OPTIMAL ECONOMIC GROWTH AND ENERGY POLICY

by

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ABSTRACT

This study is concerned with the long-term interactions between the energy markets and the aggregate determinants of economic growth. The framework of analysis is provided by a macroeconomic energy model. The structure of the model is formulated drawing upon the neo-classical theories of producer and consumer behavior, the theory of general equilibrium and the theory of economic growth. There are two sectors of production corresponding to energy products and non-energy products, respectively. The output of the energy sector is disaggregated as between consumption and intermediate goods; the output of the non-energy sector is disaggregated as between consumption, intermediate, and investment goods. Technological constraints on the production processes are embodied in two cost possibility frontiers that yield the value of gross output for given factor input prices and given levels and configuration of output. Structural specification of the cost frontiers corresponds to the transcendental logarithmic functional form incorporating constant returns to scale. Technical change is assumed to be described in terms of exponential indices of factor augmentation; the property of Hicks-neutrality is therefore susceptible of being tested empirically by statistical tests of suitable parametric restrictions. Sectoral supply and demand functions are obtained using results from the theory of derived factor demand and product supply for technologies with multiple outputs--necessary assumptions are the existence of competitive markets and the presence of cost-minimizing behavior. The technological flow coefficients between sectors explicitly incorporate price-dependent substitution possibilities; however, because of the presence of sectoral supply functions, the consumption-investment split is not determined entirely on the demand side as in conventional input-output analysis, but rather takes place within a simultaneous process of market equilibration that generates a complete set of market-clearing prices and quantities. The derived demand for imports is obtained directly from the assumptions of cost-minimizing behavior. Household behavior is modeled based on the assumption of utility-maximizing behavior. Intertemporal and intra-temporal allocation models yield demand functions for full consumption for three commodity groups and leisure, respectively. The model is completed with equations accounting for capital accumulations in the production and household sectors, and by accounting identities and balance equations.

The model was estimated using yearly time-series data corresponding to the U.S. postwar period. Historical simulation of the complete model suggests that it provides a satisfactory explanation of historical growth patterns.

The problem of optimal growth was formulated as an optimal control problem involving the maximization of welfare subject to the behavioral and technological constraints embodied in the macroeconomic model. The set of available policy instruments were taken to include tax rates on capital income and capital property, investment tax credits, tax rates on energy consumption goods and on investment goods. The performance index for the optimal control problem was defined in terms of the present value of a discounted stream of utilities accruing to households from the consumption of energy goods, non-energy goods, capital services and leisure. The parameters of this welfare function are inferred from the estimated demand functions for the household sector, so that the welfare measure is consistent with the preferences of consumers as revealed in the historical data.

The resulting optimal control problem involves a nonlinear system, nonlinear performance index and implicit state equations. The solution of the nonlinear optimal control problem with implicit state equations necessitated the development of new computational algorithms that are based on existing algorithms of the differential dynamic programming and Min-H types. This set of algorithms have been implemented in OPCON, a general purpose computer code designed for the solution of nonlinear optimal control problems in discrete-time with implicit state transition mappings.

Finally, the results of the optimal control experiments are discussed in terms of their implications for the structure of optimal growth paths and in terms of the relative welfare gains accruing under alternative policies.

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> "Tradition requires that I absolve my critics from responsibility for all errors. Although I deeply respect tradition as a matter of principle, I see no reason to absolve them. If I have committed blunders, one or another of those learned men and women should have noticed; if they did not, then let them share the disgrace. As for my interpretation and bias, the usual disclaimer is unnecessary since no one in his right mind is likely to hold them responsible for either."
>
> - Eugene D. Genovese

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CHAPTER I

INTRODUCTION

> "...within whatever technological assumptions instinct and observation lead one to make, it is possible to pose and to answer what I have claimed to be the central question of capital theory. What is the payoff to society from an extra bit of saving transformed efficiently into capital formation?"
>
> \- Robert M. Solow

1.1 Purpose and Methodology

This is a study about energy and economic growth. Its primary aim is to identify the causal links and the behavioral interdependencies among governmental policy, aggregate economic activity and patterns of energy utilization. It attempts to develop quantitative relationships between the flow of energy resources and the structural determinants of growth, and to assess the impact of alternative outcomes of these processes from the standpoint of overall social welfare.

Within these rough boundaries, many specific questions suggest themselves. What are the interrelations between an economy's productive capacity, the demand for energy and the forces behind productivity growth? What does the rate of capital accumulation required to sustain an efficient growth path imply with regard to the consumption of energy resources? How effective is fiscal policy in promoting efficient functioning of energy markets while guiding the economy towards such a growth path? To develop a framework of inquiry that will make it possible for such questions to be meaningfully addressed is the chief goal of this work.

These questions are not entirely new and they are certainly not simple. Indeed, they differ little in substance from the central issues that have shaped the theory of economic growth throughout the past several decades. In contrast to the established tradition in that field, however, the object of our study is a concrete economy evolving over a definite historical period. As such, it is rich in empirical content. In addition, the perspective adopted in this work differs in one more respect from that of most earlier researchers in long-term macrodynamics. This distinguishing aspect consists of the explicit recognition of the crucial role of energy resources in the economic activities of production, consumption and investment and, *a fortiori*, in the growth process itself.

The analytical framework that serves as the focal point of our study is provided by a long-term macroeconometric energy model of the U.S. economy. The methodological basis of the model rests upon the neoclassical theories of producer and consumer behavior, the theory of economic growth and the theory of general equilibrium. The synthesis of a model that has its roots in these diverse fields and that, in addition, was designed to be susceptible of empirical validation, has been achieved not without a certain degree of compromise. The model focuses on the connections between energy flows and intertemporal choices between present and future consumption; it does not explicitly address those aspects of intertemporal behavior that are intrinsically due to the exhaustible nature of the energy resource base. Although it is directly tied to historical data, the macroeconomic model must be termed pre-institutional in nature and its policy implications must be evaluated in this light. Finally, the model is highly aggregative. This property, of course, guarantees an analysis unencumbered by the burdens of high dimensionality and is, in any event, intrinsic to an undertaking such as we have described. Even Mrs. Robinson -- not an apologist of neo-classicism by any account -- concedes that "a model which took account of all the variegation of reality would be of no more use than a map at a scale of one to one."[1]

In order to establish the characterization of optimal growth trajectories for our macroeconometric model, we construct an aggregate index of social welfare defined in terms of a discounted stream of utilities accruing to households. Both the rate of pure time preference, as well as the parameters of the intra-period utility function are inferred from estimated consumer demand functions and, consequently, our measure of welfare is consistent with the preferences of consumers as revealed in the historical data. Our optimal control studies, therefore, provide a unifying synthesis of the descriptive and normative aspects of growth.

The study of interactions between growth and government policy -- and, inescapably, of the role of energy resources -- within a framework such as we have described, has both theoretical and empirical interest. But -- we must ask -- can we demonstrate that the subject of economic growth carries with it some measure of direct immediacy for the prescription of public policy?

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[1] Robinson [1], p. 33

1.2 Growth, Welfare and Fiscal Policy

The nature and extent of governmental control over economic growth has been the subject of prolonged controversies among economic policy-makers. The debate has entailed both positive and normative aspects. There is the question, on the one hand, of whether in a capitalistic economy the sovereignty of the consumer prevails to the extent that it is he alone, and not the government, who can and does in fact decide the rate of investment and hence the rate of growth. Another matter in dispute, related but distinct, is whether the government ought to intervene at all in the process of choosing between present and future consumption, and, if so, in what direction and by how much. Even if it were established that the government can control the split between consumption and investment, it is argued, it need not follow that the government should in fact decide the consumption-investment mix.

The prospect of a strict laissez-faire attitude toward growth has been viewed at times with some apprehension by its opponents. James Tobin has asked:[2]

> "Can we as a nation, by political decision and governmental action, increase our rate of growth? Or must the rate of growth be regarded fatalistically, the result of uncoordinated decisions and habits of millions of consumers, businessmen and governments, uncontrollable in our kind of society except by exhortation and prayer?"

It seems safe to suggest that the government is not in fact condemned to a policy of perennial passivity toward growth. "The belief in the inability of the government to influence investment in a capitalistic economy" -- it has been argued by Phelps [3] -- "flies in the face of

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[2]Tobin [2], pp. 10-11

modern as well as classical economic theory." Paul Samuelson was prepared to go even further:[3]

> "With proper fiscal and monetary policies, our economy can have full employment and whatever rate of capital formation and growth it wants."

Whether this pronouncement is overly optimistic will remain a matter of debate. Nonetheless, the weight of the evidence suggests that the discretionary powers of government to influence the rate of growth extend beyond the specific controls over research, tangible capital expenditures and education. Full employment can be achieved by a combination of low taxes (budgetary deficit) and high interest rates <u>or</u> high taxes (budgetary surplus) and low interest rates, as Phelphs [4] puts it:

> "If the government chooses high taxes and low interest rates, consumption expenditures by the heavily taxed households will be small while the low interest rates will stimulate large investment expenditures and hence produce a high rate of growth."

If we accept that the growth rate is susceptible of being controlled at least to a certain degree, the extent to which the government should exercise this power is still not immediately obvious. Does it at least follow that this power should be exercised at all? In other words, is there a need for a growth policy?

This question can be rephrased in yet another way: are freely functioning markets efficient in making intertemporal allocations?

The advocates of a laissez-faire posture predicated upon the prevalence of consumer sovereignty would reply that indeed they are. But as Phelps has pointed out, "The fundamental error of this proposition, viewed from

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[3] Samuelson [5], pp. 229-234.

the standpoint of contemporary fiscal and monetary theory, arises from its neglect of the role played by government taxation."[4] Indeed, critics of such a passive approach to economic growth would contend that "the market solution in principle cannot provide any reliable indication of consumer preferences, since the government may have biased the solution one way or the other."[5] They would further point out that "the market solution can never be relied upon to express consumers' true preferences for growth because the market solution is contaminated by the way in which the government elects to use its fiscal and monetary controls."[6]

But if freely operating markets fail to insure a growth path in accord with consumer preferences solely because of the distortions introduced by the government's own policies, would it not be possible to design these policies with the specific intent of neutralizing such distortions and restoring the full virtues of the market solution? Wouldn't the necessity for an active growth policy therefore disappear for reasons not unlike those put forth by the supporters of the proposition of consumer sovereignty?

It is precisely this category of questions that has been investigated by Phelps [3] under what he termed the principle of fiscal neutrality:

> "We shall say that tax policy is neutral if it produces the same allocation of resources (among consumption goods, among people, and between aggregate consumption and investment) as would be produced if there was no government treasury, hence no taxes and government debt, but only a government agency to conscript resources for use in the production of public goods and an agency to redistribute wealth so as to achieve the desired distribution of lifetime income."

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[4]Phelps [4], p. 75.

[5]Phelps [3], p. 13.

[6]Ibid., p. 13

The practical difficulties with such a strategy of fiscal neutrality are obvious. But if it were implementable, would it inevitably lead to a Pareto-optimal growth path in the sense of an efficient intertemporal allocation of resources?

Such a result would indeed hold -- as Phelps points out in his own analysis -- "if a competitive equilibrium is attained; if there is complete information about future as well as current prices and also perfect information about current and future supplies of public goods; if producers have complete information about the future as well as the current technology; if there are no externalities in production; if consumers know their tastes and their preferences are unchanging over time; if there are no externalities in consumption other than the public goods whose production we take as given."[7] That is to say, this set of arguments -- which are familiar examples of market failure in a static context -- are also valid objections against a laissez-faire approach to the problems of intertemporal choice and apply, by extension, to a strategy of fiscal neutrality towards growth. Let us examine them in some detail.

The objection relating to the myopic perception of consumer preferences was advanced by Pigou.[8]

> "Generally speaking, everybody prefers present pleasures or satisfactions to future pleasures or satisfactions of equal magnitude, even when the latter are perfectly certain to occur...This implies only that our telescopic faculty is defective, and that we, therefore, see future pleasures, as it were, on a diminished scale.... [People] distribute their resources between the present, the near

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[7]Phelps [4], pp. 81-82.

[8]Pigou [6], pp. 24-25.

> future and the remote future on the basis of a wholly irrational preference."

Pigou's argument implies that there is no feasible consumption program which the individual would not at some point in his lifetime prefer to exchange for some other feasible consumption program. If indeed this claim is correct -- and it is not obvious that it is empirically testable -- then it would follow that there is no Pareto optimum because there is no set of consistent preferences. The implications of this proposition would clearly extend beyond the rebuttal of the market process as an efficient allocation mechanism.

The absence of comprehensive future markets is a serious objection to the extent that it prevents both consumers and producers from accurately forecasting the course of future prices. As Graaff points out:[9]

> "Let us abstract from such familiar difficulties as external effects in production, the dependence of production functions upon the distribution of wealth, or the presence of monopoly. Let us assume, that is to say, conditions completely favorable to the satisfactory working of the price mechanism. The amount of saving any one household undertakes (out of a given income, and at a given interest rate) will depend upon the goods and services it expects those savings to be able to purchase in future years -- upon the expected level of prices. If it holds a part of its savings in the form of bonds, expected interest rates will enter the picture; if in the form of money, the general level of prices; if in the form of durable commodities, relative prices.

Uncertainty arising from imperfect forecasting of future prices is also present in production decisions:[10]

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[9]Graaff [7], p. 103.

[10]Ibid., p. 104.

> "No individual entrepreneur can estimate, on the basis of [current] market data alone, the productivity of investment until the investment plans of other entrepreneurs are determined... We have here something analogous to external effects in production -- but again something quite different from true external effects, since it works through the price system and has nothing to do with the interrelation of production functions in a technological sense."

The point here is that the profitability of investments cannot be known until entrepreneurs know future prices, and these future prices cannot be known until they know what other entrepreneurs will invest. Koopmans has referred to this phenomenon as <u>secondary uncertainty</u>:[1]

> "In a rough and intuitive judgment the secondary uncertainty arising from lack of communication, that is from one decision maker having no way of finding out what the concurrent decisions and plans made by others (or merely of knowing suitable aggregate measures of such decisions or plans), is quantitatively at least as important as the primary uncertainty arising from random acts of nature and unpredictable changes in consumer preferences."

Another condition that may cause departures from an efficient growth path is the presence of <u>monopolistic or oligopolistic</u> market structures. The major consequence of this from the standpoint of the growth process is that firms will not invest up to the point where the rate of return on investment is equal to the rate of return available to savers.

Finally, it is frequently argued that a <u>laissez-faire</u> doctrine of growth will not lead to efficient intertemporal allocations because of the <u>divergence between social and individual levels of risk</u> (Solow [8], Tobin [9]), because of <u>external returns to investment</u> in R&D and externalities arising from learning effects in tangible capital investment (Arrow [10], Solow [8]) and because of intertemporal <u>externalities in</u>

[1] Koopmans [11], pp. 162-163.

consumption arising from interpersonal jointness of preferences (Sen [12], Marglin [13]).

The above set of arguments against strictly passive or else neutral strategies toward growth, it has been suggested, demonstrate the failure of market processes in guaranteeing a Pareto-optimal growth path and point to the necessity of an active growth policy. Indeed, when taken as a whole, they constitute a strong case in favor of a departure from fiscal neutrality. They do not provide, of course, any insight on what guidelines ought to be followed in designing a growth strategy.

Analytical studies of such normative questions are scarce, primarily because models of economic growth have traditionally been cast in an abstract framework, focusing on questions far removed from the prescription of public policy.* Typically, growth theorizing has been concerned with the expansion rates required to fulfill certain long-term equilibrium conditions -- as in the study of Golden Rule steady-state paths (Phelps [14], Swan [15]). In traditional neoclassical models, moreover, growth rates are entirely determined by population growth; fiscal policy might conceivably be used to promote capital formation, but this results only in a changed capital-labor ratio and exerts no impact upon the long-term growth path. In recent years, fiscal policy parameters have been included in neoclassical growth models by some authors (e.g. Cornwall [16], Sato [17] and [18]), but they have been concerned for the most part with distributional issues or with the speed of adjustment, rather than with the properties of the growth path itself.

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*For comprehensive surveys of mathematical models of economic growth, the reader is referred to Hahn and Matthews [19] and Britto [20].

The most promising framework for the analysis of fiscal policy and its impact on growth -- a framework, we might add, upon which the present study builds -- is the one developed by Jorgenson [21] and his students (Christensen [22], Hudson [23]). The work of Jorgenson in the econometrics of economic growth has succeeded in linking neoclassical growth theory with empirically-based methodologies of model-building and with the realities of fiscal policy and its implications for the problem of intertemporal choice.

1.3 Overview

The structure of our macroeconomic energy model is presented in Chapter II. Our modeling approach reflects the fact that changes in technologies in the energy sector or introduction of policy initiatives which affect equilibrium in the energy markets will have consequences for other sectors of the economy and will therefore affect the overall level and composition of economic growth. Attention is focused on the properties and configuration of alternative paths of potential output rather than on questions related to the levels of capacity utilization or employment, as in short-term stabilization models. A full set of market-clearing prices and quantities is computed within a simultaneous process of market equilibration. This property makes it possible to establish a link between the macroeconomic relationships in the model and economic quantities susceptible of interpretation from the standpoint of micro-economic analysis. Indeed, although the model is aggregative, the introduction of a higher degree of sectoral detail would entail no new conceptual difficulties.

Chapter III is devoted to the estimation of the behavioral equations in our macroeconomic model and presents the results of statistical tests of alternative hypotheses on the structure of technology and the structure of consumer preferences. The hypothesis of separability between inputs and outputs in the characterization of technology is decisively rejected. Price elasticities of demand for factors of production, as well as Allen-Uzawa partial elasticities of substitution and transformation are computed and the implied patterns of substitutability and complementarity are examined. Statistical tests of various hypotheses concerning the configuration of technological change are reported; the commonly adopted hypothesis of Hicks neutrality is rejected and our results suggest the presence of energy-using biases in the structure of technical progress over the period 1948-1971. The remaining sections deal with the estimation of the model of consumer behavior. From the estimated parameters for the inter-temporal allocation model, we derive the implied values for the rate of social discount. Estimated values of the discount rate range between 2.02% and 7.98%, depending on the underlying assumptions about the structure of inter-temporal preferences. Finally, several hypotheses about the structure of intra-temporal consumer preferences are investigated, including groupwise separability among the various commodity groups.

In Chapter IV we present the final form of our macroeconometric energy model, including the full list of equations and variables. The results of historical simulation of the model throughout the period 1948-1971 are reported. In terms of the accuracy of these ex-post forecasts, the performance of the model is judged satisfactory. We then perform a set of simulations corresponding to the effects of changes in selected tax variables. An analysis of these simulation results highlights the strongly interrelated nature of the variables in our general equilibrium model. A comparison of these results with studies of similar tax measures based on partial equilibrium suggests the conclusion that a partial equilibrium analysis leads to an overprediction of the effects of the tax changes because it neglects the stabilizing forces arising from the feedback interrelationships among variables. Finally, we perform a set of tax policy experiments under two alternative assumptions about government financing: budget-balancing and deficit-financing fiscal packages. Detailed analysis of tax measures aimed at stimulating investment expenditures show that the impact on actual investment levels crucially depends on the configuration of the entire budgetary scheme.

In Chapter V, we develop a framework of analysis that makes it possible to compute optimal growth paths under alternative strategies, within the context of our macroeconomic energy model. We discuss the specification of a measure of economic welfare which is consistent with the behavioral assumption underlying our model and with the preferences of consumers as revealed in the historical data. The resulting infinite-horizon problem is reduced to a finite-horizon problem by constructing a valuation of terminal capital stock that is commensurate with the underlying social welfare function. Given the objective function and the behavioral and technological constraints embodied in our macroeconomic model, the problem of optimal growth is formulated as a nonlinear optimal control problem with implicit state equations. The numerical solution of this optimal control problem demands the development of new computational algorithms that are generalizations of existing algorithms in the theory of optimal control. These algorithms are incorporated into OPCON, a general purpose software package for the solution of nonlinear optimal control problems with nonlinear objective functions. Finally, we present the results of a set of numerical optimization experiments that yield insight into the structure of optimal growth paths and the relative welfare gains accruing under alternative policies.

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CHAPTER II

A LONG-TERM MACROECONOMIC ENERGY MODEL: THEORETICAL FOUNDATIONS

CHAPTER II

A LONG-TERM MACROECONOMIC ENERGY MODEL: THEORETICAL FOUNDATIONS

"Theories are nets: only he who casts will catch."

- Novalis

2.1 Introduction

In this chapter we discuss the theoretical foundations of a policy-oriented macroeconometric energy model designed to capture the long-run dynamic interactions between the U.S. energy sector and the aggregate determinants of economic growth.

Our primary objective in the development of our macroeconomic model has been the formulation of an integrated and consistent framework of analysis that would relate the market mechanisms for energy products, non-energy products, and primary factors of production to the fundamental process underlying the determination of economic growth: the link between current capital formation and future productive capacity.

This objective has demanded a new approach to macroeconomic energy modeling. The reasons that have determined this necessity are several.

In the first place, traditional approaches to macroeconometric model-building are all based to some extent on dynamic versions of the Keynesian multiplier and do not provide a suitable framework for the analysis of interactions between the economy and the energy sector. As Paul Samuelson has observed:*

> "The usual macroeconomic model is not built in such a way that one can fit into it changing assumptions about a micrceconomic availability of something like oil, or energy generally. Neither Keynes nor Irving Fisher gave us ways of handling an event of this type. Yet the forecaster must somehow adjust his system and his thinking to allow for this new limitation on supply. What is he to do?"

Secondly, models of economic growth with their inescapable emphasis on determinants of long-run supply, have seldom been developed through the stages of econometric implementation. In sharp contrast to their short-term quarterly counterparts, long-term growth models have mostly remained in the theoretical literature, rarely undergoing the test of empirical validation. Perhaps the scarcity of econometrically implemented long-term models can be explained in part by the overriding concern with short-term forecasts throughout the last two decades and in part by the fact that the causal mechanisms behind the processes of capital accumulation and technological innovation -- both central to any theory of growth -- are not completely understood. In any event, empirical studies of economic growth, such as those of Kendrick [2], Denison [3] and Jorgenson and Griliches [4], have usually focused on the explanation of specific properties of long-run economic behavior, such as variations in the levels of productivity.

The third and final reason that has dictated the necessity for a new modeling framework lies in the fact that in those few instances where econometric models incorporate some form of supply constraints,

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*Samuelson [1].

the treatment of intermediate inputs is absent or else present only in rudimentary form. Indeed, the majority of studies that investigate the interactions between the energy sector and the overall economy have typically centered on isolated quantitative features as in the analysis of energy-GNP ratios by Schurr et al. [5], Darmstadter et al. [6], Netschert [7] and the more rigorous study by Berndt and Wood [8].

Recent disruptions in the domestic and international energy markets have highlighted the necessity of long-term planning in a sphere of economic activity as critically dependent on a depletable resource base as is the energy sector. It has come to be generally recognized that the formulation of a judicious energy policy for the long-term requires the explicit consideration of supply and resource constraints that fall outside the traditional Keynesian framework. The shortcomings of conventional modeling approaches discussed above have come to command substantial attention in the recent past.

As a result, interest in the development of macroeconomic models suitable for the evaluation of long-range energy policy has increased. Foremost among these research efforts is the path-breaking work of Jorgenson [9] and Hudson and Jorgenson [10]. In these studies, projections from a one-sector growth model are fed into an inter-industry model that establishes the connection between intermediate demands and final demands as in conventional input-output analysis. The growth model, implemented by Hudson [11], was based on the formulation of Christensen [12], [13], and Jorgenson [14]. It constituted one of the first theoretically robust long-run models to be empirically validated. The inter-industry model possesses a significant innovative feature: the technological flow coefficients are endogenously determined as a function of relative prices. Such a treatment of technological coefficients can be found in the literature on neoclassical multisector models (Morishima [15]), but the formulation of Jorgenson and his co-workers is unique in that it constitutes the first consistent empirical implementation of such a system of derived demand functions.

In developing the macroeconomic energy model described in this chapter, we have taken several features of the Hudson-Jorgenson model as a point of departure. For example, we have maintained the neoclassical assumption of equality between savings and investment, rather than allowing for disequilibrium as in the growth models of the Keynes-Wicksell type (Fischer [16]). There are, however, significant differences between our model and that of Hudson-Jorgenson. The growth model in Hudson and Jorgenson [10] does not include an energy sector and the inter-industry model is entirely static. The fundamental novelty of our approach to macroeconomic energy modeling lies in the complete integration of the sectoral capital accumulation dynamics and the sectoral production structure into the growth process itself.

2.2 Overview of the Model

The underlying theoretical basis for our macroeconomic energy model is the neoclassical theory of general equilibrium. There are three basic constituents of the general equilibrium problem:

(i) Producer Behavior. Given some specification of technologically feasible combinations of inputs and outputs, producers attempt to acquire factor services and produce goods in such a way as to maximize their flow of profit.

(ii) Consumer Behavior. Given some representation of consumer preferences, households attempt to offer factor services and purchase goods in such a way as to attain a maximum level of utility flow.

(iii) Market Adjustment Process. Given the demand and supply functions resulting from the characterization of producer and consumer behavior, market forces determine an adjustment process toward a set of prices for goods and factor services that clear all goods and factor markets.

Our macroeconomic model has been formulated within this basic structure, incorporating fully endogenous treatment of the production and household sectors. The role of each individual decision unit in the overall system can be established in a straighforward way. Households acting as price takers develop decisions attempting to arrive at preferred positions subject to expenditure constraints and given price data; producers acting as price takers develop decisions attempting to achieve maximum profit subject to technological constraints and given price data. Analysis of these decisions yields results describing the manner in which individual decisions are affected by changes in price data taken as given. Processes of market adjustment then alter prices until all the foregoing decisions are mutually consistent and markets clear.

The other two major components of the model are the government and foreign sectors. The government sector has its revenue generated by the tax structure and the tax bases but its expenditure is largely exogenous. Demand for imports is generated as part of the system of derived factor demands in the production sector but the rest of the foreign trade sector is exogenous to the model.

The structure of production in our model incorporates two production sectors corresponding to energy and non-energy products, respectively. The model includes five markets for products, two markets for capital services and one market for labor services. The products are supplied by the energy and non-energy sectors and used by all sectors; factors are supplied by the household sector and used by the two production sectors. The rate of capital accumulation and the rate of increase of wealth are also determined within the model and, together with the rate of technological progress, establish the dynamic evolution of the system. Identities that relate the income and expenditure flows across the various product categories and balance equations that summarize the conditions for market equilibrium complete the structure of the model.

The formulation of our model falls within the tradition of general equilibrium models because we assume that the supply and demand schedules for each good and service determine all prices and quantities transacted within a simultaneous process of market equilibration.

Our formulation is neoclassical because we postulate that the behavioral characteristics of the basic decision units can be described in terms of maximizing behavior in the presence of appropriate constraints.

Having selected this general framework, we now must make specific assumptions about the levels of aggregation in the various markets and across the various product categories, we must select concrete mechanisms for the embodiment of our behavioral assumptions and we must choose specific functional forms leading to relationships between variables suitable for empirical implementation. This we proceed to do in the remainder of this chapter.

First, we present brief overviews of the sub-models corresponding to the structure of production, household behavior and capital accumulation.

2.2.1 Production Structure

The characterization of technology in macroeconomic modeling has traditionally been approached in terms of an aggregate production function specifying the level of value-added output attainable from a given combination of primary factor inputs. More often than not, intermediate inputs have been entirely neglected or, alternatively, have been accorded superficial treatment in an input-output framework embodying the assumption of a Leontief technology of fixed proportions. In models of economic growth, a value-added specification of technology has tended to facilitate focusing on the fundamental questions concerning the allocation of output as between consumption and investment. Our approach to the description of technology in the context of our macroeconomic energy model consists of explicitly incorporating intermediate inputs in the production processes while at the same time maintaining the specification of supply constraints on the consumption-investment split from the output side.

Our production model consists of two sectors producing energy products and non-energy products, respectively. Both the energy and non-energy sectors have five inputs: capital services, labor services, energy products, non-energy products and imports. The output of the energy sector consists of energy consumption goods and energy intermediate goods as joint products. The output of the non-energy sector consists of non-energy consumption goods, non-energy intermediate goods and investment goods produced as joint products.

Our specification of the structure of production can be contrasted with that of the conventional two-sector model in which each sector produces a single homogeneous output and the consumption-investment split is determined entirely on the demand side.

More specifically, we describe the technological constraints on the production possibilities of the two sectors in terms of two cost possibility frontiers incorporating the assumption of constant returns to scale. Using results from the duality theory of production, we are able to specify derived factor demand functions for each of the five inputs of the two sectors; correspondingly, we specify derived product supply functions for each output of the two sectors. In the specification of the cost frontiers, explicit account is taken of shifts in production possibilities across time brought about by technical change; the resulting parameters that characterize this time variation can be interpreted as exponential indices of factor augmentation.

2.2.2 Household Behavior

Our model for the household sector is based on a characterization of consumer preferences in terms of utility maximization. Consumers are assumed to make their spending-saving and work-leisure decisions as if to maximize the sum of present and discounted future utilities subject to a suitable budget constraint. Under the assumption of inter-temporal separability of preferences, we derive a consistent system of demand functions for leisure, energy products, non-energy products and capital services. Following the system of accounts developed by Christensen and Jorgenson [17], we take purchases of consumer durables to be part of investment expenditures and we include an annual service value which is imputed to these consumer durables as part of consumption expenditures. The demand function for leisure, when taken jointly with the total time endowment of the household sector yields a supply function for labor services.

Our overall consumption model is decomposed into two stages:

(a) The inter-temporal allocation model specified in terms of an inter-temporal utility function with time-varying preferences. This inter-temporal model resolves the allocation of full consumption between the present and the future, where full consumption is defined as an aggregate index composed of leisure and an arbitrary commodity bundle. The inter-temporal utility function explicitly depends on the subjective rate of social discount. The associated budget constraint depends on non-human wealth at the end of the previous period plus potential human wealth, which consists of potential time resources evaluated at the expected after-tax wage rate and of expected transfer payments discounted to the present. Under suitable assumptions on the expected rates of growth of the wage rate, total time resources and the level of transfer payments, it is possible to derive an explicit expression for the present value of the total resources of the household sector. A readily estimable demand function for full consumption is obtained.

(b) The intra-temporal allocation model which specifies behavioral relations for the allocation of full consumption within a given period among leisure, energy products, non-energy products and capital services. The appropriate budget constraint is given by the value of full consumption for the period under consideration. The assumption of utility- maximizing behavior leads to the derivation of a readily estimable set of relationships for the relative expenditure shares of leisure and the three commodity groups.

2.2.3 Capital Accumulation

Capital services are supplied to the production processes of the energy and non-energy sectors by their respective capital stocks composed of producer durables, non-residential structures, inventories and land. Capital goods are assumed to be malleable, i.e. the *ex-post* substitution possibilities between capital and other factors are assumed to be equal to the *ex-ante* substitution possibilities. The stock of capital is assumed to be non-transferable between sectors. The dynamics of capital accumulation in each sector is governed by the sectoral rates of replacement and by investment demand for expansion. The level of capital services are related to the sectoral capital stocks by the efficiency of capital in each sector. The sum of gross investment across sectors must equal total investment expenditures by the private domestic sector. The price of capital services in each sector is derived from the equality between the asset price and the discounted value of its services; the service price of capital turns out to be related to the price of the capital asset group in that sector, the sectoral depreciation rate, the sectoral rate of return, the tax rate on capital income and the tax rate on capital property.

2.3 The Structure of Production

Our objective in this section is to develop a complete specification of the technological constraints on the production possibilities of the energy and non-energy sectors in our macroeconomic model.

2.3.1 Background

Most existing macroeconomic models treat technology in a rudimentary way. Demand for factors of production is typically tied to an underlying aggregate production function relating capital and labor inputs to output measured in terms of value-added. In some short-term models even the derived demand for capital is absent and the level of output depends on labor input alone. Prices are determined endogenously for many categories of goods but their interdependence is not specified by an explicit model of technology.

Our point of departure in the characterization of producer behavior is the modern theory of cost and production. Our goal is to arrive at a specification that will capture the full range of interdependencies between prices, technology and relative factor and product intensities. To this end, it will be necessary to choose a specific conceptual structure for producer behavior that can be embodied in empirically relevant functional forms.

The modern theory of cost and production admits at least two entirely equivalent specifications of producer behavior for technologies with multiple inputs and multiple outputs. In terms of quantities, a complete model of production includes the production possibility frontier and the necessary conditions for producer equilibrium. Under the assumptions of competitive markets and constant returns to scale, this model implies the existence of a <u>cost possibility frontier</u> defining the set of product intensities and factor prices consistent with a certain value of total output and of a corresponding <u>price possibility frontier</u> defining the set of prices consistent with zero profits. Both the cost and price possibility frontiers when taken jointly with the conditions determining product and factor intensities are dual to the production possibility frontier and the conditions for producer equilibrium.

It is this duality between prices and quantities that we will exploit in formulating the behavioral relationships for derived factor demands and derived product supplies in our model of production. This concept of duality is a natural extension of the familiar duality between cost and production functions in microeconomic analysis.

The use of the concept of duality in the economic analysis of production originated with the work of Hotelling* on optimal taxation rules in 1932. Hotelling proposed the notion of a "price potential" function to characterize producer behavior and used it as a basis to develop a detailed system of interrelated supply and demand functions. Similar ideas can be found in the work of Hicks [18] and, in the context of consumer behavior, the study of Roy [19] on expenditure systems. The ideas of Hotelling were formalized by Shepard in 1953 in his pioneering work on cost and production.** Shepard's lemma asserts that, for a given state of technology, the gradient of the dual function is equal to the supply and demand correspondences. This constitutes the cornerstone of the modern theories of derived factor demand and product supply and has had a profound influence in econometric studies of producer behavior. Shepard also proved the fundamental result in duality theory concerning the recoverability of the production possibilities set starting from the cost function corresponding to the underlying technology. It is this theorem, later extended by Uzawa [20] and others, that justifies the use of the cost function in the characterization of producer behavior.

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*Hotelling [21].

**Shepard [22].

In an independent path of inquiry, Samuelson in 1962 introduced a function dual to the macroeconomic production function in the neo-classical theory of growth and termed it the factor-price frontier. Referred to also as the surrogate production function, it defines the set of values of the wage rate and the rate of interest consistent with a certain level of technology. The factor-price frontier has been discussed in the context of capital theory by Bruno [23] and Burmeister and Kuga [24].

In recent years, the above set of results has been generalized to the case of technologies exhibiting multiple inputs and multiple outputs. McFadden [25] has extended the duality properties of the cost and production frontiers to the case of several outputs. The price possibility frontier or profit function which is a generalization of the factor-price frontier to the case of several outputs has received attention by several workers: Diewert [26] has established necessary and sufficient conditions on transformation and profit functions for duality to hold; Lau [27] has investigated the relationships between separability and non-jointness assumptions in transformation and profit functions, respectively; Christensen, Jorgenson and Lau [28] have conducted extensive empirical tests of symmetry and group-wise and commodity-wise additivity for profit functions corresponding to aggregate U.S. data. Hall [29] has studied the properties of the cost possibility frontier -- which he refers to as the joint cost function -- and has investigated the implications of non-jointness and functional separability.

Empirical implementation of the conceptual structure afforded by duality for technologies with several inputs lagged substantially behind the theoretical developments. The primary reason for this was the absence of suitable functional forms. Direct generalizations of functional forms used extensively in the two-factor case -- e.g. the Cobb-Douglas (Cobb and Douglas [30]) and the CES (Arrow, Chenery, Minhas, and Solow [31]) -- imposed severe restrictions in the multiple-factor case as was noted by Uzawa [32] and McFadden [33]. For example, the Cobb-Douglas form requires that the elasticities of substitution between any pairs of inputs be unity. The CES, in the multiple input case, requires that the elasticities of substitution between all input pairs be constant and equal. Various generalizations of the CES have been suggested; for example the two-level CES form introduced by Sato [34] and Uzawa [32], impose the restrictions that the Allen partial elasticities of substitution between all factors from different input groups be equal, or be equal to unity, respectively.

As a result of these highly restrictive conditions, interest focused on specifying functional forms that placed no a priori restrictions on substitution possibilities between factors. This interest culminated in the development of two very flexible functional forms. Diewert [35] proposed a form that he termed the Generalized Leontief Production Function. This function is a quadratic form in the quantities of inputs elevated to the power one-half. It reduces to the Leontief fixed input ratios production function as a special case. Christensen, Jorgenson and Lau [28], [36] proposed the Transcendental

Logarithmic -- or translog -- functional form. This form is linear-quadratic in the logarithms of inputs. It reduces to the multiple-input Cobb-Douglas form as a special case. For any point in input space there exists a member of the translog family that can attain an arbitrary set of elasticities of substitution. Separability and aggregation properties can be imposed on the translog form as empirically testable parametric restrictions.

This flexibility of the translog form has already led to a number of applications to the econometric analysis of production. Berndt and Christensen [37] investigated separability and aggregation properties between structures and equipment as components of capital inputs in U.S. manufacturing. Woodland [38] studied substitution possibilities between structures, equipment and labor for selected industries in Canadian manufacturing. Humphrey and Moroney [39] used translog cost and production functions to study substitution possibilities between natural resource inputs and primary factor inputs for U.S. manufacturing. Berndt and Wood [40] investigated substitution patterns between energy, materials and primary factor inputs in U.S. manufacturing. On the basis of tests of separability between primary factors and other inputs, they concluded that a value-added specification of technology was not supported by the data. Toews [41] studied substitution patterns among inputs and rates of technological progress for twelve U.S. manufacturing industries.

This latter set of studies constitute attempts to model producer behavior taking explicit account of intermediate inputs and of the interactions between prices and technology. Nonetheless, they fall short of establishing the link between the individual producing sectors and aggregate economic activity.

The inter-industry model of Hudson-Jorgenson [10] is another important example of the application of the translog functional form. The result is an elegant empirical implementation of the neoclassical multisector model of production. Several earlier attempts to introduce price-dependence in the technological coefficients of an input-output structure -- see, for example, Theil and Tilanus [42] and Johansen [43] -- had been predominantly based on an ad-hoc characterization of substitution possibilities. The computation of equilibrium prices is carried out in the Hudson-Jorgenson inter-industry model by invoking the Non-Substitution theorem -- Samuelson [44] -- and thus decoupling the production structure from the product demand side. The resulting computational savings are attained not without some cost: Hudson-Jorgenson must assume non-jointness in production and in addition must assume transferability of capital stock across sectors, both conditions being necessary for Non-Substitution to hold.

2.3.2 Description of Technology

The technological constraints on the production possibilities of the energy and non-energy sectors are assumed to be described by the transformation functions

$$T_E(x_E;y_E) = 0$$

and

$$T_N(x_N;y_N) = 0$$

where x_E, x_N and y_E, y_N are input and output vectors, respectively. We assume that T_E and T_N are defined and continuous for all non-negative inputs and outputs and that the input requirement sets are closed and strictly convex. It follows -- McFadden [ ], Diewert [ ] -- that there exist cost functions $V_E(y_E;w_E)$ and $V_N(y_N;w_N)$ differentiable in the input price vectors w_E and w_N and defined by

$$V_E(y_E,w_E) = \min_{x_E} \{<w_E,x_E> : T_E(x_E;y_E)=0\}$$

and

$$V_N(y_N;w_N) = \min_{Y_E} \{<w_N,x_N> : T_N(x_N;y_N)=0\}$$

For given input price vectors and a certain configuration of output, the quantities $V_E=V_E(y_E;w_E)$ and $V_N=V_N(y_N;w_N)$ represent the minimum cost of production achievable subject to the respective technological constraints in the energy and non-energy sectors. We refer to the functions V_E and V_N as the <u>cost possibility frontiers</u> for the energy and non-energy sectors respectively. We now state the following

<u>Theorem</u> (Diewert [ ], p. 489)

The cost frontiers V_E and V_N satisfy the following conditions:

(i) V_E and V_N are <u>positive real valued functions</u>, defined and finite for all finite positive output vectors and positive input price vectors.

(ii) V_E and V_N are nondecreasing left continuous functions in output for all positive input price vectors.

(iii) V_E and V_N are nondecreasing functions in the input price vectors.

(iv) V_E and V_N are linear homogeneous in input prices for all positive values of output.

(v) V_E and V_N are concave functions in input prices for all positive values of output.

The above conditions are sufficient to guarantee the duality properties of the cost frontiers: i.e. they can be used to infer properties of the underlying technology and indeed, to reconstruct the original production possibility sets from which they were originally derived.

We now define the appropriate set of variables intervening in the two cost frontiers. For the energy sector, we define the input price vector as

$$w_E=(p_{KE},p_{LE},p_{EME},p_{NME},p_{RE})$$

where

p_{KE} - implicit deflator, purchases of capital services by the energy sector

p_{LE} - implicit deflator, purchases of labor services by the energy sector

p_{EME} - implicit deflator, intermediate energy products purchased by the energy sector

p_{NME} - implicit deflator, intermediate non-energy products purchased by the energy sector

p_{RE} - implicit deflator, competitive energy imports

We define the output vector as $y_E = (EC,EM)$, where

EC - supply of energy consumption products

EM - supply of energy intermediate products

For the non-energy sector we define, analogously, the input price vector as

$$w_N = (p_{KN}, p_{LN}, p_{EMN}, p_{NMN}, p_{RN})$$

where

p_{KN} - implicit deflator, purchases of capital services by the non-energy sector

p_{LN} - implicit deflator, purchases of labor services by the non-energy sector

p_{EMN} - implicit deflator, intermediate energy products purchased by the non-energy sector

p_{NMN} - implicit deflator, intermediate non-energy products purchased by the non-energy sector

p_{RN} - implicit deflator, competitive non-energy products

The output vector is defined as $y_N = (NC,NM,NI)$, where

NC - supply of non-energy consumption products

NM - supply of non-energy intermediate products

NI - supply of investment goods

To account for shifts in the production possibilities sets due to technological change, we adopt a factor augmentation model, described in detail in section 2.3.4, which establishes an explicit time dependence of the cost frontiers. It follows that the two cost possibility frontiers can be written as:

$$V_E = V_E(EC, EM, p_{KE}, p_{LE}, p_{EME}, p_{NME}, p_{RE}, t)$$

$$V_N = V_N(NC, NM, NI, p_{KN}, p_{LN}, p_{EMN}, p_{NMN}, p_{RN}, t)$$

The specification described above has several advantages from the standpoint of our overall model. Using results from the duality theory of production, we are able to obtain readily estimable expressions for the input cost shares and output expenditure shares. Moreover, the explicit incorporation of competitive imports within the overall input structure allows us to obtain import demand equations using the theory of derived factor demand. A satisfactory treatment of demand for imports is important for our objective of studying long-term policy implications of energy requirements. Our approach is less restrictive than the more conventional treatment of imports as either final goods that enter the utility function of consumers or as intermediate goods which are separable from primary factors in the productive process. Our derivation of import demands as cost-minimizing input requirement levels is analogous to the approach followed by Burgess [45] in studying aggregate import demands in the context of a value-added specification of technology.

More importantly for the study of the growth process, the specification of the production structure given by the two cost frontiers above, enables us to resolve the allocation of output as between consumption and investment without resorting to an aggregate value-added production function. This is accomplished by disaggregating the output of each sector among consumption, intermediate and -- for the non-energy sector -- investment goods. In a conventional two-sector model, the configuration of demands would entirely determine the allocation of output. Such a specification would provide no way of explaining changes in the relative prices of various categories of final demand. From the standpoint of policy instruments, in addition, our specification affords greater flexibility in allowing separate consideration of tax rates for each category of final demand in both sectors.

In selecting a specific functional form for the two cost possibility frontiers, a major consideration was to adopt a form that would admit arbitrary elasticities of substitution and transformation between pairs of inputs and outputs. We have selected the transcendental logarithmic form introduced by Christensen, Jorgenson and Lau [28] which meets the above requirements. If V_E and V_N, as before, denote the value of gross output of the energy and non-energy sectors, the translog specification of the cost frontiers results in the expressions

$$\ln V_E = f_E(EC,EM) + g_E(p_{KE},p_{LE},p_{EME},p_{NME},p_{RE},t)$$

$$+ h_E(EC,EM,p_{KE},p_{LE},p_{EME},p_{NME},p_{RE},t)$$

$$\ln V_N = f_N(NC,NM,NI) + g_N(p_{KN},p_{LN},p_{EMN},p_{NMN},p_{RN},t)$$
$$+ h_N(NC,NM,NI,p_{KN},p_{LN},p_{EMN},p_{NMN},p_{RN},t)$$

where f_E, g_E, h_E, f_N, g_N and h_N are linear-quadratic in the logarithms of arguments.

Rather than imposing separability between inputs and outputs -- i.e. assuming $h_E = h_N = 0$ -- as a maintained hypothesis, we will regard it as a hypothesis to be tested. Although the majority of econometric studies of production have enforced input-output separability a priori, we must state that a strong argument against enforced separability can be made on theoretical grounds for models with more than one productive sector. It has been demonstrated by Hall [29] that if a technology is separable it can be only trivially non-joint: i.e. there exists effectively a single sector of production. This result is analogous to that obtained by Morishima and Murata [46] who proved that the Non-Substitution theorem does not hold for a multi-sector technology with non-transferable capital stock. To show the essential equivalence of both results note that the Non-Substitution property states that the choice of technique is independent of the configuration of final demands: i.e. there is a unique set of equilibrium prices regardless of the composition of output. It can be easily seen that input-output separability implies that the marginal rates of substitution between pairs of factor inputs are independent of the configuration of output: i.e. non-substitution holds and therefore capital stock must be transferable between sectors. In summary, enforced separability between inputs and outputs in a multi-sector technology is an assumption that raises legitimate theoretical questions.

With the inclusion of explicit time-dependence to account for technological change in the manner described in Section 2.3.4, the translog cost frontier for the energy sector takes the form

$$\ln V_E = \alpha_0^E + \sum_{i=1}^{m} \alpha_i^E \ln y_i^E + \sum_{j=1}^{n} \beta_j^E \ln w_j^E$$

$$+ 1/2 \sum_{i=1}^{m} \sum_{j=1}^{m} \delta_{ij}^E \ln y_i^E \ln y_j^E$$

$$+ 1/2 \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij}^E \ln w_i^E \ln w_j^E$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \rho_{ij}^E \ln y_i^E \ln w_j^E + \sum_{i=1}^{n} \gamma_{it}^E \ln w_i^E \cdot t$$

$$+ \sum_{j=1}^{m} \delta_{it}^E \ln y_i^E \cdot t + 1/2\, \gamma_{tt}^E \cdot t^2 \qquad (2.1)$$

with m=2 and n=5.

Similarly, for the non-energy sector, the translog cost frontier is of the form:

$$\ln V_N = \alpha_0^N + \sum_{i=1}^{m} \alpha_i^N \ln y_i^N + \sum_{j=1}^{n} \beta_j^N \ln w_j^N$$

$$+ 1/2 \sum_{i=1}^{m} \sum_{j=1}^{m} \delta_{ij}^N \ln y_i^N \ln y_j^N$$

$$+ 1/2 \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij}^N \ln w_i^N \ln w_j^N$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \rho_{ij}^{N} \ln y_i^N \ln w_j^N + \sum_{i=1}^{n} \gamma_{it}^{N} \ln w_i^N \cdot t$$

$$+ \sum_{j=1}^{m} \delta_{it}^{N} \ln y_i^N \cdot t + 1/2\, \gamma_{tt}^{N} \cdot t^2 \qquad (2.2)$$

with m=3 and n=5.

To meet the first duality requirement of Theorem 2.1, the cost frontiers must satisfy

$$V^E\ (y^E;w^E) \geq 0$$

$$V^N\ (y^N;w^N) \geq 0 \qquad (2.3)$$

for $y^E \geq 0$, $w^E \geq 0$, $y^N \geq 0$, $w^N \geq 0$.

To meet the second duality requirement we must have

$$\partial \ln V_E/\partial \ln y_i^E = \alpha_i^E + \sum_{j=1}^{m} \delta_{ij}^{E} \ln y_j^E$$

$$+ \sum_{j=1}^{n} \rho_{ij}^{E} \ln w_j^E + \delta_{it}^{E} \cdot t \geq 0$$

and

$$\partial \ln V_N/\partial \ln y_i^E = \alpha_i^N + \sum_{j=1}^{m} \delta_{ij}^{N} \ln y_j^N$$

$$+ \sum_{j=1}^{n} \rho_{ij}^{N} \ln w_j^N + \delta_{it}^{N} \cdot t \geq 0 \qquad (2.4)$$

for i=1,2,...m and $w_E \geq 0$, $w_N \geq 0$ and must approach infinity as y_i^E,

y_i^N go to infinity.

To meet the third duality requirement, we must have

$$\partial \ln V_E/\partial \ln w_i^E = \beta_i^E + \sum_{j=1}^{n} \gamma_{ij}^E \ln w_i^E$$

$$+ \sum_{j=1}^{m} \rho_{ij}^E \ln y_j^E + \gamma_{it}^E \cdot t \geq 0$$

and

$$\partial \ln V_N/\partial \ln w_i^N = \beta_i^N + \sum_{j=1}^{n} \gamma_{ij}^N \ln w_i^N$$

$$+ \sum_{j=1}^{m} \rho_{ij}^N \ln y_j^N + \gamma_{it}^N \cdot t \geq 0 \qquad (2.5)$$

for i=1,2,...,n and $y_i^E \geq 0$, $y_i^N \geq 0$.

To meet the fourth duality requirement, that of linear homogeneity in input prices, the following restrictions must hold:

$$\sum_{i=1}^{n} \beta_i^E = 1 \quad \sum_{j=1}^{n} \gamma_{ij}^E = 0, \; i=1,2,...,n$$

$$\sum_{j=1}^{m} \rho_{ij}^E = 0, \; i=1,2,...,n$$

and

$$\sum_{i=1}^{n} \beta_i^N = 1 \quad \sum_{j=1}^{n} \gamma_{ij}^N = 0, \; i=1,2,...,n$$

$$\sum_{j=1}^{m} \rho_{ij}^{N} = 0, \; i=1,2,\ldots,n \tag{2.6}$$

Finally the concavity of the cost frontiers requires the Hessian matrices of second-order derivatives with respect to input prices to be negative semi-definite; i.e. we must have

$$z \, H_V^E \, z \leq 0$$

and

$$z \, H_V^N \, z \leq 0 \tag{2.7}$$

for $z > 0$ and $H_V^E = \left[\dfrac{\partial^2 V_E}{\partial p_i \partial p_j} \right]$; $H_V^N = \left[\dfrac{\partial^2 V_N}{\partial p_i \partial p_j} \right]$

2.3.3 Derived Factor Demand and Product Supply

The specification of the behavioral relationships for derived factor demands and product supplies in our model of production is based on the following:

Duality Lemma (Shepard-Uzawa-McFadden-Diewert)

If the cost frontiers V_E and V_N satisfy the conditions 2.2 through 2.7 above, then the cost-minimizing input requirement for each factor is equal to the derived factor demand obtained from the cost frontier; i.e.:

$$x_i^E = \frac{\partial V_E}{\partial w_i^E} \qquad i = 1,\ldots,5 \tag{2.8}$$

and

$$x_i^N = \frac{\partial V_N}{\partial w_i^E} \qquad i = 1,\ldots,5 \tag{2.9}$$

Furthermore, the partial derivatives of the cost frontiers with respect to output levels yield the marginal costs and are equal to the corresponding prices of output; i.e.:

$$p_j^E = \frac{\partial V_E}{\partial y_j^E} \qquad j = 1, 2 \tag{2.11}$$

$$p_j^N = \frac{\partial V_N}{\partial y_j^N} \qquad j = 1, 2, 3 \tag{2.11}$$

Applying the duality lemma to the translog cost frontiers, it follows that logarithmic differentiation of the frontiers with respect to factor prices yields expressions for the input cost shares. Analogously, logarithmic differentiation of the cost frontiers with respect to output quantities yields expressions for the revenue shares. The following sets of cost and revenue shares are obtained for the energy sector:

$$M_i^E = \frac{\partial \ln V^E}{\partial \ln w_i^E} = \beta_i^E + \sum_{j=1}^{m} \rho_{ij}^E \ln y_j^E + \sum_{j=1}^{n} \gamma_{ij} \ln w_j^E + \gamma_{it}^E \cdot t$$

$$i = 1,\ldots,n$$

$$F_j^E = \frac{\partial \ln V^E}{\partial \ln y_j^E} = \alpha_j^E + \sum_{i=1}^{m} \delta_{ij}^E \ln y_i + \sum_{i=1}^{n} \rho_{ij}^E \ln w_i^E + \delta_{jt}^E \cdot t$$

$$j = 1,\ldots,m \qquad (2.12)$$

Similarly for the non-energy sector, we have

$$M_i^N = \frac{\partial \ln V^N}{\partial \ln w_i^N} = \beta_i^N + \sum_{j=1}^{m} \rho_{ij}^N \ln y_j^N + \sum_{j=1}^{n} \gamma_{ij} \ln w_j^N + \gamma_{it}^N \cdot t$$

$$i = 1,\ldots,n$$

$$F_j^N = \frac{\partial \ln V^N}{\partial \ln y_j^N} = \alpha_j^N + \sum_{i=1}^{m} \delta_{ij}^N \ln y_i + \sum_{i=1}^{n} \rho_{ij}^N \ln w_i^N + \delta_{jt}^N \cdot t$$

$$j = 1,\ldots,m \qquad (2.13)$$

2.3.4 Technological Change

In this section we present the theoretical basis for the specification of our model of technological change. First, we briefly review some earlier studies on technical change and productivity trends.

(a) Background

The theoretical formalization of the notion of technical change in the context of growth theory can be traced back to the work of Hicks [47] and Harrod [48]. Hicks and Harrod approached the question of technical progress as an index number problem and were concerned with the problem of quantifying the effects of shifts in the production frontiers in a way appropriate for the measurement of productivity changes.

Empirical investigation of the sources of productivity growth did not rest on a firm conceptual basis until the pioneering work of Solow [49]. The results obtained by Solow appeared to confirm earlier observations by Abramowitz [50] to the effect that only a small portion of increases in measured output could be explained by corresponding changes in productive inputs. Solow based his studies on a neoclassical production function with two primary factor inputs and he assumed that technical change could be characterized by a single index taken to be equal to the index of total factor productivity.

It followed from Solow's definition that the index of technological change accounted not only for the consequences of innovations in technological know-how, but in addition, it incorporated the effects of quality improvements in physical inputs, non-constant returns to scale, external economies, measurement and aggregation biases and other influences. A large portion of the empirical literature on productivity trends during the past decade and a half has been aimed at adjusting the data and the specification of the models so as to correct for these biases and obtain a measure of "true" technological change. Several researchers have reported that a more careful specification of the input structure and of the measurement of factor intensities tends to produce a lower measure of the rate of technological change. Nevertheless, certain indeterminacies remain unresolved and the only undisputed conclusion appears to be that the alternative estimates of total factor productivity are "highly sensitive to the choice of conventions for measuring real factor inputs."*

It should be emphasized that none of the empirically relevant theories of technological change that have been put forth at the macroeconomic level can be regarded as truly endogenous. The decomposition of variations in total factor productivity into shifts occurring in the direction of the various particular inputs, as in the models of the factor-augmentation type, is an important first step but it predominantly addresses the question of the consequences rather than the causes of technological change. Kennedy's induced innovation hypothesis based on the innovation possibility frontier [51], Solow's factor embodiment model [52] and Arrow's learning-by-doing hypothesis [53] all represent useful contributions in the direction of developing an endogenous theory but have not been found to be fully satisfactory. A truly endogenous theory of technological change will have to integrate aspects pertaining to the effects of relative prices and interest rates on the rate and timing of technical change, as in the work of Lucas [54] and Nordhaus [55], the marginal social product and the extent of external economies resulting from research and development expenditures, as in the studies of Griliches [56] and Nordhaus [57], the speed and timing of the diffusion of new technologies, as treated by Salter [58] and Mansfield [59] and the effects of inter-industry structure as a source of external economies resulting in increased productivity, as in the studies of Massell [60] and Denison [61].

*Christensen and Jorgenson [62].

The vast majority of empirical studies of technical change have been carried out in the context of a value-added specification of technology and under the assumption of Hicks neutrality. The exceptions are Binswanger [63] who formulated a multi-factor model suitable for testing share-augmenting biases using a translog cost function and U.S. agricultural data, Toews [41] who studies rates of technological change in twelve U.S. manufacturing industries that utilize natural resource inputs, and Berndt and Wood [64], who formulated a factor-augmentation model for the U.S. manufacturing sector.

The specification of our model of technological change will be such as to enable us to identify biases in the rates of factor augmentation and to regard the property of Hicks neutrality as a hypothesis to be tested rather than as a maintained hypothesis. Our formulation is based on the model developed by Berndt and Jorgenson [65].

(b) Factor Augmentation, Factor Price Diminution and the Translog Cost Frontier

From the standpoint of the production function, factor augmentation models have been formulated under the assumption that technical change occurs at a constant exponential rate; i.e.

$$x^{*}_{it} = x_{it}\, e^{\mu_i t} \qquad (2.14)$$

where

x^{*}_{it} = services of input i at time t measured in augmented units.

x_{it} = services of input i at time t measured in natural units

μ_i = constant exponential rate of augmentation for input i

t = time measured with respect to a fixed reference point

From the standpoint of the dual cost function, the appropriate characterization of this model of technical change is in terms of factor price diminution. The corresponding specification for factor price diminishing technical change is

$$w^*_{it} = w_{it}\, e^{\lambda_i t} \qquad i = 1,\ldots,n \qquad (2.15)$$

where w^*_{it} and w_{it} are the prices associated with x^*_{it} and x_{it}, respectively, and λ_i is the constant rate of price diminution for input i.

We note that since equality must be preserved between the value of total input and the value of total output, it must be true that

$$w^*_i\, x^*_i = w_i\, x_i \qquad i = 1,\ldots,n \qquad (2.16)$$

i.e. the value of an input must be independent of the measure of factor augmentation. For this to hold, it is necessary that $\lambda_i = -\mu_i$, i.e., the rate of price diminution is the negative of the rate of factor augmentation.

We can now proceed to reinterpret the expressions for the translog cost frontiers of Eqs. (2.1) and (2.2) which are given in terms of prices corresponding to inputs measured in natural units. The time dependence in both cost frontiers can be thought of as having originated from a model formulated in terms of prices subject to a constant rate of diminution as in Eq. (2.15), i.e.

$$V_E\ (EC,EM,p^*_{KE},p^*_{LE},p^*_{EME},p^*_{NME},p^*_{RE})$$

$$= V_E\ (EC,EM,p_{KE},p_{LE},p_{EME},p_{NME},p_{RE},t) \qquad (2.17)$$

and

$$V_N\ (NC,NM,NI,p^*_{KN},p^*_{LN},p^*_{EMN},p^*_{NMN},p^*_{RN})$$

$$= V_N(NC,NM,NI,p_{KN},p_{LN},p_{EMN},p_{NMN},p_{RN},t) \qquad (2.18)$$

Assuming a model of factor price diminishing technical change of the form

$$p^*_{iE} = p_{iE}\, e^{\lambda^E_i} \qquad i = K,L,E,N,R$$

and

$$p^*_{iN} = p_{iN}\, e^{\lambda^N_i} \qquad i = K,L,E,N,R \qquad (2.19)$$

we obtain the following relationships between the exponential rates of price diminution and the parameters associated with the time variable in Eqs. (2.1) and (2.2)

$$\gamma^E_{Kt} = \gamma_{KK}\,\lambda^E_K + \gamma_{KL}\,\lambda^E_L + \gamma_{KE}\,\lambda^E_E + \gamma_{KN}\,\lambda^E_N + \gamma_{KR}\,\lambda^E_R$$

$$\gamma^E_{Lt} = \gamma_{LK}\,\lambda^E_K + \gamma_{LL}\,\lambda^E_L + \gamma_{LE}\,\lambda^E_E + \gamma_{LN}\,\lambda^E_N + \gamma_{LR}\,\lambda^E_R$$

$$\gamma^E_{Et} = \gamma_{EK}\,\lambda^E_K + \gamma_{EL}\,\lambda^E_L + \gamma_{EE}\,\lambda^E_E + \gamma_{EN}\,\lambda^E_N + \gamma_{ER}\,\lambda^E_E$$

$$\gamma^E_{Nt} = \gamma_{NK}\,\lambda^E_K + \gamma_{NL}\,\lambda^E_L + \gamma_{NE}\,\lambda^E_E + \gamma_{NN}\,\lambda^E_N + \gamma_{NR}\,\lambda^E_R$$

$$\gamma^E_{Rt} = \gamma_{RK}\,\lambda^E_K + \gamma_{RL}\,\lambda^E_L + \gamma_{RE}\,\lambda^E_E + \gamma_{RN}\,\lambda^E_N + \gamma_{RR}\,\lambda^E_R$$

$$\delta^E_{Ct} = \rho_{CK}\,\lambda^E_K + \rho_{CL}\,\lambda^E_L + \rho_{CE}\,\lambda^E_E + \rho_{CN}\,\lambda^E_N + \rho_{CR}\,\lambda^E_R$$

$$\delta^E_{Mt} = \rho_{MK}\,\lambda^E_K + \rho_{ML}\,\lambda^E_L + \rho_{ME}\,\lambda^E_E + \rho_{MN}\,\lambda^E_N + \rho_{MR}\,\lambda^E_R \qquad (2.20)$$

and

$$\gamma^N_{Kt} = \gamma_{KK}\,\lambda^N_K + \gamma_{KL}\,\lambda^N_L + \gamma_{KE}\,\lambda^N_E + \gamma_{KN}\,\lambda^N_N + \gamma_{KR}\,\lambda^N_R$$

$$\gamma^N_{Lt} = \gamma_{LK}\,\lambda^N_K + \gamma_{LL}\,\lambda^N_L + \gamma_{LE}\,\lambda^N_E + \gamma_{LN}\,\lambda^N_N + \gamma_{LR}\,\lambda^N_R$$

$$\gamma^N_{Et} = \gamma_{EK}\,\lambda^N_K + \gamma_{EL}\,\lambda^N_L + \gamma_{EE}\,\lambda^N_E + \gamma_{EN}\,\lambda^N_N + \gamma_{ER}\,\lambda^N_R$$

$$\gamma^N_{Nt} = \gamma_{NK}\,\lambda^N_K + \gamma_{NL}\,\lambda^N_L + \gamma_{NE}\,\lambda^N_E + \gamma_{NN}\,\lambda^N_N + \gamma_{NR}\,\lambda^N_R$$

$$\gamma^N_{Rt} = \gamma_{RK}\,\lambda^N_K + \gamma_{RL}\,\lambda^N_L + \gamma_{RE}\,\lambda^N_E + \gamma_{RN}\,\lambda^N_N + \gamma_{RR}\,\lambda^N_R$$

$$\delta_{Ct}^{N} = \rho_{CK}\,\lambda_{K}^{N} + \rho_{CL}\,\lambda_{L}^{N} + \rho_{CE}\,\lambda_{E}^{N} + \rho_{CN}\,\lambda_{N}^{N} + \rho_{CR}\,\lambda_{R}^{N}$$

$$\delta_{Mt}^{N} = \rho_{MK}\,\lambda_{K}^{N} + \rho_{ML}\,\lambda_{L}^{N} + \rho_{ME}\,\lambda_{E}^{N} + \rho_{MN}\,\rho_{N}^{N} + \rho_{MR}\,\lambda_{R}^{N} \qquad (2.21)$$

2.4 The Structure of Consumer Behavior

This section describes the theoretical background and the functional specification of the behavioral relationships embodying the structure of consumer preferences, which constitute the basis for our model of the household sector.

