RIGID DESIGNATION, SCOPE AND MODALITY

by

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ABSTRACT

This thesis is a study of rigid designation. In the ten years since Kripke called attention to the notion, a variety of claims have been made about rigid designators. Many of the claims are problematic, and some of them are in direct conflict with others. The effect is to raise doubts about the notion. The principal goal of this thesis is a clear statement of what rigid designators are and what is special about them.

The first part of the thesis offers a definition of rigid designation and defends it against others that have been proposed. The concern is whether there is a distinct notion here at all. I conclude that there is an intuitively well-motivated, theoretically interesting notion of rigid designation. Furthermore, it can be defined perspicuously in terms of necessity *de re*. I argue that any adequate definition must at least resort to such a modal operator, but it need not employ the full paraphernalia of possible worlds.

The second part of the thesis explores the logic of singular terms in modal contexts. The goal is to determine the logical properties that rigid designators have and other singular terms lack. To this end, various formal modal languages are emphasized. I conclude that rigid designators have a number of important properties--e.g. the substitutivity of identity holds for them. But they do not have some of the properties that they are often claimed to have. In particular, differences in the scope of a rigid designator in a modal context can affect truth-value. Thus rigid designators are not scope neutral.

Much of the second part of the thesis is devoted to accounting for why rigid designators might be thought to be scope neutral. This concern leads to some interesting conclusions about the regimentation of *de re* modality. For instance, the tendency to liken rigid designators to the constants of standard logic is found to be a major source of trouble. I show that constants like those in standard logic cannot be introduced in the formal modal languages considered in the thesis without in the

process rendering these languages philosophically uninteresting. Similarly, a previously unexamined approach to quantified modal logic is found to conform unusually well with intuitions about the scope characteristics of rigid designators, and with essentialist intuitions generally. I indicate ways in which this approach to regimenting _de re_ modality, which distinguishes between normal and _recherché_ predicates, is more promising than other approaches.

Acknowledgments

This thesis grew out of a reading course I had with Richard Cartwright during the spring of 1976. I suspect that neither the questions addressed nor the perspective from which they are viewed would have occurred to me were it not for him. I only hope that the thesis begins to reflect his standards of rigour and excellence.

Professor Cartwright and Professor George Boolos, the other member of my committee, made a number of helpful comments on an earlier version of the thesis. Also, several marked improvements were made to the last chapter as a result of conversations I had with my fellow student, Drew Christie. His questions and comments made a large difference. And India Smith, as she has for a long time, patiently helped me to express my ideas more clearly.

Finally, I wish to thank Sylvain Bromberger and Jerry Katz for their encouragement and support throughout the time I have been a student at M.I.T. Their friendship did much to make this thesis possible.

Contents

Chapter I

Introduction

Saul Kripke's lectures "Naming and Necessity" and "Identity and Necessity" have attracted a good deal of philosophic attention since their publication. One of the central notions in the lectures is rigid designation. The idea of a rigid designator is easy to motivate. Many singular referring expressions can switch reference when they occur in counterfactual contexts. For example, 'the inventor of bifocals' does not denote Benjamin Franklin on one reading of 'the inventor of bifocals might have been born in the ninth century'. Rigid designators do not suffer from such vagaries of reference. Thus, they would provide a way of maintaining reference to specific objects even in counterfactual and modal statements.

A pivotal thesis of Kripke's lectures is that proper names in natural language are rigid designators. Many of his most important arguments turn on this thesis. It is not altogether surprising, therefore, that a good deal of controversy has developed around the thesis. Unfortunately, this controversy has been less than illuminating. One difficulty with it is that different disputants seem to take the thesis to mean quite different things. One reason for this is that there are a number of related notions with which rigid designation might be confused, and there have been a number of related claims about proper names that might be confused with the claim Kripke is making. It is also in part a result of the brevity with which Kripke introduces the notion. He gives a possible worlds characterization ("call something a <u>rigid designator</u> if in any possible world it designates the same object"), a single paradigm ('the square root of 25'), and an intuitive test involving a schema that is open to different readings. I think Kripke's point comes through clearly. Still, given the leverage provided by the thesis that names are rigid, he might well have anticipated a tendency to misconstrue it.

There is a need, then, to clarify the notion of rigid designation. Such clarification will be my immediate concern in this dissertation. I will not as such address the question whether proper names are rigid. Indeed, I will give comparatively little attention to natural language. Rather, the goal will be a clear statement of what rigid designators are and of what properties they have in suitable regimented languages.

Following the publication of Kripke's lectures, a number of distinct definitions have been offered of rigid designation, and a number of conflicting claims have been made about the logical properties of rigid designators. The natural question to ask in these circumstances is whether there is a specific intuitive notion of rigid designation at all. I argue in the next chapter that the answer to this question is yes--there is a naturally motivated, theoretically interesting notion of rigid designation, indeed the one that Kripke points out. I offer a formally explicit definition of rigid designation and then show that this definition isolates the naturally motivated, theoretically interesting notion from other closely related ones. This definition is then defended at length against others in the literature.

Given a formally explicit definition of rigid designation, the obvious next question is, what are the distinctive logical properties of rigid designators. The definition put forward in the next chapter uses '$\Box$', the necessity operator of modal logic. No formal interpretation of '$\Box$' is given in the chapter, however, so that its use there is primarily for perspicuity. But equipped with a definition framed in the syntax of quantified modal logic and facing a question about logical properties, the natural approach is to concentrate on the logic that rigid designators, so defined, have within quantified modal logic. This is the approach taken in the long final chapter.

Quantified modal logic remains controversial. One source of this controversy is the complaint that "it leads us back into the metaphysical jungle of Aristotelian essentialism." But as we will see, this complaint has little force for someone studying rigid designation; rigid designation is part of this same jungle. Indeed, Aristotelian essentialism amounts to the view that certain predicates are necessarily true of some individuals and not true at all of others. If the goal is to find a regimented framework in which to express such claims prior to evaluating them, then the appropriate stance to adopt toward the complaint against essentialism is that it should be held in abeyance. This is the stance I adopt throughout. But this stance does not eliminate the controversy surrounding quantified modal logic. Having granted that the goal is a framework in which to express essentialist claims, the question remains whether the framework of quantified modal logic is adequate. In particular, is a single necessity operator--e.g. '$\Box$'--sufficient for expressing both claims that certain sentences are necessarily true and also claims that certain predicates are necessarily true of certain individuals? There are reasons to suspect not. Efforts to formalize quantified modal logic have more often than not yielded theorems like '$\Diamond(\exists x)Fx \supset (\exists x)\Diamond Fx$' and '$(\exists x)\Box Fx \supset \Box(\exists x)Fx$'--the so-called Barcan and Buridan formulas--that are unacceptable to most proponents of essentialism. Indeed, many quantified modal logics in the literature contain as theorems denials of such basic essentialist claims as (i) there are contingently existing objects and (ii) predicates can be necessarily true of contingently existing objects. In addition to these worries, concerns have been expressed about the interpretation of free variables in modal contexts. The issue raised is whether one can coherently quantify into modal contexts. Part of its force comes from doubts about the logic of singular terms in quantified modal logic.

The question whether the framework of quantified modal logic is adequate remains in the background throughout most of the long chapter on the logic of rigid designators. But it is nevertheless the question of ultimate concern. Of course, our approach to it is somewhat indirect. The immediate question concerns the logic of singular reference within the formal setting of quantified modal logic. But evidence that the behavior of singular terms in this setting is idiosyncratic or counterintuitive is evidence against the adequacy of quantified modal logic. In particular, quantified modal logic is put in doubt to the extent that the logical properties of rigid designators in it are not compatible with those that rigid designators have in informal settings.

Because of this question in the background, the logic of rigid designators turns out to be more interesting than one might have anticipated. As expected, the substitutivity of identity holds for rigid designators. But contrary to claims prevalent in the literature, rigid designators are not "scope neutral"--i.e., the truth-value of modal sentences containing rigid designators is often sensitive to the scope of these designators. Much of the long final chapter is devoted to determining the significance of this seeming discrepancy with informal expectations. Two important results emerge. First, philosophically interesting modal quantificational languages cannot contain constants of the kind found in standard logic. The tendency to think otherwise is a major source of confusion both about rigid designation and about quantified modal logic. Second, there is a modal quantificational framework that conforms well both with essentialist intuitions and with informal expectations about the scope characteristics of rigid designators. This framework appears to be the most promising of those that have been proposed for regimenting essentialist reasoning. Hence, instead of challenging the adequacy of quantified modal logic, our results lend a particular version of it support. The logic of singular terms in modal contexts appears to be perspicuously captured by this quantified modal logic.

Chapter II

Rigid Designation and Its Variants

Michael Slote claims that in "Identity and Necessity" and "Naming and Necessity" Kripke has blurred a distinction between two kinds of rigid designation.[1] Slote calls the one kind *inclusively rigid* and the other *exclusively rigid*. On his account, a designator is *inclusively rigid* if and only if, should it pick out a certain object in some one possible world, then it picks out that same object in every other possible world in which that object exists. By contrast, a designator α is *exclusively rigid* if and only if it designates some actual object and ⌜it is not logically possible that α be something other than the thing that in fact is α⌝ is true.

In "Naming and Necessity" Kripke says, "call something a *rigid designator* if in any possible world it designates the same object;"[2] and he adds that of course the object in question need not exist in every possible world. This specification corresponds to Slote's condition for inclusive rigidity. Yet on the very next page, in arguing intuitively that proper names are rigid designators, Kripke says, "although the man (Nixon) might not have been the President, it is not the case that he might not have been Nixon." This phrasing of an intuitive test for rigid designation suggests Slote's condition for *exclusive rigidity*. Accordingly, if Slote is correct that the two kinds of rigid designation are distinct, then there is some evidence that Kripke has blurred the distinction.

Slote is not alone in complaining about Kripke's specification of what a rigid designator is. In calling attention to the ambiguity of Kripke's initial specification, Hugh Chandler says that Kripke may be offering any of the following definitions:[3]

Definition I: Given a term that, in the real world, designates an object, the term is a "rigid designator" if, and only if, every possible world in which that object exists is one with respect to which the term designates the object.

Definition II: Given a term that, in the real world, designates an object, the term is a "rigid designator" if, and only if, every possible world with respect to which the term designates at all is one with respect to which it designates that object.

Definition III: Given a term that, in the real world, designates an object, the term is a "rigid designator" if, and only if, (1) every possible world in which that object exists is one with respect to which the term designates the object, and (2) every possible world with respect to which the term designates at all is one with respect to which it designates that object.

Chandler does not go on to discuss the relative merits of these three alternatives, for he is concerned with a different question.

Slote's and Chandler's remarks raise the question whether we have a clear notion of rigid designation at all. It is one thing not to have successfully defined a clear notion and quite another not to have a clear notion to define. Which is true in the case of rigid designation? This is the central concern of this chapter. I shall begin with Slote's distinction between inclusive and exclusive rigidity. Once that distinction is clear, I will consider among other topics the relation between Slote's two conditions and Chandler's three definitions. For the moment I will not be concerned with whether proper names or any other linguistic entities are in fact rigid designators.[4]

I

A difficulty with Slote's statement of the condition for exclusive rigidity is that substitution instances of the schema it relies on are open to more than one reading. In particular, substitution instances in which 'α' is replaced by a definite description suffer from the standard sort of scope ambiguity. Including the phrase 'in fact' in the schema does not eliminate such ambiguities; it merely calls attention to the need to pick out the relevant reading. Thus Slote would better have said that α is an exclusively rigid designator if and only if α picks out some actual entity and ⌜it is not logically possible that α be something other than the thing that in fact is α⌝ is true on a certain reading. But what is this reading? Consider the following definition:

> An instance of the schema
>
> $(\iota x)(\cdots x \cdots)$
>
> is an <u>exclusively rigid designator</u> if and only if the corresponding instance of the schema (1)
>
> $(\exists x)\left[(\forall y)(\cdots y \cdots \equiv y{=}x) \;\&\; \Box(\forall y)(\cdots y \cdots \supset y{=}x)\right]$
>
> is true.

It eliminates the ambiguities. And it seems to capture what Slote wants. On the one hand, according to it 'the inventor of bifocals' is not exclusively rigid. For Thomas Jefferson might have been the inventor of bifocals. On the other hand, according to it 'the thing which is identical with Scott' appears to be exclusively rigid. For intuitively the following appears to be necessary: if y is the thing which is identical with Scott, then y is identical with Scott.[5]

Two points need to be made about (1). First, defining exclusive rigidity for definite descriptions rather than for terms in general need not involve any loss of generality. Quine's method of eliminating names from regimented languages--e.g., use the predicate 'Nixonizes' and a definite description to eliminate 'Nixon'--gives us one way for (1) to handle names.[6] Or we might say, for example, that 'Nixon' is a rigid designator only if the definite description 'the individual that is identical with Nixon' is. By framing the definition in terms of definite descriptions, we can eliminate scope ambiguities completely.

The second point about (1) is that it employs modality de re. That is, the modal operator occurs within the scope of the existential quantifier. Consequently, whether (1) yields a precise definition depends in part on whether we can make sense of modality de re. I do not want to worry about this here. For now let us assume that an adequate account of modality de re can be given. The force of the governing schema of (1) is then clear. Its existential quantifier requires that there exist an actual individual, and its modal clause requires that something be necessarily true of that individual. Of course, if the reader insists, the schema is open to possible world interpretation. I am reluctant to introduce such an interpretation at this point, not only because there is controversy as to which possible world interpretation of quantified modal logic is best, but also because I am uncertain whether a possible world interpretation does anything to clarify this schema. I have framed the definition in terms of a schema with a modal operator rather than in terms of possible worlds in order to remain neutral on questions of how to explicate modality de re.

How are we to state with comparable precision Slote's condition for inclusive rigidity? First note that his statement of this condition, unlike his statement of the condition for exclusive rigidity, does not expressly require the designator to designate an actual individual.[7]

Nevertheless, to keep the conditions parallel with one another, I will assume that inclusive rigidity does require this of a designator. We might then mistakenly try to exploit the parallelism by proposing the following:

> An instance of the schema
>
> $(\iota x)(\cdots x \cdots)$
>
> is an <u>inclusively rigid designator</u> if and only if the corresponding instance of the schema (2)
>
> $(\exists x)[(\forall y)(\cdots y \cdots \equiv y=x) \ \& \ \Box(\forall y)(y=x \supset \cdots y \cdots)]$
>
> is true.

The attractive feature of this proposal is that the modal clause in the schema is the converse of the modal clause in the schema for exclusive rigidity. Thus, if the proposal were right, we could readily account for any blurring of the distinction between the two kinds of rigidity. And we could construct a further kind of rigid designation, as follows:

> An instance of the schema
>
> $(\iota x)(\cdots x \cdots)$
>
> is a <u>completely rigid designator</u> if and only if the corresponding instance of the schema (3)
>
> $(\exists x)[(\forall y)(\cdots y \cdots \equiv y=x) \ \& \ \Box(\forall y)(\cdots y \cdots \equiv y=x)]$
>
> is true.

This third kind of rigid designation will yet be of interest.[8] But (2) will not do as a definition of inclusive rigidity, for the last clause is not strong enough. In possible worlds terminology, for a definite description to be inclusively rigid, not only must it be true of an object in every possible world in which that object exists, but it must be <u>uniquely</u> true of that object in every such world. The modal clause of the governing schema of (2), unlike those of (1) and (3), does not require uniqueness.

The following definition secures the desired uniqueness:

An instance of the schema

$(\iota x)(\cdots x \cdots)$

is an <u>inclusively rigid designator</u> if and only if the corresponding instance of the schema (4)

$$(\exists x)\left[(\forall y)(\cdots y \cdots \equiv y{=}x) \ \& \ \Box(\forall y)(y{=}x \supset (\cdots y \cdots \ \& \ (\forall w)(\cdots w \cdots \supset w{=}x)))\right]$$

is true.[9]

The sort of possible world interpretation which Slote adopts in his book indicates that the governing schema of (4) is what he wants. On the one hand, according to (4) 'the inventor of bifocals' is not inclusively rigid. For in some possible world Franklin exists and in that world is not the inventor of bifocals. On the other hand, according to it 'the thing which is identical with Scott' appears to be inclusively rigid. For intuitively if in a possible world there is an individual that is identical with the actual Scott, then in that world that individual is uniquely the thing which is identical with Scott.

Four governing schemata have been singled out in (1) through (4):

$$(\exists x)\left[(\forall y)(\cdots y \cdots \equiv y{=}x) \ \& \ \Box(\forall y)(\cdots y \cdots \supset y{=}x)\right] \quad (5)$$

$$(\exists x)\left[(\forall y)(\cdots y \cdots \equiv y{=}x) \ \& \ \Box(\forall y)(y{=}x \supset \cdots y \cdots)\right] \quad (6)$$

$$(\exists x)\left[(\forall y)(\cdots y \cdots \equiv y{=}x) \ \& \ \Box(\forall y)(\cdots y \cdots \equiv y{=}x)\right] \quad (7)$$

$$(\exists x)\left[(\forall y)(\cdots y \cdots \equiv y{=}x) \ \& \ \Box(\forall y)(y{=}x \supset (\cdots y \cdots \ \& \ (\forall w)(\cdots w \cdots \supset w{=}x)))\right] \quad (8)$$

What are the relations among these four? As already remarked, (6) does not entail (8), though obviously (8) entails (6).[10] (7) entails each of the others, but is not entailed by any one of them. That is, if a designator is completely rigid, then it is both inclusively and exclusively rigid. Furthermore, since the conjunction of (8) and (5) entails (7), a designator which is both inclusively and exclusively rigid is completely rigid. What remains, then, is to examine the examples Slote uses to argue that an exclusively rigid designator need not be inclusively rigid, and conversely.

Slote's example of an exclusively rigid designator that is not inclusively rigid is 'the being that is identical with Nixon and also a politician'. Intuitively it satisfies (1).[11] For, letting 'n' stand for 'Nixon' and 'P' for 'is a politician', (9) is intuitively true:

$$\Box(\forall y)\left[(y{=}n \ \& \ Py) \supset y{=}n\right] \qquad (9)$$

But this designator does not similarly satisfy (4), for intuitively (10) is false:

$$\Box(\forall y)\left[y{=}n \supset (y{=}n \ \& \ Py \ \& \ (\forall w)((w{=}n \ \& \ Pw) \supset w{=}n))\right] \qquad (10)$$

(10) is false because, of course, Nixon might have existed and yet not been a politician. We agree with Slote, then, that an exclusively rigid designator need not be inclusively rigid.

Slote's example of an inclusively rigid designator that is not exclusively rigid is 'the being that came from Harry', where 'Harry' is the name of the particular sperm from which Slote himself originated. The reason it is not exclusively rigid is that the sperm in question might have united with a different egg and thereby yielded an individual other than Michael Slote. Thus (11) is intuitively false, where 's' stands for 'Michael Slote' and 'H' for 'comes from Harry':

$$\Box(\forall y)\left[Hy \supset y{=}s\right] \qquad (11)$$

The question then is whether, as Slote claims, 'the being that came from Harry' is inclusively rigid. Slote bases his claim on the supposition that his coming from Harry is one of his essential attributes. I will not quarrel with this supposition here. But notice that even if this attribute is essential to him, it may not be essentially unique to him. Slote might have had an identical twin, and then two different individuals would have come from Harry.[12] Hence, 'the being that came from Harry' is not inclusively rigid. It does not satisfy (4), although it does satisfy the mistaken definition of inclusive rigidity, (2).

Is there an inclusively rigid designator that is not exclusively rigid? We might try to get around my objection by modifying Slote's example to read 'the first-born being that came from Harry'. But this does not help. For, though the attribute invoked is one that would hold uniquely if at all, there is no reason to think that it is an essential attribute. Isn't it logically possible for identical twins to be born in either order?

To get a clear example, Slote needs an attribute that on the one hand holds essentially and essentially uniquely of a certain object, but on the other hand might hold of some other object were the first object not to exist. In possible world terms, Slote requires an attribute that (i) holds uniquely of some object in every possible world in which that object exists and (ii) holds of some other object in some other possible world. It would be nice to find an everyday attribute of this sort since contrived examples are less compelling. But I have not found one. Of course, this should not be surprising. It is a direct reflection of the problem central to Kripke's lectures, viz. the problem of fixing the reference of an expression that is rigidly to designate a contingent entity. We seem not to know of an everyday attribute that is essential and essentially unique to a contingent object. If we did, we could readily fix the reference of a name of that object, and in the process give that name a sense.[13]

Nevertheless, we can get a designator that on the intuitive interpretation of logical necessity is inclusively, but not exclusively rigid. In particular, we can construct an example in parallel with the one Slote gives us of a designator which is exclusively, but not inclusively rigid. The example may seem awkward, but it accomplishes what we want, and it is not without precedent in the literature. Consider the designator, 'the thing which is identical with Nixon, or should nothing be identical with Nixon, the thing which is identical with 0'. The idea here springs from Frege's suggestion that failure of reference be eliminated from regimented languages by stipulating that otherwise denotationless names denote 0.[14] In symbolic notation the example might be expressed as follows, where 'n' stands for 'Nixon':

$$(\iota x)\left[x=n \vee (\sim(\exists z)(z=n) \ \& \ x=0)\right] \tag{12}$$

However, this formulation will unfortunately require us to adopt a free-logic if its second clause is not to be vacuous. A better formulation is therefore as follows, where 'N' stands for 'Nixonizes':[15]

$$(\iota x)\left[Nx \vee (\sim(\exists! z)(Nz) \ \& \ x=0)\right] \tag{13}$$

This designator is not exclusively rigid since it is presumably possible for Nixon not to have existed, in which case it would designate 0 rather than Nixon. Thus (14) is intuitively false:

$$\Box(\forall y)\left[(Ny \vee (\sim(\exists! z)(Nz) \ \& \ y=0)) \supset y=n\right] \tag{14}$$

But since (15) seems true, this designator appears to be inclusively rigid:

$$\Box(\forall y)\Big[y{=}n \supset \big[(Ny \vee (\sim(\exists! z)(Nz) \ \& \ y{=}0)) \ \& \ (\forall w)((Nw \vee (\sim(\exists! z)(Nz) \ \& \ w{=}0)) \supset w{=}n\big]\Big] \quad (15)$$

That is, intuitively it is necessary that if Nixon exists, then the description holds of him and of him uniquely.

We therefore agree with Slote that an inclusively rigid designator need not be exclusively rigid. This example, together with the parallel one that Slote gave us earlier, shows that neither an inclusively nor an exclusively rigid designator need be completely rigid. The three notions are distinct.

II

Slote's proposed distinction has led us to define three different notions of rigid designation--inclusive, exclusive, and complete rigidity. The three correspond to Chandler's alternative definitions of rigid designation, quoted earlier. The correspondence is easy to see once we give the usual possible world interpretation to the governing schemata of (4), (1), and (3). For consider a definite description that successfully picks out a certain object in the actual world. Such a description is inclusively rigid provided that the actually designated object uniquely satisfies the description in every possible world in which it exists. What inclusive rigidity leaves open is whether some other object satisfies the description in a possible world in which the actually designated object does not exist. By contrast, a definite description is exclusively rigid provided that in no possible world does any object other than the actually designated object satisfy the description. What exclusive rigidity leaves open is whether there are possible worlds in which the actually designated object exists, but fails to satisfy the description. Finally, a definite description is completely rigid provided that the actually designated object satisfies the description in every possible world in which it exists and in no possible world does any other object satisfy the description. The only differences between these characterizations and Chandler's are matters of wording. I persist in the view that neutral definitions in terms of a de re modal operator are preferable to definitions in terms of possible worlds. Still, the correspondence between Chandler's and our definitions is interesting. It is evidence that our definitions have been stated correctly, and it is evidence that we have successfully identified the principal notions of rigid designation.

An obvious question, given these three notions, is whether any one of them is of more interest than the others. Neither Chandler nor Slote addresses this question. In offering three possible definitions of rigid designation, Chandler prompts the question. But instead of choosing one, he develops a separate argument around each of them for an independent point. Slote, having called our attention to a potential difficulty, proceeds to restrict his attention to what we have called completely rigid designation.

Of course, since what is of more interest from one point of view may not be from another, the question is vague as it stands. The way in which Kripke introduces the notion of rigid designation suggests two more precise questions. First, Kripke introduces the notion by appealing to an intuitive distinction between rigid and nonrigid designators, as illustrated by some paradigms. This suggests the question, which of our three notions best approximates the intuitive notion Kripke invokes? Second, Kripke introduces the notion in order to explicate certain features of reference. This suggests the question, which of our notions is most useful for explicating these features? Which, if any, of our notions promises to be central to the theory of reference? Since intuitive notions sometimes prove not to be best for developing a theory, these two questions need not receive the same answer. Furthermore, neither question is trivially answered by citing Kripke's paradigms. All three notions fit them. His paradigm of a rigid designator, 'the square root of 25', satisfies (1), (3), and (4); and his paradigm of a nonrigid designator, 'the inventor of bifocals', satisfies none of them.[16] We will have to look deeper for answers.

It seems clear to me that, though the questions are distinct, the answer to both is the same, viz., completely rigid designation. That is, I think that (3), the definition of complete rigidity, provides the best definition of the intuitive notion Kripke's examples invoke. And I think that the notion of complete rigidity is the most useful of the three in addressing questions about reference. I have four arguments for these claims. The structure of each argument is the same. The initial step is to show that complete rigidity has an attribute that exclusive and inclusive rigidity lack. This step usually requires most of the effort. The shorter, concluding step divides into two parts. One part shows how its having the attribute in question is evidence that complete rigidity is the intuitive notion; the other part shows how the same thing is evidence that complete rigidity is the notion of principal theoretical interest.

One argument is that all examples I have found of designators which are exclusively or inclusively, but not completely rigid seem contrived. 'The thing which is identical with Nixon and a politician' and 'the thing which is identical with Nixon or, should nothing be identical with Nixon, the thing which is identical with 0', even when less awkwardly phrased, do not represent ordinary ways of designating things. Examples like these do not favor exclusive or inclusive rigidity. Of course, this line of argument would be more compelling if I could show that the only possible examples of exclusively or inclusively rigid designators which are not completely rigid are contrived. We could then argue that complete rigidity is more likely the intuitive notion Kripke's paradigms invoke, since whatever the principle for projecting from these paradigms is, the projection should extend to as few contrived cases as possible. Similarly, we could argue that complete rigidity is more likely to be central to the theory of reference, for normal, not contrived cases comprise the principal explicative burden of a theory. Unfortunately, I cannot show that the only examples are contrived. Still, this line of argument suggests that complete rigidity is the better founded of the notions and that the others are weakened offshoots.

A more compelling line of argument considers how other notions of designation give rise to that of a rigid designator. The idea is that the way in which they naturally give rise to it will be found under close inspection to imply that it is the same as completely rigid designation.

The basic notion is that of a designator simpliciter--i.e., a singular referring expression which in fact picks out exactly one actual individual. We can define this notion in parallel with our earlier definitions as follows:

> An instance of the schema
>
> $(\iota x)(\ldots x \ldots)$
>
> is a <u>designator</u> (<u>simpliciter</u>) if and only if the corresponding instance of the schema
>
> $(\exists x)(\forall y)(\ldots y \ldots \equiv y=x)$
>
> is true. (16)

Of course, a singular referring expression may in fact designate something without it being necessary that it designate anything. This suggests the notion of an unfailing designator--i.e., a singular referring expression which is guaranteed to pick out some individual or other. We can define this notion as follows:

> An instance of the schema
>
> $(\iota x)(\ldots x \ldots)$
>
> is an <u>unfailing designator</u> if and only if the corresponding instance of the schema
>
> $\Box(\exists x)(\forall y)(\ldots y \ldots \equiv y=x)$
>
> is true. (17)

But again, a singular referring expression may be guaranteed to designate something without it being necessary that it designate the particular individual it now does. This suggests a further notion, <u>viz.</u>, that of a singular referring expression which in some sense is guaranteed to designate the individual it now designates. The notion we get at in this way, I submit, is rigid designation.

Roughly, then, the idea of the argument is to motivate the notion of a rigid designator, much as Kripke does, by considering designation in general vis-a-vis logically possible counterfactual situations. However, we need to develop this approach at some length and with some care before it can help us to choose among our three kinds of rigid designation. In particular, we need to answer the question, how is what an expression might have designated pertinent at all?

First, however, we need to make more precise the possibilities these other kinds of designation leave open. Assume we are given a singular referring expression that is a designator in a language L--one like 'the x such that Fx'. Assume further that the rules governing L, especially those governing reference, remain fixed. And now consider the ways in which things might have been different. One possibility that designation simpliciter leaves open is that things might have been different in such a way that 'the x such that Fx' would have designated some other object even though the object it now designates would still have existed. For example, suppose that Franklin had concentrated on politics rather than science and Jefferson had developed bifocal lenses. Then 'the inventor of bifocals' would have designated Jefferson, and yet Franklin would still have existed. A second possibility left open is that things might have been different in such a way that 'the x such that Fx' would have designated nothing even though the object it now designates would still have existed. For example, suppose that Franklin had concentrated on politics and that eyeglasses had never been conceived of. Then 'the inventor of bifocals' would have designated nothing, and yet Franklin would still have existed. A third possibility left open is that things might have been different in such a way that 'the x such that Fx' would still have designated something even though the object it now designates would not have existed. For example, suppose Franklin had never been born and that Jefferson had developed bifocal lenses. Then 'the inventor of bifocals' would have designated Jefferson and Franklin would not have existed.

Other possibilities left open by designation need not concern us here. Note that unfailing designation also leaves the first and third possibilities open. For a singular referring expression can be an unfailing designator without having to designate the object it does. And if the object it now designates had not existed, it would have had to designate some other object to have been unfailing. Finally, in saying that designation leaves a certain possibility open, I do not mean that all designators leave it open. I mean only that a singular referring expression can be a designator and yet the possiblity in question remain open.

What difference do these open possibilities make? And how do they give rise to a notion of rigid designation? They would seem to make no difference if the reference of every singular referring expression, regardless of context, were determined solely on the basis of what is actually the case. For then, regardless of the context in which it occurs, a designator would always in practice designate the same thing. What it might have designated or what it would have designated if things had been different would be just a matter of curiosity. And the open possibilities would probably not give rise to a notion of rigid designation.

However, some languages, notably English, have contexts in which the reference of singular referring expressions can also be determined on the basis of what might have been or would have been the case. Consider the following ambiguous sentences:

> The inventor of bifocals might have been the third President of the United States. (18)
>
> If Jefferson had devoted more time to optics, the inventor of bifocals would have been the third President of the United States. (19)

One way in which they are ambiguous is that in both 'the inventor of bifocals' can be taken to refer either to Franklin or, for example, to Jefferson.[17] If Franklin, then its reference is being determined on the basis of what is actually the case; but if Jefferson, then its reference is being determined on the basis of what might have or would have been the case.[18] Now, if the reference of singular referring expressions in some contexts can be determined on the basis of counterfactual conditions, then what a designator might have or would have designated is not just a matter of curiosity. Given our open possibilities, a designator could in practice denote something else. The open possibilities would then make a difference--e.g., they could lead to a certain type of referential ambiguity.

The idea, then is to focus on languages having contexts in which the reference of singular referring expressions can be determined either on the basis of the actual situation or on the basis of a counterfactual situation.[19] We need to examine the consequences of the possibilities left open by designation simpliciter and unfailing designation in such languages to see how these possibilities give rise to a notion of rigid designation.

We can fix our ideas by concentrating on contexts of the following form:

> Suppose p had been the case; even so the x such that Fx would have been G. (20)

The detailed workings of reference in such contexts in English need not concern us here. We will simply assume that in such contexts the reference of 'the x such that Fx' can be determined on either of two bases:

Basis 1: What is actually the case.

Basis 2: What would have been the case if p had been true.[20]

Now consider the three open possibilities sketched earlier. In the first a designator would have designated something other than what it now does even though the latter would still have existed. But then a sentence of the form of (20) could be true if the reference of 'the x such that Fx' were determined on Basis 1 and not true if it were determined on Basis 2. To illustrate, consider the following example:

> Suppose Jefferson had developed bifocals and Franklin had concentrated on politics; even so, the inventor of bifocals would have been a delegate to the Federal Convention.[20] (21)

Presumably the second clause of (21) is true if 'the inventor of bifocals' is taken to refer to the person who actually invented bifocals, Franklin; and it is not true if the phrase is taken to refer to the person who would have invented bifocals in the supposed counterfactual situation, Jefferson.

In the second open possibility, a designator would have failed to designate anything at all even though the object it now designates would still have existed. But then a sentence of the form of (20) could be true if the reference of 'the x such that Fx' were determined on Basis 1 and not true if it were determined on Basis 2.[22] To illustrate:

> Suppose human eyesight had never needed correction and eyeglasses had never been invented; even so, the inventor of bifocals would have been a delegate to the Federal Convention. (22)

Presumably the second clause of (22) is true if 'the inventor of bifocals' is taken to refer to the person who actually invented bifocals; and it is not true if the phrase is taken to refer to someone who would have invented bifocals in the supposed counterfactual situation.

Finally, in the third open possibility, a designator would have designated something even though the object it now designates would not have existed. But then a sentence of the form of (20) could be true if the reference of 'the x such that Fx' were determined on Basis 2, and not true if it were determined on Basis 1. To illustrate:

> Suppose the Franklins had had no children and Jefferson had developed bifocals; even so, the inventor of bifocals would have been in France during the Federal Convention. (23)

Presumably the second clause of (23) is true if 'the inventor of bifocals' is taken to refer to Jefferson; and it is not true if the phrase is taken to refer to Franklin.

Thus in languages of the specified type designation <u>simpliciter</u> and unfailing designation leave open the possibility that certain sentences --e.g., descriptions of counterfactual situations--will have contrasting truth-values when the reference of designators occurring in them is determined on one basis rather than another. This possibility calls attention to a special kind of designator, one that, regardless of the basis on which its reference is determined, never leads to contrasting truth-values. This special kind of designator, I take it, is a rigid designator.

We now have a natural way of motivating rigid designation. This way requires some comments before we continue with the argument. First, note that according to our characterization it is not the point of a rigid designator to eliminate contrasting truth-values by forcing reference to be determined on only one basis. Rather, the point is to eliminate contrasting truth-values while still permitting reference to be determined on different bases. This helps to explain why I have characterized rigid designation as a way of eliminating contrasting truth-values rather than as a way of eliminating ambiguities. For assume that 'the x such that x Franklinized' rigidly designates Benjamin Franklin, and consider the following:

> Suppose the Franklins had had no children; even so, the x such that x Franklinized would have been a delegate to the Federal Convention. (24)

To me the second clause of (24) still seems open to two readings. On one reading it says that there is exactly one individual who Franklinized and this individual would have been a delegate even in the supposed situation. On the other it says that even in the supposed situation there would have been an individual who alone Franklinized and who was a delegate.[23] If so, (24) has two readings, and these correspond to two bases on which the reference of 'the x such that x Franklinized' can be determined. The rigid designator would thus not eliminate the ambiguity. But since (24) is presumably not true on either reading, it would eliminate contrasting truth-values.

Second, note that we have motivated rigid designation only for a certain type of language. The point of distinguishing the rigid designators of a language is lost according to our characterization unless the language has some contexts in which the reference of singular referring expressions can be determined on different bases. Now, I suspect that a language does not have to be of this type. Even in English it is not obvious that much expressive power would be lost if the reference of singular referring expressions were invariably determined on the basis of what is actually the case. But if so, rigid designation is not a particularly fundamental theoretical notion. A philosophically ideal language could avoid the need for it. This may be one reason why rigid designation has only recently come to attract so much attention.

Finally note that our characterization universally quantifies over contexts of a language. A rigid designator provides a way of designating an object without causing <u>any</u> sentence to have contrasting truth-values when the reference of the designator is determined on one basis rather than another. Now, in practice confusion stemming from a referential ambiguity in a single sentence can be reduced by pragmatic factors or by using a more refined designator. For example, (25) is less confusing than (21):

> Suppose Jefferson had developed bifocals and Franklin had concentrated on politics; even so, the person who invented bifocals and experimented with lightning would have been a delegate to the Federal Convention. (25)

Yet 'the person who invented bifocals and experimented with lightning' is no more a rigid designator than 'the inventor of bifocals'. We can thus deal with a source of referential ambiguity in practice on a case by case basis, as the ambiguity arises. We rarely, if ever, need one way of dealing with it that works across the entire language.[24] Rigid designation might therefore be thought of as a practical notion carried to a theoretical limit. This may be another reason why it has only recently attracted attention.

If the comments of the last two paragraphs are correct, there is neither a compelling theoretical nor a compelling practical reason for rigid designators. Rigid designation is motivated by a type of referential ambiguity that does not have to occur and that, when it does, can be handled in other ways. In a way this is as it should be. For if rigid designators were somehow mandated, there should be little controversy over proper names being rigid, since they are the only candidates we have for rigidly designating contingent objects.

Let us return to the argument. So far we have a natural way of motivating a notion of rigid designation. A rigid designator is one that does not cause any sentence to have a contrasting truth-value when the reference of the designator is determined on the basis of what might have been or would have been the case instead of on the basis of what is the case. What conditions must a designator satisfy to achieve this? One condition is: regardless of how things might have been, the designator would not have denoted any object other than the now designated one. Otherwise, in those circumstances in which it would have denoted another object, something would have been true of this other object that would not have been true of the now designated object. This is the point of the first and third open possibilities discussed earlier. Another condition is: regardless of how things might have been, if the now designated object had still existed, the designator would have denoted it. Otherwise, in those circumstances in which it would have failed to denote the object, something would still have been true of the object. This is the point of the second open possiblity.

Now, these two conditions are precisely the ones satisfied by complete rigidity. And they are not satisfied by exclusive or inclusive rigidity. Exclusive rigidity fails to satisfy the second condition. It leaves open the possibility that the designator would have denoted nothing even though the now designated object would still have existed. Thus a sentence of the form, 'The x such that Fx would have been G', could be true if the reference of the exclusively rigid designator, 'the x such that Fx', were determined on the basis of what is the case, and not true if its reference were determined on the basis of what would have been the case. Similarly, inclusive rigidity fails to satisfy the first condition. It leaves open the possibility that the designator would have denoted another object if the now designated object had not existed. Thus a sentence of the form, 'The x such that Fx would have been G' could be true if the reference of the inclusively rigid designator, 'the x such that Fx', were determined on the basis of what would have been the case, and not true if its reference were determined on the basis of what is the case. Accordingly, if the point of rigid designators is to avoid contrasting truth-values of the sort sketched above, then rigid designation is completely rigid designation.

The preceding analysis is meant to expand on Kripke's remarks. It is worth noting that at one point Kripke expressly indicates that what he means by 'rigid designation' is what we have called 'completely rigid designation':

> All I mean is that in any possible world where the object in question does exist, in any situation where the object would exist, we use the designator in question to designate that object. In a situation where the object does not exist, then we should say that the designator has no referent and that the object in question so designated does not exist.[25]

Thus the upshot of our long second argument is compatible with Kripke.

So far the argument has shown that one natural way of motivating rigid designation leads to complete rather than to exclusive or inclusive rigidity. It remains to show how this bears on the claim that complete rigidity is both the intuitive and the more central theoretical notion. First, whichever intuitive notion Kripke is invoking, the principle for projecting from his paradigms must be one for which there is a natural motivation. Otherwise, the projected notion is not intuitive. The preceding analysis has yielded a naturally motivated principle for projecting from his paradigms. And what this principle projects, among our alternatives, is completely rigid designation. Thus, in the absence of a different natural principle, our argument provides evidence that complete rigidity is the intuitive notion. Similarly, whichever notion we make central to a theory of reference, it should be useful for explaining those features of reference that give rise to notions of rigid designation. According to the preceding analysis, a major item to be explained is why certain designators do and others do not cause sentences to have contrasting truth-values when their reference is determined on one basis instead of another. But the designators that cause this are not completely rigid; and those that do not are the completely rigid ones. Our argument thus provides evidence that complete rigidity is the more useful notion in addressing questions about reference.

However, on neither point is this argument conclusive. For we have not shown that our way of motivating rigid designation is the most natural one. Nor have we shown that the features of reference we used to motivate rigid designation are the most important features for a theory to explain. Still, until we have an alternative analysis of how other notions of designation give rise to that of a rigid designator, our second argument will remain compelling.

III

Of the arguments I have in favor of complete rigidity, the preceding one is probably the most important. Still, two further lines of argument deserve consideration. The first of these focuses on the problem of defining subordinate notions of rigid designation.[26] The idea is that complete rigidity yields better definitions. Kripke calls attention to the notion of a strongly rigid designator--i.e., a rigid designator which, because it designates a necessary entity, cannot fail to designate that entity.[27] If we identify rigidity with complete rigidity, we can define this notion as follows:

> An instance of the schema
>
> $(\iota x)(\cdots x \cdots)$
>
> is a <u>strongly rigid designator</u> if and only if
>
> the corresponding instance of the schema (25)

$$(\exists x)[(\forall y)(\cdots y\cdots \equiv y{=}x)\ \&\ \Box(\forall y)(\cdots y\cdots \equiv y{=}x)]\ \&\ \Box(\exists x)[\cdots x\cdots]$$

is true.

