DESIGN, CONSTRUCTION, AND TESTING OF A COMBINED MAGNETIC LEVITATION AND PROPULSION SYSTEM

by

MICHAEL ATLAS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
BACHELOR OF SCIENCE

at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

MAY, 1977

Signature of Author ..........................................................
Department of Mechanical Engineering, May 8, 1977

Certified by .................................................................
Thesis Supervisor

Accepted by ...............................................................
Chairman, Departmental Committee on Theses

JUL 7 1977
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>4</td>
</tr>
<tr>
<td>1. Introduction</td>
<td>5</td>
</tr>
<tr>
<td>2. Thesis Objectives</td>
<td>8</td>
</tr>
<tr>
<td>3. Experimental Apparatus</td>
<td>9</td>
</tr>
<tr>
<td>4. Theoretical Analysis and Design</td>
<td>25</td>
</tr>
<tr>
<td>5. Experimental Results and Discussion</td>
<td>34</td>
</tr>
<tr>
<td>6. Conclusions and Recommendations</td>
<td>64</td>
</tr>
<tr>
<td>7. Acknowledgements</td>
<td>66</td>
</tr>
<tr>
<td>8. References</td>
<td>67</td>
</tr>
<tr>
<td>Appendix</td>
<td>68</td>
</tr>
</tbody>
</table>
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cross-section of typical attractive magnetic levitation system.</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>Schematics of rail and magnet core.</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>Winding arrangement of magnet.</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>Photograph of completed magnet assembly.</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>Photograph of rail and load cell assembly.</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>Photograph of magnet and rail in place.</td>
<td>18</td>
</tr>
<tr>
<td>7</td>
<td>Photograph of whole test setup.</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>Schematic of amplifier-feedback circuit.</td>
<td>22</td>
</tr>
<tr>
<td>9</td>
<td>Sample current path in secondary.</td>
<td>31</td>
</tr>
<tr>
<td>10-31</td>
<td>Plots of vertical and horizontal forces measured in tests.</td>
<td>38-59</td>
</tr>
<tr>
<td>32</td>
<td>Comparison of theoretical and measured horizontal forces.</td>
<td>60</td>
</tr>
<tr>
<td>33</td>
<td>Comparison of measured values of vertical force at different frequencies.</td>
<td>61</td>
</tr>
<tr>
<td>34</td>
<td>Values of $\frac{B_s^2}{B_p}$ predicted by equation 18</td>
<td>62</td>
</tr>
<tr>
<td>35</td>
<td>Ratio of horizontal to vertical force at several frequencies.</td>
<td>63</td>
</tr>
<tr>
<td>36</td>
<td>Schematic of phase-splitting circuit.</td>
<td>69</td>
</tr>
</tbody>
</table>
DESIGN, CONSTRUCTION, AND TESTING OF A COMBINED MAGNETIC LEVITATION AND PROPULSION SYSTEM

by

MICHAEL ATLAS

Submitted to the Department of Mechanical Engineering on May 12, 1977, in partial fulfillment of the requirements for the Degree of Bachelor of Science.

ABSTRACT

A combined magnetic levitation and propulsion system of the linear induction motor type was analyzed to predict the vertical and horizontal forces. An experimental model was built with the aid of the analysis and measurements of these forces were made. Measurements were taken at magnet to rail gaps of from .015" to .2" and at power supply frequencies from D.C. to 120 Hz.

The results of the tests were compared with the results of the analysis and were found to agree fairly well at the larger gaps and the lower frequencies. At the smaller gaps and higher frequencies the measured forces were markedly lower than the predicted values. The possible explanation given and discussed is that the secondary magnetic field produced by the current flow induced in the rail by the primary field nullifies some of that field.
1. **Introduction.**

There are presently two types of magnetic suspensions for high speed rail type vehicles under extensive study. These are the repulsive type, in which the vehicle is suspended above a track; and the attractive type, in which the vehicle is suspended below a track.

In the repulsive type of suspension coils in the vehicle set up a magnetic field that induces currents in coils that are set into the track. These currents produce a magnetic field that opposes the one set up by the primary, creating lift. The drawbacks of repulsive systems are as follows: 1. No lift is produced at zero speed, therefore wheels must be used for suspension at speeds below the "takeoff" speed (typically around 30 mph). 2. For efficient operation the coils in the vehicle must be superconducting, thus necessitating refrigeration equipment. 3. As vehicle speed increases, magnetic "drag" also increases resulting in large propulsion requirements for high speed use. 4. At present a separate propulsion system is required. The problem here is that repulsive system gaps tend to be larger than the gaps at which linear motors work best.