2.4.1 Background

Our point of departure for the specification of a model for the household sector is the characterization of consumer behavior in terms of satisfaction maximization given prices, preferences and resources. The first step in our approach is the development of a framework in which consumer preferences are assumed to be embodied in a differentiable cardinal utility function. Next, we decompose the overall maximization problem into a multi-level structure focusing on the inter-temporal and intra-temporal preference structures of consumers. The inter-temporal model establishes the allocation of wealth as between the current period and all future periods. The rate of return on wealth provides the link between forward and future prices so that the inter-temporal allocation function can be viewed as providing the basis for a theory of saving for the household sector. The intra-temporal model leads to a set of demand functions for energy, non-energy, capital services and leisure that characterize the expenditure and working patterns of households given the total level of expenditures in the current period. We complete the specification of our model for the household sector by expressing the resulting behavioral relationships in terms of functional forms suitable for empirical implementation.

Our selection of the neoclassical theory of consumer behavior as the basis for our model is at variance with traditional practice in macroeconomic modeling, which has tended to rely on ad-hoc specifications of consumer demand functions rather than on the utility maximization hypothesis so prevalent in microeconomic analysis. Moreover, empirical studies of consumer behavior in the past have assumed either that households maximize a static utility function in each period without reference to other time periods -- as in demand studies of specific commodities -- or else that inter-temporal allocation can be expressed in terms of a utility function having single commodity aggregates as arguments -- as in the studies referred to as the modern theories of consumption.

The choice of a cardinal utility function as the basis for our model was a consequence of several considerations. On theoretical grounds, utility maximization can be shown to be consistent with the leading theories of consumer behavior: e.g. the theory of revealed preference (Samuelson [66]), the life-cycle hypothesis (Modigliani and Ando [67]) and the permanent income hypothesis (Friedman [68]). The Keynesian expenditure function (Keynes [69]), the Cambridge savings hypothesis (Robinson [70]) and the Pigou effect (Pigou [71]) can also be derived as special cases from the postulate of utility-maximizing behavior. Furthermore, a binary choice relation can be shown to imply behavior patterns derivable from an underlying utility function (Debreu [72], Houthakker [73]).

On empirical grounds, the approach we have selected leads to an empirically estimable set of demand functions. Finally, a cardinal utility function has the additional advantage of permitting, under appropriate conditions, the inter-temporal aggregation of utility. This property will be especially useful when we use the utility function of the household sector as the basis for the specification of the welfare functional in the optimal growth studies of Chapter 5.

The nature of the inter-temporal allocation problem resulting from the multi-level decomposition of utility maximization is entirely analogous to that considered by Koopmans [74] in his studies of preference structures defined on infinite consumption programs arising in the context of the theory of optimal economic growth. Koopmans introduced the terms immediate utility and prospective utility to refer to the preference structure associated with choices between consumption in the present and consumption in the entire future.

The methodological basis for our hierarchical decomposition into inter-temporal and intra-temporal components is rooted in a series of results on functional structure and consumer budgeting that has evolved during the past three decades. The conceptual framework can be traced back to the work of Leontief [75] and Sono [76] on functional structure and to the subsequent studies on functional separability by

Strotz [77], [78] and Gorman [79] in the context of consumer budgeting. These studies on separability and the analysis of the restrictions on the system of demand functions that they imply, have been subsequently extended by Goldman and Uzawa [80], Green [81] and Blackorby, Lady, Nissen and Russell [82]. The notion of strong recursiveness was introduced by Lady and Nissen [83] in order to extend the treatment of separability to the case of asymmetric structures induced by the time ordering of the arguments of the utility function. This succession of studies have culminated in a set of results that have important implications for both the question of hierarchical consistency -- i.e. whether the two-level decomposition will yield the same allocation patterns as the original global maximization -- and for the question of inter-temporal consistency -- i.e. whether the structure of inter-temporal preferences is such that it can be inferred from ex-post observations of consumer behavior. The study of functional structure also has important implications for the issue of inter-period aggregation of utility, as studied by Koopmans [74] and Morishima [75]. Several of these various sets of results will be reviewed in sub-section 2.4.2.

The specification of functional forms for our model of the household sector is based on the trascendental logarithmic utility functions introduced by Christensen, Jorgenson and Lau [85]. The translog function has been employed in empirical studies of consumer behavior by Jorgenson and Lau [86], Jorgenson [9], [87] and Christensen and Manser [88].

2.4.2 Utility, Functional Separability and Aggregation

In this section we review several results from the theory of consumer behavior that will be required in establishing necessary and sufficient conditions for the validity of our multi-level characterization of consumer preferences described in 2.4.3. First we review the implications of separability restrictions in terms of the existence of theoretically consistent price and quantity indices. We then examine the implications of functional structure on the hierarchical consistency and the inter-temporal consistency of the overall allocation problem. Finally we present some results on inter-period aggregation of utility that are generalizations of earlier results obtained by Koopmans [74] and Morishima [84].

The underlying motivation for the examination of these questions stems from our assumption that decisions faced by households can be stated in terms of selecting a consumption program

$$X^t = [x_{1t}, x_{2t}, \ldots, x_{1t+1}, x_{2t+1}, \ldots] \tag{2.22}$$

so as to maximize a cardinal utility $U(X^t)$ subject to the constraint

$$\sum_{\tau=0}^{\infty} \sum_{i=1}^{n} p_{it+\tau} \, x_{it+\tau} = M_t \tag{2.23}$$

where M_t is the present value of total resources of the household sector and $p_{it+\tau}$ are expected forward prices.

Before proceeding with our review of functional structure and consumer budgeting, we require some preliminary notions.

Let $Z = \{z: z = (z_1, z_2, \ldots), z_i \in \mathbb{R}^+\}$

and Γ Z. Let $U: \Gamma \to \mathbb{R}^+$ be a continuous, non-decreasing, strictly quasi-concave <u>utility function</u> mapping Γ into the non-negative real line.

<u>Definition 2.1</u>

Let $\Lambda = \{1, 2, \ldots\}$ denote the index set of variables in U and let $\{\Lambda^s, \Lambda^c\}$ be a partition of Λ. The group of variables Λ^s is said to be <u>separable</u> from the k-th variable Λ_k in U if and only if for any set Z_s^* of values of the variables in Λ_s, the correspondence

$$B\,(Z_s^*, z_c) = \{z_s \in \Gamma_s: U\,(z_s, z_c) \geq U\,(Z_s^*, z_c)\}$$

is independent of z_k, the value of the k-th variable.

If U is differentiable, the definition implies that the marginal rates of substitution

$$M_{ij} = \frac{\partial U/\partial z_i}{\partial U/\partial z_j}$$

be independent of z_k for all $i, j \in \Lambda_s$. If U is twice differentiable, this is equivalent to the condition that

$$\frac{\partial M_{ij}}{\partial z_k} = 0$$

for all $i, j \in \Lambda_s$.

Let Λ be partitioned into a set of disjoint sectors

$\Pi = \{\Lambda_1, \Lambda_2, \ldots\}.$

The following definitions will be useful in the sequel.

Definition 2.2

Consumer preferences are said to be weakly separable with respect to a partition Π if every sector $\Lambda_i \in \Pi$ is separable from the variables in all the other sectors.

Definition 2.3

Consumer preferences are said to be strongly separable with respect to a partition Π if any proper subset of Π is separable from its complement.

These properties of the structure of consumer preferences can be shown to imply specific restrictions on the forms of the utility functions that embody these preferences.

Theorem 2.2 (Leontief-Goldman-Uzawa)

Consumer preferences are weakly separable if and only if there exist continuous functions F, f_1, f_2, ... such that

$$U(z_1, z_2, \ldots) = F(f_1(z_1), f_2(s_2), \ldots)$$

Theorem 2.3 (Goldman-Uzawa)

Consumer preferences are strongly separable if and only if there exist continuous functions F, f_1, f_2, ... such that

$$U(z_1, z_2, \ldots) = F(f_1(z_1) + f_2(z_2) + \ldots)$$

Definition 2.4

If in theorems 2.2 and 2.3 the category satisfaction functions f_1, f_2, ... are homothetic, then the utility function is said to be weakly homothetically separable and strongly homothetically separable, respectively.

Definition 2.5

Consider a utility function $U(Z) = U(z_1, z_2, \ldots)$ with index set $\Lambda = \{1, 2, \ldots\}$ and let $\Pi = \{\Lambda_1, \Lambda_2, \ldots\}$ be a partition of Λ.

Consumer preferences are said to have the property of strong additive price aggregation with respect to the partition Π if there exist price and quantity indices

$$p_{\lambda_i} = f_i \; (p_{z_{\lambda_{ij}}}, \; j=1, 2, \ldots, n_i) \tag{2.24}$$

and

$$\underline{z}_{\lambda_i} = g_i \; (z_{\lambda_{ij}}, \; j=1, 2, \ldots n_i) \tag{2.25}$$

such that

$$\sum_{j=1}^{n_i} p_{\lambda_{ij}} \, z_{\lambda_{ij}} = s_i = p_{\lambda_i} \, \underline{z}_{\lambda_i}$$

for all $i = 1, 2, \ldots$.

Theorem 2.4 (Blackorby-Lady-Nissen-Russell)

A necessary and sufficient condition for consumer preferences to have the property of strong additive price aggregation with respect to a partition Π is that they be weakly homothetically separable with respect to Π.

Definition 2.6

Consider a utility maximization problem defined by the utility function $U(Z) = U(z_1, z_2, \ldots)$ and the budget constraint

$$\sum_i p_{z_i} \; z_i = M$$

Let $\Lambda = \{1, 2, \ldots\}$ be the index set of U and $\Pi = \{\Lambda_1, \Lambda_2, \ldots\}$ a partition of Λ. Assume that U is weakly separable with respect to Λ. A decomposition of this maximization problem with respect to Π is a pair (P1, P2) of maximization problems defined by

P1 - Maximize $U(\underline{z}_{\lambda_1}, \underline{z}_{\lambda_2}, \ldots)$

subject to

$$\sum_i p_{\lambda_i} \underline{z}_{\lambda_i} = M$$

where $p_{\lambda_i} = f_i$ and $z_{\lambda_i} = g_i$ as in Eqs. (2.24) and (2.25)

P2 - Maximize $u_{\lambda_i} \; (z_{\lambda_{i1}}, z_{\lambda_{i2}}, \ldots z_{\lambda_{ini}})$

subject to

$$\sum_{j=1}^{n_j} p_{\lambda_{ij}} z_{\lambda_{ij}} = p_{\lambda_i} \underline{z}_{\lambda_i}$$

for i = 1, 2, ..., where

$$U(\underline{z}_{\lambda_1}, \underline{z}_{\lambda_2}, \ldots) = f(u_{\lambda_1} (\underline{z}_{\lambda_1}), u_{\lambda_2} (\underline{z}_{\lambda_2}), \ldots,)$$

Definition 2.7

Let a utility maximization problem (U,M) be given. Let Z^* be the corresponding optimal allocation . Given a partition Π and a decomposition (P_1, P_2), consumer preferences are said to be hierarchically consistent with respect to the partition Π if the solutions to the sub-problems P_1 and P_2, i.e.

$$\underline{z}^* = (\underline{z}^*_1, \underline{z}^*_2, \ldots)$$

and

$$z^*_{\lambda_i} (\underline{z}^*_\lambda) = (z^*_{\lambda_{i1}} (\underline{z}^*), z^*_{\lambda_{i2}} (\underline{z}^*), \ldots z_{\lambda_{in_i}} (\underline{z}^*))$$

$$i = 1, 2, \ldots$$

satisfy

$$(z^*_{\lambda_1} (\underline{z}^*), z^*_{\lambda_2} (\underline{z}^*), \ldots) = z^*$$

Theorem 2.5 (Blackorby-Lady-Nissen-Russell)

Consumer preferences are hierarchically consistent with respect to a partition Π if and only if they are weakly homothetically separable with respect to Π.

The preceeding set of results have focused on the multi-level structure of consumer preferences and the associated conditions for consistent aggregation. In the remainder of this section we will discuss conditions regarding the inter-temporal structure of consumer preferences.

Assume given a utility function of the form

$$U = U(x_{1t}, x_{2t}, \ldots, x_{nt}, x_{1t+1}, x_{2t+1}, \ldots, x_{nt+1}, \ldots$$
$$x_{1t+\tau}, x_{2t+\tau}, \ldots x_{nt+\tau})$$

and let U be weakly separable with respect to the ordered partition $\Lambda = \{\Lambda_t, \Lambda_{t+1}, \ldots, \Lambda_{t+\tau}\}$ induced by the time index τ, i.e. $\Lambda_\tau = \{1t+\tau, 2t+\tau, \ldots, nt+\tau\}$. We rewrite U as

$$U = U(x_t, x_{t+1}, \ldots, x_{t+\tau}, \ldots)$$

A <u>consumption program</u> X is an infinite sequence of consumption vectors: $X = \{x_t, x_{t+1}, \ldots, x_{t+\tau}, \ldots\}$. A <u>consumption segment</u> $X_{\tau_2}^{\tau_1}$ is a sub-program starting at time τ_1 and ending at time τ_2-1, i.e. $X_{\tau_2}^{\tau_1} = \{x_{t+\tau_1}, x_{t+\tau_1+1}, \ldots, x_{t+\tau_2-1}\}$.

The consumption segments $X_{\tau_2}^{0}$ and $X_{\infty}^{\tau_1}$ will be denoted by X_{τ_2} and X^{τ_1} respectively; thus $X = X_{\infty}^{0}$.

The utility function of order τ, u_τ, is the restriction of U to the subset generated by the segments $X_{\tau-1}$, i.e.

$$u_1 = u_1 (x_t)$$

$$u_2 = u_2 (x_t, x_{t+1})$$

$$u_\tau = u_\tau (x_t, x_{t+1}, \ldots, x_{t+\tau-1})$$

Similarly, the utility function of co-order τ, u^τ is the restriction of U to the subset generated by the segments X^τ, i.e.

$$u^\tau = u^\tau (x_{t+\tau}, x_{t+\tau+1}, \ldots)$$

Given the ordered partition $\Pi = \{\Lambda_t, \Lambda_{t+1}, \ldots\}$ the continuation of Λ_τ is the subset $\Pi_\tau = \{\Lambda_{t+\tau}, \Lambda_{t+\tau+1}, \ldots\}$; the segment corresponding to this continuation is, of course X^τ.

Definition 2.8

Given a utility function U and an ordered partition $\Pi = \{\Lambda_t, \Lambda_{t+1}, \ldots$, consumer preferences are paid to be strongly recursive with respect to Π if and only if each continuation Π_τ, $\tau = 1, 2, \ldots$ is separable from the variables in the preceding sectors $\Lambda_t, \Lambda_{t+1}, \ldots, \Lambda_{t+\tau-1}$.

The condition of strong recursiveness is less restrictive than that of strong separability and was proposed by Lady and Nissen [83] in order to account for the asymmetry introduced by the element of time. Similar notions had been discussed earlier by Koopmans [74] and others. The following definition relates to a notion that has important implications for the empirical implementation of an intertemporal system of consumer preferences.

Definition 2.9

Given a representation of consumer preferences in terms of a utility function U, let

$$X^{\tau_1} = [x_{t+\tau_1}, x_{t+\tau_1+1}, \ldots]$$

and

$$\underline{X}^{\tau_1} = [\underline{x}_{t+\tau_1}, \underline{x}_{t+\tau_2+1}, \ldots]$$

be segments such that for the utility functions of co-order τ_1, the following holds:

$$u^{\tau_1}(X^{\tau_1}) = u^{\tau_1}(\underline{X}^{\tau_1})$$

Given any two segments X_{τ_1} and $\underline{X}_{\tau_1}$, define the consumption programs $X = [X_{\tau_1}, X^{\tau_1}]$ and $\underline{X} = [\underline{X}_{\tau_1}, \underline{X}^{\tau_1}]$.

Consumer preferences are said to be <u>intertemporally</u> <u>consistent</u> if for $\tau_1 = 1, 2, \ldots$ the condition

$$U(X) < U(\underline{X})$$

$$(\text{resp. } U(X) = U(\underline{X}),\ U(X) > U(\underline{X}))$$

holds if and only if

$$u_{\tau_1}\ (X_{\tau_1}) < u_{\tau_1}\ (\underline{X}_{\tau_1})$$

$$(\text{resp. } u_{\tau_1}\ (X_{\tau_1}) = u_{\tau_1}\ (\underline{X}_{\tau_1}),\ u_{\tau_1}\ (X_{\ 1}) > u_{\tau_1}\ (\underline{X}_{\tau_1}))$$

<u>Theorem 2.6</u> (Blackorby-Nissen-Primont-Russell)

A necessary and sufficient condition for consumer preferences to be inter-temporally consistent is that they be strongly recursive with respect to the partition induced by the time ordering.

<u>Definition 2.10</u>

Given a representation of consumer preferences in terms of a utility function U, let

$$X_{\tau_1} = [x_t,\ x_{t+1},\ \ldots,\ x_{t+\tau_1}]$$

and

$$\underline{X}_{\tau_1} = [\underline{x}_t,\ \underline{x}_{t+1},\ \ldots,\ \underline{x}_{t+\tau_1}]$$

be segments such that for the utility functions of order τ_1 the following holds:

$$u_{\tau_1}\ (X_{\tau_1}) = u_{\tau_1}\ (\underline{X}_{\tau_1})$$

Given any two segments X^{τ_1} and $\underline{X}^{\tau_1}$, define the consumption programs

$X = [X_{\tau_1}, X^{\tau_1}]$ and $\underline{X} = [\underline{X}_{\tau_1}, \underline{X}^{\tau_1}]$

Consumer preferences are said to be stationary if for $\tau_1=1,2,\ldots,$ the condition

$U(X) \leq U(\underline{X})$

holds if and only if

$u^{\tau_1}(X^{\tau_1}) \leq u^{\tau_1}(\underline{X}^{\tau_1})$

The following theorem on inter-period aggregation of utility is slightly less restrictive than earlier results obtained by Koopmans and Morishima and gives sufficient conditions for a representation of an intertemporal structure of consumer preferences that will be especially usefully in the study of optimal growth paths in Chapter 5.

Theorem 2.7

Given a representation of consumer preferences in terms of a utility function U, there exist functions $u_\tau\ (x_{t+\tau})$ such that

$$U(X) = \sum_{\tau=0}^{\infty} \frac{1}{\eta^\tau} u_\tau\ (x_{t+\tau}) \qquad \eta > 0$$

whenever consumer preferences are strongly recursive and stationary.

Proof.- From Theorem 2.6 it follows that consumer preferences are intertemporally consistent. The proof then follows that of Koopmans [74] and Morishima [84] with strong recursiveness substituting for strong separability.

The conditions of the following theorem are still less restrictive in terms of functional structure and the additional requirement on price variations is often satisfied by expected forward prices.

Theorem 2.8

If an inter-temporal utility function $U = U(x_t, x_{t+1}, \ldots, x_{t+\tau}, \ldots)$ is stationary, weakly separable and forward prices $(p_{t+1}, p_{t+2}, \ldots, p_{t+\tau}, \ldots)$ are always proportional then there exist functions $u_\tau(x_{t+\tau})$ such that

$$U(X) = \sum_{\tau=0}^{\infty} \frac{1}{\eta^\tau} u_\tau(x_{t+\tau}) \qquad \eta > 0$$

Proof: -

The assumption that forward prices change proportionally implies by using the Leontief-Hicks composite-good theorem (Wold [89]) that all future goods can be regarded as a single good for $\tau > 0$, i.e. we can write

$$U(X) = U(x_t, f_1(X^1))$$

By similar arguments we can express $f_1(X^1)$ as

$$f_1(X^1) = f_1(x_{t+1}, x_{t+2}, \ldots) = f_1(x_{t+1}, f_2(X^2))$$

Repeating the argument, we can construct a sequence f_i, $i=1,2,\ldots$ such that

$$f_i = f_i(x_{t+i}, f_{i+1}(X^{i+1})) \qquad i=1,2,\ldots$$

It follows from Lady and Nissen [83] that U is strongly recursive. The proof is complete by invoking Theorem 2.7

2.4.3 Hierarchical Decomposition of Utility Maximization

The point of departure for our model of the household sector is the assumption that the decisions faced by consumers in period t can be characterized in terms of the maximization of the utility function

$$U = U(x_t, x_{t+1}, \ldots, x_{t+\tau}, \ldots) \tag{2.26}$$

where

$$x_t = (CE_t/P_t, CN_t/P_t, CK_t/P_t, LJ_t/P_t)$$

and

CE - Energy consumption goods purchased by households

CN - Non-energy consumption goods purchased by households

CK - Capital services supplied to households

LJ - Leisure time of the household sector

P - Population

subject to the budget constraint:

$$M_t = \sum_{\tau=0}^{\infty} \beta_{t+\tau} \left(p_{CE}^{t+\tau} CE_{t+\tau} + p_{CN}^{t+\tau} CN_{t+\tau} + p_{CK}^{t+\tau} CK_{t+\tau} + p_{L}^{t+\tau} LJ_{t+\tau}\right) \tag{2.27}$$

where $\beta_{t+\tau}$ is the discount factor for the household sector at time t in the future, and

p_{CE} - Implicit deflator, energy consumption goods purchased by households

p_{CN} - Implicit deflator, n‹ ‹-energy consumption goods purchased by households

p_{CK} - Implicit deflator, capital services supplied to households

p_L - Implicit deflator, supply of labor services

We observe that the leisure time available to households is evaluated at the opportunity cost of time resources, taken to be equal to the after-tax wage rate p_L.

The above specification can be justified either by regarding the household sector as an economic entity by itself or by interpreting the arguments of the utility function as variables that have been aggregated over all households and then expressed in per capita terms. The choice of an infinite planning horizon is a result of viewing the household sector as an entity which lives perpetually although the composition of its membership constantly changes.

We assume that consumer preferences are stationary and that they are weakly homothetically separable with respect to the partition Π induced by the time index. If follows from Theorem 2.4 that consumer preferences have the property of strong additive price aggregation: i.e. there exist price and quantity indices

$$p_F^t = f_t\ (p_{CE}^t,\ p_{CN}^t,\ p_{CK}^t,\ p_L^t)$$

$$F_t = g_t\ (CE_t,\ CN_t,\ CK_t,\ LJ_t) \qquad (2.28)$$

such that

$$U = \underline{U}(F_t/P_t, F_{t+1}/P_{t+1} \cdots , F_{t+\tau}/P_{t+\tau}) \qquad (2.29)$$

and

$$p_F^{t+\tau} F_{t+\tau} = B_{t+\tau} = p_{CE}^{t+\tau} CE_{t+\tau} + p_{CN}^{t+\tau} CN_{t+\tau} + p_{CK}^{t+\tau} CK_{t+\tau} + p_L^{t+\tau} LJ_{t+\tau} \qquad (2.30)$$

The indices p_F^t and F_t are referred to as the price and quantity indices of full consumption in period t, respectively.

From Theorem 2.5 it follows that a hierarchical decomposition of the utility maximization problem given by Eqs. 2.26 and 2.27 is hierarchically consistent. We therefore define the following utility maximization problems:

Inter-temporal Allocation Problem

Find a full consumption path

$$F^t = (F_t, F_{t+1}, \ldots, F_{t+\tau}, \ldots)$$

that maximizes $U = \underline{U}(F_t/P_t, F_{t+1}/P_{t+1}, \ldots, F_{t+\tau}/P_{t+\tau}, \ldots)$ subject to:

$$\sum_{\tau=0}^{\infty} \beta_{t+\tau} \, p_F^{t+\tau} \, F_{t+\tau} = M_t \qquad (2.31)$$

Intra-Temporal Allocation Problem

Find a consumption vector

$$C^{t+\tau} = (CE_{t+\tau}, CN_{t+\tau}, CK_{t+\tau}, LJ_{t+\tau})$$

that maximizes for $\tau = 0, 1, \ldots$

$$U_{t+\tau} = U_{t+\tau}(CE_{t+\tau}/P_{t+\tau}, CN_{t+\tau}/P_{t+\tau}, CK_{t+\tau}/P_{t+\tau}, LJ_{t+\tau}/P_{t+\tau})$$

subject to the constraint given by Eq. 2.30.

2.4.4 The Inter-Temporal Allocation Model

Our first step in deriving an estimable specification of the inter-temporal allocation model will be to express the inter-temporal budget constraint (2.31) in terms of observable data. The second step will be to select a specific functional form for the utility function embodying inter-temporal preferences.

The budget constraint for period t is given by non-human wealth at the end of the previous period, plus the discounted value of all present and future time endowments plus the discounted value of all present and future transfer payments. Thus if the discounted value of full consumption throughout the planning horizon is

$$\sum_{\tau=0}^{\infty} \beta_{t+\tau}\, p_F^{t+\tau}\, F_{t+\tau} = M_t \qquad (2.32)$$

then

$$M_t = W_{t-1} + \sum_{\tau=0}^{\infty} \beta_{t+\tau}\, (p_L^{t+\tau}\, LH_{t+\tau} + EL_{t+\tau}) \qquad (2.33)$$

where

$$\beta_t = (1/1+MW)$$

$$\beta_{t+\tau} = (1/1+\gamma)^{\tau}\ \beta_t \qquad \tau = 1, 2, \ldots \qquad (2.34)$$

and

W - Private national wealth

p_L - Implicit deflator, supply of labor services

LH - Total time endowment of the household sector

EL - Government transfer payments to households

MW - Nominal rate of return, private national wealth

In order to express the budget constraint in terms of variables in the current period, we must make assumptions about the expected future prices, expected future time endowments and expected future transfer payments. In particular, we assume:

(a) Future prices are the same as present prices:

$$p_{CE}^{t+\tau} = p_{CE}^{t}$$

$$p_{CN}^{t+\tau} = p_{CN}^{t}$$

$$p_{CK}^{t+\tau} = p_{CK}^{t}$$

$$p_{L}^{t+\tau} = p_{L}^{t} \qquad \tau = 1, 2, \ldots \qquad (2.35)$$

(b) Future time endowments grow geometrically:

$$LH_{t+\tau} = (1+\eta)^{\tau}\ LH_t \qquad \tau = 1, 2, \ldots \qquad (2.36)$$

(c) Future transfer payments grow geometrically at the same constant rate :

$$EL_{t+\tau} = (1+\eta)^{\tau} EL_t \qquad \tau = 1, 2, \ldots \qquad (2.37)$$

Using these assumptions, together with the expression for future discount rates given by Eq. (2.34) we can rewrite the budget constraint (2.33) as follows:

$$M_t = (1 + MW)\, W_{t-1} + \mu[p_L^t\, LH_t + EL_t] \qquad (2.38)$$

where

$$\mu = \sum_{\tau=0}^{\infty} \left(\frac{1}{1+\gamma}\right) \quad (1+\eta)^{\tau} = \frac{1+\gamma}{\gamma-\eta} \qquad (2.39)$$

Because of the assumption about constant future prices given by 2.35, it follows that all forward prices change in direct proportion to the future discount rates. The conditions of Theorem 2.8 are therefore satisfied and we can write

$$U(F^t) = \sum_{\tau=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{\tau} u_{\tau}\, (F_{t+\tau}\, / P_{t+\tau}) \qquad (2.40)$$

where δ is the rate of pure time preference; i.e. the subjective rate of social discount.

We will assume that inter-temporal preferences can be adequately approximated by a neutral transcendental logarithmic utility function. Following the convention adopted by Christensen, Jorgenson and Lau [ ] for direct utility functions, we approximate the negative of the logarithm of the utility function as follows:

$$-\ln U(F^t) = \sum_{\tau=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{\tau} \left[\alpha_0 \ln \frac{F_{t+\tau}}{P_{t+\tau}} + \frac{1}{2}\beta_{00} \left(\ln \frac{F_{t+\tau}}{P_{t+\tau}}\right)^2 + \beta_{\tau t} \ln \frac{F_{t+\tau}}{P_{t+\tau}} \cdot t\right] \quad (2.41)$$

The translog approximation must satisfy the monotonicity and quasi-convexity restrictions imposed by the theory of demand. Maximization of utility subject to the budget constraint leads to the following identity by applying the first-order conditions for stationarity to the Lagrangian:

$$\frac{\partial \ln U}{\partial \ln F_t} = \frac{p_F^t F_t}{M_t} \sum_{\tau=0}^{\infty} \frac{\partial \ln U}{\partial \ln F_{t+\tau}} \quad (2.42)$$

Applying this relationship to the translog utility function given in (2.41), we obtain the following allocation equation:

$$\frac{p_F^t F_t}{M_t} = \frac{\alpha_0 + \beta_{00} \ln(F_t/P_t) + \beta_{0t} \cdot t}{\alpha_M + \beta_{M0} \ln(F_t/P_t) + \beta_{Mt} \cdot t} \quad (2.43)$$

where

$$\alpha_M = \sum_{\tau=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{\tau} \alpha_0 = \left(\frac{1+\delta}{\delta}\right) \alpha_0 \quad (2.44)$$

$$\beta_{M0} = \sum_{\tau=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{\tau} \beta_{00} = \left(\frac{1+\delta}{\delta}\right) \beta_{00} \quad (2.45)$$

and

$$\beta_{Mt} = \sum_{\tau=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{\tau} \beta_{\tau t} \quad (2.46)$$

Finally, substituting the expression for M_t from (2.38) into (2.44), we obtain

$$p_F^t F_t = \frac{\alpha_0 + \beta_{00} \ln(F_t/P_t) + \beta_{0t} \cdot t}{\alpha_M + \beta_{M0} \ln(F_t/P_t) + \beta_{Mt} \cdot t} \cdot [(1+MW)W_{t-1} + \mu(p_L^t LH_t + EL_t)] \tag{2.47}$$

This form of the inter-temporal allocation function gives the value of wealth allocated to consumption in the current period as a function of observable quantities and of parameters to be estimated.

2.4.5 The Intra-Temporal Allocation Model

The intra-temporal allocation model is aimed at explaining the decisions made by consumers in the current period t concerning their expenditure levels for energy products CE, non-energy CN, capital services CK and leisure LJ, given the level of total current expenditures $B_t = p_F^t F_t$ and the prices of the three commodity groups and labor.

We assume that the intra-period utility function

$$U_t = U_t(CE_t/P_t, CN_t/P_t, CK_t/P_t, LJ_t/P_t) \tag{2.48}$$

is a function of per capita levels of consumption and leisure and can be approximated by a direct transcendental logarithmic utility function with time-varying preferences. Specifically we express the negative of the logarithm of the intra-period utility function as:

$$-\ln U_t = \alpha_0^C + \alpha_E^C \ln CE_t/P_t + \alpha_N^C \ln CN_t/P_t + \alpha_K^C \ln CK_t/P_t$$
$$+ \alpha_J^C \ln LJ_t/P_t + \tfrac{1}{2}[\beta_{EE}^C(\ln CE_t/P_t)^2 + \beta_{EN}^C \ln CE_t/P_t \ln CN_t/P_t$$
$$+ \dots + \beta_{tt}^C \cdot t^2] \tag{2.49}$$

Applying the necessary conditions for stationarity to Eq. (2.49) subject to the budget constraint

$$p^t_{CE}\ CE_t + p^t_{CN}\ CN_t + p^t_{CK}\ CK_t + p^t_L\ LJ_t = B_t = p^t_F F_t \qquad (2.50)$$

we obtain the following expressions for the expenditure shares of each commodity group and leisure:

$$\frac{p^t_{CE} CE_t}{B_t} = \frac{\alpha^C_E + \sum_{i=1}^{4} \beta^C_{Ei} \ln x^i_t + \beta^C_{Et} \cdot t}{\alpha^C_M + \sum_{i=1}^{4} \beta^C_{Mi} \ln x^i_t + \beta^C_{Mt} \cdot t} \qquad (2.51)$$

$$\frac{p^t_{CN} CN_t}{B_t} = \frac{\alpha^C_N + \sum_{i=1}^{4} \beta^C_{Ni} \ln x^i_t + \beta^C_{Nt} \cdot t}{\alpha^C_M + \sum_{i=1}^{4} \beta^C_{Mi} \ln x^i_t + \beta^C_{Mt} \cdot t} \qquad (2.52)$$

$$\frac{p^t_{CK} CK_t}{B_t} = \frac{\alpha^C_K + \sum_{i=1}^{4} \beta^C_{Ki} \ln x^i_t + \beta^C_{Nt} \cdot t}{\alpha^C_M + \sum_{i=1}^{4} \beta^C_{Mi} \ln x^i_t + \beta^C_{Mt} \cdot t} \qquad (2.53)$$

$$\frac{p^t_L LJ_t}{B_t} = \frac{\alpha^C_J + \sum_{i=1}^{4} \beta^C_{Ji} \ln x^i_t + \beta^C_{Jt} \cdot t}{\alpha^C_M + \sum_{i=1}^{4} \beta^C_{Mi} \ln x^i_t + \beta^C_{Mt} \cdot t} \qquad (2.54)$$

where x^i_t refer to the per capita values of the three commodity groups and leisure, i.e.

$$x_t = (CE_t/P_t,\ CN_t/P_t,\ CK_t/P_t,\ LJ_t/P_t) \qquad (2.55)$$

The above set of expenditure share equations complete the specification of the intra-temporal allocation model. The parameters to be extimated must be consistent with the monotonicity and quasi-convexity restrictions imposed by the theory of demand and, in addition, the expenditure shares must add up to unity at each observation point. The explicit form of these restrictions will be presented in Chapter III.

2.5 Capital Accumulation

The dynamics of capital accumulation in the two productive sectors are governed by the sectoral rates of replacement and by sectoral gross investment. Capital stock in the energy and non-energy sectors is each defined as an aggregate index composed of the stocks of producer durables, non-residential equipment, inventories and land. Stocks in each sub-aggregate are computed using the perpetual inventory method and rates of replacement are obtained assuming geometrically declining rates of capital efficiency. The resulting capital accumulation equations for the two production sectors are the following:

$$KSE = (1 - DE) \cdot KSE(-1) + INE \qquad (2.56)$$

where

KSE - Capital stock, energy sector

DE - Rate of replacement, energy sector

INE - Gross investment, energy sector

and

$$KSN = (1 - DN) \cdot KSN(-1) + INN \qquad (2.57)$$

where

KSN - Capital stock, non-energy sector

DN - Rate of replacement, non-energy sector

INN - Gross investment, non-energy sector

The equations of capital accumulation for the household sector are analogous. For the stock of consumer durables, we have

$$KCD = (1 - DCD) \cdot KCD(-1) + ICD \qquad (2.58)$$

where

KCD - Stock of consumer durables, household sector

DCD - Rate of replacement, consumer durables

ICD - Gross investment in consumer durables

with an analogous equation for residential structures:

$$KRS = (1 - DRS) \cdot KRS(-1) + IRS \qquad (2.59)$$

where

KRS - Capital stock, residential structures

DRS - Rate of replacement, residential structures

IRS - Gross investment in residential structures

The model of capital stocks and flows is completed by the following relationships which determine the levels of capital services supplied by the capital stocks held by the production and household sectors and relate the sectoral gross investment levels to the level of total purchases of investment goods by the private domestic sector:

$$KE = QKE \cdot KSE(-1) \tag{2.60}$$

$$KN = QKN \cdot KSN(-1) \tag{2.61}$$

$$CK = QKS \cdot KRS(-1) + QKD \cdot KCD(-1) \tag{2.62}$$

$$INE = ZINE \cdot I \tag{2.63}$$

$$INN = ZINN \cdot I \tag{2.64}$$

$$IRS = ZRS \cdot I \tag{2.65}$$

$$ICD = ZCD \cdot I \tag{2.66}$$

where

KE - Capital services, energy sector

QKE - Quality of capital, energy sector

KN - Capital services, non-energy sector

QKN - Capital services, non-energy sector

ZINN - Scaling variable, gross investment in non-energy sector to total investment

ZINE - Scaling variable, gross investment in energy sector to total investment

ZRS - Scaling variable, gross investment in residential structures to total investment

ZCD - Scaling variable, gross investment in consumer durables to total investment

IT - Gross private domestic investment

sector

In order to complete the characterization of capital inputs, we must select a method for the imputation of the rental value of owner-utilized assets. Our point of departure is the identity that relates the price of an asset to the discounted value of its services. The value of capital service input is equal to the value of property compensation in the absence of taxes. Also, in the absence of tax policies, the price of capital services of an asset group is equal to the opportunity cost of the asset group plus its depreciation. In order to incorporate the effect of taxes, we employ the formulation developed by Christensen and Jorgenson [ ]. Specifically, for tangible fixed assets held by the energy sector, the price of capital services is:

$$p_{KE} = (1/QKE)\cdot\left[\frac{1 - TKE\cdot PVDE - ITCE}{1 - TKE}\right][ME\cdot p_I(-1) + DE\cdot p_I - CAG] + TPE\cdot p_I \qquad (2.67)$$

where

- TKE - Effective tax rate on capital income, energy sector
- TPE - Effective tax rate on capital property, energy sector
- PVDE - Present value of depreciation deductions for tax purposes per dollar of investment, energy sector
- ITCE - Effective investment tax credit, energy sector
- ME - Rate of return on fixed tangible assets, energy sector
- DE - Rate of replacement, energy sector
- CAG - Effective rate of capital gains per dollar of fixed assets

held

PI - Implicit deflator, purchases of investment goods

QKE - Quality of capital, energy sector

For the price of capital services in the non-energy sector, the appropriate expression is entirely analogous:

$$p_{KN} = (1/QKN)\cdot\left[\frac{1 - TKN\cdot PVDN - ITCN}{1 - TKN}\right][MN\cdot p_I(-1) + DN\cdot p_I - CAG] + TPN\cdot p_I \qquad (2.68)$$

where

TKN - Effective tax rate on capital income, non-energy sector

TPN - Effective tax rate on capital property, non-energy sector

PVDE - Present value of depreciation deductions for tax purposes per dollar of investment, non-energy sector

ITCN - Effective investment tax credit, non-energy sector

MN - Rate of return on fixed tangible assets, non-energy sector

DN - Rate of replacement, non-energy sector

CAG - Effective rate of capital gains per dollar of fixed assets held

p_I - Implicit deflator, purchases of investment goods

QKN - Quality of capital, non-energy sector

For capital services supplied to the household sector, the corresponding prices given by:

$$p_{CK} = (1/QCK)\ [MC \cdot p_I(-1) + DC \cdot p_I - CAG] + TPC \cdot p_I \qquad (2.69)$$

where

TPC - Effective tax rate on capital property, household sector

MC - Rate of return, fixed tangible assets, household sector

DC - Rate of replacement, household sector

CAG - Effective rate of capital gains per dollar of fixed assets held

QCK - Quality of capital, household sector

2.6 Income-Expenditure Identities

In order to complete the specification of the model, we must introduce accounting relationships that stipulate the equality between revenues and outlays for each sector and each product category.

For each of the two producing sectors, the value of output must equal the value of input:

$$p_{EC}\ EC + p_{EM}\ EM = p_{KE}\ KE + p_{LE}\ LE + p_{NME}\ NME + p_{EME}\ EME + p_{RE}\ RE \qquad (2.70)$$

where

p_{EC} - Implicit deflator, supply of energy consumption products

EC - Supply of energy consumption products

p_{EM} - Implicit deflator, supply of energy intermediate products

EM - Supply of energy intermediate products

p_{KE} - Implicit deflator, capital services, energy sector

KE - Capital services, energy sector

p_{LE} - Implicit deflator, labor services purchased by the energy sector

LE - Labor services purchased by the energy sector

p_{NME} - Implicit deflator, non-energy intermediate products purchased by the energy sector

NME - Non-energy intermediate products purchased by the energy sector

p_{EME} - Implicit deflator, energy intermediate products purchased by the energy sector

EME - Energy intermediate products purchased by the energy sector

p_{RE} - Implicit deflator, competitive imports of energy products

RE - Competitive imports of energy products

A similar identity holds for the non-energy sector:

$$p_{NC}\, NC + p_{NM}\, NM + p_{NI}\, NI = p_{KN}\, KN + p_{LN}\, LN + p_{EMN}\, EMN + p_{NMN}\, NMN + p_{RN}\, RN \qquad (2.71)$$

where

p_{NC} - Implicit deflator, supply of non-energy consumption products

NC - Supply of non-energy consumption products

p_{NM} - Implicit deflator, supply of non-energy intermediate products

NM - Supply of non-energy intermediate products

p_{NI} - Implicit deflator, supply of investment goods

NI - Supply of investment goods

p_{KN} - Implicit deflator, capital services, non-energy sector

KN - Capital services, non-energy sector

p_{LN} - Implicit deflator, purchases of labor services by the non-energy sector

LN - Labor services purchased by the non-energy sector

p_{EMN} - Implicit deflator, energy intermediate products purchased by the non-energy sector

EMN - Energy intermediate products purchased by the non-energy sector

p_{NMN} - Implicit deflator, non-energy intermediate products purchased by the non-energy sector

NMN - Non-energy intermediate products purchased by the non-energy sector

p_{RN} - Implicit deflator, competitive imports of non-energy products

RN - Competitive imports of non-energy products

The value of energy consumption goods purchased by the final users must equal the gross revenue on sales of energy consumption goods by the energy sector:

$$(1 + TCE)\, p_{EC}\, EC = p_{CE}\, CE + p_{STE}\, STE + p_{GE}\, GE + p_{EXE}\, EXE \qquad (2.72)$$

where

p_{EC} - Implicit deflator, supply of energy consumption goods

TCE - Effective tax rate, energy consumption goods

EC - Supply of energy consumption goods

p_{CE} - Implicit deflator, energy consumption goods purchased by households

CE - Energy consumption goods purchased by households

p_{STE} - Implicit deflator, energy consumption goods delivered to business inventories

STE - Energy consumption goods delivered to business inventories

p_{GE} - Implicit deflator, government purchases of energy consumption goods

GE - Government purchases of energy consumption goods

p_{EXE} - Implicit deflator, gross exports of energy consumption goods

EXE - Gross exports of energy consumption goods

Similarly for non-energy consumption goods there must be equality between revenues and expenditures:

$$(1 + TCN)\ p_{NC}\ NC = p_{CND}\ CND + p_{STN}\ STN + p_{GN}\ GN + p_{EXN}\ EXN \tag{2.73}$$

where

p_{NC} - Implicit deflator, supply of non-energy consumption products

NC - Supply of non-energy consumption products

TCN - Effective tax rate, non-energy consumption products

p_{CND} - Implicit deflator, non-energy domestic consumption products purchased by households

CND - Non-energy domestic consumption products purchased by households

p_{STN} - Implicit deflator, non-energy products delivered to business inventories

STN - Non-energy consumption products delivered to business inventories

p_{GN} - Implicit deflator, government purchases of non-energy consumption products

GN - Government purchases of non-energy consumption products

p_{EXN} - Implicit deflator, gross exports of non-energy consumption products

EXN - Gross exports of non-energy consumption products

p_{CIM} - Implicit deflator, imports of non-energy consumption goods

CIM - Imports of non-energy consumption goods

The revenues received by the suppliers of investment goods, plus the corresponding sales taxes, must equal the total outlays on investment goods by the private, government and foreign sectors:

$$(1 + TI)\, p_{NI}\, NI + p_{IMI}\, IMI + p_{STE}\, STE + p_{STN}\, STN = p_I\, I + p_{GI}\, GI + p_{EXI}\, EXI \qquad (2.74)$$

where

p_{NI} - Implicit deflator, supply of investment goods

NI - Supply of investment goods

p_I - Implicit deflator, gross private domestic investment

I - Gross private domestic investment

p_{GI} - Implicit deflator, purchases of investment goods by the government sector

GI - Purchases of investment goods by the government sector

TI - Effective tax rate, purchases of investment goods

p_{IMI} - Implicit deflator, imports of investment goods

IMI - Imports of investment goods

p_{STE} - Implicit deflator, inventory investment of energy products

STE - Inventory investment of energy products

p_{STN} - Implicit deflator, inventory investment of non-energy product

STN - Inventory investment of non-energy products

p_{EXI} - Implicit deflator, gross exports of investment goods

EXI - Gross exports of investment goods

The income-expenditure identities must also hold for intermediate products:

$$p_{EM}\,EM = p_{EMN}\,EMN + p_{EME}\,EME \qquad (2.75)$$

where

p_{EM} - Implicit deflator, supply of energy intermediate products

EM - Supply of energy intermediate products

p_{EMN} - Implicit deflator, energy intermediate products purchased by the non-energy sector

EMN - Energy intermediate products purchased by the non-energy sector

p_{EME} - Implicit deflator, energy intermediate products purchased by the energy sector

EME - Energy intermediate products purchased by the energy sector

and

$$p_{NM}\ NM = p_{NME}\ NME + p_{NMN}\ NMN \qquad (2.76)$$

where

p_{NM} - Implicit deflator, supply of non-energy intermediate products

NM - Supply of non-energy intermediate products

p_{NME} - Implicit deflator, non-energy intermediate products purchased by the energy sector

NME - Non-energy intermediate products purchased by the energy sector

p_{NMN} - Implicit deflator, non-energy intermediate products purchased by the non-energy sector

NMN - Non-energy intermediate products purchased by the non-energy sector

The total value of saving by the private domestic sector plus the government and foreign saving in the U.S. must equal the value of gross private domestic investment:

$$S = p_I\ I + p_G\ (G - G(-1)) + p_R\ (R - R(-1)) \qquad (2.77)$$

where

S - Gross private domestic saving

p_I - Implicit deflator, gross private domestic investment

I - Gross private domestic investment

G - Net claims on government by private domestic sector

p_G - Implicit deflator, net claims on government by private domestic sector

R - Net claims on rest of the world by private domestic sector

p_R - Implicit deflator, net claims on rest of the world by private domestic sector

The total value of wealth is the value of fixed tangible assets plus the value of net claims on the government and foreign sectors:

$$W = p_I\ (KSE + KSN + KRS + KCD)\ ZW + p_G\ G + p_R\ R \qquad (2.78)$$

where

p_I - Implicit deflator, gross private domestic investment

KSE - Capital stock, energy sector

KSN - Capital stock, non-energy sector

KRS - Capital stock, residential structures

KCD - Capital stock, consumer durables

ZW - Aggregation variable, capital stock to wealth

p_G - Implicit deflator, net claims on government sector

G - Net claims on government sector

p_R - Implicit deflator, net claims on foreign sector

R - Net claim on foreign sector

W - Private domestic wealth

Expenditures on labor services must equal the receipts by members of the labor force minus the corresponding income tax:

$$p_L \, L = (1 - TL) \, (p_{LE} \, LE + p_{LN} \, LN + p_{LG} \, LG + p_{LR} \, LR) \qquad (2.79)$$

where

p_L - Implicit deflator, supply of labor services

L - Supply of labor services by the household sector

p_{LE} - Implicit deflator, labor services purchased by the energy sector

LE - Labor services purchased by the energy sector

p_{LN} - Implicit deflator, labor services purchased by the non-energy sector

LN - Labor services purchased by the non-energy sector

p_{LG} - Implicit deflator, labor services purchased by the government sector

LG - Labor services purchased by the government sector

p_{LR} - Implicit deflator, labor services purchased by the foreign sector

LR - Labor services purchased by the foreign sector

Finally, the following identities relate the sectoral prices of labor services to the supply price of labor and intermediate input deflator to intermediate supply prices:

$$p_{LE} = ZLE \cdot p_L$$

$$p_{LN} = ZLN \cdot p_L$$

$$p_{EMN} = ZNE \cdot p_{EM}$$

$$p_{NME} = ZEN \cdot p_{NM} \qquad (2.80)$$

where

p_L - Implicit deflator, supply of labor services

p_{LE} - Implicit deflator, labor services purchased by the energy sector

p_{LN} - Implicit deflator, labor services purchased by the non-energy sector

ZLE - Scaling variable, price of labor services to price of labor service purchased by the energy sector

ZLN - Scaling variable, price of labor services to price of labor service purchased by the non-energy sector

p_{EMN} - Implicit deflator, intermediate energy products purchased by the non-energy sector

p_{NME} - Implicit deflator, intermediate non-energy products purchased by the energy sector

ZNE - Scaling variable, price of energy intermediate goods to price of energy purchased by the non-energy sector

ZEN - Scaling variable, price of non-energy intermediate goods to price of non-energy goods purchased by the energy sector

p_{EM} - Implicit deflator, energy intermediate products

p_{NM} - Implicit deflator, non-energy intermediate products

The following identity relates to the nominal rate of return on wealth to the sectoral rates of return:

$$MW \cdot [KSE(-1) + KSN(-1) + KCD(-1) + KRS(-1)]$$
$$= ZRN \cdot [ME \cdot KSE(-1) + MN \cdot KSN(-1)] \qquad (2.81)$$

where

MW - Nominal rate of return on wealth

KSE - Capital stock, energy sector

KSN - Capital stock, non-energy sector

KCD - Capital stock, consumer durables

KRS - Capital stock, residential structures

ZRN - Scaling variable, sectoral rates of return to nominal rate of return on wealth

ME - Nominal rate of return on fixed tangible assets, energy sector

MN - Nominal rate of return on fixed tangible assets, non-energy sector

Finally, the quantity index of full consumption for the household sector is defined as an aggregate of the quantity indices for leisure, non-energy goods, energy goods and capital services purchased by households:

$$F = 0.8225\ LJ + 1.0786\ CK + 0.9349\ CN + 2.4956\ CE + 0.8559^{*} \qquad (2.82)$$

*This linear aggregation equation is an approximation to the Divisia aggregate index obtained by regressing the Divisia index for F against the four quantity components. For this OLS regression, $R^2 = 1.00$.

where

F - Full consumption, household sector

CK - Capital services supplied to households

CN - Non-energy consumption goods purchased by households

CE - Energy consumption goods purchased by households

LJ - Leisure time available to households

2.7 Balance Equations

As a result of the process of market equilibration, equality between quantities supplied and quantities demanded must hold for each product category in the model.

The supply of energy consumption goods must equal the amounts used by households, government, the foreign sector, and business inventories:

$$EC = CE + STE + GE + EXE \qquad (2.83)$$

where

EC - Energy consumption products supplied by the energy sector

CE - Energy consumption products purchased by households

STE - Energy consumption products delivered to business inventories

GE - Energy consumption products purchased by the government

EXE - Exports of energy consumption products

Non-energy consumption products supplied by the domestic and foreign sectors must be used up by households, business inventories, and the government and foreign sectors:

$$NC + CIM = CN + STN + GN + EXN \qquad (2.84)$$

where

NC - Non-energy consumption products supplied by the non-energy sector

CIM - Imports of non-energy consumption goods

CN - Non-energy consumption goods purchased by households

STN - Non-energy consumption goods delivered to business inventories

GN - Non-energy consumption goods purchased by the government sector

EXN - Exports of non-energy consumption goods

Investment goods supplied by the domestic non-energy sector, plus imported investment goods and inventory investment in the two production sectors must equal gross private domestic investment plus investment goods purchased by the government and foreign sectors. Gross private domestic investment is itself composed of gross investment in the production and household sectors:

$$NI + STE + STN = I + GNI + EXI \qquad (2.85)$$

$$IMI + I = INE + INN + ICD + IRS \qquad (2.86)$$

where

NI - Supply of investment goods by the non-energy sector

IMI - Imports of investment goods

STE - Energy consumption goods delivered to business inventories

STN - Non-energy consumption goods delivered to business inventories

I - Gross private domestic investment

GNI - Investment goods purchased by the government sector

EXI - Exports of investment goods

INE - Gross investment, energy sector

INN - Gross investment, non-energy sector

ICD - Gross investment, consumer durables

IRS - Gross investment, residential structures

Clearing of the intermediate goods markets implies the following equalities between supply and demand:

$$NM = NME + NMN \tag{2.87}$$

$$EM = EME + EMN \tag{2.88}$$

where

NM - Supply of non-energy intermediate products

NME - Non-energy intermediate products purchased by the energy sector

NMN - Non-energy intermediate products purchased by the non-energy sector

EM - Supply of energy intermediate products

EME - Energy intermediate products purchased by the energy sector

EMN - Energy intermediate products purchased by the non-energy sector

The supply of labor must be used up exactly by the two production sector, by the government sector and the foreign sector and by the fraction of workers that remain unemployed:

$$L = LE + LN + LG + LR + LU \tag{2.89}$$

where

L - Supply of labor by the household sector

LE - Labor services purchased by the energy sector

LN - Labor services purchased by the non-energy sector

LG - Labor services purchased by the government sector

LR - Labor services purchased by the foreign sector

LU - Unemployed labor

The total time resources of the household sector must be exhausted between leisure and labor:

$$LH = LJ + L \tag{2.90}$$

where

LH - Time endowment of the household sector

LJ - Leisure time of the household sector

L - Supply of labor by the household sector

Total purchases of non-energy consumption goods by households equal the quantities of domestic and imported goods purchased:

$$CN = CND + CIM \tag{2.91}$$

where

CN - Non-energy consumption goods purchased by households

CND - Domestic non-energy consumption goods purchased by households

CIM - Imported non-energy consumption goods purchased by households

2.8 Summary

This chapter has described the behavioral assumptions which underlie the specification of the individual equations in our macro-economic energy model. The next chapter addresses the econometric questions surrounding the empirical implementation of the model.

Our long-term macroeconomic model has three major components: the sub-model of producer behavior, the sub-model of consumer behavior and the sub-model of investment and capital accumulation. The key interactions between these three sectors are diagrammatically represented in Fig. 2-1.

The two production sub-sectors lead to derived demand relationships for the following factor inputs:

Energy Sector	Non-Energy Sector
- Capital Services (KE)	- Capital Services (KN)
- Labor Services (LE)	- Labor Services (LN)
- Energy Intermediate Products (EME)	- Energy Intermediate Products (EMN)
- Non-Energy Intermediate Products (NME)	- Non-Energy Intermediate Products (NMN)
- Imports (RE)	- Imports (RN)

Correspondingly, the behavioral assumptions underlying our production sub-model yield derived supply relationships for the following products:

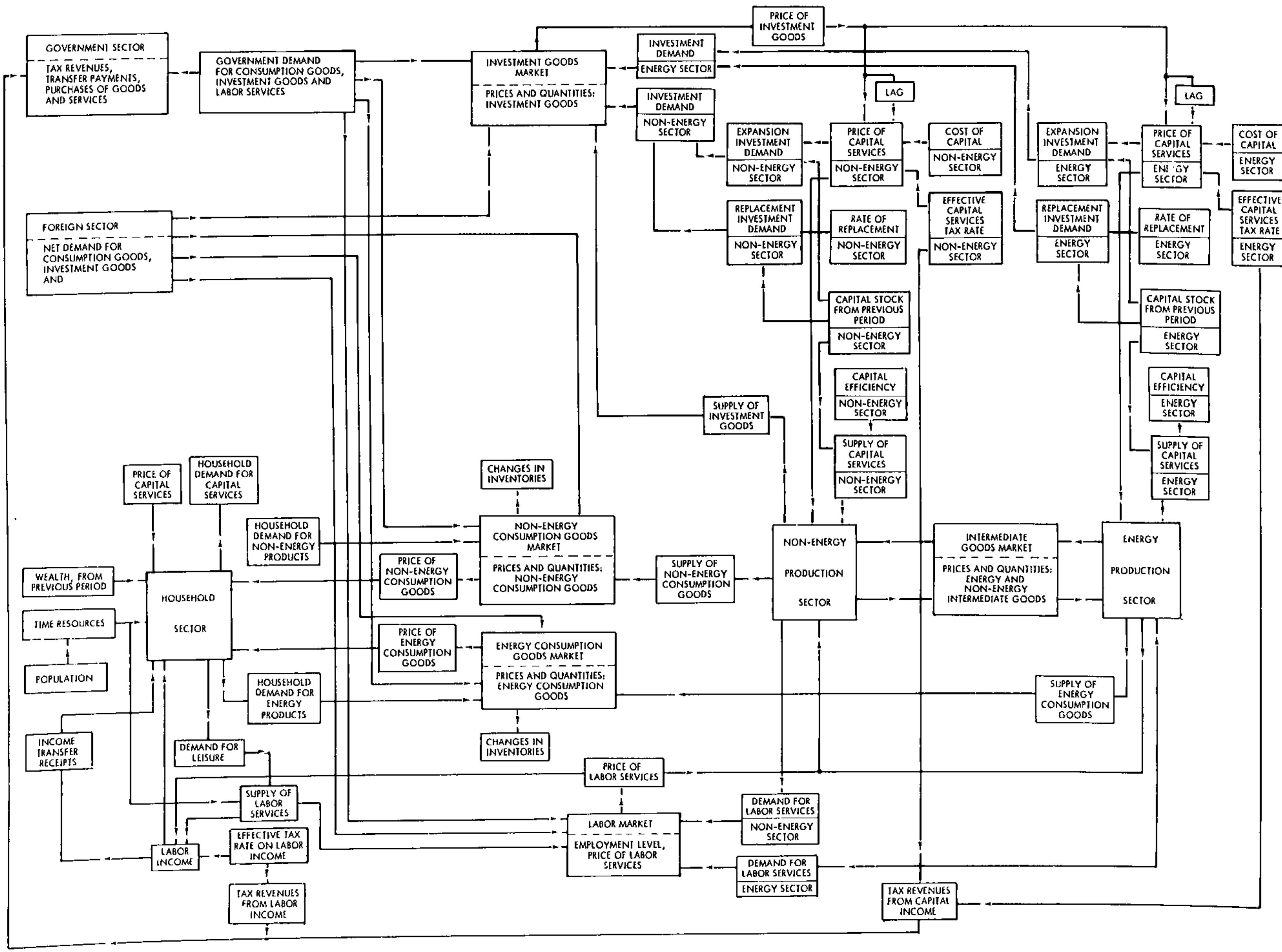

Fig. 2-1: Long-Term Macroeconomic Energy Model

Energy Sector

- Energy Consumption Products (EC)
- Energy Intermediate Products (EM)

Non-Energy Sector

- Non-Energy Consumption Products (NC)
- Non-Energy Intermediate Products (NM)
- Investment Goods (NI)

The sub-model of consumer behavior determines the demands of households for energy consumption goods (CE), non-energy consumption goods (CN), capital services (CK) and leisure (LJ). The demand for leisure yields a supply schedule for labor services when considered jointly with the total time endowment of the household sector. The third component of the model serves to determine the demand for investment, the sectoral rates of return and, through the accumulation and capital efficiency relationships, the supply of capital services. In addition to the market-clearing conditions for the various product categories, the production and household sectors are linked through the sectoral rates of return which are determinants of capital service prices and, through the nominal rate of return on wealth, of the budget constraint of the household sector.

The model attempts to characterize the interrelations between levels of energy utilization and overall economic performance. Since it is primarily a long-term growth model, it focuses attention on the properties of alternative paths of potential output rather than on questions related to the levels of capacity utilization and employment. The model draws upon the body of neoclassical theory: i.e. representations of the decisions of individual economic agents are derived from postulated assumptions of constrained maximizing behavior. A full set of market-clearing prices and quantities are computed within a simultaneous process of market equilibration: i.e. it follows the tradition of *general equilibrium* models. This property makes it possible to establish a link between the macroeconomic relationships in the model and economic magnitudes susceptible of interpretation from the vantage point of microeconomic analysis. Indeed, although the model is aggregative, the introduction of a higher degree of sectoral detail would entail no new conceptual difficulties.

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CHAPTER III

ECONOMETRIC SPECIFICATION AND EMPIRICAL VALIDATION

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"I have no data yet. It is a capital mistake to theorize before one has data. Insensibly, one begins to twist facts to suit theories instead of theories to suit facts."

- Sir Arthur Conan Doyle

The present chapter is devoted to the econometric specification of the behavioral relationships in the production and household sectors of our macroeconomic energy model and to a presentation of our empirical results. Specifically, we discuss the estimation of the various sets of simultaneous equation systems corresponding to the various model components and we discuss the implications of the resulting parameter estimates in terms of substitution possibilities among factors of production and among commodity groups, respectively. In addition, we present the results of a series of statistical tests of various behavioral hypotheses about the structure of production and the structure of consumer preferences.

