THEORY AND APPLICATIONS OF REAR-WHEEL STEERING IN THE DESIGN OF MAN-POWERED LAND VEHICLES

by

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Submitted to the Department of Mechanical Engineering on May 20, 1977 in partial fulfillment of the requirements for the Degree of Bachelor of Science.

ABSTRACT

The goal of this project was to develop an improved bicycle design. Various riding positions were studied, and the semi-recumbent position was selected as being the most suitable. A theoretical analysis of bicycle stability resulted in the design of a bicycle with rear-wheel steering and front-wheel drive.

A second application of rear-wheel steering and semi-recumbent rider position was the design of a man-powered utility tractor for use on a farm. A working prototype is currently under construction, fabricated from tubular aluminum alloy, and heli-arc welded together.

INTRODUCTION

Problem Statement

I have been an avid bicyclist for some years now. In that time I have found much to criticize in the modern day touring bicycle, in terms of comfort, safety, and mechanical efficiency. Furthermore, the design is a poor one in view of its inability to adequately complement the human body's power output.

Statement of Purpose

My purpose is to develop a bicycle with characteristics tailored to the requirements of the long-distance tourer. These include a small frontal profile (and hence less air-resistance), protection from the elements, and improved safety and comfort. This results in a less fatiguing ride typified by greater range and speed.

Project History

My interest led to a U.R.O.P.-funded project started during the summer of 1974, aimed at developing an improved bicycle design. My observations of various unorthodox designs showed one thing in common - a complete lack of concern for the rider. Designers of man-powered equipment have traditionally designed the machine first, and then attempted to "deform" the operator into or onto it. Such schemes have proven grossly inefficient.

Design Philosophy

These facts shaped my design philosophy, which states that the physiological requirements of the "human engine" are the most important design criteria. Therefore, my work proceeded from a physiological viewpoint. Once an optimal riding position was determined, I set out to design a machine to best incorporate it.

Study of Riding Positions

Air resistance is one of the greatest deterrents against a bicyclist. I therefore avoided the traditional upright riding position with its inherently large frontal area. Instead, the recumbent mode of riding represented an attractive alternative position. (see Figure 1).

I began a search of the relevant literature and uncovered a wealth of information.

P.O. Astrand and B. Saltin (1961), in their paper on oxygen uptake of muscular activity, found that pedalling in the near horizontal position was only about 86% as effective, from the point of view of efficiency of muscle usage, as the normal upright position. This is a poor riding position, since to be practical, a shoulder rest must be used to enable the rider to transmit any appreciable force to the pedals. Such a condition is equivalent to that imposed by standing with a weight on the shoulders, which results in tiredness, although no weight is actually moved.

The recumbent position is also poor in terms of respiration. It favors the development of periodic breathing, and therefore of anoxaemia, a reduction of the normal amount of oxygen in the blood. The cause of this phenomena is not completely clear, but is probably due to the increased resistance thrown on the dia-

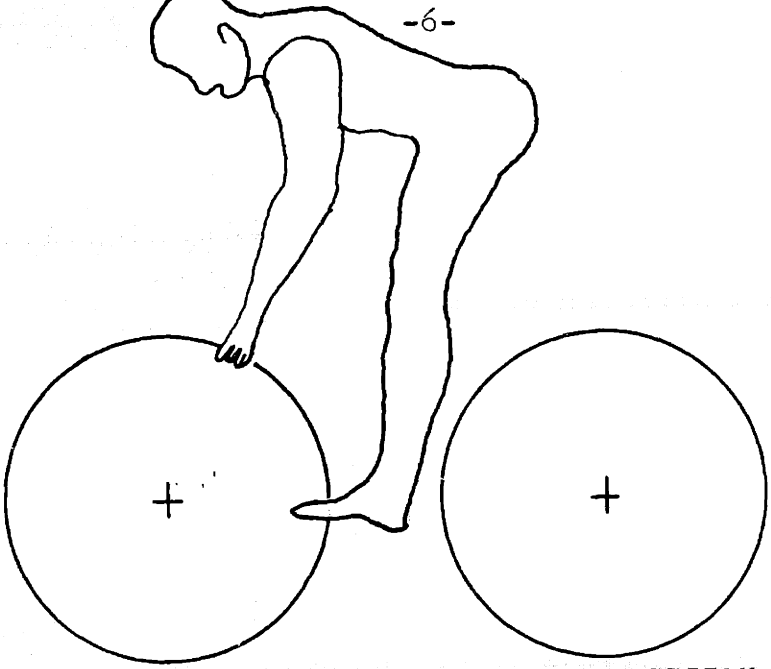


FIGURE 1A - CONVENTIONAL RIDING POSITION

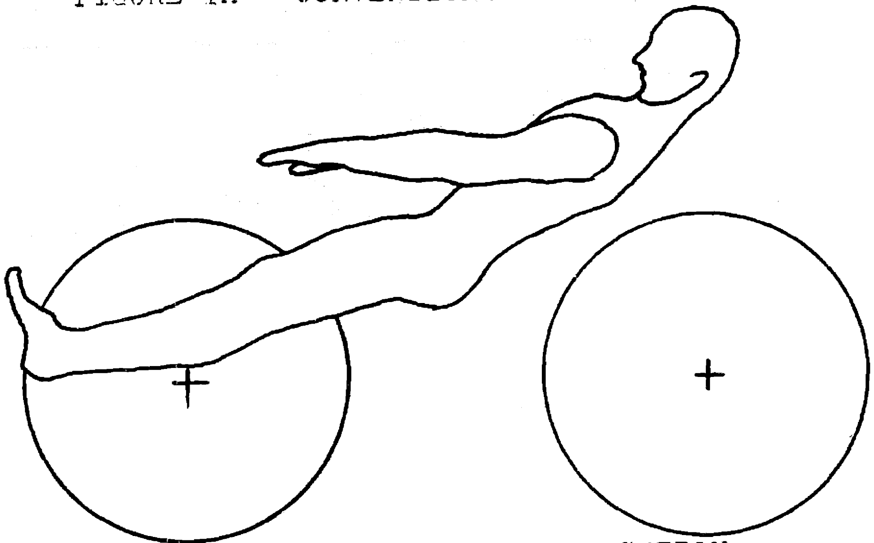


FIGURE 1B - RECUMBENT POSITION

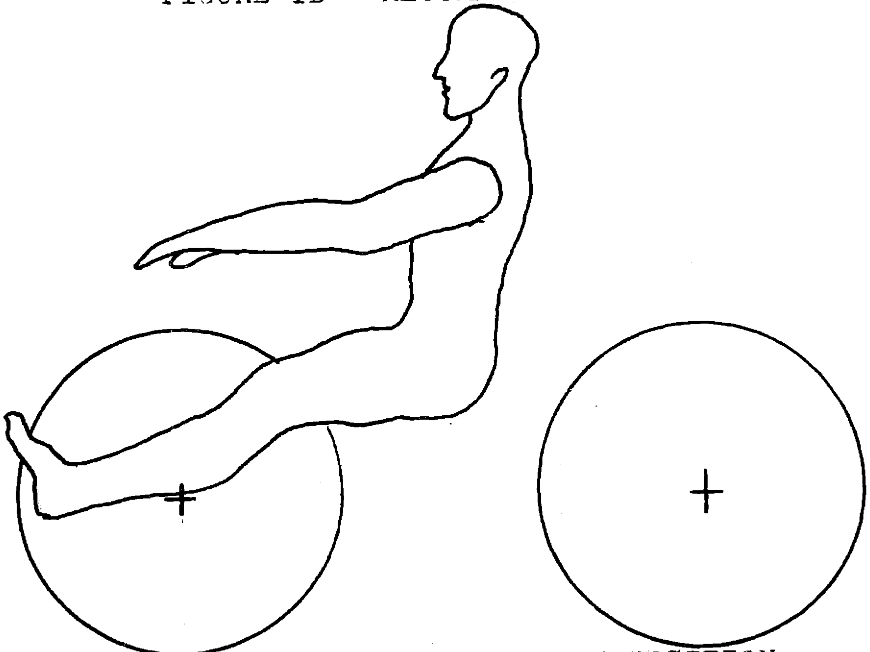


FIGURE 1C - SEMI-RECUMBENT POSITION

phragm due to the weight of the internal organs.

Although early riders of recumbent bicycles (1930's) complained of back and knee strain, these same riders broke records for short track distances. This is significant, considering the inefficiencies inherent in the recumbent position. Apparently the decreased air resistance of the lower riding position outweighed the drawbacks of the less advantageous use of the leg muscles, for at least short periods of time.

I reasoned that a semi-recumbent position would do away with most of the problems connected with the recumbent position. It must be payed for in increased air resistance as compared to the recumbent position, but still remains much less so than conventional riding positions.

In the semi-recumbent position, the rider's back is vertical or nearly so, and the legs horizontal or slightly downward. This is similar to rotating the conventional riding position about sixty degrees.

In this position, respiration is as efficient as in a conventional position. In addition, knee and back strain are minimized. With a properly constructed seat and lower back support, the rider can push with a great deal of force and require less energy to maintain a steadily seated position even under difficult riding conditions. William Bruml (1971), in his thesis on "Force Optimization of a Bicycle Pedal Path", concluded that "the distance between the foot and the hip has a greater effect on the force exerted than does the angle between the leg and the torso."

Harrison (1970) experimented with different body positions and found a 10% increase in power output for a "recumbent-rowing" body position over that of a conventional riding position.

Experiments by Kyle and Mastropaolo (1975) showed that the semi-recumbent position was 98% as efficient as conventional positions, but due to its smaller frontal profile required only 76% of the power for a given speed.

Mr. Dan Henry of Long Island, New York, built a semi-recumbent over fifteen years ago, and has traveled over 100,000 miles on it. He claims that the semi-recumbent design is highly efficient for long-distance travel (see Figure 2).

These facts clearly show the advantages of a semi-recumbent cycling mode, but what muscle groups should be used? The muscles of the legs, thighs, and lower back are generally utilized as the prime source of human power. What effect do the hands have when used in conjunction with the legs? R.B. Andrews (1966) found that, rather surprisingly, two muscle activities could be carried out simultaneously with a small gain in total power output for a given oxygen consumption. However, when one realizes the acrobatics necessary for the rider to work hand cranks and steer simultaneously, the advantages are not at all obvious. E.S. Krendel (1960) found that although simultaneous usage of hands and legs has value in some situations, the energy losses of a bicycle rider in trying to balance, more than offset any value in using hand cranks.

