## by

Yongyut Manichaikul
S.B., Massachusetts Institute of Technology
(1972)
S.M., Massachusetts Institute of Technology
(1973)
E.E., Massachusetts Institute of Technology
(1975)

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To my parents, wife, and daughter
by

Yongyut Manichaikul
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#### Abstract

The development of a new industrial electric load modeling methodology based on detailed physical/economic analyses of individual manufacturing firms is reported. Emphasis is given to the following three basic aspects of a firm and its electric load. They are: (1) stochastic Aspects of load (2) Product flow and storage aspects of a firm, and (3) Economic consideration

The idea of modeling the load of each piece of equipment, using one of a finite number of elementary random process models, is introduced. In particular, a two-state Markov process model in continuous time is extensively used. Its parameters are given in terms of four familiar quantities that have physical meaning -- the installed capacity in kW , the fraction of load used when on, the fraction of time on, and the number of starts per hour.

The product flow and storage aspect deals with the interconnection of material storages and the production processes. The physical structure of a firm and the related flow and storage capacities are important here. Insights can be gained into how firms schedule their production processes. Questions of how many machines belonging to a certain group can be treated as a single block and modeled using an elementary random process model can be answered.

The economic aspect deals with the electric rate structures and the other operating costs of a plant that vary with the time of day (such as the wage differential of a worker). In the analysis, the monthly electric cost is treated and expressed as a well defined nonlinear mathematical function. By comparing the possible reduction in the monthly electric cost due to rescheduling of a production process with the extra cost incurred due to wage differentials, etc., one can develop a very simple but powerful method: comparison of the kW per person of a production process with the breakeven kW per person of the firm. Many issues are analyzed using this type of comparison.


(ABSTRACT, continued)

The modeling methodology is tested out using actual rate structures and data collected from seven industrial customers of the New England Electric System Companies. Based on the installed capacities ( kW ) and how each piece of equipment is being used, one can develop an electric load model of each firm. The time-varying expected load and the autocorrelation function of the residual computed using the model compare favorably with those computed using time-series data.

The model developed based on the industrial load modeling methodology as described here is capable of addressing those "what if" types of questions concerning rate structure design, load management, and many other issues of interest. For many studies, the detailed model and extensive data are not always needed. The level of sophistication and the type of data required are functions of problems and issues considered. Some aspects related to the design of rate structures are considered. Many additional issues are introduced as possible candidates for study, but are not analyzed.

THESIS SUPERVISOR: Fred C. Schweppe
TITLE: Professor of Electrical Engineering

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## CHAPTER I

INTRODUCTION

### 1.1 General Background

The objective of this research is to develop a new industrial load modeling methodology that could be used by utilities, their industrial customers, regulatory agencies, and academicians to study various "what if" types of issues that are of current interest. For instance, each of these groups would like to know the effect of a change in rate structure or new technology on their load or load shape. The conventional time series [ 1,2 ], autoregressive moving average [ 2 ], and econometric [ 3,4 ] models based on aggregate hypothesized mathematical functions fitted to historical data are found to be inadequate in addressing this type of issue. The reason is that the time-series model gives only predictions based on extrapolation of past data; the auto-regressive moving average types of models include only weather effects; and the econometric typesof models use very simple-minded model structure, which is unable to account for many of the actual cause-and-effect types of relationships.

A new cause-and-effect type of model is needed. Detailed physical/ economic analysis of each individual piece of equipment that contributes to the total electric load is one possible approach. However, when following this approach, the amount of equipment that must be studied becomes very large. Therefore, insight must be gained into the underlying processes. For end users in the industrial plants, ways must be found of treating their pieces of equipment as groups. This is the approach that will be taken in this report.

Electric load models are generally developed for a limited geographical area, such as the service area of a utility. The electric load of a service area could be divided into residential, commercial, and industrial sectors, etc. However, such a division is quite arbitrary and difficult to define.

An electric power system consists of interconnected generators, transmission lines, and loads. The level of generation needed, in MW or MWH at a certain time, is determined by the amount of power that is needed, the load. There have been various types of sophisticated models developed for generators and transmission lines. Compared to generator and transmission line models, the study of electric load models has received very little attention in the past. The standard practice has been to use complicated models for generators and transmission 1 ines, and simple-minded models for load, when an electric power system is being studied.

Utility companies have been trying to optimize their long-term operation by careful planning of their new capacities, and their shortterm operation by using unit commitment scheduling, economic dispatch, and automatic control of generation. In other words, utilities have been trying to optimize their generation and transmission systems to satisfy load. Until recently, however, they have not tried to control or manage their load, the part of the power system that is the least understood. Therefore, proper understanding and management of electric load, as well as of generators and transmission systems, is desirable.

It has become more and more important and urgent for utilities to manage
their loads and adopt policies that will lead them in this direction, for the following reasons. The first is that the investment cost for new capacity to satisfy added new demand in the future, and to replace old or outworn equipment, has risen to the point where it has become difficult for utilities to finance what is necessary. There have been many attempts, by various groups of people, to stop new construction of nuclear power plants and hydro projects, so in the not too distant future, many utilities will be faced with insufficient capacity to satisfy the demand. The cost of fuel -- whether nuclear, fossil, or coal, has been rising at a very fast pace. The above reasons indicate that it is becoming more and more important to optimize the operation of electric power systems. Therefore, previously overlooked load management techniques, such as control and change of peak, load shape, etc., are becoming more and more attractive.

Realizing the truth of the above, some utilities have been shifting some effort into managing their load. Regulatory agencies, such as the department of public utilities, are also pushing more utilities to adopt time-of-day rates. President Carter, in his national energy policy plan, has asked more utilities to step up their effort to control and manage load by adopting time-of-day rate, cogeneration, and other helpful policies. However, most of the present load-management policies are implemented by using a trial-and-error method, because very little understanding of load and no causal load models exist today.

In this research, the industrial sector load is singled out for study for the following reasons. The industrial sector is responsible
for about twenty five percent of the utilities' revenue and about one quarter to one third of electric energy consumed [ 5 ]. However, industrial customers differ from each other and are limited in number. The residential sector has been studied by Woodard [ 6 ] who has developed the structure for a cause-and-effect type of model for this sector. Many utilities have gathered extensive data for their residential and commercial sectors. The industrial sector is the least understood, conceptually. For instance, one can try to study a typical residential home belonging to a certain income group in a certain geographical area. A lot of statistical data can be used, as the number of residences of this group are large. But it is difficult to find a typical industry; statistical data for the industrial sector are difficult and dangerous to apply.

A general methodology for the detailed physical/economic industrial load model development is presented in this report. It is based on detailed analysis of the various components of load of a firm, and the factors that affect the loads. The electric load of each machine or piece of equipment is treated as a random process. The pieces of equipment are then grouped together, based on the flow and storage (physical) structure of the particular industrial plant. When the production output and raw material input are given a satisfactory operating schedule, a corresponding electric load shape can be found. When an analysis is made of the labor cost and the electric cost, based on a given rate structure, an optimum schedule can be obtained, hence an optimum load shape for that plant.

For the rest of this chapter, we will discuss various perspectives related to industrial electric load modeling. Section 1.2 reviews some past load models and points out some of their drawbacks. Section 1.3 introduces quantities and issues that are important to the development of industrial electric load modeling methodology. Various quantities are defined and issues are discussed.

Section 1.4 describes the type of data available or that can be found. It is important for a model builder to know the type of data that are available for the determination of his model structure and parameters. Section 1.5 discusses the philosophy of the industrial electric load modeling methodology that is being developed. The basic approach to the industrial electric load modeling being used in this document is introduced in section 1.6. Section 1.7 points out some of the objectives of this research project, and section 1.8 gives a summary of results and contributions of this research.

### 1.2 Review of Past Work

Various types of electric load models exist and are being used by utilities, regulatory agencies and load researchers. There are generally two classes of load models. The first class, load as a function of voltage and frequency, is important for power system stability studies [7, 8, 9]. It is not the subject with which we deal in this document. The second class, load as a function of time and other environmental and economic variables, is used for forecasting load $[2,3,4,6,1 \mathrm{~g}$. In this work, we are dealing with the second class. Numerous papers have been published on the second class of load models; however, only a few selected review papers will be used for the following discussion.

### 1.2.1 Peak Load Models [2, 11]

These types of models, as reviewed by Galiana [2]
represent peak load as follows:

$$
\begin{equation*}
P=B+F(W) \tag{1.2.1}
\end{equation*}
$$

where $P$ is the peak load, $B$ is the based load, $W$ is the weather variable, such as temperature and humidity, and $F$ is a nonlinear function of weather variables. These types of models are used to forecast peak load. However, they are not very useful for answering "what if" types of questions.

### 1.2.2 Time-Series Load Models [1,2,12,13,14]

In this type of model, the load at time $t$ of the day or week, $Z(t)$ is represented by

$$
\begin{equation*}
z(t)=\sum_{i=1}^{m} a_{i} f_{i}(t)+Z_{o}(t) \tag{1.2.2}
\end{equation*}
$$

where $Z_{0}(t)$ may represent the long-term load behavior, e.g., a seasonal or weekly trend: $f_{i}(t)$ are sinusoidal functions with basic periods of 168 hours for weekly models, or 24 hours for daily models. $a_{i}$, the Fourier series coefficient, could be found by basic estimating procedure [ 1,15 ] from the measured time series data. This type of model is good only for forecasting load based on the extrapolation of time series data.

### 1.2.3 Dynamic Models [ $2,16,17,18,19]$

The dynamic models represent load at time $t, Z(t)$,
as

$$
z(t)=Z_{0}(t)+\sum_{i=1}^{\ell} a_{i} f_{i}(t)+y(t)
$$

and

$$
\begin{equation*}
y(t)=\sum_{k=1}^{m} d_{k} y(t-k)+\sum_{j=1}^{m} e_{j} u(t-k)+w(t) \tag{1.2.4}
\end{equation*}
$$

where $a_{i}, f_{i}(t), Z_{o}(t)$ are as described in the time-series model. There are various forms for $y(t)$. However, for our purpose here, $y(t)$ is represented by an autoregressive moving-average type of model. $u(t)$ represents the weather variable, and $w(t)$ the white noise zero mean-random process. This type of model is useful for time-series load forecasting under changing $u(t)$, which is a function of weather. But it is very difficult to use for answering "what if" types of questions not related to weather.

### 1.2.4 Econometric Load Mode1 [3, 4, 20]

A different approach to modeling demand of electricity was taken by econometricians. Simple econometric models as used by Honthakker [ 20] for residential sectors as described by Taylor [4] are as follows:

$$
\begin{equation*}
x=a M+\frac{b}{p}+c g+d h+E-1.25 \tag{1.2.5}
\end{equation*}
$$

or

$$
\begin{equation*}
\ell n x=\alpha \ell n M+\beta \ell n p+\gamma \ell n g+\delta \ell n h+\varepsilon^{\prime} \tag{1.2.6}
\end{equation*}
$$

where

$$
\begin{aligned}
\mathrm{x} & =\text { average annual electricity consumption per customer } \\
M & =\text { average money income per household } \\
\mathrm{g} & =\text { marginal price of gas, and } p=\text { marginal price of electricity } \\
h & =\text { average holdings of heavy domestic equipment per customer } \\
\varepsilon, \varepsilon^{\prime} & =\text { random error terms. }
\end{aligned}
$$

The econometric model described above is a simple dase of an econometric load model. However, it serves the purpose of giving us some idea of what type of model structure econometric types of load models have. Taylor reviewed work done by many other workers. The degree of complexity of these models varies and the details are different, but the basic methods and the general model structures are essentially the same.

In the econometric type of model, the main effort is involved with estimating the coefficients from data for the linear equation (1.2.5) or the log linear equation (1.2.6). The weaknesses of these types of mode1s, as can be seen, are: first, the assumptions that the mode1 equations are linear or $\log$ linear. Second, the demand is referred to the annual average energy consumption. In some cases, the peak demand is considered. The concept of load shape is not used and the actual rate structure for electricity is not used in these types of models.

None of the models described above are sufficient for the many different types of studies needed today. The time series type of model has the weakness of not being causal. The autoregressive moving average type of dynamic model is causal only for studying weather effects or variables that can be identified from past data. The econometric model has the fault of being too simple-minded and too aggregated. It could not be used to study the effect of a change in rate structure into a new type of rate that is drastically different from the present existing rate, because rate structure can be very complicated, compared to the level of complexity that is possible for the econometric method.

There have been attempts by a few utilities to make surveys of their customers and various other types of information. A lot of data from the residential, and some from the commercial, sector have been gathered by utilities. However, a plain statistical survey without a proper understanding of the subject has its dangers. For instance, one utility made a survey study of its large industrial and commercial customers and reported [ 21 ] the result of the survey in which the following question had been asked:
"If a time-of-day rate is chosen, will you be willing to shift load?"

The result of this survey was that many large industrial and commercial customers were outraged; some replied with angry letters. Some commercial building owners, supermarket managers, dentists, doctors, and lawyers, etc. did not like the idea of having to work at night for about a $10 \phi$ saving per hour per person.

It is clear that much of the information obtained is distorted. Such an answer is to be expected if a survey is made: (1) without an industrial model to guide the surveyor; (2) without a rate structure having the exact number and parameters for the rate included in the survey questionnaire.

This research was preceded and motivated by the following research projects done at MIT. Galiana [18,22] considered short-term demand modeling. His is a weather-dependent model which features hour-by-hour load covering up to one week. Although this work did not go into detailed modeling, as described here, the hypothesized structure
for load was based on engineering analysis of the load
Woodard [ 6 ] started with Galiana's results, and built a complete residential and commercial structure based explicitly on a detailed physical analysis of the load. The model structure for various usage classes was developed in detail. For instance, residential air-conditioning demand was modeled by the dynamics of an "equivalent house" with heat-transfer coefficients, thermostatic settings, and weather conditions explicitly modeled.

Peterson [23] addressed general interfuel competition modeling. His work involved the consideration of stochastic models for the decision-making process. A computer progran called GPSIE [24] (General Purpose System Identifier and Evaluator) was developed. GPSIE and related ideas which have been developed are useful to the parameter estimation and model verification aspect of modeling.

### 1.3 Quantities and Issues of Importance

The development of mathematical industrial electric-1oad models is guided by the following: the type of issues one would like to examine; the type of questions one would like to ask; the type of data available for estimating parameters and validating the models; the type of quantitative relationship that can be developed.

### 1.3.1 Time Axis

Two types of time axes are of interest: continuous time, $t$, and discrete time index, $n$.

Define $\mathrm{t}=\mathrm{n} \Delta$
where n is the discrete time index, and
$\Delta$ is the discrete time step interval.
$P(n)=\frac{1}{\Delta} \int_{\left(n-\frac{1}{2}\right) \Delta}^{\left(n+\frac{1}{2}\right) \Delta} P(\mathrm{t}) d t \quad$.
Electric power demand $P(t)$ occurs in continuous time.
For the purpose of computing the monthly electric charge of an industrial customer, a time interval of 15 minutes is used by New England Electric System and $\mathrm{P}(\mathrm{n})$ for the entire plant represents the 15 minute average power that is needed between $t=\left(n-\frac{1}{2}\right) \Delta$ and $t=\left(n+\frac{1}{2}\right) \Delta$.

### 1.3.2 Load Shape

The load shape for the whole service area is used by utility personnel for unit commitment scheduling and capacity planning. Load shape consists of knowing $P(n)$ hourly or every 15 minutes for the next 24 hours or the next week for the entire service area. The total demand or load shape of the utility of a service area is the sum of the loads of each of the sectors involved.

### 1.3.3 Service Area

Service area is a term used to limit the geographical boundary of the model to be developed. It could range from a portion of a utility's territory to an area served by many utilities. For example, it could range from the load served by a step-down transformer to the load of a power pool.

### 1.3.4 Industrial Load

Industrial load, as described previously, is one of the several sectors of a utility's load. It consists of the sum of the power demands of all the manufacturing firms located inside a service area of interest.

### 1.3.5 Load

Load could mean different things when used in different contexts, or by different people. In this document, "load" is used to mean:
(1) power demand in continuous time, $P(t)$
(2) hourly energy usage or 15 -minute by 15 -minute average power, $P(n)$
(3) peak load, which is used to mean the daily peak, weekly peak, monthly peak or yearly peak of $P(n)$.

### 1.3.6 Important Quantities for Computing Month1y Charge

For the purpose of computing the monthly charge of an industrial customer, one must know what type of rate is being used. The description of different types of rate structure is given in Appendix A. The following quantities are important for computing monthly charge of an industrial customer.

Define D: Peak demand of all 15 -minute average demands $P(n)$ of a month.

The charges to an industrial customer are computed monthly or bimonthly, therefore this is also a time scale of interest.

Another quantity that is important is the monthly energy usage, denoted as $\varepsilon(\mathrm{in}$ kwh) per month:
$\varepsilon=\sum_{\mathrm{n}} \mathrm{P}(\mathrm{n})$ for all n belonging in the month.
For the time-of-day type of rate, energy is divided into peak and off-peak energy used during the month, as described in Appendix A. 4. Define $\varepsilon_{1}$ : peak hours of energy usage in a month (in kwh/month).

Then

$$
\varepsilon_{1}=\sum_{n^{\prime}} P\left(n^{\prime}\right) \quad \text { where } n^{\prime} \text { belong to all peak hours of a }
$$ month

Define $\varepsilon_{2}$ : off-peak hours of energy usage of a month (in kwh/month).
Then $\varepsilon_{2}=\sum_{n^{\prime \prime}} P\left(n^{\prime \prime}\right) ; n^{\prime \prime}$ belong to all off-peak hours of a month.
1.3.7 Load Shape vs Peak and Energy

From the definition of peak, energy, and load shape, we can see that two different load shapes could have the same peak and energy. Consider the simple case:

Peak demand $D=\max _{n} P(n)$
$\varepsilon=\sum_{n} P(n) \quad$ for all $n$ belonging to a month.
Load shape is a sequence of $P(n)$ for all $n$ belonging to a month. $\mathrm{P}(\mathrm{n}), \mathrm{n}=1,2, \ldots=\mathrm{P}(1), \mathrm{P}(2), \mathrm{P}(3), \mathrm{P}(4) \ldots \mathrm{P}(4 \times 24 \times 30)$, assuming a 30 -day month.

Construct a new sequence $P^{\prime}(n)$, which has $P(1)$ and a certain $P\left(n^{\prime}\right)$ interchanged in position. $\mathrm{P}^{\prime}(\mathrm{n}) \neq \mathrm{P}(\mathrm{n})$,
but $D=\max _{n} P(n)=\max _{n} P^{\prime}(n)$

$$
\varepsilon=\sum_{n} P(n)=\sum_{n} P^{\prime}(n) .
$$

The above is just one example showing that two different load shapes could have the same peak and energy. There are an infinite number of possible load shapes that will have the same peak and energy.

The foregoing arguments are applicable to the demand of one customer of a utility or to the total load of the utility.

For issues such as unit commitment scheduling, long-term system planning, fuel cost of generation, etc., we do not use the peak and energy of a firm or of the total load directly. Rather, load shape is important. There are many good reasons for this, such as:
(1) The peak of each company might occur at a different time than the system peak of the utility as a whole.
(2) The cost of generation is different for two different load shapes that have the same peak and energy usage for a month, or for a year.
(3) The unit commitment scheduling of generators is based on the load shape.
(4) The load shape will tell us if we have a sharp peak, lasting for one or two hours, which is easier to cope with using various types of load management techniques, or a plateau lasting for several hours, which is difficult to cope with.

### 1.3.8 Qualitative Load Shape or Load

A qualitative understanding or feeling of how load or load shape relates to other parameters or variables is important because it guides us in the load modeling effort. Load shape, peak of the month, and energy usage of the month are functions of the individual company, the day of the week, the month, the production level, the weather, labor cost differentials, and rate structure.

For instance, it is well known that the load shape, peak, etc. of a large steel plant is different from that of a small steel plant, or of a textile plant. Load shape on Monday is different from load shape on Sunday. A cold Monday is different from a hot Monday, because of the space conditioning equipment. Load and load shape depend on the producttion level, because during a high production period, more machines are running for a longer period of time. The production level might be a function of the general economic condition, or climate. The load and load shape depends also on the schedule of the operation of machines. For instance, a firm's managers could change the firm's load shape by rescheduling equipment to be operated only at night.

Qualitative understanding is not sufficient for decision-making, because different answers might arise which are usually conflicting and inconsistent. Therefore, one could easily be led into making the wrong policy or decision.

### 1.4 Type of Data Available

When trying to understand a phenomenon or to develop a model, one must check the theory or model with the observations or data and with important physical relationships. This is called parameter estimation and model validation in formal system science language. The types of data that are available and the types of physical relationships that could be used will influence the type of model that is developed. The types of data available are:
1.4.1 Instantaneous Current and Power Measurement

Graphical records of instantaneous current of the entire plant, or a portion of the plant, can be made. However, one should be aware of the fact that the mechanical recording meters have a response time. To find the real power from the direct current measurement, one must know the level of voltage and the power angle.
1.4.2 Fifteen-Minute Average Load

Time series data of 15 -minute average load of a large customer or a portion of its load is taken for computation of monthly charges. There are various ways of measuring and recording these data.
1.4.3 Installed kW Data of Machine

Another possible type of data is that gathered by reading from the name plates of machines and equipment used in a plant, learned by a site visit.
1.4.4 Usage Patterns and Utilization of Machines

These types of data could be found by asking the workers, or managers, or engineers in a plant how and when the machines are being used. These types of data could also be found by observing how the machines and equipment are being used. The types of information found thereby are:
(a) Percentage of time on during a certain work shift
(b) Percentage of load when on
(c) Number of starts per shift, etc.
1.4.5 Structure of the Firm

This type of information tells how processes are connected or how they are related to each other. For example, a printing plant has to print the pages first, put them together in a certain manner, and then bind and cut the pages to a certain specification.

### 1.4.6 Economic Conditions

This type of information involves the level of production at a certain time, the number of labor steps needed for each process, the wage differential of labor, and the rate structure used by the firm; this is gained by a plant visit and by asking questions.

### 1.5 Philosophy of the Industrial Load Modeling Methodology

To develop a model that could be used to answer "what if" types of questions we must be able to write a set of cause-and-effect mathematical relationships and fit them to the observations or data.

As described in the previous section, there are two types of data available. The first is the direct measurement of the 15 -minute by 15-minute average power demand of a plant, or of many plants, or of a fraction of a plant. From this, it can be seen that this curve has to be fitted to the power demand of a large number of machines added together. It is obvious that it will be impossible to deduce the time shape of the power demand of each individual machine from one set of such measurements. In formal system science language, this probiem is called an "unobservable problem'. To solve it, the power demand of all machines must be measured. However, we have overlooked the second type of information, which consists of the electric stock or name plate reading of each machine, and how each is being used. This could be thought of as a set of pseudomeasurements. The combination of the two types of data will allow us to specify all the states; therefore, with the added pseudo-observation, it becomes an observable problem.

### 1.6 Basic Approach to Industrial Load Modeling

The basic approach to industrial load modeling to be followed in this document consists of many steps. The first is to decompose the industrial sector load into each individual company, or plant, of concern. Each plant is studied separately. This is important, because it is very difficult to make generalizations of industrial load at the utility level without first analyzing each plant.

The second step is to hypothesize a general model structure. The following chapters will give the detailed model structure that could be adapted to any plant or firm to be studied. The model structure consists of two parts: the physical model, and the economic consideration. There are two aspects to the physical model The stochastic aspects of Chapter II account for the random nature of each piece of the equipment involved or for a group of equipment. The flow and storage aspect, of Chapter III, allows us to group various types of equipment together and consider them as a block. The economic aspect, Chapter VI, gives us a chance to analyze the cost of operation by balancing between the electric cost and the labor cost differential and other types of costs we are willing to study.

In this detailed, hypothesized model structure, the total demand of a firm is broken down, first, into the demand due to each piece of equipment. For each piece of equipment, a stochastic model is developed -first in continuous time. This is then transformed into discrete time, using a mathematical transformation as explained in Chapter II. Chapter III then describes how the machines could be grouped together.

The third step is to follow the three substeps of the general procedure to model an individual plant, as shown in Fig. 1.6.1. Substep 1


Fig. 1.6.1 Three Substeps to Model Development
hypothesizes a structure based on the general one developed in Chapters II, III, and VI adapted to the plant or firm of interest. Substep 2 estimates all the parameters of interest, and Substep 3 verifies the model by checking the output from the mode1, with observations. If the performance of the model in comparing the theoretical result for the expected value of load, the autocorrelation function of the residuals with that obtained from time series data, is satisfactory, then the model may be accepted. Otherwise, we must go back to the beginning substep by hypothesizing a new structure, then go through all three substeps. This procedure will continue until a satisfactory model is developed.

Figure 1.6.2 shows an industrial load model for a utility that is developed for rate structure design or analysis.


Fig. 1.6.2 Industrial Load Mode1 Structure

### 1.7 Objectives of This Research

The objectives of this research project are:
(1) To gain understanding and insight into the basic processes that are responsible for the electric load and load shape. The understanding also includes the quantitative relations between electric load and other variables that can be developed.
(2) To develop an industrial load-modeling methodology that could be used to develop self-consistent load models. For instance, related quantities such as peak demand, energy, load shape, are treated as being related. In fact, the output "load" from this type of model would be treated as a stochastic process in continuous or discrete time, with variable intervals. Many models that are currently in the field treat
the peak demand, energy, and load shape separately.
Another reason why this type of model is self-consistent is that it is physically based. The model would also allow a user to go into any level of detail desired.
(3) To guide and help other model builders to understand the underlying processes of load, because this research has developed a general conceptual framework for modeling industrial load, starting out from detailed analysis of the underlying processes.
(4) To develop modeling methodology that is conceptually simple and straightforward so that others, such as academicians, other researchers, govermental agencies, industrial plant managers and utility customer service personnel could understand and use the methodological results. For instance, to develop an industrial load model for a utility, data from a few hundred industrial customers might have to be gathered, which would involve many utilities' personnel who would develop models
and gather data. The model is useful to these people only if it is understandable.
(5) To verify the conceptual idea developed for industrial load modeling methodology with actual data. In this research, the rate structures used are from New England Electric Systems, and the seven industrial plants used are the customers of the operating companies of New England Electric System.

### 1.8 Summary of Results and Contributions

The main contribution of this research work consists of the development of the general industrial load modeling methodology. It could have been labeled "the theory of the industrial electric load", or "the analysis of the industrial electric load". It allows one to understand the industrial electric load and to gain insight into various relationships and problems.

The model that is developed is new and radically different from all past load models. The industrial load model developed is capable of addressing many issues of current interest, such as peak load pricing, rescheduleing of work shifts, and various other policy questions related to load management. It is based on detailed physical/economic analysis and is verified using the actual rate structures and data from seven industrial customers of the New England Electric Systems Companies.

The modeling methodology considered the three main aspects of a firm and its load. They are:
(1) stochastic aspects of load
(2) product flow/storage aspect of a firm
(3) econonic considerations.

A summary of each chapter is given below. Chapters II, III, and VI discuss the above three basic aspects of a firm and its load. Many of the ideas discussed in these three chapters are new with respect to the industrial electric load modeling.

In Chapter II, the idea of elementary random process models is introduced. There are a finite number of random process models that have to be considered. The discrete-state (zero and one) continuoustime Markov process model and the binomial-process model are discussed. The Markov process model is extensively used. Its parameters are given in four familiar quantities that have physical meaning. They are: the installed capacity in kW , the fraction of load used when on, the fraction of time on, and the number of starts per hour. The idea of an equivalent model is introduced. If the 15 -minute average sample load is of interest, many processes can be modeled as a two-state Markov process. Finally, it is pointed out that, under favorable conditions, the change in peak demand due to rescheduling a process is equal to the product of installed capcity, the percent load when on, and the percent of time on of that process.

Chapter III considers the interconnection of production processes and storages with associated constraints. The concept introduced in this chapter is used mostly as a conceptual guide, but is helpful in gaining insights into industrial load. It helps us understand how processes are coupled. The idea of treating many machines belonging to a process collectively as a block is developed.

Chapter IV considers the problem of analyzing time-series data. The variances and the expected values associated with the sample mean and the time-average autocorrelation function of the residual load are derived. The methodology and the verification of the model is discussed.

In Chapter V, we discuss the load model of each of the seven industrial firms. They are:
(1) Small Plastics Company
(2) Brush Company
(3) Abrasive Company
(4) Soap Company
(5) Foundry Company
(6) Printing Company
and (7) Consumer Product Company
The data for the first four companies of the above list were gathered in the summer and fall of 1977. These companies were analyzed using ideas from Chapters II and III. Their load models were developed. The time-varying expected load and the autocorrelation function of the residual load were computed using the models. They compared favorably with the sample mean load and the time-average sample autocorrelation function of the residual.

The data from the remaining three companies were gathered in the summer of 1976. The load models for Foundry and Printing Companies were developed, but are not analyzed in detail because only limited data were available.

Chapter VI deals with the economic aspect of the model. The monthly electric cost is treated as a well defined, nonlinear mathematical function. By comparing the possible reduction in monthly electric cost due to rescheduling of a production process with the extra cost incurred due to wage differential, etc., one can develop a simple but powerful
method: comparison of the $\mathrm{kW} /$ person of a production process with the breakeven $\mathrm{kW} / \mathrm{person}$ of the firm. Many issues are analyzed using this type of comparison.

Chapter VII introduces many possible issues that can be studied using the physical/economic analysis of the load modeling methodology. Some issues related to the design of electric rate structures are considered; other issues introduced are not analyzed.

Chapter VIII gives the conclusion and some suggestions for possible areas of research.

## CHAPTER II

PHYSICAL MODEL STRUCTURE: STOCHASTIC ASPECTS
Electric power demand of an industrial plant, or the industrial sector, or the total electric load of a utility, can be broken down into a mean value component (deterministic), namely, the daily and weekly cycles, and a random component, namely, the residua1. This can be seen by looking at the measured electric power demand curve. The total load is the sum of the electric power demand of much equipment and many machines, where each can be broken down into a component that is deterministic and another which is random. These pieces of equipment could each be modeled as a random process.

Consider a lathe in a machine shop. The exact time when the lathe will be turned on or off is unknown, because the operation of the lathe is random. But the expected value of the usage of the lathe, such as the percent of time it is on, or the percent of load when it is on, can be determined. Similarly, the transition probabilities associated with the lathe for making transitions from off to on and from on to off can be determined. The expected usage and the transition probabilities are parameters that will be needed to specify a two-state Markov process which will be described in section 2.4 of this chapter. In the latter part of this chapter, it will be shown how these parameters could be estimated and derived from observation.

It should be noted that when we say a production process or a machine can be modeled as an elementary random process model, we mean that it can be approximated as the respective random process model for our purpose.

### 2.1 Elemental Electric Stock and Utilization Factor

The electric power demand of a machine ( $\alpha^{\text {th }}$ ) can be written as

$$
\begin{equation*}
P_{\alpha}(t)=X_{\alpha} L_{\alpha} u_{\alpha}(t) \tag{2.1.1}
\end{equation*}
$$

for continuous time; where $X_{\alpha}$ is the kW rating of the $\alpha$ th machine, and can be found by reading the name plate on the machine. $X_{\alpha}$ will be called the 'elemental capital stock'. $L_{\alpha}$ is the fraction of $X_{\alpha}$ that is being used when the machine is on, and $u_{\alpha}(t)$ is the elemental utilization factor. $u_{\alpha}(t)$ is usually a random process.

The basic modeling approach used here is to examine the nature of the demand of each element, and then hypothesize a detailed structure involving a random process model for each element. The parameters of the random process are functions of time and other exogeneous variables. Two examples of the exogeneous variables are the weather and the economic condition. The exogeneous variable can be denoted as $M(t)$, which is itself a random process. The vector $M(t)$ could be a vector of dimension larger than one, with each dimension represented by a random process. For the purpose of this document, $M(t)$ is treated as deterministic and given. For some cases, $M(t)$ would simply consist of a list describing the exogeneous variables -- for instance, the weather is cold or the economic condition is average. A more elegant approach would be to treat $M(t)$ as a set of random processes and define all the statistical parameters of the random process for the electric load to be conditional quantities which are conditioned on $M(t)$. However, this complicates the notation unnecessarily.

### 2.2 Instantaneous Continuous-Time Demand Model

The electric power demand of a machine or piece of equipment is first developed in continuous time for two reasons: first, we know that the
actual electric power demand occurs in continous time; second, the continuous time model is simple and the most natural way to develop many types of random process models.

We have

$$
\begin{equation*}
P_{\alpha}(t)=x_{\alpha} L_{\alpha} u_{\alpha}(t) \tag{2.2.1}
\end{equation*}
$$

for the $\alpha^{\text {th }}$ machine, as described previously. $X_{\alpha}$ the electric stock and $L_{\alpha}$ the percent load when on are assumed to be independent of time, and

$$
0 \leq u_{\alpha}(t) \leq 1
$$

$u_{\alpha}(t)$ can be modeled as a stochastic process. All the random processes that are needed to model all the different types of machines could be reduced to a finite set of basic random processes. In this document, these basic processes will be called 'elementary processes'. The set of elementary processes in continuous time are:
(1) finite state Markov process
(2) multinomial process
(3) periodic square wave of random period, width, height, phase
(4) some combinations of the above processes

Some examples of the mathematics of these elementary processes are given in section 2.4 \& 2.6 of this chapter. Machines that could be modeled as finite-state Markov processes include lathes, milling machines, grinders, and many other on-and-off types of processes or groups of processes. On the other hand, some repetitive manufacturing machines (air conditioners, etc.) could be modeled as periodic square waves of random phase, height, width, and period. Some machines could be modeled as binomial or multinomial processes for the purpose of deciding if they will be on or off during a shift.

The electric power demand of a plant at any instant of time can be represented as a sum of the demand of all the individual machines being operated:

$$
\begin{equation*}
\mathrm{P}_{\mathrm{T}}(\mathrm{t})=\sum_{\alpha} \mathrm{P}_{\alpha}(\mathrm{t}) \tag{2.2.2}
\end{equation*}
$$

where $P_{T}(t)$ is the total plant demand, $P_{\alpha}(t)$ is the demand of the $\alpha^{\text {th }}$ machine of the plant at time $t$. Each $P_{\alpha}(t)$ is a random process, as described above.

When each $\mathrm{P}_{\alpha}(\mathrm{t})$ is small, and the sum of the power demands of a group of tightly coupled machines is small compared to the total demand, $\mathrm{P}_{\mathrm{T}}(\mathrm{t})$, then the central limit theorem can be invoked. When the central limit theorem is applied, we know that the probability density function of $\mathrm{P}_{\mathrm{T}}(\mathrm{t})$ at any time t , approaches normal. For the purpose of most utilities' planning of peak capacity or billing applications, the expected value of load and the associated variance at different times would be sufficient. If $P_{T}(t)$ is normal, such quantities as load duration curve, energy usage per month, likely peak demand per month, can be easily deduced if the expected values and variance at each point in time is available.

The above quantities do not completely describe the load in a statistical sense, because they do not give any information about the time structure of the fluctuation of the load. To improve on the statistical description of the load, we need the expected value as well as the autocorrelation function of the residual -- the difference between the measured load and the expected load. There are many reasons for this. First, since the total utility's load (and to some extent, the total plant load) is the sum of a lot of small independent loads, we expect this total load to be
a normal (Gaussian) random process.
This is not always true. However, in most modeling practice , this is assumed to be true unless proven otherwise. If the random process is normal, it is completely described by its expected value and the autocorrelation of the residuals.

Second is the practicality reason. Mainly, we can compute the autocorrelation function and the second-order statistics of most well known random processes. However, the higher order statistics become very difficult to compute from theory, as well as from time-series data. The third reason involves tradeoffs. Should all our modeling effort be spent in the issues of higher order statistics, or should we concentrate on other aspects of the modeling that might be more important?

Mathematically, the expected load and the autocorrelation function of the residuals are as follows for the total load:

$$
\begin{equation*}
\mathrm{m}_{\mathrm{T}}(\mathrm{t})=\mathrm{E}\left[\mathrm{P}_{\mathrm{T}}(\mathrm{t})\right] \tag{2.2.3}
\end{equation*}
$$

where $m_{T}(t)$ is the expected value of the total load;
$r_{T}(t)=P_{T}(t)-m_{T}(t)$
$r_{T}(t)$ is the residual.

$$
\begin{equation*}
\mathrm{R}_{\mathrm{T}}\left(\mathrm{t}_{1}, \mathrm{t}_{2}\right)=\mathrm{E}\left[\mathrm{r}_{\mathrm{T}}\left(\mathrm{t}_{1}\right) \mathrm{r}_{\mathrm{T}}\left(\mathrm{t}_{2}\right)\right] \tag{2.2.5}
\end{equation*}
$$

where $R_{T}\left(t_{1}, t_{2}\right)$ is the autocorrelation function of the residuals.
Similarly, $m_{\alpha}(t), r_{\alpha}(t)$ and $R_{\alpha}\left(t_{1}, t_{2}\right)$ can be derived for the $\alpha{ }^{t h}$ machine of the plant.

If we assume that the residual of the total load is in a stationary state, and that the $\alpha^{\text {th }}$ components are independent, then we have

$$
\begin{align*}
& \mathrm{m}_{\mathrm{T}}(\mathrm{t})=\sum_{\alpha} \mathrm{m}_{\alpha}(\mathrm{t})  \tag{2.2.6}\\
& \mathrm{R}_{\mathrm{T}}(\tau)=\sum_{\alpha} \mathrm{R}_{\alpha}(\tau) \tag{2.2.7}
\end{align*}
$$

where $\tau=t_{1}-t_{2}$.
In actuality, the residuals are rarely stationary or independent. However, for most cases, they can be treated as locally stationary, i.e., they can be treated as stationary over a short time span. For example, many of the manufacturing machines are in a stationary state during an eight-hour shift. All the plastic molding machines as described in Appendix C1 and C2 and a large synthetic soap process (Appendix C4) are found to be in stationary state for all time during all weekdays, from Monday through Friday.

When the residuals are stationary, but the operation of the machines is not independent, we have

$$
\begin{align*}
& m_{T}(t)=\sum_{\alpha} m_{\alpha}(\tau) \quad \text { as before } \\
& \mathrm{R}_{\mathrm{T}}(\tau)=\sum_{\alpha} \mathrm{R}_{\alpha}(\tau)+\sum_{\alpha} \sum_{\beta} Q_{\alpha \beta}(\tau) \tag{2.2.8}
\end{align*}
$$

where $Q_{\alpha \beta}(\tau)=E\left[r_{\alpha}\left(t_{1}\right) r_{\beta}\left(t_{2}\right)\right]$ and $\tau=t_{1}-t_{2}$.
$Q_{\alpha \beta}(\tau)$ is called the 'cross-correlation function'. Since the manufacturing plant has hundreds of machines, there might be a very large number of $Q_{\alpha \beta}(\tau)$ to worry about, so this is not practical for industrial load modeling. The problem could be overcome when all the machines that
are coupled are grouped together and treated as a block. The issue of how to do this will be discussed in the next chapter.

### 2.3 Continuous Time to Discrete Time Transformation

The stochastic model of each machine is developed in continuous time. The actual power demand of the machine is also in continuous time. However, for computational purposes, the measurement is the time average sample of the demand. Therefore, for our model, a mathematical computation for transformation from continuous to discrete time has to be developed. Consider a stationary random process $u_{\alpha}(t)$ in continuous time; $u_{\alpha}(t)$ is transformed into $u_{\alpha}[n]$ as follows:


$$
\begin{align*}
& \tilde{u}_{\alpha}(t)=u_{\alpha}(t) \Theta h(t)^{+}  \tag{2.3.1}\\
& \text {where } h(t)= \begin{cases}\frac{1}{\Delta} & \text { for all } 0<t<\Delta \\
0 & \text { otherwise }\end{cases} \tag{2.3.2}
\end{align*}
$$

$$
\begin{equation*}
u_{\alpha}[n]=\left.\tilde{u}_{\alpha}(t)\right|_{t=n \Delta} \tag{2,3.3}
\end{equation*}
$$

$u_{\alpha}[n]$ is the time average sample; e.g., 15 -minute average power demand measurement that is used for the computation of rate.

+ Note that the symbol $\underbrace{*}_{\sim}$ is used to represent the convolutional operation, or

$$
x(t) \circledast y(t)=\int_{-\infty}^{\infty} x(\xi) y(t-\xi) d \xi
$$

The transformation of the mean and the autocorrelation of the residuals follows the following formula [ 25,26 ].

Assume $u_{\alpha}(t)$ to be in stationary state; then
$E\left\{u_{\alpha}[n]\right\}=E\left\{\tilde{u}_{\alpha}(t)\right\}=E\left\{u_{\alpha}(t)\right\}$
$\tilde{R}_{\alpha}(t)=R_{\alpha}(t) \otimes T(t)$
and

$$
\begin{equation*}
T(t)=h(t) \circledast h(t) \tag{2.3.6}
\end{equation*}
$$



$$
\begin{equation*}
\mathrm{R}_{\alpha}[\mathrm{m}]=\left.\tilde{R}_{\alpha}(\mathrm{t})\right|_{t=m \Delta} \tag{2,3.7}
\end{equation*}
$$

Therefore, $\mathrm{R}_{\alpha}[\mathrm{m}]=\left.\mathrm{R}_{\alpha}(\mathrm{t}) \nsim \mathrm{T}(\mathrm{t})\right|_{\mathrm{t}=\mathrm{m} \mathrm{\Delta}}$

### 2.4 Two State Markov Process Models

Consider a stationary two-state Markov process as described in Drake [27]. Conceptually, it could be represented by a diagram as follows:


Define:

$$
\begin{aligned}
p_{1}(t): & \text { probability that the system is in state } 1 \text {, at time } t, \\
p_{2}(t): & \text { probability that the system is in state } 2 \text {, at time } t . \\
\lambda_{12} \Delta t: & \text { probability that if the system is in State } 1 \text { at time } t, \\
& \text { it will make a transition to state } 2 \text { by time } t+\Delta t .
\end{aligned}
$$

$p_{1}(t), p_{2}(t)$ can be described by the following two equations:

$$
\begin{align*}
& \frac{d p_{1}(t)}{d t}=-\left(\lambda_{12}+\lambda_{21}\right) p_{1}(t)+\lambda_{21}  \tag{2.4.1}\\
& p_{1}(t)+p_{2}(t)=1 . \tag{2.4.2}
\end{align*}
$$

The solutions for these equations are

$$
\begin{align*}
& p_{1}(t)=\left(p_{1}(0)-\frac{\lambda_{21}}{\lambda_{12}+\lambda_{21}}\right) e^{-\left(\lambda_{12}+\lambda_{21}\right) t}+\frac{\lambda_{21}}{\lambda_{12}+\lambda_{21}}  \tag{2.4.3}\\
& p_{2}(t)=1-p_{1}(t) . \tag{2.4.4}
\end{align*}
$$

The steady-state values for $p_{1}(t)$ and $p_{2}(t)$ are

$$
\begin{align*}
& p_{1}(t \rightarrow \infty)=\frac{\lambda_{21}}{\lambda_{12}+\lambda_{21}}  \tag{2.4.5}\\
& p_{2}(t \rightarrow \infty)=\frac{\lambda_{12}}{\lambda_{12}+\lambda_{21}} \tag{2.4.6}
\end{align*}
$$

Suppose a manufacturing machine is modeled as a two-state Markov process. Let the power demand of the machine be zero when it is off (in State 1) and let the power demand be XL when the machine is in state 2. X is the kw installed capacity and L is the fraction of X being used when the machine is on (State 2). L could be called the 'percentage load' when the machine is on.

$$
\text { Define: } \begin{align*}
& \lambda=\left(\lambda_{12}+\lambda_{21}\right)  \tag{2.4.7}\\
a & =\frac{\lambda_{12}}{\lambda_{12}+\lambda_{21}}=p_{2}(t \rightarrow \infty) \tag{2.4.8}
\end{align*}
$$

Then the expected value of the load and the autocorrelation function of the residual could be written as follows:

$$
\begin{align*}
& \mathrm{E}[\mathrm{P}(\mathrm{t})]=\mathrm{XLa} \\
& \mathrm{R}(\tau)=(\mathrm{XL})^{2} \mathrm{a}(1-\mathrm{a}) \mathrm{e}^{-} \cdot \lambda|\tau|  \tag{2.4.10}\\
& \\
& \text { for all } \tau
\end{align*}
$$

$(\lambda)^{-1}$ can be considered as the time constant of the autocorrelation function of the residual, $R(\tau)$. a is the percentage time that the machine is on; $\lambda_{12}$ and $\lambda_{21}$, and thus $\lambda$, could be found if we know the following two quantities:
a: as described before in Eq. 2.4.8
$\eta$ : number of starts per hour
$n$ can be computed from data or knowledge of, say, the number of times a machine is turned on and off during a shift, or during a week, etc. $\eta$ is a rate and has a unit of (1/hour). At steady state, the process is in stationary state. The average rate of transition, $\gamma$, of a stationary two-state Markov process as given in Drake [27 ] could be derived as:

$$
\begin{gather*}
\gamma \Delta t=\frac{1}{2}\left\{a_{21} \Delta t+(1-a) \lambda_{12} \Delta t\right\}  \tag{2.4.11}\\
\text { where } a=p_{2}(t \rightarrow \infty)=\frac{\lambda_{12}}{\lambda_{12}+\lambda_{21}}  \tag{2.4.12}\\
(1-a)=p_{1}(t \rightarrow \infty)=\frac{\lambda_{21}}{\lambda_{12}+\lambda_{21}} \tag{2.4.13}
\end{gather*}
$$

which gives

$$
\begin{equation*}
\gamma=2 \frac{\lambda_{12} \lambda_{21}}{\lambda_{12}+\lambda_{21}} \tag{2.4.14}
\end{equation*}
$$

But, $\eta=\gamma / 2$. For every up transition, there is a down transition.
Substituting Eq. (2.4.12) and (2.4.13) into (2.4.14) and (2.4.7), we have

$$
\begin{align*}
& \lambda_{21}=\frac{\eta}{a}  \tag{2.4.15}\\
& \lambda_{12}=\frac{\eta}{(1-a)} \tag{2.4.16}
\end{align*}
$$

and $\quad \lambda=\frac{\eta}{a(1-a)} \quad$.

The autocorrelation function of the residual then becomes

$$
\begin{equation*}
R(\tau)=(X L)^{2} a(1-a) e^{-[\eta / a(1-a)]|\tau|} \tag{2.4.18}
\end{equation*}
$$

For the case of a time-average sample with time interval $\Delta, E\{P[n]\}$, $R[m]$ could be found using the transformation as given in section 2.3 of this chapter. They are:

$$
\begin{align*}
& E\{P[n]\}=x L a \quad \text { (as before) }  \tag{2.4.19}\\
& R[m]=\left\{\begin{array}{l}
R_{0} \quad \text { for } m=0 \\
\mathrm{Ce}^{-\lambda \Delta|m|} \text { for } m=1,2, \ldots
\end{array}\right. \tag{2.4.20}
\end{align*}
$$

where

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{o}}=(\mathrm{XL})^{2} \frac{2(1-a) a}{(\lambda \Delta)^{2}}\left[1-(1+\lambda \Delta) e^{-\lambda \Delta}\right] \\
& \mathrm{C}=(\mathrm{XL})^{2} \frac{2(1-a) a}{(\lambda \Delta)^{2}}[\cosh (\lambda \Delta)-1] \\
& \lambda=\frac{\eta}{\mathrm{a}(1-\mathrm{a})}
\end{aligned}
$$

$\Delta$, the sampling time interval.F or the rate structures as described in Appendix A, $\Delta$ is 15 minutes.

Most machines have utilization factors whose statistics depend on time of day and the exogeneous variable, $M$, such as the meteorology and economic condition. Therefore, we may write

$$
\begin{aligned}
& a=a(n \mid M) \\
& \lambda=\lambda(n \mid M)
\end{aligned}
$$

where $a(n \mid M)$ and $\lambda(n \mid M)$ are functions of the time index of interest, as well as the other exogeneous variable, M.

By substituting $a[n \mid M]$ and $\lambda[n \mid M]$ for a machine $\alpha$ into Eqs. (2.4.19) and (2.4.20), $E\left\{P_{\alpha}[n \mid M]\right\}$ and $R_{\alpha}[n \mid M]$ can be easily obtained. We then have the following:

$$
\begin{align*}
& \mathrm{P}_{\mathrm{T}}[\mathrm{n} \mid \mathrm{M}]=\sum_{\alpha} \mathrm{P}_{\alpha}[\mathrm{n} \mid M]  \tag{2.4.21}\\
& \mathrm{E}_{\mathrm{L}}\left\{\mathrm{P}_{\mathrm{T}}[\mathrm{n}] \mid M\right\}=\sum_{\alpha} E\left\{\mathrm{P}_{\alpha}[\mathrm{n}] \mid M\right\}  \tag{2.4.22}\\
& R_{T}[\mathrm{n} \mid M]=\sum_{\alpha} R_{\alpha}[n \mid M]+\sum_{\alpha} \sum_{\beta} Q_{\alpha \beta}[n \mid M] \tag{2.4.23}
\end{align*}
$$

This relationships can be generalized to the utility level, and we have

$$
P_{S}[n]=P_{R}[n]+P_{C}[n]+P_{I}[n]+P_{T}[n]+\ldots \ldots
$$

where the $\mathrm{P}^{\prime}$ s represent the load for the service area, residential sector, commercial sector, industrial sector, and transportation sector, etc. Each individual sector can be further represented as a sum of load from many customers whose load is the sum of loads of each individual piece of equipment. $\quad P_{S}[n \mid M], E P_{S}[n \mid M]$ and $R_{S}[n \mid M]$ for the entire service area can be found using the same procedure as shown in Eqs. (2.4.21), (2.4.22) and (2.4.23).

