EXPANSION AND PRICING CRITERIA
FOR PORTS USING A TWO STAGE QUEUING MODEL

by

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ABSTRACT

The queuing theory approach to the analysis of ports is developed further for the traditional one stage queuing model, and extended to cover storage space as well as berths. The indirect cargo transfer operation is modeled as a two stage process: ship to berth, and berth to storage. For a ship to be unloaded, both the berth and a suitable amount of storage space must be free. Considering both the berths and the storage spaces as servers, a two stage queuing model is developed (assuming the assumptions of the $M/M/n/\infty/FIFO$ queuing model hold at each stage), and expected wait time is computed. Then the cost of the delays is balanced against the cost of berths and storage space to determine the socially optimal port expansion strategy.

Then using microeconomic theory, optimal berth and storage occupancy charges are derived considering the difference between social and private marginal cost created by the congestion effect.

The algorithm that finds the optimal number of berths and of storage spaces, as well as nine other programs applicable to port planning, have been coded on a programmable calculator and documented* for easy use by others. Because in an analysis mean service time at both the berth and the storage area is usually not known with certainty, the programs parametrically vary the service times over any range desired by the user to give the decision maker a matrix of optimal expansion strategies for varying mean service times.

Thesis Supervisor: Ernst G. Frankel

Title: Professor of Marine Systems

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Finally, the author would like to dedicate this thesis to his most devoted and loving wife, Mafalda Romero, whose moral support, care, understanding and kindness made possible the culmination of the prsent thesis.

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CHAPTER I

Introduction

1.1. Models in Port Planning

The techniques of port planning have come a long way from the simple treatment of the capacity of specific port elements, usually aimed at meeting single demand, to the present-day approaches of multiobjective, multipurpose integrated planning of large port systems.

Accordingly, the computational techniques available for port planning have evolved from numerical and graphical methods, appropriate for hand calculation and a variety of computer-based mathematical models, capable of analyzing with unprecedented detail the physical and economic behavior of complex port systems.

Planners are called upon to provide decision-makers with suggestions as to which elements should be built, where, and when and to what sizes, and how they should be operated to achieve the desired objectives, which may include increasing national income, regional development, environmental quality, and so on.

The answer to these questions, even for relatively small problems, involves the analysis of a number of possible alternative combinations of types and sizes of the element, time of execution, etc. In many cases, the number of alternatives may, by far, exceed the planner's capacity to evaluate them all. Therefore, planners have turned to the development of computer-based mathematical models in an attempt to overcome the complexities and time-consuming aspects of the analysis. The models developed in the field of port planning might be classified into two general categories:

1. optimization models; and
2. simulation models.

In general, optimization models are descriptive and prescriptive in nature. They are descriptive in the sense that they necessarily incorporate mathematical relationships which to some extent translate the particular physical, economic, and political and social aspects of the behavior system. They are also prescriptive because, through an objective function which measures the efficiency of the alternatives in meeting the objectives, they generate the solution which is meant to be optimal, at least in the framework of the mathematical problem formulated.

The optimization models are usually based on existing algorithms or theories, formulated to solve special types of problems characterized by the nature of their mathematical relationship. These algorithms or theories often face a severe direct limitation on the optimization models with respect to size of the problem they can handle, the type of mathematical relationship they accept, or indirect limitations, through the data that is required.

Among these theories (algorithms), the one with most widespread application in port planning has been queuing theory, mainly for its close relation to port system and and its relatively low cost. Simulation models, on the other hand, are only descriptive in nature. They are not bound to the restrictions of the optimization models in the nature of the mathematical relationships allowed that translate the real time behavior of the system, or its economic responses. They are limited mainly by the data available, the size of the computer to be used, and time and budget constraints. They can, therefore, include a more realistic representation of the problem at hand, and provide a far more accurate answer to the physical and economic responses of the system under a variety of external conditions. But they cannot do more than evaluate these physical and economic responses for a given system configuration. In the face of a generally infinite number of potential solutions, they lack the mechanism that can enhance improvement of their present system configuration toward an optimal solution, which is the ultimate objective of any planning effort.

1.2 Goals of the Present Study

Within the framework of the "optimization models", specifically those using queuing theory, there is room for simplification, incorporation of a more realistic representation of port system configuration, and additional uses of the model result.

The main goals of the present study are the development of a new approach to port expansion planning, using one and two stage queuing models, that is perhaps more systematic, realistic and general than the traditional approach; and the development of a port occupancy charges model using queuing models in combination with microeconomic theory.

A further goal of this study has been to develop analysis tools that require only data that is generally available, have quick response time, do not require much computation capacity and that can be used by an analyst or technician without his being intimately acquainted with queuing models. The need for such tools has been addressed in a research project at MIT (Responsive Analysis Method Project, or RAMP) 1/ in which a number of programs were developed for programmable pocket calculators , dealing mostly with urban transportation analysis. In like manner, the analysis procedures developed in this study have been programmed for pocket calculators and meet the requirements of simple data, simple user instructions and clear output interpretation. Thus in many cases port planners can have all the computational power necessary for a quick, accurate analysis of port planning with a pocket calculator (present cost are less than $400). The result of this study seem to indicate that these goals were at least in large part achieved.

1/ See reference 51

CHAPTER II

Application of Queuing Theory to Ports - General Review

2.1 Introduction

Although the application of queuing theory to the analysis and planning of port facilities is quite recent since the middle sixties, it is now one of the tools that port planners and designers use more frequently.

There are many reasons that such a tool has gained acceptance. One of them is that queuing theory has served to present, in clear and challenging form, one of the basic problems in the control of operational systems. The simple inverse relationship between the delay in getting served and the fraction of time the service facility is idle is one of the most obvious examples of the dangers of sub-optimization. If the service facility is used with little or no idle time, the delays imposed on customers become large; if the arriving units are to be served with little or no delay, the service facility must be idle an appreciable fraction of the time. A desire to find a solution to this basic dilemma has caused a proliferation of queuing theory, with well established and general results. Another important reason is that port operation systems and the design variables (number of berths, cranes, storage areas, etc.) associated with the analysis and/or planning of port facilities fit very well into the framework of a queuing model. Furthermore, as will be demonstrated later, the basic assumptions over which queuing models are based are in accordance with real port situations.

I will present in this chapter a general review of the structure of queuing models, the main assumptions involved, and the formulation and equations of the M/M/n/∞/FIFO queuing model adapted to the parameters related to the port failities and to the use of programmable calculators.

2.2 Characteristics of Queuing Systems

Any queuing system is characterized by the arrival of customers to a facility demanding service, the servers rendered, the rules used to select customer for service (if any), and the system capacity.

A schematically representation of two queuing systems is shown in Figures 2.1 and 2.2. Such systems, as well as any other queuing system, are defined when their basic elements are fully specified, i.e when

- the arrival pattern
- the service pattern
- number of servers
- system capacity
- queue discipline

are given and/or assumed.

2.3 Queuing Notation

Since there exists a proliferation of queuing models, a more or less international convention of symbolic identification of queuing models has been developed (See reference 15).

According to the specification of the above elements, queuing models are identified as follows in the A/B/N/C/d model, where

A = symbol for the interarrival time distribution

B = symbol for the service time distribution

N = number of servers

C = system capacity (maximum queue length)

d = queue discipline.

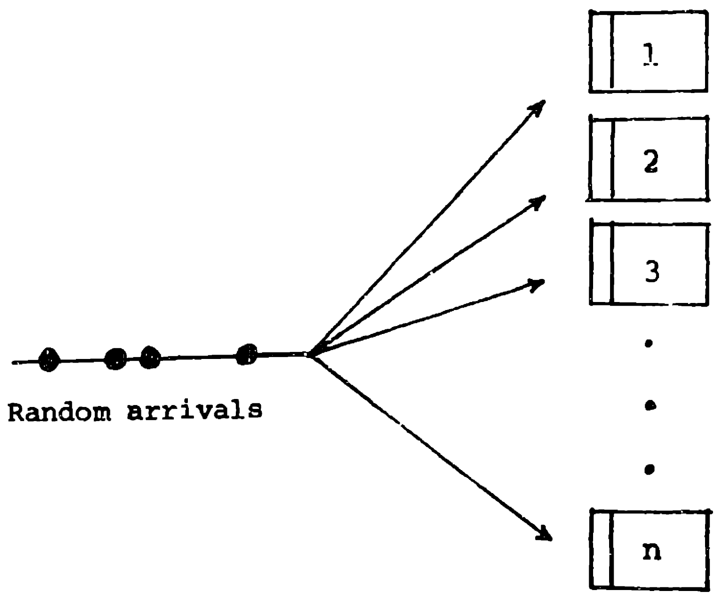


Figure 2.1 Parallel Servers-Multichannel Queuing System

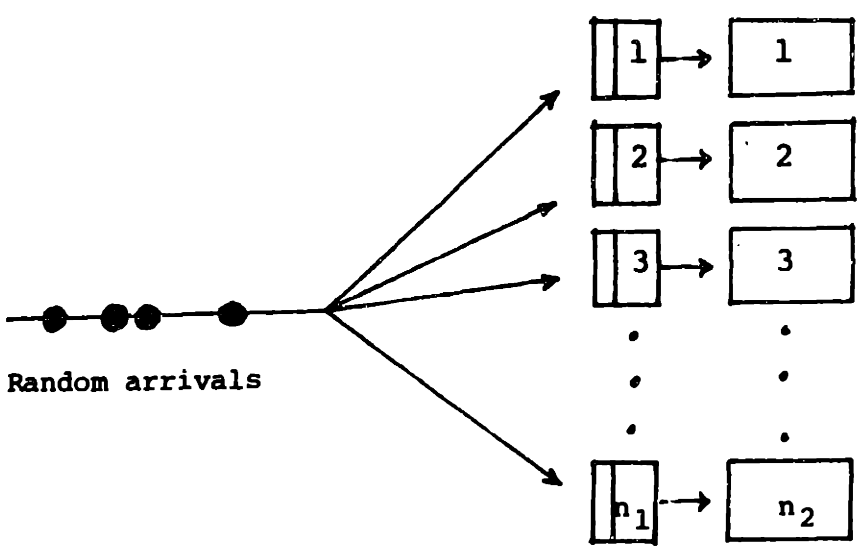


Figure 2.2 Series (Tandem) Servers-Multichannel Queuing System

Table 2.1 Queuing Notation presents a list of the most common symbols used to identify queuing models.

2.4 General Assumptions

There are many alternative assumptions that can be made about the various elements of a queuing system as can be seen from the list on Table 2.1.

With respect to ports, the assumptions about ship arrival and service time distributions are the most relevant and crucial. (Actually there are two important aspects that should be considered before using any queuing model, namely the identification of the relevant service stations in a port and whether or not the service stations are truly cooperative.We assume that they are truly cooperative, in this work) In this way the number of service stations can be specified according to the actual layout of the port, and the queue discipline according to the operational rule used. In most of the cases the capacity of the system ("waiting room" capacity) is assumed to be infinite as seems to be the case.

A general procedure to select one of the several queuing models possible is shown in Figure 2.3. This procedure involves a great deal of data analysis and is one of the critical stages in an analysis and planning of port facilities using queuing theory.

It is worth noting that in specifying each one of the basic elements in this way a simplification of the real situation is inevitable. However, this is in no way different from what occurs in any other type of model, and represents the price that one must be willing to pay in order to have a tractable model.

Table 2.1 Queuing Notation 1/

Element	Symbol	Description
Interarrival-time distribution (A)	M	Exponential
	D	Deterministic
	E_k	Erlang type k (k=1,2,...)
	GI	General Independent
Service-time distribution (B)	M	Exponential
	D	Deterministic
	E_k	Erlang type k (k=1,2,...)
	G	General
Number of service stations(parallel) (N)	n	1,2,...,n
System capacity (C) (Queue size limit)		1,2,...
Queue discipline (d)	FIFO	First in, First out
	LIFO	Last in, First out
	SIRO	Service in random order
	PRI	Priority
	GD	General discipline

1/ Source: Queuing Systems, Vol I by Kleinrock, Leonard

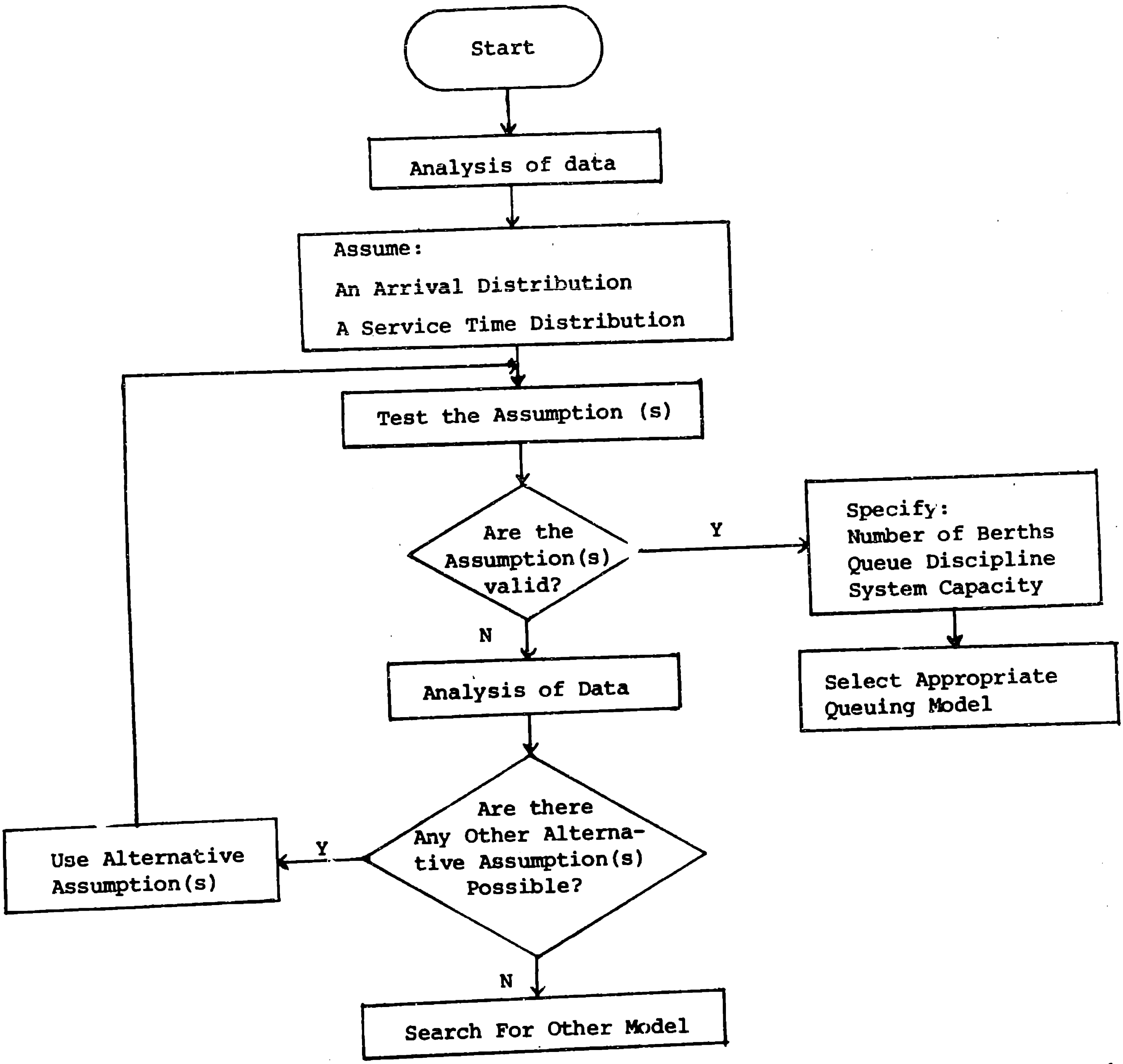


Figure 2.3 <u>A Procedure to Select a Queuing Model</u>

2.5 Port-Related Assumptions

As was pointed out in the last section, there are many alternative assumptions that can be made about the principle elements of a queuing system, most critically the assumptions concerning arrival and service time distribution. We will deal throughout this thesis mainly with the M/M/n/∞/FIFO model. That is, we are assuming that ships arrive at random, implying that the distribution of arrivals is described by the Poisson distribution, and that the duration of the service times is random, fitting the negative-exponential distribution. The validation of these assumptions depends on the particular situation of each individual port and is not in any way generally valid. However, in a fairly large number of cases (references 1,2,3,4,5) it seems that the Poisson and negative-exponential distribution are good approximations to real port situations.

It can be said that in general ships arrive at ports in a random fashion. However, this assumption may not be valid for passenger liners, container ships, ro-ro ships and tankers, which need special berths and follow more or less strict schedules. Therefore, we exclude from our analysis the facilities associated with them.

The assumption regarding the service time stipulates that the time required to serve a ship is independent of the time required to serve the ship just serviced, and does not influence the service time of the next ship.

The service time is regarded as random since there are numerous random factors affecting it. This assumption would be unrealistic if there were functional relationships between the service time and a small number of variables; however, this does not seem to be the case. Although widely accepted, this assumption has been challenged by some authors; however a complete refutation of it has not been shown so far.

There is some evidence that assuming the service time distribution to be "negative-exponential" will result in a little bit conservative (higher) estimates of some queuing measures, such as the waiting time, in comparison with the result that can be obtained assuming an Erlang Type K (k = 2 or higher).

Anyhow, we need to keep in mind that each set of assumptions needs to be tested in every particular case.

2.6 Test of Assumptions

The principle assumptions (arrival and service time distributions) can be verified in several ways. Among them the following two are commonly used:

1. Verify that the ship arrival distribution is Poisson and the service time (including all relevant aspects) is exponential. The data needed to carry out this are:
   a. an estimated mean arrival rate during certain periods (considered as a sample)
   b. an estimated mean service rate. Alternatively, the mean time between successive arrivals and ship service completions can be used.

2. Verify that the distribution of number of ships in port in a given period (sample) fits the distribution implied by the model. For this we need the same information as above plus the number of berths used in that period to provide services.

2.6.1. The Arrival Distribution Test

If the ships arrive at random, implying that the distribution of arrivals is described by a Poisson distribution, we can define p(V) as the probability of V ships arriving in port as:

$$p(v) = \frac{(\lambda)^{v} \; e^{-\lambda}}{v!} \tag{2.1}$$

where

λ = mean number of arrivals in unit time

v = as described before.

The expected frequency F(V) of V ships arriving at a port in a given period T can be expressed as:

$$F(v) = Tp(v) \tag{2.2}$$

Equation 2.1 is known as the Poisson distribution, which have a mean and variance equal to λ . Since we do not know which specific Poisson distribution to expect, i.e. which value of λ to use, we use for λ the mean of the sample:

$$\lambda = \frac{\sum_{v=0}^{\infty} V \, f(v)}{T} \tag{2.3}$$

where

$f(v)$ = the observed number of days, in period T (sample), that v, ships arrived at port.

In order to decide whether or not the ship arrival data constitute a sample from a population with Poisson distribution, at some significant level, α , we use the chi-square (χ^2) test by defining a statistic as

$$\chi^2 = \sum_{i=1}^{r} \frac{[F(v_i) - f(v_i)]^2}{F(v_i)} \tag{2.4}$$

where

r = number of categories within which the sample values fall

v_i = value(s) of v associated with category i

If certain criteria are met, this statistic has approximately a chi-square distribution with f degrees of freedom, where

$f = r-2$.

Then if we consider as the null hypothesis that arrivals are Poisson distributed, we reject the null hypothesis at significance level α if the calculated value of χ^2 exceeds the value of $\chi^2_{\alpha,f}$

As we pointed out in chapter one, several programs for programmable calculators (TI-59 in this case) have been developed through this thesis to compute most of the formulas involved in the application of the M/M/n model to ports. One of these programs "Port Traffic χ^2 Goodness of Fit Test Ships Arrival Distribution" is described in Appendix I.

An example follows to show how we can test the ship arrival distribution and at the same time the output of the program mentioned above.

Example 2.1

The following data of ship arrivals during a year at the hypothetical port is given in Table 2.2, and used as input in the program. The output is shown in Table 2.3.

Table 2.2 Ship Arrivals at a Port

Number of Ship Arrivals (v)	Number of Days That ,v Ships Arrive at Port (observed)
0	0
1	3
2	10
3	21
4	43
5	56
6	61
7	57
8	42
9	28
10	20
11	12
12	8
13	4
14	0
14	0

From Table 2.4 and with 5% level of significance, we have

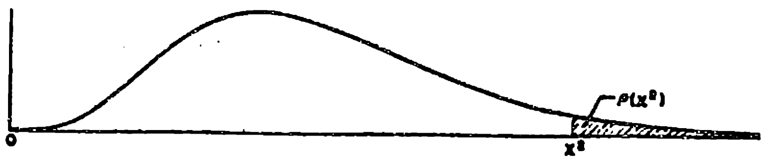
$$\chi^2_{.05,10} = 18.31$$

Since χ^2 computed (3.210392) does not exceed this value, we have no reason to reject the hypothesis that the ship arrivals distribution is Poisson.

Table 2.3 Ship Arrival Distribution Test. Program Output

Number of ship arrivals, v	Observed number of days that v ships arrive at a port W_v	Computed (Poisson) Number of days that v ships arrive at port W_v	Average ship arrivals $\bar{v}$
0.	0.	.5510151048	
1.	3.	3.579338733	
2.	10.	11.62547984	
3.	21.	25.17261433	
4.	43.	40.87963602	
5.	56.	53.10992712	6.495890
6.	61.	57.49937772	
7.	57.	53.35852234	
8.	42.	43.32633892	
9.	28.	31.27149735	
10.	20.	20.31362198	
11.	12.	11.99591475	
12.	8.	6.493678964	
13.	4.	3.244786686	
14.	0.	2.578204864	

Grouped Category	Number of ship arrivals	Observed Data grouped $f(v_i)$	Computed Data Grouped $F(v_i)$	Degrees of freedom	χ^2
	0.	-1.	-1.		
	1.	-1.	-1.		
1.	2.	13.	15.75502868		
2.	3.	21.	25.17261433		
3.	4.	43.	40.87963602		
4.	5.	56.	53.10992712		
5.	6.	61.	57.49937772		
6.	7.	57.	53.35852234	10.	3.210392
7.	8.	42.	43.32633892		
8.	9.	28.	31.27149735		
9.	10.	20.	20.31362198		
10.	11.	12.	11.99591475		
11.	12.	8.	6.493678964		
12.	13.	4.	5.82299155		
	14.	-1.	-1.		



$$P(\chi^2) = \int_{\chi^2}^{\infty} \frac{1}{\left(\frac{f-2}{2}\right)! \, 2^{f/2}} (\chi^2)^{(f-2)/2} e^{-\chi^2/2} \, d(\chi^2)$$

f \ $P(\chi^2)$	.995	.990	.975	.950	.900	.750	.500	.250	.100	.050	.025	.010	.005
1	3927×10^{-8}	1571×10^{-7}	9821×10^{-7}	3932×10^{-6}	0.01579	0.1015	0.4549	1.323	2.706	3.841	5.024	6.635	7.879
2	0.01003	0.02010	0.05064	0.1026	.2107	.5754	1.386	2.773	4.605	5.991	7.378	9.210	10.60
3	.07172	.1148	.2158	.3518	.5844	1.213	2.366	4.108	6.251	7.815	9.348	11.34	12.84
4	.2070	.2971	.4844	.7107	1.064	1.923	3.357	5.385	7.779	9.488	11.14	13.28	14.86
5	.4117	.5543	.8312	1.145	1.610	2.675	4.351	6.626	9.236	11.07	12.83	15.09	16.75
6	.6757	.8721	1.237	1.635	2.204	3.455	5.348	7.841	10.64	12.59	14.45	16.81	18.55
7	.9893	1.239	1.690	2.167	2.833	4.255	6.346	9.037	12.02	14.07	16.01	18.48	20.28
8	1.344	1.646	2.180	2.733	3.490	5.071	7.344	10.22	13.36	15.51	17.53	20.09	21.96
9	1.735	2.088	2.700	3.325	4.168	5.899	8.343	11.39	14.68	16.92	19.02	21.67	23.59
10	2.156	2.558	3.247	3.940	4.865	6.737	9.342	12.55	15.99	18.31	20.48	23.21	25.19
11	2.603	3.053	3.816	4.575	5.578	7.584	10.34	13.70	17.28	19.68	21.92	24.72	26.76
12	3.074	3.571	4.404	5.226	6.304	8.438	11.34	14.85	18.55	21.03	23.34	26.22	28.30
13	3.565	4.107	5.009	5.892	7.042	9.299	12.34	15.98	19.81	22.36	24.74	27.69	29.82
14	4.075	4.660	5.629	6.571	7.790	10.17	13.34	17.12	21.06	23.68	26.12	29.14	31.32
15	4.601	5.229	6.262	7.261	8.547	11.04	14.34	18.25	22.31	25.00	27.49	30.58	32.80
16	5.142	5.812	6.908	7.962	9.312	11.91	15.34	19.37	23.54	26.30	28.85	32.00	34.27
17	5.697	6.408	7.564	8.672	10.09	12.79	16.34	20.49	24.77	27.59	30.19	33.41	35.72
18	6.265	7.015	8.231	9.390	10.86	13.68	17.34	21.60	25.99	28.87	31.53	34.81	37.16
19	6.844	7.633	8.907	10.12	11.65	14.56	18.34	22.72	27.20	30.14	32.85	36.19	38.58
20	7.434	8.260	9.591	10.85	12.44	15.45	19.34	23.83	28.41	31.41	34.17	37.57	40.00
21	8.034	8.897	10.28	11.59	13.24	16.34	20.34	24.93	29.62	32.67	35.48	38.93	41.40
22	8.643	9.542	10.98	12.34	14.04	17.24	21.34	26.04	30.81	33.92	36.78	40.29	42.80
23	9.260	10.20	11.69	13.09	14.85	18.14	22.34	27.14	32.01	35.17	38.08	41.64	44.18
24	9.886	10.86	12.40	13.85	15.66	19.04	23.34	28.24	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	19.94	24.34	29.34	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	20.84	25.34	30.43	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	21.75	26.34	31.53	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	22.66	27.34	32.62	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	23.57	28.34	33.71	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	24.48	29.34	34.80	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	33.66	39.34	45.62	51.80	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	42.94	49.33	56.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	52.29	59.33	66.98	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	61.70	69.33	77.58	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	71.14	79.33	88.13	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	80.62	89.33	98.65	107.56	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	90.13	99.33	109.14	118.50	124.34	129.56	135.81	140.17
x	−2.576	−2.326	−1.960	−1.645	−1.282	−0.6745	0.0000	+0.6745	+1.282	+1.645	+1.960	+2.326	+2.576

Table 2.4 <u>Upper Percentage Points of the χ^2 Distribution</u>

Source: <u>Statistic Manual</u> by Crow, Davis and Maxfield.

Even at 10% level of significance, we cannot reject the hypothesis, since

$$\chi^2_{.10,10} = 15.99$$

Therefore in this hypothetical case, and in several real life situations (references 1,2,3,4,5) the ship arrivals distribution can be said that it follows a Poisson distribution.

It is worth noting that we use the level of significance and the degrees of freedom to reject or accept the hypothesis that ship arrivals followed a Poisson distribution. In many studies there exists a little bit of ambiguity about the meaning of the level of significance. Instead of testing the hypothesis, they define some ranges of "good" and "bad" fit which are subject to some controversy.

Also it is very important to note that we are talking about the ships arrival distribution and not of the distribution of ships at port, which is completely different to the distribution mentioned above; for it one cannot use Equation 2.1 for a test. I make this clarification since this is a common mistake made in many cases. In the next section the appropriate formula to test the distribution of ships at port will be presented.

2.6.2 The Ships Distribution Test

Under the assumptions that ships arrive at random, implying that the distribution of arrivals is described by the Poisson distribution, and that the service time is a random variable fitting the negative-exponential distribution, and that we are dealing with a multi-channel queuing system with parallel service stations, no limitation in the queue size and FIFO queue discipline, queuing theory provides us with well established formulas. As was pointed out, we need to know, in this case, the arrival rate, the mean service time and the number of berths involved. Then from queuing theory we find that $p(n_s)$, the probability of n_s ships present in port (both those waiting for service and being served) at any given time is expressed as:

$$p(n_s) = \begin{cases} \dfrac{(n\rho)^{n_s}\, p(0)}{n_s!} & \text{if} \quad n_s \leq n_s \\ \dfrac{(\rho)^{n_s}\, n^n}{n\,!}\, p(0) & \text{if } n < n_s < \infty \end{cases} \qquad (2.5)$$

where

ρ = berths utilization factor = $\dfrac{\lambda s}{n}$

n = number of berths

s = mean service time

λ, n_s as defined before, and

$$p(0) = \left[\sum_{i=0}^{n-1} \frac{(n\rho)^i}{i\,!} + \frac{(n\rho)^n}{n\,!(1-\rho)} \right]^{-1} \qquad (2.6)$$

Also, we can check the probability, $p(v_q)$ of v_q ships waiting for berths (in queue) and the probability $p(v_b)$ of v_b ships at berths as follows:

$$p(v_q) = \begin{cases} \displaystyle\sum_{i=0}^{n} \frac{(n\rho)^i}{i\,!}\, p(0) & \text{for } v_q = 0 \\ \dfrac{(\rho)^{(n+v_q)}\, n^n}{n\,!}\, p(0) & \text{for } v_q > 0 \end{cases} \qquad (2.7)$$

and

$$p(v_b) = \begin{cases} p(0) & \text{if } v_b = 0 \\ \dfrac{(n\,\rho)^{v_b}}{v_b!}\, p(0) & \text{if } 0 < v_b > n \end{cases} \qquad (2.8)$$

An identical test to the one described in Section 2.6.1 can be done by applying Equation (2.6) instead of Equation (2.1). The parameter needed to complete the specification is ρ . A program and the user instruction ("Port Traffic χ^2 Goodness of Fit Test - Ships Distribution at Port) is shown in Appendix I. The outputs are similar to the first program.

2.7 <u>The Service Time Distribution Test</u>

If we let

t = class interval of the time that ships spent at berth

(difference between berthing and deberthing time)

we can define the following theoretical distribution

$$f(t) = e^{-t/t_o} \qquad (2.9)$$

where

t_o = mean duration of the service time between successive arrivals.

An identical test can be carried out using Equation (2.9) and the equation of the χ^2 statistic. Alternatively we can run a linear regression to decide whether or not the distribution service time is "negative-exponential".

Theoretically, a "negative-exponential distribution of the service time can be expressed as:

$$F(t) = 1 - e^{-t/t_m} \qquad (2.10)$$

where

t_m = the mean service time obtained from a log-linear regression.

$F(t)$=the accumulative distribution computed from the observed data.

Then, we can rearrange Equation (2.10) as follows:

$$1 - F(t) = e^{-t/t_m}$$

and define

$$y = \ln [1 - F(t)] = -t/t_m$$

A regression of the form

$$y = \alpha t \qquad (2.11)$$

where

$$\alpha = -1/t_m$$

can be performed and the value of t_m compared with t_o; also the usual statistic associated with regression analysis can be used to support our assumption.

2.8 Structure of the Queuing Model

There is a lot of information about the port system that we can get from queuing models. A detail list of the most important information is given below:

- The probability of delays, i.e.the average fraction of demand that is not given immediate service.

- The probability of a waiting-time in excess of some given level , i.e. for any fixed value of delay we can find the probability that the waiting-time of a ship exceeds that value.
- The average number of ships waiting,both in the system and in queue.
- The average waiting-time,averaged over all demand, or over delayed demand only.
- Service stations (berths,etc) perfomance: average number of service stations occupied and idle and service stations occupancy rate.

