A GEOLOGICAL PREDICTION AND UPDATING MODEL IN TUNNELING

by

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ABSTRACT

Uncertainty in predicting geological conditions in tunneling often leads to design for the worst anticipated conditions and thus conservative design-construction approaches. By adapting design to the conditions encountered during construction potential savings are possible, however, only if changes from one design to another do not cause costs that exceed the savings. Optimization of design-construction has thus to consider the variability of geologic conditions.

In this thesis a probabilistic geological prediction and updating model based on the concept of Markov processes is developed. Probabilistic distributions of the geological parameters and "ground classes" ahead of the tunnel face are developed. These distributions are modified by observations made at points along the tunnel axis ahead of the tunnel face. Before tunnel construction the basic elements (transition probabilities and transition intensity coefficients) of the prediction model can be estimated using frequency data and/or subjective expert knowledge. As tunnel construction proceeds records of the geological parameters along the excavated tunnel are made and the estimates of the basic elements can be updated. A case study was made in which the state probabilities of the geological parameters were calculated. Since the parameters considered were probabilistically independent, ground class probabilities were readily calculated.

With the probabilistic description of geology, optimization of design-construction procedures prior to construction and optimal adaptation during construction becomes possible.

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List of Symbols

The geological prediction model considers several geological parameters each of which can take several states. Each parameter state has an extent and probability of existence at a certain position along the tunnel. Due to these complications a system of symbols employing subscripts and superscripts has to be used.

Random variables are denoted by capital letters while their particular realizations are usually denoted by the corresponding small letters. Matrices are capitalized and underlined. Vectors are either underlined or written with a top bar. Most symbols are self-explanatory when the context is considered.

Symbol	Definition
$\underline{A}_X$	transition intensity matrix of X
c_{Xi}	transition intensity coefficient of the state i of X
D	random variable denoting Degree of Jointing
E	random variable denoting Degree of Weathering
ES	excavation and support process
$F_{HXi}(h)$	cumulative probability density function of HX_i
$F'_{HXi}(h)$	updated cumulative probability density function of HX_i
$f^{d}_{HXi=k}(h)$	updated probability density function of HX_i based on a deterministic observation k (=i)

$f^{d}_{HXi \neq k}(h)$	updated probability density function of HXi based on a deterministic observation k (≠i)
$f_{HXi}(h)$	probability density function of HX_i
$f^{K}_{HXi}(h)$	updated probability density function of HX_i based on a combination of several deterministic observations
$f^{N}_{HXi}(h)$	updated probability density function of HX_i based on a combination of non-deterministic observations
$f^{n}_{HXi}(h)$	updated probability density function of HX_i based on a non-deterministic observation
$f'_{HXi}(h)$	updated probability density function of HX_i
$\overline{g}_{ij}$	j^{th} vector inside GCi
GC	ground class
H_m	competing hypothesis m
HX_i	random variable representing the extent of the state i of X; also used as a particular realization
h	particular realization of HX_i
K_m	the m^{th} combination of states at several points
K_{tm}	state at l_t in K_m
k_t	state at point l_t
L_m	likelihood of H_m
l	length, co-ordinate of a point
NC	number of categories
n	total number of states of X
p_{tm}	probability of having state m at l_t
p^{dc}_{Xij}	updated transition probability based on a deterministic observation and with the condition that $HX_i = h_s$
p^{d}_{Xij}	updated transition probability of X from states i to j based on a deterministic

	observation
p^{Kc}_{Xij}	updated transition probability based on a combination of deterministic observations and with the condition that $HX_i = h_s$
p^{K}_{Xij}	updated transition probability of X from states i to j based on a combination of deterministic observations
p^{Nc}_{Xij}	updated transition probability based on a combination of non-deterministic observations and with the condition that $HX_i = h_s$
p^{nc}_{Xij}	updated transition probability based on a non-deterministic observation and with the condition that $HX_i = h_s$
p^{N}_{Xij}	updated transition probability of X from states i to j based on a combination of non-deterministic observations
p^{n}_{Xij}	updated transition probability of X from states i to j based on a non-deterministic observation
p_{Xij}	transition probability of X from states i to j
P_m	probability that H_m is true
P'_m	updated probability that H_m is true
R	random variable Rock Type
RQD	Rock Quality Designation
r	particular realization of R
$\underline{S}_X$	limiting state probability vector of X
$\underline{S}_X(u)$	row vector of state probabilities at a distance u from present point
TBM	tunnel boring machine
$\underline{V}_X(u)$	interval transition probability matrix
$v^{d}_{Xij}(u)$	updated interval transition probability based on a deterministic observation
$v^{ds}_{Xij}(u)$	updated interval transition probability

	based on several deterministic observations
$v^{n}_{Xij}(u)$	updated interval transition probability based on a non-deterministic observation
$v^{ns}_{Xij}(u)$	updated interval transition probability based on several non-deterministic observations
$v_{Xij}(u)$	interval transition probability
v_{Xj}	limiting state probability
W	random variable denoting Availability of Water
X	a geological parameter as a random variable
XI	a geological parameter as a random variable
x	particular realization of X
xin	total number of states of XI
Y	a geological parameter as a random variable

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CHAPTER I

INTRODUCTION

Tunneling involves a high degree of uncertainty arising from the unknown geological conditions underground and a lack of precise understanding of the ground-structure interactions. This uncertainty often translates into a high cost of tunneling. Since underground construction such as tunneling and mining is expected to increase to even higher volumes (estimated $40 billion in the United States alone) in the near future, efficient tunneling methodologies must be developed. As a first step to efficiently solve problems of cost estimation and optimization, a geological prediction and updating model in tunneling is developed.

In conventional tunneling methods, there is usually only one (or a very few) excavation and support design options for the entire tunnel. This design has to be determined before tunnel construction and hence a conservative design has to be adopted based on the worst expected geological conditions in the tunnel. This conservatism evidently leads to unecessarily high costs of tunneling. A new approach has been developed which is generally known as the observational or adaptable method. In the observational method different excavation and support processes are used for different sections of the tunnel, based on technical and economic considerations. The aim of this approach is to minimize the expected cost of tunnel construction.

Since decisions on choosing among a number of excavation and support processes for different sections of the tunnel have to be made, cost optimization in the observational method can be much more complicated. The role of geological prediction is especially important in this method where uncertainties must be considered. With probabilistic prediction methods, construction planning before and during tunneling can be carried out systematically and optimal strategies can be found.

At the present time geological prediction is usually in the form of a "best estimate" which represents the conditions most likely to exist around the proposed tunnel. This prediction is based on the results of exploration, established geological information and inferences made by geologists. The main disadvantage is that uncertainty is not considered explicitly. Since "unanticipated" geologies occur from time to time, this approach is not quite sufficient for cost estimation and construction planning.

An improvement is made in the Tunnel Cost Model (Moavenzadeh et al, 1978) which considers the effect of geological uncertainty on cost estimation. In this model the tunnel profile is divided into segments inside each of which only one "geologic unit" is assumed to exist (a geologic unit is a set of geological conditions which dictates certain excavation and support processes.) This assumption of having only one geologic unit inside each segment is obviously easily violated. If a pre-determined tunnel segment is not extremely short, there is no reason why the geological conditions should be the same within that segment. In addition, there is no systematic procedure through which geological predictions can be updated as tunnel construction proceeds. Therefore although the Tunnel Cost Model can be a satisfactory tool for cost estimation before tunnel excavation, it cannot be used for construction planning in search for optimal (cost-minimizing) strategies.

In this thesis a more powerful probabilistic geological prediction and updating model using the Markov process is proposed. In Chapter II, technical and economic considerations required for cost optimization are examined. The concept of "ground classes" (a set of geological conditions which dictates certain excavation and support processes) for individual sections is presented. Chapter III introduces the Markov process concept adopted by the geological prediction model. In Chapter IV the geological prediction model is developed. The reasons and supporting evidence for choosing the Markov model are presented. Applications of the prediction model and the problem of parameter interdependences are discussed. Chapter V considers estimations of transition intensity coefficients and transition probabilities (the basic elements of a Markov process) to be used in applying the prediction model in practice. Both frequency-based and subjective probability derivations are discussed. Chapter VI shows how the coefficients and probabilities used in the prediction model can be updated when excavation proceeds and more geological information is gathered. To exemplify some actual applications of the concepts developed in this thesis, a case study on the construction of a water tunnel (7662 feet long) is made in Chapter VII. Chapter VIII concludes the thesis.

CHAPTER II

THE PERFORMANCE MODEL AND COST OPTIMIZATION

2.1 Introduction

Tunnel construction and planning are greatly affected by the ground conditions along the tunnel axis. Since geological conditions often cannot be determined before excavation, it seems promising to apply methods of decision analysis under uncertainty so that the expected total construction cost is minimized. Before describing formal procedures for cost optimization the general tunnel construction process and the observational tunneling method are described.

2.1.1 General tunnel construction process

Two of the main components of tunnel construction are excavation and support placement. Excavation is the removal of rock and/or soil by hand, by drilling and blasting, by machinery or by combinations of methods. It is done in cycles (rounds) that can have a length between less than one meter to about four meters depending on standup time and on equipment characteristics. After one or more rounds the excavation process is stopped to allow for the application of initial supports to the newly excavated part of the tunnel. After the initial support is placed excavation is resumed and the cycle is repeated. At some distance from the face the tunnel (wher excavation is taking place) the final support is applied. In cases where the geological conditions are of high quality, no initial and somtimes no final supports are required.

2.1.2 Observational tunneling method

In conventional tunnel methods, the excavation and support processes (full face or partial face excavation, types of initial support, methods of installation) are to a large extent predetermined before the construction of the tunnel starts (or at least before the "production phase of construction.) Only minor changes of these processes are possible during construction. Consequently the choice of excavation and support processes are often based on the worst expected geological conditions because no or limited adaptation is possible and hence over-conservatism is often inevitable. In contrast the observational tunneling method allows for adaptation of design during tunneling. Basically the design is optimized in situ by adapting it to the observed geological conditions. The observational tunneling method is composed of the following steps :

1. Exploration --- available information on the particular geology of the tunnel area is collected and additional geotechnical exploration is carried out to describe the engineering properties of the tunnel ground. (This step is

common to both "conventional" and observational methods.)

2. Preliminary design --- alternate excavation procedures and support designs for different geological conditions classified in terms of ground classes (see section 2.2) are developed. These alternate designs can be modified when more experience is gained during tunneling. New designs can be added when unexpected geological conditions occur.

3. Tunnel construction --- appropriate excavation and support processes are selected for each round based on observations and monitoring (see below) in the preceding tunnel sections. Thus, technically and economically optimal excavation and support procedures are chosen.

4. Observation and monitoring --- geological conditions of newly excavated parts of the tunnel are observed and recorded. Observation of geological conditions forms the basis for ground classification and the above-mentioned selection of excavation and support procedures. Monitoring of deformations helps to maintain safety and to get a better understanding of the ground-structure behaviour. Monitoring will thus indicate if the support performs as anticipated or if the design has to be changed.

5. Adaptation to particular geological conditions --- the appropriate excavation and support method for each round is chosen. Also design is modified based on results of monitoring. New designs are added if necessary.

6. Steps 3 to 5 are performed simultaneously and repetitively during tunneling until construction is

finished.

The main reason for using the observational method instead of the conventional one is that over-conservatism can be minimized and hence the tunnel construction costs can be lowered. In addition, the flexibility of the observational approach makes it easier to cope with unexpectedly adverse situations.

2.2 The performance model

A technical understanding of the interaction of the tunnel ground with different excavation and support processes is essential in the choice of them. The performance model is introduced to describe how a tunnel section with given geological conditions will perform as a certain excavation and support process is applied.

The performance model expresses the performance of a section of the tunnel as a function of the geological conditions and of the excavation and support processes applied to that section i.e.

$$\overline{p} = f(\overline{g}, \overline{e}, \overline{s}) \quad ..... \quad (2.1)$$

where $\overline{p}$ is the vector of performance parameters such as :

(1) Ground behaviour during excavation (i.e. overbreaks.)

(2) Convergence of tunnel at a fixed distance after initial support is applied.

(3) Convergence rate at a fixed time after initial support

is applied.

(4) Afterbreaks.

(5) Support performance (e.g. amount of displacement.)

(6) Water inflow during excavation and application of initial support.

(7) Water inflow after application of initial support.

(8) Time required for construction (including time spent in excavation and applying initial support.)

$\overline{g}$ includes all the relevant geological parameters which affect the choice of excavation and support processes and the performance of the tunnel after construction. g may include the following geological parameters :

(1) Rock type.

(2) Faulting.

(3) Degree of jointing (or RQD.)

(4) Availability of ground water at tunnel grade.

(5) Overburden.

(6) Soil type (e.g. soils with different degrees of cohesiveness, mixed face - boulders and soil.)

Parameters (1) to (5) are more important in hard rock tunneling particularly at great depths while parameters (4) to (6) are more important in soft ground tunneling at shallow depths.

$\overline{e}$ is the vector of excavation parameters such as :

(1) Excavation method (e.g. Tunnel Boring Machine,

shield, drill and blast, or cut and cover.)

(2) Round length.

(3) Amount of over-excavation.

$\overline{s}$ contains the support parameters such as :

(1) Initial support (e.g. steel ribs and lagging, rock bolts with a certain spacing, shotcrete with a certain thickness (e.g. 3 in, 5 in), liner plates with a certain thickness and bolt spacing, or segmental lining.)

(2) Final support (e.g. shotcrete with a certain thickness, cast-in-place concrete with a certain thickness, or segmental lining.)

(3) Face support (e.g. no face support, breasting, or shotcrete.)

(4) Invert support.

(5) Initial support distance delay (i.e. distance between tunnel face and section where initial support is applied.)

(6) Initial support time delay (i.e. time between finishing of excavation and application of initial support.)

(7) Final support time delay.

When examining the above parameters, it can be seen that some of them are qualitative while the others can be expressed quantitatively. It is often convenient to discretize some of the quantitative parameters (if their states are not already expressed in discrete terms.) For example, convergence of the tunnel can be expressed by values such as 2 inches, 4 inches, 6 inches, 8 inches or greater, but not 1.54 inches. Thus, for example, "c (the symbol for convergence) =3" can mean that the convergence is 6 inches. Thus, basically quantitative parameters can also be expressed qualitaively (qualitative parameters are always discrete.) For example, c=1 can mean that the convergence is small while c=5 can mean that the convergence is intolerable. In the case of geological parameters, only discrete states will be used in the geological prediction model (see Chapter 4) because geological parameters are either qualitative (e.g. rock type, faulting, cohesiveness of soil and even sometimes the degree of jointing) or discrete values are sufficiently accurate for technical considerations.

2.3 Cost optimization

As discussed in section 2.1, one wants to optimize the construction of a tunnel section economically and technically. Thus, for given geological conditions, a combination of excavation and support processes should be selected which results in the smallest cost while the performance of the tunnel, as derived from relation (2.1), will be satisfactory. Satisractory performance essentially means that the tunnel is usable and that sufficient safety against collapse is maintained. The optimal excavation and support processes can be chosen independently for each tunnel section (round) as will be discussed in section 2.3.1. Ideally, and as described in section 2.3.2, cost optimization should involve the entire tunnel where the choice of excavation and support methods for a section (round) is also affected by the conditions in the other sections.

2.3.1 Optimization of individual sections (rounds) - "ground classes"

If cost optimization is carried out for every geological condition g (there is a finite number of possible geological conditions since all geological parameters are discrete), it will be found that different sets of geological conditions require different optimal combinations of excavation and support processes. It is often convenient to denote these sets of geological conditions by ground classes (GC) such that if a particular geological condition g1 belongs to a certain GC, the excavation-support process (ES) corresponding to this GC is optimal for g1. Consequently, the optimal excavation-support process corresponding to the ground class GCi is denoted by ESi.

The phrase "ground class" is in fact borrowed from the terminolgy of the New Austrian Tunneling Method (NATM) (see Steiner, 1979) where a certain ground class would dictate a certain excavation- support method and a "section" is a round of excavation. In the NATM there are usually about 6 or 7 ground classes.

2.3.2 Optimization involving the entire tunnel - "cost of change"

So far only the cost optimization for individual sections was discussed. In fact using ESi for GCi is the optimal choice only if a single section is considered. If the construction of the entire tunnel is considered, complications arise because a certain ES requires man-power, machinery, and set-up time before it can be applied. When a currently used ES has to be replaced by another ES for a new section, the change will involve additional costs needed to replace or modify equipment and procedures.

Example : (see Figure 2.1)

GC	3	2	1	
Section	21	22	23	

Figure 2.1 Effect of cost of change.

Given : - 3 ground classes and C1 (unit cost of ES1) < C2 < C3;

- ES3 can also be applied to GC1 and GC2 with technically satisfactory performance.
- ES2 can also be applied to GC1 with technically satisfactory performance.

Determine : the optimal choice of ES in sections 21, 22, and 23.

It is probably not justified to use ES2 for section 22 because the cost of change from ES3 to ES2 may well exceed the saving in constructing section 22 (which is short.) Therefore the optimal strategy for this part of the tunnel is to continue using ES3 through section 22 and then change to ES1 for section 23.

Thus for a given ground class profile, the optimal strategy for choosing the ES for each section has to be obtained from considering the construction cost of each section and the costs of change. In this way the total construction cost of the tunnel is minimized as a result of overall planning.

2.3.3 Uncertainties considered

As was mentioned before, the high costs of tunneling spring in part from uncertainties in geology and construction which essentially include :

(a) Model uncertainty --- the interaction of the ground with different excavation and support processes is usually not known accurately i.e. there is uncertainty in the performance relation (2.1.) Probably a performance relation derived from experiment and theory is applied. The relation can be updated when construction proceeds and more observations are made (see Rollin, 1979.)

(b) Geological uncertainty --- the "ground class profile" is usually not known deterministically before the tunnel is excavated. Hence the optimal strategy as described in section 2.3.2 cannot be obtained before excavation.

(c) Construction uncertainty --- the time and cost required for applying a certain ES vary due to factors such as machinery breakdowns, strikes and resource market fluctuations.

Due to the above three main types of uncertainties, the minimum construction cost of the tunnel cannot be estimated deterministically. But if appropriate probabilistic models are used to take these uncertainties into account, an "optimal strategy" can be found which minimizes the expected cost of construction. The main purpose of this research is to find an appropriate probabilistic model for geological uncertainty which can be incorporated into the expected cost optimization to find the optimal strategy.

CHAPTER III

THE MARKOV PROCESS

3.1 Introduction

After examining the effect of ground conditions on tunnel construction decisions (Chapter II), the basic concepts underlying the geological prediction model (which will be introduced in Chapter 4) are presented in this chapter. Since geological prediction involves not only ground parameters but also their respective locations, the concept of stochastic processes has to be used. A stochastic process involves random variables which are functions of a "time" parameter. For example, the number of people N in a queue can be regarded as a random variable which depends on time. Thus at a given time t, the number of people is a random function N(t) which has a certain probability distribution $P_N(n,t)$.

An example is shown in Fig. 3.1 where $P_N(n,t)$ is the PMF (probability mass function) of N(t). $P_N(n,t_1)$ and $P(n,t_2)$ are shown as PMF's of N at times t_1 and t_2 respectively. Thus for example at $t=t_1$, P[N=2] = 0.25 while at $t=t_2$, P[N=2] = 0.5.

The Markov process (Howard, 1971; Veneziano, 1980; Cox and Miller, 1965) is one of the best known stochastic processes and is sophisticated enough to deal with complex systems, like the geologic environment. The characteristic

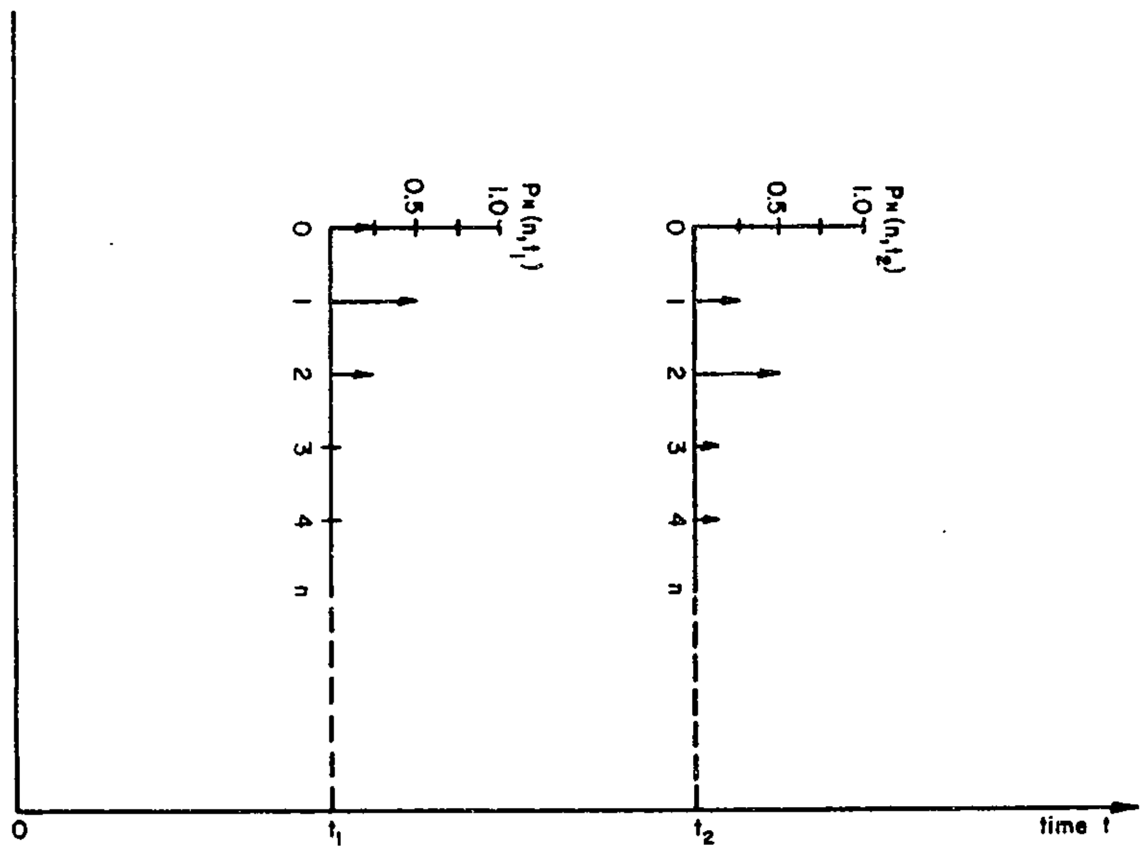


Figure 3.1 Example of a stochastic process

of the Markov process is that of a single-step memory : past history apart from the most recent event is neglected in forming predictions about the future. This is a very restrictive condition but the most recent step should usually be the most important step for forming predictions about the future. In fact a significant advantage for assuming a single-step memory instead of a multiple-step memory is that probability calculations are considerably simpler and full probability distributions can often be found.

For a probability distribution $P_X(x,t)$ to be governed by a Markov process, the condition holds that

$$P_X(x,t_{i+1} \mid x(t_i),\ x(t_{i-1}),\ \ldots)$$
$$= P_X(x,t_{i+1} \mid x(t_i)) \quad \ldots\ldots \quad (3.1)$$

where $x(t_i)$, $x(t_{i-1})$, ... are the outcomes of the random variables $X(t_i)$, $X(t_{i-1})$, ... respectively and $t_{i+1} > t_i > t_{i-1} > \ldots$ Thus the history of the past events except the most recent one has no effect on the probability distribution of the random variable at a later time.

In the previous example about the number of people in a queue, if $P_X(n,t)$ obeys the Markov process, then the prediction on $N(t^*)$ (i.e. $P_N(n,t^*)$) depends only on the number of people known at a time most recent to t^* and not on the number at any other time before.

In this thesis "time" is equivalent to the position along the tunnel axis where position is identified by the distance l from a certain fixed point (e.g. the portal of the tunnel.) The situation is shown in Fig. 3.2 where the direction of the advance of construction is the positive direction of l.

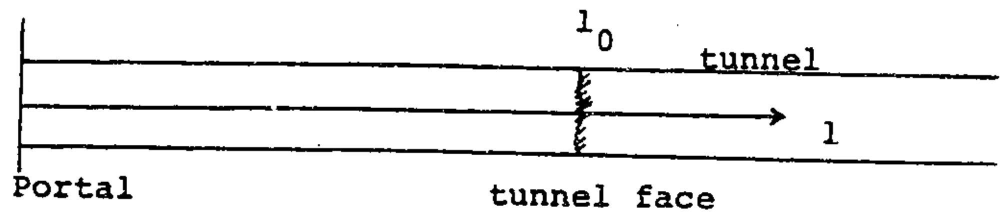


Figure 3.2 Definition of "time" in the Markov process.

3.2 Basic elements of the Markov process

Central to the Markov process are the concepts of state, state transition, and extent. These three basic elements are introduced in the following sub- sections.

3.2.1 State

The states of a random variable are the possible values that it can take. For example, for the ground parameter "Rock Type", the parameter states r can be defined as follows :

r	Definition
1	Schist
2	Metaquartzite
3	Diorite
4	Quartzite

such that "r=3" means that the state of rock type is Diorite.

3.2.2 State transition

A ground parameter X at a certain position l can be regarded as a random variable X(l). As l increases from 0, X(l) changes its value (see Fig. 3.3.) Each of these changes is called a state transition. If at a certain position x(l)=i, the probability that the next state is j is P_{Xij}, the transition probability from state i to state j. For example, if in Fig. 3.2 $x(l_0)=1$, then the probability that the next state is 2 is P_{X12}. Since the "next state" is always assumed to be different from the present state, $P_{Xii}=0$.

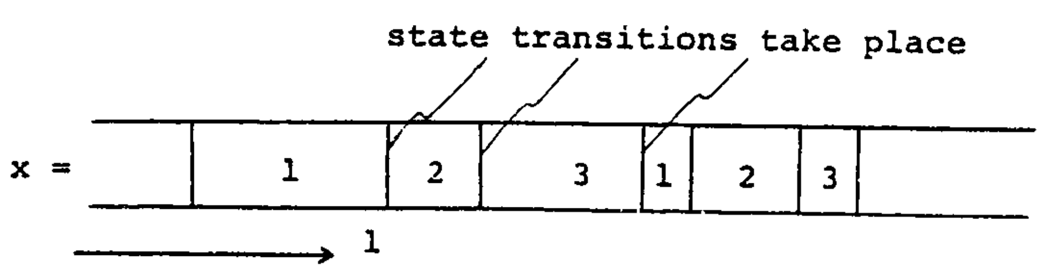


Figure 3.3 State transitions.

3.2.3 Extent

After a parameter X(l) has entered into a certain state i at l_0, the interval for which X will remain in state i is called the extent HX_i of state i at l_0. HX_i can be thought of as the "horizontal thickness" of state i and is depicted in Fig. 3.4 ($k \neq i \neq j$.)

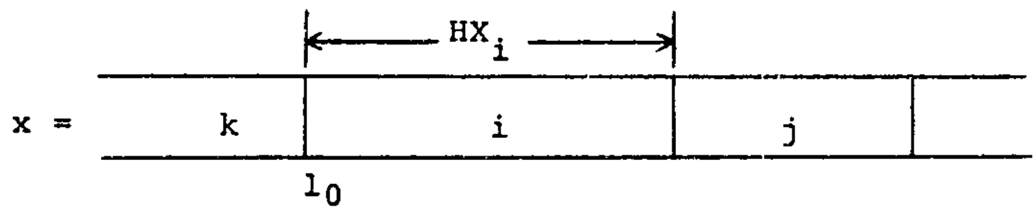


Figure 3.4 Extent.

For the continuous space Markov process [in which space (i.e. position) is measured with a continuous scale] considered in this thesis, the transition intensity coefficient c_{Xi} of state i of parameter X can be defined such that $c_{Xi}\,dl$ is the probability that a state transition is made (i.e. the extent terminates) within the infinitesimal interval dl, given that state i exists at the beginning of the interval (Fig. 3.5.) Thus the probability that a transition occurs within the interval from $l=l_1$ to $l=l_1+dl$ is $c_{Xi}\,dl$.

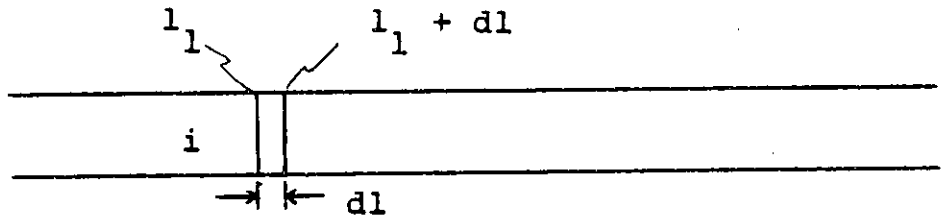


Figure 3.5 Probability of making a transition

Neglecting the small probability that there is more than one transition within dl, the PDF of HX_i can be derived by first considering the CDF (cumulative density funtion) of HX_i:

$$F_{HXi}(h) = P[HX_i < h] \quad \ldots\ldots \quad (3.2)$$

If h is divided into m equal segments of infinitesimal length dl each, then

$$\begin{aligned} P[HX_i > h] &= P[\text{no transition occurs within } h] \\ &= P[\text{no transition occurs within each of the m segments}] \\ &= \lim_{m \to \infty} (1 - c_{Xi}\ dl)^m \\ &= \lim_{m \to \infty} (1 - c_{Xi}\ h/m)^m \\ &= e^{-c_{Xi} h} \end{aligned}$$

From (3.2), $P[HX_i > h] = 1 - F_{HXi}(h)$

Hence $F_{HXi}(h) = 1 - e^{-c_{Xi} h} \quad \ldots\ldots \quad (3.3)$

By differentiating both sides of (3.3), the PDF of extent HX_i is given by

$$f_{HXi}(h) = c_{Xi}\, e^{-c_{Xi} h} \quad \ldots\ldots \quad (3.4)$$

which is the familiar exponential distribution with mean $1/c_{Xi}$ and standard deviation $1/c_{Xi}$. In other words, the extent of a state is exponentially distributed under the single-step memory assumption of the Markov process.

3.2.4 Intermediate summary

The elements (state, state transition and extent) of the continuous space Markov process have been introduced. The assumption of a single-step memory leads to transition probabilities P_{Xij} and exponential distributions of state extents (3.4).

3.3 State prediction at a future point

Based on the Markov process concept the probability of a parameter X being in a certain state at a future point can be calculated. This probability is of great interest since the state prediction of X at a point ahead of the tunnel face is often desired. The situation is depicted in Fig. 3.6 in which the probability of X being in state j at an interval u from the tunnel face l_0 is wanted, given $x(l_0)=i$. This probability cannot be found easily since within the interval u any number of transitions (including no transitions) can take place. It is therefore expedient to introduce matrix notations which express calculations in a compact form.

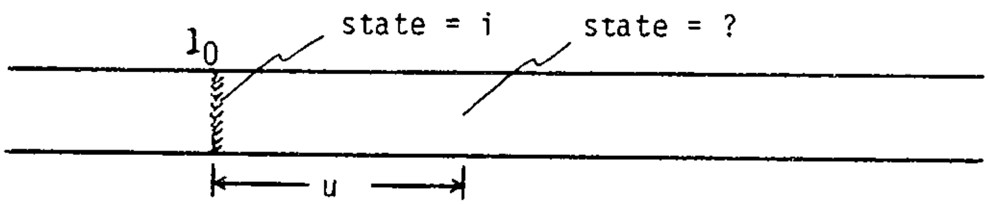


Figure 3.6 State prediction.

As will be shown later one needs for predicting states at "future points" the transition intensity matrix $\underline{A}_X$ of parameter X such that

$$\underline{A}_X = \{a_{Xij}\}$$

where $a_{Xij} = \begin{cases} -c_{Xi} & (i=j) \\ c_{Xi}\, P_{Xij} & (i \neq j) \end{cases}$

Hence

$$\underline{A}_X = \begin{pmatrix} -c_{X1} & c_{X1} P_{X12} & c_{X1} P_{X13} & \cdots & c_{X1} P_{X1n} \\ c_{X2} P_{X21} & -c_{X2} & c_{X2} P_{X23} & \cdots & c_{X2} P_{X2n} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ c_{Xn} P_{Xn1} & \cdots & & & -c_{Xn} \end{pmatrix} \quad \ldots\ldots (3.5)$$

$\underline{A}_X$ contains c_{Xi} and P_{Xij} and hence defines the Markov process completely. $\underline{A}_X$ is especially useful in making state probability predictions which are discussed in the following sub-sections.

3.3.1 Interval transition probability (see Veneziano, 1980)

For the situation shown in Fig. 3.6, given that the parameter X is in state i at the tunnel face, the probability that X is in state j at a distance u behind the tunnel face is required. This problem of state prediction can be solved by introducing the interval transition probability matrix

$$\underline{V}_X(u) = \left\{ v_{Xij}(u) \right\}$$

where

$v_{Xij}(u)$ = P[X will be in state j after an interval u given the present state is i]

Generally, $\underline{V}_X(u)$ satisfies the forward Kolmogorov differential equation,

$$\frac{d\, \underline{V}_X(u)}{du} = \underline{V}_X(u)\, \underline{A}_X \quad \ldots\ldots (3.6)$$

To prove (3.6), let du be a small interval.

$$v_{Xij}(u+du)$$

$$= v_{Xij}(u)\ v_{Xjj}(du) + \sum_{k\neq j} v_{Xik}(u)\ v_{Xkj}(du)$$

$$= v_{Xij}(u)\ (1-c_{Xi}\, du) + \sum_{k\neq j} v_{Xik}(u)\ a_{Xkj}\, du$$

Thus $v_{Xij}(u+du) - v_{Xij}(u)$

$$= -v_{Xij}(u)\ c_{Xi}\, du + \sum_{k\neq j} v_{Xik}(u)\ a_{Xkj}\, du$$

Dividing both sides by du and taking the limit as du approaches zero,

$$\frac{d\, v_{Xij}(u)}{du} = -v_{Xij}(u)\, c_{Xi} + \sum_{k \neq j} v_{Xik}(u)\, a_{kj} \quad \ldots\ldots (3.7)$$

Equation (3.7) is identical to (3.6) which is in matrix form.

The solution of (3.6) can be written as

$$\underline{V}_X(u) = \exp [u\, \underline{A}_X]$$
$$= I + u\, \underline{A}_X + 1/2\, u^2 \underline{A}_X^2 + \ldots$$
$$+ 1/m!\, u^m \underline{A}_X^m + \ldots\ldots \quad (3.8)$$

In practical cases this series may converge very quickly and one can use only a few terms to get satisfactory accuracy. If convergence is not quick or high accuracy is needed, one can use the spectral resolution of $\underline{A}_X$ (Cox and Miller, 1965, pp. 183 - 184) such that

$$\underline{A}_X = \underline{B}\, \text{diag}\, (l1,\, l2,\, \ldots\, ln)\, \underline{C}^T$$

where l1, l2, ... ln are eigenvalues of $\underline{A}_X$ and l1 = 0. The matrices $\underline{B}$ and $\underline{C}^T$ are formed from the left and right eigenvalues of $\underline{A}_X$ with the condition

$$\underline{B}\, \underline{C}^T = \underline{I}$$

Hence $\underline{V}_X(u) = \exp [\underline{A}_X\, u]$

$$= \underline{B}\, \text{diag}\, (e^{l1\, u}, \ldots\, e^{ln\, u})\, \underline{C}^T$$

Another way to find a closed-form expression for $\underline{V}_X(u)$ is by using exponential transforms (see Howard, 1971, p.710.) Howard also showed that $v_{Xij}(u)$ is equal to the sum of a constant (the limiting state probability; see section 3.3.3) and (n-1) terms such that

$$v_{Xij}(u) = v_{Xj} + k1\, e^{l1\, u} + \ldots + kn\, e^{ln\, u} \quad \ldots\ldots (3.9)$$

where v_{Xj} = limiting state probability of state j,

$k1, k2, \ldots kn$ = constants,

and $l2, l3, \ldots ln$ = eigenvalues with negative real parts.