3.1 Estimation of the Model of Producer Behavior

In this section we present the empirical results for our model of producer behavior. We first describe the sources and preparation of data for the energy and non-energy sectors. The stochastic specification of the model of production in terms of cost and revenue shares is then discussed and the resulting parameter estimates are presented together with the results of statistical tests on various separability hypotheses about the structure of the underlying technology. Tests for the presence of autocorrelated disturbances in the estimating share equations are implemented by making use of the autoregressive transformation for singular equation systems introduced by Berndt and Savin [1]. Tests for the concavity of the estimated cost frontiers are developed by means of a reparametrization procedure based on the Cholesky factorization of the Hessian matrices of the cost frontiers evaluated at the point of approximation. The implications of our empirical results in terms of the substitution possibilities among factors of production are evaluated by computing the Allen-Uzawa partial elasticities of substitution and transformation. The section is concluded by presenting the results of tests for the presence of biases in the rates of technological change in both the energy and non-energy sectors.

3.1.1 Data Sources and Preparation

In order to provide the appropriate data base required for the empirical implementation of our model of producer behavior, we have constructed time series data corresponding to price and quantity components of factor inputs and the various output categories in the energy and non-energy sectors of the U.S. private domestic economy for the years 1947 through 1971.

The energy sector is defined to consist of all industries contained within the following categories:

E1 : Coal Mining

E2 : Crude petroleum and Natural Gas production

E3 : Petroleum refining and related industries

E4 : Electric Utilities

E5 : Gas Utilities

The non-energy sector is defined to be composed of all industries classified within the following categories:

N1 : Agriculture, Non-fuel Mining and Residential Construction

N2 : Manufacturing, excluding petroleum refining

N3 : Transportation

N4 : Communications, Trade and Services

An extensive system of accounts measuring inter-industry transactions of goods and services from 25 producing sectors to 10 consuming sectors and 5 categories of final demand was developed by Faucett Associates for the Ford Foundation Energy Policy Project and is described in Faucett [2]. This set of accounts includes inter-industry flow tables in both constant and current dollars that are based on data from the annual Bureau of Mines Minerals Yearbook, the Census of Mineral Industries (1954, 1958, 1963 and 1967), the Census of Manufacturers (1947, 1954, 1963 and 1967), the U.S. Department of Commerce Input-Output Tables (1947, 1948, 1963), Annual Surveys of Manufacturers and other secondary sources. We construct annual price and quantity indices for energy intermediate inputs, non-energy intermediate inputs and imports in both the energy and non-energy sectors based on this set of accounts.

Based on the tables developed by Faucett, we construct annual quantity indices of energy inputs into both the energy and non-energy sectors as Divisia quantity indices of coal, crude petroleum, refined petroleum products, natural gas and electricity purchased by industries in the two respective production sectors. We then compute the value of energy purchases as the sum of current dollar purchases of these five energy types. Finally we define the price index of energy intermediate inputs into each sector implicitly as the ratio of the current dollar value to the constant dollar value of energy purchases.

Annual quantity indices of non-energy intermediate inputs into the two production sectors are constructed from the Faucett inter-industry tables as Divisia quantity indices of non-energy intermediate good purchases from agriculture, non-fuel mining and construction; manufacturing excluding petroleum products; transportation and communications, trade and services. We then compute the value of total non-energy intermediate good purchases as the sum of current dollar purchases of all these non-energy intermediate goods. Finally, we define the price indices implicitly as the current dollar value divided by the constant dollar value of non-energy purchases.

Constant and current dollar series of imports to the energy sector are constructed by Divisia aggregation of energy product imports into the five energy categories composing the energy sector. Similarly, indices of non-energy imports to the non-energy sector are constructed by Divisia aggregation of imported non-energy products into the four categories which compose the non-energy sector. The corresponding price indices are defined implicitly as before.

Our next step in the development of the data base was the construction of time series for the price and quantity components of capital service inputs into the energy and non-energy sectors. Imputation of a rental or service price for the use of a capital asset is the major problem in the separation of capital service flows into price and quantity components. The relationship between the value of an asset and the discounted value of its services served as a basis for our imputation. Annual price and quantity indices for capital services inputs were developed for the five energy sectors E1 through E5 and the four non-energy sectors N1 through N4. The methodology followed in the development of these time series closely paralled that of Christensen and Jorgenson [3]. The starting point was the development of capital stock and service price data for each of the four asset groups in each sector: producer durables, non-residential structures, inventories and land. Capital service price indices for each asset group were constructed following established procedures that incorporate the effects of the effective tax rate on capital income, the effective tax rate on capital property, the rate of return on fixed assets, the rate of replacement, the present value of depreciation allowances, the rate of capital gains and, where applicable, the investment tax credit. The quantity of capital services in each of the five energy sectors and four non-energy sectors were obtained by Divisia aggregation of the flows of the four asset groups. The sector capital service price was computed implicitly from the constant dollar Divisia index and the current dollar value figure. Finally, the data corresponding to the five energy sectors and the four non-energy sectors were aggregated to yield the price and quantity indices of capital service inputs into our energy sector and our non-energy sector, respectively.

Annual data on price and quantity indices of labor service inputs to the four non-energy sectors N1 through N4 and the five energy sectors E1 through E5 were developed using four categories of data as the point of departure: number of employed persons, hours worked per week, weeks worked per year and wages earned per hour. These original time series data were obtained from publications of the U.S. Bureau of the Census, Department of Commerce and the Bureau of Labor Statistics, Department of Labor. The detailed steps followed in arriving at the appropriate aggregate compensation measures are described in Gollop [4]. After adjusting for the earnings imputed to uncompensated family workers included in [4], Divisia indices were computed for the quantity of labor inputs into the energy sector by aggregation of E1 through E5 and to the non-energy sector by aggregation over N1 through N4. Price indices for labor inputs to the energy and non-energy sector were computed implicitly in terms of the current and constant dollar values of labor inputs.

Annual quantity and price indices for the various output categories of the energy and non-energy sectors were developed in several steps. First, it was necessary to modify the original matrices reported in Faucett [2] which included expenditures on consumer durables in the category of personal consumption expenditures. Since we choose to follow the system of accounts developed by Christensen and Jorgenson [5], expenditures on consumer durables were treated as an investment flow component of gross private domestic investment. Service flows imputed to the stock of consumer durables were allocated to personal consumption expenditures. Next, we allocated total purchases of the government and foreign sectors as between consumption and investment in proportion to the aggregate quantities purchased of consumption and investment goods by each sector respectively -- the latter figures taken from the national accounts developed by Christensen and Jorgenson [5]. By Divisia aggregation, we then computed the quantity indices for energy consumption goods, non-energy consumption goods and investment goods. The quantity indices for energy intermediate products and non-energy intermediate products were computed by Divisia aggregation of the corresponding flow components of the inter-industry matrices reported in Faucett [2]. Finally, price indices were defined as implicit deflators in each case, in terms of the corresponding constant dollar and current dollar figures.

The price indices for inputs to the energy sector are tabulated in Table 3-1. We note that the greatest rate of growth over the period 1947-1971 occurred for the price of labor inputs:

	P_{KE}	P_{LE}	P_{EME}	P_{NME}	P_{RE}
Average growth rate p.a. 1947-1971	1.8%	5.2%	1.3%	2.1%	1.0%

From Table 3-2 which shows the price deflators for the inputs to the non-energy sector we also note that the prices of capital service inputs, energy and non-energy intermediate inputs and those of imports rose less rapidly over 1947-1971 than the price of labor inputs:

	P_{KN}	P_{LN}	P_{EMN}	P_{NME}	P_{RN}
Average growth rate p.a. 1947-1971	2.1%	5.2%	0.03%	1.9%	1.0%

Quantity indices for inputs to the energy sector are shown in Table 3-3. We observe that the rate of growth of imported inputs in real terms was greater than that of any other input over the period 1947-1971 and that both capital service inputs and intermediate energy inputs grew an approximate total of 300% over the same period:

TABLE 3-1: INPUT PRICE INDICES, U.S. ENERGY SECTOR, 1947-1971

Year	p_{KE}	p_{LE}	p_{EME}	p_{NME}	p_{RE}
1947	0.8319	0.5744	0.6599	0.7030	0.8267
1948	1.1110	0.6424	0.8225	0.7859	0.8799
1949	0.9028	0.6412	0.7983	0.8063	0.8347
1950	0.9657	0.7060	0.7882	0.8009	0.9178
1951	1.0536	0.7284	0.8148	0.8480	1.1523
1952	0.9905	0.7558	0.8374	0.8794	1.1087
1953	1.0274	0.7678	0.8669	0.9014	1.0432
1954	1.0609	0.8401	0.9097	0.9135	1.0654
1955	1.1255	0.8858	0.9422	0.9674	1.0520
1956	1.1506	0.9270	0.9551	1.0055	1.0686
1957	1.0729	0.9785	0.9866	0.9714	1.0767
1958	1.0000	1.0000	1.0000	1.0000	1.0000
1959	1.0517	1.0472	0.9973	0.9539	0.9795
1960	1.0984	1.0349	1.0232	1.0316	0.9881
1961	1.1095	1.0810	1.0391	1.0413	0.9674
1962	1.1302	1.1094	1.0419	1.0489	0.9371
1963	1.1616	1.1573	1.0296	0.9965	0.9382
1964	1.1958	1.2162	1.0334	1.0736	0.9614
1965	1.2370	1.2596	1.0276	1.1019	0.9707
1966	1.2613	1.3323	1.0281	1.1259	1.0025
1967	1.3275	1.3980	1.0271	1.1855	1.0036
1968	1.3309	1.4866	1.0391	1.1862	1.0097
1969	1.2766	1.6418	1.0721	1.4365	1.0393
1970	1.3172	1.7828	1.1187	1.4479	1.1229
1971	1.3560	1.8763	1.2222	1.3836	1.1826

Table 3-2: INPUT PRICE INDICES, U.S. NON-ENERGY SECTOR, 1947-1971

Year	p_{KN}	p_{LN}	p_{EMN}	p_{NMN}	p_{RN}
1947	0.8580	0.5608	0.7928	0.7450	0.8197
1948	0.9378	0.6017	0.9386	0.7996	0.8884
1949	0.8437	0.6223	0.8553	0.7951	0.8399
1950	0.9853	0.6653	0.8653	0.8258	0.9135
1951	1.0473	0.6854	0.9026	0.8926	1.1498
1952	0.9915	0.7170	0.9247	0.8919	1.1073
1953	0.9914	0.7469	0.9963	0.8922	1.0424
1954	0.9848	0.8100	0.9303	0.9048	1.0644
1955	1.0660	0.8611	1.0220	0.9399	1.0502
1956	1.0219	0.9013	1.0757	0.9730	1.0670
1957	1.0161	0.9561	0.9827	0.9781	1.0758
1958	1.0000	1.0000	1.0000	1.0000	1.0000
1959	1.1053	1.0344	0.9885	0.9768	0.9796
1960	1.0952	1.0827	1.0175	1.0144	0.9880
1961	1.0790	1.1084	0.9630	1.0207	0.9676
1962	1.1125	1.1297	0.9445	1.0267	0.9378
1963	1.1227	1.1634	0.8989	1.0052	0.9386
1964	1.1691	1.2224	0.9338	1.0373	0.9616
1965	1.2323	1.2569	0.9792	1.0585	0.9715
1966	1.2568	1.3375	0.9179	1.0966	1.0037
1967	1.2298	1.3950	0.9028	1.1086	1.0045
1968	1.2816	1.4934	0.9160	1.1161	1.0106
1969	1.2860	1.6006	0.9267	1.2387	1.0401
1970	1.2529	1.7523	0.8965	1.2373	1.1233
1971	1.3723	1.8500	1.0588	1.2197	1.1833

TABLE 3-3: INPUT QUANTITY INDICES*, U.S. ENERGY SECTOR, 1947-1971

Year	K	L	E	N	R
1947	6.4612	8.2026	10.7070	7.6780	0.6020
1948	6.9404	7.4477	11.9030	6.3670	0.7650
1949	7.7166	7.7331	11.4610	6.5480	0.8450
1950	8.1926	6.5557	12.1620	8.0560	1.0130
1951	8.6207	8.4364	13.4400	7.5350	0.9760
1952	9.2531	8.5111	13.6360	7.7890	1.0930
1953	9.7535	8.8779	14.6810	8.0280	1.2770
1954	10.2632	7.7754	14.7180	7.5750	1.1620
1955	10.7905	7.7029	15.3310	8.2810	1.3930
1956	11.4435	8.1605	16.3510	8.8120	1.5390
1957	12.1331	8.3363	17.5360	10.7900	1.7950
1958	12.7941	8.1529	17.0970	10.6150	1.7250
1959	13.1244	8.1311	18.0810	11.9860	1.8500
1960	13.4417	8.7226	18.5220	11.9060	1.7010
1961	13.7392	8.4796	18.9510	12.7310	1.7240
1962	14.0479	8.4652	19.9240	13.6300	2.0110
1963	14.2953	8.4501	21.1720	14.5390	2.1580
1964	14.3946	8.3397	21.7740	14.1500	2.1650
1965	14.7920	8.4374	22.5060	14.6240	2.2400
1966	15.1650	8.5215	24.1870	15.4090	2.6990
1967	15.6053	8.5376	25.5380	15.9140	2.6950
1968	16.3052	8.6705	27.1110	15.9700	3.3270
1969	16.9940	8.4109	28.5060	15.0470	3.6160
1970	18.0345	8.0119	30.2430	16.4350	3.7920
1971	18.7289	8.0831	30.2760	18.7950	4.4890

* in billions of 1958 dollars

TABLE 3-4: INPUT QUANTITY INDICES*, U.S. NON-ENERGY SECTOR, 1974-1971

Year	K	L	E	N	R
1947	104.9730	215.7920	10.8600	285.6260	7.9090
1948	108.6850	222.4120	9.9420	252.8320	8.7890
1949	115.0760	261.5350	10.7310	264.5500	8.2000
1950	117.0610	206.9810	11.6940	292.5370	9.5150
1951	123.8960	242.7530	12.2960	302.1160	9.5210
1952	130.4890	255.5010	12.2060	312.7570	9.8340
1953	133.1760	262.9550	13.4290	335.8660	10.5920
1954	136.4900	235.9120	13.0750	322.8400	9.7350
1955	140.5550	239.8900	12.7920	345.9120	11.0280
1956	148.0720	246.1040	13.5280	358.2400	11.9110
1957	155.7800	244.9430	15.4780	363.8270	11.9720
1958	164.2590	233.6240	15.3470	348.6040	12.5400
1959	164.8680	241.8710	16.5950	380.2990	14.8230
1960	170.5230	241.7170	17.3620	387.0690	14.3540
1961	175.6190	245.5460	18.8410	390.5460	14.0060
1962	181.1270	262.5630	20.5170	413.5250	15.9190
1963	187.8110	270.0390	22.1200	433.7720	16.5700
1964	194.7020	273.7480	22.6520	456.2790	17.6230
1965	201.3720	284.7230	23.0140	492.2100	20.3340
1966	212.2320	300.0270	26.1780	509.5040	24.1680
1967	223.1810	311.5110	28.4160	526.1270	26.1100
1968	231.5520	320.5870	29.8240	560.5570	31.1700
1969	240.7360	326.8890	31.9310	562.8890	34.2750
1970	250.7020	316.7650	36.3410	544.8480	35.6480
1971	253.5850	321.3020	33.9200	576.4650	37.0820

* in billions of 1958 dollars

TABLE 3-5: OUTPUT PRICE AND QUANTITY* INDICES, U.S. ENERGY SECTOR, 1947-1971

Year	p_{EC}	p_{EM}	EC	EM
1947	0.7954	0.7268	9.2962	21.5670
1948	0.9306	0.8753	9.4988	21.8550
1949	0.9022	0.9259	9.6766	22.1920
1950	0.9133	0.8260	10.7352	23.8560
1951	0.9431	0.8567	12.3477	25.7360
1952	0.9242	0.8786	13.3854	25.8420
1953	0.9611	0.8762	14.0505	28.1100
1954	0.9370	0.9194	14.3066	27.7930
1955	0.9171	0.9785	16.7588	28.1230
1956	0.9201	1.0097	18.1358	29.8790
1957	0.9280	0.9848	19.8048	33.0140
1958	0.9036	1.0000	19.8532	32.4440
1959	0.9021	0.9931	21.2386	34.6760
1960	0.8900	1.0204	22.9848	35.8840
1961	0.8836	1.0012	23.9813	37.7920
1962	0.8913	0.9925	24.7626	40.4410
1963	0.8817	0.9628	26.0998	43.2920
1964	0.8498	0.9826	27.6256	44.4260
1965	0.8701	1.0031	28.3601	45.5200
1966	0.8803	0.9708	30.1108	50.3650
1967	0.8932	0.9616	31.9871	53.9540
1968	0.8687	0.9746	34.0442	56.9350
1969	0.8775	0.9953	35.6663	60.4370
1970	0.8887	0.9489	37.7149	66.5840
1971	0.9311	1.1359	38.6416	64.1960

* in billions of 1958 dollars

TABLE 3-6: OUTPUT PRICE AND QUANTITY* INDICES, U.S. NON-ENERGY SECTOR, 1947-1971

Year	p_{NC}	p_{NM}	p_{NI}	NC	NM	NI
1947	0.7350	0.7439	0.7051	190.1080	293.3040	114.7760
1948	0.7824	0.7992	0.7604	200.1080	259.1990	120.0930
1949	0.7553	0.7953	0.7677	195.0200	271.0980	124.1750
1950	0.7718	0.8251	0.7326	207.4810	300.5930	143.6950
1951	0.8305	0.8915	0.8264	221.0920	309.6510	155.0890
1952	0.8456	0.8915	0.8451	223.8930	320.5450	163.9760
1953	0.8450	0.8924	0.8471	235.5440	343.8940	171.2850
1954	0.8662	0.9049	0.8465	226.9690	330.4150	170.7160
1955	0.8775	0.9405	0.8582	244.1480	354.1930	185.0210
1956	0.8985	0.9737	0.8959	248.2210	367.0520	188.0840
1957	0.9292	0.9779	0.9331	253.3750	374.6170	187.1710
1958	0.9539	1.0000	0.9549	257.0910	359.2190	177.9230
1959	0.9709	0.9761	0.9675	276.8110	392.2850	189.3030
1960	0.9912	1.0149	0.9704	283.7320	398.9750	192.4910
1961	1.0014	1.0213	0.9816	290.7830	403.2770	192.4340
1962	1.0092	1.0274	0.9852	311.6880	427.1550	206.6880
1963	1.0276	1.0049	0.9860	321.9510	448.3110	218.2000
1964	1.0494	1.0383	0.9953	339.6170	470.4290	229.8620
1965	1.0684	1.0597	0.9999	363.5150	506.8340	243.8240
1966	1.1076	1.0974	1.0206	391.5050	524.9130	259.9750
1967	1.1492	1.1108	1.0435	402.8120	542.0410	267.4580
1968	1.2100	1.1180	1.0587	424.1360	576.5270	285.4560
1969	1.2916	1.2438	1.0677	434.8370	577.9360	294.8180
1970	1.3781	1.2434	1.0944	436.2130	561.2830	289.5430
1971	1.4882	1.2248	1.0777	451.8940	565.2600	300.2930

*in billions of 1958 dollars

	KE	LE	EME	NME	RE
Average input 1947-1971	12.2	8.3	16.9	11.9	1.9
Average growth rate p.a. 1947-1971	8.0%	0.0%	8.0%	4.6%	13.5%

Table 3-4 shows the quantity indices of inputs into the non-energy sector over the period 1947-1971. For this sector also, we note that the greatest growth rate of any input in real terms corresponds to imports:

	KN	LN	EMN	NMN	RN
Average input 1947-1971	174.3	259.8	18.1	383.7	14.8
Average growth rate p.a. 1947-1971	6.0%	1.9%	8.1%	4.1%	15.0%

The price and quantity indices of the two output categories in the energy sector -- energy consumption products and energy intermediate products are shown in Table 3-5. Approximate average values and growth rates are tabulated below:

	p_{EC}	p_{EM}	EC	EM
Average output 1947-1971	--	--	20.9	35.2
Average growth rate p.a. 1947-1971	-1.5%	0.5%	12.6%	8.1%

Annual price and quantity indices for the three product categories of the non-energy sector -- non-energy consumption goods, non-energy intermediate goods and investment goods -- are tabulated in Table 3-6. The following are approximate average values and growth rates:

	P_{NC}	P_{NM}	P_{NI}	NC	NM	NI
Average output 1947-1971	--	--	--	284.7	398.9	190.1
Average growth rate p.a. 1947-1971	3.8%	2.6%	2.1%	5.5%	3.6%	6.4%

We tabulate below some representative figures corresponding to the ratios in real terms between various inputs to the energy sector and the output quantity index defined as the Divisia aggregate of inputs and denoted by YE:

	KE/YE	LE/YE	KE/LE	EME/YE
1947	0.22	0.28	0.80	0.33
1958	0.21	0.16	1.40	0.31
1971	0.19	0.08	3.20	0.29

Representative values of capital-output, labor-output, capital-labor and energy-output ratios for the U.S. energy sector for 1947, 1958 and 1971

We observe that the capital-output ratios and the energy-output ratios have shown slight declines over the period 1947-1971 and that the labor-output ratio has decreased very significantly over the same period.

The corresponding values for the non-energy sector are tabulated below where the output quantity index YN was constructed as the Divisia aggregate of the input quantity indices:

	KN/YN	LN/YN	KN/LN	EMN/YN
1947	0.21	0.38	0.55	0.016
1958	0.19	0.28	0.65	0.018
1971	0.19	0.25	0.76	0.024

Representative values of capital-output, labor-output, capital-labor and energy-output ratios for the U.S. non-energy sector for 1947, 1958 and 1971

It can be observed that the values of the capital-output and labor-output ratios have declined slightly whereas the energy-output ratio shows a small increase over the period 1947-1971. We also note that the capital-labor ratio in the energy sector has been consistently larger than that of the non-energy sector, and further, that the difference in these two ratios has markedly increased over the period considered.

3.1.2 Econometric Specification: Cost and Revenue Shares

The econometric specification of our model of production is based on the systems of cost and revenue share equations for the energy and non-energy sectors described in section 2.3.3. We recall that these two sets of share equations were derived from the translog cost frontiers under the assumptions of perfectly competitive product and factor markets.

In the energy sector there are five input equations, each corresponding to the cost shares M_K^E, M_L^E, M_E^E, M_N^E, and M_R^E and two output equations corresponding to the revenue shares F_C^E and F_M^E. The systems

Table 3-7: INPUT COST SHARES, U.S. ENERGY SECTOR, 1947-1971

Year	Total Input Cost*	Cost Shares K	L	E	N	R
1947	23.0468	0.233199	0.204431	0.306573	0.234203	0.021594
1948	27.9705	0.275679	0.171046	0.350313	0.178896	0.024065
1949	27.0593	0.257462	0.183237	0.338121	0.195114	0.026066
1950	29.5079	0.268115	0.156857	0.324866	0.218655	0.031508
1951	33.6932	0.269576	0.182384	0.325018	0.189643	0.033379
1952	35.0774	0.261271	0.183379	0.325531	0.195272	0.034547
1953	38.1326	0.262781	0.178758	0.333756	0.189771	0.034935
1954	38.9677	0.279427	0.167635	0.343591	0.177577	0.031770
1955	42.8893	0.283166	0.159088	0.336794	0.186784	0.034168
1956	46.8539	0.281020	0.161462	0.333309	0.189108	0.035100
1957	50.8891	0.255792	0.160290	0.339750	0.205966	0.037978
1958	50.3840	0.253932	0.161815	0.339334	0.210682	0.034237
1959	53.5957	0.257541	0.158873	0.336448	0.213328	0.033810
1960	56.7057	0.260361	0.159191	0.334212	0.216596	0.029640
1961	59.0256	0.258243	0.155289	0.333618	0.224594	0.028256
1962	62.2078	0.255226	0.150960	0.333701	0.229818	0.030294
1963	64.6954	0.256664	0.151155	0.336943	0.223943	0.031006
1964	67.1301	0.256416	0.151092	0.335189	0.226299	0.031006
1965	70.3406	0.260124	0.151090	0.328788	0.229088	0.030912
1966	75.4023	0.253676	0.150569	0.329786	0.230086	0.035884
1967	80.4527	0.257499	0.148354	0.326031	0.234498	0.033619
1968	85.0649	0.255114	0.151528	0.331171	0.222696	0.039491
1969	91.4383	0.237261	0.151023	0.334228	0.236389	0.041100
1970	96.6951	0.245669	0.147722	0.316479	0.246095	0.044036
1971	108.8790	0.233249	0.139294	0.339858	0.238841	0.048758

* in billions of current dollars

Table 3-8: OUTPUT REVENUE SHARES, U.S. ENERGY SECTOR, 1947-1971

Year	Total Output Revenue*	EC	EM
1947	23.0468	0.320821	0.680151
1948	27.9705	0.316016	0.683934
1949	27.0593	0.322643	0.67731
1950	29.5079	0.33225	0.667785
1951	33.6932	0.345612	0.654413
1952	35.0774	0.352672	0.647302
1953	38.1326	0.354132	0.64588
1954	38.9677	0.344026	0.655738
1955	42.8893	0.358367	0.641611
1956	46.8539	0.356137	0.643893
1957	50.8891	0.361143	0.638865
1958	50.3840	0.356065	0.643934
1959	53.5957	0.357473	0.64252
1960	56.7057	0.360738	0.645747
1961	59.0256	0.358998	0.641007
1962	62.2078	0.354788	0.64521
1963	64.6954	0.355707	0.644286
1964	67.1301	0.349725	0.650285
1965	70.3406	0.35083	0.649162
1966	75.4023	0.351546	0.648461
1967	80.4527	0.35512	0.6449
1968	85.0649	0.347658	0.652323
1969	91.4383	0.342288	0.657839
1970	96.6951	0.34661	0.653411
1971	108.8790	0.33044	0.669715

* in billions of current dollars

Table 3-9: INPUT COST SHARES. U.S. NON-ENERGY SECTOR, 1947-1971

Year	Total Input Cost*	Cost Shares K	L	E	N	R
1947	438.968	0.205171	0.275693	0.019614	0.484754	0.001134
1948	455.062	0.223989	0.294090	0.020506	0.444257	0.001479
1949	458.252	0.211874	0.294054	0.020029	0.459013	0.001539
1950	513.425	0.224644	0.268197	0.019708	0.470521	0.001811
1951	587.842	0.220727	0.283027	0.018880	0.458743	0.001913
1952	613.700	0.210817	0.298513	0.018392	0.454535	0.001975
1953	651.021	0.202796	0.301670	0.018282	0.460292	0.002046
1954	640.136	0.209971	0.298522	0.019002	0.456318	0.001934
1955	706.178	0.212168	0.283027	0.018513	0.460397	0.002075
1956	748.966	0.202027	0.296176	0.019430	0.465398	0.002196
1957	776.430	0.203870	0.301624	0.019590	0.458328	0.002489
1958	774.374	0.212118	0.301694	0.019819	0.450175	0.002228
1959	834.830	0.218290	0.299694	0.019650	0.444972	0.002171
1960	872.953	0.213944	0.299786	0.020237	0.449787	0.001925
1961	891.985	0.212437	0.305126	0.020341	0.446902	0.001870
1962	956.989	0.210566	0.309937	0.020249	0.443648	0.001969
1963	996.488	0.211600	0.315274	0.019954	0.437564	0.002032
1964	1073.657	0.212012	0.311676	0.019701	0.440828	0.001939
1965	1169.304	0.212219	0.306047	0.019272	0.445568	0.001860
1966	1275.037	0.209200	0.314730	0.018846	0.438200	0.002122
1967	1344.162	0.204191	0.323288	0.019085	0.433924	0.002012
1968	1459.992	0.203261	0.327931	0.018712	0.428521	0.002301
1969	1595.305	0.194064	0.327978	0.018548	0.437064	0.002356
1970	1615.957	0.194383	0.343499	0.020161	0.417177	0.002635
1971	1725.282	0.201696	0.344519	0.020817	0.407536	0.003077

* in billions of current dollars

Table 3-10: OUTPUT REVENUE SHARES, U.S. NON-ENERGY SECTOR, 1947-1971

Year	Total Output Revenue*	NC	NM	NI
1947	438.968	0.318587	0.49705	0.184363
1948	455.062	0.344064	0.455253	0.200683
1949	458.252	0.321431	0.470534	0.208034
1950	513.425	0.311874	0.483087	0.205039
1951	587.842	0.312353	0.469613	0.218034
1952	613.700	0.308478	0.465696	0.225826
1953	651.021	0.305717	0.471407	0.222875
1954	640.136	0.307108	0.467128	0.225764
1955	706.178	0.303389	0.471741	0.22487
1956	748.966	0.297785	0.477228	0.224987
1957	776.430	0.303228	0.471827	0.224944
1958	774.374	0.316698	0.463883	0.219419
1959	834.830	0.32193	0.458668	0.219402
1960	872.953	0.322165	0.463856	0.213979
1961	891.985	0.326457	0.461764	0.211778
1962	956.989	0.328613	0.458587	0.2128
1963	996.488	0.331987	0.452103	0.21591
1964	1073.657	0.331936	0.454977	0.213086
1965	1169.304	0.33214	0.459348	0.208511
1966	1275.037	0.340091	0.451807	0.208102
1967	1344.162	0.3444	0.447959	0.207641
1968	1459.992	0.351506	0.441496	0.206997
1969	1595.305	0.352055	0.450613	0.197331
1970	1615.957	0.371994	0.431903	0.196104
1971	1725.282	0.389808	0.422608	0.187584

* in billions of current dollars

$$
\begin{bmatrix} M_K^E \\ M_L^E \\ M_E^E \\ M_N^E \\ F_C^E \\ S_t^E \end{bmatrix}
=
\begin{bmatrix} \beta_K^E \\ \beta_L^E \\ \beta_E^E \\ \beta_N^E \\ \alpha_C^E \\ \beta_t^E \end{bmatrix}
+
\begin{bmatrix}
\gamma_{KK}^E & \gamma_{KL}^E & \gamma_{KE}^E & \gamma_{KN}^E & \rho_{CK}^E & \gamma_{Kt}^E \\
\gamma_{KL}^E & \gamma_{LL}^E & \gamma_{LE}^E & \gamma_{LN}^E & \rho_{CL}^E & \gamma_{Lt}^E \\
\gamma_{KE}^E & \gamma_{LE}^E & \gamma_{EE}^E & \gamma_{EN}^E & \rho_{CE}^E & \gamma_{Et}^E \\
\gamma_{KN}^E & \gamma_{LN}^E & \gamma_{EN}^E & \gamma_{NN}^E & \rho_{CN}^E & \gamma_{Nt}^E \\
\rho_{CK}^E & \rho_{LC}^E & \rho_{CE}^E & \rho_{CN}^E & \delta_{CC}^E & \rho_{Ct}^E \\
\gamma_{Kt}^E & \gamma_{Lt}^E & \gamma_{Et}^E & \gamma_{Nt}^E & \rho_{Ct}^E & \gamma_{tt}^E
\end{bmatrix}
\begin{bmatrix} \ln(p_{KE}/p_{RE}) \\ \ln(p_{LE}/p_{RE}) \\ \ln(p_{EM}^E/p_{RE}) \\ \ln(p_{NM}^E/p_{RE}) \\ \ln(EC/EC) \\ t \end{bmatrix}
+
\begin{bmatrix} \varepsilon_K^E \\ \varepsilon_L^E \\ \varepsilon_E^E \\ \varepsilon_N^E \\ \varepsilon_C^E \\ \varepsilon_t^E \end{bmatrix}
$$

Table 3-11: MULTIVARIATE SHARE ESTIMATING SYSTEM FOR THE TRANSLOG COST FRONTIER, U.S. ENERGY SECTOR of cost and revenue shares are coupled since we do not impose input-output separability a priori. The values of the cost and revenue shares for the energy sector are tabulated in Tables 3-7 and 3-8 for the period 1947-1971.

In the non-energy sector there are also five input equations associated with the cost shares M_K^N, M_L^N, M_E^N, M_N^N, M_R^N and three output equations associated with the revenue shares F_C^N, F_M^N, and F_I^N. The values of the cost and revenue shares for the non-energy sector for the years 1947-1971 are tabulated in Tables 3-9 and 3-10.

The Energy Sector

The specific form of the seven share equations for the energy production sector was given in Eqs. 2-12 in Chapter II. Because of the cross-terms involving input prices in the revenue shares and output quantities in the cost shares, both systems must be estimated simultaneously. The unconstrained system of seven shares involves 63 unknown parameters.

Assuming a well-behaved technology leading to cost frontiers with continuous first-order derivatives, it follows that the Hessian matrices are symmetric and therefore the following symmetry constraints must hold:

$$\rho_{ij}^E = \rho_{ji}^E \qquad i = K, L, E, N, R \qquad j = C, M \qquad (3.1)$$

$$\gamma_{ij}^E = \gamma_{ji}^E \qquad i = K, L, E, N, R \qquad j = K, L, E, N, R \quad (3.2)$$

$$\delta^{E}_{CM} = \delta^{E}_{MC} \tag{3.3}$$

With symmetry imposed, the number of unknown parameters is reduced to 41.

From the assumption of competitive product and factor markets, it follows that there are zero excess profits and that the sector's output product is totally distributed to the factors of production, i.e. the input cost shares must add up to unity:

$$M^{E}_{K} + M^{E}_{L} + M^{E}_{E} + M^{E}_{N} + M^{E}_{R} = 1 \tag{3.4}$$

Since this identity must hold at every observation point, it implies the following parametric restrictions:

$$\beta^{E}_{K} + \beta^{E}_{L} + \beta^{E}_{E} + \beta^{E}_{N} + \beta^{E}_{R} = L$$

$$\gamma^{E}_{KK} + \gamma^{E}_{KL} + \gamma^{E}_{KE} + \gamma^{E}_{KN} + \gamma^{E}_{KR} = 0$$

$$\gamma^{E}_{KL} + \gamma^{E}_{LL} + \gamma^{E}_{LE} + \gamma^{E}_{LN} + \gamma^{E}_{LR} = 0$$

$$\gamma^{E}_{KE} + \gamma^{E}_{LE} + \gamma^{E}_{EE} + \gamma^{E}_{EN} + \gamma^{E}_{ER} = 0$$

$$\gamma^{E}_{KN} + \gamma^{E}_{LN} + \gamma^{E}_{EN} + \gamma^{E}_{NN} + \gamma^{E}_{NR} = 0$$

$$\gamma^{E}_{KR} + \gamma^{E}_{LR} + \gamma^{E}_{ER} + \gamma^{E}_{NR} + \gamma^{E}_{RR} = 0$$

$$\rho^{E}_{CK} + \rho^{E}_{CL} + \rho^{E}_{CE} + \rho^{E}_{CN} + \rho^{E}_{CR} = 0$$

$$\rho^{E}_{MK} + \rho^{E}_{ML} + \rho^{E}_{ME} + \rho^{E}_{MN} + \rho^{E}_{MR} = 0 \tag{3.5}$$

From the assumption of constant returns to scale (CRTS), it follows that the output revenue shares must add up to unity:

$$F_C^E + F_M^E = 1 \tag{3.6}$$

This identity must also hold at every observation point and it implies the following parametric restrictions:

$$\begin{aligned}
\alpha_C^E + \alpha_M^E &= 1 \\
\rho_{CK}^E + \rho_{MK}^E &= 0 \\
\rho_{CL}^E + \rho_{ML}^E &= 0 \\
\rho_{CE}^E + \rho_{ME}^E &= 0 \\
\rho_{CN}^E + \rho_{MN}^E &= 0 \\
\rho_{CR}^E + \rho_{MR}^E &= 0 \\
\delta_{CC}^E + \delta_{MC}^E &= 0 \\
\delta_{CM}^E + \delta_{MM}^E &= 0
\end{aligned} \tag{3.7}$$

With these additional constraints imposed, the total number of parameters remaining to be determined is reduced to 25.

Because of the identities (3.4) and (3.6), the system of cost and revenue shares is doubly over-identified and in order to avoid singularity of the covariance matrix it is necessary to drop one input share equation and one output share equation from the complete system. We arbitrarily drop the cost share equation corresponding to M_R^E and the revenue share corresponding to F_M^E. The method of estimation will later be chosen so that the resulting parameter estimates are invariant with respect to the equations deleted.

In examining the resulting set of share equations, we observe that the parameters corresponding to the time variable, i.e. γ^E_{Kt}, γ^E_{Lt}, γ^E_{Et}, γ^E_{Nt}, ρ^E_{Ct}, are not identifiable. We must adjoin to the existing set of estimating share equations an equation that reflects the nature of the <u>time variation</u> of the cost frontier.

The total time variation of the cost frontier of the energy sector can be decomposed as follows:

$$\frac{1}{V_E}\dot{V}_E = \frac{\partial V_E}{\partial t} + \frac{\partial V_E}{\partial \ln EC}(\ln \dot{EC}) + \frac{\partial V_E}{\partial \ln EM}(\ln \dot{EM}) + \frac{\partial V_E}{\partial \ln p_{KE}}(\ln \dot{p}_{KE})$$

$$+ \frac{\partial V_E}{\partial \ln p_{LE}}(\ln \dot{p}_{LE}) + \frac{\partial V_E}{\partial \ln p_{EME}}(\ln \dot{p}_{EME})$$

$$+ \frac{\partial V_E}{\partial \ln p_{NME}}(\ln \dot{p}_{NME}) + \frac{\partial V_E}{\partial \ln p_{RE}}(\ln \dot{p}_{RE}) \qquad (3.8)$$

The last seven terms in the right hand side of this equation represent the contributions to the total time variation attributable to shifts in the various input prices and in the configuration of output. The first term of the right hand side -- the partial derivative of V_E with respect to time -- can be interpreted as the rate of technological change, i.e. as shifts in the cost frontier due to structural changes unexplained by shifts in the endogenous input and output components.

An expression for the rate of technological change can be obtained by formally taking the partial derivative of the cost frontier -- Eq. (2.1) -- with respect to time:

$$\frac{\partial V_E}{\partial t} = [\beta_t^E + \gamma_{Kt}^E \ln p_{KE} + \gamma_{Lt}^E \ln p_{LE} + \gamma_{Et}^E \ln p_{EME} + \gamma_{Nt}^E \ln p_{NME}$$

$$+ \gamma_{Rt}^E \ln p_{RE} + \delta_{Ct}^E \ln EC + \delta_{Mt}^E \ln EM + \gamma_{tt}^E \cdot t] \quad (3.9)$$

This expression can serve as the additional estimating equation sought for, provided we can construct an appropriate time-series for the dependent variable. Denoting the rate of cost diminution attributable to technological change by S_t^E, we obtain from Eq. (3.8) the following expression:

$$S_t^E = \frac{\partial V_E}{\partial t} = \frac{1}{V_E} \dot{V}_E - \frac{\partial V_E}{\partial EC} \dot{EC} - \frac{\partial V_E}{\partial EM} \dot{EM} - \frac{\partial V_E}{\partial p_{KE}} \dot{p}_{KE} - \frac{\partial V_E}{\partial p_{LE}} \dot{p}_{LE}$$

$$- \frac{\partial V_E}{\partial p_{EME}} \dot{p}_{EME} - \frac{\partial V_E}{\partial p_{NME}} \dot{p}_{NME} - \frac{\partial V_E}{\partial p_{RE}} \dot{p}_{RE} \qquad (3.10)$$

Making use of the fact that V_E is of the translog form this becomes

$$S_t^E = \frac{\dot{V}_E}{V_E} - \frac{p_{EC}EC}{V_E} \frac{\dot{EC}}{EC} - \frac{p_{EM}EM}{V_E} \frac{\dot{EM}}{EM} - \frac{p_{KE}KE}{V_E} \frac{\dot{p}_{KE}}{p_{KE}} - \frac{p_{LE}LE}{V_E} \frac{\dot{p}_{LE}}{p_{LE}}$$

$$- \frac{p_{EME}EME}{V_E} \frac{\dot{p}_{EME}}{p_{EME}} - \frac{p_{NME}NME}{V_E} \frac{\dot{p}_{NME}}{p_{NME}} - \frac{p_{RE}RE}{V_E} \dot{RE} \frac{p_{RE}}{p_{RE}} \qquad (3.11)$$

Since the relevant data are available at discrete points in time, we must approximate the terms in the right-hand side of Eq. 3.11 by appropriate index numbers defined in terms of observable magnitudes. We approximate the rates of growth by the period to period charges in logarithms, i.e.:

$$\frac{\dot{V}_E}{V_E} = \ln V_E^t - \ln y_E^{t-1} \tag{3.12}$$

and we approximate the weighted rates of growth by the same quantity times the arithmetic average of the relative shares in the two periods:

$$M_i \frac{\dot{p}_i}{p_i} = \frac{1}{2} [M_i^t + M_i^{t-1}][\ln p_i^t - \ln p_i^{t-1}] \tag{3.13}$$

$$i = KE, LE, EME, NME, RE$$

$$F_i \frac{\dot{y}_i}{y_i} = \frac{1}{2} [F_i^t + F_i^{t-1}][\ln y_i^t - \ln p_i^{t-1}] \tag{3.14}$$

$$i = 1,2;\; y_1 = EC;\; y_2 = EM$$

These index numbers were suggested as discrete approximations to Divisia index numbers by Tornquist [27]. With these approximations, the rate of total cost diminution due to technological change can be expressed in terms of observable quantities as follows:

$$S_t^E = \ln V_E^t - \ln V_E^{t-1} - \sum_{i=1}^{5} \frac{1}{2} [M_i^t + M_i^{t-1}][\ln p_i^t - \ln p_i^{t-1}] - \sum_{j=1}^{2} \frac{1}{2} [F_j^t + F_j^{t-1}][\ln y_j^t - \ln y_j^{t-1}] \tag{3.15}$$

After having deleted the share equations associated with M_R^E and F_M^E to insure non-singularity of the estimating system, there remained five share equations. We now adjoin equation 3.9 to the system, where the time-series corresponding to the dependent variable is computed according to equation 3.15 above. The final set of equations to be estimated comprises six share equations and 27 unknown parameters. The complete set of estimating equations is shown in Table 3.11 where all the parametric restrictions implied by the symmetry conditions and CRTS have been incorporated. It has been assumed that departures from the set of factor demands implied by cost-minimizing behavior are attributable to random errors and can be adequately represented by the vector of stochastic disturbances $\underline{\varepsilon}^E$.

The Non-Energy Sector

The production model for the non-energy sector comprises the three revenue share equations corresponding to non-energy consumption goods, non-energy intermediate goods, and investment goods and the five cost share equations corresponding to each of the five factor inputs. This system of eight coupled share equations involves 80 unknown parameters in its unconstrained form -- see Eqs. 2-13.

The symmetry constraints on the translog cost frontier for the non-energy sector imply the following parametric restrictions:

$$\rho^N_{ij} = \rho^N_{ij} \qquad i = K, L, E, N, R \qquad j = C, M, I \qquad (3.16)$$

$$\gamma^N_{ij} = \gamma^N_{ij} \qquad i = K, L, E, N, R \qquad j = K, L, E, N, R \quad (3.17)$$

$$\delta^N_{ij} = \delta^N_{ij} \qquad i = C, M, I \qquad j = C, M, I \qquad (3.18)$$

With symmetry imposed, the number of parameters to be determined is reduced to 52.

As before, the assumption of perfectly competitive markets and of constant returns to scale imply that the input cost shares and the output revenue shares must add up to unity:

$$M_K^N + M_L^N + M_E^N + M_N^N + M_R^N = 1 \tag{3.19}$$

$$F_C^N + F_M^N + F_I^N = 1 \tag{3.20}$$

Since the identity given by Eq. 3.19 must hold at every observation point, the following restrictions on the parameters of the cost frontiers hold:

$$\begin{aligned}
&\beta_K^N + \beta_L^N + \beta_E^N + \beta_N^N + \beta_R^N = 1 \\
&\gamma_{KK}^N + \gamma_{KL}^N + \gamma_{KE}^N + \gamma_{KN}^N + \gamma_{KR}^N = 0 \\
&\gamma_{KL}^N + \gamma_{LL}^N + \gamma_{LE}^N + \gamma_{LN}^N + \gamma_{LR}^N = 0 \\
&\gamma_{KE}^N + \gamma_{LE}^N + \gamma_{EE}^N + \gamma_{EN}^N + \gamma_{ER}^N = 0 \\
&\gamma_{KN}^N + \gamma_{LN}^N + \gamma_{EN}^N + \gamma_{NN}^N + \gamma_{NR}^N = 0 \\
&\gamma_{KR}^N + \gamma_{LR}^N + \gamma_{ER}^N + \gamma_{NR}^N + \gamma_{RR}^N = 0 \\
&\rho_{CK}^N + \rho_{CL}^N + \rho_{CE}^N + \rho_{CN}^N + \rho_{CR}^N = 0 \\
&\rho_{MK}^N + \rho_{ML}^N + \rho_{ME}^N + \rho_{MN}^N + \rho_{MR}^N = 0 \\
&\rho_{IK}^N + \rho_{IL}^N + \rho_{IE}^N + \rho_{IN}^N + \rho_{IR}^N = 0
\end{aligned} \tag{3.21}$$

Similarly the constraint on the revenue shares given by Eq. 3.20

must hold at every observation point and this leads to the following parametric restrictions:

$$\alpha_C^N + \alpha_M^N + \alpha_I^N = 1$$

$$\rho_{CK}^N + \rho_{MK}^N + \rho_{IK}^N = 0$$

$$\rho_{CL}^N + \rho_{ML}^N + \rho_{IL}^N = 0$$

$$\rho_{CE}^N + \rho_{ML}^N + \rho_{IL}^N = 0$$

$$\rho_{CN}^N + \rho_{MN}^N + \rho_{IN}^N = 0$$

$$\rho_{CR}^N + \rho_{MR}^N + \rho_{IR}^N = 0$$

$$\delta_{CC}^N + \delta_{MC}^N + \delta_{IC}^N = 0$$

$$\delta_{MC}^N + \delta_{MM}^N + \delta_{IM}^N = 0$$

$$\delta_{CI}^N + \delta_{MI}^N + \delta_{II}^N = 0 \qquad (3.22)$$

With these additional restrictions imposed, the total number of parameters to be determined is reduced to 34.

The restrictions implied by identities 3.19 and 3.20 determine that the full system of eight share equations is doubly singular: we must therefore delete one input share and one output share from the estimating system in order to insure non-singularity of the covariance matrix. We arbitrarily choose to delete the cost share equation associated with M_R^N and the revenue share equation corresponding to F_I^N.

For the parameters associated with the time variable to be identifiable, it is necessary to adjoin to the set of six share equations an equation reflecting the time variation of the cost frontier V_N. In exact analogy with the derivation described above for the energy sector, we formally take the partial derivative of V_N -- given in equation (2.2) -- with respect to time, obtaining the expression:

$$\frac{\partial V_N}{\partial t} = [\beta_t^N + \gamma_{Kt}^N \ln p_{KN} + \gamma_{Lt}^N \ln p_{LN} + \gamma_{Et}^N \ln p_{EMN} + \gamma_{Nt}^N \ln p_{NMN} + \gamma_{Rt}^N \ln p_{RN} + \delta_{Ct}^N \ln NC + \delta_{Mt}^N \ln NM + \delta_{It}^N \ln NI + + \gamma_{tt}^N \cdot t] \quad (3.23)$$

As before, the partial derivative of V_N with respect to time represents the rate of cost diminution attributable to technological change -- we denote it by S_t^N. An expression for S_t^N in terms of the rates of change of input prices and output quantities can be obtained by totally differentiating V_N with respect to time and taking the translog functional form into consideration:

$$S_t^N = \frac{\dot{V}_N}{V_N} - \frac{p_{NC}NC}{V_N}\frac{\dot{NC}}{NC} - \frac{p_{NM}NM}{V_N}\frac{\dot{NM}}{NM} - \frac{p_{NI}NI}{V_N}\frac{\dot{NI}}{NI} - \frac{p_{KN}KN}{V_N}\frac{\dot{p}_{KN}}{p_{KN}} - \frac{p_{LN}LN}{V_N}\frac{\dot{p}_{LN}}{p_{LN}} - \frac{p_{EMN}EMN}{V_N}\frac{\dot{p}_{EMN}}{p_{EMN}} - \frac{p_{NMN}NMN}{V_N}\frac{\dot{p}_{NMN}}{p_{NMN}} - \frac{p_{RN}RN}{V_N}\frac{\dot{p}_{RN}}{p_{RN}} \quad (3.24)$$

Again we approximate the rates of change in the right-hand side of

$$
\begin{bmatrix} M_K^N \\ M_L^N \\ M_E^N \\ M_N^N \\ F_C^N \\ F_M^N \\ S_t^N \end{bmatrix}
=
\begin{bmatrix} \beta_K^N \\ \beta_L^N \\ \beta_E^N \\ \beta_N^N \\ \alpha_C^N \\ \alpha_M^N \\ \beta_t^N \end{bmatrix}
+
\begin{bmatrix}
\gamma_{KK}^N & \gamma_{KL}^N & \gamma_{KE}^N & \gamma_{KN}^N & \rho_{CK}^N & \rho_{MK}^N & \gamma_{Kt}^N \\
\gamma_{KL}^N & \gamma_{LL}^N & \gamma_{LE}^N & \gamma_{LN}^N & \rho_{CL}^N & \rho_{ML}^N & \gamma_{Lt}^N \\
\gamma_{KE}^N & \gamma_{LE}^N & \gamma_{EE}^N & \gamma_{EN}^N & \rho_{CE}^N & \rho_{ME}^N & \gamma_{Et}^N \\
\gamma_{KN}^N & \gamma_{LN}^N & \gamma_{EN}^N & \gamma_{NN}^N & \rho_{CN}^N & \rho_{MN}^N & \gamma_{Nt}^N \\
\rho_{CK}^N & \rho_{CL}^N & \rho_{CE}^N & \rho_{CN}^N & \delta_{CC}^N & \delta_{MC}^N & \rho_{Ct}^N \\
\rho_{MK}^N & \rho_{ML}^N & \rho_{ME}^N & \rho_{MN}^N & \delta_{MC}^N & \delta_{MM}^N & \rho_{Mt}^N \\
\gamma_{Kt}^N & \gamma_{Lt}^N & \gamma_{Et}^N & \gamma_{Nt}^N & \rho_{Ct}^N & \rho_{Mt}^N & \gamma_{tt}^N
\end{bmatrix}
\begin{bmatrix} \ln(p_{KN}/p_{RN}) \\ \ln(p_{LN}/p_{RN}) \\ \ln(p_{EM}^N/p_{RN}) \\ \ln(p_{NM}^N/p_{RN}) \\ \ln(NC/NI) \\ \ln(NM/NI) \\ t \end{bmatrix}
+
\begin{bmatrix} \varepsilon_K^N \\ \varepsilon_L^N \\ \varepsilon_E^N \\ \varepsilon_N^N \\ \varepsilon_C^N \\ \varepsilon_M^N \\ \varepsilon_t^N \end{bmatrix}
$$

Table 3-12: MULTIVARIATE SHARE ESTIMATING SYSTEM FOR THE TRANSLOG COST FRONTIER, U.S. NON-ENERGY SECTOR

(3.24) by discrete Divisia index numbers so that a time-series for S_t^N can be generated in terms of observable quantities:

$$S_t^N = \ln V_N^t - \ln V_N^{t-1} - \sum_{i=1}^{5} \frac{1}{2} [M_i^t + M_i^{t-1}][\ln p_i^t - \ln p_i^{t-1}] - \sum_{j=1}^{3} \frac{1}{2} [F_j^t + F_j^{t-1}][\ln y_j^t - \ln y_j^{t-1}] \tag{3.25}$$

The final set of estimating equations for the non-energy production sector includes four input cost shares and two output shares -- i.e. the original eight equations minus the two equations deleted to avoid singularity -- and Eq. (3.23) representing the rate of change of cost diminution attributable to technological change. The values of the dependent variable for the latter equation are computed as shown in Eq. (3.25). The final number of parameters to be determined in the seven estimating equations is 36.

In Table 3.12 we show the final estimating system for the translog cost frontier of the non-energy production sector, incorporating all the parametric restrictions implied by the symmetry constraints and CRTS. Again we postulate that the vector of stochastic disturbances $\underline{\varepsilon}^N$ represents departures from cost minimizing behavior due to random errors.

3.1.3 Parameter Estimation and Hypothesis Testing

We have fitted the parameters of the translog cost frontiers for both the energy and non-energy production sectors employing the stochastic specifications outlined above. We employ an iterative version of the three-stage least-squares estimator proposed by Zellner and Theil [6]. This I3SLS estimator is consistent and asymptotically equivalent to the maximum likelihood estimator (Malinvaud [7]) although in finite samples it provides parameter estimates that can differ from the full information maximum likelihood estimates (Dhrymes [8]). Berndt [9] has shown that the I3SLS estimator preserves the invariance property of the Iterative Zellner Efficient Estimator brought forth in the studies of Pollak and Wales [10] and the work of Barten [11] on maximum likelihood estimators. This property is important for our purposes since it implies that the I3SLS parameter estimates obtained by dropping one equation from the estimating system are invariant to the equation ommitted.

We have estimated the parameters in the two production models under alternative sets of restrictions on the structure of the underlying technology. We present empirical results of statistical tests conducted on the validity of the alternative hypotheses. Alternative specifications on the structure of the stochastic disturbances are also hypothesized and tested.

The parameter estimates for the energy and non-energy production sectors are shown in Tables 3-13 and 3-14, respectively. Estimates were obtained for observations during the period 1948-1971 under six different assumptions on the structure of the production models. The estimation and testing of hypotheses were conducted in the fol-

Table 3-13: PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER, U.S. ENERGY SECTOR*

	Symmetry	Symmetry, Input/Output Separability	Symmetry, I/O Separability Berndt-Savin Autoreg. Transf.	Symmetry, {K,L},{E,N} Separability	Symmetry, {K,L,N},{E} Separability	Symmetry, I/O Separability {K,L},{E,N} Separability
β_K^E	0.2753 (95.7547)	0.25079 (210.7720)	0.2495 (42.3290)	0.2700 (92.6381)	0.2679 (100.3210)	0.2564 (245.2830)
γ_{KK}^E	0.0791 (11.7280)	0.0853 (10.2730)	0.1390 (7.9235)	0.0338 (5.5812)	0.0792 (10.9070)	0.03953 (6.0597)
γ_{KL}^E	-0.0086 (-1.3712)	-0.0093 (-1.4059)	-0.1345 (-2.3422)	-0.0342 (-5.5435)	-0.0134 (-2.1896)	-0.0257 (-3.6445)
γ_{KE}^E	-0.0284 (-4.9355)	-0.0146 (-2.8378)	-0.0039 (-0.3525)			
γ_{KN}^E	-0.0338 (-4.1480)	-0.0423 (-4.8424)	-0.0808 (-5.4326)		-0.0619 (-10.0568)	
ρ_{CK}^E	0.0410 (8.6288)			0.0218 (4.9267)	0.0268 (6.4822)	
γ_{Kt}^E	-0.0012 (-5.4644)	-0.0005 (-2.4551)	0.0003 (0.2959)	-0.0005 (-2.4-80)	-0.0006 (-3.3994)	-0.0006 (-1.8997)
β_L^E	0.1495 (42.3136)	0.1609 (166.0870)	0.1637 (42.8270)	0.1532 (50.3595)	0.1538 (56.8704)	0.1635 (161.7640)

* asymptotic t-ratios in parentheses

Table 3-13 (cont.): PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER, U.S. ENERGY SECTOR

	Symmetry	Symmetry, Input/Output Separability	Symmetry, I/O Separability Berndt-Savin Autoreg. Transf.	Symmetry, {K,L},{E,N} Separability	Symmetry, {K,L,N},{E} Separability	Symmetry, I/O Separability {K,L},{E,N} Separability
γ^E_{LL}	-0.1282 (-8.2182)	-0.1151 (-8.4189)	-0.1229 (-3.3086)	-0.0199 (-2.5924)	-0.1173 (-9.7123)	-0.0279 (-3.2435)
γ^E_{LE}	0.0165 (1.6942)	-0.0005 (-0.0874)	0.0319 (1.3086)			
γ^E_{LN}	0.0615 (6.5411)	0.0743 (7.8413)	0.0922 (3.3914)		0.0735 (8.0276)	
ρ^E_{CL}	-0.0199 (-3.1514)			-0.0191 (-3.8540)	-0.0124 (-2.8622)	
γ^E_{Lt}	0.0026 (5.1751)	0.0017 (4.5246)	0.0020 (1.8963)	-0.0000 (-0.3664)	0.0020 (6.2659)	-0.0001 (-0.3928)
β^E_E	0.3271 (95.6482)	0.3389 (417.6250)	0.3388 (85.2590)	0.3337 (111.1310)	0.3253 (120.5730)	0.3368 (636.7290)
γ^E_{EE}	0.05949 (6.05501)	0.0440 (6.9134)	0.0272 (1.5471)	0.0371 (4.6902)	0.0610 (25.9166)	0.0302 (4.8425)
γ^E_{EN}	0.0127 (1.2808)	0.0197 (2.3785)	-0.0042 (-0.2533)	0.0304 (3.2865)		0.0265 (3.5038)

Table 3-13 (cont.): PARAMETER ESTIMATES OF TRANSLOG COST FRONTIER, U.S. ENERGY SECTOR

	Symmetry	Symmetry, Input/Output Separability	Symmetry, I/O Separability Berndt-Savin Autoreg. Transf.	Symmetry, {K,L},{E,N} Separability	Symmetry, {K,L,N},{E} Separability	Symmetry, I/O Separability {K,L},{E,N} Separability
ρ^E_{CE}	-0.0240 (-3.8803)			-0.0057 (-1.0840)	-0.0214 (-4.6799)	
γ^E_{Et}	-0.0010 (-3.0099)	-0.0008 (-4.2490)	-0.0016 (-2.2217)	-0.0010 (-6.6034)	-0.0005 (-5.7295)	-0.0010 (-9.8376)
β^E_N	0.2058 (47.0433)	0.2157 (165.5280)	0.2145 (27.5970)	0.1982 (50.1742)	0.2108 (54.1191)	0.2104 (233.2090)
γ^E_{NN}	-0.0439 (-2.7152)	-0.0563 (-3.6847)	-0.0074 (-0.2121)	-0.0393 (-3.4437)	-0.0132 (-1.3791)	-0.0379 (-3.9117)
ρ^E_{CL}	-0.0140 (-1.9300)			-0.0205 (-3.0307)	-0.0104 (-1.6249)	
γ^E_{Nt}	0.0011 (3.3359)	0.0005 (1.8953)	0.0001 (0.1054)	0.0030 (15.5659)	0.0004 (1.6985)	0.0026 (16.3939)
α^E_C	0.4253 (114.9790)	0.3852 (143.4700)		0.4230 (131.1800)	0.4248 (128.8700)	0.3890 (147.6670)
$\rho\ \sigma^E_{CC}$	0.1426 (22.5899)	0.0655 (14.4706)		0.4239 (24.8280)	0.1387 (25.1705)	0.0720 (16.2346)

Table 3-13 (cont.): PARAMETER ESTIMATES OF TRANSLOG COST FRONTIER, U.S. ENERGY SECTOR

	Symmetry	Symmetry, Input/Output Separability	Symmetry, I/O Separability Berndt-Savin Autoreg. Transf.	Symmetry, {K,L},{E,N} Separability	Symmetry, {K,L,N},{E} Separability	Symmetry, I/O Separability {K,L},{E,N} Separability
$\rho\sigma^E_{Ct}$	-0.0004 (-1.9672)			-0.0002 (-1.4998)	-0.0006 (-3.7401)	
α^E_t	-0.0131 (-7.0705)	-0.0121 (-6.5189)	-0.0119 (-2.0545)	-0.0131 (-7.0622)	-0.0133 (-7.1704)	-0.0120 (-6.3663)
γ^E_{tt}	-0.0003 (-1.1514)	-0.0007 (-3.1700)	-0.0005 (-0.5688)	-0.0002 (-0.9210)	-0.0002 (-1.0603)	-0.0008 (-3.6093)

Table 3-14: PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER, U.S. NON-ENERGY SECTOR*

	Symmetry	Symmetry, Input/Output Separability	Symmetry, I/O Separability Berndt-Savin Autoreg. Transf.	Symmetry, {K,L},{E,N} Separability	Symmetry, {K,L,N},{E} Separability	Symmetry, I/O Separability {K,L},{E,N} Separability
β_K^N	0.2211 (69.7538)	0.2064 (253.7420)	0.2189 (11.2900)	0.1950 (106.8990)	0.2102 (71.3580)	0.2035 (327.0520)
γ_{KK}^N	0.0356 (5.3629)	0.0412 (6.1688)	0.0230 (1.4535)	0.0452 (7.9154)	0.0559 (9.7707)	0.0717 (16.7186)
γ_{KL}^N	-0.0039 (-0.5199)	-0.0439 (-7.2627)	0.0120 (0.6950)	-0.0477 (-9.7238)	-0.0276 (-4.1750)	-0.0621 (-13.8353)
γ_{KE}^N	-0.0067 (-10.1069)	-0.0047 (-6.7681)	-0.0040 (-2.1729)			
γ_{KN}^N	-0.0294 (-6.5187)	0.0126 (10.7307)	-0.0273 (-1.7362)		-0.0261 (-6.3232)	
ρ_{CK}^N	0.0445 (6.5187)			0.0452 (7.4409)	0.0479 (7.7180)	
γ_{Kt}^N	-0.0007 (-3.0714)	0.0003 (1.9069)	-0.0021 (-1.3261)	0.0006 (4.1405)	-0.0000 (-0.0153)	0.0009 (6.5606)
β_L^N	0.3704 (66.3491)	0.3101 (286.8760)	0.2851 (9.1273)	0.2879 (108.2880)	0.3771 (69.6366)	0.3119 (312.8410)

* asymptotic t-ratios in parentheses

Table 3-14 (cont.): PARAMETER ESTIMATES OF TRANSLOG COST FRONTIER, U.S. NON-ENERGY SECTOR

	Symmetry	Symmetry, Input/Output Separability	Symmetry, I/O Separability Berndt-Savin Autoreg. Transf.	Symmetry, {K,L},{E,N} Separability	Symmetry, {K,L,N},{E} Separability	Symmetry, I/O Separability {K,L},{E,N} Separability
γ^N_{LL}	0.1447 (12.1069)	0.0331 (5.7593)	0.1110 (3.1067)	-0.0000 (-0.0144)	0.1629 (14.3524)	0.0649 (13.3163)
γ^N_{LE}	0.0059 (5.5782)	0.0045 (6.3141)	0.0068 (1.5169)			
γ^N_{LN}	-0.1591 (-19.6263)	0.0120 (9.0457)	-0.1151 (-3.5925)		-0.1550 (-19.2717)	
ρ^N_{CL}	0.1381 (17.2173)			0.0890 (11.6551)	0.1310 (17.4340)	
γ^N_{Lt}	-0.0021 (-5.5843)	-0.0000 (-0.0524)	0.0010 (0.3639)	0.0026 (15.3624)	-0.0027 (-7.3255)	-0.0009 (-5.9015)
β^N_E	0.0186 (28.0909)	0.0202 (224.492)	0.0197 (19.8970)	0.0173 (46.8508)	0.0175 (46.7985)	0.0198 (315.4430)
γ^N_{EE}	0.0032 (7.7510)	0.0029 (6.8842)	0.0091 (5.3391)	0.0042 (9.2318)	0.0041 (12.3894)	0.0039 (8.5046)
γ^N_{EN}	-0.0023 (-2.3005)	0.0006 (0.9917)	-0.0058 (-1.3283)	-0.0003 (-0.5037)		0.0005 (1.0456)

Table 3-14 (cont.): PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER, U.S. NON-ENERGY SECTOR

	Symmetry	Symmetry, Input/Output Separability	Symmetry, I/O Separability Berndt-Savin Autoreg. Transf.	Symmetry, {K,L},{E,N} Separability	Symmetry, {K,L,N},{E} Separability	Symmetry, I/O Separability {K,L},{E,N} Separability
ρ^N_{CE}	0.0087 (11.8729)			0.0050 (5.4212)	0.0045 (5.4631)	
γ^N_{Et}	-0.0001 (-3.6996)	-0.0001 (-7.5613)	-0.0004 (-0.2548)	0.0000 (1.3654)	0.0000 (1.6246)	-0.0000 (-5.8148)
β^N_N	0.3630 (75.0815)	0.4452 (413.2880)	0.4626 (27.6330)	0.4532 (254.2680)	0.3647 (75.7555)	0.4463 (426.8850)
γ^N_{NN}	0.2242 (27.7487)	-0.0261 (-19.9183)	0.1567 (4.3533)	0.0633 (28.7733)	0.2158 (27.2688)	-0.0041 (-4.4040)
ρ^N_{CN}	-0.2228 (-46.4328)			-0.1800 (-30.2946)	-0.2171 (-46.4444)	
γ^N_{Nt}	0.0030 (11.3049)	-0.0004 (-9.8414)	-0.0002 (-0.1017)	-0.0021 (-43.1741)	0.0028 (10.9731)	-0.0001 (-7.3834)
α^N_C	0.3513 (89.7174)	0.2993 (140.4330)		0.3334 (88.3876)	0.3558 (89.9587)	0.2926 (138.468)
δ^N_{CC}	0.3409 (33.6529)	0.1026 (23.5441)		0.3142 (31.4521)	0.3243 (33.9793)	

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Table 3-14 (cont.): PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER, U.S. NON-ENERGY SECTOR

	Symmetry	Symmetry, Input/Output Separability	Symmetry, I/O Separability Berndt-Savin Autoreg. Transf.	Symmetry, {K,L},{E,N} Separability	Symmetry, {K,L,N},{E} Separability	Symmetry, I/O Separability {K,L},{E,N} Separability
ρ^N_{Ct}	-0.0018 (-7.2929)			-0.0003 (-1.2142)	-0.0016 (-6.8693)	
α^N_t	-0.0089 (-6.9770)	-0.0078 (-6.1042)	-0.0083 (-1.8402)	-0.0061 (-4.8372)	-0.0091 (-7.1161)	
γ^N_{tt}	0.0004 (2.2639)	0.0006 (3.9725)	0.0005 (0.9451)	0.0002 (1.3649)	0.0004 (2.3438)	
ρ^N_{MK}	-0.0436 (-10.1285)			-0.0086 (-12.4241)	-0.0325 (-8.0590)	
ρ^N_{ML}	-0.1655 (-20.5461)			-0.0209 (-18.7536)	-0.1683 (-21.4140)	
ρ^N_{ME}	-0.0023 (-2.2529)			0.0007 (1.3052)	0.0006 (1.2306)	
ρ^N_{MN}	0.2422 (31.7825)			0.0919 (43.1507)	0.2366 (31.3161)	
δ^N_{MC}	-0.2230 (-46.1551)	-0.0137 (-16.6955)	-0.2200 (-35.9343)	-0.1850 (-33.4104)	-0.2215 (-47.6369)	-0.0053 (-5.7031)

Table 3-14 (cont.): PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER, U.S. NON-ENERGY SECTOR

	Symmetry	Symmetry, Input/Output Separability	Symmetry, I/O Separability Berndt-Savin Autoreg. Transf.	Symmetry, {K,L},{E,N} Separability	Symmetry, {K,L,N},{E} Separability	Symmetry, I/O Separability {K,L},{E,N} Separability
α^N_M	0.3666 (76.9127)	0.4470 (312.590)	0.4646 (980.8690)	0.4502 (240.065)	0.3663 (78.0040)	0.4544 (306.0950)
δ^N_{MM}	0.2572 (34.2071)	0.0246 (14.6217)	0.2560 (32.0044)	0.1177 (44.5325)	0.2556 (34.5482)	0.0096 (5.2665)
δ^N_{Mt}	0.0033 (12.7612)		0.0035 (13.1420)	-0.0013 (-21.6726)	0.0034 (13.3473)	

lowing sequence: (i) symmetry -- taken as a maintained hypothesis, (ii) input-output separability, (iii) autocorrelation in the stochastic disturbances, (iv) separability between {K, L} and {E, N}, (v) separability between {E} and {K, L, N}, and (vi) composite separability between inputs and outputs and between {K, L} and {E, N}.

