MATHEMATICAL ABILITY, INABILITY AND DISABILITY

By

MICHAEL HUGH NELSON

L.R.C.P., [Edin.]; L.R.C.S. [Edin.]; L.R.C.P. and S. [Glasg.], 1959
F.R.C.P., Canada, 1970
M.S., Harvard University, 1971
M.R.C. Psych., United Kingdom, 1972

SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF

SCIENCE

At The

MASSACHUSETTS INSTITUTE OF

TECHNOLOGY

June, 1977

Signature of Author . . . .
Alfred P. Sloan School of Management, May 6, 1977

Certified by . . . .
Thesis Supervisor

Accepted by . . . .
Chairman, Departmental Committee on Graduate Students

MATHEMATICAL ABILITY, INABILITY AND DISABILITY

By

Michael Hugh Nelson

Submitted to the Alfred P. Sloan School of Management on May 6, 1977, in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

The importance of mathematics to modern management is delineated. Definitions of mathematical ability are presented. Developmental theories, especially with regard to mathematical ability and cognition are described. The question of possible biological determinants of differential achievement in mathematical performance between the sexes are discussed. The evidence is reviewed concerning the causation of high verbal-quantitative discrepancies in individuals. Language factors in mathematics teaching and learning are considered. A review is made of the possible roles of anxiety, attitudes and personality traits in mathematical ability and inability. The socio-cultural factors affecting women's performance in mathematics are surveyed. The contribution of the inter-play of cognitive style, teacher's attitudes and skills, and various teaching modes are considered in the light of their possible impact in generating difficulties in mathematics learning. Mathematics remedial clinics are described. A summary of current findings and suggestions for further research are presented. An hypothesis for the causation of mathematical inability is offered. Finally, some of the implications of these findings for management are briefly discussed.

Thesis Supervisor: Edgar H. Schein
Title: Professor of Management

ACKNOWLEDGEMENT

In the area of mathematics and biology the author would like to acknowledge his debt to the work of Julia Sherman and Laurie J. Harris. Their writings provided for him sign-posts through the extensive and confusing literature on this subject, and readers interested in reading further are strongly recommended to these original papers.

For guidance in the area of social factors affecting the mathematical performance of women the author would like to acknowledge the work of Lynn H. Fox, and of Elizabeth Fennema.

For advice and generous time, the author would like to thank Ms. Lorelei Brush, formerly associated with the Wellesley Mathematics Remedial Teaching Program; and Sister Viterbo McCarthy of Regis College, Wellesley, Mass.

Finally, the author would like to thank Dr. Alan Natopoff and Mr. Jack Reichson for their kindness, patience and skill in re-introducing him to mathematics and helping him to begin to overcome a near life-long disability.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

Mathematical ability, its nature, its nurture, its incidence - is of prime interest to modern societies where science occupies a key position. There are few areas where mathematics do not impinge at some stage. Individuals in these society who have no mathematical aptitude or ability are seriously limited in the exercise of their skills and talents - a loss both to themselves and to the community.

Mathematics is thus an essential element in the education of the citizen and in the development of the community as a whole. It is not surprising therefore that the literature on the subject is extensive. The subject is of interest to educators, psychologists and neuroscientists and has been approached in a variety of ways. This paper does not attempt a total review of the field - it does attempt however to touch on certain aspects relating to the nature of mathematical ability and mathematical disability.

From the viewpoint of industry and management there are a number of more specific concerns - the gulf between the mathematically and scientifically trained technologist and the non-mathematical "intuitive" manager; the application of science to management itself as opposed to the things that are being managed; the recruitment into industry of minority groups, including women, who for reasons that will be touched on, have shown deficiencies in mathematical abilities - the list can be extended. In addition the nature of modern commerce and industry demands, increasingly, a highly trained work force so that the quality and skills of entrants as well as their more specific educability becomes a management issue of the first importance.

CHAPTER II

DEFINITION OF MATHEMATICAL ABILITY

Lovell[1] defined mathematics laconically as "the study of structures or of systematic patterns of relationship." Mathematical ability therefore is the ability to manipulate these structures and patterns.

Werdelin[2] defines mathematical ability as "the ability to understand the nature of mathematical [and similar] problems, symbols, methods, and proofs; to learn them, to retain them in the memory, and to reproduce them; to combine them with other problems, symbols, methods and proofs, and to use them when solving mathematical [and similar] tasks."

---

[1] K. Lovell, Intellectual Growth & Understanding Mathematics, Eric Information Analyses Center for Science Education, February 1971, ED049069.

[2] I. Werdelin, The Mathematical Ability: Experimental and Factorial Studies, Lund, Sweden, C.W.K. Gleerup 1958.

Piaget[1] defines mathematical ability as "the ability to understand classes and relations" explicitly differentiating them from computational abilities.

Krutetskij[2] from his observations of school children, defined the components of mathematical ability as

1. an aptitude for the formalized perception of mathematical material, for swiftly grasping the formal structure of the problem;

2. an aptitude for logical reasoning in terms of numerical and spatial relationships and for the symbolism of figures and signs;

3. the ability to generalize mathematical units, relationships and operations quickly and broadly;

---

[1] Piaget's Theory of Intellectual Development, H. Ginsburg and S. Opper, Prentice-Hall, Inc., 1969.

[2] V. A. Krutetskij, The Psychology of Mathematical Aptitudes of School Children, Educational Publishing House, Moscow, March 1969, Eric ED060227.

4. the ability to take short cuts in mathematical reasoning and to think via generalized structures;

5. the possession of flexibility of abstract thought processes in mathematical activity;

6. the striving for clarity, simplicity and economy in solutions;

7. the ability to re-orient thought processes quickly and easily;

8. mathematics memory [a synoptic memory for mathematical relations, standard features, schemes of reasoning and of mathematical proof, and methods of solving problems and the principles of approaching them];

9. a "general synthetic component" - defined as the mathematical orientation of the brain. This is a distinctive organization of mind that finds expression in an effort to reduce

> the phenomena of the world to mathematical forms and to perceive them on the plane of mathematical and logical categories.

At the same time Krutetskij postulates that several components commonly regarded as necessary for mathematical aptitude are in fact neutral; among these he included: speed of thought process, calculating ability, memory for numbers and formulae, a gift for spatial representation, the ability to visualize abstract mathematical relationships and functional dependencies.

Krutetskij's view is that the extent to which each of these components is present determines the type of mathematical mind which develops - a point which emphasizes that mathematical thinking and ability is not an homogeneous entity and that there are several different types of mathematical minds [Poincare[1]].

Other definitions of mathematical ability abound in the

---

[1] H. Poincaré, "Mathematical Creation" in B. Ghiselin [ed.] The Creative Process, New American Library, New York, 1952.

literature but generally range along two dimensions. One tendency is to match abilities with different kinds of mathematical manipulations - geometry, algebra, calculus, etc.; he other is to define abilities according to theoretical processes.

Factor Analysis

It is a basic assumption of the statistical method of factor analysis that performance in any field such as the cognitive depends neither on one undifferentiated ability nor upon a completely chaotic conglomeration of separate abilities. Thus, as Dale[1] says, it is assumed that cognitive ability consists of a number of different factors, traits, faculties or powers, each of which is elicited by a variety of different tests or problems. Faced with a large set of correlations the task of explaining all the complex inter-relations is almost impossible. The first objective therefore of factor analysis is simplification, and the second is to find a set of abilities which will be fewer in number and more fundamental in nature than the original tests. Dale went on:

---

[1] W. Dale, "Factor Analysis," Psychometric Monographs, 1940.

> ...it is not necessary to make any assumption regarding the fundamental nature of the factors or what produced them. Each factor may be unitary as it may consist of a large number of separate causes which act together in a coherent and unitary manner. The factors may be produced by genetic differences or may be due to training. All that is assumed is that a cause or group of causes, however produced, acts as a functional or operational unit. In such an analysis it is obvious that every subject is assumed to possess every factor. The fact that this assumption may not be justified is not a serious handicap to factorial methods. The factor scores may indicate a great deal of the factor, or very little of it.[1]

Thurstone[2] pointed out that it should not be assumed that there is anything final about primary factors, that no one knows how many primary mental abilities there are, but for every new factor added it must be shown factorially to be distinct from factors already found to be a functional unity. A factor therefore may be elicited by many tests and is certainly further divisible - however, it behaves as a functional unity that is strongly present in some tests and almost completely absent in others.

While the factor theory was developed out of a dissatisfaction

---

[1] W. Dale, "Factor Analysis," Psychometric Monographs, No. 3, 1940.

[2] L. L. Thurstone, "Primary Mental Abilities," Psychometric Monographs, No. 1, 1938.

with the general intelligence [g] theory of Spearmen, neither Thurstone - the first major proponent of the method - nor following workers have considered special abilities to be inconsistent with the presence of a general intelligence. Thurstone[1] in his factor analytic studies of the primary mental abilities identified [1] the verbal-comprehension factor V; [2] the word fluency factor W; [3] the space factor S; [4] the number factor N; [5] the memorizing factor M; [6] the inductive or reasoning factor R; and [7] the perceptual speed factor P. Thurstone described the number factor N as being "that factor involved in the doing of simple arithmetical tasks [but not to be found in a test simply because it contains numbers]." Arithmetical reasoning tests with statement problems were also found to involve the number factor to some degree as well as other factors such as the inductive and the verbal. He pointed out that quantitative thinking can be non-numerical and in some subjects also may be non-spatial - a fact that other workers have confirmed,[2] and which later workers have documented in their studies of the different

---

[1] Ibid.

[2] J. E. Murray, "An Analysis of Geometric Ability," Journal of Educational Psychology, Vol. 40, 1949.

mathematical abilities between the sexes.[1] A direct quote from Thurstone touches issues which have yet to be resolved and which will be addressed again later in this review. He says

> ...the insistence of the number factor makes it almost certain that it represents a unique ability, but one is puzzled about its psychological or genetic character in view of the fact that calculation is more naturally thought of as a cultural rather than a biological factor. There seems to be some evidence for the genetic interpretation of number facility as an inherited trait. Occasionally this ability is found to be extremely conspicuous even at an early age, and it seems then to be more or less independent of other abilities. Some number freaks are well-endowed in other abilities, but some are otherwise mediocre. The fact that occasional feeble minded individuals possess some degree of number ability indicates further that it constitutes a more or less independent mental ability. Of interest for this problem is the frequent occurrence of considerable mental ability in the verbal factors combined with what seems to be a blind spot for numerical and logical relations. Literary people are not infrequently of this type. In studying this problem we should keep in mind the possibility that "number" as such may not adequately describe this factor psychologically or genetically, that the factor N may be more basic and general than number, and that the number tests constitute good examples of it.[2]

---

[1] P. S. Very, "Differential Factor Structures in Mathematical Ability," Genetic Psychology Monographs, 1957.

[2] Thurstone, "Primary Mental Abilities."

Early on then, one of the principal workers in this field recognized [a] the question of the relationship of spatial ability to the demonstration of mathematical ability; [b] the possibility of an inherited component in mathematical ability; [c] the fact that some persons with high verbal abilities have low logico-mathematical abilities; [d] that certain aspects of mathematical ability can be seen to be highly developed in individuals with otherwise low intelligence; and [e] mathematical ability can appear extremely early in life.

In his analysis of the number factor itself Thurstone also described the tests that he considered the factor contributed to; they were [1] number code, [2] multiplication, [3] subtraction, [4] addition, [5] division, [6] tabular completion, [7] numerical judgement, and [8] arithmetical reasoning. In addition, the tests of tabular completion, numerical judgement and arithmetical reasoning all required verbal and logical factors. Later workers continued the further analysis of the number factor itself in particular, Very. Very's[1] methodology was to conduct factor analytic studies on adults - rather than on school children - and in addition, to compare males and females in relation to this particular

---

[1] Very, "Differential Factor Structures in Mathematical Ability."

variable. Very pointed out that research had indicated that certain abilities had been found to have no place in a definition of mathematical ability - namely computational facility,[1] and spatial ability,[2] but that one major area had consistently been found to have a significant relationship with mathematical proficiency - namely reasoning abilities. He pointed out that there are a number of these abilities, that some required reasoning with numbers, while others were more abstract and required the ability to find and evaluate relationships. Thus mathematical ability he considered a construct composed of a number of separate but interrelated factors, each factor not yet being clearly isolated but including those abilities essential to quantitative thinking, and, in addition to this, in a more abstract sense, the ability to discover, manipulate and evaluate relationships.

Using a battery then of 30 different tests which he considered were applicable to mathematical ability, he tested a cohort of

---

[1] R. H. Coleman, "An Analysis of Certain Components of Mathematical Ability and an Attempt to Predict Mathematical Achievement in a Specific Situation," Doctoral Diss. Indiana University, Ann Arbor, Michigan, University Microfilms, 1956 No. 17945.

[2] Murray, "An Analysis of Geometric Ability."

approximately equal numbers of men and women of college age, and identified factors similar to those found in previous studies by other workers. However he found interesting and significant configural differences between the sexes. In general, he found that males appeared to have a greater number of mathematical abilities than did females, and these abilities were more specific and more clearly outlined than in females whose factoral make-up were more general in nature. In spite of the fact that females are significantly superior in verbal ability they were found to be generally inferior in those abilities more closely associated with mathematical processes. Very found therefore that in the area of mathematics ability [excluding the peripheral factors of numerical ability] females revealed fewer abilities than did males. He found four reasoning factors in females, while in males were found four reasoning factors, an intuitive factor and a digit flexibility factor. While the four reasoning factors in the females were difficult to define, the factors in males were sufficiently delineated to be called arithmetic, deductive, inductive and general, with much less overlap of tests on different factors. Spatial ability in females was found to be significantly constricted as compared to males who were found to have three such factors - each describing a slightly different ability. The two additional spatial factors that he identified he called spatial visualization I - an ability to visualize spatial relationships with the added necessity of inductive and deductive processes for superior performance; and spatial visualization II - the common element of which was spatial visualization in the abstract sense and which required the ability to judge distances, directions, heights and the like. Very concluded that although his study replicated the usual total group findings, and that further differentiation of the factorial configuration by sex had revealed a differing factorial structure, nevertheless he felt that his findings must remain suggestive and tentative only. This was since the study had not been replicated, that many of the tests were unstable or unproven in factorial structure, and that finally, there was no one approved method of extracting and interpreting the factors.

Other investigators have also analyzed mathematical ability into several different factors. Coleman[1] distinguished between two types of abilities, a computational or "mechanical arithmetic" ability, and a higher order mathematical ability involving "reflective intelligence" or deductive reasoning; Rusch[2] identified three subfactors to the number

---

[1] Coleman, "An Analysis of Certain Components of Mathematical Ability and an Attempt to Predict Mathematical Achievement in a Specific Situation."

[2] C. E. Rusch, "An Analysis of Arithmetic Achievement in Grades Four, Six and Eight," [Doctoral Diss. University of Wisconsin], Ann Arbor Michigan University Microfilm, 1957, No. 57,3568.

factor - abstraction, analysis, and application; Kline[1] found five factors - verbal comprehension, deductive reasoning, algebraic manipulative skill, number ability, and adaptability to a new task; Woolridge[2] labeled the factors he found as algebra achievement, mathematical apttitude, verbal aptitude, and number; and the five factors identified by Werdelin[3] he named general reasoning, deductive reasoning, and numerical, spatial and verbal comprehension.

Beginning then with Thurstone[4] and down to recent times Very,[5] work has continued on the attempted refinement of the constituents of mathematical ability. Other workers have, of course, contributed

---

[1]W. E. Kline, "A Synthesis of Two Factor Analyses of Intermediate Algebra," [Doctoral Diss. Princeton University], Ann Arbor, Michigan University Microfilm, 1960, No. 60,5012.

[2]E. T. Woolridge, "Factorial Study of Changes in Ability Patterns of Students in College Algebra," [Doctoral Diss. University of Nebraska], Ann Arbor Michigan University Microfilm, 1964, No. 11,945.

[3]I. Werdelin, "A Synthesis of Two Factor Analyses of Problem Solving in Mathematics," Didakometry, No. 8, 1966.

[4]Thurstone, "Primary Mental Abilities."

[5]Very, "Differential Factor Structures in Mathematical Ability."

extensively [French[1], Guilford,[2] Swineford,[3] and Taylor,[4]] but the general approach and the kinds of specific results that have been reached are exemplified by the description above.

The overlap of findings in these studies in noteworthy. Furthermore, it is notable that the "number factor" has been reported by several to be unrelated to other mathematical ability factors. As importantly a positive relationship between mathematical ability and general intelligence has been documented by many investigators, [Wrigley[5]

---

[1] J. W. French, The Description of Aptitude and Achievement Factors in Terms of Rotated Factors, University of Chicago Press, Chicago, Illinois, 1951.

[2] J. P. Guilford, "Structure of Intellect Model: Its Uses and Implications," Psychological Laboratory Report No. 24, University of Southern California, Los Angeles, California, 1960.

[3] F. Swineford, "A Number Factor," Journal of Educational Psychology, No. 40, 1949.

[4] C. Taylor, "Factors of Mathematical Aptitude," Diss. Abstract, 1973.

[5] J. Wrigley, "Factorial Nature of Ability in Elementary Mathematics," British Journal of Educational Psychology, 1958.

Pirr[1] and Shire[2]] although it is held by some that mathematical ability is not identical to general intellectual ability. This subject will be expanded upon below when the relationship of mathematical abilities to verbal abilities and general intelligence is considered.

---

[1] P. M. Pirr, "Intellectual Variables in Achievement in Modern Algebra," [Doctoral Diss., Catholic University of America,] Ann Arbor, Michigan University Microfilms, 1967, No. 67-183.

[2] A. Shire, "Relationship Between Arithmetic Achievement and Term Performance on the Revised Stanford-Binet Scale," Arithmetic Teacher, 1961.

CHAPTER III

DEVELOPMENTAL THEORY

The two mainstreams that currently exert the major influence on theories of intelligence are the psychometric and the Piagetian. Piaget's theory[1] divides intellectual development into four major periods: sensorimotor [birth to two years]; preoperational [two years to seven years]; concrete operational [seven years to eleven years]; and formal operational [eleven years and above]. Workers have emphasized that the age limits of each stage are only approximate and are subject to wide individual variations. Piaget himself shares this view of the flexibility of age norms and says that they are probably influenced by individual differences in physical and social environments. Central to the Piagetian theory is the idea that there is a regular order of succession of stages, regardless of the particular ages at which they appear. Each stage is characterized by a relatively stable structure that incorporates earlier

---

[1] Piaget's Theory of Intellectual Development, Ginsburg and S. Opper.

structures into a higher synthesis. Thus Piaget is concerned with the evaluation of general ways of knowing and considers them to be the products of interaction between the developing neural substrate of the brain and the environment.