I concluded that for long-distance travel, a semi-recumbent riding position was the most feasible, due to its effective use of the rider's energy and small frontal profile.

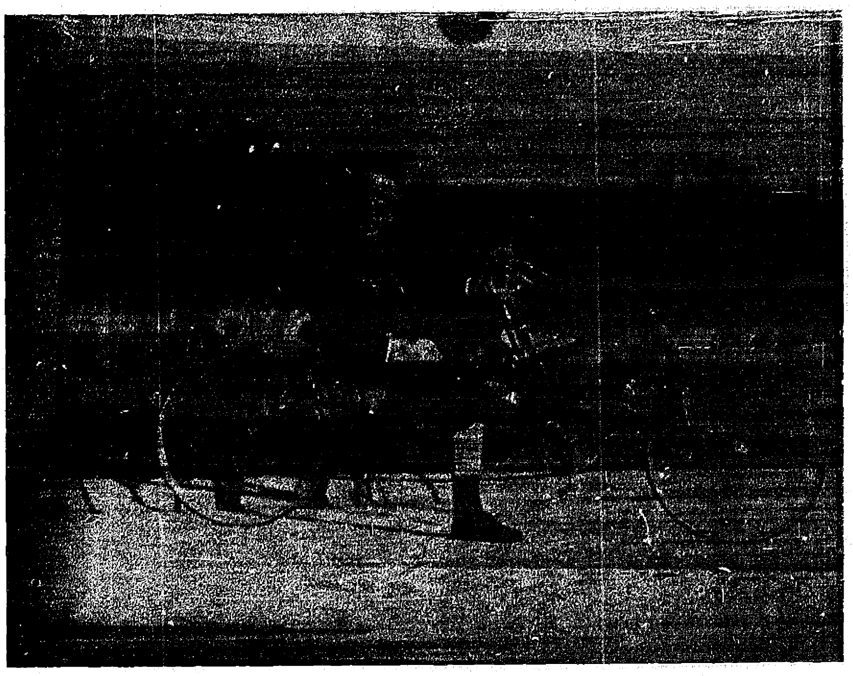


FIGURE 2 - DAN HENRY'S SEMI-RECUMBENT BICYCLE

INTENTIONAL DUPLICATE EXPOSURE

FIGURE 2 - DAN HENRY'S SEMI-RECUMBENT BICYCLE

Application #1 - Design of Bicycle

My job, then, was to design a machine which would effectively complement the semi-recumbent position. For stability, it was desirable to bring the rider's center of gravity as low to the ground as possible. The geometry of the pedals prevented the pedal-crank axle from being lower than 15 inches above the ground. The seat had to be at least 3½ inches higher to insure adequate blood circulation. This is a very important point, as born out by the example of F. Wilkie (1974), in his first attempt at building a recumbent bicycle, "The Green Planet Special I". In that case, the crank-axle height was six inches higher than the seat. As a result, riding for even short periods became tiring due to the obvious impairment of circulation (see Figure 3).

A conventional wheelbase was employed in order to avoid the problems that early recumbents faced. With their "man in the middle", these vehicles had very long wheelbases,(see Figure 4). Consequently, the light loading on the front wheel caused it to leave the ground periodically, particularly on an incline.

A final consideration was the placement of the rider. For optimal performance, 55% of the rider's weight should be carried by the front wheel.

A serious design problem arose. How can the rider be placed without seriously interfering with the steering of the front wheel? Fred Wilkie, in building his second recumbent, "The Green Planet Special II", partially solved this problem by bringing the rider forward, raising the seat somewhat, and using a small, sixteen-inch diameter front wheel (see Figure 5). This caused

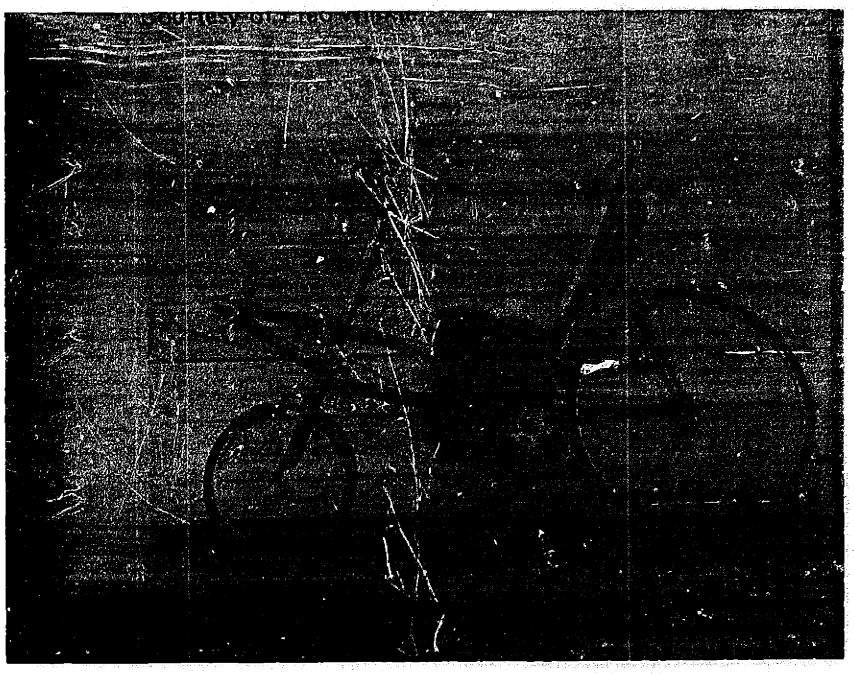
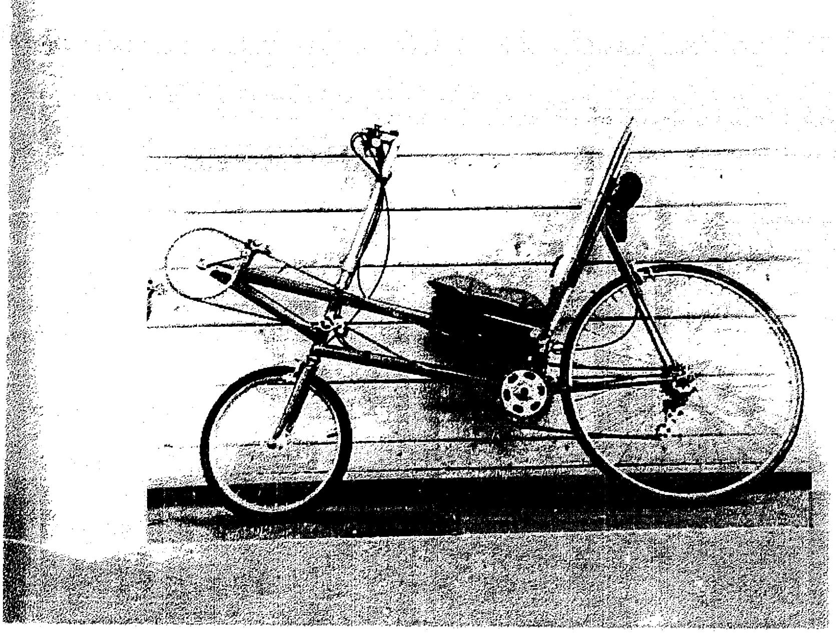


FIGURE 3 - FRED WILKIE'S GREEN PLANET SPECIAL I

Courtesy of Fred Willkie.



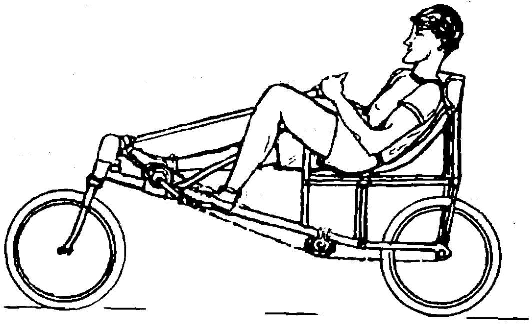
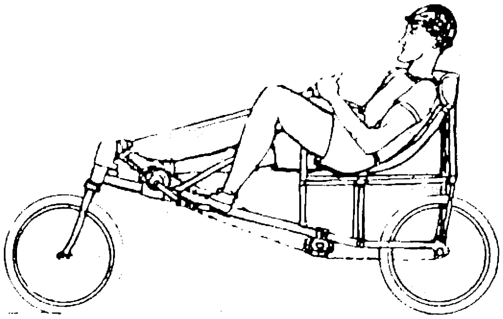


FIGURE 4 - EARLY RECUMBENT DESIGN

FIGURE 5 - GREEN PLANET SPECIAL II (In Foreground)



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problems with heavy front-wheel loading, and forced the rider into a physiologically inefficient position. In addition, the small front wheel had greater rolling resistance.

My solution was to use rear-wheel steering and front-wheel drive.

ANALYSIS OF REAR-WHEEL STEERING

Very little information appears in the literature about rear-wheel steering of land vehicles. I had to understand the principles behind the steering and stability of bicycles.