All the derived formulas that follow apply to ports for which the assumptions of an M/M/n/∞/FIFO queuing model are valid.

2.8.1 The Probability of Delay

The probability p(D),that a delay occurs:

$$p(D)= \frac{(n\rho)^n}{n!\ (1-\rho)} P_{(o)} \qquad (2.12)$$

The probability that a delay occurs means the probability that there are n or more ships in the port and hence an arrival must wait

Equation 2.12 can be derived from Equation 2.8 by setting $v_b=n$

2.8.2 The Expected Number of Ships in Queue

The expected number of ships in queue, $\bar{w}_q$, can be expressed as:

$$\bar{w}_q= p(D)\ \frac{\rho}{1-\rho} \qquad (2.13)$$

2.8.3 The Expected Number of Ships in Port

The expected number of ships in port (waiting for service and in service) , $\bar{w}$, is :

$$\bar{w} = n\rho + \bar{w}_q \tag{2.14}$$

2.8.4 The Expected Waiting Time in Queue

The expected waiting time in queue, w_q, is given as:

$$w_q = p(D) \frac{s}{n(1 - \rho)} \tag{2.15}$$

2.8.5 The Expected Waiting Time in the System

The expected waiting time for all demand, w, is

$$w = s \; [\; p(D) \frac{1}{n(1 - \rho)} + 1 \;] \tag{2.16}$$

2.8.6 The Expected Value of Total Delay

The expected value (in time) of total delay, TW, is

$$TW = \lambda w \tag{2.17}$$

2.8.7 The Marginal Waiting Time

The marginal waiting time,MWT,is

$$MWT = \frac{\delta (TW)}{\delta \lambda}$$

$$= s\left\{\left[\frac{(n\rho)^n}{n(1-\rho)\; n! \left[(1-\rho)\sum_{i=0}^{n-1}\frac{(n\rho)^i}{i!} + \frac{(n\rho)^n}{n!}\right]}\right]\left[\frac{1}{(1-\rho)} + n(1-\rho) + n\rho\left(\frac{\sum_{i=0}^{n-1}\frac{(n\rho)^i}{i!}}{(1-\rho)\sum_{i=0}^{n-1}\frac{(n\rho)^i}{i!} + \frac{(n\rho)^n}{n!}}\right)\right] + 1\right\} \tag{2.18}$$

2.8.8 The Expected Waiting Time for Those Who Wait

The expected waiting time for those ships that are deleyed, q, is expressed as

$$q = \frac{s}{n(1 - \rho)} \tag{2.19}$$

2.8.9 The Expected Number of Berths Occupied

The expected number of berths occupied, n_o, is

$$n_o = n\rho \tag{2.20}$$

2.8.10 The Expected Number of Idle Berths

The expected number of idle berths, n_i, is

$$n_i = n(1 - \rho) \tag{2.21}$$

2.9 Comments

In Appendix I a set of programs to compute most of the above Equations is shown.

Almost all of these Equations involve a great deal of calculations or the use of tables or diagrams. The programs developed made it very easy to compute quickly and accurately each one of the above formulas.

Many impacts due changes in port operations can be evaluated with only a few keystrokes. Although those programs are only a by-product of this study, special care was taken to made them very functional and useable.

CHAPTER III

Queuing Models: The Traditional Approach

3.1 Introduction

The application of queuing theory to ports is relevant for several reasons. The two main problems to which queuing theory is ultimately devoted as expressed in Chapter II are:

1. How many service stations are to provided to meet a given demand for service?
2. What is the optimal capacity in accordance with specific values of the relevant variables?

In ports the question of the optimal number of berths and the optimal capacity can be regarded as the main expansion problem. The traditional approach to this problem using queuing models, specifically the M/M/n model, will be discussed in this chapter, and an algorithm based on an optimal expansion criterion derived from the application of the queuing model to ports will be presented. One of the reasons for the inclusion of this model can be found in the following remark from Jan de Weille and Anadarop Ray (Ref. 25):

> "Though simple, the model has considerable analytical value, and has already proven effective in analysing a number of ports in developing countries. It is easy and inexpensive to apply and therefore has a high payoff as an analytical technique."

3.2 The Traditional Approach

The traditional approach to finding the optimum number of berths has followed the expansion criterion defined as follows.

The optimum number of berths is the number of berths for which the annual cost of the time that ships spend waiting for a berth and related facilities plus the annual cost of providing those berths and related facilities is minimized. Therefore the cost function can be expressed as:

$$TCOST = T \{ cn + V [W_q(n)]\} \tag{3.1}$$

where

T = period of time (usually a year)

n = number of berths

c = average berth cost per unit time (derived from berth construction cost)

V = average ship waiting time cost

$W_q(n)$ = Expected queuing (waiting) time

Some authors prefer to use a measure of berth idle time and use a so-called berth idle time cost. This assumption can be challenged in many ways, but as will be shown, its use has only an impact on the magnitude of the cost function and not on the optimum number of berths.

Figure 3.1 depicts the relationship of the cost function and the number of berths.

3.3 Optimum Number of Berths

I will use the cost function defined by Equation (3.1) and the M/M/n queuing model described in the last chapter to derive an "optimum expansion strategy" or expansion criterion. Substituting Equation (1.23) into (3.1) we have

$$TCOST = T \{ cn + V [p(0) \frac{s}{n(1-\rho)}]\} \tag{3.2}$$

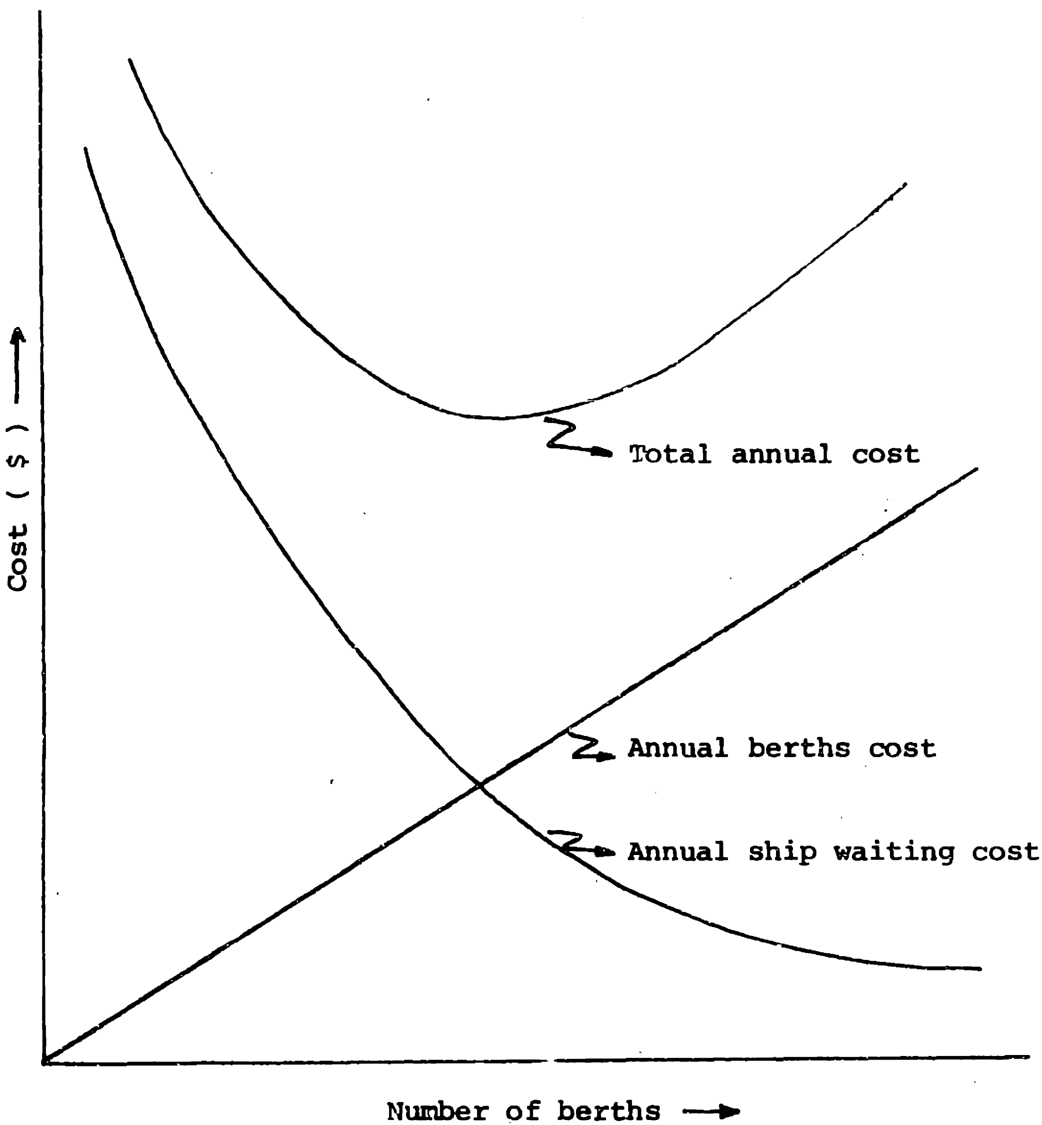


Figure 3.1 Total Annual Cost vs Number of Berths

and substituting Equations (1.7) into (1.10) and (1.10) into (3.2) yields:

$$TCOST = T \{ cn + V [\frac{(n\rho)^n}{n!(1-\rho)} \times \frac{1}{\sum_{i=0}^{n-1} \frac{(n\rho)^i}{i!} + \frac{(n\rho)^n}{n!(1-\rho)}} \times \frac{s}{n(1-\rho)}] \}$$

$$= T \{ cn + V [\frac{(n\rho)^n}{(n\rho)^n + n!(1-\rho) \sum_{i=0}^{n-1} \frac{(n\rho)^i}{i!}} \times \frac{s}{n(1-\rho)}] \} \quad (3.3)$$

It can be seen that

cn increases as n increases,

$\frac{(n\rho)^n}{(n\rho)^n + n!(1-\rho) \sum_{i=0}^{n-1} \frac{(n\rho)^i}{i!}}$ decreases as n increases, and

$\frac{s}{n(1-\rho)}$ decreases as n increases.

Therefore the cost function has a unique minimum.

Provided that the total cost function is continuously differentiable with respect to n, the efficiency condition as to the optimum number of berths, n, is obtained by taking the partial derivative of Equation (3.1) with respect to n and setting the result equal to zero:

$$\frac{\partial TCOST}{\partial n} = 0 \quad (3.4)$$

This becomes

$$c + V \frac{\partial W_q(n)}{\partial n} = 0$$

or

$$\frac{\partial W_q(n)}{\partial n} = - \frac{c}{V} \tag{3.5}$$

However, since we are talking about choosing the number of berths an indivisibility problem is unavoidable. (One way of getting around this problem is considering total berth length instead of number of berths, but the analysis becomes more complicated.) Generally speaking, factor indivisibility should not be disregarded where an addition of a unit of a factor constitutes a relatively substantial additional cost.

In order to deal with the indivisibility problem we can get a good approximation of Equation (3.5) by defining:

$$\frac{\partial W_q(n)}{\partial n} \simeq W_q(n + 1) - W_q(n) \tag{3.6}$$

Then substituting Equation (3.6) into (3.5) and multiplying both sides by -1, and changing the equality sign (since we now have an approximation), we have

$$W_q(n) - W_q(n+1) \geqslant \frac{c}{V} \tag{3.7}$$

Another way to find the optimum value of n is evaluating the cost function for sequential values of n. Then the optimum is found when:

$$TCOST(n-1) > TCOST(n) < TCOST(n+1) \tag{3.8}$$

This relation is shown in Figure 3.2.

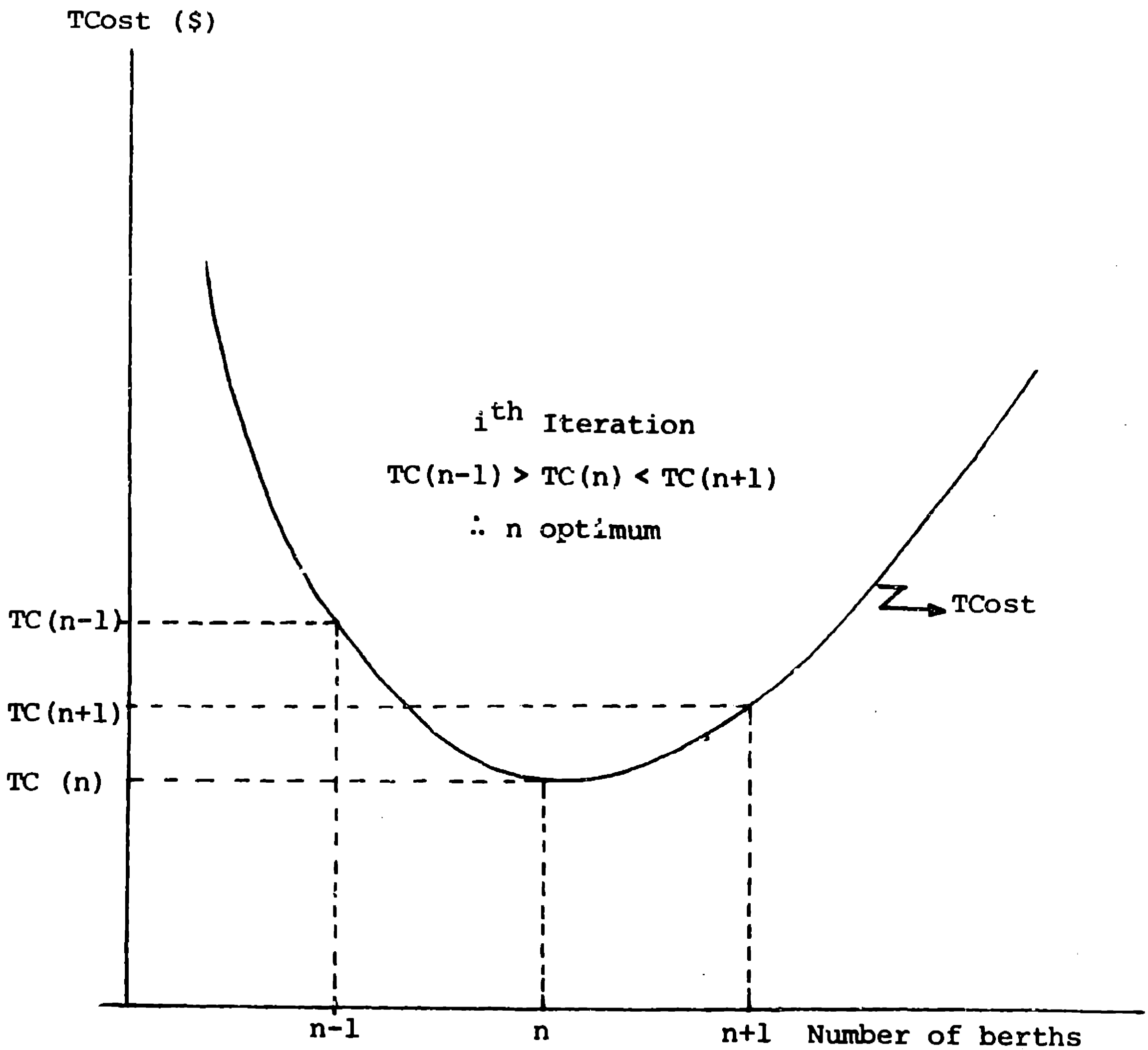


Figure 3.2 Cost Function Evaluation

It is very easy to show that both approaches yield the same result. Equation (3.8) can be expressed as:

$$c(n-1) + V(W_q(n-1)) > cn + V(W_q(n)) < c(n+1) + V(W_q(n+1)) \qquad (3.9)$$

Setting $V[W_q(n=0)] = \infty$, and after some algebraic manipulation, we obtain

$$V[\, W_q(n) - W_q(n+1)] \geqslant c \qquad (3.10)$$ [1/]

or

$$W_q(n) - W_q(n+1) \geqslant \frac{c}{V} \qquad (3.11)$$

Our expansion criterion model is defined by Equation (3.7), which can be paraphrased as:

> "Expand (construct a new berth and related facilities) if the benefit from reduction in waiting time exceeds the annual cost of the expansion."

Equation (3.7) is more general since a table or graph can be constructed as a function of the "cost index," c/V, defined as the ratio of berth

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[1/] A similar equation was used by Fook-Wah Ng in his thesis Analytical Model for an Offshore Port Facility Reception. However, he committed a tremendous mistake in multiplying the right hand side of the inequality by the service time, i.e.,

$$V \quad [W_q(n) - W_q(n+1)] \quad > \quad sc$$

which makes the equations and its results meaningless.

cost (including the cost of related facilities) to the cost of ship waiting time.

An important aspect of this formulation is that it will yield the same results, using, for the berth cost function, either the expected number of idle berths or the total berth cost, provided that the cost of an idle berth is related to construction cost only. In the former, Equation (1.22) will be expressed as

$$TCOST = cn(1-\rho) + V[W_q(n)]$$

$$= cn - cn \frac{\lambda s}{n} + V[W_q(n)] = cn - c\lambda s + V[W_q(n)]$$

where

$n(1-\rho)$ = expected number of idle berths

and

$$\frac{\partial TCOST}{\partial n} = c + V \frac{\partial W_q(n)}{\partial n}$$

which is identical to Equation (3.4). Of course, the magnitude of the cost function will be different, but the optimum number of berths will remain the same in both cases.

3.4 Port Expansion - Optimization Algorithm (I)

As part of this thesis, the following algorithm has been developed to find the optimum number of berths according to the expansion criterion as defined in Equation (3.7) in a more simple and systematic way. Since average service time is not known with certainty, it is useful to find the optimum number of berths, n*, for many values of average service time, s. Therefore this algorithm is a parametric analysis computing n* for increasing values of s.

1. Set range of service times to be used

   Define:

   s_o = initial value of s (service time)

   Δs = the increment of s to be used (>0)

   k = the number of values of s desired

2. Specify parameters

   λ = arrival rate

   c/V = cost ratio

   set flag if cost function is to be evaluated for n*

3. Initialize parametric analysis

   i = 0

   n = 2

4. Initialize search for n*

   n = n-2 1/

   W_q (old) = ∞

5. n = n + 1

6. Test if n = 0, if yes, go to 5, else continue

7. Compute $\rho = s\lambda/n$. If $\rho \geq 1$, print n and ρ and go to 5; else continue.

1/ Begins search for optimal n at 1 less than n* for the last value of s, since n* monotonically increases with s.

8. Compute $W_q = \dfrac{s(n\rho)^n}{n!(1-\rho)^2 \sum_{i=0}^{n-1} \frac{(n\rho)^i}{i!} + n(1-\rho)(n\rho)^n}$

9. Compute $\Delta W_q = W_q(\text{old}) - W_q$

10. If $W_q > c/V$, set $W_q(\text{old}) = W_q$ and go to 5; else continue.

11. For present s, $n^* = n-1$ is optimal.
Print n^*.

12. If flag is set, compute and print TCOST(n^*); else continue.

13. Set $i = i + 1$
$s = s + \Delta s$

14. If $i < k$, go to 4; else stop.

The program based in this algorithm using the TI-59 is shown in Appendix II

3.5 Implication of the Port Expansion Model

The above port expansion model can be thought of as a maximization of net benefit, regardless of whom the beneficiaries will be, and as Jan de Weille, et. al.point out, from a national point of view this would seem to be the right approach, provided that all benefits accrue to the national economy. Moreover he said that this will be true if the shipping industry is operating under fully competitive conditions which force it to pass on cost reduction in the form of freight rate reductions and therefore all benefits will accrue to the national economy.

On the other hand if monopolistic conditions exist in the shipping industry , some presure may be needed to obtain freight rate reductions, or else additional port charges might be introduced to capture part of the profit. Any way it will be expected that after the investment in expansion, sooner or later cost reduction will be reflected in freight rate reduction (eliminating cost penalties) and both parties will get their share of the benefit.

CHAPTER IV

The Two Stage Queuing Model: A New Approach

4.1 Introduction

The single stage character of the model presented in Chapter III is all right as far as direct transfer of cargo between ships and land transport vehicles is concerned. This is a single stage process. But in many ports, the direct transfer of cargo represents only a small percentage of all cargo movements. The indirect route, i.e. the transfer of import cargo from the hold of the ship to the transit shed, and after a lapse of time to the hold of a land transport vehicle (and vice versa for export cargo), is more dominant.

The justification for still considering one-stage models is the following. The main function of transit storage is to make the operation of loading and unloading ships and inland vehicles independent of one another. This waiting-room function will be performed perfectly only when the transit storage holding capacity is practically unlimited.

However, a "waiting-room" between two process stages is hardly ever of literally unlimited capacity (although if the cost of waiting-room holding capacity is very small in relation to the cost of production in the preceding and succeeding process stages the holding capacity might be considered unlimited for practical purposes). A large amount of evidence against the last assumption can be found in the literature.It is rare that storage space within the port area is sufficiently ample never to be exhausted.

So long as port storage is not of unlimited capacity the performance of the two service stages will be connected. When the storage happens to be filled by cargo (because something is holding up the production in the second stage), the preceding service stage cannot pass on cargo which has been served, but this cargo has to remain in the first stage (usually at the apron), blocking the service stations for following ships. If nothing is done to relieve the original course of delay in the second stage, the production of the first stage will eventually go down to the level of output of the second stage.

If the service time considered in the one stage model includes the delays of the other links, the results can be considered to be a reasonable approximation of the real system. However, it is possible to model the two link involved, and have a better representation of the real system.

There is an important aspect of the storage area that will permit us to build up our model, namely that the time that a consignment (cargo) spends in the transit storage area is not primarily determined by the capacity of the following service stage, but by the time that elapses before it is collected by the importer (or in the case of export cargo, the interval between receipt of a consignment at the port land-side and the arrival of the applicable ship), i.e. the cargo dwell time. This fundamental fact is taken most adequately into account by making transit storage a process stage in its own right.

The storage area then plays both the role of waiting-room in the ordinary sense of the word and that of as a service station in the sense which is relevant in queuing models. The dwell time is determined independently of what is going on in the other service stages; that is why the storage area is analytically analogous to a proper service stage rather than just an in-between waiting-room. Hence, the dwell time is to be viewed as service time rather than queuing time. This way of dealing with the storage area is a new approach to port planning which will be described in the next section.

4.2 The Two Stages Multi-Channel Queuing Model

In this analysis we use the same assumptions as previously, that ships arrive at random following a Poisson distribution and that the service time is negative-exponential.

However, in this two-stage model we assume that ships may not load/unload their cargo unless there is both an available berth and available storage space. Then the expected queuing time in the two stage multi channel system, can be derived as follow:

Let

W_q = expected queuing time of all ships

p1 = probability that every relevant service station in the first stage (cranes, berths, etc) are occupied when a ship arrives.

p2 = probability that no storage space is available (Storage space is defined as the area needed to store an average ship load).

q1 = expected time before a service station in the first stage is free on occasions when a ship is delayed at the first stage

q2 = expected time before enough space is cleared on occasions when a ship is delayed at the second stage.

Then the joint probability that a ship will be delayed can be expressed as:

$$p(\text{Delay}) = p1 + p2 - p1p2 \qquad (4.1)$$

Now delays can occour under the following combinations:

1.- The stations in the first stage are fully occupied,and the stations in the second stage are not.

2.- The stations in the second stage are fully occupied, and the stations in the first stage are not.

3.- The stations in both stages are fully occupied.

In case 1, the expected delay is simple q1. In case 2, the expected delay is q2. In case 3, the expected delay is the maximum of q1 and q2. (An arriving ship will have to wait until there is both free space at a berth <u>and</u> in storage. If the berth becomes free before storage is free, it will still have to wait for storage to be freed, and viceversa; so the maximum delay between q1 and q2 controls).

Summing the expected delays in each case times their probability of occurance, we get the total expected queuing time as:

$$W_q = (p1 - p1p2)q1 + (p2 - p1p2)q2 + p1p2[\max (q1,q2)]$$

$$= p1q1 + p2q2 - p1p2[q1 + q2 - \max (q1,q2)]$$

This can also be written as

$$W_q = p1q1 + p2q2 - p1p2[\min (q1,q2)] \quad \underline{1/} \qquad (4.2)$$

(Since if q1 is maximum the third term became p1p2[q2],and if q2 is maximum,p1p2[q1], i.e. p1p2 is multiply by the minimum (q1,q2)).

Equation 4.2 is valid under the following general assumptions:

1.- The whole storage area can be used by all cargo.

2.- Arrivals of ship-loads are Poisson distributed.

3.- Service time in both stages is negative-exponential

<u>1/</u> A similar Equation was used by Wah Ng (Ref.50).
However he commited a mistake using as the third term a unweighted average of q1 and q2 (q1+q2/2), which mean that if q1=2 and q2=14 days,a ship will wait 14 days. Actually it will wait 14 days [Max (q1,q2)],since is necesary that both a berth and storage space is available.

distributed.

The expression for q1 and q2 is shown in Equation (2.19).

In the two stage case, there is a separate number of service stations and service rate for each stage,given below:

s1 = mean service time in the first stage.

n1 = number of service stations in the first stage.

s2 = mean service time in the second stage.

n2 = number of service stations in the second stage.

From the arrival rate, λ , utilization factors for each stage can be computed:

$$\rho 1 = \frac{\lambda\ s1}{n1} \qquad\qquad \rho 2 = \frac{\lambda\ s2}{n2} \tag{4.3}$$

From Equation (2.19) the mean queuing time is found:

$$q1 = \frac{s1}{n1(1 - \rho 1)} \qquad\qquad q2 = \frac{s2}{n2(1 - \rho 2)} \tag{4.4}$$

Similarly, Equation (2.12) can be adapted to give the exact expression for p1 and p2 :

$$p1 = \frac{(n1\rho 1)^{n1}}{n1!(1-\rho 1)\sum_{i=0}^{n1-1}\frac{(n1\rho 1)^{i}}{i!} + (n1\rho 1)^{n1}}$$

$$p2 = \frac{(n2\rho 2)^{n2}}{n2!(1-\rho 2)\sum_{i=0}^{n2-1}\frac{(n1\rho 1)^{i}}{i!} + (n2\rho 2)^{n2}} \tag{4.5}$$

Substituting the values from Equations (4.4) and (4.5) into Equation (4.2), the average queuing time in the two stage model can be computed.

4.3 Port Expansion - Optimization Model. The Two Stage Case

The purpose of the model analysis of this section is to show, in principle, how the optimum capacity and output are determined when the objective is to maximize net social benefit. By optimum capacity I mean the optimum number of berths (or other relevent stations at the first stage) and optimum amount of storage space.

In the next section I will use the model (Optimization Model) to derive " optimum port occupancy charges " as an effort to show that there exist additional uses of queuing models in the port context.

The model that I will propose is still very general and some important aspects are left out of consideration in order to keep a degree of simplicity and clarity in the exposition.

Let us start defining the cost function:

$$TCost = F + bQ + c1n1 + c2n2 + V\lambda(W_q + s1) \qquad (4.6)$$

where

F = Fixed cost

b = operating cost

Q = Throughput volume

c1 = Cost per unit time of berth and related facilities.

c2 = Cost per unit time of storage area.

V = Average cost per unit time of ships at port.

n1 and n2 as before.

In order to fit the problem to the theoretical format, the storage area is measured by the number of sub-areas, each of which is sufficient to accomodate an average shipload. Therefore each of these sub-areas is defined as a service station, yielding n2 service stations in the second stage.

We will follow the same scheme presented in Chapter II (The traditional approach)to derived the Expansion Criteria for the two stages queuing model. The main difference from the traditional approach is that in this case the trade-off between queuing cost of ships is made not only against berth construction cost but also against storage space cost, i.e. a new dimension has been added to the model to incorporate a more realistic port system.

Provided that the total cost function (TCost) is continuously differentiable with respect to n1 and n2, the efficiency conditions as to the number of services stations,n1 and n2, are obtained by taking the partial derivatives of TCost with respect to n1 and n2 and setting them equal to zero (as we did in Chapter II).

From Equation (4.6) we have

$$TCost = F + bQ + c1n1 + c2n2 + V\lambda W_q + V\lambda s1$$

$$\frac{\delta(TCost)}{\delta n1} = c1 + V \frac{\delta(W_q)}{\delta n1}$$

$$\frac{\delta(TCost)}{\delta n2} = c2 + V \frac{\delta(W_q)}{\delta n2}$$

(Recall that $W_q = g\ (n1,n2,s1,s2,\lambda)$.

In order to find the minimum we set:

$$\frac{\delta(TCost)}{\delta n1} = 0$$

$$\frac{\delta(TCost)}{\delta n2} = 0$$

Then

$$c1 + V\frac{\delta[W_q(n1,n2,s1,s2,\lambda)]}{\delta n1} = 0$$

$$c2 + V\frac{\delta[W_q(n1,n2,s1,s2,\lambda)]}{\delta n2} = 0$$

or (4.7)

$$\frac{\delta[W_q(n1,n2,s1,s2,\lambda)]}{\delta n1} = -\ c1/V$$

$$\frac{\delta[W_q(n1,n2,s1,s2,\lambda)]}{\delta n2} = -\ c2/V$$

Again we get a good approximation of the above Equations (4.7) by defining

$$\frac{\delta[W_q(n1,n2,s1,s2,\lambda)]}{\delta n1} = W_q(n1+1,n2,s1,s2,\lambda) - W_q(n1,n2,s1,s2,\lambda)$$

(4.8)

$$\frac{\delta[W_q(n1,n2,s1,s2,\lambda)]}{\delta n2} = W_q(n1,n2+1,s1,s2,\lambda) - W_q(n1,n2,s1.s2,\lambda)$$

Now subtituting Equation (4.8) into (4.7), multiplying both side by -1, and changing to inequality sing, we obtain our <u>Expansion Criteria</u>

$$W_q(n1,n2,s1,s2,\lambda) - W_q(n1+1,n2,s1,s2,\lambda) \leq c1/V$$

$$W_q(n1,n2,s1,s2,\lambda) - W_q(n1,n2+1,s1,s2,\lambda) \leq c2/V$$

Solving simultaneously both Equations (using Equation (4.2)), we get the optimum number of berth and the optimum number of storage spaces. In the next section an algorithm to find the optimum values of n1 and n2 will be developed.

4.4 Port Expansion - Optimization Algorithm (II)

This is an algorithm to find the optimum number of service stations n1 and n2 , for parametrically varying service times s1 and s2. Service times are increased by increments $\Delta s1$ and $\Delta s2$. The results are a table of optimal number of service stations for K values of s1, beginning with initial value s_1i [s1i,..., s1i + $\Delta s1(K-1)$] and L values of s2, beginning with initial value s2i [s2i,..., s2i + $\Delta s2(L-1)$].