### 2.5 Equivalent Two-State Markov Process Models for Machines

In Section 2.4 of this chapter, a two-state (zero or one) Markov process model of a machine was presented. From the modeling point of view, we are interested in which types of machines can be modeled as a twostate 0 or 1 Markov process in continuous time, if one is interested only in the sample of 15 -minute time-average power demand of the machine. Figure 2.5.1 shows the four different cases of power demand of machines in continuous time, $\mathrm{P}(\mathrm{t})$, and how each case is transformed into timeaverage power demand, $\widetilde{P}(t)$ in continuous time and time-average sample in discrete time, $P$ [ n$]$. The first of the four cases of Fig. 2.5.1 in continuous time is truly a 0 or 1 process. Under the condition that its transition behaves as the two-state Markov process (described in section 2.4 ), then this first case would be a classic two-state $0-1$ Markov process.

Cases 2,3 , and 4 are not the truly $0-1$ type of process. The off state is zero. However, the on state is not just one single, discrete state, but two states, as in cases 2 and 3 , or continuous states as in case 4. Case 2 could represent the load of a compressor or an airconditioner which has a free-running and loaded type of cycle when the machine is on. Case 4 could represent the load of a machine that drifts when it is in the on state. It could also represent the load of a large group of machines where each is running intermittently when the group is on. However, when we look at their time average $P(t)$, cases 2,3 , and 4 look very similar to the output of the linear time-invariant system LTI, being driven by a zero-one Markov process type of load, as given in Case 1.

In other words, we are saying that cyclical and drift-types of fast fluctuation are being filtered out by the LTI system acting as a


| Cose No. | $P_{\alpha}(t)$ | $\widetilde{P}_{\alpha}(\dagger)$ | $P_{a}[n]$ |
| :---: | :---: | :---: | :---: |
| 1 | $\underbrace{P_{1}(t)}_{0}$ |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |

Fig. 2.5.1
low-pass filter. Therefore, if we are interested only in $\widetilde{\mathrm{P}}_{\alpha}(\mathrm{t})$ and $P_{\alpha}[n]$ of cases 2,3 , and 4 , we can model their continuous-time load $P_{\alpha}(t)$ as $P_{1}(t)$ with proper parameters $X, L, \eta$ and $a, P_{2}(t), P_{3}(t)$ and $P_{4}(t)$ can be equivalently modeled as the load of a two-state, 0-1 Markov process, if we are interested only in their $\tilde{\mathrm{P}}_{\alpha}(\mathrm{t})$ and $\mathrm{P}_{\alpha}[\mathrm{n}]$. The above argument could be easily generalized to cover the two-state Markov processes that have values for its two states differing from 0 or 1 .

### 2.6 Binomial Process Model

Instead of looking at a multinomial process, we will look at a binomial process for the sake of simplicity. Suppose we have a manufacturing machine that can be modeled as a binomial process.

Let p: probability of the machine being on,
$\mathrm{q}:$ probability of the machine being off.
$p=1-q$
XL: power used when the machine is on.
We then have

$$
\begin{align*}
& E\{P(t)\}=X L p  \tag{2.6.1}\\
& R(0)=(X L)^{2} p(1-p) \tag{2.6.2}
\end{align*}
$$

$P(t)$ for a binomial process is a constant during a given shift. The reason for this is that the binomial process model is used to model the decision-making process, ie, whether or not a machine is to be turned on or used during a given shift. Note that the binomial process is not useful to model the possible changes in the 15 -minute by 15 -minute power demand of a machine, because people do not look at their watches every 15 minutes and decide if a machine is to be turned on.

The binomial process has a time step of 8 hours, i.e., a decision of whether or not to use a machine is made independently for each step. Also, the instant of time for transition, i.e., the time instant of the beginning and end of a work shift, is also given. The autocorrelation function of the residual is

$$
R\left(t_{1}, t_{2}\right)= \begin{cases}(X L)^{2} p(1-p) & \text { for }(n-1) T<t_{1}, t_{2}<n T \\ 0 & \text { otherwise } .\end{cases}
$$

for any given $n$. When $t_{1}$ and $t_{2}$ both belong to the same shift, the autocorrelation is nonzero. $T$ is the time duration of a shift. This is the semirandom binary process as discussed in Papoulis [ 25]. However, in this research project, the residual load is assumed to be in a stationary state, as this is thought to be a satisfactory assumption for many cases.

Another reason is: the length of the time-series data is insufficient to compute the autocorrelation function of two given time instances, $R\left(t_{1}, t_{2}\right)$. Under the stationary assumption, the autocorrelation function of the residual is computed from the time-series data as shown in Chapter IV and the resultant $R(\tau)$ for the binomial process is

$$
R(\tau)=\left\{\begin{array}{lr}
\left(1-\frac{|\tau|}{8}\right)(X L)^{2} p(1-p) & \text { for }|\tau|<8 \text { hours } \\
0 & \text { otherwise. }
\end{array}\right.
$$

This is the case of the random binary process as discussed in Papoulis [25]. Note that for the computation of the autocorrelation function
under the stationary assumption the time instance for a shift change ( the 8:00 am, 4:00 pm, 12:00 midnight, etc.) has been disregarded. The knowledge of $R(\tau)$ as described above is important because we would like to know what to expect from the component of load that behaves as a binomial process when the residual is being handled as a stationary process (see Chapter IV).

The autocorrelation function of the residual for the 15 -minute time-average sample load can be found using $R(\tau)$, as above, for the continuous time case, and then transforming it into the discrete-time autocorrelation by using the transformation given in section 2.3. The expected value would remain unchanged for the discrete-time case. However, the autocorrelation function of the residuals will be slightly changed. Since there are 32 samples of 15 -minute average samples in an eight-hour time interval, $R[m]$ can be approximated as samples of $R(\tau)$ as follows:

$$
R[m] \simeq\left\{\begin{array}{cc}
\left(1-\frac{|m|}{32}\right)(X L)^{2} p(1-p) & \text { for } \quad|m|<32 \\
0 & \text { otherwise. }
\end{array}\right.
$$

### 2.7 Binomial Process Equivalent Model

The following cases of usage of machines can be modeled as a binomial process.

Case 1. A machine that is being operated constantly during an 8hour work shift if it was decided to use that machine during that shift; otherwise, the machine is off.

Case 2. A machine that is running intermittently when it has been decided to use it during a certain shift; otherwise, the machine is off. For this case, if the fluctuation is very fast compared to a 15 -minute time interval, we have Power usage and probability of being on and off as:

```
P(off) = 0
    P (on) = XLa' where a' = (a|shift is on)
    prob (off) = (1-p)
    prob (on) = p
```

We then have
$\mathrm{E}\{\mathrm{P}(\mathrm{t})\}=\mathrm{XLa}{ }^{\prime} \mathrm{p}$.
Assuming the fluctuation is fast compared to the 15 -minute time step, we have

$$
R[0] \simeq\left(X L a^{\prime}\right)^{2} p(1-p)
$$

Case 3. A group of machines in which each individual machine is run intermittently when the group is to be used. If the fluctuation of load is fast compared to a l5-minute time interval, the expected power demand curve may be found as follows:

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{T}}(\mathrm{off})=0 \\
& \mathrm{P}_{\mathrm{T}}(\mathrm{on})=\sum_{\mathrm{i}} \mathrm{X}_{\mathrm{i}} \mathrm{~L}_{\mathrm{i}} \mathrm{a}_{\mathrm{i}}^{\prime} \text { where } \mathrm{a}^{\prime} \text { is as defined above. } \\
& E\left\{P_{T}(t)\right\}=\left(\sum_{i} X_{i} L_{i} a_{i}^{\prime}\right) p \\
& R_{T}[0]=\left(\sum_{i} X_{i} L_{i} a_{i}^{r}\right) p(1-p)
\end{aligned}
$$

where $\mathrm{P}_{\mathrm{T}}$ is used to denote the total power demand of the group, and $p$ is the probability that the group will be on.

### 2.8 Determination of Parameters

It was pointed out that the total demand of power during a work shift is the sum of the power demand of each individual piece of equipment, namely:

$$
\begin{aligned}
& \mathrm{E}\left[\mathrm{P}_{\mathrm{T}}\right]=\sum_{\alpha} \mathrm{X}_{\alpha} \mathrm{L}_{\alpha} \mathrm{a}_{\alpha} \\
& \mathrm{R}_{\mathrm{T}}[\mathrm{~m}]=\sum_{\alpha} \mathrm{R}_{\alpha}[\mathrm{m}]
\end{aligned}
$$

$R_{\alpha}[m]$ is as described in the previous sections of this chapter.
In this section, we give a discussion of how the parameters needed to determine $E\left[P_{T}\right]$ and $R_{T}[m]$ are found. $X_{\alpha}$, the installed $k W$, can be found by reading the nameplates of the machines involved, such as the kW rating of a motor. $X_{\alpha}$ for the lighting of a large manufacturing area, such as a machine shop, can also be determined if some idea of the floor space and lighting level are known. This level can be found by asking questions, or from a handbook [28]. However, in some cases it might be easier to simply count the light bulbs involved. Another approximate method of finding $X$ is to know the transformer size, and how much load is generally connected to the transformer. The engineer who designs and chooses the transformer size will have a good idea of what is the ratio of actual load connected to the transformer and the transformer size KVA. Generally, though, the KVA is not a very good approximation.

In some cases the installed capacity, $X_{\alpha}$, by itself cannot be found. However, when the machine is on, $X_{\alpha} L_{\alpha}$ the load can be deduced from direct measurement of the current on one of the phases. The power demand that is thus obtained represents the complex power and has to be multiplied
by a load factor, $L_{f}$, to get the real power.
The decision of whether or not a certain machine should be modeled as a two-state Markov process, as a binomial process, or other processes, can be made based on the description by the managers, the workers, or by consulting the company's records or logs for that machine. Suppose it has been decided that a machine can be modeled as a two-state Markov process, then the parameter $\left(\mathrm{X}_{\alpha} \mathrm{L}_{\alpha}\right)$ can be found as in the last paragraph. The percentage time on, $a$, and the number of starts per hour, $\eta$, can be easily found from the company's records or by asking questions. Similarly, the probability of a machine being in use during a given shift, $p$, can be found if the machine is modeld as a binomial process. 2.9 Periodic Square Wave of Random Phase, Period, Height, and Width

Air conditioners and compressor types of load can be modeled as periodic square waves of random phase, period, height, and width. This type of process might be modeled as a semi-Markov process. But, for our purposes, the random periodic square wave process was not analyzed because we do not feel it is very important with respect to our study.
2.10 Theoretical Computation of Energy Usage, Peak Demand and the Load Duration Curve of a Month

If time-series data are available for a plant, its peak demand, energyusage, and load duration curve can easily be found. In many real situations, only the peak demand and energy usage for a month is recorded, and time series data are not available. For this case, we would like to know how to compute the peak demand, energy usage, and load duration curves from the load model, so that a comparison between the theoretical and the measured quantities can be made.

In the next three chapters, it will be shown how the expected load and the autocorrelation function for a shift can easily be found from the load model for each of the three shifts of a day. Suppose we also have the expected load and the autocorrelation function of the residual for weekends; then we can compute the probability density function $f(p)$ for a random sample of the 15 -minute average load for a month, $P$, by using Bayes' theorem

$$
f(P)=\sum_{i} h_{i} f_{i}(P)
$$

for $\mathrm{i}=1,2,3,4$ representing the first, second, and third and weekend shifts, where

$$
f_{i}(P)=\frac{1}{\sqrt{2 \pi} \sigma_{i}} e^{-\left(P-m_{i}\right)^{2} / 2 \sigma_{i}}
$$

assuming that the residual of a shift is at least normal in the first order.

$$
\begin{aligned}
& m_{i}=E\{P \mid \text { shift } i\} \\
& \sigma_{i}=\sqrt{R_{i}[0]}
\end{aligned}
$$

$m_{i}$ is the expected 15 -minute average sample load for shift $i$, and $R_{i}[0]$ is the covariance of the residual load at shift $i$.

$$
h_{i}=\frac{\# \text { of } 15 \text {-minute intervals in shift } i}{\# \text { of } 15 \text {-minute intervals in a month }} .
$$

For a typical company,

$$
h_{1}=h_{2}=h_{3}=\frac{8 \times 22}{30 \times 24} \quad=0.2444 \ldots \ldots
$$

There are 22 working days in a month and eight working hours per day.

$$
h_{1}+h_{2}+h_{3}+h_{4}=1
$$

and therfore,

$$
h_{4}=0.26666 \ldots
$$

It should be noted that we have used a 30 -day month with 22 working days for the computation. Therefore, in actuality, the $h_{i}$ will change slightly from month to month.

The probability distribution function, $\mathrm{F}(\mathrm{P})$, for a random 15 -minute average sample load for a month is

$$
\begin{aligned}
F(P) & =\int_{-\infty}^{P} f(\xi) d \xi \\
& =\int_{-\infty}^{P} \sum_{i} h_{i} f_{i}(\xi) d \xi \\
\text { for } i & =1,2,3,4 \quad \text { as before. }
\end{aligned}
$$

As we know,
$0 \leq F(P) \leq 1$. It should be noted that load duration curves can be easily obtained by rotating the curve $F(P)$ and relabeling the axes.

A second point that should be noted is that in the above derivation we have made the assumption that the expected load during a shift is a constant. In actuality, this is only approximately true. A third point is that the 15 -minute average sample load during a shift does not always have Gaussian distribution. For many cases, the above derivation for $F(P)$ will not be valid, and $F(P)$ has to be derived individually for special cases.

Let $F(P)=\delta$. Then $\delta$ has the physical interpretation of being the fraction of time where the 15 -minute average sample load of a month is below $P$. Define $G(P)=1-F(P)$. Then, $G(P)=1-\delta$, and $1-\delta$ has the physical interpretation of being the fraction of time where the l5-minute average sample load of a month is above P . Therefore, $\mathrm{P}=$ $\mathrm{G}^{-1}(1-\delta)$ is the load duration curve that is derived.

The energy usage of the month can be found easily by first obtaining the area under the curve $\mathrm{G}^{-1}(1-\delta)$ and multiplying it by the number of hours in a month. Alternatively, the energy usage of the month, $\varepsilon$, can be found by using the following formula without using the load duration curve:

$$
\varepsilon=\sum_{i} h_{i} E\{P \mid \operatorname{shift} i\} \times \text { No. of hours in a month }
$$

where $i=1,2,3,4$ as defined before. The likely 15 -minute average peak, D, for the month can be found from the following formula*

$$
D=G^{-1}\left(1-\frac{1}{2880}\right)
$$

Note that there are 2,880 15-minute time intervals in one month.
For some cases, the monthly load duration curve is not available. The peak demand of a month can then be estimated as follows, assuming

[^0]that the expected load of a shift is constant and the residual load is Gaussian, at least in the first order, and the 15 -minute average samples are independent.

Case 1 Suppose the expected loads and variance of the residual are the same for the first, second, and third shifts. The weekend's load is very sma1l. The 15 -minute average sample peak demand, $D$, of the month is

$$
D=E\{P \mid \text { shift } 1\}+3.30 \sigma_{1}
$$

where $E\{P \mid$ shift 1$\}=E\{P \mid$ shift 2$\}=E\{P \mid \operatorname{shift} 3\}$
and

$$
\sigma_{1}=\sigma_{2}=\sigma_{3} \text { as given. }
$$

The factor 3.30 is obtained from the table for normal probability function, and represents the point where the power demand, $P[n]$, has a $50 \%$ chance of being above $D$ for one out of 2,112 samples. The 2,112 samples are the number of 15 -minute average sample loads for all 22 working days in one month.

Case 2 Suppose the expected load for the first shift is much higher than that of the second, third, and weekend shifts. For this case, the peak demand, $D$, is

$$
D=E\{P \mid \text { shift } \quad 1\}+2.96 \sigma_{1}
$$

where the factor 2.96 is the point where the power demand $P[n]$ has a $50 \%$ chance of being above $D$ for one out of 704 samples; 704 is the number of 15 -minute average sample loads for the first shift of all the working days in a month.

Therefore, we have $\mathrm{D}=\mathrm{E}\left\{\mathrm{P} \mid 1^{\text {St }}\right.$ shift $\}+\mathrm{ko}_{1}$, where k is a function of the particular case of interest, and depends on the number of independent samples being considered. However, from sections 2.4 and 2.6 of this chapter and from Chapter $V$, we know that the 15 -minute average sample load is correlated in time, and therefore not independent. Consider a hypothetical case in which all the 15 -minute average sample loads of each hour are completely correlated, i.e., the four samples belonging to each hour have exactly the same value. In this siutation; then, the first case, as discussed before, would only have $25 \%$ as many indpendent samples during all the working hours of a month, or 2112/4=528 samples. Then $k$ for the case 1 discussed previously would become 2.91 instead of 3.30. The following table shows how $k$ changes as a function of the number of indepdendent samples involved.

| $\%$ of original <br> independent samples | number of <br> independent samples | k |
| :---: | :---: | :---: |
| 100 | 2112 | 3.30 |
| 25 | 528 | 2.91 |
| 10 | 211 | 2.60 |
| 5 | 106 | 2.35 |
| 1 | 21 | 1.67 |

It can be seen that $k$ changes very slowly when the number of independent samples changes. It is interesting to compute the $k$ for the four companies that were studied in the summer of 1977. The following table shows what the $k$ values are for the four companies, using $D$ computed from the time series data available and $E\{P \mid 1$ st shift $\}, \sigma_{1}$ from Chapter $V$.

| Company | $\mathrm{E}\left\{\mathrm{P} \mid 1^{\text {st }}\right.$ Shf. $\}$ <br> (in kW) | $\sigma_{1}$ <br> (in kW) | D <br> (in kW) | k |
| :--- | :---: | :---: | :---: | :---: |
| Small Plastics Co. | 760 | 47.5 | 870 | 2.32 |
| Brush Co. | 2800 | 125.0 | 3180 | 3.04 |
| Abrasive Co. | 3800 | 75.0 | 4100 | 4.00 |
| Soap Co. | 2600 | 550.0 | 3500 | 1.64 |

The formula $D=E\left\{P \mid 1^{\text {st }}\right.$ shift $\}+\mathrm{Ko}_{1}$ is only a rough approximation, for several reasons. First, it is based on the assumption that the residual is normal, at least in the first-order statistics. Second, the samples are independent, and third, $E\left\{P \mid 1^{\text {st }}\right.$ shift $\}$ is a constant for all hours of the first shift. We know that these assumptions do not always hold true, and the formula is a rough approximation. However, the above formula and the exercise that we went through offer us insight into how peak demand, $D$, is related to the expected load and variance of the residual load.

### 2.11 Change in Peak Demand Due to Rescheduling

In this section we will be using the formula for deriving peak demand:

$$
D=E\left\{P \mid 1^{\text {st }} \text { Shift }\right\}+k \sigma_{1}
$$

to find out how peak demand will change due to rescheduling. The following hypothetical example will be used.

Suppose we have a manufacturing plant with N number of identical manufacturing machines. The parameters $X, L, a$ and $\eta$ are identical for all machines and each machine behaves as an independent two-state Markov process, as described in section 2.4. Suppose the machines are running only during the first shift. We then have

$$
\begin{aligned}
E\left\{P \mid 1^{\text {st }} \text { Shift }\right\} & =N X L a \\
\sigma_{1} & \simeq X L \sqrt{N a(1-a)}
\end{aligned}
$$

$\sigma_{1}$ is approximate because it depends on $\eta$ when $\eta$ is large compared to $1 / \Delta$ as described in section 2.4. However, for this case we do not want $\eta$ to be too small compared to $1 / \Delta$ because $\mathrm{P}[\mathrm{n}]$ can become highly correlated in time, as discussed in section 2.4. Therefore, the example here is valid only for $\eta$ lying within a certain range, where $\eta$ is smaller than $1 / \Delta$ but not by more than an order of magnitude. Using the formula for $D$ and substituting for $E\left\{P \mid 1^{\text {st }}\right.$ Shift $\}$ and $\sigma_{1}$ we have

$$
D=N X L a+k X L \sqrt{N a(1-a)}
$$

Suppose one machine is being rescheduled to be operated during the second shift, then the resultant $D^{\prime}$ is

$$
D^{\prime}=(N-1) X L a+k X L \sqrt{(N-1) a(1-a)} .
$$

For N sufficiently large, k will remain unchanged; therefore the change in peak demand is

$$
\left(D-D^{\prime}\right)=X L\{a+k \sqrt{a(1-a)}(\sqrt{N}-\sqrt{N-1})\}
$$

Define

$$
\gamma=\frac{k \sqrt{a(1-a)}(\sqrt{N}-\sqrt{N-1})}{a}
$$

then $\left(D-D^{\prime}\right)=X \operatorname{La}(1+\gamma)$

At the limit where N is large or a is close to 1 , then $\gamma$ becomes very small. At this limit, ( $D-D^{\prime}$ ) can be approximated as

$$
\left(D-D^{\prime}\right) \simeq X L a \quad .
$$

The above limit does not always hold true from each industrial customer's point of view. But from the utility's point of view, N is almost always large enough so that the above limit is nearly always true.

For some welding equipment, or hoists, a is much smaller than one and therefore the above approximation is not good unless there are very large numbers of such equipment. For plastics molding types of machines, as described in Chapter $V$ and Appendix $C$, a is close to 1 and therefore the above approximation is quite good.

We can see that ( $D-D^{\prime}$ ) is bounded as follows:

$$
X L \geq\left(D-D^{\prime}\right) \geq X L a
$$

The example given in this section is an idealized case. The assumptions made do not always hold; however, this case does provide us with insight into how the peak demand is changed due to rescheduling.

### 2.12 Discussion and Perspectives

In this chapter, we have developed the necessary tools needed for modeling electric load as a random process. It was discussed that the expected value and the autocorrelation function of the residual for the total load can be found by summing up the respective quantities for each piece of equipment when they are independent. Cross-correlation functions between pieces of equipment must be considered if they are not independent. One way of getting around this problem is to use the structural relationship of Chapter III to group many pieces of equipment into a block that is usually independent of another block. If the total load is a normal process, then the expected value and the autocorrelation function of the residual will completely specify the total load. These ideas can be extended to utility level.

The models are developed in continuous time. The load of the con-tinuous-time model is transformed into time-average sample load in discrete time, to be consistent with time-series samples of 15 -minute average load measured and used by utilities.

The idea of modeling a piece of equipment as one of finite number of elementary random process models is introduced. Two such models discussed are: the discrete-state continuous-time Markov process model, and the binomial process model. The Markov process model is extensively used in this report. It is expressed in the form in which its parameters are given in four familiar quantities with physical meaning -- the installed capacity kW , the fraction of load used when on, the fraction of time on,
and the number of starts per hour. The idea of equivalent models is introduced. Many machines can be equivalent and be modeled as a twostate Zero/one Markov process in continuous time, when one is interested only in the 15 -minute average sample load. The Binomial process model is discussed but is not used in this research.

Section 2.10 discusses the computation of the energy usage, peak demand, and load duration curves of a month. Using some of the results from section 2.10 , we can show that when the number of pieces of equipment is large, and no one piece is dominant, the change in peak due to rescheduling a piece of equipment is equal to XLa. This result is used in the discussions of Chapter VI.

CHAPTER III
PHYSICAL MODEL STRUCTURE: PRODUCT FLOW AND STORAGE ASPECTS

In this chapter, it will be shown how $m_{T}(t \mid m)$ and $R_{T}(t \mid m)$ for a company's total load can be found in a practical manner when all the pieces of equipment are not independent. The structural relationship of a company, such as the product flow and storage aspect, is exploited. In sections 3.2 and 3.4, we discuss how the problem of finding the detailed autocorrelation and crosscorrelation functions of the residual load of the individual machine can be overcome.

### 3.1 Description of the Physical Model: Theory

The manufacturing plant of a firm is an entity which has input in the form of capital, raw material, energy, labor, and production schedules; and output in the form of products that the firm can market or use in producing other products.

To develop an hourly (or 15 -minute by 15 -minute) electric load model of a manufacturing plant, we have to identify all the devices that consume electric power (light bulbs, motors, etc.) and find out when and how each of them is being used. To study each of these devices separately would be impractical. In this section we describe an appropriate way of aggregating these devices into groups of similar electric characteristics and usage patterns.

In a manufacturing plant, raw materials are being received and stored in raw material storage areas, and final products are being shipped out from finished inventory storage areas. There are many other intermediate storage areas in between. The storage areas are interconnected, so that materials can flow from one storage area to the next, where the materials usually go through some kind of processes, thereby changing their form as they move from one storage area to the other.

All the levels of material and rates of flow are controlled by managerial decisions and the physical constraints of the plant. Managerial decisions are influenced by availability of labor, raw material, capital, demand for final products, economic considerations, and other information. To illustrate the possibilities of the material flow approach, we present a hypothetical manufacturing plant with N interconnected storages described by the following difference equation:

$$
\begin{gather*}
\frac{1}{\Delta}\left\{S_{i}[n+1]-S_{i}[n]\right\}=\sum_{k=0}^{N} F R_{k i}^{r}[n]-\sum_{k=0}^{N} F R_{i k}^{1}[n] \quad 1=1,2, \ldots N  \tag{3.1.1}\\
\mathrm{FR}_{\mathrm{No}}^{\ell}=C_{1} \quad \text { output requirement, and } \\
\mathrm{FR}_{\mathrm{O} 1}^{\mathrm{r}}=C_{2} \quad \text { input requirement are given. }
\end{gather*}
$$

where
$S_{i}[n]$ is the level of material in the storage ' $i$ ' at time $n$
$F R_{i k}^{1}[n]$ is the rate of material leaving storage ' $i$ ' to storage ' $k$ ' at time $n$ (quantity/hour)
$\mathrm{FR}_{\mathrm{ki}}^{\mathrm{r}}[\mathrm{n}]$ is the rate of material received at storage 'i' from storage ' k ' at time n (quantity/hour)
$n$ is the $1 / 4$-hour time index and $\Delta$ is $1 / 4$ hour.
Both the storage levels and the flow rates are constrained by the capacity of the plant, equipment, and the processes involved.

$$
\begin{align*}
& 0<\mathrm{S}_{\mathrm{i}}<\mathrm{S}_{\mathrm{i}}^{\max } \\
& 0 \leq \mathrm{FR}_{\mathrm{ik}}^{1} \leq \mathrm{FR}_{\mathrm{ik}}^{1 \mathrm{max}}  \tag{3.1.2}\\
& 0 \leq \mathrm{FR}_{\mathrm{ki}}^{\mathrm{r}} \leq \mathrm{FR}_{\mathrm{ki}}^{\mathrm{r}} \max
\end{align*}
$$

$S_{i}^{\max }, \mathrm{FR}_{\mathrm{ik}}^{\mathrm{l}} \mathrm{max}, \mathrm{FR}_{\mathrm{ki}}^{\mathrm{r}} \max$ are the physical constraints of the firm. We can interpret the above equations as the rate of change of the level of material in storage 'i.' being equal to the difference between the sums of the rates of material arriving to storage ' $i$ ' from all the other storage areas, and the rates of material leaving storage 'i' to all the other storage areas.

Using the described mode1, it is possible to find a feasible schedule for each section of the firm that satisfies the given flow and storage constraints at a particular level of output. The feasible solution gives the number of shifts per day needed at each section of a manufacturing plant. In general, depending on the interconnection of storage areas, more than one feasible schedule can be obtained from the above physical model. However, it should be realized that in an actual firm, there are other considerations that must be taken into account. For instance:
(a) An employee normally works for 8 hours/day in one stretch at 5 days/week.
(b) An employee cannot be assigned to different jobs (jobs are specialized).
(c) Day-to-day operation and scheduling of a manufacturing plant do not change drastically.

These considerations, together with the flow and storage constraints, will reduce the number of feasible schedules to a realistic set for the physical model.

### 3.2 Physical Mode1 Structure: Electric Power Demand Aspects

A feasible schedule that satisfies the above constraints, relationships, and requirements, can be found for all the production processes. Based on the feasible schedule, a reasonable electric power demand curve $P(n / M)$ can be found. $M$ includes all the exogenous variables plus the schedule that was chosen. For the rest of this chapter, $M$ is assumed to be implicit and will be suppressed in our notation.

Total electric power utilized at a particular time (hour h) under a certain given $M$ can be described as follows:

$$
\begin{equation*}
P_{T}[n]=\sum_{j} \sum_{i} P_{i j}[n]+\sum_{k} P_{k}[n] \tag{3.2.1}
\end{equation*}
$$

where $n$ is used to represent a particular time, $n$, of day $d$, month $m$, and year $y$, for simplicity. $P_{i j}[n]$ represents the electric power demand of the end users (motors, light bulbs, etc.) associated with the flow of material from storage $i$ to storage $j$ and $P_{k}[n]$ represents the electric power demand of the end users which do not have any direct relationship with the flow of material (e.g., offices, toolrooms, some machine shops, etc.). $\quad P_{i j}[n] \neq P_{j i}[n]$ for most cases.

Since material usually flows only in one direction, if

$$
P_{i j}[n] \neq 0 \quad \text { for } F R_{i j}[n] \text { positive, }
$$

then

$$
P_{j i}[n]=0 \text { for most cases, }
$$

which says that there is no material flowing backward from storage $j$ to storage i. The purpose of the above three equations is to show that a manufacturing machine or process will let product flow mainly in one direction, i.e., $\mathrm{FR}_{\mathrm{ij}}[\mathrm{n}] \neq \mathrm{FR}_{\mathrm{ji}}[\mathrm{n}]$ and $\mathrm{P}_{\mathrm{ij}}[\mathrm{n}] \neq \mathrm{P}_{\mathrm{ji}}[\mathrm{n}]$.

$$
\begin{equation*}
P_{i j}[n]=\sum_{\ell=1}^{L} u_{i j}^{\ell}[n] X_{i j}^{\ell}[n] \tag{3.2.2}
\end{equation*}
$$

where $\ell$ represents the sumnation over subgroups of electrical stock which belong to the group of electrical appliances associated with the flow of material from storage $i$ to storage $j$.

$$
\sum_{\ell=1}^{L} x_{i j}^{\ell}[n]=x_{i j}[n]
$$

$X_{i j}[n]$ is the total capacity of the end users between storage $i$ and storage $j$; $u_{i j}^{\ell}[n]$ is the utilization factor of the $\ell^{\text {th }}$ subgroup of end users between storage $i$ and storage $j$. For example, each of the subgroup might represent the lighting load, the airconditioning load, and the load for a production process.

By definition,

$$
0 \leq u_{i j}^{\ell} \leq 1
$$

Further we can proceed to show

$$
x_{i j}^{\ell}=\sum_{\alpha} x_{i j}^{\ell \alpha}
$$

where $X_{i j}^{\ell \alpha}$ is the electric capacity ( $k w$ ) of the $\alpha^{\text {th }}$ machine belonging to subgroup \& of the group between storage $i$ and storage $j$. $u_{i j}^{\ell}[n]$ is a random process:

$$
E\left\{u_{i j}^{\ell}[n] \left\lvert\, \begin{array}{c}
\text { subgroup }_{\text {is on }}^{\ell}
\end{array}\right.\right\}=\frac{1}{x_{i j}^{\ell}} \sum_{\alpha} X_{i j}^{\ell \alpha} E\left\{u_{i j}^{\ell \alpha}[n] \left\lvert\, \begin{array}{c}
\text { subgroup } \ell  \tag{3.2.4}\\
\text { is on }
\end{array}\right.\right\}
$$

where $0 \leq u_{i j}^{\ell \alpha}[n] \leq 1$ and

$$
E\left\{u_{i j}^{\ell}[n] \left\lvert\, \begin{array}{c}
\operatorname{subgroup}_{\text {is on }} \ell_{1}
\end{array}\right.\right\} \text { and } E\left\{u_{i j}^{\ell \alpha}[n] \left\lvert\, \begin{array}{c}
\text { subgroup } \ell \\
\text { is on }
\end{array}\right.\right\}
$$

can be estimated. These estimates would involve honest guesses by the modeler or the engineers as to what are the values of $L, a, \eta$ for twostate Markov types of process as discussed in $\$ 2.4$ of Chapter II and of p, $q$ for the binomial type of process as discussed in 92.6 of Chapter II.
 puted using the formula as given in Chapter II. We can then proceed to find

$$
\begin{equation*}
E\left\{u_{i j}^{\ell}[n] \mid S\right\}=\frac{1}{x_{i j}^{l}} \sum_{\alpha} x_{i j}^{\ell \alpha} E\left\{u_{i j}^{l \alpha}[n] \mid S\right\} \tag{3.2.5}
\end{equation*}
$$

where $S$ is used to represent the fact that the subgroup is on or off. From Eq. (3.2.2), we can then write

$$
\begin{equation*}
E\left\{P_{i j}[n] \mid S\right\}=\sum_{\ell=1} X_{i j}^{\ell} E\left\{u_{i j}^{\ell}[n] \mid S\right\} \tag{3,2.6}
\end{equation*}
$$

where $S$ would specify whether the group is on or off. The load of a subgroup or a group of machines can be treated as a two-state Markov process or a binomial process, as discussed in $\S 2.5$ and 2.7 of Chapter II. However,

$$
\begin{equation*}
\left.E\left\{u_{i j}^{\ell}[n]\right\}=f\left(\mathrm{FR}_{\mathrm{ij}}[\mathrm{n}]\right), \text { etc. }\right) \tag{3.2.7}
\end{equation*}
$$

where "etc." might include ambient temperature, time of day, or other external effects. That is, $E\left\{u_{i j}^{\ell}[n]\right\}$ is a function of the material flow between storage $i$ and $j$, but could also be a function of other variables. In this report, the functional dependence of $E\left[u_{i j}^{\ell}[n]\right\}$ on flow rate and temperature, etc., is not overly emphasized. Only a simple relationship will be used. For example, the lighting would generally be 0 or 1 , depending on whether or not the area is being used. The airconditioning will be on in the summer.

$$
P_{k}[n] \text { is the electric power demand of group ' } k \text { ' of the plant }
$$ which does not relate directly to production rates. For example, the electric power demand of machine shops and offices may not depend directly on the rate of production or the internal material flow rates. Again,

$$
\begin{equation*}
P_{k}[n]=\sum_{\ell=1}^{L_{k}} u_{k}^{\ell}[n] X_{k}^{\ell}[n] \tag{3.2.8}
\end{equation*}
$$

and

$$
x_{k}[n]=\sum_{\ell=1}^{L_{k}} x_{k}^{\ell}[n]
$$

where $X_{k}[n]$ is the electric strock of group ' $k$ '. $u_{k}^{\ell}[n]$ is the utilization factor of the $\ell^{\text {th }}$ subgroup of $X_{k}$ at time $n$. Again,

$$
\begin{aligned}
& 0 \leq u_{k}^{\ell}[n] \leq 1 \quad, \text { as before. } \\
& E\left\{P_{k}[n] \mid S\right\}=\sum_{\ell} x_{k}^{\ell} E\left\{u_{k}^{\ell}[n] \mid S\right\}
\end{aligned}
$$

as before. Since $S$ is specified by the exogenous variable plus the schedule, $M$, we have the total expected power demand $P_{T}[n]$; using Eqs. (3.2.1), (3.2.2), (3.2.4), (3.2.6) and (3.2.8) as follows:

$$
\begin{aligned}
E_{E}\left[P_{T}[n] \mid M\right\}= & \sum_{i} \sum_{j} \sum_{\ell}^{L_{i j}} X_{i j}^{\ell} \dot{E}\left\{u_{i j}^{\ell}[n] \mid M\right\} \\
& +\sum_{k} \sum_{\ell}^{L} x_{k}^{\ell} E\left\{u_{k}^{\ell}[n] \mid M\right\}
\end{aligned}
$$

It should be emphasized here that the key to finding the total electric power demand $\quad \mathrm{P}_{\mathrm{T}}[\mathrm{n}]$ of a plant at time n is to find the power demand of each subgroup under a given condition, M, which specifies the material and product input and output, weather, as well as all the other relevant conditions.
$R_{T}(m \mid M)$ could also be found for a given $M$ using a very similar method. Although conceptually it is very easy to find the expected value and autocorrelation of the total power demand for a given $M$, in practice there are many complications. These arise from the fact that one usually finds the power demand of a subgroup, given that the subgroup is on. But when or how the subgroup is being used depends on many factors.

For instance, how does each subgroup relate to the other subgroups in the same group or in another group? To understand this, we must go back to the flow and storage aspect of a firm. For example, machines could comprise a tightly coupled production line that must be turned on and off simultaneously. On the other hand, if two production processes are separated by a large storage area, then the two processes are independent of each other. Some possible cases are presented below, but there could be many others.

Case 1. The whole group or subgroups are either on or off for the whole shift, in a regular manner. For instance, a certain subgroup might be on during the first shift, but off during the third.

Case 2. The whole group or subgroups could fall into a number of possible states (or levels of demand) during any shift -- for example see the binomial process model of Chapter II.

Case 3. The whole group could be on or off within the time of one shift, acting as a two-state Markov process. A Manager might decide to use a group of machines which are running in a random manner when the group is on, intermittently.

All the conceivable cases for load of a subgroup can be modeled as one of the elementary random processes described in Chapter II, at least approximately.

Suppose a group of machines belonging to a production process between storage $i$ and $j$ is tightly coupled and modeled as a two-state Markov process in continuous time, as discussed in sections 2.4 and 2.5. Such statistics as the expected load, and the autocorrelation function of the residual for the group can easily be found if the $X, L, \eta$, and a for the group are known. For many cases it may be possible to deduce (XL) from measurement. Since the product, and not the values for X and L , are important for determining the statistics of interest, no difficulty will be encountered with the value of (XL) measured. The alternative way of deriving the expected load and the autocorrelation function of the residual load is first to find the statistics for each machine, including the crosscorrelation, $Q_{\alpha \beta}$, of the machines in the group. However, this will create unnecessary trouble. This is one example of how the problem of computing the crosscorrelation function could be overcome using the structural information that the machines belonging to the group are tightly coupled and treating the group as one block.

### 3.3 Hypothetical Case Example of Flow and Storage Model

To illustrate the approach which has been developed, consider the example shown in the flow diagram of a hypothetical manufacturing firm, in Fig. 3.3.1 (next page).

$$
\begin{aligned}
& \mathrm{FR}_{01}^{\max }=10 \text { tons/hr. } \quad \mathrm{S}_{1}^{\max }=200 \text { tons } \\
& \mathrm{FR}_{12 \#_{1}}^{\max }=5 \text { tons } / \mathrm{hr} . \quad \mathrm{S}_{2}^{\max }=20 \text { tons } \\
& \mathrm{FR}_{12 \#_{2}}^{\max }=5 \text { tons } / \mathrm{hr} \quad \mathrm{~S}_{3}^{\max }=150 \text { tons } \\
& \mathrm{FR}_{23}^{\max }=20 \text { tons } / \mathrm{hr} \\
& \mathrm{FR}_{30}^{\max }=30 \text { tons } / \mathrm{hr}
\end{aligned}
$$

Then, for a particular level of output, a feasible schedule of the firm can be found as follows:

Suppose 150 tons/day of output is required, a feasible solution for the hyporthetical manufacturing firm is :

$$
\begin{aligned}
& \text { Let } H W D_{i j} \triangleq \text { working hrs/day required for the } i \rightarrow j \text { section } \\
& \text { of the manufacturing firm } \\
& H W D_{01} \geq \frac{\text { Leve1 of output }}{F R_{i j}^{\max }} \\
& H W D_{01} \geq 15 \mathrm{hrs} / \text { day } \sim 2 \text { shifts } \\
& \mathrm{HWD}_{12} \#_{1}+\mathrm{HWD}_{12} \#_{2} \geq 30 \mathrm{hrs} / \text { day } \\
& 22 \text { shifts for each section \#1 and \#2, or other } \\
& \text { combinations of shifts. }
\end{aligned}
$$



$$
\begin{aligned}
& \mathrm{HWD}_{23} \geq 7 \frac{1}{2} \mathrm{hrs} / \text { day } \sim 1 \text { shift } \\
& \mathrm{HWD}_{30} \geq 5 \mathrm{hrs} / \text { day } \sim 1 \text { shift }
\end{aligned}
$$

It should also be noted that there is coupling between the section from storage $2 \rightarrow 3$ and the sections from storage $1 \rightarrow 2$ because storage 2 has a very small capacity compared to the material output required per day. So the schedule of the section from storage $2 \rightarrow 3$ must be the same as that from storage $1 \rightarrow 2$. It can be seen that the firm's schedule is not unique, because, first, different combinations of shifts can be obtained for the two sections from storage $1 \rightarrow 2$; second, the required shifts can be scheduled to be operated during different hours of the day. The total electric power demand of the manufacturing firm, for a given set of schedules at time $n$, can then be described as follows:

$$
\begin{gathered}
\mathrm{P}_{\mathrm{T}}[\mathrm{n}]=\mathrm{P}_{01}[\mathrm{n}]+\mathrm{P}_{12 \# 1}[\mathrm{n}]+\mathrm{P}_{12 \# 2}[\mathrm{n}]+\mathrm{P}_{23}[\mathrm{n}]+\mathrm{P}_{30}[\mathrm{n}] \\
+\mathrm{P}_{4}[\mathrm{n}]+\mathrm{P}_{5}[\mathrm{n}] .
\end{gathered}
$$

Note that M will give us a set of schedules for each section or department of the plant; i.e., when, and for how long, a certain department has to be in operation. We also know from 3.2 and Chapter II, that conditional load can be computed if we know when and for how long a certain department is on. Therefore, the expected value and the autocorrelation function of the residuals for $\mathrm{P}_{\mathrm{T}}[\mathrm{n}]$ can easily be computed. The examples of this type of computation for the seven firms under study are given in Chapter V. The examples of how to handle the crosscorrelation of the residual between processes that are coupled will be discussed in section 3.4.

### 3.4 Different Types of Plants

From the flow and storage structure described previously, manufacturing plants can be divided into roughly two types.
(1) Coupled Case

This case includes plants that make a final product which has to go through many different production processes. The processes are in series, and are connected by storage areas. That is to say, an intermediate product from one particular process is stored for a certain length of time before it is subjected to another production process. Plants which fall into this category include automobile plants, soap plants, cement plants, steel plants, etc.

When some of the storages are small compared to the material produced in a day by a process that is feeding it into storage, or that is taking material from storage, then these processes that are connected to the storage can be called 'coupled'. One extreme case is when two production processes are in series, with no storage in between. For such a case, when one process is on, the other process also must be on. Such a situation may cause problems to the manager trying to reschedule one process and not the other.
(2) Uncoupled Case

Consider the manufacturing plant described above, in which the processes are coupled. Suppose the manager is willing to buy or build a large storage area and add it to his present storage, which is small compared to the production rates; in this case, the two processes that were previously coupled could be made uncoupled. The key here is to have very
large areas. For example, a knife has to go through many processing stages before it is considered finished. However, since knives are small, they usually do not present a storage problem for day-to-day operation. A knife company's processes are therefore uncoupled.

Processes could be uncoupled because the company's products must go through only one process, or use only one piece of equipment. For example, in a plastic injection molding firm, each employee runs a machine which takes in raw material as input, and puts out a finished product. This is a classic case of parallel production.

When processes are not coupled, one could reschedule one process to a different time without having to consider its effect on other processes.

In general, most plants are mixed in the sense that they have some, but not all, coupled processes.

In section 3.2 it was pointed out that a group of tightly coupled machines can be modeled as an elementary random process. The expected load and the autocorrelation function of the residual load for the entire machine group) can be directly computed without considering the individual machine. If two groups of machines are uncoupled, they are independent in the statistical sense. When they are independent, the crosscorrelation function of the residual load for the two groups is zero. However, if two groups are coupled, then they are likely to be dependent in the statistical sense.

This type of coupled process might be modeled as a four-state semiMarkov process. Two coupled processes can be made independent by
specifying a set of feasible schedules for each of the groups. This set of schedules can be obtained from the plant managers. In some cases, two groups of dependent processes can be made independent by introducing pseudo (XL) for one group. For example, when a group of machines with large electric capacity, say 500 kW when on, is coupled to a group of smaller capacity, say 50 kW when on. Suppose the group of machines with 50 kW capacity is on $50 \%$ of the time when the 500 kW -capacity group is on, and only $10 \%$ of the time when the 500 -kW-capacity group is off. Instead of modeling these two processes as two dependent processes, they could be modeled as two independent processes, with one pseudo-process of 520 kW when on, and another of 5 kW at continuous operation.

### 3.5 Adaption of Theory to Modeling of Actual Plants

In this chapter, it was pointed out that an industrial plant can be modeled using flow and storage ideas. This kind of model, when available, would offer powerful insights into how production processes and machines are coupled or how the processes can be scheduled. But such a model is not always available for many actual plants. In some cases, the plant might be too complicated to model, or the flow rates and storage capacities might not be well defined, or might be unknown.

For example, what is the flow rate of a machine of a company that makes plastic cups or knives of various sizes? Theses types of flow rates are difficult to quantify because the products are not homogeneous. The amount of machine time needed is not always proportional to the weight of the products, or the sizes. In contrast, a cement grinding machine and an iron foundry have production rates easier to quantify. In many cases, the flow rates and storage capacity of a company might be measured indirectly; such measurement might consist of a set of statements as explained in the following example:


We have two production processes, $A$ and $B$, connected by a storage area. Suppose the proper information for describing the processes are, say, that process A must operate 10 hours/day, and B must operate 8 hours/day. Also, the storage has capacity of storing products produced by A for 6 hours when $B$ is off, etc. This type of information is easier to obtain from the
management's records. This type of information is usually derived, even when the more quantitative flow rates and storage capacities types of data are available.

It should be noted that most of the detailed models for production processes and random processes are not always needed if one is trying to model the electric load of a production plant. In many cases, partially aggregated electric stocks and usage patterns for groups of machines is sufficient. However, in the development of load modeling methodology, it is important to start with a detailed analysis, so that the model is consistent and important aspects are captured.

### 3.6 Discussion and Perspectives

In the flow and storage aspect of the load model, we considered the physical structure of a firm. A manufacturing plant can be described as an interconnection of a set of storage areas and production processes. Each storage and flow process has an associated constraint. When we know how much final production is needed, then the flow and storage aspect of the firm gives a set of feasible schedules. The number of shifts needed per day for each production process is included in this schedule.

The flow/storage aspect of the model is used mostly as a conceptual guide. But, it provides us with many insights, such as
(1) How processes are coupled
(2) How a group of machines can be treated collectively as one block
(3) How feasible schedules can be found
(4) How firms are different from each other.

The flow/storage aspect is one important aspect of industrial electric load models, because without this type of understanding, the problem of modeling the electric load of each firm seems conceptually very difficult and hopeless.

## CHAPTER IV

TIME SERIES ANALYSIS AND MODEL VERIFICATION

The main purpose of this chapter is to show how the industrial load model can be verified. Time series analysis of load was done because quantities, such as the sample mean of load and the time-average sample autocorrelation function, are needed for model verification. The total electric load of a plant is a random process. It is a fluctuating quantity in time. For example, see the load profiles for different companies as given in Chapter V. It is not meaningful to reproduce the fluctuating electric load curve from the theory or the model. The correct procedure is to compare the expected load and the autocorrelation function derived from the model as described in Chapters II and III with the sample mean of load and the time-average sample autocorrelation function computed from time-series data.

In section 4.1, the procedure of how to compute the time-average sample from the time series data is given. Section 4.2 discusses the general mathematical representation and analysis of time-series load data. In section 4.3 the discussion of the requirements and effects of data length on the standard deviation of the sample mean, and of the sample autocorrelation function, is given. Section 4.4 talks about methods for the verification of the model used in this research, and discusses the philosophy and possible improvement of the model verification procedure.

### 4.1 Analysis of the 15 -Minute Average Time Series Load Data

The 15 -minute time-series load at time $\mathrm{n}, \mathrm{Z}[\mathrm{n}]$, can be divided into the daily or weekly cyclical component, $m[n]$ and the residual, $r[n]$, i.e.:

$$
Z[n]=m[n]+r[n]
$$

In this document, time-series analysis is done under the assumption that the daily cyclical component is important for the weekdays. The weekly cycle is ignored in this discussion, because, first, there is only a limited amount of data available for each company. Secondly, three out of four of the companies that were studied in detail do not have a weekly cycle type of variation from one weekday to another. Other, more complicated but theoretically possible analyses are discussed in section 4.3. The computer programs used for the computation of the present section can be found in Appendix D.
4.1.1 Computation of the 15-Minute Average Sample Means of a Daily Cycle Model

Let us hypothesize a daily cycle model with the time series load at $n, Z[n]$, given as

$$
\mathrm{Z}[\mathrm{n}]=\mathrm{m}_{\mathrm{d}}[\mathrm{n}]+\mathrm{r}_{\mathrm{d}}[\mathrm{n}]
$$

The sample mean $\hat{m}_{d}[n]$ and the residual $\hat{r}_{d}[n]$ computed from time-series data for the daily cycle model are given below.