R.A. Wilson-Jones (1951) showed that if a bicycle is to be inherently stable, it must be designed so that any tendency to fall to either side will cause it to steer to that side, so that a horizontal centrifugal force will maintain equilibrium. When the bicycle is restored to the upright position the steering must automatically return to the straight-ahead position. What causes the bicycle to steer into the direction of a fall? It is the weight carried on the front tire due to the bicycle and rider. Figure 6A shows the conditions on the front wheel when the bicycle is falling over but hasn't yet started to turn in the required direction. There is no centrifugal force and the weight is directed vertically downwards so that the reaction of the ground on the tire is vertically upwards. This reaction $\vec{r}$, is resolved into two forces - $\overrightarrow{ab}$ normal to the plane of the wheel, and $\overrightarrow{cb}$ in the plane of the wheel. The force $\overrightarrow{ab}$ acts at a point inside the contact area of the tire and ground, and therefore at a point behind the steering axis. As a result, force $\overrightarrow{ab}$ tends

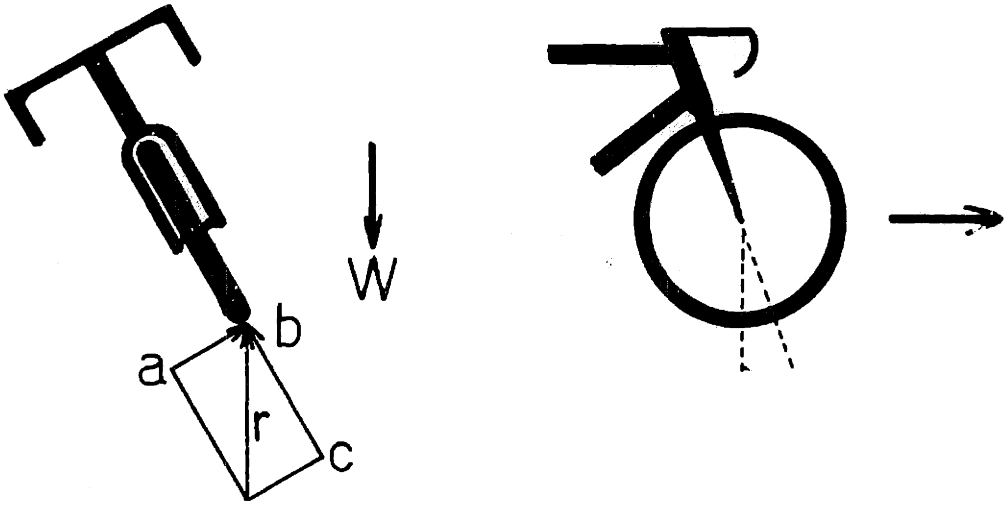

FIGURE 6A - BICYCLE FALLING, STEERING STRAIGHT

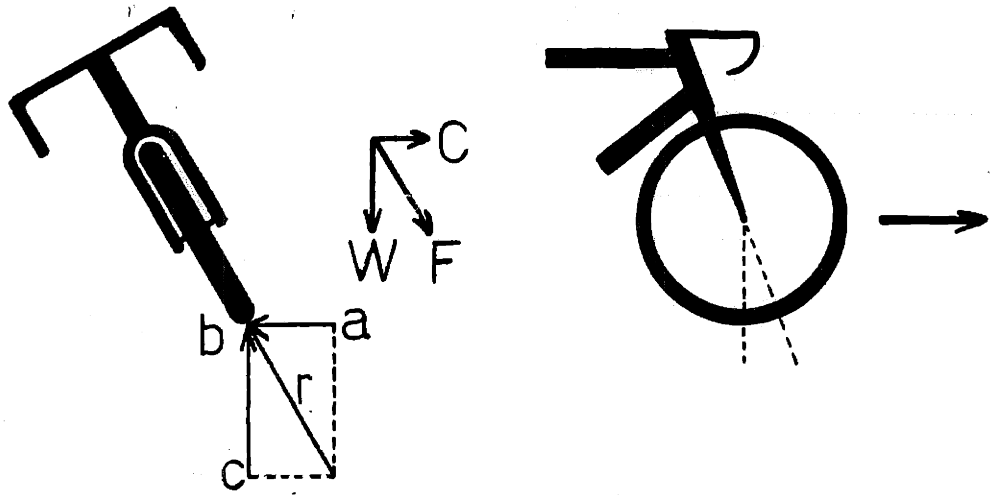

FIGURE 6B - BICYCLE FALLING, STEERING INTO A CURVED PATH

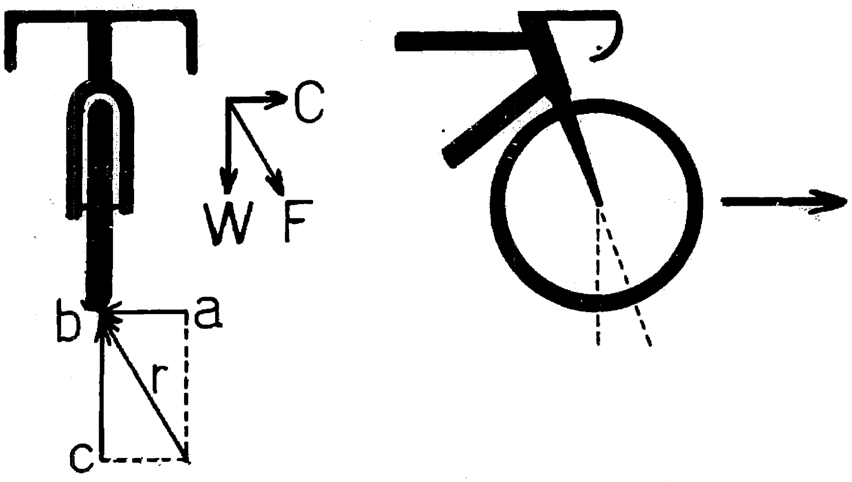

FIGURE 6C - BICYCLE RETURNED TO UPRIGHT POSITION to turn the steering in the required direction.

Once the steering has been turned and the bicycle is moving round a curved path, the conditions at the point of contact are completely changed (see Figure 6B). The resultant of the weight on the front wheel plus centrifugal force now acts in the plane of the wheel so that the reaction of the ground on the tire is also in the plane of the wheel. This reaction is resolved into a vertical force $\overrightarrow{cb}$ and a horizontal force $\overrightarrow{ab}$, which provides the necessary cornering force. There is now, however, no reaction normal to the plane of the wheel, and therefore no tendency for the wheel to either turn further or to return to the straight-ahead position.

What then causes the bicycle to return to the upright position? Generally the rider will lean in the appropriate direction. In addition, gyrostatic action appears to be responsible; as soon as the steering turns to one side, the turning of the front wheel about the steering axis creates a gyrostatic torque on the wheel, which tends to return to the upright position.

Once the bicycle is upright but is still following a curved path, other conditions exist (see Figure 6C). The resultant of weight plus centrifugal force is now at an angle to the plane of the wheel and so is the reaction of the ground on the tire. Force $\overrightarrow{ab}$ is now normal to the plane of the wheel and since it acts at a point behind the steering axis it tends to return the steering to the straight-ahead position.

The trail of the front wheel converts the side force into a turning moment. It is therefore a key factor in maintaining the stability of a bicycle. Trail is the distance between the point of contact of the front wheel and the head angle as projected to the ground (see Figure 7). As the trail is increased, stability (and therefore sluggishness) increases (Jones, 1970). This is true of all conventional bicycles (see Figure 8).

I feel that this theory would equally apply to a rear-wheel steered bicycle. The physical situation is similar for the two configurations, with the exception of steering orientation. In a rear-wheel steered bicycle the rear wheel must steer in the opposite direction. (In other words, to initiate a right hand turn, the rear wheel must turn to the left).

In light of the theory this could be accomplished by reversing the trail, and therefore having the steering axis fall <u>behind</u> the ground contact point (see Figure 9).

Unfortunately, the physics behind the steering of ground vehicles is not that simple. Other factors must be taken into account; in particular, that of slip angles. (Olley, 1947).

Consider a front-wheel steered bicycle about to make a turn (see Figure 10A). At A, the front wheel is supposed to be suddenly turned to the steering angle shown, and subsequently held in that position. The front tire experiences a side force $\vec{F}_1$ due to the slip angle. The effect of this force is to reduce the turning moment, which occurs whenever the bicycle leans into a turn. An angular acceleration occurs in the horizontal plane of the bicycle.

Since in general the radius of gyration k, of the rider-bicycle combination is less than the square root of (ab), the

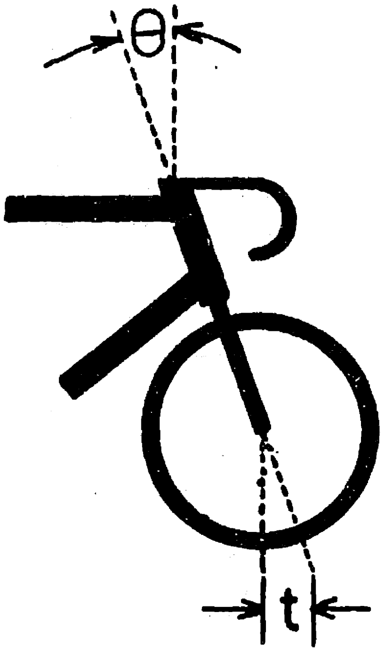

FIGURE 7 - DEFINITION OF TRAIL

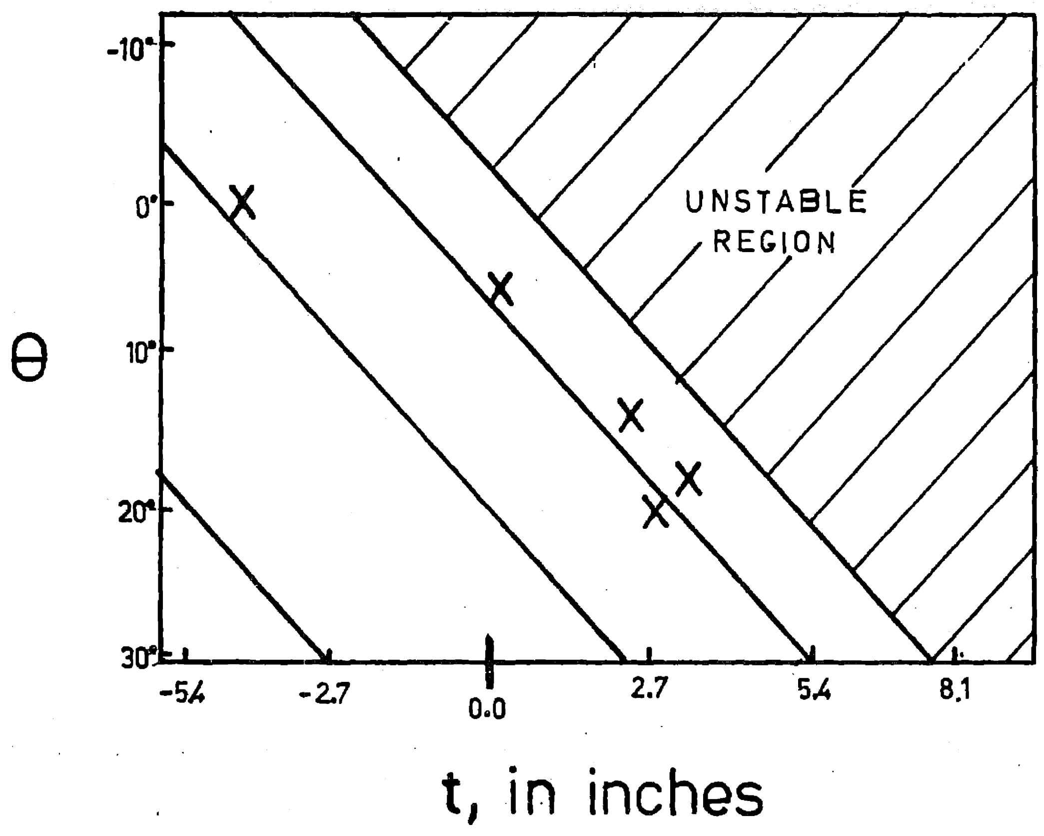

FIGURE 8 - EFFECT OF TRAIL LENGTH
(X Points Represent Conventional Bicycles)