1. Set range of services time to be used. Defined

   Initial value of s1 (berth service time)

   Initial value of s2 (Storage service time)

   $\Delta s1$, the increment to be used for s1

   $\Delta s2$, the increment to be used for s2

   K, the number of s1 desired

   L, the number of s2 desired

2. Specify the parameters.

λ , the arrival rate

c1/V , ratio of berth cost to ship cost

c2/V , ratio of storage area cost to ship cost

Let

j be a counter for s2

i be a counter for s1

3. Initialize for present value of s1.

s2 = s2 initial

Initialize $n1 = s1\lambda/a$

$n2^{*} = s2\lambda/b$ (a and b are arbitrary constant)

4. Initialize for present value of s2

Set $n1^{*}(\text{old}) = C$ ($C < 0$)

$n2^{*}(\text{old}) = n2^{*}$

*** For Present Value $n2^{*}$, find optimum $n1^{*}$ ***

5. Compute $\rho 2 = s2\lambda / n2^{*}$

6. If $\rho 2 \geqq 1$ Set $n2^{*} = n2^{*} + 1$, go to 5. Else Continue

7. Compute

$$p2 = \frac{(n2\rho 2)^{n2}}{n2!\,(1-\rho 2)}\, p(0)$$

$$q2 = \frac{s2}{n2\ (1-\rho 2)}$$

8. Compute $\rho 1 = s1\ \lambda/\ n1$. If $\rho 1 < 1$, go to 11 . Else Continue

9. Set $W_q 1 = \infty$; $W_q 1 = Z_1$

$n1 = n1 + 1$

10. Compute $\rho 1$. If $\rho 1 \geqq 1$, go to 9. Else go to 14

11. Compute

$$p1 = \frac{(n1\rho 1\ )^{n1}}{n1!\,(1-\rho 1)}\, p(0)$$

$$q1 = \frac{s1}{n1\ (1-\rho 1)}$$

$$W_q 1 = p1q1 - p1p2[\ \min\ (q1,q2)\ ] = z_1'$$

12. Set $z_1 = z_1'$

    $n1 = n1 + 1$

13. Compute $\rho 1$

14. Compute z_1'

15. If $\Delta z_1 = z_1 - z_1' \geq c1/V$,continue (search upward); else go to 20 (search downward)

16. *** Search Upward ***

    Set $z_1 = z_1'$

    $n1 = n1 + 1$

17. Compute $\rho 1$ and z_1'

18. If $\Delta z_1 \geq c1/V$, go to 16,else continue

19. For present $n2^*$, $n1^* = n1 - 1$ is optimal. Go to 27

20. *** Search Downward ***

    Set $n1 = n1 - 1$ (Original value of n1 for this iterarion)

21. If $n1 = 1$ go to 27 ($n1=1$ is optimal,since for $n1=0$ $z_1' = \infty$)

    Else continue

22. Set $n1 = n1 - 1$

23. Compute $\rho 1$. If $\rho 1 \geq 1$ go to 26 (Optimal found,since if $\rho 1 \geq 1$, $z_1' = \infty$). Else continue

24. Compute z_1'

25. If $-\Delta z_1' \leq c1/V$ go to 21. Else continue

26. For present $n2^*$, $n1^* = n1 + 1$,is optimal

27. *** Optimum found ***

Set $n1 = n1^*$

$p1 = p1(n1^*)$

$q1 = q1(n1^*)$

28. If $n1^* = n1^*$(old) go to 50. Else set n1*(old)=n1* continue

*** For Present Value $n1^*$, find optimum $n2^*$ ***

29. Compute $\rho 2$. If $\rho 2 < 1$, go to 32 ; else continue

30. Set $W_q 2 = \infty = z_2$

$n2 = n2 + 1$

31. Compute $\rho 2$. If $\rho 2 \geq 1$ go to 30 ; else go to 35

32. Compute p2 and q2 (as defined above).

Compute $W_q 2 = p2q2 - p1p2 [\min (q1,q2)] = z_2'$

33. Set $z_2 = z_2'$

$n2 = n2 + 1$

34. Compute $\rho 2$

35. Compute z_2'

36. If $\Delta z_2 = z_2 - z_2' \geq c2/V$ continue (search Upward); else go to 41 (Search Downward)

37. *** Search Upward ***

Set $z_2 = z_2'$

$n2 = n2 + 1$

38. Compute $\rho 2$ and z_2'

39. If $\Delta z_2 \geq c2/V$, go to 37 . Else continue

40. For present $n1^*$, $n2^* = n2 - 1$, is optimal. Go to 48

41. *** Search Downward ***

Set n2 = n2 - 1 (Original value of n2 for this iterarion)

42. If n2 = 1 go to 48 (n2 = 1 is optimal, since for $n2=0, z_2' = \infty$)

Else continue

43. Set n2 = n2 - 1

44. Compute ρ2. If $\rho 2 \geq 1$ go to 47 (Optimal found, since if $\rho 2 \geq 1, z_2' = \infty$)

45. Compute z_2'

46. If $-\Delta z_2 \leq c2/V$ go to 42 ; else continue

47. For present $n1^*$, $n2^* = n2 + 1$, is optimal

48. *** Optimum found ***

Set $n2 = n2^*$

$p2 = p2(n2^*)$

$q2 = q2(n2^*)$

49. If $n2^* = n2^*(\text{old})$ continue (Search is ended). Else set $n2^*(\text{old}) = n2^*$ and go to 5

50. For present s1 and s2 , optimum $n1^*$ and $n2^*$ have been found.

51. Increment s2 and counter

Set $s2 = s2 + \Delta s2$

$j = j + 1$

52. If j < L go to 4 ,else continue

53. Increment s1 and counter

Set $s1 = s1 + \Delta s1$

$i = i + 1$

54. If $i < K$ go to 4. Else STOP ,all values of s1 and s2 have been evaluated.

The program based in this algorithm using the TI-59 programmable calculator is shown in Appendix III

4.5 Optimum Port Charges - An Analytical Model

In reference (45),Dr Shneerson points out that port charges which are consistent with the factor combination and output volume which result from net social benefit maximization can be called "Optimum port charges". In this section we will develop an analytical model to compute port charges based on the results of the expansion criteria model developed in las section. In many past studies queuing models have been used to compute mainly the optimum number of berths or optimum capacity of a port; we will show in this section that our model can also be used to derive "port charges". Some aspects of port related charges such as stevedoring will be left out of this analysis . Nevertheless the results obtained from this analysis are meaningful and applicable

4.5.1 Basic Consideration

We already have incorporated in the model presented in section 4.3 some measures of demand volume (factor combination) and established a "design volume " indirectly from the net social benefit maximization , through the use of the cost function and the expansion criteria model. The former can be interpreted as the products $\lambda s1$,$\lambda s2$,used internally in the Equations to represent the total service-day of ships, x and the total transit storage-days of the cargo, y. These are used to calculate the expansion paths of the number of service stations n1 and n2. The latter is associated with the optimum values of n1 and n2.

4.5.2 Port charges Model

Under marginal-input pricing policy, every occupant of a service station should be charged for the expected queuing cost caused to other succeeding ships in order to raise the private cost faced by the individual owners of ships and cargo to the level of social marginal cost.

From Equation 4.2 we can compute the average queuing time per service-day for a ship as:

$$W_q/x$$

where

W_q and x as defined above.

Now if we increase the arrival rate, this will increase x and raise W_q/x (W_q will increase at least at the same rate as x)

Then the marginal queuing time (the additional queuing time caused by an additional ship) is:

$$\frac{\delta\ (W_q/x)}{\delta x} = \frac{\delta(\ W_q)/\ \delta x}{x} - \frac{W_q}{x^2}$$

The additional queuing time caused to all ships by a unit increase in x is:

$$\Delta TW_q = x\ \frac{\delta (W_q/x)}{\delta x} = \frac{\delta\ W_q}{\delta\ x} - \frac{W_q}{x} \qquad (4.9)$$

Finally the optimal berth occupancy charge is obtained by multiplying Equation(4.9)by the average value of ship's time V, so that

$$\text{Optimal berth occupancy charges} = V\left[\frac{\delta(W_q)}{\delta x} - \frac{W_q}{x}\right] \qquad (4.10)$$

Similarly , the optimal storage occupancy charge, can be obtained following the same argument as follow:

$$\frac{\delta(W_q/x)}{\delta y} = \frac{\delta(W_q)}{x\,\delta\,y}$$

and the total addtional queuing time of ships caused by a unit increase in y , is

$$\Delta TW_q = x\,\frac{\delta(W_q)}{x\,\delta y} = \frac{\delta(W_q)}{\delta y} \qquad (4.11)$$

Again the optimal storage occupancy charge is obtained by multiplying Equation (4.11) by the average value of ship's time V, so that:

$$\text{Optimal Storage Occupancy charge} = V\left[\frac{\delta(W_q)}{\delta y}\right]$$

The main difference between these two charges is that the berth occupancy charge is set equal to the social marginal cost minus the private marginal cost, and the storage occupancy charge only to the social marginal cost.

Since Equations (4.13) and (4.16) are extremely difficult to compute, I will use an alternative method, "the average cost of the marginal plant" to establish the port charges under marginal-input pricing policy which is a practical method of deriving marginal cost using the ratio of an incremental cost to the corresponding increment of output as proxy for the social marginal cost.

The validity of this method rests on the following condition: The quality of the level of services (product) has to remain the same in the original situation, and in the situation after a capacity (station) addition has been made. When the level of services changes (normally, improves) or capacity is expanded this method can be correctly applied as follows: if the original situation can be assumed to be an optimum, it is not necessary to translate quality of service to user costs, even if an improvement (or impairment) of the level of service actually occurs as a result of a capacity (station) addition. One can calculate the incremental cost per unit of the additional output in the hypothetical case where the level of service remains constant after a capacity (station) addition (reference 34).

In our case, if the utilization factor were kept constant after adding another service station, which would make the incremental port capacity cost per unit of additional output equal to the average port capacity cost, it is clear that the level of service would improve, i.e. the average queuing time would fall. Therefore, in order to apply the method mentioned above, we let an increase in the utilization factor accompanying the addition of another service station, which is chosen such that the average queuing time remains unchanged.

Therefore, we need to find the level of service for which the average queuing time is the same before and after the addition of a service station.

Then our berth occupancy charge can be derived using Equation (4.2) as follows:

$$\frac{W_q(x^*, y, n_1+1, n_2)}{x^*} - \frac{W_q(x, y, n_1, n_2)}{x} = 0 \qquad (4.17)$$

Then the optimal berth occupancy charge $= \dfrac{c_1}{\Delta x}$ (4.18)

where

c_1 = the incremental cost of a service station in the first stage and

$\Delta x = x^* - x$

and the storage occupancy charge as

$$\frac{W_q(x, y^*, n_1, n_2+1)}{x} - \frac{W_q(x, y, n_1, n_2)}{x} = 0$$

or

$$W_q(x, y^*, n_1, n_2+1) - W_q(x, y, n_1, n_2) = 0 \qquad (4.19)$$

Then the optimal storage charge is

$$\text{optimal storage charge} = \frac{c_2}{\Delta y} \qquad (4.20)$$

where

c_2 = the incremental cost of a service station in the second stage, and

$$\Delta y = y^* - y$$

There are not systematic differences between the results obtained using Equations (4.18) and (4.20) and the results obtained from Equations (4.13) and (4.16).

These marginal costs due to queuing should be added to the other costs of berth and storage use (labor, etc.) to obtain port charges that reflect social marginal cost, in order to have the most efficient use of port facilities to maximize net social benefit.

To find Δx, Δy, we use Equations (4.2), (4.17) and (4.19), the results from the expansion criteria model described in Section 4.2 and the half interval method to find the roots of Equations (4.17) and (4.19). as well as the algorithm described in the next section.

4.5.3. Port Occupancy Charge: Optimization Algorithm

With the algorithm described below is possible to find the principal factor: Δx (Δy) to establish an optimum berth (storage) occupancy charge as defined by Equation 4.18 (4.20).

The berth (storage) occupancy factor is allowed to increase when an addition of a service station in the first (second) stage occour, such that the average queuing time remain the same before and after the expansion.

1. Specify parameters.

   λ, the arrival rate

n_1, number of berth

n_2, number of storage spaces

s_1, average service time (first stage) before expansion.

s_2, average service time (second stage) before expansion.

$n_1+1(n_2+1)$ number of berths (storage spaces) after expansion.

2. Initialize for present values of s_1 and s_2.

$s_1 = s_1$ initial

$s_2 = s_2$ initial

$n_1 = n_1$ initial

$n_2 = n_2$ initial

3. For present value of s_1, s_2, n_1, n_2, and λ : Compute

$$x = s_1\lambda \ ; \ (y = s_2\lambda)$$

$$\rho_1 = x/n_1 \ ; \ (\rho_2 = y/n_2)$$

$$p_1 = \frac{(n_1\rho_1)^{n_1}}{n_1!(1-\rho_1)} p(0) \quad \left[p_2 = \frac{(n_2\rho_2)^{n_2}}{n_2!(1-\rho_2)} p(0) \right]$$

$$q_1 = s_1/n_1(1-\rho_1) \quad \left(q_2 = s_2/n_2(1-\rho_2) \right)$$

4. Compute actual average queuing time:

$$W_q = p_1q_1 + p_2q_2 - p_1p_2 \ [\ \mathrm{Min}\ (q_1, q_2)\ ]$$

$$\text{Set} \quad Z = W_q/x \qquad (Z' = W_q)$$

5. Define:

$$f(x^*) = W_q(n_1+1,\ n_2,\ x^*,\ y)/x^* \ - Z$$

$$[f(y^*) = W_q(n_1,\ n_2+1,\ x,\ y^*) \ - Z']$$

6. Set lower bound

$$\rho_{1L} = x/n_1+1 \qquad (\rho_{2L} = y/n_2+1)$$

7. Set upper bound

$\rho_{1U} = .999$ $(\rho_{2U} = .999)$

8. Specify accuracy desired

$\varepsilon = a \quad (a \geq 0)$

9. Compute 1/ :

$f(x_{1L}) \quad ; \quad f(x_{1U})$

If $f(x_{1L}) \; f(x_{1U}) \geq 0$ stop, there is no root in the interval defined. Else continue

10. Compute:

$f((x_{1L} + x_{1U})/2)$

If $f((x_{1L} + x_{1U})/2) = 0 \quad x^* = \dfrac{x_{1L} + x_{1U}}{2}$, print the results.

Else continue

11. If $f(x_{1L}) \; f((x_{1L} + x_{1U})/2) \geq 0$ Set $x_{1L} = \dfrac{x_{1L} + x_{1U}}{2}$ and go to 13.

Else continue

12. If $f(x_{1U}) \; f((x_{1L} + x_{1U})/2) \geq 0$ Set $x_{1U} = \dfrac{x_{1L} + x_{1U}}{2}$ and go to 14.

Else continue

13. If $x_{1L} \leq \varepsilon$ stop $x^* = x_{1L}$. Print results. Else go to 10

14. If $x_{1U} \leq \varepsilon$ stop $x^* = x_{1U}$. Print results. Else go to 10

Appendix IV shows the details of a program for TI-59 to compute port occupancy charges based in the above algorithm. Table 4.1 show a sample of the program output.

1/ The same procedure is used with y. For clarity sake we omited it.

Table 4.1 Port Occupancy Charges. Program Output

```
.2739726027     λ
         4.     N2
         6.     S2

         3.     N1
         4.     S1

 1.095890411     X

    WT(N)/X
0.424897428

 2.007615432    X*

    WT(N+1)/X+ΔX
.4249092544

.9117250207     ΔX


.2739726027     /
         5.     N2
         7.     S2

         3.     N1
         4.     S1

 1.095890411     X

    WT(N)/X
.3127820805

 1.899061982    X*

    WT(N+1)/X+ΔX
.3127376729

.8031715707     ΔX
```

a. Berths

```
.2739726027     λ
         3.     N1
         4.     S1

         4.     N2
         6.     S2

1.643835616     Y

   WT(N)
0.465641017

   2.211366     Y*

   WT(N+1)
.4656433969

.5675303838     ΔY


.2739726027     /
         3.     N1
         4.     S1

         5.     N2
         7.     S2

1.917808219     Y

   WT(N)
.3427748827

2.468737485     Y*

   WT(N+1)
.3427757858

.5509292655     ΔY
```

b. Storage

Table 4.1 Port Occupancy Charges. Program Output

.2739726027	λ
4.	N2
6.	S2
3.	N1
4.	S1
1.095890411	X
WT(N)/X	
0.424897428	
2.007615432	X*
WT(N+1)/X+ΔX	
.4249092544	
.9117250207	ΔX
.2739726027	/
5.	N2
7.	S2
3.	N1
4.	S1
1.095890411	X
WT(N)/X	
.3127820805	
1.899061982	X*
WT(N+1)/X+ΔX	
.3127376729	
.8031715707	ΔX

a. Berths

.2739726027	λ
3.	N1
4.	S1
4.	N2
6.	S2
1.643835616	Y
WT(N)	
0.465641017	
2.211366	Y*
WT(N+1)	
.4656433969	
.5675303838	ΔY
.2739726027	/
3.	N1
4.	S1
5.	N2
7.	S2
1.917808219	Y
WT(N)	
.3427748827	
2.468737485	Y*
WT(N+1)	
.3427757858	
.5509292655	ΔY

b. Storage

CHAPTER V

Study of Hypothetical Cases

5.1 Introduction

Based on the queuing models described in Chapters 3 and 4 I will examine two hypothetical cases.

The basic data for the first case was taken from the paper by Jan de Weille and Anondarup Ray, "The Optimum Port Capacity" (Reference 25). The data in the second case corresponds to a case used by Fook-Wah Ng in his thesis, "Analytical Model for an Offshore Port Facility Reception" (Reference 50). Unfortunately it was not possible to make a comparison between the results of this study and those of de Weille and Ray since they did not present results for the expansion problem. With the second case a good comparison cannot be entirely achieved because of the error committed by Wah Ng (see footnote, pp.32). However, the availability of the data and the desire to show the applicability of the models were still incentive enough for the presentation of this chapter.

5.2 Study of the First Hypothetical Case

In Reference 25, de Weille and Ray used as the value of waiting time of ships time of ships \$1500/day (1974 dollars) for a 10,000-ton ship as indicative of the order of magnitude involved. They have not attempted to present orders of magnitude involved in berth construction cost since this will differ from port to port, but they use in their case a berth construction cost of \$1 million. Amortizing this cost over a period of

20 years at a discount rate of 10% (as in Reference 25) gives a c.r.f. (capital recovery factor) value of 0.11746. Therefore the cost of berth per day is approximately

$$= 1.0 \times 10^6 \times 0.11746 \times 1/365$$

$$= 321$$

or say $325 per day. Therefore we can define

$$c = \$325 \text{ per day}$$

$$V = \$1500 \text{ per day, and}$$

$$c/V = 0.217$$

Using this cost ratio, the optimum number of berths was computed for different values of service time s (and annual berth-days required) and different arrival rates, with the program Expansion Criteria: One Stage Queuing Model presented in Appendix 2. Tables 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, and 5.7 show the relevant results and Table 5.8 a sample of the program output. The expansion path for optimum number of berths (n*) is plotted (from the data in Tables 5.1 - 5.7) as shown in Figures 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, and 5.10. In order to test the sensitivity to the cost ratio of the various operational characteristics, the optimum number of berths of c/V = .10833 and for c/V = .4333 was also calculated and are tabulated in Tables 5.1 - 5.7.

As expected, when the value of ship waiting time becomes higher (due to a lower cost ratio), the optimum number of berths increases,

Table 5.1 Optimum Number of Berths-Annual Berth-day Required

Annual Ship Arrival	S (Days/Ship)	Annual Berths-Days Required	n		$W_q(n)$	N Optimum C/V .108	N Optimum C/V .217	N Optimum C/V .433
100	1	100	1	.274	.377	2	2	1
			2	.137	.019			
			3	.091	.001			
	2	200	1	.548	2.424	3	2	2
			2	.274	.162			
			3	.183	.016			
	3	300	2	.411	.610	3	3	3
			3	.274	.077			
			4	.205	.010			
	4	400	2	.548	1.716	4	3	3
			3	.365	.239			
			4	.274	.038			
	5	500	3	.457	.590	4	4	4
			4	.342	.107			
			5	.274	.019			
200	1	200	1	.548	1.212	2	2	2
			2	.274	.081			
			3	.183	.008			
	2	400	2	.548	.858	3	3	3
			3	.365	.119			
			4	.274	.019			
	3	600	3	.548	.643	4	4	4
			4	.411	.125			
			5	.329	.026			
	4	800	4	.548	.497	5	5	5
			5	.438	.118			
			6	.365	.028			
	5	1000	5	.548	.391	6	6	5
			6	.457	.106			
			7	.391	.029			

Table 5.2 Optimum Number of Berths-Annual Berth-day Required

Annual Ship Arrival	S (Days/Ship)	Annual Berths-Day Required	n		$W_q(n)$	N Optimum C/V .108	C/V .217	C/V .433
300	1	300	1	.822	4.615	3	2	2
			2	.411	.203			
			3	.274	.026			
	2	600	3	.548	.429	4	4	3
			4	.411	.083			
			5	.329	.017			
	3	900	4	.616	.603	6	4	4
			5	.493	.147			
			6	.411	.038			
	4	1200	5	.658	.732	7	6	6
			6	.548	.208			
			7	.470	.062			
	5	1500	6	.658	.732	8	7	7
			7	.587	.261			
			8	.514	.087			
400	1	400	2	.548	.429	3	3	2
			3	.365	.060			
			4	.274	.010			
	2	800	3	.731	1.331	5	4	4
			4	.548	.248			
			5	.438	.059			
	3	1200	5	.658	.549	7	6	5
			6	.548	.156			
			7	.470	.047			
	4	1600	6	.731	.948	8	7	7
			7	.626	.300			
			8	.548	.102			
	5	2000	8	.685	.491	10	9	8
			9	.609	.181			
			10	.548	.068			

Table 5.3 Optimum Number of Berths-Annual Berth-day Required

Annual Ship Arrival	S (Days/Ship)	Annual Berths-Day Required	n		$W_q(n)$	n Optimum C/V .108	n Optimum C/V .217	n Optimum C/V .433
500	1	500	2	.685	.884	3	3	3
			3	.457	.118			
			4	.342	.021			
	2	1000	4	.685	.644	6	5	5
			5	.548	.156			
			6	.457	.042			
	3	1500	6	.685	.496	7	7	7
			7	.587	.157			
			8	.514	.052			
	4	2000	8	.685	.393	9	9	8
			9	.609	.145			
			10	.548	.054			
	5	2500	9	.761	.817	11	10	10
			10	.685	.317			
			11	.623	.129			
600	1	600	2	.822	2.082	4	3	3
			3	.548	.214			
			4	.411	.042			
	2	1200	5	.658	.366	6	6	5
			6	.548	.104			
			7	.470	.031			
	3	1800	7	.705	.447	8	8	7
			8	.6.6	.154			
			9	.548	.055			
	4	2400	9	.731	.487	11	10	9
			10	.658	.190			
			11	.598	.077			
	5	3000	11	.747	.504	13	12	11
			12	.685	.216			
			13	.632	.095			

Table 5.4 Optimum Number of Berths-Annual Berth-day Required

Annual Ship Arrival	S (Days/Ship)	Annual Berths-Day Required	n		$W_q(n)$	n Optimum C/V .108	n Optimum C/V .217	n Optimum C/V .433
700	1	700	3	.639	.376	4	4	3
			4	.479	.074			
			5	.384	.017			
	2	1400	5	.767	.846	7	6	6
			6	.639	.228			
			7	.548	.072			
	3	2100	8	.719	.403	9	9	8
			9	.639	.149			
			10	.575	.057			
	4	2800	10	.767	.584	12	11	10
			11	.697	.238			
			12	.639	.101			
	5	3500	12	.799	.761	14	13	12
			13	.738	.332			
			14	.685	.152			
800	1	800	3	.731	.666	4	4	4
			4	.548	.124			
			5	.438	.030			
	2	1600	6	.731	.479	7	7	6
			7	.626	.149			
			8	.548	.051			
	3	2400	9	.731	.365	10	10	9
			10	.658	.143			
			11	.598	.058			
	4	3200	11	.797	.683	13	12	11
			12	.731	.287			
			13	.674	.127			
	5	4000	14	.783	.488	16	15	14
			15	.731	.229			
			16	.685	.111			

Table 5.5 Optimum Number of Berths-Annual Berth-day Required

Annual Ship Arrival	s	Annual Berths-Days Required	n		$W_q(n)$	n Optimum C/V .108	n Optimum C/V .217	n Optimum C/V .433
900	1	900	3	.822	1.279	5	4	4
			4	.616	.201			
			5	.493	.049			
	2	1800	6	.822	1.054	8	7	7
			7	.705	.298			
			8	.616	.103			
	3	2700	9	.822	.902	11	10	10
			10	.740	.332			
			11	.672	.136			
	4	3600	12	.822	.787	14	13	13
			13	.759	.338			
			14	.705	.155			
	5	4500	15	.822	.696	17	16	15
			16	.771	.330			
			17	.725	.164			
1000	1	1000	4	.685	.322	5	5	4
			5	.548	.078			
			6	.457	.021			
	2	2000	7	.783	.593	9	8	7
			8	.685	.196			
			9	.609	.072			
	3	3000	10	.822	.775	12	11	11
			11	.747	.302			
			12	.685	.129			
	4	4000	13	.843	.897	15	14	14
			14	.783	.390			
			15	.731	.184			
	5	5000	17	.806	.462	19	18	17
			18	.761	.232			
			19	.721	.120			

Table 5.6 Optimum Number of Berths . General Results

Annual Ship Arrival	s Day Ship	Annual Berths - Day Required				n optimum c/V .10833	n optimum c/V .21667	n optimum c/V .43333
100	6	600				5	4	4
	7	700				5	5	3
	8	800				6	5	5
	9	900				6	6	5
	10	1000				7	6	6
200	6	1200				7	7	6
	7	1400				8	7	7
	8	1600				9	8	7
	9	1800				9	9	8
	10	2000				10	10	9
300	6	1800				9	8	8
	7	2100				10	9	9
	8	2400				11	11	10
	9	2700				12	12	11
	10	3000				13	13	12
400	6	2400				11	10	10
	7	2800				12	12	11
	8	3200				14	13	12
	9	3600				15	14	13
	10	4000				17	16	15
500	6	3000				13	12	11
	7	3500				15	14	13
	88	4000				16	15	14
	9	4500				18	17	16
	10	5000				20	19	18
600	6	3600				15	14	13
	7	4200				17	16	15
	8	4800				19	18	17
	9	5400				21	20	19
	10	6000				23	21	20

Table 5.7 Optimum Number of Berths. General Results

Annual Ship Arrival	s Day Ship	Annual Berths-Day Required				n optimum c/V .10833	n optimum c/V .21667	n optimum c/V .43333
700	6	4200				16	16	13
	7	4900				19	18	17
	8	5600				21	20	19
	9	6300				23	22	21
	10	7000				25	24	23
800	6	4800				18	17	16
	7	5600				21	20	19
	8	6400				23	22	21
	9	7200				26	25	24
	10	8000				28	27	26
900	6	5400				20	19	18
	7	6300				23	22	21
	8	7200				26	25	23
	9	8100				29	27	26
	10	9000				31	30	29
1000	6	6000				22	21	20
	7	7000				25	24	23
	8	8000				28	27	26
	9	9000				31	30	29
	10	10000				34	33	32

Table 5.8 Expansion Criteria-One Stage Model. Program Output

```
.2739726027      /
         1.      S
         1.      ΔS
         5.      K

         1.      S

         1.      N
.2739726027      UTI
.3773584906      WT
999999999.6      ΔWT

         2.      N
.1369863014      UTI
.0191241155      WT
.3582343751      ΔWT

         3.      N
.0913242009      UTI
.0010519968      WT
.0180721187      ΔWT

    OPT    N*
         2.

247720.4532      TC

         2.      S

         1.      N
.5479452055      UTI
2.424242424      WT
999999997.6      ΔWT

         2.      N
.2739726027      UTI
.1623047271      WT
2.261937697      ΔWT

         3.      N
.1826484018      UTI
.0158015601      WT
.1465031671      ΔWT

    OPT    N*
         2.

326111.8381      TC

         3.      S

         1.      N
.8219178082      UTI
13.84615385      WT
999999986.1      ΔWT

         2.      N
.4109589041      UTI
0.609618424      WT
13.23653542      ΔWT

         3.      N
.2739726027      UTI
.0767595528      WT
.5328588712      ΔWT

         4.      N
.2054794521      UTI
.0099254304      WT
.0668341225      ΔWT

    OPT    N*
         3.

397900.8552      TC

         4.      S

         2.      N
.5479452055      UTI
1.716277822      WT
999999998.3      ΔWT

         3.      N
.3652968037      UTI
.2386660989      WT
1.477611724      ΔWT

         4.      N
.2739726027      UTI
.0380229371      WT
.2006431618      ΔWT

    OPT    N*
         3.

486544.6891      TC

         5.      S

         2.      N
.6849315068      UTI
4.418522446      WT
999999995.6      ΔWT

         3.      N
.4566210046      UTI
.5903392632      WT
3.828183183      ΔWT

         4.      N
.3424657534      UTI
.1071294032      WT
.4832098601      ΔWT

         5.      N
.2739726027      UTI
.0193620595      WT
.0877673436      ΔWT

    OPT    N*
         4.

533153.3482      TC
```