3.3.2 State probabilities

According to the results of section 3.3.1, if a parameter X is in state i at l , the probability that X will be in state j at (l_0+u) is $v_{Xij}(u)$. When the state of X at l_0 is not known deterministically but only a PMF $P_X(x)$ at l_0 is given, the probability of finding state j at an interval u later can still be found. Let $s_{Xj}(u)$ be the probability of having state j at (l_0+u) :

$$\begin{aligned} s_{Xj}(u) &= P[\text{X is in state j after an interval u}] \\ &= \sum_{i=1}^{n} P_X(i)\, P[\text{X is in state j after an interval u given present state is i}] \\ &= \sum_{i=1}^{n} P_X(i)\, v_{Xij}(u) \\ &= \sum_{i=1}^{n} s_{Xi}(0)\, v_{Xij}(u) \quad \ldots\ldots (3.10a) \end{aligned}$$

since $s_{Xi}(0) = P_X(i)$ by definition of $s_{Xj}(u)$ above.

To express equation (3.10a) in a more compact form, let $\underline{S}_X(u)$ be the row vector of state probabilities such that

$$\underline{S}_X(u) = (s_{X1}(u)\ s_{X2}(u)\ \ldots\ s_{Xn}(u)).$$

Then (3.10a) can be expressed as

$$\begin{aligned} \underline{S}_X(u) &= \underline{S}_X(0)\, \underline{V}_X(u) \\ &= \underline{S}_X(0)\, \exp[\underline{A}_X u] \quad \ldots\ldots (3.10b) \end{aligned}$$

3.3.3 Limiting state probabilities

As the interval u increases, the effect of the present state on the probabilities of future states at an interval u later becomes smaller and smaller. When u approaches infinity, the probability of finding a certain state j at an interval u later becomes a limiting constant and is independent of the present state i. This limiting constant is called a limiting state probability v_{Xj} and is given by

$$v_{Xj} = \lim_{u \to \infty} v_{Xij}(u)$$

Furthermore, let $\underline{S}_X = (v_{X1}\ v_{X2}\ \ldots\ v_{Xn})$ be the limiting state probability vector. When the transition intensity matrix $\underline{A}_X$ is given, v_{Xj} can be found by first differentiating (3.10b) with respect to u :

$$\frac{d\,\underline{S}_X(u)}{du} = \underline{S}_X(0)\,\frac{d\ \exp[\underline{A}_X u]}{du}$$

$$= \underline{S}_X(0)\ \exp[\underline{A}_X u]\ \underline{A}_X$$

$$= \underline{S}_X(u)\ \underline{A}_X \quad \ldots\ldots (3.11)$$

As u approaches infinity, $\underline{S}_X(u)$ approaches $\underline{S}_X$ and (3.11) becomes $\frac{d}{du}\underline{S}_X = \underline{S}_X\underline{A}_X$. Since $\frac{d}{du}\underline{S}_X = 0$ ($\underline{S}_X$ is constant),

$$\underline{S}_X\underline{A}_X = 0$$

i.e.

$$v_{X1}(-c_{X1}) + v_{X2}(c_{X2}P_{X21}) + \ldots v_{Xn}(c_{Xn}P_{Xn1}) = 0$$
$$v_{X1}(c_{X1}P_{X12}) + v_{X2}(-c_{X2}) + \ldots v_{Xn}(c_{Xn}P_{Xn2}) = 0$$
$$\vdots \qquad\qquad\qquad\qquad \vdots$$
$$v_{X1}(c_{X1}P_{X1n}) + v_{X2}(c_{X2}P_{X2n}) + \ldots v_{Xn}(-c_{Xn}) = 0 \quad \ldots\ldots (3.12)$$

Equations (3.12) are linearly dependent since when all the equations are added together, the left-hand-side vanishes (the coefficients of v_{Xi} vanish) and is identically equal to the right-hand-side. One more equation is thus needed which is

$$v_{X1} + v_{X2} + \ldots + v_{Xn} = 1, \quad \ldots\ldots (3.13)$$

since the parameter can occupy one and only one state at a time.

Thus solving (n-1) equations from (3.12) simultaneously with (3.13) will give the values of v_{Xi}. On the other hand, if $\underline{V}_X(u)$ is already found in a closed form (section 3.3.1), then v_{Xi} can easily be found by taking the limit as u approaches infinity.

The physical significance of v_{Xi} is that it is the relative percentage of the occurence of state j. If in a certain region state j (e.g. Granite) of a parameter X (e.g. Rock Type) occurs 70% of the time, $v_{Xj} = 0.7$. For a tunnel of length L in such a region, the expected total length of Granite is $v_{Xj} L = 0.7\ L$.

3.4 Summary

Based on the elementary concepts of the Markov process introduced in section 3.2, probabilistic state predictions of a parameter X at a certain interval u after the present point can be calculated. As the interval u increases, the state probabilities are less dependent on the present situation. In particular, the interval transition probability $v_{Xij}(u)$ approaches a constant v_{Xj} (the limiting state probability) as u approaches infinity. In addition, since the transition intensity coefficients and the transition probabilities mentioned so far are regarded as constants (independent of "time", or l), the Markov process is said to be "homogeneous".

CHAPTER IV

THE GEOLOGICAL PREDICTION MODEL

4.1 Introduction

As was shown in Chapter II, geological conditions are an essential factor in selecting excavation and support methods. Since usually little or none of the geological conditions ahead of the tunnel face are known, it is desirable to predict them in a manner reflecting uncertainty. Methods of decision analysis under uncertainty can then be used to minimize the expected cost of tunneling. How the method of decision analysis can solve the problem of choosing excavation and support processes (ES) can best be shown by a simplified example :

Problem : choosing an ES for a short tunnel of length L ft. at 100 ft. below ground level.

General geology of tunnel region : 50 to 200 feet of clayey soil in contact with a metamorphic rock.

Geological uncertainty : since the tunnel is short, it is assumed that the whole tunnel is either in clayey soil or metamorphic rock.

Ground class classification : according to an established performance relation (section 2.2), several ES's are found technically satisfactory for tunneling in the metamorphic rock. The cheapest (optimal) one among them is [drilling and blasting, 5 inch shotcrete and steel sets]. The optimal ES for tunneling in the clayey soil is [tunnel boring machine, 3 inch shotcrete]. Therefore the ground class classification is :

GC1 = clayey soil

ES1 = [tunnel boring machine, 3 inch shotcrete]

GC2 = metamorphic rock

ES2 = [drilling and blasting, 5 inch shotcrete and steel sets]

Other information : for ES1, set-up cost = S1, unit cost = C1 (dollars per unit length); for ES2, set-up cost = S2, unit cost = C2; cost of change from ES1 to ES2 = C12; cost of change from ES2 to ES1 = C21; S1 > S2; C1 < C2.

After an ES is chosen, excavation starts and after a short length the actual geological conditions (either rock or soil) can be determined. If ES2 was chosen and if the tunnel is found to be in rock, then ES2 will be used for the tunnel and the total cost is (S2 + C2*L). If after choosing ES2 the tunnel is actually in soil, it is cheaper to change to ES1 and the total cost is (S2 + C21 + C1*L). If ES1 was chosen and if the tunnel is found to be in soil, then ES1 will be used for the entire tunnel and total cost = (S1 + C1*L). If after choosing ES1 the tunnel is actually in rock, ES2 has to used (ES1 is not technically satisfactory for GC2) and the total cost is (S1 + C12 + C2*L). These considerations are summarised in the decision tree shown in

Fig. 4.1.

This problem of selecting an optimal ES cannot be solved rationally without considering the geological uncertainty involved. To quantify the geological uncertainty, a (probabilistic) geological prediction model can be used. Suppose according to the results of the geological prediction model the probability that the tunnel is in rock is p. Then a decision analysis under uncertainty can be carried out by calculating the expected cost associated with choosing each ES.

Expected cost of choosing ES1

E1= P[tunnel is in soil]*(total cost when tunnel is in soil)

+P[tunnel is in rock]*(total cost when tunnel is in rock)

$= p*(S1 + C1*L) + (1 - p)*(S1 + C12 + C2*L)$

Expected cost of choosing ES2

E2= P[tunnel is in soil]*(total cost when tunnel is in soil)

+P[tunnel is in rock]*(total cost when tunnel is in rock)

$= p*(S2 + C21 + C1*L) + (1 - p)*(S2 + C2*L)$

The expected cost of choosing a certain ES can be regarded as the average cost of choosing that ES in a large number of similar tunnel projects. Thus the ES with a lower expected cost should be chosen.

In actual tunnel projects complications arise because there are other important geological parameters (e.g. Faulting, RQD, Availability of Water) in addition to Rock

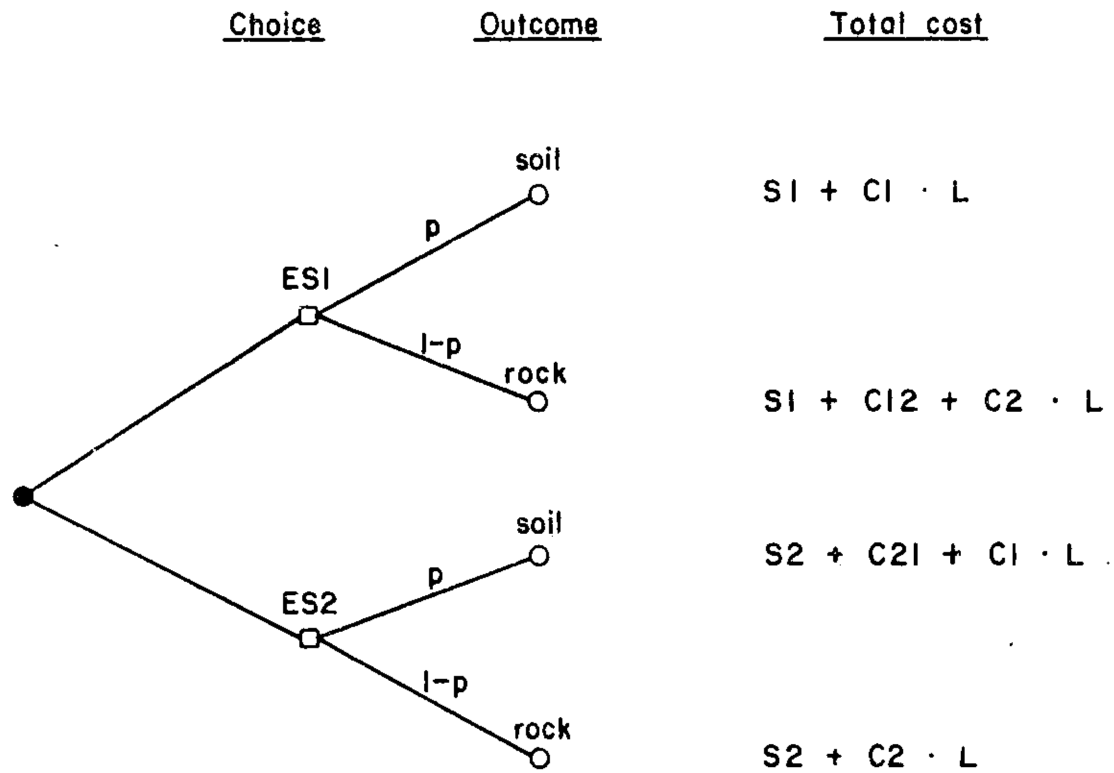


Figure 4.1 Decision tree for choice of ES

Type. Hence more ground classes are used (e.g. in the New Austrian Tunneling Method typically there are 6 or 7 ground classes.) Another complication is that there are usually many sections within the tunnel having different ground classes (the previous example is about a case with one section only.) In the example above the geological prediction is just the value of p but generally many more predictions are required. For example for planning purposes (e.g. resource and equipment mobilisation, cost estimation) very often the following questions need to be answered :

Given that the face of the tunnel is in a certain state (e.g. Granite) of a certain state of a parameter X (Rock Type),

(1) How long will the present state persist ?

(2) What is the next state ?

(3) What is the state at a certain distance ahead of the tunnel face ?

The geological prediction model developed in this thesis will be used to answer these three common questions probabilistically. By modeling the random variable X with a Markov process, the answers to these questions are given by (1) extent distributions (section 4.2.2), (2) transition probabilities (section 4.2.3), and (3) interval transition probabilities (section 4.2.1) respectively. [Among these three probability distributions, (3) will be discussed first because the updatings of (1) and (2) based on point observations have to make use of (3).]

Another very important use of the geological prediction model is that the extent distributions and the transition probabilities of a parameter X can be used to simulate profiles of the states of X in the unexcavated part of the tunnel (section 4.5.1.) When all the parameter profiles are simulated they can be combined to form a ground class profile (section 4.5.2.) After a sufficient number of ground class profiles are simulated, a certain construction strategy can be carried out for each profile. Examples of construction strategies are (assuming that there are 7 ground classes) :

(1) Conventional method --- use ES7 for the whole tunnel.

(2) Start with ES5; change to ES7 when GC6 or GC7 is encountered and then keep using ES7 for the rest of the tunnel.

(3) Change immediately to the corresponding ES whenever a new GC is encountered.

(These three construction strategies serve as simple examples only.)

When one of the above strategies is carried out for a simulated profile, a total cost for tunneling can be calculated. After the total costs are calculated for all the simulated profiles, the mean cost (and the standard deviation) of using that strategy can be calculated. If all the three strategies are tried in term, the best (optimal) strategy can be chosen based on minimum total cost and/or standard deviation of total cost. Thus expected cost optimization can be achieved through this straightforward method.

In the following parts of this introductory section (4.1) the theoretical development of the geological prediction model will be discussed. The reasons for choosing the Markov process (Chapter III) for modeling all geological parameters are presented, together with the advantages and disadvantages. Section 4.2 presents the geological prediction model and its applications under actual conditions. A discussion in section 4.3 shows how all the geological parameter predictions can be combined to form predictions on the ground classes. The problem of parameter interdependence and a proposed solution will be presented in section 4.4. Monte Carlo simulation of the tunnel profile will be discussed in section 4.5. In section 4.6 the chapter summary will be given.

4.1.1 Requirements on the prediction model

As E. E. Wahlstrom suggested (Robinson, 1972), in addition to the particular exploration of the site in question, a knowledge of the regional geology, the geologic history of the area, and thorough appreciation and understanding of the way in which rocks respond to changing geological environments, may be equally important.

Therefore the first requirement on the prediction model is that both the general and particular geological knowledge about the tunnel site should be utilized to yield the predictions. The general information about the tunnel site will remain essentially unchanged as tunnel construction proceeds while the particular information increases when more records are obtained during construction. It is therefore disirable that the predictions about the geologies of the unexcavated parts of the tunnel can be updated based on new observations. Furthermore, since subjective judgment is often necessary in geological predictions, the prediction and updating processes should be capable of incorporating subjective assessments; subjective biases, however, should be minimized.

The prediction model should include all geological parameters affecting tunnel performance considerably (e.g. "Color of Rock" by itself should not be included), such as those given in section 2.2. The prediction model should therefore have the flexibility of including unexpected but important parameters encountered during tunneling. Most importantly, the model should be capable of simulating possible tunnel geology profiles to facilitate overall construction planning. The profiles thus generated should not contradict the general expectations about the profile, which means : (1) each generated profile should not contradict observations on the parameters known before construction; (2) most of the generated profiles should not deviate considerably from the general geology of the tunnel region.

All these requirements for the geological prediction model can be summarised as follows :

(a) Tunnel profiles generated by the prediction model should be compatible with general expectations of the actual profile.

(b) The knowledge on both the general and particular geology of the tunnel region should be incorporated.

(c) Predictions can be updated as excavation proceeds and more information is gathered.

(d) The prediction and updating processes should be capable of including subjective judgment when necessary.

(e) The prediction model should include all relevant parameters and the entire ranges of their possible states. However, when unexpected important parameters are encountered, the model should be capable of including them also.

4.1.2 Reasons for adopting the Markov process

The Markov process model provides good solutions for the five requirements stated in section 4.1.1. Specifically the Markov model satisfies these requirements in the following manner :

(a) Tunnel profiles generated by the prediction model should be compatible with general expectations of the actual profile.

This requirement implies that the underlying concept of the prediction model should correspond to or at least be compatible with the actual situation. Whether geologic processes generally take place according to the Markov process is still an open question. However, observed thickness distributions of lithologic units show that they are either lognormally or exponentially distributed (identical to geometrical distribution when a discrete space approach is used.) Exponential (or geometric) distributions on the other hand are characteristic of the Markov process. Krumbein and Dacey proposed a simple genetic process model of sedimentation which leads to a geometric distribution of lithologic unit thicknesses. The derived geometric distribution is in fact the "discrete-time" analog of the exponential extent distribution of section 3.2.3.

The form of extent distribution was examined using the recorded extents of sections with various degrees of jointing in one of the Seabrook water tunnels. Degree of jointing was expressed as RQD (with states low, medium, and high) and the lengths (extents) of different sections in each state were recorded. The recorded extent distributions of medium and high RQD sections were fitted with exponential distributions and then tested by Chi-square tests (see Appendix A.) The results of the two tests confirm the possibility of an exponential extent distribution. It should be noted that the appropriateness of using transition probabilities cannot be tested likewise. For a parameter with n states, there are (n - 2n) independent transition probabilities. These (n - 2n) probabilities can always be chosen so that they fit any data set of actual transitions perfectly since the data set also has (n - 2n) independent values only (see section 5.2.)

Since at present geologic processes usually are not fully understood and since there are indications that some geologic processes (concerning lithologic unit thicknesses and RQD unit thicknesses) do show exponential extent distributions, the Markov model seems satisfactory. The prediction model is thus compatible with several of the more important aspects of actual geology.

(b) The knowledge on both the general and particular geology of the tunnel region should be incorporated.

Assuming that a parameter X in the tunnel region actually obeys the Markov process in the direction of the tunnel axis, the transition probabilities P and transition intensity coeficients c can be assessed from recorded frequency data or expert knowledge of geologists about X (as will be shown in sections 5.2 and 5.3.) Thus the knowledge of the regional geology is incorporated.

Particular geological knowledge of the tunnel area consists of known facts and exploration results about areas in the vicinity of the tunnel axis. Usually explorations include geologic mapping, geophysical investigations, trenching and core drilling. This kind of information can be regarded as "observations" of the parameter at different positions along the tunnel axis. If such an observation can determine the state of a parameter, then a deterministic statement (e.g. "the rock type at l=1000 ft. is Diorite") can be made at the point of observation. If the observation is non-deterministic, subjective judgment is needed (see (d) below) and only probabilistic statements about the parameter at the place of observation can be made. Examples of how these observations can be incorporated in the prediction model will be shown in section 4.2.

Thus general geological knowledge is incorporated when the values of P_{Xij} and c_{Xi} are assessed. Particular geological knowledge in the form of records from the excavated part of the tunnel is also used to update predictions as shown in (c) below.

(c) Predictions can be updated as excavation proceeds and more information is gathered.

Suppose a parameter X is in state i at the tunnel face. The probability of X being in state j at an interval u ahead of the tunnel face is given by the interval transition probability $v_{Xij}(u)$ (section 3.3.1.) As excavation proceeds the above probability changes (is updated) because u decreases and i may change. Thus predictions are continuously updated.

Another higher level of updating is that of the geological prediction model itself (Chapter 6) :

1) If the transition probabilities P_{Xij} and transition intensity coefficients c_{Xi} are estimated from a set of existing data, new data derived from the excavated part of the tunnel can be pooled with the existing data and new estimates can be calculated.

2) If P_{Xij} and c_{Xi} are originally established by subjective judgment, they can be updated using the concept of "competing hypotheses". If different geologists or different opinions of a geologist are consulted, several estimates (c_{1Xi}, c_{2Xi}, ... c_{yXi}) of a transition intensity coefficient can be established. Each of these y values represents a "competing hypothesis" H_m (m = 1, 2, ... y) which has a probability of being true P_m (see Fig. 4.2.) At first each P_m is assigned a value 1/y (i.e. a vague prior is used.) Then before tunnel excavation they are updated based on available records using Bayesian updating. The weighted mean

$$c_{Xi} = P_1 c_{1Xi} + P_2 c_{2Xi} + \ldots + P_y c_{yXi}$$

is used in the geological prediction model. As construction starts and new records on extents of state i are taken, the likelihood of each competing hypothesis is calculated. Then

H_1 : c_{Xi} equals c_{1Xi} ;

H_2 : c_{Xi} equals c_{2Xi};

$\cdot \qquad \cdot$

$\cdot \qquad \cdot$

$\cdot \qquad \cdot$

H_y : c_{Xi} equals c_{yXi}.

$P[H_m \text{ is true}] = p_m$, $m=1, \ldots y$

The weighted mean

$$c_{Xi} = P_1 c_{1Xi} + P_2 c_{2Xi} + \ldots + P_y c_{yXi}$$

is used in the geological prediction model.

When each P_m is updated, c_{Xi} is updated.

Fig. 4.2 Concept of competing hypothesis.

P_m (and thus c_{Xi}) are again updated using the Bayesian technique (section 6.3.) The updating of transition probabilities which are established by the subjective judgment method is similar. When a "row" of transition probabilities P_{Xi1}, P_{Xi2}, ... P_{Xin} is estimated, different opinions can be used to yield different rows of probabilites. Each of these rows represents a competing hypothesis whose probability of being true can be updated based on new records of transitions (section 6.3.) Predictions can therefore be updated based on new information from the excavated part of the tunnel.

(d) The prediction and updating processes should be capable of including subjective judgment when necessary.

If the amount of existing data is not sufficient to form best estimates of the transition probabilities P_{Xij} and transition intensity coefficients c_{Xi} of a parameter X, subjective judgment can be used instead (see section 5.3.) Another important use of subjective judgment is in the formation of non-deterministic observations at places ahead of the tunnel face. At a certain point the state of a parameter X is known with uncertainty due to imperfect (non-deterministic) explorations and geological inferences. A PMF of X can be established subjectively at that point. This PMF is regarded as the posterior (final) probability distribution of X at that point (while the prior is the original prediction of the geological prediction model.) Using Bayesian updating (or conditional probabilities) this PMF can be incorporated into the probability analyses as shown in section 4.2.

Hence subjective judgment can be used to establish the transition probabilities and the transition intensity coefficients. It can also be used to form non-deterministic observations ahead of the tunnel face.

(e) The prediction model should include all relevant parameters and the entire ranges of their possible states. However, when unexpected important parameters are encountered, the model should be capable of including them also.

When a new and important parameter X is encountered in the course of construction, the corresponding transition probabilities and transition intensity coefficients can be established in the same way as the other parameters. Thus unexpected important parameters can also be included.

It was shown above that the geological prediction model using the Markov process can satisfactorily fulfil the five requirements listed in section 4.1.1. However, there are several important assumptions associated with the Markov process adopted in the prediction model. These assumptions together with their respective advantages and disadvantages are discussed in section 4.1.3 below.

4.1.3 Assumptions --- their advantages and disadvantages

4.1.3.1 Single-step memory

In order to adopt the Markov process concept, a single-step memory has to be assumed. This assumption implies that probabilistic predictions depend only on the most recent step, which is usually the most important step in the past history. In the case of a tunnel, it means that geological predictions of a parameter depend only on the state of the parameter at the tunnel face and not on those states at points preceding the tunnel face. The advantage is that calculations become simpler and manageable with this assumption.

The disadvantage is that in some cases past history (apart from the most recent step) which may also be important in forming predictions is not used. As a simple but extreme case the cyclic structure shown in Fig. 4.3 can be considered. Assuming a single-step memory, the best value that can be assessed for P_{X23} is 0.5, which actually corresponds to the cyclic process because if the present state is 2, 50 out of a 100 times it will happen that the next state is 3. But if one more step of past history (i.e. a double-step memory) is used, the prediction model obviously becomes superior to the previous one because by "remembering" the present and the preceding steps, the next state can be determined. For example, if states 1 and 2 are encountered in succession, the probability that the next

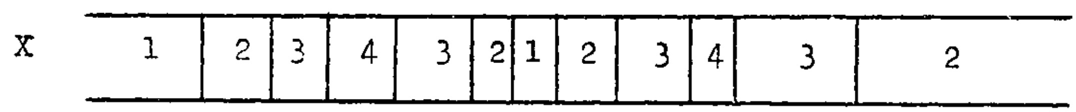


Figure 4.3 Example of a cyclic structure.

state is 3 is 1.0. This defect of the single- step memory can be lessened using subjective judgment (e.g. if state 1 and then state 2 are encountered, an "observation" is added subjectively which states that the next state is observed to be 3.)

To summarise, the assumption of a single-step memory greatly simplifies calculations but some "predicting power" may be sacrificed in cases where past history apart from the most recent step is also important in forming predictions.

4.1.3.2 Regional homogeneity

The Markov process used in the geological prediction model is assumed to be homogeneous i.e. P_{Xij} and c_{Xi} are constants independent of position l. There are two cases in which this simplifying assumption has to be modified. The first case is that of a tunnel crossing terrains of very different geologies. P_{Xij} and c_{Xi} of a parameter X may be significantly different in some of these terrains. Each of these terrains should be treated as an "homogeneous region" inside which X is governed by an homogeneous Markov process. For example, if X represents Rock Type and the tunnel goes through a sedimentary rock terrain and then an igneous rock terrain (see Fig. 4.4), different values of P_{Xij} and c_{Xi} (i.e. different transition intensity matrices) have to be used in these two terrains.

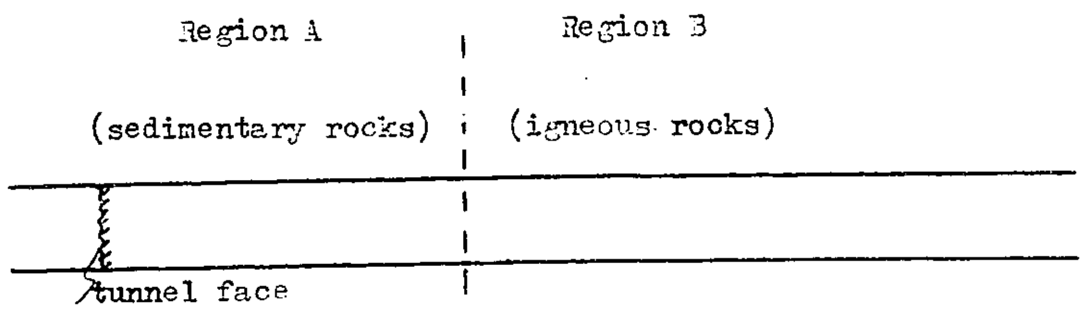


Figure 4.4 Tunnel crossing terrains of different geologies.

The second case is that, if P_{Xij} and c_{Xi} at a certain position depend on the state of another parameter Y at that position, then the Markov process for X cannot be homogeneous throughout the entire tunnel (unless Y is in the same state throughout the entire tunnel, which is unlikely.) Therefore in regions where different states of Y exist, different transition intensity matrices for X have to be used. An example is the case where X represents "Degree of Jointing" and Y "Rock Type". In a certain tunnel region the degree of jointing may vary strongly with rock types. Thus different transition intensity matrices for X have to be used in regions with different rock types. Each of these regions is an "homogeneous region" for X. This case of parameter interdependence can be neglected if it is weak (e.g. the different transition intensity matrices for X in regions where different states of Y exist are approximately equal.) The advantage is that only one transition intensity matrix needs to be established for each parameter and calculations to predict the ground classes at a certain point are greatly simplified and manageable (section 4.3.) If parameter interdependence is significant and cannot be neglected, the problem and its solution are discussed in section 4.4.

4.1.3.3 Intercommunication of states

In the prediction model it is also assumed for simplicity that there is intercommunication between every two states i and j. This means that if $x(l_0)=i$, then there is a non-zero probability that $x(l_0 + u) = j$. This means that there are no "transient states" which have essentially no probability of occuring after a great distance from the present position. This assumption seems to be reasonable within the context of this thesis : there is no reason why a certain state cannot occur at a great distance from the present position. For example, if the degree of jointing at the tunnel face is high, there is no reason why it cannot be low at a great distance ahead.

4.2 The model and its applications

After the development of the geological prediction model in section 4.1, the model and some of its applications are presented in this section. Basically the Markov process (with the assumptions given in section 4.1.3) is used to model all relevant geological parameters which exist along the tunnel axis. The "time" parameter in the Markov process is equivalent to the distance measured along the tunnel axis from a fixed point (e.g. the portal; see Fig. 4.5.)

Once the transition probabilities P_{Xij} and the transition intensity coeffients c_{Xi} of a parameter X are established, predictions on the states (in the form of the interval transition probabilities $v_{Xij}(u)$ and the transition probabilities P_{Xij}) and state extents (extent distributions) can be calculated.

In addition, if appropriate simplifying assumptions are made, simple empirical prediction rules can be established. An example is a "proximity rule" (see Lindner, 1975) which gives the probability of finding a state at a point given the same state is found at a certain distance from that point. In Appendix B the "proximity rule" is derived using simplifying assumptions and approximations.

In cases where there are point "observations" on parameter X ahead of the tunnel face, $v_{Xij}(u)$, P_{Xij} and the extent distributions will be modified (updated.) These point observations are generally the results of geologic mapping, geophysical explorations, trenching, core drilling and subjective judgment. [These point observations are different from the data (in the form of transition chains) from which P_{Xij} and c_{Xi} are established before tunneling.] An example of such observations is shown in Fig. 4.5 :

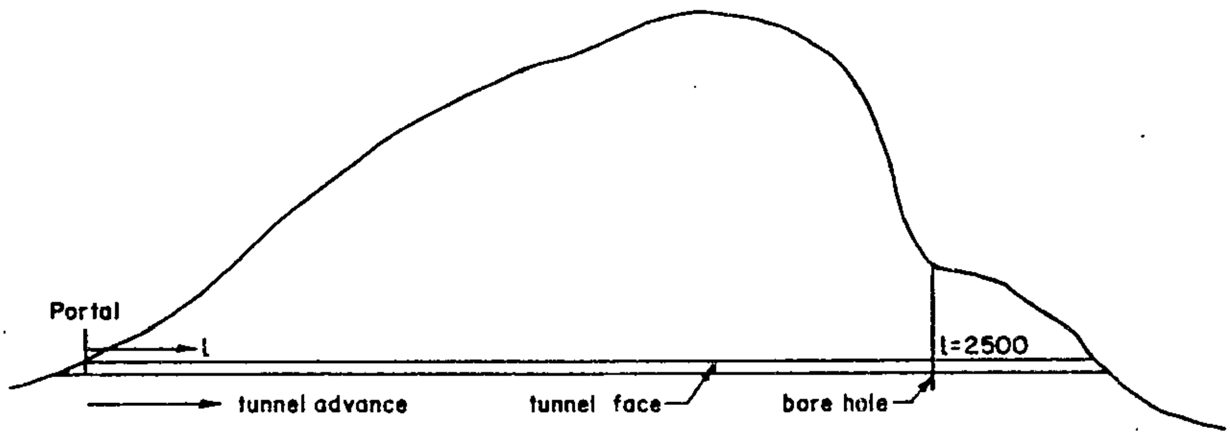


Figure 4.5 Observation using bore drilling

A borehole was drilled to explore the ground conditions at a distance of 2500 ft. along the tunnel axis (i.e. at l = 2500) from the portal. At the intersection of the bore hole and the tunnel axis it was found that the rock type was Granite and that the rock was moist.

Hence a deterministic observation on Rock Type is made at l = 2500. On the other hand, the exploration result on the availability of water is imperfect and subjective judgment is needed. Observing the geologic environment around l = 2500 and other information including records from the excavated part of the tunnel, the following PMF is subjectively assigned to the Availability of Water W at l = 2500 :

P[low availability of water] = P[w(2500)=1] = 0.2

P[medium availability of water] = P[w(2500)=2] = 0.6

P[high availability of water] = P[w(2500)=3] = 0.2

The above deterministic observation on Rock Type can now be used to "update" (or improve) the predictions on its states and state extents. Also the non-deterministic observation on Availability of Water can be used to update the predictions on its states and state extents. The details of updating based on different kinds and combinations of observations are presented in sections 4.2.1 to 4.2.3 below.

4.2.1 State prediction at a point

4.2.1.1 No observations

This is the base case. At the face l_0 (Fig. 4.6) of the tunnel, the state of a parameter X is i. The probability that at position l ($> l_0$) the state is j is given by the interval transition probability (section 3.3.1)

$$P[x(l)=j \mid x(l_0)=i] = v_{Xij}(l-l_0)$$

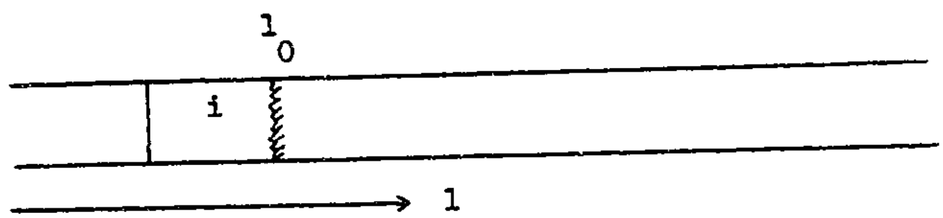


Figure 4.6 State prediction at position l.

4.2.1.2 One deterministic observation

There is a deterministic observation at l_1 ($x(l_1)=k$, see Fig. 4.7.) The probability that at l ($>l_0$) the state is j is given by the updated interval transition probability

$$v^d_{Xij}(l-l_0) = P[x(l)=j \mid x(l_0)=i,\ x(l_1)=k]$$

(The superscript "d" stands for "deterministic".)

For $l_0 \leq l < l_1$,

$$v^d_{Xij}(l-l_0)$$

$$= \frac{P[x(l)=j \mid x(l_0)=i]\ P[x(l_1)=k \mid x(l_0)=i,\ x(l)=j]}{P[x(l_1)=k \mid x(l_0)=i]} \qquad \ldots\ldots (4.1)$$

Since

$$P[x(l)=j \mid x(l_0)=i] = v_{Xij}(l-l_0),$$

$$P[x(l_1)=k \mid x(l_0)=i,\ x(l)=j] = P[x(l_1)=k \mid x(l)=j] = v_{Xjk}(l_1-l) \quad \text{and}$$

$$P[x(l_1)=k \mid x(l_0)=i] = v_{Xik}(l_1-l_0),$$

(4.1) becomes

$$v^d_{Xij}(l-l_0) = \frac{v_{Xij}(l-l_0)\ v_{Xjk}(l_1-l)}{v_{Xik}(l_1-l_0)}$$

For $l > l_1$,

$$v^d_{Xij}(l-l_0) = P[x(l)=j \mid x(l_0)=i,\ x(l_1)=k]$$
$$= P[x(l)=j \mid x(l_1)=k]$$
$$= v_{Xkj}(l-l_1)$$

Thus

$$v^d_{Xij}(l-l_0) = \begin{cases} \dfrac{v_{Xij}(l-l_0)\ v_{Xjk}(l_1-l)}{v_{Xik}(l_1-l_0)} & (l_0 \leq l < l_1) \\ v_{Xkj}(l-l_1) & (l \geq l_1) \end{cases} \quad \text{..... (4.2)}$$

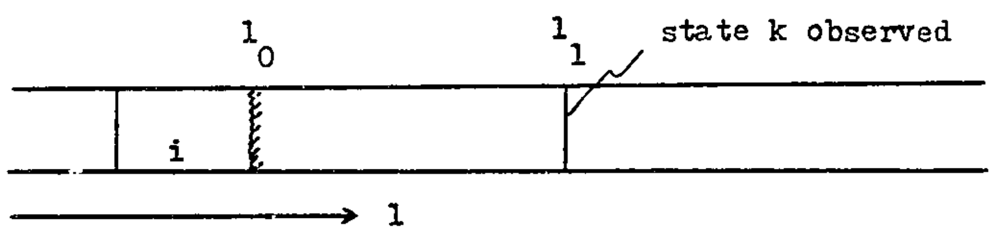


Figure 4.7 Case with one deterministic observation.