For Piaget, thinking is an action which early on overtly, and later covertly, transforms one reality state into another, and he addressed specifically the development of mathematical ability.[1] He gives details of experiments and data relating to the ability of the child to differentiate and manipulate the ordinal and cardinal aspects of number, and to the beginnings of the ability to comprehend the basic additive and multiplicative properties of number. In the realm of logico-mathematical experience he considers that knowledge comes not from the objects themselves but from actions performed on objects. An implication of this scheme of successive stages is that when the absorption of a mathematical idea is beyond the level of logical thought which the child has reached, the idea is only partially learned, or learned with much difficulty and tenuously held.

---

[1] J. Piaget, and A. Szeminska, The Childs Conception of Number, Humanities Press, New York, 1952.

Brown,[1] using Piagetian tests studied the growth of children's understanding of the properties of the set of natural numbers and concluded that children pass through a number of stages in grasping each property and that understanding is reached at the following ages in children of average ability: closure at 7, identity at 7-8, commutativity at 8-9, associativity at 8-9 and distributivity at 10-11. However, his opinion was that performance could be advanced or retarded by up to four years compared with the norm.

By and large the stages in the development of cognitive structures proposed by Piaget have been validated by other workers, but with the emphasis that the age range for the elaboration of a particular structure is considerable even in children of comparable background.

Piagetian observation and theory also has something to say about the development of other conceptual capabilities necessary for the exercise of the logico-mathematical ability. According to Piagetian

---

[1] P. G. Brown, "Tests of Development in Childrens Understanding of the Laws of Natural Numbers," Math Education Thesis, University of Manchester, 1969.

theory therefore the growth of spatial concepts [Piaget and Inhelder[1]] and of geometrical concepts [Piaget, Inhelder and Szeminska[2]] arises at about the same time as logico-arithmetic operations but are distinguished from them in that they involve proximity and continuity. Spatial and geometrical concepts, Piaget says, are not derived from an apprehension of the physical properties of objects, but from actions performed on objects in thought. A conceptual space emerges in children from 7 years onwards. Lunzer[3] found that not until the age of formal operational thought at around age 14 were the majority of those he tested able to dissociate area and perimeter of square/rectangle, and realize that under certain circumstances area is conserved and not perimeter, while under other circumstances the reverse is true. The concept of volume in its three aspects, internal volume, volume as occupied space, and complementary or displaced volume, was found not to be well

---

[1] J. Piaget and B. Inhelder, The Childs Conception of Space, Routledge and Kegan Paul, London, 1956.

[2] J. Piaget, B. Inhelder, and A. Szeminska, The Childs Conception of Geometry, Basic Books, New York, 1960.

[3] E. A. Lunzer, "Formal Reasoning," Development in Human Learning, Vol. 2, E. A. Lunzer and J.F. Morris, eds. Staple Press, London, 1968.

understood even at the end of elementary school. Displacement volume is not well understood until the emergence of formal operational thought.

Although a child can use time words early, it is not until ages 8-9 that he can begin to carry out temporal operations such as putting events into a sequence according to their order of succession, mark off intervals of time between ordered points on a time scale, and choose some time interval as a unit and use it as a unit of measuring for some other time intervals.

From around 12 years in the brightest pupils and from ages 14-15 in average children the emergence of formal operational thought is seen. The chief characteristic of such thinking is the ability to invert reality and possibility, thereby leading to the ability to use a combinational system and to the appearance of hypothetico-deductive thought.[1] The child can now structure relations between relations, as in, say, the recognition of the equivalence of two ratios. However, metrical proportion in non-geometrical form depends eventually on the emergence of the growth of formal thought. Many pupils may not manage this kind of

---

[1] Lovell, Intellectual Growth and Understanding Mathematics.

operation until 15 years of age, and some apparently never. Allied to proportion is quantitative probability, which studies have shown [Shepler[1]] also requires the development of the capability for formal thought. Work has, in addition, been done on pupils understanding of mathematical function and mathematical proof. Reynolds'[2] conclusions were that Piaget's formulations regarding stages of thinking accounts for a good deal of the variability in the nature of pupil responses, for replies indicative of concrete operational thought appeared regularly while responses indicative of formal operational thought increased with age. The same comments, the necessity for the development of the capability of formal operational thought, applies to the concepts of point and limit - two further notions of fundamental importance in mathematics.

At this time, Piagetian theory continues to provide the most powerful theoretical and experimental tool in the exploration of cognitive development in general, and in the elucidation of specific abilities like the mathematical in particular.

---

[1] J. L. Shepler, "A Study of Parts of the Development of a Unit in Probability and Statistics for the Elementary School," Technical Report 105, Madison, Wisconsin Res. and Development Center for Cognitive Learning, 1969.

[2] J. Reynolds, "The Development of the Concept of Mathematical Proof in Grammer School Pupils," Ph.D. Thesis University of Nottingham, 1967.

This focus on mathematical ability, however, has its mirror image - mathematical inability. The marked inability in this specific area of some persons of average or high intelligence and high verbal ability is of considerable interest. This phenomenon was commented on by Thurstone very early on. One class of the relatively less able has been touched on and will be explored further in this review - namely women. Fierabends review of research on psychological problems in mathematical education[1] refers to nineteen investigations concerned with the relationship of general intelligence and special abilities to mathematics learning and concluded that

> ...studies in this area appear to agree on the importance of a general intellectual factor of ability in mathematics, but the investigation of specific abilities is not conclusive and the approach to this problem is perhaps not as meaningful as it could be..... The question remains unanswered as to whether all persons of sufficient general intelligence have equal potential for mathematics, or whether there may not exist some special abilities factors or conceptual approaches which are specific to the field of mathematics or perhaps to creativity in mathematics thinking...

---

[1] R. L. Fierabend, Review of Research on Psychological Problems in Mathematical Education, U.S. Office of Education, Cooperative Research Monograph No. 3, 1960.

A different way of asking this question would be to ask, in light of the Piagetian model, are there factors which can effectively interfere with the orderly progression of the unfolding of the various developmental cognitive stages, and, if so, what are they? Studies by Piaget and his co-workers support the observations of other workers that high mathematical ability is closely associated with high intelligence - but that both intelligence and mathematical ability [logico-mathematical ability] are highly correlated with abstract reasoning capability. A study by Keating[1] of mathematical precocious youth is worth detailing for the insights that it suggests on the relationships between the developmental stages of Piaget, intelligence, and mathematical ability, and for the suggestions that arise from it for future research. In Keatings study, four groups of boys were tested both by psychometric tests and by Piagetian tests. The groups were [1] 7th grade high-intelligence boys[7B], [2] 7th grade average-intelligence boys[7A], [3] 5th grade high intelligence boys[5B], and [4] 5th grade average-intelligence boys[5A]. The order of results on the psychometric tests was found to be 7B = 5B > 7A > 5A. On the

---

[1] D. P. Keating, "A Piagetian Approach to Intellectual Precocity," Intellectual Talent - Research and Development, John Hopkins University Press, Baltimore, 1976.

Piagetian tests the order was 7B > 5B > 7A > 5A. Thus the hypothesis that Keating put forward that high intelligence as measured by psychometric testing implies developmental precocity in reasoning was supported in this study. The presence of precocious reasoning was confirmed both by the psychometric tests and the Piagetian. Importantly, it was noted that the younger high intelligence [7B] boys were at a more advanced cognitive developmental level than the older average-intelligence [7A] boys, both on the psychometric tests of abstract reasoning ability and on the Piagetian tests evaluating cognitive development. The question of the contribution of nature and nurture is here at its most provocative. Since then, according to Piaget, cognitive development proceeds as an interaction between the organism and the environment, the brighter individual is at an advantage in moving through successive stages more quickly and is therefore involved in more varied and interesting ineractions with the environment. Keating emphasized in the study the differences between precocity in moving from stage to stage, and precocity within each stage noting that he thoughtprecocity in the former case to be less pronounced than in the latter, and that the latter - precocity within the stage - was likely to be more dependent on environmental stimulation. Thus, although precocity through stages is present, it is not as marked as precocity within stages. It may be at this point that hereditary factors bear more heavily. It need hardly be pointed out that if development can be enhanced by appropriate or enriched environments, it can also be retarded by the opposite. Thus if, as said earlier, a mathematical idea is presented to a child that is beyond the level of logical thought which the child has reached, the idea is either partially learned, or learned with much difficulty. In addition, the possibility for emotional disturbances around mathematical learning is increased. This is a fruitful area of study and may suggest approaches to those of high or average intelligence and good verbal abilities, but low quantitative achievement.

CHAPTER IV

MATHEMATICS AND BIOLOGY

The causes of differential mathematical ability, in particular the differences in achievement between men and women, have been sought for by some in differential biological endowment. Such biological variables have included spatial ability, cerebral lateralization, hormonal differences and hereditary differences. Each of these is considered briefly below:

A. Spatial Ability

Males do better than females on a variety of spatial tests and factor analytic studies have more spatial elements emerging in the factoring of males' abilities as compared to females.[1] These tests of spatial ability include the differentiation of embedded figures; certain visual coding tasks; geometric problems; cube-painting and cube-cutting problems; visual and tactile maze learning; map-reading; left-right discrimination;

[1] Very, "Differential Factor Structures in Mathematical Ability."

and certain logical conservation tasks having visuo-spatial components.[1] Since mathematics involves such diverse activities as computation, mathematical problem solving, integration and topology, it is obvious that the quality and type of cognitive skills involved differs with the task-spatial skills being important to integration and topology for example, but not to computation. It has however been proposed that sex-related differences in space perception might partly account for sex discrepancies in mathematics performance.[2] In an investigation of this question, for both sexes, both vocabulary and spatial visualization were found to be moderately correlated with mathematics achievement in grades 6-12.[3] The correlation between spatial visualization and mathematics achievement was consistent

---

[1] L. J. Harris, "Interaction of Experimental and Neurological Factors in the Patterning of Human Abilities," Eric Document Reproduction Service [EDRS], ED119854.

[2] J. Sherman, "Problem of Sex Differences in Space Perception and Aspects of Intellectual Functioning," Psychological Review, 1967, p. 74.

[3] J. Sherman and E. Fennema, "Distribution of Spatial Visualization and Mathematical Problem Solving Scores. Submitted for publication, 1976.

from sub-sample to sub-sample of overall 2000 students. Others like Murray[1] believed that spatial visualization is not related to mathematics achievement, his point being that it cannot be assumed that geometry is based primarily on spatial skill. Murray's test of geometry appeared to be in fact quite verbal in character. Hyde, Geiringer and Yen[2] felt, on the contrary, that sex-related differences in mathematical problem solving could be accounted for by sex-related differences in spatial disposition. The Fennema and Sherman study[3] showed that while sex-related differences in mathematics achievement could be interpreted as being caused by sex-related differences in space perception, socio-cultural factors appeared to be a more weighty explanation for the different achievement of the sexes in mathematics [see also below "Women and Mathematics"]. Thus

---

[1] Murray, "An Analysis of Geometric Ability."

[2] J. S. Hyde, E. R. Geiringer and W. M. Yen, on the Empirical Relation Between Spatial Ability and Sex Differences in Other Aspects of Cognitive Performance, Multivariate Behavioral Research, 1975, p. 10.

[3] Sherman, Fennema, "Distribution of Spatial Utilization and Mathematical Problem Solving Scores."

in this study, in two high schools where differences in mathematics achievement were found, differences on the spatial task were found in only one school while differences favoring males were found in at least half the eight socio-cultural factors at both schools. Moreover, a sex-related difference on the spatial task was found at a third school, but was not accompanied by a difference in mathematics achievement. Some studies moreover have failed to find sex-related differences in space perception.[1] This suggests that differences in spatial abilities between the sexes are not immutable.

Considering the matter from another viewpoint, Garrett[2] proposed a developmental theory of intelligence based on the differentiation of ability with age, from a fairly unified and general ability to a loosely organized group of abilities or factors. This "Age Differentiation Hypothesis"

---

[1] E. E. Maccoby and C. N. Jacklin, Psychology of Sex Differences, Stanford University Press, Palo Alto, California, 1974.

[2] H. E. Garrett, "A Developmental Theory of Intelligence," American Psychologist, 1946.

states [a] that abilities emerge at particular points in maturation and become increasingly specific and refined with age; [b] that certain abilities present at one age level should not be present at some earlier age; and that [c] it is possible to demonstrate this process through factor analysis of performance. Dye and Very,[1] based on their factor analytic studies, concluded that sex differences in ability have generally indicated male superiority in reasoning and mathematics and female superiority in verbal ability and perceptual speed. In addition, they claim that age differentiation does in fact occur and that this differentiation process - from a general ability to a host of specific abilities - is greater in males and more specific. Age differentiation of ability then is characterized by an increase in the number of factors present at successive stages and by the concurrent increased clarity or specificity of such factors. In summary, they say that reasoning factors are more

---

[1] N. W. Dye and P.S. Very, "Growth Changes in Factorial Structure by Age and Sex," Genetic Psychology Monographs, 1968,

numerous and better defined in males than in females, and that the greater specificity of factors is particularly evident in the areas of inductive, arithmetic and general reasoning. An earlier paper by Very[1] showed, in addition, that females had less spatial factors than males.

Although then this evidence is convincing, no mechanism was put forward as an explanation for these findings other than an implicitly biological cause. It is, perhaps, equally plausible, or even likely in light of the clearly documented weighty cultural factors, to consider that a differential factor structure of this order, particularly in the light of the somewhat ambiguous evidence offered above, to be the result of markedly different environmental factors which have hindered differentiation. It would be interesting to compare factor analytic studies of competent male and female mature mathematicians.

---

[1] Very, "Differential Factor Structures in Mathematical Ability."

B. Cerebral Lateralization

An alternative set of hypotheses to account for the more developed spatial function in males as compared to females calls upon sex-related differences in brain lateralization. For normal, right handed persons, the two hemispheres of the brain are thought to specialize in somewhat different functions - the left hemisphere in verbal, analytic tasks and the right in spatial, gestalt tasks. Sex-differences in brain lateralization refers then to the degree of completeness of the establishment of left hemisphere dominance for verbal, analytic function and dominance of the right for spatial function. According to this neurological model, some brains are more specialized than others for spatial analysis and these further specialized brains are more frequently male than female. Concurrently, it is fairly well accepted that left cerebral dominance for verbal function is established earlier in females.[1] This conclusion is based

---

[1] Maccoby and Jacklin, Psychology of Sex Differences.

not only on data showing more precocious verbal development in females but also on tachistoscopic studies and studies of dichotic listening.[1] Such studies take advantage of the fact that material presented in one hemi-field of space goes initially to the opposite hemisphere.

Several varying hypotheses have been put forward to explain the differences in brain lateralization between the sexes. Buffery and Gray[2] hypothesized that because dominance of the left hemisphere for verbal function is attained earlier in girls, this leads to less bilateral representation of spatial function. They believe that bilateral spatial representation is beneficial to superior spatial functioning and hence that lack of this in females would account for their poorer spatial performance. Levy[3] hypothesized that since

---

[1] D. Kimura, "Functional Assymetry of the Brain in Dichotic Listening," Cortex, 1967, III.

[2] A.W.H. Buffery and J. A. Gray, "Sex Differences in the Development of Spatial and Linguistic Skills," Gender Differences, C. Ounstead and D.C. Taylor, eds., William & Wilkins Company, Baltimore, 1972.

[3] J. Levy, "Lateral Specialization of the Human Brain," The Biology of Behavior, J.A. Kiger, ed., Oregon Univ. Press, 1972.

females, like left handed males, are more likely to have verbal function located in both hemispheres of the brain, and since it is likely that maximal verbal and spatial function is attained when hemispheres are specialized, then females would be likely to have poorer spatial function. Thus because females are supposed to be more bilateral in verbal function, the spatial function of the right hemisphere is interfered with. Harshman and Remington[1] suggest that while females start out as more lateralized in language than males, this is only because females mature faster. Males eventually surpass females and as adults are more lateralized for both verbal and spatial function. This hypothesis also assumes that lateralization is more favorable.

Sherman[2] hypothesized that the precocious verbal development of females caused them to develop a

---

[1] R. A. Harshman and R. Remington, "Sex, Language and the Brain: Part I: A Review of the Literature on Adult Sex Differences in Lateralization," UCLA Working Papers in Phonetics, 1976.

[2] Sherman, "Problem of Sex Differences in Space Perception and Aspects of Intellectual Functioning."

preference for a verbal approach, and consequently to neglect a non-verbal approach to problem solving. She thought that females appear not only to have a preference for a verbal approach, but a preference for a left hemisphere approach, that they tend to rely on the left hemisphere not only for verbal function but for part of spatial function. This she felt was reinforced by their verbal education and sex-role expectations.

Sherman[1] has critically evaluated each of these hypotheses. With regard to the Buffery and Gray theory while accepting that there is good evidence that left cerebral dominance for verbal function is established earlier in females, she concluded that they had presented no independent evidence to indicate that bilateral representation of spatial function is associated with superior function. Commenting on the Levy study, she concluded that present

---

[1] J. Sherman, "Effects of Biological Factors on Sex-Related Differences in Mathematics Achievement," Womens Research Institute of Wisconsin, Inc., National Institute of Education, Project Contract No. 400-70-0113, 1976.

data does not support a conclusion that females have more bilateral representation than males, while accepting as plausible the hypothesis that cerebral hemispheric specialization may lead to maximal functioning. On the Harshman and Remington study, she reiterates that there is no convincing evidence that females are more verbally bilateral than males, but in addition comments that while there are several converging lines of evidence indicating that for females more than males the left hemisphere is more likely to be involved in performing spatial tasks, this is not necessarily the same as saying that females are more spatially bilateral, nor that one can infer that the brains of normal members of the two sexes are organized differently. Sherman's view then, is that the evidence for greater bilaterality of verbal functioning in adult females is lacking but that the evidence does support the position that for human females the left cerebral hemisphere is more involved in spatial functioning than is the case for males and that males rely more on their right hemisphere for spatial function than do females. In addition, she feels that there is some support for the opinion that for females the left hemisphere tends in general to be the preferred. This preference may have a biological basis, or on the contrary may be a function of educational and cultural factors. Kagan[1] offers a similar idea. He says that since anatomical and psychological systems mature earlier in the girl than in the boy, perhaps the normal dominance relation of left over right hemisphere becomes established earlier in the girl. If language functions of the left temporal cortex then elaborate earlier in the female, experience is more likely to be transformed into linguistic structures early in development at the expense of other categories of representation. Kagan goes on to say that the equipotentiality of the hemispheres in young boys should lead to a more even cerebral development of mental abilities, and spatial skills - which are non-linguistic and preferentially elaborated in the right hemisphere - should

---

[1] J. Kagan, Change and Continuity in Infancy, John Wiley & Sons, Inc., New York, 1971.

develop to higher levels in boys than in girls.