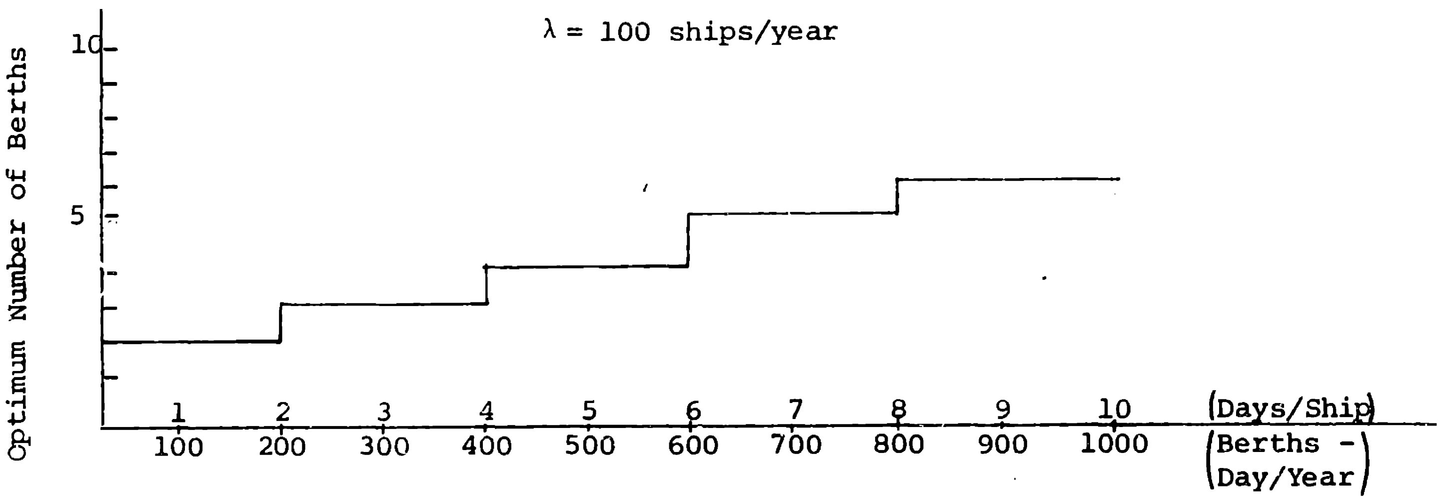

Figure 5.1 Optimum Number of Berths vs. Annual Berths-Day Required

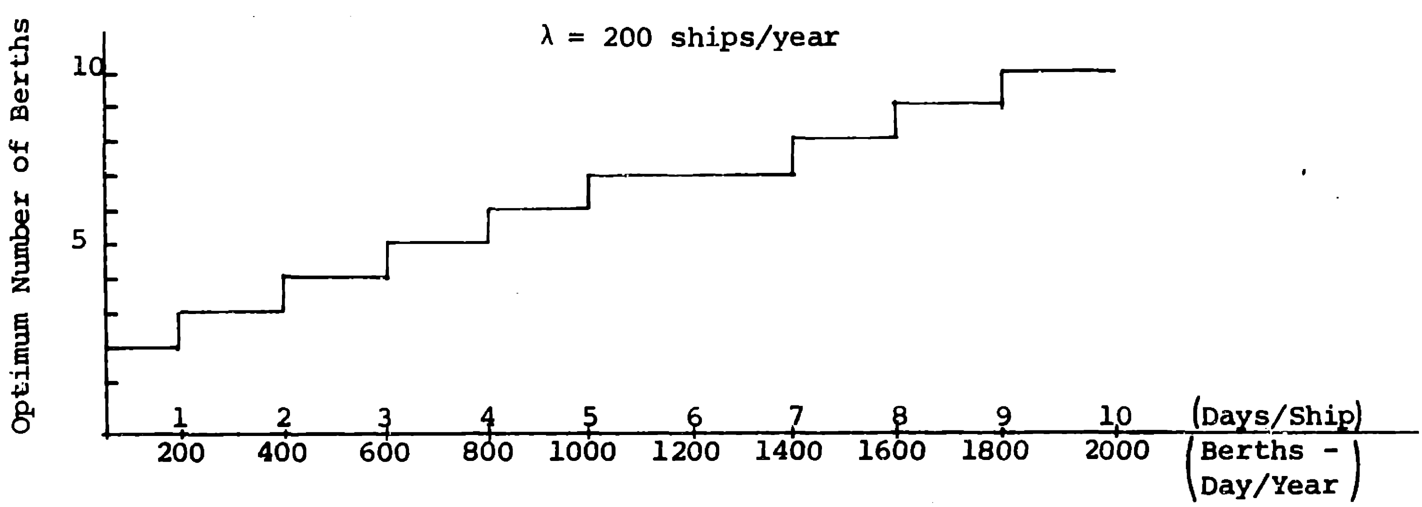

Figure 5.2 Optimum Number of Berths vs. Annual Berths-Day Required

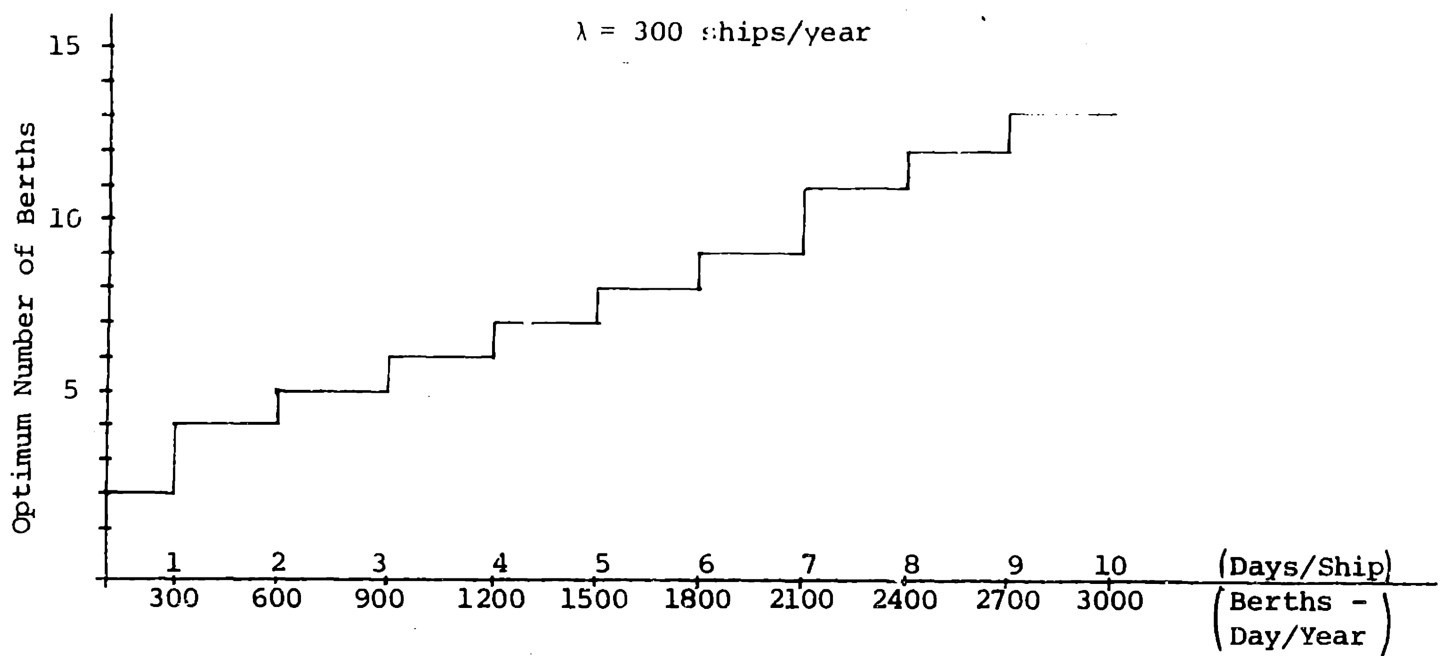

Figure 5.3 Optimum Number of Berths vs. Annual Berths-Day Required

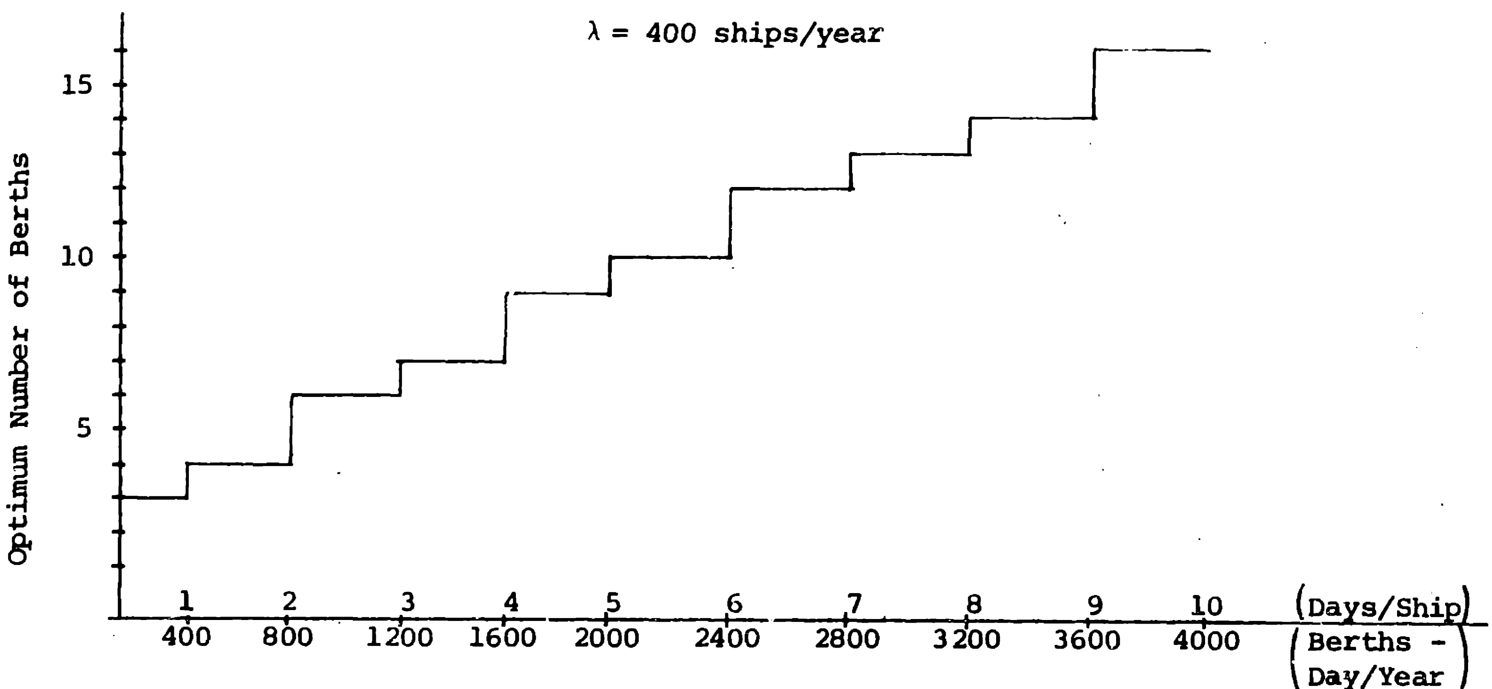

Figure 5.4 Optimum Number of Berths vs. Annual Berths-Day Required

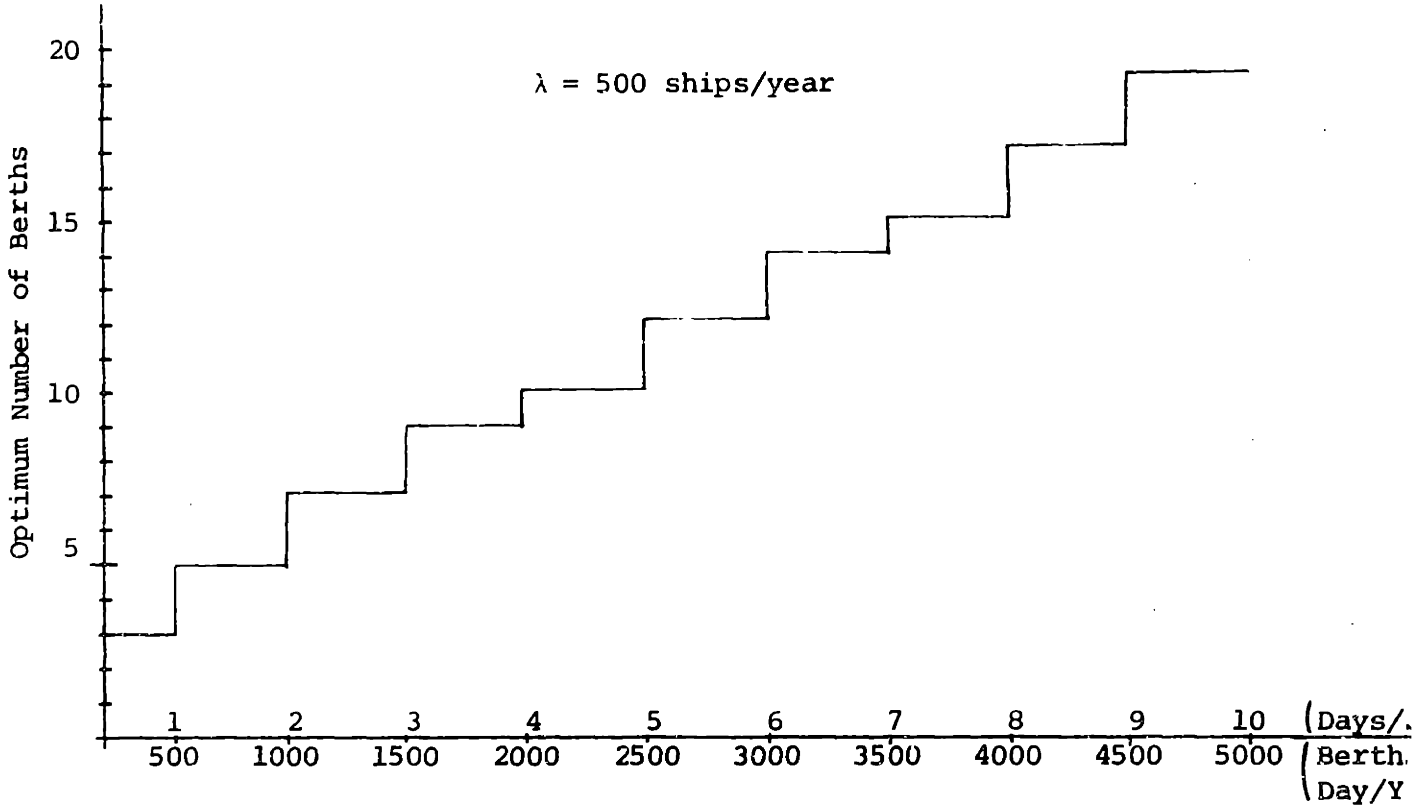

Figure 5.5 Optimum Number of Berths vs. Annual Berths-Day Required

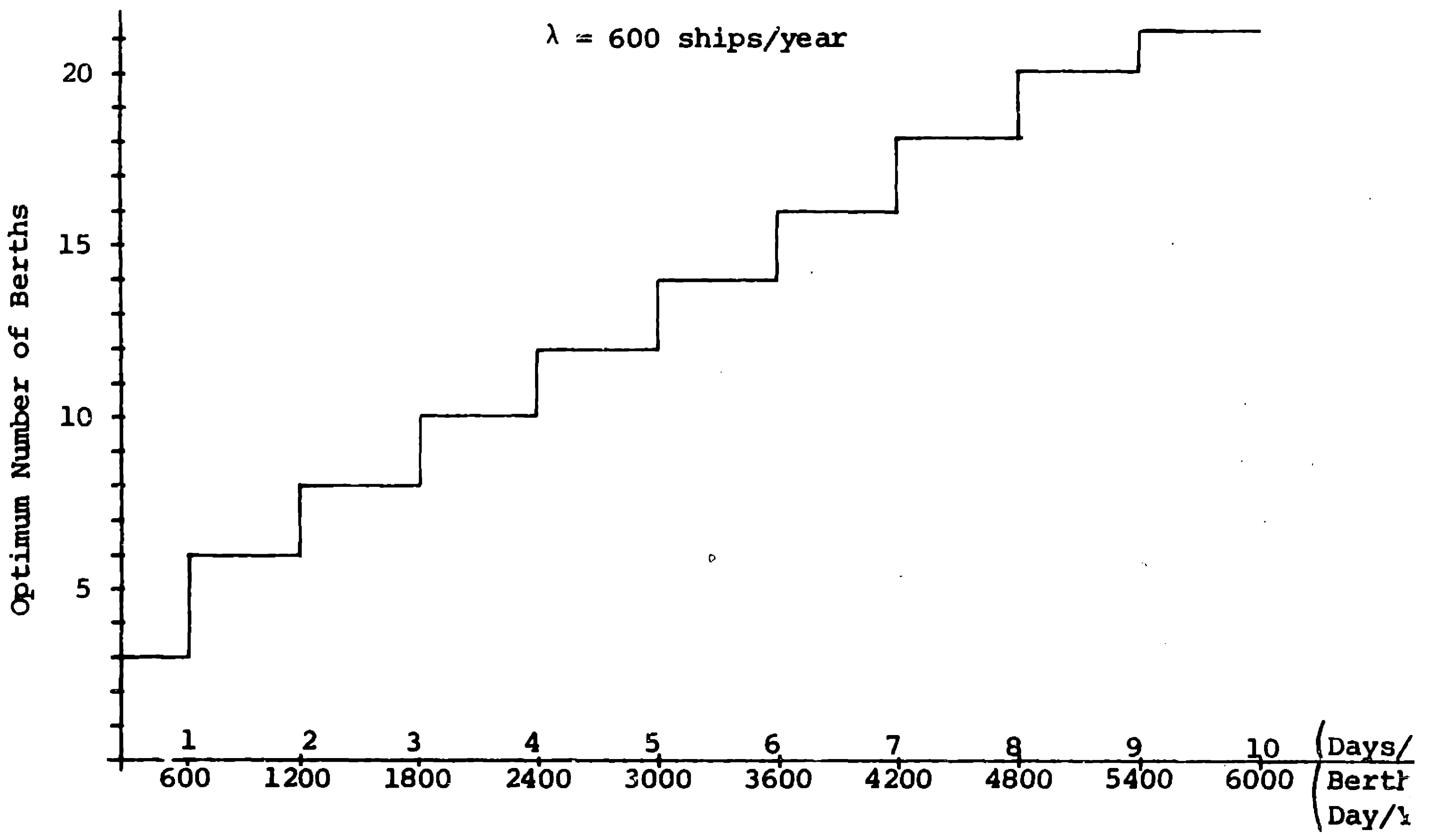

Figure 5.6 Optimum Number of Berths vs. Annual Berths-Day Required

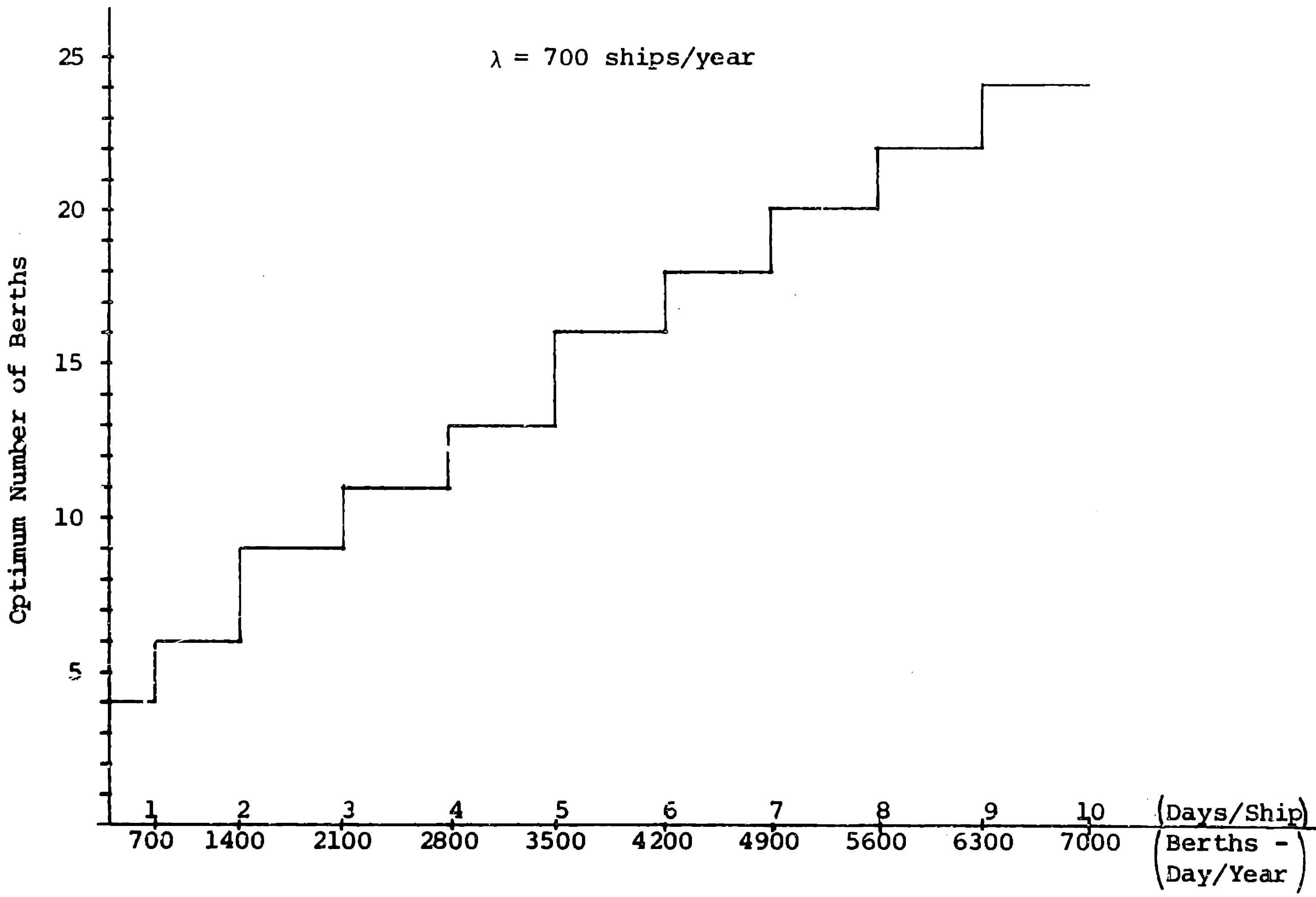

Figure 5.7 Optimum Number of Berths vs. Annual Berths-Day Required

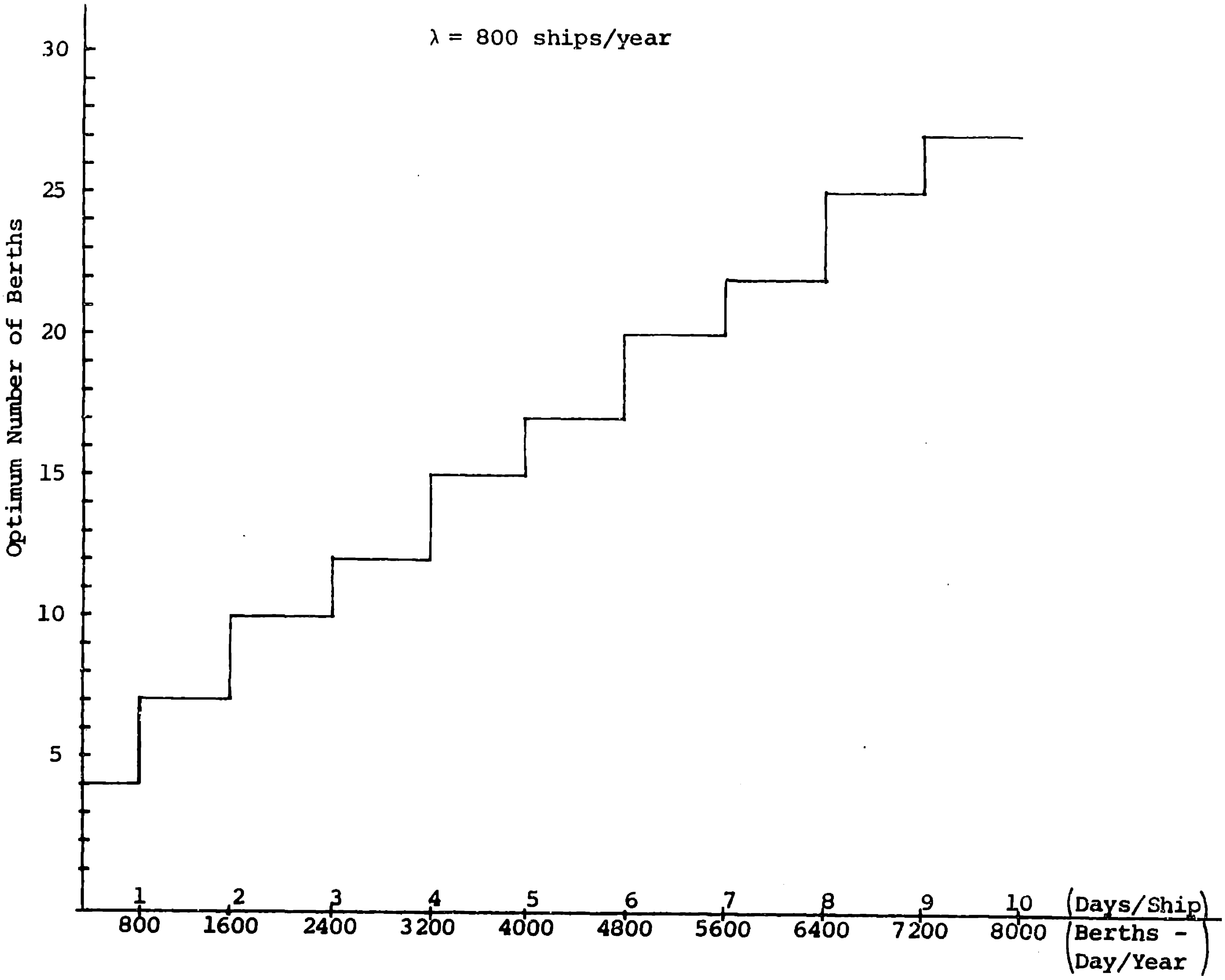

Figure 5.8 Optimum Number of Berths vs. Annual Berths-Day Required

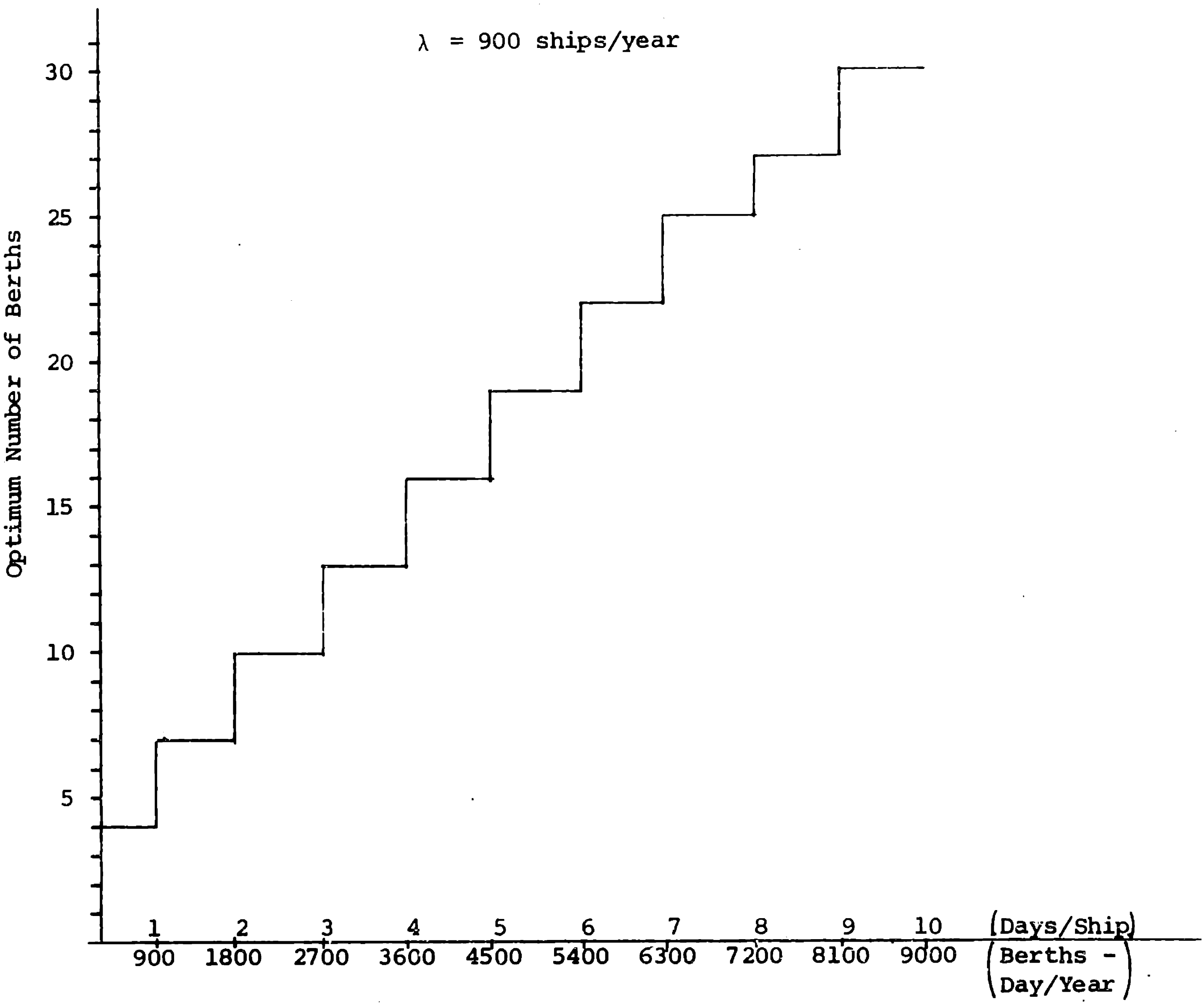

Figure 5.9 Optimum Number of Berths vs. Annual Berths-Day Required

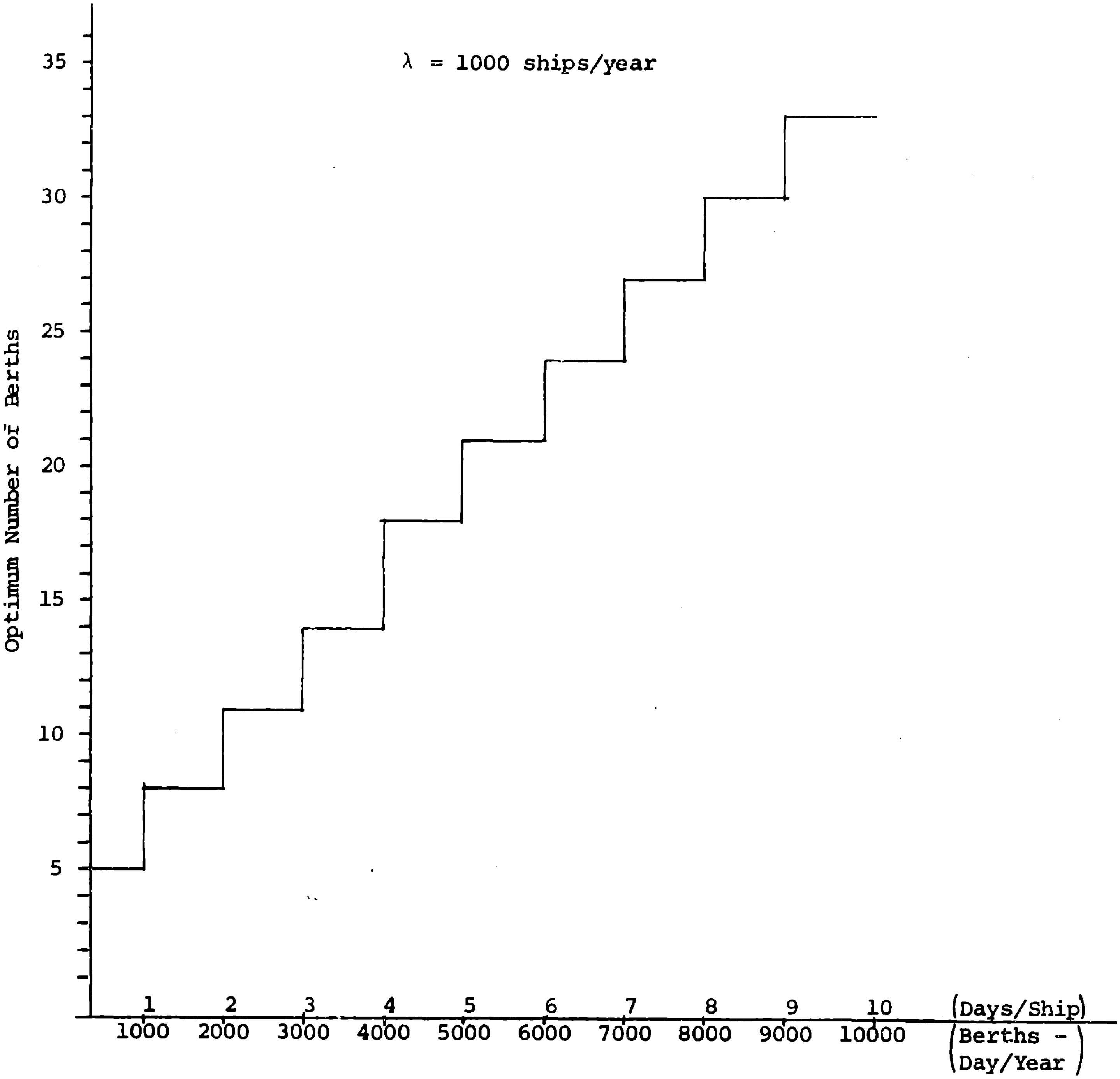

Figure 5.10 Optimum Number of Berths vs. Annual Berths-Day Required

since basically our model is a tradeoff of ship cost and berth cost.

