ON THE EFFECT OF REDUNDANCY ON THE MULTIPLE ACCESS BROADCAST CHANNEL

by

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B.Sc., University of Nigeria
(1975)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF

MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

(FEBRUARY, 1979)

Signature of Author . . . .
Department of Electrical Engineering and Computer
Science, January 12, 1979

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Chairman, Departmental Committee on Graduate Studies

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Submitted to the Department of Electrical Engineering
and Computer Science on January 12, 1979, in partial fulfillment of the requirements for the Degree of Master of Science.

ABSTRACT

We consider strategies for transmitting packets over a multiple access broadcast channel. In the strategies considered, a message is formatted into K packets and N-K redundant packets are added to these K packets to form what we call a superpacket. We propose a simple coding scheme that sets these N packets of a superpacket into a special temporal pattern. With a superpacket constructed in this manner, it is possible to reconstruct packets which are lost through collision on the channel with the packets of other superpackets. We compare the throughput of our scheme with that of the slotted Aloha scheme. We use as our retransmission strategy the conflict resolving tree algorithm of Capetanakis.

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Title: Professor of Electrical Engineering

ACKNOWLEDGEMENTS

I wish to express my sincere gratitude to Professor James L. Massey who started me on this research before leaving for UCLA. He guided me through the first half of the research. Professor Gallager graciously encouraged and guided me throughout the remaining part of the research. I am greatly indebted to him.

I wish to thank my wife, Christina, for her encouragement to me throughout the period of the research. Finally, I wish to thank Terry Crossley who typed this thesis, and to thank Arthur Giordani for doing the drawings.

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*Dedicated to Christiana^2 (mother and wife)
and Benedict Ibe (father).*

CHAPTER 1

INTRODUCTION

1.1 The Problem

A broadcast channel is a communication channel in which a signal generated at one information source can be received by many receivers. Examples of a broadcast channel include a satellite channel and a ground radio network. A major issue in designing any communication channel is to ensure that a signal from one source passes through the channel without encountering any interference from signals from other sources. In other words, the channel is to be so designed that many individual signals can be transmitted simultaneously over that channel.

Two popular techniques used in solving this problem are frequency division multiplex (FDM) and time division multiplex (TDM). In FDM the channel frequency spectrum is partitioned into many frequency bands and each source is allocated one such band. In TDM the channel time is divided into time slots and one slot is dedicated to each source. If these sources transmit very frequently, then the channel utilization will be high. However, in many applications the messages transmitted may be short in duration and infrequent in occurrence; that is, the applications have low duty cycle, as in a time-shared application or an inquiry-response system. Under such situations, dedicating a frequency band or a time slot to each source will be uneconomical. It may, therefore, be appropriate to allow sources to transmit their messages at will, whenever they have any messages to transmit, using the entire channel frequency or time. Because of the random nature of the message generation process it may turn out that the probability of two or more sources attempting to transmit simultaneously is very low. This technique in which many sources transmit at will and randomly over a common channel is called the random access technique. The random access technique, FDM and TDM are all forms of the multiple access technique; that is a technique in which two or more sources share the channel at the same time. The type of multiple access technique we are concerned with here is the random access technique which is finding a wide application in data transmission.

In this technique, when a source has a message to transmit it formats the message into packets. A packet will include the source and destination addresses and other overhead information such as parity-check bits for error detection and error correction. The packet is then burst out onto the channel, which from now on we assume is a satellite channel. The satellite receives the packet and, acting as a transponder, beams it back to the earth where it can be received by the sources. By being able to receive its transmitted packet a source can then know whether or not its transmission was successful. When two or more sources transmit their packets at the same time, the packets will collide on the channel and the intelligibility of each will be destroyed. Such packets which suffer a collision are rescheduled for transmission. The main issue in this technique then is how to formulate transmission and retransmission strategies that produce acceptable levels of performance.

The first analysis of this type of multiple access broadcast scheme was carried out by Abramson [1] in the so-called pure Aloha scheme. He made the following assumptions.

1. There are infinitely many sources in the system.
2. Each packet has a fixed length of τ seconds.
3. The starting times of all packets newly generated by the sources form a stationary Poisson point process with mean λ_n.
4. When a collision occurs the packets are not retransmitted immediately; they are retransmitted randomly after a given time has elapsed. The starting times of all retransmitted packets are also assumed to form a stationary Poisson point process with mean λ_r.

Since the sum of two independent stationary Poisson point processes is a Poisson random variable, assumptions (3) and (4) then imply that the starting times of all packets presented to the channel for transmission form a stationary Poisson point process with mean $\lambda = \lambda_n + \lambda_r$. With these assumptions Abramson obtained a throughput of 0.184. Roberts [8] suggested that considerable improvements could be made if the channel is divided into slots whose lengths are τ seconds and the packets are synchronized to these slots. With the same assumptions made by Abramson he obtained a throughput of 0.368 packets/slot, twice that for the pure Aloha. This later version is known as the slotted Aloha.

However, the Aloha scheme is unstable. For a large number of sources the Poisson assumption made for the newly generated packets may be valid. However, the Poisson assumption made for the retransmitted packets can only be valid as long as the traffic is low. When the system is operating near the saturation point the assumption is no longer valid. This is so because, due to the high packet rate, collisions become so frequent that very few transmissions are successful. More sources are in the retransmission mode, more collisions occur and soon the system breaks down as eventually no successful transmission is made.

Several schemes have been proposed for increasing the throughput of the Aloha scheme, but unfortunately these have not cured the stability problem of the scheme. Some of these throughput improvement schemes include the dynamic reservation schemes proposed by Crowther and others [4], Binders [2], and Roberts [9]; and the carrier sense scheme proposed by Kleinrock and Tobagi [5]. The binary tree algorithm proposed by Capetanakis [3] is a scheme which is not only superior to the Aloha scheme in terms of thoughput but is also stable. We shall discuss this scheme in Chapter 3.

All the above schemes deal with the transmission of one packet at a time, the irredundant transmission. When a source uses the multiple access broadcast channel to transmit a long message it is possible to transmit the entire message at a time by breaking it into many packets and encoding the message in such a way that a number of these packets lost due to collision on the channel may be reconstructed at the receiver. We consider here a simple way in which this encoding may be done by introducing redundancy to the message. This idea was first proposed by Massey [6] and we shall study this scheme by comparing its performance with that of the slotted Aloha scheme. We shall also use the binary tree algorithm to resolve conflicts that arise in the transmission.

1.2 The Superpacket Structure

Let K information packets be formed from a message which is to be transmitted. Let (N-K) parity packets be generated from these K information packets by some operations performed on them, such as the modulo-2 addition of a number of them. We define the N-packet structure so formed as a superpacket. Suppose now that we have a spatial arrangement of the N packets in a manner that their starting times form a simple difference-set. A simple difference-set is a set of N integers $\{i_1, i_2, \ldots., i_N\}$ such that the set of N(N-1) differences $\{i_j-i_k | j \neq k\}$ are all distinct [6]. Thus, let the spatial pattern of the superpacket be specified by the index vector $\{i_1, i_2, \ldots., i_N\}$, where $0 \leq i, < i_2 < \ldots < i_N$. That is, if the starting time of the superpacket is t then the j^{th} packet in the superpacket starts at time $t+i_j$. The structure of a superpacket is shown in Figure 1.1. Here N=3, K=2 and the parity packet is the modulo-2 sum of the two information packets. The superpacket has the index vector (0, 1, 3) and τ is the length of one channel slot.

The starting time of a superpacket is synchronized to the beginning of a channel slot. When more than one packet is present in any slot at the same time the intelligibility of each packet will be destroyed. For an irredundant transmission, the transmitted packets

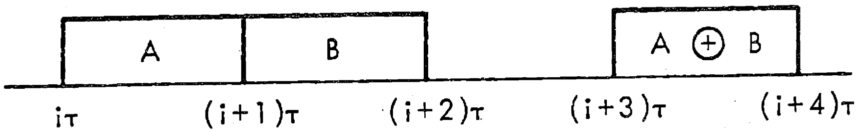


Figure 1.1 An Example of a Superpacket: N=3, k=2.

will not be accurately received at their destinations; they will have to be retransmitted. However, for a redundant transmission using the superpacket configuration described, it is possible to reconstruct a lost packet from other correctly received packets. It is also possible that the lost packet cannot be reconstructed at the receiver, in which case we assume that the whole superpacket will have to be retransmitted.

We have chosen the index vector of the superpacket so that its components form a simple difference-set for the following reasons.

Proposition 1: When and only when the components of the index vector form a simple difference-set any two colliding transmitted superpackets collide either in all N components or in only one component packet.

Proof (Massey [6])

Suppose first that the components of the index vector form a simple difference-set and that a superpacket starting in slot j collides with a superpacket starting in slot j' of the channel in at least two packets. Then for some integers p, q, r and s we have that

$$\begin{aligned} j + i_p &= j' + i_q \\ j + i_r &= j' + i_s \end{aligned} \qquad (1)$$

That is, $i_p - i_r = i_q - i_s$

But the defining property of a simple difference-set then implies that p=q and r=s, and hence that j=j', so the superpackets collide in all N packets. Conversely, if the indices do not form a simple difference-set, there exist $p \neq q$ such that $i_p - i_r = i_q - i_s$, and hence there exist $j \neq j'$ such that (1) is satisfied. But this implies that two superpackets can collide in more than one, but less than N packets. Q.E.D.

Proposition 2: For a given packet of a given transmitted superpacket, there are exactly N-1 starting slots for a transmitted superpacket that collides with the given superpacket in only the given packet. Moreover, these N-1 slots are disjoint from the corresponding N-1 slots for any other packet of the given superpacket.

To state this proposition in another way, consider the j^{th} packet of a given transmitted superpacket. A single-packet collision of this superpacket and any other transmitted superpacket implies that the starting times of the two superpackets are different. By Proposition 1 if their starting times are the same then and only then will they collide in all N packets. Excluding, therefore, this particular case, Proposition 2 states that there are N-1 other slots in which another superpacket can start and cause a collision with the j^{th} packet of the current superpacket. Also the slots in which that superpacket can start and cause a collision with the j^{th} packet of the current superpacket are different from the slots in which the superpacket can start and cause a single packet collision with any other packet of the current superpacket. The proof is thus obvious and we shall omit it.

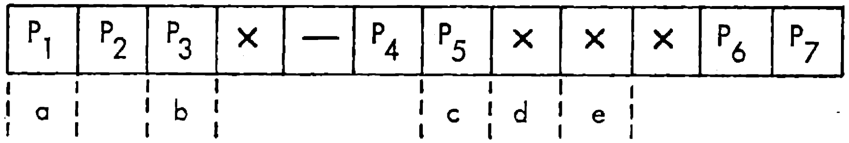


Figure 1.2 An Example of a Received Packet Sequence.

1.3 Reconstructing the Superpacket at the Receiver

Generally a receiver can reconstruct a superpacket if any K or more packets of the superpacket are correctly received. In this section we shall restrict our discussion to the superpackets with index vector (0, 1, 3). Then if any two or all packets of the superpacket are correctly received, the receiver can reconstruct the superpacket.

If each packet of a superpacket carries a number indicating its position in the superpacket, the problem of reconstructing the superpacket at the receiver becomes trivial. However, in the absence of such position identity it is still possible to reconstruct a superpacket which has not lost more than one packet due to collision on the channel.