In an attractive system normal electromagnets mounted in the vehicle exert an upward force towards a ferromagnetic rail situated above. This type of system is depicted
in Figure 1. This type of system is inherently unstable but this can be remedied through the use of active magnet control systems.

One of the limitations of a magnetic suspension is the problem of propelling a vehicle that has no physical contact with the rail it rides over or under. Many proposed systems use a normal magnet for lift and a double-sided linear induction motor for propulsion. The obvious drawback of two separate magnet systems is the increased complexity and weight of the system. It has been proposed to combine the suspension and propulsion functions by using one magnet of the single-sided linear induction motor type. This system would require only one magnet to provide both lift and propulsion thus lowering vehicle weight and thus increasing the load carrying capability.

It is the intent of this thesis to study the properties of such a combined suspension system with the intent of developing the analytic tools needed to make reliable predictions of the performance of one.
Figure 1- Cross-section of typical attractive magnetic levitation system.
2. **Thesis Objectives.**

1. Develop an analytical model of the system as an aide in designing the magnet and rail and in predicting their performance.

2. Construct the test magnet and a section of test rail of the linear induction type.

3. Assemble the equipment and instrumentation necessary to run static force tests on the test magnet and rail.

4. Perform static tests of vertical and horizontal (lift and propulsive) forces under various conditions, i.e. different frequencies, currents, and gaps.

5. Compile the results and compare them with the analytical predictions.

6. Try to explain any major discrepancies or unexpected results.
3. **Experimental Apparatus.**

The first part of the apparatus consisted of the magnet and the rail, both of which I designed and constructed myself. The second part consisted of the equipment for generating three-phase current at the desired frequencies and the force measuring and recording equipment. I shall deal with each piece of apparatus in turn.

The magnet core was constructed from eight 1/8-inch thick sheets of mild steel. The steel was bolted together and the slots for the magnet coils were milled. The outline and dimensions of the core are given in Figure 2.

The reason for composing the core of a lamination of thinner sheets instead of a solid block was to reduce the magnetic losses due to induced eddy currents during A.C. operation of the magnet. After milling was complete the core was disassembled and the separate laminations were painted with Glyptol, a special insulating paint produced by General Electric for use on magnet and motor windings. This was to insulate the laminations from each other after their assembly. Two 1-inch square aluminum bars were made into combination cooling fins and heat sinks for the magnet by milling ½-inch slots at ½-inch intervals along their length.

Four bolts were used to simultaneously hold the cooling fins in close contact with the magnet core and to
**Figure 2**- Bottom- schematic of magnet core, Top- schematic of rail core and "squirrel cage".
hold the core laminations themselves together. After assembly the core was again painted with Glyptal to help prevent possible shorting of the coils due to contact with the core.

The fifteen magnet coils were each wound individually on a special form. The form was chucked in a lathe for the actual winding with a mechanical counter to keep track of the number of winds. Each coil was composed of 200 turns of #26 enamel insulated magnet wire. The coils were each tied with twine before removing from the form to help them keep their shape. Cardboard liners were placed in the slots in the core and the coils were laid in place and connected in a manner to give a winding arrangement as shown in the schematic in Figure 3.

It can be seen that the winding pattern repeats every six slots so that six slots produces two poles (one positive and one negative). Thus we have a magnet with a winding that requires 3 slots/pole. Being as there are three phases the magnet winding is of the type known as 1 slot/pole/phase. Furthermore, the winding is of the full chord type in that each coil spans three slots, or one pole pitch (reversal). This results in the current in the coils (which are of the same phase) that share slots traveling in the same direction, thus increasing the current density and the field strength.
After the coils were checked for continuity and possible shorting the core and winding assembly was again coated with Glyptal as a protective measure. A photograph of the finished magnet and cooling fin assembly is shown in Figure 4.

The entire magnet and fin assembly was then bolted to an aluminum plate which was in turn bolted to an aluminum clamp designed to attach to the overhead support bars on a Milwaukee type horizontal milling machine. The use of aluminum was to insure adequate magnetic decoupling from the steel machine. The magnet assembly is shown in place on the milling machine in the photographs in Figures 6 and 7.