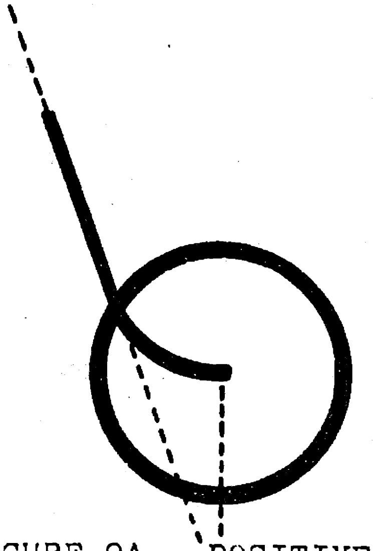

FIGURE 9A - POSITIVE TRAIL

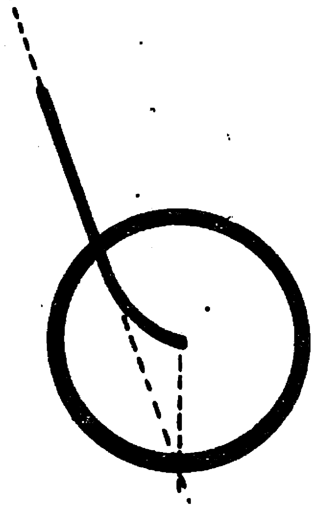

FIGURE 9B - NEUTRAL TRAIL

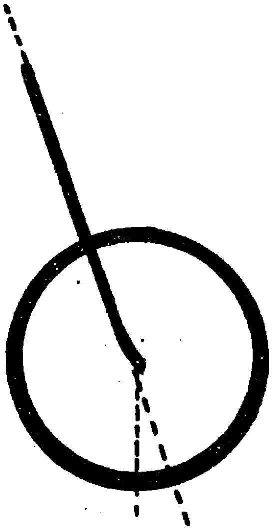

FIGURE 9C - NEGATIVE TRAIL

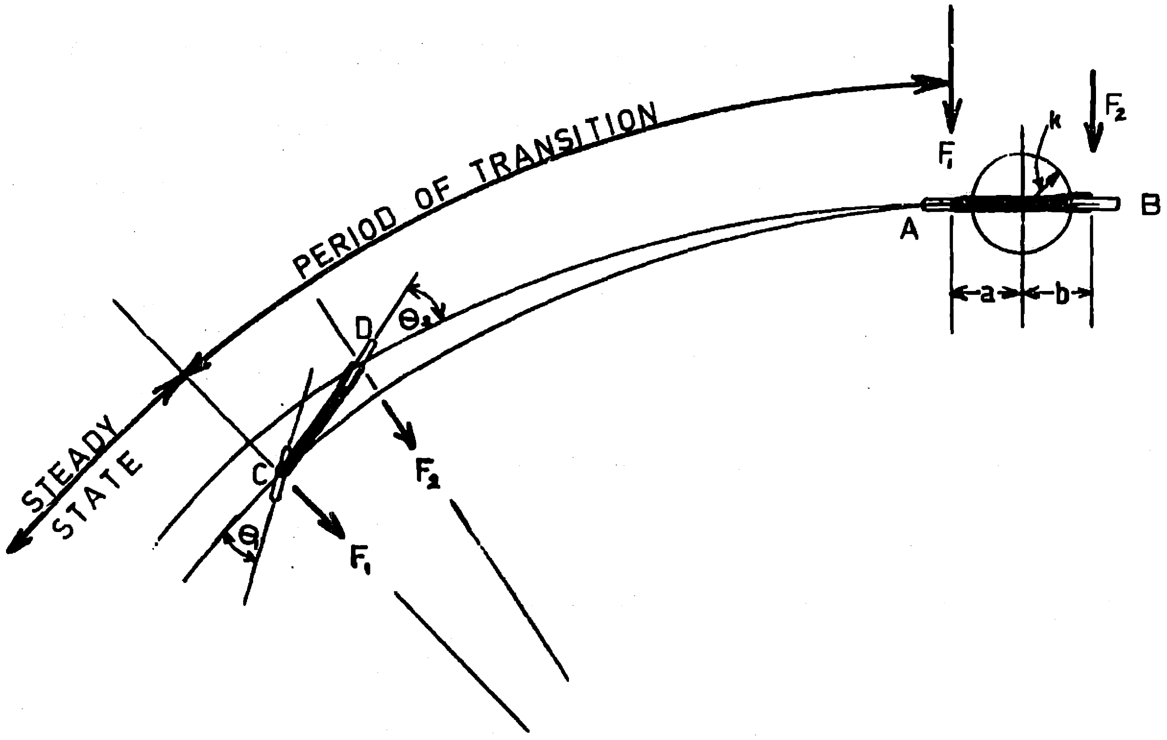

FIGURE 10A - CAR ENTERING A TURN

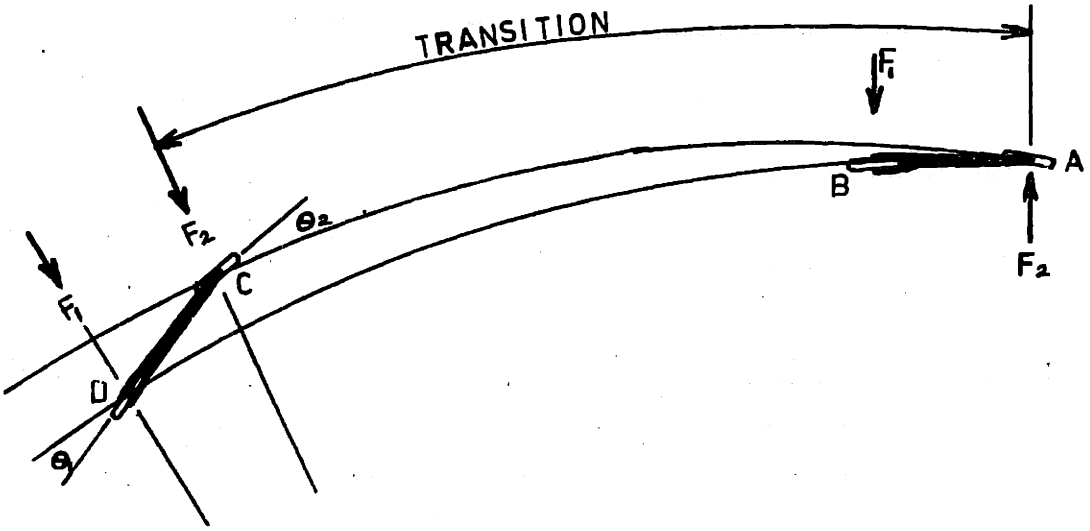

FIGURE 10B - CAR ENTERING A TURN, REAR STEERING rear wheel also experiences a side force $\vec{F}_2$ in the same direction. Force $\vec{F}_2$ increases as the rear end follows the arc $\overset{\frown}{BD}$, until at about point D, the force reaches its ultimate value of $F_1(a/b)$. At this point, angular acceleration ceases, the transition period is over, and the bicycle proceeds around the curved path in a condition of equilibrium and nosed in at an angle $\angle\theta_1$.

In the case of rear-wheel steering the situation is critically different. Consider a rear-wheel steered bicycle about to make a turn (see Figure 10B). At A, the rear wheel is turned to a steering angle which is maintained. At first the rear wheel is accelerated outwardly by force $\vec{F}_2$, which initially directs the rear tire to the right. One effect of this force is to <u>increase</u> the turning moment caused by the bicycles' leaning into the turn. Then, the rear end moves outwardly along arc $\overset{\frown}{AC}$. The front wheel, with the body of the bicycle, slowly turns into position until at D it has attained slip-angle θ_1 , sufficient to produce the necessary centripetal force $\vec{F}_1$. Since the front wheel can't be steered, the attitude angle of the rear-wheel steered bicycle must be as great as that of the front-steering bicycle <u>plus the steering angle</u>. It is therefore nosed in much more on a turn. (All things being equal, rear-wheel steering bicycles typically have a 30% smaller turning radius than conventional front-steering bicycles). The force on the rear tire $\vec{F}_2$, is now directed inwards, so that somewhere between A and C the side force on the rear tire has reversed its direction. Its effect now is to <u>reduce</u> the turning moment on the tire.

This means that between A and C the rider has to make leaning and steering corrections, in order to compensate. Therefore a rear-wheel steered bicycle should have little or no trail, in order to minimize the effects of slip angles.

METHOD

Development of Prototype

I constructed a prototype in order to determine the feasibility of my design and to test the theory of bicycle stability as it applies to rear-wheel steering.

The prototype was designed around a single piece, main tubular member of 2.0 inch O.D. 1020 seamless steel, with a wall thickness of 0.065 inches (16 gauge). (See Figure 11). This was the thinnest tubing readily available at the time; it results in a theoretical safety factor of five. The main tube was terminated by a three-inch long, 2 1/8 inch O.D. tube, which was welded in place perpendicular to the main tube. This contained the bearings which supported the fork.

The fork was constructed from a modified standard bicycle fork. The fork was straightened, cut down to fit a 20 inch wheel, and triangular blades with multiple slits cut into them were welded on the ends. This was done in order to experiment with various trails.

The various components were attached to aluminum alloy brackets, which could slide along the main member. The main member was keyed and the brackets slotted to maintain rigidity and alignment of the components when clamped into place.

