Essays on Expectations and Exchange Rate Volatility

by

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ABSTRACT

Chapter I, "An Empirical Investigation of the Martingale
Property of Foreign Exchange Futures Prices," employs a large body
of data to rigorously test the hypothesis that past changes in the
price of a foreign exchange futures contract cannot be used to pre-
dict future values of the contract beyond any information contained
in the current price. It is statistically necessary to correct for
any heteroskedasticity caused by Samuelson's "law of increasing
volatility of a maturing futures contract."

The second two essays build on Professor Dornbusch's model
of expectations and exchange rate dynamics. That model of an
economy with flexible exhange rates and perfect capital markets

views the exhange rate as a variable determined in an efficient market. The goods market can experience disequilibrium and price adjustment is sticky. The second essays generalizes this model and permits an anticipated series of permanent and transitory shocks, in addition to contemporaneous shocks. Given imperfections in the price equation, a long anticipated monetary shock will still generate real effects even after it occurs.

The third chapter reveals the isomorphism between sticky prices and sticky expectations in these models. A backwards and forwards solution to Dornbusch-type models is derived. It is shown how rational expectations can be explicity retrospective, as well as forward looking, even when the usual restriction of continuous domestic price adjustment is relaxed.

Thesis Supervisor: Rudiger Dornbusch, Professor of Economics

ACKNOWLEDGEMENTS

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CHAPTER I

AN EMPIRICAL INVESTIGATION OF
FOREIGN EXCHANGE FUTURES PRICES

Summary

This paper is an empirical investigation of weak form market efficiency in foreign exchange futures markets. It differs from previous work in that it uses futures prices rather than forward rates for the data set. In order to obtain covariance stationarity, an attempt was made to correct for heteroskedasticity with respect to time as suggested by Samuelson's theoretical observation on the "law of increasing volatility of a maturing futures contract."

Daily and weekly observations for sixteen contracts covering six currencies over a three-year period are used. The only formal test for efficiency presented is a joint test on the significance of the empirical correlogram coefficients. Generally, the null hypothesis of no serial correlation in the daily and weekly first differences is not rejected.

Introduction

The theoretical motivation for this empirical investigation is based on work by Samuelson (1965). In his article "Proof that properly anticipated prices fluctuate randomly," he rigorously derives the martingale property of a series of futures prices for a contract maturing at a given date. One implication is that under certain conditions the first differences of the series will be serially uncorrelated.

The formal test used to detect serial correlation is the Box-Pierce ("portmanteau") joint test on the correlogram coefficients. The test

requires covariance stationarity in the series. The fact that this con-
dition may well be violated was noted in Samuelson's 1965 paper. He la-
bels this phenomenon: "The law of increasing volatility of a maturing
futures contract." The martingale property itself derives from the as-
sumption that the futures prices are formed rationally based on knowledge
of the underlying stochastic process governing the spot rates. If, in
addition, we assume that the underlying stochastic process of the spot
rates is stationary, then the futures contract will be more volatile in
its period-to-period price changes at times near its expiration date than
it will when the contract has a long term to maturity. This potential
source of heteroskedasticity is investigated prior to the formation of
the empirical correlograms.

The first section of the paper presents summary statistics on the
data. In the next section, the empirical methodology is given. This in-
cludes a discussion of the "portmanteau" test and the problem of hetero-
skedasticity in the period-to-period changes in the futures prices. The
third section presents the empirical results. The final section contains
interpretations and suggests possible extentions. The data are discussed
in an appendix.

I. Description of the Data

Sixteen futures contracts were analyzed using data from the *International Monetary Market Yearbook* (see Appendix).

The contracts terminate in June 1975, June 1976, and June 1977 for the British pound, Swiss franc, Mexican peso, Canadian dollar, and German mark. A June 1977 Japanese yen contract is included. In all cases, a fixed amount of foreign currency is exchanged, while the dollar price is allowed to fluctuate.

The number of observations differs from contract to contract, as trading starts in some currencies earlier than in others, and prices are not listed on days where no trading takes place. Still, the sample sizes are quite large. Both daily and weekly first differences were used.

Table I presents certain summary statistics for each of the contracts. In comparing statistics for different currencies, remember that the contracts are of somewhat different size. Indeed, within the life of a given contract, the variance may be related to the dollar size of the contract at any given time, so that the ex-ante daily variance changes from period to period.

The range and standard deviation statistics reflect considerable variability, particularly since contracts may be purchased on the margin. The sample standard deviations are fairly stable across contract years. However, it should be noted that series in some financial markets have appeared to arise from a class of stable Pareto-Levy distributions where the variance may not exist. This

TABLE I

Currency	British Pound			Swiss Franc		
Expiration Date	June '75	June '76	June '77	June '75	June '76	June '77
Unit Contract		-- 25,000 --			-- 125,000 Swiss francs --	
Number of Trading Days Where Settlement Price Listed	158	217	215	238	308	200
Standard Deviation (Dollars Per Contract Per Day)	267	300	236	336	246	175
Price Range (High - Low) (Dollars/Contract)	9400	14,500	7100	16,750	7262	5425
Sample Variance* for First Half of Contract	306	320	318	302	260	199
Sample Variance* for Second Half of Contract	236	296	100	356	233	144
Closing Price ($)	56,975	44,410	42,475	50,287	50,150	50,162

*Twenty central observations omitted.

TABLE I (cont.)

Currency	Mexican Peso			German Mark		
Expiration Date	June '75	June '76	June '77**	June '75	June '76	June '77
Unit Contract	-- 1,000,000 pesos --			-- 125,000 marks --		
Number of Trading Days Where Settlement Price Listed	222	239	319	215	261	195
Standard Deviation (Dollars Per Contract Per Day)	167	253	1467	318	247	172
Price Range (High - Low) (Dollars/Contract)	10,670	6550	43,400	11,687	7750	4150
Sample Variance* for First Half of Contract	242	207	614	340	308	205
Sample Variance* for Second Half of Contract	129	303	1858	289	178	121
Closing Price ($)	80,000	80,030	43,540	53,575	48,512	53,012

*Twenty central observations omitted.

**Throughout, the Mexico 1977 series includes the period of the peso devaluation. Obviously, a normal approximation to the underlying distribution will not be satisfactory.

TABLE I (cont.)

Currency	Canadian Dollar			Japanese Yen
Expiration Date	June '75	June '76	June '77	June '77
Unit Contract	-- 100,000 Canadian dollars --			-- 12,500,000 yen --
Number of Trading Days Where Settlement Price Listed	96	123	158	107
Standard Deviation (Dollars Per Contract Per Day)	247	164	298	169
Price Range (High - Low) (Dollars/Contract)	6620	5390	6910	4275
Sample Variance* for First Half of Contract	255	171	351	157
Sample Variance* for Second Half of Contract	238	159	218	158
Closing Price ($)	97,500	102,540	94,550	45,965

*Twenty central observations omitted.

hypothesis is not tested, although later, in Table II, statistics on the kurtosis and skewness of the normalized series are presented.

It is generally held that foreign exchange futures prices are more stable than spot prices. In evaluating the evidence, it is for some purposes important to remember to compare only the variance of the unanticipated change in the spot rate with the variance of the futures rate.

The half-sample standard deviations are also of interest, because although the martingale result in its simplest form posits a constant first moment, it does not require the variance of the process to remain constant over time, an important condition for our empirical test. This is discussed in the next section. For the time being, note that the sample variance for the half-sample farther from the expiration date generally has a larger variance.

II. Methodology

Samuelson's martingale result implies that the first dif-
ference of the series of futures rates will form a series which
is serially uncorrelated at any lag, although not necessarily
independent. If it were further possible to assume that the
mean and variance of the period-to-period price changes were
constant, it would be natural to test the assumption of no serial
correlation at any lag by a joint test on the empirical correlogram
coefficients. Then, for large samples $\hat{r}(k)$, the correlogram
coefficient at lag k, is distributed Normal with mean 0, $\sigma^2 =$
n - k/n(n+2) $\simeq$ 1/n.[1] If we have a specific alternative hypothesis,
it is possible to calculate the alternative theoretical correlo-
gram, and then perform a likelihood ratio test. Otherwise,
while the series of individual correlogram coefficients may be
of interest, a formal test for serial correlation must be based
on a joint test of the significance of the estimated autocorre-
lations. The formal test used in this study is the "portmanteau"
test:

$$n \sum_{k=1}^{L} r(k) \sim \chi^2 (L-p-q) \quad ,$$

where n = number of observations,

p,q = number of parameters of the fitted AR(MA) process

(in our case, p = q = 0).

See Box-Pierce (1970), or Box-Jenkins (1976).

In some respects, the decision to use such a broad alternative

hypothesis is quite conservative. If, for example, the market reacts quickly, but not instantaneously, to new information, we might expect to find a small low-order moving-average process in the series of first differences.[2] If the phenomenon is minor relative to the variability of the series, even a large sample may not allow us to detect it without using a less general test. Also, our method is not particularly sensitive to periodic movements in the series. (Our test is better when the theoretical correlogram is damped.) A diagnostic check on the periodogram coefficients would be more sensitive to such periodic movements. These considerations point to limitations in the power of our test.

An important school of thought, due to Alexander, argues that we should use a test which will be more sensitive to non-linear relationships among the series. These tests were not employed because the statistical theory behind them is not sufficiently well developed.

Another consideration is robustness, and here the correlogram coefficients can be quite sensitive to outliers in the sample. Even the asymptotic normality of the series is not assured. The martingale theorem tells us only that the series is uncorrelated, not necessarily independent or even m-dependent. Also, some writers have argued that data in financial markets may be generated from a class of Pareto-Levy distributions where the variance is not finite. The issue of asymptotic normality is important.[3] There is a danger that we will understate the probability of a Type I error. It does not necessarily follow that a non-parametric test

would have been better. Our null hypothesis implies lack of serial correlation, not independence, and therefore the correlogram test is the appropriate one.

Throughout this study, contracts for different years for the same currency are not stacked, although this is commonly done in similar work on commodity futures[4]. First of all, this would require eliminating some 25 percent of the data points used, as it would be eminently unsatisfactory to use overlapping contracts. The main issue is the specification of the alternative hypothesis and its effect on the power of the test. True, even under the null hypothesis of no serial correlation in the intra-contract first differences, it would be a mistake to take differences across contracts. This would introduce a bias into the correlogram coefficients unless the spot rate itself is a martingale[5]. But this problem is easily negotiated by omitting differences across contracts. The real problem results from the formulation of the alternative hypothesis.

For example, suppose there is a pattern of serial correlation in the underlying stochastic process of the differenced futures price series. What matters is whether this process is stable from contract year to contract year.

Suppose we have n contracts, each with k+1 equi-spaced observations. The contracts are all for the same currency, but for different years. Let e_j be the k-vector of intra-contract first differences for the jth contract. The usual alternative hypothesis, when contracts are grouped, is presumably that the

whole time-ordered series e_j^i: $i = 1,\ldots,k$; $j = 1,\ldots,n$ follows some stationary process. Then this implies, for example, $\text{cov}(e_j^1,e_j^2) = \text{cov}(e_j^1,e_{j-1}^k)$. This assumption is particulary unsatisfactory when only nine or ten months of data per contract are available.

A second alternative hypothesis would be that each contract year followed the same ARMA process, with the same parameters. Thus $\text{cov}(e_j^i,e_j^{i-1}) - \text{cov}(e_{j-z}^i,e_{j-z}^{i-1})$, but $\underline{\text{across}}$ contract covariances are zero: $\text{cov}(e_j^i,e_g^w) = 0$, $g \neq j$. One could make a profit by estimating the parameters from earlier contracts, and then applying them to the current year's contracts once some of the current year's price changes had been observed.

A third case would be that contracts for different years have nothing to do with each other, but each follows its own serially correlated process.

The first case is the least general form of the alternative hypothesis. If the second and third cases are permitted, and they are consistent with the methods of some chartists,[6] our correlogram coefficients will be biased under the alternative hypothesis. This will have an adverse effect on the power of our test. Thus contracts for the same currency for different years were not stacked.

Stationarity of the First and Second Moments

Covariance stationarity under the null hypothesis is necessary for the "portmanteau" test, but is not an implication of

Samuelson's martingale result. Samuelson's result that the variability of the price of a given futures contract will ultimately decrease as we move farther and farther from the expiration date is termed: "the law of increasing volatility of a maturing futures contract." This result follows from the additional assumption that the stochastic process of the spot rates themselves is ergodic and not explosive. A simple example illustrates the idea. Suppose the spot rates follow the process:[7]

$$S_t = aS_{t-1} + u_t \qquad (1)$$

where the u's are serially uncorrelated with constant variance, σ^2, and mean 0. $a < 1$ is necessary for the process to be strongly damped.

Let $F_{t+T,t}(F_\Theta, \Theta-T)$ be the T-period ahead futures rate at time $t(\Theta-T)$. Further, assume as in Samuelson (1965) that

$$F_{t+T,t} = E_t(S_{t+T}) \; ,$$

where E_t is the expectation at time t.

$$E_t(S_{t+T}) = a^T S_t \qquad \text{and} \qquad (2)$$

$$E_{t-1}(E_t(S_{t+T}) - E_{t-1}(S_{t+T}))^2 = a^{2T}\sigma^2 =$$

$$\text{VAR } \Delta F_{t+T,t}$$

and $\qquad \dfrac{\partial \text{ var } \Delta F_{\Theta,\Theta-T}}{\partial T} < 0 \; .$

Even if the underlying process is damped, it is possible for

the variances to begin decreasing again very close to the maturity date of the contract. Take the nth order difference equation:

$$S_t = c_1 S_{t-1} + c_2 S_{t-2} + \cdots c_n S_{t-n} + u_t \qquad (3)$$

with characteristic roots all strictly less than 1.

Then an unanticipated shock $(u_{\Theta-1})$ in the period just before the contract matures will not have its full first round effect on the system in one period. Samuelson (1976) points out the example

$$S_t = .1 S_{t-1} + .2 S_{t-2} + u_t . \qquad (4)$$

To calculate the sequence of variances of period to period changes in a futures contract for nth order recursive system such as (3), note

$$Var = (E_{\Theta-T}(S_\Theta) - E_{\Theta-T-1}(S_\Theta))^2 =$$

$$Var \; \Delta F_{\Theta,\Theta-T} = w_T^2 \sigma^2$$

where the w's follow the same autoregressive process as the S's, with the initial condition $(w_0, w_{-1}, \ldots, w_{-n+1})(1, 0, \ldots, 0)$.[8]

Of course, it is entirely possible that the spot rates do not follow such a stable process. The economist may want to believe that real prices follow an ergodic distribution, returning to a probabilistic equilibrium if we look far enough into the future. But the exchange rate is the ratio of two nominal variables, and the argument for stability is less compelling. (Inflation rates between two countries could in principle differ for indefi-

nitely long periods.)

While this phenomenon is of interest in itself, it is also important for the econometrician, in his attempts to achieve a covariance stationary series. We might expect the phenomenon to be difficult to detect, not only because the variances may decrease in the end as in (4), but because empirically it is difficult to sort this effect out from the effects of other variables (such as trading volume) on the variance of the futures price series, particularly as the theory is not well defined. We can only hope that the excluded variables are independent of time (volume, for one, is certainly not), or that their biasing effects cancel out. Both seem a bit much to hope for.

A previous study of commodity futures (Rutledge) failed to detect the relationship of futures price variability with time. There, an attempt was made to allow the u's in an equation such as (1) to have non-constant variance. The method of holding changes in the spot rate constant was not entirely satisfactory, however, as it is correct only to hold the variance of the unanticipated changes in the spot rate constant.

My methods, which as in (Rutledge) allowed the variance to increase or decrease with time (nothing was held constant), also failed to detect the phenomenon. This failure does not interfere with the primary goal of this study, which was to test for serial correlation in the series. The formal method was one suggested by Glejser (Johnston, pg. 220) to correct for heteroskedasticity. The specification used was $\log |\Delta F_t| - a + b \log T + u_t$. Only those

series where a two-tailed test rejected the null hypothesis of no
significant relationship with T (time) were corrected for hetero-
skedasticity. (For several reasons it was somewhat inappropriate
to regard each test separately.) The June 1975 and June 1977
British pound contract, the June 1977 Canadian dollar contract,
the June 1976 German mark contract, and the June 1975 Mexican
peso contracts all show decreasing variance as the contract matured.
The 1977 Mexican peso showed increasing variance with respect to
time. (As mentioned in a footnote, I have not treated this series
specially, although it includes the period of the major devaluation.)

These normalizations help to achieve stationarity, but prob-
ably don't greatly influence our priors on the question of futures
contract volatility with respect to time to maturity.

Table II presents summary statistics on the normalized series.
The expected value of the kurtosis statistic for an exactly
normal distribution is 3. This statistic is very sensitive to
outliers. Other similar data on financial markets have often
shown kurtosis statistics greater than 3, implying fat tails
relative to a normal distribution. These series show kurtosis
statistics above and below 3 (though this does not imply that
any of these series is close to normal.)

The question of whether or not the first moment of the
differenced series will be constant under the null hypothesis is
discussed in the final section.

Table II : Summary statistics on the empirical distri-
bution of the normalized series of daily and weekly first
differences

Country and Contract	Kurtosis		Skewness	
	Daily	Weekly	Daily	Weekly
British pound				
June 1975	2.7	2.3	.10	.29
June 1976	2.6	1.8	-.20	-.43
June 1977	6.1	3.9	-1.01	-.05
Swiss Franc				
June 1975	3.0	2.2	-.02	-.15
June 1976	3.1	2.4	.03	.06
June 1977	4.2	2.6	.29	-.89
Canadian Dollar				
June 1975	4.1	9.3	-.33	-.42
June 1976	.9	1.8	-.49	.29
June 1977	1.1	1.6	-.48	-1.05
German Mark				
June 1975	3.0	3.6	.00	.01
June 1976	1.1	1.0	.09	.08
June 1977	3.2	2.4	-.12	-.03
Mexican Peso				
June 1975	5.7	16.0	-.09	-.85
June 1976	6.5	12.9	.02	.90
June 1977	142.0	5.8	-10.00	-1.24
Japanese Yen				
June 1977	.8	1.6	.15	.24

Table III: Box-Pierce (Portmanteau) Test on the Correlogram

Country and Contract	Daily Differences		Weekly Differences
	Q(24)	Q(36)	Q(12)
British Pound			
June 1975	22.3	34.6	4.6
June 1976	39.3	50.4	12.5
June 1977	46.8	62.7	23.2
Swiss Franc			
June 1975	34.3	47.9	8.3
June 1976	17.7	24.2	8.9
June 1977	20.3	26.6	8.0
Canadian Dollar			
June 1975	18.5	24.8	10.8
June 1976	23.7	46.2	18.7
June 1977	32.5	39.1	11.0
German Mark			
June 1975	23.7	38.3	12.8
June 1976	24.0	44.0	9.8
June 1977	27.7	39.1	15.3
Mexican Peso			
June 1975	40.8	68.6	14.5
June 1976	31.2	40.4	9.6
June 1977	5.8	8.0	16.6
Japanese Yen			
June 1977	15.5	26.2	10.7

$$Q(k) = T \ (\ T+2) \sum_{j=1}^{k} r(j)/(T-j) \quad , \quad Q(k) \sim \chi^2(k)$$

where T= N-1, where N is the number of observations in the
futures price series.

Pr.$(\chi^2(12)) > 14.8 =.75$ Pr$(\chi^2(24)) > 29.2=.75$ Pr$(\chi^2(36))$
$> 17.3 =.90$ $> 34.4=.90$ $> 41 = .75$
$> 19.7 =.95$ $> 37.7=.95$ $> 46 = .90$
$> 24.7 =.99$ $> 44.3=.99$ $> 51 = .95$
                                          $> 58 = .99$

III. Empirical Results

Table III lists the Box-Pierce statistics for both daily
and weekly differences for all of the series. As described in
Part I, the Box-Pierce (portmanteau) test is a joint test of the
significance of the empirical correlogram coefficients. The
weekly data are less sensitive to technical factors such as daily
price change limits, etc. The weekly differences are not weekly
averages of daily differences. As Working showed, this would
introduce a degree of serial correlation even if the daily
differences are serially uncorrelated.

There is a certain degree of arbitrariness involved in
choosing the number of correlogram coefficients to be included in
the Box-Pierce test. It was decided to use the first thirty-six
autocorrelations for the daily data and the first twelve auto-
correlations for the weekly data, but Q(24) is also presented
in Table III for the daily differences, as a descriptive statistic
on the stability of this test. Table IV actually gives the
empirical correlograms for the first 36 autocorrelations for all
the daily series. (The weekly coefficients are available from the
author, but are not presented for reasons of space.)

Examining Table III, one can determine that for two of the
contracts, Britain 1977 and Mexico 1975, the daily Q statistics
are high even at the 1% significance level. The Britain 1976
and Switzerland 1975 are borderline, depending on the significance
level chosen. The Canada 1976 and the Germany 1976 have large

Q(36), but not Q(24).

For the weekly data, only for the Britain 1977 contract is the null hypothesis rejected at the 5% level of significance. The Canada 1976 is rejected at the 10% level of significance, but not at the 5% level. Overall, the results tend not to reject the null hypothesis of no serial correlation in the price changes.

The correlograms for the daily differenced series are presented in Table IV. The reader may examine these graphs to look at the significance of the individual coefficients, particularly of the lower order autocorrelations. The outer lines represent two standard deviation units from the mean under the null hypothesis of no serial correlation. In interpreting these results, it is useful to remember that the theoretical correlogram for a first-order moving-average process will go to zero after the first autocorrelation, while the autoregressive coefficients would dampen gradually. The opposite is true for a first-order autoregressive process. It is not our purpose here to "fit" the series. However, it may be true that the correlograms are also consistent with a theory which posits a small moving-average process, as markets adjust rapidly, but not instantaneously, to new information. No likelihood ratio tests are performed. As explained earlier, it is preferable not to choose a specific alternative hypothesis.

It is interesting to note that the empirical correlogram coefficients show little stability from year to year for contracts on the same currency. Still, the underlying variance of these
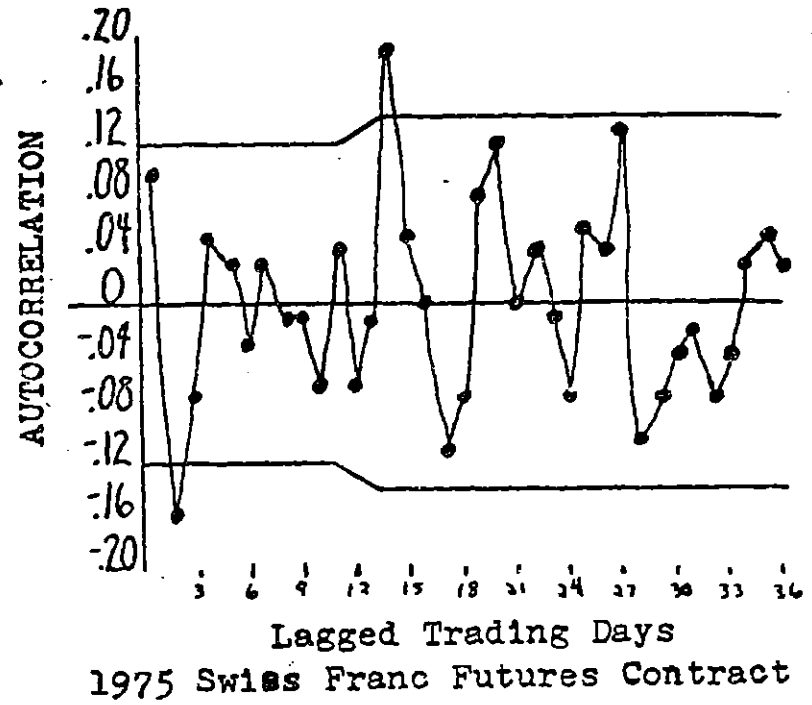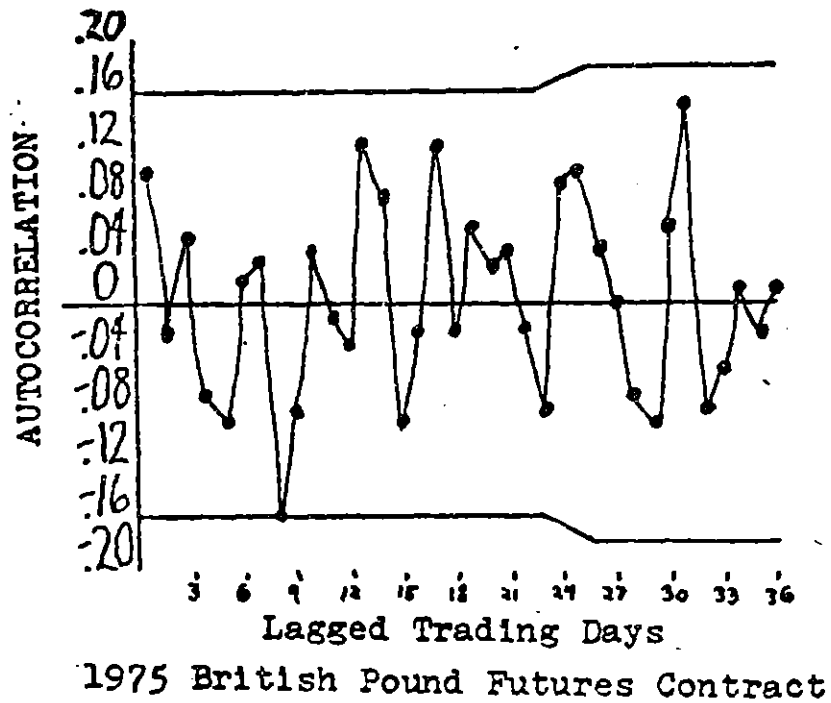
Table IV

1975 British Pound Futures Contract
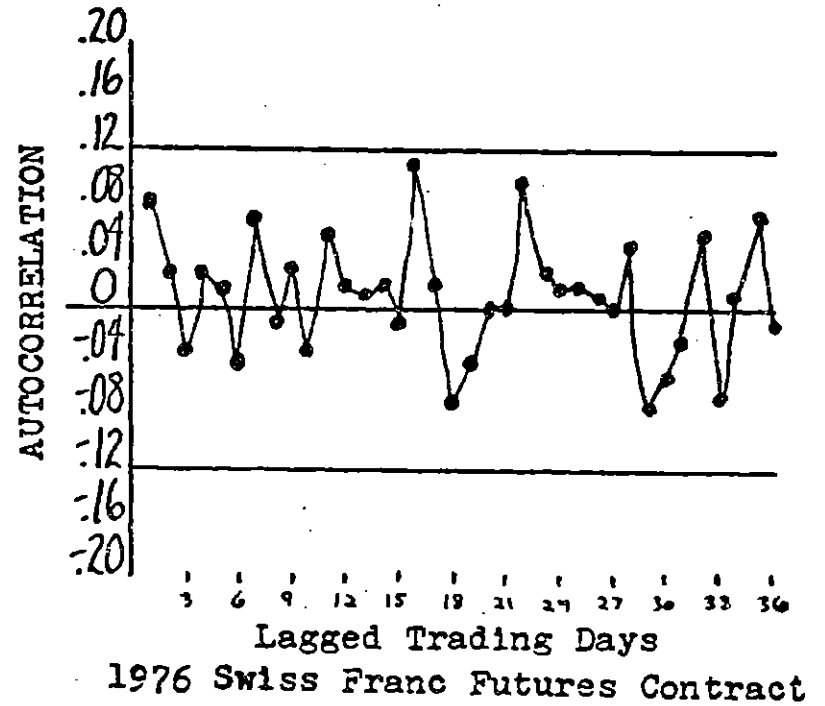
1975 Swiss Franc Futures Contract

Table IV (continued)

Lagged Trading Days
1976 British Pound Futures Contract

Lagged Trading Days
1976 Swiss Franc Futures Contract

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Table IV (continued)

1977 British Pound Futures Contract

1977 Swiss Franc Futures Contract

Table IV (continued)

1975 German Mark Futures Contract

1975 Mexican Peso Futures Contract

Table IV (continued)

Lagged Trading Days
1976 German Mark Futures Contract

Lagged Trading Days
1976 Mexican Peso Futures Contract

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Table IV (continued)

1977 German Mark Futures Contract

1977 Mexican Peso Futures Contract

Table IV (continued)

Lagged Trading Days

1975 Canadian Dollar Futures Contract

Lagged Trading Days

1977 Canadian Dollar Futures Contract

Table IV   (continued)

1976 Canadian Dollar Futures Contract

1977 Japanese Yen Futures Contract

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tests is sufficiently high that the informed reader may not weight

the evidence heavily against his priors on the subject.

