factor of safety. It is possible to estimate slope reliability as the probability that failure will occur, rather in the slope. Through simulation, probabilities of failure of the individual wedges are obtainable. Geometric joint parameters affecting wedge reliability are assigned distributions and are randomly generated in each realization of the simulation. A factor of safety is then calculated for each realization. The probability of wedge failure is simply the percentage of realizations with factors of safety less than one.

1.4 Objective

The objective of this work is to develop a model of slope reliability that accounts for the effect of two distinct joint sets. This is an additional step towards the development of a complete reliability model which takes into consideration the stochastic character of geometric parameters, resistances, persistence and water pressure as well as computation of an overall probability of slope failure.

Chapter 2 will briefly review current methods by which jointed rock slopes are analyzed and a deterministic resistance model for the resistance of any failure path, involving either one of the two joint sets or both joint sets within a rock slope. Chapter 3 presents the probability model for failures in slopes containing two sets of parallel joints. The model is based on the calculation of resistance in Chapter 2 and on a procedure to simulate joint geometry. In Chapter 4, a demonstration run will be presented. In Chapter 5, the program will be used to conduct an extensive parametric sensitivity analysis that will assess the influence of the various parameters on the slope reliability. Chapter 6 will conclude with design recommendations based on the results of the preceeding chapter and recommendations for future research.

CHAPTER 2

DETERMINISTIC ANALYSIS AND FAILURE PATHS IN SLOPES WITH TWO PARALLEL SETS OF JOINTS

2.1 Introduction

In this chapter a realistic mechanical model will be developed to determine the stability of any potential failure surface within slopes of the type shown in Figures 2.1 and 2.2, with two parallel joint sets. Failure surfaces to be analyzed are either "in-plane" or "en echelon" and typically include both jointed and intact rock sections.

This chapter will also present the combined effect of joint intact rock on rock mass stability as well as limit equilibrium models which are used to derive resistance along non-continuous joint planes.

2.2 A Deterministic Mechanical Model (Lajtai's Direct Shear Model)

Because joint shearing resistance is generally several orders of magnitude smaller than intact resistance, discontinuities are generally found to totally govern the performance of a risk slope.

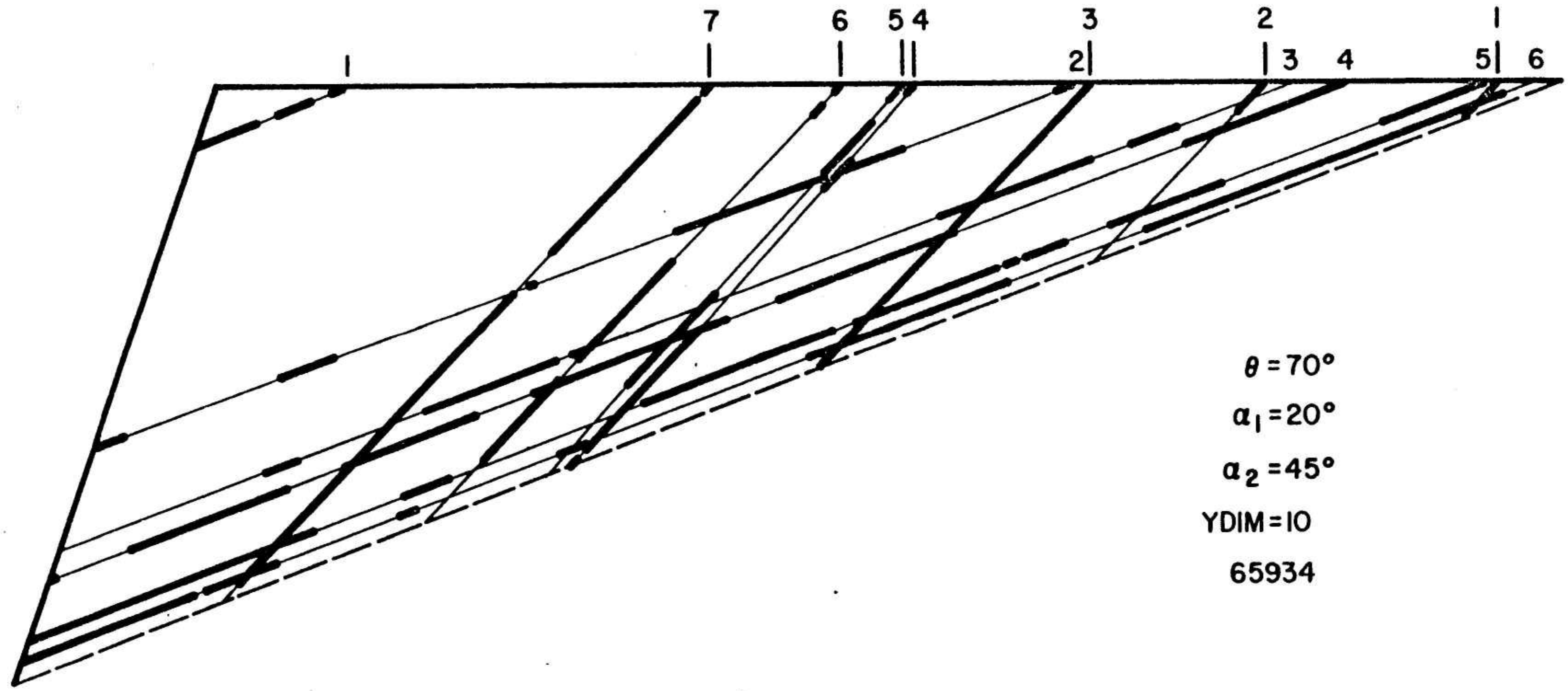


FIGURE 2.1 CASE I

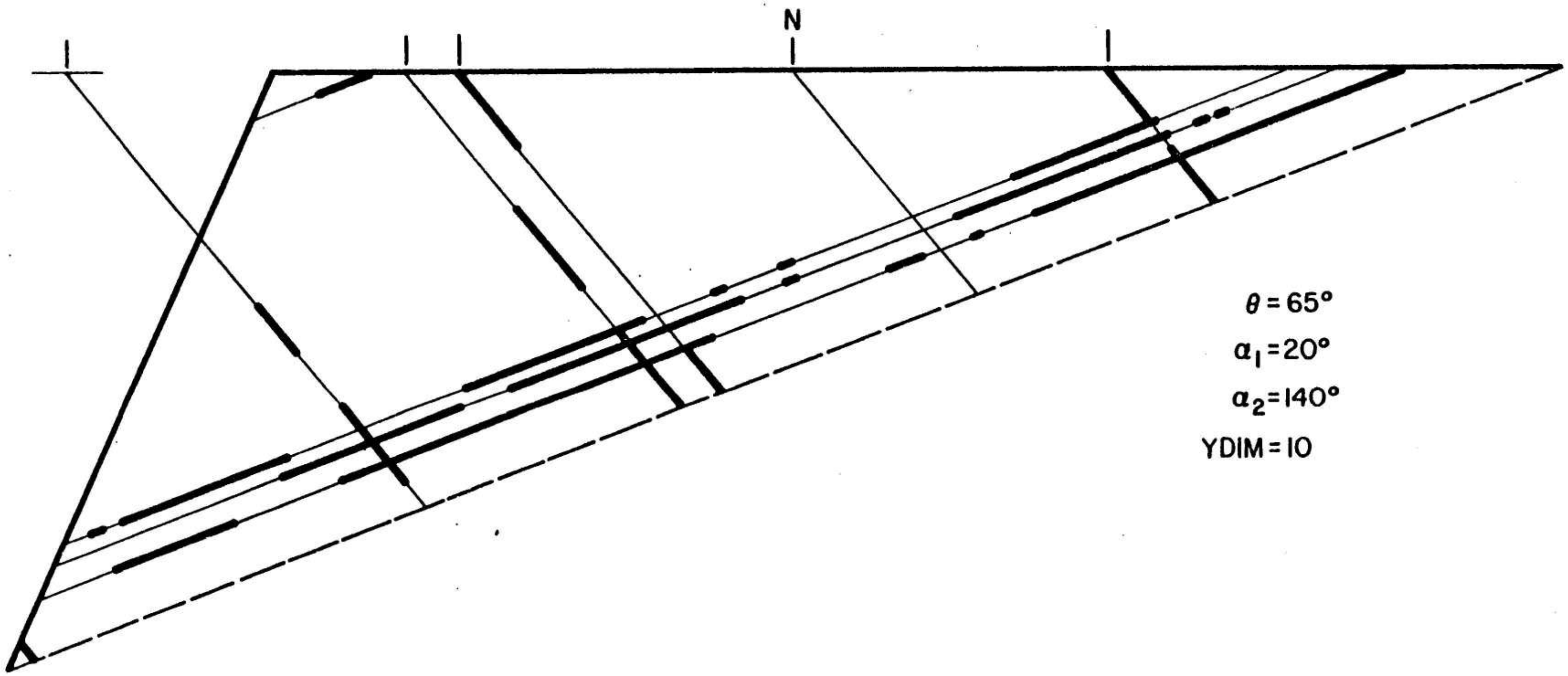
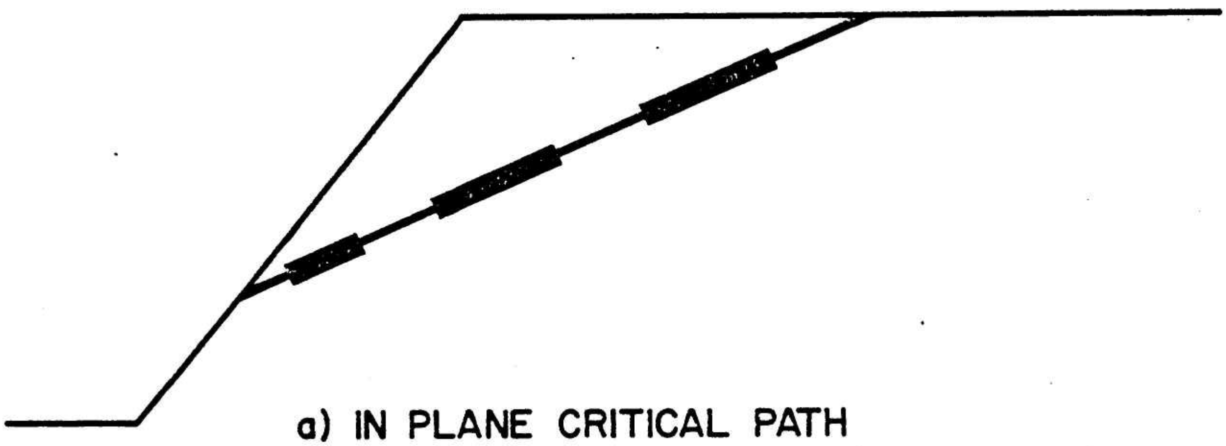
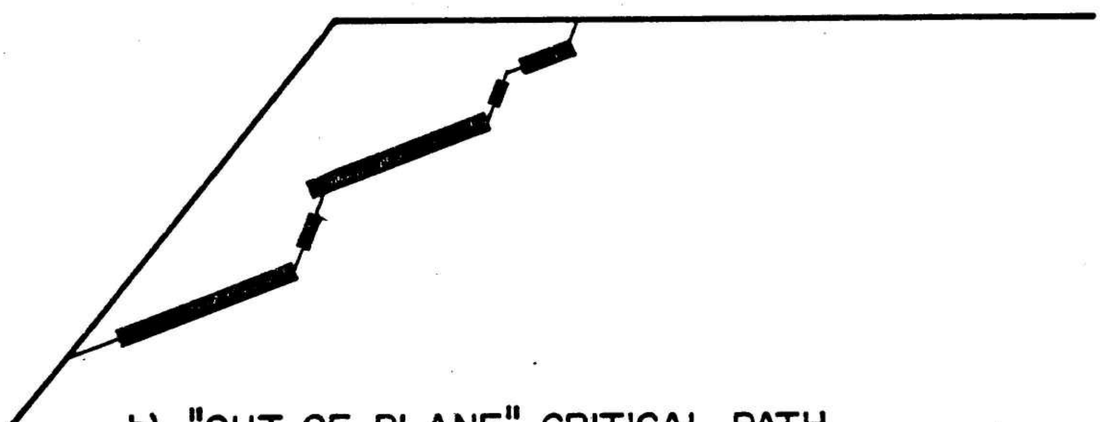


FIGURE 2.2 CASE 2



a) IN PLANE CRITICAL PATH



b) "OUT OF PLANE" CRITICAL PATH

FIGURE 2.3 CRITICAL PATHS

The model presented here was proposed by Lajtai and used by O'Reilly (1980) in analyzing a slope with a single set of parallel joints. We shall consider here the same slope with an additional parallel joint set. The physical behavior of the rock and jointed rock will basically be the same. Movement will always be assumed to take place in the direction of the shallower angled joints, and consequently there will be no frictional resistance between joint surfaces of the second joint set as relative movement will be away from these surfaces.

Lajtai suggests that failure resulting from stress applied on an intact rock bridge connecting two open joints is one of direct shear as shown in Figure 2.4. According to Lajtai, normal stresses in the direction of jointing can be assumed to be zero at failure. He bases this assumption on the fact that a truly "open" joint cannot transmit stresses to the surrounding intact rock.

Failure of intact rock bridges in a slope can be visualized as shown in Figures 2.5(a) and 2.5(b). In Figure 2.5(a) a rock bridge in an otherwise continuous joint plane is about to fail. Failure is assumed to involve the rigid body motion of rock overlying the joint planes down-dip in the direction of jointing of the first joint set. set. The direct shear assumption is again attractive because, the mechanism involved in failing a block of intact rock in a direct shear device, is analogous to the mechanism of failure of an intact rock bridge within a slope. Both involve forced rigid body motion

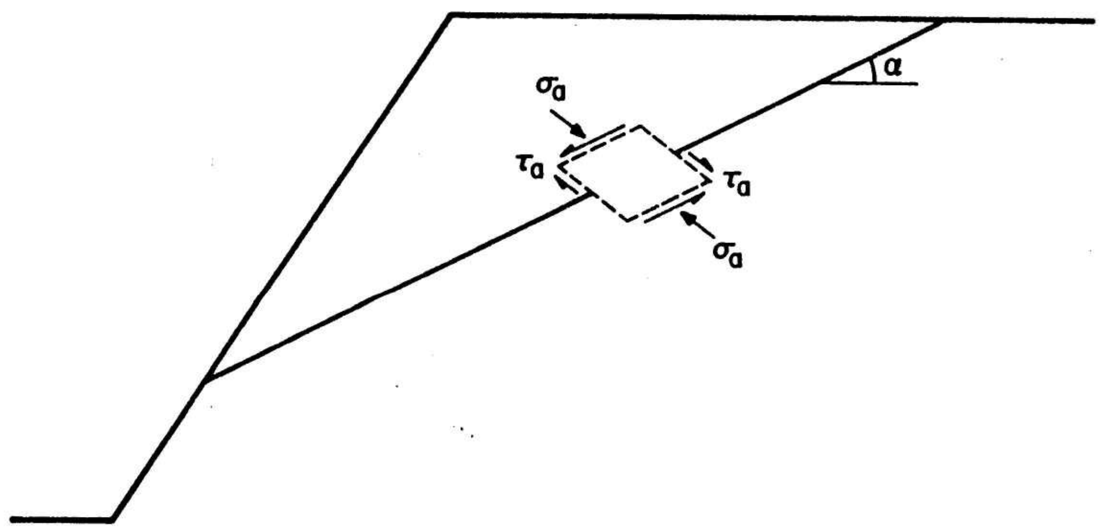
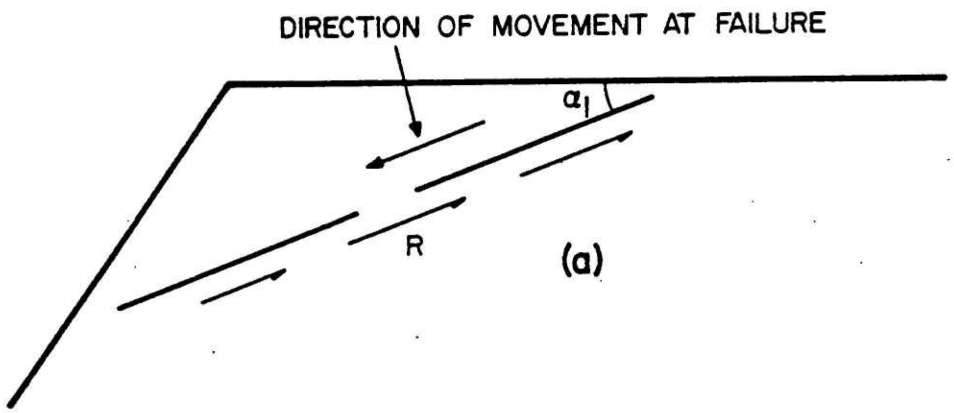


FIGURE 2.4 DIRECT SHEAR STATE OF STRESS IN ROCK BRIDGE AS PROPOSED BY LAJTAI (1969)



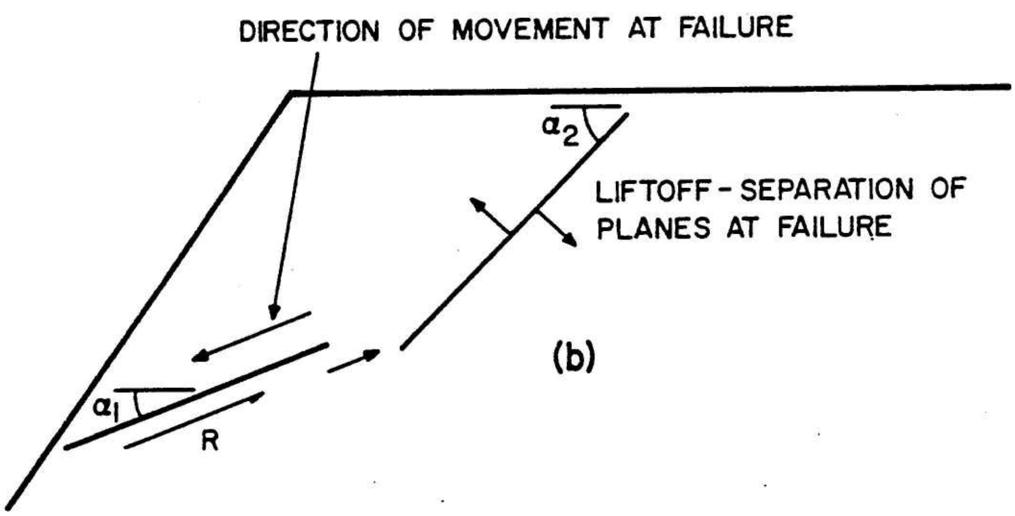
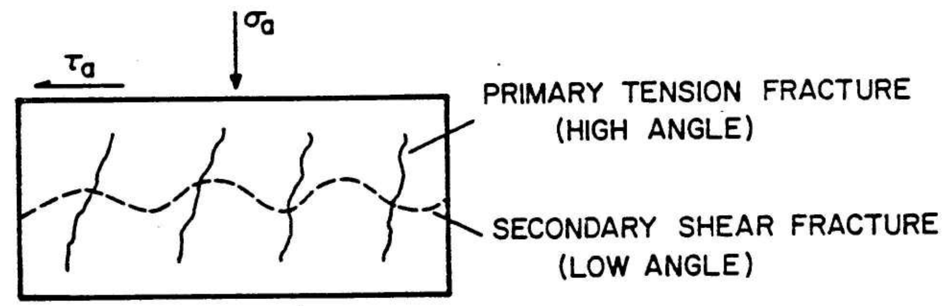


FIGURE 2.5 (a) & (b) FAILURE MECHANISM OF INTACT ROCK BRIDGE IN SLOPE

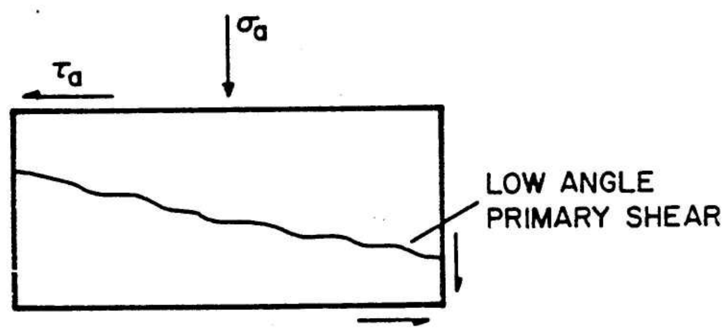
along a predefined plane under an approximately constant stress, normal to the joint palne. In a slope with two joint sets, the joints of the second set transmit no stresses of failure to the surrounding intact rock. In an actual slope, normal stresses are, more or less, fixed by the weight of rock overburden.

When a direct shear test is performed at low levels of stress applied normal to the joint plane, Lajtai states that the maximum shear resistance can be distinguished as one of two modes. At relatively low stress levels, the application of shear stress, in the direction of the first joint set, can lead to a minimum principal stress (σ_3) equal to the tensile strength of the intact rock. In this mode, failure occurs as tensile fractures develop at the high angles in the direction of the first joint set. (Refer to Figure 2.6(a).) These tensile fractures occur when τ_a, (peak shear stress in the enforced direction) is mobilized. This is followed by shearing in the direction of the first joint set at residual stress levels. (Figure 2.6(c)

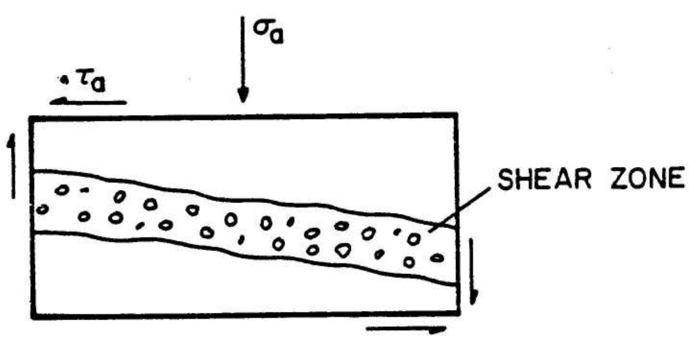
At higher normal stress levels, the minimal principal stress does not exceed tensile strength. In this case, failure occurs when stress (τ), in the direction of the first joint set, equals the shear resistance defined by the Coulomb failure criterion (approximately equal to twice the tensile strength). In this second mode, shear fractures develop at the moment of peak applied shear stress. (See Figure 2.6(b)). Shear fracture develops sub-parallel to the enforced direction. Again as in the tensile mode, peak behavior is followed by shearing which is parallel to the enforced direction at residual shear stress levels.



(a) FAILURE IN TENSION



(b) FAILURE IN PURE SHEAR



(c) FAILURE AT ULTIMATE STRENGTH

FIGURE 2.6 DIRECT SHEAR FAILURE MODES
LAJTAI (1969)

Graphically, referring to Figure 2.7, the center of the Mohr stress circle remains constant at $\sigma_{a/2}$ as the applied shear stress is raised from its initial value of zero to its value at failure. Applied shear stress can be increased only until the Mohr circle becomes tangent to the failure envelope. At relatively low values of σ_a, this point of tangency is at $\sigma = -T_s$, $\tau = 0$ and thus failure occurs as tension fractures initiate, (mode 1). As σ_a is increased, the centers of the Mohr circles move further out along the σ axis away from the τ - axis. Beyond a certain value of σ_a, increasing applied shear stress to failure results in the point of tangency lying on the linear portion of the envelope. (See Figure 2. 8).

Summarizing; if the point of tangency lies along the parabola of failure envelope, failure is by tensile fracturing (mode 1). If the point of tangency is on the linear portion, failure will be by shear fracturing (mode 2). After either mode, post peak shear resistance drops to stress dependent residual values due to secondary shearing in the enforced direction. Analytically, expressions of peak shear resistance for intact rock in direct shear will now be presented for each of the above described failure modes.

Lajtai described mode 1 by the following expression:

$$\tau_a = [T_s (T_s - \sigma_a)]^{\frac{1}{2}} \qquad \text{(Eq. 2.1)}$$

Plotting τ_a as a function of σ_a leads to what Lajtai terms as the direct shear parabola (tension). (See Figure 2.9a).

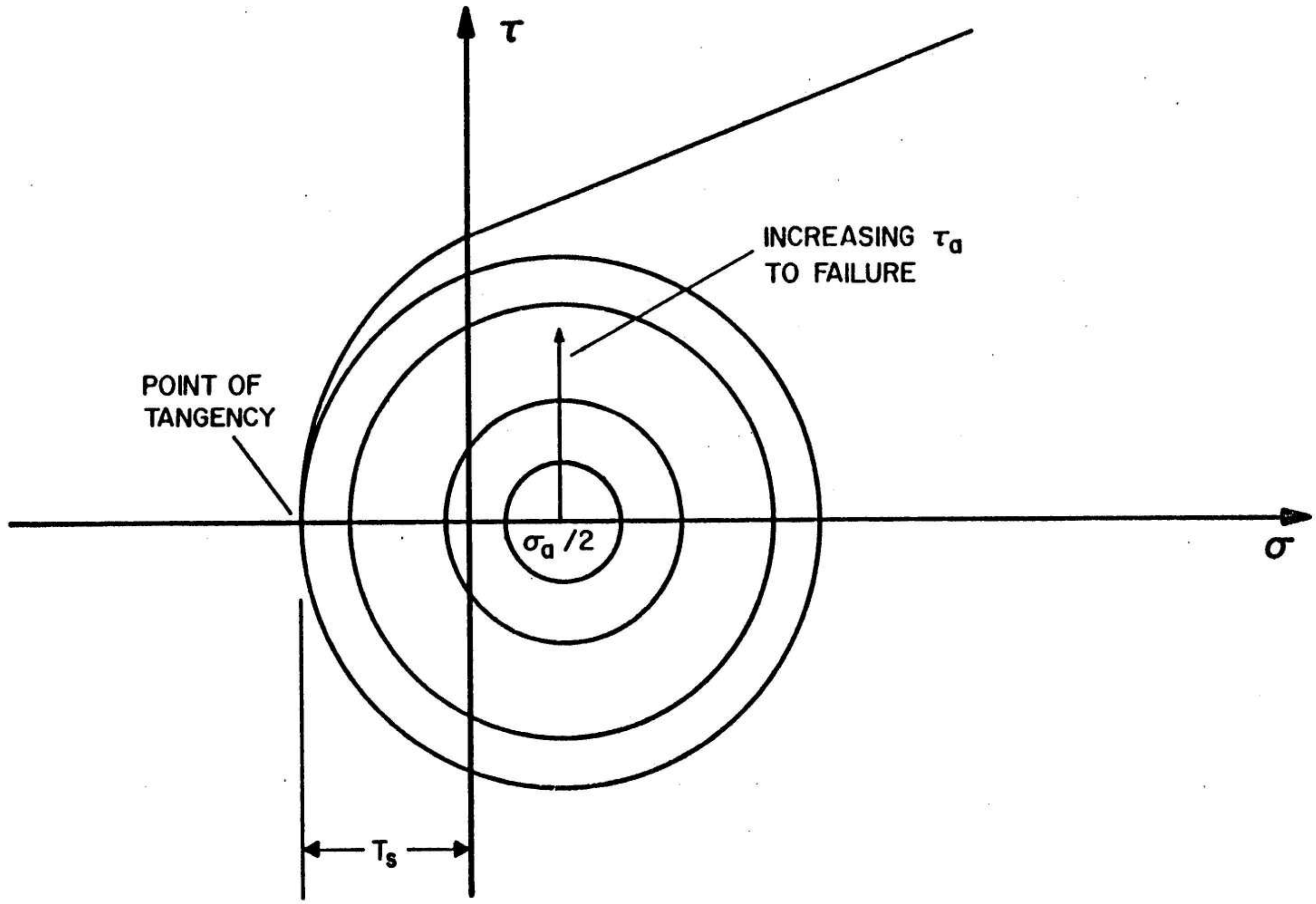


FIGURE 2.7 MOHR'S CIRCLE - FAILURE BY TENSILE FRACTURING
(MODE I)

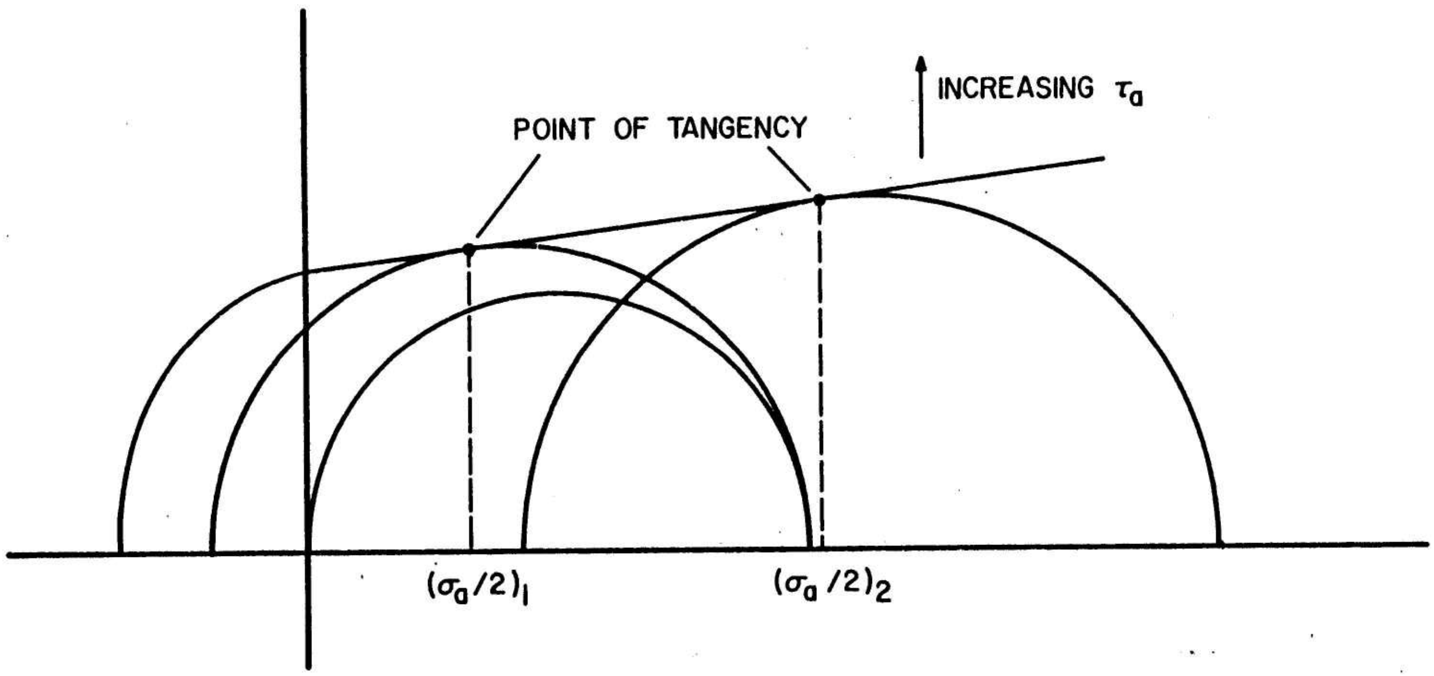


FIGURE 2.8 MOHR'S CIRCLE - FAILURE BY SHEAR FRACTURING (MODE 2)

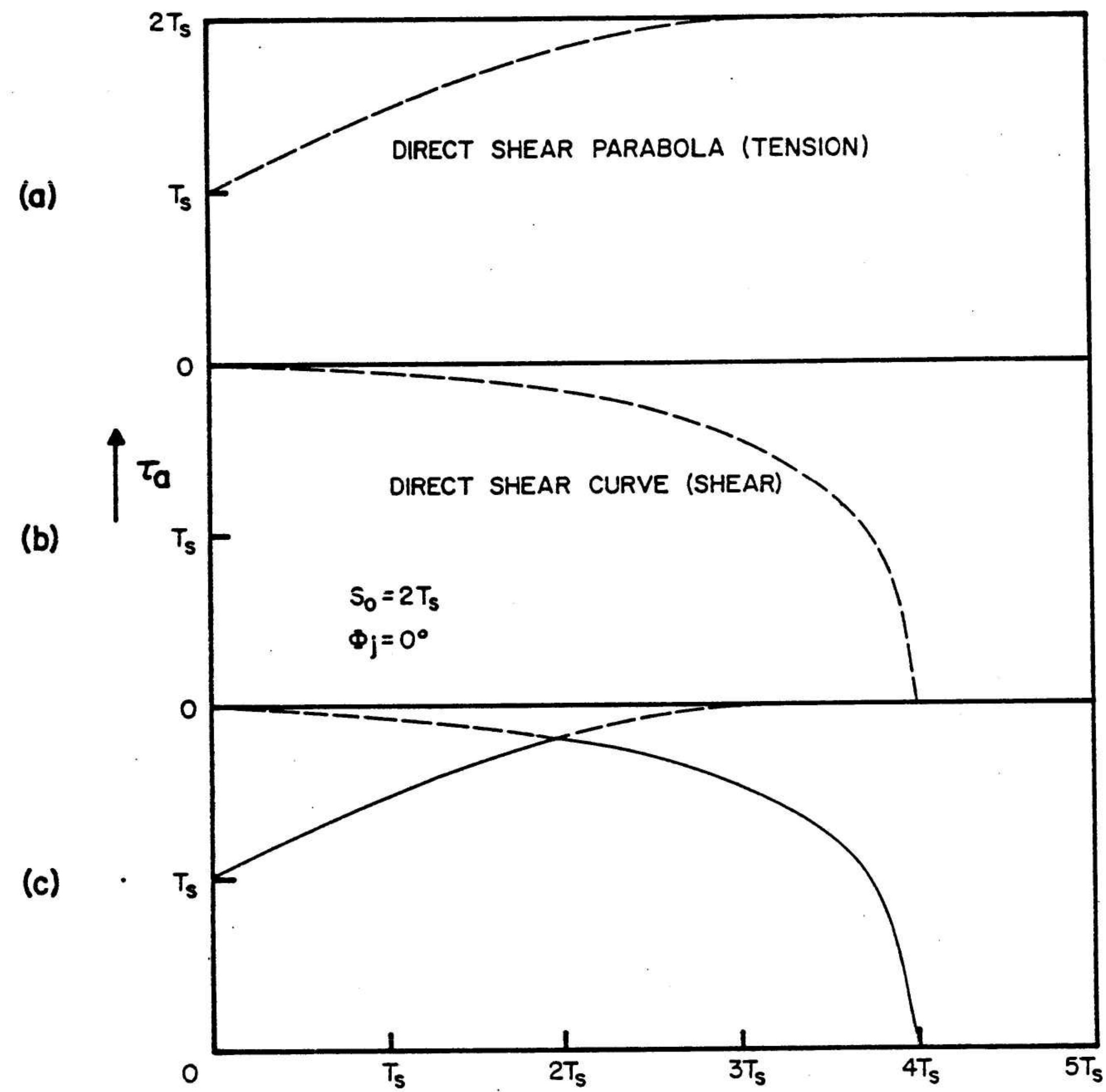


FIGURE 2.9 DEVELOPMENT OF COMPOSITE LIMIT CURVES AFTER LAJTAI (1969)

At high stress levels of σ_a close to σ_c, we get what is referred to as the third direct shear failure mode, i.e., failure at ultimate strength. This develops by formation of a zone of crushed material in the direction of jointing.

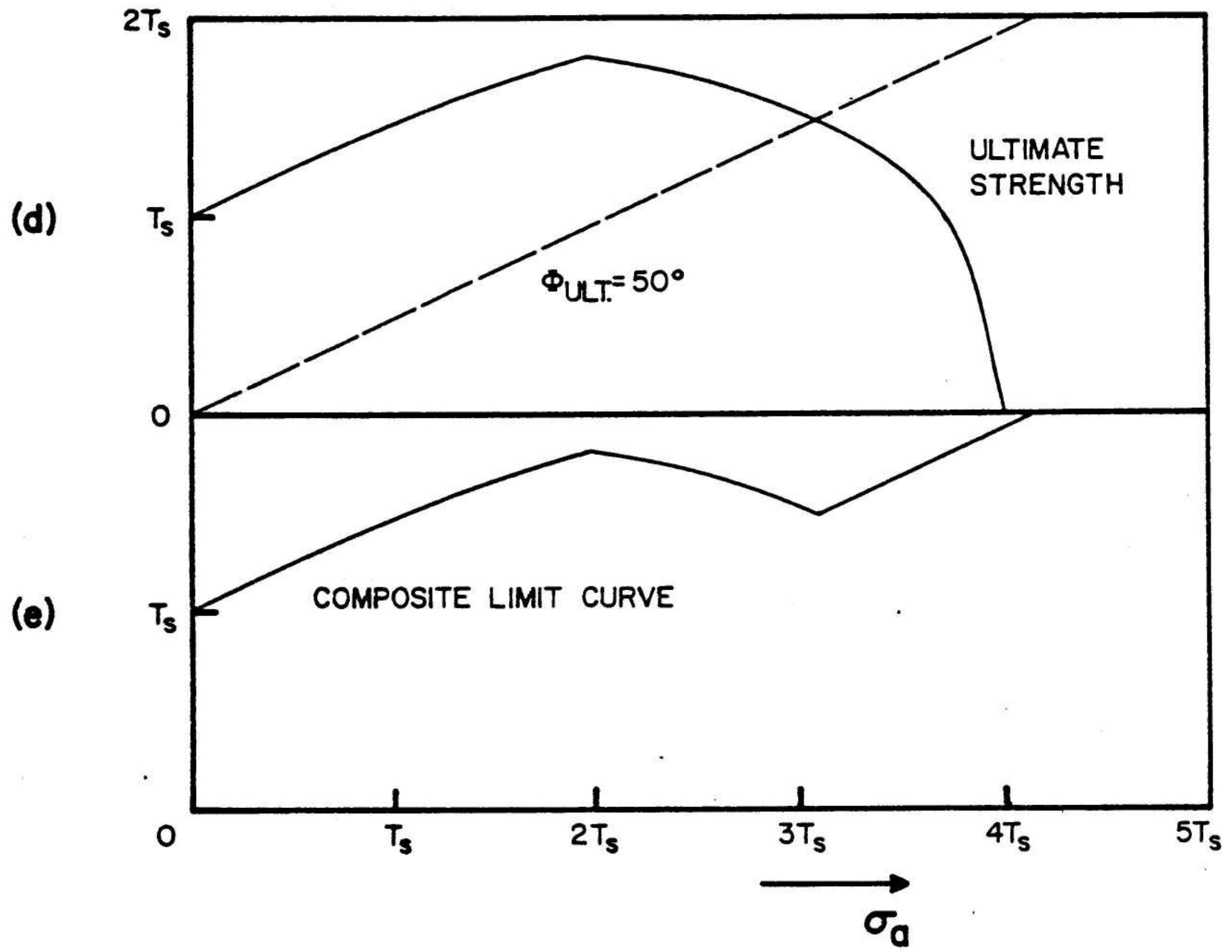
This can be described by the following expression established by Lajtai:

$$\tau_{ult} = \sigma_a \tan \Phi_{ult} \qquad \text{(Eq. 2.2)}$$

where Φ_{ult} is the friction angle of the crushed rock. Equation 2.2 is plotted in with the composite curve of Figures 2.9c and 2.9d. Lajtai finally superimposes the ultimate strength (Eq. 2.2) curve to the composite curve to yield what is termed as the "composite limit curve" shown in Figure 2.9c.

By superimposing Figures 2.9a, 2.9b and 2.9c, we get a composite curve which gives, for any applied value of normal stress, the peak shear stress τ_a at which failure of the intact rock occurs in tension of shear.

Lajtai's mechanical model has the characteristics of being deterministic in the sense that it requires a pre-set value on joint plane persistence. It is not possible to derive a single persistence value that can be used in obtaining acceptable slope reliability values from a deterministic resistance model. The complexity increases when another set of joints of a different orientation exists. This de-



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2.3 Previous Research

The fundamental probabilistic approach was first utilized by Glynn 1979 in his model (JOINTSIM) which incorporates a deterministic mechanical model and a Monte Carlo simulation program. His program generates joints randomly within a rectangular block in a stress field. Joint generation is based on assumed exponential distributions of the joint plane spacing, joint length and rock bridge length about their mean values. In each realization the program finds the paths of minimum safety margin and the associated apparent persistences.

Values of apparent persistence for all realizations form a distribution whose mean and standard deviations are determined at the conclusion of the entire simulation. Glynn found that block stability is strongly dependent on the geometric properties of the block and the joints (block dimensions, mean joint length, mean rock bridge length, mean joint plane spacing) and is relatively insensitive to the ambient stress field and variations in the rock strength parameters.

"JOINTSIM" has the limitation of analyzing a block, not an actual slope. Its algorithm has a drawback of influencing failure paths. "Jointsim" artifically constrains the failure path thus distorting the inclination of the failure path and apparent persistence. This is a noticeable limitation especially when joint planes are closely spaced. This and other drawbacks were alleviated by O'Reilly (1980) in his model which is a much closer model of the actual jointed rock slope.

Briefly, O'Reilly's model reduces the major shortcomings encountered in Glynn's model. It is a probabilistic model for the simple two dimensional cases of a single set of slope parallel joints (See Fig. 2.10). It is a combination of probabilistic simulation approaches, and deterministic models developed to its date.

2.4 The Mechanical (deterministic) Model "TALAL"

In the remaining parts of this chapter the mechanical model for the slope with two joint sets is developed and described. It will be incorporated to determine the stability of any potential failure surface within slopes of the type previously shown in Fig. 2.1.

This thesis follows the same guidelines and criteria previously used to establish the mechanical models developed by Glynn and O'Reilly. In a typical realization, two quantities are computed for a potential failure path. One is the force resisting downward movement of the rock overlying that potential failure path, namely resistance. The other is the force component in the direction of sliding, tending to displace the overlying rock, namely the driving force.

The method of slices is used to determine driving and resisting forces. Figure 2.11 is an illustration of this method. The rock overlying a path is divided into a series of vertical slices. Slices can be bound by either joint or intact rock bridges. Here, a failure is assumed to take place as a rigid body movement of material over-

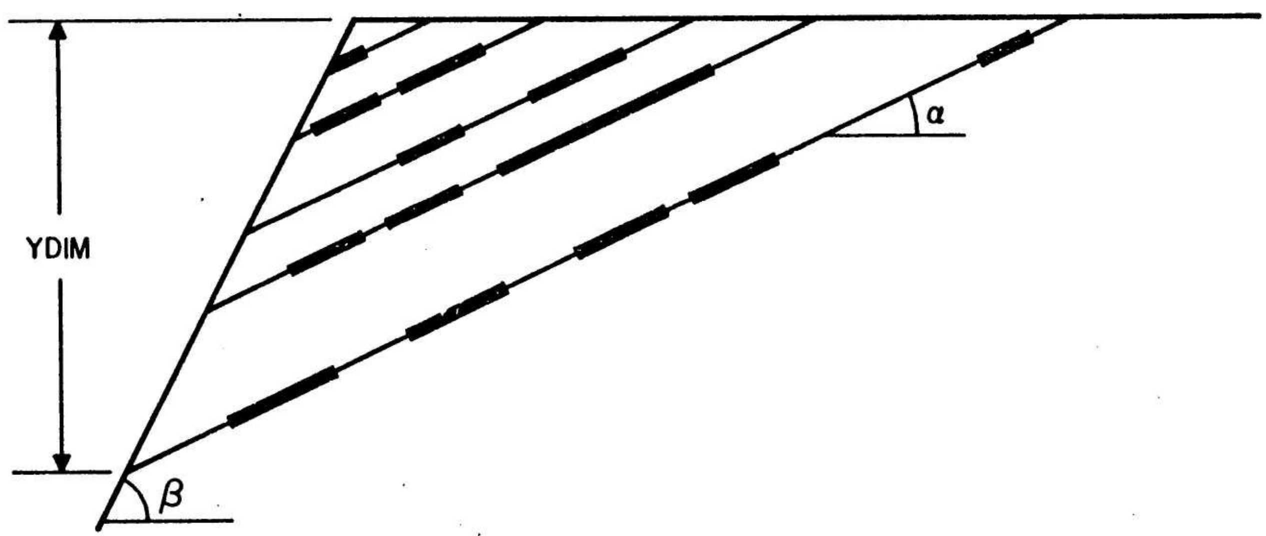


FIGURE 2.10 SLOPE GEOMETRY SINGLE SET OF SLOPE PARALLEL JOINTS

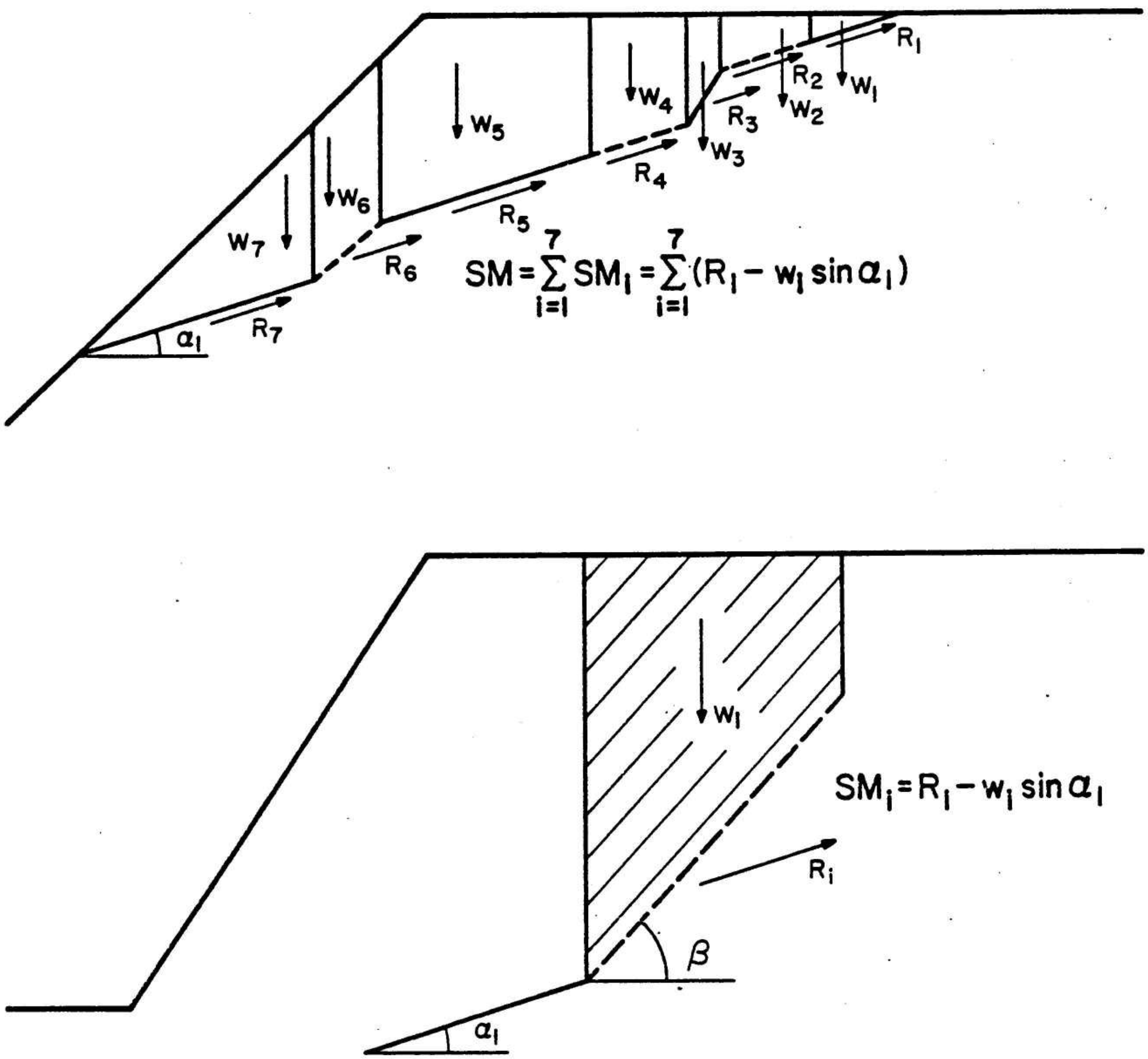


FIGURE 2.11 METHOD OF SLICES

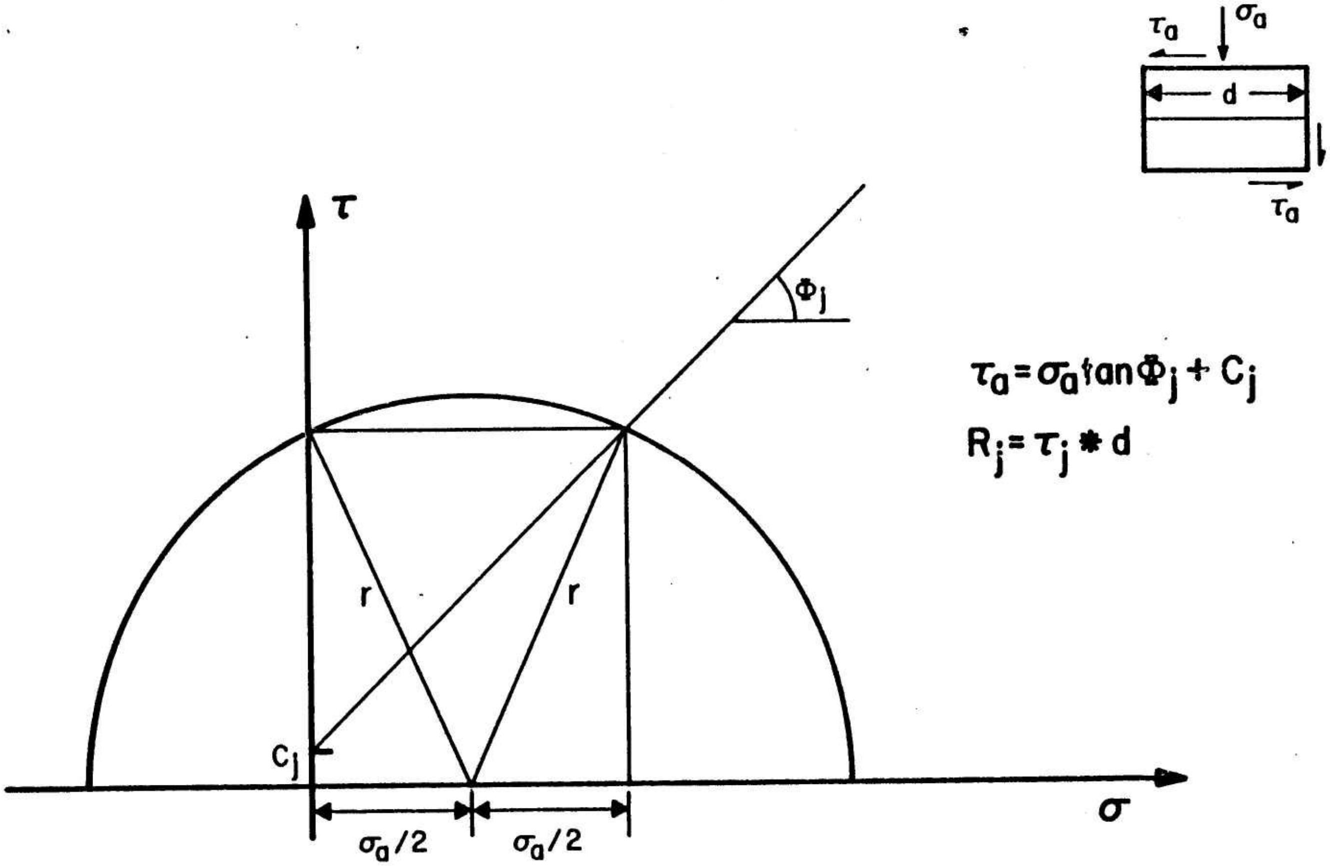


FIGURE 2.12 MOHR'S CIRCLE - JOINT PLANE FAILURE (SET I)

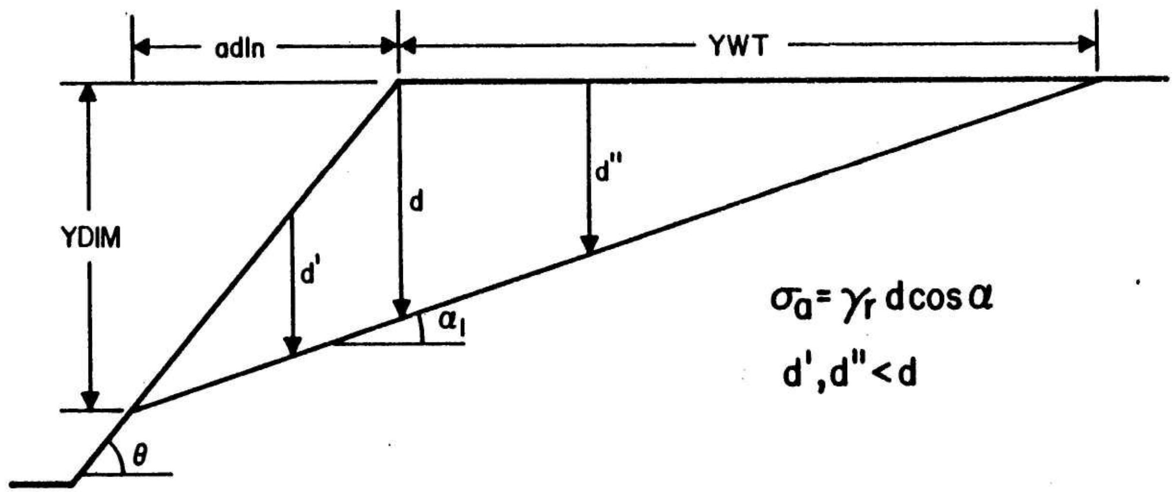


FIGURE 2.13 MAXIMUM STRESS LOCATION IN SECTION OF SLOPE

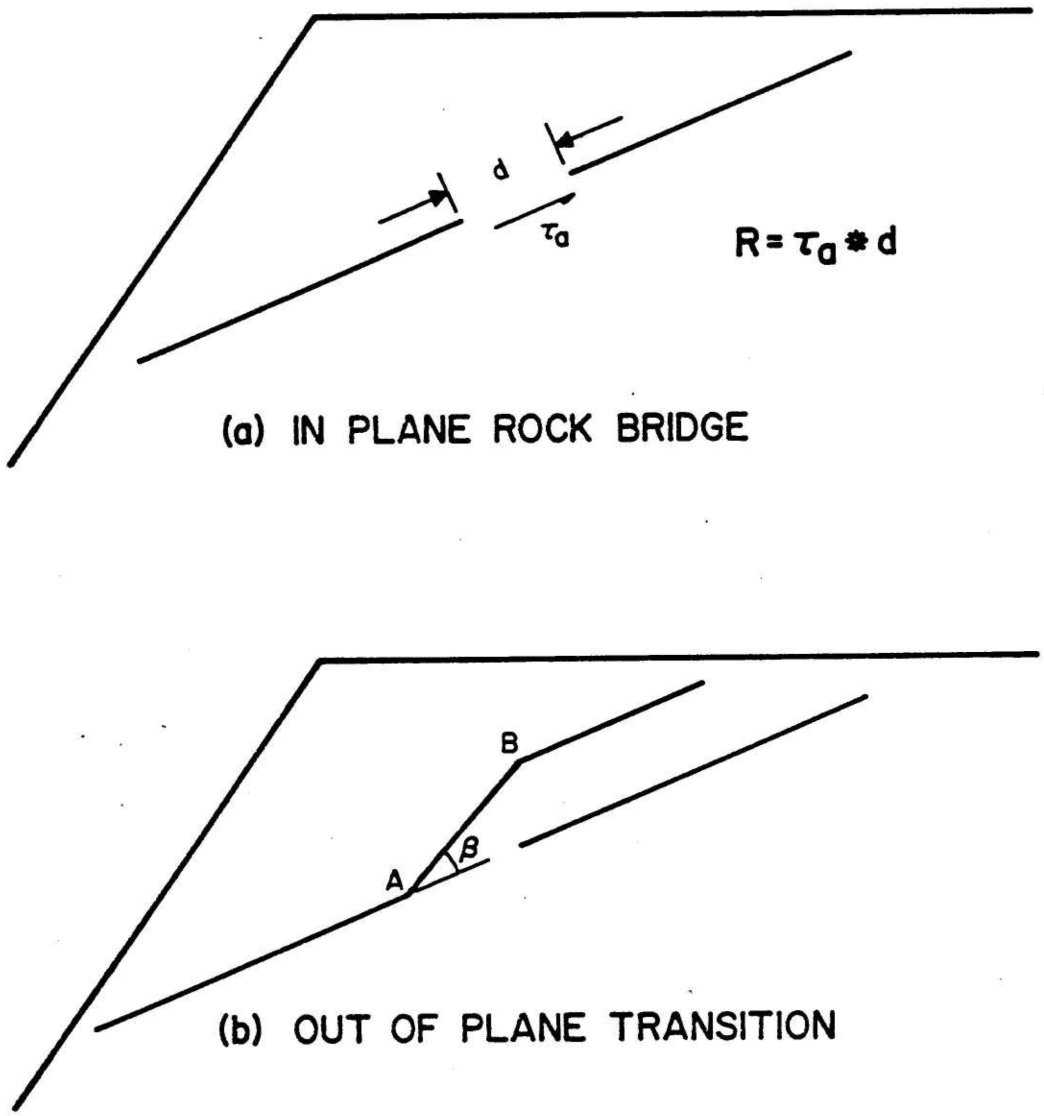


FIGURE 2.14 IN PLANE AND OUT OF PLANE TRANSITIONS

lying the down slope path in the direction of jointing of the first joint set. Total driving force (DF) and total resistance (R)along the path are given by the following relationships:

$$DF = \Sigma_i W_i \sin \alpha 1 \qquad \text{(Eq. 2.3)}$$

$$R = \Sigma_i R_i \qquad \text{(Eq. 2.4)}$$

where W_i is the weight of a slice and R_i is the peak shear force mobilized by the portion of the path underlying the i-th slice. The safety margin SM for each slice can be defined as the difference between resisting and driving forces for that slice:

$$SM_i = R_i - W_i \sin \alpha 1 \qquad \text{(Eq. 2.5)}$$

Thus, the total safety margin SM along the path is given by:

$$SM = \Sigma_i (R_i - W_i \sin \alpha 1) = R - DF \qquad \text{(Eq. 2.5)}$$

Failure occurs when SM $\leq$ 0 (i.e., when the driving force equals or exceeds resistance).

In the following section, the failure mechanism is described. Methods to calculate resistance of the intact rock bridge for "in plane" and "en - echelon" transitions are discussed in detail. Methods to calculate the rock weight overlying a failure path, are also discussed.

Finally, the section concludes with a summary of the deterministic model.

2.5 The Failure Mechanism

In the case of two joint sets, failure is assumed to occur as a downslope movement of a rigid body of rock which is bounded by a failure path consisting of joints, fractured rock, as well as the slope face and the top free surface. Due to the fact that movement is always downslope along the first joint set inclination, joints of the second joint set have, consequently, no shear resistance. In other words, the first set dominates the direction of rock failure sliding while the second set only enhances this to occur.

This thesis is not concerned with cases where intact rock strength parameters are so low that the failure mechanism approaches that of soils (e.g., clay shales) where jointing does not influence the kinematics of slope failure. Limitations also exist since certain combinations of stress field magnitude and orientation (relative to joint inclination) can also lead to failure of intact rock before full joint resistance can be mobilized. Experimental model test by Einstein (1970) show that, even in very strong rock, failure planes are formed without being influenced by discontinuities in the rock.

According to O'Reilly (1980), two basic failure mechanisms in intact rock can describe the failure of a rock slope. The first, Type One, is the most relevant to the research being carried out.

It is referred to as translational sliding parallel to jointing. It is the mode originally assumed, especially when joint resistance is significantly lower than intact rock resistance. The second, Type Two, is that in which shearing is independent of the existing joints. This occurs when the stress required to propagate cracks through intact rock is less than that required to fail joints. Type Two failure may be due to a combination of factors such as weak rock in any stress field, or strong rock in an unfavorable stress field. The decrease in the percentage of intact rock in a potential failure path, is much greater for a slope with second set joints than for a slope with single set joints. Hence, the importance of Type One failure becomes obvious as the amount of intact rock in the failure path becomes less. As a result stability of the slope becomes increasingly dependent on availability of the intact rock.

2.6 "In Plane" and "Out of Plane" Tensile Failures of Intact Rock Bridges

For low stress values ($\sigma_a < 2T_s$), peak shear resistance is mobilized at the moment when tensile fractures develop at an angle θ_t from α 1. θ_t should be approximately equal to 45° for low σ_a values relative to the intact rock tensile strength (See Fig. 2.15) as is expected for Mode Two failure to occur. Actual failure occurs, when a continuous fracture develops in the direction of jointing. After tensile fracturing takes place, a secondary progressive shear fracture follows. (See Fig. 2.15).

The analogy between the direct shear test and "in plane" intact rock bridges is acceptable at low stresses, and thus the resistance

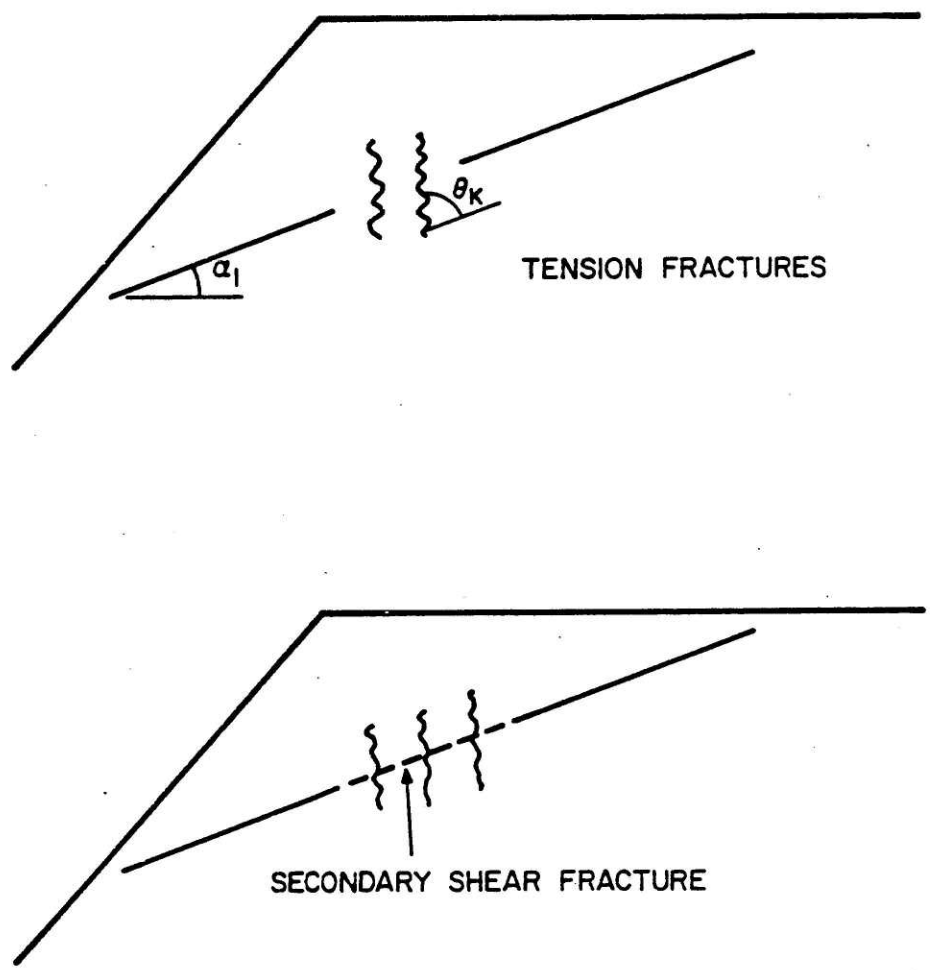


FIGURE 2.15 FAILURE MECHANISMS - DOWN SLOPE MOVEMENT AFTER SECONDARY SHEARS DEVELOP

for in-plane transitions is:

$$R = \tau_a \times d$$

where d is the rock bridge length shown in Figure 2.14a and τ_a is given by the following relation from Lajtai:

$$\tau_a = [T_s\,(T_s - \sigma_a)]^{1/2}$$

and where $-2T_s$ may be replaced by C_r to yield:

$$\left(\frac{C_r}{2}\right) \quad \frac{\sigma_a}{2} \qquad ^{1/2}$$

Out of plane transitions, occur when a continuous fracture develops at an angle to a first joint plane up to a point on another plane, i.e., rock bridges involve a transition from one joint to another in an overlying plane at an angle β , (greater than $\alpha 1$). In such cases criteria established previously apply. However, the assumption here is that the block of rock containing a rock bridge at the moment of failure is in direct shear with zero normal stress in the direction of jointing of the first set. (See Figure 2.16).

Transitions at β values that exceed the angle of tensile fracturing (i.e., $\beta \geq \theta_t + \quad 1$) are referred to as high angle transitions. In such situations, a fracture can develop connecting the discontinuities immediately through a tensile fracture without requiring secondary shearing;

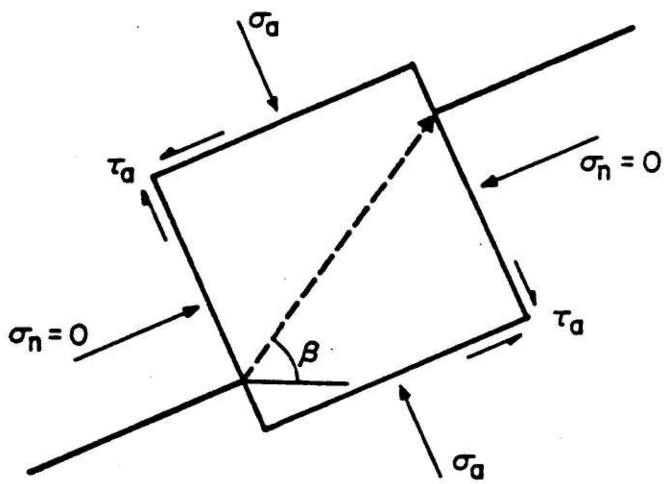


FIGURE 2.16 FAILURE STRESS STATE FOR "OUT OF PLANE" TRANSITIONS

it is assumed that the tensile strength T_s of intact rock is mobilized along the path segment. The tensile force, acting in direction of jointing of the first joint set, is the peak shear resistance for high angle transition.

For "low angle transitions", i.e., $\beta < (\theta_t + \alpha 1)$ failure will occur as "in plane" failure. Peak resistance will be mobilized at the moment of tensile fracturing followed by rupture when secondary shears form a continuous fracture connecting the tips of joints forming the bridge, (See Fig. 2.18). Peak resistance is given by:

$$R = \tau_a \cdot d \qquad \text{(Eq. 2.10)}$$

where d is the distance between joint tips defining the bridge and τ_a is the peak shear mobilized in the direction of jointing. Notice that "in plane" transition (discussed earlier) is one case of the general "low angle transition".

As mentioned previously, the driving force is due to the overburden weight. DF_i is taken as the component of weight over a particular path in the direction of jointing of the first set. (See Fig. 2.19.)

$$DF_i = W_i \cdot \sin \alpha 1$$

Weight calculations are shown in Figures 2.20 - 2.34. Generally, three types of calculations can be distinguished: the first is for transition paths lying to the right of the slope apex, the second is for those lying beneath the slope apex, and the third is for those

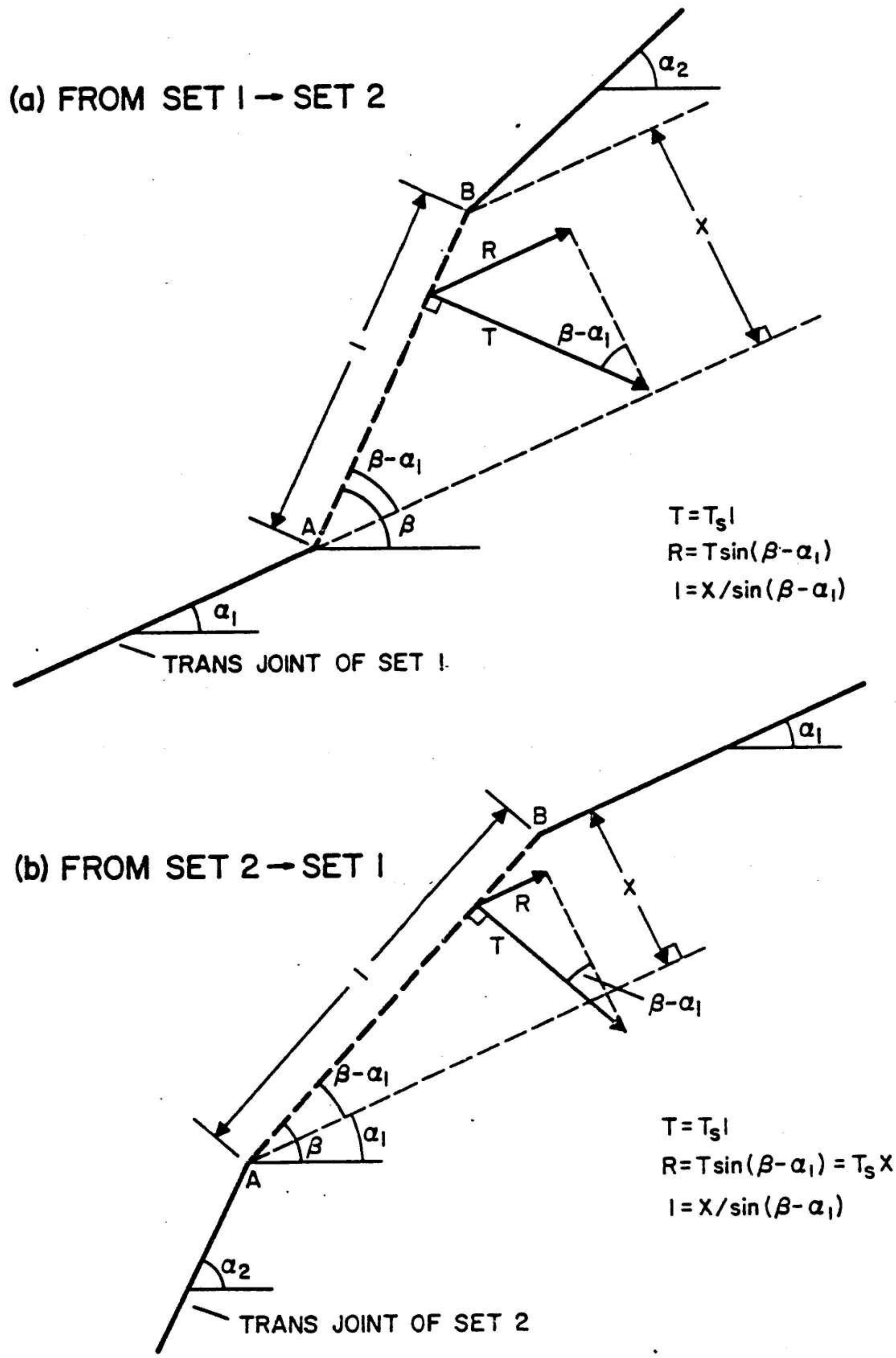


FIGURE 2.17 INTACT ROCK RESISTANCE FOR $\beta \geq \theta_t + \alpha_1$ TRANSITIONS

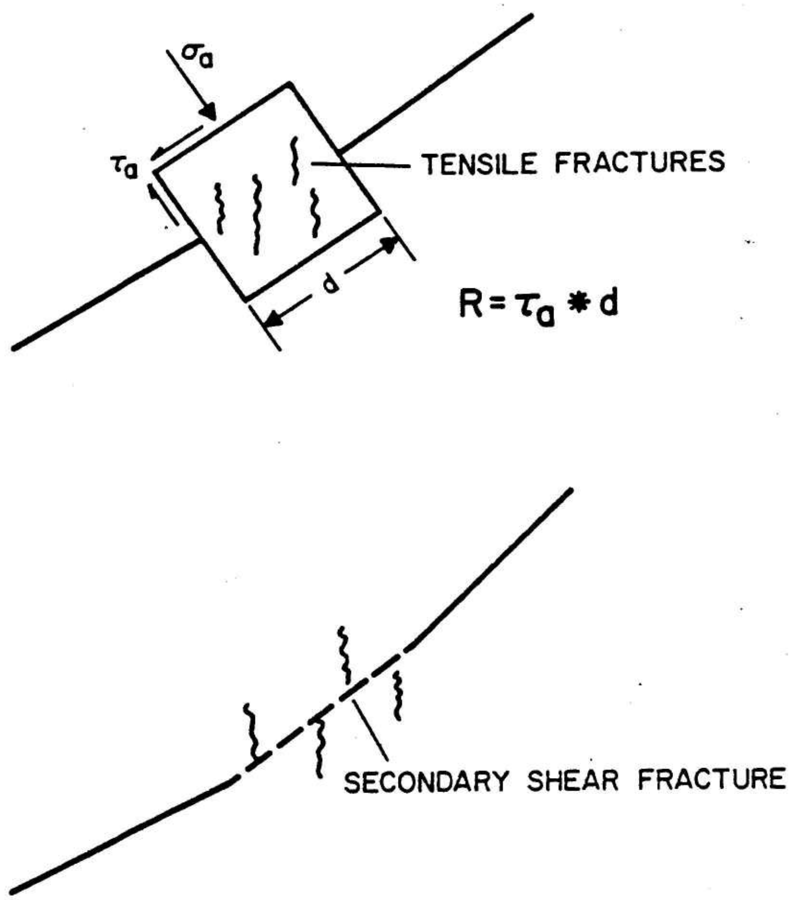


FIGURE 2.18 INTACT ROCK RESISTANCE FOR "LOW ANGLE" TRANSITIONS $\beta < \theta_t + \alpha_1$

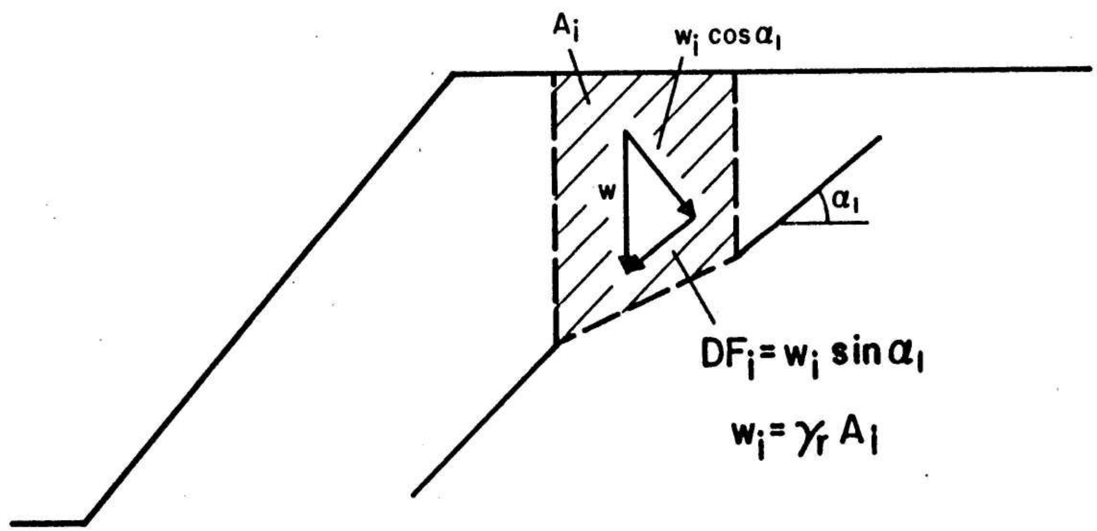
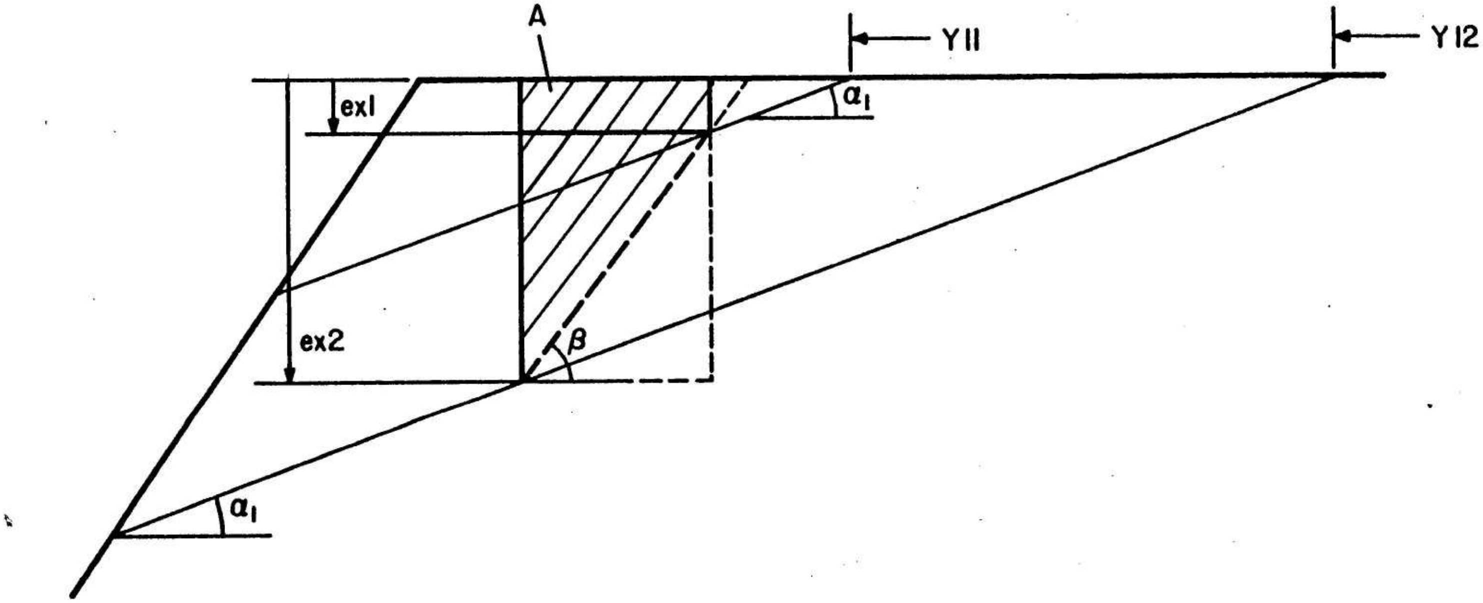


FIGURE 2.19 DRIVING FORCE, DF_i, FOR SLICE i

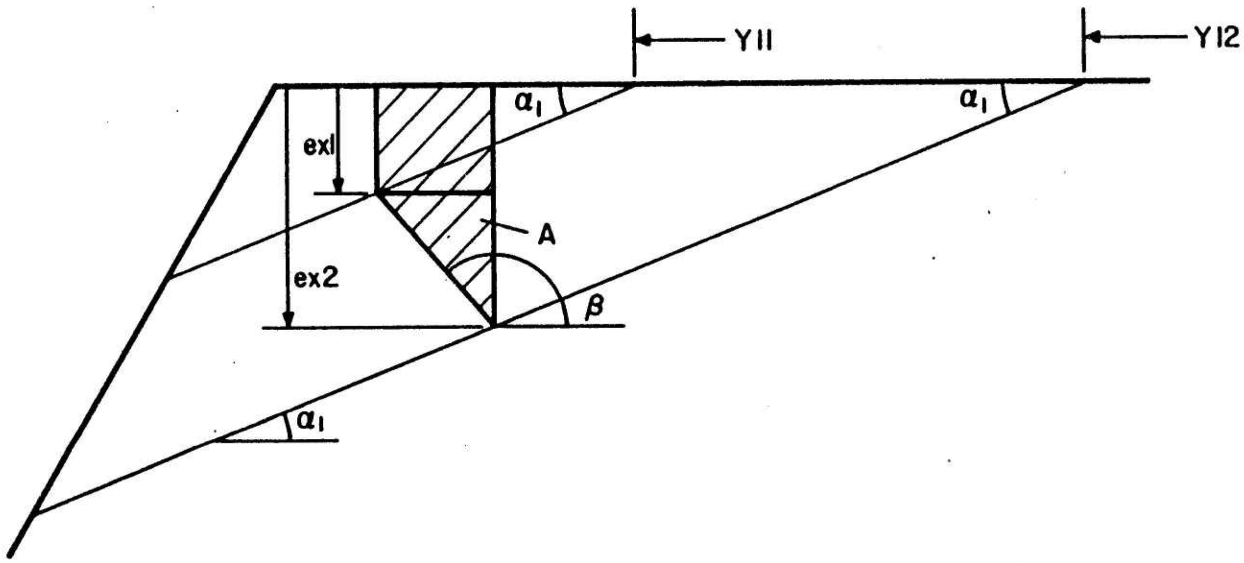
lying to the left of the slope apex, i.e., beneath the slope face. Each of these distinguishable groups can be subdivided into two groups; those with transition angles less than 90 degrees and those with transition angles greater or equal to 90 degrees.



$$A = \frac{1}{2}\left[(ex2/\tan\alpha_1) + Y12 - Y11 - (ex1/\tan\alpha_1)\right](ex2 + ex1)$$

$$\beta = \left\{\tan^{-1}\left[(ex2 - ex1)/(ex2/\tan\alpha_1 - ex1/\tan\alpha_1 + Y12 - Y11)\right]\right\}$$

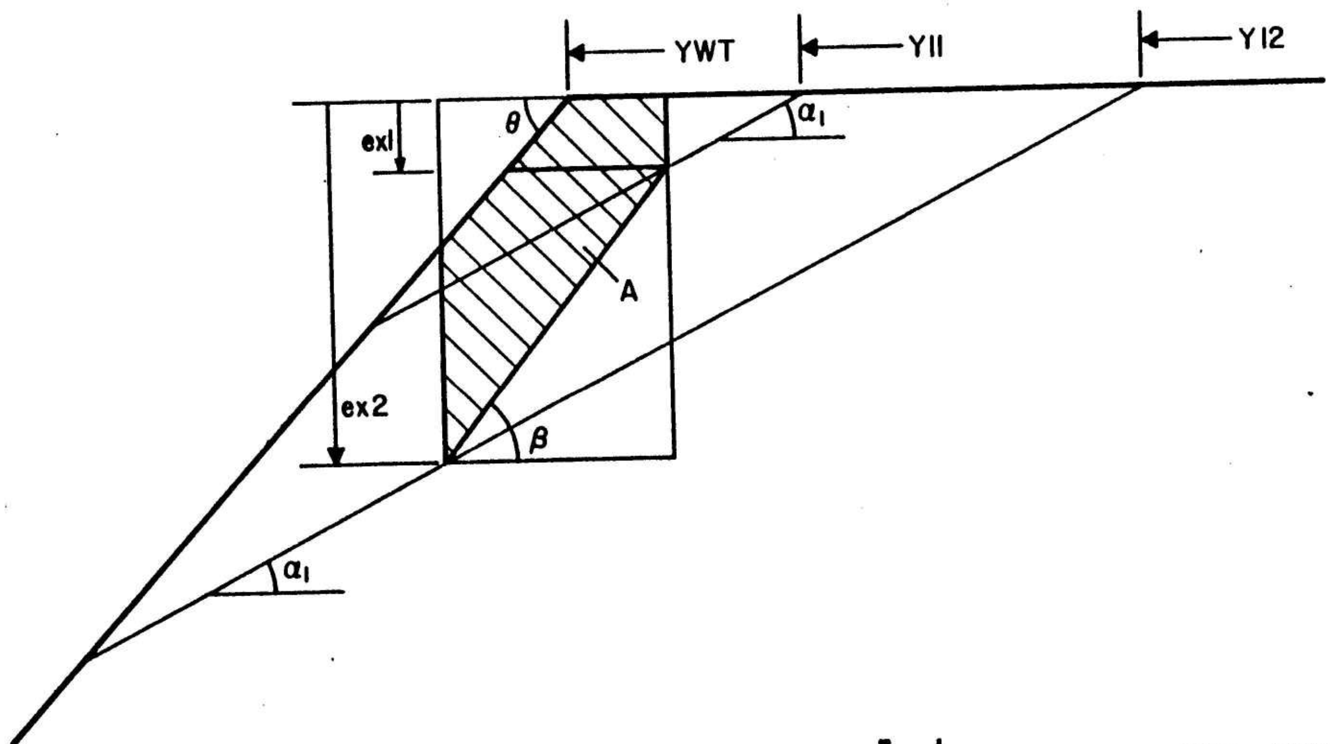
FIGURE 2.20 WEIGHT CALCULATIONS: SLICE TO THE RIGHT OF SLOPE APEX
$\beta<90$ TRANSITION WITHIN JOINT SET 1



$$|A| = \text{abs}\left\{\frac{1}{2}(ex2+ex1)\left[(ex2/\tan\alpha_1)+Y12-Y11-(ex1/\tan\alpha_1)\right]\right\}$$

$$\beta = 180-\left\{\tan^{-1}\left[(ex2-ex1)/\text{abs}(ex2/\tan\alpha_1+Y12-Y11-ex1/\tan\alpha_1)\right]\right\}$$

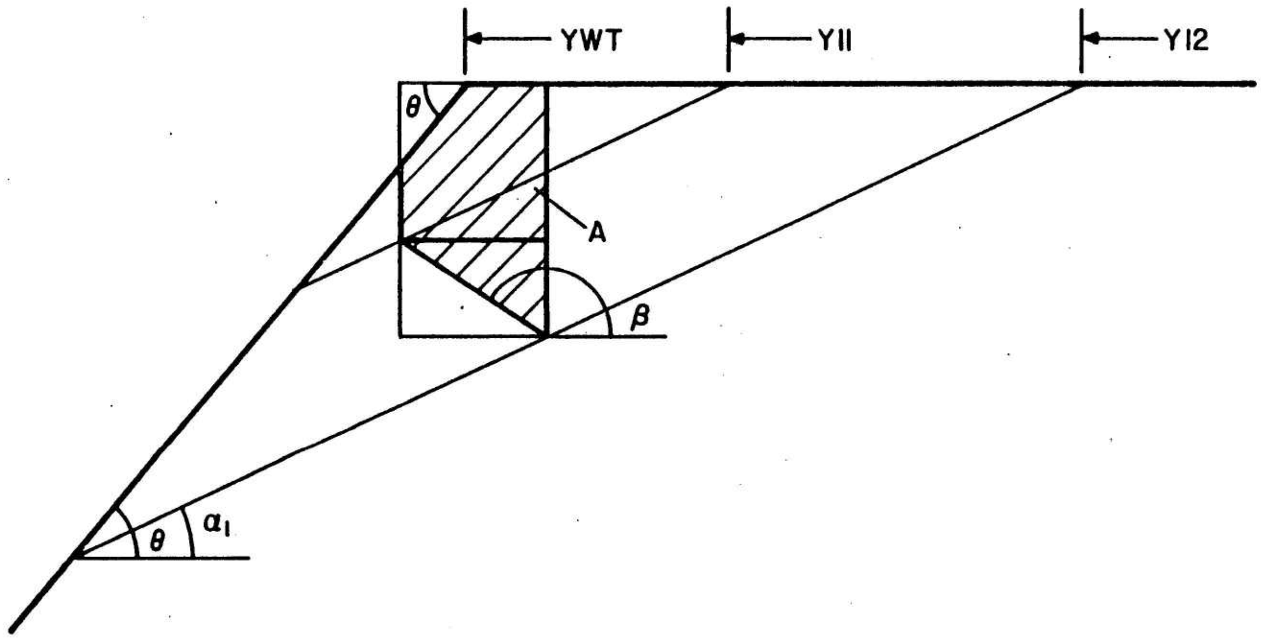
FIGURE 2.21 WEIGHT CALCULATIONS: SLICE TO THE RIGHT OF SLOPE APEX $\beta > 90$ TRANSITION WITHIN JOINT SET 1



$$A=\frac{1}{2}(ex2+ex1)\left[ex2/\tan\alpha_1+Y12-Y11-ex1/\tan\alpha_1\right]-\frac{1}{2}(ex2/\tan\alpha_1+Y12-YWT)^2\tan\theta$$

$$\beta=\tan^{-1}(ex2-ex1)/(ex2/\tan\alpha_1+Y12-Y11-ex1/\tan\alpha_1)$$

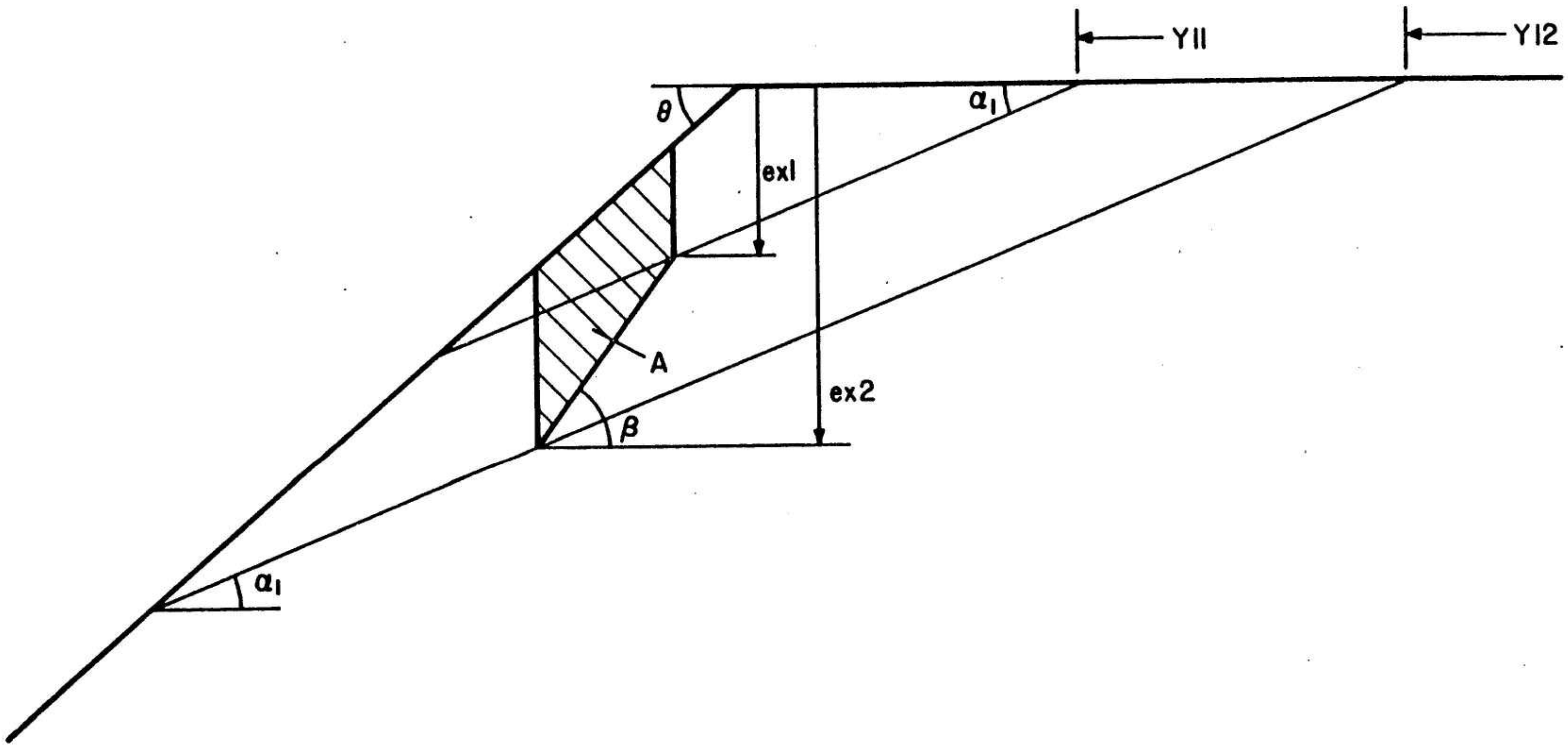
FIGURE 2.22 WEIGHT CALCULATIONS: SLICE BENEATH SLOPE APEX $\beta<90$ TRANSITION WITHIN JOINT SET 1



$$A = \mathrm{abs}\left\{\frac{1}{2}(ex2+ex1)(ex2/\tan\alpha_1+Y12-Y11-ex1/\tan\alpha_1)+\frac{1}{2}(ex1/\tan\alpha_1+Y11-YWT)^2\tan\theta\right\}$$

$$\beta = 180-\tan^{-1}\left\{\mathrm{abs}\left[(ex2-ex1)/\mathrm{abs}(ex2/\tan\alpha_1+Y12-Y11-ex1/\tan\alpha_1)\right]\right\}$$

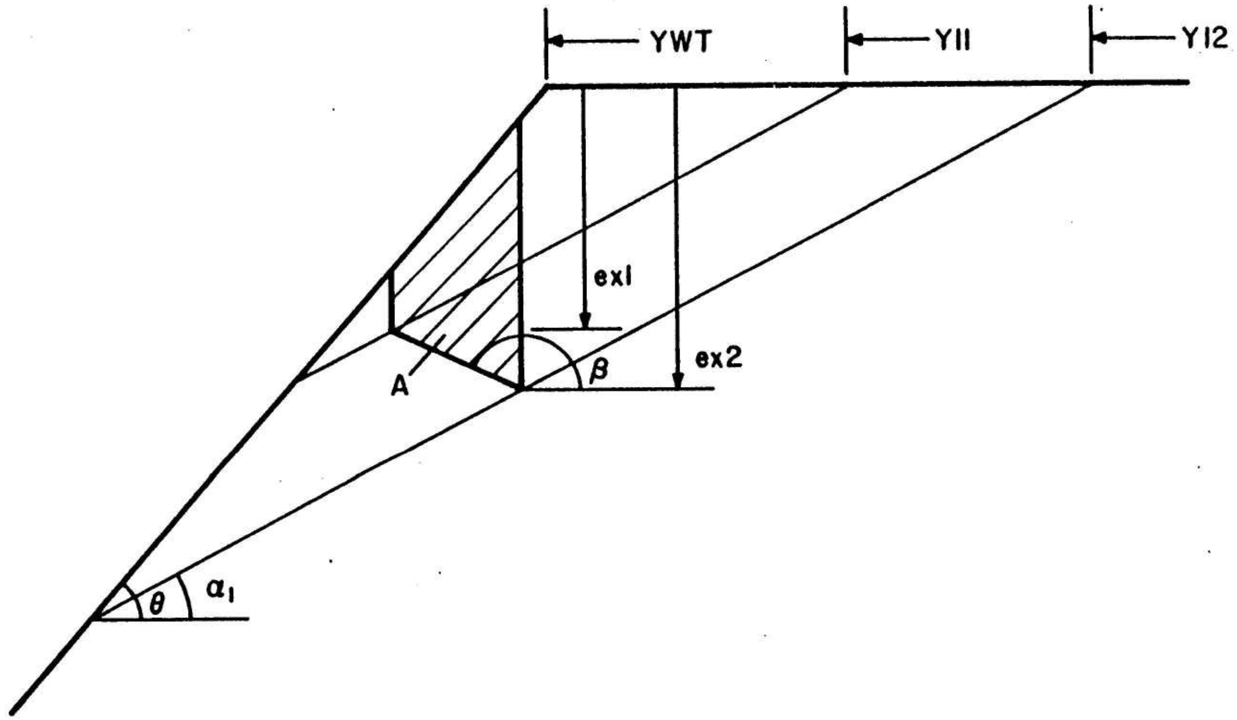
FIGURE 2.23 WEIGHT CALCULATIONS: SLICE BENEATH SLOPE APEX $\beta > 90$ TRANSITIONS WITHIN JOINT SET 1



$$A = \frac{1}{2}(ex2 + ex1)(ex2/\tan\alpha_1 + Y12 - Y11 - ex1/\tan\alpha_1) - \frac{1}{2}\tan\theta\left[(ex2/\tan\alpha_1 + Y12 - YWT)^2 - (ex1/\tan\alpha_1 + Y11 - YWT)^2\right]$$

$$\beta = \tan^{-1}\left[(ex2 - ex1)/(ex2/\tan\alpha_1 + Y12 - Y11 - ex1/\tan\alpha_1)\right]$$