A more basic question is why does lateralization proceed as it does in nearly all brains - to one hemisphere for language and to the other for visuo-spatial relations - even though this does not appear to be equally so for males and females? Based on work with brain-damaged war veterans Semmes[1] suggests that this difference - language in the left hemisphere and spatial abilities in the right - may be a function of the fact that speech requires fine and very precisely replicable movements and an exact degree of control that can only be organized via a focal representation of elementary functions characteristic of the left cerebral hemisphere. By contrast, diffuse organization of the right hemisphere would be advantageous for spatial abilities. Thus compared with speech functions that may depend on a

---

[1] J. Semmes, "Hemispheric Localization: A Possible Clue to Mechanisms," Neuropsychologia, 1968.

high degree of convergence of like elements, spatial functions might depend instead on the convergence of unlike elements - visual, kinaesthetic, vestibular and others. In addition, other workers have suggested that the left hemisphere operates more by analyzing stimulus information sequentially, abstracting out the relevent details to which it attaches verbal labels, while the right seems more concerned with overall stimulus configuration, and organizing and processing information in terms of gestalts or wholes. [Levy-Agresti & Sperry[1] and Zangwill[2].] Levy-Agresti and Sperry make a further proposal which has some considerable interest for this inquiry into differences of mathematical ability, namely that "the left hemisphere... would be inadequate for the more rapid complex synthesis achieved by the right hemisphere..." and that there may

---

[1] J. Levy-Agresti and R. Sperry, "Differential Perceptual Capacities in Major and Minor Hemispheres," Proceedings of National Academy of Sciences, 1968.

[2] O. Zangwill, Cerebral Dominance and Its Relation to Psychological Functioning, Charles C. Thomas, Springfield, Illinois, 1960.

be "...a basic incompatibility of language functions on the one hand and synthetic perceptual functions on the other." Does this observation or hypothesis have any bearing on the nature of significant verbal quantitative differences in the average or high intelligence male who has severe mathematical disability?

C. Hormonal Theories

Some workers have proposed an hormonal basis for cognitive differences between the sexes. Thus it is held that a critical role is played by sex steroid hormones. For example, it has been argued that proficiency in spatial tasks is linked to the presence of testosterone - thus males who are insensitive to testosterone - the so-called testicular feminization syndrome - show a deficit in spatial skill.[1] Bock and Kolakowski[2] also examined such subjects - XY

---

[1] J. Money, A. A. Erhardt and D. W. Masica, "Fetal Feminization Induced by Androgen Insensitivity in the Testicular Feminizing Syndrome," John Hopkins Medical Journal, 1968.

[2] R. D. Bock and D. Kolakowski, "Further Evidence of Sex-Linked Major Gene Influence on Human Spatial Visualizing Ability," American Journal of Human Genetics, 1973.

males unresponsive to androgens and suggested further that a minimum androgen level, specifically testosterone is required for spatial ability. However, analysis of this study shows that although the subjects did have a lower performance IQ [testing spatial ability] than verbal, it was still well above normal. Rosenfield[1] suggested that the capacity of the normal female to express the spatial trait depends upon the production of ovarian testosterone above some threshold level. Harris[2] proposed that female hormones set into action the "spatial" and "verbal" blueprints for organization of the cerebral hemispheres that originate in embryonic or fetal neurohormonal events. This work, however, has been criticised and Sherman[3] documented many methodological flaws, although there is evidence that in lower animals during a species

---

[1] R. L. Rosenfield, "Plasma Testosterone binding-globulin and Indices of the Concentration of Unbound Plasma-androgens in Normal and Hirsute Subjects," Journal of Clinical Encrocrinology,1971.

[2] Harris, "Interaction of Experiential and Neurological Factors in the Patterning of Human Abilities."

[3] Sherman, "Effects of Biological Factors on Sex-Related Differences in Mathematics Achievement."

specific critical period very early in development the hormones do differentially organize the brain for male or female behavior.

Normally in humans some fetal androgenic substance is responsible for the sexual differentiation to maleness. Some authors have reported therefore on "natural experiments" in which fetuses have been exposed to hormones at what were considered "critical" times. Dalton[1] found that female offspring of mothers who had received injections of progesterone during pregnancy scored significantly higher than a group of controls on general aptitude tests. Erhardt and Money[2] reported that fetally androgenized girls had unusually high IQ's. Yalom, Green and Fisk[3] reported that prenatal exposure to

---

[1] K. Dalton, "Ante-natal Progesterone and Intelligence," British Journal of Psychiatry, 1968.

[2] A.A. Erhardt and J. Money, "Progestin Induced Hermaphroditism," Journal of Sex Research, 1967.

[3] I. D. Yalom, R. Green and N. Fisk, "Prenatal Exposure to Female Hormones," Archives of General Psychiatry, 1973.

exogeneously administered estrogen and small amounts of progestins was associated with a lower performance on a test of spatial ability. This study also had methodological flaws, inasmuch as its subjects were children of severely diabetic mothers and as such notoriously subject to brain injury and birth complications.

Much of the purported evidence concerning sex hormones and intellectual functioning is confusing and in fact little is known as to how sex hormones actually do influence nervous system activity. Furthermore, even if relationships between the two are established it is still not known whether the relationship is causal. Broverman, Klaiber, Kobayaski, and Vogel[1], for example, conceptualized cognitive functioning to be the result of an interplay between two competing systems - the adrenergic and the cholinergic and hypothesized that sex differences are reflections of differences in relationships between adrenergic activating

---

[1] D. M. Broverman, E. L. Klaiber, Y. Kobayashi and W. Vogel, "Roles of Activation and Inhibition in Sex Differences in Cognitive Abilities," Psychological Review, 1968.

and cholinergic inhibitory neural processes which in turn are sensitive to the sex hormones, androgens and estrogens. On the face of it the theory has a good deal of internal inconsistency and has been severly criticized by Singer and Montgomery[1] and also by Sherman.[2] After characterizing the Broverman hypothesis as "containing factual error, conceptual distortions" and "...as having very little empirical support," Sherman makes an assessment of the whole subject by saying that on reviewing the evidence there is very little that can be safely concluded about sex hormones and intellectual functioning at this time. Harris[3] is rather more sanguine, and after his review of the evidence, says that it would seem that spatial ability requires at least some minimum level of androgen, and that the evidence for human beings suggests a relationship

---

[1] G. Singer and R. Montgomery, "Comment on Role of Activation and Inhibition in Sex Differences in Cognitive Abilities," Psychological Review, 1968.

[2] Sherman, "Effects of Biological Factors on Sex-related Differences in Mathematics Achievement."

[3] Harris, "Interaction of Experiential and Neurological Factors in the Patterning of Human Abilities."

between sex steroid level and spatial skill, but one that is highly complex and different for males and females.

D. Heredity

Burt[1] examined the correlations between various sibling combinations on mathematical ability. In descending order the correlations were highest in identical twins reared together, moving to identical twins reared apart, non-identical twins reared together, siblings reared together, siblings reared apart, and, finally, unrelated children reared together. Noticeable was the fact not only of the high correlation between the abilities of identical twins reared apart - which speaks strongly to an hereditary component in mathematical ability - but also of the comparatively significant correlation [even though only approximately half of that of the identical twins reared apart] of the unrelated children reared together - which speaks to not insignificant

---

[1]C. Burt, "The Inheritance of Mental Ability," *American Psychologist,* 1958,

environmental factors. Stafford[1] cautions on estimates of heritability and pointed out that they are particularly susceptible to mis - and over-interpretation when based upon kinship relations other than twins. He emphasized that they are sensitive not only to the interaction effects with the environment and the reliability of the measurements used, but also to mating patterns and to phenotypic frequency. He administered tests of quantitative reasoning to 300 pairs of twins [50 male monozygous [MZ], 69 female monozygous [MZ], 25 male dizygous [DZ], 56 female dizygous [DZ] and 110 hetero-dizygous [male-female] pairs ages 12-18. He found higher correlations in mathematical abilities among MZ females and MZ males than among DZ females and DZ males. Stafford thus concluded than an underlying hereditary component for proficiency in quantitative reasoning seems to be supported by his data as it was by the data from other twin

---

[1] R. E. Stafford, "Hereditary and Environmental Components of Quantitative Reasoning," Review of Educational Research, 1972.

studies.[1]

Stafford however went further in his study and put forward the theory that having a proficiency in quantitative reasoning was due to the presence of a sex-linked recessive gene. He also proposed the same mechanism for spatial visualization.[2] This theory - the x-linked or sex-linked theory of the inheritance of cognitive skills - holds that high potential is carried as a recessive characteristic on the x-chromosone. However Sherman[3] pointed out that if the x-linked hypothesis were true one would expect improved performance on spatial ability in females compared to males, not inferior performance, since the recessive gene would have a full chance to express itself. Furthermore, if the hypothesis were true, at least one third of high ability

---

[1] S. G. Vandenberg,"The Nature and Nurture of Intelligence," in D.C. Glass [ed.] Genetics: Biology and Behavior, Rockefeller University Press, New York, 1968.

[2] R. E. Stafford, "Sex Differences in Spatial Visualization an Evidence of Sex-linked Inheritance," Perceptual and Motor Skills, 1961.

[3] J. Sherman, "Problem of Sex Differences in Space Perception and Aspects of Intellectual Functioning," Psychological Review, 1967.

mathematicians should be female. That this is very far from the truth points to the presence of other significant factors.

In an attempt to clarify the issue further, Sherman and Fennema[1] examined the Mental Arithmetic Problems score distribution of 161 males and 152 females and found that the gene frequency estimates for mathematical problem solving were grossly out of line with theoretical predictions. Yen[2] studied sibling correlations and within sex score distributions for four spatial tests in a population of 2508 high school students and found that while some of the results supported the x-linked hypothesis, most did not. Similarly, Bouchard[3] obtained spatial visualization scores on members of 200 families and, while finding large sex differences favoring males in both parents and children, he found that familial

---

[1] Sherman and Fennema, Distribution of Spatial Utilization and Mathematical Problem Solving Scores.

[2] W. M. Yen, "Sex-Linked Major Gene Influence on Selected Types of Spatial Performance," Behavior Genetics, 1975.

[3] T. Bouchard, "Sex Differences in Human Spatial Ability: Not an X-Linked Recessive Effect," in press, 1976.

correlations did not order themselves as predicted from a theory that human spatial visualization is under the control of an x-linked recessive gene.

In sum, then, the evidence does not seem to support a sex-linked mechanism of inheritance for either mathematical problem solving or spatial ability, although there is some support for an hereditary component to proficiency in quantitative reasoning.

CHAPTER V

VERBAL QUANTITATIVE [VQ] DIFFERENCES

Before examining the findings concerning persons with large discrepancies between their verbal and quantitative abilities, it is important to point out that a positive relationship between verbal ability and achievement in mathematics has been documented widely, and that although many students with high verbal ability do poorly in mathematics, competent mathematicians have high verbal abilities.[1] In studies employing a variety of ability measures, tests of mathematical ability not only have significant correlations with several other tests in the battery, such as spatial relationships, numerical reasoning, and logical reasoning, but also with verbal concepts and total language factors.[2] In fact, a common theme of these multi-variable correlational studies is the

---

[1] L.R. Aiken, "Verbal Factors and Mathematics Learning," Journal for Research in Math Education, November 1971.

[2] R.D. Muscio, "Factors Related to Quantitative Understanding in the Sixth Grade," Arithmetic Teacher, 1962.

importance of verbal ability, and this has been documented for a wide range of school grades. Cottrell[1] reported a correlation of .86 between arithmetic achievement and reading achievement in third graders; Erickson[2] calculated a similar correlation in sixth graders; and Ivanoff, Deware and Praem[3] in ninth graders. Aiken[4] made an important point when he indicated that the correlations between scores on reading tests and mathematics tests are not due simply to the overlap which these tests share with general intelligence. He said that despite the hypothesis that the correlations between verbal and mathematical ability were due to the common contribution of general intelligence and that although general intelligence could seem to account for a large portion of the variance common to both verbal and mathematical ability, a significant portion is

---

[1]R. S. Cottrell, "A Study of Selected Language Factors Associated with Arithmetic Achievement of 3rd Grade Students," Doctoral Diss. Syracuse University. Ann Arbor, Mich., University Microfilms, 1968, No. 68-5505.

[2]L. H. Erickson, "Certain Ability Factors and Their Effect on Arithmetic Achievement," Arithmetic Teacher, 1958.

[3]J. M. Ivanoff, E. T. DeWare and O. Praem, "Use of Discriminant Analysis for Selecting Students for Ninth Grade Algebra or General Mathematics," Mathematics Teacher, 1965.

[4]L. R. Aiken, "Intellective Variables and Mathematics Achievement," Journal of School Psychology, 1971.

left unexplained. Muscio[1] maintained that high achievement in mathematics depends upon both high general intelligence and high verbal ability.

Turning then to differential cognitive abilities we can begin to survey what has been said about the development of persons with high verbal ability, normal or high intelligence, and low mathematical abilities. As McCarthy[2] observes, among the scattered studies that have either incidentally or deliberately inquired about the pertinent correlates of verbal-quantitative discrepancies, no single theory has prevailed in the literature, and that several longitudinal studies have noted the relative stability over time of the more marked intra-individual V-Q differences. Tyler[3] compared the Primary Mental Abilities scores of children in grade one and retested them at grade four; Meyer[4] carried out the same studies

---

[1] Muscio, "Factors Related to Quantitative Understanding in the Sixth Grade."

[2] S. V. McCarthy, "Differential V-Q Ability: 20 Years Later," Review of Educational Research, 1975.

[3] L. E. Tyler, Individual Differences: Abilities and Motivational Directions, Appleton-Century-Crofts, 1974.

[4] W. J. Meyer, "The Stability of Patterns of Primary Mental Abilities Among Jr. and High School Students," Education and Psychology Measurements, 1960.

in children in grade eight and when they were in grade twelve; Nichols[1] retested college students. All these studies reaffirmed the stability of the patterns and suggest that a theory to account for this must include not only hereditary and biological givens, but also aptitudinal inclinations toward differentiation shaped and re-inforced by familial, social and societal systems. Maccoby[2] puts the same concept into different words:

> ...a child's intellectual abilities, such as the ability to read quickly and with understanding, the ability to solve arithmetical problems or to reason abstractly, are not solely a function of innate endowment, but also a function of emotional and personality development.....However the evidence for relationships between intellect and personality is scattered and incomplete, and theoretical formulations which would enable us to predict in detail are poorly developed.

There is a consistency in the low but significant relations that differential verbal-quantitative ability shows with the motivational or affective aspects of personality. This is the answer to the question as to

---

[1] R. E. Nichols, "Effects of Various College Characteristics on Student Aptitude Scores," Journal of Educational Psychology, 1964.

[2] E. E. Maccoby and L. Rau Differential Cognitive Abilities, Co-operative Research Program of Office of Education. Dept. Health, Education and Welfare, Stanford, 1962.

whether groups defined on the basis of differential V-Q ability are as equally well defined on the basis of preferences, values, attitudes and drives. Cleveland and Bosworth[1] obtained a number of measures - intelligence, achievement, personality, aptitude and socioeconomic level - on 282 sixth graders. For both sexes and for all socioeconomic levels, higher achievers in mathematics, compared to low achievers, made scores more indicative of a psychologically healty personality. Similar results were reported by Fischer[2], that students who had disabilities in mathematics tended to show more maladjustments, rebelliousness and conflicts than did high achievers in mathematics. Sanders, et al[3] examined male college students with discrepancies in their ability scores. They found that high quantitative men, by comparison with the high verbal group, were characterized by less autonomy, more affiliation, less dominance,

---

[1] G. A. Cleveland and D. L. Bosworth, "A Study of Certain Psychological and Sociological Characteristics as Related to Arithmetic Achievement," Arithmetic Teacher, 1967.

[2] E. C. Fischer, "A Study of the Relationship Between Selected Personality Factors and Special Disability in Mathematics," Doctoral Diss., University of Alabama. Ann Arbor, Michigan, University Microfilms, 1968, No. 68-1039.

[3] E. Sanders, R. B. Mefferd and W. H. Brown, "Verbal-Quantitative Ability and Certain Personality and Metabolic Characteristics of Male College Students," Educ. Psychol. Measurements, 1960.

more endurance, and less rejection of authority. The verbal men tended to be sedentary, the quantitative athletic. An earlier study by Cattell[1] during World War II showed that, although the correlations were small, there was some likelihood that a man who does well on the verbal section of the graduate record examination will be shy, unsociable, sensitive and moody; and that a man who tests high on the quantitative tests will be opposite. In general Cattell saw the high-verbal man being somewhat more emotionally unstable than the high quantitative man. Altus[2] identified 43 items from the MMPI that effectively discriminated between verbal and quantitative subjects among 200 college women. He thought that it was possible to describe the quantitative women as conventional people, orthodox in belief, exhibiting anxious and dysphoric tendencies, admitting to a more frequent attraction to their own sex, and displaying a clearcut antipathy toward the verbal. Using the same tests Altus[3] later examined

---

[1] R. B. Cattell, "Personality Traits Associated with Abilities, II: With Verbal and Mathematical Abilities," Journal Educ. Psychol. 1945.

[2] W. D. Altus, "Personality Conetates of Q-L Variability as the Ace," Journal of Consulting Psychology, 1952.

[3] W. D. Altus, "Q-L Variability, MMPI Responses of College Males," Journal of Consulting Psychology, 1958.

200 college males and investigated the extent to which V-Q discrepancies would be reflected in certain personality variables and found that there were 22 statements that differentiated between V and Q subjects at significant levels. These indicated that V men were more at ease with the opposite sex and more socially dominant, and that V men linked pursuits for which Q men expressed antipathy; i.e. cooking, reading, poetry and dramatics. Dana, Dahlke and Mueller[1] compared groups with extreme V and Q scores and control groups and found that V subjects, regardless of sex, evidenced more projection, repression and distortion in their thinking, and had a more subjective orientation than Q subjects.

Other investigators have taken a somewhat different approach to the comparison of persons with either high verbal or high quantitative abilities, and have attempted to discover the effects of childrearing and early socialization. Maccoby and Rau[2] compared fifth grade boys and girls who were high on Q with those who were high on V. They found that

---

[1] R. H. Dana, A. E. Dahlke and D. Mueller, "Intra-Individual Verbal-Numerical Discrepancies and Personality," American Psychologist, 1959.