4.2.1.3 One non-deterministic observation

In the case that the observation at l_1 is non-deterministic (see Fig. 4.8) but is expressed in the probabilistic form (n is the total number of states)

$$P[x(l_1)=m] = p_{1m} \quad (m = 1, 2, \ldots n), \quad \text{..... (4.3)}$$

$v_{Xij}(l-l_0)$ is updated to $v^n_{Xij}(l-l_0)$ where (the superscript "n" stands for "non-deterministic")

$$v^n_{Xij}(l-l_0) = \sum_{k=1}^{n} P[x(l_1)=k]\ P[x(l)=j \mid x(l_0)=i, x(l_1)=k]$$

$$= \sum_{k=1}^{n} p_{lk}\ v^d_{Xij}(l-l_0) \quad \ldots\ldots \quad (4.4)$$

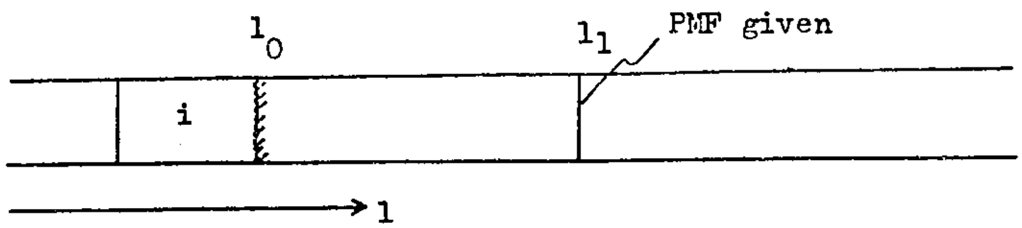


Figure 4.8 Case with a non-deterministic observation.

4.2.1.4 Several deterministic observations

When there are several determinisic observations at l_1, l_2, ... l_s (Fig. 4.9) such that

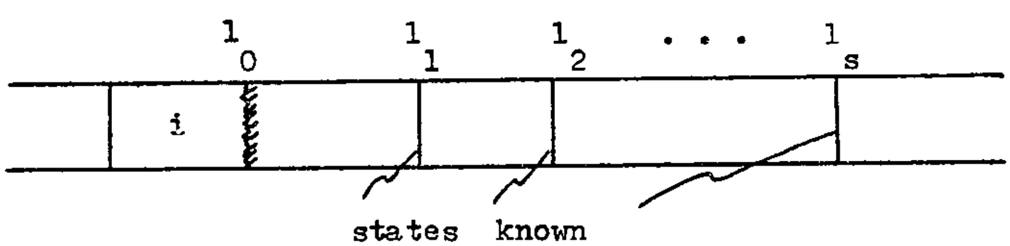


Figure 4.9 Several deterministic observations.

$$x(l_t) = k_t \qquad (t=1,\ 2,\ \ldots s), \quad \ldots\ldots \quad (4.5)$$

$v_{Xij}(l-l_0)$ is updated to $v^{ds}_{Xij}(l-l_0)$ ("ds" stands for deterministic-several.) Due to the assumption of a single-step memory, $v^{ds}_{Xij}(l-l_0)$ is dependent on the known states immediately preceding and following the position l. Thus for $l_0 \leq l < l_1$,

$$v^{ds}_{Xij}(l-l_0) = P[x(l)=j \mid x(l_0)=i,\ x(l_1)=k_1]$$

which is the same probability given by (4.1) with $k=k_1$. Again because of single-step memory,

for $l_{t-1} \leq l < l_t \quad (t = 2, 3, \ldots s)$,

$$v^{ds}_{Xij}(l-l_0) = P[x(l)=j \mid x(l_{t-1})=k_{t-1}, x(l_t)=k_t]$$

$$= \frac{P[x(l)=j \mid x(l_{t-1})=k_{t-1}]\ P[x(l_t)=k_t \mid x(l_{t-1})=k_{t-1}, x(l)=j]}{P[x(l_t)=k_t \mid x(l_{t-1})=k_{t-1}]}$$

$$= \frac{v_{Xkt-1j}(l-l_{t-1})\ P[x(l_t)=k_t \mid x(l)=j]}{v_{Xkt-1kt}(l_t-l_{t-1})}$$

$$= \frac{v_{Xkt-1j}(l-l_{t-1})\ v_{Xjkt}(l_t-l)}{v_{Xkt-1kt}(l_t-l_{t-1})}$$

For $l \geq l_s$,

$$v^{ds}_{Xij}(l-l_0) = P[x(l)=j \mid x(l_s)=k_s]$$

$$= v_{Xksj}(l-l_s)$$

Thus to sum up (k is equal to i, the state at the tunnel face),

$$v^{ds}_{Xij}(l-l_0) = \begin{cases} \dfrac{v_{Xkt-1j}(l-l_{t-1})\ v_{Xjkt}(l_t-l)}{v_{Xkt-1kt}(l_t-l_{t-1})} & (l_{t-1} \leq l < l_t) \\ & (t=1,\ldots,s) \\ v_{Xksj}(l-l_s) & (l \geq l_s) \end{cases} \qquad \ldots\ldots (4.6)$$

4.2.1.5 Several non-deterministic observations

There are s non-deterministic observations at l_1, l_2,

... l_s (see Fig. 4.10) which are given by the PMF's

$$P[x(l_t)=m] = p_{tm} \quad (m=1,\ldots n)$$
$$(t=1,\ldots s) \quad \ldots\ldots \quad (4.7)$$

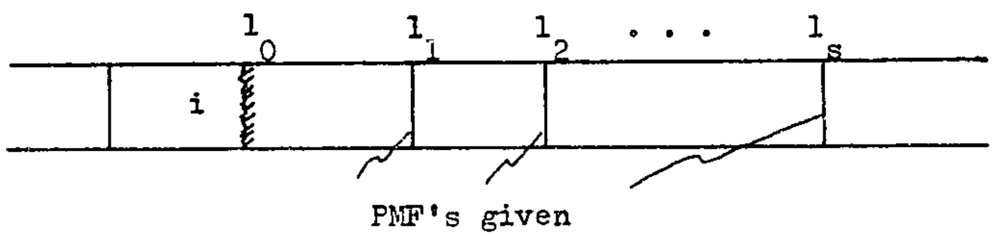


Figure 4.10 Several non-deterministic observations.

$v_{Xij}(l-l_0)$ is updated to $v^{ns}_{Xij}(l-l_0)$ which depends on the non-deterministic observations immediately preceding and following position l ("ns" stands for "non- deterministic-several") :

For $l_{t-1} \leqslant l < l_t$ (t=1,2...s),

$$v^{ns}_{Xij}(l-l_0) = P[x(l)=j \mid \text{Observations at } l_{t-1} \text{ and } l_t]$$

$$= \sum_{m=1}^{n} p_{tm-1}\, P[x(l)=j \mid x(l_{t-1})=m, \text{observation at } l_t]$$

(let $p_{0i} = 1$ and $p_{0r} = 0$ for $r = i$)

$$= \sum_{m=1}^{n} p_{t-1m} \sum_{k=1}^{n} p_{tk}\, P[x(l)=j \mid x(l_{t-1})=m, x(l_t)=k]$$

where

$$P[x(l)=j \mid x(l_{t-1})=m, x(l_t)=k]$$

$$= \frac{P[x(l)=j \mid x(l_{t-1})=m]\; P[x(l_t)=k \mid x(l_{t-1})=m, x(l)=j]}{P[x(l_t)=k \mid x(l_{t-1})=m]}$$

$$= \frac{v_{Xmj}(l-l_{t-1})\; v_{Xjk}(l_t-l)}{v_{Xmk}(l_t-l_{t-1})}$$

For $l \geqslant l_s$,

$$v^{ns}_{Xij}(l-l_0) = P[x(l)=j \mid \text{Observation at } l_s]$$

$$= \sum_{k=1}^{n} p_{sk} P[x(l)=j \mid x(l_s)=k]$$

$$= \sum_{k=1}^{n} p_{sk}\ v_{Xkj}(l-l_s)$$

To sum up,

$$v^{ns}_{Xij}(l-l_0)$$

$$= \begin{cases} \displaystyle\sum_{m=1}^{n} p_{t-1m} \sum_{k=1}^{n} \frac{p_{tk} v_{Xmj}(l-l_{t-1})\ v_{Xjk}(l_t-l)}{v_{Xmk}(l_t-l_{t-1})} \\ \quad (l_{t-1} \leq l < l_t;\ t = 1, \ldots s) \\ \displaystyle\sum_{k=1}^{n} p_{sk}\ v_{Xkj}(l-l_s) \quad (l \geq l_s) \end{cases} \quad \ldots\ldots \quad (4.8)$$

4.2.2 Extent distribution

4.2.2.1 No observations

This is the base case. At a certain point l_e a parameter X enters into state i (see Fig. 4.11.) The CDF of the extent HX_i at l_e is given by (section 3.2)

$$F_{HXi}(h) = 1 - e^{-c_{Xi} h} \quad \ldots\ldots \quad (4.9)$$

The PDF is

$$f_{HXi}(h) = c_{Xi} e^{-c_{Xi} h} \quad \ldots\ldots \quad (4.10)$$

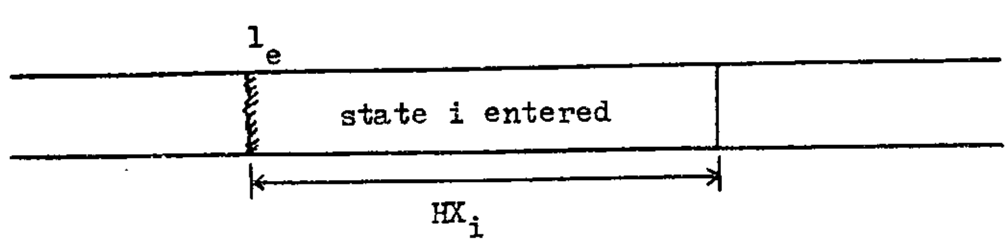


Figure 4.11 Transition to state i encountered.

If excavation continues up to $l = l_0 > l_e$ (see Fig. 4.12), and state i still persists, the updated CDF is

$$F'_{HXi}(h) = P[HX_i \leq h \mid HX_i \geq l_0 - l_e]$$

For $h > l_0 - l_e$,

$$F'_{HXi}(h) = \frac{P[HX_i \leq h \text{ and } HX_i \geq l_0 - l_e]}{P[HX_i \geq l_0 - l_e]}$$

$$= \frac{F_{HXi}(h) - F_{HXi}(l_0 - l_e)}{1 - F_{HXi}(l_0 - l_e)}$$

$$= \frac{1 - e^{-c_{Xi} h} - [1 - e^{-c_{Xi}(l_0 - l_e)}]}{1 - [1 - e^{-c_{Xi}(l_0 - l_e)}]}$$

$$= 1 - e^{-c_{Xi}[h-(l_0-l_e)]}$$

Therefore $F'_{HXi}(h)$

$$= \begin{cases} 0 & (h \leq l_0 - l_e) \\ 1 - e^{-c_{Xi}[h-(l_0-l_e)]} & (h > l_0 - l_e) \end{cases} \quad \ldots\ldots (4.11)$$

and $f'_{HXi}(h)$

$$= \begin{cases} 0 & (h \leq l_0 - l_e) \\ c_{Xi} e^{-c_{Xi}[h-(l_0-l_e)]} & (h > l_0 - l_e) \end{cases} \quad \ldots\ldots (4.12)$$

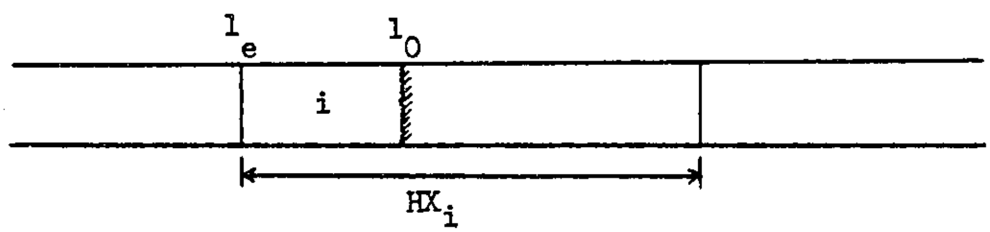
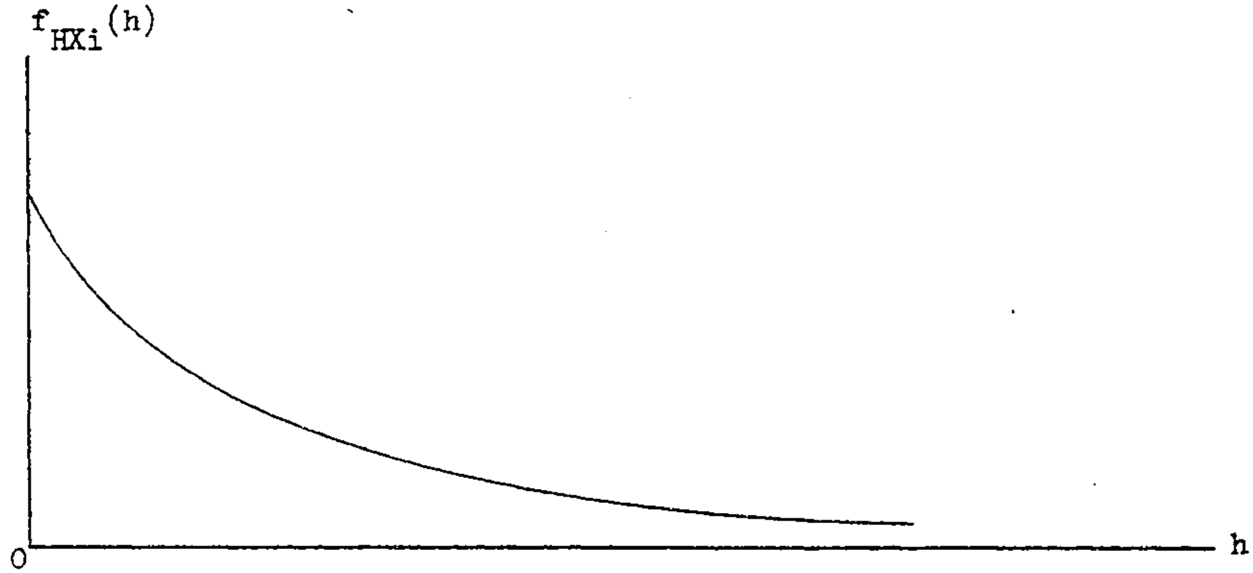
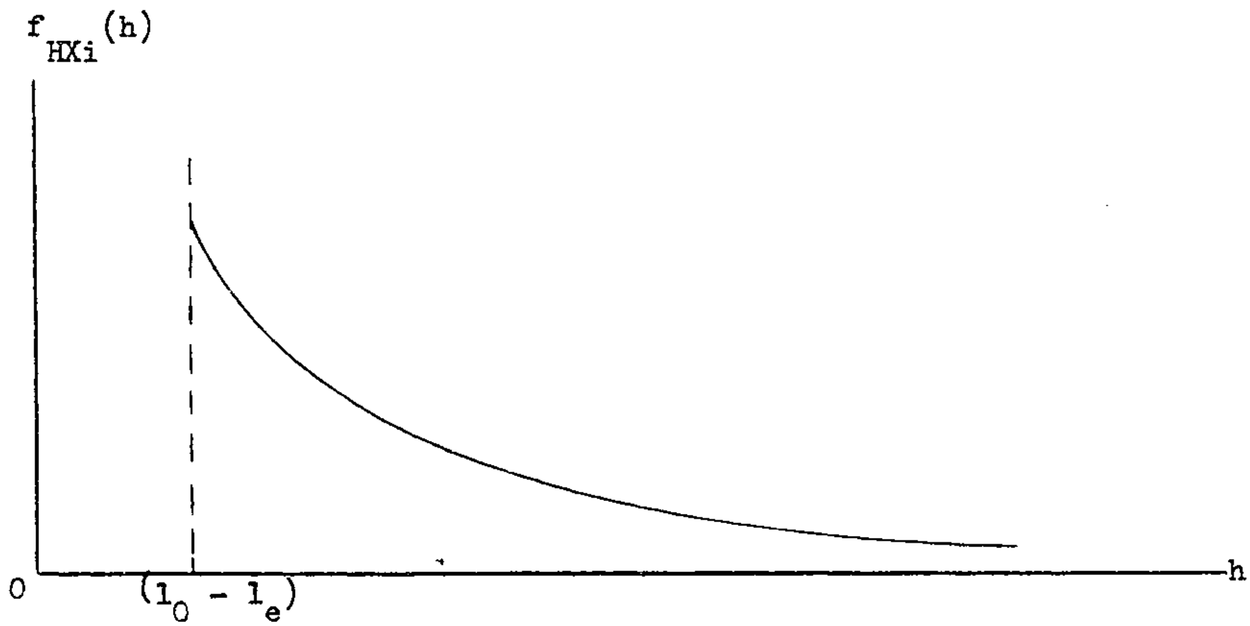


Figure 4.12 State i persists as excavation proceeds.

Thus the exponential shapes of the CDF and PDF are maintained with a shift of magnitude $(l_0 - l_e)$ (see Fig. 4.13.) the PDF vanishes for $h < l_0 - l_e$ because the extent



(a) PDF of HX_i as parameter enters state i at l_e.



(b) Updated PDF of HX_i as excavation proceeds from l_e to l_0 and no transition occurs.

Figure 4.13 Comparison of prior and updated PDF's of HX_i.

must be greater than that value (Fig. 4.12.)

4.2.2.2 One deterministic observation

If there is a determinisic observation as shown in Fig. 4.14 and $k \neq i$, $f'_{HXi}(h)$ is updated to

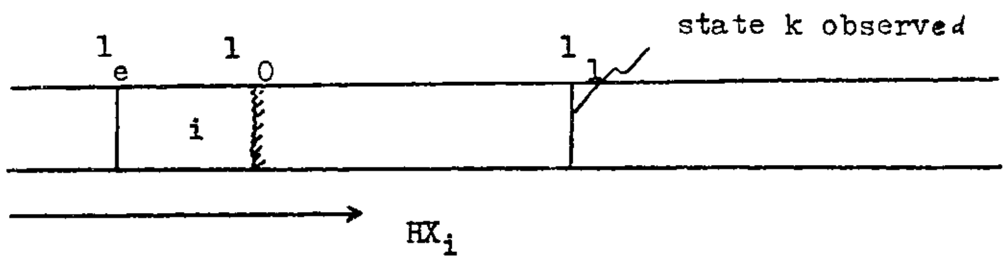


Figure 4.14 Case with one deterministic observation.

$$f^{d}_{HXi\neq k}(h) = \frac{f'_{HXi}(h)\ P[x(l_1)=k \mid HX_i=h]}{P[x(l_1)=k \mid x(l_0)=i]}$$

where $P[x(l_1)=k \mid HX_i=h]$

$= P[x(l_1)=k \mid \text{state i exits at } (l_e+h)]$.

After state i exits, X can enter any other state b and can then be in state k at l_1,

$$P[x(l_1)=k \mid HX_i=h] = \sum_{b=1}^{n} P_{Xib}\ v_{Xbk}(l_1-l_e-h)$$

Therefore for $l_0-l_e \leq h < l_1-l_e$,

$$f^{d}_{HXi\neq k}(h) = \frac{f'_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\ v_{Xbk}(l_1-l_e-h)}{v_{Xik}(l_1-l_0)} \quad \ldots\ldots \quad (4.13)$$

For $h < l_0-l_e$ or $h \geq l_1-l_e$,

$$f^{d}_{HXi\neq k}(h) = 0 \quad \ldots\ldots \quad (4.14)$$

If k=i, there is a possibility that state i will persist past the point l_1. The updated PDF is found using Bayesian updating :

$$f^{d}_{HXi=k}(h) = C\, f'_{HXi}(h)\ [\text{likelihood of observation} \mid HX_i = h] \qquad \ldots\ldots \quad (4.15)$$

[likelihood of observation$\mid HX_i = h$]

$$= P[x(l_1)=i \mid \text{state } i \text{ exits at } (l_e+h)]$$

$$= \begin{cases} \sum_{b=1}^{n} P_{Xib}\ v_{Xbi}(l_1 - l_e - h) & (l_0 - l_e \leq h < l_1 - l_e) \\ 1 & (h \geq l_1 - l_e) \end{cases}$$

where P_{Xib} is the transition probability of parameter X from state i to state b.

Therefore, from (4.15),

$$f^{d}_{HXi=k}(h) = \begin{cases} 0 & (h < l_0 - l_e) \\ C\, f'_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\ v_{Xbi}(l_1 - l_e - h) & (l_0 - l_e \leq h < l_1 - l_e) \\ C\, f'_{HXi}(h) & (h \geq l_1 - l_e) \end{cases} \qquad \ldots\ldots \quad (4.16)$$

C is a normalising constant such that

$$\int_0^{\infty} f^{d}_{HXi=k}(h)\, dh = 1$$

which implies

$$C = \left[\int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\, v_{Xbi}(l_1 - l_e - h)\, dh + \int_{l_1 - l_e}^{\infty} f'_{HXi}(h)\, dh \right]^{-1}$$

where $\displaystyle\int_{l_1 - l_e}^{\infty} f'_{HXi}(h)\, dh$

$$= F'_{HXi}(\infty) - F'_{HXi}(l_1 - l_e)$$

$$= 1 - [1 - e^{-c_{Xi}(l_1-l_e)}] = e^{-c_{Xi}(l_1-l_e)}$$

$$\text{Therefore } C = \left[\int_{l_0-l_e}^{l_1-l_e} f'_{HXi}(h) \sum_{b=1}^{n} p_{Xib}\, v_{Xbi}(l_1-l_e-h)\, dh + e^{-c_{Xi}(l_1-l_e)} \right]^{-1} \qquad \text{..... (4.17)}$$

If $v_{Xij}(u)$ is already found in closed form using exponential transforms or spectral resolution (see section 3.3.1), then (4.17) can be calculated in a closed form.

As an illustration, the shapes of $f'_{HXi}(h)$, $f^d_{HXi \neq k}(h)$ and $f^d_{HXi=k}(h)$ are plotted in Fig. 4.15 for comparison. $f'_{HXi}(h)$ is a shifted exponential distribution given by (4.12). It has zero probability density for $h \leq l_0-l_e$ because the tunnel face is at l_0 and the extent of state i must be greater than (l_0-l_e). $f^d_{HXi \neq k}(h)$ is the updated extent distribution given that the observation at l_1 is k ($\neq$i.) It has zero probability density for $h > l_1-l_e$ because HX_i cannot persist up to point l_1 at which the state is k ($\neq$i.) $f^d_{HXi=k}(h)$ is the updated extent distribution given that the observation at l_1 is k (=i.) There is a non-zero probability that $HX_i > l_1-l_e$ because HX_i can persist past point l_1. There is a jump at $h = l_1-l_e$ due to the observation at l_1. As HX_i approaches (l_1-l_e) from the left (i.e. from lower values), there is a decreasing probability that HX_i ends before l_1. The probability that HX_i ends at any point near to l_1 given that X is in state i again at l_1 is small : X has to make at least one further transition between that point and l_1 to

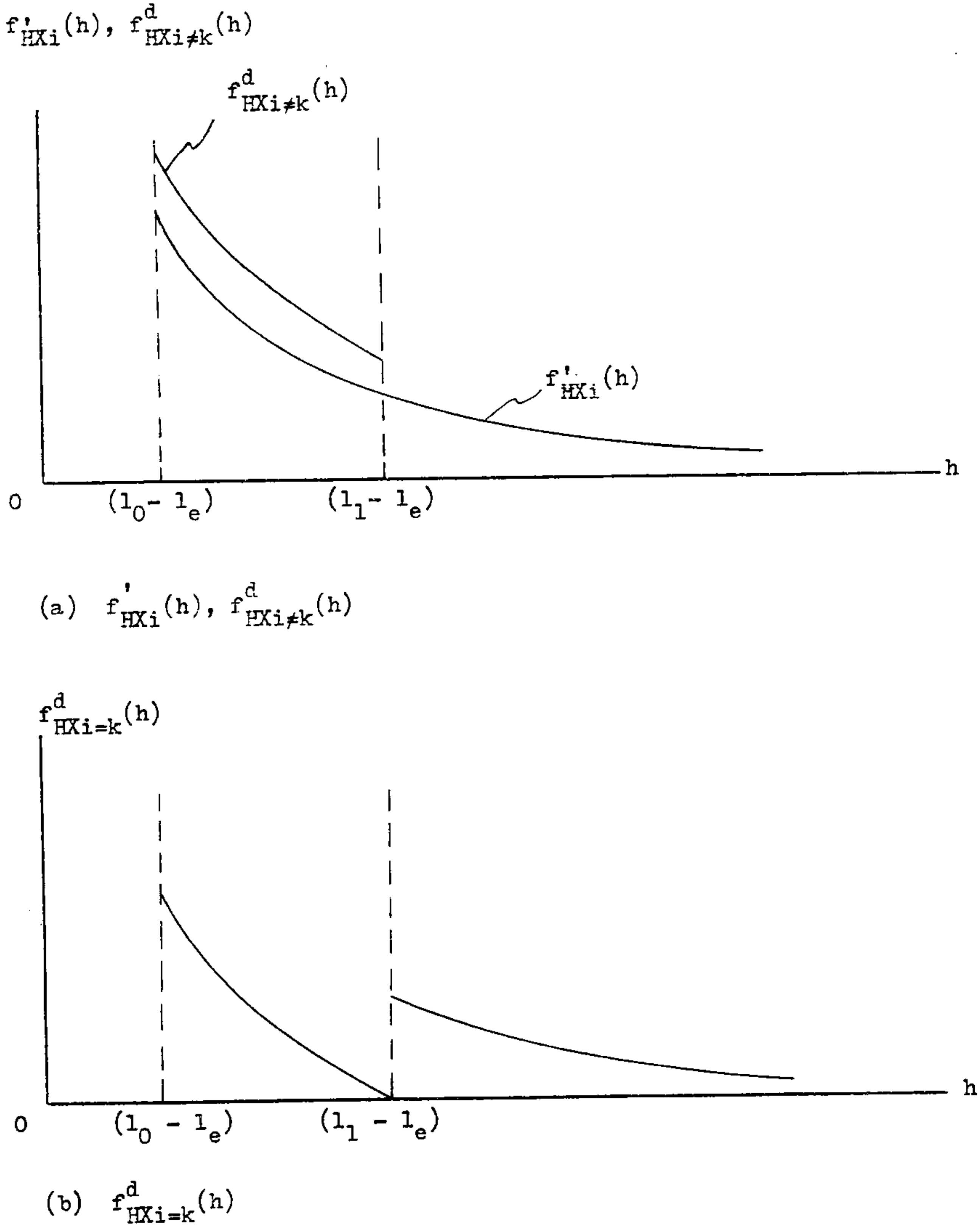


Figure 4.15 Shapes of different updated PDF's of extent.

go back to state i. This fact can easily be seen by taking the limit as h approaches (l_1-l_e) from the left in (4.16) :

$$\lim_{h\to(l_1-l_e)^-} f^{d}_{HXi=k}(h) = C\, f_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\, v_{Xbi}(0)$$

$$= C\, f'_{HXi}(h)\ [\sum_{\substack{b=1\\ b\neq i}}^{n} P_{Xib}\, v_{Xbi}(0) + P_{Xii}\, v_{Xii}(0)]$$

$$= C\, f'_{HXi}(h)\ (0 + 0) = 0$$

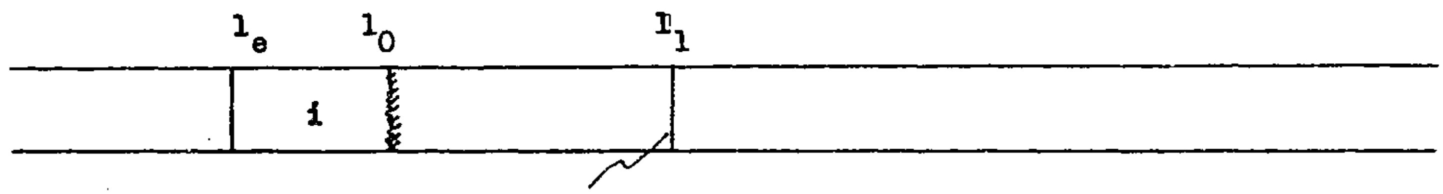
Therefore the probability density approaches zero as h approaches (l_1-l_e) from the left. For $h \geqslant (l_1-l_e)$, state i at l_1 is "connected" with state i at l_e. There is a non-zero probability that this happens and so the probability density jumps from zero to some finite value at l_1. The extent distribution after (l_1-l_e) again takes an exponential shape, as can readily be seen in (4.15).

4.2.2.3 One non-deterministic observation

When the observation at l_1 is non-deterministic as shown in Fig. 16, the PDF of extent is updated to $f^{n}_{HXi}(h)$ which is given by

$$f^{n}_{HXi}(h) = P[x(l_1)=1]\ (\text{updated PDF}|\,x(l_1)=1)$$
$$+ \ldots$$
$$+ P[x(l_1)=n]\ (\text{updated PDF}|\,x(l_1)=n)$$
$$= \sum_{k=1}^{n} p_{1k}\ (\text{updated PDF}|\,x(l_1)=k)$$

$$= \sum_{\substack{k=1\\ k\neq i}}^{n} p_{1k}(\text{updated PDF}|\,x(l_1)=k\neq i) + p_{1i}(\text{updated PDF } x(l_1)=i)$$



PMF given :

$$P[x(l_1) = m] = p_{1m}$$

$(m = 1, 2, \ldots n)$

where n = total number of states of X.

Figure 4.16 Case with one non-deterministic observation.

Using the two updated PDF's derived in (4.13) and (4.15),

$$f^{n}_{HXi}(h) = \sum_{\substack{k=1 \\ k \neq i}}^{n} p_{1k} f^{d}_{HXi \neq k}(h) + p_{1i}\, f^{d}_{HXi=k}(h) \qquad \ldots\ldots (4.18)$$

4.2.2.4 Several deterministic observations

There is a combination (K) of s deterministic observations (see Fig. 4.17) such that

$$x(l_t) = k_t \qquad (t=1,2,\ldots s.)$$

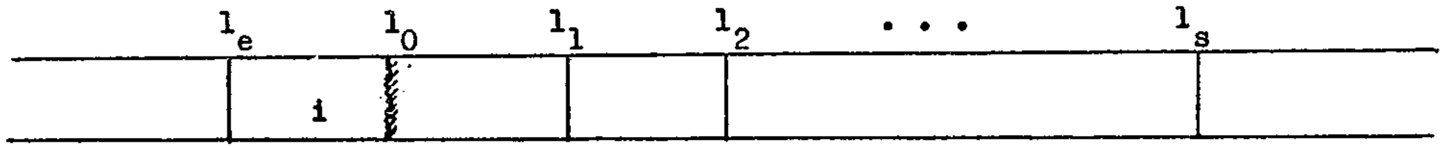
It should be noted that if $x(l_t) = k_t \neq i$, then the extent cannot persist past point l_t and observations at points after l_t have no effect on the PDF of HX_i. The updated PDF when a combination K of observations is given is denoted by $f^{K}_{HXi}(h)$.