IV. Sources of Bias and Possible Extensions

In Part II, we noted that the correlogram test required the time series to be stationary in both its first and second moments. Having considered the issue of variance stationarity earlier, we now turn to the question of the first moment, and ways in which its behavior could bias our results.
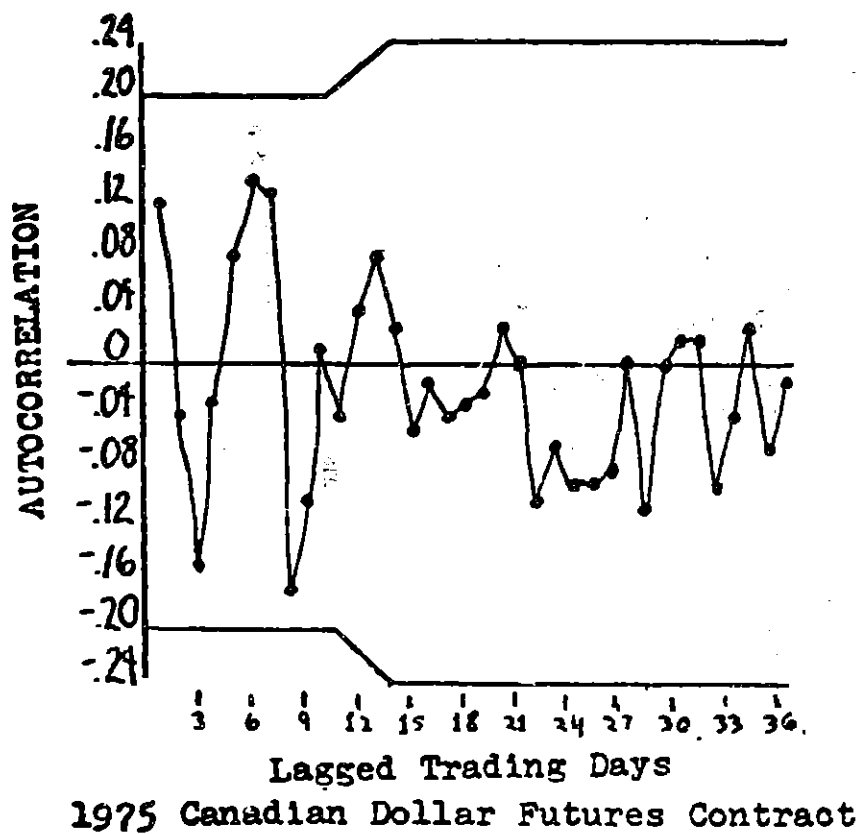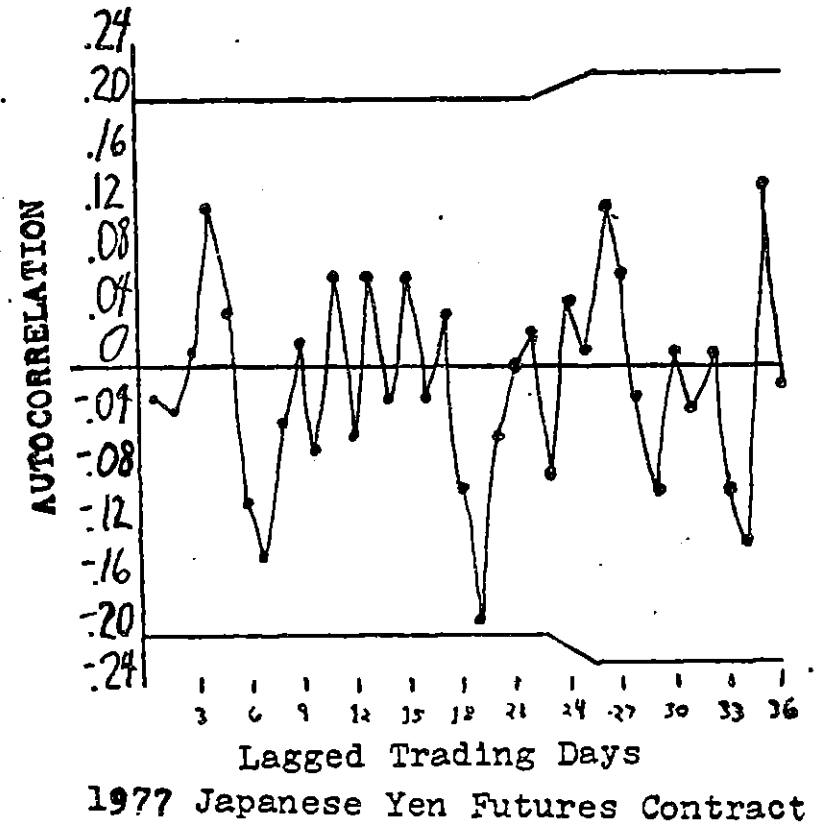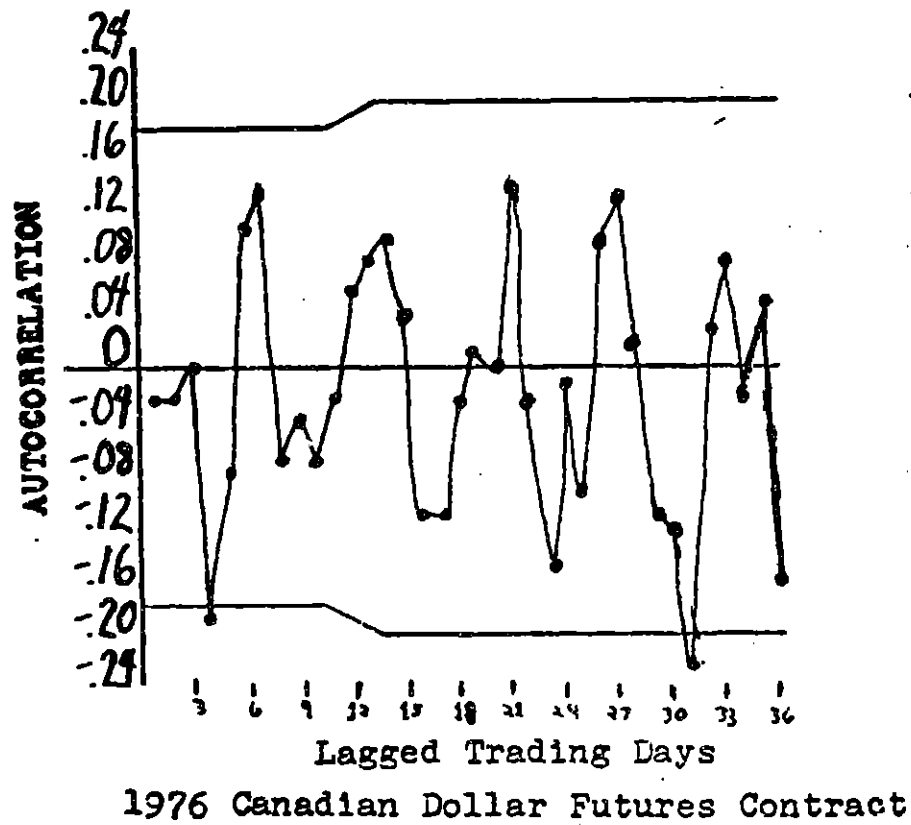
An assumption used in the most basic version of Samuelson's model is that:

$$F_{t+T,t} = E_t(S_{t+T}) \quad , \tag{5}$$

where $F_{t+T,t}$ is the T-period ahead futures rate at time t, and S_t is the spot rate at time t.

In this case, it turns out that the mean of the series of first differences will be zero. This is more than the correlogram test requires under H_0. It is only necessary that the mean of the series of first differences be constant. Condition (5), or even more weakly, the unbiasedness of the futures price as a predictor of the future spot price, is not directly tested in this paper. The usual forward rate data are more appropriate for this purpose, since the mean of a series of first differences depends only on the endpoints of the original series and the number of observations.

Deviations from (5) may be due to a risk premium, interest on a margin deposit, or due to Jensen's inequality.

Consider

$$F_{\Theta,\Theta-T} = F_{\Theta-T}(S_\Theta) + r(T) \quad , \tag{6}$$

where $r(T)$ is a risk premium, $r' > 0$.

Now, if $r'' = 0$, the mean of the differenced series will be uncorrelated. However, if $r'' \neq 0$, then although the series is uncorrelated, it is not stationary in mean. The empirical correlogram, which calculates a constant mean (unless an appropriate correction is made), will show positive serial correlation.

If the series exhibits mean percentage drift (as in the case considered in Samuelson (1965), where money is tied up in holding the contract), again the correlogram will be biased and show serial correlation unless the appropriate normalization is made.

A further problem, which is commonly seen elsewhere, is that the exchange rate is the ratio of two nominal variables. Since $E(1/x) \geq 1/E(x)$ by Jensen's inequality, we cannot simultaneously have (5) and

$$1/F_{t+T,t} = E(1/S_{t+T}) \qquad .^9 \qquad (7)$$

Clearly, even in a model for commodity futures, (5) requires that the dollar price of whatever is being maximized be constant. The convexity term which will enter the expected value arbitrage condition is separable from the ris' premium. Again, this will not bias our tests if the term is linearly related to time, or if it is very small.

We have already seen that the property of lack of serial correlation is quite general and does not necessarily require

condition (5) to hold. A further example of this is

$$F_{t+T,t} = E(S_{t+T}) + e_t \quad ,$$

where e_t is a serially uncorrelated random variable with mean zero. It is also uncorrelated with innovations in the spot rate. Here the futures price is a noisy indicator of expectations. The mean of the series of price changes is zero, and it is serially uncorrelated. Thus our correlogram test examines a necessary but not sufficient condition for weak form market efficiency.

Table V

Correlation Matrix for Weekly First Differences (Contemporaneous)

For June 1977 Contract

	Switzerland	Canada	Britain	Germany	Japan	Mexico
Switzerland	1.00					
Canada	.59	1.00				
Britain	-.34	-.17	1.00			
Germany	.15	.39	-.28	1.00		
Japan	.14	.03	-.35	.46	1.00	
Mexico	.01	.06	.46	.21	.06	1.00

These are sample correlations of the original price series. This table is the cross-correlogram at lag zero.

V. Possible Extensions

An immediate extension is to calculate a joint test, based on the correlogram coefficients for all contracts for all the contracts for a given year. As explained in the Appendix, it is only possible to do this with any confidence for the weekly data. To derive the test statistic, it is necessary to note that the contemporaneous innovations in the different series are not uncorrelated. For one thing, they all share uncertainty about the dollar. In Table V, the sample correlation matrix for contemporaneous weekly differences for the June 1977 series is presented. While this matrix is of considerable interest in itself, it is also important because it shows that our tests on different currency contracts within the same year are not independent. In fact, the empirical correlogram coefficients for lags of the same order are correlated.

Another natural extension along these lines would involve calculation of cross-correlograms. This procedure requires a larger data set and would work better if we reconciled ourselves to grouping contracts for different years.

Footnotes

1. For a further discussion, see G.E.P. Box and D.A. Pierce, "Distribution of residual autocorrelations in auto-regressive-integrated moving average time series models," Journal of the American Statistical Association, No. 64, p. 1509-15, 1970.

2. This alternative hypothesis is mentioned in Arnold B. Larson, "Measurement of a random process in futures prices," in Paul Cootner (ed.), The Random Character of Stock Market Prices (Cambridge: M.I.T. Press, 1964).

3. Slow convergence to normality due to fat tails in the distribution is also important. One reason that the series may have large tails is the small probability of a major event such as devaluation of the peso.

4. See, for example, Katherine Dusak, "Futures trading and investor returns: An investigation of commodity market risk premiums," Journal of Political Economy, No. 81, Nov./Dec. 1973, pp. 1387-1406. Because so many more years' data are available in the case of commodity futures, the tradeoff in statistical power is more in favor of grouping contracts from different years. The use of bi-weekly observations as in her study is desirable as it entirely avoids problems such as daily price limits. Of course, bi-weekly observations will not be as sensitive to low-order serial correlation in the daily price changes.

5. That including first differences across the linked contracts would introduce a bias into the correlogram coefficients unless

the spot rate itself is serially uncorrelated is an obvious proposition which is best illustrated by example.

Suppose the spot rate follows a first-order autoregressive process:

$$S_t = aS_{t-1} + u_t \quad,$$

where u_t is serially uncorrelated with mean zero and variance σ^2. Assume the futures price is unbiased.

Now take a series of consecutive non-overlapping futures contracts; n contracts, each with k observations. Now take first differences of the stacked series. Clearly a first difference within the jth contract, between the i and i-1 observation on that contract will be equal to u_{kj+i}. A first difference across the j and j-1 contract will equal

$$a^{k-1}u_{kj+1} + \sum_{i=1}^{jk} (a^{j(k+1)-i} - a^{jk-i})u_i +$$

$$(a^{j(k+1)} - a^{jk})S_0 \quad.$$

(Here we assume that the realized spot rate is included in each series, but leaving it out will not qualitatively affect the result.) where S_0 is the spot rate prevailing at period zero. This difference is obviously correlated with all past differences. Defining ΔF_t to be the change in price between two periods,

$$E(\Delta F_t, \Delta F_{t-1}) = \frac{a^k-1}{k} \sigma^2 \tag{F1}$$

This expression is larger if we use larger (discrete) time

intervals between price observations on the underlying series. (Weekly, instead of daily observations.) If we had taken only k/z observations per contract (k/z an integer), the above expression becomes:

$$\frac{a^k-1}{k/z} \sigma^2 (1 + a^2 + \ldots + a^{2z}) \tag{F1'}$$

The bias in the empirical correlogram of this non-stationary series is a considerably clumsier expression, as the mean is no longer zero.

6. See the <u>Commodity Trading Manual</u>, Chicago Board of Trade, 1973.

7. This example is only illustrative. A more reasonable process for exchange rates might be

$$\log S_t = a \log S_{t-1} + u_t \quad,$$

where $u_t \sim N(-1/2\sigma^2, \sigma^2)$ and is serially uncorrelated.

8. Using similar notation to Samuelson (1976), let

$$E(S_t|t-T-1) - E(S_t|t-T) = W_t U_{t-T}$$

where $E(S_t|t-T)$ is the expectation of the spot price at time t, given past spot prices up to and including S_{t-T}. Assume $W_T U_{t-T} = F_{t,t-T-1} - F_{t,t-T}$, the change in the futures price.

$$= E(\sum_{i=1}^{n} c_i S_{t-i}|t-T-1) - E(\sum_{i=1}^{n} c_i S_{t-i}|t-T)$$

$$= \sum_{i=1}^{n} c_i [E(S_{t-i}|t-T-1) - E(S_{t-i}|t-T)]$$

$$= \sum_{i=1}^{n} c_i W_{T-i} U_{t-T} = W_T U_{t-T} \quad .$$

(since u_{t-T} affects each future price linearly with the effect (w_{T-i}) only depending on the time in between.) So

$$W_T = \Sigma c_i W_{T-i} \quad .$$

The initial conditions follow from the fact that innovations occurring after the maturity date of the contract will have no effect. Note that this difference equation is indexed "backwards" relative to calendar time.

9. For the case of foreign exchange markets, this is sometimes called Siegel's paradox. See J. Siege, "Risk, interest rates and the forward exchange," <u>Quarterly Journal of Economics</u>, LXXXVI (May 1972) and comments on that article by McCulloch and Roper in the same journal (February 1975).

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Appendix I: A Note on the Data

The basic data for this study are taken from the International Monetary Market Yearbook's 1974-75, 1975-76, and 1976-77 editions. Data for the June futures contracts were taken from each issue. The currencies included were the most heavily traded. Trading in Japanese yen increased significantly in 1977, so it was only included for that year. The price used was the settlement clearinghouse price, which is an end-of-the-day quotation. The series are discontinuous. During some periods for some currencies, trading did not take place every day. So the "daily" differences of this study are period-to-period differences. The weekly differences are at intervals of seven days, or six to eight days if an observation was missing.

Each currency is subject to a system of daily price change limits (which are released on some occasions, such as the Mexican devaluation). The basic limits are $1250 for the British pound and $750 for all the other currencies. These limits are systematically released when on any two successive days, trading on a contract for any expiration date settles at the limit in the same direction. (Not necessarily the same contract month: seven delivery dates besides June are traded.) The use of daily data has drawbacks for this reason.

Appendix II: On the Empirical Relevance of Siegel's Paradox

It is a consequence of Jensen's inequality that if an investor is an unbiased predictor of the future dollar price of pounds spot rate, then he will be a biased predictor of the future pound price of dollars spot rate. Thus the assumption that the futures rate is equal to the expected value of the future spot rate under risk neutrality and "rational" expectations cannot be entirely satisfactory. This phenomenon and its empirical consequences have been discussed in the QJE (Siegel, McCulloch, Roper). There it was resolved that although theoretically valid, "Siegel's paradox" was of no practical relevance. The conclusion was that it would take hundreds of years of data to discern the phenomenon.

The purpose of this note is essentially a negative one. Here it is shown that the empirical relevance of Siegel's paradox cannot be dismissed on a priori grounds without making more specific distributional assumptions. Naturally we are not stating that there is any problem in defining the correct arbitrage conditions which will arise from any particular maximizing model of the foreign exchange market. For example, arbitrage conditions may show that the futures rate will depend upon the expected value of the future spot rate, a risk premium, and a convexity term (the Jensen's inequality term). Can the econometrician ignore the convexity term?

Siegel (1972) uses a Taylor expansion of $1/x$ in the neighborhood of $E(x)$ in order to produce an approximate expression for

$E(1/x) - 1/E(x)$, the Jensen's inequality term. This is Siegel's equation (5). This expression is reached by taking expectations of both sides of the Taylor expansion and assuming the remainder term to be small. McCulloch (1975) manipulates Siegel's expression to form his equation (2); he then asks how many observations it would take to distinguish the convexity term from zero, given empirical estimates of the variance. Let us consider their method. The Taylor expansion gives:

$$\frac{1}{x} = \frac{1}{E(x)} - \frac{(x - E(x))}{E(x)^2} + \frac{(x - E(x))^2}{E(x)^3} + R, \qquad (1)$$

where $R = \dfrac{-(X - E(x))^3}{E(x)^4}$

and $X = kx + (1-k)E(x) \quad 1 \geq k \geq 0$.

If we take expectations of both sides and ignore the remainder term, we obtain:

$$E(\frac{1}{x}) - \frac{1}{E(x)} \simeq \frac{Var(x)}{E(x)^3} \quad .$$

As is well known (for example, from portfolio analysis), this can be mathematically erroneous. The Taylor approximation is a local one in the neighborhood of $E(x)$, whereas the expectations integral is global. There is no guarantee that the term being ignored is actually of a smaller order of magnitude than the included terms without further assumptions (such as that the distribution of x is "compact" -- that is, that the uncertainty about the

mean is sufficiently small; large changes are almost ruled out).

After taking expectations, the remainder term will be of
the form:

$$E(R) = \sum_{n=3}^{\infty} \frac{(-1)^n M_n(x)}{E(x)^{n+1}} \quad , \tag{3}$$

where $M_n(x)$ is the nth moment about the mean. Clearly
this series will diverge for probability distributions where
$E(\frac{1}{x})$ fails to exist. In general, it will depend on the higher
moments of the distribution, which are extremely sensitive to
outliers. It is easy to find cases where use of the approximation
(2) will be misleading.

Consider a Gamma distribution with parameters α and β.

$$F(Y;\alpha,\beta) \sim \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^Y x^{\alpha-1} e^{-\beta x} \, dx \ . \tag{4}$$

In this case:

$$E(\frac{1}{x}) - \frac{1}{E(x)} = \frac{\beta}{\alpha-1} - \frac{\beta}{\alpha} = \frac{\beta}{\alpha(\alpha-1)} \tag{5}$$

and

$$\frac{Var(x)}{E(x)^3} = \frac{\beta}{\alpha^2} \ .$$

Then the difference between (5) and (6) is:

$$\frac{\beta}{\alpha^2(\alpha-1)} \ . \tag{7}$$

Now α is > 1, or else the $E(\frac{1}{x})$ would not exist (the Gamma function
would diverge). Still, for low α, (7) can be quite large. For α

near 1, (7) can be made arbitrarily large for any mean.

Not every example turns out this way, of course. A similar exercise for the lognormal distribution will show that $E(\frac{1}{x}) - \frac{1}{E(x)} = \frac{Var(x)}{E(x)^3}$ exactly. Both are equal to:

$$e^{-\mu + 1/2\sigma^2} - e^{-\mu - 1/2\sigma^2} \quad , \sigma^2 > 0 \quad . \tag{8}$$

The lognormal distribution arises from the transformation $y = \log x$, where $y \sim N(\mu, \sigma^2)$.[1]

The moments given in Siegel's expression for the convexity term are themselves nominal quantities. The variance of the dollar/pound rate may be quite small at the same time that the variance of the pound/dollar rate is quite large. The Siegel formula normalizes the variance by the mean cubed. With the mean in the neighburhood of 1, we can give another illustration, this time considering:[2]

$$P(X = 1) = .99 \ , \ P(X = .01) = .01 \quad . \tag{9}$$

Then $E(\frac{1}{x}) - \frac{1}{E(x)} = 1.99 - 1.01 = .98$.

$$\frac{Var(x)}{E(x)^3} \approx .01 \quad .$$

The expression for the remainder term in Siegel's formula shows that the convexity term can be quite sensitive to outliers.

In summary, the point of this note is not so much to demonstrate the importance of the convexity term, as to show that the

resolution given to the problem in the literature is inadequate. The problem is not a theoretical one, but an empirical one. Its importance will vary from case to case. One way to resolve exactly the theoretical problem is to make specific assumptions about the underlying random variables and their distribution functions.

Footnotes

1. Clearly our analysis shows that there are many cases where

the variance divided by the mean cubed is a good approximation

for the convexity term. There still may be cases where the variance

divided by the mean cubed is large.

2. This example illustrates that the size of the convexity term

depends heavily on points near the origin. Roughly speaking,

this corresponds to cases where the variance of 1/x is large.

Such a probability distribution could also arise through risk of

default.

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CHAPTER II

ANTICIPATED AND TRANSITORY SHOCKS

IN A MODEL OF

EXCHANGE RATE DYNAMICS

Introduction

The framework for the analysis of this paper is taken from Rudiger Dornbusch's 1976 _JPE_ paper, "Expectations and Exchange Rate Dynamics." There he develops a simple macroeconomic framework for studying exchange rate movements under perfect capital mobility. The model is consistent with rational expectations, yet suggestive of the enormous exchange rate volatility experienced under flexible rates. Slow adjustment of goods markets relative to assets markets causes the economy to overshoot the new long run equilibrium exchange rate in response to an unanticipated increase in the money supply.

The purpose of this paper is to analyze the implications of anticipated and transitory shocks to the economy. This extension is natural and important because not all of the continual shocks which buffet the economy come as surprises. Particular attention is given to the impact effect of news of a future shock, and the position of the economy when the shock actually takes place. The term structure of exchange rates and interest rates, as well as the expected future course of inflation, depend to a large degree on the extent to which the shock is anticipated. The usual dichotomy between monetary and real shocks is also important here. Another issue discussed is how the economy reacts to an anticipated series of shocks.

This paper builds on recent work by Wilson and Blanchard.[1] It is distinguished by an analytical treatment which permits sharp-

er and more general results. Wilson has analyzed the case of
an anticipated increase in the money supply in the basic Dorn-
busch model. Blanchard has derived the effects of announced
monetary and fiscal policy in a related framework. The basic
. del is presented in the first section of the paper, where the
case of an unanticipated increase in the money supply is examined.
This shock is used for purposes of discussion and comparison
throughout, although the formal solution is more general and can
be used to analyze supply shocks, export demand shocks, and world
money market shocks among others. Different formulations of the
model are presented, such as allowing for short run adjustment
of output, in which case overshooting need not occur.

Next we turn to the case of an anticipated increase in the
money supply. This paper solves for the discount rate, which
determines the impact effect of a future change. The term struc-
ture of exchange rates and the rate of inflation differ from the
unanticipated shock of the first section, especially during the
transition period prior to the actual money supply increase. The
same long run equilibrium is reached in both cases.

It might be thought that under perfect foresight, an antici-
pated change will have no effects when it actually takes place,
so long as it is anticipated far enough in advance. While it
turns out to be true that a long anticipated change will have no
discrete impact on the endogenous variables (with the exception of
the interest rate), it will cause an important change in the rate
of depreciation and inflation. In general, the economy will not

be very near the eventual long run equilibrium when the change actually takes place. This result derives from the assumption that prices adjust to disequilibrium in current markets. It occurs despite the fact that initial exchange rate adjustment is determined by expectations about the future path of the economy. The result is reasonably robust to other specifications of the model.

In the next section, we consider the cases of a transitory decrease in export demand and a transitory increase in the money supply. The latter is decomposed into the effects of an unanticipated increase in the money supply, and an anticipated decrease of the same magnitude. Even though the shock causes an initial depreciation and worsening of the terms of trade, the exchange rate will appreciate beyond its initial equilibrium value before finally returning to it. As this process is initially accompanied by inflation, the economy enjoys improved terms of trade at the transition point when the money supply decreases back. This quasi-cyclical adjustment is not due to the usual reasons in such dynamic models (relative speeds of adjustment to disequilibrium in different markets), but rather is due to the buffeting of the economy by anticipated discrete changes in exogenous variables.

Finally, we consider the more general case of how the economy reacts to an anticipated series of shocks. Once again, the path of prices and exchange rates decomposes neatly into the independent effects of each shock. The impact effect is the sum of all the discounted jumps. Once a shock has actually taken place, any effects other than those on the long run equilibrium price

level and exchange rate are eventually dampened out. In the case of money supply increases, it is quite possible for the subsequent dynamics to strongly resemble the case of an unanticipated money supply increase, where rising prices are accompanied by an appreciating exchange rate.

A formal solution of the various exercises given in the paper is found in the mathematical appendix.

I. Unanticipated Shocks in the Dornbusch Model

The Dornbusch model characterizes a small, open economy where capital is perfectly mobile. Therefore, interest rates, net of depreciation, are equalized. The country faces a given world price of imports. Domestic output is an imperfect substitute for imported goods, so their relative and absolute price is determined endogenously. The capital stock is exogenous and fixed. In the short run, asset market equilibrium dominates the behavior of the system. Prices adjust slowly and cannot make discrete movements. The exchange rate can make unanticipated discrete movements, but cannot make anticipated discrete movements because that would allow an infinite arbitrage profit.

We set out the model, using the notation of Dornbusch. Aggregate demand is given by equation 1.

$$\ln D = u + \delta(e-p) + \gamma y - \sigma r \tag{1}$$

where e,p,y are respectively the log of the exchange rate, price level, and income. Initially, assume income is fixed. r is the domestic nominal interest rate, and u is a shift parameter. The exchange rate channel turns out to be an important means through which monetary policy can act on aggregate demand. The use of a real interest rate $r-\dot{p}$ is shown to have little qualitative impact on the results.

Domestic prices are assumed to adjust to excess demand in the goods market.

$$\dot{p} = \ln(D/Y) = \Pi[u + \delta(e-p) + (\gamma-1)y - \sigma r] \qquad (2)$$

where Π is the speed of adjustment coefficient.

The domestic interest rate is determined in the money market, which is characterized by a standard demand for money equation. (Below m is the log of the nominal money supply.)

$$-\lambda r + \phi y = m - \alpha p - (1-\alpha)e \qquad (3)$$

In the above equation, we use a weighted average of domestic prices and import prices, where the weights α and $1-\alpha$, are the expenditure shares of domestic and imported goods. (We have set the foreign price level equal to unity, so $p^* = 0$.) The polar case of $\alpha = 1$ is used in Dornbusch, and as he points out, this assumption does not affect his qualitative results. It can become important when we deal with <u>anticipated</u> shocks.