Consider, for example, a receiver that has received a packet sequence shown in Figure 1.2. The P_i indicate packets that were correctly received; X indicates a collision and a dash indicates a position where nothing was received. Since each receiver knows the superpacket configuration, then assuming nothing was received for a long time before P_1 was received, the receiver will associate P_1 and P_2 with one superpacket. P_3 then is the first packet of a second superpacket; its second packet was involved in the first collision with the redundant packet of the first superpacket. P_4 then is the redundant packet of this second superpacket. Hence the receiver can reconstruct the first two superpackets. The receiver will identify P_5 as the first packet of a third superpacket. Unfortunately three other collisions have obliterated the two other packets of the superpacket and the first two packets of each of two other superpackets. Precisely, the second packet of this third superpacket collided with the first packet of a fourth superpacket at the position marked d. And the redundant packet of the third superpacket collided with the second packet of a fifth superpacket. The second packet of the fourth superpacket collided with the first packet of the fifth superpacket. P_6 and P_7 are the redundant packets of the fourth and fifth superpackets respectively. Thus the third, fourth, and fifth superpackets cannot be reconstructed at the receiver. The alphabets a, b, ..., e indicate the starting packets of the five superpackets. When many collisions have occurred it may turn out that the superpackets are not reconstructible at the receiver, probably due to loss of two or more packets of each superpacket or due to lack of proper identification of the packets.

CHAPTER 2

THE SLOTTED ALOHA TYPE OF TRANSMISSION

2.1 Assumptions

As we stated earlier, the slotted Aloha scheme is unstable. However, we wish to analyze our scheme when used in the slotted Aloha type of transmission in order to compare its performance with that of the slotted Aloha. We shall make the same assumptions made in the analysis of the slotted Aloha. These include:

(a) there are infinitely many sources in the system;

(b) each packet of the superpacket has a fixed length of τ seconds, the length of each channel slot;

(c) The starting times of all newly generated superpackets form a stationary Poisson point process with mean λ_n;

(d) the starting times of all retransmitted superpackets form a stationary Poisson point process with mean λ_r.

Because the sum of two independent stationary Poisson point processes is a stationary Poisson random variable, assumptions (c) and (d) then imply that the starting times of all superpackets presented to the channel for transmission form a stationary Poisson point process with mean $\lambda = \lambda_n + \lambda_r$. We shall also assume that if the channel receives any packet of the superpacket it acknowledges receipt of that packet by broadcasting it; the same source which transmitted the packet can also hear the broadcast. After transmitting a superpacket the transmitting source waits for a time long enough to receive an acknowledgement for the last packet of the superpacket. If the number of acknowledgements received by the source after this time-out is less than K, the number of information packets in the superpacket, then the entire superpacket is retransmitted randomly. Otherwise, the message transmission is taken to be successful. The process of retransmission and waiting for acknowledgements is repeated until the right number of acknowledgements is received, assuming the system does not break down.

2.2 Analysis of the Scheme

Let N = the number of packets in a superpacket,

K = the number of information packets in the superpacket,

λ = the channel superpacket rate; i.e., all the superpackets (both newly generated and those being retransmitted) arrive at the channel at the rate of λ per slot,

λ_p = the channel packet rate = $N\lambda$.

P_1 = Prob[a given transmission of a superpacket will be successful].

Then P_1K = the expected number of correctly received information packets per attempted superpacket transmission.

A performance measure of the system that we are interested in is the system throughput; that is the expected number of information packets per slot that are successfully transmitted per attempted superpacket transmission. Thus the system throughput, λ_o, is given by

$$\lambda_o = P_1K\lambda \quad \text{information packets per slot}$$

$$= \frac{P_1K\lambda_p}{N}$$

Let A = the event that there is no "direct hit" on the current superpacket.

By a direct hit we mean that two or more superpackets start in the same slot and so completely render one another unintelligible. Similarly, by an indirect hit we mean that two superpackets whose starting times are different collide with each other. According to Proposition 1, an indirect hit gives rise to a loss of only one packet of a superpacket involved in the hit.

Let B = the event that (N-K) or fewer indirect hits are made on the current superpacket.

Then P(A) = Prob[no other superpacket started at the time the transmission of the given superpacket started]

$$= e^{-\lambda}$$

$$= e^{-\lambda p/N}$$

For any packet of the superpacket, the probability of an indirect hit, p, is given by

$$p = 1-e^{-\lambda (N-1)}$$

where $e^{-\lambda (N-1)}$ = Prob[no superpacket starts in any of the (N-1) slots that would cause a collision with a given packet of a transmitted superpacket].

$$\text{Hence } P(B) = \sum_{i=0}^{N-K} \binom{N}{i} p^{i} (1-p)^{N-i}$$

Events A and B are independent and therefore,

$$P_1 = P(A \cap B) = P(A)\ P(B)$$

CASE 1

$$K = N-1; \text{ i.e., one redundant packet.}$$

$$P(B) = \sum_{i=0}^{1} \binom{N}{i} p^{i}(1-p)^{N-i}$$

$$= Ne^{-\lambda(N-1)^2} - (N-1)e^{-\lambda N(N-1)}; \quad N \geq 2.$$

$$= Ne^{-\lambda p(N-1)^2/N} - (N-1)e^{-\lambda p(N-1)}$$

$$P_1 = P(A)\ P(B)$$

$$= e^{-\lambda p/N}\ [Ne^{-\lambda p(N-1)^2/N} - (N-1)e^{-\lambda p(N-1)}]$$

$$\lambda_o = \frac{P_1 K\ \lambda_p}{N} = \frac{P_1(N-1)\lambda_p}{N}$$

$$= \lambda_p[(N-1)\exp\{-\frac{\lambda_p}{N}(N^2-2N+2)\} - \frac{(N-1)^2}{N} \exp\{-\frac{\lambda_p}{N}(N^2-N+1)\}]$$

For the slotted Aloha scheme

$$P_1 = e^{-\lambda p}$$

$$\lambda_o = P_1\lambda_p = \lambda_p e^{-\lambda p}$$

Values of P_1 and λ_o are given in Appendix 2.1A for some values of N and λ_p. Corresponding values are given for the slotted Aloha for comparison.

CASE 2:

$$K = N-2;\text{ i.e., two redundant packets}$$

$$P(B) = \sum_{i=0}^{2} \binom{N}{i} p^{i}(1-p)^{N-i}; \quad N \geq 3$$

$$= \frac{1}{2}N\ (N-1)\ \exp\left[\frac{-\lambda_p (N-1)(N-2)}{N}\right] - N(N-2)\exp\left[\frac{-\lambda_p (N-1)^2}{N}\right]$$

$$+ \frac{1}{2}\ (N-1)(N-2)\ \exp\ [-\lambda_p (N-1)]$$

$$\lambda_o = \frac{P_1 K\ \lambda_p}{N} = \frac{P_1 (N-2)\lambda_p}{N}$$

$$= \frac{P(A)\ P(B)\ (N-2)\lambda_p}{N}$$

$$= \frac{\lambda_p e^{-\lambda_p/N}\ P(B)\ (N-2)}{N}$$

Values of P_1 and λ_o are given in Appendix 2.1B for some values of N and λ_p.

CASE 3:

$$K = N-3; \quad \text{i.e., three redundant packets}$$

$$P(B) = \sum_{i=0}^{3} \binom{N}{i} p^{i} (1-p)^{N-i}; \quad N \geq 4$$

$$= \frac{1}{6} N (N-1)(N-2) \exp[- \frac{\lambda_p (N-1)(N-3)}{N}]$$

$$- \frac{1}{2} N(N-1)(N-3) \exp [-\frac{\lambda p(N-1)(N-2)}{N}]$$

$$+ \frac{1}{2} N(N-2)(N-3) \exp[-\frac{\lambda p(N-1)^2}{N}]$$

$$- \frac{1}{6} (N-1)(N-2)(N-3) \exp [-\lambda_p(N-1)]$$

$$\lambda_o = \frac{P_1 K \lambda_p}{N} = \frac{P(A) \ P(B) \ (N-3) \lambda_p}{N}$$

$$= \frac{\lambda_p \ P(B) \ (N-3) \exp \ (-\frac{\lambda_p}{N})}{N}$$

Values of P_1 and λ_o are given in Appendix 2.1C for some values of N and λ_p.

2.3 The Use of A Different Simple Difference-set for Each Source

Theorem 2.1 Let $D = \{i_1, i_2, \ldots, i_{MN}\}$ be a simple difference-set. Then any M simple difference-sets of N digits whose union is D have the property that if they are used for the spatial patterns of M sources, any collision of superpackets from two of these sources will be a single packet collision.

Proof: Suppose a superpacket from one source starting in slot j collides with a superpacket from a different source starting in slot j' in at least two packets. Then for some integers p, q, r and s

$$\left.\begin{aligned} j + i_p &= j' + i_q \\ j + i_r &= j' + i_s \end{aligned}\right\} (*)$$

Thus $i_p - i_r = i_q - i_s$

By the definition of a simple difference-set p=q and r=s. From (*) it follows that j=j'; hence the two superpackets are identically constructed and must come from the same source, which contradicts the supposition. Thus the two superpackets cannot collide in more than one packet.

Q.E.D.

This theorem implies that the probability of the successful transmission of any superpacket is the probability that (N-K) or fewer packets of the superpacket are lost through indirect hits on the superpacket. Direct hits are impossible.

Thus P_1 = Prob [successful transmission of a superpacket]

= Prob [(N-K) or fewer packets are lost through indirect hits]

$$= \sum_{i=0}^{N-K} \binom{N}{i} p^i (1-p)^{N-i}$$

where $p = 1 - e^{-\lambda N}$

$e^{-\lambda N}$ = the probability that no superpacket starts in any of the N slots that would cause a collision with a given packet of a transmitted superpacket.

CASE 1:

$$K = N-1; \quad \text{i.e., one redundant packet}$$

$$P_1 = \sum_{i=0}^{1} \binom{N}{i} p^i (1-p)^{N-i}; \quad N \geqslant 2$$

$$= N \exp[-\lambda N(N-1)] - (N-1) \exp[-\lambda N^2]$$

$$= N \exp [-\lambda_p(N-1)] - (N-1) \exp (-\lambda_p N) \qquad (*)$$

$$\lambda_o = \frac{P_1 K \lambda_p}{N} = \frac{\lambda_p (N-1) P_1}{N}$$

Theorem 2.2 For $K = N-1$, $P_1(N)$ decreases monotonically for $N = 2,3, \ldots$ when $\lambda_p \geqslant 0$, where $P_1(N)$ is the probability of the successful transmission of a superpacket given as a function of N, the number of packets in the superpacket.

Proof: $P_1(N) = N\exp [-\lambda_p(N-1)] - (N-1) \exp(-\lambda_p N)$

(see (*)).

$$P_1(N)-P_1(N-1) = N \exp[-\lambda_p(N-1)] - (N-1) \exp(-\lambda_p N)$$

$$-(N-1) \exp[-\lambda_p(N-2)] + (N-2) \exp[-\lambda_p(N-1)]$$

$$= 2(N-1) \exp[-\lambda_p(N-1)] - (N-1)\{\exp(-\lambda_p N) + \exp[-\lambda_p(N-2)]\}$$

$$= - (N-1) \exp(-\lambda_p N) [1 -2\exp(\lambda_p) + \exp(2\lambda_p)]$$

$$= - (N-1) \exp(-\lambda_p N) [1 - \exp(\lambda_p)]^2$$

$$< 0 \text{ for } \lambda_p \geqslant 0 \text{ and } N \geqslant 2.$$

Q.E.D.

Values of λ_o and P_1 are given for selected values of λ_p and N in Appendix 2.2A.

CASE 2:

$$K = N-2; \quad \text{i.e., two redundant packets.}$$

$$P_1 = \sum_{i=0}^{2} \binom{N}{i} p^i (1-p)^{N-i}; \quad N \geq 3.$$

$$= \frac{1}{2} N(N-1) \exp[-\lambda_p (N-2)] \; -N(N-2)\exp[-\lambda_p (N-1)]$$

$$+ \frac{1}{2}(N-1)(N-2)\exp(-\lambda_p N)$$

$$\lambda_o = \frac{P_1 K \lambda_p}{N} = \frac{\lambda_p (N-2) P_1}{N}$$

Values of λ_o and P_1 are given in Appendix 2.2B for selected values of λ_p and N.