The correlative notion is that of a weakly rigid designator--i.e., a rigid designator which, because it designates a contingent entity, can fail to designate that entity. We can define this notion as follows:

An instance of the schema

$$(\iota x)(\cdots x\cdots)$$

is a <u>weakly rigid designator</u> if and only if the corresponding instance of the schema (26)

$$(\exists x)[(\forall y)(\cdots y\cdots \equiv y{=}x)\ \&\ \Box(\forall y)(\cdots y\cdots \equiv y{=}x)]\ \&\ \sim\Box(\exists x)[\cdots x\cdots]$$

is true.

These definitions have several virtues. They bifurcate the class of rigid designators (so construed). They are simple and transparent. And they are expressly of the genus-difference type. That is, their governing schemata are obtained by conjoining a clause to the schema for completely rigid designation; they involve no modification of the clause corresponding to that schema.

What happens if we instead identify rigidity with exclusive or inclusive rigidity? When we conjoin the same clauses to the schema governing exclusive rigidity, we get the following:

$$(\exists x)[(\forall y)(\cdots y\cdots \equiv y{=}x)\ \&\ \Box(\forall y)(\cdots y\cdots \supset y{=}x)]\ \&\ \Box(\exists x)[\cdots x\cdots] \quad (27)$$

$$(\exists x)[(\forall y)(\cdots y\cdots \equiv y{=}x)\ \&\ \Box(\forall y)(\cdots y\cdots \supset y{=}x)]\ \&\ \sim\Box(\exists x)[\cdots x\cdots] \quad (28)$$

Now (27) defines a notion of strong rigidity. For it entails both that the designator cannot fail to designate the object it does and that the object is necessary. But (28) does not define a notion of weak rigidity because it does not entail that the designated object is contingent. 'The number which is the square of 3 and which numbers the planets' satisfies (28) and yet denotes a necessary entity. To get a notion of weak rigidity we need a more complicated conjoined clause:

$$(\exists x)\left[(\forall y)(\cdots y\cdots \equiv y{=}x)\ \&\ \Box(\forall y)(\cdots y\cdots \supset y{=}x)\right]\ \&\ (\exists x)\left[\cdots x\cdots\ \&\ {\sim}\Box(\exists y)(y{=}x)\right] \tag{29}$$

By analogous reasoning, we need (30) and (31) to construct definitions of strong and weak rigidity out of inclusive rigidity:

$$(\exists x)\left[(\forall y)(\cdots y\cdots \equiv y{=}x)\ \&\ \Box(\forall y)(y{=}x \supset (\cdots y\cdots\ \&\ (\forall w)(\cdots w\cdots \supset w{=}x)))\right]\ \&\ (\exists x)\left[\cdots x\cdots\ \&\ \Box(\forall y)(y{=}x)\right] \tag{30}$$

$$(\exists x)\left[(\forall y)(\cdots y\cdots \equiv y{=}x)\ \&\ \Box(\forall y)(y{=}x \supset (\cdots y\cdots\ \&\ (\forall w)(\cdots w\cdots \supset w{=}x)))\right]\ \&\ {\sim}\Box(\exists x)\left[\cdots x\cdots\right] \tag{31}$$

The two pairs of definitions so obtained are less perspicuous than (25) and (26). In neither case are the correlative schemata--(27) and (29) or (30) and (31)--duals of one another. In neither case do they bifurcate the class of rigid designators (so construed). Nor are they so simple and transparent as the schemata in (25) and (26). In particular, the added clauses in (29) and (30) are more complicated. There are simple alternatives to (29) and (30); for example, (32) is equivalent to but simpler than (29):

$$(\exists x)\left[(\forall y)(\cdots y\cdots \equiv y{=}x)\ \&\ \Box(\forall y)(\cdots y\cdots \supset y{=}x)\ \&\ {\sim}\Box(\forall y)(y{=}x)\right] \tag{32}$$

But this simplicity is gained at the sacrifice of an expressly genus-difference form of definition. That is, unlike the other schemata, (32) does not leave the rigidity clause intact.

Grant me that complete rigidity yields better definitions of these subordinate notions. How does this show which notion is the intuitive one and which is theoretically more central? On the second point, one of the roles of a central notion in a theory is to provide definitions of subordinate notions. If better definitions are obtained from one rendering of a fundamental notion than from others, then in the absence of opposing reasons, considerations of simplicity and elegance make this rendering preferable. Thus, in the absence of opposing reasons, the argument shows that complete rigidity is the preferable theoretical rendering of rigid designation. On the first point, the correlative notions of strong and weak rigidity are intuitively closely related to rigid designation. If rigidity is identified with complete rigidity, then the relation is indeed close since the two bifurcate it. But the relation is not so close if rigidity is instead identified with either of the other two. The argument thus offers further evidence that complete rigidity is the intuitive notion. However, considerations of simplicity and elegance--whether with respect to our intuitions or with respect to a theory--are not conclusive unless other factors do not outweigh them. We have not shown that there are no overriding factors here.

The three preceding arguments have indicated what we lose with exclusive and inclusive rigidity. We must admit more contrived examples We must give up a natural way of motivating rigidity out of other notions of designation. And we must accept less elegant definitions of some subordinate notions. The final argument examines what we must give up when we identify rigidity with complete rigidity. If we lose more with complete than with exclusive or inclusive rigidity, then we should reconsider.

We appear to lose little in choosing complete over exclusive rigidity. An exclusively rigid designator never would denote any other object. But, it may include superfluous details in its description component--i.e., details not needed to insure that it would never denote anything else. And these details can affect whether it would refer in some counterfactual situations. For example, Slote's 'the being that is identical with Nixon and a politician' includes such a detail. The addition of extra details can be carried to extremes. It is logically possible to incorporate enough details that the exclusively rigid designator would lack reference in any but the actual situation. By contrast, completely rigid designators contain no superfluous details that could ever affect their reference.[28] Thus, with complete rigidity we must give up a license for imprecision which, given our ignorance of essential attributes, we might want to retain. But surely complete rigidity is none the worse for this. We want the central notions of a theory to determine matters exactly. We have found here that in one respect complete rigidity will distinguish between the crucial and the superfluous more directly than exclusive rigidity. Thus, what complete rigidity loses _vis-a-vis_ exclusive rigidity does not detract from its usefulness in theories. Furthermore, it is scarcely an intuitive feature of rigidity that a rigid designator might lack reference in any but the actual situation. Therefore, what complete rigidity loses _vis-a-vis_ exclusive rigidity does not make it any the less intuitive.

We appear to lose more in choosing complete over inclusive rigidity. For we have to give up devices like Frege's for eliminating all failure of reference. 'The x such that x is identical with Nixon or, should Nixon not exist, is identical with 0' illustrates Frege's device. As we found earlier, it is inclusively, but not completely rigid. Of course, inclusively rigid designators need not be unfailing. But inclusive rigidity does permit us to make all designators in a regimented language unfailing. Complete rigidity does not. Thus, if we permit completely rigid designators of contingent objects in a language, we will have to face failure of reference problems that we might otherwise avoid. Now, since devices like Frege's are scarcely intuitive, in losing them we are not sacrificing an intuitive feature of rigidity. Hence, the claim that complete rigidity is the intuitive notion is not being challenged here. Rather, the challenge is to the claim that complete rigidity should be made central to the theory of reference. The objection is that in so doing we increase the burden on the theory. It must now treat failure of reference as a basic rather than as a peripheral phenomenon. The point can be made in another way. It is reasonable to contend that all the central notions of a theory of reference should be reflected in a philosophically ideal, regimented language. Now, if complete rigidity is central, then such a regimented language can have failure of reference problems when reference is determined on counterfactual bases. But failure of reference problems are the very ones that Frege thought a mathematically ideal, regimented language should not have.

Can we nevertheless justify making complete rigidity central? I think so. Our long second argument showed that in languages of a certain type unfailing designators can lead to contrasting truth-values. We find now that in languages of this type completely rigid designators can lead to failures of reference (i.e., when their reference is determined on counterfactual bases). Of course, we may not want a regimented language to have contexts in which the reference of singular referring expressions can be determined on any but the actual basis. But if the language has such contexts, then problems of one sort or another will have to be faced. On the one hand, we can avoid failure of reference problems if we make all designators unfailing. But then we will have no way of circumventing contrasting truth-value problems. On the other hand, we can circumvent contrasting truth-value problems by having a completely rigid designator for every object the language refers to singularly. But then we will require means for handling cases in which a designator would lack reference if its reference were determined on a counterfactual basis.

Now, the theory of reference should show us that regimented languages of the indicated type involve a trade-off between contrasting truth-value problems and failure of reference problems. The theory should indicate both that a choice must be made and what the choice will affect. But the theory can do this better if both unfailing designation and completely rigid designation are central notions in it. It can then explicate directly the trade-off between using an unfailing designator and using a completely rigid designator to designate a contingent entity. The theory cannot explicate this trade-off so directly if instead inclusively rigid designation is made central. Thus, as before, what complete rigidity loses _vis-a-vis_ inclusive rigidity does not detract from its usefulness in theories.

In sum, our fourth argument has shown that the obvious things we have to give up when we identify rigidity with complete rigidity are just as well given up. Sacrificing them does not detract from either the intuitive quality of the explicative power of rigid designation. However, again, the argument is not conclusive. We have not shown that the features we have discussed are the only ones lost in choosing complete over exclusive and inclusive rigidity. Nevertheless, until other features lost with complete rigidity are put forward, the argument will carry weight.

IV

None of the four arguments offered above for complete rigidity is conclusive. But together they make a strong case. We can restate their conclusion formally by means of the following definitions:

An instance of the schema
$(\iota x)(\cdots x \cdots)$
is a rigid designator if and only if (33)
the corresponding instance of the schema
$(\exists x)[(\forall y)(\cdots y \cdots \equiv y{=}x) \ \& \ \Box(\forall y)(\cdots y \cdots \equiv y{=}x)]$
is true.

An instance of the schema
$(\iota x)(\cdots x \cdots)$
is a nonrigid designator if and only if (34)
the corresponding instance of the schema
$(\exists x)[(\forall y)(\cdots y \cdots \equiv y{=}x) \ \& \sim\Box(\forall y)(\cdots y \cdots \equiv y{=}x)]$
is true.

Various subordinate notions can be defined. Exclusive and inclusive rigidity, as defined by (1) and (4), can be viewed as degenerate notions of rigidity, obtained by weakening its requirements. Strong and weak rigidity have already been defined by (25) and (26). We should note that a strongly rigid designator is an unfailing rigid designator, and vice versa. Another subordinate notion is that of a conditionally rigid designator--i.e., a singular referring expression which, if it designates something, is a rigid designator. One way to define this notion is via the material conditional:

> An instance of the schema
>
> $(\iota x)(\cdots x \cdots)$
>
> is a <u>conditionally rigid designator</u> if and only if the corresponding instance of the schema (35)
>
> $(\exists x)[(\forall y)(\cdots y \cdots \equiv y{=}x) \supset (\exists x)[\Box(y)(\cdots y \cdots \equiv y{=}x)]$
>
> is true.

On this definition, all singular referring expressions which happen not to designate are conditionally rigid designators. A stronger notion of conditional rigidity is obtained if we instead use the strict implication connective. But even with this all inconsistent singular referring expressions are conditionally rigid designators. A yet stronger notion is thus desirable, although we must leave the point here. A final subordinate notion is that of a potentially rigid designator--i.e., a singular referring expression which can be a rigid designator:

> An instance of the schema
>
> $(\iota x)(\cdots x \cdots)$
>
> is a <u>potentially rigid designator</u> if and only if the corresponding instance of the schema (36)
>
> $\Diamond(\exists x)[(\forall y)(\cdots y \cdots \equiv y{=}x) \;\&\; \Box(\forall y)(\cdots y \cdots \equiv y{=}x)]$
>
> is true.

These are the principal notions related to rigid designation.

Undoubtedly some people will want to identify rigid designation with potential rigidity rather than with complete rigidity. Their rationale derives from a possible world reading of its governing schema. The standard possible world interpretation for the schema is: in some possible world there exists exactly one object of which the description is true, and necessarily of this object and only of this object is the description true. This interpretation reads just like the one for complete rigidity except that 'in some possible world' replaces 'in the actual world'. In other words, so the rationale continues, the only difference between a completely rigid designator and a potentially rigid designator is whether the object described exists in the actual world. This suggests that a potentially rigid designator designates--indeed, rigidly designates--the object that satisfies its description, even though that object is a possible-yet-not-actual object. But then why restrict rigid designators to those that designate actual objects? The contribution which a designator makes to determining whether a sentence is true on a specific world is essentially the same whether the world is actual or just possible. Hence, to restrict rigid designators to those designating actual objects reflects an arbitrariness that the theory of language can ill afford.

Slote promotes this view when he characterizes an inclusively rigid designator as one that, should it pick out a certain entity in some one possible world, picks out the same entity in every other possible world in which that entity exists. Indeed, the view seems natural for anyone who accepts possible-yet-not-actual objects existing in possible, but non-actual worlds. Moreover, once one is inclined to the view, other considerations appear to recommend it. For example, on the possible world reading, potential rigidity seems a straightforward generalization of complete rigidity. More general characterizations of a notion are preferable. Similarly, if rigidity is identified with potential rigidity, then whether a singular referring expression is a rigid designator is a matter of logic, not a matter of contingent fact. And we would prefer to categorize linguistic items on formal grounds, as independently as possible of contingent facts. Finally, if complete rigidity promises to yield an account of denoting proper names, then potential rigidity promises to yield an account of proper names in general, including fictional names. And we would prefer a unified account of names.

All these considerations not withstanding, I think that rigid designation should not be identified with potential rigidity. Indeed, I think that there are decisive arguments against identifying the two.[29] However, for now I want to take a more moderate line, one requiring a less ambitious argument. I want to leave open the question whether potential rigidity is the well-motivated, straightforward generalization of complete rigidity that the two preceding paragraphs suggest. We can leave this open and still claim that rigidity is complete rigidity. For there is too much that is problematic in the rationale for identifying rigidity with potential rigidity. First, the possible world interpretation of the schema is open to dispute. Not only are possible world style interpretations of quantified modal formulas still controversial when taken literally, but there may also be more than one reasonable possible world interpretation of this schema.[30] Second, even if we accept the possible world interpretation of the schema, perhaps we need not accept such controversial entities as possible-yet-not-actual objects. Plantinga, for example, would agree with the possible world interpretation of the schema, yet he rejects such objects.[31]

Third, we can even grant both the possible world interpretation and the existence of nonactual objects, and yet deny that a potentially rigid designator designates anything. For suppose some singular referring expression is potentially rigid--i.e., suppose that necessarily its description is satisfied by exactly one possible, nonactual object. It is still not clear that the expression denotes the object. To secure reference, a possible world in which the object exists and rules for picking out the object in that world may also be needed. This point can be made in another way. The condition set down by the governing schema of complete rigidity is necessary and sufficient for rigidly designating an object in the actual world. Still, this schema may succeed only because of background rules governing reference that do not apply in the case of nonactual worlds. If so, then why should we conclude that the schema for potential rigidity gives a condition necessary and sufficient for rigidly designating an object in a nonactual world? We know too little about how reference is determined, especially in the case of possible worlds, to accept the generalization of rigid designation to nonactual objects uncritically.[32] Finally, even granting all the points we have questioned and also granting that proper names are rigid designators, fictional names need not be (generalized) rigid designators. If proper names are rigid, they are so in part by virtue of the way their reference gets fixed--e.g., by baptism. But it is not clear whether the reference of a fictional name can be comparably fixed.[33]

Such difficulties indicate that potential rigidity is not the intuitive notion of rigid designation. Furthermore, they discourage us from taking potential rather than complete rigidity as a central notion in the theory of reference. A better strategy is evident. For the present we should make complete rigidity the central notion. This will permit us to build a theory on comparatively firm grounds. Potential rigidity should for now be regarded as a related, but subordinate notion. As the theory develops, we can re-examine the status of potential rigidity. As we come to know more about singular reference, we may have grounds for extending rigid designation to include potentially rigid designators. I predict not. But this way we can handle the issue sensibly.

Let us recapitulate. We began by questioning whether we have a clear notion of rigid designation at all. Slote and Chandler provoked the question by calling our attention to three conflicting plausible definitions of rigidity. Our main accomplishment has been to put the doubts raised by Slote and Chandler to rest. Among the notions related to rigid designation, one--complete rigidity--is both the intuitive notion Kripke invokes and the notion of principal theoretical interest. The other notions--exclusive and inclusive rigidity and, for that matter, potential rigidity--are degenerate forms of rigid designation, obtained by weakening its conditions in one way or another. We have thus dispensed with the concern that provoked the original question. However, we have not yet quite answered the question. Our formal definition of rigidity, (33), contains an unexplicated *de re* modal operator. Whether *de re* necessity itself is clear remains controversial. Hence, our definition will not convince everyone that we indeed have a clear notion of rigid designation.

V

The *de re* modal operator in our definition of rigid designation is not necessarily a flaw. The notion may well be inherently modal, for better or worse at Quine's third grade of modal involvement. However, if it is not, then our definition is at best misleading. We thus have cause to be interested in whether rigid designation can be defined without resorting to modal notions. From the discussion so far, one would suspect not. But not all of the definitions proposed in the literature are modal. Christopher Peacocke has proposed a definition which is meant to be, and which on the surface appears to be nonmodal.[34] His definition for the case of a language L free of indexicals and ambiguity is as follows:

> A singular term t is a rigid designator in L if and only if there is an object x such that for any sentence G(t) in which t occurs, the truth condition for G(t) is that $\langle x \rangle$ satisfy G( ).[35] (37)

The idea behind this definition is that of "a certain object entering the truth-conditions of all the sentences of the language in which t occurs."[36]

If Peacocke has succeeded here, we will have to reconsider our definition--and we will probably have to abandon it. But has he succeeded? Does (37) provide an adequate definition of rigidity without implicitly or covertly relying on modal notions? This is the principal question of this section of the chapter.

Before addressing this question we should discuss a direct objection Peacocke lodges against a definition like ours. His main complaint against Kripke's definition is that it "quantifies over possible worlds and appeals to transworld identity."[37] But there is a simple way to avoid this complaint and still employ a modal definition: one can define rigid designation, as we have, in terms of uninterpreted modal operators. Peacocke calls attention to this move, and he responds to it. Specifically, he responds to the following definition of this type:

> A singular term t is a <u>rigid designator</u> in L if and only if it is true in L that nothing else might have been t or, more carefully, if and only if
> $\ulcorner(\exists x)[x=t \,\&\, {\sim}\Diamond(\exists y)(y=t \,\&\, y\neq x)]\urcorner$
> is true in L.[38] (38)

He contends that the indicated condition is neither necessary nor sufficient for rigid designation.

> This criterion is not necessary, for it presumes that the object language is capable of defining a possibility operator, which is not, intuitively, required for a language to contain rigid designators. It is not sufficient either; for if t is, intuitively, a rigid designator, then so, by this criterion, is $(\iota x)(x=t \,\&\, p)$, for any true sentence replacing p.[39]

How broad a conclusion he expects us to draw from this argument is not clear. The conclusion suggested is that no necessary and sufficient condition for rigidity can be stated in terms of uninterpreted modal operators. By intimation, then, the argument challenges our definition.

Part of what is going on here should be evident. The modal clause of the condition can be rewritten as follows:

$$\Box(\forall y)[y=t \supset y=x] \qquad (39)$$

Thus the condition that Peacocke says is not sufficient is just the condition for exclusive rigidity. And indeed, the schema he uses to argue that the condition is not sufficient is transparently akin to 'the x such that x is Nixon and a politician'--Slote's example of a designator which is exclusively, but not inclusively rigid. Accordingly, Peacocke's argument does not show that our condition, the condition for complete rigidity, is not sufficient.

Furthermore, his argument against modal definitions being necessary rests on a mistake. The definition should not require truth in L. The correct form of definition has the condition expressed in the metalanguage and requires simply that the instances of the schema be true. To see the absurdity of requiring truth in L, observe what happens if L employs for example intuitionistic rather than classical negation or if L has no negation connective at all: then the condition expressed is not necessary for rigidity in L even if L does contain a possibility operator. Of course, once the form of the definition has been corrected, there is nothing left to the argument that L must contain a possibility operator for the condition to be necessary. Consequently, Peacocke has not shown that our condition--and *a fortiori* the condition for exclusive rigidity--is not necessary for rigid designation. If his paper is to undermine our definition, the success of his alternative definition will have to do the undermining.

But does (37) successfully define rigid designation without indirectly relying on modal notions? As (37) stands, the question is difficult to answer. The meaning of the key phrase, 'the truth condition for G(t)', is not clear. To answer the question, we need to rephrase (37) to make the criterion it sets down explicit and clear. An obvious way to rephrase it is as follows:

> A singular term t is a *rigid designator* in L if and only if there is an object x such that for any sentence G(t) in which t occurs, G(t) is true if and only if $\langle x \rangle$ satisfies G ( ). (40)

(40) should be contrasted with the modal definition that is like it save for the last clause reading, '*necessarily* G(t) is true if and only if <x> satisfies G( ).' The logical connective in the last clause of (40) is the material biconditional. Hence (40) is clearly nonmodal. The question is whether it succeeds in capturing rigidity.

Consider first the case of a purely extensional language, L. Let 'the F' be short in L for 'the x such that x invented bifocals.' According to (40), 'the F' is a rigid designator in L. For let G(t) be a sentence of L. Since L is purely extensional, substitution of co-referring expressions preserves truth in L. Hence, if G(the F) is true, then <Franklin> satisties G( ); and if <Franklin> satisfies G( ), then G(the F) is true. In other words, in an extensional language something is true of Franklin if and only if it is true of Franklin as picked out by a description. The argument clearly generalizes: every denoting definite description in a purely extensional language satisfies (40). (40) is therefore too weak. In the case of extensional languages, it would have us relinquish the distinction between designators and rigid designators.

A similar result holds for certain nonextensional languages. Let L' be a language like L except for having modal operators at Quine's third grade of modal involvement. As before, let 'the F' be short for 'the x such that x invented bifocals.' Now consider the open sentence G( )--'necessarily x is identical with Franklin'--derived from 'necessarily Franklin is identical with Franklin.' Under what conditions is G(the F) true? It depends on how the scope ambiguity in G(the F) is resolved. G(the F) can be read as (41) or (42):

$$\Box(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ (x=\text{Franklin})] \tag{41}$$

$$(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ \Box(x=\text{Franklin})] \tag{42}$$

As is evident on inspection, (42) is true if and only if <Franklin> satisfies G( ). Moreover, the reasoning does not depend on our choice of G( ). Hence, if 'the F' is always read with wide scope _vis-a-vis_ modal operators in L', then for any open sentence H( ), H(the F) is true if and only if <Franklin> satisfies H( ). The argument clearly generalizes: every denoting definite description always read with wide scope _vis-a-vis_ modal operators in L' satisfies (40). Again (40) is too weak. In the case of certain nonextensional languages it could force us to concede that even some paradigmatic nonrigid designators are rigid.

The arguments of the last two paragraphs exploit a point we made when discussing how other notions give rise to rigid designation. The gist of (40) is to require of t that it designate the same object in every sentence in which it occurs. But as we noted earlier, in the right sort of language a singular referring expression may invariably designate the same object and yet not be a rigid designator. For example, consider a language in which the reference of every singular referring expression, regardless of context, is determined solely on the basis of what is actually the case. In such a language, any singular referring expression that happens to denote will invariably denote the same object. This is the point the arguments exploit. In a purely extensional language, the reference of Russellian definite descriptions is determined on the basis of what is actually the case; and to read a definite description with wide scope in a modal language is tantamount to determining its reference on the basis of what is actually the case. But just because a singular referring expression invariably designates the same object does not mean that it is a rigid designator. To think so is to confuse a motive for having rigid designators with the criterion that distinguishes them. A rigid designator is one that, even if things other than language had been different, would have denoted the object it now does, provided that object would still have existed; and it would in any case have denoted no other object. 'The inventor of bifocals' is not rigid because it would have denoted Jefferson if he had invented bifocals. And whether 'the inventor of bifocals' occurs in a purely extensional language or is always read with wide scope does not alter this fact. (40) is too weak, therefore, because it fails to impose a constraint on what t would have denoted if things had been different. If (37) is to define rigidity, the explication of 'the truth condition for G(t)' will have to yield such a constraint.

Peacocke notwithstanding, the obvious way to impose such a constraint is to use modal notions. But this must be done with care. For consider the strengthened version of (40) in which the connective in the last clause is the strict biconditional--i.e., the version in which the last clause reads 'necessarily G(t) is true if and only if <x> satisfies G( ).' This definition imposes on t the constraint that, even if things had been different, still G(t) would be true if and only if <x> were to satisfy G( ). The strengthened definition thus indeed imposes a constraint on what t would have denoted if things had been different. The trouble is that it imposes too strong a constraint. A singular term t will satisfy the modified definition only if t bears to the object it denotes a relation that is *essential* to this object. But no terms of languages of interest bear to the objects they denote a relation that is essential to these objects.[40] Hence, none will satisfy the modified definition. Strengthening (40) this way accordingly does not yield an alternative to our definition of rigidity. The modification goes too far. A rigid designator is one that would have denoted no other object even if things other than language had been different. The strengthened version of (40) drops the italicized qualification.[41]

We originally introduced (40) as a plausible way of spelling out the meaning of 'the truth condition for G(t)' in Peacocke's first definition of rigidity, (37). Now (40) is not something that Peacocke proposes. To the contrary, he expressly notes that the troublesome phrase must be explicated "in a way that prevents the notion of a truth-condition of a sentence from collapsing into that of a material equivalent of it."[42] Nevertheless, (40) and the strengthened version of it have served a purpose. They have helped us to see what the problem is in rephrasing (37). The basic problem is to impose the proper constraint on what t would have denoted if things had been different. An adequately rephrased definition must be stronger than (40), but weaker than the strengthened version of it. For Peacocke, there is of course an added dimension to the problem: the constraint must be imposed without resorting to modal notions.

How does Peacocke end up rephrasing (37)? It would not be helpful to quote his final definition here. In spelling out what he means by 'the truth condition for G(t)', he introduces elaborate technical apparatus. Some aspects of the apparatus remain rough. Much of it is designed to accommodate languages that include indexicals. To present his apparatus in sufficient detail to make his final definition intelligible would take us far afield. Happily, however, not every aspect of his apparatus is crucial from our point of view. Accordingly, instead of quoting his definition and providing the background it requires, we will sketch it only in enough detail to permit us to assess it as an alternative to ours.

Peacocke's strategy is to replace the notion of truth-conditions for sentences with the notion of a "Tarski-Davidson" truth theory for a language. Indeed, his final definition gives a criterion not for a term's being a rigid designator, but for its being treated as a rigid designator by such a truth theory. His account of truth theories is largely standard.[43] A truth theory contains for each sentence A of the object language a preferred derivation terminating in a theorem of the form

$$\overline{A} \text{ is true} \equiv B$$

where $\overline{A}$ is a structural-descriptive name of A, and B is a suitable translation of A in the metalanguage. (For example, if A contains no "semantical" predicates like 'true' and 'satisfies', then B must contain none to be suitable). The preferred derivation for A is the shortest one yielding a theorem of the required form. The resources used to effect such derivations are the usual ones: a function that translates the predicates and constants of the object language into predicates and constants of the metalanguage; a definition of truth in terms of satisfaction; axioms that recursively characterize what it is for a sequence of objects s to satisfy a sentence; and an evaluation function val that, given a sequence of objects and a term of the proper sort, picks out an object. The evaluation function is used at the base of the recursion to characterize satisfaction for atomic predications. That is, it enters derivations via biconditionals like

$$\text{s satisfies } \overline{P\tau} \equiv P'\text{val}(s,\tau)$$

where $\overline{P\tau}$ is a structural-descriptive name, P' is the translation of the object language predicate in the metalanguage, and val(s,τ) is the object from s assigned to τ.

Peacocke's account is distinctive primarily in that his evaluation function provides for indexicals. It is defined roughly as follows:

$$\text{val}(s,\tau) = \begin{cases} \text{the } i^{th} \text{ object in s if } \tau \text{ is the } i^{th} \text{ variable of the object language.} \\ \text{the } k^{th} \text{ object demonstrated by the speaker at the time the sentence is uttered if } \tau \text{ is the } k^{th} \text{ placeholder of the corresponding reduced sentence.} \\ \text{the object denoted by the translation of } \tau \text{ if } \tau \text{ is a constant.} \end{cases}$$

(The reduced sentence is obtained from the original sentence by replacing anaphorically distinct indexicals with distinct special terms called "placeholders.")

Peacocke's final rephrasing of (37) fixes on preferred derivations. Consider all of the sentences G(t) in which a term t occurs. According to Peacocke, a truth theory treats t as a rigid designator if and only if t gets treated in a certain way in the preferred derivations for these sentences. Stated informally, the treatment of t must satisfy two conditions:

i. The evaluation function must be applied either to t or to a term that exactly corresponds to t (e.g., a placeholder).

ii. The evaluation function must not treat t as a variable.[44]

For a first order language without indexicals, Peacocke's definition boils down to this: a truth theory treats t as a rigid designator if and only if the preferred derivation for each sentence G(t) in which t occurs treats t as a constant.[45]

The whole point of this definition, as far as I can see, is to deny that Russellian definite descriptions are rigid.[46] A truth theory treats a singular term as a Russellian definite description if and only if the term becomes expanded in the usual way during the course of preferred derivations. But once expanded, there remains no term to which the evaluation function becomes applied that exactly corresponds to the original term. The evaluation function ends up being applied to terms--namely variables--that represent parts of a Russellian definite description, and not to a term that corresponds to the entire description. Consequently, no Russellian definite description will satisfy Peacocke's final definition of rigidity.

This will not do. The paradigm of a rigid designator, 'the square root of 25', is a Russellian definite description.[47] In other words, if there are any rigid designators, then certain Russellian definite descriptions are among them. Hence, Peacocke's proposed definition does not capture the notion dubbed "rigid designation" by Kripke. Peacocke's efforts seem directed toward a different notion, one for which he has infelicitously chosen the name Kripke used. Peacocke seems preoccupied with the notion of a term's occurring only in singular term positions in the underlying logical form of any sentence. He speaks of "genuine singular terms"[48] and of "a criterion of rigid designation or genuine reference,"[49] where 'genuine reference' is presumably meant to contrast with reference that includes descriptive or predicative elements. Peacocke may have adequately characterized such a notion. We have not provided enough details of his account to discuss the matter. He may even have given a sufficient condition for rigidity, although arguments are clearly needed to support this claim. But he has definitely not given a necessary condition for rigidity. Consequently, his final definition is not an alternative to ours. Nor is it in any way evidence that (37) can be rephrased to capture rigidity successfully without resorting to modal notions.

VI

Peacocke is not alone in taking rigid designation to be akin to pure reference. In "On Predicating Proper Names" Michael Lockwood speaks of "the Kripke-Donnellan conception of proper names as "rigid designators" or purely referential devices."[50] As the wording suggests, Lockwood considers Kripke's distinction between rigid and nonrigid designators to be closely related to Donnellan's distinction between referential and attributive uses of singular terms.[51] But he does not think that the two distinctions are quite the same. For example, he grants that 'the cube of 408' is rigid even when used attributively. Still, he hastens to add that "the rigidity of 'the cube of 408' here stems from an irrelevant source....Here we have a term whose rigidity is insured, quite independently of the speaker's intentions, by the necessity of a certain mathematical proposition."[52] On his view, then, referential uses of singular terms form the salient subclass of rigidly designating uses. Cases in which a singular term is used attributively and yet rigidly are aberrations. Nevertheless, having conceded that modal considerations make 'the cube of 408' always rigid, Lockwood might be expected to adopt a modal definition of rigidity. But instead, he offers a nonmodal definition along lines suggested to him by Peacocke:

> Let S(t) be a sentence, containing a definite singular term t. In uttering this sentence, by way of making an assertion, a person will be using t as a rigid designator if and only if it is a necessary and sufficient condition of the truth of what S(t) is being used to assert that x satisfy the predicate expressed by the context S( ).[53] (43)

This definition is like (40) except that Lockwood drops the universal quantification over sentences in order to extend his definition to languages with ambiguities. Since (43) is a weakened version of (40), it is open to the objections we gave earlier. Notice here that our (40) is not entirely a straw man. Once one chooses (in Peacocke's words) to base rigid designation on "the idea of a certain object entering the truth conditions" of the sentences in which the designating term occurs, (40) lurks nearby.

It is difficult to see why anyone would confuse Kripke's rigid designation with a notion of pure reference. In "Identity and Necessity" and "Naming and Necessity" Kripke does claim both that proper names are rigid designators and that proper names are nonconnotative.[54] But these two claims need not amount to the same thing. Similarly, in one footnote he uses the phrase, "both a 'referential' (rigid) and a non-rigid reading of the description."[55] But the scare-quotes around 'referential' should warn against quick conclusions. Besides, the textual evidence against identifying rigid designation with pure reference is clear and overwhelming. As we have said repeatedly, Kripke's paradigm of rigid designation is not purely referential. Also, in a footnote in "Identity and Necessity" he plainly indicates that Russellian definite descriptions can be rigid:

> Some logicians have been interested in the question of the conditions under which, in an intensional context, a description with small scope is equivalent to the same one with large scope. One of the virtues of a Russellian treatment of descriptions in modal logic is that the answer (roughly that the description be a 'rigid designator' in the sense of this lecture) then often follows from the other postulates for quantified modal logic.[56]

Furthermore, there is clear textual evidence against identifying Kripke's distinction between rigid and nonrigid designators with Donnellan's distinction between referential and attributive uses of definite descriptions. In "Naming and Necessity" Kripke calls attention to Donnellan's distinction. But he then elects to confine his remarks to attributive uses--i.e., to uses for which 'the referent of the description' means "the object uniquely satisfying the conditions in the definite description."[57] Donnellan's referential uses of definite descriptions may well be rigidly designating uses, as Peacocke and Lockwood think. But nowhere in "Identity and Necessity" or "Naming and Necessity" does Kripke make this claim.[58]

Perhaps the tendency to confuse rigid designation with pure reference comes from taking proper names to be paradigms of rigidity.

This in turn may be prompted by confusing the thesis that proper names are rigid with the claim that they are nonconnotative. These latter mistakes are serious. They deserve comment even though it requires a brief digression. For these mistakes significantly misrepresent what Kripke is doing in "Identity and Necessity" and "Naming and Necessity." Most notably, they rob some of his central arguments of their force.

For example, in both works Kripke argues that "an identity statement between names, if true at all, is necessarily true"--e.g., if 'Hesperus is identical with Phosphorus' is true, then 'If Hesperus exists, Hesperus is identical with Phosphorus' is necessarily true.[59] His argument has the following form:[60]

(i) Denoting proper names are rigid designators.
(ii) Any identity statement between rigid designators, if true, is necessarily true. (44)
(iii) ∴ Any identity statement between denoting proper names, if true, is necessarily true.

The notion of necessary truth here is the weak notion exhibited in the above example. Kripke's defense of premiss (ii) is straightforward.[61] A rigid designator denotes the same object in every possible world in which that object exists, and it denotes no other object in any possible world. Hence, if two rigid designators denote the same object in the actual world, then they denote the same object in every possible world in which that object exists and they denote no other objects in any possible world. The premiss can also be defended without explicitly resorting to possible world reasoning. From (iv) and (v) below and our definition of rigidity, (33), we can derive (vi) via "standard", intuitive modal reasoning (e.g., within less controversial parts of standard modal predicate logics):[62]

(iv) '$\imath xFx$' and '$\imath yGy$' are rigid designators

(v) $\imath xFx = \imath yGy$

(vi) $\Box[((\exists x)Fx \vee (\exists y)Gy) \supset (\exists! x)(\exists! y)(Fx \,\&\, Gy \,\&\, x=y)]$

Accordingly, in (44) the premiss that carries the weight of the argument is (i). If one is to disagree with Kripke's conclusion, then short of objecting generally to standard modal reasoning, one must disagree with the thesis that proper names are rigid designators.

This thesis seems no less crucial to Kripke's argument against strong versions of the cluster theory.[63] At least it is crucial to my reconstruction of the argument. Let X be a denoting proper name and let the ϕ's be the properties in the cluster that, according to the theory, determines the reference of X. Strong versions of the theory are those that include (i):

(i) ⌜If X exists, then X has most of the ϕ's⌝ expresses a necessary truth.

Kripke contends that the proponent of a cluster theory must grant some lattitude in the choice of X and the ϕ's. In particular, they can be so chosen that, for someone's idiolect, the ϕ's all hold contingently (if at all) of the individual having most of them in the actual world. Kripke makes the point this way: "it does not seem that it should be trivially true on the basis of a theory of proper names" that, for example, some of the properties commonly attributed to Aristotle are properties that are essential to him.[64] Accordingly, X can be so chosen that the proponent of (i) must allow a stipulation of (iii) along with his stipulation of (ii):[65]

(ii) In the actual world the individual denoted by X, and only this individual, has most of the ϕ's.

(iii) The individual that has most of the ϕ's in the actual world might still have existed and yet had none of the ϕ's.

I take the form of the argument to be that, since proper names are rigid designators, such an X is a counterexample to (i). We thus add one more premiss:

(iv) Denoting proper names are rigid designators.

Since X is a denoting proper name, from (iv) we get (v), and from (ii) and (v) we get (vi):

(v) X is a rigid designator.

(vi) With respect to any possible world w, X designates the individual that has most of the ϕ's in the actual world, provided this individual exists in w.

But (vii) is a consequence of (i) and (vi):

(vii) With respect to any possible world w, if the individual that has most of the ϕ's in the actual world exists in w, then this individual has most of the ϕ's in w.

And (vii) contradicts (iii) since (iii) can be paraphrased as (viii):

(viii) There is some possible world w in which the individual that has most of the ϕ's in the actual world exists and yet has none of the ϕ's.

Therefore, since (ii) and (iii) are true for X by stipulation, (i) is false.

Now without (iv) we do not get (vi). But without (vi), (i), (ii), and (iii) are compatible. They can be reconciled with one another in the context of counterpart theory. That is, if with respect to every possible world w, X denotes the individual (if any) that has most of the ϕ's in w, then (i), (ii), and (iii) are clearly compatible.[66] Accordingly, like the argument on the necessity of identity statements, the argument in "Naming and Necessity" against (i) seems to hinge on the thesis that proper names are rigid. If the proponent of (i) is to respond, then short of resisting the stipulation of (iii) or short of objecting to the modal reasoning, he must disagree with this thesis.[67]

Of course, the thesis that names are rigid is not the only major claim about names put forward in "Identity and Necessity" and "Naming and Necessity." Prominent in the lectures are other claims needed to support this thesis. For example, there is the claim that proper names are nonconnotative. The exact claim here is not clear. For present purposes the following approximation will suffice: proper names are not synonymous with any definite descriptions (or clusters of descriptions) that are suitable for determining their reference. The "suitability" restriction is meant to exclude descriptions like 'the individual identical with Benjamin Franklin'.[68] Now, we seem to know of no description, suitable for picking out an entity not yet named, that is necessarily and necessarily uniquely true of any contingent entity. Hence, if a proper name of a contingent entity were synonymous with a description (or a cluster), the description (or cluster) in question would have to be contingently or contingently uniquely true of the individual picked out. But then this proper name would not be a rigid designator.[69] Similar reasoning applies to the claim that the reference of a proper name can be fixed via a description without giving the name a meaning. Kripke concedes that the reference of some names may be fixed via descriptions that are only contingently or contingently uniquely true of the individuals names. But if fixing reference in this way were to give names meaning, then those names whose reference is so fixed would not be rigid.[70]

Accordingly, given that we do not know of suitable descriptions that are necessarily and necessarily uniquely true of contingent entities, and given that the reference of some names may nonetheless be fixed via contingent marks of the entities names, Kripke needs these other claims to retain the thesis that names are rigid. But these claims are nevertheless logically independent of this thesis. They could be false, and yet it be true. For, it is logically possible for the reference of each proper name to be fixed by means of a description that is necessarily and necessarily uniquely true of the individual named. In this case proper names would be rigid even if they were connotative and even if the way in which their reference is fixed gave them their meaning. Consequently these other claims should not be confused with the thesis that proper names are rigid. Only the latter is crucial to the two arguments just discussed.

