# Assessing Quantum Magnetometry as an Emerging Detection Modality for Strategic Anti-Submarine Warfare

by

Liam J. Coy

Submitted to the Department of Nuclear Science and Engineering
in partial fulfillment of the requirements for the degree of

BACHELOR OF SCIENCE IN ENGINEERING AS RECOMMENDED BY THE
DEPARTMENT OF NUCLEAR SCIENCE AND ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May 2025

Authored by:     Liam J. Coy
                 Department of Nuclear Science and Engineering
                 May 14, 2025

Certified by:    R. Scott Kemp
                 Associate Professor of Nuclear Science and Engineering, Thesis Supervisor

Accepted by:     Jacopo Buongiorno
                 Battelle Energy Alliance Professor in Nuclear Science and Engineering
                 Undergraduate Officer, Department of Nuclear Science and Engineering

# Assessing Quantum Magnetometry as an Emerging Detection Modality for Strategic Anti-Submarine Warfare

by

Liam J. Coy

Submitted to the Department of Nuclear Science and Engineering
on May 14, 2025 in partial fulfillment of the requirements for the degree of

BACHELOR OF SCIENCE IN ENGINEERING AS RECOMMENDED BY THE
DEPARTMENT OF NUCLEAR SCIENCE AND ENGINEERING

## ABSTRACT

Emerging technologies have been a subject of much consternation in the nuclear deterrence communities as people fear they might erode secure second strike capabilities. One such emerging technology is the idea of 'quantum magnetometry'—magnetic field sensors which use quantum principles in order to obtain more precise measurements. Nuclear submarines can be detected by the magnetic distortions they cause in the background Earth magnetic field. Quantum magnetometers could enable more precise measurements of such distortions. However, the lack of certainty around the potential for this emerging technology has led to a lack of clarity in policy circles. This thesis explores some limits on the impact of quantum magnetometry in the context of strategic anti-submarine warfare (ASW). It does this in two parts. First, it provides a survey of quantum magnetometry technologies and developments. Second, it characterizes the magnetic anomaly associated with a nuclear submarine (according to the best unclassified estimates of key parameters). It finds that while quantum magnetometers may indeed result in more sensitive magnetic anomaly detectors, their impact on strategic ASW will be limited. Firstly, the magnetic anomaly associated with a submarine scales as the inverse cube of the distance from the submarine. Thus, a ten-fold decrease in the minimum field necessary to detect a submarine would only provide a slightly more than two-fold increase in detection range. Improvements in detection range would have to be quite significant to have any strategic impact, due to the vast areas of the ocean required to search to find submarines. Secondly, magnetometers are limited by more factors than solely their sensitivity. There are signal processing issues involved in determining whether a change in measured magnetic field is as a result of a target or some form of environmental noise. As such, while quantum magnetometry may indeed improve submarine detection capabilities, it is unlikely to do so in a manner that impactfully destabilizes nuclear deterrence.

Thesis supervisor: R. Scott Kemp
Title: Associate Professor of Nuclear Science and Engineering

# Contents

# List of Figures

# List of Tables

# Chapter 1

# Introduction

Nuclear deterrence is considered a strategic necessity by countries that maintain nuclear arsenals, with nuclear forces perceived as the "supreme guarantor" of security [1, 2]. The U.S. nuclear arsenal consists of three legs: nuclear-armed bombers, intercontinental ballistic missiles, and nuclear-armed submarines. Each element of this nuclear 'triad' is perceived to provide different strategic benefits [3]. The main benefit of the submarine leg of the triad is perceived to be their survivability—submarines are difficult to detect and, therefore, are seen as deterring a preemptive strike by securing second-strike capability. However, technological advancements have sparked concern about the future survivability of current nuclear submarines [4]. Several authors have discussed the possible detection of submarines through improvements in detection technologies [4, 5]. Moreover, this concept of an asymmetric disadvantage in detection capabilities has been promulgated by certain defense media outlets as a potential cause for alarm [6, 7]. The argument is that emerging technologies may allow for increased detection capabilities, allowing submarines to be tracked and destroyed. Under such a regime, submarines would no longer be considered as effective a method of deterrence, as their survivability would be weakened.

One such detection technology that is being considered is quantum magnetometry, a catch-all term for magnetic field sensors that use quantum mechanical effects to obtain sensitivities

beyond the capabilities of classical magnetometers [8]. While quantum magnetometers are in use in other fields, the technical feasibility and practical implications of the breadth of quantum magnetometry approaches in the context of strategic anti-submarine warfare (ASW) remain largely unexplored. This thesis aims to investigate quantum magnetometry in the use-case of detecting submerged submarines, providing reasonable bounds on the possible detection ranges one might expect the technology to practically be able to achieve. By determining a reasonable expectation for the detection range of quantum magnetometers in a strategic ASW context, claims surrounding the possible impact of quantum magnetometry on nuclear deterrence can be more properly understood and addressed.

In order to determine the potential uses of quantum magnetometers in strategic ASW, this study will employ a multi-faceted approach. First, the technology itself will be analyzed. A variety of quantum magnetometers will be considered, in order to determine physical limitations on the sensitivity and precision of the capabilities of quantum magnetometers in detecting fluctuations in magnetic fields. Second, the magnetic signatures relevant to strategic ASW will be explored. The magnetic field disturbances produced by current nuclear-armed submarines will be modelled through electromagnetic calculations and computational methods.

This study will extend previous discussions of individual magnetometer technologies by considering their suitability specifically for submarine detection. By providing a technical analysis, this study will provide a foundation for determining the validity of arguments that quantum magnetometry will be a destabilizing technology in the context of nuclear deterrence. As a result, this study will contribute to informed decision-making in defense policy and strategic planning. Furthermore, it will provide a framework for similar analyses of other emergent sensing technologies and their potential impacts on fields such as strategic ASW.

# Chapter 2

# Background

## 2.1  Nuclear deterrence

A major purpose of the U.S. nuclear arsenal, according to the Nuclear Posture Reviews of successive administrations, is to deter nuclear attack against the United States [9–11]. U.S. nuclear weapons theoretically accomplish this goal by deterring through threat of punishment. The logic of deterrence theory is that actors undertake an action if they determine it to be beneficial to do so. In other words, "if they expect the costs to exceed the benefits" [12]. Deterrence by threat of punishment increases the costs associated with an action by threatening retaliation if that action occurs. If an adversary launches a nuclear attack against the United States, the logic goes, it will be punished by a nuclear response.

An element of establishing a 'credible' deterrent is convincing the other actor that the costs of the action will actually be increased. As such, the concept of a secure second strike is one of the key aspects of modern nuclear deterrence. Specifically, a threat to retaliate with nuclear weapons would not be a credible deterrent to a state that believed it could avoid such a retaliation by destroying the weapons before they were used. Such a strike is termed a disarming first strike.

Having a secure second strike capability (similar terms used in the literature include

'assured second strike' and 'assured retaliation', among others [13]) means that a country has a high degree of certainty that such a disarming first strike could not be successfully completed against it. That is, regardless of the type of attack, the country could emerge with sufficient surviving forces to launch a response. This capability is considered central to the stability of nuclear deterrence [14].

### 2.1.1   The role of submarines

Submarine-launched ballistic missiles (SLBMs) are considered the most difficult component of the strategic nuclear forces deployed by the United States to destroy. As such, they are considered central to the United States' secure second strike capabilities [14]. This is primarily due to the difficulty involved with finding submarines in the ocean. If detected, a submarine is not especially difficult to destroy—methods include the use of attack submarines, missiles with torpedo warheads, and unmanned underwater vehicles [15]. However, finding a submarine in the wide area of ocean it may be operating in is very difficult. A strategy primer from the U.S. Naval Postgraduate School described it as 'a cost-ineffective search', likening a submarine in the ocean to the proverbial 'needle in the haystack' [16].

Indeed, SLBM range is such that, to ensure retaliation is impossible, a state would have to guarantee that no enemy submarines were present in an incredibly large area of ocean. For instance, the maximum range of the Trident II SLBMs U.S. Ohio class submarines are armed with (and U.S. Columbia class submarines will be armed with) is approximately 12000 km [17]. Therefore, if China were to attempt to launch a disarming first-strike on the United States, they would need to remove any U.S. SSBNs within 12000 km of any target of concern. For reference, a 12000 km radius around Beijing includes the majority of the Pacific and Indian Oceans, as well as the Mediterranean Sea and portions of the eastern Atlantic Ocean. To launch a disarming first strike and be certain of destroying the United States' second-strike capability, China would therefore have to search and safeguard an area millions of square kilometers large.

## 2.2 Methods of submarine detection

Various submarine detection methods have been deployed, with each approach tracking different signals emitted from undersea vessels. Acoustic sensing involves tracking submarines based on the sounds they emit as they move and operate underwater, and has historically been the centerpiece of submarine detection [13]. Optical satellite imagery and RADAR have been used to detect surfaced submarines. Detection modalities like synthetic aperture radar (SAR) and light detection and ranging (LiDAR) are emerging as non-acoustic methods of detection, each with their own benefits and challenges [18].

Another non-acoustic signal associated with a submarine is it's 'magnetic anomaly'. Materials interact with magnetic fields differently according to their magnetic permeabilities and any ferromagnetic properties. The Earth has a magnetic field, and submarines are made of metals that create detectable anomalies in the Earth's local magnetic fields (the mechanism and extent of this process is explored in chapter 4). As such, devices capable of measuring local magnetic fields can, with *a priori* knowledge of the background field in a location, determine if a submarine is likely to be in the area. These devices are called magnetometers.