[2] Maccoby and Rau, Differential Cognitive Abilities.

high Q boys and girls were well-liked and sociable, highly attentive and tolerant of ambiguity, while by contrast the high V children of both sexes were intolerant of ambiguity, highly distractible, easily disrupted and high in aggression. On the basis of these findings they put forward the theory that different cognitive operations demanded different kinds of strength in ego organization, and that mathematical ability required a greater capacity to maintain tension and to shut out distracting stimuli than verbal ability required. The corollary of this hypothesis is that mathematical ability is the most abstract form of reasoning, and hence the latest acquired in mans evolution; hence its expression should not only be closely related to the capacity for inhibition for motor discharge and impulse expression - especially anxiety - in general, but that high anxiety would act as a disorganizing force and would get in the way of mathematical learning. Nelson and Maccoby[1] examined the hypothesis that conflict-ridden parent-child relations produced the V profile. They examined two samples of Stanford University freshmen who had either high discrepancy scores on the verbal aptitude tests or the mathematical aptitude tests. The V

[1] E. Nelson and E. Maccoby "The Relationship Between Social Development and Differential Aptitudes on the Scholastic Aptitude Test," Merrill-Palmer Quarterly, 1966.

subjects of both sexes reported more frequent disruption of family status. Carlsmith[1] reported similar results for subjects who had experienced early separation from their father, and those who had not. Q subjects suffered no separation. V subjects did. Ferguson and Maccoby[2] assessed the interpersonal correlates of differential V-Q ability and found that the V pattern was associated with a continued and somewhat discordant dependency on adults and lowered social interaction with peers, whereas the Q pattern was related to assertiveness, interpersonal acceptance, and a more appropriate level of dependency.

---

[1] L. Carlsmith, "Effect of Early Father Absence on Scholastic Aptitude," Harvard Educational Review, 1964.

[2] L. R. Ferguson and E. Maccoby, "Interpersonal Correlates of Differential Abilities," Child Development, 1966.

CHAPTER VI

LANGUAGE FACTORS, INTELLIGENCE TESTS AND MATHEMATICS

As mentioned at the beginning of the previous chapter, although many persons with a high verbal ability may have a low quantitative ability, nevertheless the converse does not hold true. To attain competence in mathematics a high level of verbal ability is necessary. Thus it is generally recognized that not only do linguistic abilities affect performance in mathematics, but that morevover mathematics itself is a specialized language. Reading ability is positively related to scores on tests of arithmetic problem solving.[1] The general finding of studies examining the correlation between general reading ability, arithmetic achievement and specific reading skills is that there is a moderate positive correlation between reading and mathematics achievement and that

---

[1] L. R. Aiken, "Verbal Factors and Mathematics: A Review of Research," Journal for Research in Mathematical Education, November 1971.

this applies to the whole range of school grades.[1] A cross-sectional study of the relationships of various aspects of linguistic ability to performance on a variety of mathematical tasks shows that reading vocabulary, reading comprehension, mechanics of English and spelling have higher correlations with arithmetic reasoning than with arithmetic fundamentals, although the correlations of these four linguistic tests with arithmetic fundamentals are also sizeable.[2] Martin found likewise that the partial correlation between reading comprehension and problem solving abilities was higher than the correlation between computational ability and problem solving ability.

In addition to the positive correlation between scores on mathematics and scores on linguistic abilities, there is also a positive correlation between scores on mathematics and scores for general intelligence.

---

[1] J. T. Johnson, "On The Nature of Problem Solving in Arithmetic," Journal of Educational Research, 1949.

[2] Aiken, "Verbal Factors and Mathematics: A Review of Research."

[3] M. D. Martin, "Reading Comprehension, Abstract Verbal Reasoning and Computation on Factors in Arithmetic Problem Solving," Doctoral Diss. State University of Iowa, Ann Arbor Michigan University Microfilms, 1964, No. 64-3395.

Wrigley[1] in fact concluded that high general intelligence is the first requirement for success in mathematics, and that the positive correlation between measures of verbal and mathematical abilities can be explained by the joint relationship of these two variables to general intelligence. Alternatively, it can be argued that differences in reading ability may serve to explain the positive correlation between scores on mathematics tests and general intelligence tests, and that although general intelligence can account for a substantial portion of the variability shared by verbal and mathematical abilities, a significant degree of overlap between the two remains unexplained. Muscio[2] thus maintained that arithmetic achievement depends on both general intelligence and verbal ability. Anastasi[3] in her comments on the study of mathematically and scientifically precocious youth makes the comment that the high level of mathematical development identified in the study required a certain minimum level of

---

[1] J. Wrigley, "Factorial Nature of Ability in Elementary Mathematics," British Journal of Educational Psychology, 1958.

[2] Muscio, "Factors Related to Quantititative Understanding in the Sixth Grade."

[3] N. Anastasi, "Commentary on the Precocity Project," Mathematical Talent: Discovery, Description and Development, J.C. Stanley, D. P. Keating and L. Fox [eds.], Johns Hopkins University Press, 1974.

of superiority in verbal ability also. Thus she noted that verbal ability is also required for ordinary formal instruction in all subjects including mathematics, and that because verbal language - both oral and written - is the principal means of communication and cultural transmission, verbal aptitude tests have generally proved to be the best predictors of performance in most academic courses. She therefore hypothesizes that this is one of the reasons why verbal aptitude plays such a large part in tests of general intelligence. Mathematical talent in the absence of normal superior verbal development presents the picture of the typical idiot Savant. Such a person may perform spectacular feats of mathematical computation, but within a narrowly limited context and with little reference either to practical or theoretical implications. These mathematical talents may be accompanied by a Stamford-Binet IQ of 50 or 60 - a score reflecting largely deficient verbal development. Thus Rife and Snyder[1] reported on a mentally retarded man who

> ....could give the square root of any number running into four figures in an average of four seconds, and the cube root of any number running into six figures in six seconds. When he was asked how many grains of corn

---

[1] D. C. Rife, and H. Snyder, "Studies in Human Inheritance VI," __Human Biology__, 1931.

> there would be in any one of sixty-four boxes with one in the first, two in the second, etc., he gave the figures correctly for the forty-eighth in six seconds.

Anastasi went on to point out, however, that although verbal ability was undoubtedly a prerequisite for mathematical ability - as opposed to computational ability - nevertheless one of the findings in the study raised some interesting questions. She pointed out that although the subjects scored extraordinarily high on their mathematical aptitude tests - to such a degree as to be considered highly precocious - nevertheless their scores on the verbal aptitude tests were nearly half a standard deviation lower. Keating[1] explained this finding by attributing the difference to regression towards the mean, but did not give an estimate of expected regression. He also suggested that mathematics is a "closed system" and can be developed without any necessary external reference. Much of verbal learning on the other hand concerns information about external reality, and some amount of social interaction seems essential. Thus word tests and, in some cases, tests of verbal reasoning sample a good deal of general information. Verbal abilities therefore cover a broader and more

---

[1] J. C. Stanley, D. P. Keating, and L. H. Fox [eds.], Mathematical Talent - Discovery, Description and Development, Johns Hopkins University Press, 1974.

varied territory than do mathematical abilities and their development requires more external contacts. Mathematics learning is a step-by-step procedure and hindrance at one level can prevent further development.

Among investigators of the relationship between mathematical abilities and specific linguistic abilities particular attention has been paid to vocabulary and syntax. Treacy[1] found that a knowledge of vocabulary is important in solving mathematics problems, and Linville[2] concluded that both synctatic structure and vocabulary level, with vocabulary perhaps being most crucial, are important variables in solving verbal arithmetic problems. It was also found that pupils of higher general ability and/or reading ability made significantly higher scores on arithmetic problems than children of lower ability. In fact, training in mathematics vocabulary as a specific strategy has been shown to contribute to reading, arithmetic computation and arithmetic reasoning [Lyda &

---

[1] J. P. Treacy, "The Relationship of Reading Skills to the Ability to Solve Arithmetic Problems," Journal of Educational Research, 1944.

[2] W. J. Linville, "The Effects of Syntax and Vocabulary Upon the Difficulty of Verbal Arithmetic Problems with Fourth Grade Students," [Doctoral Diss. Indiana University], Ann Arbor, Mich. University Microfilms, 1970, No. 70-7957.

Duncan[1], and VanderLinde[2]].

Following on this observation have been specific efforts to teach reading instruction in mathematics[3] with some improved achievement in those exposed to these methods compared to those not. Methods have been devised to break down verbal mathematical problems in a structured and formalized way [Taschow[4] and Dahmus[5]] and persons who have been taught to follow these methods have indeed improved their mathematical performance. However, these results are also open to other interpretation inasmuch as they could not only be considered as tutoring in analysis and logic, but might also be helping persons anxious about mathematics to

---

[1] W. J. Lyda and F. H. Duncan, "Quantitative Vocabulary and Problem Solving," Arithmetic Teacher, 1967.

[2] L. F. VanderLinde, "Does the Study of Quantitative Vocabulary Improve Problem Solving," Elementary School Journal, 1964.

[3] N. W. Earp, "Observations of Teaching Reading in Mathematics," Journal of Reading, 1970.

[4] H. G. Taschow, "Reading Improvement in Mathematics," Reading Improvement, 1969.

[5] M. E. Dahmus, "How to Teach Verbal Problems," School Science and Mathematics, 1970.

approach the subject in such a way as to aid their mastery and thus reduce anxiety.

The concept that mathematics is itself a formalized language arises naturally from the idea of teaching reading in mathematics as an aid to mastery of the subject. Certainly it is most doubtful if mathematics educators commonly view the subject in this manner as judged by the pace and manner of teaching of mathematics to children. Much of mathematics is taught as if each new concept is self-explanatory once it has been demonstrated and mastery is implicitly viewed as being a product of understanding only - not familiarity, repetition, connection with qualifying factors, and incorporation. Emphasis on these aspects of learning - as in learning a language - would have considerable effect on changing teaching methods [Madden[1], Ausubel and Robinson[2]].

---

[1]R. Madden, "New Directions in the Measurement of Mathematical Ability," Arithmetic Teacher, 1966.

[2]D. P. Ausubel and F. C. Robinson, School Learning: An Introduction to Educational Psychology, New York: Holt, Rinehart and Winston, 1969.

CHAPTER VII

ANXIETY AND MATHEMATICS

In 1957 Dreger and Aiken[1] reported on a study to detect the presence of a syndrome of emotional reactions to arithmetic and mathematics. They called this syndrome "Number Anxiety." A current name is mathophobia. They hypothesized that [1] Number Anxiety exists as a factor separate from general anxiety; [2] Number Anxiety is not related to intelligence, although probably related negatively to high Q scores; and [3] persons with high number anxiety will tend to make lower scores in mathematics than others, even though the difference in intelligence between the two groups is not significantly different. All three hypotheses were substantially supported. Alpert, Stellwagon and Becker[2] constructed separate tests for facilitating anxiety and debilitating anxiety - anxiety,

---

[1] R. M. Dreger and L. R. Aiken, "The Identification of Number Anxiety in a College Population," Journal of Educational Psychology, 1957.

[2] R. Alpert, G. Stellwagon and D. Becker, "Psychological Factors in Mathematics Education," Newsletter 15, School Math Study Group, Stanford University, 1963.

that is, different in degree. They found that both test batteries were significantly correlated with achievement, but in opposite directions. The interaction between the level of text anxiety and the degree of difficulty of a mathematics test was documented by Jonnson[1]. He found a significant interaction between level of test anxiety and the difficulty of the test - highly anxious students doing the more difficult test doing most poorly.

The relationship of anxiety and mathematics learning has been explored by a variety of methods with instruments designed to measure anxiety in general or mathematical anxiety in particular. An example of the latter is the Mathematical Anxiety rating scale [Suinn, Edie, Nicoletti and Spinelli[2]] - a 98 item inventory of situations that might arouse mathematics anxiety. Callahan and Glennon[3] conclude their study by the statement that "anxiety and mathematics are related. In general

---

[1] H. A. Jonsson, "Interaction of Test Anxiety and Test Difficulty in Mathematics Problem Solving Performance," [Doctoral Diss: University of California, Berkely], Ann Arbor, Mich. University Microfilms, 1966, No. 65-13514.

[2] R. M. Suinn, C. A. Edie, J. Nicoletti and P. R. Spinelli, "The MARS, A Measure of Mathematic Anxiety," Journal of Clinical Psychology, 1972.

[3] L. G. Callahan and V. J. Glennon, Elementary School Mathematics: A Guide to Current Research, Assoc. for Supervision and Curriculum Development, 197 , Washington, D.C.

high anxiety is associated with lower achievement in mathematics." During grade levels 4-10 Crosswhite[1] reported anxieties about mathematics increasing in girls. Fennema[2] states that although confidence and anxiety have been defined as separate traits it appears that, in relation to mathematics, they are very similar. In her study she noted that a high rating on the confidence scale correlated highly with a low rating on the anxiety scale. With regard to the fact that there are sex-related differences in the confidence - anxiety dimension she noted that it appears reasonable to believe that lower confidence, or greater anxiety on the part of females is an important variable which helps explain sex-related differences in mathematics studying. In the Fennema study, at each grade level from 6-12 boys were significantly more confident in their abilities to do mathematics than girls. In addition, confidence in learning mathematics and achievement were more highly correlated than any other variable and achievement. Other studies have also reported on the

---

[1] F. J. Crosswhite, "Correlates of Attitudes Towards Mathematics," NLSNA Report No. 20 in Investigation in Mathematics Education, 1957.

[2] E. Fennema, "Influences of Selected Cognitive, Affective and Educational Variables on Sex-Related Differences in Mathematics Learning and Studying," Department of Curriculum and Instruction, University of Wisconsin, Madison, October 1976.

relationship of high anxiety and poor performance in mathematics. Lynn[1] from his investigations of children's progress in both primary and secondary schools, found that anxious children were penalized particularly in arithmetic, a finding supported by Biggs.[2] Unzer[3] was of the opinion that skill in mathematics demands "a certain elasticity of approach and flexibility of reasoning which are impaired by blockages due to anxiety."

The recognition of how widespread is the phenomenon of anxiety around the performance and studying of mathematics has become more general in recent years, particularly due to the fact that anxiety, unlike other differential factors, is seen to be amenable to remedy. In consequence mathematics clinics have been established in various parts of the country, and although it is yet too early to make specific statements about the results, the outlook is encouraging. In particular, efforts have been

---

[1] R. Lynn, "Temperamental Characteristics Related to Disparity of Attainment in Reading and Arithmetic," British Journal of Educational Psychology, 1957.

[2] I.B. Biggs, "The Teaching of Mathematics II - Attitudes to Arithmetic-Number Anxiety," Educational Review, 1959.

[3] E. A. Unzer, "Aggressive and Withdrawing Children in the Normal School, II - Disparity in Attainment," British Journal of Educational Psychology, 1960.

directed towards the elucidation and treatment of mathematics anxiety in women. This in itself is a reflection of two broad underlying social trends in American society - the first is the increasing reliance on technology and science, or rather scientific methods, in all fields of endeavor, the second is the concomitant changes in the roles of the sexes. Clearly the two are not unconnected. Less emphasis is being paid to minorities and to males who are mathematically unable although the problem with regard to minorities is being tackled in some small way. The programs for women do indeed stress minority women as one of their concerns, but as women, not as persons coming from socially disadvantaged groups. This paper will not attempt to evaluate cultural, linguistic and national factors that might go into the production of relative disabilities in mathematics. The literature is extensive. A good starting point for those interested is the report of a symposium sponsored by UNESCO in cooperation with the International Commission on Mathematical Instruction and the Center for Educational Instruction Overseas in 1974.[1]

Interestingly enough, the literature on adult male disability

---

[1] UNESCO, Interaction Between Linguistics and Mathematical Education, Final Report of the Symposium sponsored by UNESCO, CEDU and ICMI, Nairobi, Kenya, September 1974.

in mathematics is relatively sparse and certainly there seems to be a paucity of well defined studies. Studies on children are of course numerous and many have been mentioned in the course of this review.

With regard to the genesis and nature of math anxiety in women, which can be taken as a paradigm for mathematics anxiety in general - the comments and observations of Kogelman[1], a mathematician and psychiatric social worker, who has been investigating the dynamics and etiology of mathematics anxiety in women, are illuminating. Although the women interviewed were all high achievers in other areas, Kogelman found that in math-related activities they exhibited many of the symptoms which would usually be associated with a phobia or inhibition of ego-function. These symptoms were found to stem from conflicts over feminine identification and from abivalence about detail. Another suggested cause has been the observation that anxiety about success is increased when females enter fields where they must compete with men - particularly when female socialization practices have emphasized the "un-femininity" of assertiveness in general. Thus one study showed that of the high achievers

---

[1] S. Kogelman, "Dynamics and Aetiology of Math Anxiety in Women," [Masters Thesis, Smith College School of Social Work, Boston, 1975.]

in mathematics, by far the majority were rebellious about female role expectations and low achievers were conforming to those expectations.

Not all authorities agree that mathematics avoidance is necessarily the result of mathematics anxiety and that it is psychological in origin. Poor or unimaginative teaching is put forward as the cause. However, the scenario for the consequences of poor and unimaginative teaching most likely and most commonly, is a gradual falling off in mathematics performance despite increasing school and social pressures to do well in mathematics; an increasing emotional reaction to the inability in the face of, or continuing demand for, performance; lowered self-esteem based on perceived failure; and finally, a career-choice consciously selected to avoid mathematics and science-based subjects. This scenario applies to men, not only women. Sells[1] has pointed out the central place of mathematical inability as a critical filter for women in the job market-clearly a central concern for those wishing to expand the social roles of women. "t needs to be re-enforced that the same scenario applies equally to males. Fuller comment will be made below on math remedial clinics.

---

[1] L. Sells, "High School Mathematics as the Critical Filter in the Job Market," quoted in the bibliography of the Wellesley-Wesleyan Math Project 1976-77.

CHAPTER VIII

ATTITUDES, PERSONALITY & MATHEMATICS

It is a logical step to go on from a consideration of anxiety around mathematics to a consideration of attitudes. Although the two subjects are commonly discussed separately in the literature, there is clearly a dynamic interplay between the two, as well as with innate ability and socio-cultural factors. In light of the discussion about mathematics anxiety, it is not surprising that many studies have found that differences in both attitudes and achievement in mathematics are frequently found to favor boys over girls [Keeves[1] and Simpson[2]], and that greater interest and more favorable attitudes towards mathematics on the part of males have been found in other countries as well.[1] Aiken[3] points out that lack

---

[1] J. P. Keeves, "Differences Between the Sexes in Mathematics and Science Courses," International Review of Education, 1973.

[2] C. J. Simpson, "The Effect of Laboratory Instruction on the Achievement and Attitudes of Slow Learners in Mathematics," [Doctoral Diss., Lehigh University, 1973], Diss. Abstract Int. 1974, 6959A-6960A. University Microfilms No. 74-11357.

[3] L. R. Aiken, "Update on Attitudes and Other Effective Variables in Learning Mathematics," Review of Educational Research, Spring 1976, Vol. 46.

of success in mathematics leading towards unfavorable attitudes may be due in part to lack of innate ability, but that since there are disparities between countries in the magnitude of the attitude differences between boys and girls, biological explanations are insufficient. He emphasizes the important role of differing socio-cultural expectations and reinforcement schedules, complemented by same-sex modeling, that must be taken into account.