CASE 3:

$$K = N-3; \quad \text{three redundant packets.}$$

$$P_1 = \sum_{i=0}^{3} \binom{N}{i} p^i (1-p)^{N-i}; \quad N \geq 4$$

$$= \frac{1}{6} N(N-1)(N-2)\exp[-\lambda_p (N-3)] - \frac{1}{2} N(N-1)(N-3)\exp[-\lambda_p (N-2)]$$

$$+ \frac{1}{2} N(N-2)(N-3)\exp[-\lambda_p (N-1)] - \frac{1}{6}(N-1)(N-2)(N-3)\exp(-\lambda_p N)$$

$$\lambda_o = \frac{P_1 K \lambda_p}{N} = \frac{(N-3)\lambda_p P_1}{N}$$

Values of λ_o and P_1 for selected values values of N and λ_p are given in Appendix 2.2C.

2.4 Finite Number of Groups of Sources

In the previous section we considered the situation where each source has a different simple difference-set. However, as the size of the set D increases, the delay of the superpackets increases rapidly. We, therefore, consider a situation where we have η groups of sources and the member sources of each group have superpackets with the same index vector; the index vector of one group is different from those of other groups. We further make the following assumptions and remarks.

(1) There are the same number of sources in each group and the total number of sources is infinite.

(2) Direct hits on a given superpacket are only possible from members of the same group as the source transmitting the superpacket.

(3) Indirect hits on a given superpacket are possible from all members of the system.

(4) The overall channel superpacket rate $= \lambda$,
the overall channel packet rate $= \lambda_p$.

(5) Adding a fixed integer to every element of a simple difference-set results in another simple difference-set. No generality is lost, therefore, in considering only simple difference-sets containing the element 0. That is, we assume that in all the groups $i_1 = 0$. For example, consider the simple difference-set D=(0, 1, 3, 7, 12, 20). If we wish to generate two groups of sources from D, then these groups will have the index vectors (0, 1, 3) and (7, 12, 20). Adding -7 to every element of the second group gives (0, 5, 13) which is still a simple difference-set that is different from (0, 1, 3).

We shall restrict our analysis to the case of K=N-1.

Let P_1 = the probability that a given transmission of a superpacket will be successful.

= the probability that one or no packet of the given superpacket is lost through collision.

= the probability that there is no direct hit on the superpacket <u>and</u> no more than one packet is lost through indirect hits.

Let A = the event that there is no direct hit on the given superpacket.

B = the event that no more than one packet of the superpacket is lost through indirect hits.

Then events A and B are independent and so

$$P_1 = P(A)\ P(B)$$

For any given superpacket the probability of an indirect hit on a given packet from members of the same group as the superpacket is

$$q_1 = 1 - \exp[-\frac{1}{\eta}\lambda(N-1)]$$
$$= 1 - \exp[-\frac{\lambda p(N-1)}{\eta N}]$$

where we have assumed that the number of all superpackets presented to the channel for transmission is a Poisson random variable with mean λ. Note also that η is the number of groups in the system. The probability of an indirect hit from members of other groups is

$$q_2 = 1 - \exp[-N\lambda(\frac{\eta-1}{\eta})]$$

$$= 1 - \exp[-\frac{(\eta-1)}{\eta}\ \lambda_p]$$

For any given superpacket the probability of a direct hit is

$$q = 1 -\exp[-\frac{\lambda}{\eta}]$$

$$1 -\exp(-\frac{\lambda p}{\eta N})$$

Hence $P(A) \;= 1 - q = \exp\;(-\frac{\lambda p}{\eta N})$

The event B is composed as follows:

B_1 = the event that there are no indirect hits on the given superpacket;

B_2 = the event that there is one indirect hit on the given superpacket from members of other groups and none from members of the same group as the superpacket;

B_3 = the event that there is one indirect hit on the given superpacket from members of the same group as the superpacket and none from members of other groups.

Since events B_1, B_2 and B_3 are disjoint we may write

$$P(B) = P(B_1) + P(B_2) + P(B_3)$$

Let us also define the following events.

C_1 = the event that there is no indirect hit on the given superpacket from members of the same group as the superpacket,

C_2 = the event that there is no indirect hit on the given superpacket from members of other groups,

D_1 = the event that there is one indirect hit on the given superpacket from members of the same group as the superpacket,

D_2 = the event that there is one indirect hit on the given superpacket from members of other groups.

Then $B_1 = C_1 \cap C_2$

$B_2 = C_1 \cap D_2$

$B_3 = C_2 \cap D_1$

Events C_1 and C_2 are independent; events C_1 and D_2 are also independent, and events C_2 and D_1 are independent. Hence

$$\begin{aligned} P(B) &= P(C_1)\ P(C_2) + P(C_1)\ P(D_2) + P(C_2)\ P(D_1) \\ &= (1-q_1)^N\ (1-q_2)^N + (1-q_1)^N \binom{N}{1} q_2 (1-q_2)^{N-1} \\ &\qquad + (1-q_2) \binom{N}{1} q_1 (1-q_1)^{N-1} \\ &= N\ \exp[-\lambda_p(N-1)] - (2N-1)\exp[-\lambda_p(\frac{\eta N-1}{\eta})] \\ &\qquad + N\exp[-\lambda_p(\frac{\eta N^2-2N+1}{\eta N})] \end{aligned}$$

$$P(A) = \exp\ (-\frac{\lambda_p}{\eta N})$$

$$\therefore P_1 = P(A)\ P(B)$$

$$\lambda_o = (\frac{N-1}{N})\lambda_p\ P_1$$

As a check, (a) $\lim\limits_{\eta \to 1} P_1 = N\exp[-\frac{\lambda_p}{N}(N^2-2N+2)]$

$$-(N-1)\exp[-\frac{\lambda_p}{N}(N^2-N+1)]$$

which is the same result obtained in Case 1 of Section 2.2;

(b) $\lim\limits_{\eta \to \infty} P_1 = N\exp[-\lambda_p(n-1)] - (N-1)\exp(-\lambda_p N)$

which is the same result obtained in Case 1 of Section 2.3.

Consider the special case of N = 2. Then

$$P_1 = 2\ \exp[-\frac{\lambda p}{2\eta}\ (2\eta+1)] - 3\exp[-\frac{\lambda p}{2\eta}\ (4\eta-1)]$$

$$+ 2\exp[-\frac{\lambda p}{2\eta}\ (4\eta-2)]$$

$$\lambda_o = \frac{1}{2}\ \lambda_p\ P_1$$

Values of P_1 and λ_o are given in Appendix 2.3 for some selected values of η and λ_p.

APPENDIX 2

Values of λ and P_1

A2.1 All Superpackets Have The Same Configuration

A2.1A K = N-1

	N = 2		N = 3		N = 4		Slotted Aloha	
λ_p	P_1	λ_o	P_1	λ_o	P_1	λ_o	P_1	λ_o
0.1	0.9490	0.0474	0.9557	0.0637	0.9476	0.0711	0.9048	0.0905
0.2	0.8966	0.0897	0.8954	0.0094	0.8660	0.1290	0.8187	0.1637
0.4	0.7918	0.1584	0.7538	0.2010	0.6539	0.1962	0.6703	0.2681
0.6	0.6911	0.2073	0.6104	0.2442	0.4657	0.2096	0.5488	0.3293
0.8	0.5975	0.2390	0.4815	0.2568	0.3185	0.1911	0.4493	0.3595
1.0	0.5126	0.2563	0.3727	0.2485	0.2120	0.1590	0.3679	0.3679
1.2	0.4371	0.2623	0.2844	0.2275	0.1384	0.1246	0.3012	0.3614
1.4	0.3707	0.2595	0.2146	0.2003	0.0891	0.0935	0.2466	0.3452
1.6	0.3131	0.2505	0.1606	0.1713			0.2019	0.3230
1.8	0.2634	0.2371	0.1194	0.1432			0.1653	0.2975
2.0	0.2209	0.2209	0.0882	0.1176			0.1353	0.2706
2.2	0.1847	0.2032					0.1108	0.2437
2.4	0.1541	0.1849					0.0907	0.2177
2.6	0.1283	0.1668					0.0743	0.1931
2.8	0.1066	0.1493					0.0608	0.1703
3.0	0.0885	0.1327					0.0498	0.1498

A2.1B: K = N-2

	N = 3		N = 4		N = 5		Slotted Aloha	
λ_p	P_1	λ_o	P_1	λ_o	P_1	λ_o	P_1	λ_o
0.1	0.9670	0.0322	0.9730	0.0487	0.09762	0.0586	0.9048	0.0905
0.2	0.9337	0.0622	0.9420	0.0942	0.9362	0.1123	0.8187	0.1636
0.4	0.8639	0.1152	0.8541	0.1708	0.8028	0.1927	0.6703	0.2681
0.6	0.7894	0.1579	0.7414	0.2224	0.6337	0.2281	0.5488	0.3293
0.8	0.7118	0.1898	0.6197	0.2479	0.4696	0.2254	0.4493	0.3595
1.0	0.6340	0.2113	0.5023	0.2511	0.3321	0.1993	0.3679	0.3679
1.2	0.5584	0.2234	0.3972	0.2383	0.2268	0.1633	0.3012	0.3614
1.4	0.4870	0.2273	0.3079	0.2155	0.1508	0.1267	0.2466	0.3452
1.6	0.4212	0.2246	0.2349	0.1879	0.0982	0.0943	0.2019	0.3230
1.8	0.3615	0.2169	0.1769	0.1592			0.1653	0.2975
2.0	0.3084	0.2056	0.1318	0.1318			0.1353	0.2706
2.2	0.2616	0.1919	0.0973	0.1071			0.1108	0.2437
2.4	0.2209	0.1767					0.0907	0.2177
2.6	0.1858	0.1610					0.0743	0.1931
2.8	0.1557	0.1453					0.0608	0.1703
3.0	0.130.	0.1301					0.0498	0.1494

A2.1C: K = N-3

	N = 4		N = 5		N = 6		Slotted Aloha	
λ_p	P_1	λ_o	P_1	λ_o	P_1	λ_o	P_1	λ_o
0.1	0.9753	0.0244	0.9800	0.0392	0.9829	0.0491	0.9048	0.0905
0.2	0.9509	0.0475	0.9588	0.0767	0.9610	0.0961	0.8187	0.1637
0.4	0.9008	0.0901	0.9028	0.1445	0.8811	0.1762	0.6603	0.2681
0.6	0.8459	0.1269	0.8218	0.1972	0.7507	0.2252	0.5488	0.3293
0.8	0.7848	0.1570	0.7199	0.2304	0.5960	0.2384	0.4493	0.3595
1.0	0.7184	0.1796	0.6081	0.2433	0.4462	0.2231	0.3679	0.3679
1.2	0.6489	0.1947	0.4978	0.2390	0.3187	0.1912	0.3012	0.3614
1.4	0.5788	0.2026	0.3969	0.2222	0.2194	0.1536	0.2466	0.3452
1.6	0.5105	0.2042	0.3095	0.1981	0.1467	0.1173	0.2019	0.3230
1.8	0.4456	0.2005	0.2370	0.1706	0.0958	0.0862	0.1653	0.2975
2.0	0.3856	0.1928	0.1787	0.1430			0.1353	0.2706
2.2	0.3311	0.1821	0.1331	0.1171			0.1108	0.2437
2.4	0.2824	0.1694	0.0981	0.0942			0.0907	0.2177
2.6	0.2395	0.1557					0.0743	0.1931
2.8	0.2021	0.01415					0.0608	0.1703
3.0	0.1698	0.1274					0.0494	0.1494