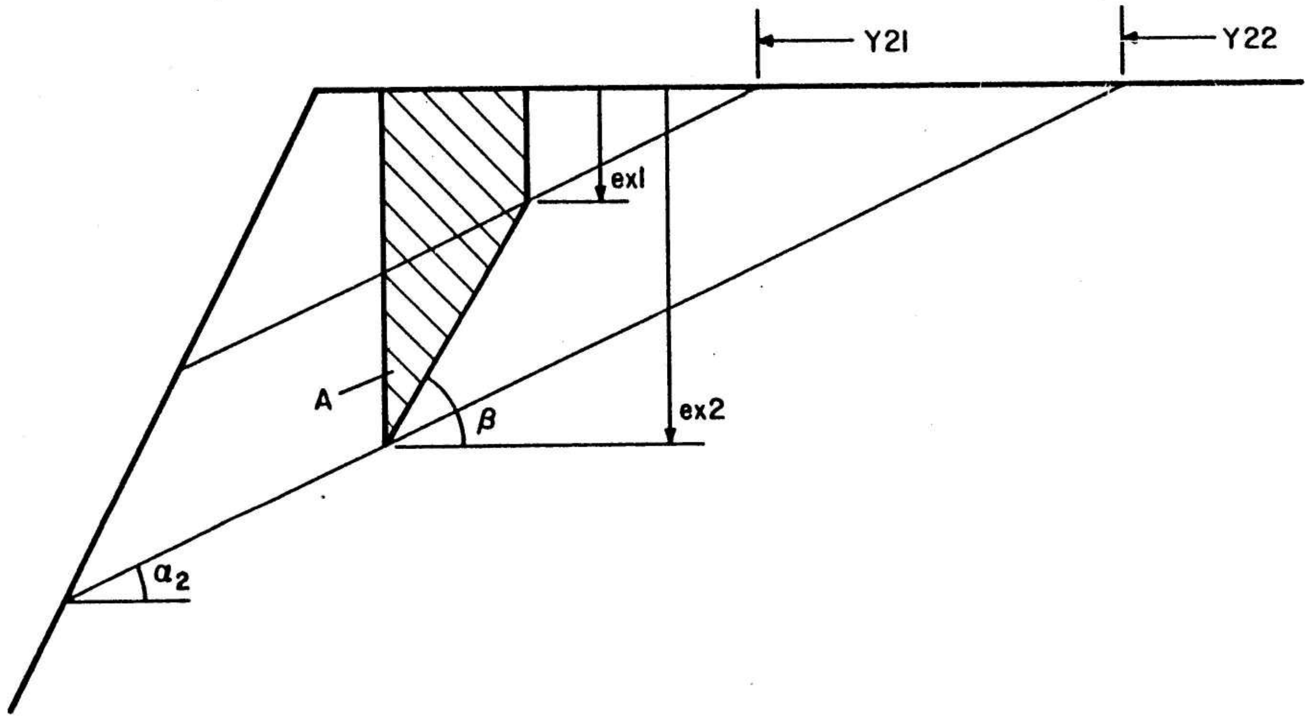
FIGURE 2.24 WEIGHT CALCULATIONS: SLICE TO THE LEFT OF SLOPE APEX β<90 TRANSITIONS WITHIN JOINT SET I



$$|A| = \text{abs}\left\{\tfrac{1}{2}(ex1+ex2)(ex2/\tan\alpha_1+Y12-Y11-ex1/\tan\alpha_1)-\tfrac{1}{2}\tan\theta\left[(ex2/\tan\alpha_1 +Y12-YWT)^2-(ex1/\tan\alpha_1+Y11-YWT)^2\right]\right\}$$

$$\beta = 180-\tan^{-1}\left[(ex2-ex1)/\text{abs}(ex2/\tan\alpha_1+Y12-Y11-ex1/\tan\alpha_1)\right]$$

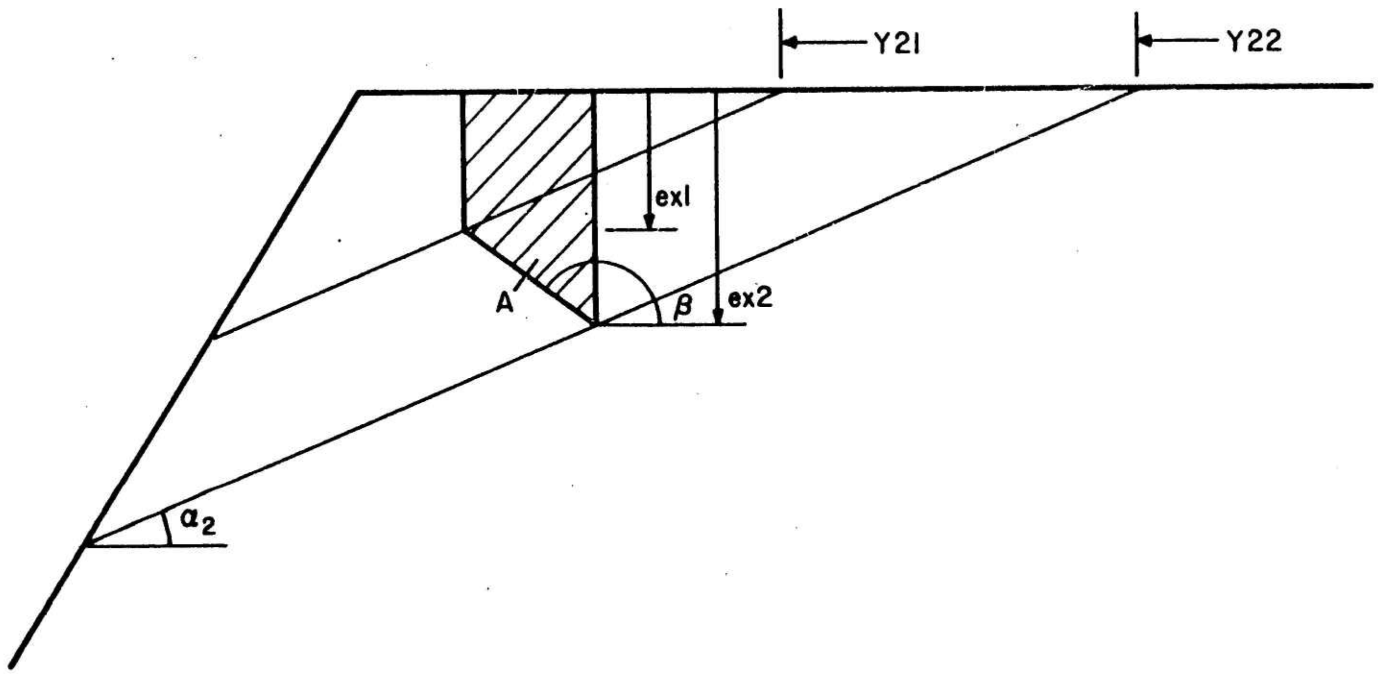
FIGURE 2.25 WEIGHT CALCULATIONS: SLICE TO THE LEFT OF SLOPE APEX $\beta > 90$ TRANSITIONS WITHIN JOINT SET I



$$A = \frac{1}{2}\left[(ex2/\tan\alpha_2) + Y22 - Y21 - (ex1/\tan\alpha_2)\right](ex2 + ex1)$$

$$\beta = \tan^{-1}\left[(ex2 - ex1)/(ex2/\tan\alpha_2 - ex1/\tan\alpha_2 + Y22 - Y21)\right]$$

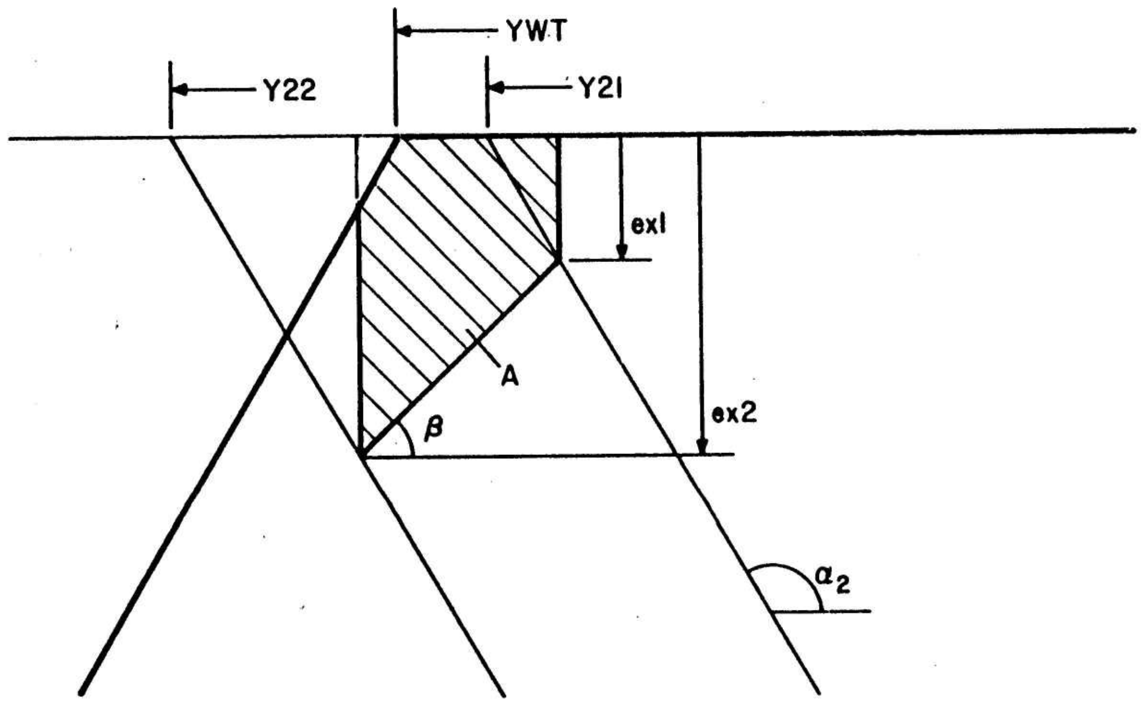
FIGURE 2.26 WEIGHT CALCULATIONS: SLICE TO THE RIGHT OF SLOPE APEX $\beta < 90$ TRANSITIONS WITHIN JOINT SET 2



$$A = \mathrm{abs}\left\{\frac{1}{2}(ex2+exl)\left[ex2/\tan\alpha_2 + Y22 - Y21 - exl/\tan\alpha_2\right]\right\}$$

$$\beta = 180 - \left\{\tan^{-1}\left[(ex2-exl)/\mathrm{abs}(ex2/\tan\alpha_2 + Y22 - Y21 - exl/\tan\alpha_2)\right]\right\}$$

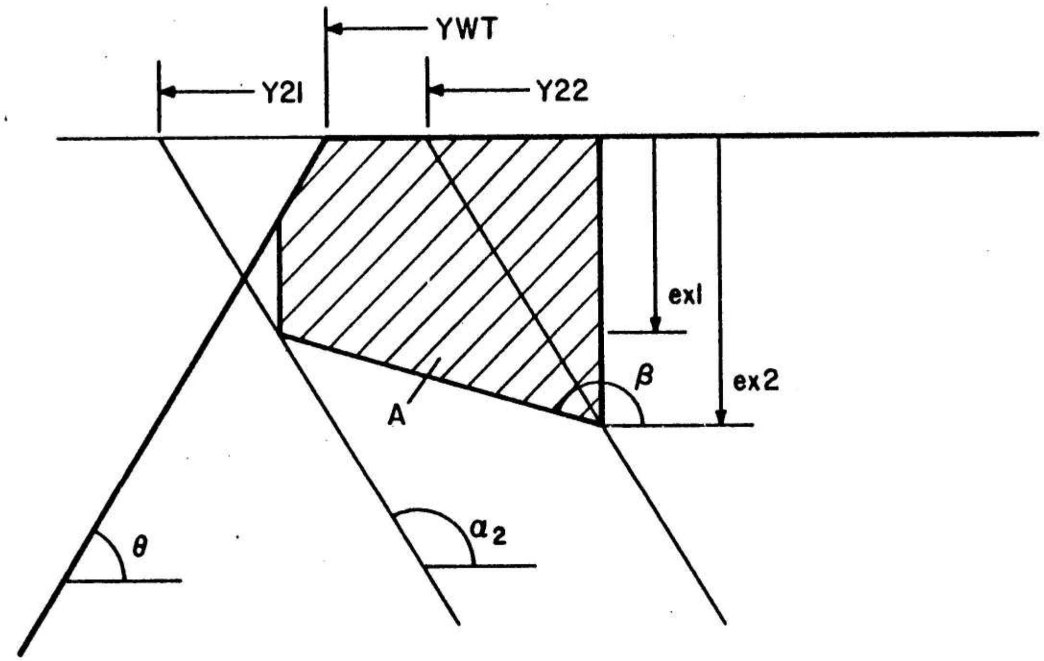
FIGURE 2.27 WEIGHT CALCULATIONS: SLICE TO THE RIGHT OF SLOPE APEX
$\beta > 90$ TRANSITIONS WITHIN JOINT SET 2



$$A=\frac{1}{2}(ex2+exl)(ex2/\tan\alpha_2+Y22-Y2l-exl/\tan\alpha_2)-\frac{1}{2}(ex2/\tan\alpha_2+Y22-YWT)^2\tan\theta$$

$$\beta=\tan^{-1}\left[(ex2-exl)/(ex2/\tan\alpha_2+Y22-Y2l-exl/\tan\alpha_2)\right]$$

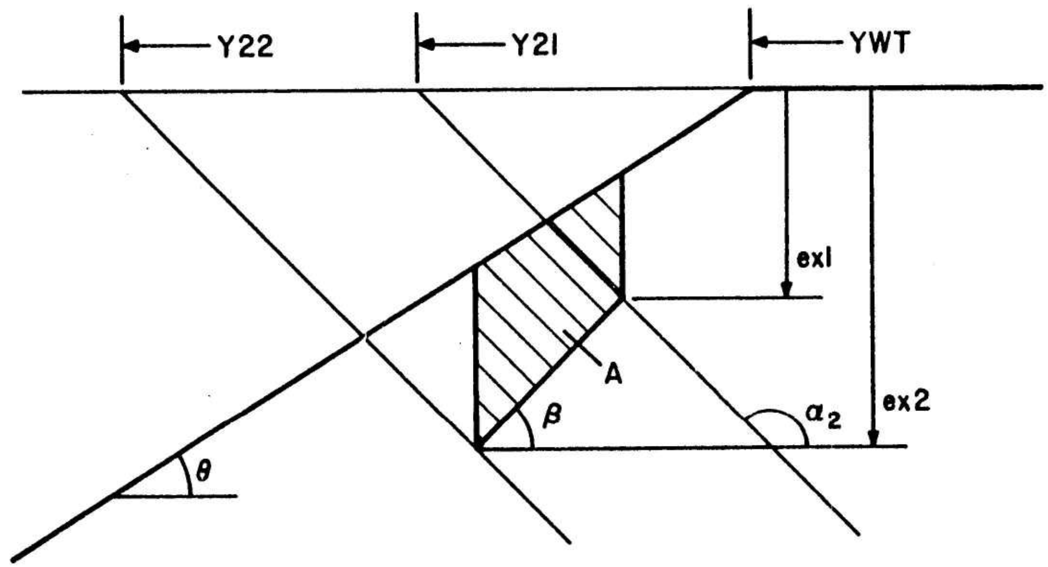
FIGURE 2.28 WEIGHT CALCULATIONS: SLICE BENEATH SLOPE APEX $\beta<90$
$\alpha_2>90$ TRANSITIONS WITHIN JOINT SET 2



$$A = \mathrm{abs}\left\{\frac{1}{2}(ex2+ex1)(ex2/\tan\alpha_2+Y22-Y21-ex1/\tan\alpha_2)+\frac{1}{2}(ex1/\tan\alpha_2+Y21-YWT)^2\tan\theta\right\}$$

$$\beta = 180-\tan^{-1}\left\{\mathrm{abs}\left[ex2-ex1\ /\mathrm{abs}(ex2/\tan\alpha_2+Y22-Y21-ex1/\tan\alpha_2)\right]\right\}$$

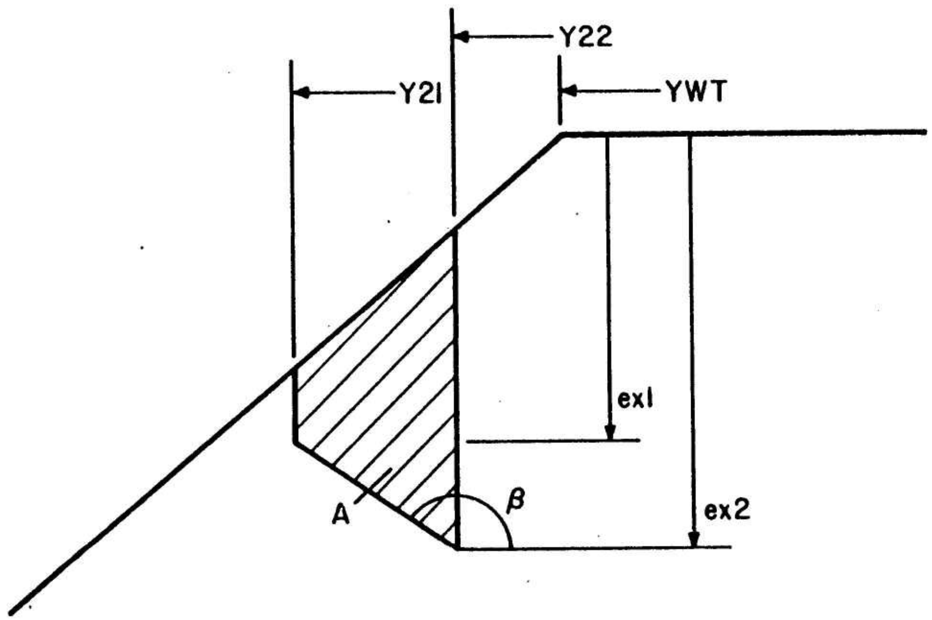
FIGURE 2.29 WEIGHT CALCULATIONS: SLICE BENEATH SLOPE APEX $\beta>90$ $\alpha_2>90$ TRANSITIONS WITHIN JOINT SET 2



$$A=\frac{1}{2}(ex2+exl)(ex2/\tan\alpha_2+Y22-Y2l-exl/\tan\alpha_2)-\frac{1}{2}\tan\theta\left[(ex2/\tan\alpha_1+Y22-YWT)^2-(exl/\tan\alpha_1+Y2l-YWT)^2\right]$$

$$\beta=\tan^{-1}\left[(ex2-exl)/(ex2/\tan\alpha_2+Y22-Y2l-exl/\tan\alpha_2)\right]$$

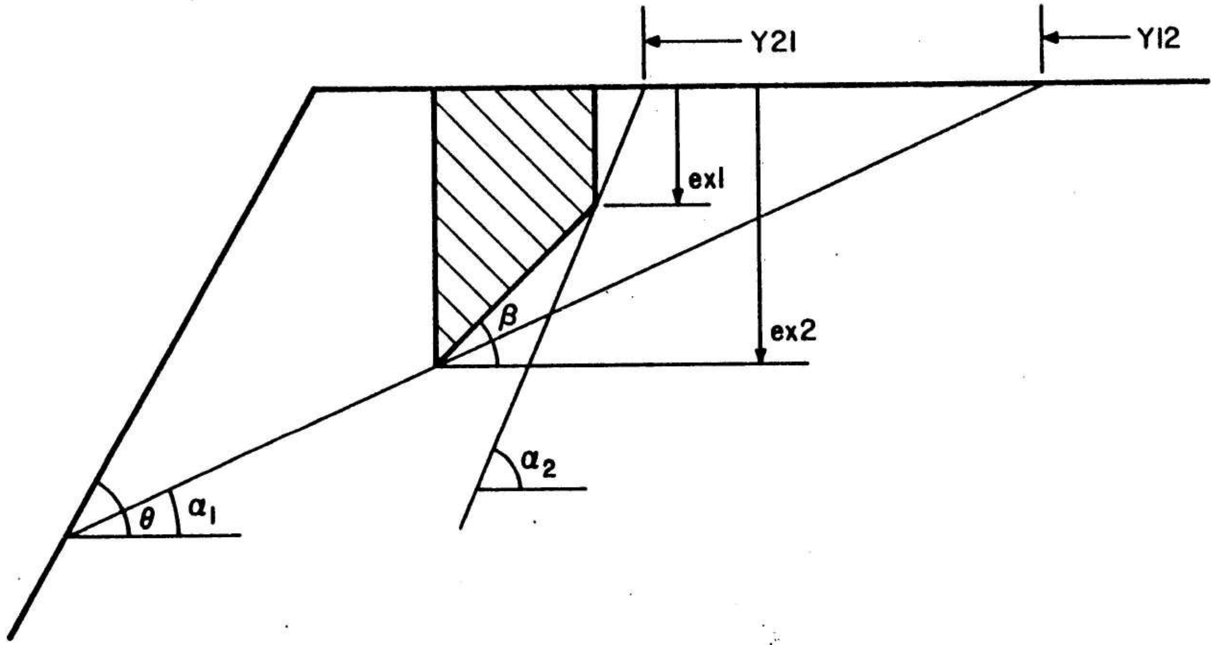
FIGURE 2.30 WEIGHT CALCULATIONS: SLICE TO THE LEFT OF SLOPE APEX $\beta<90$ $\alpha_2>90$ TRANSITIONS WITHIN JOINT SET 2



$$A = \mathrm{abs}\left\{\frac{1}{2}(ex2+ex1)(ex2/\tan\alpha_2+Y22-Y21-ex1/\tan\alpha_2)-\frac{1}{2}\tan\theta\left[(ex2/\tan\alpha_2+Y22-YWT)^2-(ex1/\tan\alpha_1+Y21-YWT)^2\right]\right\}$$

$$\beta = 180-\tan^{-1}\left[(ex2-ex1)/\mathrm{abs}(ex2/\tan\alpha_1+Y22-Y21-ex1/\tan\alpha_2)\right]$$

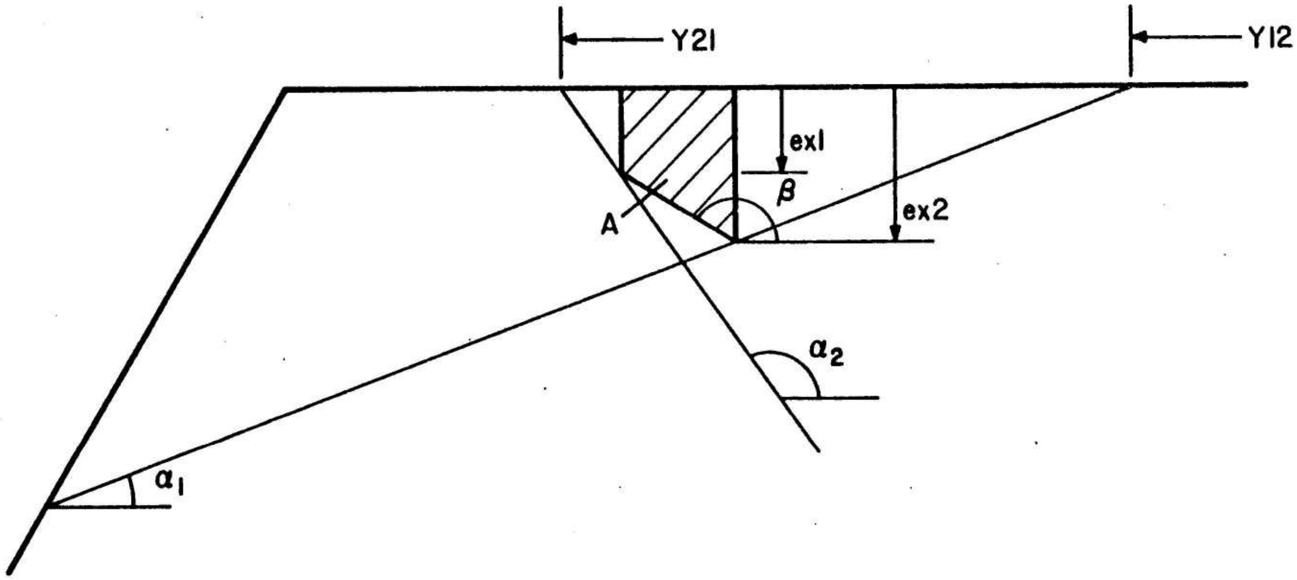
FIGURE 2.31 WEIGHT CALCULATIONS: SLICE TO THE LEFT OF SLOPE APEX $\beta>90$ $\alpha_2=90$ TRANSITION WITHIN JOINT SET 2



$$A=\frac{1}{2}(ex2+ex1)(ex2/\tan\alpha_1+Y12-Y21-ex1/\tan\alpha_2)$$

$$\beta=\tan^{-1}\left[(ex2-ex1)/(ex2/\tan\alpha_1+Y12-Y21-ex1/\tan\alpha_2)\right]$$

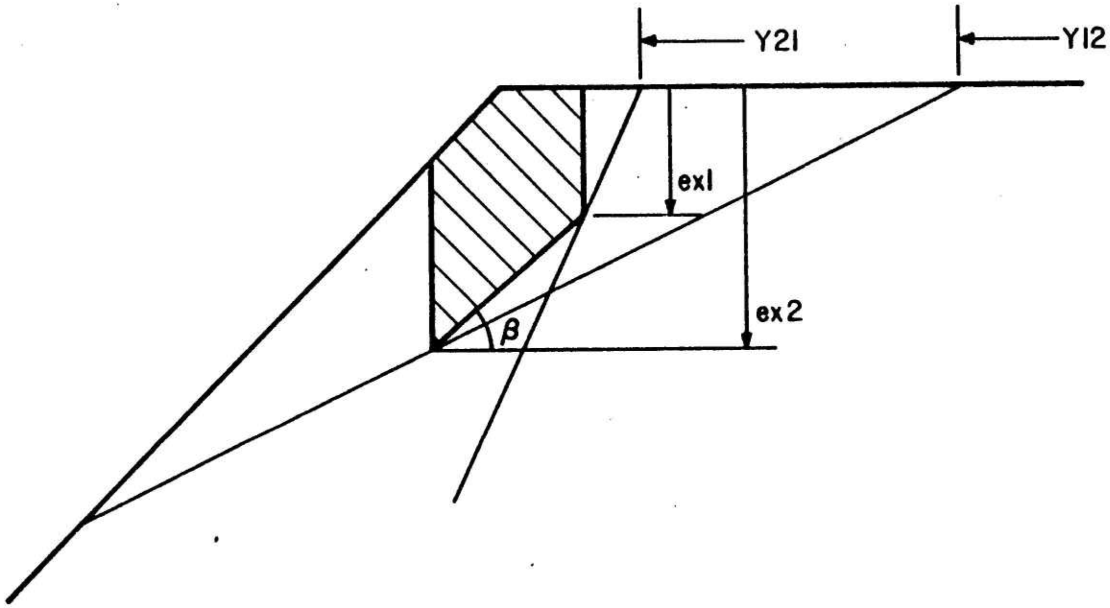
FIGURE 2.32 WEIGHT CALCULATIONS: SLICE TO THE RIGHT OF SLOPE APEX
$\beta<90$ $\alpha_2<90$ TRANSITION FROM SET $1\rightarrow2$



$$|A| = \mathrm{abs}\left[\frac{1}{2}(ex2+ex1)(ex2/\tan\alpha_1 + Y12 - Y21 - ex1/\tan\alpha_2)\right]$$

$$\beta = 180 - \tan^{-1}\left[(ex2-ex1)/\mathrm{abs}(ex2/\tan\alpha_1 + Y12 - Y21 - ex1/\tan\alpha_2)\right]$$

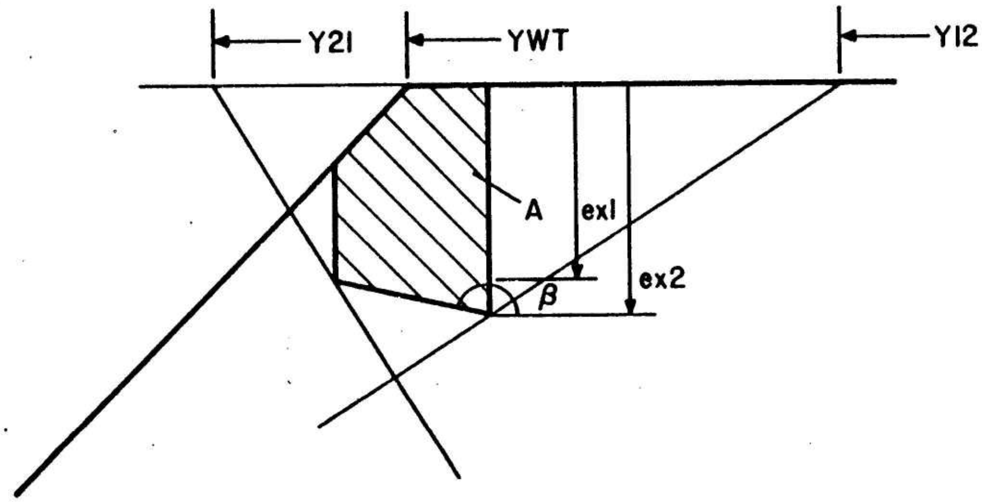
FIGURE 2.33 WEIGHT CALCULATIONS: SLICE TO THE RIGHT OF SLOPE APEX
$\beta > 90$ $\alpha_2 > 90$ TRANSITION FROM SET 1 → 2



$$A = \frac{1}{2}(ex2 + ex1)(ex2/\tan\alpha_1 + Y12 - Y21 - ex1/\tan\alpha_2) - \frac{1}{2}\tan\theta\,(ex2/\tan\alpha_1 + Y12 - YWT)^2$$

$$\beta = \tan^{-1}\left[(ex2 - ex1)/(ex2/\tan\alpha_1 + Y12 - Y21 - ex1/\tan\alpha_2)\right]$$

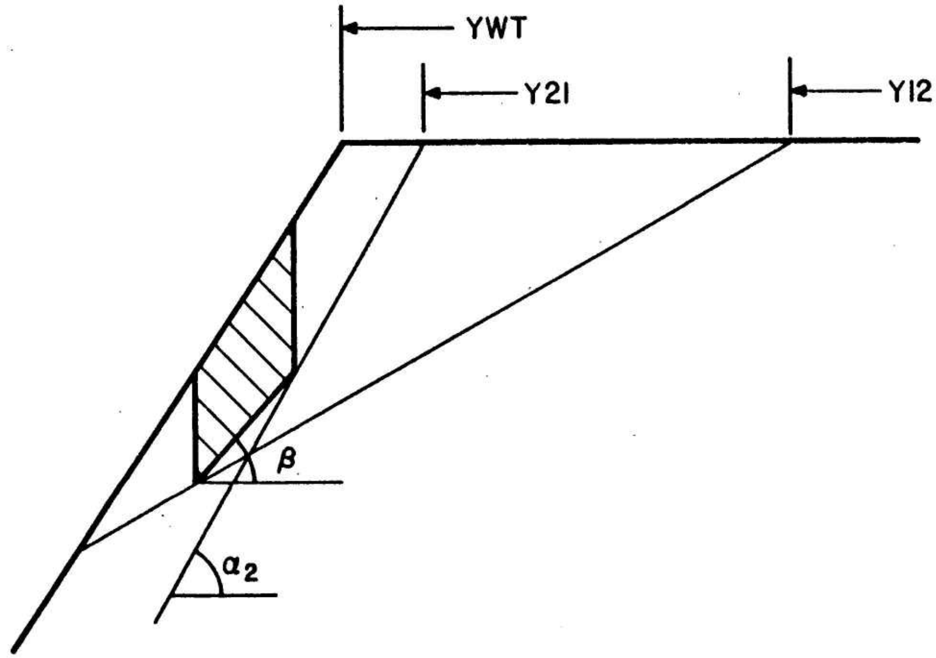
FIGURE 2.34 WEIGHT CALCULATIONS: SLICE BENEATH SLOPE APEX $\beta < 90$ $\alpha_2 < 90$ TRANSITION FROM SET $1 \rightarrow 2$



$$A=\frac{1}{2}(ex2+ex1)(ex2/\tan\alpha_1+Y12-Y21-ex1/\tan\alpha_2)+\frac{1}{2}\tan\theta\,(ex2/\tan\alpha_2+Y21-YWT)^2$$

$$\beta=180-\tan^{-1}\left[(ex2-ex1)/\mathrm{abs}(ex2/\tan\alpha_1+Y12-Y21-ex1/\tan\alpha_2)\right]$$

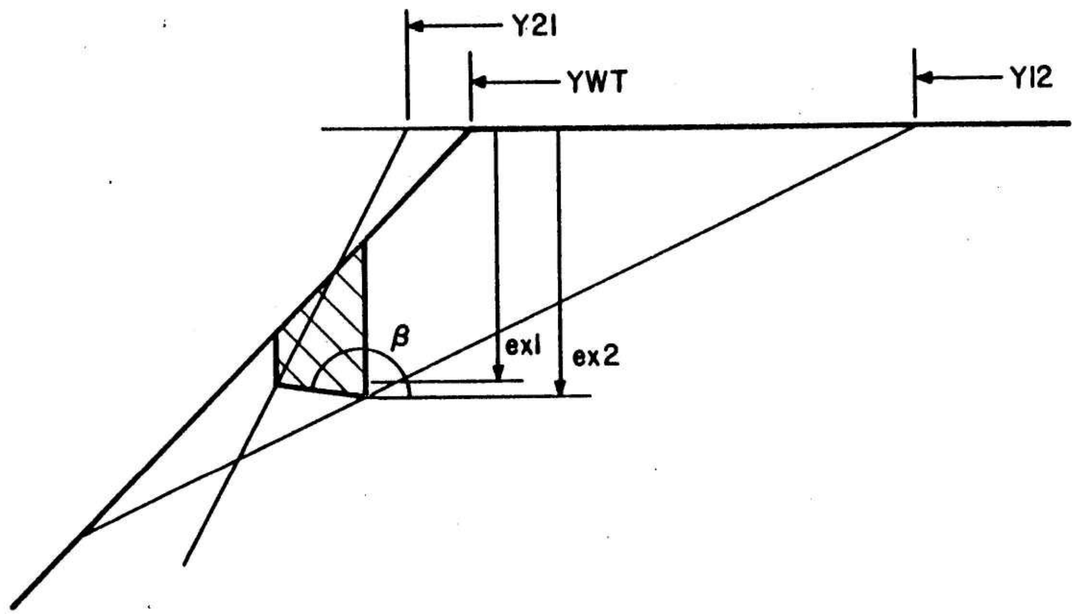
FIGURE 2.35 WEIGHT CALCULATIONS: SLICE BENEATH SLOPE APEX $\beta>90$
$\alpha_2>90$ TRANSITION FROM SET $1\rightarrow2$



$$A = \frac{1}{2}(ex2+ex1)(ex2/\tan\alpha_1 + Y12 - Y21 - ex1/\tan\alpha_2) - \frac{1}{2}\tan\theta\left[(ex2/\tan\alpha_1 + Y12 - YWT)^2 - (ex1/\tan\alpha_2 + Y21 - YWT)^2\right]$$

$$\beta = \tan^{-1}\left[(ex2-ex1)/(ex2/\tan\alpha_1 + Y12 - Y21 - ex1/\tan\alpha_2)\right]$$

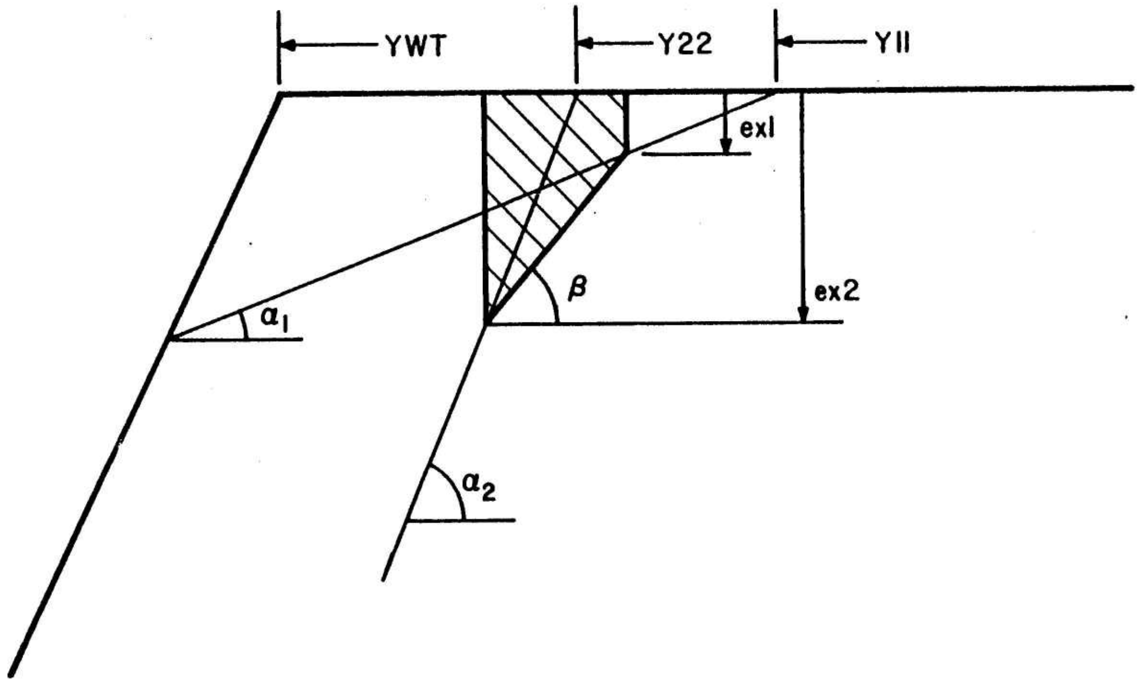
FIGURE 2.36 WEIGHT CALCULATIONS: SLICE TO THE LEFT OF SLOPE APEX
$\beta<90$ $\alpha_2<90$ TRANSITION FROM SET 1 → 2



$$A=\frac{1}{2}(ex2+exl)(ex2/\tan\alpha_1+Y12-Y21-exl/\tan\alpha_2)-\frac{1}{2}\tan\theta\Big[(ex2/\tan\alpha_1+Y12-YWT)^2-(exl/\tan\alpha_2-Y21-YWT)^2\Big]$$

$$\beta=180-\tan^{-1}\Big[(ex2-exl)/\mathrm{abs}(ex2/\tan\alpha_1+Y12-Y21-exl/\tan\alpha_2)\Big]$$

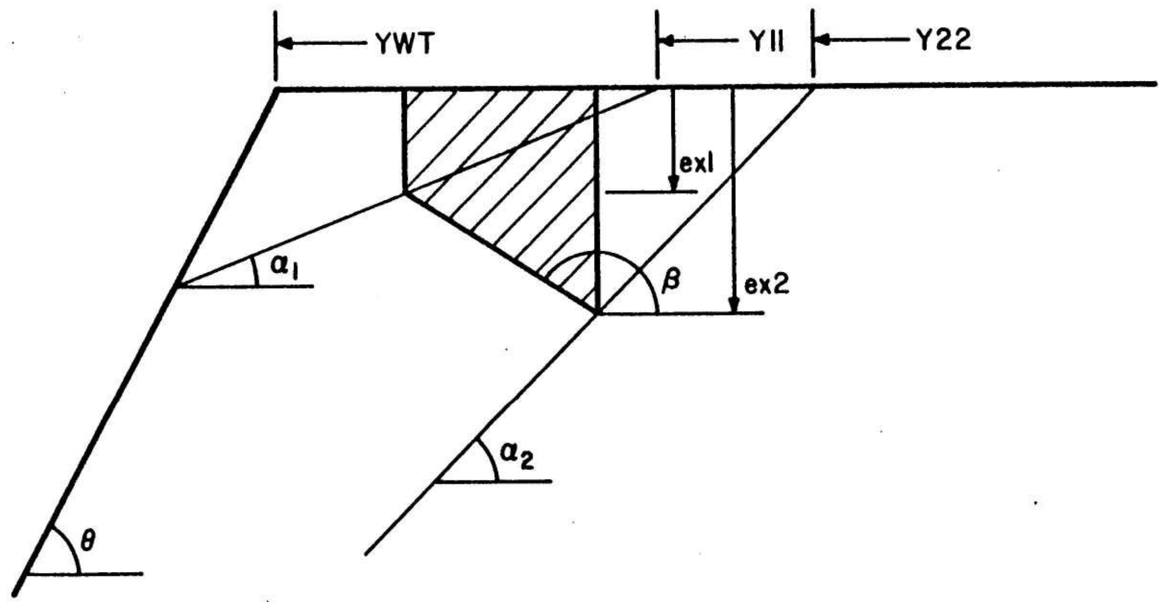
FIGURE 2.37 WEIGHT CALCULATIONS: SLICE TO THE LEFT OF SLOPE APEX $\beta>90$ $\alpha_2<90$ TRANSITION FROM SET $1\rightarrow2$



$$A = \frac{1}{2}(ex2 + ex1)(ex2/\tan\alpha_2 + Y22 - Y11 - ex1/\tan\alpha_1)$$

$$\beta = \tan^{-1}\left[(ex2 - ex1)/(ex2/\tan\alpha_2 + Y22 - Y11 - ex1/\tan\alpha_1)\right]$$

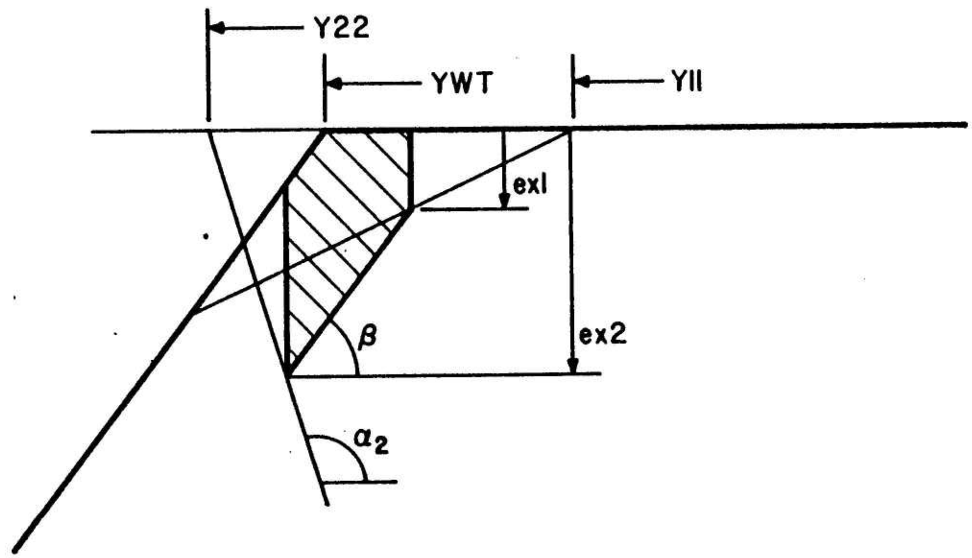
FIGURE 2.38 WEIGHT CALCULATIONS: SLICE TO THE RIGHT OF SLOPE APEX $\beta < 90$ $\alpha_2 < 90$ TRANSITION FROM SET $2 \rightarrow 1$



$$A = \text{abs}\left[\frac{1}{2}(ex2 + ex1)(ex2/\tan\alpha_2 + Y22 - Y11 - ex1/\tan\alpha_1)\right]$$

$$\beta = 180 - \tan^{-1}\left[(ex2 - ex1)/\text{abs}(ex2/\tan\alpha_2 + Y22 - Y11 - ex1/\tan\alpha_1)\right]$$

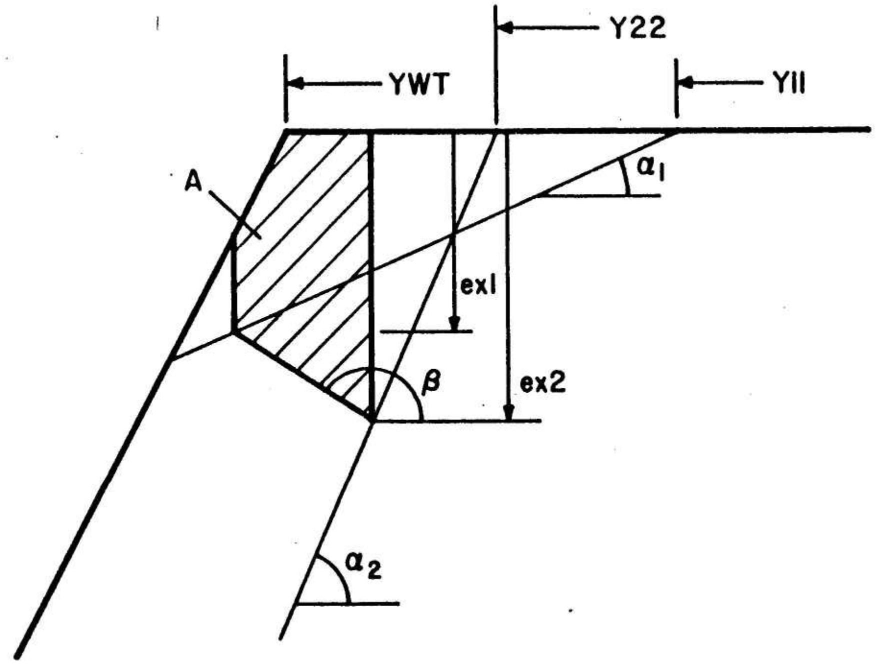
FIGURE 2.39 WEIGHT CALCULATIONS: SLICE TO THE LEFT OF SLOPE APEX
$\beta > 90$ $\alpha_2 < 90$ TRANSITION FROM SET 2 → 1



$$A = \frac{1}{2}(ex2 - ex1)(ex2/\tan\alpha_2 + Y22 - Y11 - ex1/\tan\alpha_1) - \frac{1}{2}\tan\theta\,(ex2/\tan\alpha_2 + Y22 - YWT)^2$$

$$\beta = \tan^{-1}\left[(ex2 - ex1)/\mathrm{abs}(ex2/\tan\alpha_2 + Y22 - Y11 - ex1/\tan\alpha_1)\right]$$

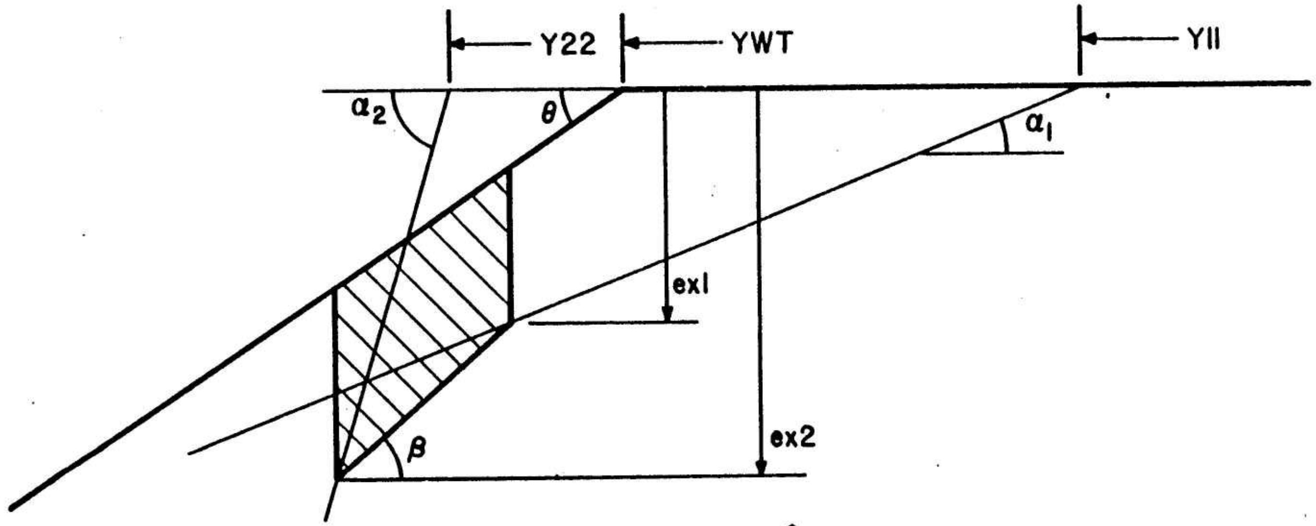
FIGURE 2.40 WEIGHT CALCULATIONS: SLICE BENEATH SLOPE APEX $\beta<90$
$\alpha_2>90$ TRANSITION FROM SET $2 \rightarrow 1$



$$A = \mathrm{abs}\left[\frac{1}{2}(ex2+ex1)(ex2/\tan\alpha_2 + Y22 - Y11 - ex1/\tan\alpha_1) + \frac{1}{2}\tan\theta\,(ex1/\tan\alpha_1 + Y11 - YWT)^2\right]$$

$$\beta = 180 - \tan^{-1}\left[(ex2-ex1)/\mathrm{abs}(ex2/\tan\alpha_2 + Y22 - Y11 - ex1/\tan\alpha_1)\right]$$

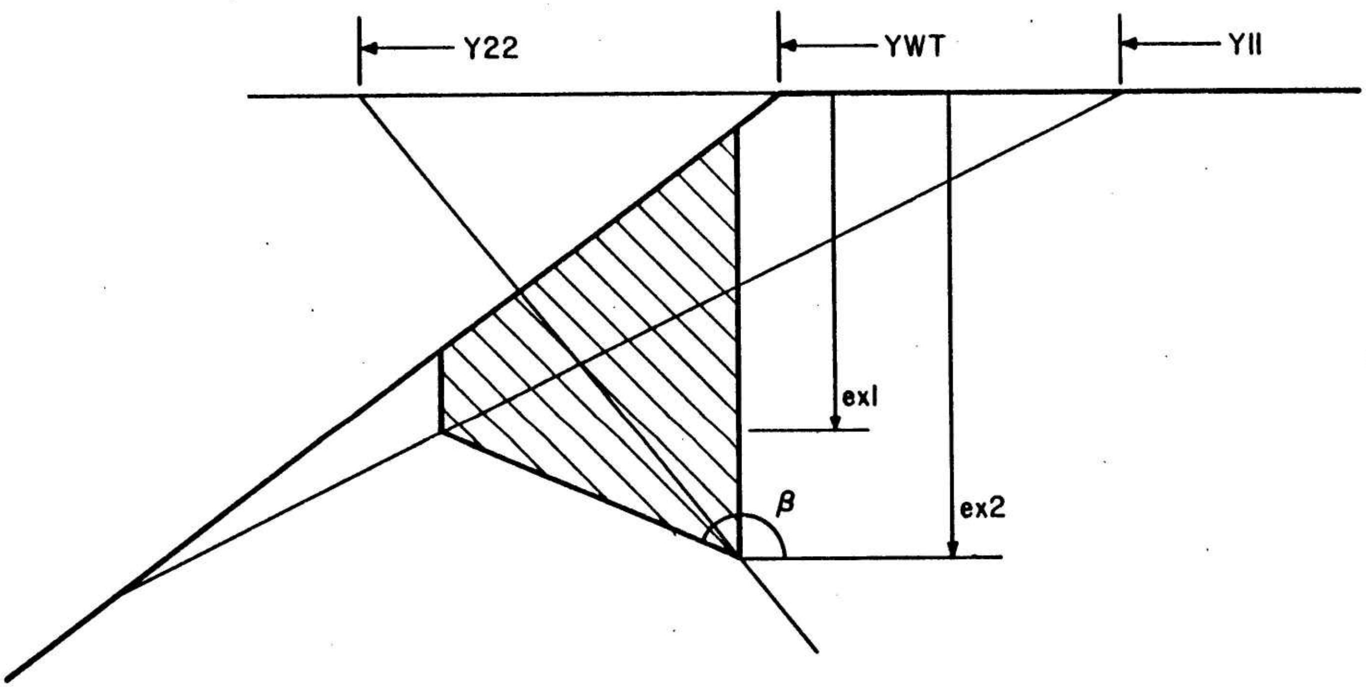
FIGURE 2.41 WEIGHT CALCULATIONS: SLICE BENEATH SLOPE APEX $\beta > 90$ $\alpha_2 < 90$ TRANSITION FROM SET $2 \rightarrow 1$



$$A=\frac{1}{2}(ex1+ex2)(ex2/\tan\alpha_2+Y22-Y11-ex1/\tan\alpha_1)-\frac{1}{2}\tan\theta\Big[(ex2/\tan\alpha_2+Y22-Y11)^2 - (ex1/\tan\alpha_1+Y11-YWT)^2\Big]$$

$$\beta=\tan^{-1}\Big[(ex2-ex1)/(ex2/\tan\alpha_2+Y22-Y11-ex1/\tan\alpha_1)\Big]$$

FIGURE 2.42 WEIGHT CALCULATIONS: SLICE TO THE LEFT OF SLOPE APEX $\beta<90$ $\alpha_2<90$ TRANSITION FROM SET 2 → 1



$$A = \frac{1}{2}(ex1+ex2)(ex2/\tan\alpha_2 + Y22 - Y11 - ex1/\tan\alpha_1) - \frac{1}{2}\tan\theta\left[(ex2/\tan\alpha_2 + Y22 - YWT)^2 - (ex1/\tan\alpha_1 + Y11 - YWT)^2\right]$$

$$\beta = 180 - \tan^{-1}\left[(ex2 - ex1)/\mathrm{abs}(ex2/\tan\alpha_2 + Y22 - Y11 - ex1/\tan\alpha_1)\right]$$

FIGURE 2.43 WEIGHT CALCULATIONS: SLICE TO THE LEFT OF SLOPE APEX
$\beta > 90$ $\alpha_2 < 90$ TRANSITION FROM SET 2 → 1

CHAPTER 3

THE PROBABILISTIC MODEL - COMPUTER PROGRAMS

3.1 Introduction

An analysis, of the stability of slopes with two distinct joint sets is performed through the probabilistic model. The program is a modification of O'Reilly's (1980) model which was developed to handle the situation with a single set of joints. Similar to O'Reilly's model, "Talal" is a Monte Carlo simulation of the rock slope being examined in this thesis. In each realization, jointing patterns are generated stochastically based on distributions of the joint parameters (i.e., joint length - set 1, rock bridge length - set 1, joint plane spacing - set 1, joint length - set 2, rock bridge length - set 2, joint plane spacing - set 2) compiled from previously collected data of joint surveys. For each realization, the program finds for every joint plane of the first set exiting on the slope face, a "critical path" beginning at the slope face, passing through the slope till the free surface of which the safety margin SM (the difference between the sums of resisting and driving forces) is a minimum. Critical paths may be planar or may involve transitions to an overlying joint plane. Transitions, between a joint plane of the first set and points in the second joint set, may take place only in that region of the slope above the joint plane of the first set (See Fig. 2.19 - 2.43).

The program divides the slope into intervals of equal height (see Fig. 3.1). In each realization the program stores the safety margins of all critical paths that fall within such an interval. Together, the numerous individual SM values form distributions within each interval. After the last realization takes place, the mean and standard deviation of SM for each interval are calculated. Also found for each interval is the probability of joint plane failure $(P_F)_i$ given by:

$$(P_F)_i = N_{Fi} \mid N_{Ti} \qquad \text{(Eq. 3.1)}$$

where (N_{Fi}); is the number of critical paths in the i-th height interval for which SM $\leq$ 0, while $(N_T)_i$ is the total number of critical paths in the interval. By independently evaluating $(P_F)_i$ and the distribution of SM for each interval, the program is capable of evaluating reliability as a function of slope depth.

3.2 Programming

Stochastic Generation of Joint Geometry

Three fixed input parameters define the geometry of the slope to be analyzed, the slope height (ydim), the slope angle (θ) and the inclination of the first joint set (α 1). (see Fig. 3.2). The upper free surface is always taken to be horizontal. Joint patterns for each joint set, i.e., joint plane spacing, length of joint segments and length of rock-bridges between adjacent joint segments in a given joint plane, are expressed as exponential distributions about mean

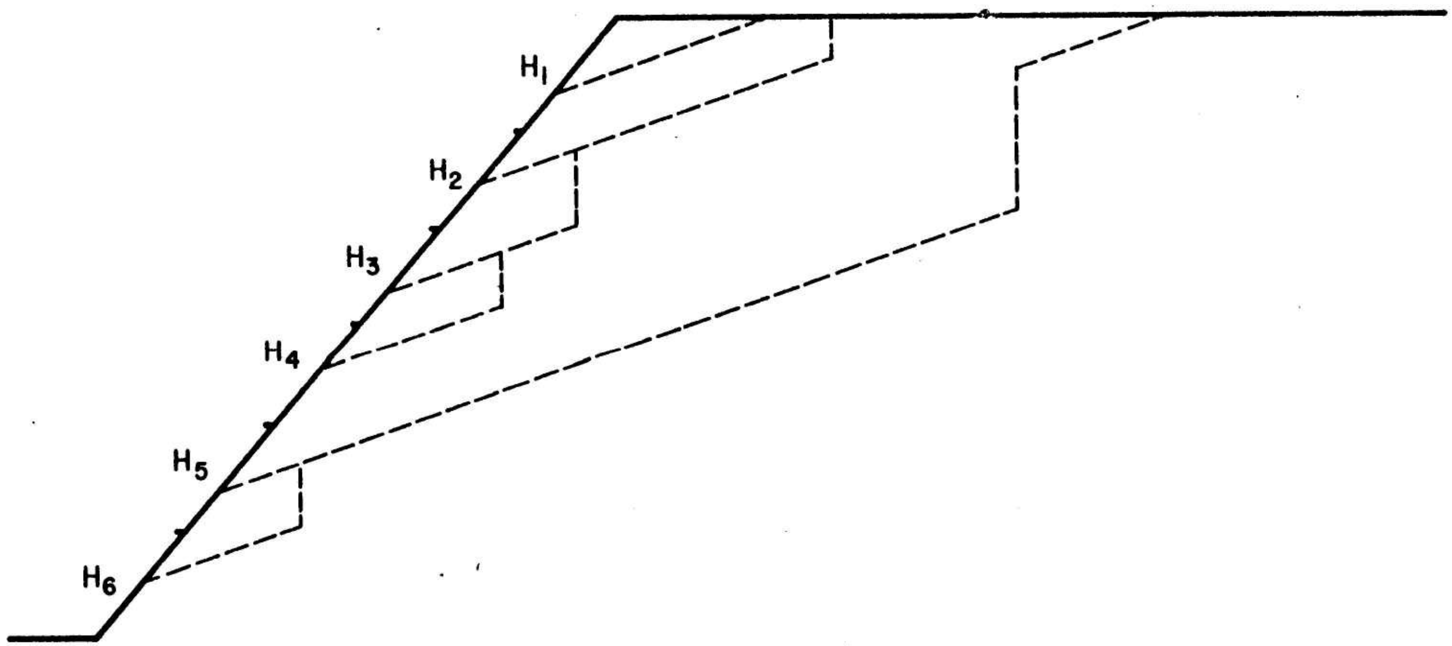


FIGURE 3.1 HEIGHT INTERVALS H_i

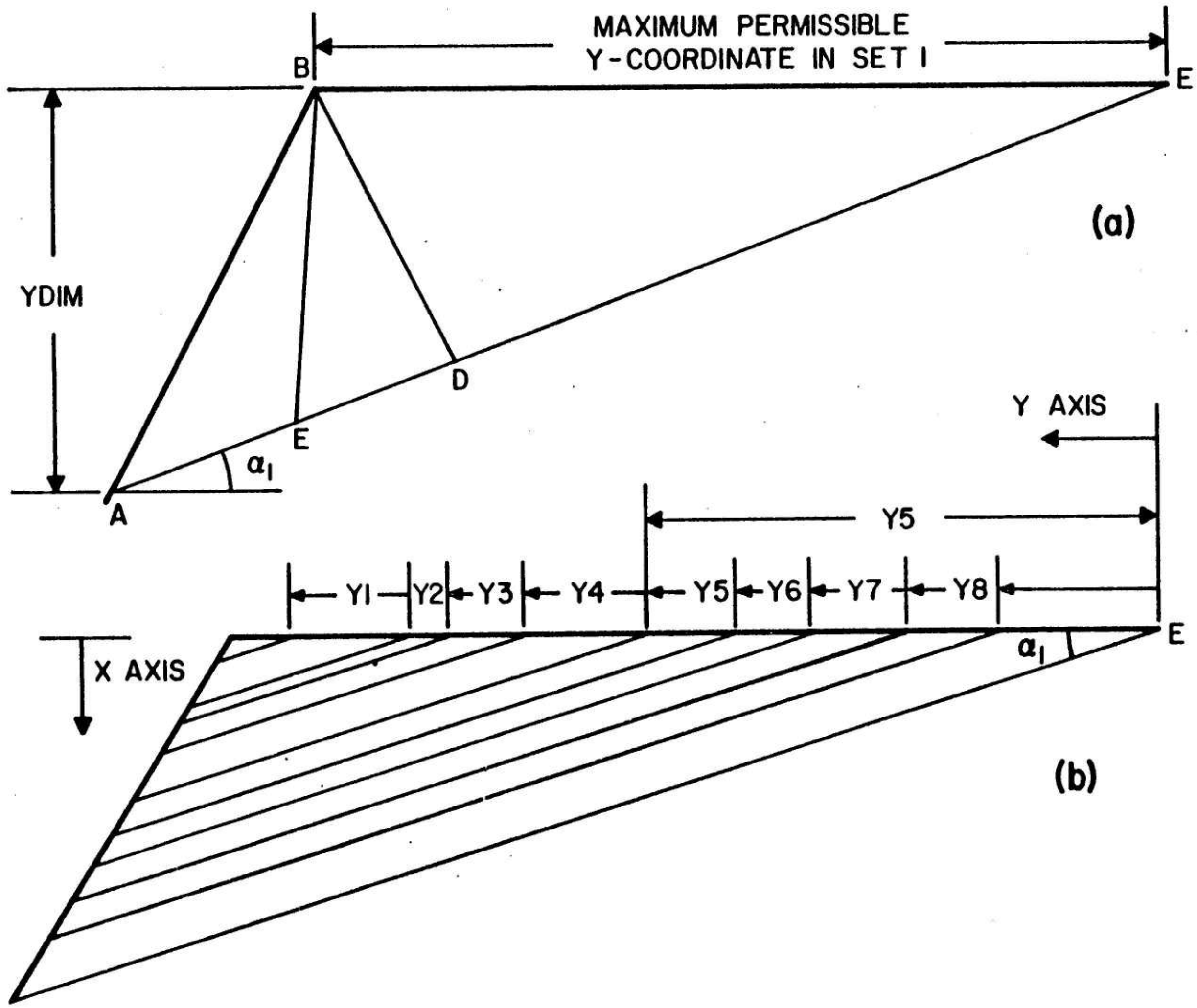


FIGURE 3.2 STOCHASTIC GENERATION OF JOINT PLANES, SET I

values; the latter specified by the user. The distribution of jointing patterns is simulated by the number of realizations (the more the realizations, the better the simulation). Generation of joint planes in each realization for joint set 1 begins at the slope apex and works its way back towards point E (Fig. 3.2b), the limit to generating any additional joint planes. This procedure is the same as the one utilized by O'Reilly (1980), where generation begins at the slope apex and works until the exit point of the i-th joint plane exceeds slope depth. The difference in the two procedures is in the storage method and axes used. Maximum permissible Y - coordinate in set 1, (ywt) is given by: (Fig. 3.2).

$$ywt = (ydim/\sin\alpha 1) - (ydim/\tan\theta)$$

values of Y (i) are generated until Y (i) reaches a value less or equal to zero. Joint planes with a negative Y (i) are not considered since they do not exit on the exposed slope face (see Fig. 3.2).

Next, joint segments are generated while assuming the exponential distribution of joint segment and rock bridge lengths. The location of each joint tip, in a given joint plane, is determined by its depth cjoint 1 (i,j) below the upper free surface (see Fig.3.3). The orientation of each joint plane is fixed and any point can be defined in terms of the joint plane it belongs to and an x - coordinate. For example, a cjoint 1 (3,2) equal to 10 stands for the second coordinate point of the third joint plane in the first joint set as measured from the free surface and equals 10 units of length.

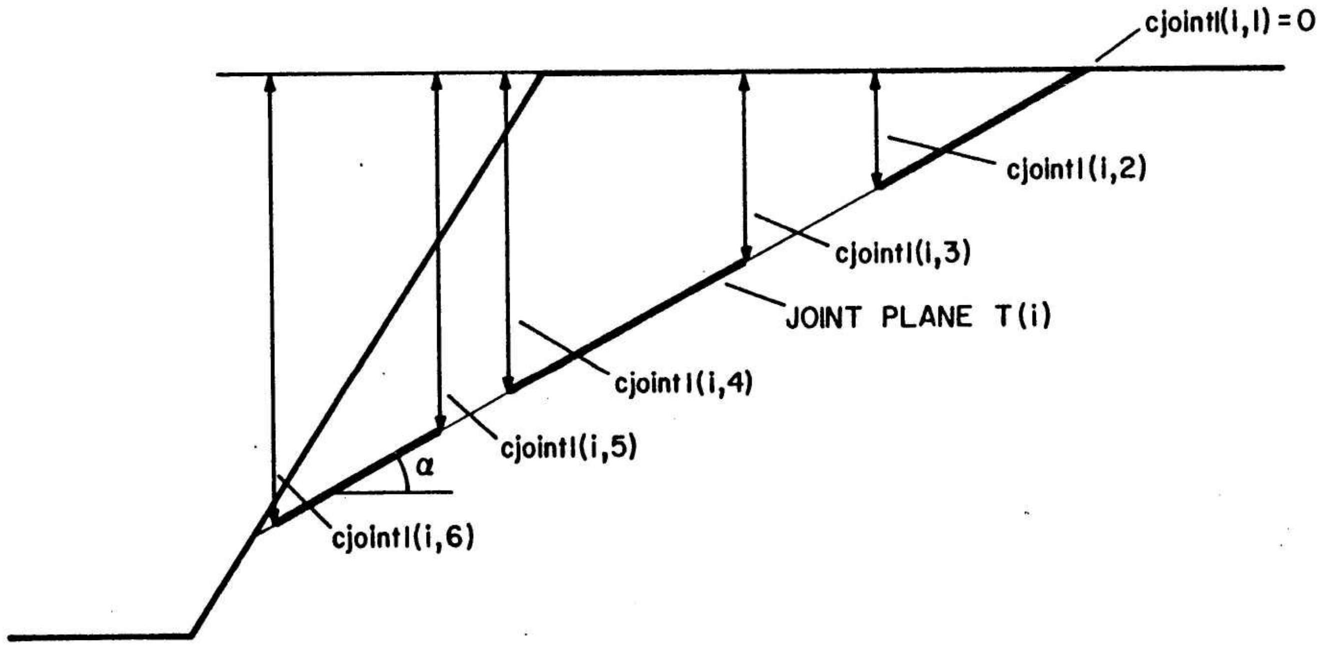
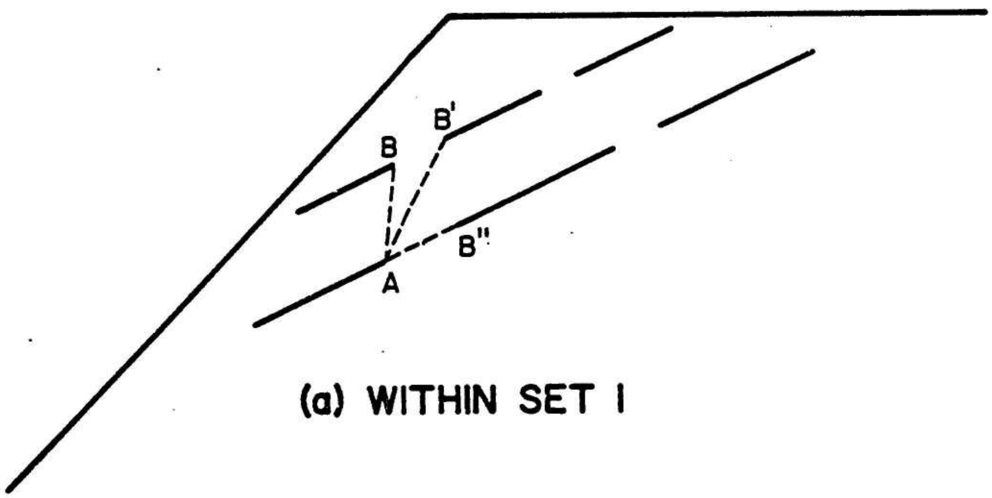
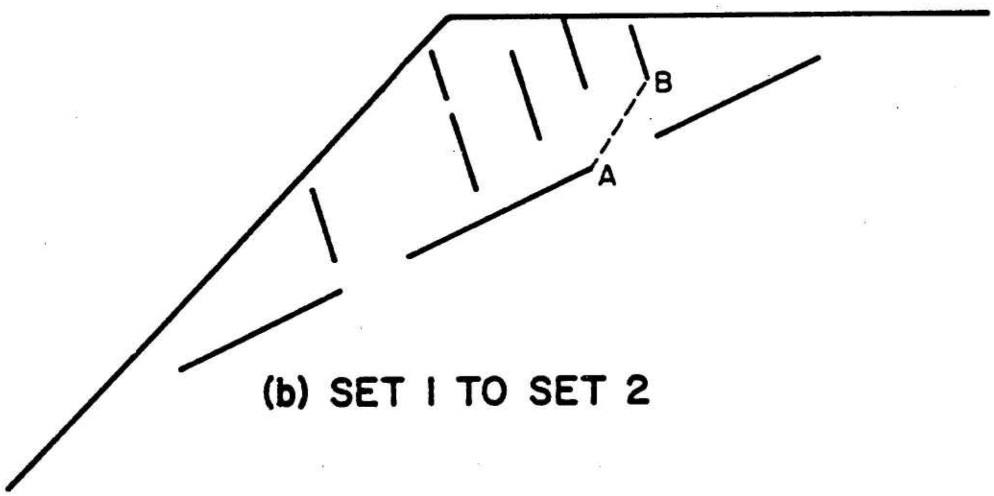


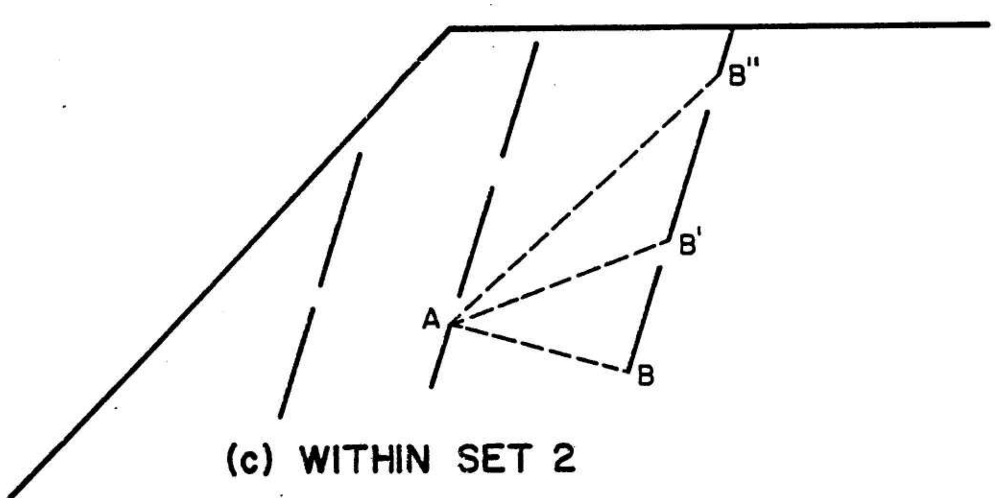
FIGURE 3.3 STOCHASTIC GENERATION OF JOINT SEGMENTS OF JOINT SET I



(a) WITHIN SET 1



(b) SET 1 TO SET 2



(c) WITHIN SET 2

FIGURE 3.4 JOINT TIPS AS "NODAL" POINTS

The coordinate points on a typical first set joint plane are generated by the program and are referred to as the dynamic programming points. These points are compiled from three data sources. The first source being the set of points on a plane which define the x-coordinates of the intersection of that plane with the free surface and slope face and all the x-coordinates of the joint segments on that plane (See Fig. 3.12a). The second source being the set of points of intersection of the joint plane with lines drawn from the right hand tips of joints of underlying planes at ($45° + \alpha 1$) (See Fig. 3.12b). Finally, the third source being the set of points of intersection of the joint plane with planes of the second set (See Fig. 3.12c). Figure 3.12d shows all points superimposed on the plane being examined. Dynamic programming plane points serve as potential transition nodes. Transition may take place in plane or out of plane to a point above the plane which contains the dynamic programming plane point. Transitions from a point on a first set joint plane to a point anywhere below that plane are not permissible (see Fig. 3.13a).

Angles of transition within the second set are never less than the first joint set inclination ($\alpha 1$), nor greater than the greater of either 180° or the sum of the second set inclination ($\alpha 2$) and 90° (See Fig. 3.14a). Transitions within the second set are only permissible between two adjacent planes (See Fig. 3.14b).

Restrictions concerning the inclination of paths between the two joint sets are the same as those for paths within the first joint set. Generally, a line segment connecting nodes on the critical path must be between the angle of the first set inclination ($\alpha 1$), and the sum of the first set inclination ($\alpha 1$) plus 45° except for transitions within the second set (See Fig. 3.15).

Some of the dynamic programming plane points and some of the second set points can only be ends of a transition path within the plane they are in. Such points are left ends of discontinuities of both sets and points within the first set of discontinuities that are not intersection points with the second set (See Fig. 3.15).

In each realization, the number of intersection points of each first set plane with second set planes is stored. This number can be imagined to represent traces of possible critical paths. As expected, a critical path would most probably follow the paths cutting through the second set joints as can be seen from the lines connecting nodes 1, 2, 3 and 4 in Figure 3.16b. A look at Figure 3.7 may give some more insight into the effect of a jointed region bounded by a first set plane and either the free surface or another first set plane. In Figure 3.7, Region A is unjointed, hence the intersection point there is irrelevant. However, Region C, in the same figure, is partially jointed and thus one would expect a transition through that region. However, Region B is fully jointed and will act as a path between planes 1 and 2 as it is the weakest possible transition from 1 to 2 (e.g., path; b-ii-i).

The program establishes the critical paths by finding the lowest possible safety margins between every node and the free surface.

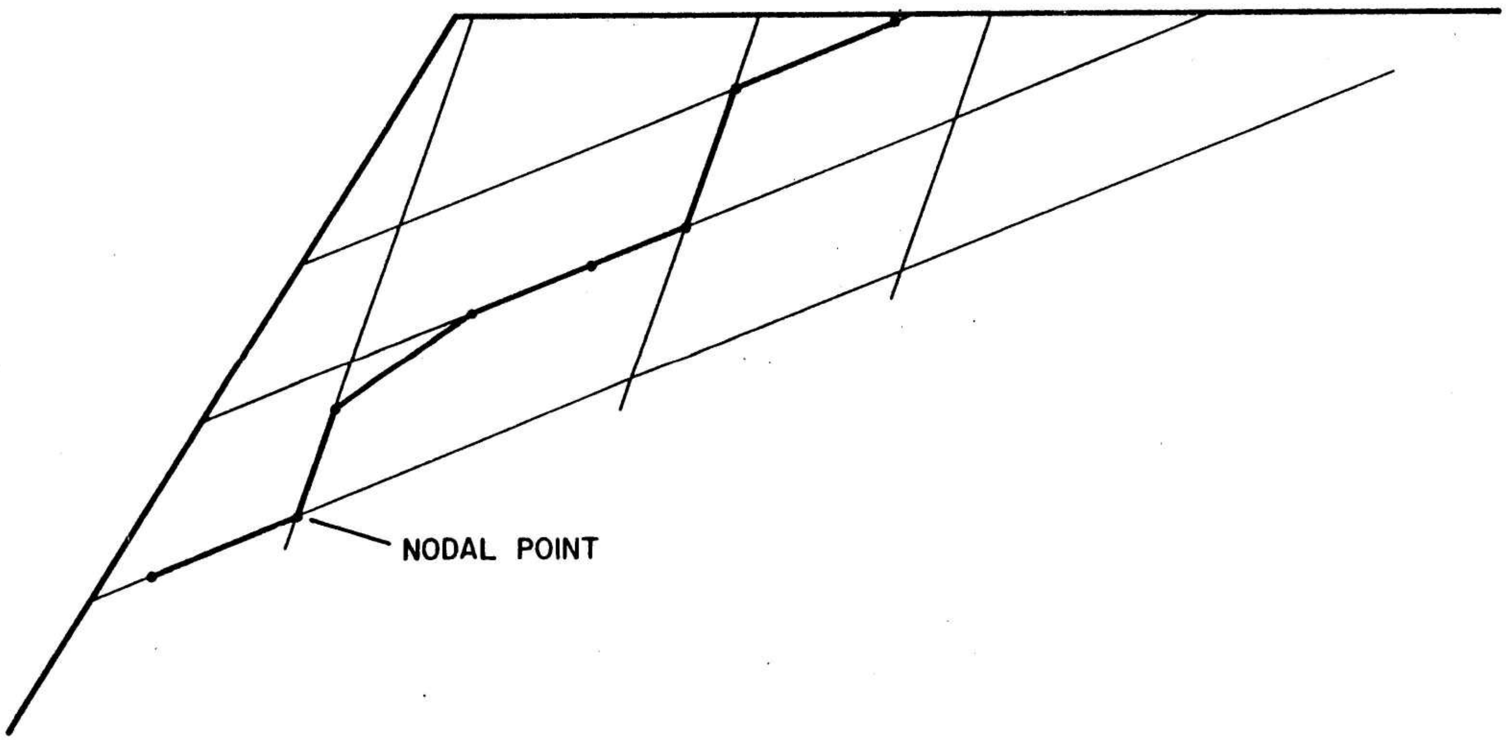


FIGURE 3.5 CRITICAL PATH COMPOSED OF NODAL POINTS

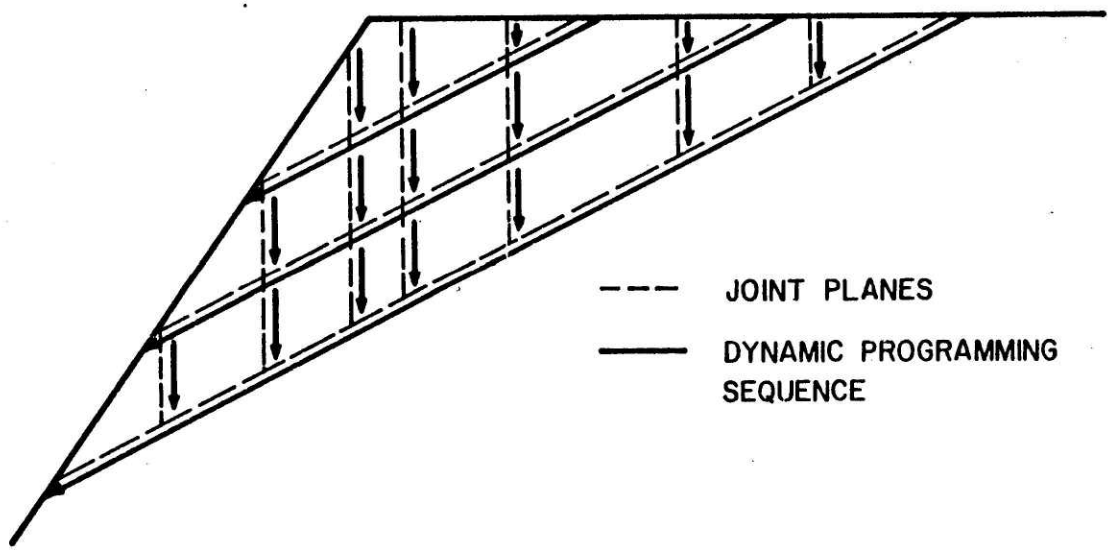


FIGURE 3.6 SEQUENCE IN WHICH SAFETY MARGINS OF "NODES" ARE COMPUTED

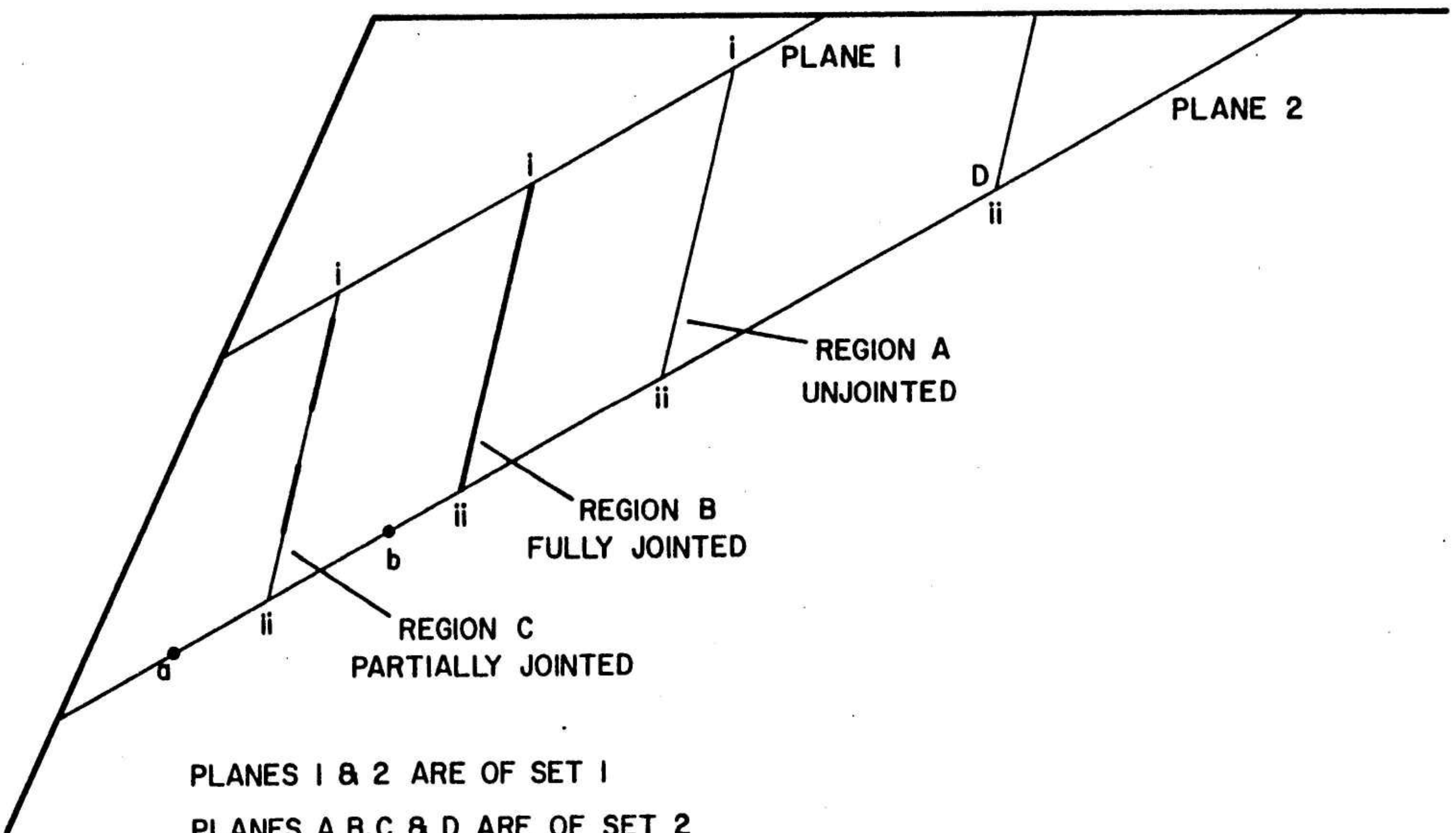


FIGURE 3.7 POINTS OF INTERSECTION

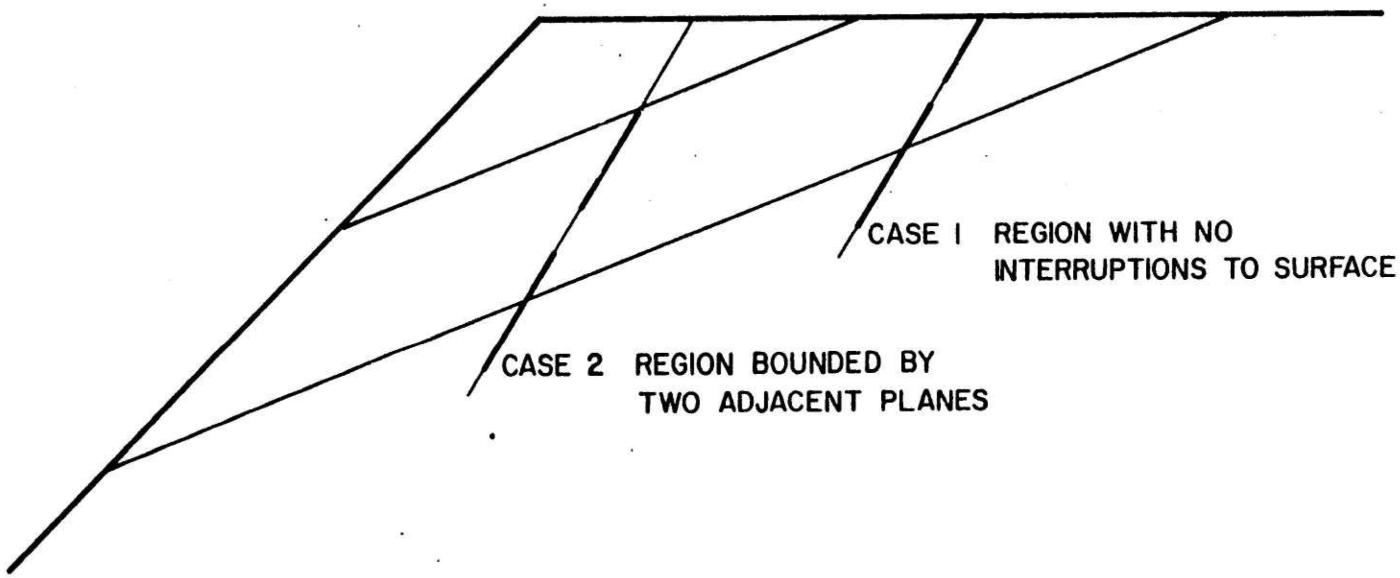


FIGURE 3.8 CASES OF TYPES OF BOUNDING REGIONS

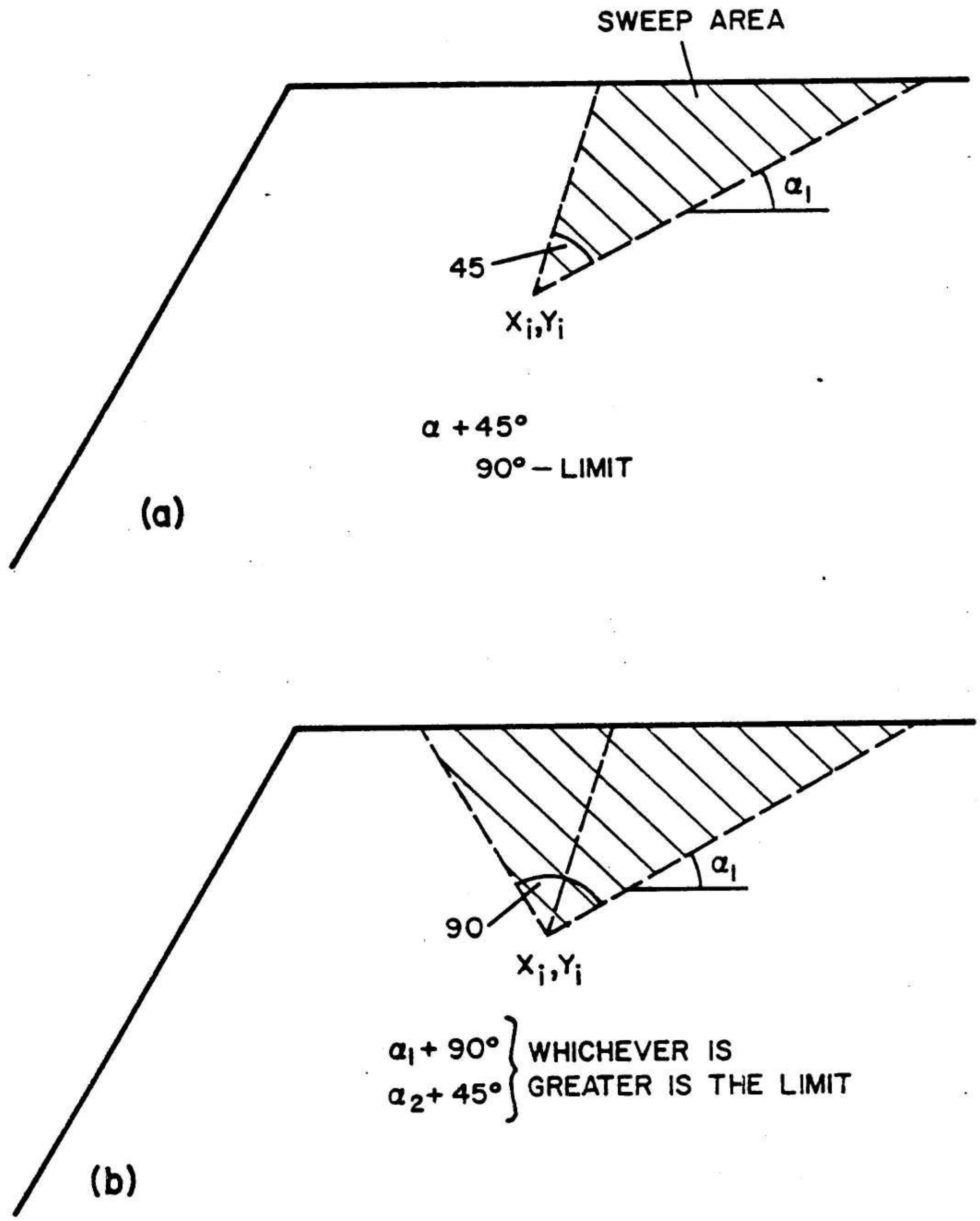


FIGURE 3.9 JOINT SET 1,2 EXAMPLE SWEEPS ARE FOR ARBITRARY NODAL POINT (X_i, Y_i)

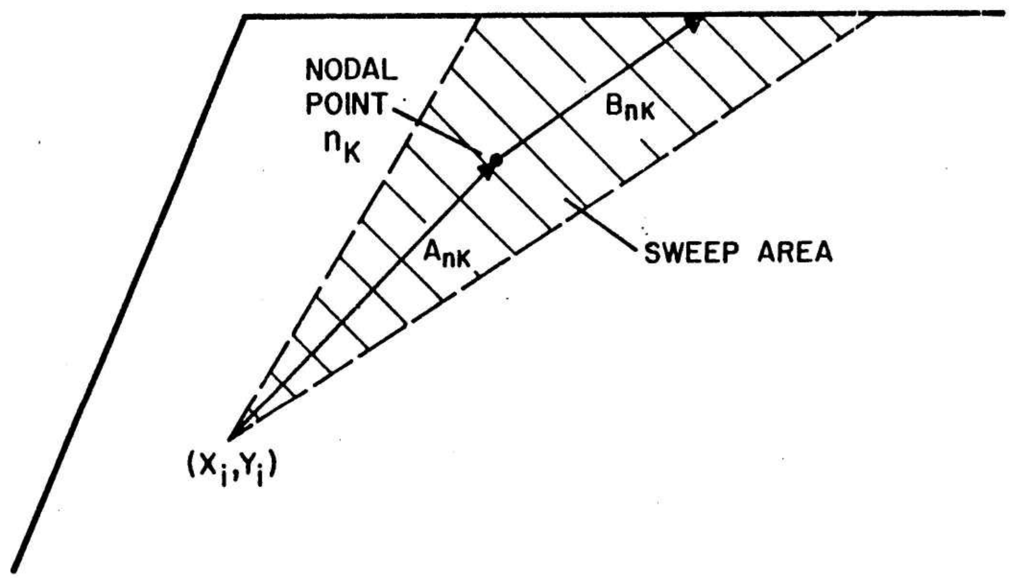


FIGURE 3.10 SAFETY MARGIN COMPONENTS (A_{nK}, B_{nK})

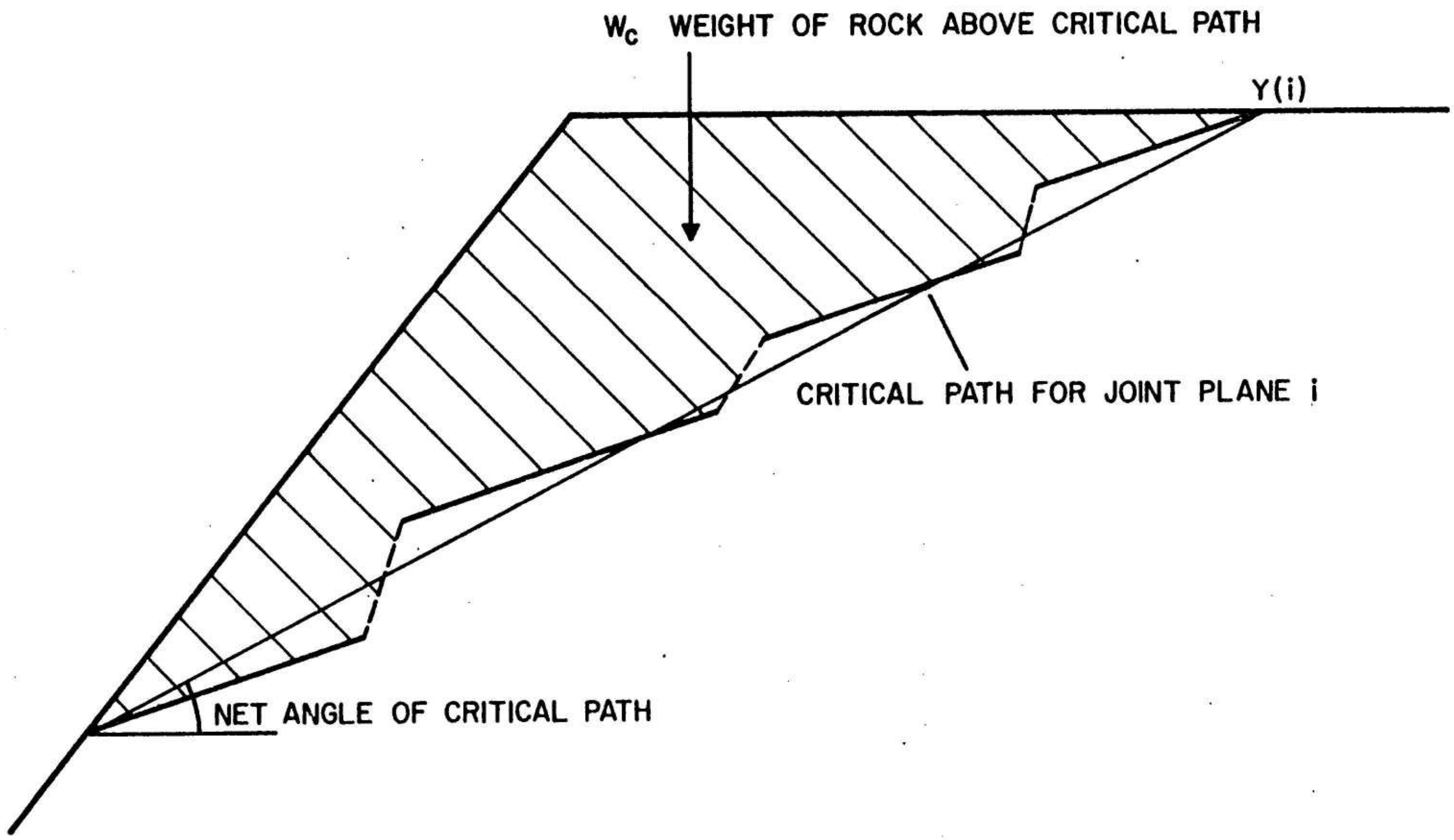


FIGURE 3.11 GEOMETRY OF CRITICAL PATH

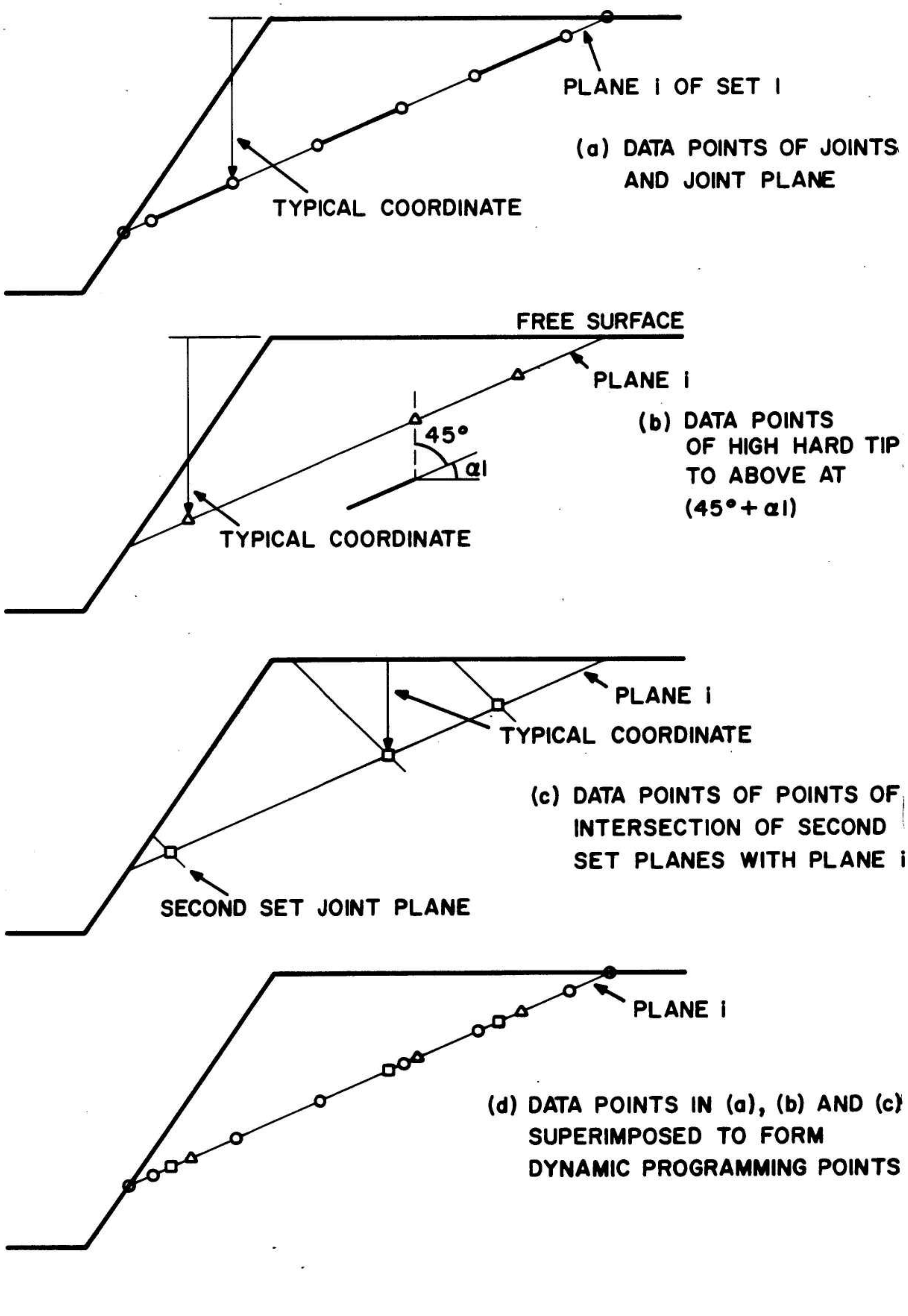
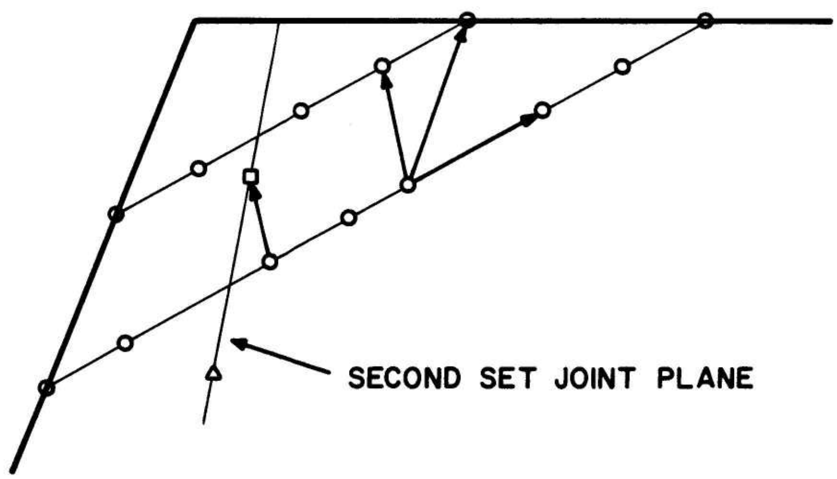
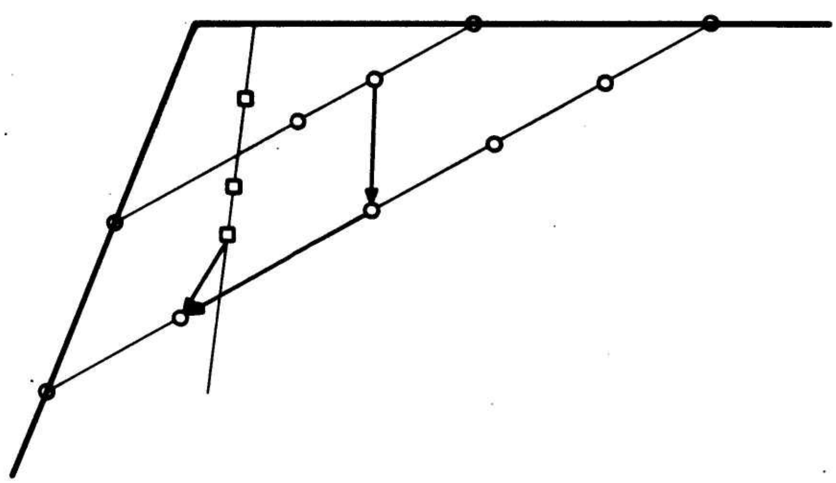