Let us construct a new time-series load $Z^{\prime}[n]$ from $Z[n]$ that was recorded. $Z^{\prime}[\mathrm{n}]$ will not have load data for Saturday and Sunday. $\mathrm{Z}^{\prime}[\mathrm{n}]$ is the time series load for Monday through Friday of each week, over
several weeks.
Let $N_{o d}$ be the number of weekdays whose time-series load is to be used. Then by definition, we write

$$
\hat{m}_{d}[n]=\frac{1}{N_{o d}} \sum_{k=0}^{N_{o d}^{-1}} Z^{\prime}[n+96 k]
$$

(There are 96 quarter-hours in a day.) $\hat{m}_{d}[n]$ represents the daily cycle load and is periodic of period 96 , the number of 15 -minute intervals in a day, when weekends are excluded in our time-series data.

$$
\hat{m}_{d}[n]=\hat{m}_{d}[n+96 m]
$$

for $m, n$ integers.
4.1.2 Computation of the Residual of the Daily Cycle Model

By definition,

$$
\hat{\mathrm{r}}_{\mathrm{d}}[\mathrm{n}]=\left\{\mathrm{Z}^{\prime}[\mathrm{n}]-\hat{\mathrm{m}}_{\mathrm{d}}[\mathrm{n}]\right\}
$$

$\hat{r}_{d}[n]$ is assumed to be a stationary random process at least within a shift. For the four cases studied in detail in Chapter V, three of the firms are found to be approximately stationary during all the hours of each weekday.

### 4.1.3 Computation of the Time-Average Sample Autocorrelation

## Function of the Residual

Suppose $\hat{\mathrm{r}}_{\mathrm{d}}[\mathrm{n}]$ is stationary during all the hours of all weekdays, then $\hat{R}_{d}[m]$, which is the time-average sample autocorrelation function, can then be written as

$$
\hat{R}_{d}[\xi]=\frac{1}{N_{o s}-\xi} \quad N_{o s}-\xi \hat{n}_{\mathrm{N}=1} \hat{r}_{d}[n] \hat{r}_{d}[n+\xi]
$$

where $N_{o s}$ is the number of sample points and $N_{o S}=N_{o d} \times 96 ; m$ and $\xi$ are integers.

For the case where $r_{d}[n]$ is stationary only over a short time span of each day, such as all the first shift, the autocorrelation function that is valid during this time interval can be computed by using another new set of time-series load $Z^{\prime \prime}[n]$, which consists of the load from the particular interval of interest. We will not, however, be dealing with this case of a non-stationary random process.
4.2 General Mathematical Representation and Analysis of Time-Series Data

Assume a simple case in which time-series data $Z[n]$ can be written as

$$
z[n]=\sum_{k=1}^{3}\left\{a_{k} \sin k \omega_{0} n+b_{k} \cos k \omega_{0} n\right\}+c+r[n]
$$

Note that it is assumed that only six Fourier series coefficients are needed.

There are many ways to generalize this case, but our objective is only to present the mathematical formulation extendable to other cases of interest. Woodard [ 6 ] discussed the idea of representing the load of a day in terms of Fourier series whose coefficients are slowly time-varying, and whose weekly variation is specified by coefficients from a weekly block, which in turn is specified by coefficients from a yearly block.

In this section we will discuss how the random aspect of the load can be incorporated into the derivation of the state space representation.

The advantage is that once a problem is formulated into a state space form, one could use some well known mathematical techniques, such as least-square fitting, or Kalman filtering methods as described, for example, in Schweppe [ 15 ] to estimate all the parameters of interest. Define

$$
\begin{aligned}
& H_{f}[n]=\left[\begin{array}{llllll}
\sin \omega_{0} n & \sin 2 \omega_{0} n & \sin 3 \omega_{o} n & \cos \omega_{o} n & \cos 2 \omega_{0} n & \cos 3 \omega_{o} n
\end{array}\right] \\
& x_{f}^{T}[n]=\left(a_{1}[n] a_{2}[n] a_{3}[n] b_{1}[n] b_{2}[n] b_{3}[n] \quad c[n]\right) .
\end{aligned}
$$

In state space form, we then have

$$
\begin{aligned}
& x_{f}[n+1]=x_{f}[n] \\
& Z[n]=H_{f}[n] x_{f}[n]+r[n]
\end{aligned}
$$

$r[n]$ is the correlated residual. Therefore, to represent it in standard white-noise form, we can augment the state as follows.

Define the following state-space relation:

$$
\mathrm{x}_{\mathrm{r}}[\mathrm{n}+1]=\Phi_{\mathrm{r}} \mathrm{x}_{\mathrm{r}}[\mathrm{n}]+\underline{\mathrm{w}}[\mathrm{n}]
$$

where

$$
\Phi_{\mathrm{r}}=\left[\begin{array}{lll}
\phi_{1} & 0 & 0 \\
0 & \phi_{2} & 0 \\
0 & 0 & \phi_{3}
\end{array}\right]
$$

if we assume that $\mathrm{x}_{\mathrm{r}}[\mathrm{n}]$ is a three-state vector, for simplicity.
$r$ [ $n$ ] can then be represented as

$$
\mathrm{r}[\mathrm{n}]=\mathrm{x}_{\mathrm{r}}[\mathrm{n}]
$$

Suppose

$$
E\left\{\underline{w}[n] \underline{w}^{T}[n]\right\}=\left[\begin{array}{lll}
q_{1} & 0 & 0 \\
0 & q_{2} & 0 \\
0 & 0 & q_{3}
\end{array}\right]
$$

which is equivalent to

$$
\begin{aligned}
R_{r}[\tau] & =\sum_{i=1}^{3} q_{i} e^{-\pi_{i}|\tau|} \quad \text { where } \phi_{i}=e^{-\pi_{i}} \\
\tau & =1,2,3, \ldots
\end{aligned}
$$

where $\pi_{i}$ are prechosen. $\pi_{i}$ is equivalent to $\lambda_{i}$ as discussed in Chapter II. Therefore we know how to estimate them. Other methods of choosing $\pi_{i}$ are discussed in Lanning and Batting [ 30 ] or in Woodard [ 29 ]. We then have

$$
\begin{gathered}
{\left[\begin{array}{c}
x_{f}[n+1] \\
x_{r}[n+1]
\end{array}\right]=\left[\begin{array}{cc}
I & 0 \\
0 & \Phi_{r}
\end{array}\right]\left[\begin{array}{c}
x_{f}[n] \\
x_{r}[n]
\end{array}\right]+\left[\begin{array}{c}
0 \\
\underline{w}[n]
\end{array}\right]} \\
z[n]=\left[\begin{array}{ll}
H_{f}[n] & I
\end{array}\right]\left[\begin{array}{c}
x_{f}[n] \\
x_{r}[n]
\end{array}\right]
\end{gathered}
$$

Therefore, in standard white-noise form, we have:

$$
\begin{aligned}
& x[\mathrm{n}+1]=\Phi \mathrm{x}[\mathrm{n}]+\mathrm{Gw}[\mathrm{n}] \\
& \mathrm{Z}[\mathrm{n}]=\mathrm{H}[\mathrm{n}] \mathrm{x}[\mathrm{n}]
\end{aligned}
$$

where

$$
\begin{aligned}
& \mathrm{x}[\mathrm{n}]=\left[\begin{array}{c}
\mathrm{x}_{\mathrm{f}}[\mathrm{n}] \\
\mathrm{x}_{\mathrm{r}}[\mathrm{n}]
\end{array}\right], \\
& \Phi=\left[\begin{array}{ll}
\mathrm{I} & 0 \\
0 & \Phi_{\mathrm{r}}
\end{array}\right] \quad \mathrm{G}=\left[\begin{array}{ll}
0 & 0 \\
0 & \mathrm{I}
\end{array}\right], \quad \mathrm{H}[\mathrm{n}]=\left[\mathrm{H}_{\mathrm{f}}[\mathrm{n}] \mathrm{I}\right] \quad .
\end{aligned}
$$

### 4.3 Analysis of the Variance of the Sample Mean and the Variance of

 Time-Average Sample Autocorrelation Function4.3.1 The Expected Value and the Variance of the Sample Mean $Z^{\prime}[\mathrm{n}]$, as described previously, is constructed from $Z[n]$ by throwing away the time-series load data for Saturday and Sunday. This is done because the weekend load has different statistics from those of weekdays. The daily sample mean from the daily cycle model, as described in section 4.1, is

$$
\hat{m}_{d}[n]=\frac{1}{N_{o d}} \sum_{i=0}^{N_{o d^{\prime}}} Z^{\prime}[n+i 96]
$$

where $\mathrm{N}_{\mathrm{od}}$ is the number of working days that is being considered.
It is easy to show that the expected value of the sample mean is the true mean itself:

$$
\begin{aligned}
E\left\{\hat{m}_{d}[n]\right\} & =\frac{1}{N_{o d}} \sum_{i=0}^{N_{o d}-1} E\left\{Z^{\prime}[n+i 96]\right\} \\
& =\frac{1}{N_{o d}} \sum_{i=0}^{N_{O d^{-}}-1} \mu_{d}[n]=\mu_{d}[n]
\end{aligned}
$$

where $\mu_{d}[n]$ is the true mean or the expected load for a certain time index, $n$, of a day and $\mu_{d}[n]=\mu_{d}[n+96]$.

The variance of the sample mean, $\sigma_{m}$, can be derived as follows. By definition,

$$
\sigma_{\mathrm{m}}^{2}=E\left\{\left[\hat{m}_{\mathrm{d}}[\mathrm{n}]-\mu_{\mathrm{d}}[\mathrm{n}]\right]^{2}\right\}
$$

Let us define $a_{i}=Z^{\prime}\left[n^{+}(i-1) 96\right]-\mu_{d}[n]$;

Then, $\sigma_{m}^{2}=\frac{1}{\left(N_{o d}\right)^{2}} \sum_{i=1}^{N_{\text {od }}} \sum_{j=1}^{N_{\text {od }}} E\left\{a_{i} a_{j}\right\} \quad$.
Assume that the residual load is a random process in stationary state for all hours of a weekday, and that $\phi[\xi]$ is the true autocorrelation function at time difference $\xi$.

By definition,

$$
E\left\{a_{i} a_{j}\right\}=\phi[(i-j) 96]
$$

Therefore,

$$
\left.\sigma_{m}^{2}=\bar{N}_{\text {od }}\right)^{2} \sum_{i=1}^{N_{\text {od }}} \sum_{j=1}^{N_{\text {od }}} \phi[(i-j) 96]
$$

Using the fact that $\phi$ is an even function, we have

$$
\sigma_{m}^{2}=\frac{1}{N_{\mathrm{od}}} \phi(0)+\frac{2}{\left(\mathrm{~N}_{\mathrm{od}}\right)^{2}} \sum_{i=1}^{N_{\mathrm{od}^{-1}}^{-1}\left(N_{o d}-i\right) \phi(96 i)}
$$

Remember, there are 96 quarter-hours per day.
The following table 4.3 .1 shows the value for $\sigma_{m}$ and $\left(\sigma_{m} / \hat{m}\right)$ for the four companies under consideration. The values for $\phi(0), \phi(96)$ and $\phi(96 \times 2), N_{\text {od }}$ are taken from Chapter $V$; $\hat{m}$ for the first shift is used in computing $\left(\sigma_{m} / \hat{m}\right)$. The theoretically computed values for the autocorrelation are used for $\phi$.

| Company | $\left\lvert\, \begin{aligned} & \phi[0] \\ & \left(\text { in } \mathrm{kW}^{2}\right) \end{aligned}\right.$ | $\begin{aligned} & \phi[96] \\ & \left(\text { in } k W^{2}\right) \end{aligned}$ | $\begin{aligned} & \phi[192] \\ & \left(\text { in } \mathrm{kW}^{2}\right) \end{aligned}$ | $\mathrm{N}_{\text {od }}$ | $\begin{gathered} \sigma_{\mathrm{m}} \\ (\mathrm{in} \mathrm{~kW}) \end{gathered}$ | $\hat{m}$ for lst Sh. (in kW) | $\begin{aligned} & \left(\sigma_{m} / \hat{m}\right) \\ & (\text { in } \%) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Small Plastics | cs 2500 | 500 | 125 | 30 | 11 | 770 | 1.4\% |
| Brush Co. | 13200 | 1320 | 0 | 30 | 23 | 2700 | 0.9\% |
| Soap Co. | 302500 | 0 | 0 | 30 | 100 | 2500 | 4.0\% |
| Abrasive | 6400 | 0 | 0 | 9 | 27 | 3800 | 0.7\% |

Table 4.3.1
The Variance of the Sample Mean and Related Information for Various Companies

### 4.3.2 The Expected Value and Variance of the Time-Average Sample

## Autocorrelation Function

The continuous time version of this present subsection is discussed in Lanning and Battin [ 30 ]. A discrete time version is given in Wilks [ 31 ]. However, Wilks did not consider the case where the expected load or the sample mean is periodic and varying with time, as is the case here.

Let $N_{\text {Os }}$ be the number of sample points, and let $\quad N_{o s}=N_{o d} \times 96$. $N_{\text {od }}$ is the number of weekdays of interest, as defined in the previous subsection. The time-average sample autocorrelation function $R[\xi]$ is

$$
\hat{R}[\xi]=\frac{1}{N_{o s}-\xi} \sum_{n=1}\{\hat{r}[n] \hat{r}[n+\xi]\}
$$

where

$$
\hat{r}[n]=z^{\prime}[n]-\hat{m}_{d}[n]
$$

$Z^{\prime}[\mathrm{n}]$ and $\hat{m}_{d}[\mathrm{n}]$ are as defined before. Note that we are considering only the case in which the residual load is in stationary state during all weekdays. Subscript $d$ has been dropped from $\hat{R}[\xi]$ and $\hat{r}[n]$ for notational convenience. By taking the expectation of $\hat{R}[\xi]$, we have

$$
E\{\hat{R}[\xi]\}=\frac{1}{N_{o S}-\xi} \sum_{n=1}^{N_{o s}^{-} \xi} E\{\hat{r}[n] \hat{r}[n+\xi]\}
$$

Let $\mu_{d}[\mathrm{n}]$ and $\phi[\xi]$ be the true expected load and the true autocorrelation function of the residual, and let

$$
\alpha[\mathrm{n}]=\mathrm{Z}^{\prime}[\mathrm{n}]-\mu_{\mathrm{d}}[\mathrm{n}]
$$

and

$$
\beta[\mathrm{n}]=\hat{m}_{\mathrm{d}}[\mathrm{n}]-\mu_{\mathrm{d}}[\mathrm{n}]
$$

Then

$$
\begin{aligned}
E\{\hat{r}[n] \hat{r}[n+\xi] & =E\{(\alpha[n]-\beta[n])(\alpha[n+\xi]-\beta[n+\xi])\} \\
& =E\{\alpha[n] \alpha[n+\xi]-\alpha[n] \beta[n+\xi]-\beta[n] \alpha[n+\xi]+\beta[n] \beta[n+\xi]\}
\end{aligned}
$$

Let $i$ be the label of the weekday that is of interest, and let $i=0$ represent the day that $n$ belongs to, i.e., if $n=965$, then $n$ belongs to the eleventh day of the time series data $Z^{\prime}[n]$, and therefore the eleventh day is labeled as the day $i=0$. For this case, the first day is $i=-10$. For an arbitrary $n$, we have

$$
\begin{aligned}
& E\{\alpha[n] \alpha[n+\xi]\}=\phi[\xi] \\
& E\{\alpha[n] \beta[n+\xi]\}=\frac{1}{N_{o d}}\left\{\sum_{i=1-k}^{N_{o d}^{-k}} \phi[\xi+96 i]\right\}
\end{aligned}
$$

Note that $k$ is the order of the weekday that $n$ belongs to; there are 96 quarter hours in a day.

$$
\begin{aligned}
& E\{\beta[n] \alpha[n+\xi]\}=\frac{1}{N_{o d}}\left\{\sum_{i=1-k}^{N_{o d}-k} \phi[\xi+96 i]\right\} \\
& E\{\beta[n] \beta[n+\xi]\}=\frac{1}{N_{o d}}\left\{-\phi[\xi]+\frac{2}{N_{o d}} \sum_{i=1-k}^{N_{o d}-k}\left(N_{o d}-i\right) \phi[\xi+96 i]\right\}
\end{aligned}
$$

By first finding $E\{\hat{r}[n] \hat{r}[n+\xi]\}$ from the above expressions and substituting into the expression for $E\{\hat{R}[\xi]\}$, we have

$$
\mathrm{E}\{\hat{\mathrm{R}}[\xi]\}=\frac{1}{\mathrm{~N}_{\mathrm{os}}-\xi} \sum_{\mathrm{n}=1}^{\mathrm{N}_{o s}-\xi}\left\{\phi[\xi]\left(1-\frac{1}{\mathrm{~N}_{\mathrm{od}}}\right)-\frac{1}{\left(\mathrm{~N}_{\mathrm{od}}\right)^{2}} \sum_{i=1-k}^{N_{o d}-\mathrm{k}} 2 i \phi[\xi+96 i]\right\}
$$

If $N_{o s}$ is very large compared to $\xi$ and $N_{o d}$ is large, then

$$
E\{\hat{\mathrm{R}}[\xi]\} \simeq \phi[\xi]\left(1-\frac{1}{\mathrm{~N}_{\mathrm{od}}}\right)-\frac{1}{\left(\mathrm{~N}_{\mathrm{od}}\right)^{3}} \sum_{\mathrm{k}=1}^{\mathrm{N}_{\mathrm{od}}} \sum_{i=1-k}^{\mathrm{N}_{\mathrm{od}^{-k}}} 2 i \phi[\xi+96 i]
$$

The above relationship can be made exact if one is willing to throw away some data points of $Z^{\prime}[n]$ and $\hat{r}[n]$ by always using the same number of points for $\hat{r}[n] \hat{r}[n+\xi]$, which is an integer multiple of 96 . Let the second term of the right-hand side of the above relationship be $\theta[\xi]$, then

$$
\theta[\xi]=\frac{1}{\left(\mathrm{~N}_{\mathrm{od}}\right)^{3}} \sum_{\mathrm{k}=1}^{\mathrm{N}_{\mathrm{od}}} \sum_{i=1-\mathrm{k}}^{\mathrm{N}_{\mathrm{od}}-\mathrm{k}} 2 \mathrm{i} \phi[\xi+96 \mathrm{i}]
$$

which is equal to

$$
\theta[\xi]=\frac{1}{\left(N_{o d}\right)^{3}} \sum_{i=-N_{o d}}^{N_{\text {od }}} 2 i\left(N_{o d}-i\right) \phi[\xi+96 i]
$$

Assume that the maximum time scale of interest for $\xi$ is 196 quarter hours, or 48 hours, and we then have

$$
\theta[\xi] \leq \frac{2 N_{o d}}{\left(\mathrm{~N}_{\mathrm{od}}{ }^{3}\right.}\left\{\sum_{\mathrm{k}=-\infty}^{\infty}(\mathrm{k}+\delta) \phi[96 \mathrm{k}]+\phi[0]\right\}
$$

where

$$
\delta= \begin{cases}0 & \text { for } \quad|\xi| \leq 96 \\ 1 & \text { for } 96 \leq|\xi| \leq 192\end{cases}
$$

For the purpose of this research project, it can be seen in Chapter $V$ that the time constants for $\phi[\xi]$ are less than one day. See Table 4.3.2 for the effective time constant of $\phi[\xi]$ for the various cases of interest. The knowledge of how fast $\phi[\xi]$ approaches zero as $\xi$ becomes large is needed to see that $\phi[\xi]$ approaches zero for large $N_{o d}$. Therefore, each case must be examined individually.

Suppose $\phi[\xi]=q_{1} e^{-\xi / K_{1}}$,

Then

$$
\theta[\xi]<\frac{2 q_{1}}{\left(N_{o d}\right)^{2}}\left\{\sum_{k=-\infty}^{\infty} k e^{-96 k / K_{1}}+2 \delta \sum_{k=0}^{\infty} e^{-96 k / K_{1}}+(1-\delta)\right\}
$$

The two terms on the right-hand side are bounded as follows:

$$
\sum_{\mathrm{k}=-\infty}^{\infty} \mathrm{ke}^{-96 \mathrm{k} / \mathrm{K}_{1}}{ }^{1} \quad 2 \int_{0}^{\infty} \mathrm{xe}^{-96 \mathrm{x} / \mathrm{K}_{1}} \mathrm{dx}=2\left(\frac{\mathrm{~K}_{1}}{96}\right)^{2}
$$

and

$$
\sum_{k=0}^{\infty} e^{-96 k / K_{1}}=\frac{1}{1-e^{-96 / K_{1}}}
$$

which gives

$$
\frac{\theta[\xi]}{\phi[0]}<\frac{4}{\left(\frac{N_{o d}}{o d}\left\{\left(\frac{K_{1}}{96}\right)^{2}+\frac{2 \delta}{1-e^{-96 / K_{1}}}+(1-\delta)\right\} . . . ~ . ~ . ~\right.}
$$

For the purpose of this report as shown in Chapter V, each $\phi[\xi]$ has more than one time constant, so effective time constant $\mathrm{T}_{\mathrm{e}}$ would be used for $K_{1} / 4$, i.e., $\phi\left[4 \mathrm{~T}_{\mathrm{e}}\right]=\phi[0] \mathrm{e}^{-1}$. Note that $K_{1}, \xi_{\max }, N_{\mathrm{od}}$ are given in quarter hours, therefore they have to be divided by four to convert them into hours. Table 4.3.2 gives us some idea of the bound for $\theta[\xi]$ for the four cases of interest.

| Company | Maximum <br> $\xi / 4$ <br> (hour) | $\mathrm{K}_{1} / 4$ <br> (hour) | $\mathrm{N}_{\text {od }}$ | for $\frac{\xi}{4} \leq 96$ <br> $\theta[\xi] \leq$ | for $96 \leq \xi \leq 192$ <br> $\theta[\xi] \leq$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Smal1 <br> Plastics | 50 | 12.5 | 30 | $\frac{\phi[0]}{197}$ | $\frac{\phi[0]}{86}$ |
| Brush | 50 | 7.5 | 30 | $\frac{\phi[0]}{205}$ | $\frac{\phi[0]}{103}$ |
| Soap | 50 | 4.5 | 30 | $\frac{\phi[0]}{217}$ | $\frac{\phi[0]}{110}$ |
| Abrasive | 8 | 3 | 9 | $\frac{\phi\left[\frac{0]}{20}\right.}{}$ |  |

Table 4.3.2
$\theta[\xi]$ is bounded by a number which is relatively small compared to $\phi[0]$ as shown in Fig. 4.3.2. for the four cases of interest when $\xi$ is relatively small. Further, from the inequality for $\theta[\xi] / \phi[0]$ given previously it can be seen that $\theta[\xi] / \phi[0]$ approaches zero as $N_{\text {od }}$ becomes large.

The variance of $\hat{R}[\xi]$ can be expressed as

$$
\sigma_{\hat{R}}^{2}[\xi]=\hat{E}\left\{(\phi[\xi]-\hat{R}[\xi])^{2}\right\}
$$

where $\phi[\xi]$ and $\hat{R}[\xi]$ are the true and the time-average sample autocorrelation function of the residual. The mathematical manipulation involved is unmanageable, so the following reasonable assumptions are made. First, the residual load is assumed to be a gaussian stationary random process. This is a reasonable assumption when the total load is the sum of a large number of independent loads and none of the loads is large enough
to be dominant, as explained in Chapter II. Second, instead of the sample mean $\hat{m}_{d}[n]$, the true mean $\mu_{d}[n]$ is used. This is a reasonable assumption when the samples or days of interest are large (described in subsection 4.3.1). Under the second assumption, $r[n]$ is used instead of $\hat{r}[n]$, as follows:

$$
r[n]=Z^{\prime}[n]-\mu_{d}[n]
$$

which gives

$$
\begin{aligned}
& \sigma_{R}^{\hat{R}[\xi]}=\phi^{2}[\xi]+\phi[\xi] E\left\{\frac{1}{N_{o s}^{-\xi}} \sum_{N_{0 s}-\xi}^{N_{o s}-\xi} r[n] r[n+\xi]\right\} \\
& +E\left\{\left(\frac{1}{N_{\text {os }}-\xi} \sum_{\mathrm{n}=1} \mathrm{r}[\mathrm{n}] \mathrm{r}[\mathrm{n}+\xi]\right)^{2}\right\}
\end{aligned}
$$

Since $\mu_{d}[n]$ is used, the middle term of the right hand side of the above relation can be simplifed to give

$$
\begin{aligned}
& \sigma_{\hat{R}}^{2}[\xi]=\phi^{2}[\xi]-2 \phi^{2}[\xi]+ \\
& \frac{1}{\left(N_{o s}-\xi\right)} \sum_{j=1}^{N_{o s}} \sum_{i=1}^{N_{o s}-\xi} E\{r[i] r[i+\xi] r[j] r[j+\xi]\}
\end{aligned}
$$

Note that to find $\sigma_{\hat{R}}^{2}[\xi]$, we must take the expected value of the product of four random variables. However, $r[n]$ is assumed to be a gaussian random process, therefore all the four random variables are jointly gaussian. The formula for handling the expected value of the product of a large number of jointly gaussian random variables is given in Lanning and Battin [ 23 ]. They also show how the formula is derived. However, for our
purpose here, their result will be used without repeating the proof. For the case of four jointly gaussian random variables $x_{1}, x_{2}, x_{3}$ and $x_{4}$, we have

$$
\mathrm{E}\left\{\mathrm{x}_{1} \mathrm{x}_{2} \mathrm{x}_{3} \mathrm{x}_{4}\right\}=\mathrm{m}_{12} \mathrm{~m}_{34}+\mathrm{m}_{13} \mathrm{~m}_{24}+\mathrm{m}_{14} \mathrm{~m}_{23}
$$

where

$$
m_{i j}=E\left\{x_{i} x_{j}\right\}
$$

By substituting $r[i], r[i+\xi], r[j], r[j+\xi]$ for $x_{1}, x_{2}, x_{3}$, and $x_{4}$, we have

$$
\begin{gathered}
E\{r[i] r[i+\xi] r[j] r[j+\xi]\}=\phi[\xi] \phi[\xi]+\phi[i-j] \phi[i-j] \\
+\phi[i-j-\xi] \phi[i-j+\xi] .
\end{gathered}
$$

Note that we have used the facts that $r[n]$ is a stationary gaussian random process and that $\phi[\xi]$ is an even function of $\xi$. Substituting the above relationship into the expression for $\sigma^{2} \hat{R}[\xi]$, we have

$$
\begin{aligned}
\sigma^{2} \hat{R}[\xi]=-\phi^{2}[\xi]+ & \frac{1}{\left(N_{o s}-\xi\right)^{2}} \sum_{j=1}^{N_{o s}-\xi} \sum_{i=1}^{N_{o s}-\xi}\left\{\phi^{2}[\xi]+\phi^{2}[i-j]\right. \\
& +\phi[i-j-\xi] \phi[i-j+\xi]\}
\end{aligned}
$$

Let $k=i-j$. We then have

$$
\sigma_{R}^{2} \hat{R}[\xi]=\frac{1}{\left(N_{o S^{-}}-\xi\right)^{2}} \sum_{j=1}^{N_{o s}-\xi} \sum_{k=1-j}^{N_{o s}-\xi-j}\left\{\phi^{2}[k]+\phi[k-\xi] \phi[k+\xi]\right\} .
$$

Let $f(k, \xi)=\phi\{k]+\phi[k-\xi] \phi[k+\xi]$.
Since $\phi[k]$ is an even function of $k$, then $f(k, \xi)$ is an even function of $k$ and $\quad \xi . \quad \sigma^{2} \hat{R}[\xi]$ then becomes

$$
\sigma_{\hat{R}[\xi]}^{2}=\left\{\frac{2}{\left(N_{o s}-\xi\right)^{2}} \sum_{k=1}^{N_{o s}-\xi-1}\left(N_{o s}-\xi-k\right) f(k, \xi)\right\}+\frac{\left(N_{o s}-\xi\right)}{\left(N_{o s}-\xi\right)^{2}} f(0, \xi)
$$

which gives

$$
\sigma_{R[\xi]}^{2 \hat{R}}=\frac{f(0, \xi)}{\left(N_{o s}-\xi\right)}+\frac{2}{\left(N_{o s}-\xi\right)} \sum_{k=1}^{N_{o s}-\xi-1}\left(1-\frac{k}{N_{o s}-\xi}\right) f(k, \xi)
$$

A bound for $\sigma_{R[\xi]}^{2}$ can be found. First note that

$$
\sigma^{2} \hat{R}[\xi] \leq \frac{f(0, \xi)}{\left.N_{o s}-\xi\right)}+\frac{2}{\left(N_{o s}-\xi\right)} \sum_{k=1}^{N_{o s}-\xi-1} f(k, \xi)
$$

which is

$$
\begin{aligned}
\sigma^{2} \hat{\mathrm{R}}[\xi] & \leq \frac{1}{\left(\mathrm{~N}_{\mathrm{OS}}-\xi\right)}\left(\phi^{2}[0]+\phi^{2}[\xi]\right)+\frac{2}{\left(\mathrm{~N}_{\mathrm{OS}}-\xi\right)} \sum_{\mathrm{k}=1}^{\mathrm{N}_{\mathrm{os}}^{-\xi-1}} \phi^{2}[\mathrm{k}] \\
& +\frac{2}{\left(\mathrm{~N}_{\mathrm{os}}-\xi\right)} \sum_{\mathrm{os}} \sum_{=}^{-\xi-1} \phi[\mathrm{k}-\xi] \phi[\mathrm{k}+\xi]
\end{aligned}
$$

Usịng Schwartz inequality as described, for example, in Feller [ 26] or Davenport [ 32 ], we have

$$
\sum_{k=1}^{\mathrm{N}_{\mathrm{os}}-\xi-1} \phi[k-\xi] \phi[k+\xi] \leq\left(\sum_{k=1}^{\mathrm{N}_{\mathrm{os}}-\xi-1} \phi^{2}[k-\xi] \sum_{\mathrm{k}}^{=} \mathrm{N}_{\mathrm{os}}-\xi-1 \phi^{2}[k+\xi]\right)^{1 / 2}
$$

Suppose $\sum_{k=0} \phi^{2}[k]$ is finite, then the right-hand side of the above inequality is bounded by $\sum_{k=0}^{\infty} \phi^{2}[k] . \sum_{k=0}^{\infty} \phi^{2}[k]$ is finite for a real physical quantity, such as load or signal, etc. For $\phi[k]$ of a two-state Markov process or binomial process (as described in Chapter II), this condition is always satisfied. Hence, $\sigma^{2} \hat{R}[\xi]$ is bounded as

$$
\sigma^{2} \hat{R}[\xi] \leq \frac{1}{\left(N_{O S}-\xi\right)}\left(\phi^{2}[0]+\phi^{2}[\xi]\right)+\frac{4}{\left(N_{O S}-\xi\right)} \sum_{k=1}^{\infty} \phi^{2}[k] .
$$

Suppose $\phi[k]=q_{1} e^{-k / K_{1}}$,
Then $\quad \sigma^{2} \hat{R}[\xi] \leq \frac{q_{1}{ }^{2}}{\bar{N}_{o s}-\xi}\left(1+e^{-|\xi| / K_{1}}+4 \sum_{k=0}^{\infty} e^{-2|k| / K_{1}}\right)$
which gives

$$
\sigma^{2} \hat{R}[\xi] \leq \frac{\mathrm{q}_{1}^{2}}{\mathrm{~N}_{\mathrm{os}}-\xi}\left(1+\mathrm{e}^{-|\xi| / \mathrm{K}_{1}}+\frac{4}{1-\mathrm{e}^{-2 / \mathrm{K}_{1}}}\right)
$$

Since $e^{-2|\xi| / K_{1}} \leq 1$ for all $\xi$, we have

$$
\sigma_{\hat{R}[\xi]}^{2} \leq \frac{2 q_{1}^{2}}{\mathrm{Nos}^{-} \xi}\left(1+\frac{2}{1-\mathrm{e}^{-2 / \mathrm{K}_{1}}}\right)
$$

Again, using the effective time constants for $\phi[\xi]$ of various cases as explained before, the normalized $\sigma_{R}^{2} \hat{R}[\xi]$ is given in Table 4.3.3 for cases of interest. See Chapter $V$ for how $T_{e}, N_{o s}, \xi_{\max }$ are obtained.

| Company | $\begin{aligned} & \mathrm{T}_{\mathrm{e}}=\mathrm{K}_{1} / 4 \\ & \text { (hours) } \end{aligned}$ | $\begin{aligned} & \mathrm{N}_{\mathrm{OS}} / 4 \\ & \text { (hours) } \end{aligned}$ | $\frac{\xi_{\max }}{4}$ | $\frac{\sigma^{2} \hat{R}[\xi]}{\phi[0]} \leq$ | $\begin{aligned} & \frac{\sigma \hat{\mathrm{R}}[\xi]}{\mathrm{V}_{\phi}[0]} \\ & (\mathrm{in} \%) \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Small Plastics | 12.5 | 720 | 50 | 3.9 | 19.7 |
| Brush | 7.5 | 720 | 50 | 2.4 | 15.5 |
| Soap | 4.5 | 720 | 50 | 1.5 | 12.2 |
| Abrasive | 3 | 216 | 8 | 3.4 | 18.4 |

Table 4.3.3

For most purposes where $K_{1} \gg 2$ and $N_{o s} \gg \xi_{\max }$, a very simple expression for the bound of $\sigma^{2} \hat{R}[\xi]$ can be derived as follows:

$$
\text { Since } e^{-2 / K_{I}} \simeq 1-\frac{2}{K_{1}} \text { for } K_{l} \gg 2
$$

and

$$
N_{o s}-\xi_{\max } \simeq N_{o s} \quad \text { for } N_{o s} \gg \quad \xi_{\max }
$$

we then have

$$
\sigma^{2} \hat{\mathrm{R}}[\xi] / \phi[0] \leq \frac{2 \mathrm{~K}_{\mathrm{I}}}{\mathrm{~N}_{\mathrm{os}}}
$$

### 4.4 Methodology and Philosophy of Model Verification

The model verification as described in this section involves mainly the comparison of the theoretical expected load and the theoretical autocorrelation function with their time-average counterpart derived from time series data as described in Table 4.4.1

$$
\begin{gathered}
\text { Theory } \\
{\mathrm{E}\left\{\mathrm{P}_{\mathrm{T}} \mid \text { shift i }\right\}}^{\sigma_{\mathrm{T}}(\text { shift i) }} \\
\mathrm{R}_{\mathrm{T}}[\mathrm{~m} \mid \text { shift i }]
\end{gathered}
$$

Measured
$\hat{\mathrm{m}}_{\mathrm{T}}$ (shift i)
$\hat{\sigma}_{\mathrm{T}}($ shift i)
$\hat{\mathrm{R}}_{\mathrm{T}}[\mathrm{m} \mid \operatorname{shift}$ i]

Table 4.4.1 Comparison of Theoretical and Measured Quantities
Where $i=1,2,3,4$ represent the first, second, third, and weekend shifts. The first line of Table 4.4 .1 represents the expected load, the second line represent the standard deviation of the residual load of a shift $i$, and the third line represents the autocorrelation functions. The theoretical quantities are derived by using the theory, as given in Chapters II and III, the inventory and usage data of equipment collected, and the computer program given in Appendix D. The measured quantities were computed from time-series data using the formula given in section 4.1 of this chapter and the computer program in Appendix D. Chapter $V$ gives us the actual case studies comparison of the six out of seven industrial customers that are being studied. However, the statistical study and comparison are made only for four cases. The difference between the theory and the measured expected load, standard deviation and autocorrelation function were found to be about 10 percent. This is within the bound for the variance of the quantities described above, as
derived in section 4.3 .
For the remaining two companies, only rough comparisons were made to see if the load shape, the load during a certain shift, the approximate energy usage, and the peak load of a month derived from theory were consistent and agree with the measured quantities.

In all the above cases, the theoretical quantities were based on our knowledge of $X, L$, a and $\eta$ for each machine and for each shift being considered, if the machine was modeled as a two-state Markov process. Correction of Parameters:
$\mathrm{m}_{\mathrm{T}}[\mathrm{n}], \sigma_{\mathrm{T}}$ and $\mathrm{R}_{\mathrm{T}}[\mathrm{m}]$ derived from the model are corrected in two different ways. The first is to readjust $m_{T}[n], \sigma_{T}$, and $R_{T}[m]$ so that these quantities, during a given shift, agree reasonably well with the measured quantity at least within the predicted variance given in subsections 4.3 .1 and 4.3.2. If this is not the case, the parameters L , $a$, and $\eta$ have to be changed so that the statistical curves for the theoretical and measured cases agree. The second way is to readjust the parameters so that the expected value varies from one time index to another, that is to make $m_{T}$ a function of the 15 minute average time index, so that it will reflect the actual variation of load within a shift.

If $m^{\prime \prime}[n]$ and $R_{\alpha}^{\prime \prime}[0]$ are the corrected quantities for $\alpha^{\text {th }}$ machine, then

$$
m_{\alpha}^{\prime \prime}[n]=m_{\alpha}[n]+G_{\alpha}^{\prime}\left[\hat{m}_{T}[n]-m_{T}[n]\right\}
$$

where

$$
\mathrm{G}_{\alpha}=\frac{\mathrm{R}_{\alpha}[0]}{\mathrm{R}_{\mathrm{T}}[0]}
$$

Similarly,

$$
\mathrm{R}_{\alpha}^{\prime \prime}[0]=\mathrm{R}_{\alpha}[0]+\mathrm{G}_{\alpha}\left\{\hat{\mathrm{R}}_{\mathrm{T}}[0]-\mathrm{R}_{\mathrm{T}}[0]\right\}
$$

The choice of $G_{\alpha}$ here includes only the variance of the residual loads. To be exact, it is desirable to include the error and variance associated with the measurements, guesses and estimates of parameters that are made. This is an area where further research is desirable.

The transition rate $\lambda_{\alpha}$ could also be corrected. Correction of it is more involved, because $\lambda_{\alpha}$ 's are not additive. Note that $m_{\alpha}[\mathrm{n}]$ is constant over a time duration of one shift ( 8 hours). However, $m_{\alpha}^{\prime \prime}[n]$ varies from one 15 -minute time index to another. The corrected parameters $m_{\alpha}^{\prime \prime}[n], R_{\alpha}^{\prime \prime}[0]$ and $R_{\alpha}^{\prime \prime}[m]$ represent the estimated parameters based on all available information.

When one is trying to change the parameters so that the unexpected load, the standard deviation of the load, and the autocorrelation function derived from the model could be fitted to the measured values, one should retain the relationships of the above quantities with $x, L, a$, and $n$ as described in Chapter II. For example, if a process is a two-state Markov process, then we can write

$$
\begin{aligned}
& E\{P[n]\}=X L a \\
& R[0]=\sigma^{2}=(X L)^{2} a(1-a) \\
& \frac{1}{\lambda}=\frac{a(1-a)}{n} .
\end{aligned}
$$

The above three relationships are for the continuous time case, as given in section 2.4 and the second and third relationships are represented graphically in Fig. 4.4.1. Section 2.4 also gives the $\sigma^{2}$ and $1 / \lambda$ for the


Fig. 4.4.1a


Fig. 4.4.1b
discrete time case for the time-average sample load $P[n]$. This will not be discussed again here.

The correction of parameters is an area where further work is desirable in how to efficiently choose $G_{\alpha}$. Formal parameter estimation methods using least-square fitting or maximum likelihood estimates are desirable. Work on formulating the problem and developing the algorithm could be carried out. The random process model used should not be limited to twostate Markov process; other types of random process models should be explored.

Another possible way of verifying the model is to check it, as well as its output, with the managers of each respective plant. A check can be made to see if the model structure, approximate parameters used, and many of the assumptions made in the model developmental stage are reasonable and correct. If some type of prediction on possible rescheduling of processes and change in load shape is made by the model, then this type of prediction can also be verified by checking with plant managers. The prediction of the change in load shape and other quantities under a changed schedule can be checked by comparing the predicted new load shape with the measured load shape under a given type of new schedule. This needs more field work and was not done in the present research project. It is an area where utilities or individual customers can continue to do more work. In some cases, the direct-load measurements and records for a section or group of equipment can be used in estimating and evaluating the parameters $X, L, n$, a etc. for the processes.

At present, many parameters are guessed values or estimates made independently. Therefore, one possible thing that could be done is to find out if the model and parameters are self-consistent. In general, for many cases, there is an infinite set of model structures and parameters that will be consistent with the given time-series data. The maximum likelihood method can be used to find the best model strucure and best parameters for the model.

### 4.5 Discussion and Perspectives

In this chapter, we discussed how the sample mean and the timeaverage autocorrelation function of the residual can be computed from timeseries data of load. Only the daily cycle variation for week days is considered, because there is not enough data to study the weekly cycle variation. The general mathematical structure of representing time-series load in the standard state space white-noise form is discussed. However, this mathematical structure is not used in any of the computations for this research project.

The expected value and the variance of the sample mean and the timeaverage sample autocorrelation function of the residual is analysed. The mathematical derivation is carried out because the specific results required for this research are not available in the form needed.

The verification of model consists of comparing the expected load and the time-average sample autocorrelation function, computed using the model with that computed using time-series data. Formal parameter estimation and validity testing methods were not used. This is felt to be unnecessary, because only limited time-series data were available.

## CHAPTER V

## APPLICATION OF THE MODELING METHODOLOGY: PHYSICAL MODEL

In this chapter, the idea and approach described in Chapters II, III, and IV will be used. In particular, it will be shown that if the electric stock data, the utilization of the electric stock, and the schedule of a manufacturing plant are available (which amounts to the fact that the vector M is given), the total hourly electric load shape for the plant can be obtained by using the approach described in Chapters II and III. Further, it will be shown that the expected load shape of the entire plant derived from the physical model agrees with and is consistent with the sample mean load shape computed from the time series data. The autocorre1ation function of the residual, computed from the model, is compared with that computed from the time series data.

For all cases, the assumption is made that the expected demand for each section is constant, or the random process is stationary, over the span of at least an eight hour shift. This simplifies the analysis. In actuality, the load of a particular shift of a particular section starts to increase one or two hours before the shift begins, and stays approximately constant during the shift. There are variations of load during lunch and coffee breaks during an eight-hour shift. For load forecasting when hourly details are required, the hourly detail variations of the demand for electric power for each section of the plant is needed. However, for many studies including the effect of electric rate structure on electric power demand load shape of a firm, the hourly details for each section are not necessary. Meaningful results can be obtained if only the constant
eight-hour block of expected demand is available for each plant section, even though the hourly detail of the load shape for each section is not available.

For the rest of this chapter, data used are taken from Appendices C.l to C.7. The installed capacity type of data is obtained from plant visits or companies' own survey records. The percentage of load on, L , is assumed to be 0.8 for most machines, unless it is found to be something different from data or records obtained from company personnel. For lighting, it is assumed to be 1 . The reason that L is smaller than 1 for most equipment is that engineers generally choose a motor of slightly larger capacity than needed, to allow for a safety margin.

L for the large plastic molding machines of Small Plastics and Brush Companies are deduced from direct measurements. XL for the tower of Soap Company is computed from survey data; however, it did check out reasonably well with direct measurements. Abrasive Company gave its L for most machines. The percentage of time on, $a$, and the number of starts/hour, $\eta$, are computed from company records or from guesses. For example, Small Plastic, Soap, and Abrasive Company have good records of their machine utilization.

It is important to note that all the guesses made in this document will be identified by using a small ' $g$ ' following the data. For instance, if $\mathrm{L}=0.8$ is a guess in a given Table, we would write it as $\mathrm{L}=0.8 \mathrm{~g}$. Otherwise, it can be assumed that a number or parameter is taken from records, nameplate readings, or direct measurement. Table 5.0.1 shows a list of symbols used with the value of parameters in various tables to identify how each parameter or number has been obtained.

Another fact to be noted is that the six cases considered in this chapter follow two different patterns. The data for the first four cases Small Plastics, Brush, Abrasive, and Soap Companies, were gathered during the summer of 1977. The data for the last two cases - Foundry and Printing Companies - were gathered during the summer of 1976. The type of data gathered, and the kind of analysis made for the cases of $\$ 5.1$ to $\S 5.4$ are superior to those of $\$ 5.5$ \& 5.6 . This is due to the improvement of our understanding of the industrial electric load modeling toward the latter part of this research project. The data for the last two companies (that gathered in 1976) are included because they are informative in their own right.

There is insufficient data from Knife Company for their utilization factor, La, for each group of equipment during each shift, so it is not analyzed in this chapter. However, it will be used for analysis in other chapters whenever possible.
$\left.\begin{array}{cc}\text { Symbol } & \text { Explanation of the Symbol } \\ \ell & \text { Guess } \\ \mathbf{r} & \begin{array}{c}\text { Company's log; for example, this could be the } \\ \text { actual log of a foreman or manager, etc. } \\ \text { Company's record, or publication; manager's or } \\ \text { engineer's estimate made for other purposes. }\end{array} \\ \text { D irect Measurement - or deduced from other } \\ \text { measurements }\end{array}\right]$

Table 5.0.1

### 5.1 Small Plastics Company

The description of this company is given in Appendix C.1. This company has flow and storage structure and equipment similar to that of Brush Company, because both are in the injection extmsion molding business. However, Brush Company is much larger than Small Plastics Company in terms of the sizes of their machinery and the level of electric power consumed. Figure 5.1.1 shows the measured time series load of Small Plastics Company for a typical week.

There are two major types of processes that consume electricity those processes that relate to production and those which are independent of production. The machines or equipment that are related to production include large plastic molding machines and small assembly line types of machines. There is a total of 23 plastic molding machines set up in parallel operation; therefore they are independent of each other. As explained in Appendix C.1, these plastics molding machines operate during all weekdays, three shifts per day. They are interrupted or shut down at a random time when a machine has a problem, or when a machine has finished producing a certain order. They could each be modeled as a two-state, stationary Markov process during all weekdays (see App. C.1). The necessary parameters for this model are $X, L, a$, and $\eta$ as explained in Chapter II, and X, $a, n$ for five months is given in Table C.1.3 of App. C.l. These parameters are derived from the company's detailed record of machine hours for each molding machine from January, 1977 to May, 1977.
$\mathrm{L}=0.44$ is used. This value of L is deduced from direct measurement of the cuurent of one of the three phases (See App. C.1). The equipment that does not related directly to production includes the lighting, heating, and air conditioning types of machinery.

Table 5.1.1 of this section and Table C.1.1 and C.1.3 of App. C. 1 show how the expected load for each group of equipment can be found for the first, second, and third shift. Note that we are assuming that the expected power demand is constant during each shift. When all the load for each group during each shift are summed together, we will have the expected load for each shift. The expected load for the lighting, airconditioning, compressors and small machines, varies slightly with each shift. However, the expected load for all large plastics machines which are on are assumed to be constant during the 24 hours/day five days/week for all working weekdays, and off during weekends.

Figure 5.1 .2 shows the comparison of the 15 -minute by 15 -minute expected load for a day, derived from the model as described above, with the sample mean load derived by using time series data of 30 weekdays of six weeks. The computer program, as given in Appendix D, is useful to compute the sample mean load. The daily cycle model as discussed in Chapter IV is used to compute the sample mean. The theoretical curve is computed without using a computer. Note that the theoretical curves do not have hour-by-hour variations. For the third shift, the model of Case 1 of Table 5.1 .1 shows higher constant expected demand than the expected load computed from time series data. The discrepancy was found
to be caused by the fact that we overlooked the firm's personnel problem for the third shift. The firm has trouble with workers quitting their jobs from the third shift, so that replacements must be found for the absentees. Therefore, some of the plastic molding machines have to be shut off during the third shift occasionally, as there are insufficient workers to tend them. The plant manager suggested that the third shift load for the total of all the plastic molding machines is on the average $10 \%$ lower than the total load for all such machines during the first and second shifts. The theoretical expected load for each of the three shifts that has been adjusted to account for this fact is given in Case 2 of Table 5.1.1, and is used for comparison between the theoretical and the measured expected load, as shown in Fig. 5.1.2.

Figure 5.1 .3 shows the residual load for five weekdays, the typical week. Figure 5.1 .4 gives the comparison of the theoretical and the timeaverage sample autocorrelation function for Small Plastics Company. The parameters $\mathrm{X}, \mathrm{L}, \mathrm{a}, \mathrm{n}$ for large plastic molding machines are obtained from C.1.3 of App. C.I. Instead of $\eta$, the numbers of starts or interruptions per month, $(=n \times 22 \times 24)$ is used, assuming a month of 22 working days. The lighting and air conditioning load, and the load of other small machines are not included in the computation of the theoretical autocorrelation function of the total residual load of this plant. The reasons are: (1) the variation of load in time is small for the slowly varying component of load which will not be filtered out by the 15 -minute averaging process; (2) the product XL itself might be small. The time-average sample autocorrelation function computed from
time series data was computed based on the daily cycle model described in subsection 4.1 .3 of Chapter IV and the computer program of Appendix D. Time series data of 30 weekdays were used.