Studies mentioned earlier in this review noted the correlation of more disturbed personality development in children and adults with low quantitative ability as compared with those of competent or high quantitative ability [Cattell[1], Maccoby and Rau[2]]. Later studies have also reported that attitudes towards mathematics and achievement in the subject are related to a number of personality variables indicative of good adjustment [Naylor and Gaudry[3], Swafford[4]]. This finding, if corroborated

---

[1] Cattell, "Personality Traits Associated with Abilities, II: With Verbal and Mathematical Abilities."

[2] Maccoby and Rau, Differential Cognitive Abilities.

[3] F. D. Naylor and E. Gaudry, "The Relationship of Adjustment, Anxiety and Intelligence to Mathematics Performance," Journal of Educational Research, 1973.

[4] J. O. Swafford, " A Study of the Relationship Between Personality and Achievement in Mathematics," [Doct. Diss. University of Georgia, 1969] Diss. Abstr. Int. 1970, 30, 5353A.

further, is perhaps a significant finding taken in conjunction with other points noted in this review - namely the presence of anxiety around mathematics in the mathematically unable, the developmental stages of Piaget, and the fact that abstract reasoning - the hallmark of mathematics - is critically vulnerable to disorganization under severe degrees of arousal. Here is fruitful ground for a coherent hypothesis and remedial approaches.

Some investigators have made attempts to correlate personality or personality variables with mathematics ability. Cleveland and Bosworth[1] obtained a number of measures and concluded that for both sexes and all socio-economic levels, higher achievers in mathematics made scores more indicative of a psychologically healthy personality than did low achievers. Fischer[2] reported similarly, concluding that students who did less well in mathematics than was expected from their intelligence

---

[1] G. A. Cleveland and D. L. Bosworth, "A Study of Certain Psychological and Sociological Characteristics as Related to Arithmetic Achievement," Arithmetic Teacher, 1967.

[2] E. C. Fischer, "A Study of the Relationship Between Selected Personality Factors and Special Disability in Mathematics," [Doctoral Diss. University of Alabama] Ann Arbor, Mich. University Microfilms 1968, No. 68-1039.

scores tended to show more maladjustments than did high achievers. Trown[1] considered the introversion-extroversion variable in its effect on performance in mathematics and Lewis and Peng-Sim[2] found that extroverted boys and introverted girls perform higher in their own sex group. This is in line with the general finding that well-adjusted boys do well in mathematics, but rebellious and maladjusted girls - rebelling from sex stereotypes - also do well. This study also comments on the neuroticism-stability dimension quoting further studies equating higher abilities with stability. However, the authors also point out that the pattern is not uniform, that stability seems to be an asset up to the age of 16, but that at later stages a different picture emerges suggesting that for college students neuroticism may have a facilitating effect.

Even if personality assessment was a more rigorous science than it is, it is clear that situational and environmental factors play a

---

[1] E. A. Trown, "Some Evidence on the Interaction Between Teaching Strategy and Personality," British Journal of Educational Psychology, 1970.

[2] D. L. Lewis and K. Peng-Sim, "Personality and Performance in Elementary Mathematics with Special Reference to Item Type," British Journal of Education, 1973.

large part in forming personality quite apart from hereditary traits. Apart from broad classifications of personality variables such as those mentioned above - extroversion and introversion, neuroticism and stability - more carefully designed studies of the personalities of creative mathematicians, as well as studies of biographies and autobiographies - will probably point to as many differences as to similarities in the personalities of persons with high mathematical abilities.

CHAPTER IX

MATHEMATICS AND WOMEN

The basic fact concerning mathematics and the performance of mathematics by women in American society is that women do much less well in mathematics than men. Two broad categories of explanation have been put forward to account for these well-documented facts - the biological and the socio-cultural.

The biological arguments have been considered earlier in this paper; what follows therefore is a consideration of the socio-cultural factors.

The question as to whether there is an innate difference between the sexes as to the ability to think mathematically is not the same as to whether achievement to date has been the same in both sexes. There is much evidence to show for example that in American society women have achieved significantly less in the mathematics field than have men. This does not answer the question, however, as to whether there is an innate difference - biologically based. The weight of current evidence is, if anything, leaning towards the conclusion that there is no significant inborn difference between men and women, or at best it remains an open question.

The question of achievement is more easily documented. Thus for the period 1972-1975 the percentage of Ph.D.'s earned by women was below ten percent for the following fields: geography, astronomy, economics, mathematics, computer science, applied mathematics, geology, atmospheric science, physics, engineering, and operations research.[1] At every rung of the ladder leading to the Ph.D. in mathematics, the attrition rate has been higher for women than for men. Thus, during the past four decades, about 7 percent of the mathematics Ph.D.'s were earned by women,[2] although there now appears to be a slight upswing. Thus the figure for 1972-75 is 9.1% and for 1974-75, 10%. For all doctorate granting mathematics departments in the United States women

---

[1] J. L. McCarthy and D. Wolfle, "Doctorates Granted to Women and Minority Group Members," Science 189, 1975.

[2] V. H. Larney, "Female Mathematicians, Where Are You," American Mathematical Monthly, Vol. 80, 1973.

comprised 4.5% of the tenured faculty.[1] Out of 27 research departments in the years 1975-76, 3% of the faculty were women. At the most prestigious schools women represent only 1.6% of the full professors.[2]

At the entering stage of university life, Sells[3] found that, in a random sample of freshmen, 57% of the men had taken four full years of mathematics, compared with 8% of the girls. She pointed out that the four years mathematics sequence is required for admission to that mathematics course which in turn is required for majoring in every field at the university except the traditionally female. In addition, she points out that there is a strong and statistically significant relationship between having a one-year college mathematics requirement in the curriculum and having less than one-third of the degrées in the department earned by women.

Taking this line of inquiry further back in development and

---

[1] "American Mathematical Society 19th Annual AMS Survey," Notices of the American Mathematical Society, 22, 1975.

[2] J. Ernest, "Mathematics and Sex," American Mathematical Monthly, 1976.

[3] L. Sells, "Sex and Discipline Differences in Doctoral Attrition," Ph.D. thesis, 1975, University of California, Berkeley.

schooling, Fennema[1] has summarized studies related to preschool subjects. She pointed out that although one authoritative study[2] on a review of evidence had generally concluded that boys will achieve higher than girls on tests dealing with mathematical reasoning, and another[3] had concluded that there were no significant differences between the sexes in arithmetic achievement before the seventh grade, but that boys surpass girls after the seventh grade, nevertheless, in a review of nine studies, three reported no significant differences between the mathematical knowledge of three, four or five year old boys and girls, one study showed girls' performance significantly better, and in five others, involving a variety of measures, no significant differences were found. From the results of these studies Fennema thus concluded that it seemed reasonable to believe that there were no consistent differences in learning mathematics by boys and girls in the early years.

---

[1] E. Fennema, "Mathematics Learning and the Sexes," Journal for Research in Mathematics Education, 1974.

[2] V. J. Glennan and L. C. Callahan, A Guide to Current Research: Elementary School Mathematics, Association for Supervision and Curriculum Development, Washington, D.C. 1968.

[3] M. N. Suydam and C. A. Riedesel, Interpretative Study of Research and Development in Elementary School Mathematics, Vol. 1, Project No. 8-8586, 1969, Dept. Health, Education and Welfare.

However, the picture becomes more confused in consideration of the pre-adolescent and adolescent years. Reviewing twenty studies that concerned themselves with these age groups she said that the conclusion is inescapable that boys of these populations learned the mathematics measured by these tests better than did the girls. Thus, although the girls appeared to have done slightly better at younger ages, by the time puberty was established, boys were out-performing girls at all levels in tests of cognitive complexity. Hilton and Berglund[1] found no significant differences between males and females at grade five. However, when the same subjects were tested at grades seven, nine and eleven, significant differences favoring males appeared. However, other studies [Alexander[2] D'Augustine[3] and Sowder[4]] showed no such significant differences.

---

[1] T. L. Hilton and G. W. Berglund, Sex Differences in Mathematics Achievement - A Longitudinal Study, Princeton, N.J. Educational Testing Service, 1971.

[2] V. Alexander, "Sex Differences in Seventh Grade Problem Solving," School Science and Mathematics, 1962.

[3] C. H. D'Augustine, "Factors Relating to Achievement With Selected Topics in Geometry and Topology," Arithmetic Teacher, 1966.

[4] L. Sowder, "Performance on Some Discovery Tasks Grades 4-7," Journal for Research in Education, 1971.

When evidence concerning differential abilities of boys and girls of high school age are considered, some important sociological factors have to be taken into account, firstly that lower ability boys tend to drop out so that not only are samples of high school boys more homogeneous as far as ability is concerned, but also the samples are skewed; secondly, girls do not elect mathematics courses as often as boys. Backman[1] using standaridized achievement tests found significant differences in favor of boys; Easterday and Easterday[2] found significant differences in favor of girls, and Bhushan, Jeffryes and Wakamura[3] found no significant differences.

Beginning then around the onset of puberty there is a documented falling off in mathematics achievement and, more importantly perhaps, mathematics involvement on the part of females. While biological causes for lack of achievement in mathematics have been involved - ranging from

---

[1] H. E. Backman, "Patterns of Mental Abilities: Ethnic, Socio-Economic and Sex Differences," American Educ. Research Journal, 1972.

[2] K. Easterday and H. Easterday, "Ninth Grade Algebra, Programmed Instruction and Sex Differences: An Experiment," Mathematics Teacher, 1968.

[3] V. Bhushan, J. Jeffreys and I. Wakamura, "Large Group Instruction in Mathematics Under Flexible Scheduling," Mathematics Teacher, 1968.

spatial ability, to hormonal effects, to brain laterality, there is no question but that subtle and not so subtle psychological and environmental forces militate against girls and women staying in the science and mathematics stream. The perception of the usefulness of mathematics for future educational and career plans and the support or lack of support from significant others appear to be the major factors associated with women's decisions to elect or not to elect advanced courses in mathematics. These factors are in turn influenced by the sterotype of mathematics as a male domain. Other factors are attitudes toward mathematics, feelings of self-confidence, and values. Certain educational policies tend to reinforce sex-role sterotypes while some practices may promote greater course-taking and achievement.[1]

Perceptions of the relevance of mathematics to future careers begins early and several studies show that females elect not to take mathematics courses because they do not see them as relevant to their careers.[2]

---

[1] L. H. Fox, "The Effects of Sex Role Socialization on Mathematics Participation and Achievement," Paper prepared for Education and Work Group Career Awareness Division, National Int. of Education, U.S. Dept. HEW, Contract No. FN17-420-76-0114, December 1976.

[2] E. W. Haven, "Factors Associated with the Selection of Advanced Academic Mathematics by Girls in High School," [Diss. Abstr. Inter. 1971 32 1747A].

[Fennema & Sherman[1]]. Hilton and Berglund[2] concluded that sex differences in the perception of mathematics as useful resulted from the sex-typing of mathematics and careers. This sex difference in career choice is found early[3]. Later on women anticipate conflict between home-making and a marriage, on the one hand, and a career on the other. Ory and Helfrich[4] found that females who held non-traditional role stereotypes were more likely to opt for professional careers than those who held traditional view of sex-roles. In addition, these girls are likely to score higher on mathematical aptitude tests than less career oriented girls.[5] Furthermore, the evidence is strong that the absence of role models for women and the

---

[1] E. Fennema and J. Sherman, "Sex Related Differences in Mathematics Achievement, Spatial Visualization and Socio-Cultural Factors," Journal of Educational Research, in press.

[2] Hilton and Berglund, Sex Differences in Mathematics Achievement - A Longitudinal Study.

[3] W. R. Loofts, "Sex Differences in the Expression of Vocational Aspirations by Elementary School Children," Developmental Psychology, 1971, 5 [2] 366.

[4] J. C. Ory and L. M. Helfrich, "A Study of Individual Characteristics and Career Aspirations," Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, Calif. April, 1976.

[5] H. S. Astin, "Career Development of Girls During the High School Year," Journal of Counseling Psychology, 1968, 15 [6].

additional presence of negative stereotypes contribute significantly to the lack of interest in scientific careers. Levine[1] found that parents held lower educational aspirations for daughters than for sons, and Casserly[2] also found that parents had limited aspirations for daughters. There is evidence to show that support and encouragement from parents is crucial for girls in their decisions to elect or not to elect mathematics courses in high school.[3]

These stereotypes are not only reinforced by peers but it seems that stereotyping increases as the child progresses through school until, in adolescence, girls may perceive real pressures against achievement in mathematics. In addition, teachers appear to have different expectations for girls than for boys at all levels, including high school, [Ernest[4]

---

[1] M. Levine, "Identification of Reasons Why Qualified Women Do Not Pursue Mathematical Careers," Report to the National Science Foundation, August 1976.

[2] P. L. Casserly, "An Assessment of Factors Affecting Female Participation in Advanced Placement Programs in Mathematics, Chemistry and Physics," Report to the National Science Foundation, July 1975.

[3] Haven, "Factors Associated with the Selection of Advanced Academic Mathematics by Girls in High School."

[4] Ernest, "Mathematics and Sex."

Levine[1]] - a finding not surprising when it is considered that there is no reason to exclude teachers from cultural norms more or less than any other group.

Importantly, Ernest reported that 41 percent of a small, primarily female sample, of teachers believed boys did better in mathematics than girls, while no teacher felt girls did better than boys. A very negative attitude towards mathematics is often linked to bad experiences with a teacher, and this, of course, is not confined to girls. However, for girls it is likely to be the mathematically able who are discouraged, while for boys it is the mathematically less able. This is a reflection of the belief that mathematics is a male domain - a belief that is at the very core of the issue of sex-role socialization and mathematics achievement. This belief is reinforced in its turn by the availability of models, or to be more exact, the non-availability in the case of women. Mathematics teachers tend to be males and there is a marked tendency for more advanced mathematics courses to be taught by males.

Traditionally the physical sciences and mathematics have been male

---

[1] Levine, "Identification of Reasons Why Qualified Women Do Not Pursue Mathematical Careers."

provinces, and from an early age boys are expected to be interested in these subjects. While the concept of mathematics as unfeminine may not be as prevalent as it once was prejudice against girls entering the field still exists, or is seen to exist by girls entering the field. The mutually reinforcing and cumulative effect of these factors is quite clear. Gifted girls are often reluctant to accelerate their progress in mathematics because of fear of negative social consequences, primarily peer rejection. Fox[1] and Romer[2] have said that those who have a high fear of success were the least likely to develop their intellectual potential in situations requiring competition with men.

Taking all these factors into account, it is not surprising that self-confidence with respect to mathematics differs for males and females. Kaminski, et al[3] found sex differences on self-concept and confidence as

---

[1] L. H. Fox, "Facilitating the Development of Mathematical Talent in Young Women," [Diss. Abstract. Int. 1974 [b] 35. 3553B.] University Microfilm No. 74-29027.

[2] N. Romer, "The Motive to Avoid Success and Its Effect on Performance in School-Age Males and Females," Developmental Psychology, 1975, 11[b].

[3] D. M. Kaminski, et al, "Why Females Don't Like Mathematics: The Effect of Parental Expectations," Paper presented at the 1976 Sociological Association Meeting, New York, August 1976.

early as grade eight, Fennema and Sherman[1] documented the same in high school populations and Erlick and LeBold[2] in college students. Lack of confidence and low self-concept in performance of a task give rise to anxiety in that task. Mathematics anxiety in women is well-documented - itself a potent source of dysfunction. Levine[3] reported that guidance counselors notice that even when girls do well in mathematics they do not perceive of themselves as competent.

In conclusion, it can be said that the evidence that at least the greater part of the cause of women achieving less in mathematics as opposed to men is socio-cultural, rather than biological, seems to be convincing. The evidence is clear that subtle and not so subtle messages are given by parents, teachers, peers and counselors and ultimately society at large, that mathematics is a male domain. Women entering it,

---

[1] Fennema and Sherman, "Sex Related Differences in Mathematics Achievement, Spatial Visualization and Socio-Cultural Factors."

[2] A. C. Erlick and W. K. LeBold, "Factors Influencing the Science Career Plans of High School Students," Measurement and Research Center, Purdue University, Indiana, 1975.

[3] Levine, "Identification of Reasons Why Qualified Women Do Not Pursue Mathematical Careers."

at least until recently, have had to fight societal norms and to come to grips with the questioning and nonacceptance of their feminine sex-role by significant others. The evidence for these factors is unequivocal, the evidence for a significant biological cause is equivocal at best.

CHAPTER X

COGNITIVE STYLE, TEACHING AND TEACHING THEORIES

Cognitive style has been defined as that characteristic mode an individual uses in processing information.[1, 2] Davis and Klausmeier[3] describe the "analytic" and the "global" as two characteristic styles - the analytic are subjects who analyze and differentiate the components of a stimulus complex, and the global are those who fail to do this and respond to the stimulus as a whole. Based on these findings some writers have suggested that one style might be preferable to another in terms of

---

[1]R. D. Odom, C. W. McKintyre and G. S. Neale, "The Influence of Cognitive Style of Perceptual Learning," Child Development, 1971.

[2]J. Kagan, H. A. Moss, and I. E. Sigel, "Psychological Significance of Styles of Conceptualization," the Monograph of the Society for Research in Child Development, 1963, 28[86].

[3]J. K. Davis and H. J. Klausmeier, "Cognitive Style and Concept Identification as a Function of Complexity and Training Procedures," Journal of Educational Psychology, 1970.

performance on a variety of tasks and a study by Thorrell[1] did support the hypothesis that an analytic style is preferable to a global one in terms of learning performance. Thorell also did suggest that differences in cognitive style may be a significant factor in determining the most effective instruction programs. Hudson[2] has written extensively on "convergers" and "divergers" - a converger being defined as one who is substantially better at intelligence tests, at reasoning down to a single answer, and a diverger as one who is better at open-ended tasks, at managing ambiguity. The former prefer the scientific, the latter prefer the arts and humanities. As Hudson points out, the convergence/divergence dimension is a measure of bias, not of level, of ability, and in particular it was noted that convergers show a bias of ability towards questions expressed in numbers and patterns, rather than words. The bias among the divergers is the reverse.

Other studies in memory and learning showed differences between "visualizers" and "non-visualizers".[3] These, and other, "cognitive styles" are

---

[1] J. G. Thorell, "Individual Differences in Cognitive Style and the Guidance Variable in Instruction," presented at the Annual Meeting of the American Educational Research Association, 1974, EDRS ED106705.

[2] L. Hudson, Contrary Imaginations, Methuen & Co. London, 1966.