FIGURE 3.12 DYNAMIC PROGRAMMING PLANE POINTS



(a) TYPICAL ROCK SLOPE WITH JOINT PLANES - DYNAMIC PROGRAMMING POINTS AND PERMISSIBLE TRANSITIONS SHOWN



(b) TYPICAL ROCK SLOPE WITH JOINT PLANES - DYNAMIC PROGRAMMING POINTS AND NON-PERMISSIBLE TRANSITIONS SHOWN

FIGURE 3.13 PERMISSIBLE AND NON-PERMISSIBLE TRANSITIONS FROM DYNAMIC PROGRAMMING PLANE POINTS

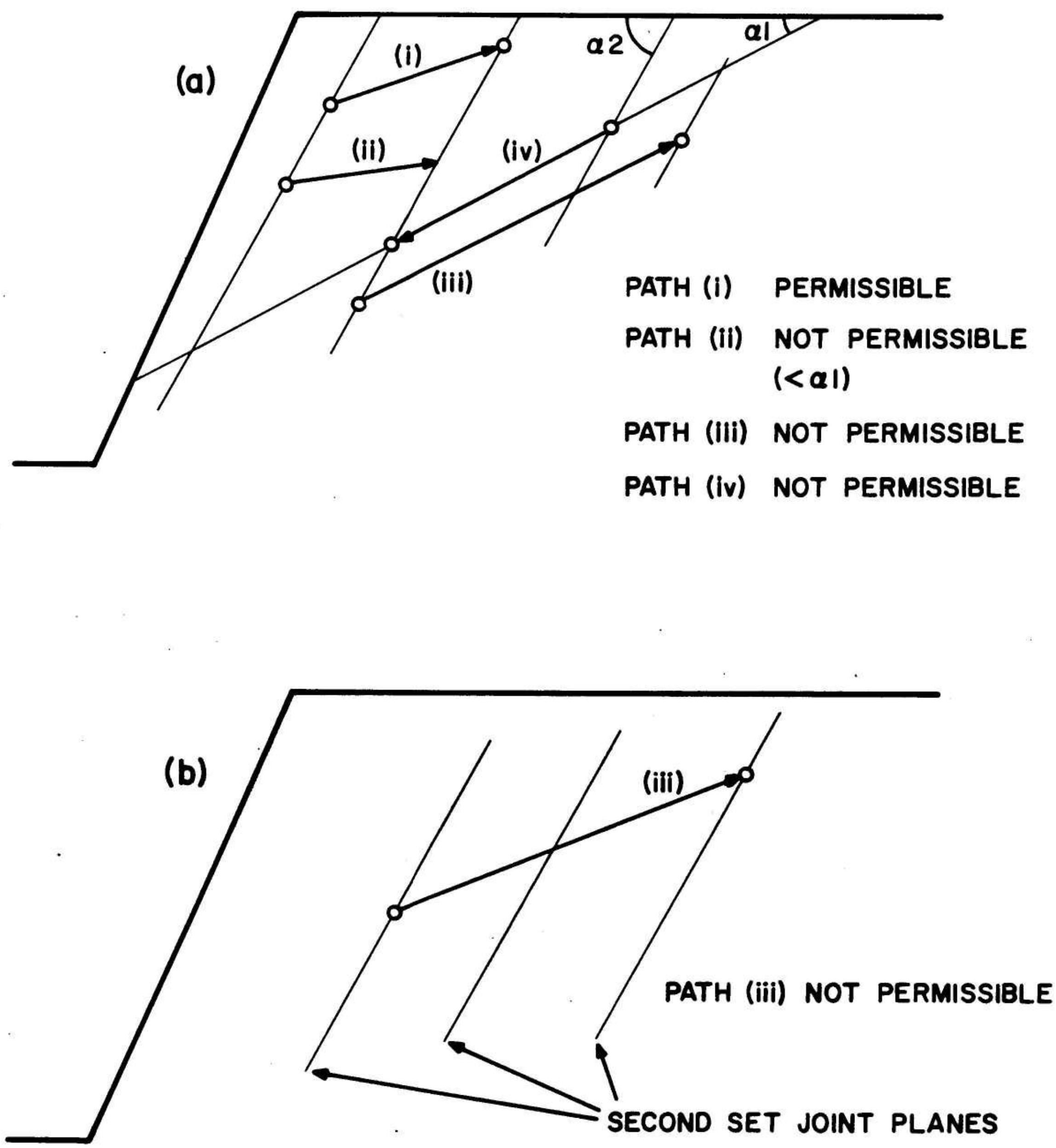


FIGURE 3.14 SECOND SET TRANSITIONS

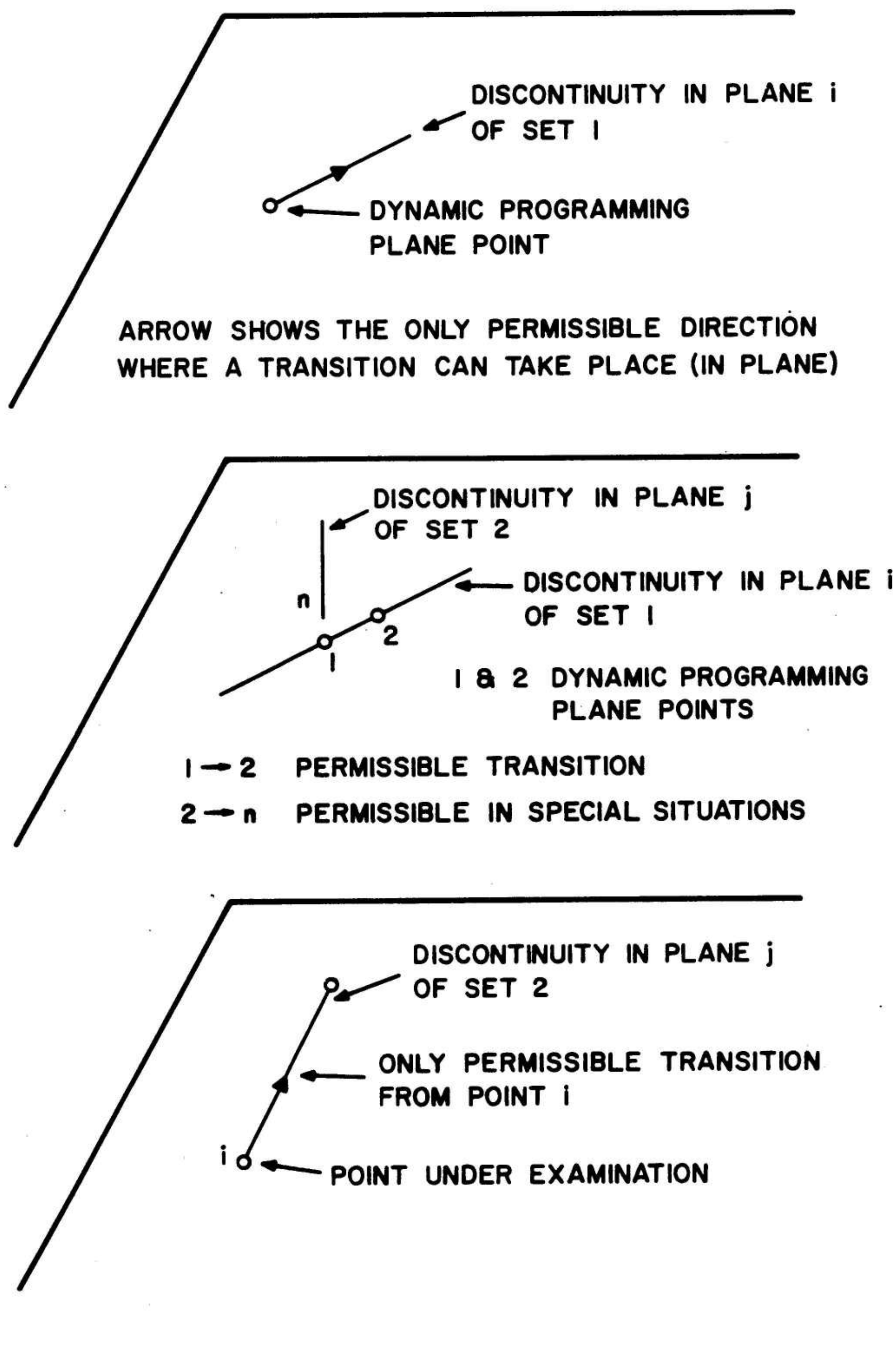


FIGURE 3.15 POSSIBLE ENDS OF TRANSITION PATHS

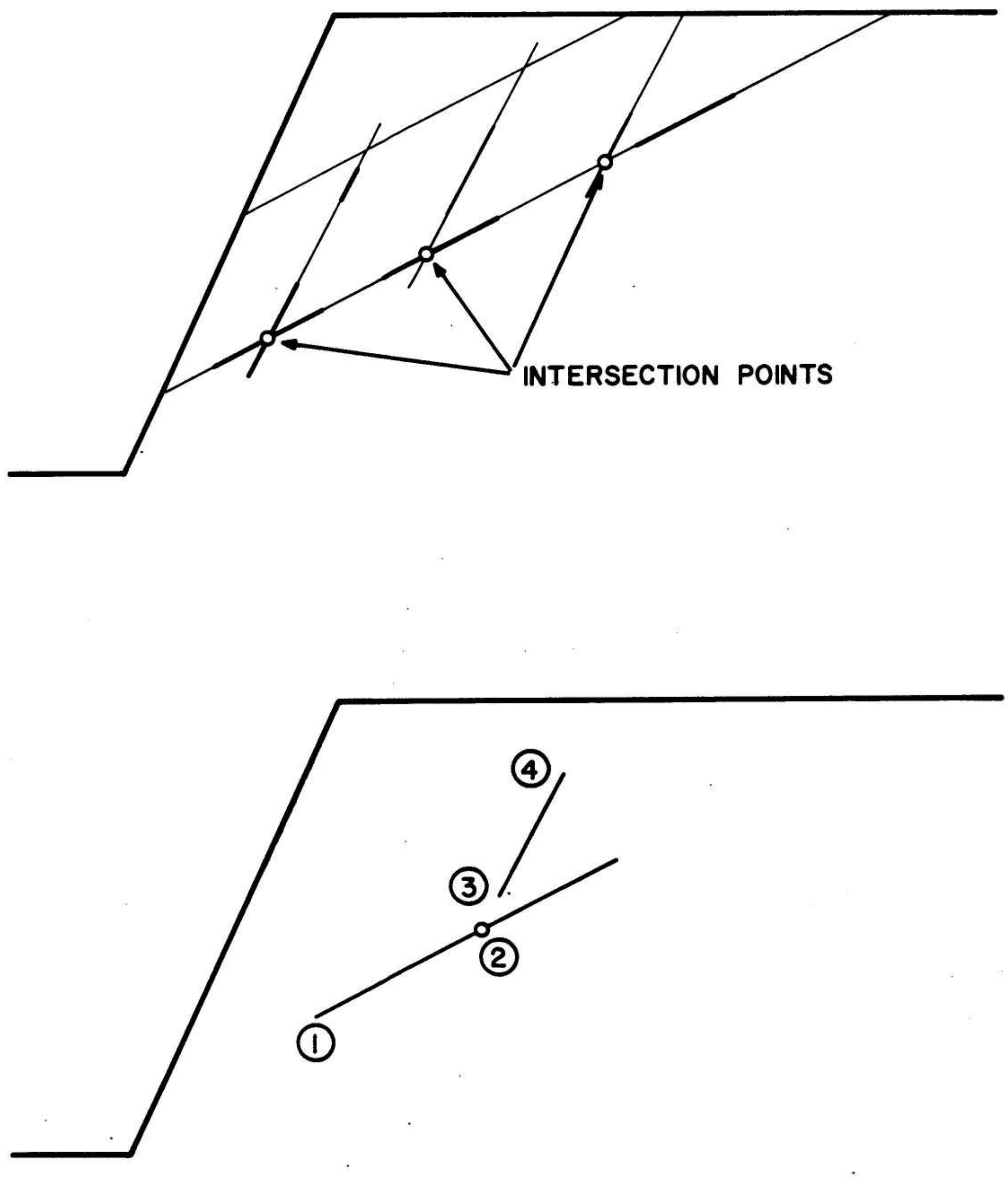
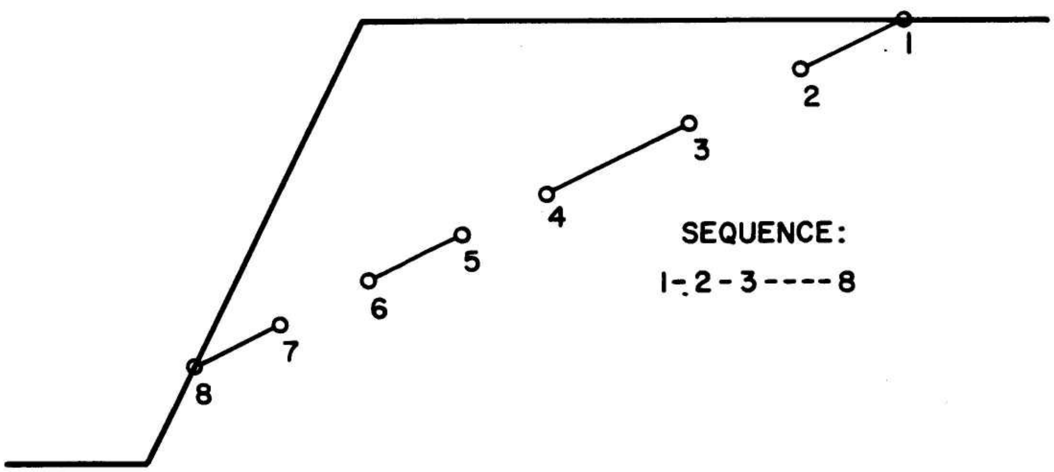
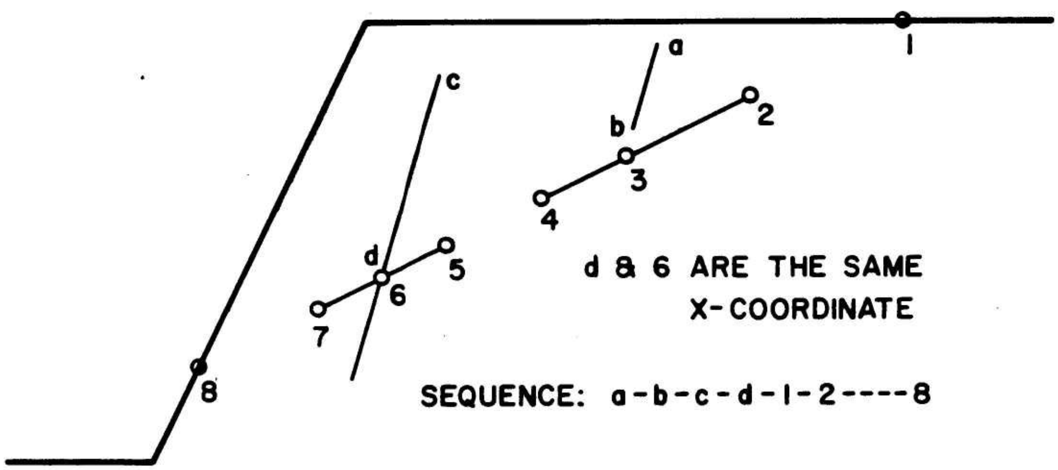


FIGURE 3.16 POINTS OF INTERSECTION ON SET ONE

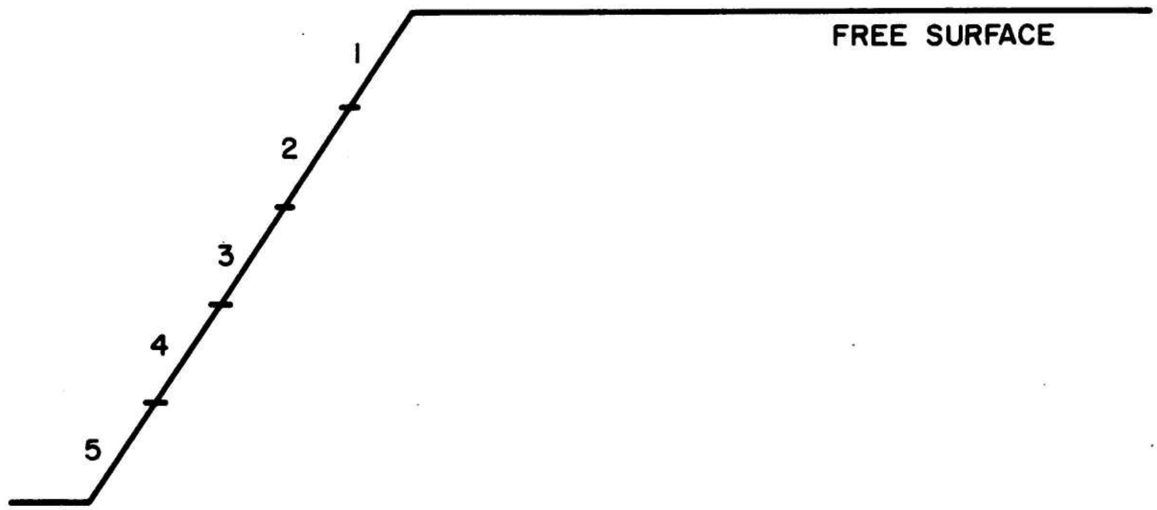


(a) JOINT PLANE WITH NO POINTS OF INTERSECTION WITH THE SECOND SET

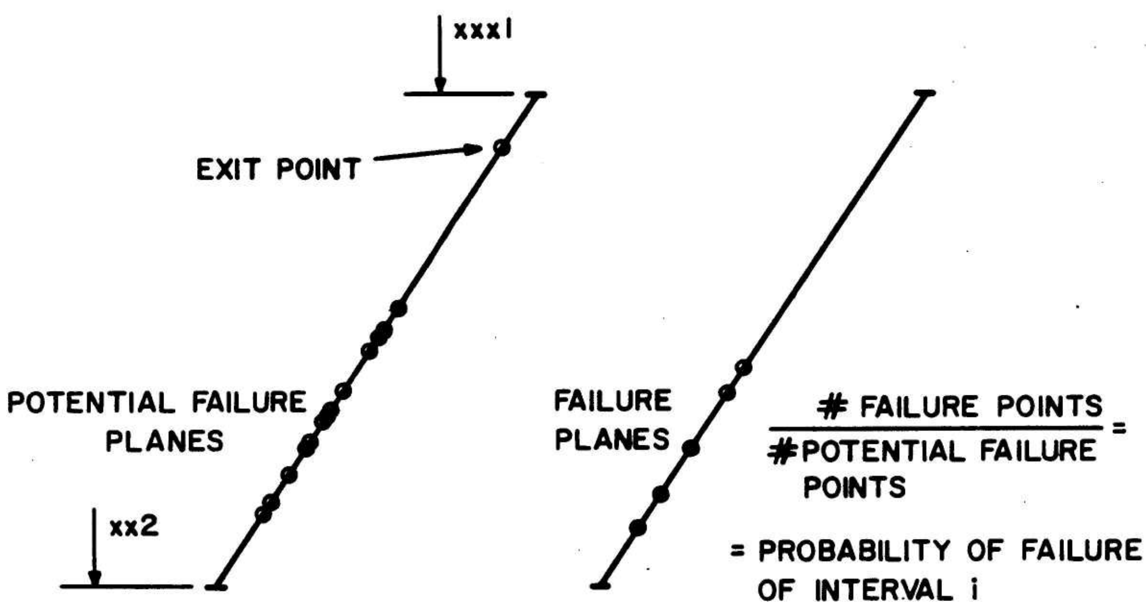


(b) JOINT PLANE WITH POINTS OF INTERSECTION WITH THE SECOND SET

FIGURE 3.17 SEQUENCE IN THE ALGORITHM



(a) EXAMPLE OF A SLOPE WITH A FACE DIVIDED INTO FIVE EQUAL PARTS



(b) TYPICAL INTERVAL WITH EXIT POINTS OF POTENTIAL FAILURE PLANES

FIGURE 3.18 SLOPE FACE INTERVALS AND THE PROBABILITY OF FAILURE

The algorithm begins with the top joint plane and evaluates the minimum safety margin of each dynamic programming plane point moving from the top (intersection of the joint plane with the free surface) to the last (intersection of the plane with the slope face) (See Figure 3.17a). If and when the plane has points of intersection with set 2, the program will start by evaluating minimum safety margins for points on the planes of set 2 that intersect set 1 (See Fig. 3.17b). These points should be in the region between the first set plane being analyzed and the free surface if those second set planes extend to the free surface (Case I) without intersecting other first set planes. Otherwise these points that lie in the region are bounded by the plane being analyzed and the first set plane above it (Case II) (See Fig. 3.8). Consequently, for any given node, the program checks possible transitions to overlying nodes within a given "sweep area" bounded by the kinematic restrictions imposed as described previously, depending on whether the point belongs to joint set 1 or set 2. The sweep area for a nodal point of set 1, point (X_i, Y_i), is shown in Fig. 3.9a; for set 2, point (X_i, Y_i), it is shown in Fig. 3.9b. The program computes the safety margin of transitions to all nodal points within this area using the mechanical model presented previously in Chapter 2. The order in which nodes are considered is always from shallow to deep joint planes, and down dip within each plane.

Referring to Figure 3.10, the safety margin SM of any path from (S_i, Y_i) to the free surface has two components (A_n, B_n), A_n is the safety margin from (S_i, Y_i) to a nodal point (n) while B_n is the safety margin from that nodal point to the free surface:

$$SM_n = A_n + B_n$$

thus, the minimum safety margin SM (i, j) for the point (X_i, Y_i) is the one for which the above sum is a minimum. In this manner the minimum safety margin for each nodal point within the slope is systematically found.

The safety margin, of the point of intersection of each joint plane of set 1 with the slope face, is the minimum safety margin for the path originating at that point - and rising to the free surface. The coordinates of nodal points of the path, yielding this minimum safety margin, determine the critical path for that joint plane. The program then calculates the weight of rock overlying the critical path as well as the net angle θ_c of the critical path (See Fig. 3.11). The critical path should not be considered a failure path unless the calculated safety margin is zero or less. In each realization, several critical paths could result, some of which may be failure path(s). For a number of realizations simulating a joint spacing and length distributions, one can obtain a distribution of the ratio of failure paths to critical for each interval on the slope face (See Fig. 3.18b).

3.3 Program Limitations

Before moving on to a detailed discussion of program input and output it is important to outline the limitations of the program in its present form.

(a) - The analysis is two dimensional.

- Joint persistence parallel to the strike of the slope is not considered (i.e., it is assumed to be 100% - a conservative assumption).
- Side wall resistance transverse to slope strike is assumed to be neglible. In other words, a better way to visualize the slope is to imagine that we have a model with the present dimensions and generated joint patterns, as described, 1-length unit thick in the third dimension, considering all joints to extend from one face to the other (See Fig. 3.19).

(b) - Joint and slope geometry is limited to that shown in Fig. 2.1 and 2.2 with joint set 1 always less than 90 degrees and the upper surface always horizontal.

(c) - As a Monte Carlo simulation program, the program is based on a deterministic resistance algorithm. Thus, the program is only a reliable as the deterministic algorithm presented in the previous chapter.

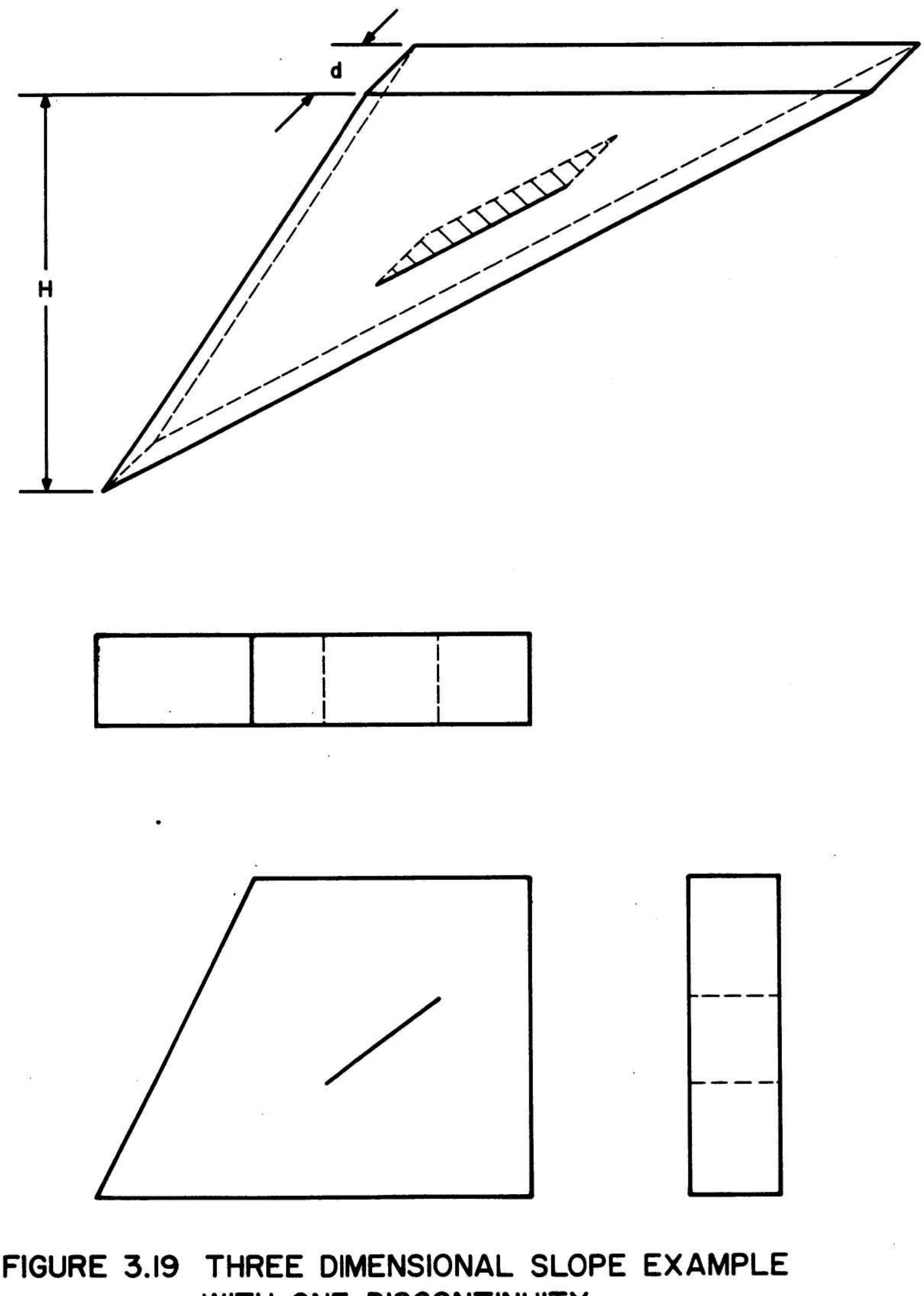


FIGURE 3.19 THREE DIMENSIONAL SLOPE EXAMPLE WITH ONE DISCONTINUITY

(d) - Application of the program depends on reliable measurement of joint geometry distribution parameters (i.e., mean values of joint length, rock bridge length and joint plane spacing).

The above limitations should be kept in mind specifically with regard to results presented in the parametric study. Data analyzed and results arrived at are mainly for research purposes until more refinements have been performed (e.g., three dimensional analysis - end conditions) and comparisons with field conditions and case histories have been thoroughly investigated.

CHAPTER 4
THE PROGRAM

4.1 Introduction

The purpose of this chapter is to portray the programming details of the model by discussing a sample program run. The main goals are to understand the capabilities and limitations of the program, and to establish an understanding for proper interpretation of the output data which is used in the sensitivity analysis in Chapter 5.

The sample run is described and shown on the next pages and is divided into two main parts. The first being the input data which is user specified. The second being the output generated by the program.

4.2 Sample Input

Since the program is implemented on an interactive system, the program will systematically "ask" for the required inputs. For example, if the computer asks for values of x, y, and z the program will print, "input, x,y,z,". The printer will start a new line and await the user to input the three values. After this is carried out the program moves to the next group of inputs "asking" for their values. This process continues until the user has specified all input values required.

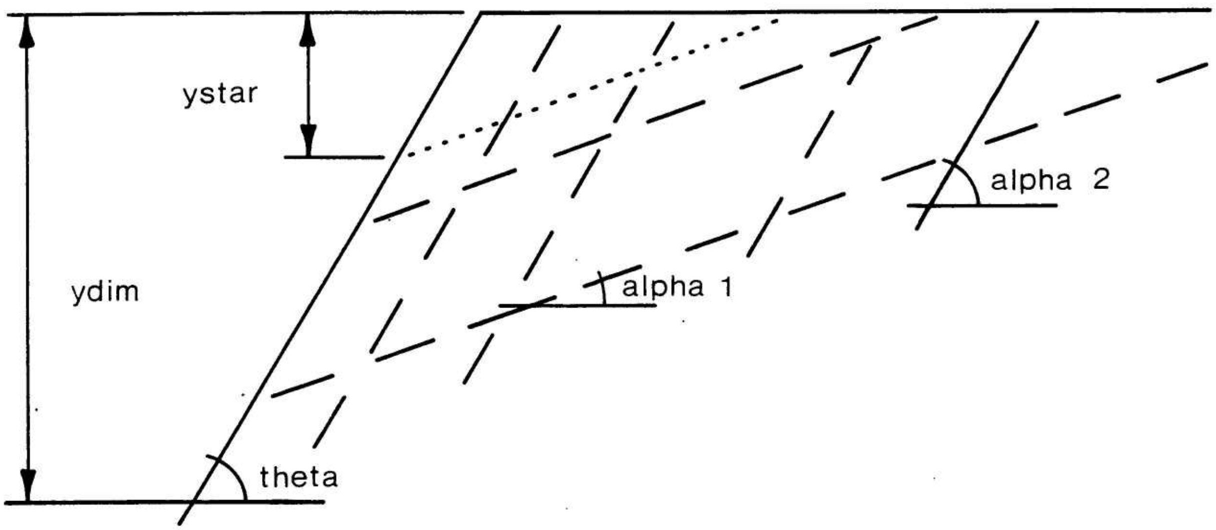


Figure 4.1 (a) Slope Geometry Parameters

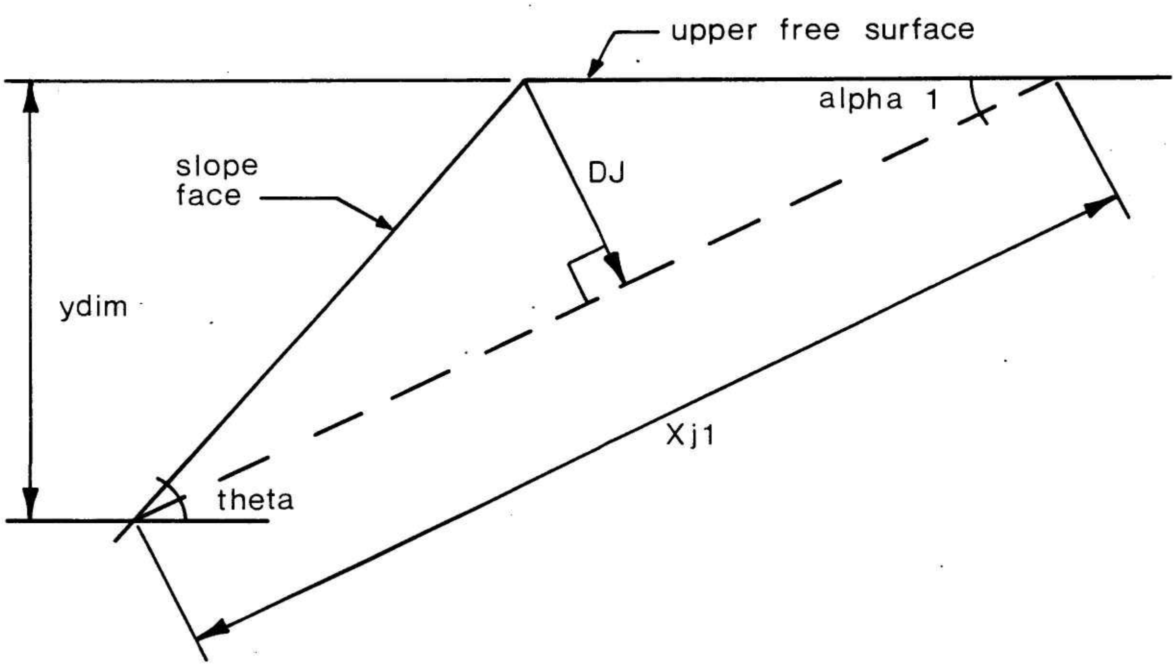


Figure 4.1 (b) Variables Defining Slope Geometry

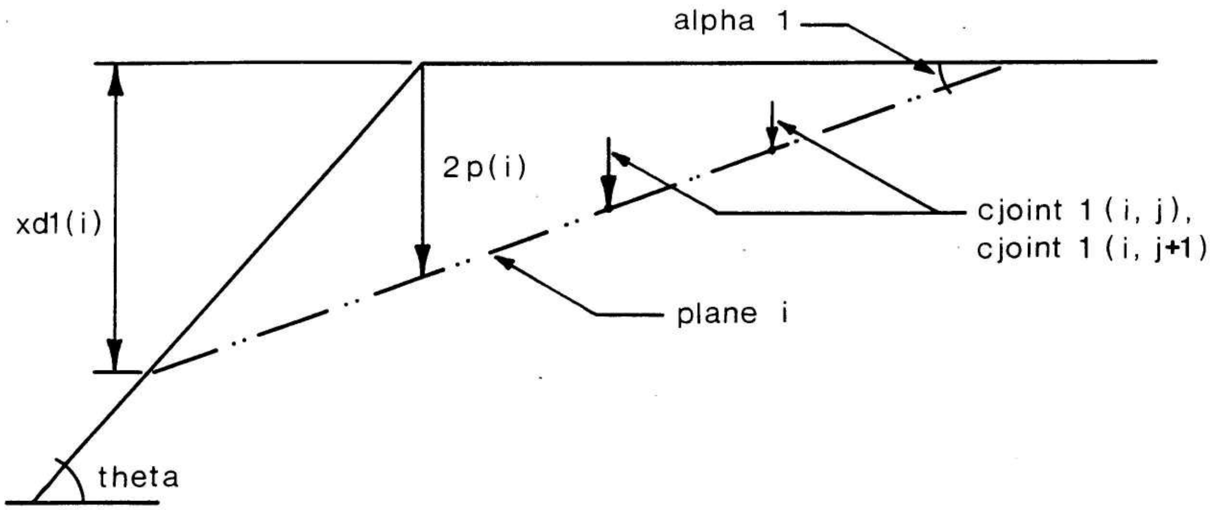


Figure 4.2 (a) Maximum Possible x-coordinate for Plane i of Set One

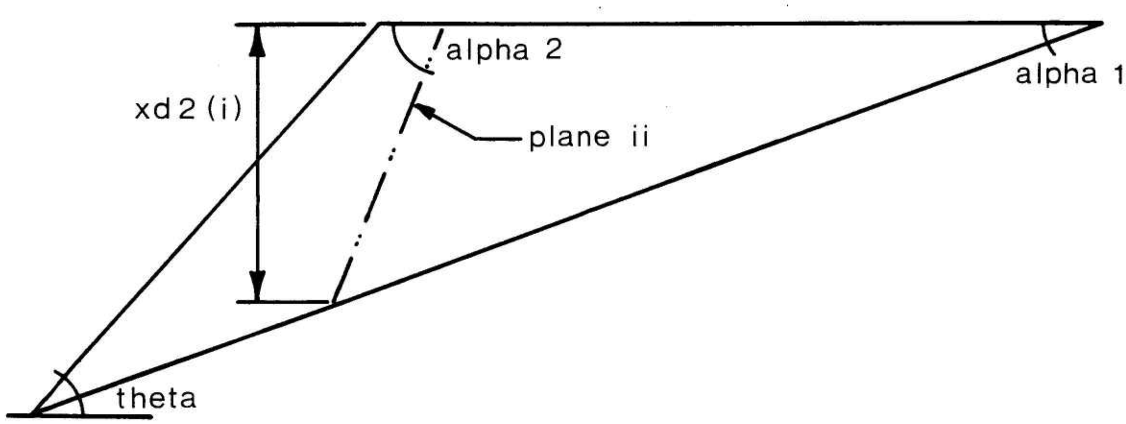
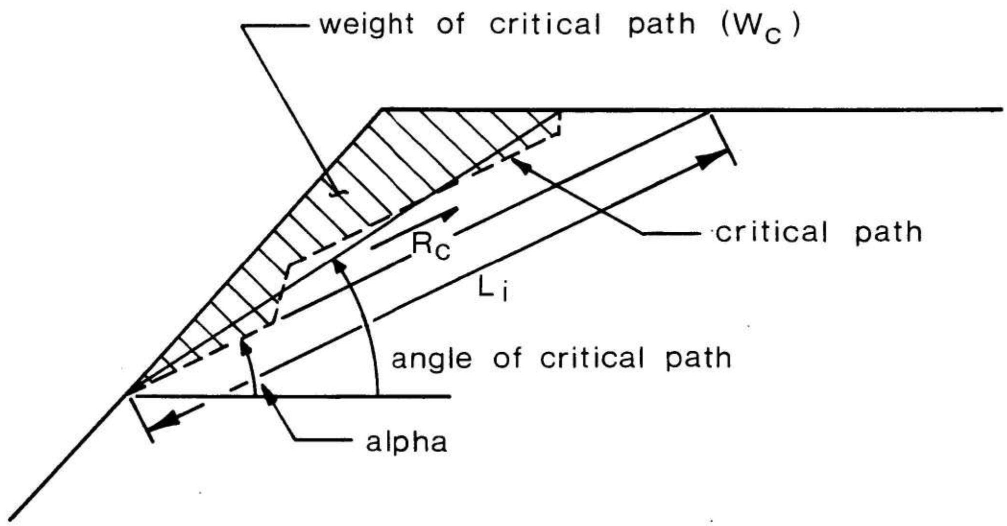


Figure 4.2 (b) Maximum Possible x-coordinate for Plane ii of Joint Set Two



$$SM_i = R_C - W_C \sin \text{alpha}$$

$$USM_i = SM_i / L_i$$

Figure 4.3 Critical Path for Joint Plane i

In this section each input variable will be defined. Along with the definition, permissible or recommended ranges as well as the specific values used in the sample run, will be given. As with any program, the user must always make sure that units are consistent.

theta

The angle in degrees of the slope face relative to the horizontal. It may vary between 0 and 90 degrees (See Fig. 4.4a).

alpha 1

The angle of joint planes of the first set relative to the horizontal. It is assumed that the range of alpha 1 is between zero degrees and 60 degrees, and less than theta (See Fig. 4.4a).

alpha 2

The angle of joint planes of the second set relative to the horizontal. To remain within program limitations, the range of values of alpha 2 should be greater than alpha 1 (a must) and less or equal to180 degrees. For the present, values not greater than 90 degrees are considered for alpha 2 (See. Fig. 4.4a).

ydim

The vertical height of the slope and input in units of length. Theoretically ydim can be set to any positive value, however, due to storage limitations and different rock behavior at relatively high stress levels, ydim must be kept less than 300 feet for most combinations of input parameters. (See Fig. 4.4a).

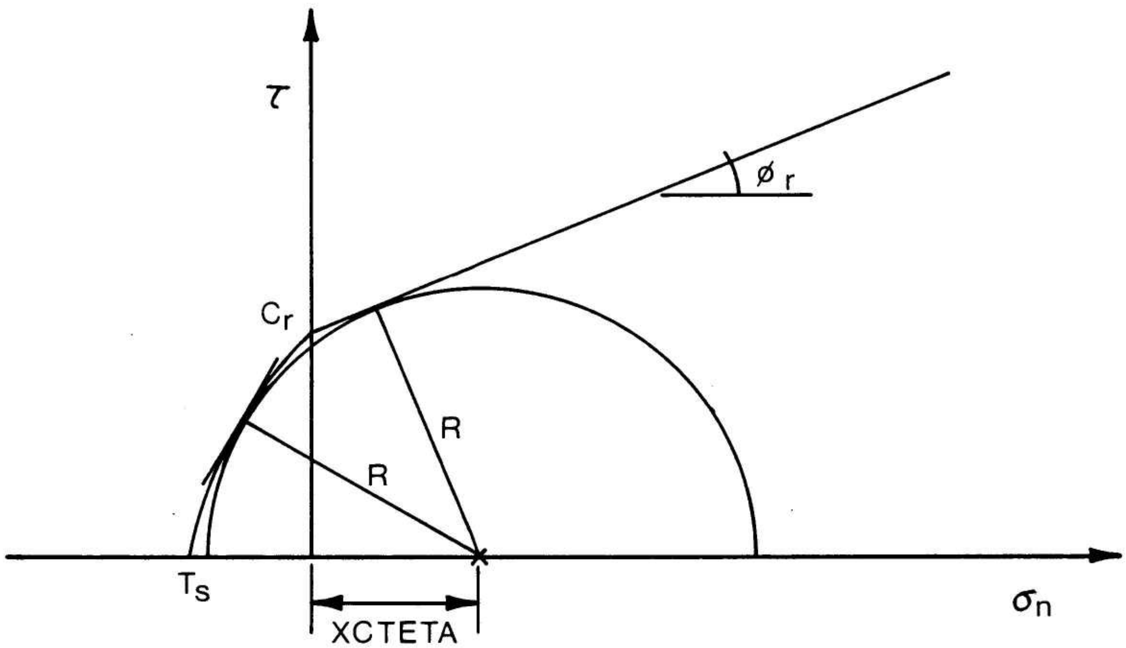


Figure 4.4 Circle of Dual Tangency

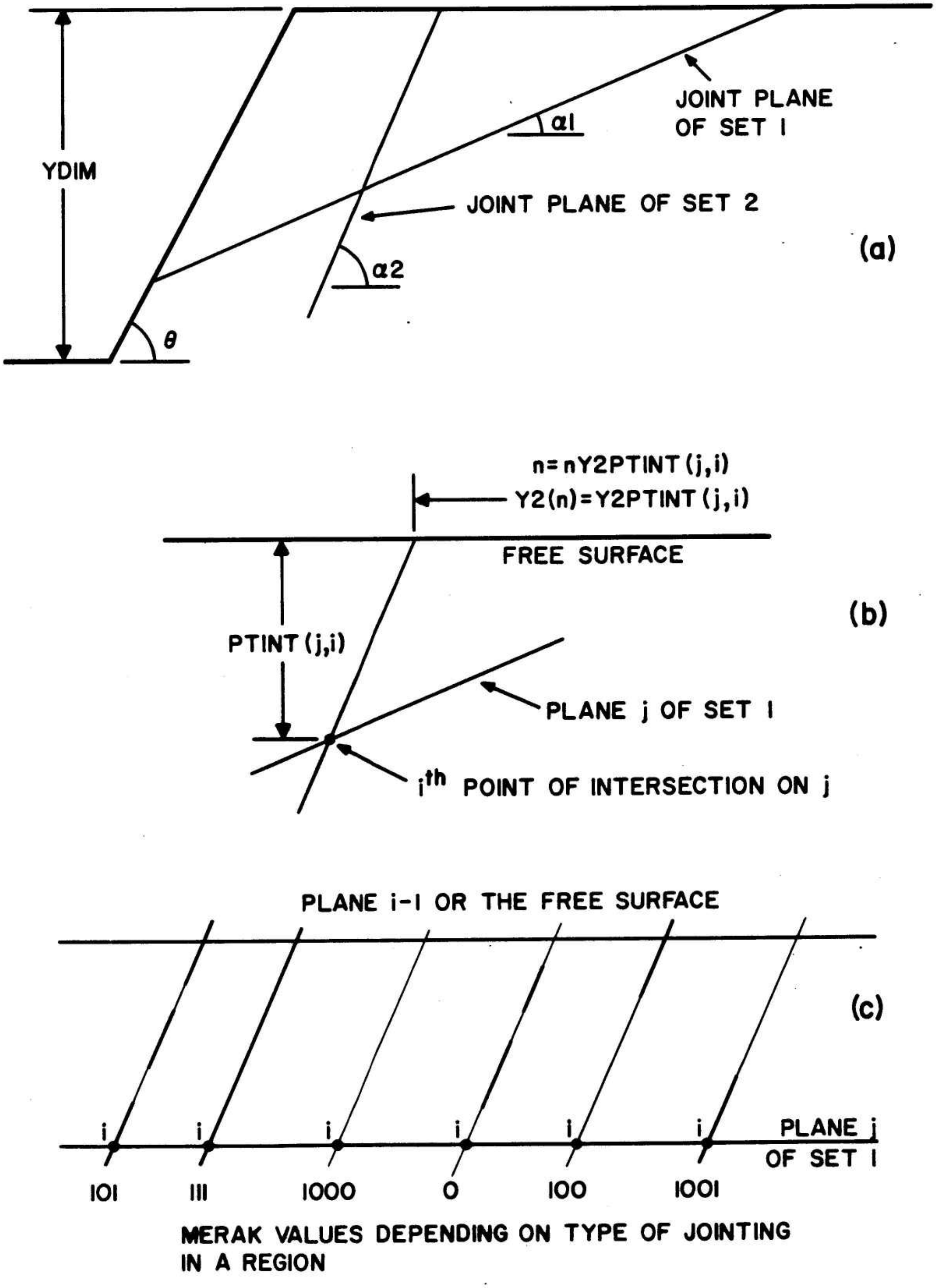


FIGURE 4.4 VARIABLES USED IN THE COMPUTER PROGRAM

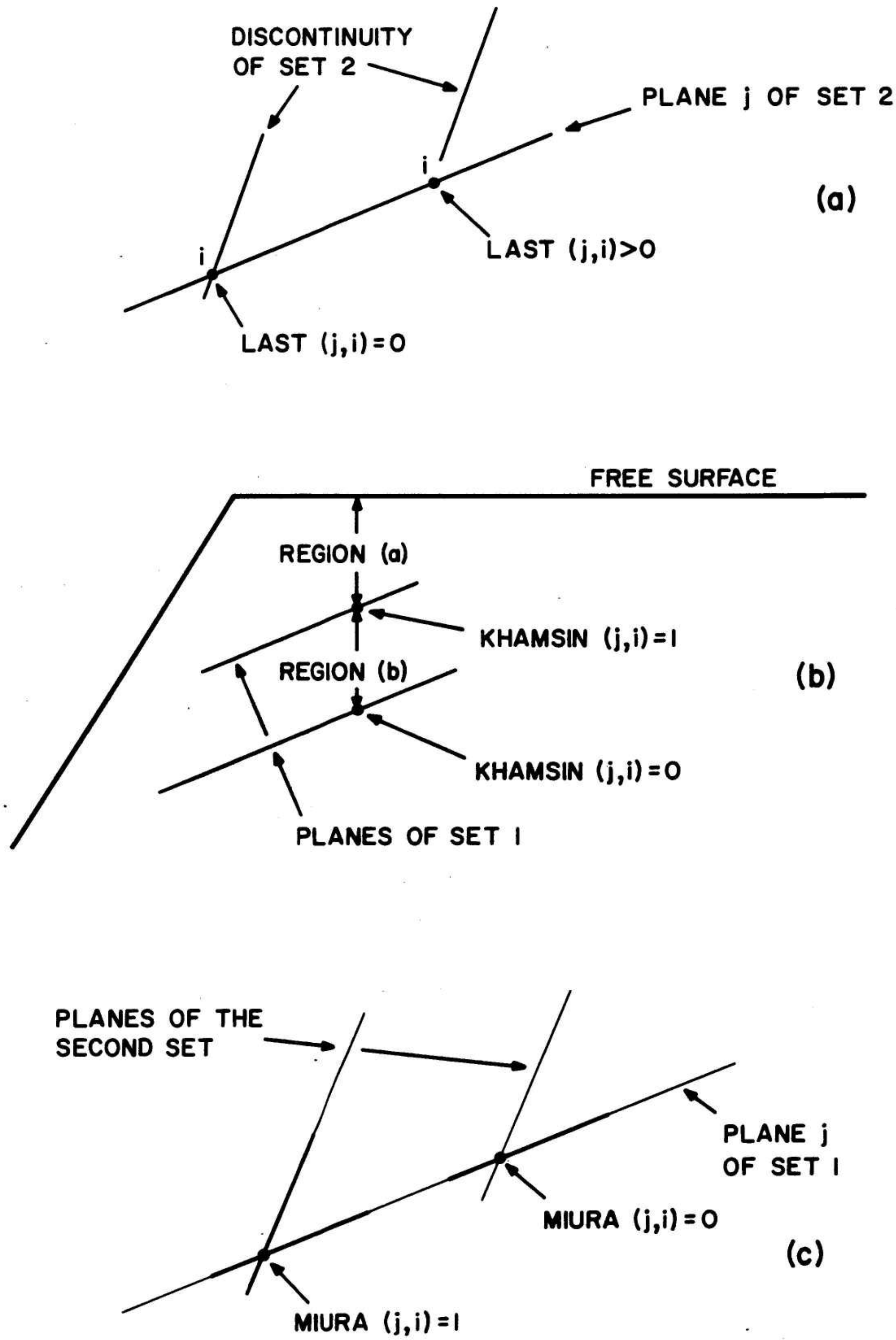


FIGURE 4.5 VARIABLES USED IN THE COMPUTER PROGRAM

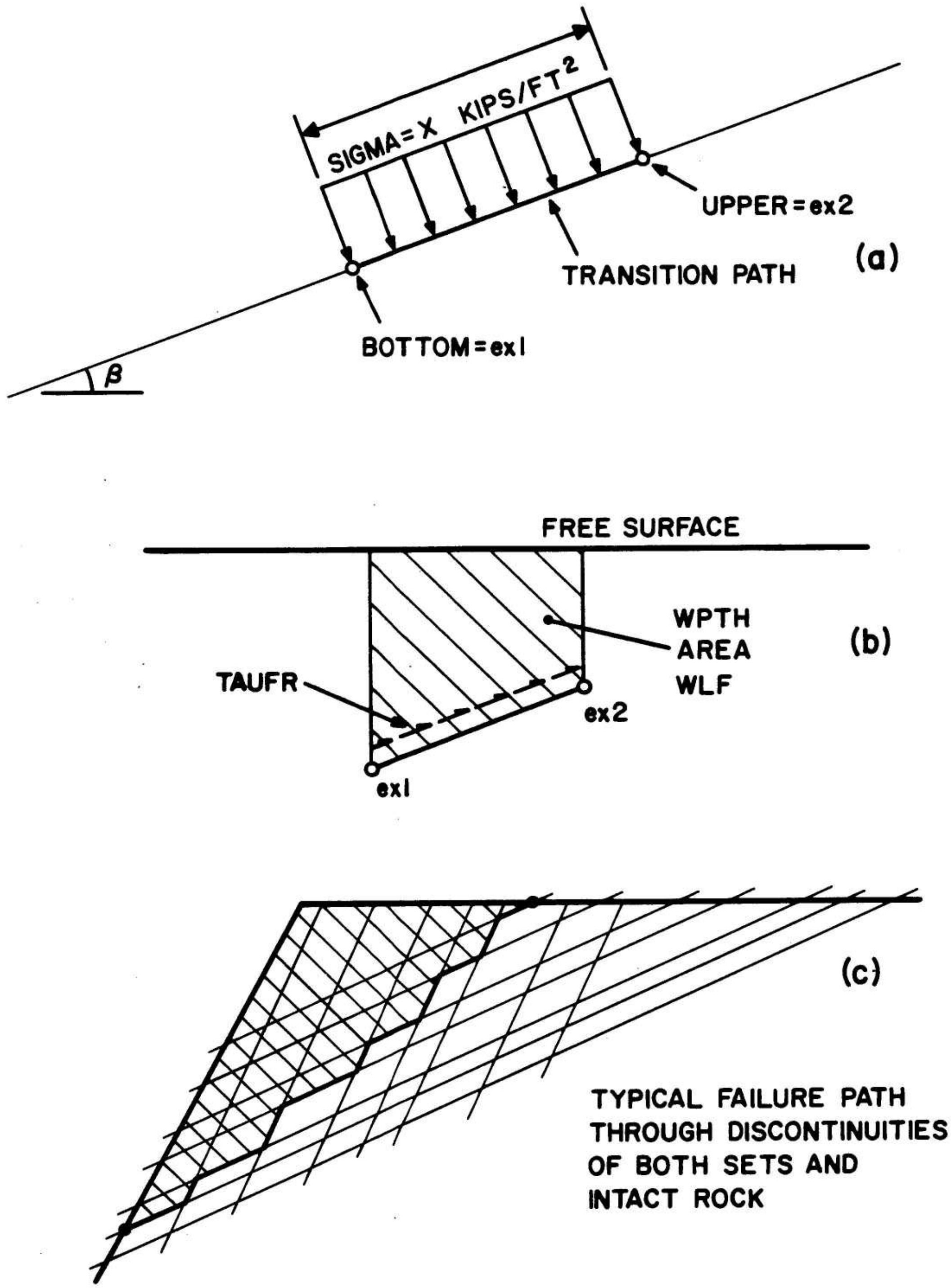


FIGURE 4.6 VARIABLES USED IN THE COMPUTER PROGRAM

ystar

In units of length, giving the x-coordinate on the slope face from where joint generation begins. The purpose of ystar is to allow study of deeper slopes when the storage limitations related to ydim are prohibiting. To analyze the entire slope, ystar is set to zero.

phijt

The friction angle of joint segments in degrees. It may be considered to be as friction angle mobilized by the joint at the moment of intact rock bridge failure and not necessarily the peak value.

cojt

Joint cohesion is input in units of stress and must be kept much smaller than intact rock cohesion (cork) for a realistic analysis.

phirk

Intact rock friction angle is to be input in degrees and should be kept between 0 and 45 degrees. Since the present version of the program assumes failures of intact rock bridges in tension, phirk has no influence on intact rock strength at stress levels for which ($\sigma_a < C_r$) is valid. phirk is a factor in deep slopes (> 200') and in weak rock (< 25 KSF) where failure is in the shear mode rather than the tension mode.

cork

Intact cohesion, "cork" is to be input in units of stress. It must be large enough so that nowhere within the slope is it exceeded by the normal stress on any joint plane. This assures that all intact rock failures are in pure tension. If "cork" is too low for the slope depth considered, the program will print out a warning to this effect. Keeping "cork" greater than 25 Ksf for slopes up to 250' should alleviate this problem for most typical slope configurations. Lower values of cork can be used for shallower slopes.

phiult

Defined as the friction angle of crushed rock at large strains in direct shear. It is to be input in degrees and must be less than or equal to the intact friction angle. Like phirk, phiult only affects intact rock shear resistance at very high stress levels. "phiult" can influence resistance only when stress levels within the slope start to approach the unconfined compressive strength of the intact rock. At such stress levels, it is unlikely that the mode of failure assumed in the model analysis is applicable. Once again the user is cautioned against using this program for analysis of slopes in which cork is exceeded by joint normal stress anywhere in the slope.

gamr

"gamr" is defined as the unit weight of intact rock and is given in units of weight (force) per unit volume.

sp31, sp32

The average spacing, in units of length, between adjacent joint planes for the first and second joint sets, respectively. Since the program is limited at present to 100 joint planes, input values of sp31 must be balanced against values of ystar and ydim such that the limiting value of 100 joint planes is not exceeded.

spjtln1, spjtln2

The average lengths of joint segments within the slope for the first and second joint sets, respectively. They must be input in units of length. It is difficult to estimate the minimum values of the spacings that can be used without exceeding storage limitations since they vary with the magnitudes of other input parameters. If, in a particular realization, too many joint segments (current limit is 50 joints per plane) are generated, the program will stop operating and a message to that effect will be printed out. This is in contrast to "slopesim" where such a realization is ignored when a similar condition arises and movement to the next realization takes place causing a bias toward output parameters at the end of the run.

sprkbr1, sprkbr2

The average lengths of rock bridges for the first and second joint sets, respectively. They must be input in units of length.

iseed

The initial "seed" in generating random numbers. It can be any value grater than 0. It is used in the random generation of jointing patterns for all realizations of a particular run.

<u>ndiv</u>

An integer greater than 0 specifying the number of equal height increments into which the slope is to be divided for independent statistical evaluation of ouputs. It is recommended to set the value of "ndiv" to no less than 4 and no greater than 10.

<u>notpop</u>, <u>notpot</u>, <u>notpod</u>

Are integer input that regulate the type and amount of program output. For any of them set to 1, part of the output will be printed out. However, setting any one of the above parameters to zero will not allow the printer to print the ouput. The first, notpop, when set to 1 will print the input data in a format for easy reference; the y-coordinates of the starting and exit points of joint planes of both sets, the maximum x-coordinate for each joint plane, the x-coordinates of the joint segments in each joint plane, the right ends of first set joint segments to above projections, the x-coordinate of the points of intersection on each joint plane of the first set, with planes of the second set and finally the x-coordinates of the dynamic programming points, in ascending order, on each joint plane within the first joint set.

The second, notpot, when set to a value of 1 will output a description of each region, the type of jointing pattern within the region, the minimum safety margin of every coordinate of the joint segments of the second set, the path length, the transition angle, the minimum critical weight up to the point in question in addition to the path incremental weight and incremental safety margin contributed by the path, the x-coordinates of the transition path, the total jointed rock length and finally the total intact rock length of the critical path, up to that point. This is performed for every dynamic programming plane point on every first set joint plane in addition to all points on second set joint plane.

The third, notpod outputs the joint plane number (of the first set) and next to it the minimum safety margin.

<u>output 1</u>, <u>output 2</u>, <u>output 3</u>

When "output 1" is set to 1, the computer prints a statement indicating which realization is taking place in the computer.

When "output 2" is set to 1, the computer prints, following each realization, the dynamic programming results, namely, safety margin, persistence, weight and plane height for the dynamic programming plane points which are the points of intersection of the first set joint planes and the slope face.

When "output 3" is set to 1 and after all realizations have been carried out, the computer will print for every height interval the distributions of the angles of critical paths, the critical weights, persistences and the unit safety margin. The computer prints out the number of critical paths exiting from the height interval as well as the number of failure paths (i.e., paths with safety margins equal to zero or negative).

noreal

Is the number of realizations specified for the simulation run and should be input as an integer greater than zero up to a value of no more than 1000 which will yield the best possible results.

dmin

Must be input in units of length. Its purpose is to give the maximum allowable length of a transition path. Setting it to a large value has the effect of checking all possible paths.

njump

Is input as an integer greater than or equal to zero. It sets the number of first set joint planes above a particular point in that set from which transition paths are checked, with the purpose of finding a minimum safety margin. Setting "njump" to zero will only check points along a given joint plane and will not allow paths (or jumps) to overlying joint planes. Since the critical path seldom involves more than two or three transitions, it is advisable to set "njump" relatively low so that the number of paths checked and hence cost is kept low. Within the scope of this thesis, it is not recommended to set it to zero, nor greater than three.

4.3 Sample Output

For purposes of illustration, the sample run used in this study, consists only of a single realization. The slope geometry specified by the input parameters discussed previously as well as the joint pattern geometry for this realization, was previously described.

Although the output shown on the following pages is self explanatory, additional discussion on it will follow.

Part (a) is the print-out of the input parameters as inserted by the user, primarily, for reference or demonstration purposes.

Part (b) is a print-out of the geometry of the joint planes of the first set. It starts with the plane number and is followed by the maximum permissible x-corrdinate on the slope face as well as the y-coordinate of the joint plane followed by the x-coordinates of joint segments on that plane. Finally, a statement of the number of joints and the percent persistence of that plane is printed out.

Part (c) is self explanatory. Each plane projection of right-ends of joints to above planes is listed on the x-axis (x-coordinates).

Part (d) is the same as part (b) except that it is for planes of the second set. It starts with a printout of the maximum permissible y-coordinate where a plane can be generated.

Part (e) is a list of the x-coordinate of the points of intersection of each plane of the first set with planes of the second joint set.

Part (f) is a list of the compiled x-coordinates of the joint segments as well as the right ends to above projections and points of intersection. All are listed in ascending order as can be seen in the sample output (this list of x-coordinates is referred to as the dynamic programming plane points).

Part (g) is the output of the dynamic programming carried out on every dynamic programming plane point as well as the x-coordinates of second set discontinuities. It starts with a print out of the realization number. Next, the type of region being analyzed as well as the boundaries and type of jointing in that region are printed out. Following that, each of the dynamic points including those in the second joint set, as well as the lower and upper x-coordinates which might constitute a section of the path of a minimum safety margin, are printed out. Next, beta, the angle of transition, and the actual path length (negative if path is within a discontinuity), are printed out. Average stress on the potential path due to rock overburden and total weight of overlying rock as well as the incremental safety margin of that path which connects the two points mentioned, all follow on the same print out line.

The print out line that follows consists of the total safety margin of all the points constituting a path from the upper surface down to the point of interest and the total length of all discontinuities on that path (negative for indexing purposes only) as well as the total length of that path connecting points of minimum safety margin to the upper surface. Finally there is a print out of the weight of rock overlying the path just described.

When the exit points of planes of the first set are reached, a statement indicating the plane reference number and the total safety margin at that point, is printed out.

Part (h) is a summary of the preceding. It is a list consisting of: the number of the plane, the x-coordinate at the exit point, the angle of the critical path, the minimum safety margin, the unit safety margin, the apparent persistence and the weight of the critical path.

Part (i) is most important to the user since it is an arrangement of all the data listed in part (h) in the sample program output.

Notice that this part is an output of the statistical data for each of the height increments specified by the user. By setting "ndiv", the user divides the slope to "ndiv" parts. The values of output parameters (i.e., continuity, SM, USM, etc.) associated with the exit point of each joint plane that falls within a given interval, are stored. For each interval, the means and standard deviations of these parameters are evaluated. These statistics are based on parameter values of all exit points that fall within a given interval, independent of realization. For the sample run only one realization was carried out. This is not a sufficient number to reliably determine statistical parameters of outputs. However, results such as distributions (i.e., means and standard deviations) are more reliable when the number of critical path exit points that fall within each interval, increase with the increase in the number of realizations (user specified).

In this part, output of the total number of joints (N_{Ii}) existing within an interval and the total number that fail (N_{Fi}), i.e., ($SM \leq 0$), are also printed out. Again, as was the case for the statistics, a running count is made of N_{Fi} and N_{Ti} for each interval, i, independent of the number of realizations. From N_{Ti} and N_{Fi} the user can estimate the probability of joint plane failure P_f for each interval i:

$$P_{Fi} = N_{Fi} \mid N_{Ti} \qquad \text{(Eq. 4.1)}$$

Main Program Variables

pi	: π
alpha 1	: Inclination of the first joint set, in radians [Figure 4.1(a)].
alpha 2	: Inclination of the second joint set, in radians [Figure 4.1(a)].
theta	: Inclination of the slope face in radians [Figure 4.1(a)].
phojt	: Friction angle of jointed rock in radians.
tabht	: Tangent of joint friction angle.
phork	: Friction angle of intact rock in radians (taken as 0.0).
sinrk	: sin (phork).
cosrk	: cos (phork).
tanrk	: tan (phork).
p1 , p2	: Average joint persistence defined as (spjtln1/(spjtln1 + sprkbr1)) and (spjtln2/(spjtln2 + sprkbr2)) of the joint sets respectively.
xcteta	: Center of failure circle tangent to the parabolic and linear sections [Figure 4.4].
xj1	: y dim/sin (alpha 1). [Figure 4.1(b)].
noreal	: number of realizations that will be carried out in one run; user specified.
ystar	: perpendicular distance from apex of slope to a generated joint plane [Figure 4.1(a)].
icrotch, icar	: define number of random numbers to be generated.
yrand (i)	: random number produced to generate x - coordinates for joint planes.

Main Program Variables (cont.)

y1 (i), y2 (i)	: Y - coordinate of the i-th joint plane on the y axis of set 1 and 2, respectively.
ri	: random numbers produced by random number generator ggub.
cjoint 1(i), cjoint 2(i,j)	: For odd j: cjoint 1,2(i,j) is the depth to the right end of the (j + 1)/2 joint segment in the ith joint plane. For even j: cjoint 1,2(i,j) is the depth to the left end of the j/2 joint segment in the ith joint plane; joint set 1 and 2 respectively. [Figure 4.2(a)].
xd1 (i)	: x - coordinate of the exit point on the slope face of the ith joint plane of the first set. [Figure 4.2(a)].
xd2	: Vertical distance of exit point of the ith joint plane below the upper free surface of the second joint set; on either the slope face or the line designated xj1. [Figure 4.2(b)].
zp (i)	: Vertical distance from the apex to the ith joint plane. [Figure 4.2(a)].
npt1 (i), npt2 (i)	: Number of joint points in the ith point plane.
njoint	: Number of joint segments in joint plane under consideration.
percon1 (i), percon2 (i)	: Average percent persistence of ith joint plane of joint set one and two respectively.
sumjtln	: Sum of joint segment lengths in joint plane under consideration.
jpl1 , jpl2	: Number of joint planes in realization under consideration of set one and two respectively.
xcoor (j,i)	: x - coordinate of right end of joint segment to upper plane for use in dynamic programming.
nptint (j)	: Number of points of intersection on plane j of set one with planes of joint set two.

Main Program Variables (cont.)

ptint (j,i)	: The ith point of intersection on plane j of set one.
y2ptint (j,i)	: y coordinate of the joint plane of set two defined by the ith point of intersection on plane j of set one [Figure 4.4b].
ny2ptnt (j,i)	: Number of the plane of set two defined by the ith point of intersection on j-th plane of set one [Figure 4.4b].
dee (i)	: Vertical distance from (x = 0.00) to the starting point of joint plane i of set one.
ml (j)	: Number of dynamic programming points on plane j of set one.
plpt (j,i)	: cjointl (j,i), xcoor (j,i) and ptint (j,i) rearranged in ascending order.
vtran (j,i)	: Vertical distance between plane j and n of set one.
sml (j,i) sm2 (j,i)	: Minimum safety margin of the i-th x-coordinate of plane j in set one and two respectively, [Fig. 4.3].
sml2 (j,i)	: Minimum safety margin of the ith x-coordinate in plane j of set two which corresponds to a point of intersection on a plane of set one.
merak (j,i)	: An integer which describes an arrangement of joints within a region [Figure 4.4(c)].
last (j,i)	: Of plane j of set one and coordinate i [Figure 4.5(a)]; if last equals zero a joint segment intersects point of interest. If last is greater than zero then the number stands for the number of the end of a joint segment of set two, immediately above the dynamic programming plane point plpt (j,i).
khamsin (j,i)	: An integer, either 0 or 1; 0 for a regions between two planes of set one; a 1 for region between a plane and free surface [Figure 4.5(b)].
miura (j,i)	: An integer, either 0 or 1; 1 for a point on a first set discontinuity which is intersected by a joint segment of the second set; 0 for a point which is not intersected by a joint segment [Figure 4.5(c)].

Main Program Variables (cont.)

mpos	: Integer which determines type of path to be checked by the resistance subroutine. For mpos = 1, the subroutine considers vertical transitions to the free surface. For mpos = 0, the subroutine considers transitions through intact work to other joint planes. For mpos - 2, the subroutine considers transitions through joints of the first set. Finally for mpos = 10, the subroutine considers transitions through joints of the second set.
str	: Safety margin of a particular path under consideration
wmin1 (j,i) wmin2 (j,i) wmin12 (j,i)	: Weight of overlying "slice" above path of minimum safety margin (Fig. 4.3)
sf	: Force required to cause failure for transition path under consideration. sf is calculated in the subroutine - msaf. (sf = df - res) where df is the driving force and res is the rock resistance.
sv	: Resistance of path involving a vertical transition to the free surface. sv is calculated in the subroutine - msaf.
siga	: The average component of overburden stress perpendicular to the direction of jointing for a particular transition path (Figure 4.6(a)).
taufr	: Peak shear stress on a transition path resisting downslope motion of overlying material in the direction of jointing; calculated by msaf (Figure 4.6(b)).
wpth	: Weight of overlying "slice" of rock above transition path in question; calculated by msaf (Figure 4.6(b)).
dist	: Length of the joint plane under consideration.
usm (j)	: Unit safety margin of the exit point of the jth joint plane. (Figure 4.3).
fang (j)	: Net angle of critical path for the exit point of the jth joint plane in degrees. (Figure 4.3).

Main Program Variables (cont.)

appt (j)	: Apparent persistence of the jth joint plane.
wgt	: Weight of rock overlying joint plane under consideration.
numj (n)	: Number of joint planes in the nth height interval.
sper (n)	: Sum of joint persistences in the nth height interval.
ssqper (n)	: Sum of squares of joint persistences in nth height interval.
sfan (n)	: Sum of angles of critical paths (in degrees) in nth height interval.
ssgfan (n)	: Sum of squares of failure angles in nth height interval.
ssm (n)	: Sum of safety margins in nth height interval.
ssqsm (n)	: Sum of squares of safety margins in nth height interval.
susm (n)	: Sum of unit safety margins in nth height interval.
ssqusm (n)	: Sum of squares of unit safety margins in nth height interval.
sapp (n)	: Sum of apparent persistences in the nth height interval.
ssqapp (n)	: Sum of squares of apparent persistences in nth height interval.
swgt (n)	: Sum of the weight of rock overlying the critical paths in nth height interval.
ssqwgt (n)	: Sum of squares of weights of rock overlying the critical paths in nth height interval.
smleo (n)	: Number of critical paths in the nth height interval with safety margins less than zero.
upper	: Upper x-coordinate of a particular transition path (Figure 4.6(a)).
bottom	: Lower x-coordinate of a particular transition path (Figure 4.6(a).

Main Program Variables (cont.)

smpthr (j,i)	: Total intact rock length of a particular path up to point i of joint plane j.
smpthj (j,i)	: Total jointed rock length of a particular path up to point i of joint plane j.
smptht (j,i)	: smpthr (j,i) + smpthj (j,i)
perave (n)	: Average percent persistence of joint planes in the nth height interval.
sdper (n)	: Standard deviation of percent persistence of joint planes in the nth height interval.
fanave (n)	: Average angle of critical path (in degrees) in the nth height interval.
sdfan (n)	: Standard deviation of angles of critical path (in degrees) in the nth height interval.
snave (n)	: Average safety margin of critical paths in the nth height interval.
sdsm (n)	: Standard deviation of safety margins of joint planes in the n-th height interval.
usmave (n)	: Average unit safety margin of joint planes in the n-th height interval.
sdusm (n)	: Standard deviation of unit safety margin of joint planes in the n-th height interval.
appave (n)	: Average of apparent persistence of joint planes in the n-th height interval.
sdapp (n)	: Standard deviation of apparent persistence of joint planes in n-th height interval.
wgtave (n)	: Average weight of rock overlying critical paths of joint planes in n-th height interval.
sdwgt (n)	: Standard deviation of rock weight overlying critical paths of joints in the n-th height interval.
xxx1	: Top x-coordinate of height interval under consideration (Figure 3.18(b)).
xx2	: Lower x-coordinate of height interval under consideration (Figure 3.18(b)).

Variables in the Subroutine Msaf

beta	:	Angle of transition path in degrees measured from horizontal (Figure 4.6a).
beto	:	Beta in radians.
galo 1,2	:	Angles to be used in the subroutine. Either can be alpha 1 and alpha 2 depending on the type of the path
why 1,2	:	y-coordinates of plane(s) under consideration for a particular path of transition (Figure 2.19).
ex 1,2	:	x-coordinates of ends of transition path (Fig. 4.6a).
area	:	Area of rock overlying transition path (Fig. 4.6b).
wlf	:	Weight of rock overlying transition path = Area x gamr.
tenang	:	Angle of tension fracture measured from the joint inclination angle.
res	:	Resistance of the transition path to shear in the direction of jointing.
tanfjt	:	Shear resistance of jointed transition path.
rult	:	Resistance of intact rock at ultimate strength.

```
1,30p
        common/logo/ r(200),yrand(100),y1(100),y2(100),fans(100),usm(100),per
\cave(100),fanave(100),sdfan(100),smave(100),sdsm(100),usmave(100),sdusm(100),a
\cppave(100),sdper(100)
        common/lser/ wmin1(80,60),wmin12(80,60),wmin2(80,60),m(50),app(50),sd
\cwgt(50),wgtave(50),sdapp(50)
        common/causa/ ssqper(50),sper(50),sfan(50),ssqfan(50),ssm(50),ssqsm(5
\c0),susm(50),ssqusm(50),sapp(50),ssqapp(50),swgt(50),ssqwgt(50),smleo(50),numj
\c(50),sm1(100,100),m1(100),vtran(100,100)
        common/pizza/ ptint(75,120),plpt(75,120),xcoor(75,120),cjoint1(75,120
\c),cjoint2(75,120),y2ptint(75,120),ny2ptnt(75,120)
        common/kuss/  npt1(60),nptint(60),npt2(60),percon1(60),percon2(60),zp
\c(60),xd1(60),xd2(60)
        common/alto/ dee(35),last(50,100),miura(50,50),sm2(80,100),merak(40,2
\c00),khamsin(80,100),crement(80),nmplpt(70,70),sm12(80,100),smpthj1(80,100),sm
\cptht1(80,100),smpthr1(80,100),smptht2(80,100),smpthr2(80,100),smpthj2(80,100)
\c,smptht12(80,100),smpthr12(80,100),smpthj12(80,100)
3745    format(5x,'Joint Plane ',i2,6x,'Safety Margin ',f8.2)
3746    format(5x,'Safety Margin =',f7.2,5x,'Jointed Rock;Sum =',f7.2,5x,'Cri
\ctical Path Length =',f7.2,5x,'Critical Weight =',f7.2)
3747    format(5x,'In Joint Transition Within Plane ',i3,5x,'Of Set ',i1)
3748    format(5x,'From Plane ',i3,5x,'In Set ',i1,5x,'Reference Point ',i3,7
\cx,'Up To Plane ',i3,5x,'Of Set ',i1,5x,'Reference Point ',i3)
3749    format(/,1x,'Lower x-coordinate = ',f6.2,6x,'Upper x-coordinate = ',f
\c6.2)
3750    format(5x,'Beta = ',f5.2,5x,'Path = ',f7.2,5x,'Stress = ',f7.2,5x,' W
\ceight = ',f7.2,5x,' S.F.(path) =',f7.2)
465     format(/,3x,'******* Realization Number ',i3)
467     format(/5x,'There Are No Points Of Intersection On This Plane',/)
466     format(/5x,'No Joints On Plane',i3,4x,'Of The Second Joint Set')
469     format(/5x,'Region Starts On Slope Face To Plane',i3,5x,'ycoordinate
\c=',f6.2)
468     format(/5x,'Region  From  Free Surface  To Plane',i3,5x,'y-coordinate
\c = ',f6.2)
470     format(/5x,'Region Between Planes',i3,'   And ',i3,5x,'  y-coordinate
\c = ',f6.2)
491     format(5x,' Joint(s)  In  Between  Only')
492     format(10x,' No Second Set Joints')
493     format(10x,' Continuous Joint Throughout ')
494     format(5x,' Joint Intersects Top Point')
495     format(5x,' Joint Intersects Bottom Point')
1008    format(///,10x,'Slope  Angle: ',f4.1,' degrees')
1010    format(10x,'Slope Hight: ', f6.1,' feet')
1013    format(10x,'Rock Unit Weight: ',f6.2)
1011    format(10x,'First  Joint Set Inclination: ',f5.1,' degrees')
1009    format(10x,'Second Joint Set Inclination: ',f5.1,' degrees')
1012    format(//,5x,'Strength Parameters',/,10x,'Phi (Joint) = ',f7.2,' degr
\cees',/,10x,'Cohesion (Joint) =',f7.2,/,10x,'Phi (rock) = ',f7.2,' degrees',/,
\c10x,'Cohesion (rock) = ',f7.2)
1015    format(//,5x,'Path Criteria:  ',/,10x,'Minimum Spacing =',f6.1,/,10x,
\c'Maximum Transition =',i3,///)
```

```
31,70p
1014      format(//5x,'Distributional Parameters;Joint Set One: '/10x,'Mean Pla
\cne Spacing =',f7.2,/10x,'Mean Joint Spacing =',f7.2,/10x,'Mean Joint Length
\c=',f7.2)
1004      format(//5x,'Distributional Parameters;Joint Set Two: '/10x,'Mean Pla
\cne Spacing =',f7.2,/10x,'Mean Joint Spacing =',f7.2,/10x,'Mean Joint Length
\c=',f7.2)
1016      format(//,10x,'Number of Realizatins:',i5)
1090      format(10x,'Point Considered:',//,t20,'Joint Plane = ',12x,
          &i5,/,t20,'y-coordinate = ',9x,f10.3,/,t20,'x-coordinate = ',9x,
          &f10.3,///,' Path no.',8x,'-- Junction Point --',7x,'Angle ',
          &4x,'Strength',4x,'Strength',4x,'Strength',6x,'Minor Prin.',4x,
          &'Tangent To',/,14x,'Jt. Pl.',3x,'y-coor',4x,'x-coor',3x,
          &'To Jct.',4x,'To Jt. Pt.',4x,'To Edge',6x,'Total',10x,'Stress',/)
1092      format(i5,7x,i5,2x,f10.3,f10.3,5x,f6.2,4x,f8.2,4x,f8.2,4x,f8.2,4x,f7.
\c2,4x,f7.2)
1098      format(1x,'The Minimum Safety Margin For y-coor = ',f10.3,'And x-coor
\c = ',f10.3,' is ',f10.2)
1126      format(t25,' ( To Free Surface ) ')
1127      format(i5,4x,' Vert. Transition to Free Surface ',5x,' 90 ',4x,f8.2,4
\cx,'0.0 ',4x,f8.2,7x,f7.2)
1017      format(//,10x,'Initial Random Number:',i8)
1130      format(//,10x,'Realization Number  ',i4)
1963      format(//,10x,'--Joint Set One:Dynamic Programmicng Plane Points--')
1131      format(/20x,'Number of Joint Planes Is = ',i3)
1132      format(/1x,'Joint',9x,'Percent',9x,'Angle Of',16x,'Minimum',9x,'Unit'
\c,13x,'Apparent',9x,'Weight Of',/,' Plane',7x,'Continuity',7x,'Critical Path',
\c8x,'Safety Margin',5x,'Safety Margin',5x,'Persistence',6x,'Critical Path',5x,
\c'Height')
1133      format(i4,11x,f6.2,14x,f5.2,9x,f11.2,6x,f11.2,12x,f6.2,9x,f10.2,7x,f6
\c.2)
1135      format(//7x,'The Slope Hight Of  ',f6.1,5x,' Is Divided Into Incremen
\cts Of ',f6.1,5x,' For Statistical Analysis')
1138      format(//,20x,'Interval From  ',f8.1,4x,'To  ',f8.1,//,
          &5x,'The Number Of Joints Is',i5,//,5x,
          &'Percent Continuity:',/,15x,'Average = ',f6.2,/,15x,
          &'Standard Deviation = ',f6.2,//,5x,'Net Angle of Critical Path:',
          &/,15x,'Average = ',f6.2,/,15x,'Standard Deviation = ',f6.2,//,
          &5x,'Safety Margin:',/,15x,'Average = ',f12.1,/,15x,
          &'Standard Deviation = ',f10.1,//,5x,'Unit Safety Margin:',/,
          &15x,'Average =' ,f12.1,/,15x,'Standard Deviation = ',f10.1,//,
          &5x,'Apparent Persistence:',/,15x,'Average = ',f6.2,/,15x,
          &'Standard Deviation = ',f6.2,//,5x,'Weight of Critical Path :',
          &/,15x,'Average = ',f12.2,/,15x,'Standard Deviation = ',f10.2,
          &//,5x,'The Number Of Joints With Safety Margins < 0.0 Is ',
          &2x,f5.0)
          print,"input theta,alpha1,alpha2,ydim"
          read(5,999)theta,alpha1,alpha2,ydim
          print,"input ystar,ndiv"
          read(5,999)ystar,ndiv
          print,"input phijt,cojt,phirk,cork,phiult"
          read(5,999)phijt,cojt,phirk,cork,phiult
          print,"input samr"
```