5.3 Study of the Second Hypothetical Case

In Reference 50, Wah Ng examines a hypothetical case of an offshore terminal having an annual throughput of 20 million tons of fertilizer using a fleet of 250,000 D.W.T. carriers. He used the following data:

1. Arrival rate, $\lambda = 0.22$ ships/day
2. Ship waiting cost, $V = \$560/\text{hr}$
3. Cost of berth, $c_1 = \$230/\text{hr}$
4. Cost of storage (land) $c_2 = \$89/\text{hr}$
5. Cost ratios: $c_1/V = 0.41$

   $c_2/V = 0.1589$

Even though his investment criteria model is faulty as well as his queuing model (proposed also in Reference 43) (see footnote, pp. 41) and that the likelihood of violation of the main assumptions involved in our model is high (since the schedule of ship arrivals and servicing is usually highly organized in an offshore terminal), I will still use the above data to compare the results obtained by Wah Ng and also to show how the model performs the optimization more systematically and simply.

Given the limitations of the approach used in Reference 50, the optimization of the storage space was made by fixing the value of s_1 and n_1^* as follows:

$$s_1 = n_1^* = 2$$

Then for s_2 = 1 to 60, the optimum n_2* were computed. The relevant results from the program Expansion Criteria: Two Stage Queuing Model, presented in Appendix 3, and the above data are shown in Table 5.9 (including the results of Reference 50). Table 5.10 shows a portion of the program results.

Table 5.11 shows the results of a small example (example 5.1) using the following data:

<u>Example 5.1</u>

Data:

Arrival rate λ	=	2.4 ships/day
s_1 initial	=	3 days
s_2 initial	=	5 days
Δs_2	=	15 days
c_1/V	=	0.1
c_2/V	=	0.5

5.4 Analysis of the Results

The results from the first hypothetical case are as expected. Since the model is a tradeoff of ship cost and berth cost, when the service time increases up to some level,the addition of more service stations is needed in order to maintain the balance in cost. Also the sensitivity analysis shows that a decrease in the cost ratio (an increase in ship waiting cost) implies an increase in the number of service stations as we can expect (and viceversa).

In addition, the values for the average queuing time and the berth utilization factor, which are part of the program output (see

Table 5.9 Optimum Number of Storage Spaces

Data: λ = .22 Ship/day, s_1= 2 days/ship, n_1= 2 berths
$c_1/V = 0.4$, $c_2/V = 0.1589$

Storage Service Time, s_2	Optimum n_2		Storage Service Time, s_2	Optimum n_2	
	Computed	Referece 50		Computed	Reference 50
1	2	1	26	11	8
2	2	2	27	11	8
3	3	2	28	12	8
4	3	2	29	12	-
5	4	3	30	12	9
6	4	3	31	13	9
7	5	3	32	13	9
8	5	4	33	13	10
9	5	4	34	14	10
10	6	4	35	14	10
11	6	4	36	14	10
12	6	5	37	15	10
13	7	5	38	15	11
14	7	5	39	15	11
15	8	5	40	15	11
16	8	6	41	16	11
17	8	6	42	16	-
18	9	6	43	16	12
19	9	6	44	17	12
20	9	7	45	17	12
21	10	7	46	17	12
22	10	7	47	17	-
23	10	7	48	18	13
24	10	-	49	18	13
25	11	8	50	18	13

Table 5.10 <u>Expansion Criteria-Two Stage Model. Program Output</u>

```
         2.   S1
         1.   S2
         1.   N2
       0.22   UTI

N2*
         1.

         2.   N1
       0.22   UTI
.0793442623   WT

         3.   N1
.1466666667   UTI
.0065264786   WT

         1.   N1
       0.44   UTI
1.447326007   WT

N1*
         2.

         1.   N2
       0.22   UTI
.2596721311   WT

         2.   N2
       0.11   UTI
.0112763786   WT

         3.   N2
.0733333333   UTI
.0005089646   WT

N2*
         2.

         2.   N1
       0.22   UTI
.1007515885   WT

         3.   N1
.1466666667   UTI
.0082361005   WT

         1.   N1
       0.44   UTI
1.566039362   WT

OPT
         2.   N1
         2.   N2

         2.   S1
         2.   S2
         1.   N2
       0.44   UTI

N2*
         1.

         2.   N1
       0.22   UTI
.0569651114   WT

         3.   N1
.1466666667   UTI
.0046856769   WT

         1.   N1
       0.44   UTI
       0.88   WT

N1*
         2.

         1.   N2
       0.44   UTI
 1.52667027   WT

         2.   N2
       0.22   UTI
0.093652244   WT

         3.   N2
.1466666667   UTI
.0077033845   WT

N2*
         2.

         2.   N1
       0.22   UTI
0.093652244   WT

         3.   N1
.1466666667   UTI
.0077033845   WT

         1.   N1
       0.44   UTI
 1.52667027   WT

OPT
         2.   N1
         2.   N2

         2.   S1
         3.   S2
         1.   N2
       0.66   UTI

N2*
         1.
```

...Continue Table 5.10

2.	N1	3.	N1
0.22	UTI	.1466666667	UTI
.0345859605	WT	.0081022223	WT
3.	N1	1.	N1
.1466666667	UTI	0.44	UTI
.0028448753	WT	1.553558984	WT
1.	N1		
0.44	UTI	OPT	
.5342857143	WT	2.	N1
		3.	N2

N1*
2.

1.	N2
0.66	UTI
5.756391959	WT
2.	N2
0.33	UTI
.3499673541	WT
3.	N2
0.22	UTI
.0373903148	WT
4.	N2
0.165	UTI
.0040460171	WT

N2*
3.

2.	N1
0.22	UTI
.0985010285	WT

Table 5.11 Example 5.1 Results

3.	S1
5.	S2
14.	N2
.8571428571	UTI
N2*	
14.	
9.	N1
0.8	UTI
.3733641563	WT
10.	N1
0.72	UTI
.1408301736	WT
11.	N1
.6545454545	UTI
.0578313742	WT
N1*	
10.	
14.	N2
.8571428571	UTI
1.073375231	WT
15.	N2
0.8	UTI
.4452541384	WT
16.	N2
0.75	UTI
.2001308468	WT
N2*	
15.	
10.	N1
0.72	UTI
.1849884709	WT
11.	N1
.6545454545	UTI
.0759648108	WT
12.	N1
0.6	UTI
.0317752391	WT
N1*	
11.	
15.	N2
0.8	UTI
.4963687375	WT
16.	N2
0.75	UTI
.2328909484	WT
14.	N2
.8571428571	UTI
1.150514691	WT
OPT	
11.	N1
15.	N2

... Continue Table 5.11

3.	S1
20.	S2
54.	N2
.8888888889	UTI
N2*	
54.	
11.	N1
.6545454545	UTI
0.077960145	WT
12.	N1
0.6	UTI
.0326098653	WT
10.	N1
0.72	UTI
.1898474817	WT
N1*	
11.	
54.	N2
.8888888889	UTI
.9707396945	WT
55.	N2
.8727272727	UTI
.6553683974	WT
53.	N2
.9056603774	UTI
1.46415322	WT
52.	N2
.9230769231	UTI
2.278153002	WT
N2*	
53.	

11.	N1
.6545454545	UTI
.0695655206	WT
12.	N1
0.6	UTI
.0290984869	WT
10.	N1
0.72	UTI
.1694050068	WT
9.	N1
0.8	UTI
.4491207803	WT
N1*	
10.	
53.	N2
.9056603774	UTI
1.403854447	WT
54.	N2
.8888888889	UTI
0.922488772	WT
52.	N2
.9230769231	UTI
2.203523681	WT
OPT	
10.	N1
53.	N2

table 5.8), can be used to plot them against the service time and the optimum number berths.Also the probability of delay in each case can be retrived from its storage register.

In the second case, the results shows that due to a misestimate expansion criteria as well as que ... model an under-estimation of the storage space required were obtained in reference 50. The correct results are tabulated in the second column in table 5.9.. Again the average queuing time (of the joint process) and the utilization factors (for the service stations in both stages)can be used to plot them against the service times and the optimum number of berths and storage spaces. The probability of delay attributed to each stage can be retrived from the storage registers.

Since the optimization of both the number of berths and storage spaces, is carried out simultaneously meaningful results about a port system modeled as a two stage system are obtained as is shown below.

Example 5.1 was used to show the dangers of fixing the parameters of one stage in order to optimized the number of service station in the other one. If the level of service in the second stage (in this example) is expected to get worse the utilization of the service stations in the first satge will drop down and eventually one or more stations will stand idle 100 % of their time. This is reflected in the results, if we consider the initial condition given as normal, the optimum number of berth and storage spaces are: 11 and 15 respectively. Now if we let the service time in the second stage to increase up to 20 days to represent a deterioration in the level of service the optimum number of berth is reduced to 10, also the optimum number of storage spaces is enormously increased as we expected.

This is one of the main adventages of our model which I did not find in any of the actual analytic models used in port planning.

CHAPTER VI

Conclusions and Recommendations for Future Research

The basic purpose of the present study was the development of a new approach to port expansion using the M/M/n/ /FIFO queuing model in a more systematic, realistic and general way than the traditional approach. As an extension of this new approach a port occupancy charge model was developed to show an additional use of queuing model results in combination with a microeconomic model.

Parallel to this was the goal that the analysis procedures used in the study be programmed on programmable pocket calculators for easy use requiring simple input data, user instructions, and user output interpretation.

The optimum number of berths using a one stage queuing model is found using an optimization algorithm (developed by the author) which was written in terms of the demand for port service, given by the arrival rate, and the cost ratio, defined as the berth cost divided by ship waiting cost. The optimization is made parametrically over a wide range of service times (since the service time is not known with certainty) using different service time increments (both specified by the user).

The results obtained in a case study evidenced the capability of the optimization algorithm and the program working together in an effort to find an optimal solution for the configuration of a port system represented in one stage.

The procedure developed for the estimation of the optimum number of berths using the one stage queuing model has the following uncommon feature: it requires only knowledge of arrival rate and cost ratio.

The specification of service times and for service stations is no longer needed (as in some traditional approaches; see Reference 1).

The single stage character of the above model is all right as far as direct transfer of cargo between ships and land transport vehicles is concerned. However, in reality the indirect route, i.e. the transfer of import cargo from the hold of the ship to the transit shed, and after a lapse of time to the hold of a land transport vehicle (and vice versa for export cargo) is more dominant (the direct transfer represents a small percentage, if any).

The indirect route can be modeled as a two stage process: one corresponding to ship-apron transfers, involving the berth-ship interface, and the other to the apron-storage transfer. The basic point of this approach is that as long as port storage is not of unlimited capacity the performance of the two service stages will be connected. When the storage happens to be filled with cargo, the preceding service stage cannot pass on cargo which has been served, but this cargo has to remain in the first stage (usually at the apron) blocking the service station for subsequent ships. Therefore the storage area plays both the role of waiting room in the ordinary sense of the word and that of a service station. Hence the cargo dwell time is to be viewed as the service time rather that as queuing time.

This way of dealing with the storage area is a new approach to port expansion planning and was incorporated in the derivation of the two stage queuing model.

The same set of assumptions as for the M/M/n/ /FIFO queuing model were retained in the two stage model; however as prerequisite to starting the load/unload operation both a berth and storage space need to be available (implying that all berths and storage are interchangeable, i.e. can provide service to any customer).

The optimum number of berths and storage spaces is found simultaneously using an optimization algorithm (developed by the author) and the two stage queuing model which was written in the same manner as the first optimization algorithm.

Once the cost parameters of berths, storage spaces and ships (cost ratios) have been established, the optimum number of berths and storage spaces for a given arrival rate and for a wide range of service times in both stations can be found.

The results obtained from both algorithms can be used for the following:

- To check whether the existing number of berths and/or storage spaces is adequate to service the existing flow of traffic
- To find a good estimate of the order of magnitude of expansion needed to service future demand
- To assess the impact of changes of the service times on port capacity
- To measure actual port performance.

Finally an attempt to develop port occupancy charges from the results of the two stage queuing model and marginal cost using the ratio of an incremental cost to the corresponding increment of output as proxy for the social marginal cost, derived from the application of the "average cost of a marginal plant" method in combination with the half interval method of numerical analysis.

The validity of this model rests on the condition that the quality of the level of service has to remain the same as in the original situation after a capacity (station) addition has been made. The basic principle to derive this model was taken from Dr. Dan Shneerson's work on port economics. It gives an approximate but simple approach to establish berth and storage occupance charges. The model is still very general and more work needs to be done in order to incorporate some further items (stevedoring, etc.) that are factors in the determination of port charges. However, the results can be considered as a good approximation of marginal-input pricing resulting from a net social benefit maximization.

As a by-product of this study a complete set of programs for programmable pocket calculators has been developed to tackle various problems encountered with the M/M/n/ /FIFO queuing model (verification of assumptions, port traffic distribution, delays, etc.) which normally are solved by computer.

In summary, a systematic general approach to port expansion planning using one and two stage queuing models was developed in this study. The introduction of programmable calculators to this field promises great dividends for the future; indeed a research project to develop this kind of simple to use, responsive analysis tools may be quite fruitful.

Future work in this area could include:

1. Incorporation of a third stage into the model corresponding to storage-land transport cargo transfer involving the port-hinterland interface.

2. Adaptation of the present two stage model to an optimal scheduling of port improvements or expansion to cater to growing traffic congestion.

3. Development in the two stage queuing model the capabilities necessary for its operation without the assumption of interchangeable berths and storage.

4. Extension to the program set to incorporate other queuing models or time-staging models.

5. Further study in port pricing.

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Appendix I

I.1 Port Traffic χ^2 Goodness of Fit Test for Ship Arrival Distribution

Purpose: This program computes the χ^2 statistic, derived from Equation (2.4):

$$\chi^2 = \sum_{i=1}^{r} \frac{[F(Vi) - f(Vi)]^2}{F(Vi)}$$

The user inputs the actual observed frequencies for V = 0, 1, 2, . . . , V_{max} and the program computes the expected frequencies from equation (2.2):

$$F(V) = T\,P(V)$$

where $$\left(P(V) = \frac{(\lambda)^V e^{-\lambda}}{V!} \right)$$

It then groups both the observed and expected frequencies so that the expected frequency of each group is at least five. Using these categories it computes the χ^2 statistic as shown above.

As output it prints:

1. Input data
2. Expected frequencies
3. The average arrival rate
4. The grouped categories
5. The degrees of freedom
6. The χ^2 statistic value

Table I.1 Program Port Traffic χ^2 Goodness of Fit Test for Ship Arrival Distribution.

000	76	LBL	050	01	1	100	99	PRT	150	75	-
001	11	A	051	00	0	101	42	STO	151	76	LBL
002	85	+	052	42	STO	102	35	35	152	65	×
003	01	1	053	01	01	103	22	INV	153	43	RCL
004	95	=	054	43	RCL	104	44	SUM	154	59	59
005	42	STO	055	06	06	105	59	59	155	72	ST*
006	06	06	056	42	STO	106	03	3	156	03	03
007	42	STO	057	00	00	107	06	6	157	99	PRT
008	00	00	058	98	ADV	108	42	STO	158	98	ADV
009	01	1	059	76	LBL	109	03	03	159	43	RCL
010	00	0	060	85	+	110	43	RCL	160	03	03
011	42	STO	061	43	RCL	111	06	06	161	42	STO
012	01	01	062	07	07	112	75	-	162	07	07
013	76	LBL	063	65	×	113	02	2	163	69	OP
014	98	ADV	064	73	RC*	114	95	=	164	23	23
015	00	0	065	01	01	115	42	STO	165	75	-
016	72	ST*	066	44	SUM	116	00	00	166	02	2
017	01	01	067	03	03	117	01	1	167	05	5
018	69	OP	068	85	+	118	42	STO	168	95	=
019	21	21	069	69	OP	119	01	01	169	42	STO
020	97	DSZ	070	21	21	120	42	STO	170	01	01
021	00	00	071	69	OP	121	02	02	171	42	STO
022	98	ADV	072	27	27	122	76	LBL	172	00	00
023	01	1	073	97	DSZ	123	75	-	173	69	OP
024	00	0	074	00	00	124	43	RCL	174	21	21
025	42	STO	075	85	+	125	09	09	175	05	5
026	01	01	076	00	0	126	65	×	176	32	X:T
027	00	0	077	95	=	127	43	RCL	177	76	LBL
028	91	R/S	078	55	÷	128	08	08	178	94	+/-
029	76	LBL	079	43	RCL	129	45	Y×	179	69	OP
030	12	B	080	03	03	130	43	RCL	180	33	33
031	72	ST*	081	95	=	131	01	01	181	69	OP
032	01	01	082	42	STO	132	49	PRD	182	31	31
033	99	PRT	083	08	08	133	02	02	183	69	OP
034	69	OP	084	98	ADV	134	55	÷	184	30	30
035	21	21	085	99	PRT	135	43	RCL	185	69	OP
036	43	RCL	086	94	+/-	136	02	02	186	37	37
037	01	01	087	22	INV	137	95	=	187	73	RC*
038	75	-	088	23	LNX	138	22	INV	188	03	03
039	01	1	089	65	×	139	44	SUM	189	77	GE
040	00	0	090	43	RCL	140	59	59	190	60	DEG
041	95	=	091	03	03	141	72	ST*	191	01	1
042	91	R/S	092	98	ADV	142	03	03	192	94	+/-
043	76	LBL	093	99	PRT	143	99	PRT	193	63	EX*
044	13	C	094	42	STO	144	69	OP	194	03	03
045	00	0	095	59	59	145	21	21	195	74	SM*
046	42	STO	096	95	=	146	69	OP	196	07	07
047	07	07	097	42	STO	147	23	23	197	01	1
048	42	STO	098	09	09	148	97	DSZ	198	94	+/-
049	03	03	099	98	ADV	149	00	00	199	63	EX*

... Continue Table I.1

200	01	01
201	74	SM*
202	00	00
203	61	GTO
204	94	+/-
205	76	LBL
206	60	DEG
207	73	RC*
208	07	07
209	22	INV
210	77	GE
211	94	+/-
212	09	9
213	42	STO
214	01	01
215	01	1
216	00	0
217	42	STO
218	00	00
219	03	3
220	04	4
221	42	STO
222	03	03
223	03	3
224	05	5
225	42	STO
226	07	07
227	76	LBL
228	80	GRD
229	69	OP
230	20	20
231	69	OP
232	21	21
233	69	OP
234	23	23
235	69	OP
236	27	27
237	73	RC*
238	03	03
239	77	GE
240	70	RAD
241	01	1
242	94	+/-
243	63	EX*
244	03	03
245	74	SM*
246	07	07
247	01	1
248	94	+/-
249	63	EX*
250	01	01
251	74	SM*
252	00	00
253	61	GTO
254	80	GRD
255	76	LBL
256	70	RAD
257	73	RC*
258	07	07
259	22	INV
260	77	GE
261	80	GRD
262	98	ADV
263	01	1
264	00	0
265	42	STO
266	01	01
267	43	RCL
268	06	06
269	42	STO
270	00	00
271	29	CP
272	02	2
273	94	+/-
274	42	STO
275	07	07
276	76	LBL
277	33	X²
278	73	RC*
279	01	01
280	69	OP
281	21	21
282	22	INV
283	77	GE
284	44	SUM
285	69	OP
286	27	27
287	76	LBL
288	44	SUM
289	99	PRT
290	97	DSZ
291	00	00
292	33	X²
293	98	ADV
294	43	RCL
295	06	06
296	42	STO
297	00	00
298	03	3
299	05	5
300	42	STO
301	01	01
302	76	LBL
303	34	ΓX
304	73	RC*
305	01	01
306	99	PRT
307	69	OP
308	21	21
309	97	DSZ
310	00	00
311	34	ΓX
312	98	ADV
313	43	RCL
314	07	07
315	99	PRT
316	01	1
317	00	0
318	42	STO
319	01	01
320	03	3
321	05	5
322	42	STO
323	03	03
324	43	RCL
325	06	06
326	42	STO
327	00	00
328	76	LBL
329	32	X:T
330	53	(
331	73	RC*
332	01	01
333	75	-
334	73	RC*
335	03	03
336	54	)
337	33	X²
338	55	÷
339	73	RC*
340	03	03
341	85	+
342	69	OP
343	21	21
344	69	OP
345	23	23
346	97	DSZ
347	00	00
348	32	X:T
349	00	0
350	95	=
351	99	PRT
352	98	ADV
353	91	R/S

User Instructions

Step	Procedure	Enter	Press		Print
1.	Load the program				
2.	a. Enter maximum number of arrivals on any one day, M	M		A	
	b. Enter observed frequency W_v (number of days v ships arrived), from v = 0 to v = M.	W_0		B	W_0
		W_1		B	W_1
		⋮			
		W_M		B	W_M
3.	Compute			C	
	a. Average arrival rate $\overline{v}$				$\overline{v}$
	b. Period of time considered (in days)				T
	c. Expected frequencies				W_0'
					W_1'
					⋮
					$1 - \Sigma^{v} W_v'$
	d. Revised vector f(v) and F(v) (with -1 in null entries) as grouped into r categories criteria for grouping categories (expected) $\geq$ 5 elements				$f(v_0)$
					$f(v_1)$
					⋮
					$f(v_r)$
					$F(v_0) \to g\ W_0'$
					$F(v_1) \to g\ W'$
					⋮
					$F(v_r) \to g\ W_M+1$
4.	Degrees of freedom				f
5.	χ^2 STATISTIC				χ^2

Port Traffic χ^2 Goodness of Fit Test, I (Poisson)

Registers Used

0	Index
1	v
2	
3	
4	
5	T
6	M
7	Index
8	$\bar{v}$
9	
10-34	W_v, $f(v_i)$
35-59	$W_v'^4$, $F(v_i)$

I.2 Port Traffic χ^2 Goodness of Fit Test for Distribution of Ships in Port

Purpose: This program computes the χ^2 statistic, derived from Equation (2.4):

$$\chi^2 = \sum_{i=1}^{r} \frac{(F(V_i) - f(V_i))^2}{F(V_i)}$$

where $F(V_i)$ = expected frequency of V_i ships in port

$f(V_i)$ = observed frequency of V_i ships in port

It is structured in a way that can serve to print the basic data needed to carry out the χ^2 goodness of fit test about the distribution of ships in port. The user inputs the observed frequencies, the number of berths and berth utilization, and the program prints out the expected frequencies, derived from the theory of the M/M/n queuing model as follows:

$F(v_i) = T\,P(v_i)$ where

$$P(v) = \begin{cases} \dfrac{(n\rho)^v}{v!} P(o) & \text{if } 1 \le v \le n \\[2ex] \dfrac{(\rho)^v n^n}{n!} P(o) & \text{if } n < v < \infty \end{cases}$$

Then after grouping the data according to the criteria that the expected frequency of each category must be at least 5, it computes and prints the degree of freedom and the χ^2 statistic.

Table I.2 Program Port Traffic χ^2 Goodness of Fit Test for Distribution of Ships in Port

000	76	LBL	050	12	B	100	03	03	150	22	INV
001	15	E	051	72	ST*	101	95	=	151	49	PRD
002	42	STO	052	01	01	102	98	ADV	152	02	02
003	00	00	053	99	PRT	103	99	PRT	153	97	DSZ
004	29	CP	054	69	OP	104	43	RCL	154	07	07
005	67	EQ	055	21	21	105	08	08	155	43	RCL
006	23	LNX	056	43	RCL	106	99	PRT	156	76	LBL
007	76	LBL	057	01	01	107	65	×	157	48	EXC
008	28	LOG	058	75	-	108	43	RCL	158	01	1
009	43	RCL	059	01	1	109	09	09	159	85	+
010	00	00	060	00	0	110	99	PRT	160	43	RCL
011	65	×	061	95	=	111	95	=	161	05	05
012	97	DSZ	062	91	R/S	112	42	STO	162	45	Y^X
013	00	00	063	76	LBL	113	05	05	163	43	RCL
014	28	LOG	064	13	C	114	43	RCL	164	08	08
015	76	LBL	065	00	0	115	08	08	165	55	÷
016	23	LNX	066	42	STO	116	15	E	166	43	RCL
017	01	1	067	07	07	117	42	STO	167	04	04
018	95	=	068	42	STO	118	04	04	168	55	÷
019	92	RTN	069	03	03	119	43	RCL	169	53	(
020	76	LBL	070	01	1	120	08	08	170	01	1
021	11	A	071	00	0	121	75	-	171	75	-
022	85	+	072	42	STO	122	01	1	172	43	RCL
023	01	1	073	01	01	123	95	=	173	09	09
024	95	=	074	43	RCL	124	42	STO	174	95	=
025	42	STO	075	06	06	125	07	07	175	35	1/X
026	06	06	076	42	STO	126	29	CP	176	65	×
027	42	STO	077	00	00	127	67	EQ	177	43	RCL
028	00	00	078	98	ADV	128	48	EXC	178	03	03
029	01	1	079	76	LBL	129	43	RCL	179	98	ADV
030	00	0	080	85	+	130	04	04	180	99	PRT
031	42	STO	081	43	RCL	131	55	÷	181	42	STO
032	01	01	082	07	07	132	43	RCL	182	59	59
033	76	LBL	083	65	×	133	08	08	183	95	=
034	98	ADV	084	73	RC*	134	95	=	184	42	STO
035	00	0	085	01	01	135	42	STO	185	35	35
036	72	ST*	086	44	SUM	136	02	02	186	98	ADV
037	01	01	087	03	03	137	76	LBL	187	99	PRT
038	69	OP	088	85	+	138	43	RCL	188	22	INV
039	21	21	089	69	OP	139	43	RCL	189	44	SUM
040	97	DSZ	090	21	21	140	05	05	190	59	59
041	00	00	091	69	OP	141	45	Y^X	191	03	3
042	98	ADV	092	27	27	142	43	RCL	192	06	6
043	01	1	093	97	DSZ	143	07	07	193	42	STO
044	00	0	094	00	00	144	55	÷	194	03	03
045	42	STO	095	85	+	145	43	RCL	195	43	RCL
046	01	01	096	00	0	146	02	02	196	06	06
047	00	0	097	95	=	147	85	+	197	75	-
048	91	R/S	098	55	÷	148	43	RCL	198	02	2
049	76	LBL	099	43	RCL	149	07	07	199	95	=

... Continue Table I.2

200	42	STO	250	01	01	300	21	21
201	00	00	251	65	×	301	05	5
202	43	RCL	252	43	RCL	302	32	X:T
203	08	08	253	08	08	303	76	LBL
204	42	STO	254	45	Y×	304	94	+/-
205	07	07	255	43	RCL	305	69	OP
206	01	1	256	08	08	306	33	33
207	42	STO	257	55	÷	307	69	OP
208	01	01	258	43	RCL	308	31	31
209	42	STO	259	04	04	309	69	OP
210	02	02	260	65	×	310	30	30
211	76	LBL	261	43	RCL	311	69	OP
212	75	-	262	35	35	312	37	37
213	43	RCL	263	95	=	313	73	RC*
214	05	05	264	22	INV	314	03	03
215	45	Y×	265	44	SUM	315	77	GE
216	43	RCL	266	59	59	316	60	DEG
217	01	01	267	72	ST*	317	01	1
218	49	PRD	268	03	03	318	94	+/-
219	02	02	269	99	PRT	319	63	EX*
220	55	÷	270	69	OP	320	03	03
221	43	RCL	271	21	21	321	74	SM*
222	02	02	272	69	OP	322	07	07
223	65	×	273	23	23	323	01	1
224	43	RCL	274	97	DSZ	324	94	+/-
225	35	35	275	00	00	325	63	EX*
226	95	=	276	55	÷	326	01	01
227	22	INV	277	76	LBL	327	74	SM*
228	44	SUM	278	65	×	328	00	00
229	59	59	279	43	RCL	329	61	GTO
230	72	ST*	280	59	59	330	94	+/-
231	03	03	281	72	ST*	331	76	LBL
232	99	PRT	282	03	03	332	60	DEG
233	69	OP	283	99	PRT	333	73	RC*
234	21	21	284	98	ADV	334	07	07
235	69	OP	285	43	RCL	335	22	INV
236	23	23	286	03	03	336	77	GE
237	22	INV	287	42	STO	337	94	+/-
238	97	DSZ	288	07	07	338	09	9
239	00	00	289	69	OP	339	42	STO
240	65	×	290	23	23	340	01	01
241	97	DSZ	291	75	-	341	01	1
242	07	07	292	02	2	342	00	0
243	75	-	293	05	5	343	42	STO
244	76	LBL	294	95	=	344	00	00
245	55	÷	295	42	STO	345	03	3
246	43	RCL	296	01	01	346	04	4
247	09	09	297	42	STO	347	42	STO
248	45	Y×	298	00	00	348	03	03
249	43	RCL	299	69	OP	349	03	3

... **Continue Table I.2**

350	05	5	400	42	STD	450	43	RCL
351	42	STD	401	07	07	451	06	06
352	07	07	402	76	LBL	452	42	STD
353	76	LBL	403	33	X²	453	00	00
354	80	GRD	404	73	RC*	454	76	LBL
355	69	DP	405	01	01	455	32	X:T
356	20	20	406	69	DP	456	53	(
357	69	DP	407	21	21	457	73	RC*
358	21	21	408	22	INV	458	01	01
359	69	DP	409	77	GE	459	75	-
360	23	23	410	44	SUM	460	73	RC*
361	69	DP	411	69	DP	461	03	03
362	27	27	412	27	27	462	54	)
363	73	RC*	413	76	LBL	463	33	X²
364	03	03	414	44	SUM	464	55	÷
365	77	GE	415	99	PRT	465	73	RC*
366	70	RAD	416	97	DSZ	466	03	03
367	01	1	417	00	00	467	85	+
368	94	+/-	418	33	X²	468	69	DP
369	63	EX*	419	98	ADV	469	21	21
370	03	03	420	43	RCL	470	69	DP
371	74	SM*	421	06	06	471	23	23
372	07	07	422	42	STD	472	97	DSZ
373	01	1	423	00	00	473	00	00
374	94	+/-	424	03	3	474	32	X:T
375	63	EX*	425	05	5	475	00	0
376	01	01	426	42	STD	476	95	=
377	74	SM*	427	01	01	477	99	PRT
378	00	00	428	76	LBL	478	98	ADV
379	61	GTD	429	34	ΓX	479	91	R/S
380	80	GRD	430	73	RC*			
381	76	LBL	431	01	01			
382	70	RAD	432	99	PRT			
383	73	RC*	433	69	DP			
384	07	07	434	21	21			
385	22	INV	435	97	DSZ			
386	77	GE	436	00	00			
387	80	GRD	437	34	ΓX			
388	98	ADV	438	98	ADV			
389	01	1	439	43	RCL			
390	00	0	440	07	07			
391	42	STD	441	99	PRT			
392	01	01	442	01	1			
393	43	RCL	443	00	0			
394	06	06	444	42	STD			
395	42	STD	445	01	01			
396	00	00	446	03	3			
397	29	CP	447	05	5			
398	02	2	448	42	STD			
399	94	+/-	449	03	03			

User Instructions

Step	Procedure	Enter	Press		Print
1.	Load program.				
2.	Enter number of berths.	n	STO	08	
	Enter berth utilization factor.	ρ	STO	09	
3.	Enter maximum number of ships in port on any day, of categories, $v_{max} = M$	M		A	
4.	Enter observed frequency W_v				
	for $x = 0, \ldots, v_{max}$	W_0		B	W_0
	(W_v = # of days that v ships were in port)	W_1		B	W_1
		⋮			⋮
		W_m		B	W_m
5.	Compute and print output			C	
a.	print $\bar{v}$ (expected number of ships in port), n, ρ				$\bar{v}$ n ρ
b.	print T (period of time considered)				T
c.	print expected frequencies				W_0' W_1' ⋮ W_m' $1-W_m'$

User Instructions

Step	Procedure	Enter	Press		Print
1.	Load program.				
2.	Enter number of berths.	n	STO	08	
	Enter berth utilization factor.	ρ	STO	09	
3.	Enter maximum number of ships in port on any day, of categories, v_{max} = M	M		A	
4.	Enter observed frequency W_v for x = 0,..., v_{max}	W_0		B	W_0
	(W_v = # of days that v ships were in port)	W_1		B	W_1
		⋮			⋮
		W_m		B	W_m
5.	Compute and print output			C	
a.	print $\bar{v}$ (expected number of ships in port), n, ρ				$\bar{v}$ n ρ
b.	print T (period of time considered)				T
c.	print expected frequencies				W_0' W_1' ⋮ W_m' $1-W_m'$

Step	Procedure	Enter	Press	Print
	⋮			
d.	Print revised vectors W_i and W_i', as grouped into r categories (with -1 in null entries). (Criteria for grouping number of entries in each category > 5)			$f(v_1)$ $f(v_2)$ ⋮ $f(v_r)$ $F(v_1)$ $F(v_2)$ ⋮ $F(v_r)$
e.	Print degrees of freedom			f
f.	Print χ^2 statistic			χ^2

χ^2 Registers

0	Index to 10-34
1	V
2	i
3	$n\rho$
4	ρ
5	T
6	M
7	counter
8	n
9	s
10-34	W_v, $f(v_i)$
35-59	W_v', $F(v_i)$

χ^2 Registers

0	Index to 10-34
1	V
2	i
3	$n\rho$
4	ρ
5	T
6	M
7	counter
8	n
9	s
10-34	W_v, $f(v_i)$
35-59	W_v', $F(v_i)$

I.3 Port Traffic Distribution

This program computes p(v), the probability of v ships present in port (waiting for service and being served) at any given time, and $p_q(v)$, the probability of v ships waiting for berths (in queue), given by the equations:

$$p(v) = \begin{cases} \dfrac{(n\rho)^v}{v!}\, p(0) & \text{if } 1 \le v \le n \\[2ex] \dfrac{(\rho)^v\, n^n}{n!}\, p(0) & \text{if } v > n \end{cases}$$

and

$$p_q(v) = \begin{cases} \left[\sum_{i=0}^{n} \dfrac{(n\rho)^i}{i!}\right] p(0) & \text{for } v = 0 \\[2ex] \dfrac{(\rho)^{(n+v)}\, n^n}{n!}\, p(0) & \text{for } v > 0 \end{cases}$$

where

$$p(0) = \frac{1}{\sum_{i=0}^{n-1} \dfrac{(n\rho)^i}{i!} + \dfrac{(n\rho)^n}{n!(1-\rho)}}$$

The user is free to specify any value of $n \le 55$ and $\rho < 1$. The program prints the values of p(v) and/or $p_q(v)$ for v = 0,1,..., v_{max}, where v_{max} is specified by the user.