A2.2 Each Superpacket Differently Constructed

A2.2A K = N-1

	N = 2		N = 3		N = 4		Slotted Aloha	
λ_p	P_1	λ_o	P_1	λ_o	P_1	λ_o	P_1	λ_o
0.1	0.9909	0.0495	0.9746	0.0650	0.9523	0.0714	0.9048	0.0905
0.2	0.9671	0.0967	0.9133	0.1218	0.8473	0.1271	0.8187	0.1637
0.4	0.8913	0.1783	0.7456	0.1988	0.5991	0.1797	0.6703	0.2681
0.6	0.7964	0.2389	0.5730	0.2292	0.3890	0.1751	0.5488	0.3293
0.8	0.6968	0.2787	0.4243	0.2262	0.2406	0.1444	0.4493	0.3595
1.0	0.6004	0.3002	0.3064	0.2043	0.1442	0.1082	0.3679	0.3679
1.2	0.5117	0.3070	0.2175	0.1740	0.0846	0.0761	0.3012	0.3614
1.4	0.4324	0.3027	0.1524	0.1423			0.2466	0.3452
1.6	0.3630	0.2904	0.1058	0.1129			0.2019	0.3230
1.8	0.3033	0.2729					0.1653	0.2975
2.0	0.2524	0.2524					0.1353	0.2706
2.2	0.2093	0.2303					0.1108	0.2437
2.4	0.1732	0.2078					0.0907	0.2177
2.6	0.1430	0.1859					0.0743	0.1931
2.8	0.1179	0.1651					0.0608	0.1703
3.0	0.0971	0.1456					0.0498	0.1494

A2.2B: K = N-2

	N = 3		N = 4		N = 5		Slotted Aloha	
λ_p	P_1	λ_o	P_1	λ_o	P_1	λ_o	P_1	λ_o
0.1	0.9991	0.0333	0.9968	0.0498	0.9926	0.0596	0.9048	0.0905
0.2	0.9940	0.0663	0.9794	0.0979	0.9555	0.1147	0.8187	0.1637
0.4	0.9642	0.1286	0.8921	0.1784	0.7955	0.1909	0.6703	0.2681
0.6	0.9082	0.1816	0.7569	0.2271	0.5909	0.2127	0.5488	0.3293
0.8	0.8330	0.2221	0.6079	0.2432	0.4056	0.1947	0.4493	0.3595
1.0	0.7474	0.2491	0.4687	0.2343	0.2636	0.1581	0.3679	0.3679
1.2	0.6588	0.2635	0.3504	0.2102	0.1647	0.1186	0.3012	0.3614
1.4	0.5724	0.2671	0.2566	0.1792	0.1000	0.0840	0.2466	0.3452
1.6	0.4916	0.2622	0.1837	0.1470			0.2019	0.3230
1.8	0.4184	0.2511	0.1300	0.1170			0.1653	0.2975
2.0	0.3535	0.2357	0.0911	0.0911			0.1353	0.2706
2.2	0.2969	0.2178					0.1108	0.2437
2.4	0.2483	0.1986					0.0907	0.2177
2.6	0.2067	0.1791					0.0743	0.1931
2.8	0.1716	0.1601					0.0608	0.1703
3.0	0.1420	0.1420					0.0498	0.1494
3.2	0.1174	0.1252					0.0408	0.1304

A2.2C: K = N-3

	N = 4		N = 5		N = 6		Slotted Aloha	
λ_p	P_1	λ_o	P_1	λ_o	P_1	λ_o	P_1	λ_o
0.1	0.9999	0.0250	0.9996	0.0400	0.9989	0.0499	0.9048	0.0905
0.2	0.9989	0.0499	0.9954	0.0796	0.9881	0.0988	0.8187	0.1637
0.4	0.9882	0.0988	0.9565	0.1530	0.9034	0.1870	0.6703	0.2681
0.6	0.9586	0.1438	0.8676	0.2082	0.7428	0.2228	0.5488	0.3293
0.8	0.9080	0.1816	0.7428	0.2377	0.5571	0.2229	0.4493	0.3595
1.0	0.8403	0.2101	0.6054	0.2422	0.3893	0.1947	0.3679	0.3679
1.2	0.7615	0.2285	0.4742	0.2276	0.2579	0.1547	0.3012	0.3614
1.4	0.6778	0.2372	0.3600	0.2016	0.1641	0.1149	0.2466	0.3452
1.6	0.5943	0.2377	0.2666	0.1706	0.1012	0.0810	0.2019	0.3230
1.8	0.5146	0.2316	0.1936	0.1394			0.1653	0.2975
2.0	0.4410	0.2205	0.1384	0.1107			0.1353	0.2706
2.2	0.3748	0.2062	0.0978	0.0860			0.1108	0.2438
2.4	0.3164	0.1898					0.0907	0.2177
2.6	0.2652	0.1726					0.0743	0.1931
2.8	0.2219	0.1554					0.0608	0.1703
3.0	0.1848	0.1386					0.0498	0.1494
3.2	0.1533	0.1227					0.0408	0.1304
3.4	0.1270	0.1079					0.0334	0.1135
3.6	0.1049	0.0944					0.0273	0.0984

A2.3 Finite Number of Groups of Sources: The Case of K = N-1 N = 2

	$\eta = 1$		$\eta = 2$		$\eta = 3$		$\eta = 4$	
λ_p	P_1	λ_o	P_1	λ_o	P_1	λ_o	P_1	λ_o
0.1	0.9490	0.0474	0.9680	0.0484	0.9753	0.0488	0.9790	0.0490
0.2	0.8966	0.0897	0.9252	0.0925	0.9377	0.0938	0.9445	0.0945
0.4	0.7918	0.1584	0.8209	0.1642	0.8401	0.1680	0.8513	0.1703
0.6	0.6911	0.2073	0.7081	0.2124	0.7303	0.2191	0.7442	0.2233
0.8	0.5975	0.2390	0.5984	0.2393	0.6216	0.2486	0.6369	0.2548
1.0	0.5126	0.2563	0.4979	0.2490	0.5209	0.2605	0.5368	0.2684
1.2	0.4371	0.2633	0.4095	0.2457	0.4315	0.2589	0.4472	0.2683
1.4	0.3707	0.2595	0.3336	0.2335	0.3541	0.2479	0.3693	0.2585
1.6	0.3131	0.2505	0.2697	0.2157	0.2886	0.2309	0.3029	0.2423
1.8	0.2634	0.2371	0.2167	0.1950	0.2338	0.2105	0.2470	0.2223
2.0	0.2209	0.2209	0.1732	0.1732	0.1886	0.1886	0.2006	0.2006
2.2	0.1847	0.2032	0.1378	0.1516	0.1516	0.1667	0.1624	0.1786
2.4	0.1541	0.1849	0.1092	0.1311	0.1214	0.1457	0.1311	0.1573
2.6	0.1283	0.1668	0.0863	0.1122	0.0970	0.1261	0.1056	0.1372
2.8	0.1066	0.1493	0.0680	0.0953	0.0774	0.1083	0.0849	0.1188
3.0	0.0885	0.1327	0.0535	0.0803	0.0616	0.0924	0.0681	0.1022

	$\eta = 5$		$\eta = 6$		$\eta = 7$		$\eta = 8$	
λ_p	P_1	λ_o	P_1	λ_o	P_1	λ_o	P_1	λ_o
0.1	0.9813	0.0491	0.9829	0.0491	0.9840	0.0492	0.9849	0.0492
0.2	0.9488	0.0949	0.9517	0.0952	0.9538	0.0954	0.9554	0.0955
0.4	0.8586	0.1717	0.8636	0.1727	0.8673	0.1735	0.8702	0.1740
0.6	0.7534	0.2260	0.7599	0.2280	0.7647	0.2294	0.7684	0.2305
0.8	0.6473	0.2589	0.6546	0.2619	0.6601	0.2640	0.6643	0.2657
1.0	0.5476	0.2738	0.5554	0.2777	0.5612	0.2806	0.5657	0.2829
1.2	0.4581	0.2748	0.4659	0.2795	0.4718	0.2831	0.4763	0.2858
1.4	0.3798	0.2659	0.3875	0.2712	0.3932	0.2752	0.3976	0.2783
1.6	0.3129	0.2503	0.3201	0.2561	0.3256	0.2604	0.3298	0.2638
1.8	0.2563	0.2307	0.2631	0.2368	0.2682	0.2413	0.2721	0.2449
2.0	0.2091	0.2091	0.2153	0.2153	0.2200	0.2200	0.2236	0.2236
2.2	0.1701	0.1871	0.1757	0.1932	0.1799	0.1979	0.1832	0.2015
2.4	0.1379	0.1655	0.1429	0.1715	0.1467	0.1761	0.1497	0.1796
2.6	0.1116	0.1451	0.1161	0.1509	0.1194	0.1553	0.1221	0.1587
2.8	0.0902	0.1263	0.0941	0.1317	0.0971	0.1359	0.0994	0.1391
3.0	0.0728	0.1091	0.0762	0.1143	0.0788	0.1181	0.0808	0.1212

	$\eta = 9$		$\eta = 10$		$\eta = 20$		$\eta = \infty$	
λ_p	P_1	λ_o	P_1	λ_o	P_1	λ_o	P_1	λ_o
0.1	0.9855	0.0493	0.9861	0.0493	0.9885	0.0494	0.9909	0.0495
0.2	0.9567	0.0957	0.9577	0.0958	0.9624	0.0962	0.9671	0.0967
0.4	0.8724	0.1745	0.8742	0.1748	0.8826	0.1765	0.8913	0.1783
0.6	0.7713	0.2314	0.0737	0.2321	0.7848	0.2353	0.7964	0.2389
0.8	0.6677	0.2671	0.6704	0.2682	0.6832	0.2733	0.6968	0.2789
1.0	0.5693	0.2846	0.5722	0.2861	0.5859	0.2929	0.6004	0.3002
1.2	0.4799	0.2880	0.4829	0.2897	0.4968	0.2981	0.5117	0.3070
1.4	0.4012	0.2808	0.4041	0.2829	0.4177	0.2924	0.4324	0.3027
1.6	0.3332	0.2665	0.3359	0.2688	0.3490	0.2792	0.3630	0.2904
1.8	0.2753	0.2478	0.2779	0.2501	0.2901	0.2661	0.3033	0.2729
2.0	0.2265	0.2265	0.2289	0.2289	0.2402	0.2402	0.2524	0.2524
2.2	0.1858	0.2044	0.1880	0.2068	0.1982	0.2181	0.2093	0.2303
2.4	0.1521	0.1825	0.1540	0.1848	0.1632	0.1959	0.1732	0.2078
2.6	0.1242	0.1614	0.1259	0.1637	0.1341	0.1743	0.1430	0.1859
2.8	0.1012	0.1417	0.1028	0.1439	0.1100	0.1540	0.1179	0.1651
3.0	0.0824	0.1236	0.0838	0.1256	0.0901	0.1352	0.0971	0.1456

CHAPTER 3

THE BINARY TREE ALGORITHM TYPE OF TRANSMISSION

The binary tree algorithm was proposed by Capetanakis [3] for an irredundant transmission. As a transmission strategy it is both stable and has a higher throughput (up to 0.43) than the slotted Aloha strategy. We shall discuss this algorithm as it was originally proposed and then use it as a retransmission strategy for the super-packets with the index vector (0, 1, 3).

3.1 The Irredundant Scheme

The channel slots are divided into slot pairs. When a source has a message packet to transmit, it flips a coin. If the coin comes up heads, the source transmits the packet in the first slot of the next slot pair; otherwise it transmits the packet in the second slot of the slot pair. If the source is the only member of the system that has a packet in that slot, then the packet will be accurately received. However, if two or more sources transmitted their packets in that slot, then a collision occurs and each packet involved in the collision is rendered unintelligible and has to be retransmitted. Assume that the number of packets arriving at the channel is a Poisson random variable with mean λ.

When a conflict occurs, all sources stop their transmissions at the end of that slot pair in which the conflict occurred. A conflict may arise from a collision in either of the slots of the slot pair; it may also arise from collisions in both slots. If it is a single slot collision, the sources involved in that collision will use the algorithm described below to resolve their conflict. If the conflict involves collisions in both slots, then the sources involved in the collision in the first slot (the "heads") will resolve their conflict first. At the end of that the sources involved in the collision in the second slot (the "Tails") will then resolve their conflict in the same manner.