### 2.2.1 Magnetometry

Magnetometers function by exploiting responses to magnetic fields. The most familiar type of magnetometer to the general population is a compass—a magnetized strip of metal allowed to freely align itself with the horizontal direction of the Earth's magnetic field. The types of magnetometers used in magnetic anomaly detection (often called magnetic anomaly detectors, or MADs) tend to measure the magnitude of a magnetic field at a point. Magnetometers have a long history of being used in ASW. The fluxgate magnetometer was developed in the late 1930s and subsequently used as a way of detecting submarines during the second world war [19, 20]. Magnetometry is still in use today to detect submarines. For instance, the U.S. Navy, through Lockheed Martin, will have CAE Magnetic Anomaly Detection-Extended Role

MADs installed in their "primary anti-submarine warfare helicopter" [21].

There are a variety of figures of merit to assess the performance and ability of a magnetometer. These include the sample rate (number of readings output per second), bandwidth (ability to respond to rapid changes in field), resolution (smallest change in output the magnetometer can report), absolute error (average difference between measured field and true field), drift (change in the absolute error over time), and dynamic range (range of magnetic fields over which a magnetometer can provide measurements) [20, 22]. Particularly relevant metrics for use in ASW are a magnetometer's SWAP-C (size, weight and power requirements, as well as cost), as well as whether the magnetometer is a vector magnetometer or a scalar magnetometer [23]. Vector magnetometers, such as fluxgates, measure the component of the magnetic field in a specific direction. Scalar magnetometers instead measure the magnitude of the magnetic field at a point.

Most relevant to a magnetometer's performance in an ASW context is its sensitivity, which we define here as the smallest change in field measurement the magnetometer can resolve. Unfortunately, despite being a common specification offered for commercial magnetometers, the term is not consistently defined [20]. This is as the sensitivity of a magnetometer is a function of its resolution, which is determined primarily by the electronics of the machine, and its noise (which, confusingly, some authors itself call sensitivity). The noise of a magnetometer is a measure of the variation in measurement that a magnetometer reports not caused by actual variations in the external field [20]. Noise specifications of commercial magnetometers are reported in $T/\sqrt{Hz}$, where the frequency in Hz refers to the bandwidth used. As such, operating a given machine at a higher bandwidth will increase the noise associated with its measurements [22].

Currently existing magnetometers are capable of sensitivities on the femtotesla to picotesla scale when operating at a bandwidth of 1 Hz. The CAE magnetometer designed for ASW purposes mentioned above is of a design that typically achieves noise in the range of 0.3-1.0 $pT/\sqrt{Hz}$ [20]. CAE claims their MAD is "capable of detecting anomalies within a range

of 1,200 meters" [24].

It is worth noting that quantum magnetometry, despite being frequently described as an 'emerging technology', does exist and is currently used for a variety of purposes. For instance, the CAE MAD is technically a quantum magnetometer itself, relying on spin states of optically pumped helium [24]. As such, any improvements potentially offered by quantum magnetometers will likely be evolutionary rather than revolutionary [13].

# Chapter 3

# Quantum Magnetometry

This chapter assesses the improvements quantum magnetometry may potentially provide to magnetic anomaly detectors. First, this chapter describes how a quantum sensor functions. Next, it categorizes the types of quantum magnetometers under development and determines feasible near-term capabilities based on estimates from research literature.

## 3.1 Quantum sensing

According to a review by Degen, Reinhard, and Cappellaro, quantum sensing refers to the use of a quantum system, quantum properties, or quantum phenomena to perform a measurement of a physical quantity [25]. In said review, they define four necessary attributes for a quantum system to function as a quantum sensor:

(1) The quantum system has discrete, resolvable energy levels. Specifically, we assume it to be a two-level system (or an ensemble of two-level systems) with a lower energy state $|0\rangle$ and an upper energy state $|1\rangle$ that are separated by a transition energy $E = \hbar\omega_0$

(2) It must be possible to initialize the quantum system into a well-known state and to read out its state.

(3) The quantum system can be coherently manipulated, typically by time-dependent fields. This condition is not strictly required for all protocols; examples that fall outside of this criterion are continuous-wave spectroscopy or relaxation rate measurements.

(4) The quantum system interacts with a relevant physical quantity $V(t)$, such as an electric or magnetic field [which] leads to a shift of the quantum system's energy levels or to transitions between energy levels [25].

There are a wide variety of quantum sensors under development. There are two main methods of categorization used. In areas focused on potential applications of the sensors, categorization is often based on the type of target measured. Alternatively, in discussions more connected to the technical foundation of the sensors, categorization can be done based on the type of quantum systems used [26]. Given that this thesis examines quantum magnetometers in particular, and thus is already distinguishing based on the type of target measured, the natural further categorization is based on the type of quantum system used.

## 3.2 Types of Quantum Magnetometers

In the context of quantum magnetometry, this categorization is done based on the specific two-level system, or 'qubit', that the device uses. This section discusses five different types of qubit platforms that are being explored for use in magnetometry: (1) defect qubits, (2) atomic vapor neutral atom qubits, (3) cold neutral atom qubits, (4) trapped ion qubits, and (5) superconducting qubits [25, 27–35]. There are other types of qubits that could theoretically be the basis for future magnetometers—for instance, using muons and a qubit based on muonic spin—but considerably less research has been done on those sensor types [25]. Thus, it is unlikely that any quantum magnetometer outside of these five categories could be relevant in the short-to-medium future. Modern quantum magnetometers are typically scalar magnetometers, with exceptions such as SQUIDs that may be vector magnetometers

instead [13].

Defect qubits use isolated electron spins in a solid. A common form is a Nitrogen-Vacancy (NV) color centers in diamonds (hence NV-center magnetometry) [36]. NV-centers are the general choice for defect qubit systems as they "can be individually addressed, optically polarized and detected, and exhibit excellent coherence properties even at room temperature" [27]. The electron associated with the defect has a distinct change in energy structure associated with its spin when under the influence of a magnetic field (known as the Zeeman effect). Detecting this relative energy shift induced by a magnetic field $\vec{B}$ allows for a precise determination of the applied magnetic field.

Neutral atom qubits function similarly, but detect the energy shift between spin levels of a neutral atom. Alkali atoms are often used as they are sensitive to a wide range of physical phenomena and can be measured and manipulated with existing technologies and techniques [25]. There are two main methods of manifesting this behavior: through the use of 'thermal' or 'atomic' vapors; and through hypercooled atoms. Atomic vapor neutral atom qubits can operate at or above room temperature, as they simply consist of a gas of atoms spin polarized by an optical pump beam [25]. The most sensitive of these designs are made by pushing the relaxation time of the atoms—that is, the time between exposure to the magnetic field and resulting measurement—to the second to minute range. These designs are referred to as 'spin exchange relaxation free' (SERF) sensors [37].

With advancement of laser-cooling and cryogenic techniques, cold neutral atom sensors have also been developed. They operate under similar principles to atomic vapor neutral atom qubits—utilizing the spin of these neutral atoms—but can achieve longer manipulation and interrogation times for individual atoms [13]. Quantum magnetometers using cold neutral atom qubits do have significant systems control requirements, due to the difficulties involved in cooling the atoms down to ultra-low temperatures. For instance, cold-atom vector magnetometry requires the atoms to be cooled to the 10-100 µK range [38].

Trapped ion qubits are formed by trapping ions in a vacuum using electric or magnetic

fields. They have historically been used for electrometry (the measurement of electric fields), but recent research has demonstrated that these qubits can also measure magnetic fields [32]. This can be done by measuring the results of different interactions between the spin sublevels of the ion and applied magnetic fields [33].

Whereas these other systems use subatomic or atomic particles as the basis of their qubits, superconducting qubits are made from macroscopic circuits instead. The qubits may represent a number of circuit parameters, including the flux in the circulating currents (flux qubits) or the charge distribution in the circuit (charge qubits) [13]. This category of quantum magnetometers thus can operate in a slightly different manner from the others. For instance, instead of measuring magnetic fields based on the shifts between quantized spin levels, superconducting qubits may operate based on the magnetic flux through a current loop being quantized [34].

While they are of a distinct type to the spin-based qubits, there is a long history of research into superconducting qubits. Indeed, one of the most developed forms of quantum sensor currently is the superconducting quantum interference device (SQUID). These are interferometry-based sensors that measure the interference imposed by measurement targets as flux induced in superconducting circuit loops [13]. The effective area of the loop is not necessarily known, so commercial SQUIDs are calibrated by exposure to samples of known magnetic moment [34].