If state i is not observed at all of the points l_1, l_2, ... l_s, let l_t be the first point where state i is not observed i.e. $x(l_t)=k_t \neq i$ and $k_{t-1} = k_{t-2} = \ldots = k_1 = i$. The updated PDF is given by

$$f^{K}_{HXi}(h) = \begin{cases} C\, f'_{HXi}(h)\ [\text{likelihood of K given } HX_i = h] & (l_0 - l_e < h < l_t - l_e) \\ 0 & (h < l_0 - l_e \text{ or } h > l_t - l_e) \end{cases} \qquad \ldots\ldots (4.19)$$

where [likelihood of K given $HX_i = h$]

$$= P[x(l_1)=k_1, \ldots x(l_s)=k_s \mid HX_i = h]$$



The state at l_t ($t = 1, 2, \ldots s$) is known :

$$x(l_t) = k_t$$

Figure 4.17 Case with s deterministic observations.

$$= \begin{cases} \sum_{b=1}^{n} P_{Xib} v_{Xbk1}(l_1 - l_e - h)\, v_{Xk1k2}(l_2 - l_1) \ldots v_{Xks-1ks}(l_s - l_{s-1}) & (l_0 - l_e \leq h < l_1 - l_e) \\ \sum_{b=1}^{n} P_{Xib}\, v_{Xbk2}(l_2 - l_e - h)\, v_{Xk2k3}(l_3 - l_2) \ldots v_{Xks-1ks}(l_s - l_{s-1}) & (l_1 - l_e \leq h < l_2 - l_e) \\ \vdots & \vdots \\ \sum_{b=1}^{n} P_{Xib}\, v_{Xbkt}(l_t - l_e - h)\, v_{Xktkt+1}(l_{t+1} - l_t) \ldots v_{Xks-1ks}(l_s - l_{s-1}) & (l_{t-1} - l_e \leq h < l_t - l_e) \end{cases}$$

Since $v_{Xktkt+1}(l_{t+1} - l_t) \ \ldots \ v_{Xks-1ks}(l_s - l_{s-1})$ is a common factor in each of the above expressions, let

$$C_K = C\, v_{Xktkt+1}(l_{t+1} - l_t) \ \ldots \ v_{Xks-1ks}(l_s - l_{s-1}).$$

Then from (4.19) and replacing $k_1, k_2, \ldots k_{t-1}$ by i,

$$f^{K}_{HXi}(h) = \begin{cases} 0 & (h < l_0 - l_e) \\ C_K\, f'_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\, v_{Xbi}(l_1 - l_e - h)\, v_{Xii}(l_2 - l_1) \ldots v_{Xikt}(l_t - l_{t-1}) & (l_0 - l_e \leq h < l_1 - l_e) \\ C_K\, f'_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\, v_{Xbi}(l_2 - l_e - h)\, v_{Xii}(l_3 - l_2) \ldots v_{Xikt}(l_t - l_{t-1}) & (l_1 - l_e \leq h < l_2 - l_e) \\ \vdots & \vdots \\ C_K\, f'_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\, v_{Xbkt}(l_t - l_e - h) & (l_{t-1} - l_e \leq h < l_t - l_e) \\ 0 & (h \geq l_t - l_e) \end{cases} \quad \ldots\ldots \quad (4.20)$$

where C_K = normalising constant

$$= \Big[\int_{l_0-l_e}^{l_1-l_e} f'_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\, v_{Xbi}(l_1-l_e-h) \ldots v_{Xikt}(l_t-l_{t-1})\, dh$$

$$+ \quad \ldots$$

$$+ \int_{l_{t-1}-l_e}^{l_t-l_e} f'_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\, v_{Xbkt}(l_t-l_e-h)\, dh \Big]^{-1} \quad \ldots\ldots \quad (4.21)$$

If state i is observed at all the points l_1, l_2, ... l_s (i.e. $k_1=k_2= \ldots =k_s= i$), then

$$f^K_{HXi}(h) = \begin{cases} C_K\, f'_{HXi}(h)\ [\text{likelihood of K given } HX_i = h] & (h \geq l_0 - l_e) \\ 0 & (h < l_0 - l_e) \end{cases} \quad \ldots\ldots \quad (4.22)$$

where [likelihood of K given HX_i =h]

$$= \begin{cases} \sum_{b=1}^{n} P_{Xib}\, v_{Xbi}(l_1-l_e-h)\, v_{Xii}(l_2-l_1)\ldots v_{Xii}(l_s-l_{s-1}) & (l_0-l_e \leq h < l_1-l_e) \\ \sum_{b=1}^{n} P_{Xib}\, v_{Xbi}(l_2-l_e-h)\, v_{Xii}(l_3-l_2)\ldots v_{Xii}(l_s-l_{s-1}) & (l_1-l_e \leq h < l_2-l_e) \\ \vdots & \vdots \\ \sum_{b=1}^{n} P_{Xib}\, v_{Xbi}(l_s-l_e-h) & (l_{s-1}-l_e < h < l_s-l_e) \\ 1 & (h \geq l_s-l_e) \end{cases}$$

Then from (4.22),

$$f^{K}_{HXi}(h) = \begin{cases} 0 & (h < l_0 - l_e) \\ C_K\, f'_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\, v_{Xbi}(l_1 - l_e - h) \ldots v_{Xii}(l_s - l_{s-1}) & (l_0 - l_e \leqslant h < l_1 - l_e) \\ C_K\, f'_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\, v_{Xbi}(l_2 - l_e - h) \ldots v_{Xii}(l_s - l_{s-1}) & (l_1 - l_e \leqslant h < l_2 - l_e) \\ \vdots & \vdots \\ C_K\, f'_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\, v_{Xbi}(l_s - l_e - h) & (l_{s-1} - l_e \leqslant h < l_s - l_e) \\ C_K\, f'_{HXi}(h) & (h \geqslant l_s - l_e) \end{cases} \quad \ldots\ldots \quad (4.23)$$

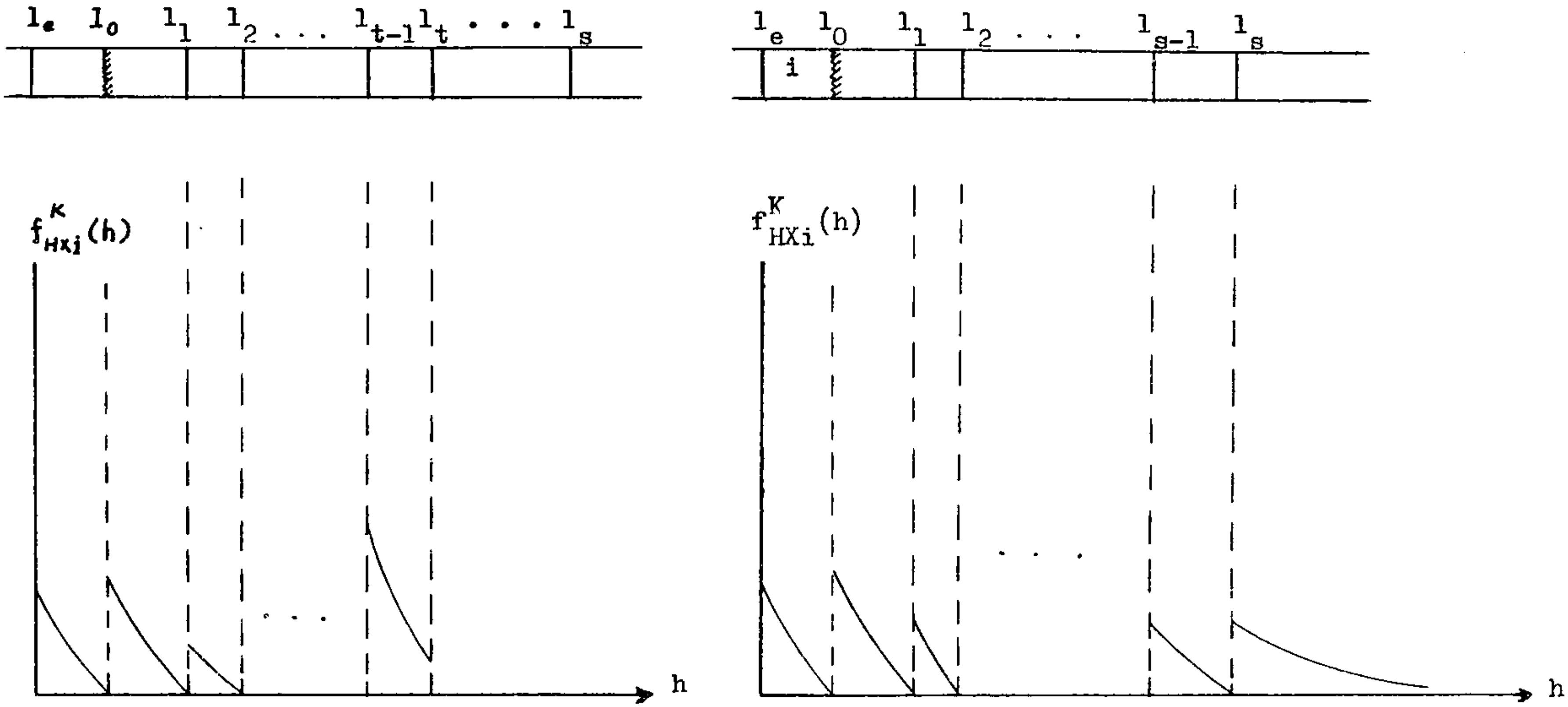
where C_K = normalising constant

$$= \left[\int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h) \sum_{b=1}^{n} P_{Xib}\, v_{Xbi}(l_1 - l_e - h) \ldots v_{Xii}(l_s - l_{s-1})\, dh \right.$$

$$+ \quad \ldots$$

$$\left. + \int_{l_s - l_e}^{\infty} f'_{HXi}(h)\, dh \right]^{-1} \quad \ldots\ldots \quad (4.24)$$

The shapes of the updated PDF's are shown in Fig. 4.18. In (4.20) state i is observed at l_1, $l_2 \ldots l_{t-1}$ while in (4.23) state i is observed at all points l_1, ... l_s. In both PDF's the probability density drops to zero when h approaches each point of observation (where the state is i) from the left. The probability that HX_i ends near to a point where the state is observed to i is small because if it does so X has to make at least one further transition to go back to state i at the point of observation. The main difference between (4.20) and (4.23)



(a) $k_1=k_2= \ldots =k_{t-1}=i\neq k_t$

(b) $k_1=k_2= \ldots =k_s$

Figure 4.18 Updated extent distribution given several determinisitc observations.

is that (4.23) has a "tail" which extends to infinity while (4.20) has zero probability density for $h > l_t$ because $k_t \neq i$.

4.2.2.5 Several non-deterministic observations

When there are several non-deterministic observations as shown in Fig. 4.19, the extent distribution is updated to $f^N_{HXi}(h)$ which can be found by considering all combinations of observations on X at l_1, l_2, ... l_s. There are n x n x ... n = $(n)^s$ different combinations altogether. Let K_m (m = 1, 2, ... $(n)^s$) represent a combination such that

$x(l_1) = K_{m1}$, $x(l_2) = K_{m2}$, ..., $x(l_s) = K_{ms}$. According to the PMF's given by the observations,

$$P[K_m \text{ occurs}] = p_{Km}$$

$$= P[x(l_1)=K_{m1},\ x(l_2)=K_{m2}, \ldots,\ x(l_s)=K_{ms}]$$

$$= p_{1Km1}\ p_{2Km2} \cdots p_{sKms}$$

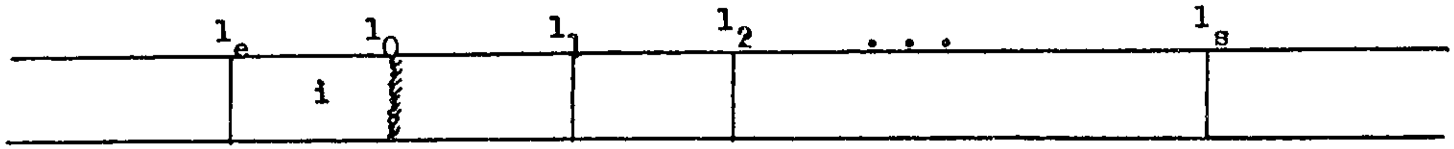
Then

$$f^N_{HXi}(h) = \sum_{m=1}^{n^s} P[K_m \text{ occurs}] \text{ (updated PDF given } K_m \text{ occurs)}$$

$$= \sum_{m=1}^{n^s} p_{Km}\ f^{Km}_{HXi}(h) \qquad \ldots\ldots \quad (4.25)$$

If $K_{m1} = K_{m2} = \ldots = K_{ms} = i$, then (4.23) is used for $f^{Km}_{HXi}(h)$. Otherwise (4.20) is used.

4.2.3 Prediction of next state - transition probability



The observation at 1_t $(t = 1, 2, \ldots s)$ is non-deterministic and is given by the PMF

$$P\left[x(1_t) = m\right] = p_{tm}$$

$$(m = 1, 2, \ldots n)$$

where n = total number of states of X.

Figure 4.19 Case with s non-deterministic observations.

4.2.3.1 No observations

This is the base case. Given that the state of a parameter X at the tunnel face is i, the probability that the next state is j is P_{Xij} (see section 3.2.2) which is a basic component of the Markov process.

4.2.3.2 One deterministic observation

If there is a deterministic observation ahead of the tunnel face as shown in Fig. 4.20, the transition probability P_{Xij} is updated to P^d_{Xij} where

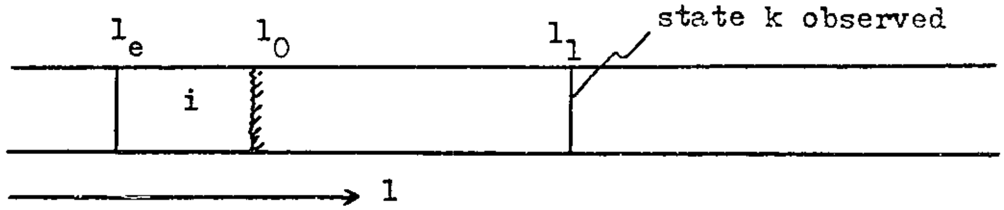


Figure 4.20 Case with one deterministic observation.

$$P^d_{Xij} = P[\text{next state (after i) is } j \mid x(l_1)=k]$$

$$= C\, P_{Xij} [\text{likelihood of } x(l_1) \mid \text{next state is } j] \quad \ldots\ldots \quad (4.26)$$

The next state (after i) is the state just following state i.

In the case that k≠i,

[likelihood of $x(l_1)$ | next state is j]

= P[extent of state i at l_e terminates at some point

between l_0 and l_1 and the next state is j

and $x(l_1)$ = k | next state is j]

$$= \int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h)\ v_{Xjk}(l_1 - l_e - h)\ dh$$

Therefore from (4.26)

$$P^d_{Xij} = C\ P_{Xij} \int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h)\ v_{Xjk}(l_1 - l_e - h)\ dh \quad \ldots\ldots (4.27)$$

where C = normalising constant

$$= \left[\sum_{j=1}^{n} P_{Xij} \int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h)\ v_{Xjk}(l_1 - l_e - h)\ dh \right]^{-1} \quad \ldots\ldots (4.28)$$

In the case that k = i,

[likelihood of $x(l_1)$ | next state is j]

= P[extent of state i at $l_e \geqslant l_1 - l_e$]

+ P[extent of state i at l_e terminates at some point between l_0 and l_1 and the next state is j and $x(l_1) = i$ | next state is j]

$$= \int_{l_1 - l_e}^{\infty} f'_{HXi}(h)\ dh + \int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h)\ v_{Xji}(l_1 - l_e - h)\ dh$$

$$= e^{-c_{Xi}(l_1 - l_0)} + \int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h)\ v_{Xji}(l_1 - l_e - h)\ dh$$

Therefore from (4.26),

$$P^d_{Xij}$$

$$= C\ P_{Xij} \left[e^{-c_{Xi}(l_1 - l_0)} + \int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h)\ v_{Xji}(l_1 - l_e - h)\ dh \right] \quad \ldots\ldots (4.29)$$

where C = normalising constant

$$= \left\{ \sum_{j=1}^{n} P_{Xij} \left[e^{-c_{Xi}(l_1 - l_0)} + \int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h) v_{Xji}(l_1 - l_e - h) dh \right] \right\}^{-1} \quad \ldots\ldots (4.30)$$

4.2.3.3 One non-deterministic observation

When there is a non-deterministic observation as shown in Fig. 4.21, P_{Xij} is updated to P^{n}_{Xij} where

$$P^{n}_{Xij} = \sum_{k=1}^{n} P[x(l_1)=k]\ P[\text{next state is } j \mid x(l_1)=k]$$

$$= \sum_{k=1}^{n} p_{1k}\ P^{d}_{Xij} \qquad \ldots\ldots \quad (4.31)$$

4.2.3.4 Several deterministic observations

When there is a combination (K) of s deterministic observations (see Fig. 4.22), the updated transition probability is denoted by P^{K}_{Xij}. If state i is not observed at all of the points l_1, l_2, ... l_s, let l_t be the first point where state i is not observed i.e. $x(l_t) = k_t \neq i$ and $k_{t-1} = k_{t-2} = \ldots = k_1 = i$. The updated transition probability is

$P^{K}_{Xij} = C\ P_{Xij}$ [likelihood of K given next state is j]

where [likelihood of K given next state is j]

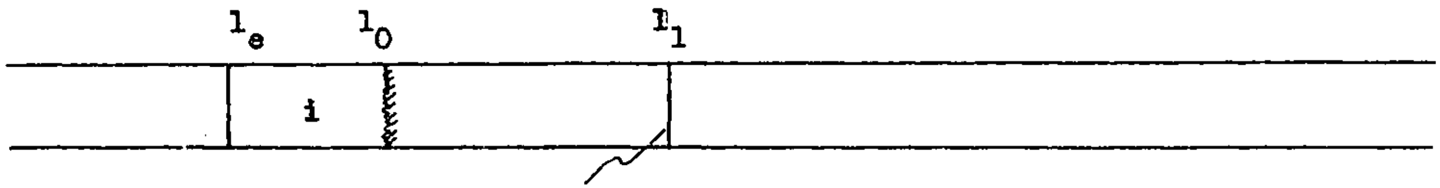
$= P[x(l_1) = k_1,\ x(l_2) = k_2,\ \ldots\ x(l_t) = k_t \mid \text{next state is } j]$

= P[(extent HX_i terminates between l_0 and l_1 and K occurs)

or (HX_i terminates between l_1 and l_2 and K occurs) or ...

or (HX_i terminates between l_{t-1} and l_t and K occurs)

given next state is j]



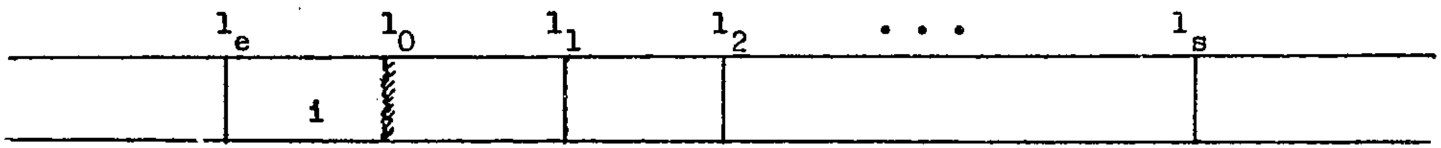
PMF given :

$P[x(l_1)] = m = p_{1m}$

$(m = 1, 2, \ldots n)$

where n = total number of states of X.

Figure 4.2.1 Case with one non-deterministic observation.



The state at l_t $(t = 1, 2, \ldots s)$ is known :

$$x(l_t) = k_t$$

Figure 4.22 Case with s deterministic observations.

$$= \int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h)\, v_{Xji}(l_1 - l_e - h)\, dh\, v_{Xii}(l_2 - l_1) \ldots v_{Xktkt+1}(l_{t+1} - l_t) \ldots v_{Xks-1ks}(l_s - l_{s-1})$$

$$+ \ldots$$

$$+ \int_{l_{t-1}}^{l_t} f'_{HXi}(h)\, v_{Xjkt}(l_t - l_e - h)\, dh\, v_{Xktkt+1}(l_{t+1} - l_t) \ldots v_{Xks-1ks}(l_s - l_{s-1})$$

Since $v_{Xktkt+1}(l_{t+1} - l_t) \ldots v_{Xks-1ks}(l_s - l_{s-1})$ is a common factor, let $C_K = C\, v_{Xktkt+1}(l_{t+1} - l_t) \ldots v_{Xks-1ks}(l_s - l_{s-1})$.

Then from (4.32),

$$P^K_{Xij}$$

$$= C_K P_{Xij} \Big[\int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h)\, v_{Xji}(l_1 - l_e - h)\, dh\, v_{Xii}(l_2 - l_1) \ldots v_{Xikt}(l_t - l_{t-1})$$

$$+ \ldots$$

$$+ \int_{l_{t-1}}^{l_t} f'_{HXi}(h)\, v_{Xjkt}(l_t - l_e - h)\, dh \Big] \quad \ldots\ldots (4.33)$$

where C_K = normalising constant

$$= \left\{ \sum_{j=1}^{n} P_{Xij} \Big[\int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h)\, v_{Xji}(l_1 - l_e - h)\, dh\, v_{Xii}(l_2 - l_1) \ldots v_{Xikt}(l_t - l_{t-1}) \right.$$

$$+ \ldots$$

$$\left. + \int_{l_{t-1}}^{l_t} f'_{HXi}(h)\, v_{Xjkt}(l_t - l_e - h)\, dh \Big] \right\}^{-1} \quad \ldots\ldots (4.34)$$

If state i is observed at all points $l_1, l_2 \ldots l_s$, then

[likelihood of K given next state is j]

$= P[$(extent HX_i terminates after l_s)

or (extent terminates between l_0 and l_1 and K occurs)

or ...

or (HX_i terminates between l_{s-1} and l_s and K occurs)

given next state is j]

$$= \int_{l_s - l_e}^{\infty} f'_{HXi}(h)\, dh$$
$$+ \int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h)\, v_{Xji}(l_1 - l_e - h)\, dh\; v_{Xii}(l_2 - l_1) \ldots v_{Xii}(l_s - l_{s-1})$$
$$+ \ldots + \int_{l_{s-1}}^{l_s} f'_{HXi}(h)\, v_{Xji}(l_s - l_{s-1})\, dh$$

Then from (4.32) and using C_K in place of C

$$P^K_{Xij}$$
$$= C_K P_{Xij} [e^{-c_{Xi}(l_s - l_0)}$$
$$+ \int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h)\, v_{Xji}(l_1 - l_e - h)\, dh\; v_{Xii}(l_2 - l_1) \ldots v_{Xii}(l_s - l_{s-1})$$
$$+ \ldots$$
$$+ \int_{l_{s-1}}^{l_s} f'_{HXi}(h)\, v_{Xji}(l_s - l_{s-1})\, dh] \quad \ldots\ldots (4.35)$$

where C_K = normalising constant

$$= \Bigg\{ \sum_{j=1}^{n} P_{Xij} [e^{-c_{Xi}(l_s - l_0)}$$
$$+ \int_{l_0 - l_e}^{l_1 - l_e} f'_{HXi}(h)\, v_{Xji}(l_1 - l_e - h)\, dh\; v_{Xii}(l_2 - l_1) \ldots v_{Xii}(l_s - l_{s-1})$$
$$+ \ldots$$
$$+ \int_{l_{s-1}}^{l_s} f'_{HXi}(h)\, v_{Xji}(l_s - l_{s-1})\, dh] \Bigg\}^{-1} \quad \ldots\ldots (4.36)$$

4.2.3.5 Several non-deterministic observations

When there are several non-deterministic observations as shown in Fig. 4.23, the transition probability is updated to P^{N}_{Xij} which can be found by considering all combinations of observations at l_1, l_2, ... l_s. There are $n \times n \times \ldots n = (n)^s$ different combinations altogether. Let K_m (m=1, 2, ... n) be a combination such that

$x(l_1)=K_{m1},\ x(l_2)=K_{m2},\ \ldots\ x(l_s)=K_{ms}.$

According to the PMF's given by the observations,

$P[K_m \text{ occurs}] = p_{Km}$

$= P[x(l_1)=K_{m1},\ x(l_2)=K_{m2},\ \ldots,\ x(l_s)=K_{ms}]$

$= p_{1Km1}\ p_{2Km2} \cdots p_{sKms}$

Then

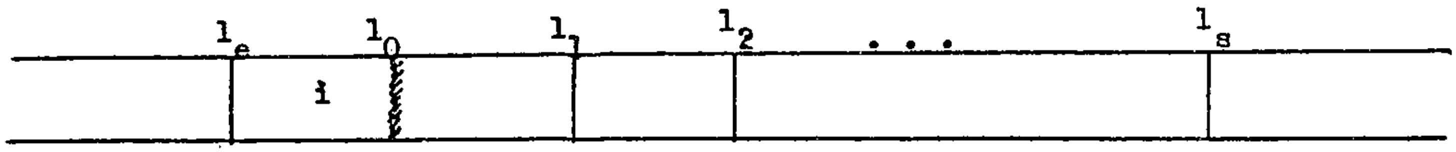
$$P^{N}_{Xij} = \sum_{i=1}^{n^s} P[K_m \text{ occurs}] \text{ (updated PMF given } K_m \text{ occurs)}$$

$$= \sum_{i=1}^{n^s} p_{Km}\, P^{Km}_{Xij} \qquad \ldots\ldots \quad (4.37)$$

If $K_{m1}=K_{m2}=\ldots=K_{ms}=i$, then (4.35) is used for P^{Km}_{Xij}. Otherwise (4.33) is used.

4.3 Ground class formation

So far some important calculations concerning a single geological parameter have been discussed. Subsequently, all the parameters predicted have to be combined to yield ground class predictions. Let $\overline{g}(l)$ be the vector of geological parameters at point l :



The observation at l_t $(t = 1, 2, \ldots s)$ is non-deterministic and is given by the PMF

$$P\left[x(l_t) = m\right] = p_{tm}$$

$(m = 1, 2, \ldots n)$

where n = total number of states of X.

Figure 4.23 Case with s non-deterministic observations.

$\overline{g}(l) = (r(l)\ f(l)\ d(l)\ w(l)\ \ldots)$,

where r(l), f(l), d(l), w(l) ... are the states of Rock Type, Faulting, Degree of Jointing, Availability of Water ... at l respectively. Thus given $\overline{g}(l)$ the ground class at l can be determined. The probability that the ground class at a point is GCi is P[$\overline{g}(l)$ belongs to GCi]. Suppose according to the ground class classification a certain ground class GCi contains the geological vectors $\overline{g}_{i1}$, $\overline{g}_{i2}$, ... $\overline{g}_{im}$ such that

$$\overline{g}_{ij} = (r_{ij}\ f_{ij}\ d_{ij}\ w_{ij}\ \ldots)$$
$$(j = 1, 2, \ldots m.)$$

For example GC1 contains 2 vectors :

$$\overline{g}_{11} = (r_{11}\ \ f_{11}\ \ d_{11}\ \ w_{11})$$
$$= (1\quad 1\quad 1\quad 1)$$
$$\overline{g}_{12} = (r_{12}\ \ f_{12}\ \ d_{12}\ \ w_{12})$$
$$= (2\quad 1\quad 1\quad 1)$$

where r = 1 means Diorite;

r = 2 means Quartzite;

f = 1 means no faulting;

d = 1 means low degree of jointing;

w = 1 means low availability of water.

The probability of having GC1 at l is then

P[$\overline{g}(l)$ belongs to GC1]

$= P[\overline{g}(l) = \overline{g}_{11} \text{ or } \overline{g}(l) = \overline{g}_{12}]$

$= P[\overline{g}(l) = \overline{g}_{11}] + P[\overline{g}(l) = \overline{g}_{12}]$

$= P[r(l)=1 \text{ and } f(l)=1 \text{ and } d(l)=1 \text{ and } w(l)=1]$

$+ P[r(l)=2 \text{ and } f(l)=1 \text{ and } d(l)=1 \text{ and } w(l)=1]$

If the above parameters are independent,

$P[\bar{g}(1)$ belongs to GC1]

$$= P[r(1)=1]*P[f(1)=1]*P[d(1)=1]*P[w(1)=1]$$
$$+ P[r(1)=2]*P[f(1)=1]*P[d(1)=1]*P[w(1)=1] \quad .....(4.38)$$

Thus ground class probability calculations can be reduced to that of single parameters in the case with independent parameters. For interdependent parameters the problem is much more complicated and is discussed in section 4.4 below.

4.4 Parameter interdependences

When the list of geological parameters (see section 2.2) is examined, it can be seen that there may be probabilistic interdependences (correlation) among most of them. For example, in the tunnel region faulting may occur more frequently in one rock type than in another. Thus the probability prediction of one parameter can also depend on the status of the other parameters. This implies that the transition intensity matrix of a parameter may depend on the states of other parameters.

When this problem of parameter interdependences has to be incorporated considerable complications arise. One approximate solution to this problem is to assume a hierarchy of dependences in which the parameters are arranged in order of decreasing average extents. The average extent of a parameter is defined to be the overall average of the average extents of its states i.e. average extent of X = $(1/c_{X1} + 1/c_{X2} + \ldots + 1/c_{Xn})/n$. Suppose the hierarchy-sequence is X1, X2, X3, ... XN where the average extent of XI is greater than that of XJ for I < J. Then the assumption of hierarchy of dependences states that XI is probabilistically independent of XJ while XJ may depend on XI for I < J. Thus X1 is an "independent" random variable while X2 may depend on X1 and X3 may depend on X1 and/or X2 and so on. Us ally it should be sufficient to assume a certain parameter XJ to be dependent on at most two other parameters higher on the list of hierarchy. Examples of possible parameter interdependences are :

(1) High availability of water exists in a region of high degree of jointing more frequentoy i.e. ground water is more "available" to the tunnel in highly jointed rock than usual.

(2) Degree of jointing in a certain rock may usually be higher than that in another.

It should be noted that the assumption of hierarchy of dependences is intrinsically contradictory because if XJ is dependent on XI then XI should also depend on XJ. An example is : given that gas is found more frequently in Schist than in other rocks, then, if gas is found in a certain place, the rock at that place is more probable to be Schist than usual. However, the hierarchy assumption can be shown to be a reasonable approximation by considering two parameters with different average extents as shown in Fig. 4.24. In a region where x1 = 1, X2 is governed by a Markov process with a certain transition intensity matrix $\underline{A}_{X2,1}$ while in another region where x1 = 2, X2 is governed by another Markov process with $\underline{A}_{X2,2}$. However, the dependence of $\underline{A}_{X1}$ on X2 is much less significant since a state of X1 can easily outlast several states of X2 and the dependence is weakened.

In assuming a hierarchy of dependences, the probability calculations (states, extents) for X1 are the same as before (where there were no dependences) but those for X2 still depend on the states of X1. For the situation in Fig. 4.24, probability calculations concerning X2 are grouped into 3 regions separated by l_1, l_2, l_3 and l_4 as shown. For region 1, $\underline{A}_{X2,1}$ is used up to point l_2. Then another Markov process for X2 starts at l_2 and $\underline{A}_{X2,3}$ is used, while in region 3 $\underline{A}_{X2,2}$ is employed. Thus regions 1, 2, and 3 are three different "homogeneous regions" for X2 (recall that $\underline{A}_X$ is constant in a homogeneous region of X.) With the hierarchy assumption, ground class profiles can be simulated using Monte Carlo simulation methods introduced in section 4.5 below.

4.5 Monte Carlo simulation of tunnel profile

Another very useful result of the prediction model is

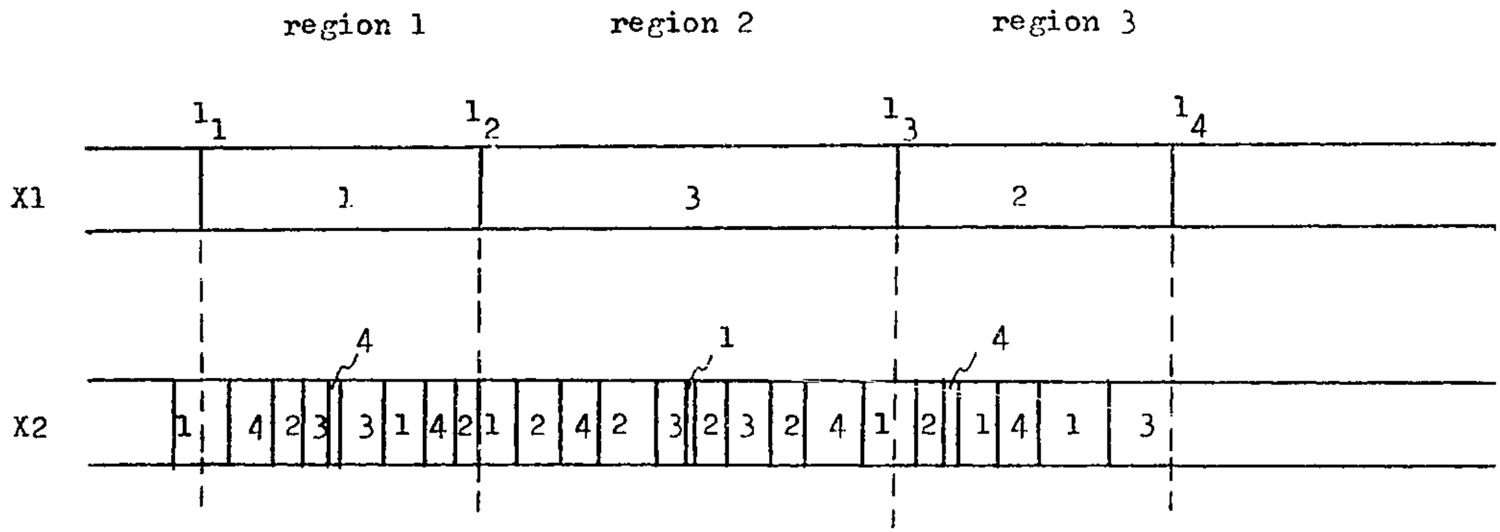


Figure 4.24 Two parameters with different average extents.

the straight- forward Monte Carlo simulation of the tunnel profile. The concept of the Monte Carlo method is simple : a large number of experiments on a random variable X are carried out according a given probability distribution of X. The outcomes of the experiments should follow approximately the same distribution as the given probability distribution. For example, if X is the numner on the top face of an unbiased dice after it is thrown, the PMF of X follows a uniform distribution :

$$P_X(x) = 1/6 = \text{constant}$$
$$(x = 1,\ 2,\ \ldots\ 6)$$

If the dice is thrown 1000 times and the outcome of each throw (experiment) is recorded, the outcomes should also follow approximately a uniform distribution i.e. the number of times each number (1, 2, ...6) comes up is about 167. In actual simulations no physical experiment is necessary but 1000 random numbers are generated instead. Six categories Ci (i = 1, 2, ... 6) are set up such that the probability of a random number generated being in Ci is $P_X(i)$ (= 1/6 in this case.) After each random number is generated it is inspected to determine to which category it belongs. If it belongs to Ci then the outcome of the experiment is i. Thus generating 1000 random numbers is the same as actually throwing the dice the same number of times. By examining the distribution of the outcomes of the 1000 experiments some statistics (e.g. mean, standard deviation) of X can be derived.

If X is continuous (e.g. a state extent), it has to be discretized and its PDF is converted to the corresponding PMF before simulations can be made. Monte Carlo methods are used to simulate parameter and ground class profiles in sections 4.5.1 and 4.5.2 respectively.

4.5.1 Parameter profile simulation

If a parameter X enters state i at l_e as shown in Fig. 4.25(a), the unknown part of the X-profile can be simulated by first simulating the extent at l_e. Then the next state (after i) j is simulated and then its extent also. The next state after j can then be simulated and the process is repeated up to the end of the tunnel (see Fig. 4.25.)

The PDF used to simulate HX_i is given by (4.12), (4.13), (4.16), (4.18), (4.20), (4.23), or (4.25), depending on the kinds of observations ahead of the tunnel face. The PMF used to simulate the next state after state i is given by P_{Xij} :

$$P[\text{next state} = j] = P_{Xij}$$

When there are observations ahead of the tunnel face, the PMF's of the next state used are not the same as (4.27), (4.29), (4.31), (4.33), (4.35) or (4.37) because now there is one more condition : the extent of state i given by simulation is known. The required PMF's with different kinds of observations are derived in Appendix C which can be used to simulate the next state j.

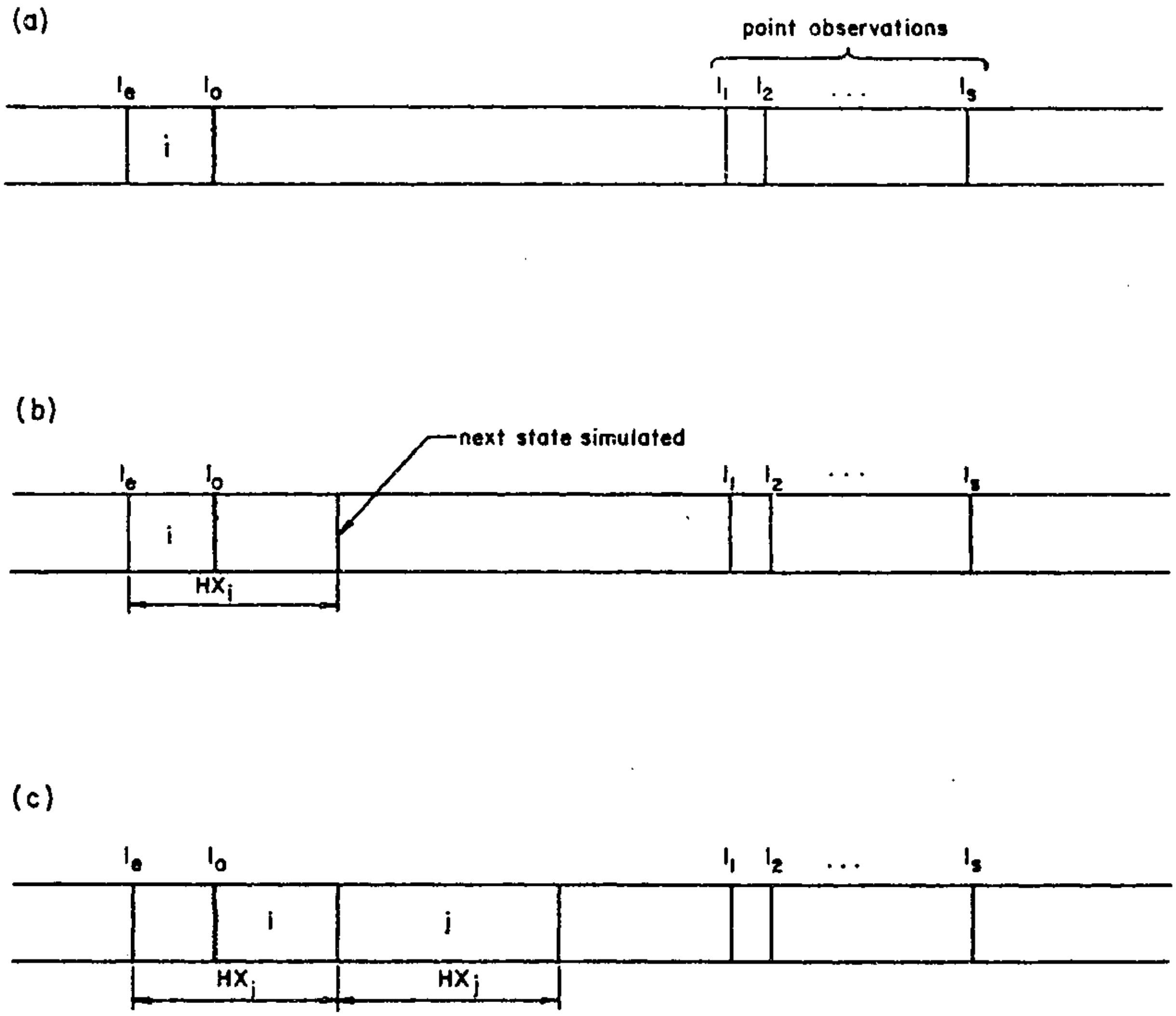


Figure 4.25 Steps in simulation of parameter profile: extent of present state, next state, and the extent of next state

4.5.2 Ground class profile simulation

If parameter interdependences are weak and can be neglected, all the parameter profiles are simulated independently and then combined to form a GC-profile (see Fig. 4.26.) If the parameters are correlated, the hierarchy of dependences is assumed and the parameter profiles are simulated one by one, starting with the independent parameter at the top of the list of hierarchy and moving down. After the first independent parameter (X1) profile is simulated, the homogeneous regions for X2 are determined and a X2-profile can be simulated in each homogeneous region. After X1 and X2-profiles are simulated X3-profile can be simulated and so on until all the parameter profiles are simulated. Then they can be combined to form a GC profile (Fig. 4.26.)