Likelihood Ratio Tests

To implement the statistical tests on the validity of the above hypotheses, we employ test statistics based on the likelihood ratic λ, where

$$\lambda = \frac{\max_{\Omega} L}{\operatorname{Max}_{\Omega_0} L} \qquad (3.26)$$

The likelihood ratio is the ratio of the maximum value of the likelihood function for the production model Ω without restriction to the maximum value of the likelihood function for the model Ω_0 subject to restriction. For normally distributed disturbances, the likelihood ratio is equal to the ratio of the determinant of the restricted estimator of the variance-covariance matrix of the disturbances to the determinant of the unrestricted estimator, each raised to the power -(n/2).

Our test statistic for each set of restrictions is based on minus twice the logarithm of the likelihood ratio, or:

$$-2 \ln \lambda = n(\ln |\Sigma_{\Omega_0}| - \ln |\Sigma_{\Omega}|) \qquad (3.27)$$

where Σ_{Ω_0} is the restricted estimator of the variance-covariance matrix and Σ_{Ω} is the unrestricted estimator. Under the null hypothesis the likelihood ratio test statistic is asymptotically distributed as chi-squared with a number of degrees of freedom n equal to the number of restrictions to be tested. We employ the asymptotic distribution of the likelihood ratio test statistic for tests of hypothesis. Economic studies employing likelihood ratio test statistics in multivariate equation systems include Byron [12] and Woodland [13]. For a comparison of the relative power of the Wald, likelihood ratio and Lagrange multiplier test statistics, see Berndt and Savin [14].

Tests of Separability between Inputs and Outputs

A technology is said to be separable with respect to a partitioning between inputs and outputs if the production possibility frontier is groupwise additive between inputs and outputs. In terms of a cost frontier $C(y, w)$, a necessary and sufficient condition for separability is that it admit a representation of the form $C(y, w) = F(y) \cdot G(w)$ -- see e.g., Hall [15]. For the translog specification of the cost frontiers, separability implies that all the interaction terms must vanish identically, i.e.:

$$\rho_{ij}^{E} = \rho_{ij}^{N} = 0 \qquad \text{for all } i, j \qquad (3.28)$$

We implement tests for the null hypothesis of separability between inputs and outputs in terms of the parametric restrictions given in Eq. 3.28.

We assign a level of significance of 0.005 to this test. The results for the energy production sector are as follows:

Degrees of freedom = 5

Critical value = 16.7496

Test Statistic = 50.338

For the non-energy production sector, the results of the test are:

Degrees of freedom = 10

Critical value = 25.1882

Test Statistic = 44.843

In both instances, the null hypothesis of separability between inputs and outputs is rejected. This result conforms with the theoretical arguments presented in Section 2.3.2 regarding the implications of separability assumptions in multiple-output technologies. As was pointed out by Hall [15], enforced separability eliminates an important feature of multi-sector and more elaborate technologies -- the dependence of output price ratios on factor prices. We note also that using a value-added production function, Burgess [16] was unable to reject the hypothesis of separability between inputs and outputs.

Tests of Autocorrelation in the Stochastic Disturbances

The assumption underlying the stochastic specification of the systems of cost and revenue shares shown in Tables 3-11 and 3-12 is that the stochastic disturbance vectors $\underline{\varepsilon}^N$ and $\underline{\varepsilon}^E$ are uncorrelated across time. We have conducted a limited set of tests for the null hypothesis of zero autocorrelation as against a first-order multivariate autoregressive process. These tests were conducted only for the input shares systems under the assumption of input-output separability. Thus, although the results of our tests reject the hypothesis of no autocorrelation, it is likely that the results would be less conclusive for the case without the separability assumption: it seems plausible that for the tests we conducted, the presence of autocorrelated disturbances can be explained at least in part by the fact that the effects of the composition of output on the time-variation of the input shares are neglected under the assumption of separability.

We implemented our statistical tests for the presence of autocorrelated disturbances by making use of the multivariate autoregressive transformation introduced by Berndt and Savin [1]. Berndt and Savin showed that the adding-up constraint on the share equations imposes restrictions on the parameters of the autoregressive process. When these restrictions are not imposed, the specification of the model is conditional on the equation deleted and the estimates are not consistent.

If the original system of equations is of the form

$$p_t = Hq_t + v_t$$

and it is assumed that the disturbance vector v_t corresponds to a first-order autoregressive process of the form

$$v_t = Rv_{t-1} + \varepsilon_t$$

where ε_t is a zero-mean uncorrelated process, then consistent estimates of H can be obtained by estimating the transformed system

$$\underline{p}_t = \underline{R}\, \underline{p}_{t-1} + H\, \underline{q}_t - \underline{R}\, H\, \underline{q}_{t-1} + \underline{\varepsilon}_t$$

where the underlined variables signify that one of the equations has been deleted. By virtue of the restrictions implied by the adding-up constraints, R can be recovered from the reduced matrix $\underline{R}$. We assign a level of significance of 0.005 to the tests.

Test Results for the Energy Sector

Degrees of freedom = 21

Critical value = 41.4010

Test Statistic = 72.34

Test Results for the Non-energy Sector

Degrees of freedom = 22

Critical value = 42.7956

Test Statistic = 68.52

Tests of Groupwise Separability Between {K, L} and {E, N}

Our next set of tests corresponds to functional separability restrictions between the input group composed of capital and labor and the input group composed of energy and materials. Weak separability between primary factor inputs and intermediate inputs has been discussed by Arrow [17] as a justification for the existence of a value added specification of technology. This question was also investigated empirically by Berndt and Wood [18].

For our translog cost frontiers, inputs K and L will be weakly separable from E, N if and only if

$$M_K^i \gamma_{LE}^i - M_L^i \gamma_{KE}^i = 0 \qquad i = E, N$$

and

$$M_K^i \gamma_{LN}^i - M_L^i \gamma_{KN}^i = 0 \qquad i = E, N \qquad (3.29)$$

One of the ways in which these conditions can be satisfied is if the following equalities hold:

$$\gamma_{KE}^E = \gamma_{LE}^E = \gamma_{KN}^E = \gamma_{LN}^E = 0$$

$$\gamma_{KE}^N = \gamma_{LE}^N = \gamma_{KN}^N = \gamma_{LN}^N = 0 \qquad (3.30)$$

These conditions have been referred to as the <u>linear separability restrictions</u> by Berndt and Wood [18] and correspond to the notion of <u>explicit groupwise separability</u> used by Jorgenson [19] in the context of consumer demand studies. We define the null hypothesis of groupwise separability between {K, L} and {E, N} in terms of the equalities 3.30. We assign a level of significance of 0.005 to these tests.

<u>Test Results for the Energy Sector</u>

Degrees of freedom = 4

Critical value = 14.8602

Test Statistic = 33.36

<u>Test Results for the Non-Energy Sector</u>

Degrees of freedom = 4

Critical value = 14.8602

Test Statistic = 25.08

In both instances the separability hypothesis is rejected. This result suggests that a value-added specification of technology may not be suitable for an econometric model intended to study the effects of energy use on aggregate exonomic performance.

Tests of Groupwise Separability between {E} and {K, L, N}

We next implement statistical tests of the hypothesis of groupwise separability between the input group formed by capital, labor, and non-energy and the input group composed of energy.

The restrictions that define the null hypothesis of separability between {E} and {K, L, N} are:

$$\gamma_{KE}^{E} = \gamma_{LE}^{E} = \gamma_{EN}^{E} = 0$$

$$\gamma_{KE}^{N} = \gamma_{LE}^{N} = \gamma_{EN}^{N} = 0 \qquad (3.31)$$

We assign a significance level of 0.005 to these tests.

Test Results for the Energy Sector

Degrees of freedom = 3

Critical value = 12.8386

Test Statistic = 13.534

Test Results for the Non-Energy Sector

Degrees of freedom = 3

Critical value = 12.8386

Test Statistic = 9.610

The results indicate that the null hypothesis can be rejected for the energy sector but cannot be rejected for the non-energy sector at the chosen level of significance.

Tests of Groupwise Separability between {K, L} and {E, N} given Input-Output Separability

The final set of tests involves separability between the input group composed of {K, L} and the input group composed of {E, N} given separability between inputs and outputs.

Test Results for the Energy Sector

Degrees of freedom = 4

Critical value = 14.8602

Test Statistic = 22.344

Test Results for the Non-Energy Sector

Degrees of freedom = 4

Critical value = 14.8602

Test Statistic = 13.383

We notice that for the non-energy sector, a maintained hypo-

Table 3-15: LIKELIHOOD RATIO TESTS ON THE STRUCTURE OF TECHNOLOGY, U.S. ENERGY SECTOR

	Degrees of Freedom	Critical Value	Test Statistic
Separability between Inputs and Outputs	5	16.74	50.33
No Autcorrelation in Share Equations	21	41.40	72.34
Groupwise Separability between {K,L} and {E,N}	4	14.86	33.36
Groupwise Separability between {K,L,N} and {E}	3	12.83	13.53
Input-Output Separability and Groupwise Separability between {K,L} and {E,N}	4	14.86	22.34

Table 3-16: LIKELIHOOD RATIO TESTS ON THE STRUCTURE OF TECHNOLOGY, U.S. NON-ENERGY SECTOR

	Degrees of Freedom	Critical Value	Test Statistic
Separability between Inputs and Outputs	10	25.18	44.84
No Autcorrelation in Share Equations	22	42.79	68.52
Groupwise Separability between {K,L} and {E,N}	4	14.86	25.08
Groupwise Separability between {K,L,N} and {E}	3	12.83	9.60
Input-Output Separability and Groupwise Separability between {K,L} and {E,N}	4	14.86	13.83

thesis of input-output separability would lead to the acceptance of separability between {K, L} and {E, N} whereas the latter was rejected for the more complete specification discussed above.

3.1.4 Concavity Tests of the Cost Frontiers

The conditions that must be satisfied by a cost frontier for it to be consistent with a well-behaved technology were described in Section 2.3.2. The positivity conditions require that the input demand functions and output supply functions be strictly positive. These conditions will be satisfied if the fitted cost and revenue shares are strictly positive. We have verified that the fitted shares based on our I3SLS estimates satisfy the positivity conditions at each annual observation for all of the alternative specifications described in the previous section.

In order to verify the concavity of the cost frontiers with respect to input prices, we must establish the negative semidefiniteness of the Hessian matrices of second-order partial derivatives with respect to input prices.

The functional form of the translog approsimations to the cost frontiers were given in Eqs. (2.1) and (2.2) in Chapter II. Denoting the right-hand sides of these two equations by Φ_E and Φ_N, respectively, we have:

$$\ln V_E = \Phi_E$$

and

$$\ln V_N = \Phi_N \tag{3.32}$$

so that

$$V_E = \exp\,[\Phi_E] \tag{3.33-a}$$

and

$$V_N = \exp\,[\Phi_N] \tag{3.33-b}$$

Let $[H_V^k]_{ij}$, k = E, N denote a representative element in the i-th row and j-th column of the Hessian matrices H_V^E and H_V^N, respectively.

Since by definition

$$[H_V^k]_{ij} = \frac{\partial V_k}{\partial w_i \partial w_j} \quad , \; k = E,\, N \tag{3.34}$$

we obtain the following expression by differentiating Eqs. 3.33 twice:

$$\begin{aligned}[H_V^k]_{ij} = {} & \exp\,[\Phi_k] \cdot (1/w_i^k\, w_j^k)\, \gamma_{ij}^k \\ & + (1/w_i^k\, w_j^k)\, [\beta_i^k + \sum_{j=1}^{n} \gamma_{ij}^k \ln w_j^k + \sum_{j=1}^{m} \rho_{ij}^k \ln y_j^k + \gamma_{it}^k \cdot t] \\ & [\beta_j^k + \sum_{i=1}^{n} \gamma_{ij}^k \ln w_i^k + \sum_{i=1}^{m} \rho_{ij}^k \ln y_i^k + \gamma_{jt}^k \cdot t]\, \exp\,[\Phi_k] \\ & k = E,\, N \end{aligned} \tag{3.35}$$

The elements of the diagonal take the form

$$[H_V^k]_{ii} = \exp[\Phi_k] \left(-\frac{1}{w_i^2}\right) [\beta_i^k + \sum_{j=1}^{n} \gamma_{ij}^k \ln w_j^k$$

$$+ \sum_{j=1}^{m} \rho_{ij}^k \ln y_j^k + \gamma_{it}^k \cdot t] + \frac{1}{w_i} \cdot \gamma_{ij} \exp[\Phi_k]$$

$$+ \exp[\Phi_k] \cdot \frac{1}{w_i} [\beta_i^k + \sum_{j=1}^{n} \gamma_{ij}^k \ln w_j^k + \sum_{j=1}^{m} \rho_{ij} \ln y_j^k + \gamma_{it}^k \cdot t]^2$$

$$k = E, N \qquad (3.36)$$

We wish to evaluate the Hessian matrices at the point of approximation of the translog frontiers which is determined by the base year t_0; in our case t_0 = 1958. In the base year, t = 0 by definition, $w_i = 1$, i = 1, ..., n by construction of the price indices and $y_j = 1$, j = 1, ..., m because we assume that the output quantity indices have been normalized accordingly. It then follows from Eqs. (3.35) and (3.36) that the elements of the Hessian matrices are given by

$$[H_V^E]_{ij}^{\;t=t_0} = \gamma_{ij}^E + \beta_i^E \beta_j^E$$

$$[H_V^N]_{ij}^{\;t=t_0} = \gamma_{ij}^N + \beta_i^N \beta_j^N \qquad (3.37)$$

and

$$[H_V^E]_{ii}^{\;t=t_0} = \gamma_{ii}^E + \beta_i^E (\beta_i^E - 1)$$

$$[H_V^N]_{ii}^{\;t=t_0} = \gamma_{ii}^N + \beta_i^N (\beta_i^N - 1) \qquad (3.38)$$

In order for the Hessian matrices to be negative semi-definite and thus insure the concavity of the cost frontiers, it is necessary and sufficient that they have non-positive eigenvalues. To test these conditions empirically, we must be able to evaluate the eigenvalues of the Hessian in terms of the estimated parameters of the translog approximation.

To accomplish this, we observe that if the Hessian matrices are negative semi-definite, then they admit a Cholesky factorization of the form (Gantmacher [20]):

$$H_V^k = L_k \, D_k \, L_k' \qquad k = E, N \tag{3.39}$$

where L_k, $k = E, N$ is a unit lower triangular matrix:

$$L_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \lambda_{LK}^k & 1 & 0 & 0 & 0 \\ \lambda_{EK}^k & \lambda_{EL}^k & 1 & 0 & 0 \\ \lambda_{NK}^k & \lambda_{NL}^k & \lambda_{NE}^k & 1 & 0 \\ \lambda_{RK}^k & \lambda_{RL}^k & \lambda_{RE}^k & \lambda_{RN}^k & 1 \end{bmatrix} \qquad k = E, N \tag{3.40}$$

and D_k, $k = E, N$ is a diagonal matrix with non-negative elements:

$$D_k = \begin{bmatrix} \delta_K^k & 0 & 0 & 0 & 0 \\ 0 & \delta_L^k & 0 & 0 & 0 \\ 0 & 0 & \delta_E^k & 0 & 0 \\ 0 & 0 & 0 & \delta_N^k & 0 \\ 0 & 0 & 0 & 0 & \delta_R^k \end{bmatrix}$$

$$k = E, N \qquad (3.41)$$

In order to express the concavity restrictions we reparametrize the coefficients in the estimating share equations as follows:

$$\gamma_{KK}^k = \delta_K^k - \beta_K^k \cdot (\beta_K^k - 1)$$

$$\gamma_{KL}^k = \delta_K^k \cdot \lambda_{LK}^k - \beta_K^k \cdot \beta_L^k$$

$$\gamma_{KE}^k = \delta_K^k \cdot \lambda_{EK}^k - \beta_K^k \cdot \beta_E^k$$

$$\gamma_{LL}^k = \delta_K^k \cdot (\lambda_{LK}^k)^2 + \delta_L^k - \beta_L^k \cdot (\beta_L^k - 1)$$

$$\gamma_{LE}^k = \delta_K^k \cdot \lambda_{LK}^k \cdot \lambda_{EK}^k + \delta_L^k \cdot (\lambda_{EL}^k) - \beta_L^k \cdot \beta_E^k$$

$$\gamma_{EE}^k = \delta_K^k \cdot (\lambda_{EK}^k)^2 + \delta_L^k \cdot (\lambda_{EL}^k)^2 - \beta_E^k \cdot (\beta_E^k - 1) + \delta_E^k$$

$$\gamma_{KN}^k = \lambda_{NK}^k \cdot \delta_K^k - \beta_K^k \cdot \beta_N^k$$

$$\gamma_{LN}^k = \lambda_{LK}^k \cdot \lambda_{NK}^k \cdot \delta_K^k + \lambda_{NL}^k \cdot \delta_L^k - \beta_L^k \cdot \beta_N^k$$

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*This reparametrization procedure is based on Lau [21].

Table 3-17: ESTIMATES OF THE EIGENVALUES OF THE HESSIAN MATRICES OF THE TRANSLOG COST FRONTIER: U.S. ENERGY SECTOR

	Symmetry	Symmetry Input/Output Separability	Symmetry {K, L}, {E, N} Separability	Symmetry {K, L, N}, {E} Separability	Symmetry I/O Separability {K, L}, {E, N} Separability
δ_K^E	-0.1203	-0.1025	-0.1632	-0.1169	-0.1511
δ_L^E	-0.2466	-0.2408	-0.1494	-0.2409	-0.1630
δ_E^E	-0.1016	-0.1082	-0.1151	-0.0726	-0.1183
δ_N^E	-0.0108	-0.0003	-0.0075	-0.0088	0.0005

Table 3-18: ESTIMATES OF THE EIGENVALUES OF HESSIAN MATRICES OF THE TRANSLOG COST FRONTIER: U.S. NON-ENERGY SECTOR

	Symmetry	Symmetry Input/Output Separability	Symmetry {K, L}, {E, N} Separability	Symmetry {K, L, N}, {E} Separability	Symmetry I/O Separability {K, L}, {E, N} Separability
δ_K^N	-0.1290	-0.1226	-0.1199	-0.1050	-0.0899
δ_L^N	-0.0400	-0.1775	-0.2109	-0.0349	-0.1495
δ_E^N	-0.0141	-0.0162	-0.0147	-0.0134	-0.0151
δ_N^N	0.0146	-0.0035	0.0128	0.0215	-0.0028

TABLE 3-19: PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER WITH CONCAVITY RESTRICTIONS, U.S. NON-ENERGY SECTOR*

β^N_K	0.2007 (70.0290)	γ^N_{Lt}	-0.0022 (-8.3031)
γ^N_{KK}	0.0351	β^N_E	0.0182 (27.6252)
γ^N_{KL}	-0.0042	γ^N_{EE}	0.3200
γ^N_{KE}	-0.0073	γ^N_{EN}	-0.0016
γ^N_{KN}	-0.0288	ρ^N_{CE}	0.0088 (12.1511)
ρ^N_{CK}	0.044451 (6.5621)	γ^N_{Et}	-0.0001 (-3.1139)
γ^N_{Kt}	-0.0007 (-3.0339)	β^N_N	0.3630 (144.8070)
β^N_L	0.3705 (97.2699)	γ^N_{NN}	0.2246
γ^N_{LL}	0.145853	ρ^N_{CN}	-0.2233 (-54.2730)
γ^N_{LE}	0.0054	γ^N_{Nt}	0.0030 (25.7553)
γ^N_{LN}	-0.1598	α^N_C	0.3515 (92.7581)
ρ^N_{CL}	0.1386 (19.2750)	δ^N_{CC}	0.3414 (34.5058)

*asymptotic t-ratios in parentheses

Table 3-19 (cont.): PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER WITH CONCAVITY RESTRICTIONS, U.S. NON-ENERGY SECTOR*

ρ^N_{Ct}	-0.0019 (-8.1107)		
α^N_t	-0.0089 (-7.0167)		
γ^N_{tt}	0.0004 (2.2733)		
ρ^N_{MK}	-0.0429 (-10.0269)		
ρ^N_{ML}	-0.1660 (-50.8858)		
ρ^N_{ME}	-0.0017 (-1.7243)		
ρ^N_{MN}	0.2429 (85.2872)		
δ^N_{MC}	-0.2236 (-54.0977)		
α^N_M	0.3667 (137.0210)		
δ^N_{MM}	0.2573 (80.0773)		
δ^N_{Mt}	0.0034 (26.6373)		

*asymptotic t-ratios in parentheses

$$\gamma_{EN}^k = \lambda_{EK}^k \cdot \lambda_{NK}^k \cdot \delta_K^k + \lambda_{NL}^k \cdot \lambda_{EL}^k \cdot \delta_L^k + \lambda_{NE}^k \cdot \delta_E^k - \beta_E^k \cdot \beta_N^k$$

$$\gamma_{NN}^k = (\lambda_{NK}^k)^2 \cdot \delta_K^k + (\lambda_{NL}^k)^2 \cdot \delta_L^k + (\lambda_{NE}^k)^2 \cdot \delta_E^k + \delta_N^k - \beta_N^k \cdot (\beta_N^k - 1)$$

$$k = E,\ N \qquad (3.42)$$

The effect of this reparametrization is to replace the ten parameters γ_{KK}^k, γ_{KL}^k, γ_{KE}^k, γ_{LL}^k, γ_{LE}^k, γ_{EE}^k, γ_{KN}^k, γ_{LN}^k, γ_{EN}^k, γ_{NN}^k, $k = E, N$ by the new parameters λ_{LK}^k, λ_{EK}^k, λ_{EL}^k, λ_{NK}^k, λ_{NL}^k, λ_{NE}^k, δ_K^k, δ_L^k, δ_E^k, δ_N^k, $k = E, N$.

The symmetry conditions together with the conditions defining the remaining parameters imply:

$$1 + \lambda_{LK}^k + \lambda_{EK}^k + \lambda_{NK}^k + \lambda_{RK}^k = 0$$

$$1 + \lambda_{EL}^k + \lambda_{NL}^k + \lambda_{RL}^k = 0$$

$$1 + \lambda_{NE}^k \quad \lambda_{RE}^k = 0$$

$$1 + \lambda_{RN}^k = 0$$

$$\delta_R^k = 0$$

$$k = E,N \qquad (3.43)$$

In terms of the new parameters, necessary and sufficient conditions for concavity are:

$$\delta_K^k \leq 0$$

$$\delta_L^k \leq 0$$

$$\delta_E^k \leq 0$$

$$\delta_N^k \leq 0$$

We have estimated the reparametrized systems of cost shares for both the energy and non-energy sectors in order to verify the concavity property.

As can be seen from Table 3-17, in the case of the energy sector, all the eigenvalues of the Hessian matrices satisfy the non-positivity conditions necessary to insure concavity. This holds under all five alternative hypotheses on the structure of technology. Asymptotic t-ratios for the four estimated eigenvalues with only symmetry restrictions imposed are -12.12, -17.88, -12.80, and -4.31, respectively.

The corresponding estimates of the eigenvalues for the non-energy sector are shown in Table 3-18. We notice that the concavity conditions are not satisfied except for the two cases that include the assumptions of input-output separability. The statistical significance of these violations is not negligible -- asymptotic t-ratios for the estimated eigenvalues in the case of only symmetry imposed are -18.21, -4.03, -17.43, and 4.52 -- so that we chose to re-estimate the translog cost frontier under the restrictions implied by symmetry and with the additional conditions for concavity imposed. In this case, we impose the concavity restriction by setting $\delta_N^N = 0$ and re-estimating the remaining parameters. The resulting estimated parameters and implied estimates are shown in Table 3-19. The values of the estimated eigenvalues for this constrained case were:

δ_N^N	δ_L^N	δ_E^N
-0.1290 (-19.41)	-0.03996 (-5.43)	-0.0143 (-19.55)

3.1.5 Substitutability and Complementarity in the U.S. Energy and Non-Energy Production Sectors

In this section we analyze our empirical results on the structure of technology for the energy and non-energy sectors in terms of their implications about the substitution possibilities between factors of production and between categories of output. Given the estimated parameters of the translog cost frontier, we compute the price elasticities of demand for factors of production and the Allen-Uzawa partial elasticities of substitution and transformation. We also compute elasticities of factor demands with respect to the composition of output for both the energy and non-energy sectors.

For a technology described in terms of a cost frontier $V = V(y,w)$ with output y and input prices w, the partial elasticities of substitution between inputs i and j are given by the expression (Allen [22], Uzawa [23]):

$$\sigma_{ij} = \frac{V\,V_{ij}}{V_i V_j}$$

where $V_i = \frac{\partial V}{\partial w_i}$, $V_{ij} = \frac{\partial^2 V}{w_i w_j}$

and, by definition, $\sigma_{ij} = \sigma_{ji}$. For the translog cost frontiers, the Allen-Uzawa partial elasticities of substitution are given by:

$$\sigma_{ii}^k = \frac{\gamma_{ii}^k + (M_i^k)^2 - M_i^k}{(M_i^k)^2}$$

$i = K, L, E, N, R$

$k = E, N$

$$\sigma_{ij}^k = \frac{\gamma_{ij}^k + M_i^k M_j^k}{M_i^k M_j^k}$$

$i, j = K, L, N, E, R, i \neq j$

$k = E, N$ (3.45)

The price elasticity of demand for factors of production, η_{ij} is defined as

$$\eta_{ij} = \frac{\partial \ln x_i}{\partial \ln w_i}$$

where output quantities and all other input prices are fixed. The Allen-Uzawa partial elasticities of substitution are related to the price elasticities of demand for factors of production:

$$\eta_{ij}^k = M_j^k \sigma_{ij}^k \qquad (3.46)$$

$k = E, N$

Therefore, in general, $\eta^k_{ij} \neq \eta^k_{ij}$ even though $\sigma^k_{ij} = \sigma^k_{ji}$

The price elasticities of demand for factors of production are tabulated for selected years in tables 3-20 and 3-21 for the energy and non-energy sectors. It is worthwhile to list representative figures for the own-price elasticities and compare the corresponding substitution possibilities for the two sectors:

Energy Sector: Own-Price Demand Elasticities

η^E_{KK}	η^E_{LL}	η^E_{EE}	η^E_{NN}	η^E_{RR}
-0.43	-1.64	-0.48	-0.98	-0.75

Non-Energy Sector: Own-Price Demand Elasticities

η^N_{KK}	η^N_{LL}	η^N_{EE}	η^N_{NN}	η^N_{RR}
-0.62	-0.22	-0.82	-0.05	-0.07

In comparing the own-price elasticities between the two sectors, we observe that the degree of price-responsiveness of demand for a given factor is inversely related to the relative intensities of the factor in the two corresponding sectors, which is a result consistent with an economy operating in an efficient region. Thus, we note that the demand elasticities for capital and energy inputs are lower in the energy sector than in the non-energy sector. Conversely, the elasticities of demand for labor and non-energy inputs are substantially higher in the energy sector.

Table 3-20: FACTOR DEMAND ELASTICITIES,
U.S. ENERGY SECTOR

	η^E_{KK}	η^E_{KL}	η^E_{KE}	η^E_{KN}	η^E_{KR}
1950	-0.436	0.004	0.026	0.002	0.000
1960	-0.435	0.003	0.025	0.004	0.000
1970	-0.429	0.002	0.024	0.005	0.000

	η^E_{LK}	η^E_{LL}	η^E_{LE}	η^E_{LN}	η^E_{LR}
1950	0.004	-1.601	0.050	0.021	0.000
1960	0.003	-1.641	0.050	0.029	0.000
1970	0.002	-1.725	0.049	0.037	0.000

	η^E_{EK}	η^E_{EL}	η^E_{EE}	η^E_{EN}	η^E_{ER}
1950	0.021	0.025	-0.485	0.008	0.000
1960	0.019	0.023	-0.486	0.011	0.000
1970	0.016	0.021	-0.487	0.015	0.000

	η^E_{NK}	η^E_{NL}	η^E_{NE}	η^E_{NN}	η^E_{NR}
1950	0.003	0.018	0.015	-1.03	0.000
1960	0.004	0.020	0.017	-0.98	0.000
1970	0.005	0.023	0.022	-0.94	0.000

	η^E_{RK}	η^E_{RL}	η^E_{RE}	η^E_{RN}	η^E_{RR}
1950	0.000	0.002	-0.001	0.000	-0.764
1960	0.000	0.002	-0.001	0.000	-0.759
1970	0.000	0.002	-0.001	0.000	-0.802

Table 3-21: FACTOR DEMAND ELASTICITIES,
U.S. NON-ENERGY SECTOR

	η^N_{KK}	η^N_{KL}	η^N_{KE}	η^N_{KN}	η^N_{KR}
1950	-0.621	0.021	0.000	0.061	0.000
1960	-0.623	0.027	0.000	0.062	0.000
1970	-0.624	0.036	0.000	0.047	0.000

	η^N_{LK}	η^N_{LL}	η^N_{LE}	η^N_{LN}	η^N_{LR}
1950	0.021	-0.228	0.000	-0.020	0.000
1960	0.027	-0.220	0.000	-0.013	0.000
1970	0.036	-0.232	0.000	-0.009	0.000

	η^N_{EK}	η^N_{EL}	η^N_{EE}	η^N_{EN}	η^N_{ER}
1950	0.000	0.000	-0.812	0.081	0.000
1960	0.000	0.000	-0.820	0.074	0.000
1970	0.000	0.000	-0.821	0.058	0.000

	η^N_{NK}	η^N_{NL}	η^N_{NE}	η^N_{NN}	η^N_{NR}
1950	0.033	-0.012	0.003	-0.054	0.000
1960	0.028	-0.009	0.003	-0.053	0.000
1970	0.023	-0.007	0.002	-0.046	0.000

	η^N_{RK}	η^N_{RL}	η^N_{RE}	η^N_{RN}	η^N_{RR}
1950	0.001	0.001	0.000	-0.001	-0.094
1960	0.001	0.001	0.000	-0.001	-0.072
1970	0.001	0.001	0.000	-0.001	-0.273

Table 3-22: ALLEN-UZAWA PARTIAL ELASTICITIES OF SUBSTITUTION, U.S. ENERGY SECTOR

	σ^E_{KK}	σ^E_{KL}	σ^E_{KE}	σ^E_{KN}	σ^E_{KR}
1950	-1.25	0.030	0.048	0.013	0.000
1960	-1.28	0.025	0.044	0.017	0.000
1970	-1.34	0.021	0.036	0.023	0.000

	σ^E_{LL}	σ^E_{LE}	σ^E_{LN}	σ^E_{LR}
1950	-8.91	0.123	0.112	0.009
1960	-9.75	0.125	0.130	0.009
1970	-11.08	0.125	0.163	0.014

	σ^E_{EE}	σ^E_{EN}	σ^E_{ER}
1950	-1.29	0.042	-0.002
1960	-1.28	0.049	-0.002
1970	-1.29	0.063	-0.003

	σ^E_{NN}	σ^E_{NR}
1950	-5.24	0.001
1960	-4.65	0.001
1970	-3.87	0.002

	σ^E_{RR}
1950	-18.21
1960	-18.32
1970	-15.58

Table 3-23: ALLEN-UZAWA PARTIAL ELASTICITIES OF SUBSTITUTION, U.S. NON-ENERGY SECTOR

	σ^N_{KK}	σ^N_{KL}	σ^N_{KE}	σ^N_{KN}	σ^N_{KR}
1950	-2.93	0.078	-0.0003	0.154	0.000
1960	-3.10	0.090	-0.0003	0.142	0.000
1970	-3.17	0.109	-0.0004	0.118	0.000

		σ^N_{LL}	σ^N_{LE}	σ^N_{LN}	σ^N_{LR}
1950		-0.760	0.0007	-0.041	0.001
1960		-0.749	0.0007	-0.029	0.001
1970		-0.714	0.0007	-0.021	0.001

			σ^N_{EE}	σ^N_{EN}	σ^N_{ER}
1950			-43.71	0.171	0.001
1960			-41.61	0.162	0.001
1970			-41.40	0.137	0.002

				σ^N_{NN}	σ^N_{NR}
1950				-0.107	-0.001
1960				-0.107	-0.001
1970				-0.099	-0.001

					σ^N_{RR}
1950					-2.03
1960					-0.69
1970					-9.13

The Allen-Uzawa partial elasticities of substitution were computed for both the energy and non-energy sectors and are shown in tables 3-22 and 3-23, respectively. Our empirical results based on the translog specification indicate that the structure of technology in the energy sector is characterized by a certain degree of substitutability between all pairs of factor inputs, with the exception of energy and imports that exhibit slight complementarity ($\sigma^{E}_{ER} \simeq 0.002$). In particular, we note that capital and energy inputs are slightly substitutable ($\sigma^{E}_{KE} \simeq 0.044$).

In the non-energy sector, capital and energy inputs can be seen to be slightly complementary ($\sigma^{N}_{KE} = -0.0003$). Complementarity is also exhibited between labor and non-energy inputs and between non-energy inputs and imports. All other pairs of factor inputs are substitutable.

The translog cost frontier estimated without the assumption of separability between inputs and outputs permit the marginal rate of transformation between various outputs to be affected by changes in the composition of input. The elasticity of transformation between a pair of outputs is defined as the negative of the percentage change in the output mix divided by the percentage change in the ratio of output prices. In terms of the parameters of the translog cost frontiers, the elasticity of transformation between outputs i and j takes the form

$$\tau^{k}_{ij} = - (F^{k}_{i} F^{k}_{j})/(\delta^{k}_{ij} (1 - F^{k}_{i} F^{k}_{j})) \qquad (3.47)$$

where F^{k}_{i}, F^{k}_{j} are the corresponding fitted values of the revenue shares.

The computed values for the two sectors are tabulated below:

Energy Sector: Elasticities of Transformation

	τ_{CM}^{E}
1950	-2.29
1960	-2.40
1970	-2.28

Non-Energy Sector: Elasticities of Transformation

	τ_{CM}^{N}	τ_{CI}^{N}	τ_{MI}^{N}
1950	0.76	0.54	3.87
1960	0.79	0.56	3.60
1970	0.82	0.60	3.25

Since we did not impose *a priori* restrictions on the separability between inputs and outputs in the cost frontiers, we can compute the elasticities of demand for factors with respect to output levels. From the maintained hypothesis of constant returns to scale, it follows that the elasticity of demand for all factors with respect to a change in the level but not the composition of output is unity. This does not imply that the elasticities of factor demands with respect to a change in one output are unity, and it suggests that we compare the effects of a change in the composition of output on factor demands. Consider, for example, the percentage increase in demand for the j-th factor when the amount of the i-th output is increased by one percent, holding factor prices fixed. In terms of the parameters and fitted cost and revenue shares, we have

$$\theta_{ij}^{k} = \frac{y_i}{x_j}\frac{\delta x_j}{\delta y_i} = (M_j^k F_i^k + \rho_{ij}^k)/M_j^k \text{ , } k = E, N \qquad (3.48)$$

where M_j^k and F_i^k are the fitted values of the cost and revenue shares, respectively.

We note that under the assumption of separability between inputs and outputs, i.e., $\rho_{ij}^k = 0$, the above expression for the output elasticities of factor demands reduces to the revenue share of the corresponding output, for all factor inputs.

The output elasticities of factor demands are shown in Tables 3-24 and 3-25, for the energy and non-energy sectors, respectively.

In the energy sector, the average values of the revenue shares for consumption and intermediate goods are approximately 0.35 and 0.65, as can be seen from Table 3-8. From Table 3-24, we observe that neglecting the cross-terms in the cost frontier -- i.e., adopting input-output separability would have led to overestimating the demand elasticities for labor, energy and non-energy with respect to the output of consumption goods and to underestimating the corresponding elasticities for capital and imports. Exactly the reverse statements would hold for the elasticities of factor demands with respect to the output level of intermediate goods. We also conclude that a shift in the composition of output towards consumption goods and away from intermediate goods would lead to an increase in demand for capital services and a decrease in demand for labor, energy and non-energy inputs, and imports.

The average values of revenue shares in the non-energy sector are

TABLE 3-24 ELASTICITIES OF FACTOR DEMANDS
WITH RESPECT TO OUTPUT INTENSITIES, U.S. ENERGY SECTOR

	θ^E_{CK}	θ^E_{CL}	θ^E_{CE}	θ^E_{CN}	θ^E_{CR}
1950	0.542	0.230	0.259	0.248	0.436
1960	0.577	0.251	0.286	0.282	0.464
1970	0.571	0.212	0.256	0.262	0.402

	θ^E_{MK}	θ^E_{ML}	θ^E_{ME}	θ^E_{MN}	θ^E_{MR}
1950	0.457	0.769	0.740	0.751	0.564
1960	0.422	0.748	0.713	0.717	0.536
1970	0.428	0.787	0.743	0.737	0.592

TABLE 3-25 ELASTICITIES OF FACTOR DEMANDS WITH RESPECT TO OUTPUT INTENSITIES, U.S. NON-ENERGY SECTOR

	θ^N_{CK}	θ^N_{CL}	θ^N_{CE}	θ^N_{CN}	θ^N_{CR}
1950	0.528	0.786	0.768	-0.191	2.81
1960	0.559	0.772	0.762	-0.187	2.54
1970	0.598	0.766	0.794	-0.192	2.08

	θ^N_{MK}	θ^N_{ML}	θ^N_{ME}	θ^N_{MN}	θ^N_{MR}
1950	0.275	-0.109	0.473	1.00	-1.79
1960	0.254	-0.079	0.463	1.00	-1.53
1970	0.220	-0.060	0.433	1.01	-1.12

	θ^N_{IK}	θ^N_{IL}	θ^N_{IE}	θ^N_{IN}	θ^N_{IR}
1950	0.195	0.323	-0.242	0.191	-0.02
1960	0.185	0.307	-0.225	0.181	-0.01
1970	0.180	0.293	-0.227	0.174	-0.04

approximately 0.32 for consumption goods, 0.46 for intermediate goods, and 0.22 for investment goods. An examination of Table 3-25 suggests that a maintained hypothesis of input-output separability would lead to erroneous estimates of the elasticities of factor demands with respect to output. We conclude that a shift in the composition of output toward consumption goods and away from intermediate goods would increase the demands for capital, labor energy inputs, and imports, and decrease the demand for non-energy inputs; similarly, a shift toward investment goods and away from consumption goods would increase the demand for non-energy inputs and decrease the demand for capital, labor, energy inputs, and imports; finally, a shift towards intermediate goods and away from investment goods would increase the demand for capital and energy inputs and decrease the demand for labor, non-energy inputs and imports.

3.1.6 Technological Change: Factor Augmentation Biases

Our analysis of technological change in the U.S. energy and non-energy production sectors is based on the explanation of the rate of total cost diminution defined as the negative of the growth rate of total factor productivity. The rates of total cost diminution for each of the two sectors are denoted by S^E and S^N, respectively, and are computed by means of a discrete Divisia approximation as described in section 3.1.2. The values of the rate of total cost diminution for the energy and non-energy sectors are shown in Table 3-26 for the period 1948-1971. The average annual rates for this period were -0.0133 and -0.0067, respectively; thus the average rate of increase of total

TABLE 3-26: ANNUAL RATES OF TOTAL COST DIMINUTION, U.S. ENERGY AND NON-ENERGY SECTORS, 1948-1971

Year	Energy Sector S^E	Non-Energy Sector S^N
1948	0.0030	-0.0259
1949	0.0042	0.0064
1950	-0.0272	-0.0571
1951	-0.0234	0.0224
1952	0.0448	0.0098
1953	-0.0145	-0.0098
1954	-0.0210	-0.0166
1955	-0.0145	-0.0286
1956	-0.0046	0.0113
1957	-0.0072	0.0039
1958	0.0067	0.0043
1959	-0.0137	-0.0239
1960	-0.0290	-0.0040
1961	-0.0245	0.0034
1962	-0.0128	-0.0080
1963	-0.0212	-0.0047
1964	-0.0335	-0.0151
1965	0.0033	-0.0135
1966	-0.0368	-0.0061
1967	-0.0330	0.0093
1968	-0.0150	-0.0101
1969	-0.0432	0.0029
1970	-0.0316	0.0021
1971	0.0659	-0.0156

factor productivity in the energy sector was double the average rate in the non-energy sector. The value of the augmentation rate for the non-energy sector (0.67%) is close to estimates for U.S. manufacturing obtained by Star [24] (0.59%), Gollop [4] (0.84%) and Berndt and Wood [25] (0.60%).

The empirical results discussed in section 3.1.3 assumed a parametrization of the structure of technology such that the effects of technological change were embodied in the parameters

$$\gamma^{E}_{it}, \quad \gamma^{N}_{it} \qquad i = K,L,E,N,R$$

$$\delta^{E}_{it} \qquad i = C,M$$

$$\delta^{N}_{it} \qquad i = C,I,M$$

These parameters can be interpreted as factor-share and output-share augmentation indices. In particular since they appear in the share equations as coefficients that multiply the time index, it follows that γ^{E}_{Kt}, for example, can be taken as the average yearly rate of capital-share-augmenting technological change. We remark that the net effect of technical change on a given factor share depends on the technological substitution parameters as well as on the rates of factor augmentation.

Summarized below are the estimated values and corresponding t-statistics of the factor-share augmentation parameters from Tables 3-13 and 3-19.

Energy Sector		Non-Energy Sector	
γ^{E}_{Kt}	-0.0012 (-5.4644)	γ^{N}_{Kt}	-0.0007 (-3.0339)
γ^{E}_{Lt}	0.0026 (5.1751)	γ^{N}_{Lt}	-0.0022 (-8.3031)
γ^{E}_{Et}	-0.0010 (-3.0099)	γ^{N}_{Et}	-0.0001 (-3.1139)
γ^{E}_{Nt}	0.0011 (3.3359)	γ^{N}_{Nt}	0.0030 (25.7553)

We observe that, in the energy sector, the pattern of technical change over the period 1948-1971 appears to have been capital-share and energy-share saving and labor-share and non-energy-share using. Our results also suggest that, in the non-energy sector, technological change over this period has been of the share saving type for capital, labor and energy inputs and has been non-energy input share using.

The above analysis is useful in that it allows us to examine the effects of technical change on equilibrium factor cost shares. However, in order to identify biases in the rates of input price diminution, it is necessary to reparametrize and reestimate the translog share equations subject to the restriction that technical change is of the factor augmenting (i.e. price diminuting) form.

The estimating system incorporating this restriction is obtained by substituting the expressions in Egs (2-20) and (2-21) into the cost and revenue share equations; in addition, the estimating equations corresponding to the rates of total cost diminution are replaced by:

$$S_t^E = M_K^E \lambda_K^E + M_L^E \lambda_L^E + M_E^E \lambda_E^E + M_N^E \lambda_N^E + M_R^E \lambda_R^E$$

and

$$S_t^N = M_K^N \lambda_K^N + M_L^N \lambda_L^N + M_E^N \lambda_E^N + M_N^N \lambda_N^N + M_R^N \lambda_R^N \qquad (3.49)$$

Equations (3.49) are obtained by using Egs (2.20), (2.21) and noting that

$$\beta_t^E = \beta_K^E \cdot \lambda_K^E + \beta_L^E \cdot \lambda_L^E + \beta_E^E \cdot \lambda_E^E + \beta_N^E \cdot \lambda_N^E + \beta_R^E \cdot \lambda_R^E$$

$$\beta_t^N = \beta_K^N \cdot \lambda_K^N + \beta_L^N \cdot \lambda_L^N + \beta_E^N \cdot \lambda_E^N + \beta_N^N \cdot \lambda_N^N + \beta_R^N \cdot \lambda_R^N \qquad (3.50)$$

and

$$\gamma_{tt}^E = \gamma_{Kt}^E \lambda_K^E + \gamma_{Lt}^E \lambda_L^E + \gamma_{Et}^E \lambda_E^E + \gamma_{Nt}^E \lambda_N^E + \gamma_{Rt}^E \lambda_R^E$$

$$\gamma_{tt}^N = \gamma_{Kt}^N \lambda_K^N + \gamma_{Lt}^N \lambda_L^N + \gamma_{Et}^N \lambda_E^N + \gamma_{Nt}^N \lambda_N^N + \gamma_{Rt}^N \lambda_R^N \qquad (3.51)$$

We remark that in the absence of equations (3.49) the condition of CRTS would imply that one of the indices of input price diminution would remain unidentified.

TABLE 3-27: PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER UNDER ALTERNATIVE HYPOTHESES ON TECHNOLOGICAL CHANGE, U.S. ENERGY SECTOR *

	Factor Augmenting Tech. Change	Hicks-Neutral Technological Change	Solow-Neutral Technological Change	Harrod-Neutral Technological Change
β_K^E	0.2784 (108.57)	0.2738 (123.79)	0.2810 (155.22)	0.2730 (118.64)
γ_{KK}^E	0.0760 (11.840)	0.1091 (18.075)	0.0705 (12.895)	0.1005 (15.756)
γ_{KL}^E	-0.0064 (-1.058)	-0.0400 (-12.442)	0.0092 (1.932)	-0.0353 (-9.468)
γ_{KE}^E	-0.0277 (-4.875)	-0.0126 (-2.682)	-0.0331 (-7.093)	-0.0190 (-3.602)
γ_{KN}^E	-0.0351 (-4.389)	-0.0568 (-7.691)	-0.0389 (-6.843)	-0.0446 (-5.590)
γ_{CK}^E	0.0456 (10.44)	0.0442 (10.60)	0.0497 (14.06)	0.0427 (10.152)
β_L^E	0.1460 (45.694)	0.1496 (80.215)	0.1608 (64.212)	0.1503 (74.501)
γ_{LL}^E	-0.1285 (-8.215)	-0.0574 (-16.962)	-0.0559 (-11.240)	-0.0440 (-8.142)
γ_{LE}^E	0.0185 (1.903)	-0.0100 (-2.817)	-0.0334 (-5.953)	-0.0092 (-2.994)
γ_{LN}^E	0.0579 (6.206)	0.0835 (13.213)	0.0636 (8.926)	0.0713 (9.016)
ρ_{CL}^E	-0.0253 (-4.323)	-0.0260 (-7.815)	0.0036 (0.7188)	-0.0244 (-6.588)
β_E^E	0.3249 (99.35)	0.3300 (147.92)	0.3189 (115.00)	0.3294 (139.83)
γ_{EE}^E	0.0642 (6.640)	0.0651 (10.762)	0.0911 (11.617)	0.0631 (9.666)

* asymptotic t-ratios in parentheses

TABLE 3-27 (Cont.): PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER UNDER ALTERNATIVE HYPOTHESES ON TECHNOLOGICAL CHANGE, U.S. ENERGY SECTOR

	Factor Augmenting Tech. Change	Hicks-Neutral Technological Change	Solow-Neutral Technological Change	Harrod-Neutral Technological Change
γ^E_{EN}	0.0053 (0.5607)	-0.0002 (-0.0248)	0.0111 (1.327)	0.0075 (0.837)
ρ^E_{CE}	-0.0279 (-4.721)	-0.0153 (-3.605)	-0.0401 (-7.342)	-0.0191 (-4.296)
β^E_N	0.2094 (52.2435)	0.2095 (54.101)	0.2041 (55.072)	0.2089 (48.812)
γ^E_{NN}	-0.0311 (-2.014)	-0.0236 (-1.476)	-0.0401 (-2.648)	-0.0374 (-2.260)
ρ^E_{CL}	-0.0254 (-4.323)	-0.0260 (-7.815)	0.0036 (0.7188)	-0.0244 (-6.588)
α^E_C	0.4234 (118.95)	0.4117 (134.93)	0.4257 (124.88)	0.4132 (134.34)
ρ^E_{CC}	0.1407 (22.693)	0.1199 (21.626)	0.1466 (23.888)	0.1251 (22.363)
$\underline{\lambda}^E_K$ *	0.0596 (-5.924)	0.0	-0.0268 (-8.156)	
$\underline{\lambda}^E_L$ *	-0.0494 (-6.606)	0.0		0.0082 (1.980)
$\underline{\lambda}^E_E$ *	-0.0221 (-4.007)	0.0		
$\underline{\lambda}^E_N$ *	-0.0622 (-3.8010)	0.0		
λ^E_R	0.0306 (3.382)	-0.0133 (-7.386)		

* $\underline{\lambda}^E_K = \lambda^E_K - \lambda^E_R$, $\underline{\lambda}^E_L - \lambda^E_L - \lambda^E_R$, $\underline{\lambda}^E_E = \lambda^E_E - \lambda^E_R$, $\underline{\lambda}^E_N = \lambda^E_N - \lambda^E_R$

TABLE 3-28: PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER UNDER ALTERNATIVE HYPOTHESES ON TECHNOLOGICAL CHANGE, U.S. NON-ENERGY SECTOR *

	Factor Augmenting Tech. Change	Hicks-Neutral Technological Change	Solow-Neutral Technological Change	Harrod-Neutral Technological Change
β^N_K	0.2170 (85.337)	0.1989 (76.247)	0.2206 (74.972)	0.2058 (83.969)
γ^N_{KK}	0.0270 (4.513)	0.0290 (5.159)	0.0332 (8.936)	0.0502 (8.826)
γ^N_{KL}	-0.0069 (-1.437)	-0.0268 (-15.404)	-0.0004 (-0.1619)	-0.0435 (-11.170)
γ^N_{KE}	-0.0063 (-9.774)	-0.0083 (-12.968)	-0.0091 (-13.961)	-0.0069 (-11.135)
γ^N_{KN}	-0.0236 (-6.615)	0.0002 (0.0578)	-0.0281 (-6.139)	-0.0064 (-1.923)
ρ^N_{CK}	0.0445 (6.849)	0.0403 (8.184)	0.0427 (12.022)	0.0167 (2.892)
β^N_L	0.3524 (91.597)	0.3190 (336.19)	0.3245 (144.05)	0.3437 (132.02)
γ^N_{LL}	0.1139 (12.535)	0.0751 (35.477)	0.0705 (30.331)	0.1537 (19.799)
γ^N_{LE}	0.0083 (10.649)	0.0021 (8.730)	-0.0040 (-5.834)	0.0039 (8.560)
γ^N_{LN}	-0.1391 (-20.144)	-0.0665 (-50.781)	-0.0845 (-22.184)	-0.1381 (-22.668)
ρ^N_{CL}	0.1411 (21.863)	0.0830 (35.480)	0.1131 (32.921)	0.1587 (22.471)
β^N_E	0.0201 (38.474)	0.0173) (43.703)	0.0140 (21.905)	0.0189 (46.914)
γ^N_{EE}	0.0026 (7.534)	0.0037 (9.963)	0.0050 (11.033)	0.0036 (10.346)

* asymptotic t-ratios in parentheses

TABLE 3-28 (Cont.): PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER UNDER ALTERNATIVE HYPOTHESES ON TECHNOLOGICAL CHANGE, U.S. NON-ENERGY SECTOR

	Factor Augmenting Tech. Change	Hicks-Neutral Technological Change	Solow-Neutral Technological Change	Harrod-Neutral Technological Change
γ_{EN}^{N}	-0.0034 (-3.468)	0.0010 (1.373)	0.0052 (4.973)	-0.0009 (-1.121)
ρ_{CE}^{N}	0.0085 (11.511)	0.0097 (13.293)	0.0089 (11.342)	0.0086 (11.745)
β_{N}^{N}	0.3757 (94.714)	0.4313 (219.07)	0.4095 (102.97)	0.3931 (120.22)
γ_{NN}^{N}	0.2089 (28.515)	0.1105 (30.485)	0.1501 (21.438)	0.1901 (28.929)
ρ_{CN}^{N}	-0.2248 (-52.336)	-0.1739 (-47.529)	-0.2001 (-42.466)	-0.2226 (-47.510)
α_{C}^{N}	0.3529 (100.04)	0.3249 (97.407)	0.3371 (89.080)	0.3486 (91.396)
δ_{CC}^{N}	0.3413 (33.953)	0.3181 (36.530)	0.3385 (42.416)	0.3617 (38.390)
ρ_{MK}^{N}	-0.0369 (-11.389)	-0.0096 (-2.763)	-0.0420 (-10.058)	-0.0108 (-3.641)
ρ_{ML}^{N}	-0.1421 (-21.43)	-0.0618 (-46.225)	-0.0892 (-22.207)	-0.1327 (-22.200)
ρ_{ME}^{N}	-0.0043 (-4.860)	-0.0008 (-1.144)	0.0041 (3.962)	-0.0027 (-3.920)
ρ_{MN}^{N}	0.2252 (32.859)	0.1179 (36.285)	0.1661 (25.368)	0.1969 (31.986)
δ_{MC}^{N}	-0.2254 (-53.864)	-0.1744 (-50.143)	-0.2020 (-45.212)	-0.2259 (-48.121)
α_{M}^{N}	0.3812 (101.99)	0.4380 (219.460)	0.4105 (105.636)	0.4057 (129.787)

TABLE 3-28 (Cont.): PARAMETER ESTIMATES OF THE TRANSLOG COST FRONTIER UNDER ALTERNATIVE HYPOTHESES ON TECHNOLOGICAL CHANGE, U.S. NON-ENERGY SECTOR

	Factor Augmenting Tech. Change	Hicks-Neutral Technological Change	Solow-Neutral Technological Change	Harrod-Neutral Technological Change
δ^N_{MM}	0.2375 (35.672)	0.1281 (37.580)	0.1853 (28.38)	0.2000 (32.961)
$\underline{\lambda}^N_K$ *	-0.0372 (-7.695)	0.0	-0.0252 (-12.33)	
$\underline{\lambda}^N_L$ *	-0.0622 (-8.1990)	0.0		-0.01658 (-24.487)
$\underline{\lambda}^N_E$ *	0.0991 (5.097)	0.0		
$\underline{\lambda}^N_N$ *	-0.0312 (-6.613)	0.0		
λ^N_R	0.0329 (6.371)	-0.0067 (-5.386)		

* $\underline{\lambda}^N_K = \lambda^N_K - \lambda^N_R$

$\underline{\lambda}^N_L = \lambda^N_L - \lambda^N_R$

$\underline{\lambda}^N_E = \lambda^N_E - \lambda^N_R$

$\underline{\lambda}^N_N = \lambda^N_N - \lambda^N_R$

Parameter estimates of the translog cost frontiers under alternative hypotheses on the structure of technological change are shown in Tables 3-27 and 3-28.

Statistical tests of the various hypotheses were implemented using the likelihood ratio test. The results for the null hypothesis of factor augmenting -- i.e. price diminuting -- technical change as against the translog specification with symmetry alone imposed (at the 0.005 significance level) were:

<u>Energy Sector</u>

Degrees of Freedom	=	3
Critical Value	=	12.383
Test Statistic	=	0.842

<u>Non-Energy Sector</u>

Degrees of Freedom	=	4
Critical Value	=	14.860
Test Statistic	=	1.970

The hypothesis that technological change has been of the factor augmenting type over the period 1974-1971 is thus decisively accepted for both sectors. The implied estimates of the exponential indices of price diminution are summarized below:

Energy Sector

λ_K^E	λ_L^E	λ_E^E	λ_N^E	λ_R^E
-0.0290	-0.0288	0.0085	-0.0316	0.0306
(4.543)	(4.962)	(3.789)	(3.614)	(3.382)

Non-Energy Sector

λ_K^N	λ_L^N	λ_E^N	λ_N^N	λ_R^N
-0.0043	-0.0297	0.1320	0.0017	0.0329
(-7.210)	(-7.948)	(5.842)	(6.516)	(6.371)

We note that in the energy sector, technological change has been capital, labor and non-energy input saving and that the annual rate of factor augmentation for these three inputs is close to 3.0%. In the non-energy sector, technical change has been predominantly labor saving with an augmentation rate of 2.9%. In both sectors, the exponential indices of price diminution corresponding to energy inputs are positive, indicating that technical change has been energy using. The implication that is suggested by these results is that the increase in industrial demand for energy over the period 1948-1971 can be partly explained by the presence of energy using biases.