In Tables C.1.2 and C.1.3, data of the parameters for the two-state Markov processes for five months were given separately in five groups. $\mathrm{R}_{\mathrm{T}}[\mathrm{m}]$, the autocorrelation function computed from the model and shown in Fig. 5.1.4, is:

$$
\mathrm{R}_{\mathrm{T}}[\mathrm{~m}]=\sum_{j=1}^{5} \frac{1}{5} \mathrm{R}_{\mathrm{Tj}}[\mathrm{~m}]
$$

where $j=1,2,3,4,5$ correspond to the months of January, February, March, April, and May of 1977.

The comparison of theory and measured autocorrelation function shows that

$$
\begin{array}{ll}
\sigma_{\mathrm{T}}=\sqrt{\mathrm{R}(0)}=45 \mathrm{~kW} & \text { from theory } \\
\hat{\sigma}_{\mathrm{T}}=\sqrt{\hat{\mathrm{R}}(0)}=50 \mathrm{~kW} & \text { from measurements. }
\end{array}
$$

The time shapes of the normalized $\mathrm{R}_{\mathrm{T}}[\mathrm{m}]$ and $\hat{\mathrm{R}}_{\mathrm{T}}[\mathrm{m}]$ look quite similar. Note that these two curves were derived with a single trial. The theoretical curve was based on data taken directly from the company's log for X , a, $\eta$ and direct measurement for $L$. The time-average sample curve was based on data for 30 weekdays. There were no guesses or adjustments of parameters. The theoretical and the measured curves for the expected load and the autocorrelation function agree very well, within the variance or the error bound given in Tables 4.3.1, 4.3.2, and 4.3.3. of Chapter IV.

When computing the expected load from theory, some guesses for L and a of supporting machinery were made, as shown in Table C.1.1. However, no guesses were involved with respect to all X , L , and a for the large plastic molding machines as shown in Table C.1.3. An adjustment of the third shift load for large plastic machines was made, at the suggestion of the plant managers.

| Name of Equipment or <br> Group | lst Shift <br> XLa <br> (in kW) | 2nd Shift <br> XLa <br> (in kW) | 3rd Shift <br> XLA <br> (in kW) |
| :--- | :--- | :--- | :--- |
| Case 1 | 296.53 |  |  |
| Subtotal from Table C.1.1 <br> for all the supporting <br> equipment |  |  |  |
| Subtotal for all the large <br> plastic molding machines <br> (see Table C.1.3) | 459.98 | 250.95 | 250.95 |
| Case I TOTAL: | 756.51 | 710.93 | 710.93 |


| Case 2 |  |  |  |
| :--- | :---: | :---: | :---: |
| Subtotal for all the <br> supporting equipment | 296.53 | 250.95 | 250.95 |
| Subtotal for all the <br> large plastic molding <br> machines* | 459.98 | 459.98 | 413.98 |

* Note that $459.98 \times 0.9=413.98$

Table 5.1.1


Figure 5.1.1

Note: Hour zero corresponds to 7:00 AM of a Monday.


Figure 5.1.2

Note: Hour zero corresponds to 7:00 am


Figure 5.1.3
Note: Hour zero corresponds to 7:00 am


Figure 5.1.4

$$
\begin{aligned}
& \sigma_{\mathrm{T}}=\sqrt{\mathrm{R}_{\mathrm{T}}[0]}=45 \mathrm{~kW} \text { from theory } \\
& \hat{\sigma}_{\mathrm{T}}=\sqrt{\hat{\mathrm{R}}_{\mathrm{T}}[0]}=50 \mathrm{~kW} \text { from time-series data }
\end{aligned}
$$

### 5.2 Brush Company

The detailed description of Brush Company is given in Appendix C. 2 . This company is similar to Small Plastic Company in that both are manufacturers of extrusion molded plastics products; the difference is that Brush Company has very large numbers of asembly type machines for making toothbrushes located in Building 690, as described in Appendix C.2. Brush Company also has higher peak demand, as well as higher usage of energy compared to Small Plastics Company.

The flow and storage structure of this company is simple. The large plastic molding machines are all in parallel, therefore independent of each other. Some of the small assembly machines are in series. But their electric installed capacity is small, therefore we need not worry about the coupling of these processes and their effect on the time structure of the total power demand.

The equipment can be divided into two groups: that which is productionrelated, and that component which is production-independent. The component that relates to production includes load from the large molding machines and that from the small assembly-line types of machines. The parameters $X, L, a, \eta$ for each large machine are given in Table 5.2.2. Each of the large molding machines can be modeled as a two-state Markov process in continuous time from the viewpoint of finding the 15 -minute average power demand (theoretically), as explained in Appendix C.l for Small P1astics Company and Appendix C .2 for Brush Company. X , the installed capacity for each molding machine, is obtained from Table C.2.1 of Appendix C.2; L
the percent of load when on, is obtained by assuming that it is the same as for the large molding machines of Small Plastics Company; $a$, the percent of time on, and $\eta$ the number of starts/hour, are obtained from guesses based on conversation with the managers of the plant.

This company does not keep detailed records of machine usage data as does Small Plastics Company. However, they explained that these machines go through many production cycles for each 15 -minute interval, and are running for about 80 to $90 \%$ of the time during weekdays from Monday through Friday, and are interrupted one or two times/week. So, we can come up with a set of guesses that are consistent with what the managers said about their machines, and that are similar to the parameters of Small Plastics Company.

The following is assumed for the parameters as given in Table 5.2.2. a, the percent of time on, varies from $70 \%$ for the smaller machines to $95 \%$ for the larger machines. $\eta \times 22 \times 24$, the number of interruptions per month, varies from 15 per month for small machines to only two per month for larger machines. The weekend shut-down is not considered an interruption here. The parameters for smaller assembly type of machines in Bldg. 690 are given in Table 5.2.2.

The pieces of equipment which are independent of the production process include lighting, heating, airconditioning, compressors -- are shown in Table C.2.2. Many of these types of loads are assumed to have minimal effect of $R_{T}[m]$, the theoretically computed autocorrelation function for total load; they are therefore excluded from the computation.

To find the expected load for the entire plant, first it is assumed that the expected load for each group of equipment is constant during a shift. The total expected load during a shift is the sum of the expected load of each group. Table 5.2.1 shows how the total expected power demand for each group, and then for the entire plant during each shift, can be found from theory. The expected load XLa for each large plastics molding machine can be deduced from information given in Table 5.2.2 and for each group of supporting equipment in Table C.2.2 of Appendix C.2. Note that four of the large machines, \#4, 5, 42, and 29 , were not included in the computation of the expected load, to reflect the fact that there are a few machines being shut off for a long period of time for maintenance or other purposes.

Figure 5.2.1 shows the total load profile of Brush Company for a typical week. In Fig. 5.2.2, the theoretically computed expected daily load profile is compared with the daily sample mean load profile computed from the time series data of 30 week days. The daily cycle model of section 4.1 and the computer programs of Appendix $D$ are used for computing the sample mean load. The theoretical curve is computed using the formula given in Chapter II and III by hand.

To find the autocorrelation function of the residual from the model or theory, we first assume that the autocorrelation function of the residual of the plant is due only to the plastic molding machines. The smaller, assembly-type machines generate the very fast fluctuating component given in Fig. 5.2.3, but this component is small compared to the theoretical autocorrelation function of the total load.

The large machines are observed to be in a stationary state during all hours of week days and each is modeled as a two-state Markov process. Using the parameters for $X, L, a, \eta$ as given in Table 5.2.2, the theoretical autocorrelation function for the entire plant can be computed using the computer program described in Appendix D.

Time series data of 30 week days were used to compute the timeaverage sample autocorrelation function of the residual. Figure 5.2.3 shows the residual load for a typical week during all five week days. It was found that the standard deviations computed are

$$
\sigma_{\mathrm{T}}=\mathrm{R}_{\mathrm{T}}(0)=115 \mathrm{~kW} \text { from theory }
$$

and

$$
\hat{\sigma}_{\mathrm{T}}=\mathrm{R}_{\mathrm{T}}(0)=135 \mathrm{~kW} \text { measured. }
$$

The shape of the normalized autocorrelation function from theory agrees very well with the time-average sample autocorrelation computed from time series data, as can be seen in Fig. 5.2.4. The variance of the sample mean $\sigma_{\mathrm{m}}^{\hat{m}}$ taken from Table 4.3.1 and that of the time-average sample autocorrelation function $\sigma_{\hat{R}}$ from Table 4.3.3 gives

$$
\begin{aligned}
& \frac{\sigma_{\mathrm{m}}^{\wedge}}{\hat{\mathrm{m}}} \cong 0.9 \% \\
& \frac{\sigma_{\hat{R}}^{\hat{R}}[\xi]}{} \cong 15.5 \% .
\end{aligned}
$$

If we also take these variances into account, then we can conclude that the theory and measured curves agree very well for both the expected loads and the autocorrelation functions.

In the above computation of the theoretical curve shown in Figs. 5.1 .2 and 5.1.4, a lot of guessed parameter values were used. However, it is interesting to see how a set of guessed parameters can be obtained based on some knowledge of how the pieces of equipment are being used. This type of knowledge, such as $L$, $a, n$ is obtained from conversing with plant managers. It is also interesting to see how the theoretical and measured curves agree, which says that the statistics of the time series load can be duplicated reasonably by the theoretical model. Improvement on the model and the parameters can be made by plant managers and plant engineers. This could be done by carefully checking the guessed parameters given in Tables 5.2.2 and C.2.2, to see what type of changes are to be made. Improvement on guessed parameters for $\mathrm{X}, \mathrm{L}, \mathrm{a}$ and could be made by keeping logs and records of some machines' usage time that are important, or by making direct measurements as described in Appendix C. 1.

| Name of Equipment and <br> Source of Data | Ist Shift <br> XLa <br> (in kW) | 2nd Shift <br> XLa <br> (in kW) | 3rd Shift <br> XLa <br> (in kW) |
| :--- | ---: | ---: | ---: |

Large Plastics Molding Machines
(from Table 5.2.2)

| Machines | \#3,7,15,17 | 97.75 | 97.75 | 97.75 |
| :---: | :---: | :---: | :---: | :---: |
| Machines | $\begin{gathered} \# 24,23,36,2,8 \\ 20,21,22 \end{gathered}$ | 268.44 | 268.44 | 268.44 |
| Machines | $\begin{aligned} & \# 25,26,27,28,30, \\ & 31,35,45 \end{aligned}$ | 302.64 | 302.64 | 302.64 |
| Machines | $\begin{gathered} \# 46,47,32,33, \\ 38,39 \end{gathered}$ | 596.71 | 596.71 | 596.71 |

Supporting Equipment
(from Table C.2.2)

B1dgs. 1, 1A,2,3,3A,3B $4,4 \mathrm{~A} \quad 380.00$
277.90
85.72

Bldgs. $6,6 \mathrm{~A}, 6 \mathrm{~B}, 6 \mathrm{C}$
419.40
354.70
354.70

Bldgs. 11, 63
45.05
43.61
62.50

B1dg. 690
546.07
465.67
357.36

XYZ Company
(as described in App. C.2)

Total

| 250.00 | 250.00 | 250.00 |
| :---: | :---: | :---: |
| 2906.03 | 2657.42 | 2375.82 |

Table 5.2.1

| Name of Machine ID \# | \# of Mach <br> in group | $\begin{gathered} \text { Subtotal } \\ X \\ (\mathrm{~kW}) \end{gathered}$ | $\left.\begin{array}{r} n \times 21 \\ \times \times 24 \\ (\# \text { of } \text { Int } \\ \text { per mo. }) \end{array} \right\rvert\,$ | $\begin{gathered} \text { n } \\ \text { (No. of } \\ \text { starts/hr) } \end{gathered}$ | a <br> Fraction of time on (in frac) | L <br> Fraction of Load when on (in frac) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 1 | 75.94 | 15 g | 0.0297 g | 0.7 g | 0.45 d |
| 7 | 1 | 78.13 | 15 g | 0.0297 g | 0.7 g | 0.45 d |
| 15 | 1 | 78.13 | 15 g | 0.0297 g | 0.7 g | 0.45d |
| 17 | 1 | 78.13 | 15 g | 0.0297 g | 0.7 g | 0.45d |
| 24 | 1 | 88.66 | 8 g | 0.0159 g | 0.8 g | 0.45d |
| 23 | 1 | 88.66 | 8 g | 0.0159 g | 0.8 g | 0.45 d |
| 36 | 1 | 86.20 | 8 g | 0.0159 g | 0.8 g | 0.45d |
| 2 | 1 | 93.41 | 8 g | 0.0159 g | 0.8 g | 0.45d |
| 8 | 1 | 94.39 | 6 g | 0.0119 g | 0.8 g | 0.45 d |
| 22 | 1 | 98.12 | 6 g | 0.0119 g | 0.8 g | 0.45 d |
| 21 | 1 | 98.12 | 6 g | 0.0119 g | 0.8 g | 0.45d |
| 20 | 1 | 98.12 | 6 g | 0.0119 g | 0.8 g | 0.45d |
| 25 | 1 | 93.41 | 6 g | 0.0119 g | 0.9 g | 0.45d |
| 26 | 1 | 93.41 | 6 g | 0.0119 g | 0.9 g | 0.45d |
| 27 | 1 | 93.41 | 6 g | 0.0119 g | 0.9 g | 0.45d |
| 28 | 1 | 93.41 | 6g | 0.0119 g | 0.9 g | 0.45d |
| 30 | 1 | 93.41 | 4 g | 0.0079 g | 0.9 g | 0.45 d |
| 31 | 1 | 93.41 | 4 g | 0.0079 g | 0.9 g | 0.45d |
| 35 | 1 | 93.41 | 4 g | 0.0079 g | 0.9 g | 0.45 d |
| 45 | 1 | 93.41 | 4 g | 0.0079 g | 0.9 g | 0.45d |
| 4 | 1 | 104.24 | 2 g | 0.0040 g | 0.9 g | 0.45 d |
| 5 | 1 | 104.24 | 2 g | 0.0040 g | 0.9 g | 0.45 d |
| 42 | 1 | 104.24 | 2 g | 0.0040 g | 0.9 g | 0.45d |
| 29 | 1 | 124.16 | 2 g | 0.0040 g | 0.9 g | 0.45 d |
| 46 | 1 | 150.75 | 4 g | 0.0079 g | 0.95g | 0.45 d |
| 47 | 1 | 150.75 | 4 g | 0.0079 g | 0.95 g | 0.45 d |
| 32 | 1 | 183.05 | 4 g | 0.0079 g | 0.95 g | 0.45d |
| 33 | 1 | 295.11 | 4 g | 0.0079 g | 0.95 g | 0.45 d |
| 39 | 1 | 246.46 | 2 g | 0.0040 g | 0.95 g | 0.45 d |
| 38 | 1 | 367.70 | 2 g | 0.0040 g | 0.95 g | 0.45d |

Table 5.2.2

| Name of | No of <br> Machines <br> in Grp. | Subtotal <br> (kW) | $\eta \times 21$ <br> $\times 24$ | $\eta$ <br> (\# of starts <br> per hr.) | (fract.) | (fract.) |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |


| Heating <br> ovens <br> Dryers | 8 | 82.40 | 0.25 g | 0.6 g | 0.8 g |
| :--- | :--- | ---: | :--- | :--- | :--- |
| Grinders | 3 | 57.82 | 0.50 g | 0.6 g | 0.8 g |
| Electric <br> over G painter | 2 | 88.61 | 0.50 g | 0.8 g | 0.8 g |
| Furnace | 1 | 49.88 | 0.25 g | 0.8 g | 0.8 g |
| Machine shop | 5 | 37.30 | 1.00 g | 0.8 g | 0.4 g |
| Charger | 3 | 37.11 | 0.25 g | 0.8 g | 0.8 g |
| Nylon mach. | 2 | 152.42 | 0.25 g | 0.8 g | 0.8 g |
| Nylon dept. | 2 | 92.90 | 0.25 g | 0.8 g | 0.8 g |

Bldg 690

| Group 1 | 85 | 143.90 | 0.50 g | 0.8 g | 0.5 g |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Group 2 | 40 | 81.78 | 8.00 g | 0.8 g | 0.5 g |
| Group 3 | 4 | 45.25 | 0.25 g | 0.8 g | 0.5 g |
| Group 4 | 20 | 112.88 | 20.00 g | 0.8 g | 0.5 g |
| Group 5 | 17 | 96.04 | 1.00 g | 0.8 g | 0.5 g |
| Group 6 | 20 | 47.22 | 20.00 g | 0.8 g | 0.5 g |

Table 5.2 .2 (continued)


Figure 5.2.1

Note: Hour zero corresponds to 7:00 am, Monday


Figure 5.2.2.

Note: Hour zero corresponds to 7:00 am


Figure 5.2.3

Note: Hour zero corresponds to 7:00 am, Monday


Figure 5.2.4

$$
\begin{aligned}
& \frac{\sigma_{\mathrm{m}}^{\hat{m}}}{\frac{\mathrm{~m}}{\sim}} \stackrel{\approx}{=} 0.9 \% \\
& \frac{\sigma_{\hat{R}}^{\hat{R}}[\xi]}{\sqrt{\phi(0)}} \cong 15.5 \%
\end{aligned}
$$

### 5.3 Abrasive Company

The detailed description of Abrasive Company is given in Appendix C.3. This company can be considered to be a large machine shop, from their operation and scheduling point of view. The company has approximately 740,000 square feet of space. The floor area is very large, and lighting makes up about half of the monthly peak load and more than half of the energy usage.

Table 5.3.1 gives the total load for each shift, which includes the lightling load of each department during each shift, and the load of other equipment. The lighting load of 3 watts per square foot is used for most areas; the details of the lighting load in relation to the level of lighting and floor space is given in Table C.3.1 of Appendix C.3. The load of the other equipment was taken from company records. One sheet out of roughly 20 sheets of such a record is given in Table C.3.2. Table 5.3.2 gives the parameters of machines that are important to the computation of the autocorrelation function of the residual load. The parameters $X, L, \eta$ and a for groups of machines are given in Table 5.3.2. They are taken directly from the company records. It should be noted that very few guesses of parameters were made.

With this type of data, one can see that the expected load of the entire plant could, theoretically, be derived by first finding the load from each department of the plant for each shift, then summing them up accordingly. The expected load of each department is assumed to be constant during each shift.

[^1]Figure 5.3.1 gives the total load of the company for a typical week; Fig. 5.3.2 shows the comparison of the daily expected load, derived theoretically, with the sample mean computed from time series data. Time series data for nine days were used, the nine days being Tuesday, Wednesday, and Thursday of three consecutive weeks. Monday and Friday were omitted because the statistics of load for these days is very different from the rest of the week days, as can be seen in the load profile for a typical week shown in Fig. 5.3.1.

It can be seen that the general trend of the two curves, the theoretical and the measured, agree reasonably well. However, the theoretical curve cannot account for the finer variations of the expected load, such as coffee breaks and lunch breaks. Since the employees of this company are working a $4 \frac{1}{2}$-day work week, the theoretical expected demand curve is drawn as lasting nine hours for the first and second shifts.

We will assume that the autocorrelation function of the residual depends mainly on the manufacturing machines. Since many of these are not running during all three shifts, the random process of the residual is not stationary for 24 hours per day for all week days, as in the two previous cases and the case of Soap Company. However, the process can be considered as being stationary during a given shift, from day to day.

Although the total residual load for this company is assumed to be in the stationary state for a time scale over one shift, the computation of the time average autocorrelation function for a shift is not made, because the time-series data for nine week days are available, and are insufficient to compute the time average autocorrelation function of a shift.

The standard deviations of the residual load are

|  | $\sigma_{\mathrm{T}}=79 \mathrm{~kW}$ | from theory |
| :--- | :--- | :--- |
| and $\quad \hat{\sigma}_{\mathrm{T}}=69 \mathrm{~kW}$ | measured . |  |

The variances of the sample mean, $\hat{\sigma}_{\hat{m}}^{\hat{m}}$, and the time-average sample autocorrelation function, $\sigma_{\hat{R}[\xi]}$, from Tables 4.3.1 and 4.3.3 gives:

$$
\begin{gathered}
\frac{\sigma_{\mathrm{m}}}{\mathrm{~m}} \simeq 0.7 \% \\
\frac{\sigma_{\hat{R}[\xi]}^{\hat{R}}[ }{\sqrt{\phi[\hat{0}]}} \simeq 18.4 \%
\end{gathered}
$$

| Name of Department | Name of Subgroup | \# of Shifts per day | $\begin{aligned} & \text { lst Shf. } \\ & \text { XLa } \\ & \text { (kW) } \end{aligned}$ | $\begin{gathered} \text { 2nd Shf } \\ \text { XLa } \\ (\mathrm{kW}) \end{gathered}$ | $\begin{aligned} & \text { 3rd Shf. } \\ & \text { XLa } \\ & \text { (kW) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Maintenance 1 | Lights | 2 | 108.01 | 108.01 | 10.80 |
| Maintenance | Other Equip. | 2 | 333.04 | 261.78 | 261.78 |
| Maintenance 2 | Lights | 1 |  |  |  |
| Grading | Lights | 2 | 87.93 | 87.93 | 8.79 |
| Vitrified | Lights | 3 | 172.23 | 172.23 | 172.23 |
| Vitrified | Other Equip. | 3 | 265.38 | 184.48 | 117.06 |
| Boiler | Lights | 2 | 12.90 | 12.90 | 1.29 |
| $T$ \& B | Lights | 2 | 142.65 | 142.65 | 14.27 |
| T \& B | Other Equip. | 2 | 256.86 | 190.76 | 0 |
| Shipping | Lights | 1 | 65.73 | 13.20 | 13.20 |
| R $\mathcal{G} E$ | Lights | 1 | 142.80 | 14.28 | 14.28 |
| $R \& E \quad 0$ | Other Equip. | 1 | 210.31 | 91.51 | 91.51 |
| Organic | Lights | 3 | 92.64 | 92.64 | 92.64 |
| Organic 0 | Other Equip. | 3 | 496.66 | 408.10 | 325.06 |
| Snagger |  | 1 | 158.07 | 15.81 | 15.81 |
| Resinoid | Lights | 3 | 172.20 | 172.20 | 172.20 |
| Raisehup | Lights | 2 | 49.47 | 49.47 | 4.95 |
| Cafeteria | Lights | 3 | 13.35 | 13.35 | 13.35 |
| Offices (1A, 1B) | ) Lights | 1 | 48.99 | 4.90 | 4.90 |
| Packing | Lights | 2 | 59.10 | 59.10 | 5.91 |
| String Winding \& Rubber | Lights | 1 | 54.60 | 54.60 | 54.60 |
| String Winding <br> \& Rubber | Other Equip. | 1 | 22.39 | 0 | 0 |
| Storage |  | 3 | 4.40 | 4.40 | 4.40 |
| Machine Shop | Lights | I | 63.45 | 6.35 | 6.35 |
| Machine Shop | Other Equip. |  | 12.9 | 0 | 0 |
| Freight Hse. | Lights | 3 | 1.12 | 1.12 | 1.12 |
| Warehouse | Lights | 3 | 112.98 | 112.98 | 112.98 |
| Bldgs. 32,17,28 | 8 Lights | 1 | 43.19 | 86.4 | 86.4 |
| Barlett | Lights | 1 | 62.10 | 12.42 | 12.42 |

Table 5.3.1

| Name ${ }_{\text {dept }}$ of | Name of Subgroup | \# Shfts. per day | 1st Shft XLa | 2nd Shft XLa | $\begin{aligned} & \text { 3rd Sh } \\ & \text { XLa } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Ovens |  |  | 266.76 | 266.76 | 266.76 |
| Wash plant | Other Equip. |  | 84.60 | 8.46 | 8.46 |
| Bond |  |  | 22.38 | 22.38 | 0 |
| Mounted Pt. |  |  | 16.79 | 16.79 | 16.79 |
|  | TOTAL: |  | 3812.45 | 2610.20 | 1832.55 |

Table 5.3.1 Continued

Table 5.3.2


NOTE that equipment which is not included in this list is considered to have little or no effect on the total plant's autocorrelation function.
$+\sqrt{\bar{w}}$ is as explained in the text of this section.

Table 5.3.2 (continued)

| Mach. ID | Type $\quad$ ¢ | ubtotal <br> (kW) | $\begin{aligned} & \mathrm{L} \times \sqrt{\mathrm{w}} \\ & \% / 100 \end{aligned}$ | No. of machines in group | $\begin{gathered} \pi \\ \# / \mathrm{hr} \end{gathered}$ | $\frac{a}{\text { (fraction) }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 737 | Rubber mill | 37.30 | 0.46g | 1 | 1.875 r | 0.10 r |
| 741 | Colander | 37.30 | 0.46 g | 1 | 2.500 r | 0.10 r |
| 886 | Mixing Rolls | 29.84 | 0.46 g | 1 | 1.250 r | 0.5 r |
| 1140 | Colander | 18.65 | 0.46 g | 1 | 5.000 r | 0.10 r |
| 2306 | Mixer | 29.84 | 0.46 g | 1 | 1.250 r | 0.50 r |
| 3945 | Rubber Mill | 37.30 | 0.46 g | 1 | 1.875 r | 0.10 r |
| 4090 | Colander | 18.65 | 0.46 g | 1 | 2.500 r | 0.10 r |
| 5870 | Rubber Mill | 44.76 | 0.46 g | 1 | 1.250 r | 0.10 r |
| $\left.\begin{array}{l} 1622 \\ 4256 \end{array}\right)$ | Grinder | 62.66 | 0.46 g | 2 | 0.500 r | 0.50 r |
| 4808 | Grinder | 20.14 | 0.46g | 1 | 0.500 r | 0.50 r |
| $\begin{aligned} & 5807 \\ & 6323 \end{aligned}$ | Grinder | 37.30 | 0.46 g | 2 | 0.250r | 0.50 r |
| 6686 | Vert. Boring Mill | 122.38 | 0.46 g | 1 | 0.500 r | 0.50 r |
| 968 | Hi-Press Water pump | 18.65 | 0.80 g | 1 | 0.500 r | 0.80 r |
| 6519 | Hi-Press Water pump | 93.25 | 0.80 g | 1 | 0.500 r | 0.80 r |
| 6527 | Lo-Press " | 44.76 | 0.80 g | 1 | 0.500 r | 0.80 r |
| $\begin{aligned} & 3096 \\ & 4349 \end{aligned}$ | Air compressor | 111.90 | 0.80 g | 2 | 1.00 g | 0.90r |
| 5582 | Air compressor | 93.25 | 0.80 g | 1 | 1.00 g | 0.90 r |
|  | Reclaim VT | 36.93 | 0.46 g | 1 | 0.125 r | 0.60 r |
|  | Reclaim RG | 67.14 | 0.46 g | 1 | 0.125 r | 0.60 r |
| 2569 | Sinter Abrasive drying oven | 33.57 | 0.46 g | 1 | $0.125 r$ | 0.85 r |
| 2496 | Extruder | 18.65 | 0.46 g | 1 | 0.125 r | 0.65 r |
|  | Sweco | 22.38 | 0.43 g | 1 | 0.125 r | 0.75 r |
|  | $\begin{gathered} (788,937,3221, \\ 5571) \end{gathered}$ | 54.83 | 0.65g | 4 | $0.125 r$ | 0.40 r |
| $(1735,4)$ | Hanchet grinder $010,5493)$ | 94.0 | 0.65g | 3 | 1.000 r | 0.40 r |

Table 5.3.2. (continued)

| Mach. ID | Group | $\begin{gathered} \text { Subtotal } \\ X \\ (\mathrm{~kW}) \end{gathered}$ | $\begin{aligned} & \mathrm{L} \times \sqrt{\mathrm{W}} \\ & \% / 100 \end{aligned}$ | No. of machines in group | $\stackrel{\eta}{\# / \text { hour }}$ | $\underset{\text { (fraction) }}{a}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6751 | Blanchard grinder | r 31.33 | 0.46 g | 1 | 0.5 r | 0.40r |
| 6505 | Shot Siding $\&$ Rotex | 32.08 | 0.46 g | 1 | 0.125 r | 0.40 r |
| 6461 | Power supply for speed tester | 82.50 | 0.46 g | 1 | 0.250 r | 0.40 r |
| 1613 | Power supply for speed tester | 18.65 | 0.65 g | 1 | 0.125 r | $0.25 r$ |
| 2306 | Power supply for speed tester | 74.60 | 0.65 r | 1 | 0.125 r | 0.20 r |
| 3432 | Power supply for speed tester | 29.84 | 0.65 g | 1 | 0.125 r | 0.25 r |
| 4777 | Grinder | 41.03 | 0.80 g | 1 | $0.001 r$ | 0.0027 r |
| 5845,6261,6834 |  | 76.09 | 0.80 g | 3 | 0.004 r | 0.0212 r |
|  | Ovens (elec.) | 240.0 | 0.60 g | 6 | 0.0416 g | 0.80 g |
|  | Misc. machines | 109.0 | 0.80 g | 15 | 1.0 g | 0.10 g |
|  | Induct, heater | $\begin{aligned} & 51.44 \\ & \text { KVA } \end{aligned}$ | 0.6 g | 1 | 1.0 g | 0.20 g |
|  | Bal. of machines | 149.0 | 0.8 g | 9 | 0.25 g | 0.10 g |
|  | DC welding | 214.0 | 0.35 g | 2 | 1.0 g | 0.30 g |
|  | Rest of machines | 174.0 | 0.45 g | 7 | 1.0 g | 0.30 g |
|  | Grinder | 93.25 | 0.45 g | 1 | 0.125 g | 0.10 g |
|  | Speed tester | 15.0 | 0.45 g | 3 | 0.125 g | 0.10 g |
|  | Hydraulic Compressor | 18.65 | 0.45 g | 1 | 0.125 g | 0.10 g |
| A25 | Misc. Group 1 | 41.0 | 0.80 g | 5 | 0.5 g | 0.60 g |
| A26 | Misc. Group 2 | 32.0 | 0.80 g | 6 | 0.5 g | 0.40 g |



Figure 5.3.1


Figure 5:3.2

Note: Zero hour corresponds to 7:00 am


Figure 5.3.3

Note: Zero hour corresponds tc 7:00 am, Monday


Figure 5.3.4

$$
\begin{aligned}
& \sigma_{\mathrm{T}}=\sqrt{\mathrm{R}_{\mathrm{T}}[0]}=79 \mathrm{~kW} \quad \text { from theory } \\
& \hat{\sigma}_{\mathrm{T}}=\sqrt{\hat{\mathrm{R}}_{\mathrm{T}}[0]}=69 \mathrm{~kW} \text { from time-series data. }
\end{aligned}
$$

### 5.4 Soap Company

The detailed description of Soap Company is given in Appendix C. 4. The flow and storage diagram is given in Fig. C.4.1 of that Appendix. This is a company that has some of the production processes coupled to each other, as described in Chapter III.

Most of the electric power and energy is used by the company's pro-duction-related processes. Table 5.4.1 gives the XLa during each of the three shifts per day of the department involved. Each of the (XL) for a department is derived by
$(X L)=\sum_{\ell} X^{\ell} L^{\ell}\left(a^{\ell} \mid\right.$ when department or group is on)
\& refers to a subgroup.
Note that the subscripts $i, j$ are dropped from the notation, for the sake of convenience. Therefore, instead of writing $X_{i j}$, we will write simply $X$. The expression ( $a^{\ell} \mid$ when group is on) means the percentage of time the individual machine is on, given that the group is on. Another way of estimating (XL) is by measuring the transformer current when the group or department is on. The KVA obtained from the measured current usage is multipled by 0.8 , the load factor, to account for the phase angle and reactive power. The parameter values for (XL) and XLa are as given in Tables 5.4.1 and 5.4.2 are the average or the compromise of the value for XL and XLa derived using the above two methods as given in Table C. 4.5 of Appendix C.5.

Figure 5.4.1 gives the load profile for a typical week; Fig. 5.4.2 shows the comparison of the expected load curve for a day, derived from theory with the sample mean load profile for a day. The theoretical expected load for a day was derived by summing up all the XLa for each
department during each shift, as given in Table 5.4.2. The sample mean was computed using data from 30 week days and the formula given in Sec. 4.1. The computer program, as given in Appendix D, is used for computing the sample mean. Although the theoretical curve gives constant expected load during each shift, good agreement and consistency can be found in the above comparison.

To find the autocorrelation function of the residual for the total load, the following approximations and assumptions are made. The first assumption is that the synthetic tower's load is generated by a two-state Markov process; the second is that the operation of the packaging lines $\# 1,5,9,10,11$ and 12 are not coupled to the operation of the synthetic tower. The third assumption is that the autocorrelation function of the residual of the total plant load is approximately equal to that of the synthetic tower alone. In other words, the effect of the rest of the plant's load on the autocorrelation function of the residual was assumed to be minimal. Each of these assumptions will be explained and justified below.

An example of how one group of machines can be acting as a block and its behavior taken as a group can sometimes be modeled as a two-state Markov process is given in Case 4 of section 2.5 of Chapter II. The synthetic tower of Soap Company is found to behave in such a manner.

The coupling between the tower and the packaging lines Nos. 1, 5, 9, 10 , and 11 is weak because of two reasons. First, the storage between the tower and the packaging line has enough capacity to hold five to six hours'
worth of the material produced by the tower when the packaging line is off. Second, the tower is being used for approximately 10 hours per day on the average for a week day. However, the storage constraint for this particular storage becomes active only if we try to run the tower for six hours continuously or more during the off-peak hours, as described in rate schedules (see Appendix A) without running the packaging line simultaneously. Another constraint is the fact that the tower must be shut down every few hours for the purpose of changing to a different product line.

The effect of the packaging line on the tower's load when it is on is taken into account to give a conditional effective load for the tower when it is on. (XL) for the tower is approximately 1150 kW and a $\simeq 0.40$. This will give us a variance of the residual load for the tower as

$$
\begin{aligned}
\sigma_{\text {tower }}^{2} & =(\mathrm{XL})^{2} \mathrm{a}(1-\mathrm{a}) \\
& \simeq 317,400 \mathrm{~kW}^{2}
\end{aligned}
$$

The $\sigma^{2}$ tower $\simeq 317,400 \mathrm{~kW}^{2}$ is much larger than the variance of the residual of total of the rest of the plant -- which is estimated to be less than $10,000 \mathrm{~kW}^{2}$. We can see that by not including the rest of the plant, we have introduced a $3 \%$ error into the variance of the residual load plant or $1.5 \%$ error into the standard deviation of the plant's residual load. This is negligible compared to the $14 \%$ error on the standard deviation error that can be expected from the time average sample computation of the autocorrelation function (see section 4.3.2 of Chapter IV).

Figure 5.4 .3 gives the residual load of the total plant and Fig. 5.4.4 the comparison of the autocorrelation function of the residual of the total load, derived from theory as explained above, and the timeaverage sample derived using the time series data. The theoretical curve, $\mathrm{R}_{\mathrm{T}}[\mathrm{m}]$, was computed based on parameters given in Table 5.4.2. $\mathrm{R}_{\mathrm{T}}[\mathrm{m}]$ is a weight autocorrelation function of three months, and can be shown as follows.

$$
\mathrm{R}_{\mathrm{T}}[\mathrm{~m}]=\frac{1}{3} \mathrm{R}_{\text {TMay }}[\mathrm{m}]+\frac{1}{3} \mathrm{R}_{\mathrm{TJune}}{ }^{[\mathrm{m}]}+\frac{1}{3} \mathrm{R}_{\mathrm{TJu} \mathrm{I}^{2}}[\mathrm{~m}]
$$

The comparison of the standard deviation of the residual loads is:

$$
\begin{aligned}
& \sqrt{\hat{\mathrm{R}}[0]}=\hat{\sigma}_{\mathrm{T}}=546 \mathrm{~kW} \quad \text { from measured time-average sample } \\
& \sqrt{\mathrm{R}[0]}=\sigma_{\mathrm{T}}=558 \mathrm{~kW} \quad \text { from theory. }
\end{aligned}
$$

The shape of the theory and measured normalized autocorrelation curve agree very well for $m / 4$ less than five hours. For $m / T$ greater than five hours, along the wing of the autocorrelation function, the theoretical curve approaches zero, while the measured curve oscillates.

| $:$ | lst Shft. <br> Adjusted <br> XLa <br> (kW) | 2nd Shft. <br> Adjusted <br> XLa <br> $(\mathrm{kW})$ | 3rd Shft. <br> Adjusted <br> XLa <br> (kW) | Source <br> of <br> Department |
| :--- | :---: | :---: | :---: | :---: |


| Tower Load that is <br> Independent of <br> Operation | 186.57 | 202.82 | 202.82 | Table C.4.1 |
| :--- | :---: | :---: | :---: | :---: |
| Tower Load that is On <br> Only When Tower is <br> in Operation | 462.03 | 462.03 | 462.03 |  <br> 5.4.2 |
| Soap Process | 459.43 | 459.43 | 459.43 | Table C.4.5 |
| Bldg. \& Mechanical (128) | 175.86 | 158.01 | 158.01 | " |
| Sewage Pump (134) | 15.56 | 13.51 | 13.51 | " |
| Shipping and <br> Warehouse (180) | 105.94 | 105.18 | 155.20 | " |
| Soap Tower \& Making | 714.77 | 580.54 | 270.55 | " |
| Shop | 42.14 | 22.46 | 12.50 | " |
| Bar Soap Packing | 167.71 | 167.71 | 59.62 | $"$ |
| Bar Soap Refrig'n. | 165.91 | 165.91 | 165.91 | " |
| Synthetic Granule <br> Packing (PSG) | 62.52 | 62.52 |  | Table C.4.3 |
| PSG Support | 144.55 | 144.55 | 117.99 | " |

Total $\quad$| 2703.88 2544.56 2077.46 |
| :--- |

Table 5.4.1

| Month | 0.8 KVA <br> When On | XL | Adjusted <br> XL | a <br> in frac.) | $n$ <br> (\#/hour) |
| :--- | :---: | :---: | :---: | :---: | :---: |


| May 1977 | 1064.16 d | 1158.58 r | 1111.37 | $0.4339 \ell$ | $0.0595 \ell$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| June 1977 | 1064.16 d | 1158.58 r | 1111.37 | $0.4535 \ell$ | $0.0587 \ell$ |
| July 1977 | 1064.16 d | 1158.58 r | 1111.37 | $0.3598 \ell$ | $0.0438 \ell$ |

Table 5.4.2

Parameters of the Component of Load of the Tower that is Intermittent


Figure 5.4.1

Note: Zero hour corresponds to 7:00 am, Monday


Figure 5.4.2

Note: Zero hour corresponds to 7:00 am


Figure 5.4.3

Note: Zero hour corresponds to 7:00 am, Monday


Figure 5.4.4

$$
\begin{aligned}
& \sigma_{\mathrm{T}}=546 \mathrm{~kW} \text { from measurements } \\
& \hat{\sigma}_{\mathrm{T}}=558 \mathrm{~kW} \text { from theory }
\end{aligned}
$$

### 5.5 Foundry Company

The detailed description of this company is given in Appendix C.5. The plant is divided into sections, such as: machine shop east, furnaces, machine shop west, offices, etc. The electric stock in each section is divided into subgroups which have similar usage patterns. To each of these subgroups an expected utilization factor is assigned for the day, evening, night shifts and weekends (see Table 5.2). These utilization factors are guesses based on our knowledge of the plant obtained during site visits and the procedure described in Chapters II and III.

The foundry has to be treated as a special case, because the furnaces are operated in such a manner as to take advantage of the offpeak night hours (see Appendix A.2). This company agreed to the increased night use, as described in Appendix A.2. The two furnaces require a total of 580 kW to maintain them at the temperature of molten steel. Additional power is needed for melting steel. For this analysis, it is assumed that the 1720 kW additional power is needed to melt steel during the hours from 10:00 PM to 1:00 AM and from 2:00 AM to 7:00 AM for all week days. The increased night-use hours are described in Appendix A. 2.

The expected electric load of each subgroup for a particular section at a particular time is the product of the electric stock of this subgroup and the respective expected utilization factor, which depends on the work shift. The electric load for each plant section is the sum of the expected loads of the respective subgroups, and the expected total load of the whole plant is the sum of the expected electric load
of each plant section. Except for the load at the furnaces, the expected loads for all the other sections of the plant are derived by using the approach described in the previous section.

Figure 5.5.1 gives the comparison of the theoretically derived expected load of a week, with the actual load of one typical week. Note that the form of data and analysis for this company is not as complete as the previous four cases, because Foundry Company was studied during the summer of 1976 at the early stage of the analytical and theoretical development of the load-modeling methodology.

Table 5.5.1
Electric Stock and the Associated Utilization Factors
for Different Sections of Foundry Company

Symbols Used:
$\mathrm{X}=$ the electric stock for each subgroup in KW ; superscript $\ell$ has been dropped for notational convenience.
$\mathrm{L} a=$ the expected utilization factor of X during a shift.
$\mathrm{E}\{\mathrm{P}\}=$ the expected power demand


[^2]| Subgroup | $\begin{gathered} x \\ (\mathrm{~kW}) \end{gathered}$ | $1 s t$ L a | 2nd Sh. L a | 3rd Sh. L a | 1st Sh. E $\{\mathrm{P}\}$ $(\mathrm{kW})$ | ```2nd Sh. E {P } (kW)``` | $\begin{aligned} & \text { 3rd Sh } \\ & \mathrm{E}\{\mathrm{P}\} \\ & (\mathrm{kW}) \end{aligned}$ | ```Weekend E{P} (kW)``` |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FOUNDRY (BLDGS. 9A,9B,9C,9D ¢ 9) |  |  |  |  |  |  |  |  |
| LightHoistSand mov'g | 86.9 | Ig | 1 g | 1 g | 86.9 | 86.9 | 86.9 | 4 g |
|  | 190.04 | . 1 g | 0.07 g | 0.01 g | 19.0 | 13.3 | 1.9 | 0 g |
|  | 141.7 | .15 g | .15 g | 0 g | 21.5 | 21.5 | 0 | 0 g |
| Subtotal $\quad 127.4 \quad 121.3 \quad 88.8 \quad 4$ |  |  |  |  |  |  |  |  |
| OFFICES $¢$ WOODSHOP $\mathcal{G}$ MISCELLANEOUS (BLDGS. $1,1 \mathrm{~B}, 1 \mathrm{C}, 24,25,24 \mathrm{~B}, 35,31$ ) |  |  |  |  |  |  |  |  |
| Light <br> Machines <br> Hoist | 29.9955.9 | 1 g0.5 g0.1 g | 0.05 g0 g0 | 0.05 g0 g0 | 29.94.55.5 | 1.500 | 1.500 | 1.5 g0 g0 |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| Subtotal $\quad 39.9$ 1.5 1.51 .5 |  |  |  |  |  |  |  |  |
| Exhaust fans 93.25 kW fans ( 125 HP ) |  |  |  |  | 93.25 g | 75 g | 50 g | 0 g |
|  |  |  |  |  | $93.25 \mathrm{~g} \quad 93.25 \mathrm{~g}$ |  | $93.25 \mathrm{~g} \mathrm{93.25g}$ |  |
| Electric H O Heater |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| \& Pump 62.3 |  |  |  |  | 62.3 g | 62.3 g | 62.3 g | 62.3 g |
| $\begin{aligned} & \text { Dust Co11. } 111.9 \\ & (150 \mathrm{HP}) \end{aligned}$ |  |  |  |  | 111.9 g | 75.0 g | 50.0 g | 0 g |
| X (in kW) |  |  |  |  |  |  |  |  |
| Air condition- <br> ing (summer) |  | 98.4 kW | running during 1 st shift only |  |  |  |  |  |
| Heating (Electric (winter) |  | 100 kW | running at all times during winter |  |  |  |  |  |

Table 5.5.1 (continued)


### 5.6 Printing Company

The detailed description of this company is given in Appendix C.6. It can be divided into the expected electric power demand of the electric stock in Buildings A, B, and C. When the physical model of Printing Company shown in Fig. C. 2 of Appendix C. 6 is used as a guide to derive the total electric load of the plant, it can be seen that electric stocks from Bldg. A can be treated as a group and divided into subgroups for lighting, fans, machines, etc. Building $A$ has supporting equipment for the plant that does not directly relate to the rate of flow of paper through the production steps and processes involved. Building C consists of storage areas, and except for the waste baling operation, electric power is needed only for lighting. Building $B$ is the main production building; it has four printing presses and three different sets of binders, cutters, and packaging equipment as shown in Fig. C.1.l. The production level and schedules are different from day to day, and for different times of the year; therefore, different cases or schedules or M's are considered for summer and winter in this study.

The schedule of the four cases for this company is shown in Table 5.6.1; they were constructed as some of the representative cases for operation during different times of year. The electric stock data were taken during plant visits and the expected utilization factors are pure guesses, based on the procedure as described in Chapter II.

The theoretical expected load for each shift and for weekends resulted from the above four cases (see Table 5.6); it gives such results as: the level of demand and energy usage during an eight-hour shift, that
is consistent with the measured data on demand and energy usage of a month, obtained from the billing data (see Table C.l.1).

This company was studied in the summer of 1976. At that time, the modeling methodology was not developed to the present stage, so the analysis is different from that of the first four cases.

Table 5.6.1

Electric Power Demand of a Particular Section During a Particular Shift for Printing Company

| Subgroup | X (kW) | $\begin{aligned} & \text { Ist Sh. } \\ & \text { La } \end{aligned}$ | $\begin{aligned} & \text { 2nd Sh. } \\ & \text { L a } \end{aligned}$ | $\begin{aligned} & \text { 3d Sh. } \\ & \mathrm{L} \text { a } \end{aligned}$ | $\begin{gathered} \text { 1st Sh. } \\ E\{P\} \\ (\mathrm{kW}) \end{gathered}$ | $\begin{gathered} \text { 2nd Sh. } \\ E\{P\} \\ (\mathrm{kW}) \end{gathered}$ | $\begin{aligned} & \text { 3d. SH } \\ & \text { E\{P\}} \\ & (k W) \end{aligned}$ | $\begin{aligned} & \text { Weekend } \\ & \mathrm{E}\{\mathrm{P}\} \\ & (\mathrm{kW}) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BUILDING A |  |  |  |  |  |  |  |  |
| Lighting | 27.6 | 1 g | . 05 g | . 05 g | 27.6 | 1.4 | 1.4 | 1.4 g |
| Machines | 73.62 | 0.5 g | 0g | 0 g | 36.8 | 0 | 0 | 0 g |
| Fans | 5.2 | 1 g | 0 g | 0 g | 5.2 | 0 | 0 | 0 g |
|  |  |  | Subtotal |  | 69.63 | 1.4 | 1.41 .4 |  |

BUILDING B

t- to suck out rejects.