The rail was constructed of 1/8-inch steel sheets in the same manner as the core of the magnet. Schematics of the rail are given in Figure 2. The slots in the rail are not filled with windings as in the magnet but instead an electrical path is provided by a piece of 1/4-inch thick aluminum which had slots milled in it so that it would fit over the teeth of the rail and into the slots. With the aluminum in place the rail is the linear equivalent of the rotor in a normal "squirrel cage" type rotary induction motor. Like the magnet, the rail is secured by means of a series of aluminum plates and blocks to ensure decoupling from the load cell to which it is attached.
It should be noted that the spacing of the teeth and slots in the rail is different from the spacing in the magnet. This is to ensure that (on a real vehicle) as the vehicle moves with respect to the rail the total amount of steel in the high portions (the teeth) of the magnet and the rail that is in alignment at any moment will stay relatively constant. If the spacings were the same for the magnet and rail there would be moments when all of the teeth were lined up with each other and there would be moments when all of the teeth on the magnet were lined up with slots on the rail. Obviously this would result in a great variation of both vertical and horizontal forces as a function of the vehicle’s linear position.

In rotary machines the similar problem is often countered by skewing the slots on the rotor so that they are not quite parallel with the slots in the stator. In this way each tooth on either the rotor or the stator is always lined up partially with a tooth and partially with a slot on the other. Thus the variation in the total lined up tooth area between the two as they move relative to each other is reduced.

In the experimental magnet and rail the average area of teeth that will be aligned at any one time is found by lining up (either with the actual device or better yet on paper) any arbitrary tooth pair and then computing the
amount of overlap in each succeeding tooth pair until the overlap pattern begins to repeat. In my magnet and rail five teeth on the magnet take up about the same amount of space as six teeth on the rail. The area obtained in this manner comes out to be 2.516 inches square.

The rail was mounted on an existing load cell capable of measuring force in both the vertical and horizontal directions. The cell was of the semiconductor strain gage type with internal compensation to offset the effects of moment inputs. Deflection was negligible for the forces I was dealing with as the cell was originally designed to withstand forces of up to 2000 lbs with little deflection and the maximum developed in my tests was on the order of 100 lbs. The load cell was constructed for use as a milling machine dynamometer so it had been designed to attach to a milling machine bed and no further fixture was needed for me to use it. A closeup view of the rail and the load cell is given by the photograph in Figure 5. The rail and cell are also shown in place on the milling machine in the photographs in Figures 6 and 7.

The cell was electrically excited with the carrier signal from a Sanborn model 321 dual-channel chart recorder which also recorded the vertical and horizontal forces. The recorder was calibrated by means of spring scales to give an output of .4 ounces per millimeter of
Figure 5- Rail and load cell assembly.
Figure 6- Closeup of magnet and rail mounted on milling machine.
pen deflection on the vertical scale and .6 ounces per millimeter on the horizontal scale when set on the most sensitive range. Attenuations of 1, 2, 5, 10, 20, 50, 100, and 200 were available. By using the "monitor output" of the recorder and displaying the output voltage on an oscilloscope it was possible to measure values down to about .1 ounce on either the vertical or horizontal scales.

During use the calibration of the cell was checked occasionally using the spring scales. If the readings began to differ by more than about 5% the recorder was recalibrated. Assuming that the spring scales were accurate to about 5% (they were initially checked with a set of weights and found to be within this range) it can be decided that the overall measuring accuracy is probably within a range of ±10% of the actual value. This value is valid only for those readings that were over about .3 ounces though. Below that level the amount of noise made it more difficult to ascertain the actual values. For these low level measurements I would figure the possible error to rise to ±25% or maybe even more at the very low (about .1 ounce) levels.

Zero drift on the recorder was found to be a problem on the most sensitive scale and necessitated repeated adjustment to retain accuracy.

To supply the magnet it was required to be able to
produce three-phase power at frequencies between 10 and 120 hertz. It was decided to start by producing a low power three-phase voltage signal and then using that to control the 3 Kepco Bipolar Power Amplifiers that were available. The amplifiers, of which was rated at ±72 volts at ±5 amps, the other 2 at ±36 volts at ±5 amps, are essentially very high power operational amplifiers and can be treated as such. They are equipped with internal variable feedback resistors and zero offset capabilities.