Consider now the consequences of taking proper names to be paradigms of Kripke's notion of rigid designation. If they are paradigms, then any explication of rigidity will be constrained by their having to satisfy it. Hence, if they are paradigms, the claim that they are rigid is trivially true (if anything is rigid). But arguments central to "Identity and Necessity" and "Naming and Necessity" turn on this claim. If this claim is trivially true, these arguments can amount to little more than bare assertions of their conclusions. Therefore, to take proper names to be paradigms of rigidity, as Peacocke and Lockwood perhaps do, is to rob some of the central arguments in the text of their force. Even if Peacocke and Lockwood do not take names to be paradigms of rigidity, I suspect that they have underestimated how much weight the thesis that proper names are rigid carries in Kripke's lectures. It is a strong thesis. It says that in salient respects proper names are like certain definite descriptions, viz. those that are necessarily and necessarily uniquely true of the objects they denote. Thus to think that no Russellian descriptions are rigid or even to think that rigidity is akin to pure reference is to distort and weaken the thesis. The pivotal role this thesis plays in the lectures is strong evidence that Peacocke and Lockwood have misconstrued Kripke's notion.[71]

VII

The preceding digression should have put Peacocke's definition to rest. Since Russellian definite descriptions can clearly be rigid if anything can, Peacocke's definition is wrong. But what about his approach? Can the apparatus he uses to rephrase (37) provide a nonmodal definition of rigidity? That is, can rigid designators be precisely characterized in terms of the way they are handled in the preferred derivations of Tarski-Davidson truth theories?

The answer appears to be no, if the preferred derivations do no more work than they do in standard Tarski-Davidson truth theories. In a standard theory the working steps of preferred derivations give satisfaction conditions for molecular constituents of sentences in terms of the components of these constituents. Every preferred derivation thus constitutes a step-by-step compositional analysis of what the theory takes to be the logical form of a sentence. Accordingly, how preferred derivations handle a term depends first on what the theory takes to be the term's logical form and second on what the theory takes to be the way the term contributes to the logical form of sentences. But as we have noted, the difference between rigid and nonrigid designators does not seem to be a matter of their logical form or of the way they contribute to the logical form of sentences. Some Russellian descriptions are rigid, and some are not. The differences between those that are and those that are not fail to be systematically reflected in the logical form of sentences. How then are rigid designators to be discriminated in preferred derivations?

This objection can be made precise. Consider "Russellian English", a language like English except that it is restricted to purely extensional, first order sentences and all definite descriptions in it are read as Russellian descriptions. Both 'the number of pardoned Presidents' and 'the number of even primes' occur in it. A standard truth theory for it will construe these two comparably--i.e., the preferred derivations will handle them comparably. Yet 'the number of pardoned Presidents' is intuitively nonrigid, and 'the number of even primes' is intuitively rigid. That one is rigid, and not the other, is not altered by their occurring in Russellian English rather than in English. The same point can be made about other syntactically like pairs: e.g., 'the number of books in Aristotle's *Metaphysics*' and 'the number of modalities in Lewis's S4', or 'the set of golden mountains' and 'the set of composite primes'. Now, rigid designators can be precisely characterized in terms of how they are handled in the preferred derivations of standard truth theories only if the preferred derivations of each such theory handle rigid designators in a way that is distinctively different from the way they handle nonrigid designators. The truth theory for Russellian English fails this necessary condition. Accordingly, rigid designation cannot be defined on the basis of the way terms are handled in the preferred derivations of standard Tarski-Davidson truth theories.

Can Peacocke's approach be saved by extending Tarski-Davidson truth theories? That is, can the work done in preferred derivations be increased to the point where they might handle rigid designators distinctively? Perhaps. However, even if it can, this seems an ill-motivated way of saving the approach. Consider how truth theories would have to be extended. Standard preferred derivations do not handle rigid designators distinctively for a good reason. A standard Tarski-Davidson truth theory is a purely linguistic theory. It does not tell us which sentences are true; it only gives us the conditions under which sentences are true. But the difference between definite descriptions that denote and those that do not--and _a fortiori_ the difference between those that rigidly designate and those that do not--is in crucial respects extralinguistic. Hence, preferred derivations will not be able to handle rigid designators distinctively unless they include steps warranted by extralinguistic considerations. In other words, to make Peacocke's approach work, the domain of Tarski-Davidson truth theories will have to be extended to various extralinguistic matters.

Of what significance is the apparent failure of Peacocke's approach? Should we draw any conclusion from it? To answer these questions, we must examine the reason for framing the definition in terms of what happens in preferred derivations. Why pursue this indirect way of using the notion of a truth theory? A definition like (45) is an obvious, more direct approach to using this notion to render (37) precise:

> A singular term t of a language L is treated as a <u>rigid designator</u> by a truth theory of L if and only if there is an object x such that, according to the truth theory, any sentence G(t) in which t occurs is true if and only if $\langle x \rangle$ satisfies G( ). (45)

(45) has some virtues. It is not modal. Its notion of truth conditions is not obscure. And it does not require preferred derivations to handle rigid designators distinctively. It has, however, a decisive fault. It is open to the same objections as (40). For instance, every denoting definite description of a purely extensional language satisfies it.

Peacocke expressly recognizes this.[72] He turns to a definition in terms of preferred derivations in the effort to get a criterion that is not satisfied by nonrigidly designating definite descriptions. His appeal to preferred derivations is accordingly in response to a primary desideratum. It is intended to accomplish without modality what the modal clauses of other definitions accomplish, <u>viz.</u> to impose the proper constraint on what a term would have denoted if things besides language had been different. Earlier we identified the problem of satisfying this desideratum as the problem that is basic to rephrasing (37) without using modal notions. Indeed, we examined (40) just to bring this problem out. It is easy to eliminate the obscurity of (37). The problem is to avoid resorting to modal notions and yet formulate the constraint that excludes nonrigidly designating definite descriptions. The significance of the failure of Peacocke's approach is thus apparent. The failure puts us back where we began, with an obscure (37) and an inadequate (40).

What conclusion should we draw from the failure of Peacocke's approach? The issue is whether (37) can be rephrased somehow to yield an exact criterion of rigidity without resorting to modal notions. So far we have shown only that Peacocke has given no evidence that it can. We have not yet shown that it cannot. However, we do have some evidence pointing to the stronger conclusion. There is a pattern to the failures of the various attempts to rephrase (37). Both Peacocke's approach and definitions like (40) and (45) fail in the same respect: they do not mark the distinction between rigidly and nonrigidly designating definite descriptions. If a denoting definite description could have denoted some object other than the one it does had things other than language been different, then it is a nonrigidly designating definite description. If, to the contrary, it could have denoted no other object and would still have denoted the object it does provided that object still existed, then it is a rigidly designating definite description.[73]

The modal clause in our definition, (33), specifically marks this distinction. It requires rigidly designating definite descriptions to be necessarily and necessarily uniquely true of the objects of which they are true. The modal clauses in the definitions of exclusive and inclusive rigidity, (1) and (4), also mark this distinction, although they draw the boundary at slightly different places. For a definite description to be rigid, it must satisfy some modal condition--either ours or one much like it. The attempts to rephrase (37) simply fail to come up with a nonmodal condition that will be satisfied only if a modal condition like ours is satisfied. These attempts thus fail exactly where they should if there is no way to rephrase (37) successfully without resorting to modal notions. The pattern of failure hence gives us some evidence confirming our suspicion that (37) cannot be rephrased successfully without using modal notions.

Two questions prompted our looking at Peacocke's paper in detail. First, is his definition of rigid designation in any way preferable to the one we defended at length in the last section? Our conclusion is no. Second, and more important, is there reason to doubt that rigid designation is a _de re_ modal notion, as our definition and the others we examined earlier imply? Our study of Peacocke has given us no reason to doubt and some reason to believe that rigid designation is an intrinsically modal notion. But is the modality _de re_ modality? The need for modality is clearest in connection with discriminating between rigidly and non-rigidly designating definite descriptions. We have found that this distinction can be marked naturally by a _de re_ modal condition. But does it have to be? Can it instead be marked by means of a _de dicto_ modal condition? That is, for each denoting definite description, is there a proposition such that the description is rigid if and only if the proposition is necessarily true?

An obvious candidate for such a _de dicto_ condition is given in (46):

> A denoting definite description of the form
> the x such that Fx
> is a rigid designator if and only if the proposition
> expressed by the corresponding sentence of the form (46)

the object that happens to satisfy F satisfies
and uniquely satisfies F
is necessarily true.

Symbolically, the condition is:

$$\underline{F(\iota xFx)\ \&\ (\forall y)(Fy \equiv y=(\iota xFx))}\ \text{is necessarily true.} \tag{47}$$

This condition admits of two readings, depending on whether we interpret 'necessarily true' in the usual strong way or in the weak way Kripke suggests. Consider the strong reading first, as given by (48), with the definite description operators having narrow scope:

$$\Box\left[F(\iota xFx)\ \&\ (\forall y)(Fy \equiv y=(\iota xFx))\right] \tag{48}$$

Once the description operators are expanded, (48) simplifies to (49):

$$\Box(\exists x)\left[Fx\ \&\ (\forall y)(Fy \equiv y=x)\right] \tag{49}$$

But (49) amounts to nothing more than an instance of the schema governing "unfailing designation," as defined by (17). The strong reading of (47) fails to mark the distinction between rigidly and nonrigidly designating descriptions for the same reason that unfailing designation is not extensionally equivalent to rigid designation. (49) says that the predicate F must be satisfied by some object; but it fails to say that the predicate must be satisfied by the very object that happens to satisfy it.

Consider now the weak reading of (47), in which 'necessarily true' is taken roughly as 'necessarily true provided the object that happens to satisfy F exists'. The condition in this case is given by (50), again with the description operators having narrow scope:

$$\Box\left[(\exists x)(Fx\ \&\ (\forall y)(Fy \equiv y=x)) \supset (F(\iota xFx)\ \&\ (\forall y)(Fy \equiv y=(\iota xFx)))\right] \tag{50}$$

Once the description operators are expanded and the expression simplified, the necessitated clause is found to be a tautology:

$$[(\exists x)(Fx \,\&\, (\forall y)(Fy \equiv y=x)) \supset (\exists x)(Fx \,\&\, (\forall y)(Fy \equiv y=x))] \quad (51)$$

Hence, the weak reading of (47) fails to mark the distinction between rigidly and nonrigidly designating descriptions since every definite description satisfied it. (51) can be revised to avoid this objection:

$$[(\exists x)(Fx \,\&\, (\forall y)(Fy \equiv y=x)) \supset \Box(\exists x)(Fx \,\&\, (\forall y)(Fy \equiv y=x))] \quad (52)$$

But (52) is like (49) in failing to say that the predicate F must be satisfied by the very object that happens to satisfy it.

Thus the simplest proposals for a de dicto distinction between rigidly and nonrigidly designating definite descriptions do not work. Perhaps there is a subtler way to construct a de dicto condition marking the distinction. I do not see how to close off this possibility, though we should keep in mind the failure of other attempts to replace de re with de dicto modality.[74] At any rate, our use of de re necessity in (33) is naturally motivated. It is the natural way to mark the distinction between rigidly and nonrigidly designating descriptions. By contrast, the attempts we have examined to fashion a nonmodal or a de dicto distinction fail to specify that the description must be satisfied by the very same object that happens to satisfy it. The de re modal distinction succeeds where these approaches fail. This was the upshot of the long argument in Section II: the de re modal aspect of rigid designation specifically shuts off possibilities left open by the nonmodal notion of designation simpliciter and by the de dicto modal notion of unfailing designation.

The conclusion is clear. All the evidence we have is that, for better or worse, rigid designation is intrinsically a de re modal notion. Earlier we concluded that (33) is clearly preferable among the suggested *de re* modal definitions of rigidity. We now further conclude that (33) is the preferred definition among those in the literature generally. (33) is the best working definition we have.

Notes

1 Michael A. Slote, Metaphysics and Essence, (New York: New York University Press, 1975), pp. 72-75.

2 Saul Kripke, "Naming and Necessity," Semantics of Natural Language, ed. Donald Davidson and Gilbert Harman, (Dordrecht: Reidel, 1972), p. 269.

3 Hugh S. Chandler, "Rigid Designation," The Journal of Philosophy, LXXII, (July 17, 1975), pp. 363-369.

4 Indeed, I will never address the question whether proper names are rigid designators, though in Section VI I will examine the significance of claiming that they are. Throughout this chapter and the next I will be concerned with rigid designation in regimented, not in natural languages.

5 Kripke carefully calls his arguments that proper names are rigid designators "intuitive." I use the term here for the same reason. A nonintuitive argument for something's being rigid is hard to come by at this stage.

6 Willard Quine, Methods of Logic, (New York: Holt, Rinehart, and Winston, 1959), pp. 220-224.

7 The question whether an expression is a designator only if it designates an actual individual will be addressed more forthrightly at the end of Section IV.

8 Section II of the paper will examine complete rigidity at some length. By the way, the evident redundancy of the schema for complete rigidity is intentional and will be retained throughout. It serves two purposes. It lends perspicuity. And it avoids prejudging questions about quantified modal logic.

9 The schema for inclusive rigidity in the text does not require the description to be necessarily true of at most one individual should the now designated individual not exist. Hence, there is a slightly

stronger notion of inclusive rigidity, governed by the following schema:

$$(\exists x)\left[(\forall y)(\cdots y \cdots \equiv y{=}x) \;\&\; \Box(\forall y)(y{=}x \supset \cdots y \cdots)\right] \;\& \\ \Box\left[(\exists w)(\cdots w \cdots) \supset (\exists w)(\cdots w \cdots \;\&\; (\forall z)(\cdots z \cdots \supset z{=}w))\right]$$

Which of the two schemata one prefers will make no difference to our arguments.

10 Strictly speaking, the substitution instances of the schemata entail one another, not the schemata.

11 As before, we say 'intuitively' here to avoid overstatement.

12 Identical twins come from the same zygote. A clone of Slote would provide another example.

13 E.g., '7' is the name of the successor of 6.

14 Gottlob Frege, "Sense and Reference," Translations from the Philosophical Writings of Gottlob Frege, (Oxford: Blackwell, 1960), p. 70. I owe the example to Richard Cartwright.

15 There is another way out of using free-logic. Assume that being human is essential to Nixon and let the designator be 'the x such that x=Nixon or should there be no humans x=0'.

16 Saul Kripke, "Identity and Necessity," Identity and Individuation, ed. Milton K. Munitz, (New York: New York University Press, 1971), p. 144.

17 We say 'for example, to Jefferson' because the specific counterfactual basis on which reference is to be determined is not explicitly clear, given (18) or (19) alone. We seem to require pragmatic factors as well to identify the counterfactual situation in question.

18 The ambiguity here is usually called one of scope. As we shall see in the next chapter, this is somewhat misleading. The ambiguity is one of reference. In our regimented language we make the two readings explicit by means of scope distinctions. But all the scope distinctions amount to is a way of specifying unambiguously the basis on which reference is to be determined.

19 In talking this way, we are leaving open the question whether the expression refers at all and the question of how its reference gets fixed.

20 Presumably, then, p must contain much more detail than it would appear to in the examples to follow since it must suffice to fix the reference of 'the x such that Fx' in the counterfactual situation in question.

21 The Federal Convention is now popularly known as the Constitutional Convention. Jefferson was in France while the Constitution was being drafted in Philadelphia.

22 We say 'not true' here to avoid having to choose between saying 'false' and 'lacking in truth value'. For present purposes the relevant truth-value distinction is between true and not true.

23 In other words, the scope ambiguity persists.

24 This observation was suggested by Jerry Katz.

25 Kripke, op. cit., p. 146.

26 This line of argument was suggested by Ali Akhtar.

27 Kripke, "Naming and Necessity," p. 270.

28 A completely rigid designator may have a redundant description component and hence have superfluous details. For example, 'the number which is the successor of 8 and the square of 3' contains more detail than is needed for it to designate 9 rigidly. But the superfluous details of a completely rigid designator are ones that can never affect reference.

29 One argument will be given in the next chapter.

30 Several points are in controversy here: whether to take a realistic, in contrast to heuristic, view of possible world interpretations; whether to interpret the possibility operator followed by the existential quantifier as existence of an object in some possible world; and what the truth conditions are for the formula.

31 Alvin Plantinga, The Nature of Necessity, (Oxford: Oxford U. Press, 1974), Chapter VII.

32 In the addenda to "Naming and Necessity" Kripke calls attention to the problem noted here. He says, "Similarly, I hold the metaphysical view that, granted that there is no Sherlock Holmes, one cannot say of any possible person that he would have been Sherlock Holmes, had he existed. Several distinct possible people, and even actual ones such as Darwin or Jack the Ripper, might have performed the exploits of Holmes, but there is none of whom we can say that he would have been Holmes had he performed these exploits. For if so, which one?" (p. 764.) This question--for if so, which one?--is the one we are claiming may well remain open even if there are possible, nonactual objects satisfying the description of a potentially rigid designator in some possible world.

33 Cf. Kripke, ibid., p. 764.

34 Christopher Peacocke, "Proper Names, Reference, and Rigid Designation," in Meaning, Reference, and Necessity, ed. Simon Blackburn, (Cambridge: Cambridge U. Press, 1975), pp. 109-132.

35 Ibid., p. 110. The constraint against indexicals is removed later in Peacocke's article (as we shall see), but the constraint against ambiguities remains.

36 Ibid.

37 Ibid.

38 Ibid., p. 114. Peacocke uses 't' as a name of a term and as a term. I trust the reader can sort out any potential use-mention confusions deriving from this double duty.

39 Ibid. Again I trust the reader can sort out any potential use-mention confusions in the quoted passage.

40 I qualify this remark because perhaps there is a language in which the denotation relation is the identity relation, in which case terms would bear essential relations to their denotata.

41 Expressing such an intermediate definition in the framework of (40) appears to be no small task; but our definition, (33), is just such an intermediate, stated in a less confining framework.

42 Peacocke, op. cit., p. 115.

43 The presentation in the text is rather sketchy. A more thorough presentation of the standard account can be found in "On the Frame of Reference," by John Wallace, in Semantics of Natural Language, ed. Harman and Davidson.

44 Two phrases in this characterization make it informal: 'corresponds to t' and 'as a variable'. Correspondence is problematic because constituents like 'Ft' may be replaced by constituents like 'Gt & Ht' during a preferred derivation. Exact correspondence is not easily made formally precise because of such expansions. Treatment of a term as a variable in a preferred derivation is more readily made formally precise, but somewhat tediously. I trust the characterization in the text is adequate for the purposes at hand. For those interested, Peacocke's actual definition is as follows:

> Truth theory T treats expression α of language L as a rigid designator iff for any sentence $G(\alpha)$ of L containing α, given as premisses specifications of the objects demonstrated by person p at time t, then: in any maximally short derivation in T from those premisses of a target biconditional of the form
>
> $$T(\overline{G(\alpha)},p,t) \equiv A$$
>
> where A does not contain sats, the evaluation function of T is applied to some expression e (e.g. α itself or a place-holder) which occupies the place of this occurrence of α via the application of the satisfaction axiom for the atomic predicate in which the given occurrence of α features as argument in the original sentence $G(\alpha)$; where the evaluation is such that either given p, t, and the fixing of the indexical referents, there is a sequence of objects s_0 such that evaluation of e only with respect to s_0 occurs in maximally short derivations, or e is such that the result of evaluating it with respect to any sequence of objects is always the same. (P. 125.)

The next to last clause is intended to distinguish between variables and indexicals, and the final clause, between variables and constants.

The definition assumes that the original sentence is expressed in terms of atomic predicates, so that the problem alluded to above is avoided.

45 Presumably, Peacocke wants to restrict his definition to denoting constants. However, if we follow Church's terminological conventions, the added qualification is gratuitous. (Cf. Church, Introduction to Mathematical Logic, (Princeton: Princeton U. Press, 1956), p. 4.

46 Restricting this claim to Russellian definite descriptions is crucial. Peacocke distinguishes between two kinds of definite descriptions, Russellian and "entity-invoking." The entity invoking use of a definite description is something I am not altogether clear about. Peacocke says the following: "if, in an utterance of "the F is G", what is strictly and literally said would equally and appropriately be said by an utterance of "that F is G", then "the F" functioned as a rigid designator. I shall label this an entity-invoking use of the description." (Op. cit., p. 117.) He introduces the apparatus for indexicals in order to account for entity-invoking uses of definite descriptions being rigid while Russellian uses are not. That Peacocke definitely wants to deny that Russellian definite descriptions are rigid can be seen at many places in his article; for example, see p. 110, p. 117ff, p. 122ff.

47 I am being somewhat presumptuous here in claiming that Kripke intended 'the square root of 25' to be Russellian. Some people may want to claim that it is for example Strawsonian. But notice that either way it comes out to be rigid, if anything is rigid. (This is subject to our earlier proviso that Kripke is talking about the domain of positive integers.) Kripke's use in no way seems entity-invoking. Indeed, throughout the lectures, as we shall see shortly, Kripke shows a distinct preference for reading definite descriptions in Russell's way.

48 Peacocke, op. cit., p. 126.

49 Ibid., p. 119.

50 Michael Lockwood, "On Predicating Proper Names," Philosophical Review, LXXXIV, (October, 1975), pp. 471-498.

51 Donnellan's distinction is between referential and attributive uses of definite descriptions. Lockwood extends the distinction, and we follow him for the sake of discussion. Lockwood wants to claim that proper names occur in attributive uses.

52 Ibid., p. 487.

53 Cf. ibid., pp. 485 and 487.

54 What Kripke expressly claims in "Naming and Necessity" is that Mill is "more-or-less right about 'singular' names" (p. 322), and he later says that "the present view endorses Mill's view of singular terms." (P. 327.) I have chosen to express this position in terms of Mill's 'nonconnotative'; I do not think that Kripke uses this term.

55 Kripke, "Identity and Necessity," p. 149n.

56 Ibid., p. 140n. I presume that this footnote is where Peacocke's idea that rigid designators are scopeless comes from. But scopeless in what sense? Let '$\imath xFx$' be a rigid designator. Consider '$\Box G(\imath xFx)$'. This admits of three readings:

$(\exists x)((\forall y)(Fy \equiv y=x)\ \&\ \Box Gx)$

$(\exists x)\Box((\forall y)(Fy \equiv y=x)\ \&\ Gx)$

$\Box(\exists x)((\forall y)(Fy \equiv y=x)\ \&\ Gx)$

Surely it is not Kripke's view that these three have the same truth value in all instances. For the last can be false even when the others are true on standard interpretations of '$\Box$'. We will return to this footnote and the question of the relation between scope and rigid designation in some detail in the next chapter.

57 Kripke, "Naming and Necessity," p. 254. Kripke is careful in presenting Donnellan's distinction. He does not generalize it in the manner of Lockwood and Peacocke.

58 Elsewhere ("Identity and Necessity," p. 149n), in considering the view that definite descriptions in English can be used both rigidly and nonrigidly, Kripke remarks that those who call the nonrigid or innermost scope reading of a description "attributive" are "following

Donnellan, perhaps loosely." He also distinguishes between Donnellan's distinction and the distinction between the purportedly alternative readings of definite descriptions in English.

59 Kripke, "Naming and Necessity," p. 310. The form of the identity statement is given on p. 311.

60 Cf., for example, "Identity and Necessity," p. 154.

61 Cf. ibid. or "Naming and Necessity," p. 306.

62 (vii) and (viii) can also be derived from (iv) and (v):

(vii) $(\exists x)(\exists y)\Box((\forall z)(Fz \equiv z=x)\ \&\ (\forall z)(Gz \equiv z=y)\ \&\ x=y)$

(viii) $(\exists x)(\exists y)((\forall z)(Fz \equiv z=x\ \&\ (\forall z)(Gz \equiv z=y)\ \&\ \Box x=y)$

In some respects (vii) captures Kripke's weak sense of necessity as well as (vi) does. But (vi) has modality de dicto, whereas (vii) has modality de re; and Kripke seems to want to claim that if an identity statement is true, then a certain de dicto statement involving identity, viz. (vi), is true.

63 The argument to which I refer is presented compactly in "Naming and Necessity," pp. 287-289. It is initially developed in pp. 270-281.

64 Cf. ibid., p. 287. Also, on p. 279, Kripke remarks, "Many people just have some vague cluster of his most famous achievements. Not only each of these singly, but the possession of the entire disjunction of these properties, is just a contingent fact about Aristotle."

65 A version of the argument that gives the proponent of the cluster theory less room to maneuver is obtained if (i) is replaced by (i′) and (iii) by (iii′), with appropriate adjustments elsewhere:

(i′) ⌜If X exists, then X and X alone has most of the ϕ's⌝ expresses a necessary truth.

(iii′) The individual that has most of the ϕ's in the actual world might have existed and yet not been the individual that has and alone has most of the ϕ's.

The proponent of the cluster theory will find the stipulation that (iii′) is true for some X harder to dispute than the stipulation that (iii) is true.

66 I take this to explain why Kripke argues against counterpart theory while developing his case against (i).

67 I remind the reader that resisting the stipulation of (iii) is not an easy way out, if only because a like argument can be run off of (i′) and (iii′), and the stipulation of (iii′) is hard to resist.

68 I do not see how to characterize those descriptions that are not suitable for determining the reference of names. This is the respect in which the claim has been only approximately stated.

69 This is an argument that the proper names of contingent entities would not be rigid if they were so synonymous. In "Naming and Necessity" Kripke extends the claim to names of necessary entities like π (Cf. p. 278). However, he does not give an argument. Furthermore, as he notes, whether the claim is true of names of necessary entities does not matter to the lectures.

70 Kripke's causal "picture" of fixing the reference of proper names also supports the thesis that proper names are rigid. For it suggests a way of accounting for how the reference of proper names can be determined so that they would still denote the same object, if they denote anything, even had things been different.

71 Of course, arguments in "Identity and Necessity' and "Naming and Necessity" pertaining to common names turn on a like thesis, viz. that they too are rigid designators.

72 Cf. Peacocke, op. cit., p. 116. Peacocke offers the following criterion that closely resembles that of (45):

$$(\exists\alpha)(\text{for all sentences } \overline{G(t)} \text{ of } L \vdash_{T_L} T(\overline{G(t)}) \equiv \langle\alpha\rangle \text{ sats } \overline{G(\xi)})$$

His main reason for rejecting this is as follows: "Provided T_L is cast in a free logic, it is possible to write out a truth theory for a first-order extensional language that evaluates definite descriptions directly (as terms), and which contains as theorems sentences of the form

$$T(\overline{G(\iota x)Fx}) \equiv \langle(\iota x)Fx\rangle \text{ sats } \overline{G(\xi)}\text{"}$$

He also objects to it on the grounds that it will exclude his entity-invoking uses of definite descriptions--the ones for which he introduces his apparatus for indexicals. His final moves are thus in response to these two objections to the above criterion, and hence indirectly to (45).

73 I here leave open the question whether definite descriptions that are exclusively or inclusively, but not completely rigid should be counted as rigid or nonrigid. The important issue concerns modality. Exclusively, inclusively, and completely rigid descriptions must all satisfy some modal condition.

74 Cf. M. J. Cresswell, "The Elimination of De Re Modalities," The Journal of Symbolic Logic, Vol. 34, No. 3 (September, 1969), pp. 329-330; and Richard Cartwright, "Some Remarks on Essentialism," The Journal of Philosophy, LXV, No. 20 (October 24, 1968), pp. 615-626.

Chapter III

Scope and Rigid Designation

I

The relationship between rigid designation and the scope of modal operators is a matter of controversy. There is general agreement that rigid designators have scope-related characteristics that distinguish them from nonrigid designators. But what these characteristics are is disputed. On the one hand, some hold that rigid designators always have only wide scope with respect to modal operators. For example, in arguing that proper names are not rigid designators, Michael Dummett contends that rigid designators always have wide scope, whereas names can have either wide or narrow scope.[1] Tyler Burge appears to agree with Dummett about rigid designators since he says that they are always taken to have referentially transparent position in modal contexts.[2] On the other hand, some hold that rigid designators can have either wide or narrow scope with respect to modal operators; but they add that these scope differences, unlike the corresponding ones for nonrigid designators, do not affect truth-value. Thus, Christopher Peacocke says that the truth conditions of modal sentences containing rigid designators are the same whether these designators are read with wide or narrow scope.[3] Michael Slote says that rigid designators enable one to argue with appropriate existence qualifications from modality _de dicto_ to modality _de re_ and conversely.[4] If we view the difference between modality _de dicto_ and modality _de re_ as a difference in the scope of designators _vis-a-vis_ modal operators, then Slote is suggesting that, with appropriate existence qualifications, rigid designators can be taken to have wide or narrow scope in modal contexts _salva veritate_. Leonard Linsky is explicit on this point. He remarks that the de re/de dicto distinction for modal propositions containing rigid designators collapses.[5] He then gives as a necessary and sufficient condition for a designator α to be rigid that, for all atomic $\phi(\xi)$, $\ulcorner\Box\phi(\alpha)\urcorner$ have the same truth-value whether α be taken to have wide or narrow scope.[6]

Kripke sides with Peacocke, Slote, and Linsky. In the course of a footnote in "Identity and Necessity," he comments:

> Some logicians have been interested in the question of the conditions under which, in an intensional context, a description with small scope is equivalent to the same one with large scope. One of the virtues of a Russellian treatment of descriptions in modal logic is that the answer (roughly that the description be a 'rigid designator' in the sense of this lecture) then often follows from the other postulates for quantified modal logic; no special postulates are needed, as in Hintikka's treatment. Even if descriptions are taken as primitive, special postulation of when scope is irrelevant can often be deduced from more basic axioms.[7]

These comments are reminiscent of the discussion of theorems 14.3 to 14.34 in Principia Mathematica. There Whitehead and Russell show that "when $E!(\imath x)(\phi x)$, the scope of $(\imath x)(\phi x)$ does not matter to the truth-value of any proposition in which $(\imath x)(\phi x)$ occurs" in an extensional context.[8]

There is an obvious name to be given to singular terms satisfying the condition that Whitehead and Russell identify:

> An instance of the schema
>
> $(\imath x)(\cdots x \cdots)$
>
> is a designator if and only if the corresponding instance of the schema (1)
>
> $(\exists x)(\forall y)(\cdots y \cdots \equiv y=x)$
>
> is true.

Suppose α is a definite description in a first order quantificational language all of whose sentential connectives are truth-functional. Let A be any sentence in which α occurs with its scope unmarked; and let B and B' be any two sentences obtained from A via Russellian expansions of α. (In the cases of interest B and B' will reflect different construals of the scope of α in A.) What Whitehead and Russell show is that if α is a designator, then B and B' are materially equivalent. One claim Kripke makes in the quoted footnote is that when certain modal sentential connectives are added to the quantificational language, the analogue of the Whitehead-Russell condition is (roughly) that α be a rigid designator.

This claim about rigid designators is comparatively precise. At least it is precise if we ignore the qualifier 'roughly' and if we specify which modal languages we are talking about. We are best off deciding whether it is correct first. We can subsequently return to the general question of the relationship between scope distinctions and rigid designation.

For now, then, we will adopt Russell's treatment of definite descriptions. The question is whether rigidly designating definite descriptions in modal languages are the analogue of designating definite descriptions in truth-functional languages. We will consider two families of modal languages, one described in detail and the other alluded to in passing in Kripke's "Semantical Considerations on Modal Logic." We will consider the scope of definite descriptions vis-a-vis the four distinct higher modalities of S5, symbolized by '$\Box$', '$\Diamond$', '$\sim\Box$', and '$\sim\Diamond$'. We will also consider weak necessity and contingency operators corresponding to the weak notion of necessity Kripke employs in "Naming and Necessity" and "Identity and Necessity." The question is whether rigidly designating definite descriptions can be read with contrasting scope with respect to these operators without affecting truth-value.

The answer is slightly surprising. The analogue of the Whitehead-Russell condition turns out not to be rigid designation, but to be what Kripke calls "strongly rigid designation." In the case of the languages Kripke discusses in detail in "Semantical Considerations on Modal Logic," rigid designation is not sufficient for truth-value to be unaffected by scope for any of the six modal operators we consider. In other words, rigid designators as a class exhibit no salient scope-related characteristics in the modal languages most discussed in the recent literature. However, they do exhibit salient scope-related characteristics in the modal languages Kripke alludes to in passing. Even in the case of these languages, rigid designation is not sufficient to make truth-value immune to scope for the strong necessity and contingency operators, '$\Box$' and '$\sim\Box$'; and in a wide range of contexts it is not sufficient to do so for any of the six operators. Nevertheless, in what may appropriately be called "simple standard" contexts in these languages, rigid designation is sufficient for truth-value to be unaffected by scope for all of the operators we consider except strong necessity and contingency. Kripke undoubtedly meant to exclude these strong operators in his original claim. With suitable qualifications about contexts, then, Kripke's claim is correct if taken as applying to the languages he alludes to in passing. But it is incorrect if taken as applying to the languages we more often associate with him.

Developed in detail, these results are more significant than they at first may appear to be. Scope considerations have sometimes been advanced in an effort to motivate the notion of rigid designation. One consequence of our results is that, while scope considerations can be used to motivate the stricter notion of strongly rigid designation, they are an ill-conceived way of motivating the notion of rigid designation. Substitution of co-referring designators in modal contexts _salva veritate_ and elimination of a type of referential ambiguity in counter-factual descriptions provide much sounder bases for motivating rigidity _simpliciter_. Rigid designators are not the logical analogue of standard constants in modal languages; strongly rigid designators are.

How then are we to account for the fact that rigid designators are widely thought to have distinctive scope-related characteristics in modal contexts? The most striking consequences of our results are in response to this question. If we construe informal essentialist talk along the lines of the formal languages Kripke develops in detail in "Semantical Considerations on Modal Logic," we have no way of accounting for the mistaken views other than to attribute them to confusion. But if we construe this talk along the lines of the formal languages Kripke alludes to in passing, we can readily account for them. For, so long as necessity and contingency are taken weakly, rigid designators have just the right sort of scope-related characteristics in these other languages. The preferred regimentation of essentialist talk remains a matter of controversy. A second consequence of our results is that intuitions about the scope behavior of rigid designators are evidence in favor of these other languages. This, along with other points brought out below, indicates that these languages deserve more attention than they have yet received. They may well be the best formal languages for regimenting essentialist talk.

These languages and the contrast between weak and strong necessity and contingency account for many mistakes about the logic of rigid designators. But they do not account for all of them. A further source of error is the predilection to view constants as paradigms of rigid designation. Constants in standard logic have three sets of features:

i. A distinct logic, involving scope and instantiation as well as the substitutivity of identity.
ii. Distinct referential features, best summarized by the claim that constants are comparable to denoting proper names in natural languages.
iii. A distinct relation to free variables, often expressed by the claim that free variables are like generalized constants, but perhaps better expressed by the claim that constants are like fixed-valued free variables.

A third consequence of our results is that in philosophically interesting modal languages constants can have at most one of these three sets of features. Mistakes result from thinking that they can have more--mistakes not just about rigid designation, but also about the construal of free variables in modal contexts. To put the point differently, philosophically interesting modal languages cannot include constants of the sort found in standard nonmodal languages. The tendency to think otherwise is a special source of trouble in quantified modal logic.

Since the chapter is long, a brief outline of the rest of it is called for. The next three sections pose the question about the "scope neutrality" of rigid designators in formal languages in a full and precise way. The fifth section then answers this question in detail, and the sixth applies the answer to the claim Kripke makes in the footnote quoted earlier. The following three sections critically examine a variety of claims that have been made about the logic of rigid designators. Finally, the tenth section addresses the problem of constants in quantified modal logic, and the last section offers a few remarks about the regimentation of de re modality.

II

Our initial question is not yet precise. We need to specify which modal languages we will be considering, and we need to give a formal criterion under which definite descriptions are rigid designators. Some may object to the latter on the grounds that definite descriptions--particularly Russellian definite descriptions--are never rigid designators. The footnote from "Identity and Necessity" quoted earlier is clear evidence that Kripke thinks they can be. Also, his paradigm of a rigid designator is 'the square root of 25', and nothing in his lectures discourages us from taking it to be Russellian. If further justification is wanted, our criterion and its consequences will show that one can talk coherently about rigidly designating definite descriptions.

We here require only a necessary and sufficient condition for definite descriptions to be rigid designators. Criteria for the rigidity of other kinds of singular terms do not matter. Indeed, since the modal languages Kripke discusses in "Semantical Considerations on Modal Logic" lack constants, contextually expanded definite descriptions are the only means for singular reference available in them. Still, we should note in passing that the criterion we give for definite descriptions may suffice for other kinds of singular terms as well. For example, Quine's method for eliminating names from regimented languages via special predicates like 'Socratizes' offers a means for applying our criterion to names. If Quine is correct about the eliminability of names, a criterion for the rigidity of definite descriptions may be all that will ever be needed.[9]

We will use the formal criterion for rigidity developed and defended in the preceding chapter:

> An instance of the schema
>
> $(\iota x)(\cdots x\cdots)$
>
> is a <u>rigid designator</u> if and only if the corresponding instance of the schema (2)
>
> $(\exists x)[(\forall y)(\cdots y\cdots \equiv y{=}x)\ \&\ \Box(\forall y)(\cdots y\cdots \equiv y{=}x)]$
>
> is true.[10]

The criterion requires several comments. First, although as stated the criterion applies literally only to expressions containing an ι-operator, it is meant to apply to definite descriptions in general, and not just to those that happen to be symbolized in this customary way. Second, the redundant initial clause of the schema is included to emphasize that on this criterion a definite description must be a designator in order to be a rigid designator. In this respect, the criterion should be contrasted with one in which '$\Diamond$' is prefixed to the entire schema. Third, since '$\Box$' occurs within the scope of a quantifier, it is of course to be taken as expressing necessity <u>de re</u>. If our criterion is correct, rigid designation is, for better or worse, a <u>de re</u> modal notion. Fourth, the instances of the defining schema are formulas in the metalanguage. They need not be formulas in the object language. Thus the criterion says that a certain expression in the object language is a rigid designator just in case a certain related expression in the metalanguage is true. The criterion therefore applies to definite descriptions in nonmodal as well as in modal languages.[11] Of course, when the criterion is applied to a modal quantificational language, homophonic translation may make the instances of the defining schema formulas in the object language too. We will be taking advantage of such homophonic translation below. Finally, as the criterion stands, '$\Box$' in the schema remains an uninterpreted modal operator, to be read 'it is necessary that' or 'necessarily'. Thus according to the criterion, a definite description is a rigid designator if and only if, informally speaking, there exists an object that necessarily and necessarily uniquely satisfies the description.[12] Here the interpretation of 'necessarily' is left open. In the context of a specific modal language, '$\Box$' in the schema will receive a more precise interpretation. In particular, in the context of the modal languages that we will be considering, the criterion for rigid designation will admit of a "possible world" model-theoretic interpretation.

Rigid designation should be distinguished from what Kripke calls "strongly" rigid designation:

> An instance of the schema
>
> $(\imath x)(\cdots x \cdots)$
>
> is a <u>strongly rigid designator</u> if and only if the corresponding instance of the schema (3)
>
> $(\exists x)[(\forall y)(\cdots y \cdots \equiv y=x)\ \&\ \Box(\forall y)(\cdots y \cdots \equiv y=x)]\ \&$
> $\Box(\exists x)(\cdots x \cdots)$
>
> is true.

A strongly rigid designator thus rigidly designates a necessarily existing object. This notion will be important below.

There are several arguments in favor of our criterion. One is based on Kripke's account of rigidity. On the standard possible world interpretations of our criterion, a designator is rigid just in case there is an actual object that it designates with respect to every possible world in which that object exists; and with respect to other worlds it designates nothing. But this is precisely what Kripke says in "Identity and Necessity."[13] A second argument is based on Kripke's intuitive test for distinguishing rigid designators. Our criterion exactly captures the test he means to use, namely that ⌜α might not have been α⌝ be false on the reading in which the first occurrence of α has wide scope and the second, narrow scope with respect to the modal operator.[14] Another argument concerns descriptions of counterfactual situations. The reference of singular terms in such descriptions can be determined either on the basis of the actual situation or on the basis of the supposed counterfactual situation. Our criterion singles out essentially just those designators whose reference can be determined on either basis without affecting the truth of the counterfactual descriptions in which they appear. Other arguments turn on more formal considerations. For example, as (3) shows, our criterion yields a perspicuous formal statement of the distinction between strongly and weakly rigid designators.

Such arguments in support of (2) have been developed in considerable detail in the preceding chapter. Other proposed criteria for rigidity have also been examined there. For present purposes, the preceding sketches of the arguments will have to suffice.

III

Next we have to identify the modal quantificational languages we will be considering. A word of warning is needed here. We will rely on possible world model theory to characterize the languages. Hence, there will be much talk below of possible worlds, of objects existing in possible worlds, and the like. Nothing we do will require any of this talk to be taken literally. It is only an heuristic substitute for the purely mathematical characterizations of the languages that the model-theoretic definitions of truth provide. The model-theoretic definitions themselves are given in an appendix. Model theory will enable us to determine whether certain formulas are valid in the different languages. Of course, if a formula is valid in a language and the logic of that language is axiomatizable, then the formula is derivable from the axioms. Accordingly, our proofs of validity and invalidity will also have a bearing on a further claim Kripke makes in the footnote quoted earlier, namely that with a Russellian treatment of descriptions, the adequacy of rigid designation for eliminating scope difficulties "often follows from the other postulates for quantified modal logic."