Interest parity always holds.

$$r = r^* + x, \qquad (4)$$

where x is the expected rate of depreciation of the domestic currency, and r^* is the foreign interest rate, which is given exogenously. Dornbusch assumes that expected exchange rate depreciation will be a linear function of the distance of the exchange rate from the equilibrium exchange rate, and then shows that such regressive expectations are rational in this model. Here, we impose perfect foresight directly.

$$x = \dot{e}, \qquad (5)$$

where $\dot{e}$ is the rate of depreciation of the exchange rate. This is necessary when we take into account anticipated shocks. Regressive expectations will only coincide with rational expectations when the economy is not anticipating further known shocks.[2]

The instantaneous forward premium is given by $\dot{e}$. The M-period forward premium can be determined from the expected sequence for short term forward premiums. By interest parity, this premium will equal the rate of interest on a domestic discount bond, which pays a single balloon payment after M periods, minus the foreign interest rate.

$$\dot{E}_M = R_M - r^*, \qquad R_M = \frac{1}{M} \int_0^M r(t) \, dt \qquad (6)$$

where $\dot{E}_M$ is the M period forward premium, and R_M is the rate of interest on a domestic discount bond. These long term rates are determined recursively, and do not feed back into the rest of the model. (They are not included in Dornbusch's model.) They show how the term structure of exchange rate expectations is reflected in current long rates.

In the long run, $\dot{e} = 0$, $\dot{p} = 0$, and $r = r^{*}$[3], so we can solve for the equilibrium exchange rate and price level, using equations 1, 2, 3, 4, and 5.

$$\bar{p} = (1-\alpha)/\delta \ [u + (\gamma-1)y + \sigma r^*] + \lambda r^* - \phi y + m \qquad (7)$$

$$\bar{e} = -\alpha/\delta \ [u + (\gamma-1)y + \sigma r^*] + \lambda r^* - \phi y + m \qquad (8)$$

In the case where the weight of the domestic good price in the

deflator for real money balances is unity ($\alpha = 1$), these reduce to:

$$\bar{p} = m - \phi y + \lambda r^* \qquad (7')$$

$$\bar{e} = m + 1/\delta \ [-u - (\delta\phi+\gamma-1)y + (\delta\lambda+\sigma)r^*] \qquad (8')$$

Note the long run homogeneity of both functions in the money supply. Unlike in the Mundell-Fleming world, the effects of monetary policy in this full employment model are only transitory. Note also how the long run equilibrium exchange rate is determined by both real and monetary factors. In comparative statics exercises, both real and monetary factors operate through their effects on the equilibrium exchange rate and price level.

The equations of motion of the system are given by:

$$\dot{p} = -\pi[\delta+\sigma\alpha/\lambda](p_t-\bar{p}) + \pi[\delta-(1-\alpha)\sigma/\lambda](e_t-\bar{e}) \qquad (9)$$

$$\dot{e} = \alpha/\lambda \ (p_t-\bar{p}) + (1-\alpha)/\lambda \ (e_t-\bar{e}) \qquad (10)$$
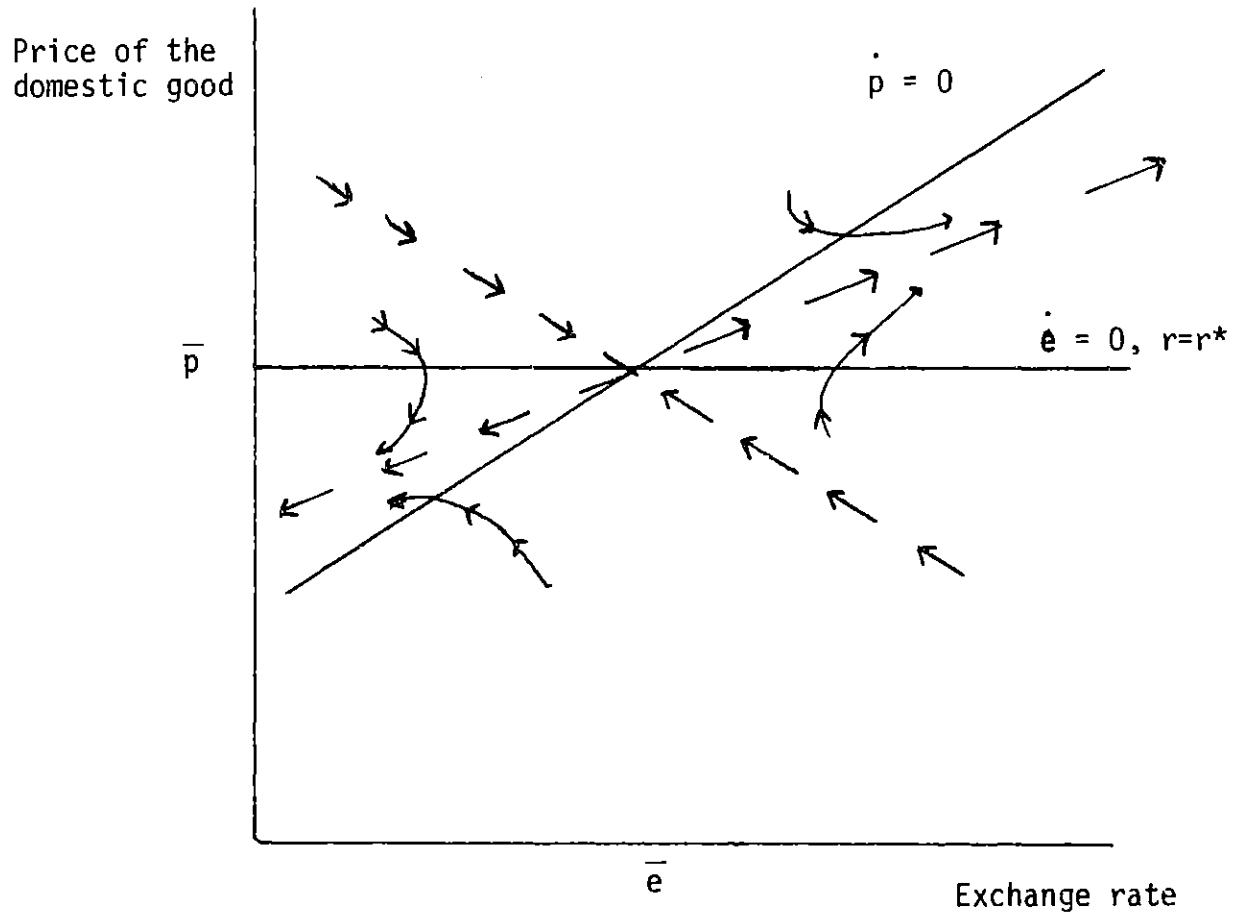
In the polar case where $\alpha=1$, exchange rate depreciation will be a linear function of the distance of the price level from the equilibrium price level, even off the perfect foresight path considered by Dornbusch. There, $\dot{e}$ is also a linear function of $(e_t - \bar{e})$, as will be shown later.

The roots of the characteristic equation

$$\theta^2 + [\pi(\delta + \frac{\sigma\alpha}{\lambda}) - \frac{(1-\alpha)}{\lambda}]\theta - \pi\delta/\lambda \text{ are given by}$$

$$\theta_1 = z + (z +\pi\delta/\lambda)^{1/2} > 0 \ ,$$

$$\theta_2 = z - (z +\pi\delta/\lambda)^{1/2} < 0, \text{ where } z \equiv \frac{\pi- (\delta+\frac{\sigma\alpha}{\lambda}) + \frac{1-\alpha}{\lambda}}{2}$$

FIGURE I: Output is fixed and $\alpha = 1$

Price of the domestic good

$\dot{p} = 0$

$\overline{p}$

$\dot{e} = 0$, $r = r^*$

$\overline{e}$

Exchange rate

Solving for the characteristic vectors, the general solution has the form:

$$(p_t - \bar{p}) = \frac{c1}{\alpha}(\lambda\Theta1 - (1-\alpha))e^{\Theta1 t} + \tag{11}$$

$$\frac{c2}{\alpha}(\lambda\Theta2 - (1-\alpha))e^{\Theta2 t}$$

$$(e_t - \bar{e}) = c1e^{\Theta1 t} + c2e^{\Theta2 t} \tag{12}$$

where c1 and c2 are arbitrary constants depending on the initial conditions.

As only one of the two roots is less than zero and therefore stable, the equilibrium is a saddle point. Figure 1 describes the disequilibrium motion of the system in the case where $\alpha=1$, and import prices do not enter into the deflator for real money balances. The $\dot{e} = 0$ schedule is horizontal, as there is only one price level that equates the domestic interest rate with the world interest rate. The $\dot{p} = 0$ schedule has slope less than unity. The exchange rate increases aggregate demand through the terms of trade effect. The price level dampens aggregate demand both through the terms of trade effect and through domestic interest rates. (To use a real interest rate, $r - \dot{p}$, in the aggregate demand function, replace Π by $\rho = \Pi/(1-\sigma\Pi)$. $\rho > 0$ is required for stability.)[4]

In Figure II, we consider a case where α is not one, and where higher exchange rates actually dampen aggregate demand. This occurs if $\frac{\sigma}{\lambda}(1-\alpha) > \delta$. In this case, a higher exchange rate raises the cost of imported goods and lowers real money balances. This

FIGURE II: A higher e reduces aggregate demand
through its effect on real balances.
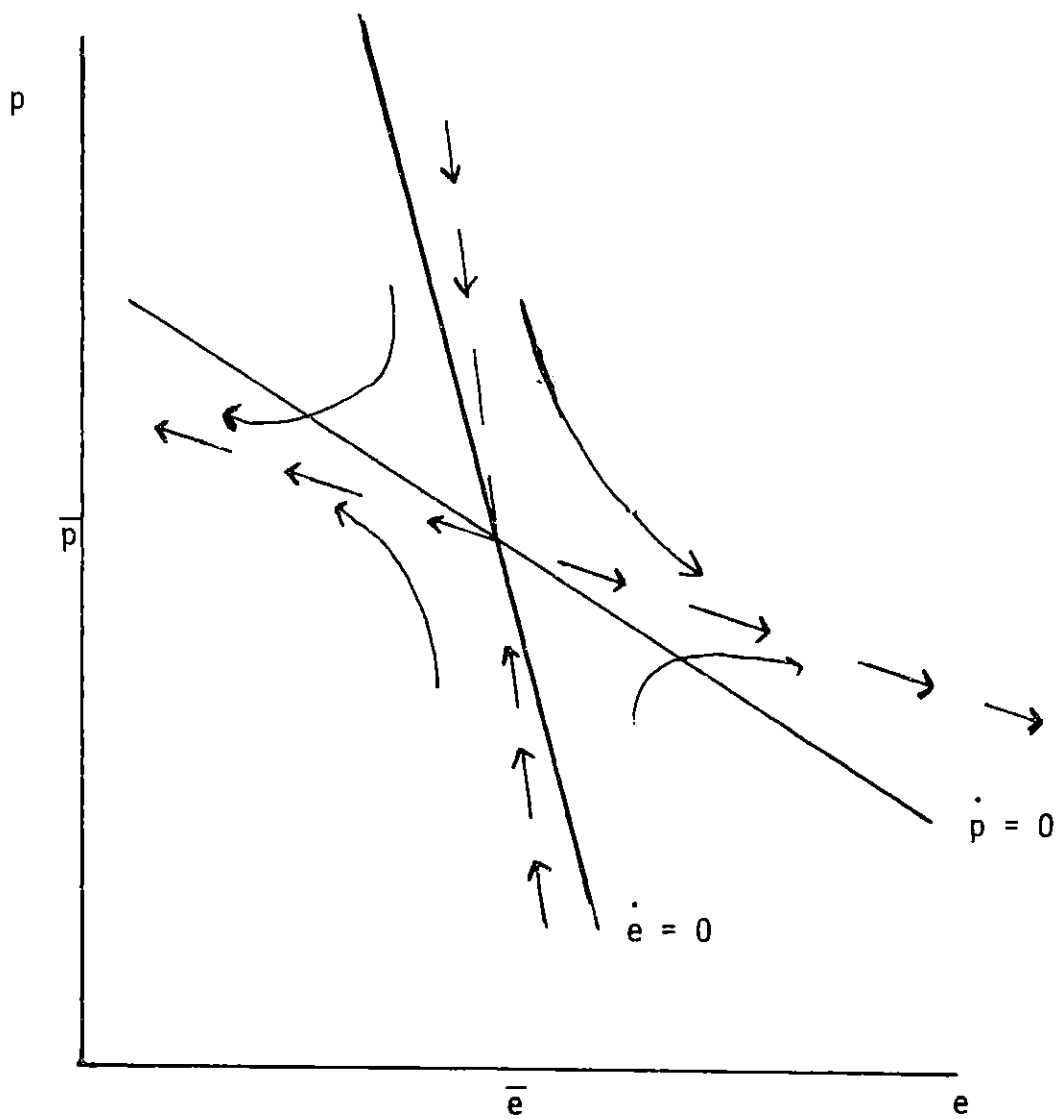
leads to higher interest rates. The dampening effect of interest rates on aggregate demand is sufficiently large as to outweigh the expansionary effect through the terms of trade. The $\dot{e} = 0$, or $r = r^*$, schedule is now downward sloping (for any positive α less than one). It has a more negative slope than the $\dot{p} = 0$ schedule. (See equations 9 and 10.)

Finally, Dornbusch considers the case where short run adjustment of output is permitted, although in the long run output returns to its full employment level. Set $Y = D$ and replace the price adjustment equation by

$$\dot{p} = \pi (y_t - \bar{y}) \tag{13}$$

The details of this case are set out in the Appendix. The slope of the $\dot{p} = 0$ schedule is unaffected by this modification (since along it $y = \bar{y}$). The $\dot{e} = 0$ schedule may have positive slope if higher prices, holding the interest rate constant, create an excess supply of real money balances. To keep the domestic interest rate equal to the world interest rate, the exchange rate must depreciate. This corresponds to the case considered by Dornbusch, where short run overshooting of the long run equilibrium exchange rate in response to an increase in the money supply does not occur. This case is more likely when the weight of domestic goods prices in the deflator for real money balances is small, for large price and income elasticities of aggregate demand, and for a large income elasticity in the demand for real balances. Figure III illustrates this

FIGURE III : Short run variations in real output
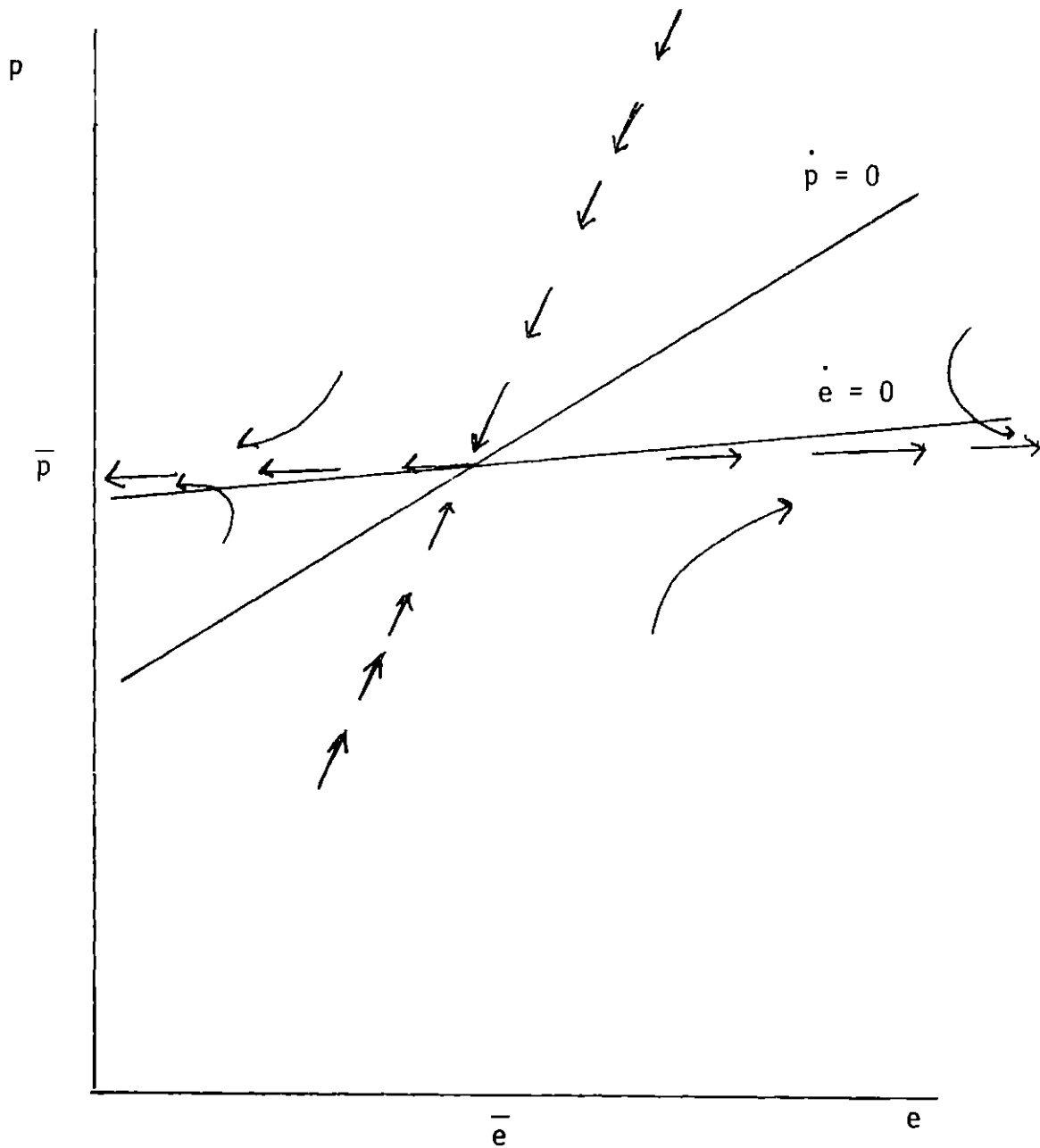
case, as the system can be reduced to the p,e plane.

We are now ready to perform the hypothetical experiment of an unanticipated increase in the money supply. Both $\bar{p}$ and $\bar{e}$ change by Δm due to the first-degree homogeneity of both functions in the money supply. Let Δm be the increase in money.

$$\bar{p}' = \bar{p} + \Delta m \quad ,$$

$$\bar{e}' = \bar{e} + \Delta m \quad ,$$

where $\bar{p}'$, $\bar{e}'$ are the new long run equilibrium price level and exchange rate respectively.

We first consider the case where output is fixed at its full-employment level, and imports do not enter the real balance deflator.

Because the price level cannot change by a discrete amount instantaneously[5] an increase in nominal money is an increase in real money balances in the short run. Therefore the domestic interest rate falls. In the long run, the price level must rise and the exchange rate must depreciate in proportion to the increase in the money supply. But since the domestic interest rate has fallen below the world interest rate, asset market equilibrium dictates that the exchange rate be appreciating in the short run. Therefore the exchange rate must instantaneously overshoot its long run equilibrium[6] and the economy has worsened terms of trade in the short run. The expansionary effect of the depreciated exchange rate causes prices to rise, as the exchange rate appreciates and the interest rate rises back to the world level until the new equilibrium is reached.

We can formally solve for the path of the economy using equations 11 and 12, and by specifying two initial conditions. One is that $p(0) = \bar{p}$, so that the price level cannot jump instantaneously. The other assumption is that the economy moves to the stable path. Agents do not evaluate possibilities which lead to economically infeasible results. (It is sufficient to assume that the interest rate is bounded.) This second condition requires that cl in equations 11 and 12, the arbitrary condition corresponding to the unstable root, is zero. One can solve to find:

$$e(0) - \bar{e} = \Delta m \, (1 - \alpha/[\lambda\Theta2-(1-\alpha)]) \qquad (14)$$
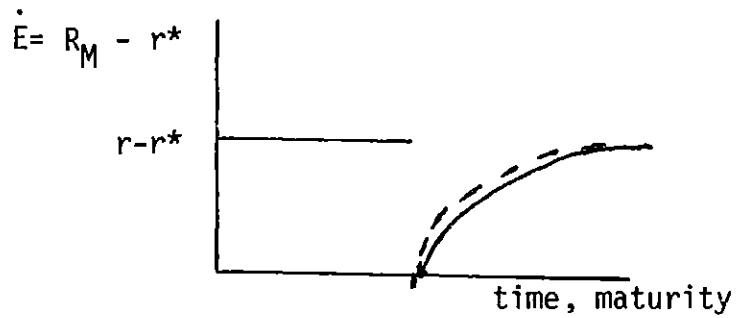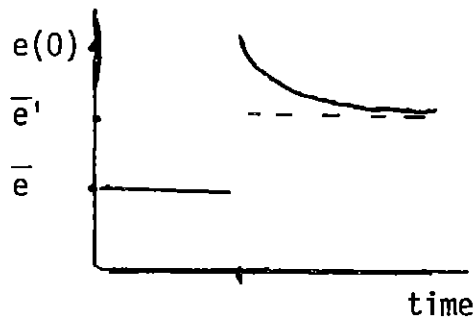
Since cl in equation 12 is zero, we have

$$\dot{e}(t) = \Theta2(e_t - \bar{e}) \, . \qquad (15)$$

In (14), setting $\alpha = 1$, we get Dornbusch's overshooting result. (Here $\Theta2$ is the negative of his Θ.) If α is less than one, overshooting will still occur, but it will be dampened. The exchange rate must still depreciate by more than Δm. If it only depreciated by Δm, interest rates would still lie below world interest rates because the share of imports in the deflator for real money balances is less than one. But that would mean the exchange rate would have to be appreciating, and it could not reach the new long run equilibrium. Therefore overshooting also occurs in that case. The result in equation 15 confirms Dornbusch's assumption that along the stable path exchange rate depreciation is a linear function of $(e_t - \bar{e})$.

Factors which accelerate the speed of adjustment reduce the degree of overshooting. These include a large coefficient of price adjustment and large elasticities. Figure IV illustrates the case of an unanticipated increase in the money supply when α, the weight of domestic goods prices in the real balance deflator, is 1. Figure V illustrates the case where $\alpha < 1$ and a higher e lowers aggregate demand. ($\frac{\sigma}{\lambda}(1-\alpha) > \delta$.) Overshooting is dampened, but it is essentially the same.

Finally we present the case where output can adjust in the short run (corresponding to Figure III). If, as in Figure III, higher prices depress output sufficiently so higher prices lead to an excess supply of real balances at constant interest rates, overshooting will not occur.[8] This is illustrated in Figure VI. In this case, if the exchange rate initially increased in proportion to the increase in the money supply, interest rates would go up. Therefore the exchange rate undershoots and depreciates to the new equilibrium as prices rise. Output initially expands, but then contracts back to full employment output.

FIGURE IV : After an unanticipated increase in the money
supply, the exchange rate overshoots its long
run equilibrium value. [7]

$\bar{e}$ is the old long run
equilibrium.

$\bar{e}'$is the new long run
equilibrium.

e(0) is the exchange
rate immediately after
the shock.

$\dot{E}= R_M - r^*$

$r-r^*$
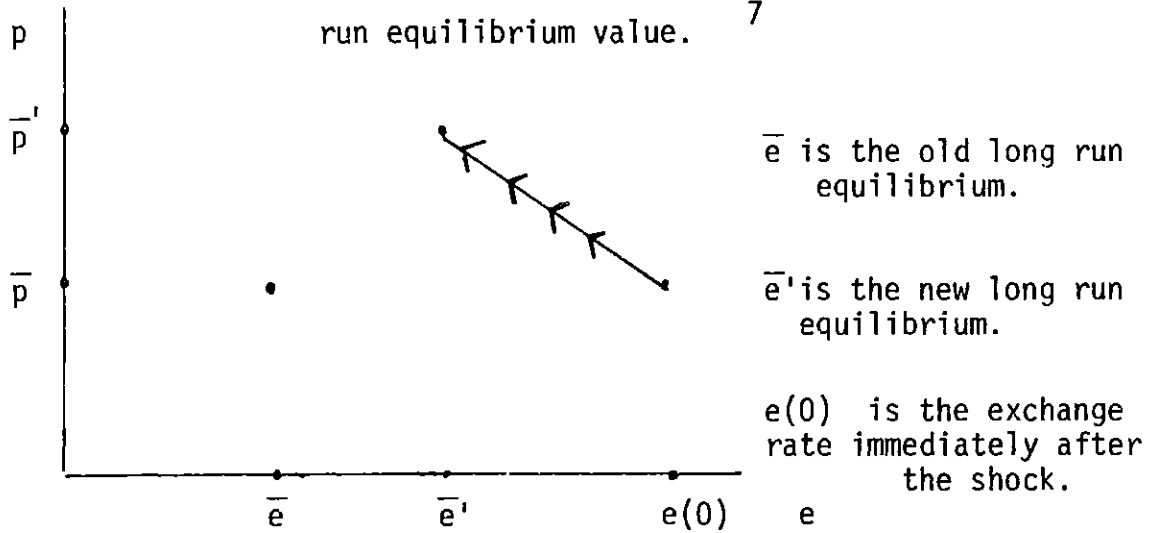
Legend: ----------- $\dot{E}_M$ is the M-period forward
premium.

_____ $r-r^*, \dot{e}$

FIGURE V : When import prices enter the deflator
for real balances, overshooting is
dampened.
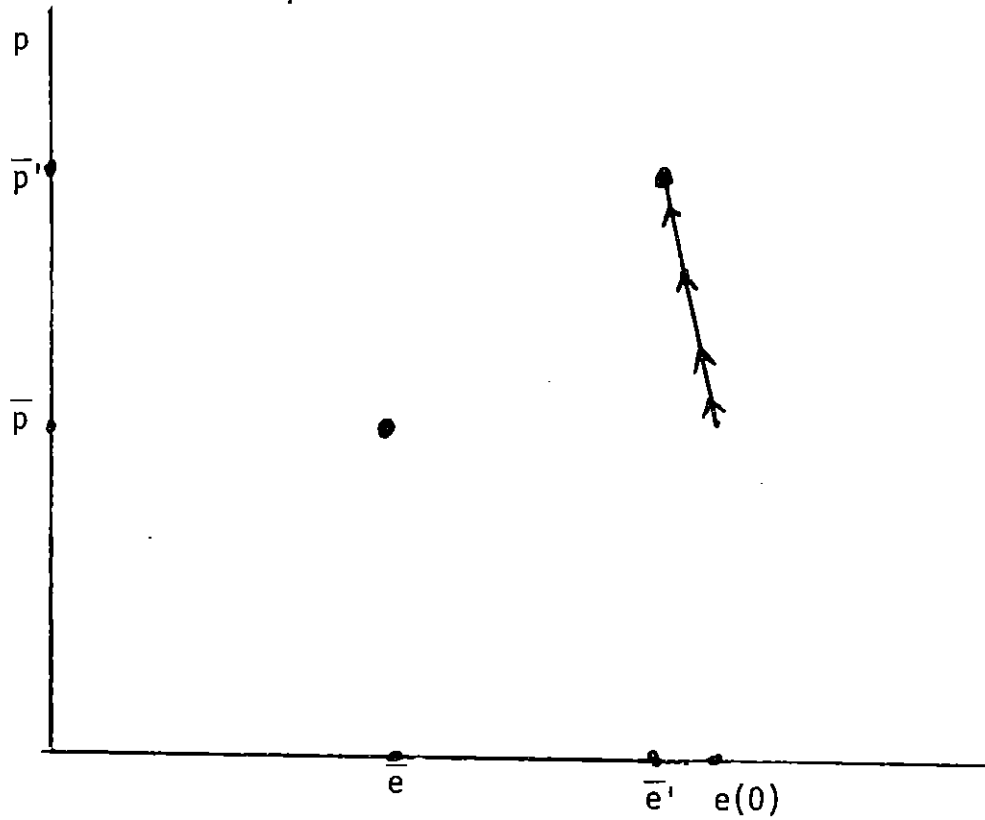
Legend : $\bar{e}'$ is the new long run equilibrium.
$e(0)$ is the exchange rate immediately
after the unanticipated money increase.

FIGURE VI : Output is demand determined in the short run.