FIGURE 11 - AUTHOR'S SEMI-RECUMBENT BICYCLE

As a result of this method of construction, all important geometric parameters could be easily adjusted such as: wheelbase, trail, seat position (fore and aft, up and down), pedal-crank axle position (fore and aft, up and down), and handlebar-steering position (fore and aft, up and down).

The steering column was connected to the rear fork via 1/16 inch thick stainless-steel cable, strung in such a way as to reverse the orientation of the rear wheel with respect to the handlebars. The seat was originally constructed much like a lawn chair, by attaching nylon webbing across a skeleton of steel tubing. This proved uncomfortable, and was replaced by a standard saddle and separate backrest. Standard bicycle components were used wherever possible to simplify construction.

A final version was designed, but never built.(see Figure 12). The primary attribute of the design is its weight (about 25 pounds). This was achieved by using welded aluminum-alloy construction, and minimizing adjustability of components.

Testing

The prototype was originally set-up as follows: wheelbase-45 inches, pedal axle height-17 inches above ground level, seat-centrally located and 24 inches above ground, backrest-set to incline towards the rear 10 degrees from the vertical.

Field testing proved to be very interesting. My first attempts to ride the beast met with failure. Adjustments in the trail from 1 to 5 inches seemed to make no apparent change in the stability. I attributed this to my inability to notice such

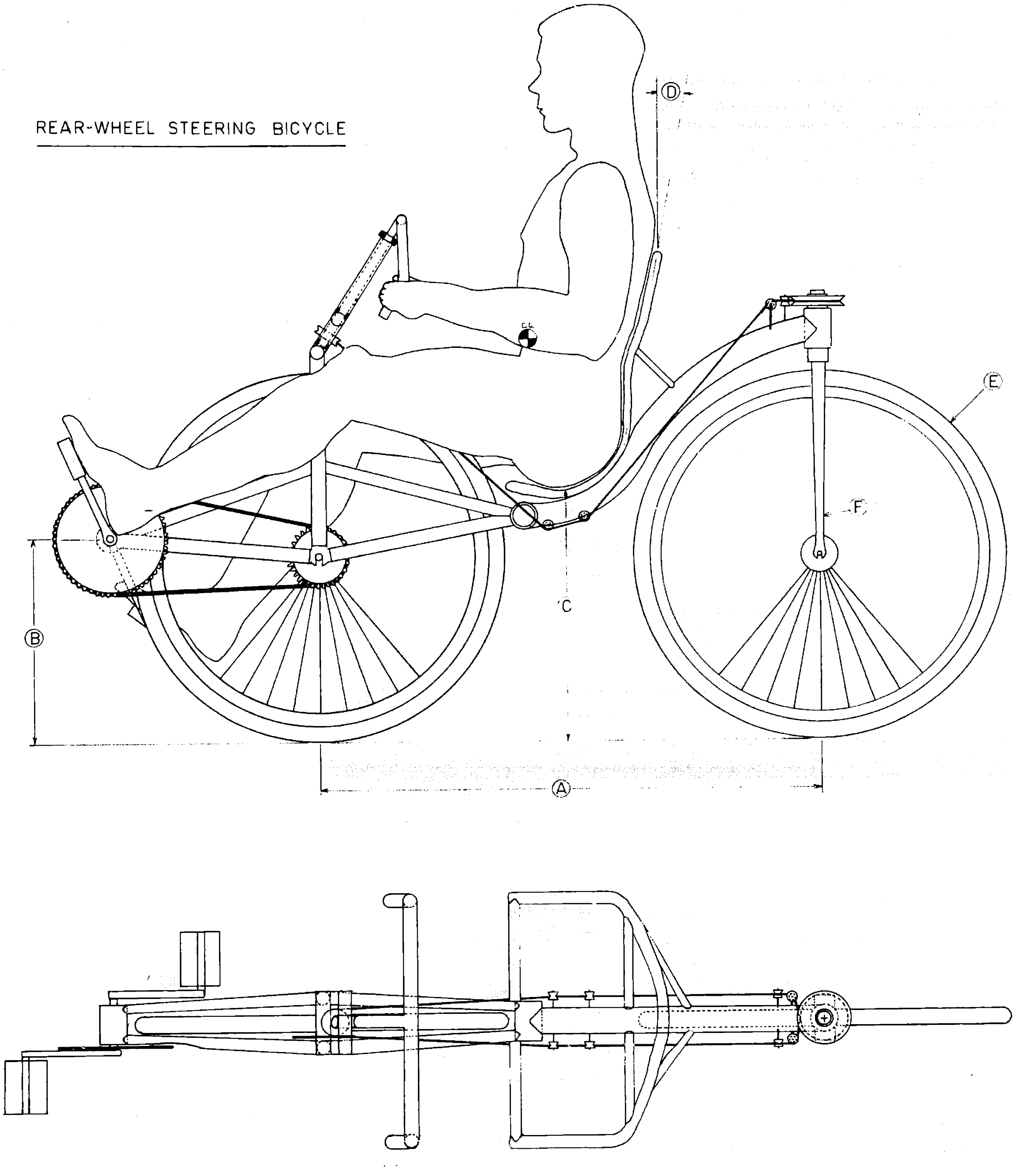

FIGURE 12 - REAR-WHEEL STEERING BICYCLE, FINAL VERSION

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changes while keeping my balance.

The immediate problem was the sensitivity of the steering. A small change in the handlebars resulted in a very sharp turn of the entire bicycle. Any hand movements on the bars were amplified. This is because rear-wheel steering produces a much sharper turn for a given angular displacement of the steering wheel, as compared to conventional schemes.

The steering was subsequently geared down to eliminate this problem. This was done by attaching pulleys to the steering-column and fork, and running the cable around them. Through trial and error, a gear reduction of 2½ to 1 was found to be the most satisfactory.

Afterwards, riding proved much easier. The steering was no longer unmanageable, and I could now pay attention to the effects of trail variation.

An increase in the trail did nothing to improve stability or handling. On the contrary, as a turn was initiated, the longer trail tended to cause oversteering. Extending the trail to a maximum of 8 inches only worsened the problem. The best results were obtained by setting the trail to its minimum value of one inch.

I then reversed the fork 180 degrees, so that the steering axis would fall in front of the ground contact point, as in a conventional bicycle. Increasing the trail surprisingly did not make handling worse, as expected, but rather prevented oversteering from occurring at the beginning of a turn. However, once the turn had been completed the bicycle seemed to be reluctant to pull out of the turn and return to a straight course. The best results with the reversed fork were obtained when the trail was set to its minimum value of one inch.

One major handling problem remained. As I pedaled through a turn, the angular acceleration of the bicycle around its center of gravity tended to knock me off balance. This made it difficult to make accurate leaning and steering corrections. On a suggestion by Professor Woodie Flowers, I decreased the wheelbase to 35 inches. This reduced the problem significantly, without adversely affecting steering sensitivity.

What remained was to practice in order to gain proficiency at controlling the bicycle.

Results

The test results confirmed the theory. The reversing force due to slip angles significantly influenced steering. That was why handling improved when a minimal trail was used. The results obtained by varying the trail and reversing the fork also made sense in light of the slip-angle theory.

I also found that there is a great deal of man-machine interaction involved in riding the bicycle. Experiences with other "pilots" shed light on the psychological factors that influence the rider. Most of these people relied heavily on visual cues. The long, main frame member protruding out in front of the rider caused considerable confusion. As the bicycle began a right turn for example, the front end of the frame would first dip to the left before aiming to the right. As a result these people would invariably steer the handlebars in the wrong direction.

The prototype was adequate for determining the feasibility of a semi-recumbent, rear-wheel steered bicycle. The final weight was within the limits of commercially available bicycles (around 45 pounds). Some problems may have resulted from the use of cable to transmit steering commands to the rear wheel, as they introduced an element of frictional damping into the steering system. On the other hand, the damping seemed to make the rider's job easier.

CONCLUSIONS

The objectives of this project have been met. The prototype represents a radical departure from conventional bicycle designs. As such, it eliminates most of the problems inherent in these designs. The combination of semi-recumbent rider position, front-wheel drive, and rear-wheel steering combine to create a safe, comfortable, and efficient bicycle. The semi-recumbent position is safe, as it is the rider's legs, and not his head, that first make contact in a collision. By virtue of the backrest, the rider can exert greater force on the pedals for short bursts of power. Less energy is wasted in maintaining one's position on the saddle. As an added plus, difficult maneuvers requiring tight turns at high speeds can be instituted with relative ease.

However, the bicycle is a difficult one to master. It requires a different set of reflexes to control properly. The rider is therefore faced with the dual task of forgetting or ignoring a set of old reflexes associated with conventional bike riding, and adopting a new set. Therefore a person who has never ridden a bicycle may find it easier to learn to ride the prototype.

Note that in order to optimize handling characteristics, the trail had to be kept negligible. This was done to minimize the effects of slip angles. As a result, there is no inherent force present to maintain stability. The rider is called upon to exert a much greater degree of control than with a conventional bicycle.

This requires a great deal of concentration, which detracts from riding pleasure.

I believe that with continued practice, it may be possible for a rider to achieve a level of riding ease comparable to a conventional bicycle.

APPLICATION #2 - MAN-POWERED TRACTOR

Background

The second application of a rear-wheel steering, man-powered vehicle was suggested by Rodale Press, Inc. They are interested in the advancement of alternative energy sources, such as man-power.

Purpose

My goal was to develop a man-powered utility tractor, capable of light duties on a farm, such as cutting grass, hauling supplies, pumping water, and supplying power to a drill.

Feasibility Study - Pedaling vs. Pushing

I felt it was necessary to first determine whether it was advantageous to pedal a tractor, as opposed to push it (as in the case of a wheel-barrow). Data collected by Dean (1965) concerning the energy expenditure of walking, indicated that a vehicle with wheels larger than 13 inches in diameter is more easily pedaled than pushed over either hilly or flat terrain. (See Figure 13).

Method

Rider Position: The results of my study of riding positions were equally applicable to the tractor. Therefore, a semi-recumbent position was chosen.