The amplifiers were set up to be voltage controlled but the voltage that was feedback to be compared with the input was not the output voltage as might normally be the case. Instead a 1 ohm precision resistor was placed in the circuit going to each phase's coils and the voltage across this resistor was fed back. In this way it was possible to ensure that the current waveform would be sinusoidal regardless of the impedance characteristics of the magnet itself. A schematic of the amplifier-magnet circuit is given in Figure 8.

The low power three-phase signal was obtained by first starting with the sinusoidal output from a Hewlett Packard Low Frequency Oscillator. The output was set to be about ±5 volts. This output was used as the A-phase input to the power amplifiers. This signal was also passed through
Figure 8- Schematic of power amplifier-magnet feedback circuit for one phase.
an RC circuit in which the resistance was adjusted to produce a voltage across the capacitor that was of magnitude \( \frac{1}{2} \) that of the input and with a phase lag of 60°. This signal was passed through an operational amplifier (part of a small analog computer that was available) with a gain of -2, producing a signal with magnitude equal to that of the input but with a phase lead of 120°. This signal was used as the B-phase input to the amplifiers. The A and B phase signals were also added and inverted using a second operational amplifier of gain -1. The output of this amplifier was a signal of the same magnitude as that of the A-phase but with a phase lead of 240°. This signal was used as the C-phase input. A schematic of the circuit used to do this and an analysis of it is included in the Appendix.

In practice the oscillator was first set on the desired frequency and the signal was displayed on an oscilloscope capable of simultaneously displaying three signals. The output of the first op-amp was also displayed and the resistor in the RC circuit was adjusted until the output was of the same magnitude as the oscillator signal. The output of the second op-amp was then also displayed and the gain of that op-amp was adjusted until its output was of the same magnitude as the other two signals. The signals could also be inspected to ensure that they were in-
deed each 120° apart.

Once the input to the power amplifiers were set in this way the voltage across each of the three precision resistors was displayed on a separate channel of the oscilloscope. The current in each phase could then be adjusted using the gain control on the power amplifier. Any D.C. component would be displayed and could be eliminated using the internal offset adjustment.

The oscillator, the power supplies, the analog computer, the oscilloscope, the chart recorder, and the magnet and rail are all visible in the photograph in Figure 7.

As mentioned before, both the magnet and the rail were securely attached respectively to the support bars and the table of a Milwaukee horizontal milling machine. The movable table allowed setting the gap with an accuracy of .001 inch and also provided a very rigid base for the tests. The zero gap reading on the milling machine was obtained by bringing the table up until contact was made between the magnet and rail as evidenced by the vertical measurement from the load cell. The rail was also adjusted in its mount in the load cell to ensure that the rail and magnet were parallel to each other.
4. Theoretical Analysis and Design.

What follows is that part of the analysis that was done before construction of the rail and magnet was begun. The results of the analysis were used as an aid in the design of the rail and magnet to insure both that the available power supplies would be properly matched to the magnet and that the expected forces would be within the measuring sensitivity of the available force transducer.

A. Determination of voltage requirements.

The inductance for a magnet in which the major reluctance is assumed to be due to an air gap is 2:

\[
L = \frac{N^2 u_o A_c}{l} \tag{1}
\]

Where:

- \( N \) = number turns of wire
- \( u_o \) = permeability of air = \( 4\pi \times 10^{-7} \)
- \( A_c \) = surface area of steel in opposing faces
- \( l \) = air gap

It is worth mentioning now that throughout this analysis all units are in the MKS (meters, kilograms, seconds) system.

\( A_c \), the surface area of steel in opposing faces, is an average value per coil found by taking the total area of steel in the primary which is lined up with steel in the secondary (as described in section 3) and multiplying
by 3/17. This is because there are 17 teeth in my primary and each coil spans 3 teeth. The total area, \( A_t \), is \( 0.001623 \, \text{m}^2 (2.516 \, \text{in}^2) \) resulting in an \( A_c \) of \( 0.0002864 \, \text{m}^2 \). The number of turns per coil is 200, as stated before. Substituting into equation 1 we get:

\[
L_{\text{coil}} = \frac{1.4396 \times 10^{-5}}{1} \text{ henries}
\]