[3] M. F. Stuart, Neurophysiological Insights in Teaching, Palo Alto, California, Pacific Books, 1963.

clearly an important consideration for teaching to the degree that students learn more effectively when they are taught in their preferred mode - that modality in which he or she has the greater aptitude. Thus this notion assumes that for an individual having a particular pattern of abilities certain techniques of instruction are more effective than others.[1] Some studies have failed to find these "aptitude-treatment interaction effects" significant; others have been more positive. Behr[2] studied college-age students in an elementary mathematics course and found a correlation between semantic aptitude and an ability to learn arithmetic best by verbal instruction. There was also a tendency for diagrammatic instruction to aid those more who have a high visual ability. Kropp, Nelson and King[3] found that those with more deductive and inductive reasoning ability learned better from a deductive presentation of basic concepts.

---

[1] L. J. Cronbach, "How Can Instruction be Adapted to Individual Differences," Learning and Individual Differences, R. M. Gagre, ed., Bobbs-Merrill, Columbus, Ohio, 1967.

[2] M. J. Behr, "A Study of Interaction Between Structure of Intellect Factors and Two Methods of Presenting Concepts of Medium Seven Arithmetic," Doctoral Thesis, Tallahassee, Florida State University, Diss. Abstr. 28. 1693-A, 1967.

[3] R. P. Kropp, W. H. Nelsen and F. J. King, Identification and Definition of Subject Matter Content Variables Related to Human Aptitudes, Co-operative Research Project No. 2117, Florida State University, 1967.

Arising from this work there are clear implications for the relationship between methods of teaching and testing. Thus Duncan and Hartley[1] found that subjects who were tested in the same form in which they were taught, did significantly better than those who had to translate information from one mode to another; furthermore, efforts to train individuals in the mode which they are deficient in have been largely unsuccessful.[2]

A consistent theme of all the literature on the variety of approaches to the teaching of mathematics is the quality of the teacher. A creative teacher produces creative students. An interested teacher produces interested students. Poor teachers produce students who are turned away from the subject. Torrence and Parent[3] reported that the teacher's effectiveness

---

[1] C. Duncan and J. Hartley, "The Effect of Mode of Presentation and Recall on a Simple Learning Task," Programmed Learning & Educational Technology, 6, 1969.

[2] J. Goins, Visual Perceptual Abilities and Early Reading Progress, University of Chicago Press, No. 37, February 1958.

[3] E. P. Torrance and E. Parent, Characteristics of Mathematics Teachers That Affect Students Learning, Report No. CRP-1020, September 1966, U.S. Office of Education.

had a positive influence on student attitudes. Perkin[1] found there was a strong interaction between teacher attitude and understanding in their effects on student achievement. Despite these two examples of research into the effect on teachers and teaching methods of achievement, Aiken[2] points out that although much has been written on the subject many of the investigations have asked unanswerable or at least unmanageable questions. Rather than demonstrating that a given method did not have a differentially greater effect on attitude and achievement than another, he says that all that the results of certain investigations demonstrate is that a particular lesson presented in a particular way to a particular class did not improve scores on a selected measure more than another lesson in another class. Aiken goes on to summarize the findings of these kinds of investigations in mathematics education as follows: [a] modern mathematics programs do not improve attitudes more than traditional ones; [b] compared to regular classes continuous progress classes do not have a different effect

---

[1] A. S. Perkin, "Teacher Understanding and Attitude, and Student Achievement and Attitude in 7th Grade Mathematics," [Doctoral Diss. New York University], Ann Arbor, Michigan University Microfilms, 1966, No. 65-6584.

[2] Aiken, "Update on Attitudes and Other Effective Variables in Learning Mathematics."

on attitudes towards mathematics; [c] discovery methods are not superior to expository methods; [d] neither follow-up instructions nor flexible scheduling improve attitudes more than traditional instruction; [e] an individualized approach to instruction sometimes has a more positive effect on attitudes than a traditional approach, but that sometimes no difference in the effects of the two types of programs is found.

A basically different approach from the analysis of teacher approaches and teaching methods are mathematics teaching programs based on some specific model of cognitive development, particularly the Piagetian. Thus suggestions arising from the former include the encouragement of questions, verbalization, training in problem solving and logic, attempts to break up incorrect mental sets, or a stress on the student/teacher relationship. The latter methods attempt instruction on the basis of a developmental theory. Most of this work derives from Piaget via Bruner[1, 2] and has been commonly known as the "learning by discovery"

---

[1] J. Bruner, The Process of Education, Cambridge, Mass., Harvard University Press, 1960.

[2] J. Bruner, Toward a Theory of Instruction, Cambridge, Mass., Belknap Press, 1966.

method. Scott[1] attempted to distil the basic principles of this new psychology in ten statements:

1. The structure of mathematics should be stressed at all levels.

2. Children are capable of learning more abstract and more complex concepts when the relationships between concepts are stressed.

3. Existing elementary arithmetic programs may be severely condensed because children are capable of learning concepts at much earlier stages than previously thought.

4. Any concept may be taught a child of any age... if one is able to find the proper language for expressing the concept.

5. The inductive approach or the discovery method is logically productive and should enhance

---

[1] L. Scott, Trends in Elementary School Mathematics, Rand McNally & Company, Chicago, Illinois, 1966.

learning and retention.

6. The major objective of the program is the development of independent and creative thinking processes.

7. Human learning seems to pass through the stages of pre-operations, concrete operations, and formal operations.

8. Growth of understanding is dependent upon concept exploration through challenging apparatus and concrete materials and cannot be restricted to mere symbolic manipulations.

9. Teaching mathematical skills is a tidying up of concepts developed through discovery rather than a step-by-step process of memorization.

10. Practical application of isolated concepts... are valuable to reinforcement and retention.

Piaget himself does not subscribe to this interpretation of his work as put forward by Bruner - especially disagreeing with the assertion that "any subject can be taught effectively...to any child at any stage of development." Bruner, however, seems to be saying that ideas which are usually expressed in symbolic forms only can be taught to children who have not reached this stage of thinking if, and only if, the translator can put the idea in non-symbolic forms for assimilation by the child. Piaget himself remains quite dubious over the attempts to accelerate cognitive development that are reflected in many contemporary mathematics and science curricula. This subject was touched upon earlier in this paper when theories of development were discussed. Experimental studies to date on mathematically precocious children, for example, have seemed to show that though acceleration can or has occurred within the cognitive stages, no significant hastening has been found in overall unfolding of cognitive development.

In contradistinction to Bruner is the work of Gagné[1] who is an advocate of an approach to instruction that is the antithesis of discovery. It has been called variously "guided learning," "expository learning" or "reception learning." In accordance with this method, if the desired end

---

[1] R. M. Gagné, The Condition of Learning, Holt Rinehart & Winter, New York, 1966.

point is problem-solving ability, then Gagné considers that the learner must first know certain principles; to understand these principles he must know specific concepts; pre-requisite to these are particular simple association or facts discriminated from each other in a distinctive manner; and he then continues the analysis until reaching the fundamental building blocks of learning - operationally conditioned responses. Thus Gagné, having determined what the pupil has already mastered, can then determine what is to be taught. A very structured and specific teaching program can then be developed.

Another learning theorist Ausubel[1] also argues against the discovery method in favor of a systematically guided expostion in the process of education. The opposite of discovery learning must be, then, according to Ausubel, meaningful verbal learning. The key is a careful sequencing of instruction so that any unit taught is clearly related to those which preceded it. Ausubel sees no reason why problem-solving activity must precede the internalization of new facts or concepts. If the material can be meaningfully organized by the instructor, the need for student discovery

---

[1] D. P. Ausubel, Educational Psychology: A Cognitive View, Holt, Rinehart & Winston, New York, 1968.

is removed and the process of learning rendered more efficient. Thus Ausubel, like Gagné, emphasizes the great importance of systematically guided exposition in the process of education and the fact that he considers the continuity between the learners' existing cognitive structure and the new material to be learned as the condition that makes the new material meaningful.

As Shulman summarizes,[1] for Bruner the emphasis is on the kinds of processes learned by the student, in contrast to the specific subject matter that he may acquire; Gagné, although in substantial agreement with Bruner on the priority of process over product, is more concerned with the teaching of the rules or intellectual skills that are relevant to particular subjects. For Gagné, the objectives of instruction are intellectual skills that can be specified in operational terms, can be task-analyzed, and then taught. Ausubel strongly rejects the notion that any kind of process, be it strategy or skill, should hold priority. He remains a militant advocate of mastering well-organized bodies of subject-matter as the most important goal, and the most effective means of education.

---

[1] L. S. Shulman, "Psychology and Mathematics Education," Mathematics Education, 69th Year Book of the National Society for the Study of Education, University of Chicago Press, 1970.

Piaget's increasing influence on teaching in general and on teaching mathematics in particular deserves an over-all summing up. Gallagher[1] has summarized Piaget's scheme well and has suggested five major themes in Piaget's work:

> 1. Continuous and progressive changes take place in the structures of behavior and thought in the developing child.
>
> 2. Successive structures make their appearance in a fixed order.
>
> 3. The nature of accommodation [adaptive change to outer circumstances] suggests that the rate of development is, to a considerable degree, a function of the child's encounters with his environment.
>
> 4. Thought processes are conceived to originate

---

[1] J. J. Gallagher, "Productive Thinking," Review of Child Development Research, Hoffman and Hoffman eds., Russell Sage Foundation, N. Y., 1964.

through a series of internalizing actions. Intelligence increases as thought processes are loosened from their basis in perception and action and thereby become reversible, transitive, associative and so on.

5. A close relationship exists between thought processes and properties of formal logic.

Piaget's emphasis upon action as a prerequisite to the internalization of cognitive operations has stimulated the focus upon direct manipulation of mathematically relevant materials in the early grades. His description of cognitive development has reinforced tendencies to emphasize pupil-initiated problem-solving activities as a major vehicle of mathematics instruction. His characterization of the number-related concepts understood by children of different ages has influenced understanding of what children at different stages can be expected to learn. To determine, then, whether a child is ready to learn a particular concept or principle, the structure of what is to be taught is analyzed and compared to what is already known about the cognitive structure of a child of that age. If the two structures are not consonant, further maturation has to take place. Bruner's[1] development of this theme has led to attempts at training and acceleration and has been touched upon earlier. His suggestion is that the conception of readiness must include not only the child, but the subject matter as well, and that the basic principles of a discipline can be manipulated to be able to be understood by the child in terms of his current stage of cognitive development - manipulatively, visually, or as formal symbolic representation. The possible effect on teaching practice of such beliefs are self-evident, though the problems of readiness, early learning and acceleration are less complex for the proponents of expository teaching than for the proponents of discovery.

A review or analysis of the mathematically disabled from the standpoints of any of these learning theories outlined might be expected to give some fruitful clues not only to remediation but to prevention. In view of the step-wise development of cognition, for example, it would be expected that anything preventing or hindering an uninterrupted evolution would lead to progressive inabilities and such causes could be expected to flow from either home environment factors or teaching factors or both. Furthermore, allied to the conception of readiness - the right principle

---

[1] Bruner, The Process of Education.

in the right form for the right cognitive stage at the right time - is that of aptitude. A child's or student's aptitude is usually seen as given and as setting a theoretical limit to the level of complexity or abstraction. Carroll[1] on the contrary, has put forward the view that aptitude is the amount of time required by the learner to attain mastery of a learning task, and that the logic that flows from this is that levels of achievement should be set as the constant and time as the variable, not vice versa, as is currently the case in school systems. Following this suggestion, Herriot[2] reported that slow-learning students achieve as well as average students when they are allowed substantially more time than is usually allotted. If confirmed, this further finding also has important implications not only for teaching, but for prevention and remediation.

---

[1] J. Carroll, "A Model of School Learning," Teachers College Record, LXIV, 1963.

[2] S. T. Herriot, "The Slow Learner Project," SMSG Report No. 5, Stanford University, 1967.

CHAPTER XI

MATHEMATICS REMEDIAL CLINICS

Originating largely from the thrust generated by the current movement concerning the changing role of women in American society has been a close analysis of the position of women in the job market in general, and in academics in particular. Ernest[1] details an investigation into the current status of women, both in their performance in mathematics, and in their recruitment into mathematics faculties and finds them grossly under-represented. In addition, it has already been pointed out that mathematics, or a competence in the subject, is a critical filter in the job market for positions in fields other than those traditionally open to women, and that if women are indeed to have greater career options, then the roadblock in the way of mathematical competency has to be overcome. To this end, two colleges in the New England area - Wellesley College and Wesleyan University - undertook to set up "math clinics" - primarily

---

[1] Ernest, "Mathematics and Sex."

aimed at women. These were both founded by means of grants and the purpose of their programs is to develop programs for reducing anxiety about mathematics and remediating mathematical skills when necessary in order to encourage students to enter regular mathematics courses and mathematics related careers. At Wellesley the course is under the mathematics department, is directed at those students who have shied away from quantitative courses, and is designed and taught in a manner to lessen anxiety about mathematics. The course - entitled "A Discovery Course in Elementary Mathematics and its Applications" - was offered on an experimental basis in the Spring of 1976 to a group of students, 14 of whom were freshmen. It met for two 70 minute periods each week with one two-hour meeting weekly. The course was team-taught by two members of the mathematics department with a senior mathematics major acting as tutor in the two-hour meeting. The course is designed to require little background and to stress mathematical reasoning and its application. It is being conducted as a discussion group in which students discover mathematical structure in several fields, including some not often recognized as mathematical in nature. Topics studied included network analysis, mathematics in music and art, graphing and interpretation of data and game theory.

A verbatim description of the course is worth setting out in full as it does give a good impression of how classes are actually conducted:

> The course is conducted as a discussion group without lectures or text. The instructors present the students with problems to solve that, as far as the students know, have nothing to do with mathematics. The goal of the students is to solve the problem while that of the instructors is to help the students discover the mathematical structure behind the problem. Technical terminology is avoided in stating the problems and is introduced only when the need arises. The role of the instructors is to guide the discussion, referee disagreements, provide hints, and get the students to work out their ideas in front of the class. Rather than declare a correct solution or not, they ask the class to decide. In particular they avoid acting as authority figures who are presenting mathematics as a revelation of divine wisdom. The students see mathematics as a subject in which it is possible to make conjectures, argue opinions and make discoveries. In short, they see it as a living subject, not a dead one.

The two-hour meeting was used to offer remedial work on mathematical skills. Further plans also included the conscious use of women as tutors in order to provide role models.

At Wesleyan there are also plans to extend and enlarge the diagnostic, therapeutic and remedial activities that were established in the first year, with the goal of learning more about the various aspects of mathematical anxiety in undergraduates and adults - both men and women. Included among these activities are interviews, testing math-anxious individuals for facility in algebra, extent of spatial intuition and psychological attitudes towards mathematics and science, intervention in the form of credit and noncredit courses tailored to meet the needs of some of these individuals for specific information and general support; and evaluation of these efforts. The clinic believes that it is beginning to develop useful techniques in two particular aspects of the study; the first is in how to cope with the poor self-image and lack of self-confidence common to a considerable number of students with poor or poorly-developed mathematical ability; and, secondly, how to offer the necessary review and remedial mathematics to students in a nonthreatening format without demanding an unreasonable time commitment.

A similar remedial program at Mills College in Oakland, California reported on its progress after two years.[1] In a student population size of 850, the percentage of students enrolled in pre-calculus increased 133% [from 27 to 63], the percentage starting calculus increased 68% [from 67 to 96], and the percentage of students enrolled in regular

---

[1] Description of the Elementary Course at Wellesley College, available from Wellesley College, Department of Mathematics, Wellesley, Mass., 02181.

mathematics and computer science courses increased 58% [from 324 to 513].

A closer study of the two remedial clinics at Wellesley and Wesleyan reveals that even among projects that have ostensibly the same aim, goals can be sufficiently different as to make actual teaching procedures, and hence outcome, somewhat different. This is clearly important from the viewpoint of the pupil who- already having suffered failure in earlier experiences with mathematics - could only be further impeded if his or her experience with a specifically remedial program were also unsuccessful.

At Wesleyan the goal was to review high school mathematics in order to prepare students for further quantitatively-oriented courses. A textbook was used in addition to structured assignments and tests. In addition, allowance was made for much individualized instruction by maintaining small class sizes and a manageable pace of instruction. At Wellesley the goals were different in that the course is designed to allow students to discover creative experiences. Rather than reviewing high school mathematics, students in this course discover the principles of calculus and advanced algebra. There is no text, and there are no tests. Rather the group is set problems which they solve together. The students reported that the course is successful in providing them with a positive group experience and in convincing them that mathematics can be exciting. At Wesleyan, in contrast, the students reported a gain in a sense of mathematical competence, enjoyed a flexible pace of instruction and felt that they had succeeded in obtaining useful mathematical tools. They do not think very highly of mathematics as a creative endeavor. Wellesley students did not feel confident about their mathematical skills and felt unsure that it would be enjoyable if they chose to take a course in the regular curriculum.

Clearly these two programs reflect the two different teaching philosophies that were discussed in the previous section - the discovery approach of Bruner, and the expository approach of Gagné and Ausubel, and clearly they arrived at two somewhat different end points. Common to both programs, however, was the diminishing of anxiety that students felt as the unfamiliar became more familiar, and as a sense of mastery was achieved. In consequence, both programs increased mathematical capability. The lack of structure in one, and the lack of general interest in mathematics provided by the program in the other, were both quoted by the pupils themselves as deficiencies of those particular programs. The logical conclusion is that the optimum design would be a judicious mix.

An n of 1

In conjunction with the writing of this paper the author enrolled in a mathematics remedial clinic conducted at the University of Massachusetts campus by Natopoff - an MIT faculty member who teaches in the Harvard-MIT Program in Health Services and Technology and who also has a theory of brain function in relation to learning mathematics.[1] From personal observation Natopoff's teaching strategy is heavily based upon the expository method. Starting with basic maneuvers the students are taught simple but powerful strategies that they are encouraged, nay coerced, to become manipulatively familiar with. The strategies are simple and are additive, one strategy building upon another. Following demonstration of the strategy the class is rehearsed, sometimes by exercises, sometimes by rote, sometimes by voice chanting. Interspersed with classes are tutorial hours open to all who feel they need help. At the same time attendance at classes is not only mandatory - but it soon becomes clear that missing classes results in missing, or not mastering, essential strategies. In addition, a text is provided which is written by Natopoff, and

---

[1] A. Natopoff, "The Consideration of Evolutionary Conservatism Toward a Theory of the Human Brain," Perspectives in Biology and Medicine, 1967.

which explains and demonstrates each strategy in step-by-step detail. The overall effect is the gradual gaining of a set of tools which enable the previously mathematically illiterate - myself in this instance - to obtain a sense of mastery and then enjoyment. Arising from the sense of confidence comes the ability to approach the more theoretical aspects.