Symbols Used: $\quad X=$ electric stock for each subgroup (in $k W$ ); superscript $\ell$ has been dropped for notational convenience.
$L a=$ the expected utilization factor of $X^{\ell}$ during the $k^{\text {th }}$ shift.
${ }^{S} S_{1}=\left\{\begin{array}{l}l \\ \text { when someone }\end{array}\right.$ is working in the building
1 when nobody is working in the building
$* * S_{2}=\left\{\begin{array}{l}1 \text { when a cutter is on } \\ 0 \text { when all cutters are off }\end{array}\right.$

Table 5.6.1 (continued)

|  | $\begin{gathered} X \\ (k W) \end{gathered}$ | E\{u\|on \} | E\{u\} $0 f f\}$ | $\begin{aligned} & \mathrm{E}\{\mathrm{P} \text { \|on }\} \\ & (\mathrm{kW}) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| BUILDING B |  |  |  |  |
| Printing Press \#1 |  |  |  |  |
| Other Motors | 11.19 | 1 g | Og | 11.19 |
|  |  | Subtotal |  | 123.09 |
| Printing Press \#2 (same as \#1) |  |  |  |  |
| Printing Press \#3 |  |  |  |  |
| Main Drive $(2 \times 100 \mathrm{HP})$ | 149.2 | 1 g | 0 g | 149.2 |
| Other Motors | 14.9 | 1 g | 0 g | 14.9 |
|  |  | Subtotal |  | 164.1 |
| Printing Press \#4 |  |  |  |  |
| Main Drive $(2 \times 100 \mathrm{HP})$ | 149.2 | $1 g$ | 0g | 149.2 |
| Other motors | 22.3 | 1 g | 0g | 22.3 |
|  |  | Subtotal |  | 171.5 |
| Binder, Cutter ¢ Packaging \#1 |  |  |  |  |
| Motors for machines | 48.1 | 0.75 g | 0 g | 36.1 |
| Packaging machine | 30 | 0.50 g | 0 g | 15 |
|  |  | Subtotal |  | 51.1 |
| Binder, Cutter ¢ Packaging \#2 (same as \#1) |  |  |  |  |
| Binder, Cutter \& Packaging \#3 |  |  |  |  |
| Motors for machines | 28.0 | 0.75 g | 0 g | 21 |
| Packaging machine | 30 | 0.50 g | 0 g | 15 |
|  |  | Subtotal |  | 36 |

Table 5.6 .1 (continued)

| BUILDING C | $\begin{gathered} X \\ (\mathrm{~kW}) \\ \hline \end{gathered}$ | E(u\|on) | $E(u)$ | $E(P \mid o n)$ |
| :---: | :---: | :---: | :---: | :---: |
| Warehouse |  |  |  |  |
| Lighting | 10.5 | 1 | 1 | 1 |
| Waste Baling |  |  |  |  |
| Hogger drive <br> Ist blower 100.9 1 0 100.9 |  |  |  |  |
| Balers and |  |  |  |  |
|  |  |  | total | 132.6 (kW) |


|  | $\begin{gathered} X \\ (\mathrm{~kW}) \end{gathered}$ | $\begin{gathered} 1 \mathrm{st} \\ \text { Shift } \\ \mathrm{L} \text { a } \\ \hline \end{gathered}$ | 2nd <br> Shift <br> L a | 3rd <br> Shift <br> L a |  | 2 nSh . E(P) (kW) | 3 r.d Sh E(P) ( kW ) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Air conditioning |  |  |  |  |  |  |  |  |
| for all buildings <br> in summer | 80.53 | 19 | 0.4 g | 0.4 g | 80.53 | 32.2 | 32.2 | 32.2 g |

Table 5.6.2

Case Studies for Printing Co. During Different Seasons

## Case 1

## Summer

2 Printing Presses (\#1 and \#3) on for the first shift
1 PrintingPress (\#3) on for the second shift
\#1 and \#2 Cutter, Binder, \& Packaging machine on for first shift
Waste Baler on for first and second shifts

Total Power Demand for $\quad \frac{\text { First Shft }}{817.04} \cdot \frac{2 \text { nd Shft }}{470.34} \cdot \frac{\text { 3rd Shft }}{96.38} \cdot \frac{\text { Weekend }}{96.38}$

Case 2
Sunmer
2 Printing Presses (\#1 and \#2) for first shift
\#3 Cutter and Binder on for first shift
Waste Baling on first and second shifts

Total Power Demand for $\quad$ 1st Shft. 2nd Shft. 3rd Shft. Weekend (in kW)
$\begin{array}{llll}709.8 & 218.98 & 96.38 & 96.38\end{array}$

Case 3
Summer
\#1 and \#2 Printing Presses on for first shift
\#1 Printing Press on for second shift
\#1 and \#3 Binder, Cutter \& Packaging Machine on for first shift Waste Baling on first and second shifts

| Total Power Demand for | 1st Shft. | 2nd Shft. | 3rd Shft. | Weekend |
| :---: | :---: | :---: | :---: | :---: |
| (in kW ) | 801.9 | 378.23 | 96.38 | 96.38 |

## Table 5.6.2 Continued

## Case 4

Winter (Very little electric heating)
\#1, \#2, and \#3 Printing Presses on for first shift
\#1 and \#2 Printing Presses on for both second and third shifts
\#1, \#2, \#3 Binders, cutters \& Packing Machines on for first shift Waste Baling - first and 2nd shifts
Total Power Demand
$\quad$ (in kW) $\frac{\text { 1st Shift }}{895.6} \quad \frac{\text { 2nd Shift }}{50.132} \quad \frac{\text { 3rd Shift }}{368.72} \quad \frac{\text { Weekend }}{64.18}$

Tab1e 5.6.3
Electric Power and Energy Usage Data of Printing Company's Printing Plant for the Year 1975

| Month | Demand <br> $\mathrm{kW} / 0.8 \mathrm{KVA}$ | Effective <br> Demand | Ratio of <br> $\mathrm{kW} / \mathrm{KVA}$ | Energy <br> Usage in <br> KWH | Hour Used <br> Energy <br> Usage Eff. <br> Demand (Hr) |
| :--- | ---: | :---: | :---: | :---: | :---: |


| January | $954 / 888$ | 954 | 0.859 | 318,600 | 333 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| February | $909 / 864$ | 909 | 0.842 | 324,900 | 357 |
| March | $864 / 816$ | 864 | 0.847 | 290,700 | 336 |
| Apri1 | $864 / 816$ | 864 | 0.847 | 288,000 | 333 |
| May | $918 / 840$ | 918 | 0.874 | 344,700 | 375 |
| June | $909 / 864$ | 909 | 0.842 | 273,600 | 300 |
| July | $783 / 720$ | 783 | 0.870 | 293,400 | 374 |
| August | $753 / 696$ | 753 | 0.865 | 233,200 | 296 |
| September | $828 / 792$ | 828 | 0.836 | 334,800 | 404 |
| October | $891 / 816$ | 891 | 0.873 | 340,500 | 348 |
| November | $873 / 840$ | 873 | 0.831 | 342,000 | 391 |
| December | $882 /$ | 882 |  | 342,000 | 387 |

### 5.7 Discussion and Perspectives

This chapter deals with the physical model and analysis of the six companies analyzed. The idea from the two aspects of the physical model the stochastic aspect of Chapter II and the flow/storage aspect of Chapter III -- is used to model and analyze each company.

The two-state (zero-one) Markov process model in continuous times was extensively used for computing the expected loads and the autocorrelation function of the residuals for the four companies studied in the summer of 1977. They are described in sections 5.1 to 5.4. The parameters need for modeling each piece of equipment as a Markov process are $\mathrm{X}, \mathrm{L}, \mathrm{a}, \eta$ as described in Section 2.4 of Chapter II. $X$, the installed capacity in kW , is found by reading the nameplates of the machines. $L, a$, and $n$ are found from the company's records, or logs, or they could be guessed values. These parameters are found for all large machines that are turned on and off at completely random times; each machine is then modeled as a Markov process.

If the Markov process model is used, the autocorrelation function of the residual of each piece of equipment is as given in Eq. (2.4.20). If these pieces of equipment are independent of each other, then the autocorrelation function of the residual of the total load can be approximated and found by summing up all the autocorrelation functions of each piece of large equipment. The expected load for each piece is XLa, as given in Eq. (2.4.19). The expected load of the whole plant is the sum of all the expected loads for all equipment for the entire plant.

The idea discussed in Chapter III is used as a conceptual guide. For example, from flow/storage types of considerations, it can be concluded that the plastic molding machines of Small Plastics Company are independent of each other. The same holds true for Brush Company. From the flow/storage type of consideration, all the equipment belonging to the tower of Soap Company is treated as a single block and modeled as a two-state Markov process, as described in Chapter II.

The expected loads and the autocorrelation functions of the residual for each of the four companies studied is computed from the model. These two quantities compared favorably with those computed using timeseries data. The comparison agrees within the range of variances for the sample mean and the time-average sample autocorrelation function, as described in Chapter IV.

The conclusion of this chapter is that it is feasible to develop a satisfactory industrial electric load model for each industrial firm, based on physical data -- $X, L, a, \eta$ for each piece of equipment, how processes are coupled, etc. The model gives statistics for load that compare well with the statistics computed from time-series data.

It should be noted that many of the parameters used are guesses. Therefore, the model can be modified and improved by company personnel. Improved guesses for parameters could be found. (XL) for some crucial equipment could be measured directly, and logs kept to derive a and $\eta$.

## CHAPTER VI

## MODEL STRUCTURE: ECONOMIC CONSIDERATIONS

In Chapters II, III, IV and V, it was shown that the physical load modeling methodology could be used to derive the total expected load of a plant, and the related autocorrelation function of the residual load. The theoretically derived expected load and autocorrelation was found to have good agreement with the sample mean and the time-average sample autocorrelation function derived from time series data. However, in these chapters it was not explained why a particular schedule out of a set of an infinite number of possible schedules was the one chosen by the managers.

Here in Chapter VI it will be shown that the economically attractive schedules of a manufacturing plant can be determined based on economic considerations of two types of costs which are functions of the time of day. The demand charge or time-dependent energy costs discourage the plant manager from using electricity at the plant's or utility's peak demand hours. (See Appendix A and 56.1 , this chapter). The wage of a firm's employee generally varies with the time of day. For example, the wage at Foundry Company, compared with the first shift, is $20 \%$ higher for the third shift, $15 \%$ higher for the second shift.

Sections 6.1 and 6.2 show how an incremental monthly charge of electricity, under the present rate structure with respect to demand, along with the time-of-day wage, can be used to explain how an optimal schedule of a firm that incurred the least monetary cost can be found. In § 6.2 we introduce a very important concept which considers the ratio of the load of each production process or department during a shift, and the number of employees working during that shift, in $k W$ per person. The consideration
of $\mathrm{kW} / \mathrm{person}$ of a particular section of the plant greatly simplifies the firm's scheduling problem. Instead of having to consider the scheduling problem of a firm for all sections simultaneously, it is now possible to take each section individually. Parts 6.3 and 6.4 of this chapter use the approach described in $\$ 6.2$ to explain why each manufacturing firm being studied operates its plant according to a certain schedule. Section 6.5 explains how $\mathrm{kW} /$ person as a concept can be used by managers as an aid in deciding when to buy new equipment; 86.6 gives the net change in the electric power demand of a utility when an industrial customer reschedules its operation, and 56.7 discusses the social implications due to the changes in a firm's schedule. Sections 6.8 and 6.9 show the effect of the present $H$ rate, the new time-of-day rate, as well as other hypothetical rate structures, on a manufacturing firm's scheduling. 56.10 describes how the cost of moving a kW of the coincident electric power demand from peak demand hours of a utility's load, can be computed quantitatively for a manufacturing firm.
6.1 Analysis of the Monthly Electrical Charges and the Schedule of

## A Manufacturing Firm

The manufacturing firm as a large power user is generally charged according to a few special rates. Appendix A shows one of these rates, Rate $H$, being used by the Massachusetts Electric Company and its new proposed rate, Rate $X$. The monthly charge for electricity according to Rate H for a particular month consists of a demand charge, which is a highly nonlinear function of the monthly peak demand, and the energy charge, which is another highly nonlinear function of the monthly peak demand, plus the energy usage of that particular month. Therefore,
under these rates, there are two types of incremental monthly electric charges to the firm. Note that not all the industrial customers are charged under the $H$ Rate. The proposed Rate $X$ is a linear function of peak demand, peak hour energy usage, and off-peak hourly usage plus a constant customer charge and a fuel charge. (The definition of these terms can be found in Appendix A.) An important concept that will be introduced here is the fact that the monthly charges under H Rate or X Rate are well defined functions and can be represented mathematically as functions of peak demand, energy usage, peak hour energy usage, offpeak hourly energy usage, etc.

For instance, MCH, the monthly charge under Rate H , and MCX, the same under the proposed Rate X (as given in Appendix A) are as follows:

$$
\begin{equation*}
\operatorname{MCH}(\varepsilon, D)=f_{d}(D)+f_{e}(\varepsilon, D) \tag{6.1.6}
\end{equation*}
$$

where $f_{d}$ is the demand charge and $f_{e}$ is the energy charge; both are nonlinear functions.
$\operatorname{MCX}\left(\varepsilon_{1}, \varepsilon_{2}, D\right)=100+\alpha_{\text {seasonal }} D+\beta_{1} \varepsilon_{1}+\beta_{2} \varepsilon_{2}+$ fuel charge

The advantage of representing the monthly charges as mathematical functions is that the rates can be analyzed and compared in various ways in which we may be interested.

The first step in analyzing the $H$ Rate is to find its two possible partial derivatives, i.e., the incremental charges.
(1) $\frac{\partial M C H(E, D)}{\partial D}$ - incremental monthly charge for electricity with respect to a unit change in peak demand keeping the energy usage as a constant,
(2) $\frac{\partial \mathrm{MCH}(\varepsilon, \mathrm{D})}{\partial \varepsilon}$ - incremental monthly charge for electricity with respect to a unit change in the amount of the energy usage keeping the peak demand as a constant.

To find the two incremental charges, we can proceed, using Eq. (6.1.6) as follows:

$$
\begin{align*}
& \frac{\partial M C H(\varepsilon, D)}{\partial D}=\frac{\partial f_{d}(D)}{\partial D}+\frac{\partial f_{e}(\varepsilon, D)}{\partial D}  \tag{6.1.3}\\
& \frac{\partial M C H(\varepsilon, D)}{\partial \varepsilon}=\frac{\partial f_{e}(\varepsilon, D)}{\partial \varepsilon} \tag{6.1.4}
\end{align*}
$$

From Eq. (6.1.2) we can see that the incremental monthly charges for the proposed Rate X are:

$$
\begin{align*}
& \frac{\partial M C X}{\partial D}=\alpha_{\text {seasonal }}  \tag{6.1.5}\\
& \frac{\partial M C X}{\partial \varepsilon_{I}}=\beta_{1}  \tag{6.1.6}\\
& \frac{\partial M C X}{\partial \varepsilon_{2}}=\beta_{2} \tag{6.1.7}
\end{align*}
$$

For the purpose of the following discussion, most of the derivations are done using the $H$ Rate. However, the result can be generalized to Rate X . and any other type of conceivable rates.
$\partial M C H / \partial D$ is useful for a manager who is trying to evaluate changes in the total plant cost by changing a day shift to operation at night, or vice versa. Consider the case of a company with a peak demand of electricity occurring during the day. To reduce the demand charge of his electric bill, he could change a particular section or department of his
plant (e.g., a machine shop, a crusher) which is operating during the day to be operated at night, when the demand for electricity of the firm is low. Of course, the new schedule must satisfy the physical constraints such as flow and storage capacities, which were described in Chapter III. Assume that such a change in schedule will change the peak demand for electricity only for the month, with little or no change at all in the amount of electrical energy used. The rescheduling of the section from the day shift to the night shift will reduce the demand, thereby reducing the demand charge. For a company working under time-of-day rate, such a rescheduling will also reduce $\varepsilon_{1}$ and increase $\varepsilon_{2}$. Therefore, there is an additional saving, due to the saving from the energy price differential.

However, other costs will increase, mainly because the employees of the plant are paid at a higher rate to work at night. If the possible reduction in kW of peak demand, as well as the number of employees involved is known, it is possible to develop a rough, but very fast and convenient, rule for evaluating the merit of such a change in schedule as follows:

The total cost for the month to operate the plant can be described as

$$
\begin{equation*}
\mathrm{TMC}=\mathrm{MC}+\mathrm{OMC} \tag{6.1.8}
\end{equation*}
$$

where
TMC is the total monthly cost
MC is the monthly charge for electricity
OMC is the other operating costs for the month.
The change in total cost due to a schedule can be expressed as follows:

```
        monthly monthly
        savings extra cost
```

$\triangle \mathrm{TMC}=\triangle \mathrm{MC}+\triangle \mathrm{OMC} \quad$.

Assuming that only the peak demand for electricity of the month is changed with the energy usage remaining constant, we can rewrite the above expression as

$$
\begin{equation*}
\Delta T M C=\int_{D_{0}}^{D_{o}^{\prime}} \frac{\partial M C H(\varepsilon, D)}{\partial D} d D+\Delta O M C \tag{6.1.10}
\end{equation*}
$$

where $D_{o}^{\prime}$ indicates a new peak demand for a plant under H Rate. Note that the $\operatorname{\partial MCH}(\varepsilon, D) / \partial D$ used is valid only for a company under Rate $H$. For a company under rate $X, \quad \operatorname{MCX}\left(\varepsilon_{1}, \varepsilon_{2}\right.$, $\left.D\right) / \partial D$ plus a term due to savings from the energy rate differential must be used.

We can proceed further with the analysis by assuming that the change in the other monthly operating cost, $\triangle O M C$, is mainly due to the change in the hourly rate of employees:

$$
\begin{equation*}
\Delta R_{i j} \triangleq R_{i j}^{2}-R_{i j}^{l}=R_{i j}^{3}-R_{i j}^{1} \tag{6.1.11}
\end{equation*}
$$

Note that $R_{i j}^{2}=R_{i j}^{3}$ is assumed, which says that the hourly wage for employees working during the second shift is the same as the hourly wage paid during the third shift, which gives

$$
\begin{equation*}
\Delta O M C=\Delta R_{i j} \times H W_{i j} \times L_{i j} \tag{6.1.12}
\end{equation*}
$$

where
$H W_{i j}$ - working hours/person/month for the section $i \rightarrow j$.
$L_{i j}$ - number of persons for the section $i \rightarrow j$.
for the rescheduling of the section $i \rightarrow j$.

Substituting $\triangle O M C$ from Eq. (6.1.12) into Eq. (6.1.11), for a company under H Rate, we have

$$
\begin{equation*}
\Delta T M C=\int_{D_{0}}^{D_{0}^{\prime}} \frac{\partial M C H(\varepsilon, D)}{\partial D} d D+\Delta R_{i j} \times H W_{i j} \times L_{i j} \tag{6.1.13}
\end{equation*}
$$

For the new schedule to incur a net saving, $\triangle T M C \leq 0$ is required. This gives

$$
\begin{equation*}
-\int_{D_{0}}^{D^{\prime}} \frac{\partial M C H(\varepsilon, D)}{\partial D} d D \geq \Delta R_{i j} \times H W_{i j} \times L_{i j} \tag{6.1.14}
\end{equation*}
$$

Note that in Eq. (6.1.14), the term $\int_{D_{0}}^{\circ} \frac{\partial M C H(\varepsilon, D)}{\partial D} d D$ is negative for the cases of interest which represents the fact that there is a saving or reduction in monthly charges of electricity from rescheduling. Equation (6.1.14) gives the first necessary condition for a manager to consider rescheduling of a section to save on operating cost.

Using the argument and approaches described above, we can find a schedule for each section of the entire plant which will incur the least cost for a given level of material output for a manufacturing plant; this is called the "optimal feasible solution".

### 6.2 Decision Rules for Rescheduling in a Plant

$$
\text { In Eq. (6.1.14) of } \S 6.1 \text {, if it is assumed that the change in } D_{0}
$$ is small, the following expression is obtained for a firm under $H$ Rates:

$$
\begin{equation*}
-\left.\frac{\partial M C H(\varepsilon, D)}{\partial D}\right|_{\varepsilon_{0}, D_{o}}\left(D_{o}^{\prime}-D_{o}\right) \geq \Delta R_{i j} \times H W_{i j} \times L_{i j} \tag{6.2.1}
\end{equation*}
$$

$H W_{i j}$ is the number of working hours per employee per month; $L_{i j}$ is the number of employees at the section $i \rightarrow j$ for a particular shift. Note that this is just an approximation, because $\partial \mathrm{MCH} / \partial \mathrm{D}$ contains many discontinuous points. See Appendix A. The above equation can be rewritten to yield:

$$
\begin{equation*}
\frac{\left(D_{o}-D_{o}^{\prime}\right)}{L_{i j}} \geq\left.\frac{\Delta R_{i j} \times H W_{i j}}{\frac{\partial M C H(E, D)}{\partial D}}\right|_{\varepsilon_{o}, D_{o}} \tag{6.2.2}
\end{equation*}
$$

which is the new form of the first necessary condition that a particular section has to satisfy before it can be considered for rescheduling to achieve a net savings in its operating cost.

$$
\begin{equation*}
\text { Define } A_{i j} \triangleq \frac{\left(D_{o}-D_{o}^{\prime}\right)}{L_{i j}}=\frac{X_{i j} \alpha_{i j}}{L_{i j}} \tag{6.2.3}
\end{equation*}
$$

where $\alpha_{i j} \simeq$ La can be shown using the argument shown in Chapter II if the load of section $i \rightarrow j$ is not dominant compared to the plant's total electric load. Note that in Chapters II and III, La was assumed to be a constant. $A_{i j}$ has the unit of (kW/person). Under H rate, $A_{i j}^{e}$ can be computed as follows:

$$
\begin{aligned}
A_{i j}^{e} & \triangleq \frac{\Delta R_{i j} \times H W_{i j}}{\left.\frac{\partial M C H}{\partial D}\right|_{\varepsilon_{0}, D_{o}}} \\
& =\frac{\text { Additional 1abor cost/person due to rescheduling (\$/pers.) }}{\text { Saving per kW reduction in demand due to rescheduling }} \begin{array}{l}
(\$ / \mathrm{kW})
\end{array}
\end{aligned}
$$

where $A_{i j}^{e}$ is the break-even $k W$ of electrical demand per person.
For a first-round analysis which is approximate, we can proceed and develop the following rules that can be used to determine if a section of a manufacturing plant can be rescheduled to save on its operating cost. These rules were derived based on the first necessary condition for economically rescheduling their operation by the plant managers.
(1) if $A_{i j}=\frac{X_{i j} \alpha_{i j}}{L_{i j}} \geq A_{i j}^{e} \quad$,
then adopt the idea for further study on possible rescheduling of the shift from day to night.
(2) If $A_{i j}=\frac{X_{i j}^{\alpha}{ }_{i j}}{L_{i j}}<A_{i j}^{e}$,
reject the idea.
(3) Clearly, if $\frac{X_{i j}}{L_{i j}}<A_{i j}^{e}$,
then

$$
A_{i j}<A_{i j}^{e} \quad, \text { because } A_{i j} \leq \frac{X_{i j}}{L_{i j}}
$$

therefore, reject the idea.
$A_{i j}$ and $A_{i j}^{e}$ are important and useful concepts to deal with, and $A_{i j}^{e}$ can be rewritten as follows:

$$
A_{i j}^{e}=\left(\frac{H W_{i j}}{\left.\frac{\partial M C H}{\partial D}\right|_{\varepsilon_{0}, D}}\right)\left(\Delta R_{i j}\right)
$$

It can be seen that $A_{i j}^{e}$ is linearly dependent on the wage differential $\Delta \mathrm{R}_{\mathrm{ij}}$. However, the coefficient ( $\mathrm{HW} \mathrm{ij}_{\mathrm{ij}} / \partial \mathrm{MCH} / \partial \mathrm{D}$ ) is a nonlinear function of the peak demand and energy usage because of $\partial \mathrm{MCH} / \partial \mathrm{D}$.

If the company is under $X$ Rate instead of $H$ Rate, then to derive $A_{i j}^{e}$ , the dollar saving per month due to reduction in one kW of demand requires the use of

$$
\frac{\partial M C X}{\partial D}+\left(\frac{\partial M C X}{\partial \varepsilon_{1}}-\frac{\partial M C X}{\partial \varepsilon_{2}}\right) \mathrm{NHI}
$$

where NHI is the number of hours involved per month that change from peak hour operation to off-peak hour operation and $\left(\frac{\partial M C X}{\partial \varepsilon_{1}}-\frac{\partial M C X}{\partial \varepsilon_{2}}\right)$ NHI is the dollar saving per month derived from energy rate differential due to rescheduling of a 1 kW constant expected power demand from the first shift to the second or third shift, for a month, as given in Table 6.3.3 of the next section.

### 6.3 Determination of the Optimal Schedule of a Manufacturing Plant: Numerical Example

Case 1: Assume that all companies are under $H$ rate, as described in Appendices A. 1 and A. 2 to simplify the analysis. Tables 6.3.1 and 6.3 .2 give some of the $A_{i j}$ and $A_{i j} \quad$ for some equipment and processes of the seven companies.

The above approach could be used to explain why Foundry Company runs the furnaces for melting at night, and all the other sections during the day, unless a particular section is under multiple shift Consumer Product Company*, Printing Company* and Abrasive Company run all their operations during the day, unless there is a multipleshift operation for a certain process or department. The need for the multiple-shift operations at a particular section is a function of: (1) the demand of the plant's material output, and (2) the flows and other constraints of a particular section of the plant which can be explained by the physical model described in Chapter IV.

The waste baling operation at Printing Company has $A_{i j}$ slightly greater than $A_{i j}^{e}$. For the waste baling operation, $A_{i j} \approx 66$ and $A_{i j}^{e} \simeq 34, \mathrm{~kW} /$ person, based on $\Delta R_{i j}=\$ 0.60 /$ person/hour. However, the waste baling operation at Printing Company is operated during the first and second shifts, although it might have been possible to save money by operating during the second and third shifts. This does not contradict the analysis, because the possible saving is approximately equal to $\$ 200 /$ month. However, in actuality, under Rate $G$, the saving is smaller.

[^3]In the computation of $A_{i j}$ for Printing Company, the factor (La) is not included in computing the expected load. If this is included, the $A_{i j}$ will be smaller, and therefore the saving will be smaller.

It can be seen that the waste baler of Printing Company deserves further study, but the amount of savings involved might be insufficient for the firm to change their operation to the third shift, under $H$ rate, assuming it is a two-shift operation.

Consumer Product Company schedules most of its operations during the first shift, except for the heat-treating furances which operate 24 hours/day. There is no wage differential for different shifts, according to the company's policy, because only a very small percentage of employees work at night. Further consideration is needed to determine if sections of Consumer Product Company can be operated at night. First, if more employees are to be assigned to work at night, an incentive in the form of a higher wage is needed. Second, one should be aware of the fact that Consumer Preduct Company has had difficulty finding skilled and semi-skilled employees; this will limit them from running much of their operations at night. However, if one assume that a wage differential of, say, $\Delta R_{i j}=\$ .50 /$ hour/person between the workers of the first and second shift is necessary, a breakeven $A_{i j}^{e}$. of $50 \mathrm{~kW} /$ person is reached. But all sections of this company have their $A_{i j}$ ( $k W /$ person) much lower than $50 \mathrm{~kW} /$ person (see Appendix C.7), so it is clear that this company will be unable to effect a saving by rescheduling its operations from day to night shifts under $H$ rate.

Abrasive Company has most of its larger production processes in the category of $A_{i j} \simeq 10$ to $20 \mathrm{~kW} /$ person. Therefore, none of the processes are economically attractive for rescheduling under $H$ rate, because the breakeven $A_{i j}^{e} \simeq 25$, which is larger than its $A_{i j}$.

Sma11 Plastics Company has some of its power-intensive plastics molding machines ranging from $A_{i j}=7-40 \mathrm{~kW} /$ person. $\Delta R_{i j}=0.25 \$ /$ person per hour and its breakeven $A_{i j}^{e}=12.5 \mathrm{~kW} /$ person. So it can be seen that a few of their processes have economic potential for rescheduling. However, they are running three shifts day, therefore there cannot be any rescheduling.

Brush Company has its power-intensive plastics molding machines ranging from $A_{i j}=20$ to $150 \mathrm{~kW} /$ person. $\Delta R_{i j} \simeq 0.25 \$ /$ person/hour, and $A_{i j}^{e}=$ $12.5 \mathrm{~kW} /$ person. Many of its processes have economic potential to be rescheduled, but since they, too, run three shifts per day, there can be no rescheduling.

Soap Company has $\mathrm{A}_{\mathrm{ij}}=200 \mathrm{~kW} /$ person for its "tower". Its breakeven $A_{i j}^{e} \simeq 21 \mathrm{~kW} /$ person. However, the tower is running three shifts per day, intermittently, so although it has economic potential for rescheduling, it is not physically possible.

Table 6.3.1
Some $A_{i j}$ of the Seven Manufacturing Firms*

| Section or Department | $\begin{gathered} \mathrm{A}_{\mathrm{ij}} \\ (\mathrm{~kW} / \text { person }) \end{gathered}$ | Schedule of Operation |
| :---: | :---: | :---: |
| Furnace for melting steel at Foundry Company | 1700 or greater | 3rd shift |
| Typical machine shop for all firms | from 2 to < 20 | day shift or multiple shifts |
| Printing press at Printing Co. | $\sim 20$ | day or multiple shift |
| Waste baling at Printing Co. | ₹66 | day or multiple shift |
| Plastics extrusion molding at Small Plastics Co. | 7 to 40 | three shifts/day, five days/week intermittently |
| Plastics extrusion molding at Brush Co. | 20 to 150 | three shifts/day, five days/week intermittently |
| Tower (synthetic soap process) at Soap Co. | $\sim 200$ | three shifts/day, five days/week intermittently |
| Large machines at Abrasive Co. | 10 to 20 | day or multiple shifts |

* Note that when computing the $A_{i j}$ of Soap, Abrasive, Small P1astics and Brush Companies, the factors $L$ and a were included, However, when computing $A_{i j}$ for Printing, Consumer Product and Foundary Cos. the factors L and a were not always included.

Table 6.3.2

|  | Foundry <br> Company | Consumer Product Co. | Printing Company | $\begin{aligned} & \text { Small } \\ & \text { Plastics } \\ & \text { Co. } \end{aligned}$ | Brush Company | Soap Company | Abrasive Company |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (thousands of KWH) | 1000 | 350 | 320 | 410 | 1500 | 1250 | 1800 |
| $\begin{gathered} D_{0} \\ (\text { in } k W) \end{gathered}$ | 1600 | 1500 | 810 | 850 | 3400 | 3500 | 4300 |
| $\begin{aligned} & H U_{0} \\ = & \varepsilon_{0} / D_{0} \end{aligned}$ <br> (in hour/month) | 650 | 230 | 395 | 480 | 440 | 370 | 425 |
| $\begin{aligned} & \left.\frac{\partial \mathrm{MCH}}{\partial \mathrm{D}}\right\|_{\varepsilon_{O}, D_{O}} \\ & (\text { in } \$ / \mathrm{KWH}) \end{aligned}$ | 3.79 | 1.76 | 3.14 | 3.54 | 3.54 | 3.14 | 3.54 |
| ```1st to 3rd Shift \DeltaR ij ($/person/hour)``` | 1.00 | 0.5* | 0.60 | 0.25 | 0.25 | 0.375 | 0.5* |
| Breakeven (kW/person) $A_{i j}$ | 46 | 50 | 34 | 12.5 | 12.5 | 21 | 25 |

Suppose all the seven companies were under Rate $X$, as described in Appendices A. 3 and A. 4.

Let $S$ be savings in dollars/month for reducing 1 kW of the expected load from the first shift. Then

$$
S=\frac{\partial M C X}{\partial D}+\left(\begin{array}{ll}
\frac{\partial M C X}{\partial \varepsilon_{1}} & \frac{\partial M C X}{\partial \varepsilon_{2}}
\end{array}\right) \mathrm{NHI}
$$

as described in Section 6.2. Table 6.3 .3 gives the savings $S$ for different types of rescheduling during different seasons. To compute the NHI, we have assumed that the company is under a schedule in which shift changes occur at 8:00 $\mathrm{AM}, 4: 00 \mathrm{PM}$, and 12:00 midnight.

Figure 6.3.1 gives the breakeven curves $A_{i j}^{e}$ as a function of $\Delta R$, the wages differential, for different types of rescheduling for different seasons under $X$ rate. Note that:

$$
A_{i j}^{e}=\frac{\Delta R_{i j} \times H W_{i j}}{S}
$$

where $\Delta R_{i j}$ and $H W_{i j}$ are as described in Section 6.1. Table 6.3.4 gives the $A_{i j}^{e}$ of each company for different types of rescheduling during different seasons under $X$ rate.

$\qquad$

| Winter | $4.938(\$ / \mathrm{kW})$ |
| :--- | :--- |
| Sumner | $7.358 \quad "$ |

\{
Winter
Summer
$6.368(\$ / \mathrm{kW})$ Summer
8.788 "
lst to 3rd Shift

2nd to 3rd Shift
$1.43(\$ / \mathrm{kW})$

Savings in $\$ / \mathrm{kW}$ per Month (based on X-01 Rate of Appendix A.4)

Table 6.3.3

Savings $S$, due to rescheduling of a constant 1 kW of expected power demand from an 8 -hour shift, or 8 KWH per shift for all 22 working days of a month.

| Company | $\begin{aligned} & 1 \mathrm{st}-2 \mathrm{nd} \text { Sh. } \\ & \Delta \mathrm{R} \\ & (\$ / \mathrm{hr} \text {-pers }) \end{aligned}$ | $\begin{gathered} \text { lst-3rd Sh. } \\ \Delta R \\ (\$ / \mathrm{hr} \text {-pers }) \end{gathered}$ | $\begin{aligned} & \mathrm{A}_{\mathrm{ij}}^{\mathrm{e}} \text { for } \\ & \text { lst-2nd } \\ & \text { Winter } \\ & (\mathrm{kW} / \mathrm{pers}) \end{aligned}$ | $\begin{gathered} \mathrm{A}_{\mathrm{ij}}^{\mathrm{e}} \text { for } \\ 1 \text { st-2nd } \\ \text { Sunmer } \\ (\mathrm{kW} / \mathrm{per}) \end{gathered}$ | $\left\{\begin{array}{c} A_{i j}^{e} \text { for } \\ \text { lst-2nd } \\ \text { Winter } \\ (\mathrm{kW} / \mathrm{per}) \end{array}\right.$ | $\begin{gathered} \mathrm{A}_{\mathrm{ij}}^{\mathrm{e}} \text { for } \\ 1 \mathrm{st}-2 \mathrm{nd} \\ \text { Summer } \\ \mathrm{ck} / \mathrm{per} .) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Foundry Co. | 0.75 | 1.00 | 27 | 18 | 28 | 20 |
| Consumer <br> Product Co. | 0.50* | 0.50* | 18 | 12 | 14 | 10 |
| Printing Co. | 0.45 | 0.60 | 16 | 11 | 17 | 12 |
| Small Plastics Co. | 0.15 | 0.25 | 6 | 4 | 7 | 5 |
| Brush Co. | 0.15 | 0.25 | 6 | 4 | 7 | 5 |
| Soap Co. | 0.3 | 0.38 | 11 | 8 | 11 | 8 |
| Abrasive Co. | 0.5* | 0.5* | 18 | 12 | 14 | 10 |

Table 6.3.4

### 6.4 Example of Optimizing Schedule Under Production Growth

What is the best schedule for Foundry Company (see Appendix A.5) when the production increases? The answer to this question involves both the physical model and the economic considerations that have been discussed. Its furnaces each have storage capacities of 15 tons of molten steel and a maximum melting rate of $2^{\frac{1}{2}}$ tons per hour. It takes 400 KWH of electrical energy to melt a ton of steel. At present, Foundry Company's operation is as follows: during the 3rd shift, it melts about 20 tons of steel per day on the average, at the rate of 2 tons per hour for each furnace, using 1600-1700 KW of electricity for both furnaces during melting hours. The furnaces are operated by a single person working the third shift. Two other workers on the first shift pour the . molten steel into molds. Foundry Company would not have to make any schedule changes for melting up to 30 tons/day or 150 tons/week of steel, but if more than 30 tons/day is to be melted, additional consideration is needed to find an optimal schedule. The following options are open to the manager:
(1) To melt steel during the day on week days
(2) To melt steel at peak capacity of 5 tons/hour at all reduced demand hours, pouring simultaneously at night, except weekends.
(3) To melt steel on Saturdays and Sundays during off-peak hours. The following analysis will be done for the company under $H$ rate. Assume that an additional 75 tons of molten steel above the 150 tons/week is needed.

* The example as given in this chapter was derived based on the assumptions that increased night use (off peak) hours, as described in App. A.2, are from 10:00 PM to 7:00 AM for all weekdays. This time interval changes with time and is determined by the utility to suits its needs.

Note that the wage for the first shift is $\$ 5 /$ hour/person, that the second shift pays 1.15 times the first-shift wage, and the third shift 1.20 times that of the first shift; the wage for Saturday is 1.5 times that of weekdays, and for Sunday, twice the regular weekday rate.)

Option 1 involves melting and pouring during the day, which would mean an additional cost for an increase in demand, plus the additional labor cost for a furnace operator. An additional shift running, with one fornace during the day would be able to increase the melting capacity to 45 tons/day, which means 75 tons/week more steel.

| Added cost/month for additional |  |
| ---: | :--- |
| capacity of 75 ton/week: | $\simeq$Cost/month for one man at regular <br> wage |
|  | +Added demand charge for electricity <br> per month |
|  | +Added energy charge for electricity <br> per month |
|  | $\simeq 5 \times 176$ (\$/month) |
|  | $+800 \times 3.79$ (\$/month) |
|  | + added energy charge/month |
|  | $\simeq 880+3032$ (\$/month) |
|  | + added energy charge/month |
|  | $\simeq 4912+$ added energy charge /month |
| in (\$/month) |  |

Option 2 Involves added cost for two additional foundry men on the third shift and the added demand charge for melting the 1 ton/hour more steel, which consumes an additional 400 kW of demand. Since this additional demand occurs during the third shift, for billing purposes only, one-half of the added 400 kW of demand will be used (see Appdices A. 4 and C.5).

Added cost per month for additional
75 tons/week $\quad \simeq$ added cost/month for two men on the third shift (wages for 5 days/week)

+ added demand charge/month
+ added energy charge/month
$\simeq 2 \times 1.2 \times 5 \times 176+200 \times 3.79+$ added energy charge/month (\$/month)
$=2112+758+$ added energy charge/month (\$/month)
$\simeq 2870+$ added energy charge/month (\$/month).

Option 3. In this option, one furnace operator will work from 7:00 AM to 7:00 PM on Sunday, in addition to the regular operation. Two men wi.11 work from 8:00 AM to 7:00 PM on Saturday, 12:00 Noon to 8:00 PM on Sunday, and one furnace operator could melt steel on Saturday, Sunday, and Monday mornings if needed.

Added cost per month for additional $=$ two men on Saturday for 4 days $/ \mathrm{mo}$.
75 tons/week

+ four men, Sunday, 4 days/month
+ added demand charge/month
+ added energy charge/month
$\simeq 2 \times 1.5 \times 5 \times 8 \times 4+4 \times 2 \times 5 \times 8 \times 4$
$+0+$ added energy charge/month (\$/month) demand charge
$\simeq 480+1280+$ added energy charge/month (\$/month)
$\simeq 1760+$ added energy charge/month (\$/month).

We can see that Option 3 gives the cheapest alternative, and can therefore conclude that if the company's business grows and the demand for molten steel grows, then the economically attractive option is for the company to use the melting furnaces during weekends.

This example shows that the physical/economic load modeling methodology as described in this document can be used by utilities and their customers themselves to find out what type of options large industrial power users have when production and business grow at a given rate.

If the above computations for the three options were carried out for the company under $X$ rate, then it can be seen that option 3, that for melting steel during weekends, becomes even more attractive, because the demand charge under X rate is higher and there is also an energy rate differential (see Table 6.3.3). The actual computation is similar to that shown above, and will not be repeated.

### 6.5 Guide to New Investment Decisions

The described approach (see $\S 6.2$ ) gives an indication of how new investment decisions can be made. For example, when a firm wants to expand its production and needs new capacity, should it: (1) buy new machines and operate them during the first shift, or (2) run the existing machines for a second or third shift, instead of buying new machinery? The rules for making the decisions for these possible alternatives are as follows:
(1) If $A_{i j} \ll A_{i j}^{e}$, then it is not necessary to consider the effect of a savings on the monthly electric charge; buy the additional equipment if needed, and operate it during the first shift.
(2) If $A_{i j} \approx A_{i j}^{e}$, then consider the effect of savings on the monthly electric charge before buying new equipment. It might be cheaper to use the existing equipment for the second or third shift.
(3) If $A_{i j}>A_{i j}^{e}$, then do NOT buy new equipment. It is cheaper to run the existing equipment for second or third shifts.

In the above analysis, the interest cost of financing the purchase of equipment was not included. If this cost is also added in, then under the second and third alternatives, it will be more attractive for an industrial company not to invest in new equipment, but to run the old equipment for 1onger hours.

It should be noted that the decision rules given in this section are first-round approximations. The actual investment decisions might involve many other important issues. However, for many situations, this type of rough guide is helpful.

### 6.6 Net Demand Change Due to Production Rescheduling

In this section, the effect of rescheduling an industrial employee on the load of a utility will be analyzed. If an industrial customer reschedules one of his operations or production processes from the first to the second or third shift, then his expected demand would go down. The net effect on the utility's load during the first shift would include the reduction of the expected demand due to reduction of the expected demand of this industrial customer; the increase in the expected residential demand due to the fact that the employee would be at home, using his appliances and other equipment.

Define the following:
$A_{i j}^{n}[n] \triangleq$ the net changes in the electric power demand of a utility at hour $n$ (of the day shift) due to changes in the schedule of an employee from a particular $i \rightarrow j$ section of a manufacturing plant.
$\mathrm{A}^{\mathrm{r}}[\mathrm{n}] \triangleq$ the kW per person in the residential sector at hour n (of the day shift). This is the amount of electric power a person from the residential sector would use in hour n (day shift).
$A_{i j}^{n}=\quad$ can be expressed as follows:
$A_{i j}^{n}[n]=A_{i j}-A^{r}[n]$.
Let us assume that on the average, each person from the residential sector is responsible for less than 2 kW of the utility's peak. We know that $\mathrm{A}_{\mathrm{ij}}$ which are economically rescheduled has to be larger than $A_{i j}^{e}$. This is
the necessary condition for economic rescheduling of an operation. Under H Rate, $A_{i j}^{e}$ is approximately $50 \mathrm{~kW} /$ person.

Define $A_{i j}^{\prime}=$ the $\mathrm{kW} /$ person of a process that could be economically rescheduled.
$A_{i j}^{\prime \prime}=$ the $\mathrm{kW} /$ person of a process that could not be economically rescheduled.

We then have

$$
\begin{aligned}
& A^{r} \leq 2(\mathrm{~kW} / \text { person }) \\
& A_{i j}^{\prime \prime} \lesssim 50(\mathrm{~kW} / \text { person }) \\
& A_{i j}^{\prime}>50(\mathrm{~kW} / \text { person })
\end{aligned}
$$

It can be seen that $A_{i j}^{\prime} \gg A^{r}$ under $H$ Rate; this condition will still be true under X Rate.

Therefore

$$
A_{i j}^{n}[n] \simeq A_{i j}
$$

The conclusion here is that if a person could be economically rescheduled, then the effect on the total system load, due to his usage of power at his home, is minimal.

### 6.7 Social Implication Due to Production Rescheduling

Suppose X kW of demand from the industrial sector of a utility during the first shift can economically be rescheduled by its industrial cusomters to the third shift. What is the number of people that have to be rescheduled from the first to the third shift?

Define

> NPC $\triangleq$ the number of people that are being economically rescheduled by industrial managers from the first to the third shift for the entire service area of a utility.
> $A_{i j}^{\prime} \triangleq$ those production processes with $\mathrm{kW} /$ person that could economically be rescheduled, or $A_{i j}>A_{i j}^{e}$.
> $A_{i j}^{\prime} \min _{\underline{m}}$ the smallest $A_{i j}^{\prime}$ that has $\mathrm{kW} /$ person that could be economically rescheduled.

Then NPC can be expressed as:

$$
N P C \leq \frac{X}{A_{i j}^{\prime} \min }
$$

If the following values are assumed:

$$
\begin{gathered}
\mathrm{X}=100 \mathrm{MW} \\
\mathrm{~A}_{\substack{\prime \\
\mathrm{min}}}^{\mathrm{min}}=50 \mathrm{~kW} / \text { person, } \\
\text { then } \mathrm{NPC}<\frac{100 \times 106}{.5 \times 10^{6}}
\end{gathered}
$$

which gives NPC $<2000$ people.

However, the actual $A_{i j}^{\prime}$ for processes that are being economically rescheduled are much larger than the 50 kW per person. For example, the $A_{i j}$ for the furnaces of Foundry Company is 1700 kW per person. Printing Company has not considered rescheduling its waste baling operation, although it has an $A_{i j}$ of $66 \mathrm{~kW} /$ person. Therfore, the number of persons that may have to be rescheduled economically from the first to the third shift for the entire industrial sector of a utility (by the industrial managers) is at best a few hundred and at worst a few thousand for the reduction of 100 MW of demand from the peak load of a utility. Note that the industrial managers could reschedule only if the process would save them money.

A good estimate can be obtained if reliable data of all the industrail customers of a utility is available. If this type of data is available, a simple count of the number of people using more than some Ailj kW of power within the entire service area would give the number of people that will be affected.

### 6.8 Effect of the Present H Rate on Production Schedules

The present rate structure for electricity is one of the factors that make the present electric load shape of each manufacturing firm the way it is.

The $\partial M C H / \partial D$ of the present rate structure influences the load shape of each firm by influencing the managers . to get a flat or evenly distributed electric power demand (load) shape, or one with effective flat billing demand when the rate for increased night use is also considered. For some cases, the effect of the present rate structure on a firm's load shape is good for the utility, because it forces the customer firm to reduce its usage of electricity during the peak hours of the utility. In some other cases, the effect of the present rate structure is bad for the utility, because it forces some customer firms to increase their electricity usage during peak hours. This will be shown, using the hypothetical firm, as follows:

Consider a manufacturing firm with only single shift operations. Assume that the firm has three departments: Dept. 1 consumed 300 kW of constant power during the shift, and was operated by only one employee. Dept. 2 consumer 200 kW of constant power and was operated by 20 employees. Dept. 3 consumed 150 kW of constant power, and was operated by 30 employees We can see that

$$
\begin{aligned}
& A_{1}=200(\mathrm{~kW} / \text { person }) \\
& \mathrm{A}_{2}=200 / 20=10(\mathrm{~kW} / \text { person }) \\
& A_{3}=150 / 30=5(\mathrm{~kW} / \text { person })
\end{aligned}
$$

Suppose $\partial \mathrm{MCH} / \partial \mathrm{D}=0$ (there was no demand charge);
then all the section will be scheduled to be operated during the first shift, because the monthly electric charge will be unchanged, regardless of what the peak demand is and the labor cost is lowest during the first shift. But in reality, the demand charge is not zero under $H$ Rate; if demand charge is not zero, then the breakeven $A^{e}$ can be computed easily.

Suppose the $A^{e} \simeq 100 \mathrm{~kW} /$ person is obtained. Then, using the decision rules shown in 6.2 , we know that Dept. I should be rescheduled to operate during the second shift, and that Depts. 2 and 3 should operate during the first shift, because

$$
\begin{aligned}
& A_{1}=200>A^{\mathrm{e}}=100(\mathrm{~kW} / \text { person }) \\
& \mathrm{A}_{2}=10<\mathrm{A}^{\mathrm{e}}=100(\mathrm{~kW} / \text { person }) \\
& A_{3}=5<\mathrm{A}^{\mathrm{e}}=100 \quad(\mathrm{~kW} / \text { person })
\end{aligned}
$$

In Appendix B, it is clear that the utility peak during a peak day occurs from 2:00 to $3: 00 \mathrm{pm}$ in the summer, and from 6:00 to $7: 00 \mathrm{pm}$ in the winter. Therefore, the rescheduling of Dept. 1 from the first to the second shift is good for the system's load in summer, because the peak would be reduced. But it is bad for the system load in winter, as its peak would increase because Dept. 1 is rescheduled to operate during the second shift.

### 6.9 Effect of a General Time-of-Day Rate Structure on Production Schedules

The discussion in this section is based on a hypothetical time-of-day pricing structure in order to keep it general. A very general time-of-day rate structure for a monthly charge can be presented as a function of the hourly demand for each hour of the month. For example, monthly charges MC for a manufacturing firm can be shown as

$$
\begin{equation*}
M C=f(d[1], d[2], d[3], \ldots \ldots, d[4 \times 24 \times 30]) \tag{6.9.1}
\end{equation*}
$$

where $d[n]$ is the 15 -minute by 15 -minute demand of the firm at time $n$ of the month.

A simpler but still general version of the time-of-day rate structure is obtainable by dividing the time of day into different intervals. For example, MC can be shown as

$$
\begin{equation*}
M C=f\left(d_{1}, d_{2}, d_{3}, d_{4}, \varepsilon_{1}, \varepsilon_{2}, \varepsilon_{3}, \varepsilon_{4}\right) \tag{6.9.2}
\end{equation*}
$$

where $d$ is the actual demand during $\mathrm{k}^{\text {th }}$ time interval of the day.
$\varepsilon_{k}$ is the actual energy usage during $k^{\text {th }}$ time interval of the day
$k=1,2,3,4 \ldots$ can be used to represent the first, second, third, and weekend shifts, etc. of the day. Note that, in general, the utility could partition the time interval according to its needs.

To further simplify the time-of-day rate structure, consider the special case which is a modified version of the present $H$ Rate structure whose monthly charge, MC , for a given $\varepsilon$ and $D$ is show below:

$$
\begin{equation*}
M C=f_{d}(D)+f_{e}(\varepsilon, D) \tag{6.9.3}
\end{equation*}
$$

where

$$
\begin{aligned}
& D=\max \left\{\alpha_{1} d_{1}, \quad \alpha_{2} d_{2}, \alpha_{3} d_{3}, \alpha_{4} d_{4}\right\} \\
& \varepsilon=\beta_{1} \varepsilon_{1}+\beta_{2} \varepsilon_{2}+\beta_{3} \varepsilon_{3}+\beta_{4} \varepsilon_{4} \\
& \alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4} \text { and } \beta_{1}, \beta_{2}, \beta_{3}, \beta_{4}
\end{aligned}
$$

are the respective weighting factors. $d_{1}, d_{2}, d_{3} d_{4}$ and $\varepsilon_{1}, \varepsilon_{2}, \varepsilon_{3}, \varepsilon_{4}$ are as described above. The different time-of-day rates of energy charges will encourage the manufacturing firm to use electricity during low-price hours when the total system demand is low, and not to use electricity during high-price hours when the total system demand is high. The $\partial \mathrm{MC} / \partial \mathrm{D}$ is needed to ensure that the customer firms keep their demand as low as possible. Suppose, if $\partial M C / \partial D=0$ is used for the new rate structure, a company such as Foundry Company might be melting steel during the day occasionally for a short interval of time, say an hour or less.

The effects of different electric rate structures on the schedule of a manufacturing firm are discussed below using the modified version of the present rate H as shown in Eq. (6.9.3) for simplicity.