71,120p

```
        read(5,999)samr
        print,"input sp31,sp32,spjtln1,spjtln2,sprkbr1,sprkbr2"
        read(5,999)sp31,sp32,spjtln1,spjtln2,sprkbr1,sprkbr2
        print,"input iseed"
        read(5,999)iseed
        print,"notpop,notpot,notpod"
        read(5,999)notpop,notpot,notpod
        print,"input output1,output2,output3"
        read(5,999)output1,output2,output3
        print,"input noreal,distmn,njump"
        read(5,999)noreal,distmn,njump
999     format(v)
        if(notpop.eq.0)go to 6850
        write(6,1008)theta
        write(6,1010)ydim
        write(6,1011)alpha1
        write(6,1009)alpha2
        write(6,1012)phijt,cojt,phirk,cork
        write(6,1013)samr
        write(6,1014)sp31,spjtln1,sprkbr1
        write(6,1004)sp32,spjtln2,sprkbr2
        write(6,1017)iseed
        write(6,1016)noreal
        write(6,1015)distmn,njump
6850    continue
        pi=3.141592
        alpho1=alpha1*pi/180.
        alpho2=alpha2*pi/180.
        xj1=ydim/sin(alpho1)
        theto=theta*pi/180.
        phojt=phijt
        tanjt=tan(phojt)
        phork=phirk*pi/180.
        sinrk=sin(phork)
        cosrk=cos(phork)
        tanrk=tan(phork)
        xcteta=cork*((1.-sinrk*cosrk)-(1.-(2.*cosrk*sinrk))**.5)/sinrk**2.
        cota1=1./(tan(theto-alpho1))
        cotb1=1./(tan(alpho1))
        dj1=xj1/(cota1+cotb1)
        ysum1=ystar
        ysum2=ystar
        p1=spjtln1/(spjtln1+sprkbr1)
        p2=spjtin2/(spjtln2+sprkbr2)
        if(theta.eq.90.0)adln=0.0
        if(theta.lt.90.0)adln=ydim/(tan(theto))
        ytot=xj1*cos(alpho1)
        ylim=0.0
        ywt=(ydim/tan(alpho1))-(ydim/tan(theto))
        if(alpha2.lt.90.0)ylim=ydim/tan(alpho2)
```

```
121,165p
          ymax=ytot-ylim
          if(ylim.gt.adln)ymax=ytot-adln
          if(alpha2.eq.90.0)ymax=ytot
          if((alpha2.gt.90.0).and.(alpha2.lt.180.0))ymax=ytot+(ydim/tan(((180.0
\c-alpha2)/180.0)*pi))
          if(alpha2.eq.180.)ymax=ydim
27        format(//,5x,'Maximum Allowable y-coordinate : ymax = ',f7.2,//)
          do 55 n=1,ndiv
          sper(n)=0.
          ssqper(n)=0.
          sfan(n)=0.
          ssqfan(n)=0.
          ssm(n)=0.
          ssqsm(n)=0.
          susm(n)=0.
          ssqusm(n)=0.
          sapp(n)=0.
          ssqapp(n)=0.
          swst(n)=0.
          ssqwst(n)=0.
          smleo(n)=0.
          numj(n)=0
 55       continue
          do 1960 mm=1,noreal
          if(output1.ne.0)write(6,465)mm
          ysum1=ystar
          ysum2=0.00
          nreal=nreal+1
          icrotch=100
          call ssub(iseed,icrotch,r)
          do 30 i=1,icrotch
          yrand(i)=r(i)
 30       continue
 31       do 199 i=1,50
          ysum1=ysum1+(sp31)*alog(1./(1.-yrand(i)))
          y1(i)=ytot-((ysum1/sin(alpho1))+adln)
          if(y1(i).lt.0.0)go to 201
 23       format(x,'Joint Plane',i3,7x,'Max x-coordinate =',f7.2,7x,'y-coordina
\cte =',f7.2)
 155      icar=200
          call ssub(iseed,icar,r)
          if(r(200).lt.p1)cjoint1(i,1)=0.0
          if(r(200).ge.p1)cjoint1(i,1)=sprkbr1*alog(1./r(1))*sin(alpho1)
          xd1(i)=ysum1*(xj1/dj1)*sin(alpho1)
          xdim=xd1(i)
          if(notpop.ne.0)write(6,23)i,xdim,y1(i)
          zp(i)=ysum1/cos(alpho1)
```

```
166,210p
        if(cjoint1(i,1).ge.xdim)cjoint1(i,1)=xdim
        if(notpop.ne.0)write(6,320)cjoint1(i,1)
        if(cjoint1(i,1).eq.xdim)go to 150
        do 80 j=2,50,2
        cjoint1(i,j)=cjoint1(i,j-1)+spjtln1*alog(1./r(j))*sin(alpho1)
        if(cjoint1(i,j).ge.xdim)go to 90
        if(notpop.ne.0)write(6,320)cjoint1(i,j)
        cjoint1(i,j+1)=cjoint1(i,j)+sprkbr1*alog(1./r(j+1))*sin(alpho1)
        if(cjoint1(i,j+1).ge.xdim)go to 120
        if(notpop.ne.0)write(6,320)cjoint1(i,j+1)
80      continue
90      cjoint1(i,j)=xdim
        if(notpop.ne.0)write(6,320)cjoint1(i,j)
        npt1(i)=j
        njoint=j/2
        nrn=j+1
        go to 150
120     cjoint1(i,j+1)=xdim
        if(notpop.ne.0)write(6,320)cjoint1(i,j+1)
        npt1(i)=j+1
        njoint=j/2.
        nrn=j+2
150     continue
        percon1(i)=0.
        if(cjoint1(i,1).eq.xdim)njoint=0
        if(cjoint1(i,1).eq.xdim)npt1(i)=1
        m1(i)=npt1(i)
        sumjtln=0.0
        if(npt1(i).eq.1)go to 159
        do 160 j=1,njoint
        nnjt=2*j
        sumjtln=sumjtln+cjoint1(i,nnjt)-cjoint1(i,nnjt-1)
160     continue
159     continue
        percon1(i)=sumjtln*100./xdim
        if(njoint.gt.0)go to 888
        if(notpop.ne.0)write(6,1288)
        go to 190
888     if(notpop.ne.0)write(6,128)njoint
1288    format(17x,' No Joints On This Plane ')
128     format(17x,'Number of Joints On This Plane Is ',i3)
190     if(notpop.ne.0)write(6,28)percon1(i)
28      format(17x,'Average Percent Continuity Is ',f6.2,//)
199     continue
201     jpl1=i-1
```

```
211,255p
          do 40 i=1,jpl1
          do 41 j=1,m1(i)
          plpt(i,j)=cjoint1(i,j)
41        continue
40        continue
          gam=(45.0-alpha1)*pi/180.0
          mp1=jpl1-1
          if(notpop.ne.0)write(6,665)
          do 351 l=1,mp1
          m2=l+1
          k=0
          do 350 i=m2,jpl1
          do 349 j=1,npt1(i),2
          pdis=(y1(l)-y1(i))*tan(alpho1)
          adinc=0.0
          if(alpha1.lt.45.0)adinc=(((y1(l)-y1(i))*(sin(alpho1)/cos(45.0*pi/180.
\c0)))*cos(gam))-pdis
          if((pdis+adinc).gt.cjoint1(i,j))go to 349
          if(cjoint1(i,j).eq.xd1(i)) go to  350
          ppp=cjoint1(i,j)-pdis-adinc
          if(ppp.gt.xd1(l))go to 350
          k=k+1
          xcoor(l,k)=ppp
          if(notpop.ne.0.and.k.eq.1)write(6,1289)l
665       format(10x,'--Joint Set One:Joint Right End To Above Projections--')
          if(notpop.ne.0)write(6,320)xcoor(l,k)
666       format(1x,'l=',i3,8x,'k=',i3,8x,'xcoor(l,k)=',f8.2)
349       continue
350       continue
          m1(l)=m1(l)+k
          do 42 n=(npt1(l)+1),m1(l)
          plpt(l,n)=xcoor(l,(n-npt1(l)))
42        continue
351       continue
          if(notpop.ne.0)write(6,27)ymax
          do 198 k=51,icrotch
          i=k-50
          ysum2=ysum2+(sp32)*alog(1./(1.-yrand(k)))
          if(alpha2.lt.180.0)y2(i)=ysum2/sin(alpho2)
          if(y2(i).gt.ymax)go to 202
          if(alpha2.lt.90.0)go to 11
          if(alpha2.eq.90.0)go to 12
          if((alpha2.lt.180.0).and.(alpha2.gt.90.0))go to 13
          if(alpha2.eq.180.0)go to 14
11        gammo=alpho1
          if((adln.lt.(ydim/tan(alpho2))).and.(y2(i).gt.(~~ytot~~-(ydim/tan(alpho2)
\c))))gammo=theto
```

```
256,295p
        xd2(i)=(((ytot-y2(i))*sin(gammo)-(ydim*cos(gammo)))*sin(alpho2))/sin(
\cgammo-alpho2)
        if((alpha2.le.theta).and.(y2(i).eq.(ytot-(ydim/tan(alpho2)))))go to 1
\c5
        if((theta.eq.90.0).and.(y2(i).gt.(ytot-(ydim/tan(alpho2)))))xd2(i)=(y
\ctot-y2(i))*tan(alpho2)
        go to 70
15      xd2(i)=ydim
        go to 70
12      xd2(i)=y2(i)*tan(alpho1)
        go to 70
13      xd2(i)=(y2(i)/ymax)*(ydim/sin(((180.0-alpha2)/180.0)*pi))*sin(((180.0
\c-alpha2)/180.0)*pi)
        go to 70
14      xd2(i)=ysum2
        y2(i)=0.0
70      continue
        if(notpop.ne.0)write(6,23)i,xd2(i),y2(i)
154     icar2=200
        call ggub(iseed,icar2,r)
        if(alpha2.eq.180.0)go to 308
        if(alpha2.gt.90.0)go to 365
        if((theta.eq.90.0).and.(alpha2.eq.90.0))go to 17
        dd=(((ydim*cos(theto))-((ytot-y2(i))*sin(theto)))/(sin(alpho2-theto))
\c)*(sin(alpho2))
        go to 18
365     dd=((y2(i)-ytot+adln)/(ymax-ytot+adln))*ydim
        go to 18
17      dd=0.0
18      if((r(200).lt.p2).and.(y2(i).gt.(ytot-adln)))cjoint2(i,1)=dd
        if((r(200).lt.p2).and.(y2(i).le.(ytot-adln)))cjoint2(i,1)=0.0
        if((r(200).ge.p2).and.(y2(i).le.(ytot-adln)))cjoint2(i,1)=sprkbr2*alo
\cg(1./r(1))*sin(alpho2)
        if((r(200).ge.p2).and.(y2(i).gt.(ytot-adln)))cjoint2(i,1)=(sprkbr2*al
\cog(1./r(1))*sin(alpho2))+dd
308     continue
        xdim=xd2(i)
        if(cjoint2(i,1).ge.xdim)cjoint2(i,1)=xdim
        if((y2(i).lt.(ytot-adln)).or.(dd.lt.0.0))go to 34
34      if(notpop.ne.0)write(6,320)cjoint2(i,1)
        if(cjoint2(i,1).eq.xdim)go to 151
        do 81 j=2,50,2
        cjoint2(i,j)=cjoint2(i,j-1)+spjtln2*alog(1./r(j))*sin(alpho2)
        if(cjoint2(i,j).ge.xdim)go to 91
        if(notpop.ne.0)write(6,320)cjoint2(i,j)
        cjoint2(i,j+1)=cjoint2(i,j)+sprkbr2*alog(1./r(j+1))*sin(alpho2)
        if(cjoint2(i,j+1).ge.xdim)go to 121
```

```
296,340p
         if(notpop.ne.0)write(6,320)cjoint2(i,j+1)
81       continue
91       cjoint2(i,j)=xdim
         if(notpop.ne.0)write(6,320)cjoint2(i,j)
         npt2(i)=j
         njoint=j/2
         nrn=j+1
         goto 151
121      cjoint2(i,j+1)=xdim
         if(notpop.ne.0)write(6,320)cjoint2(i,j+1)
         npt2(i)=j+1
         njoint=j/2
         nrn=j/2
151      continue
         percon2(i)=0.0
         if(cjoint2(i,1).eq.xdim)njoint=0
         if(cjoint2(i,1).eq.xdim)npt2(i)=1
         sumjtln=0.0
         dee(i)=0.0
         if((dd.ge.0.0).and.(y2(i).gt.(ytot-adln))) dee(i)=dd
         if(npt2(i).eq.1) go to 162
         do 161 j=1,njoint
         nnjt=2*j
         sumjtln=sumjtln+cjoint2(i,nnjt)-cjoint2(i,nnjt-1)
161      continue
162      continue
         percon2(i)=sumjtln*(100./(xdim-dee(i)))
         if(njoint.gt.0)go to 889
         if(notpop.ne.0)write(6,1288)
         go to 191
889      if(notpop.ne.0)write(6,128)njoint
191      if(notpop.ne.0) write(6,28)percon2(i)
198      continue
202      jpl2=i-1
         if(jpl1.le.1)go to 46
1289     format(/30x,'plane ',i3)
1287     format(10x,'--Joint Set One:Pts. of Intersection--')
         if(notpop.ne.0)write(6,1287)
         do 45 i=1,jpl1
         if(notpop.ne.0)write(6,1289)i
         k=0
         do 44 j=1,jpl2
         if(y1(i).gt.y2(j))go to 44
         dummy=(y2(j)-y1(i))*(cos(alpho1)+(sin(alpho1)/tan(alpho2-alpho1)))*si
\cn(alpho1)
         if(dummy.gt.xd1(i))go to 44
```

```
341,385p
          k=k+1
          ptint(i,k)=dummy
          y2ptint(i,k)=y2(j)
          ny2ptnt(i,k)=j
          if(notpop.ne.0)write(6,320)dummy
320       format(27x,f7.2)
44        continue
          do 43 n=(m1(i)+1),(m1(i)+k)
          plpt(i,n)=ptint(i,(n-m1(i)))
43        continue
          m1(i)=m1(i)+k
          nptint(i)=k
45        continue
46        do 10 i=1,jpl1
          do 20 j=1,(m1(i)-1)
          do 33 k=(j+1),m1(i)
          dummy=plpt(i,j)
          if(plpt(i,j).le.plpt(i,k))go to 33
          plpt(i,j)=plpt(i,k)
          plpt(i,k)=dummy
33        continue
20        continue
10        continue
          if(notpop.ne.0)write(6,1963)
          do 111 i=1,jpl1
          kak=1
          if(notpop.ne.0)write(6,1289)i
          do 222 j=1,m1(i)
          if(notpop.ne.0.0)write(6,320)plpt(i,j)
          if(nptint(i).eq.0)go to 370
          if(plpt(i,j).eq.ptint(i,kak))nmplpt(i,kak)=j
          if(plpt(i,j).eq.ptint(i,kak))kak=kak+1
370       continue
222       continue
111       continue
431       do 450 i=1,jpl1
          sm1(i,1)=0.0
450       continue
          do 500 j=1,jpl1
          mmmm=j+1
          if(mmmm.gt.jpl1)go to 500
          do 501 n=mmmm,jpl1
          vtran(j,n)=(y1(j)-y1(n))*tan(alpho1)
501       continue
500       continue
```

```
386,427p
          vtran(1,1)=xd1(1)
          k=1
          mpos=0
c
c                   The Following Routine Computes TheValues Of
c                   The Minimum Safety Margins Required To Initiate
c                   Failure Among The Various Joint PlanesAccording
c                   To The Dynamic Programming Algorithm Preset
c
          do 998 j=1,jpl1
          wcrit=0.0
          str=0.0
c
c                   Routine For Paths Involving Joint Set Two
c
          if(nptint(j).eq.0.and.notpot.ne.0) write(6,467)
          if(nptint(j).eq.0) go to 502
          do 503 int=1,nptint(j)
          merak(j,int)=0
          if(percon2(ny2ptnt(j,int)).eq.0.0.and.notpot.ne.0) write(6,466)ny2ptn
\ct(j,int)
          if(percon2(ny2ptnt(j,int)).eq.0.0)go to 503
          mpos=10
          if((j.gt.1.and.y2ptint(j,int).le.y1(j-1).and.y2ptint(j,int).ge.y1(j))
\c.or.(dee(ny2ptnt(j,int)).ge.xd1(j-1).and.dee(ny2ptnt(j,int)).le.xd1(j)).or.(j
\c.eq.1)) go to 498
c
c                   Routine For Paths Within A Region BoundedBy
c                   Two Joint Planes Of The First Joint Set
c
          khamsin(j,int)=1
          if(notpot.ne.0) write(6,470)j,j-1,y2ptint(j,int)
          crement(j)=vtran(j-1,j)*cos(alpho1)*sin(alpho2)/sin((alpha2-alpha1)*(
\cpi/180.0))
          do 499 n=1,npt2(ny2ptnt(j,int)),2
          if((cjoint2(ny2ptnt(j,int),n).gt.ptint(j,int)).and.(n.eq.1)) go to 41
\c4
          go to 412
414       if(notpot.ne.0) write(6,492)
          merak(j,int)=1000
          if(cjoint2(ny2ptnt(j,int),n+1).gt.ptint(j,int)) go to 503
412       if(cjoint2(ny2ptnt(j,int),n).lt.(ptint(j,int)-crement(j)).and.cjoint2
\c(ny2ptnt(j,int),n+1).lt.(ptint(j,int)-crement(j)).and.((n+2).gt.npt2(ny2ptnt(
\cj,int)).or.cjoint2(ny2ptnt(j,int),n+2).gt.ptint(j,int))) go to 432
          go to 410
432       if(notpot.ne.0)write(6,492)
          merak(j,int)=1000
          if(cjoint2(ny2ptnt(j,int),n+1).gt.ptint(j,int)) go to 503
410       if(cjoint2(ny2ptnt(j,int),n).le.(ptint(j,int)-crement(j)).and.cjoint2
\c(ny2ptnt(j,int),n+1).ge.ptint(j,int))go to 433
```

```
470,512p
         call msaf(cjoint2(ny2ptnt(j,int),n21),0.0,y2ptint(j,int),point,0.0,y2
\cptint(j,int),mpos,samr,cork,phork,cojt,phojt,phoult,theto,alpho1,alpho2,sinrk
\c,cosrk,tanrk,xcteta,pi,beta,sf,sv,sisa,taufr,wpth,ywt)
         if(point.ne.(ptint(j,int)-crement(j)))go to 1999
         do 2000 kok=1,nptint(j-1)
         if((abs(ptint(j-1,kok)-point)).lt.(.001))go to 2001
2000     continue
2001     esem=sm1(j-1,nmplpt(j-1,kok))
1999     if(point.eq.cjoint2(ny2ptnt(j,int),n21-1))esem=sm2(ny2ptnt(j,int),n21
\c-1)
         if(islero.eq.0)go to 3500
         if((sf+esem).gt.dummy2)go to 3600
3500     islero=1
         if(point.eq.(ptint(j,int)-crement(j)))go to 5300
         nuplane=ny2ptnt(j,int)
         nsetu=2
         nref=n21-1
         go to 5301
5300     nuplane=j-1
         nsetu=1
         nref=nmplpt(j-1,kok)
5301     bottom=cjoint2(ny2ptnt(j,int),n21)
         upper=point
         dummy1=beta
         dummy2=sf+esem
         dummy3=sisa
         dummy4=wpth
         dummy5=wpth+wmin1(j-1,kok)
         if(point.eq.cjoint2(ny2ptnt(j,int),n21-1))dummy5=wpth+wmin2(ny2ptnt(j
\c,int),n21-1)
         dummy6=sf
3600     continue
         do 3007 kkk=1,njump
         if((j-kkk).le.0)go to 3008
         if(m1(j-kkk).eq.1)go to 3007
         do 3006 n3=1,m1(j-kkk)
         horiz=y2ptint(j,int)+(cjoint2(ny2ptnt(j,int),n21)/tan(alpho2))-y1(j-k
\ckk)-(plpt((j-kkk),n3)/tan(alpho1))
         verti=cjoint2(ny2ptnt(j,int),n21)-plpt((j-kkk),n3)
         if((((horiz**2.0)+(verti**2.0))**0.50).gt.distmn)go to 3007
         ang21=atan(abs(verti/horiz))
         if(ang21.eq.alpho2)go to 3006
         if(ang21.gt.(90.0*(pi/180.0)))go to 3007
         mpos=0
         call msaf(cjoint2(ny2ptnt(j,int),n21),0.0,y2ptint(j,int),plpt((j-kkk)
\c,n3),y1(j-kkk),0.0,mpos,samr,cork,phork,cojt,phojt,phoult,theto,alpho1,alpho2
\c,sinrk,cosrk,tanrk,xcteta,pi,beta,sf,sv,sisa,taufr,wpth,ywt)
         if(beta.gt.90.0)go to 3007
         if(islero.eq.0)go to 3501
         if((sf+sm1(j-kkk,n3)).gt.dummy2)go to 3601
```

```
513,553p
3501      islero=1
          nuplane=j-KKK
          nsetu=1
          nref=n3
          bottom=cjoint2(ny2ptnt(j,int),n21)
          upper=pipt((j-KKK),n3)
          dummy1=beta
          dummy2=sf+sm1(j-KKK,n3)
          dummy3=sisa
          dummy4=wpth
          dummy5=wpth+wmin1(j-KKK,n3)
          dummy6=sf
3601      continue
3006      continue
3007      continue
3008      continue
          if((int-1).eq.0)go to 3010
          if(merak(j,int-1).eq.111.or.merak(j,int-1).eq.1000)go to 3010
          tlimit=0.0
          if(Khamsin(j,int-1).eq.1)tlimit=(ptint(j,int-1)-crement(j))
          do 3009 n2n=2,npt2(ny2ptnt(j,int-1)),2
          if(cjoint2(ny2ptnt(j,int-1),n2n).lt.tlimit)go to 3009
          if(((cjoint2(ny2ptnt(j,int),n21)-cjoint2(ny2ptnt(j,int-1),n2n))/tan(a
\clpha1)).lt.(y2ptint(j,int)-y2ptint(j,int-1)))go to 3010
          mpos=0
          call msaf(cjoint2(ny2ptnt(j,int),n21),0.0,y2ptint(j,int),cjoint2(ny2p
\ctnt(j,int-1),n2n),0.0,y2ptint(j,int-1),mpos,samr,corK,phorK,coJt,phoJt,phoult
\c,theto,alpha1,alpha2,sinrK,cosrK,tanrK,xcteta,pi,beta,sf,sv,sisa,taufr,wpth,y
\cwt)
          if(islero.eq.0)go to 3502
          if((sf+sm2(ny2ptnt(j,int-1),n2n)).gt.dummy2)go to 3602
3502      islero=1
          nuplane=ny2ptnt(j,int-1)
          nsetu=2
          nref=n2n
          bottom=cjoint2(ny2ptnt(j,int),n21)
          upper=cjoint2(ny2ptnt(j,int-1),n2n)
          dummy1=beta
          dummy2=sf+sm2(ny2ptnt(j,int-1),n2n)
          dummy3=sisa
          dummy4=wpth
          dummy5=wpth+wmin2(ny2ptnt(j,int-1),n2n)
          dummy6=sf
3602      continue
3009      continue
```

```
554,596p
3010      if(notpot.ne.0)write(6,3749)bottom,upper
          path=(bottom-upper)/sin(dummy1*(pi/180.))
          if(notpot.ne.0)write(6,3748)ny2ptnt(j,int),2,n21,nuplane,nsetu,nref
          if(notpot.ne.0)write(6,3750)dummy1,path,dummy3,dummy4,dummy6
          if(nsetu.eq.1)go to 3522
          smpthr2(ny2ptnt(j,int),n21)=path+smpthr2(nuplane,nref)
          sm2(ny2ptnt(j,int),n21)=dummy2
          smptht2(ny2ptnt(j,int),n21)=path+smptht2(nuplane,nref)
          go to 3523
3522      smpthr2(ny2ptnt(j,int),n21)=path+smpthr1(nuplane,nref)
          smptht2(ny2ptnt(j,int),n21)=path+smptht1(nuplane,nref)
3523      smpthj2(ny2ptnt(j,int),n21)=smpthr2(ny2ptnt(j,int),n21)-smptht2(ny2pt
\cnt(j,int),n21)
          sm2(ny2ptnt(j,int),n21)=dummy2
          if(notpot.ne.0)write(6,3746)sm2(ny2ptnt(j,int),n21),smpthj2(ny2ptnt(j
\c,int),n21),smptht2(ny2ptnt(j,int),n21),dummy5
          go to 3003
3005      point=cjoint2(ny2ptnt(j,int),n21-1)
          if(n21.eq.ksb)point=(ptint(j,int)-crement(j))
          start=cjoint2(ny2ptnt(j,int),n21)
          if(cjoint2(ny2ptnt(j,int),n21).gt.ptint(j,int))start=ptint(j,int)
          mpos=10
          call msaf(start,0.0,y2ptint(j,int),point,0.0,y2ptint(j,int),mpos,gamr
\c,cork,phork,cojt,phojt,phoult,theto,alpho1,alpho2,sinrk,cosrk,tanrk,xcteta,pi
\c,beta,sf,sv,siga,taufr,wpth,ywt)
          if(point.eq.(ptint(j,int)-crement(j)))go to 1998
          esemu=sm2(ny2ptnt(j,int),n21-1)
          wminu=wmin2(ny2ptnt(j,int),n21-1)
          go to 1997
1998      continue
          do  8000 kaka=1,(m1(j-1))
          if(point.gt.plpt(j-1,kaka))go to 8000
          go to 8001
8000      continue
8001      kaka=kaka-1
          esemu=sm1(j-1,kaka)
          wminu=wmin1(j-1,kaka)
1997      if(start.eq.ptint(j,int))go to 1996
          sm2(ny2ptnt(j,int),n21)=sf+esemu
          wmin2(ny2ptnt(j,int),n21)=wpth+wminu
          esem=sf+esemu
          wmin=wpth+wminu
          go to 1995
1996      do 8002 koko=1,m1(j)
          if(plpt(j,koko).lt.ptint(j,int))go to 8002
          go to 8003
8002      continue
```

```
597,636p
8003      sm12(j,KoKo)=sf+esemu
          wmin12(j,KoKo)=wpth+wminu
          wmin=wmin12(j,KoKo)
          esem=sm12(j,KoKo)
1995      continue
          if(notpot.ne.0)write(6,3749)start,point
          path=((start-point)/sin(beta*(pi/180.)))*(-1.0)
          if(notpot.ne.0)write(6,3747)ny2ptnt(j,int),2
          if(notpot.ne.0)write(6,3750)beta,path,siga,wpth,sf
          if(point.gt.(ptint(j,int)-crement(j)).and.start.lt.ptint(j,int))smpth
\ct2(ny2ptnt(j,int),n21)=(abs(path))+smptht2(ny2ptnt(j,int),n21-1)
          if(point.gt.(ptint(j,int)-crement(j)).and.start.lt.ptint(j,int))smpth
\cr2(ny2ptnt(j,int),n21)=smpthr2(ny2ptnt(j,int),n21-1)
          if(point.eq.(ptint(j,int)-crement(j)).and.start.lt.ptint(j,int))smpth
\ct2(ny2ptnt(j,int),n21)=(abs(path))+smptht1(j-1,KaKa)
          if(point.eq.(ptint(j,int)-crement(j)).and.start.lt.ptint(j,int))smpth
\cr2(ny2ptnt(j,int),n21)=smpthr1(j-1,KaKa)
          if(start.lt.ptint(j,int))psem=smptht2(ny2ptnt(j,int),n21)
          if(start.lt.ptint(j,int))smpthj2(ny2ptnt(j,int),n21)=(smptht2(ny2ptnt
\c(j,int),n21)-smpthr2(ny2ptnt(j,int),n21))*(-1.0)
          smpthj=smpthj2(ny2ptnt(j,int),n21)
          if(start.lt.ptint(j,int))sm2(ny2ptnt(j,int),n21)=esem
          if(start.lt.ptint(j,int))wmin2(ny2ptnt(j,int),n21)=wmin
          if(start.lt.ptint(j,nt))go to 6170
          if(point.gt.(ptint(j,int)-crement(j)))smptht12(j,KoKo)=(abs(path))+sm
\cptht2(ny2ptnt(j,int),n21-1)
          if(point.gt.(ptint(j,int)-crement(j)))smpthr12(j,KoKo)=smpthr2(ny2ptn
\ct(j,int),n21-1)
          if(point.eq.(ptint(j,int)-crement(j)))smptht12(j,KoKo)=(abs(path))+sm
\cptht1(j-1,KaKa)
          if(point.eq.(ptint(j,int)-crement(j)))smpthr12(j,KoKo)=smpthr1(j-1,Ka
\cKa)
          psem=smptht12(j,KoKo)
          smpthj12(j,KoKo)=(smptht12(j,KoKo)-smpthr12(j,KoKo))*(-1.0)
          smpthj=smpthj12(j,KoKo)
          sm12(j,KoKo)=esem
          wmin12(j,KoKo)=wmin
6170      continue
          if(notpot.ne.0)write(6,3746)esem,smpthj,psem,wmin
3003      continue
3300      last(j,int)=0
          if(merak(j,int).eq.100.or.merak(j,int).eq.0)last(j,int)=(n21-1)
          if(last(j,int).eq.0)go to 3222
          if(((-1)**last(j,int)).lt.0)print,"ERROR 1"
          if(cjoint2(ny2ptnt(j,int),last(j,int)).lt.(ptint(j,int)-crement(j)).o
\cr.cjoint2(ny2ptnt(j,int),last(j,int)).gt.ptint(j,int))print,"ERROR 2"
3222      continue
          go to 503
c
c                   Routine For Paths Within A Region Bounded
```

```
637,677p
c                    By The Free Surface And A Joint Plane  Of
c                                Set Two
c
498       if(notpot.ne.0.and.dee(ny2ptnt(j,int)).eq.0.0)write(6,468)j,y2ptint(j
\c,int)
          if(notpot.ne.0.and.dee(ny2ptnt(j,int)).gt.0.0)write(6,469)j,y2ptint(j
\c,int)
          khamsin(j,int)=0
          do 497 n=1,npt2(ny2ptnt(j,int)),2
          if(cjoint2(ny2ptnt(j,int),n).gt.ptint(j,int).and.(n.eq.1))go to 415
          go to  416
415       if(notpot.ne.0)write(6,492)
          merak(j,int)=1000
          if(cjoint2(ny2ptnt(j,int),n+1).gt.ptint(j,int))go to 503
416       if(cjoint2(ny2ptnt(j,int),n).eq.dee(ny2ptnt(j,int)).and.(cjoint2(ny2p
\ctnt(j,int),n+1).ge.ptint(j,int)))go to 411
          go to 444
411       if(notpot.ne.0)write(6,493)
          merak(j,int)=111
          if(cjoint2(ny2ptnt(j,int),n+1).ge.ptint(j,int))go to 4000
444       if((cjoint2(ny2ptnt(j,int),n).eq.dee(ny2ptnt(j,int))).and.(cjoint2(ny
\c2ptnt(j,int),n+1).lt.ptint(j,int))) go to 417
          go to 446
417       if(notpot.ne.0)write(6,494)
          merak(j,int)=100
446       if(cjoint2(ny2ptnt(j,int),n).le.ptint(j,int).and.(cjoint2(ny2ptnt(j,i
\cnt),n+1).ge.ptint(j,int)))go to 418
          go to 497
418       if(notpot.ne.0)write(6,495)
          if(merak(j,int).eq.100)merak(j,int)=101
          if(merak(j,int).ne.101)merak(j,int)=1001
          if(cjoint2(ny2ptnt(j,int),n+1).ge.ptint(j,int)) go to 4000
497       continue
4000      if(merak(j,int).eq.0.and.notpot.ne.0)write(6,491)
          if(merak(j,int).eq.1000) go to 496
          do 4118 n2=1,npt2(ny2ptnt(j,int))
          islero=0
          nuplane=0
          nsetu=0
          nref=0
          bottom=0.0
          upper=0.0
          dummy1=0.0
          dummy2=0.0
          dummy3=0.0
          dummy4=0.0
```

```
678,719p
        dummy5=0.0
        dummy6=0.0
        if(n2.gt.1) go to 4002
        if(cjoint2(ny2ptnt(j,int),n2).gt.0.0) go to 4001
        mpos=10
        n2=2
        point=cjoint2(ny2ptnt(j,int),n2)
        if(cjoint2(ny2ptnt(j,int),n2).gt.ptint(j,int))point=ptint(j,int)
        call msaf(point,0.0,y2ptint(j,int),dee(ny2ptnt(j,int)),0.0,y2ptint(j,
\cint),mpos,gamr,cork,phork,cojt,phojt,phoult,theto,alpho1,alpho2,sinrk,cosrk,t
\canrk,xcteta,pi,beta,sf,sv,sisa,taufr,wpth,ywt)
        path=((point-dee(ny2ptnt(j,int)))/sin(beta*(pi/180.0)))*(-1.0)
        if(point.eq.ptint(j,int))sm12(j,nmplpt(j,int))=sf
        if(point.eq.ptint(j,int))wmin12(j,nmplpt(j,int))=wpth
        if(point.lt.ptint(j,int))sm2(ny2ptnt(j,int),n2)=sf
        if(point.lt.ptint(j,int))wmin2(ny2ptnt(j,int),n2)=wpth
        if(point.lt.ptint(j,int))smpthj2(ny2ptnt(j,int),n2)=path
        if(point.lt.ptint(j,int))smptht2(ny2ptnt(j,int),n2)=path*(-1.0)
        if(point.lt.ptint(j,int))smpthr2(ny2ptnt(j,int),n2)=0.0
        if(point.eq.ptint(j,int))smpthr12(ny2ptnt(j,int),n2)=0.0
        if(point.eq.ptint(j,int))smpthj12(j,nmplpt(j,int))=path
        if(point.eq.ptint(j,int))smptht12(j,nmplpt(j,int))=path*(-1.0)
        if(dee(ny2ptnt(j,int)).lt.0.0)go to 4006
        if(notpot.ne.0)write(6,3749)point,dee(ny2ptnt(j,int))
        if(notpot.ne.0)write(6,3747)ny2ptnt(j,int),2
        if(notpot.ne.0)write(6,3750)beta,path,sisa,wpth,sf
        if(notpot.ne.0)write(6,3746)sf,path,(abs(path)),wpth
        go to 4118
4001    mpos=1
        if(dee(ny2ptnt(j,int)).lt.0.0)go to 4006
        point=cjoint2(ny2ptnt(j,int),n2)
        call msaf(cjoint2(ny2ptnt(j,int),n2),0.0,y2ptint(j,int),dee(ny2ptnt(j
\c,int)),0.0,y2ptint(j,int),mpos,gamr,cork,phork,cojt,phojt,phoult,theto,alpho1
\c,alpho2,sinrk,cosrk,tanrk,xcteta,pi,beta,sf,sv,sisa,taufr,wpth,ywt)
        islero=1
        nuplane=ny2ptnt(j,int)
        nsetu=2
        nref=0
        bottom=cjoint2(ny2ptnt(j,int),n2)
        upper=dee(ny2ptnt(j,int))
        dummy1=beta
        dummy2=sf
        dummy3=sisa
        dummy4=wpth
        if(int.eq.1)go to 4006
        if(merak(j,int-1).eq.111.or.merak(j,int-1).eq.1000)go to 4006
```

```
720,761p
        if(y2ptint(j,int).gt.y1(j))go to 4009
4006    if(dee(ny2ptnt(j,int)).ge.-0.9)go to 6006
        print,"ERROR",j,int,dee(ny2ptnt(j,int))
        go to 1901
6006    if(notpot.ne.0)write(6,3749)point,dee(ny2ptnt(j,int))
        path=(point-dee(ny2ptnt(j,int)))/sin(beta*(pi/180.0))
        if(notpot.ne.0)write(6,3750)beta,path,sisa,wpth,sf
        if(notpot.ne.0)write(6,3746)sf,0.0,path,wpth
        smpthr2(ny2ptnt(j,int),n2)=path
        smptht2(ny2ptnt(j,int),n2)=path
        sm2(ny2ptnt(j,int),n2)=sf
        wmin2(ny2ptnt(j,int),n2)=wpth
        go to 4118
4002    if((-1.0)**n2)4003,4003,4004
4003    if(cjoint2(ny2ptnt(j,int),n2).gt.ptint(j,int)) go to 496
        mpos=0
        point=cjoint2(ny2ptnt(j,int),n2)
        call msaf(cjoint2(ny2ptnt(j,int),n2),0.0,y2ptint(j,int),cjoint2(ny2pt
\cnt(j,int),n2-1),0.0,y2ptint(j,int),mpos,gamr,cork,phork,cojt,phojt,phoult-the
\cto,alpho1,alpho2,sinrk,cosrk,tanrk,xcteta,pi,beta,sf,sv,sisa,taufr,wpth,ywt)
        if(islero.eq.0)go to 4500
        if((sf+sm2(ny2ptnt(j,int),n2-1)).gt.dummy2)go to 4501
4500    islero=1
        nuplane=ny2ptnt(j,int)
        nsetu=2
        nref=n2-1
        bottom=point
        upper=cjoint2(ny2ptnt(j,int),n2-1)
        dummy1=beta
        dummy2=sf+sm2(ny2ptnt(j,int),n2-1)
        dummy3=sisa
        dummy4=wpth
        dummy5=wpth+wmin2(ny2ptnt(j,int),n2-1)
        dummy6=sf
4501    continue
4007    if(int.eq.1)go to 4008
        if(merak(j,int-1).eq.111.or.merak(j,int-1).eq.1000)go to 4008
4009    do 4005 nt=2,npt2(ny2ptnt(j,int-1)),2
        if(cjoint2(ny2ptnt(j,int-1),nt).gt.ptint(j,int-1).or.cjoint2(ny2ptnt(
\cj,int-1),nt).gt.cjoint2(ny2ptnt(j,int),n2)) go to 4008
        ambdo=atan((cjoint2(ny2ptnt(j,int),n2)-cjoint2(ny2ptnt(j,int-1),nt))/
\c(y2ptint(j,int)-y2ptint(j,int-1)+((cjoint2(ny2ptnt(j,int),n2)-cjoint2(ny2ptnt
\c(j,int-1),nt))/tan(alpho2))))
        ambda=ambdo*(180.0/pi)
        if(ambda.lt.alpha1)go to 4008
        mpos=0
        point=cjoint2(ny2ptnt(j,int),n2)
```

```
762,803p
          call msaf(cjoint2(ny2ptnt(j,int),n2),0.0,y2ptint(j,int),cjoint2(ny2pt
\cnt(j,int-1),nt),0.0,y2ptint(j,int-1),mpos,samr,cork,phork,cojt,phojt,phoult,t
\cheto,alpho1,alpho2,sinrk,cosrk,tanrk,xcteta,pi,beta,sf,sv,sisa,taufr,wpth,ywt
\c)
4109      if(islero.eq.0)go to 4509
          if(sf.gt.dummy2)go to 4510
4509      islero=1
          nuplane=ny2ptnt(j,int-1)
          nsetu=2
          nref=nt
          bottom=cjoint2(ny2ptnt(j,int),n2)
          upper=cjoint2(ny2ptnt(j,int-1),nt)
          dummy1=beta
          dummy2=sf
          dummy3=sisa
          dummy4=wpth
          dummy5=wpth
          dummy6=sf
4510      continue
4005      continue
          go to 4008
4004      mpos=10
          point=cjoint2(ny2ptnt(j,int),n2)
          if(cjoint2(ny2ptnt(j,int),n2).ge.ptint(j,int)) point=ptint(j,int)
          call msaf(point,0.0,y2ptint(j,int),cjoint2(ny2ptnt(j,int),n2-1),0.0,y
\c2ptint(j,int),mpos,samr,cork,phork,cojt,phojt,phoult,theto,alpho1,alpho2,sinr
\ck,cosrk,tanrk,xcteta,pi,beta,sf,sv,sisa,taufr,wpth,ywt)
          if(notpot.ne.0)write(6,3749)point,cjoint2(ny2ptnt(j,int),n2-1)
          path=((point-cjoint2(ny2ptnt(j,int),n2-1))/sin(beta*(pi/180.0)))*(-1.
\c0)
          if(notpot.ne.0)write(6,3747)ny2ptnt(j,int),2
          if(notpot.ne.0)write(6,3750)beta,path,sisa,wpth,sf
          if(point.eq.ptint(j,int))go to 6700
          sm2(ny2ptnt(j,int),n2)=sf+sm2(ny2ptnt(j,int),n2-1)
          wmin2(ny2ptnt(j,int),n2)=wpth+wmin2(ny2ptnt(j,int),n2-1)
          smpthj2(ny2ptnt(j,int),n2)=path+smpthj2(ny2ptnt(j,int),n2-1)
          smptht2(ny2ptnt(j,int),n2)=(abs(path))+smptht2(ny2ptnt(j,int),n2-1)
          smpthr2(ny2ptnt(j,int),n2)=smptht2(ny2ptnt(j,int),n2)+smpthj2(ny2ptnt
\c(j,int),n2)
          smpthj=smpthj2(ny2ptnt(j,int),n2)
          smptht=smptht2(ny2ptnt(j,int),n2)
          esem=sm2(ny2ptnt(j,int),n2)
          wmin=wmin2(ny2ptnt(j,int),n2)
          go to 6701
6700      sm12(j,nmplpt(j,int))=sf+sm2(ny2ptnt(j,int),n2-1)
          wmin12(j,nmplpt(j,int))=wpth+wmin2(ny2ptnt(j,int),n2-1)
          smpthj12(j,nmplpt(j,int))=path+smpthj2(ny2ptnt(j,int),n2-1)
          smptht12(j,nmplpt(j,int))=(abs(path))+smptht2(ny2ptnt(j,int),n2-1)
          smpthj=smpthj12(j,nmplpt(j,int))
```

```
804,845p
        smptht=smptht12(j,nmplpt(j,int))
        smpthr12(j,nmplpt(j,int))=smptht+smpthj
        esem=sm12(j,nmplpt(j,int))
        wmin=wmin12(j,nmplpt(j,int))
6701    if(notpot.ne.0)write(6,3746)esem,smpthj,smptht,wmin
        go to 4118
4008    continue
        if(notpot.ne.0)write(6,3749)bottom,upper
        path=((bottom-upper)/sin(dummy1*(pi/180.0)))
        if(notpot.ne.0)write(6,3748)ny2ptnt(j,int),2,n2,nuplane,nsetu,nref
        if(notpot.ne.0)write(6,3750)dummy1,path,dummy3,dummy4,dummy6
        if(nuplane.lt.ny2ptnt(j,int))smptht2(ny2ptnt(j,int),n2)=smptht2(nupla
\cne,nt)+path
        if(nuplane.lt.ny2ptnt(j,int))smpthr2(ny2ptnt(j,int),n2)=smpthr2(nupla
\cne,nt)+path
        if(nuplane.eq.ny2ptnt(j,int).and.nref.eq.0)smptht2(ny2ptnt(j,int),n2)
\c=path
        if(nuplane.eq.ny2ptnt(j,int).and.nref.eq.0)smpthr2(ny2ptnt(j,int),n2)
\c=path
        if(nuplane.eq.ny2ptnt(j,int).and.nref.gt.0)smptht2(ny2ptnt(j,int),n2)
\c=path+smptht2(ny2ptnt(j,int),n2-1)
        if(nuplane.eq.ny2ptnt(j,int).and.nref.gt.0)smpthr2(ny2ptnt(j,int),n2)
\c=path+smpthr2(ny2ptnt(j,int),n2-1)
        smpthj2(ny2ptnt(j,int),n2)=smpthr2(ny2ptnt(j,int),n2)-smptht2(ny2ptnt
\c(j,int),n2)
        sm2(ny2ptnt(j,int),n2)=dummy2
        wmin2(ny2ptnt(j,int),n2)=dummy5
        if(notpot.ne.0)write(6,3746)dummy2,smpthj2(ny2ptnt(j,int),n2),smptht2
\c(ny2ptnt(j,int),n2),dummy5
4118    continue
496     last(j,int)=0
        if(merak(j,int).eq.100.or.merak(j,int).eq.0)last(j,int)=(n2-1)
        if(last(j,int).eq.0)go to 3333
        if(((-1)**last(j,int)).lt.0)print,"ERROR 1"
        if(cjoint2(ny2ptnt(j,int),last(j,int)).gt.ptint(j,int))print,"ERROR 2
\c"
3333    continue
503     continue
502     continue
        str=0.0
c
c                 Routine For Paths Within A Region Bounded
c                 By A Point On A Joint Plane Of Joint Set
c                 Two And A Point Of Either Set  Above   It
c
        do 950 i=1,m1(j)
        islero=0
        nuplane=0
        nsetu=0
        nref=0
        bottom=0.0
```

```
846,887p
        upper=0.0
        dummy1=0.0
        dummy2=0.0
        dummy3=0.0
        special=0.0
        dummy4=0.0
        dummy5=0.0
        dummy6=0.0
        nen=0
        if(plpt(j,i).eq.0.0)go to 950
        miura(j,i)=0
        if(nptint(j).eq.0)go to 3080
        do 3070 int=1,nptint(j)
        if(ptint(j,int).eq.plpt(j,i))go to 3071
        if(int.eq.nptint(j))go to 3072
3070    continue
3071    if(merak(j,int).eq.1000)go to 3072
        miura(j,i)=1
        if(merak(j,int).eq.0.or.merak(j,int).eq.100.or.merak(j,int).eq.1000)g
\co to 3072
        do 6801 nunu=1,npt2(ny2ptnt(j,int)),2
        if(cjoint2(ny2ptnt(j,int),nunu+1).lt.ptint(j,int))go to 6801
        nref=nunu
        go to 6802
6801    continue
6802    upper=cjoint2(ny2ptnt(j,int),nref)
        bottom=ptint(j,int)
        islero=1
        nuplane=ny2ptnt(j,int)
        nsetu=2
        special=6.0
        mpos=10
        call msaf(bottom,0.0,y2ptint(j,int),upper,0.0,y2ptint(j,int),mpos,gam
\cr,cork,phork,cojt,phojt,phoult,theto,alpho1,alpho2,sinrk,cosrk,tanrk,xcteta,p
\ci,beta,sf,sv,sisa,taufr,wpth,ywt)
        dummy1=beta
        dummy2=sm12(j,i)
        dummy3=sisa
        dummy4=wpth
        dummy5=wmin12(j,i)
        dummy6=sf
3072    continue
        do 3077 inte=1,nptint(j)
        if(miura(j,i).eq.1.and.(merak(j,int).eq.1001.or.merak(j,int).eq.101.o
\cr.merak(j,int).eq.111))go to 3080
        if(merak(j,inte).eq.1000.or.merak(j,inte).eq.111)go to 3077
```

```
888,930p
        if(y2ptint(j,inte).gt.((plpt(j,i)/tan(alpho1))+y1(j)))go to 3080
        do 3979 nnn=2,npt2(ny2ptnt(j,inte)),2
        if(cjoint2(ny2ptnt(j,inte),nnn).gt.ptint(j,inte))go to 3077
        if(((cjoint2(ny2ptnt(j,inte),nnn)/tan(alpho2))+y2ptint(j,inte)).gt.((
\cplpt(j,i)/tan(alpho1))+y1(j)))go to 3077
        verti=(plpt(j,i)-cjoint2(ny2ptnt(j,inte),nnn))
        horiz=(((plpt(j,i)/tan(alpho1))+y1(j))-(y2ptint(j,inte)+(cjoint2(ny2p
\ctnt(j,inte),nnn)/tan(alpho2))))
        if(((verti**2.+horiz**2.)**.5).gt.distmn)go to 3077
        if(plpt(j,i).gt.ptint(j,inte))go to 3077
        do 3900 nen=2,npt1(j),2
        if(plpt(j,i).eq.cjoint1(j,nen))go to 3077
        if(plpt(j,i).lt.cjoint1(j,nen))go to 3901
3900    continue
3901    continue
        point=cjoint2(ny2ptnt(j,inte),nnn)
        mpos=0
        call msaf(plpt(j,i),y1(j),0.0,point,0.0,y2(ny2ptnt(j,inte)),mpos,gamr
\c,cork,phork,cojt,phojt,phoult,theto,alpho1,alpho2,sinrk,cosrk,tanrk,xcteta,pi
\c,beta,sf,sv,sisa,tanfr,wpth,ywt)
        if(islero.eq.0)go to 4110
        if((sf+sm2(ny2ptnt(j,inte),nnn)).gt.dummy2)go to 4111
4110    islero=1
        nuplane=ny2ptnt(j,inte)
        nsetu=2
        nref=nnn
        bottom=plpt(j,i)
        upper=point
        dummy1=beta
        dummy2=(sf+sm2(ny2ptnt(j,inte),nnn))
        dummy3=sisa
        dummy4=wpth
        dummy5=wpth+wmin2(ny2ptnt(j,inte),nnn)
        dummy6=sf
4111    continue
3979    continue
3077    continue
3080    continue
c
c                   routine for paths with only in plane transitions allowed
c
        if(plpt(j,i).ge.cjoint1(j,1)) go to 510
        if(i.eq.1)point=0.0
        if(plpt(j,i).eq.0.0)go to 950
        if(i.gt.1)point=plpt(j,i-1)
        mpos=0
        call msaf(plpt(j,i),y1(j),0.0,point,y1(j),0.0,mpos,gamr,cork,phork,co
\cjt,phojt,phoult,theto,alpho1,alpho2,sinrk,cosrk,tanrk,xcteta,pi,beta,sf,sv,si
\csa,tanfr,wpth,ywt)
```

```
S31,977p
         esem=0.0
         wmin=0.0
         if(point.eq.0.0)esem=sm1(j,i-1)
         if(point.eq.(0.0))wmin=wmin1(j,i-1)
         if(islero.eq.0)go to 4112
         if((sf+esem).gt.dummy2)go to 4114
4112     islero=1
         nuplane=j
         nsetu=1
         nref=1
         if(i.ge.2)nref=i-1
         bottom=plpt(j,i)
         upper=point
         dummy1=beta
         dummy2=sf+esem
         dummy3=sisa
         dummy4=wpth
         dummy5=wpth+wmin
         dummy6=sf
4114     continue
         go to 949
510      continue
         do 560 n=1,npt1(j)
         if(plpt(j,i).gt.cjoint1(j,n)) go to  560
c
c                  In Plane And To Free Surface Transitions
c
         if((-1.0)**n)520,520,530
520      continue
         mpos=0
         if(plpt(j,i).eq.cjoint1(j,n).and.plpt(j,i).le.zp(j)) mpos=1
         point=0.0
         if(i.gt.1)point=plpt(j,i-1)
         call msaf(plpt(j,i),y1(j),0.0,point,y1(j),0.0,mpos,gamr,cork,phork,co
\cjt,phojt,phoult,theto,alpho1,alpho2,sinrk,cosrk,tanrk,xcteta,pi,beta,sf,sv,si
\cga,taufr,wpth,ywt)
         if(point.eq.0.0)esem=0.0
         if(point.eq.0.0)wmin=0.0
         if(point.gt.0.0)esem=sm1(j,i-1)
         if(point.gt.0.0)wmin=wmin1(j,i-1)
         if(islero.eq.0)go to 4630
         if((sf+esem).gt.dummy2)go to 4631
4630     islero=1
         nuplane=j
         nsetu=1
         nref=i-1
         bottom=plpt(j,i)
         upper=plpt(j,i-1)
         dummy1=beta
```

```
978,1020p
          dummy2=sf+esem
          dummy3=sisa
          dummy4=wpth
          dummy5=wpth+wmin
          dummy6=sf
4631      continue
          if(str.gt.sv.and.mpos.eq.1)str=sv
          if(j.eq.1.or.miura(j,i).eq.1)go to 949
          if(vtran(j-1,j).gt.plpt(j,i))go to 949
          do 3090 kub=1,njump
          if((j-kub).le.0)go to 3092
          do 3091 kin=1,(ml(j-kub)-1)
          if(plpt(j-kub,kin).gt.(plpt(j,i)-vtran(j-1,j)))go to 3092
          mpos=0
          call msaf(plpt(j,i),yl(j),0.0,plpt(j-kub,kin),yl(j-kub),0.0,mpos,gamr
\c,cork,phork,cojt,phojt,phoult,theto,alpho1,alpho2,sinrk,cosrk,tanrk,xcteta,pi
\c,beta,sf,sv,sisa,taufr,wpth,ywt)
          if(islero.eq.0)go to 4632
          if((sf+sml(j-kub,kin)).gt.dummy2)go to 4633
4632      islero=1
          nuplane=j-kub
          nsetu=1
          nref=kin
          bottom=plpt(j,i)
          upper=plpt(j-kub,kin)
          dummy1=beta
          dummy2=(sf+sml(j-kub,kin))
          dummy3=sisa
          dummy4=wpth
          dummy5=wpth+wminl(j-kub,kin)
          dummy6=sf
4633      continue
3091      continue
3090      continue
3092      go to 949
530       continue
          mpos=2
          call msaf(plpt(j,i),yl(j),0.0,plpt(j,i-1),yl(j),0.0,mpos,gamr,cork,ph
\cork,cojt,phojt,phoult,theto,alpho1,alpho2,sinrk,cosrk,tanrk,xcteta,pi,beta,sf
\c,sv,sisa,taufr,wpth,ywt)
          path=((plpt(j,i)-plpt(j,i-1))/sin(beta*(pi/180.0)))*(-1.0)
          smpthj=0.0
          if((i-1).ge.1)smpthj=smpthjl(j,(i-1))
          smpthjl(j,i)=path+smpthj
          smptht=0.0
          if((i-1).ge.1)smptht=smpthtl(j,i-1)
          smpthtl(j,i)=(abs(path))+smptht
```

```
1021,1064p
        smpthr1(j,i)=smptht1(j,i)+smpthj1(j,i)
        sm1(j,i)=sf+sm1(j,i-1)
        wmin1(j,i)=wpth+wmin1(j,i-1)
        if(miura(j,i).eq.0)go to 2501
        if(merak(j,int).eq.100.or.merak(j,int).eq.0)go to 2501
2502    if(sm1(j,i).lt.sm12(j,i))go to 2501
        smpthj1(j,i)=smpthj12(j,i)
        sm1(j,i)=sm12(j,i)
        wmin1(j,i)=wmin12(j,i)
        smptht1(j,i)=smptht12(j,i)
        smpthr1(j,i)=smptht1(j,i)+smpthj1(j,i)
        if(merak(j,int).ne.111)go to 6503
        if(khamsin(j,int).eq.0)path=plpt(j,i)/sin(alpho2)
        if(khamsin(j,int).eq.1)path=crement(j)/sin(alpho2)
        go to 6504
6503    do 6501 khari=1,npt2(ny2ptnt(j,int)),2
        if (cjoint2(ny2ptnt(j,int),khari).lt.ptint(j,int).and.cjoint2(ny2ptnt
\c(j,int),khari+1).lt.ptint(j,int))go to 6501
        go to 6502
6501    continue
6502    path=(plpt(j,i)-cjoint2(ny2ptnt(j,int),khari))/sin(alpho2)
6504    continue
        path=path*(-1.0)
        bottom=plpt(j,i)
        upper=(plpt(j,i)+(path*sin(alpho2)))
        if(upper.lt.(-0.1))print,"ERROR 4",j,i,int,ny2ptnt(j,int),khari,upper
\c,cjoint2(ny2ptnt(j,int),khari)
        if(upper.lt.(-0.9))go to 1901
        call msaf(plpt(j,i),0.0,y2ptint(j,int),upper,0.0,y2ptint(j,int),mpos,
\cgamr,cork,phork,cojt,phojt,phoult,theto,alpho1,alpho2,sinrk,cosrk,tanrk,xctet
\cg,pi,beta,sf,sv,siga,taufr,wpth,ywt)
        if(j.eq.2.and.i.eq.5)print,upper
        if(notpot.ne.0)write(6,3749)bottom,upper
        if(notpot.ne.0)write(6,3747)ny2ptnt(j,int),2
        if(notpot.ne.0)write(6,3750)beta,path,siga,wpth,sf
        if(notpot.ne.0)write(6,3746)sm1(j,i),smpthj1(j,i),smptht1(j,i),wmin1(
\cj,i)
        if(notpod.ne.0.and.i.eq.m1(j))write(6,3745)j,sm1(j,i)
        go  to 950
2501    if(notpot.ne.0)write(6,3749)plpt(j,i),plpt(j,i-1)
        if(notpot.ne.0)write(6,3747)j,1
        if(notpot.ne.0)write(6,3750)beta,path,siga,wpth,sf
        if(notpot.ne.0)write(6,3746)sm1(j,i),smpthj1(j,i),smptht1(j,i),wmin1(
\cj,i)
        if(notpod.ne.0.and.i.eq.m1(j))write(6,3745)j,sm1(j,i)
        go to 950
930     continue
949     if(notpot.ne.0)write(6,3749)bottom,upper
        if(notpot.ne.0)write(6,3748)j,1,i,nuplane,nsetu,nref
        path=((bottom-upper)/sin(dummy1*(pi/180.0)))
```

```
1065,1107p
          smpthr=0.0
          smptht=0.0
          if(special.ne.6.0.or.(alpha2-dummy1).gt.(.01))go to 6900
          smpthj1(j,i)=smpthj12(j,i)
          smptht1(j,i)=smptht12(j,i)
          sm1(j,i)=sm12(j,i)
          wmin1(j,i)=wmin12(j,i)
          smpthr1(j,i)=smptht1(j,i)+smpthj1(j,i)
          do 9176,mardi=1,nptint(j)
          if(plpt(j,i).eq.ptint(j,mardi))go to 9177
          go to 9176
9177      if((merak(j,mardi).eq.101.or.merak(j,mardi).eq.1001.or.merak(j,mardi)
\c.eq.111).and.path.gt.(0.0))path=path*(-1.0)
          go to 9178
9176      continue
9178      continue
          go to 6901
6900      if(nsetu.eq.1.and.i.gt.1)smpthr=smpthr1(nuplane,nref)
          if(nsetu.eq.1.and.i.gt.1)smptht=smptht1(nuplane,nref)
          if(nsetu.eq.2.and.i.gt.1)smpthr=smpthr2(nuplane,nref)
          if(nsetu.eq.2.and.i.gt.1)smptht=smptht2(nuplane,nref)
          smpthr1(j,i)=path+smpthr
          smptht1(j,i)=path+smptht
6901      if(notpot.ne.0)write(6,3750)dummy1,path,dummy3,dummy4,dummy6
          smpthj1(j,i)=(smptht1(j,i)-smpthr1(j,i))*(-1.0)
          sm1(j,i)=dummy2
          wmin1(j,i)=dummy5
          if(notpot.ne.0)write(6,3746)sm1(j,i),smpthj1(j,i),smptht1(j,i),wmin1(
\cj,i)
          if(notpod.ne.0.and.i.eq.m1(j))write(6,3745)j,sm1(j,i)
950       mpos=0
c
c                   The Following Calculates The Unit Safety Margin
c
          dist=xd1(j)/sin(alpho1)
          usm(j)=sm1(j,m1(j))/dist
c
c                   The Following Calculates The Angle
c                        Of The Critical Path
c
          fang(j)=1.0000
          if(output3.eq.1) go to 1981
          fang(j)=(atan(1.0/(((wmin1(j,m1(j))/gamr)/0.5*(xd1(j)**2.0)))+(1.0/ta
\cn(theto))))*(180.0/pi)
1981      dfact=ndiv
c
```

```
1108,1154p
 c                  The Following Routine Calculates The Apparent
 c                  Persistence Of The Critical Path
 c
          wgt=.5*gamr*xd1(j)*xd1(j)*(1./tan(alpho1)-1./tan(theto))
          sigg=wgt*cos(alpho1)*sin(alpho1)/xd1(j)
          rr=sigg/cork
          cc=2./((2.*rr+1.)**.5-2.*rr*tan(phojt))
          acrit=100.*(1.-rr*cc*(tan(alpho1)-tan(phojt)))
          app(j)=acrit-100.*cc*usm(j)/cork
 c
 c                  The Following Routine Divides The Slope Into Hight
 c                  Increments And Performs A Statistical Analysis On
 c                  The Various Parameters Previously Calculated
 c
          dxd=ydim/dfact
          do 980  ndx=1,ndiv
          xndx=ndx
          if(xd1(j).gt.dxd*xndx) go to 980
          numj(ndx)=numj(ndx)+1
          sper(ndx)=sper(ndx)+percon1(j)
          ssqper(ndx)=ssqper(ndx)+percon1(j)**2.
          sfan(ndx)=sfan(ndx)+fang(j)
          ssqfan(ndx)=ssqfan(ndx)+fang(j)**2.
          ssm(ndx)=ssm(ndx)+sm1(j,m1(j))
          ssqsm(ndx)=ssqsm(ndx)+sm1(j,m1(j))**2.
          susm(ndx)=susm(ndx)+usm(j)
          ssqusm(ndx)=ssqusm(ndx)+usm(j)**2.
          sapp(ndx)=sapp(ndx)+app(j)
          ssqapp(ndx)=ssqapp(ndx)+app(j)**2.
          swgt(ndx)=swgt(ndx)+wmin1(j,m1(j))
          ssqwgt(ndx)=ssqwgt(ndx)+wmin1(j,m1(j))**2.
          if(sm1(j,m1(j)).le.(0.0))smleo(ndx)=smleo(ndx)+1
          go to 982
 980      continue
 982      continue
 998      continue
          if(output2.eq.0)go to 1505
          write(6,1131)jp11
          write(6,1132)
          do 1500 j=1,jp11
          write(6,1133)j,percon1(j),fang(j),sm1(j,m1(j)),usm(j),app(j),wmin1(j,
 \cm1(j)),xd1(j)
 1500     continue
 1505     continue
          if(mm.lt.noreal)go to 1960
          if(output3.ne.0)write(6,1135)ydim,dxd
          do 1900 n=1,ndiv
          if(numj(n).lt.2) go to 1900
```

```
1155,1186p
          sddiv=numj(n)-1
          perave(n)=sper(n)/numj(n)
          perss=abs(ssqper(n)-sper(n)*perave(n))
          sdper(n)=(perss/sddiv)**.5
          fanave(n)=sfan(n)/numj(n)
          fanss=abs(ssqfan(n)-sfan(n)*fanave(n))
          sdfan(n)=   (fanss/sddiv)**.5
          smave(n)=ssm(n)/numj(n)
          smss=abs(ssqsm(n)-ssm(n)*smave(n))
          sdsm(n)=(smss/sddiv)**.5
          usmave(n)=susm(n)/numj(n)
          usmss=abs(ssqusm(n)-susm(n)*usmave(n))
          sdusm(n)=(usmss/sddiv)**.5
          appave(n)=sapp(n)/numj(n)
          appss=abs(ssqapp(n)-sapp(n)*appave(n))
          sdapp(n)=(appss/sddiv)**.5
          wgtave(n)=swgt(n)/numj(n)
          wgtss=abs(ssqwgt(n)-swgt(n)*wgtave(n))
          sdwgt(n)=(wgtss/sddiv)**.5
          xxx1=(n-1)*dxd
          xxx2=n*dxd
          if(ydim.eq.10.)go to  8954
          if(gamr.eq..151)go to 8954
          if(xxx1.lt.90.0)go to 8955
8954      if(output3.ne.0)write(6,1138)xxx1,xxx2,numj(n),perave(n),sdper(n),fan
\cave(n),sdfan(n),smave(n),sdsm(n),usmave(n),sdusm(n),appave(n),sdapp(n),wgtave
\c(n),sdwgt(n),smleo(n)
8955      continue
1900      continue
1960      continue
1966      continue
1901      stop
          end
c
```

```
1187,1230p
c                    The Following Subroutine Calculates Resistance And Safety
c                 Marsin For Transition Paths Checked In The Dynamic Programming
c
        subroutine msaf(ex2,y12,y22,ex1,y11,y21,mpos,samr,cork,phork,cojt,pho
\cjt,phoult,theto,alpho1,alpho2,sinrk,cosrk,tanrk,xcteta,pi,beta,sf,sv,sisa,tau
\cfr,wpth,ywt)
        pi=3.141593
c                    For paths within the second joint set
        if((y12.eq.0.).and.(y11.eq.0.)) go to 604
c                    For paths within the first joint set
        if((y22.eq.0.).and.(y21.eq.0.)) go to 603
c                    path from first set point up to a second set point
        if((y11.eq.0.).and.(y22.eq.0.)) go to 602
c                    path from second set point up to a first set point
        if((y12.eq.0.).and.(y21.eq.0.)) go to 601
604     whyl=y21
        why2=y22
        salo1=alpho2
        salo2=alpho2
        go to 605
603     whyl=y11
        why2=y12
        salo1=alpho1
        salo2=alpho1
        go to 605
602     whyl=y21
        why2=y12
        salo1=alpho2
        salo2=alpho1
        go to 605
601     whyl=y11
        why2=y22
        salo1=alpho1
        salo2=alpho2
605     poco=((ex2/tan(salo2))+why2-why1-(ex1/tan(salo1)))
        ar=.5*(ex1+ex2)
c                    For paths to the right of the slope apex
        if(((ex2/tan(salo2)+why2).le.ywt).and.((ex1/tan(salo1)+why1).le.ywt))
\c go to 606
c                    For paths under the slope apex
        if(((((ex1/tan(salo1))+why1).le.ywt).and.(((ex2/tan(salo2))+why2).gt.
\cywt)).or.((((ex1/tan(salo1))+why1).gt.ywt).and.(((ex2/tan(salo2))+why2).le.yw
\ct))) go to 607
c                    For paths to the left of the slope apex
        if((((ex1/tan(salo1))+why1).gt.ywt).and.(((ex2/tan(salo2))+why2).gt.y
\cwt)) go to 608
606     area=ar*poco
        go to 609
607     area=ar*poco
        if(area.gt.0.) area=area-.5*tan(theto)*(((ex2/tan(salo2))+why2-ywt)**
\c2.)
```

```
1231,1277p
        if(area.lt.0.) area=area+.5*tan(theto)*(((ex1/tan(salo1))+why1-ywt)**
\c2.)
        go to 609
608     area=(ar*poco)-(.5*tan(theto)*((((ex2/tan(salo2))+why2-ywt)**2.)-(((e
\cx1/tan(salo1))+why1-ywt)**2.)))
609     if(area.gt.0.0) beta=((atan(abs((ex2-ex1)/poco)))*(180./pi))
        if(area.eq.0.0) beta=90.0
        if(area.lt.0.0) beta=180.-((atan(abs((ex2-ex1)/poco)))*(180./pi))
        beto=beta*(pi/180.)
        dist=abs(poco/cos(alpho1))
        wlf=area*gamr
        if(dist.lt.(0.0))sisa=.0001
        if(dist.gt.(0.00))sisa=abs(wlf*(cos(alpho1))/dist)
        dt=2.*xcteta
        if(sisa.le.cork) rad=.5*(sisa+cork)
        if(sisa.gt.cork.and.sisa.le.dt) rad=(cork*sisa)**.5
        if(sisa.gt.dt) rad=((sisa/2.)+(cork/tanrk))*sinrk
        xct=sisa/2.
        taufr=(rad**2.-((sisa/2.)**2.))**.5
        if(rad.le.xct)taufr=0.0
        if(mpos.gt.1) go to 650
        if(sisa.gt.cork) go to 640
c
c               This Part Examines Failures In Pure Tension
c
        if(sisa.eq.0.0) sisa=0.00001
        tenang=.5*(pi-atan(2.*taufr/sisa))
        if(beto.le.(alpho1+tenang)) res=taufr*dist*cos(beto-alpho1)
        if(beto.gt.(alpho1+tenang)) res=(cork/2.)*(abs(ex2-ex1)*cos(alpho1))
        go to 660
c
c               This Part Examines Failures In Shear
c
640     res=taufr*dist*cos(beto-alpho1)
        go to 660
c
c               This Part Examines Joint Failures
c
650     taufjt=cojt+sisa*tan(phojt)
        if(taufjt.gt.taufr) taufjt=taufr
        if(mpos.eq.10)taufjt=0.0
        res=taufjt*dist
        taufr=taufjt
        go to 670
c
c               This Part Checks Failures At Ultimate Strength
c
660     rult=sisa*tan(phoult)*dist*cos(beto-alpho1)
        if(rult.gt.res) res=rult
```

```
1278,$p
670       sf=res-wlf*sin(alphol)
          wpth=wlf
          if(mpos.ne.1) go to 700
          sv=ex2*(cork/2.)*cos(alphol)
          svs=ex2*(cork/2.)*sin(alphol)
          if(svs.gt.sv)sv=svs
700       return
          end


w

q
r 15:24 11.495 469
```

```
talal
input theta,alphal,alpha2,ydim
75 25 55 10
input ystar,ndiv
0 10
input phijt,cojt,phirk,cork,phiult
0 0 30 25 30
input samr
.150
input sp31,sp32,spjtln1,spjtln2,sprkbr1,sprkbr2
3 3  4 4 4 4
input iseed
63934
notpop,notpot,notpod
1 1 1
input output1,output2,output3
1 1 1
input noreal,distmn,njump
1 20 2
```

(a)

```
     Slope  Angle: 75.0 degrees
     Slope Hight:   10.0 feet
     First  Joint Set Inclination:  25.0 degrees
     Second Joint Set Inclination:  55.0 degrees


Strength Parameters
     Phi (joint) =     0.00 degrees
     Cohesion (joint) =   0.00
     Phi (rock) =   30.00 degrees
     Cohesion (rock) =   25.00
     Rock Unit Weight:   0.15


Distributional Parameters;Joint Set One:
     Mean Plane Spacing =   3.00
     Mean Joint Spacing =   4.00
     Mean Joint Length  =   4.00


Distributional Parameters;Joint Set Two:
     Mean Plane Spacing =   3.00
     Mean Joint Spacing =   4.00
     Mean Joint Length  =   4.00
```

```
Initial Random Number: 65934

Number of Realizatins:   1

Path Criteria:
    Minimum Spacing =  20.0
    Maximum Transition =  2
```

(a) cont'd

```
******* Realization Number   1
Joint Plane  1       Max x-coordinate =   2.75        y-coordinate =  13.61
                         0.00
                         0.69
                         2.75
                 Number of Joints On This Plane Is   1
                 Average Percent Continuity Is  25.24

Joint Plane  2       Max x-coordinate =   9.08        y-coordinate =   1.73
                         4.41
                         6.08
                         9.03
                         9.08
                 Number of Joints On This Plane Is   2
                 Average Percent Continuity Is  18.84

Joint Plane  3       Max x-coordinate =   9.40        y-coordinate =   1.13
                         0.00
                         2.54
                         7.90
                         9.40
                 Number of Joints On This Plane Is   2
                 Average Percent Continuity Is  42.99
```

(b)

```
    --Joint Set One:Joint Right End To Above Projections--

                         plane   1
                         2.36
                         0.69

                         plane   2
                         7.56
```

(c)

```
Maximum Allowable y-coordinate : ymax =   18.77


Joint Plane  1        Max x-coordinate =   0.17        y-coordinate =   0.25
                            0.17
                     No Joints On This Plane
                    Average Percent Continuity Is   0.00


Joint Plane  2        Max x-coordinate =   3.86        y-coordinate =   5.58
                            0.00
                            3.86
                    Number of Joints On This Plane Is   1
                    Average Percent Continuity Is 100.00


Joint Plane  3        Max x-coordinate =   5.24        y-coordinate =   7.57
                            0.00
                            1.84
                            4.56
                            5.03
                            5.24
                    Number of Joints On This Plane Is   2
                    Average Percent Continuity Is  44.01


Joint Plane  4        Max x-coordinate =   8.36        y-coordinate =  12.07
                            0.00
                            8.36
                    Number of Joints On This Plane Is   1
                    Average Percent Continuity Is 100.00
```

(d)

Joint Plane 5 Max x-coordinate = 2.77 y-coordinate = 17.57
0.57
2.26
2.77
Number of Joints On This Plane Is 1
Average Percent Continuity Is 61.02

(d) cont'd

--Joint Set One:Pts. of Intersection--

plane 1
2.74

plane 2
2.66
4.04
7.16

plane 3
3.08
4.46
7.58

(e)

--Joint Set One:Dynamic Programmicns Plane Points--

plane 1
0.00
0.69
0.89
2.36
2.74
2.75

plane 2
2.66
4.04
4.41
6.08
7.16
7.56
9.03
9.08

plane 3
0.00
2.54
3.08
4.46
7.58
7.90
9.40

(f)

Region From Free Surface To Plane 1 y-coordinate = 17.57
Joint(s) In Between Only

Lower x-coordinate = 0.57 Upper x-coordinate = 0.00
Beta = 55.00 Path = 0.70 Stress = 0.04 Weight = 0.02 S.F.(path) = 4.81
Safety Margin = 4.81 Jointed Rock;Sum = 0.00 Critical Path Length = 0.70 Critical Weight = 0.02

Lower x-coordinate = 2.26 Upper x-coordinate = 0.57
In Joint Transition Within Plane 5 Of Set 2
Beta = 55.00 Path = -2.06 Stress = 0.15 Weight = 0.21 S.F.(path) = -0.09
Safety Margin = 4.72 Jointed Rock;Sum = -2.06 Critical Path Length = 2.76 Critical Weight = 0.23

Lower x-coordinate = 0.69 Upper x-coordinate = 0.00
In Joint Transition Within Plane 1 Of Set 1
Beta = 25.00 Path = -1.64 Stress = 0.04 Weight = 0.08 S.F.(path) = -0.03
Safety Margin = -0.03 Jointed Rock;Sum = -1.64 Critical Path Length = 1.64 Critical Weight = 0.08

Lower x-coordinate = 0.89 Upper x-coordinate = 0.69
From Plane 1 In Set 1 Reference Point 3 Up To Plane 1 Of Set 1 Reference Point 2
Beta = 25.00 Path = 0.46 Stress = 0.10 Weight = 0.05 S.F.(path) = 5.72
Safety Margin = 5.69 Jointed Rock;Sum = -1.64 Critical Path Length = 2.10 Critical Weight = 0.13

Lower x-coordinate = 2.36 Upper x-coordinate = 0.89
From Plane 1 In Set 1 Reference Point 4 Up To Plane 1 Of Set 1 Reference Point 3
Beta = 25.00 Path = 3.49 Stress = 0.20 Weight = 0.77 S.F.(path) = 43.60
Safety Margin = 49.29 Jointed Rock;Sum = -1.64 Critical Path Length = 5.58 Critical Weight = 0.90

Lower x-coordinate = 2.74 Upper x-coordinate = 2.26
From Plane 1 In Set 1 Reference Point 5 Up To Plane 5 Of Set 2 Reference Point 2
Beta = 55.00 Path = 0.58 Stress = 0.05 Weight = 0.02 S.F.(path) = 3.96
Safety Margin = 8.67 Jointed Rock;Sum = -2.06 Critical Path Length = 3.34 Critical Weight = 0.25

(g)

```
Lower x-coordinate =   2.75      Upper x-coordinate =   2.74
    From Plane   1      In Set 1      Reference Point   6       Up To Plane   1     Of Set 1     Reference Point   5
    Beta = 25.00      Path =     0.02      Stress =     0.00      Weight =     0.00      S.F.(path) =   0.22
    Safety Margin =   8.89     Jointed Rock;Sum =  -2.06     Critical Path Length =   3.36     Critical Weight =   0.25
    Joint Plane  1      Safety Margin      8.89

    Region  From  Free Surface  To Plane  2     y-coordinate =   5.58
          Continuous Joint Throughout

Lower x-coordinate =   2.66      Upper x-coordinate =   0.00
    In Joint Transition Within Plane   2     Of Set 2
    Beta = 55.00      Path =    -3.25      Stress =     0.16      Weight =     0.37      S.F.(path) =  -0.16
    Safety Margin =  -0.16     Jointed Rock;Sum =  -3.25     Critical Path Length =   3.25     Critical Weight =   0.37

    Region  From  Free Surface  To Plane  2     y-coordinate =   7.57
     Joint Intersects Top Point

Lower x-coordinate =   1.84      Upper x-coordinate =   0.00
    In Joint Transition Within Plane   3     Of Set 2
    Beta = 55.00      Path =    -2.24      Stress =     0.11      Weight =     0.18      S.F.(path) =  -0.07
    Safety Margin =  -0.07     Jointed Rock;Sum =  -2.24     Critical Path Length =   2.24     Critical Weight =   0.18

    Region  From  Free Surface  To Plane  2     y-coordinate =  12.07
          Continuous Joint Throughout

Lower x-coordinate =   7.16      Upper x-coordinate =   0.00
    In Joint Transition Within Plane   4     Of Set 2
    Beta = 55.00      Path =    -8.74      Stress =     0.44      Weight =     2.69      S.F.(path) =  -1.14
    Safety Margin =  -1.14     Jointed Rock;Sum =  -8.74     Critical Path Length =   8.74     Critical Weight =   2.69

Lower x-coordinate =   2.66      Upper x-coordinate =   0.00
    From Plane   2      In Set 1      Reference Point   1       Up To Plane   2     Of Set 2     Reference Point   1
    Beta = 55.00      Path =    -3.25      Stress =     0.16      Weight =     0.37      S.F.(path) =  -0.16
    Safety Margin =  -0.16     Jointed Rock;Sum =  -3.25     Critical Path Length =   3.25     Critical Weight =   0.37
```

(g)
cont'd

```
Lower x-coordinate =   4.04      Upper x-coordinate =   1.84
   From Plane   2     In Set 1     Reference Point   2       Up To Plane   3     Of Set 2     Reference Point   2
   Beta = 55.00     Path =     2.69     Stress =     0.36      Weight =     0.68      S.F.(path) =  18.42
   Safety Margin =  18.34     Jointed Rock;Sum =  -2.24     Critical Path Length =    4.93     Critical Weight =    0.86

Lower x-coordinate =   4.41      Upper x-coordinate =   4.04
   From Plane   2     In Set 1     Reference Point   3       Up To Plane   2     Of Set 1     Reference Point   2
   Beta = 25.00     Path =     0.88     Stress =     0.52      Weight =     0.50      S.F.(path) =  10.96
   Safety Margin =  29.31     Jointed Rock;Sum =  -2.24     Critical Path Length =    5.81     Critical Weight =    1.36

Lower x-coordinate =   6.08      Upper x-coordinate =   4.41
   In Joint Transition Within Plane   2     Of Set 1
   Beta = 25.00     Path =    -3.94     Stress =     0.65      Weight =     2.81      S.F.(path) =  -1.19
   Safety Margin =  28.12     Jointed Rock;Sum =  -6.18     Critical Path Length =    9.75     Critical Weight =    4.17

Lower x-coordinate =   7.16      Upper x-coordinate =   0.00
   From Plane   2     In Set 1     Reference Point   5       Up To Plane   4     Of Set 2     Reference Point   1
   Beta = 55.00     Path =    -8.74     Stress =     0.44      Weight =     2.69      S.F.(path) =  -1.14
   Safety Margin =  -1.14     Jointed Rock;Sum =  -8.74     Critical Path Length =    8.74     Critical Weight =    2.69

Lower x-coordinate =   7.56      Upper x-coordinate =   7.16
   From Plane   2     In Set 1     Reference Point   6       Up To Plane   2     Of Set 1     Reference Point   5
   Beta = 25.00     Path =     0.95     Stress =     0.91      Weight =     0.95      S.F.(path) =  11.88
   Safety Margin =  10.75     Jointed Rock;Sum =  -8.74     Critical Path Length =    9.69     Critical Weight =    3.64

Lower x-coordinate =   9.03      Upper x-coordinate =   7.56
   From Plane   2     In Set 1     Reference Point   7       Up To Plane   2     Of Set 1     Reference Point   6
   Beta = 25.00     Path =     3.49     Stress =     0.62      Weight =     2.40      S.F.(path) =  43.64
   Safety Margin =  54.39     Jointed Rock;Sum =  -8.74     Critical Path Length =   13.18     Critical Weight =    6.04
```

(g)
cont'd

Lower x-coordinate = 9.08 Upper x-coordinate = 9.03
In Joint Transition Within Plane 2 Of Set 1
Beta = 25.00 Path = -0.10 Stress = 0.02 Weight = 0.00 S.F.(path) = -0.00
Safety Margin = 54.39 Jointed Rock;Sum = -8.84 Critical Path Length = 13.28 Critical Weight = 6.05
Joint Plane 2 Safety Margin 54.39

Region Between Planes 3 And 2 y-coordinate = 5.58
Continuous Joint Throughout

Lower x-coordinate = 3.08 Upper x-coordinate = 2.66
In Joint Transition Within Plane 2 Of Set 2
Beta = 55.00 Path = -0.51 Stress = 0.35 Weight = 0.13 S.F.(path) = -0.05
Safety Margin = -0.21 Jointed Rock;Sum = -3.76 Critical Path Length = 3.76 Critical Weight = 0.00

Region Between Planes 3 And 2 y-coordinate = 7.57
No Second Set Joints

Region Between Planes 3 And 2 y-coordinate = 12.07
Continuous Joint Throughout

Lower x-coordinate = 7.58 Upper x-coordinate = 7.16
In Joint Transition Within Plane 4 Of Set 2
Beta = 55.00 Path = -0.51 Stress = 0.91 Weight = 0.32 S.F.(path) = -0.14
Safety Margin = -1.27 Jointed Rock;Sum = -9.25 Critical Path Length = 9.25 Critical Weight = 0.00

Lower x-coordinate = 2.54 Upper x-coordinate = 0.00
In Joint Transition Within Plane 3 Of Set 1
Beta = 25.00 Path = -6.01 Stress = 0.16 Weight = 1.04 S.F.(path) = -0.44
Safety Margin = -0.44 Jointed Rock;Sum = -6.01 Critical Path Length = 6.01 Critical Weight = 1.04

Lower x-coordinate = 3.08 Upper x-coordinate = 0.00
From Plane 3 In Set 1 Reference Point 3 Up To Plane 2 Of Set 2 Reference Point 1
Beta = 55.00 Path = -3.76 Stress = 0.19 Weight = 0.50 S.F.(path) = -0.21
Safety Margin = -0.21 Jointed Rock;Sum = -3.76 Critical Path Length = 3.76 Critical Weight = 0.00

(g)
contd

```
Lower x-coordinate =   4.46      Upper x-coordinate =   4.04
   From Plane   3     In Set 1     Reference Point  4       Up To Plane   2     Of Set 1     Reference Point   2
   Beta = 55.00     Path =    0.51     Stress =    0.52     Weight =    0.19     S.F.(path) =   3.48
   Safety Margin =  21.82     Jointed Rock;Sum =  -2.24     Critical Path Length =   5.44     Critical Weight =    1.04

Lower x-coordinate =   7.58      Upper x-coordinate =   0.00
   From Plane   3     In Set 1     Reference Point   5      Up To Plane   4     Of Set 2     Reference Point   1
   Beta = 55.00     Path =   -9.25     Stress =    0.47     Weight =    3.01     S.F.(path) =  -1.27
   Safety Margin =  -1.27     Jointed Rock;Sum =  -9.25     Critical Path Length =   9.25     Critical Weight =    0.00

Lower x-coordinate =   7.90      Upper x-coordinate =   7.58
   From Plane   3     In Set 1     Reference Point   6      Up To Plane   3     Of Set 1     Reference Point   5
   Beta = 25.00     Path =    0.76     Stress =    0.95     Weight =    0.80     S.F.(path) =   9.55
   Safety Margin =   8.28     Jointed Rock;Sum =  -9.25     Critical Path Length =  10.01     Critical Weight =    0.80

Lower x-coordinate =   9.40      Upper x-coordinate =   7.90
   In Joint Transition Within Plane   3     Of Set 1
   Beta = 25.00     Path =   -3.55     Stress =    0.61     Weight =    2.40     S.F.(path) =  -1.01
   Safety Margin =   7.26     Jointed Rock;Sum = -12.80     Critical Path Length =  13.56     Critical Weight =    3.20
   Joint Plane  3     Safety Margin     7.26
```

(g) cont'd

```
Number of Joint Planes Is =   3
```

Joint Plane	Percent Continuity	Angle Of Critical Path	Minimum Safety Margin	Unit Safety Margin	Apparent Persistence	Weight Of Critical Path	Height
1	25.24	1.00	8.89	1.37	88.57	0.25	2.75
2	18.84	1.00	54.39	2.53	78.34	6.05	9.08
3	42.59	1.00	7.26	0.33	95.59	3.20	9.40

```
QUIT
r 17:24 0.070 0 level 2
```

(h)

The Slope Hight Of 10.0 Is Divided Into Increments Of 1.0 For Statistical Analysis

Interval From 0.0 To 1.0

The Number Of Joints Is 4

Percent Continuity:
Average = 45.84
Standard Deviation = 53.36

Net Angle of Critical Path:
Average = 1.00
Standard Deviation = 0.00

Safety Margin:
Average = 9.5
Standard Deviation = 13.7

Unit Safety Margin:
Average = 6.5
Standard Deviation = 6.4

Apparent Persistence:
Average = 48.12
Standard Deviation = 50.89

Weight of Critical Path :
Average = 0.66
Standard Deviation = 1.21

The Number Of Joints With Safety Margins < 0.0 Is 1.

(i)

CHAPTER 5

RELIABILITY OF SLOPES CONTAINING A SINGLE SET OF SLOPE PARALLEL JOINTS

5.1 Introduction

Slopes with two sets of random joints of the type shown in Figure 2.1 and 2.2 are rather commonly encountered, especially in sedimentary formations. Their relative simplicity makes it possible to analyze reliability taking into account all the governing parameters (geometric and mechanical properties) that affect slope stability. The computer program described previously is specifically aimed at analyzing slopes of these types.

In order to run the program, joint spacings and length distributions are needed in addition to the deterministic values of joint attitudes and mechanical properties. The program output includes the probability distribution of the safety margin from which the probability of failure (probability that the safety margin is negative) can be derived. The present chapter evaluates sensitivities of the probability of failure with respect to the main input parameters. Such a study is important for two reasons:

1. It provides further insight into slope safety.
2. It may make possible simplified procedures for slope reliability analysis.

The main results are presented first, followed by a brief description of input parameters, geometric and mechanical that are relevant to slope safety. The parametric study constitutes the body of the chapter and is followed by a summary of results.

5.2 Main Conclusions of Parametric Analysis

The main conclusions from the parametric study can be summarized as follows:

1. Intact rock cohesion (C_r) is the parameter which has the strongest influence on rock slope stability. Its effect is noticeable in all runs in which C_r was varied. This is not surprising since the failure algorithm assumes intact rock bridges to fail in tension with small joint friction angle. Actual slope failures seem to support this model feature.

2. Mean joint length of the first set at high persistence values has a strong effect on rock slope stability. This already established in previous research, sensitivity of safety, to mean joint length is slightly magnified here due to the presence of two joint sets.

3. When comparing results for a slope with a single joint set and one containing two joint sets, lower safety values due to the second set, though not detrimental to slope stability.

5.3 Geometric Parameters

The geometry of slopes of the type shown in Figures 2.1 and 2.2 is described by four deterministic parameters and six stochastic parameters. The four deterministic parameters are: slope height, slope face angle, and angles of inclination of the two joint sets.

Slope height (Y_{dim}) gives the vertical distance from the slope apex to the foot of the slope. Slope angle (θ) is the angle between the slope face and the horizontal. The angles of joint set inclinations ($\alpha 1$, $\alpha 2$) are the angles between joint planes and the horizontal.

Six more parameters are necessary to completely specify the probabilistic model of joints in the first joint set: mean joint length ($\overline{JL}1$), mean rock bridge length ($\overline{RB}1$) and mean joint plane spacing ($\overline{SP}1$). Similarly for the second joint set: mean joint length ($\overline{JL}2$), mean rock bridge length ($\overline{RB}2$) and mean joint plane spacing ($\overline{SP}2$). Actual joint lengths, rock bridge lengths and joint plane spacings are generated stochastically within the program by assuming that all uncertain geometric parameters are independent exponential distributions. Some comments on these six input parameters and on derived parameters such as mean joint plane persistence of set one ($\overline{K}1$) and set two ($\overline{K}2$), mean joint intensity of set one ($\overline{I}1$) and set two ($\overline{I}2$), are given next.

Mean joint length is simply the average length of joint segments for each set. In practice, this parameter needs to be estimated from joint survey data. The model assumes that joint lengths are exponentially distributed about their mean values $\overline{JL}1$ and $\overline{JL}2$ for joint sets 1 and 2, respectively. Mean joint plane persistence ($\overline{K}$) has previously been estimated to be:

$$K = \overline{JL} / (\overline{JL} + \overline{RB})$$

where $\overline{JL}$ is mean joint length and $\overline{RB}$ is mean rock bridge length.

The parametric study does not include equivalent consideration of

$\overline{RB}_1$ and $\overline{RB}_2$

Rock bridge lengths, like joint lengths, are assumed to be exponentially distributed about the mean values, $\overline{RB}_1$ and $\overline{RB}_2$.

The mean joint plane spacing $\overline{SP}$ is the average spacing between two adjacent planes within a joint set. Like $\overline{JL}$ and $\overline{RB}$, joint plane spacings are assumed to be exponentially distributed about the mean values, $\overline{SP}1$ and $\overline{SP}2$.

Joint intensities can be derived from other input parameters as

$$\overline{I}1 = \overline{K}1 \,/\, \overline{SP}1 \qquad\qquad \overline{I}2 = \overline{K}2 \,/\, \overline{SP}2$$

Strength Parameters

Five parameters completely specify intact rock and joint resistance properties within the slope: intact rock cohesion (C_r), intact rock friction angle (Φ_r), joint cohesion (C_j), joint friction angle (Φ_j) and ultimate friction angle (Φ_{ult}). Of these, C_r and Φ_j are most critical with respect to reliability.

Intact rock cohesion (C_r) is defined as the intersection of the linear portion of the intact rock failure envelope with the shear stress axis. C_r is assumed to be twice the tensile strength (T_s) of the intact rock - see Figure 5.1.

> At relatively low stresses (low compared to C_r) within a rock slope, intact rock resistance is a function of the cohesive (tensile) component of resistance. Only at relatively high stress levels can the frictional component of resistance (Φ_r) play a role. One of the basic assumptions of the model is that the state of stress within the slope is low compared to C_r. For most slopes in which depth does not exceed 150' and C_r is greater than 25 Ksf, the low stress assumption is valid and intact rock resistance is essentially independent of Φ_r (O'Reilly, 1980).

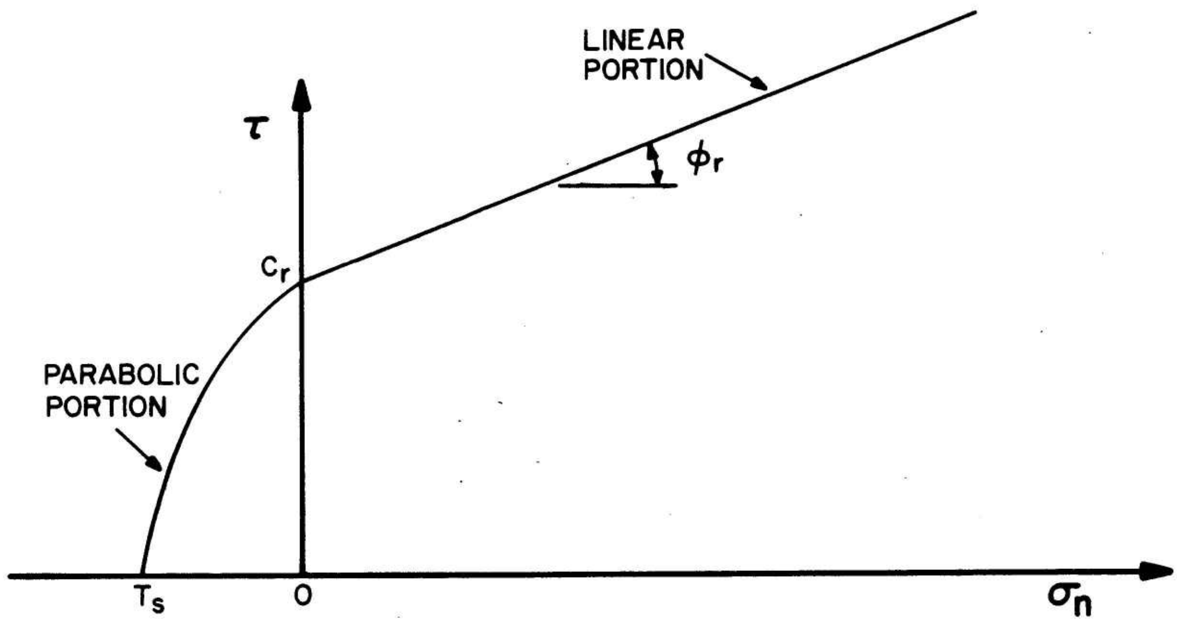


FIGURE 5.1 INTACT ROCK FAILURE ENVELOPE

The ultimate friction angle (Φ_{ult}) is important in the calculation of intact rock resistance only for high stress levels, higher than those at which Φ_r becomes significant. One may therefore conclude that the ultimate friction angle (Φ_{ult}) does not affect reliability in the stress range under consideration.

Joint friction angle (Φ_j), which is the angle of the joint failure envelope, is the parameter that generally determines the resistance properties of joints; see Figure 5.2.

Since rock bridges ususally fail at low strains before joint frictional resistance is fully mobilized, it may not be wise to depend on frictional resistance for the purpose of determining stability. Rather, it is avisable to use reduced values of Φ_j, at least for the purpose of sensitivity analysis.

Joint cohesion (C_j) is defined as the intersection of the joint failure envelope with the shear stress axis (see Figure 5.2). Unless joints are filled with cohesive material, joints do not possess a true cohesive component of resistance since they are unable to resist tensile stresses. Consequently, C_j may be set equal to zero.

The unit weight of intact rock (γ_r) affects reliability through its influence on resistance and driving forces. In all cases analyzed here, γ_r has been set equal to 0.15 KSF. The effect of varying γ_r was not studied because of its small variability compared to other parameters.

5.4 Dependence of Reliability on Various Slope Parameters

The computer program of Chapter IV does not directly give the probability of slope failure. Rather, it calculates the probability that an

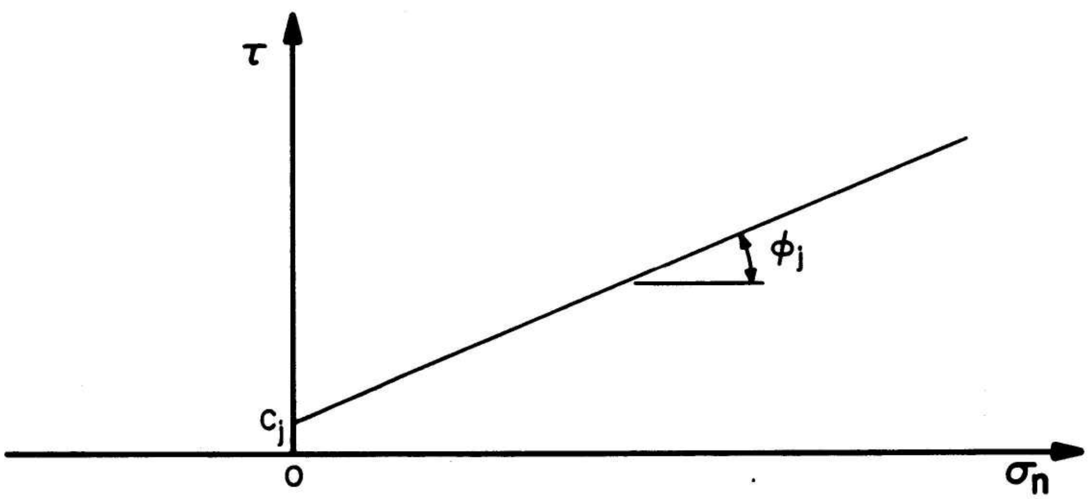


FIGURE 5.2 JOINT FAILURE ENVELOPE

unstable portion of the slope exists. Calculating P_f values as a function of slope depth allows the designer to more closely relate failure probability P_f to failure costs.

The probability of failure is defined as the probability of a joint plane exiting within a sepcific height interval and that the exit point is part of a failure path.

5.4.1 Probability of Failure Derived from Safety Margins

The safety margin of a given path through the slope is defined as the difference between the resisting (R) and driving forces (DF) along that path.

The driving force is simply the component of overburden weight acting parallel to jointing of the first joint set:

$$SM = R - DF \tag{5.1}$$

The force mobilized to resist the driving force, R, is derived from two sources: resistance from the intact rock bridges (or transitions), R_r and resistance from the jointed portion of the plane, R_j:

$$R + R_j + R_r \tag{5.2}$$

As discussed before, intact rock transitions can be in the form of low angle transitions (including in-plane) and high angle transitions. Thus resistance R_r is the sume of the respective components:

$$R_r = R_{rL} + R_{rH} \tag{5.3}$$

where the low angle component R_{rL} is given by

$$R_{rL} = \tau_\alpha \times d_L \tag{5.4}$$

in which d_L is the total length of low angle transitions and τ_α is the peak shear mobilized for low angle transitions. The latter quantity is given by

$$\tau_\alpha = \frac{1}{2} C_r \sqrt{2c + 1} \tag{5.5}$$

where C_r is the intact rock cohesive strength and $c = \sigma_\alpha / C_r$ with σ_α the stress acting normal to the joint plane.
The parameter τ_α can be estimated from

$$\tau_\alpha = \frac{1}{2} C_r \sqrt{2c + 1} \approx \frac{1}{2} C_5 \approx T_s \tag{5.6}$$

where T_s is the tensile strength of intact rock.
Consequently, the total resistance contribution due to low angle transitions through intact rock is approximately

$$R_{rL} \approx T_s \; d_L \tag{5.7}$$

The second component of intact rock shear resistance, R_{rH}, is derived from all high angle transitions between joint planes. Resistance for high angle transitions has been derived previously (Chapter 2) as

$$R_{rH} = T_s \; \chi \tag{5.8}$$

where χ is the distance separating the joint planes between which transition takes place (for transitions between planes of the first set, other than in plane transitions).

If the sume of the lengths, χ, of all such transitions along a given path is d_H, the total resistance derived from high angle transitions is simply

$$R_{rH} = T_s \; d_H \tag{5.9}$$

Thus the total resistance of intact bridges can be expressed as

$$R_r = R_{rL} + R_{rH} = T_s \; (d_L + d_H) \qquad 5.10$$

where d_L and d_H are independent of the strength parameters C_r and Φ_j. Joint resistance (R_j) can be expressed in a simple form:

$$R_j = W'\cos\alpha 1 \;\; \tan \; \Phi_j \tag{5.11}$$

The quantity W' is the weight of rock that overlies the jointed portion of the path, as shown in Figure 5.3. Hence, W' is not larger than the actual weight of rock (W) overlying the critical path. W' is a geometric property of the path.

The safety margin of a path can be derived from the above expressions:

$$SM = R_r + R_j - W \sin\alpha 1 = T_s \; (d_L + d_H) + W'\cos\alpha 1 \tan \; \Phi_j - W\sin\alpha 1 \tag{5.12}$$

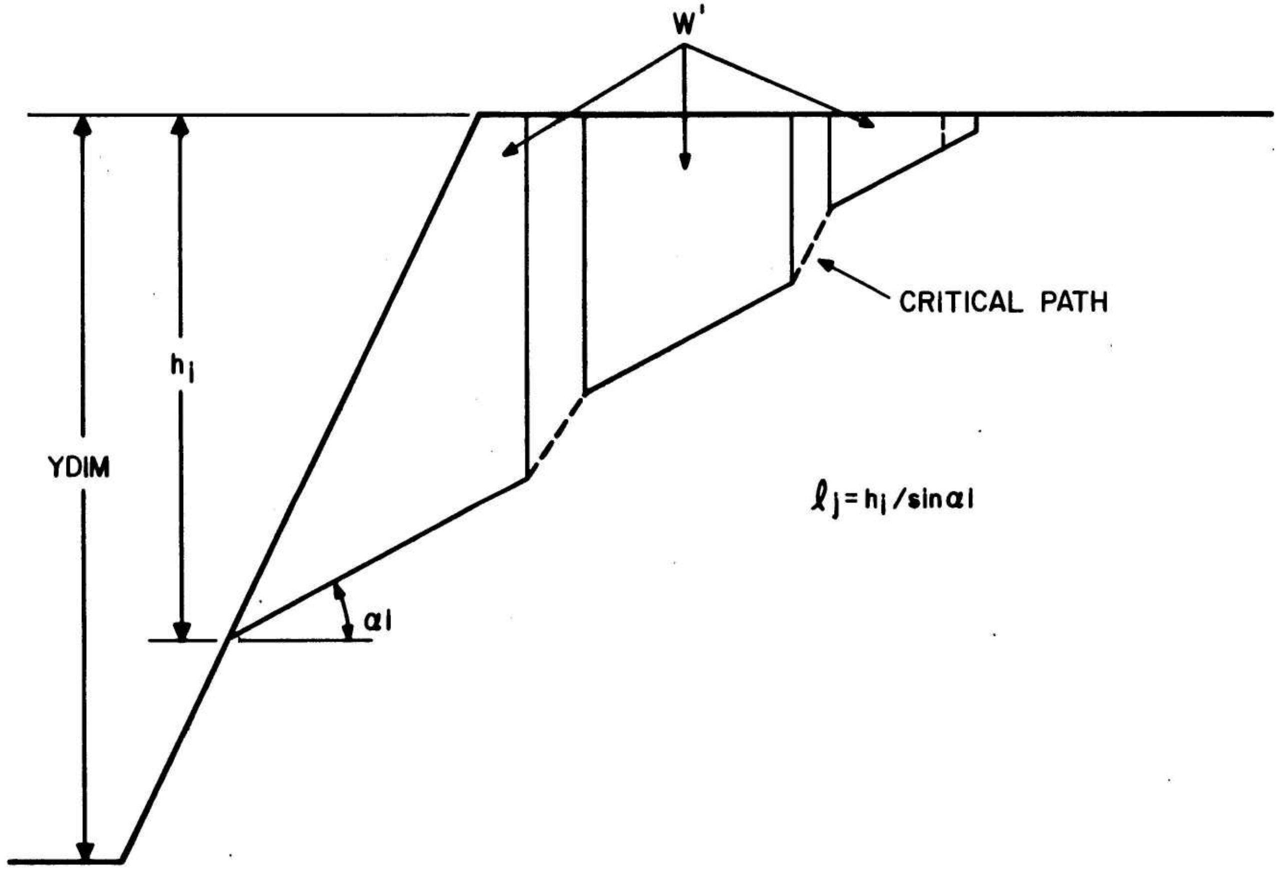


FIGURE 5.3 EFFECTIVE WEIGHT (W') OVERLYING JOINTED PORTION OF PATH

Finally, the unit safety margin is defined here as the safety margin divided by the length (ℓ_j) of the joint plane

$$USM = SM/\ell_j \tag{5.13}$$

where the length ℓ_j depends on the exit height (h) of the joint plane on the slope face - (see Figure 5.3):

$$\ell_j = h_i/\sin\alpha 1$$

Probability of Failure

For each exit point (i.e., for each joint plane) in one realization, a number of critical paths are possible, but only one is most critical. Also for each given realization of the jointing pattern, there are usually a number of exit points and hence of critical paths within a given height interval. The probability of failure, P_f, within that interval is the percentage of those critical paths with zero or negative safety margins; quantitatively

$$P_f = \{(\text{Number of Critical Paths})\ SM \leq 0\ /\ (\text{Total Number Critical Paths})\} \times 100$$

5.5 Parametric Study

5.5.1 Introduction

The parametric study is carried out by varying the parameters, one at a time, and observing the effect on slope reliability. Each parameter is described in a separate subsection in this chapter, and for purposes of clarity, each subsection has the same basic structure:

1. Each subsection begins by defining the input variable whose effect on reliability is under consideration. Program outputs are then given and conclusions are drawn. Runs are divided into groups called "cases", each case consisting of a number of runs with different simulated realizations of joint patterns. For each group of cases, all parameters are held constant, except for the parameter in question.

2. Next, the effect of varying the parameter on the probabilities of failure at various depths are examined. Relevant data plots are included.

3. A particular height interval is selected to examine how the probability of failure of this interval varies within each case. The height interval selected in all cases is the interval from 90 to 100 ft. This is also the deepest interval in the analysis and is usually the most sensitive to parameter variations.

For each case consisting of a given set of input parameters all are held constant except one. The probability of failure is plotted as a function of the parameter under consideration.

4. Each subsection ends with a summary of results which stress the practical significance and relative importance of the variable with respect to slope safety.

The input parameters examined are, in order of treatment: intact rock cohesion (C_r), joint friction angle (Φ_j). Then for joint set one: mean joint length ($\overline{JL1}$), mean joint plane persistence ($\overline{K}1$), mean joint plane spacing ($\overline{SP1}$) and joint intensity ($\overline{I1}$), and for joint set two: mean joint length ($\overline{JL2}$), mean joint plane persistence ($\overline{K}2$), mean joint plane spacing ($\overline{SP2}$) and joint intensity (I2). Finally we will examine the angle of slope face inclination (θ), the angle of joint inclination for set one ($\alpha 1$) and the angle of joint inclination for set two ($\alpha 2$). Variables and their range of values used in the parametric study are listed in Table 5.1.

The effect of varying each parameter is measured in terms of the mean and standard deviation of the safety margin ($\overline{SM}$, $\tilde{SM}$), the unit safety margin ($\overline{USM}$, $U\tilde{S}M$) and the apparent persistence ($\overline{K}_a$, $\tilde{K}_a$). The distribution characteristics are used to derive the probability failure. Most of the sensitivity results will be presented as relations between distribution characteristics of this type and vertical distance from the slope apex to the midpoint of a height interval. P_f as a function of depth is also calculated.

Three functions are noticed in the following analysis. Namely, critical persistence (K_c), the index of reliability (β) and the probability of a joint plane which is 100% continuous at a given depth (P_1).

Critical persistence (K_c) is defined as the persistence required along a joint plane at a given depth to yield a zero safety margin, SM. Solving for the critical persistence (K_c) of this plane requires the calculations of d_c, the critical rock bridge length, such that SM = 0.

TABLE 5.1

RANGES IN INPUT PARAMETER VALUES

PARAMETER	RANGE OF VALUES
Intact Rock Cohesion, C_r	8 - 500 Ksf
Joint Friction Angel, Φ_j	0 - 40°
Mean Discontinuity Length (set 1), $\overline{JL1}$	10 - 40'
Mean Persistence-Constant Joint Plane Spacing (set 1) $\overline{K1}\ \theta\ \lvert\overline{SP1}\rvert$	10 - 80%
Mean Joint Plane Spacing - (set 1), $\overline{SP1}$	2 - 10'
Mean Persistence - Constant Intensity (set 1), $\overline{K1}\ \theta\ \lvert\overline{I1}\rvert$	10 - 50%
Mean Discontinuity Length (set 2), $\overline{JL2}$	10 - 40'
Mean Persistence-Constant Joint Plane Spacing (set 2), $\overline{K2}\ \theta\ \lvert\overline{SP2}\rvert$	10 - 80%
Mean Joint Plane Spacing-(set 2), $\overline{SP2}$	2 - 15'
Mean Persistence-Constant Intensity (set 2), $\overline{K2}\ \theta\ \lvert\overline{I2}\rvert$	10 - 50%
Rock Slope Face Inclination, θ	50 - 90°
Joint Plane Inclination (set 1)-$\alpha 1$	10 - 80°
Joint Plane Inclination (set 2)-$\alpha 2$	11 - 180°
Intact Rock Friction Angle Φ_R	30°- 40°

K_c may be calculated from the following relation (from O'Reilly - 1980):

$$K_c = \left(1 - \frac{d_c}{d_j}\right) 100$$

where d_c is calculated as (from O'Reilly - 1980):

$$d_c = \frac{2W\ (\sin\alpha 1 - \cos\alpha 1 \tan \Phi_j)}{C_r\ (\sqrt{2c + 1} - 2c \tan \Phi_j)}$$

and by rearranging variables we get (from O'Reilly - 1980):

$$K_c = 1 - \frac{2c\ (\tan \alpha 1 - \tan \Phi j)}{\sqrt{2c + 1} - 2c \tan \Phi_j} \quad x\ 100$$

where C_r is intact rock cohesion, ℓ_j is the joint plane length, d_c is the length of intact rock bridges along the joint plane, σ_a is defined as the average stress of overburden weight applied normal to the joint plane. Quantitatively (referring to Fig. 5.3):

$\sigma_a = W\cos\alpha 1/\ell_j$ where W is calculated as:

$$W = \frac{1}{2}\ \alpha_r\ h^2\ (1/\tan\alpha 1 - 1/\tan \Phi)$$

and σ_r is rock unit weight while ℓ_j is quantitatively defined as:

$$\ell_j = h/\sin\alpha 1$$

and c is α_a/C_r

The reliability index (β) is the difference between the critical persistence and the mean apparent persistence measured in terms of standard deviations of apparent persistence. A negative reliability index implies an unsafe slope while a positive index implies a stable slope. β can be calculated as follows:

$$\beta = \frac{K_c - \overline{k}_a}{\tilde{K}_a}$$

In other words, β is the number of standard deviations between the critical state and the most likely state; the latter obtained from a model run.

The theoretical lower bound probability of failure, P_1, is calculated by the following equation (from O'Reilly - 1980):

$$P = \overline{K}[\exp(-h/JL1*\sin\alpha 1)]$$

where $\overline{K}$ is the mean joint plane persistence.

This closed form equation is to be used as an approximation for predicting slope reliability, However, the admissability of such usage will be examined in the parametric study.

5.5.2 Effect of Intact Rock Cohesion on Slope Reliability

Intact rock cohesion (C_r) has been defined as the intercept of the linear portion of the intact rock failure envelope with the shear stress axis. It is assumed to equal twice the intact rock tensile strength (T_s). Since failure of a rock bridge is assumed to occur in tension, C_r is the only parameter needed to calculate intact rock bridge resistance.

Four cases, each consisting of three runs, are analyzed to examine the effect of intact rock cohesion on rock slope reliability. The range of values for C_r is varied from 25 to 500 ksf. Following is a brief description of each case:

Case #1: Mean joint persistence ($\overline{K}$) is set equal to 50 percent and mean joint length ($\overline{JL}$) is set equal to 40 feet.

Case #2: Same as the above except that mean joint persistence ($\overline{K}$) is increased to 75 percent.

Case #3: Identical to Case #1 except that the mean joint length of the first set ($\overline{JL1}$) is reduced to 15 feet.

Case #4: Also identical to Case #1 except that the mean joint length of the second joint set ($\overline{JL2}$) is reduced to 15 feet.

Thus, the influence of intact rock cohesion is studied in cases when joint persistences are moderate and joint lengths are high (Case #1), when joint persistences are high and joint lengths are high (Case #2), when joint persistences are moderate and the mean joint length of the first set is small while the mean joint length of the second set is high (Case #3) and finally when persistences are moderate and mean joint lengths in both sets are low (Case #4).

The Effect of Intact Rock Cohesion (C_r) on the Probability of Failure P_f (h)

The effect of varying C_r on P_f (h) while holding all other input parameters constant is schematically shown in Figure 5.4. For a given h, program output showed that the probability of failure increases when C_r decreases. The probability of failure also increases as a result of increasing the driving forces which increase with depth due to the overburden weight.

The Probability of Failure as a Function of Intact Rock Cohesion

The influence of C_r can be seen clearly in Figure 5.5 where the number of realizations and the jointing patterns are kept constant such that changes in results are caused by variations in C_r. In Figure 5.5 P_f is plotted as a function of C_r for the four cases (1, 2, 3 and 4) described previously. In comparison to O'Reilly's (1980) findings, P_f here, is approximately a linear function of C_r. For slopes with a single set of joints, the probability of failure is less sensitive to changes in cohesion of intact rock (O'Reilly 1980). From program output, P_f increases rapidly as C_r decreases below 100 ksf. For C_r greater than 100 ksf, the probability of failure decreases from approximately 35% to 3% when C_r equals 500 ksf.