We will be considering two families of modal languages. One consists of those languages that admit of the definition of truth Kripke gives in detail in "Semantical Considerations on Modal Logic."[14] This definition has two distinctive features. First, the truth-value of a quantified formula with respect to a given possible world is evaluated with the quantified variable ranging only over the objects that exist in that world. Second, the function that assigns _T_ and _F_ to formulas with respect to possible worlds (given an assignment of objects from the universe of discourse to the variables) is a complete function. Thus, if a predicate is not satisfied with respect to a possible world by an object in the universe of discourse, then the complement of this predicate is satisfied by the object with respect to that world, even if the object fails to exist in it. An example will help. Consider a possible world in which Socrates does not exist. Even with respect to this world, Socrates is either in the extension of '① is a philosopher' or in the extension of '① is not a philosopher'.[16] Moreover, given the first distinctive feature of the truth definition, Socrates may be in the extension of '① is a philosopher' with respect to this world, and yet the formula '(∃x)(x is a philosopher)' still be false with respect to it. Another consequence of the truth definition is that an object satisfies an open formula '□Gx' only if it is in the extension assigned to 'G①' with respect to every accessible world, even those in which it fails to exist.

We will call the members of this family "the K-languages." They differ from one another model-theoretically by having different admissible model-structures. First, they do not place the same restrictions on the accessibility relation among possible worlds. For some of the languages the relation only has to be reflexive, in the manner of T, while for others it has to be, for instance, symmetric and transitive as well, in the manner of S5. Second, the languages do not place the same restrictions on which objects exist in different worlds. Among the K-languages is the naive extension of S5 in which both the Barcan and the converse Barcan formulas are valid.[17] This language requires the same objects to exist in all worlds. Also among the K-languages are the ones Kripke sets out in

"Semantical Considerations on Modal Logic." In these neither the Barcan nor the converse Barcan formulas are valid. These languages place no restrictions on which objects exist in different worlds (other than that each object in the universe of discourse exists in at least one world). The quantified version of T for which Kripke specifies axioms is the weakest of the K-languages.[18] Its valid formulas are included among those of all of the other K-languages.

Our second family of modal languages springs from an alternative truth definition Kripke mentions while discussing the converse Barcan formulas.[19] Unlike the definition just reviewed, this one renders these formulas--'$\Box(\forall x)(Fx) \supset (\forall x)(\Box Fx)$' and '$(\exists x)(\Diamond Fx) \supset \Diamond(\exists x)(Fx)$'--valid even for model-structures in which there are no special restrictions on which objects exist in different worlds. This definition differs from the other one in two ways. First, only objects that exist in a possible world satisfy atomic predicates or their complements with respect to that world. Thus, for these languages, the function that assigns *T* and *F* to formulas with respect to worlds need not be complete. For some assignments of objects from the universe of discourse to variables, the function may fail to assign either *T* or *F* to certain formulas for some worlds. The second respect in which this truth definition is different compensates for the first. Instead of ⌜$\Box$A⌝ being assigned *T* just in case A is assigned *T* with respect to every accessible world, ⌜$\Box$A⌝ is assigned *T* (if it is assigned a value at all) just in case A is assigned *F* with respect to no accessible world. This treatment of '$\Box$' leaves the usual "de Morgan relations" between '$\Box$' and '$\Diamond$' intact. But on this definition, unlike the other, an existing object satisfies an open formula '$\Box$Gx' provided merely that it is in the extension assigned to 'G①' with respect to all accessible worlds in which it exists.

The striking feature of this alternative truth definition, then, is that it allows what might be called "satisfaction gaps." That is, open formulas may fail to have a truth-value with respect to a world for some assignments of objects to variables. For example, when Socrates is assigned to 'x', the formula 'x is a philosopher' will not have a truth-value with respect to any possible world in which Socrates does not exist. However, since quantified variables still range only over the objects that exist in a world, every closed formula will still have a truth-value with respect to every possible world. Hence the phrase "satisfaction gaps" rather than "truth-value gaps."

Our second family consists not of languages that satisfy this alternative truth definition, but of ones that satisfy a slightly modified version of it. The trouble with the truth definition as it stands is that too many of the languages satisfying it are philosophically deviant.[20] To appreciate this, consider the closed formula '$(\exists x)(\sim\Box(\exists y)(y=x))$'. Normally this is taken as asserting that at least one object exists whose existence is contingent. In terms of possible worlds, it is taken as asserting that at least one object that exists in the actual world fails to exist in some other accessible world. But it cannot be taken this way when it occurs in a language that satisfies the alternative truth definition. For '$(\exists x)(\sim\Box(\exists y)(y=x))$' would not be true in such a language even when some object that exists in the actual world fails to exist in some accessible world. In order for this closed formula to be true in such a language, the open formula '$\Box(\exists y)(y=x)$' would have to be false of some object--say Socrates--that exists in the actual world. But this open formula is false when Socrates is assigned to 'x' only if there is some accessible world with respect to which the open formula '$(\exists y)(y=x)$' is false when Socrates is assigned to 'x'. This last formula, however, is never false of any object with respect to any world in such a language. For, according to the alternative truth definition, when Socrates is assigned to 'x', '$(\exists y)(y=x)$' is true with respect to those worlds in which Socrates exists, and it lacks a truth-value with respect to all other worlds. In other words, regardless of which objects exist in which worlds, '$(\forall x)(\Box(\exists y)(y=x))$' is true in all languages satisfying the alternative truth definition as it stands. But then in these languages '$(\forall x)(\Box(\exists y)(y=x)$' cannot be taken as it normally is, _viz._ as asserting that every object is a necessary existent.

This point can be generalized. The usual way of constructing predicates for contingent and necessary existence does not work for the languages in question because in them 'y=x' has a truth-value only when the objects assigned to both variables exist. But the point does not hang on the identity predicate. Let '$E\Phi$' be an existence predicate--i.e. let 'Ex' be true of an object with respect to a world just in case the object exists in that world. Then the natural way to express contingent and necessary existence is via formulas like '$\sim\Box Ex$' and '$\Box Ex$', respectively. But such formulas do not express contingent and necessary existence in languages satisfying the alternative truth definition. For according to this definition, 'Ex' will lack a truth value with respect to any world in which the object assigned to 'x' does not exist. Thus 'Ex' will never be false of any object with respect to any world. Hence '$\Box Ex$' will be true of every object that exists in the actual world, regardless of which other worlds these objects exist in. According to this truth definition, then, '$(\forall x)(\Box Ex \equiv Ex)$' is logically true and '$(\exists x)(\sim\Box Ex)$' is logically false. But surely then '$\sim\Box Ex$' and '$\Box Ex$' should not be taken as expressing contingent and necessary existence in these languages.

This is not to say that languages satisfying the alternative truth definition cannot express contingent and necessary existence at all. For let 'Cx' be true of an object with respect to a world just in case the object exists in that world and not in some accessible world. Then 'Cx' will express contingent existence. The trouble is that the modal relationship between existence and contingent existence--as expressed by '$(\forall x)(Cx \equiv \sim\Box Ex)$'--cannot be affirmed in these languages (except vacuously, when there are no contingent objects). It is the inability to affirm this relationship in these languages that makes them philosophically deviant.

Again, this is not to say that the existence and the contingent existence predicates will not be related to one another in the right way in these languages. In fact, the open formula 'Cx' will be true of an object with respect to a world if and only if the open formula 'Ex' is not true of that object with respect to some accessible world. Therefore, the requisite modal relationship between 'Cx' and 'Ex' can be affirmed in a suitable metalanguage. The trouble is that this relationship cannot be (non-vacuously) affirmed in the languages themselves. This is what makes them philosophically deviant. The incongruity of the object languages and metalanguages in this regard underscores the deviance of the object languages.

The deviance of these languages would be of less concern here if it did not extend to our criterion for rigid designation. But it is easy to see that it does. Intuitively, a designator is rigid only if it would in no circumstances designate any object other than the one it designates in the present circumstances. In possible world terms, a definite description, '$(\iota x)(Fx)$' is a rigid designator only if, with respect to every accessible world, the open formula '$(\forall y)(Fy \equiv y=x)$' is true of no object other than the one it is true of with respect to the actual world. But if our criterion for rigidity is interpreted in accordance with the alternative truth definition, it fails to secure this requirement. For let 'Gx' be uniquely true of some contingent object--say Socrates--with respect to every world in which that object exists; and let it be uniquely true of some other object--say 0--with respect to every other world.[21] Since by hypothesis '$(\iota x)(Gx)$' will designate the number 0 with respect to some accessible world, it does not satisfy the indicated requirement for rigidity. Yet it does satisfy our criterion for rigidity when that criterion is interpreted in accordance with the alternative truth definition. For according to this definition, '$\Box(\forall y)(Gy \equiv y=x)$' will be true of Socrates since, with respect to every accessible world, '$(\forall y)(Gy \equiv y=x)$' either will be true or will lack a truth-value when Socrates is assigned to 'x'. In other words, our criterion, so interpreted, fails to require the definite description to denote nothing with respect to worlds in which the designated object does not exist.

The point can be put in another way. When '$(\exists x)(\Box(\forall y)(Fy \equiv y=x))$' is taken to be a sentence in a language that (non-trivially) satisfies the alternative truth definition, it does not express the necessary and sufficient condition under which '(ɿx)(Fx)' is a rigid designator. Worse, no sentence in such a language can express this condition. For no sentence in such a language can express the requirement that the definite description denote nothing with respect to worlds in which the designated object does not exist. These languages simply have no way to require either that something be true of or that something be false of an object with respect to a world in which it does not exist. They have no way of saying anything about an object with respect to a world in which it does not exist.

Of course, the inexpressibility of the necessary and sufficient condition for rigidity does not prevent these languages from containing rigid designators. Their rigid designators can even be picked out by our criterion when it is expressed in an appropriate metalanguage. Hence we could perfectly well study the scope-related characteristics of rigid designators in these languages. But we will not do so. Instead we will turn to languages that satisfy a slightly modified version of the truth definition. Although a little more complicated, these languages avoid the shortcomings we have been discussing. Moreover, we can turn to them without sacrificing anything of note. In particular, our study of rigid designators in them will straightforwardly determine the scope-related characteristics of rigid designators in the languages satisfying the unmodified alternative truth definition.

The way to change the truth definition is evident. The identity predicate should be handled differently from the standard predicates. The basic change we need is for the case in which the object assigned to but one of the variables does not exist. Specifically, an open formula of the form 'y=x' should be false with respect to a world if the object assigned to one of the variables exists in that world and the object assigned to the other variable does not. One way to accomplish this change is to have 'y=x' be true with respect to a world so long as the same object is assigned to both variables, and false otherwise. The identity predicate would then be handled exactly as it is in the K-languages, so that 'x=x' would invariably be assigned T with respect to all worlds. Another way to accomplish the basic change is to introduce it directly, but still have 'y=x' lack a truth-value with respect to worlds in which the objects assigned to both variables do not exist. This way is somewhat more in keeping with the spirit of the alternative truth definition since the self-identity predicate would still be neither true nor false of nonexisting objects.[22] For our present purposes, the choice between these two ways of achieving the basic change is of no consequence. We will therefore leave the choice open, though where definiteness is needed, we will proceed as if identity is handled in the second way.

The change, regardless of how it is made, is easy to motivate. It is not so peculiar to hold that 'y=x' is true when the same object is assigned to both variables, even should the object not exist. At worst, this is comparable to saying that 'Athena is identical with Minerva' is true. Equally, it is not so peculiar to hold that 'x=x' and 'y=x' lack a truth-value when the objects assigned to the variables do not exist. At worst, this is comparable to saying that 'Pegasus is identical with Pegasus' and 'Athena is identical with Minerva' lack a truth-value. By contrast, it is definitely peculiar to hold that 'y=x' lacks a truth-value when the object assigned to 'y' exists and the object assigned to 'x' does not.

This is comparable to saying that 'Socrates is identical with Pegasus' is not false. If an object exists in a world, then surely it is distinct from any object that does not exist in that world. Indeed, for 'y=x' to lack a truth-value in this case is so peculiar that one is likely not to notice that the alternative truth definition Kripke mentions requires this of the identity predicate. And in missing it, one is likely also to miss the philosophic deviance of the languages satisfying this truth definition.

This one change will remedy the points of deviance we called attention to. In all the languages we are considering, binding a variable automatically restricts its range with respect to each world to just those objects that exist in that world. Hence, with the truth definition changed to treat '=' as we have suggested, the open formula '$(\exists y)(y=x)$' will never exhibit any satisfaction gaps. Furthermore, since it will be false wherever it had no truth-value before, '$\Box(\exists y)(y=x)$' and '$\sim\Box(\exists y)(y=x)$' will now successfully express necessary and contingent existence. Similarly, with the truth definition changed, open formulas of the form '$(\forall y)(Fy \equiv y=x)$' will exhibit no satisfaction gaps. As a consequence, our criterion for rigidity will no longer fail to express the proper necessary and sufficient condition. In particular, '$(\iota x)(Fx)$' will now satisfy our criterion only if, with respect to every accessible world, '$(\forall y)(Fy \equiv y=x)$' is true of no object other than the object it is true of in the actual world. Indeed, with the change, our criterion will express basically the same necessary and sufficient condition for rigidity in all of the languages we will be considering. Of course, different languages have different admissible models. But consider a class of admissible models in which the existent objects that 'Fx' is true of with respect to each world remain fixed. On these models '(∃x)(□(∀y)(Fy ≡ y=x))' will be true according to the revised alternative truth definition if and only if it is true according to the truth definition for the K-languages.[23]

The change does complicate matters. The new languages contain two distinct kinds of open formulas, one standard and the other not. The standard open formulas have a truth-value with respect to a world just in case the objects assigned to their free variables exist in that world. Open formulas of this kind are what the alternative truth definition was set up to provide. By contrast, the nonstandard or special open formulas are permitted to have a truth-value with respect to worlds in which an object assigned to one of their free variables does not exist. The nonstandard open formulas thus do not display satisfaction gaps in at least some circumstances in which the standard ones must. The open formulas discussed in the preceding paragraph, '(∃y)(y=x)' and '(∀y)(Fy ≡ y=x)', are nonstandard in the extreme since they never display any satisfaction gaps. Corresponding to the distinction between standard and special open formulas is a distinction between standard and special predicates. Thus 'F①' and '(F①& G①)' are standard predicates, while '(∃y)(y=①)' and '(∀y)(Fy ≡ y=①)' are special.

The distinction between the two kinds of predicates becomes significant only in the case of de re modal contexts. So long as the variables associated with a predicate are not bound from outside the scope of a modal operator, any closed formula containing the predicate will have the same truth-value whether the predicate is taken to be standard or special. But when a variable is bound from outside the scope of a modal operator, a closed formula can have one truth-value if the predicate is taken to be standard and a contrasting truth-value if it is taken to be special. Suppose, for example, that 'FΦ' and '$\hat{F}$Φ' have identical extensions with respect to each world, where 'FΦ' is standard and '$\hat{F}$Φ' is not. Then, as we have seen, '$(\exists x)(\Box Fx)$' can be true even when '$(\exists x)(\Box \hat{F}x)$' is false. The standard predicates thus stand out in de re modal contexts. Indeed, what the alternative truth definition gives us is a family of languages whose standard predicates exhibit a logic in de re modal contexts distinctly different from that exhibited by the predicates of the K-languages.

The contrast in the handling of modality de re in the two families is more significant than it may first appear to be. To see this, consider the notion of essential predication. Essential properties are usually deemed to be those an object has to have if it exists at all. In a footnote in "Identity and Necessity," Kripke remarks that "an exception must be made for existence itself; on the definition given, existence would be trivially essential. We should regard existence as essential to an object only if the object necessarily exists. Perhaps there are other recherché properties, involving existence, for which the definition is similarly objectionable."[24] (I take it that existence predicates also underlie Kripke's opposition to the converse Barcan schema insofar as he wants to allow '$\Box(\forall x)(\exists y)(x=y)$' to be true without having '$(\forall x)\Box(\exists y)(y=x)$' be true.)

In the present context it is better to talk not about essential properties, but about predicates being essentially true of an object. Suppose we say that 'FΦ' is essentially true of an object just in case '□FΦ' is true of it. Then a standard predicate in the languages satisfying the alternative truth definition will be essentially true of an object if and only if it has to be true of the object if the object exists at all. In other words, the standard predicates will conform to the usual way of defining 'essential' in metaphysics. But the nonstandard predicates will not. (Nor will K-language predicates.) In particular, the existence predicate, '(∃y)(y=①)', will be essentially true of just those objects that necessarily exist. Perhaps the fact that the usual definition fits the standard predicates in the new languages, and not the nonstandard predicates, is no accident. Perhaps these languages closely mirror our usual way of talking about essential properties. If so, this is a key feature that makes them philosophically interesting.

There is a further, related virtue to the languages that satisfy the alternative truth definition. Many who talk about essential properties also want to hold that a relation can be "internal" to one relatum and not to the other. For example, suppose '① is the offspring of ②' is true of a pair of objects, s and t. Then many want to hold that the relational predicate '① is the offspring of t' is essentially true of s, but 's is the offspring of ①' is not essentially true of t. Given the definition of 'is essentially true of' proposed in the preceding paragraph, such an asymmetry is straightforwardly expressible in the languages satisfying the alternative truth definition (presuming that '① is the offspring of ②' is a standard predicate). But the asymmetry is not expressible in the K-languages except under a definition of 'is essentially true of' on which the existence predicate will be essentially true of everything.

The languages of our second family contain an unlimited number of nonstandard as well as standard predicates. However, the only nonstandard atomic predicate we will require them to contain is the identity predicate. Perhaps it is the only nonstandard atomic predicate there is any reason for them to contain. But whether it is will not matter here. We will permit the languages to include any number of other nonstandard atomic predicates, so long as all of them are marked in the syntax as nonstandard. They can be marked by resorting to special symbols, as we have done with '=', or by using qualified predicate letters, as we did with '$\hat{F}$' in the preceding paragraph. Unmarked atomic predicates will always be taken to be standard. Marking the nonstandard atomic predicates in the syntax will enable us to take advantage of the following restricted rule of substitution:

> Suppose a logically true closed formula A contains one or more occurrences of an unmarked n-place atomic predicate σ. Then any closed formula obtained from A by replacing all occurrences of σ with an n-place standard predicate is also logically true.

Since the standard and nonstandard predicates differ logically in _de re_ modal contexts, the languages of our second family, unlike the K-languages, will not in general allow an unrestricted rule of substitution. But this restricted rule will suffice for our purposes.

Let me summarize. Our second family consists of modal languages that satisfy the modified version of the alternative truth definition Kripke calls attention to. In these languages $\ulcorner \Box A \urcorner$ is assigned _T_ (should it be assigned a value at all) if and only if A is assigned _F_ with respect to no accessible world. But $\ulcorner \Box A \urcorner$ could just as well be handled in this way in the K-languages too. The real contrast between the two families comes from their handling atomic predicates differently. Unlike the truth definition Kripke develops in detail, our other truth definition distinguishes between two kinds of atomic predicates, standard and nonstandard. On this truth definition, an atomic open formula formed with a standard predicate is assigned _T_ or _F_ with respect to a world if and only if the objects assigned to its free variables exist in that world. As a result, the standard open formulas of these other languages, unlike those of the K-languages, display satisfaction gaps. That is, they display satisfaction gaps unless the same objects exist in every world.

The nonstandard predicates of these languages contrast less sharply with the predicates of the K-languages. Their principal and perhaps their only nonstandard atomic predicate is the identity predicate. This predicate can be handled in either of two ways without affecting any of our results. On the one hand, it can be treated as a K-language predicate. In this case, an open formula of the form 'y=x' is assigned _T_ with respect to a world _w_ if and only if the same object is assigned to both variables; and it is assigned _F_ otherwise. On the other hand, it can be treated more in keeping with the standard predicates. In this case, an open formula of the form 'y=x' is assigned _T_ with respect to a world _w_ if and only if the same object is assigned to both variables and this object exists in _w_; it is assigned _F_ with respect to _w_ if and only if different objects are assigned to the variables and at least one of these objects exists in _w_; and it is assigned neither _T_ nor _F_ if and only if the object or objects assigned to the variables do not exist in _w_. Either way, as a result of the special handling of the identity predicate (and of any other nonstandard atomic predicates), the languages contain various nonstandard open formulas--open formulas that may have a truth-value even with respect to worlds in which the objects assigned to their free variables do not exist. These nonstandard open formulas do not have to be entirely comparable to the open formulas of the K-languages. They can still display satisfaction gaps, as 'y=x' would on the second approach to identity when the objects assigned to 'x' and 'y' do not exist. But some nonstandard open formulas--e.g. '$(\exists y)(y=x)$' and '$(\forall y)(Fy \equiv y=x)$'--display no satisfaction gaps in these languages. The predicates corresponding to these open formulas are completely comparable to the predicates of the K-languages.

Since our second family of languages derives in some part from the work of Prior and Hintikka, we will call its members "the PH-languages." Like the K-languages, they differ from one another model theoretically in placing different restrictions on the accessibility relation and on the existence of objects from world to world. One of the PH-languages is the naive extension of S5 that requires the same objects to exist in all worlds. Consequently, our two families overlap at their strong ends. But they do not overlap at their weak ends. The weakest of the PH-languages is one for which the accessibility relation is just reflexive and no restriction is placed on which objects exist in different worlds (again, other than that each object exist in at least one world). This language is not a K-language since '$(\forall x)(\Box(Fx \supset (\exists y)(Fy)) \supset (\Box Fx \supset \Box(\exists y)(Fy)))$'--i.e., Kripke's distribution axiom schema--is not valid in it.[25] Furthermore, the K-languages Kripke sets out in "Semantical Considerations on Modal Logic" are not PH-languages since '$(\exists x)(\Diamond Fx) \supset \Diamond(\exists x)(Fx)$' is not valid in them. The validity of '$(\exists x)(\Diamond Fx) \supset \Diamond(\exists x)(Fx)$' and '$\Box(\forall x)(Fx) \supset (\forall x)(\Box Fx)$'--

i.e. the converse Barcan formulas with standard predicates--is a salient feature of all PH-languages. Notice, however, that the first version of the converse Barcan formula is not valid when the predicate is the non-existence predicate, '$\sim(\exists y)(y=\textcircled{0})$'; and the second version is not valid when the predicate is the existence predicate, '$(\exists y)(y=\textcircled{0})$'. The validity of the converse Barcan formulas with standard predicates and the invalidity of them with such *recherché* predicates as existence is no small virtue of most of the PH-languages. In this respect these languages conform with intuitions in a way that none of the K-languages do. This is another feature of the PH-family that makes it philosophically interesting.

IV

We can now make our principal question precise. We want to know whether rigidly designating definite descriptions can be read with contrasting scope without affecting truth-value. Let 'μ' be replaceable by '$\Box$', '$\sim\Box$', '$\Diamond$', '$\sim\Diamond$', '$\boxminus$', and '$\sim\boxminus$'--where '$\boxminus$', to be defined in the next section, corresponds to Kripke's weak notion of necessity. Consider formulas of the following form:

$$(\exists x)\left[(\forall y)(Fy \equiv y=x) \;\&\; \Box(\forall y)(Fy \equiv y=x)\right] \supset \left[\mu(\exists x)((\forall y)(Fy \equiv y=x) \& Gx) \equiv (\exists x)((\forall y)(Fy \equiv y=x) \;\&\; \mu Gx)\right] \tag{4}$$

The schema embodies the following claim: if a Russellian definite description is a rigid designator, then a secondary occurrence of it *vis-a-vis* μ will yield the same truth-value as the corresponding primary occurrence. We ask our principal question of each of our two families of languages separately: are all formulas of the form of (4) valid in all languages of the family? Different answers give rise to different subsequent questions. If the answer is yes, then will a weaker satisfiable antecedent still secure validity across the board? If the answer is no, then for which modal operators and for which members of the family does validity fail? Further, how must the antecedent of (4) be strengthened to secure validity across the board for the family?

The point of considering the two families separately will become evident as we proceed. But a few words are needed now about the way we put the issue. Suppose first that some formulas of the form of (4) are not universally valid in a family. Then rigid designation would not be the analogue of the Whitehead-Russell condition in the case of modal languages generally since it would not be the analogue in the case of certain PH- or K-languages. It would remain then to show that this result is not just a consequence of some idiosyncratic feature that should disqualify these particular languages from consideration. This is the point of our subsequent questions should the answer to our principal question be no.

Now suppose instead that all formulas of the form of (4) are universally valid in a family. Of what significance would such a positive result be? To answer we need to make some terminology precise. Suppose 'ϑ' stands for an operator, modal or extensional. A definite description, '$(\imath x)(Fx)$', will be said to have a <u>primary occurrence vis-a-vis ϑ</u> in formulas of the form

$$(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ \vartheta Gx]$$

Correspondingly, it will be said to have a <u>secondary occurrence vis-a-vis ϑ</u>

in formulas of the form

$$\nu(\exists x)\left[(\forall y)(Fy \equiv y=x) \ \& \ Gx\right]$$

When formulas of the indicated forms occur embedded in further contexts, '$(\iota x)(Fx)$' will have neither a primary nor a secondary occurrence vis-a-vis ν (though it will still have wide and narrow scope vis-a-vis ν). We are thus reserving the notions of primary and secondary occurrence vis-a-vis ν to the appropriate Russellian expansions of nonembedded formulas of the form $\ulcorner\nu\Psi((\iota x)(Fx))\urcorner$.[26] The notions cease to be pertinent once $\ulcorner\nu\Psi((\iota x)(Fx))\urcorner$ is embedded in a further context.

Suppose now that all formulas of the form of (4) are valid throughout a family. The significance of such a result would depend on the family. In the case of the K-family, we could conclude that any secondary occurrence of a rigid designator vis-a-vis a modal operator will yield the same truth-value as the corresponding primary occurrence. But in the case of the PH-family, a positive result for (4) would generalize only to rigid designators occurring in standard contexts--i.e., to occurrences as in $\ulcorner\phi((\iota x)(Fx))\urcorner$, where $\phi(\xi)$ is a standard open formula. That is, we could only conclude that a secondary occurrence of a rigid designator vis-a-vis a modal operator in any standard context in a PH-language will yield the same truth-value as the corresponding primary occurrence. The question then would be whether rigidity is sufficient to assure like truth-values for corresponding primary and secondary occurrences vis-a-vis modal operators in nonstandard contexts. However, since the most extreme nonstandard predicates in the PH-languages are just like K-language predicates, this question will turn out to be answered when we answer our principal question for the K-family. Hence, our way of addressing the issue will end up covering all cases of concern for non-embedded formulas.

For either modal family, then, a positive result for (4) would be of some significance. Nevertheless, obtaining a positive result for (4) for a family would not be tantamount to showing that rigid designation is the analogue of the Whitehead-Russell condition for the family. To show this, we would have to extend the generalizations discussed in the preceding paragraph to cases in which $\ulcorner\mu\Psi((\iota x)(Fx))\urcorner$ is embedded in further contexts. For only then could we conclude that the scope of a rigid designator will never affect the truth-value of any formula in any language of the family. Of course, the generalizations can be extended straightforwardly to cases of $\ulcorner\mu\Psi((\iota x)(Fx))\urcorner$ embedded in extensional contexts. For materially equivalent formulas can be substituted for one another in extensional contexts <u>salva veritate</u>. However, they cannot in general be substituted for one another <u>salva veritate</u> in modal contexts. Hence, we have no offhand reason to think that the generalizations of the preceding paragraph can be extended to cases of $\ulcorner\mu\Psi((\iota x)(Fx))\urcorner$ embedded in modal contexts. Rather than resolve this matter now, I want to postpone considering cases of $\ulcorner\mu\Psi((\iota x)(Fx))\urcorner$ embedded in modal contexts until the end of the next section of the paper. By then the question whether rigid designation is the analogue of the Whitehead-Russell condition will be moot for both of the modal families under discussion.

A positive result for (4) would be of considerable interest even if it cannot be extended to cases of $\ulcorner\mu\Psi((\iota x)(Fx))\urcorner$ embedded in modal contexts. Modal formulas embedded in modal contexts are scarcely the basis for our intuitions about rigid designation. An important question to raise, given a positive result for (4) is whether it can be used to motivate the notion of rigid designation. This is the point of asking about weaker antecedents when the answer to our principal question is yes. Indeed, this question of weaker antecedents is worth pursuing even should all formulas of the form of (4) be universally valid in a family for just some one modal operator. Because of this, we will want to look at each modal operator separately.

V

The best way to attack our principal question is to examine each combination of operator and family in turn. The detail we generate as we proceed in this way may at the time seem cumbersome. But in the long run it will help us to develop a number of points, both about rigid designation in the formal languages we are considering and about the relationship between scope and rigid designation generally.

Consider first '$\Diamond$' and the PH-languages. Suppose '$(\iota x)(Fx)$' is a rigid designator. Then (5) has the same truth-value as (6) in every PH-language:[27]

$$(\exists x)\left[(\forall y)(Fy \equiv y{=}x) \ \& \ \Diamond Gx\right] \tag{5}$$

$$\Diamond(\exists x)\left[(\forall y)(Fy \equiv y{=}x) \ \& \ Gx\right] \tag{6}$$

(5) is true in a PH-language if and only if in the actual world there exists an object that satisfies 'Fx' uniquely, and this object satisfies 'Gx' with respect to some accessible world in which it exists. (6) is true in a PH-language if and only if in some accessible world there exists

an object that satisfies both 'Fx' uniquely and 'Gx'. But '(℩x)(Fx)' is a rigid designator. Hence, in the actual world there exists an object that satisfies 'Fx' with respect to every accessible world in which it exists, and in no accessible world does there exist any other object that satisfies 'Fx'. Thus, because '(℩x)(Fx)' is a rigid designator, both (5) and (6) are true in a PH-language if and only if the object satisfying 'Fx' in the actual world satisfies 'Gx' with respect to some accessible world in which it exists.

The reasoning here is general. When '◇' replaces 'μ', all formulas of the form of (4) are valid in every PH-language. Consequently, in all standard contexts in PH-languages, any secondary occurrence of a rigid designator <u>vis-a-vis</u> '◇' yields the same truth-value as the corresponding primary occurrence.

Can the antecedent of (4) be weakened, yet the schema obtained when '◇' replaces 'μ' still be valid in all PH-languages? First, suppose '(℩x)(Fx)' is not a designator. Then (5) is false regardless of the extensions assigned to 'GΦ'. But (6) is then false regardless of the extensions assigned to 'GΦ' just in case no accessible world contains an object that satisfies 'Fx' uniquely. Thus there is a way to weaken the antecedent of (4) and still have a schema valid in all PH-languages when '◇' replaces 'μ'. For let the antecedent be (7):

$$(\exists x)\left[(\forall y)(Fy \equiv y=x) \;\&\; \Box(\forall y)(Fy \equiv y=x)\right] \vee \sim\Diamond(\exists x)(\forall y)(Fy \equiv y=x) \quad (7)$$

However, weakening the antecedent in any other way so that it no longer entails that '(℩x)(Fx)' is a designator will yield a schema that is not valid in many PH-languages. In particular, weakening it to allow '(℩x)(Fx)' to single out a "possible-yet-not-actual" object will yield a schema that is not valid in most PH-languages.[28] As will become clear below, the schema thus obtained is also not valid in most K-languages.

These results are of some note. Weakening our criterion for rigid designation by prefixing '◇' to it will simply insure that, in most of the languages we are considering, rigid designators fail to render scope ambiguities innocuous. Those who prefer such a weaker criterion for rigidity should be mindful of this. Similarly, weakening the criterion in the manner of (7) will simply insure that rigid designators fail to render ordinary truth-functional scope ambiguities innocuous. If rigid designation is to be an extrapolation of the Whitehead-Russell condition to modal languages, then rigid designators must at least satisfy this condition. Accordingly, we will give no further consideration below to (7).

Next suppose '(ιx)(Fx)' is a designator, but not a rigid designator. We need to consider the several different ways in which this can happen. The last way we consider will be the sole exception to the general pattern.

First, suppose the object satisfying 'Fx' in the actual world fails to satisfy 'Fx' with respect to some other accessible world in which it exists. Then (5) could be true even though (6) is false. For suppose 'G①' is a contrary of 'F①'. Then (6) will be false since in no accessible world will there be an object that satisfies both 'Fx' and 'Gx' Nevertheless, the object that satisfies 'Fx' in the actual world might exist and satisfy 'Gx' in some accessible world in which it does not satisfy 'Fx'. Thus (5) could still be true.

Next, suppose '$(\iota x)(Fx)$' is a nonrigid designator by virtue of one of more "errant" objects satisfying 'Fx'. That is, suppose that (i) the object satisfying 'Fx' in the actual world properly satisfies 'Fx' with respect to every accessible world in which it exists; yet (ii) some other object--i.e. an "errant" object--both exists and satisfies 'Fx' in some accessible world. There are two cases to consider. On the one hand, it may be the case that in some accessible world there is an errant object that satisfies 'Fx' uniquely. Then (6) could be true even though (5) is false. For suppose the object that satisfies 'Fx' in the actual world does not satisfy 'Gx' with respect to any accessible world in which it exists. Then (5) will be false. Nevertheless, an errant object might satisfy 'Gx' in an accessible world in which it is the sole existing object that satisfies 'Fx'. Thus (6) could still be true.

On the other hand, it may be the case that in every accessible world in which an errant object satisfies 'Fx', more than one object satisfies 'Fx'. Again there are two cases to consider. First, the object that satisfies 'Fx' in the actual world may exist and satisfy 'Fx' in an accessible world in which an errant object does the same. Then (5) could be true even though (6) is false. For suppose 'Gx' is satisfied only in those accessible worlds in which there exists more than one object satisfying 'Fx'. Then (6) will be false. Nevertheless, the object that satisfies 'Fx' in the actual world may exist and satisfy 'Gx' in one of these worlds. Thus (5) could still be true.

Second, the object that satisfies 'Fx' in the actual world may not exist in the accessible worlds in which multiple errant objects satisfy 'Fx'. This final case furnishes the only exception. In it accessible worlds fall into three distinct groups. Some contain the object that satisfies 'Fx' in the actual world; in these worlds, it and it alone satisfies 'Fx'. Other worlds contain more than one object satisfying 'Fx', but they do not contain the object that does so in the actual world. Finally, perhaps some accessible worlds contain neither an object satisfying 'Fx' nor the object that does so in the actual world. Only worlds in the first group can render either (5) or (6) true in any PH-language. From this it is easy to see that both (5) and (6) are true in a PH-language if and only if the object that satisfies 'Fx' in the actual world satisfies 'Gx' in some accessible world. In this one case, then, (5) and (6) must match in truth-value in all PH-languages even though '$(\iota x)(Fx)$' is a nonrigid designator.

In sum, weakening the antecedent of (4) in the manner of (8) yields a schema that is still valid in all PH-languages when '$\Diamond$' replaces 'μ':

$$(\exists x)\{(\forall y)(Fy \equiv y=x) \ \& \ \Box[(\forall y)(Fy \equiv y=x) \vee [(\sim(\exists y)(y=x) \ \& \ (\exists y)(Fy)) \supset (\exists y)(\exists z)(Fy \ \& \ Fz \ \& \ y \neq z)]]\} \tag{8}$$

Moreover, should the antecedent be further weakened, yet still entail that '$(\iota x)(Fx)$' is a designator, then the resulting schema would be invalid in some PH-languages. Let us call a nonrigid designator '$(\iota x)(Fx)$' a <u>semi-rigid designator</u> just in case (8) is true. A semi-rigid designator is a nonrigid designator that picks out the same object with respect to every accessible world in which it exists, but fails to pick out a single object with respect to other accessible worlds. Perhaps not all semi-rigid designators are curiosities. Consider 'the individual that grew from r', where 'r' names the zygote from which

Socrates developed. Some claim that it is essential to Socrates that he grew from the zygote he did. One might also plausibly claim that had this zygote produced identical twins, Socrates would not have been one of them. If both claims are correct, then 'the individual that grew from r' is a semi-rigid designator.

Earlier, we showed that if '(ɿx)(Fx)' is a rigid designator, then (5) and (6) must match in truth-value in every PH-language. We have now extended the result to semi-rigid designators, and we have shown that it holds for designators only of these two kinds. We have thus answered our questions for '◇' and the PH-languages. Every secondary occurrence of a designator vis-a-vis '◇' is guaranteed to yield the same truth-value as the corresponding primary occurrence in standard contexts in all PH-languages if and only if the designator is either rigid or semi-rigid.

This pairing of rigid and semi-rigid designators will continue to hold throughout our results. Furthermore, the natural ways of motivating the notion of rigidity do not isolate it from semi-rigidity.[29] From a theoretical standpoint, then, semi-rigid designation is not entirely a curiosity.

Now consider '◇' and the K-languages. The possible world interpretation of our criterion for rigid designation is the same in both families of languages. Nevertheless, the rigidity of '(ɿx)(Fx)' is not sufficient to assure that (5) and (6) agree in truth-value in every K-language. Unlike the PH-languages, the K-languages generally permit an open formula like 'Gx' to be satisfied with respect to a world even by objects that do not exist in that world. As a result, even when '(ɿx)(Fx)' is a rigid designator, (5) could be true in some K-languages while (6) is false. For suppose that the object rigidly designated by '$(\iota x)(Fx)$' does not satisfy 'Gx' with respect to any accessible world in which it exists. Then (6) will be false since in no accessible world will there exist an object that satisfies both 'Fx' and 'Gx'. But the designated object may still satisfy 'Gx' with respect to some accessible world in which it does not exist. Thus (5) could still be true. Accordingly, the schema obtained from (4) when '$\Diamond$' replaces 'μ' is not valid in some K-languages.

The counter-model to the schema exploits the fact that the converse Barcan formulas are invalid in some K-languages. Any counter-model to '$(\exists x)(\Diamond Gx) \supset \Diamond(\exists x)(Gx)$' furnishes a counter-model to the schema so long as one of the relevant objects that exists and satisfies '$\Diamond Gx$' in the actual world can be rigidly designated. Furthermore, if '$(\iota x)(Fx)$' is rigid, (5) and (6) can differ in truth-value in a K-language only if '$(\exists x)(\Diamond Gx)$' can be true without '$\Diamond(\exists x)Gx$' being true. Therefore, the schema obtained from (4) when '$\Diamond$' replaces 'μ' is valid in just those K-languages in which the converse Barcan formulas are valid. Kripke has shown that these formulas are invalid unless the K-language requires each object to exist in every world accessible from any world in which it exists.[30] Our counter-model thus equally exploits the fact that '$(\iota x)(Fx)$' can designate a "contingently existing" object--i.e., an object that fails to exist in some accessible world. This tells us how to strengthen the antecedent of (4) to obtain a schema that is valid in all K-languages. The object rigidly designated by '$(\iota x)(Fx)$' must be required to exist in all accessible worlds. In other words, the antecedent must entail that '$(\iota x)(Fx)$' is a strongly rigid designator.

This completes the answers to our questions in the case of '◇' and the K-languages. Every secondary occurrence of a designator vis-a-vis '◇' is guaranteed to yield the same truth-value as the corresponding primary occurrence if and only if the designator is strongly rigid. Simple rigidity does not suffice unless the converse Barcan formulas are valid.

As remarked earlier, the most extreme of the nonstandard predicates of the PH-languages are just like K-language predicates. The other nonstandard predicates fall between these and the standard predicates--i.e., they display some satisfaction gaps, but not everywhere that standard predicates would. A distinctive feature of all of the nonstandard predicates is that the converse Barcan formulas formed with them are invalid in those PH-languages that permit contingently existing objects. In particular, '$\Diamond(\exists x){\sim}(\exists y)(y=x)$' is invariably logically false, while '$(\exists x)\Diamond{\sim}(\exists y)(y=x)$' is true so long as there is at least one contingently existing object. Accordingly, the preceding results for the K-languages also give us results for the nonstandard contexts in the PH-languages. The rigidity of a designator is not enough to make truth-value unaffected by the scope of '◇' in nonstandard contexts in many PH-languages. The designator must be strongly rigid. In other words, every secondary occurrence of a designator vis-a-vis '◇' is guaranteed to yield the same truth-value as the corresponding primary occurrence in all contexts in all PH-languages if and only if the designator is strongly rigid. Our earlier positive result for '◇' and the PH-languages holds for rigid and semi-rigid designators only in standard contexts. When this restriction is dropped, the result no longer holds.