## 3.3   Feasible near-term improvements

Recalling the discussion of magnetometer figures of merit in Ch 2, many of these metrics have less to do with the system interacting with the magnetic field and more with the electronics surrounding said system. The main improvements quantum sensing technologies can offer over existing magnetometers are in sensitivity and potentially SWaP metrics. Features such as the bandwidth and sample rate of the magnetometer will not be directly impacted by the type of

Table 3.1: Table of near-term capabilities of quantum magnetometers

| Sensor Type | Qubit | Order of Magnitude Noise (T/$\sqrt{\text{Hz}}$) | Operability and Deployment Constraints |
|---|---|---|---|
| NV-Center Diamond | Defect | $10^{-12}$ | Requires better diamond fabrication and characterization techniques; sensitive to crystal alignment and EM interference [28] |
| Optically Pumped Helium | Atomic vapor neutral atom | $10^{-12}$ | Currently used for ASW MADs by CAE systems; somewhat bulky [24, 40] |
| SERF | Atomic vapor neutral atom | $10^{-16}$ | Very limited bandwidth and operational range; requires temperature control and shielding [13] |
| Cold atom | Cold neutral atom | $10^{-10}$ | Nascent technique; demands microkelvin cooling (vacuum systems); power-intensive [41] |
| Trapped ion | Trapped ion | $10^{-10}$ | Generally complex infrastructure; portable systems under development [42] |
| SQUID | Superconducting | $10^{-15}$ | Vector magnetometer. Also requires some cryogenic capabilities. Good SWaP metrics [34] |

quantum sensing implemented (as the spins and currents will freely respond to changes in magnetic field). Certain other figures, such as the dynamic range, would be impacted by the choice of quantum system—however, not in a manner that would be relevant to submarine detection [39]. Thus, for the purposes of this thesis, the main points of comparison will be internal noise and qualitative comments on constraints and factors that may affect SWaP metrics.

It is worth noting that these many of these noise figures are still below those of current state-of-the-art magnetometers (as outlined in 2.2.1), even before accounting for operability concerns.

# Chapter 4

# Submarine Magnetic Signature

This chapter contains a description of the magnetic anomaly a submarine produces while in the ocean. Any material of will exhibit an induced temporary magnetization under the influence of an external field, proportional to the difference in magnetic susceptibility between it and the environment. Ferromagnetic materials, such as the steel most submarines are made of, may also become permanently magnetized when exposed to the Earth's magnetic field. These effects cause the submarine to produce a local distortion of the magnetic field around it. At distances much larger than the scale of the submarine, this distortion can be modeled as originating from a point dipole, with the induced moment determined by qualities of the submarine (shape, material, tonnage) and the external magnetic field at that point. By the principle of superposition, the total magnetic field at a point near the submarine is the vector sum of the magnetic field sourced from the submarine and the background magnetic field. Scalar magnetometers measure solely the magnitude of this total magnetic field, whereas a vector magnetometer measures the magnetic field in a particular direction. For purposes of detection, the magnetic anomaly is the difference between the value of the background magnetic field without any additional source present and the value measured.

## 4.1 Determination of a submarine's dipole moment

We begin with the idea that an external magnetic field will induce a magnetization in a magnetizable object [43]. For an ellipsoid, such as a submarine, in a constant external magnetic field, the induced dipole will be related to the applied field by a collection of 'demagnetization coefficients' determined by the shape of the ellipsoid. These coefficients describe the reduction in the field inside the ellipse due to the geometrical constraints of the boundary. Specifically, the magnetization in the direction of one of the principle axes of the ellipse, $M_i$ is given by:

$$M_i = \frac{\chi}{1 + \chi N_i} \cdot \frac{B_i}{\mu} \tag{4.1}$$

Where $B_i$ is the external field in that direction if there weren't an ellipsoid present, $\mu$ is the magnetic permeability of the environment, $\chi$ is the magnetic susceptibility of the material of the ellipsoid, and $N_i$ is the demagnetization coefficient associated with that axis [44, 45].

A material of constant magnetization $\vec{M}$ and volume $V$ has a total magnetic dipole moment $\vec{m} = \vec{M}V$. The magnetic field associated with a dipole of moment $\vec{m}$ located at the origin of some coordinate system is given by the formula [43]:

$$\vec{B}(\vec{r}) = \frac{\mu}{4\pi} \cdot \frac{3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m}}{r^3} \tag{4.2}$$

### 4.1.1 Background earth magnetic field

In order to ascertain the magnetic signature of a submarine, it is necessary to have a model for the background magnetic field of the Earth. Firstly, this is the external field that induces the magnetic dipole moment in the submarine. Secondly, a magnetometer measures the total magnetic field at a point—that is, the vector sum of the submarine magnetic field, the background magnetic field, and any other potential magnetic signals. As such, the possible

signal that can be measured may depend on the relative orientation of the magnetic field sourced from the submarine and the background field. For instance, a scalar magnetometer would measure the magnitude of the resultant sum. If that resultant sum has the same magnitude as the background field otherwise would, the scalar magnetometer would record no anomaly (as is seen later in this chapter in the 0 contour line on Fig. 4.4).

For the purposes of the qualitative analyses of submarine signatures presented in this paper, the centered dipole model of the Earth's magnetic field is used, using coefficients from the International Geomagnetic Reference Field (IGRF) [46]. Under this dipole approximation, the Earth's magnetic field is treated as emanating from a point dipole at the center of the Earth. This point dipole has magnitude 7.7 x $10^{22}$ A·m$^2$, pointed in the direction of the coordinates 80.7° N and 72.8° W.

More accurate, higher-order models exist. The International Geomagnetic Reference Field itself has 196 spherical harmonic coefficients $g_n^m$ and $h_n^m$, determined based on spherical harmonic analysis of magnetic field data [46]. This allows for somewhat finer resolution of somewhat smaller-scale internal signals—however, even these models are not detailed enough to describe small-scale geomagnetic anomalies. For actual detection purposes, more precise measurement of reference magnetic fields can be performed. The dipole model is sufficient for our purposes exploring the magnetic anomaly produced by a submarine experiencing Earth's magnetic field.

## 4.1.2   Calculation of dipole moment

Given the position, heading, and a model for the composition of the submarine, we can calculate the demagnetization coefficients and thus the induced dipole moment of the submarine. We model a Columbia-class submarine as an ellipsoid of length 560ft $\approx$ 171m, height and width 43ft $\approx$ 13m, a submerged displacement of 20815 tons, and an effective magnetic susceptibility equivalent to that of a shell of HY-80 steel ($\chi \approx 0.5$) [17]. The normalized demagnetization coefficients of a prolate spheroid (an ellipsoid where the two shorter axes

are equal in length) are given by:

$$N_l = \frac{1}{m^2 - 1} \left[ \frac{m}{2\sqrt{m^2 - 1}} \cdot \ln \left( \frac{m + \sqrt{m^2 - 1}}{m - \sqrt{m^2 - 1}} \right) - 1 \right] \qquad (4.3)$$

for the major axis, and

$$N_a = N_v = \frac{1}{2(m^2 - 1)} \left[ \frac{-1}{2\sqrt{m^2 - 1}} \cdot \ln \left( \frac{m + \sqrt{m^2 - 1}}{m - \sqrt{m^2 - 1}} \right) + m \right] \qquad (4.4)$$

for the minor axes. In these equations, $m$ is the ratio between the length of the major axis and the length of the shorter axes [47].

Then, using Eq 4.1 and $\vec{m} = \vec{M}V$, the induced dipole moment of the submarine can be calculated. Due to the different demagnetization coefficients for the longitudinal and athwartship axes, the induced magnetization is a function of the relative bearing of the submarine and the declination, or horizontal direction, of the magnetic field at that location. Specifically, the submarine will have a maximum induced moment when its longer axis is pointing in the same direction as the magnetic field, and a minimum induced moment when the submarine bearing and magnetic field declination are offset by 90°. Figs. 4.1 and 4.2 show how the induced moment of such a submarine at a specific geographic location varies with its heading. Note that at 25° N, 58° E, the declination of the magnetic field is approximately 172° from geographic north.

**Permanent Moment**

A submarine made of ferromagnetic material can also have a permanent magnetization as a function of the magnetic history of the material. Similarly to the induced moment, the highest potential permanent moment is in the longitudinal direction [48]. If a submarine is exposed to an external magnetic field in a consistent direction for a long period of time, the permanent moment will build up asymptotically. However, when at sea, the heading of the submarine may vary, leading to the horizontal component of the external field being applied
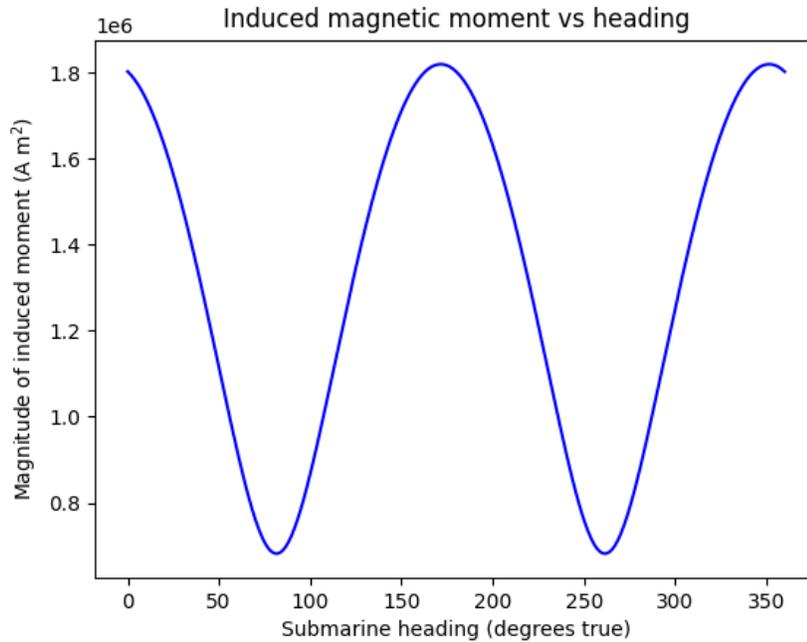
Figure 4.1: The magnitude of the induced magnetic moment in a submarine located at 25°
N, 58° E, and a depth of 200m.

.

in all directions. As such, "it appears to be impossible, except for peculiar special cases, to
be able to predict the permanent longitudinal magnetic moment of a submarine" [48].