Tractor Design: One of my aims was to keep the tractor light in

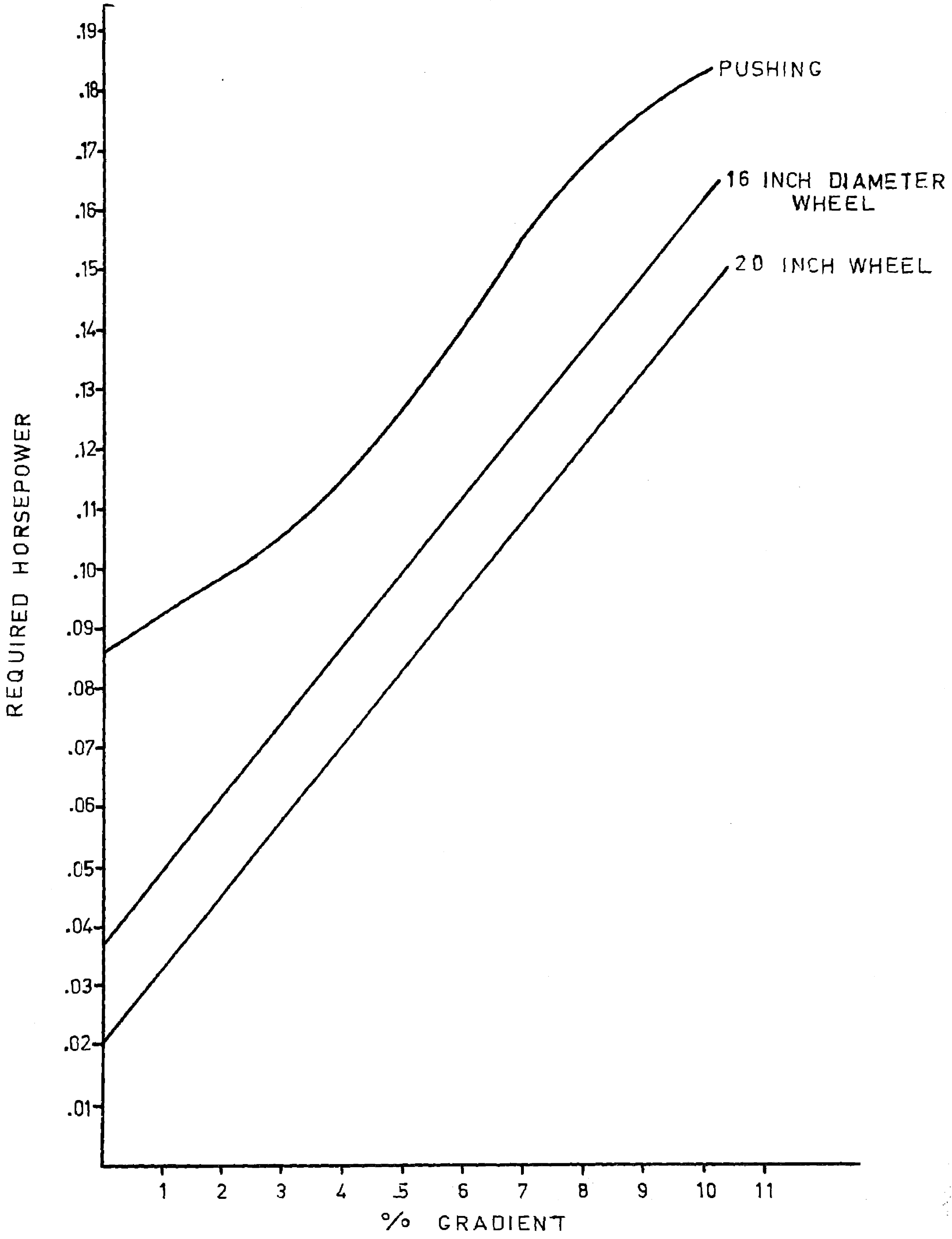

FIGURE 13 - COMPARISON OF POWER NEEDED TO PEDAL AND PUSH TRACTOR UP VARIOUS GRADIENTS

weight. I decided to use three wheels instead of the conventional four. This offers no stability problems as long as correct tricycle design practice is followed (Rice, 1972). Calculations indicate that a tractor with a 45 inch wheelbase and 36 inch wide track, can safely maneuver a 250 pound rider around a 6½ foot radius turning circle, at the maximum operating speed of 7 m.p.h. These were the dimensions I used in the design.

I chose single, rear-wheel steering and dual front-wheel drive because of its excellent maneuvering capability in confined areas, and simplicity of operation. (See Figure 14).

Work by Fuller (1937) indicated that for a single, rear-wheel steered, three-wheeled vehicle, maneuverability and stability can be maximized by locating the center of gravity of vehicle at a point 1/5 L in back of the front axle . (L=wheelbase).

The tractor body was constructed from 2inch O.D., 6061-T6 aluminum alloy, with a wall thickness of 0.065 inches (16 gauge). The various tubular sections making up the frame were heli-arc welded together. This was done to minimize weight while maximizing frame strength and rigidity. Calculations indicate that without re-heat-treatment after welding, the frame would still have an adequate safety factor of strength.

In order to maintain the correct center of gravity of the rider in relation to the tractor, the seat position was fixed. Instead, the pedal position was made adjustable.

Standard bicycle components were used where possible.

<u>Drive</u>: A chain drive was used to transmit power between the pedals and the two front driving wheels. The front drive shaft was split