4.6 Summary

The development of the geological prediction model is presented in section 4.1. The Markov process concept is found to be a satisfactory solution to the general requirements of a geological prediction model which are :

(a) Tunnel profiles generated by the prediction model should be compatible with general expectations of the actual profile.

(b) The knowledge on both the general and particular geology of the tunnel region should be incorporated.

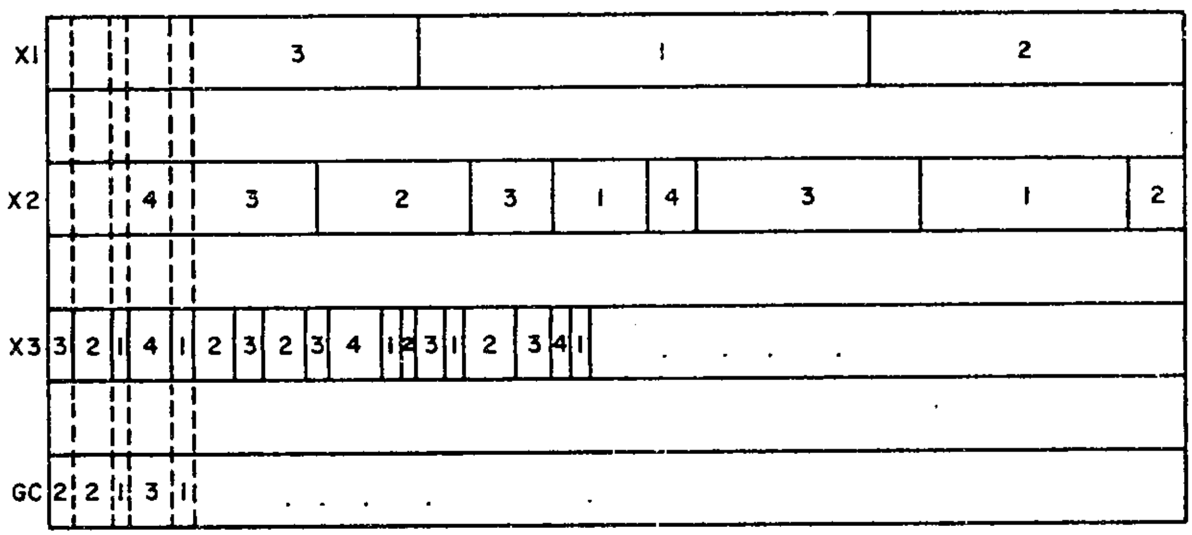


Figure 4.26 Combination of parameter profile to form GC profile

(c) Predictions can be updated as excavation proceeds and more information is gathered.

(d) The prediction and updating processes should be capable of including subjective judgment when necessary.

(e) The prediction model should include all relevant parameters and the entire ranges of their possible states. However, when unexpected important parameters are encountered, the model should be capable of including them also.

The Markov model is then adopted and its assumptions are presented in section 4.1.3, together with the advantages and disadvantages.

The prediction model and its applications are presented in section 4.2. the "time" parameter in the Markov process is equivalent to the distance measured along the tunnel axis from a fixed point such as the portal (Fig. 4.5.) State predictions at a certain point ahead of the tunnel face are given by interval transition probabilities. After the tunnel enters into a certain state, the length in which the tunnel will remain in the same state is predicted probabilistically by extent distributions. The probability of running into a certain state following the state at the tunnel face is given by the transition probabilities. All these three probability distributions (interval transition probability, extent distribution and transition probability) are modified (updated) when there are "observations" of the parameter ahead of the tunnel face. These observations can be "deterministic" (the state is determined at the point of observation) or "non- determinstic" (only the PMF of the parameter at the point of observation is known.) The updated expressions for these probability distributions based on these observations are also formulated.

In section 4.3 probability calculations involving ground classes are presented. Since a ground class is a set of geological vectors (a combination of parameter states which dictates the geological condition) , its PMF can be calculated from the PMF's of the parameters.

The problem of parameter interdependences is discussed in section 4.4. A "hierarchy of dependences" is assumed in which the parameters are arranged in order of decreasing average extent. The transition intensity coefficients and transition probabilities of a parameter in the list may depend on the states of the parameters higher in the list.

The simulation of parameter profiles is presented in section 4.5.1. The extent of the state at the tunnel face is first simulated using expressions for extent distributions given in section 4.2.2. Then the next state (after the state at the tunnel face) is simulated using the expressions for transition probabilities given in Appendix C. The simulation processes are repeated until the entire parameter profile is simulated. If the parameters are independent, simulated ground class profiles (section 4.5.2) can be directly obtained from individually simulated parameter profiles. If the parameters are interdependent, the profile of the first parameter on the hierarchy list is simulated first, then the second parameter profile is simulated and so on. When all the parameter profiles have been simulated, the ground class profile can be obtained. On the whole, simulation methods can be a solution to many complicated cases, especially when there are parameter interdependences.

Chapter V

INPUT REQUIRED FOR THE GEOLOGICAL PREDICTION MODEL

5.1 Introduction

Before the geological prediction model (developed in Chapter IV) is used to make probabilistic predictions, the input required for the model have to be derived. The necessary inputs to the model for a parameter X are the transition probabilities P_{Xij} and the transition intensity coefficients c_{Xi}. Generally there are two ways of assesing the values of c_{Xi} and P_{Xij} : the frequency- based method and the subjective judgment method. These two methods are discussed respectively in sections 5.2 and 5.3 below.

5.2 Frequency-based method

If there are sufficient relevant data, c_{Xi} and P_{Xij} can be estimated directly. The data are relevant if they are recorded in regions in which the same Markov process as that around the tunnel axis governs. Therefore the recorded data should be from a region of similar geology as that of the tunnel. The best data should be from the tunnel region and measured in the direction of the tunnel advance at tunnel grade depth. The amount of data is regarded as sufficient if the statistical significance of a given set of probability values can be tested. Thus to set up the best estimates of P_{Xi1} ,P_{Xi2} ,... P_{Xin},the required number of transitions recorded is usually about 10(n - 1) (n is the total number of states of X; see section 6.2.2.) To calculate the best estimate of c_{Xi}, at least about 10 extents of state i have to be recorded (see section 6.2.1.)

There are two main sources of data : maps and existing tunnel profiles from regions with similar geology as the tunnel region. When maps are used, a line parallel to the tunnel axis is drawn (see Fig. 5.1.) The states of the parameters encountered by the line are recorded and converted into a "transition chain" of the parameters. If the parameters considered are X1, X2, ... XN in order of decreasing extents, the form of a transition chain is shown in Fig. 5.2. If more data (transition chains) are desired, other parallel lines can be drawn, but they have to be at a distance far enough from each other so that essentially the same data are not recorded twice. When there are existing tunnel profiles from regions with similar geology as the tunnel region, it should be noted that only those tunnels with a directon approximately equal to that of the proposed tunnel should be used. Each tunnel profile is regarded as a transition chain.

After all the relevant data are collected in the form of transition chains $\hat{c}_{Xi}$ and $\hat{P}_{Xij}$ (the best estimates of c_{Xi} and P_{Xij} respectively) of a parameter X can readily be calculated. When the parameters are probabilistically independent, each parameter is treated individually (section

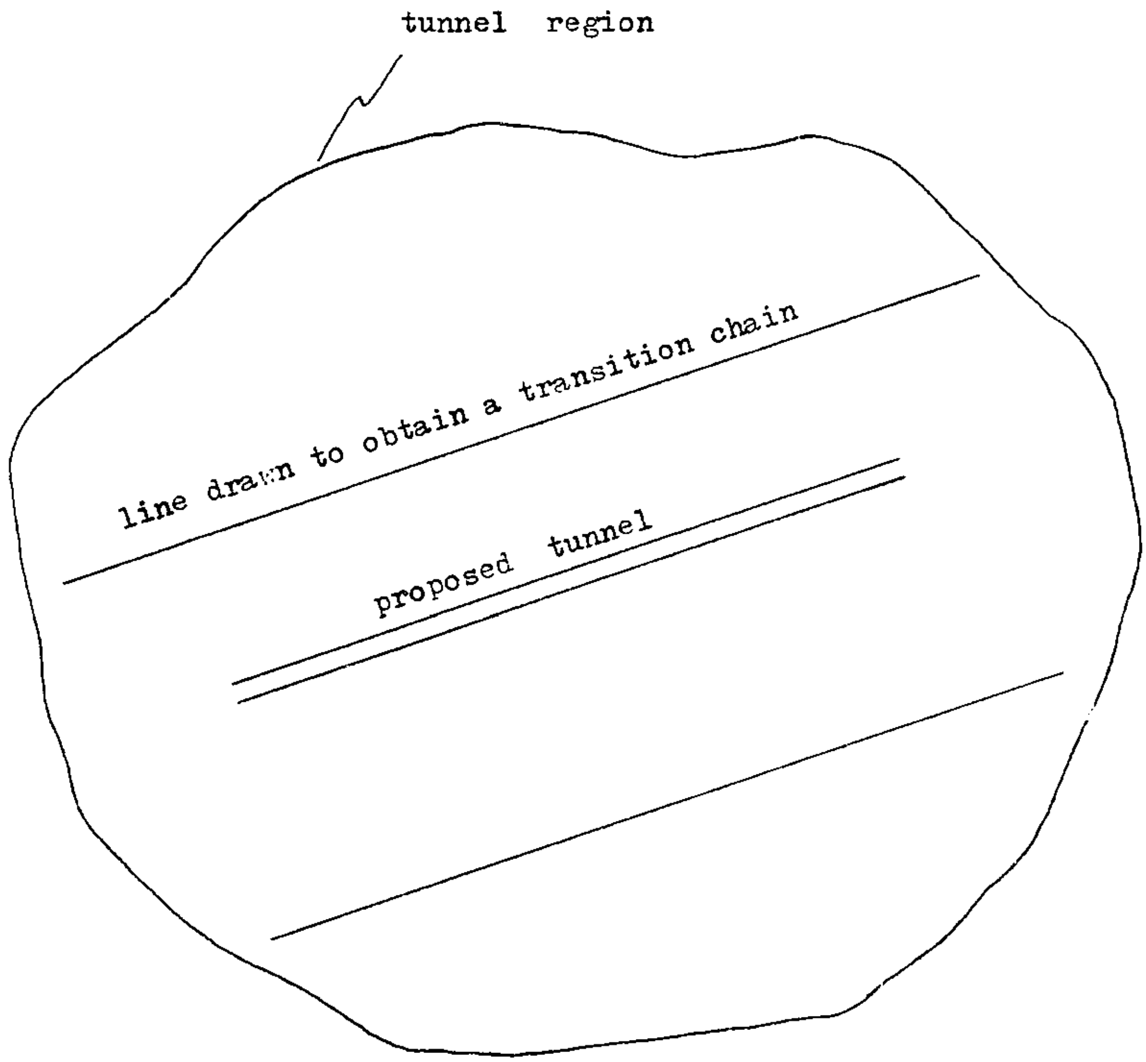


Figure 5.1 To obtain data from a map.

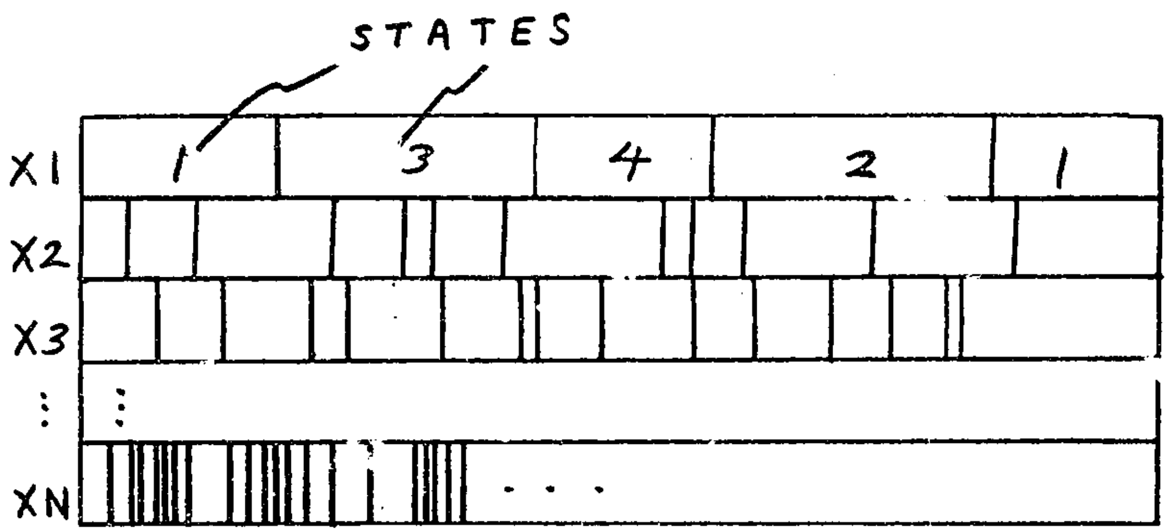


Figure 5.2 A transition chain for X1, X2, ... XN.

5.2.1.) When there are parameter interdependences, a hierarchy of dependences is assumed and parameters X1, X2, ... XN are treated successively (section 5.2.2.)

5.2.1 Independent parameters

To calculate $\hat{c}_{Xi}$, the extents of state i of parameter X from the transition chains are considered. From this sample of extents the average extent can be calculated. Since the extent of state i is exponentially distributed (section 3.2.3) $\hat{c}_{Xi}$ can be taken as the reciprocal of the average extent. Another method is to take the reciprocal of the sample standard deviation as $\hat{c}_{Xi}$. It should be better to use both methods by taking the average of the calculated values of $\hat{c}_{Xi}$.

$\hat{P}_{Xij}$ can be calculated by considering the transitions made by parameter X in a given set of transition chains. Let F_{Xij} be the number of times that a transition is made from state i to state j in the set of transition chains (Fig.5.2.) The number of times that state i appears as the first state of a "transition pair" (i.e. two consecutive states in a transition chain) is B_{Xi} where

$$B_{Xi} = F_{Xi1} + F_{Xi2} + \dots + F_{Xin} \quad \dots\dots \quad (5.1)$$

Therefore the best estimate of P_{Xij} is

$$\hat{P}_{Xij} = F_{Xij}/B_{Xi} \quad \dots\dots \quad (5.2)$$

For i=1, (5.2) can be used to form the best estimates of a "row" of transition probabilities P_{X11} (=0), P_{X12}, ...

P_{xin}. The remaining (n-1) rows of transition probabilities are similarly estimated. The least amount of data required to evaluate the best estimates of a row of transition probabilities is usually about 10(n-1) recorded transitions because the same number of recorded transitions is required to test the statistical significance of a row of transition probabilities (see section 6.2.2.) This means that B_{xi} has to be at least about 10(n-1) so that P_{xi1}, P_{xi2}, ... P_{xin} can be estimated by the frequency-based method. If B_{xi} is less than 10(n-1), the row of transition probabilities should be estimated by the subjective judgment method (section 5.3.)

An interesting point to make is that for a given set of transition chains the recorded frequencies F_{xij} are not independent. Let G_{xi} be the number of times that i appears as the second state of a transition pair. Then

-If state i is never situated at the beginning or end of a transition chain, then B_{xi} is equal to G_{xi} which means that

$$F_{xi1} + F_{xi2} + \ldots + F_{xin} = F_{x1i} + F_{x2i} + \ldots + F_{xni}$$

-If state i appears B_i times at the beginning and E_i times at the end of the chains, then

$$B_{xi} = G_{xi} + B_i - E_i$$

i.e.

$$F_{xi1} + \ldots + F_{xin} = F_{x1i} + \ldots + F_{xni} + B_i - E_i$$

In either of the two cases above there is a linear dependence among (i.e. a linear equation involving) F_{Xi1}, F_{Xi2}, ... F_{Xin}, F_{X1i}, ... F_{Xni}. Therefore for n states there will be n such linear dependences and the number of independent transition frequencies inside a transition chain

$= $ number of non-zero F_{Xij}'s $- n$

$= (n^2 - n) - n = n^2 - 2n$

Since there are also n linear dependences among transition probabilities in the form

$P_{Xi1} + P_{Xi2} + \ldots + P_{Xin} = 1$, ... (5.4)

the number of independent transition probabilities

$= $ number of non-zero P_{Xij}'s $- n$

$= (n^2 - n) - n = n^2 - 2n$.

Therefore the best estimates of P_{Xij} (5.2) fit the data set perfectly. This result invalidates any significance tests using a single set of data (transition chains) to test the appropriateness of the transition probability concept with the assumption of a single-step memory.

5.2.2. Independent parameters

When there are probabilistic interdependences among the parameters, a hierarchy of dependences is assumed (section 4.4.) The parameters are arranged in order of decreasing average extents : X1, X2, ... XN. Since X1 is regarded as an independent parameter, $\hat{c}_{Xi}$ and $\hat{P}_{Xij}$ are evaluated as shown in section 5.2.1. For the other parameters if XI is dependent on XJ, $\hat{P}_{XIij}$ and $\hat{c}_{XIi}$ will have different values in regions with different states of XJ. For example, Degree of Jointing (XI) may depend on Rock Type (XJ). Thus for regions with different rock types, different transition probabilities and transiton intensity coefficients have to be evaluated for Degree of Jointing. Each of these regions is a homogeneous region for XI and $\hat{P}_{XIij}$ and $\hat{c}_{XIi}$ have to be evaluated from the transition chains in the region.

5.3 Subjective judgment method

In actual situations the amount of data available may not be sufficient for the frequency- based method to be used. Under such situation in addition to data expert knowledge (subjective judgment) of the geologists who are familiar with the geology of the tunnel region should be incorporated.

5.3.1 Independent parameters

5.3.1.1 Transition intensity coefficients

To assess the value of c_{Xi}, a geologist (or group of geologists) is basically asked such a question :

"What is the average extent of state i of parameter X in this region at the tunnel grade ?" ..... (Q1)

A concrete example is

"If there are several lengths of granite along the tunnel axis, what would be the average length ? "

c_{Xi} is the taken to be the reciprocal of the estimated average length.

5.3.1.2 Transition probabilities

To estimate P_{Xi1}, P_{Xi2}, ... P_{Xin}, a geologist is basically asked such a question (to assist the geologist, a profile of the tunnel region such as that of Fig 5.3 should be shown also) :

"If state i of X occurs at a certain place along the tunnel axis, what is the probability that the next state is j ?" ..... (Q2)

(The place where state i occurs can be at any location along the tunnel axis; therefore the answer should be independent of location.)

An example is

"If there is a length of low RQD rock along the tunnel axis, what is the probability that the rock next to it is of high RQD ?"

Another example where the term "probability" is avoided is

"If there is length of low RQD rock along the tunnel axis, how many times out of a hundred would it happen that the rock next to it is of high RQD ?"

If the direct assessment of a probability value is difficult, an indirect mode of encoding is for the geologist to choose between two bets (Fig. 5.4.) The geologist is

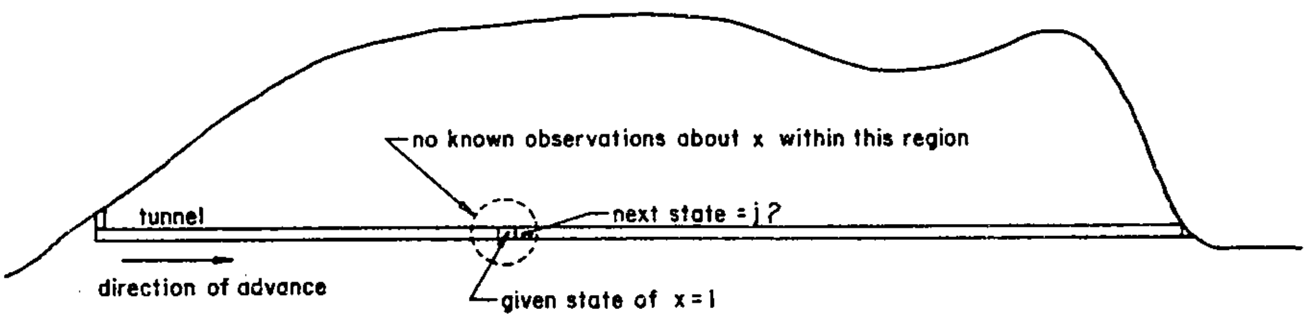


Figure 5.3 Subjective assessment of Pxij

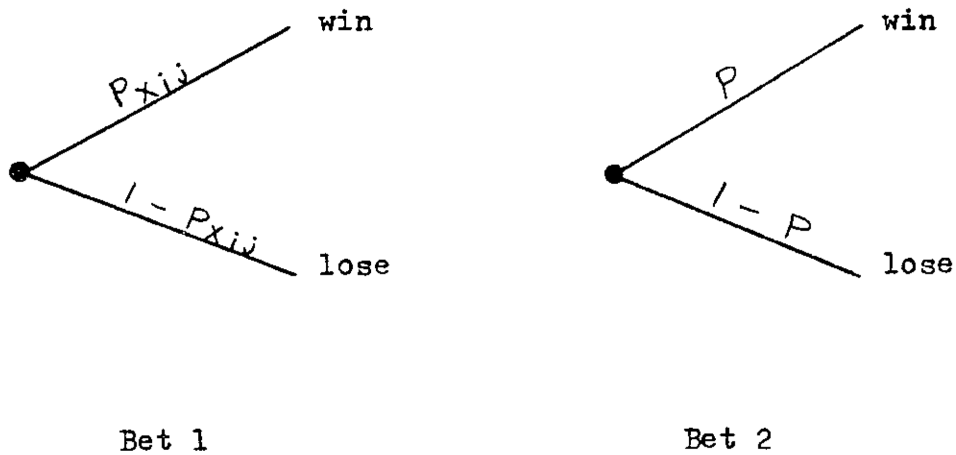


Figure 5.4 Indirect probability encoding.

told that the first bet has a probability of winning equal to the answer to (Q2) (i.e. P_{Xij}.) The second bet has a probability of winning equal to P which is varied until the geologist shows indifference in choosing between the two bets. Fig 5.4 is shown to the geologist and a series of questions are asked for different values of P :

"If the probability of winning the second bet is P, which bet would you prefer ?"

If the geologist prefers bet 1, it means that $P_{Xij} > P$ and if he prefers bet 2, $P > P_{Xij}$. If he is indifferent between the two bets, P is taken to be equal to P_{Xij}. It is better to start the sereis of questions with alternately high and low values of P. An example of the encoding process where seven questions are asked to reach the required probability is :

P	.95	.05	.80
Choice of bet	2 (definitely)	1 (definitely)	2
P	.20	.70	.40
Choice of bet	1 (definitely)	2	1
P	.60		
Choice of bet	(indifferent)		

Thus $P_{Xij} = .6$ in the above example. To help the geologist to "visualise" the probability P, a probability wheel (see Spetzler and Holstein, 1974) may be used. An important precaution when asking (Q2) is to make sure that there are no known observations of the states of X near (e.g. within a distance of $1/c_{Xi}$) the place where state i is assumed to occur (see Fig 5.3.) If there are such observations, the estimate of P_{Xij} would be affected and may not represent the general situation in the tunnel region.

For a parameter X with n states, when a row of transition probabilities (P_{Xi1}, P_{Xi2}, ... P_{Xin}) is being assessed, (n-2) questions of type (Q2) are required. For example, to assess (P_{X21}, P_{X22}, ... P_{X25}) (5 - 2 = 3) questions have to be asked to get P_{X21}, P_{X23}, P_{X24}. The remaining 2 probabilities are given by :

$$P_{X22} = 0$$

$$P_{X25} = 1 - P_{X21} - P_{X23} - P_{X24}$$

When these (n-2) questions are being asked, the geologist may have difficulty in answering some of them. After all the questions to assess the n rows of transition probabilities are asked, let m be the number of questions that cannot be answered by the geologist (i.e. there are m "missing" probabilities.) To obtain these m missing probabilities, the values of v_{Xi} (the limiting probability of state i, which is also the relative percentage of occurence of state i in the region; see section 3.3.3) are assessed. n additional questions for the values of v_{Xi} are asked (i = 1, 2 , 3, ..., n) :

"What is the probability of having state i of parameter X at any given point along the tunnel axis ?" ... (Q3) or alternatively,

"What is the relative percentage of the occurence of state i of parameter X along the tunnel axis ? "

..... (Q4)

After v_{X_i} (i=1,2,....,n) is estimated, the sum (v_{X1} + v_{X2} + ... + v_{Xn}) should be checked. If the sum is not equal to 1, each of the estimate v_{X_i} can be determined by dividing by the sum so that the new sum will be equal to 1.

When the values of v_{X_i} have been determined, equations (3.12) are used to infer the values of $P_{X_{ij}}$:

$$
\begin{aligned}
v_{X1}(-c_{X1}) + v_{X2}(c_{X2}\,P_{X21}) + \ldots + v_{Xn}(c_{Xn}\,P_{Xn1}) &= 0 \\
v_{X1}(c_{X1}\,P_{X12}) + v_{X2}(-c_{X2}) + \ldots + v_{Xn}(c_{Xn}\,P_{Xn2}) &= 0 \\
\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad & \vdots \\
v_{X1}(c_{X1}\,P_{X1n}) + v_{X2}(c_{X2}\,P_{X2n}) + \ldots + v_{Xn}(-c_{Xn}) &= 0
\end{aligned}
\qquad \ldots\ldots (3.12)
$$

Since there are (n-1) independent equations in (3.12), when $m \leq n-1$, all the m missing probabilities can be calculated. When $m > n-1$, (m-n+1) additional questions are still needed. Each of these additional questions can be formulated to derive the ratio of $P_{X_{iu}}$ to $P_{X_{iv}}$, at least one of which is a missing probability. Questions of the following type can be asked :

"If state i of X occurs at a certain place along the tunnel axis and the state next to it is either u or v, what is the probability that it is u ?" ... (Q5)

Suppose the answer is q_{uv}, then

$$\frac{P_{Xiu}}{P_{Xiv}} = \frac{q_{uv}}{1 - q_{uv}} \quad ..... \quad (5.5)$$

When (m-n+1) questions of the form (Q5) are answered, the (m-n+1) corresponding results (5.5) are used together with (3.12) to obtain the m missing probabilities. Equations (3.12) and (5.5) are linear in P_{Xij} and so the solutions should be easy to obtain.

5.3.2 Interdependent parameters

If significant (whether something is "significant" or not is to be determined by the geologist subjectively) parameter interdependences are suspected, the parameters are arranged in decreasing average extents : X1, X2, ... XN and questionings are carried out for each parameter in the same order. Starting from X1 (the "independent parameter"), questions (Q1), (Q2) and if appropriate (Q3) to (Q5) are asked to assess the values of P_{X1ij} and c_{X1i}.

After P_{X1ij} and c_{X1i} are established the dependence of X2 on X1 is tested. To test the dependence of c_{X2i} on X1 x2n (= total number of states of X2) of the following type of questions are asked (i = 1, 2, ... x2n) :

"If the places where state i of X2 exists are in different states of X1, would there be significant differences in the average extents of state i of

X2 ?" ..... (Q6)

To test the dependence of P_{X2ij} on X1, (x2n - 2 x2n) (= total number of independent transition probabilities) questions of the type are asked :

"If the places where state i of X2 exists are in different state of X1, would there be significant differences in the probability that the state next to state i is j ?" ..... (Q7)

If the answer to any one of the questions (Q6) and (Q7) is "yes", then X2 is probabilistically dependent on X1. Different sets of P_{X2ij} and c_{X2i} are established for regions in different states of X1 by using the methods [questions (Q1) to (Q5)] of section 5.3.1. The amount of effort needed is less than proportional to the number of sets of P_{X2ij} and c_{X2i} required : after the first set is established the other sets should be easier to assess due to the repetition of procedures. If the answers to all the questions (Q6) and (Q7) are negative, then X2 is indepedent of X1 and P_{X2ij} and c_{X2i} can be established independently.

After the transition probabilities and transition intensity coeficients of X1 and X2 are established, the dependences of X3 on X1 and X3 on X2 are tested similarly as the dependence of X2 on X1. If X3 is only dependent on one of them, different sets of P_{X3ij} and c_{X3i} are established for regions in different states of the correlated parameter.

If X3 is dependent on both parameters, different sets of P_{X3ij} and c_{X3i} are established for regions in different combinations of states of X1 and X2. An example is that RQD may depend on the combination of Rock Type and Faulting : average extent of high RQD rocks is much greater in a region with quartzite- no faulting than one with granite- faulting. But if X3 is not dependent on X2 or X1, it is treated as an independent parameter.

After the transition probabilities and transition intensity coefficients are established for X1, X2 and X3, X4 is considered. The dependences of X4 on X1, X2 and X3 are tested similarly and different sets of P_{X4ij} and c_{X4i} are established if there are dependences. Similar procedures are applied to X5, ... XN until all required transition probabilities and transition intensity coefficients are encoded.

Summary

In this chapter the procedures for assessing the basic inputs (transition probabilities and transition intensity coefficients) required for the geological prediction model are presented. Both a frequency- based method (when there is sufficient data) and a subjective judgment method can be used. For the case of independent parameters, each parameter is treated individually. For the case of interdependent parameters, a hierarchy of dependences (section 4.4) is assumed and the parameters are treated successively, starting with the one with the largest average extent.

CHAPTER VI

UPDATING OF THE GEOLOGICAL PREDICTION MODEL

6.1 Introduction

As tunnel construction proceeds the states and extents of different geological parameters in the newly excavated part of the tunnel are recorded. This information can be expressed as a "transition chain" of parameters X1, X2, ... XN (in decreasing order of average extents) as shown in Fig. 6.1. This information must be relevant to the problem of geological prediction in the unexcavated part of the tunnel since it comes from an excavated part of the same tunnel. Therefore it is desirable to update (refine) the geological prediction model (i.e. to update the transition probabilities and transition intensity coefficients used in the model) based on this information so that it may be in better correspondence with the actual geology of the unexcavated part of the tunnel.

Before tunnel excavation starts, the transition probabilities and transition intensity coefficients are estimated by the methods presented in Chapter V. The geological prediction model can then be used to form probabilistic predictions about the geological parameters ahead of the tunnel face. After a certain length of the tunnel is excavated, records of the geological parameters in the newly excavated part can be used to update the estimates

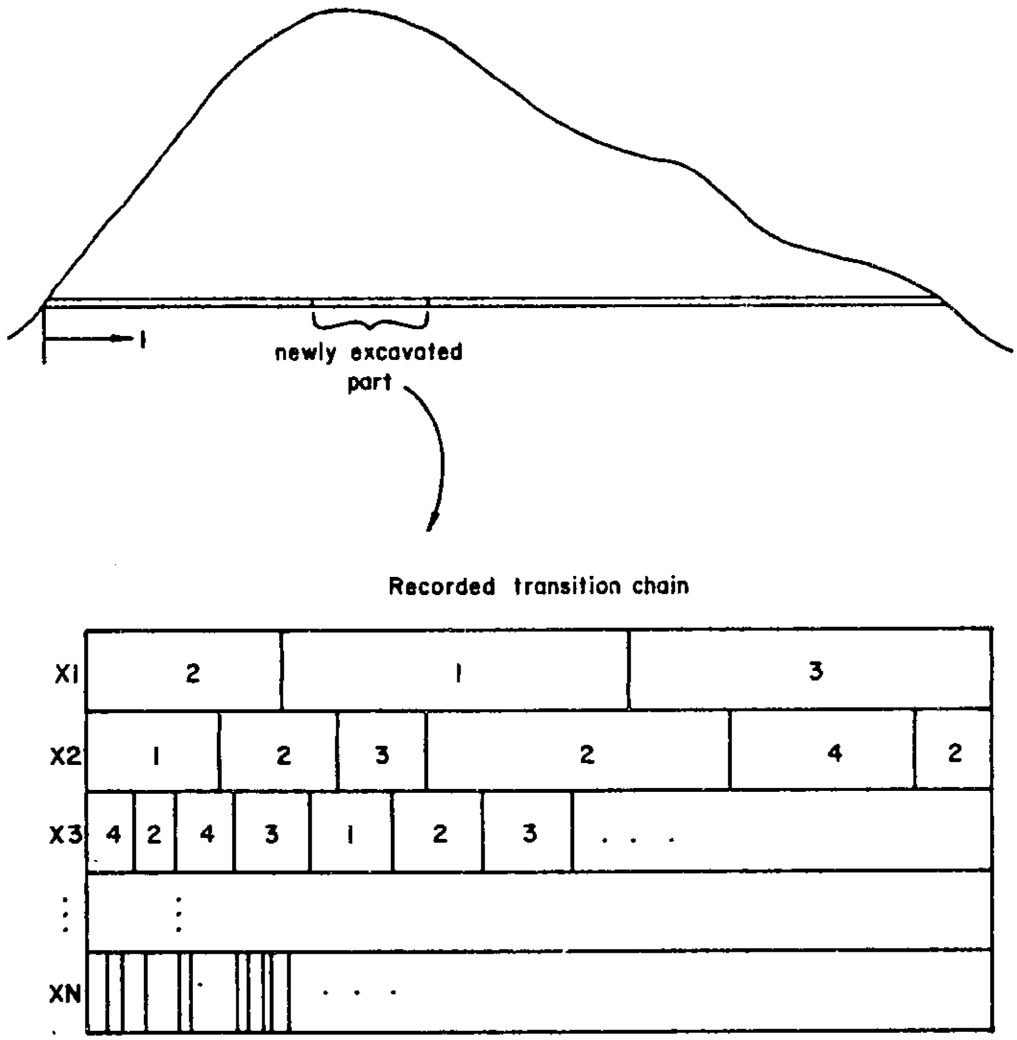


Figure 6.1 A recorded transition chain taken from a newly excavated part of the tunnel

of the transition probabilities and intensity coefficients. This updating is intended to modify the geological prediction model so that it may give better predictions in the unexcavated part of the tunnel. This updating process can be repeated as another length of the tunnel is excavated and new records taken. A typical updating process is presented in section 6.2 where the frequency- based method (section 5.2) is used. When the subjective judgment method (section 5.3) is used the procedures for updating are presented in section 6.3. The updating processes presented in sections 6.2 and 6.3 are equally applicable to independent or interdependent parameters.

6.2 Frequency-based method

6.2.1 Updating of a transition intensity coefficient

When the frequency- based method is used to establish a transition intensity coefficient, its best estimate $\hat{c}_{XKi}$ is calculated from a sample of extents extracted from an existing set of transition chains (section 5.2.) Updating is done by adding the newly recorded extents of state i to the existing sample and re- calculating the best estimate.

If the transition intensity coefficient c_{XKiu} being in use deviates considerably from the newly recorded extents of state i, updating of the coefficient should not be done simply by adding the new information to the existing one but by treating it separately. If such a significant deviation is suspected, a conventional Chi-square test (see Cornell, 1970; Appendix A) should be used to test the exponential extent distribution (see section 3.2.3.)

$$f_{HXKi}(h) = c_{XKiu}\, e^{-c_{XKiu} h}$$

based on the new records. Since the degrees of freedom of the Chi-square statistic is equal to (NC - 1) (NC is the total number of categories), NC is required to be at least 2. Therefore at least 10 (= 2x5) extents are required since each category should have an expected frequency of 5 or above. If the Chi-square test result is positive, the transition intensity coefficient can be updated as mentioned above. If the test result is negative, c_{XKiu} is rejected and the best estimate of c_{XKi} is calculated based on the new records only. This is intended to ensure that the transition intensity coefficient used is up-to-date because the new records should usually correspond better with the geology of the remaining part of the tunnel.