The resistance per coil, \( R_{\text{coil}} \), is easily found by consulting published tables of wire resistance\(^3\). In this case the total length per coil is approximately 190 ft. of \#26 wire, which has a resistance of 45 ohms/1000 ft. Therefore we get:

\[
R_{\text{coil}} = 4.5 \text{ ohms}
\]

The total impedance per coil, \( X_{\text{coil}} \), is then:

\[
X_{\text{coil}} = \sqrt{R_{\text{coil}}^2 + (f2\pi L_{\text{coil}})^2}
\]  \hspace{1cm} (2)

Where \( f \) is the supply voltage frequency in hertz. The coils in each phase are connected in series so that the impedance per phase, \( X_{\text{phase}} \), is just five times the impedance for a single coil. For sinusoidal voltage, \( V \), the resulting current is just:

\[
I = \frac{V}{X_{\text{phase}}}
\]  \hspace{1cm} (3)

This equation was used to insure that at the higher frequencies and smaller gaps I would still be able to get
enough current to flow to provide measurable forces as predicted by the following analysis. (the power supplies were limited to ±36 volts)

B. Vertical force prediction.

For a given current I (amps), the magnetic flux density B (webers/m²) for a steel cored magnet with the major reluctance in an air gap l(m) is:

\[ B = \frac{NIu_0}{l} \]  \hspace{1cm} (4)

The attractive force F (newtons) between two pieces of steel with a flux density B between them is:

\[ F = \frac{AtB^2}{2u_0} \]  \hspace{1cm} (5)

Combining equations 4 and 5 we get the force equation that was used in vertical force predictions:

\[ F = \frac{AtN^2I^2u_0}{2l^2} \]  \hspace{1cm} (6)

Now, for a three-phase magnet wound as shown in Figure B, the current in adjacent coils of the same phase runs in the same direction in the slot that they share, thus increasing the current density in the slot and thereby increasing the flux density and the force. The outer coil of wire of each phase does not gain this advantage though. Also, due to the fact that the windings them-
selves are wound in layers, all of the coils could not possibly be in direct contact with the steel core. Thus the magnetic flux density would be less than that predicted by equation 4. After taking all of this into account I decided to leave equation 6 intact. For D.C. operation the current I is in D.C. amps and for A.C. operation the current used is RMS amps.

Although equation 6 may be off in predicting actual forces it should serve well to predict the dependence of the force developed on the current and gap. If the physical values for the experimental magnet are substituted into equation 6 the result is:

\[ F = 4.08 \times 10^{-4} \frac{I^2}{12} \text{ newtons} \]  
\[ = 1.47 \times 10^{-3} \frac{I^2}{12} \text{ ounces} \]

Values obtained from this equation are plotted in Figure 10 along with force data from D.C. measurements.

C. Horizontal force prediction.

When a nonmagnetic conductor is moved through a perpendicular magnetic field at a speed \( V_{el} \) a voltage is induced in the conductor according to Faraday's Law. This voltage, \( V_{ind} \), is found by the relation:

\[ V_{ind} = BV_{el}w \]  

Where \( w \) is the length of the conductor in the field in
the direction perpendicular to both the magnetic field
and the direction in which the conductor is moving.

For this thesis all tests were made in a situation
in which the magnet and rail were held stationary with
respect to each other. What does move in this case is
not the conductor but the magnetic field. In a three-
phase winding arrangement as shown in Figure 3 a trav-
elling magnetic wave is set up which has essentially a
sinusoidal shape and which will have a distance between
one peak and the following trough of \( p \), where \( p \) is the
pole pitch.\(^5\) The pole pitch is the distance from the
center of a positive coil of one phase to the center of
the adjacent negative coil of the same phase.

I will thus redefine \( V_{el} \) to be the velocity at which
the magnetic traveling wave passes the conductor. For
the case in which both the conductor and the magnet are
standing still this will be the speed of the traveling
wave itself. Thus:

\[
V_{el} = 2pf
\]

In this motor the conductors are in the form of an
aluminum "squirrel cage" type set up (Figure 2). The vol-
tage set up in one of the crossbars will cause a current
to flow. It is necessary to decide on the path the cur-
rent will follow in order to determine the impedance of
the path. At first one might assume that the current
would flow in the shortest possible path, from one cross-
bar through the shorting bars to the adjacent crossbar on
either side. However, the distribution of induced volt-
ages in the crossbars is proportional to the distribution
of the magnetic flux density (which is sinusoidal with a
peak to next peak length of 2p). The current must flow
between those crossbars with opposite induced voltages.
This is not between adjacent bars but is between those
that are separated by a distance p. The spacing of the
crossbars on my secondary is such that the current will
have to follow the path shown in Figure 9.