Legend:

$\dot{E}_M$ is the M period forward premium (discount)

II. An Anticipated Permanent Shock

Consider an anticipated shock which will take place T periods in the future. For example, an unanticipated announcement is made at time 0 that the money s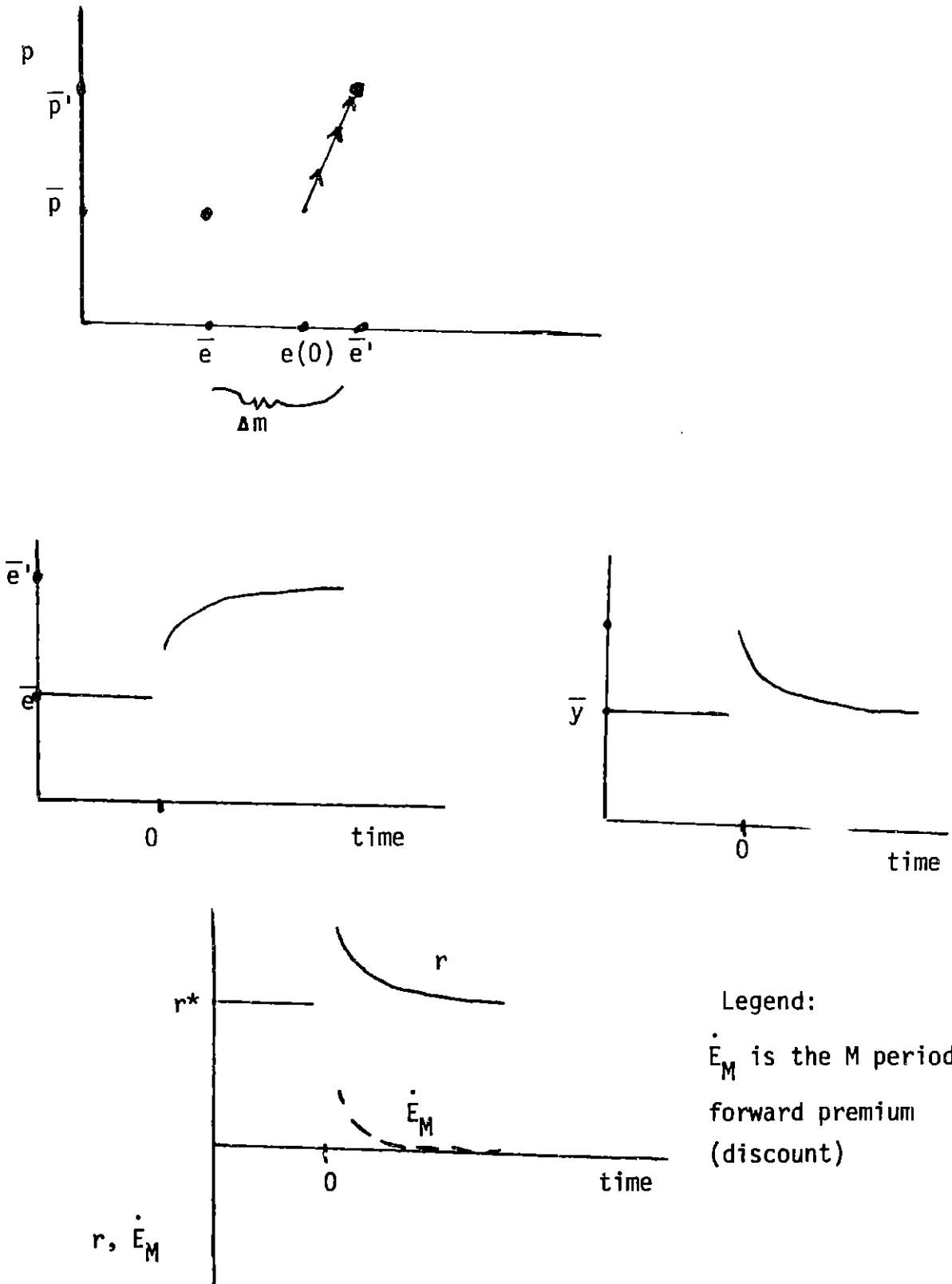upply will be increased by Δm at time T. This classic case will be used as our example, although it is not more difficult to treat real shocks, or shocks that effect $\bar{e}$ and $\bar{p}$ differentially and therefore change the long run terms of trade. We will only work through the simplest case algebraically, allowing the interested reader to see the appendix for the case where a price index is used as a deflator for real money balances, and where short run output adjustment is permitted.

By assumption, the economy must eventually reach the new long run equilibrium corresponding to a higher nominal money supply, a higher price level, and a depreciated exchange rate. This will only be possible if the economy lies on the single stable path at the time the money supply is actually increased. This is the same path the economy jumps to immediately in the case of an unanticipated increase in the money supply. We can use equations 11 and 12, together with the stability restriction that $c1 = 0$, to solve for that path:[9]

$$(p_t - \bar{p}') = \lambda\theta_2(e_t - \bar{e}), \quad t \geq T \tag{16}$$

where we have set $\alpha = 1$, and $\bar{e}' = \bar{e} + \Delta\bar{e}$, $p' = \bar{p} + \Delta\bar{p}$. $\bar{p}'$, $\bar{e}'$ are the new long run equilibrium price level and exchange rate

respectively, and $\Delta\bar{p} = \Delta\bar{e} = \Delta\bar{m}$.

The first initial condition is provided by the fact that the economy cannot jump to this path at time T, but must meet it at the transition point when the money supply is increased. The exchange rate can make unanticipated discrete jumps, but cannot make anticipated discrete jumps without infinite interest rates. The price level can never move suddenly, and this provides also a second initial condition, that $p(0) = \bar{p}$ or $p(0) - \bar{p}' = -\Delta m$. Evaluating at time T, and rewriting (16) as

$$(p_T - \bar{p}) = \lambda\Theta_2(e_T - \bar{e}) + \Delta p - \lambda\Theta_2\Delta\bar{e}, \tag{16'}$$

we can solve for c1. Then use c1, evaluate the system (equations 11 and 12) at time zero, solving for c2 by noting that the price level is still at $\bar{p}$. Thus we can completely solve the path of the economy.

Solving yields:

$$e(0) - \bar{e} = (\Delta\bar{e} - \Delta\bar{p}/\lambda\Theta_2)\ e^{-\Theta 1 T} \tag{17}$$

$$= \Delta m(1 - 1/\lambda\Theta_2)\bar{e}^{\Theta 1 T}$$

Letting $T \to 0$, which gives the case of an unanticipated increase in the money supply, we have Dornbusch's overshooting result. (This is equation 14 in Section I, setting α, the weight of the price of the domestic good in the deflator for real money balances, equal to unity.)

Equation 17 is seen to be the discounted future jump.

Recall that $\Theta 1 > 0$, and $\Theta 2 < 0$. It is easy to establish:

$$\frac{d\ e(0)}{dT} < 0\ ,\quad \frac{d\ e(0)}{d\Delta m} > 0\ .$$

The initial jump is smaller, the farther in the future the money supply is actually going to increase, and the smaller the projected change in the money supply. Immediate overshooting of the new long run equilibrium occurs if $(1 - 1/\lambda\Theta_2) > e^{\Theta 1 T}$. This depends on T as shown above, and on the parameters of $\Theta 1$ and $\Theta 2$, which we shall proceed to investigate. First, the same factors which make a jump in response to an unanticipated increase in the money supply large cause a large initial movement.

$$1 - 1/\lambda\Theta_2 = 1 + 1/[\Pi(\sigma+\delta\lambda)/2 + \tag{19}$$

$$(\frac{\Pi^2(\sigma+\delta\lambda)^2}{4} + \Pi\delta\lambda)^{1/2}]$$

A small coefficient of price adjustment and small elasticities serve to accentuate the jump. Anticipated slow adjustment magnifies the jump. Now, examining the discount rate:

$$\Theta 1 = -\pi(\sigma+\lambda\delta)/2\lambda + (\Pi^2[(\sigma+\lambda\delta)/2\lambda]^2 + \Pi\delta/\lambda)^{1/2} \tag{20}$$

The speed of adjustment coefficient, Π, raises the discount rate and reduces the initial jump. It serves to speed up price adjustment and thereby accelerates exchange rate adjustment. The economy cannot jump as much initially for large Π or it will overshoot the stable path. The relative price coefficient, δ, also

unambiguously serves to speed up price adjustment and therefore
raises the discount rate. A large interest response of money
demand, λ, lowers the discount rate, and aggravates the initial
change in the exchange rate. This is because a large λ dampens
interest rate adjustments and thereby the rate of exchange rate
depreciation. Thus asset market equilibrium does not require
the exchange rate to move as much during the transition period
prior to the actual increase in the money supply. A large interest
elasticity of aggregate demand also tends to accentuate the
initial jump in the exchange rate. Because the money supply
does not actually increase until time T, rising prices caused
by the effects of the depreciated exchange rate on aggregate
demand cause real money balances to fall and the interest rate to
rise. As it rises, the exchange rate depreciates faster and faster.
A large interest elasticity of aggregate demand serves as a feed-
back channel which dampens the price increases by reducing aggre-
gate demand. This in turn causes interest rates to rise more
slowly, and the exchange rate to move less. So large interest
elasticities accentuate the jump through the discount rate. Since
interest elasticities lower both the discount rate and the jump
itself, which factor dominates? Clearly for large T, they
accentuate the jump, as the discount rate is exponential. The
discount rate is bounded, since it is easy to show that $\lambda\Theta1$ is
less than the slope of the $\dot{p} = 0$ schedule.

It is still clear that if T is mildly large, there will be
little impact effect. In that case the economy will stay near

the current equilibrium for a long time.

Given the initial depreciation of the exchange rate and worsened terms of trade, prices begin to rise. This lowers real balances, raises the interest rate and causes the exchange rate to depreciate. Will the new long run equilibrium with higher prices and a depreciated exchange rate be reached at just the time the money supply actually increases? We solve for e(T):

$$e(T) - \bar{e} = \Delta m(1 - 1/\lambda\Theta_2) \frac{(\Theta_2 - \Theta_1 e^{(\Theta_2 - \Theta_1)T})}{(\Theta_2 - \Theta_1)} \quad (21)^*$$

$$\equiv \Psi(\Delta m, T)$$

The exchange rate must overshoot its long run equilibrium value, $\bar{e}' = \bar{e} + \Delta m$. This follows since $\lambda\Theta_1 < 1$ and $\Theta_2 < 0$. Thus $e(T) - \bar{e} > \Delta m = \Delta\bar{e}$. We quickly establish that $\frac{d\ e(T) - \bar{e}}{d\ \Delta m} > 0$ and $\frac{d\ e(T) - \bar{e}'}{d\ \Delta m} > 0$. Thus the larger the change in the money supply, the greater the percentage difference between the exchange rate at the time the money supply actually increases and the eventual long run equilibrium. Also, since d e(T)/dT < 0, the farther in the future the change takes place, the more the economy will have adjusted to it. In Figure VII, we illustrate the case of an anticipated increase in the money supply.

---

* For the more general case where $\bar{e}$ and $\bar{p}$ do not have to increase by the same amount, we have:

$$\Psi(\Delta\bar{p}, \Delta\bar{e}, T) \equiv \frac{(\Delta\bar{p} - \lambda\Theta_2\Delta\bar{e})}{\lambda(\Theta_1 - \Theta_2)} \left[1 - \frac{\Theta_1}{\Theta_2} e^{(\Theta_2 - \Theta_1)T}\right].$$

FIGURE VII : An anticipated increase in the money supply
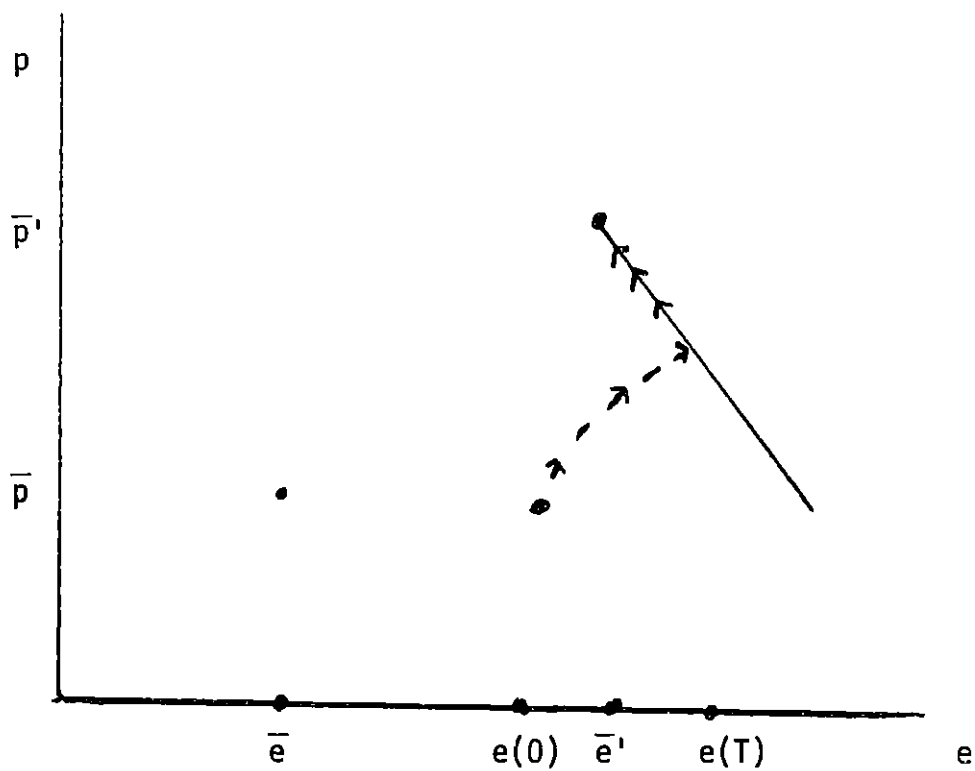
The money supply increase is announced at time 0, and
implemented at time T. ē' is the new long run
equilibrium.

It can be shown that the economy will not have completely adjusted to the anticipated money supply increase at the time it actually takes place, even if it was known about a long time in advance.[10] This happens because although the exchange rate initially jumps based on future interaction between the goods market and the money market, prices always adjust to reduce current goods market disequilibrium. Since price changes increase interest rates linearly through the money demand equation, they accelerate exchange rate changes.

Inspection of Figure VII might lead one to conjecture that this property only exists because domestic nominal interest rates are used in the aggregate demand equation. The use of $r - \dot{p}$ will have no qualitative effect, since the $\dot{p} = 0$ schedule will still have a slope of less than unity. Since it cannot be crossed during the transition period, we cannot end up near the new equilibrium at time T. To see that this is not essential for the result, substitute the world rate of interest into the aggregate demand function, so that the $\dot{p} = 0$ schedule will have a slope of unity. Even here it is simple to show that nothing restricts the economy to be near the new equilibrium before time T. $(p_t - \bar{p})/(e_t - \bar{e})$ increases with t during the transition period and reaches a maximum of $\lambda \theta_1$.[11] Complete adjustment will take place prior to the actual money supply increase only if Π, δ and λ are large. Otherwise complete adjustment is impossible prior to the money supply increase.

This result is in sharp contrast to the common presumption that if an announcement is made today of a change very far in the

future, the economy will have completely adjusted to it before it actually takes place. Here further price inflation and exchange rate depreciation will be necessary after time T. Thus the terms of trade remain worsened until the final equilibrium is reached. "Anticipated money" matters here.

Finally, we can determine the behavior of interest rates and depreciation:

$$\dot{e}_t = \frac{\Theta_1 \Delta m (1 - \lambda \Theta_2)}{\lambda(\Theta_1 - \Theta_2)} \ (e^{\Theta_1 (t-T)} - e^{\Theta_2 t - \Theta_1 T}), \qquad (22)$$

$$0 \leq t < T \ .$$

$\dot{e}(0) = 0$. Until the price level actually rises, the domestic interest rate remains equal to the world interest rate. Since $\ddot{e} > 0$, the interest rate will rise at an increasing rate. After the money supply increases, the interest rate falls below the world rate and then begins to rise at a decreasing rate. The path of the interest rate and the corresponding term structure of forward premiums are given in Figure VIII. $\dot{E}_M$ is the M period forward premium.

Allowing imports to enter into the deflator for real money balances reduces the initial jump. The increase in the exchange rate lowers real money balances because the money supply has not yet increased. This raises interest rates, tending to speed up movement of the exchange rate through interest parity and to dampen price increases. Higher exchange rates increase aggregate demand through the terms of trade channel, but dampen aggregate demand

FIGURE VIII : The term structure of interest rates and
exchange rates in the presence of an
anticipated money shock.
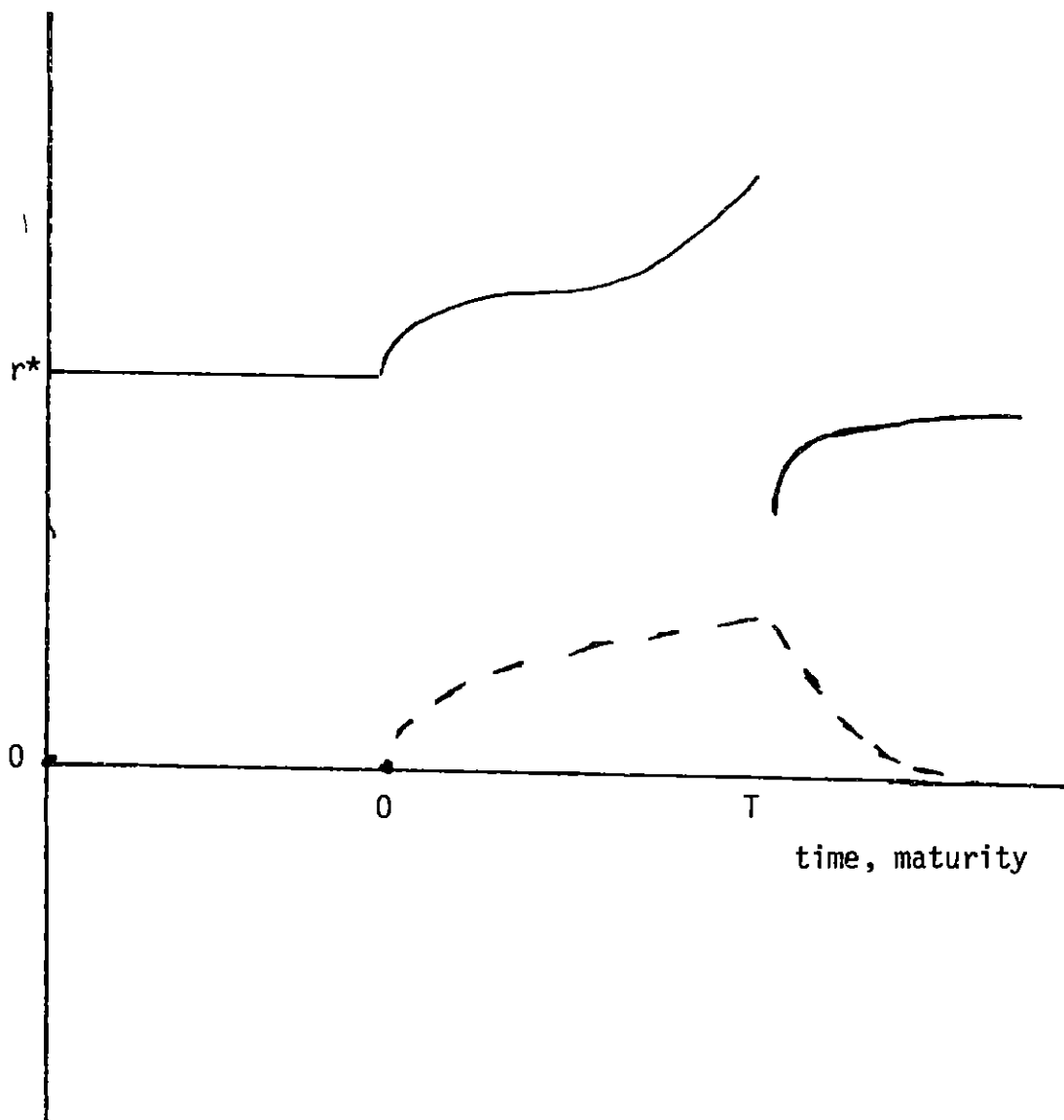

Legend : $----$ $E_M = R_M - r^*$

_____ $r = r^* + \dot{e}$

through higher interest rates. In fact, it is possible that rising interest rates will lead to declining aggregate demand and therefore falling prices during the transition period. This corresponds to the case considered in Figure II, where $\frac{\sigma(1-\alpha)}{\lambda} > \delta$. In this case, it can be shown that $\frac{d\ e(0)}{dT} > 0$, $\frac{d\ e(T)}{dT} > 0$. The longer in advance the announcement is made, the smaller the initial movement as before. However, in this "perverse" case, the exchange rate will lie farther from the long run equilibrium exchange, and prices will lie farther from the long run equilibrium price level at the time the money supply actually increases the longer in advance it was anticipated. (Both e(T), p(T) are bounded.) This case is illustrated in Figure IX.

In the case with short run adjustment in output, treated in the Appendix, price and exchange rate behavior is similar to the first case considered. After an initial jump in output, output may initially fall, but then will begin to rise. When the money supply increase actually takes place, rising prices and an appreciating exchange rate cause output to fall, returning to its equilibrium full employment level.

This "perverse" case where the domestic price level falls in anticipation of an inflationary shock is quite similar to a result obtained by Blanchard in his paper "Output, the Stock Market and Interest Rates." There, output initially decreases in anticipation of expansionary fiscal policy in one case.

FIGURE IX
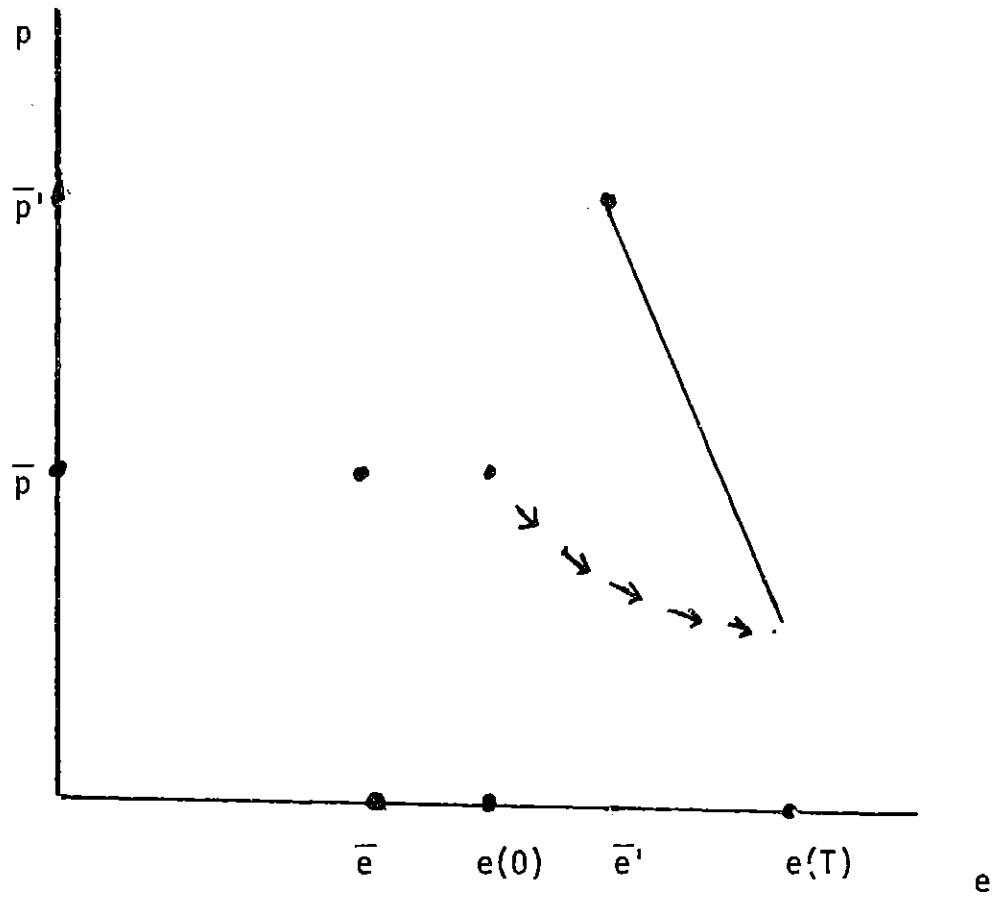
III. Transitory Shocks

The next case dealt with involves discrete transitory shocks to the economy -- a temporarily increased demand for exports and a temporary change in the world rate of interest are two examples. The analysis is again focused on the case of a (transitory) money supply increase, mainly for purposes of comparison.

Again the economy starts from a position of equilibrium. There will be a transitory shock in one of the exogenous variables, which is known to last for only T periods. In addition, it is assumed that the economy eventually returns to long run equilibrium. Let $\bar{e}$, $\bar{p}$ be the initial and long run equilibrium exchange rate and price level, respectively. Let $\bar{e}'$ and $\bar{p}'$ govern the short run dynamics.

$$\bar{e} = \bar{e}' - \Delta\bar{e} \quad , \quad \bar{p} = \bar{p}' - \Delta\bar{p} \quad .$$

We solve in a manner exactly analogous to the case of an anticipated change. We restrict analysis to the simplest case,[*]

$$p_t - \bar{p} = \lambda\Theta_2(e_t - \bar{e}) \quad , \quad t \geq T \tag{23}$$

Solving for the initial exchange rate change:

$$e(0) - \bar{e} = \Delta\bar{p}(e^{-\Theta_1 T} - 1)/\lambda\Theta_2 + \Delta\bar{e}(1 - e^{-\Theta_1 T}) \tag{24}$$

The exchange rate may even overshoot the temporary equilibrium

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[*] The analysis of the case where short run output adjustment is permitted is solved in the appendix of this paper.

initially. The direction of the impact effect is as expected. A transitory inflationary shock initially depreciates the exchange rate. That is, for $\Delta\bar{p}$, $\Delta\bar{e} > 0$, $e(0) > \bar{e}$. It is easy to establish:

$$\frac{d\;e(0)}{d\bar{T}} > 0 \quad , \quad \frac{d\;e(0)}{d\;\bar{p}} > 0 \quad , \quad \frac{d\;e(0)}{d\;\bar{e}} > 0 \quad .$$

The impact effect can be decomposed into the effect of a permanent unanticipated shock together with an anticipated shock of equal and opposite magnitude. This is easy to see when we look at the case of a transitory money supply increase.

$$e(0) - \bar{e} = \Delta m(1 - 1/\lambda\Theta_2)(1 - e^{-\Theta_1 T}) \tag{25}$$

Compare this with equations 14 and 21 to confirm the decomposition. Also, note that as $T \to 0$, $e(0) \to \bar{e}$, and as $T \to \infty$, we have the Dornbusch result for permanent changes in m.

Another polar case would be a temporary decrease in demand for our exports. (This is modeled by a change in u, the shift factor in aggregate demand.) Then we have:

$$e(0) - \bar{e} = \Delta\bar{e}(1 - e^{-\Theta_1 T}) \; , \tag{26}$$

where $\Delta\bar{e} = \frac{1}{\delta} \Delta u$

It turns out that in this case the path of the economy is identical to the case where the economy was initially at a lower level of export demand and experienced an anticipated increase in export demand. This symmetry derives from the instantaneous adjustment of the exchange rate, while no long run price level

adjustment is necessary.

Now we turn to the subsequent dynamics, solving for e(T) in the general case:[13]

$$e(T) - \bar{e}' = \frac{\Delta\bar{p} - \lambda\Theta_2\Delta\bar{e}}{\lambda(\Theta_1-\Theta_2)} \left[\frac{\Theta_1}{\Theta_2} e^{(\Theta_2 - \Theta_1)} - 1 \right] - \frac{\Delta\bar{p}}{\lambda\Theta_2} e^{\Theta_2 T} \quad (27)$$

$$= - \Psi(\Delta\bar{p},\Delta\bar{e},T) - \frac{\Delta\bar{p}}{\lambda\Theta_2} e^{\Theta_2 T} \quad * \qquad (27')$$

The last term in this equation corresponds to the distance from the new equilibrium exchange rate T periods after an instantaneous jump to the perfect foresight line, which would correspond to the transitory equilibrium. The first term is recognized as the position of the exchange rate from initial equilibrium T periods after the announcement of a shock due to take place in T periods, whicl will affect the long run equilibrium by $-\Delta\bar{e}$, $-\Delta p$.