The striking conclusion about '$\Diamond$' is the contrast between the two families of languages. (9) is valid in all PH-languages, but not in all K-languages:

$$(\exists x)\left[(\forall y)(Fy \equiv y=x) \;\&\; \Box(\forall y)(Fy \equiv y=x)\right] \supset \left[\Diamond(\exists x)((\forall y)(Fy \equiv y=x) \;\&\; Gx) \equiv (\exists x)((\forall y)(Fy \equiv y=x) \;\&\; \Diamond Gx)\right] \quad (9)$$

The K-languages in which it is not valid are among the philosophically more interesting in that they sanction rigid designation of contingent objects. To obtain a schema valid in all K-languages, the antecedent of (9) must be strengthened, as in (1), to entail that '$(\iota x)(Fx)$' is a strongly rigid designator:

$$\left\{(\exists x)\left[(\forall y)(Fy \equiv y=x) \;\&\; \Box(\forall y)(Fy \equiv y=x)\right] \;\&\; \Box(\exists x)(Fx)\right\} \supset \left[\Diamond(\exists x)((\forall y)(Fy \equiv y=x) \;\&\; Gx) \equiv (\exists x)((\forall y)(Fy \equiv y=x) \;\&\; \Diamond Gx)\right] \quad (10)$$

Next consider '$\Box$' and the PH-languages. Suppose '$(\iota x)(Fx)$' is a rigid designator. Even so, (11) and (12) need not match in truth-value in some PH-languages:

$$(\exists x)\left[(\forall y)(Fy \equiv y=x) \;\&\; \Box Gx\right] \quad (11)$$

$$\Box(\exists x)\left[(\forall y)(Fy \equiv y=x) \;\&\; Gx\right] \quad (12)$$

For suppose the rigidly designated object fails to exist in some accessible world. Since '$(\iota x)(Fx)$' is rigid, no object existing in that world will satisfy 'Fx'. Thus (12) will be false. Nevertheless, the object that satisfies 'Fx' in the actual world may satisfy 'Gx' in every accessible world in which it exists. Thus (11) could still be true. For that matter, (11) could still be true in the K-languages. For the object that exists and satisfies 'Fx' in the actual world may satisfy 'Gx' with respect to all accessible worlds, including those in which it does not exist. But (12) will be false in K-languages when the rigidly designated object does not exist in all accessible worlds. Therefore, for neither family of languages is rigidity sufficient to guarantee that differences in the scope of a designator with respect to '$\Box$' do not affect truth-value. Both in some PH-languages and in some K-languages, the schema obtained from (4) when '$\Box$' replaces 'μ' is not valid.

The counter-models to the schema in this case require '$(\imath x)(Fx)$' to designate a "contingently existing" object. Indeed, when '$(\imath x)(Fx)$' is rigid, (11) and (12) can differ in truth-value in any of the languages we are considering if and only if '$\Diamond\sim(\exists x)(Fx)$' is true. Presumably, any of the languages that permits contingently existing objects also permits rigid designation of some of them. If so, the schema is valid in just those PH- and K-languages that bar contingent objects. But in these languages rigid designation is tantamount to strongly rigid designation. This shows that to obtain an always valid schema when '$\Box$' replaces 'μ', the antecedent of (4) must be strengthened to entail that '$(\imath x)(Fx)$' is a strongly rigid designator. If '$(\imath x)(Fx)$' is strongly rigid, then both (11) and (12) are true in any PH- or K-language just in case the object that satisfies 'Fx' in the actual world satisfies 'Gx' with respect to every accessible world whatever. Thus, in all of the languages we are considering, every secondary occurrence of a designator vis-a-vis '$\Box$' is guaranteed to yield the same truth-value as the corresponding primary occurrence if and only if the designator is strongly rigid.[31]

In the case of '$\Box$', then, the results are the same for both families of modal languages. (13) is not valid in those languages in each family that sanction rigid designation of contingent objects; but

(14) is valid in every one of the languages:

$$(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ \Box(\forall y)(Fy \equiv y=x)] \supset [\Box(\exists x)((\forall y)(Fy \equiv y=x) \ \& \ Gx) \equiv (\exists x)((\forall y)(Fy \equiv y=x) \ \& \ \Box Gx)] \quad (13)$$

$$\{(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ \Box(\forall y)(Fy \equiv y=x)] \ \& \ \Box(\exists x)(Fx)\} \supset [\Box(\exists x)((\forall y)(Fy \equiv y=x) \ \& \ Gx) \equiv (\exists x)((\forall y)(Fy \equiv y=x) \ \& \ \Box Gx)] \quad (14)$$

This conclusion is so easily established that it is unclear why anyone might think that rigidity alone would suffice. The only hope I see for holding that rigid designators are neutral with respect to the scope of '$\Box$' when they designate contingent objects is to adopt a free-logic in which definite descriptions are not Russellian. But Kripke is clearly not talking about such logics in the footnote quoted earlier. Rather, he must be excepting '$\Box$' from his claims.

Our conclusions about '$\Box$' must be circumscribed to prevent confusion. As interpreted in the languages we are considering, '$\Box$' expresses necessity _de re_ in (11) and _de dicto_ in (12). So far we have shown that, even if '$(\imath x)(Fx)$' is a rigid designator, (11) and (12) need not be materially equivalent. However, this is not to say that no purely _de dicto_ modal formula is guaranteed to have the same truth-value as (11) when '$(\imath x)(Fx)$' is rigid. To the contrary, in all PH-languages--though, as we shall see, not in all K-languages--(15) is such a formula:

$$\Box[(\exists! x)(Fx) \supset (\exists x)((\forall y)(Fy \equiv y=x) \ \& \ Gx)] \quad (15)$$

For suppose '$(\imath x)(Fx)$' rigidly designates the object s. With respect to any accessible world in which s does not exist, the bracketed portion of (15) is true by virtue of the falsity of its antecedent. Therefore, when '$(\imath x)(Fx)$' is rigid, (15), like (11), is true in a PH-language if and only if s satisfies 'Gx' with respect to every accessible world in which it exists.

Furthermore, if '(ɿx)(Fx)' is a nonrigid designator, then unless it is semi-rigid, (11) and (15) need not agree in truth-value in some PH-languages. Case-by-case reasoning like that which led us to identify semi-rigidity during the discussion of (5) and (6) will establish this result. (Simply reversing truth and falsity in the models used to contrast (5) and (6) will yield models on which (11) and (15) contrast in truth-value.) But we do not have to proceed exhaustively through all of the types of nonrigid designators to see why the result holds. The upshot of the case-by-case reasoning is that, if '(ɿx)(Fx)' is a designator, it must meet two conditions before (11) and (15)--or (5) and (6)--are guaranteed to agree in truth-value in all PH-languages. First, the designated object must satisfy 'Fx' uniquely with respect to every accessible world in which it exists. Second, there must be no object that satisfies 'Fx' uniquely with respect to any other accessible world. Rigid and semi-rigid designators are the only ones that meet both conditions. They differ merely in the way they meet the second condition. Rigid designators meet it because no existing object satisfies 'Fx' with respect to accessible worlds in which the designated object does not exist; semi-rigid designators meet it because no <u>one</u> existing object satisfies 'Fx' with respect to these worlds. Nonrigid designators of other types fail to meet one of the two conditions and thereby open certain PH-languages to models on which (11) and (15) contrast in truth-value.

(15) is reminiscent of the weak notion of necessity that Kripke uses in "Identity and Necessity" and "Naming and Necessity." The weak

notion is illustrated by parsing (16) as (17):

Necessarily, Hesperus is self-identical. (16)

'Hesperus is self-identical, if it exists' is a necessary truth. (17)

The antecedent in (17), like that in (15), renders the conditional trivially true should the designating expression in the consequent fail to denote. Let us use '$\boxminus$' to represent such a weak necessity operator. $\ulcorner \boxminus A \urcorner$ is to be read along the lines of 'it is necessary that A is true unless a designating expression in A lacks reference'. The general approach to characterizing '$\boxminus$' model-theoretically is obvious: $\ulcorner \boxminus A \urcorner$ is assigned T just in case A is assigned F with respect to no accessible world in which there exist objects denoted by the designating expressions in A. The difficulty lies in picking out the designating expressions in A after, for example, definite descriptions have been expanded in the manner of Russell. To a first approximation, '$(\iota x)(Fx)$' is a designating expression in A if and only if there is an open formula $\phi(\xi)$ such that $\ulcorner A \equiv \phi((\iota x)(Fx))\urcorner$ is valid when '$(\iota x)(Fx)$' has maximal scope. Several refinements are needed, but so far as I can see, they can be introduced only rather clumsily.[32] Fortunately, the approximate characterization will suffice for our present purposes.

Two minor consequences of this way of characterizing the designating expressions in a formula should be noted. First, free variables are not designating expressions. Second, a definite description can be a designating expression in a constituent of a formula without being a designating expression in the overall formula. For example, '$(\iota x)(Fx)$'

is not a designating expression in '$(\exists!x)(Fx) \supset (\exists x)((\forall y)(Fy \equiv y=x) \ \&\ Gx)$', though it clearly is one in the consequent of this formula.

With '$\boxminus$' thus characterized, (18) and (19) are respectively equivalent to (11) and (15) in every language we are considering:

$$(\exists x)\left[(\forall y)(Fy \equiv y=x) \ \&\ \boxminus Gx\right] \tag{18}$$

$$\boxminus(\exists x)\left[(\forall y)(Fy \equiv y=x) \ \&\ Gx\right] \tag{19}$$

Moreover, '$G\oslash$' can be replaced by any predicate, standard or nonstandard, that contains no designating expressions, and the equivalences between (11) and (18) and (15) and (19) will continue to hold in all PH- and K-languages.

Given such equivalences, our earlier results for (11) and (15) imply that (18) and (19) must match in truth-value in every PH-language if '$(\iota x)(Fx)$' is a rigid or semi-rigid designator. Specifically, when '$(\iota x)(Fx)$' is rigid or semi-rigid, both (18) and (19) are true in any PH-language just in case the designated object satisfies 'Gx' with respect to every accessible world in which it exists. In the PH-languages, therefore, (18) and (19) are related to one another in the same way as (5) and (6). Furthermore, since '$G\oslash$' is in the scope of '$\boxminus$' in both (18) and (19), this result holds for any standard predicate replacing '$G\oslash$'. Thus with regard to scope, '$\boxminus$' is just like '$\Diamond$' in the PH-languages. (20), the schema obtained from (4) when '$\boxminus$' replaces 'μ', is valid in all of these languages:

$$(\exists x)\left[(\forall y)(Fy \equiv y=x) \ \&\ \Box(\forall y)(Fy \equiv y=x)\right] \supset \left[\boxminus(\exists x)((\forall y)(Fy \equiv y=x) \ \&\ Gx) \equiv (\exists x)((\forall y)(Fy \equiv y=x) \ \&\ \boxminus Gx)\right. \tag{20}$$

More generally, every secondary occurrence of a designator <u>vis-a-vis</u> '$\boxminus$' is guaranteed to yield the same truth-value as the corresponding primary occurrence in standard contexts in all PH-languages if and only if the designator is either rigid or semi-rigid.

'⊟' also behaves like '◇' with regard to scope in the K-languages. Even when '$(\iota x)(Fx)$' is a rigid designator, (11) and (15) need not agree in truth-value in some K-languages. For suppose the rigidly designated object satisfies 'Gx' with respect to every accessible world in which it exists. Then (15) will be true. Nevertheless, the designated object might fail to satisfy 'Gx' with respect to an accessible world in which it does not exist. Thus in some K-languages (11) could still be false. But (11) and (15) are respectively model-theoretically equivalent to (18) and (19) in every K-language. Hence, even when '$(\iota x)(Fx)$' is a rigid designator, (18) and (19) need not match in truth-value in some K-languages.

The contrast between the K-languages and the PH-languages is accordingly the same for '⊟' as for '◇'. (20) is valid in all PH-languages, but it is not valid in those K-languages that permit rigid designation of contingent objects. As before, to get a schema that is valid in all K-languages, the antecedent of (20) must be strengthened, as in (21), to entail that '$(\iota x)(Fx)$' is a strongly rigid designator:

$$\begin{array}{l}\{(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ \Box(\forall y)(Fy \equiv y=x)] \ \& \ \Box(\exists x)(Fx)\} \supset \\ \quad [\boxminus(\exists x)((\forall y)(Fy \equiv y=x) \ \& \ Gx) \equiv (\exists x)((\forall y)(Fy \equiv y=x) \ \& \ \boxminus Gx)]\end{array} \qquad (21)$$

More generally, every secondary occurrence of a designator <u>vis-a-vis</u> '⊟' is guaranteed to yield the same truth-value in all K-languages as the corresponding primary occurrence if and only if the designator is strongly rigid.[33]

(15) was first introduced as an example of a purely <u>de dicto</u> modal formula that is materially equivalent in every PH-language to

(11), a purely de re modal formula, when '(ιx)(Fx)' is a rigid designator. But (15) is not an example of such a formula in the case of the K-languages. In fact, if '(ιx)(Fx)' is a designator, then unless it is strongly rigid, no purely de dicto formula of the form ⌜□A⌝ will be materially equivalent to (11) in every K-language.[34] For consider the K-languages in which '(ιx)(Fx)' rigidly designates a contingent object, s. In these languages the truth of (11) will depend in part on whether s satisfies 'Gx' with respect to worlds in which it does not exist. Hence, if ⌜□A⌝ were materially equivalent to (11) in all of these languages, its truth would also have to depend on whether s satisfies 'Gx' with respect to such worlds. However, there is no way for the truth of ⌜□A⌝ to depend on this. Since ⌜□A⌝ is by supposition a purely de dicto modal formula, A contains no free variables. And, with respect to any particular world, the quantified variables in A range only over objects that exist in that world. Hence, no variable in A will range over s with respect to a world in which it does not exist. Consequently, with respect to every such world, the truth of A will be independent of whether s satisfies 'Gx'. But then the truth conditions for ⌜□A⌝ will differ from those for (11) in the K-languages in which '(ιx)(Fx)' rigidly designates s.[35]

Accordingly, no purely de dicto modal formula of the form ⌜□A⌝ will be materially equivalent to (11) in all K-languages when '(ιx)(Fx)' is a rigid, but not a strongly rigid designator. Of course, if it is strongly rigid, then (12) is a formula of the desired form that is equivalent to (11) in all K-languages. Again, however, the important result is the contrast between the two families of modal languages. In standard contexts rigid designators bridge the truth-value gap between de re and de dicto (strong) necessity in all PH-languages; but they do not do so in all K-languages.

The relationship between (11) and (15) provides some further results. (11) is model-theoretically equivalent to (22) in all of the languages we are considering:

$$(\exists x)\left[(\forall y)(Fy \equiv y{=}x) \ \& \ {\sim}\Diamond{\sim}\, Gx\right] \tag{22}$$

(15) is likewise equivalent to (23), which in turn simplifies to (24):

$${\sim}\Diamond{\sim}\left\{{\sim}(\exists x)(\forall y)(Fy \equiv y{=}x) \ \mathrm{v} \ (\exists x)\left[(\forall y)(Fy \equiv y{=}x) \ \& \ Gx\right]\right\} \tag{23}$$

$${\sim}\Diamond(\exists x)\left[(\forall y)(Fy \equiv y{=}x) \ \& \ {\sim}Gx\right] \tag{24}$$

With no loss of generality, 'H' can be substituted for '~G' in (22) and (24) to get (25) and (26):

$$(\exists x)\left[(\forall y)(Fy \equiv y{=}x) \ \& \ {\sim}\Diamond Hx\right] \tag{25}$$

$${\sim}\Diamond(\exists x)\left[(\forall y)(Fy \equiv y{=}x) \ \& \ Hx\right] \tag{26}$$

Therefore, (25) and (26) must be related in the same way in the various languages as (11) and (15). That is, if '$(\iota x)(Fx)$' is a designator, then (25) and (26) must match in truth-value in all PH-languages if and only if '$(\iota x)(Fx)$' is rigid or semi-rigid. By contrast, they must match in truth-value in all K-languages if and only if it is strongly rigid. Therefore, (27)--the schema obtained from (4) when the impossibility operator, '$\sim\Diamond$', replaces 'μ'--is valid in all PH-languages, but not in all K-languages:

$$\begin{aligned}&(\exists x)\left[(\forall y)(Fy \equiv y{=}x) \ \& \ \Box(\forall y)(Fy \equiv y{=}x)\right] \supset \\ &\quad\left[{\sim}\Diamond(\exists x)((\forall y)(Fy \equiv y{=}x) \ \& \ Gx) \equiv (\exists x)((\forall y)(Fy \equiv y{=}x) \ \& \ {\sim}\Diamond Gx)\right]\end{aligned} \tag{27}$$

To get a schema that is valid in all K-languages, the antecedent of (27) must be strengthened to entail that '$(\iota x)(Fx)$' is strongly rigid.

The contrast between the PH- and the K-languages is thus the same for '$\sim\Diamond$' as for '$\Diamond$'. Every secondary occurrence of a designator *vis-a-vis* '$\sim\Diamond$' is guaranteed to yield the same truth-value as the corresponding primary occurrence in standard contexts in all PH-languages if and only if the designator is either rigid or semi-rigid. But to guarantee matching truth-values in all K-languages, the designator must be strongly rigid.

It is instructive to make the same move with (12) as we made with (11) and (15). Just as (15) is model-theoretically equivalent to (24) in all of the languages we are considering, (12) is equivalent to (28):

$$\sim\Diamond\sim(\exists x)\left[(\forall y)(Fy \equiv y=x) \;\&\; Gx\right] \tag{28}$$

(28) should not be confused with (24), for the second occurrence of '$\sim$' has wide scope in (28) and narrow scope in (24). Since (15) is also equivalent to (19), the contrast between (28) and (24) parallels that between (12) and (19). The contrast between (28) and (24) thus shows that, in the languages we are considering, the difference between the *de dicto* '$\Box$' and the *de dicto* '$\boxminus$' is akin to the difference between outer and inner negation.

That '$\sim\Diamond$' behaves in the same way with regard to scope as '$\Diamond$' should not be surprising. Interchanging truth and falsity in our arguments about '$\Diamond$' will trivially yield parallel arguments about '$\sim\Diamond$'. This point holds equally for '$\sim\Box$' and '$\sim\boxminus$'. That is, in the languages we are considering, the strong and weak contingency operators behave respectively in the same way with regard to scope as the strong and weak necessity operators. Thus, neither in all PH- nor in all K-languages is (29) valid:

$$(\exists x)\left[(\forall y)(Fy \equiv y=x) \;\&\; \Box(\forall y)(Fy \equiv y=x)\right] \supset \left[\sim\Box(\exists x)((\forall y)(Fy \equiv y=x) \;\&\; Gx) \equiv (\exists x)((\forall y)(Fy \equiv y=x) \;\&\; \sim\Box Gx)\right] \tag{29}$$

To get a schema that is valid in all of the languages in either family, the antecedent of (29) must be strengthened to entail that '$(\iota x)(Fx)$' is a strongly rigid designator. Hence, in the languages we are considering, every secondary occurrence of a designator vis-a-vis '$\sim\Box$' is guaranteed to yield the same truth-value as the corresponding primary occurrence if and only if the designator is strongly rigid.

In the case of '$\sim\boxminus$', we get the usual contrast between the two families of languages. (30) is valid in all PH-languages, but not in all K-languages:

$$(\exists x)\left[(\forall y)(Fy \equiv y=x) \;\&\; \Box(\forall y)(Fy \equiv y=x)\right] \supset \left[\sim\boxminus(\exists x)((\forall y)(Fy \equiv y=x) \;\&\; Gx) \equiv (\exists x)((\forall y)(Fy \equiv y=x) \;\&\; \sim\boxminus Gx)\right] \tag{30}$$

To obtain a schema that is valid in all K-languages, the antecedent of (30) must be strengthened to entail that '$(\iota x)(Fx)$' is a strongly rigid designator. More generally, every secondary occurrence of a designator vis-a-vis '$\sim\boxminus$' is guaranteed to yield the same truth-value as the corresponding primary occurrence in standard contexts in all PH-languages if and only if the designator is rigid or semi-rigid. But the designator must be strongly rigid to guarantee matching truth-values in all K-languages.

The results for '$\sim\Diamond$', '$\sim\Box$', and '$\sim\boxminus$' thus add little to our earlier findings. However, the fact that '$\sim\Diamond$' and '$\sim\Diamond\sim$' do not have the same scope characteristics in the PH-languages does call attention to something we have so far ignored. If '$(\iota x)(Fx)$' is a designator, then (31) and (32) must have the same truth-value:

$$(\exists x)[(\forall y)(Fy \equiv y{=}x) \ \& \ {\sim}Gx] \tag{31}$$

$$\sim(\exists x)[(\forall y)(Fy \equiv y{=}x) \ \& \ Gx] \tag{32}$$

But when either '$\Diamond$' or '$\Box$' is prefixed to both (31) and (32), '$(\iota x)(Fx)$' must be a strongly rigid designator to assure that the resulting two formulas match in truth-value. Thus, for example, (33) will be trivially false while (34) will be true in all those PH- and K-languages in which '$(\iota x)(Fx)$' is a rigid, but not strongly rigid designator:

$$\Diamond(\exists x)[(\forall y)(Fy \equiv y{=}x) \ \& \ {\sim}Fx] \tag{33}$$

$$\Diamond{\sim}(\exists x)[(\forall y)(Fy \equiv y{=}x) \ \& \ Fx] \tag{34}$$

Similarly, if '$\overline{Fx}$' is a logical contrary of 'Fx', then (35) will be false and (36) will be trivially true in these languages:

$$\Box(\exists x)[(\forall y)(Fy \equiv y{=}x) \ \& \ {\sim}\overline{Fx}] \tag{35}$$

$$\Box{\sim}(\exists x)[(\forall y)(Fy \equiv y{=}x) \ \& \ \overline{Fx}] \tag{36}$$

Of course, if '$(\iota x)(Fx)$' is a strongly rigid designator, (33) and (34) will both be false, and (35) and (36) will both be true in every language we are considering.

'$\sim$' is not the only operator that has different scope characteristics when it occurs embedded in modal contexts. Earlier we showed that (5) and (6)--i.e., '$(\exists x)[(\forall y)(Fy \equiv y{=}x) \ \& \ \Diamond Gx]$' and '$\Diamond(\exists x)[(\forall y)(Fy \equiv y{=}x) \ \& \ Gx]$'--must match in truth-value in all PH-languages when '$(\iota x)(Fx)$' is a rigid designator. But (37) and (38) need not match in truth-value in all PH-languages when '$(\iota x)(Fx)$' is rigid:

$$\Box(\exists x)[(\forall y)(Fy \equiv y{=}x) \ \& \ \Diamond Gx] \tag{37}$$

$$\Box\Diamond(\exists x)[(\forall y)(Fy \equiv y{=}x) \ \& \ Gx] \tag{38}$$

For example, consider PH-languages in which the actual world is accessible from all other worlds. Suppose the object designated by '$(\iota x)(Fx)$' satisfies 'Gx' only with respect to the actual world. Then (38) would be true; but (37) would be false unless the designated object exists in all accessible worlds. In other words, (37) and (38) need not match in truth-value in the PH-languages in question unless '$(\iota x)(Fx)$' is a strongly rigid designator. However, if it is strongly rigid, then (37) and (38) will match in truth-value in all PH-languages, for then '$(\exists x)[(\forall y)(Fy \equiv y=x)\ \&\ \Diamond Gx]$' and '$\Diamond(\exists x)[(\forall y)(Fy \equiv y=x)\ \&\ Gx]$' will match in truth-value with respect to every accessible world.[36]

The contrast between (37) and (38) when '$(\iota x)(Fx)$' is rigid, but not strongly rigid, resolves a matter we left open at the end of the preceding section of the paper. The positive results we have obtained for the scope of rigid designators *vis-a-vis* modal operators in the PH-languages are limited. They do not generalize to the case of modal formulas embedded in modal contexts. They do not generalize any more than Whitehead's and Russell's positive results for the scope of designators *vis-a-vis* truth-functional operators generalize to the case of truth-functional formulas embedded in modal contexts. Of course, such positive results do generalize to the case of formulas embedded in extensional contexts. But this generalization requires only that '$(\exists x)[(\forall y)(Fy \equiv y=x)\ \&\ \nu Gx]$' and '$\nu(\exists x)[(\forall y)(Fy \equiv y=x)\ \&\ Gx]$' always be assigned matching values with respect to the actual world.[37] By contrast, the generalization to embeddings in modal contexts requires that '$(\exists x)[(\forall y)(Fy \equiv y=x)\ \&\ \nu Gx]$' and '$\nu(\exists x)[(\forall y)(Fy \equiv y=x)\ \&\ Gx]$' always be assigned matching values with respect to every accessible world. In the PH-languages, as well as in the K-languages, this last requirement is met if and only if '$(\iota x)(Fx)$' is a strongly rigid designator.

This conclusion can be stated formally. Let 'ϑ' stand for an operator, modal or extensional. Then all formulas of the form of (39) are valid both in all PH-languages and in all K-languages:

$$\{(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ \Box(\forall y)(Fy \equiv y=x)] \ \& \ \Box(\exists x)(Fx)\} \supset \Box\{\vartheta(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ Gx] \equiv (\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ \vartheta Gx]\} \quad (39)$$

However, if the antecedent of (39) is weakened so that it no longer entails that '$(\iota x)(Fx)$' is a strongly rigid designator, then some formulas of the resulting form will not be valid in some PH-languages and in some K-languages.

What condition, then must a definite description satisfy to assure that its scope never affects the truth-value of any formula in any language in one of our modal families? The answer is the same for both of our families: the definite description must be a strongly rigid designator. Earlier one might have thought that strong rigidity is the answer in the case of standard contexts in the PH-languages only because of our insistence on interpreting '$\Box$' strongly. But the contrast between (33) and (34), which do not contain '$\Box$', shows otherwise. The requirement of strong rigidity comes from deeper considerations. Our crucial examples have all turned on the same model-theoretical feature of the PH- and K-languages, viz., that with respect to any world, bound variables range only over the objects existing in that world. If one wants to weaken the requirement of strong rigidity, this feature is the appropriate one to abandon.

VI

The principal question we have been addressing is whether all formulas of the form of (4) are valid in either all PH- or all K-languages:

$$(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ \Box(\forall y)(Fy \equiv y=x)] \supset [\mu(\exists x)((\forall y)(Fy \equiv y=x) \ \& \ Gx) \equiv (\exists x)((\forall y)(Fy \equiv y=x) \ \& \ \mu Gx)] \quad (4)$$

The answer for both families of languages is no. We found that all such formulas are valid in a PH- or K-language just in case (40) is valid in that language:

$$(\exists x)\Box(\forall y)(Fy \equiv y=x) \supset \Box(\exists x)(Fx) \quad (40)$$

But (40) is not valid in PH- and K-languages that permit rigid designation (in the sense of (2)) of objects whose existence is contingent.

The validity of (40) is required for the validity of (4) because of the way the PH- and K-languages treat variables occurring in contexts governed by '$\Box$'. For suppose α rigidly designates a contingent object in a PH- or K-language in which (40) is not valid. Then in this language an open formula $\Psi(\xi)$ can be necessarily true of the object α designates without the corresponding closed formula $\Psi(\alpha)$ being a necessary truth. This can happen because the PH- and K-languages treat variables that are bound from within the scope of '$\Box$' differently from those that are not. In particular, in the PH- and K-languages that countenance contingent objects, '$(\exists x)(\Box Gx)$' can be true without '$\Box(\exists x)(Gx)$' being true. Of course, this treatment of variables does not lack motive. It is the basis for claiming that the PH- and K-languages need only one modal operator to capture both <u>de re</u> and <u>de dicto</u> necessity.

The requirement that (40) be valid provides the answer to a follow-up question we posed--viz., how must the antecedent of (4) be strengthened to transform the schema into one that is valid for all modal operators in all PH- or all K-languages? The answer for both families of languages is that the antecedent must entail that '$(\iota x)(Fx)$' is a strongly rigid designator.

Our principal question was put forward as a step toward answering another question: with regard to scope distinctions, are rigidly designating Russellian definite descriptions the analogue in modal languages of designating Russellian definite descriptions in truth-functional languages? Unless our definition of rigid designation is radically in error, again the answer is no. In the case of PH-like or K-like modal extensions of standard quantificational languages, the analogue of Whitehead's and Russell's "scope equivalency" condition is not that the definite description be a rigid designator, but that it be a strongly rigid designator. This result is worth stating precisely. Suppose α is a definite description in a PH- or K-language. Let A be any sentence in which α occurs with its scope unmarked; and let B and B' be any two sentences obtained from A via Russellian expansions of α. Then B and B' are guaranteed to be materially equivalent just in case α is a strongly rigid designator.

Although the general result is the same for both families of languages, the results in the case of individual modal operators are different. This is not surprising. The two families construe occurrences of modal operators outside of the scope of quantifiers in basically the same way; but they construe occurrences inside quantifiers differently.

To appreciate the difference, consider a PH- and a K-language with the same vocabulary, both of which countenance contingent objects. Let them be sufficiently comparable to one another that both can be interpreted on all the same models, with like extensions assigned to their atomic predicates with respect to every world. Then, on individual models, some formulas like '$(\exists x)(\Box Gx)$'--and hence also like '$(\exists x)(\boxminus Gx)$'--will be true in the PH-language, yet false in the K-language; and some formulas like '$(\exists x)(\Diamond Gx)$' will be true in the K-language, yet false in the PH-language. However, on each model the truth-value of any formula like '$\mu(\exists x)(Gx)$' will be the same in both languages. Moreover, on each model the same definite descriptions will be rigid designators in both languages. Therefore, the relationship between rigid designators and the scope of modal operators must not be the same in the two languages.

In the K-family, designators are "scope neutral" in all extensional contexts, and strongly rigid designators are "scope neutral" in all contexts, But as a class, rigid designators--i.e., rigid Russellian definite descriptions--exhibit no distinctive scope-related characteristics. A secondary occurrence of a designator <u>vis-a-vis</u> <u>any</u> of the modal operators we are considering need not yield the same truth-value as the corresponding primary occurrence in all K-languages unless the designator is strongly rigid. Specifically, (4) is invalid regardless of the modal operator replacing 'μ' in just those K-languages in which the converse Barcan formulas are invalid. Accordingly, the failure of rigid designators to exhibit distinctive scope characteristics across the family of K-languages can be viewed as a consequence of the converse Barcan formulas being invalid in some of these languages. But the model theory for the family provides another, perhaps more instructive way of viewing it. Rigidly designating definite descriptions have an important model-theoretical characteristic in all the languages we are considering: only with respect to those worlds in which an object exists does it fulfill a definite description that rigidly designates it. However, whether an object satisfies 'μGx' in the K-languages may depend on all accessible worlds, including those in which it does not exist. Consequently, in the K-languages a rigid designator may fail to pick out the designated object with respect to some of the worlds it must in order to block contrasts in truth-value associated with scope ambiguities. Metaphorically speaking, the reference of a rigid designator may not extend widely enough in the K-languages.

The model-theoretical situation is different in the PH-family. Whether an object satisfies 'μGx', where 'Gx' is standard, does not depend on worlds in which it fails to exist. Instead, it depends on just those worlds with respect to which the object fulfills any definite description that rigidly designates it. A rigidly designating definite description is thus an apt way of referring to an object in standard contexts in the PH-languages. Of course, the more privileged status of rigid designators in the family of PH-languages can also be viewed as a consequence of the converse Barcan formulas being valid in all of these languages. But regardless of how the matter is viewed, (4) is valid in all PH-languages for many modal operators. Among the modal operators we are considering, the only exceptions are '$\Box$' and '$\sim\Box$'. With these exceptions, every secondary occurrence of a rigid designator _vis-a-vis_ the modal operators we are considering is guaranteed to yield the same truth-value as the corresponding primary occurrence in standard contexts in all PH-languages.

Thus, as a class, rigid designators do exhibit some special scope-related characteristics in the PH-family. But these characteristics are nonetheless not distinctive since in this family semi-rigid designators exhibit the very same characteristics. Therefore, their scope characteristics cannot be used to differentiate rigid designators even in the case of the PH-languages.

Among the claims Kripke makes in the footnote we quoted at the beginning of the paper are: (i) ambiguities in the scope of a definite description have no effect on truth-value in modal contexts if the definite description is a rigid designator; (ii) roughly, the condition that a definite description must satisfy for it to be thus scope neutral is that it be a rigid designator; and (iii) when definite descriptions are treated in the manner of Russell, (i) and (ii) often follow from the other postulates of quantified modal logic. One difficulty in assessing these claims is that it is not clear which modal quantificational languages Kripke had in mind. All three claims are true of those strong languages that form the intersection of the PH- and K-families. But these languages are not of much philosophic interest, since their domains of discourse are restricted to objects whose existence is necessary. Moreover, the claims, though true, are somewhat misleading in the case of these languages. The distinction between rigid and strongly rigid designation is too important to be glossed over. Yet the claims are true of rigid designators in these extreme languages just because rigidity in them is tantamount to strong rigidity. Hence, if these are the languages Kripke had in mind, it would have been more appropriate for him to phrase his claims to be about strongly rigid designators.

Kripke's claims are of more note if they are about rigid designators in languages in which rigidity and strong rigidity diverge. But which languages of this sort might he have had in mind? Even with qualifications his claims are not in the least true of rigid designators in any K-language of this sort. In such K-languages ambiguities in the scope of a rigid designator can affect truth-value regardless of which of the customary modal operators forms the modal context. In particular, then, the claims are in no way true of the minimal K-language that Kripke presents in detail in "Semantical Considerations on Modal Logic." To be true of this language, the claims would have to single out not rigid, but strongly rigid designators.

Strictly speaking, the claims are also not true of rigid designators in any PH-language in which rigidity and strong rigidity diverge. Again, they would have to be modified to single out strongly rigid designators. Nevertheless, the claims are in a sense more true for these PH-languages than they are for the corresponding K-languages. For they can be made to hold for rigid designators in these PH-languages by adding four qualifications. First, scope ambiguities occurring embedded within broader modal contexts must be excepted in (i) and (ii). Second, the claims must be restricted to definite descriptions occurring in contexts formed with standard predicates. Third, '$\Box$' and '$\sim\Box$' must be excepted. Finally, the 'roughly' in (ii) must allow for the fact that semi-rigid designators have the same scope-related characteristics as rigid designators. With all of these qualifications, (i), (ii), and (iii) become true in the case of the PH-languages in which rigidity and strong rigidity diverge.

Perhaps, then, these are the languages Kripke had in mind. However, if they are, he again seems open to criticism. For the requisite qualifications are far too significant to have been left unstated. Hence, as far as I can see, unless Kripke had entirely different formal languages in mind, his claims about scope and rigid designators are at best misleading. But notice that once (i), (ii), and (iii) are revised to become claims about strongly rigid designators, they become true for <u>all</u> of the formal languages we have considered. Revising the claims in this way is clearly the most reasonable move to make.

VII

At the very beginning of this chapter I called attention to a number of conflicting claims regarding scope distinctions and rigid designation. How do our results for the PH- and K-languages bear on these comparatively broad claims? One advantage of focusing on precise questions about specific formal languages, as we have been doing, is that answers are often forthcoming. Now we must face a disadvantage. Drawing conclusions of general interest from our results will require some potentially controversial intervening assumptions. For example, we will have to assume that our definition of rigid designation is correct. I see no problem here, but others--e.g., Peacocke and Dummett--may.[38] We will also have to assume that name-free regimented languages employing Russell's treatment of definite descriptions are not mere contrivances, of no general significance.[39] Since our results are for definite descriptions that in fact denote, perhaps Russell's treatment will not be so controversial here as it sometimes has been made out to be in discussions of vacuous reference. Nevertheless, because many of our results have turned on the existential implications of Russell's treatment, the possibility of alternative treatments of definite descriptions is clearly germane.

Finally, we will have to assume that at least one of our two language families achieves a tolerable representation of relevant modal notions. The burgeoning list of formal modal languages in the literature indicates that some will object to this assumption.[40] Both of our families represent _de dicto_ modalities in basically the same way; but, as we have noted, they treat _de re_ modalities differently. To draw conclusions of general interest, we will have to assume that their representation of _de dicto_ modalities is acceptable and that at least one of the families represents _de re_ modalities adequately. Each of the families has something to be said for it in regard to these assumptions. On the one hand, the K-family derives from the most straightforward way of joining axioms for normal modal propositional logic with ones for standard quantificational logic without in the process automatically validating the Barcan formulas.[41] Hence, its easily motivated formalization weighs in favor of the K-family. On the other hand, as I tried to indicate in passing while defining it, the PH-family avoids the salient counterintuitive ramifications of other approaches to formalizing _de re_ and _de dicto_ modalities without resorting to separate operators.[42] Hence, its apparent fit with our informal intuitions weighs in favor of the PH-family. It is accordingly not unreasonable to assume that at least one of our families achieves a tolerable representation of the relevant modal notions. But until some agreement on quantified modal logic is reached in the literature, this assumption is likely to be controversial. In particular, the possibility of needing separate operators for de re and de dicto modalities cannot yet be dismissed.

Too much space would be needed to defend these various intervening assumptions here. Since none of them is eccentric, I suggest that we put debate over them aside for now and turn to the ramifications of our results, taken at face value. I will return to these assumptions briefly at the end of the paper.

Consider first Linsky's claim that a designator α is rigid if and only if, for all atomic $\phi(\xi)$, $\ulcorner\Box\phi(\alpha)\urcorner$ has the same truth-value whether α is taken to have wide or narrow scope. If he intends $\ulcorner\Box\phi(\alpha)\urcorner$ to be construed as it would be in either the PH- or the K-languages, then he is mistaken.[43] His condition is generally necessary and sufficient not for α to be rigid, but for it to be strongly rigid. Indeed, so long as '$\Box$' expresses strong necessity in a language that treats singular terms in Russell's way, Linsky's condition will not be both necessary and sufficient for α to be rigid unless the language also requires that nothing be de re necessarily true of any contingent object.

Short of radical moves, I do not see how Linsky can save this claim. Perhaps it can be saved by shifting to nonstandard languages--e.g., to languages based on a free-logic or to languages in which a Strawsonian treatment of singular terms leads to truth-value gaps. But Linsky does not seem prepared to abandon Russell's treatment of definite descriptions. Nor would I abandon it just to save this one claim about rigid designation. Maybe he would prefer to interpret '$\Box$' as expressing weak necessity. If his condition were stated in terms of $\ulcorner \boxminus \phi(\alpha) \urcorner$ instead of $\ulcorner \Box \phi(\alpha) \urcorner$, then it would be a necessary condition for the rigidity of α in the case of PH-like languages. But it would still not be a necessary condition in the case of K-like languages. And even in the case of PH-like languages, it would not be a sufficient condition since semi-rigid designators would also satisfy it. Our results are clear on this point. If the modalities are represented tolerably in either the PH-languages or the K-languages, then no scope-based condition will be both necessary and sufficient for a Russellian definite description to be a rigid designator. Scope considerations are ideal for demarcating strongly rigid designators; but they cannot serve to demarcate rigid designators.

Next, consider Linsky's claim that rigid designators collapse the _de re/de dicto_ distinction. If he means by this what he appears to, _viz._ that ambiguities in the scope of a rigid designator never affect truth-value, then he is mistaken as before. But this claim he can save. To see this, consider Slote's similar, but more guarded suggestion that, with appropriate existence qualifications, inferences from _de dicto_ to _de re_ and vice versa are legitimate when rigid designators are used. Unfortunately, as it stands this suggestion puts no limits on what can count as an appropriate existence qualification. Thus if Slote means to require the rigid designators to be strongly rigid, then his suggestion is true, but misleading and uninteresting. There are, however, interpretations under which the suggestion is true and interesting. For example, if Slote means only to be excepting strong necessity and contingency in favor of their weak counterparts, then although his suggestion is false in the case of K-like languages, it is true in the case of standard contexts in PH-like languages. Even better, it is true in the case of standard contexts in PH-like languages, although again not in the case of K-like languages, if by 'appropriate existence qualifications' he simply means to require that an antecedent clause like the one in (15) be introduced into de dicto cases involving strong necessity and contingency. Thus, if taken to be about PH-like and not K-like languages, Slote's suggestion is correct when suitably interpreted. Rigid designators do license inferences across the de re/de dicto boundary in PH-like languages. Hence, as Linsky claims, they do in a certain sense collapse the de re/de dicto distinction in these languages.[44] Again, however, they are not alone in doing so, for semi-rigid designators license the same inferences.

Slote's suggestion is open to a more interesting interpretation than those just considered. As last interpreted, it authorized clauses like the one in (15) to be introduced into the premisses as well as the conclusions of inferences. This enabled us to exploit our finding that every purely de re modal formula involving a rigid designator in a standard context is materially equivalent in PH-languages to a purely de dicto modal formula. But suppose now that we allow such existence qualifications to be introduced only into the conclusion of an inference, and never into the premiss. Is Slote's suggestion correct under this restriction? That is, can de re and de dicto conclusions, perhaps incorporating such existence qualifications, be respectively inferred from unqualified de dicto and de re premisses when rigid designators are used?