When all sources have stopped their transmissions due to the occurrence of a collision, the sources involved in the collision in the first slot (assuming a two-slot conflict, otherwise that group involved in the conflict just follows this algorithm) will flip coins. Those sources whose coins comes up heads will transmit their packets in the first slot of the next slot pair while those whose coins come up tails will transmit their packets in the second slot of the slot pair. If collisions occur again, then those sources that previously obtained tails in their toss will suspend their transmission at the end of that slot pair, assuming a collision occurred in their slot. If a collision occurred in the first slot then the members that transmitted in that slot will flip coins again and those whose coins come up heads will transmit in the first slot of the next slot pair and those whose coins come up tails will transmit in the second slot. If the collision occurred only in the second slot then it is members of this group that will take the above step in resolving their conflict. The process of coin flipping and transmission continues if a collision occurs at any stage until all the members of the first group have successfully transmitted their packets. If a collision also occurred in the second slot of the original slot pair in which the conflict arose, then the members of that group who were involved in the collision will also use the same algorithm to resolve their conflict. Otherwise, the conflict resolution ends when the members in the first slot finish transmitting their packets. While the conflict resolution is in progress, no new packets are transmitted; all newly arriving packets will be transmitted in the next epoch. An epoch is the interval of conflict resolution, given that a conflict occurred; otherwise it is a pair of slots. We also define an algorithmic step as consisting of the transmissions taken in a pair of slots, the observation of the outcomes of those transmissions, and the decision as to what action to take in the next slot pair.

A typical binary tree for a conflict involving five sources is shown in Figure 3.1.

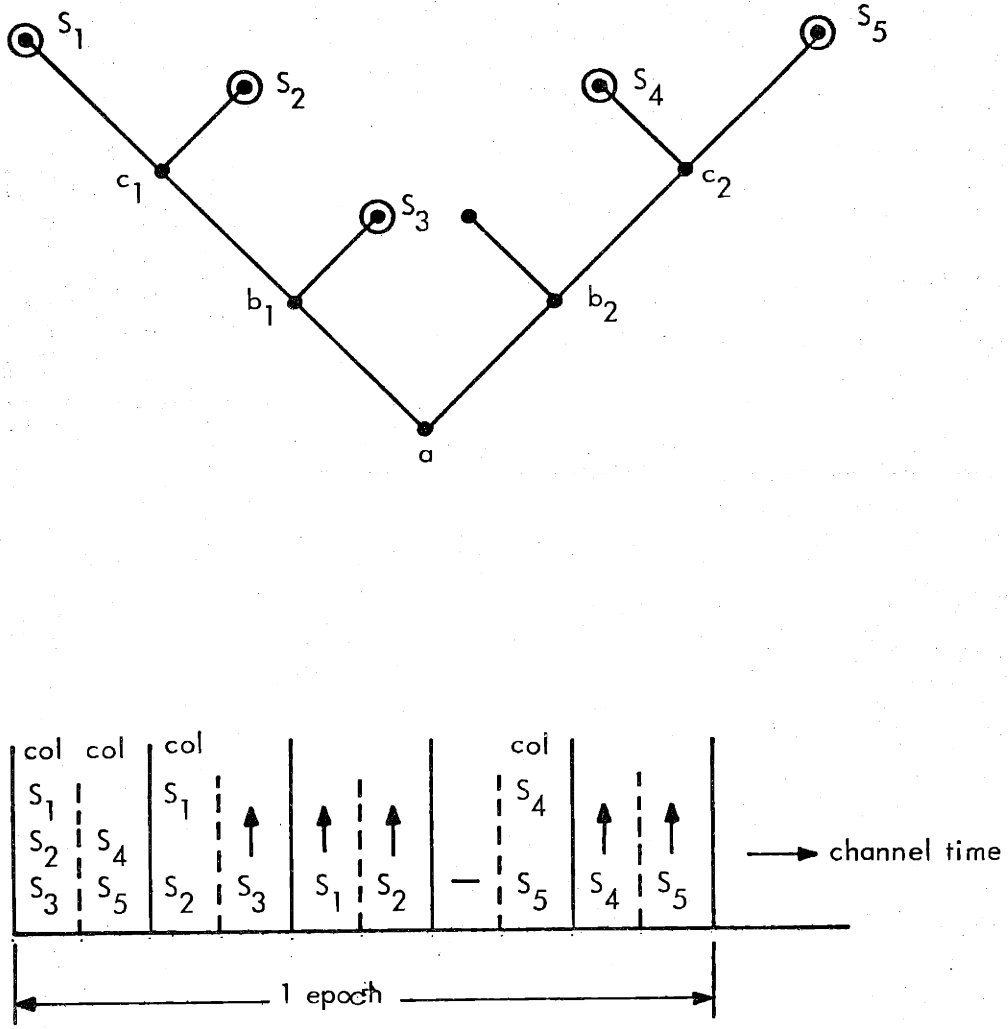


Figure 3.1 An Example of a Binary Tree

The left branch represents packets from those sources that obtained heads in their original coin toss, and the right branch represents packets from sources that obtained tails in their original toss. Point [a] is the beginning of the epoch during which all the five sources flip coins together. Sources S_1, S_2 and S_3 obtained heads while sources S_4 and S_5 obtained tails. Hence a collision occurred in both groups. At this point $S4$ and S_5 suspended their transmissions while S_1, S_2 and S_3 tossed again at point [b_1]. Here S_1 and S_2 obtained heads and hence generated a collision while S_3 obtained tails and had a successful transmission. At point [c_1] S_1 and S_2 tossed again; S_1 obtained heads while S_2 obtained tails. Both sources had successful transmissions, and at that point all members in the left branch of the tree finished resolving their conflict. The sources in the right branch then took over with S_4 and S_5 tossing at point [b_2]. Both obtained tails again; no one transmitted in the first slot and the two transmitted in the second slot and so another collision occurred. At point [c_2] they tossed again S_4 obtained heads and S_5 obtained tails. Both sources had successful transmissions. That marked the end of the epoch.

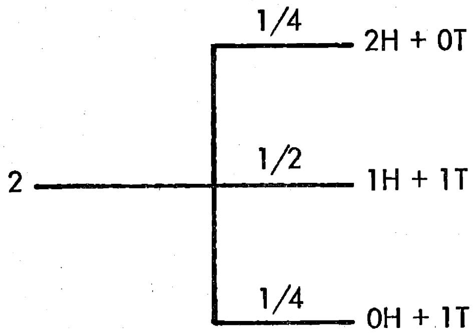
3.1.1 Approximate Analysis of the Strategy (Massey [7])

The following is an approximate analysis of the above scheme; the accurate analysis was carried out in [3].

Let L_i = the expected number of slots required to resolve a conflict involving i packets in the first slot of the slot pair, including the slot where the conflict occurred.

Clearly $L_o = 1$ and $L_1 = 1$.

To compute L_i for $i \geqslant 2$ we display the outcomes of the different tosses graphically as shown below.

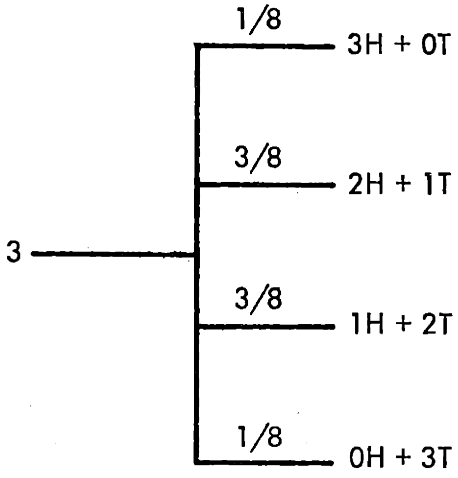


Together with the original slot in which the collision occured we obtain the following:

$$L_2 = 1 + \frac{1}{2}(L_2+L_o) + \frac{1}{2}(L_1+L_1)$$

$$\therefore \quad \frac{1}{2}L_2 = 1 + {}^3/2 = {}^5/2$$

$$\therefore \quad \boxed{L_2 = 5 \text{ slots}}$$



Thus $L_3 = 1 + \frac{1}{4}(L_3+L_o) + \frac{3}{4}(L_2+L_1)$

$$\therefore \frac{3}{4} L_3 = 1 + \frac{1}{4} + \frac{15}{4} + \frac{3}{4} = \frac{23}{4}$$

$$\therefore \boxed{L_3 = 7.667 \text{ slots}}$$

Similarly, $L_4 = 1 + \frac{1}{8}(L_4+L_o) + \frac{1}{2}(L_3+L_1) + \frac{3}{8}(L_2+L_2)$

$$\frac{7}{8}L_4 = 1 + \frac{1}{8} + \frac{23}{6} + \frac{1}{2} + \frac{15}{4} = \frac{221}{24}$$

$$\therefore \boxed{L_4 = 10.524 \text{ slots}}$$

$$L_5 = 1 + \frac{1}{16}(L_5+L_o) + \frac{5}{16}(L_4+L_1) + \frac{10}{16}(L_3+L_2)$$

$$\therefore \frac{15}{16}L_5 = 1 + \frac{1}{16} + \frac{5 \times 221}{16 \times 21} + \frac{5}{16} + \frac{230}{48} + \frac{50}{16}$$

$$\therefore \boxed{L_5 = 13.419 \text{ slots}}$$

Theorem 3.1 $L_i \lesssim 3i-1$ for $i \gtrsim 2$.

Proof:
$$L_\eta = 1 + 2\sum_{i=0}^{\eta} p_i L_i$$

where $p_i = 2^{-\eta}\binom{\eta}{i} = p_{\eta-i}$, binomial process with $p = 1/2$.

Hence $\sum_{i=0}^{\eta} ip_i = \sum_{i=1}^{\eta} ip_i = \frac{1}{2}\eta$

$$\sum_{i=0}^{\eta} p_i = 1$$

We use induction on η; we have already verified the theorem for $\eta=2$. Assume the theorem is true for all $i < \eta$. Then

$$L_\eta = 1 + 2p_oL_o + 2p_1L_i + 2\sum_{i=2}^{\eta-1} p_1L_1 + 2p_oL_\eta$$

$$\leq 1 + 2p_o\_2p_1 + 2p_oL_\eta + 2\sum_{i=2}^{\eta-1}(3i-1)p_1$$

$$\therefore\ L_\eta\ (1-2p_o) \lesssim 1 + 2p_o+2p_1+6\sum_{i=1}^{\eta} ip_i - 6p_1-6\eta p_o$$

$$- 2\sum_{i=0}^{\eta} p_i + 4p_o + 2p_1$$

$$= 1 + 3\eta -2 + 6p_o - 6\eta p_o - 2p_1$$

$$= 3\eta(1-2p_o) - (1-2p_o) - (2p_1-4p_o)$$

$$\therefore\ L_\eta \lesssim 3\eta -1 - \left\{\frac{2p_1-4p_o}{1 - 2p_o}\right\}\ ;\ \eta \gtrsim 2$$

$$= 3\eta-1 - \left\{\frac{2\eta-4}{2^\eta-2}\right\}$$

$$\lesssim 3\eta-1 \text{ for } \eta \gtrsim 2$$

Q.E.D.