The linear combination of these two conflicting terms gives the position of the exchange rate when the shock actually takes place. Note that for $\Delta\bar{e} > 0$, $\Delta\bar{p} > 0$ the first term in (27') will be negative and the second term will be positive. For a transitory money supply increase, substitute $\Delta m = \Delta\bar{p} = \Delta\bar{e}$ to get:

$$e(T) - \bar{e}' = - \Psi(\Delta m, T) - \frac{\Delta m}{\lambda\Theta_2} e^{\Theta_2 T} \qquad (28)$$

---

* Ψ is defined as in equation 21.

$$\Psi(\Delta\bar{p},\Delta\bar{e}, T) \equiv \frac{(\Delta\bar{p} - \lambda\Theta_2\Delta\bar{e})}{\lambda(\Theta_1-\Theta_2)} (1 - \frac{\Theta_1}{\Theta_2} e^{(\Theta_2-\Theta_1)T})$$

or

$$e(T) - \bar{e} = - \Psi(\Delta m, T) + \Delta m(1 - (e^{\Theta_2 T}/\lambda\Theta_2)) \quad {}^* \quad (29)$$

Here $e(T) - \bar{e} < 0$, and we must have quasi-cyclical adjustment.[14] The exchange rate initially depreciates so that there can be the expectation of an appreciation as the temporarily higher money supply lowers the domestic interest rate. The two channels of feedback into aggregate demand of the higher exchange rate and a lower interest rate put upward pressure on domestic prices. Prices rise, but then may begin to fall as the terms of trade improve for the home country. Prices cannot fall below their original level without causing instability when the money supply decreases again. Therefore, when the domestic money supply decreases back to its original level, the domestic interest rate will rise above the world interest rate and necessitate an appreciation of the exchange rate. A sufficient but not necessary condition for quasi-cyclical adjustment is[15] $\Delta\bar{p} > \dfrac{\delta}{(\delta + \sigma/\lambda)} \Delta\bar{e}$, $\Delta\bar{e} > 0$, $\Delta p > 0$. Here, the cyclical approach to equilibrium comes from anticipated shocks, which buffet the economy due to sluggish price adjustment. This is as opposed to the more common case in macroeconomics, where cyclical adjustment is the result of differential speeds of adjustment to disequilibrium in different markets, which can lead to a cyclical path.

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* Note $\Psi(-\Delta m, T) = - \Psi(\Delta m, T)$.

FIGURE X : A transitory increase in the
money supply.

e(T) $\bar{e}$ e(0) $\bar{e}'$
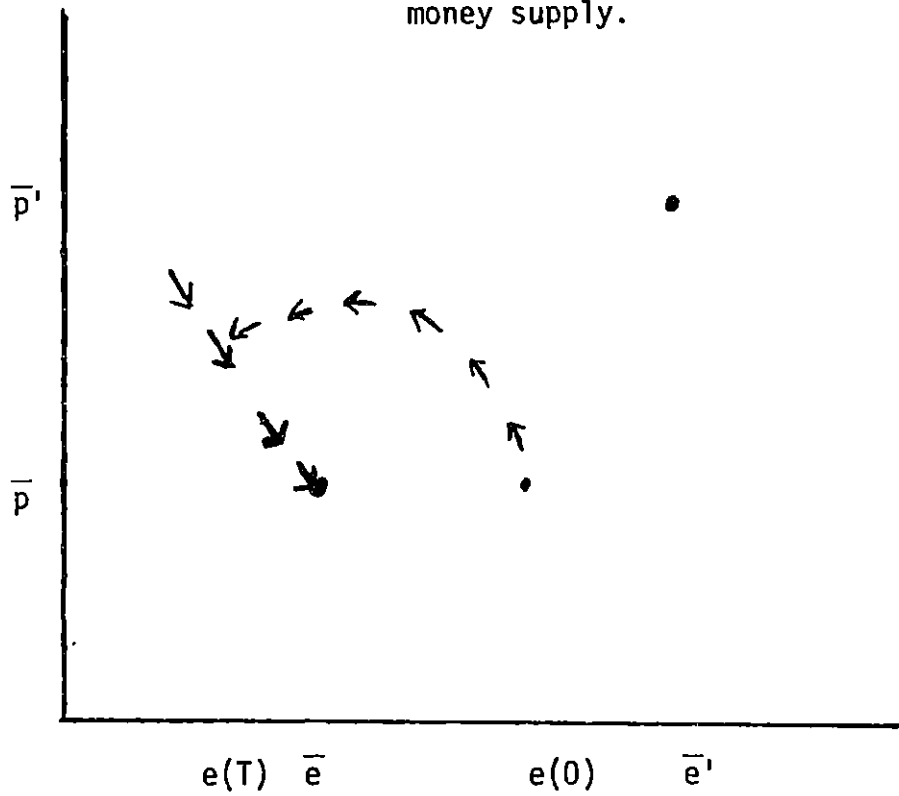
The term structure of interest rates and forward premiums at time 0
for forward contracts of differing maturity, M, are given below.

time, maturity (M)
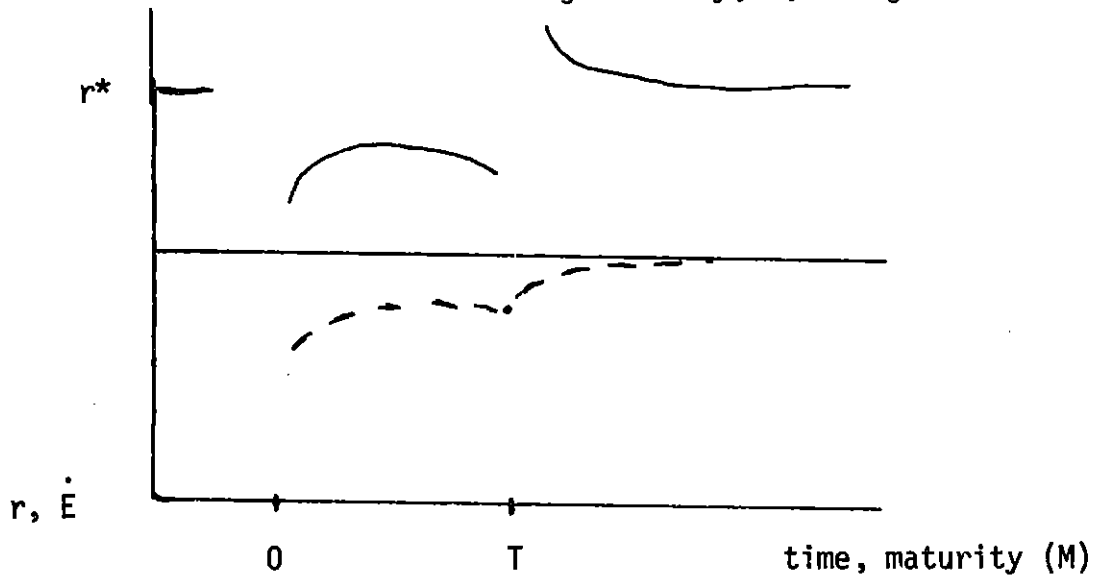
Legend :   - - - - - - - $\dot{E}_M = R_M - r^*$ _____ $r = \dot{e} + r^*$

Returning to the case of a transitory decrease in export demand, $\Delta\bar{e} = -\frac{1}{\delta}\Delta u$, $\Delta u < 0$:

$$e(T) - \bar{e}' = -\Psi(\Delta\bar{p}, \Delta\bar{e}, T) = \qquad (30)$$

$$-\Psi(0, -\frac{1}{\delta}\Delta u, T), \quad \bar{e}' = \bar{e} + \Delta\bar{e} \quad,$$

where Ψ is as defined in equation 21.

Here $e(T) - \bar{e} > 0$. The path to equilibrium involves continual appreciation. During the transition the interest rate falls at an increasing rate. There is no discrete jump in the interest rate when the transitory demand decreases, but it remains below the world interest rate. As prices rise, it begins to rise at a decreasing rate, eventually returning to the world rate. Note that $e(T) - \bar{e}'$ exactly corresponds to where it would be had the economy originally been at equilibrium $\bar{e}'$ and experienced an anticipated increase in export demand. This neat symmetry comes from the infinite flexibility of the exchange rate and the fact that no long run price adjustment is necessary.

Finally, we note that for large T the economy moves very near the path it would follow if the transitory change (of any type) were permanent. In the transition period prices, the exchange rate, and the interest rate pass very near the transitory equilibrium, and in fact spend a long time there. This quasi-turnpike property results from the saddle-point properties of the system. For large T it is naturally difficult to distinguish a transitory shock from a permanent one. Even if T is small, but

FIGURE XI : A transitory decrease in export demand
which occurs at time 0 and terminates
at time T.
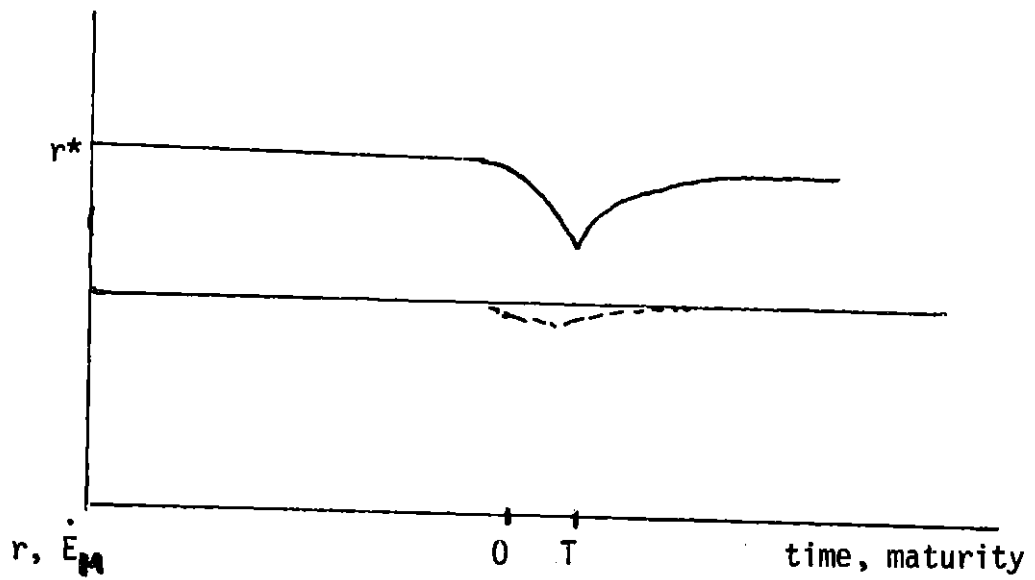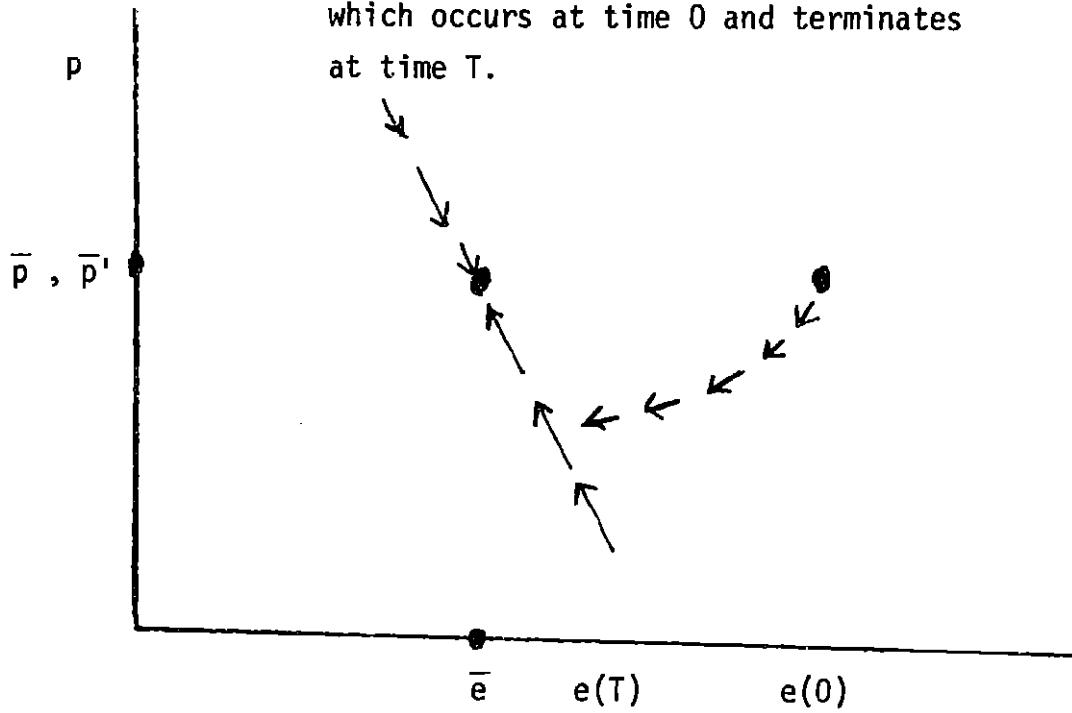
the discount rate, Θ_1, is large, it will be difficult to tell between the two cases in the initial periods.

IV. Anticipated Shocks in Different Periods

This section examines the case where the economy anticipates a series of shocks. For example, the money supply is expected to increase in period T, and again in period T + K. This case subsumes the possibility of an anticipated transitory change. The marginal benefit of examining the more general case is twofold. First, there is a straightforward generalization of the earlier results. A series of anticipated changes (we assume news of all of them is received at time 0) have an additive impact effect, with each change discounted by its distance into the future. It is again a property of Dornbusch's model that the subsequent path of the exchange rate and prices decomposes into the separate effects of each shock. Of considerable interest is the effect of multiple shocks on the dynamics and path of adjustment. It is shown that sometimes the behavior of prices and the exchange rate will closely resemble the equilibrium path resulting from a single unanticipated shock long before the last anticipated shock takes place. This is in contrast to our earlier result that showed the path following news of a future increase in the money supply differed sharply from the path following an actual increase in the money supply. In the latter case, we observe rising prices and a depreciating exchange rate during the transition period. With two anticipated money supply increases, we may observe rising prices and an appreciating exchange rate after the first actual increase. This will happen of course, if

the initial shock is sufficient to at least temporarily raise real money balances above their initial level.

Incidentally, although a complete closed form solution is only presented for the case of two anticipated changes, the solution algorithm easily generalizes and in fact closed form solution of the n-change case is not difficult.

Let $\bar{e}$, $\bar{p}$, be the logs of the initial equilibrium exchange rate and price level, respectively. Let $\Delta 1\bar{p}$, $\Delta 1\bar{e}$ be the change in temporary equilibrium which will take place at time T. They increase due to the first anticipated increase in the (log of the) money supply, and they are equal to it. At time T+K, there will be an additional change of $\Delta 2\bar{p}$, $\Delta 2\bar{e}$. So $\bar{e}' = \bar{e} + \Delta 1\bar{e}$, $e'' = \bar{e}' + \Delta 2\bar{e}$, and $\bar{p}' = \bar{p} + \Delta 1\bar{p}$, $p'' = \bar{p}' + \Delta 2\bar{p}$.

Additional initial conditions are provided by transversality conditions. The price level can never make discrete movements and the exchange rate only can at time 0, when it is not expected. Solving backwards, we find that at time T, the economy must lie on a line parallel to the perfect foresight line which passes through the final long run equilibrium. Solving for e(0) - $\bar{e}$:

$$e(0) - \bar{e} = (\Delta 1\bar{e} - \Delta 1\bar{p}/\lambda\Theta_2)e^{-\Theta_1 T} + \qquad (31)$$

$$(\Delta 2\bar{e} - \Delta 2\bar{p}/\lambda\Theta_2)e^{-\Theta_1 (T+K)} \quad .$$

Simply substitute in $\Delta 1m$, $\Delta 2m$ appropriately to get the case of money supply changes (e.g., $\Delta 1m = \Delta 1\bar{p} = \Delta 1\bar{e}$). The generalization of this expression to n changes is immediate.

The general solution of the system is presented in the Appendix. Below, the transition points are presented, as they are of particular interest.

$$e(T) - \bar{e} = \Psi\ (\Delta 1\bar{p},\ \Delta 1\bar{e},\ T) + \Psi\ (\Delta 2\bar{p},\ \Delta 2\bar{e},\ T)e^{-\Theta_1 K} \qquad (32)$$

where Ψ is defined in equation 21 for the case of money supply shocks, and at the bottom of page 77 for more general shocks.

The exchange rate at time T lies at a point consistent with the first shock, which actually takes place at time T, plus the discounted influence of the second shock. Note that $\Psi(\Delta 1\bar{p},\ \Delta 1\bar{e},\ T)$ is precisely the point where the exchange rate would be if the second shock were never going to take place. Naturally, if the second shock has zero magnitude or else lies very far in the future, the second term disappears, and we have the result for a single anticipated shock.

$$e(T + K) - \bar{e}' = \Psi(\Delta 1\bar{p},\ \Delta 1\bar{e},\ T)e^{\Theta_2 K} - \Delta 1\bar{e}(e^{\Theta_2 K}) + \qquad (33)$$

$$\Psi(\Delta 2\bar{p},\ \Delta 2\bar{e},\ T + K)$$

The last term in (33) is again just the transition point if only the second shock took place. Letting $(\Delta 1\bar{p},\ \Delta 1\bar{e}) \to 0$, the first two terms disappear and $\bar{e}' \to \bar{e}$, since $\bar{e}' = \bar{e} + \Delta 1\bar{e}$.

The analysis follows very much along the lines of the earlier sections, though there are important additions to note.

For example, whether the exchange rate is appreciating or depreciating will depend on the magnitude of the total increase

in the money supply. Whenever real money balances lie below their
original level, then in the fixed output case we are considering
here, the exchange rate must be depreciating. Suppose the first
money supply increase is relatively large and takes place soon
Then, although prices are rising due to the initial depreciation
and its effects on aggregate demand, they do not rise enough to
cancel out the first money supply increase. The fall in the
interest rate causes the exchange rate to appreciate. Prices
rise monotonically and the terms of trade improve as the exchange
rate appreciates. If the second money supply increase does not
occur for a while, prices may rise enough so that real balances
again fall below their original level and the exchange rate
again begins to depreciate. In Figure XII, we illustrate
such a case and the corresponding term structure of interest
rates and exchange rates. Here there is not such a sharp
dichotomy between the behavior of the economy in the case of an
anticipated shock as opposed to an unanticipated shock. Another
case is given in Figure XIII.

Once again, we have from the saddlepoint properties of
the equilibrium the result that if the second change does not
take place for a very long time, the economy spends most of
its time near the transitory equilibrium, $\bar{p}'$, $\bar{e}'$, corresponding
to the first money supply increase.

Finally, it follows from (33) that after an anticipated
shock actually takes place, the overshooting due to that shock
is eventually dampened out. In the long run, its only contribution

FIGURE XII : At time zero, it is announced that the
money supply will be increased by $\Delta_1 m$ at
time T, and $\Delta_2 m$ at time T+K.   Here, the
first shock occurs relatively soon, and is
relatively large.
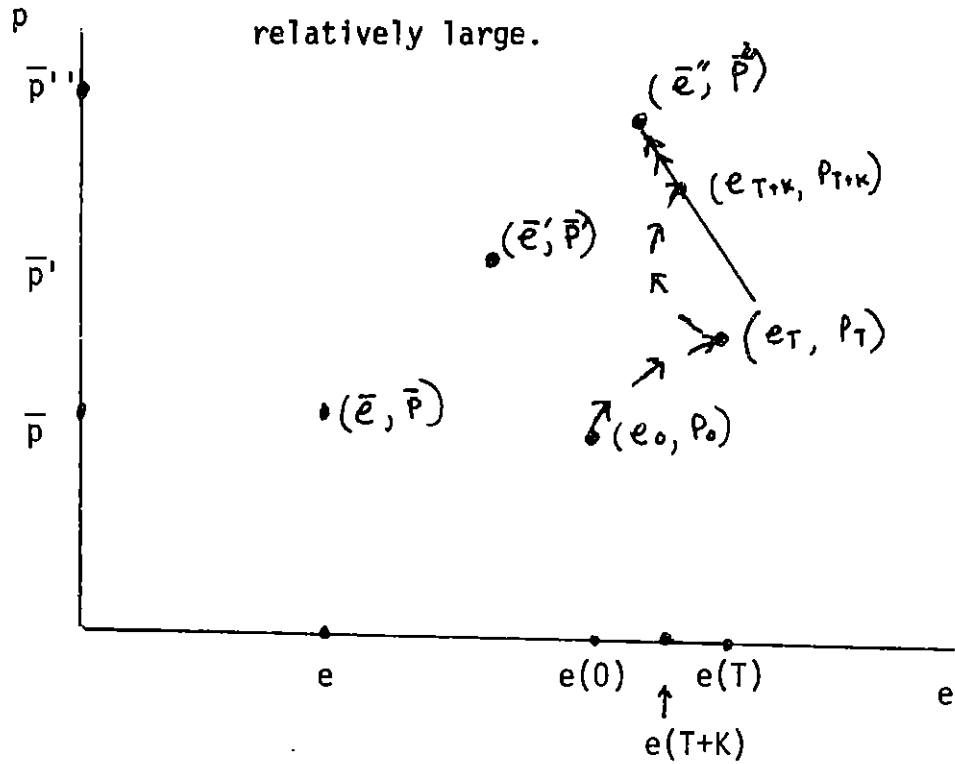
The term structure of interest rates and
forward premia.
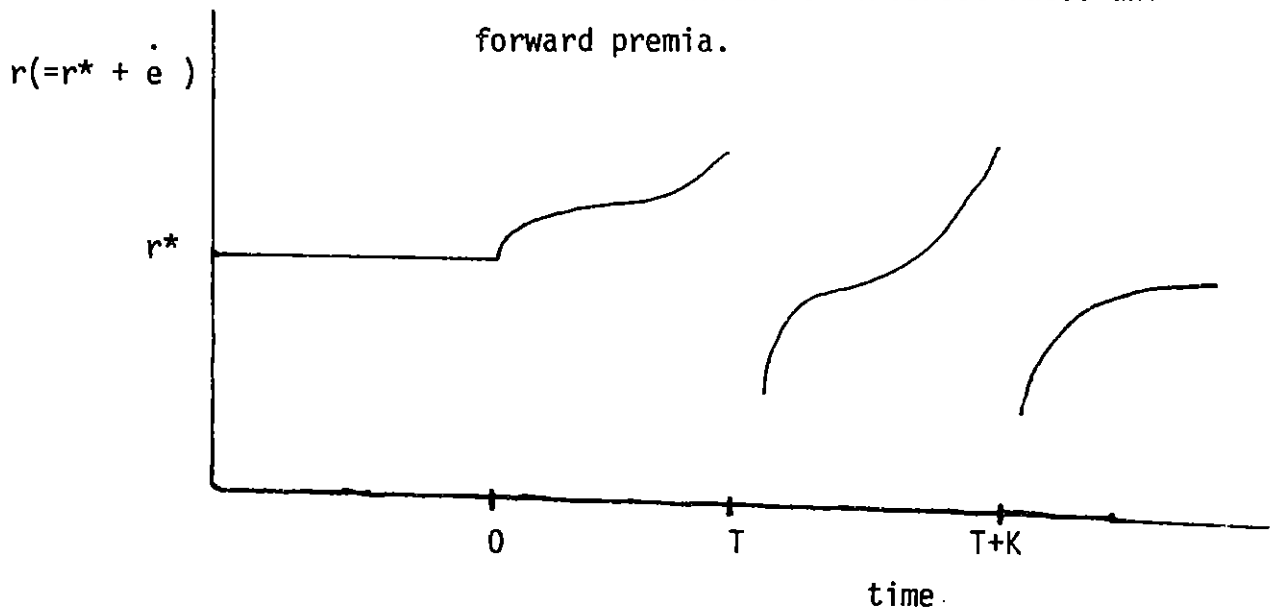
FIGURE XIII : Case II, in the event two money supply increases are anticipated.

$\bar{p}''$

$(\bar{e}''.\bar{p}'')$

$\bar{p}'$

$(\bar{e}',\bar{p}')$

$\bar{p}$

$(\bar{e},\bar{p})$

e   e(0) e(T) e(T+K)

r,

$\dot{r*+e}$

r*

0   T   T+K   time
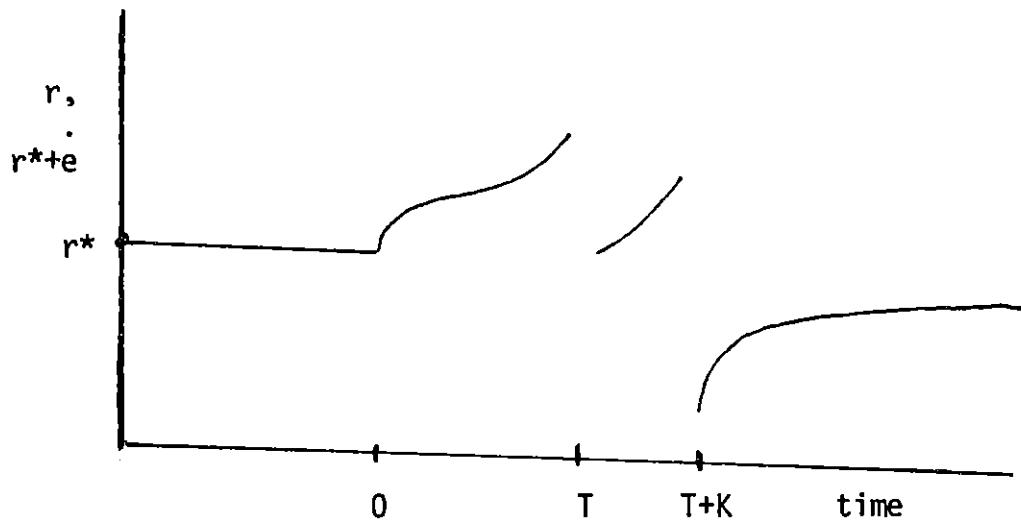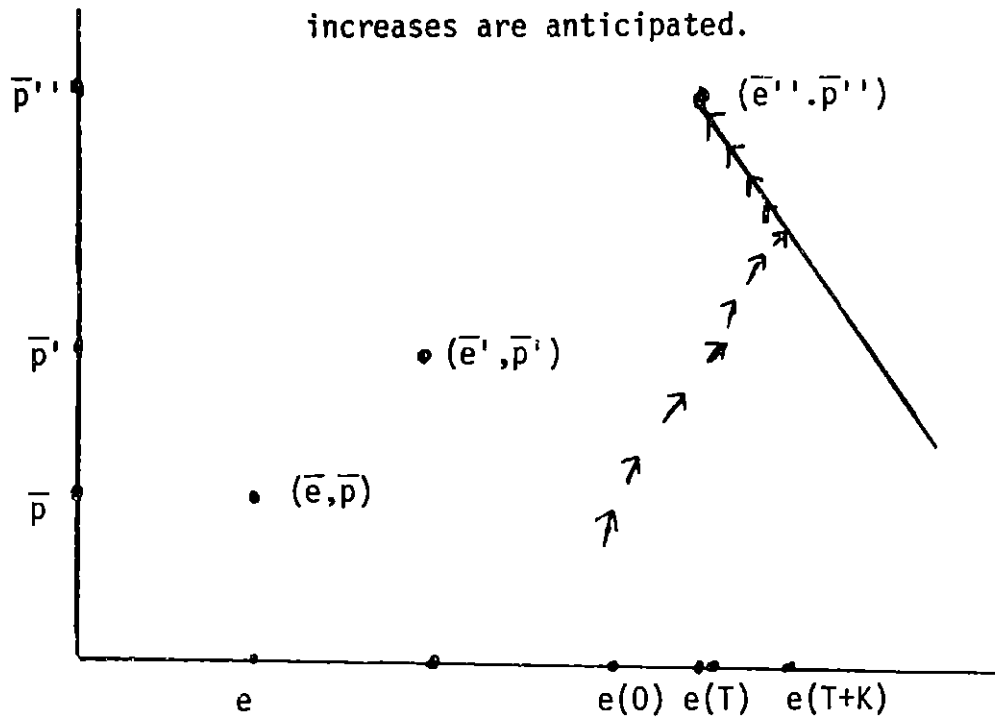
is to additively change the long run $\bar{e}$, $\bar{p}$.