The next statistical test corresponded to the hypothesis that technical change was Hicks neutral, i.e. that all rates of price diminution are equal:

$$\lambda_K^E = \lambda_L^E = \lambda_E^E = \lambda_N^E = \lambda_R^E$$

$$\lambda_K^N = \lambda_L^N = \lambda_E^N = \lambda_N^N = \lambda_R^N$$

Note that this implies that technical change is also neutral with respect to factor shares. The results for the null hypothesis of Hicks neutrality at the 0.005 significance level were:

<u>Energy Sector</u>

Degrees of Freedom = 4

Critical Value = 14.860

Test Statistic = 16.80

<u>Non-Energy Sector</u>

Degrees of Freedom = 4

Critical Value = 14.860

Test Statistic = 27.02

We thus reject the hypothesis of Hicks neutrality for both the energy and non-energy sectors. The common rate of price diminution was found to be -0.0133 (-7.348) for the energy sector and -0.0067 (-5.386) for the non-energy sector.

The final tests involve the hypotheses of Solow-neutral technical change--i.e. that productivity growth is purely of the capital-augmenting type--and of Harrod-neutral technical change--i.e. that factor augmentation is null for all inputs except labor.

Tests for the null hypothesis of Solow neutrality as against factor augmenting technical change were implemented by setting all exponential indices to 0 except λ_K^E and λ_K^N; the results at the 0.005 significance level were:

<u>Energy Sector</u>

Degrees of Freedom	=	4
Critical Value	=	14.860
Test Statistic	=	10.36

<u>Non-Energy Sector</u>

Degrees of Freedom	=	4
Critical Value	=	14.860
Test Statistic	=	22.62

Thus the hypothesis of Solow-neutrality is rejected in the non-energy sector whereas in the energy sector we are unable to reject it.

The results for the null hypothesis of Harrod-neutrality as against factor augmenting technical change--implemented by setting all exponential indices to 0 except λ_L^E and λ_L^N -- were:

<u>Energy Sector</u>

Degrees of Freedom	=	4
Critical Value	=	14.860
Test Statistic	=	22.22

<u>Non-Energy Sector</u>

Degrees of Freedom	=	4
Critical Value	=	14.860
Test Statistic	=	17.60

We thus reject the hypothesis of Harrod-neutrality in technical change in both the energy and non-energy sectors.

The above series of tests suggest the following set of conclusions. First, to the extent that technological progress has occurred in the U.S. energy and non-energy production sectors over the period of 1948-1971, it has been introduced through the factors of production and not through some independent force acting over time. Second, technical change has been predominantly labor saving and energy using in the non-energy sector and capital, labor and non-energy saving in the energy sector. Third, our results do not support the hypothesis of Hicks neutrality which is commonly adopted by empirical researchers.

We should point out that the technical change biases identified with our model need not necessarily persist into the future and therefore our results are of limited value from the standpoint of technology forecasting; for this purpose it would be necessary to have a fully endogenous model of technical change with the characteristics outlined in Section 2.3.4. Such a model based on an endogenous theory would make it possible to better address questions such as the induced innovation hypothesis or issues related to governmental energy policy such as the enactment of investment tax credits applying only to energy-conserving capital or the promotion of R & D funding policies aimed at producing energy-saving biases in technological change.

3.2 Estimation of the Model of Consumer Behavior

In this section we present the empirical results for our model of consumer behavior. We first discuss the sources of data and the preparation of our set of accounts on personal consumption expenditures. Next we describe the stochastic specification of the model of the household sector in terms of the expenditure shares corresponding to the inter-temporal and intra-temporal sub-models, respectively. After presenting the parameter estimates for both sub-models, we examine their implications in terms of the possibilities for substitution among commodity groups and in terms of the validity of alternative separability hypotheses about the structure of consumer preferences.

3.2.1 Data Sources and Preparation

The empirical implementation of our model of consumer behavior required the construction of time series data corresponding to price and quantity components of leisure and three commodity groups: energy goods, non-energy goods and capital services; we constructed these time series for the U.S. household sector for the period 1947-1971.

Price and quantity components of personal consumption expenditures on energy were constructed as Divisia aggregates of consumption by households of refined petroleum products, electricity and natural gas. Consumption of non-energy products was defined as an aggregate Divisia index of consumption expenditures for the products of sectors N1, N2, N3 and N4 defined in section 3.1.1 and of imported consumer goods; consumer durables were not included in this category. Price and quantity components of capital services consumed by households were constructed by aggregating over the corresponding series of capital services from consumer durables and residential structures. The quantity index for leisure was constructed taking as a basis the total time endowment of the household sector and the supply of labor; the price index for leisure was taken to be equal to the after-tax wage rate.

The price and quantity components of full consumption to be used in the inter-temporal allocation model were constructed by Divisia aggregation of the corresponding series for leisure and the three commodity groups. The time series for full consumption are tabulated in Table 3-29.

	P_F	F/P
Average Value 1947-1971	--	12.16
Average Growth Rate p.a. 1947-1971	6.9	1.1%

Price indices for the three commodity groups and leisure are tabulated in Tables 3-30; quantity indices expressed in per capita terms are shown in Table 3-31; both these sets of data are summarized below:

	P_L	P_{CN}	P_{CK}	P_{CE}
Average Growth Rate p.a. 1947-1971	7%	2.3%	2.4%	0.0%

TABLE 3-29: PRICE AND QUANTITY COMPONENTS,

FULL CONSUMPTION PER CAPITA, 1947-1971

Year	p_F	F/P
1947	0.607979	11.3183
1948	0.653533	11.4391
1949	0.678514	11.5033
1950	0.724671	11.6102
1951	0.723553	11.6257
1952	0.74498	11.6504
1953	0.771534	11.7476
1954	0.832242	11.8312
1955	0.877986	11.818
1956	0.911579	11.8829
1957	0.956609	11.9986
1958	1.	12.1682
1959	1.03346	12.2594
1960	1.07044	12.3278
1961	1.09539	12.4478
1962	1.13884	12.5146
1963	1.14496	12.6959
1964	1.21012	12.8643
1965	1.24042	13.0018
1966	1.29952	13.2184
1967	1.34294	13.4254
1968	1.41733	13.6615
1969	1.49497	13.886
1970	1.62106	14.0617
1971	1.71874	14.2594

TABLE 3-30: PRICE INDICES OF CONSUMPTION BY U.S. HOUSEHOLDS, 1947-1971

Year	P_L	P_{CN}	P_{CK}	P_{CE}
1947	0.524696	0.797251	0.8208	0.8555
1948	0.565074	0.843783	0.7895	1.0002
1949	0.596125	0.823441	0.6684	0.9566
1950	0.637924	0.835753	0.855	0.9629
1951	0.63175	0.886219	0.8421	0.9851
1952	0.650301	0.913857	0.8754	0.9774
1953	0.675668	0.918971	0.931	1.0145
1954	0.73855	0.925951	0.9374	1.9929
1955	0.783713	0.930959	1.0135	0.9956
1956	0.81852	0.943762	0.9884	1.0121
1957	0.863579	0.972002	0.9735	1.0146
1958	0.905656	1.	1.	1.
1959	0.936416	1.01514	1.0874	1.001
1960	0.972524	1.0351	1.12	0.9802
1961	0.998223	1.04148	1.1165	1.008
1962	1.04246	1.0468	1.1608	0.9967
1963	1.04598	1.06046	1.2029	0.9863
1964	1.11321	1.07407	1.2389	0.9511
1965	1.14325	1.0821	1.2734	0.9756
1966	1.20095	1.11547	1.323	0.976
1967	1.24413	1.15061	1.3054	0.9827
1968	1.31855	1.2002	1.311	0.9611
1969	1.39195	1.25513	1.4008	0.969
1970	1.52284	1.31779	1.3619	0.9738
1971	1.61586	1.37869	1.474	1.0188

TABLE 3-31: CONSUMPTION QUANTITY INDICES PER CAPITA, U.S. HOUSEHOLDS, 1947-1971

Year	LJ/P	CN/P	CK/P	CE/P
1947	10.9581	1.14646	0.240144	0.051135
1948	10.0636	1.14738	0.256399	0.052998
1949	11.118	1.14197	0.273078	0.056586
1950	11.1896	1.15799	0.289215	0.062509
1951	11.1645	1.15804	0.315513	0.067435
1952	11.1366	1.18056	0.328429	0.072243
1953	11.202	1.2018	0.336844	0.075532
1954	11.2791	1.2004	0.349625	0.077427
1955	11.1979	1.23625	0.360674	0.084039
1956	11.2112	1.26283	0.38068	0.086516
1957	11.3172	1.25433	0.391641	0.093095
1958	11.485	.26696	0.399967	0.096811
1959	11.5356	1.30966	0.401518	0.919331
1960	11.5782	1.32007	0.41187	0.110563
1961	11.6884	1.33049	0.419482	0.112534
1962	11.7203	1.36305	0.423828	0.115763
1963	11.8712	1.39524	0.434269	0.118973
1964	11.9829	1.44634	0.447952	0.125717
1965	12.0565	1.50365	0.464692	0.128501
1966	12.2178	1.54986	0.486437	0.13588
1967	12.3801	1.58741	0.507906	0.140398
1968	12.5627	1.63759	0.525356	0.150882
1969	12.7476	1.6674	0.548898	0.159114
1970	12.8952	1.68355	0.571376	0.165868
1971	13.0592	1.71973	0.585902	0.169463

	LJ/P	CN/P	CK/P	CE/P
Average Value 1947-1971	11.69	1.31	0.39	0.09
Average Growth Rate p.a. 1947-1971	1.2%	2.0%	5.8%	8.5%

3.2.2 Estimation of the Inter-Temporal Model

The estimating equation for the inter-temporal allocation sub-model is obtained from Eg. 2.47 by defining the share of present full consumption in current wealth

$$S_t^F = p_F \cdot F/ (1 + MW)\, W_{t-1} \tag{3.52}$$

which leads to the share equation

$$S_t^F = \frac{\alpha_0 + \beta_{00} \ln (F_t/P_t) + \beta_{0t} \cdot t}{\alpha_M + \beta_{M0} \ln (F_t/P_t) + \beta_{MT} \cdot t} \quad \frac{[\, 1 + \mu(p_L^t \cdot LH_t + EL_t)\,]}{(1 + MW) \cdot W_{t-1}}$$

TABLE 3-32: PARAMETER ESTIMATES OF
INTER-TEMPORAL TRANSLOG UTILITY FUNCTION*

	Homothetic Separability	Explicit Homotheticity	Neutrality	Homogeneity	Neutral Homogeneity	Neutral Linear Logarithmic Utility
α_0	-0.0250 (24.82)	-0.07425 (-3.761)	-0.0197 (-26.22)	-0.0248 (-43.45)	-0.0245 (-176.41)	-0.0724 (-116.95)
β_{00}	0.0095 (24.80)		0.0073 (43.67)	0.0092 (57.18)	0.0091 (66.26)	
β_{0t}	-0.0001 (-24.63)	0.0254 (3.806)	-0.000 (-21.74)			
β_{M0}	0.3828 (71.47)		0.3732 (27.31)	0.3716 (60.19)	0.3732 (0.000)**	
β_{Mt}	-0.0019 (49.23)	0.3426 (97.65)		0.0004 (49.23)		
μ	39.82 (24.43)	12.746 (3.574)	50.72 (0.000)**	40.34 (0.000)**	40.34 (24.58)	13.09 (110.39)

*Asymptotic t-ratios in parentheses

**Singular likelihood function

Since this equation is homogeneous of degree zero in the parameters, we adopt the normalization $\alpha_M = -1$, which leaves 6 parameters to be estimated.

The restrictions implied by the theory of demand are those of monotonicity, i.e.

$$\alpha_0 \leq 0 \qquad (3.54)$$

and quasi-convexity which can be shown to imply the parametric restriction

$$\beta_{00} - \alpha_0 \geq 0 \qquad (3.55)$$

In table 3-32 we present I3SLS estimates of the parameters of the translog utility function under alternative hypotheses on the inter-temporal structure of consumer preferences. The most general specification incorporates a maintained hypothesis of homothetic separability, which was a necessary assumption for the derivation of the inter-temporal allocation model in Section 2.4.4. Next we assume that inter-temporal preferences are explicitly homothetic which implies the parametric restriction

$$\beta_{00} = \beta_{M0} = 0$$

The next three specifications incorporate both separately and jointly the restrictions of neutrality, i.e., $\beta_{Mt} = 0$, and homogeneity, i.e., $\beta_{0t} = 0$.

Finally, we estimate the share equation under the assumption that the utility function is neutral linear logarithmic. The results of likelihood ratio tests of each of the five alternative hypotheses as against a homothetically separable utility function are shown below for a significance level of 0.005

	Degrees of Freedom	Critical Value	Test Statistic
Explicit Homotheticity	2	10.60	17.24
Neutrality	1	7.87	3.39
Homogeneity	2	10.60	4.12
Neutral Homogeneity	2	10.60	6.51
Neutral Linear Logarithmic Utility	4	14.86	18.12

We remark that the fitted values of the parameters of the translog utility function satisfy the monotonicity and quasi-convexity restrictions given by 3.54 and 3.55.

Given the estimated parameters for each of the above six specifications, we can compute the implied estimates of the social discount rate δ by using Eg. 2.44:

$$\frac{1+\delta}{\delta} = -\frac{1}{\alpha_0}$$

from which we obtain

$$\delta = \frac{-\alpha_0}{1+\alpha_0}$$

We tabulate below the implied estimates of the social discount rate corresponding to each of the alternative hypotheses on the intertemporal structure of consumer preferences:

	α_o	Discount Rate δ
Homothetic Separability	-0.0250	2.66%
Explicit Homotheticity	-0.742	7.98%
Neutrality	-0.0197	2.02%
Homogeneity	-0.0248	2.62%
Neutral Homogeneity	-0.0245	2.57%
Neutral Linear Logarithmic Utility	-0.0724	7.80%

We remark that the above set estimates of the social rate of discount is well within the range of values that might be derived from observable magnitudes of market behavior, as well as values of the discount rate that are commonly used in practice. The market rate of interest faced by households wishing to borrow funds -- e.g. the mortgage rate -- is in the neighborhood of 8.00%; the yield on long-term government securities -- commonly used as the discount rate in cost-benefit analysis of water resource projects -- is currently at 6-1/4%. The Department of the Interior recommends the use of an 8% rate for internal purposes, while the Office of Management and Budget specifies a 10% rate of discount for projects to be included in budgetary requests. Dorfman [26], using a constant-elasticity function defined on per capita aggregate consumption, concludes that the social rate of discount is "no greater than 3%."

3.2.3 Estimation of the Intra-Temporal Model

The estimating system for the intra-temporal allocation model is derived from the expenditure share equations for leisure, energy, non-energy and capital services given in Equations 2.51-2.54 of Chapter 2.

Since the equations for the budget shares are homogeneous of degree zero in the parameters, normalization of the parameters is required for estimation. We adopt the normalization

$$\alpha_M^C + \alpha_J^C + \alpha_N^C + \alpha_K^C + \alpha_E^C = -1 \qquad (3.56)$$

which leaves 28 parameters to be estimated in the unrestricted model. The additional identities given by:

$$\beta_{JJ}^C + \beta_{JN}^C + \beta_{JK}^C + \beta_{JE}^C = \beta_{MJ}^C$$

$$\beta_{NJ}^C + \beta_{NN}^C + \beta_{NK}^C + \beta_{NE}^C = \beta_{MN}^C$$

$$\beta_{KJ}^C + \beta_{KN}^C + \beta_{KK}^C + \beta_{KE}^C = \beta_{MK}^C$$

$$\beta_{EJ}^C + \beta_{EN}^C + \beta_{EK}^C + \beta_{EE}^C = \beta_{ME}^C$$

$$\beta^C_{Jt} + \beta^C_{Nt} + \beta^C_{Kt} + \beta^C_{Et} = \beta^C_{Mt} \qquad (3.57)$$

reduce the number of parameters to be estimated to 23.

Since the negative of the logarithm of the direct translog utility function is twice differentiable in the logarithms of the quantities consumed, it follows that the Hessian of the utility function is symmetric; this condition implies the following set of restrictions on the parameters:

$$\beta^C_{ij} = \beta^C_{ji} \qquad i,j = J,N,K,E \quad i\ i \neq j \qquad (3.58)$$

These symmetry restrictions further reduce the total number of parameters to be estimated to 20.

The budget constraint implies that the sum of all expenditure shares must be equal to unity, i.e.

$$M^C_J + M^C_N + M^C_K + M^C_E = 1 \qquad (3.59)$$

where M^C_i is the budget share of the i-th commodity group.

The stochastic specification for the estimating share system is based on the assumption of additive disturbances that are serially uncorrelated. The estimating system for the intra-temporal allocation model in terms of per capita quantities is shown in Table 3-33. Because of the budget constraint (3.59), it follows that only three share equations are necessary to estimate the entire system, since the disturbances across the four equations are constrained to add up to zero.

The resulting I3SLS estimates for the parameters of the direct intra-temporal translog utility function with symmetry alone imposed are shown in Table 3-34.

Table 3-33: MULTIVARIATE ESTIMATING SYSTEM OF EXPENDITURE SHARES, INTRA-TEMPORAL MODEL OF CONSUMER BEHAVIOR

$$M_J^C = \frac{\alpha_J + \beta_{JJ}^C \ln(LJ/P) + \beta_{JN}^C \ln(CN/P) + \beta_{JK}^C \ln(CK/P) + \beta_{JE}^C \ln(CE/P) + \beta_{Jt}^C \cdot t}{-1 + \beta_{MJ}^C \ln(LJ/P) + \beta_{MN}^C \ln(CN/P) + \beta_{MK}^C \ln(CK/P) + \beta_{ME}^C \ln(CE/P) + \beta_{Mt}^C \cdot t} + \varepsilon_t^J$$

$$M_N^C = \frac{\alpha_N^C + \beta_{JN}^C \ln(LJ/P) + \beta_{NN}^C \ln(CN/P) + \beta_{NK}^C \ln(CK/P) + \beta_{NE}^C \ln(CE/P) + \beta_{Nt}^C \cdot t}{-1 + \beta_{MJ}^C \ln(LJ/P) + \beta_{MN}^C \ln(CN/P) + \beta_{MK}^C \ln(CK/P) + \beta_{ME}^C \ln(CE/P) + \beta_{Mt}^C \, t} + \varepsilon_t^N$$

$$M_K^C = \frac{\alpha_K^C + \beta_{JK}^C \ln(LJ/P) + \beta_{NK}^C \ln(CN/P) + \beta_{KK}^C \ln(CK/P) + \beta_{KE}^C \ln(CE/P) + \beta_{Kt}^C \cdot t}{-1 + \beta_{MJ}^C \ln(LJ/P) + \beta_{MN}^C \ln(CN/P) + \beta_{MK}^C \ln(CK/P) + \beta_{ME}^C \ln(CE/P) + \beta_{MT}^C \, t} + \varepsilon_t^K$$

Table 3-34: PARAMETER ESTIMATES OF INTRA-TEMPORAL DIRECT TRANSLOG UTILITY FUNCTION*

	Symmetry	Neutrality	Homotheticity	Homogeneity
α_J^C	-0.2458 (-4.785)	-0.4200 (-7.2921)	-0.3879 (-75.44)	-0.4102 (-4.625)
β_{JJ}^C	-1.6498 (-67.04)	-0.9913 (-19.512)	-0.1601 (89.278)	-0.1990 (-5.762)
β_{JN}^C	-0.1152 (-5.546)	-0.0106 (-0.4365)	0.0805 (67.185)	0.1389 (4.990)
β_{JK}^C	0.0373 (277.90)	0.0395 (33.485)	0.0555 (169.715)	0.0417 (3.681)
β_{JE}^C	-1.4062 (-1812.82)	-0.9226 (-25.968)	0.0175 (20.130)	-0.0216 (-1.237)
β_{JT}^C	-0.0084 (-22.708)		-0.0854 (-224.05)	-0.0024 (-2.780)
β_{MJ}^C	-1.7103 (-311.44)	-0.9489 (-18.300)		
β_{MN}^C	-0.2789 (-18.637)	-0.0291 (-1.311)		
β_{MK}^C	-0.0332 (-46.722)	0.0127 (2.304)		
β_{ME}^C	-1.7029 (0.0)**	-1.0632 (-29.792)		
β_{MT}^C	-0.0026 (-7.110)	-0.0002 (-29.792)	-0.0959 (-250.215)	
α_N^C	-0.3979 (-8.025)	-0.3140 (-7.957)	-0.3719 (-100.28)	-0.3221 (-4.051)
β_{NN}^C	-0.0760 (-10.752)	0.0154 (1.570)	-0.0179 (0.0)**	-0.1188 (-4.708)

*asymptotic t-ratios in parentheses.
**singular likelihood function.

Table 3-34 (cont.): PARAMETER ESTIMATES OF INTRA-TEMPORAL DIRECT UTILITY FUNCTION*

	Symmetry	Neutrality	Homotheticity	Homogeneity
β^C_{NK}	-0.0847 (0.0)**	-0.0374 (-9.377)	-0.0625 (-62.43)	-0.0257 (-3.623)
β^C_{NE}	-0.2087 (0.0)**	-0.0897 (0.0)**	-0.0065 (0.0)**	-0.0508 (3.416)
β^C_{NT}	0.0052 (0.0)**		-0.0071 (0.0)**	0.0008 (1.030)
α^C_{K}	-0.2689 (0.0)**	-0.2070 (0.0)**	-0.1704 (0.0)**	-0.2112 (-9.815)
β^C_{KK}	0.0092 (0.0)**	0.0072 (4.555)	0.0028 (2.573)	-0.0112 (-2.035)
β^C_{KE}	-0.0707 (0.0)**	-0.0416 (0.0)**	-0.0080 (-10.370)	-0.0303 (-7.640)
β^C_{KT}	0.0006 (0.0)**		-0.0024 (0.0)**	0.0016 (0.0)**

**singular likelihood function.

3.2.4 The Structure of Consumer Preferences and Substitution Possibilities

In this section we report on the results of statistical tests designed to investigate alternative hypotheses about the structure of consumer preferences in the U.S. household sector. The tests are conducted under a maintained assumption of utility-maximizing behavior with a functional specification derived from a direct translog utility function, as described in the previous section. We will also present numerical estimates of price and income elasticities which provide insight into the possibilities for substitution among commodity groups.

We implement tests on the structure of consumer preferences by using the likelihood ratio test applied to the corresponding parametric restrictions on the translog utility.

We test for neutrality of consumer preferences by testing the parametric restrictions $\beta^C_{Jt} = 0$, $\beta^C_{Nt} = 0$, $\beta^C_{Kt} = 0$. The results of the test for this hypothesis as against the alternative hypothesis of a translog utility function with symmetry alone imposed are, at the 0.005 level of significance:

Degrees of freedom = 3

Critical value = 12.38

Test statistic = 14.0

We test for homotheticity of consumer preferences by testing the parametric restrictions $\beta^C_{MJ} = 0$, $\beta^C_{MN} = 0$, $\beta^C_{MK} = 0$, $\beta^C_{ME} = 0$. The results for the test of homotheticity as against symmetry alone, at the 0.005 level of significance, are:

Degrees of freedom = 4

Critical value = 14.86

Test statistic = 4.03

We test for homogeneity of consumer preferences by testing the parametric restrictions $\beta^C_{MJ} = 0$, $\beta^C_{MN} = 0$, $\beta^C_{MK} = 0$, $\beta^C_{ME} = 0$. The results for the test of homogeneity as against the alternative hypothesis of a translog utility function with symmetry alone imposed, at the 0.005 level of significance are:

Degrees of freedom = 5

Critical value = 16.74

Test statistic = 22.03

The parameter estimates corresponding to the hypothesis of neutrality, homotheticity and homogeneity are shown in Table 4-34. The results of the above set of tests lead us to reject the hypothesis of neutrality and homogeneity, while we are unable to reject the hypothesis of homotheticity. We remark that the parametric restrictions adopted to implement the tests correspond to explicit separability restrictions in the nomenclature used by Jorgenson and Lau [19]; according to their terminology the above set of hypotheses correspond to an explicitly neutral, explicitly homothetic and explicitly homogeneous direct translog utility function, respectively.

The final set of structural hypotheses we wish to investigate correspond to groupwise separability among commodity groups. We test for groupwise separability between LJ and {CN,CK,CE} by testing the parametric restrictions $\beta^C_{JN} = 0$, $\beta^C_{JK} = 0$, $\beta^C_{JE} = 0$. We test for groupwise separability between CN and {LJ, CK, CE} by testing the parametric restrictions $\beta^C_{JN} = 0$, $\beta^C_{NK} = 0$, $\beta^C_{NE} = 0$. We test for groupwise separability

Table 3-35: LIKELIHOOD RATIO TESTS OF GROUPWISE SEPARABILITY HYPOTHESES ON THE STRUCTURE OF CONSUMER PREFERENCES

Groupwise Separability between	Degrees of Freedom	Critical Value*	Test Statistic
LJ and {CN, CK, CE}	3	12.38	16.38
CN and {LJ, CK, CE}	3	12.38	14.42
CK and {LJ, CN, CE}	3	12.38	16.12
CE and {LJ, CN, CK}	3	12.38	4.64

*0.005 significance level

between CK and {LJ, CN, CE} by testing the parametric restrictions $\beta^C_{JK} = 0$, $\beta^C_{NK} = 0$, $\beta^C_{KE} = 0$. We test for separability between {CE} and {LJ, CN, CK} by testing the parametric restrictions $\beta^C_{JE} = 0$, $\beta^C_{NE} = 0$, $\beta^C_{KE} = 0$. The results of likelihood ratio tests of these separability hypotheses are shown in Table 3-35. We observe that all groupwise separability hypotheses can be rejected at the 0.005 significance level with the exception of groupwise separability between CE and {LJ, CN, CK}.

Finally we compute the implied elasticities of demand for each commodity group with respect to prices and income given the estimated parameters. For our direct translog utility function the own-price elasticity of the i-th commodity group is given by:

$$\eta^C_{ii} = \frac{1}{\left(-1 + \dfrac{\beta^C_{ii}/M^C_i - \beta^C_{Mi}}{-1 + \sum_{k=1}^{4} \beta^C_{Mk} \ln (x_k)}\right)}$$

where x = (LJ/P, CN/P, CK/P, CE/P)

Similarly, the cross-price elasticity of demand for the i-th commodity group with respect to the price of the j-th commodity group is given by:

$$\eta^C_{ij} = \frac{-1 + \sum_{k=1}^{4} \beta^C_{Mk} \ln(x_k)}{\beta^C_{ij}/M^C_i - \beta^C_{Mj}}$$

where x = (LJ/P, CN/P, CK/P, CE/P)

Table 3-36: PRICE AND INCOME ELASTICITIES OF COMMODITY GROUPS, INTRA-TEMPORAL MODEL OF CONSUMER BEHAVIOR

Price Elasticities

	η^C_{JJ}	η^C_{JN}	η^C_{JK}	η^C_{JE}
1950	-2.724	-0.380	-0.210	-0.098
1960	-1.177	-0.098	-0.051	-0.036
1970	-1.090	-0.057	-0.034	-0.026

		η^C_{NN}	η^C_{NK}	η^C_{NE}
1950		-18.130	1.980	0.165
1960		-1.432	0.640	0.198
1970		-1.243	0.329	0.160

			η^C_{KK}	η^C_{KE}
1950			-0.529	1.960
1960			-0.833	0.232
1970			-0.893	0.121

				η^C_{EE}
1950				-0.184
1960				-0.468
1970				-0.593

Income Elasticities

	η^C_{JB}	η^C_{NB}	η^C_{KB}	η^C_{EB}
1950	3.435	---	-3.213	-1.843
1960	1.362	0.692	0.012	0.074
1970	1.207	0.817	0.477	0.338

The income elasticity for the i-th commodity group is computed by using the identity

$$\eta^{C}_{iB} = - \sum_{j=1}^{4} \eta_{ij}$$

Income and price elasticities are tabulated in Table 3-36. We observe slight substitutability between non-energy, capital and energy, while leisure and the other three commodity groups exhibit slight complementarity. We also note increasing income elasticities for CE, CN and CK throughout the period 1948-1971 while the income elasticity of leisure decreased during the same period. We summarize below typical values of the own-price elasticities of demand throughout the period 1948-1971.

Own-Price Elasticities of Demand

η^{C}_{JJ}	η^{C}_{NN}	η^{C}_{KK}	η^{C}_{EE}
-1.225	-1.534	-0.799	-0.421

3.3 Summary

This chapter has discussed the estimation of the behavioral equations in our macroeconomic energy model and has presented the results of statistical tests of alternative hypotheses on the structure of technology and the structure of consumer preferences.

The translog specification of the cost frontiers appears to provide a satisfactory characterization of technology for the U.S. energy and non-energy sectors. The unrestricted estimates satisfy the concavity constraints implied by the theory of cost and production; the single exception turns out to be statistically insignificant. The hypothesis of separability between inputs and outputs is decisively rejected for both the energy and non-energy sectors. An important consequence of this result is that the elasticities of factor demands with respect to output intensities are not equal to the corresponding cost shares, as in the separable case. Price elasticities of demand for factors of production, as well as Allen-Uzawa partial elasticities of substitution are computed and the implied patterns of substitutability and complementarity are examined for the two production sectors. Statistical tests of various hypotheses concerning the structure of technological change are reported; the commonly adopted hypothesis of Hicks neutrality is rejected and our results reveal the presence of energy-using biases in technical progress occuring during the period 1948-1971.

The remaining sections deal with the estimation of the model of consumer behavior. From the estimated parameters for the inter-temporal allocation model, we derive the implied values for the rate of social discount. The estimated values of the discount rate range between 2.02% and 7.98%, depending on the underlying assumptions about the structure of inter-temporal preferences. Finally, several structural assumptions about the intra-temporal translog utility function are investigated, including groupwise separability among the various commodity groups.

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CHAPTER IV

SIMULATION: ENERGY MARKETS AND ECONOMIC PERFORMANCE

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> "The skeptic will say:
>
> - It may well be true that this system of equations is reasonable from a logical standpoint. But this does not prove that it corresponds to nature.
>
> You are right, dear skeptic. Experience alone can decide on truth."
>
> - Albert Einstein

4.1 Introduction

In this chapter we present the final form of our long-term macroeconomic energy model; we examine its dynamic behavior both in terms of its ability to replicate historical growth patterns as well as from the standpoint of its capabilities as a tool for policy analysis.

The previous chapter was devoted to the empirical validation of individual components of the overall model. The results we obtained were important because they suggested that the specification of our behavioral equations was satisfactory in terms of conventional measures of goodness of fit and in terms of the implied values of static measures of price responsiveness such as demand elasticities. However, the model is intended to be regarded as a whole, and it is not until we investigate its dynamic performance in its final form that we can evaluate whether it has successfully captured the links between the forces of economic growth and the patterns of energy utilization.

In the next section, we present the complete model and we list the full set of equations and describe the corresponding variables. Section 4.3 is devoted to a presentation of historical simulations of the complete model for the period 1948-1971. In section 4.4 we investigate the impact of selected fiscal policy instruments on relative prices, the levels of energy consumption and the configuration of national output. Special emphasis is given to a discussion of the interconnected nature of the endogenous variables of the model. We show that analyses based on partial equilibrium representations which do not take account of the simultaneous interdependence and feedback relationships among variables, can lead to inaccurate inferences about the effects of given tax policies on the performance of the energy markets. Indeed, not even general equilibrium analysis necessarily guarantees an accurate evaluation of the effects of a selected tax measure. As we demonstrate in section 4.5, the manner in which a particular tax reduction or surcharge reflects itself on the levels of government debt or government expenditures can have a crucial impact on the ultimate effect of the tax measure itself. It follows that the consequences of a change in the rates of selected tax instruments cannot be correctly assessed unless the change in the tax rate is accompanied by the specification of a complete fiscal package.

Although the simulation experiments described in this chapter do not constitute an attempt at conducting an exhaustive policy study, they yield valuable insight into the potentialities of the model for such applications.

4.2 The Complete Model: Equations and Variables

The final form of the macroeconomic energy model is presented in the following pages which provide a list of all the equations in the model. The complete model includes 83 estimated coefficients and 127 variables, of which 59 are endogenous, 2 definitional and 66 exogenous. Of the 61 equations in the model, 17 are behavioral equations, 2 are definitional and 42 are identities, aggregation and balance equations. In order to provide the reader with a guide for cross-referencing between the list of equations in the following pages and the derivations in Chapter 2, we give the following tabulation that refers each individual equation to the corresponding form derived in Chapter 2:

1.- *	21.- (2.88)	41.- (2.58)
2.- *	22.- (2.72)	42.- (2.59)
3.- (2.12)	23.- (2.73)	43.- (2.86)
4.- (2.12)	24.- (2.74)	44.- (2.78)
5.- (2.12)	25.- (2.75)	45.- (2.47)
6.- (2.12)	26.- (2.76)	46.- **
7.- (2.13)	27.- (2.80)	47.- **
8.- (2.13)	28.- (2.80)	48.- **
9.- (2.13)	29.- (2.89)	49.- (2.62)
10.- (2.13)	30.- (2.79)	50.- (2.52)
11.- (2.70)	31.- (2.67)	51.- (2.53)
12.- (2.71)	32.- (2.68)	52.- (2.54)
13.- (2.12)	33.- (2.60)	53.- **
14.- (2.13)	34.- (2.61)	54.- **
15.- (2.13)	35.- (2.63)	55.- **
16.- (2.85)	36.- (2.64)	56.- **
17.- (2.2)	37.- (2.65)	57.- (2.91)
18.- (2.84)	38.- (2.66)	58.- (2.81)
19.- (2.87)	39.- (2.56)	59.- (2.82)
20.- (2.83)	40.- (2.57)	60.- (2.77)
		61.- (2.50)

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* Definitional Equations

** Equations relating total to per capita quantity indices

EQUATIONS IN LONG-TERM MACROECONOMIC ENERGY MODEL

1. $VE^* == p_{EC}.EC + p_{EM}.EM$

2. $VN^* == p_{NC}.NC + p_{NI}.NI + p_{NM}.NM$

3. $p_{KE}.KE/VE = 0.2753 + 0.0792 \ln(p_{KE}/p_{RE}) - 0.0086 \ln(p_{LE}/p_{RE})$
$- 0.0284 \ln(p_{EME}/p_{RE}) - 0.0338 \ln(p_{NME}/p_{RE})$
$+ 0.0410 \ln(EC/EM) - 0.0013.t$

4. $p_{LE}.LE/VE = 0.1495 - 0.0086 \ln(p_{KE}/p_{RE}) - 0.1283 \ln(p_{LE}/p_{RE})$
$+ 0.0165 \ln(p_{EME}/p_{RE}) + 0.0615 \ln(p_{NME}/p_{RE})$
$- 0.0199 \ln(EC/EM) + 0.0026.t$

5. $p_{EME}.EME/VE = 0.3272 - 0.0284 \ln(p_{KE}/p_{RE}) + 0.01655 \ln(p_{LE}/p_{RE})$
$+ 0.0595 \ln(p_{EME}/p_{RE}) + 0.0127 (p_{NME}/p_{RE})$
$- 0.0240 \ln(EC/EM) - 0.0010.t$

6. $p_{NME}.NME/VE = 0.2059 - 0.0338 \ln(p_{KE}/p_{RE}) + 0.0615 \ln(p_{LE}/p_{RE})$
$+ 0.0127 \ln(p_{EME}/p_{RE}) - 0.0439 \ln(p_{NME}/p_{RE})$
$- 0.0140 \ln(EC/EM) + 0.0011.t$

7. $p_{KN}.KN/VN = 0.2211 + 0.0351 \ln(p_{KN}/p_{RN}) - 0.0042 \ln(p_{LN}/p_{RN})$
$- 0.0073 \ln(p_{EMN}/p_{RN}) - 0.0288 \ln(p_{NMN}/p_{RN}) + 0.0444 \ln(NC/NI)$
$- 0.0429 \ln(NM/NI) - 0.0007.t$

8. $p_{LN}.LN/VN = 0.3705 - 0.0042 \ln(p_{KN}/p_{RN}) - 0.1458 \ln(p_{LN}/p_{RN})$
$+ 0.0053 \ln(p_{EMN}/p_{RN}) - 0.1598 \ln(p_{NMN}/p_{RN})$
$+ 0.1386 \ln(NC/NI) - 0.1660 \ln(NM/NI) - 0.0022.t$

*VE and VN are definitional or dummy variables introduced to simplify the analytical form of subsequent equations.

EQUATIONS IN LONG-TERM MACROECONOMIC ENERGY MODEL (continued)

9. $p_{EMN} \cdot EMN/VN = 0.0182 - 0.0073 \ln(p_{KN}/p_{RN}) + 0.0054 \ln(p_{LN}/p_{RN})$
$+ 0.0032 \ln(p_{EMN}/p_{RN}) - 0.0023 \ln(p_{NMN}/p_{RN})$
$+ 0.0088 \ln(NC/NI) - 0.0017 \ln(NM/NI) - 0.0001.t$

10. $p_{NMN} \cdot NMN/VN = 0.3631 - 0.0288 \ln(p_{KN}/p_{RN}) - 0.1599 \ln(p_{LN}/p_{RN})$
$- 0.0023 \ln(p_{EMN}/p_{RN}) + 0.2246 \ln(p_{NMN}/p_{RN})$
$- 0.2233 \ln(NC/NI) + 0.2425 \ln(NM/NI) + 0.0030.t$

11. $p_{EC} \cdot EC + p_{EM} \cdot EM = p_{KE} \cdot KE + p_{LE} \cdot LE + p_{EME} \cdot EME + p_{NME} \cdot NME + p_{RE} \cdot RE$

12. $p_{NC} \cdot NC + p_{NM} \cdot NM + p_{NI} \cdot NI = p_{KN} \cdot KN = p_{LN} \cdot LN + p_{EMN} \cdot EMN + p_{NMN} \cdot NMN + p_{RN} \cdot RN$

13. $p_{EC} \cdot EC/VE = 0.4253 + 0.1426 \ln(EC/EM) + 0.0410 \ln(p_{KE}/p_{RE})$
$- 0.0199 \ln(p_{LE}/p_{RE}) - 0.0240 \ln(p_{EME}/p_{RE})$
$- 0.0140 \ln(p_{NME}/p_{RE}) - 0.005.t$

14. $p_{NC} \cdot NC/VN = 0.3516 + 0.3415 \ln(NC/NI) - 0.2236 \ln(NM/NI)$
$+ 0.0444 \ln(p_{KN}/p_{RN}) + 0.1386 \ln(p_{LN}/p_{RN})$
$+ 0.0088 \ln(p_{EMN}/p_{RN}) - 0.2233 \ln(p_{NMN}/p_{RN}) - 0.0019.t$

15. $p_{NM} \cdot NM/VN = 0.3667 - 0.2236 \ln(NC/NI) + 0.2574 \ln(NM/NI)$
$- 0.0429 \ln(p_{KN}/p_{RN}) - 0.1660 \ln(p_{LN}/p_{RN})$
$- 0.0018 \ln(p_{EMN}/p_{RN}) + 0.2425 \ln(p_{NMN}/p_{RN}) + 0.0034.t$

16. $NI + STE + STN = I + GI + EXI$

17. $NI = \exp [\frac{1}{0.2817} [\ln(VN(-1)) - 0.3516 \ln(NC(-1))$

EQUATIONS IN LONG-TERM MACROECONOMIC ENERGY MODEL (continued)

$$- 0.3516 \ln(NC(-1)) - 0.3667 \ln(NM(-1))$$

$$- 0.2211 \ln(p_{KN}(-1)) - 0.3705 \ln(p_{LN}(-1))$$

$$- 0.0182 \ln(p_{EMN}(-1))$$

$$- 0.3631 \ln(p_{NMN}(-1))]]$$

18. $NC + CIM = CN + STN + GN + EXN$

19. $NM = NME + NMN$

20. $EC = CE + STE + GE + EXE$

21. $EM = EME + EMN$

22. $(1 + TCE) \cdot p_{EC} \cdot EC = p_{CE} \cdot CE + p_{STE} \cdot STE + p_{GE} \cdot GE + p_{EXE} \cdot EXE$

23. $(1 + TCN) \cdot p_{NC} \cdot NC = p_{CND} \cdot CND + p_{STN} \cdot STN + p_{GN} \cdot GN + p_{EXN} \cdot EXN$

24. $(1 + TI) \cdot p_{NI} \cdot NI = p_{IMI} \cdot IMI + p_{STE} \cdot STE + p_{STN} \cdot STN + p_{I} \cdot I + p_{GI} \cdot GI + p_{EXI} \cdot EXI$

25. $p_{EM} \cdot EM = p_{EME} \cdot EME + p_{EMN} \cdot EMN$

26. $p_{NM} \cdot NM = p_{NME} \cdot NME + p_{NMN} \cdot NMN$

27. $p_{EMN} = ZNE \cdot p_{EM}$

28. $p_{NME} = ZEN \cdot p_{NM}$

EQUATIONS IN LONG-TERM MACROECONOMIC ENERGY MODEL (continued)

29. $L = LE + LN + LG + LR + LR$

30. $p_L.L = (1-TL)\ (p_{LE}.LE + p_{LN}.LN + p_{LG}.LG + p_{LR}.LR)$

31. $p_{KE} = (1/QKE).[\frac{1-TKE/PVDE-ITCE}{1-TKE}]$

$[ME.p_I(-1) + DE.p_I - CAG] + TPE.p_I$

32. $p_{KN} = (1/QKN).[\frac{1-TKN\cdot PVDN-ITCN}{1-TKN}]\ [MN.p_I(-1) + DN.p_I - CAG + TPN.p_I]$

33. $KE = QKE.KSE(-1)$

34. $KN = QKN.KSN(-1)$

35. $INE = ZINE.I$

36. $INN = ZINN.I$

37. $ICD = ZICD.I$

38. $IRS = ZIRS.I$

39. $KSE = (1-DE).KSE(-1) + INE$

40. $KSN = (1-DN).KSN(-1) + INN$

41. $KCD = (1-DCD).KCD(-1) + ICD$

42. $KRS = (1-DRS).KRS(-1) + IRS$

43. $IMI + I = INE + INN + ICD + IRS$

EQUATIONS IN LONG-TERM MACROECONOMIC ENERGY MODEL (continued)

44. $W = p_I(KSE + KSN + KRS + KCD).ZW + p_G.G + p_R.R$

45. $p_F.F/P = [-0.0742 + 0.0254.t] .$
$[(1 + MW).HW(-1) + 12.7458(p_L.HLH + EL)] / (-1 + 0.343.t)$

46. $HLH = LH/P$

47. $HW = W/P$

48. $HEL = EL/P$

49. $KC = QKS.KRS(-1) + QKD.KCD(-1)$

50. $p_L.LJ/p_F.F = -(-0.4102 - 0.1991.\ln(HLJ) + 0.1389.\ln(HCN)$
$+ 0.0417 \ln(HCK) - 0.0216 \ln(HCE) - 0.0024.t)$

51. $p_{CN}.CN/p_F.F = -(-0.3221 + 0.1389.\ln(HLJ) - 0.1188.\ln(HCN)$
$- 0.0257.\ln(HCK) + 0.0508 \ln(HCE) + 0.0008.t)$

52. $p_{CK}.HCK/p_F.F = -(-0.2113 + 0.0418 \ln(HLJ) - 0.0257.\ln(HCN)$
$- 0.0113 \ln(HCK) - 0.0303 \ln(HCE) + 0.0017.t)$

53. $LJ = HLJ.P$

54. $CK = HCK.P$

55. $CE = HCE.P$

56. $CN = HCN.P$

57. $CND = CN-CIM$

VARIABLES IN LONG-TERM MACROECONOMIC ENERGY MODEL (continued)

58. MW.(KSE(-1) + KSN(-1) + KCD(-1) + KRS(-1)

= ZRN.(ME.KSE(-1) + MN.KSN(-1))

59. F = 0.8225 LJ = 1.0787 CK + 0.9349 CN + 2.4957 CE + 0.8559

60. $S = p_I.I + p_G(G-G(-1)) + P_R(R-R(-1))$

61. $P_F = 1/F\ (p_L\ LJ + p_{CN}.CN + p_{CE}.CE + p_{CK}.CK)$

VARIABLES IN LONG-TERM MACROECONOMIC ENERGY MODEL

-Quantities

*NMN - Non-energy intermediate products purchased by the non-energy sector

*EMN - Energy intermediate products purchased by the non-energy sector

*NME - Non-energy intermediate products purchased by the energy sector

*EME - Energy intermediate products purchased by the energy sector

*RN - Competitive imports of energy products

*RE - Competitive imports of non-energy products

*CE - Energy consumption goods purchased by households

*CN - Non-energy consumption goods purchased by households

*KC - Capital services supplied to households by stock of consumer durables and residential structures

*KE - Capital services supplied to the energy sector by its capital stock

*KN - Capital services supplied to the non-energy sector by its capital stock

*KSE - Capital stock of producer durables, non-residential structures, inventories and land, energy sector

*KSN - Capital stock of producer durables, non-residential structures, non-energy sector, inventories and land, non-energy sector

*LE - Labor services purchased by the energy sector

*LN - Labor services purchased by the non-energy sector

*EC - Energy consumption products supplied by the domestic energy sector

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*Asterisk denotes endogenous variable

VARIABLES IN LONG-TERM MACROECONOMIC ENERGY MODEL (continued)

*EM - Energy intermediate products supplied by the domestic energy sector

*NC - Non-energy consumption products supplied by the domestic energy sector

*NM - Non-energy intermediate products supplied by the domestic non-energy sector

*NI - Investment goods supplied by the domestic non-energy sector

*I - Gross private domestic investment

*INE - Gross investment in the energy sector

*INN - Gross investment in the non-energy sector

*IRS - Gross investment in residential structures

*ICD - Gross investment in consumer durables

*KRS - Capital stock, residential structures

*KCD - Capital stock, consumer durables

CIM - Direct imports of consumption goods by the private domestic sector

*CND - Total purchases of domestic non-energy consumption goods by the private domestic sector

STN - Non-energy consumption products delivered to business inventories

STE - Energy consumption products delivered to business inventories

GN - Government purchases of domestic non-energy consumption products

*IMIN - Direct imports of investment goods by the private domestic sector

GI - Government purchases of domestic investment goods

GE - Government purchases of energy consumption products

EXN - Gross exports of non-energy consumption products

EXI - Gross exports of investment goods

EXE - Gross exports of energy consumption products

VARIABLES IN LONG-TERM MACROECONOMIC ENERGY MODEL (continued)

R - Net claims on rest of world by the private domestic sector

G - Net claims on government by the private domestic sector

LH - Total availability of time resources to the household sector

EL - Transfer payments to the household sector

LU - Rate of unemployment

*L - Supply of labor services by the household sector

*LJ - Leisure time of the household sector

P - Total population

*HEL - Transfer payments, per capita

*HCE - Consumption expenditures by households on energy products, per capita

*HCN - Consumption expenditures by households on domestic non-energy products, per capita

*HCK - Consumption expenditures by households on capital services, per capita

*HLJ - Leisure available to households, per capita

LR - Labor services purchased by the foreign sector

LG - Labor services purchased by the government sector

*HLH - Total availability of time resources to households, per capita

*F - Full consumption, household sector

-Prices

p_{NMN} - Implicit deflator, non-energy intermediate products purchased by the non-energy sector

*p_{EMN} - Implicit deflator, energy intermediate products purchased by the non-energy sector

p_{NME} - Implicit deflator, non-energy intermediate products purchased by the energy sector

VARIABLES IN LONG-TERM MACROECONOMIC ENERGY MODEL (continued)

$*p_{EME}$ - Implicit deflator, energy intermediate products purchased by the energy sector

p_{RE} - Implicit deflator, competitive imports of energy products

p_{RN} - Implicit deflator, competitive imports of non-energy products

$*p_{CE}$ - Implicit deflator, energy consumption goods purchased by households

$*p_{CN}$ - Implicit deflator, non-energy consumption goods purchased by households

$*p_{CK}$ - Implicit deflator, capital services supplied to households by consumer durables and residential structures

$*p_{KE}$ - Implicit deflator, capital services, energy sector

$*p_{KN}$ - Implicit deflator, capital services, non-energy sector

$*p_{LE}$ - Implicit deflator, labor services purchased by the energy sector

$*p_{LN}$ - Implicit deflator, labor services purchased by the non-energy sector

$*p_{EC}$ - Implicit deflator, energy consumption products supplied by the domestic energy sector

$*p_{EM}$ - Implicit deflator, energy intermediate products supplied by the domestic energy sector

$*p_{NC}$ - Implicit deflator, non-energy consumption products supplied by the domestic energy sector

$*p_{NM}$ - Implicit deflator, non-energy intermediate products supplied by the domestic non-energy sector

$*p_{NI}$ - Implicit deflator, investment goods supplied by the domestic non-energy sector

$*p_{I}$ - Implicit deflator, gross private domestic investment

$*p_{CND}$ - Implicit deflator, total purchases of domestic non-energy consumption goods by the private domestic sector

p_{STN} - Implicit deflator non-energy consumption products delivered to business inventories

VARIABLES IN LONG-TERM MACROECONOMIC ENERGY MODEL (continued)

p_{STE} - Implicit deflator, energy consumption products delivered to business inventories

p_{GN} - Implicit deflator, government purchases of domestic non-energy consumption products

p_{GI} - Implicit deflator, government purchases of domestic investment goods

p_{GE} - Implicit deflator, government purchases of energy consumption products

p_G - Implicit deflator, net claims on government by the private domestic sector

p_R - Implicit deflator on rest of world by the private domestic sector

p_{EXN} - Implicit deflator, gross exports of non-energy consumption products

p_{EXI} - Implicit deflator, gross exports of investment goods

p_{EXE} - Implicit deflator, gross exports of energy consumption products

*p_L - Implicit deflator, supply of labor services

p_{LG} - Implicit deflator, labor services purchased by the government sector

p_{LR} - Implicit deflator, labor services purchased by the foreign sector

p_F - Implicit deflator, full consumption by the household sector

-Financial Variables

DN - Rate of replacement, capital stock, non-energy sector

DE - Rate of replacement, capital stock, energy sector

DRS - Rate of replacement, capital stock, residential structures

DCD - Rate of replacement, capital stock, consumer durables

*MN - Nominal rate of return, fixed tangible assets, non-energy sector

*ME - Nominal rate of return, fixed tangible assets, energy sector

VARIABLES IN LONG-TERM MACROECONOMIC ENERGY MODEL (continued)

*MW	-	Nominal rate of return on wealth, household sector
*W	-	Private national wealth
*S	-	Gross private national saving
ITCE	-	Effective investment tax credit, energy sector
ITCN	-	Effective investment tax credit, non-energy sector
*HW	-	Private national wealth, per capita
PVDE	-	Present value of depreciation deductions for tax purposes per dollar of investment, non-energy sector
PVDN	-	Present value of depreciation deductions for tax purposes per dollar of investment, non-energy sector
CAG	-	Effective rate of capital gain per dollar of fixed assets held

-Tax Variables

TCE	-	Effective tax rate, energy consumption goods
TCN	-	Effective tax rate, non-energy consumption goods
TI	-	Effective tax rate, investment goods
TKE	-	Effective tax rate on capital income, energy sector
TKN	-	Effective tax rate on capital income, non-energy sector
TL	-	Effective tax rate, labor services
TPE	-	Effective tax rate on capital property, energy sector
TPN	-	Effective tax rate on capital property, non-energy sector

-Aggregation and Scaling Variables

ZNE	-	Price of intermediate energy products to price of intermediate energy products purchased by the non-energy sector
ZEN	-	Price of intermediate non-energy products to price of intermediate non-energy products purchased by the energy sector
ZW	-	Capital stock to wealth

VARIABLES IN LONG-TERM MACROECONOMIC ENERGY MODEL (continued)

QKN - Quality of capital, non-energy sector

QKE - Quality of capital, energy sector

QCK - Quality of capital, household sector gross

ZINE - Gross private domestic investment to gross investment in the energy sector

ZINN - Gross private domestic investment to gross investment in the non-energy sector

ZIRS - Gross private domestic investment to gross investment in res. structures

ZICD - Gross private domestic investment to gross investment in consumer durables

ZRN - Nominal rate of return, energy and non-energy sectors, to nominal rate of return on wealth

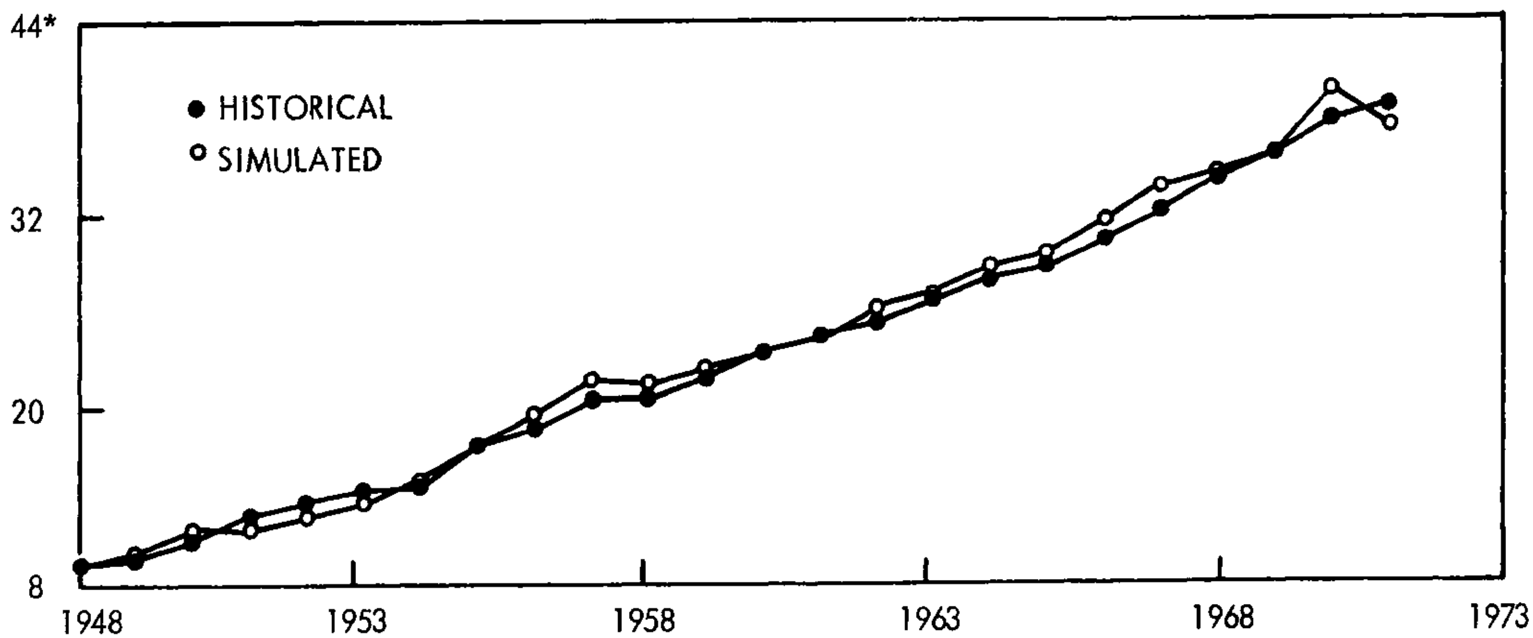
AL - Index of total cost diminution, non-energy sector, lagged one period

4.3 Historical Simulation: 1948-1971

A fully dynamic simulation of the complete model was performed for the period 1948-1971 with all exogenous variables set at their historical values. The simulated time paths of key endogenous variables together with the corresponding historical values are plotted in Figs. 4-1 through 4-46; error statistics including actual and percentage mean and R.M.S. deviations are also tabulated for each variable.

This simulation provides a crucial test of the realism and validity of the model. On the basis of the results exhibited in the following pages, the ability of the model to capture the chief causal mechanisms of aggregate economic performance must be judged highly satisfactory. The simulated values of both price and quantity variables closely track the corresponding historical time-paths. This result could not have been predicted solely on the basis of goodness-of-fit of individual behavioral equations since the simulation of the model entails the repeated solution of a set of 61 simultaneous nonlinear equations with a high degree of interdependence among contemporaneous variables.