The total resistance to the current flow will be the
sum of the resistances of the crossbars and the shorting
bars. The resistance of each section is:

\[ R = \frac{L \rho}{A} \]  \hspace{1cm} (10)

Where \( \rho \) is the resistivity of the material (aluminum), \( L \)
is the length of the path, and \( A \) is the cross sectional
area of the path through that section. For the path shown
the total resistance is found to be \( 9.2 \times 10^{-5} \) ohms.

Now we can use equation 1 again to find the inductance
of the secondary. Plugging in values we obtain:

\[ L_{sec} = \frac{3.6 \times 10^{-10}}{1} \text{ henries} \]  \hspace{1cm} (11)
Figure 9 - Sample current path in "squirrel cage" secondary.
These values of resistance and inductance are for a single coil. The total secondary impedance per coil can be found by using equation 2.

For the conductor moving in the magnetic field, as mentioned before, if a current $I_s$ flows in it (due to the induced voltage) the magnetic field will exert a force on it equal to:

$$F = I_s wB$$  \hspace{1cm} (12)

The current will be related to the induced voltage by the relationship:

$$I_s = \frac{V_{se}}{X} = \frac{BVelw}{X}$$  \hspace{1cm} (13)

Where $X$ is the impedance of the secondary coils.

Each coil is made up of essentially four parts— the two shorting bars and the two crossbars that make up each current loop. The two shorting bars are parallel to the direction of motion of the magnetic field and thus do not contribute anything to the horizontal force. The two crossbars however are perpendicular to the direction of motion and will thus each contribute some force. That the two bars each contribute a force in the same direction despite the fact that the current in the two bars of each loop must travel in opposite directions is due to the further fact that when one bar is located in a section of "positive" magnetic field the other bar in the pair is located
in a section of "negative" magnetic field. The total force contributed per coil will thus be twice the value obtained by combining equations 12 and 13. The result is:

\[ F_c = \frac{2B^2wV_{el}}{X} \]  
(14)

The total force developed for the whole magnet will be the value just given times the number of coils per phase. For this magnet there are five coils per phase. The final equation for the force in the horizontal direction is found using equations 14, 9, 4, and 2 to be:

\[ F_h = \frac{10N^2I^2u_0^2w^22pf}{12\sqrt{R^2 + \frac{L_0se(f)^2(2\pi)^2}{12}}} \]  
(15)

While \( N \) and \( I \) refer to the magnet, \( l \) is the magnet to rail gap, and the rest of the variables refer to the rail. If values given before are substituted into this equation the result is:

\[ F_h = \frac{3.74 \times 10^{-11} fI^2}{12\sqrt{8.46 \times 10^{-9} + 5.1076 \times 10^{-10}f^2}} \]  
(16)

Where \( F_h \) is in newtons. To transform to ounces just multiply by a factor of 3.6. A comparison of values obtained from this equation for a current of .5 amps versus experimental data for a current of .5 amps is given in Figure 32.
Experimental Results and Discussion.

The results of the vertical and horizontal force measurements are given in Figures 10 through 33. I will run through all of these and try to point up what I feel are the most important features.

The plots are arranged in sets of four (except for the D.C. measurements for which there are only two). The first two plots in each group are vertical force measurements. They show the same data but differ in that in one the data is plotted versus gap and in the other the data is plotted versus current. This is done to show the dependence on each of these two variables. The second two plots in each group are the same as the first two except that they show horizontal force measurements. Each set of four is taken at a different magnet current frequency. The frequencies covered are 10, 30, 60, 90, and 120 hertz.

Figure 10 plots vertical force for D.C. currents vs. gap. It also contains values predicted by equation 7. The prediction appears to be fairly good except that the slope of the experimental data is a bit less than the inverse square law slope (-2 on log-log paper) predicted by equation 7. Figure 11 shows the same thing vs. current. It can be seen that the experimental values closely follow the square law predicted by equation 7.