At a given depth (h=90-100"), comparing cases 2 & 4 with case 3 shows that decreasing the mean joint length of the first set ($\overline{JL1}$) does not effect the

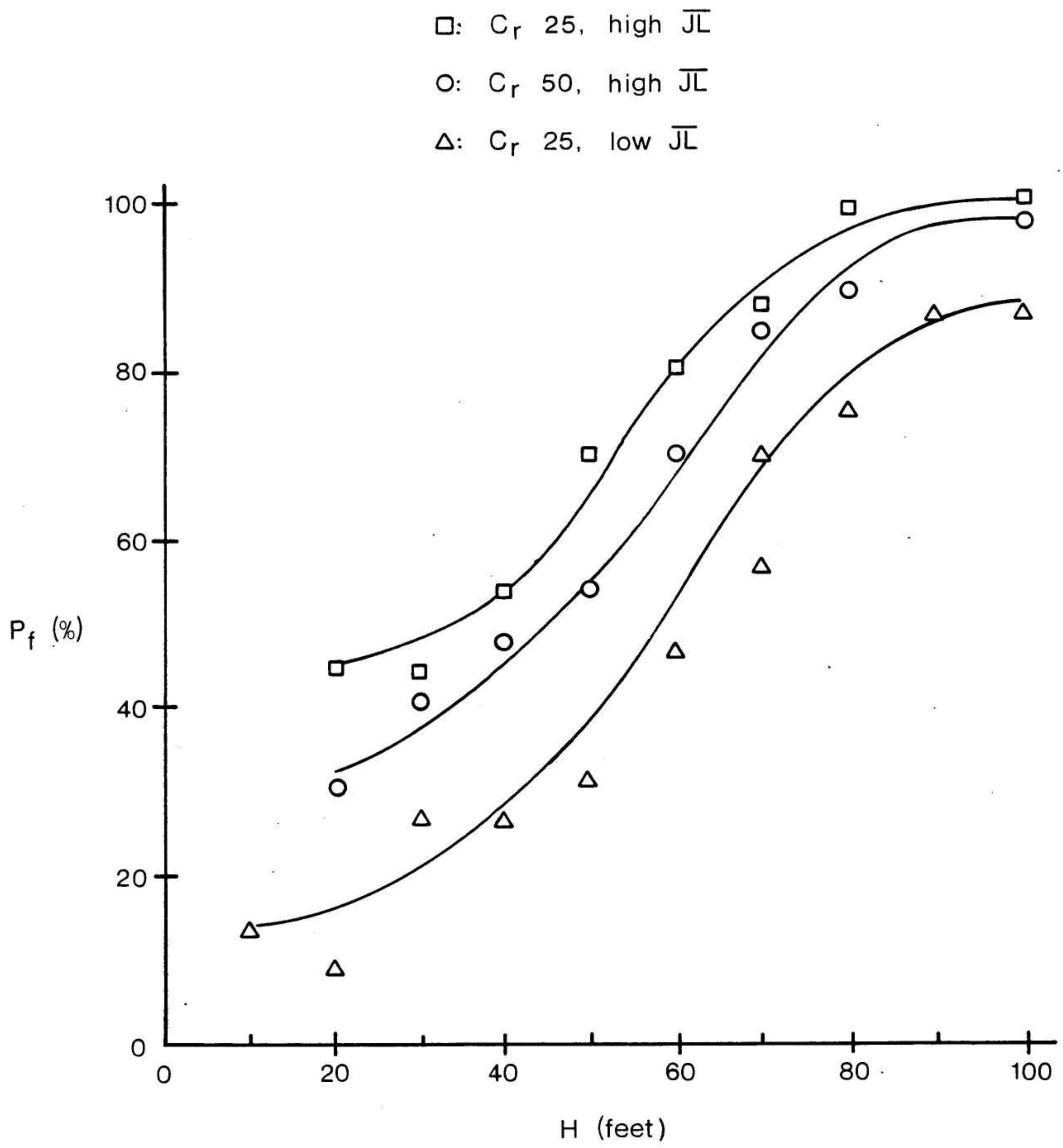


Figure 5.4 Effect of Intact Rock Cohesion (C_r) on the Probability of Failure $P_f(h)$

C_r = 25-500 ksf Θ = 80° $\alpha 1$ = 30° $\alpha 2$ = 60°

H = 90'-100'

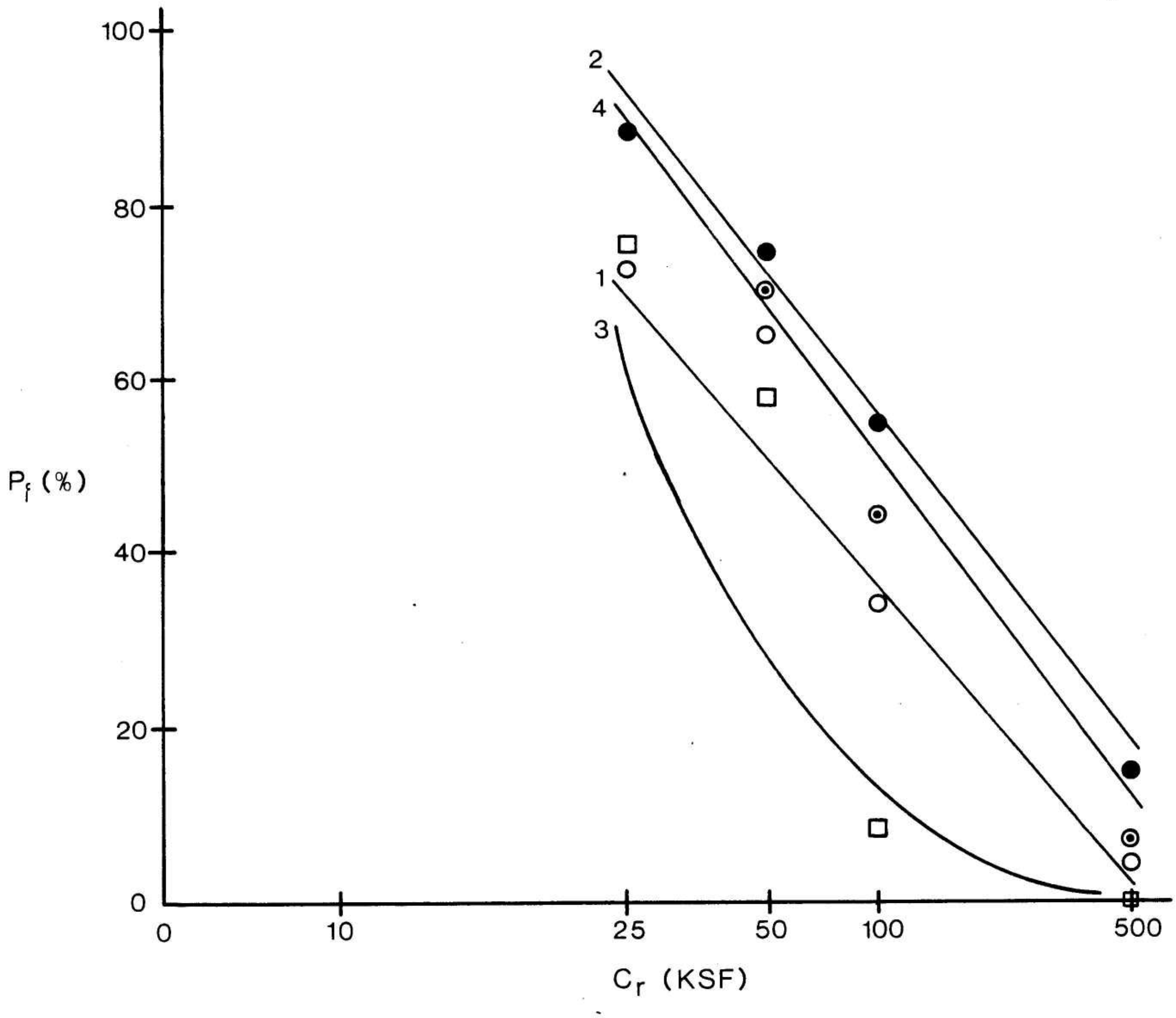


Figure 5.5 P_f as a Function of Intact Rock Cohesion (C_r)

dependence of P_f on C_r except at lower $\overline{JL1}$ (Case #3). This may be due to the presence of relatively long joints of the second set which increase the persistence of potential failure paths. One may expect slopes with two joint sets to be more sensitive to changes in C_r. From Figure 5.5, the influence of C_r on reliability (P_f) can be summarized by the following:

1. As expected, decreasing C_r has the effect of increasing P_f at any given height h. The magnitude of this increase grows substantially with depth.
2. Beyond a certain value, depending on other input parameters, C_r no longer has a significant effect on P_f and P_f approaches in value the probability of existence of a 100 percent persistent joint. For failure to occur the case with high joint persistences and long joints, C_r must be in the high range (500 ksf) so that the probability of failure corresponds to the probability of a fully persistent joint. For moderate joint persistence and low joint lengths, C_r must be greater than approximately 100 ksf for the above condition to take place.

Effect of Intact Rock Cohesion on Apparent Persistence

In Figures 5.6 through 5.9, mean apparent persistence ($\overline{K}_a$), mean plus one standard deviation of apparent persistence ($\overline{K}_a + \tilde{K}_a$) and critical persistence (K_c), are plotted as a function of C_r for the same four cases described previously. In all, computer output revealed that

C_r = 25-500 ksf Θ = 80° $\alpha 1$ = 30° $\alpha 2$ = 60°

H = 90-100' Φ_j = 15° $\overline{JL}$ = 40' $\overline{SP}$ = 15'

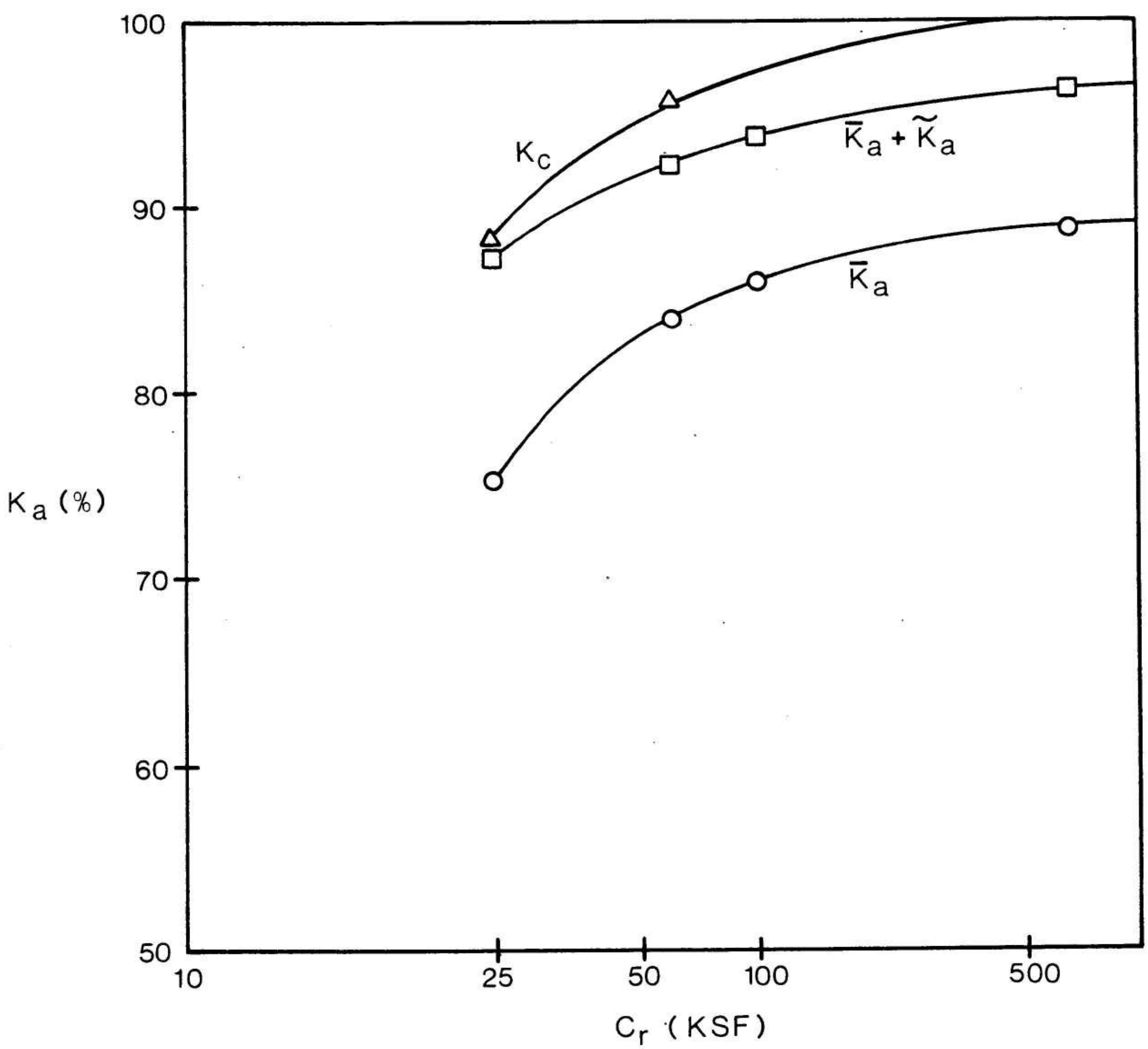


Figure 5.6 Effect of Rock Cohesion (C_r) on Apparent Persistence (K_a), Case 1

C_r = 25-500 ksf Θ = 80° $\alpha 1$ = 30° $\alpha 2$ = 60°

H = 90-100' Φ_j = 15° $\overline{JL}$ = 40' $\overline{SP}$ = 10'

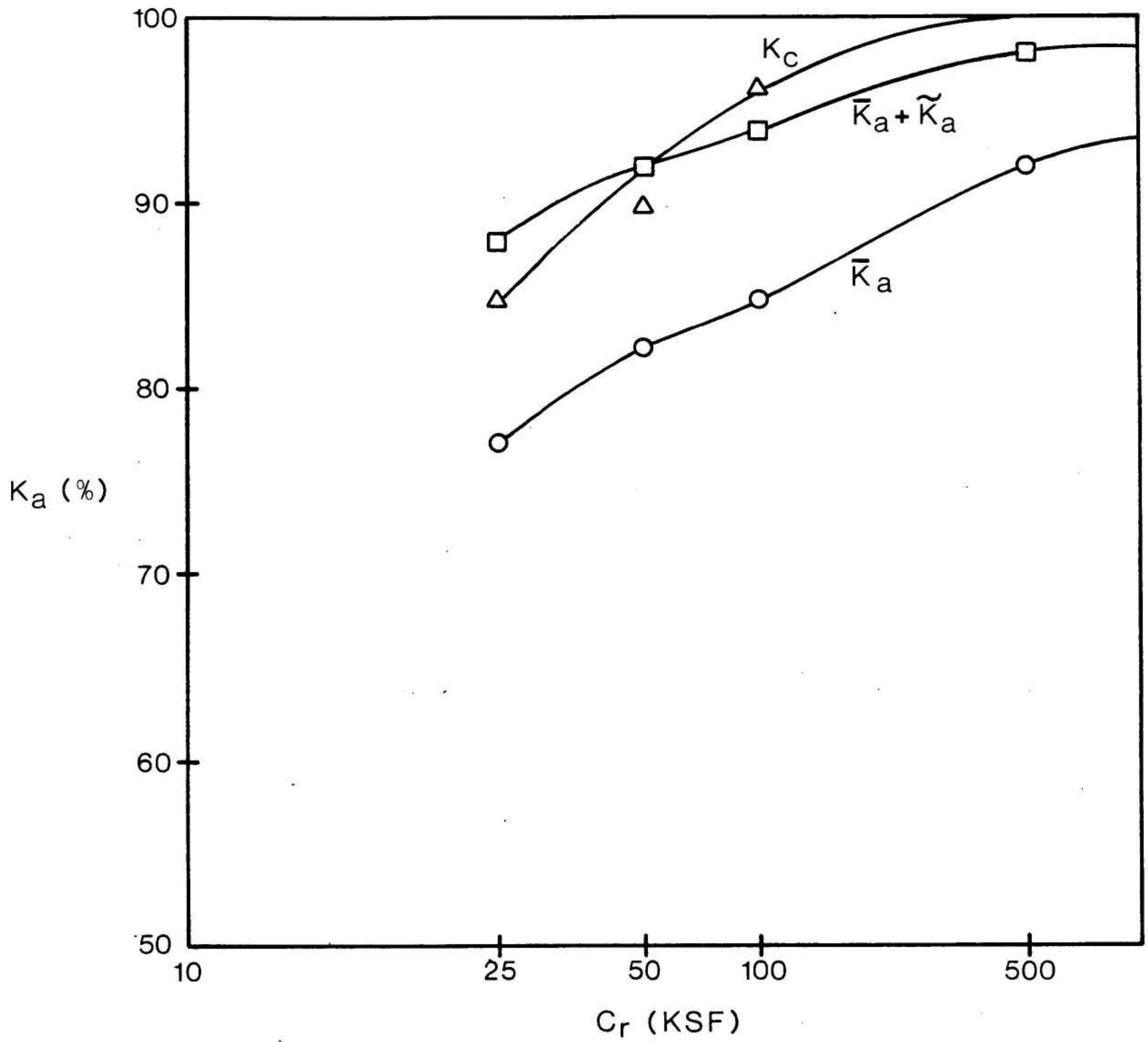


Figure 5.7   Effect of Rock Cohesion (C_r) on Apparent Persistence (K_a), Case 2

C_r = 25-500 ksf Θ = 80° $\alpha 1$ = 30° $\alpha 2$ = 60°

H = 90-100' Φ_j = 15° $\overline{JL}$ 1 = 15 $\overline{JL}$ 2 = 40'

SP = 10'

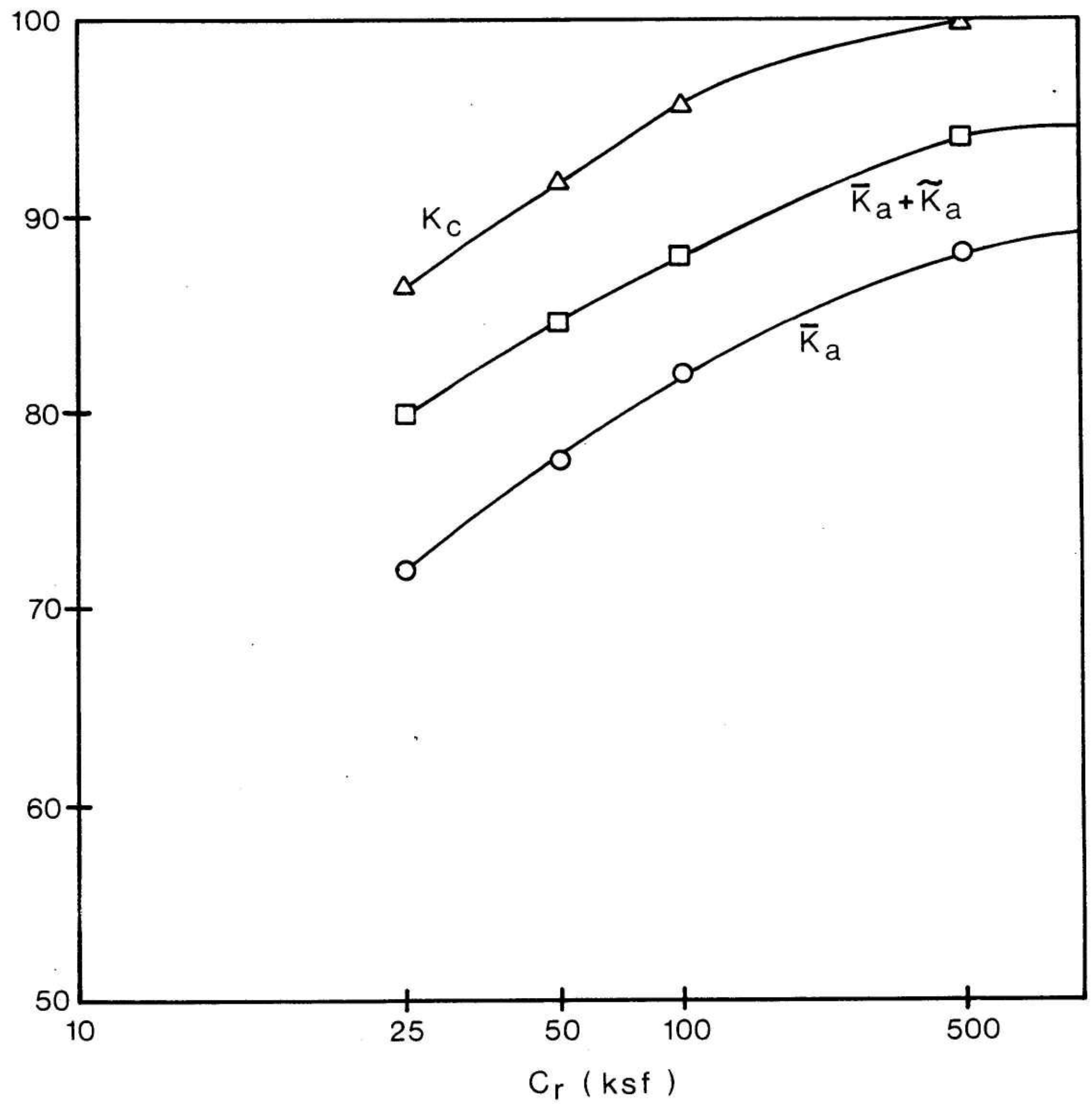


Figure 5.8 Effect of Rock Cohesion (C_r) on Apparent Persistence (K_a), Case 3

C_r = 25-500 ksf Θ = 80° $\alpha 1$ = 30° $\alpha 2$ = 60°

H = 90-100' Φ_j = 15° $\overline{JL}$ 1 = 40' $\overline{JL}$ 2 = 15

$\overline{SPRKBR}$ 1 = $\overline{SPRKBR}$ 2 = 40'

$\overline{SP}$ 1 = $\overline{SP}$ 2 = 10'

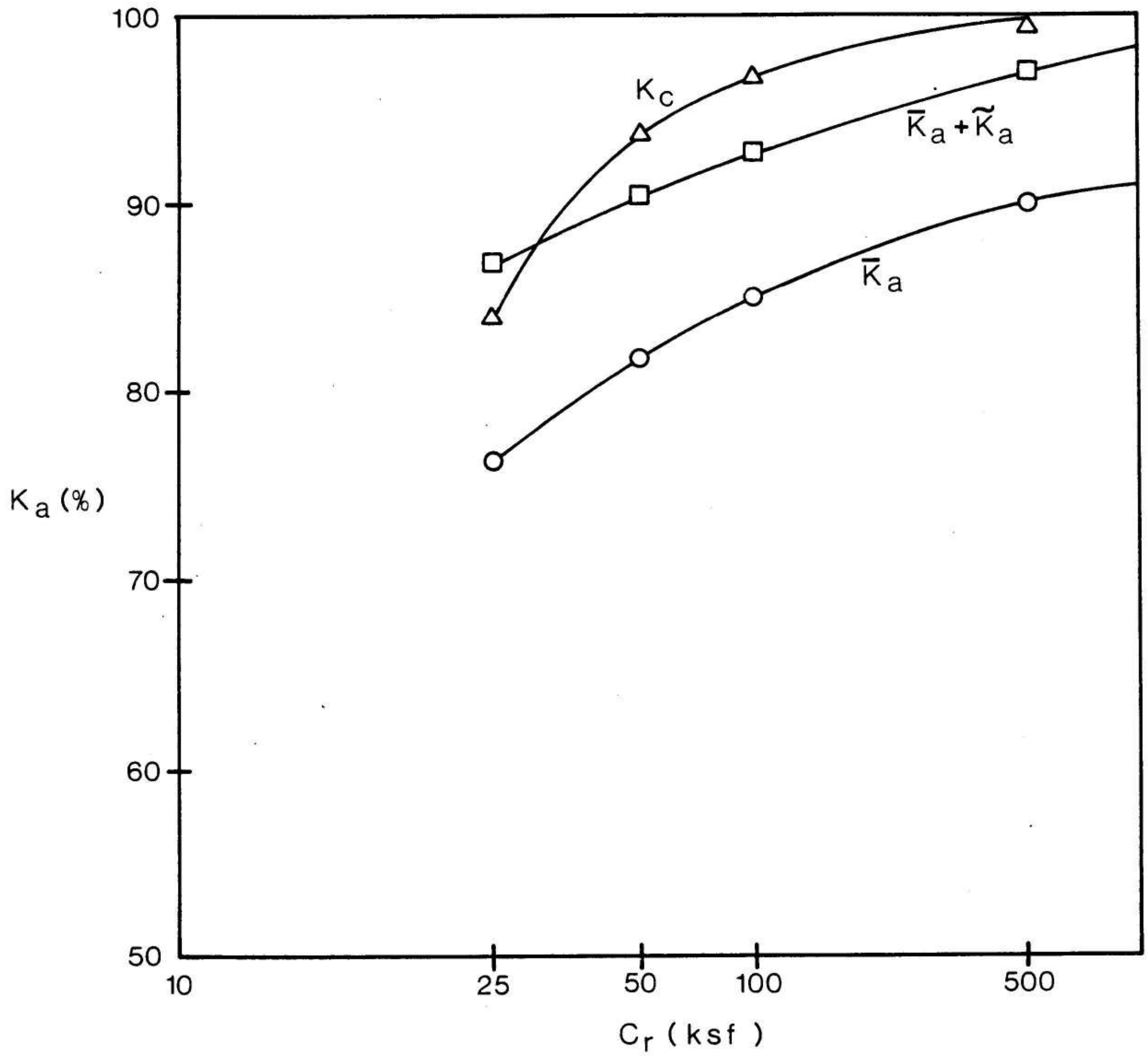


Figure 5.9 Effect of Rock Cohesion (C_r) on Apparent Persistence (K_a), Case 4

variations in $\overline{K}_a$ and $\tilde{K}_a$ are semi-sensitive to changes in C_r. Sensitivity appears to increase slightly as C_r is decreased. Insensitivity to C_r is due to the fact that apparent persistence is essentially a geometric property of the critical path. It is more noticeable in the case where the slope has a single joint set (O'Reilly- 1980). Typically, varying C_r does not significantly affect the geometry of the critical path especially in slopes with a single joint set, however, the distribution of K_a is affected.

Also plotted in each case is the critical persistence (K_c) as a function of C_r. As K_c approaches K_a, one would expect P_f to increase substantially. When studying the plots of all four cases, one finds that K_c is furthest from $\overline{K}_a$ in Case #3 (Figure 5.8) which consists of runs with short-joint lengths (JL1=15). Consequently, one concludes that Case #3 is the most reliable while Case #2 is the most unreliable ($\overline{K1}$=73%, $\overline{JL1}$=40'), thus implying the highest probability of failure.

As discussed previously, the use of reliability index (β) values gives a more quantitative description of how $\overline{K}_a$, $\tilde{K}_a$ and K_c interact to affect reliability. At depths greater than approximately 50 feet.

$$P_f = f(\beta)$$

$$\beta = \frac{K_a - K_c}{\tilde{K}_a}$$

The plots of β vs. C_r (Figure 5.10) for the three cases (#1, 2 and 4) under consideration, at a depth interval of 90-100, show that reli-

C_r = 25-500 ksf Θ = 80° $\alpha 1$ = 30° $\alpha 2$ = 60°

H = 90-100' Φ_j = 15° $\overline{JL}$ = 40' $\overline{SP}$ = 15'

$\overline{K}$ = 50%

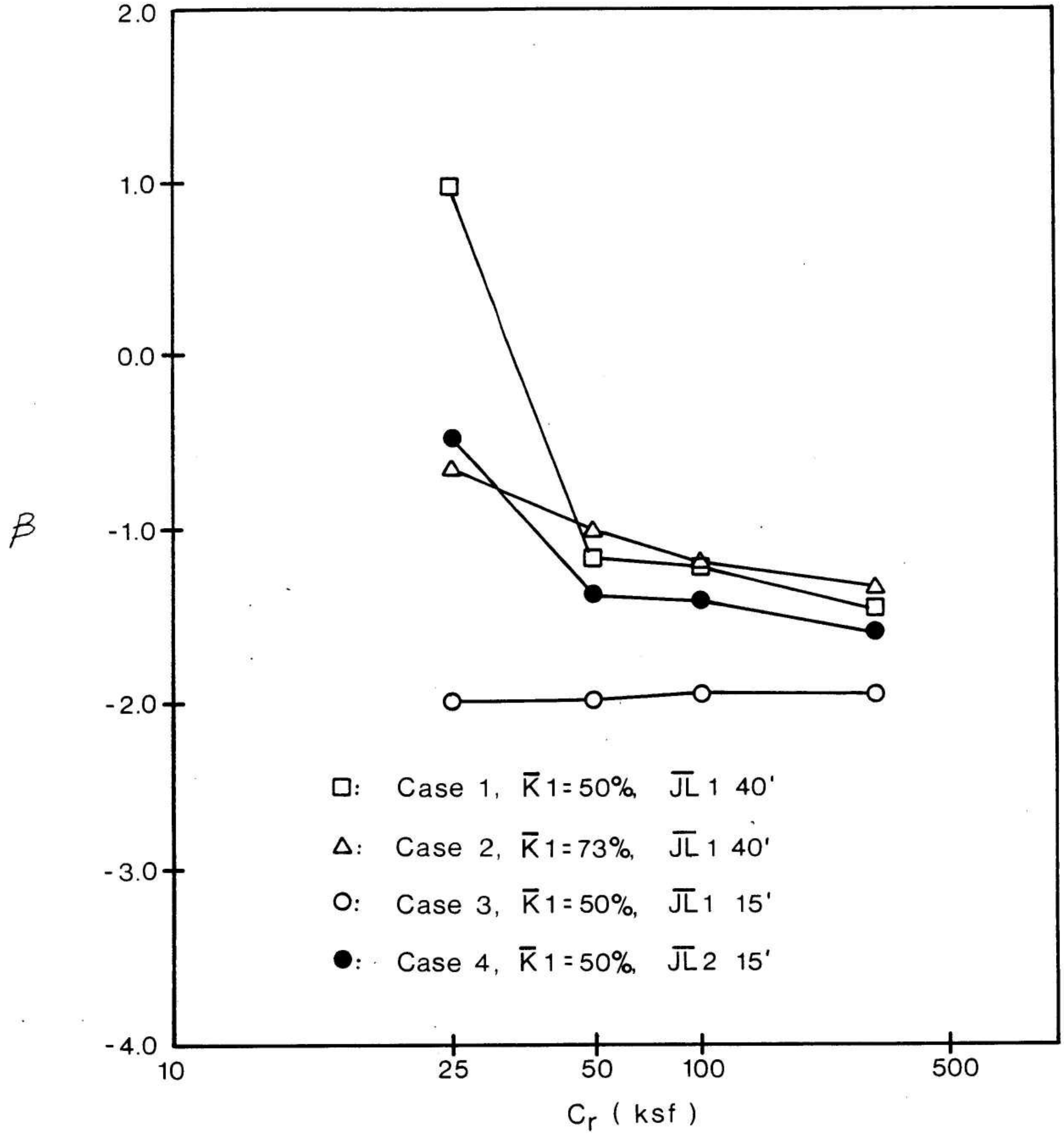


Figure 5.10 Index of Reliability (β) as a Function of Rock Cohesion (C_r)

ability (β) increases with increasing C_r. Case #3 is clearly the most reliable.

In many situations, β values can be used to give reasonable estimates of P_f without going through an entire simulation process. In cases where $\overline{K}_a$ and $\tilde{K}_a$ change very little when the parameter of interest is varied, values can be estimated from a single model run simply by assuming that both are independent of the parameter analyzed (C_r here). As for the critical persistence (K_c), which is a function of the parameter being analyzed (C_r here too), its value can be calculated from the following closed form equation:

$$K_c = 1 - \frac{2c\ (\tan \alpha l - \tan \Phi_j)}{\sqrt{2c + 1} - 2c \tan \Phi_j} \times 100$$

Once K_c is determined, β values can quickly be found as a function of C_r without making additional lengthy simulations.

SUMMARY - Effects of Intact Rock Cohesion on Apparent Persistence

The influence of C_r on the distribution of K_a at a given depth can be summarized as follows:

1. Increasing C_r results in an increase in $\overline{K}_a$ and a slight decrease of $\tilde{K}_a$ at deep slope intervals. Mean apparent persistence sensitivity to C_r increases with depth and with decreasing C_r.

2. At a deep slope interval, variations in $\overline{K}_a$ as a function of C_r are moderate, approximately 5 to 12 percent. Variations in $\tilde{K}_a$ are much less than those in $\overline{K}_a$. In the case of a slope with a single joint set, similar variations are practically non-existent.
3. For a given height interval, it is possible to calculate the reliability index (β) for a wide range of C_r values from the output of a single simulation.

5.5.3 Effect of Joint Friction Angle (Φj) on Slope Reliability

In most program runs, Φ_j is set equal to zero. A reason for this is that sensitivity of the probability of failure (P_f) can be better examined if P_f is high (> 90%). This is done by keeping low the other resistance parameter (Φ_j). Later on, when studying the effects of the other parameters, both C_r and Φ_j are kept at low values so that failure probabilities are high and the influence of the examined parameter on stability could be demonstrated with better precision.

Another reason for setting Φ_j equal to zero has to do with rock mechanics. In the process of calculating total resistance, peak intact rock strength and peak joint strength are fully mobilized. However, this can be unconservative since peak shear resistance along joints is generally mobilized at strains higher than that for intact rock (O'Reilly-1980). Also, by keeping Φ_j equal to zero throughout, one achieves the additional benefit of being able to exclude the effects of water pressures on slope reliability analysis without being unconservative. In other words, the component that could be affected by water pressure should and is here set to zero. Future work is expected to provide information and procedures for including water pressures in reliability analyses.

In some situations, Φ_j can have a large effect on slope stability. This is particularly true in weak rock (C_r< 100ksf) as is shown next. The influence of variation of Φ_j on slope reliability is examined in Cases 5, through 8. Detailed plots using computer output data to establish understanding on the influence of Φ_j on rock slope reliability are shown in Figures 5.11 through 5.18.

<u>The Effect of Joint Fricition Angle (Φj) on the Probability of Failure P_f (h)</u>

The effects of varying Φ_j on P_f (h) are shown in Figure 5.12. As expected, when Φ_j approaches the angle of the first set inclination (α1), the probability of failure approaches that of a joint plane being 100 percent persistent. This fact holds independently of all other jointing and strength parameters. Hence, decreasing Φ_j has the effect of increasing P_f at all values of h. In all cases, the influences of Φ_j increases with depth.

Figure 5.11 presents P_f as a function of Φ_j for the deep interval from 90-100 feet. For very low intact rock cohesion (25ksf), program output shows that P_f is very sensitive to variations in Φ_j. For high

C_r = 25-100 ksf Θ = 80° $\alpha 1$ = 40° $\alpha 2$ = 70°

H = 90-100' Φ_j = 0° -40° $\overline{JL}$ 1 = $\overline{JL}$ 2 = 20'

$\overline{SP}$ 1 = $\overline{SP}$ 2 = 12'

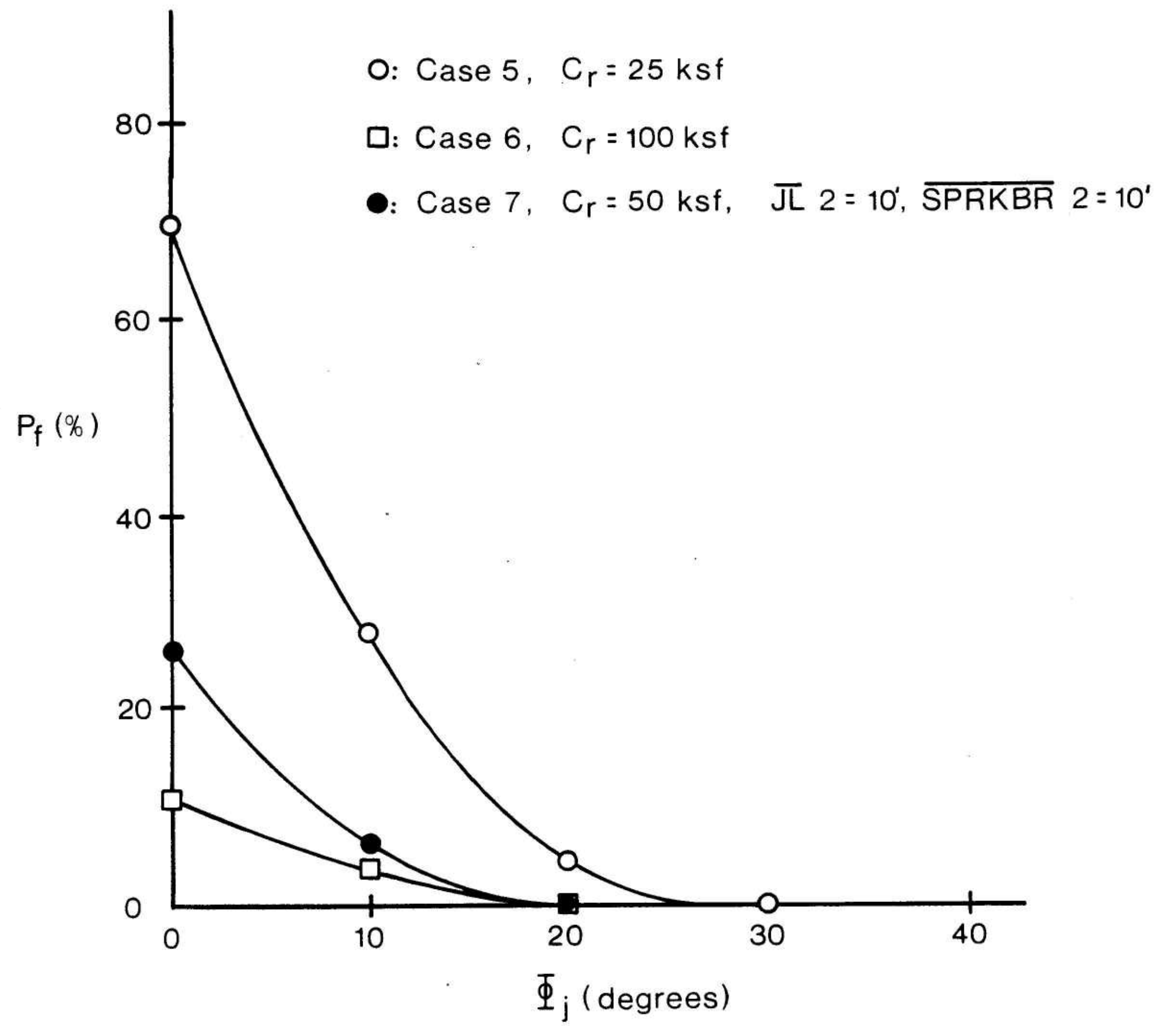


Figure 5.11 P_f as a Function of Joint Friction Angle (Φ_j)

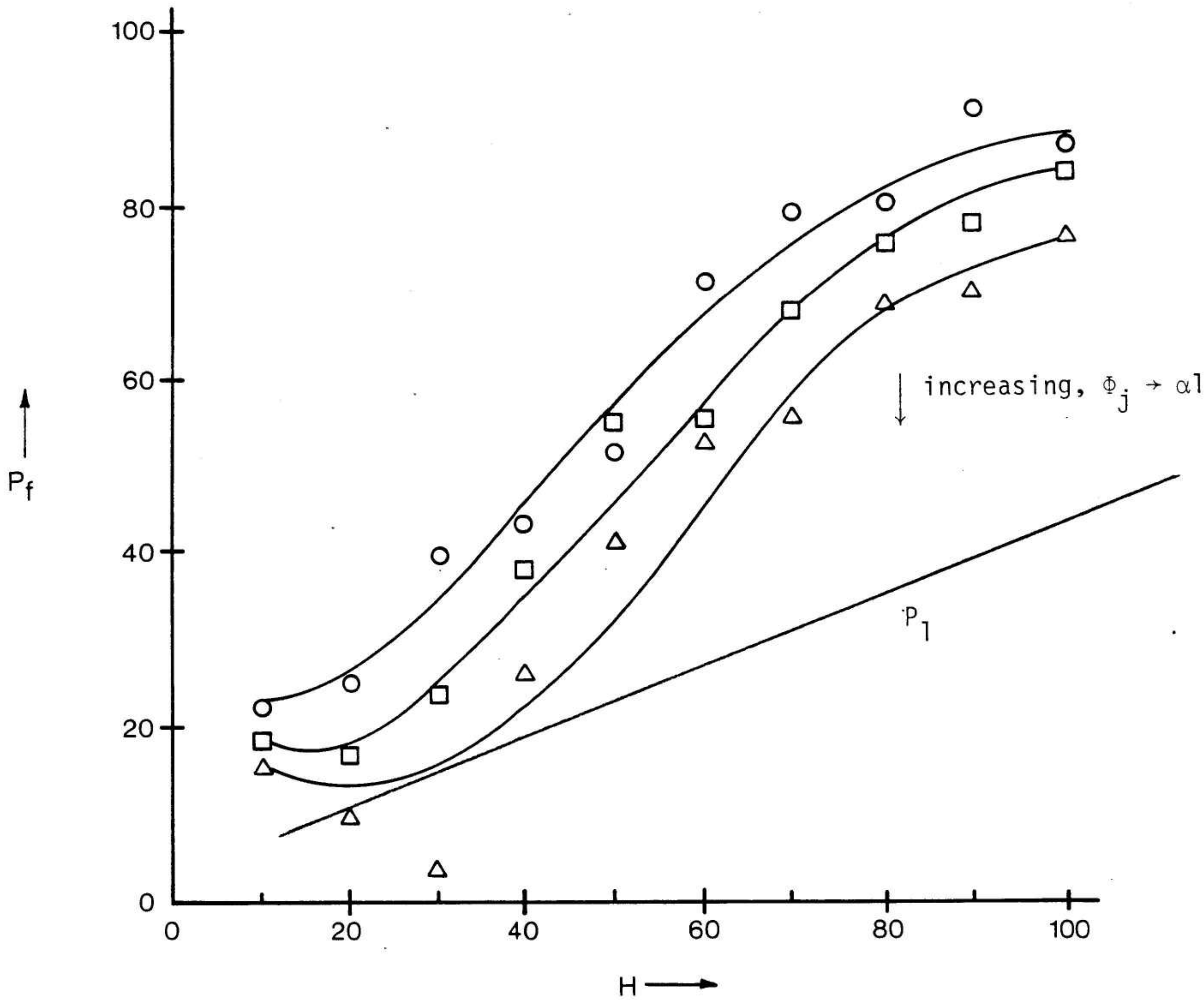


Figure 5.12 Effect of Joint Friction Angle (Φ_j) on P_f (h) (Qualitative)

C_r = 25 ksf Θ = 60° α = 40°

H = 90-100' Φ_j = 0-40° $\overline{JL}$ = 40' $\overline{SP}$ = 5'

No second joint set.

* Notice complete independence of K_a from changes in Φ_j.

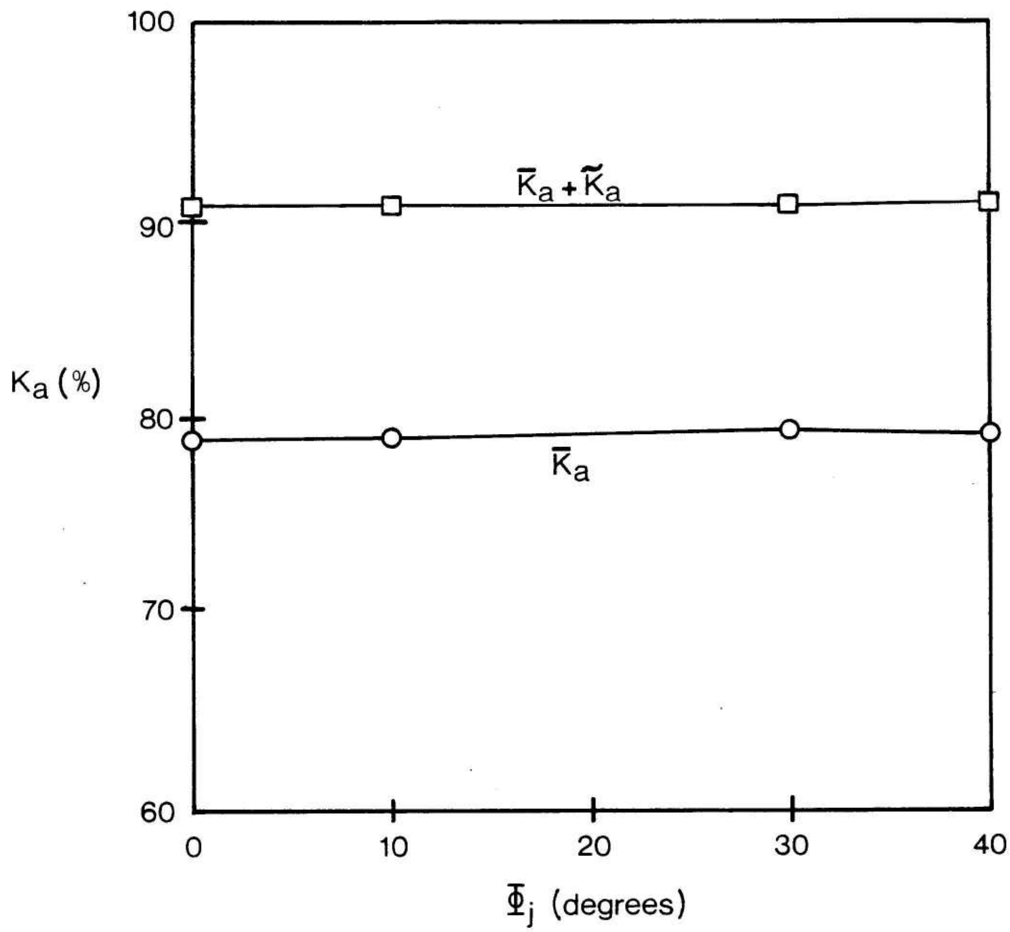


Figure 5.13 Effect of Joint Friction Angle (Φ_j) on Apparent Persistence (K_a)

(Figure 6.35) O'Reilly- 1980

cohesion (100 skf), the data plots whose neglible sensitivity to variations in Φ_j. In Figure 5.11, one may notice that in rock with moderate joint friction angles (10 -20°) and a high mean joint length first set, reliability is high in depths of up to 100 feet.

Effect of Joint Friction Angle (Φ_j) on Apparent Persistence (K_a)

Figures 5.13 through 5.18 are plots of distributions of K_a at various Φ_j. Figure 5.13 is from O'Reilly - 1980 for comparison. Figure 5.14 shows results from runs in which C_r is kept at 25 ksf while Φ_j is varied between 0 and 40 degrees. Sensitivity of $\overline{K}_a$ to variations in Φ_j is moderate. In all, $\tilde{K}_a$ decreases as Φ_j is increased.

In cases 5, 6, and 7, the critical persistence curve intersects the $\overline{K}_a$ curve at Φ_j approximately equal to 25° which is halfway in the range. This implies that at least 50 percent of the critical paths are failure paths.

At higher cohesion values, program output reveals that sensitvity of apparent persistence (K_a) to variations in Φ_j decreases. Contrast to the findings of O'Reilly (Fig. 5.13), Figures 5.14 through 5.17 show some sensitivity of $\overline{K}_a$ and $\tilde{K}_a$ to variations in Φ_j. This may be explained by critical paths having high persistences and thus implying lesser dependence on intact rock cohesion and greater dependence on the joint friction angle.

Reliability index (β) values, derived for the data points in Figures 5.14 to 5.17, are plotted in Figure 5.18. As Φ_j increases, the plots of β vs. Φ_j for cases 5, 6 and 7 level off. This indicates that as Φ_j increases, reliability increases and becomes less and less

C_r = 25 ksf Θ = 80° $\alpha 1$ = 40° $\alpha 2$ = 70°

H = 90-100' Φ_j = 0-40° $\overline{JL}$ = 20' $\overline{SP}$ = 12'

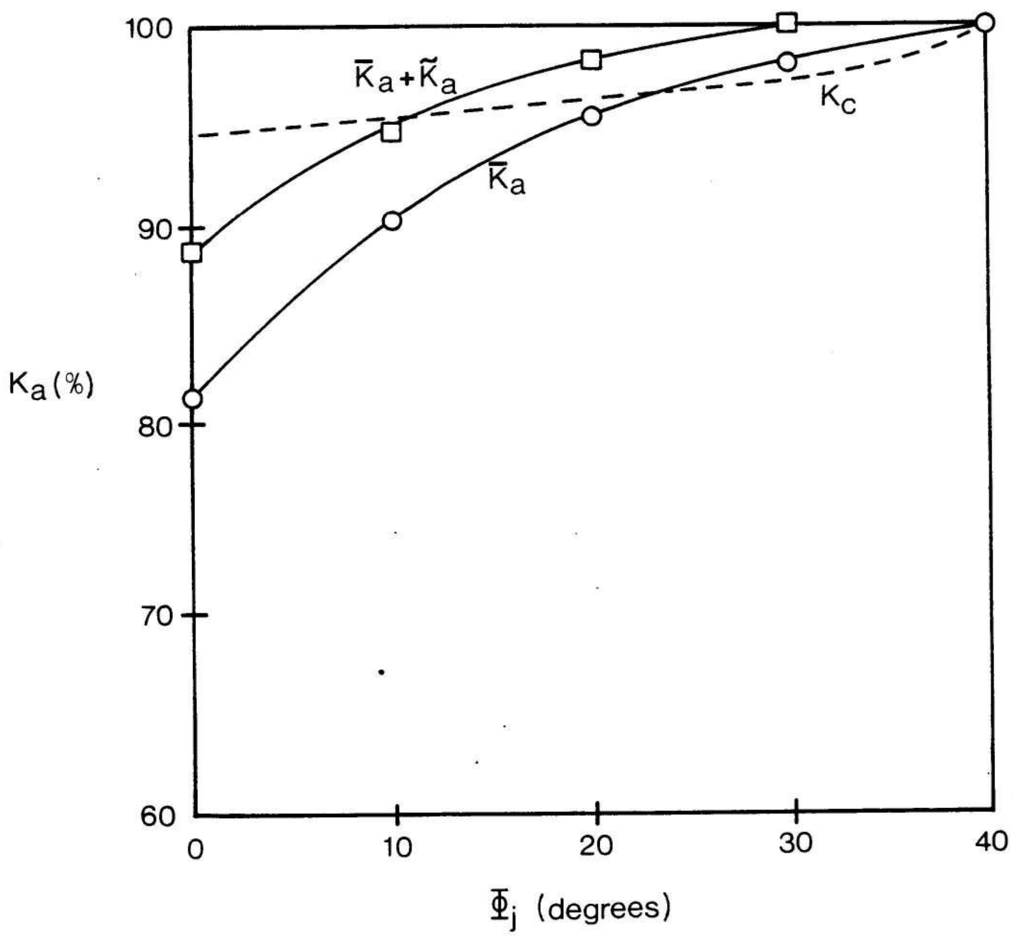


Figure 5.14 Effect of Joint Friction Angle (Φ_j) on Apparent Persistence (K_a), Case 5

C_r = 100 ksf Θ = 80° $\alpha 1$ = 40° $\alpha 2$ = 70°

H = 90-100' Φ_j = 0-40° $\overline{JL}$ = 20' $\overline{SP}$ = 12'

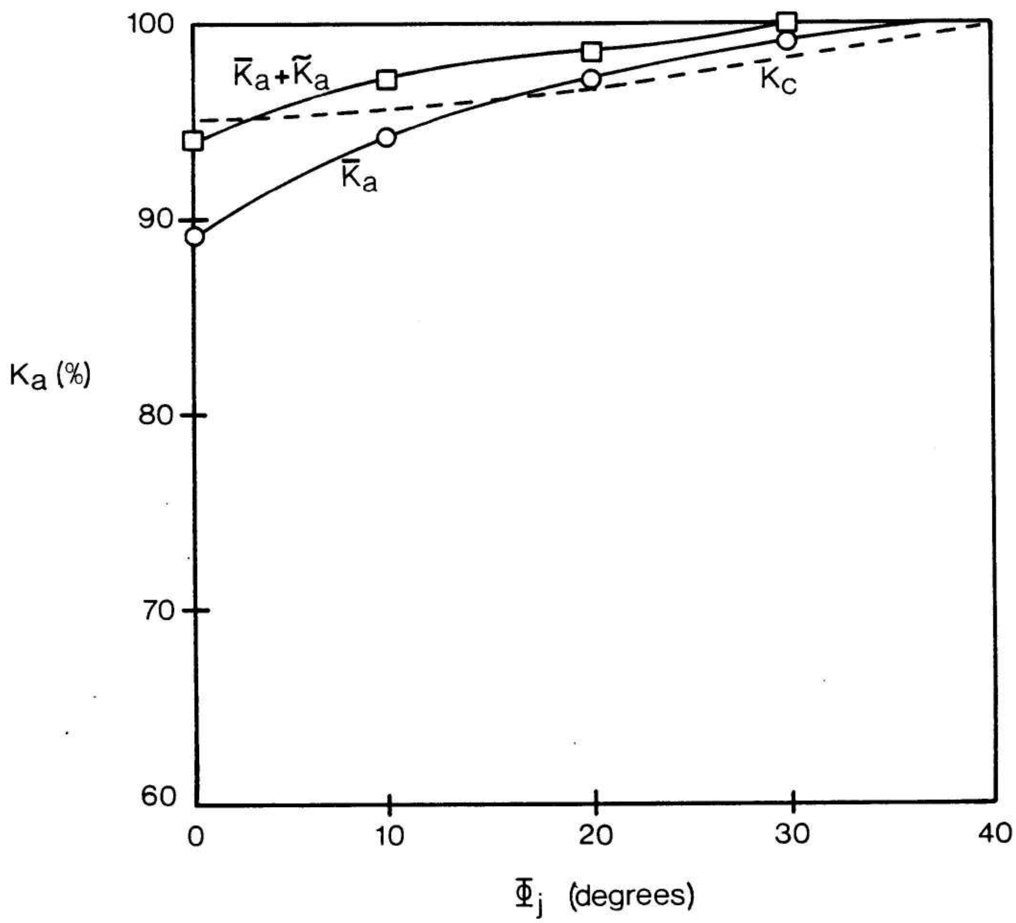


Figure 5.15 Effect of Joint Friction Angle (Φ_j) on Apparent Persistence (K_a), Case 6

C_r = 50 ksf Θ = 80° $\alpha 1$ = 40° $\alpha 2$ = 70°

H = 90-100' Φ_j = 0-40° $\overline{JL}$ 1 = 20' $\overline{JL}$ 2 = 10'

$\overline{SP}$ = 12'

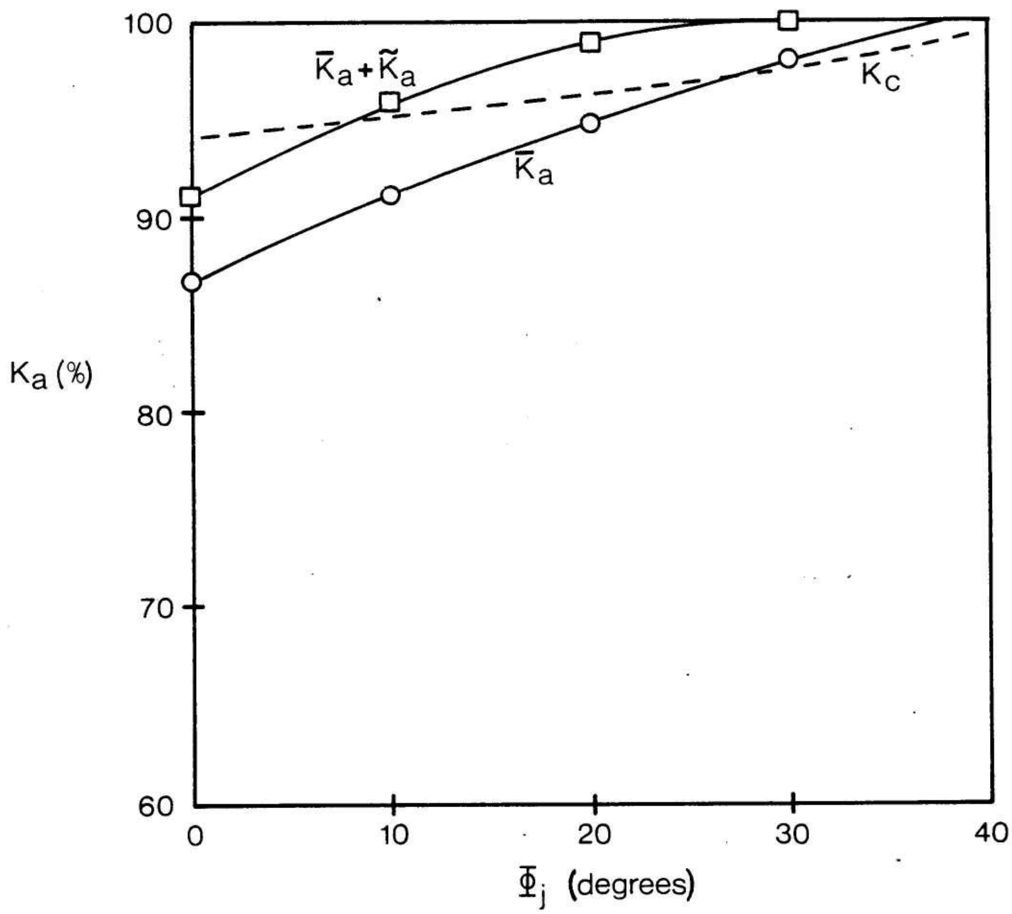


Figure 5.16 Effect of Joint Friction Angle (Φ_j) on Apparent Persistence (K_a), Case 7

C_r = 50 ksf Θ = 80° $\alpha 1$ = 35° $\alpha 2$ = 70°

H =90-100' Φ_j = 0-35° $\overline{JL}$ = 30' $\overline{SP}$ =8'

$\overline{K}$ = 50%

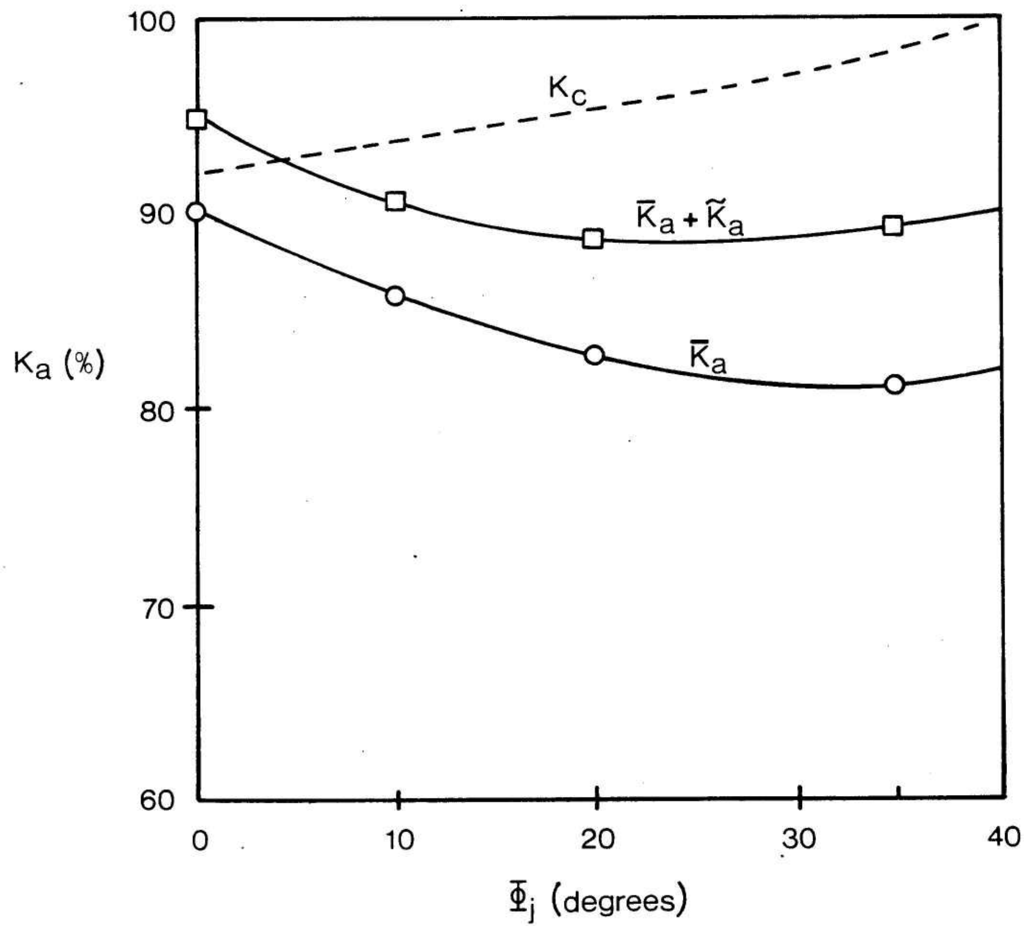


Figure 5.17 Effect of Joint Friction Angle (Φ_j) on Apparent Persistence (K_a), Case 8

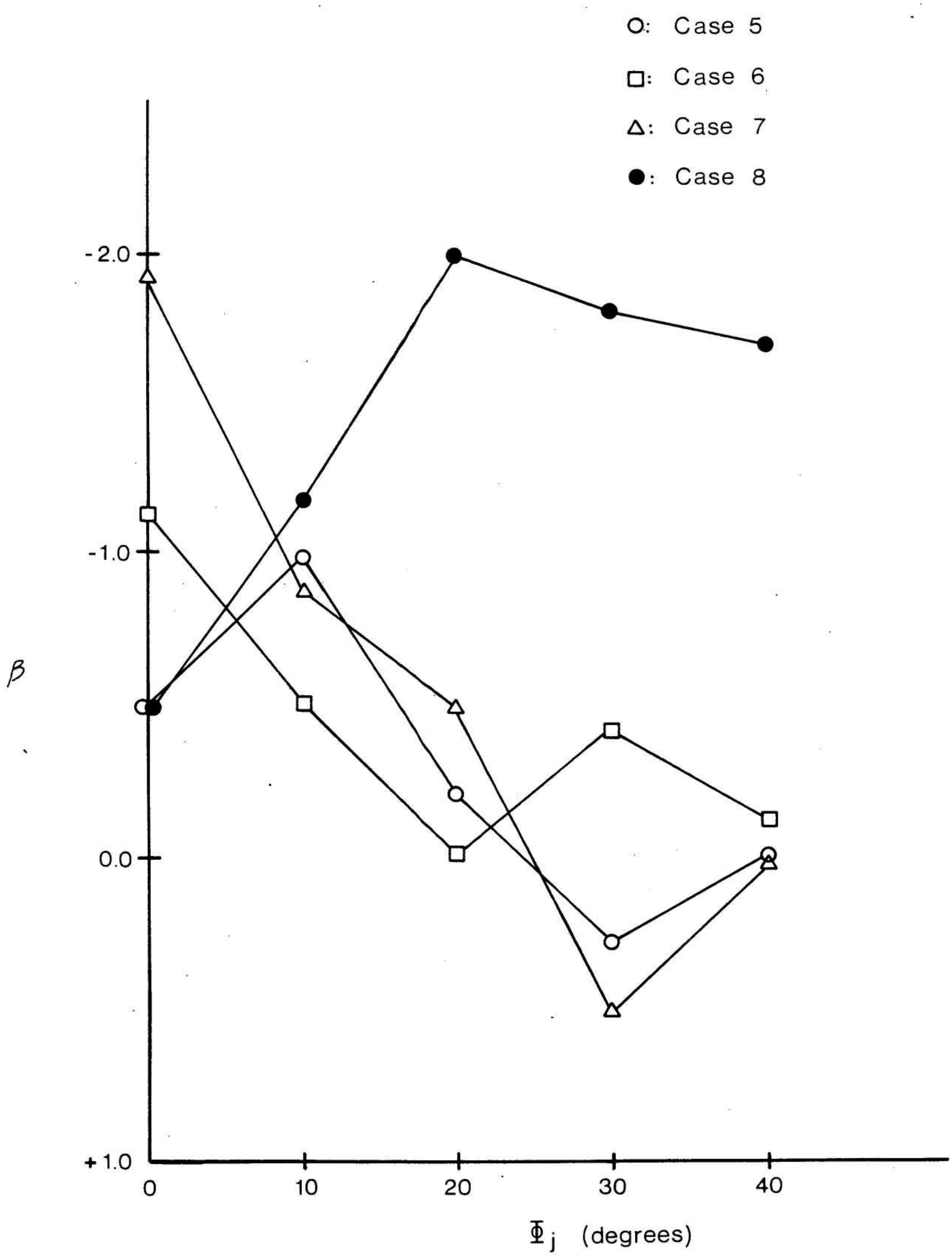


Figure 5.18 Index of Reliability (β) as a Function of Joint Friction Angle (Φ_j)

dependent on intact rock strength.

Summary - Effects of Φj

The effect of Φ_j on reliability can be summarized as follows:

1. Φ_j has a strong influence on the reliability of slopes in weak rock (i.e., less than 50 ksf).
2. As Φ_j increases, while approaching first joint inclination, the influence of intact rock strength decreases until P_f (h) is approximately equal to the probability of a joint being 100 percent persistent (P_1).
3. Both $\overline{K}_a$ and $\tilde{K}_a$ can be assumed to be dependent on Φ_j. This is a result of the critical paths being less dependent on intact rock cohesion (C_r) as a result of high persistences.

5.5.4 Effect of First Set Mean Joint Length ($\overline{JL1}$)

In a rock slope, joint lengths of the first set are assumed to be exponentially distributed about their mean - $\overline{JL1}$. Effects of varying $\overline{JL1}$ on rock slope reliability are examined by varying intact rock cohesion and varying mean joint plane spacing of the second joint set.

Effect of First Set Mean Joint Length ($\overline{JL1}$) on the Probability of Failure P_f (h)

From looking at Figure 5.19, one can see that the probability of failure increases with depth. In all depths, computer output reveals that the probability of failure increases when increasing the mean joint length ($\overline{JL1}$). Generally, increasing $\overline{JL1}$ at any depth does not considerably increase the probability of failure except in the deep sections of the slope (i.e., greater than 80 feet).

The Probability of Failure (P_f) as a Function of First Set Mean Joint Length ($\overline{JL1}$)

Figure 5.21 shows plots of P_f vs. $\overline{JL1}$ for cases 6, 7 and 8 at a depth of 90-100 feet. From program output, decreasing intact rock cohesion increases the dependence of P_f on variations in $\overline{JL1}$ as the figure clearly shows. The curve for C_r equal 100 ksf, is approximately equivalent to the 100 percent persistence curve for all $\overline{JL1}$. In other words, as C_r approaches the value of 100 ksf, failures are expected to occur along 100 percent persistent planes. A similar trend may be seen in the findings of O'Reilly - 1980, as can be seen in Fig. 5.20.

Effect of First Set Mean Joint Length ($\overline{JL1}$) on Apparent Persistence and the Reliability Index

Plots of mean apparent persistence $\overline{K}_a$ and mean apparent persistence plus one standard deviation of apparent persistence ($\overline{K}_a + \tilde{K}_a$) for cases 8, 9, and 10 are shown in Figures 5.24 through 5.26. Figure 5.23 (figure 6.41 from O'Reilly - 1980) is included for comparison purposes. In all cases the variations, of $\overline{K}_a$ and $\tilde{K}_a$ with varying $\overline{JL1}$, are small. Therefore, for practical purposes $\overline{K}_a$ and $\tilde{K}_a$ may be assumed to be constant. Thus, the reliability index values (β) may be obtained by one simulation for each

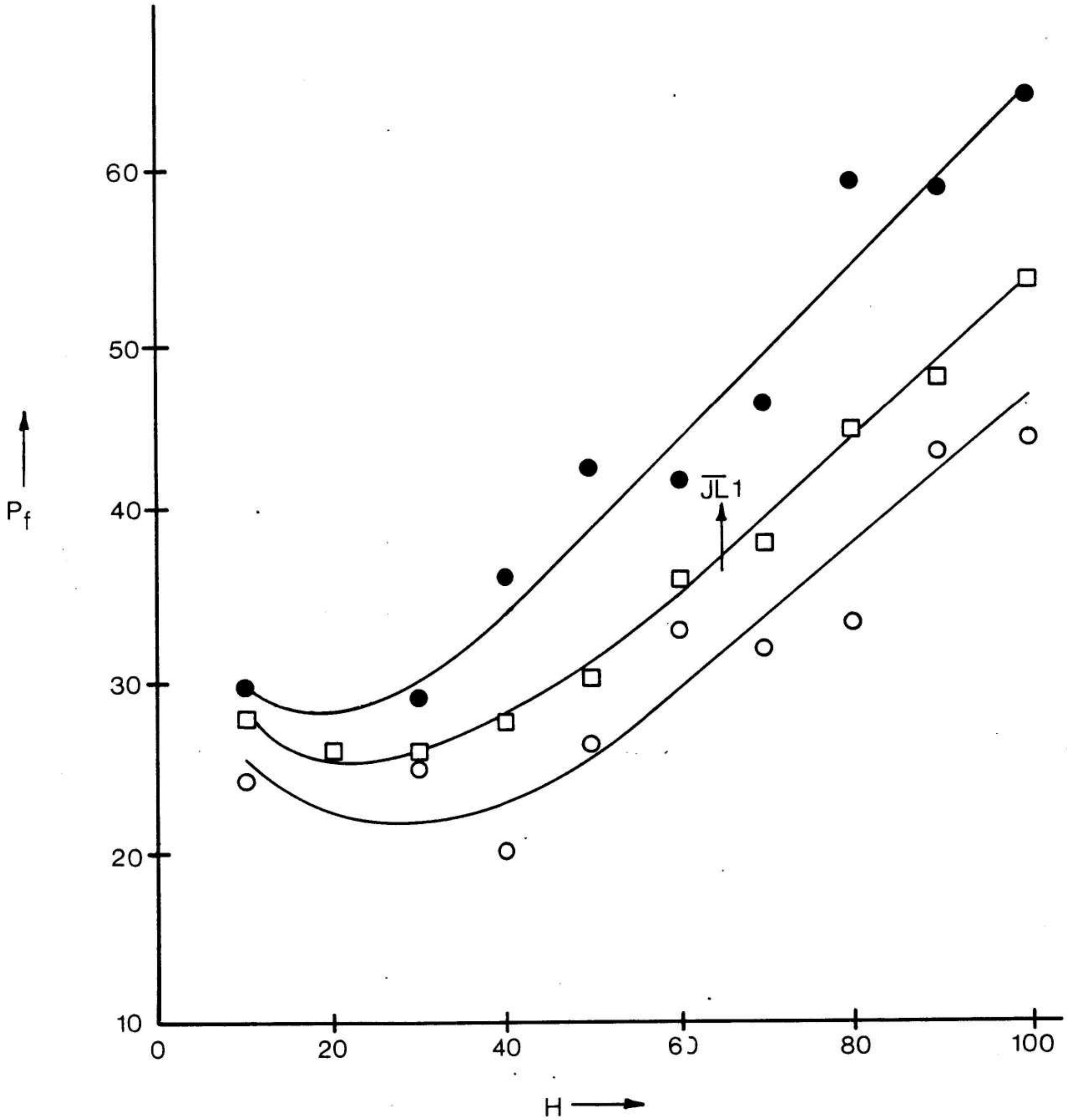


Figure 5.19 Effect of Mean Joint Length (Set 1) $\overline{JL}$ 1 on P_f (h)

C_r = 25-100 ksf Θ = 60° α = 40°

H = 90-100' Φ_j = 0° $\overline{JL}$ = 10-40'   SP = 5'

$\overline{K}$ = 50%

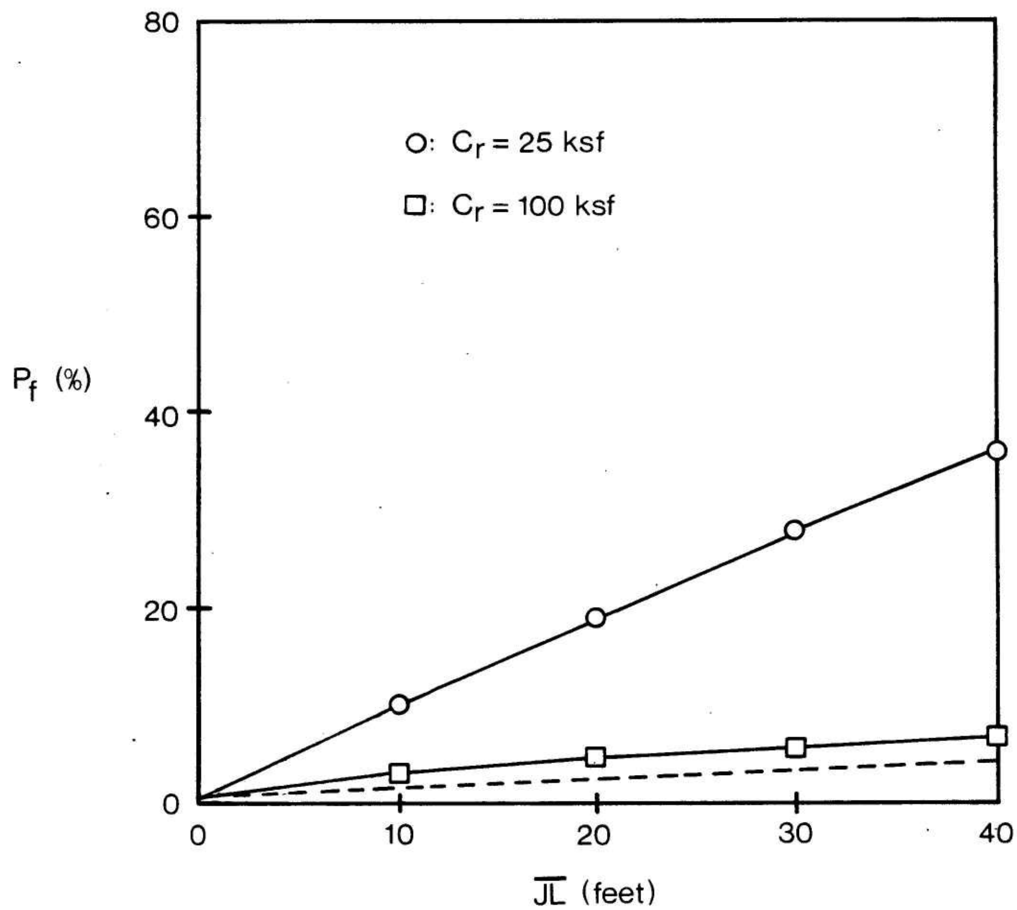


Figure 5.20 P_f as a Function of Mean Joint Length ($\overline{JL}$)

(From O'Reilly-1980)

C_r = 25-100 ksf $\quad \Theta$ = 80° $\quad \alpha 1$ = 20° $\quad \alpha 2$ = 70°

H = 90-100' $\quad \Phi_j$ = 0° $\quad \overline{JL}$ 1 = 10-40' $\quad$ SP = 10'

$\overline{K}$ 1 = 50% $\quad \overline{K}$ 2 = 50%

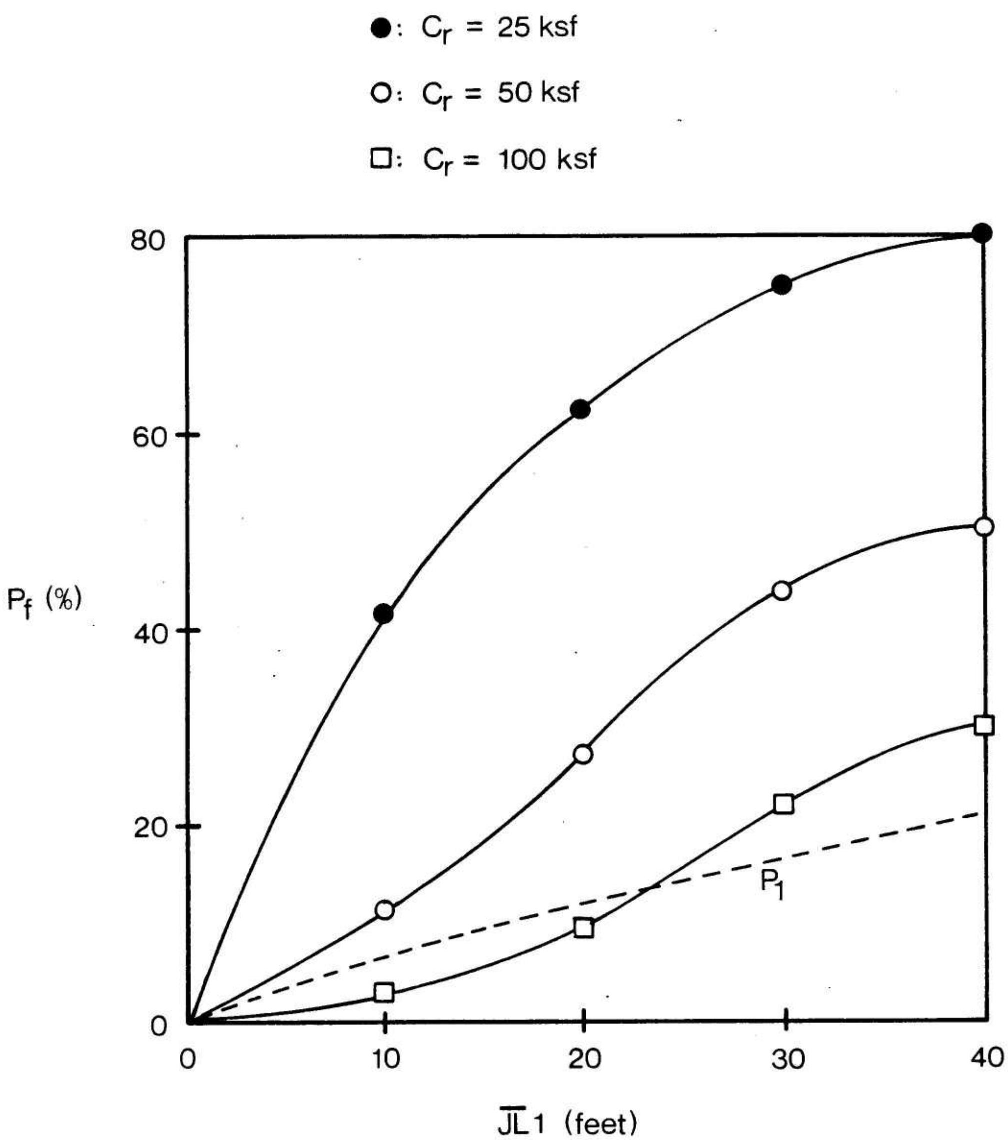


Figure 5.21 $\quad P_f$ as a Function of Mean Joint length ($\overline{JL}$ 1) (Varying C)

C_r = 50 ksf Θ = 80° $\alpha 1$ = 20° $\alpha 2$ = 70°

H = 90-100' Φ_j = 0° $\overline{JL}$ 1 = 10-40' $\overline{SP}$ 1 = 10'

$\overline{K}$ = 50% $\overline{SP}$ 2 = 10-30'

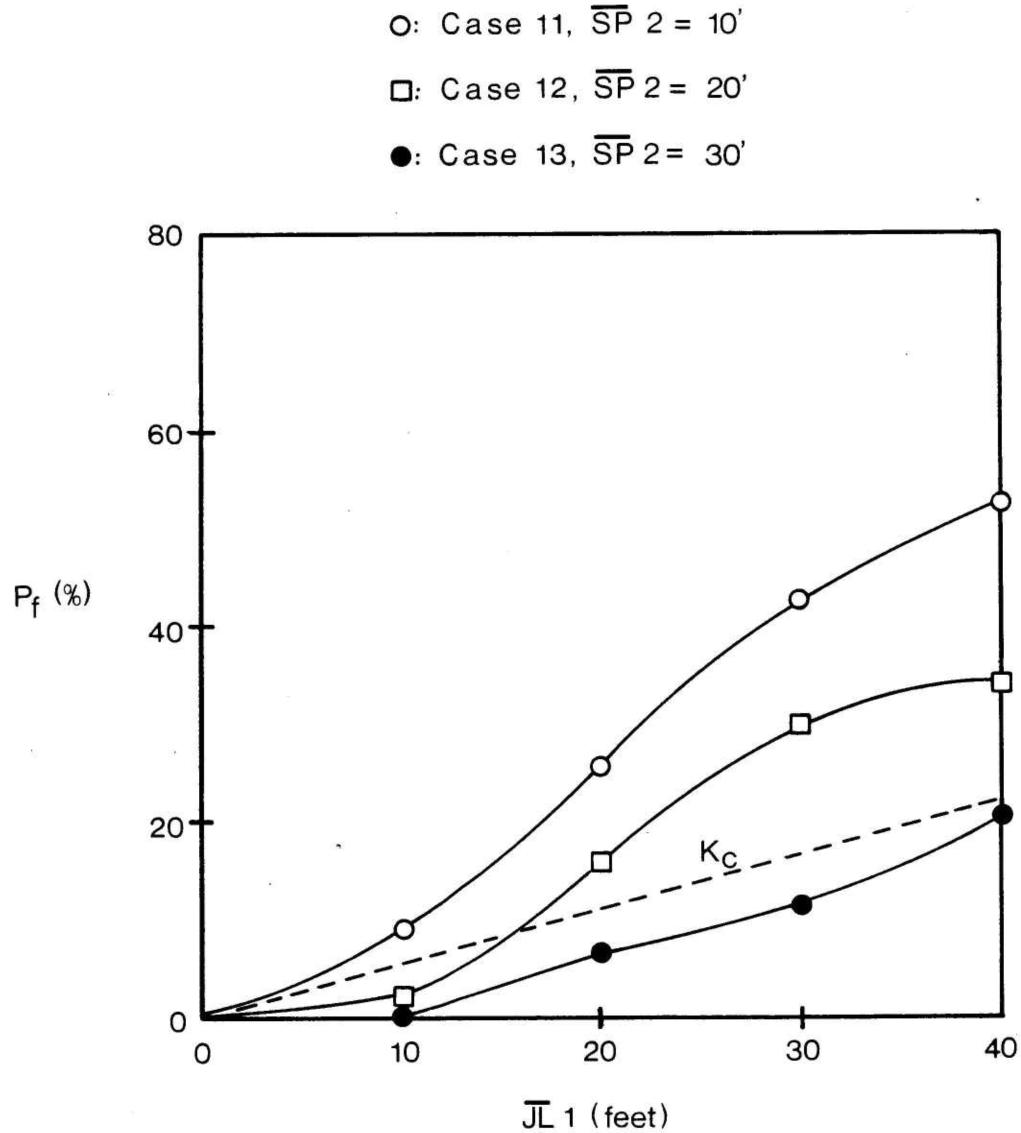


Figure 5.22 P_f as a Function of Mean Joint Length (Set 1, $\overline{JL}$ 1), Effect of Varying Mean Joint Plane Spacing ($\overline{SP}$ 2)

C_r = 100 ksf Θ = 60°

H = 90-100' Φ_j = 0° $\overline{JL}$ = 10-40' $\overline{SP}$ = 5'

$\overline{K}$ = 50%

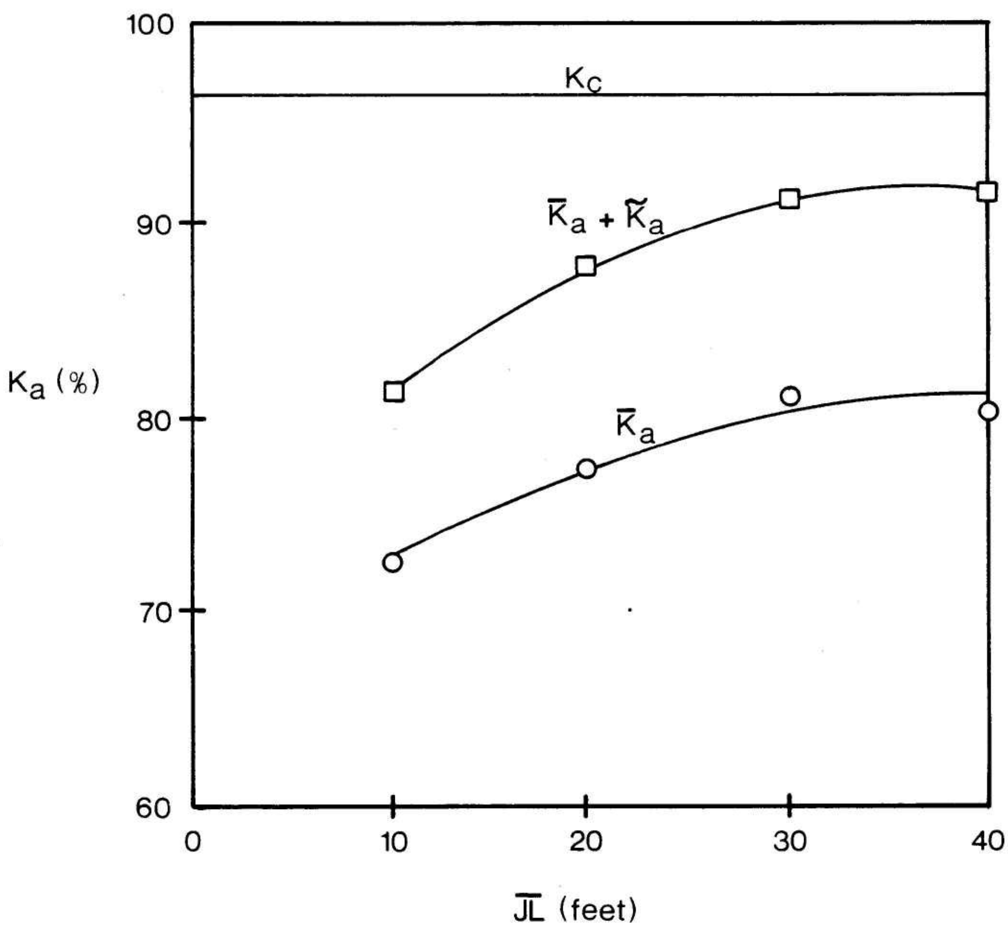


Figure 5.23   Effect of Mean Joint Length ($\overline{JL}$) on Apparent Persistence (K_a)

(Figure 6.41) O'Reilly-1980

$\overline{K} = 50\%$ $\Theta = 80°$ $\alpha 1 = 20°$ $\alpha 2 = 70°$

$H = 90\text{-}100'$ $\Phi_j = 0°$ $\overline{JL}\ 1 = 10\text{-}40'$ $\overline{SP} = 10'$

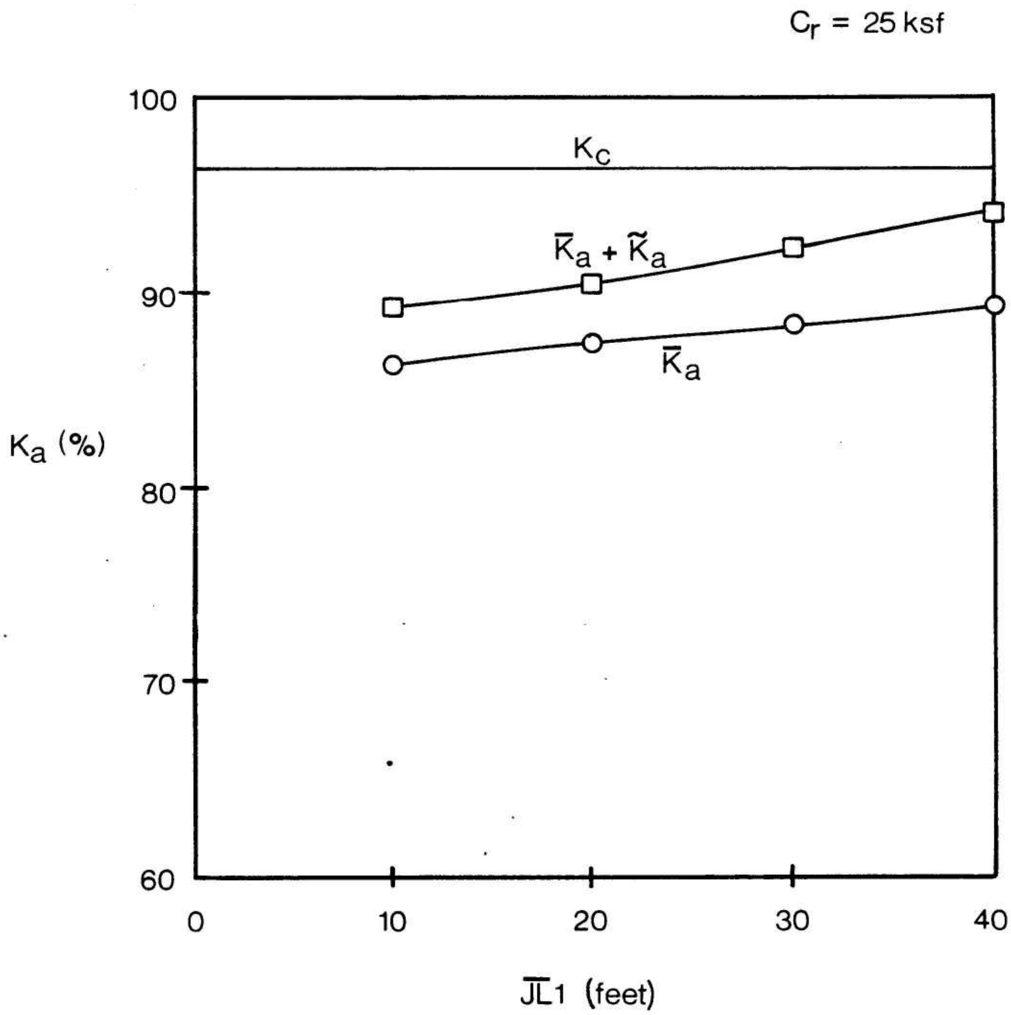


Figure 5.24 Effect of Mean Joint Length (Set 1, $\overline{JL}$ 1) on Apparent Persistence (K_a), Case 8

$\overline{K} = 50\%$ $\Theta = 80°$ $\alpha 1 = 20°$ $\alpha 2 = 70°$

$H = 90\text{-}100'$ $\overline{\Phi}_j = 0°$ $\overline{JL}\ 1 = 10\text{-}40'$ $SP = 10'$

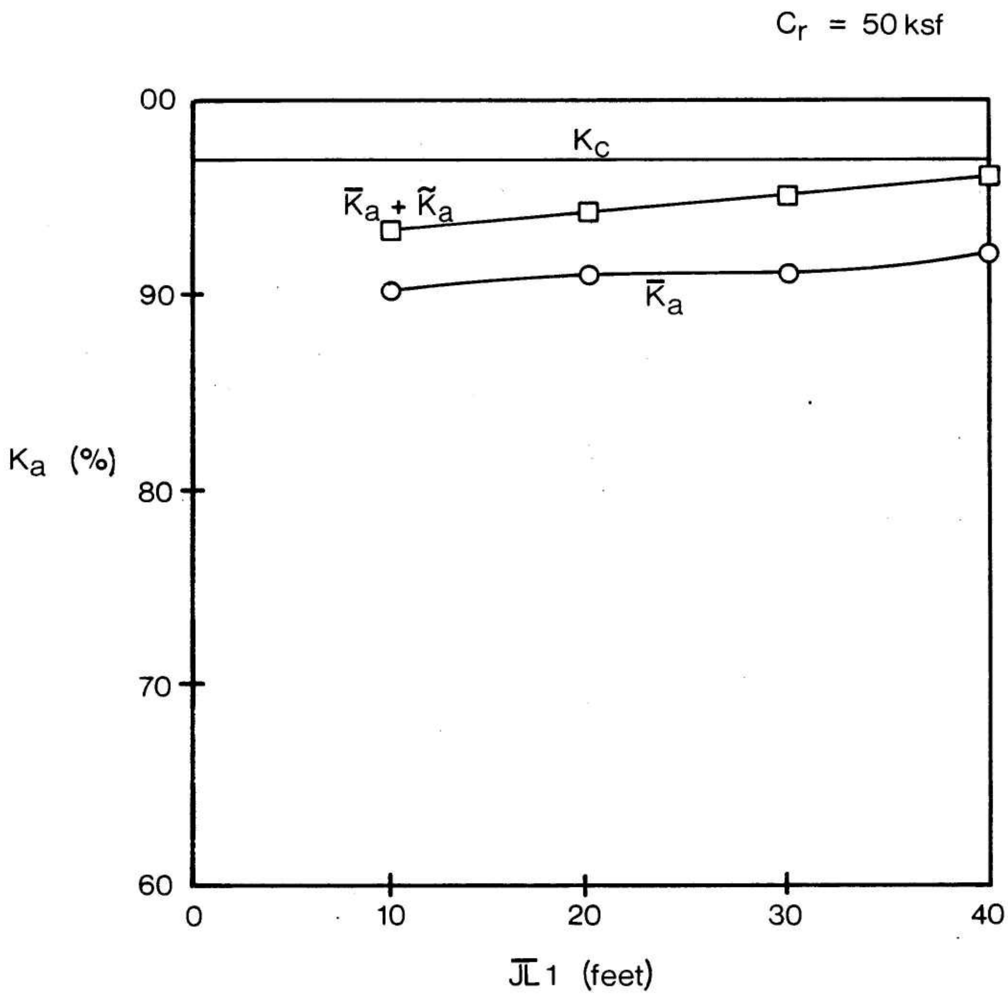


Figure 5.25 Effect of Mean Joint Length (Set 1, $\overline{JL}$ 1) on Apparent Persistence (K_a), Case 9

$\overline{K} = 50\%$ $\Theta = 80°$ $\alpha 1 = 20°$ $\alpha 2 = 70°$

$H = 90\text{-}100'$ $\Phi_j = 0°$ $\overline{JL}\ 1 = 10\text{-}40'$ $SP = 10'$

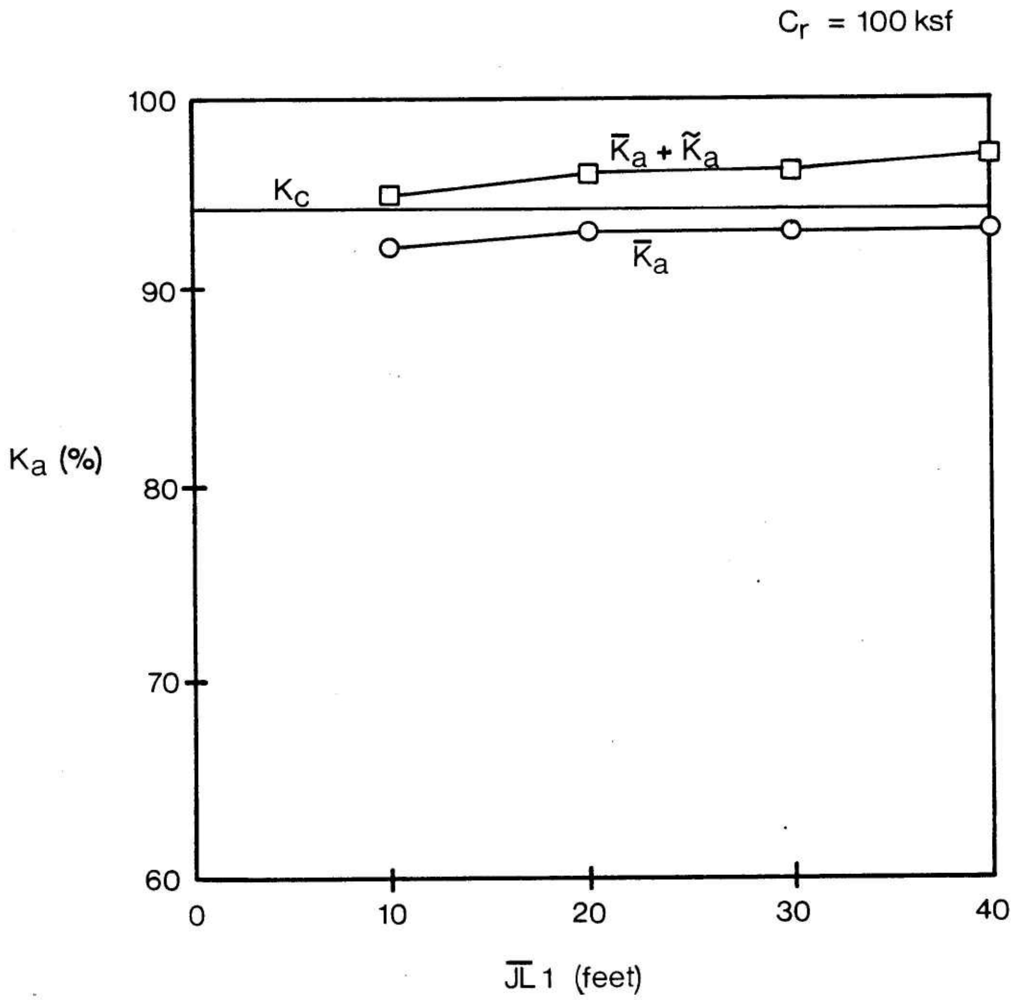


Figure 5.26 Effect of Mean Joint Length (Set 1, $\overline{JL}$ 1) on Apperent Persistence (K_a), Case 10

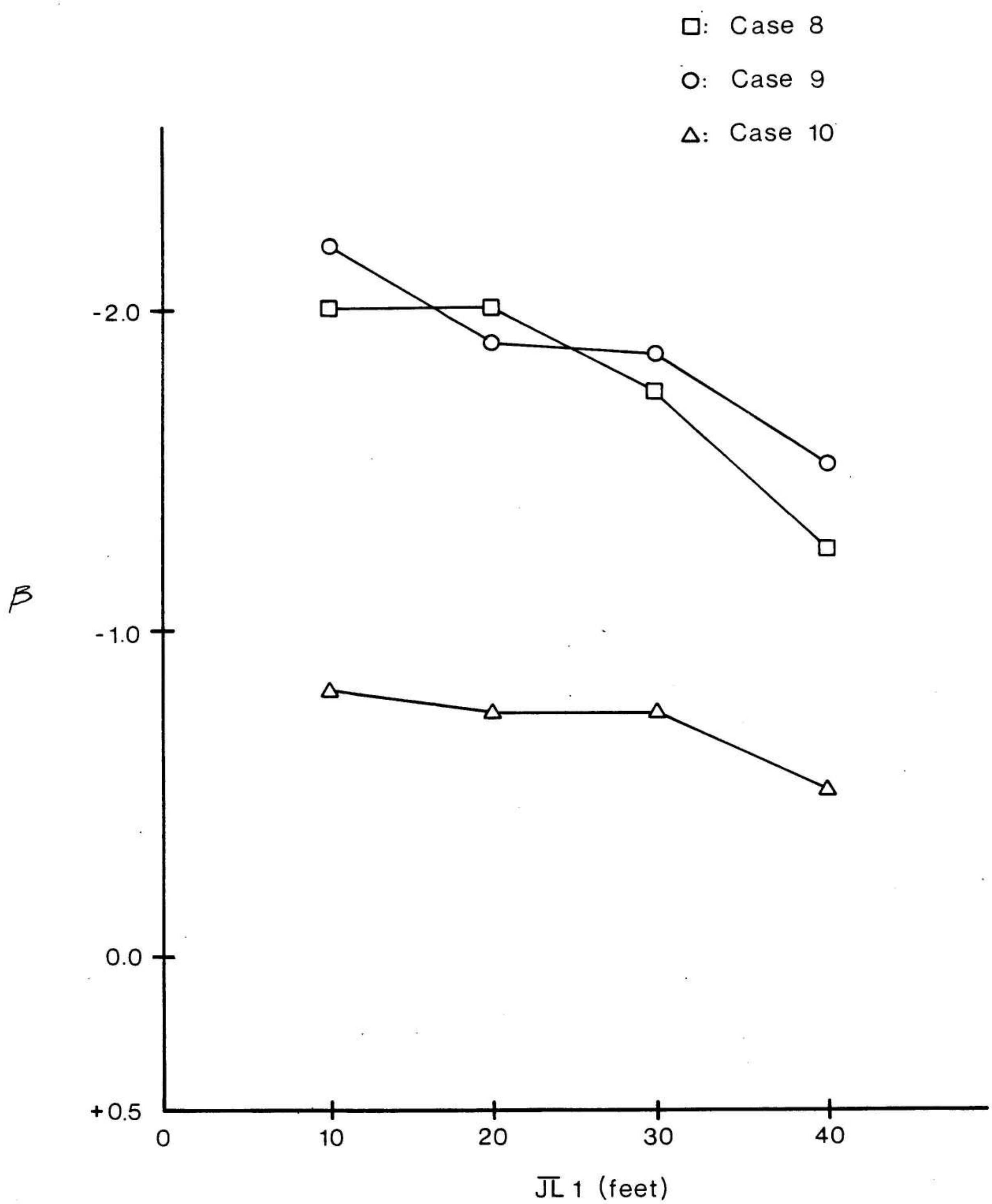


Figure 5.27 Index of Reliability (β) as a Function Mean First Set Joint Length ($\overline{JL}$ 1)

case. As expected, β values are higher for the case when C_r equal to 100 ksf (See Figure 5.27). Notice for example that β values for C_r equal 100 ksf is the most reliable (highest β - Case 10).