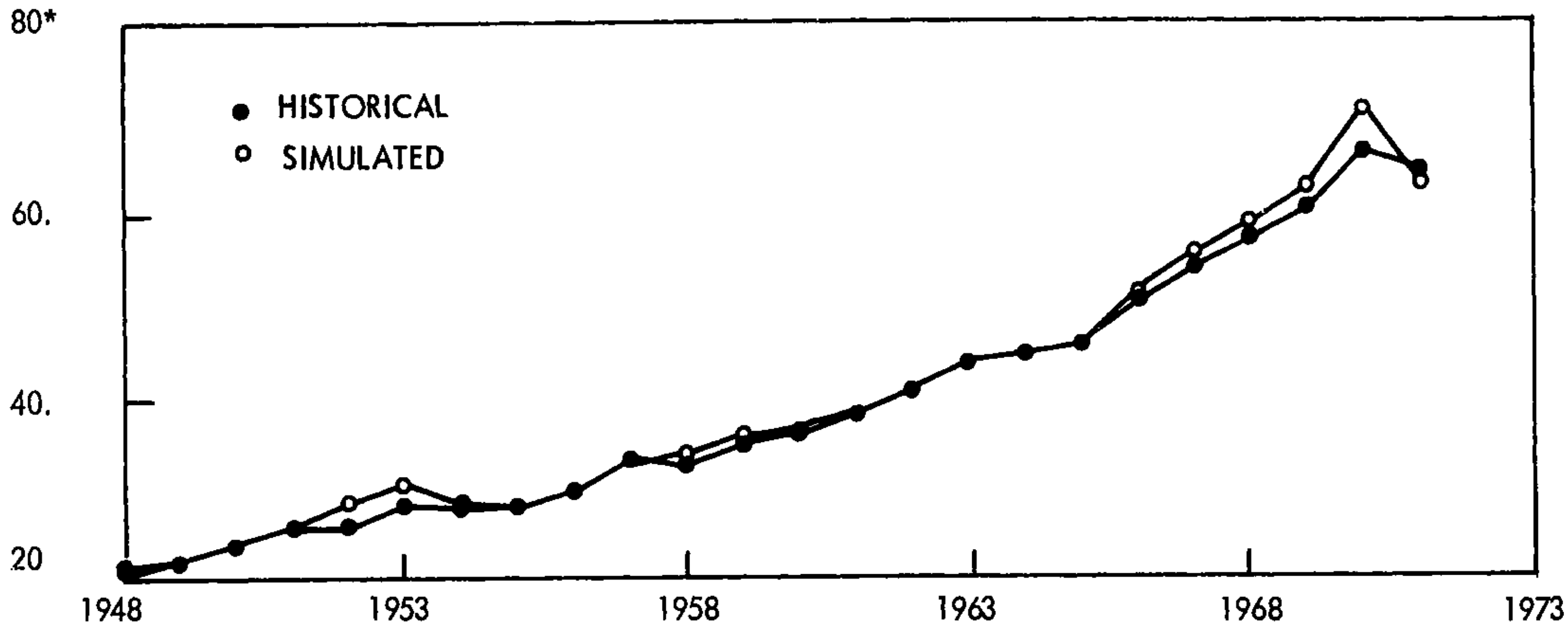
The year 1952 and, to a smaller extent, 1953, shows above average deviations of simulated vs. historical values for a few endogenous variables -- e.g. EC, NM, NC, EM, p_{KN}, p_{KE}. The higher than usual errors in these years can be largely explained by the fact that governmental intervention during the Korean War created regulatory constraints that signified a departure from a market economy upon which our model is predicated. In addition, the expansionary pressures of the Korean War, and the subsequent recession caused sharp fluctuations in the levels of



* in billions of 1958 dollars

					ACTUAL	%
HISTORICAL MEAN	225.738	ERROR STATISTICS	-	MEAN	0.3238	1.1535
				RMS	0.9175	4.6641

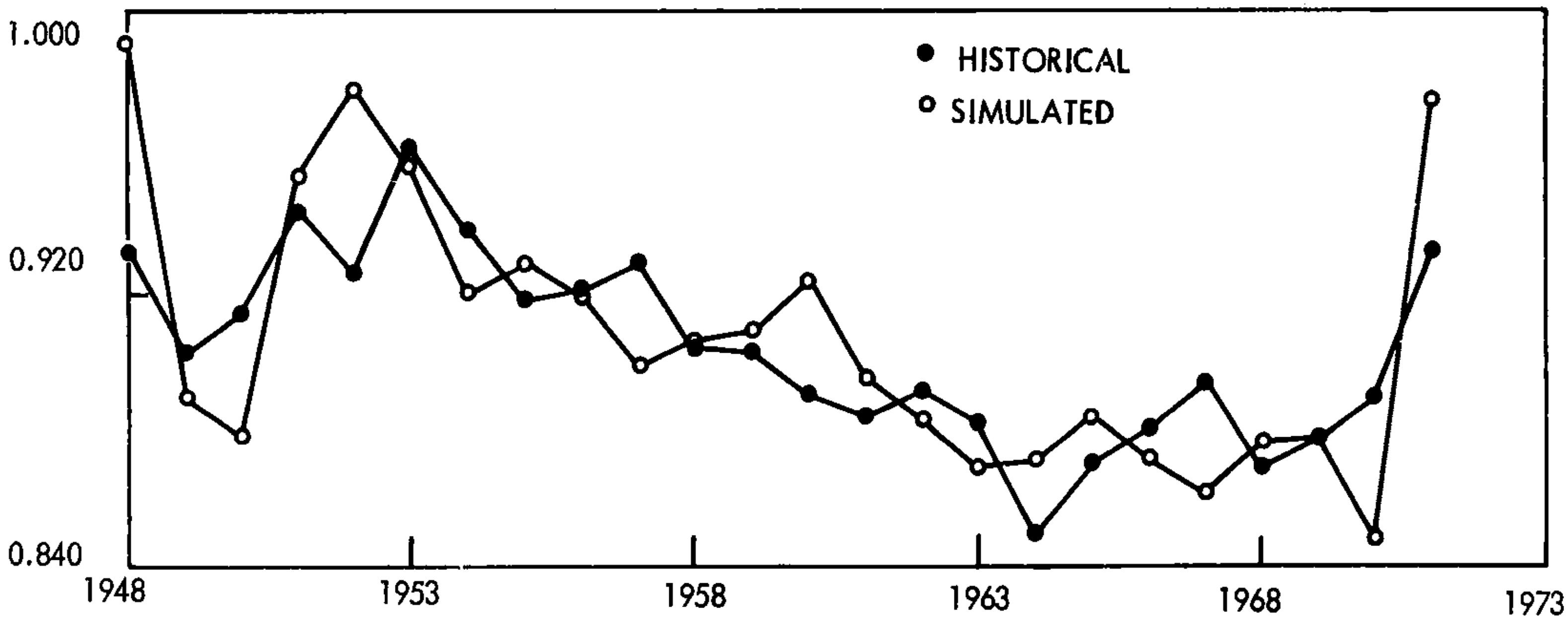
FIG. 4-1: EC - ENERGY CONSUMPTION PRODUCTS SUPPLIED BY U.S. ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

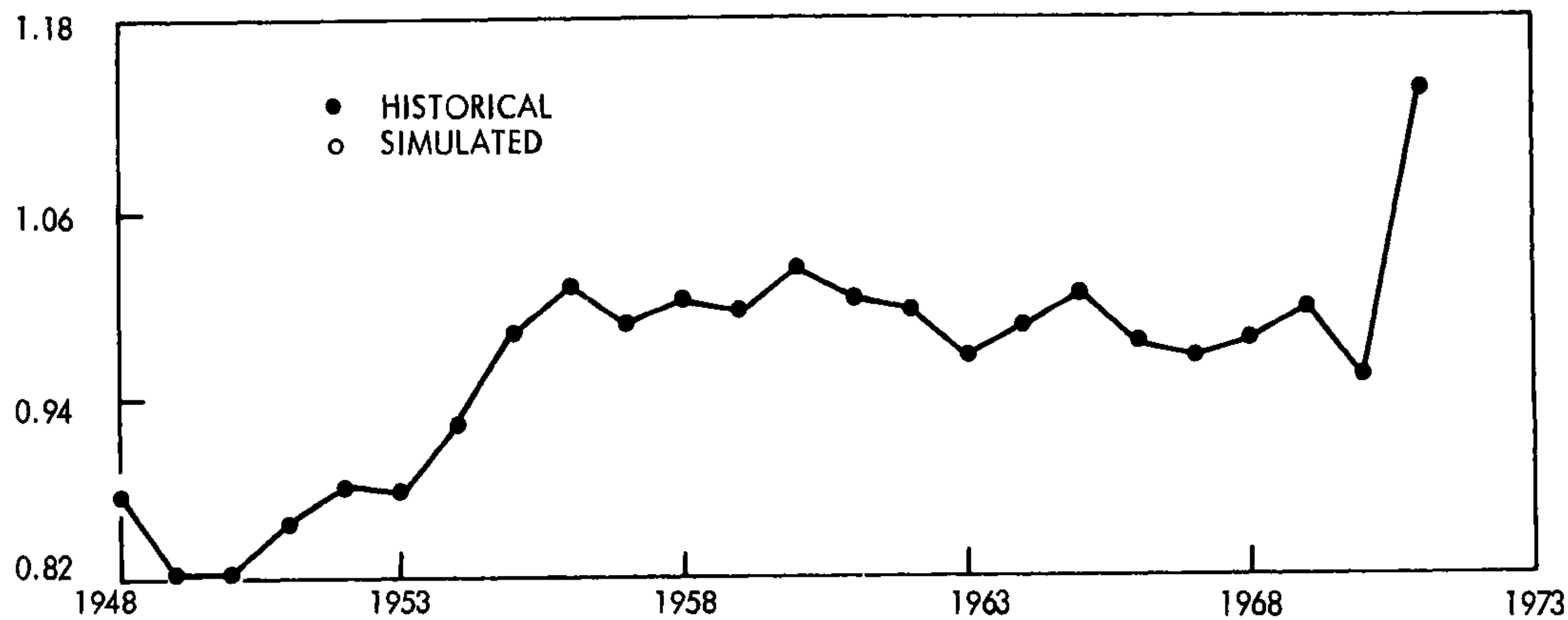
					ACTUAL	%
HISTORICAL MEAN	38.889	ERROR STATISTICS	-	MEAN	0.6303	1.4831
				RMS	1.4066	3.6174

FIG. 4-2: EM - ENERGY INTERMEDIATE PRODUCTS SUPPLIED BY U.S. ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



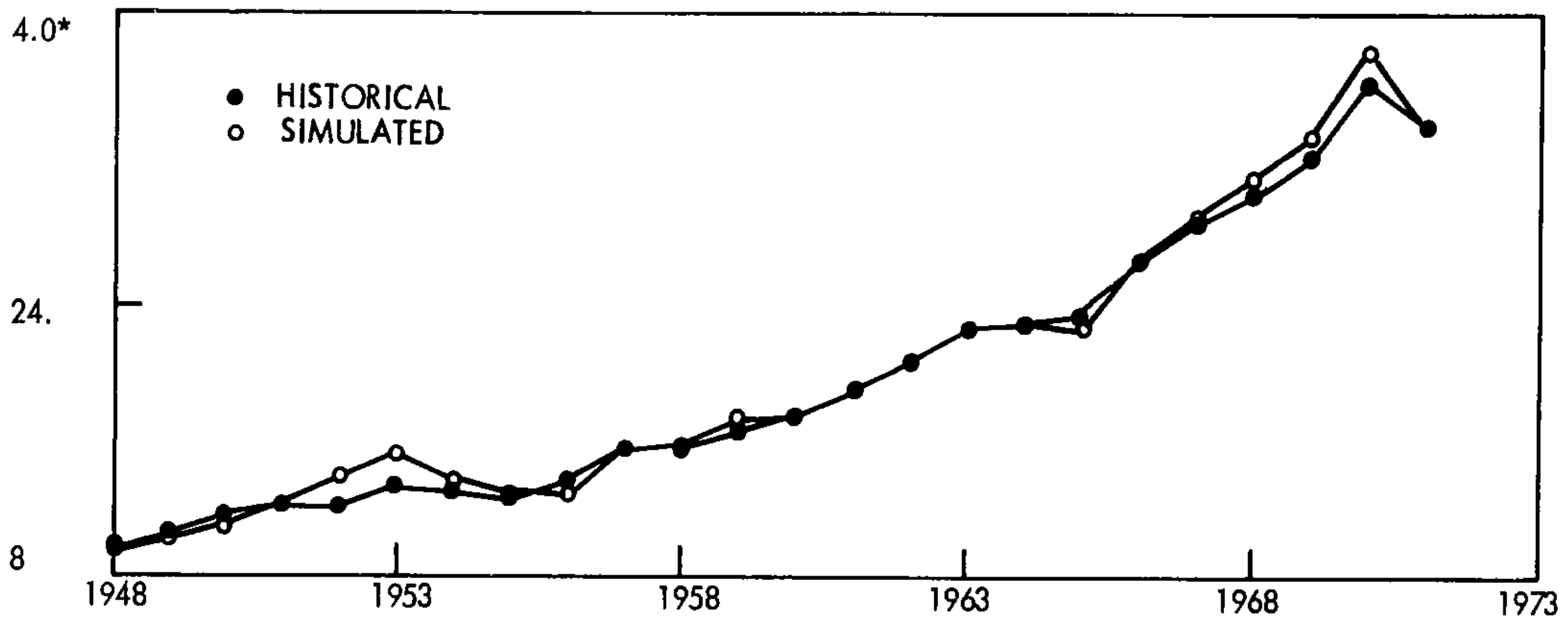
				ACTUAL	%
HISTORICAL MEAN	0.9036	ERROR STATISTICS -	MEAN	0.0025	0.2732
			RMS	0.0261	2.8613

FIG. 4-3: PEC - PRICE INDEX, ENERGY CONSUMPTION PRODUCTS SUPPLIED BY U.S. ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



				ACTUAL	%
HISTORICAL MEAN	0.9572	ERROR STATISTICS -	MEAN	0.0001	0.0331
			RMS	0.0013	0.1531

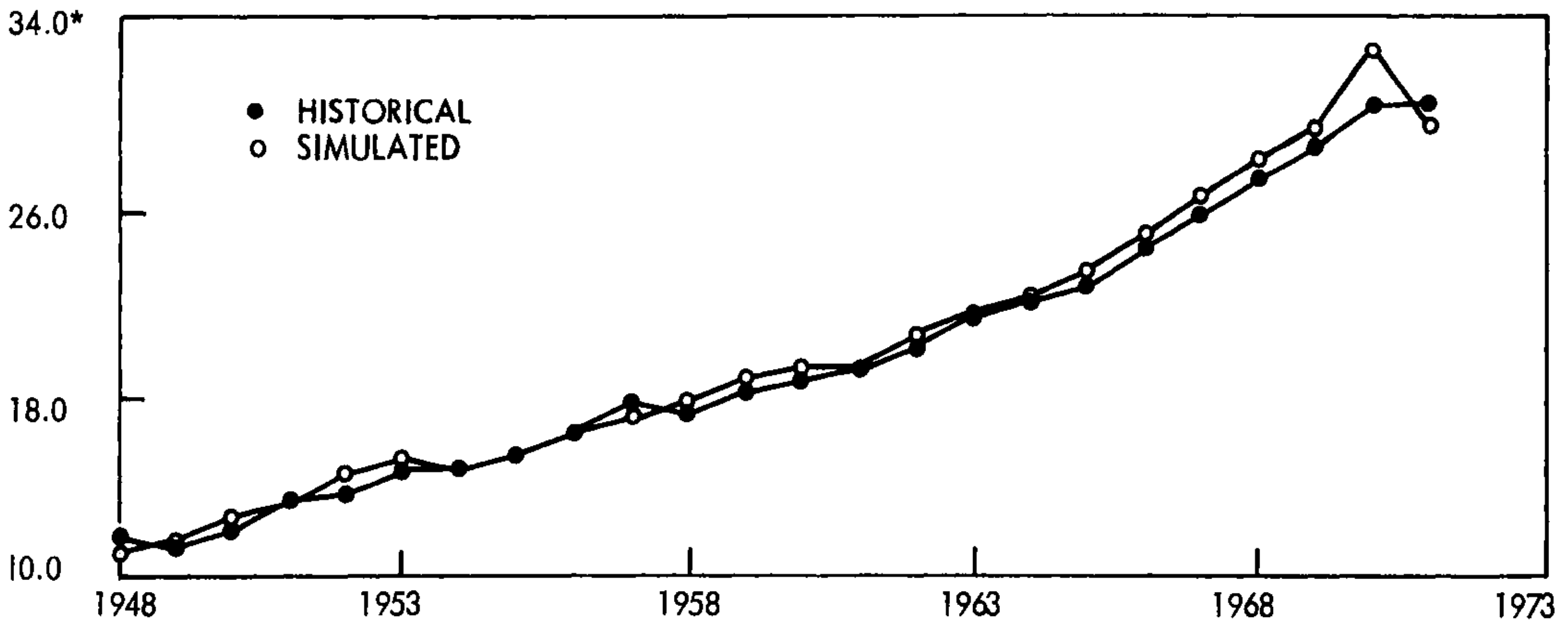
FIG. 4-4: PEM - PRICE INDEX, ENERGY INTERMEDIATE PRODUCTS SUPPLIED BY U.S. ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

					ACTUAL	%
HISTORICAL MEAN	19.5095	ERROR STATISTICS	-	MEAN	0.2895	1.4468
				RMS	0.7937	4.8616

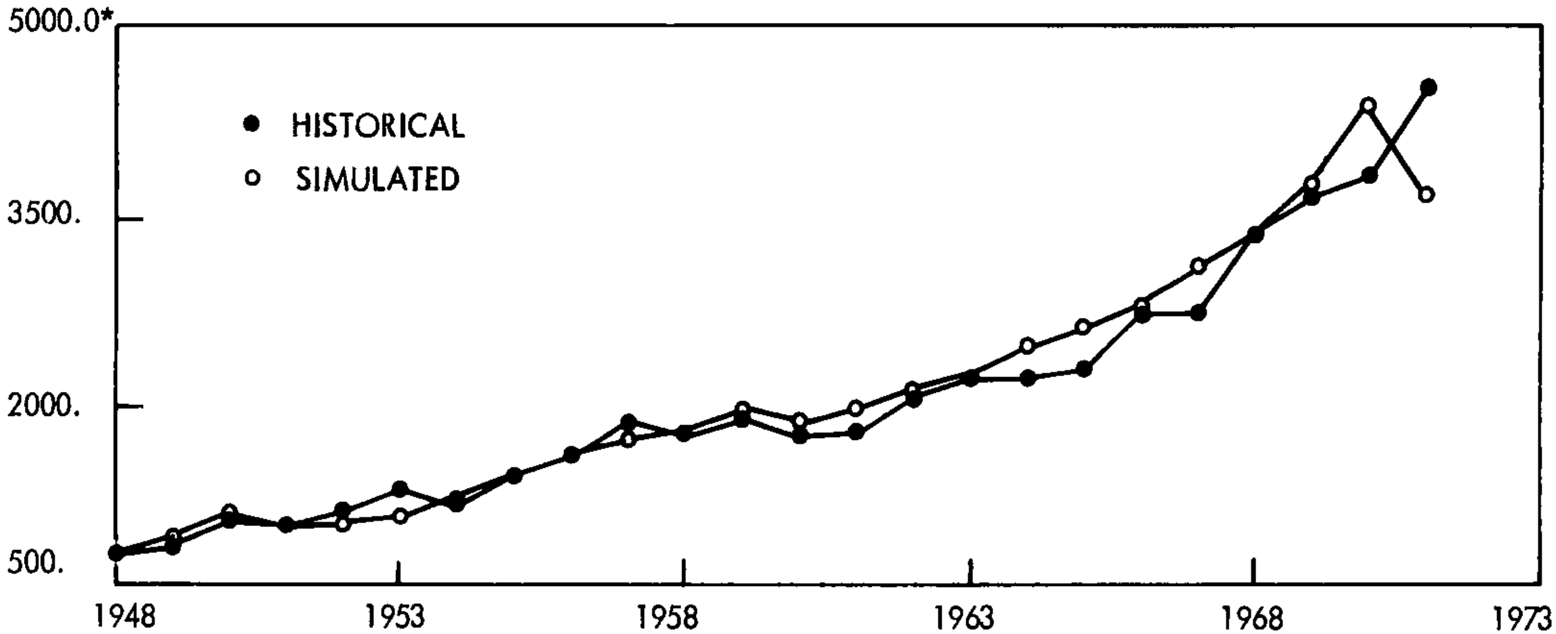
FIG. 4-5: EMN - ENERGY INTERMEDIATE PRODUCTS PURCHASED BY U.S. NON-ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



* in billions of 1958 dollars

					ACTUAL	%
HISTORICAL MEAN	19.3799	ERROR STATISTICS	-	MEAN	0.3487	1.5869
				RMS	0.7558	3.5352

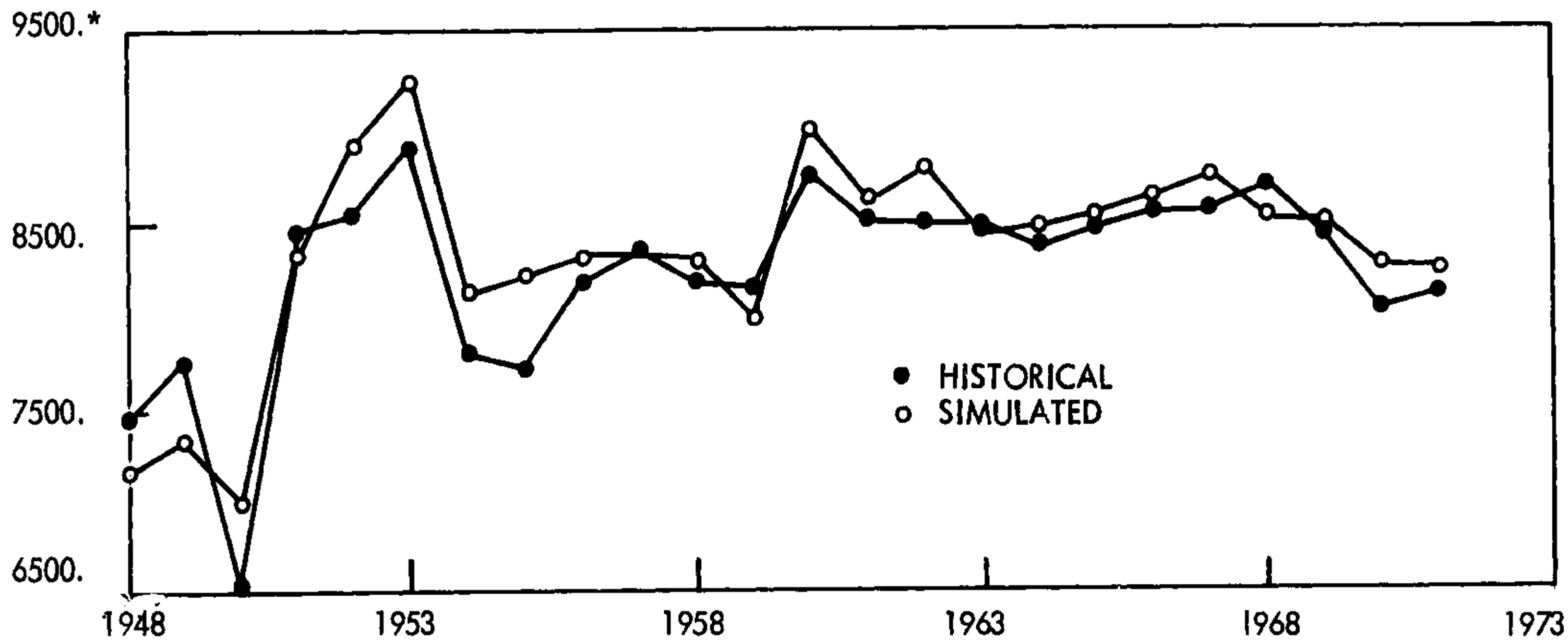
FIG. 4-6: EME - ENERGY INTERMEDIATE PRODUCTS PURCHASED BY U.S. ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

				ACTUAL	%
HISTORICAL MEAN	2002.08	ERROR STATISTICS -	MEAN	43.47	2.091
			RMS	256.71	9.075

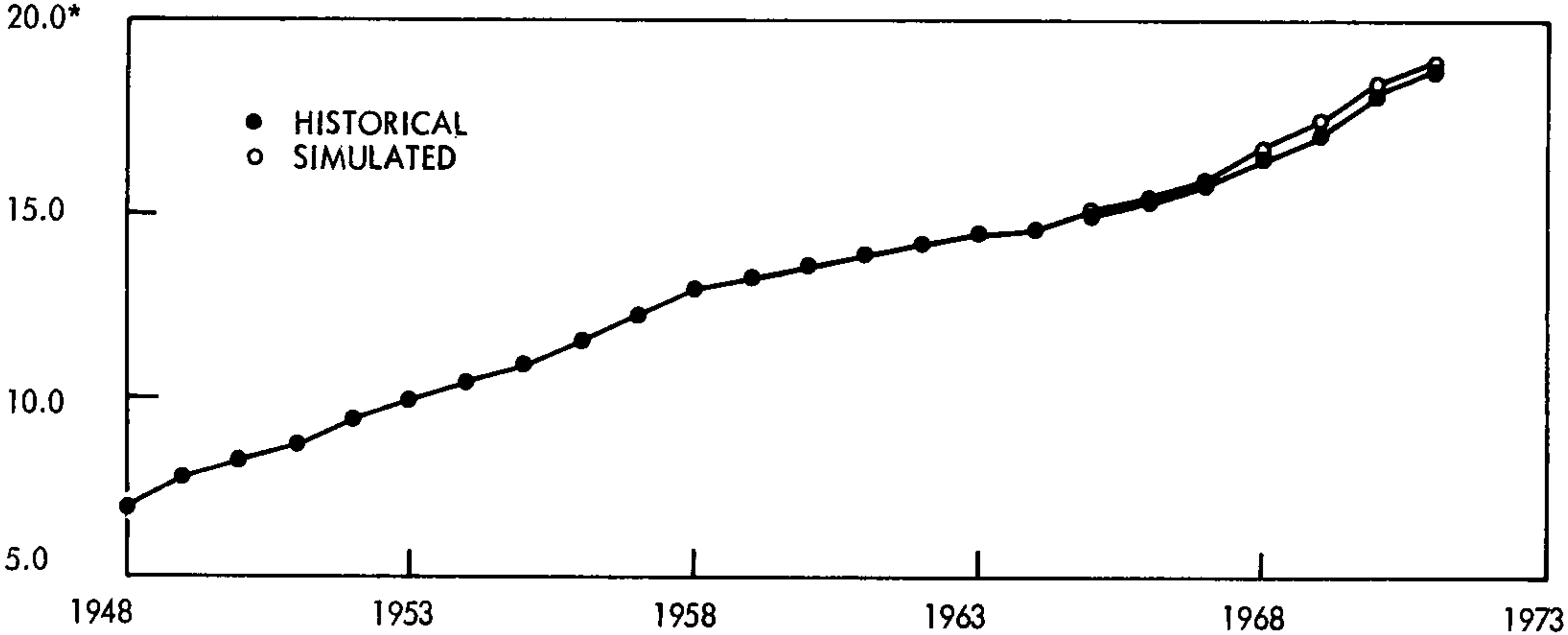
FIG. 4-7: RE - COMPETITIVE IMPORTS OF ENERGY PRODUCTS, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

				ACTUAL	%
HISTORICAL MEAN	8206.28	ERROR STATISTICS -	MEAN	101.242	1.2545
			RMS	245.445	3.1309

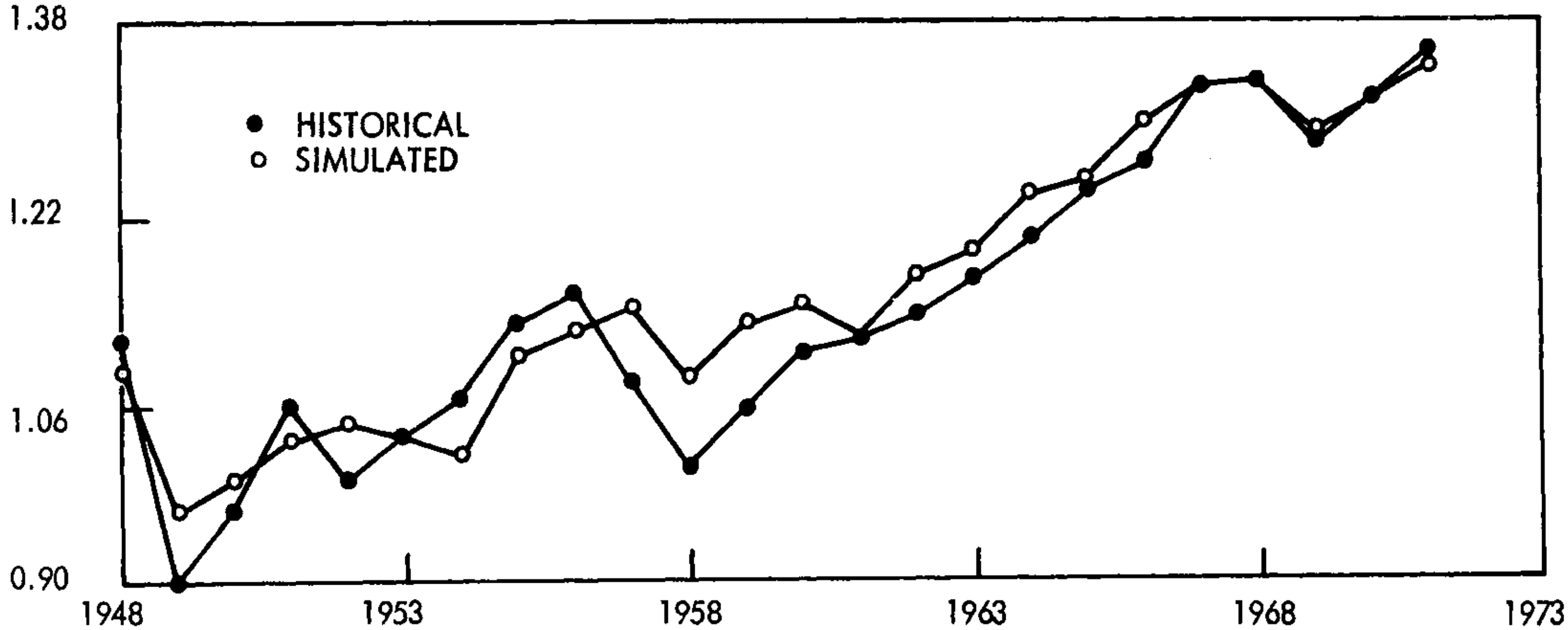
FIG. 4-8: LE – LABOR SERVICES PURCHASED BY U.S. ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

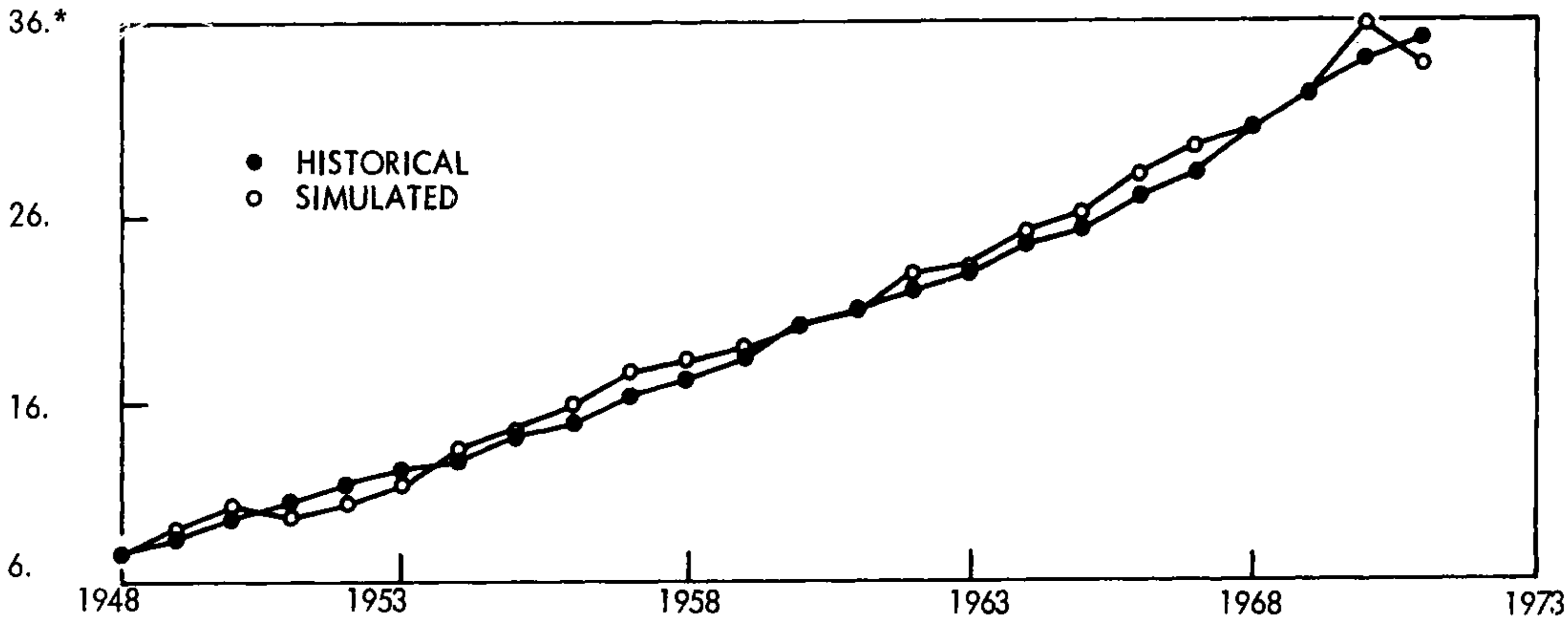
					ACTUAL	%
HISTORICAL MEAN	12.7737	ERROR STATISTICS	-	MEAN	0.0543	0.3078
				RMS	0.1497	0.9943

FIG. 4-9: KE - CAPITAL SERVICES SUPPLIED TO U.S. ENERGY SECTOR, HISTORICAL VS SIMULATED



				ACTUAL	%
HISTORICAL MEAN	1.1381	ERROR STATISTICS -	MEAN	0.0152	1.4837
			RMS	0.0384	3.6813

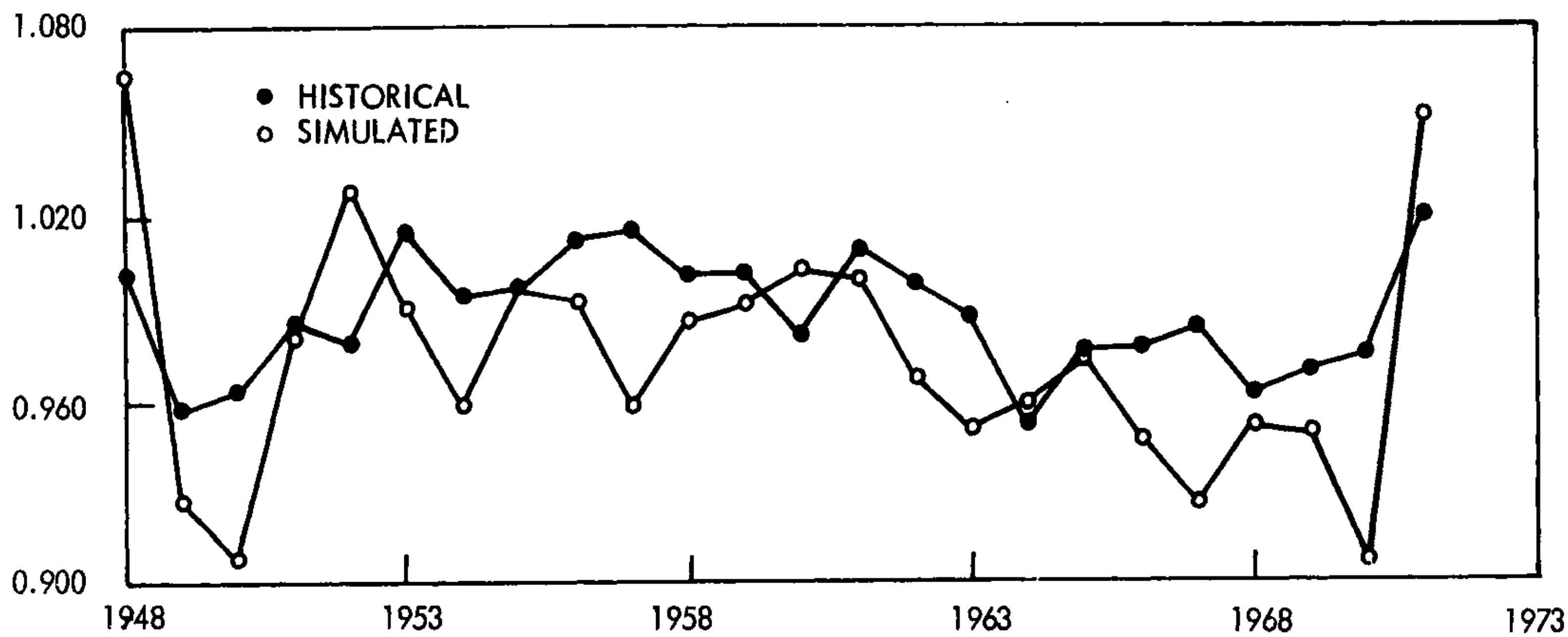
FIG. 4-10: PKE - PRICE OF CAPITAL SERVICES, U.S. ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

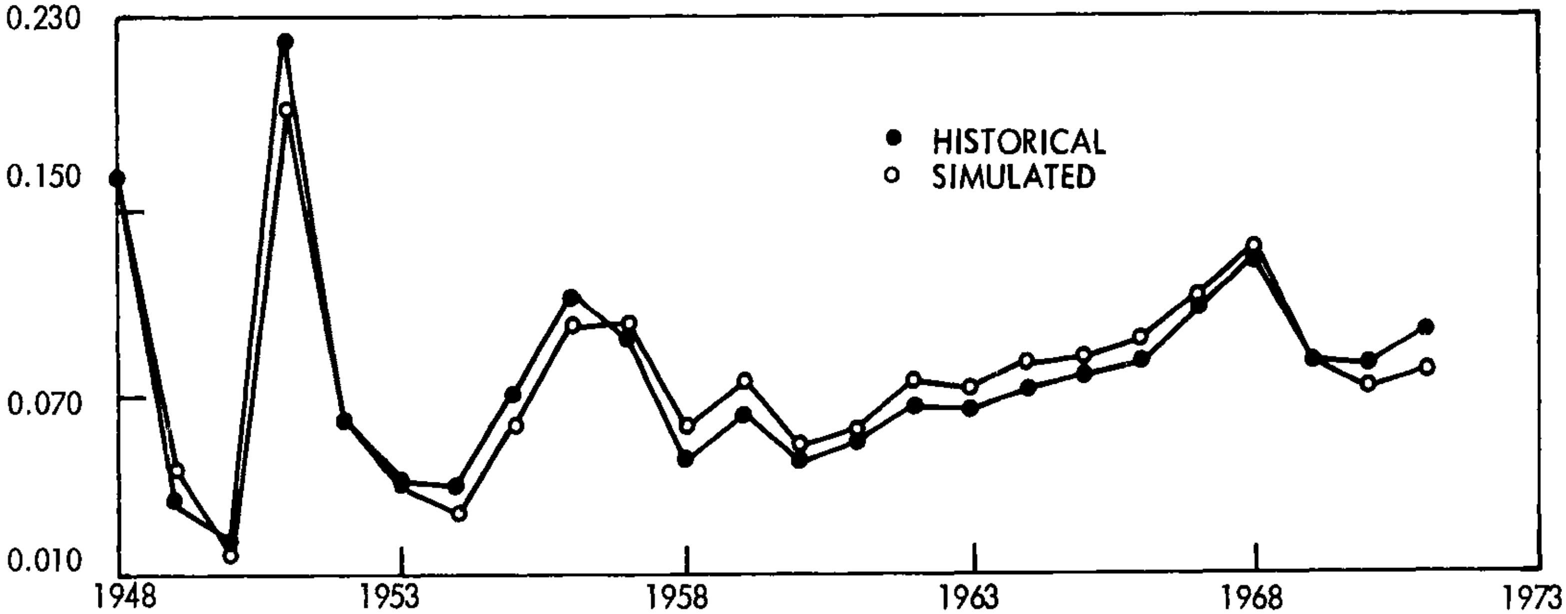
				ACTUAL	%
HISTORICAL MEAN	19.659	ERROR STATISTICS	- MEAN	0.3239	1.3248
			RMS	0.9175	5.4570

FIG. 4-11: CE - ENERGY CONSUMPTION PRODUCTS PURCHASED BY U.S. HOUSEHOLDS, HISTORICAL VS SIMULATED, 1948-1971



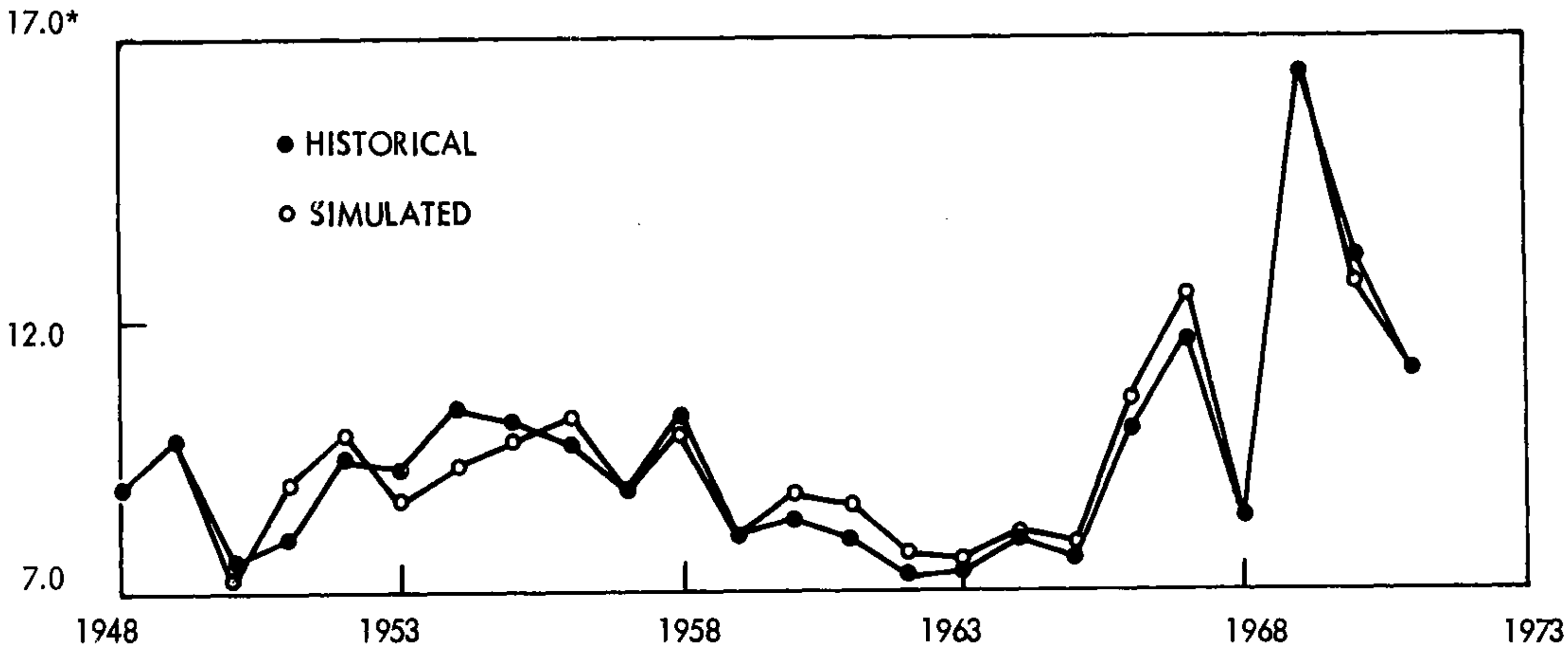
				ACTUAL	%
HISTORICAL MEAN	0.9871	ERROR STATISTICS -	MEAN	0.0146	-1.4919
			RMS	0.0353	3.5825

FIG. 4-12: PCE - PRICE INDEX, ENERGY CONSUMPTION PRODUCTS PURCHASED BY U.S. HOUSEHOLDS, HISTORICAL VS SIMULATED, 1948-1971



				ACTUAL	%
HISTORICAL MEAN	0.0758	ERROR STATISTICS -	MEAN	0.00004	-1.4572
			RMS	0.0114	32.0939

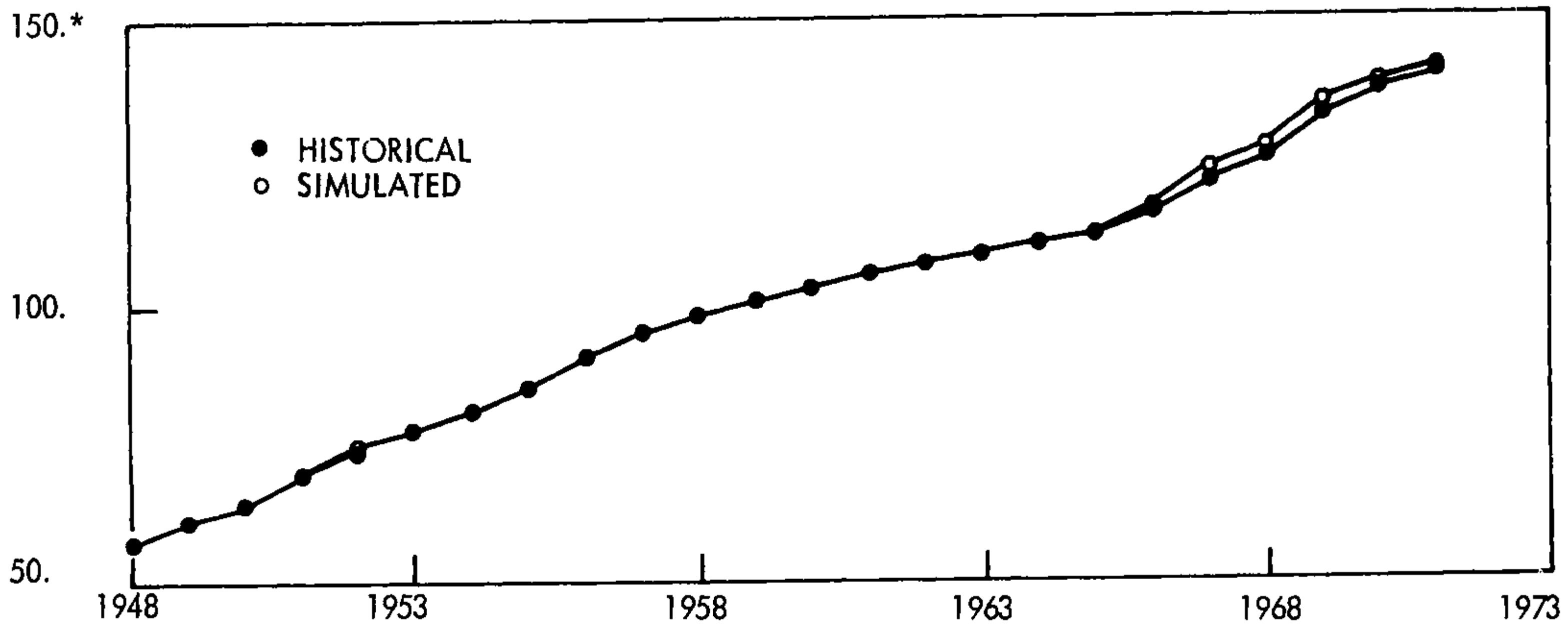
FIG. 4-13: ME - NOMINAL RATE OF RETURN, U.S. ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

				ACTUAL	%
HISTORICAL MEAN	9.4665	ERROR STATISTICS -	MEAN	0.0857	1.1492
			RMS	0.4641	5.0240

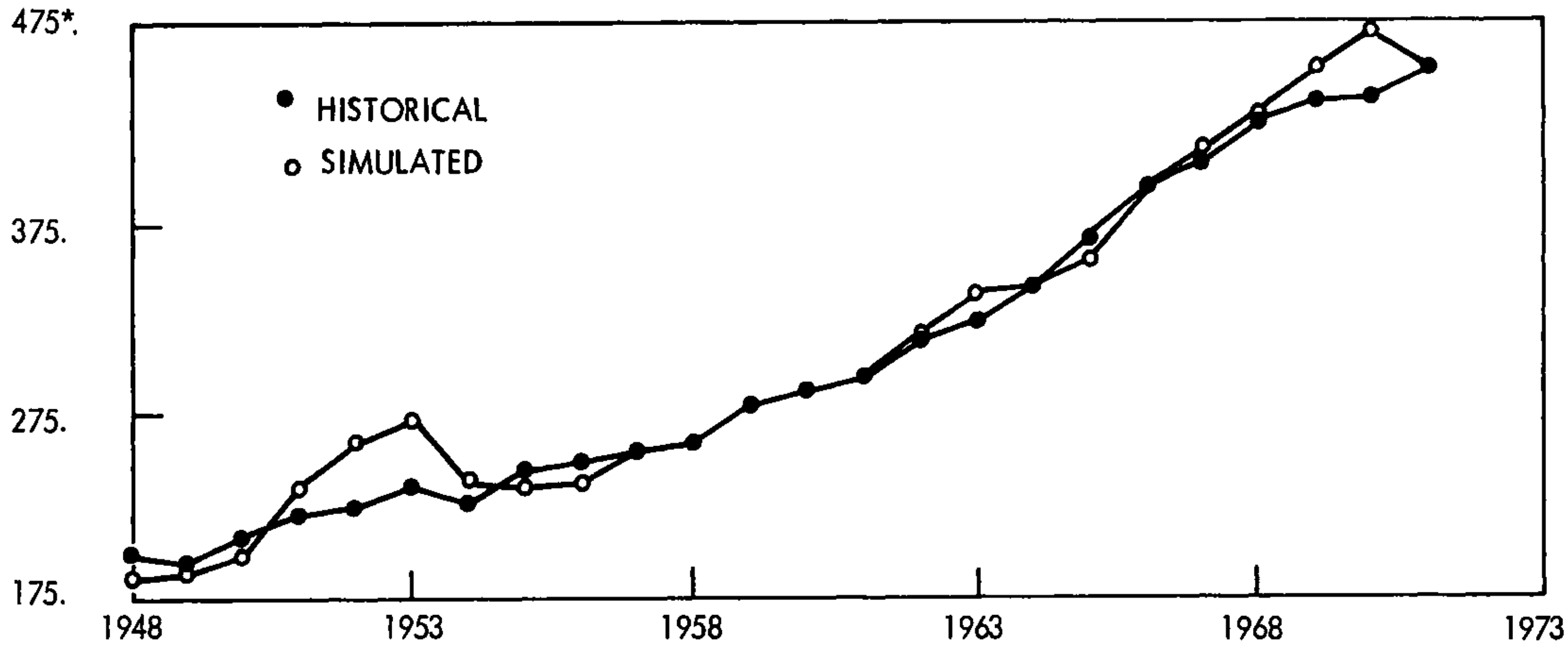
FIG. 4-14: INE - GROSS INVESTMENT, U.S. ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

					ACTUAL	%
HISTORICAL MEAN	98.5034	ERROR STATISTICS	-	MEAN	0.4633	0.3522
				RMS	1.1489	1.0177

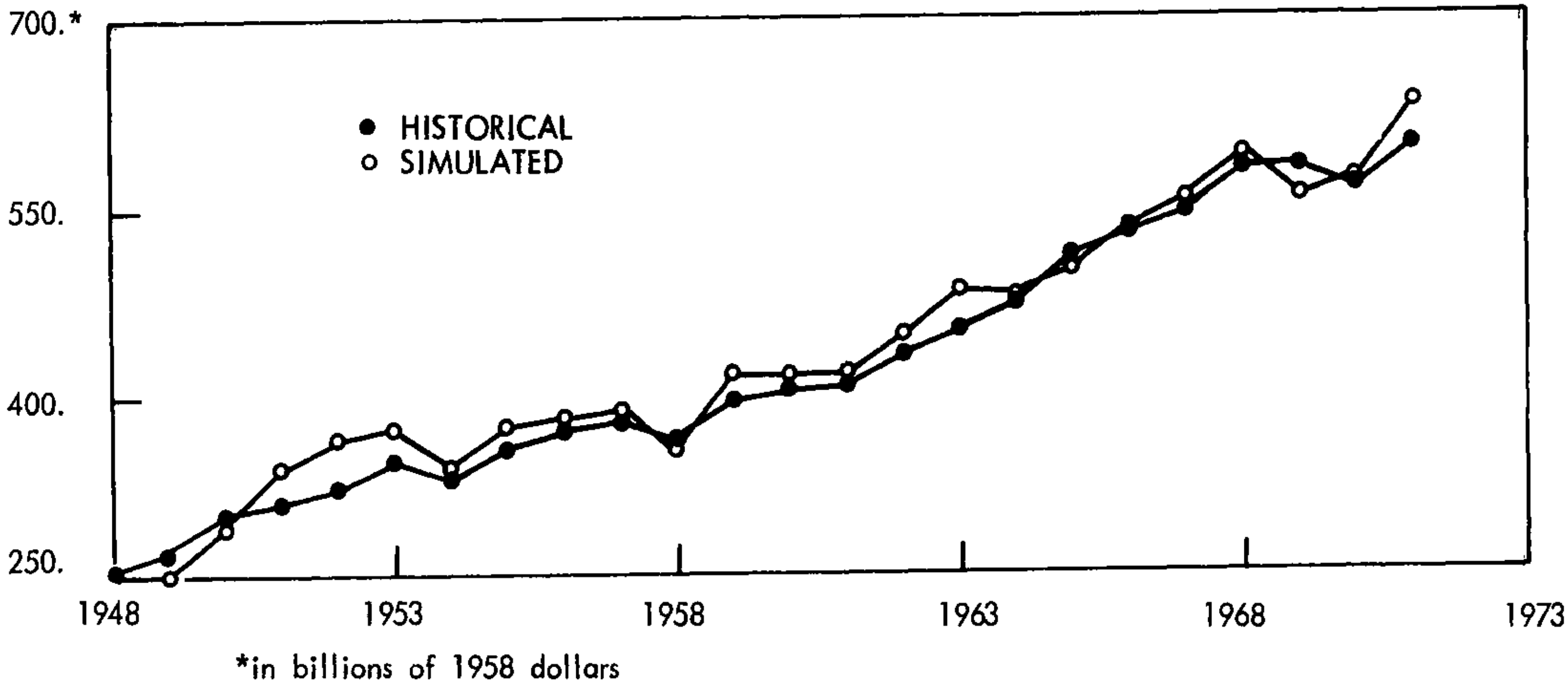
FIG. 4-15: KSE - CAPITAL STOCK, U.S. ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

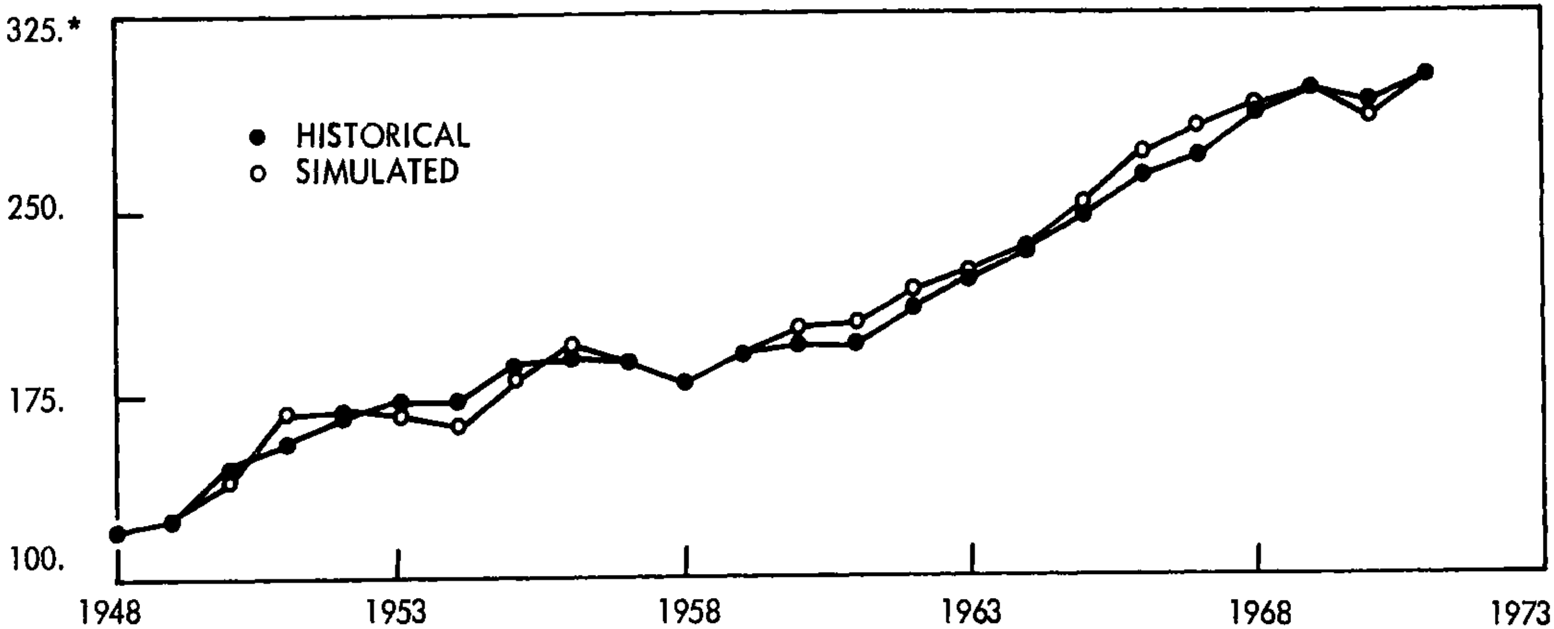
				ACTUAL	%
HISTORICAL MEAN	301.768	ERROR STATISTICS -	MEAN	4.644	1.433
			RMS	14.816	5.555

FIG. 4-16: NC - NON-ENERGY CONSUMPTION PRODUCTS SUPPLIED BY U.S. NON-ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1973



					ACTUAL	%
HISTORICAL MEAN	417.321	ERROR STATISTICS	-	MEAN	9.3566	2.2706
				RMS	18.7478	4.9894

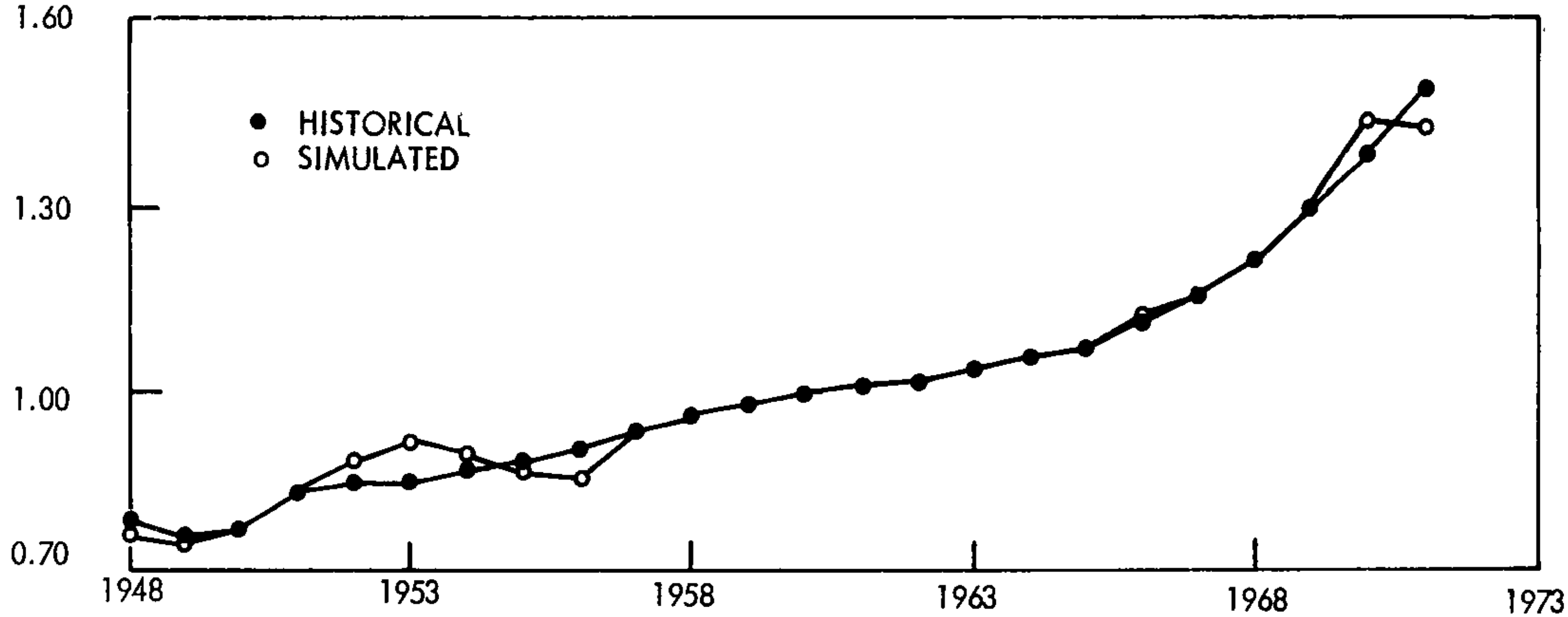
FIG. 4-17: NM - NON-ENERGY INTERMEDIATE PRODUCTS SUPPLIED BY U.S. NON-ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

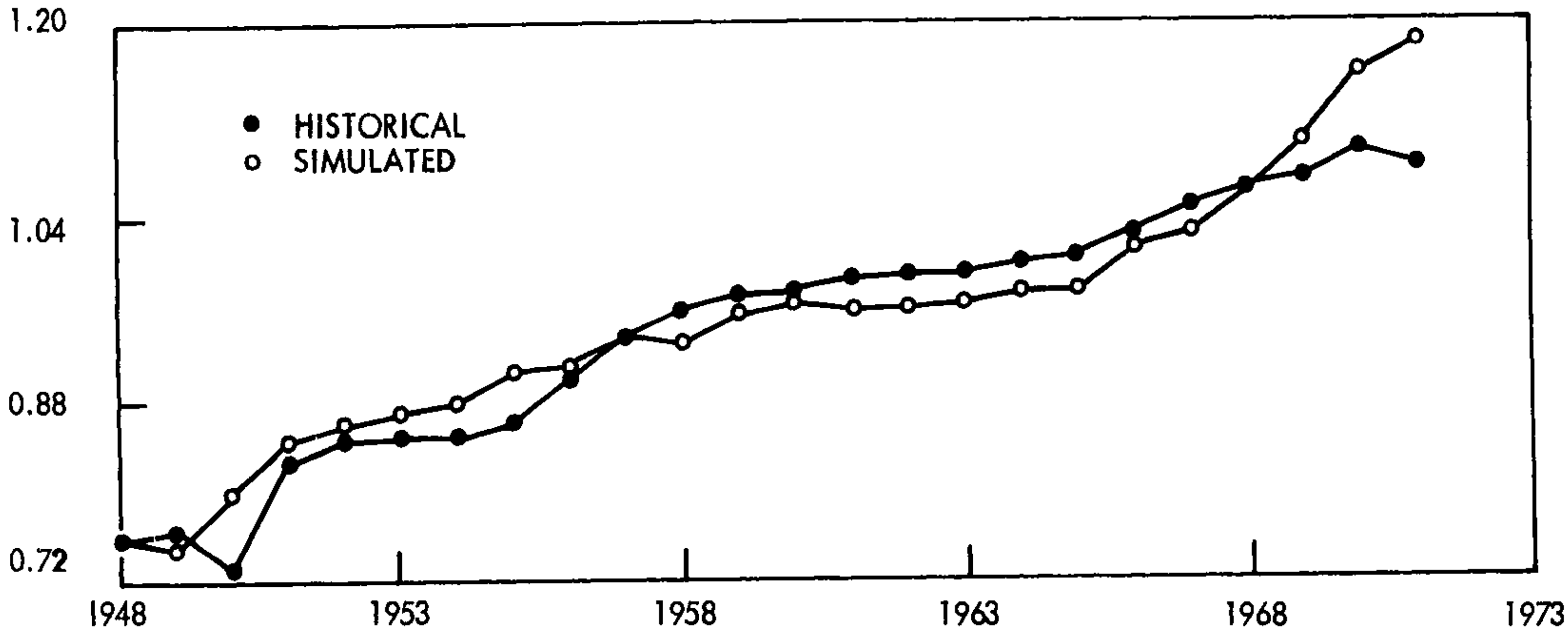
					ACTUAL	%
HISTORICAL MEAN	206.565	ERROR STATISTICS	-	MEAN	1.7015	0.7180
				RMS	6.0733	3.0995

FIG. 4-18: NI - INVESTMENT GOODS SUPPLIED BY U.S. NON-ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



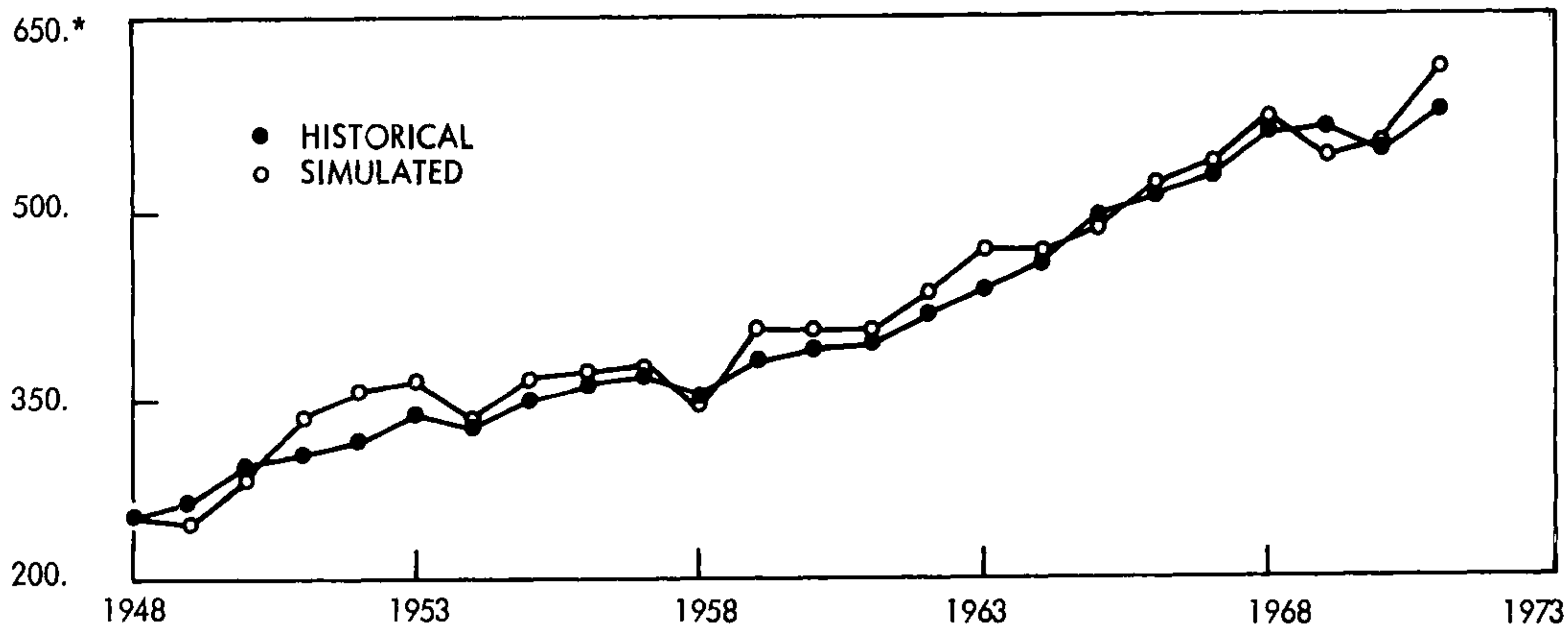
				ACTUAL	%
HISTORICAL MEAN	1.0040	ERROR STATISTICS -	MEAN	0.0031	0.3404
			RMS	0.0277	2.7446