Table I.3 <u>Program Port Traffic Distribution</u>

000	76	LBL	050	43	RCL	100	06	6	150	76	LBL
001	15	E	051	43	RCL	101	69	OP	151	18	C'
002	53	(	052	05	05	102	04	04	152	43	RCL
003	42	STO	053	45	Y^x	103	43	RCL	153	09	09
004	00	00	054	43	RCL	104	18	18	154	45	Y^x
005	29	CP	055	07	07	105	69	OP	155	43	RCL
006	67	EQ	056	55	÷	106	06	06	156	04	04
007	85	+	057	43	RCL	107	98	ADV	157	65	×
008	76	LBL	058	13	13	108	43	RCL	158	43	RCL
009	75	-	059	85	+	109	08	08	159	08	08
010	43	RCL	060	43	RCL	110	42	STO	160	45	Y^x
011	00	00	061	07	07	111	01	01	161	43	RCL
012	65	×	062	22	INV	112	01	1	162	08	08
013	97	DSZ	063	49	PRD	113	42	STO	163	55	÷
014	00	00	064	13	13	114	04	04	164	43	RCL
015	75	-	065	97	DSZ	115	42	STO	165	10	10
016	76	LBL	066	07	07	116	00	00	166	65	×
017	85	+	067	43	RCL	117	76	LBL	167	43	RCL
018	01	1	068	76	LBL	118	19	D'	168	18	18
019	54	)	069	48	EXC	119	43	RCL	169	95	=
020	92	RTN	070	01	1	120	05	05	170	99	PRT
021	76	LBL	071	85	+	121	45	Y^x	171	69	OP
022	10	E'	072	43	RCL	122	43	RCL	172	24	24
023	43	RCL	073	05	05	123	04	04	173	97	DSZ
024	08	08	074	45	Y^x	124	49	PRD	174	01	01
025	65	×	075	43	RCL	125	00	00	175	18	C'
026	43	RCL	076	08	08	126	55	÷	176	91	R/S
027	09	09	077	55	÷	127	43	RCL	177	76	LBL
028	95	=	078	43	RCL	128	00	00	178	11	A
029	42	STO	079	10	10	129	65	×	179	42	STO
030	05	05	080	55	÷	130	43	RCL	180	08	08
031	43	RCL	081	53	(	131	18	18	181	91	R/S
032	08	08	082	01	1	132	95	=	182	42	STO
033	75	-	083	75	-	133	99	PRT	183	09	09
034	01	1	084	43	RCL	134	69	OP	184	91	R/S
035	95	=	085	09	09	135	24	24	185	76	LBL
036	42	STO	086	95	=	136	97	DSZ	186	12	B
037	07	07	087	35	1/X	137	01	01	187	42	STO
038	29	CP	088	42	STO	138	19	D'	188	06	06
039	67	EQ	089	18	18	139	76	LBL	189	91	R/S
040	48	EXC	090	92	RTN	140	65	×	190	76	LBL
041	43	RCL	091	76	LBL	141	98	ADV	191	13	C
042	10	10	092	70	RAD	142	43	RCL	192	43	RCL
043	55	÷	093	03	3	143	06	06	193	08	08
044	43	RCL	094	03	3	144	75	-	194	15	E
045	08	08	095	05	5	145	43	RCL	195	42	STO
046	95	=	096	05	5	146	08	08	196	10	10
047	42	STO	097	00	0	147	95	=	197	10	E'
048	13	13	098	01	1	148	42	STO	198	61	GTO
049	76	LBL	099	05	5	149	01	01	199	70	RAD

...Continue Table I.3

200	98	ADV
201	91	R/S
202	76	LBL
203	16	A'
204	43	RCL
205	08	08
206	15	E
207	42	STO
208	10	10
209	43	RCL
210	08	08
211	42	STO
212	01	01
213	01	1
214	42	STO
215	04	04
216	42	STO
217	00	00
218	10	E'
219	76	LBL
220	30	TAN
221	43	RCL
222	05	05
223	45	Y×
224	43	RCL
225	04	04
226	49	PRD
227	00	00
228	55	÷
229	43	RCL
230	00	00
231	85	+
232	69	OP
233	24	24
234	97	DSZ
235	01	01
236	30	TAN
237	01	1
238	95	=
239	65	×
240	43	RCL
241	18	18
242	95	=
243	42	STO
244	21	21
245	98	ADV
246	69	OP
247	00	00
248	04	4
249	02	2
250	04	4
251	03	3
252	03	3
253	04	4
254	06	6
255	04	4
256	00	0
257	01	1
258	69	OP
259	02	02
260	69	OP
261	05	05
262	43	RCL
263	21	21
264	99	PRT
265	98	ADV
266	76	LBL
267	39	COS
268	43	RCL
269	06	06
270	75	-
271	43	RCL
272	08	08
273	95	=
274	42	STO
275	01	01
276	43	RCL
277	08	08
278	85	+
279	01	1
280	95	=
281	42	STO
282	04	04
283	18	C'
284	91	R/S
285	00	0
286	00	0
287	00	0
288	00	0
289	00	0
290	00	0
291	00	0

User Instructions

Step	Procedure	Enter	Press		Print
1.	Load program.				
2.	Enter number of berths.	n	A		
3.	Enter utilization factor.	ρ	R/S		
4.	To compute p(v) (probability of v ships in port):				
a.	Enter number of values of v (# of ships in port) desired	M	B		
b.	Compute and print p(v) for v = 0,1,..., M-1		C		p(0) p(1) ⋮ p(M-1)
5.	To compute $p_q(v)$ (probability of v ships in queue):				
a.	Enter number of values of v (# of ships in queue) desired	M	B		
b.	Compute and print $p_q(v)$ for v = 0, 1,..., M-1		2nd	A'	$p_q(0)$ $p_q(1)$ ⋮ $p_q(M-1)$

Registers Used

00	Used in n!
01	counter
02	
03	
04	counter
05	n p
06	m
07	used
08	n
09	p
10	n!
11	
12	
13	used
14	
15	
16	
17	
18	p(0)
19	
20	
21	used

I.4 Probability of Delay in Multi-Channel Facility

Purpose: Computes and prints the probability of delay in a multi-channel facility, as a function of the number of servers (n) and the utilization factor (ρ), for Poisson arrivals and negative-exponential service times.

The program can be used in three ways:

1. It can be used to obtain a list of the probability of delay, p(D), for values of n (number of berths) from 2 up to 55 and values of ρ (berths utilization factor) between 0.1 and .99. For each value of n, starting from n = 2, and increasing by one (for $2 \leq n \leq 20$), the program computes p(D) varying ρ from 0.1 to 0.8 by increments of $\Delta\rho = 0.1$ and from 0.8 to 0.95 by increments of $\Delta\rho = 0.5$, and finally for $\rho = .99$. For values of n > 20 the increment Δn becomes 5 (up to n = 55).
2. It is possible to compute specific values of p(D) for given n and ρ.
3. Any set of p(D), for any increment Δn and a range of ρ with any increment $\Delta\rho$, can be computed to analyze specific situations.

Table I.4 <u>Program Probability of Delay in Multichannel Facility</u>

000	76	LBL	046	13	13	092	95	=	138	01	1
001	15	E	047	76	LBL	093	99	PRT	139	69	OP
002	53	(	048	43	RCL	094	92	RTN	140	04	04
003	42	STO	049	43	RCL	095	76	LBL	141	43	RCL
004	00	00	050	05	05	096	11	A	142	08	08
005	29	CP	051	45	Y×	097	42	STO	143	69	OP
006	67	EQ	052	43	RCL	098	08	08	144	06	06
007	85	+	053	07	07	099	91	R/S	145	15	E
008	76	LBL	054	55	÷	100	42	STO	146	42	STO
009	75	-	055	43	RCL	101	03	03	147	10	10
010	43	RCL	056	13	13	102	91	R/S	148	98	ADV
011	00	00	057	85	+	103	76	LBL	149	76	LBL
012	65	×	058	43	RCL	104	16	A'	150	14	D
013	97	DSZ	059	07	07	105	42	STO	151	10	E'
014	00	00	060	22	INV	106	14	14	152	43	RCL
015	75	-	061	49	PRD	107	91	R/S	153	15	15
016	76	LBL	062	13	13	108	42	STO	154	44	SUM
017	85	+	063	97	DSZ	109	06	06	155	09	09
018	01	1	064	07	07	110	91	R/S	156	97	DSZ
019	54	)	065	43	RCL	111	76	LBL	157	02	02
020	92	RTN	066	01	1	112	19	D'	158	14	D
021	76	LBL	067	95	=	113	42	STO	159	69	OP
022	10	E'	068	65	×	114	09	09	160	28	28
023	43	RCL	069	43	RCL	115	42	STO	161	97	DSZ
024	08	08	070	10	10	116	16	16	162	03	03
025	65	×	071	65	×	117	91	R/S	163	13	C
026	43	RCL	072	53	(	118	42	STO	164	69	OP
027	09	09	073	01	1	119	15	15	165	38	38
028	99	PRT	074	75	-	120	91	R/S	166	43	RCL
029	95	=	075	43	RCL	121	42	STO	167	14	14
030	42	STO	076	09	09	122	02	02	168	44	SUM
031	05	05	077	54	)	123	42	STO	169	08	08
032	43	RCL	078	85	+	124	17	17	170	97	DSZ
033	08	08	079	43	RCL	125	91	R/S	171	06	06
034	75	-	080	05	05	126	76	LBL	172	13	C
035	01	1	081	45	Y×	127	13	C	173	92	RTN
036	95	=	082	43	RCL	128	43	RCL	174	98	ADV
037	42	STO	083	08	08	129	16	16	175	76	LBL
038	07	07	084	95	=	130	42	STO	176	12	B
039	43	RCL	085	35	1/X	131	09	09	177	42	STO
040	10	10	086	65	×	132	43	RCL	178	08	08
041	55	÷	087	43	RCL	133	17	17	179	03	3
042	43	RCL	088	05	05	134	42	STO	180	01	1
043	08	08	089	45	Y×	135	02	02	181	69	OP
044	95	=	090	43	RCL	136	98	ADV	182	04	04
045	42	STO	091	08	08	137	03	3	183	43	RCL

... Continue Table I.4

184	08	08	230	70	RAD
185	69	OP	231	10	E'
186	06	06	232	93	.
187	15	E	233	01	1
188	42	STO	234	44	SUM
189	10	10	235	09	09
190	91	R/S	236	97	DSZ
191	42	STO	237	02	02
192	09	09	238	70	RAD
193	10	E'	239	93	.
194	91	R/S	240	08	8
195	76	LBL	241	05	5
196	18	C'	242	42	STO
197	02	2	243	09	09
198	42	STO	244	10	E'
199	08	08	245	93	.
200	01	1	246	09	9
201	09	9	247	42	STO
202	42	STO	248	09	09
203	03	03	249	10	E'
204	08	8	250	93	.
205	42	STO	251	09	9
206	06	06	252	05	5
207	76	LBL	253	42	STO
208	60	DEG	254	09	09
209	93	.	255	10	E'
210	01	1	256	93	.
211	42	STO	257	09	9
212	09	09	258	09	9
213	08	8	259	42	STO
214	42	STO	260	09	09
215	02	02	261	10	E'
216	98	ADV	262	69	OP
217	03	3	263	28	28
218	01	1	264	97	DSZ
219	69	OP	265	03	03
220	04	04	266	60	DEG
221	43	RCL	267	04	4
222	08	08	268	44	SUM
223	69	OP	269	08	08
224	06	06	270	97	DSZ
225	15	E	271	06	06
226	42	STO	272	60	DEG
227	10	10	273	92	RTN
228	98	ADV	274	98	ADV
229	76	LBL	275	00	0

User Instructions

Step	Procedure	Enter	Press		Print
1.	Load program.				
2.	Clear memories.		2nd	CMs	
3.	Compute and print list of p(D) for $2 \leq n \leq 55$. Varying ρ from 0.1 to 0.99 (repeated for each value of n).				
	a. Print number of berths, n				n
	b. Print berth utilization factor, ρ, starting at $\rho = 0.1$				ρ
	c. Print p(D) If $\rho \leq .99$, $\rho = \rho + \Delta\rho$, go to b; else $n = n + \Delta n$, go to a				p(D)
4.	Compute and print specific values of p(D).				
	a. Clear memories		2nd	CMs	
	b. Enter value of n ($2 \leq n \leq 55$)	n		B	
	c. Enter value of ρ ($0<\rho<1$)	ρ		R/S	
	d. Print number of berths, n				n
	e. Print berth utilization factor, ρ				ρ
	f. Print p(D)				p(D)
	g. Repeat steps a to c for different values of n and ρ				
5.	Compute and print a set of p(D)				
	a. Clear memories		2nd	CMs	
	b. Set limits				
	1. Enter initial value of n	n initial		A	
	2. For increment $\Delta n = 1$, number of values, J, of n desired (including n initial) (Note: $n_{MAX} \leq 55$)	J [1/]		R/S	

1/ Default Value = 1

Step	Procedure	Enter	Press		Print
	:				
	3. Defined increment (Δn)' desired	(Δn)'	2nd	A'	
	4. For increment (Δn)', number of values, K, of n desired	K 2/		R/S	
	5. Enter initial value of ρ	ρ initial	2nd	D'	
	6. Enter any increment Δρ desired	Δρ 2/		R/S	
	7. Enter number of values, L, of ρ desired	L 1/		R/S	
	c. Print number of berths, n				n
	d. Print berth utilization factor ρ				ρ
	e. Print p(D) If $\rho \leq \rho$ initial + $L \Delta \rho$, $\rho = \rho + \Delta\rho$ go to d else $n = n + \Delta n$ or $n = n + (\Delta n)'$ go to c				p(D)
	f. Repeat step a to e for different set				

1/ Default Value = 1
2/ Default Value = 0

Registers Used

00	Used in n!
01	Not used
02	Counter
03	Counter
04	Not used
05	n ρ
06	Counter
07	i
08	η, n initial
09	ρ
10	n!
11	Not used
12	Not used
13	(n - 1)!
14	(Δn)'
15	Δρ
16	ρ initial
17	Counter
18-59	Returned

I.5 Expected Waiting Time in Multi-Channel Facility

Purpose: This program computes the expected waiting time in queue and the expected time in the system, in multiples of the average service time, and the average total time in the system in a multi-channel facility derived from the M/M/n/∞/FIFO queuing model.

The features of the program are:

1. It can be used to obtain a complete list of the expected waiting time in queue, W_q/s, the expected time in the system, W/s (in multiples of the service time, s) and the average total time in the system TW, for values of n (number of berths) from 1 up to 55, and values of ρ (berth utilization factor) between 0.1 and .99. For each value of n, starting from 1 and increasing by one (until n = 20), the program computes W_q/s, W/s or TW (as specified by the user), varying ρ from 0.1 to 0.8 by increment $\Delta\rho$ = .1, and from 0.8 to 0.95 by increment $\Delta\rho$ = 0.05 and finally for ρ = .99. For values of n > 20 the increment Δn becomes 5 (up to n = 55).
2. It is possible to compute only specific values of W_q/s, W/s or TW, for given n and ρ.
3. A set of W_q/s, W/s or TW, for a range of n and ρ with any desired increments Δn and $\Delta\rho$, can be computed to analyze specific situations.

Table I.5 Program Expected Waiting Time in Multichannel Facility

000	76	LBL	050	76	LBL	100	43	RCL	150	06	06
001	15	E	051	43	RCL	101	09	09	151	91	R/S
002	53	(	052	43	RCL	102	54	)	152	76	LBL
003	42	STO	053	05	05	103	95	=	153	19	D'
004	00	00	054	45	Y^x	104	35	1/X	154	42	STO
005	29	CP	055	43	RCL	105	65	×	155	09	09
006	67	EQ	056	07	07	106	43	RCL	156	42	STO
007	85	+	057	55	÷	107	05	05	157	16	16
008	76	LBL	058	43	RCL	108	45	Y^x	158	91	R/S
009	75	-	059	13	13	109	43	RCL	159	42	STO
010	43	RCL	060	85	+	110	08	08	160	15	15
011	00	00	061	43	RCL	111	85	+	161	91	R/S
012	65	×	062	07	07	112	43	RCL	162	42	STO
013	97	DSZ	063	22	INV	113	04	04	163	02	02
014	00	00	064	49	PRD	114	95	=	164	42	STO
015	75	-	065	13	13	115	42	STO	165	17	17
016	76	LBL	066	97	DSZ	116	12	12	166	91	R/S
017	85	+	067	07	07	117	43	RCL	167	76	LBL
018	01	1	068	43	RCL	118	11	11	168	13	C
019	54	)	069	76	LBL	119	29	CP	169	43	RCL
020	92	RTN	070	48	EXC	120	67	EQ	170	16	16
021	76	LBL	071	01	1	121	80	GRD	171	42	STO
022	10	E'	072	95	=	122	43	RCL	172	09	09
023	43	RCL	073	65	×	123	12	12	173	43	RCL
024	08	08	074	43	RCL	124	65	×	174	17	17
025	65	×	075	08	08	125	43	RCL	175	42	STO
026	43	RCL	076	65	×	126	05	05	176	02	02
027	09	09	077	43	RCL	127	95	=	177	98	ADV
028	99	PRT	078	10	10	128	99	PRT	178	03	3
029	95	=	079	65	×	129	92	RTN	179	01	1
030	42	STO	080	53	(	130	76	LBL	180	69	OP
031	05	05	081	01	1	131	80	GRD	181	04	04
032	43	RCL	082	75	-	132	43	RCL	182	43	RCL
033	08	08	083	43	RCL	133	12	12	183	08	08
034	75	-	084	09	09	134	99	PRT	184	69	OP
035	01	1	085	54	)	135	92	RTN	185	06	06
036	95	=	086	33	X^2	136	76	LBL	186	15	E
037	42	STO	087	85	+	137	11	A	187	42	STO
038	07	07	088	43	RCL	138	42	STO	188	10	10
039	29	CP	089	08	08	139	08	08	189	98	ADV
040	67	EQ	090	65	×	140	91	R/S	190	76	LBL
041	48	EXC	091	43	RCL	141	42	STO	191	14	D
042	43	RCL	092	05	05	142	03	03	192	10	E'
043	10	10	093	45	Y^x	143	91	R/S	193	43	RCL
044	55	÷	094	43	RCL	144	76	LBL	194	15	15
045	43	RCL	095	08	08	145	16	A'	195	44	SUM
046	08	08	096	65	×	146	42	STO	196	09	09
047	95	=	097	53	(	147	14	14	197	97	DSZ
048	42	STO	098	01	1	148	91	R/S	198	02	02
049	13	13	099	75	-	149	42	STO	199	14	D

... Continue Table I.5

200	69	OP
201	28	28
202	97	DSZ
203	03	03
204	13	C
205	69	OP
206	38	38
207	43	RCL
208	14	14
209	44	SUM
210	08	08
211	97	DSZ
212	06	06
213	13	C
214	92	RTN
215	98	ADV
216	76	LBL
217	12	B
218	42	STO
219	08	08
220	03	3
221	01	1
222	69	OP
223	04	04
224	43	RCL
225	08	08
226	69	OP
227	06	06
228	15	E
229	42	STO
230	10	10
231	91	R/S
232	42	STO
233	09	09
234	10	E'
235	91	R/S
236	76	LBL
237	18	C'
238	01	1
239	42	STO
240	08	08
241	01	1
242	09	9
243	42	STO
244	03	03
245	08	8
246	42	STO
247	06	06
248	76	LBL
249	60	DEG
250	93	.
251	01	1
252	42	STO
253	09	09
254	08	8
255	42	STO
256	02	02
257	98	ADV
258	03	3
259	01	1
260	69	OP
261	04	04
262	43	RCL
263	08	08
264	69	OP
265	06	06
266	15	E
267	42	STO
268	10	10
269	98	ADV
270	76	LBL
271	70	RAD
272	10	E'
273	93	.
274	01	1
275	44	SUM
276	09	09
277	97	DSZ
278	02	02
279	70	RAD
280	93	.
281	08	8
282	05	5
283	42	STO
284	09	09
285	10	E'
286	93	.
287	09	9
288	42	STO
289	09	09
290	10	E'
291	93	.
292	09	9
293	05	5
294	42	STO
295	09	09
296	10	E'
297	93	.
298	09	9
299	09	9
300	42	STO
301	09	09
302	10	E'
303	69	OP
304	28	28
305	97	DSZ
306	03	03
307	60	DEG
308	04	4
309	44	SUM
310	08	08
311	97	DSZ
312	06	06
313	60	DEG
314	92	RTN
315	98	ADV

Step	Procedure	Enter	Press		Print
1.	Load program.				
2.	Clear memories.		2nd	CMs	
3.	Compute complete list of W_q/s (starting from n = 1)		2nd	C'	
	a. for ρ = 0.1				
	b. print/compute				n ρ W_q/s
	c. if $\rho \leq 0.99$, $\rho = \rho + \Delta\rho$, go to b				
	d. if n < 55, n = n + Δn, go to a				
4.	Compute complete list of W/s (the program repeats the same procedure [step 3.a. to 3.d.] explained above)	1	STO 2nd	04 C'	
	print				n ρ W/s
5.	Compute complete list of TW (the program repeats the same procedure [step 3.a. to 3.d.])	1 1	STO STO	04 11	
	print				n ρ TW

Step	Procedure	Enter	Press		Print
6.	Compute specific values of W_q/s				
	a. Clear memories		2nd	CMs	
	b. Enter values of n ($1 \leq n \leq 55$)	n		B	n
	c. Enter value of ρ ($0 < \rho < 1$)	ρ		R/S	ρ
	d. Repeat steps a to c for different values of n and ρ				W_q/s
7.	Compute specific values of W/s				
	a. Clear memories		2nd	CMs	
	b. Select subroutine	1	STO	04	
	c. Repeat steps 6.b. and 6.c.				
	d. Print				n ρ W/s
	e. Repeat steps a to c for different values of n and ρ				
8.	Compute specific values of TW				
	a. Clear memories		2nd	CMs	
	b. Select subroutine	1	STO STO	04 11	
	c. Repeat steps 6.b. and 6.c.				
	d. Print				n ρ TW
	e. Repeat steps a to c for different values of n and ρ				

Step	Procedure	Enter	Press		Print
9.	Compute a set of W_q/s				
	a. Clear memories		2nd	CMs	
	b. Set limits				
	1. Enter initial value of n	n initial		A	
	2. For increment $\Delta n = 1$, number of values, J, of n desired (including n initial)		J 1/	R/s	
	3. Defined increment $(\Delta n)'$ desired	$(\Delta n)'$	2nd	A'	
	4. For increment $(\Delta n)'$, number of values, K, of n desired		K 1/	R/s	
	5. Enter initial value ρ	ρ initial	2nd	D'	
	6. Enter any increment $\Delta\rho$, desired	$\Delta\rho$		R/s	
	7. Number of values, L, of ρ desired (including ρ initial	L 2/		R/s	
	c. Compute set of W_q/s specified			C	
	d. Print (set)				n ρ W_q/s
	e. If $\rho \leq \rho + L\Delta\rho$, $\rho = \rho + \Delta\rho$, go to d				
	f. If $n \leq n + J$, $n = n + 1$, go to d				
	g. If $n \leq n + K(\Delta n)'$, $n = n + (\Delta n)'$, go to d				
	h. Repeat steps a to c for different sets of n and ρ				
	1/Default Value = 1 2/Default Value = 0				

Step	Procedure	Enter	Press		Print
10.	Compute a set of W/s				
	a. Clear memories		2nd	CMs	
	b. Select subroutine		STO	04	
	c. Repeat steps 9.b. and 9.c.				
	d. Print (set)				n
					ρ
					W/s
	e. Repeat steps a to c for different sets of n and ρ				
11.	Compute a set of TW				
	a. Clear memories		2nd	CMs	
	b. Select subroutine	1	STO	04	
			STO	11	
	c. Repeat steps 9.b. and 9.c.				
	d. Print (set)				n
					ρ
					TW
	e. Repeat steps a to c for different values of n and ρ				

Registers Used

00	Used in n!
01	Not used
02	Counter
03	Counter
04	Subroutine flag
05	nρ
06	Counter
07	i
08	n
09	ρ
10	n!
11	Subroutine flag
12	W_q/s or W/s
13	(n-1)!
14	(Δn)'
15	Δρ
16	ρ
17	Counter
18-59	Not used

I.6 Marginal Queuing Time

Purpose: This program computes the expected marginal queuing time in a multi-channel facility (in multiples of the service time, s) as a function of the number of berths, n, and the berth utilization factor ρ, derived from the M/M/n/∞/ FIFO queuing model.

The program can be used in three ways:

1. It can be used to obtain a complete list of the marginal queuing time (MQT), for values of n from 1 up to 55 and values of ρ between 0.1 and 0.99. For each value of n, starting from 1 and increasing by one (until n = 20), the program computes MQT varying ρ from 0.1 to 0.95 by increments Δρ = 0.05, and finally for ρ = 0.99. For values of n > 20, the increment Δn becomes 5 (up to n = 55).
2. It is possible to compute only specific values of MQT for given n and ρ.
3. A set of MQT, for a range of n and ρ with any desired increments Δn and Δρ, can be computed to analyze specific situations.