However, the randomizing effect that diminishes the buildup of permanent horizontal
moment does not always apply to the vertical moment. If a submarine primarily operates in
the northern hemisphere, it would consistently experience an external magnetic field with
a vertical component directed downward, leading to a build-up of magnetization in the
downward direction. Conversely, a submarine primarily operating in the southern hemisphere
would gain a permanent vertical magnetization in the upward direction.

Nevertheless, for the purposes of this thesis, the permanent magnetization of submarines
will be mostly ignored. While a report presented for the US Naval Air War center argues
that "the permanent magnetic moment... must be considered in any signature modeling
effort", the way the report handles permanent magnetization is simply by assigning possible
values for the components of the permanent moment and examining the consequences [48].
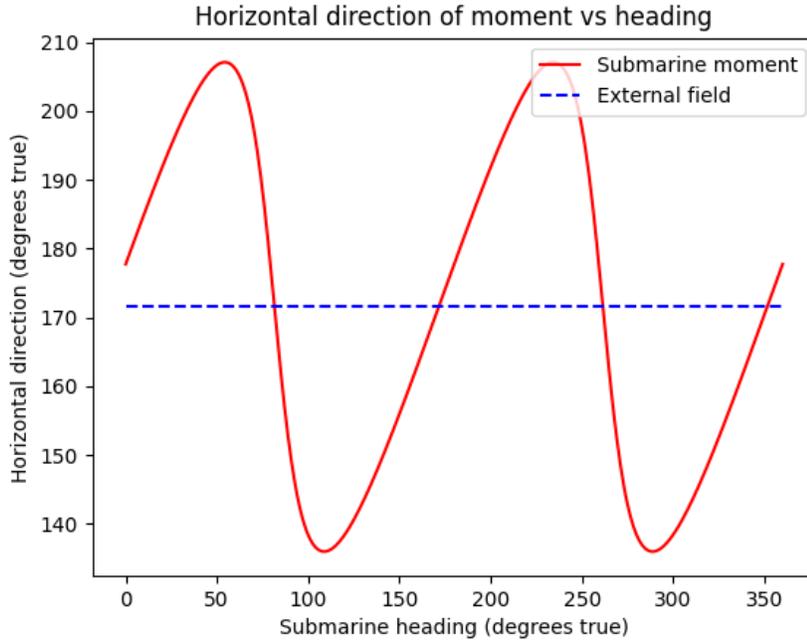
Figure 4.2: The horizontal direction of induced moment in said submarine, compared to the magnetic field declination. .

Indeed, The effects of the permanent moment are second-order. This is partly as submarines undergo degaussing procedures or 'deperming' in order to minimize such effects. This process results in a decrease of the permanent moment that lasts for a time-scale of months before "components of the permanent magnetization recover to values comparable to those before the deperming treatment" [49, 50]. The author of the report admits "it is very unlikely the permanent moment would ever exceed, or even equal, the induced moment" [48]. In the event that magnetometry were to become a more significant detection modality for strategic ASW, submarines could undergo this deperming process more frequently. Moreover, in this process the ship can be left with a vertical permanent magnetization that is counter to the assumed induced vertical component of magnetic fields it will subsequently be exposed to [49]. This enables even the build-up of the permanent vertical moment, unavoidable by shifting heading, to be kept minimal.

## 4.2 Magnetic Anomaly

To demonstrate the type of magnetic anomaly that could be detected, consider a Columbia-class submarine located in the Pacific ocean. Let this submarine be at the coordinates 20°N, 160°W, a depth of 150m, and a heading of 315°T. Assume the permanent moment of the submarine to be negligible. If $\vec{m}_s$ is the induced magnetic moment of the submarine, calculated as in 4.1, then the magnetic field at a point $\vec{r}$ is given by the vector sum of the two dipole fields:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \cdot \left( \frac{3(\vec{m}_s \cdot \hat{r})\hat{r} - \vec{m}_s}{r^3} + \frac{3(\vec{r} - \vec{r}_E) \cdot \vec{m}_E(\vec{r} - \vec{r}_E)}{|\vec{r} - \vec{r}_E|^5} - \frac{\vec{m}_E}{|\vec{r} - \vec{r}_E|^3} \right) \qquad (4.5)$$

Where $\vec{m}_E$ is the Earth's dipole moment, $\vec{r}_E$ is the displacement of the center of the Earth from the submarine, and the equation is given in submarine-fixed coordinates, meaning $\vec{r}_{sub} = \vec{0}$. Under these conditions, $\vec{m}_s$ evaluates to $-1.65\text{x}10^6 \text{A·m}^2$ in the longitudinal direction, $-3.2\text{x}10^5 \text{A·m}^2$ in the athwartship direction, and $5.2\text{x}10^5 \text{A·m}^2$ upwards. The following plots show the magnitude of the magnetic anomaly at an altitude of 1000m (Fig 4.3), and the magnitude of the magnetic anomaly detectable by a scalar magnetometer at that altitude (Fig 4.4).

A scalar magnetometer measures the difference in magnitude between the two fields, not the magnitude of the difference. This distinction between the signal observable to a scalar magnetometer and the actual change in the magnetic field can be made more clear by plots from the perspective of a detector flying above the water (Figs 4.5 and 4.6). These plots compare the (absolute value of) the difference in magnitude between the measured field and the background field to the magnitude of the difference between the fields. Before processing of the background earth magnetic field, the raw data captured by a scalar magnetometer may look more similar to Fig 4.7 (the altitude of the theoretical detector is reduced in this plot to make the effect of the anomaly more clear).
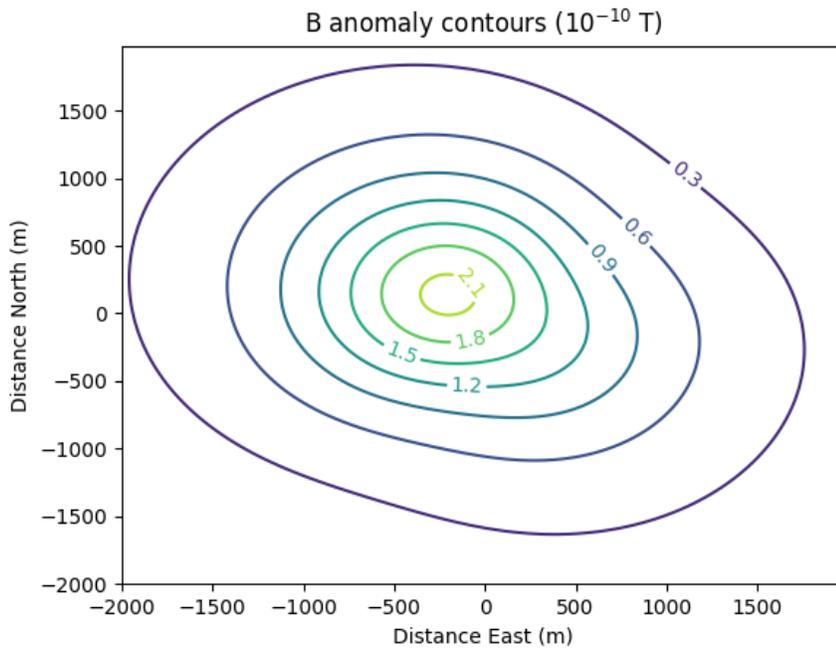
Figure 4.3: Contour plot of the vector difference between the anomalous $\vec{B}$ field and the background Earth field at an altitude of 1000m
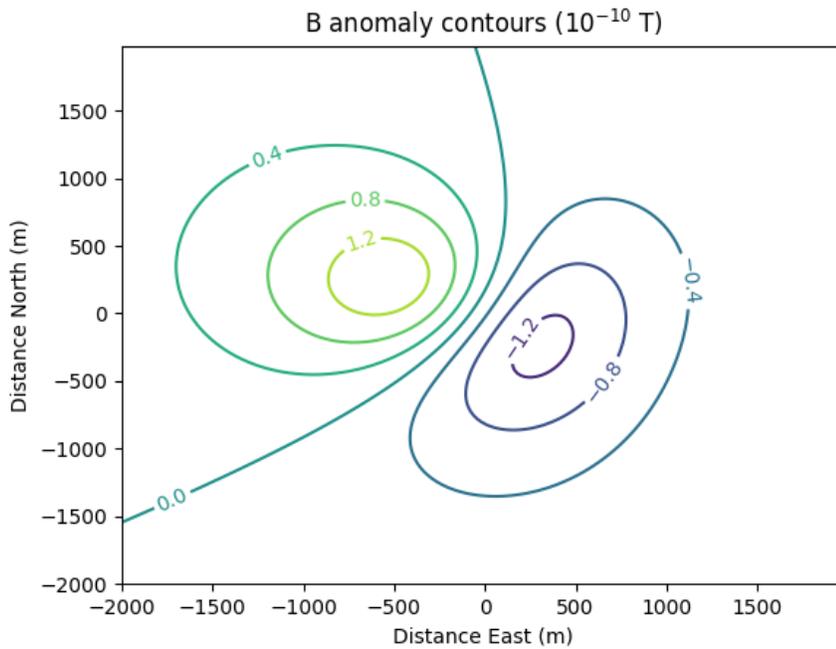