FIG. 4-19: PNC - PRICE INDEX, NON-ENERGY CONSUMPTION GOODS SUPPLIED BY U.S. NON-ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



				ACTUAL	%
HISTORICAL MEAN	0.9382	ERROR STATISTICS -	MEAN	0.0054	0.6428
			RMS	0.0356	3.7314

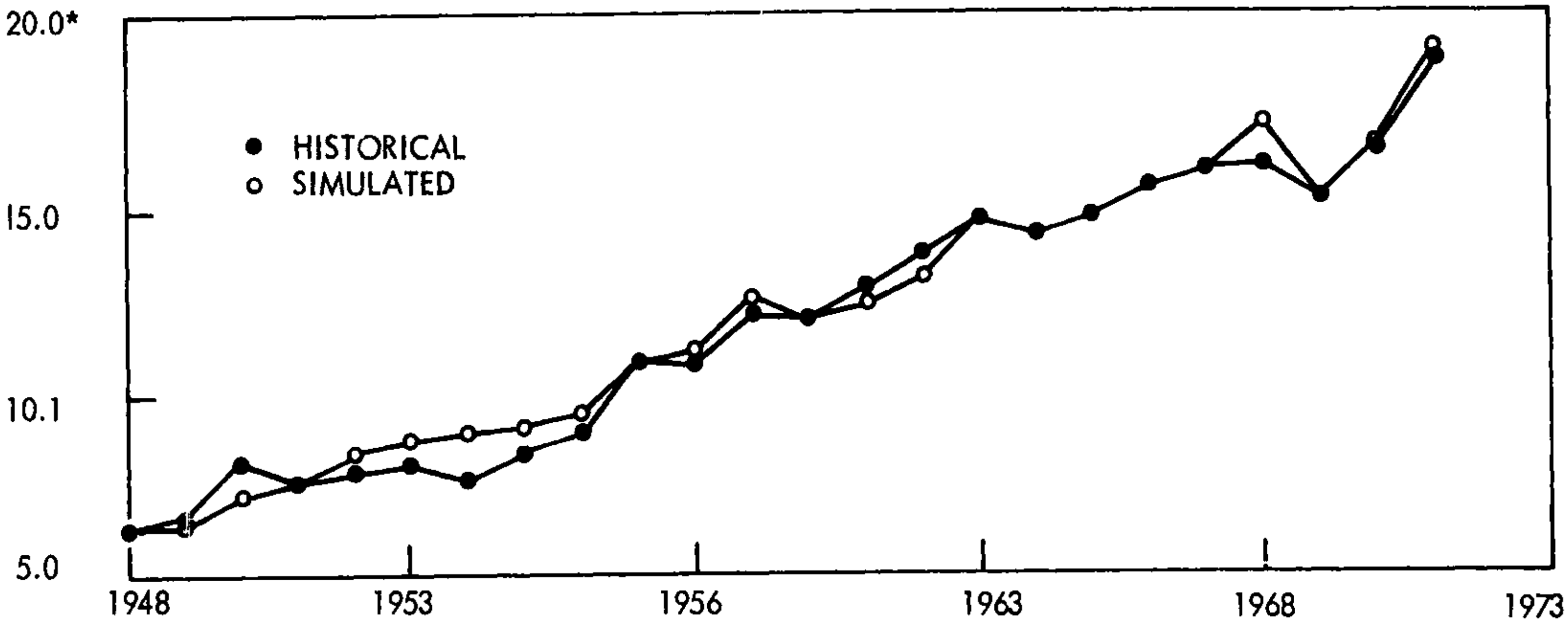
FIG. 4-20: PNI – PRICE INDEX, INVESTMENT GOODS SUPPLIED BY U.S. NON-ENERGY, SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

					ACTUAL	%
HISTORICAL MEAN	405.590	ERROR STATISTICS	-	MEAN	0.2142	2.3008
				RMS	18.5498	5.0617

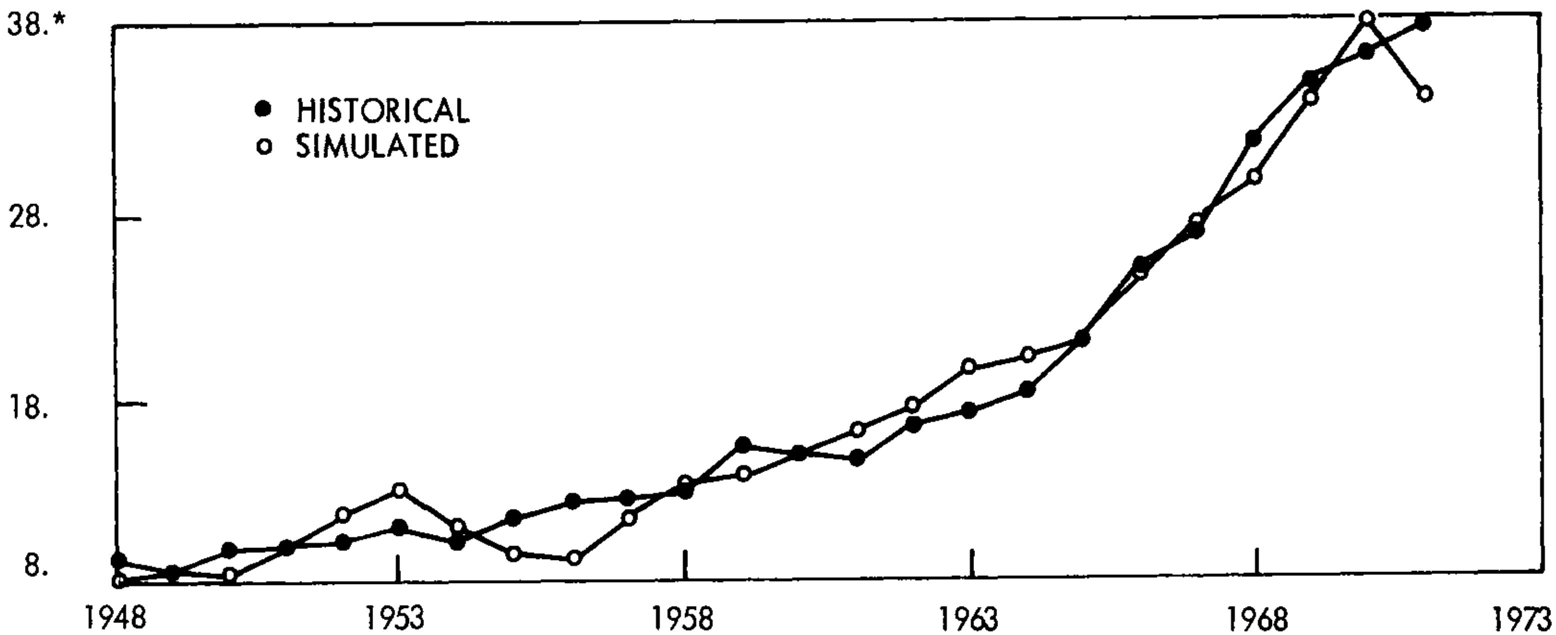
FIG. 4-21: NMN - NON-ENERGY INTERMEDIATE PRODUCTS PURCHASED BY U.S. NON-ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

					ACTUAL	%
HISTORICAL MEAN	11.7305	ERROR STATISTICS	-	MEAN	0.1424	1.4897
				RMS	0.5074	5.4669

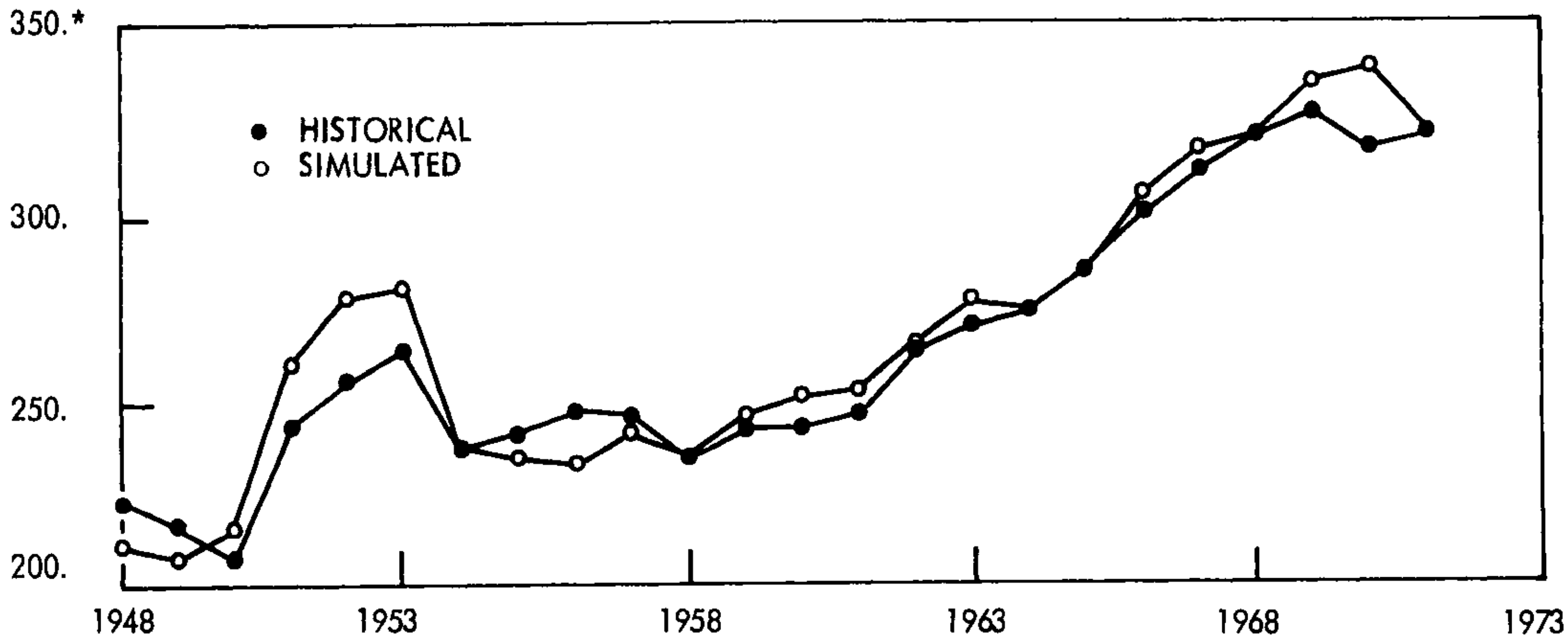
FIG. 4-22: NME - NON-ENERGY INTERMEDIATE PRODUCTS PURCHASED BY U.S. ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

					ACTUAL	%
HISTORICAL MEAN	17.3216	ERROR STATISTICS	-	MEAN	-0.2256	-1.3572
				RMS	1.6361	11.1383

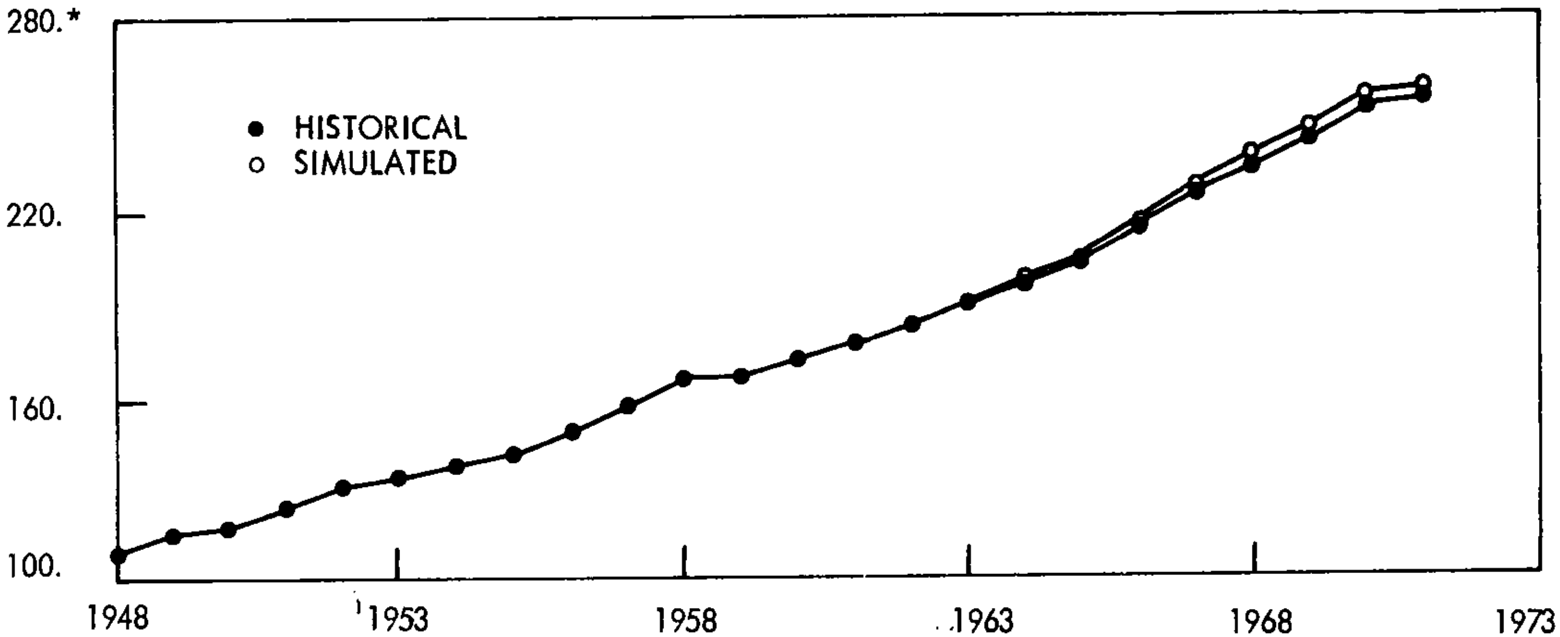
FIG. 4-23: RN - COMPETITIVE IMPORTS OF NON-ENERGY PRODUCTS, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

				ACTUAL	%
HISTORICAL MEAN	263.537	ERROR STATISTICS -	MEAN	3.773	1.3299
			RMS	9.914	3.8430

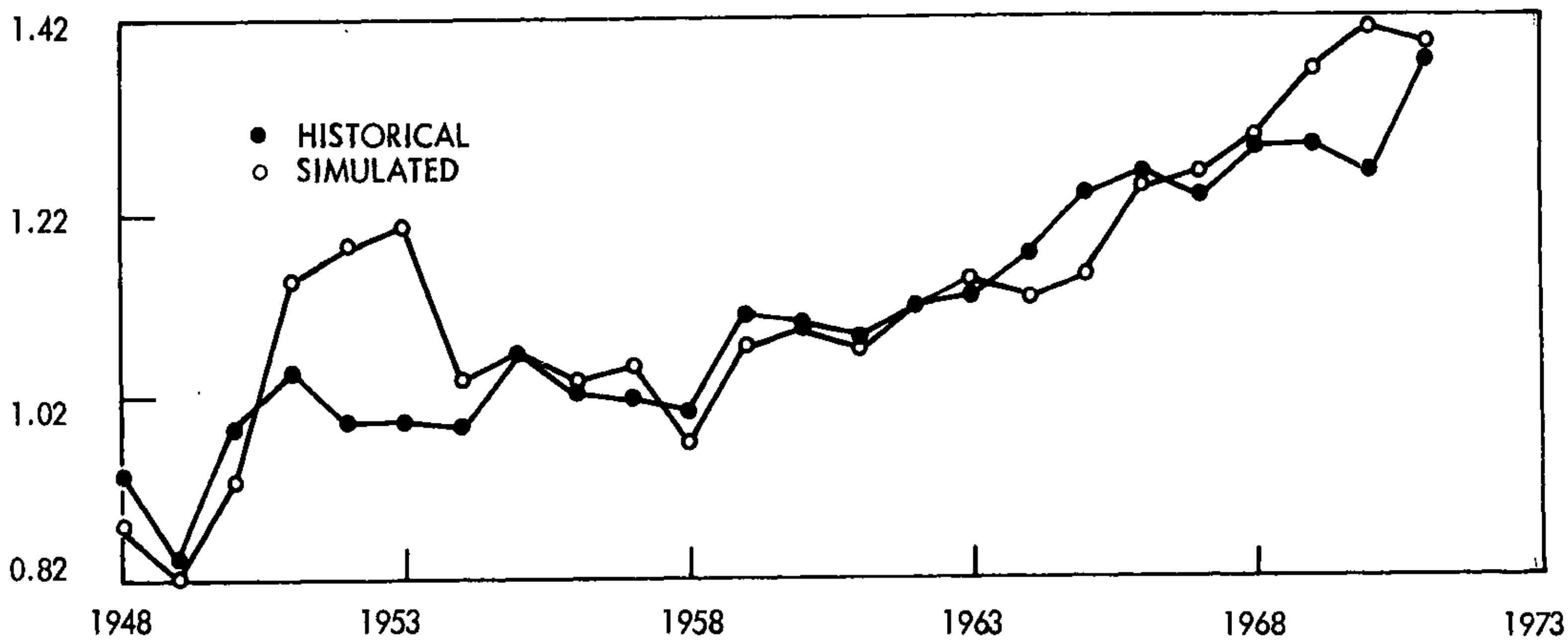
FIG. 4-24: LN - LABOR SERVICES PURCHASED BY U.S. NON-ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

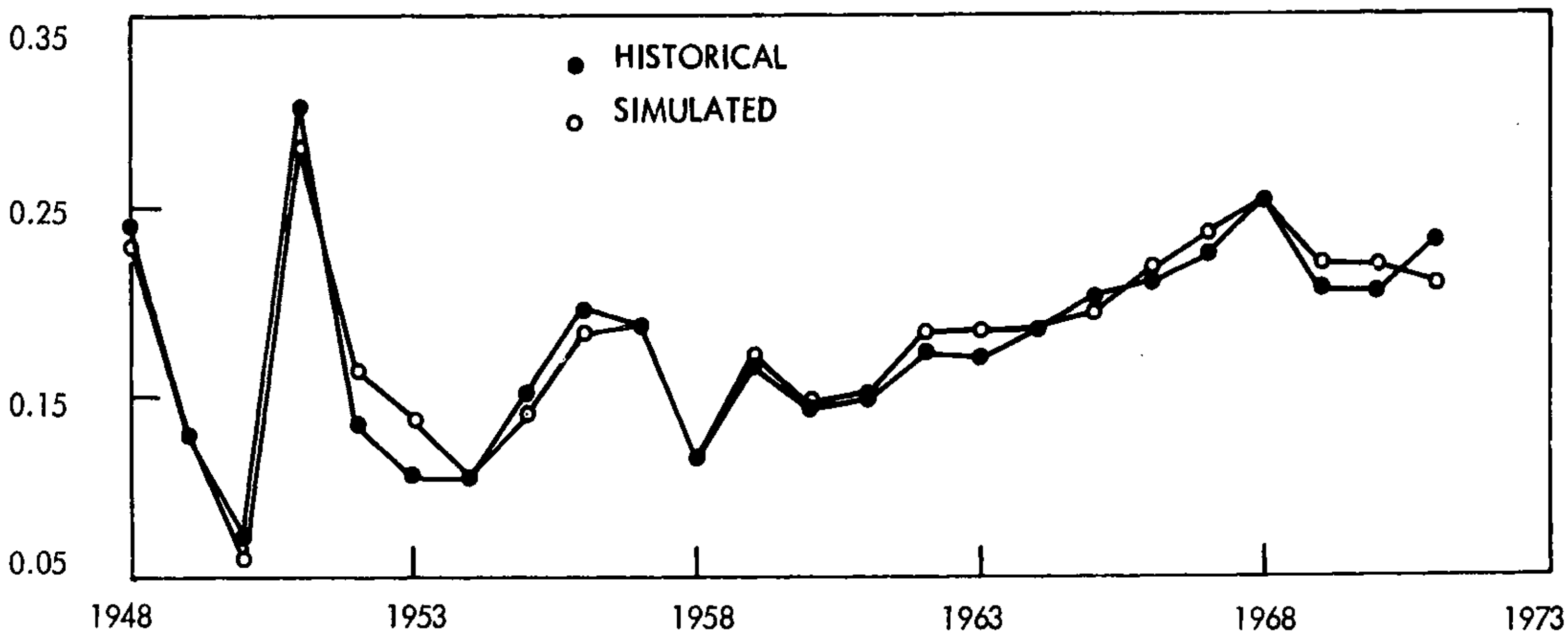
					ACTUAL	%
HISTORICAL MEAN	173.398	ERROR STATISTICS	-	MEAN	0.9778	0.4114
				RMS	1.9896	0.8860

FIG. 4-25: KN - CAPITAL SERVICES APPLIED TO U.S. NON-ENERGY SECTOR; HISTORICAL VS SIMULATED, 1948-1971



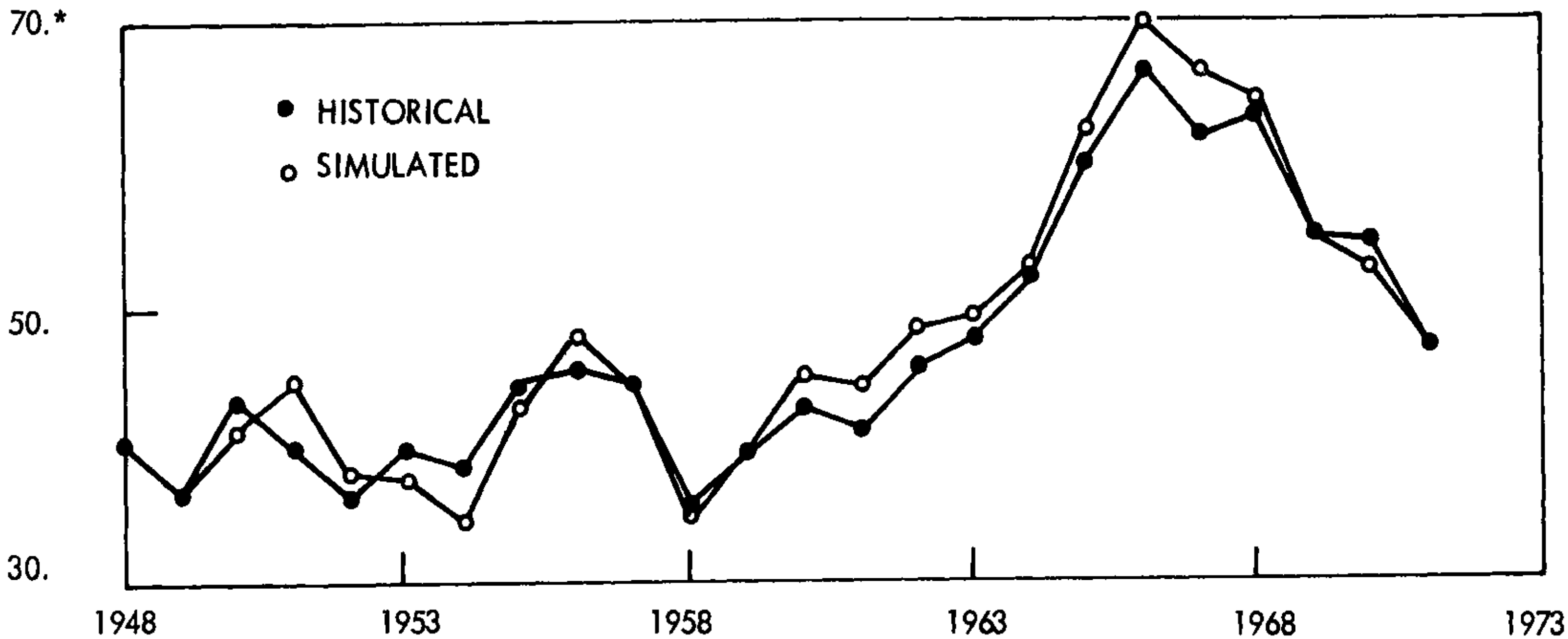
				ACTUAL	%
HISTORICAL MEAN	1.1033	ERROR STATISTICS –	MEAN	0.0218	2.0049
			RMS	0.0780	7.4127

FIG. 4-26: PKN - PRICE OF CAPITAL SERVICES, U.S. NON-ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



				ACTUAL	%
HISTORICAL MEAN	0.1753	ERROR STATISTICS -	MEAN	0.0020	1.7374
			RMS	0.0133	9.5464

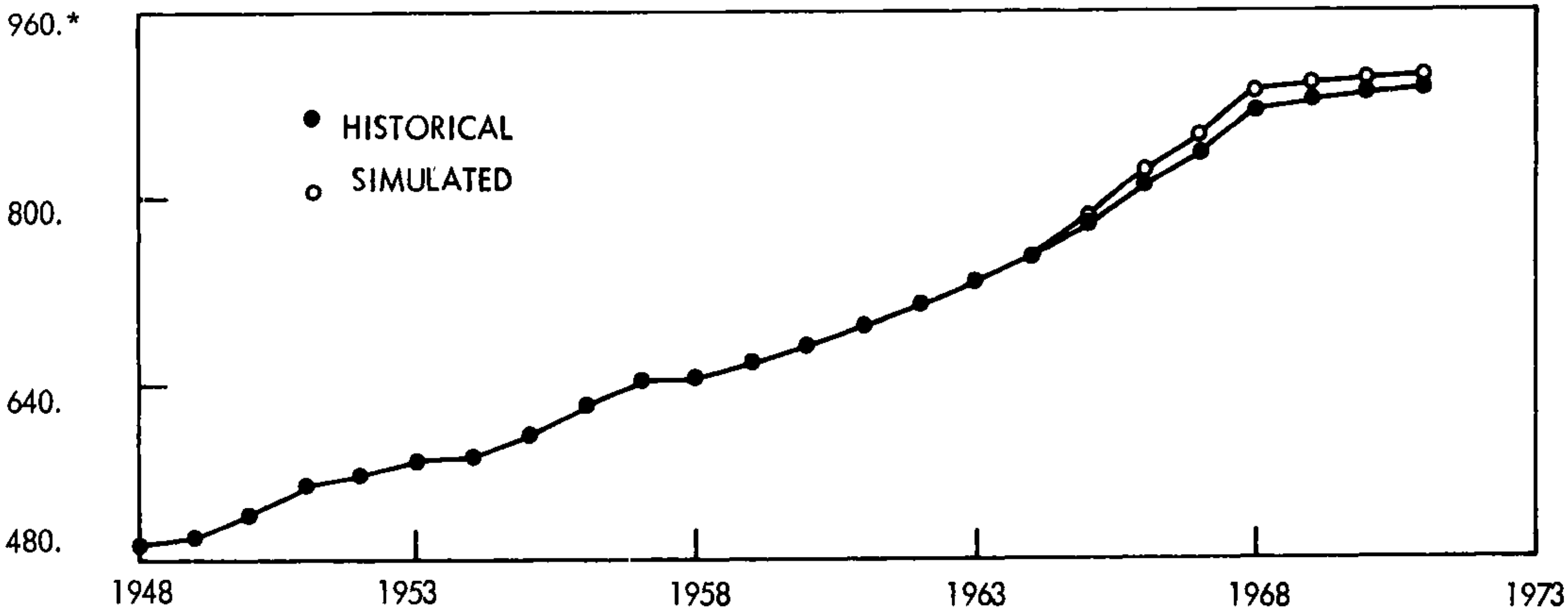
FIG. 4-27: MN - NOMINAL RATE OF RETURN, U.S. NON-ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

				ACTUAL	%
HISTORICAL MEAN	46.5922	ERROR STATISTICS -	MEAN	0.6486	1.1493
			RMS	2.2769	5.0240

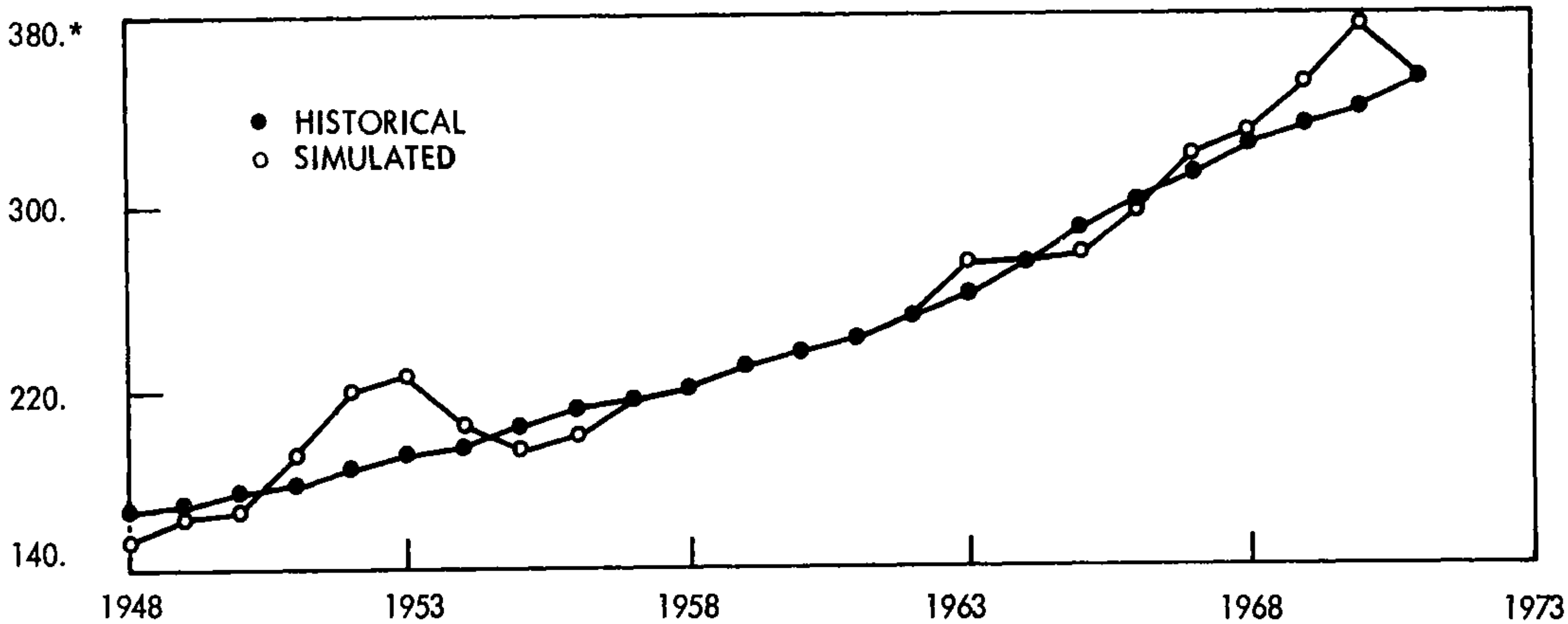
FIG. 4-28: INN - GROSS INVESTMENT, U.S. NON-ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

					ACTUAL	%
HISTORICAL MEAN	681.739	ERROR STATISTICS	-	MEAN	3.9845	0.4660
				RMS	7.5707	0.9255

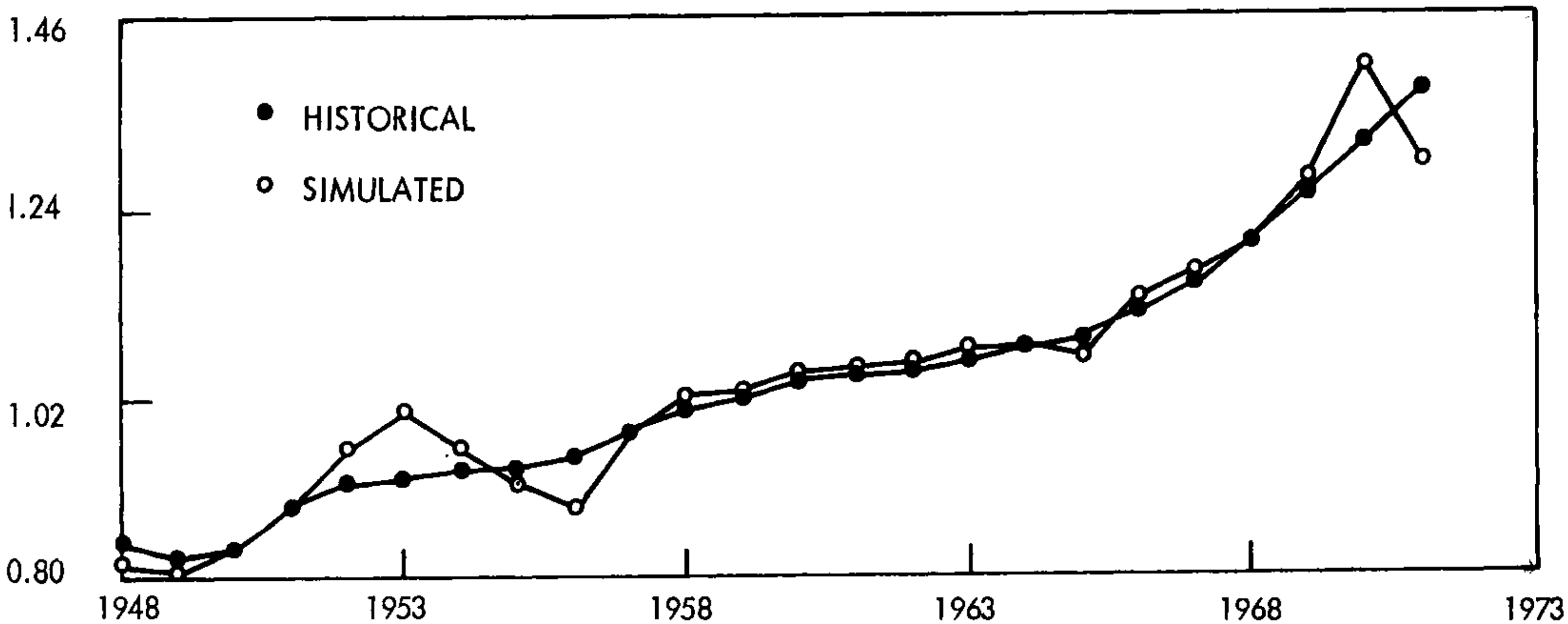
FIG. 4-29: KSN - CAPITAL STOCK, U.S. NON-ENERGY SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

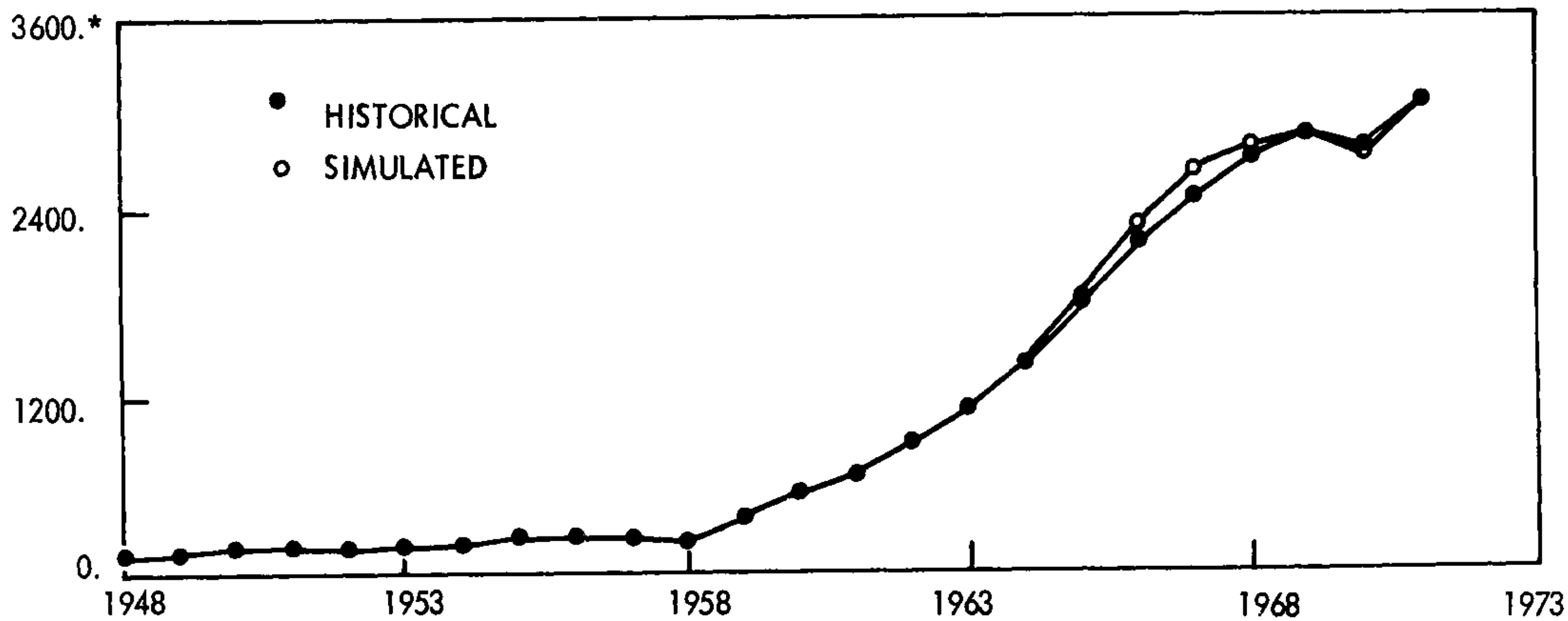
				ACTUAL	%
HISTORICAL MEAN	243.092	ERROR STATISTICS -	MEAN	4.644	1.8282
			RMS	14.816	6.8346

FIG. 4-30: CN - NON-ENERGY CONSUMPTION GOODS PURCHASED BY U.S. HOUSEHOLDS, HISTORICAL VS SIMULATED, 1948-1971



				ACTUAL	%
HISTORICAL MEAN	1.0313	ERROR STATISTICS -	MEAN	0.0045	0.4287
			RMS	0.0361	3.3429

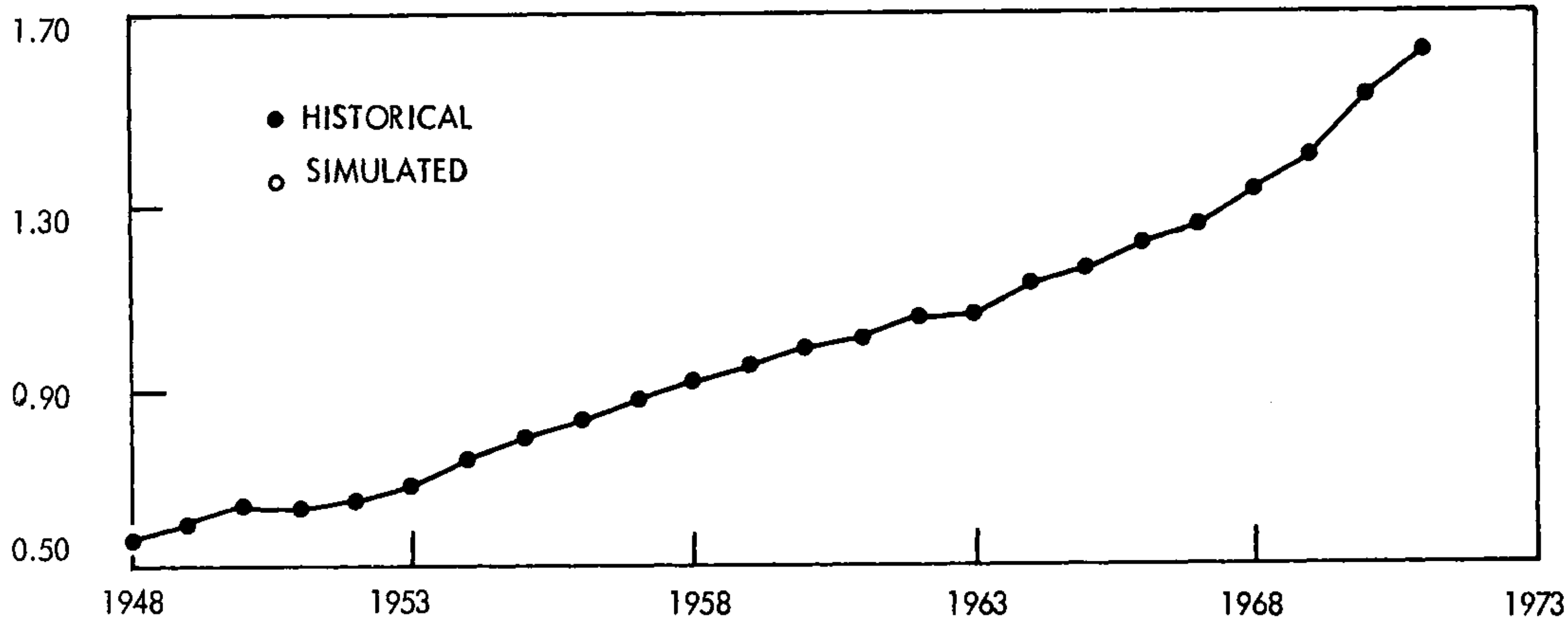
FIG. 4-31: PCN - PRICE INDEX, NON-ENERGY CONSUMPTION GOODS PURCHASED BY U.S. HOUSEHOLDS, HISTORICAL VS SIMULATED, 1948β1971



*in billions of 1958 dollars

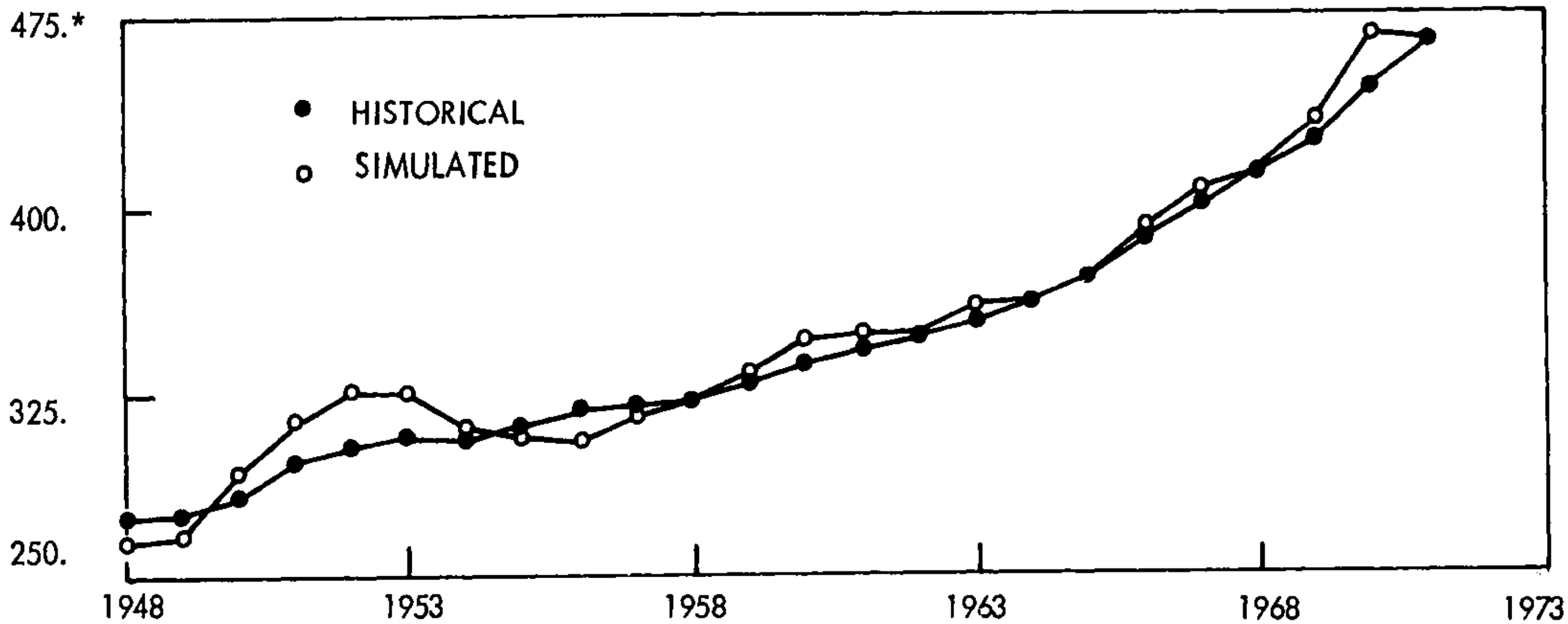
				ACTUAL	%
HISTORICAL MEAN	1031.12	ERROR STATISTICS	- MEAN	18.5365	1.1366
			RMS	53.2711	5.0309

FIG. 4-32: IMIN - DIRECT IMPORTS OF INVESTMENT GOODS, HISTORICAL VS SIMULATED, 1948-1971



					ACTUAL	%
HISTORICAL MEAN	0.9755	ERROR STATISTICS	-	MEAN	0.00006	0.0038
				RMS	0.0004	0.0438

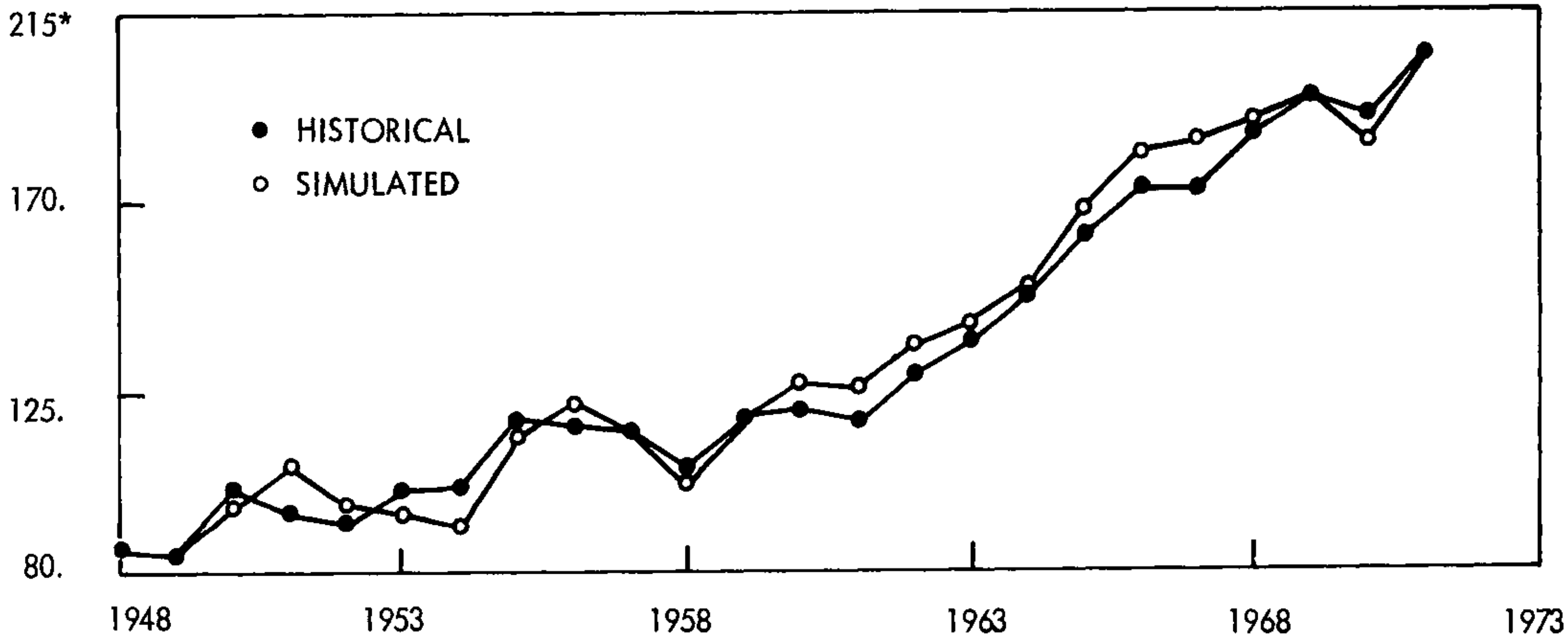
FIG. 4-33: PL - PRICE INDEX, SUPPLY OF LABOR SERVICES, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

				ACTUAL	%
HISTORICAL MEAN	344.752	ERROR STATISTICS -	MEAN	3.874	1.0656
			RMS	10.037	3.0956

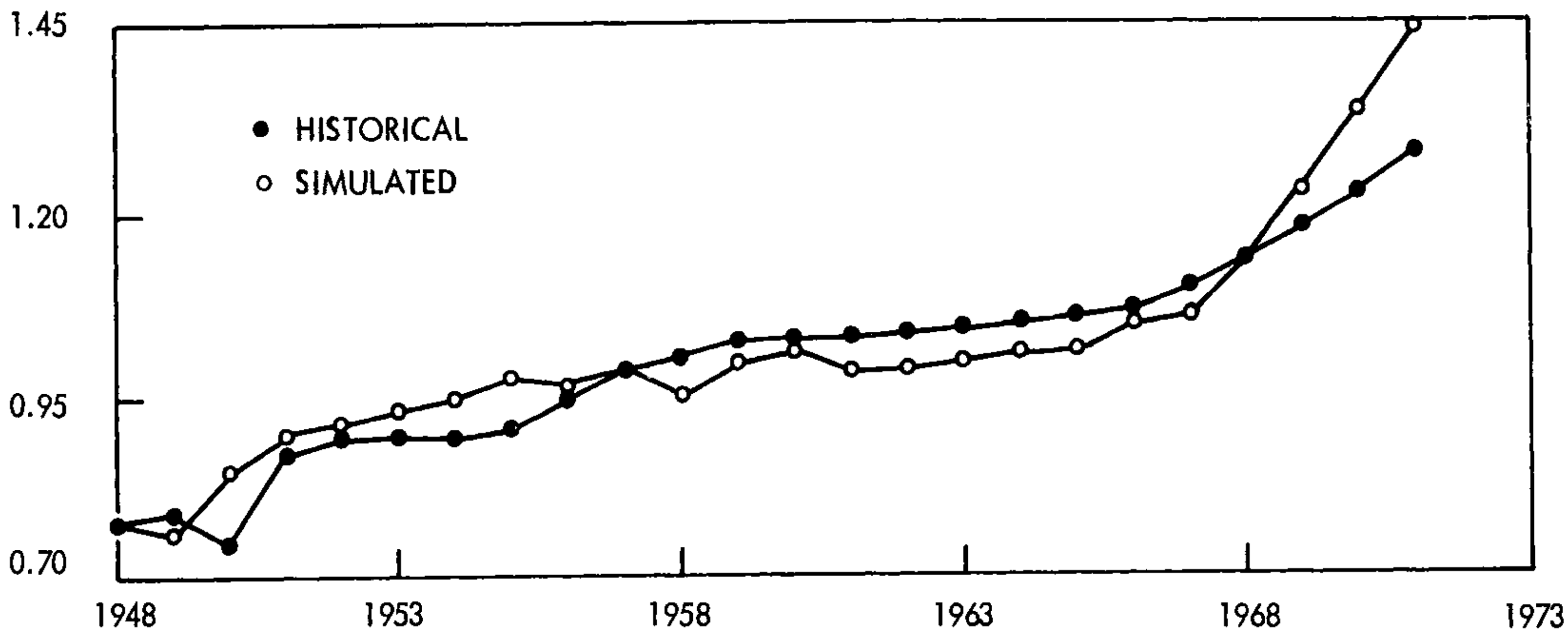
FIG. 4-34: L - SUPPLY OF LABOR BY U.S. HOUSEHOLDS, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

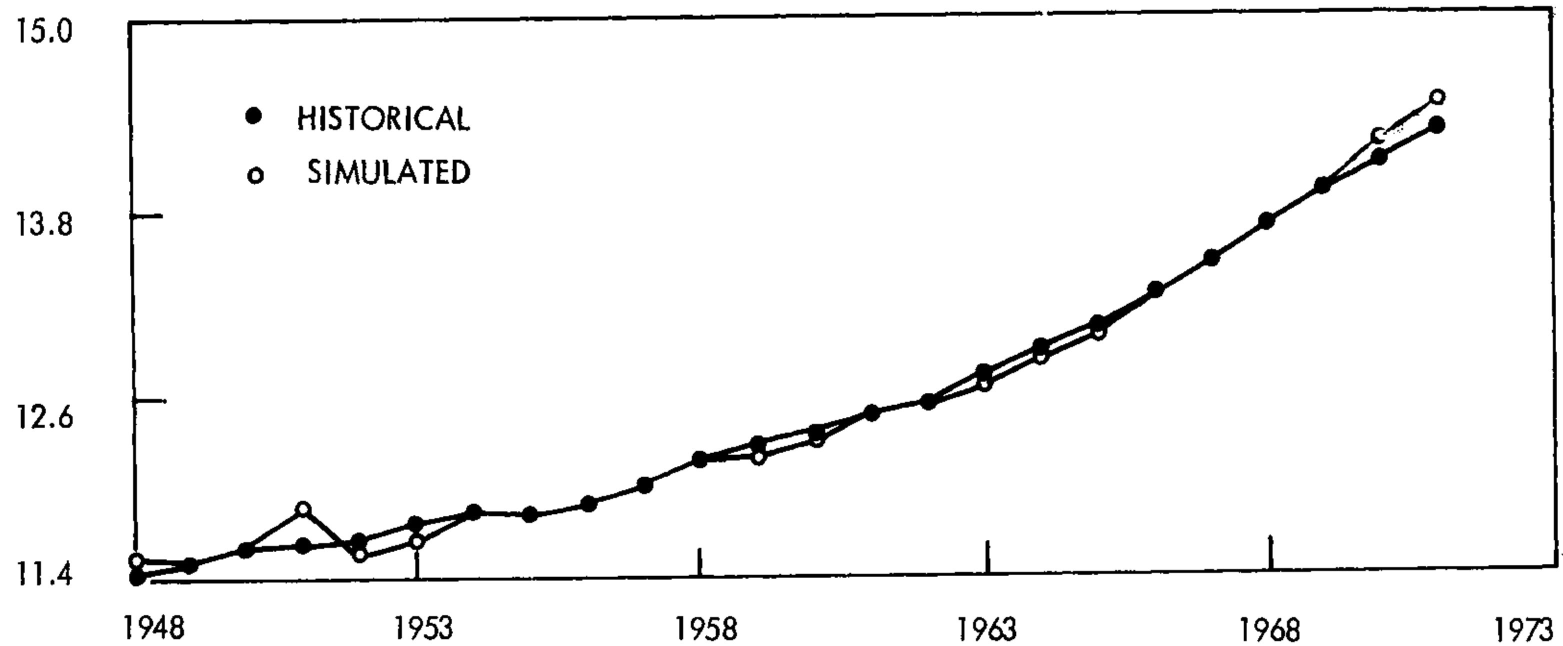
				ACTUAL	%
HISTORICAL MEAN	131.336	ERROR STATISTICS -	MEAN	1.7015	1.1493
			RMS	6.0733	5.0240

FIG. 4-35: I - GROSS PRIVATE DOMESTIC INVESTMENT, HISTORICAL VS SIMULATED, 1948-1971



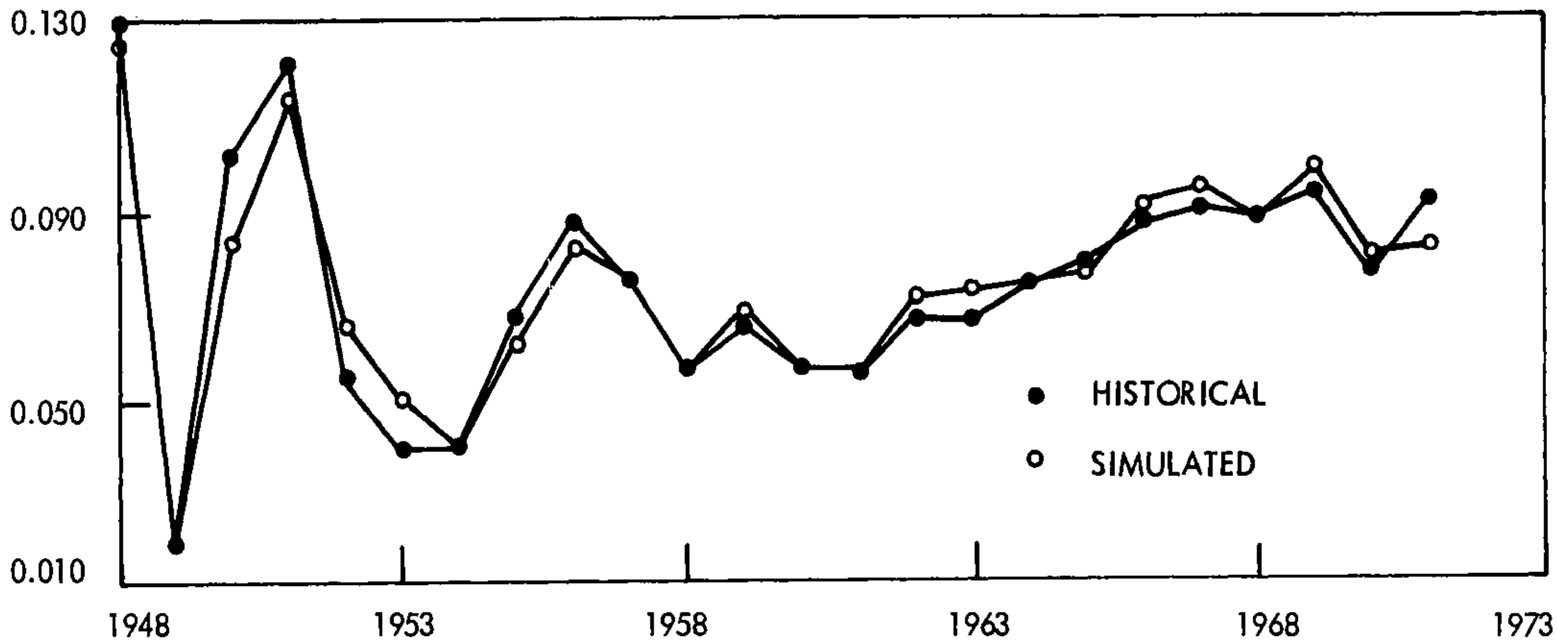
				ACTUAL	%
HISTORICAL MEAN	0.9955	ERROR STATISTICS –	MEAN	0.0086	0.8900
			RMS	0.0573	5.5306

FIG. 4-36: PIN - PRICE INDEX, GROSS PRIVATE DOMESTIC INVESTMENT, HISTORICAL VS SIMULATED, 1948-1971



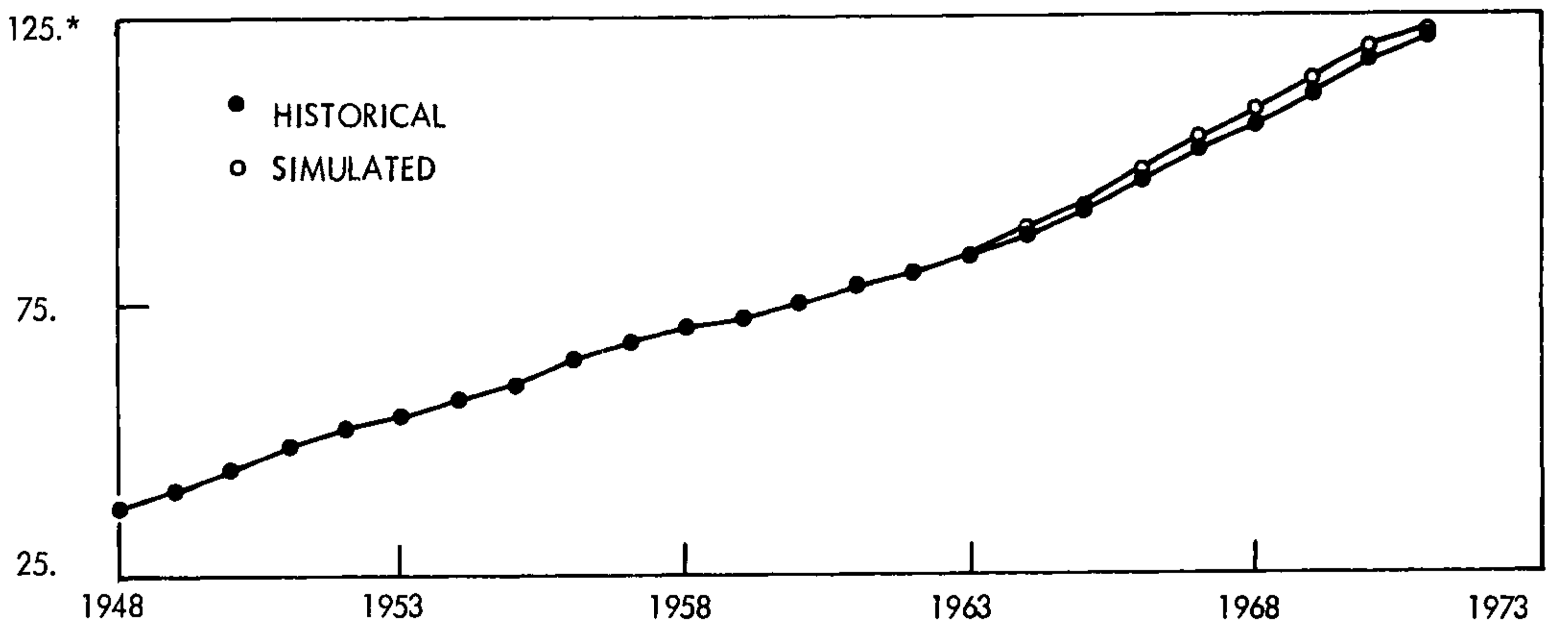
				ACTUAL	%
HISTORICAL MEAN	12.4958	ERROR STATISTICS -	MEAN	0.0043	0.3013
			RMS	0.0856	0.6883