Table I.6 Program Marginal Queuing Time

```
000  76  LBL     050  08  08      100  04  04      150  91  R/S
001  15  E       051  95  =       101  95  =       151  76  LBL
002  53  (       052  42  STO     102  35  1/X     152  19  D'
003  42  STO     053  13  13      103  65  ×       153  42  STO
004  00  00      054  76  LBL     104  43  RCL     154  09  09
005  29  CP      055  43  RCL     105  05  05      155  42  STO
006  67  EQ      056  43  RCL     106  45  Y^X     156  16  16
007  85  +       057  05  05      107  43  RCL     157  91  R/S
008  76  LBL     058  45  Y^X     108  08  08      158  42  STO
009  75  -       059  43  RCL     109  65  ×       159  15  15
010  43  RCL     060  07  07      110  53  (       160  91  R/S
011  00  00      061  55  ÷       111  43  RCL     161  42  STO
012  65  ×       062  43  RCL     112  04  04      162  02  02
013  97  DSZ     063  13  13      113  35  1/X     163  42  STO
014  00  00      064  85  +       114  85  +       164  17  17
015  75  -       065  43  RCL     115  43  RCL     165  91  R/S
016  76  LBL     066  07  07      116  08  08      166  76  LBL
017  85  +       067  22  INV     117  65  ×       167  13  C
018  01  1       068  49  PRD     118  43  RCL     168  43  RCL
019  54  )       069  13  13      119  04  04      169  16  16
020  92  RTN     070  97  DSZ     120  85  +       170  42  STO
021  76  LBL     071  07  07      121  43  RCL     171  09  09
022  10  E'      072  43  RCL     122  05  05      172  43  RCL
023  43  RCL     073  01  1       123  65  ×       173  17  17
024  08  08      074  95  =       124  43  RCL     174  42  STO
025  65  ×       075  42  STO     125  11  11      175  02  02
026  43  RCL     076  11  11      126  55  ÷       176  98  ADV
027  09  09      077  65  ×       127  43  RCL     177  03  3
028  99  PRT     078  43  RCL     128  12  12      178  01  1
029  95  =       079  04  04      129  54  )       179  69  OP
030  42  STO     080  85  +       130  85  +       180  04  04
031  05  05      081  43  RCL     131  01  1       181  43  RCL
032  01  1       082  05  05      132  95  =       182  08  08
033  75  -       083  45  Y^X     133  99  PRT     183  69  OP
034  43  RCL     084  43  RCL     134  92  RTN     184  06  06
035  09  09      085  08  08      135  76  LBL     185  15  E
036  95  =       086  55  ÷       136  11  A       186  42  STO
037  42  STO     087  43  RCL     137  42  STO     187  10  10
038  04  04      088  10  10      138  08  08      188  98  ADV
039  43  RCL     089  95  =       139  91  R/S     189  76  LBL
040  08  08      090  42  STO     140  42  STO     190  14  D
041  75  -       091  12  12      141  03  03      191  10  E'
042  01  1       092  65  ×       142  91  R/S     192  43  RCL
043  95  =       093  43  RCL     143  76  LBL     193  15  15
044  42  STO     094  10  10      144  16  A'      194  44  SUM
045  07  07      095  65  ×       145  42  STO     195  09  09
046  43  RCL     096  43  RCL     146  14  14      196  97  DSZ
047  10  10      097  08  08      147  91  R/S     197  02  02
048  55  ÷       098  65  ×       148  42  STO     198  14  D
049  43  RCL     099  43  RCL     149  06  06      199  69  OP
```

... Continue Table I.6

200	28	28
201	97	DSZ
202	03	03
203	13	C
204	69	OP
205	38	38
206	43	RCL
207	14	14
208	44	SUM
209	08	08
210	97	DSZ
211	06	06
212	13	C
213	92	RTN
214	98	ADV
215	76	LBL
216	12	B
217	42	STO
218	08	08
219	03	3
220	01	1
221	69	OP
222	04	04
223	43	RCL
224	08	08
225	69	OP
226	06	06
227	15	E
228	42	STO
229	10	10
230	91	R/S
231	42	STO
232	09	09
233	10	E'
234	91	R/S
235	76	LBL
236	18	C'
237	02	2
238	42	STO
239	08	08
240	01	1
241	09	9
242	42	STO
243	03	03
244	08	8
245	42	STO
246	06	06
247	76	LBL
248	60	DEG
249	93	.
250	01	1
251	42	STO
252	09	09
253	08	8
254	42	STO
255	02	02
256	98	ADV
257	03	3
258	01	1
259	69	OP
260	04	04
261	43	RCL
262	08	08
263	69	OP
264	06	06
265	15	E
266	42	STO
267	10	10
268	98	ADV
269	76	LBL
270	70	RAD
271	10	E'
272	93	.
273	01	1
274	44	SUM
275	09	09
276	97	DSZ
277	02	02
278	70	RAD
279	93	.
280	08	8
281	05	5
282	42	STO
283	09	09
284	10	E'
285	93	.
286	09	9
287	42	STO
288	09	09
289	10	E'
290	93	.
291	09	9
292	05	5
293	42	STO
294	09	09
295	10	E'
296	93	.
297	09	9
298	09	9
299	42	STO
300	09	09
301	10	E'
302	69	OP
303	28	28
304	97	DSZ
305	03	03
306	60	DEG
307	04	4
308	44	SUM
309	08	08
310	97	DSZ
311	06	06
312	60	DEG
313	92	RTN
314	98	ADV
315	93	.
316	01	1
317	42	STO
318	09	09
319	08	8
320	42	STO
321	02	02
322	01	1
323	42	STO
324	08	08
325	98	ADV
326	03	3
327	01	1
328	69	OP
329	04	04
330	43	RCL
331	08	08
332	69	OP
333	06	06
334	98	ADV
335	76	LBL
336	80	GRD
337	17	B'
338	93	.
339	01	1
340	44	SUM
341	09	09
342	97	DSZ
343	02	02
344	80	GRD
345	93	.
346	08	8
347	05	5
348	42	STO
349	09	09
350	17	B'
351	93	.
352	09	9
353	42	STO
354	09	09
355	17	B'
356	93	.
357	09	9
358	05	5
359	42	STO
360	09	09
361	17	B'
362	93	.
363	09	9
364	09	9
365	42	STO
366	09	09
367	17	B'
368	91	R/S
369	76	LBL
370	17	B'
371	43	RCL
372	09	09
373	99	PRT
374	65	×
375	53	(
376	02	2
377	75	-
378	43	RCL
379	09	09
380	54	)
381	55	÷
382	53	(
383	01	1
384	75	-
385	43	RCL
386	09	09
387	54	)
388	33	X^2
389	95	=
390	99	PRT
391	92	RTN

User Instructions

Step	Procedure	Enter	Press		Print
1.	Load program				
2.	Clear memories		2nd	CMs	
3.	Compute complete list of MQT (starting from n = 2)				
	a. for ρ = 0.1				
	b. Print/compute				n
					ρ
					MQT
	c. If ρ < 0.99, ρ = ρ + Δρ, go to b				
	d. If n < 55, n = n + Δn, go to a				
	e. For n = 1		GTO	315	
				R/s	
4.	Compute specific values of MQT				
	a. Clear memories		2nd	CMs	
	b. Enter value of n	n		B	n
	c. Enter value of ρ	ρ		R/s	ρ
					MQT
	d. Repeat steps a to c for different values of n and ρ				
5.	Compute a set of MQT				
	a. Clear memories		2nd	CMs	
	b. Set limits				
	1. Enter initial value of n	n initial		A	
	2. For increment Δn = 1, number of values, J, of n desired (including n initial)	J		R/S	

Step	Procedure	Enter	Press		Print
	3. Defined increment $(\Delta n)'$ desired	$(\Delta n)'$	2nd	A'	
	4. For increment $(\Delta n)'$, number of values, K, of n desired	K		R/s	
	5. Enter initial value ρ	ρ initial	2nd	D'	
	6. Enter any increment Δρ desired	Δρ		R/s	
	7. Number of values, L, of ρ desired (including ρ initial)	L		R/s	
	c. Compute set of MQT specified			C	
	d. Print (set)				n ρ MQT
	e. If $\rho \leq \rho + L\Delta\rho$, $\rho = \rho + \Delta\rho$, go to d				
	f. If $n \leq n + J$, $n = n + 1$ go to d				
	g. If $n \leq n + K(\Delta n)'$, $n = n + (\Delta n)'$, go to d				
	h. Repeat steps a to c for different sets of n and ρ				

Registers Used

00	Used in n!
01	Not used
02	Counter
03	Counter
04	$(1 - \rho)$
05	$n\rho$
06	K
07	i
08	n
09	ρ
10	n!
11	Used
12	Used
13	(n-1)!
14	$(\Delta n)'$
15	$\Delta\rho$
16	ρ
17	L
18-59	Not used

Appendix II

II.1 Program Expansion Criteria: One Stage Queuing Model

Purpose: This program searches for the optimal number of berths in a port, assuming Poisson arrivals with arrival rate λ, and negative-exponential service time, s, distribution. The service times over which the optimization is made vary from s_1 to s_k by intervals Δs (specified by the user). The optimal number of berths is obtained using a cost function defined as follows:

$$TC = T\left\{cn + V\,[Wq(n)]\right\}$$

where

c = berth cost per unit time (construction cost)

V = ship waiting time cost per unit time

η = number of berths

Wq(n) = Expected ship waiting time (derived from a M/m/n queuing model)

T = period of time considered

and the expansion criteria given by

$$\frac{\partial TC}{\partial \eta} = 0$$
$$= c + V\,\frac{\partial[Wq(n)]}{\partial \eta}$$

i.e.

$$\frac{\partial Wq(n)}{\partial n} = -\frac{c}{V}$$

This is approximated by

$$Wq(n) - Wq(n+1) \leq c/V \qquad \text{(Expansion Criteria)}$$

The program computes $Wq(n) - Wq(n+1)$ for increasing values of n until this difference is less than (or equal) c/V. Then for the given service time, s, print the optimal number of berths n^*.

The program is quite flexible and the user only needs to specify the cost ratio (i.e. the ratio of berth cost to ship waiting time cost) and the arrival rate. A wide range of values of service time can be specified as well as different increments of the service time. Also, if the user has information about the values of c and V, the berth cost and ship waiting time cost, he can specify that the program print the total cost per unit time for the optimal number of berths.

Table II.1 <u>Program Expansion Criteria-One Stage Queuing Model</u>

000	76	LBL	050	43	RCL	100	08	08	150	92	RTN
001	15	E	051	08	08	101	65	×	151	76	LBL
002	53	(	052	95	=	102	53	(	152	11	A
003	42	STO	053	42	STO	103	01	1	153	42	STO
004	00	00	054	13	13	104	75	-	154	15	15
005	29	CP	055	76	LBL	105	43	RCL	155	32	X:T
006	67	EQ	056	43	RCL	106	09	09	156	03	3
007	85	+	057	43	RCL	107	54	)	157	42	STO
008	76	LBL	058	05	05	108	95	=	158	08	08
009	75	-	059	45	Y^x	109	35	1/X	159	01	1
010	43	RCL	060	43	RCL	110	65	×	160	42	STO
011	00	00	061	07	07	111	43	RCL	161	04	04
012	65	×	062	55	÷	112	05	05	162	06	6
013	97	DSZ	063	43	RCL	113	45	Y^x	163	03	3
014	00	00	064	13	13	114	43	RCL	164	69	OP
015	75	-	065	85	+	115	08	08	165	04	04
016	76	LBL	066	43	RCL	116	65	×	166	43	RCL
017	85	+	067	07	07	117	43	RCL	167	16	16
018	01	1	068	22	INV	118	15	15	168	69	OP
019	54	)	069	49	PRD	119	95	=	169	06	06
020	92	RTN	070	13	13	120	69	OP	170	03	3
021	76	LBL	071	97	DSZ	121	06	06	171	06	6
022	10	E'	072	07	07	122	92	RTN	172	69	OP
023	04	4	073	43	RCL	123	76	LBL	173	04	04
024	03	3	074	01	1	124	18	C'	174	32	X:T
025	03	3	075	95	=	125	03	3	175	69	OP
026	07	7	076	65	×	126	07	7	176	06	06
027	69	OP	077	76	LBL	127	01	1	177	91	R/S
028	04	04	078	48	EXC	128	05	5	178	76	LBL
029	43	RCL	079	43	RCL	129	69	OP	179	16	A'
030	08	08	080	08	08	130	04	04	180	42	STO
031	65	×	081	65	×	131	43	RCL	181	04	04
032	43	RCL	082	43	RCL	132	21	21	182	91	R/S
033	09	09	083	10	10	133	65	×	183	76	LBL
034	95	=	084	65	×	134	43	RCL	184	12	B
035	42	STO	085	53	(	135	25	25	185	42	STO
036	05	05	086	01	1	136	85	+	186	03	03
037	43	RCL	087	75	-	137	43	RCL	187	32	X:T
038	08	08	088	43	RCL	138	22	22	188	07	7
039	75	-	089	09	09	139	65	×	189	05	5
040	01	1	090	54	)	140	43	RCL	190	03	3
041	95	=	091	33	X^2	141	24	24	191	06	6
042	42	STO	092	85	+	142	95	=	192	69	OP
043	07	07	093	43	RCL	143	65	×	193	04	04
044	29	CP	094	08	08	144	03	3	194	43	RCL
045	67	EQ	095	65	×	145	06	6	195	04	04
046	48	EXC	096	43	RCL	146	05	5	196	69	OP
047	43	RCL	097	05	05	147	95	=	197	06	06
048	10	10	098	45	Y^x	148	69	OP	198	02	2
049	55	÷	099	43	RCL	149	06	06	199	06	6

... Continue Table II.1

200	69	OP	250	69	OP	300	69	OP
201	04	04	251	04	04	301	06	06
202	32	X:T	252	01	1	302	77	GE
203	69	OP	253	32	X:T	303	42	STO
204	06	06	254	43	RCL	304	43	RCL
205	98	ADV	255	15	15	305	08	08
206	91	R/S	256	65	×	306	75	-
207	76	LBL	257	43	RCL	307	01	1
208	13	C	258	16	16	308	95	=
209	98	ADV	259	55	÷	309	42	STO
210	98	ADV	260	42	STO	310	25	25
211	03	3	261	19	19	311	98	ADV
212	06	6	262	43	RCL	312	69	OP
213	69	OP	263	08	08	313	00	00
214	04	04	264	95	=	314	03	3
215	43	RCL	265	42	STO	315	02	2
216	15	15	266	09	09	316	03	3
217	69	OP	267	69	OP	317	03	3
218	06	06	268	06	06	318	03	3
219	03	3	269	77	GE	319	07	7
220	22	INV	270	42	STO	320	69	OP
221	44	SUM	271	43	RCL	321	02	02
222	08	08	272	08	08	322	03	3
223	09	9	273	15	E	323	01	1
224	22	INV	274	42	STO	324	05	5
225	28	LOG	275	10	10	325	01	1
226	42	STO	276	10	E'	326	69	OP
227	18	18	277	32	X:T	327	03	03
228	76	LBL	278	43	RCL	328	69	OP
229	42	STO	279	20	20	329	05	05
230	03	3	280	32	X:T	330	43	RCL
231	01	1	281	75	-	331	25	25
232	69	OP	282	48	EXC	332	99	PRT
233	04	04	283	18	18	333	98	ADV
234	98	ADV	284	42	STO	334	29	CP
235	69	OP	285	24	24	335	43	RCL
236	28	28	286	95	=	336	23	23
237	43	RCL	287	94	+/-	337	67	EQ
238	08	08	288	42	STO	338	14	D
239	29	CP	289	26	26	339	18	C'
240	67	EQ	290	07	7	340	76	LBL
241	42	STO	291	05	5	341	14	D
242	69	OP	292	04	4	342	43	RCL
243	06	06	293	03	3	343	04	04
244	04	4	294	03	3	344	44	SUM
245	01	1	295	07	7	345	15	15
246	03	3	296	69	OP	346	97	DSZ
247	07	7	297	04	04	347	03	03
248	02	2	298	43	RCL	348	13	C
249	04	4	299	26	26	349	91	R/S

User Instructions

Step	Procedure	Enter	Press		Print
1.	Load program				
2.	Enter data				
	a. The arrival rate, λ	λ	STO	16	
	b. Cost ratio c/V	c/V	STO	20	
	c. Berth cost per unit time	c [1/]	STO	21	
	d. Ship waiting time cost per unit time	V [1/]	STO	22	
3.	Specify if you want to evaluate cost function (c and V must be entered in Step 2)	1	STO	23	
4.	Set limits of parametric analysis				
	a. Specify initial service time	s		A	λ
					s
	b. Desired increment, Δs	Δs [2/]	2nd	A'	
	c. Number of values, K, of s desired	K		B	Δs
					K
5.	Compute and print results (repeated for each value of s)			C	
	a. Print number of berths, n				n
	b. Print berth utilization factor, ρ If $\rho \geq 1$, $n = n + 1$, go to a				ρ
	c. Print waiting time, WT				WT

[1/] c and V are optional. They need to be entered when evaluation of cost function is desired.

[2/] Default value = 1

Step	Procedure	Enter	Press		Print
	d. Print change in WT, ΔWT				ΔWT
	If ΔWT $\geq$ c/V go to e				
	else:				
	print optimal n				n*
	print total cost if desired				TC*
	Increment s, go to a				
	e. Increment n, go to a				

Registers Used

00	Used in n!	16	λ
01	Not used	17	not used
02	Not used	18	Wq new
03	K	19	sλ
04	Δs	20	c/V
05	nρ	21	c
06	Not used	22	V
07	Used	23	Used to specify TΣ Evaluation
08	n	24	Wq
09	ρ	25	n opt
10	n!	26	Δ Wq
11	Not used	27	Not used
12	Not used	28	Not used
13	Used in n!	29	Not used
14	Not used		
15	s		

Appendix III

III.1 Expansion Criteria: Two Stage Queuing Model

Purpose: This program searches for the optimal number of berths and storage spaces (defined as the area needed to store an average shipload) in a port, assuming Poisson arrivals with arrival rate λ, and negative-exponential service times distribution. The service times over which the optimization is made vary from s_1 (initial) to s_{1k} by intervals Δs_1 (specify by the user) and for each s_1 (the average service time at the first stage), s_2 (the average service time at the second stage) is varied from s_2 (initial) to s_{2L} by intervals Δs_2 (specify by the user also), using the following cost function:

$$TC = F + c_1 n_1 + c_2 n_2 + V W_q\ (n_1, n_2, x_1, x_2, \lambda) + V\lambda s_1$$

where

F = fixed cost

c_1 = cost per unit time of berth and related facilities

c_2 = cost per unit time of storage area

V = average cost per unit time of ships at port

n_1 = number of berths

n_2 = number of storage spaces

W_q = expected queuing time of all ships

x_1 = total service-day of ships required ($= \lambda s_1$)

x_2 = total transit storage-day of cargo required ($= \lambda s_2$)

λ = arrival rate

The optimal number of berths n_1^* and the optimal number of storage spaces n_2^* are found when:

$$\frac{\partial TC}{\partial n_1} = 0 \quad \text{and} \quad \frac{\partial TC}{\partial n_2} = 0 , \quad \text{i.e.}$$

when

$$\frac{\partial W_q(n_1 , n_2 , x_1 , x_2 , \lambda)}{\partial n_1} = - c_1/V$$

and

$$\frac{\partial W_q(n_1 , n_2 , x_1 , x_2 , \lambda)}{\partial n_2} = - c_2/V$$

Approximated by

$$W_q (n_1 , n_2 , x_1 , x_2 , \lambda) - W_q (n_1 + 1, n_2 , x_1 , x_2 , \lambda) \geq c_1/V$$

and

$$W_q (n_1 , n_2 , x_1 , x_2 , \lambda) - W_q (n_1 , n_2 + 1, x_1 , x_2 , \lambda) \geq c_2/X$$

The program starts by guessing a value for n_1 and n_2 for a given s_1 and s_2, then varying n_1 and varying n_2 will satisfy both equations when the ΔW_q are less than c_1/V and c_2/X, respectively.

Then for the given s_1 and s_2 print n_1^* and n_2^* optimals.

The program is quite flexible and the user only needs to specify the cost ratios, c_1/V and c_2/V and the arrival rate λ ; a wide range of values of s_1 and s_2 can be specified, as well as different increments for both s_1 and s_2.

Table III.1 <u>Program Expansion Criteria-Two Stage Queuing Model</u>

000	76	LBL	050	04	04	100	18	18	150	18	18
001	18	C'	051	43	RCL	101	65	×	151	55	÷
002	85	+	052	18	18	102	43	RCL	152	53	(
003	03	3	053	75	-	103	19	19	153	01	1
004	01	1	054	01	1	104	54	)	154	75	-
005	00	0	055	95	=	105	45	Y^X	155	43	RCL
006	00	0	056	42	STO	106	43	RCL	156	19	19
007	95	=	057	03	03	107	18	18	157	95	=
008	69	OP	058	29	CP	108	55	÷	158	48	EXC
009	04	04	059	67	EQ	109	43	RCL	159	25	25
010	43	RCL	060	48	EXC	110	02	02	160	42	STO
011	18	18	061	43	RCL	111	55	÷	161	20	20
012	69	OP	062	02	02	112	53	(	162	92	RTN
013	06	06	063	55	÷	113	01	1	163	76	LBL
014	42	STO	064	43	RCL	114	75	-	164	10	E'
015	02	02	065	18	18	115	43	RCL	165	43	RCL
016	29	CP	066	95	=	116	19	19	166	25	25
017	67	EQ	067	42	STO	117	95	=	167	42	STO
018	85	+	068	13	13	118	65	×	168	02	02
019	76	LBL	069	76	LBL	119	43	RCL	169	32	X:T
020	75	-	070	43	RCL	120	02	02	170	43	RCL
021	43	RCL	071	53	(	121	65	×	171	24	24
022	02	02	072	43	RCL	122	53	(	172	77	GE
023	65	×	073	18	18	123	01	1	173	87	IFF
024	97	DSZ	074	65	×	124	75	-	174	42	STO
025	02	02	075	43	RCL	125	43	RCL	175	02	02
026	75	-	076	19	19	126	19	19	176	76	LBL
027	76	LBL	077	54	)	127	54	)	177	87	IFF
028	85	+	078	45	Y^X	128	95	=	178	43	RCL
029	01	1	079	43	RCL	129	35	1/X	179	26	26
030	95	=	080	03	03	130	65	×	180	65	×
031	42	STO	081	55	÷	131	53	(	181	43	RCL
032	02	02	082	43	RCL	132	43	RCL	182	25	25
033	04	4	083	13	13	133	18	18	183	75	-
034	01	1	084	85	+	134	65	×	184	43	RCL
035	03	3	085	43	RCL	135	43	RCL	185	26	26
036	07	7	086	03	03	136	19	19	186	65	×
037	02	2	087	22	INV	137	54	)	187	43	RCL
038	04	4	088	49	PRD	138	45	Y^X	188	29	29
039	69	OP	089	13	13	139	43	RCL	189	65	×
040	04	04	090	97	DSZ	140	18	18	190	43	RCL
041	43	RCL	091	03	03	141	95	=	191	02	02
042	19	19	092	43	RCL	142	48	EXC	192	95	=
043	69	OP	093	76	LBL	143	26	26	193	69	OP
044	06	06	094	48	EXC	144	42	STO	194	06	06
045	04	4	095	01	1	145	10	10	195	98	ADV
046	03	3	096	95	=	146	43	RCL	196	92	RTN
047	03	3	097	85	+	147	21	21	197	76	LBL
048	07	7	098	53	(	148	55	÷	198	14	D
049	69	OP	099	43	RCL	149	43	RCL	199	43	RCL

... Continue Table III.1

200	28	28	250	04	4	300	42	STO	350	03	3
201	65	×	251	95	=	301	19	19	351	05	5
202	43	RCL	252	59	INT	302	22	INV	352	01	1
203	06	06	253	85	+	303	77	GE	353	69	OP
204	42	STO	254	01	1	304	97	DSZ	354	02	02
205	21	21	255	95	=	305	69	OP	355	69	OP
206	55	÷	256	42	STO	306	25	25	356	05	05
207	43	RCL	257	04	04	307	61	GTO	357	43	RCL
208	04	04	258	76	LBL	308	99	PRT	358	05	05
209	42	STO	259	90	LST	309	76	LBL	359	99	PRT
210	18	18	260	09	9	310	97	DSZ	360	98	ADV
211	95	=	261	94	+/-	311	98	ADV	361	98	ADV
212	42	STO	262	42	STO	312	03	3	362	01	1
213	08	08	263	14	14	313	06	6	363	32	X:T
214	42	STO	264	43	RCL	314	00	0	364	14	D
215	19	19	265	28	28	315	02	2	365	77	GE
216	92	RTN	266	65	×	316	69	OP	366	80	GRD
217	76	LBL	267	43	RCL	317	04	04	367	02	2
218	19	D'	268	07	07	318	43	RCL	368	18	C'
219	43	RCL	269	55	÷	319	06	06	369	10	E'
220	28	28	270	93	.	320	69	OP	370	42	STO
221	65	×	271	07	7	321	06	06	371	22	22
222	43	RCL	272	95	=	322	03	3	372	69	OP
223	07	07	273	59	INT	323	06	6	373	24	24
224	42	STO	274	85	+	324	00	0	374	14	D
225	21	21	275	01	1	325	03	3	375	76	LBL
226	55	÷	276	95	=	326	69	OP	376	70	RAD
227	43	RCL	277	42	STO	327	04	04	377	02	2
228	05	05	278	05	05	328	43	RCL	378	18	C'
229	42	STO	279	42	STO	329	07	07	379	10	E'
230	18	18	280	15	15	330	69	OP	380	42	STO
231	95	=	281	42	STO	331	06	06	381	23	23
232	42	STO	282	18	18	332	03	3	382	94	+/-
233	09	09	283	76	LBL	333	18	C'	383	85	+
234	42	STO	284	99	PRT	334	43	RCL	384	43	RCL
235	19	19	285	01	1	335	26	26	385	11	11
236	92	RTN	286	32	X:T	336	42	STO	386	32	X:T
237	76	LBL	287	43	RCL	337	29	29	387	43	RCL
238	13	C	288	28	28	338	43	RCL	388	22	22
239	43	RCL	289	65	×	339	25	25	389	95	=
240	27	27	290	43	RCL	340	42	STO	390	77	GE
241	42	STO	291	07	07	341	24	24	391	60	DEG
242	01	01	292	42	STO	342	76	LBL	392	69	OP
243	43	RCL	293	21	21	343	17	B'	393	34	34
244	28	28	294	55	÷	344	69	OP	394	43	RCL
245	65	×	295	43	RCL	345	00	00	395	10	10
246	43	RCL	296	05	05	346	98	ADV	396	42	STO
247	06	06	297	95	=	347	03	3	397	26	26
248	55	÷	298	42	STO	348	01	1	398	43	RCL
249	93	.	299	09	09	349	00	0	399	20	20

... Continue Table III.1

400	42	STO	450	29	29	500	43	RCL	550	32	X:T
401	25	25	451	43	RCL	501	22	22	551	43	RCL
402	76	LBL	452	20	20	502	95	=	552	15	15
403	50	I×I	453	42	STO	503	77	GE	553	67	EQ
404	43	RCL	454	24	24	504	23	LNX	554	30	TAN
405	04	04	455	76	LBL	505	69	OP	555	43	RCL
406	32	X:T	456	16	A'	506	35	35	556	05	05
407	01	1	457	69	OP	507	43	RCL	557	42	STO
408	67	EQ	458	00	00	508	10	10	558	15	15
409	57	ENG	459	03	3	509	42	STO	559	19	D'
410	69	OP	460	01	1	510	26	26	560	43	RCL
411	34	34	461	00	0	511	43	RCL	561	10	10
412	01	1	462	02	2	512	20	20	562	42	STO
413	32	X:T	463	05	5	513	42	STO	563	29	29
414	14	D	464	01	1	514	25	25	564	43	RCL
415	77	GE	465	69	OP	515	76	LBL	565	20	20
416	65	×	466	02	02	516	59	INT	566	42	STO
417	02	2	467	98	ADV	517	43	RCL	567	24	24
418	18	C'	468	69	OP	518	05	05	568	61	GTO
419	10	E'	469	05	05	519	32	X:T	569	17	B'
420	75	-	470	43	RCL	520	01	1	570	76	LBL
421	48	EXC	471	04	04	521	67	EQ	571	30	TAN
422	22	22	472	99	PRT	522	49	PRD	572	98	ADV
423	95	=	473	98	ADV	523	69	OP	573	69	OP
424	32	X:T	474	98	ADV	524	35	35	574	00	00
425	43	RCL	475	01	1	525	01	1	575	03	3
426	11	11	476	32	X:T	526	32	X:T	576	02	2
427	77	GE	477	19	D'	527	19	D'	577	03	3
428	50	I×I	478	77	GE	528	77	GE	578	03	3
429	76	LBL	479	89	π	529	34	ΓX	579	03	3
430	65	×	480	03	3	530	03	3	580	07	7
431	69	OP	481	18	C'	531	18	C'	581	69	OP
432	24	24	482	10	E'	532	10	E'	582	02	02
433	76	LBL	483	42	STO	533	75	-	583	69	OP
434	57	ENG	484	22	22	534	48	EXC	584	05	05
435	43	RCL	485	69	OP	535	22	22	585	03	3
436	04	04	486	25	25	536	95	=	586	01	1
437	32	X:T	487	19	D'	537	32	X:T	587	00	0
438	43	RCL	488	76	LBL	538	43	RCL	588	02	2
439	14	14	489	79	x̄	539	12	12	589	69	OP
440	67	EQ	490	03	3	540	77	GE	590	04	04
441	30	TAN	491	18	C'	541	59	INT	591	43	RCL
442	43	RCL	492	10	E'	542	76	LBL	592	04	04
443	04	04	493	42	STO	543	34	ΓX	593	69	OP
444	42	STO	494	23	23	544	69	OP	594	06	06
445	14	14	495	94	+/-	545	25	25	595	03	3
446	14	D	496	85	+	546	76	LBL	596	01	1
447	43	RCL	497	43	RCL	547	49	PRD	597	00	0
448	10	10	498	12	12	548	43	RCL	598	03	3
449	42	STO	499	32	X:T	549	05	05	599	69	OP

... Continue Table III.1

600	04	04
601	43	RCL
602	05	05
603	69	OP
604	06	06
605	43	RCL
606	17	17
607	44	SUM
608	07	07
609	97	DSZ
610	01	01
611	90	LST
612	98	ADV
613	43	RCL
614	16	16
615	44	SUM
616	06	06
617	43	RCL
618	17	17
619	65	×
620	43	RCL
621	27	27
622	95	=
623	22	INV
624	44	SUM
625	07	07
626	97	DSZ
627	00	00
628	13	C
629	91	R/S
630	76	LBL
631	60	DEG
632	69	OP
633	24	24
634	14	D
635	02	2
636	18	C'
637	10	E'
638	75	-
639	48	EXC
640	23	23
641	95	=
642	94	+/-
643	32	X:T
644	43	RCL
645	11	11
646	32	X:T
647	77	GE
648	60	DEG
649	69	OP
650	34	34
651	61	GTO
652	57	ENG
653	76	LBL
654	23	LNX
655	69	OP
656	25	25
657	19	D'
658	03	3
659	18	C'
660	10	E'
661	75	-
662	48	EXC
663	23	23
664	95	=
665	94	+/-
666	32	X:T
667	43	RCL
668	12	12
669	32	X:T
670	77	GE
671	23	LNX
672	69	OP
673	35	35
674	61	GTO
675	49	PRD
676	76	LBL
677	80	GRD
678	09	9
679	22	INV
680	28	LOG
681	42	STO
682	22	22
683	69	OP
684	24	24
685	14	D
686	77	GE
687	80	GRD
688	61	GTO
689	70	RAD
690	76	LBL
691	89	π
692	09	9
693	22	INV
694	28	LOG
695	42	STO
696	22	22
697	69	OP
698	25	25
699	19	D'
700	77	GE
701	89	π
702	61	GTO
703	79	$\bar{x}$

User Instructions

Step	Procedure	Enter	Press		Print
1.	Load program				
	a. Set partitioning	3	2nd	Op	
			17		
2.	Enter data				
	a. The arrival rate λ	λ	STO	28	
	b. Berth-ship cost ratio c_1/V	c_1/V	STO	11	
	c. Storage-ship cost ratio c_2/V	c_2/V	STO	12	
3.	Set limits of parametric analysis				
	a. Specify s_1 initial	s_1 initial	STO	06	
	b. Desired increment Δs_1	Δs_1 1/	STO	16	
	c. Specify s_2 initial	s_2 initial	STO	07	
	d. Desired increment Δs_2	Δs_2 1/	STO	17	
	e. Number of values, K, of s_1 desired	K 2/	STO	00	
	f. Number of values, L, of s_2 desired	L 2/	STO	27	
4.	Compute and print results (for given s_1 and s_2 varying n_1 and n_2)			C	
	a. Print parameters values				s_1
					s_2
					n_2
					ρ_2
	1/ Default value = 0				n_2^*
	2/ Default value = 1				

Step	Procedure	Enter	Press		Print
	b. For present n_2^*				
	Print (until $\Delta W_q \leq c_1/V$)				n_1
					1
					W_q
	c. When $\Delta W_q \leq c_1/V$				
	print				n_1^*
	If $n_1^* = n_1$ old and				
	$n_2^* = n_2$ old				
	print optimals n_1 and n_2				n_1^* n_2^*
	d. For present n_1^* print				
	(until $\Delta W_q \leq c_2/V$)				n_2
					2
					W_q
	e. When $\Delta W_q \leq c_2/V$				n_2^*
	print				
	If $n_2^* = n_2$ old and n_1^*				
	n_1 old				
	print optimals n_1 and n_2				n_1^* n_2^*
	f. Set $s_2 = s_2 + \Delta s_2$				
	$j = i + 1$				
	If $j \leq L$ go to a				
	g. Set $s_1 = s_1 + \Delta s_1$				
	$i = i + 1$				
	If $i \leq K$ go to a				

<u>Registers Used</u>

00	Counter on s_1(K)	15	Last n_2
00	Counter on s_2(L)	16	Δs_1
02	Used in n!	17	Δs_2
03	Counter	18	n
04	n_1	19	ρ
05	n_2	20	q_1 or q_2
06	s_1	21	s
07	s_2	22	W_q (old)
08	ρ_1	23	W_q (new)
09	ρ_2	24	q_1 or q_2
10	p_1 or p_2	25	q_1 or q_2
11	c_1/V	26	p_1 or p_2
12	c_2/V	27	L
13	(n-1)!	28	λ
14	Last n_1	29	p_1 or p_2

Appendix IV

IV.1 Berth Occupancy Charge: Two Stage Queuing Model

Purpose: This program searches the principal factor

$$\Delta x = x^* - x$$

to establish an optimum berth occupancy charge, using the following expression:

$$\text{optimum berth occupancy charge} = \frac{c_1}{\Delta x}$$

where

c_1 = incremental cost of a service station (berth) in the first stage

Δx as above

The program carries out the optimization finding the level of service x^* for which the average queuing time is the same before and after the addition of a service station (berth) allowing the berth utilization factor (ρ_1) to vary as required by the average cost of a marginal plant method, used as a proxy for the social marginal cost minus the private marginal cost, which is found when

$$\frac{W_q\,(n_1+1\,,\,n_2\,,\,x^*\,,\,y)}{x^*} - \frac{W_q\,(n_1\,,\,n_2\,,\,x\,,\,y)}{x} = 0$$