Several factors become increasingly clear to the author as he goes through this process. The first, and certainly not the least, is the intensity and the disorganizing effect of anxiety upon performance; the second is the re-awakening and the re-experiencing of the internalized rage and frustration which were so familiar from his childhood and the accompanying loss of self-confidence and self-regard; the third is the reinforced recognition of the need for pacing and, closely allied to this, the specific need to put the material into assimilable form so that it fits to the appropriate cognitive style; the fourth is the unassimilability of certain textbooks of mathematics which, with all the will in the world and an intelligence in the normal range, the author finds to be incomprehensible; the fifth is the enormous usefulness of that part of the course which treats mathematics specifically as a language, and hence makes an effort to drill in translation of use of mathematical terms.

The author furthermore continues to ponder on the relevance of that aspect of learning that, as far as he is aware, has been little explored in the literature on mathematics teaching - that of personality structure. Not only are the factors of motivation, models of identification, and reinforcements, important variables for success in learning mathematics - but it is also possible in his view that certain forms of personality might be inhibitory or facilitatory. For example, an over-concern or under-concern with detail may lead to being bound to certain superficial aspects of mathematical thought, in addition to being quite closed off from those aspects - the more creative - which involve ambiguity and lack of structure. In the author's own case, he finds that his compulsive personality structure is more than offset by a deep distaste for detail, the psychodynamic underpinning of which he has not yet successfully approached and certainly not conquered.

CHAPTER XII

SUMMARY, AN HYPOTHESIS, AND SUGGESTIONS FOR FURTHER RESEARCH

1. Factor analysis has separated out a set of mental abilities which act as a functional or operational unit and which are associated with mathematical ability. Factor analysis says nothing about the fundamental nature of this set of abilities.

2. Computational ability is part of, but not synonymous with, mathematical ability.

3. Mathematical ability has an underlying hereditary component.

4. Whether mathematical ability is synonymous with intelligence, or whether intelligence is a necessary but insufficient factor for the presence of mathematical ability is unresolved.

5. High mathematical ability is associated with high intelligence. Whether all those with average or high intelligence, given appropriate teaching and lack of emotional hindrance, are capable of competent or high mathematical performance

is an unresolved question.

6. The most powerful current theory for the investigation of cognition in general, and special abilities including the mathematical, in particular, is that of Piaget. Piaget's theory divides intellectual development into four successive stages. They are: sensorimotor; pre-operational; concrete operational; and formal operational.

7. Some individuals demonstrate a precocious ability to perform mathematics. This ability is associated with high intelligence which in turn is associated with a more rapid rate of progress within developmental stages rather than through developmental stages.

8. High intelligence is associated with developmental precocity in reasoning.

9. Precocity within the developmental stage may be associated with environmental factors; precocity through stages may be associated with hereditary factors. The question is unresolved.

10. Most studies have demonstrated that women have less spatial abilities than men, although others have not confirmed this.

11. It is postulated frequently that the cause of the lesser achievement of women in mathematics is due to the fact that they have less spatial abilities and that this is a biological given. It is possible that spatial abilities do not appear nor differentiate to the same extent as in men because of the effect of socio-cultural factors on cognition.

12. Women have to date achieved less in the aggregate, than men, in mathematics. There are many socio-cultural factors to account for this.

13. Spatial ability is only relevant for certain forms of mathematics.

14. Left cerebral dominance for verbal function is established earlier for females.

15. Females show higher verbal abilities than males early in life; later this difference disappears.

16. With regard to cerebral lateralization, it is possible that for females more than for males, the left hemisphere is more likely to be involved in the performance of spatial tasks. This may have a biological basis but may be conversely, a function of educational and cultural factors.

17. There is no good evidence to indicate that bilateral cerebral hemisphere representation of spatial function is associated with superior function, or that females have more bilateral representation than males.

18. The relationship, if any, between sex hormones and intellectual functioning remains unclear.

19. The evidence does not seem to support an x-linked basis of inheritance for mathematical problem-solving or spatial ability.

20. Despite evidence put forward for differences in brain organization between the sexes, the conclusion that the brains of normal members of the two sexes are organized differently seems to be premature.

21. Mathematical ability is associated with high verbal ability.

High verbal ability may be accompanied by mathematical inability.

22. Males with high verbal abilities and low mathematical abilities seem to have more personality and emotional disturbances than males with high mathematical ability.

23. Males with low achievement in mathematics and females with high achievement show evidence of more personality and emotional disturbance than same-sex counterparts.

24. There is evidence that socio-cultural factors, different for each sex, contribute heavily to mathematical inability in persons of normal intelligence.

25. Anxiety of sufficiently high level can hinder the acquiring of mathematical skills.

26. Mathematics, both because of its progressive structure - one step being necessary for the comprehension of the next - and because of its initmate relationship with abstract reasoning, is particularly vulnerable to the disorganization of thought caused by high levels of anxiety.

27. The fit between cognitive style and teaching method is important, as are teacher attitudes and skills, for the successful learning of mathematics. A poor fit between any or all of these factors, as well as a poor congruence between the material presented and the stage of intellectual development of the pupil leads to poor performance, diminished abilities, lowered self-regard, and emotional disturbance around mathematical performance.

28. Remedial teaching of adults can re-motivate the de-motivated, overcome emotional withdrawal and anxiety, and lead to improved performance.

An Hypothesis

As McCarthy[1] pointed out no single theory in literature has prevailed concerning the pertinent correlates of verbal-quantitative differences.

Based on the findings above, then, the author somewhat

---

[1] S. Viterbo McCarthy, "Differential V-Q Ability: 20 Years Later," Review of Education Research, 1975.

hesitantly offers the following over-all hypothesis concerning the development of differential mathematical ability in persons of the same cultural group; each part of this hypothesis is testable.

1. The ability to perform mathematically correlates precisely with the degree of intelligence, given

   a. there is no acquired emotional impediment, and

   b. motivation.

2. Mathematical ability incorporates the most abstract form of thought; abstract thought is the last form of cognition to evolve both in the individual and in the species; therefore mathematical ability, as is abstract thought, the first to be disorganized and the most vulnerable to organic brain impairment and high anxiety states.

3. There are no biologically based differences in cognition between the sexes.

4. Motivation is acquired or diminished by interpersonal and social factors.

5. Emotional impediment - the end result of disorganizing anxiety - is acquired.

6. The earlier in development emotional disability is acquired, the more severely impaired is mathematical ability.

7. Lack of congruence between stage of cognitive development and teaching method may result in acquired anxiety and decreased motivation leading to impaired mathematical performance.

Suggestions For Further Research

As can be seen from a cursory review of the literature, much emphasis has been devoted to a review of differences between the sexes. It is the author's impression that these studies are in general of better design and quality than much of the work which deals with other aspects of the problem of differential mathematical ability. At this stage, it would seem to be more fruitful to concentrate on intra-sex differences thus removing what is at best a contentious variable.

In view of the subtle, yet pervasive and powerful, socio-cultural factors that militate yet against mathematical performance by women, and which will still confuse the results of new research, it would be especially interesting to design properly controlled studies to answer at least the following questions about the differences between the mathematically able and mathematically unable male.

1. Are there any differences in spatial ability between the mathematically able and the mathematically unable male of equivalent age and IQ?

2. Using Piagetian tests are there differences between the mathematically able and the less able in same age groups? It would be particularly interesting to compare a cohort of upper range achievement boys and lower range achievement boys of same age exposed to the same teachers and to compare the stages of cognitive development and the congruence of teaching methods in each cohort.

3. Are there significant differences between the mathematically able and the mathematically unable in terms of personality structure, personality traits, or cognitive style?

4. Are there particular personality structures or traits which contribute to, or hinder, mathematical performance.

5. Can the observation that the mathematically unable seem to be more disturbed than the mathematically able be further documented.

6. Can it be shown that emotional disturbance at an earlier age is more disruptive than emotional disturbance at a later age and can this be correlated with impaired mathematical performance?

7. Have the mathematically unable been exposed to significantly more poor teaching?

8. What is the influence of supportive role models in early childhood. Are there significant differences between the mathematically able and unable?

The list could be continued - each however, if able to be satisfactorily answered, would contribute to the unravelling of the two central questions:

1. Is the high verbal-low quantitative configuration in the presence of normal intelligence a biological, hereditary given, or is it an acquired state?

2. Is the ability to reason abstractly synonymous with mathematical ability?

The tentative opinion of this author is that the high-verbal low-quantitative configuration is an acquired state; and that the abilities to reason abstractly and to perform mathematically are synonymous.

CHAPTER XIII

IMPLICATIONS FOR MANAGEMENT

There are two broad reasons as to why management should be interested in the quantitative skills, or lack of them, of staff at all levels. The first is enlightened self-interest, the second, social responsibility.

Enlightened self-interest is based on the recognition that more and more is the successful conduct of industry and commerce based on the intelligent application of quantitative methods - and that his applies not only to the production of goods and services, but to the art and science of management itself. For the manager therefore, the area of operations research is where the mathematically and scientifically trained technologist works increasingly with the non-mathematical manager. The gulf between these two classes has traditionally been wide and difficult to bridge. The environment is forcing a change. In addition, at levels below management there is an increasing need for a highly-skilled work-force - a work-force which not only can absorb new methods, but initiate them. As competition for the scarcest of resources - capable people - increases, management will need more and more to develop its skilled staff from its own house.

With regard to social responsibility, American society as a whole is increasingly committed to drawing the under-privileged and the minorities into the mainstream of American life. The mainstream in the area of work is becoming steadily more science-based. There is a decreasing need for the unskilled - who then find it increasingly difficult to maintain themselves much less to contribute to the resources of the community. Hence, to fulfill its responsibilities to these groups, management could, and in the author's opinion should, address the question of quantitative skills. Although experience with mathematical remedial clinics for adults is short, the results to date are extremely promising - as confirmed both by the early results of these clinics and by personal experience. On-the-job training should include mathematical skills. Such programs need careful design, proper staffing, and a commitment of time and funds. The investment would be certain to yield a good rate of return.

BIBLIOGRAPHY

BOOKS

1. Anastasi, A. "Commentary on the Precocity Project." in Mathematical Talent: Discovery, Description and Development. J.C. Stanley, D. P. Keating and L. Fox eds.

2. Ausubel, D.P. Educational Psychology: A Cognitive View. New York: Holt, Rinehart & Winston. 1968.

3. Ausubel, D.P. and Robinson, F.C. School Learning: An Introduction to Educational Psychology. New York: Holt, Rinehart & Winston. 1969.

4. Bruner, J. The Process of Education. Cambridge, Mass: Harvard University Press. 1960.

5. Bruner, J. Toward a Theory of Instruction. Cambridge, Mass: Belknap Press. 1966.

6. Buffery, A.W.H. and Gray, J.A. "Sex Differences in the Development of Spatial & Linguistic Skills." Gender Differences. Ounstead, C. & Taylor D.C. eds. Baltimore: William & Wilkins Co. 1972.

7. Callahan, L.G. and Glennan, V.J. Elementary School Mathematics: A Guide to Current Research. Washington D.C. Association for Supervision and Curriculum Development. 1978.

8. Cronbach, L.J. "How Can Instruction be Adapted to Individual Differences." Learning and Individual Differences. R. M. Gagre, ed. Columbus, Ohio: Bobbs-Merrill. 1967.

9. Fierabend, R.L. Review of Research on Psychological Problems in Mathematical Education. U.S. Office of Education. Cooperative Research Monograph No. 3. 1960.

10. French, J.W. The Description of Aptitude and Achievement Factors in Terms of Rotated Factors. Chicago, Ill.: University Chicago Press. 1951.

11. Gagne, R.M. The Conditions of Learning. New York: Holt, Rinehart & Winston. 1968.

12. Gallagher, J.J. "Productive Thinking." Review of Child Development Research. Hoffman & Hoffman, eds. Russell Sage Foundation, N.Y. 1964.

13. Ginsburg, H. and Opper S. eds. Piaget's Theory of Intellectual Development. Prentice-Hall Inc. 1969.

14. Glennan, V.J. and Callahan, L.G. A Guide to Current Research: Elementary School Mathematics. Assoc. for Supervision and Curriculum Development. Washington, D.C. 1968.

15. Goins, J. Visual Perceptual Abilities and Early Reading Progress. University of Chicago Press. No. 37. February 1958.

16. Hilton, T.L. and Berglund, G.W. Sex Differences in Mathematics Achievement - a Longitudinal Study. Princeton, N.J. Educational Testing Service. 1971.

17. Hudson, L. Contrary Imaginations. London: Methuen & Co. 1966.

18. Kagan, J. Change and Continuity in Infancy. John Wiley & Sons, Inc. 1971.

19. Keating, D. P. "A Piagetian Approach to Intellectual Precocity." Intellectual Talent - Research & Development. Baltimore: John Hopkins University Press. 1976.

20. Levy, J. "Lateral Specialization of the Human Brain." The Biology of Behavior. J.A. Kiger, ed. Oregon University Press. 1972.

21. Maccoby, E.E. and Jacklin, C.N. Psychology of Sex Differences. Palo Alto, California: Stanford University Press. 1974.

22. Piaget, J. and Inhelder, B. The Child's Conception of Space. London: Routledge & Kegan Paul. 1956.

23. Piaget, J.; Inhelder, B. and Szeminska, A. The Child's Conception of Geometry. New York: Basic Books. 1960

24. Piaget, J. and Szeminska, A. The Child's Conception of Number. New York: Humanities Press. 1952.

25. Poincare, H. "Mathematical Creation." The Creative Process. B. Ghiselin, ed. New York: New American Library. 1952.

26. Saad, L.G. and Storer, W.O. Understanding in Mathematics. Edinburgh: Oliver & Boyd. 1960.

27. Scott, L. Trends in Elementary School Mathematics. Chicago, Illinois: Rand McNally & Co. 1966.

28. Shulman, L.S. "Psychology and Mathematics Education." Mathematics Education. 69th Year Book of the National Society for the Study of Education. University of Chicago Press. 1970.

29. Stanley, J.C., ed. "Intellectual Precocity." Mathematical Talent. Johns Hopkins University Press. 1974.

30. Stanley, J.C.; Keating, D.P. and Fox, L.H., eds. Mathematical Talent - Discovery, Description and Development. Johns Hopkins University Press. 1974.

31. Stuart, M.F. Neurophysiological Insights in Teaching. Palo Alto, Calif.: Pacific Books. 1963.

32. Tyler, L.E. Individual Differences: Ability and Motivational Directions. Appleton-Century-Crofts. 1974.

33. Vandenberg, S.G. "The Nature and Nurture of Intelligence." Genetics: Biology and Behavior. D.C. Glass, ed. New York: Rockefeller University Press. 1968.

34. Werdelin, I. The Mathematical Ability: Experimental and Factorial Studies. Lund, Sweden: C.W.K. Gleerup. 1958.

35. Zangwill, O. Cerebral Dominance and Its Relation to Psychological Functioning. Springfield, Illinois: C.C. Thomas. 1960.

ARTICLES

1. Aiken, L.R. "Intellectual Variables and Mathematics Achievement." Journal of School Psychology. Vol. 9. 1971.

2. Aiken, L.R. "Update on Attitudes and Other Affective Variables in Learning Mathematics." Review of Educational Research. Vol. 66. 1973.

3. Aiken, L.R. "Verbal Factors and Mathematics: A Review of Research." Journal for Research in Mathematical Education. November 1971.

4. Aiken, L.R. "Verbal Factors and Mathematics Learning." Journal for Research in Mathematical Education. November 1971.

5. Alexander, V. "Sex Differences in Seventh Grade Problem Solving." School Science & Mathematics. Vol. 62. 1962.

6. Altus, W.D. "Personality Correlates of Q-L Variability on the ACE." Journal of Consulting Psychology. Vol. 16. 1952.

7. Altus, W.D. "Q-L Variability, MMPI Responses of College Males." Journal of Consulting Psychology. Vol. 22. 1958.

8. "American Mathematical Society, 19th Annual AMS Survey." Notices of the American Mathematical Society. 22. 1975.

9. Astin, H.S. "Career Development of Girls During the High School Years." Journal of Counseling Psychology. Vol. 15. 1968.

10. Backman, M.E. "Patterns of Mental Abilities: Ethnic, Socio-Economic and Sex Differences." American Educational Research Journal. Vol. 9. 1972.

11. Bhushan, V.; Jeffreys, J. and Wakamura, I. "Large Group Instruction in Mathematics Under Flexible Scheduling." Mathematics Teacher. Vol. 61. 1968.

12. Biggs, I.B. "The Teaching of Mathematics II - Attitudes to Arithmetic - Number Anxiety." Educational Research. Vol. 1. 1959.

13. Bock, R.D. and Kolakowski, D. "Further Evidence of Sex-Linked Major Gene Influence on Human Spatial Visualizing Ability." American Journal of Human Genetics. Vol. 25. 1973.

14. Broverman, D.M.; Klaiber, E.L.; Kobayashi, Y. and Vogel, W. "Roles of Activation and Inhibition in Sex Differences in Cognitive Abilities." Psychological Review. Vol. 75. 1968.

15. Burt, C. "The Inheritance of Mental Ability." American Psychologist. Vol. 13, 1958.

16. Carlsmith, L. "Effect of Early Father Absence on Scholastic Aptitude." Harvard Educational Review. Vol. 34. 1964.

17. Carroll, J. "A Model of School Learning." Teachers College Record. LXIV. 1963.

18. Cattell, R.B. "Personality Traits Associated with Abilities, II: With Verbal and Mathematical Abilities." Journal of Educational Psychology. Vol. 36. 1945.

19. Cleveland, G.A. and Bosworth, D.L. "A Study of Certain Psychological and Sociological Characteristics as Related to Arithmetic Achievement." Arithmetic Teacher. Vol. 14. 1967.

20. Crosswhite, F.J. "Correlates of Attitudes Towards Mathematics." NLSMA Report No. 20. Investigation in Mathematics Education. Vol. 8. 1975.

21. D'Augustine, C. H. "Factors Relating to Achievement with Selected Topics in Geometry and Topology." Arithmetic Teacher. Vol. 13. 1966.

22. Dahmus, M.E. "How to Teach Verbal Problems." School Science and Mathematics. Vol. 70. 1970.

23. Dale, W. "Factor Analysis." Psychometric Monographs No. 3. 1940.

24. Dalton, K. "Ante-natal Progesterone and Intelligence." British Journal of Psychiatry. Vol. 114. 1968.

25. Dana, R.H.; Dahlke, A.E. and Mueller, D. "Intra-Individual Verbal-Numerical Discrepancies and Personality." American Psychologist. Vol. 14. 1959.

26. Davis, J.K. and Klausmier, H.J. "Cognitive Style and Concept Identification as a Function of Complexity and Training Procedures." Journal of Educational Psychology Vol. 61. 1970.

27. Duncan, C. and Hartley, J. "The Effect of Mode of Presentation and Recall on a Simple Learning Task." Programmed Learning and Educational Technology. Vol. 6. 1969.