Summary - Effects of Mean Joint Length of Set One ($\overline{JL1}$)

The effects of $\overline{JL1}$ on reliability can be summarized as follows:

1. At any given depth in the slope, increasing $\overline{JL1}$ decreases reliability.
2. Unlike the case for a slope with a single set, $\overline{K}_a$ and $\tilde{K}_a$ are sensitive to variations in $\overline{JL1}$. As a result, β values can be obtained from a single model run.

5.5.5 Effect of First Set Mean Joint Plane Persistence ($\overline{K1}$) at Constant Mean Joint Plane Spacing ($\overline{SP1}$)

Mean joint persistence ($\overline{K}$) is estimated by the two input parameters, the mean joint length ($\overline{JL1}$) and the mean rock bridge length ($\overline{RB}$) as follows:

$$\overline{K} = [\ \overline{JL1} \ / \ (\overline{JL1} + \overline{RB})\ ] \times 100$$

Mean joint plane persistence ($\overline{K}$), an input parameter, should not be confused with the apparent persistence (K_a) which is an output parameter. $\overline{K}$ of a joint set, is the average percentage of joint segment lengths expected along any joint plane within that joint set. Actual joint plane persistence along any plane can vary dramatically from this mean value. With all this in mind, the following section examines the effect of varying the mean joint plane persistence while holding constant the joint plane spacing ($\overline{SP1}$). The estimate of the first set joint plane intensity is defined as:

$$\overline{I1} = \overline{K1} / \overline{SP1}$$

I1 varies in proportion to $\overline{K1}$. In the section which follows, the effect of varying $\overline{K}$ at a constant $\overline{I}$ is examined.

The influence of $\overline{K1}$ (at constant $\overline{SP1}$) is studied in two cases. Common input parameters are listed at the top of the pages containing figures 5.29 and 5.30. In case #14, C_r equals 25 ksf while in case #15, C_r equals 100 ksf.

Effect of First Set Mean Joint Persistence (K1) on the Probability of Failure Pf (h)

The effect of increasing mean joint plane persistence ($\overline{K1}$) on the probability of failure is shown in Figure 5.28. At any depth, the value of P_f increases with increasing K1. At depths greater than 30 feet, the probability of failure (P_f) increases with increasing depth.

The Probability of Failure (Pf) as a Function of Mean Apparent Persistence ($\overline{K1}$)

Figure 5.29 is the plot of the probability of failure (P_f) as a function of mean joint plane persistence ($\overline{K1}$). In case 14, P_f increases from 10 to 100 percent for $\overline{K1}$ values of 12 to 80 percent. O'Reilly found that the upper limit was 100 percent at 100 percent $\overline{K1}$. This means that for low cohesion values (C_r=25) a 100% persistent plane will definitely fail. With respect to case 15 at hand, a similar trend may be observed although at a slower rate due to the higher cohesion (100ksf). As a result of the introduction of a second set, the smaller first joint set per-

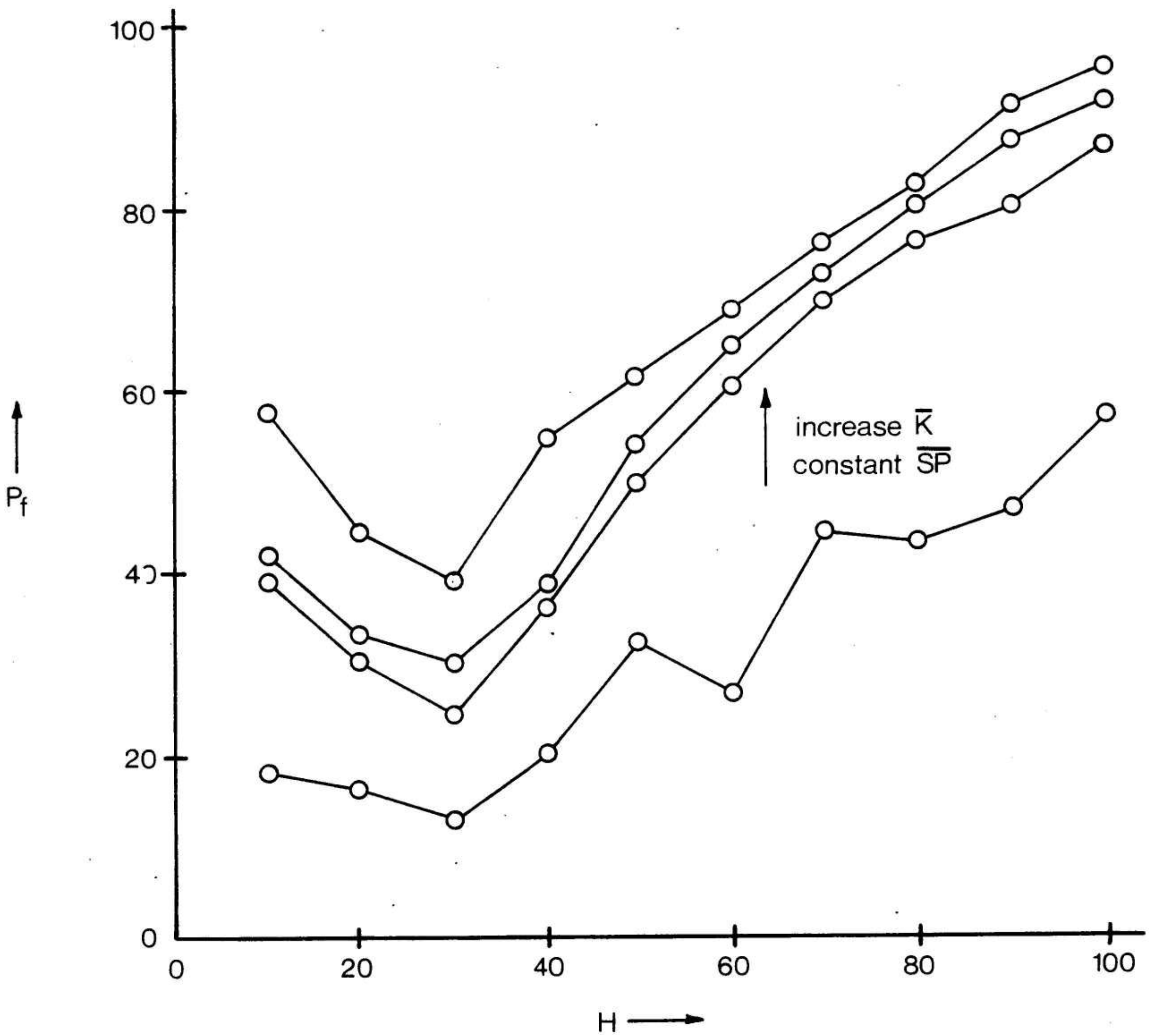


Figure 5.28 Effect of Mean Joint Plane Persistence ($\overline{K}$) on P_f (h), Constant Joint Plane Spacing

C_r = 25-100 ksf   = 80   1 = 20   2 = 70

H = 90-100' $\overline{JL}$ 1 = $\overline{JL}$ 2 = 40' $\overline{SP}$ 1 = $\overline{SP}$ 2 = 12'

$\overline{K}$ = 25-80%

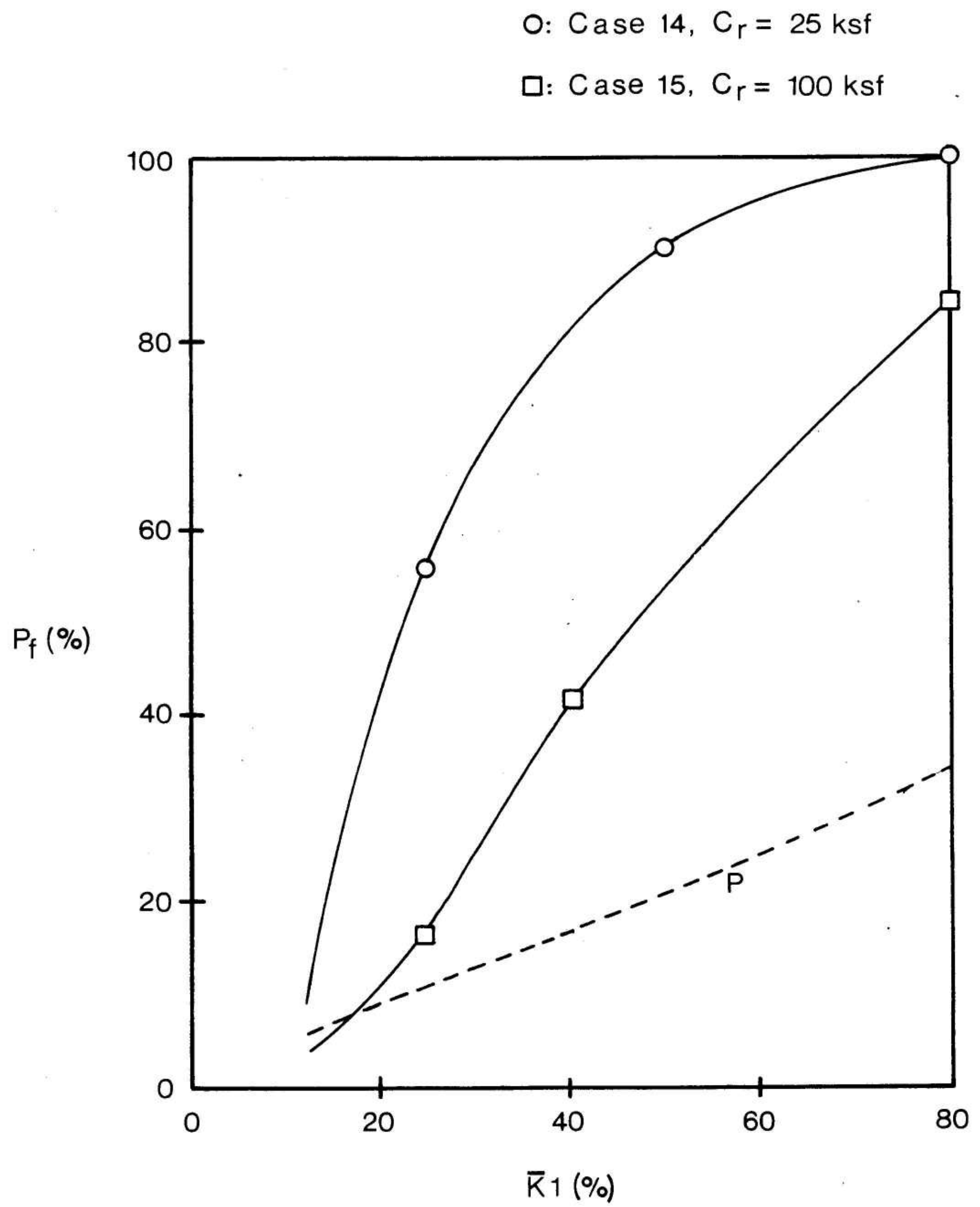


Figure 5.29 P_f as a Function of Mean Joint Plane Persistence ($\overline{K}$), Constant Mean Joint Plane Spacing

$C_r = 25$ ksf $\Theta = 60°$ $\alpha = 40°$

$H = 90\text{-}100'$ $\Phi_j = 0°$ $\overline{JL} = 40°$ $\overline{SP} = 5'$

$\overline{K} = 25\text{-}73\%$

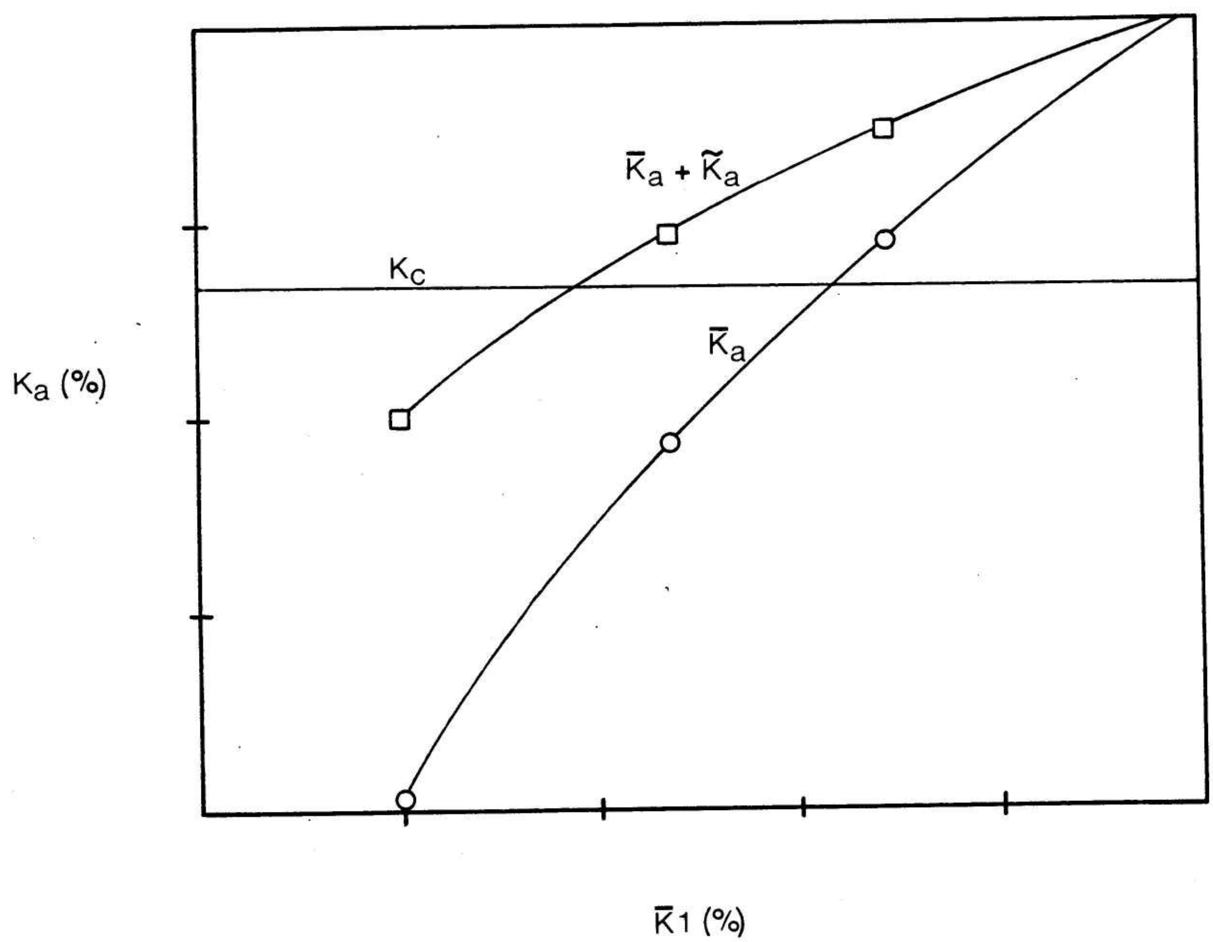


Figure 5.30 Effect of Mean Joint Plane Persistence ($\overline{K}$) on Apparent Persistence ($\overline{K}_a$)
(Figure 6.46) O'Reilly-1980

$\overline{K}$ = 25-80% Θ = 80° $\alpha 1$ = 20° $\alpha 2$ = 70°

H = 90-100' Φ_j = 0° $\overline{JL}$ = 40' $\overline{SP}$ 1 = 5'

$\overline{SP}$ 2 = 12

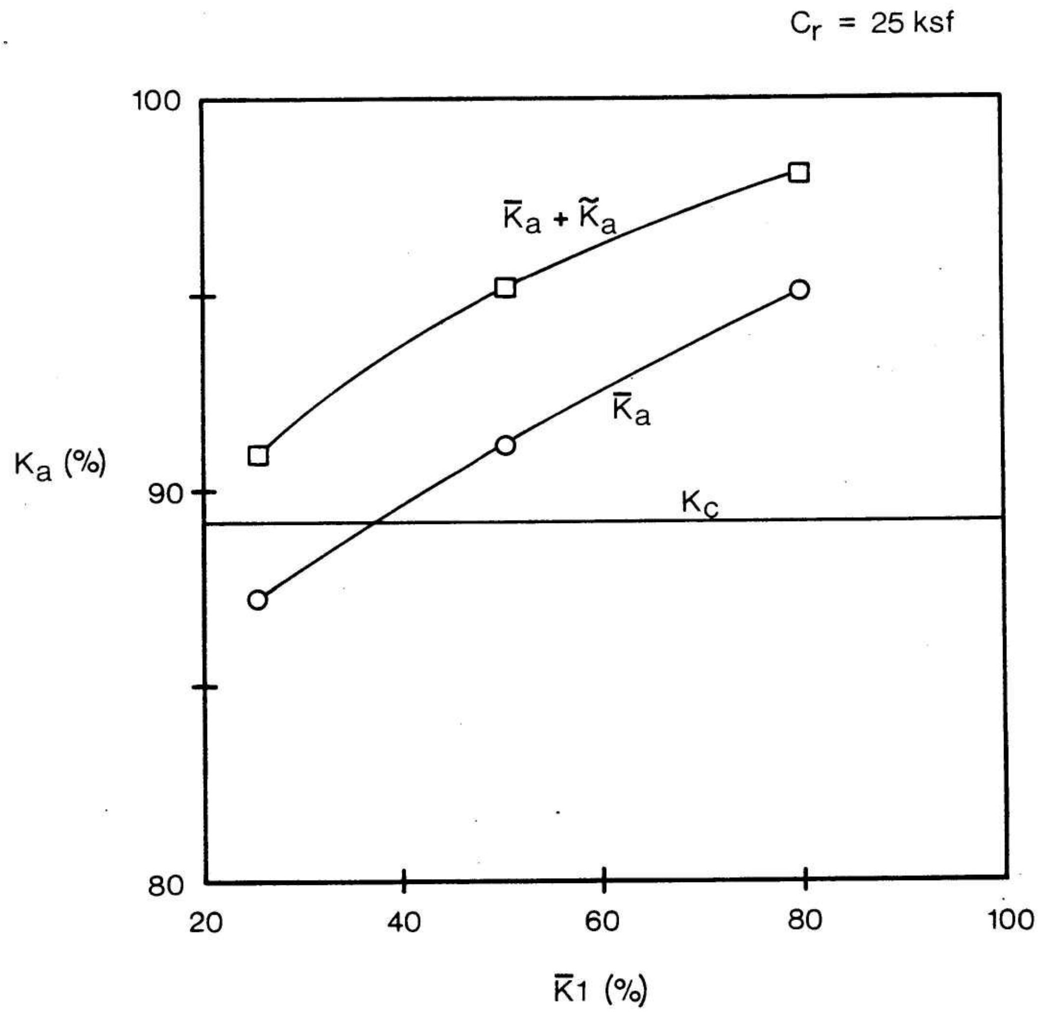


Figure 5.31 Effect of Mean Joint Plane Persistence ($\overline{K}$) on Apparent Persistence ($\overline{K}_a$), Case 14

$K = 25\text{-}80\%$ $\Theta = 80°$ $\alpha 1 = 20°$ $\alpha 2 = 70°$

$H = 90\text{-}100'$ $\Phi_j = 0°$ $\overline{JL} = 40'$ $\overline{SP}\ 1 = 5'$

$\overline{SP}\ 2 = 12'$

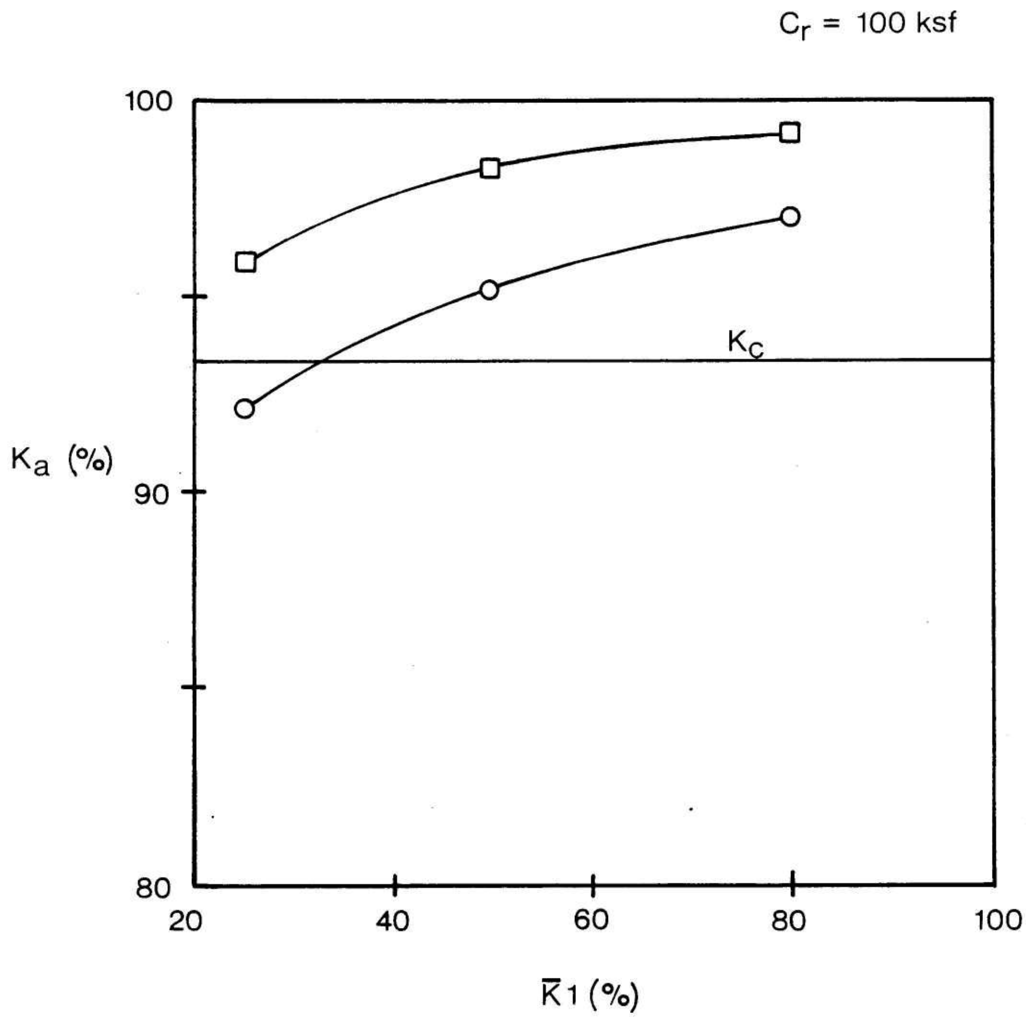


Figure 5.32 Effect of Mean Joint Plane Persistence ($\overline{K}$) on Apparent Persistence ($\overline{K}_a$), Case 15

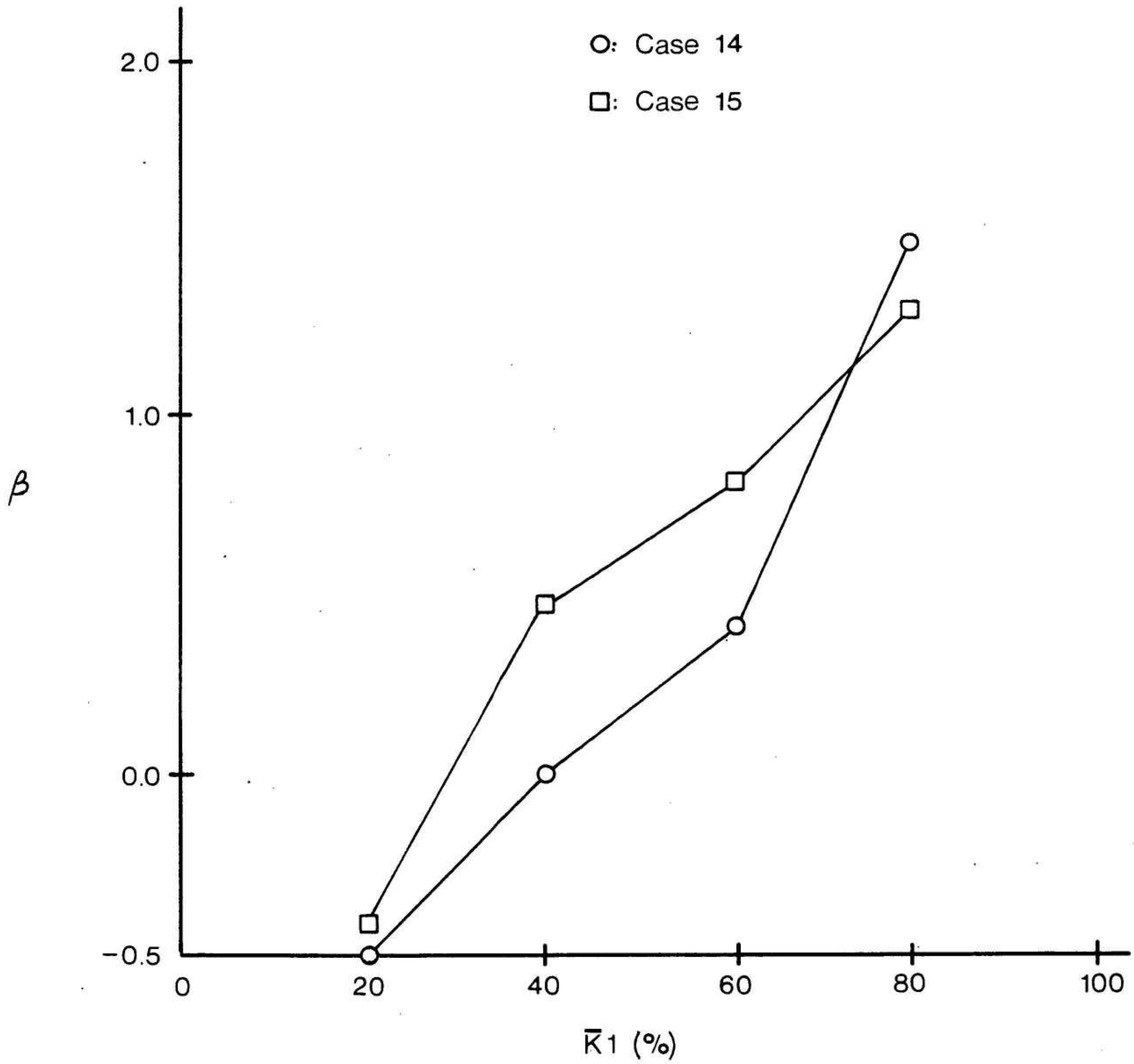


Figure 5.33 Index of Reliability (β) as a Function of Mean Joint Plane Persistence of First Set ($\bar{K}$ 1), Constant Joint Plane Spacing

sistence required to induce failure is possibly due to the higher persistence of critical paths. Consequently, smaller driving forces are required to change a critical path into a failure path.

The Effect of First Set Mean Joint Persistence ($\overline{K1}$) on Apparent Persistence ($\overline{K}_a$)

The effect of varying mean joint plane persistence (K1) on apparent persistence can be seen in Fig. 5.31 and 5.32, Fig. 5.30 (from O'Reilly '80) is included for comparison purposes. Computer output revealed that sensitivity of K_a to variations in $\overline{K1}$ is not as dramatic in a slope with two joint sets as it is in a slope with a single set.

The effect of increasing $\overline{K1}$ on K_a is shown in Fig. 5.32. For the case of a slope with a single joint set (O'Reilly-1980), reliability is more sensitive to changes in $\overline{K}_1$ than for cases with two joint sets. In the former, senstivity of K_a to variations in K_1 decreases with increasing cohesion as program output shows. Fig. 5.33 is a plot of reliability, expressed in terms of β. Reliability increases with increasing K_1 for both cases examined. Reliability is not much affected by variations in cohesion in the range considered perhaps due to the fact that other parameters overcame the unstable conditions that cohesion variations would cause.

5.5.6 Effect of Mean Joint Plane Spacing of the First Set ($\overline{SP1}$) at Constant Persistence

The effect of mean joint plane spacing of the first set is examined in three cases, 16, 17 and 18. The range of values of $\overline{SP1}$ is varied from 3 to 12 feet.

Case 16 examines the influence of $\overline{SP1}$ on slopes with weak rock (C_r = 25 ksf) and long mean joint length of the first set ($\overline{JL1}$ = 40').

Case 17 is similar to case 16 except that C_r is set to 100 ksf.

Case 18 is similar to case 16 except that $\overline{JL1}$ is reduced to 20 feet.

Effect of Mean Joint Plane Spacing ($\overline{SP1}$) on the Probability of Failure P_f (h)

Figure 5.34 is a plot of the probability of failure as a function of depth. At any depth, and as one may expect, the probability of failure increases with decreasing mean joint plane spacing. The probability of failure increases with increasing depth. Also shown in Fig. 5.34 is the probability of a 100% persistent plane (P_1) which decreases with depth, contrary to the curves that are obtained from the model since in the equation defining P_1, the probability of failure is directly proportional to mean joint length.

For a slope with either a single joint set or two joint sets, Fig. 5.35-36, program output data shows that an increase in the mean joint plane spacing of the first set causes a decrease in P_f (h). A greater $\overline{SP1}$ causes failure paths to be more in plane in the case of a single joint set. In the case of two joint sets, an increase in $\overline{SP1}$ causes failure paths to be more "in plane" and to use joints of the second set for transitions to above planes when such discontinuities exist in a way that satisfy the algorithm.

The Probability of Failure (P_f) as a Function of Mean Joint Plane Spacing of the First Set ($\overline{SP1}$)

The probability of failure P_f as a function of mean joint plane spacing of the first set ($\overline{SP1}$) is shown in Figure 5.36. In all cases (16, 17 and 18), P_f decreases linearly with increasing $\overline{SP1}$. Also

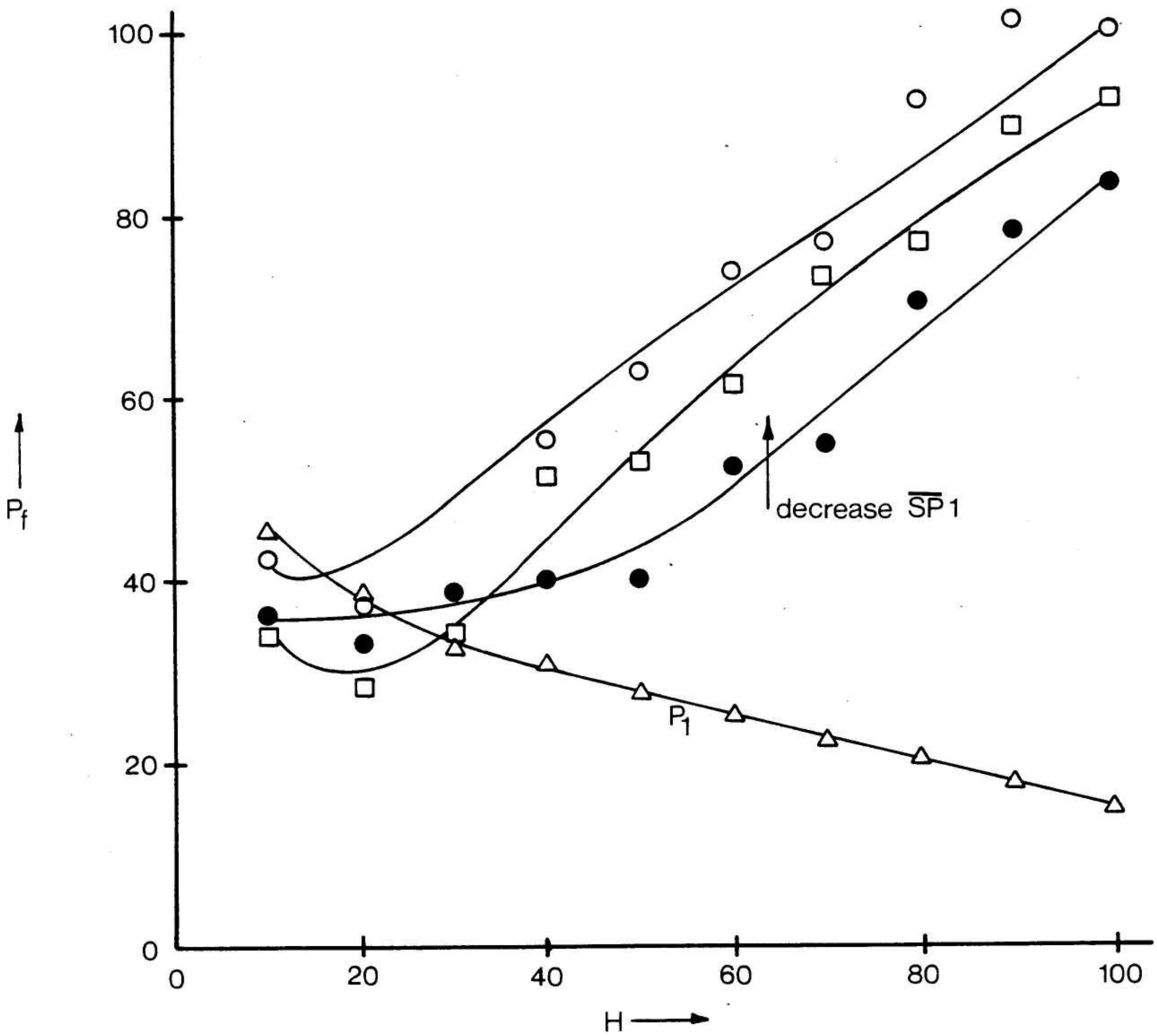


Figure 5.34 Effect of Mean Joint Plane Spacing (Set 1, $\overline{SP}$ 1) on the Probability of Filure with Depth P_f (h), Constant Mean Persistence of Set 1 ($\overline{K}$ 1)

C_r = 25-100 ksf Θ = 60° α = 40°

H = 90-100' Φ_j = 0° $\overline{JL}$ = 20-40' $\overline{SP}$ = 2.5-10'

$\overline{K}$ = 50%

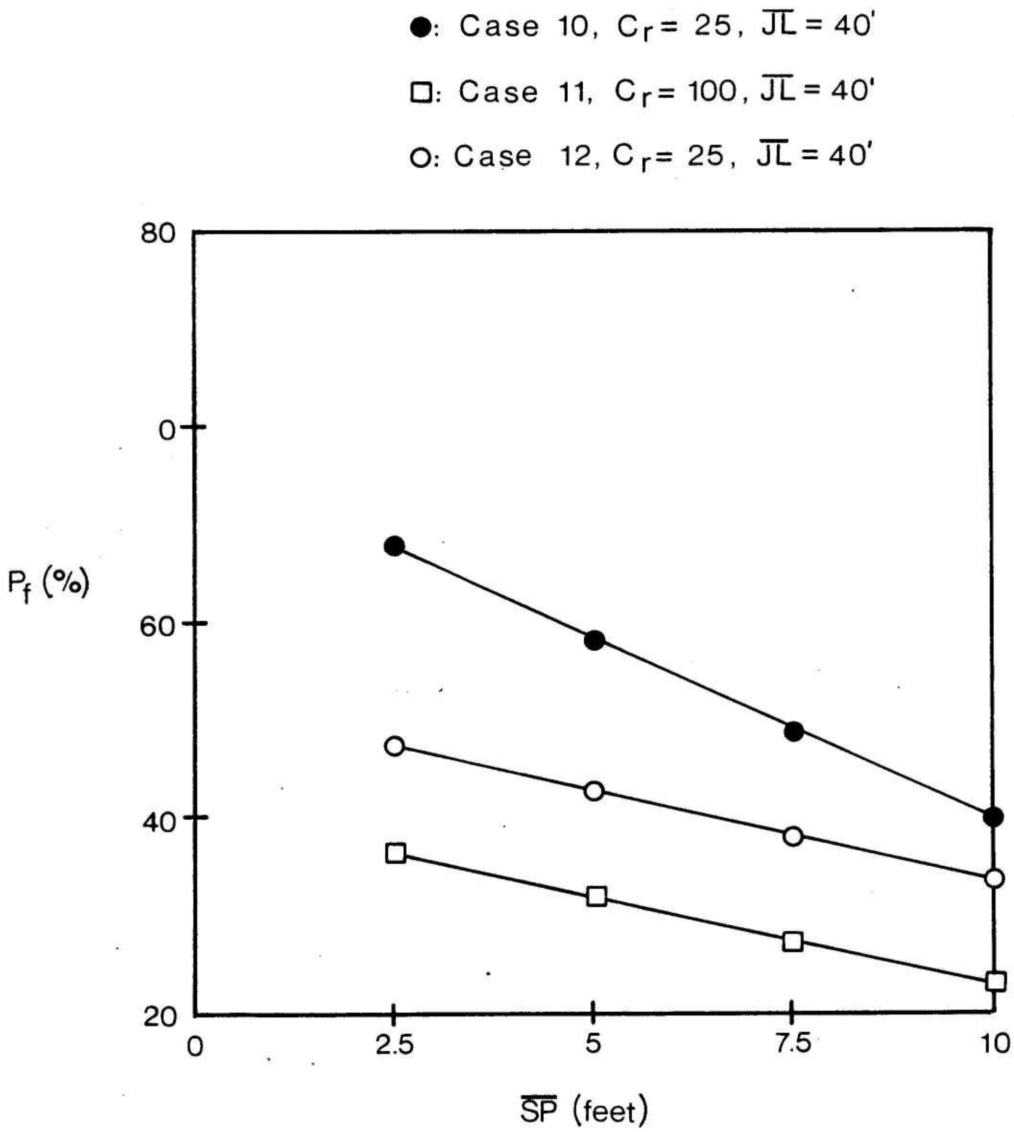


Figure 5.35 P_f as a Function of Mean Joint Spacing ($\overline{SP}$ 1), Constant $\overline{K}$

(O'Reilly-1980)

C_r = 30-90 ksf Θ = 80° $\alpha 1$ = 20° $\alpha 2$ = 70°

H = 90-100' Φ_j = 0° $\overline{JL}$ 1 = 20-40' $\overline{SP}$ 1 = 3-12'

$\overline{K}$ = 50% $\overline{JL}$ 2 = 30' $\overline{SP}$ 2 = 8'

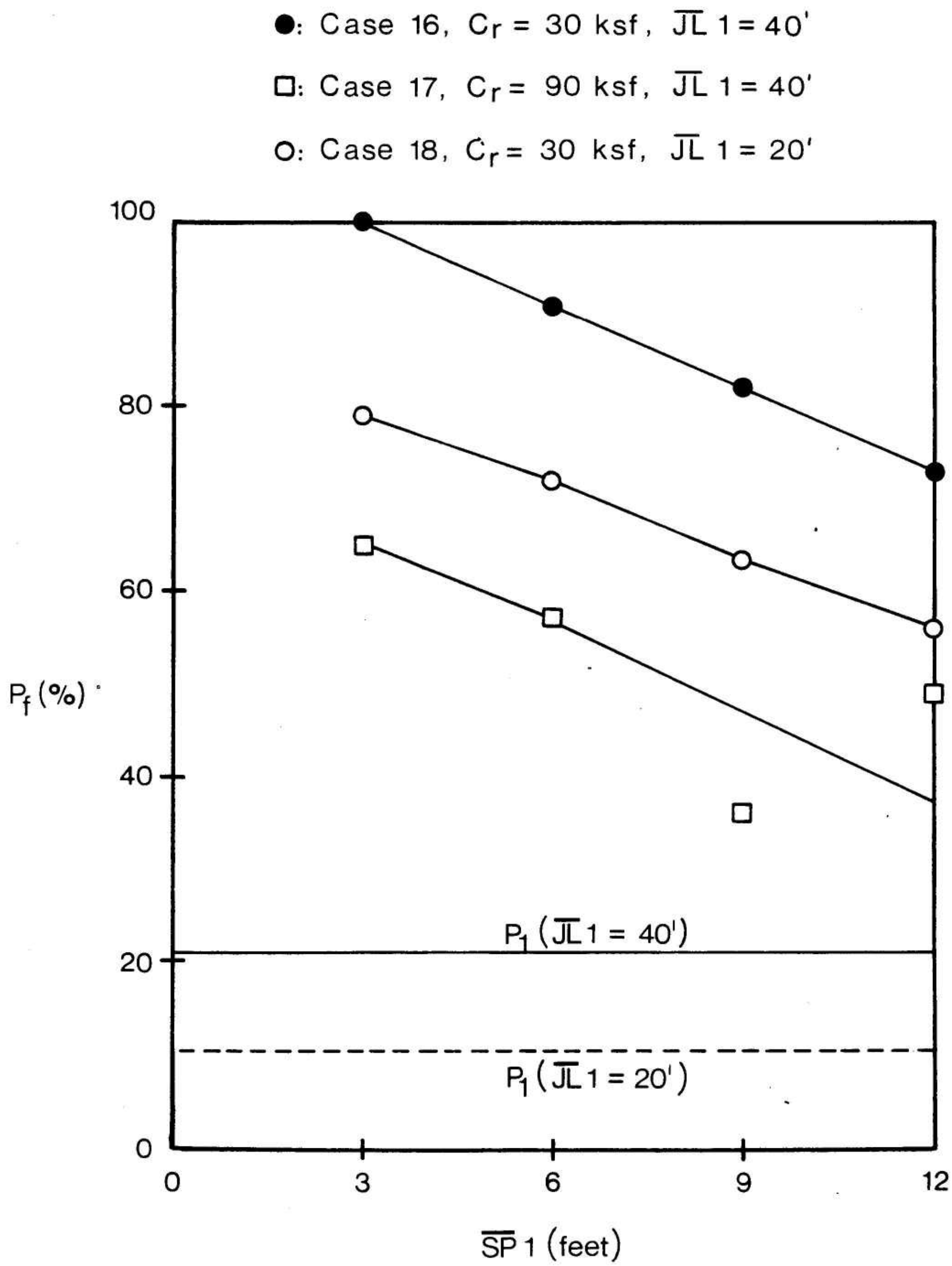


Figure 5.36 P_f as a Function of Mean Joint Plane Spacing ($\overline{SP}$ 1), Constant $\overline{K}$ 1

plotted are the lower limit expressions P_1 as functions of $\overline{SP1}$ for various $\overline{JL1}$. As one may expect, the probability of failure curves are higher and further away from the theoretical curves (P1) for slopes with two sets than for slopes with a single set. This may be the result of a higher discontinuity concentration per unit area of the slope's cross-section. The slopes of the three curves are identical and the probability of failure for cases with low intact rock cohesion is higher for a specific $\overline{SP1}$. Also, as one may expect, program output shows that decreasing the mean joint length decreases the probability of failure. This can be seen by comparing cases 16 and 18.

Effect of First Set Mean Joint Place Spacing ($\overline{SP1}$) on Apparent Persistence ($\overline{K}_a$)

The effect of varying the mean joint spacing ($\overline{SP1}$) on apparent persistence ($\overline{K}_a$) is shown in Figures 5.37 through 5.39. Shown in the curves is the critical persistence for each case. Program data indicates that mean apparent persistence ($\overline{K}_a$) is approximately constant for all values of $\overline{SP1}$. Also constant in each case is the standard deviation of apparent persistence ($\tilde{K}_a$). As a result, reliability index values (β) can be calculated from a single simulation for each case shown in Figure 5.40.

5.5.7 Effect of First Set Mean Joint Plane Persistence (K1) at Constant Intensity (I1)

Intensity of a joint set is defined as the average jointing per unit-sectional area of the rock slope (O'Reilly-1980). In the present study, intensity in either set is defined as follows:

C_r = 30 ksf Θ = 80° $\alpha 1$ = 20° $\alpha 2$ = 70°

$\overline{K}$ = 50% Φ_j = 0° $\overline{JL}$ 1 = 40' $\overline{SP}$ 1 = 3-12'

$\overline{JL}$ 2 = 30' $\overline{SP}$ 2 = 8'

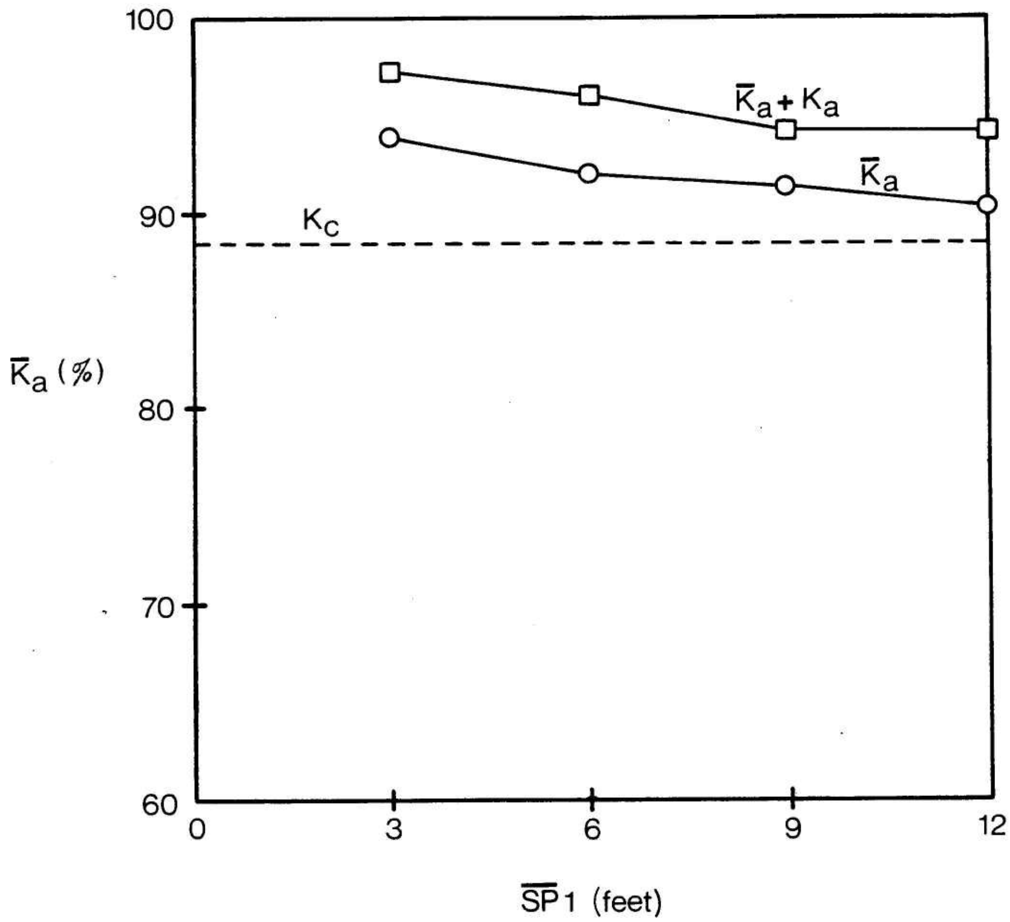


Figure 5.37 Effect of Mean Joint Spacing (Set 1, $\overline{SP}$ 1) on Apparent Persistence ($\overline{K}_a$), Constant $\overline{K}$ 1, Case 16

C_r = 90 ksf Θ = 80° $\alpha 1$ = 20° $\alpha 2$ = 70°

H = 90-100' Φ_j = 0° $\overline{JL}$ 1 = 40' $\overline{SP}$ 1 = 3-12'

$\overline{K}$ = 50% $\overline{JL}$ 2 = 30' $\overline{SP}$ 2 = 8'

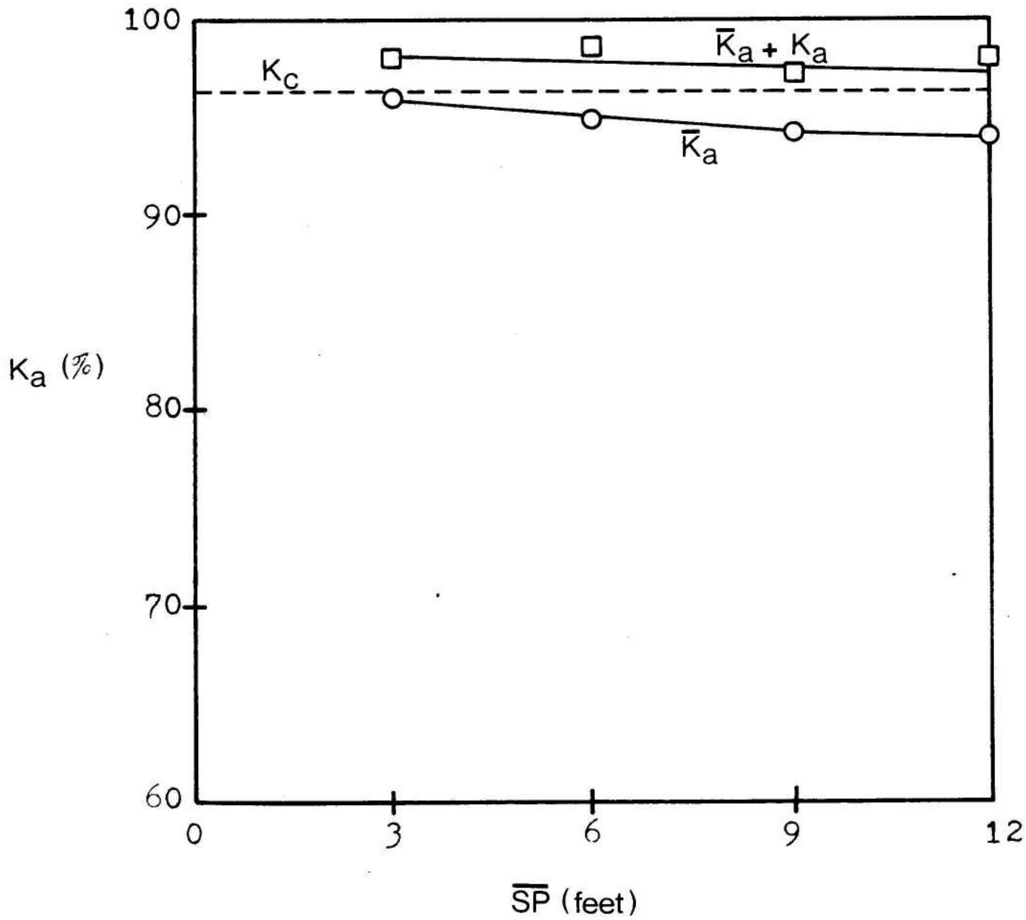


Figure 5.38 Effect of Mean Joint Plane Spacing ($\overline{SP}$ 1) on Apparent Persistence (K_a), Constant $\overline{K}$ 1, Case 17

C_r = 30 ksf Θ = 80° $\alpha 1$ = 20° $\alpha 2$ = 70°

H = 90-100' Φ_j = 0° $\overline{JL}$ 1 = 20' $\overline{SP}$ 1 = 3-12'

$\overline{K}$ = 50% $\overline{JL}$ 2 = 30' $\overline{SP}$ 2 = 8'

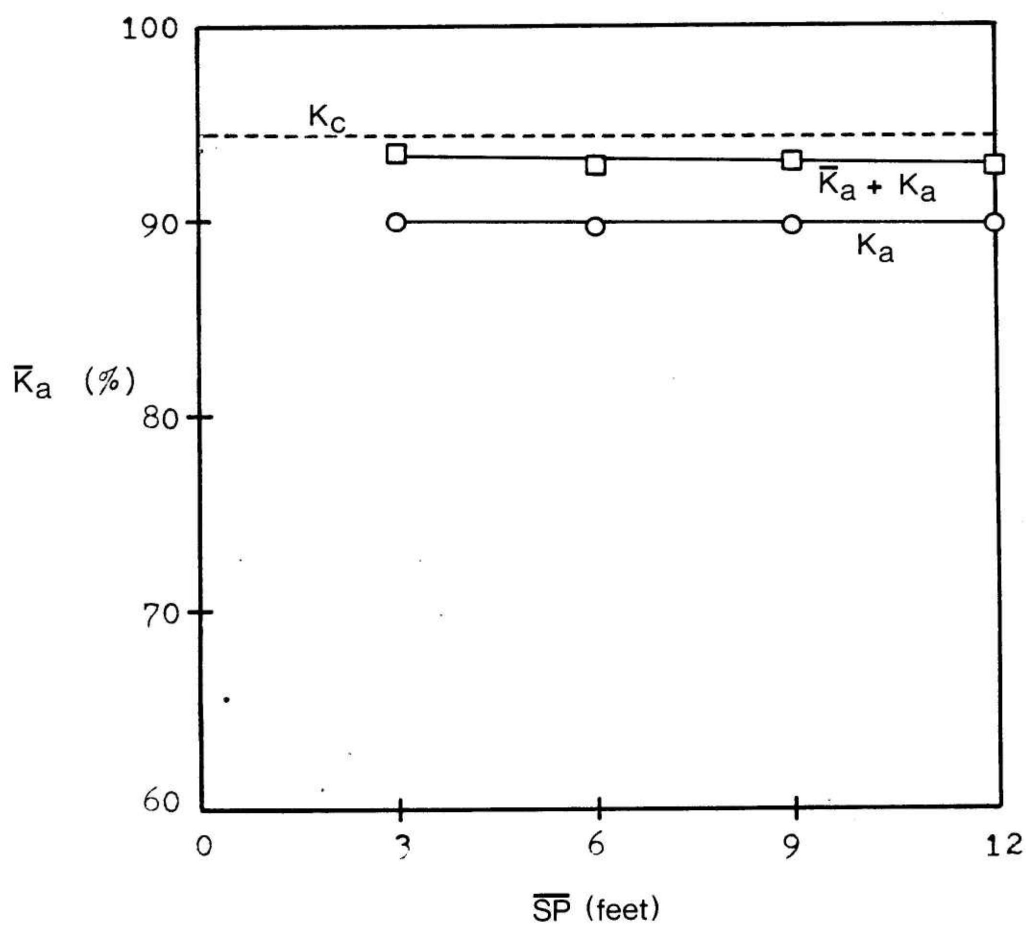


Figure 5.39 Effect of Mean Joint Plane Spacing ($\overline{SP}$ 1) on Apparent Persistence (K_a), Constant $\overline{K}$ 1, Case 18

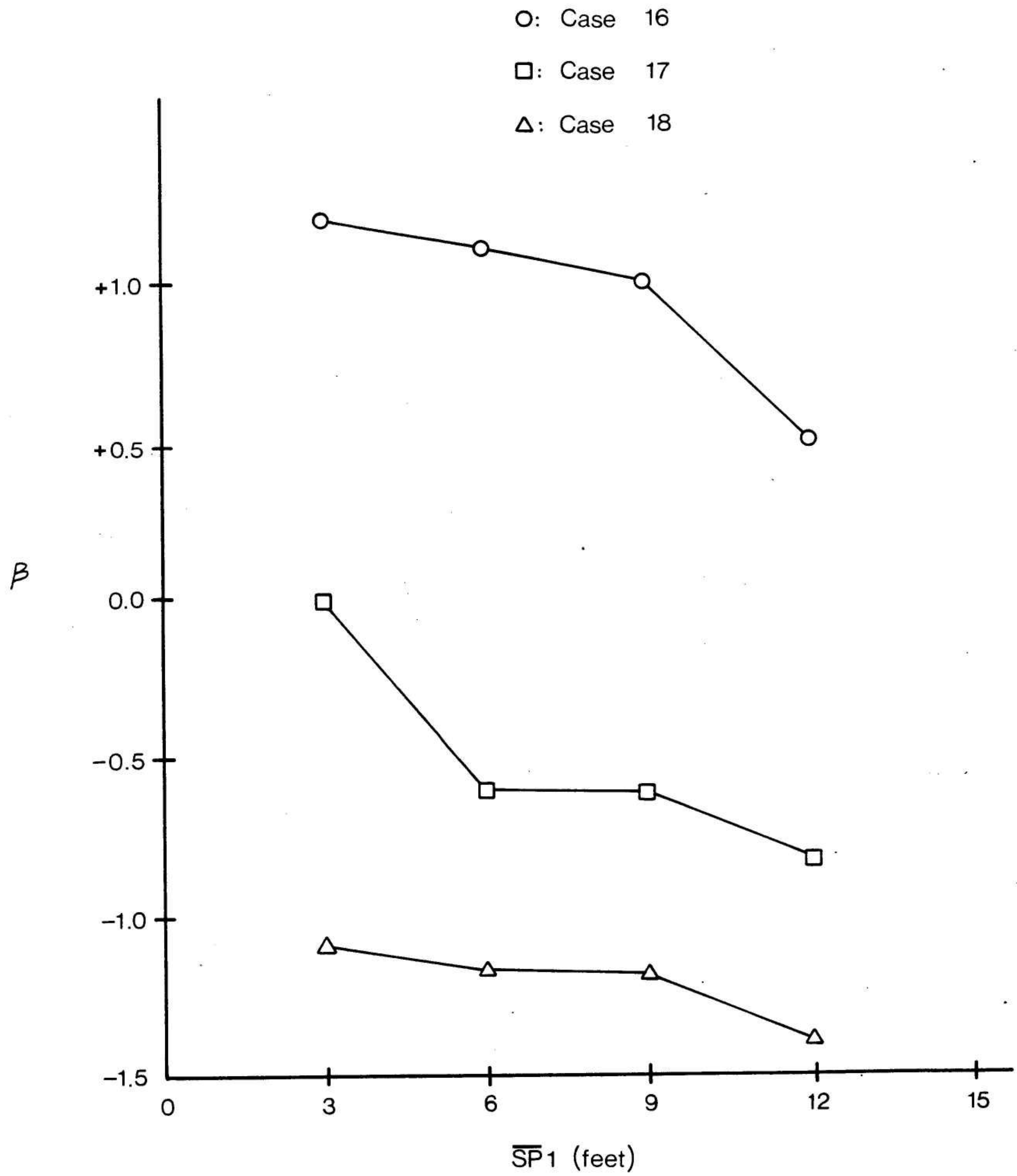


Figure 5.40 Index of Reliability (β) as a Function of Mean Joint Plane Spacing of the First Set ($\overline{SP}$ 1), Constant Persistence

$$\overline{I1} = \overline{K1} / \overline{SP1} \qquad \overline{I2} = \overline{K2} / \overline{SP2} \tag{5.17}$$

The sum of the above quantities, $\overline{I}$, yields the total length of joint segments in a given unit area of the slope's cross-section.

$$\overline{I} = \overline{I1} + \overline{I2} \tag{5.18}$$

An increase in joint intensity results when $\overline{K}$ is increased or $\overline{SP}$ is decreased in Equations 5.17 or 5.18. Not surprisingly, results of the parametric study discussed previously have shown that this increase in intensity, whether achieved through an increase in $\overline{K}$ or a decrease in $\overline{SP}$, has similar effects on P_f , $\overline{K}_a$ and $\tilde{K}_a$. Thus, it would be desirable to relate slope reliability to jointing intensity. The extent to which this can be done is explored in 3 cases, (19, 20 and 21).

In the 3 cases examined, $\overline{I1}$ is kept constant (5.0/ft.) $\overline{K1}$ and $\overline{SP1}$ are varied such that their ratio is constant. By so doing, it is possible to examine to what extent joint plane reliability is a function of intensity alone rather than of the separate component $\overline{K1}$ and $\overline{SP1}$. Besides having common parameters, differences between cases are as follows:

Case 19 had C_r equal 25 ksf and $\overline{JL1}$ equal 40'.

Case 20 had C_r equal 8 ksf and $\overline{JL1}$ equal 40'.

Case 21 had C_r equal 25 ksf and $\overline{JL1}$ equal 20'.

With increasing depth, the probability of failure increases gradually. At any depth, the probability of failure increases when increasing either SP1 or K1 and that increase is greater at deeper intervals in the slope.

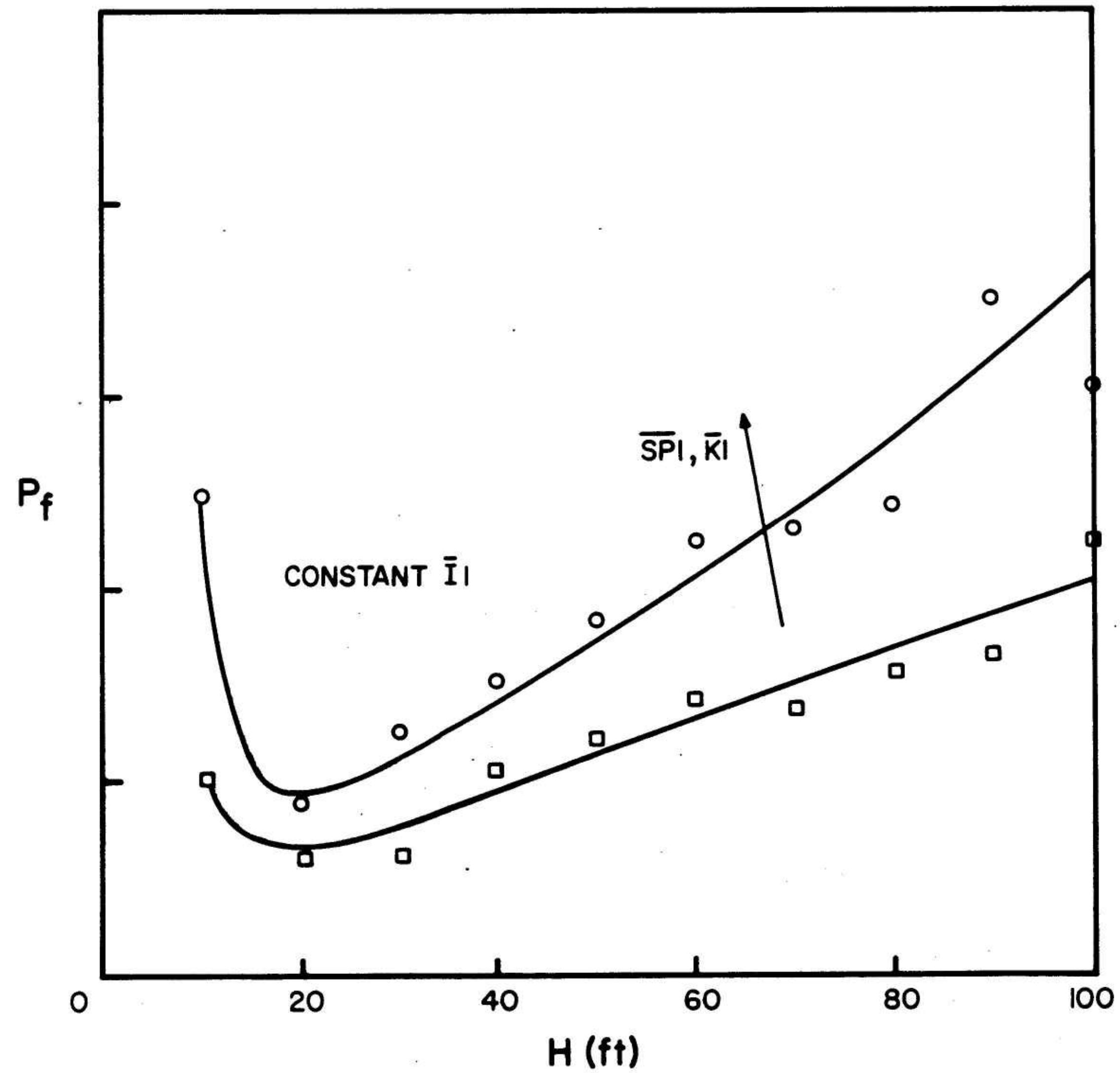


FIGURE 5.41 EFFECT OF MEAN PERSISTENCE ($\overline{K}I$) ON $P_f(h)$, CONSTANT JOINT INTENSITY $\overline{I}I$

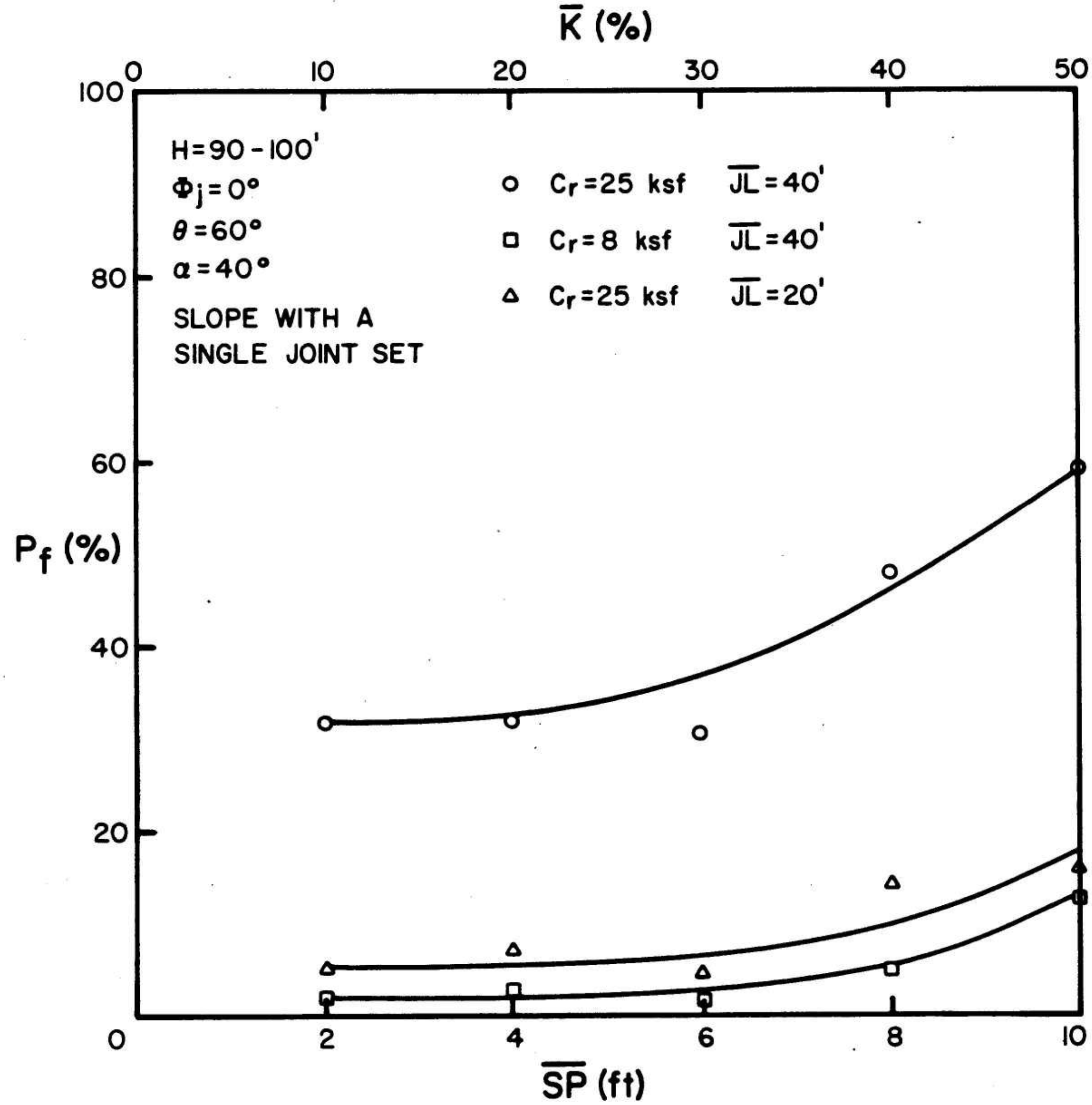


FIGURE 5.42 P_f AS A FUNCTION MEAN JOINT PLANE PERSISTENCE $\overline{K}$, CONSTANT JOINT INTENSITY $\overline{I}$ (FIGURE 6.55-OREILLY-1980)

Effect of Persistence (Set 1 and Constant Intensity) on the Probability of Failure P_f (h)

The effect of varying only $\overline{SP1}$ and $\overline{K1}$ while holding $\overline{I1}$ (and all other input parameters) constant, is shown in Fig. 5.43. Figure 5.42 is included for demonstration purposes. Computer output reveals that at any depth, increasing both $\overline{K1}$ and $\overline{SP1}$ (keeping $\overline{I1}$ constant) increases P_f. However, as can be seen in the figure, the magnitude of this increase tends to steadily decrease with increasing depth. Thus at deep intervals (i.e., greater than approximately 60 feet), P_f (h) becomes independent of $\overline{K1}$ and $\overline{SP1}$ and is only a function of their ratio (I1). For the case of a single joint set, Fig. 5.42, failure probabilities are lower than those shown in Fig. 5.43. This is possibly due to sensitivity to joint length of the first set rather than cohesion.

The Probability of Failure (P_f) as a Function of Persistence ($\overline{K1}$) - Constant Intensity (I1)

Figure 5.45 is a plot of the probability of failure (P_f) vs. persistence ($\overline{K1}$) and mean joint plane spacing of set one - ($\overline{SP1}$) for the three cases mentioned above in which intensity ($\overline{I1}$) is kept constant at I1 = 5. Other input parameters remain constant and are listed in the figure. In all cases, the probability of failure (for the depth interval from 90 to 100 feet) increases with increasing persistence ($\overline{K1}$) and of joint plane spacing ($\overline{SP1}$).

Effect of Persistence (at Constant $\overline{I1}$) on Apparent Persistence ($\overline{K_a}$) and the Reliability Index (β)

Each of Figures 5.44 through 5.46 contains plots of mean apparent persistence ($\overline{K_a}$), mean plus one standard deviation apparent persistence

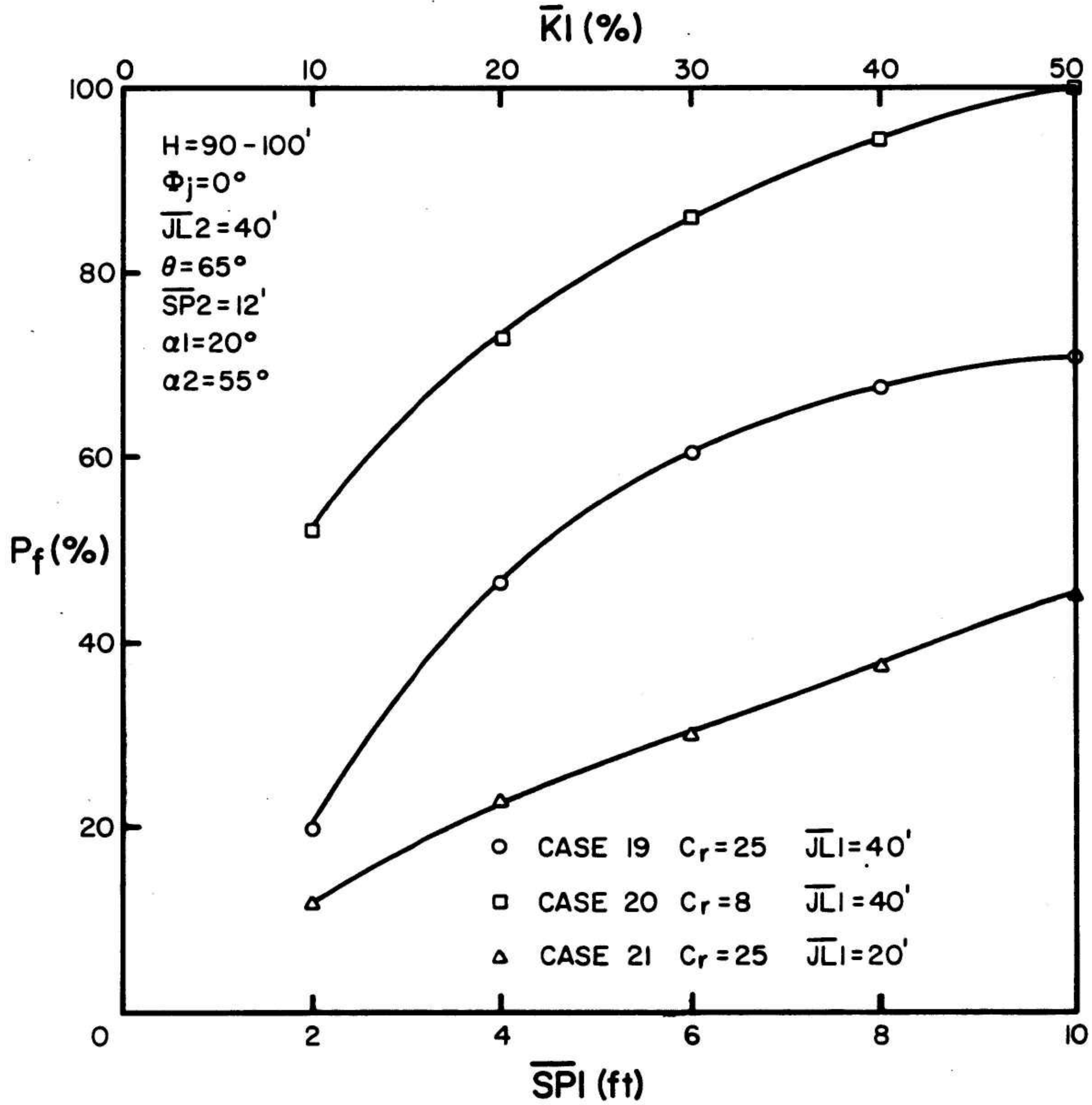


FIGURE 5.43 P_f AS A FUNCTION OF MEAN JOINT PLANE PERSISTENCE ($\overline{K}1$) AT CONSTANT MEAN JOINT INTENSITY ($\overline{I}1$)

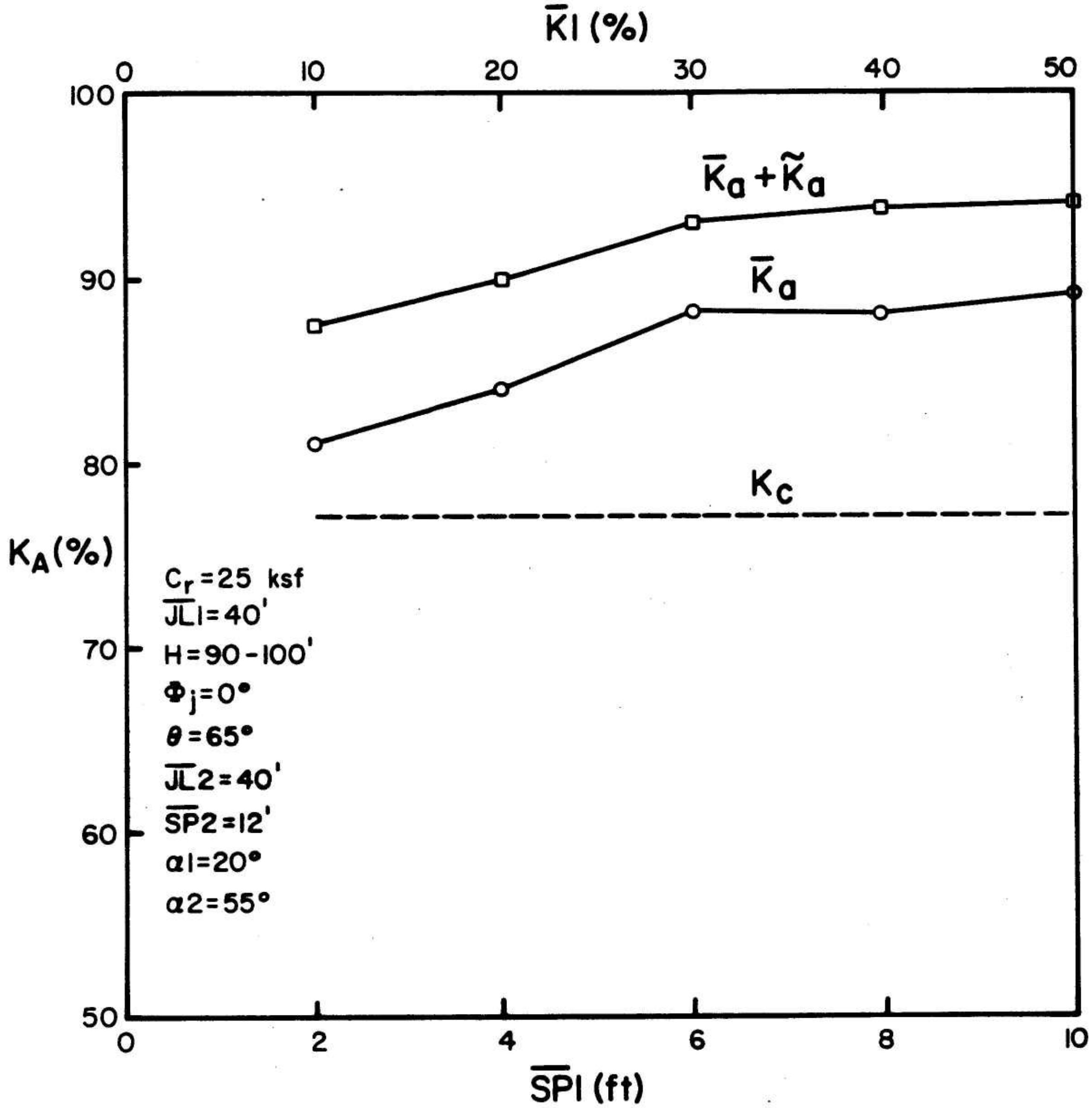


FIGURE 5.44 VARIABILITY IN APPARENT PERSISTENCE (K_A) AT CONSTANT JOINT INTENSITY CASE 19

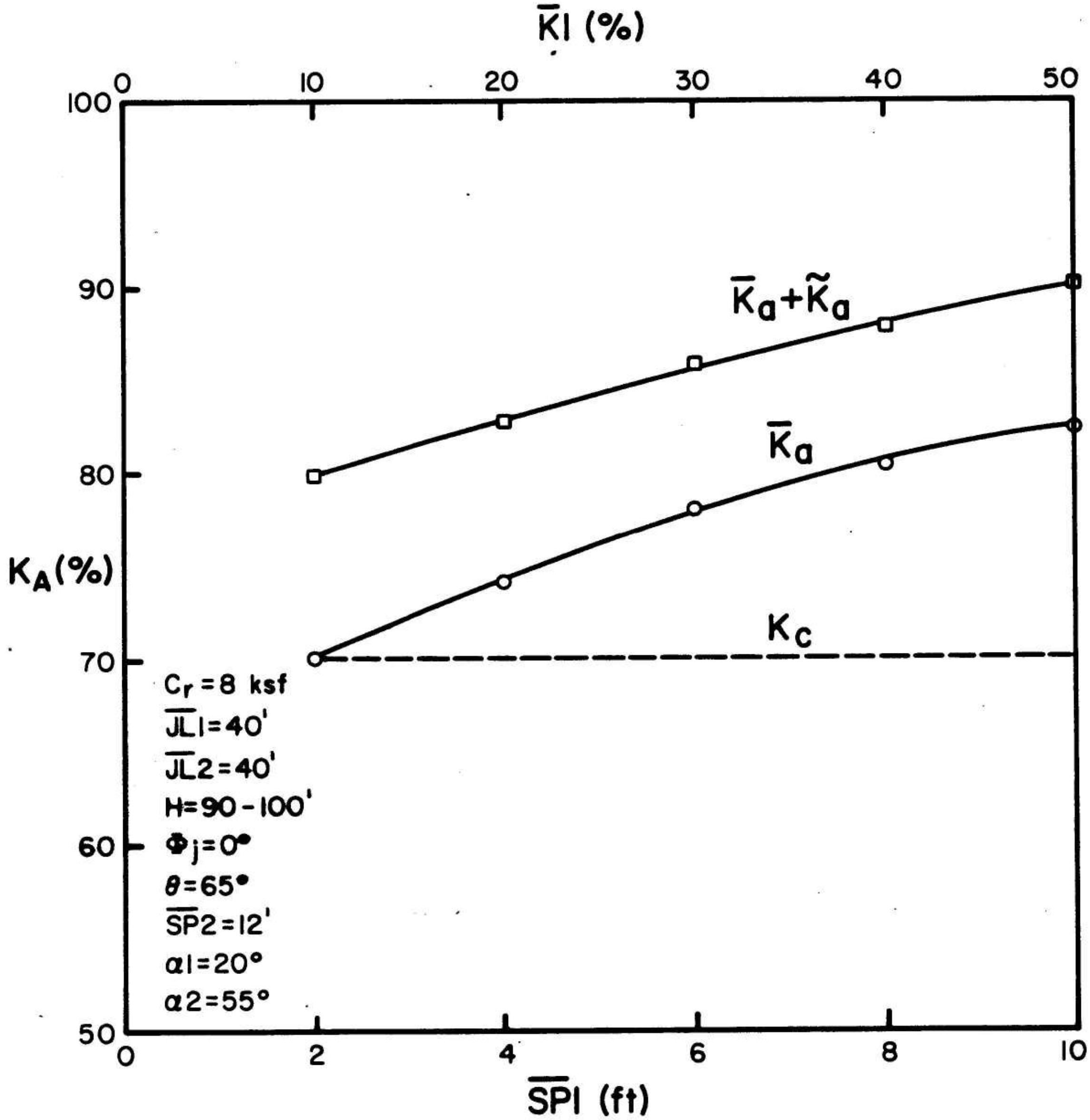


FIGURE 5.45 VARIABILITY IN APPARENT PERSISTENCE (K_a) AT CONSTANT INTENSITY ($\overline{I}1$) CASE 20

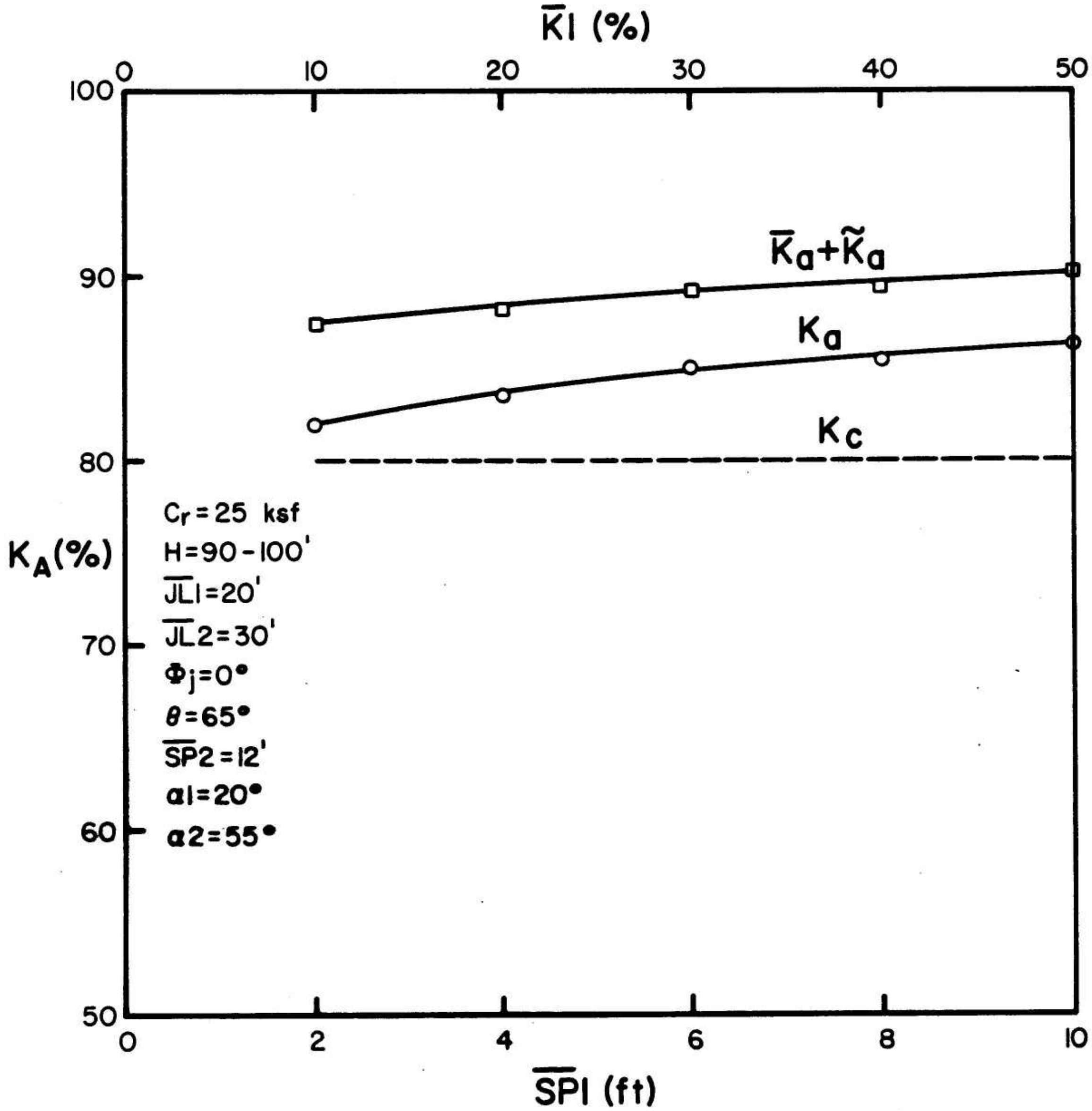


FIGURE 5.46 VARIABILITY IN APPARENT PERSISTENCE (K_A) AT CONSTANT JOINT INTENSITY ($\bar{I}1$) CASE 21

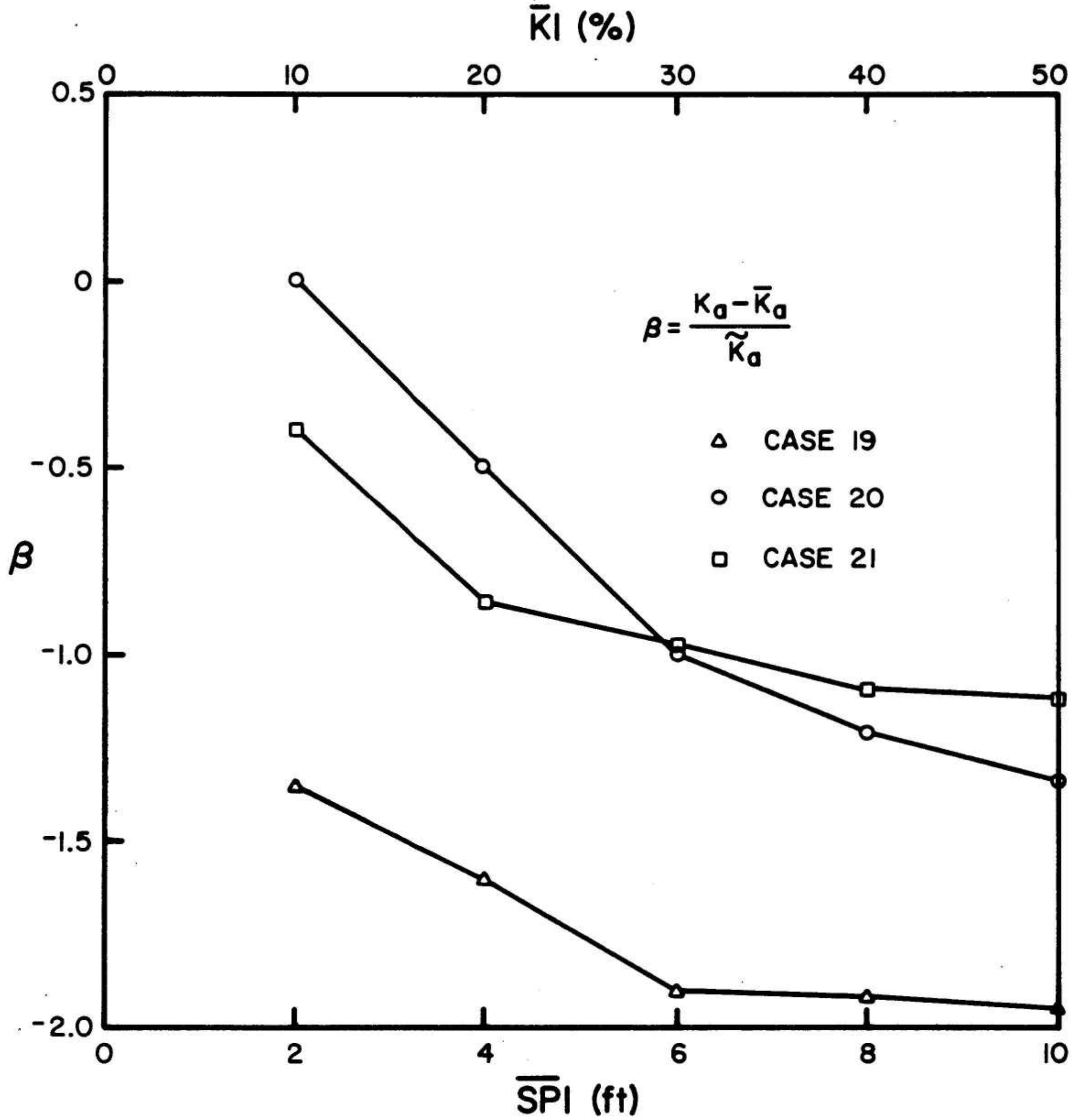


FIGURE 5.47 INDEX OF RELIABILITY AS A FUNCTION OF PERSISTENCE (OR SPACING) OF THE FIRST SET - CONSTANT INTENSITY ($\bar{I}1$)

$(\overline{K}_a + \tilde{K}_a)$ and critical persistence (K_c) for each of cases (19, 20 and 21). When both persistence and joint plane spacing are reduced and the intensity of the first set is maintained, a moderate decrease in K_a occurs. Figures 5.44 through 5.46 also show that standard deviation of apparent persistence ($\tilde{K}_a$), remains essentially constant when $\overline{K1}$ and $\overline{SP1}$ are varied while keeping intensity constant.

The net result, of a decreasing $\overline{K}_a$ and a more or less constant $\tilde{K}_a$ and a constant K_c (the critical persistence, K_c, is independent of $\overline{K1}$ and $\overline{SP1}$) on joint plane reliability, as expressed by β values, is shown in Figure 5.47.

Plots of β as a function of $\overline{K1}$ (and $\overline{SP1}$) show similar trends; an increase in β (and thus reliability) as $\overline{K1}$ and $\overline{SP1}$ are reduced. At values of $\overline{K1}$ below 30 percent (or $\overline{SP1}$ equal to 6 feet), reliability decreases at a faster rate. Examining slopes with a single joint set reveals a limit beyond which reliability would be a function of intensity ($\overline{I1}$) only. However, this condition is not noticeable in the present cases.

5.5.8 Effect of Second Set Mean Joint Length ($\overline{JL2}$)

Joint lengths of the second set are assumed to be exponentially distributed about $\overline{JL2}$. The effect of $\overline{JL2}$ on reliability is studied in two cases with common input parameters and is listed in Figure 5.50.

Case #22: Intact rock cohesion is set to 25 ksf

Case #23: Intact rock cohesion is set to 100 ksf

Two additional cases (24 and 25), mainly for observational purposes, are included. The difference is in the second set mean joint plane spacing ($\overline{SP2}$) for each case. Case 24 considers effects when $\overline{SP2}$ is set to 5 feet while case 25 considers effects when $\overline{SP2}$ is set to 15'. For cases 22 and 23 this parameter is set to 10 feet.

Effect of Second Set Mean Joint Length (JL1) on the Probability of Failure Pf (h)

Although the effect of increasing second set mean joint length ($\overline{JL2}$) at any depth (h) in the slope is not significant, it is not trivial. This can be seen by comparing P_f (h) plots within each of the three cases shown in Figure 5.48. Like most other cases, P_f decreases with increasing h up until approximately 30 feet beyond which P_f increases with increasing depth. As a result, an approximately constant trend of P_f as a function of h occurs for the entire range of $\overline{JL2}$ values examined.

The plot of the probability of 100 percent persistence (P1) is also shown in Figure 5.48. P1 is not sensitive to changes in $\overline{JL2}$ and only varies with changes in depth. This is due to the fact that P1 is not a function of $\overline{JL2}$ and thus the probability of failure (P_f) is not much affected by changes in $\overline{JL2}$ as compared to the results of the analysis of $\overline{JL1}$.

Also shown in Figure 5.48 is the variation of the mean apparent persistence as a function of depth (h) for the three runs. $\overline{K}_a$ (h) is almost constant for all cases. Figure 5.49 is a plot of the variation of the standard deviation of apparent persistence as a function of depth.