Summary and Extensions

This paper adds to a literature which attempts to reconcile large changes in exchange rates with rational expectations. It explores the consequences of anticipated shocks and anticipated series of shocks. On a technical level, it is novel in that it solves exactly to what extent future shocks are discounted in a small macroeconomic model, and that it analyzes the case of a series of shocks, all of which were anticipated. It also shows how anticipations of future events, such as money supply increases, are reflected in the term structure of interest rates and forward premiums on contracts of differing durations.

The assumption that prices adjust gradually in response to current excess aggregate demand has the implication that a long anticipated increase in the money supply will still affect nominal variables, the exchange rate and prices, at the time it actually occurs. It will also cause continued changes in the terms of trade, a real variable. This type of result occurs even though expectations are rational (perfect) and the exchange rate is allowed to jump to take into account future interactions between the goods market and asset markets. This result is quite robust to small changes in the specification, such as using the world interest rate instead of a domestic nominal or real rate in the aggregate demand function.

Transitory shocks are shown to sometimes cause quasi-cyclical adjustment. In response to an inflationary transitory shock, the

exchange rate will initially depreciate, but then may appreciate beyond its original value before returning to it.

Footnotes

1. This paper was directly and indirectly motivated by Charles A. Wilson's work. Some of the results obtained are analogous to those obtained in Olivier Blanchard, "Output, the stock market and interest rates," November 1978, Harvard (mimeo). That paper also gives a very lucid presentation of the assumptions implicit in a Dornbusch-type model.

2. This modification is also used by Wilson. In this model, $\dot{e}$ will be a linear function of $(p_t - \bar{p})$ when only the domestic price level enters into the deflator for real money balances. It is only after no further shocks are anticipated that it will be a linear function of $(e_t - \bar{e})$. This has important consequences for empirical work based on the Dornbusch model, if anticipated and transitory shocks are to be taken into account. In reduced for exchange rate equations such as in Jeffrey Frankel (1979), which are based on the Dornbusch model,

$$e - m - m^* - \phi(y - y^*) - 1/\theta(r - r^*) + (\tfrac{1}{\theta} + \lambda)(\pi - \pi^*)$$

it can be shown that one implication is that relative price level terms should be included.

3. We are abstracting from steady state inflation.

4. This is noted in Dornbusch (1976).

5. The path of prices is continuous by assumption. Relative to the path of the exchange rate, this is an economically reasonable assumption.

6. Under perfect capital mobility, incipient capital flows cause

instantaneous exchange rate adjustment, to avoid an infinite expected profit.

7. Compare with Figure I for the lines of motion.

8. See Part I of the Appendix.

9. Part II of the Appendix carefully shows how to derive the results, and how to solve the cases where short run output adjustment is permitted, or where a price index is used in the deflator for real money balances.

10. This follows by taking the limit as $T \to \infty$ in (21). $\lambda\Theta_1$ has a maximum equal to the slope of the $\dot{p} = 0$ schedule, $\frac{\delta\lambda}{\delta\lambda + \sigma}$.

$$\lambda\Theta_1 = \frac{\Pi\delta}{\Theta_1 + \Pi(\delta + \sigma/\lambda)} \quad , \quad \Theta_1 > 0 \quad .$$

Even if $\sigma = 0$, so that the domestic interest rate does not enter into the aggregate demand schedule, $\lambda\Theta_1$ will be less than 1 unless its parameters become very large. Simple algebraic manipulation of equation 21 will show that this is a sufficient condition to prove the result.

11. Both c1 and c2 are positive (see Appendix) in equations 11 and 12 in this case for $0 \leq t < T$. Take the ratio of (11) to (12) and the statement follows by simple inspection or differentiation.

12. The point of inflection in the path of interest rates between time 0 and T occurs when $\ddot{p} = 0$. That such a point occurs can be proved directly by differentiating equation 11 twice with respect to time, and noting that $c1 > 0$, $c2 > 0$, $\Theta_1 > 0$, $\Theta_2 < 0, |\Theta_2| > \Theta_1$.

13. $- \Psi(\Delta\bar{p}, \Delta\bar{e}, T) = \Psi(-\Delta\bar{p}, -\Delta\bar{e}, T)$.

14. This follows by noting in (29) that $e(T) - e = 0$, for $T = 0$,

and $e(T) - \bar{e} \xrightarrow[T \to \infty]{} \Delta m \left(\dfrac{\lambda\Theta_2 - 1}{\lambda\Theta_1 - \lambda\Theta_2} + 1 \right) < 0$, since $\lambda\Theta_1 < 1$, and

that $e(T) - e$ is monotonically decreasing in T.

15. That is equivalent to the condition that the $\dot{p} = 0$ schedule

shifts upwards.

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Appendix

Part I

Here the system of the first section is solved when short run changes in output are permitted. Let $Y = D$ and $\dot{p} = \mu(y_t - \bar{y})$. We will assume $\alpha = 1$ here. $\mu \equiv 1/(1 - \gamma) > 0$.

Clearly the long run equilibrium exchange rate and price level remain unchanged by this modification. The system can be reduced to p,e space:

$$\begin{bmatrix} \dot{p} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} -\Pi\mu(\delta + \sigma\Delta) & \Pi\mu(\delta - \sigma\gamma) \\ \Delta & \gamma \end{bmatrix} \begin{bmatrix} p_t - \bar{p} \\ e_t - \bar{e} \end{bmatrix} \quad \text{(A1)}$$

where $\Delta = \dfrac{1 - \phi\mu\delta}{\lambda + \phi\mu\delta}$, and $\gamma = \dfrac{\phi\mu\delta}{\lambda + \sigma\phi\mu}$.

These equations are solved for the characteristic roots and vectors. The general solution is:

$$p_t - \bar{p} = c1J1e^{\Theta_1 t} + c2J2e^{\Theta_2 t} \quad \text{(A2)}$$

$$e_t - \bar{e} = c1e^{\Theta_1 t} + c2e^{\Theta_2 t} \quad \text{(A3)}$$

where $J1 = \dfrac{\Theta_i - \gamma}{\Delta} = \dfrac{\Pi\mu[\delta - \sigma\gamma]}{\Theta_i + \Pi\mu[\delta + \sigma\Delta]}$

$$\Theta_1 = \frac{z}{2} + \left(\frac{z^2}{4} + \frac{\Pi\mu\delta}{\lambda + \sigma\phi\mu} \right)^{1/2} > 0$$

$$\Theta_2 = \frac{z}{2} - \left(\frac{z^2}{4} + \frac{\Pi\mu\delta}{\lambda + \sigma\phi\mu} \right)^{1/2} < 0$$

$$z \equiv \frac{\mu[\phi\delta - \Pi\Delta\lambda - \Pi\mu\sigma]}{\lambda + \sigma\mu\delta}$$

The slope of the $\dot{e} = 0$ schedule is:

$$\frac{-\gamma}{\Delta} = \frac{-\phi\mu\delta}{1 - \phi\mu\delta} \qquad \text{(A4)}$$

This will be upward sloping if $1- \phi\mu\delta < 0$, $\mu \equiv 1/1-\gamma$, which is the case where the exchange rate will not overshoot in response to an unanticipated increase in the money supply. It is easy to show that the slope of the $\dot{p} = 0$ schedule simplifies to $\frac{\delta\lambda}{\delta\lambda + \sigma}$, which is the same as in the case where output is fixed.

The various exercises performed in the text are solved in an identical manner for this case. Simply substitute the characteristic roots and vectors of (A1) in place of the corresponding elements. For example, in the basic model where output is fixed and only domestic goods prices enter into the deflator for real money balances, the elements corresponding to J1 and J2 are $\lambda\Theta_1$ and $\lambda\Theta_2$, respectively.

Once two initial conditions are specified, we can again solve for the two arbitrary constants, c1 and c2. This completely specifies the system. $\dot{e}$ and $\dot{p}$ can be found by differentiating equations A3 and A2 respectively with respect to time, t. $y(t)$ is given by:

$$y(t) - \bar{y} = v\delta(e_t - \bar{e}) - v(\delta + \frac{\sigma}{\lambda})(p_t - \bar{p}) \qquad \text{(A5)}$$

where $v \equiv \frac{\lambda\mu}{\lambda + \mu\delta\phi}$

$$\dot{y} = v\delta\dot{e} - v(\delta + \frac{\sigma}{\lambda})\dot{p} \quad .$$

$$\delta\dot{e} = (\delta + \frac{\sigma}{\lambda})\dot{p} \quad \text{along } \dot{y} = 0. \qquad \text{(A6)}$$

In response to a long anticipated increase in the money supply, the system asymptotes to J1, the characteristic root vector corresponding to the unstable root. There $\delta\dot{e} > (\delta + \frac{\sigma}{\lambda})\dot{p}$, so $\dot{y} > 0$. After the initial jump in response to the jump in the exchange rate, output may initially fall if $(\delta + \frac{\sigma}{\lambda})\dot{p} > \delta\dot{e}$. When the money supply actually increases, output falls monotonically back to $\bar{y}$.

An anticipated increase in the money supply with short run adjustment in output. Case where overshooting does not occur.
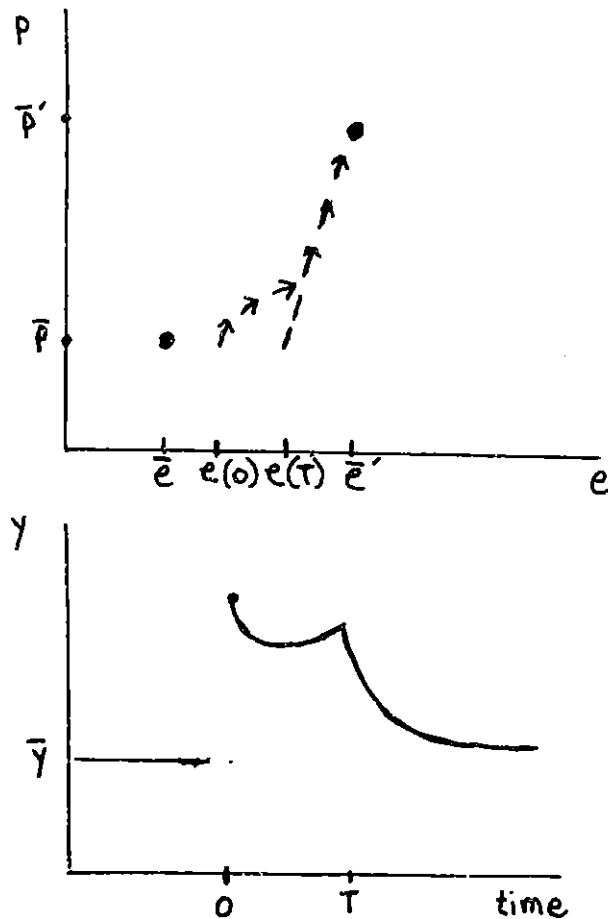
Part II

The general solution to the model of the text is given by equations 11 and 12. (In the case with short run adjustment in output, equations A2 and A3 of the Appendix give the general solution.) To do the various exercises in the text, it is only necessary to solve for the arbitrary constants, which will change whenever a shock actually hits the system.

$$(p_t - \bar{p}) = c1K1e^{\Theta_1 t} + c2K2e^{\Theta_2 t} \tag{11}$$

$$(e_t - \bar{e}) = c1e^{\Theta_1 t} + c2e^{\Theta_2 t}, \tag{12}$$

where $K_i = \dfrac{\lambda\Theta_i - (1 - \alpha)}{\alpha}$

Recall that $\Theta_1 > 0$, and $\Theta_2 < 0$. By imposing the assumption that the economy eventually reaches the final long run equilibrium, we obtain the condition that c1 = 0, after time T when the last shock takes place. Multiply (12) by K2 and subtract it from (11) to obtain:

$$(p_t - \bar{p}') = K2(e_t - \bar{e}') , \quad t \geq T . \tag{A7}$$

At the same points where c1 and c2 change, $\bar{p}$ and $\bar{e}$ also change. Here, $\bar{p}'$ and $\bar{e}'$ are the long run equilibria after all the shocks have taken place.

To solve for c1 in the period prior to T, subtract K2 times (12) from (11) to obtain:

$$(p_t - \bar{p}) - K2(e_t - \bar{e}) = c1(K1 - K2)e^{\Theta_1 t} . \tag{A8}$$

Expand equation A7 by substituting $\bar{p}' = \bar{p} + \bar{\bar{p}}$, $\bar{e}' = \bar{e} + \bar{\bar{e}}$.

Substitute into (A8) to obtain $c1 = \frac{(\Delta\bar{p} - K2\Delta\bar{e})}{(K1 - K2)} e^{-\theta_1 t}$ by eval-

uating both equations at time T, and noting that they must meet

because no anticipated jumps are possible. In the case of a single

anticipated shock, we obtain our other initial condition by eval-

uating (11) at time 0, so that $c2 = -c1(K1/K2)$. We can now find

the exact point where (A7) and (A8) meet by evaluating (11) and (12)

at time T, using c1, c2, $(p - \bar{p})$, $(e_T - \bar{e})$. To do the case with

variable output, use J1 and J2 and the roots define on the first

page of the Appendix in place of K1, K2, etc. We can find the term

structure of exchange rates by differentiating (12) with respect to

time.

The case of a transitory change is analogous. This time

$\bar{p}'$ and $\bar{e}'$ are relevant in the transition period, and $\bar{p}$, $\bar{e}$ in the

long run. c1 is the negative of its value in the previous case.

Noting that the price level is at $\bar{p}$ at time 0, $c2 = \frac{-\Delta\bar{p}}{K2} - \frac{c1K1}{K2}$.

Remember that for money supply shocks, $\Delta\bar{p} = \Delta\bar{e} = \Delta m$.

The solution algorithm in the general case where many

shocks are anticipated is identical. Work backwards from the

terminal condition analogous to (A7). Solve backwards using the

transversality conditions and find the initial exchange rate by

noting that the price level at time 0 will be equal to the initial

equilibrium. In the process, we will have solved for c1 in each

regime. Now that we have c2 for the first regime, we can solve

forwards for all the remaining c2's and we will have completely

specified the path of the system. With two anticipated shocks

$(\Delta 1\bar{e}, \Delta 1\bar{p})$, $(\Delta 2\bar{e}, \Delta 2\bar{p})$,

$$c1'' = 0 \quad , \quad t \geq T + K$$

$$c1 = \frac{(\Delta 1\bar{p} - K2\Delta 1\bar{e})e^{-\Theta_1 T} + (\Delta 2\bar{p} - K2\Delta 2\bar{e})e^{-\Theta_1(T+K)}}{K1 - K2} \quad ,$$

$$c2 = \frac{-c1K1}{K2} \quad , \quad 0 \leq t < T \quad .$$

$$c1' = \frac{(\Delta 2\bar{p} - K2\Delta 2\bar{e})e^{-\Theta_1(T+K)}}{K1 - K2} \quad ,$$

$$c2 = \frac{1}{K1-K2} \left[(\Delta 1\bar{p} - K2\Delta 1\bar{e})e^{-\Theta_2 T} - \right.$$

$$\frac{K1}{K2}\left((\Delta 1\bar{p} - K2\Delta 1\bar{e})e^{-\Theta_1 T} + (\Delta 2\bar{p} - K2\Delta 2\bar{e})e^{-\Theta_1(T+K)}\right) \right] -$$

$$\Delta 1\bar{e}\, e^{-\Theta_2 T} \quad , \quad T \leq t \leq T+K$$

The decomposition theorem of the text may be obtained by simple substitution. It results from the linearity of the system. The general case may be solved by induction.

CHAPTER III

THE ROLE OF THE EXPECTATIONS-AUGMENTED
PHILLIPS CURVE AND THE ASSUMPTION OF
STRICTLY CONTINUOS DOMESTIC PRICE
MOVEMENTS IN DORNBUSCH-FRANKEL MODELS
OF EXCHANGE RATE DYNAMICS

Chapter III

THE ROLE OF THE EXPECTATIONS-AUGMENTED PHILLIPS

CURVE AND THE ASSUMPTION OF STRICTLY CONTINUOUS

DOMESTIC PRICE MOVEMENTS IN DORNBUSCH-FRANKEL MODELS

OF EXCHANGE-RATE DYNAMICS

Introduction

This paper introduces a new notion of "sticky rational expecta-
tions," which can arise in Dornbusch-type models of price and
exchange rate dynamics. It is shown that in rational expectations
solutions to these models, current levels of the endogenous variables
may be expressed as weighted averages of past and future behavior
of the exogenous variables.

When the economy experiences an unanticipated shock, adjustment
to the new long-run equilibrium will not necessarily be immediate
even when the price level and the exchange rate can both respond
discontinuously. Complete adjustment will only occur immediately
if either:

a.) Agents "forget" historical values of the exogenous
variables and form expectations as if they had always experienced
the new long-run equilibrium, or

b   Agents assume that the goods market will clear instantane-
ously.

Sticky rational expectations equilibria arise when agents use observed historical levels of exogenous variables in evaluating the retrospective component of current endogenous variables, and expected future values of the exogenous variables in the forward looking component.

Sticky rational expectations can lead to some overshooting even in the absence of continuous domestic price adjustment. The standard overshooting result which occurs when the domestic price level cannot "jump" even unanticipatedly can be decomposed into overshooting due to sticy prices.

Dornbusch-Frankel models employ a price equation which adjusts for disequilibrium in the goods market and that rate of price adjustment necessary to keep the economy in a steady state if it is already there. The second section generalizes Frankel's model, where the only expected change is a constant growth rate of the money supply, to the case where relative incomes, interest rates, and money supplies follow homogenously nonstationary processes. For empirical work with reduced form exchange rate equations, it is necessary to use a procedure which involves estimating the pattern of the exogenous variables.

The final section briefly considers issues arising when uncertainty is explicitly incorporated into the model.

I. Backward and Forward Solutions in Rational Expectations Models of Exchange Rate Determination.

This section shows how the celebrated "overshooting" result can occur even in a world where price movements can respond discontinuously to exogenous shocks. This is important because large price movements often accompany large exogenous shocks, even in economies experiencing gradual price adjustment in response to excess demand.

Because agents live in an economy which often experiences excess demand or excess supply, and have learned to expect prices to respond slowly to these "normal" conditions, it is not surprising that there exist expectational equilibriums which involve market disequilibrium. We will take a simplified version of the Dornbusch model used in the previous chapter, and show that the usual forward solution involves a special additional mathematical and economic assumption. The point is best illustrated by dropping Dornbusch's assumption of temporarily fixed prices and freely flexible exchange rates. Instead, we allow both the exchange rate and the price level to respond to exogenous shocks. By taking the full-blown backwards and forwards solution to this problem, we see that the universal presumption that the exchange rate and price level would immediately adjust to their respective equilibrium levels need not be correct.[1] It will only be true under the additional constraint that market participants assume that all markets clear whenever the price level and exchange rate are allowed to jump. Relaxing this assumption

permits a rational-expectations equilibrium where prices do not instantaneously respond fully to the shock. It is the retrospective, or backward looking, component of expectations which leads to some overshooting of the exchange rate.

The backward looking component of the rational expectations solution for the current levels of prices and the exchange rate is generated by the sticky price equation (assumption). Burmeister, Flood and Turnovsky (1978) point out that pure forward looking expectations are not an automatic consequence of the rational expectations hypothesis as it is usually employed:

> "-in this stable case, even <u>certain</u> information concerning future changes in exogenous variables does not affect current behavior, in contrast to the apparently widespread belief that rational expectations require expected future changes to be reflected in current behavior. However, this belief is founded upon the properties of 'inherently unstable' models."

Blanchard (1979) has emphasized the fragility of the logic of rational expectations models which impose the assumption of pure forward looking agents. He presents the monetary model:

$$(a) p_t = (1/1+\alpha) m_t + (\alpha/1+\alpha)_t p_{t+1} \text{ , where } _t p_{t+1} = E_t (P_{t+1})$$

He considers various rationales for using the forward solution to the above equation:

"-, a related argument can be disposed of. The argument runs as follows:

Equation (a) only includes p_t, $_tp_{t+1}$, and m_t, none of them necessarily depending on the past. It is therefore hard to see why the past should affect either p_t or $_tp_{t+1}$. This implies the forward solution should be chosen.

Equation (a), however is an incomplete description of the economy without an explicit expectation mechanism. If agents assume that p_t depends on past m_t's in the way indicated by equation (b)," -(which has a forward and backward component) -"then they will be rational and p_t will depend on past m_t's."

The argument that the pure forward solution must be "optimal"

is seen to be tautological in the context of these macro-models.

Here we argue that there is an isomorphism between models with sticky

prices and models with sticky expectations.

By adding the assumption that prices cannot move discontinuously

in response to shocks, we find that the standard overshooting result

can be decomposed into a component which is due to sticky expecta-

tions, and a component which is due to sticky prices. Under cer-

tainty, the subsequent movement to equilibrium follows the usual

path. Now, however, movement along this path can be attributed both

to the gradual adjustment of the expectational equilibrium, and to

movement which is the result of the initial sticky price level.

While many writers have given models where real effects of monetary

shocks result from either adaptive expectations or sticky prices, it

is interesting how this model can contain sticky expectations only

because sticky prices give rise to a partly backwards-looking solution

which remains even after we use up the assumption of convergent expectations.

Later in this section, we will reconsider a modified Dornbusch model as proposed by Frankel, which assumes a world of constant inflation. Here too, we find an alternative rational-expectations path. This example is then easily generalized.

We will use a model similar to that of the second chapter.

I.1: The Model

(1) $m-p=-\lambda r+\phi y$ Money market equilibrium

(2) $r=r^*+x$ Interest parity condition

(3) $x=\dot{e}$ Rational expectations

(4) $\dot{p}=\delta(e-p)+\ell(m,y,r^*,e,p)$ Expectations-augmented

Phillips curve.

goods market disequilibrium (excess demand)

(5) $(\dot{e}, \dot{p})=(\frac{de}{dt}, \frac{dp}{dt})$

m, p, y are the logrithms of the domestic money supply, the domestic price level, and real income. r and r^* are the respective levels of the domestic and foreign nominal-interest rates. Domestic and foreign bonds are perfect substitutes, and under perfect-capital markets, the home country produces a good domestically, which is an imperfect substitute for the imported good. e is the log of the

exchange rate, defined as the domestic price of foreign currency. x is the expected rate of depreciation.

In equation (3) we impose rational expectations directly. A common technique is to assume expectations about the exchange rate are adaptive. Dornbusch and Frankel give models where these expectations may coincide with rational expectations. This solution technique conceals the existence of the backwards-looking solution, and thus we do not employ it here. Equation (4) is the expectations-augmented Phillips curve. Here we assume that excess demand depends only on the terms of trade. We have suppressed the foreign price level. It would be possible to allow an equilibrium terms of trade that varies over time by introducing an exogenous variable into the excess demand term in (4). See Mussa (1976). We have chosen this model for maximum simplicity. For the moment we will set the $\dot{z}$ term equal to zero. In Dornbusch's model, the exogenous variables are constant in the steady state. The $\dot{z}$ term will be of crucial significance when we look at more general Dornbusch models. Finally, we will assume income is exogenous, which allows us to make our main points while greatly simplifying the mathematics.[2]

I. 2 Deriving the General Backwards and Forwards Solution

When the exogenous variables, y, r^*, m are constant, we can solve for the equilibrium exchange rate and price level by imposing the condition that $\dot{p}=\dot{e}=0$. Thus:

(6) $$\overline{p}=\overline{e}=\overline{m}+\lambda r^*+\phi y$$

We immediately see that the exchange rate and price level are proportional in the long run to the money supply.

$$\Delta\overline{e}=\Delta\overline{m}=\Delta\overline{p}$$

We purposefully omit Dornbusch and Frankel's technique of expanding these already log-linear equations around their long-run steady-state equilibria, and directly set down the set of simultaneous first-order differential equations for e and p.

(7) $$\dot{p}=\delta(e-p)$$ (Setting $\dot{z}$ equal to zero)

(8) $$\dot{e}=1/_\lambda p - 1/_\lambda (m+\lambda r^*-\phi y)$$

The roots of the homogenous part of these equations are given by:

$$\theta_1=-\delta/2+ \left(\delta^2+4\delta/_\lambda\right)^{1/2} /2 \quad >0$$

$$\theta_2=\delta/2- \left(\delta^2+4\delta/_\lambda\right)^{1/2} /2 \quad <0$$

The equilibrium is a strict saddle point. (These roots are the same as in Frankel's adaptation of the model.)

Therefore, the general solution to the homogenous equations is given by:

$$p_t = c1\lambda\theta1e^{\theta1t} \quad + \quad c2\lambda\theta2e^{\theta2t}$$

$$e_t = c1e^{\theta1t} \quad + \quad c2e^{\theta2t}$$

where $\frac{\delta}{\delta+\theta_i} = \lambda\theta_i$ are the characteristic vectors.

The crucial step comes in finding the particular solution given by equations (1-4). This is done in the appendix, which draws heavily on Burmeister, Flood and Turnovsky (1977) for correct application of the convergent-expectations assumption. We also use the assumption of convergent expectations in order to apply the restriction $c1=0$, so the economy chooses the stable branch. This assumption is discussed in the previous chapter. In solving his single equation model with one positive characteristic root, Blanchard employs the method of undetermined coefficients. Here, without assuming specific forms for the behavior for m, r^*, and y, it is necessary to solve this type of system by means of the method of variation of parameters. Later we will find it convenient to solve for each variable by a single second-order differential equation. The general solution to equations (1-4), as derived in the appendix, is given below.

$$(9) \qquad p_t = c1 \cdot \lambda \theta 1 e^{\theta 1 t} + c2 \cdot \lambda \theta_2^{\theta 2 t} + \lambda \theta 1 v1 + \lambda \theta 2 v2$$

$$(10) \qquad e_t = c1 \cdot e^{\theta 1 t} + c1 \cdot e^{\theta 2 t} + v1 + v2$$

where we can use the assumption of convergent expectations to set one of the arbitrary constants, $c1$, equal to zero. $v1$ and $v2$ are given by:

$$v1 = \frac{1}{\lambda(\theta 1 - \theta 2)} \int_{+\infty}^{t} \theta 2 (m + \lambda r^* - \phi y) e^{\theta 1 (t-s)} \, ds$$

$$v2 = \frac{1}{\lambda(\theta 1 - \theta 2)} \int_{-\infty}^{t} -\theta 1 (m + \lambda r^* - \phi y) e^{\theta 2 (t-s)} \, ds$$

Let

$$\lambda \theta 1 v1 + \lambda \theta 2 v2 \equiv \tilde{p} \quad , \qquad v1 + v2 \equiv \tilde{e}$$

We have determined the appropriate bounds of integration by imposing the assumption of stationary expectations. Under certainty, this solves the backward-forward problem for each particular root. The integral associated with the negative root looks backward in time, and the integral associated with the positive root looks forward in time. In Blanchard, part I, the solution only as a single positive root, so backwards-looking expectations can be ignored on the equilibrium path under certainty. Mussa (1978) gives a

rational-expectations model with both a positive root and a negative root. In his model, the backward solution disappears. The backward root plays an important role in his solution when discontinuous price movements are ruled out.

In the modified Dornbusch model, we define the sticky expectations equilibrium $(\tilde{e}, \tilde{p})$ as the solution to (9) and (10) which imposes the initial conditions c_1, c_2 equal to zero. Prices and exchange rates move (perhaps discontinuously in response to discontinuous shocks) to the current perceived equilibrium value.

(11) $$\tilde{p}_t \equiv \lambda \theta v_1 + \lambda \theta v_2$$

(12) $$\tilde{e}_t \equiv v_1 + v_2$$

I.3. A Money Supply Shock and the Classic Overshooting Exercise.

Note that agents are fully aware of current, future and past levels of the exogenous variables, and in this world of sticky prices, they rationally believe that the current levels of the endogenous variables are determined by all three considerations. The solution given did not presuppose that m is constant. We assume now that m is occasionally subjected to random shocks with mean zero. Thus we can comfortably retain the form of the Phillips curve with $\dot{z}=0$.