Figure 4.4: Contour plot of the difference in magnitude of the measured $\vec{B}$ field and the background Earth field
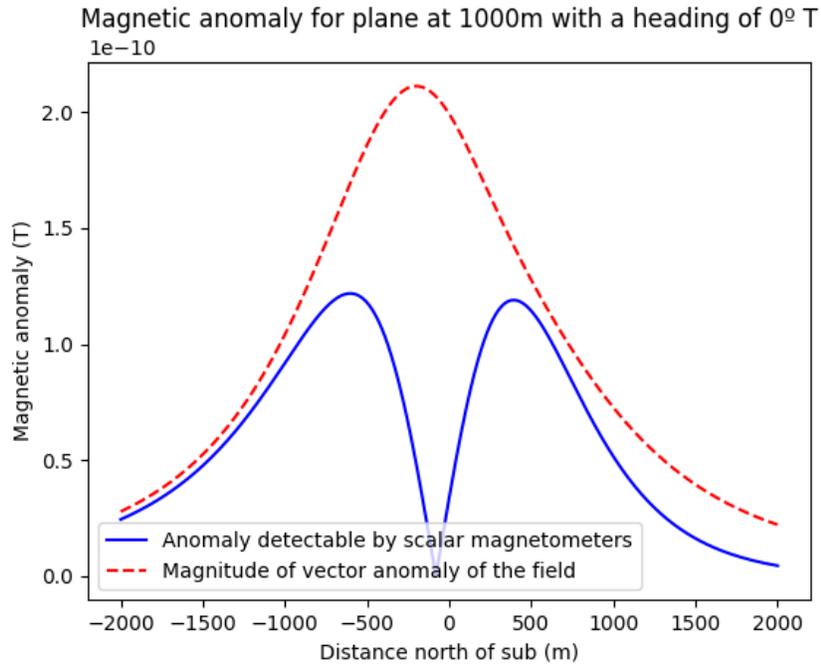
Figure 4.5: Magnitude of magnetic anomaly detectable by a scalar magnetometer on a south to north flight path passing directly over the submarine
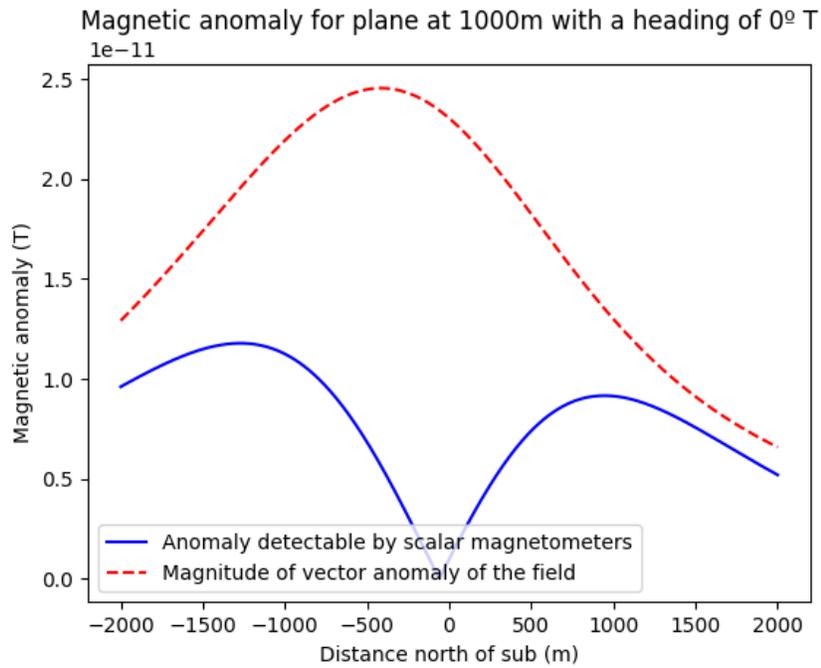


Figure 4.6: Magnitude of magnetic anomaly detectable by a scalar magnetometer on a south to north flight path passing 2 kilometers east of the submarine
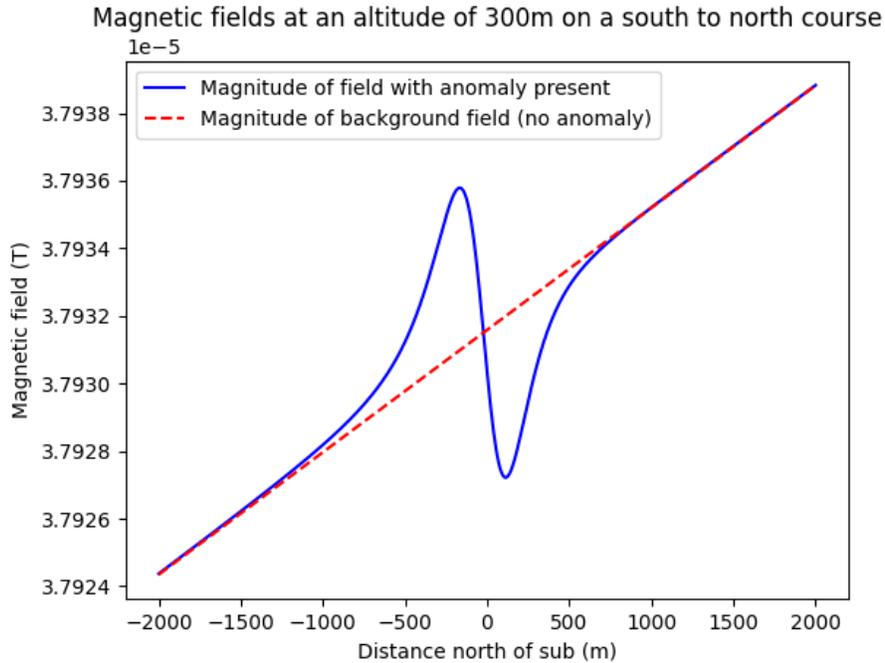
Figure 4.7: Magnitude of magnetic field at an altitude of 300 m on a south to north course path passing 2 passing directly over the submarine

## 4.2.1   Signal minimization techniques

There are ways in which submarines may reduce their magnetic signatures. Some of these methods are already in use—for instance, the degaussing that occurs in Magnetic Silencing Facilities (otherwise referred to as 'deperming' facilities) to reduce permanent magnetization. As seen in Figs 4.1 and 4.2, the heading of a submarine impacts the magnitude and direction of the induced moment. In turn, this also affects the magnetic signal that may potentially be detected. If we return to the submarine in the Pacific Ocean used previously in this section, the following graphs depict the magnitude of the magnetic anomaly detectable by a scalar magnetometer at an altitude of 1000m if the submarine is oriented parallel (Fig 4.8) or perpendicular (Fig 4.9) to the declination of the magnetic field.

There are numerous other methods of magnetic quieting, both more complex and more effective. A full discussion of possible approaches is not the focus of this thesis, but if magnetometer detection capabilities were to significantly increase, signal reduction possibilities could
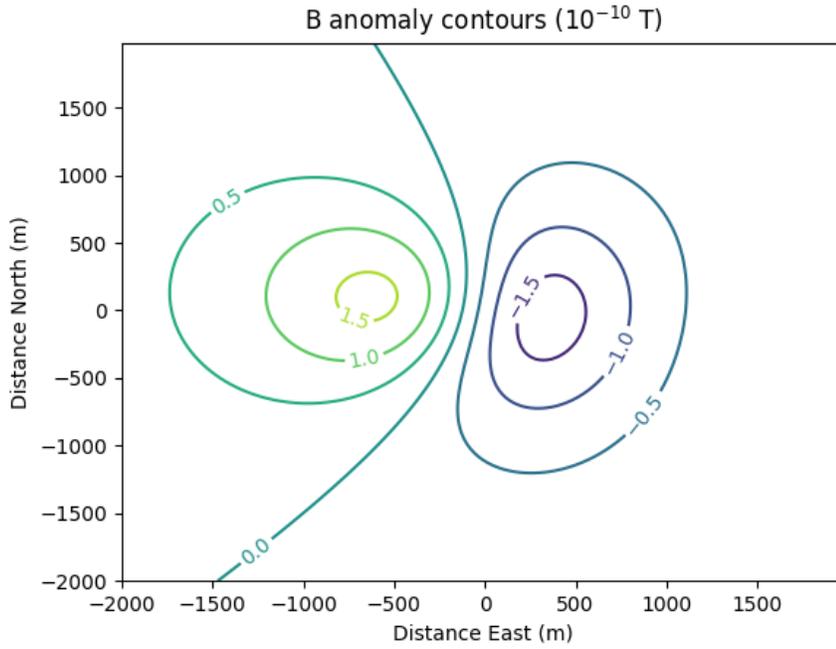
Figure 4.8: Scalar anomaly at an altitude of 1000 m given the submarine is parallel to the background magnetic field declination—in this case, at a heading of 172° or 352° T



Figure 4.9: Scalar anomaly at an altitude of 1000 m given the submarine is perpendicular to the background magnetic field declination—in this case, at a heading of 82° or 262° T

be explored. Immediate possibilities include using less magnetically permeable material in the construction of SSBNs (certain Soviet submarines were made of titanium, for instance [51]) or implementing electrical coils on the submarines themselves to create a cloaking field [50].