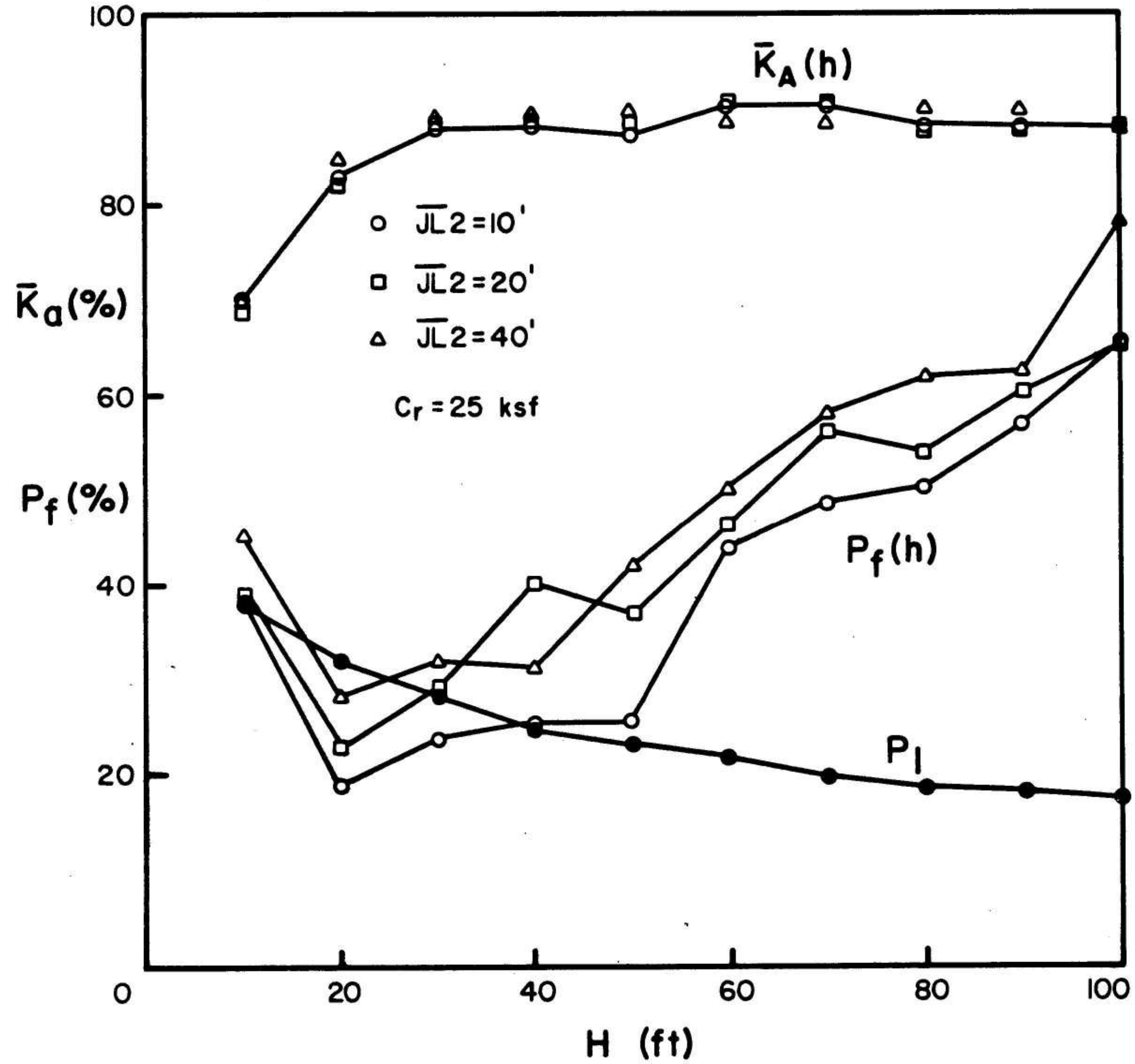


FIGURE 5.48 EFFECT OF MEAN JOINT LENGTH (JOINT SET 2) ON $P_f(h)$

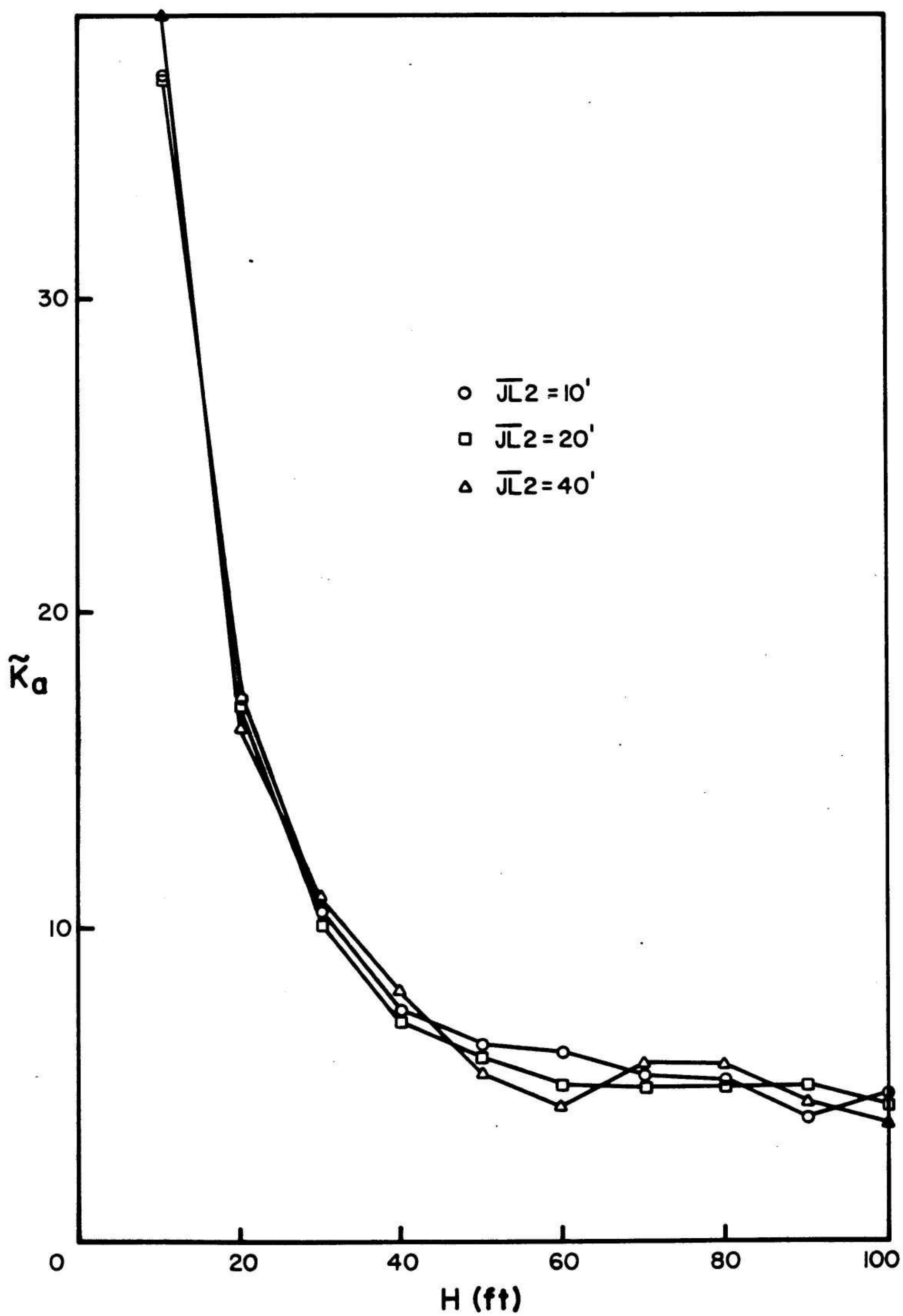


FIGURE 5.49 VARIATION IN THE VALUE OF STANDARD DEVIATION OF APPARENT PERSISTENCE WITH DEPTH

$\overline{K}_a$ (h) starts at very high values and levels off at depths greater than approximately 60 feet. For all cases and at any depth, $\overline{K}_a$ and $\tilde{K}_a$ curves almost overlap.

The Probability of Failure (Pf) as a Function of Second Set Mean Joint Length ($\overline{JL2}$)

Figure 5.50 shows plots of P_f vs. $\overline{JL2}$ for cases 22 and 23 at a depth of 90-100 feet. For low intact rock cohesion (C_r = 25 ksf), the effect of varying $\overline{JL2}$ on P_f is noticeable for $\overline{JL2}$ values in excess of 20 feet. At higher C_r (100 ksf), that effect is erratic and tends to increase slightly with $\overline{JL2}$. In cases 24 and 25, the effect of varying $\overline{JL2}$ on P_f for different mean second set joint plane spacing is hardly noticeable except at high $\overline{JL2}$. This can be seen in Figure 5.51.

Effect of Second Set Mean Joint Length ($\overline{JL2}$) on Apparent Persistence and the Reliability Index (β)

Plots for mean, mean plus one standard deviation of apparent persistence ($\overline{K}_a$, $\overline{K}_a + \tilde{K}_a$) and critical persistence (K_c), are shown for cases 22 and 23 in Figures 5.52 and 5.53, respectively.

Program output reveals that variations in second set mean joint length ($\overline{JL2}$) have no influence on $\overline{K}_a$ and $\tilde{K}_a$ and thus, the reliability index (β) can be obtained through one simulation. As one may expect, β values for smaller C_r are smaller (See Figure 5.54) and consequently reliability is lower (case 23).

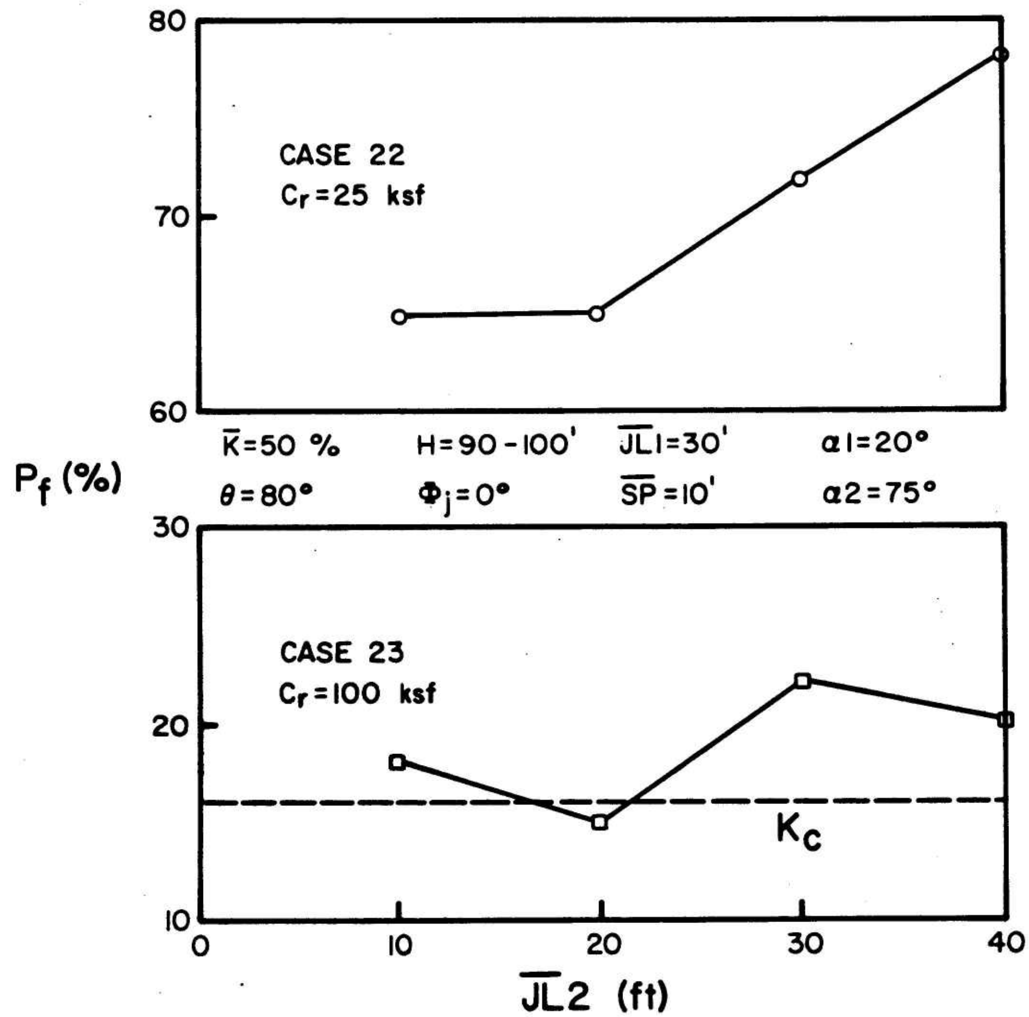


FIGURE 5.50 P_f AS A FUNCTION OF MEAN JOINT LENGTH (SET 2 - $\overline{JL2}$) (VARYING C_r)

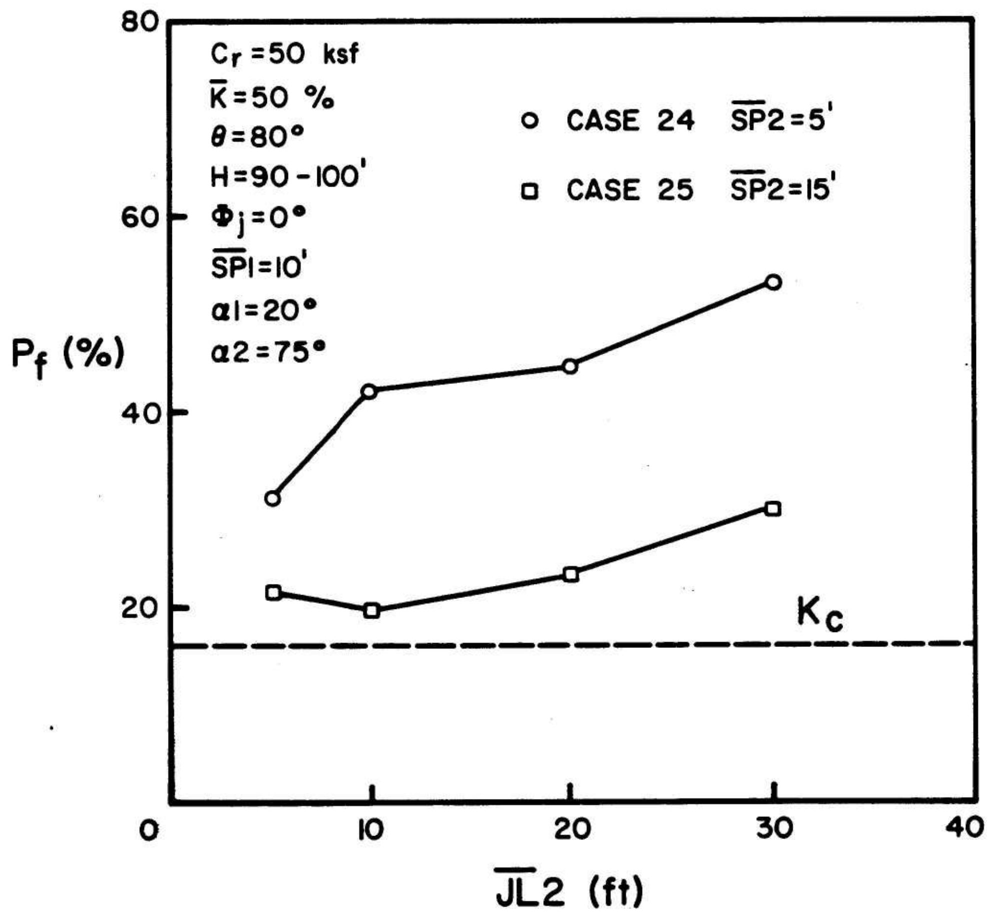


FIGURE 5.51 P_f AS A FUNCTION OF MEAN JOINT LENGTH (SET 2 - $\overline{JL2}$) EFFECT OF VARYING $\overline{SP2}$ (MEAN JOINT PLANE SPACING - SET 2)

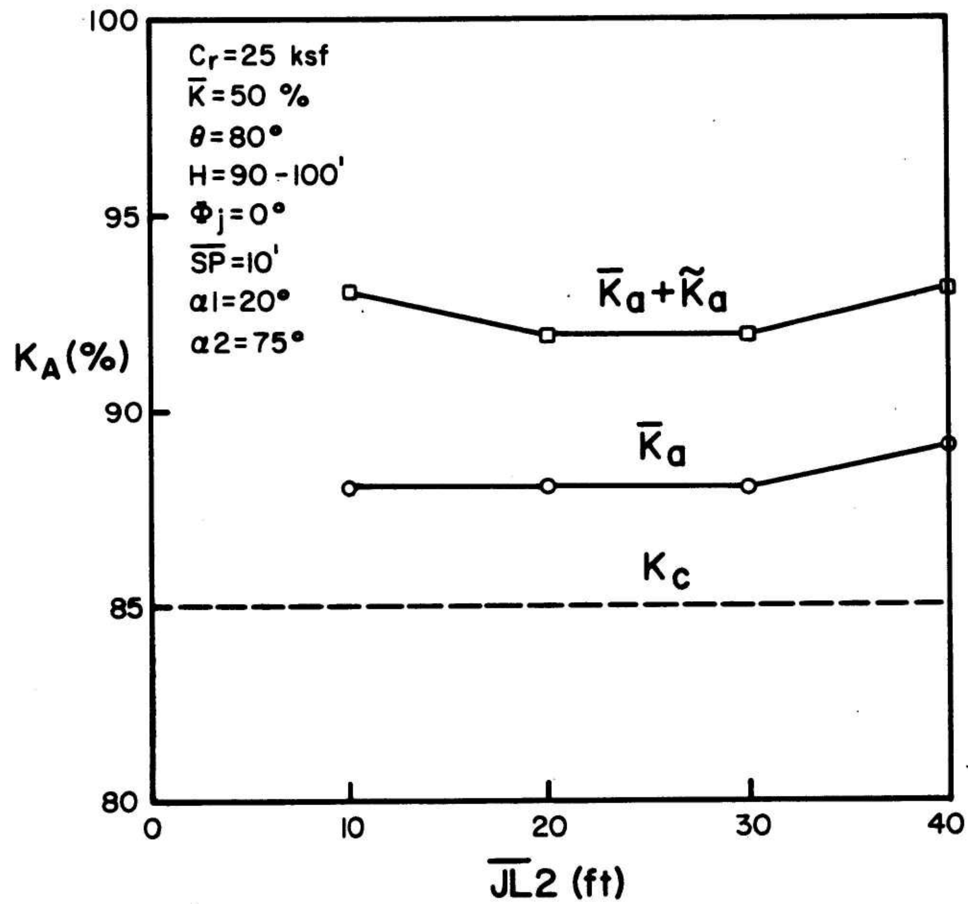


FIGURE 5.52 EFFECT OF MEAN JOINT LENGTH (SET 2 - $\overline{JL}$2) ON APPARENT PERSISTENCE (K_a) CASE 22

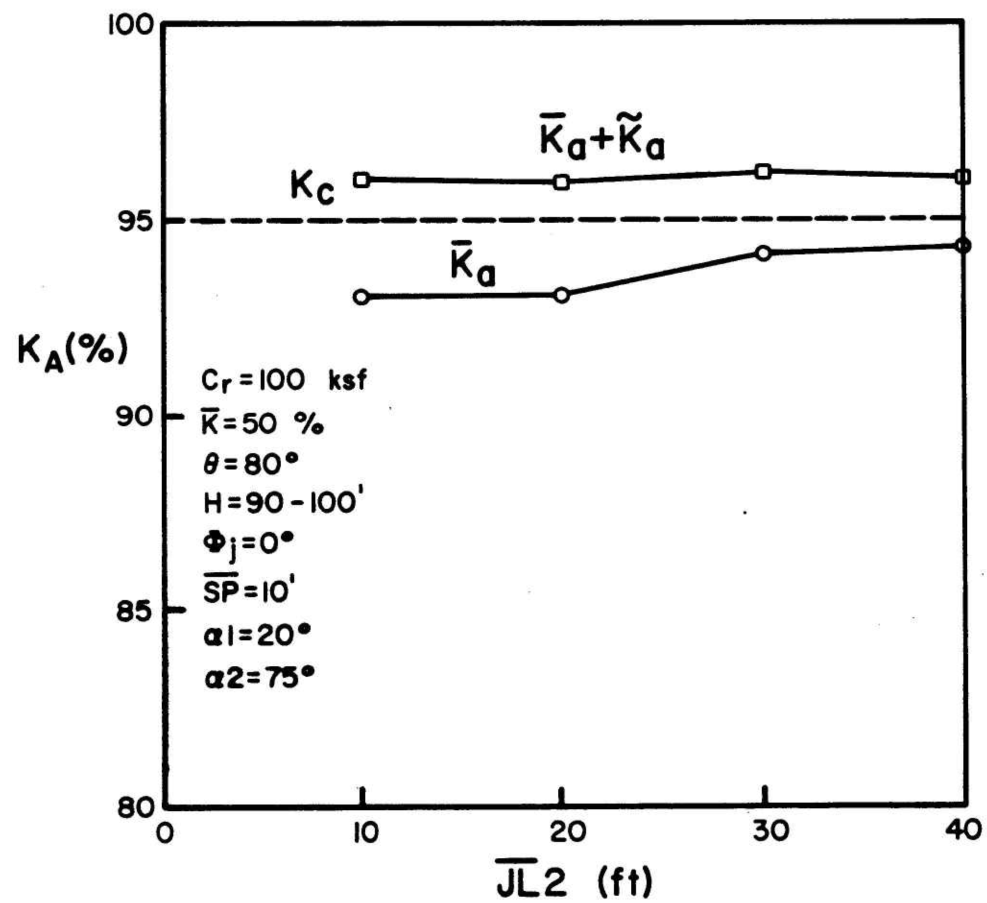


FIGURE 5.53 EFFECT OF MEAN JOINT LENGTH (SET 2 - $\overline{JL}2$) ON APPARENT PERSISTENCE K_a - CASE 23

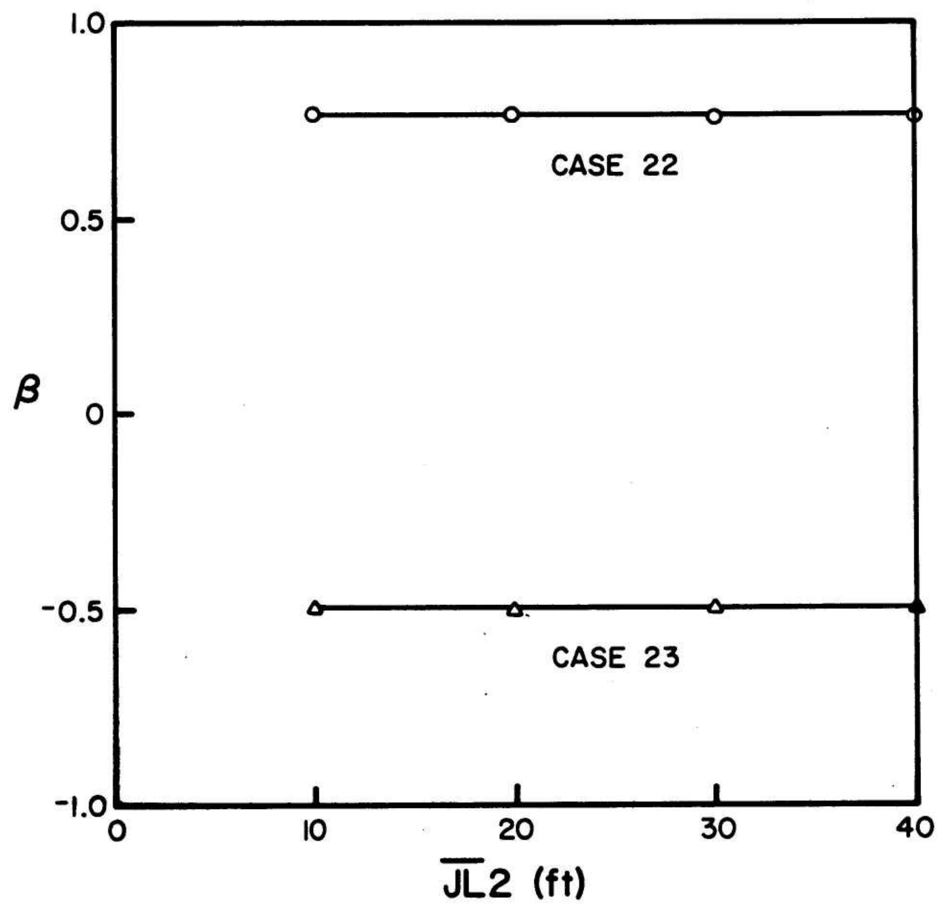


FIGURE 5.54 RELIABILITY INDEX (β) AS A FUNCTION OF THE MEAN JOINT LENGTH OF THE SECOND SET ($\overline{JL}2$)

5.5.9 Effect of Second Set Mean Joint Plane Persistence ($\overline{K}2$) at Constant Spacing on Slope Reliability

Second set mean joint plane persistence ($\overline{K}2$) is estimated by the input parameters second set mean joint length ($\overline{JL}2$) and mean rock bridge length ($\overline{RB}2$) as follows:

$$\overline{K}2 = [\overline{JL}2 / (\overline{JL}2 + \overline{RB}2)] \times 100$$

The influence of $\overline{K}2$ (at constant $\overline{SP}$) is studied in cases 26 and 27 with common input parameters listed in Figure 5.57. In case 26, C_r equals 25 ksf while in case 27 it equals 50 ksf.

Effect of Second Set Mean Persistence ($\overline{K}2$) on the Probability of Failure P_f (h)

The effect of increasing second set mean joint plane persistence ($\overline{K}2$) on the probability of failure P_f (h) is shown in Figure 5.55 for various $\overline{K}2$. The value of P_f, at any given value of h, increases with increasing $\overline{K}2$. For any $\overline{K}2$ value, P_f increases with depth especially at depths greater than approximately 50 feet.

Also shown in Figure 5.55 is mean apparent persistence ($\overline{K}_a$) as a function of depth. For depths approximately greater than 20 feet, the value of $\overline{K}_a$ levels off for all values of $\overline{K}2$. Similar to this is the variation of standard deviation of apparent persistence as a function of depth (See Figure 5.56). Beyond depths of 40 feet, $\tilde{K}_a$ as a function of depth is constant and may be obtained from a single simulation for practically any depth.

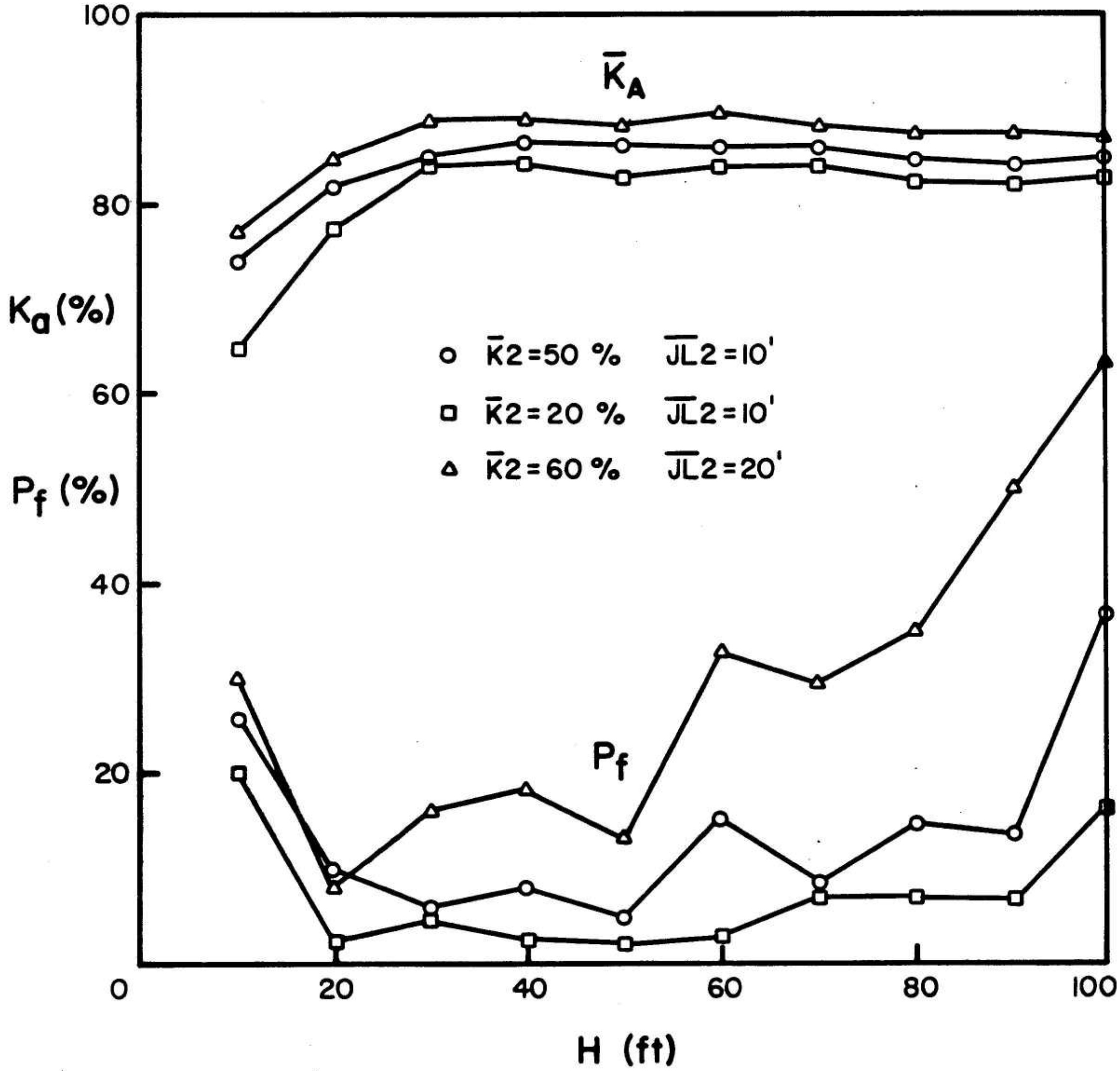


FIGURE 5.55 EFFECT OF MEAN JOINT PLANE PERSISTENCE (SET 2 - $\bar{K}2$) ON $P_f(h)$ CONSTANT MEAN JOINT PLANE SPACING

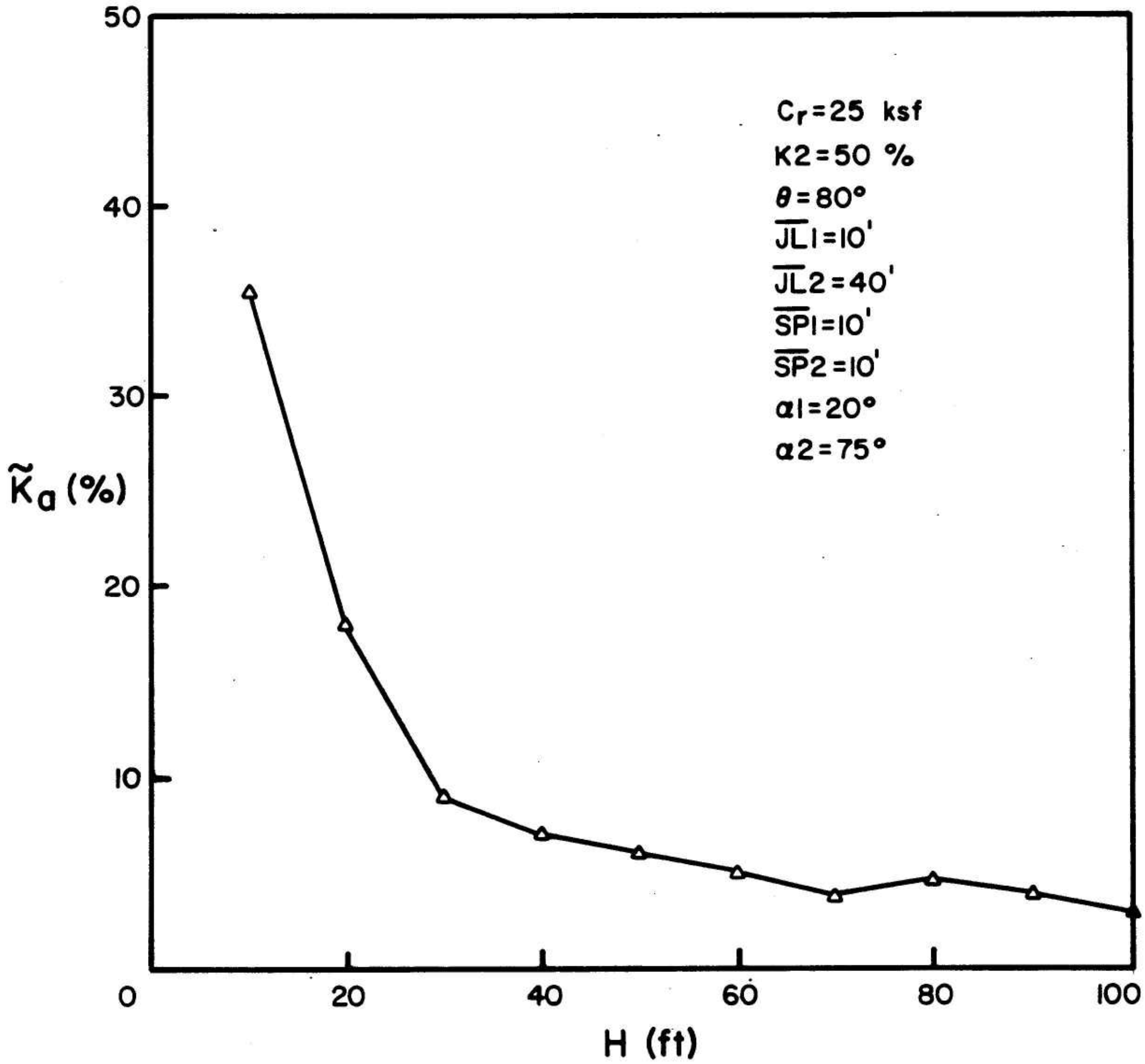


FIGURE 5.56 VARIATION OF STANDARD DEVIATION OF APPARENT PERSISTENCE WITH DEPTH

The Probability of Failure (Pf) as a Function of Second Set Mean Persistence ($\overline{K}2$)

In Figure 5.57 P_f is plotted as a function of second set mean joint plane persistence ($\overline{K}2$). Also plotted is the probability of a plane being 100 percent persistence (P1) as a function of $\overline{K}2$ for all the input paramters listed. Computer output shows that cases 26 and 27 P_f increases from a value of 5 at $\overline{K}2$ equal to 20 percent up to a value for $\overline{K}2$ of 40 percent where C_r P_f levels off at approximately 20 percent for the case with C_r equal to 25 ksf and a P_f of approximately 60 percent for the weaker rock (case 26). This implies that the effects of the second set have a limit which depends on the input paramters used.

Effects on Apparent Persistence (K_a) and the Reliability Index (β)

Figures 5.58 and 5.59 are plots of mean ($\overline{K}_a$) and mean plus one standard deviation ($\overline{K}_a$ + $\tilde{K}_a$) of apparent persistence and critical persistence for cases 26 and 27, respectively. For practical purposes, $\overline{K}_a$ and $\tilde{K}_a$ may be assumed to be constant for varying second set mean persistence ($\overline{K}2$). For C_r equal to 25 ksf, the effect of reliability can be clearly seen as $\overline{K}_a$ approaches and exceeds K_c with increasing $\overline{K}2$. At $\overline{K}2$ equal 30 percent, $\overline{K}_a$ = K_c and one may expect a P_f value close to 50 percent as computer output data shows.

Figure 5.60 is a plot of the reliability index (β) as a function of second set persistence ($\overline{K}2$). For values greater than 40 percent for second set peristence, β values are approximately constant. Reliability is slightly more sensitive to $\overline{K}2$ when rock strength (C_r) is low (See

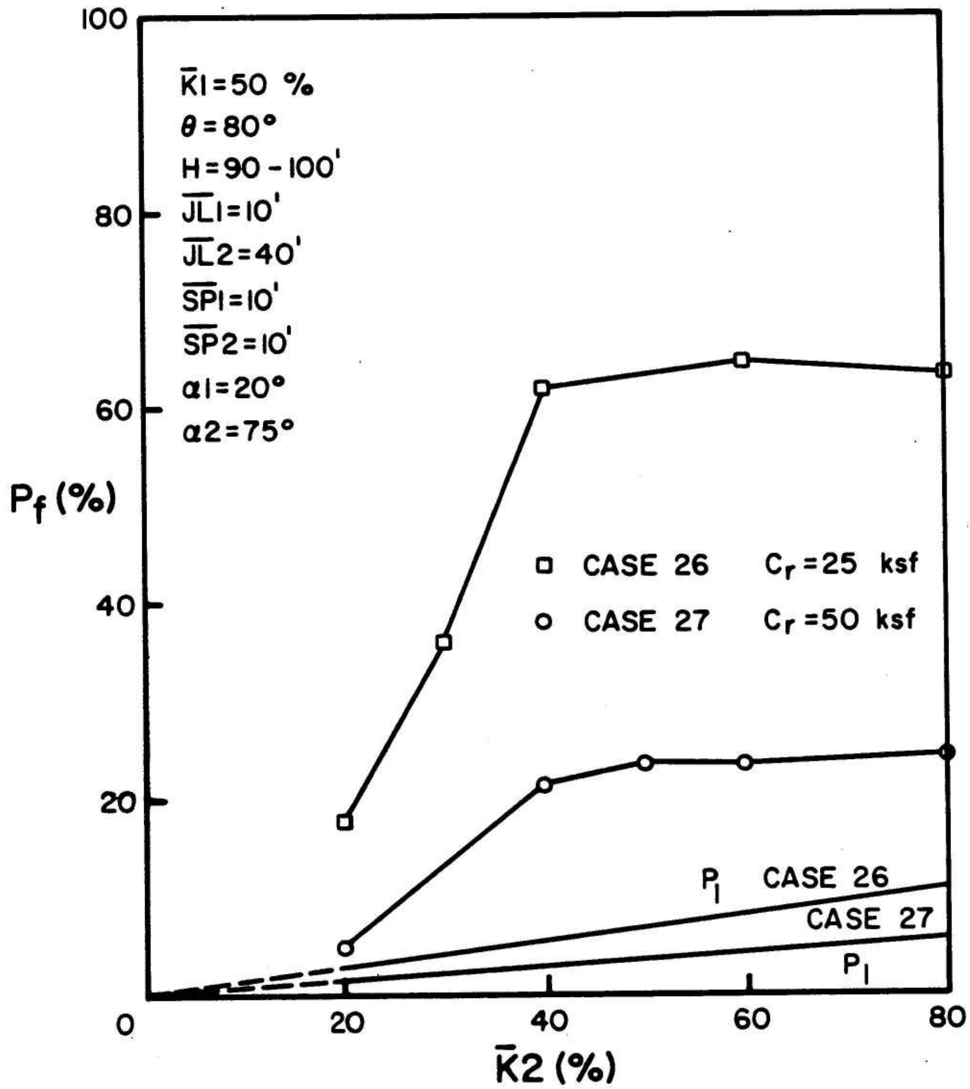


FIGURE 5.57 P_f AS A FUNCTION OF MEAN JOINT PLANE PERSISTENCE $\bar{K}2$

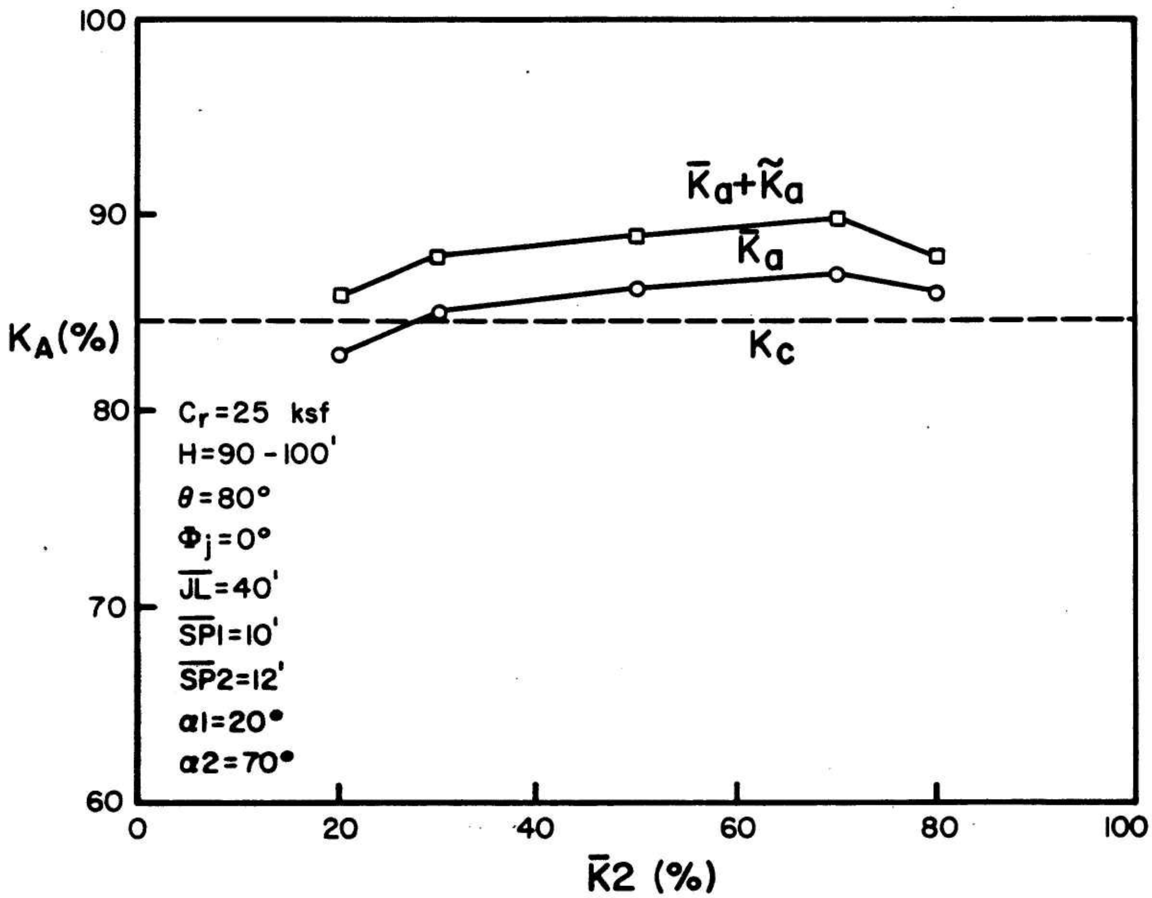


FIGURE 5.58 EFFECT OF MEAN JOINT PLANE PERSISTENCE ($\bar{K}2$) ON APPARENT PERSISTENCE K_a - CASE 26

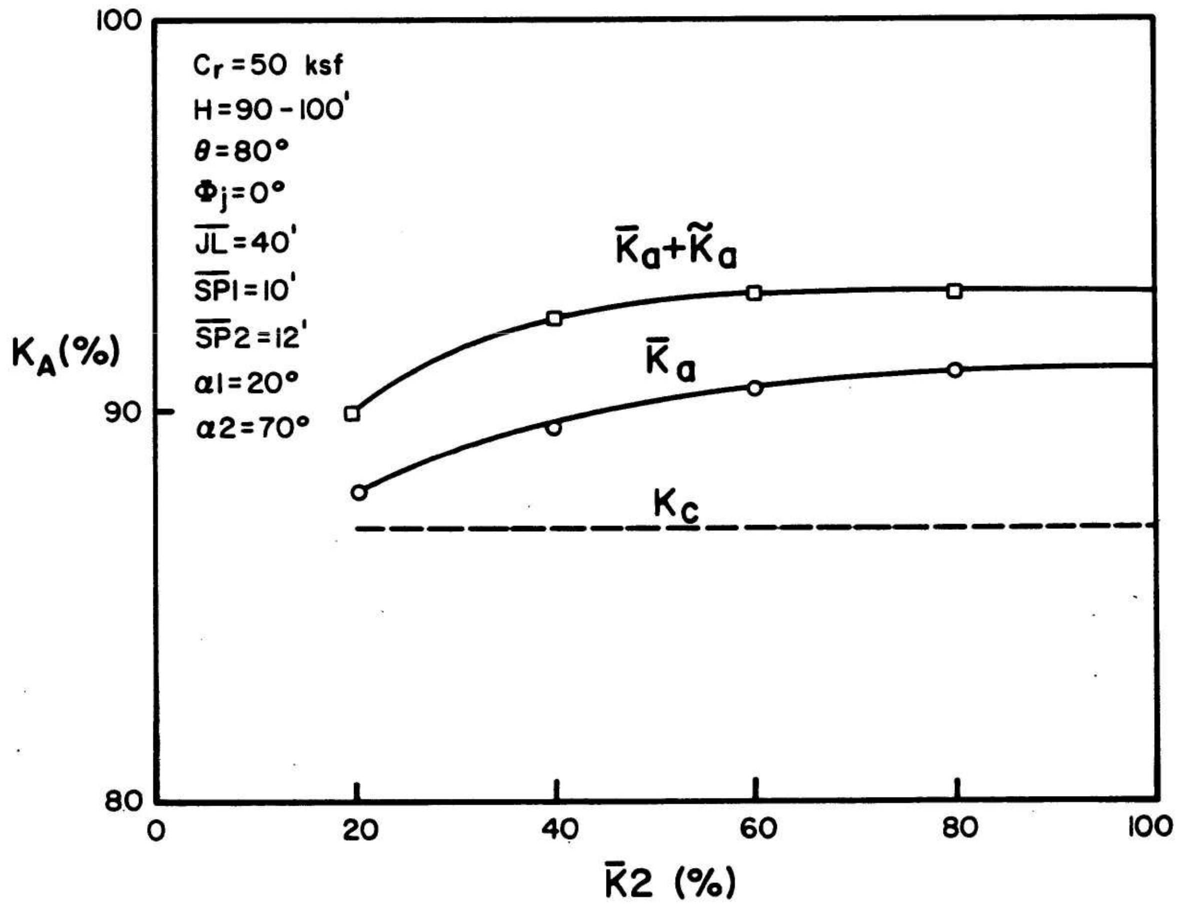


FIGURE 5.59 EFFECT OF MEAN JOINT PLANE PERSISTENCE ($\bar{K}2$-SET 2) ON APPARENT PERSISTENCE $\bar{K}_a$-CASE 27

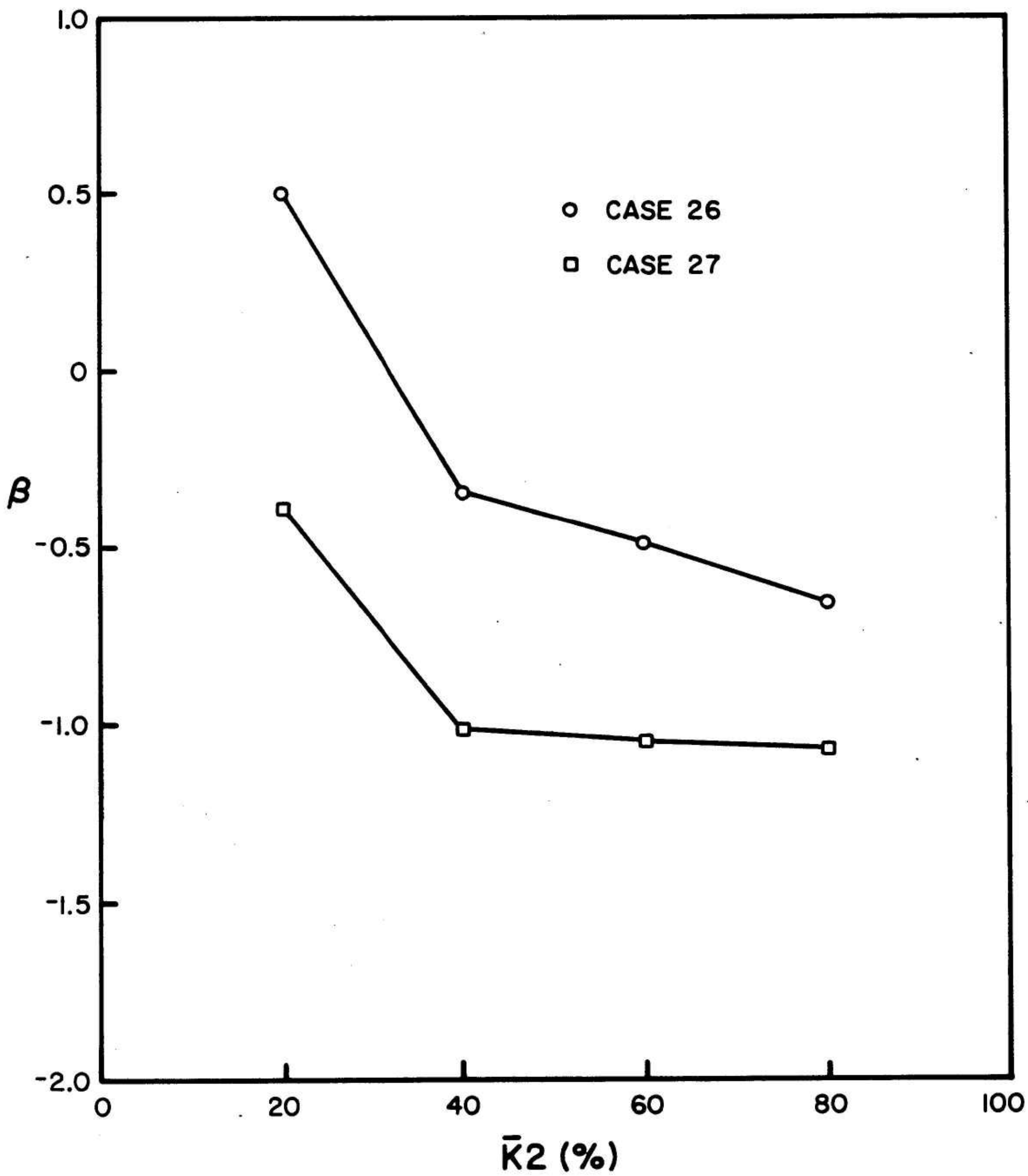


FIGURE 5.60 RELIABILITY INDEX (β) AS A FUNCTION OF VARYING SECOND SET PERSISTENCE ($\bar{K}2$)

case 26.

5.5.10 Effects of Second Set Mean Joint Plane Spacing ($\overline{SP2}$)

The effect of second set mean joint plane spacing is studied in two separate cases with common input parameters lised in Figure 5.63. In both cases, $\overline{SP2}$ varies between 2 and 10 feet.

Case #28: Examines the influence of $\overline{SP2}$ in slopes with weak rock (C_r = 25 ksf)

Case #29: Examines the influence of $\overline{SP2}$ in slopes with strong rock (C_r = 100 ksf)

Effects of Second Set Mean Joint Spacing (SP2) on the Probability of Failure P_f (h)

The effect of varying $\overline{SP2}$ on the probability of failure is shown in Figure 5.61. Except for the shallow depths, increasing $\overline{SP2}$ has the effect of increasing P_f. One can see that the influence of $\overline{SP2}$ on P_f (h) is most pronounced in weak rock with long mean joint lengths of the second set (case in which $\overline{SP2}$ = 6 and C_r = 25 ksf).

As $\overline{SP2}$ decreases, the P_f (h) curve begins to approach the lower limit curve (the probability of being 100 percent persistent). For rock with high $\overline{K1}$ and low strength (C_r) joint planes will commonly fail

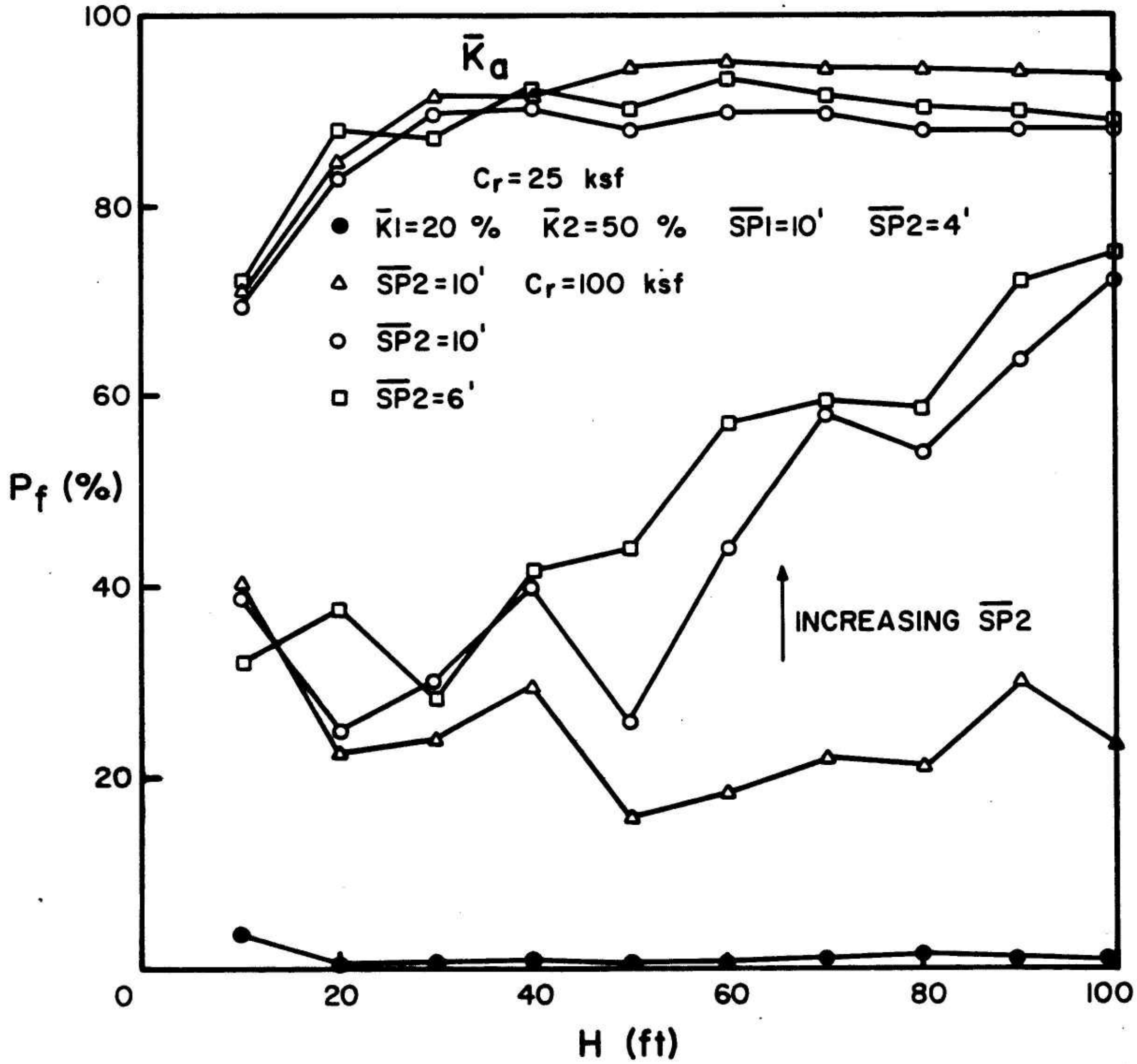


FIGURE 5.61 EFFECT OF MEAN JOINT PLANE SPACING ($\overline{SP2}$ - SET 2) ON $P_f(h)$

without transitions and P will be high even if $\overline{SP}$ of either set is high (e.g., greater than 20 feet).

The mean apparent persistence ($\overline{K}_a$) is insensitive to variations in $\overline{SP}2$ at any depth. Mean apparent persistence remains constant with depth as may be seen in Figure 5.61. Figure 5.62 is a plot of the standard deviation of apparent persistence ($\tilde{K}_a$) as a function of depth (h). For depths greater than about 40 feet, $\tilde{K}_a$ reaches a relatively constant value. Finally, at any depth $\tilde{K}_a$ shows no sensitivity to variations in $\overline{SP}2$.

The Probability of Failure (P_f) as a Function of Second Set Mean Joint Plane Spacing ($\overline{SP}2$)

For constant mean joint segments lengths ($\overline{JL}$), the influence of second set mean joint plane spacing ($\overline{SP}2$) on the probability of failure (P_f), in the depth interval from 90 to 100 feet is examined in cases, 28 and 29, as shown in Figure 5.63. Case 28 analyzes a slope with weak rock (C_r = 25 ksf) while case 29 analyzes a slope with strong rock (C_r = 100 ksf). For each case, P_f is approximately constant and shows a slight decrease for $\overline{SP}2$ greater than 8 feet. As C_r increases beyond C_r equal 100 ksf, one would expect that P_f would eventually become equal to P_1 for all $\overline{SP}2$.

Effect of Varying Second Set Mean Joint Plane Spacing ($\overline{SP}2$) on Apparent Persistence (K_a)

From model runs, variations in $\overline{SP}2$ have no effect on neither the

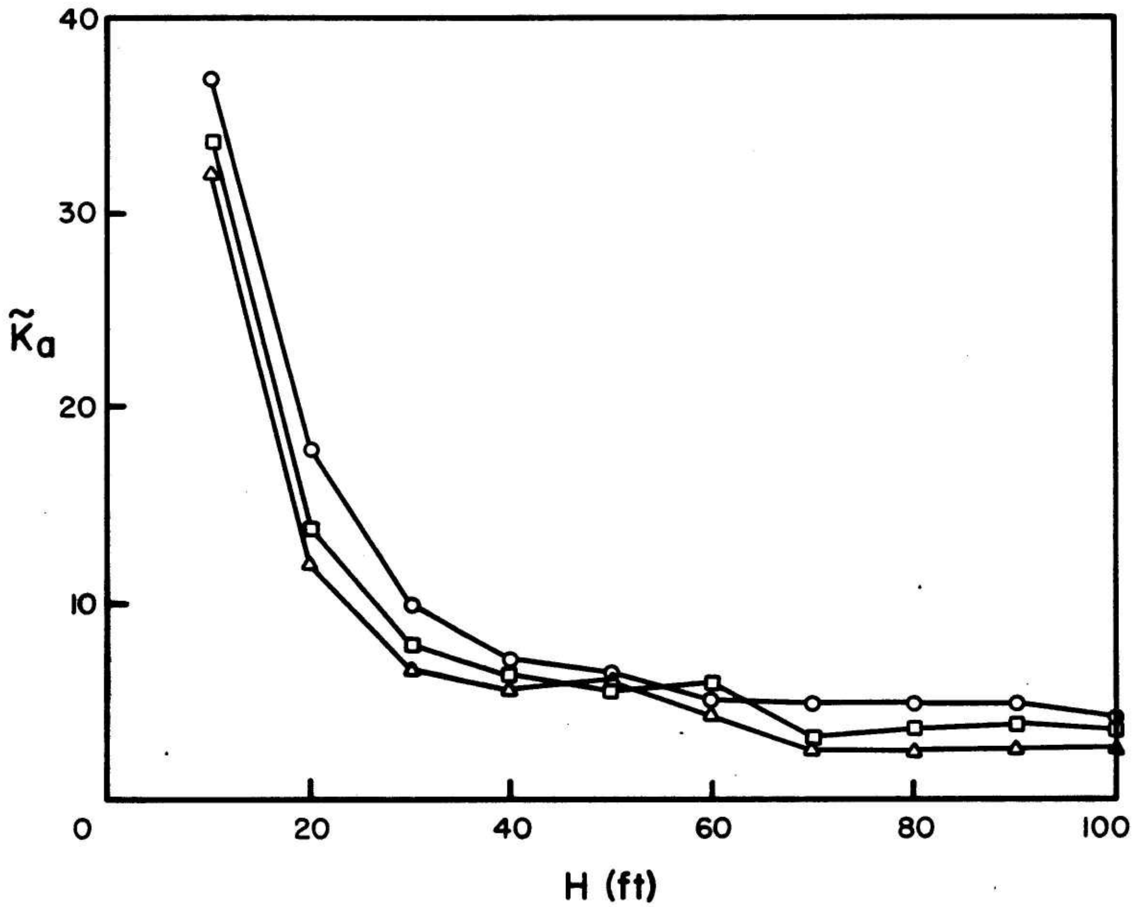


FIGURE 5.62 VARIATION OF STANDARD DEVIATION OF APPARENT PERSISTENCE WITH DEPTH

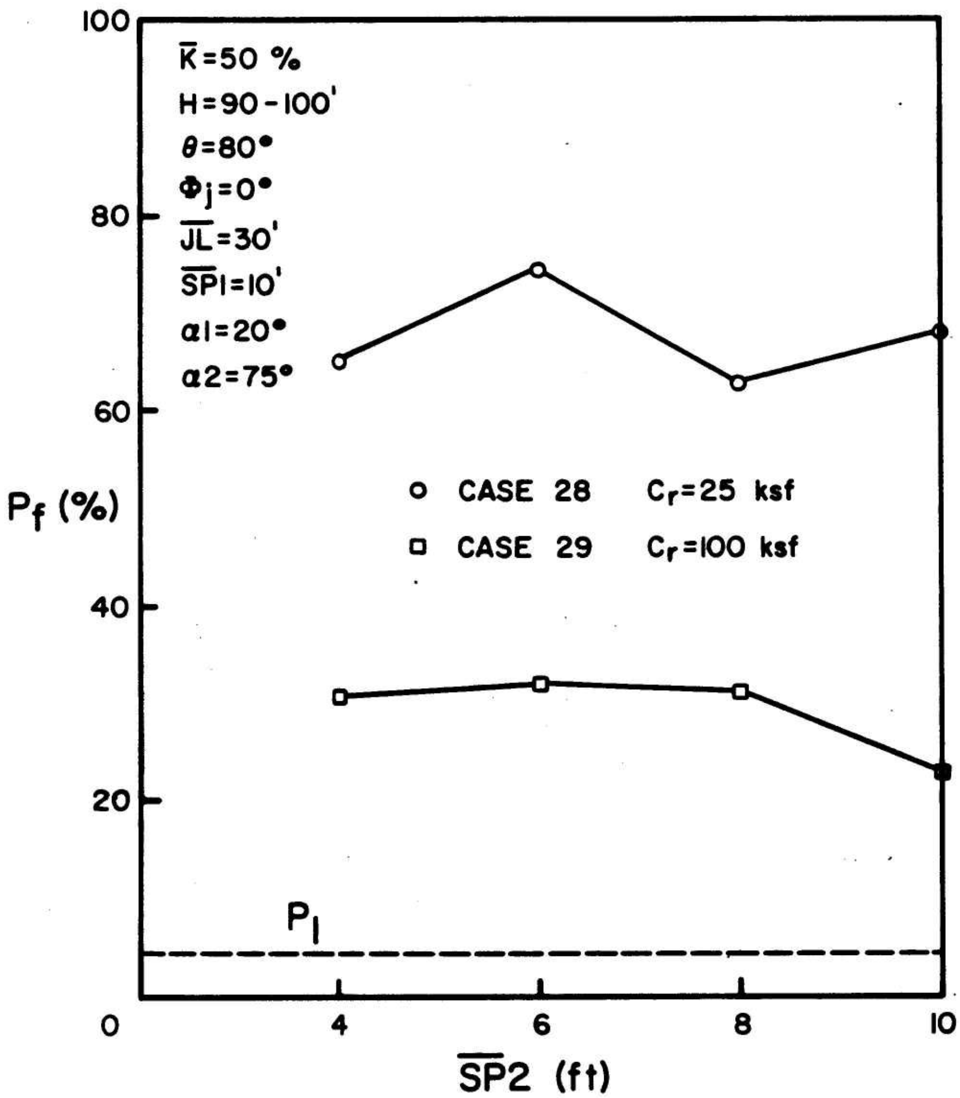


FIGURE 5.63 P_f AS A FUNCTION OF MEAN JOINT SPACING (SET 2 - $\overline{SP2}$) CONSTANT $\overline{K}$

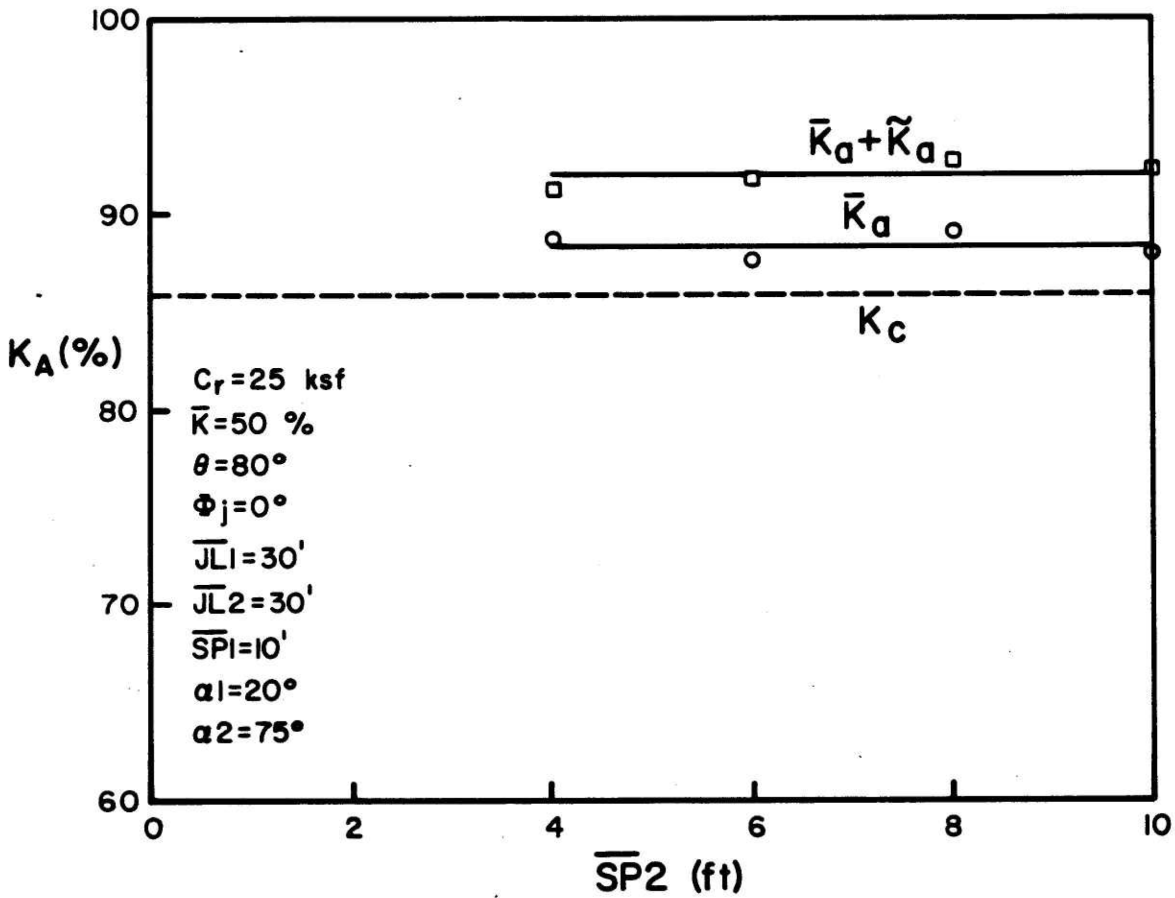


FIGURE 5.64 EFFECT OF MEAN JOINT SPACING (SET 2-$\overline{SP2}$) ON APPARENT PERSISTENCE (K_a) CONSTANT $\bar{K}$ - CASE 28

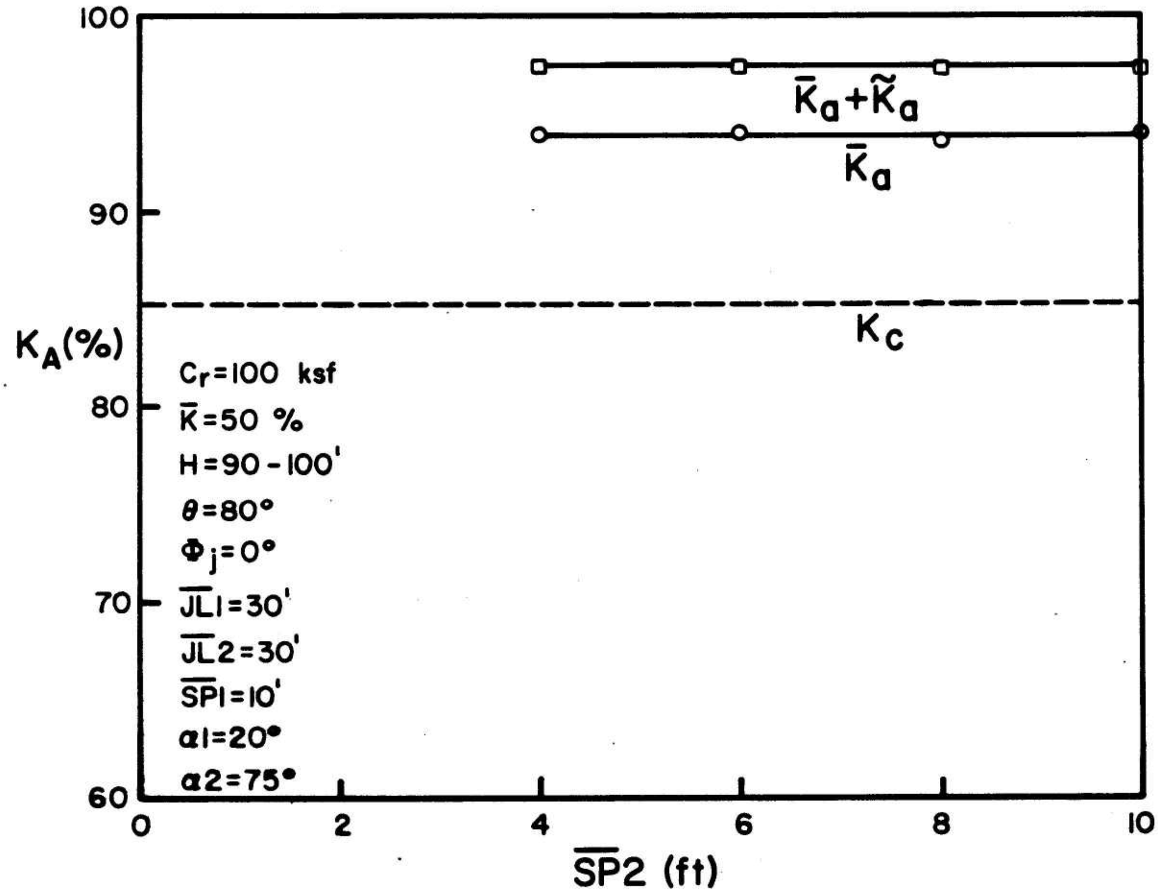


FIGURE 5.65 EFFECT OF MEAN JOINT SPACING (SET 2 - $\overline{SP2}$) ON APPARENT PERSISTENCE (K_a) CONSTANT $\overline{K}$ - CASE 29

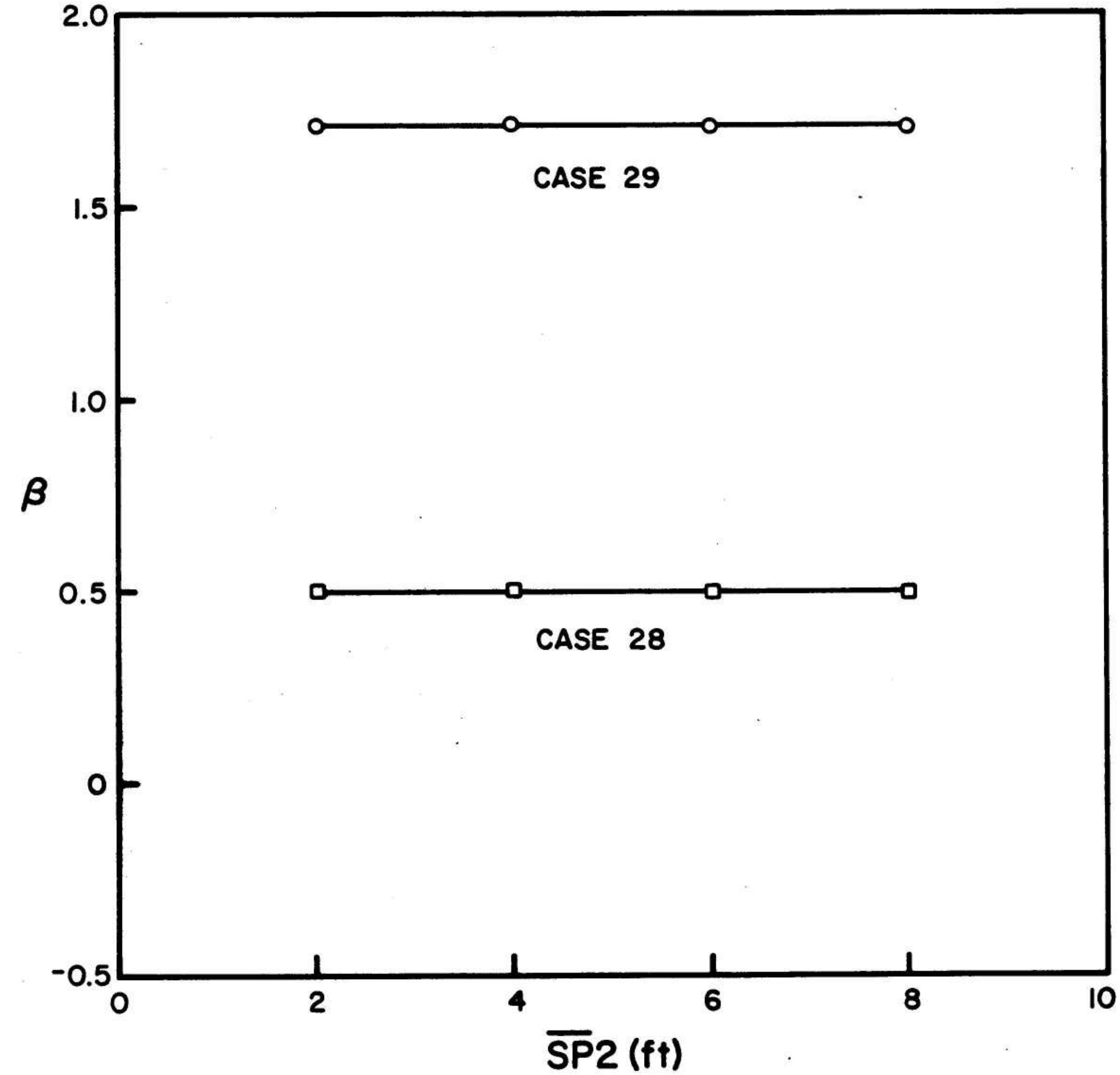


FIGURE 5.66 RELIABILITY INDEX (β) AS A FUNCTION OF SECOND SET MEAN JOINT PLANE SPACING $\overline{SP2}$

mean apparent persistence nor the standard deviation of apparent persistence. Figures 5.64 and 5.65 are plots of mean apparent persistence ($\overline{K}_a$) and mean plus one standard deviation of apparent persistence as a function of $\overline{SP2}$ for cases 28 and 29. All plots show constant $\overline{K}_a$ and $\tilde{K}_a$ as $\overline{SP2}$ varies from 2 to 10 feet. The effect of this variation on reliability can be studied from the plots of β values shown in Figure 5.66.

$$\beta = \frac{\overline{K}_a - \overline{K}_c}{\tilde{K}_a}$$

Since K_c is independent of $\overline{SP2}$ and remains constant for each case, reliability remains constant with varying $\overline{SP2}$. From styidng the P_f ($\overline{SP2}$) plots, one concludes that case 28 is the least reliable, since it has both the lowest reliability (highest β) values and the highest P_f values over the range of $\overline{SP2}$ values examined.

As one may expect, decreasing $\overline{SP2}$ reduces reliability but this is only noticeable at spacings less or equal to approximately 8 feet within the parameters used in this section.

5.5.11 Effects of Second Set Persistence ($\overline{K2}$) on Constant Joint Intensity ($\overline{I2}$)

The influence of estimated second set persistence ($\overline{K2}$) at constant intensity $\overline{I2}$, is examined in cases (30, 31 and 32). In all cases, $\overline{I2}$ is kept constant ($\overline{I2}$ = 5). It is possible to examine to joint plane reliability as a function of intensity alone rather than as a function of the separate components $\overline{K2}$ and $\overline{SP2}$. All cases had the common input parameters listed in Figure 5.69.

Case #30: Cohesion (C_r) is set to 25 ksf and first set mean joint length ($\overline{JL2}$) is set to 20 feet.

Case #31: Cohesion is set to 8 ksf and $\overline{JL1}$ is as above.

Case #32: Cohesion is set to 25 ksf and $\overline{JL1}$ is set to 5 feet.

<u>Effect of Second Set Persistence ($\overline{K2}$) at Constant Intensity ($\overline{I2}$) on the Probability of Failure Pf (h)</u>

The effect of varying only $\overline{SP2}$ and $\overline{K2}$ while holding $\overline{I2}$ (and all other input parameters) constant is shown in Figure 5.67. Increasing both $\overline{K2}$ and $\overline{SP2}$ by the same percentage (keeping $\overline{I2}$ constant) increases P_f at any depth. However, as shown in Figure 5.67, the magnitude of this increase tends to decrease with depth. Often at depth, P_f (h) becomes independent of $\overline{K2}$ and $\overline{SP2}$, and is a function of only their ratio; ($\overline{I2}$) as indicated by the intersection of the P_f curves.

Shown in Figure 5.67 is the variation of mean apparent persistence as a function of depth which becomes independent of the K2 values as computer output shows.

Figure 5.68 is a plot of standard deviation of apparent persistence ($\tilde{K}_a$) as a function of depth (h). Up to a depth of approximately 50 feet, $\tilde{K}_a$ is highly sensitive to changes in depth.

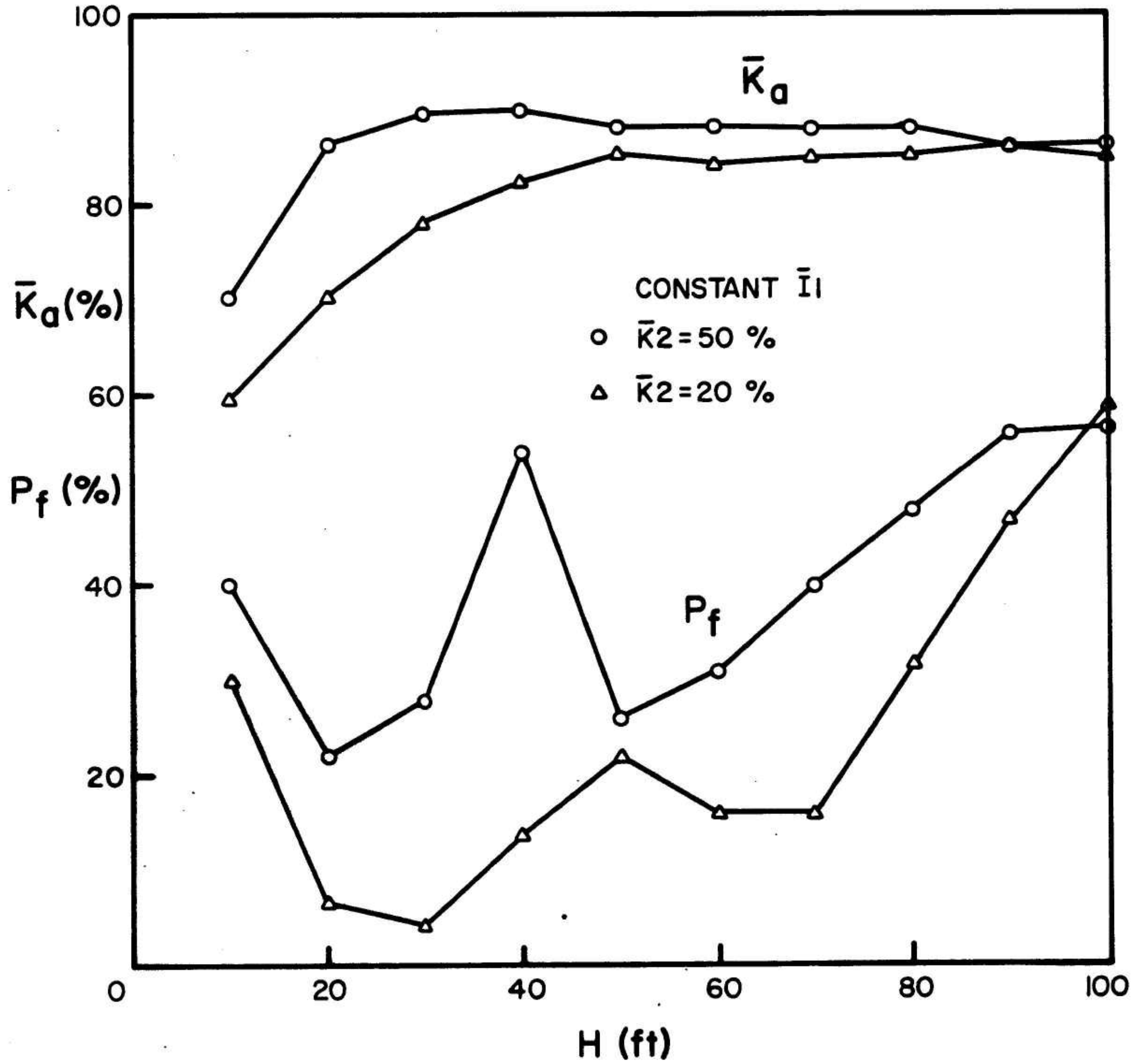


FIGURE 5.67 EFFECT OF MEAN PERSISTENCE ($\bar{K}2$) ON THE PROBABILITY OF JOINT PLANE FAILURE $P_f(h)$ AT CONSTANT SECOND JOINT PLANE INTENSITY $\bar{I}2$

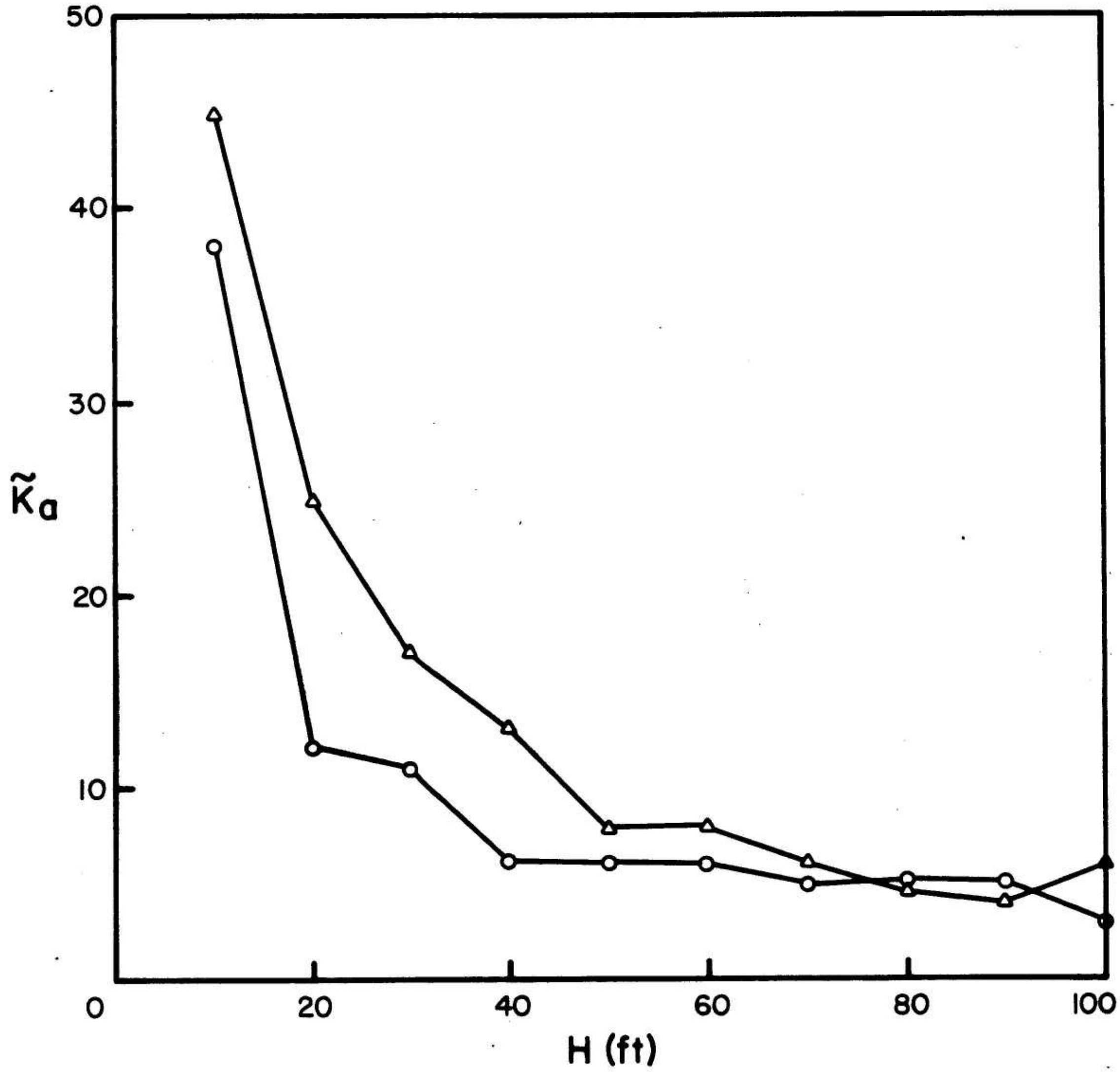


FIGURE 5.68 VARIATION OF STANDARD DEVIATION OF APPARENT PERSISTENCE ($\tilde{K}_a$) WITH DEPTH (h)

Effect of Second Set Persistence ($\overline{K}2$) on the Probability of Failure (P_f)

Figure 5.69 is a plot of the probability of failure P_f as a function of $\overline{K}2$ (and $\overline{SP}2$) for the three cases mentioned above (30, 31 32) in which $\overline{I}2$ was kept constant ($\overline{I}2$ = 5). Other input parameters are also kept constant as shown in Figure 5.69. In all cases, P_f increases with increasing $\overline{K}2$ (and $\overline{SP}2$). For very weak rock (C_r = 8 ksf), P_f seems to be independent of $\overline{K}2$ (or $\overline{SP}2$) and simply a function of $\overline{I}2$ only. With less accuracy, the same may be said for cases 30 and 32 as can be seen in Figure 5.69. As expected, when $\overline{K}2$ and $\overline{SP}2$ approach zero, joint planes are no longer defined. At such a point, only joint intensity can then describe jointing within an area of rock. Hence the probability of failure becomes a function of that ratio.

Effect on Apparent Persistence (K_a) and the Reliability Index (β)

Figures 5.70 and 5.71 are plots of mean apparent persistence ($\overline{K}_a$) and mean plus one standard deviation of apparent persistence ($\overline{K}_a + \tilde{K}_a$) of cases 30 and 31, respectively. In both cases, and over the range of values considered, $\overline{K}_a$ and $\tilde{K}_a$ may be assumed constant. This indicates that $\overline{K}_a$ and $\tilde{K}_a$ may be assumed to be functions of intensity which is constant in this section.

For both cases, K_c is constant and it is neither a function of $\overline{K}2$ and $\overline{SP}2$. Joint plane reliability can be expressed by β values as shown in Figure 5.72. Recall:

$$\beta = [(K_c - K_a) / K_a].$$

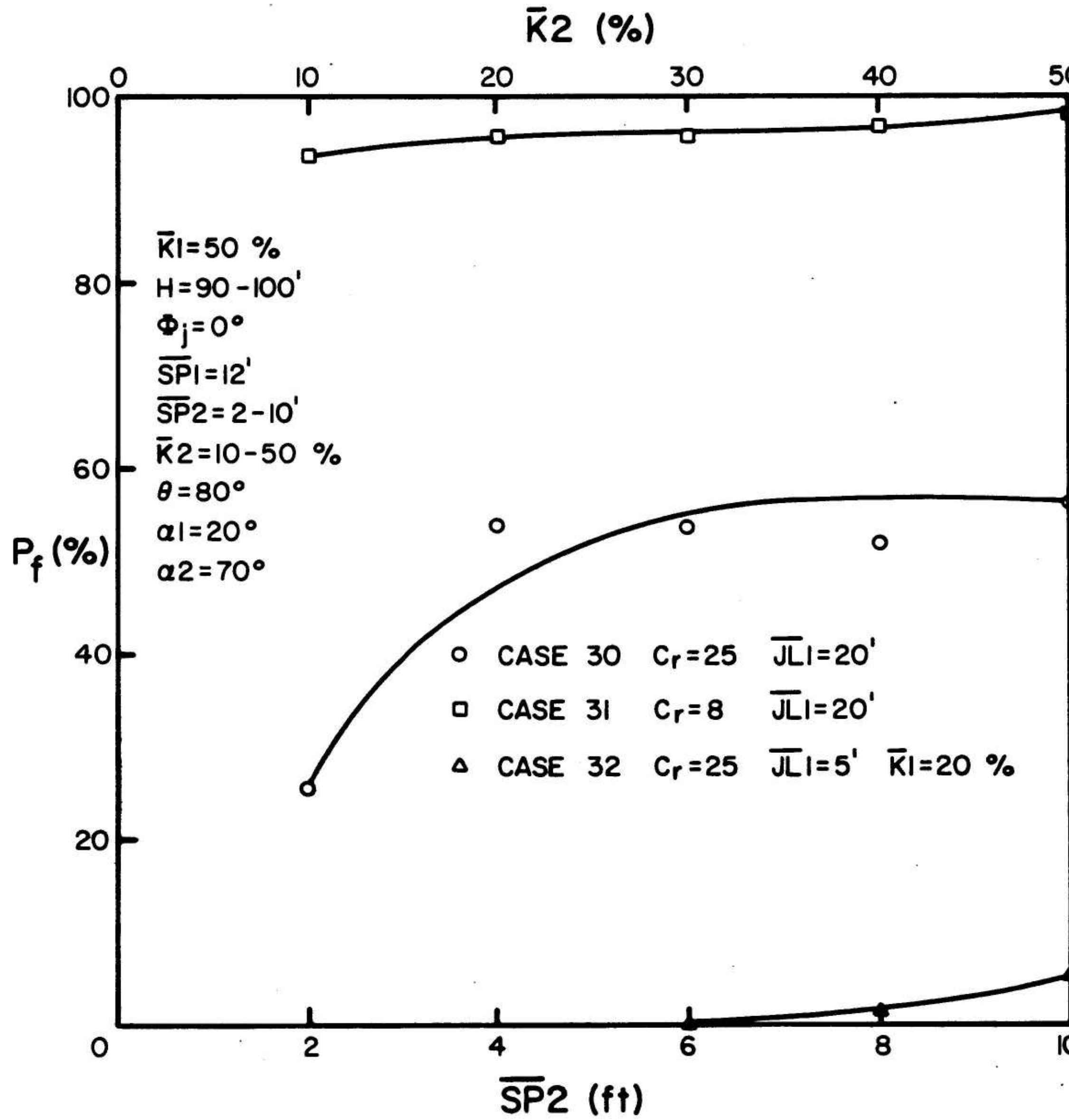


FIGURE 5.69 THE PROBABILITY OF FAILURE (P_f) AS A FUNCTION OF $\bar{K}2$ AND $\overline{SP}2$ WITH CONSTANT INTENSITY ($\bar{I}2$)

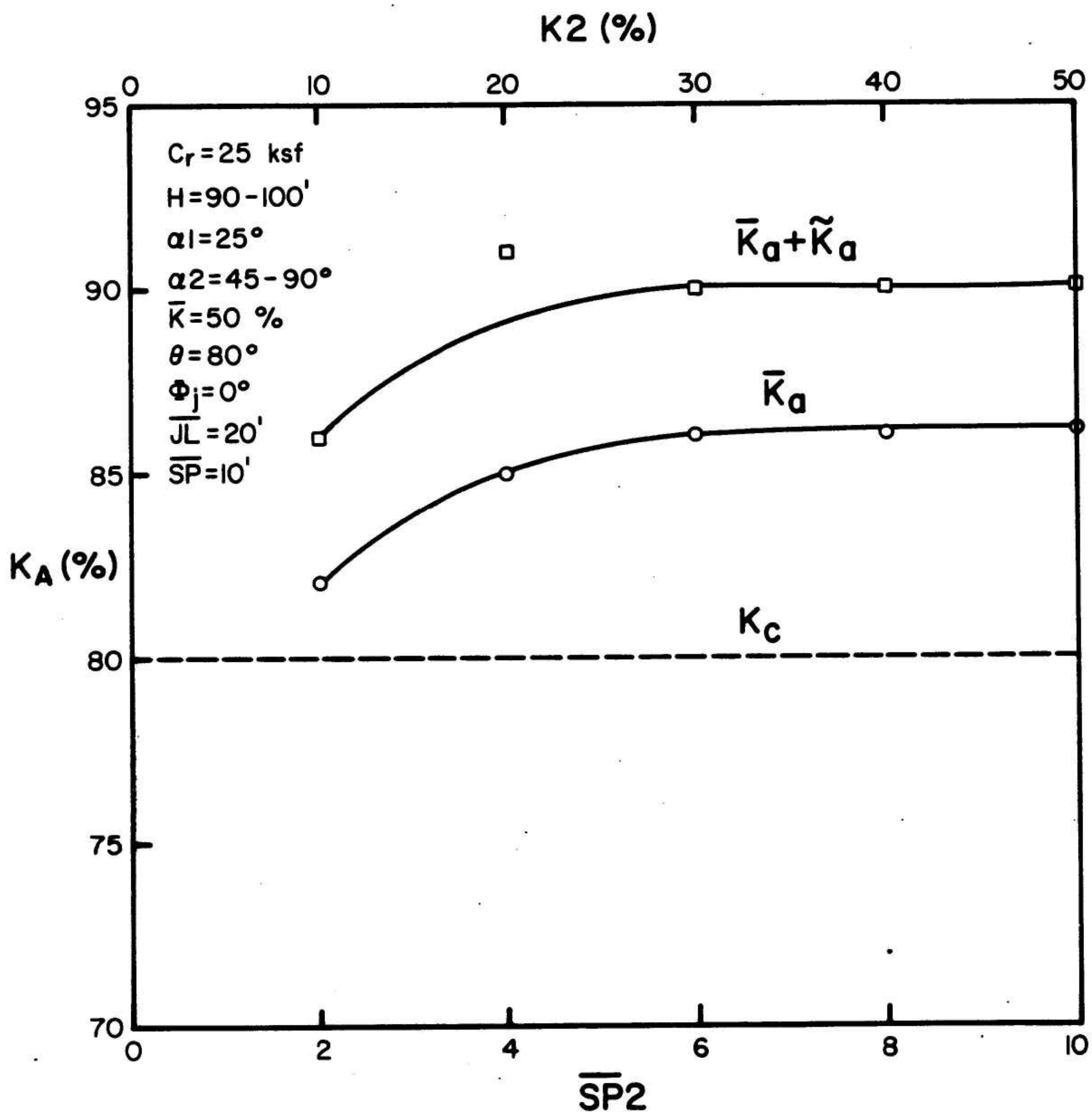


FIGURE 5.70 EFFECT OF JOINT SET 2 PERSISTENCE ($\bar{K}2$) ON APPARENT PERSISTENCE (K_a) AT CONSTANT INTENSITY ($\bar{I}2$)

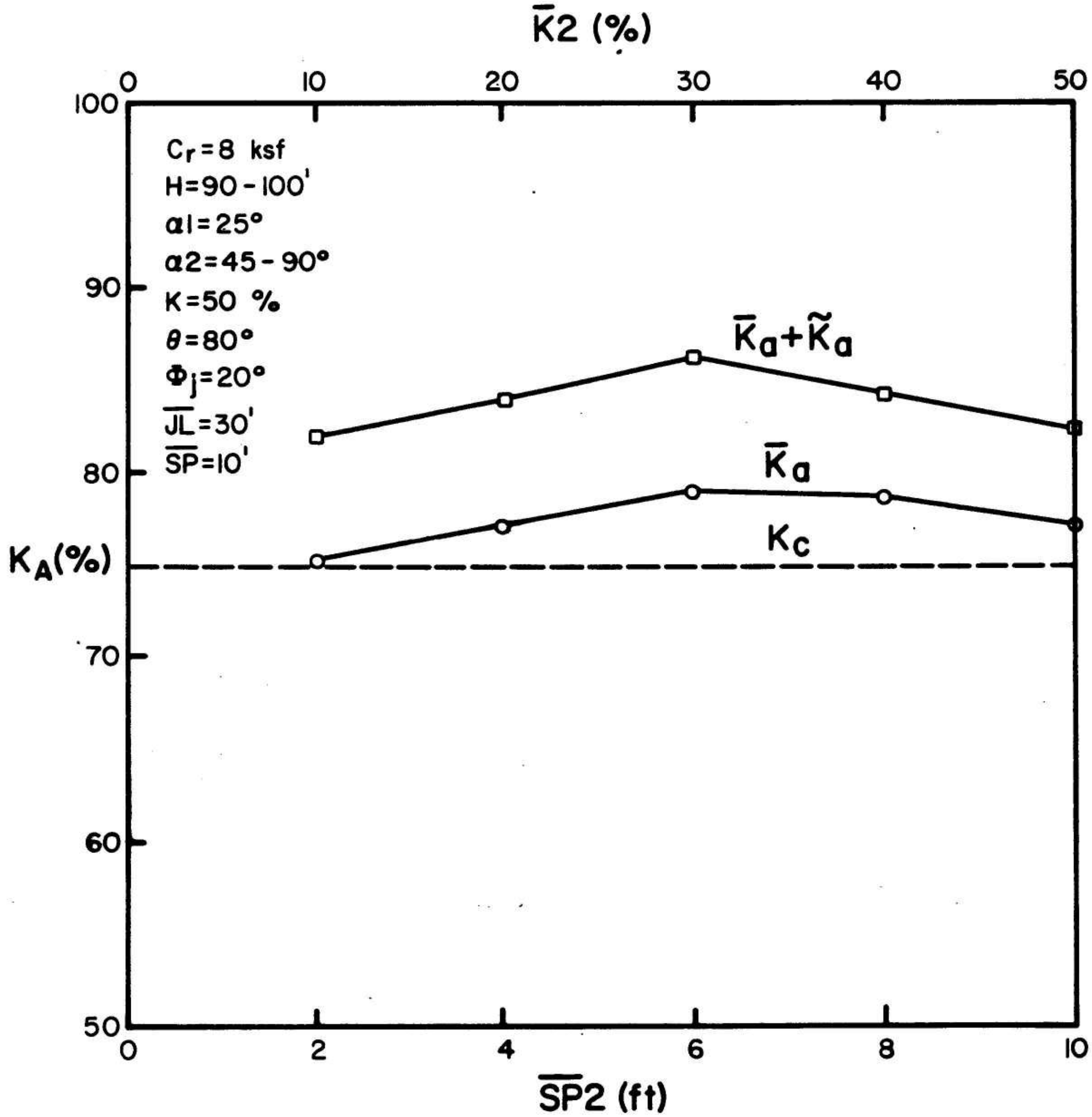


FIGURE 5.71 EFFECT OF SECOND JOINT SET PERSISTENCE ($\overline{K}2$) ON APPARENT PERSISTENCE (K_a) CASE 31

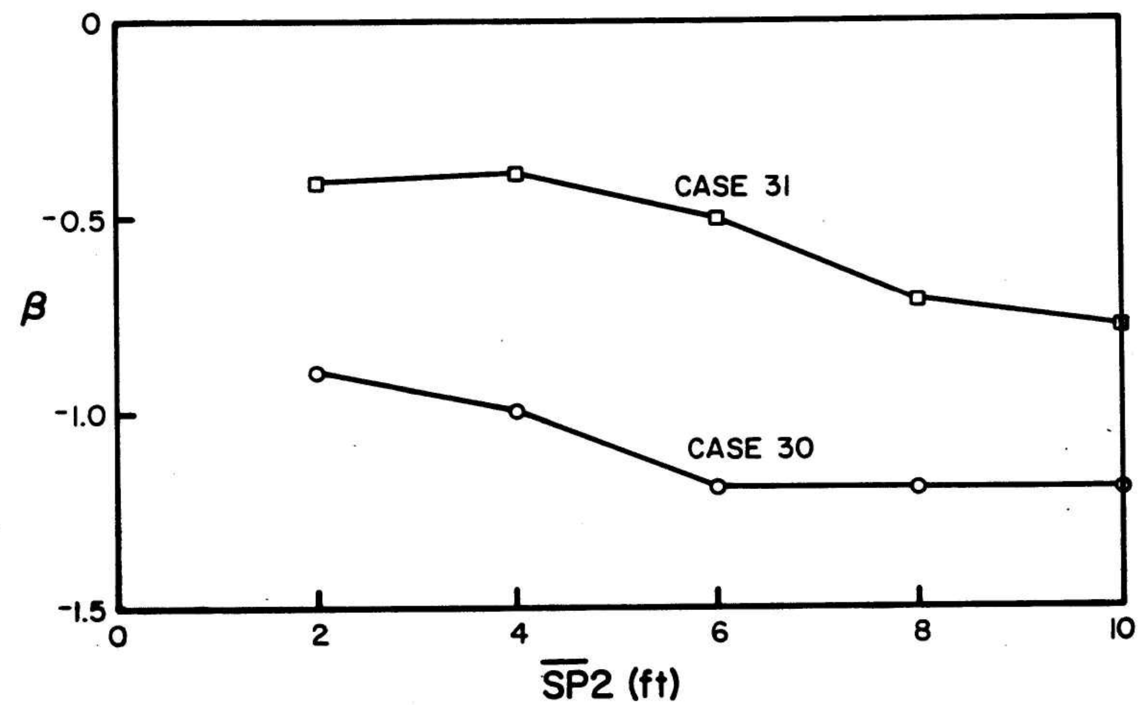


FIGURE 5.72 RELIABILITY INDEX (β) AS A FUNCTION OF SECOND SET MEAN JOINT PLANE SPACING ($\overline{SP2}$)

Plots of β as a function of $\overline{K2}$ (and $\overline{SP2}$) for cases 30 and 31 are given in Figure 5.72 and show identical trends; an increase in β (and thus reliability) as $\overline{K2}$ and $\overline{SP2}$ are increased. However, as $\overline{SP2}$ and $\overline{K2}$ are reduced below values of 6 feet and 30 percent respectively. Note, however, that for different jointing intensities and different combinations of input parameters, the limit values where β becomes indpendent of $\overline{K2}$ (and $\overline{SP2}$) may be different.