This version of Slote's suggestion is worth making more precise. As we can see from what has gone before, permitting the categories of de re and de dicto modality to include occurrences of modal operators embedded within modal contexts will complicate the issue without improving the suggestion's chances of being correct. Hence we will restrict these categories for present purposes to occurrences of modal operators that are not themselves within the scope of a modal operator. Even so, such occurrences can be simultaneously de re with respect to one designator (or variable) and de dicto with respect to another--e.g. as in '$(\exists x)[(\forall w)(Fw \equiv w=x)$ & $\Box(\exists y)((\forall w)(Gw \equiv w=y)$ & $Hxy)]$'. Hence, we need to construe Slote's suggestion to be about inferences between de re occurrences of an operator with respect to a designator and de dicto occurrences of that operator with respect to the same designator. Finally, we need to restrict appropriate existence qualifications to clauses like the one in (15) that, when introduced into a de re case, add no variables bound from within the scope of the modal operator, and when introduced into a de dicto case, add no variables bound from outside the scope of the operator. The idea behind this last restriction is to bar the existence qualifications from making a de re case more de dicto or a de dicto case more de re than it would be without the added existence qualification.

I suspect that the version of Slote's suggestion we get with these restrictions is the one he had in mind. It is clearly of some interest. But is it correct? Of course, it is false just as before in the case of K-like languages. Inferences between de re and de dicto cases involving a rigid designator are generally not warranted in these languages unless the designator is strongly rigid. In particular, our earlier results for the K-languages show that if the designator is not strongly rigid, then even with existence qualifications of the sort specified, we cannot infer *de dicto* from *de re* strong necessity.[45] Equally then, this version of the suggestion is not true in the case of nonstandard contexts in PH-like languages. What is slightly surprising is that it is also not true in the case of standard contexts in these languages. It is almost true. Existence qualifications of the sort specified are all that is needed in this case to license the inferences from *de re* to *de dicto*. (Our earlier results show that the existence qualifications are not even needed except when going from *de re* to *de dicto* strong necessity and contingency.) Moreover, with one notable exception, the inferences from *de dicto* to *de re* are legitimate in this case--indeed, without existence qualifications. The exception is strong contingency. In PH-like languages, as well as in K-like languages, such existence qualifications are not enough to license inferences from *de dicto* to *de re* cases involving rigid designators and strong contingency. A formula like (41) may be true in a PH-language (or in a K-language) only because '$(\iota x)(Fx)$' is not a strongly rigid designator:

$$\sim\Box(\exists x)\left[(\forall y)(Fy \equiv y{=}x)\ \&\ Gx\right] \tag{41}$$

Accordingly, the form of qualification needed to license an inference to a corresponding *de re* formula is not like the one in (15), but like the one in (42):

$$\Box(\exists x)(Fx) \supset (\exists x)\left[(\forall y)(Fy \equiv y{=}x)\ \&\ \sim\Box Gx\right] \tag{42}$$

This qualification, however, is tantamount to requiring the rigid designator '$(\iota x)(Fx)$' to be strongly rigid.

So, the most interesting version of Slote's suggestion turns out to be false not just for K-like languages, but even for standard contexts in PH-like languages. Even with allowances for nonstandard predicates, in neither kind of language can one always infer a conclusion of opposite modality from an unqualified de re or de dicto premiss involving a rigid designator. In both kinds of languages such inferences are universally legitimate only if the designator is strongly rigid. In a certain important sense, then, rigid designators do not collapse the de re/de dicto distinction in either K-like or PH-like languages.

Sentences of the sort represented schematically in (43) through (46) are commonly held to be open to two readings, one with the modal phrase taken to be expressing modality de dicto and the other, modality de re:

The x such that Fx had to be G. (43)

The x such that Fx did not have to be G. (44)

The x such that Fx might have been G. (45)

The x such that Fx could not have been G. (46)

What do our results show about how the alternative readings are related to one another? In order for our results to show anything about such sentences, we must first stipulate that the definite descriptions in them can be construed in the manner of Russell without unacceptable distortion. We must also stipulate that the treatment of madality de dicto in the PH- and K-languages is adequate to capture the de dicto readings of the sentences. Then, if we ignore unsatisfiable definite descriptions, our results show three things. First, if the de re readings are construed as in the K-languages, then the definite descriptions in the sentences must be strongly rigid designators in order to guarantee that the alternative readings agree in truth-value. This conclusion also holds if the de re readings are construed as in the PH-languages and 'G' is allowed to stand for a nonstandard predicate. Second, even if the de re readings are construed as in the PH-languages and 'G' is taken to stand for a standard predicate, the definite descriptions still must be strongly rigid to guarantee like truth-values if the modal phrases in sentences like (43) and (44) are taken to express strong necessity and contingency. Finally, however, if the modal phrases in sentences like (43) and (44) are taken to express weak necessity and contingency and if the de re readings are construed as in the PH-languages with 'G' standing for a standard predicate, then to guarantee like truth-values it is both sufficient and necessary that the definite descriptions be rigid or semi-rigid designators.

Intuitively, using rigid designators seems the right move to make to avoid problems stemming from the de re/de dicto ambiguity of sentences of the sort schematized in (41) through (44). Such intuitions gain support from our results. But they do so only if four potentially controversial points are acceded to, not the least of which is the adoption of a PH-like construal of the de re readings.

With some stretching, these last results can be brought to bear on Peacocke's claim that the truth conditions of modal sentences containing rigid designators are the same whether these designators are read with wide or narrow scope. It is unclear whether he intends this claim to hold when the alternative readings are both embedded within a modal context. If he does, then his view is thoroughly incompatible with our findings for the PH- and K-languages. Moreover, such a broad claim seems in trouble independently of formal languages. Consider, for example, (47) and (48), which employ Peacocke's method of exhibiting scope:

It might have been the case that: Heath is not human (47)

It might not have been the case that: Heath is human. (48)

These two appear to me to have different truth-conditions insofar as (48), unlike (47), can be true simply because Heath's existence is contingent. I suspect Peacocke would agree since in dealing with a related example he feels it necessary to stipulate in a footnote that the 'not' following the colon has narrow scope with respect to the designator.[46] At any rate, none of his examples involve alternative readings embedded within broader modal contexts. Hence, it is neither unreasonable nor uncharitable for us to exclude such cases from his claim.

This still leaves us with Peacocke's idiosyncratic use of 'rigid designator'. His rigid designators are closer to Russell's logically proper names than to Kripke's rigid designators. Thus for him the paradigm of a rigid designator in a regimented language is a constant, and all Russellian definite descriptions are paradigmatically nonrigid. I will discuss the scope characteristics of constants in modal languages in two later sections. For reasons that will become increasingly clear then, I think that Peacocke's claim in no way turns on the special syntax of constants. The examples he offers to illustrate and defend his claim support this contention. These examples indicate that for him differences in the scope of a rigid designator in a modal sentence amount basically to what others have identified as the de re/de dicto distinction.[47] Furthermore, thanks to Quine's method for eliminating them, there seems to be no crucial reason to have constants in any regimented language. I am therefore going to take Peacocke's claim not to be about constants as such, but to be about special definite descriptions that have the crucial semantic characteristics of constants.

Once we agree that certain special definite descriptions--call them "rigid designators"--can do the semantic work of constants, Peacocke's claim takes on a different guise. He can then be appropriately viewed as claiming that the truth conditions of alternative readings of sentences of the sort schematized in (43) through (46) are the same provided that the definite descriptions in them are of the special type. Furthermore, once we allow special definite descriptions to replace constants, little motivation remains for restricting rigid designation as severely as Peacocke does. Instead, we can interpret his claim to be about rigid designators in our sense. Then, since he treats singular terms in a standard rather than in a Strawsonian or a free-logic way, our conclusions about (43) through (46) become pertinent. Specifically, Peacocke's claim is correct if he intends such sentences to be construed in the manner of standard predications in the PH-languages and if he uniformly adopts the weak reading of modal phrases expressing necessity and contingency. But if he allows such phrases to express strong necessity or contingency or if he intends such sentences to be construed in the manner of the K-languages, then his claim is wrong.

In putting his claim forward, Peacocke relies primarily on our intuitions about some examples. What he says about these examples gains support from our results. Again, however, it gains this support only after certain major points concerning the semantics of modal sentences have been acceded to--points that are not transparently at issue in his examples. Admittedly, then, our results do not offer much in the way of an argument on behalf of Peacocke. A more promising move to make at this juncture is to turn the argument on its head. Our review of Peacocke's claim, like our review of Slote's and Linsky's, suggests that our informal intuitions about the scope characteristics of rigid designators ought to be taken as evidence favoring a PH-like construal of modal sentences.

This last point deserves emphasis. Several philosophers--Kripke, Linsky, Slote, and Peacocke among them--have claimed that rigid designators are "scope neutral" in modal contexts. An adequate account of the scope characteristics of rigid designators should do more than just determine whether this claim is mistaken. It should also make clear why philosophers would think that the claim is correct. Much of our attention in this and the two preceding sections has been devoted to the latter question. Part of the answer--a tendency to disregard modal sentences embedded in further modal contexts--is of little philosophical interest. A more significant part of the answer is the inclination to read informal necessity and contingency statements weakly. This helps to explain why obvious counterexamples involving '$\Box$' and '$\sim\Box$' escape notice. But it is still not enough to explain why someone would think that rigid designators are scope neutral. The main part of the answer thus seems to lie in the PH-family. If we construe modal sentences in the manner of the K-languages, we have no way to explain the scope neutrality claim other than to write it off as a blunder. If, however, we construe modal sentences in the manner of the PH-languages, we can readily explain why it is reasonable to have thought that rigid designators are scope neutral. This, as I said, is evidence that a PH-like construal is more in keeping with our informal intuitions about rigid designation. And this in turn is evidence that the PH-family offers the more attractive approach to regimenting de re modality.

VIII

So much for those who think that rigid designators, like other singular terms, can have wide or narrow scope with respect to modal operators. What about those, such as Burge and Dummett, who think that rigid designators have a peculiarly restricted logical syntax? Although our approach to this claim will have to be different, we will again be concerned with two questions. The immediate question is whether the claim is mistaken; but the more interesting question is what would lead someone into thinking that it is correct. Differences between the PH-family and the K-family have no bearing on either of these questions. Hence, we will have to look elsewhere for answers. This will be our central concern throughout the next three sections. It will lead us to another important factor that has caused confusion both about rigid designation and more significantly about the regimentation of de re modality.

The first question, then, is whether Burge and Dummett are correct in claiming that rigid designators have a restricted logical syntax. Here, our formal results show little. True, in PH- and K-languages rigid designators (as we have defined them) have the same logical syntax as all other singular terms. Hence, on the surface our results stand opposed to views like those of Burge and Dummett. But that they do so was guaranteed beforehand. From the outset we asked our principal question only of modal quantificational languages in which all singular terms have the same logical syntax. Taken by themselves, therefore, our results are either at cross-purposes with Burge and Dummett, or they beg questions of concern to them.

Even so, we can develop an indirect line of argument against Burge and Dummett. First, consider Dummett's position that a designator is rigid just in case it always has wide scope with respect to modal operators.[48] One consequence of this view is that no PH- or K-language contains any rigid designators at all. Indeed, since Dummett holds that names can have wide or narrow scope with respect to modal operators, he must believe that virtually no formal modal language that has been put forward contains any rigid designators. Rigid designation thus becomes a rather empty notion on his view. Moreover, whether a designator is rigid becomes an issue about its syntax within a given language, and not an issue about the relationship between it and the object it denotes. Thus, should there be a language in which 'the inventor of bifocals' is always read with wide scope _vis-a-vis_ modal operators, then even this paradigmatically nonrigid designator will be rigid in that language, even though it be accidental to Benjamin Franklin that he invented bifocals. Worse still, Dummett's view is incompatible with Kripke's intuitive test for rigid designation. For, according to Kripke's test, α is a rigid designator provided that $\ulcorner\alpha$ might not have been $\alpha\urcorner$ is false on a reading in which the first occurrence of α has wide scope and the second, <u>narrow</u> scope with respect to the modal operator.[49]

In sum, Dummett's view departs radically form the usual approach to rigid designation. As such, it calls for justification. Our results, however, are evidence that any such justification will be difficult to come by. For our results show that a reasonable account of rigid designation is possible without compromising, much less abandoning, the usual approach. Our definition of rigidity is a formal restatement of Kripke's test. On our definition, 'the inventor of bifocals' is a rigid designator in any language only if it is essential to Benjamin Franklin that he (uniquely) invented bifocals. And our definition does not outlaw rigid designators in standard formal modal languages. What reason can there be, then, for adopting Dummett's radical alternative?

One requirement Dummett places on his characterization of rigid designation is that it be free of "the metaphor of possible worlds." Another is that in sentences with modal operators a rigid designator still refer to the object it refers to in sentences with no modal operators. In the case of modal languages that treat bound variables in the manner of the PH- and K-languages, Dummett's characterization meets the second requirement trivially. For when a designator occurs with wide scope with respect to modal operators in such languages, its reference is determined in the very same way as when it occurs in a nonmodal context.

By contrast, we do not insist that rigid designators have wide scope. Nevertheless, on our characterization a rigid designator that has narrow scope with respect to a modal operator still refers to no object other than the one it refers to when it has wide scope. Thus, on our characterization, if a rigid designator occurs in a modal sentence, then whenever it has reference, it refers to the same object as it does when it occurs in a nonmodal sentence. The qualification allows for loss of reference in the case of modal contexts that require the object in question not to exist. With this one qualification, then, our characterization meets Dummett's second requirement. Moreover, it meets it less trivially. On our characterization, a nonrigid definite description fails to be rigid not by virtue of special rules governing the logical syntax of individual singular terms, but because the description is only accidentally true of or accidentally uniquely true of the object it happens to be true of.

Dummett's syntactic characterization of rigidity is obviously free of the metaphor of possible worlds. Though it may be less obvious, our characterization is free of this metaphor too. Of course, our characterization does include a modal operator. But to use a modal operator in explicating a notion is by no means to invoke all the paraphernalia of possible worlds. Modal operators in their own right provide far less expressive power than is employed in possible world talk. For example, there is no way to express 'there is an x that is an F with respect to exactly two possible worlds' in any PH- or K-language. Similarly, 'only with respect to the actual world is there an x that is an F' cannot be expressed.[50] Those of us who resist viewing possible worlds as anything more than a heuristic for a model theory are particularly skeptical about counting over these worlds and referring to specific ones of them. But most formalisms with modal operators are like the PH- and K-languages in having no way either to count over such worlds or to refer to them individually. Modal operators as such, then, should not be confused with "the metaphor of possible worlds."[51]

Moreover, resorting to a modal operator in characterizing rigid designation may be unavoidable. We have no reason to think that modal notions can be reduced to nonmodal ones. Hence, it is entirely appropriate for a characterization of rigid designation to include a modal operator if the notion is, as it appears to be, intrinsically modal. As far as I can see, then, unless Dummett has requirements besides those he mentions, he has no basis for objecting to our characterization of rigidity.

The contrast between Dummett's version of rigid designation and ours can be sharpened by considering counterfactuals like (49):

> Suppose the row of tomato plants had been planted in reverse order; even so, the fourth plant from the right would have had the greatest yield. (49)

'The fourth plant from the right' is referentially ambiguous in (49). Its reference can be based either on what is actually the case, so that it denotes the tomato plant that is now fourth from the right, or on what is counterfactually supposed to be the case, so that it denotes the one that would have been fourth from the right had the row been reversed. Furthermore, which one it denotes may affect the truth-value of (49). (Perhaps their specific locations is the factor controlling the relative yields of the plants in question.) Now both we and Dummett would insist that, unlike 'the fourth plant from the right', rigid designators exhibit a certain invariance of reference in such counterfactual contexts. Dummett would have rigid designators do this by having their reference always based on what is actually the case. On his view, there would be special rules applying to some, but not to all, singular terms. These rules would block the reference of these terms from ever being based on what is counterfactually supposed to be the case. In effect, then, he would have the grammar of the language mark some singular terms with the feature of referential invariance, and others not.

On our view, by contrast, the reference of any singular term occurring in a position like 'the fourth plant from the right' in (49) could still be determined on either of the two bases. But on either basis, a rigid designator, unlike other singular terms, would always be assigned the same referent whenever a referent is assigned to it at all. Rigid designators would be thus referentially invariant not because of special grammatical rules governing the way their reference is determined, but because of the nature of the semantic relation they bear to the objects they denote. That is, according to our version of rigidity, but not Dummett's, the reference of rigid designators is in certain respects insulated from the vagaries of how things happen to be. Even if things besides language had been different, a rigid designator would still have denoted the object it now does, had that object still existed; and regardless, it would have denoted no other object. Consequently, a rigid designator in a counterfactual like (49) can have its reference determined on either of the two bases without giving rise to a true reading on one basis and an untrue one on the other. This is an important feature of rigid designators, one that any characterization of them should secure. But why tamper with the syntax of singular terms when it can be secured in a theoretically more interesting, less artificial way?

The argument against Dummett's position, then, is directed at the claim that rigid designators always have wide scope with respect to modal operators, and not at the considerations that lie behind this claim. I agree with him that rigid designators provide a safe, neutral--i.e., rigid--way of referring to objects in counterfactual contexts like (49). Indeed, this feature can be exploited to motivate the notion of rigid designation in the first place. It is easy to show that counterfactual contexts like (49) engender referential ambiguities and that these ambiguities can affect truth-value. The idea of a special kind of designator that never yields conflicting truth-values in any such context is then a natural one. This is much the way Kripke motivates rigid designation in "Naming and Necessity" and "Identity and Necessity," although in this case he resorts to talk of possible worlds where in general he prefers to let the counterfactuals speak for themselves. Because it does not rely on technical, formal considerations, this way of motivating the notion has a comparatively direct and broad intuitive appeal. It is the best way I have found to motivate the notion.[52] Still, it is not perfect. Since semi-rigid designators have the very same sort of referential invariance, it does not quite succeed in picking out rigid designators alone.

The argument against Burge closely resembles the one against Dummett. Burge holds that, unlike other singular terms, rigid designators occur only in referentially transparent positions. Specifically, he says that what seems crucial to rigid designation "is that, in modal and related contexts, a term in the surface syntax always be taken in the semantical representation to have referentially transparent position--or Frege's 'customary reference'."[53] Of course, in the PH- and K-languages the distinction between surface syntax and semantical representation collapses once definite descriptions are expanded or, what amounts to almost the same thing, once their scope is marked. Furthermore, in these languages every singular term can occur within the scope of '□'; and all singular term positions within the scope of '□' are referentially opaque. Consequently, if Burge is correct, the PH- and K-languages contain no rigid designators whatever.

Indeed, if Burge is correct, there are no rigid designators in most every formal modal language that has been put forward. For even those formal modal languages that contain constants generally permit them to occur in referentially opaque positions--e.g., in de dicto necessary formulas. Burge's position is thus unusual. Perhaps he differs from others in thinking that rigid designation is not so much a logical as a linguistic notion, and therefore should not be exemplified in formal languages designed to exhibit principles of logic. Or he may think that a different kind of formal modal language is needed to capture the logic of reference in modal contexts. Either way, his position calls for justification. But here our results become pertinent, for they challenge whether any compelling justification can be given. Our results show that a reasonable account of rigid designation is possible without abandoning the usual approach. Why then pursue an unusual approach? What reason can there be for insisting that rigid designators--in Kripke's sense--occur only in referentially transparent positions?

Burge's remark is too brief for us to decide what he had in mind. Nevertheless, his mention of Frege's "customary reference" does suggest something. Frege attributes failures of the substitutivity of identity to loss of customary reference. On Frege's view, then, co-referring designators that always have customary reference can always be substituted for one another salva veritate. Hence, Burge's intended point may only have been that co-referring rigid designators can be substituted for one another in modal contexts salva veritate. As the reader can readily verify, in both every PH-language and every K-language, co-referring designators that are rigid in our sense can be substituted for one another in any context without disturbing truth-value. In this respect, unlike in others we have noted, rigid designators behave the same way in the K-languages as in the PH-languages. The substitutivity of identity throughout each of these languages is a striking feature of rigid designators. It is not, however, a feature that distinguishes them, for semi-rigid designators exhibit it too.[54] (We have yet to come upon a more felicitous basis for defining rigid designation than the "essentialist" one we used in (2).)

The principle of the substitutivity of identity for PH- and K-languages is worth stating precisely:

For all singular terms α and β and all formulas A and A' in a PH- or K-language, if

i. α is a rigid or semi-rigid designator

ii. β is a rigid or semi-rigid designator (50)

iii. $\ulcorner \alpha = \beta \urcorner$ is true

iv. A is like A' save for having an occurrence of α where A' has an occurrence of β

then A' is true only if A is true.

Of course, conditions (i) and (ii) can be dropped if the occurrences of α and β in A and A' are not within the scope of a modal operator.

An important consequence of (50) is that, in Kripke's words, true identity statements between rigid designators are (weakly) necessarily true. That is, for all singular terms α and β in a PH- or K-language, if $\ulcorner \alpha = \beta \urcorner$ is true and α and β are rigid designators, then the <u>de dicto</u> formula $\ulcorner \boxminus \alpha = \beta \urcorner$ is true. This follows from (50) because the <u>de dicto</u> formula $\ulcorner \boxminus \alpha = \alpha \urcorner$ is valid in all of these languages.[55] The same, however, cannot be said for '$\Box$'. True identity statements between rigid designators need not be strongly necessarily true, for the <u>de dicto</u> formula $\ulcorner \Box \alpha = \alpha \urcorner$ is not true unless α is a strongly rigid designator.[56] A correct way of putting the point in terms of '$\Box$' is that (51) is valid in every PH- and K-language (where the scope of each definite description is bounded by the brackets within which it occurs):

$$\begin{aligned} &\{(\exists x)\Box(\forall y)(Fy \equiv y{=}x) \ \& \ (\exists w)\Box(\forall y)(Gy \equiv y{=}w) \ \& \\ &\quad [(\iota x)(Fx)=(\iota w)(Gw)]\} \supset \Box\{[(\exists! x)(Fx) \ \vee \\ &\quad (\exists! w)(Gw)] \supset [(\iota x)(Fx)=(\iota w)(Gw)]\} \end{aligned} \tag{51}$$

This is another striking feature of rigid designators in the languages we are considering. Again, however, it is not a feature that distinguishes them. For if $\ulcorner \alpha = \beta \urcorner$ is true and either or both of α and β are semi-rigid instead of rigid, then on our understanding of '$\boxminus$', the <u>de dicto</u> formula $\ulcorner \boxminus \alpha = \beta \urcorner$ is still true.[57]

The last two paragraphs have described features rigid designators have in the PH- and K-languages. But we need not have confined the claims to these languages. On our characterization, a rigid designator is formed with a definite description predicate--i.e., one of the sort

'$(\forall y)(Fy \equiv y=\text{①})$'--that is special: the object denoted by a rigid designator could not exist and fail to have the predicate in question be true of it; and no other object could exist and have the predicate be true of it. But then the definite description predicates of co-referring rigid designators must be strictly equivalent. So long as strictly equivalent predicates can always be substituted for one another in modal contexts without disturbing truth-value, so too co-referring rigid designators can always be substituted for one another in modal contexts without disturbing truth value. That true identity statements between rigid designators are always (weakly) necessarily true then follows.

If the substitutability of co-referring rigid designators was what Burge had in mind, then our argument against him parallels our argument against Dummett. That is, our argument against his position is directed at the claim that rigid designators occur only in referentially transparent position, and not at the considerations that lie behind this claim. I agree that co-referring rigid designators can be substituted for one another in modal contexts _salva veritate_. Indeed, this is another feature that can be exploited to motivate the notion of rigidity. As Quine has amply illustrated, it is easy to show that _de dicto_ modal contexts resist the substitutivity of identity. The idea of a special kind of designator for which the substitutivity of identity in modal contexts is preserved is then a natural one. As before, however, this way of motivating the notion does not do everything we would like it to do. Since semi-rigid and rigid designators have the same substitution characteristics, it too does not quite succeed in picking out rigid designators alone.[58]

IX

The substitutivity of identity is not to be confused with "scope neutrality." The condition co-referring designators must satisfy to be everywhere substitutable for one another *salva veritate* is not the same as the condition designators must satisfy for differences in their scope never to affect truth-value. Of course, the two conditions do amount to the same thing for extensional languages, and for virtually all intensional languages scope neutrality implies substitutivity.[59] These facts alone, however, should not lead anyone to conclude that the two conditions are equivalent. The main source of such a mistaken conclusion in the case of modal languages is, I suspect, a tendency to liken rigid designators to the constants of standard logic. It is natural to conceive of rigid designators as those singular terms in modal languages that behave in essentially the same way constants behave in standard quantificational languages. The trouble comes from thinking that the various special features constants have in standard languages automatically go hand in hand with one another. The substitutivity of identity and scope neutrality are two such features. Constants are scope neutral in standard languages not just in the trivial sense that their scope is syntactically unmarked, but in the important sense that, regardless of what operator replaces 'ϑ', all formulas of the forms of both (52) and (53) are valid:

$$\vartheta Fc \equiv (\exists x)(x=c \;\&\; \vartheta Fx) \tag{52}$$

$$\vartheta Fc \equiv \vartheta(\exists x)(x=c \;\&\; Fx) \tag{53}$$

Our formal results show clearly that substitutivity and scope neutrality do not amount to the same thing in the PH- and K-families.

Rigid and semi-rigid designators satisfy the condition for the substitutivity of identity, but only strongly rigid designators satisfy the condition for scope neutrality. This conclusion need not be confined to the PH- and K-families. We have already seen that the substitutivity of identity in modal contexts is a general trait of rigid and semi-rigid designators. A similar point holds for scope neutrality and strongly rigid designators. Intuitively, (54) is true, but (55) is false:

The planet Hesperus exists and is necessarily self-identical. (54)

It is necessarily the case that the planet Hesperus exists and is self-identical. (55)

On the natural reading of the formal notation, (56) is true, while (57) is false unless '$(\iota x)(Hx)$' denotes a necessarily existing object:

$$(\exists x)\left[(\forall y)(Hy \equiv y=x) \ \& \ \Box x=x\right] \tag{56}$$

$$\Box(\exists x)\left[(\forall y)(Hy \equiv y=x) \ \& \ x=x\right] \tag{57}$$

From intuitive considerations alone, then, we should expect only strongly rigid designators to be scope neutral. Indeed, on the natural reading, (58) is true, while (59) is false unless 'c' denotes a necessarily existing object:

$$(\exists x)(x=c \ \& \ \Box x=x) \tag{58}$$

$$\Box(\exists x)(x=c \ \& \ x=x) \tag{59}$$

Hence, we should not expect even constants to be scope neutral in modal languages unless we are prepared to require them to denote necessarily existing objects.

Since scope neutrality and the substitutivity of identity do not amount to the same thing in modal languages, care is needed in likening rigid designators to the constants of standard logic. The idea of a special kind of designator that behaves in modal languages in essentially the way constants do in standard languages is undoubtedly natural. But it is a poor idea nonetheless since it induces confusion with strong rigidity. For that matter, using standard constants as guides to fashion any special category in modal languages risks confusion. Given our conclusion about (58) and (59), we have reason to doubt that any singular terms in modal languages have all of the semantic features constants characteristically have in standard languages. But if this is correct, the idea of a modal counterpart of standard constants cannot be used without qualifications even to motivate the notion of strongly rigid designation.[60]

Several factors have led to confusion in the literature on rigid designation. Up to now we have concentrated on two of these factors--the tendency to lose sight of the distinction between weak and strong necessity and the failure to notice how much rides on the contrast between PH-like and K-like languages. I suspect, however, that the principal source of confusion in the literature is a predilection to think of rigid designators as having all of the semantic features that constants have in standard logic. As we will see, this predilection has ramifications not just for rigid designation, but for quantified modal logic generally.

Likening rigid designators to standard constants has prompted confusion where one would least expect to find it. Consider Quine's remarks about rigid designators and quantified modal logic in his recent article, "Intensions Revisited."[61] Noting difficulties with substituting constants for variables in contexts involving '$\Box$', he recommends that constants be dropped in favor of special definite descriptions, which "can be defined away in essentially Russell's way." He then asks "how singular terms fare when restored definitionally as descriptions."[62] Specifically, he asks what the conditions for languages with modal operators are under which singular terms qualify first for the substitutivity of identity and second for instantiation of quantifications. Problems with substitutivity in such languages are well known from his earlier papers on modal logic. The problem he calls attention to with instantiation (which he says "is under the same wraps as the substitutivity of identity"[63]) is that invalid inferences like those from (60) to (61) and from (62) to (63) must be distinguished from the parallel valid ones:

$$(\forall x)[x \text{ is a number} \supset (\Box(5 < x) \vee \Box(5 \geq x))] \quad (60)$$

$$\Box(5 < \text{number of planets}) \vee \Box(5 \geq \text{number of planets}) \quad (61)$$

$$(5 < \text{number of planets}) \mathbin{\&} {\sim}\Box(5 < \text{number of planets}) \quad (62)$$

$$(\exists x)[5 < x \mathbin{\&} {\sim}\Box(5 < x)] \quad (63)$$

Quine's answer is that what qualifies a term "for the instantial role in steps of universal instantiation and existential generalization in modal contexts" is that it be a rigid designator.[64] He then adds that rigid designators also "lend themselves in pairs to the substitutivity of simple identity."[65] Thus, the impression--if not the claim--that he leaves us with is that in languages with modal operators singular terms qualify for instantiation and the substitutivity of identity just in case they are rigid designators.

Before responding to this, I need to comment on the way Quine characterizes rigid designators. He remarks that "a rigid designator differs from others in that it picks out its object by essential traits.

It designates the object in all possible worlds in which it exists."[66] But his formal characterization is that α is a rigid designator just in case $\ulcorner(\exists x)\Box(x=\alpha)\urcorner$ is true. α must have narrow scope in this expression to avoid having every designator be rigid. Thus his condition is that '$(\iota x)(Fx)$' is a rigid designator if and only if (64) is true:

$$(\exists x)\Box(\exists w)\left[(\forall y)(Fy \equiv y=w) \ \& \ w=x\right] \tag{64}$$

Since he does not indicate otherwise, I assume that he intends variables bound from within the scope of a modal operator to be handled as they usually are--i.e., as they are in the PH- and K-languages. But then (64) represents not the condition under which '$(\iota x)(Fx)$' is a rigid designator, but that under which it is a strongly rigid designator. Perhaps the trouble here is that Quine intended '$\Box$' to express a weaker necessity, as in (65):

$$(\exists x)\Box\left\{\left[(\exists w)(w=x) \ \vee \ (\exists! w)(Fw)\right] \supset (\exists w)\left[(\forall y)(Fy \equiv y=x) \ \& \ w=x\right]\right\} \tag{65}$$

But (65) too does not represent the condition under which '$(\iota x)(Fx)$' is a rigid designator; rather, it gives the condition under which '$(\iota x)(Fx)$' is either a semi-rigid or a rigid designator. On either plausible reading, then, Quine's formal characterization fails to pick out what Kripke has called "rigid designators."

Now consider Quine's remarks about instantiation and the substitutivity of identity. Universally instantiating as in (61) involves a shift from <u>de re</u> to <u>de dicto</u> and hence an implied shift in scope. To put the point more graphically, from (66) we can always infer (67), but not always (68):

$$(\exists! x)(Fx) \ \& \ (\forall x)(\ Gx) \tag{66}$$

$$(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ \Box Gx] \tag{67}$$

$$\Box(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ Gx] \tag{68}$$

Similarly, existentially generalizing as in (63) involves an implied shift in scope, since we can always infer (69) from (70), but not always from (71):

$$(\exists x)(\sim\Box Gx) \tag{69}$$

$$(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \sim\Box Gx] \tag{70}$$

$$\sim\Box(\exists x)[(\forall y)(Fy \equiv y=x) \ \& \ Gx] \tag{71}$$

The inferences from (67) to (68) and from (71) to (70) are precisely the ones we found were problematic in both the PH- and K-families unless '$(\iota x)(Fx)$' is a strongly rigid designator. Therefore, as Quine sets up the instantiation problem for modal languages, a designator qualifies for the instantial role in universal instantiation and existential generalization if and only if it is strongly rigid. This is the formal condition he gives, interpreted as in (64). By contrast, as we have seen, co-referring designators qualify for the substitutivity of identity if and only if they are rigid or semi-rigid. This is the formal condition he gives, interpreted as in (65). But (64) and (65) are by no means equivalent to one another, and neither represents the condition under which '$(\iota x)(Fx)$' is a rigid designator. I can only conclude that Quine is confused about rigid designation.

What has misled him? The argument of the preceding paragraph turned on the fact that rigid designators are not scope neutral. Nevertheless, I am not sure whether the main source of difficulty is his assuming that rigid designators are scope neutral or his assuming that they have another feature constants have in standard non-modal languages, viz. being like "fixed-valued free variables." Viewing constants as fixed-valued free variables is to turn upside-down--or, perhaps better, right side up--the familiar idea that free variables are "generalized names." The difference between free variables and constants in standard logic is that a free variable takes on different values, while the value of a constant does not change. So long as the value of a free variable remains the same, the difference disappears. A constant in standard logic is accordingly just like a free variable whose value has become fixed. In particular, in standard nonmodal languages an open formula 'Fx' is true of the object denoted by 'c' if and only if the corresponding closed formula 'Fc' is true.[67] Accordingly, given a specific assignment of objects to variables, an open formula in these languages is akin to a closed formula in which suitably denoting constants replace its free variables. If an open formula is true when certain objects are assigned to its free variables, then so long as there are constants denoting these objects, there will be a corresponding true closed formula of the indicated sort. The special relationship between free variables and standard constants, summarized in the claim that constants are just like fixed-valued free variables, thus secures a comparable special relationship between certain open and closed formulas. This latter relationship is, of course, widely exploited in a variety of formal methods in standard logic.

One virtue of this way of viewing the constants of standard logic is that it helps in conceptualizing and explaining universal instantiation and existential generalization. Both of these forms of inference, on this view, are mediated by an intervening step, consisting of an open formula whose relevant free variable has as its value the object denoted by the instantial constant. For example, from the truth of 'Fc', we first infer that 'Fx' is true of the object denoted by 'c'; given that free variables range only over existing objects, we then infer the truth of '$(\exists x)(Fx)$' from the fact that 'Fx' is true for at least one value of x. The parallel analysis of universal instantiation is obvious. Given the interpretation of the quantifiers, then, the instantial role of constants in standard logic can be viewed as a concomitant of their being like fixed-valued free variables.

As we saw in the discussion of (66) through (71), rigidity does not always qualify a designator for the instantial role in universal instantiation and existential generalization. It does not do so even in the case of the PH- and K-languages in which free variables range only over existing objects. By the argument of the preceding paragraph, however, fixed-valued free variables would be instantiable in these latter languages. Therefore, in the languages we are considering, rigid designators are not just like fixed-valued free variables. Free variables in the PH- and K-languages always occur in what Quine calls referential position. Consequently, even when the value of a free variable has been fixed, open formulas in which it occurs never express pure modality de dicto. To say that $\ulcorner\sim\Box\Psi(\xi)\urcorner$ is true when a certain object is the value of the variable is just to say that $\ulcorner\sim\Box\Psi(\xi)\urcorner$ is true of that object. By contrast, $\ulcorner\sim\Box\Psi((\iota x)(Fx))\urcorner$ can be read as expressing pure modality de dicto even when '$(\iota x)(Fx)$' is rigid. Thus, as we saw, $\ulcorner\sim\Box\Psi((\iota x)(Fx))\urcorner$ can be true, yet $\ulcorner\sim\Box\Psi(\xi)\urcorner$ not be true of the object rigidly designated by '$(\iota x)(Fx)$'. This simple point clearly established our conclusion. In modal languages of the sort we are considering, fixed-valued free variables would be like rigid designators in having invariant reference, and co-referring fixed-valued free variables could be everywhere substituted for one another *salva veritate*. Rigid designators, however, provide something that fixed-valued free variables would not, *viz.* the power to express purely *de dicto* strong necessity and contingency statements, like (68) and (71), in which the singular terms have invariant reference and qualify for the substitutivity of identity.[68]

At one point Quine says that variables "figure only *de re*,"[69] and at another he says that rigid designators enjoy "*de re* privileges in a *de dicto* setting."[70] This suggests that he does not think of rigid designators as being just like fixed-valued free variables, so that his claim about their instantial role must amount to the claim that they are scope neutral. Still, he nowhere says that they are scope neutral, and when illustrating their supposed instantial role, as in (60) through (63), he simply exchanges the bound variable and singular term. This suggests, to the contrary, that he does think of rigid designators along the lines of fixed-valued free variables. But regardless of what has misled Quine, the important thing to notice here is how two quite different ideas--that rigid designators are scope neutral and that they are like fixed-valued free variables--both can lead to the mistaken conclusion that they have the instantial characteristics of standard constants. At the beginning of the paper I pointed out that the literature on the scope behavior of rigid designators divides into two camps, one holding that they are scope neutral, and the other, in effect, that they always occur de re. We now see that both camps may well spring from the same mistaken belief about the instantial behavior of rigid designators. If the idea that rigid designators should be instantiable in modal contexts in the way that constants are in nonmodal ones is given up, perhaps the two camps and the controversy between them will disappear.

At any rate, the idea that rigid designators are like fixed-valued free variables is undoubtedly what lies behind both Dummett's claim that they always have wide scope and Burge's claim that they always occur in referential position. The upshot of our argument against Dummett and Burge is that in modal languages like those in the PH- and K-families, which contain no constants, no singular terms whatever are just like fixed-valued free variables.[71] Even strongly rigid designators, which have the instantial characteristics of standard constants, occur in referentially opaque de dicto as well as referentially transparent de re positions, and hence are not exactly like fixed-valued free variables.

We have seen how four features constants characteristically have in standard logic cease to go hand in hand with one another in modal logics. In modal languages of the sort we have considered, the conditions under which singular terms (i) lend themselves to the substitutivity of identity, (ii) are scope neutral, (iii) qualify for the instantial role in universal instantiation and existential generalization, and (iv) are like fixed-valued free variables, though still related, are none entirely extensionally equivalent. An obvious conclusion to draw is that rigid designators should not be conceived of as modal counterparts of the constants of standard logic. Constants should not even be taken to be paradigms of rigidity, as Peacocke takes them. The notion of rigidity is better developed in other ways, employing modal languages that have no constants.

X

Needless to say, then, I agree with Quine when he suggests that allowing constants in regimented modal languages only engenders confusion.[72] Still, since constants are so widely used, the difficulties we have called attention to do invite the question, how are constants best introduced into modal languages? In particular, how are they best introduced into the PH- and K-languages? Evidently constants cannot have all of the features in these languages that they have in standard nonmodal languages. Which combinations of these features can they have?

We should specify the features more carefully before trying to answer these questions. Substitutivity of identity for constants amounts to all formulas of the form of (72) being valid:

$$c=d \supset (Gc \supset Gd) \tag{72}$$

Scope neutrality requires all formulas of the form of (73) to be valid, regardless of the operator replacing 'ϑ':

$$(\exists x)(x=c \;\&\; \vartheta Gx) \equiv \vartheta(\exists x)(x=c \;\&\; Gx) \tag{73}$$

So long as strictly equivalent formulas can be substituted for one another in all contexts _salva veritate_, scope neutrality can also be expressed by the requirement that all formulas of the form of (74) be valid:[73]

$$Gc \equiv (\exists x)(x=c \;\&\; Gx) \tag{74}$$

The validity of these last formulas of course represents the "existential presupposition" of standard constants. The instantial role of constants in universal instantiation and existential generalization requires all formulas of the forms of (75) and (76) respectively to be valid:[74]

$$(\forall x)(Gx) \supset Gc \tag{75}$$

$$Gc \supset (\exists x)(Gx) \tag{76}$$

But this alone does not make constants instantiable in the stricter sense Quine invokes with (60) through (63). For them to be instantiable in this sense, all formulas of the form of (75) and (76) must be valid when the constants in them are construed to have minimal scope.

The requirements for constants to be like fixed-valued free variables are not so simple to spell out. The general idea is clear: a formula like 'Gc' is to be construed just as the corresponding open formula 'Gx' is when the object denoted by 'c' is the one assigned to 'x'. Thus, as remarked earlier, part of what is implicated is that (77) hold on all interpretations:

> For all formulas $\phi(\xi)$, $\phi(\xi)$ is true of the object denoted by the constant α if and only if $\phi(\alpha)$ is true. (77)

Constants are not entirely like fixed-valued free variables, however, unless also they can be introduced in effect by fixing the value of a free variable. Hence, (78) should also hold on all interpretations:

> Suppose α is a constant letter that has not yet been assigned an object. If $\phi(\xi)$ is a formula that is true of at least one object, then α can be introduced as a constant that denotes an object of which $\phi(\xi)$ is true (without introducing an inconsistency). (78)

(77) and (78) together are the conditions that lie behind the use of constants as "witnesses" in Henkin-style completeness proofs for standard quantificational logic; the constants themselves or equivalence classes of them become the objects in the domains of the canonical models.[75] Hence, those inclined to prefer Henkin-style proofs will want (77) and (78) to hold. These conditions also lie behind the use of constants in inferences involving existential instantiation and, though less obviously, universal generalization.[76] When (77) and (78) hold for a language, they comprise a comparatively weak sufficient condition for introducing new constants. In this regard, (78) should be contrasted with, for example, (79), which probably comes closer to the condition under which new names are introduced into natural languages:

> Suppose α is a constant letter that has not yet been assigned an object. If $\Phi(\xi)$ is a formula such that $\ulcorner(\exists!\xi)(\Phi(\xi))\urcorner$ is true, then α can be introduced as a constant that rigidly designates the object of which $\Phi(\xi)$ is true (without introducing any inconsistency). (79)

One last feature of standard constants should be noted. They resemble natural language names not just in being syntactivally atomic, but also in often having ambiguous scope. For example, '$\sim$Gc' can be read as predicating '$\sim G\Phi$' of the object denoted by 'c' or as the negation of the closed formula 'Gc'. Of course, so long as constants are scope neutral, such ambiguities are innocuous.