Figures 12 through 31 are composed of the sets of four
plots per frequency as stated before. It can be seen that throughout all of them the square law current reliance holds up pretty well. Looking at the plots of vertical force versus gap though, one discovers that the slope of the experimental data begins to drop off more and more at the lower gaps as the frequency increases. This can be easily seen in Figure 33 in which I have replotted the force data for a current of .5 amps (.354 RMS) at the various frequencies at which measurements were taken. As can be seen, at larger gap settings the slopes appear to approach the same asymptote as the slope of the D.C. readings. This implies that some extra effect is coming into play at the smaller gaps and that this effect becomes more pronounced as the frequency increases.

Even more dramatic is the effect that increasing frequency has on the horizontal force measurements. Figure 32 gives a comparison of experimentally measured values and theoretical values for a current of .5 amps versus gap. As the frequency increases, the deviation, both in slope and in actual value, becomes larger and larger at the lower gaps. At the higher gaps the slopes begin to agree more and more and the predicted values get closer. Obviously whatever is affecting the vertical forces is also effecting the horizontal forces. I have one possible explanation for this effect, which I will now present.
This motor is of the induction type which means that it derives its force by means of currents induced in the secondary. These currents set up a magnetic field that will oppose the field set up by the primary coils. The analysis that was previously done does not take this secondary field into account. In the analysis that follows I will try to get some qualitative idea of the relative size of this secondary field as a function of frequency and gap to see if its characteristics could possibly explain the results measured.

Using equation 13 for the current in the secondary and using equation 4 for the magnetic flux density for a steel-cored magnet with an air gap we can get a qualitative equation that gives us a value for the secondary field, $B_s$ that is set up. I say qualitative rather than quantitative because I would say now that equation 4 probably does not really hold for this situation where there is another field already existing in the gap. The equation obtained in this manner is:

$$B_s = \frac{B_p^2 pf w u_o N}{1 \times} \quad (17)$$

Where $B_p$ is the primary field flux density, $N$ is the number of turns of conductor in the secondary coils (1 for the case of a squirrel cage rotor), and $X$ is the impedance of the secondary. Substituting in the values of the variables
for the test magnet and rail we can obtain the following equation for the ratio of \( B_s \) to \( B_p \):

\[
\frac{B_s}{B_p} = 3.66 \times 10^{-9} \frac{f}{1 \sqrt{8.46 \times 10^{-9} + 5.1076 \times 10^{-18} f^2}}
\]  

(13)

Because both the vertical and horizontal force equations have factors of \( B^2 \) in them I have squared the values produced by this equation. These results are plotted in Figure 34. As can be seen, the relative size of \( B_s \) to \( B_p \) increases with frequency and decreases with increasing gap. This fits in well with trying to explain the experimental results.

As a matter of interest I have plotted the ratio of horizontal to vertical forces for the different frequencies. This appears in Figure 35. The drawn in line is merely an indication of the general trend of the values, it does not presume to be a curve fit. It can be seen that especially at the larger gaps and higher frequencies that the amount of propulsion generated can be a significant proportion of the vertical force.
Figure 10 - D.C. current

$F_v$ vs. gap, plus theoretical results.
Figure 11- D.C. current

$F_v$ vs. D.C. current, plus theoretical results.
Figure 12- 10 hertz

$F_v$ vs. gap.
Figure 14- 10 hertz

$F_h$ vs. gap.
$F_h$ (ounces)

Figure 15 - 10 hertz

$F_h$ vs. peak current.

Current (Peak amps)
Figure 16- 30 hertz

Fv vs. gap.
Figure 17 - 30 hertz

$F_V$ vs. peak current.
Figure 18 - 30 hertz

$F_h$ vs. gap.
Figure 19-30 hertz
$F_h$ vs. peak current.
Figure 20 - 60 hertz

$F_v$ vs. gap.
Figure 21 - 60 hertz

$F_v$ vs. peak current.
Figure 23- 60 hertz

$F_h$ vs. peak current.
Figure 24- 90 hertz

$F_v$ vs. gap.
Figure 25—90 hertz

$F_y$ vs. peak current.
Figure 26 - 90 hertz

$F_h$ vs. gap.