6.2.2 updating of transition probabilities

It was mentioned in section 5.2.1 that a row of transition probabilities (P_{Xi1}, P_{Xi2}, ... P_{Xin}) can be estimated by recording F_{Xi1} (the number of times that a transition is made from state i to state 1), F_{Xi2}, ... F_{Xin} from an existing set of tansition chains of parameter X. The best estimate can be updated by adding the transition chain of X in the newly excavated part of the tunnel to the existing set : F_{Xij} is increased by the number of times that a transition is made from state i to state j in the new transiton chain.

If the row of transition probabilities in use $(P_{Xi1u}\ P_{Xi2u}\ \ldots\ P_{Xinu})$ deviates considerably from the newly recorded transition frequency distribution, updating of the row should not be done by simply adding the new information to the existing one but by treating it separately. If such a significant deviation is suspected, a Chi-square test can be carried out. Let NF_{Xij} be the number of times that X makes a transition from state i to state j in the new transition chain and

$$NB_{Xi} = NF_{Xi1} + NF_{Xi2} + \ldots + NF_{Xin}$$

(NB_{Xi} is the number of times that state i appears as the first state of a transition pair in the new transition chain.)

The expected frequency of transitions from state i to state j is

$$E_{Xij} = P_{Xiju}\, NB_{Xi}$$

and the Chi-square statistic is

$$C_{Xi} = \sum_{j=1}^{n} \frac{(E_{Xij} - NF_{Xij})}{E_{Xij}}$$

The number of categories is NC (= n-1) and so the degrees of freedom is (n-2). To ensure that the result of the Chi-square test is reliable, each E_{Xij} should not be less than 5 (see Lumsden, 1971.) Therefore the minimum number of recorded transitions (with i as the first state) is five times the number of categories (= 5(n-1).) 10(n-1) transitions may be required in actual situation.

If the result of the Chi-square test is positive, $P_{Xij\mu}$ can be updated as mentioned above. If the result is negative, $P_{Xij\mu}$ is rejected and the best estimates of the transition probabilities based on the new records (NF_{Xij}) only are used instead. This ensures that the best estimates used are up-to-date.

6.3 Subjective judgment method

When the amount of existing data is not sufficient for the frequency- based method to be used, a transition intensity coefficient (or a row of transition probabilities) has to be assessed by the subjective judgment method. [Recall that about 10 recorded extents are sufficient to establish a transition intensity coefficient and at least 5(n-1) recorded transitions are sufficient to establish a row of transition probabilities; see section 6.2.] In this case the updating is not as easy because the new records are frequency data while the coeficient or the row of probabilities have been established subjectively. If the new records together with the existing (frequency data) records are sufficient, the transition intensity coefficient (or the row of transition probabilities) can be re-established by the frequency- based method. Further updating can then be done as shown in section 6.2. If the two sets of records (of frequency data) together are still not sufficient, the updating can be carried out subjectively or by using the Bayesian updating technique.

In subjective updating, the transition intensity coefficient (or the row of transition probabilities) is re-assessed by the same geologist who assisted in establishing it. Since the geologist, with the help of the new records, should have become more familiar with the geology of the tunnel region, the probabilistic assessments he makes should become more reliable and in this way the geological prediction model is updated. However, care must be taken to guard against possible subjective biases towards the new records. The new records are recent events which are more "available" and hence may be given more weight than optimal (see Spetzler and Holstein, 1974.) Another way to update the model is the concept of "competing hypotheses" which enables the geological prediction model to be updated by the Bayesian technique. The concept of "competing hypotheses" is introduced in section 6.3.1 below.

6.3.1 "Competing hypotheses"

In section 5.3 the subjective judgment method is used to establish a transition intensity coefficient (or a row of transition probabilities.) Such a coefficient (or row) is in fact the outcome of a "hypothesis" because it depends on the particular opinions of the geologist (or group of geologists) questioned. If different opinions of a geologist (or group of geologists) are enlisted, different hypotheses (the competing hypotheses) are established. In the concept of "competing hypotheses" it is assumed that one and only one of these hypotheses is "true" (i.e. exactly corresponding to the geologic processes of the tunnel region) and the probability of hypothesis H_m being true is denoted by P_m. The coefficient (or row) used is then a weighted mean of the estimates based on the competing hypotheses :

$$c_{Xi} = P_1 c_{1Xi} + P_2 c_{2Xi} + \dots P_y c_{yXi}, \quad \dots\dots (6.1)$$

and

$$\begin{aligned} &(P_{Xi1}\ P_{Xi2}\ \dots\ P_{Xin}) \\ = &\ P_1 (P_{1Xi1}\ P_{1Xi2}\ \dots\ P_{1Xin}) \\ &+ P_2 (P_{2Xi1}\ P_{2Xi2}\ \dots\ P_{2Xin}) \\ &+ \dots \\ &+ P_z (P_{zXi1}\ P_{zXi2}\ \dots\ P_{zXin}) \quad \dots\dots (6.2) \end{aligned}$$

where y is the number of competing hypotheses for c_{Xi} and z the number of hypotheses for the row $(P_{Xi1}\ P_{Xi2}\ \dots\ P_{Xin})$. (n is the number of states of X.)

Among the competing hypotheses for c_{Xi}, at least the upper bound, the lower bound and the best estimate should be included. Thus y is at least three. For a row of transition probabilities, the number of hypotheses should be at least n, one of which being the best estimate. The reason can be made clear by considering an example where n=4. In this example the number of independent transition probabilities in the row is (n-2) = 2. Let these 2 probabilities be P_{X12} and P_{X13}. These 2 values can be represented by a graph as shown in Fig. 6.2(a). Suppose the true values are at $C_*(P_{*X12}, P_{*X13})$ and the best estimate is at C_1 which is hopefully "near" to C_*. The other estimates C_2, ... C_z are at positions surrounding the best estimate. If z is less than n(4), it can be seen that the chance of including C_* in the convex region formed by the estimates is small (Fig 6.2(a).) If C_* is not enclosed by the convex region, the row of transition probabilities given by (6.2) can never be updated to the true row because this row always falls inside the convex region formed by C_1, C_2, ... C_z due to the nature of the weighting factors P_m:

$$P_1 + P_2 + \ldots + P_z = 1 \quad \ldots\ldots (6.3)$$

$$P_m \geqslant 0.$$

On the other hand, if z is at least 4, the chance of enclosing C_* is much greater and updating may yield a row of transition probabilities near to C_* (Fig 6.2(b).)

The updating procedures for a transition intensity coefficient is presented in section 6.3.2. Updating of a row of transition probabilities is similar and is presented in section 6.3.3.

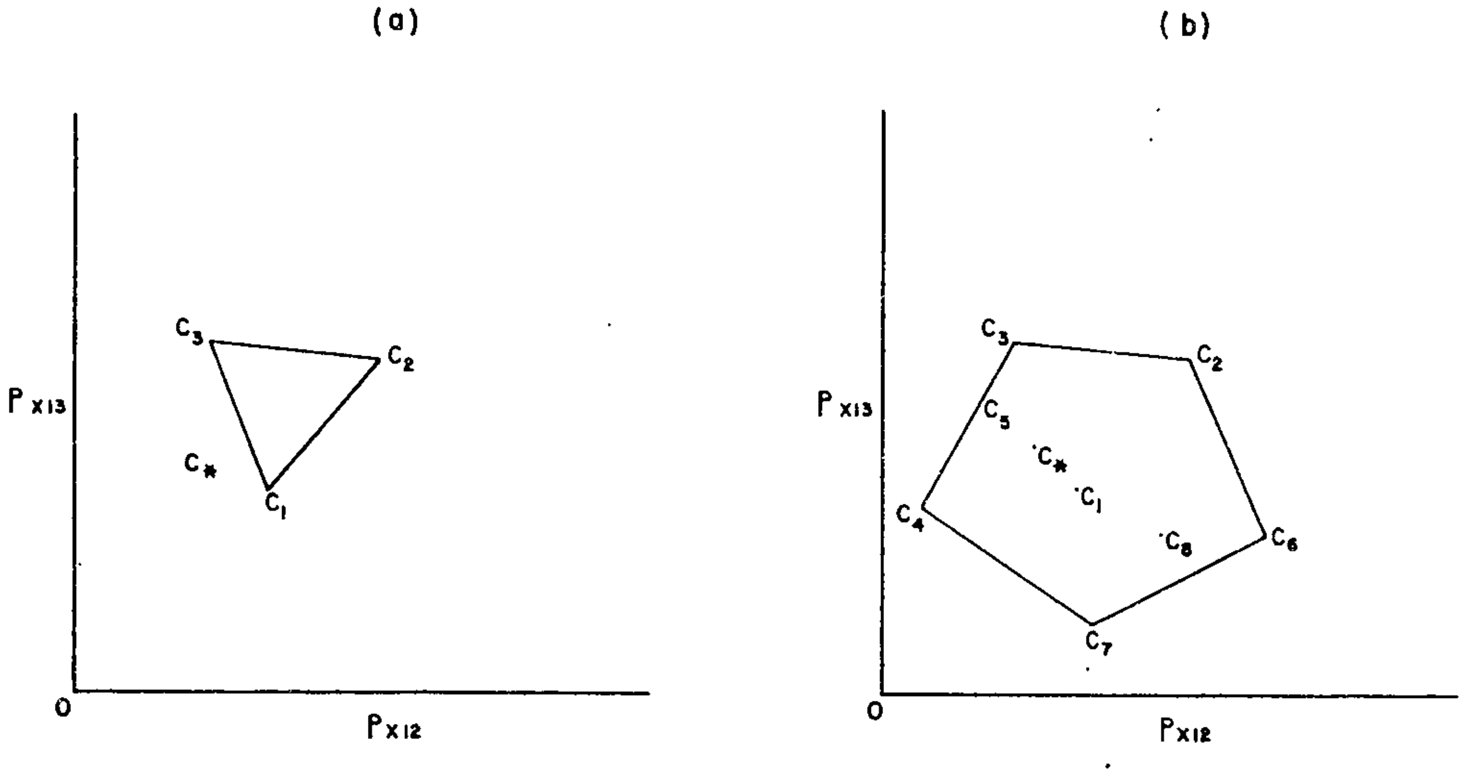


Figure 6.2 (a) number of hypotheses = 3
(b) number of hypotheses = 8

6.3.2 Updating of a transition intensity coefficient

Before construction of the tunnel starts, y competing hypotheses for c_{Xi} are set up based on different opinions of the geologists. The coefficient corresponding to hypothesis H_m is c_{mXi}. The probability P_m of H_m being true can be assessed subjectively or a vague prior (i.e. $P_m = 1/y$) is assumed and then is updated by existing data before tunnel construction. To update P_m to P'_m based on a sample of recorded extents the Bayesian technique is used :

$$P'_m \propto P_m \text{ [likelihood of } H_m]$$

$$= K P_m L_m \quad \ldots\ldots \quad (6.4)$$

where K is the proportionality constant. Since hypothesis H_m states that the extent HX_i is exponentially distributed with coefficient c_{mXi}, its likelihood is

$$L_m = f_{mHXi}(h_1)\ f_{mHXi}(h_2)\ \ldots\ f_{mHXi}(h_e) \quad \ldots\ldots \quad (6.5)$$

where h_1, h_2, ... h_e are the e extents in the sample. Since one and only one of the hypotheses is true,

$$P'_1 + P'_2 + \ldots + p'_y = 1$$

and so from (6.4),

$$K P_1 L_1 + K P_2 L_2 + \ldots K P_y L_y = 1.$$

Hence

$$K = 1/(P_1 L_1 + \ldots + P_y L_y) \quad \ldots\ldots \quad (6.6)$$

and from (6.4) again,

$$P'_m = P_m L_m / (P_1 L_1 + P_2 L_2 + \ldots + P_y L_y) \quad \ldots\ldots \quad (6.7)$$

As P_m is updated to P'_m, c_{Xi} is also updated according to

(6.1).

The time before actual tunnel construction is started is called the "initial stage". The "first stage" starts with tunnel construction and ends at a point when the tunnel records up to that point are used to further update P_m. Thus generally the recorded extents in the n^{th} stage are used for the n^{th} updating and the new value of c_{Xi} is used in the $(n+1)^{th}$ stage. The prior in the n^{th} updating is the values of P_m used in the n^{th} stage and the posterior P'_m is calculated according to (6.7).

6.3.3 Updating of transition probabilities

The updating of a row of transition probabilities is similar to that of a transition intensity coefficient. z competing hypotheses are set up before tunnel construction which results in z rows of transition probabilities. The probability P_m of H_m being true can be assessed subjectively or a vague prior is assumed and then is updated by existing data before tunnel construction.

Generally updating in each stage is carried out using the Bayesian technique :

$$P'_m \propto P_m [\text{likelihood of } H_m]$$
$$= K\,P_m\,L_m \quad \ldots\ldots \quad (6.8)$$

Suppose in the same stage the number of transitions X makes from state i to state j is SF_{Xij}. Then the likelihood of

H_m is

$$L_m = P[\text{The recorded transitions take place} | H_m]$$

$$= (P_{mxi1})^{SF_{xi1}} \ \ldots \ (P_{mxi2})^{SF_{xin}} \ldots\ldots \quad (6.9)$$

(P_{mii} is not included in the right-hand-side because it is always zero by definition.)

Then P'_m is calculated as shown in section 6.3.2 and is given by

$$P'_m = P_m L_m / (P_1 L_1 + P_2 L_2 + \ldots \ P_z L_z) \ \ldots\ldots \quad (6.10)$$

The row of transition probability is thus updated according to (6.2).

6.4 Summary

In this chapter the updating of the transition intensity coefficients and transition probabilities to be used in the geological prediction model is discussed. When the frequency- based method is applied to estimate a transition intensity coefficient (or a row of transition probabilities) the estimate is calculated from an existing set of frequency records (section 5.2.) As tunnel excavation proceeds the new records are added to the existing set of records and the estimates are re-calculated (updated.)

When the subjective judgment method is used several estimates of a transition intensity coefficient (or a row of transition probabilities) are established based on different opinions (competing hypotheses.) Each hypothesis has a probability P_m of being true and the estimate to be used in the model is the weighted mean of the estimates with P_m as the weighting factor. As tunnel construction proceeds the new records made are used to calculate the likelihoods of H_m. P_m is then updated by the Bayesian technique and in this way the estimate to be used in the model is updated.

CHAPTER VII

A CASE STUDY

7.1 Introduction

The discharge water tunnel of the Seabrook power station, NH, is used for an example application of the proposed geological prediction and updating model. The actual discharge tunnel is over 15,000 feet (2.8 miles, see Fig. 7.1) long. Only the western portion (7662 feet long) from borehole ADT-1 (l=0) to ADT-42 (l=7662) is used for the example (Fig 7.1.)

7.1.1 General geology

The bedrock types in the region include metamorphic rocks of the Kittery formation, igneous rocks of the Newburyport pluton and intrusive diabase dikes. Due to the complicated processes of formation, the spatial relationships between the rock types are sometimes very irregular (see Rand, 1974 for a detailed description of regional geology and history.)

7.1.2 Geological parameters

According to Moavenzadeh et al (1976), the most important geological parameters affecting the tunnel are Rock Type, RQD, Joint Orientation, Major Defects, Water Inflow, Hardness, Exploratory Drill Penetration Rate,

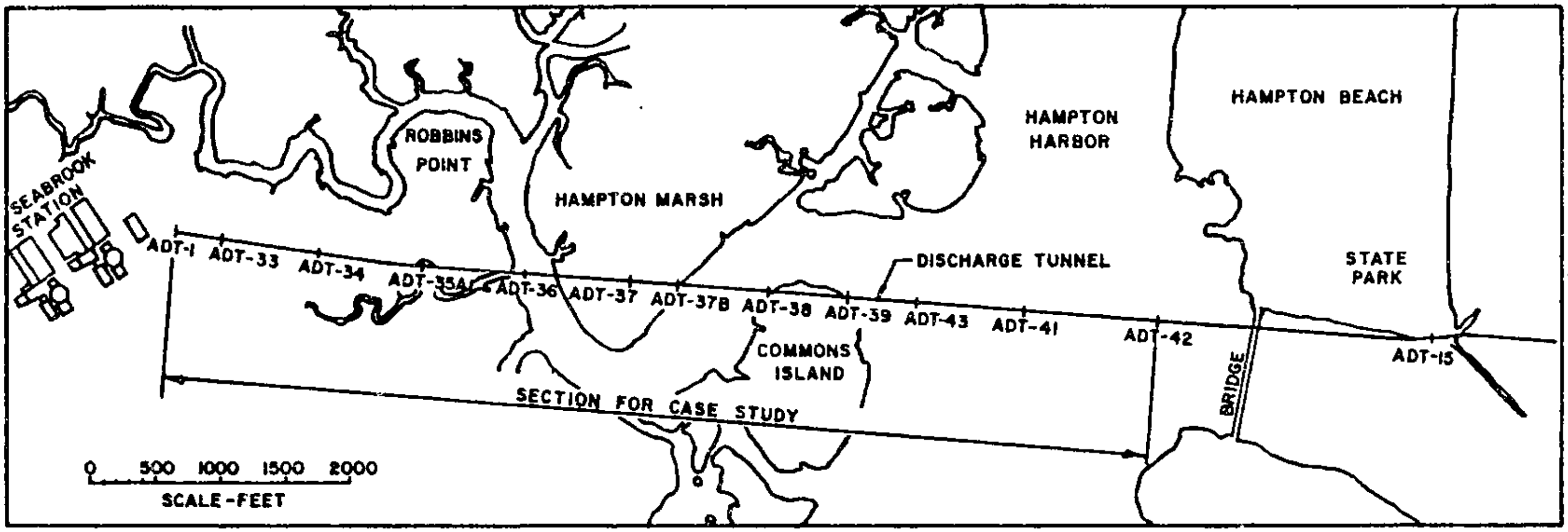


Fig. 7.1 Discharge water tunnel of the Seabrook Station, N.H.

Compressive Strength, and Foliation. Since this example serves as a simplified demonstration, only four most important parameters are chosen:

(a) Rock Type (R)

Rock Type is a useful categorizing parameter because it allows one to draw certain conclusions on other parameters such as Joint Orientation and Foliation. The rock types considered include Schist, Metaquartzite, Diorite and Quartzite. Diabase dikes are neglected because they are thin and their effect on tunneling performance* may be small. The states of Rock Type (as a random variable R) are defined in Table 7.1 (a).

(b) RQD (D)

RQD (Rock Quality Designation) is commonly used as a quantitative measure of the degree of jointing which directly affects tunneling performance. The states of this parameter are defined in Table 7.1 (b).

(c) Degree of Weathering (E)

Severe weathering is found in some zones in the tunnel region. It is detrimental to tunneling performance in a similar way as other major defects such as faults and clay seams. The states of this parameter are defined in Table

* Tunneling performance refers to the performance of the excavated opening and of the supported tunnel; see section 2.2.

(a) Rock Type (R)

r	Definition
1	Schist
2	Metaquartzite
3	Diorite
4	Quartzite

(b) RQD (D)

d	Definition	
1	High	75-100%
2	Medium	25-75%
3	Low	0-25%

(c) Degree of Weathering (E)

e	Definition
1	Not Severe
2	Severe

(d) Availability of Water (W)

d	Definition
1	Low
2	Medium
3	High

Table 7.1 Definition of parameter states.

7.1 (c).

(d) Availability of Water (W)

This parameter is called "Availability of Water" instead of "Water Inflow" because the flow of water into the tunnel depends not only on geological conditions but also on the excavation and support methods. Therefore Water Inflow is a performance parameter (see section 2.2) but not a geological parameter. On the other hand, the availability of water is a geological property and indicates the potential water inflow into the tunnel. The states of this parameter are defined in Table 7.1 (d).

7.1.3 Ground classes

Now that the states of the four geological parameters have been defined (section 7.1.2), it can be seen that there are a total of 72 (= 4x3x2x3) vectors of geological parameters. For example (2 3 2 3) is a geological vector which means Metaquartzite, low RQD, severe weathering and high availability of water.

The next step is to establish the "ground classes" (sets of geological vectors) needed for the cost optimization of individual sections as described in section 2.3.1. (Cost Optimization involving the entire tunnel was out of the scope of this case study.) The performance model (a technical relationship describing the ground- structure behavior for certain geological conditions) used is based on the expert knowledge and experience of a geotechnical engineer (i.e Prof. Einstein.) Five ground classes (GC) are established and the corresponding excavation and support processes (ES) are listed in Table 7.2. The geological vectors corresponding to each GC are shown in Table 7.3. Table 7.3 is in fact a simplified ground class classification table because only some of the 72 geological vectors are included. The corresponding ground classes of the remaining geological vectors can be assigned by a conservative approach. For example, a geological vector (not listed in Table 7.3) is taken to be in GC3 if the geological conditions as indicated by this vector are "better" than those of the vectors listed in GC3 but are "worse" than those listed in GC2. Using this approach the GC of the remaining geological vectors are defined and the full GC classification table is shown in Table 7.4.

7.2 Derivation of input to the model

After having defined the states of the four geological parameters, the basic input (transition intensity coefficients and transition probabilities) to the geological prediction model can be derived. Since no frequency records on state transitions and extents are available, the subjective judgment method (section 5.3) is used. According to the expert (Prof. Einstein) consulted, parameter

GC	Excavation	Support
1	TBM, cycle length = 5'	None (except spot bolting)
2	TBM, cycle length = 4'	None (except spot bolting)
3	TBM, cycle length = 5'	Bolts 5 x 5', 3" shotcrete
4	TBM, cycle length = 3', extensive pumping	Bolts 3 x 3', 6" shotcrete
5	Multiple drift with drilling and blasting, pumping	Bolts 3 x 3', 8" shotcrete, light steelsets with 3' spacing

* The cycle length is reduced due to the hardness of Quartzite.

Table 7.2 Excavation and support processes corresponding to the ground classes.

GC	r	d	e	w
1	1,2,3	1	1	1,2
2	4	1	1	1,2
3	1,2,3,4	2	1	2
4	1,2,3,4	3	1	3
5	1,2,3,4	3	2	1,2,3

Table 7.3 "Simplified" GC classification table.

GC	r	d	e	w
1	1,2,3	1	1	1,2
2	4	1	1	1,2
3	1,2,3,4	2	1	2
	1,2,3,4	2	1	1
4	1,2,3,4	3	1	3
	1,2,3,4	2	1	3
	1,2,3,4	1	1	3
	1,2,3,4	3	1	2
	1,2,3,4	3	1	1
5	1,2,3,4	3	2	1,2,3
	1,2,3,4	2	2	1,2,3
	1,2,3,4	1	2	1,2,3

Table 7.4 GC classification table.

interdependences are not significant and so the four parameter are assumed to be independent. Since the geologies of the Western portion of the discharge tunnel are believed to be similar (homogeneous) throughout, that portion is regarded as an homogeneous region for each of the four parameters. This implies that the values of the transition intensity coefficients and transition probabilities remain the same throughout that portion of the tunnel. The details of the derivations of the transition intensity coefficients and transition probabilities are presented in sections 7.2.1 and 7.2.2 respectively.

Although there are no frequency data on transitions and extents, there are many point observations in the form of boreholes along the tunnel axis. After the drilled cores from these boreholes were inspected, subjective judgment was used to arrive at PMF's of the parameter states at the points of observation. For example, at borehole ADT-1 ($l=0$) the following PMF was designated for Rock Type:

$P[r=1] = .00$

$P[r=2] = .00$

$P[r=3] = 1.00$

$P[r=4] = .00$

This PMF in fact denotes a deterministic observation i.e. the rock type at $l=0$ is observed to be Diorite. Throughout the length of the western portion considered, there are 13 boreholes altogether. Therefore for each parameter there are 13 observations which are shown in Tables 7.5 to 7.8.

7.2.1 Transition intensity coefficients

The transition intensity coefficient of each parameter state is estimated according to the questioning procedures described in section 5.3.1.1. In order that the updating (section 6.3.2) of the estimate of c_{Xi} can be done, several (usually 3) "hypotheses" are set up for each coefficient. The detailed procedures can be made clear by considering how estimates for c_{R1} were derived :

(1) The geologist was asked the following question:

"The tunnel goes through several lengths of Schist. What could be the average length? Give a best estimate. Also give the upper and lower bounds on the average length."

To answer this question the geologist produced and used an estimated profile of rock types along the tunnel (Fig. 7.2.) By taking in this profile the average of the lengths of Schist existing at the tunnel axis the best estimate was determined. Then the upper and lower bounds were determined by considering how much could the average length deviate from the best estimate. (This procedure of using an estimated profile is not mandatory and the geologist could derive directly the estimates through subjective judgment.)

(2) The three estimates were :

Bore-hole	l (feet)	State Probabilities 1	2	3	4
1	0	.0	.0	1.0	.0
33	341	.0	.0	1.0	.0
2	717	.0	.2	.8	.0
34	1239	.0	.5	.0	.5
35A	1945	.0	.0	.2	.8
36	2788	.0	.0	.0	1.0
37	3566	.8	.0	.0	.2
37B	4010	1.0	.0	.0	.0
38	4659	.0	.0	.0	1.0
39	5256	.0	.0	.0	1.0
43	5785	1.0	.0	.0	.0
41	6604	.0	.0	.9	.1
42	7662	.0	.0	.0	1.0

Table 7.5 13 observations on R.

Bore-		State Probabilities		
hole	l (feet)	1	2	3
1	0	.5	.5	.0
33	341	1.0	.0	.0
2	717	1.0	.0	.0
34	1239	.5	.5	.0
35A	1945	.2	.8	.0
36	2788	.2	.0	.8
37	3566	.5	.5	.0
37B	4010	.0	.2	.8
38	4659	.0	.0	1.0
39	5256	1.0	.0	.0
43	5785	.8	.2	.0
41	6604	.8	.2	.0
42	7662	1.0	.0	.0

Table 7.6 13 observations on D.

Bore-hole	l (feet)	State Probabilities 1	2
1	0	1.0	.0
33	341	1.0	.0
2	717	1.0	.0
34	1239	1.0	.0
35A	1945	.6	.4
36	2788	.2	.8
37	3566	.5	.5
37B	4010	.2	.8
38	4659	.2	.8
39	5256	.8	.2
43	5785	.6	.4
41	6604	.8	.2
42	7662	1.0	.0

Table 7.7 13 observations on E.

Bore-hole	l (feet)	State Probabilities 1	2	3
1	0	1.0	.0	.0
33	341	1.0	.0	.0
2	717	1.0	.0	.0
34	1239	.5	.0	.5
35A	1945	.4	.0	.6
36	2788	.8	.0	.2
37	3566	.5	.0	.5
37B	4010	.2	.0	.8
38	4659	.6	.2	.2
39	5256	.2	.8	.0
43	5785	1.0	.0	.0
41	6604	.6	.4	.0
42	7662	1.0	.0	.0

Table 7.8 13 observations on W.

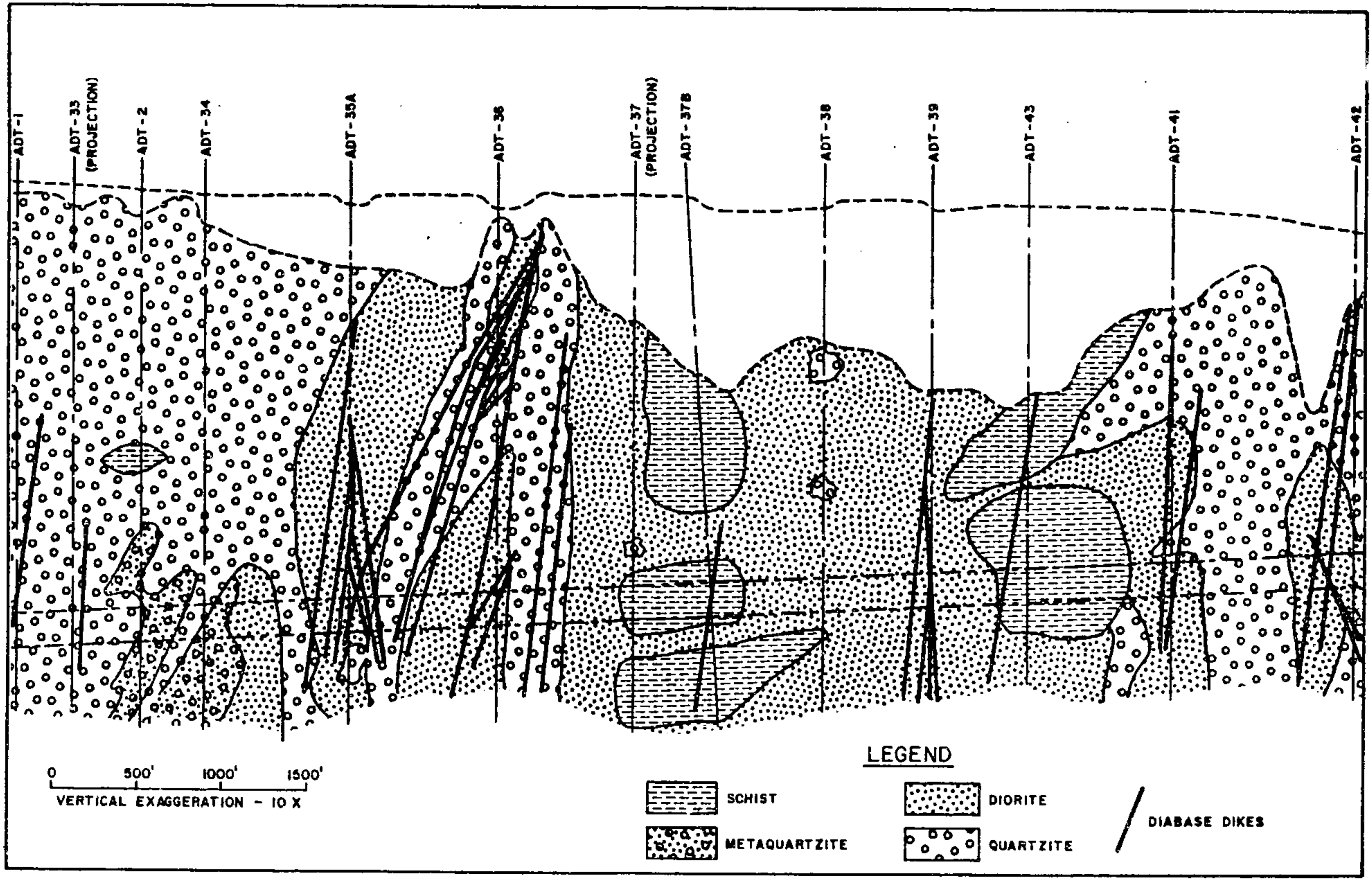


Fig. 7.2 Estimated Rock Type profile

H_1 : best estimate = 750'

H_2 : upper bound = 1000'

H_3 : lower bound = 550'

Therefore

$c_{1R1} = 1/750 = .00133$

$c_{2R1} = 1/1000 = .00100$

$c_{3R1} = 1/550 = .00182$

Similar questioning procedures were carried out for Metaquartzite, Diorite and Quartzite. The other parameters were considered and the results are summarised in Table 7.9.

7.2.2 Transition probabilities

The transition probabilities of each parameter were estimated according to the questioning procedures given in section 5.3.1.2. In order that the updating (section 6.3.3) of each estimated row of transition probabilities could be done, several "hypotheses" were set up for each row. The detailed procedures can be made clear by considering how estimates for P_{R1j} (j=1, 2, 3, 4) were derived:

(1) The geologist was asked the following question:

"If the tunnel runs into Schist at some point, how many times out of a hundred will it run into (i) Metaquartzite, (ii) Diorite, (iii) Quartzite next? At first give the best estimates of the three frequencies. Then assume different view points and theories to give additional sets of estimates."

m	1	2	3
c_{mR1}	.00133	.00100	.00182
R2	.00800	.00667	.01000
R3	.00286	.00167	.00333
R4	.00250	.00167	.00333
D1	.00222	.00143	.00333
D2	.00333	.00182	.00667
D3	.00154	.00125	.00286
E1	.000444	.000435	.000465
E2	.00167	.00200	.00118
W1	.000833	.000769	.000870
W2	.00500	.00400	.00100
W3	.00167	.00143	.00200

Table 7.9 Transition intensity coefficients of all the parameters based on different competing hypotheses.

(2) The geologist came up with three sets (in fact it would be better to have four sets; see section 6.3.3) of estimates based on three hypotheses (one based on an estimated profile and the other two on different opinions.) The estimates were divided by 100 to give the following probabilities:

m	1	2	3
P_{mR12}	.05	.00	.00
P_{mR13}	.20	.50	.00
P_{mR14}	.75	.50	1.00

Similar questioning procedures were carried out for the other three states of Rock Type. Then the other parameters were considered similarly and the results are summarised in Tables 7.10 and 7.11. (There was no need to assess the transition probabilities of E which had only two states.)

7.3 Parameter probability profiles

Cost optimization involving the entire unexcavated tunnel requires the PMF's of each of the four parameters at equally spaced points along the unexcavated tunnel. These PMF's of each parameter constitute the "probability profile" of that parameter. The PMF's can be calculated using the expression for the interval transition probabilities given in section 4.2.1.5. The most general expression is given by (4.8) where several non- deterministic observations ahead of the tunnel face provide information on the parameter.

m	1	2	3
$P_{mR\|12}$	.05	.00	.00
13	.20	.50	.00
14	.75	.50	1.00
21	.05	.00	.00
23	.60	.50	.40
24	.35	.50	.60
31	.05	.00	.00
32	.20	.10	.30
34	.75	.90	.70
41	.20	.20	.30
42	.10	.30	.10
43	.70	.50	.60

Table 7.10 Transition probabilities of R based on different hypotheses.

m	1	2	3
$P_{mD\|12}$	1.00	.85	*
13	.00	.15	
21	.75	.90	
23	.25	.10	
31	.50	.00	1.00
32	.50	1.00	.00
$P_{mW\|12}$	.50	.75	.40
13	.50	.25	.60
21	1.00	.80	
23	.00	.20	
31	1.00	.80	
32	.00	.20	

* In fact it would be better to have three (= number of states) hypotheses.

See section 6.3.3.

Table 7.11 Transition probabilities of D and W based on different hypotheses.

(These observations are in form of specified PMF's of the parameter at specific points of observations along the unexcavated tunnel, e.g. the result of geological exploration such as boring.)

The actual calculations of the parameter probability profiles are done by a Fortran computer program STATEP (for State Prediction) according to (4.8). (See Appendix D for user's manual and examples.) Spectral resolutions of the transition intensity matrices (section 3.3.1) are used to calculate the interval transition probabilities. In this context the eigenvalues and eigenvectors of the matrices have to be calculated, which is accomplished with the subroutine EIGRF of the International Mathematical and Statistical Library. (This subroutine is automatically called in STATEP.)

In the following parts of this section it will be shown how the parameter probability profiles are established and how they are updated once additional information from the excavated part of the tunnel becomes available. This is done by dividing the tunnel into two sections. In the first section from l=0 to l=4010 only the information that is available prior to tunnel construction is used. The associated probability calculations are the "first stage calculations" of the parameter probability profiles. For the second section of the tunnel (l=4010 to l=7662), the parameter probability profiles are updated with the information obtained from the first excavated part of the tunnel. (The associated calculations are the "second stage calculations".)

7.3.1 First stage calculations

The transition intensity coefficients and transition probabilities to be used in the first stage are based on the competing hypotheses and frequency data (if any) available before tunnel construction. (In this case study there was no such data available.) A vague prior is assumed for each of the competing hypotheses H_m.