3.1.2 Throughput Consideration:

Let L_j = the length of the j^{th} epoch in slot pairs. Since we assumed that the number of packets arriving at the channel is a Poisson random variable with mean λ packets per slot, the expected number of packets involved in the conflict resolution process in the j^{th} epoch, given that the length of the (j-1)st epoch is m, is $2\lambda m$. And since there are two groups of sources in the system we must have that

$$E(\ell_j|\ell_{j-1} = m) = 2 \sum_{i=0}^{\infty} P_i \left(\frac{L_i}{2}\right) = \sum_{i=0}^{\infty} P_i L_i,$$

where we have divided L_i by 2 to convert it to slot pairs, and we have multiplied the sum by 2 to recognize the two groups mentioned above. Also

$$P_i = \frac{\left(\frac{2\lambda m}{2}\right)^i}{i!} e^{-2\lambda m/2}$$

$$= \frac{(\lambda m)^i}{i!} e^{-\lambda m}, \quad i = 0, 1, 2 \ldots$$

$$\therefore \; E(\ell_j|\ell_{j-1} = m) = P_o L_o + P_1 L_1 + \sum_{i=2}^{\infty} P_i L_i$$

$$\leq P_o + P_1 + \sum_{i=2}^{\infty} (3i-1) P_i$$

$$= P_o + P_1 + 3 \sum_{i=1}^{\infty} iP_i - 3P_1 - \sum_{i=0}^{\infty} P_i + P_o + P_1$$

$$= 3\lambda m - 1 + e^{-\lambda m}(2-\lambda m) \quad \ldots \quad (3.1)$$

Theorem 3.2 (Capetanakis [3])

Let ℓ_j be a positive integer corresponding to the state of a Markov chain after the j^{th} transition. Also assume that for some constants a and b, $0 \lesssim a < 1$,

$$E(\ell_{j+1} | \ell_j) \leq a(\ell_j - 1) + b \qquad (**)$$

then $$\lim_{j\to\infty} E(\ell_j) \lesssim \frac{b-a}{1-a} \qquad (3.2)$$

Proof: The proof for this theorem can be found in the above reference. It consists of multiplying both sides of (**) by $p(\ell_j)$ and summing over ℓ_j and obtaining the steady-state solution as $\ell_j \to \infty$. The theorem then states that $E(\ell)$ is finite.

Thus from Equation (3.1)

$$E(\ell_j | \ell_{j-1} = m) \lesssim 3\lambda m - 1 + e^{-\lambda m} (2-\lambda m)$$
$$= 3\lambda(m-1) + 3\lambda - 1 + e^{-\lambda m}(2-\lambda m)$$

Let $a = 3\lambda$, where $\lambda < {}^1/_3$,

and $$b = \max_{\lambda m}\{3\lambda - 1 + e^{-\lambda m}(2-\lambda m)\}$$
$$= 3\lambda - 1 + 2$$
$$= 3\lambda + 1$$

Then $E(\ell) \lesssim \frac{3\lambda+1-3\lambda}{1-3\lambda} = \frac{1}{1-3\lambda}$.

This is finite for $\lambda < {}^1/3$. Thus the maximum stable throughput is at least ${}^1/3$. An exact analysis shows that it is ${}^1/2.88$ (see [3]). In

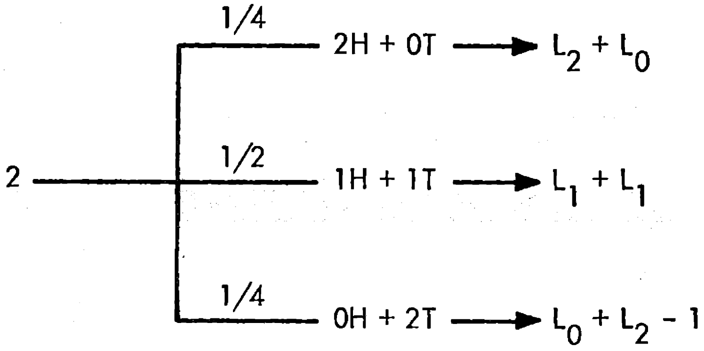
Appendix 3A we have a tabulation of λ against different values of m. From this table we see that this approximate analysis gives a maximum value of $\lambda = 0.423$ as against 0.43 obtained in the accurate analysis in [3]. However, this is an unstable maximum.

3.1.3 Avoiding Situations of Obvious Conflict

It is possible to eliminate those situations where a collision is sure to occur. If no transmission was heard in the first slot after a conflict, then all sources involved in that conflict obtained tails in their toss and will certainly collide again if they transmit in the second slot. Under such a situation, the sources involved in the conflict will not transmit in the second slot. Instead they will toss again and continue the process of conflict resolution from there.

Let L_i = the expected number of slots required to resolve a conflict involving i sources in the first slot, including the slot where the conflict occurred.

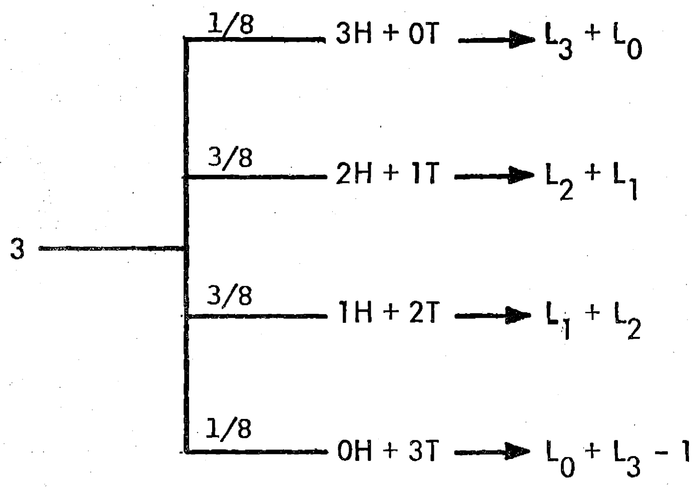
Then $L_o = L_1 = 1$ slot.



That is, $L_2 = 1 + \frac{1}{4}\ (L_2+L_o) + \frac{1}{2}(L_1+L_1) + \frac{1}{4}(L_o+L_2-1)$

$$\therefore \frac{1}{2}L_2 = 1 + {}^1/2 + 1 - {}^1/4 = 2.25$$

$$\therefore \boxed{L_2 = 4.5 \text{ slots}}$$



$$\therefore L_3 = 1 + \frac{1}{4}(L_3+L_o) + \frac{3}{4}(L_2+L_1) - \frac{1}{8}$$

$$\cdot\ \frac{3}{4}L_3 = 1 + \frac{1}{4} + \frac{3}{4} + \frac{13.5}{4} - \frac{1}{8} = \frac{21}{4}$$

$$\boxed{L_3 = 7 \text{ slots}}$$

Similarly, $L_4 = 1 + \frac{1}{8}(L_4+L_o) + \frac{1}{2}(L_3+L_1) + \frac{3}{8}(L_2+L_1) - \frac{1}{16}$

$$\therefore \frac{7}{8}L_4 = 1 + 1/8 + 4 + 27/8 - 1/16 = \frac{67.5}{8}$$

$$\therefore \boxed{L_4 = 9.6429 \text{ slots}}$$

$$L_5 = 1+\frac{1}{16}(L_5+L_o) + \frac{5}{16}(L_4+L_1) + \frac{10}{16}(L_3+L_2) - \frac{1}{32}$$

or $\boxed{L_5 = 12.3143 \text{ slots}}$

Theorem 3.3

$L_i \leq 2.75i-1$ for $i \geq 2$.

Proof: $L_\eta = 1 + 2 \sum_{i=0}^{\eta} pi^{Li} - p_o$

$$= 1 + 2 \sum_{i=0}^{\eta-1} pi^{Li} + 2p_oL_n - p_o$$

We use induction on $\eta = 2$. Assume the theorem is true for all $i < \eta$. Then

$$L_\eta = 1+2p_oL_o + 2p_1L_1 + 2 \sum_{i=2}^{\eta-1} p_iL_i + 2p_oL_n - p_o$$

$$\leq 1 + p_o + 2p_1 + 2P_oL_\eta + 2 \sum_{i=2}^{\eta-1} (2.75i-1)p_i$$

$$\therefore L_\eta(1-2p_o) \leq 1+p_o + 2p_i + 5.5 \sum_{i=1}^{\eta} ip_i - 5.5p_1 - 5.5_\eta\, p_o$$

$$- 2 \sum_{i=0}^{\eta} p_i + 4p_o + 2p_i$$

$$\therefore\ L_\eta(1-2p_o) \leq 2.75\eta-1 -5.5\eta p_o + 5p_o - 1.5p_1$$

$$= 2.75\ (1-2p_o) - (1-2p_o) - (1.5p_1-3p_o)$$

$$\therefore\ L_\eta \leq 2.75\eta-1 - \{\frac{1.5p_1-3p_o}{1-2p_o}\}$$

$$= 2.75\eta-1 - \{\frac{1.5\eta-3}{2^\eta-2}\}$$

$$\leq 2.75\eta-1 \quad \text{for } \eta \geq 2.$$

Q.E.D.

And from the results of Section 3.1.2,

$$E(\ell_j|\ell_{j-1} = m) = \sum_{i=0}^{\infty} P_iL_i;\ \text{where } P_i = \frac{(\lambda m)^i}{i!} e^{-\lambda m}$$

$$= P_oL_o + P_iL_i + \sum_{i=2}^{\infty} P_iL_i$$

$$\leq P_o + P_1 + \sum_{i=2}^{\infty}(2.75i-1)P_i$$

$$= P_o + P_1 + 2.75 \sum_{i=1}^{\infty} iP_i - 2,75P_1$$

$$- \sum_{i=0}^{\infty} P_i + P_o + P_1$$

$$= 2.75\lambda m -1 + 2P_o -0.75P_1$$

$$= 2.75\lambda m -1 +e^{-\lambda m} (2 - 0.75\lambda m)$$

Thus $E(\ell_j | \ell_{j-1} = m) \leq 2.75\lambda(m-1) + 2.75\lambda - 1 + e^{-\lambda m}(2 - 0.75\lambda_m)$

Let $\quad a - 2.75\lambda$, where $\lambda < 1/2.75$

$$b = \max_{\lambda m}\{2.75\lambda - 1 + e^{-\lambda m}(2 - 0.75\lambda m)\}$$

$$= 2.75\lambda - 1 + 2 = 2.75\lambda + 1$$

Then $E(\ell_j | \ell_{j-1} = m) \leq 2.75\lambda(m-1) + \{2.75\lambda + 1\}$

From theorem 3.2, therefore, we must have that

$$E(\ell) \leq \frac{b-a}{1-a} = \frac{1}{1-2.75\lambda}$$

Thus the maximum stable throughout is at least $1/2.75 = 0.3636$. An exact analysis was not carried out in [3] for this strategy. In Appendix 3B, we have given a tabulation of the values of m and the corresponding values of λ. From the table it is seen that an instantaneous maximum throughput of at least 0.456 can be obtained with this scheme.

3.2 Retransmission Strategy For The Superpackets

In Section 3.1 we gave an overview of the binary tree algorithm and used an approximate analysis to compute the throughput of the scheme. In this section we propose how the algorithm can be used as a retransmission strategy for the superpackets with index vector (0, 1, 3).

We assume that the number of superpackets generated by the sources and presented to the channel for transmission is a Poisson random variable with mean λ superpackets per slot. The superpackets are thus randomly generated and transmitted as they are generated. To simplify the analysis we assume the satellite has memory and keeps track of the number of packets of each superpacket that sustain collision when the system is in the free-running mode. When this number exceeds one for any superpacket, the number of parity packets, the satellite sends a message to all the sources that a collision has occurred. At this point all sources stop their transmissions. All the sources that were transmitting their superpackets when the order to stop transmission came will then start tossing coins as described for the irredundant transmission.

Two patterns for formatting the superpacket for retransmission are considered. In the first scheme, the superpacket structure is preserved and one algorithmic step spans five slots. In the second scheme the sources transmit only the information packets and one algorithmic step spans four slots. When the system is in the retransmission mode, the channel slots will be partitioned into superslots. A superslot consists of the number of slots that are used in one algorithmic step. Thus in Scheme 1 a superslot comprises the five slots that are used in one algorithmic step, and in Scheme 2 a superslot comprises only four slots. These schemes are graphically shown in Figures 3.2 and 3.3 respectively.