In the original Dornbusch overshooting exercise, it is assumed that m, r^* and y have no trend and that e_t and p_t have fully adjusted to their long run equilibrium values $\bar{e}$, $\bar{p}$. $\bar{p}=\bar{m}+\lambda r^*-\phi y=\bar{e}$

There is then an unexpected permanent increase in m to m', so $m'=m+\Delta m$. Now $\bar{e}(m')=\bar{p}(m')=\bar{m}+\lambda r^*-\phi y$. By expanding the differential equations around their long-run equilibrium values, he essentially finds:

$$p_t=c_1\lambda\theta 1e^{\theta 1t}+c_2\lambda\theta 2e^{\theta 2t}+p'(m')$$

$$e_t=c_1e^{\theta 1t}+c_2e^{\theta 2t}+e'(m')$$

Setting c_1 equal to zero by convergent expectations, and by imposing the second initial condition $p(t)=\bar{p}(m)$, he obtains the overshooting result $e(t)=\bar{e}'(m')\frac{1}{\lambda\theta 2}$ which is greater than $\bar{e}$, since $\theta 2$ is negative.

We will repeat this experiment using our more general solutions (9) and (10). In Appendix II, certain intuitively plausible properties of those solutions are examined. In particular, it is demonstrated that when the exogenous variables remain constant for long periods, then the sticky expectations equilibria converge to the usual long-run equilibrium given by Dornbusch.

Now take the Dornbusch exercise of an unanticipated money-supply shock which takes place at time t, such that $m'=m+\Delta m$, where m' is the new higher money supply. People are assumed to correctly remember

what the money supply was and to know what it is and will be. First, we relax the assumption that the price level is fixed at the time of the shock, and replace it by the assumption that it moves directly to its sticky expectations equilibrium value as defined by (11). In other words, we impose the initial condition that c equals zero.

(13) $\quad \tilde{\tilde{p}}_t = \lambda\theta 1 v1(m', r^*, y) + \lambda\theta 2 v2(m, r^*, y)$

(14) $\quad \tilde{\tilde{e}}_t = v1(m', r^*, y) + v2(m, r^*, y)$

Rational retrospective expectations evaluate v2 at the historically given m. We can expand v2 by substituting m=m'-Δm to obtain:

$$[k \gtreqless 0] \quad \tilde{\tilde{p}}_{t+k} = m' + \lambda r^* - \phi y - \frac{\theta 2}{(\theta 1 - \theta 2)} \int_{-\infty}^{t} -\theta 1 \Delta m e^{\theta 2(t-s)} \, ds \quad = \bar{p}' - \rho \Delta m$$

where

$$\rho = \frac{\theta 1}{(\theta 1 - \theta 2)} e^{\theta 2 k} \quad, \quad \text{so} \quad \bar{p} \leq \tilde{\tilde{p}}_{t+k} \leq \bar{\bar{p}}, \quad \tilde{\tilde{e}}_{t+k} = m' + \lambda r - \phi y \frac{-1}{(\theta 1 - \theta 2)\lambda} \cdot \int_{-\infty}^{t}$$

$$-\theta 1 \Delta m e^{\theta 2(t-s)} \, ds = \bar{e}' + h\Delta m$$

where

$$h = \frac{-\theta 1}{\lambda \theta 2(\theta 1 - \theta 2)} \cdot e^{\theta 2 k} \quad, \quad \text{so} \quad \tilde{\tilde{e}} \gtreqless \bar{e}' > \bar{e}$$

(15) $\quad p_{t+k} = \bar{p}' - \Delta m \frac{1}{(\theta 1 - \theta 2)} e^{\theta 2 k} \quad [k \gtreqless 0]$

(16) $(e_{t+k}-\bar{e}')=\frac{1}{\lambda\theta 2}(p_{t+k}-\bar{p}')$

In the Dornbusch exercise, $p(t)=\bar{p}$ and $e(t)=\bar{e}+\Delta m(1-\frac{1}{\lambda\theta 2})$.

Since $m+\Delta m=m'$, we see that the price level rises initially, but by less than the change in m, and that the exchange rate <u>overshoots</u> its new equilibrium, but by <u>less</u> than the amount it would rise if the price level could not jump freely.

Over time, as people observe the higher money supply over a period of time, the expectational equilibrium price level adjusts:

(17) $\bar{\overset{\approx}{p}}_{t+k}=\frac{\Delta m\delta/\lambda}{(\theta 1-\theta 2)}\ e^{\theta 1 k}>0 \quad [k\overset{\geq}{=}0]$

(18) $\dot{\overset{\approx}{e}}=\frac{1}{\lambda\theta 2}\dot{\overset{\approx}{p}}<0$

We also note that $\dot{\overset{\approx}{e}}=\theta 2(\overset{\approx}{e}-\bar{e}')$, so that adaptive expectations are rational.

Because people have always observed the old money supply, they do not immediately move to a point where the price level rises enough to keep real-money balances unchanged. The exchange rate must adjust to overshoot its long-run equilibrium. This happens since higher real-money balances lower the domestic nominal interest rate. It is readily seen that the solution where p and e jump immediately to their long-run equilibrium values happens only if people "forget" that the money supply was ever anything other than the new higher

m', and substitute that value into the backward integrals. With fully flexible prices, there is one other assumption which would bring p and e immediately and permanently to their long-run equilibrium values. That is the assumption that markets must clear instantaneously if prices are fully flexible. If this is what people think, then they will set c2 in equations (9) and (10) so that

$$c2^* = \frac{\theta 1}{\lambda \theta 2 (\theta 1 - \theta 2)} \, \Delta m \cdot e^{-\theta 2 t} \quad \text{so that for } t+k \gtreqless t$$

(19) $\quad p_{t+k} = \lambda \theta c2^* e^{\theta 2 k} + \tilde{\tilde{p}} = \bar{p}'$ for all $t+k \gtreqless t$

(20) $\quad e_{t+k} = c2^* e^{\theta 2 k} + \tilde{\tilde{e}} = \bar{e}'$ for all $t+k \gtreqless t$

As expectations adjust, this added term will cancel out at the same rate, and the economy will remain at the new long-run equilibrium.

To reach the expectational equilibrium after a money shock, we have assumed that prices and exchange rates can both jump discontinuously in response to unanticipated shocks. We have assumed convergent expectations, so that in the long run the shock has no real effects. But in order to move immediately to the new long-run equilibrium, we must make the additional assumption that either people forget what the money supply used to be or that they are convinced that markets clear instantaneously when prices can jump. The first assumption is

clearly unsatisfactory. The alternative is not unreasonable, but it is not compelling. Agents live in an economy which may often experience disequilibrium, since it has slow price adjustment. The alternative path we propose is reasonable both economically and mathematically.

Both the exchange rate and the price level are formed from a weighted average of the forward-looking integral and the backward-looking integral in the current expectational equilibrium solution. Naturally, the relative weight of the forward integral is inversely proportional to θ_1/θ_2. That is because the positive root serves as the forward discount factor, and the negative root serves as the backward discount factor. The more sticky prices are, and the more persistent goods-market disequilibrium tends to be (i.e., the smaller δ) the greater the weight on the backward solution. As δ becomes large, the backward-looking integral begins to have a very small weight. People do not expect much disequilibrium then.

Now let us add the assumption that prices cannot jump discontinuously. We set $c1=o$ by convergent expectations, and solve for $\hat{c}2$ by the assumption $p(t)=\bar{p}$:

(21) $$p_{(t)}=\bar{p}=\hat{c}2\lambda\theta2e^{\theta2t}+\tilde{p}_t \rightarrow \hat{c}2=[-\Delta m\frac{+\theta1\Delta m}{\lambda\theta2(\theta1-\theta2)}]e^{-\theta2t}$$

It follows readily that $e_t=\hat{c}2e^{\theta2t}+\tilde{e}=\bar{e}+\Delta m(1-\frac{1}{\lambda\theta2})$ which is

precisely the standard overshooting result. (Recall $\bar{e}=\bar{e}'-\Delta m$)

Furthermore:

(22) $\qquad [k\overset{\geq}{=}0], \dot{e}_{t+k}=\theta 2\hat{c}2e^{\theta 2}+\dot{\bar{e}}_{t+k} =\theta 2(e_{t+k}-\bar{e}')$

The first term represents the overshooting due to prices which not only adjust normally, but cannot more discontinuously even in response to discontinuous shocks. The second term represents overshooting due to sticky, but rational, expectations. Movement towards the long-run equilibrium can be decomposed into the revisions of the current expectational equilibrium exchange rate, and to adjustment from "pure" overshooting due to continuous price adjustment. Diagramatically:

FIGURE I : The position of the economy immediately following an
unanticipated increase in the money supply.

Price of
domestic
good

New long run equili-
brium with higher m

C

$\bar{p}'(m + \Delta m)$

Δm

Old long run
equilibrium with
lower money
supply

B $(\tilde{e}, \tilde{p})$

Sticky expectations equili-
brium

$\bar{p}(m)$

E

A

Standard overshooting
temporary equilibrium , when
domestic price level cannot
jump in response to shock

45°

$\bar{e}(\bar{m})$

Δm

$\bar{e}(\bar{m} + \Delta m)$

Exchange rate
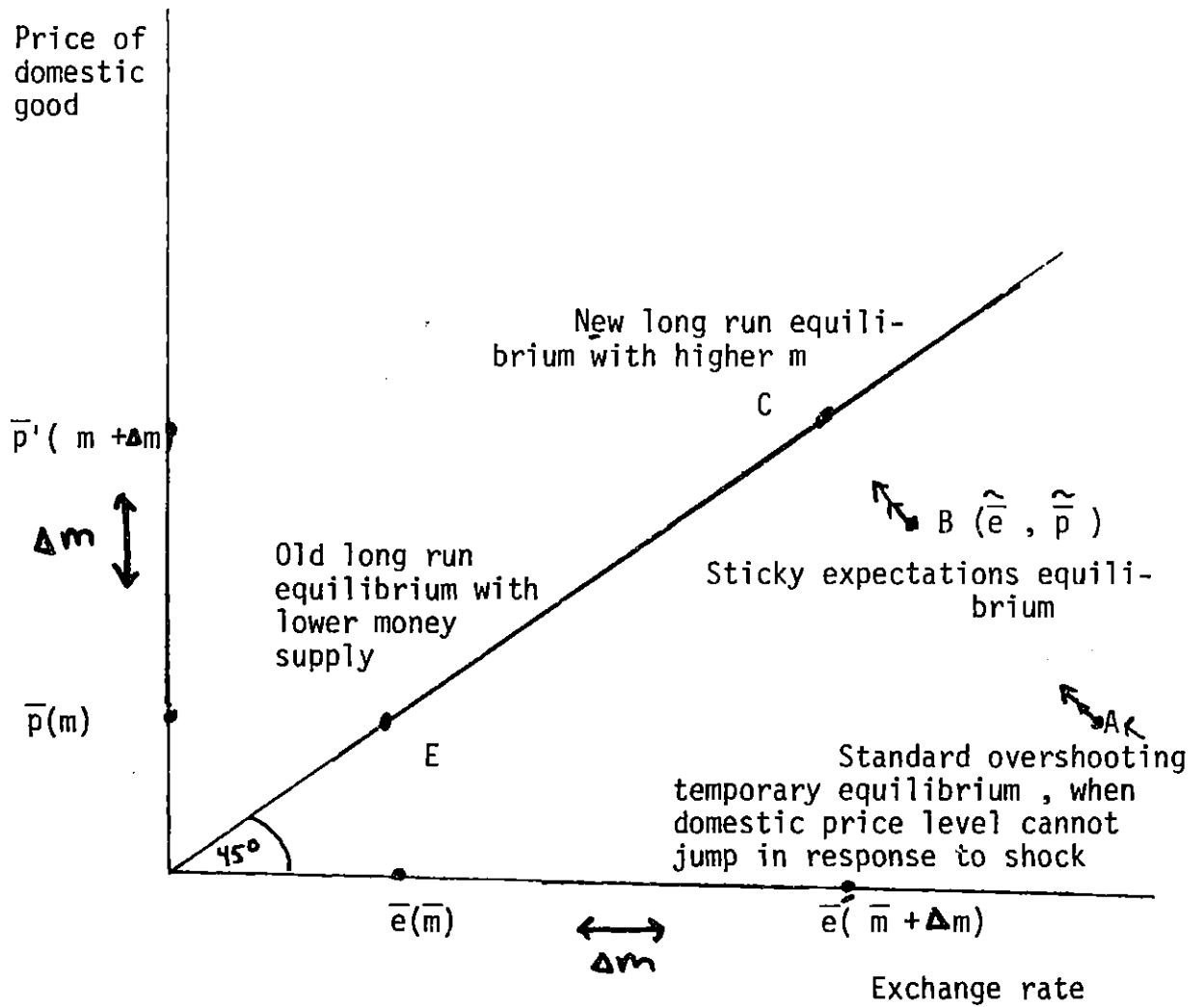
C is the point reached in the long run where both the goods market and money market are in quasi-equilibrium (the money market is always in equilibrium by assumption). B is the point of current expectational equilibrium. A is where the economy moves in the absence of discontinuous price movements. At A, there is adjustment towards B. At B, there is adjustment towards C. When the initial price level is given, then overall adjustment sums the two factors, which both orient the economy to expect a rising price level and an appreciating exchange rate.

It is a simple matter to show that the weight on the forward solution relative to the backward solution depends on the ratio of the negative root to the positive root. This in turn depends on δ and λ. When δ is very large, B is very close to C, the long-run equilibrium, as agents attach very little weight to the past. Even when δ is very small, B must lie at least halfway towards C from A. Thus, even when agents know that the goods market adjusts very slowly, and thus put a large weight on the old, lower money supply, <u>overshooting due to sticky expectations can never be even half as large as overshooting due to inflexible prices.</u> These propositions are proved below:

From (13), (14), and $\bar{m}'=m(s)_{s \geq t}, \bar{\bar{m}}=m(s)_{s<t}$

$$(23) \qquad \tilde{\tilde{e}} = \frac{\theta 2}{\theta 1} \cdot k\bar{m}' + \frac{\theta 1}{\theta 2} \cdot k\bar{\bar{m}} + \lambda r^* - \phi y$$

(24) $\quad \tilde{p}=-\lambda\theta2\cdot\overline{km}'+\lambda\theta1\cdot\overline{km}'+\lambda r^{*}-\phi y$ where $k=\frac{1}{(\theta1-\theta2)}>0$

So the weight on $\overline{m}'$ at point B is inversely proportional to $\theta1/_{\theta2}$. From (7), (8) and (15)

(15') $\quad \tilde{p}_t-\overline{p}\frac{\theta1\Delta m}{(1\theta-\theta2)}=\Delta m\ (\frac{-\delta+\left(\delta^2-4^\delta/_\lambda\right)^{1/2}}{-2\cdot\left(\delta^2-4^\delta/\lambda\right)^{1/2}})=\Delta mJ$

fixing λ, as $\delta\to\infty$, $J\to0$

fixing δ, as $\lambda\to\infty$, $J\to0$, and as $\lambda\to0$, $J\to1/2$

From (16):$[(16)'\ (\tilde{e}_{t+k}-\overline{e}')=1/_{\lambda\theta2}(\tilde{p}_{t+k}-\overline{p})]$ and (15'), we have

(25) $\quad 0\leq\tilde{e}_t-\overline{e}\frac{-\Delta m}{\lambda\theta2}\cdot1/2$

There is a fundamental intuitive interpretation of the expectational equilibrium, $(\tilde{e}$, $\tilde{p})$. It corresponds to imposing the initial condition that the exchange rate jump to the point it would h ve been at, had the shock been anticipated infinitely far in advance. In the previous chapter, we demonstrated that for a model with the price equation used here, where price adjustment corresponds only to current goods market disequilibrium, then a long anticipated money shock will always have real effects even after it takes place. Alternatively, it will always take the economy a finite amount of time after the shock actually takes place to move within any given

neighborhood of the equilibrium.

The expectational equilibrium could occur as follows. The perfectly rational participants in the exchange market ask themselves the question: "Where would the exchange be now if we hadn't been surprised by this money supply increase?" If they look at the way price adjustment would have occurred in the past, i.e., they take the price equation as given, then they will decide that the exchange rate should move to $\tilde{e}$, the expectational equilibrium. If instead, they assume that price adjustment would have been just as forward looking as they would have been, then the economy could move directly to the new long run equilibrium.

The expectational equilibrium comes about when exchange market participants think backwards about what a sensible level for the exchange rate would be. $(\tilde{e}, \tilde{p})$ also has the virtue that it will converge to the old equilibrium if agents think back in time, and to the new equilibrium if they think forwards. (The market clearing solution is unstable "backwards.")

I.4 The Dornbusch-Frankel Model with Steady State Money Growth.

We now extend the sticky-expectations interpretation to a more general class of Dornbusch models. Consider the Dornbusch-Frankel model where $\dot{z}=\dot{m}$, where $\dot{m}$ is the steady-state rate of change in the money supply. So the Phillips curve now becomes:

(26) $\dot{p}=\delta(e-p)+\dot{m}$

$\dot{p}$ now contains a term which reflects disequilibrium adjustment due to excess demand, and the adjustment which would be necessary to keep the economy in a steady state if it was already there. In the appendix we show that the solution to the model is now:

(27) $\qquad p_t = c1\lambda\theta1e^{\theta1t} + c2\lambda\theta2e^{\theta2t} + \tilde{\tilde{p}}_t$

(28) $\qquad e_t = c1e^{\theta1t} + c2e^{\theta2t} + \tilde{\tilde{e}}_t$

(27') where $\qquad \tilde{\tilde{p}}_t = \lambda\theta1v1 + \lambda\theta2v2$

(28') $\qquad \tilde{\tilde{e}}_t = v1 + v2$

and

$$v1 = \frac{1}{\lambda(\theta1-\theta2)} \int_{\infty}^{t} [\dot{m}_t + \theta2(m+\lambda r^* - \phi y)_t]e^{\theta1(t-s)} \, ds$$

$$v2 = \frac{-1}{\lambda(\theta1-\theta2)} \int_{-\infty}^{t} [\dot{m}_t + \theta1(m+\lambda r^* - \phi y)_t]e^{\theta1(t-s)} \, ds$$

The initial conditions, c1=0 and c2=0, move the economy to the current expectational equilibrium $(e_t, p_t) = (\tilde{\tilde{e}}_t, \tilde{\tilde{p}}_t)$.

In appendix II, it is demonstrated that when r^*, y are constant and m, the rate of money growth, is constant, then the sticky expectations equilibrium is the same as the forward looking long run equilibrium.

Consider now Frankel's experiment of an unanticipated increase in $\dot{m}$. We will not impose his assumption that the price level cannot respond discontinuously. Once again there exists a sticky expectations equilibrium, ($\tilde{\tilde{e}}$, $\tilde{\tilde{p}}$), where p_t rises, but not all the way to its higher equilibrium value, and the exchange rate overshoots its moving equilibrium. As before, the sticky expectations equilibrium represents the same point the economy would be at if the new money growth rate had been announced very far in advance. (Given the imperfection in the price equation, which only adjusts in response to current goods market disequilibrium, and current money supply growth.) Setting (c1, c2)=(0, 0) in (27) and (28):

$$(29) \qquad \tilde{\tilde{e}}_t = m + \lambda(r^* + \dot{m}') - \phi y - \frac{\Delta\dot{m}}{\lambda(\theta 1 - \theta 2)} \int_{-\infty}^{t} -(1 + \frac{\theta 1}{\theta 2}) e^{\theta 2(t-s)} \, ds$$

where $\dot{m}' = \dot{m} + \Delta\dot{m}$, so $\Delta\dot{m}$ is the unanticipated permanent increase in the rate of monetary growth which takes place at time t.

$$\tilde{\tilde{e}}_{t+k} = \overline{e}' - \frac{\Delta\dot{m}}{\lambda\theta 2(\theta 1 - \theta 2)} (1 + \frac{\theta 1}{\theta 2}) e^{\theta 2 k} > \overline{e}', \text{ since } \left|\frac{\theta 1}{\theta 2}\right| < 1$$

Similarly for $k \geqq 0$

(29') $\tilde{\tilde{p}}_{t+k} = \bar{p}' - \frac{\Delta \dot{m}}{(\theta 1 - \theta 2)} (1 + \frac{\theta 1}{\theta 2}) e^{\theta 2k} < \bar{p}' = m_t + \lambda (r^* + \dot{m}') - \phi y$

Analogously with our earlier example, the economy will immediately jump to the long-run equilibrium path if we impose one of the two assumptions. Either people must forget that the inflation rate was ever lower than its new level or they must assume that markets clear instantaneously when the price can jump. The latter assumption corresponds to setting $\hat{c}2 = \frac{\Delta m}{\lambda \theta 2 (\theta 1 - \theta 2)} e^{-\theta 2t} (1 + \frac{\theta 1}{\theta 2})$ in the general solution, so that

(30) $p_{t+k} = \lambda \theta 2 \hat{c} 2 e^{\theta 2k} + \tilde{\tilde{p}}_{t+k} = \bar{p}'$

for all $k \geqq 0$

(31) $e_{t+k} = \hat{c} 2 e^{\theta 2k} + \tilde{\tilde{e}}_{t+k} = \bar{e}'$

Thus the economy remains on the long-run equilibrium path. Expectations adjust at the same rate the "market-clearing shock" dampens out. When we add instead Frankel's assumption that prices cannot move discontinuously, then

(32) $p(t) = \bar{p} = \lambda \theta 2 \hat{c} 2 e^{\theta 2t} + \tilde{p}_t$ After solving for $\hat{e}2$

(33) $\hat{e}(t) = \frac{\Delta \dot{m}}{\lambda \theta 2} (\frac{1 + \frac{\theta 1}{\theta 2}}{(\theta 1 - \theta 2)} - \lambda) + \tilde{e}_t = \bar{e} + \Delta \dot{m} (\lambda + \frac{1}{\theta 2})$

where $\quad \hat{\hat{e}}_t \geq \tilde{\tilde{e}}_t \geq m_t + \lambda(r^* + m) - \phi y = \overline{e}'$

Here $\quad \hat{\hat{e}}_t$ is the standard overshooting result.

Once again we see that the initial jump breaks down into a part due to forward- and backward-looking expectations, and a part due to the continuous price-movement restriction. We again present a diagram:

FIGURE II :   An unanticipated increase in the steady-state
inflation.
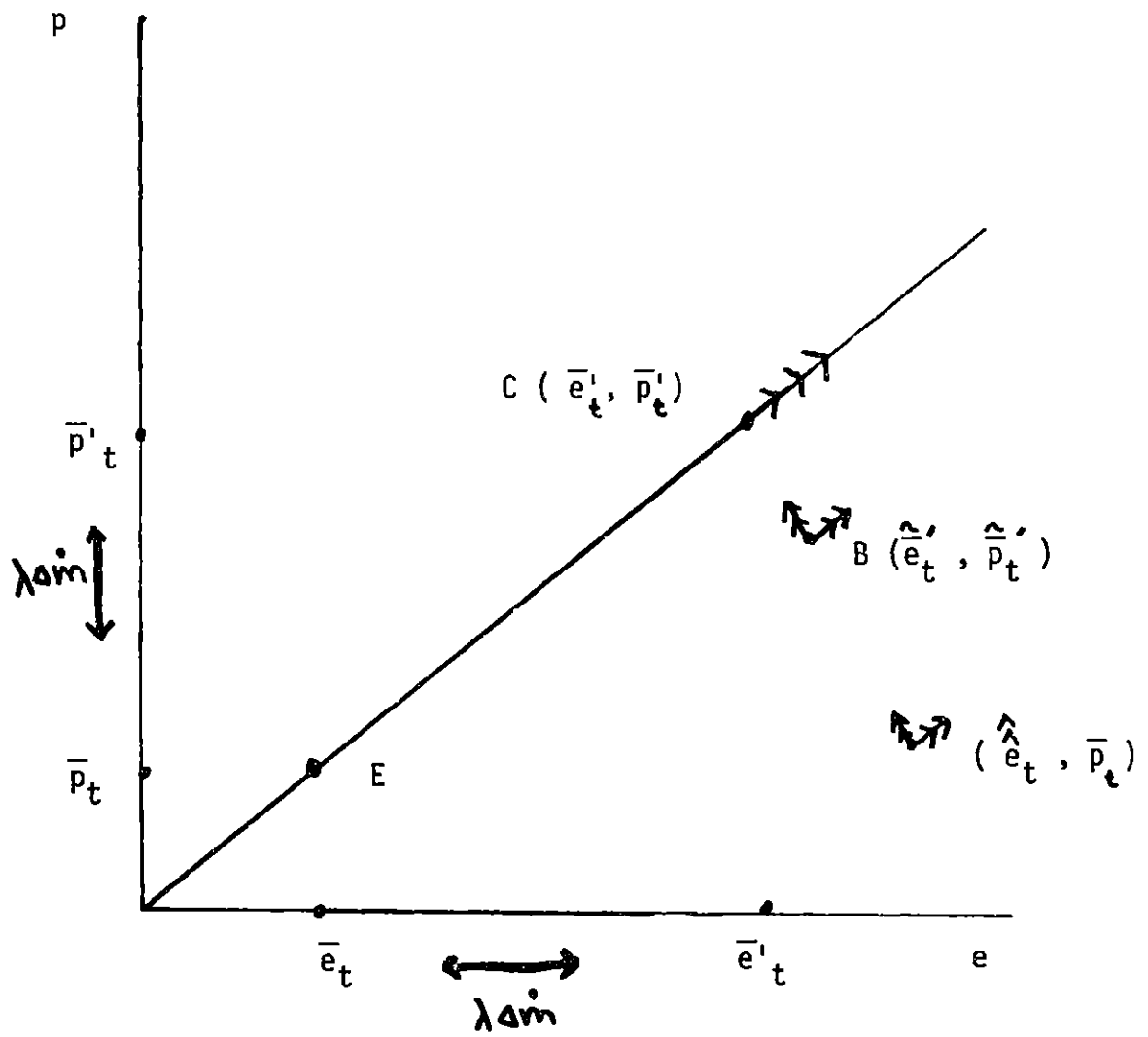
Note that at point B, the initial expectational equilibrium, people are only gradually adjusting to the effect of the higher inflation rate on the long-run equilibrium values of p and e. However, people are fully aware of m', the new higher rate of inflation, and they assume that the current expectational equilibrium also adjusts for that. To illustrate this, differentiate (31), (32):

$$(34) \qquad \dot{\tilde{e}}_{t+k} = \dot{m}' - \frac{\Delta \dot{m}}{\lambda(\theta 1 - \theta 2)} (1 + \frac{\theta 1}{\theta 2}) e^{\theta 2 k} \leq \dot{m}'$$

$$\dot{\tilde{p}}_{t+k} = \dot{m}' - \frac{\theta 2 \Delta \dot{m}}{\lambda(\theta 1 - \theta 2)} (1 + \frac{\theta 1}{\theta 2}) e^{\theta 2 k} \geq \dot{m}'$$

The first term in each equation represents the adjustment necessary to keep pace with current inflation. The second term represents expectational adjustment as people experience a higher inflation rate for a longer period.

The generalization to other Dornbusch-type models which embody steady money-supply rules is straight forward.

Frankel's model illustrates what Flood has emphasized as the proper concept of overshooting. Here the domestic inflation rate overshoots even the new higher rate of money growth. This overshooting is necessary because the demand for real balances falls in the long run because of the higher rate of inflation. This is a common feature of macro-models with a sticky adjustment equation.

It has been shown that overshooting is important in a wide class of models even in the absence of structures on discontinuous price adjustments. Overshooting from sticky, but rational, expectations is always smaller than overshooting due to temporarily fixed prices.

In the next section, we consider erratic anticipated shocks, and take a closer look at the form of the Phillips curve.

II. Expectational Adjustment and the Neutrality of Perfectly Anticipated Money.

It is a feature of the Dornbusch and Frankel models considered in the previous section that anticipated jolts to the money supply and the rate of growth of the money supply will have real effects (i.e., the terms of trade will be affected, at least temporarily). This is true even if we permit discontinuous movements in the price level and the exchange rate at the time of the announcement. The economy can follow a path such that markets will be in equilibrium forever[4] after the time of the shock, but then they will be in disequilibrium prior to the shock. (See Chapter 2.) These are perfectly sensible features of models which contain expectations-augmented Phillips curves which correct only for "normal" patterns of money-supply change. Agents are perfectly rational in the foreign-exchange market. In the goods market, however, they will only be perfectly rational when the exogenous variables follow stationary patterns through time. Later we will give the appropriate adjustment term ($\dot{z}$) for the Phillips curve for which agents will be rational in the goods market, no matter how complicated the (stationary) pattern of the exogenous variables.