Case 1 Consider the case in which the present rate structure, with different values of $\partial M C / \partial D$, is used. If the $\partial M C / \partial D$ is changed, then the $A_{i j}^{e}$, the breakeven $\mathrm{kW} /$ person of the firm, will be changed also. The opportunity for the firm to reschedule its operation is as described in 56.2 . The necessary condition for rescheduling a production process i $\rightarrow \mathrm{j}$ is:

$$
A_{i j} \geq A_{i j}^{e}=\frac{H W_{i j}}{\frac{\partial M C}{\partial D}} \times \Delta R_{i j} \text {, as given before. }
$$

Case 2 Consider the case where the $\partial M C / \partial D$ is the same as the present rate $H$, except that the incremental energy cost $\partial M C / \partial \varepsilon$ varies with time. Let $(\partial M C / \partial \varepsilon)_{k}$ be the incremental monthly charge for energy during the $\mathrm{k}^{\text {th }}$ time interval (or the $k^{\text {th }}$ shift). The necessary condition for moving a section of the plant from first to third shift will become

$$
\begin{equation*}
A_{i j} \geq \frac{H W_{i j} \quad \Delta R_{i j}^{I \rightarrow 3}}{\frac{\partial M C}{\partial D}+H W_{i j}\left[\left(\frac{\partial M C}{\partial \varepsilon}\right)_{1}-\left(\frac{\partial M C}{\partial \varepsilon}\right)_{3}\right] \gamma}=A_{i j}^{e} . \tag{6.9.4}
\end{equation*}
$$

$\gamma \simeq 1$ is a correction factor to account for the fact that for each $k W$ reduction in peak demand, the energy usage that is being rescheduled could be different from $\mathrm{HW}_{\mathrm{ij}}$. This is the case because neither the total plant nor the individual production process has a constant power demand during a shift. The power demand, as we know, is a random process, whose expected value varies from hour to hour.

Case 3 Consider the proposed $X$ Rate as described in Appendix $A$ and §6.2. We know that

$$
\begin{aligned}
& \frac{\partial M C}{\partial D}=\alpha_{\text {seasonal }}^{\prime} \\
& \frac{\partial M C}{\partial \varepsilon_{1}}=\beta_{1}^{\prime} \\
& \frac{\partial M C}{\partial \varepsilon_{2}}=\beta_{2}^{\prime}
\end{aligned}
$$

where $\alpha^{\prime}, \beta_{1}^{\prime}$ and $\beta_{2}^{\prime}$ (the primed notation) are used to separate these quantities from the $\alpha_{1}, \alpha_{2} \ldots, \beta_{1}, \beta_{2} \ldots \ldots$, used in this section, 6.9, because their time intervals are
different from $X$ rate. The necessary condition for economically rescheduling is:

$$
\begin{equation*}
A_{i j} \geq \frac{H W_{i j} \Delta R_{i j}^{l \rightarrow R}}{\alpha_{\text {seasonal }}^{\prime}+\operatorname{NHI[\beta _{1}^{\prime }-\beta _{2}^{\prime }]\gamma }}=A_{i j}^{e} \tag{6.9.5}
\end{equation*}
$$

NHI represents the number of hours of energy usage that is being rescheduled from peak hours to off-peak hours. It is different from $H_{i j}$ because the partition of the time intervals to peak and off-peak hours is different. For example, peak hours last from 7:00 am to $9: 00 \mathrm{pm}$ of a weekday, whereas the first shift of the operation of a manufacturing firm might last from 8:00 am to $4: 00 \mathrm{pm} ; \gamma$ is the correction factor as described in Case 2 of this section.

Case 4

$$
\text { Let } D=\max \left\{\alpha_{1} d_{1}, \alpha_{2} d_{2}, \quad \alpha_{3} d_{3}, \quad \alpha_{4} d_{4}\right\}
$$

$$
\varepsilon=\beta_{1} \varepsilon_{1}+\beta_{2} \varepsilon_{2}+\beta_{3} \varepsilon_{3}+\beta_{4} \varepsilon_{4}
$$

Then, for summer season when the system peak for the utility is from 2:00-3:00 pm (see Appendix B), a desirable choice for the weighting factors $\alpha$ 's and $\beta^{\prime}$ s will be:

$$
\left.\begin{array}{l}
\alpha_{1} \geq 1, \quad \beta_{1} \geq 1 \\
\alpha_{2}=1, \beta_{2}=1 \\
\alpha_{3} \leq 1, \beta_{3} \leq 1 \\
\alpha_{4} \leq 1, \beta_{4} \leq 1
\end{array}\right\} \text { for summer }
$$

But for winter season, when the system peak demand for the utility is from 5:00-6:00 pm (see Appendix B), a desirable choice for the weighting factors $\alpha$ 's and $\beta^{\prime} s$ will
be:

$$
\left.\begin{array}{ll}
\alpha_{1}=1, & \beta_{1}=1 \\
\alpha_{2} \geq 1, & \beta_{2} \geq 1 \\
\alpha_{3} \leq 1, & \beta_{3} \leq 1 \\
\alpha_{4} \leq 1, & \beta_{4} \leq 1
\end{array}\right\} \quad \text { for winter }
$$

$(\xrightarrow[\partial D]{\partial D})_{\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}, \quad \text { indicates the fact that the value of } \partial M C / \partial D \text { depends }}$ on the choices of the weighting factors $\alpha$ 's.
$\left(\frac{\partial M C}{\partial \varepsilon_{k}}\right)=\beta_{k} \frac{\partial M C}{\partial \varepsilon}$ for the $k^{\text {th }}$ shift, which gives the necessary condition for economically rescheduling as follows:
$A_{i j}^{1 \rightarrow 3} \geq \frac{\Delta R_{i j}^{1 \rightarrow 3} H W_{i j}}{\left(\frac{\partial M C}{\partial D}\right)_{\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}}+H W_{i j} \frac{\partial M C}{\partial \varepsilon}\left[\beta_{1}-\beta_{3}\right] \gamma}=A_{i j}^{e}$
for rescheduling from the first to the third shift and again, $\gamma \simeq 1$
is the correction factor, as described previously.
6.10 Electric Power "Supply Curves" of a Manufacturing Firm

In the field of microecononics, the supply curve is used for expressing the quantity of certain products available for sale as a function of the price (\$/quantity). In this section, the supply curves that will be derived are used for expressing the quantity ( kW ) of electric power demand a particular manufacturing firm is willing to "sell back" to the utility at a given "price" (\$/kW). The utility would like its customers to reduce their usage of power during the peak demand hours; for this, the utility is willing to pay the manufacturing firm a "price". "Selling back" refers to the fact that the customer will reduce its electric power usage by a certain amount of kW during certain hours of the day and time of year.

The coincident electric power demand of a firm during the tility's system peak demand hour can be described as follows:

$$
\begin{equation*}
\mathrm{d}_{\mathrm{co}}=\mathrm{d}_{\mathrm{co}_{1}}+\mathrm{d}_{\mathrm{co}_{2}} \quad \text { (approximately) } \tag{6.10.1}
\end{equation*}
$$

where $d_{c o}$ is the electric power demand of a manufacturing firm during the utility's system peak demand hour or the coincidence demand (in kW).
$\mathrm{d}_{\mathrm{CO}_{1}}$ is the component of $\mathrm{d}_{\mathrm{co}}$ that does not involve rescheduling of shifts (in kW)
$\mathrm{d}_{\mathrm{CO}_{2}}$ is the component of $\mathrm{d}_{\mathrm{co}}$ that does involve the rescheduling of shifts (in kW).

The above relation is approximate, because in actuality, $d_{c o}$ cannot be separated exactly into two components, as shown; there is some overlapping between the two components.

Let $C P_{I} \triangleq$ the cost to the firm to decrease a kW of $\mathrm{d}_{\mathrm{col}}(\$ / \mathrm{kW})$ $\mathrm{CP}_{2} \triangleq$ the cost to the firm to decrease a kW of $\mathrm{d}_{\mathrm{co} 2}(\$ / \mathrm{kW})$ $C P_{1}$ and $\mathrm{CP}_{2}$ can be viewed as the least (breakeven) prices that the firm must charge the utility for buying back each unit of demand during the system peak hours. $\mathrm{CP}_{1}$ and $\mathrm{CP}_{2}$ are increasing functions of the amount of demand available for sale by the manufacturing firm. $\mathrm{CP}_{2}$, but not $\mathrm{CP}_{1}$, can be described quantitatively. The reduction of $\mathrm{d}_{\mathrm{cO} 2}$ involves the rescheduling of workers, but for some cases the reduction can be achieved by delaying a shift or starting it a few hours earlier. The rescheduling and movement of a shift in time is possible only for the sections or machines that are under one-shift or two-shift operation, not those sections or machines under three-shift continuous operation. See Appendix $B$ for the load profiles of the peak days for a utility. They indicate that if economic considerations are neglected, the system peak load during winter could be reduced by $10 \%$ of the original peak by delaying, in industrial firms, the second shift; by changing its time from 4:00 PM- midnight to 9:00 PM-5:00 AM as a shift. This is possible, because during winter peaking days, the system load stays above the $90 \%$ level only from 4:00 to 8:00 or 9:00 PM.

During summer peaking days, however, the system load stays above the $95 \%$ level from 10:00 AM to about $4: 00 \mathrm{PM}$, so it is obvious that to
reduce the systems's peak in summer substantially, industrial firms will have to reschedule their first shift to the 2 nd or 3 rd shifts. The delaying or moving of a shift a few hours forward will be unable to reduce the system peak by more than one or two percent.
$\mathrm{CP}_{2}$ for a particular section $i \rightarrow j$ of $a$ firm can be found from the breakeven relation as follows:

$$
\begin{equation*}
\mathrm{CP}_{2 i j} \mathrm{~A}_{i j}=\Delta \mathrm{R}_{i j} H W_{i j} \tag{6.10.2}
\end{equation*}
$$

where

$$
\begin{aligned}
& A_{i j} \text { is the } k W \text { of demand per employee at section } i \text { to } j \\
& H W_{i j}^{\prime} \text { is the hour involved per month for the changing or delaying } \\
& \text { of a shift } \\
& \Delta R_{i j} \text { is the difference in wage for different times of day. }
\end{aligned}
$$

The above relation says that for a manufacturing plant to break even, the increase in cost for each employee per month due to schedule changes has to equal the saving from the monthly electric charge incurred:

$$
\begin{equation*}
\mathrm{CP}_{2 i j}=\frac{\Delta \mathrm{R}_{i j} H W_{i j}^{!}}{A_{i j}} \tag{6.10.3}
\end{equation*}
$$

$\mathrm{CP}_{2}$ for the manufacturing firms during the summer as well as winter season, will be derived in the following case studies.

Case 1.a $\quad \mathrm{CP}_{2}$ for Printing Company (see Fig. 6.10.1) involves, in the summer, the rescheduling of the first shift to the second or third shifts. Assuming it will be from the first to the third:

$$
\begin{aligned}
& \Delta R_{i j}=\left\{\begin{array}{l}
0.85 \$ / \text { person/hour for printing press } \\
0.60 \$ / \text { person/hour for the rest of the plant }
\end{array}\right. \\
& \begin{aligned}
\mathrm{HW}_{\mathrm{ij}}= & 176 \text { hour/month for rescheduling the lst shift to } \\
& \text { the third shift. }
\end{aligned} \\
& \mathrm{CP}_{2} \text { waste baling }=\frac{\$ 0.6 \times 176}{66}=1.60 \$ / \mathrm{kW} \\
& \mathrm{CP}_{2} \text { printing press }=\frac{\$ 0.85 \times 176}{20}=6.16 \$ / \mathrm{kW} \\
& \mathrm{CP}_{2} \text { cutter, binder, etc. }=\frac{\$ 0.06 \times 176}{4}=26.4 \$ / \mathrm{kW}
\end{aligned}
$$

Case 1.b $\quad \mathrm{CP}_{2}$ for Printing Company (see Fig. 6.10.2) involves, in the winter season, the delaying of the second shift from a $4: 00 \mathrm{PM}$ to midnight setup to a 9:00 PM to 5:00 AM arrangement.

$$
\left.\begin{array}{l}
\Delta R_{i j} \text { 2nd - 3rd shift }=\left\{\begin{array}{l}
\$ 0.21 \text { for printing press/person/hr } \\
\$ 0.15 \text { for the rest of the plant }
\end{array}\right. \\
H W_{i j}^{\prime}=5 \mathrm{hr} / \text { day } 22 \text { days/month }=110 \mathrm{hr} / \mathrm{month}
\end{array}\right\} \begin{aligned}
& \mathrm{CP}_{2} \text { waste baling }=\frac{\$ 0.15 \times 110}{66}=0.25 \$ / \mathrm{kW} \\
& \mathrm{CP}_{2} \text { printing press }=\frac{\$ 0.21 \times 110}{20}=1.16 \$ / \mathrm{kW} \\
& \mathrm{CP}_{2} \text { cutter, binder, etc. }=\frac{\$ 0.15 \times 110}{4}=4.13 \$ / \mathrm{kW} .
\end{aligned}
$$

A similar analysis can be performed for each of the remaining six companies to find out their electric power supply curve; however, this computation will not be repeated again here.

Figure 6.10.1



Figure 6.10.2

### 6.11 Discussion and Perspectives

The economics aspect of the industrial electric load is discussed. The idea of representing the monthly electric cost as a well defined nonlinear mathematical function is introduced. Partial derivatives of this function can be taken to yield incremental charges due to a unit change of a certain variable of interest.

By comparing the possible reduction in the monthly electric cost due to rescheduling of a process with the increase in monthly cost due to wage differential, the following condition is derived. The necessary condition that the firm have net lower cost due to rescheduling of a process is

$$
A_{i j}>A_{i j}^{e}
$$

where $A_{i j}$ is the $k W /$ person of the process, and $A_{i j}^{e}$ is the breakeven $k W / p e r-$ son of the firm computed using information from rate structures and wage differentials.

The second necessary condition is that the new schedule does not violate the flow/storage constraint of the firm.

Many issues were introduced and analyzed based on the above simple idea. It is found that the ideas described above do not give us a fixed answer, but offer powerful insights into otherwise very complicated issues.

## CHAPTER VII

POSSIBLE AREA OF APPLICATIONS

It was pointed out in the introduction of this document that the industrial load model and methodology developed could be used to study different issues of interest. Some of these issues and possible areas of application are:
(1) Rate structure design
(2) Effect of a change in exogenous conditions on load
(3) After-the-fact evaluation of the effect of a change in exogenous condition
(4) Cost and benefit evaluation of various alternatives by plant managers
(5) Customer service.

The solutions to the issues listed above might require massive data-gathering efforts. The need for data and manpower to do the survey will depend on the issues of interest, the size of the utility and the number of customers involved. In some cases, a questionnaire might have to be developed to aid the data-gathering effort. The modeling methodology developed, and the insight gained in the present research, would be invaluable to the design and preparation of such a questionnaire. It is very important to ask the right questions in a proper way so that the desired data can be obtained.

The problem of rate structure design is discussed on a reasonably detailed level using physical/economic modeling methodology and analysis. The other issues listed previously could be viewed as the same problem as the rate structure design except viewed from a different perspective.

For example, some problems will be viewed from the industrial customer's perspective. The steps leading toward solution to these other issues and problems involve the same general physical/economic modeling methodology and analysis.

### 7.1 Rate Structure Design

Electric rate is important to a utility and its customers. Quantitative methodology and analysis of the problem of rate structure design is highly desirable. Increased understanding in this area is beneficial to society as a whole because it will be helpful in the design of a rate which will allow efficient use and operation of an industrial customer's equipment and power system.

Two different types of rates for the utility under study were analyzed. The $H$ rate has a reducing block type of structure. It is described in Appendix A. 2 and analyzed in Appendix A.1. Rate X does not have the reducing block type of structure; it has a time-dependent rate for energy usage and is called "time-of-day" rate. It is described in Appendix A. 4 and analyzed in A.3. Although the two above-named rates are for only the utility under study, other utilities have similar rate structures. The analysis done in this document is general and could be used to analyze any conceivable rate or industrial customer.

An important concept introduced in section 6.2 and Appendices A.1 A. 5 is that the monthly charge under any type of rate could be described as a well-defined mathematical function of a number of variables such as peak demand, energy usage, peak energy and off-peak energy usage, etc.

Under the functional representation, the partial derivatives of the monthly charge with respect to each variable involved, could be taken. This would yield the incremental monthly charge of electricity with respect to a unit change in one of the chosen variables. This information is essential to the economic analysis that might be made.

The design of electric rate structures involves social, political, and economic considerations. The social and political issues are outside the scope of this research. We will deal with only some of the economic issues involved; those involved with rate structure design are:
(1) The general structure of rate that the utility would like to have
(2) The present cost of generation under different types of possible utility load shapes and the future cost of generation under load growth
(3) The choices of marginal cost pricing or an average cost pricing scheme, etc. that the commissioner of utilities or an executive of a utility wishes to make
(4) The level of revenue from a certain group of customers
(5) The change of revenue when the rate is changed, with the customer's load shape unchanged
(6) The level of change in each customer's load shape that can be expected.

The above list is by no means exhaustive. It does show the complexity of rate design. Items (2) and (3) above will not be considered here because, again, they are beyond the scope of this document. These are areas where research is being conducted by many other researchers. The questions posed by items (4), (5) and (6) can be answered by using a computer. The knowledge and insight gained from this research will aid the computer study, as the physical/economic model of an industrial plant and various analytical tools developed here could be used for such a study. See Fig. 1.6.2 for the block diagram associated with such a study. The computer study could be done, to a large extent, in a straightforward way.

However, for our purposes, the off-line graphical techniques and analysis will be emphasized because it allows us to learn and gain insight into the problem of rate structure design. The philosophy is that the analytical solution to a problem, when available, is usually more powerful than is computer simulation or studies. Since analytical solutions are not always possible, computer solutions have their place in various areas of research. Note that the $H$ rate and $X$ rate will be used as examples. The procedure for the graphical analysis of rates and its possible effect on an industrical customer are:
(a) To compare the possible saving under X rate over H rate. This could be done using the graphs given in Figs. A.5.1, A.5.2 and A.5.3 of Appendix A.5. Each of these figures is applicable for a given season. From this curve and a simple formula (described in Appendix A.5), we can compute the
saving without much effort. These graphs also give us an overall feel for the effect and the savings possible under X rate compared to H rate when there are no changes in operation schedule and load shape for an industrial firm.
(b) To find out the type of economically attractive options open to the industrial customer with respect to its schedule. We must look at the following three simple steps to see if a certain option is attractive. The first step is involved with the comparison of $A_{i j}$ with $A_{i j}^{e}$ as described in $\$ 6.1$ and 6.2. If $A_{i j}$ is greater than $A_{i j}^{e}$, then the process has economic potential and we can proceed to the next step. The values of $A_{i j}^{e}$ for $H$ rate are given in Table 6.3.2 and for $X$ rate in Fig. 6.3.1. $A_{i j}^{e}$ can be computed from any given real or hypothetical rate as discussed in 56.9 . The second step involves checking to find out if the production process $i \rightarrow j$ can be rescheduled without violating the flow and storage constraints of Chapter III. If so, we proceed to the third step to determine if the new load shape and peak demand after the process is rescheduled will yield a lower total cost to the plant. After going through the three basic steps, it can be determined if a given schedule is an attractive alternative for the plant.

In 56.9 , it was shown that $A_{i j}^{e}$ changes with respect to the partition of the time interval for peak and off-peak hours, as well as the coefficient for demand charge and energy charges. The $A_{i j}^{e}$ computed could be used
with the $A_{i j}$ and the three-step procedures to determine the economic attractiveness of a schedule. This type of analysis will guide the utility company personnel in their choices for rate structures and coefficients.

Two items ,(1) and (4), from the economic issues related to rate, remain to be explained. Item (4), the determination of the revenue from a certain group of customers, involves a straightforward computation. It could be easily computed using the rate, as shown in Appendix A.l, for a company under $H$ rate or the rate as shown in Appendices A. 3 and A. 4 if the company is under $X$ rate.

Item (1), the general structure of rate, is a very complex issue. However, this research has provided us with insight into some of the problems of choosing a rate structure.

In rates $H$ and $X$, described in Appedices A.1 - A.5, it can be seen that to compute the peak demand, the 15 -minute average demand is used. The formula in Chapter II gives us insight into the relationship between the peak demand and time step $\Delta$.

Another possible type of rate is called the "interruptible rate", where a utility would give its customer some type of discount or incentive for the right to disconnect a portion of the customer's load, under terms agreed upon by both parties. It is possible to design many different types of interruptible rates for a single utility. A utility might design, say, five different types of interruptible rates, each with a different kind of
incentive and right-to-interrupt the customer's load in a different way. The physical/economic analysis of the customer's cost, with respect to such interruption, would guide the utility's personnel in the design of such a rate. In particular, the "supply curve" for power as discussed in 56.10 , is useful for this purpose.

### 7.2 Effects of a Change in Exogenous Variables on Load

### 7.2.1 Effect of a Change in Economic Condition

In this document, a relationship between the economic condition and production output is excluded. Suppose the new level of production output requirement is given or determined, using some other method. One can then use the concepts of Chapters II, III, VI and $V$ to find a set of feasible and economically attractive schedules, and a set of related load shapes. This type of information is not complete, but would help a utility planner and forecaster to narrow down the set of possible effects on the system's load that must be considered.

### 7.2.2 Effect of a Change in Working Hours

Two types of working hour arrangements are of interest. They are the four-day work week, or the staggered work shift type of schedule. For these two cases, if the schedule is given, the related load shape could be easily found for a company using the analysis given in the previous chapters.

### 7.2.3 The Introduction of a Conservation Measure

Many different types of conservation measures are conceivable. The manager of a firm could take out some of its light bulbs. This would
change the electric stock for lighting. He could ask his employees not to turn on their air conditioning as often, which would change the utilization factor. The detailed physical and economic analysis of load, as explained previously, is useful for the study of these types of conservation measures.
7.2.4 The Introduction of New Technology

The analysis for this case is involved with the introduction or replacement of a piece of equipment. A physical and economic analysis could be performed to see the change in load shape and the economic attractiveness of the change.

### 7.3 Evaluation of the After Effect due to a Change in an Exogenous Condition

Being able to observe and evaluate the effect due to a change in the exogenous conditions is important to policymakers who are interested in the effectiveness of their policies. For example, how could the change in load shape due to a change in rate be measured? This situation is complicated because the change in load shape could be due to any one of the many changing factors that affect load. The problem of evaluating the aftereffect due to a change in an exogenous condition could be solved by using the physical and economic analysis to determine how each industrial process is affected by the change of a particular exogenous condition. The advantage here is that, by using the physical/economic analysis, the number of candidates that have to be watched for their changes can be reduced.

### 7.4 Cost and Benefit Evaluation of the Various Alternatives by a Plant Manager

The types of issues of interest are:
(a) Investment in new equipment
(b) Rescheduling of production processes
(c) Scheduling under production growth
(d) Load shape of a new industrial plant to be constructed. Most of these issues have been discussed in Chapter VI. They are listed again as a reminder.

### 7.5 Customer Service

This is a service provided by utilities to help their customers with problems related to electric power consumption. Clearly, the analysis and the physical/economic modeling of the industrial electric load would help these utility customer personnel to gain insight into the problems and physical/economic constraints that the utilities customer faces.

### 7.6 Discussion and Perspectives

In this chapter, many possible issues that can be studied using the physical/economic analysis are raised but not analyzed. The design of rate structure is discussed; the problems of rate design related to the physical/economic load model are analyzed to a certain extent. Results from other parts of this report which relate to this issue are pointed out.

## CHAPTER VIII

CONCLUSION AND RECOMMENDATIONS FOR FUTURE WORK

The purpose of this research study is to develop a general industrial load modeling methodology and test out the ideas using the actual electric rates and data from the seven industrial plants chosen. The main conclusion is that it is possible to develop a general methodology that can be used to model industrial electric load in a systematic way.

The modeling methodology developed was first started by analyzing each piece of equipment of a plant individually, and then aggregating them systematically, using basic ideas from the theory of random processes, systems science, and economics. The strength of the methodology rests in the fact that:
(1) Although the basic building blocks are very simple and fundamental, a very complicated plant could be modeled easily using these same basic blocks
(2) While working with the basic idea of modeling as described in this document, powerful insight into industrial electric load has been obtained
(3) The results that can be obtained are not only a computer simulation type of result, but analytical tools that can be used for planning and for decision-making.
(4) The model makes good physical sense at all levels, from the very basic building block to the highly aggregated block (group).

The other conclusion of this study is that it is feasible to develop a model for all of a utility's industrial load which can answer questions of the following types:
(1) How will a utility's total revenues change if the rate structure is changed and if industrial demand does not change?
(2) What type of changes in a utility's industrial demands can be expected if the rate structure is changed?
(3) What type of rate changes should be attempted if a utility wants to modify its industrial load demand?

This list of questions, or issues of interest, is by no means exhaustive. In Chapter VII we can see many other issues that are being considered. The model is useful for evaluating potential industrial demand responses to rate changes, primarily with respect to production scheduling (i.e., changes in working hours) and secondarily, with respect to equipment replacement. The model does not consider the effect of rate changes on production levels, competitive position in the marketplace, etc.

The industrial load demand model would consist of many individual customer modules, each of which models one particular industrial customer. Each customer module is a physical model, consisting of:
(1) A representation of the industrial plant as a set of storage areas and production flows with associated constraints on storage levels and flow rates.
(2) Approximate values of electrical stocks (installed kilowatts rating) associated with each flow and storage area.
(3) Approximate hour-by-hour utilization factors associated with
each electric stock (actual kW demand equals electric stock times utilization factor).
(4) A set of elementary random processes to model the utilization factor.
(5) The number of people associated with each storage level and flow process.

The number of people and the utilization factors are functions of the overall production level of the plant, and its scheduling.

Specification of the utilization factors yields a 15 -minute by $15-$ minute power demand model for the plant that could be used to generate the expected load and the autocorrelation function of the residual for a given time index, n. Peak demand and energy usage can then be deduced. This can be combined with any hypothesized rate structure to determine the utility's revenues (or customer cost).

In order to gain understanding of how a plant might respond to a change in rate structure, it is necessary to learn which processes within the plant satisfy the two conditions of:
(1) being able to change, subject to constraints on plant flow rates and storage leve1s; and
(2) being cost-effective to change, relative to plant costs. One especially useful approach that was developed is as follows. Consider the portion of the overall production process between "storage $i$ " and "storage $j$ ". The physical model yields

$$
A_{i j}=\frac{\text { number of kilowatts used in process i to } j}{\text { number of people employed in process } i \text { to } j}=\mathrm{kW} / \text { person. }
$$

From the rate structure and the knowledge of the salary (and other)costs associated with rescheduling a process to another time of day, it is possible to determine
$A_{i j}^{e}:$ Break-even $k W / p e r s o n$
$A_{i j}^{e}=\frac{\text { dollar cost per person due to rescheduling }}{\text { dollar savings per } k W \text { due to reducing demand }}$

An industrial plant manager will consider the possibility of rescheduling a process to reduce demand only if the actual $A_{i j}$ is appreciably greater than the breakeven $A_{i j}^{e}$. A similar type of analysis can be done to indicate how much it would cost a particular industrial customer to reduce demand at any given hour of the day, time of the year, etc. This can yield "supply curves" that indicate how much power demand an industrial concern would be willing to "sell back" to the utility as a function of the "price" the utility is willing to pay. Such information guides the determination of rate structures that may reduce demands during peak periods.

The model does not make direct predictions of changes in load due to a change in one of the exogenous conditions. The physical and economic analysis would determine only what are the set of possible schedules and load shapes that are physically feasible and economically advantageous. By simple comparison, an optimum schedule and load shape can be found. However, managers do not always adopt schedules that give them economic advantage, because the saving might be small and not worth the trouble. A rational manager will certainly not choose to reschedule his employees to work at night if there is no economic advantage or it is not feasible.

Therefore, the model and the methodology can be used to make predictions of what managers will not do, which in its own right is a very valuable piece of information.

The modeling methodology was tested out using actual rate structures and data from seven industrial customers. The expected load and the autocorrelation function derived from theory is found to have good agreement with the sample mean and the time-average sample of the autocorrelation function computed from time-series data. The parameters used for the theoretical model are based on actual records or values that are consistent with managers' descriptions. Formal statistical tests for validity, such as maximum likelihood, are desirable and are one possible area for further work.

Further analytical work is desirable in the improvement of random process models. In this document, only the two-state Markov process and the binomial process are used. There are many other possible areas for further improvement.... for example, a semi-Markov process model for load could probably be developed. The general discrete-state Markov process model, with more than one state, might be needed for some cases. For instance, a two-coupled production process might be treated as a four-state semi-Markov process. Multinomial process models are needed for some production processes. Possible areas for more work here are quite open-ended. Care should be taken in choosing an area that will be likely to be fruitful.

The present model was developed for short-term application lasting several weeks. It does not include relationshipsfor economic, social,
and weather effects on load. The short-term model is fitted to a given set of schedule and time-series data. The model structure and the conceptual framework are extendable to models that will be valid for a longer time. This is a possible area for further work.

One obvious area for work is for a utility to do a field study by gathering data from a very large number of its industrial customers. For this, it is essential to have a good questionnaire. The analysis and insight gained from this present research would be helpful in the preparation of such questionnaires.

Mailing out questionnaires to customers is not the best approach, since the questionnaires could easily be misunderstood. The best way is to have the utility's own personnel gather the data and interact with the industrial customers. SIC code listing by itself is not useful; but two firms from the same SIC code do have something in common. See, for example, Brush Company and Smal1 Plastics Company in Appendix C of this document. SIC code combined with the knowledge gained from the analysis of industrial customers would be useful for selecting candidates to be studied for certain purposes. A fraction of the customers might not be sensitive to a certain issue, so not all the industrial customers are needed for such a study.

Rate structure design is a challenging area in which to do field or analytical work. It is a very complicated and important issue to both the utilities and their customers. In Chapter VII, many of the issues related to rate structure design were brought out; they, too, deserve further study.

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## GUIDE TO THE APPENDICES

There are four groups of appendices which are described below. Appendices Group A describe and analyze two types of electric rates. There are five parts to this Appendix. Appendix A.l deals with the representation of the monthly electric cost under $H$ rate as a mathematical function. Appendix A. 2 gives the description of Rate H. Appendix A. 3 deals with the mathematical representation of Rate $X$ (time-of-day rate), and A. 4 gives the description of Rate X. A. 5 then gives us the comparison of the monthly electric cost under Rate $H$, as compared to Rate $X$.

Appendix B shows load profile for four seasonal peak days of the New England Electric System. Two of its peak winter days for two different years, and two peak summer days for two different years, are given.

Appendices Group C describe the seven industrial customers of the New England Electric System's companies that are being studied. Data from four of these industrial companies (Small Plastics Company, Brush Company, Abrasive Company and Soap Company) are described, respectively, in Appendices A. 1 to A.4. Data for these companies were gathered in the sumner and fall of 1977. The remaining three industrial companies (Foundry, Printing, and Consumer Product Companies) were studied, in the summer of 1976;
they are described in Appendices C. 5 to C.7, respectively.
Appendix $D$ describes the computer programs used in this research project for computing and plotting all the graphs shown in Chapter $V$.

## APPENDIX A. 0

The appendices A.l through A. 5 deal with the analysis and the description of two different electric rates: Rate $H$, and Rate $X$ (the time-of-day rate). A. 1 analyzes Rate $H$, which is described in A.2. The analysis involves the derivation and representation of the monthly electric cost under Rate $H$ as a piecewise linear mathematical function. It gives an exact representation of Rate $H$ in terms of a mathematical function. Appendix A. 3 analyzes Rate X , which is described in A.4. The monthly electric cost under Rate $X$ is represented as an exact mathematical function in A. 3 for winter and for summer. There is a seasonal variation of the monthly electric cost under Rate X .

Appendix A. 5 then gives a comparison of Rates H and X . In particular, the possibly saving for a company that might choose Rate $X$ over Rate H is shown graphically. These kinds of graphs for the comparison of rates are useful to utility personnel and planners.

## APPENDIX A. 1

ANALYSIS OF H RATE

The monthly bill of an industrial customer under $H$ rate as given in Appendix A. 2 can be represented as follows:

$$
\begin{equation*}
\operatorname{MCH}(\varepsilon, D)=f_{d}(D)+f_{e}(\varepsilon, D)+\text { fuel adj. } \tag{A.1.1}
\end{equation*}
$$

where

$$
\begin{aligned}
\operatorname{MCH}(\varepsilon, \mathrm{D})= & \text { monthly charge under } H \text { rate for a } \\
& \text { given } \varepsilon \text { and } \mathrm{D} \text { ( } \$ / \text { month }) \\
\mathrm{f}_{\mathrm{d}}(\mathrm{D})= & \text { monthly demand charge under } \mathrm{H} \text { rate } \\
& \text { as given in Figure A.l.1 ( } \$ / \text { month }) \\
\mathrm{f}_{\mathrm{e}}(\varepsilon, \mathrm{D})= & \text { month1y energy charge under } \mathrm{H} \text { rate } \\
& \text { as given in Figure A.1.1 ( } \$ / \text { month }) \\
\varepsilon & \text { total energy usage of the month } \\
& \text { (KWH/month) } \\
\mathrm{D} & \text { Maximum of all the } 15 \text { minute average } \\
& \text { demand of the month (KW) }
\end{aligned}
$$

We will claim that the monthly charge under $H$ rate could be represented by a constant plus two piecewise linear term as in the following Equation A.1.2. This claim will be "proven" informally
using a case example as in Equation A.1.3. The new form for monthly charge under H rate is as follows:

$$
\begin{align*}
\mathrm{MCH}(\varepsilon, \mathrm{D}) & \left.\right|_{\varepsilon_{O}, D_{O}}=A \text { constant }+\left.\frac{\partial M C H(\varepsilon, D)}{\partial D}\right|_{\varepsilon O, D O} x  \tag{D}\\
& +\left.\frac{\partial M C H(\varepsilon, D)}{\partial \varepsilon}\right|_{\varepsilon O, D o} x(\varepsilon)+\text { fuel adj } \tag{A.1.2}
\end{align*}
$$

where

$$
\begin{aligned}
\frac{\partial M C H(E, D)}{\partial D}= & \text { incremental change in the monthly charge under } \\
& H \text { rate for each unit change in demand } D \text { keeping } \\
& \text { energy usage as a constant. See Table A.l.2. } \\
\frac{\partial M C H(\varepsilon, D)}{\partial \varepsilon}= & \text { incremental change in the monthly change under } \\
& H \text { rate for each unit change in energy usage of } \\
& \text { the month keeping the demand } D \text { as a constant. } \\
& \text { See Table A.l. } 2 .
\end{aligned}
$$

The constant term needed for Equation A.1. 2 has been found and note that the fuel adjustment term is independent of demand and is strictly a linear function of energy usage. The items $\frac{\partial M C H(\varepsilon, D)}{}$ and $\frac{\partial M C H(\varepsilon, D)}{\partial \varepsilon}$ for the Equation A.1.2 is constant only for a certain range of $D_{o}$ and $\varepsilon_{o}$ of interest as shown in Table A.1.2 and Fig. A.1.1.

To show how the monthly charge can be represented as shown in Equation A.1.2, let us consider the following case. Suppose we have a case where

$$
\text { D }>500 \mathrm{~kW}
$$

and

$$
300<\varepsilon / D<400 \text { hours }
$$

then the monthly charge can be derived under $H$ rate as given in Appendix A. 2 as follows:

Demand Charge

$$
\begin{aligned}
f_{d}(D) & =820+1.54(D-500)(\$ / \text { month }) \\
& =50+1.54 D(\$ / \text { month }) \text { for } D>500
\end{aligned}
$$

Energy Charge

$$
f_{e}(\varepsilon, D)=\left\{\begin{array}{c}
2.547 \times 500 \\
+2.247 \times 500 \\
+1.937(2 D-1000) \\
+1.827(3 D-2 D) \\
+1.367\left(\frac{\varepsilon}{100}-3 D\right)(\$ / \text { month })
\end{array}\right.
$$

which is equal to

$$
f_{e}(\varepsilon, D)=400+1.6 D+0.0 .1367 \varepsilon(\$ / \text { month })
$$

Note that in the above equation for $f_{e}(\varepsilon, D)$ the coefficient for demand $D$ is 1.6 and this is the value for $\frac{\partial f_{e}(\varepsilon, D)}{\partial D}$ for $\varepsilon / D$ between 300 and 400 hours as given in Table A.l.1. The coefficient of energy usage $\varepsilon, 0.01367$ is the value of $\frac{\partial f_{e}(\varepsilon, D)}{\partial \varepsilon}$ for $\varepsilon / D$ between 300 and 400 hours as given in Figure A.1.1. The monthly charge under $H$ rate for this case can be expressed as follows:

$$
\begin{aligned}
\operatorname{MCH}(\varepsilon, D)= & f_{d}(D)+f_{e}(\varepsilon, D)+\text { fuel adjustment } \\
\operatorname{MCH}(\varepsilon, D)= & {[50+1.54 D]+[460+1.6 \mathrm{D}} \\
& +0.01367 \varepsilon]+ \text { fuel adj. (\$/month })
\end{aligned}
$$

which gives

$$
\begin{equation*}
\operatorname{MCH}(\varepsilon, D)=510+3.14 \mathrm{D}+0.01367 \varepsilon+\text { fuel adj. } \tag{A.1.3}
\end{equation*}
$$

Note that in the above equation A.1.3. the coefficient for the demand $D, 3.14$, is equal to the $\frac{\partial M C H(\varepsilon, D)}{\partial D}$ for $\varepsilon / D$ between 300 and 400 as shown in Table A.1.2 and the coefficient for energy usage $\varepsilon, 0.01367$ is equal to the $\frac{\partial M C H(\varepsilon, D)}{\partial \varepsilon}$ for $\varepsilon / D$ between 300 and 400 as shown in Table A.1.2. Also note that the constant from the equation A.1.3 is equal to 510 . Figure A.l. 2 gives a graphical explanation of the relation given by Equation A.l.3 for the particular case where $\mathrm{D} \geqq 500$, and $300 \leqq \varepsilon / \mathrm{D} \leqq 400$.




Figure A. 1.1



Figure A. 1.2

TABLE A.1.1
$\$ 820 \quad \delta(D)$ for $D_{0}=0$ $\left.\frac{\partial f_{d}(D)}{\partial D}\right|_{D_{0}}= \begin{cases}0 & \text { for } 0<D_{0}<500 \mathrm{~kW}\end{cases}$
$\$ 1.54 / \mathrm{kW} \quad$ for $500<D_{0}$

$$
\begin{aligned}
& 0 \quad \$ / \mathrm{kW} \text { for } \varepsilon_{0} / D_{o}<200 \mathrm{hr} \\
& \left.\frac{\partial f_{\mathrm{e}}(\varepsilon, D)}{\partial D}\right|_{\varepsilon_{0}, D_{o}}=\left\{\begin{array}{lll}
0.22 & \$ / \mathrm{kW} & \text { for } 200<\varepsilon_{0} / D_{0}<300 \mathrm{hr} \\
1.60 & \$ / \mathrm{kW} & \text { for } 300<\varepsilon_{0} / D_{0}<400 \mathrm{hr} \\
2.00 & \$ / \mathrm{kW} & \text { for } 400<\varepsilon_{0} / D_{0}<500 \mathrm{hr}
\end{array}\right. \\
& 2.25 \$ / \mathrm{kW} \text { for } 500<\varepsilon_{0} / D_{o} \\
& \left.\frac{\partial M C H(\varepsilon, D)}{\partial D}\right|_{\varepsilon_{0}, D_{0}}=\left.\frac{\partial f_{d}(D)}{\partial D}\right|_{D_{0}}+\left.\frac{\partial f_{e}(\varepsilon, D)}{\partial}\right|_{\varepsilon_{0}, D_{o}}
\end{aligned}
$$

TABLE A.1.2

$$
\begin{aligned}
& \operatorname{MCH}\left(\varepsilon_{0} D_{0}\right)=510+\left.\frac{\partial M C H(\varepsilon, D)}{\partial D}\right|_{\varepsilon_{0}, D_{0}} x^{\left(D_{0}\right)}+\left.\frac{\partial M C H(\varepsilon, D)}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}} x^{\left(\varepsilon_{0}\right)} \\
&+ \text { fuel adj. } \quad(\$ / \text { month })
\end{aligned}
$$

where

$$
\left.\frac{\partial M C H(\varepsilon, D)}{\partial D}\right|_{\varepsilon_{0}, D_{0}}=\left\{\begin{array}{lll}
1.54 & \$ / k W & \text { for } \varepsilon_{0} / D_{0}<200 \text { hour } \\
1.76 & \$ / k W & \text { for } 200<\varepsilon_{0} / D_{0}<300 \\
3.14 & \$ / k W & \text { for } 300<\varepsilon_{0} / D_{0}<400 \\
3.54 & \$ / k W & \text { for } 400<\varepsilon_{0} / D_{0}<500 \\
3.79 & \$ / k W & \text { for } 500<\varepsilon_{0} / D_{0}
\end{array}\right.
$$

and

$$
\left.\frac{\partial M C H(\varepsilon, D)}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}=\left.\frac{\partial f_{e}(\varepsilon, D)}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}
$$

$$
2.547 \quad \phi / \mathrm{kWH} \text { for } 0<\varepsilon_{0}<50,000
$$

$$
2.247 \quad \phi / \mathrm{kWh} \text { for } 50,000<\varepsilon_{0}<100,000
$$

$$
\left.\frac{\partial \operatorname{MCH}(\varepsilon, \mathrm{D})}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}=\left\{\begin{array}{lll}
1.937 & \phi / \mathrm{kWH} & \text { for } 100,000<\varepsilon_{0}<200 \mathrm{D} \\
1.827 & \phi / \mathrm{kWH} & \text { for } 200<\varepsilon_{0} / D_{0}<300 \\
1.367 & \phi / k W H & \text { for } 300<\varepsilon_{0} / D_{0}<400 \\
1.267 & \phi / k W h & \text { for } 400<\varepsilon_{0} / D_{0}<500 \\
1.217 & \phi / k W h & \text { for } 500<\varepsilon_{0} / D_{0}
\end{array}\right.
$$

From the explanation of the particular case above one can see that in general the monthly charge for an industrial customer under H rate can be expressed as follows:

$$
\begin{align*}
& \operatorname{MCH}\left(\varepsilon_{o}, D_{o}\right)=510+\left.\frac{\partial M C H(\varepsilon, D)}{\partial D}\right|_{\varepsilon_{0}, D_{0}} x\left(D_{o}\right) \\
& +\left.\frac{\partial M C H(\varepsilon, D)}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}} x\left(\varepsilon_{0}\right)+\text { fue1 charge } \tag{A.1.4}
\end{align*}
$$

where

are as given in A.1.2.

## APPENDIX <br> A. 2

MASSACHUSETTS ELECTRIC COMPANY
Optional Large-Power Rate H M.D.P.U. No. 376

Purchased Power Cost Adjustment No. 6

## Monthly Charge as Adjusted

Demand Charge
$\$ 820.00$
1.54 per KW
First
500 kW or less
Xcs of
500 KW of Demand

Energy Charge per KWH


Other Rate Clauses apply as usual

## MASSACHUSETTS ELECTRIC COMPANY

## OPTIONAL LARGE-POWFSR RATE H

## AVAILABILITY

This rate is availabla for all purposea except reasle. All barvice delivered at a given location ohall be billed hersurder, and all chargea 3 hat? be based on a Damand of 500 kilowatts or more. If delfvary is through more than one metar, except at the Company's option, the Monthly Charge for servire through each meter shall be computed separately under this rate.

## MONTHIT CHARGB

The Honthly Charge will be the an of the Demand and Enargy Charges. Demand Charga
$\$ 820.00$ for the first 500 kilowatts or leas of Demand,
1.54 per kilowatt of Demand in excess of 500 kilosat s .

Energy Charga
1.891 cents per kilowatt-hour for the first 50,000 kilowatt-hours,
1.591 cents per kilowatt-hour for the next 50,000 kilowatt-hours,
1.281 cents per kilowatt-hour for the excess over $100,000 \mathrm{kilowatt-hours}$.

Notwithstanding the foregoing, the following reduced prices shall apply:
1.171 cents per kilowstt-hour for all kilosistt-hours in excess of 200 kilowatt-houre per kilowatt of Demand,
. 711 cent per kilowatt-hour for all kiloustt-hours in exceas of 300 kilowat -hours per kilowatt of Demand,

MASSACHUSETTS ELECTRIC : • COMPANY

OPTIOMAL LMRGE-POWER RATE H

> .611 cent per kilowatt-hour for all kilowatt-hours in excess of 400 kilowatt-hours per kilosatt of Demand,
> .561 cent per kilowart-hour for all kilowatt-hours in excess of 500 kilowatt-hours per kilowatt of Demand.

## PURCHASED POWER COST ADJUSTMERT

The prices under this rate as set forth under "Honthly Charge" may be adjusted from tiue to time in the manner provided in the Company's Purchased Power Cost Adjustment Provisions to reflect changes occurring on or after January 1, 1971 in the Paimary Service for Resale Rate of the Company's supplier, New England Power Company.

ADJUSTMENT FOR COST OF PUEL
The amount determined under the precediag provisions shall be adjusted In accordance with the Company's Standard Fuel Clause as from time to time effective in accordance with law.

DEMAND
The Demand for each month under ordinary load conditions shall be thi: greatest of the following:
a) The greatest fifteen-minute peak occurring during such month as measured in kilowarts,
b) 809 of the greatest fifteen-minute peak occurring during such month as measured in kilovoit-amperes,
c) $80 \%$ of the greatest Demand as so deterained above during the preceding eleven months,
d) 500 kilowatts .

Any Demands established during the eleven months prior to the application of this rate shall be considered as having been eatablished under this rate.

# MASSACYUSETTS ELECTRIC COMPANY 

OPTIONAL LARGE-POWER RAIE $\%$

INCREISED NIGHI USE
If a Customer has entered into a five-year agreament for electic service guaranteeing that the Lemand shall be not less than 500 kilowatts and guaranteeing to pay for not less then 200 kilowart-hours per kilowatt of Demand each month, then any fifteen-minute peaks occurring during the periods from 9:00 P.M. to 8:00 A.M. daily and frem 8:00 A.M. to 9:00 P.M. on Saturdays, Sundays and legal holidays in Massachusetts shall be reduced by one-half for the purpose of ascertaining the Dewand as defined above. On one week's notice to the Customer the Company may change the hours specifled above provided that the aggregate weekly number of hours be not decreased.

For a Customer using the Increasad Night Use prowisions and taking aumiliary service, the minimum Demand under Auxiliary Service previsions shali be $40 \%$ of the kilovolt-ampere rating of the transforners through which service is furnished, whether auch transformers be supplied by the Company or the Customer, or, if service be not furnished through separate transformers, $40 \%$ of the kilovolt-ampere rating of the standard size of transfcrmers which would be required for such service. In no case shall the monthly Demand be less than 500 kilowatts.

## HIGK VOLTAGE METERING ADJUSTMRNT

The Company resezves the right to determine the metering installation. Where service is metered at the Company's supply line voltage, ir no case less than 2400 volts, thereby gaving the Company transformer losses, a discount of $2 \frac{1}{2} \%$ will be allowed from the amount deternined ander the preceding . provisions.

## CREDIT FOR HIGY VOLTAGE DSLIVERY

If the Customer accepts delivery at the Company's supply line voltage, not less than 2400 voits, and the Company is saved the cost of installing any transformer and associated equipment, a credit of 12 cents per kjlowatt of biling demand for such month shall be allowed agzinst the amount determined. under the preceding provisions.

## TERM OF AGREEAENT

The agreement for service under this rate will comtinue for an finitial tern of one year if electricity can be properly supplied to a Cus.toner without an uneconomic expenditure by the Company. The agrement may be terminated at any time on or after the expiration date of the initial term by twelve months' prior uritten notice.

## MASSACHUSEITS ELECIRIC

 COAPANYOPTIONAL LARGE-POWER RATE H

TERMS AND CONDITIONS
The Company's Terms and Conditions in effect from time to time, where not inconsistent with any specific provisions hereof, are a part of this rate.

Effective December 8, 1976

APPENDIX A. 3
ANALYSIS OF X RATE

The monthly charge under X rate can be represented for summer and winter separately as follows (see Appendix A.4):

For Summer

$$
\begin{align*}
\operatorname{MCX}\left(\varepsilon_{1}, \varepsilon_{2}, \mathrm{D}\right)= & 100+6.50 \mathrm{D}+0.01517 \varepsilon_{1} \\
& +0.00277 \varepsilon_{2}+\text { fuel adj. (\$/month) } \tag{A.3.1}
\end{align*}
$$

For Winter

$$
\begin{align*}
\operatorname{MCX}\left(\varepsilon_{1}, \varepsilon_{2}, D\right)= & 100+4.08 \mathrm{D}+0.01517 \varepsilon_{1} \\
& +0.0027 \varepsilon_{2}+\text { fuel adj. (\$/month) } \tag{A.3.2}
\end{align*}
$$

where
$\operatorname{MCX}\left(\varepsilon_{1}, \varepsilon_{2}, D\right)=$ the monthly charge under $X$ rate for $\varepsilon_{1}, \varepsilon_{2}$, D given. See Appendix A. 4 for the X rate.
$\varepsilon_{1} \quad=$ energy usage of the month during peak hours. See Appendix A. 4 for the definition of peak hours.
$\varepsilon_{2} \quad=$ energy usage of the month during offpeak hours. See Appendix A. 4 for the definition of off-peak hours.

D $\quad=$ maximum 15 minute average demand of the month.