$F_h$ (ounces)

$T = 0.8$

$T = 0.5$

$T = 0.25$

gap (inches)
Figure 27- 90 hertz

$F_h$ vs. peak current.
Figure 28 - 120 hertz

$F_V$ vs. gap.
Figure 29- 120 hertz

$F_V$ vs. peak current.
Figure 30 - 120 hertz

$F_h$ vs. gap.
Figure 31- 120 hertz
$F_h$ vs. peak current.
Figure 32- .5 amps peak current, comparison of experimental and theoretical values of $F_h$ at different frequencies vs. gap.
Figure 33 - .5 amps peak (.354 RMS) current, comparison of values of $F_v$ vs. gap at different frequencies.
Figure 34- values of \( \left( \frac{B_S}{B_P} \right)^2 \)
predicted by equation 18.
Figure 35 - ratio of horizontal to vertical force vs. gap at several frequencies.
6. **Conclusions and Recommendations.**

From the results presented in this thesis it is possible to conclude that vertical forces of the magnitude needed for a full size suspension system could be generated by a combined lift and propulsion magnet. For example, by using the equations given in section 4 it can be predicted that a scaled up version of the test magnet 3 feet long by 5 inches wide could support some 1100 lbs at a gap of .2 inches while drawing a current of 10 amperes.

At the higher frequencies tested the ratio of the horizontal to vertical forces was in the neighborhood of a value of unity or greater. These two results together lead to a conclusion that a combined suspension and propulsion system of the type proposed in this thesis is definitely feasible at least for low speed vehicles.

From the comparison of the results presented in this thesis with the predictions presented it is possible to conclude that the simple analysis done here would be a good start a predicting forces, but that a more thorough analysis taking secondary induced fields into account would have to be done to gain a greater predicting accuracy.

One thing that should be done in future work is to use a search coil to determine the actual flux density in the magnet gap. This would serve to either verify or suggest alterations to the analysis presented here. Also, in future
work where it will be desired to operate at larger gaps at
times it will be necessary to either obtain power supplies
capable of delivering much higher voltages or to obtain a
load cell of much greater sensitivity.

It would be profitable to repeat these experiments
using different rail styles. The type used here, the
"squirrel cage" kind, was chosen because it can be oper-
ated at high slip frequencies. This was necessary as our
test set-up was stationary. Other rail types, like the
variable reluctance or the hysteresis types, use much
simpler rails but are mainly synchronous machines. To
test these types would require the construction of a turn-
table type experimental set-up to allow the rail to move
relative to the magnet. If this was done it would prob-
ably be advisable to mount the magnet on the load cell so
that it is not necessary to instrument the turntable.
7. Acknowledgements.

I would like to acknowledge the guidance and assistance of my thesis advisor, Professor David Wormley. I would also like to thank Tiny Calogero for the use of his milling machine for the duration of the tests and Sam Marcolongo for his advise in machining the magnet and rail assemblies.
8. **References.**


4. Same as #3 above.

APPENDIX

For the circuit shown in Figure 36:

If, \[ \phi_A = K \sin(w_0t) \]

Then, \[ V_C = \frac{K}{\sqrt{1 + (w_0RC)^2}} \sin(w_0t - \tan^{-1}(w_0RC)) \]

If we adjust R so that \( w_0RC = \tan 60^\circ = 1.732 \) \( V_C \) will be:

\[ V_C = \frac{K}{2} \sin(w_0t - 60) \]

This voltage is fed into op-amp 1 which has a gain of -2. The resultant voltage, \( \phi_B \), is thus:

\[ \phi_B = -K \sin(w_0t - 60) \]

Using the relationship \( \sin x = -\sin (x + 180) \) we get:

\[ \phi_B = K \sin(w_0t + 120) \]

Which is what was wanted for the B phase.

\( R_2 \) is adjusted so that op-amp 2 is an inverting summer with unity gain. The A and B phases are added together, giving the following result:

\[ \phi_C = -K \sin(w_0t) + \sin(w_0t + 120) \]

Using a trigonometric identity for the sum of two sines we obtain:

\[ \phi_C = -K \left( 2 \sin \left( \frac{1}{2}(2w_0t + 120) \right) \cos \left( \frac{1}{2}(w_0t + 120 - w_0t) \right) \right) \]

\[ = -K \left( 2 \sin (w_0t + 60) \cos 60 \right) \]

\[ \phi_C = -K \sin(w_0t + 60) = K \sin(w_0t + 240) \]

Which is what was needed for the C phase.

-68-
Figure 36- Schematic of phase-splitting circuit.