That nonstationarity of the exogenous variables should permit anticipated real effects is a perfectly reasonable feature of a rational-expectations model where expectations are based on the past as well as the present and the future. Only when price expectations

of the future totally dominate the determination of the levels of

the levels of current variables will the economy remain in perpetual

equilibrium in the presence of nonstationary behavior of the exogenous

variables. (To be precise, we mean not homogenously nonstationary.)

In Mussa's model, when the domestic price level and the exchange

rate can jump, then the current levels of these two variables are

based soley on forward-looking expectations. Then even nonstationary

behavior of the exogenous variables will not jolt the goods market

out of equilibrium. Because his equilibrium model derives from the

Sargent-Wallace (1973) work, his pure forward-looking solution shares

the criticisms which Blanchard (1979) has made about that strand of

the literature. In particular, it is highly sensitive to the sign of

the parameter representing the elasticity of money demand with respect

to the inflation rate. Also it is sensitive to the assumption of

uncertainty versus certainty. For a much more thorough and detailed

discussion see Blanchard.

In spite of their limitations, the Dornbusch-Frankel literature

stands as an appealing compromise between the old adaptive-expectations

models and the newer forward-looking rational-expectations models.

This is particularly true for the backward and forward solutions to

their models presented here. As pointed out here and in Chapter II

their approach of directly assuming adaptive expectations in the

foreign exchange market is limited. This property will never hold

in the presence of nonstationary anticipated shocks, and causes the

omission of the backward solution.

Below we solve for the appropriate form of the steady-state adjustment term in the Phillips curve for the Dornbusch-class model presented earlier. The resulting Phillips curve will give the economy the property that anticipated changes in the exogenous variables will have no real effects, assuming they follow a homogenously nonstationary process. For convenience, we repeat equations (1) through (4) from Section I.

(1) $\qquad m-p=-\lambda r^* + \phi y \qquad$ Money market equilibrium

(2) $\qquad r = r^* + x \qquad$ Interest parity condition

(3) $\qquad x = \dot{e} \qquad$ Rational expectations

(4) $\qquad \dot{p} = \delta(e-p) + \dot{z}(m, y, r^*, e, p) \qquad$ Expectations-
augmented Phillips curve

$\qquad$ (e-p= goods market disequilibrium (excess demand))

$\qquad (\dot{e}, \dot{p}) = (de/dt, dp/dt)$

First we solve for $\dot{z}$, such that in certainty world, where the exogenous variables follow a set pattern, there will be no goods-market disequilibrium. This can be accomplished by equating the solutions for e and p (given in the appendix), and then solving the

resulting differential equation in $\dot{z}$. An alternative procedure is given below. This method is based on the fact that for goods-market equilibrium to hold, then e and p must follow equivalent second-order differential equations. By equating the forcing functions in these two equations, which will have identical roots, one again obtains a differential equation in $\dot{z}$.

Under certainty, solve for $\dot{z}$ s.t. $p_t = e_t$ for all t:

$$\dot{p} = \delta(e-p) + \dot{z}$$

$$\dot{e} = p/_\lambda - 1/_\lambda (m+\lambda r^* - \phi y)$$

Using D to denote the differential operator $(Dx = \dot{x})$:

$$\begin{bmatrix} -\dot{z} \\[2mm] 1/_\lambda (m+\lambda r^* - \phi y) \end{bmatrix} = \begin{bmatrix} -(\delta+D) & \delta \\[2mm] 1/_\lambda & -D \end{bmatrix} \begin{bmatrix} p \\[2mm] e \end{bmatrix}$$

This can be solved to give a second-order differential equation in either p or e. Thus,

(5) $\qquad \ddot{e} + \delta\dot{e} - \delta/_\lambda e = 1/_\lambda \dot{z} - 1/_\lambda (\delta+D)(m+\lambda r^* - \phi y)$

(6) $\qquad p + \delta\dot{p} - \delta/_\lambda p = D\dot{z} - \delta/_\lambda (m+\lambda r^* - \phi y)$

Naturally, both equations have identical roots:

$$\theta 1 = \frac{\delta}{2} + \frac{\left(\delta^2 + 4^{\delta/\lambda}\right)^{1/\lambda}}{2}$$

$$\theta 2 = \frac{\delta}{2} - \frac{\left(\delta^2 + 4^{\delta/\lambda}\right)^{1/\lambda}}{2}$$

For e and p to be equal over time, so that the goods market is always in equilibrium, we require that the right-hand sides of (5) and (6) be equal. Equating the two forcing functions and canceling terms yields:

(7) $\quad 1/_\lambda \dot{z} - \left(D/_\lambda\right)(m + \lambda r^* - \phi y) = D\dot{z}$, or

(7') $\quad (1/_\lambda - D)\dot{z} = D/_\lambda (m + \lambda r^* - \phi y)$, which is a differential equation in $\dot{z}$, the desired goods-market clearing steady-state adjustment term. The convergent-expectations assumption yields the solution:

(8) $\quad \dot{z} = 1/\lambda \int_t^\infty D(m + \lambda r^* - \phi y) e^{1/\lambda(t-s)} ds$

(8') $\quad = D/\lambda \int_t^\infty (m + \lambda r^* - \phi y) e^{1/\lambda(t-s)} ds$, where the procedure

of passing the D operator outside the integral sign can be made

rigorous by using integration by parts to evaluate (8) and (8').

The form of $\dot{z}$ should not be too surprising. It is the solution of the money-market equation for p when goods-market equilibrium (e=p) is imposed. (5) and (6) have the same backwards and forwards solutions as in Section I.

To illustrate (5) and (6), substitute $\dot{z}=\dot{m}$, as in Frankel's model:

(5') $\qquad \ddot{e}+\delta\dot{e}-(\delta/\lambda)e= -\delta/\lambda(m+\lambda r^{*}-\phi y)$ $\qquad$ (Imposing Frankel's assumption $\dot{y}=\dot{r}^{*}=0$)

(6') $\qquad \ddot{p}+\delta\dot{p}-(\delta/\lambda)p= -\delta/\lambda(m+\lambda r^{*}-\phi y)+\ddot{m}$

The Frankel model needs the assumption that the inflation rate is constant ($\ddot{m}=0$), so $e_t=p_t$ $\qquad \forall t$

In the Dornbusch-Frankel models, the $\dot{z}$ term in the Phillips curve provides the necessary price-level adjustment to keep the economy in a steady state, if it is already there. The powerful equilibrium properties of the Mussa model derive from a $\dot{z}$ term which keep the goods market in equilibrium, if it is already there. Below we derive an assumption analytically equivalent to Mussa's, though somewhat different in interpretation, and apply it to the simple model of this paper. Note that Mussa's assumption accentuates the overshooting phenomenon.

In the long run, the domestic and foreign real rates of interest will be equalized.

$$(2") \qquad r-r^*=\dot{p}$$

By substiting equation (2') for equations (2) and (3) in the model above, one can obtain a long-run money-market clearing condition:

$$(5") \qquad m-p=-\lambda(r^*+\dot{p})+\phi y$$

Given the current actual price level, p_t, we can define the real interest rate equalizing rate of inflation, $\bar{p}_t$, by:

$$(6") \qquad \bar{p}_t=\frac{1}{\lambda}p_t-\frac{1}{\lambda}(m_t+\lambda r^*-\phi y)$$

(Note that we are suppressing $\dot{p}^*$, and therefor r^* is the foreign real rate of interest.)

The Phillips curve now becomes:

$$(4") \qquad \dot{p}_t=\delta(e_t-p_t)+\bar{p}_t$$

The first term in (4') corrects for current goods-market disequilibrium, and the second term represents the adjustment necessary to keep real interest rates equalized. Actual real interest rates will of course only be equal when the domestic goods market is in equilibrium.

The Mussa technique is to take the goods-market equilibrium condition (here, e=p), and then define $\dot{p}$ by differentiating this equation with respect to time. The Phillips curve will then be the same as equation (4'). The solution to the Mussa-type model is given by:

$$(9'') \qquad e_t = c1e^{(1/\lambda)t} + c2e^{-\delta t} \quad -1/\lambda \int_\infty^t (m + \lambda r^* - \phi y)e^{(1/\lambda)(t-s)} ds$$

$$(10'') \qquad p_t = c1e^{(1/\lambda)t} + c2(-\lambda\delta)e^{-\delta t} \quad -1/\lambda \int_\infty^t (m + \lambda r^* - \phi y)e^{(1/\lambda)(t-s)} ds$$

The backwards-looking integral corresponding to the negative root, $-\delta$, is vanishing. Once again the assumption of convergent expectations allows us to set c1=0. If prices can jump discontinuously in response to shocks to the forward integral, then the solution curve, where c2=0, insures that no kind of anticipated money shock will have real effects (that is, assuming that the exogenous variables do not increase by an order of magnitude greater than exponential). If the domestic price level cannot jump discontinuously in response to shocks, then c2 will be non-zero, and expectations will have a backwards-looking component. As shown in Section I, Dornbusch models always have a backwards-looking component. Overshooting is greater in the Mussa model for the classic overshooting exercise than in our earlier model. That is because exchange-rate appreciation itself hinders the necessary price adjustments after the shock through

the augmented Phillips curve. Assume that m, r^*, y have been at a constant level. Let an unanticipated permanent increase in the stock of money take place. Overshooting in the Mussa model is:[5]

$$e_t - \bar{e} = \Delta m \left(1 + \frac{1}{\lambda\delta}\right) > \Delta m \left(1 + \frac{1}{\left(\lambda\delta/2\right) + \frac{\lambda}{2} \cdot \left(\delta^2 + 4\delta/\lambda\right)^{1/2}}\right)$$

where the right-hand expression gives overshooting in the Dornbusch model. Overshooting in both cases is small when λ is large.

III. Uncertainty

This section briefly considers some of the issues which arise when uncertainty is introduced explicitly. The generalization of the earlier models to cases where the exogenous variables follow stationary stochastic processes is straight-forward. The entire analysis can be shifted to expectations space. As an example, a special case using the Mussa model is considered below. As in Chapter II, the level of the money supply will occasionally be subjected to random shocks. (This does not lead to a mixed difference/differential equation, but simply to a differential equation with a discontinuous forcing function.) The linear restriction place on the exchange rate and price level at the time of each shock is that the domestic price level cannot jump.

Assume that in the Mussa model, which has the solution given in (9') and (10') of Section II, future values of the exogenous variable are unknown. As a particular example, assume r^*, y are known and constant. $\tilde{m}$ is subjected to occasional shocks, with mean zero. Suppose the most recent shock occurred at time t. The solution to the Mussa model given at the end of Section II is now:

$$(9'') \qquad _tEe_{t+k} = \hat{c}2e^{-\delta(t+k)} - 1/\lambda \int_{-\infty}^{t+k} {_tE(\tilde{m}+\lambda r^* - \phi y)e^{(1/\lambda)(t+k-s)}ds}$$

$$(10'') \quad {}_tEp_{t+k} = -\lambda\delta\hat{c}2e^{-\delta(t+k)} - 1/\lambda \int_{\infty}^{t+k} {}_tE(\tilde{m}+\lambda r^* - \phi y)e^{(1/\lambda)(t+k-s)}ds$$

where ${}_tEx_t+k$ represents the expected value of x_{t+k} at time t. $\tilde{m}$ is the uncertain money supply. $\hat{c}2$ depends on what type of initial condition we impose after the shock. (The domestic price level could be fixed, the terms of trade could be fixed, goods-market equilibrium could be reached, etc.)

Now impose the restriction that the domestic price level cannot "jump" in response to the occasional increases or decreases in the permanent level of the money supply. Define:

$$\bar{p}_h = \bar{e}_h \equiv -1/\lambda \int_{-\infty}^{h} {}_hE(\tilde{m}+\lambda r^* - \phi y)e^{(1/\lambda)(h-s)} ds$$

Assume that at time t-k, the economy was in equilibrium. Finally, define:

$$(11) \quad \Delta\bar{p}_h = -1/\lambda \int_{\infty}^{j} {}_jE(\tilde{m}+\lambda r^* \phi y)e^{(1/\lambda)(j-s)}ds + 1/\lambda \int_{\infty}^{j} {}_jE(\tilde{m}+\lambda r^* \phi y)e^{(1/\lambda)(j-s)}ds$$
$$\lim(h\leftarrow j) \qquad\qquad\qquad \lim(j\rightarrow h)$$

where the first integral is the right-hand side limit and the second integral is a left-hand side limit. Thus $\Delta\bar{p}_h$ reflects the shock occurring at time h. The shock may represent an increase in the money supply at time h (unanticipated), or a shock which brings news

of (a) future money-supply shock (s). This is the same exercise as performed in Chapter II. Here, the general solution has a neat closed-form solution.

$$(12) \qquad _tEp_t = -e^{-\delta k} \sum_{i=0}^{k} \Delta\bar{p}_{t-k+i} e^{\delta i} + \bar{p}_t$$

The exchange rate is always buffeted by each shock to a greater degree that the price level, when all shocks are to the money supply $(\Delta\bar{e}_t = \Delta\bar{p}_t)$

$$(13) \qquad _tEe_t = \frac{e^{-\delta k}}{\lambda\delta} \sum_{i=0}^{k} \Delta\bar{e}_{t-k+i} e^{\delta i} + \bar{e}_t$$

Equation (11) is the proper formalization of the idea of "new information available at time h," under the assumptions made here. Equations (12) and (13) show how disequilibrium can be generated when uncertainty is explicitly introduced. Here it is true that money-supply information buffets the exchange rate, and may cause over-shooting.

Uncertainty poses technical problems when we try to use the assumption of convergent expectations to form the appropriate bounds of integration in the expectations integrals corresponding to each of the roots of the model. As an example, consider a stationary process which is sufficiently damped that we cannot throw out the forward

solution associated with a negative root simply through the convergent-expectations assumption. Take the case where the expected value of the log of the money supply is zero. Set $\lambda r^* - \phi y = 0$. While we know that the actual money supply will almost never be zero in the future, the expected-value integral is zero. In fact, as long as the money supply has expected value zero beginning at some point in the future (say, the money supply follows a damped stochastic process), then the forward integral may be finite. Yet in the certainty world it is never finite.

A topic for further research is to show how uncertainty provides a motivation for using past levels of the exogenous variables in determining the present levels of endogenous variables, even in the absence of strictures on discontinuous price and exchange-rate movements.

Finally, the introduction of uncertainty can lead to severe problems when the assumption of continual money-market clearing is relaxed. See Burmeister-Flood-Turnovsky (1978).

SUMMARY

e first section of this paper proposed a backward- and forward-looking expectational equilibrium as a solution to a well-known model of prices and exchange rates. The solution to the Dornbusch model, or a similar one, generally requires an initial condition in addition to the assumption of convergent expectations. One natural condition is to assume that the domestic price level cannot respond instantaneously to a shock. Another is that prices and exchange rates immediately jump to their new long-run equilibrium. While any linear restriction on prices and the exchange rate will leave the economy on a stable path, most writers consider other assumptions less plausible, or even totally arbitrary. However, the assumption that prices are able to respond instantaneously allows a third natural solution, the expectational equilibrium. There agents choose a general solution to the dynamic model governing the economy, which would make sense even in the face of anticipated, erratic shocks.

The expectational equilibrium rationally weighs the past and the future in the determination of current variables. Overshooting may occur to some extent, even if prices can jump. As long as agents do not forget what happened in the past, or assume that the goods market will automatically clear in the face of an unanticipated shock (even though it cannot always remain in equilibrium in the face of an anticipated shock), they may be assumed to choose the expectational (temporary) equilibrium. In the long-run, as

as expectations adjust, the economy will reach the normal long-run equilibrium. The standard exercise, which assumes sticky prices, leads to overshooting which can be decomposed into overshooting due to the temporarily fixed price-level assumption, and overshooting due to the stickiness of the perceived current expectational equilibrium.

This interpretation is extended to a wide class of Dornbusch models.

The second section examines the role of the expectations-augmented Phillips curves in Dornbusch-Frankel models. It derives from the appropriate expectations augmentation term corresponding to any homogenously nonstationary distribution for the exogenous variable. In the face of other types of nonstationary behavior of the exogenous variables, the real effects of perfectly anticipated monetary shocks cannot be assumed away, except in the limiting case of a pure forward-looking solution.

The final section gives some results under uncertainty. A general solution to a forward-looking model with continuous price adjustment, which experiences money-supply shocks at discrete intervals, is derived.

FOOTNOTES

[1]   The solution to the unanticipated money-supply shock in Dornbusch's "Expectations and Exchange Rate Dynamics" is usually given as:

$$F_1) \qquad (p_t - \overline{p}') = \lambda\theta2c2e^{\theta2t}$$

$$(e_t - \overline{e}') = c2e^{\theta2t}$$

where $\overline{e}'$ and $\overline{p}'$ represent the new forward-looking long-run equilibria. We have already imposed the condition $c_1=0$, so the root θ_1 has no unstable effect. Dornbusch solves these equations using the initial condition $p_t=\overline{p}$, the old price-level equilibrium. Inspection of (F_1) seems to indicate that if prices and exchange rates are freely flexible, the only natural assumption is $c_2=0$. From the perspective of this exercise, the text argues that there is an alternative initial condition and corresponding c_2 value which is not ad hoc, and also leads to some overshooting. The reason this choice of c_2 is rational is suppressed in the pure forward-looking $\overline{p}'$, $\overline{e}'$. However, the logic of the text obviously would apply to the equations (F_1). When we include anticipated shocks, a solution of the form (F_1) does not exist. Then we must use the more general answers given in the text.

2   Even in "equilibrium" in this system, the current account is going to be in surplus or deficit, and wealth is changing hands. The effects of this transfer must be considered in the long run. See Henderson.

3   See footnote (1), where the more common representation of the solution is given. There, the assumption that under "flexible" prices and exchange rates the economy would move to the current perceived expectational equilibrium amounts to assigning a specific non-zero value to c_2. The text explains why this particular value has a natural interpretation as the rational-expectations weighting of the forward and backward solution at time t.

4   The money markets and foreign exchange markets are always in equilibrium in these models by assumption. It is the goods market which experiences disequilibrium. By "equilibrium forever," we mean the goods market was actually in equilibrium in the past, or at least it had fully adjusted to all previous shocks by time t. In the future, expected disequilibrium is zero, although it is known that future surprises may come.

5   In a world with no secular changes, the Mussa model and Dornbusch model are virtually equivalent. Overshooting for the Mussa-type model is only greater, given the parameters δ and λ.

APPENDIX I

The basic model of the text is given by:

A_1)
$$-\dot{z} = -(D+\delta)p + \delta e$$

$$1/\lambda(m + \lambda r^* - \phi y) = p/\lambda - De \qquad \text{where } D \cdot x = \dot{x}$$

This type of equation may be solved by the method of variation of parameters. (See Shepley L. Ross, Introduction to Ordinary Differential Equations (Waltham: Ginn and Co., 1966).)

Burmeister, Flood and Turnovsky, "Perfect Forsight and the Stability of Monetary Models," carefully derives the appropriate bounds of integration for such systems. We freely employ their solution technique.

The solution to the homogenous equations is given by:

A_2)
$$e_t = c_1 e^{\theta 1 t} + c_2 e^{\theta 2 t}$$

$$p_t = \lambda \theta 1 c 1 e^{\theta 1 t} + \lambda \theta 2 c 2 e^{\theta 2 t} \quad , \text{ where } \theta 1 > 0, \ \theta 2 < 0$$

are the roots of the characteristic equation:

$$\theta^2 + \delta\theta - \delta/\lambda = 0$$

We will set $c_1 = 0$ by the assumption of convergent expectations. A particular solution to (A_1) may be found by noting that:

$A_3)$ $\qquad e_t^p = v_1(t)e^{\theta 1t} + v_2(t)e^{\theta 2t}$

$$p_t^p = \lambda \theta 1 v_1(t)e^{\theta 1t} + \lambda \theta 2 v_2(t)e^{\theta 2t}$$

is a particular solution if and only if:

$A_4)$ $\qquad \dot{v}_1(t)e^{\theta 1t} + \dot{v}_2(t)e^{\theta 2t} = (m + \lambda r^* - \phi y)(-1/\lambda)$

$$\lambda \theta 1 \dot{v}_1(t)e^{\theta 1t} + \lambda \theta 2 \dot{v}_2(t)e^{\theta 2t} = \dot{z}$$

To verify the necessity of the conditions imposed in (A_4), dif-
ferentiate e_t^p, p_t^p in (A_3), and substitute into the equations in (A_1).
The equations in (A_4) may be solved to yield:

$$\dot{v}_1(t) = \frac{e^{-\theta 1t}}{\lambda(\theta 1 - \theta 2)} \ (\dot{z} + \theta 2(m + \lambda r^* - \phi y))$$

$$\dot{v}_2(t) = \frac{e^{-\theta 2t}}{\lambda(\theta 1 - \theta 2)} - (\dot{z} + \theta 1(m + \lambda r^* - \phi y)) \quad . \ \text{Thus:}$$

$$v_1(t) = \frac{1}{\lambda(\theta 1 - \theta 2)} \int_a^t (\dot{z} + \theta 2(m + \lambda r^* - \phi y))e^{\theta 1(t-s)}ds$$

$$v_2(t) = \frac{-1}{\lambda(\theta 1 - \theta 2)} \int_b^t (\dot{z} + \theta 1(m + \lambda r^* - \phi y))e^{\theta 2(t-s)}ds$$

Because $\theta 1 > 0$, and $\theta 2 < 0$, the appropriate bounds of integration are $a = +\infty$, $b = -\infty$, under the assumption of convergent expectations. The general solution to (A_1) is given by:

$$A_5) \qquad e_t = c_1 e^{\theta 1 t} + c_2 e^{\theta 2 t} + v_1(t) + v_2(t)$$

$$p_t = \lambda \theta 1 c_1 e^{\theta 1 t} + \lambda \theta 2 c_2 e^{\theta 2 t} + \lambda \theta 1 v_1(t) + \lambda \theta 2 v_2(t)$$

Under convergent expectations, $c_1 = 0$.

APPENDIX II

We confirm that if m, y and r^* have been constant through all time and are expected to remain constant, then $\bar{p}=\bar{\tilde{p}}=\tilde{\tilde{e}}=\bar{\tilde{e}}=m+\lambda r^*-\phi y$.

Setting $m+\lambda r^*-\phi y=z$, We have:

$$A_6) \qquad \tilde{\tilde{p}}_t=\frac{\lambda\theta1}{\lambda(\theta1-\theta2)} \int_{+\infty}^{t} z_t\theta2 e^{\theta1(t-s)}ds+\frac{\lambda\theta2}{\lambda(\theta1-\theta2)} \int_{-\infty}^{t} -z_t\theta1 e^{\theta2(t-s)}ds$$

$$=\bar{z}\left(\frac{-\theta2+\theta1}{(\theta1-\theta2)}\right) = z$$

The first integral in $\tilde{\tilde{p}}$ looks forward in time. The second looks backward

$$A_7) \qquad \tilde{\tilde{e}}_t=\frac{1}{\lambda(\theta1-\theta2)} \int_{+\infty}^{t} z_t\theta2 e^{\theta1(t-s)}ds+\frac{1}{\lambda(\theta1-\theta2)} \int_{-\infty}^{t} -z_t\theta1 e^{\theta2(t-s)}$$

$$=\bar{z}\frac{\left(\frac{\theta2}{\theta1}+\frac{\theta1}{\theta2}\right)}{\lambda(\theta1-\theta2)}$$

$$=\frac{\bar{z}(\theta1+\theta2)}{\lambda\theta1\theta2}$$

Cancelling and noting $\theta1\theta2=\delta/\lambda$ and $\theta1+\theta2=-\delta$, we see $\tilde{\tilde{p}}=\tilde{\tilde{e}}=z$.

If we integrate backwards only to zero instead of minus infinity, then

$$A_8) \qquad \tilde{\tilde{e}}_t=z-\frac{\theta1}{\theta2}(\Delta z)e^{-\theta2t}$$

A_9) $\tilde{\tilde{p}}_t = z - \lambda\theta 1(\Delta z)e^{-\theta 2 t}$

This problem can easily be negotiated by setting the arbitrary constant, $c2$, appropriately, though that again imposes an implicit economic assumption.

When the rate of money _growth_ has been constant through all times and is expected to remain constant into the future, then the expectational equilibrium is equivalent to the pure forward solution Frankel gives. We first confirm that $\tilde{\tilde{p}} = \dot{\bar{m}} = \dot{\tilde{e}}$. Integrate v_1 and v_2 by parts to obtain:

$$A_{10})\qquad v_1(t) = \frac{1}{\lambda(\theta 1 - \theta 2)}\left\{ -\frac{\theta 2}{\theta 1}(m + \lambda r^* - \phi y)_t + \int_\infty^t \dot{m}(1 + \frac{\theta 2}{\theta 1})\ e^{\theta 1(t-s)}ds \right\}$$

$$A_{11})\qquad v_2(t) = \frac{1}{\lambda(\theta 1 - \theta 2)}\left\{ \frac{\theta 1}{\theta 2}(m + \lambda r^* - \phi y)_t + \int_{-\infty}^t -\dot{m}(1 + \frac{\theta 2}{\theta 1})\ e^{\theta 2(t-s)}ds \right\}$$

Differentiating $\tilde{\tilde{p}}_t = \lambda\theta 1 v_1(t) + \lambda\theta 1 v_2(t)$, and setting $\ddot{m} = \dot{r}^* = \dot{y} = 0$ (Frankel's implicit assumption that only m changes and that it changes at a constant rate over time)

A_{12}) $\tilde{\tilde{p}}_t = \frac{\dot{m}}{\lambda(\theta 1 - \theta 2)}\lambda[\theta 1 - \theta 2] = \dot{m}$ Similarly $\dot{\tilde{e}} = \dot{v}_1(t) + \dot{v}_2(t)$

$$A_{13}) \qquad \dot{\tilde{e}}_t = \frac{\dot{m}}{\lambda(\theta 1 - \theta 2)} [\theta 1^2 - \theta 2^2] = \dot{m}$$

where the proof is identical to the homogeneity proof in the model with no steady-state monetary growth. Similarly, we expect $\tilde{p} = \tilde{e} = m + \lambda(r^* + \dot{m}) - \phi y$. This can be seen by setting $\dot{\bar{p}} = \dot{\bar{e}} = \dot{\bar{m}}$, and solving for p in the money-market equation. We confirm this below:

Integrate (28') of the text by parts, noting $\ddot{m} = \dot{r}^* = \dot{y} = 0$, and group terms:

$$A_{14}) \qquad \tilde{e}_t = (m_t + \lambda r^* - \phi y) + \frac{1}{\lambda(\theta 1 - \theta 2)} [\int_{\infty}^{t} \dot{m}(1 + \frac{\theta 2}{\theta 1}) e^{\theta 1(t-s)} ds$$

$$+ \int_{-\infty}^{t} -\dot{m}(1 + \frac{\theta 1}{\theta 2}) e^{\theta 2(t-s)} ds] =$$

$$= (m_t + \lambda r^* - \phi y) + \frac{\dot{m}}{\lambda(\theta 1 - \theta 2)} [-\frac{1}{\theta 1} - \frac{\theta 1}{(\theta 1)^2} + \frac{1}{\theta 2} + \frac{\theta 1}{(\theta 2)^2}]$$

$$= (m_t + \lambda r^* = \phi y) + \dot{m} [\frac{-\frac{\delta^2}{\lambda^2}}{\lambda \frac{\delta^2}{\lambda^2}}] = m_t + \lambda(r^* + \dot{m}) - \phi y)$$

Similarly,

$$A_{15}) \qquad \tilde{p} = (m_t + \lambda r^* - \phi y) + \dot{m} [\frac{-1 - \frac{\theta 2}{\theta 1} + 1 + \frac{\theta 1}{\theta 2}}{(\theta 1 - \theta 2)}] = m_t + \lambda(r^* + \dot{m}) - \phi y$$

as one would expect.

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