How then might constants be introduced into the PH- and K-languages? I see little point in exploring approaches that abandon the substitutivity of identity. This is not to say that such approaches are invariably incoherent. Thomason's Q3 has such constants,[77] and if constants were added to Lewis's counterpart version of quantified modal logic, it would have them.[78] As these examples illustrate, however, the natural way of introducing constants while abandoning the substitutivity of identity is to treat them as logically akin to abbreviations for definite descriptions. But treating constants as abbreviations for definite descriptions, without requiring the definite descriptions to satisfy any special modal constraints, would clearly sacrifice virtually all of the other features listed in the preceding paragraphs. Since our present concern is to see how many of these features can be saved, approaches that abandon the substitutivity of identity seem not worth pursuing.

As suggested earlier, the most straightforward way of introducing constants into the PH- and K-languages would be as strongly rigid designators. This way of introducing them would save most of the features listed, so that they would retain much of the logical character of standard constants. It would obviously save the substitutivity of identity, scope neutrality, and their full instantial role in existential generalization. It would also save their full instantial role in universal instantiation provided that at least one necessarily existing object is guaranteed in every domain.[79] It would even save (77), so that in a limited respect such constants would be like fixed-valued free variables. But they would not be like fixed-valued free variables in the respect expressed by (78). Neither (78) nor (79) would hold, since in addition to the requirements they impose, an object would have to be one whose existence is necessary before a constant could denote it. This requirement of necessary existence is of course why such constants would be scope neutral and instantiable in Quine's strict sense. Constants in standard quantificational languages have these features precisely because they are required to denote existing objects. Simply put, the counterpart of this requirement in the case of the PH- and K-languages is necessary existence.

The trouble with introducing constants as strongly rigid designators is that they would be of little use or interest. For example, they would not suffice as witnesses for Henkin-style completeness proofs except in the case of the philosophically uninteresting languages that require every object to exist necessarily. Equally, they would not suffice for inferences involving existential instantiation and universal generalization except in the case of these languages. Moreover, most of the resemblance between constants and natural language names would be lost. Indeed, since it is at least disputable whether there are any necessarily existing objects, there may not even be any strongly rigid designators. So, if this is the best way of introducing constants into the PH- and K-languages, they are probably not worth the bother. Strongly rigidly designating constants would have the virtue of preserving the basic logic of standard constants. No valid sentence schemata involving constants would be lost in going from standard to modal languages, and the scope of constants would still in general be ambiguous without the ambiguity affecting truth-value. Such constants would thus especially be a reminder of the relationship between the conditions for scope neutrality in standard and in modal languages. But otherwise they would add nothing.

A second way of introducing constants into the PH- and K-languages would be as rigid designators. Rigidly designating constants would still not suffice for such purposes as Henkin-style completeness proofs in the case of languages that permit "possible-yet-not-actual" objects. But they would more closely resemble natural language names. They would qualify for the substitutivity of identity, and presumably (79) would hold for them, though not (78). Scope neutrality, however, would be lost, so that many sentences containing such constants would be nontrivially ambiguous--i.e. open to different readings that need not match in truth-value. In this respect, rigidly designating constants would be too much like natural language names. However tolerable such nontrivial ambiguities may be in natural languages, they are unacceptable in regimented languages.

The ambiguities in question could be eliminated if the scope of constants were always marked or if a convention were adopted that would always yield a univocal sentence. But marking the scope of constants in an orthographically perspicuous way is easier said than done. Anyone who is going to try to mark the scope of rigidly designating constants would undoubtedly be better off adopting Quine's strategy of replacing names with special Russellian definite descriptions. A major virtue of such special definite descriptions is that their scope can always be indicated perspicuously.

The troublesome ambiguities could be eliminated, at the sacrifice of some of the resemblance to natural language names, by adopting a convention covering the scope of constants. The obvious move is for constants always to have maximal or always minimal scope. If constants were always to have maximal scope, then they would always occur in referential position, so that modal sentences containing them would never express pure modality *de dicto*. There would still be sentences equivalent to any *de dicto* reading--e.g., '$\Box(\exists x)(x=c \,\&\, Gx)$' with the convention would be equivalent to the *de dicto* reading of '$\Box Gc$' in its absence. Also, since both constants and free variables would always occur in referential position, (77) would hold. Such rigidly designating constants would thus be instantiable in the limited sense that all formulas of the form of (75) and (76) would be valid. But of course they would not be instantiable in Quine's sense. This failure reflects a deeper shortcoming. In keeping with a point Kripke makes, if designating constants were always to have maximal scope, a formula like '$\Box Gc$' would not assert the necessity of the closed formula 'Gc'.[80] Consequently, strictly equivalent formulas could not always be substituted for one another *salva veritate*. For example, all formulas of the form of (80) would be valid, but not all of the form of (81):

$$Gc \equiv (\exists x)(x=c \,\&\, Gx) \tag{80}$$

$$\Box Gc \equiv \Box(\exists x)(x=c \,\&\, Gx) \tag{81}$$

This would be a lot to give up for so little in return.

The alternative remaining is that rigidly designating constants always be taken to have minimal scope. Again no expressive power need be lost since, for example, '$(\exists x)(x=c \,\&\, \Box Gx)$' with this convention would be equivalent to the *de re* reading of '$\Box Gc$' in its absence. Unlike the other convention, this one would not sacrifice the substitutability of strictly equivalent formulas. In particular, all formulas of the form of (81) would be valid, thereby providing a limited sort of existential generalization. But such constants would not be instantiable either in Quine's strict sense or in the looser sense exhibited by (75) and (76). Also, since $\ulcorner\Box\Psi(\xi)\urcorner$ could be true of the object denoted by α without $\ulcorner\Box\Psi(\alpha)\urcorner$ being true, (77) would not hold. As a result, such constants would not suffice for existential instantiation, universal generalization, or Henkin-style completeness proofs except in the case of the uninteresting languages that require all objects to exist necessarily. If rigidly designating constants are to be added to the PH- and K-languages, always assigning them minimal scope is probably the best way to do it. But the only thing gained would be a class of singular terms slightly resembling natural language names, and even then the logic of these names would be better captured by Quine's definite descriptions. Rigidly designating constants would add nothing to the formal logic of the languages. Accordingly, the gain would appear not to offset the attendant risk of confusion.

A third way of introducing constants into the PH- and K-languages would be as fixed-valued free variables. Much of the resemblance between constants and natural language names would be lost since, unlike names, free variables occur only in referential position. But the special relationship between free variables and constants would be saved, so that (77) and (78) would hold along with the substitutivity of identity. In the languages whose variables range only over existing--i.e. "actual"--objects, such constants would be instantiable in the loose sense. Generally speaking, however, the logic of these constants would differ markedly from the familiar standard one. Fixed-valued free variables would clearly not be scope neutral in the sense of (73). Even in the languages whose variables range only over "actual" objects, the left-hand side of (82) could be true and the right-hand side false unless all objects are required to exist necessarily:

$$(\exists x)(x=c \ \& \ \Box Gx) \equiv \Box(\exists x)(x=c \ \& \ Gx) \qquad (82)$$

Hence, with the exception of the philosophically uninteresting languages that require every object to exist necessarily, valid sentence schemata involving constants would be lost in going from standard to our modal languages. Accordingly, to the extent that preservation of standard logic is a concern, introducing constants as fixed-valued free variables would be open to objection.

In fact, such constants would have different nonstandard logics in our two families. Consider the K-family first, for which this way of introducing constants has been prevalent in the literature. Since 'Gx' is always either true or false of an object in the K-languages, 'Gc' would always be either true or false; and since 'Gx' can be true of non-existing objects, so too 'Gc' could be true, yet '$(\exists x)(x=c)$' be false. Such constants would thus have a familiar logic in the K-family, <u>viz.</u> free-logic. Hence, not all formulas of the form of (74) would be valid. In part because of this, substitution of strictly equivalent formulas would not be undermined when constants were introduced even though the constants would in effect always have maximal scope. Since they would lack "existential import," such constants would not be instantiable in the sense exhibited by (75) and (76). But modified versions of universal instantiation and existential generalization, as in (83) and (84), would hold:

$$(\forall x)(Gx)) \supset [(\exists x)(x=c) \supset Gc] \quad (83)$$

$$Gc \supset [(\exists x)(x=c) \supset (\exists x)(Gx)] \quad (84)$$

Such constants could also be used in free-logic versions of existential instantiation and universal generalization. A further virtue of introducing constants as fixed-valued free variables in the case of the K-family is that they would suffice as witnesses in Henkin-style completeness proofs.[81] The constructions for Henkin-style canonical models are comparatively straightforward. This is probably why this way of introducing constants into the K-languages is prevalent in the literature. The fact that fixing the value of a K-language variable results in a familiar free-logic constant only makes the move more palatable.

To the best of my knowledge, the logic fixed-valued free variables would have in the PH-family is not one that has been explored in the literature. Unfortunately, it would be quite complicated since free variables occur in two kinds of contexts in the PH-languages--standard and nonstandard. Occurrences of such constants in nonstandard contexts would be like occurrences in the K-languages, so that free-logic would hold for them.[82] Occurrences in standard contexts, however, would require a Strawsonian logic. As a result, once such constants were introduced into the PH-languages, the logic of closed formulas would cease to be classically two-valued. A closed formula like 'Fc' could be true, false, or lack a truth-value. The resulting complications are too involved to pursue here. For example, whether instantiation formulas like (75) and (76), with 'Gx' standard, would be valid would depend on whether 'valid' would mean "always true" or "never false."[83]

Since the logic of the constants would be both hybrid and not classically two-valued, it would be dramatically different from standard logic on two counts, even ignoring modal formulas. The question is whether the added complications would yield an adequate return. The preferred return would be simple deductive techniques or Henkin-style completeness proofs for the PH-family. In deference to those who emphasize Henkin-style proofs, let us examine this possibility; similar remarks apply to the possibility of achieving simple deductions via such rules as existential instantiation and universal generalization. Our condition on introducing constants, (78), would have to be relaxed, for as it stands it would not provide the "possible-yet-not-actual" witnesses needed to invalidate the Barcan formulas. For example, it might be modified to allow an available constant to be introduced whenever $\phi(\xi_1)$ or $\ulcorner\sim(\exists\xi_2)(\xi_2=\xi_1 \;\&\; \phi(\xi_2))\urcorner$ is true of an object. Presumably nothing would then stand in the way of equivalence classes of such constants being the needed witnesses in canonical models. But the constructions required for the models remain unclear.[84] At best, then, it is an open question whether tractable and interesting Henkin-style completeness proofs would emerge should constants be introduced as fixed-valued free variables into the PH-languages. If not, and if simple deduction techniques would also fail to emerge, then the added complications do not seem worth the bother.

These, in sum, appear to me to be the reasonable choices. Constants can be introduced as strongly rigid designators, as rigid designators, or as fixed-valued free variables. If they are introduced as strongly rigid designators, then the standard logic of constants will be preserved in the sense that the class of valid sentence schemata involving constants will be a conservative extension of the class for standard logic. But the range of values of the constants will be too narrow for them to retain either their resemblance to natural language names or their special relationship to free variables. If instead they are introduced as rigid designators, then they will resemble denoting proper names both in their range of values and in the fact that, unless a convention is adopted, their scope will often be ambiguous. But their range of values will still be too narrow to permit such things as Henkin-style completeness proofs and rules of existential instantiation and universal generalization; and they will not be instantiable and scope neutral in the way standard constants are. Finally, if they are introduced as fixed-valued free variables, then the special relationship between them and free variables will remain. But their scope will not be ambiguous, so that much of their resemblance to names will be lost; and again some valid closed formulas of standard logic will cease to be valid. The standard logic of constants, their resemblance to denoting proper names, and the relationship between them and free variables--at most one of these can be retained when adding constants to the PH- and K-languages.

Some may wish to conclude, so much the worse for modal logic. I think this would be a mistake. The appropriate conclusion, I think, is that the combination of features found in standard constants is a happy accident. Standard constants resemble denoting proper names in their range of values, in their being syntactically atomic, and in their having ambiguous scope in many contexts. Yet they enter into truth conditions in a manner equivalent to that in which free variables whose values are fixed enter. And since they are scope neutral, ambiguities in their scope are no reason for them to give way, for purposes of regimentation, to syntactically nonatomic terms whose scope can be marked perspicuously. As standard logic attests, this is a nice combination of features to find. But the lesson modal logic teaches us is that the possibility of this combination of features is a special, parochial virtue of standard logic.

Various lines of argument against quantifying into modal contexts and hence against quantified modal logic can be detected in the literature. One line objects to quantified modality on the grounds that it "leads us back into the jungle of Aristotelian essentialism."[85] Nothing that has been said in this paper has any bearing on this line of argument. Rather, we have presupposed the desirability of expressing essentialist claims in a regimented language and have inquired into the logic underlying such claims in certain regimented languages. Hence, this is not the place to respond to this line.

A second line of argument emphasizes the repeated failures to come up with a quantified modal logic that fully conforms to our pretheoretical essentialist intuitions. I sympathize with this line because it raises the issue whether _de re/de dicto_ distinctions would be captured better by resorting to separate modal operators than by relying on scope distinctions with single modal operators.[86] Here, however, some of our efforts have a bearing. The PH-languages, though initially fashioned to account for mistakes about the scope characteristics of rigid designators, appear not to have any of the counterintuitive features objected to in other modal quantificational languages. So perhaps the PH-languages vitiate this line of argument.

The line of argument that has received the most attention in the literature questions whether quantifying in is formally coherent. This line stresses the need for restrictions on the substitutivity of identity and on the instantiation of quantifiers in modal quantificational languages. The need for such restrictions is a point well taken. The issue is whether it is evidence that binding a variable from outside the scope of a modal operator is in some way incoherent. Here our efforts have their greatest bearing. As we have seen, the languages in the PH- and K-families, which of course contain no constants, are in no way formally incoherent. The class of valid closed formulas in every one of the languages in these two families is a conservative extension of the class for standard logic without constants. And differences in the scope of quantifiers _vis-a-vis_ modal operators account for the differences between referentially transparent _de re_ and referentially opaque _de dicto_ readings of informal modal statements. Quine himself concedes as much.[87] So, this line of argument must ultimately come down to the claim that standard constants cannot be coherently introduced into the philosophically interesting languages of these families. With this I agree. But this seems to me just to be evidence that one should be wary of constants when dealing with modal languages.

The combination of features found in standard constants presupposes an extensional logic and hence should not be expected in the case of intensional languages. Equally the interpretation given free variables in modal languages should not be narrowly parasitic on standard constants. The specific handling of free variables that fall within the scope of modal operators appears to be the critical factor in constructing a modal quantificational language. This is one of the more striking lessons brought out by the contrast between the PH- and the K-families. But then it is important that we be no less wary of letting standard constants lead us into a parochial treatment of free variables in modal languages.

XI

What morals should be drawn from all of this? Our basic technical result is firm. Under our definition of rigid designation, rigidly designating Russellian definite descriptions need not be scope neutral. To be scope neutral in all extensional and modal contexts, a Russellian definite description must be a strongly rigid designator. One moral I obviously want to draw involves the direct generalization of this result. Rigid designators are not invariably scope neutral; but strongly rigid designators are. Thus, strongly rigid designators, and not rigid designators, are the modal counterparts of Whitehead's and Russell's designators. As a consequence, it is a mistake to think of rigid designators as if they were logically akin to the constants of standard logic. The roots of the notion of rigid designation lie elsewhere.

Some may question whether this is the appropriate moral to draw from our basic result. Some, for instance, may prefer to conclude that rigid designation, as we have defined it, is not the important referential notion one might have thought. But surely strongly rigid designation is too restrictive a notion to be of much importance. Moreover, our rigid designators do what such designators are primarily supposed to do--i.e. they provide a way of referring rigidly to specific objects in modal and counterfactual statements. The notion of rigid designation does not stand or fall with scope neutrality. Scope considerations are, of course, extremely important in the case of modal contexts. Nevertheless, too much can be made of them.

Again, some may prefer to take our basic technical result as evidence that regimented modal languages should employ Strawsonian rather than Russellian definite descriptions. On the face of it, Strawsonian definite descriptions added to the K-languages appear likely to resemble Russellian definite descriptions in the PH-languages.[88] So, this suggestion is not entirely uninteresting. But to draw such a moral from our findings nonetheless seems to me to be a mistake. One obvious drawback to employing Strawsonian definite descriptions is that a new primitive will have to be added to quantification theory. What is to be gained in return? So far as I can see, no expressive power pertinent to the issues we have discussed would be gained. Existence qualifications of the sort we introduced in representing Kripke's weak necessity in terms of '$\Box$' can provide the same thing as Strawsonian definite descriptions would in modal contexts. And such existence qualifications have the virtue of leaving standard logic intact. It might be different if our basic result indicated a lack of expressive power in modal languages of the sort we have considered. But our result does not indicate this.

I see no reason, then, to think that the direct moral I want to draw is inappropriate. But there are other morals that I want to draw too, ones that bear on the general question of how modality de re should be represented in regimented languages. For example, consider Quine's view that proper names of natural language are best represented by special definite descriptions, and not by constants, in regimented languages. His main reason is to avoid various difficulties arising with names that lack denotation.[89] But such difficulties arise in modal contexts even with denoting names--e.g. as in 'Nixon could have failed to exist'. Furthermore, as we have seen in some detail, constants are a direct source of confusion in regimented modal languages. So, a second moral I want to draw is that Quine's method of handling proper names ought to be adopted in the case of regimented modal languages. Several things are to be gained from doing so. Our definitions of rigid designation and related notions will then cover all forms of singular terms. There will be no need for a special logic for the nonstandard constants that would have to be introduced otherwise. It will be possible to eliminate significant scope ambiguities in a perspicuous manner. And, as I trust our efforts testify, no expressive power will be lost in the process.

Lack of constants is not the only Spartan aspect of the modal languages we have considered. These languages represent the simplest extension of standard quantificational syntax to include unary modal connectives. In effect, the list of unary connectives has just been extended to include '$\Box$' and '$\Diamond$' along with '$\sim$'. As a result, scope differences are the only means available in them for expressing such contrasts as that between modality *de dicto* and *de re*. Now, we have not found this Spartan syntax unduly restrictive. To the contrary, it has proved adequate for exploring and displaying the distinctive logic of singular terms in modal contexts. Moreover, no logical difficulties have arisen from quantifying into modal contexts. Indeed, we have grounds to think that the difficulties often alluded to in this regard are ones with constants, not ones with quantifiers and free variables. So a third moral I want to draw is that there is no compelling reason to turn to a more elaborate syntax for purposes of regimenting *de re* and *de dicto* modality. In particular, there is no compelling reason to introduce different operators for the two kinds of modality.[90] Modal considerations of the sort we have dealt with do not require radical departures from standard quantificational syntax.

If we can agree about the resources needed for representing the logic of singular terms in modal contexts, then the obvious next question is, which specific regimented framework is best? In the present context the appropriate version of this question is whether the K-family or the PH-family offers the more promising approach. The two families have a number of virtues in common. Both leave standard quantificational logic (*sans* constants) intact; and both leave normal modal sentential logic intact. Also, both families contain philosophically interesting languages--e.g. languages that permit contingently existing objects to have things necessarily true of them. The choice between the two will therefore have to be based on more subtle considerations, be they formal or essentialist.

The K-family is the one most widely discussed in the philosophic literature. In part this is owing to the influence of Kripke. But even more it reflects the formal simplicity and elegance of the logics associated with the K-languages. As Kripke showed, these logics can be axiomatized in a manner strongly reminiscent of Quine's preferred axiomatization of quantification theory. This is no small virtue in the present climate of opinion toward quantified modal logic. Nevertheless, from the standpoint of philosophers interested in expressing essentialist doctrines in a regimented framework, the K-family has serious shortcomings. The failure to distinguish between normal and *recherché* predicates and the unqualified invalidity of the converse Barcan formulas in the K-languages that permit contingently existing objects do not conform well with pre-theoretical intuitions. Worse, as we indicated in Section III, certain common essentialist claims appear to be incompatible with the K-languages. For example, one is apparently forced to give up either the claim that '① exists' is not essentially true of contingently existing objects or the claim that some relations, like '① is the offspring of ②', are internal to one relatum and external to the other. And finally, the K-family fails to provide an explanation of why rigid designators have been widely held to be scope neutral.

The PH-family, on the other hand, does provide such an explanation. But of course it was devised specifically for this purpose. We needed the converse Barcan formulas to be valid in order for rigid designators to be scope neutral in a limited, yet reasonable range of modal contexts. Identity then had to be treated specially to prevent the

PH-family from being philosophically unacceptable. What is striking about the PH-family is that, though it was concocted for such a narrow purpose, it nevertheless appears to provide an adequate framework for expressing essentialist claims. In particular, it avoids the shortcomings of the K-family. And it does so through two simple moves that to some extent can be motivated on other grounds. Of all the regimented modal frameworks that have been proposed in the literature, the PH-family appears to conform best with pre-theoretical essentialist intuitions. Its only obvious shortcomings are formal--having two different kinds of predicates complicates the formal logic. But if the point is to find a regimented framework for expressing essentialist doctrines, formal considerations ought to weigh less heavily than essentialist ones.

The final moral I want to draw, then, is again in the form of a proposal: the PH-family should be viewed as offering a more promising framework than the K-family offers for regimenting _de re_ modality.

Appendix

The purpose of this appendix is to give an explicit characterization of the two families of modal quantificational languages considered in the text. The characterization is a semantic one, using the mathematical framework of "possible world" model theory. The first section sets up the framework, and the two remaining sections characterize the K-family and the PH-family respectively.

The Semantic Framework

By a qml-model structure, we mean a quadruple $Q = \langle \underline{c_0}, K, R, \mathfrak{I} \rangle$, where K is a set, $\underline{c_0} \in K$, R is a reflexive binary relation defined on K, and $\mathfrak{I}$ is a function that assigns a set to every member of K. For heuristic purposes, K is the set of possible worlds, $\underline{c_0}$ is the actual world, R is an accessibility relation, and $\mathfrak{I}(\underline{c})$ is the domain of $\underline{c}$--i.e. the set of objects that would exist were $\underline{c}$ the actual world. Different languages in a family are obtained by imposing different further restrictions on the accessibility relation and on the domain function. For example, a language that imposes no further restrictions on R and $\mathfrak{I}$ is to be contrasted with one that requires R to be an equivalence relation and $\mathfrak{I}(\underline{c}) = \mathfrak{I}(\underline{c_0})$ for all $\underline{c} \in K$.

The universe of discourse of a qml-model-structure Q is the set $\Delta = \bigcup_{\underline{c} \in K} \mathfrak{I}(\underline{c})$. Let Π be the set of predicate letters (with numerical superscripts indicating the number of places in the predicate). A function ρ defined on $\Pi \times K$ is a qml-model on a qml-model-structure Q provided that $\rho(P^n, \underline{c}) \subseteq \Delta^n$ for all $\underline{c} \in K$ and all predicate letters P^n. A qml-model is then a couple $\mathfrak{M} = \langle Q, \rho \rangle$, where Q is a qml-model-structure and ρ is a qml-model on it.

We now proceed in a customary way. A function τ from the variables of the language into the universe of discourse Δ of a qml-model $\mathfrak{M}$ is an assignment (of values to the variables) on $\mathfrak{M}$. Let Λ be the set of well-formed formulas of the language. The next step is to define a function $I_{\mathfrak{M},\tau}$ from the set $\Lambda \times K$ into $\{\underline{T}, \underline{F}\}$. This function is called the valuation induced by the assignment τ on the qml-model $\mathfrak{M}$. The valuation functions will be different for the K-family and the PH-family. In particular, the valuation functions for the K-family will be total functions, while those for the PH-family will generally be partial functions.

The difference between the PH-family and the K-family will be specified below when we specify the requirements on their respective valuation functions. Once the valuation functions have been characterized, other semantical categories can be defined in terms of them. For example, a formula A is said to be satisfied with respect to a world $\underline{c}$ by an assignment τ on a qml-model $\mathfrak{M}$ just in case $I_{\mathfrak{M},\tau}(A, \underline{c}) = \underline{T}$. A closed formula B is true on $\mathfrak{M}$ if and only if it is satisfied with respect to the actual world by every assignment on $\mathfrak{M}$--i.e. if and only if $I_{\mathfrak{M},\tau}(B, \underline{c_0}) = \underline{T}$ for all assignments τ on $\mathfrak{M}$. Finally, a closed formula B is logically true or valid just in case it is true on all qml-models.

The K-Family

A function $I_{\mathfrak{M},\tau}$ from $\Lambda \times K$ into $\{\underline{T}, \underline{F}\}$ is the K-valuation induced by the assignment τ on the qml-model $\mathfrak{M}$ provided that it satisfies the following conditions for all A, B $\in \Lambda$ and $\underline{c} \in$ K:

i. For atomic identity formulas $\ulcorner \xi_1 = \xi_2 \urcorner$, $I_{m,\tau}(\ulcorner \xi_1 = \xi_2 \urcorner, \underline{c}) = \underline{T}$ iff $\tau(\xi_1) = \tau(\xi_2)$; otherwise, $I_{m,\tau}(\ulcorner \xi_1 = \xi_2 \urcorner, \underline{c}) = \underline{F}$.

ii. For other atomic formulas $\ulcorner P^n \xi_1 \ldots \xi_n \urcorner$, $I_{m,\tau}(\ulcorner P^n \xi_1 \ldots \xi_n \urcorner, \underline{c}) = \underline{T}$ iff $\langle \tau(\xi_1), \ldots, \tau(\xi_n) \rangle \in \rho(P^n, \underline{c})$; otherwise, $I_{m,\tau}(\ulcorner P^n \xi_1 \ldots \xi_n \urcorner, \underline{c}) = \underline{F}$.

iii. $I_{m,\tau}(\ulcorner A \,\&\, B \urcorner, \underline{c}) = \underline{T}$ iff $I_{m,\tau}(A, \underline{c}) = \underline{T}$ and $I_{m,\tau}(B, \underline{c}) = \underline{T}$; otherwise, $I_{m,\tau}(\ulcorner A \,\&\, B \urcorner, \underline{c}) = \underline{F}$.

iv. $I_{m,\tau}(\ulcorner A \vee B \urcorner, \underline{c}) = \underline{T}$ iff $I_{m,\tau}(A, \underline{c}) = \underline{T}$ or $I_{m,\tau}(B, \underline{c}) = \underline{T}$; otherwise, $I_{m,\tau}(\ulcorner A \vee B \urcorner, \underline{c}) = \underline{F}$.

v. $I_{m,\tau}(\ulcorner A \supset B \urcorner, \underline{c}) = \underline{T}$ iff $I_{m,\tau}(A, \underline{c}) = \underline{F}$ or $I_{m,\tau}(B, \underline{c}) = \underline{T}$; otherwise, $I_{m,\tau}(\ulcorner A \supset B \urcorner, \underline{c}) = \underline{F}$.

vi. $I_{m,\tau}(\ulcorner A \equiv B \urcorner, \underline{c}) = \underline{T}$ iff $I_{m,\tau}(A, \underline{c}) = I_{m,\tau}(B, \underline{c})$; otherwise, $I_{m,\tau}(\ulcorner A \equiv B \urcorner, \underline{c}) = \underline{F}$.

vii. $I_{m,\tau}(\ulcorner \sim A \urcorner, \underline{c}) = \underline{T}$ iff $I_{m,\tau}(A, \underline{c}) = \underline{F}$; otherwise, $I_{m,\tau}(\ulcorner \sim A \urcorner, \underline{c}) = \underline{F}$.

viii. $I_{m,\tau}(\ulcorner \Box A \urcorner, \underline{c}) = \underline{T}$ iff for all $\underline{c}'$ such that $\underline{c}R\underline{c}'$, $I_{m,\tau}(A, \underline{c}') = \underline{T}$; otherwise, $I_{m,\tau}(\ulcorner \Box A \urcorner, \underline{c}) = \underline{F}$.

ix. $I_{m,\tau}(\ulcorner \Diamond A \urcorner, \underline{c}) = \underline{T}$ iff there is some c' such that $\underline{c}R\underline{c}'$ and $I_{m,\tau}(A, \underline{c}') = \underline{T}$; otherwise, $I_{m,\tau}(\ulcorner \Diamond A \urcorner, \underline{c}) = \underline{F}$.

x. $I_{m,\tau}(\ulcorner (\forall \xi) A \urcorner, \underline{c}) = \underline{T}$ iff for every assignment τ' that is like τ save perhaps at ξ, $\tau'(\xi) \in S(\underline{c})$ only if $I_{m,\tau'}(A, \underline{c}) = \underline{T}$; otherwise, $I_{m,\tau}(\ulcorner (\forall \xi) A \urcorner, \underline{c}) = \underline{F}$.

xi. $I_{m,\tau}(\ulcorner (\exists \xi) A \urcorner, \underline{c}) = \underline{T}$ iff there is some assignment τ' that is like τ save perhaps at ξ and for which $\tau'(\xi) \in S(\underline{c})$ and $I_{m,\tau'}(A, \underline{c}) = \underline{T}$; otherwise, $I_{m,\tau}(\ulcorner (\exists \xi) A \urcorner, \underline{c}) = \underline{F}$.

The K-languages are those languages whose valuation functions on qml-models are K-valuations.

The two important things to notice about K-valuations are first, that an open formula can be satisfied with respect to a world by an assignment that assigns "nonexisting" objects to its free variables--i.e., objects that are not in the domain of that world; and second, that when a quantificational formula is evaluated with respect to a world, quantified variables, unlike free variables, range only over the domain of that world. This contrast between free and quantified variables is also a feature of the PH-languages.

The PH-Family

A function $I_{m,\tau}$ from $\Lambda \times K$ into $\{\underline{T}, \underline{F}\}$ is the PH-valuation induced by the assignment τ on the qml-model $\mathcal{M}$ provided that it satisfies the following conditions for all A, B $\in \Lambda$ and $\underline{c} \in K$:

i. For atomic identity formulas $\ulcorner \xi_1 = \xi_2 \urcorner$, $I_{m,\tau}(\ulcorner \xi_1 = \xi_2 \urcorner, \underline{c}) = \underline{T}$ iff $\tau(\xi_1) = \tau(\xi_2)$; otherwise, $I_{m,\tau}(\ulcorner \xi_1 = \xi_2 \urcorner, \underline{c}) = \underline{F}$.

ii. For other atomic formulas $\ulcorner P^n \xi_1 \ldots \xi_n \urcorner$, $I_{m,\tau}(\ulcorner P^n \xi_1 \ldots \xi_n \urcorner, \underline{c})$ is undefined iff $\tau(\xi_1) \notin \mathcal{S}(\underline{c})$ or...or $\tau(\xi_n) \notin \mathcal{S}(\underline{c})$; otherwise, $I_{m,\tau}(\ulcorner P^n \xi_1 \ldots \xi_n \urcorner, \underline{c}) = \underline{T}$ if $\langle \tau(\xi_1), \ldots, \tau(\xi_n) \rangle \in \rho(P^n, \underline{c})$, and $I_{m,\tau}(\ulcorner P^n \xi_1 \ldots \xi_n \urcorner, \underline{c}) = \underline{F}$ if $\langle \tau(\xi_1), \ldots, \tau(\xi_n) \rangle \notin \rho(P^n, \underline{c})$.

iii. $I_{m,\tau}(\ulcorner A \,\&\, B \urcorner, \underline{c})$ is undefined iff $I_{m,\tau}(A, \underline{c})$ or $I_{m,\tau}(B, \underline{c})$ is undefined; $I_{m,\tau}(\ulcorner A \,\&\, B \urcorner, \underline{c}) = \underline{T}$ iff $I_{m,\tau}(A, \underline{c}) = \underline{T}$ and $I_{m,\tau}(B, \underline{c}) = \underline{T}$; otherwise, $I_{m,\tau}(\ulcorner A \,\&\, B \urcorner, \underline{c}) = \underline{F}$.

iv. $I_{m,\tau}(\ulcorner A \vee B \urcorner, \underline{c})$ is undefined iff $I_{m,\tau}(A, \underline{c})$ or $I_{m,\tau}(B, \underline{c})$ is undefined; $I_{m,\tau}(\ulcorner A \vee B \urcorner, \underline{c}) = \underline{T}$ iff $I_{m,\tau}(A, \underline{c}) = \underline{T}$ or $I_{m,\tau}(B, \underline{c}) = \underline{T}$; otherwise, $I_{m,\tau}(\ulcorner A \vee B \urcorner, \underline{c}) = \underline{F}$.

v. $I_{m,\tau}(\ulcorner A \supset B \urcorner, \underline{c})$ is undefined iff $I_{m,\tau}(A, \underline{c})$ or $I_{m,\tau}(B, \underline{c})$ is undefined; $I_{m,\tau}(\ulcorner A \supset B \urcorner, \underline{c}) = \underline{T}$ iff $I_{m,\tau}(A, \underline{c}) = \underline{F}$ or $I_{m,\tau}(B, \underline{c}) = \underline{T}$; otherwise, $I_{m,\tau}(\ulcorner A \supset B \urcorner, \underline{c}) = \underline{F}$.

vi. $I_{m,\tau}(\ulcorner A \equiv B \urcorner, \underline{c})$ is undefined iff $I_{m,\tau}(A, \underline{c})$ or $I_{m,\tau}(B, \underline{c})$ is undefined; otherwise, $I_{m,\tau}(\ulcorner A \equiv B \urcorner, \underline{c}) = \underline{T}$ if $I_{m,\tau}(A, \underline{c}) = I_{m,\tau}(B, \underline{c})$, and $I_{m,\tau}(\ulcorner A \equiv B \urcorner, \underline{c}) = \underline{F}$ if $I_{m,\tau}(A, \underline{c}) \neq I_{m,\tau}(B, \underline{c})$.

vii. $I_{m,\tau}(\ulcorner {\sim} A \urcorner, \underline{c})$ is undefined iff $I_{m,\tau}(A, \underline{c})$ is undefined; $I_{m,\tau}(\ulcorner {\sim} A \urcorner, \underline{c}) = \underline{T}$ iff $I_{m,\tau}(A, \underline{c}) = \underline{F}$; otherwise, $I_{m,\tau}(\ulcorner {\sim} A \urcorner, \underline{c}) = \underline{F}$.

viii. $I_{m,\tau}(\ulcorner \Box A \urcorner, \underline{c})$ is undefined iff $I_{m,\tau}(A, \underline{c})$ is undefined: $I_{m,\tau}(\ulcorner \Box A \urcorner, \underline{c}) = \underline{T}$ iff there is no $\underline{c}'$ such that $\underline{cRc}'$ and

$\mathcal{I}_{m,\tau}$(A, c) = F; otherwise, $\mathcal{I}_{m,\tau}$(⌜□A⌝, c) = F.

ix. $\mathcal{I}_{m,\tau}$(⌜◇A⌝, c) is undefined iff $\mathcal{I}_{m,\tau}$(A, c) is undefined; $\mathcal{I}_{m,\tau}$(⌜◇A⌝, c) = T iff there is some c' such that cRc' and $\mathcal{I}_{m,\tau}$(A, c) = T; otherwise, $\mathcal{I}_{m,\tau}$(⌜◇A⌝, c) = F.

x. $\mathcal{I}_{m,\tau}$(⌜(∀ξ)A⌝, c) = T iff either $\mathfrak{I}$(c) is empty and no variable other than ξ occurs freely in A or $\mathfrak{I}$(c) is not empty and for every assignment τ′ that is like τ save perhaps at ξ, τ′(ξ) ∈ $\mathfrak{I}$(c) only if $\mathcal{I}_{m,\tau'}$(A, c) = T; $\mathcal{I}_{m,\tau}$(⌜(∀ξ)A⌝, c) = F iff there is some assignment τ′ that is like τ save perhaps at ξ and for which τ′(ξ) ∈ $\mathfrak{I}$(c) and $\mathcal{I}_{m,\tau'}$(A, c) = F; otherwise, $\mathcal{I}_{m,\tau}$(⌜(∀ξ)A⌝, c) is undefined.

xi. $\mathcal{I}_{m,\tau}$(⌜(∃ξ)A⌝, c) = T iff there is some assignment τ′ that is like τ save perhaps at ξ and for which τ′(ξ) ∈ $\mathfrak{I}$(c) and $\mathcal{I}_{m,\tau'}$(A, c) = T; $\mathcal{I}_{m,\tau}$(⌜(∃ξ)A⌝, c) = F iff either $\mathfrak{I}$(c) is empty and no variable other than ξ occurs freely in A or $\mathfrak{I}$(c) is not empty and for every assignment τ′ that is like τ save perhaps at ξ, τ′(ξ) ∈ $\mathfrak{I}$(c) only if $\mathcal{I}_{m,\tau'}$(A, c) = F; otherwise, $\mathcal{I}_{m,\tau}$(⌜(∃ξ)A⌝, c) is undefined.

Clauses (x) and (xi) include special provisions for worlds with empty domains. The need is to prevent such a world from validating '(∃x)◇(∀y) F^2xy' and from invalidating '(∃x)□(∃y) F^2xy'.

The PH-family includes all those languages whose valuation functions on qml-models are PH-valuations. But we permit it to include other related languages as well. The characterization of PH-valuations presupposes that identity is the only special, nonstandard atomic predicate. As indicated in the text, however, perhaps there are reasons to include other nonstandard atomic predicates. This can be done by singling them out syntactically and adding further clauses between (i) and (ii) to cover them. Languages whose valuation functions on qml-models involve such extensions of PH-valuations are also considered PH-languages. Of course, from a logical standpoint, it would be nice if identity turns out to be the only well-motivated nonstandard atomic predicate.

The main differences between PH-valuations and K-valuations are in clauses (ii) and (viii). Clause (ii) introduces satisfaction gaps at the atomic level, and clause (viii) adjusts the handling of '$\Box$' to accommodate these gaps. Of course, nothing would be affected if the first part of clause (viii) in the definition of K-valuations were also, $\mathcal{I}_{m,\tau}(\ulcorner\Box A\urcorner, \underline{c}) = \underline{T}$ iff there is no $\underline{c}'$ such that $\underline{c}R\underline{c}'$ and $\mathcal{I}_{m,\tau}(A, \underline{c}') = \underline{F}$. Accordingly, the critical difference in the PH-valuations is clause (ii). The other clauses must then provide for satisfaction gaps, which under the valuation rules given propagate to nonatomic levels. Only open formulas, however, can fail to be assigned $\underline{T}$ or $\underline{F}$. PH-valuations never fail to assign a value to closed formulas. Hence, the term "satisfaction gaps" is appropriate.

The treatment of identity in clause (i) is the same as in K-valuations. This is the most straightforward way of treating identity, and it simplifies formal matters considerably. As mentioned in the text, however, one might instead adopt the following treatment:

i'. $\mathcal{I}_{m,\tau}(\ulcorner\xi_1 = \xi_2\urcorner, \underline{c})$ is undefined iff $\tau(\xi_1) \notin \mathcal{S}(\underline{c})$ and $\tau(\xi_2) \notin \mathcal{S}(\underline{c})$; otherwise, $\mathcal{I}_{m,\tau}(\ulcorner\xi_1 = \xi_2\urcorner, \underline{c}) = \underline{T}$ if $\tau(\xi_1) = \tau(\xi_2)$, and $\mathcal{I}_{m,\tau}(\ulcorner\xi_1 = \xi_2\urcorner, \underline{c}) = \underline{F}$ if $\tau(\xi_1) \neq \tau(\xi_2)$.

None of the conclusions in the text is affected if (i') is chosen instead of (i).