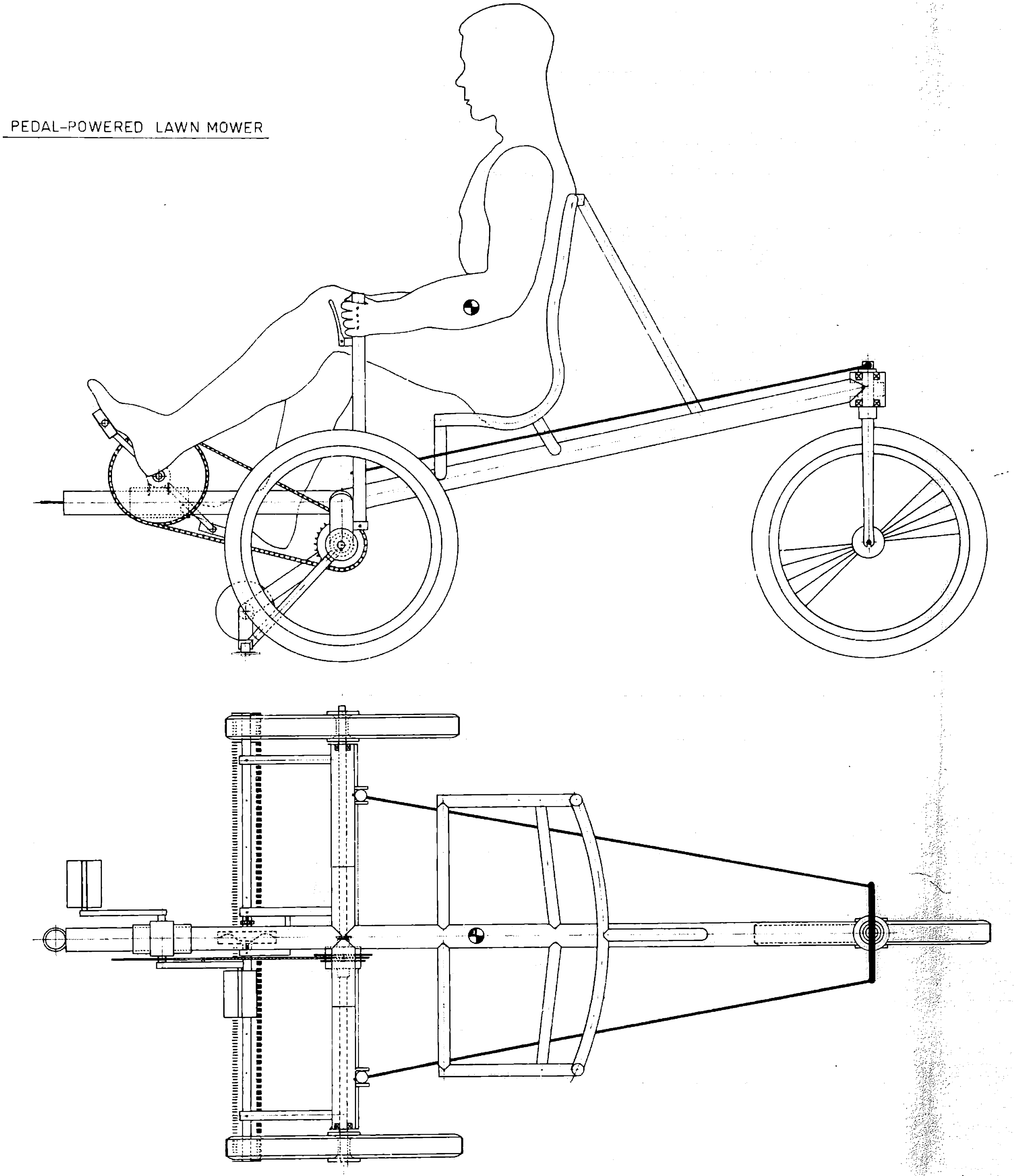

FIGURE 14A - TRACTOR DESIGN DETAILS

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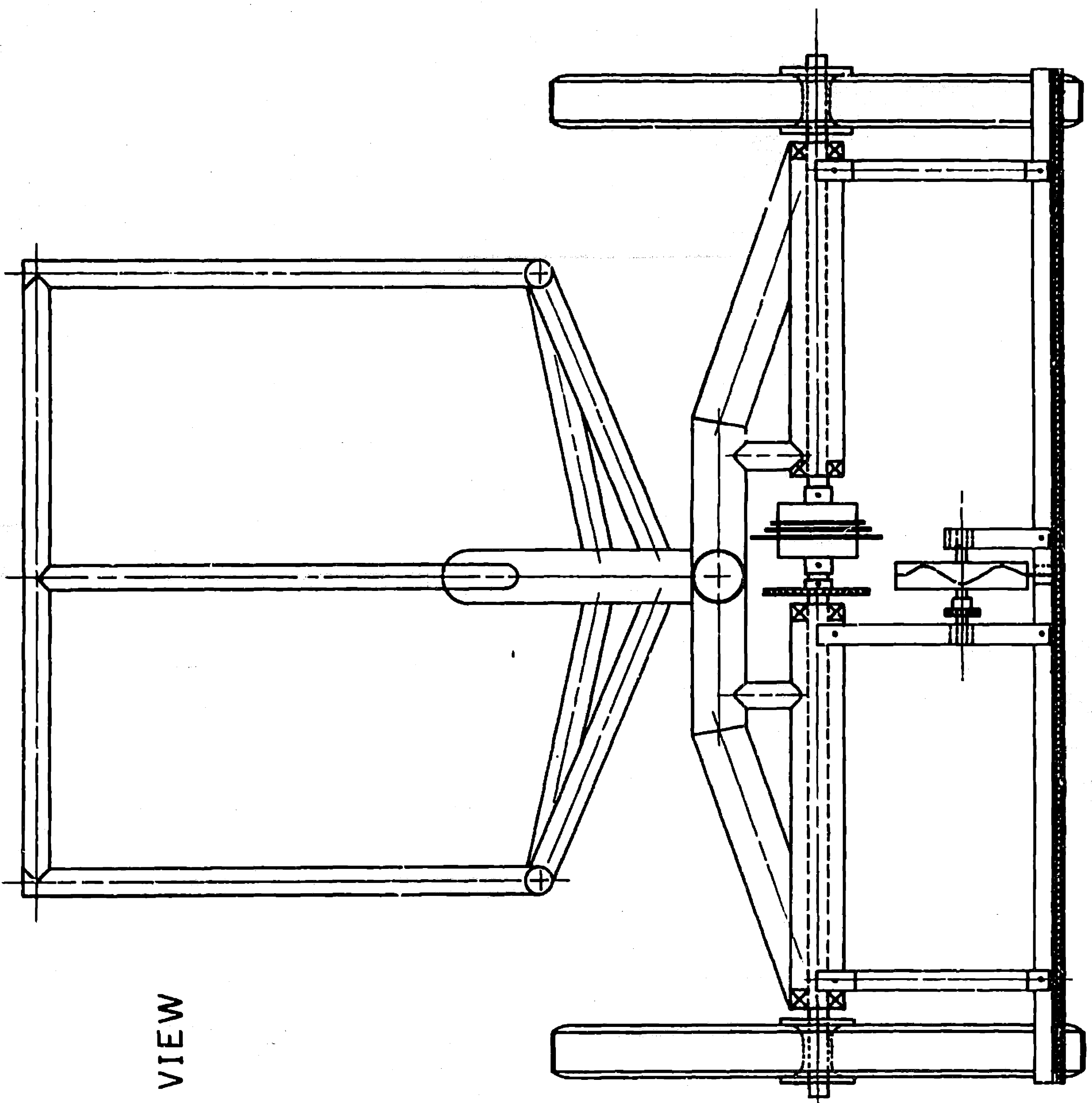
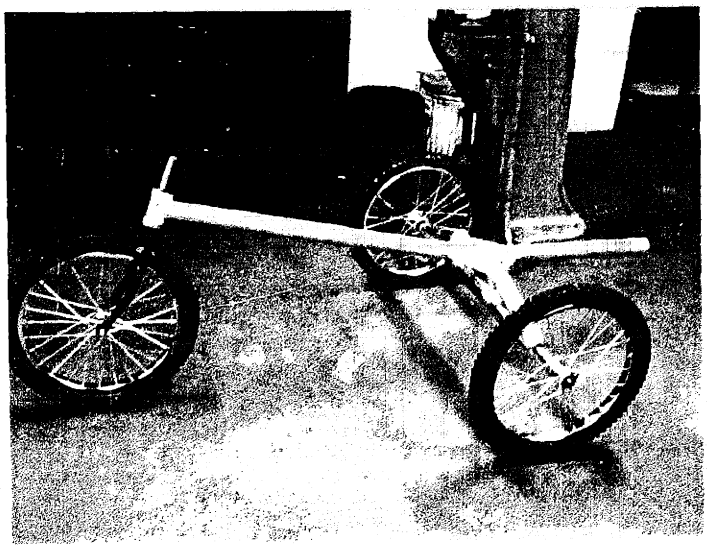


FIGURE 14B - TRACTOR, CONTINUED



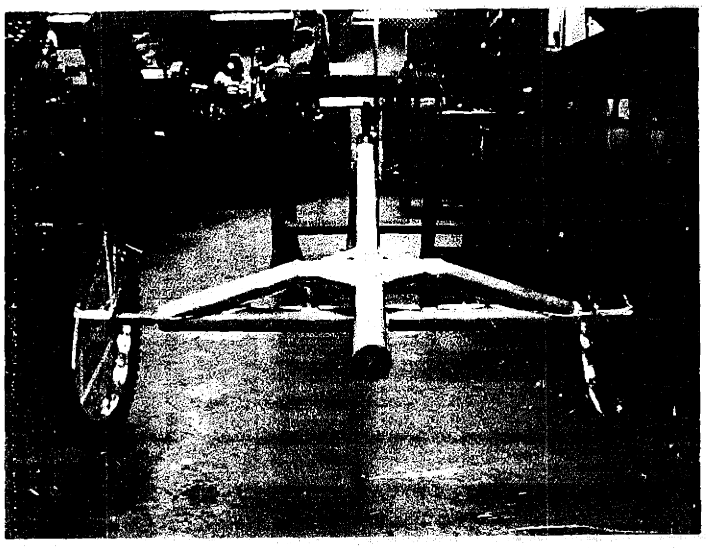


FIG. 14C – TRACTOR, CONTINUED

in the middle so that a centrally located differential unit could be mounted. The differential was modified so that three drive sprockets could be mounted directly to its body. Gear changing could be accomplished by a standard rear derailleur.

I arranged the gearing so that the tractor could be capable of traversing very steep hills (20% and greater) or transporting heavy loads. In bicycle nomenclature the range of gearing ratios was 11.7-27.5. For common pedaling rates, this corresponds to a range of 1 to 6 miles per hour (see Figure 15).

Cutting Unit: I designed a grass-cutting unit which could be attached to the tractor and driven by it. The cutter is light in weight (about 6 pounds) and features a oscillating, scissors-type motion, similar to that found in electric hedge trimmers. The motion of the cutting blade is achieved through the use of a closed, cylindrical cam with a cycloidal track. (See Figure 16).

Field Testing: Unfortunately, at the time of this writing, I have not completed construction of the tractor. As a result, there are no field test results.

Discussion: I believe the tractor design is a sound one, based on proven rear-wheel steering and semi-recumbent design concepts. I have also been successful at building a light-weight tractor. The complete prototype will weigh about 30 pounds.

Future work calls for completing construction and performing field tests to determine the usefulness of the design.