A transition intensity coefficient to be used in the first stage is calculated according to (6.1). For example (P_m = 1/3 due to vague prior; c_{mR1} is from Table 7.9),

$$
\begin{aligned}
c_{R1} &= P_1\, c_{1R1} + P_2\, c_{2R1} + P_3\, c_{3R1} \\
&= .333(.00133) + .333(.001) + .333(.00182) \\
&= .00138
\end{aligned}
$$

A row of transition probabilities to be used in the first stage is calculated according to (6.2). For example (P_m = 1/2 due to vague prior; p_{mDij} is from Table 7.11),

$$
\begin{aligned}
(p_{D21}\ p_{D22}\ p_{D23}) &= P_1(p_{1D21}\ p_{1D22}\ p_{1D23}) \\
&\quad + P_2(p_{2D21}\ p_{2D22}\ p_{2D23}) \\
&= .5\ (.75\ .00\ .25) \\
&\quad + .5\ (.90\ .00\ .10) \\
&= (.83\ .00\ .17)
\end{aligned}
$$

The transition intensity coefficients and the transition probabilities to be used in the first stage are summarised in Tables 7.12 to 7.15. The transition intensity matrix (section 3.3) for each parameter is assembled and input to STATEP, together with the observations ahead of the tunnel face given in Tables 7.5 to 7.8. The actual input (transition intensity matrices and observations) are shown in Fig. D.3 of Appendix D. STATEP then calculated the parameter probability profile for each parameter using equation (4.8) and the output is listed in Fig. D.4.

In the output shown in Fig. D.4, X1, X2, X3, and X4 denote Rock Type, RQD, Degree of Weathering and Availability of Water respectively. Thus, for the example, the probability of having state 1 (Schist) of X1 (R) at l=5100 is 0.036 while the probability of having state 4 (Quartzite) is .784.

With these parameter probability profiles it is possible to perform probability calculations involving a geological vector

$\bar{g}(l) = (r(l)\ d(l)\ e(l)\ w(l))$

as described in section 4.3. For example,

$P[\bar{g}(6000) = (1\ 1\ 1\ 1)$

$= P[r(6000)=1,\ d(6000)=1,\ e(6000)=1,\ w(6000)=1]$

$= P[r(6000)=1]\ P[d(6000)=1]\ P[e(6000)=1]\ P[w(6000)=1]$

$= (.617)\ (.661)\ (.680)\ (.899)$

(from the parameter probability profiles in Fig. D.4)

i \ j	1	2	3	4	c_{Ri}
1	.00	.02	.23	.75	.00138
2	.02	.00	.50	.48	.00822
3	.02	.20	.00	.78	.00262
4	.23	.17	.60	.00	.00250

Table 7.12 Transition probabilities (first 4 columns) and transition intensity coefficients for R used in the first stage.

i \ j	1	2	3	c_{Di}
1	.00	.93	.07	.00233
2	.83	.00	.17	.00394
3	.50	.50	.00	.00188

Table 7.13 Transition probabilities and transition intensity coefficients for D in the first stage.

i \ j	1	2	c_{Ei}
1	.00	1.00	.000448
2	1.00	.00	.00162

Table 7.14 Transition probabilities and transition intensity coefficients for E in the first stage.

i \ j	1	2	3	c_{Wi}
1	.00	.55	.45	.000824
2	.90	.00	.10	.00633
3	.90	.10	.00	.00170

Table 7.15 Transition probabilities and transition intensity coefficients for W in the first stage.

= .249

GC (ground class, a set of geological vectors requiring the same optimal excavation and support processes) probabilities can also be calculated as described in section 4.3. For example,

$p[GC$ exists at $l = 6000]$

$= P[\bar{g}(6000)=(4\ 1\ 1\ 1)$ or $\bar{g}(6000)=(4\ 1\ 1\ 2)]$

(see GC classification in Table 7.4)

$= P[r(6000)=4]\ P[d(6000)=1]\ P[e(6000)=1]\ P[w(6000)=1]$

$+ P[r(6000)=4]\ P[d(6000)=1]\ P[e(6000)=1]\ P[w(6000)=2]$

$= (.236)\ (.661)\ (.680)\ (.899)$ (From Fig. D.4)

$+ (.236)\ (.661)\ (.680)\ (.055)$

= .101

7.3.2 Second stage

The second stage starts at the time when the tunnel face reaches the position of borehole ADT-37B (l=4010). The transition intensity coefficients and transition probabilities to be used in the second stage are based on the competing hypotheses H_m and their corresponding probabilities of being true P_m. With the new frequency data (in the form of transition chains) recorded during the first stage (from l=0 to l=4010), P_m can be updated and hence the coefficients and probabilities are also updated. (Recall that P_m serve as weighting factors.)

The recorded transition chain of each parameter is generally expressed by a table instead of a figure (profile.) For example, the transition chain of Rock Type is given in Fig. 7.3. The corresponding information is listed in Table 7.16 where "h" denotes extent. The extent of state 3 at l=0 is not known because it is not known where that state started. Extent of state 1 at l=3485 is also not known because it extended up to the tunnel face (l=4010) at the end of the first stage. Tables 7.17 to 7.19 give the transition chains for RQD, Degree of Weathering, and Availability of Water respectively.

Every transition intensity coefficient is now updated according to the procedures described in section 6.3.2. The updating of c_{R3} can serve as an example (m is the number designation of hypothesis H_m):

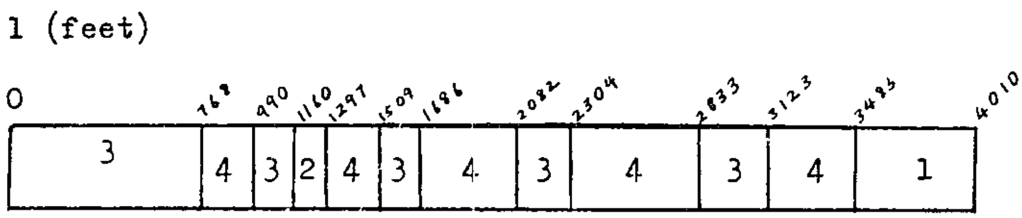
m	1	2	3
c_{mR3}	.00286	.00167	.00333
P_m	.333	.333	.333
L_m	5.85E-12	1.85E-12	7.04E-12
P'_m	.397	.126	.477

L_m is calculated according to (6.5) :

$$L_m = f_{mHR3}(h_1)\ f_{mHR3}(h_2)\ f_{mHR3}(h_3)\ f_{mHR3}(h_4),$$

since there are four recorded extents of state 3 in the first stage (Table 7.16.) For example,

$$L_1 = f_{1HR3}(170)\ f_{1HR3}(177)\ f_{1HR3}(222)\ f_{1HR3}(290)$$
$$= (.00286\ e^{-.00286(170)}) \ldots (.00286\ e^{-.00286(290)})$$



Definitions of states :

1 = Schist

2 = Metaquartzite

3 = Diorite

4 = Quartzite

Figure 7.3 Recorded transition chain of Rock Type in the first stage.

l (ft.)	r	h (ft.)
0	3	
768	4	222
990	3	170
1160	2	137
1297	4	212
1509	3	177
1686	4	396
2082	3	222
2304	4	529
2833	3	290
3123	4	362
3485	1	
4010		

Table 7.16 Recorded transition chain of Rock Type in the first stage.

l (ft.)	d	h (ft.)
0	1	
922	2	170
1092	1	111
1203	2	128
1331	1	137
1468	2	972
2440	1	169
2609	2	258
2867	3	273
3140	2	474
3614	1	93
3707	2	150
3857	3	
4010		

Table 7.17 Recorded transition chain of RQD in the first stage.

l (ft.)	e	h (ft.)
0	1	
1055	2	358
1413	1	1341
2754	2	267
3021	1	890
3911	2	
4010		

Table 7.18 Recorded transition chain of Degree of Weathering in the first stage.

l (ft.)	w	h (ft.)
0	1	
1297	3	751
2048	1	1467
3515	3	
4010		

Table 7.19 Recorded transition chain of Availability of Water in the first stage.

$= 5.85E-12$

After P_m is updated to P'_m using (6.7), the coefficient to be used in the second stage is

$$c_{R3} = P'_1 \, c_{1R3} + P'_2 \, c_{2R3} + P'_3 \, c_{3R3}$$
$$= .397(.00286) + .126(.00167) + .477(.00333)$$
$$= .00293$$

(whereas previously c_{R3} =.00263 was used in the first stage.)

Every row of transition probabilities is now updated according to the procedures stated in section 6.3.3. The updating of the second row of transition probabilities of RQD can be taken as an example :

m	1	2
P_{mD21}	.75	.90
P_{mD23}	.25	.10
P_m	.50	.50
L_m	.0198	.00656
P'_m	.751	.249

L_m was calculated according to (6.9) :

$$L_m = (P_{mD21})^4 \, (P_{mD23})^2$$

since four transitions are make from state 2 to state 1 (i.e. $SF_{D21} = 4$) and two transitions are made from state 2 to state 3 (i.e. $SF_{D23} = 2$.)

After P_m is updated to P'_m using (6.10), the row to be used in the second stage is

$$(P_{D21}\ P_{D22}\ P_{D23})$$
$$= P'_1\ (P_{1D21}\ P_{1D22}\ P_{1D23}) + P'_2\ (P_{2D21}\ P_{2D22}\ P_{2D23})$$
$$= .751(.75\ .00\ .25) + .249(.90\ .00\ .10)$$
$$= (.787\ .000\ .213)$$

The transition intensity coefficients and the transition probabilities to be used in the second stage are summarised in Tables 7.20 to 7.23. The transition intensity matrix (section 3.3) for each parameter can then be assembled and input into STATEP. Recall that, in addition to the transition intensity matrices, observations ahead of the tunnel face also need to be input into STATEP. However, since the tunnel face is now at borehole ADT-37B (l=4010), the observations at the boreholes preceding ADT-37B are not used, because only the remaining observations can affect the geological predictions. Thus only the observations at boreholes ADT-37B through ADT-42 are input into STATEP. The actual input and output are shown in Figs. D.5 and D.6 respectively.

For example, from Fig. D.6 the probability of having state 1 (Schist) of X1 (R) at l=5100 is .032 instead of .036 as it was at the start of the first stage. The difference in the probabilities is solely due to the updating of the transition intensity coefficients and transition probabilities since the observations immediately preceding

i \ j	1	2	3	4	c_{Ri}
1	.00	.02	.23	.75	.00138
2	.012	.00	.483	.505	.00819
3	.016	.203	.00	.781	.00293
4	.239	.125	.636	.00	.00221

Table 7.20 Transition probabilities and transition intensity coefficients for R used in the second stage.

i \ j	1	2	3	c_{Di}
1	.00	.954	.046	.00294
2	.787	.00	.213	.00282
3	.167	.833	.00	.00200

Table 7.21 Transition probabilities and transition intensity coefficients for D in the second stage.

i \ j	1	2	c_{Ei}
1	.00	1.00	.000448
2	1.00	.00	.00169

Table 7.22 Transition probabilities and transition intensity coefficients for E in the second stage.

i \ j	1	2	3	c_{Wi}
1	.00	.470	.530	.000824
2	.90	.00	.10	.00633
3	.911	.089	.00	.00169

Table 7.23 Transition probabilities and transition intensity coefficients for W in the second stage.

and following the point l=5100 are assumed to be unchanged from the first stage. (Recall that only these two observations can affect the state probabilities at l=5100 because of the single -step assumption; see section 4.2.1.4.)

With the updated parameter probability profiles (Fig. D.6), the probability calculations involving a geological vector $\overline{g}(l)$ can be performed as described in section 4.3. For example,

$P[\overline{g}(6000) = (1\ 1\ 1\ 1)$

$= P[r(6000)=1,\ d(6000)=1,\ e(6000)=1,\ w(6000)=1]$

$= P[r(6000)=1]\ P[d(6000)=1]\ P[e(6000)=1]\ P[w(6000)=1]$

$= (.621)\ (.576)\ (.683)\ (.897)$

$= .219$ (was .249 at start of first stage)

GC probability calculations can also be performed as described in section 4.3. For example,

P[GC exits at l=6000]

$= P[\overline{g}(6000)=(4\ 1\ 1\ 1)$ or $\overline{g}(6000)=(4\ 1\ 1\ 2)]$

(see GC classification in Table 7.4)

$= P[r(6000)=4]\ P[d(6000)=1]\ P[e(6000)=1]\ P[w(6000)=1]$

$+ P[r(6000)=4]\ P[d(6000)=1]\ P[e(6000)=1]\ P[w(6000)=2]$

$= (.241)\ (.576)\ (.683)\ (.897)$

$+ (.241)\ (.576)\ (.683)\ (.054)$ (from Fig. D.6)

$= .090$ (was .101 at start of first stage)

(Updating was applied to the second section of the tunnel using information from the first section. It should be remembered that updating as shown above should be used whenever subjectively obtained competing hypotheses are to be combined with frequency type data. Thus, if in the first section of the tunnel frequency data had been available in addition to the subjectively assessed hypotheses, such an updating would have been performed there also.)

7.4 Summary

A case study for the construction planning of a water tunnel has been presented. The general geology of the tunnel region is examined and four geological parameters (Rock Type, RQD, Degree of Weathering, Availability of Water) are considered. The ground class classification table is set up using a performance model based on past engineering experience and knowledge. The entire tunnel is considered to be "homogeneous" (having similar geology) and the geological parameters are taken to be independent of each other. The transition intensity coefficients and transition probabilities for these parameters are derived using the subjective judgment method described in section 5.3.1. Example calculations on parameter probability profiles are made.

In order to show how the transition intensity coefficients and transition probabilities (which are established using subjective judgment can be updated as described in Chapter 6, competing hypotheses are set up before tunnel construction. The probability P_m of hypothesis H_m being true is then updated by newly recorded data from the tunnel. The estimates of the transition intensity coefficients and transition probabilities are then also updated.

CHAPTER VIII

CONCLUSION

A geological prediction and updating model in tunneling has been developed in this thesis . The concept of Markov processes is used to model the states of geological parameters along a tunnel axis. Through the Markov process concept the existence and extent of the parameter states along the tunnel axis can be predicted probabilistically.

The existence of parameter states at a certain point along the unexcavated tunnel is predicted probabilistically by interval transition probabilities while the state immediatedly following the state at the tunnel face is predicted by transition probabilities. The extent of a certain state is described probabilistically by exponential extent distributions. These three probability distributions can be modified (updated) by "observations" of the parameter states ahead of the tunnel face. (e.g. If the tunnel face at station 0+50 is in Schist while Diorite is found in a boring at station 1+00, it is clear that the Schist cannot extend up to or past station 1+00. The extent distribution of the the Schist is updated so that the probability density of its extending past station 1+00 is zero.)

Another higher level of updating is that of the transition intensity coefficients and transition probabilities of the geological parameters. These coefficients and probabilities are the basic elements of the Markov process and can be estimated before tunnel construction either based on frequency data (if the amount of data is sufficient) or subjective judgment (if the amount of data is not sufficient.) In either case, the coefficients and probabilities can be updated as tunnel construction proceeds and frequency data are recorded from the newly excavated parts of the tunnel.

To show that the proposed model is a practically useful tool for geological prediction and construction planning in tunneling, the concept of ground classes and its application in cost optimization are introduced in Chapter II. A case study (Chapter VII) is presented to demonstrate the actual application of the proposed model.

Thus a practically useful geological prediction and updating model has been developed. Cost optimization and estimation of tunnel construction can be performed systematically using the probability distributions of the geological parameters considered in the model.

APPENDIX A

CHI-SQUARE TESTS ON TWO RQD EXTENT DISTRIBUTIONS

A.1 Procedure

An estimated RQD-profile of the discharge tunnel of Seabrook Station (Fig. 5, Report on TCM utilization program, Seabrook Station, Fall 1976) was examined. The RQD values along the lower boundary (invert) of the tunnel were considered and the geological parameter Degree of Jointing (RQD) was defined by:

d	RQD (%)
1	0 - 25
2	25 - 75
3	75 - 100

For the states 2 and 3, the extents were recorded and summarised in the frequency tables and histograms of Fig. A.1 and A.2 respectively. An exponential extent distribution was fitted to each frequency record. Then a number (= NC) of categories were established, keeping the expected frequency inside each category at 5 (except that the last category might have less or more than 5.) The number of degrees of freedom is NC-1-1 = NC-2. The Chi-square statistic is

$$C = \sum_{i=1}^{NC} \frac{(F_i - E_i)^2}{E_i}$$

Extent (ft.)	frequency	Extent (ft.)	frequency
0-50	5	250-300	4
50-100	7	300-350	1
100-150	5	350-400	0
150-200	4	400-450	1
200-250	0	450	1
		total	28

average extent = 167 ft.

std. deviation = 142 ft.

$$\hat{c}_{D2} = (1/167 + 1/142)/\ 2 = .00652\ \text{ft.}^{-1}$$

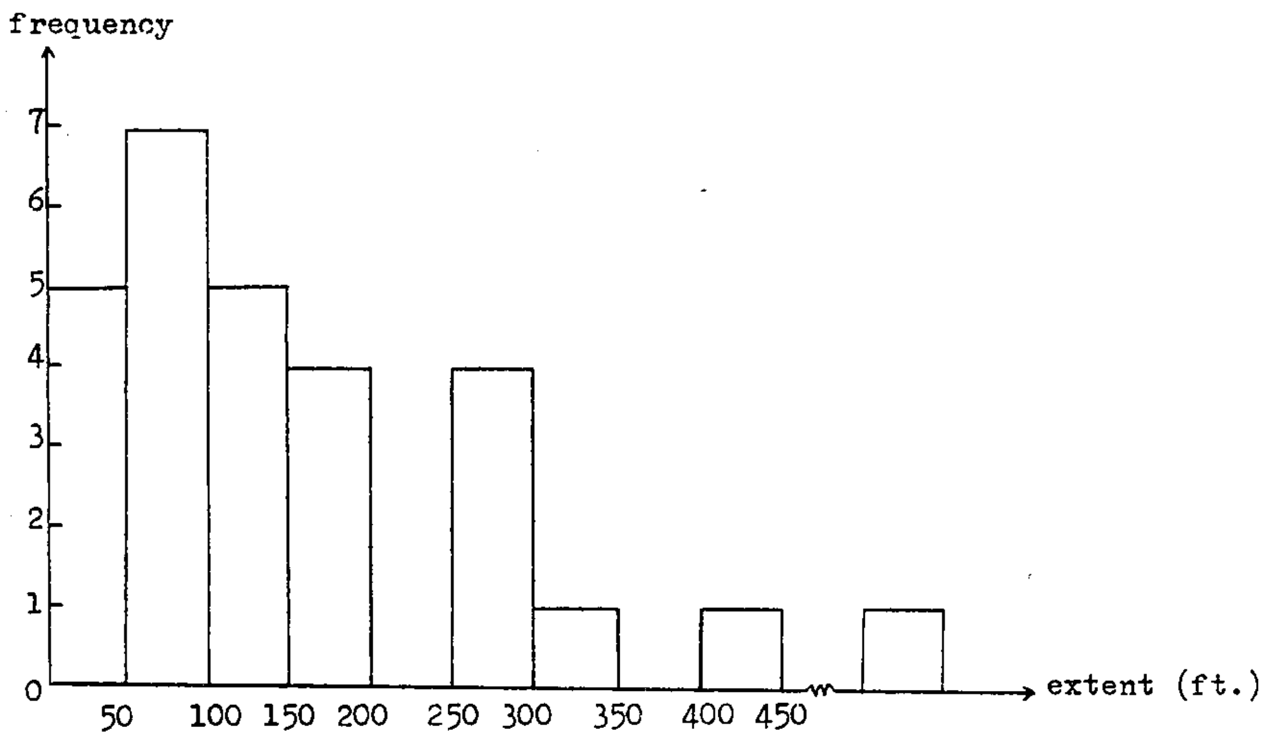


Figure A.1 Extent frequency counts and histogram of medium RQD (d = 2).

Extent (ft.)	frequency	Extent (ft.)	frequency
0-50	5	400-450	1
50-100	3	600-650	1
100-150	4	800-850	1
150-200	2	950-1000	1
200-250	1	1100-1150	1
300-350	1	1750-1800	1
350-400	1	1950-2000	1
		total =	24

average extent = 420 ft.

std. deviation = 549 ft.

$$\hat{c}_{D3} = (1/420 + 1/549)/\ 2 = .00210\ \text{ft.}^{-1}$$

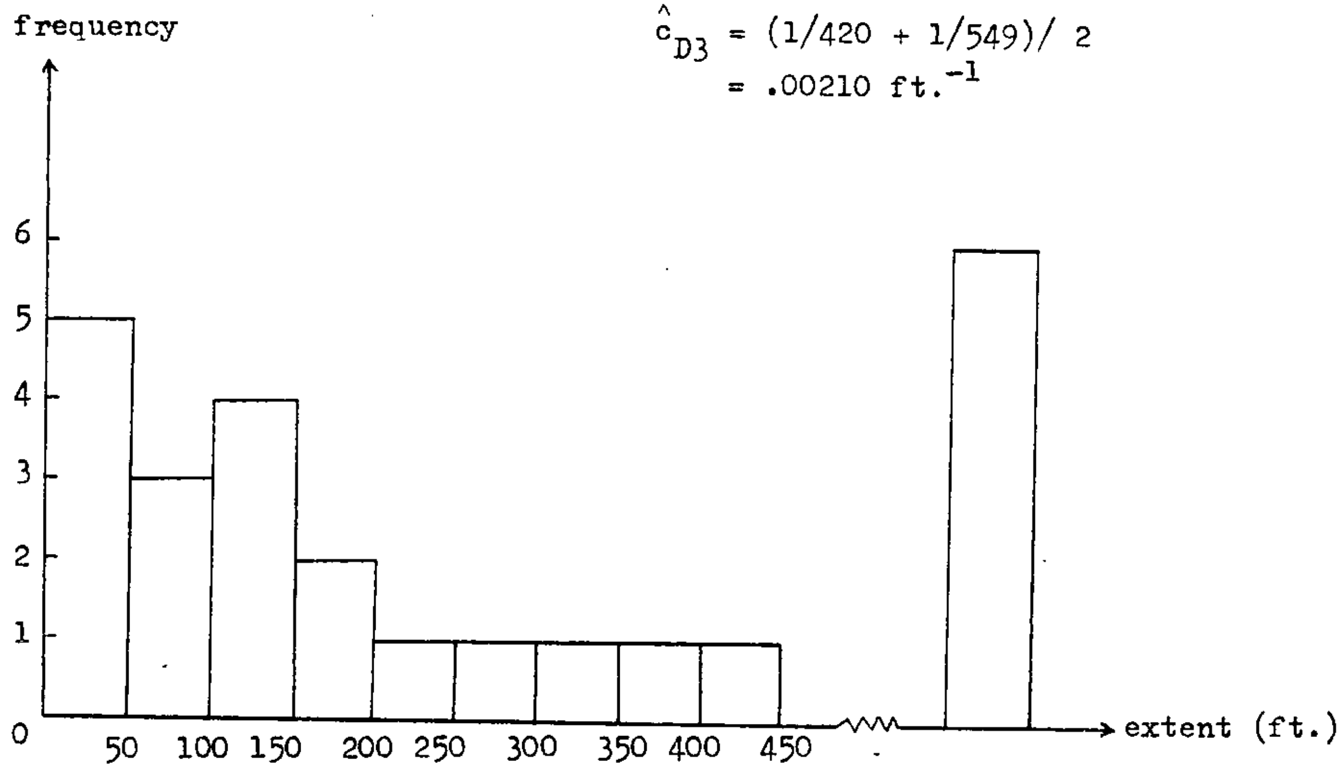


Figure A.2 Extent frequency counts and histogram of high RQD (d = 3).

where F_i = recorded frequency within category i and E_i = expected frequency within category i.

A.2 Results

For d=2 (medium RQD), the Chi-square level calculated is 0.18, meaning that if the extent distribution is really exponentially distributed, then there is 0.18 probability that the Chi-square value is greater than that calculated. For d=3, the Chi-square level is 0.16.

Hence both tests were passed with satisfaction because usually the Chi-square level required is about 0.05 only. The results confirm the assumption that the extent distribution of RQD states are exponential.

APPENDIX B

THE PROXIMITY RULE

B.1 Introduction

The proximity rule is a rule relating the states of a random variable at two locations separated by a certain distance. In the case of a tunnel the situation is shown in Fig. B.1. The random variable is a geological parameter X which has n possible states.

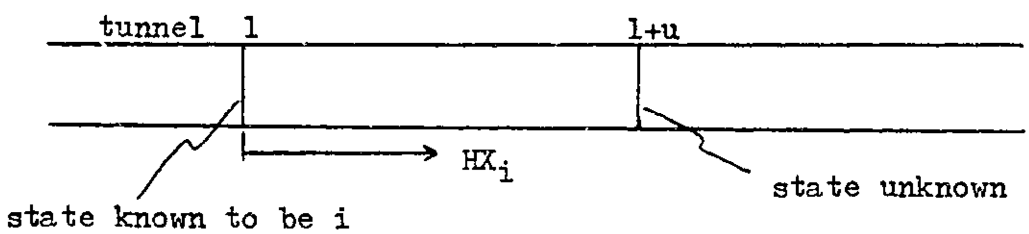


Figure B.1 Illustration of the proximity rule.

Given that x(l)=i, the proximity rule states that

$$P[x(l+u)=i \mid x(l)=i] = p + (1-p)\, e^{-au} \quad \ldots\ldots \quad (B.1)$$

where p is the prior probability of finding state i at any point along the tunnel axis and a is a certain constant. Furthermore, if x(l) is not known deterministically and P[x(l)=i] is given as q, the proximity rule states that

$$P[x(l+u)=i \mid q] = p + (q-p)\, e^{-au} \quad \ldots\ldots \quad (B.2)$$

B.2 Derivation of rule using Markov process concept

By assuming that x(l) obeys a homogeneous Markov process with n states and transition intensity coefficients (section 3.2.3) c_{X_i}, it can be shown that (B.1) is approximately true:

$P[x(l+u)=i \mid x(l)=i]$

$= P[HX_i \geq u \mid x(l)=i] + P[HX_i < u \text{ and } x(l+u)=i \mid x(l)=i]$

where HX_i is the extent of state i at l in the positive direction of l. Since

$P[HX_i \geq u \mid x(l)=i] = 1 - F_{HX_i}(u)$

where $F_{HX_i}(h)$ is the CDF of HX_i (section 3.2.3) and

$P[HX_i < u \text{ and } x(l+u)=i \mid x(l)=i]$

$= P[HX_i < u \mid x(l)=i]\ P[x(l+u)=i \mid x(l)=i \text{ and } HX_i < u]$

$\cong F_{HX_i}(u)\ p$, we have

$P[x(l+u)=i \mid x(l)=i] \cong 1 - F_{HX_i}(u) + F_{HX_i}(u)\ p$

$= 1 - (1 - e^{-c_{X_i} u}) + (1 - e^{-c_{X_i} u})\ p$

$= p + (1 - p)\ e^{-c_{X_i} u}$

which shows that (B.1) is approximately true if the constant a is taken to be c_{X_i}.

(B.2) can also be shown to be approximately true if all the states are also assumed to have the same transition intensity coefficient c :

$P[x(l+u)=i \mid q]$

$= q\ P[x(l+u)=i \mid x(l)=i] + (1-q)\ P[x(l+u)=i \mid x(l)\neq i]$

$$\cong q\left\{ p+(1-p)e^{-au} \right\} + (1-q)\ P[x(l+u)=i \mid x(l)\neq i] \qquad \text{..... (B.3)}$$

Since $P[x(l+u)=i \mid x(l)\neq i]$

$$= \sum_{m\neq i} P[x(l)=m \mid x(l)\neq i]\ P[x(l+u)=i \mid x(l)=m \text{ and } x(l)\neq i]$$

$$= \sum_{m\neq i} P[x(l)=m \mid x(l)\neq i]\ P[x(l+u)=i \mid x(l)=m]$$

and $P[x(1+u)=i \mid x(1)=m \neq i]$

$= P[HX_m < u]\ p = F_{HX_m}(u)\ p$

$= (1 - e^{-cu})\ p$, we have

$P[x(1+u)=i \mid x(1) \neq i]$

$$\cong \sum_{m \neq i} P[x(1)=m \mid x(1) \neq i]\ (1 - e^{-cu})\ p$$

$$= (1 - e^{-cu})\ p \sum_{m \neq i} P[x(1)=m \mid x(1) \neq i]$$

$$= (1 - e^{-cu})\ p$$

Therefore from (B.3),

$$P[x(1+u)=i \mid q]$$

$$\cong q \left\{ p + (1-p)e^{-au} \right\} + (1-q)\ (1-e^{-cu})\ p$$

$$= q(1-p)e^{-au} + p - p\ e^{-au} + q\ p\ e^{-au}$$

$$= p + (q - p)\ e^{-au}$$

where c is taken to be equal to a.

B.3 Practical considerations

To put the proximity rule into practical use, Lindner (1975) suggested an "exploration function" through which a can be determined by subjective judgment. However, when the concept of the Markov process is applied, with the assumptions stated above a is seen to be the transition intensity coefficient c_{x_i} (section 3.2.3) of state i. Thus the value of a is the reciprocal of the average extent of state i and can be found much easier.

APPENDIX C

UPDATED TRANSITION PROBABILITY FOR SIMULATIONS

C.1 One deterministic observation

If $HX_i = h_s$ and there is a deterministic observation ahead of the point $(l_e + h_s)$ as shown in Fig. C.1, the transition probability is updated to

$$P^{dc}_{Xij} = C\, P_{Xij}\, [\text{likelihood of } x(l_1) \mid \text{next state is } j \text{ and } HX_i = h_s] \quad \ldots\ldots \text{(C.1)}$$

where

$$[\text{likelihood of } x(l_1) \mid \text{next state is } j \text{ and } HX_i = h\ ]$$

$$= v_{Xjk}(l_1 - l_e - h_s)$$

Therefore from (C.1),

$$P^{dc}_{Xij} = C\, P_{Xij}\, v_{Xjk}(l_1 - l_e - h_s) \quad \ldots\ldots \text{(C.2)}$$

where C = normalizing constant

$$= \left\{ \sum_{j=1}^{n} P_{Xij}\, v_{Xjk}(l_1 - l_e - h_s) \right\}^{-1} \quad \ldots\ldots \text{(C.3)}$$

(n is the total number of states of X.)

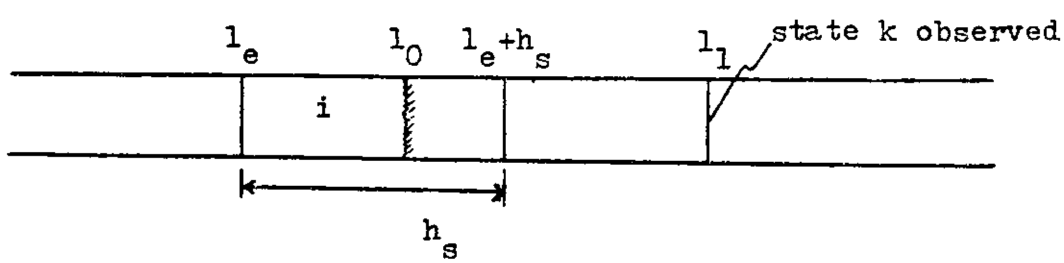


Figure C.1 Case with one deterministic observation.

C.2 One non-deterministic observation

When there is a non-deterministic observation as shown in Fig. C.2, P_{Xij} is updated to

$$P^{nc}_{Xij} = \sum_{k=1}^{n} p_{1k} \; P^{d}_{Xij} \quad \ldots\ldots \quad (C.4)$$

C.3 Several deterministic observations

There are s deterministic observations ahead of the point $(l_e + h_s)$ (see Fig. C.3.) Due to the property of a single-step memory, observations at l_2, l_3, ... l_s have no effect on P_{Xij}. Therefore P_{Xij} is updated to P^{Kc}_{Xij} as in the case with one deterministic observation (section C.1) :

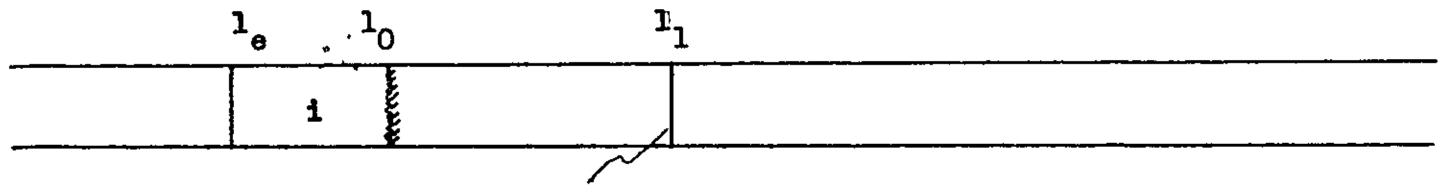
$$P^{Kc}_{Xij} = \left\{ \sum_{j=1}^{n} P_{Xij} \; v_{Xjk}(l_1 - l_e - h_s) \right\}^{-1} P_{Xij} \; v_{Xjk}(l_1 - l_e - h_s) \quad \ldots\ldots \quad (C.5)$$

where $k = k_1$ = observation at l_1.

C.4 Several non-deterministic observations

There are s non-deterministic observations ahead of the point $(l_e + h_s)$ (Fig. C.4.) Due to the property of a single-step memory, only the observation at l_1 affects P_{Xij} which is then updated to

$$P^{Nc}_{Xij} = \sum_{k=1}^{n} p_{1k} \; P^{Kc}_{Xij} \quad \ldots\ldots \quad (C.6)$$



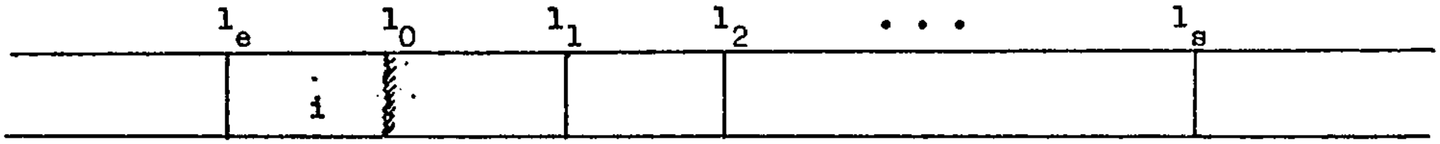
PMF given :

$P[x(l_1) = m] = p_{lm}$

$(m = 1, 2, \ldots n)$

where n = total number of states of X.

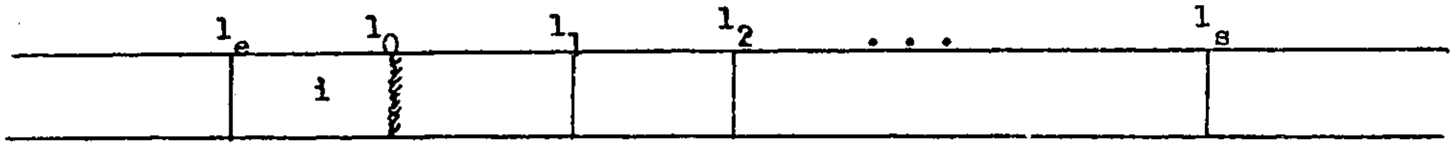
Figure C.2 Case with one non-deterministic observation.



The state at l_t $(t = 1, 2, \dots s)$ is known :

$$x(l_t) = k_t$$

Figure C.3 Case with s deterministic observations.



The observation at l_t $(t = 1, 2, \ldots s)$ is non-deterministic and is given by the PMF

$$P\left[x(l_t) = m\right] = p_{tm}$$

$(m = 1, 2, \ldots n)$

where n = total number of states of X.

Figure C.4 Case with s non-deterministic observations.

APPENDIX D

STATEP - USER'S MANUAL AND EXAMPLES

D.1 Introduction

STATEP (State Prediction) is a Fortran computer program which calculates the probability profiles of independent parameters according to the geological prediction model presented in Chapter IV. The co-ordinate system uses the station concept : a position along the tunnel axis is identified by its distance l from a fixed point such as the portal of the tunnel (Fig. D.1.) The positive direction of l is in the direction of advance of tunnel construction.