The valuation rules given above for the truth-functional connectives are the familiar "weak" three-valued ones. Thus an open formula that is a truth-functional compound is not assigned a value just in case it includes a constituent that is not assigned a value. The weak valuation rules were adopted because they promise to be easier to work with than other three-valued approaches. But none of the conclusions in the text turns on this choice. In particular, the points put forward in favor of the PH-family hold equally well, for example, for the related family obtained by adopting Kleene's "strong" valuation rules:[91]

A	B	⌜~A⌝	⌜A & B⌝	⌜A v B⌝	⌜A ⊃ B⌝	⌜A ≡ B⌝
T	T	F	T	T	T	T
T	F	F	F	T	F	F
T	-	F	-	T	-	-
F	T	T	F	T	T	F
F	F	T	F	F	T	T
F	-	T	F	-	T	-
-	T	-	-	T	T	-
-	F	-	F	-	-	-
-	-	-	-	-	-	-

Any philosophically motivated choice between such alternative valuation rules would have to be based on essentialist considerations beyond those emphasized in the text. For example, is '$F\Phi \& (\exists y)(y = \Phi)$' essentially true of every object that '$F\Phi$' is essentially true of? One good reason to prefer the weak valuation rules comes from sentences like 'Jimmy Carter had to be the offspring of Miss Lillian'. (i) expresses this straightforwardly with the weak valuation rules, but not with the strong valuation rules since with them it is false just because Miss Lillian did not have to exist.

(i) $(\exists x)((\forall y)(Jy \equiv y{=}x) \;\&\; \Box(\exists w)((\forall y)(Ly \equiv y{=}w) \;\&\; O^2xw))$

Notes

1 Michael Dummett, Frege: Philosophy of Language, (New York: Harper & Row, 1973), p. 127f.

2 Tyler Burge, "Book Review: Meaning, Reference, and Necessity. Simon Blackburn, ed." The Journal of Philosophy, LXXIV, 4 (April, 1977), p. 244.

3 Christopher Peacocke, "Proper Names, Reference, and Rigid Designation," Meaning, Reference, and Necessity, ed. Simon Blackburn, (Cambridge: Cambridge U. Press, 1975), p. 111f. As we discuss later, Peacocke's notion of a rigid designator is idiosyncratically narrow.

4 Michale A. Slote, Metaphysics and Essence, (New York: New York University Press, 1975), p. 75.

5 Leonard Linsky, Names and Descriptions, (Chicago: University of Chicago Press, 1977), p. 56ff. (Again, p. 66.)

6 Ibid., p. 59.

7 Saul Kripke, "Identity and Necessity," Identity and Individuation, ed. M. K. Munitz, (New York: New York University Press, 1971), p. 140.

8 Alfred North Whitehead and Bertrand Russell, Principia Mathematica: to *56, (Cambridge: Cambridge U. Press, 1962), p. 184ff.

9 Kripke has expressly sanctioned Quine's move in the context of discussions of a rigid designation. Cf. "Naming and Necessity," Semantics of Natural Languages, (Dordrecht: Reidel, 1972), p. 343, fn. 5.

10 '(···x···)' is here assumed to include no free variables besides 'x'. I have not examined the "rigid functors" that would result were this assumption relaxed.

11 Peacocke, op. cit., p. 114, has intimated that definitions of rigid designation that employ modal operators do not give necessary conditions for rigidity since nonmodal languages can contain rigid designators too. The remarks in the text are meant to counter this idea.

12 As we shall see, this informal characterization is not quite accurate when the schema is interpreted in accordance with the modal languages Kripke develops in detail in "Semantical Considerations on Modal Logic." A more accurate, though otherwise less felicitous characterization would be as follows: a designating definite description is a rigid designator if and only if it is impossible for (i) the object it designates to exist and not satisfy the description and (ii) for any other object to exist and satisfy the description. This point will be discussed further in footnote 23.

13 Kripke, "Identity and Necessity," p. 146.

14 Kripke is not always consistent about his intuitive test. Sometimes his test seems not to be that $\ulcorner\alpha$ might not have been $\alpha\urcorner$ be false, but that $\ulcorner\alpha$ might have been something other than the thing that in fact is $\alpha\urcorner$ be false. (*Cf.*, "Naming and Necessity," p. 270, where the alternative phrasings occur in consecutive paragraphs; or see "Identity and Necessity," p. 148.) The two phrasings differ significantly. Suppose α is a designating definite description, '$(\imath x)(Px)$'. Then the test based on $\ulcorner\alpha$ might not have been $\alpha\urcorner$ requires that (i) be false:

(i) $(\exists x)\Diamond\sim(\forall y)(Py \equiv y=x)$

By contrast, the test based on $\ulcorner\alpha$ might have been something other than the thing that in fact is $\alpha\urcorner$ requires that (ii) be false:

(ii) $(\exists x)\Diamond(\exists w)((\forall y)(Py \equiv y=w)\ \&\ \sim(w=x))$

The crucial difference between (i) and (ii) is a difference in the scope of negation. As can readily be verified, the falsity of (i) is in keeping with the criterion for rigidity we give in the text, whereas the falsity of (ii) is not. As Slote has pointed out (*op. cit.*, p. 72ff), (ii) is false if '$(\imath x)(Px)$' is taken to stand for 'the individual that is Nixon and a politician'. Hence according to the test based on $\ulcorner\alpha$ might have been something other than the thing that in fact is $\alpha\urcorner$, an intuitively nonrigid designator--'the individual that is Nixon and a politician'--would be rigid. By contrast, (i) is true if '$(\imath x)(Px)$' is taken to stand for this designator. That is, the test based on

$\ulcorner\alpha$ might not have been $\alpha\urcorner$ --and hence our criterion--excludes this intuitively nonrigid designator. These considerations lead me to conclude that the test Kripke meant to propose, his lapses not withstanding, is that $\ulcorner\alpha$ might not have been $\alpha\urcorner$ be false.

15 Kripke, "Semantical Considerations on Modal Logic," reprinted in *Reference and Modality*, ed. Leonard Linsky, "London: Oxford U. Press, 1971), p. 66f.

16 Here and elsewhere in the paper I use Quine's notation for predicates. *Cf.* W. V. Quine, *Methods of Logic*, third edition, (New York: Holt, Rinehart and Winston, 1972), p. 143ff.

17 This modal language is discussed at length under the title "LPC + S5" in G. E. Hughes and M. J. Cresswell, *An Introduction to Modal Logic*, (London: Methuen, 1972), Chapters 8 and 9, pp. 133-169.

18 Kripke, *ibid.*, p. 69.

19 *Ibid.*, p. 68.

20 The languages that require the same objects to exist in every accessible world are not deviant in the manner described (though, of course, they are philosophically objectionable on other grounds). The deviance described is easy to miss. I am indebted to Drew Christie for first calling my attention to it in regard to the criterion for rigid designation given earlier.

21 '$(\iota x)(Gx)$' will be what Slote calls an "inclusively rigid designator," as contrasted with an "exclusively rigid designator" exemplified by '$(\iota x)(Px)$' in footnote 14 (*op. cit.*, p. 72ff). As I point out in my "Rigid Designation and Its Variants," where inclusively and exclusively rigid designators are systematically contrasted with rigid designators, the example Slote gives of an inclusively rigid designator will not quite do. I am indebted to Richard Cartwright for the Fregean example I use in the text.

22 Thus 'x=x' would still be handled differently from the way it is in the K-languages. The identity predicate would not be just a K-language

predicate. But '$(\forall x)(\Box x=x)$' would continue to be logically true on the modified definition. By contrast, it would not be logically true if we required '$y=x$' to be false with respect to worlds in which the object assigned to at least one of its variables does not exist.

23 I use the phrase, "existent objects 'Fx' is true of," for good reason. We need to be careful about what our criterion for rigidity requires of a designator '$(\iota x)(Fx)$'. One thing it requires, stated in model-theoretical terms, is that the predicate '$(\forall y)(Fy \equiv y=\Phi)$' not be satisfied with respect to any accessible world by any extraneous object--i.e. by any object other than the designated object. (When I speak of an object satisfying "the definite description" or "the definite description predicate," I am alluding to a predicate of this sort.) This requirement must not be confused with a related one, *viz.* that '$F\Phi$' not be satisfied with respect to any accessible world by any extraneous object. In the case of our revised alternative truth definition, these two requirements are equivalent. This is one of several features of the languages picked out by this truth definition that makes them philosophically attractive. But the two requirements are not equivalent in the case of the truth definition for the K-languages. Any number of objects that do not exist in a world may satisfy '$F\Phi$' with respect to that world in a K-language, and yet '$(\forall y)(Fy \equiv y=\Phi)$' still be satisfied only by the designated object. K-languages are peculiar in this respect. The requirement our criterion imposes on '$F\Phi$' in their case is that it not be satisfied with respect to any accessible world by any existent extraneous object. Of course, our criterion clearly imposes this same requirement in the case of the languages picked out by our revised alternative truth definition.

There is another potential confusion here. In standard logic (i) and (ii) are equivalent:

(i) $(\exists x)(\forall y)(Fy \equiv y=x)$

(ii) $(\exists x)(Fx \& (\forall y)(Fy \supset y=x))$

One might therefore expect (iii) and (iv) to be equivalent:

(iii) $(\exists x)\Box(\forall y)(Fy \equiv y=x)$

(iv) $(\exists x)\Box(Fx \& (\forall y)(Fy \supset y=x))$

And indeed they are equivalent in the case of our revised alternative truth definition. But they are not equivalent in the case of the truth definition for the K-languages. Specifically, (iv) can be false while (iii) is true since (iv) additionally requires 'FΦ' to be true of the object in question even with respect to accessible worlds in which this object does not exist. The non-equivalence of (iii) and (iv) is another peculiarity of most K-languages. Two considerations lie behind our adopting (iii) rather than (iv) in our criterion for rigidity. First, imposing requirements with respect to worlds in which the designated object does not exist seems gratuitous to a referential notion like rigid designation. Second, if a requirement of this sort were to be imposed, a more appropriate one would be for 'FΦ' to be uniquely true of the designated object with respect to every accessible world--i.e. the requirement expressed by (v) in the K-languages:

(v) $(\exists x)\Box(Fx \,\&\, (\forall y)\Box(Fy \supset y=x))$

Even though (iii) and (v) are equivalent in the case of our revised alternative truth definition, I do not find (v) perspicuous as an expression of our intuitive notion of rigid designation. Whether these two considerations suffice to justify our choice of (iii) will not be important here. None of our results pertaining to scope will turn on this choice.

What then does our criterion for the rigidity of a designator '$(\iota x)(Fx)$' require of 'FΦ'? Stated in language that is free of model-theoretic talk about possible worlds and that is neutral with respect to all of the modal languages we are considering, our criterion requires that

a. the designated object cannot exist unless 'FΦ' is true of it.

b. no other object can both exist and have 'FΦ' be true of it.

Neither of these requirements is readily expressed in standard philosophic terms like 'essential' and 'essentially unique'. But this should not be altogether surprising. We have no reason to expect our standard philosophic terminology to be neutral with respect to all of the modal languages we are considering. Undoubtedly, some of these languages conform more closely to this terminology than others do. At this point we should be pleased just to have an informal characterization of our criterion that is neutral with respect to all of the formal languages we are considering.

24 Kripke, "Identity and Necessity," p. 151.

25 The example is Drew Christie's. He has shown that a restricted form of the distribution axiom schema holds in all PH-languages. Also, see Jaakko Hintikka, "Modality and Quantification," Models for Modalities, (Dordrecht: Reidel, 1969), p. 65.

26 Our metalinguistic convention has 'ξ' as a variable ranging over variables and 'α' as a variable ranging over singular terms. '$\phi(\xi)$' denotes an open formula in which ξ has one or more free occurrences. '$\phi(\alpha)$' denotes the formula that results from substituting α for ξ in $\phi(\xi)$, and '$\phi((\iota x)(Fx))$' denotes a formula that results from substituting '$(\iota x)(Fx)$' for ξ in $\phi(\xi)$.

27 According to our convention about the representation of predicates in the PH-languages, both '$F\Phi$' and '$G\Phi$' are standard predicates in (5) and (6). As will be evident, our conclusions about (5) and (6) will require '$G\Phi$' to be standard. But they will not require '$F\Phi$' to be standard. '$F\Phi$' occurs in (5) and (6) only within a definite description predicate, '$(\forall y)(Fy \equiv y=\Phi)$'. Whether the predicate occurring thusly is standard or not makes no difference to the truth-value of any PH-sentence in which the definite description predicate occurs. Hence, although we will use definite descriptions formed with standard predicates--e.g. '$(\iota x)(Fx)$'--when discussing the PH-languages, our results will always extend to definite descriptions formed with nonstandard predicates.

28 Some may prefer (7) to (2) as the criterion for rigid designation. Kripke's comments about 'Sherlock Holmes' in the addenda to "Naming and Necessity" (ibid., p. 764) are compatible with both of them. Furthermore, the intuitive considerations that best serve to motivate the notion of rigid designation do not discriminate between them. (7)

is, of course, less restrictive. And it has the virtue of allowing the thesis that proper names are rigid designators to stand without qualifications for fictional names, assuming Kripke's comments are correct. Still, I prefer (2). As we will see, (7) is not the only weakening of (2) that is in accord with the intuitive underpinnings of the notion of rigidity. Considerations beyond these motivating ones will be needed to settle on a definition in any case. For me, one such consideration is the incongruity of expressions like 'the round square' being rigid designators.

29 Obviously, the distinction between rigidity and semi-rigidity can be drawn. But I do not see how to motivate it except by expressly appealing to "essentialist" considerations. Such considerations seem to me more elusive than the ones usually put forward to motivate a special class of designators in modal languages. This point will be explored further in Section VIII.

30 Kripke, "Semantical Considerations on Modal Logic," <u>op. cit.</u>, p. 67f.

31 Note that this conclusion holds for the PH-languages even when the occurrences of the designator are restricted to standard contexts. The distinction between standard and nonstandard predicates is not significant when '$\Box$' replaces 'μ' in (4).

32 I think that the model-theoretical characterization of $\ulcorner \boxminus A \urcorner$ below accomplishes everything needed. Given a formula A, let σ_A be the set of satisfiable closed formulas B_i such that

i. B_i is $\ulcorner (\exists! \xi)(\phi_i(\xi)) \urcorner$, where no atomic predicate occurs more times in B_i than in A.

ii. There is an open formula $\Psi_i(\xi)$ such that $\ulcorner A \equiv (\exists \xi_1)[(\forall \xi_2)(\phi(\xi_2) \equiv \xi_2 = \xi_1)\ \&\ \Psi_i\ (\xi_1)] \urcorner$ is valid.

In the K-languages, $\ulcorner \boxminus A \urcorner$ is assigned a truth-value with respect to all worlds. In the PH-languages, it will have satisfaction gaps in the usual manner. With this proviso, one rule for assigning truth-values will serve for all of the languages: given an assignment of objects to variables, $\ulcorner \boxminus A \urcorner$ is assigned <u>T</u> with respect to a world if and only if (i) the members of σ_A are mutually satisfiable and

(ii) A is assigned _F_ with respect to no accessible world in which all members of σ_A are assigned _T_. (The first clause, or something like it, is needed to prevent such formulas as '$(\iota x)(Fx)=(\iota x)(Gx)$' from being weakly necessarily true when '$(\exists x)(Fx) \equiv \sim(\exists x)(Gx)$' is necessarily true.)

33 It is possible to construct a weaker necessity operator than '$\boxminus$' that has the desired scope-related characteristics in the K-languages. Let $\ulcorner\boxminus' A\urcorner$ be assigned _T_ in a K-language just in case A is assigned _F_ with respect to no accessible world in which (i) there exist objects denoted by the designating expressions in A and (ii) _the objects assigned to the free variables in A exist_. '$\boxminus'$' in the K-languages mirrors '$\boxminus$' in the PH-languages provided that one ignores nonstandard contexts. '$\Diamond'$' and '$\Box'$' can be similarly constructed to mirror '$\Diamond$' and '$\Box$' in the PH-languages, again excepting nonstandard contexts. '$\boxminus'$', '$\Box'$', and '$\Diamond'$' have the same scope-related characteristics in the K-languages as '$\boxminus$', '$\Box$', and '$\Diamond$' have in standard contexts in the PH-languages. However, these weakened operators in the K-languages lack many of the philosophic virtues of the corresponding operators in the PH-languages. The distinction between standard and nonstandard contexts is an important feature of the PH-languages. Thus, for example, we cannot say that '$G\Phi$' is essentially true of an object just in case '$\Box' G\Phi$' is true of it unless we concede that the existence predicate is trivially essentially true of every object. Thus '$\Box'$' is not the philosophic analogue in the K-languages of '$\Box$' in the PH-languages. '$\Box G\Phi$' in the PH-languages seems to capture the notion of an essential predicate exceptionally well. In passing, we should also note that '$\boxminus G\Phi$' in the PH-languages does not give an acceptable representation of essential predicates unless we concede that there are no relations that are "internal" to one relatum and "external" to the other. (These points are discussed in more detail in Christie and Smith, _op. cit._)

34 The requirement is that A be closed and contain no _de re_ occurrences of modal operators. The latter is needed because the following formula

is materially equivalent to (11) in every K-language when '$(\iota x)(Fx)$' is a rigid designator:

$\Box(\forall y)(y=(\iota x)(Fx) \supset \Box Gy)$

(where the definite description is read with minimal scope).

35 A corollary of this result is that, without employing de re modality, there is no way in the K-languages to refer singularly to an object with respect to worlds in which it does not exist. This holds for the PH-languages even without the stricture against de re modality.

36 A parallel point can be developed about '⊟' in the PH-languages by replacing '$\Diamond$' with '⊟' and 'Gx' with 'Fx' in (37) and (38).

37 That is, both must be assigned 'T', 'F', or nothing. (Keep in mind that extensional contexts include ones like '$(\forall x)$.........'.)

38 Both Peacocke and Dummett seem to insist on a nonmodal definition of rigid designation. Not surprisingly, the nonmodal definitions they offer turn out to be thoroughly incompatible with our definition. I discuss their definitions briefly later. A more detailed discussion, particularly of Peacocke's definition, can be found in my "Rigid Designation and Its Variants."

39 The best defense of this assumption I know of is by Quine (cf. Word and Object, (Cambridge: MIT Press, 1960), pp. 176-190). Although the point is sometimes ignored, Quine's method of reparsing names involves two steps: first, names are replaced by definite descriptions formed with special predicates and second, the definite descriptions are then treated in the manner of Russell. The latter is the controversial step. Without it, the former step is essentially notational. Thus one could have name-free languages in which definite descriptions are treated along the lines suggested by Strawson or as they are in some free-logics (e.g. as in K. Lambert and B. C. van Fraassen, Derivation and Counterexample, (Encino: Dickenson, 1972), Chapters 7 and 10). As I hope this paper illustrates, one important advantage of replacing names by definite descriptions is that it puts one in a position to

mark scope distinctions that otherwise are difficult to mark and thus are sources of confusion.

40 Our two families together cover a large fraction of the formal modal languages in the literature. Still, there are modal languages in the literature that are not in either family. For example, languages in which bound variables always range over all of the objects in the universe of discourse are neither PH- nor K-languages (unless, of course, they require the same objects to exist in every possible world). An unattractive feature of such languages is the extent to which their nonmodal fragments depart from standard quantificational theory.

The PH- and K-families should also be contrasted with the family of modal languages in which the semantics is just like Kripke's except F is assigned to an atomic open sentence with respect to every world in which an object assigned to one of its free variables does not exist. (Robert Stalnaker has promoted a variant of these languages in his "Complex Predicates," The Monist, 60, 3, (July, 1977), p. 334ff.) One unattractive feature of this family, pointed out by Kripke (in "Semantical Considerations on Modal Logic," fn. 11, p. 66), is that validity is not always preserved when nonatomic predicates are substituted for atomic ones in valid formulas. Another unattractive feature of this family is that '$(\exists x)(\Diamond\sim(\exists y)(y=x)\ \&\ \Box x=x)$' is invariably false in it, so that it precludes contingently existing objects being necessarily self-identical. Hence, being essential cannot reasonably be identified with being necessary *de re*.

41 Of course, the Barcan formulas are valid in some K-languages. But these languages are special: in the model theory, they require the universe of discourse to consist of just the objects that exist in the actual world. Informally, the Barcan formulas assert that if it is possible for '$(\exists x)(Gx)$' to be true, then there is an actual object of which 'Gx' can be true. This is a strong claim, reaching well beyond pre-theoretical intuitions. Still, given our present understanding of *de re* modalities, we cannot entirely rule it out. As matters stand, an ideal formalism for expressing alternative essentialist theories is

one that leaves the question of the validity of the Barcan formulas open. One virtue of the K-family (and the PH-family) is that it does just that. Indeed, it does more since it brings out what is at issue in the question by systematically contrasting those member languages in which the Barcan formulas are valid with those in which they are not.

The converse Barcan formulas are also not automatically validated in the K-family. Their pre-theoretical status is, however, sharply different from that of the Barcan formulas (cf. Alvin Plantinga, The Nature of Necessity, (Oxford: Oxford U. Press, 1974), p. 59). The Barcan formulas are suspect generally. But the converse Barcan formulas seem correct for most predicates. That is, it seems reasonable to grant that if there is an actual object of which 'Gx' can be true, then unless something peculiar is going on with the predicate, it is possible for '(∃x)(Gx)' to be true. The objectionable instances of the converse Barcan formulas exploit exceptional predicates such as the nonexistence predicate, '∼(∃y)(y=①)'--predicates Kripke would call "recherché." An attractive line to take with the converse Barcan formulas is to distinguish two kinds of predicates, one for which the formulas in question are universally valid and another for which they are not. This is precisely the line taken in the PH-family.

In spite of some counterintuitive features, the K-family has been dominant in the recent philosophic literature on essentialism. I suspect this is as much owing to the historical role of Kripke's formalization as it is to its elegance. Until "Some Considerations on Modal Logic," efforts to join axioms for standard quantificational logic with ones for S5 invariably ended up validating the Barcan formulas (cf. A. N. Prior, "Modality and Quantification in S5," Journal of Symbolic Logic, 21 (1956), pp. 6-62, and Hughes and Cresswell, op. cit., Ch. 8-10). This made the enterprise of quantified modal logic suspect. For it is counterintuitive for a principle making so controversial a claim to be a logical consequence of independently motivated principles that seem to make no such controversial claim. Kripke put an end to this source of suspicion (though not to others) by finding axioms for the two logics that when combined do not validate the Barcan formulas. It is not just that Kripke's formalization of quantified modal logic leaves standard quantificational logic and normal modal propositional logic intact. The formal logic underlying the PH-family does this. The point is that each of Kripke's axioms is an axiom for either standard quantificational logic or normal modal propositional logic. Hence, each of his axioms is motivated completely independently of de re modal considerations. The one restriction he introduces is that only closed formulas can be theorems, so that open formulas in proofs must be taken as abbreviations of their universal closures. But even this restriction can be motivated within standard quantificational logic (cf. Quine, Mathematical Logic, (Cambridge: Harvard U. Press, 1947), pp. 76-89). Consequently, to object to the K-family, one must argue that some basic principle of standard quantificational logic or of normal modal propositional logic requires modification in the context of quantified modal logic.

It is striking that the one restriction Kripke imposes pertains to open formulas. This reinforces a point suggested by contrasting the PH- and K-families, viz. that the treatment of open formulas is the crucial issue in quantified modal logic.

42 The PH-family has many intuitively attractive features that other formalizations have been criticized for lacking:

- i. It leaves standard quantificational logic intact.
- ii. It leaves normal modal propositional logic intact.
- iii. It allows contingently existing objects.
- iv. The Barcan formulas are not automatically valid in it.
- v. It allows predicates to be essentially and necessarily true of contingently existing objects.
- vi. It requires every object to be essentially and necessarily identical with itself.
- vii. It allows being essential to be straightforwardly defined in terms of de re necessity--viz. by equating the two.
- viii. In it, existence is essential only to necessarily existing objects.

ix. It allows relations that are internal (essential) to one of their relata and external (nonessential) to the other--i.e., relations of the sort that *being the off-spring of* is often held to be.

x. It represents the distinction between *de re* and *de dicto* modality via differences in the scope of modal operators *vis-a-vis* quantifiers, and not via separate operators for *de re* and *de dicto* modalities.

xi. Validity is preserved when nonatomic predicates are substituted for atomic ones in its valid schemata (subject to a syntactically specifiable restriction on substitution).

xii. Instances of '$(\exists x)(\Box Gx) \supset \Box(\exists x)(Gx)$' are not automatically valid in it.

xiii. The condition can be specified under which co-referring designators can be universally substituted for one another in it *salva veritate*--*viz.* that the designators be rigid or semi-rigid.

xiv. Rigid designators are "scope neutral" in a specifiable range of its contexts that can appropriately be considered the "intuitively standard" contexts.

xv. The converse Barcan formulas are valid in it in the case of normal predicates, but not in the case of a specifiable set of *recherché* predicates.

xvi. Normally, a predicate is essentially true of an object in it if and only if it is necessary that the predicate be true of the object should the object exist; but this condition is not sufficient in the case of a specifiable set of *recherché* predicates.

xvii. Normally, '$\Diamond Gx \supset \Diamond(\exists y)(Gy))$' is not false in it regardless of the object assigned to 'x'; but instances of the schema can be false in it in the case of a specifiable set of *recherché* predicates.

xviii. Normally, '$Gx \supset (\exists y)(Gy)$' is not false in it regardless of the object assigned to 'x'; but instances of the schema can be false in it in the case of a specifiable set of <u>recherché</u> predicates.

xix. It allows the two versions of the Russellian definite description predicate--$\ulcorner\Phi(\textcircled{1}) \mathbin{\&} (\forall\xi)(\Phi(\xi) \supset \xi = \textcircled{1})\urcorner$ and $\ulcorner(\forall\xi)(\Phi(\xi) \equiv \xi = \textcircled{1})\urcorner$--to be interchanged <u>salva veritate</u> unless $\Phi(\textcircled{1})$ is a <u>recherché</u> predicate.

To the best of my knowledge, every other approach to formalizing <u>de re</u> and <u>de dicto</u> modalities in the literature lacks at least one of these features.

Of course, some features in the list carry more intuitive weight than others. I have tried to list them in what I take to be roughly the order of their importance. Again, to the best of my knowledge, every other approach to formalizing <u>de re</u> and <u>de dicto</u> modalities in the literature lacks at least one of the first ten features. For example, Kripke's approach cannot have all of (vii), (viii), and (ix). For suppose <u>'Gx' is essentially true of s</u> is equated with <u>'$\Box$Gx' is true of s</u>; then the K-family will bar relations that are internal to one relatum and external to the other. Suppose instead <u>'Gx' is essentially true of s</u> is equated with <u>'$\Box((\exists y)(y=x) \supset Gx)$' is true of s</u>; then existence will be essential to contingently existing objects. Needless to say, the K-family also lacks (xiv) through (xix). Formally, the PH-family secures the features the K-family lacks chiefly through a not unreasonable revision of the distribution axiom: $\ulcorner\Box(A \supset B) \supset (\Box A \supset \Box B)\urcorner$ becomes $\ulcorner\Box(A \supset B) \supset (\Box A \supset \Box((\exists\xi)(\xi = \xi_1) \mathbin{\&} \ldots \mathbin{\&} (\exists\xi)(\xi = \xi_k) \supset B))\urcorner$, where $\xi_1, \ldots, \xi_k$ are the variables occurring free in standard contexts in A but not in B (<u>cf.</u> Christie and Smith, <u>op. cit.</u>).

Showing that the PH-family has all of the features listed does not show that it is beyond objection. Perhaps it has some yet to be noticed feature that conflicts with our informal intuitions. The list covers only those features that other formalizations have been criticized for lacking. Still, since the PH-family does have all of these features, it is not open to any of the usual objections. This fact provides a good <u>prima facie</u> case that the family fits our intuitions. It also provides a good *prima facie* case against the claim that to get an intuitively reasonable formalization of *de re* modality, one must resort to separate *de re* and *de dicto* operators (*cf.*, for example, David Wiggins, "The *De Re* Must," *Truth and Meaning*, ed. Gareth Evans and John McDowell, (Oxford: Oxford U. Press, 1976), pp. 285-312).

43 Linsky alludes to the PH-languages (*op. cit.*, p. 134), but the formal semantics he elaborates and defends is that of the K-languages, supplemented with constants (pp. 130-152).

44 This does not imply that *de re* modality can be reduced to *de dicto* modality in standard contexts in the PH-languages, for rigid designation is itself an inherently *de re* modal notion.

45 Under the imposed restrictions, the only such inferences that are warranted in K-like languages are ones from *de dicto* to *de re* possibility and strong necessity.

46 Peacocke, *op. cit.*, p. 111.

47 *Ibid.*, p. 111f.

48 Some may think that the position I attribute to Dummett, while suiting my purposes, is not entirely faithful to his text. I attribute the position to him primarily on the basis of the following passage (*op. cit.*, p. 128):

> Kripke's doctrine that proper names are rigid designators and definite descriptions non-rigid ones thus reduces to the claim that, within a modal context, the scope of a definite description should always be taken to exclude the modal operator, whereas the scope of a proper name should always be taken to include it. Even if this were so, it would not demonstrate the non-equivalence of a proper name with a definite description in any very strong sense: it would simply show that they behaved differently with respect to ad hoc conventions employed by us for determining scope. But, in any case, we have already seen that Kripke's doctrine,

> thus understood, is false on both counts. Kripke himself gives several examples in which a definite description is taken as having a scope inclusive of the modal operator; while we have noted that there are, conversely, cases in which the scope of a proper name must be taken to exclude the modal operator, a fact which is implicitly admitted by Kripke in his comments on the example of the standard metre rod.

Notice that, even though Kripke's paradigm of a rigid designator is a definite description, Dummett nonetheless attributes to him the view that all definite descriptions are nonrigid. In his haste to reject Kripke's claim that names are rigid, Dummett appears to me not to have fully grasped what Kripke is getting at. I discuss Dummett's response to Kripke in more detail in my "Rigid Designation and Its Variants."

49 Every designator would trivially satisfy Kripke's test if the two occurrences of α were taken to have the same scope, be it wide or narrow. Hence, Dummett's view completely undermines this test. It cannot even be employed to argue that 'the inventor of bifocals' is nonrigid.

50 In both the PH- and K-languages, all variables that are bound from outside the scope of modal operators range only over the objects that exist in the actual world. Nevertheless, 'there are no possible-yet-not-actual objects' cannot be expressed in any of these languages unless first a further modal device--e.g., an actuality predicate or operator--is introduced that in effect permits one to refer to other possible worlds in contrast to the actual world. '$\Box$', '$\Diamond$', and '$=$' do not enable one to talk exclusively about worlds other than the actual one.

51 Given Dummett's criticisms of the explanatory use of possible world model theory (<u>cf.</u> <u>ibid.</u>, p. 284f), I would expect him to agree that one can use modal operators independently of the possible world metaphor.

52 This way of motivating the notion proved to be especially effective in my "Rigid Designation and Its Variants," where it is used to defend the definition of rigid designation given earlier--i.e., (2)--against some alternatives that have been proposed in the literature.

53 Burge, op. cit., p. 244.

54 This point is easy to see. Substitution of '$(\forall y)(Gy \equiv y=x)$' for '$(\forall y)(Fy \equiv y=x)$' preserves truth in all modal contexts so long as '$\Box(\forall x)[(\forall y)(Fy \equiv y=x) \equiv (\forall y)(Gy \equiv y=x)]$' is true. But the latter is clearly true if '$(\iota x)(Fx)$' and '$(\iota x)(Gx)$' denote the same object with respect to every accessible world in which it exists, and they denote no object with respect to any other accessible world.

55 So long as both occurrences of α have narrow scope with respect to '$\boxminus$', $\ulcorner \boxminus \alpha = \alpha \urcorner$ is valid regardless of whether α is even a designator. If, however, one or both occurrences of α has wide scope, then $\ulcorner \boxminus \alpha = \alpha \urcorner$ is valid only if α is a designator. Of course, the substitution of β for an occurrence of α in $\ulcorner \boxminus \alpha = \alpha \urcorner$ is comparatively uninteresting unless the occurrence of α in question has narrow scope.

56 The purely de dicto formula $\ulcorner \Box \alpha = \alpha \urcorner$, which is not true unless α is a strongly rigid designator, should not be confused with the purely de re formula $\ulcorner \Box \alpha = \alpha \urcorner$, which is true even if α is only a designator.

57 On our understanding of '$\boxminus$', if $\phi(\xi)$ is 'Gx', the de dicto formula $\ulcorner \boxminus \phi((\iota x)(Fx)) \urcorner$ is equivalent to '$\Box[(\exists ! x)(Fx) \supset (\exists x)((\forall y)(Fy \equiv y=x) \; \& \; Gx)]$'. One might instead drop the uniqueness requirement from the antecedent, so that the equivalent formula would read '$\Box[(\exists x)(Fx) \supset \; .....]$'. On this understanding of '$\boxminus$', $\ulcorner \boxminus \alpha = \beta \urcorner$ would no longer follow from $\ulcorner \alpha = \beta \urcorner$ unless both α and β are rigid designators. Semi-rigidity would no longer suffice because the de dicto formula $\ulcorner \boxminus \alpha = \alpha \urcorner$ would not be true if α is a semi-rigid designator. Such an alternative construal of '$\boxminus$', while no more arbitrary than ours, is nonetheless too arbitrary to provide a natural way of motivating the notion of rigid, as opposed to semi-rigid, designation.

58 Identity statements do yield a characteristic or rigid designators that semi-rigid designators interestingly lack. If '$(\iota x)(Fx)$' and '$(\iota x)(Gx)$' are co-referring rigid designators, then '$\Box(\forall x)(Fx \equiv Gx)$' is true. But it need not be true if either or both of them are semi-rigid. This difference is nonetheless a flimsy intuitive basis for motivating rigid, as opposed to semi-rigid, designation. As the text indicates, the pair of notions together can be naturally motivated without blatantly invoking essentialist considerations. I suspect, however, that essentialist considerations like those used in our definitions of the two notions are indispensable for motivating the distinction between them.

59 Co-referring, scope neutral designators can be everywhere substituted for one another *salva veritate* unless some of their wide scope occurrences are not referentially transparent. If there is such an exception, it is clearly idiosyncratic.

60 The main difference remaining between strongly rigid designators and standard constants is that the latter can denote, if not any existing object, at least any object that can be picked out by a definite description, whereas the former can denote only necessarily existing objects. Of course, this difference would disappear if every existing object were to exist necessarily. Perhaps this is why the early quantified modal logics, like Barcan's, required every object to exist necessarily; the alternative was a quantified modal logic with no exact counterparts of standard constants.

61 W. V. Quine, "Intensions Revisited," *Midwest Studies in Philosophy, Volume II: Studies in the Philosophy of Language*, ed. Peter A. French, Theodore E. Uehling, Jr., and Howard K. Wettstein, (Morris: University of Minnesota, 1977), pp. 5-11.

62 *Ibid.*, p. 7.

63 *Ibid.*

64 *Ibid.*, p. 8.

65 *Ibid.*

66 *Ibid.*

67 *Cf.* Alonzo Church, *Introduction to Mathematical Logic*, vol. 1, (Princeton: Princeton U. Press, 1956), p. 10ff, where this view of constants is explicit. Church remarks (*ibid.*, no. 27),

> It is indeed possible, as we shall see later by particular examples, to construct languages of so restricted a vocabulary as to contain no constants, but only variables and forms. But it would seem that the most natural way to arrive at the meaning of forms which occur in these languages is by contemplating languages which are extensions of them and which do contain constants--or else, what is nearly the same thing, by allowing a temporary change in the meaning of the variables ("fixing the values of the variables") so that they become constants.

68 The phrase "purely *de dicto*" is crucial to this claim. For suppose 'c' is a fixed-valued free variable that denotes the same object as the rigid designator '$(\iota x)(Fx)$'. Then '$\Box(\exists x)(x=c \,\&\, Gx)$' would have the same truth-value as '$\Box(\exists x)[(\forall y)(Fy \equiv y=x) \,\&\, Gx]$'. But '$\Box(\exists x)(x=c \,\&\, Gx)$' would not be purely *de dicto* since 'c' has referential position in it, *ex hypothesi*. Indeed, as Kripke remarks ("Identity and Necessity," p. 139, n. 4), '$\Box(\exists x)(x=c \,\&\, Gx)$' would not be the necessitation of '$(\exists x)(x=c \,\&\, Gx)$'.

69 Quine, *op. cit.*, p. 7.

70 *Ibid.*, p. 8.

71 Contrast this with standard nonmodal languages that have no constants, in which denoting definite descriptions are like fixed-valued free variables.

72 *Ibid.*, p. 6.

73 If all formulas of the form of (74) are valid, then so too are all formulas of the form of both (i) and (ii):

(i) $\vartheta Gc \equiv (\exists x)(x=c \,\&\, \vartheta Gx)$

(ii) $Gc \equiv (\exists x)(x=c \,\&\, Gx)$

From the latter we conclude, by virtue of the substitutability of strictly equivalent formulas, that all formulas of the form of (iii) are valid:

(iii) $\nu Gc \equiv \nu(\exists x)(x=c \;\&\; Gx)$

The validity of all formulas of the form of (73) then follows from (i) and (iii).

74 For universal instantiation we here assume that domains are never empty. The adjustment needed to allow for empty domains is obvious.

75 See, for example, the discussion and use of "Henkin constants" in Jane Bridge, *Beginning Model Theory*, (Oxford: Oxford U. Press, 1977), p. 68ff.

76 See the discussion of existential instantiation and universal generalization in W. V. O. Quine, *Methods of Logic*, second edition, (New York: Holt, Rinehart, and Winston, 1959), pp. 160-167.

77 Richmond H. Thomason, "Some Completeness Results for Modal Predicate Calculi," in *Philosophical Problems in Logic*, ed. K. Lambert, (Dordrecht: D. Reidel, 1970), pp. 56-76. See note 9, p. 76, in particular.

78 David K. Lewis, "Counterpart Theory and Quantified Modal Logic," *The Journal of Philosophy*, LXV, 5 (March 7, 1968), pp. 113-126. By the way, the condition Lewis gives for designators to be scope neutral in his counterpart version of quantified modal logic (p. 121f) turns out to be strong rigidity (in the manner of (64)) if the counterpart relation is taken to be identity.

79 Universal instantiation could be revised to allow for domains in which there are no necessarily existing objects, in much the way it can be revised in standard logic to allow for empty domains. This parallelism, however, should not lead anyone to think that domains with no necessarily existing objects are the modal analogue of empty domains. The analogy holds only for constants, and then when the constants in the modal case are required to be strongly rigid designators.

80 Saul Kripke, "Identity and Necessity," p. 139, n. 4.

81 Even though the constants of Q3 are not fixed-valued free variables, Thomason's completeness proof for Q3 shows that such constants would suffice as witnesses in the case of the K-languages (op. cit.). Completeness proofs along the lines of Thomason's, but using such constants as witnesses, can be found in Hughes Leblanc, Truth-Value Semantics, (Amsterdam: North Holland Publishing Co., 1976), chapters 8 and 9. Not surprisingly, Leblanc emphasizes the free-logic character of these constants. More recently, Kit Fine has published an elegant Henkin-style completeness proof using such constants as witnesses; cf. "Model Theory for Modal Logic, Part I - The De Re/De Dicto Distinction," Journal of Philosophical Logic, 7 (1978), pp. 125-156.

82 Some simplification might be gained here were we to decide to handle '=' in the PH-languages exactly as it is handled in the K-languages. Up to this point the precise handling of 'x=y' when the objects assigned to the variables do not exist has not mattered. But if '=' were handled as in the K-languages, and '①=②' were the only nonstandard atomic predicate, then the logic of fixed-valued free variables in the PH-languages would be a little more straightforward.

83 In keeping with the interpretation of '□' in the PH-family, 'valid' would probably best mean "never false." But questions of proper formalization remain to be answered here.

84 One way to make the construction more manageable is to restrict occurrences of constants to '=' contexts and adopt a K-language treatment of '①=②'. This would avoid the loss of a classically two-valued logic for the closed formulas. But however much this might facilitate the construction needed in Henkin-style completeness proofs, it would clearly compromise the program of introducing constants as fixed-valued free variables. Rather, the constants would be a special restricted sort of fixed-valued free variable, namely one confined to '=' contexts. The subject of completeness proofs for the PH-family is explored in detail in Christie and Smith, op. cit.

85 W. V. O. Quine, "Three Grades of Modal Involvement," The Ways of Paradox and Other Essays, (Cambridge: Harvard, 1976), p. 176.

86 The issue is whether any quantificational language employing only such operators as '□' and '◇' is adequate for representing the full range of modal and essentialist talk. If there is one, then as Wiggins has illustrated (op. cit.), techniques of the λ-calculus will enable us to construct another adequate language with, for example, '□' and '◇' as closed formula operators and 'Nec' and 'Pos' as predicate modifiers.

87 Quine, "Intensions Revisited," p. 7.

88 Drew Christie has explored this suggestion. Cf. Christie and Smith, op. cit.

89 Cf. Quine, Word and Object, pp. 176-180.

90 Quine's remarks in "Intensions Revisited," p. 7, are instructive on this point.

91 Stephen Cole Kleene, Introduction to Metamathematics, (Princeton: Van Nostrand, 1962), p. 334ff.