The root of this function x^* is obtained using the half interval which is decribed in Appendix IV section Iv.3

Table IV.1 Program Berth Occupancy Charge-Two Stage Queuing Model

000	76	LBL	050	43	RCL	100	65	×	150	43	RCL
001	18	C'	051	43	RCL	101	43	RCL	151	10	10
002	43	RCL	052	39	39	102	39	39	152	65	×
003	18	18	053	45	Y^x	103	45	Y^x	153	43	RCL
004	42	STO	054	43	RCL	104	43	RCL	154	20	20
005	02	02	055	03	03	105	18	18	155	75	-
006	29	CP	056	55	÷	106	95	=	156	43	RCL
007	67	EQ	057	43	RCL	107	48	EXC	157	26	26
008	85	+	058	13	13	108	26	26	158	65	×
009	76	LBL	059	85	+	109	42	STO	159	43	RCL
010	75	-	060	43	RCL	110	10	10	160	29	29
011	43	RCL	061	03	03	111	43	RCL	161	65	×
012	02	02	062	22	INV	112	21	21	162	43	RCL
013	65	×	063	49	PRD	113	55	÷	163	02	02
014	97	DSZ	064	13	13	114	43	RCL	164	95	=
015	02	02	065	97	DSZ	115	18	18	165	92	RTN
016	75	-	066	03	03	116	55	÷	166	76	LBL
017	76	LBL	067	43	RCL	117	53	(	167	17	B'
018	85	+	068	76	LBL	118	01	1	168	19	D'
019	01	1	069	48	EXC	119	75	-	169	18	C'
020	95	=	070	01	1	120	43	RCL	170	43	RCL
021	42	STO	071	95	=	121	19	19	171	26	26
022	02	02	072	85	+	122	54	)	172	42	STO
023	43	RCL	073	43	RCL	123	95	=	173	29	29
024	18	18	074	39	39	124	48	EXC	174	43	RCL
025	65	×	075	45	Y^x	125	25	25	175	25	25
026	43	RCL	076	43	RCL	126	42	STO	176	42	STO
027	19	19	077	18	18	127	20	20	177	24	24
028	95	=	078	55	÷	128	92	RTN	178	43	RCL
029	42	STO	079	43	RCL	129	76	LBL	179	06	06
030	39	39	080	02	02	130	10	E'	180	65	×
031	43	RCL	081	55	÷	131	43	RCL	181	43	RCL
032	18	18	082	53	(	132	25	25	182	28	28
033	75	-	083	01	1	133	42	STO	183	95	=
034	01	1	084	75	-	134	02	02	184	42	STO
035	95	=	085	43	RCL	135	32	X⇌T	185	37	37
036	42	STO	086	19	19	136	43	RCL	186	43	RCL
037	03	03	087	95	=	137	24	24	187	28	28
038	29	CP	088	65	×	138	77	GE	188	65	×
039	67	EQ	089	43	RCL	139	87	IFF	189	43	RCL
040	48	EXC	090	02	02	140	42	STO	190	07	07
041	43	RCL	091	65	×	141	02	02	191	42	STO
042	02	02	092	53	(	142	76	LBL	192	21	21
043	55	÷	093	01	1	143	87	IFF	193	55	÷
044	43	RCL	094	75	-	144	43	RCL	194	43	RCL
045	18	18	095	43	RCL	145	26	26	195	05	05
046	95	=	096	19	19	146	65	×	196	42	STO
047	42	STO	097	54	)	147	43	RCL	197	18	18
048	13	13	098	95	=	148	25	25	198	95	=
049	76	LBL	099	35	1/X	149	85	+	199	42	STO

... Continue Table IV.1

200	09	09	250	06	06	300	43	RCL	350	06	06
201	42	STO	251	03	3	301	28	28	351	54	)
202	19	19	252	06	6	302	95	=	352	42	STO
203	18	C'	253	00	0	303	92	RTN	353	35	35
204	10	E'	254	02	2	304	76	LBL	354	94	+/-
205	55	÷	255	69	OP	305	16	A'	355	85	+
206	43	RCL	256	04	04	306	53	(	356	43	RCL
207	37	37	257	43	RCL	307	42	STO	357	16	16
208	95	=	258	06	06	308	06	06	358	54	)
209	42	STO	259	69	OP	309	43	RCL	359	92	RTN
210	16	16	260	06	06	310	30	30	360	91	R/S
211	98	ADV	261	98	ADV	311	42	STO	361	76	LBL
212	06	6	262	04	4	312	04	04	362	11	A
213	03	3	263	04	4	313	19	D'	363	13	C
214	69	OP	264	69	OP	314	18	C'	364	42	STO
215	04	04	265	04	04	315	43	RCL	365	01	01
216	43	RCL	266	43	RCL	316	26	26	366	92	RTN
217	28	28	267	37	37	317	42	STO	367	76	LBL
218	69	OP	268	69	OP	318	29	29	368	12	B
219	06	06	269	06	06	319	43	RCL	369	13	C
220	03	3	270	69	OP	320	25	25	370	53	(
221	01	1	271	00	00	321	42	STO	371	42	STO
222	00	0	272	98	ADV	322	24	24	372	11	11
223	03	3	273	91	R/S	323	43	RCL	373	75	-
224	69	OP	274	76	LBL	324	10	10	374	32	X:T
225	04	04	275	19	D'	325	42	STO	375	43	RCL
226	43	RCL	276	43	RCL	326	26	26	376	01	01
227	05	05	277	28	28	327	43	RCL	377	54	)
228	69	OP	278	65	×	328	20	20	378	42	STO
229	06	06	279	43	RCL	329	42	STO	379	12	12
230	03	3	280	06	06	330	25	25	380	93	.
231	06	6	281	42	STO	331	43	RCL	381	00	0
232	00	0	282	21	21	332	29	29	382	01	1
233	03	3	283	55	÷	333	42	STO	383	42	STO
234	69	OP	284	43	RCL	334	10	10	384	23	23
235	04	04	285	04	04	335	43	RCL	385	32	X:T
236	43	RCL	286	42	STO	336	24	24	386	92	RTN
237	07	07	287	18	18	337	42	STO	387	76	LBL
238	69	OP	288	95	=	338	20	20	388	14	D
239	06	06	289	42	STO	339	10	E'	389	42	STO
240	98	ADV	290	08	08	340	42	STO	390	23	23
241	03	3	291	42	STO	341	33	33	391	92	RTN
242	01	1	292	19	19	342	53	(	392	76	LBL
243	00	0	293	92	RTN	343	43	RCL	393	15	E
244	02	2	294	76	LBL	344	33	33	394	53	(
245	69	OP	295	13	C	345	55	÷	395	24	CE
246	04	04	296	65	×	346	43	RCL	396	43	RCL
247	43	RCL	297	43	RCL	347	28	28	397	11	11
248	04	04	298	30	30	348	55	÷	398	32	X:T
249	69	OP	299	55	÷	349	43	RCL	399	43	RCL

... Continue Table IV.1

400	01	01	450	43	RCL	500	69	OP	550	00	0
401	77	GE	451	15	15	501	02	02	551	02	2
402	50	I×I	452	75	-	502	06	6	552	05	5
403	42	STO	453	43	RCL	503	03	3	553	06	6
404	14	14	454	14	14	504	04	4	554	06	6
405	85	+	455	54	)	505	04	4	555	03	3
406	43	RCL	456	22	INV	506	00	0	556	04	4
407	12	12	457	77	GE	507	00	0	557	04	4
408	54	)	458	70	RAD	508	00	0	558	04	4
409	42	STO	459	53	(	509	00	0	559	07	7
410	01	01	460	43	RCL	510	00	0	560	69	OP
411	42	STO	461	17	17	511	00	0	561	03	03
412	15	15	462	16	A'	512	69	OP	562	07	7
413	16	A'	463	65	×	513	03	03	563	05	5
414	42	STO	464	43	RCL	514	69	OP	564	04	4
415	22	22	465	22	22	515	05	05	565	04	4
416	53	(	466	54	)	516	43	RCL	566	00	0
417	43	RCL	467	29	CP	517	16	16	567	00	0
418	14	14	468	67	EQ	518	99	PRT	568	00	0
419	42	STO	469	70	RAD	519	98	ADV	569	00	0
420	17	17	470	77	GE	520	04	4	570	00	0
421	16	A'	471	45	Y^X	521	04	4	571	00	0
422	29	CP	472	43	RCL	522	05	5	572	69	OP
423	67	EQ	473	17	17	523	01	1	573	04	04
424	70	RAD	474	42	STO	524	69	OP	574	69	OP
425	65	×	475	15	15	525	04	04	575	05	05
426	48	EXC	476	61	GTO	526	43	RCL	576	43	RCL
427	22	22	477	80	GRD	527	17	17	577	35	35
428	54	)	478	76	LBL	528	65	×	578	99	PRT
429	77	GE	479	45	Y^X	529	43	RCL	579	98	ADV
430	15	E	480	43	RCL	530	28	28	580	43	RCL
431	76	LBL	481	17	17	531	95	=	581	32	32
432	80	GRD	482	42	STO	532	42	STO	582	75	-
433	53	(	483	14	14	533	32	32	583	43	RCL
434	53	(	484	61	GTO	534	69	OP	584	37	37
435	43	RCL	485	80	GRD	535	06	06	585	95	=
436	14	14	486	76	LBL	536	69	OP	586	42	STO
437	85	+	487	70	RAD	537	00	00	587	31	31
438	43	RCL	488	69	OP	538	04	4	588	07	7
439	15	15	489	00	00	539	03	3	589	05	5
440	54	)	490	04	4	540	03	3	590	04	4
441	55	÷	491	03	3	541	07	7	591	04	4
442	02	2	492	03	3	542	05	5	592	69	OP
443	54	)	493	07	7	543	05	5	593	04	04
444	42	STO	494	05	5	544	03	3	594	43	RCL
445	17	17	495	05	5	545	01	1	595	31	31
446	43	RCL	496	03	3	546	04	4	596	69	OP
447	23	23	497	01	1	547	07	7	597	06	06
448	32	X:T	498	05	5	548	69	OP	598	92	RTN
449	53	(	499	06	6	549	02	02	599	76	LBL

... Continue Table IV.1

600	50	I×I
601	69	OP
602	00	00
603	03	3
604	01	1
605	03	3
606	02	2
607	00	0
608	00	0
609	03	3
610	05	5
611	03	3
612	02	2
613	69	OP
614	02	02
615	03	3
616	02	2
617	03	3
618	07	7
619	00	0
620	00	0
621	00	0
622	00	0
623	00	0
624	00	0
625	69	OP
626	03	03
627	69	OP
628	05	05
629	92	RTN

User Instructions

Step	Procedure	Enter	Press		Print
1.	Load program				
	a. Set partitioning	4	2nd 17	Op	
	b. Clear printer registers		2nd 00	Op	
2.	Enter data				
	a. The arrival rate, λ	λ	STO	28	
	b. Number of berths, n_1	n_1	STO	04	
	c. Number of storage spaces, n_2	n_2	STO	05	
	d. Average service time, s_1 (first stage)	s_1	STO	06	
	e. Average service time, s_2 (second stage	s_2	STO	07	
	f. Number of berths after expansion, n + 1	n + 1	STO	30	
3.	Compute $W_q(n_1)/X$		2nd	B'	
	Print				
	a. The arrival rate, λ				λ
	b. Number of storage spaces, n_2				n_2
	c. Average service time, s_2 (second stage)				s_2
	d. Number of berths, n_1				n_1
	e. Average service time, s_1 (first stage)				s_1
	f. Service time required, x, before expansion (first stage)				x

Step	Procedure	Enter	Press	Print
4.	Set limit of parametric analysis			
	a. Lower bound, $\rho_{1L} = \frac{s_1 \lambda}{n_1+1}$	ρ_{1L}	A	
	b. Upper bound, $\rho_{1U} = .9999$	ρ_{1U}	B	
	c. Accuracy desired, ξ	ξ	D	
5.	Compute, x^* 1/		E	
	Print			
	a. The average queuing time before expansion			$W_q(n_1)/X$
	b. The level of service x^* after expansion (that satafies condition of equal queuing time)			x^*
	c. The average queuing time after expansion			$\frac{W_q(n+1)}{x^*}$
	d. The factor Δx			Δx
6.	To change parameter(s) value(s) repeat appropriated parts od step 2. Then repet step 3 to 5.			
7.	To change analysis limit repeat appropriated step 4.			

1/ If there is no root in the interval defined, the following message will be printed: "No Root". Try another interval.

Registers Used

00	Used HIM	20	q_1 , q_2
01	Used HIM	21	s
02	Used in n!; min(q_1 , q_2)	22	Used in HIM
03	Counter	23	Used in HIM
04	n_1	24	q_1 , q_2
05	n_2	25	q_1 , q_2
06	s_1	26	p_1 , p_2
07	s_2	27	not used
08	ρ_1	28	x
09	ρ_2	29	p_1 , p_2
10	p_1, p_2	30	$n_1 = n_1 + 1$
11	Used in HIM	31	Used in HIM
12	Used in HIM	32	x^*
13	n!	33	Used in HIM
14	Used in HIM	34	Used in HIM
15	Used in HIM	35	Used in HIM
16	$W_q(n_1)$ 1x	36	Not used
17	Used in HIM	37	x
18	n	38	Not used
19	ρ	39	$n\rho$

Note: HIM = Half Interval Method

IV.2 Storage Occupancy Charge: Two Stage Queuing Model

Purpose: This program searches for the principal factor:

$$\Delta y = y^* - Y$$

to establish optimum storage occupancy charge, using the following expression:

$$\text{Optimum storage occupancy charge} = \frac{c_2}{\Delta y}$$

where

c_2 = incremental cost of a service station (storage space) in the second stage

Δy as before

The program carries cut the optimization finding the level of services y^* for which the average queuing time is the same before and after the addition of a service station (storage space), allowing the storage utilization factor (ρ_2) to vary as required by "the average cost of a marginal plant" method, used as a proxy for the social marginal cost, which is found when

$$W_q(n_1, n_2 + 1, x, y^*) - W_q(n_1, n_2, x, y) = 0$$

The root of this function y^* is obtained using "the half interval" method.

Table IV.2 Program Storage Occupancy Charge-Two Stage Queuing Model

000	76	LBL	050	43	RCL	100	65	×	150	65	×
001	18	C'	051	43	RCL	101	43	RCL	151	43	RCL
002	43	RCL	052	39	39	102	39	39	152	20	20
003	18	18	053	45	Y^x	103	45	Y^x	153	75	-
004	42	STO	054	43	RCL	104	43	RCL	154	43	RCL
005	02	02	055	03	03	105	18	18	155	26	26
006	29	CP	056	55	÷	106	95	=	156	65	×
007	67	EQ	057	43	RCL	107	48	EXC	157	43	RCL
008	85	+	058	13	13	108	10	10	158	29	29
009	76	LBL	059	85	+	109	42	STO	159	65	×
010	75	-	060	43	RCL	110	26	26	160	43	RCL
011	43	RCL	061	03	03	111	43	RCL	161	02	02
012	02	02	062	22	INV	112	21	21	162	95	=
013	65	×	063	49	PRD	113	55	÷	163	92	RTN
014	97	DSZ	064	13	13	114	43	RCL	164	76	LBL
015	02	02	065	97	DSZ	115	18	18	165	17	B'
016	75	-	066	03	03	116	55	÷	166	19	D'
017	76	LBL	067	43	RCL	117	53	(	167	18	C'
018	85	+	068	76	LBL	118	01	1	168	43	RCL
019	01	1	069	48	EXC	119	75	-	169	28	28
020	95	=	070	01	1	120	43	RCL	170	65	×
021	42	STO	071	95	=	121	19	19	171	43	RCL
022	02	02	072	85	+	122	95	=	172	06	06
023	43	RCL	073	43	RCL	123	48	EXC	173	42	STO
024	19	19	074	39	39	124	20	20	174	21	21
025	65	×	075	45	Y^x	125	42	STO	175	55	÷
026	43	RCL	076	43	RCL	126	25	25	176	43	RCL
027	18	18	077	18	18	127	76	LBL	177	04	04
028	95	=	078	55	÷	128	10	E'	178	42	STO
029	42	STO	079	43	RCL	129	43	RCL	179	18	18
030	39	39	080	02	02	130	25	25	180	95	=
031	43	RCL	081	55	÷	131	42	STO	181	42	STO
032	18	18	082	53	(	132	02	02	182	08	08
033	75	-	083	01	1	133	32	X:T	183	42	STO
034	01	1	084	75	-	134	43	RCL	184	19	19
035	95	=	085	43	RCL	135	24	24	185	18	C'
036	42	STO	086	19	19	136	77	GE	186	43	RCL
037	03	03	087	95	=	137	87	IFF	187	10	10
038	29	CP	088	65	×	138	42	STO	188	42	STO
039	67	EQ	089	43	RCL	139	02	02	189	29	29
040	48	EXC	090	02	02	140	76	LBL	190	43	RCL
041	43	RCL	091	65	×	141	87	IFF	191	20	20
042	02	02	092	53	(	142	43	RCL	192	42	STO
043	55	÷	093	01	1	143	26	26	193	24	24
044	43	RCL	094	75	-	144	65	×	194	10	E'
045	18	18	095	43	RCL	145	43	RCL	195	42	STO
046	95	=	096	19	19	146	25	25	196	16	16
047	42	STO	097	54	)	147	85	+	197	98	ADV
048	13	13	098	95	=	148	43	RCL	198	06	6
049	76	LBL	099	35	1/X	149	10	10	199	03	3

... Continue Table IV.2

200	69	OP	250	69	OP	300	30	30	350	11	11
201	04	04	251	04	04	301	42	STO	351	75	-
202	43	RCL	252	43	RCL	302	05	05	352	32	X:T
203	28	28	253	07	07	303	19	D'	353	43	RCL
204	69	OP	254	65	×	304	18	C'	354	01	01
205	06	06	255	43	RCL	305	43	RCL	355	54	)
206	03	3	256	28	28	306	26	26	356	42	STO
207	01	1	257	95	=	307	42	STO	357	12	12
208	00	0	258	42	STO	308	29	29	358	93	.
209	02	2	259	37	37	309	43	RCL	359	00	0
210	69	OP	260	69	OP	310	10	10	360	01	1
211	04	04	261	06	06	311	42	STO	361	42	STO
212	43	RCL	262	98	ADV	312	26	26	362	23	23
213	04	04	263	91	R/S	313	43	RCL	363	32	X:T
214	69	OP	264	76	LBL	314	29	29	364	92	RTN
215	06	06	265	19	D'	315	42	STO	365	76	LBL
216	03	3	266	43	RCL	316	10	10	366	14	D
217	06	6	267	28	28	317	43	RCL	367	42	STO
218	00	0	268	65	×	318	25	25	368	23	23
219	02	2	269	43	RCL	319	42	STO	369	92	RTN
220	69	OP	270	07	07	320	24	24	370	76	LBL
221	04	04	271	42	STO	321	43	RCL	371	15	E
222	43	RCL	272	21	21	322	20	20	372	53	(
223	06	06	273	55	÷	323	42	STO	373	24	CE
224	69	OP	274	43	RCL	324	25	25	374	43	RCL
225	06	06	275	05	05	325	43	RCL	375	11	11
226	98	ADV	276	42	STO	326	24	24	376	32	X:T
227	03	3	277	18	18	327	42	STO	377	43	RCL
228	01	1	278	95	=	328	20	20	378	01	01
229	00	0	279	42	STO	329	10	E'	379	77	GE
230	03	3	280	09	09	330	42	STO	380	50	I×I
231	69	OP	281	42	STO	331	35	35	381	42	STO
232	04	04	282	19	19	332	94	+/-	382	14	14
233	43	RCL	283	92	RTN	333	85	+	383	85	+
234	05	05	284	76	LBL	334	43	RCL	384	43	RCL
235	69	OP	285	13	C	335	16	16	385	12	12
236	06	06	286	65	×	336	54	)	386	54	)
237	03	3	287	43	RCL	337	92	RTN	387	42	STO
238	06	6	288	30	30	338	91	R/S	388	01	01
239	00	0	289	55	÷	339	76	LBL	389	42	STO
240	03	3	290	43	RCL	340	11	A	390	15	15
241	69	OP	291	28	28	341	13	C	391	16	A'
242	04	04	292	95	=	342	42	STO	392	42	STO
243	43	RCL	293	92	RTN	343	01	01	393	22	22
244	07	07	294	76	LBL	344	92	RTN	394	53	(
245	69	OP	295	16	A'	345	76	LBL	395	43	RCL
246	06	06	296	53	(	346	12	B	396	14	14
247	98	ADV	297	42	STO	347	13	C	397	42	STO
248	04	4	298	07	07	348	53	(	398	17	17
249	05	5	299	43	RCL	349	42	STO	399	16	A'

... Continue Table IV.2

400	29	CP
401	67	EQ
402	70	RAD
403	65	×
404	48	EXC
405	22	22
406	54	)
407	77	GE
408	15	E
409	76	LBL
410	80	GRD
411	53	(
412	53	(
413	43	RCL
414	14	14
415	85	+
416	43	RCL
417	15	15
418	54	)
419	55	÷
420	02	2
421	54	)
422	42	STO
423	17	17
424	43	RCL
425	23	23
426	32	X:T
427	53	(
428	43	RCL
429	15	15
430	75	-
431	43	RCL
432	14	14
433	54	)
434	22	INV
435	77	GE
436	70	RAD
437	53	(
438	43	RCL
439	17	17
440	16	A'
441	65	×
442	43	RCL
443	22	22
444	54	)
445	29	CP
446	67	EQ
447	70	RAD
448	77	GE
449	45	Y×
450	43	RCL
451	17	17
452	42	STO
453	15	15
454	61	GTO
455	80	GRD
456	76	LBL
457	45	Y×
458	43	RCL
459	17	17
460	42	STO
461	14	14
462	61	GTO
463	80	GRD
464	76	LBL
465	70	RAD
466	69	OP
467	00	00
468	04	4
469	03	3
470	03	3
471	07	7
472	05	5
473	05	5
474	03	3
475	01	1
476	05	5
477	06	6
478	69	OP
479	02	02
480	69	OP
481	05	05
482	43	RCL
483	16	16
484	99	PRT
485	98	ADV
486	04	4
487	05	5
488	05	5
489	01	1
490	69	OP
491	04	04
492	43	RCL
493	17	17
494	65	×
495	43	RCL
496	28	28
497	95	=
498	42	STO
499	32	32
500	69	OP
501	06	06
502	98	ADV
503	69	OP
504	00	00
505	04	4
506	03	3
507	03	3
508	07	7
509	05	5
510	05	5
511	03	3
512	01	1
513	04	4
514	07	7
515	69	OP
516	02	02
517	00	0
518	02	2
519	05	5
520	06	6
521	00	0
522	00	0
523	00	0
524	00	0
525	00	0
526	00	0
527	69	OP
528	03	03
529	69	OP
530	05	05
531	43	RCL
532	35	35
533	99	PRT
534	98	ADV
535	43	RCL
536	32	32
537	75	-
538	43	RCL
539	37	37
540	95	=
541	42	STO
542	31	31
543	07	7
544	05	5
545	04	4
546	05	5
547	69	OP
548	04	04
549	43	RCL
550	31	31
551	69	OP
552	06	06
553	98	ADV
554	92	RTN
555	76	LBL
556	50	I×I
557	69	OP
558	00	00
559	03	3
560	01	1
561	03	3
562	02	2
563	00	0
564	00	0
565	03	3
566	05	5
567	03	3
568	02	2
569	69	OP
570	02	02
571	03	3
572	02	2
573	03	3
574	07	7
575	00	0
576	00	0
577	00	0
578	00	0
579	00	0
580	00	0
581	69	OP
582	03	03
583	69	OP
584	05	05
585	92	RTN

User Instructions

Step	Procedure	Enter	Press		Print
1.	Load program				
	a. Set partitioning	4	2nd 17	Op	
	b. Clear printer registers		2nd 00	Op	
2.	Enter data				
	a. The arrival rate, λ	λ	STO	28	
	b. Number of berths, n_1	n_1	STO	04	
	c. Number of storage spaces, n_2	n_2	STO	05	
	d. Average service time s_1 (first stage)	s_1	STO	06	
	e. Average service time s_2 (second stage)	s_2	STO	07	
3.	Compute $W_q(n_2)$				
	Print				
	a. The arrival rate, λ				λ
	b. Number of berths, n_1				n_1
	c. Average service time, s_1 (first stage)				s_1
	d. Number of storage spaces, n_2				n_2
	e. Average service time, s_2 (second stage)				s_2
	f. Service time required, y, before expansion (second stage)				y
4.	Set limit of parametric analysis				
	a. Lower bound, $\rho_{2L} = \frac{s_2\lambda}{n_2+1}$	ρ_{2L}		A	
	b. Upper bound, $\rho_{2U} = .9999$	ρ_{2U}		B	

Step	Procedure	Enter	Press		Print
	c. Accuracy desired, ε	ε		D	
5.	Compute, y^* 1/			E	
	Print				
	a. The average queuing time before expansion				$W_q(n_2)$
	b. The level of services, y^* after expansion (that satisfy condition of equal queuing time)				y^*
	c. The average queuing time after expansion				$W_q(n_2+1)$
	d. The factor Δy				Δy
6.	To change parameter(s) value(s) repeat appropriated step 2. Then repeat steps 3 to 5.				
7.	To change analysis limit repeat appropriated step 4.				

1/ If there is no root in the interval defined, the following message will be printed: "No Root". Try another interval.

Registers Used

00	Used in HIM	20	q_1 , q_2
01	Used in HIM	21	s
02	Used in n!; min (q_1 , q_2)	22	Used in HIM
03	Counter	23	Used in HIM
04	n_1	24	q_1 , q_2
05	n_2	25	q_1 , q_2
06	s_1	26	p_1 , p_2
07	s_2	27	Not used
08	ρ_1	28	λ
09	ρ_2	29	p_1 , p_2
10	ρ_1 , ρ_2	30	$n_2 = n_2 + 1$
11	Used in HIM	31	Used in HIM
12	Used in HIM	32	y^*
13	n!	33	Used in HIM
14	Used in HIM	34	Used in HIM
15	Used in HIM	35	Used in HIM
16	$W_q(n_2)$	36	Not used
17	Used in HIM	37	y
18	n	38	Not used
19	ρ	39	$n\rho$

Note: HIM = Half Interval Method

IV.3 The "Half Interval Method"

It gives the root(s) of function, if values X_{L1} and X_{U1} are known (see Figure 4), such that $f(X_{L1})$ and $f(X_{U1})$ are opposite in sign. For continous functions, the value of $f((X_{L1} + X_{U1})/2)$, being the value of the function at the halfway point, will be either zero or have the sign of $f(X_{L1})$ or the sign of $f(X_{U1})$. If the value is not zero, a second pair XL_2 and XU_2 can be chosen from the three numbers X_{L1}, X_{U1} and $\frac{X_{L1} + X_{U1}}{2}$ so that $f(X_{L2})$ and $f(X_{U2})$ are opposite in sign while

$$|X_{L2} - X_{U2}| = 1/2\, |X_{L1} - X_{U1}|$$

Continuing in this manner, there is always a point α in the interval $[X_{Li}, X_{Ui}]$ for which $f(\alpha) = 0$; α is uniquely determined by the process even though the interval may contain more than one zero for $f(x)$ (we avoid this problem setting $X_{L1} = X$ (before expansion)). Because each new application of the iterative scheme reduces by half the length of the interval in X known to contain α, this procedure is called the "Half Interval Method."

In order to reduce the computation time, a degree of accuracy, ε, can be defined, such that when $f(\alpha) \leq \varepsilon$, the process stops.

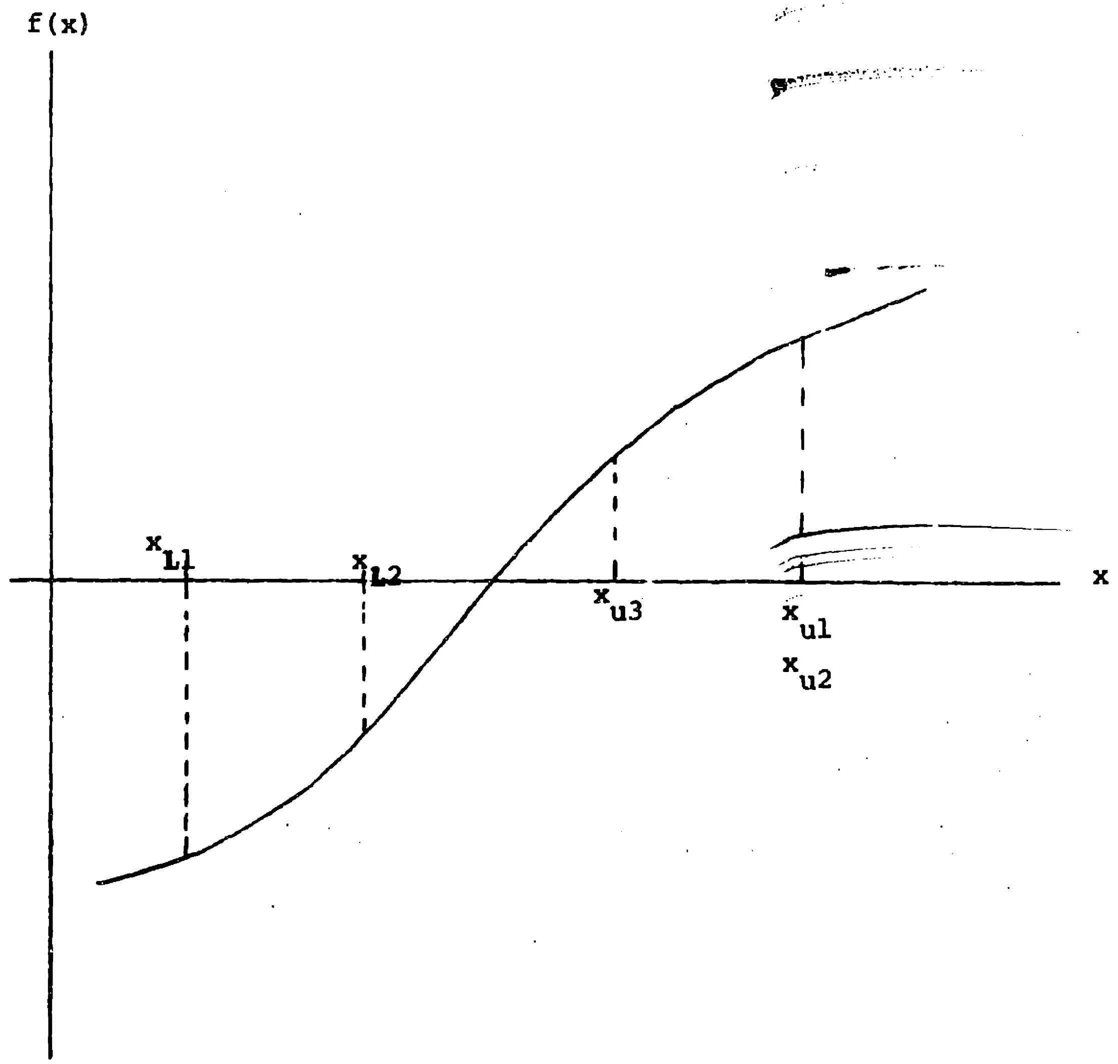

Figure IV.1 Half Interval Method

Source: Applied Numerical Method by Carnahan, Luther and Wilken (John Wiley and Sons)