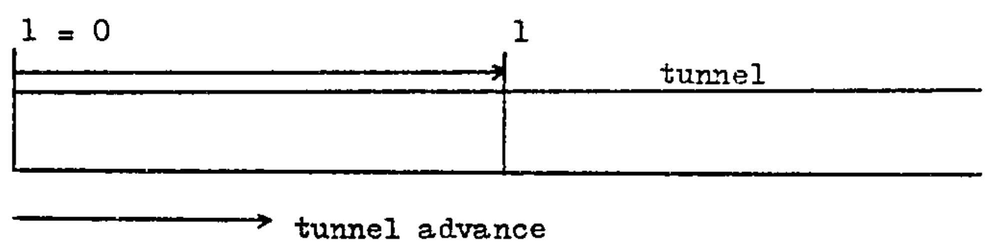


Figure D.1 Co-ordinate system.

In order to calculate the interval transition probabilities of a parameter XI, STATEP forms the spectral resolution of the transition intensity matrix of XI (section 3.3.1.) In calculating the eigenvalues and eigenvectors for the spectral resolution, the subroutine EIGRF of the International Mathematical and Statistical Library is used.

D.2 Input

The same length unit (e.g. feet) must be used throughout. STATEP takes input from a file (with free format) and writes the output into another file. If other forms of input and output are used some input and output statements must be modified. The format of the input file is described below:

<u>Line 1</u> AL, BL, SL, BP

AL is the beginning point (usually the tunnel face) of the section of the tunnel considered while BL is the end point. SL is the interval between the points at which the state probabilities of each parameter are required (see Fig. D.2). BP is the point at which the parameter probability profile begins i.e. state probabilities are calculated at BP, BP+SL, BP+2SL ... BP must precede the second observation at OL(I,2), which is the co-ordinate of the second observation on XI. The parameter probability profile is limited to the to the range (AL,BL) and the total number of points at which state probabilities are calculated is at most 100.

<u>Line 2</u> N

N is the total number of geological parameters considered. It cannot exceed 5.

<u>Line 3</u> NS(1)

NS(1) is the number of states of parameter X1.

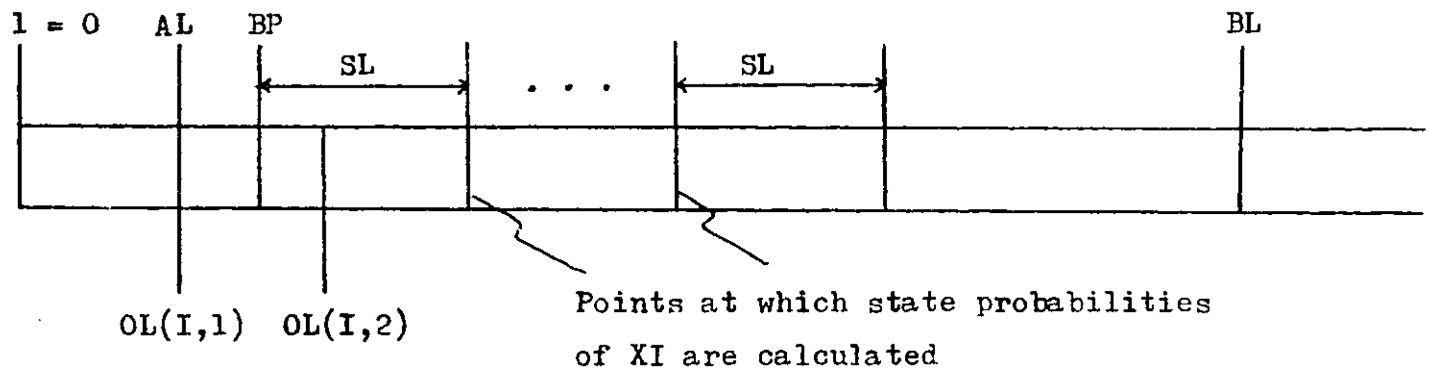


Figure D.2 Positions of different points in tunnel section.

Lines 4 through (3 + NS(1))

These NS(1) lines represent the transition intensity matrix of X1. Each line contains a row of the transition intensity matrix (see example input in Fig. D.3.)

Line (4 + NS(1)) NO(1)

NO(1) is the total number of observations on X1. It must be at least 1 and less than 20.

Lines (5 + NS(1)) through (4 + NS(1) + NO(1))

These NO(1) lines give the non-deterministic observations (which can include deterministic observations) on X1. The first observation must be at AL, which is usually the tunnel face. The first number on each line is the position of the corresponding observation and increases with the line number. The remaining NS(1) numbers on each line are the state probabilities of X1 representing the non-deterministic observation. (see lines 9 to 21 in the example input in Fig. D.3 in section D.4.1.)

Lines (5 + NS(1) + NO(1)) through

(2 + 2N + NS(1) + NO(1) + ... + NS(N) + NO(N))

The first (2 + NS(2) + NO(2)) of these lines contain NS(2), the transition intensity matrix of X2, NO(2) and the observations on X2 respectively. The similar information relating to X3 , ... , XN are input similarly in the remaining lines.

D.3 Dictionary

STATEP is written in Fortran IV. This dictionary contains the definitions of important variables, functions and subroutines listed in the order of their appearance in the executable part of the program. Input variables defined in section D.2 are not defined again here.

Variable	Definition
A(I,J,K)	The element of the transition intensity matrix of XI at the Jth row and Kth coloumn.
OL(I,J)	The position of the Jth observation of XI.
OP(I,J,K)	The probability of having state K at the Jth observation of XI.
AX	Transition intensity matrix of parameter being considered.
AXT	Transpose of AX. It is used to find the left eigenvectors of AX.
IJOB	See SUBROUTINE EIGRF.
EIGRF	See SUBROUTINE EIGRF.
ORDER	See SUBROUTINE ORDER.
WR	Complex vector containing the eigenvalues of AX.
ZR	Complex vector containing the right eigenvectors of AX.
WL	Complex vector containing the eigenvalues of AX.
ZL	Complex vector containing the left eigenvectors

	of AX.
TEMP	(i) Temporary variable used to form the transpose of ZL.
	(ii) The dot product of the Jth row of ZL and the Jth column of ZR.
RL	Point at which state probabilities are calculated.
IS	Status of RL : IS = number of observations preceding RL.
NP	Total number of points in parameter probability profile.
NPT	Total number of points in parameter probability profile according to input specifications.
IP	Number designation of point corresponding to RL.
NRO	Number of remaining observations following RL.
RLTM1	Position of the observation immediately preceding RL.
RLT	Position of the observation immediately following RL.
SP(I,IP,J)	Probability of having state J of XI at point IP.
TP1,TP2,TP3	Terms in the expression given by equation (4.8a).

P	See FUNCTION P below.
IPNOA	Point following which there are no more observations.

SUBROUTINE EIGRF (A,N,IA,IJOB,W,Z,IZ,WK,IER)

This subroutine is contained in the International Mathematical and Statistical Library. Its purpose is to calculate the eigenvalues and eigenvectors of matrix A.

Variable	Definition
A	A square matrix.
N	The input order of A.
IA	Row dimension of A as specified in the DIMENSION statement of the calling program.
IJOB	IJOB = 2 means : compute eigenvalues, eigenvectors and performance index.
W	Complex eigenvalues of A returned.
Z	Complex right eigenvectors of A returned.
IZ	Row dimension of Z as specified in the DIMENSION statement of the calling program.
WK	Work area. Length of WK must be at least 35.
IER	Error parameter.

SUBROUTINE ORDER (W,Z,NS)

The purpose of this subroutine is to re-arrange the eigenvalues in W and the eigenvectors in Z in an ascending order of magnitude of the eigenvalues.

Variable	Definition
W	Complex vector of eigenvalues.
Z	Complex array of eigenvectors.
NS	Number of states of parameter being considered.

FUNCTION P (I,J,U,NS,WR,ZR,ZL)

The purpose of this function is to calculate the interval transition probability from state I to state J using the spectral resolution of the transition intensity matrix. The underlying theory is given in section 3.3.1.

Variable	Definition
D	Complex exponential of the product of an eigenvalue and U.
NS	Number of states of the parameter.
WR	Complex vector of eigenvalues.
U	Distance between the points with states I and J.
CP	Complex interval transition probability.
ZR	Complex right eigenvectors.
I,J	States of the parameter considered.

ZL Complex left eigenvectors.

D.4 Example cases

In Chapter VII a case study was presented on the probabilistic prediction of the geological parameters along a water tunnel. The calculations of parameter probability profiles in the first and second stages (section 7.3) are presented in sections D.4.1 and D.4.2 below.

D.4.1 First stage

A parameter probability profile was calculated for each parameter at the beginning of the first stage i.e. when the tunnel face was at l=0. The first point at which state probabilities were calculated was at l=300 (length were always measured in feet.) The following points at which state probabilities were calculated were at 600, 900, ... , 7500 (i.e. SL=300.) Thus there were a total of 25 points in each parameter probability profile at which state probabilities were calculated. The input is listed in Fig. D.3 (the input data are based on Tables 7.5 to 7.8 and Tables 7.12 to 7.15) and the output in Fig. D.4. In the output shown in Fig. D.4, X1, X2, X3 and X4 denote Rock Type, RQD, Degree of Weathering and Availability of Water respectively.

						Explanations (see Dictionary, section D.3)
0.	7662.	300.	300.			AL BL SL BP
4						N
4						NS(1)
-.138E-2	.276E-4	.317E-3	.104E-2			Transition intensity matrix of R used in the first stage
.164E-3	-.822E-2	.411E-2	.395E-2			
.524E-4	.524E-3	-.262E-2	.204E-2			
.575E-3	.425E-3	.150E-2	-.25E-2			
13					NO(1)	
0.	0.	0.	1.	0.	OL(1,1) OP(1,1,1) OP(1,1,2) OP(1,1,3) OP(1,1,4)	
341.	0.	0.	1.	0.	.	
717.	0.	.2	.8	0.	.	
1239.	0.	.5	0.	.5	.	
1945.	0.	0.	.2	.8		
2788.	0.	0.	0.	1.		
3566.	.8	0.	0.	.2		
4010.	1.	0.	0.	0.		
4659.	0.	0.	0.	1.		
5256.	0.	0.	0.	1.		
5785.	1.	0.	0.	0.		
6604.	0.	0.	.9	.1		
7662.	0.	0.	0.	1.	OL(1,13) OP(1,13,1) OP(1,13,2) OP(1,13,3) OP(1,13,4)	
3						
-.233E-2	.217E-2	.163E-3				

Figure D.3 Input to STATEP (first stage).

```
.327E-2  -.394E-2  .670E-3
.940E-3  .940E-3  -.188E-2
13
0.      .5   .5   0.
341.    1.   0.   0.
717.    1.   0.   0.
1239.   .5   .5   0.
1945.   .2   .8   0.
2788.   .2   0.   .8
3566.   .5   .5   0.
4010.   0.   .2   .8
4659.   0.   0.   1.
5256.   1.   0.   0.
5785.   .8   .2   0.
6604.   .8   .2   0.
7662.   1.   0.   0.
2
-.448E-3  .448E-3
.162E-2  -.162E-2
13
0.      1.   0.
341.    1. 0.
717.    1. 0.
```

Figure D.3 (continued).

```
1239.  1. 0.
1945.  .6  .4
2788.  .2  .8
3566.  .5  .5
4010.  .2  .8
4659.  .2  .8
5256.  .8  .2
5785.  .6  .4
6604.  .8  .2
7662.  1.  0.
3
-.824E-3  .453E-3  .371E-3
.570E-2  -.633E-2  .633E-3
.153E-2 .170E-3  -.170E-2
13
0.     1.  0.  0.
341.   1.  0.  0.
717.   1.  0.  0.
1239.  .5  0.  .5
1945.  .4  0.  .6
2788.  .8  0.  .2
3566.  .5  0.  .5
4010.  .2  0.  .8
```

Figure D.3 (continued).

4659.	.6	.2	.2
5256.	.2	.8	0.
5785.	1.	0.	0.
6604.	.6	.4	0.
7662.	1.	0.	0.

Figure D.3 (continued).

states probabilities

PROBABILITY PROFILE OF X1

IP	RL	1	2	3	4
1	0.3000E+03	0.001	0.013	0.949	0.037
2	0.6000E+03	0.003	0.099	0.791	0.108
3	0.9000E+03	0.010	0.120	0.564	0.305
4	0.1200E+04	0.004	0.379	0.113	0.503
5	0.1500E+04	0.041	0.097	0.304	0.557
6	0.1800E+04	0.031	0.039	0.282	0.648
7	0.2100E+04	0.042	0.037	0.274	0.647
8	0.2400E+04	0.066	0.047	0.291	0.596
9	0.2700E+04	0.027	0.026	0.118	0.829
10	0.3000E+04	0.225	0.029	0.131	0.615
11	0.3300E+04	0.501	0.021	0.101	0.378
12	0.3600E+04	0.810	0.002	0.005	0.183
13	0.3900E+04	0.931	0.002	0.006	0.061
14	0.4200E+04	0.660	0.012	0.082	0.246
15	0.4500E+04	0.229	0.024	0.118	0.630
16	0.4800E+04	0.035	0.032	0.136	0.797
17	0.5100E+04	0.036	0.033	0.148	0.784
18	0.5400E+04	0.280	0.019	0.066	0.636
19	0.5700E+04	0.825	0.005	0.016	0.153
20	0.6000E+04	0.617	0.016	0.132	0.236
21	0.6300E+04	0.247	0.038	0.376	0.339
22	0.6600E+04	0.002	0.002	0.888	0.107

Figure D.4 Output (parameter probability profiles) in the first stage.

23	0.6900E+04	0.036	0.051	0.532	0.380
24	0.7200E+04	0.059	0.051	0.386	0.503
25	0.7500E+04	0.040	0.038	0.206	0.716

PROBABILITY PROFILE OF X2

IP	RL	1	2	3
1	0.3000E+03	0.910	0.084	0.006
2	0.6000E+03	0.869	0.123	0.009
3	0.9000E+03	0.734	0.247	0.019
4	0.1200E+04	0.533	0.460	0.008
5	0.1500E+04	0.542	0.410	0.049
6	0.1800E+04	0.423	0.542	0.036
7	0.2100E+04	0.358	0.511	0.131
8	0.2400E+04	0.339	0.310	0.351
9	0.2700E+04	0.216	0.118	0.666
10	0.3000E+04	0.323	0.193	0.484
11	0.3300E+04	0.472	0.330	0.198
12	0.3600E+04	0.473	0.479	0.049
13	0.3900E+04	0.149	0.298	0.552
14	0.4200E+04	0.076	0.137	0.787
15	0.4500E+04	0.045	0.077	0.878
16	0.4800E+04	0.198	0.143	0.660
17	0.5100E+04	0.629	0.197	0.174
18	0.5400E+04	0.798	0.188	0.014

Figure D.4 (continued).

19	0.5700E+04	0.747	0.241	0.012
20	0.6000E+04	0.661	0.302	0.037
21	0.6300E+04	0.642	0.312	0.046
22	0.6600E+04	0.796	0.203	0.001
23	0.6900E+04	0.630	0.317	0.053
24	0.7200E+04	0.620	0.314	0.066
25	0.7500E+04	0.755	0.209	0.035

PROBABILITY PROFILE OF X3

IP	RL	1	2
1	0.3000E+03	0.993	0.007
2	0.6000E+03	0.983	0.017
3	0.9000E+03	0.969	0.031
4	0.1200E+04	0.990	0.010
5	0.1500E+04	0.854	0.146
6	0.1800E+04	0.693	0.307
7	0.2100E+04	0.584	0.416
8	0.2400E+04	0.494	0.506
9	0.2700E+04	0.291	0.709
10	0.3000E+04	0.370	0.630
11	0.3300E+04	0.487	0.513
12	0.3600E+04	0.489	0.511
13	0.3900E+04	0.311	0.689
14	0.4200E+04	0.274	0.726

Figure D.4 (continued`.

15	0.4500E+04	0.266	0.734
16	0.4800E+04	0.395	0.605
17	0.5100E+04	0.686	0.314
18	0.5400E+04	0.761	0.239
19	0.5700E+04	0.646	0.354
20	0.6000E+04	0.680	0.320
21	0.6300E+04	0.751	0.249
22	0.6600E+04	0.799	0.201
23	0.6900E+04	0.824	0.176
24	0.7200E+04	0.865	0.135
25	0.7500E+04	0.938	0.062

PROBABILITY PROFILE OF X4

IP	RL	1	2	3
1	0.3000E+03	0.981	0.013	0.006
2	0.6000E+03	0.957	0.029	0.015
3	0.9000E+03	0.806	0.042	0.152
4	0.1200E+04	0.536	0.013	0.452
5	0.1500E+04	0.502	0.036	0.462
6	0.1800E+04	0.449	0.027	0.524
7	0.2100E+04	0.498	0.030	0.472
8	0.2400E+04	0.641	0.047	0.313
9	0.2700E+04	0.754	0.026	0.220
10	0.3000E+04	0.714	0.043	0.244

Figure D.4 (continued).

11	0.3300E+04	0.612	0.043	0.344
12	0.3600E+04	0.482	0.009	0.509
13	0.3900E+04	0.296	0.019	0.685
14	0.4200E+04	0.410	0.035	0.554
15	0.4500E+04	0.603	0.096	0.301
16	0.4800E+04	0.682	0.154	0.164
17	0.5100E+04	0.592	0.335	0.072
18	0.5400E+04	0.636	0.330	0.034
19	0.5700E+04	0.934	0.046	0.020
20	0.6000E+04	0.899	0.055	0.046
21	0.6300E+04	0.836	0.106	0.058
22	0.6600E+04	0.608	0.391	0.001
23	0.6900E+04	0.818	0.110	0.073
24	0.7200E+04	0.850	0.067	0.083
25	0.7500E+04	0.912	0.044	0.044

Figure D.4 (continued).

D.4.2 Second stage

A parameter probability profile was calculated for each parameter at the beginning of the second stage i.e. when the tunnel face was at l=4010. The first point at which state probabilities were calculated was at l=4050 and the following points were separated by intervals of 150 ft. Thus there were a total of 25 points in a parameter probability profile. The input data for the transition intensity matrices of the parameters are based on the updated data in Tables 7.20 to 7.23.

Since the tunnel face has advanced up to l=4010 now, the observations which can affect geological predictions are the last 6 observations shown in Tables 7.5 to 7.8. Thus only these last 6 observations are input to STATEP. The input and output to STATEP in the second stage are shown in Figs. D.5 and D.6 respectively.

Explanations (see Dictionary, section D.3)

4010.	7662.	150.	4050.	AL BL SL BP
4				N
4				NS(1)
-.138E-2	.276E-4	.317E-3	.104E-2	Transition intensity matrix of R used in the second stage
.983E-4	-.819E-2	.396E-2	.414E-2	
.469E-4	.595E-3	-.293E-2	.229E-2	
.530E-3	.272E-3	.141E-2	-.221E-2	

6					NO(1)
4010.	1.	0.	0.	0.	OL(1,1) OP(1,1,1) OP(1,1,2) OP(1,1,3) OP(1,1,4)
4659.	0.	0.	0.	1.	.
5256.	0.	0.	0.	1.	.
5785.	1.	0.	0.	0.	.
6604.	0.	0.	.9	.1	
7662.	0.	0.	0.	1.	OL(1,6) OP(1,6,1) OP(1,6,2) OP(1,6,3) OP(1,6,4)

3		
-.294E-2	.280E-2	.135E-3
.222E-2	-.282E-2	.601E-3
.334E-3	.167E-2	-.200E-2

6			
4010.	0.	.2	.8
4659.	0.	0.	1.
5256.	1.	0.	0.
5785.	.8	.2	0.

Figure D.5 Input to STATEP (second stage).

```
6604.  .8  .2  0.
7662.  1.  0.  0.
2
-.448E-3  .448E-3
.169E-2  -.169E-2
6
4010.  .2  .8
4659.  .2  .8
5256.  .8  .2
5785.  .6  .4
6604.  .8  .2
7662.  1.  0.
3
-.824E-3  .387E-3  .437E-3
.570E-2  -.633E-2  .633E-3
.154E-2  .150E-3  -.169E-2
6
4010.  .2  0.  .8
4659.  .6  .2  .2
5256.  .2  .8  0.
5785.  1.  0.  0.
6604.  .6  .4  0.
7662.  1.  0.  0.
```

Figure D.5 (continued).

PROBABILITY PROFILE OF X1 — states / probabilities

IP	RL	1	2	3	4
1	0.4050E+04	0.919	0.002	0.018	0.061
2	0.4200E+04	0.651	0.010	0.082	0.257
3	0.4350E+04	0.424	0.018	0.122	0.436
4	0.4500E+04	0.218	0.020	0.114	0.649
5	0.4650E+04	0.013	0.002	0.011	0.974
6	0.4800E+04	0.032	0.023	0.127	0.818
7	0.4950E+04	0.042	0.030	0.171	0.757
8	0.5100E+04	0.032	0.026	0.136	0.807
9	0.5250E+04	0.002	0.002	0.008	0.989
10	0.5400E+04	0.271	0.013	0.062	0.654
11	0.5550E+04	0.538	0.010	0.050	0.401
12	0.5700E+04	0.821	0.003	0.014	0.161
13	0.5850E+04	0.872	0.003	0.035	0.089
14	0.6000E+04	0.621	0.013	0.124	0.241
15	0.6150E+04	0.418	0.024	0.226	0.332
16	0.6300E+04	0.252	0.032	0.355	0.360
17	0.6450E+04	0.115	0.033	0.548	0.304
18	0.6600E+04	0.003	0.002	0.887	0.108
19	0.6750E+04	0.016	0.043	0.638	0.303
20	0.6900E+04	0.034	0.050	0.496	0.420
21	0.7050E+04	0.049	0.049	0.411	0.491
22	0.7200E+04	0.056	0.046	0.349	0.549

Figure D.6 Output (parameter probability profiles) in the second stage.

23	0.7350E+04	0.053	0.042	0.283	0.623
24	0.7500E+04	0.037	0.032	0.185	0.746
25	0.7650E+04	0.004	0.004	0.018	0.975

PROBABILITY PROFILE OF X2

IP	RL	1	2	3
1	0.4050E+04	0.016	0.196	0.788
2	0.4200E+04	0.048	0.182	0.770
3	0.4350E+04	0.049	0.158	0.793
4	0.4500E+04	0.029	0.108	0.863
5	0.465[illegible]E+04	0.001	0.008	0.990
6	0.4800E+04	0.115	0.263	0.622
7	0.4950E+04	0.290	0.378	0.332
8	0.5100E+04	0.541	0.331	0.128
9	0.5250E+04	0.976	0.021	0.003
10	0.5400E+04	0.753	0.235	0.012
11	0.5550E+04	0.670	0.313	0.017
12	0.5700E+04	0.713	0.278	0.010
13	0.5850E+04	0.703	0.285	0.011
14	0.6000E+04	0.576	0.390	0.033
15	0.6150E+04	0.531	0.425	0.044
16	0.6300E+04	0.542	0.417	0.041
17	0.6450E+04	0.616	0.359	0.026
18	0.6600E+04	0.793	0.206	0.001

Figure D.6 (continued).

19	0.6750E+04	0.613	0.359	0.028
20	0.6900E+04	0.525	0.426	0.049
21	0.7050E+04	0.494	0.446	0.060
22	0.7200E+04	0.503	0.439	0.059
23	0.7350E+04	0.557	0.397	0.046
24	0.7500E+04	0.686	0.289	0.025
25	0.7650E+04	0.967	0.032	0.002

PROBABILITY PROFILE OF X3

IP	RL	1	2
1	0.4050E+04	0.222	0.778
2	0.4200E+04	0.279	0.721
3	0.4350E+04	0.294	0.706
4	0.4500E+04	0.270	0.730
5	0.4650E+04	0.205	0.795
6	0.4800E+04	0.398	0.602
7	0.4950E+04	0.561	0.439
8	0.5100E+04	0.689	0.311
9	0.5250E+04	0.796	0.204
10	0.5400E+04	0.762	0.238
11	0.5550E+04	0.712	0.288
12	0.5700E+04	0.647	0.353
13	0.5850E+04	0.629	0.371
14	0.6000E+04	0.683	0.317

Figure D.6 (continued).

15	0.6150E+04	0.723	0.277
16	0.6300E+04	0.754	0.246
17	0.6450E+04	0.778	0.222
18	0.6600E+04	0.799	0.201
19	0.6750E+04	0.812	0.188
20	0.6900E+04	0.826	0.174
21	0.7050E+04	0.843	0.157
22	0.7200E+04	0.866	0.134
23	0.7350E+04	0.897	0.103
24	0.7500E+04	0.938	0.062
25	0.7650E+04	0.995	0.005

PROBABILITY PROFILE OF X4

IP	RL	1	2	3
1	0.4050E+04	0.250	0.008	0.741
2	0.4200E+04	0.411	0.032	0.557
3	0.4350E+04	0.530	0.053	0.417
4	0.4500E+04	0.603	0.092	0.304
5	0.4650E+04	0.604	0.190	0.206
6	0.4800E+04	0.678	0.154	0.168
7	0.4950E+04	0.686	0.186	0.129
8	0.5100E+04	0.586	0.336	0.078
9	0.5250E+04	0.225	0.772	0.004
10	0.5400E+04	0.636	0.328	0.036

Figure D.6 (continued).

11	0.5550E+04	0.825	0.134	0.041
12	0.5700E+04	0.935	0.043	0.022
13	0.5850E+04	0.958	0.021	0.021
14	0.6000E+04	0.897	0.048	0.054
15	0.6150E+04	0.863	0.067	0.070
16	0.6300E+04	0.833	0.099	0.067
17	0.6450E+04	0.774	0.179	0.046
18	0.6600E+04	0.607	0.391	0.002
19	0.6750E+04	0.762	0.186	0.051
20	0.6900E+04	0.815	0.103	0.081
21	0.7050E+04	0.833	0.072	0.095
22	0.7200E+04	0.846	0.059	0.094
23	0.7350E+04	0.869	0.051	0.080
24	0.7500E+04	0.911	0.038	0.051
25	0.7650E+04	0.991	0.004	0.005

Figure D.6 (continued).

D.5 Listing of STATEP

```
C *** THIS PROGRAM EXECUTES THE STATE PREDICTION
C     PART OF THE GEOLOGICAL PREDICTION MODEL FOR
C     INDEPENDENT PARAMETERS.
      DIMENSION NS(5),A(5,5,5),OL(5,20),QP(5,20,5),
     +          NO(5),NOUT(5),
     +          SP(5,100,5),WK(50),AX(5,5),AXT(5,5)
      COMPLEX WR(5),ZR(5,5),WL(5),
     +        ZL(5,5),TEMP
      OPEN(1,MODE="IN",FORM="FORMATTED";FILE="FILE1")
      OPEN(2,MODE="OUT",FORM="FORMATTED",FILE="FILE2")
      DO 5 J=1,5
    5 NOUT(J)=J
C *** INPUT INTERVAL AND SEGMENT LENGTH (TOTAL NO.
C     OF SEGMENTS IN INTERVAL MUST BE AT MOST 100.)
      READ(1,2)AL,BL,SL,BP
      READ(1,2)N
    2 FORMAT(V)
C *** INPUT TIM'S.
      DO 20 I=1,N
      READ(1,2)NS(I)
      DO 30 J=1,NS(I)
   30 READ(1,2)(A(I,J,K),K=1,NS(I))
C *** INPUT OBSERVATIONS FOR XI.
      READ(1,2)NO(I)
      DO 40 J=1,NO(I)
   40 READ(1,2)OL(I,J),(OP(I,J,K),K=1,NS(I))
   20 CONTINUE
C *** ONE PARAMETER IS CONSIDERED AT A TIME.
      DO 50 I=1,N
      DO 60 J=1,NS(I)
      DO 60 K=1,NS(I)
      AX(J,K)=A(I,J,K)
   60 AXT(K,J)=AX(J,K)
C *** FORM EIGENVALUES AND RIGHT EIGENVECTORS OF AX.
      IJOB=2
      CALL EIGRF(AX,NS(I),5,IJOB,WR,ZR,5,WK,IER)
      IF(IER.GT.128)WRITE(2,70)I
   70 FORMAT(//" ***** EIGENVALVES OF TIM OF X",I1,
     +   " CANNOT BE FOUND."//)
      IF(WK(1).GT.10.)WRITE(2,80)I
   80 FORMAT(//" ***** WARNING : INACCURACY IN",
     +   " ITPM OF PARAMETER X",I1//)
C *** ORDER EIGENVALUES IN INCREASING ORDER OF
C     MAGNITUDE SO THAT FIRST EIGENVALUE IS ZERO
C     AND ZR AND ZL MATCH.
      CALL ORDER(WR,ZR,NS(I))
C *** FORM EIGENVALUES AND LEFT EIGENVECTORS OF AX.
      CALL EIGRF(AXT,NS(I),5,IJOB,WL,ZL,5,WK,IER)
      IF(IER.GT.128)WRITE(2,70)I
```

```
      IF(WK(1).GT.100.)WRITE(2,80)I
      CALL ORDER(WL,ZL,NS(I))
      DO 90 J=1,NS(I)
      JM1=J-1
      DO 90 K=1,JM1
      TEMP=ZL(J,K)
      ZL(J,K)=ZL(K,J)
   90 ZL(K,J)=TEMP
C *** MAKE ZL X ZR = I
      DO 100 J=1,NS(I)
      TEMP=(0.,0.)
      DO 110 K=1,NS(I)
  110 TEMP=TEMP+ZL(J,K)*ZR(K,J)
      DO 120 K=1,NS(I)
  120 ZL(J,K)=ZL(J,K)/TEMP
  100 CONTINUE
C *** CALCULATE PROBABILITY PROFILE OF XI.
      RL=BP-SL
      IS=1
      NP=100
      NPT=(BL-BP)/SL+1
      IF(NPT.LT.NP)NP=NPT
      DO 130 IP=1,NP
      RL=RL+SL
      NRO=NO(I)-IS
      IF(NRO.EQ.0)GO TO 140
C *** INCREMENT IS APPROPRIATELY.
      DO 150 J=1,NRO
      RLTM1=OL(I,IS)
      RLT=OL(I,IS+1)
      IF(RL-RLT) 160, 170, 180
  180 IS=IS+1
  150 CONTINUE
C *** NORMAL EXIT MEANS NO MORE
C     OBSERVATIONS AHEAD OF RL.
      GO TO 140
C *** CALCULATE SP AT RL USING (4.8A).
  160 DO 190 J=1,NS(I)
      SP(I,IP,J)=0.
      DO 200 M=1,NS(I)
      DO 200 K=1,NS(I)
      PMK=OP(I,IS,M)*OP(I,IS+1,K)
      IF(PMK.EQ. 0.) GO TO 200
      TP1=P(M,J,RL-RLTM1,NS(I),WR,ZR,ZL)
      TP2=P(J,K,RLT-RL,NS(I),WR,ZR,ZL)
      TP3=P(M,K,RLT-RLTM1,NS(I),WR,ZR,ZL)
      SP(I,IP,J)=SP(I,IP,J)+PMK*TP1*TP2/TP3
  200 CONTINUE
  190 CONTINUE
```

```
      GO TO 130
C *** RL COINCIDES WITH OL(I,IS+1)
  170 DO 210 J=1,NS(I)
  210 SP(I,IP,J)=OP(I,IS+1,J)
      IS=IS+1
  130 CONTINUE
C *** OUTPUT PROBABILITY PROFILE OF XI
C     AND CONSIDER NEXT PARAMETER.
      GO TO 252
C *** NO OBSERVATIONS AHEAD OF RL.
C     (4.8B) IS USED FROM NOW ON.
  140 IPNOA=IP
      DO 220 IP=IPNOA,NP
C *** CALCULATE SP AT RL USING (4.8B).
      DO 230 J=1,NS(I)
      SP(I,IP,J)=0.
      DO 240 K=1,NS(I)
      PSK=OP(I,NO(I),K)
      IF(PSK.EQ. 0.) GO TO 240
      SP(I,IP,J)=SP(I,IP,J) +  PSK*
     1    P(K,J,RL-OL(I,NO(I)),NS(I),WR,ZR,ZL)
  240 CONTINUE
  230 CONTINUE
      RL=RL+SL
  220 CONTINUE
C *** OUPUT PROBABILITY PROFILE OF XI.
  252 WRITE(2,250)I,(NOUT(K),K=1,NS(I))
  250 FORMAT(//" PROBABILITY PROFILE OF X",I1//
     +  T6,"IP",T12,"RL",T24,I1,4(5X,I5))
      RL=BP-SL
      DO 260 IP=1,NP
      RL=RL+SL
      WRITE(2,270)IP,RL,(SP(I,IP,J),J=1,NS(I))
  270 FORMAT(/T4,I3,T8,E12.4,T24,F5.3,4(5X,F5.3))
  260 CONTINUE
   50 CONTINUE
      END
      SUBROUTINE ORDER(W,Z,NS)
      COMPLEX W(5),Z(5,5),TEMP
      DIMENSION    5)
      DO 10 I=1,.
   10 V(I)=CABS(W(I))
      NS1=NS-1
      DO 20 I=1,NS1
      J=NS-I
      DO 30 K=1,J
      KP1=K+1
      IF(V(K).LE.V(KP1))GO TO 30
      T=V(K)
```

```
      V(K)=V(KP1)
      V(KP1)=T
      TEMP=W(K)
      W(K)=W(KP1)
      W(KP1)=TEMP
      DO 40 L=1,NS
      TEMP=Z(L,K)
      Z(L,K)=Z(L,KP1)
   40 Z(L,KP1)=TEMP
   30 CONTINUE
   20 CONTINUE
      RETURN
      END
      FUNCTION P(I,J,U,NS,WR,ZR,ZL)
C *** THIS FUNCTION CALCULATES A ITP BY
C     SPECTRAL RESOLUTION OF TIM.
      COMPLEX WR(5),ZR(5,5),ZL(5,5),D(5),CP
      D(1)=(1.,0.)
      DO 10 K=2,NS
   10 D(K)=CEXP(WR(K)*U)
      CP=(0.,0.)
      DO 20 K=1,NS
   20 CP=CP+D(K)*ZR(I,K)*ZL(K,J)
      P=CABS(CP)
      RETURN
      END
```

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Mr. Chan came to M.I.T. from Hong Kong to study Civil Engineering in 1976. He started to do Undergraduate Research in Rock Mechanics in 1978 under the supervision of Professor Herbert H. Einstein. He became a Graduate Research Assistant in spring 1979 and Professors Einstein and Gregory B. Baecher were his supervisors. From Fall 1979 to Fall 1980 Mr. Chan was involved in probabilistic modeling in tunneling under the supervision of Professors Einstein and David B. Ashley. His discussions with Professor Daniele Veneziano was especially fruitful towards the conceptual formation of the research.

Education and Experience :

Student assistant in M.I.T. Undergraduate Research Opportunities Program in rock slope stability analysis (1/1978 to 1/1979);
M.I.T. Graduate Research Assistant in rock slope stability analysis and reliability assessment of offshore drilling platforms (Spring, 1979);
Bachelor of science in Civil Engineering, M.I.T., June 1979;
Graduate trainee and site engineer, mainly involved with rock slope projects (Summer, 1979);
M.I.T. Graduate Research Assistant in probabilistic approaches in tunneling (9/1979 to 10/1980);
Engineer (Soils),
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Master of Science in Civil Engineering, M.I.T., February 1981.

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SWARS (computer program to calculate rock wedge stability);
"Approach to Complete Limit Equilibrium Analysis for Rock Wedges - the Method of Artificial Supports", Rock Mechanics, 1980;
"Geological Predicton and Updating in Tunneling - a Probabilistic Approach", 22nd U.S. Symposium on Rock Mechanics, 1981.

Awards and Honor Societies :

Richard Lee Russel Award, in recognition of distinguished academic achievements (M.I.T., 1979);
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