3.2.1 Scheme 1

Here the sources that obtain heads in their toss will start their transmission in the first slot of the superslot while the sources that obtain tails will start their transmission in the second slot of the superslot. Since a superpacket can be reconstructed if one packet is lost in collision, then if a superpacket does not lose another packet through a direct hit arising from the fact that more than one source obtained heads or tails, the superpackets of Figure 3.2 can be reconstructed at the receiver.

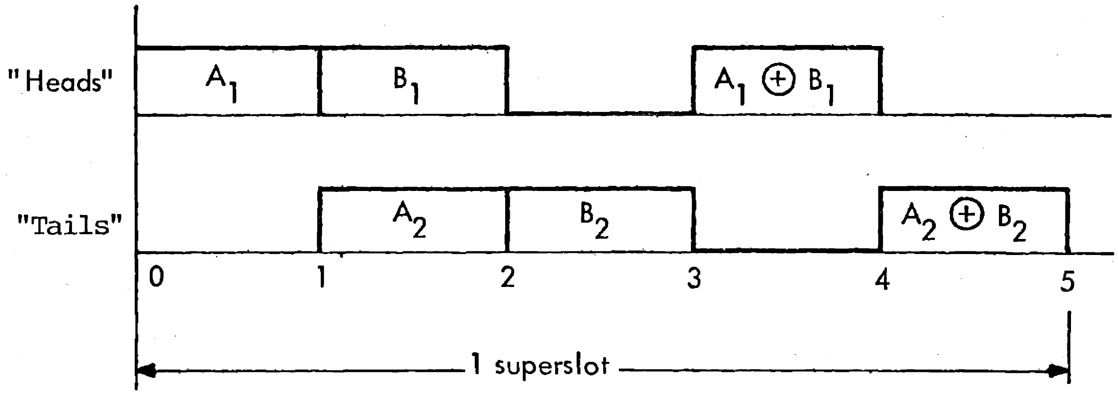


Figure 3.2 Retransmission Model For Scheme 1

3.2.2 Scheme 2

This scheme is similar to the scheme for irredundant transmission. The sources that obtain heads in their toss transmit their two information packets in the first two slots of the superslot. Sources that obtain tails transmit their two packets in the second two slots of the superslot. Thus one superslot consists of only four slots as shown.

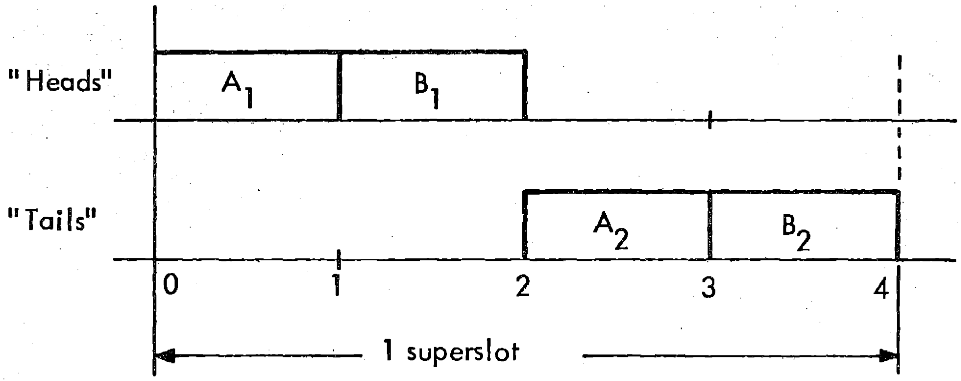
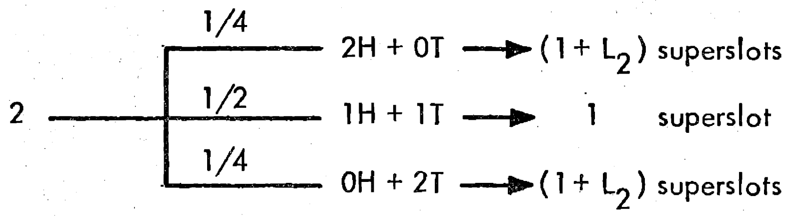


Figure 3.3 Retransmission Model for Scheme 2.

3.2.3 An Approximate Analysis of the Schemes

Let L_i = the expected number of superslots used in resolving a conflict involving i sources excluding the slots which the original conflict occurred.

Then L_i is insensitive to which of the two schemes is in use. For a collision to occur $i \geqslant 2$. Applying the same method of analysis we used for the irredundant transmission we obtain the following.



Thus $L_2 = \frac{1}{2}(1 + L_2) + \frac{1}{2}(1)$

$$\therefore \frac{1}{2}L_2 = 1$$

or $\boxed{L_2 = 2 \text{ superslots}}$

Similarly, $L_3 = \frac{1}{4}(1 + L_3) + \frac{3}{4}(1 + L_2)$

$$\therefore \frac{3}{4}L_3 = 1 + \frac{3}{4}L_2 = 5/2$$

$$\therefore \boxed{L_3 = 3.33}$$

$$L_4 = \frac{1}{8}(L_4 + 1) + \frac{1}{2}(L_3+1) + \frac{3}{8}(2L_2+1)$$

$$\therefore \frac{7}{8}L_4 = 1 + \frac{5}{3} + \frac{3}{2} = \frac{25}{6}$$

$$\therefore \boxed{L_4 = 4.762}$$

$$L_5 = \frac{1}{16}(L_5 + 1) + \frac{5}{16}(L_4+1) + \frac{10}{16}(L_3 + L_2 + 1)$$

$$\therefore \frac{15}{16}L_5 = 1 + \frac{500}{16x21} + \frac{100}{48} + \frac{20}{16}$$

or $\boxed{L_5 = 6.210}$

Theorem 3.4

$$L_i \leq 1.5i - 1 \quad \text{for } i \geq 2.$$

Proof: $L_\eta = 1 + 2 \sum_{i=2}^{\eta} p_i L_i$

where p_i is as defined as theorem 3.1

$$\therefore L_\eta = 1 + 2p_o L_\eta + 2 \sum_{i=2}^{\eta-1} p_i L_i \quad \ldots\ldots (*)$$

We use induction on η. We have verified the theorem for $\eta = 2$. Assume the theorem is true for all $i < \eta$. Then from (*)

$$L_\eta \leq 1 + 2p_o L_\eta + 2 \sum_{i=2}^{\eta-1} (1.5_i - 1)p_i$$

$$= 1 + 2p_o L_\eta + 3 \sum_{i=1}^{\eta} ip_i - 3p_1 - 3\eta p_o - 2 \sum_{i=0}^{\eta} p_i + 4p_o + 2p_1$$

$$\therefore L_\eta (1-2p_o) \leq 1 + \frac{3}{2}\eta - 2 - 3\eta p_o + 4p_o - p_1$$

$$= \frac{3}{2}\eta(1-2p_o) - (1 - 2p_o) - (p_1 - 2p_o)$$

$$\therefore L_\eta \leq 1.5\eta - 1 - \{\frac{p_1 - 2p_o}{1 - 2p_o}\}$$

$$= 1.5\eta - 1 - \{\frac{\eta - 2}{2\eta - 2}\}$$

$$\leq 1.5\eta - 1 \text{ for } \eta \geq 2$$

Q.E.D.

3.2.4 Throughput Considerations:

Let ℓ_j = the expected length in superslots of the j^{th} epoch not counting the slots where the original conflict occurred.

If we have as a rule that if no collision occurs in the superslot that immediately comes after a superslot in which a collision occurred the system reverts from the retransmission mode to the free-running mode, then it follows that

$$E(\ell_j | \ell_{j-1} = m) = \sum_{i=2}^{\infty} P_i L_i$$

where $$P_i = \frac{[K\lambda(m+1)]^i}{i!} e^{-K\lambda(m+1)}$$

$$k = \begin{cases} 5 \text{ if Scheme 1 is adopted} \\ 4 \text{ if Scheme 2 is adopted} \end{cases}$$

That is k = the number of slots in one superslot. We have added 1 to account for the fact that one superslot is used to change from the retransmission mode to the free-running mode at the end of each epoch.

$$\therefore E(\ell_j | \ell_{j-1} = m) \leqslant \sum_{i=2}^{\infty} (1.5i - 1)\, P_i$$

$$= 1.5 \sum_{i=1}^{\infty} iP_i - 1.5P_1 - \sum_{i=0}^{\infty} P_i + P_o + P_1$$

$$= 1.5K\lambda(m+1) - 0.5P_1 + P_o - 1 \; ..... \; (3.4)$$

$$= 1.5K\lambda(m-1)+3K\lambda-1+e^{-K\lambda(m+1)}\,[1-0.5K\lambda(m+1)]$$

Let $a = 1.5K\lambda$ where $\lambda < \frac{1}{1.5K}$

$$b = \max_{\lambda m}\{3\,K\lambda-1 + e^{-K\lambda(m+1)}\,[1 - 0.5K\lambda(m+1)]\}$$

$$= 3K\lambda - 1 + e^{-K\lambda}(1-0.5K\lambda)$$

Then, from theorem 3.2

$$E(\ell) \leqslant \frac{b-a}{1-a} = \frac{e^{-K\lambda}(1-0.5K\lambda)+1.5K\lambda-1}{1-1.5K\lambda}$$

which is finite for $\lambda < {}^{1}/1.5K$.

Thus the maximum stable throughput is obtained when λ is at least $\frac{1}{1.5K}$

Since λ is the channel superpacket rate, the throughput $\lambda_o = 2\lambda$, the channel information packet rate. Hence the system throughput is at least

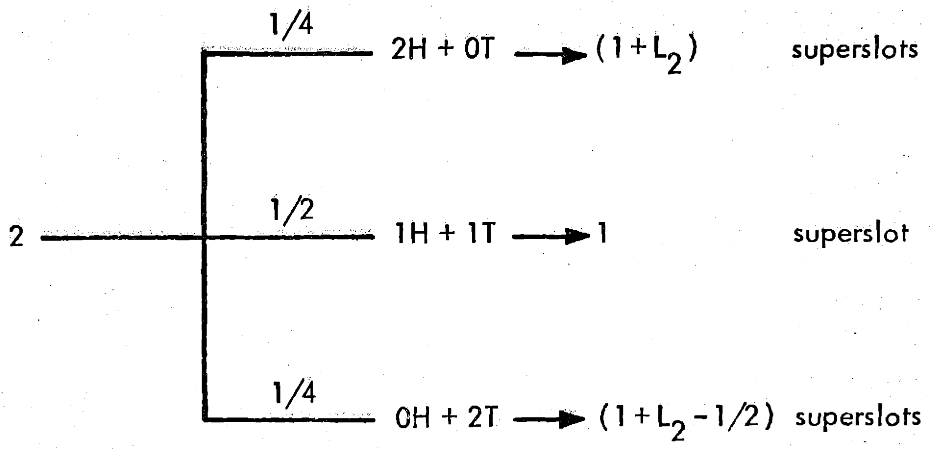
$$\lambda_o = \frac{2}{1.5K} = \begin{cases} 0.2667 \text{ for Scheme 1} \\ 0.3333 \text{ for Scheme 2.} \end{cases}$$

In Appendix 3C we have tabulated the values of λ and m obtained for these schemes.

3.2.5 Avoiding Obvious Conflicts

As in the case of the irredundant transmission it is possible to avoid these situations where a conflict is sure to occur. Consider Scheme 2. If no transmission was heard in the first half of the superslot after a conflict, then all sources involved in that conflict obtained tails in their toss. If they transmit their packets in the second half of the superslot, then they are sure to generate a collision. A strategy to adopt in such a situation is to skip transmission in the second half of the superslot and toss again. Those who obtain heads will transmit in the first half of the superslot after the superslot in which they were scheduled to transmit. And those who obtain tails transmit in the second half of that next superslot. Thus the sources use one half of a superslot to avoid an obvious conflict.

We can, as usual carry out an approximate analysis as follows.