One can further continue to find the yearly average of the month1y charge under $X$ rate assuming that $\varepsilon_{1}, \varepsilon_{2}$ and $D$ are identical for all 12 months of the year as follows:

Yearly Average

$$
\begin{aligned}
\operatorname{MCX}\left(\varepsilon_{1}, \varepsilon_{2}, D\right)= & 100+\left(\frac{8}{12} \times 4.08+\frac{4}{12} \times 6.50\right) D \\
& +0.01517 \varepsilon_{1}+0.00277 \varepsilon_{2} \\
& + \text { fuel adj. (\$/month })
\end{aligned}
$$

which gives

$$
\begin{aligned}
\operatorname{MCX}\left(\varepsilon_{1}, \varepsilon_{2}, D\right)= & 100+4.88666--D+0.01517 \varepsilon_{1} \\
& +0.00277 \varepsilon_{2}+\text { fuel adj. (\$/month) }
\end{aligned}
$$

(A.3.3)

## APPENDIX A. 4

# MASSACHUSETTS ELECTRIC COMPANY 

PROPOSED
OPTIONAL RATE X-01
(April 8, 1977)
Purchased Power Cost Adjustment No. 6 - Adjusted

Monthly Charge as Adjusted

## Customer Charge

$\$ 100.00$ per month.

Demand Charge
Billing Months of:
Winter Period
Summer Period
October-May
June-September
$\$ 4.08$ per KW
$\$ 6.50$ per KW

## Energy Charge per KWH

$$
\begin{array}{lc}
\text { On Peak } & 1.51700 ¢ \\
\text { Off Peak } & .277 ¢
\end{array}
$$

Other Rate Clauses apply as usual.

# MASSACHUSETTS ELECTRIC COMPANY 

PROPOSED
OPTIONAL RATE X-O1
(April 8, 1977)

## AVAILABILITY

This rate is available for all purposes except resale. All service delivered at a given location shall be billed hereunder, and all charges shall be based on a Demand of 500 kilowatts or more. If delivery is through more than one meter, except at the Company's option, the Monthly Charge for service through each meter shall be computed separately under this rate.

## MONTHLY CHARGE

The Monthly Charge will be the sum of the Customer, Demand and Energy Charges.

## Customer Charge

$\$ 100.00$ per month.

Demand Charge

During the Billing Months of:

Winter Period
October-May
$\$ 4.08$ per KW

## Energy Charge

Peak Hours
Off-Peak Hours

Summer Period
June-September

- $\$ 6.50$ per KW
1.290 cents per kilowati-hour
.05 cent per kilowatt-hour


# MASSACHUSETTS ELECTRIC COMPANY 

PROPOSED
OPTIONAL RATE X-OI

## PEAK AND OFF-PEAK PERIODS

Peak hours will be from 8:00 A.M. to 9:00 P.M. daily on Monday through Friday except for legal holidays in Massachusetts

Off-peak hours will be from 9:00 P.M. to 8:00 A.M. daily Monday through Friday and all day on Saturdays, Sundays, and legal holidays in Massachusetts. PURCHASED POWER COST ADJUSTMENT

The prices under this rate as set forth under "Monthly Charge" may be adjusted from time to time in the manner provided in the Company's Purchased Power Cost Adjustment Provisions to reflect changes occurring on or after June 1, 1975 in the Primary Service for Resale Rate of the Company's supplier, New England Power Company.

## ADJUSTMENT FOR COST OF FUEL

The amount determined under the preceding provisions shall be adjusted in accordance with the Company's Standard Fuel Clause as from time to time effective in accordance with law.

DEMAND
The Demand for each month under ordinary load conditions shall be the greatest of the following:
a) The greatest fifteen-minute peak occurring during the Peak Hour period within such month as measured in kilowatts,
b) $80 \%$ of the greatest fifteen-minute peak occurring during the Peak Hour period of such month as measured in kilovoltamperes,

MASSACHUSETTS ELECTRIC<br>COMPANY<br>PROPOSED<br>OPTIONAL RATE X-01

c) One-half the greatest 15 minute peak, efther KW or $80 \% \mathrm{KVA}$, occurring during the Off Peak period within such month.
d) $80 \%$ of the greatest Demand as so determined above during the preceding seven winter months when billing in the Winter Period; or $80 \%$ of the greatest Demand as so detemmined above during the preceding three summer months when billing in the Sumner Period. e) 500 kilowatts.

HIGH VOLTAGE METERING ADJUSTMENT
The Company reserves the right to determine the metering installation. Where service is metered at the Company's supply lime voltage, in no case less than 2400 volts, thereby saving the Company transformer losses, a discount of $2 \frac{1}{2} \%$ will be allowed from the amount determined under the preceding provisions.

CREDIT FOR HIGH VOLTAGE DELIVERY
If the Customer accepts delivery at the Compary's supply line voltage, not less than 2400 volts, and the Company is saved the cost of installing any transformer and associated equipment, a credit of 12 cents per kilowatt of billing demand for such month shall be allowed against the amount determined under the preceding provisions.

## TERM OF AGREEMENT

The agreement for service under this rate will continue for an initial term of one year if electricity can be properly supplied to a Customer without

# MASSACHUSETTS ELECTRIC 

PROPOSED
OPTIONAL RATE X-01
an uneconomic expenditure by the Company. The agreement may be terminated at any time on or after the expiration date of the initial term by twelve months' prior written notice:

TERMS AND CONDITIONS
The Company's Terms and Conditions in effect from time to time, where not inconsistent with any specific provisions hereof, are a part of this rate.

## Effective

APPENDIX A. 5
COMPUTATION OF MONTHLY SAVING UNDER X RATE AS COMPARED TO H RATE

Define:
$S_{x h}$ : Monthly saving for choosing $X-01$ rate instead of $H$ rate. Then

$$
\begin{equation*}
S_{x h}=\operatorname{MCH}\left(\varepsilon_{o}, D_{o}\right)-\operatorname{MCX}\left(\varepsilon_{1}, \varepsilon_{2}, D_{o}\right) \tag{A.5.1}
\end{equation*}
$$

where
$\operatorname{MCH}\left(\varepsilon_{o}, D_{o}\right)$ is as defined and given in Appendix A.l.
$\operatorname{MCX}\left(\varepsilon_{1}, \varepsilon_{2}, D_{o}\right)$ is as defined and given in Appendix A. 3.

One can rewrite $S_{x h}$, using expressions for MCH , MCX from Appendices A. 1 and A. 3 as follows:

$$
\begin{aligned}
S_{x h}= & {\left[510+\left.\frac{\partial M C H}{\partial D}\right|_{\varepsilon_{0}, D_{0}} x\left(D_{o}\right)+\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{o}}\left(\varepsilon_{o}\right)\right.} \\
& \therefore+\text { fue1 adj. }]-\left(100+\alpha D_{o}\right. \\
& \left.+0.01517 \varepsilon_{1}+0.00277 \varepsilon_{2}+\text { fuel adj. }\right)
\end{aligned}
$$

$\alpha$ is the seasonal rate coefficient for demand or the incremental monthly charge for demand under X rate. The equation above can be rewritten as

$$
\begin{gathered}
S_{x h}=410+\left(\left.\frac{\partial M C H}{\partial D}\right|_{\varepsilon_{0}, D_{0}}-\alpha\right) D_{0} \\
+\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}-0.01517\right) \varepsilon_{1}+\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}-0.00277\right) \varepsilon_{2}
\end{gathered}
$$

Let $\beta=\left(\varepsilon_{1} / \varepsilon_{2}\right)$
$\varepsilon=\varepsilon_{1}+\varepsilon_{2}$ by definition.
where $\varepsilon_{1}, \varepsilon_{2}$ are as defined in Appendices A. 3 and A.4. $\varepsilon_{1}$ and $\varepsilon_{2}$ can be expressed as follows:

$$
\begin{aligned}
& \varepsilon_{1}=\frac{\beta}{1+\beta} \varepsilon \\
& \varepsilon_{2}=\frac{1}{1+\beta} \varepsilon
\end{aligned}
$$

After substitution of $\varepsilon_{1}, \varepsilon_{2}$, equation A.5.2 becomes:

$$
\begin{gathered}
S_{x h}=410+\left(\left.\frac{\partial M C H}{\partial D}\right|_{\varepsilon_{0}, D_{0}}-\alpha\right) D \\
+\frac{1}{(1+\beta)}\left[\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}-0.01517\right) \beta+\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{o}}\right.\right. \\
-0.00277)]\left(\frac{\varepsilon}{D}\right)
\end{gathered}
$$

which gives

$$
\begin{aligned}
& \frac{S_{x h}-410}{D}=\left(\left.\frac{\partial M C H}{\partial D}\right|_{\varepsilon_{0}, D_{o}}-\alpha\right) \\
& +\frac{1}{(1+\beta)}\left[\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{o}}-0.01517\right) \beta+\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}-0.00277\right)\right]\left(\frac{\varepsilon}{D}\right) \\
& \text { Define: } \\
& y=\frac{S_{x h}-410}{D}
\end{aligned}
$$

Then

$$
\begin{gather*}
y=\left(\left.\frac{\partial M C H}{\partial D}\right|_{\varepsilon_{0}, D_{0}}-\alpha\right) \\
+\frac{1}{(1+\beta)}\left[\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}-0.01517\right) \beta+\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}-0.00277\right)\right]\left(\frac{\varepsilon}{D}\right) \tag{A.5.3}
\end{gather*}
$$

Case 1 - Summer

$$
\begin{gather*}
y_{s}=\left(\left.\frac{\partial M C H}{\partial D}\right|_{\varepsilon_{0}, D_{0}}-6.50\right) \\
+\frac{1}{(1+\beta)}\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}-0.01517\right) \beta+\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}-0.00277\right)\left(\frac{\varepsilon}{D}\right) \tag{A.5.4}
\end{gather*}
$$

Note that Equation A. 5.4 was used to generate the curves on Figure A.5.1 using the values $\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}$ and $\left.\frac{\partial M C H}{\partial D}\right|_{\varepsilon_{0}, D_{0}}$ from Table A.l. 2.
$\alpha=6.50$ is used for summer.

Case 2-Winter

$$
y_{w}=\left(\left.\frac{\partial M C H}{\partial D}\right|_{\varepsilon_{0}, D_{0}}-4.08\right)
$$

$$
\begin{equation*}
+\frac{1}{(1+\beta)}\left[\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}-0.01517\right) \beta+\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}-0.0027\right)\right]\left(\frac{\varepsilon}{D}\right) \tag{A.5.5}
\end{equation*}
$$

Note that Equation A.5.5 was used to generate the curves on Figure A.5.2 using the values $\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{o}}$ from Table A.1.2.
$\alpha=4.08$ is used for winter.

Case 3-Yearly Average

$$
\begin{gathered}
y_{a}=\left(\left.\frac{\partial M C H}{\partial D}\right|_{\varepsilon_{0}, D_{o}}-4.89\right) \\
+\frac{1}{(1+\beta)}\left[\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{o}}-0.01517\right) \beta+\left(\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{o}, D_{o}}-0.00277\right)\right]
\end{gathered}
$$

Note that Equation A. 5.6 was used to generate the curves of
Figure A.5.3 using the values $\left.\frac{\partial M C H}{\partial \varepsilon}\right|_{\varepsilon_{0}, D_{0}}$ and $\left.\frac{\partial M C H}{\partial D}\right|_{\varepsilon_{0}, D_{0}}$ from Table A.1.2.
$\alpha=4.89$ is used for yearly average.

Since $y$ was defined as:

$$
y=\frac{S_{x h}-410}{D}
$$

One can rewrite this to give

$$
\begin{equation*}
S_{x h}=410+y D \tag{A.5.7}
\end{equation*}
$$

Using the curves from Figures A.5.1, A.5.2, A.5.3, and Equation A.5.7 the monthly saving under $X$ rate over $H$ rate can be easily determined.


Figure A. 5.1


Figure A.5. 2


Figure A. 5.3

Four sets of load profiles for four separate seasonal peak days of the New England Electric System are given. The following table describes these peak days.

| Figure <br> Number | Description | Date Occurred | Hourly Ave. <br> Peak Demand <br> (MW) |
| :---: | :---: | :---: | :---: |
| B.1 | Summer Peak, 1975 | June 24, '75 | 2865 |
| B.2 | Summer Peak, 1974 | June 10, '74 | 2788 |
| B.3 | Winter Peak, 1975-76 | Dec. 19, '75 | 2990 |
| B.4 | Winter Peak, 1974-75 | Nov. 26, '74 | 2883 |


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Figure B. 2


Figure B. 3


Figure B. 4

Guide to the Descriptions of the Seven Industrial Companies Studied

The following Appendices C. 1 through C. 7 describe the seven industrial firms that are being studied. They are listed in the following table.

| Appendix <br> Number | Name of the Company | Time When Data Were Gathered |
| :---: | :---: | :---: |
| C. 1 | Small Plastics Company | Summer \& Fall of 1977 |
| C. 2 | Brush Company | Summer \& Fall of 1977 |
| C. 3 | Abrasive Company | Summer \& Fall of 1977 |
| C. 4 | Soap Company | Summer \& Fall of 1977 |
| C. 5 | Foundry Company | Summer of 1976 |
| C. 6 | Printing Company | Summer of 1976 |
| C. 7 | Consumer Product Co. | Summer of 1976 |

Data for the four companies studied in the summer and fall of 1977 are far superior to those for the three companies studied in the summer of 1976. Data were obtained from various sources. For instance, data for the installed capacity, $X$, for each piece of equipment, came from nameplate reading or from the company's records. Data for $L$, $a$, and $\eta$ or the fraction of load when on, the fraction of time on, and the number of starts/hour, respectively, were obtained from direct measurement, from company records or logs, or from guesses. In Appendices C.1-C.7, these data are described in various tables. The sources are given by following the data value with a symbol; for instance, if the value is a guess, a letter 'g' is placed next to that value. However,
in many cases, we do not use any symbol following the installed capacity data, because it came from the machine's nameplate and there was no necessity to qualify it. The symbols used to describe data are explained in the following table.

Symbo1
Explanation of Symbol
$\ell$
r
Company's record
m
Measurement

Guess
e
Estimate of company's employee

## APPENDIX C. 1

## Smal1 P1astics Company

The flow and storage diagram of this injection extrusion molding company is shown in Fig. C.l.1. There are 22 plastics molding machines, shown in Table C.1.2, in parallel operation. There are also a number of other supporting machines and pieces of equipment as shown in Table C.1.1.

Each of the plastics molding machines takes in plastic raw material, heats it to a desired temperature, and molds it into a desired shape. To make one item of the molded plastic product, the machine has to go through a cycle of heating, molding, cooling, etc. Pumps, heaters, and coolers are needed along with the machines.

For a different product, a different setup is needed. A plastics molding machine can operate for many hours, or days, for each setup without interruption. Usually, each machine starts on Monday morning and stays on until: (a) a certain job is done; (b) the machine breaks down; or (c) the following Friday evening. On the average, each machine is interrupted once or twice per week.

From Monday morning to Friday evening, a plastics molding machine can be modeled as a two-state Markov process in continuous time, from the I5-minute average-power-demand point of view. The parameters needed are $X, L, a$, and $n$ as shown in Tables C.1.2 and C.1.3. The $k W$ installed capacity, $X$, for each molding machine, can be obtained from name plate readings or from company records. The percentage of time on, $a$, and the number of starts/hour, $\eta$, of each machine as given in Table C.1.3 are taken directly from company records. The percent load when on, L, for
each molding machine is deduced from direct measurement.
Figure C.l. 2 shows the graphical records of the direct current measurement of molding machine number 24. It is the current of one of the phases during the production cycles of a particular product. It should be noted that the cycling time, and the level of electric power usage, are different for each machine under different setups producing different products. In Fig. C.l.2, each dot represents one measurement and measurements are taken every five seconds. Therefore, it can be seen that there are 180 points of measurement in 15 -minute time, and the 15 minute average current can be estimated from this graph.

Let 'I' be the 15 -minute average current. It can be seen that $I \simeq 67.5 \mathrm{Amps}$ and is relatively constant. The phase-to-phase voltage, $\tilde{V}$, is given as 600 volts. Therefore, the 15 -minute average power for machine \#24 under this particular setup can be computed using the following formula:

$$
\mathrm{P}=\sqrt{3} \times \mathrm{I} \times \tilde{\mathrm{V}} \times 0.8
$$

The factor 0.8 is the correction of the power factor and $\sqrt{3}$ is used because phase-to-phase voltage is used in the formula. $P=56.12 \mathrm{~kW}$ is obtained. $L=P / X$ is found to be $53 \%$. Note that $X$, the $k W$ capacity, is 105.60 , as given in Table C.l.2. The same type of measurements were made for machines Nos. 22, 25, and T-1 and their fractions of load, $L$, were found to be 0.55 , 0.35 , and 0.33 , respectively. The average of these four measurements gives an $L_{\text {ave }}$ equal to 0.44 . This $L_{\text {ave }}$ is used for all the plastics molding machines for the Small Plastics Company and for Brush Company of Appendix C. 2.

For lighting, L is generally assumed to be 1 . For most machines used in production that do not have very fast variation of instantaneous power in 15 minute time scales, L is assumed to be 0.8 . This is to account for the fact that, generally, a designer or an engineer chooses a motor of slightly higher rating than needed for a certain machine. For machines that are in a cycling type of operation, with cycling time periods much shorter than 15 minutes, L is assumed to be smaller than 0.8. If a certain machine's power utilization is important and must be known precisely, then $L$ has to be deduced from direct measurement. One type of such measurement was explained previously.

Sma11 Plastics Company has one employee operating each of the plastics molding machines. Once started, the machines are not interrupted even during the employees' lunch hours. There are floor workers that go to each employee running a molding machine to relieve them. The machines are interrupted only for production-reiated reasons, as described earlier.

The wage paid by this company is approximately $\$ 3.50$ per hour per person for the first shift. Workers on the second and third shifts receive $\$ 0.15$ and $\$ 0.25$ per hour more than the first shift workers, respectively. On Saturdays and Sundays, workers get $1 \frac{1}{2}$ time and two times the regular wage.

The 15 -minute-by-15-minute average electric power demand curve for one week is given in Fig. 5.1.1 of Chapter V.



Figure C.1.1


Current measurement from one phase of plastics molding machine \#24. One point of measurement is taken every five second.

Scale: Horizontal axis - 1 minute per division
Vertical axis - 250 Amps full range.
$X_{24}=105.60 \mathrm{~kW}$
$\widetilde{\mathrm{V}}=$ phase-to-phase voltage $=600$ Volts
I $=15$-minute average current from above measurement
$=65$ to 75 Amps.

Figure C. 1.2

| Quantity | Name of the Equipment | Subtotal <br> capacity X <br> (kW) | Fract'n of load when on L | 1st Shft. <br> Fract. of time <br> on <br> a | $\begin{gathered} \text { 1st. } \\ \text { shift } \\ \text { LXa } \\ (k W) \end{gathered}$ | $2^{\text {nd }}$ <br> shift <br> LXa <br> (kW) | $\begin{aligned} & \quad 3^{\text {rd }} \\ & \text { shift } \\ & \text { LXa } \\ & (\mathrm{kW}) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Lighting | 40.78 | 1 | 1 | 40.78 | 37.14 | 37.14 |
| 1 | Power loader | 3.73 | 0.8 g | 0.6 e | 1.79 | 1.79 | 1.79 |
| 13 | Grinders | 81.60 | 0.8 g | 0.6 e | 39.17 | 39.17 | 39.17 |
| 6 | Prod'n. drills | 2.24 | 0.8 g | 0.6 g | 1.08 | 1.08 | 1.08 |
| 1 | Fan | 2.24 | 0.8 g | 0.6 g | 1.08 | 1.08 | 1.08 |
| 1 | Hopper Loading | 14.40 | 0.8 g | 0.4 e | 4.61 | 4.61 | 4.61 |
| 17 | Water heaters (process) | 126.72 | 0.8 g | 0.6 e | 60.83 | 60.83 | 60.83 |
| 3 | Water coolers (30 tons) | 45.00 | 0.8 g | 0.8 g | 28.80 | 28.80 | 28.80 |
| 2 | Compressors | 55.95 | 0.8 g | 0.8 e | 35.81 | 17.90 | 17.90 |
| 1 | Warehouse pump | 5.02 | 0.8 g | 0.5 e | 2.01 | 2.01 | 2.01 |
| 1 | Air conditioner ( 10 tons) |  | g | g | 15.00 | 10.00 | 10.00 |
| 6 | Window air conditioner |  | g | g | 6.00 | 4.00 | 4.00 |
| 4 | Dehumidifiers | 20.00 | 0.8 g | 0.5 g | 8.00 | 8.00 | 8.00 |
| 15 | Assembly mach. | 15.00 | 0.8 g | 0.5 g | 6.00 | 0 | 0 |
| 25 | Assembly mach. in warehouse | 25.00 | 0.8 g | 0.5 e | 10.00 | 0 | 0 |
| 5 | Machines in shop | 11.19 | 0.8 g | 0.25 e | 2.24 | 1.12 | 1.12 |
| 3 | Water Circulation pumps | 33.57 | 0.8g | 0.8 e | 21.48 | 21.48 | 21.48 |
| 1 | Water Circulation pump | 18.65 | 0.8 g | 0.8 e | 11.94 | 11.94 | 11.94 |
| TOTAL: |  |  |  |  | 296.53 | 250.95 | 204.95 |

Table C.l.l. List of Supporting Equipment of
Small Plastics Company and Their Utilization
Data

| Machine <br> ID No. | Drives <br> (HP) | Heater <br> (kW) | Subtotal <br> X <br> $(\mathrm{kW})$ |  | Machine <br> ID No. | Drives <br> (HP) | Heater <br> $(\mathrm{kW})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 20 | 10 | 24.92 | 20 | 25 | Subtotal <br> x <br> $(\mathrm{kW})$ |  |
| 3 | 15 | 8 | 19.19 | 21 | 70 | 33 | 85.22 |
| 6 | 20 | 10 | 24.92 | 22 | 85 | 46 | 109.41 |
| 9 | 40 | 23 | 52.84 | 23 | 50 | 20 | 57.30 |
| 10 | 30 | 23 | 45.38 | 24 | 100 | 31 | 105.60 |
| 12 | 40 | 23 | 52.84 |  | 25 | 100 | 31 |
| 13 | 55 | 23 | 64.03 |  | T-1 | 55 | 23 |
| 16 | 30 | 23 | 45.38 |  | T-2 | 54 | 23 |
| 17 | 30 | 23 | 45.38 | T-3 | 40 | 23 | 52.80 |
| 18 | 30 | 23 | 45.38 | T-4 | 50 | 33 | 70.30 |
| 19 | 30 | 23 | 45.38 | T-5 | 70 | 33 | 85.22 |

Table C.1.2
List of plastics molding machines of the
Small P1astics Company and their Electric
Installed Capacities

| Machine No. | $\begin{gathered} \text { Installed } \\ \mathrm{kW} \\ \mathrm{X} \end{gathered}$ | Jan. ${ }^{\prime} 77$ |  | Feb. '77 |  | March '77 |  | Apri1 '77 |  | May ' 77 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | a* | $\begin{aligned} & n \times 21 \\ & \times 24 * * \end{aligned}$ | a | $\begin{gathered} 7 \times 20 \\ \times 24 \end{gathered}$ | a | $\begin{aligned} & n \times 22 \\ & \times 24 \end{aligned}$ | a | $\begin{aligned} & \eta \times 21 \\ & \times 24 \end{aligned}$ | a | $\left[\begin{array}{l} 7 \times 21 \\ \times 24 \end{array}\right.$ |
| 2 | 24.92 | 42.1 | 4 | 38.3 | 7 | 64.2 | 13 | 76.2 | 9 | 94.9 | 5 |
| 3 | 19.19 | 83.5 | 8 | 78.6 | 9 | 88.1 | 10 | 93.6 | 6 | 96.0 | 5 |
| 6 | 24.92 | 60.5 | 10 | 24.4 | 7 | 69.3 | 10 | 70.0 | 11 | 66.0 | 17 |
| 9 | 52.84 | 98.8 | 5 | 96.4 | 6 | 99.7 | 11 | 88.4 | 7 | 67.8 | 14 |
| 10 | 45.38 | 98.8 | 2 | 95.5 | 7 | 98.6 | 12 | 90.2 | 8 | 97.2 | 6 |
| 12 | 52.84 | 89.9 | 3 | 87.9 | 6 | 76/5 | 12 | 92.1 | 3 | 92.2 | 8 |
| 13 | 64.03 | 97.0 | 4 | 92.7 | 2 | 85.3 | 8 | 97.2 | 4 | 94.8 | 8 |
| 16 | 45.38 | 99.5 | 2 | 96.9 | 4 | 94.7 | 12 | 69.8 | 8 | 86.9 | 5 |
| 17 | 45.38 | 67.2 | 4 | 83.1 | 8 | 80.8 | 7 | 98.7 | 4 | 95.9 | 5 |
| 18 | 45.38 | 0 | 0 | 65.3 | 11 | 54.3 | 10 | 95.9 | 4 | 8.12 | 11 |
| 19 | 45.38 | 100.0 | 0 | 99.3 | 3 | 91.2 | 12 | 97.8 | 7 | 96.3 | 5 |
| 20 | 38.65 | 97.6 | 1 | 98.4 | 1 | 89.6 | 5 | 99.3 | 2 | 96.6 | 5 |
| 21 | 85.22 | 34.3 | 3 | 71.4 | 6 | 92.5 | 9 | 95.3 | 8 | 82.2 | 9 |
| 22 | 109.41 | 38.6 | 4 | 58.4 | 2 | 57.3 | 6 | 75.0 | 2 | 75.2 | 11 |
| 23 | 57.30 | 56.7 | 5 | 93.8 | 5 | 77.3 | 7 | 93.4 | 3 | 73.4 | 9 |
| 24 | 105.60 | 85.1 | 5 | 98.1 | 4 | 78.8 | 10 | 84.6 | 8 | 88.5 | 10 |
| 25 | 105.60 | 56.3 | 10 | 64.0 | 7 | 88.3 | 9 | 94.8 | 2 | 97.3 | 5 |
| T-1 | 64.03 | 93.4 | 3 | 99.8 | 1 | 96.2 | 8 | 90.9 | 3 | 93.6 | 5 |
| T-2 | 64.03 | 81.1 | 7 | 92.7 | 6 | 96.1 | 3 | 85.7 | 4 | 23.8 | 1 |
| T-3 | 52.80 | 86.8 | 7 | 100.0 | 0 | 92.8 | 5 | 81.5 | 6 | 74.4 | 9 |
| T-4 | 70.30 | 10.9 | 8 | 83.1 | 6 | 92.2 | 8 | 84.4 | 6 | 91.1 | 10 |
| T-5 | 85.22 | 58.9 | 5 | 50.0 | 4 | 70.7 | 12 | 86.2 | 8 | 91.5 | 8 |

Table C.1.3 Plastics molding machine utilization


*     - in percentage ** - no. of interruptions/month


## APPENDIX C. 2

## Brush Company

Brush Company is an extrusion plastic molding company. The flow and storage diagram is similar to that of Small Plastics Company described in Fig. C. 1.1 and will not be shown again in this appendix. The layout of its building is as shown in Fig. C.2.1. The buildings can be roughly divided into four groups.

Group 1 consists of Buildings 1 to 4 and warehouse Building 63. The main offices and some small manufacturing processes are located in this area. Group 2 consists of Building 690. Brush making types of small machine assembly processes are located in this building. Some of the products manufactured in Building 690 have to go through a few production steps to be completed. Some of the plastic products needed in this building are molded in Building Group 3.

Group 3 consits of Buildings $6,6 A, 6 B, 6 C, 11,12$, and 13. There are 30 large plastics molding machines of various sizes located in Building 6. These machines are the major users of electricity in this company. The machines are under a three-shift operation, five days a week. The utilization patterns of these machines are similar to those of the plastics molding machines of Small Plastics Company described in Appendix C.1. These machines are on between $70 \%$ to $90 \%$ of the time or more during all working days. Each machine is interrupted once or twice a week during weekdays.

Building Group 4 consists of Building 16 and a large warehouse. There is an independent company, XYZ, attached to Brush Company at this location. Brush Company rents this space to this independent company. A visit to this company was not made for this present study. XYZ company shares the
same power meter as Brush Company. The only information available about XYZ company is that it is responsible for approximately $10 \%$ of the monthly energy usage and its machines have long hours of usage. The simplest assumption that can be made for this present study is to assume that XYZ company accounts for $10 \%$ of the expected power demand and $10 \%$ of the autocovariance $\mathrm{R}_{\mathrm{T}}(0)$ during all shifts.

Table C.2.1 gives the list of the 30 plastic molding machines and their installed kW capacities. Table C. 2.2 gives the electric capacities and their utilization of the remaining machines for the entire plant.

The pay scale for Brush company is $\$ 4.00-\$ 4.50$ per hour for the first shift. The second and third shift workers get $\$ 0.15$ and $\$ 0.25$ more per hour respectively than those on the first shift. If they work on Saturday or Sunday, they will get $1 \frac{1}{2}$ time and twice the usual wage, respectively.

The 15 -minute by 15 -minute average power demand curve for Brush Company is given in Fig. C.2.1 of Chapter V.


| Machine ID NO. | Drives (HP) | Heaters <br> (kW) |  | Machine ID No. | Drives <br> (HP) | $\begin{aligned} & \text { Heater } \\ & \& \begin{array}{c} \text { chiller } \\ (\mathrm{kW}) \end{array} \end{aligned}$ | $\begin{gathered} \text { Subtota1 } \\ X \\ (\mathrm{~kW}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 85.5 | 33 | 93.41 | 27 | 85.5 | 33 | 93.41 |
| 3 | 45.5 | 42 | 75.94 | 28 | 85.5 | 33 | 93.41 |
| 4 | 95.5 | 33 | 104.24 | 29 | 115.5 | 38 | 124.16 |
| 5 | 95.5 | 33 | 104.24 | 30 | 85.5 | 33 | 93.41 |
| 7 | 60.5 | 33 | 78.13 | 31 | 85.5 | 33 | 93.41 |
| 8 | 70.5 | 41.8 | 94.39 | 32 | 140.5 | 78.23 | 183.05 |
| 15 | 60.5 | 33 | 78.13 | 33 | 250.5 | 108.24 | 295.11 |
| 17 | 60.5 | 33 | 78.13 | 35 | 85.5 | 33 | 93.41 |
| 24 | 65.5 | 39.8 | 88.6 | 36 | 55.5 | 44.8 | 86.20 |
| 22 | 75.5 | 31.8 | 88.12 | 38 | 322.5 | 113 | 367.70 |
| 21 | 75.5 | 31.8 | 88.12 | 39 | 205.5 | 93.53 | 246.46 |
| 20 | 75.5 | 31.8 | 88.12 | 46 | 90.5 | 83.24 | 150.75 |
| 23 | 65.5 | 39.8 | 88.6 | 47 | 90.5 | 83.24 | 150.75 |
| 25 | 85.5 | 33 | 93.41 | 42 | 95.5 | 33 | 104.24 |
| 26 | 85.5 | 33 | 93.41 | 45 | 85.5 | 33 | 93.41 |

Total: 1334.99
Total: $\underline{\underline{2118.13}}$

Table C.2.1

| Quant 'y. | - Item | $\begin{gathered} \mathrm{X} \\ (\mathrm{~kW}) \end{gathered}$ | L | $\begin{gathered} \text { 1st Shf.t } \\ \mathrm{a} \end{gathered}$ | $\begin{gathered} \text { lst Shft } \\ \text { XLa } \end{gathered}$ | $\begin{aligned} & \text { 2nd } \\ & \text { Shift } \\ & \text { XLa } \end{aligned}$ | 3rd Shift XLa |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (Buildings 1, 1A, 2, 3, 3A, 3B, 4, 4A (Main Bldg.) |  |  |  |  |  |  |  |
| 1 | Compressor | 55.95 | 0.8 g | 0.6 g | 26.86 | 0 | 0 |
| 4 | Air conditioner | 17.86 | 0.8 g | 0.8 g | 46.00 | 34.88 | 23.00 |
|  | Lighting | 104.00 | 0.9 g | 1.0 | 93.60 | 9.36 | 9.36 |
| 8 | Fan $G_{4}$ blowers | 25.75 | 0.8 g | 0.6 g | 12.36 | 8.24 | 0 |
| 18 | Heaters | 28.86 | 0.8 g | 0 | 0 | 0 | 0 |
|  | Waste disposal | 11.19 | 0.8 g | 0.4 g | 3.58 | 0 | 0 |
| 7 | Machines in cotton brush Rm. | 10.63 | 0.8 g | 0.4 g | 3.40 | 0 | 0 |
|  | Air condition'g for computer Rm. | 61.74 | 0.8 g | 0.5 g | 24.69 | 24.69 | 24.69 |
|  | Water cooler | 14.48 | 0.8 g | 0.5 g | 5.79 | 5.79 | 5.79 |
|  | Xerox \& SCM | 15.30 | 0.8 g | 0.2 g | 1.33 | 0 | 0 |
|  | Machines in shop | 6.71 | 0.8 g | 0.2 e | 1.07 | 0 | 0 |
| 1 | Oven in shop | 2.00 | 0.8 g | 0.2 g | 0.32 | 0 | 0 |
| 8 | Vending machines | 20.00 | 0.8 g | 0.5 g | 8.0 | 8.0 | 8.0 |
| 2 | Nylon machines | 152.42 | 0.8 g | 0.5 g | 60.97 | 60.97 | 0 |
| 7 | Dryers | 32.42 | 0.8 g | 0.5 g | 12.97 | 12.97 | 0 |
|  | Computers | 37.20 | 0.8 g | 0.5 g | 14.88 | 14.88 | 14.88 |
|  | Elevators | 5.60 | 0.9 g | 0.3 g | 1.34 | 1.34 | 0 |

Total for Main Building:
$\qquad$

Table C.2.2

| Quant. | Item | $\begin{gathered} \mathrm{X} \\ (\mathrm{~kW}) \end{gathered}$ | L | 1st Sh. | 1st Sh XLa (kW) | $\begin{gathered} \begin{array}{c} \text { 2nd } \\ \text { XLa } \\ (\mathrm{kW}) \end{array} \end{gathered}$ | $\underset{\substack{\text { XLa } \\(k W)}}{ }$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (Buildings \#6, 6A, 6B, 6C) |  |  |  |  |  |  |  |
|  | Fluoresc. Light | 45.32 | 1.0 | 1.0 | 45.32 | 45.32 | 45.32 |
|  | Mercury vapor Lamb | 11.78 | 1.0 | 1.0 | 11.78 | 11.78 | 11.78 |
|  | Fans | 41.03 | 0.8 g | 0.8 g | 26.29 | 26.29 | 26.29 |
| 4 | Heater ovens | 30.00 | 0.8 g | 0.4 g | 9.60 | 9.60 | 9.60 |
| 2 | Dryer heaters | 32.00 | 0.8g | 0.4 g | 10.24 | 10.24 | 10.24 |
| 1 | Hopper dryer | 6.0 | 0.8 g | 0.4 g | 1.92 | 1.92 | 1.92 |
| 1 | Press heater | 4.0 | 0.8 g | 0.4 g | 1.28 | 1.28 | 1.28 |
| 3 | Grinder | 40.5 | 0.8 g | 0.3 g | 9.72 | 9.72 | 9.72 |
| 3 | $7 \frac{1}{2}$-ton air cond. | 33.75 | 0.8 g | 0.8 g | 21.60 | 10.80 | 10.80 |
| 5 | 15-ton air cond. | 112.50 | 0.8 g | 0.5 g | 45.00 | 22.50 | 22.50 |
| 1 | $50-$ ton air cond. | 75.00 | 0.8 g | 0.5 g | 30.00 | 15.00 | 15.00 |
| 1 | 30-ton air cond. | 45.00 | 0.8 g | 0.5 g | 18.00 | 9.00 | 9.00 |
| 3 | Electric heater | 150.00 | 0.8 g | 0 | 0 | 0 | 0 |
| 1 | Blower for gas | 14.92 | 0.8 g | 0 | 0 | 0 | 0 |
| 16 | Washers | 192.00 | 0.8 g | 0.3 g | 46.08 | 46.08 | 46.08 |
|  | Electric oven | 39.91 | 0.8 g | 0.5 g | 15.96 | 15.96 | 15.96 |
|  | Alum. spraying | 32.80 | 0.8 g | 0.5 g | 13.12 | 13.12 | 13.12 |
|  | Furnace | 39.91 | 0.8 g | 0.5 g | 15.96 | 15.96 | 15.96 |
| 2 | Air compressors | 149.20 | 0.8 g | 0.6 g | 71.62 | 71.62 | 71.62 |
| 11. | Machines in shop | 46.25 | 018g | 0.2 g | 7.40 | 0 | 0 |
| $\begin{aligned} & \text { Total for Bldgs. } \\ & 6,6 \mathrm{~A}, 6 \mathrm{~B}, 6 \mathrm{C} \text { : } \end{aligned}$ |  |  |  |  | 419.40 | 354.70 | 354.70 |

Table C. 2.2 continued


Total for Building 690: $\quad$| $546.07465 .67 \quad 357.36$ |
| :---: |

Table C.2.2 continued

## APPENDIX C. 3

## Abrasive Company

Figure C.3.1 gives the general plant layout of Abrasive Company. This company could be viewed as a very large machine shop. It has a total floor space of more than 700 thousand square feet. The main part of the plant is as shown in the diagram. A large warehouse, located about a mile away from the plant, is not included in the diagram.

Some of the products have to go through more than one stage of processing. But from the viewpoint of day-to-day operation, the machines in this plant can be considered to be independent of each other. Therefore, the total expected power demand and the autocorrelation function of the residual for the total demand are the sum of the expected demand and the autocorrelation functions of the individual equipment.

The list of the departments, their floorspace, level of lighting, and electric power demand for lighting when the shift is on, is all given in Table C.3.1. Table C.3.2 gives the installed capacity of the machines and their usage pattern for Organic Department. More than 20 pages of such records are available. However, all this information is not included here because of its length. This type of information is used to deduce the parameters $X, L, a$, and $\eta$ as described in Chapters II and III, and used in Chapter $V$.

Table C.3.3 gives a list of production machines that have XLa (average power demand) larger than 10 kW . Some of these machines deserve further study for the possibility of rescheduling under X rate.

Time series data for a 15 -minute average power demand for the entire plant has been recorded for the summer of 1977. The load profile for one such week is given in Fig. 5.3.1 of Chapter V.


| Name of Area | Floor Area <br> $(000 ' s ~ s q . ~ f t) ~$. | Watts/Sq. Ft. | \#f Shifts <br> per day | 1st Shift <br> kW |
| :---: | :---: | :---: | :---: | :---: |


| Maintenance | 36.00 | 3.0 e | 2 |  | 108.01 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Grading | 29.31 | 3.0 e | 2 | r | 87.93 |
| Vitrified | 57.41 | 3.0 e | 3 | r | 172.23 |
| Boiler | 4.30 | 3.0 e | 3 | r | 12.90 |
| $T$ \& $B$ | 47.55 | 3.0 e | 2 | r | 142.65 |
| Shipping | 43.82 | 1.5 g | 1 | r | 65.73 |
| $R \& E$ | 47.60 | 3.0 e |  | r | 142.80 |
| Organic | 30.88 | 3.0 e | 3 | r | 92.64 |
| Snagger | 52.69 | 3.0 e | 1 | r | 158.07 |
| Resinoid | 57.40 | 3.0 e |  | r | 172.20 |
| Raisehup \& Q.C. | 16.49 | 3.0 e | 2 | r | 49.47 |
| Cafeteria | 4.45 | 3.0 e | 3 |  | 13.35 |
| Offices ( $1 \mathrm{~A}, 1 \mathrm{~B}$ ) | 16.33 | 3.0 e | 1 | r | 48.99 |
| Packing | 19.70 | 3.0 e |  | r | 59.10 |
| String winding <br> \& rubber | 18.20 | 3.0e |  | r | 54.60 |
| Storage | 16.34 | 0.3 e | 3 | r | 4.90 |
| Machine Shop | 21.15 | 3.0 e |  | r | 63.45 |
| Freight House | 3.75 | 0.3 e |  | r | 1.12 |
| Warehouse | 112.98 | 1.0 g |  | r | 112.98 |
| B1dgs. 32, 17, 28 | 28.79 | 1. 5 g |  | r | 43.19 |
| Bartlett | 41.40 | 1.5 g |  | r | 62.10 |
| Diamond | 28.20 | 3.0 e |  | r | 84.60 |

Total: 1752.97
Table C.3.1


Table C. 3.2

| Machine <br> ID No. | Name | $\begin{gathered} X \\ (\mathrm{~kW}) \end{gathered}$ | $\begin{aligned} & \mathrm{XLa} \\ & (\mathrm{~kW}) \end{aligned}$ | No. <br> Shifts per day | Number of Persons |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1360 | Press | 33.57 | 13.43 r | 2 r | 4 |
| 3211 | Press | 44.76 | 26.86 r | 2 r |  |
| 4855 | Grinder | 31.33 | 12.53 r | 1 r |  |
| 5646 | Grinder | 31.33 | 12.53r | 1 r |  |
| 6024 | Grinder | 31.33 | 12.53 r | 1 r |  |
| 6919 | Grinder | 31.33 | 12.53 r | 1 r |  |
| 4159 | $\begin{gathered} \text { Large whee } \\ \text { press } \end{gathered}$ | 44.76 | 31.33 r | 2 r | 4 |
| 4219 | Thin wheel press | 44.76 | 31.33 r | 1 r | 3 |
| 5905 | Thin wheel press | 141.74 | 85.04 r | 1 r | 4 |
| 6378 | Auto thin wheel pres $\$$ | 18.65 | 13.06r | 1 r |  |
| 6402 | Auto thin wheel pres $\$$ | 17.90 | 12.53 r | 2 r |  |
| 5161 | Thick wheel pres $\$$ | 18.65 | 13.06r | 1 r |  |
| 6743 | Thick wh. press | 18.65 | 13.06r | 2 r |  |
| 1735 | Grinder | 31.33 | 12.53 r | 2 r |  |
| 4010 | Grinder | 31.33 | 12.53 r | 2 r |  |
| 5493 | Grinder | 31.33 | 12.53 r | 2 r |  |
| 6751 | Grinder | 31.33 | 12.53 r | 1 r |  |
| 6461 | Power supply | 74.60 | 24.62r | 2 r |  |
| 2306 | ${ }_{1}$ Power supply | 29.84 | 14.92r | 2 r |  |

Table C. 3.3

## APPENDIX C. 4

## Soap Company

Figure C.4.1 shows the flow and storage diagram of the Soap Company. There are basically two types of production processes. They are the synthetic granule-making and the bar-soap-making processes. There are also processes and areas not directly related to production processes. These include the boiler house, the mechanical department, and offices.

Consider the synthetic soap granule production process. Different types of raw materials required are mixed and processed while passing through the "tower". The end product from the tower is synthetic granules which is received in granule storage and is then packed, using a few of the available packaging lines as determined by the supervisors. It is a function of the brand being produced. The synthetic granule storage has the capacity for storing six to eight-hours' worth of the material produced in the tower when the packaging lines are all off. The granules are packed and sent to the warehouse to await shipping. The tower has three shifts of people, six people per shift, working continuously. However, the tower as a whole is on only part of the time; it can be modeled as a two-state Markov process. This fact will be discussed in the latter part of this Appendix.

The soap production goes through the following steps. First, raw material such as fat, caustics, etc. are mixed and processed. The processed material is then stored in a storage area with a capacity for up to one week's worth of production. The production of different types of
soap are now involved with the material flowing through the "making" stage which consists of squeezing, mixing, and pressing. There are three "making" production lines in parallel to make different types of soap. Some granules are also produced. The soap is refrigerated. Soap bars that are produced are packed by up to four of the packing lines. Lines \#2, 3, 4, 6, and 7 are for this purpose.

Table C.4.1 gives the power demand for various floors and the supporting areas of the tower. $\mathrm{L}=0.8$ is assumed for all machines. The complete list of machines and their capacities as well as the percentage of time on when the tower is on, etc., are given us by company records. These data were takn by the company's employee; the details are not included here due to the length. The power demand of the tower can be represented as:

$$
\mathrm{P}_{\text {tower }}[\mathrm{n}]=\mathrm{P}_{\mathrm{I}}[\mathrm{n}]+\mathrm{P}_{\mathrm{D}}[\mathrm{n}]
$$

where $P_{I}[n]$ is the component that is independent of whether or not the tower is on or off. We have

$$
E\left\{P_{I}[n]\right\}=E\left\{P_{I}[n] \left\lvert\, \begin{array}{l}
\text { Tower } \\
\text { is On }
\end{array}\right.\right\}=E\left\{P_{I}[n] \left\lvert\, \begin{array}{l}
\text { Tower } \\
\text { is Off }
\end{array}\right.\right\}
$$

To determine $P_{I}[n]$ we need only the regular type of information, such as $X, L, a, n$ as described in Chapters II and III. $E\left\{P_{I}[n]\right\}$ is only a function of time or shift. $P_{D}[n]$ is the component of load that depends on whether or not the tower is on or Off. It can be modeled as the outcome of a two-state Markov process in continuous time. Note that $\mathrm{P}_{\mathrm{D}}[\mathrm{n}]$
is the sum of a very large number (hundreds) of machines. The type of information needed here are values for $X, L$, and the percentage of time the machine is on when the tower is on. From this information, we can find $E\left\{P_{D}[n] \left\lvert\, \begin{array}{l}\text { Tower } \\ \text { is On }\end{array}\right.\right\}$ to be 1158.58, as shown in Table C.4.1. $E\left\{P_{D}[n] \left\lvert\, \begin{array}{l}\text { Tower } \\ \text { is } 0 f f\end{array}\right.\right\}$ is zero. Figure C.4.2 gives more information related to the operation of the tower, such as the number of people needed per shift. There is also information leading to the derivation of the parameters a and $\eta$ for the tower to be modeled as a two-state Markov process.

Table C. 4.3 gives the power utilization for the synthetic granule packing. Again, these data are as recorded by the plant supervisor and personnel. Table C. 4.4 is the same type of data for bar-soap packing. Table C.4.3 gives the comparison of the expected power demand derived from a physical model with that derived from direct current measurement. The number of persons required for some of the departments is also given.

Table C. 4.6 gives the electric energy usage and the peak demand of Soap Company for the year 1976.

Soap Company pays an average wage of $\$ 8.00$ per hour per person for the first shift of production workers; the second and third-shift workers get about $\$ 0.30$ and $\$ 0.375$ per hour more. Saturday and Sunday workers get 1 and one-half times and twice the normal salary. The 15 -minute by 15 -minute average power demand curve for a week is given in Fig. 5.4.1 of Chapter V.


| Location | Component of Load when Tower is On (kW) | Component lst Shift XLa (kW) | Load Indep <br> 2nd Shift <br> XLa <br> (kW) | ndent of Tower 3rd Shift XLa (kW) |
| :---: | :---: | :---: | :---: | :---: |
| lst Floor | 110.29 r | 11.25 r | 22.5 r | 22.5 r |
| 2nd Floor | 119.98 r |  |  |  |
| 3rd Floor | 303.39 r |  |  |  |
| 6th Floor | 464.18 r | 47.09r | 52.09 r | 52.09 r |
| Yard MCC | 51.55 r | 17.40r | 17.40r | 17.40r |
| Unloading | 9.09 r | 77.58 r | 77.58 r | 77.58 r |
| Reclaim | 15.15 r | 33.25 r | 33.25 r | $33.25 r$ |
| Big Bin | 69.70 r |  |  |  |
| 66B Unloading | 15.53 r |  |  |  |
| Total | 1158.58 | 186.57 | 202.82 | 202.82 |

Table C.4.1

6 persons per shift
3 shifts per day
5 days per week

|  | May 1977 | June 1977 | July 1977 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| o/o time on 'a' <br> of a1l week days | 43.39 | $\ell$ | 45.35 | $\ell$ | 35.98 |
| \# of starts <br> per month | 30 | $\ell$ | 31 | $\ell$ | 21 |
| \# of working days <br> for the month | 21 | $\ell$ | 22 | $\ell$ | 20 |

Table C.4.2


Table C.4.3

Bar Soap Packing

| Line \#2 | 11.46 r | 11.46 r | 0 |
| :--- | ---: | ---: | :--- |
| Line \#3 | 14.90 r | 14.90 r | 0 |
| Line \#4 | 0.79 r | 0.79 r | 0 |
| Line \#6 | 58.91 r | 58.91 r | 32.46 r |
| Line \#7 | 81.65 r | 81.65 r | 27.16 r |
|  |  |  |  |
| Total: |  |  |  |
|  | 166.71 | 166.71 | 59.62 |

Table C.4.4

| Item | $\begin{gathered} \text { 1st Shft. } \\ \text { XLa } \\ (\mathrm{kW}) \end{gathered}$ | $\begin{gathered} \text { 2nd } \begin{array}{c} \text { Shft. } \\ \text { XLa } \\ (\mathrm{kW}) \end{array} . \end{gathered}$ | $\begin{gathered} \text { 3rd Shft. } \\ \text { XLa } \\ (\mathrm{kW}) \end{gathered}$ | $\begin{array}{cl} 0.8 & \text { KVA } \\ \text { lst } & \text { Shft } \end{array}$ | No. of People for Each Shft |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Soap Process | 519.58 r | 519.58 r | 519.58 r | 399.06 m | 5 |
| $\begin{aligned} & \text { Bldg. and } \\ & \text { Mechanical (128) } \end{aligned}$ | 218.70 | 196.50 | 196.50 | 133.02m |  |
| Sewage pumps $(134)$ | $16.56 r$ | $13.51 r$ | 13.51 r |  |  |
| Shipping and Warehouse (180) | 112.10r | 111.30 r | 164.23r | 99.77 m |  |
| Soap Granules (463) | 107.13r | 76.01 r | 8.61 r | 166.28 m |  |
| Soap Making (A1l types) | 657.29 r | 544.86 r | 280.73 r | 498.83m |  |
| Shop | 42.14 r | 22.46 r | 12.50 r |  |  |
| Bar Soap Refrigeration | 165.91 r | 165.91 r | $165.91 \mathrm{r}\left\{\begin{array}{l} \text { to } \end{array}\right.$ | $\begin{aligned} & 133.02 \mathrm{~m} \\ & 332.55 \mathrm{~m} \end{aligned}$ |  |
| Bar Soap Packing | 166.71 r | 166.71 r | $59.62 \mathrm{r}\left\{_{\text {to }}\right.$ | $\begin{aligned} & 133.02 \mathrm{~m} \\ & 266.04 \mathrm{~m} \end{aligned}$ | 10 |
| Synthetic Granule Packing | 207.07r | 207.07 r | 117.99 r |  | 8-10 |

Table C.4.5

| Month | $\begin{gathered} \text { Energy } \\ \text { Usage } \\ \left(000^{\prime} \mathrm{s} \mathrm{KWH}\right) \end{gathered}$ | Peak Demand (kW) | $\begin{gathered} 0.8 \mathrm{KVA} \\ (\mathrm{~kW}) \end{gathered}$ | Hours Used (Hr) |
| :---: | :---: | :---: | :---: | :---: |
| January | 1376.6 | 3528 | 3533 | 390 |
| February | 1275.3 | 3466 | 3533 | 368 |
| March | 1467.5 | 3379 | 3418 | 434 |
| April | 1309.5 | 3374 | 3340 | 388 |
| May | 1252.0 | 3427 | 3418 | 365 |
| June | 1305.0 | 3533 | 3533 | 369 |
| July | 1424.9 | 3643 | 3610 | 391 |
| August |  | 3634 | 3610 | 341 |
| September | 1254.0 | 3739 | 3533 | 336 |
| October | 1468.0 | 3562 | 3571 | 412 |
| November | 1223.6 | 3557 | 3494 | 344 |
| December | 1299.7 | 3658 | 3648 | 355 |

Table C.4.6

## APPENDIX C. 5

## Foundry Company

Foundry Company's main product is sluice gates for water pollution control. In addition, it also produces industrial rolls.