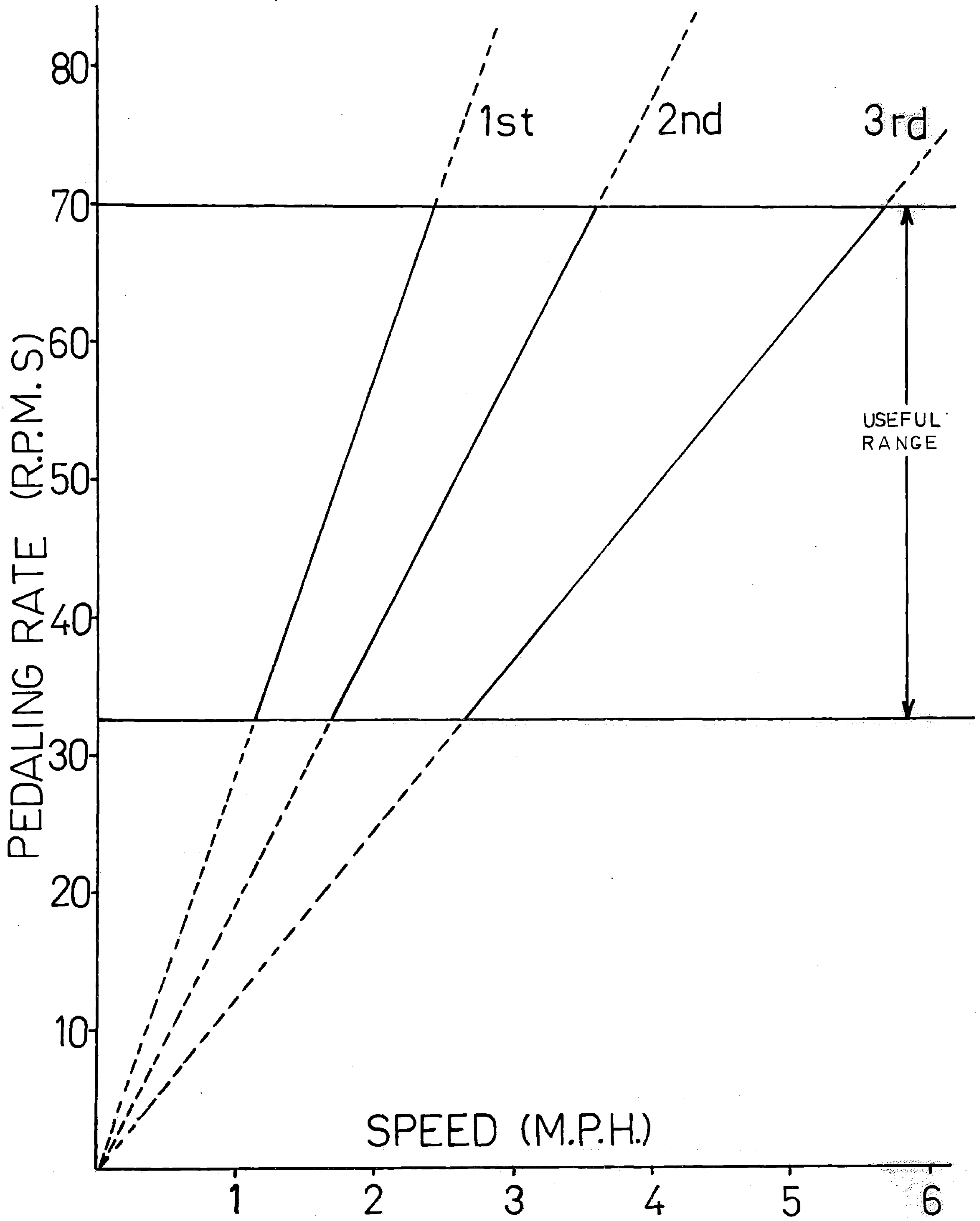
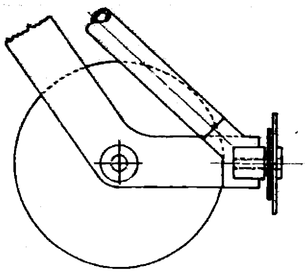
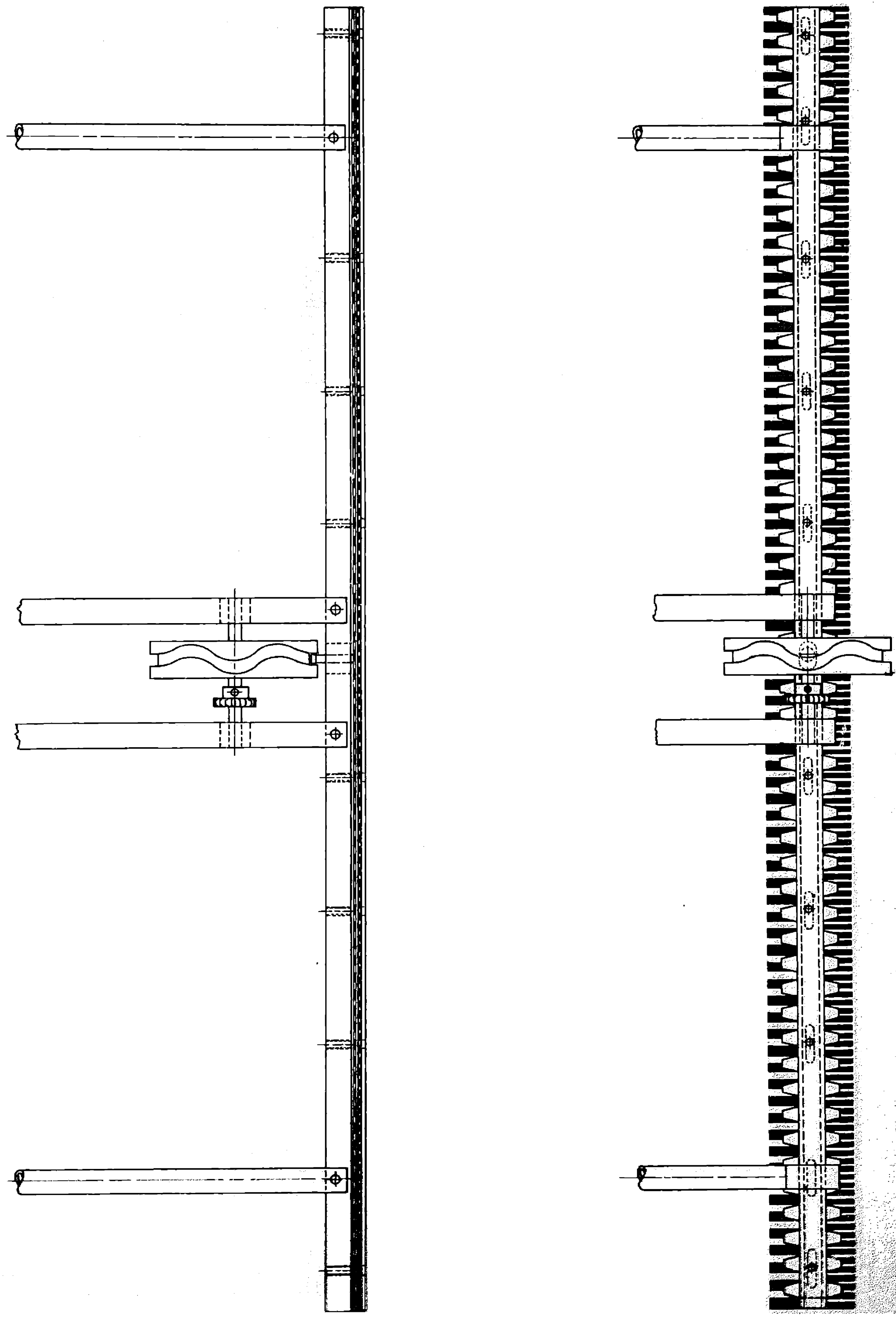


FIGURE 15 - RANGE OF GEARING FOR TRACTOR

CUTTER ASSEMBLY_





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APPENDIX

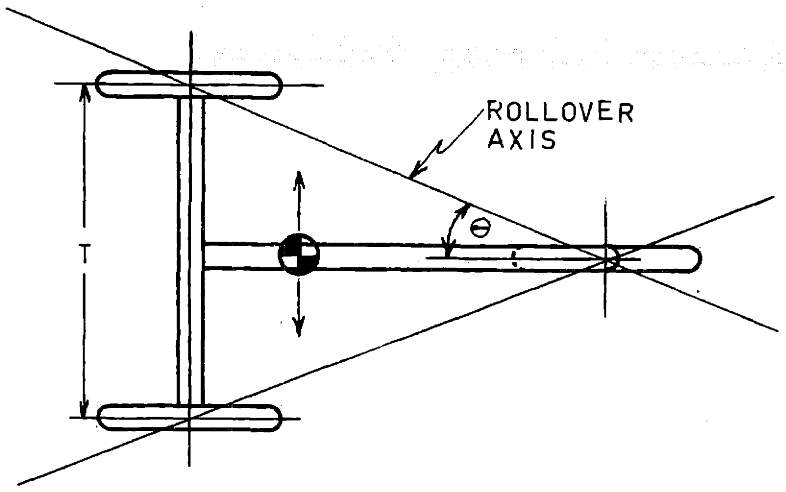
Tractor Design Specifications

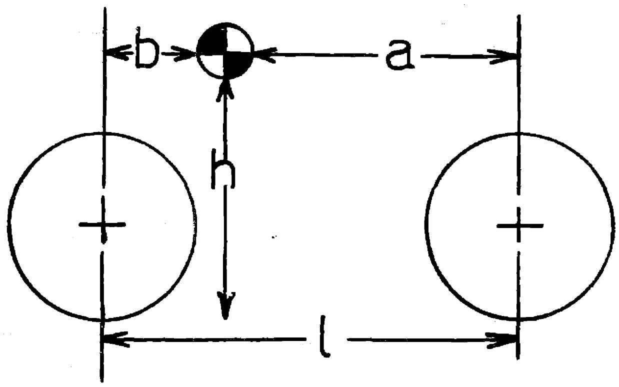
Overall Length	80.5	inches
Overall Height	40	"
Wheelbase	45	"
Front Track	36	"
Seat Height	20	"
Seat Back Inclination	5	degrees
Crank-axle Height	16	inches
Cutter Length	39.5	inches
Cutter Blade Height	1.0	inches
Weight	30	pounds
Cutter Assembly Wt.	7	"
Gearing:		
First	1:1.667	
Second	1:1.125	
Third	1:0.708	
Wheel Diameter	20	inches

Bicycle Design Specifications

Overall Length	75	inches
Wheelbase	38	"
Seat Height	19	"
Seat Back Inclination	7	degrees
Crank-axle Height	15.5	inches
Weight	25	pounds
Wheel Diameter	27	inches

Tricycle Rollover Stability Criteria





On a turn:

Upsetting Force.... mV^2/R

Restoring Force.... $W=mg$

For Stability: $\left[mga\sin\theta > \frac{mV^2h}{R}\cos\theta\right]$ OR $\left[\frac{a}{h}\tan\theta > V^2/gR\right]$

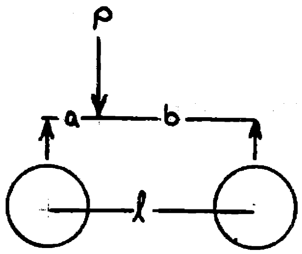
Let $\tan\theta = \frac{T}{2l}$, $\therefore \left[\frac{aT}{2hl} = \frac{V^2}{gR}\right]$, $\therefore \left[R = \frac{V^2 2hl}{gaT}\right]$

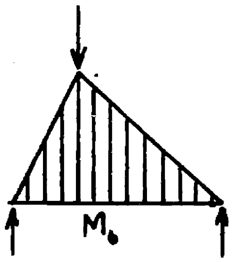
For a Maximum Operating Speed of 7 miles per hour:

$$R = \frac{(10.267 \text{ ft/sec})^2\ (2)\ (30.5)\ (45)}{(32.2 \text{ ft/sec}^2)\ (36 \text{ inches})\ (38.5)} = 6.48 \text{ Foot Turning Radius}$$

Frame Strength Calculations

Bending Stresses - Rear





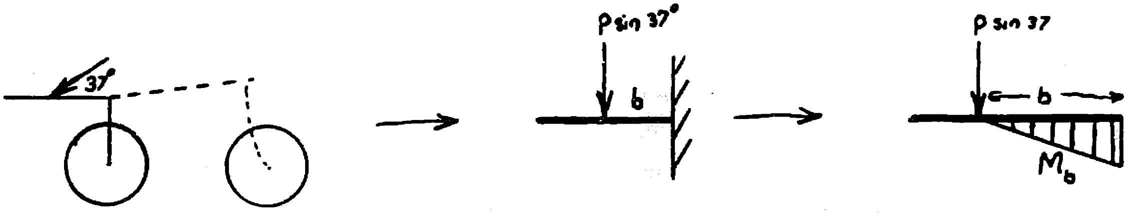
D. = 2.0 inches
d. = 1.87 "
Wall thickness = 0.065 in
I = $.049(D^4-d^4)$= 0.185 in^4
= 40000 p.s.i. (6061-T6)
= 8000 p.s.i. (6061)
P = 250 pounds
a = 9 inches
b = 36 inches
l = 45 "

$\sigma_{b\,max} = \frac{M\,c}{I}$, where $M = \frac{Pba}{l}$ = 1800 lb-in.

$\therefore \quad \sigma_b = \frac{(1800)(1)}{0.185}$ = 9730 p.s.i.

Safety Factor $= \frac{40000}{9730} = 4.11$

Bending Stresses - Front



M_{max} = Pb = $\underline{(100 \text{ pounds})(\sin 37^\circ)(15.5)}$ = 933 pound-in.

$\sigma_{b\,max} = \frac{(933)(1)}{0.185}$ = 5042 p.s.i.

Assuming aluminum has not been re-heat-treated after welding:

Safety Factor $= \frac{8000}{5042} = 1.6$