# Chapter 5

# Synthesis and Discussion

## 5.1 Estimation of detection ranges

Frequently, the detection range of magnetometers is estimated by taking the farthest distance such that the magnetic anomaly of the object to be detected is still larger than the sensitivity limit of the magnetometer. A convenient manner in which to do this is by letting $B_{min}$ equal the minimum detectable field strength by the magnetometer and simplifying the magnetic field of a dipole equation (Eq 4.1) to that along the axis of the submarine's induced moment (i.e, where $\vec{r} = r\ \hat{m}$):

$$\vec{B}(r) = \frac{\mu}{4\pi} \cdot \frac{3\vec{m} - \vec{m}}{r^3} \qquad (5.1)$$

Then, the detection distance $d$ is solved for by determining the minimum $r$ such that the magnitude of $\vec{B}$ is greater than $B_{min}$ [13]:

$$d = \left( \frac{\mu}{2\pi B_{min}} \right)^{1/3} \qquad (5.2)$$

It is clear to see that at best this approach provides an order of magnitude estimation for detection range. In addition to the already explicit simplifying assumptions, the detectable

anomaly will be less than the magnitude of the submarine's dipole field. As discussed in section 4.2, scalar magnetometers measure solely the magnitude of the magnetic field at a point, and thus the signal can be masked if the submarine dipole field at that point is in a different direction to the background earth field.

Moreover, analyses such as [13] tend to estimate $B_{min}$ based purely on qualities of the magnetometer. This process has an underlying key assumption: that the magnetic signal can be processed and separated from environmental noise. As discussed in Section 2.2.1, the $T/\sqrt{Hz}$ figure given for different varieties of magnetometers is simply a limit for how precisely a magnetometer can report the magnetic field it measures. It does not take into account what the source of the measured magnetic field is or any other external noise factors. Thus, this type of detection range estimation is an upper limit for magnetometers with given sensitivities.

For instance, already existing SQUIDs are used in clinical magnetoencephalography systems for measuring small (on the order of 100 fT $\approx 10^{-4}$ nT) stray fields of electric currents in the brain [25]. Solving Eq 5.2 with such a value for $B_{min}$ and taking $m \approx 10^6$A m$^2$ (the dipole moment of the submarine in Section 4.2) gives an approximate estimate of a detection range of 10 km. This is an order of magnitude further than the ranges claimed by modern magnetic anomaly detectors [24]. The importance of recognizing environmental noise has been explicitly recognized in some policy circles. For instance, in a technology brief overviewing non-acoustic detection technologies, Lisman notes "their high sensitivity makes SQUIDs susceptible to background noise" [18].

To demonstrate this point, consider again the submarine used in Section 4.2. A paper exploring magnetic detection of a surface ship by an airborne SQUID MAD included values for environmental noise before and after a magnetic compensation and noise suppression process. Even after this process, the magnetometer reading included random fluctuations on the order of 0.01 nT [50]. Fig 5.1 compares the anomaly detectable by a detector at an altitude of 1000 m on a path that comes within 1, 2, or 3 km of the horizontal position of the
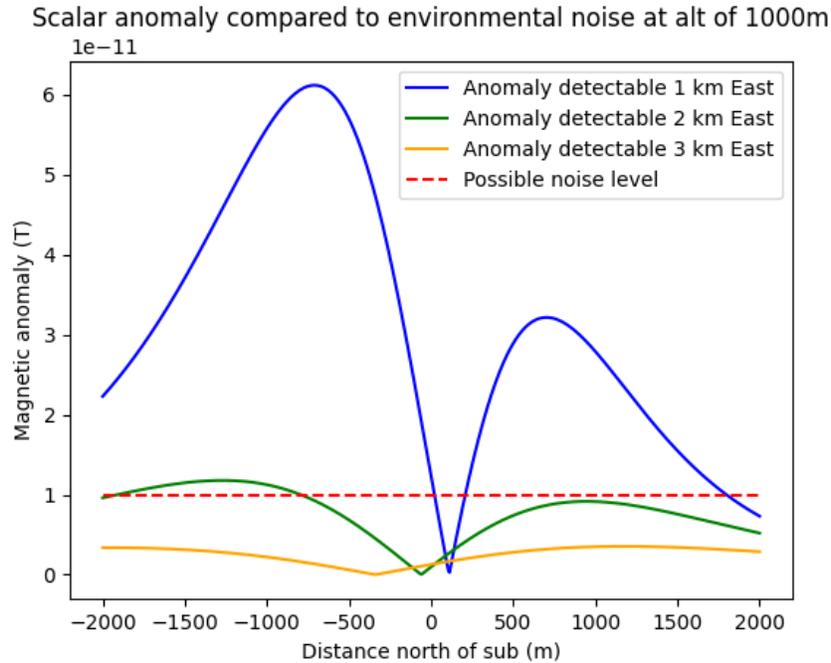
Figure 5.1: Scalar anomalies and order of magnitude environmental noise

submarine to that level of random noise.

A discussion of the signal processing necessary to properly estimate actual operational detection distances is beyond the scope of this thesis. Nevertheless, if concerns about the impact of quantum magnetometry on submarine detection continue to abound, further research into characterization of noise sources may need to be performed.

## 5.2   Potential consequences for strategic ASW

Even if we assume advances in signal processing sufficient to allow emerging quantum magnetometers to be sensitivity limited rather than limited by environmental noise, the maximum detection ranges would still be relatively small compared to the size of the ocean. Assuming the upper limit of sensitivities from Chapter 3 and the simplified (upper-limit) detection distance estimate from above, detection ranges of quantum magnetometers in the near-term future would be around 10 km at maximum. As discussed in Section 2.1.1, there

is a very large area of ocean to search and monitor. Furthermore, if sensitivities increased further, Eq. 5.2 indicates that the detection range scales with the inverse of the cube root of $B_{min}$.

To achieve the feat of 'fully transparent oceans' policymakers have expressed concerns over in analyses like [4] would require consistent monitoring of millions of square kilometers of oceans. Even assuming a detection range of 10 km, a network of approximately 1 million floating MADs would be needed [13]. A state would have severe difficulties maintaining this network, especially in a manner where they do not indicate to adversaries they are creating such a network. Interference, either natural (for instance from weather events or interactions with marine life) or unnatural (attempts of sabotage) would necessitate large upkeep and replacement requirements.

Another possibility that has been the source of some discussion is a detection strategy of 'tracking' [4, 13]. This would consist of using alternatively-sourced information about a submarine's location for targeted surveillance over smaller regions. However, such an approach would present its own operational difficulties. For instance, nuclear-armed submarines travel at speeds of up to 20 knots [17]. As such, continued tracking would require a network of high-speed drone swarms or a large enough fleet of dedicated nuclear-powered general-purpose submarines to follow all foreign submarines of interest. Moreover, an adversary using this strategy would have more difficulty assuring themselves of a disarming first strike. They would have to account for each and every submarine. Additionally, if a tracking network loses a submarine, it would require a new piece of information to begin tracking again.

Thus, even if advanced quantum sensors provided a ten-fold increase in detection range over currently deployed MAD sensors, there would still be major operational challenges that would have to be met to accomplish persistent monitoring or tracking. Any increase in detection range does technically increase the feasibility of deploying such systems, but there would be obvious signs of such a process that could signal an adversary to start developing countermeasures.

As such, the potential consequences for strategic ASW seem to not be as monumental as some reports have feared. This is due to a variety of factors, including the $d^{-\frac{1}{3}}$ scaling of magnetic anomaly with distance, potential countermeasures available to reduce induced moment, and the difficulty of isolating submarine magnetic anomalies from those caused by other sources. However, perhaps the clearest reason why quantum magnetometry will not be a 'revolutionary' emerging technology is simply that it already exists. As such, advances in submarine detection as a result of quantum magnetometry will build off of existing MADs, rather than occur in a completely novel manner.

# Chapter 6

# Conclusion

While enhanced MAD sensors using quantum magnetometry could incrementally improve SSBN detection capabilities, they do not resolve the fundamental physical and strategic barriers to guaranteed tracking or ocean-wide transparency. Sensitivity is not the key issue facing modern MADs, rather, it is the ability to determine the origin of changes in detected fields below the nanotesla scale. Moreover, the logistical scale of surveillance networks, coupled with adversarial adaptive responses, ensures that SSBNs retain significant survivability advantages. Consequently, second-strike capabilities are likely to persist as a stabilizing element of nuclear deterrence, regardless of quantum-enabled sensing advancements

## 6.1   Future work to be explored

Firstly, signal to noise questions should be further explored. Sources of environmental noise (such as geomagnetic noise, geophysical noise, and magnetic anomalies associated with non-submarine targets) should be characterized and compared to the signal derived from a submarine. This will also allow an interrogation of the signal processing questions that have largely been ignored in this thesis. Emerging technologies like machine learning may allow for better noise reduction and provide larger increases in MAD detection range.

Secondly, a similar process should be undertaken with respect to other emerging sensing

modalities like SAR and LIDAR. A technical study providing reasonable bounds on the potential impacts of these technologies would be similarly useful in informing arguments about the potential destabilizing influence of these technologies in the context of nuclear deterrence. This can further extend to use-cases separate from strategic ASW—for instance, whether quantum sensing improvements could enable significantly improved missile accuracies.

# References

[1]  The White House. *Joint Statement of the Leaders of the Five Nuclear-Weapon States on Preventing Nuclear War and Avoiding Arms Races*. en-US. Jan. 2022. URL: https://www.whitehouse.gov/briefing-room/statements-releases/2022/01/03/p5-statement-on-preventing-nuclear-war-and-avoiding-arms-races/ (visited on 12/06/2024).