5.5.12 Effect of Slope Face Angle (θ)

The slope face angle (θ) is defined as the angle between the slope face and the horizontal. The effect of varying θ on joint plane reliability is examined for three cases, 33 through 35. Within each case, θ was varied between 50 and 90 degrees while holding all other input parameters constant. Common inputs to all 3 cases are listed in Figure 5.75.

Case #33: Examined the influence of θ in weak rock (C_r = 25 ksf) and long mean joint lengths ($\overline{JL}$ = 40')

Case #34: Examined the influence of θ in stronger rock C_r = 100 ksf) and long mean joint lengths ($\overline{JL}$ = 40')

Case #35: Examined the influence of θ in weaker rock (C_r = 25 ksf) and moderate mean joint lengths ($\overline{JL}$ = 20')

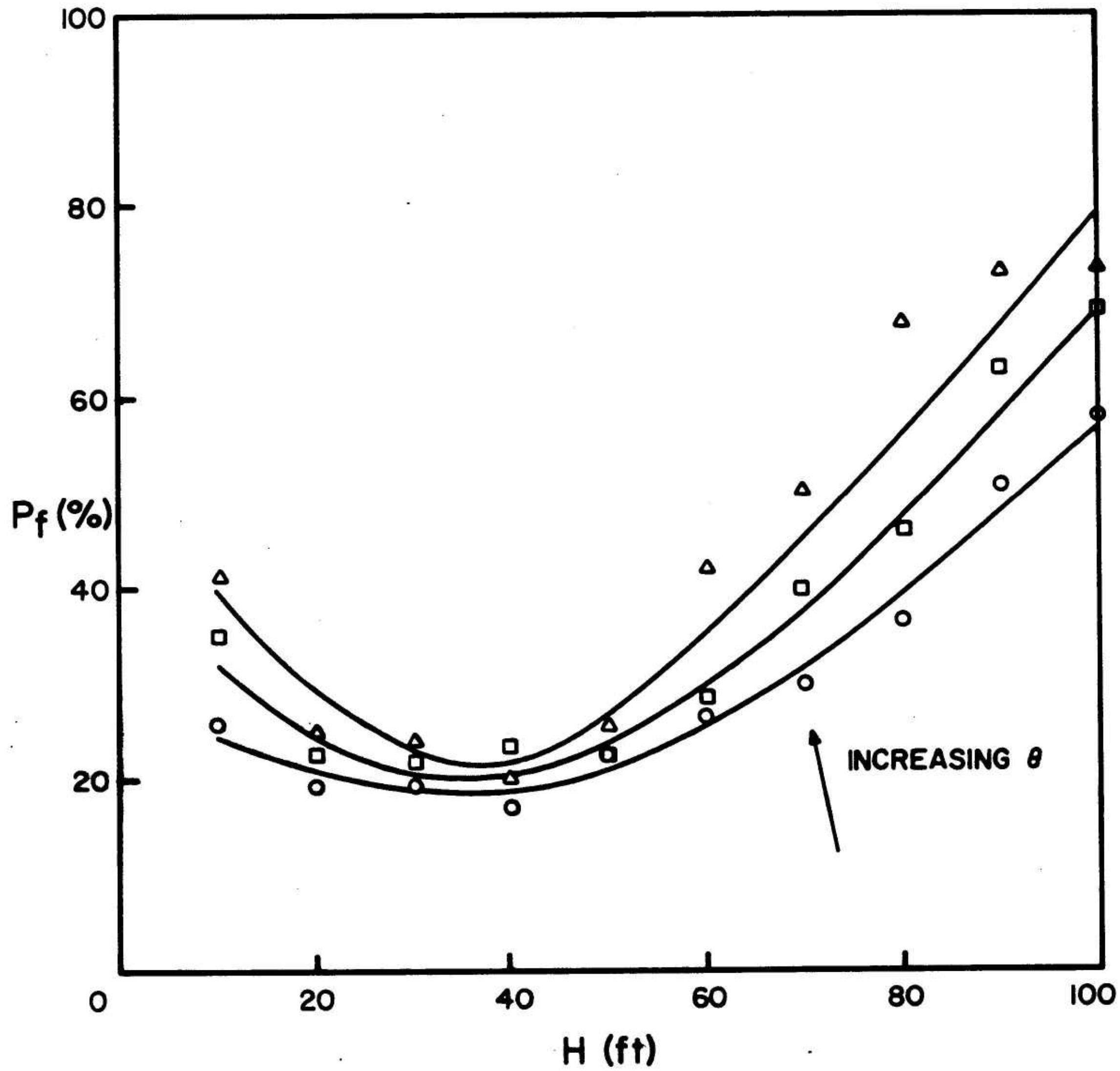


FIGURE 5.74 $P_f(h)$ AS A FUNCTION OF SLOPE ANGLE (θ)

Effect of Slope Face Angle (θ) on the Probability of Failure P_f (h)

The effect of varying the slope face angle (θ) on the probability of failure P_f (h) is shown in Figure 5.74. Increasing θ has the effect of increasing the probability of failure at any given depth (h) in the slope. The effect of θ becomes increasingly more pronounced with depth.

The Probability of Failure (P_f) as a Function of Slope Face Angle (θ)

The values of the probability of failure (P_f) at the depth interval from 90 to 100 feet are plotted in Figure 5.75 as a function of slope face angle (θ). P_f increases very gradually with increasing θ for all cases. The increase is most noticeable in weak rock (C_r = 25 ksf) and long joint segments ($\overline{JL}$ = 40'), i.e., in case #33.

As θ approaches joint plane inclination in a slope with a single joint set, P_f approaches the probability of a joint plane being 100 percent persistent. This is due to the fact that rock overlying the critical path approaches zero weight.

Due to geometric reasons, θ does not equal to first set inclination in this research anywhere at anytime.

For high intact rock cohesion (C_r = 100 ksf) and moderate mean joint lengths ($\overline{JL}$ = 20'), program output shows that a variation of P_f with θ becomes very small. This implies that for fixed C_r and $\overline{JL}$, the probability of failure may be assumed constant in the range of high values of intact rock cohesion. This indicates that at high cohesion values (>100 ksf), the probability of failure is no longer a function of

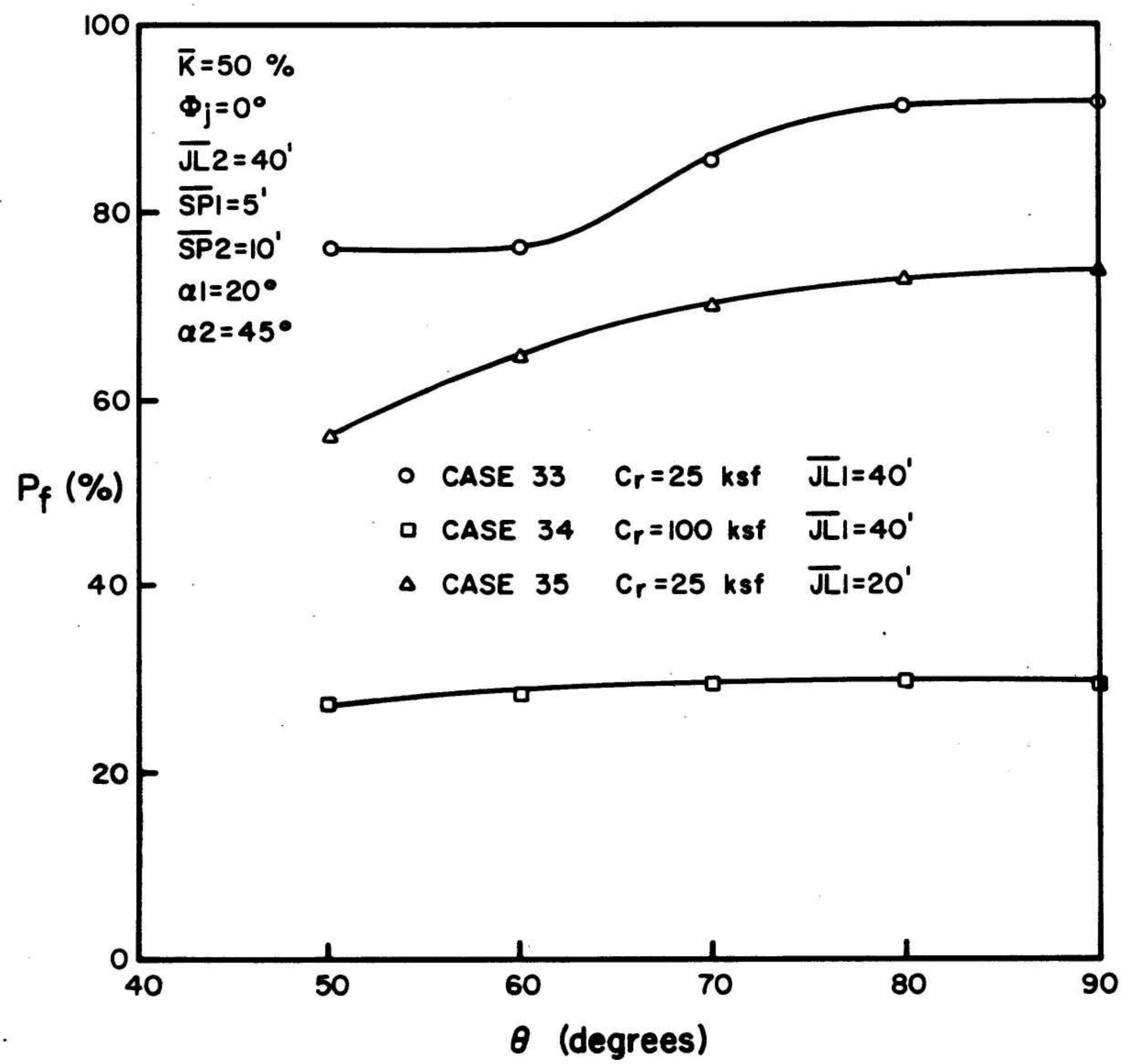


FIGURE 5.75 P_f AS A FUNCTION OF SLOPE ANGLE (θ)

slope face angle.

Effect on Apparent Persistence ($\overline{K}_a$) and the Index of Reliability (β)

Figures 5.76 through 5.78 are plots of mean, mean plus one standard deviation apparent persistence and critical persistence for cases 33, 34, and 35 for height interval from 90 to 100 feet. For all the values of θ examined, $\overline{K}_a$ and $\tilde{K}_a$ are essentially independent of θ. Insensitivity of K_a to θ variations is of particular interest because in most design situations, θ and slope height are the only slope parameters which are controlled by the designer. Once $\overline{K}_a$ and $\tilde{K}_a$ are determined from a single model run, it is possible to investigate joint plane reliability as a function of θ for a wide range of values of (all other parameters are assumed constant).

Figure 5.79 is a plot of the index of reliability (β) for cases 33 through 35. Recall that β is defined as follows:

$$\beta = \frac{K_c - \overline{K}_a}{\tilde{K}_a}$$

The insensitivity of K_a to variations in θ is useful for the same reasons as insensitivity of K_a to C_r and Φ_j is useful; it enables estimations of β and P values from a single model run for a wide range of θ values without additional lengthy simulations. Case 34 is of a slope with high reliability as one might expect when comparing it with the others.

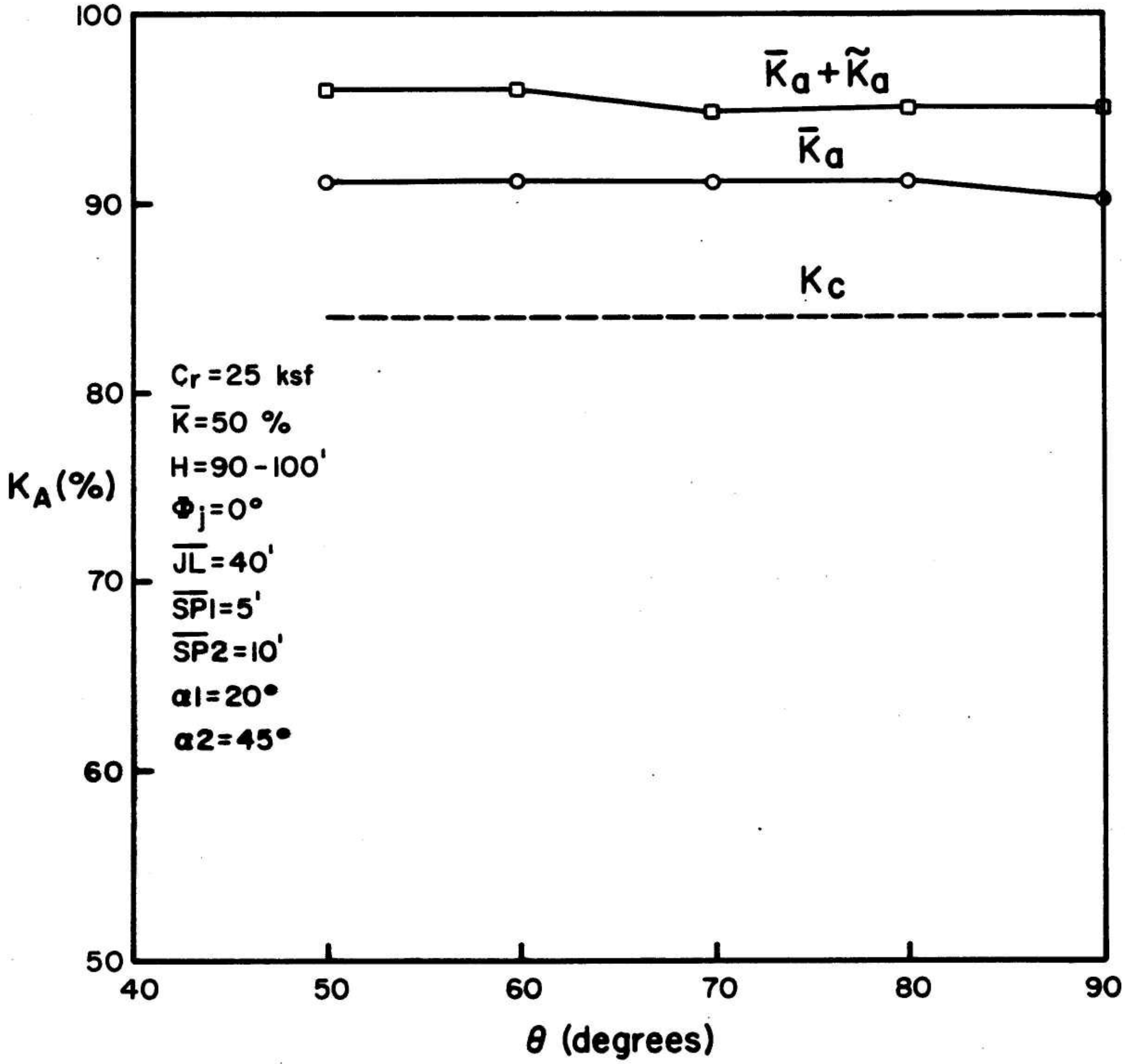


FIGURE 5.76 EFFECT OF SLOPE ANGLE ON APPARENT PERSISTENCE (K_a) - CASE 33

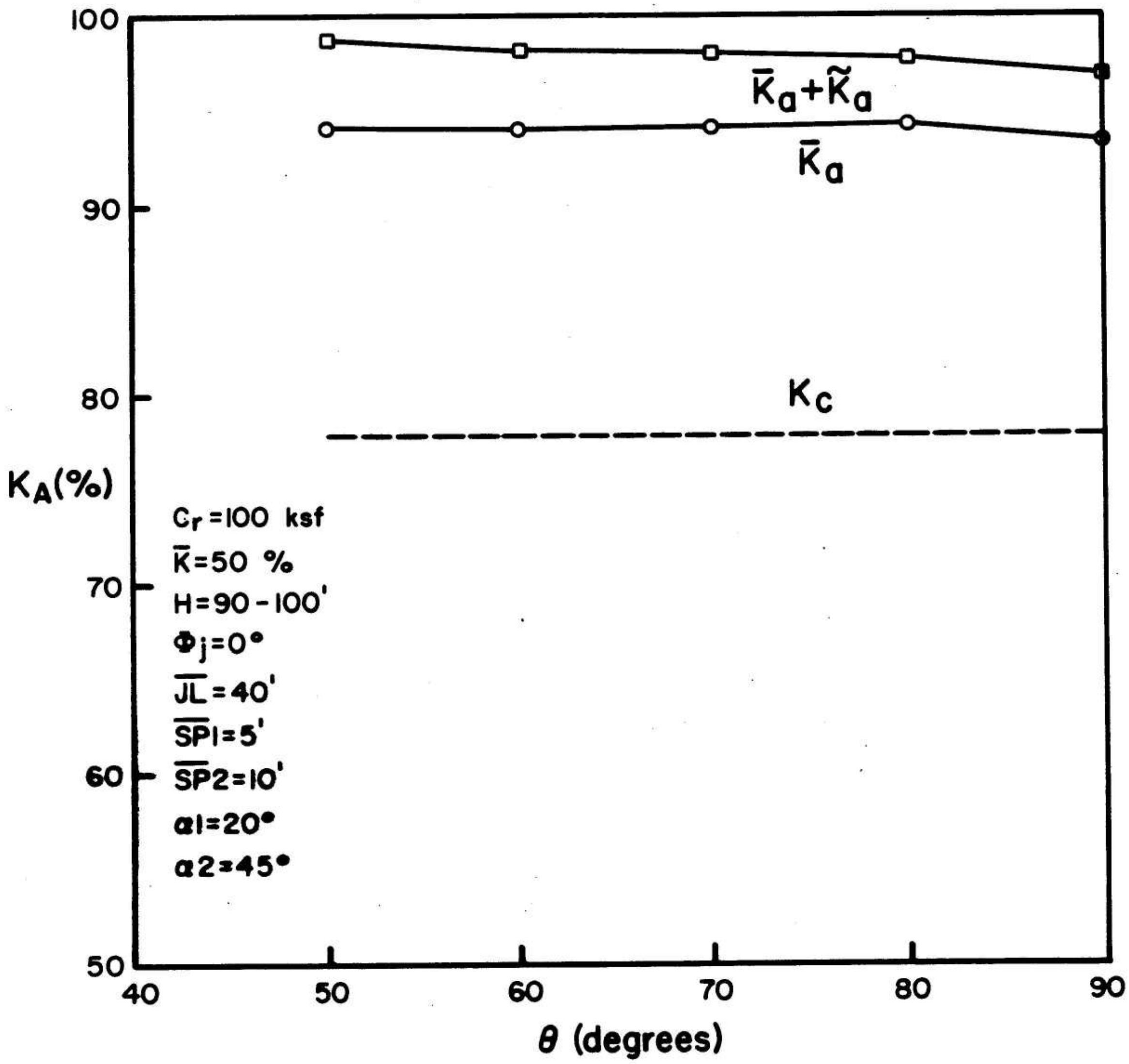


FIGURE 5.77 EFFECT OF SLOPE ANGLE (θ) ON APPARENT PERSISTENCE (K_a) - CASE 34

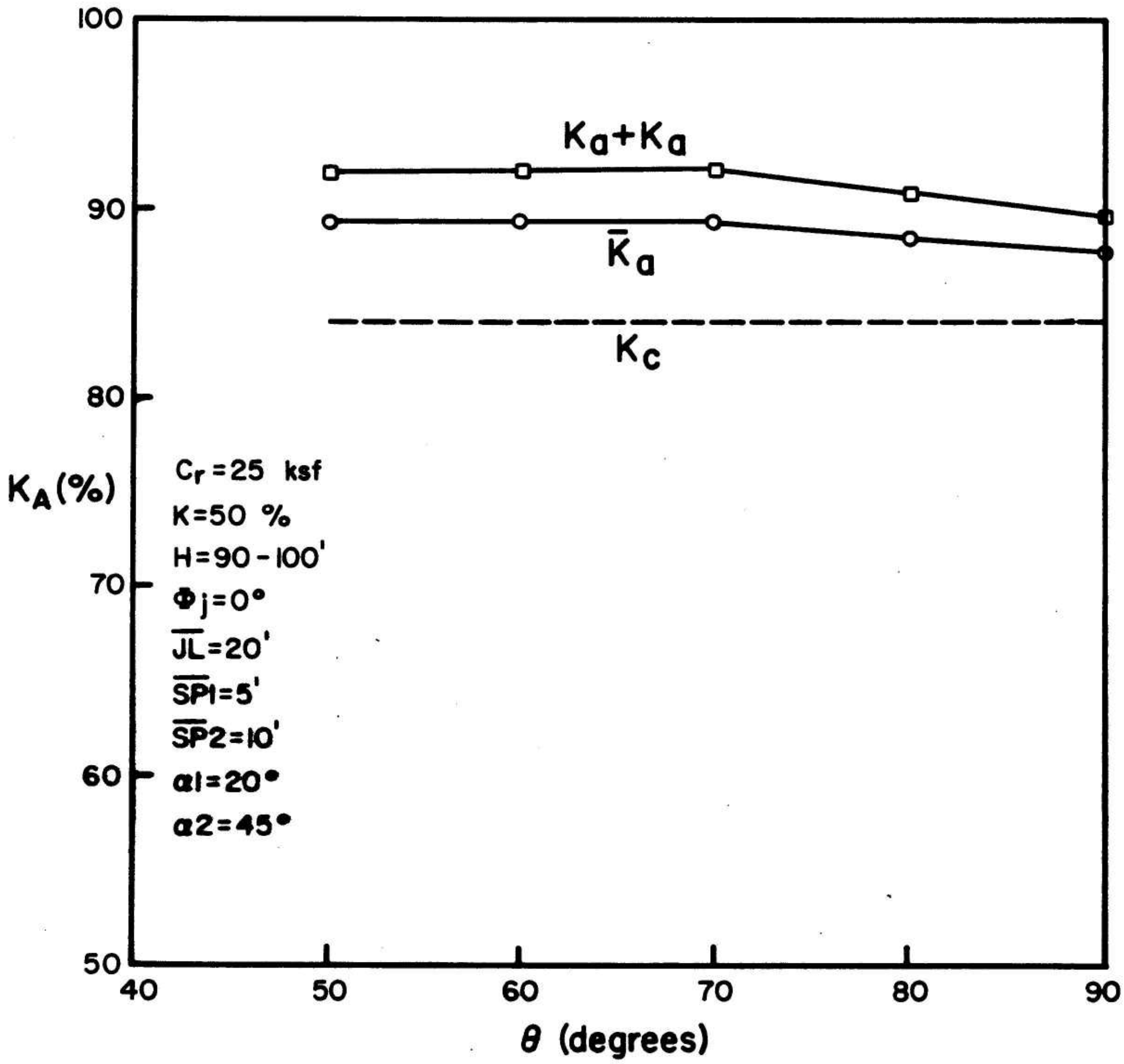


FIGURE 5.78 EFFECT OF SLOPE ANGLE (θ) ON APPARENT PERSISTENCE (K_a) - CASE 35

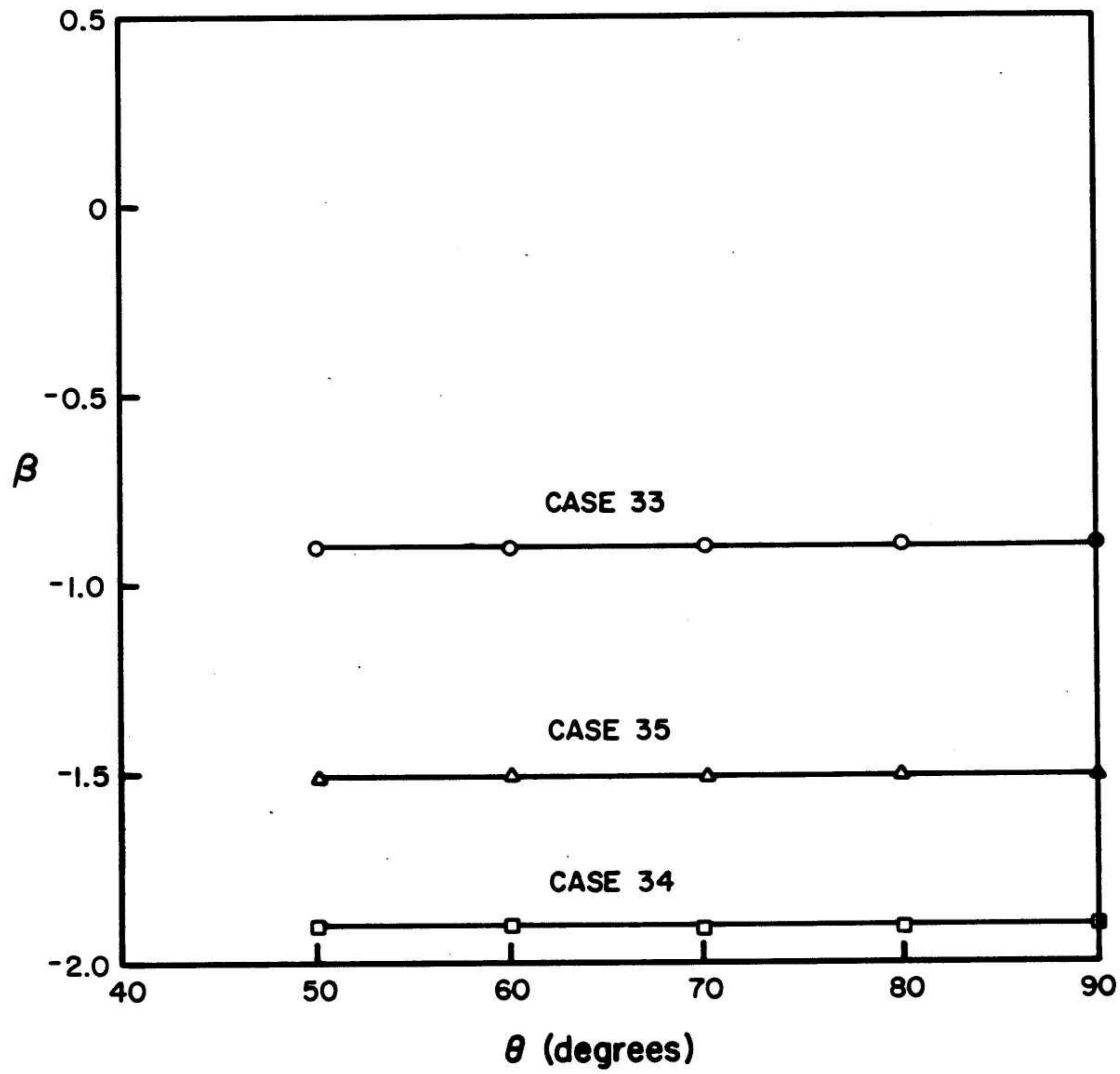


FIGURE 5.79 INDEX OF RELIABILITY (β) AS A FUNCTION OF SLOPE ANGLE (θ)

5.5.13 Effect of First Set Joint Plane Inclination ($\alpha 1$)

The influence of first set joint plane inclination ($\alpha 1$), measured between the first set joint planes and the horizontal, is examined in two cases (36 & 37). In each case, $\alpha 1$ is varied from 20 to 70 degrees while holding other parameters constant. Common parameters of both cases are given in Figures 5.82 and 5.83.

Case #36: Examined the influence of $\alpha 1$ in slopes with weak rock (C_r = 25 ksf)

Case # 37: Examined the influence of $\alpha 1$ in slopes with moderately strong rock (C_r = 100 ksf)

Effect of First Set Joint Plane Inclination ($\alpha 1$) on the Probability of Failure Pf (h)

Figure 5.80 is a plot of the probability of failure as a function of height. In the same figures, the mean apparent persistence as a function of depth, (h) is also plotted. Computer output reveals that at any hgieht, and for all $\alpha 1$, P_f is maximum for the depth interval 90 to 100 feet.

Mean apparent persistence ($\overline{K}_a$) as a function of depth is practically constant for depths greater than approximately 20 feet and for any value of $\alpha 1$. This indicates that $\overline{K}_a$ is not a function of $\alpha 1$ nor of depth (See Fig. 5.81).

The Probability of Failure (Pf) as a Function of First Set Joint Place Inclination ($\alpha 1$)

Plots of the probability of failure as a function of first set

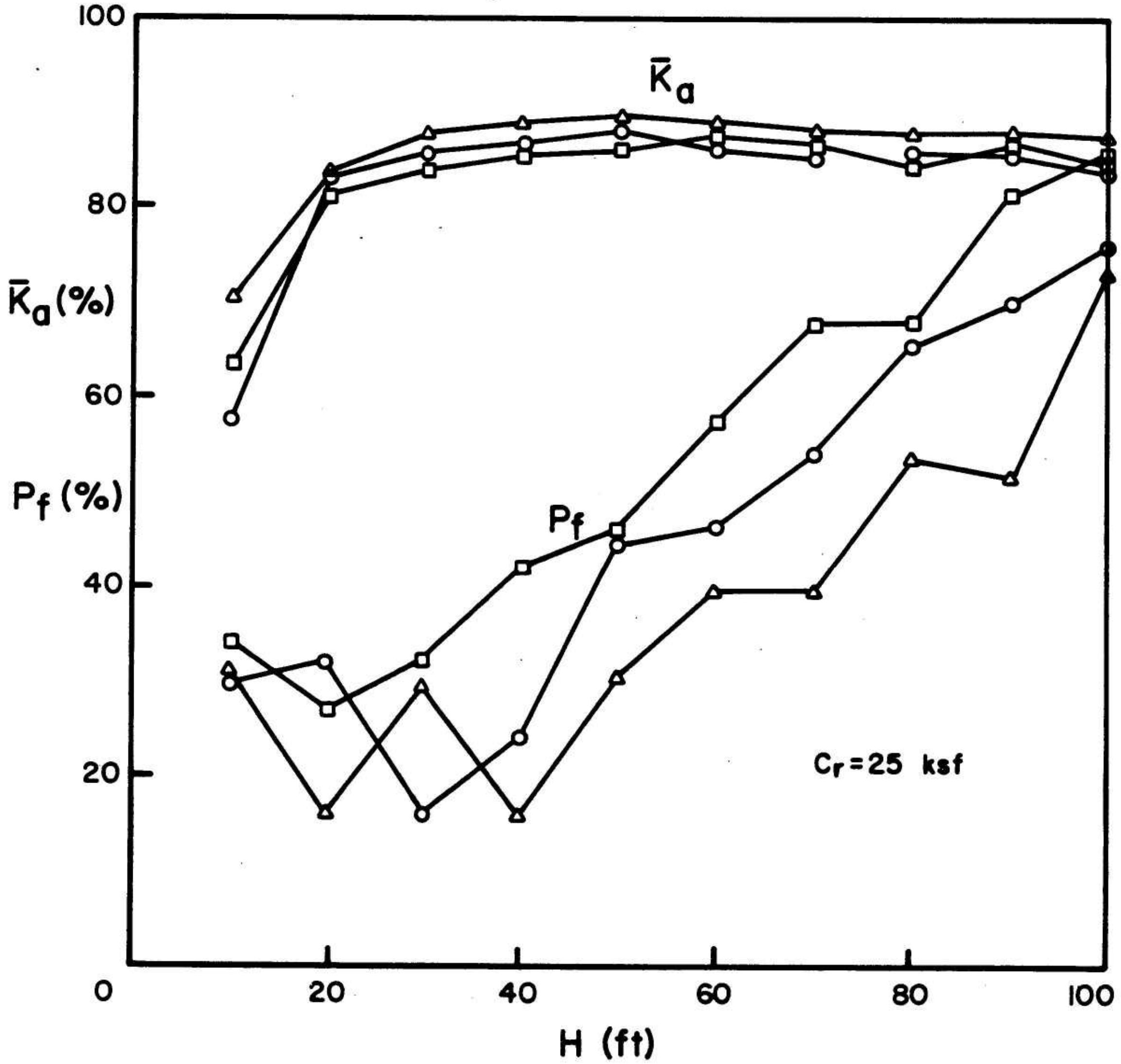


FIGURE 5.80 EFFECT OF JOINT SET 2 INCLINATION ON $P_f(h)$

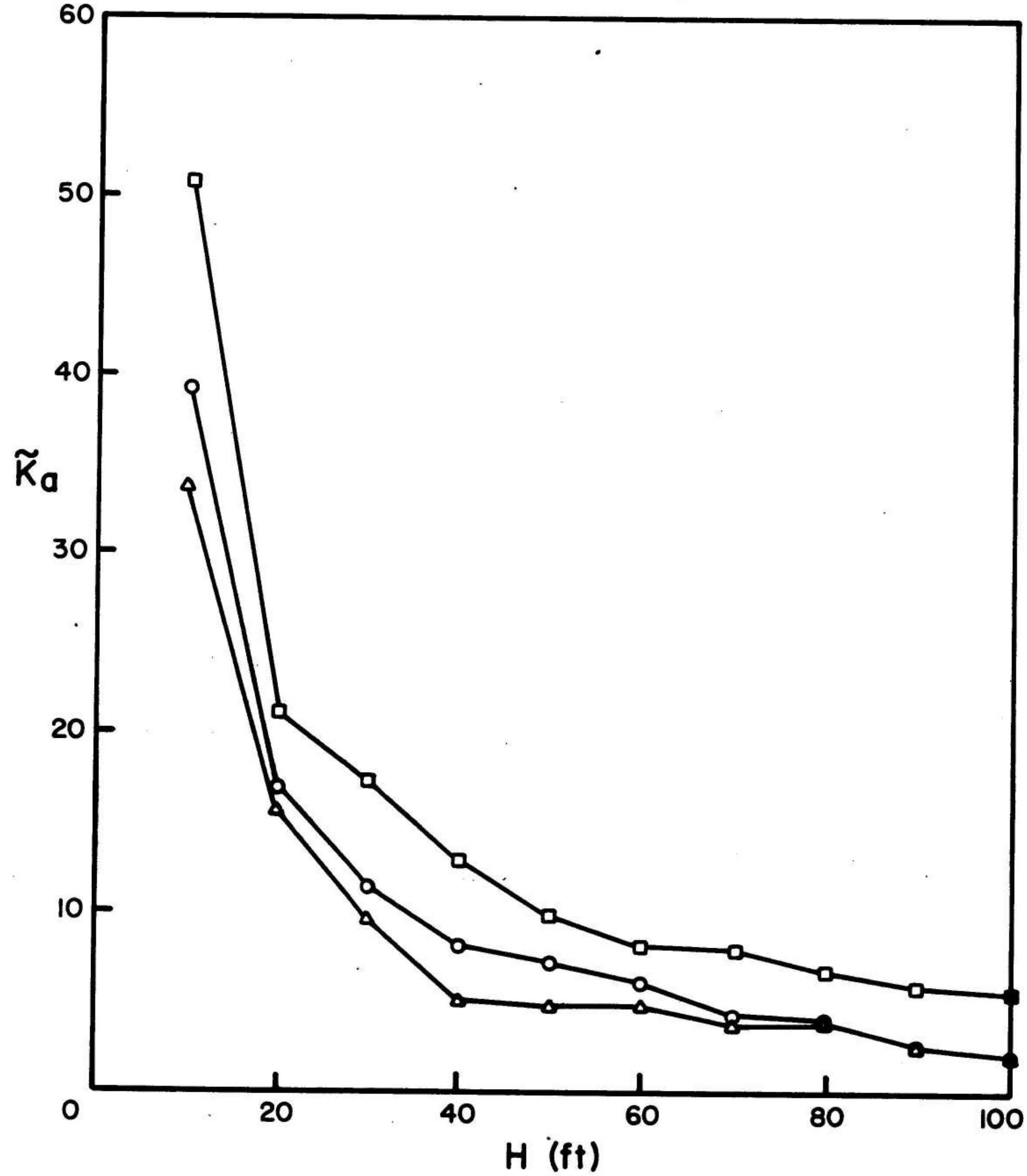


FIGURE 5.81 VARIATION OF STANDARD DEVIATION OF APPARENT PERSISTENCE ($\tilde{K}_a$) WITH DEPTH

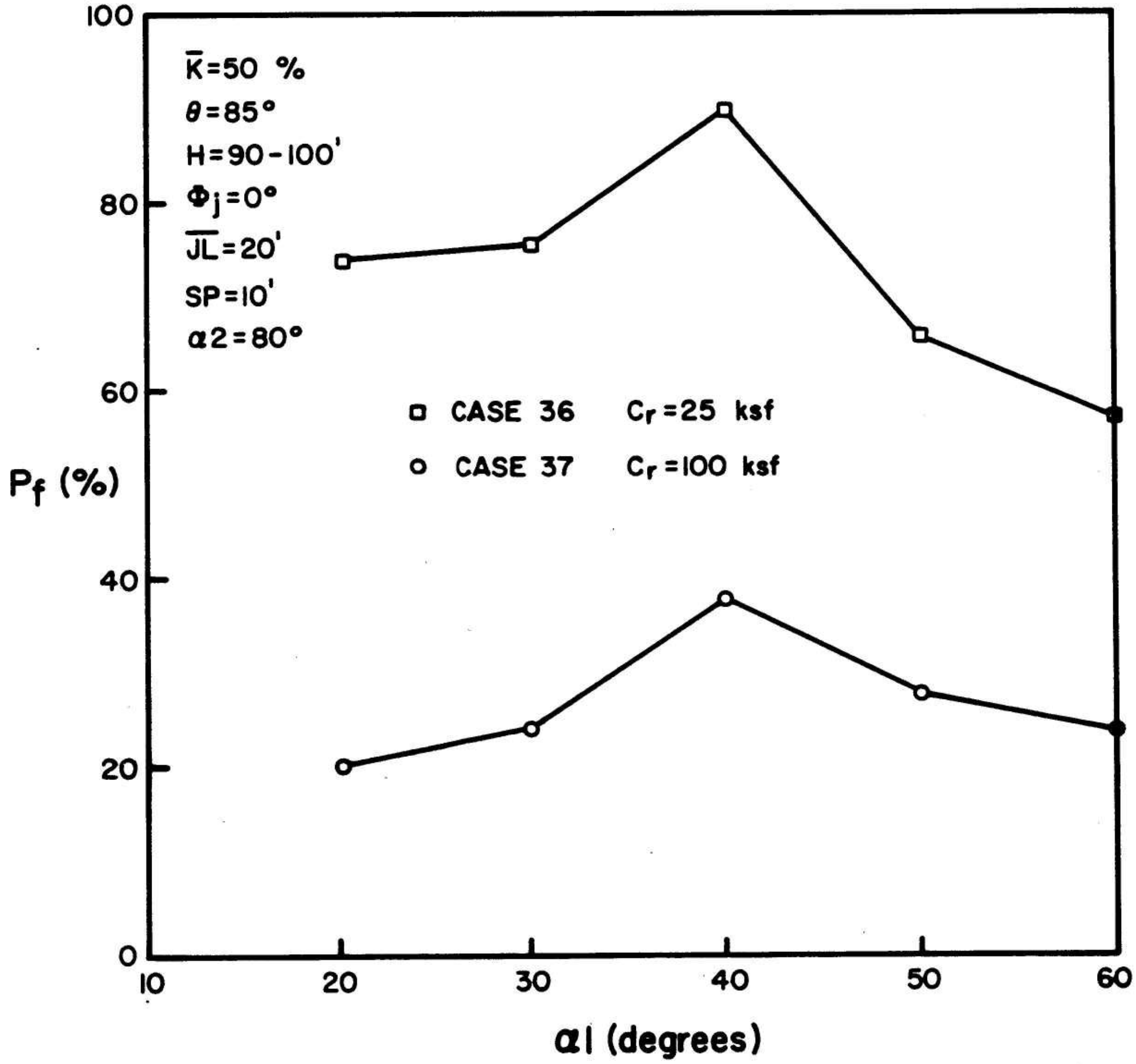


FIGURE 5.82 P_f AS A FUNCTION OF JOINT SET 1 INCLINATION (α1)

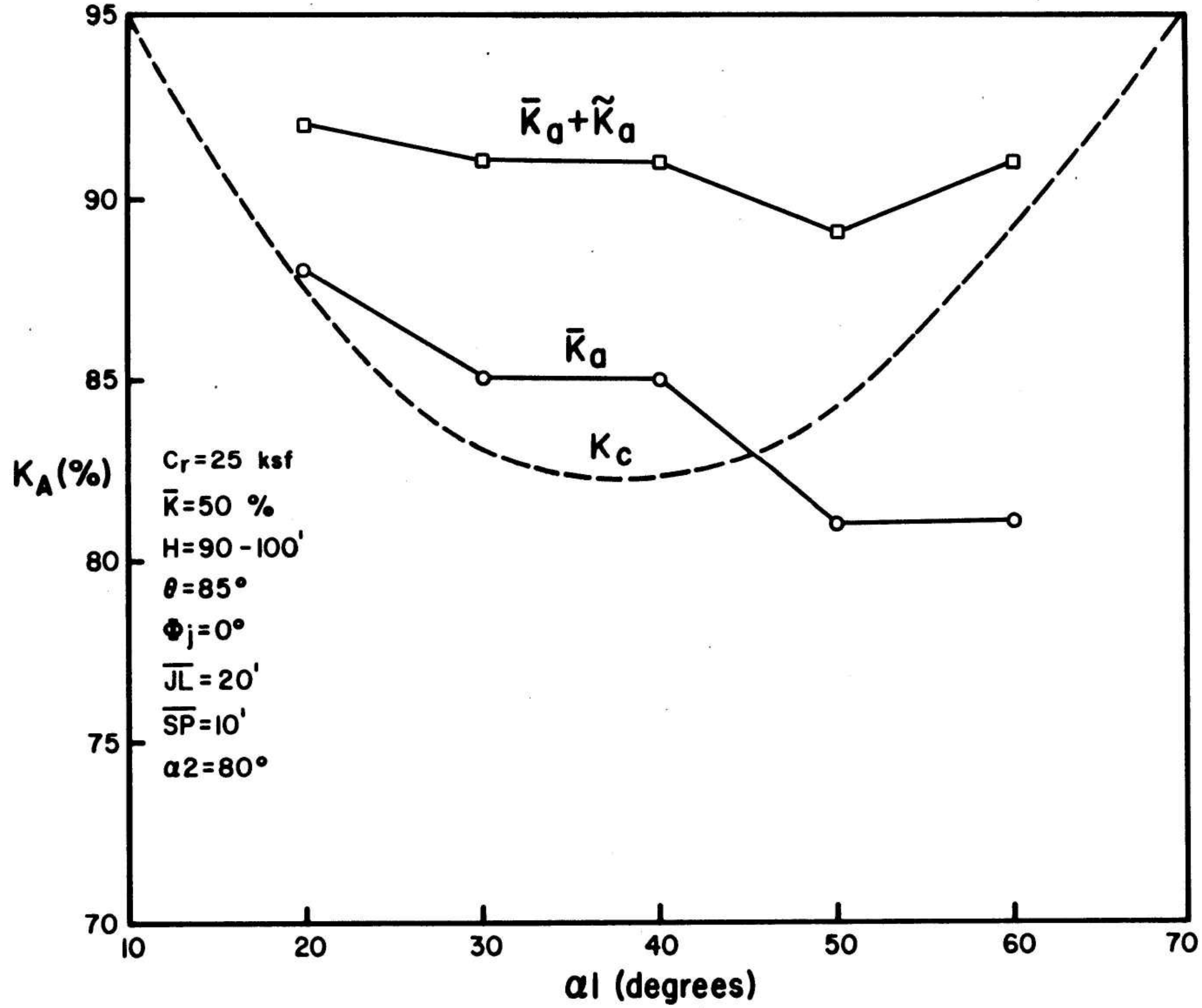


FIGURE 5.83 EFFECT OF JOINT SET 1 INCLINATION (α1) ON APPARENT PERSISTENCE (K_a) - CASE 36

inclination ($\alpha 1$) are given in Figure 5.82. Program output reveals that P_f ($\alpha 1$) is a maximum when $\alpha 1$ equals 40 to 50 degrees. Changing intact rock cohesion C_r, does not appear to influence the value of $\alpha 1$ at which P_f is maximum. This is in agreement of findings by O'Reilly-1980. supporting this fact.

By referring to Fig. 5.83 & 5.84 one may conclude the integrity of the model developed in this thesis. It is obvious from both said figures that K_a is a minimum at the angle $\alpha 1$ that coincides with the value used for the intact rock friction angle.

As expected, the weaker rock has a higher probability of failure at all values of $\alpha 1$ examined. The position of the relative maximum remains unchanged at $\alpha 1$, approximately equal to 40 degrees. While the influence of C_r is large over the entire range of $\alpha 1$, the influence of joint length becomes increasingly less significant with increasing $\alpha 1$ (discussed previously when examining $\overline{JL}$).

Effect of First Set Joint Plane Inclination ($\alpha 1$) on Apparent Persistence (K_a) and on the Index of Reliability (β)

Mean apparent persistence ($\overline{K}_a$), mean plus one standard deviation ($\overline{K}_a$ + $\tilde{K}_a$) and critical persistence (K_c) are plotted as functions of $\alpha 1$ for cases 36 and 37. Plots of both cases show similar trends with increasing $\alpha 1$; decreasing $\overline{K}_a$ and increasing $\tilde{K}_a$ where $\overline{K}_a$ + $\tilde{K}_a$ remains practically constant. When θ and $\alpha 1$ approach each other (regardless of which is held constant), transitions become less common (due to the restriction that the critical path cannot intersect the slope face). Since transitions are the mechanisms which increase $\overline{K}_a$ and reduce variability in K_a (i.e., K), then it is not surprising that by increasing $\alpha 1$ toward , $\overline{K}_a$ is reduced and $\tilde{K}_a$ is increased.

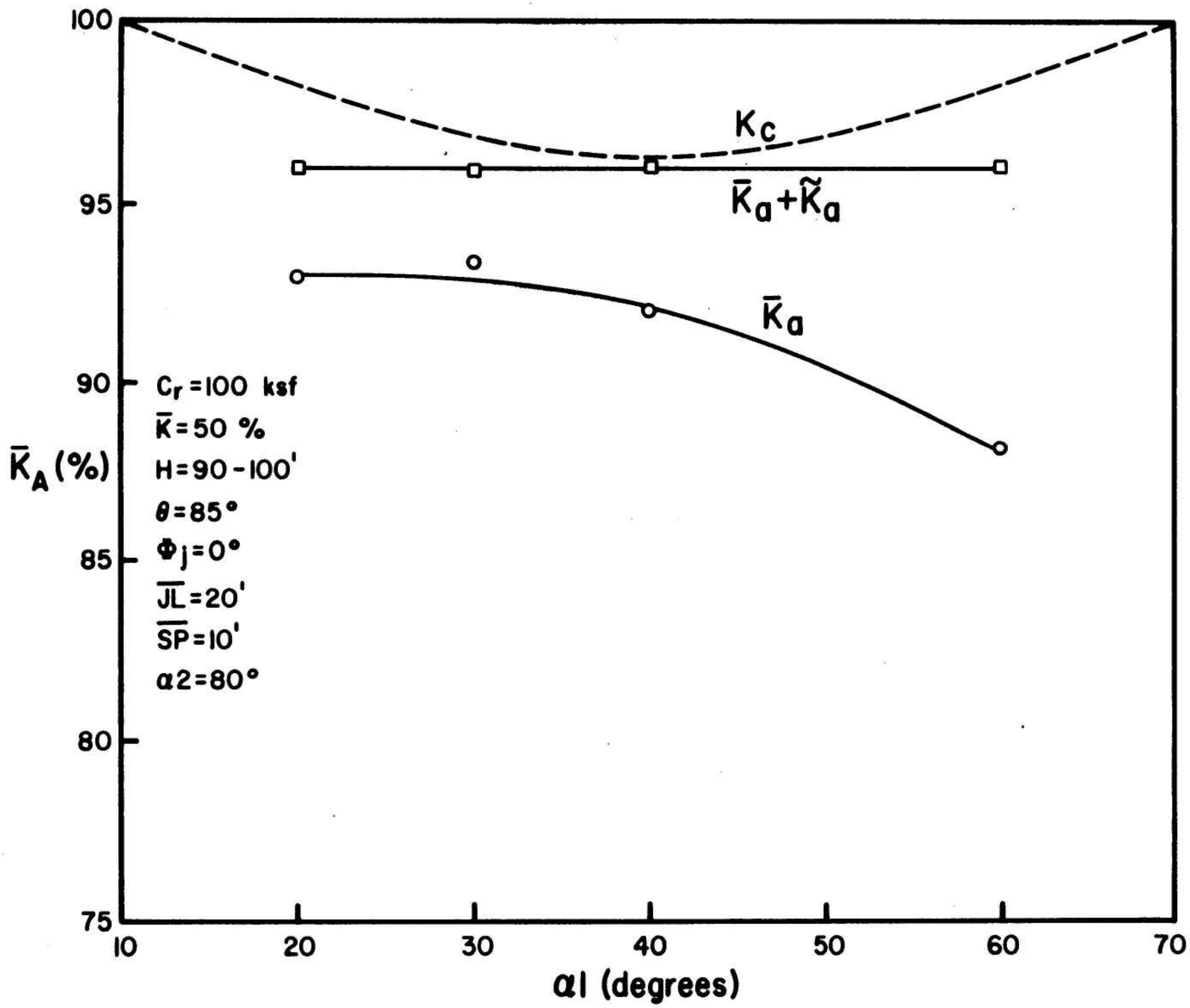


FIGURE 5.84 EFFECT OF JOINT SET 1 INCLINATION (α1) ON APPARENT PERSISTENCE (K_a) - CASE 37

How $\overline{K}_a$, $\tilde{K}_a$ and K_c interact as functions of $\alpha 1$ to influence joint plane reliability can be seen from plots of β (derived from Figures 5.83 and 5.84) as a function of $\alpha 1$. Recall that:

$$\beta = \frac{K_c - \overline{K}_a}{\tilde{K}_a}$$

In Figure 5.85, β values for the height interval 90 to 100 feet are plotted as a function of $\alpha 1$ for both cases 36 and 37. Case #37 is more reliable as one may expect for the range of $\alpha 1$ greater than 40-50 degrees.

5.5.14 Effect of Second Set Joint Plane Inclination ($\alpha 2$)

Second set joint inclination ($\alpha 2$) is the angle between joint planes of the second set and the horizontal. The influence of $\alpha 2$ on joint plane reliability was examined in three cases, 38, 39, and 40. In each case, $\alpha 2$ was varied between 30 and 80 degrees while holding all other input parameters constant. Common input parameters for all three cases are given in Figures 5.88 through 5.92.

Case #38: Examined the influence of $\alpha 2$ in slopes in weak rock (C_r = 25 ksf) and long joints ($\overline{JL}$ = 30').

Case #39: Examined the influence of $\alpha 2$ slopes in moderately strong rock (C_r = 100 ksf) and long joints ($\overline{JL}$ = 30').

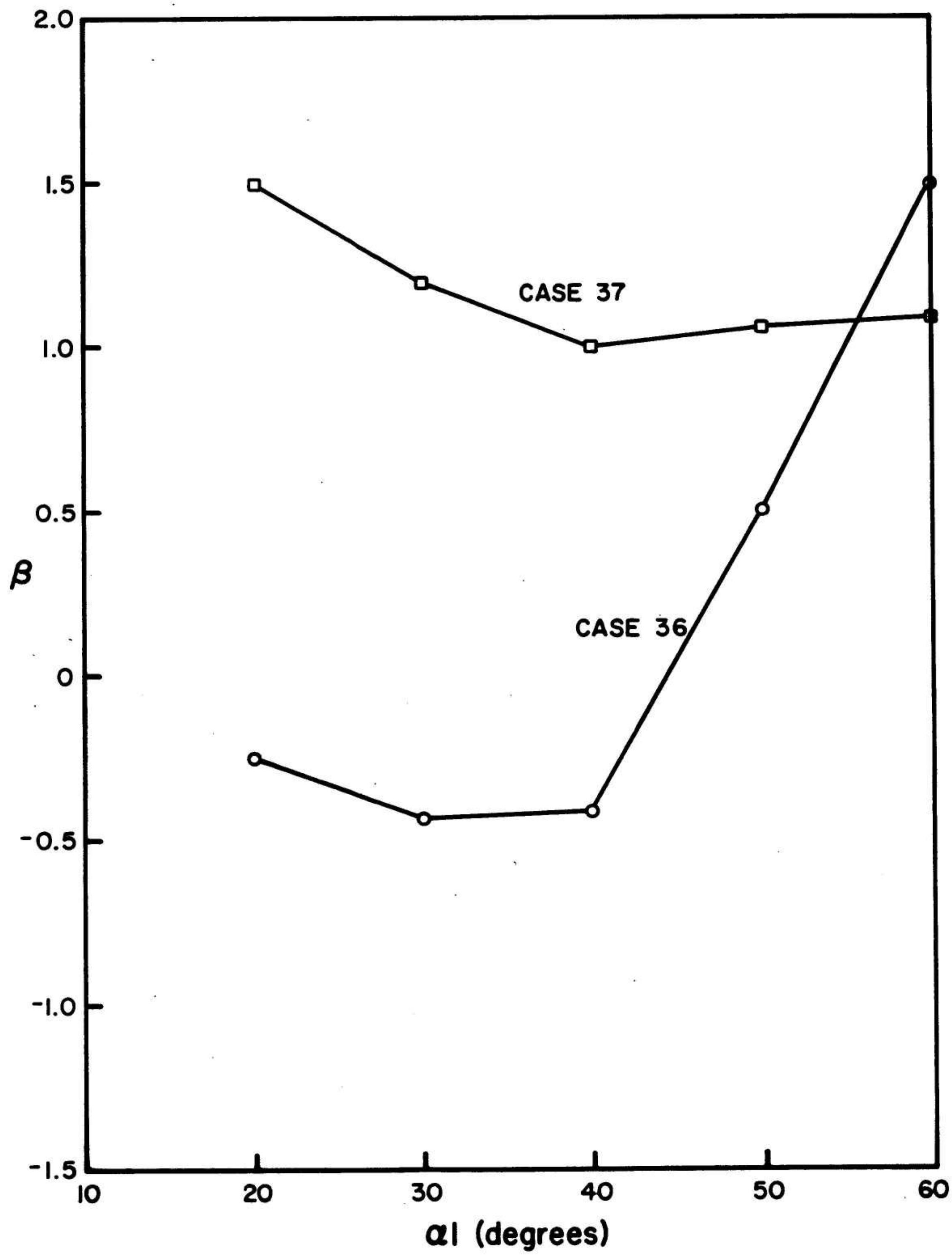


FIGURE 5.85 INDEX OF RELIABILITY (β) AS A FUNCTION OF FIRST SET JOINT PLANE INCLINATION (αl)

Case #40: Examined the influence of $\alpha 2$ in weak rock (C_r = 25 ksf) and short joints ($\overline{JL}$ = 10').

Effect of Second Set Joint Inclination ($\alpha 2$) on the Probability of Failure P_f (h)

The effect of varying $\alpha 2$ on P_f (h) is shown in Figure 5.86. For any value of $\alpha 2$, the probability of failure in any depth interval does not vary significantly. However, the probability of failure increases with depth for any $\alpha 2$. For the range of $\alpha 2$ values examined, the probability of failure is not a function of $\alpha 2$. This fact is more pronounced in the deeper intervals where data points almost overlap.

Mean apparent persistence as a function of depth is shown in Figure 5.86. At depths in excess of 20 feet, $\overline{K}_a$ becomes constant at all depths and for all $\alpha 2$ values examined. This indicates that $\overline{K}_a$ is fully independent of variation in depth of $\alpha 2$ (within at least the range of $\alpha 2$ values examined - 40° - 80°).

Variation of standard deviation of apparent persistence ($\tilde{K}_a$) as a function of depth is shown in Figure 5.87. Beyond 40' depth, $\tilde{K}_a$ is constant for any depth and for any of the $\alpha 2$ values being examined.

The Probability of Failure (P_f) as a Function of Second Set Joint Plane Inclination ($\alpha 2$)

Plots of the probability of failure as a function of second set inclination ($\alpha 2$) for each case are given in Figures 5.88 and 5.89.

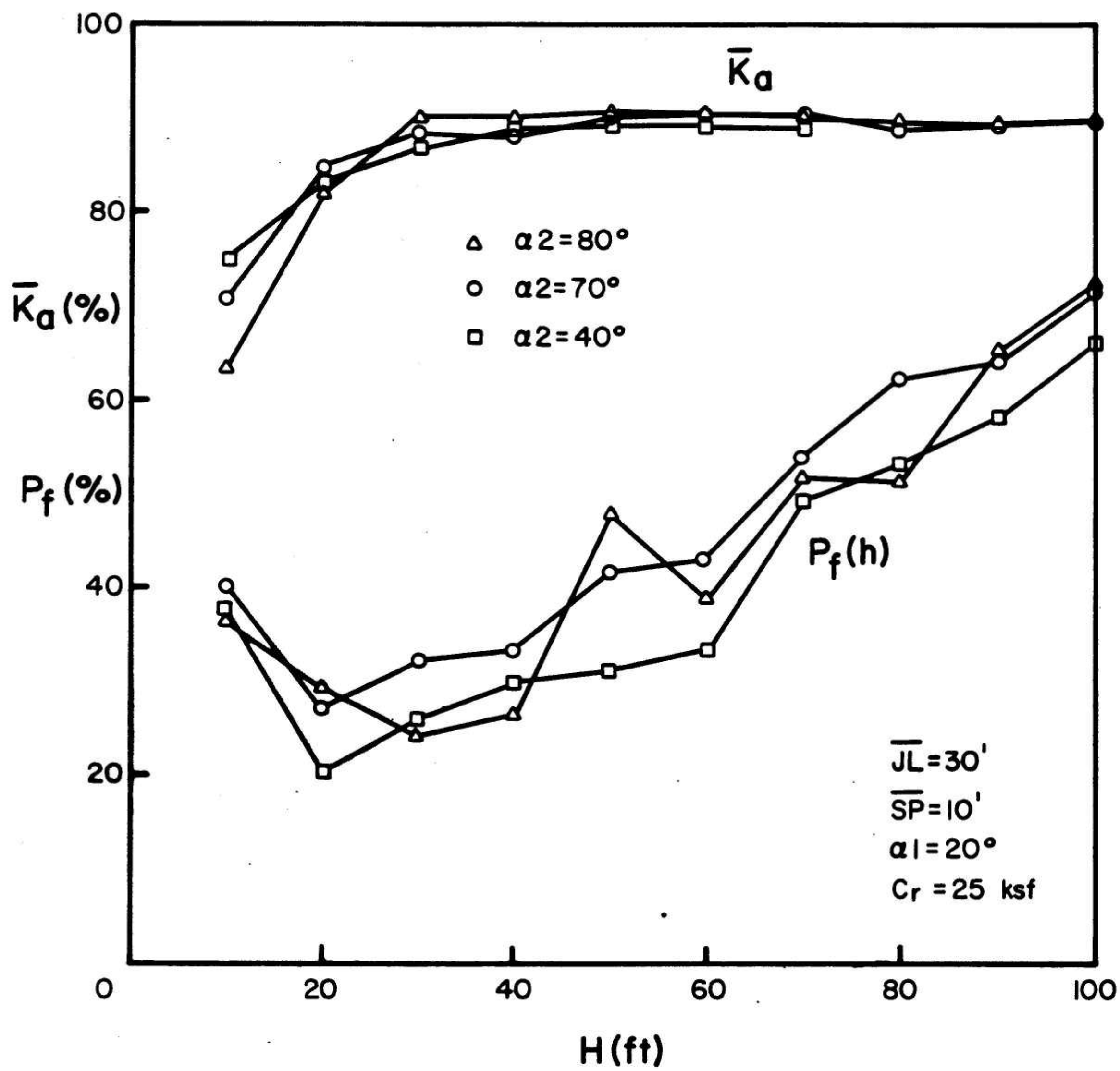


FIGURE 5.86 EFFECT OF JOINT SET 2 INCLINATION (α2) ON P_f(h)

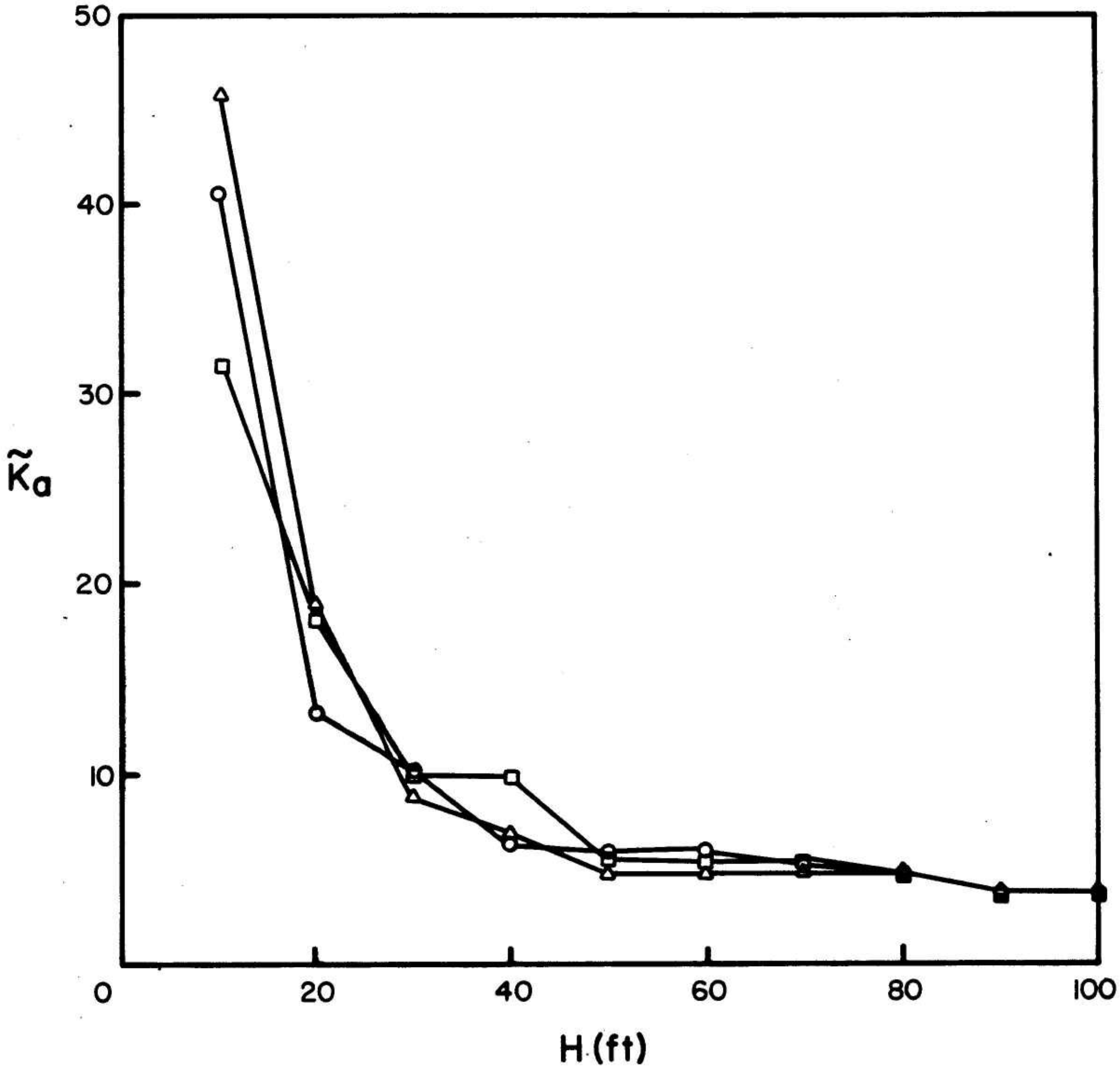


FIGURE 5.87 VARIATION OF STANDARD DEVIATION OF APPARENT PERSISTENCE AS A FUNCTION OF DEPTH

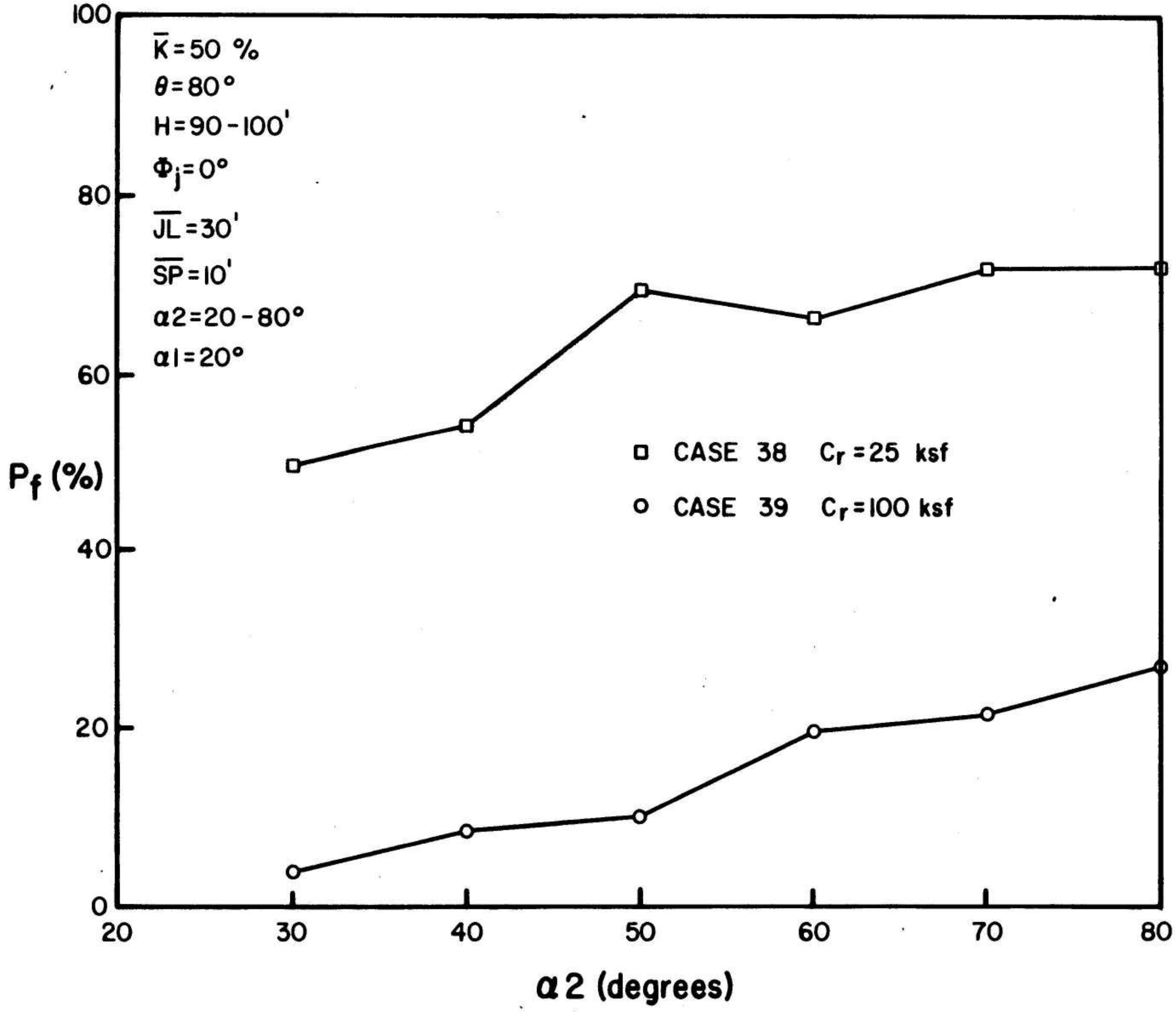


FIGURE 5.88 P_f AS A FUNCTION OF JOINT SET 2 INCLINATION (α2)

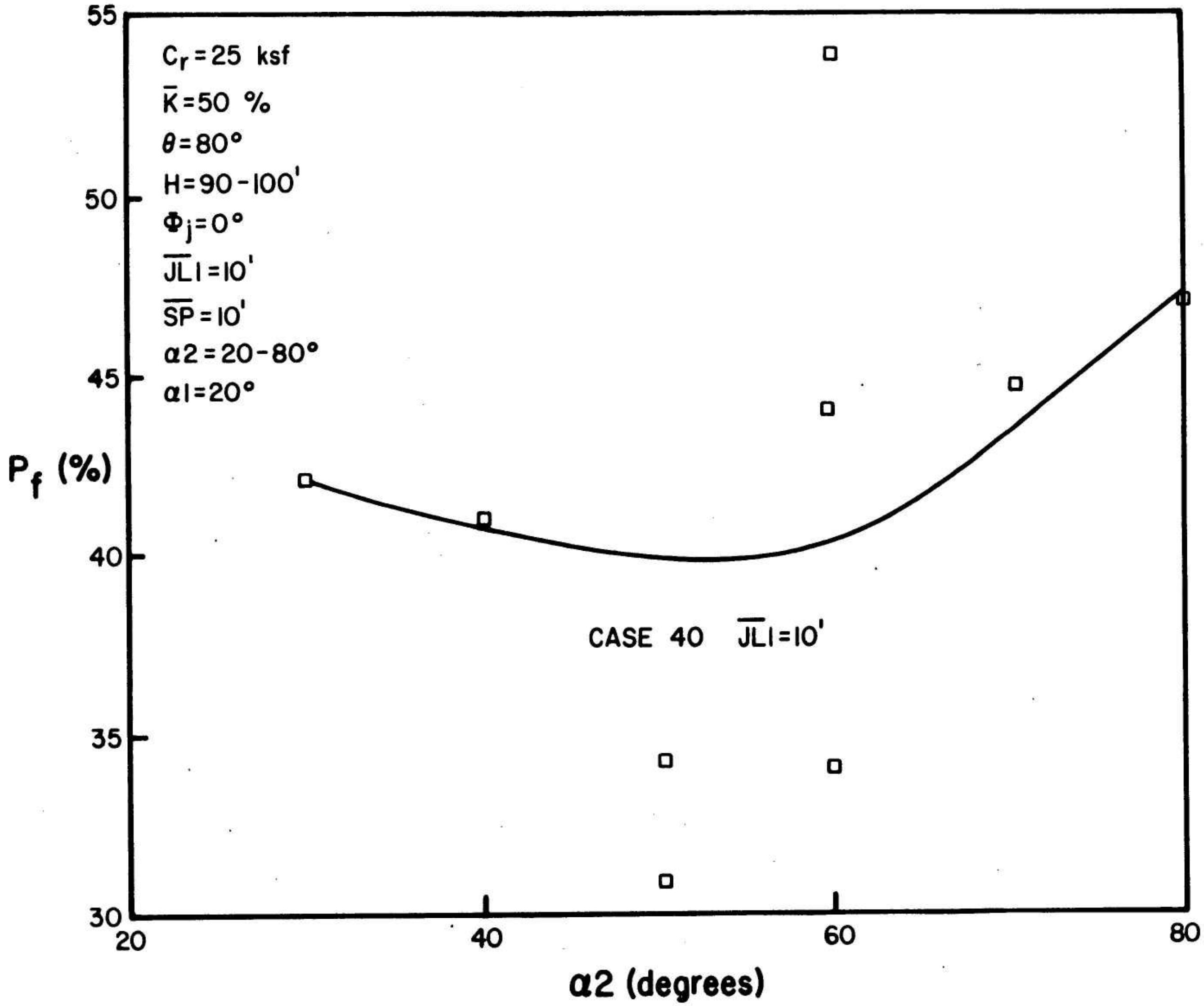


FIGURE 5.89 P_f AS A FUNCTION OF JOINT SET 2 INCLINATION ($\alpha 2$)

Program output shows that P_f tends to increase with increasing $\alpha 2$. The most unreliable case is clearly that for weak rock (C_r = 25 ksf) with long joints ($\overline{JL}$ = 40'). As expected, weakening intact rock strength and increasing joint lengths, both have strong negative influence on reliability. In each of these cases, one of these unfavorable input parameters is improved from a reliability point of view in order to examine the combined effect for a range of $\alpha 2$ values.

Figure 5.88 is a plot of cases 38 and 39. Both cases are identical except that C_r is increased to 100 ksf over the 25 ksf of case #38. Increasing C_r clearly has a large positive effect on reliability for all values of $\alpha 2$ which are less than 90°.

Figure 5.89 is a plot of case #40. Cases #40 and 38 are identical except that $\overline{JL}$ has been reduced from 40 to 10 feet. As expected from the discussion about the influence of JL on slope reliability, this decrease in $\overline{JL}$ results in an increase in joint plane reliability (decrease in P_f (h) for the values of $\alpha 2$ examined.

Effect of Second Set Joint Plane Inclination ($\alpha 2$) on Apparent Persistence (K_a) and the Index of Reliability

Figures 5.90 through 5.92 support the fact that case 39 is the safest of the three cases (38-40). This may be seen by comparing the distance between K_c and $\overline{K}_a$ in each of Figures 5.90 through 5.92 in which Figure 5.91 shows that the distance between K_c and $\overline{K}_a$ is the largest, thus the safest. Index of reliability values (β) for the three cases are shown in Figure 5.93.

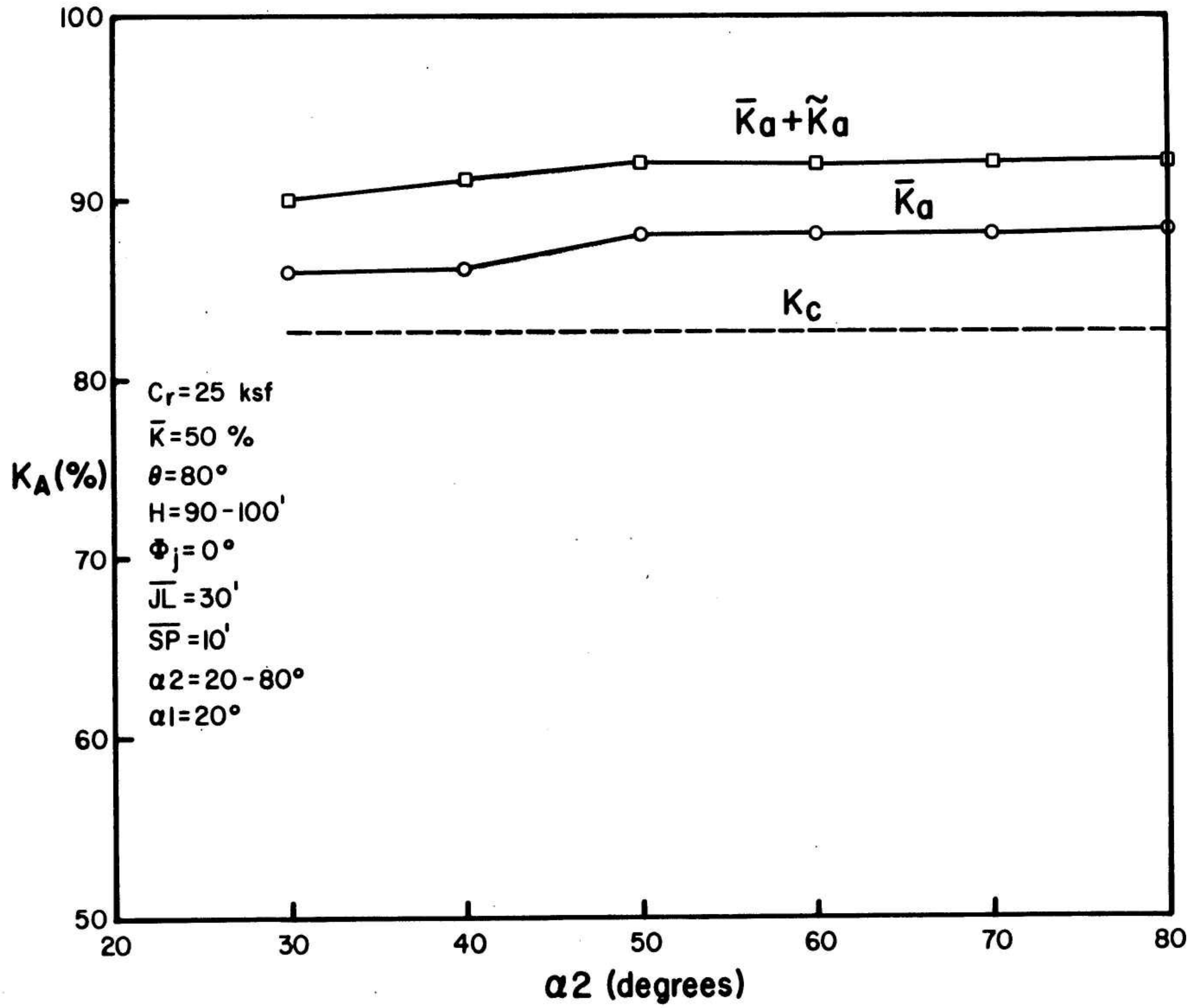


FIGURE 5.90 EFFECT OF JOINT SET 2 INCLINATION ($\alpha 2$) ON APPARENT PERSISTENCE (K_a)-CASE 38

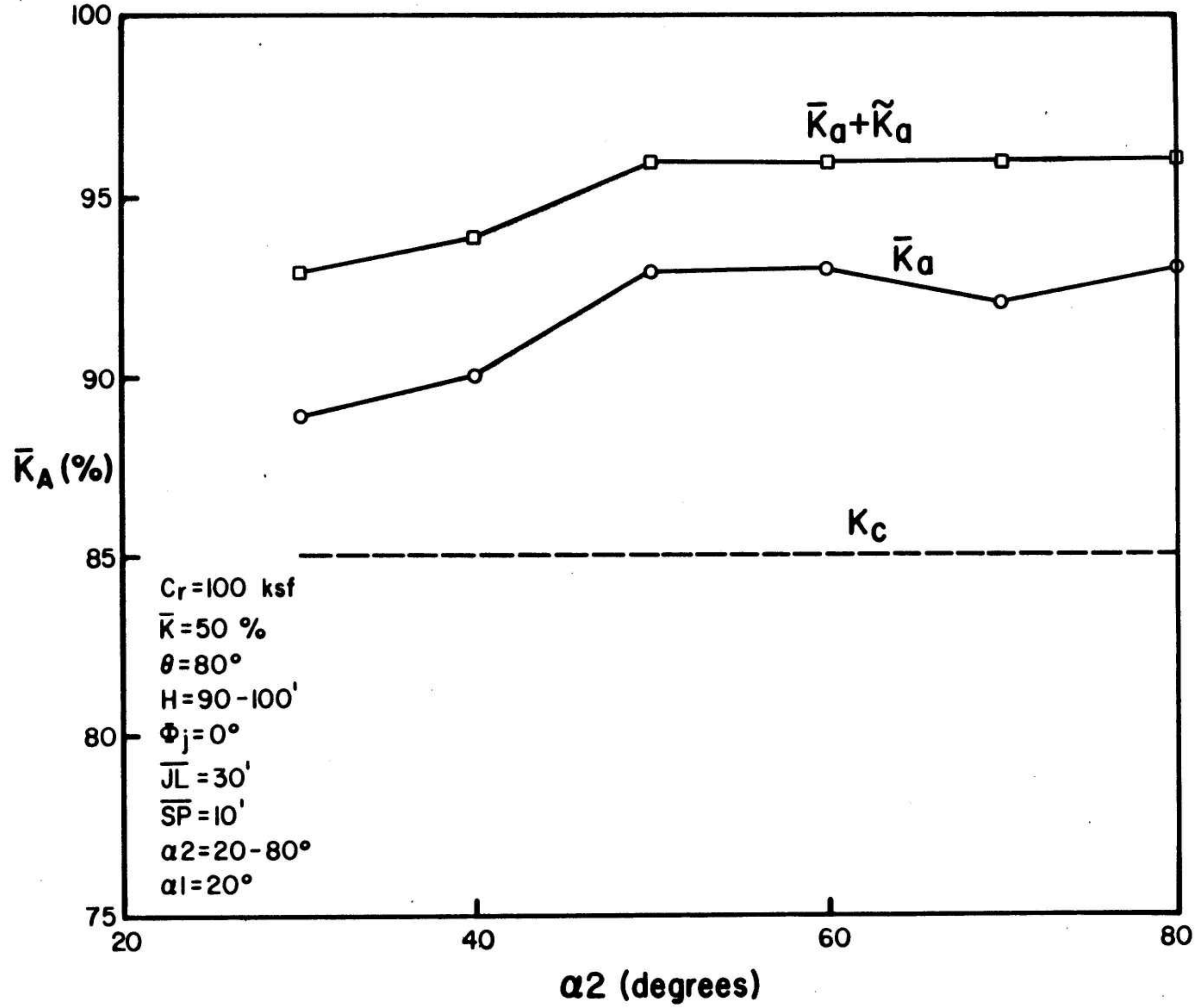


FIGURE 5.91 EFFECT OF JOINT SET 1 INCLINATION (α1) ON APPARENT PERSISTENCE (K_a) - CASE 39

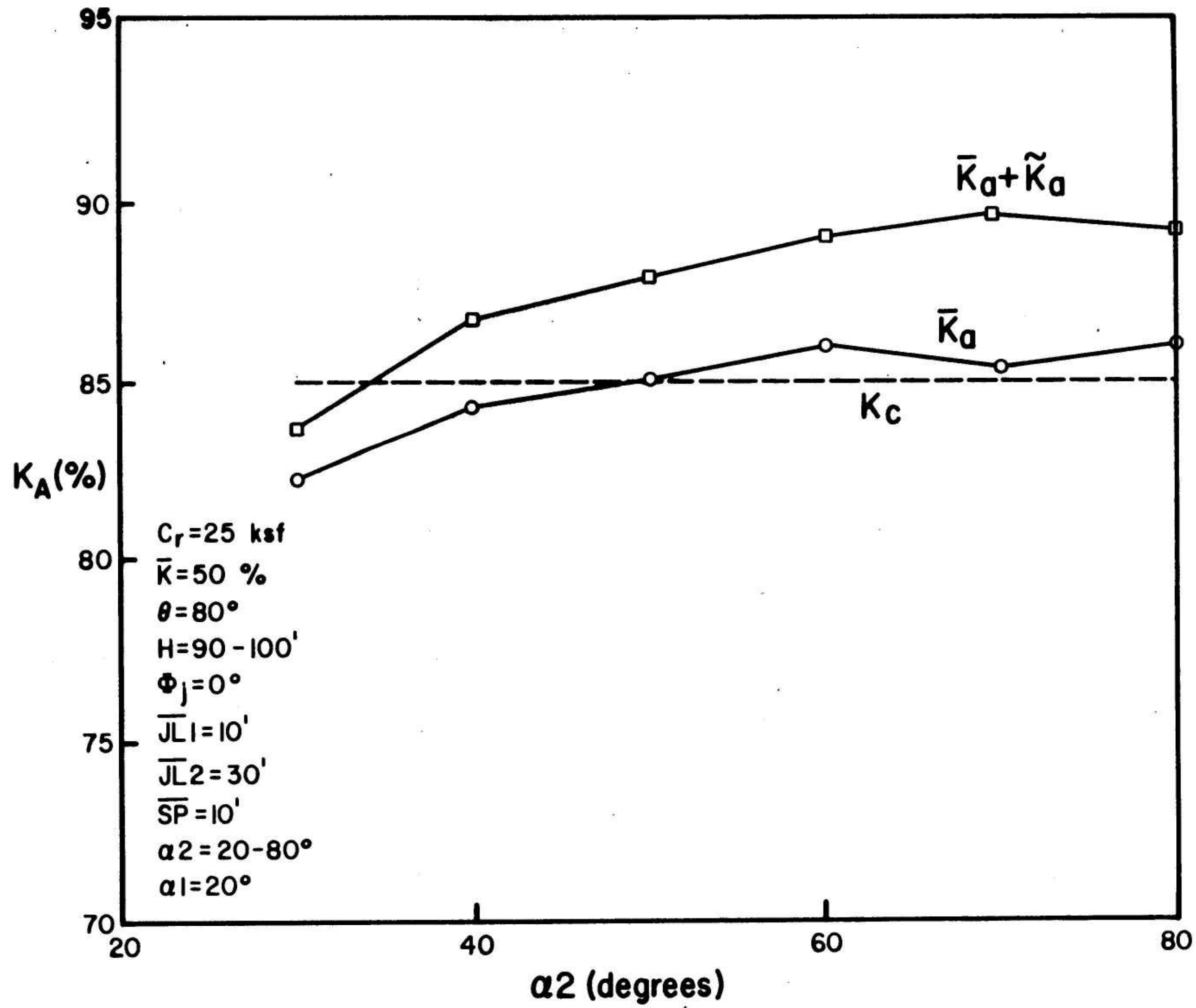


FIGURE 5.92 EFFECT OF JOINT SET 2 INCLINATION (α2) ON APPARENT PERSISTENCE (K_a) - CASE 40

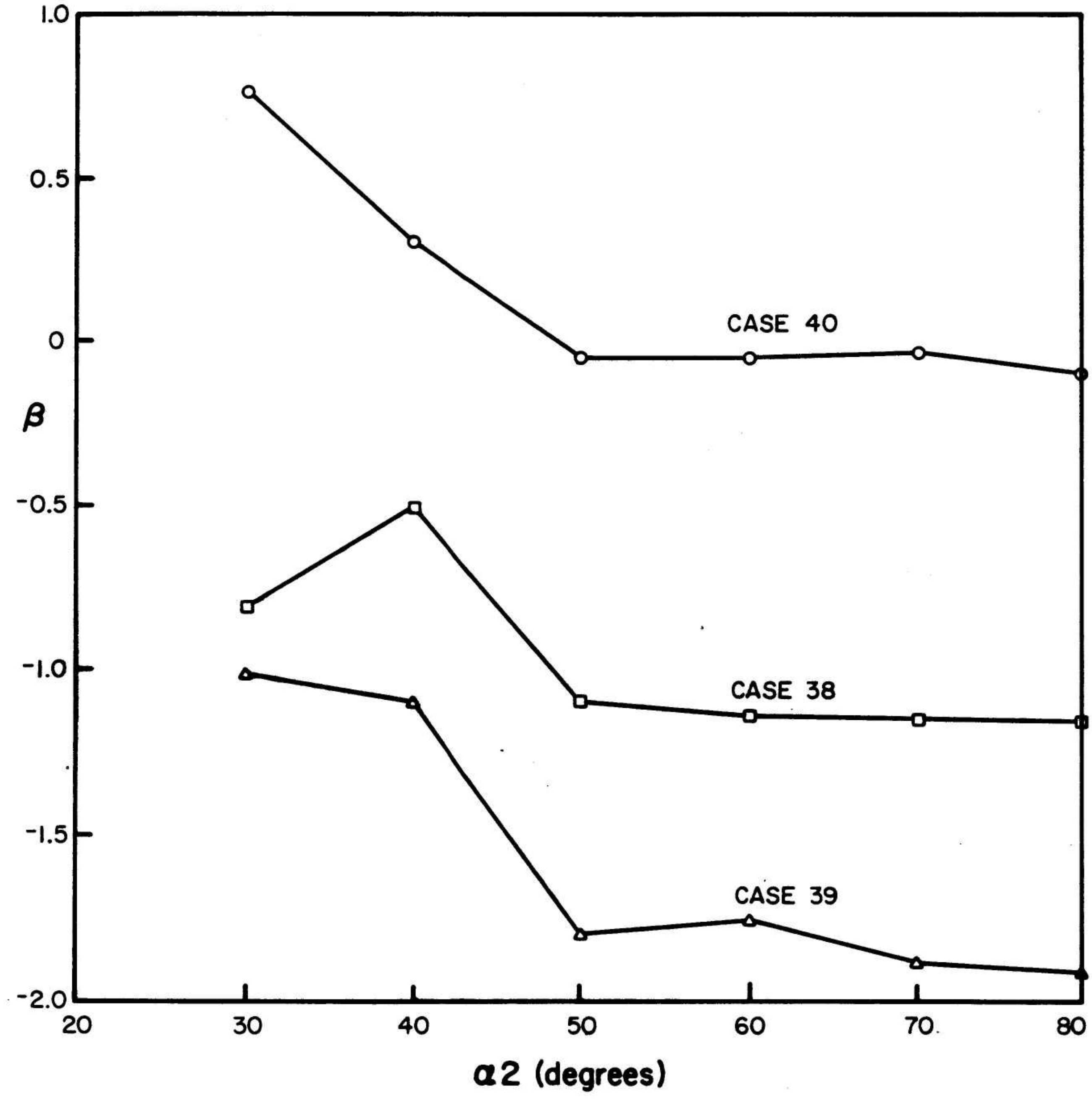


FIGURE 5.93 INDEX OF RELIABILITY (β) AS A FUNCTION OF SECOND SET JOINT PLANE INCLINATION (α2)

5.6 Parametric Study Conclusions

In this section, the major conclusions on the effects of each of the parameters that define a rock slope (geometric and mechanical) are briefly reviewed. Interaction of the various parameters that affect slope safety is described.

5.6.1 Effect of Strength Parameters: Interact Rock Cohesion (C_r) and Joint Persistence (Φ_j)

As a conclusion drawn from results of the parametric study, the parameter with the strongest influence on reliability is intact rock cohesion (C_r).

Model runs show that the path of minimum safety margin (critical path) for any joint plane existing on the slope face is almost totally independent of C_r and Φ_j. Increasing either intact rock resistance (C_r) or joint resistance along any path in the slope but does not change the location of the critical path. The safety margin and thus reliability of the critical path must also increase. However, there is a limit to the possible increase in joint place reliability. In other words, there is a point beyond which further increases in C_r and Φ_j will not yield significant further increases in reliability. The exact values of C_r and Φ_j; at which the probability of failure is equal to the probability of a joint plane being 100 percent persistent is a function of the other parameters (joint length, spacing, persistence, etc.).

At high intact rock cohesion values (C_r), program runs have shown that the probability of failure in a particular height interval is equal to the probability of a joint plane, existing in that interval, is 100 percent persistent. This holds regardless of other parameters even when joint resistance Φ_j equals zero. The study has also shown that when Φ_j is set equal to first set joint plane inclination, the probability of failure for a joint plane existing in a height interval, is approximately equal to P_1 (the probability of a joint plane being 100 percent persistent.)

An important result of the parametric study is that the distribution of apparent persistence ($\overline{K}_a$, $\tilde{K}_a$) is insensitive to strength parameter variation (C_r, Φ_j). Thus from a single model run $\overline{K}_a$ and $\tilde{K}_a$ can be generated for any combination of C_r and Φ_j; values (all other parameters held constant). By calculating the critical persistence (K_c), one can calculate the indices of reliability (β) without additional simulation. β values can then be used directly or can be converted to probability of failure values to assess joint plane reliability.

5.6.2 Effect of Slope Geometry Parameters

Slope geometry parameters are slope depth (h), slope face angle (θ) and the inclination angles of the two joint sets ($\alpha 1$ and $\alpha 2$). First set inclination ($\alpha 1$) has the greatest influence on reliability. The difference between θ and $\alpha 1$ strongly influences reliability. As $\alpha 1$ approaches θ, reliability increases as a result of a reduction in the driving force (a function of weight of rock between the critical path and slope face). In such situations, the effects casued by second set joints are minimal due to their neutral orientation (i.e., that orientation could not provoke a rock movement within the rock mass). Reliability however decreases as the difference between θ and $\alpha 1$ increases especially for $\alpha 1$ values of 40-50° and 30-70° for $\alpha 2$. Reliability increases when first joint set inclination $\alpha 1$ approaches joint frictional resistance due to the ability of joints to resist a higher percentage of the driving force.

5.6.3 Effect of Joint Geometry Parameters

Of the joint geometry parameters (mean joint plane spacings $\overline{SP}$, mean joint lengths $\overline{JL}$ and mean persistences $\overline{K}$), those with the strongest influence on joint plane reliability are the mean joint length of the first set $\overline{JL1}$ and the first set estimated mean persistence $\overline{K1}$. The effect of each becomes increasingly more pronounced with depth. Decreasing the means of joint plane spacing of both or either set have a strong effect on slope reliability, but not as severe as $\overline{JL1}$ and $\overline{K1}$.

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Model runs show that the path of minimum safety margin (critical path) for any joint plane exiting on the slope face is almost totally independent of C_r and Φ_j. Increasing either intact rock resistance (C_r) or joint resistance for a rock slope while holding all other parameters constant, does not change the locations of paths of minimum safety margin. As a consequence, the safety margin and thus reliability of those critical paths must also increase. However, there is a limit beyond which further increases in C_r and Φ_j do not yield significant further increases in reliability. Values of C_r and Φ_j which define that limit are a function of other parameters (joint length, spacing, persistence, etc.).

At high intact rock cohesion values (C_r), program runs have shown that the probability of failure in a particular height interval is equal to the probability of a joint plane, existing in that interval, is 100 percent persistent. This holds regardless of other parameters even when joint resistance Φ_j equals zero. The study has also shown that when Φ_j is set equal to first set joint plane inclination, the probability of failure for a joint plane existing in a height interval, is approximately equal to P_1 (the probability of a joint plane being 100 percent persistent.)

An important result of the parametric study is that the distribution of apparent persistence ($\overline{K}_a$, $\tilde{K}_a$) is sensitive to strength parameter variation (C_r, Φ_j). Thus from a single model run $\overline{K}_a$ and $\tilde{K}_a$ can be generated for any combination of C_r and Φ_j; values (all other parameters held constant). By calculating the critical persistence (K_c), one can calculate the indices of reliability (β) without additional simulation. β values can then be used directly or can be converted to probability of failure values to assess joint plane reliability.

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5.6.3 Effect of Joint Geometry Parameters

Of the joint geometry parameters (mean joint plane spacings $\overline{SP}$, mean joint lengths $\overline{JL}$ and mean persistences $\overline{K}$), those with the strongest influence on joint plane reliability are the mean joint length of the first set $\overline{JL1}$ and the first set estimated mean persistence $\overline{K}$. The effect of each becomes increasingly more pronounced with depth. Decreasing the means of joint plane spacing of both or either set have a strong effect on slope reliability, but not as severe as $\overline{JL1}$ and $\overline{K1}$.

The conclusion has been drawn in this thesis that the effect of a second joint set on slope availability, compared to the same slope with a single set, is minor but not marginal or trivial. However, one should not apply this statement to all rock slopes with all joint patterns; at least those not covered by the ranges established in this thesis. One should neither underestimate nor overlook the second set in a rock slope (the second set defined previously as the steeper joint pattern). Weight calculations have concluded stability of a slope with minor effect by the second set. However, potential instability does exist. A slope with two joint sets may prove to be stable when analyzed by the model developed in this thesis, but may become unstable from temperature changes (freezing and thawing), in situ water pressure changes or earthquake loads.

CHAPTER 6

DESIGN RECOMMENDATIONS

Perhaps, a matter of controversy would be whether a rock slope safety is critically effected by having two joint sets as compared to one with a single joint set. However, in the work associated with this thesis, the author believes that in most cases, the effect of having a second joint set on rock slope safety, compared to one having a single joint set, is small but never in any case trivial. An example where a second joint set causes instability is when one set is horizontal and the other vertical. Another is when the shallower set has a very low persistance and a short mean joint length and the other joint set has high persistence and a high mean joint length.

Present design methods do not take into account the distribution associated with rock slope parameters due to the complexity of such a task. A rather simple method was developed by O'Reilly - 1980 that considers results obtained by probabilistic approaches. Briefly, a slope can be classified to fall in one of three equal height intervals (zones), the zone of shallow instability, the zone of stability and the zone of deep instability. Thus the main purpose is to attempt to maximize the probability that a particular slope lies within the zone of stability (See O'Reilly -1980).

A great amount of research is yet to be carried out to establish generally acceptable and dependable methods to analyze rock slopes taking into account the respective uncertainties. The writer strongly recommends additional work and research aided with field data as often as it will be possible.