28. Dye, N.W. and Very, P.S. "Growth Changes in Factorial Structure by Age and Sex." Genetic Psychology Monographs. Vol. 78. 1968.

29. Earp, N.W. "Observations on Teaching Reading in Mathematics. Journal of Reading. Vol. 13. 1970.

30. Erhardt, A.A. and Money, J. "Progestin-Induced Hermaphroditism." Journal of Sex Research. Vol. 3. 1967.

31. Easterday, K and Easterday, H. "Ninth Grade Algebra, Programmed Instruction of Sex Differences: An Experiment." Mathematics Teacher. Vol. 61. 1968.

32. Erlenmeyer-Kimling, L. and Jarvik,L.F. "Genetics and Intelligence." Science. Vol. 142. 1963.

33. Ernest, J. "Mathematics and Sex." American Mathematics Monthly. Vol. 83. 1976.

34. Fennema, E. "Mathematics Learning and the Sexes." Journal for Research in Mathematics Education. Vol. 5. 1974.

35. Fennema, E. and Sherman, J. "Sex Related Differences in Mathematics Achievement, Spatial Visualization and Socio-Cultural Factors." Journal of Educational Research. in press.

36. Ferguson, L.R. and Maccoby, E. "Interpersonal Correlates of Differential Abilities." Child Development. Vol. 37. 1966.

37. Fox, L.H. "The Effects of Sex Role Socialization on Mathematics Participation and Achievement." Paper prepared for Education and Work Group. Career Awareness Division. National Institute of Education. U.S. Dept. of HEW. Contract No. FN17-420-0114. December 1976.

38. Garron, D.C. "Sex-Linked Recessive Inheritance of Spatial and Numerical Abilities and Turners Syndrome." Psychological Review. Vol. 77. 1970.

39. Garrett, H.E. "A Developmental Theory of Intelligence." American Psychologist. Vol. 1. 1946.

40. Guilford, J.P. "Structure of Intellect Model: Its Uses and Implications." Psychological Lab. Report. No. 24. University Of Southern California. Los Angeles, Calif. 1960.

41. Harris, L.J. "Interaction of Experiential and Neurological Factors in the Patterning of Human Abilities." Eric Document Reproduction Service [EDRS] ED119854.

42. Hyde, J.S.; Gennger, E.R. and Yen, W.M. "An Empirical Relation Between Spatial Ability and Sex Differences in Other Aspects of Cognitive Performance. Multivariate Behavioral Research. Vol. 10. 1975.

43. Johnson, J.T. "On the Nature of Problem Solving in Arithmetic." Journal of Educational Research. Vol. 43. 1949.

44. Kagan, J.; Moss, H.A.; and Sigel, I.E. "Psychological Significance of Styles of Conceptualization." The Monograph of the Society for Research in Child Development. Vol. 28. 1963.

45. Keeves, J.P. "Differences Between the Sexes in Mathematics and Science Courses." International Review of Education. Vol. 19. 1973.

46. Krutetskij, V.A. "The Psychology of Mathematical Aptitudes of School-Children. Moscow: Educational Publishing House. March 1969. Eric EDO60227.

47. Larney, V.H. "Female Mathematicians Where Are You?" American Mathematical Monthly Vol. 80. 1973.

48. Levy-Agresti, J. and Sperry. R. "Differential Perceptual Capacities in Major and Minor Hemispheres." Proc. of National Academy of Sciences. Vol. 61. 1968.

49. Lewis, D.L. and Peng-Sim, K. "Personality and Performance in Elementary Mathematics with Special Reference to Item Type." British Journal of Education. Vol. 43. 1973.

50. Loofts, W.R. "Sex Differences in the Expression of Vocational Aspirations by Elementary School Children." Developmental Psychology. Vol. 5. 1971.

51. Lovell, K. "Intellectual Growth and Understanding Mathematics." Mathematics Education Reports. ERIC Information Analysis Center. Columbus, Ohio. February 1971. EDRS EDO49069.

52. Lunzer, E.A. "Agressive and Withdrawing Children in the Normal School, II - Disparity in Attainment." British Journal Educational Psychology. Vol. 30. 1960.

53. Lunzer, E.A. "Formal Reasoning". Development in Human Learning. Vol. 2. E.A. Lunzer and J.F. Morris, eds. London: Staples Press. 1968.

54. Lyda, W.J. and Duncan F.M. "Quantitative Vocabulary and Problem Solving." Arithmetic Teacher. Vol. 14. 1967.

55. Lynn, R. "Temperamental Characteristics Related to Disparity of Attainment in Reading and Arithmetic." British Journal of Educational Psychology. Vol. 27. 1957.

56. Maccoby, E.E. and Rau, L. "Differential Cognitive Abilities," Cooperative Research Program of Office of Education. Department Health, Education and Welfare. Stanford. 1962.

57. Madden, R. "New Directions in the Measurement of Mathematical Ability." Arithmetic Teacher. Vol. 13. 1966.

58. McCarthy, J.L. and Wolfe, D. "Doctorates Granted to Women and Minority Group Members." Science. Vol. 189. 1975.

59. Meyer, W.J. "The Stability of Patterns of Primary Mental Abilities Among Jr. and High School Students." Education and Psychology Measurements. Vol. 20. 1960.

60. McCarthy, S. Viterbo. "Differential V-Q Ability: 20 Years Later." Review of Educational Research. Vol. 45. 1975.

61. Money, J.; Erhardt, A.A. and Masica, D.W. "Fetal Feminization Induced by Androgen Insensitivity in the Testicular Feminizing Syndrome." John Hopkins Medical Journal. Vol. 123. 1968.

62. Murray, J.E. "An Analysis of Geometric Ability." Journal of Educational Psychology. Vol. 40. 1949.

63. Muscio, R.D. "Factors Related to Quantitative Understanding in the Sixth Grade." Arithmetic Teacher. Vol. 9. 1962.

64. Natopoff, A. "The Consideration of Evolutionary Conservatism Toward a Theory of the Human Brain." Perspectives in Biology of Medicine. Vol. 10. 1967.

65. Naylor, F.D. and Gaudry, E. "The Relationship of Adjustment, Anxiety and Intelligence to Mathematics Performance." Journal of Educational Research. Vol. 66. 1973.

66. Nelson, E. and Maccoby, E. "The Relationship Between Social Development and Differential Aptitudes on the Scholastic Aptitude Test." Merrill-Palmer Quarterly. Vol. 12. 1966.

67. Nichols, R.E. "Effects of Various College Characteristics on Student Aptitude Scores." Journal of Educational Psychology. Vol. 55. 1964.

68. Odom, R.D.; McKintyre, C.W. and Neale, G.S. "The Influence of Cognitive Style of Perceptual Learning." Child Development. Vol. 42. 1971.

69. Rife, D.C. and Snyder H. "Studies in Human Inheritance VI." Human Biology. Vol. 3. 1931.

70. Romer, N. "The Motive to Avoid Success and Its Effect on Performance in School-age Males and Females." Developmental Psychology. Vol. 11. 1975.

71. Rosenfield, R.L. "Plasma Testosterone binding globulin and Indices of the Concentration of Unbound Plasma Androgens in Normal and Hirsute Subjects." Journal of Clinical Endoinnology. Vol. 32. 1971.

72. Sanders, E.; Mefferd, R.B. and Brown, W.H. "Verbal-Quantitative Ability and Certain Personality and Metabolic Characteristics of Male College Students." Educational Psychology Measurements. Vol. 20. 1960.

73. Sells, L. "High School Mathematics as the Critical Filter in the Job Market." Quoted in the bibliography of the Wellesley-Wesleyan Math. Project 1976-77.

74. Semmes, J. "Hemispheric Localization: A Possible Clue to Mechanisms." Neuropsychologia. Vol. 6. 1968.

75. Shepler, J.L. "A Study of Parts of the Development of a Unit in Probability and Statistics for the Elementary School." Madison, Wisconsin Research and Development Center for Cognitive Learning. Technical Report. No. 105. 1969.

76. Sherman, J. "Problem of Sex Differences in Space Perception and Aspects of Intellectual Functioning." Psychological Review. Vol. 74. 1967.

77. Shine, A. "Relationship Between Arithmetic Achievement and Term Performance on the Revised Standord-Binet Scale." Arithmetic Teacher. Vol. 8. 1961.

78. Singer, G. and Montgomery, R.G. "Comment on Role of Activation and Inhibition in Sex Differences in Cognitive Abilities." Psychological Review. Vol. 76. 1969.

79. Sowder, L. "Performance on Some Discovery Tasks Grades 4-7." Journal for Research in Education. Vol. 2. 1971.

80. Stafford, R.E. "Sex Differences in Spatial Visualization as Evidence of Sex-Linked Inheritance." Perceptual and Motor Skills. Vol. 13. 1961.

81. Stafford, R.E. "Hereditary and Environmental Components of Quantitative Reasoning." Review of Educational Research. Vol. 42. 1972.

82. Suinn, R.M.; Edie, C.A.; Nicoletti, J. and Spinelli, P.R. "The MARS, A Measure of Mathematics Anxiety." Journal of Clinical Psychology. Vol. 23. 1972.

83. Suydam, M.N. and Reidesel, C.A. "Interpretative Study of Research and Development in Elementary School Mathematics." Vol. 1. Project No. 8-0586. 1969. Dept. H.E.W.

84. Swineford, F. "A Number Factor." Journal of Educational Psychology. Vol. 40. 1949.

85. Taschow, H.G. "Reading Improvement in Mathematics." Reading Improvement. Vol. 6. 1969.

86. Thurstone, L.L. "Primary Mental Abilities." Psychometric Monograph. No. 1. 1938.

87. Torrance, E.P. and Parent, E. "Characteristics of Mathematics Leaders that Affect Students Learning." Report No. CRP-1020. September 1966. U.S. Office of Education.

88. Treach, J.P. "The Relationship of Reading Skills to the Ability to Solve Arithmetic Problems." Journal of Educational Research. Vol. 38. 1944.

89. Trown, E.A. "Some Evidence on the Interaction Between Teaching Strategy and Personality." British Journal of Educational Psychology. Vol. 40. 1970.

90. Vander Linde, L.F. "Does the Study of Quantitative Vocabulary Improve Problem Solving." Elementary School Journal. Vol. 65. 1964.

91. Very, P.S. "Differential Factor Structures in Mathematical Ability." Genetic Psychology Monographs. Vol. 75. 1967.

92. Werdelin, I. "A Synthesis of Two Factor Analyses of Problem Solving in Mathematics." Didakometry. No. 8. 1966.

93. Wrigley, J. "Factorial Nature of Ability in Elementary Mathematics." British Journal of Educational Psychology. Vol. 28. 1958.

94. Yalom, I.D., Green, R. and Fisk, N. "Prenatal Exposure to Female Hormones." Archives of General Psychiatry. Vol. 28. 1973.

95. Yen, W.M. "Sex-Linked Major Gene Influence on Selected Types of Spatial Performance." Behavior Genetics. Vol. 5. 1975.

Additional Articles

96. Dreger, R.M. and Aiken, L.R. "The Identification of Number Anxiety in a College Population." Journal of Educational Psychology. Vol. 48. 1957.

97. Erickson, L.H. "Certain Ability Factors and Their Effect on Arithmetic Achievement." Arithmetic Teacher. Vol. 5. 1958.

98. Ivanoff, J.M., DeWare, E.T. and Praem, O. "Use of Discriminant Analysis for Selecting Students for Ninth Grade Algebra or General Mathematics." Mathematics Teacher. Vol. 58. 1965.

UNPUBLISHED MATERIAL

1. Alpert, R.; Stellwagon, G. and Becker, D. "Psychological Factors in Mathematics Education." Newsletter 15. School Math Study Group. Stanford University. 1963.

2. Behr, M.J. "A Study of Interactions Between Structure of Intellect Factors and Two Methods of Presenting Concepts of Medium Seven Arithmetic." Doctoral Thesis. Tallahassee: Florida State University. Diss. Abstract 28. 1693-A. 1967.

3. Blum, L. "Educating College Women in Mathematics." A report of an action program in progress. Proceedings of the Conference "Educating Women for Science: A Continuous Spectrum." Mills College. April 24, 1976.

4. Bouchard, T. "Sex Differences in Human Spatial Ability: Not an X-linked Recessive Effect." in press. 1976.

5. Brown, P.G. "Tests of Development in Children's Understanding of the Laws of Natural Numbers." Math. Education Thesis. University of Manchester. 1969.

6. Casserly, P.L. "An Assessment of Factors Affecting Female Participation in Advanced Placement Program in Mathematics, Chemistry and Physics." Report to the National Science Foundation. July 1975.

7. Coleman, R.H. "An Analysis of Certain Components of Mathematical Abilities and an Attempt to Predict Mathematical Achievement in a Specific Situation." Diss. Abstract 1950. 16.2062.

8. Cottrell, R.S. "A Study of Selected Language Factors Associated with Arithmetic Achievement of 3rd Grade Students." Doctoral Diss. Syracuse University. [Ann Arbor, Mich. University Microfilms, 1958. No. 68-5505.]

9. Erlick, A.C. and LeBold, W.K. "Factors Influencing the Science Career Plans of High School Students." Measurement and Research Center. Purdue University, Indiana. 1975.

10. Fennema, E. "Influences of Selected Cognitive, Affective and Educational Variables on Sex-Related Differences in Mathematics Learning and Studying." Department of Curriculum and Instruction. University of Wisconsin, Madison. October 1976.

11. Fischer, E.C. "A Study of the Relationship Between Selected Personality Factors and Special Disability in Mathematics." Doctoral Diss. University of Alabama. [Ann Arbor, Mich. Microfilms, 1968. No. 68-1039.]

12. Fox, L.H. "Facilitating the Development of Mathematical Talent in Young Women." [Diss. Abstract. Int. 1974 [b] 35.3553B.] University Microfilm No. 74-29027.

13. Harshman, R.A. and Remington R. "Sex, Language and the Brain: Part I: A Review of the Literature on Adult Sex Differences in Lateralization." UCLA Working Paper in Phonetics. 31. 1976.

14. Haven, E.W. "Factors Associated with the Selection of Advanced Academic Mathematics by Girls in High School." [Diss. Abstract International 1971 32 1747A].

15. Herriott, S.T. "The Slow Learner Project." SMSC Report No. 5. Stanford University. 1967.

16. Jonsson, H.A. "Interaction of Test Anxiety and Test Difficulty in Mathematics Problem-Solving Performance." Doctoral Diss. University of California, Berkeley. [Ann Arbor, Mich. University Microfilms 1966 No. 65-13, 514.]

17. Kaminski, D.M., et.al. "Why Females Don't Like Mathematics: the Effects of Parental Expectations." Paper presented at the 1976 Sociological Association Meeting, New York. August 1976.

18. Kline, W.E. "A Synthesis of Two Factor Analyses of Intermediate Algebra." Doctoral Diss. Princeton University. [Ann Arbor, Mich. University Microfilms. 1960. No. 60,5012].

19. Kogelman, S. "Dynamics and Aetiology of Math Anxiety in Women." Masters Thesis. Smith College School of Social Work. Boston. 1975.

20. Kropp, R.P.; Nelson, W.H. and King, F.J. "Identification and Definition of Subject Matter Content Variables Related to Human Aptitudes." Cooperative Research Project No. 2117. Florida State University. 1967.

21. Levine, M. "Identification of Reasons Why Qualified Women Do Not Pursue Mathematical Careers." Report to the National Science Foundation. August 1976.

22. Linnville, W.J. "The Effects of Syntax and Vocabulary Upon The Difficulty of Verbal Arithmetic Problems with Fourth Grade Students." Doctoral Diss. Indiana University. [Ann Arbor, Mich. University Microfilms 1970 No. 70-7957.]

23. Martin, M.D. "Reading Comprehension, Abstract Verbal Reasoning and Computation as Factors in Arithmetic Problem Solving. Doct. Diss. State Univ. of Iowa. [Ann Arbor, Mich. Univ. Microfilms 1965 No. 64-3395.]

24. Ory, J.C. and Helfrich, L.M. "A Study of Individual Characteristics and Career Aspirations." Paper presented at the Annual Meeting of the American Educational Research Association. San Francisco, California. April 1976.

25. Peskin, A.S. "Teacher Understanding and Attitude and Student Achievement and Attitude in 7th Grade Mathematics." Doctoral Diss. New York University. [Ann Arbor, Mich. University Microfilms. 1966. No. 65-6584.]

26. Pirr, P.M. "Intellectual Variables in Achievement in Modern Algebra." Doctoral Diss. Catholic University of America [Ann Arbor, Mich. University Microfilms, 1967. No. 67-183.

27. Reynolds, J. "The Development of the Concept of Mathematical Proof in Grammar School Pupils." Ph.D. Thesis. University of Nottingham. 1967.

28. Rusch, C.E. "An Analysis of Arithmetic Achievement in Grade Four, Six and Eight." Doctoral Diss. University of Wisconsin. [Ann Arbor, Mich. University Microfilms 1957. No. 57,3568.]

29. Sells, L. "Sex and Discipline Differences in Doctoral Attrition Ph.D. Thesis. University of California, Berkeley. 1975

30. Sherman, J. "Effects of Biological Factors on Sex-Related Differences in Mathematics Achievement." Women's Research Inst. of Wisconsin, Inc. National Institute of Education. Project Contract No. 400-70-0113. 1976.

31. Sherman, J. and Fennema, E. "Distribution of Spatial Visualization and Mathematical Problem Solving Scores." Submitted for publication. 1976.

32. Simpson, C.J. "The Effect of Laboratory Instruction on the Achievement and Attitudes of Slow Learners in Mathematics." Doctoral Diss. Lehigh University. 1973. Diss. Abstract Int. 1974. 34. 6959A-6960A [University Microfilms No. 74-11,357.]

33. Stafford, R.E. "An Investigation of Similarities in Parent-Child Test Scores for Evidence of Hereditary Components." Doctoral Diss. Princeton University. 1963. [Diss. Abstracts 1964 11 4785-6.].

34. Swafford, J.O. "A Study of the Relationship Between Personality and Achievement in Mathematics." Doctoral Diss. University of Georgia. 1969. [Diss. Abstr. Int. 1970 30 5353A].

35. Taylor, C. "Factors of Mathematical Aptitude." Diss. Abstr. 1973.

36. Thornell, J.G. "Individual Differences in Cognitive Style and the Guidance Variable in Instruction." Presented at the Annual Meeting of the American Educational Research Association 1974 EDRS ED106705.

37. UNESCO. "Interactions Between Linguistics and Mathematical Education." Final Report of the Symposium sponsored by UNESCO, CEDO and ICMI. Nairobi, Kenya. September 1974.

38. Woolridge, E.T. "Factorial Study of Changes in Ability Patterns of Students in College Algebra." Doctoral Diss. University of Nebraska. [Ann Arbor, Mich. University Microfilms, 1964. No. 64-11,945.]