[2]  V. Narang. *"Nuclear Threats and the Role of Allies": Remarks by Acting Assistant Secretary of Defense*. en-US. Aug. 2024. URL: https://www.defense.gov/News/Speeches/Speech/Article/3858311/nuclear-threats-and-the-role-of-allies-remarks-by-acting-assistant-secretary-of/https%3A%2F%2Fwww.defense.gov%2FNews%2FSpeeches%2FSpeech%2FArticle%2F3858311%2Fnuclear-threats-and-the-role-of-allies-remarks-by-acting-assistant-secretary-of%2F (visited on 12/06/2024).

[3]  V. Narang and S. D. Sagan, eds. *The Fragile Balance of Terror: Deterrence in the New Nuclear Age*. und. Cornell Studies in Security Affairs. Ithaca: Cornell University Press, 2023. ISBN: 978-1-5017-6702-9.

[4]  K. Kubiak. *Quantum Technology and Submarine Near-Invulnerability*. en. Global Security Policy Brief. European Leadership Network, Dec. 2020.

[5]  S. J. Gamberini and L. Rubin. "Quantum Sensing's Potential Impacts on Strategic Deterrence and Modern Warfare". en. In: *Orbis* 65.2 (2021), pp. 354–368. ISSN: 00304387. DOI: 10.1016/j.orbis.2021.03.012. URL: https://linkinghub.elsevier.com/retrieve/pii/S0030438721000120 (visited on 05/06/2025).

[6]   S. Roblin. *Can China's Quantum Radar Detect Any Submarine?* en-US. Dec. 2021. URL: https://nationalinterest.org/blog/reboot/can-chinas-quantum-radar-detect-any-submarine-198560/ (visited on 05/06/2025).

[7]   J. Saballa. *China's New Drone-Mounted Tech Hunts Submarines Using Magnetics - NextGen Defense.* en-US. Section: Next-Gen Radar & Detection. Apr. 2025. URL: https://nextgendefense.com/china-drone-submarines-magnetics/,%20https://nextgendefense.com/china-drone-submarines-magnetics/ (visited on 05/06/2025).

[8]   V. Giovannetti, S. Lloyd, and L. Maccone. "Quantum Metrology". en. In: *Physical Review Letters* 96.1 (Jan. 2006), p. 010401. ISSN: 0031-9007, 1079-7114. DOI: 10.1103/PhysRevLett.96.010401. URL: https://link.aps.org/doi/10.1103/PhysRevLett.96.010401 (visited on 10/11/2024).

[9]   Defense Department. *Nuclear Posture Review Report.* Tech. rep. D 1.2:N 88/4. Apr. 2010. URL: https://dod.defense.gov/portals/1/features/defensereviews/npr/2010_nuclear_posture_review_report.pdf.

[10]  Defense Department. *Nuclear Posture Review.* Tech. rep. Feb. 2018. URL: https://media.defense.gov/2018/feb/02/2001872886/-1/-1/1/2018-nuclear-posture-review-final-report.pdf (visited on 12/31/2024).

[11]  Defense Department. *2022 Nuclear Posture Review.* Tech. rep. Oct. 2022. URL: https://media.defense.gov/2022/Oct/27/2003103845/-1/-1/1/2022-NATIONAL-DEFENSE-STRATEGY-NPR-MDR.pdf (visited on 12/31/2024).

[12]  K. A. Lieber and D. G. Press. "The New Era of Counterforce: Technological Change and the Future of Nuclear Deterrence". en. In: *International Security* 41.4 (Apr. 2017), pp. 9–49. ISSN: 0162-2889, 1531-4804. DOI: 10.1162/ISEC_a_00273. URL: https://direct.mit.edu/isec/article/41/4/9/12158/The-New-Era-of-Counterforce-Technological-Change (visited on 05/14/2025).

[13] L. E. Rand. "Schrodinger's Technology Is Here and Not: A Socio-Technical Evaluation of Quantum Sensing Implications for Nuclear Deterrence". PhD thesis. University of Maryland, College Park, 2023.

[14] K. Tsipis. *Tactical and strategic antisubmarine warfare*. eng. A SIPRI monograph. Cambridge, Mass.: MIT Press [u.a.], 1974. ISBN: 978-0-262-20031-8.

[15] B. Clark. *The Emerging Era in Undersea Warfare*. Tech. rep. CSBA6292. Washington, D.C.: Center for Strategic and Budgetary Assessments, Jan. 2015. URL: https://csbaonline.org/uploads/documents/CSBA6292_(Undersea_Warfare_Reprint)_web.pdf (visited on 05/14/2025).

[16] J. S. Breemer. *Anti-Submarine Warfare: A Strategy Primer:* en. Tech. rep. Fort Belvoir, VA: Defense Technical Information Center, July 1988. DOI: 10.21236/ADA199554. URL: http://www.dtic.mil/docs/citations/ADA199554 (visited on 05/14/2025).

[17] R. O'Rourke. *Navy Columbia (SSBN-826) Class Ballistic Missile Submarine Program: Background and Issues for Congress*. eng. legislation. Mar. 2022. URL: https://www.congress.gov/crs-product/R41129 (visited on 05/09/2025).

[18] E. Lisman. *Non-acoustic Submarine Detection*. en. Tech. rep. On The Radar: Center for Strategic & International Studies, Nov. 2019. URL: https://ontheradar.csis.org/issue-briefs/non-acoustic-submarine-detection/.

[19] G. Musmann and Y. V. Afanassiev, eds. *Fluxgate magnetometers for space research*. eng. Norderstedt: Books on Demand, 2010. ISBN: 978-3-8391-3702-4.

[20] D. C. Hovde, M. D. Prouty, I. Hrvoic, and R. E. Slocum. "Commercial magnetometers and their application". In: *Optical Magnetometry*. Ed. by D. Budker and D. F. Jackson Kimball. 1st ed. Cambridge University Press, Mar. 2013, pp. 387–405. ISBN: 978-1-107-01035-2 978-0-511-84638-0. DOI: 10.1017/CBO9780511846380.021. URL: https://www.cambridge.org/core/product/identifier/CBO9780511846380A137/type/book_part (visited on 05/13/2025).

[21] *Lockheed Martin Awards CAE Contract to Provide MAD-XR for U.S. Navy MH-60R Helicopters*. en. Nov. 2020. URL: https://www.aviationpros.com/aircraft/defense/press-release/21163509/cae-lockheed-martin-awards-cae-contract-to-provide-mad-xr-for-us-navy-mh-60r-helicopters (visited on 05/14/2025).

[22] G. Inc. *Total Field Magnetometer Performance Published Specifications and What They Mean*. Technical Report TR120. Geometrics Inc, Jan. 2000. URL: https://www.geometrics.com/wp-content/uploads/2018/10/M-TR120.pdf (visited on 05/13/2025).

[23] L.-A. Ponirakis. *(SBIR) Navy - Low Size, Weight, Power, and Cost (SWAP-C) Magnetic Anomaly Detection (MAD) System*. URL: https://www.navysbir.com/n15_2/n152-117.htm (visited on 05/14/2025).

[24] CAE. *DM044_MAD-XR-_EN_Feb2020*. en. Feb. 2020. URL: https://www.cae.com/media/documents/DM044_MAD-XR-_EN_Feb2020 (visited on 05/14/2025).

[25] C. L. Degen, F. Reinhard, and P. Cappellaro. "Quantum sensing". en. In: *Reviews of Modern Physics* 89.3 (July 2017), p. 035002. ISSN: 0034-6861, 1539-0756. DOI: 10.1103/RevModPhys.89.035002. URL: http://link.aps.org/doi/10.1103/RevModPhys.89.035002 (visited on 05/06/2025).

[26] L. Rand. *Quantum Technology: A Primer on National Security and Policy Implications*. en. Tech. rep. Lawrence Livermore National Laboratory, July 2022. URL: https://cgsr.llnl.gov/sites/cgsr/files/2024-08/Quantum-Primer_CGSR_LR_Jul18.pdf.

[27] J. M. Taylor, P. Cappellaro, L. Childress, L. Jiang, D. Budker, P. R. Hemmer, A. Yacoby, R. Walsworth, and M. D. Lukin. "High-sensitivity diamond magnetometer with nanoscale resolution". en. In: *Nature Physics* 4.10 (Oct. 2008), pp. 810–816. ISSN: 1745-2473, 1745-2481. DOI: 10.1038/nphys1075. URL: https://www.nature.com/articles/nphys1075 (visited on 05/06/2025).

[28] J. Zhang, L. Xu, G. Bian, P. Fan, M. Li, W. Liu, and H. Yuan. *Diamond Nitrogen-Vacancy Center Magnetometry: Advances and Challenges*. arXiv:2010.10231 [physics].

Oct. 2020. DOI: 10.48550/arXiv.2010.10231. URL: http://arxiv.org/abs/2010.10231 (visited on 05/06/2025).

[29]  Y. Ma et al. "Ultrasensitive SERF atomic magnetometer with a miniaturized hybrid vapor cell". en. In: *Microsystems & Nanoengineering* 10.1 (Aug. 2024), p. 121. ISSN: 2055-7434. DOI: 10.1038/s41378-024-00758-6. URL: https://www.nature.com/articles/s41378-024-00758-6 (visited on 05/06/2025).

[30]  G. A. Acosta. "Cold Atom System Development and Characterization for Atomic Clock and Magnetometer Experiments". Pages: 13 ADS Bibcode: 2023MsT.........13A. MA thesis. Delaware State University, Jan. 2023. URL: https://ui.adsabs.harvard.edu/abs/2023MsT.........13A (visited on 05/06/2025).