A brief description of its manufacturing practices and processes is as follows: steel and scrap iron are melted in two 15-ton channel-type induction furnaces in parallel operations. The furnace temperature is maintained above $2750^{\circ} \mathrm{F}$ during the day; for this, a total of 500 kW of power is needed. The furnace is operated by a single operator from $11: 00 \mathrm{pm}$ to 7:00 am for melting.

The furnace operator is allowed by the management to use up to a maximum of 2500 kW for both furnaces during melting hours; 400 kWh of electric energy is needed to melt a ton of iron. Between 20 and 30 tons per day of iron are melted during the utility's off-peak hours, between 11:00 pm and 7:00 am. During the first shift, usually in the morning, the molten steel is poured into molds made of sand, and cooled in the cooling area. The iron castings are then removed by breaking the molds. Any sand that is still stuck to the iron castings is blown off with air. The castings then go through grinding and machining processes in the West machine shop.

The products vary in size, shape, and other requirements. Therefore, most of the processes are not assembly-line operations, but are "specialorder' type operations, which are typical of the foundry industry. The East machine shop is used for making industrial rolls. The pipes used for
making rolls are purchased, not produced in the foundry. The molds and cores are produced using wood patterns from the wood shop.

Figure C.5.1 gives the flow and storage diagran for Foundry Company. Table C.5.1 gives the number of employees that are involved at some of the sections of interest for different shifts. The sections described are the furnaces, machine shop west, machine shop east, and the foundry itself. Table C.5.2 gives the detailed electric stock data of the Company. Figure C.5.2 gives the electric load profile of the Company on a Monday. The load shape for one week is depicted in Figure 5.5.1 of Chapter V.

Wages at Foundry Company are $\$ 5.00 /$ hour for the first shift workers. Second and third shift workers receive $\$ 0.75$ and $\$ 1.00$ more per hour, respectively. Saturday and Sunday wages are $1 \frac{1}{2}$ and two times the normal wage.

Table C. 5.1
Number of Employees in Each Section

Ist Shift
2nd Shift
3rd Shift

| No. of persons for <br> charging the furnace | 0 | 0 | 1 |
| :--- | :---: | :---: | :---: |
| East machine shop | 15 | $5-7$ | 0 |
| West machine shop | 40 | 25 | 12 |
| Foundry G mold- <br> cooling area | 35 | 25 | 1 |




Figure C.5.1


Figure C.5. 2

Table C.5.2
Detailed Electric Stock Data of Foundry Company

Building 1, 1B, 1C, Main Offices

| Itern | Quantity | HP or kW Each | kW Tota1 |
| :---: | :---: | :---: | :---: |
| Lighting | 480 | 0.045 kW | 21.600 kW |
| Air conditioning <br> for summer | 28 tons | 98.450 kW |  |
| Electric heating <br> for winter |  | 100.0 kW |  |
| uilding 37, Machine Shop West |  |  |  |


| Lighting (mercury |  |  |  |
| :--- | ---: | :---: | :---: |
| vapor lights) | 64 | 0.8 kW | 5.2 kW |
| Large hoist | 2 | 15 HP | 22.380 kW |
| Small hoist | 9 | 5 HP | 33.570 kW |
| Machines | 16 | 120 HP | 90.52 kW |
| No. of employees for: | first shift | second shift | 3 rd shift |
| Bldgs. 37 \& 7 B combined: | 40 | 25 | 12 |

Building 7B, Part of Machine Shop West

| Lights, fluorescent | 50 | 0.9 kW | 4.5 kW |
| :--- | ---: | :---: | ---: |
| Lights, Hg vapor | 8 | 0.4 kW | 3.2 kW |
| Large hoist | 2 | 15.0 HP | 22.38 kW |
| Small hoist | 3 | 5 HP | 11.19 kW |
| Milling machine | 2 | 5 HP | 7.46 kW |

Bui1ding 35, Stockroom
$\begin{array}{llll}\text { Lights, fluorescent } 11 & 0.09 \mathrm{~kW} & 0.99 \mathrm{~kW}\end{array}$

Building 31


Buildings 34 and 36, East Machine Shop


Building 24, Wood Shop \& Office
Fluoresc. Lights
46
Air conditioning

Building 24B, Wood Shop \& Office

| Fluoresc. Lights | 17 |
| :--- | ---: |
| Large Hoist | 5 |
| Machines | 5 |


| 0.09 kW |  | 1.53 | kW |
| ---: | :---: | ---: | :---: |
| 15 | HP | 55.95 | kW |
| 3 | HP | 9.05 | kW |

General Support for Entire Plant
Exhaust Fans
Compressor
Electric Water Heater $\mathcal{G}$ Pump

Dust Collector

| HP Total |  | kW Total |  |
| :---: | :---: | :---: | :---: |
| 125 | HP | 93.25 | kW |
| 125 | HP | 93.25 | kW |
|  |  | 62.3 | kW |
| 150 | HP | 111.9 | kW |

## APPENDIX C. 6

## Printing Company

Printing Company's product line is printing telephone directories for various towns and cities. Figure C.6.1 shows the layout of the printing plant. There are mainly three buildings, as shown. Building $B$, located in the middle of the plant, is the main building which houses most of the large electric-power-consuming machines. There are three printing presses in operation, and a new one that will be in operation in the near future. There are three sets of binding, cutting, and packaging machines, offices, a machine shop, a plate-making shop and a shipping area in Building B.

Building A is a supporting building for production; it has a proofreading area, linotypes, typesetting, files, two small printing machines and other supporting machinery.

Building C is mainly a warehouse. There are storage areas for paper, glue, and other supporting materials. There is a railroad terminal for receiving raw material, and an area for shipping out bales of waste paper which have been cut and baled in B1dg. C.

Figure C.6.2 shows the flow and storage diagram for Printing Company, a physical model suitable for the study of hourly electric load. As shown in the diagram, paper and supporting material are shipped in by rail and stored in Building C. The rolls of paper then go through one of the four printing presses, connected in parallel, which print, cut, and fold the paper. The outcome from these presses is 'signatures', which consist of ten or more sheets of printed paper folded together. These
are stored in Building $C$ to await a sufficient amount of the rest of the telephone book to be printed. At this time, all the stored signatures are transferred back to be bound together, using one of three binders, cutters, and packaging systems which bind, glue on covers, and then trim the finished books. They are then wrapped together into packages of desired size, and moved to the shipping area.

Also shown in Fig. C.6.2 are the blocks that do not relate directly to production, but need electric power for their many functions; for example, these blocks include offices, machine shops, platemaking rooms, etc.

The billing data for 1975 and detailed electric stock data for Printing Company are given in Tables C.6.1, C.6.3. Table C.6.2 gives the number of employees, wages, and flow capacities for each of the four printing presses, each of the three binders, cutters, packagers, and waste baling systems. The storage areas for paper and signatures are large, and during normal operation, it takes three weeks to fill up the signature storage. Daily production of the plant during the slow months from June to August, and the busy months from September to May, could have a ratio of one to three or four.


Figure C. 6.1


Figure C.6.2

| Month | Demand <br> kW/0.8 KVA | Eff. <br> Demand | Ratio of <br> kW/KVA | Energy <br> Usage <br> KWH | Hour Used <br> Energy Usage <br> Eff. Demand (Hr) |
| :--- | :---: | :---: | :---: | :---: | :---: |
| January | $954 / 888$ | 954 | 0.859 | 318,600 | 333 hr. |
| February | $909 / 864$ | 909 | 0.842 | 324,900 | 357 |
| March | $864 / 816$ | 864 | 0.847 | 290,700 | 336 |
| Apri1 | $864 / 816$ | 864 | 0.847 | 288,000 | 333 |
| May | $918 / 840$ | 918 | 0.874 | 344,700 | 375 |
| June | $909 / 864$ | 909 | 0.842 | 273,600 | 300 |
| July | $783 / 720$ | 783 | 0.870 | 293,400 | 374 |
| August | $753 / 696$ | 753 | 882 | 0.865 | 223,200 |

Table C.6.1
Electric Power \& Energy Usage Data of Printing Company for the Year 1975

Item
Item

Printing Press \#1

Printing Press \#2

Printing Press \#3

Printing Press \#4

Binder, Cutter, Packager \#1
Binder, Cutter, Packager \#2
Binder, Cutter, Packager \#3

Waste Baling
No. of

Employees | Maximum Flow |
| :---: |
| Rates |

$2000 \mathrm{ft} /$ min $^{1}$
$2000 \mathrm{ft} / \mathrm{min}$
$3000 \mathrm{ft} / \mathrm{min}$
$3600 \mathrm{ft} / \mathrm{min}$ 95 books/min ${ }^{2}$

117 books/min

100 books/min

35 bales/day ${ }^{3}$

Wages: $\quad \$ 5.00 /$ hour and up for the lst shift 2nd shift: $\$ 1.09$ additional to 1 st shift wage 3rd shift: 1.12 " " "
$155,000 \mathrm{ft}$. paper $=1$ ton
2900 to 1000 books $=1$ ton for large telephone book
31200 Ibs/bale.

Table C.6.2
Data on number of employees, maximum flow rate, and wage information for each printing press, binder, cutter, and packager of Printing Company

Table C.6.3 Printing Company
Building A Quantity $\quad \mathrm{kW}$ Total

Proof Reading Area ( $500 \mathrm{ft}^{2}$ )

| Lighting | 48 | 40 Watts each | = | 1.920 kW |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Air Conditioning | 5 tons |  |  |  |  |
| Linotype Area |  |  |  |  |  |
| Lighting | 27 | 80 Watts each | $=$ | 2.160 | kW |
| Linotype Machine | 21 | 1 HP each |  | 15.666 | " |
| Fans \& Blower | 6 |  | $=$ | 1.492 | " |

Type Setting \& Files Area

| Lighting | 195 | 80 Watts each | $=15.600 \mathrm{~kW}$ |
| :--- | :---: | :---: | :---: |
| Fans \& Blowers | 16 | $(1 / 4 \sim 1 / 2) \mathrm{HP}$ each | $=3.730 \mathrm{~kW}$ |

Sma11 Press \& Other Machine Area

| Lighting | 91 | 80 Watts each | $=7.280 \mathrm{~kW}$ |
| :--- | :---: | :--- | :--- |
| Presser | 2 | 20 HP each | $=29.840 \mathrm{~kW}$ |
| Other Machines | 6 | 35 HP total | $=26.210 \mathrm{~kW}$ |
| Smal1 Office $\left(300 \mathrm{ft}^{2}\right)$ | 80 Watts each | $=0.640 \mathrm{~kW}$ |  |
| Lighting | 8 | $=$ |  |
| Airconditiong | 3 tons |  | $=2.000 \mathrm{~kW}$ |
| Presser Machine | 2 | $\sim 1 \mathrm{~kW}$ each |  |

Table C. 6.3 (continued)

| Building B | Quantity |  | KW Total |
| :---: | :---: | :---: | :---: |
| Building B (General) |  |  |  |
| Lighting | 177 | 80 Watts each | 14.160 kW |
| Motor | 1 | 50 HP each | 37.300 kW |
| " | 1 | 25 HP each | 18.650 kW |
| (For fans to suck out rejected papers) |  |  |  |
| Motor for* |  |  |  |
| Compressor | 1 | 125 HP each | 93.250 kW |
| Small Compressor | 2 motors | 10 HP each | 14.980 kW |
| Small Compressor | 1 motor | 25 HP " | 18.650 kW |
| Vacuum Pump | 1 " | 25 HP " | 18.650 kW |

Machine Shop

| Lighting | 29 | 80 Watts | 2.320 kW |
| :--- | :--- | :--- | :--- |
| Machine | 6 | $(3 / 4-5 \mathrm{HP})$ each <br> 12 HP total | 8.952 kW |

Office ( $\sim 500 \mathrm{sq} . \mathrm{ft}$. )

| Lighting | 18 | 40 Watts each |
| :--- | :--- | :--- |
| Air Conditioning | $71 / 2$ tons | 0.720 kW |
| Plate Room $(\sim 550 \mathrm{sq}$. | $\mathrm{ft})$. |  |
| Lighting | 20 | 40 W each |
| Air Conditioning | 10 tons |  |
| Machine |  | 0.800 kW |

[^4]Table C. 6.3 (continued)
Building B Quantity KW Total
Printing Press 非

| Main Drive Motor | 2 | 75 HP each | 111.900 kW |
| :--- | :--- | :--- | ---: |
| Starter Motor | 2 | 7.5 HP " | 11.190 kW |
| Other Motors |  | 15 HP Total | 11.190 kW |
| Lighting | 21 | 80 Watts each | 1.680 kW |

## Printing Press \#2

Same as Printing Press \#1 (see above).

Printing Press \#3

| Lighting | 30 | 80 Watts each |  | 2.400 kW |
| :---: | :---: | :---: | :---: | :---: |
| Main Drive Motor | 2 | 100 HP | " | 149.200 kW |
| Other Motors |  | 20 HP |  | 14.9 kW |
| Printing Press 栍** |  |  |  |  |
| Main | 2 | 100 HP |  | 149.2 kW |
| Other Motor |  | 30 HP |  | 22.38 kW |

**
Under construction.

Table C. 6.3 (continued)


Cutter 非1

| Main Cutter Drive | 1 | 10 HP |
| :--- | :--- | :--- |
| Conveyor Drive | $6(1 / 2$ | 7.460 kW |
|  |  | $\mathrm{HP}-1 \mathrm{HP}) 41 / 2 \mathrm{HP}$ total |

Packaging Machine \#1

| Conveyor Drive | motors <br> $(1 / 2-1 \mathrm{HP})$ $\mathbf{3} \mathrm{HP}$ Total |
| :--- | :--- |

Packaging Machine
( 480 V 30 Amps $3 \phi$ )

Binder ${ }^{12}$
Same as Binder \#1 (see above)

Cutter
Same as Cütter $\# 2$ (see above)

Packaging Machine \#2
Same as Packaging Machine \#1

| Building B |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Quan |  | kW Total |
| Cutter \#3 |  |  |  |
| Packaging Machine \#3 |  | Same as \#1 (see above) |  |
| Binder \#3 |  |  |  |
| Main Drive | 1 | $71 / 2 \mathrm{HP}$ | 5.595 kW |
| " $\quad$ | 1 | 10 HP | 7.460 kW |
| Other Motors | 4 | $21 / 2 \mathrm{HP}$ | 1.865 kW |
| Building C (Warehouse \& Waste Baling) |  |  |  |
| Warehouse |  |  |  |
| Lighting | 42 | 250 Watt each | 10.500 kW |
| Waste Baling |  |  |  |
| Hogger Drive | 1 | 100 HP | 74.600 kW |
| Blower | 1 | 50 HP | 37.300 kW |
| Big Baler | 1 | 25 HP | 18.650 kW |
| Small " | 1 | 10 HP | 7.460 kW |

## APPENDIX C. 7

## Consumer Product Company

Consumer Product Company's produces pocket knives, and also kitchen and other cutlery products. The plant is divided into different departments, some of which are directly involved in production -- such as the grinding department, polishing department, the heat-treating department, etc. Ohters are not directly involved in production, but act as support for production, e.g., the personnel, purchasing departments, the tool room, etc.

A complete flow and storage model is not available. However, the storages are large compared to the material flow rates, so it is physically possible to reschedule production from first to second or third shift, although this has been found to be uneconomical, becasue the $A_{i j}$ ( kW demand per person) for any particular department is small (see Chapter VI for a detailed arguement).

A component of a knife must go through various production steps, as shown in Fig. C.7.1, where steel sheets made of coil undergo blanking, a process that punches out a piece of steel of a given shape from the steel sheet. The output pieces from the blanking process then go through straightening, then heat-treating, and finishing. Other parts go through only some of the steps. When enough quantities of all the necessary parts are ready, they will be assembled to produce the desired product, e.g., a pocket knife.

The electric stock data and information on employees of each department obtained from the departments of Consumer Product Company are shown in Table C.7.2; the kW of electric stock per person for each department can be found from the above information, also given in Table C.7.2.

The heat-treating furnaces are operated three shifts per day. The number of people involved in each shift is listed in Table C.7.2. All other departments are operated only one shift per day, the first shift, although employees from some departments might occasionally stay for a few hours longer than a single shift.

There is no wage differential for different shifts, in view of the fact that the majority of employees work during the day. However, if rescheduling is to be attempted, some wage differential for second and third shift has to be provided.

Table C.7.1 gives the billing data for this company, and Fig. c.7.2 gives the load profile for a summer week.



1976
1976

| Month | $\begin{array}{\|c} \mathrm{kW} \\ \text { Demand } \end{array}$ | KWH | Hour Used K.jH/kW (hour) | kW Demand | KWH | $\begin{aligned} & \text { Hour Used } \\ & \text { (hour) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Jan. | 1555.2* | 259,407 | 166.80 hr | 1512.0 | 299,000 | 197.75 |
| Feb. | 1526.4* | 360,229 | 236.00 | 1512.0 | 396,177 | 262.02 |
| Mar. | 1512.0 | 363,840 | 240.63 | 1476.0 | 392.706 | 266.06 |
| Apr. | 1512.0 | 284,631 | 188.25" | 1526.0* | 410,633 | 269.09 |
| May | 1512.0 | 277,422 | 183.48 | 1548.0 | 407,030 | 262.94 |
| June | 1440.0 | 288,230 | 200.16 | 1512.0 | 385,422 | 254.91 |
| Ju1y | 1440.0 | 263,018 | 182.65 | 1548.0 | 410,586 | 265.24 |
| Aug. | 1468.8* | 263,002 | 179.16 |  |  |  |
| Sept. | 1476.0 | 367,437 | 248.60 |  |  |  |
| oct. | 1440.0 | 342,240 | 237.67 |  |  |  |
| Nov. | 1440.0 | 342,215 | 237.65 |  |  |  |
| Dec. | 1512.0 | 371,084 | 245.43 |  |  |  |

Table C.7.1 Billing Data for Consumer Product Co. for the year 1975 \& Part of 1976.

* 0.8 KVA was used

TABLE
C. 7.2

Electric Stock and Number of Employees at Each Section of Consumer Product
Department No. 47,Toolroom

| Item | Quantity | HP Each | HP Total | X ( in kW$)$ |
| :---: | :---: | :---: | :---: | :---: |
| Motors | 16 | . 2 to 15.75 | 34.45 | 25.70 |
| Lights |  |  |  | 4.40 |
| No. of Employees $L_{i j}=30$ persons <br> Electric capacity/person $X_{i j} / L_{i j}=1 \mathrm{~kW} /$ person |  |  |  |  |

Department 5, Spr. Production
$\begin{array}{llllll}\text { Motors } & 41 & .25-10 & 153.31 & 114.37\end{array}$
Lights 3.46
No. of Employees $\quad L_{i j}=26$ persons
Electric Capacity/person $X_{i j} / L_{i j}=4.53 \mathrm{~kW} /$ person

Department 6, Blanking
$\begin{array}{lllll}\text { Motors } & 39 & .25 & \text { to } 30 & 93.22\end{array}$
Lights 4.64
No. of Employees $L_{i j}=14$ persons
E1ectric Capacity/person $X_{i j} / L_{i j}=7.41 \mathrm{~kW} /$ person
Department 8, Press
$\begin{array}{lllll}\text { Motors } & 45 & .25 \text { to } 10 & 108.83 & 77.46\end{array}$
Lights 2.40
No. of Employees $L_{i j}=14$ persons
Elec. capacity/person $X_{i j} / L_{i j}=7.41 \mathrm{~kW} /$ person
Department 16, Auto. Grinding
Motors $155 \quad .5$ to $25 \quad 854.0 \quad 637.08$
$\begin{array}{ll}\text { Lights } & 2.80\end{array}$
No. of Employees $L_{i j}=51$ persons
Electric capacity/person $X_{i j} / L_{i j}=12.55 \mathrm{~kW} /$ person

Item
Quantity HP Each
$\underline{H P \text { Total }} \mathrm{X}$ (in kW )

Department 24, Bolster

| Motors | 57 | .25 to 7.5 | 109.66 | 81.81 |
| :--- | :--- | :--- | :--- | :--- |

Lights
No. of Employees $L_{i j}=80$ persons
Electric capacity/person $X_{i j} / L_{i j}=1.09 \mathrm{~kW} /$ person
Department 10, Material \& Tubbing

| Motors | 66 | .25 | to 15 | 182.25 |
| :--- | :--- | :--- | ---: | ---: |$\quad 135.96$

Department 32, Skeleton Assembly
$\begin{array}{lllll}\text { Motors } & 69 & .25 & \text { to } 5 & 46.33\end{array}$
Lights
No. of Employees $L_{i j}=28$ persons
Electric capacity/person $X_{i j} / L_{i j}=1.50 \mathrm{~kW} /$ person

Department 33, JM Shell Wrapping

| Motors 22 | .15 to .75 | 12.48 | 9.31 |
| :--- | :--- | :--- | :--- | :--- |

Lights
8.88

No. of Employees $L_{i j}=24$ persons
Electric capacity/person $X_{i j} / L_{i j}=0.76 \mathrm{~kW} /$ person
Department 18, Dressing
$\begin{array}{lllll}\text { Motors } & 36 & .125 & \text { to } 7.5 & 99.125\end{array}$
Lights
2.92

No. of Employees $L_{i j}=20$ persons
Electric capacity/person $X_{i j} / L_{i j}=3.84 \mathrm{~kW} /$ person

Department 20, Finishing

| Motors | 152 | .25 | to 10 | 675.58 |
| :--- | :--- | :--- | ---: | ---: |
| Lights |  | 503.98 |  |  |
| No. of Employees $L_{i j}=$ | 66 persons |  |  |  |
| Electric capacity/person | $X_{i j} / L_{i j}=$ | $14.17 \mathrm{~kW} /$ person |  |  |

Department 23, Hollow Grinding
$\begin{array}{lllll}\text { Motors } & 96 & 25 & \text { to } 10 & 330.75\end{array}$
Lights 2.96

No. of Employees $L_{i j}=14$ persons
Electric capacity/person $\mathrm{X}_{\mathrm{ij}} / \mathrm{L}_{\mathrm{ij}}=17.83 \mathrm{~kW} /$ person
Department 14, Heat Treating

| Motors | 49 | .25 | to 62.5 | 664.73 |
| :--- | :--- | ---: | ---: | ---: |
| Lights |  |  | 495.89 |  |
|  |  |  | 1.76 |  |

No. of Employees : 13 on 1st shift, 4 on 2nd shift, 4 on 3rd shift.
Electric capacity/person $X_{i j} / L_{i j}=$
Furnaces See the measured hourly load shpae during 2nd and 3rd shifts.

## APPENDIX D <br> Computer Programs

This Appendix describes and lists the computer programs used for computing and plotting the graphs shown in Chapter V. All the programs are written in PL1 language and implemented on a Honeywell MULTICS System computer. They are briefly described below and shown in detail afterward. Note that the programs discussed here were written for Brush Company, but could be used for other companies with minor changes.
pg_1 : This is the program used for drawing the load profiles for a given week; time-series input data are used. The load profiles from Figs. 5.1.1, 5.2.1, 5.3.1, and 5.4 .1 were drawn using this program.
pg_dmb : This is the program used for drawing the daily 15 -minute average sample mean from time-series data, as shown in Figs. $5.1 .2,5.2 .2,5.3 .2$ and 5.4.2. The time-series input data were used. Subroutine m_dasb was called by this program. Note that the theoretical expected time-varying load, as shown in Figs. 5.1.2, 5.2.2, 5.3.2 and 5.4.2 were computed using electric capacities and utilizations data, shown in Chapter V and Appendix C. The computations were done by long hand. pg_res_dasb: The program is used for plotting the curves shown in Figs. 5.1.3, 5.2.3, 5.3.3 and 5.3.4, using time-series input data. Subroutines m_dasb and res_dasb were called by this program.
pg_Rb: This program is used for plotting the time-average sample autocorrelation functions of the residual as shown in Figs. 5.1.4, 5.2.4, 5.3.4 and 5.4.4, using time-series input data. Subroutines $m$ dasb, res_dasb, fd_b and Rd_kb were called.
th_R_br: This program is used to plot the theoretical autocorrelation function of the residual, shown in Figs. 5.1.4, 5.2.4, 5.3.4 and 5.4.4 using the electric stocks and utilization data for groups of equipment, $X, L$, a and $n$ as given in Chapter $V$ and Appendix C. Note that graphs drawn by this program were compared with those drawn by the program pg Rb . The mathematics needed for this program is discussed in Chapter II.
m_dasb: This is a subroutine for computing the sample mean. This subroutine is called by pg _1, pg _dmb, $\mathrm{pg}_{-}$res_dasb, and pg Rb . The mathematics involved is described in Chapter IV.
fd_b: This is a subroutine serving as a gating function on a filter in time. This subroutine is called by $\mathrm{pg}_{-} \mathrm{Rb}$.
res_dasb: This is a subroutine for computing the residual load. The mathematics involved is described in Chapter IV of this report. This subroutine is called by pg_res_dasb and pg_Rb.

Rd_kb: This is a subroutine for computing the time-average sample autocorrelation function. The mathematics involved is described in Chapter IV; this subroutine is called by pg Rb .

Format of Input Time Series Data

This type of 15 -minute average sample time-series data is used by all the programs for plotting graphs except th-R-br. See Chapter IV for the formulae for the time-series analysis and computation. The timeseries data for each company were originally recorded on paper tape in the form of a graph, or as an arithmetic number to be read visually. This type of data has to be punched on computer cards. Note that there are 96 data points for each day.

These data points are recorded in terms of numbers, therefore a factor "fac" is used in the program for converting them into kW. Sixteen data points are punched on each card and a 17 th data point to represent a card number, or date, etc. which is disregarded by computer programs. There are six computer cards for each day. The programs are written such that the data point for $7: 00 \mathrm{a} . \mathrm{m}$. of a Monday is read as the first data point. The computer programs are written for 15 -minute average sample time-series data for six weeks. One page of the time-series data in the form needed (for Soap Co.) is printed out and shown in Table D.1.
$0009000,000,2834460,612,613,660,650,660,688,700,682,702,702,708$ $672,695965,6809690,6809670,655,642,650,650,648,648,660,660,645$ $624,592,580,532,500,458,430,364,350,350,325,320,300,290,265,245$

 $1009093,093,090,090,088,092,092,090,090,088,090,088,090,090,090$ $090,092,090,090,086,086,09290929095.090 .0909090 \cdot 100,093.093 .110$ $102,090,085,080,080,082,080,078,080,078,078,078,078,080,078,078$ $0769078,075,072,072,078,076,076,080,076,078,078,078,080,0729076$ $072,074,074,072,079,078,080,078,078,030,078,078,075,080,075,073$ $076,076,07690789078,075,075907590789076,076,075,078,07890789078$ $078,0759074,072,072,070,072,072,072,074,074,072,074,0749072,074$ $0729078,078,077,0789078,080,080,0829082,082.0080,080,076,0809080$ $082,080,082,0829070,079,080,080,082,078,0809080,080,080,080,080$ $080,080,078,080,0809080,080,0804078,078,078,078,076,078,082,085$
 $250,253,276,255,275495,312,302,292,290,310,310,300,302,310,320$

 $684 \cdot 6959690,690,680 \cdot 602.682,6659623,5689566,572,5659595462,646$ $592985,618,624,61696164630,6059595 \cdot 633,6209597,597,597,597,582$ $575,573,570,569,576,560,568,545,550,555,556,563,532,490,445,402$ $385,387,400,400,390,380,400,462,493,502,522,50,520,530,540,540$ $553,557,553,562,570,572,570 y 629553,586,586,610,590 \cdot 632,642,642$ $642,6409600,568,535,550,490,470,480,492,492 y 502,408,498,484 y 482$ $479,482,474,590,630 \cdot 620,650,650,545,555,542,539,535 y, 532,530,562$ $562964,575 \cdot 633,6329630,644,644,630 \cdot 650,642,633,6289628,618 \cdot 608$
 475,440 y $430,3724292,285,282,292,290,298,294,304,310,330,390,417$ $443,4,45,456,456,460,470,468,460,440,470,477,503,530,510,492,512$ $525,560 \cdot 637,6159620 \cdot 640,660,660 \cdot 660,657,619,602,585,610,612,602$ $6039610.610,6109620.612 .603,593,5759590,600,595,598,580,5659560$ $535.535,568,572,570,570,570460,542,570,572,565,563,570,570,560$ $552,540,530,523,520,543 y 543: 495,460,4064380,3759362,343 y 350,363$

## Format for Electric Stocks and Their Utilization Data

This type of data is used by the program th_R br for computing the theoretical autocorrelation function of the residual. The format for the input data needed is as follows, and an example of this type of data for Abrasive Company is shown in Table D. 2.

For each group or piece of equipment, five data points are needed. The first number represents the installed kW for the piece of equipment or for the entire group. The second data point represents the fraction of load when the equipment is on, $L$, and the third point represents the number of machines in a group " n mach". The fourth number represents the number of startups per hour, $n$, and the fifth number is the percent of time the piece of equipment or the group is on, a. See Chapters II and III for further explanation.


```
/* prosram to elot the load frofile of a given week*/
ps_1: proc;
dcl (sssiny susfrint, soaw) fileg
close file (soam)y
del (fac,Whumy maxs)float bing
dcl ( A(6,7,6,17)ywm(7,6,16),Al(672),y(672), < (672)) float bing
/* A(6,7,6,17) is the four dimensional arras for the time series data*/
dcl (i, j, k, l, m,m,o...w) fiked bing
dcl m.was entry (fiwed bir, (*,7,6,17) float birig(7,6,d6)float bim)\hat{y}
** The following three sustem subroutines are used for plottins araphs */
dcl plot_ entry ( (*) float biny (*) float biny fixed biny
        fixed biruy char (1))%
dcl flot.कsetap ertry (char (*)y char (*)y char (*)y fi*ed biry
        float bin, fi%ed bim, fixed biro);
dcl wlot._sscale entry (float birig float bing float bine float bir);
Fut skiplist ( "enter moonw fac maxs Wrum")"
put skifot
set list (ro_o..w, facymaxs,Wrum);
/* rimow is the mumber of weeks considered*/
/* fac is the conversion factor into kW */
/* maxs is the maximum load expected , It is used for scaling the s axis */
/* Wromis the week number to be plotted */
```



```
\c6)
                    sok=1 to 7) do l=1 to (1..onw))%
```

```
Q1: do 1 = 1 to n_onwt
q2: do k=1 to 7%
a3: do J =: 1 to 60
Q4: do i = 1 to 16%
A(1,k,j,i) == fac*A(1,k,joi) t
end &4;
end a3:
erod a2%
enod a.t
                    n=0;
F2: So 1.= Wrumit
53: do k=1 to 7%
F4: do j=1 to 6\hat{y}
p5 : do i= =1 to 16%
                                    n= n+1%
                                    x(ri)}=n,\mp@code{m
Al(ri)=A(lyk,jyi)\hat{g}
y(r)}=\mathrm{ Al(m);
                                    end psi
                end pa;
                    end m3;
                    end F2%
            call wlot..कsetuw (" soas Co. load profile", "time in 1/4 hour",
                "power in kw", Ig OeOy 1, O)\hat{g}
            call plot_$scale (0.0, 672, 0.0, maxs)\hat{y}
            call plot_ (x, צ, 672, 2, " ")\hat{y}
        end ra_l;
```

```
* prosram for mlottinss the l5 mimute averase sammle mean of a das */
/* from the time series data based on the daily cscle model */
* see chapter 4 for the explanation of the daily escle model */
Fs_dmb: frocg
```

del (susing susfrint, soam) filey
close file (sozm)
del fac float bing

/ $A(6,7,6,17)$ is the four dimensional arrass for the time series data */
del (i, j, k, ly ngr_o_w) fiked bimy
del m_dasb entrs ( (*,7yo,17) float biny (6, 16)float binyploat biri)
/* the followins three ssstem sumpotines are used for plottins araphs */
del Flot...entrs ( (*) flost bing (*) float biny fiked bin"
fiked biny char (1))
dol whot.-\$setur entrs (char (*), char (*) y char (*), fi\%ed biny
flout bing fixed biry fixed biri)
del Flot $\$$ scale entrs (float bing float biny flozt birig float bim)
put skif list("enter fac minf")
Fut skife;
/* fac is the coriversion factor into k.w */
(* data point of value below minf is rejected (bad data) */
set list (f'scyminf)
 \c6) dok=1 to 7) do $1=1$ to 6) $) \hat{y}$

```
minF= minF/fac!
```

    \(n=0 \hat{y}\)
    call mi...rasb (A, dmyminifl)
F4: Jo $j=1$ to 6i
FO:
$d o i=1$ to $16 \hat{y}$
$n=m+1 \hat{y}$
$\therefore\left(r_{1}\right)=r_{1} \hat{y}$
$\operatorname{dim} I\left(n_{1}\right)=\operatorname{dm}(j, j) \hat{y}$
$s\left(n_{1}\right)=d m l\left(n_{n}\right) * f a c t$
end FGA
$\operatorname{erid} \mathrm{F} 4 \hat{y}$
call flot, osetup (" sozf Co. load frofile", "time in $1 / 4$ inour".
"Fower in kiw", 1, OeO, ty O)
call. Flot... $\$$ scale( $0.0,96,0,4000$ )
call Flot... (x, sy96.2." ")
erod Fscrimos

```
/*Frosmam for wlottins the residmal load */
F& res_masb: wroct
Jol (susin% suswrint, soaw) fileg;
close fille (soa&)%
Gol fac float bin\hat{y}
Gfl ( B (6, 7, 6, 17), dm (6, 16), reda (6, 5, 6, 16), redal (2880), 4 (480),
\0% (4GO)gminf) float bing
/* B(6,7,6,17) rewresents the time series date**
dci. (i, jy k.y J.y ny n...n...w) fixed ming
```



```
dol res...dash eritrs ( (*, 7,6,17) float biriy ( 6y 16) float biriy
(*, Ey Gy 16) float biny float bin)\hat{g}
/* sustem srafoics subroutime */
```



```
        fiwed birig char (1));
dcl Flot_$setug entre (char (*), char (*)y char (*)y fi`ed bimg
        floab biny fiwed biny fixer bim)%
ded flot..⿻三人口ale entrs (float biny float miny float oiny float ain):
Fut skj= Iist ("emter factor minF")\hat{y}
#et list, (facymjwF")
/* face is usect to morivert data into kW */
/* data with valme below mimF is considered to be bad data */
```

ri...O.w $=6$ 名

dok $=1$ to 7 ) foly $1=1$ to fi.....w) $)$
$m i n F=m i m F / f a c i$
$n=0 \%$

conl res riash(B, dmy redagminfo)
F2: do $1=1$ to no.o...
FB :
F4:
dok $=$ l. to 5 \%
So $j=1$ to $6 \hat{y}$
So $i=1$ to $16 \hat{y}$
$n=r i+1 \%$

end $\because 5 \hat{y}$
and $54 \hat{y}$
end mo3
and
do $n=1$ to 480
$\therefore\left(n_{1}\right)=n_{1} \hat{y}$
$\because(n)=$ regal (n)*fact
ang
call flot, \#setup (" soaf Co. residual wang "time in 1/4 hour"y


enci wes res...dashy

```
/* prosram for slottinse the autocorrelation function of residual load */
/* comfuted from the time series data*/
ps_Fib: procg
dcl (susim, susprint, soaw) fileg
close fi.le (soar)%
dcl (fac,mimF') float, bim;
/* B(6y7,6y,7)is the four dimensional arrass for the time series data */
dcl ( E (6, 7, 6, 17), dm( 6, 16), rede (6, 5,6,16), F:1 (-99:99), % (-99:99), <
\c (-99:99), F9*1(199),st(199)) float birig
del (R2(-99:99), R3(--99:99)) fi%ed bin (30:6);
```



```
dcl modasb entrs( (*, 7, 6, 17)floet bi|y(6, 16)float biryfloat bir!)
del resmdasb entru( (*) 7, 6, 17) float bing ( 6y 16) float bing
    (*,5,6,16)float biny(loat bin)\hat{y}
dcl Fod_kb ertrs (fixed bing fi*ed bing fixed bing fixed biry
(*g5,6,j6) float bim, float bimy fixed birm)%
/* the followinss are three system sumpoutines for folottins smawhs */
dcl Flot... entrs ((*) float bjry (*) float biny fiked biny
        fiked biny char (1))#
dcl plot.-$setup entrs (char (*), char (*)y char (*)y fi*ed bing
        float bing fixed biry fi%ed bin)\hat{y}
del Flot.mscale entrs (float bine float biny float bing float bin);
```

mut 5kif(2) Iist ("check Jim. of Fif) $\hat{y}$

/* al ge are used to specify the filterins time interval for the subroutirue
vofob */
/* if rothiris is to be thrown out from the time series data then ai = 0 arad as
\c: = 97 */
/* the autocorrelation function is to be samwled avers as woints */
/冰 rormilly a5 $=1$ is used */
Fut skiが

mimF = mirrf/faci

\o6)
dok=t to 7) do $1=1$ to 6)
$\mathrm{ri}=0 \%$

call res....asb ( By amy reday minf)
do $m 3=-33$ to 33 ;

FI(ma) = Fit
erió

$$
F \because(n)=F 1(n) * n^{2} \sigma * * 2
$$


$\mathrm{F} 3\left(\mathrm{r}_{1}\right)=y\left(\mathrm{r}_{\mathrm{i}}\right)$ )
erid力
so $m=1$ to (2*33 +1) $\hat{y}$
$x 1(m)=m i$
$y t(m)=4(m-(a 3+1)) \hat{y}$
end;

\e)",



End wesmity

$$
\begin{aligned}
& \text { So } n=-a 3 \text { to } 33 \text { 乡 } \\
& x\left(r_{1}\right)=r_{i}
\end{aligned}
$$

```
* prosram used for computins and plotting the autocorrelation furction */
* from the theory or the phssical model */
th_Figr:proc;
dcl (susir, suspririt, brush-th)file;
close file(brush.th)%
del (nogy rits, i,jymin)fixed bint
dcl [l(44,5) float bin (53)\hat{g}
/* n(44,5) is the 44 sroups of equipment considered */
/* 5 parameters is needed for each sious */
/* see the data format for the fhusical model in the besiminins of this Amb. */
dcl (X(44),L(44), rimach(44)yeta(44),algha(44),Fil(44),
Fo(44),C(44),lamda(44), delta)flozt bin (53)%
del (Fou(44), Rowi(44), Fou2(44), Fol(44), ay, a3)float bin(53);
dcl (F(44),RT(-100:100))float bin (53)\hat{y}
dcl(x(201),y(201))float bin (53);
dcl (FT1(-100:100),si(201))fixed bin (30,8)\hat{y}
/* three sustem subroutines for plottins arachs */
dol plot... entrs ((*) float bin (53)y(*) float bin (53)yfixed birig fixed birn
char(1));
del Flot..$setup enitrs (char(*)gchar(*),char(*),fi*ed bin, float bir, (53),
fi%ed bin, fiNed bin);
del plot,$scale entry (flost bin (53), float bin (53)y float bin (53), float bi
\en (53))\hat{g}
wut skif(2) list ("enter ruo_s not...s delta a3");
Fut skipg
set list(ros, ritsyreltaga3);
/* nos is the number of srouf irivolved; nts is the number of time poimbs */
* to be comFuted and mlotted */
/* delta is the averasing time stepy 15 mirute ig used bs the utilits */
/* the autocorrelation function is to sammled evers a3 points */
get file (brush_th) list (((I)(i,j) do j = 1 to 5) do i == 1 to nos));
Qa: do i == 1 to mos'
X(i) = L(i,i);
L(i) == L(ig2)%
rimach(i)}=\textrm{m}(\textrm{i},3)
eta(i) = g(i,4)\hat{y}
```



```
X(i)=X(i)/sart(musch(i));
end Qa;
a2: do m= -rots to motsp
RT(m)=0\hat{y}
a1: do i = 1 to mog;
if alpha(i) = 1 then alpha(i) = (1 .-. 0.0001);
if eta(i) = 0 then eta(i) = 0.0001%
if alpha(i) == 0 then alwha(i) = = 0.0001%
lamda(i) = eta(i)/(alwha(i)*(i - alfha(i)))方
Fou1(i) = (((X(i)*L(i))**2)*2*(1 - alfha(i))**afma(i) );
Fol(i)=(1amda(i)*delta)**2%
a}=(1\textrm{amda}(\textrm{j})*\mathrm{ *jelta) 方
/* the followins step is used for preventins frosram from overflowins */
if a >}30\mathrm{ then a = 30;
Fou2(i) = (1 - (1 + lamda(i)*delta)*exp(-a))%
Rou(i) = Foul(i)*Fous(i);
R1(i) = Fou(i)/Rol(i);
C(i) = Rout(i)*(cosh(a) - 1)/Fol(i)\hat{}
b= (lamda(i)*de1ta*m*a3);
* srevention of prosram from overflowins*/
if b> 30 then b = 30;
if b < (-30) then b=(-30);
if m=0 then R(i) = R1(i)%
if m<0 then R(i) = C(i) *exp(b)%
if m>o then F(i) == C(i)*exs(-b);
```



```
end al;
FTT:(m)=FFT(m);
end ax\hat{y}
a5: do n = 0 to ((2*nts));
<(n)=ng
s(ri) =FT(n, -..nts)/F゙r(O)t
s(m)=y(n);
end a5;
call slot.-$getup ("brusin...th Co. ",
"Time in 1/A hour", "autocorrelation",1,0e0,1,0);
call flot_sscale (0.0.(2*nts ),- 0.5y 1.5)%
call flot_ (x,y9rts*2 ,2," ");
wat data (RT1(O))㓪
end th_f_me%
```

```
/* subroutine to comwuted the sample mean from the time serige; data */
* from the dailu oscle model. see chapter 4 for the evelamation */
m..dasb: wroc (A, dmy minF);
col mirif float bin\hat{y}
del (A (*, 7, 6y 17), dim (6y 16)) float bin名
/* A(*,7,6,17) js the fotu dimensiomal arrass for the time series data */
ricl (i, jy k, l, nom) fjxed biri%
F2: doj=1 to 6%
w: ro j = 1 to 16%
    Clim (j, i)}=0
    ri=0;
    m= O%
F4:
F5;
    do 1=1 to 6%
dok=1 to 5%
                                    mi= mi +1\hat{g}
if A(1ykgjyi) <minF' then A(lyky,jyi)=0;
```



```
                    j.f A (I, k, jy i) =0 theri n= =n+1%
                        encic:%多
            enid =4\hat{y}
```



```
            erig 4.3%
        emcl g'2.⿱宀⿻三丨口
        returrio
        enrs m...desb%
```

* subroutine for filtering the time series data */
* to be used for cases where the residual load is not stationary */ * this subroutine is not, used for ans weekdays in this refort */

declare (al, a2, iy jy k. ly $n$ ) fixed bing declare fal (*) float birt

Fi: do $1:=1$ to 6 :
p2: bok=1 to 5 g
$\pm 3$ *
F. 4
so $j=1$ to 6 .
do $i=1$ to 16 \%
$n=n+1$ !
if ( 3.1 < ( ( $j-1) * 16)+i))$
\& $(((j-1) * 16+i)$ < 32$)$
then fol $\left(n_{1}\right)=1 . \%$
else fol ( r ) $=0$.
end 54 ;
end Fory
end :ey
end wis
return
end folm

/* see chawler \& for the mathemetice involved $\boldsymbol{w}^{*}$
resmasebt wroc ( Ay Bm: redet mirf)
del minf flost biny

del (i, jy ky 1 ) tixed biny
Fat bo $1=1$ to $6 \%$
F1各 do k =: 1 to 5 y
C2: $\operatorname{cio}, \mathrm{i}=\mathrm{l}$ to 6 \%
F3: do $=1=1$ to $16 \hat{y}$


if $A(1, k, j y$ i) $=0$ then reáa (Iy ky iy i) $=0$ :
aroje 5 \%

end $; 1 \hat{y}$
emd
returni
enci res ...assby
/* sunroutire for comfutiras the autocorrelation function of the resional */
/* from tine time serjes data */
/* see chapter 4 of text for the mathematios involved */

dectare (reda (*y 5y 6y 16) ,
F1
murest (2880) y
muress (2880)y
ffil (2880)y
fat (2880)) float anfô

Geclare foberity (fixed biry fixed biriy (*) float bin)
call fomb (aly as, fal)
$\mathrm{ra}=0 \%$
do $1=1$ to $6 \hat{y}$
do k = 1 to 5
$d o j=1$ to $6 \hat{y}$
do $\mathbf{i}=1$ to 16 多
$\mathrm{r}=\mathrm{n}=\mathrm{m}+\mathrm{y}$
muresi (r) $=$ redi (ly k, jy i)
ent $=4 \dot{y}$
end F 3
erid wat
end rid
$34=2880 \%$
no $=0$ \%
Q2: do $n=1$ to 2880\%
if ( $0<(11-35 *$ mas) $)$
\& (n - a5*ma) < (a4 - 1) then
murese (n) =" murest (n- (a5*ma))
else mures2 ( $n$ ) = 0;
ffil ( $n$ ) = (fol ( $n$ )*murest ( $n$ )*mures ( $n$ ) ) \%
if fRil(n) $=0$ then no $=-n o+1 \hat{1}$
end a2
Fi $=0$;
Q3: do $n_{1}=a 5 * a 3$ to (a4 - a5*a3)
$\mathrm{F} 1=\mathrm{Fi}+\mathrm{fR}(\mathrm{m}) / \mathrm{rog}$
end ảy
returrig
end Fideng

Yongyut Manichaikul came to the United States from Thailand in September 1968. Until June 1970, he was a student at California State University at Sacramento, California. He subsequently transferred to M.I.T., where he received the S.B., S.M., and Engineer's degrees in Electrical Engineering in 1972, 1973, and 1975, respectively.

While in graduate school at M.I.T., Mr. Manichaikul worked as a Teaching Assistant in the Department of Electrical Engineering and Computer Science. He was a Research Assistant in the Research Laboratory of Electronics from September 1972 to May 1974, in the area of the generation and amplification of nanosecond TEA $\mathrm{CO}_{2}$ laser pulses. From September 1976 to May 1978, he was a Research Assistant in the Electric Power Systems Engineering Laboratory, working in the area of industrial electric load modeling. He has also worked as a consultant for the New England Electric System, in the summer and fall of 1976 .


[^0]:    * Note that we have assumed that the 15 -minute ayerage sample loads are -independent.

[^1]:    * This is the number suggested to us by the design engineer of this company.

[^2]:    * The expected utilization for the machines in this section is assumed to be 0.5 at maximum during lst shift $\&$ weighted by the number of employees during the 2 nd and 3 rd shifts.

[^3]:    * Consumer Product and Printing Companies are under Rate G, but for our purpose here, they are treated as belonging to Rate H .

[^4]:    ※
    Undergoing Repair Job, Inoperative.