1/4
2H + 0T → $(1+L_2)$ superslots
1/2
2
1H + 1T → 1 superslot
1/4
0H + 2T → $(1+L_2-1/2)$ superslots

$$\therefore\ L_2 = \frac{1}{4}(1+L_2) + \frac{1}{2}(1) + \frac{1}{4}(1+L_2-\frac{1}{2})$$

$$= \frac{1}{2}(1+L_2) + \frac{1}{2} - \frac{1}{8}$$

$$\therefore\ \frac{1}{2}L_2 = 1 - \frac{1}{8} = 7/8$$

$$\therefore\ \boxed{L_2 = 7/4 = 1.75 \text{ superslots}}$$

Similarly, $L_3 = \frac{1}{4}(1+L_3) + \frac{3}{4}(1+L_2) - \frac{1}{8}\ (\frac{1}{2})$

$$\therefore\ \frac{3}{4}L_3 = 1 + \frac{3}{4} \text{ x } \frac{7}{4} - \frac{1}{16} = 1 + 5/4 = 9/4$$

$$\therefore\ \boxed{L_3 = 3}$$

$$L_4 = \frac{1}{8}(1+L_4) + \frac{1}{2}(1+L_3) + \frac{3}{8}(1+2L_2) - \frac{1}{16}\ (\frac{1}{2})$$

$$\therefore\ \frac{7}{8}L_4 = 1 + \frac{3}{2} + \frac{3}{4} \text{ x } \frac{7}{4} - \frac{1}{32} = \frac{60.5}{16}$$

$$\boxed{L_4 = 4.3214}$$

Theorem 3.5

$$L_i \lesssim 1.375i - 1 \text{ for } i \geq 2.$$

Proof $\quad L_\eta = 1 + \sum_{i=2}^{\eta} p_i L_i - \frac{1}{2}p_o$

where p_i is as defined in theorem 3.1. We use induction

We use induction on η. We have verified the theorem for η=2. Assume that the theorem is true for all $i<\eta$. Then

$$L_\eta = 1 + 2p_o L_\eta - \frac{1}{2}\, p_o + 2 \sum_{i=2}^{\eta-1} p_i\, L_i$$

i.e. $\quad L_\eta \lesssim 1 + 2p_o L_\eta - \frac{1}{2}p_o + 2 \sum_{i=2}^{\eta-1} (1.375i-1)p_i$

$$\therefore L_\eta(1-2p_0) \leq 1 - \frac{1}{2}p_o + 2.75 \sum_{i=1}^{\eta} ip_i - 2.75p_1 - 2.75\eta p_o$$

$$- 2 \sum_{i=0}^{\eta} p_i + 4p_o + 2p_1$$

$$= 1.375_\eta - 2.75_\eta p_o - 1 + 3.5p_o - 0.75p_1$$

$$= 1.375_\eta \ (1-2p_o) - (1-2p_o) - \{0.75p_1 - 1.5p_o\}$$

$$L\eta \leq 1.375\eta - 1 - \{\frac{0.75p_1 - 1.5p_o}{1 - 2p_o}\}$$

$$= 1.375\eta - 1 - \{\frac{0.75\eta - 1.5}{2^\eta - 2}\}$$

$$\leq 1.375\eta \quad \text{for } \eta \geq 2$$

Q.E.D.

Let ℓ_j = the expected length in superslots of the j^{th} epoch excluding the slots where the original conflict occurred.

Then by the same argument in Section 3.2.4 we have that

$$E(\ell_j|\ell_{j-1}=m) = \sum_{i=2}^{\infty} P_i L_i ; \quad P_i = \frac{[4\lambda(m+1)]^i}{i!} e^{-4\lambda(m+1)}$$

$$\leq \sum_{i=2}^{\infty} (1.375i - 1) P_i$$

$$= 1.375i \sum_{i=1}^{\infty} i P_i - 1.375P_1 - \sum_{1=0}^{\infty} P_i + P_o + P_1$$

$$= 1.375x - 1 + e^{-x}(1-0.375x) \ ..... \quad (3.5)$$

where $x = 4\lambda(m+1)$

$$E\ell_j|\ell_{j-1}=m) \leq 5.5\lambda(m+1) - 1 + e^{-4\lambda(m+1)}[1-1.5\lambda(m+1)]$$

$$= 5.5\lambda(m-1) + 11\lambda - 1 + e^{-4\lambda(m+1)}[1-1.5\lambda(m+1)]$$

Let $a = 5.5\lambda$ where $\lambda < \frac{1}{5.5}$

$$b = \max_{\lambda m} \{11\lambda-1 + e^{-4\lambda(m+1)} [1-1.5\lambda(m+1)]\}$$

$$= 11\lambda -1+e^{-4\lambda}(1-1.5\lambda)$$

Then from theorem 3.2

$$E(\ell) \lesssim \frac{b-a}{1-a} = \frac{e^{-4\lambda}(1-1.5\lambda)+ 5.5\lambda-1}{1-5.5\lambda} ,$$

which is finite for $\lambda < \frac{1}{5.5}$

Thus the maximum stable throughput is obtained when λ is at least $1/5.5 = 0.1818$.

That is, the maximum stable throughput is at least

$$2\lambda = \lambda_o = 0.3636$$

In Appendix 3D we have tabulated the values of λ for different values of m.

APPENDIX 3

Throughput Calculations for the Approximate Analyses

3A The Irredundant Scheme

From Equation (3.1) we have that

$$E(\ell_j | \ell_{j-1} = m) \leq 3x-1 + e^{-x}(2-x), \text{ where } x = \lambda m .$$

Since $E(\ell)$ is finite, then for all m,

$$m \geq 3x-1 + e^{-x}(2-x)$$

The values of m and λ satisfying this inequality are given in the table below.

m	x	$\lambda = \frac{x}{m}$
2	0.8299	0.41495
2.5	1.0575	0.42300
2.6	1.10010	0.42312
2.7	1.1420	0.42296
3	1.2640	0.42133
4	1.6437	0.41093
5	2.0000	0.40000
6	2.3443	0.39072
7	2.6822	0.38317
8	3.0165	0.37706
9	3.3491	0.37212
10	3.6807	0.36807
50	17.0000	0.34000
100	33.6666	0.33666

3B The Irredundant Scheme Avoiding Obvious Conflicts

From Equation (3.3) we have that

$$E(\ell_j | \ell_{j-1} = m) \leqslant 2.75x - 1 + e^{-x}(2 - 0.75x)$$

where $x = \lambda m$

Since $E(\ell)$ is finite, then for all m,

$$m \geqslant 2.75x - 1 + e^{-x}\ (2 - 0.75x)$$

Values of m and λ satisfying this inequality are given in the table below.

m	x	$\lambda = \frac{x}{m}$
2	0.89279	0.44640
2.5	1.13944	0.45578
2.6	1.18568	0.45603
2.7	1.23115	0.45598
3	1.36367	0.45456
4	1.77715	0.44429
5	2.16617	0.43323
6	2.54279	0.42380
7	2.91273	0.41610
8	3.27901	0.40988
9	3.64333	0.40481
10	4.00665	0.40066
50	18.54545	0.37091
100	36.72727	0.36727

3C The Redundant Scheme

From Equation (3.4) we have that

$$E(\ell_j|\ell_{j-1} = m) \leqslant 1.5x -1 + e^{-x} (1 - 0.5x), \text{ where } x = K\lambda(m+1)$$

$$K = \begin{cases} 5 \text{ of scheme 1 is used.} \\ 4 \text{ of scheme 2 is used.} \end{cases}$$

And since $E(\ell)$ is finite, then for all m,

$$m \geqslant 1.5x -1+ e^{-x} (1-0.5x)$$

In the table below we give the values of m and λ satisfying the above inequality for the two schemes.

		Scheme 1		Scheme 2	
m	x	$\lambda=\frac{x}{5(m+1)}$	$\lambda_o = 2\lambda$	$\lambda=\frac{x}{4(m+1)}$	$\lambda_o = 2\lambda$
2	2.00000	0.13333	0.26666	0.16667	0.33333
3	2.68222	0.13411	0.26822	0.16764	0.33528
3.1	2.74931	0.13411	0.26823	0.16764	0.33528
3.2	2.81627	0.13411	0.26822	0.16764	0.33527
4	3.34912	0.13396	0.26793	0.16746	0.33491
10	7.33449	0.13335	0.26671	0.16669	0.33339
50	34.00000	0.13333	0.26667	0.16667	0.33333
100	67.33333	0.33333	0.26667	0.16667	0.33333

3D The Redundant Scheme Avoiding Obvious Conflicts

From Equation (3.4) we have that

$$E(\ell_j | \ell_{j-1} = m) \leq 1.375x - 1 + e^{-x} (1 - 0.375x)$$

Since (ℓ) is finite, then for all m, the inequality holds:

$$m \geq 1.375x - 1 + e^{-x}(1 - 0.375x)$$

The values of m and λ that satisfy this relation are given in the table below.

m	x	$\lambda = \frac{x}{4(m+1)}$	$\lambda_o = 2\lambda$
1	1.36366	0.17046	0.34092
2	2.16617	0.18051	0.36103
3	2.91273	0.18205	0.36409
3.6	3.35200	0.18217	0.36435
3.7	3.42491	0.18218	0.36435
3.8	3.49776	0.18218	0.36435
3.9	3.57057	0.18217	0.36434
4	3.64333	0.18217	0.36433
5	4.36951	0.18206	0.36413
6	5.09496	0.18196	0.36393
10	8.00048	0.18183	0.36366
50	37.09090	0.18182	0.36364
100	73.45454	0.18182	0.36364

CHAPTER 4

CONCLUSION

4.1 Discussion

We have studied how the performance of the multiple access broadcast channel is affected by the use of redundant packets. In particular, we have considered the use of the superpacket in the slotted Aloha type of transmission, and the use of the binary tree contention resolving algorithm to resolve conflicts that arise in a random access transmission using the superpackets. Our measure of system performance has been the system throughput only; neither delay nor system stability was explicitly considered. From the results obtained we observe that we cannot increase the throughput above that for an irredundant scheme.

For the slotted Aloha type of transmission we observe, however, that over some range of values of λ_p, the channel packet rate, the probability of a successful transmission of the superpacket per attempted transmission is higher than that for an irredundant scheme. One obvious consequence of this then is that, over this range, the number of retransmissions required to get the message to be successfully transmitted is fewer in the case of the superpacket scheme than in the irredundant scheme. As we pointed out in Chapter 2, a major cause of the instability of the slotted Aloha system is the numerous retransmissions that are encountered in the system when the traffic is heavy. We may then conjecture that the superpacket scheme will be more "stable" than the slotted Aloha scheme. Note that slotted Aloha is unstable. Anything more stable than it may not be stable itself. We have, therefore, used the term in a relative but not absoulte sense.

In the binary tree algorithm type of transmission we observe from the values of the throughput given in the Appendix that the throughput of the superpacket scheme is somewhat constant over all possible values of m. But for the irredundant scheme, the throughput values are not as constant as noted above; the throughput reaches a maximum value and finally remains constant at a much lower value. One possible conclusion then is that the superpacket scheme is again more stable than the irredundant scheme. Thus, whereas we cannot achieve higher throughput with the superpacket scheme, we may be able to achieve a greater system stability.

4.2 Suggestion for Further Work

The above conclusion on the effect of redundancy on the stability of the multiple access broadcast channel is a mere conjecture. The slotted Aloha scheme is known to be unstable and it is not recommended that an analytical proof of the fact that the superpacket scheme may be more stable than the slotted Aloha scheme be carried out. However, the binary tree algorithm has been proved in [3] to be stable. It may be necessary, therefore, to study the stability effects of the superpacket scheme when used for the binary tree type of transmission. In particular, one may wish to consider how frequently the system switches from the free-running mode to the retransmission mode. Our conjecture is that the system will not switch as frequently as when the irredundant scheme is used.

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