[31]  T. Ruster, H. Kaufmann, M. A. Luda, V. Kaushal, C. T. Schmiegelow, F. Schmidt-Kaler, and U. G. Poschinger. "Entanglement-Based dc Magnetometry with Separated Ions". en. In: *Physical Review X* 7.3 (Sept. 2017), p. 031050. ISSN: 2160-3308. DOI: 10.1103/PhysRevX.7.031050. URL: https://link.aps.org/doi/10.1103/PhysRevX.7.031050 (visited on 05/06/2025).

[32]  W. C. Campbell and P. Hamilton. "Rotation sensing with trapped ions". In: *Journal of Physics B: Atomic, Molecular and Optical Physics* 50.6 (Mar. 2017), p. 064002. ISSN: 0953-4075, 1361-6455. DOI: 10.1088/1361-6455/aa5a8f. URL: https://iopscience.iop.org/article/10.1088/1361-6455/aa5a8f (visited on 05/06/2025).

[33]  Q. Iqbal and A. H. Nizamani. "Scalable surface ion trap design for magnetic quantum sensing and gradiometry". en. In: *Physics Open* 19 (May 2024), p. 100208. ISSN: 26660326. DOI: 10.1016/j.physo.2024.100208. URL: https://linkinghub.elsevier.com/retrieve/pii/S2666032624000061 (visited on 05/06/2025).

[34]  M. Buchner, K. Höfler, B. Henne, V. Ney, and A. Ney. "Tutorial: Basic principles, limits of detection, and pitfalls of highly sensitive SQUID magnetometry for nanomagnetism and spintronics". en. In: *Journal of Applied Physics* 124.16 (Oct. 2018), p. 161101. ISSN:

0021-8979, 1089-7550. DOI: [10.1063/1.5045299](10.1063/1.5045299). URL: [https://pubs.aip.org/jap/article/124/16/161101/1030088/Tutorial-Basic-principles-limits-of-detection-and](https://pubs.aip.org/jap/article/124/16/161101/1030088/Tutorial-Basic-principles-limits-of-detection-and) (visited on 05/06/2025).

[35] N. N. Gusarov, M. R. Perelshtein, P. J. Hakonen, and G. S. Paraoanu. "Optimized emulation of quantum magnetometry via superconducting qubits". en. In: *Physical Review A* 107.5 (May 2023), p. 052609. ISSN: 2469-9926, 2469-9934. DOI: [10.1103/PhysRevA.107.052609](10.1103/PhysRevA.107.052609). URL: [https://link.aps.org/doi/10.1103/PhysRevA.107.052609](https://link.aps.org/doi/10.1103/PhysRevA.107.052609) (visited on 05/06/2025).

[36] S. Sengottuvel, M. Mrózek, M. Sawczak, M. J. Głowacki, M. Ficek, W. Gawlik, and A. M. Wojciechowski. "Wide-field magnetometry using nitrogen-vacancy color centers with randomly oriented micro-diamonds". en. In: *Scientific Reports* 12.1 (Oct. 2022), p. 17997. ISSN: 2045-2322. DOI: [10.1038/s41598-022-22610-5](10.1038/s41598-022-22610-5). URL: [https://www.nature.com/articles/s41598-022-22610-5](https://www.nature.com/articles/s41598-022-22610-5) (visited on 05/06/2025).

[37] M. V. Balabas, T. Karaulanov, M. P. Ledbetter, and D. Budker. "Polarized Alkali-Metal Vapor with Minute-Long Transverse Spin-Relaxation Time". In: *Phys. Rev. Lett.* 105 (7 Aug. 2010), p. 070801. DOI: [10.1103/PhysRevLett.105.070801](10.1103/PhysRevLett.105.070801). URL: [https://link.aps.org/doi/10.1103/PhysRevLett.105.070801](https://link.aps.org/doi/10.1103/PhysRevLett.105.070801).

[38] K. Sycz, A. M. Wojciechowski, and W. Gawlik. "Atomic-state diagnostics and optimization in cold-atom experiments". en. In: *Scientific Reports* 8.1 (Feb. 2018), p. 2805. ISSN: 2045-2322. DOI: [10.1038/s41598-018-20522-x](10.1038/s41598-018-20522-x). URL: [https://www.nature.com/articles/s41598-018-20522-x](https://www.nature.com/articles/s41598-018-20522-x) (visited on 05/06/2025).

[39] J. S. Bennett, B. E. Vyhnalek, H. Greenall, E. M. Bridge, F. Gotardo, S. Forstner, G. I. Harris, F. A. Miranda, and W. P. Bowen. "Precision Magnetometers for Aerospace Applications: A Review". en. In: *Sensors* 21.16 (Aug. 2021), p. 5568. ISSN: 1424-8220. DOI: [10.3390/s21165568](10.3390/s21165568). URL: [https://www.mdpi.com/1424-8220/21/16/5568](https://www.mdpi.com/1424-8220/21/16/5568) (visited on 05/06/2025).

[40]  G. Oelsner, V. Schultze, R. IJsselsteijn, and R. Stolz. "Performance analysis of an optically pumped magnetometer in Earth's magnetic field". en. In: *EPJ Quantum Technology* 6.1 (Dec. 2019), p. 6. ISSN: 2662-4400, 2196-0763. DOI: 10.1140/epjqt/s40507-019-0076-9. URL: https://epjquantumtechnology.springeropen.com/articles/10.1140/epjqt/s40507-019-0076-9 (visited on 05/14/2025).

[41]  Y. Cohen, K. Jadeja, S. Sula, M. Venturelli, C. Deans, L. Marmugi, and F. Renzoni. "A cold atom radio-frequency magnetometer". en. In: *Applied Physics Letters* 114.7 (Feb. 2019), p. 073505. ISSN: 0003-6951, 1077-3118. DOI: 10.1063/1.5084004. URL: https://pubs.aip.org/apl/article/114/7/073505/37107/A-cold-atom-radio-frequency-magnetometer (visited on 05/14/2025).

[42]  E. R. Potter. "Development and demonstration of a high bandwidth, ultra sensitive trapped ion magnetometer". In: (Sept. 2019). URL: https://sussex.figshare.com/articles/thesis/Development_and_demonstration_of_a_high_bandwidth_ultra_sensitive_trapped_ion_magnetometer/23472335.

[43]  D. J. Griffiths. *Introduction to electrodynamics*. eng. 4. ed., international ed. Always learning. Boston: Pearson, 2013. ISBN: 978-0-321-85656-2 978-0-321-84781-2.

[44]  N. Bort-Soldevila, J. Cunill-Subiranas, N. Del-Valle, W. Wieczorek, G. Higgins, M. Trupke, and C. Navau. *Modelling magnetically-levitated superconducting ellipsoids, cylinders and cuboids for quantum magnetomechanics*. arXiv:2404.05710 [physics]. Apr. 2024. DOI: 10.48550/arXiv.2404.05710. URL: http://arxiv.org/abs/2404.05710 (visited on 05/09/2025).

[45]  J. A. Cape and J. M. Zimmerman. "Magnetization of Ellipsoidal Superconductors". en. In: *Physical Review* 153.2 (Jan. 1967), pp. 416–421. ISSN: 0031-899X. DOI: 10.1103/PhysRev.153.416. URL: https://link.aps.org/doi/10.1103/PhysRev.153.416 (visited on 04/29/2025).

[46]  P. Alken et al. "International Geomagnetic Reference Field: the thirteenth generation". en. In: *Earth, Planets and Space* 73.1 (Dec. 2021), p. 49. ISSN: 1880-5981. DOI: 10.1186/s40623-020-01288-x. URL: https://earth-planets-space.springeropen.com/articles/10.1186/s40623-020-01288-x (visited on 05/09/2025).

[47]  J. A. Osborn. "Demagnetizing Factors of the General Ellipsoid". en. In: *Physical Review* 67.11-12 (June 1945), pp. 351–357. ISSN: 0031-899X. DOI: 10.1103/PhysRev.67.351. URL: https://link.aps.org/doi/10.1103/PhysRev.67.351 (visited on 05/09/2025).

[48]  P. M. Moser. *Magnetic Signatures of Submarines I.* Tech. rep. N62269-91-C-0533. Warminster, PA: Pacific-Sierra Research Corporation for the Naval Air Warfare Center, June 1994. URL: https://apps.dtic.mil/sti/pdfs/AD1012958.pdf (visited on 05/09/2025).

[49]  C. Isa, Hasril Nain, N. Hassanudin, A. Rauf, A. Manap, Roslan Slamatt, and Hambali Anuar. *AN OVERVIEW OF SHIP MAGNETIC SIGNATURE AND SILENCING TECHNOLOGIES.* 2019. DOI: 10.13140/RG.2.2.14643.58401. URL: http://rgdoi.net/10.13140/RG.2.2.14643.58401 (visited on 05/09/2025).

[50]  M. Hirota. "High Temperature Superconducting (HTS) Cable Application to Ship Deperming Work". In: *Journal of Physics: Conference Series* 1559.1 (June 2020), p. 012132. ISSN: 1742-6588, 1742-6596. DOI: 10.1088/1742-6596/1559/1/012132. URL: https://iopscience.iop.org/article/10.1088/1742-6596/1559/1/012132 (visited on 05/09/2025).

[51]  I. Bellany. "Sea power and the Soviet submarine forces". en. In: *Survival* 24.1 (Jan. 1982), pp. 2–8. ISSN: 0039-6338, 1468-2699. DOI: 10.1080/00396338208442007. URL: http://www.tandfonline.com/doi/abs/10.1080/00396338208442007 (visited on 05/09/2025).