FIG. 4-37: FULL CONSUMPTION, U.S. HOUSEHOLDS, HISTORICAL VS SIMULATED 1948-1971



					ACTUAL	%
HISTORICAL MEAN	0.0773	ERROR STATISTICS	-	MEAN	0.00002	1.6166
				RMS	0.0065	9.3070

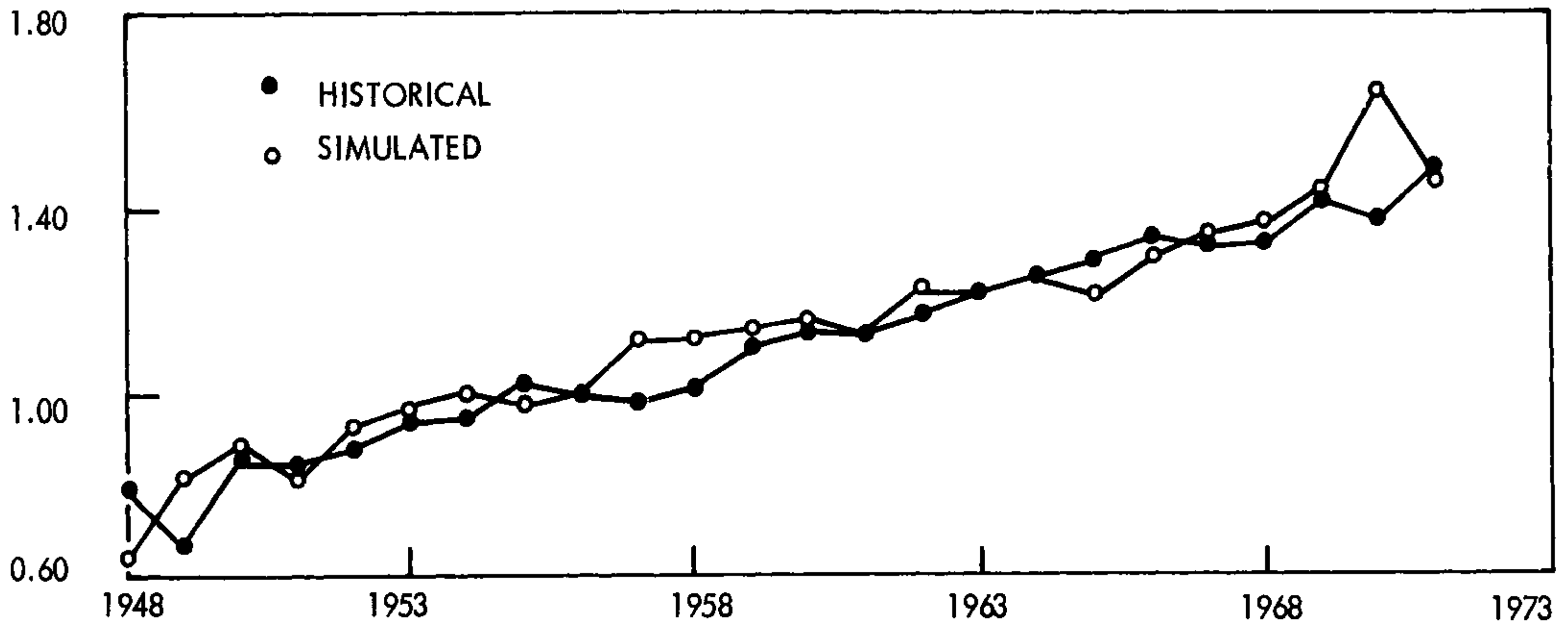
FIG. 4-38: MW - NOMINAL RATE OF RETURN ON WEALTH, HISTORICAL VS SIMULATED 1948-1971



*in billions of 1958 dollars

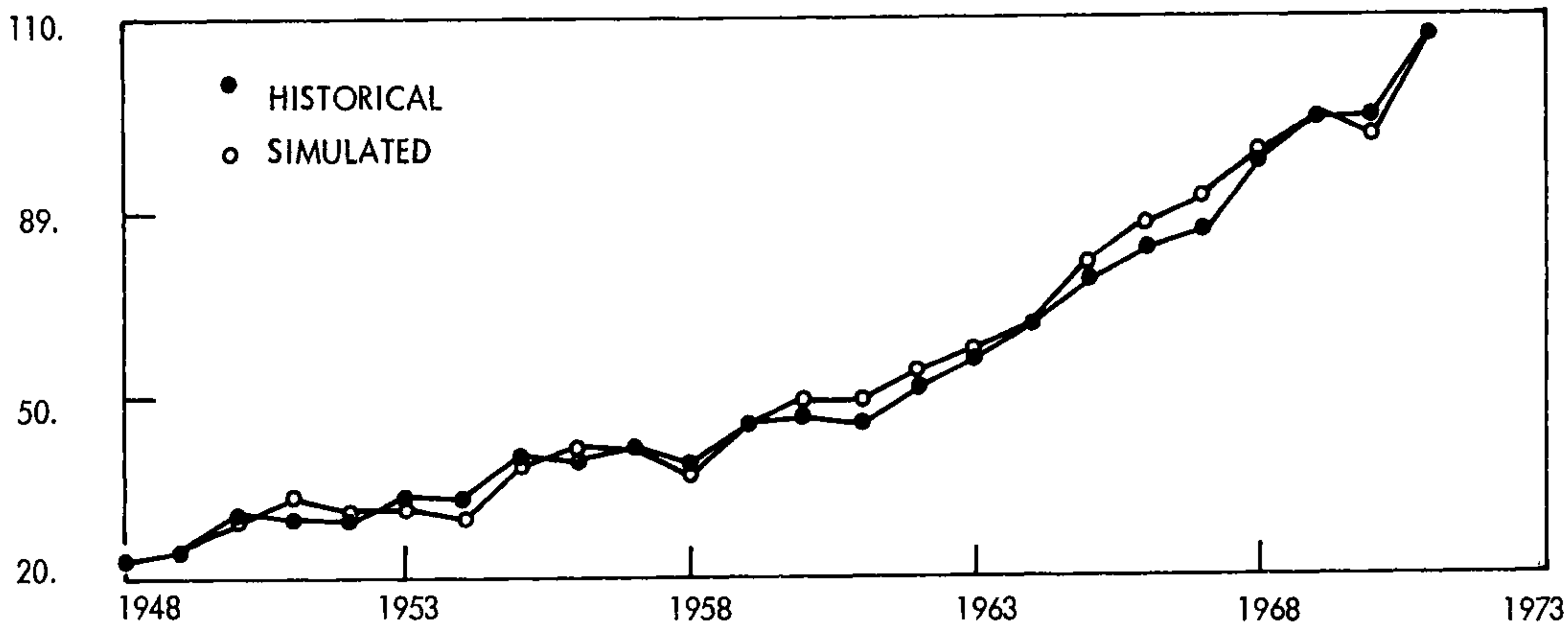
				ACTUAL	%
HISTORICAL MEAN	75.306	ERROR STATISTICS -	MEAN	0.6553	0.6103
			RMS	1.3288	1.3695

FIG. 4-39: CK - CAPITAL SERVICES SUPPLIED TO U.S. HOUSEHOLDS, HISTORICAL VS SIMULATED, 1948-1971



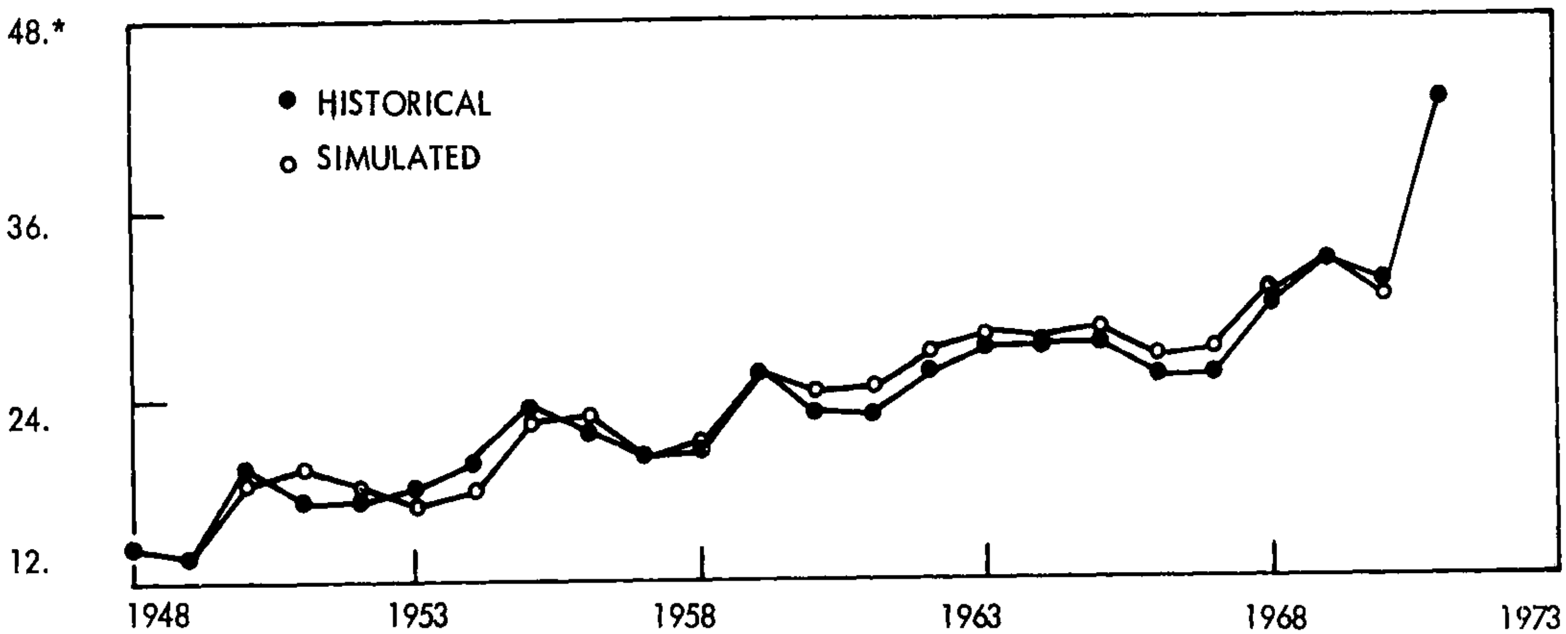
				ACTUAL	%
HISTORICAL MEAN	1.0937	ERROR STATISTICS -	MEAN	0.0262	2.5335
			RMS	0.0859	8.6416

FIG. 4-40: PCK - PRICE OF CAPITAL SERVICES, U.S. HOUSEHOLD SECTOR, HISTORICAL VS SIMULATED, 1948-1971



				ACTUAL	%
HISTORICAL MEAN	52.563	ERROR STATISTICS –	MEAN	0.7075	1.1493
			RMS	2.3480	5.0240

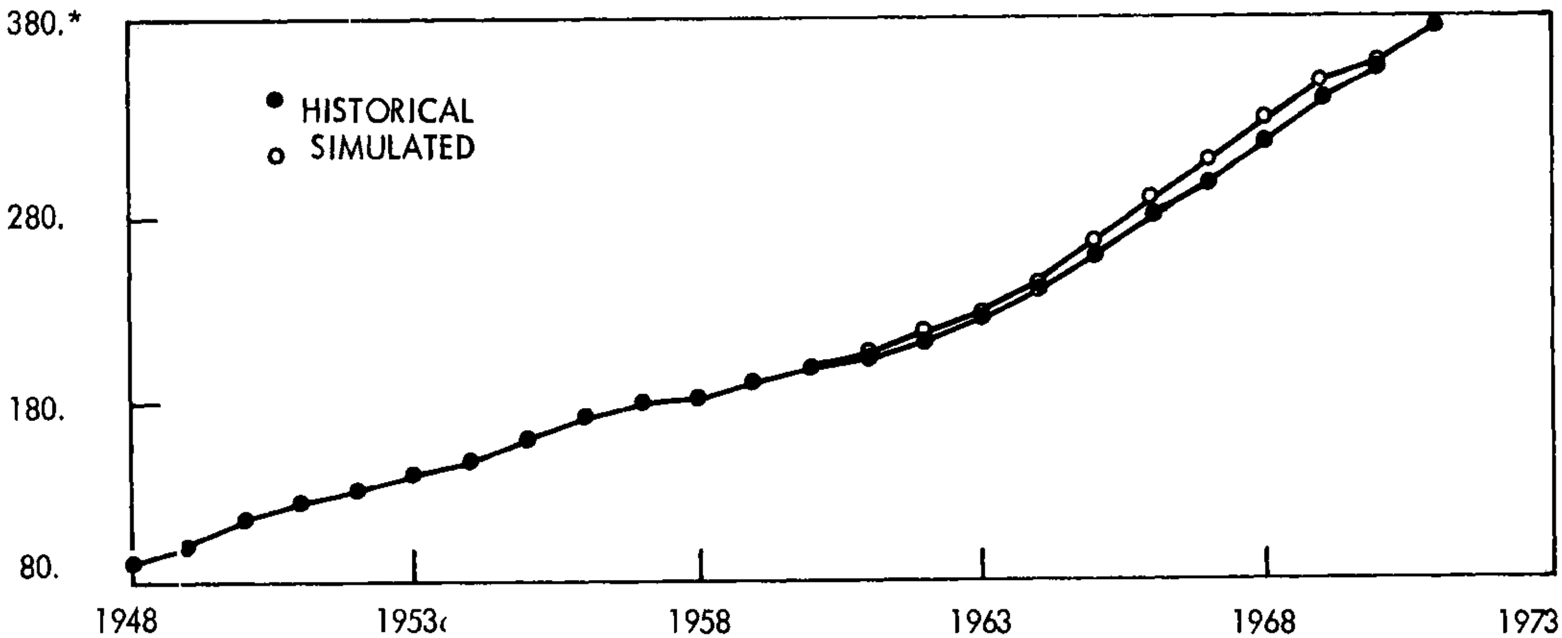
FIG. 4-41: ICD - GROSS INVESTMENT IN CONSUMER DURABLES BY U.S. HOUSEHOLDS, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

					ACTUAL	%
HISTORICAL MEAN	23.7453	ERROR STATISTICS	-	MEAN	0.2782	1.1492
				RMS	1.0771	5.0240

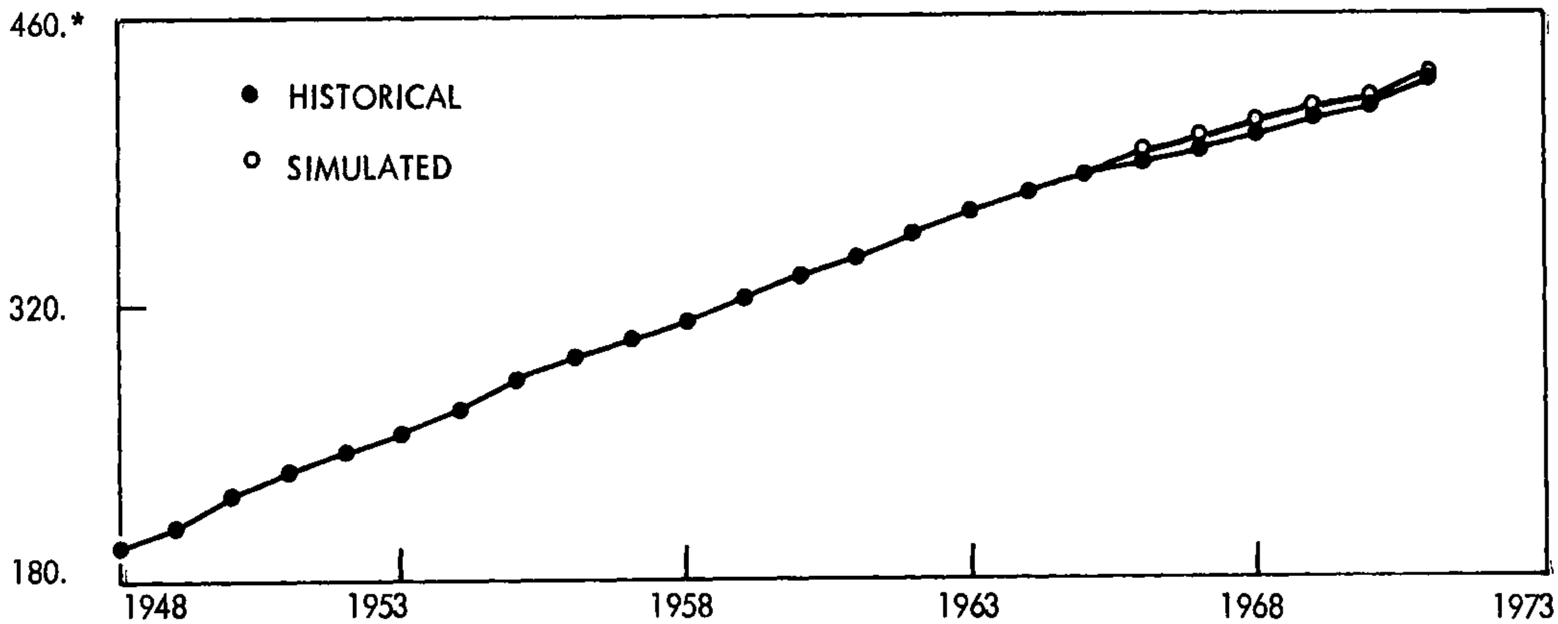
FIG. 4-42: IRS - GROSS INVESTMENT IN RESIDENTIAL STRUCTURES BY U.S. HOUSEHOLDS, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

					ACTUAL	%
HISTORICAL MEAN	207.815	ERROR STATISTICS	-	MEAN	2.8636	0.9853
				RMS	5.5212	2.1736

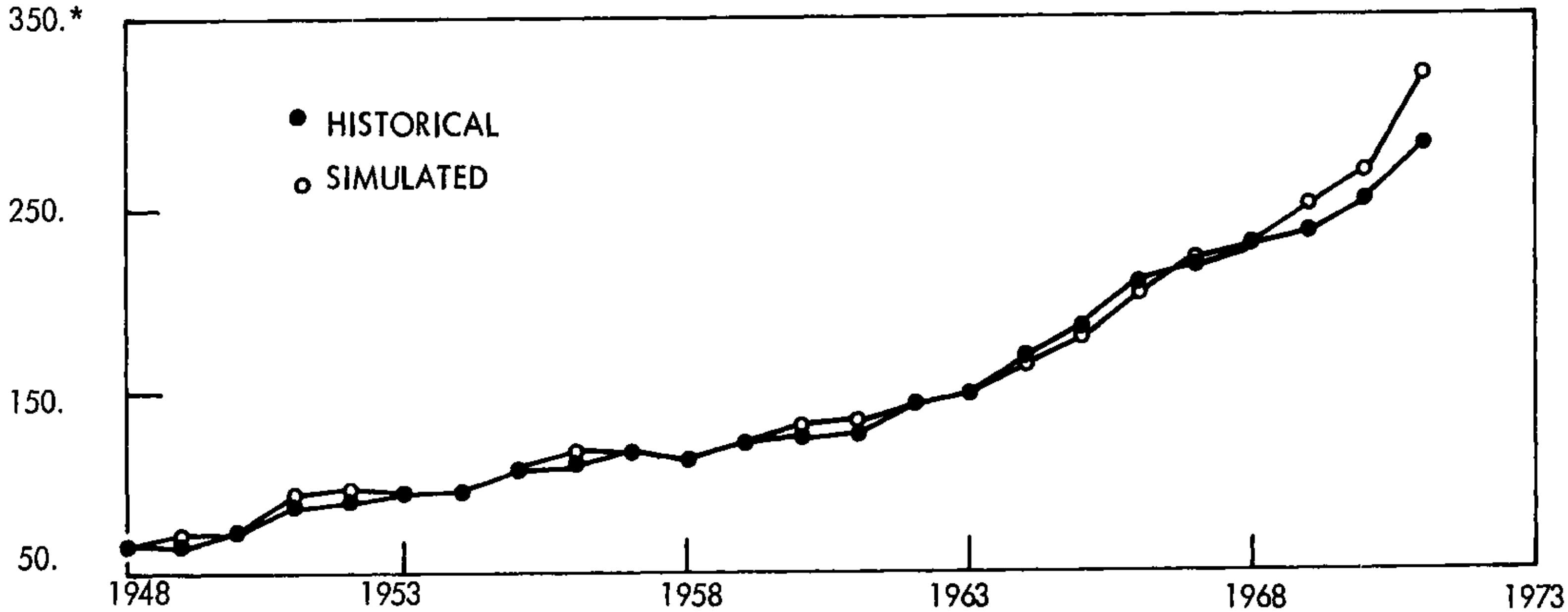
FIG. 4-43: KCD - CAPITAL STOCK OF CONSUMER DURABLES, U.S. HOUSEHOLD SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of 1958 dollars

				ACTUAL	%
HISTORICAL MEAN	320.711	ERROR STATISTICS -	MEAN	1.6937	0.4127
			RMS	3.3950	0.8936

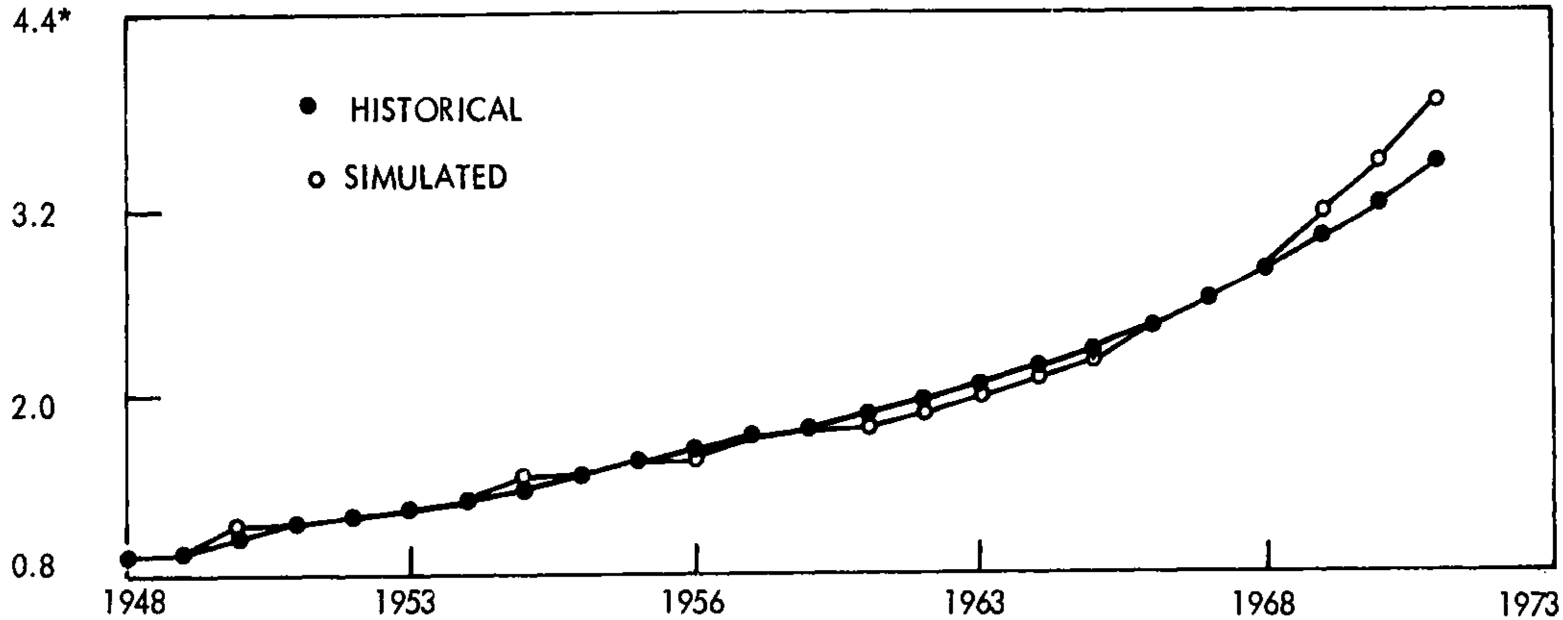
FIG. 4-44: KRS - CAPITAL STOCK OF RESIDENTIAL STRUCTURES, U.S. HOUSEHOLD SECTOR, HISTORICAL VS SIMULATED, 1948-1971



*in billions of current dollars

					ACTUAL	%
HISTORICAL MEAN	142.943	ERROR STATISTICS	-	MEAN	3.6169	2.0994
				RMS	9.5346	4.8479

FIG. 4-45: S - GROSS PRIVATE DOMESTIC SAVING, HISTORICAL VS SIMULATED, 1948-1971



*in trillions of 1958 dollars

				ACTUAL	%
HISTORICAL MEAN	1.8466	ERROR STATISTICS -	MEAN	0.0294	1.1633
			RMS	0.1158	4.6000

FIG. 4-46: W - PRIVATE DOMESTIC WEALTH, HISTORICAL VS SIMULATED, 1948-1971

capacity utilization which are not adequately reflected in the configuration of the long-run equilibrium. It is reasonable to expect that explicit incorporation of a disequilibrium representation of short-term adjustment processes would result in a more accurate explanation of the behavior of price and output levels during these two years.

The sectoral rates of return and the nominal rate of return on wealth show somewhat high R.M.S. errors but, on the whole, track the historical paths satisfactorily through the various peaks and troughs. The quantity of competitive energy imports is significantly over-predicted especially in the period 1960-1971. This can be attributed to the implementation of the oil import quota that maintained historical energy import levels below the values that would have prevailed in the absence of such protectionary measures.

An examination of the simulation results shows that the model has succeeded in closely replicating the actual time-paths of the endogenous variable over an extended historical period spanning a fairly broad range of conditions of the overall economy. We conclude that our behavioral assumptions, our empirical specification and the final form of the model constitute a satisfactory basis for analyzing long-term interactions between energy and economic growth.

4.4 Tax Policy in a Model of General Equilibrium

Our aim in this and the following section is to investigate the dynamic response of the model to changes in selected policy variables. The simulation experiments described were designed with the intent of exploring the capabilities of the model as a tool for policy analysis by highlighting the degree and nature of interrelations among variables.

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The results we discuss should be evaluated in this light rather than as an attempt to prescribe any specific set of policy actions.

First, we performed five different simulation runs, each reflecting the model's response to a given change in the rate of a selected tax variable. Each simulation is described in detail below and the results are summarized in Tables 4-1 through 4-6. An examination of these results provides valuable insight into the structure of the model. In order to further illustrate the general equilibrium properties of the model, we selected one particular tax measure -- a reduction in the capital income tax rate -- and contrasted the results of the simulation with the implications that would follow from a partial equilibrium analysis of the same tax measure.

The five simulation runs were:

1. - Surcharge of 8% on the tax rate on energy consumption goods
2. - Surcharge of 2% in the tax rate on labor income
3. - Surcharge of 8% in the tax rate on investment goods
4. - Reduction of 1% in the tax rate on capital property, energy sector
5. - Reduction of 1% in the tax rate on capital property, non-energy sector.

Each of these surcharges and reductions were applied to the historical tax rates uniformly throughout the period 1948-1971. We summarize the results below:

Simulation 1 (8% surcharge on the tax rate on energy consumption goods):

The tax variable TCE enters the model in equation 22 from which it is apparent that the tax on energy consumption goods serves as a wedge between the producers' price p_{EC} and the consumers' price p_{CE}. From

Table 4-1: SIMULATION 1: SURCHARGE OF 8% ON THE TAX RATE ON ENERGY CONSUMPTION GOODS

		1948	1955	1960	1965	1971
EC	Hist. Tax	8907.	16965.	22394.	28798.	36624.
	New Tax	8868.	16846.	22151.	28430.	36124.
	Net change	-38.4	-118.	-243.8	-367.1	-499.5
	% change	-0.43	-0.70	-1.08	-1.27	-1.36
EM	Hist. Tax	20452.	28057.	35894.	45010.	61975.
	New Tax	20307.	27865.	35538.	44576.	61408.
	Net change	-144.8	-191.4	-356.3	-434.1	-566.2
	% change	-0.70	-0.68	-0.99	-0.96	-0.91
F/P	Hist. Tax	11.52	11.85	12.253	12.938	14.438
	New Tax	11.53	11.85	12.252	12.934	14.429
	Net change	0.01	0.0	-0.001	-0.004	-0.008
	% change	0.061	0.0	-0.005	-0.033	-0.058
HW	Hist. Tax	6.411	8.499	9.402	11.024	18.373
	New Tax	6.394	8.460	9.340	10.947	18.261
	Net change	-0.017	-0.039	-0.061	-0.077	-0.112
	% change	-0.267	-0.466	-0.652	-0.697	-0.610
CGNP*	Hist. Tax	0.6230	0.5719	0.6150	0.6209	0.6586
	New Tax	0.6173	0.5694	0.6109	0.6184	0.6575
	Net change	-0.0067	-0.0025	-0.0041	-0.0025	-0.0011
	% change	-1.10	-0.48	-0.81	-0.45	-0.22
IGNP*	Hist. Tax	0.3769	0.4280	0.3850	0.3790	0.3413
	New Tax	0.3826	0.4305	0.3890	0.3815	0.3424
	Net change	0.0057	0.0025	0.0040	0.0025	0.0011
	% change	1.23	0.54	1.05	0.57	0.33
ECT*	Hist. Tax	0.3285	0.3636	0.3602	0.3603	0.3373
	New Tax	0.3286	0.3629	0.3591	0.3585	0.3353
	Net change	0.0001	-0.0007	-0.0011	-0.0018	-0.0020
	% change	0.00	-0.20	-0.33	-0.51	-0.56
EGNP*	Hist. Tax	0.0365	0.0416	0.0421	0.0403	0.0349
	New Tax	0.0367	0.0417	0.0421	0.0400	0.0346
	Net change	0.0002	0.0001	0.000	-0.0003	-0.0020
	% change	0.00	0.43	0.00	-1.43	-1.48
BTU*	Hist. Tax	21.042	31.410	40.724	51.473	69.245
	New Tax	20.741	30.931	39.801	50.266	67.641
	Net change	-0.301	-0.479	-0.923	-1.207	-1.604
	% change	-1.51	-1.50	-2.20	-2.23	-2.31

*see addendum to Tables 4-1/4-5 for a description of these quantities

Table 4-1 we note that, as expected, the tax surcharge shifts the composition of output towards investment goods and away from consumption goods although the shift is greatest at the beginning of the period and becomes progressively smaller as the economy adjusts to the reduction in energy demand. The output of the energy sector shows average percentage reductions of -0.91% and -0.89% in consumption and intermediate goods, respectively, with the reductions becoming progressively greater towards the end of the period 1948-1971. However, the price of energy intermediate goods is almost unchanged and the price of energy consumption goods is reduced an average of -0.15% throughout the period; consequently, the share of consumption goods in total energy produced decreases by up to -0.56% in 1971. Total energy consumed measured in B.T.U.'s decreases by approximately 2 percent throughout the period. Full consumption by households decreases an average of -0.006%with a growing decline towards the end of the period.

Dynamic elasticities with respect to TCE of:

	1948	1955	1965	1971
EC	-0.116	-0.192	-0.350	-0.376
EM	-0.197	-0.189	-0.268	-0.255
KE	0.0	-0.070	-0.146	-0.195
KN	0.0	-0.043	-0.120	-0.169
P_{KE}	-0.259	-0.188	-01246	-0.183
P_{KN}	-0.287	-0.233	-0.209	-0.109
P_{CE}	0.261	0.306	0.474	0.450
P_{EC}	-0.079	-0.032	-0.012	0.008
CE	-0.144	-0.231	-0.396	-0.416

Simulation 2 (2% surcharge on the tax rate on labor income):

The immediate effect of a surcharge on the labor income tax is to

Table 4-2: SIMULATION 2: SURCHARGE OF 2% ON THE TAX RATE ON LABOR INCOME

		1948	1955	1960	1965	1971
EC	Hist. Tax	8907.	16965.	22394.	28798.	36624.
	New Tax	8863.	16890.	22289.	28680.	36472.
	Net change	-44.2	-75.0	-105.7	-117.3	-152.0
	% change	-0.49	-0.44	-0.47	-0.40	-0.41
EM	Hist. Tax	20452.	28057.	35894.	45010.	61975.
	New Tax	20478.	28082.	35934.	45047.	62049.
	Net change	26.32	24.86	40.31	37.21	74.37
	% change	0.12	0.08	0.11	0.08	0.12
F/P	Hist. Tax	11.523	11.850	12.253	12.938	14.438
	New Tax	11.520	11.849	12.251	12.937	14.437
	Net change	-0.003	-0.001	-0.002	-0.001	-0.001
	% change	-0.021	-0.009	-0.015	-0.009	-0.004
HW	Hist. Tax	6.411	8.499	9.402	11.024	18.372
	New Tax	6.415	8.507	9.411	11.034	18.391
	Net change	0.004	0.008	0.009	0.010	0.018
	% change	0.057	0.096	0.096	0.090	0.102
CGNP*	Hist. Tax	0.6230	0.5719	0.6150	0.6209	0.6586
	New Tax	0.6203	0.5707	0.6130	0.6197	0.6581
	Net change	-0.0027	-0.0012	-0.0020	-0.0012	-0.0005
	% change	-0.43	-0.20	-0.32	-0.19	-0.075
IGNP*	Hist. Tax	0.3769	0.4280	0.3850	0.3790	0.3413
	New Tax	0.3796	0.4292	0.3869	0.3802	0.3418
	Net change	0.0027	0.0012	0.0019	0.0012	0.0005
	% change	0.71	0.28	0.49	0.31	0.14
ECT*	Hist. Tax	0.3285	0.3636	0.3602	0.3603	0.3373
	New Tax	0.3281	0.3628	0.3591	0.3589	0.3358
	Net change	-0.0004	-0.0008	-0.0011	-0.0014	-0.0015
	% change	-0.12	-0.24	-0.30	-0.38	-0.44
EGNP*	Hist. Tax	0.0365	0.0416	0.0421	0.0403	0.0349
	New Tax	0.0365	0.0416	0.0420	0.0400	0.0346
	Net change	0.00	0.00	-0.0001	-0.0003	-0.0003
	% change	0.0	0.0	-0.23	-0.74	-0.85
BTU*	Hist. Tax	21.042	31.410	40.724	51.473	69.245
	New Tax	20.875	31.136	40.201	50.388	68.349
	Net change	-0.167	-0.274	-0.523	-1.085	-1.896
	% change	-0.79	-0.87	-1.28	-2.10	-2.73

*see addendum to Tables 4-1/4-5 for a description of these quantities

decrease the supply price of labor services according to equation 30. This affects the value of present full consumption as determined by the inter-temporal allocation equation 45, and, in turn, the quantities demanded by households of energy goods CE, non-energy goods CN and capital services CK. As can be seen from Tables 4-2 and 4-6, the effect of the tax increase on the economy is mildly contractionary; of course, the mechanisms that cause the reductions in aggregate demand are different, as we have seen, to those that would be present in a stabilization model with a detailed representation of the short-run dynamics of the labor market. There is a shift in the composition of national output towards investment and away from consumption, with the shift showing a declining trend towards the end of the period. The level of energy consumed throughout the period 1948-1971 measured in B.T.U.'s declines an average of -1.3%.

Dynamic Elasticities with respect to TL of:

	1948	1955	1965	1971
EC	-0.064	-0.044	-0.041	-0.060
EM	0.248	0.221	0.204	0.208
KE	0.0	-0.045	-0.056	-0.075
KN	0.0	0.026	0.050	0.066
p_{KE}	0.065	0.152	0.165	0.210
p_{KN}	-0.152	-0.100	-0.035	-0.045
p_{CE}	-0.130	-0.094	-0.065	-0.055
p_{EC}	-0.103	-0.075	-0.060	-0.050
CE	0.308	0.265	0.230	0.230

Simulation 3 (8% surcharge on the tax rate of investment goods):

The tax rate on investment goods enters the model in equation 24 directly affecting the demand price of investment, p_I. Tracing the propagation of a higher tax TI through the model, we observe that the increase in P_I results in lower rates of return ME and MN and, therefore, in a lower MW. This affects the budget constraint of households and thus, the quantities demanded CE, CN and CK; in turn, of course, these changes affect the equilibrium values of NI and p_{NI}, among others, thus establishing a feedback into equation 24. In examining the results summarized in Tables 4-3 and 4-6, we note that although gross investment decreases, as expected, the value share of investment in GNP becomes greater as a result of proportionally greater reductions in the demands CE and CN. The output of the energy sector shows decreases in both consumption and intermediate goods averaging -1.126% and -1.131%, respectively, with the decline becoming gradually more pronounced throughout the period. Full consumption declines initially but later increases in response to changes in the relative prices of present vs. future consumption. Average reductions in B.T.U.'s consumed amount to -1.05% for the period 1948-1971.

Dynamic Elasticities with respect to TI of:

	1948	1955	1965	1971
EC	0.032	0.033	0.031	0.032
EM	-0.008	-0.003	-0.004	-0.005
KE	0.0	-0.005	-0.006	-0.008
KN	0.0	-0.003	-0.005	-0.007
p_{KE}	0.008	0.023	0.025	0.030
p_{KN}	-0.019	-0.011	-0.005	-0.005
p_{CE}	-0.016	-0.013	-0.009	-0.007
p_{EC}	-0.013	-0.010	-0.008	-0.007
CE	0.038	0.040	0.035	0.036

Table 4-3: SIMULATION 3: SURCHARGE OF 8% ON THE TAX RATE ON INVESTMENT GOODS

		1948	1955	1960	1965	1971
EC	Hist. Tax	8907.	16965.	22394.	28798.	36624.
	New Tax	8885.	16920.	22333.	28727.	36528.
	Net change	-21.9	-44.7	-61.6	-70.6	-95.8
	% change	-0.24	-0.26	-0.27	-0.24	-0.26
EM	Hist. Tax	20452.	28057.	35894.	45010.	61975.
	New Tax	20465.	28065.	35909.	45023.	61999.
	Net change	13.2	7.9	15.3	13.2	24.3
	% change	0.064	0.028	0.042	0.029	0.039
F/P	Hist. Tax	11.523	11.850	12.253	12.938	14.438
	New Tax	11.521	11.853	12.255	12.940	14.440
	Net change	-0.001	0.002	0.001	0.002	0.002
	% change	-0.011	0.021	0.014	0.015	0.017
HW	Hist. Tax	6.411	8.499	8.402	11.024	18.372
	New Tax	6.439	8.541	9.453	11.079	18.459
	Net change	0.027	0.042	0.051	0.054	0.087
	% change	0.435	0.496	0.543	0.498	0.473
CGNP*	Hist. Tax	0.6230	0.5719	0.6150	0.6209	0.6586
	New Tax	0.6199	0.5705	0.6127	0.6196	0.6580
	Net change	-0.0031	-0.0014	-0.0023	-0.0013	-0.0006
	% change	-0.51	-0.30	-0.39	-0.22	-0.12
IGNP*	Hist. Tax	0.3769	0.4280	0.3850	0.3790	0.3413
	New Tax	0.3800	0.4294	0.3872	0.3803	0.3419
	Net change	0.0031	0.0014	0.0022	0.0013	0.0006
	% change	0.79	0.33	0.58	0.29	0.18
ECT*	Hist. Tax	0.3285	0.3636	0.3602	0.3603	0.3373
	New Tax	0.3286	0.3631	0.3596	0.3593	0.3362
	Net change	0.0001	-0.0005	-0.0006	-0.0010	-0.0011
	% change	0.00	-0.14	-0.18	-0.31	-0.30
EGNP*	Hist. Tax	0.0365	0.0416	0.0421	0.0403	0.0349
	New Tax	0.0366	0.0416	0.0421	0.0401	0.0347
	Net change	0.0001	0.0000	0.0000	-0.0002	-0.0002
	% change	0.27	0.00	0.00	-0.46	-0.57
BTU*	Hist. Tax	21.042	31.410	40.724	51.473	69.245
	New Tax	20.876	31.139	40.205	50.794	68.339
	Net change	-0.166	-0.271	-0.519	-0.679	-0.806
	% change	-0.85	-0.86	-1.20	-1.23	-1.17

*see addendum to Tables 4-1/4-5 for a description of these quantities

<u>Simulation 4</u> (1% reduction in the tax rate on capital property, energy sector):

The tax rate on capital property for the energy sector, TPE, enters the model in equation 31. The reduction in TPE has the initial effect of increasing the rate of return ME which in turn affects MW and thus the budget equation for the household sector. Because of the interrelated nature of the variables in our general equilibrium model, the demand price of investment goods P_I which feeds back into equation 31, is also modified in the simultaneous process of market equilibration. The outcome of the interactions is such that it brings about a somewhat unexpected result: the reduction in TPE which might appear to apply strong pressure in the direction of stimulating investment, in effect results in only slightly higher values of gross investment. The implications of this result will be explored in greater depth in Section 4-5. As can be seen from Tables 4-4, the consumption of energy measures in B.T.U.'s decreases an average of -1.1% throughout the period 1948-1971.

<u>Dynamic elasticities with respect to TPE of</u>:

	1948	1955	1965	1971
EC	0.005	0.005	0.004	0.004
EM	-0.001	-0.001	-0.001	-0.001
KE	0.0	-0.001	-0.001	-0.001
KN	0.0	-0.001	-0.001	-0.001
P_{KE}	0.001	0.003	0.003	0.004
P_{KN}	-0.003	-0.002	-0.001	-0.001
P_{CE}	-0.003	-0.002	-0.001	-0.001
P_{EC}	-0.002	-0.002	-0.001	-0.001
CE	0.006	0.005	0.005	0.005

Table 4-4: SIMULATION 4: REDUCTION OF 1% IN THE TAX RATE ON CAPITAL PROPERTY, U.S. ENERGY SECTOR

		1948	1955	1960	1965	1971
EC	Hist. Tax	8907.	16965.	22394.	28798.	36624.
	New Tax	8885.	16927.	22341.	28738.	36546.
	Net change	-22.4	-38.4	-53.8	-59.9	-78.0
	% change	-0.25	-0.22	-0.24	-0.21	-0.21
EM	Hist. Tax	20452.	28057.	35894.	45010.	61975.
	New Tax	20464.	28068.	35913.	45028.	62010.
	Net change	12.4	11.5	19.12	17.8	35.6
	% change	0.06	0.04	0.05	0.04	0.05
F/P	Hist. Tax	11.523	11.850	12.253	12.938	14.438
	New Tax	11.522	11.850	12.252	12.938	14.438
	Net change	-0.001	0.0	-0.001	0.0	0.0
	% change	-0.007	0.0	-0.005	0.0	0.0
HW	Hist. Tax	6.411	8.499	9.402	11.024	18.372
	New Tax	6.413	8.503	9.406	11.029	18.381
	Net change	0.002	0.004	0.004	0.005	0.009
	% change	0.027	0.046	0.046	0.043	0.049
CGNP*	Hist. Tax	0.6230	0.5719	0.6150	0.6209	0.6586
	New Tax	0.6198	0.5705	0.6127	0.6195	0.6580
	Net change	-0.0032	-0.0014	-0.0023	-0.0014	-0.0006
	% change	-0.84	-0.24	-0.37	-0.22	-0.09
IGNP*	Hist. Tax	0.3769	0.4280	0.3850	0.3790	0.3413
	New Tax	0.3801	0.4294	0.3872	0.3804	0.3419
	Net change	0.0032	0.0014	0.0022	0.0014	0.0006
	% change	0.84	0.32	0.57	0.36	0.17
ECT*	Hist. Tax	0.3285	0.3636	0.3602	0.3603	0.3373
	New Tax	0.3286	0.3632	0.3596	0.3593	0.3362
	Net change	0.0001	-0.0004	-0.0006	-0.0010	-0.0011
	% change	0.03	-0.12	-0.18	-0.30	-0.33
EGNP*	Hist. Tax	0.0365	0.0416	0.0421	0.0403	0.0349
	New Tax	0.0366	0.0417	0.0421	0.0401	0.0347
	Net change	0.0001	0.0001	0.0000	-0.0002	-0.0002
	% change	0.27	0.24	0.00	-0.46	-0.57
BTU*	Hist. Tax	21.042	31.410	40.724	51.473	69.245
	New Tax	20.876	31.145	40.212	50.803	68.358
	Net change	-0.166	-0.265	-0.512	-0.670	-0.887
	% change	-0.78	-0.84	-1.25	-1.30	-1.28

*see addendum to Tables 4-1/4-5 for a description of these quantities

Simulation 5 (1% reduction in the tax rate on capital property, non-energy sector):

The initial impact of the reduction in the tax variable TPN, through equation 32, is to increase the rate of return of the non-energy sector, MN; in turn, this causes MW to increase through equation 58. From that point on, the repercussions of the tax change are qualitatively very similar to those that followed the reduction in TPE, because the predominant impact is subsumed in the change in MW. If there were sectoral investment demand functions that explicitly depended on the sectoral rates of return, then the difference in the behavior of the model resulting from changes in TPE and TPN, respectively, would be somewhat more pronounced. Again we observe the somewhat unexpected effects on investment expenditures of the private sector--this is discussed in 4-5 below. From Table 4-5 we note a reduction of in the supply of energy consumption products of -0.27% in 1948 which decreases to -0.24% by 1971. The decrease in total energy consumption measured in B.T.U.'s is similar to that in Simulation 4 during the initial part of the period, but is significantly larger, -2.73%, by 1971.

Dynamic elasticities with respect to TPN of:

	1948	1955	1965	1971
EC	0.005	0.005	0.005	0.005
EM	-0.001	-0.001	-0.001	-0.001
KE	0.0	-0.001	-0.001	-0.001
P_{KE}	0.002	0.003	0.004	0.005
P_{KN}	-0.002	-0.002	-0.001	-0.001
P_{CE}	-0.002	-0.002	-0.001	-0.001
P_{EC}	-0.002	-0.002	-0.001	-0.001
CE	0.009	0.006	0.005	0.005

TABLE 4-5: SIMULATION 5: REDUCTION OF 1% IN THE TAX RATE ON CAPITAL PROPERTY, U.S. NON-ENERGY SECTOR

		1948	1955	1960	1965	1971
EC	Hist. Tax	8907.	16965.	22394.	28798.	36624.
	New Tax	8882.	16923.	22336.	28731.	36534.
	Net change	-24.7	-41.8	-58.2	-66.2	-89.6
	% change	-0.27	-0.24	-0.26	-0.23	-0.24
EM	Hist. Tax	20452.	28057.	35894.	45010.	61975.
	New Tax	20461.	28067.	35911.	45025.	62003.
	Net change	8.96	9.95	16.88	14.99	28.54
	% change	0.043	0.035	0.047	0.033	0.046
F/P	Hist. Tax	11.523	11.850	12.253	12.938	14.438
	New Tax	11.524	11.851	12.254	12.939	14.439
	Net change	0.001	0.001	0.001	0.001	0.001
	% change	0.013	0.011	0.004	0.007	0.011
	Hist. Tax	6.411	8.499	9.402	11.024	18.372
	New Tax	6.413	8.503	9.405	11.028	18.380
	Net change	0.002	0.003	0.003	0.004	0.007
	% change	0.021	0.041	0.040	0.037	0.042
CGNP*	Hist. Tax	0.6230	0.5719	0.6150	0.6209	0.6586
	New Tax	0.6203	0.5707	0.6130	0.6197	0.6581
	Net change	-0.0027	-0.0012	-0.0020	-0.0012	-0.0005
	% change	-0.43	-0.20	-0.32	-0.19	-0.075
IGNP*	Hist. Tax	0.3769	0.4280	0.3850	0.3790	0.3413
	New Tax	0.3796	0.4292	0.3869	0.3802	0.3418
	Net change	0.0027	0.0012	0.0019	0.0012	0.0005
	% change	0.71	0.28	0.49	0.31	0.14
ECT*	Hist. Tax	0.3285	0.3636	0.3602	0.3603	0.3373
	New Tax	0.3281	0.3628	0.3591	0.3589	0.3358
	Net change	-0.0004	-0.0008	-0.0011	-0.0014	-0.0015
	% change	-0.12	-0.24	-0.30	-0.38	-0.44
EGNP*	Hist. Tax	0.0365	0.0416	0.0421	0.0403	0.0349
	New Tax	0.0365	0.0416	0.0420	0.0400	0.0346
	Net change	0.00	0.00	-0.0001	-0.0003	-0.0003
	% change	0.0	0.0	-0.23	-0.74	-0.85
BTU*	Hist. Tax	21.042	31.410	40.724	51.473	69.245
	New Tax	20.875	31.136	40.201	50.388	68.349
	Net change	-0.167	-0.274	-0.523	-1.085	-1.896
	% change	-0.79	-0.87	-1.28	-2.10	-2.73

*See addendum to Tables 4-1/4-5 for a description of these quantities

ADDENDUM TO TABLES 4-1 THROUGH 4-5

DESCRIPTION OF VARIABLES

EC, EM, F, P, HW -- As defined in section 4-2.

CGNP - Value share of consumption in Gross National Product

IGNP - Value share of investment in Gross National Product

ECT - Value share of energy consumption goods in total energy output

EGNP - Value share of energy products in Gross National Product

BTU - Total energy consumption measured in quadrillions of B.T.U.'s

Table 4-6: AVERAGE PERCENTAGE CHANGES IN ENDOGENOUS VARIABLES, 1948-1971, SIMULATIONS 1 THROUGH 5

	Average percentage change in selected endogenous variables from historical simulation values 1948-1971				
Variable	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
P_{KE}	-0.877	-0.296	-0.175	-0.151	-0.165
EM	-0.897	0.156	0.060	0.074	0.065
P_{CN}	-0.451	0.179	0.086	0.088	0.087
P_{EMN}	-0.959	-0.001	-0.001	-0.001	-0.001
P_{KN}	-0.933	0.261	0.117	0.127	0.121
P_{EM}	-0.002	-0.001	-0.001	-0.001	-0.001
EC	-0.909	-0.501	-0.290	-0.257	-0.279
P_{EC}	-0.155	0.188	0.097	0.094	0.096
RN	-0.141	0.538	0.258	0.265	0.262
EMN	-0.914	0.250	0.110	0.121	0.114
NMN	-1.101	0.197	0.077	0.095	0.083
LN	-0.947	0.299	0.135	0.146	0.139
KN	-0.277	0.069	0.028	0.033	0.029
NM	-1.093	0.194	0.076	0.093	0.082
NI	-0.418	0.098	0.040	0.047	0.042
P_{NI}	-0.234	0.034	0.013	0.016	0.014
NC	-0.978	0.301	0.135	0.147	0.140
P_{NC}	-0.352	0.144	0.069	0.071	0.070
RE	-0.731	-0.277	-0.165	-0.141	-0.157
NME	-0.819	0.082	0.022	0.038	0.028
EME	-0.884	0.064	0.012	0.028	0.017

Table 4-6 (cont.): AVERAGE PERCENTAGE CHANGES IN ENDOGENOUS VARIABLES, 1948-1971, SIMULATIONS 1 THROUGH 5

	Average percentage change in selected endogenous variables from historical simulation values 1948-1971				
Variable	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
LE	-0.907	0.090	0.025	0.041	0.031
KE	-0.353	0.091	0.037	0.044	0.039
p_{CE}	1.336	0.214	0.109	0.106	0.108
CE	-1.045	-0.587	-0.336	-0.298	-0.324
CN	-1.204	0.369	0.166	0.180	0.171
p_I	-0.364	0.054	0.608	0.025	0.022
I	-0.661	0.156	0.064	0.076	0.067
L	-0.751	0.236	0.106	0.115	0.109
p_L	0.001	-0.424	-0.212	-0.212	-0.212
ME	0.736	-0.273	-0.575	-3.155	0.006
MN	-0.707	0.254	-0.661	0.124	4.643
KSE	-0.383	0.098	0.040	0.047	0.042
KSN	-0.303	0.074	0.030	0.036	0.032
INE	-0.661	0.157	0.064	0.076	0.067
INN	-0.661	0.157	0.064	0.076	0.067
ICD	-0.661	0.157	0.064	0.076	0.067
IMIN	-0.661	0.157	0.064	0.076	0.067
IRS	-0.661	0.157	0.064	0.076	0.067
S	-0.967	0.199	0.639	0.096	0.085
LJ	0.199	-0.060	0.002	-0.026	-0.007
KCD	-0.539	0.127	0.052	0.062	0.005

Table 4-6 (cont.): AVERAGE PERCENTAGE CHANGES IN ENDOGENOUS VARIABLES, 1948-1971, SIMULATIONS 1 THROUGH 5

	Average percentage change in selected endogenous variables from historical simulation values 1948-1971				
Variable	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
KRS	-0.315	0.077	0.032	0.037	0.033
W	-0.586	0.114	0.525	0.055	0.048
CK	-0.370	0.089	0.036	0.043	0.038
P_{CK}	-1.873	-0.633	-0.379	-0.325	-0.362
MW	-0.689	0.196	-0.722	0.814	4.402
F	-0.006	-0.017	0.013	-0.006	0.006

Partial vs. General Equilibrium

The final experiment to be described in this section is aimed at illustrating the limitations of partial equilibrium analysis in evaluating the impact of a given tax measure. Suppose we wish to assess the effect of a 1% reduction in the capital property tax rate for the energy sector on the equilibrium values of the derived demands for factors of production in the energy sector. A partial equilibrium model can be constructed from the four equilibrium share equations 3, 4, 5 and 6 of our macroeconomic energy model. These four equations, by transposition of variables, can be interpreted as derived demand functions: by regarding prices as exogenous, one can compute the equilibrium values of the demands for capital services, labor services, energy and non-energy intermediate inputs. The impact of the tax reduction could then be evaluated by computing a new capital service price time series corresponding to the reduced tax rate, substituting this new series into the four equation models, and computing the new set of equilibrium demands. Indeed, we have conducted this experiment and report on its results below.

Table 4-7 shows time series for capital service prices corresponding to historical values, the partial equilibrium effects of a 1% reduction in TPE computed by using equation 31 directly, and the general equilibrium effects of the same tax measure on p_{KE}, taken directly from the results of simulation 4 with the complete macroeconomic model. We note that the values of p_{KE} computed by direct substitution into equation 31 greatly overestimate the effect of the tax reduction on the equilibrium values of capital service prices. This is due to the fact that the partial equilibrium analysis doesn't take into account the rise in the rate of return

TABLE 4-7: EFFECT OF 1% REDUCTION IN CAPITAL PROPERTY TAX ON CAPITAL SERVICE PRICE UNDER PARTIAL AND GENERAL EQUILIBRIUM

	p_{KE} Historical	p_{KE} 1% Reduction in TPE Partial Equilibrium	p_{KE} 1% Reduction in TPE General Equilibrium
1948	1.1110	1.0532	1.0687
1949	0.9028	0.8444	0.9518
1950	0.9657	0.9098	0.9799
1951	1.0536	0.9885	1.0142
1952	0.9905	0.9238	1.0254
1953	1.0274	0.9597	1.0164
1954	1.0609	0.9942	1.0002
1955	1.1255	1.0582	1.0873
1956	1.1506	1.0805	1.1076
1957	1.0729	0.9995	1.1271
1958	1.0000	0.9262	1.0653
1959	1.0517	0.9758	1.1138
1960	1.0984	1.0222	1.1270
1961	1.1095	1.0332	1.0987
1962	1.1302	1.0532	1.1538
1963	1.1616	1.0844	1.1715
1964	1.1958	1.1174	1.2221
1965	1.2370	1.1587	1.2339
1966	1.2613	1.1831	1.2846
1967	1.3275	1.2467	1.3173
1968	1.3309	1.2470	1.3134
1969	1.2766	1.1900	1.2751
1970	1.3172	1.2269	1.3010
1971	1.3560	1.2624	1.3321

TABLE 4-8: DYNAMIC ELASTICITIES OF DEMANDS FOR FACTORS OF PRODUCTION WITH RESPECT TO TPE, UNDER PARTIAL AND GENERAL EQUILIBRIUM

		Dynamic elasticities of demands with respect to TPE	
		Partial Equilibrium	General Equilibrium
KE	1948	-0.0366	0.0
	1952	-0.1743	-0.0001
	1956	-0.0493	-0.0010
	1960	-0.1588	-0.0011
	1964	-0.1436	-0.0012
	1968	-0.0858	-0.0013
	1971	-0.0836	-0.0015
LE	1948	-0.0026	-0.0007
	1952	-0.0116	-0.0005
	1956	-0.0036	-0.0002
	1960	-0.0118	-0.0004
	1964	-0.0115	-0.0003
	1968	-0.0073	-0.0007
	1971	-0.0074	-0.0006
EME	1948	-0.0019	-0.0021
	1952	-0.0026	-0.0071
	1956	-0.0017	-0.0016
	1960	-0.0021	-0.0020
	1964	-0.0018	-0.0016
	1968	-0.0016	-0.0022
	1971	-0.0014	-0.0018
NME	1948	-0.0042	-0.0005
	1952	-0.0051	-0.0034
	1956	-0.0034	-0.0001
	1960	-0.0039	-0.0002
	1964	-0.0033	-0.0001
	1968	-0.0028	-0.0003
	1971	-0.0022	-0.0004

for the energy sector, which acts as a stabilizing force through the lagged value RE(-1) in equation 31.

Table 4-8 presents a tabulation of dynamic elasticities of demand for factors of production in the energy sector, corresponding to both partial and general equilibrium analyses. The column corresponding to general equilibrium was constructed directly from the results of simulation 4. The partial equilibrium results were computed using the partial equilibrium sub-model described above. The difference between the two sets of dynamic elasticities can be explained as follows: as discussed above, the tax reduction augments the rate of return in the energy sector, resulting in a smaller decrease in the equilibrium values of the capital service price p_{KE}. The nominal rate of return on wealth, however, decreases as a result of lowered values of RN; this shows up in reduced demand by households and, ultimately, in lower equilibrium levels for the derived demands for factors of production. The net result is a change in equilibrium factor inputs substantially lower than that computed under partial equilibrium.

The somewhat unexpected results for the general equilibrium case tend to support the conjectures of Christensen [1], Harberger [2] and others that an analysis of investment incentives based on partial equilibrium may be misleading. Our results suggest that the interactions between rates of return, demand price of investment goods and capital service prices are such that they produce stabilizing forces that tend to offset the effects of the tax reduction. As we shall see in the next section, this observation is not insensitive to the composition of the entire fiscal package adopted by the government in implementing the tax change. We conclude, nevertheless, that an analysis based on partial equilibrium may tend to greatly overstate the effectiveness of tax policies to influence the equilibrium demands for factors of production and, in some instances, to predict incorrectly not only the magnitude but even the direction of this influence.

4.5 Government Debt, Government Expenditures and Fiscal Policy

The simulation experiments reported in the last section were conducted with the levels of government expenditures and government debt fixed exogenously at their historical values. In this section, the extent to which the effectiveness of a given tax measure is influenced by the configuration of the entire fiscal package adopted by the government will be examined in greater detail.

It has long been recognized that the selection of a budget-balancing fiscal package or, alternatively, deficit financing through the sale of government debt to the household sector, can decisively alter the effectiveness of a given tax measure. In the case of tax measures intended to stimulate investment expenditures, this question has been examined, for example, by Taubman and Wales [3]. Because of the complementarity between capital and energy revealed in the dynamic simulations of our general equilibrium model, discrepancies in the effects of such investment incentives in the face of different fiscal packages will effect themselves in the levels of energy demanded for industrial production.

In this section we evaluate the impact of alternative tax policies under two alternative government financing schemes.

Fiscal Plan A: Deficit Financing

Under this strategy, it is assumed that a tax reduction or surcharge is not accompanied by offsetting revenue or expenditure measures. Rather, the foregone or added revenues result in changes in the government deficit that are covered by the sale of government debt to the household sector. We implement this financial strategy of the government in the context of our macroeconomic energy model by introducing government deficit DEF and government debt G as endogenous variables and by adding the following two equations to our macroeconomic model:

62A. $$\begin{aligned} DEF = {} & TKE \cdot p_{KE} \cdot KE + TKN \cdot p_{KN} \cdot KN \\ & + TCE \cdot p_{EC} \cdot EC + TCN \cdot p_{NC} \cdot NC \\ & + TI \cdot p_{NI} \cdot NI + TL \cdot p_L \cdot L \\ & + TPE \cdot p_I(-1) \cdot KSE(-1) \\ & + TPN \cdot p_I(-1) \cdot KSN(-1) \\ & + TPH \cdot p_I(-1) \cdot [KRS(-1) + KCD(-1) \\ & - [p_{LG} \cdot LG + p_{GE} \cdot GE + p_{GN} \cdot GN + p_{GI} \cdot GI + MG \cdot p_G(-1) \cdot G(-1)] \end{aligned}$$

where

TPH - Effective tax rate on capital property, household sector.

MG - Effective rate of return on government deb.

and

63A. $$p_G \cdot G = p_G \cdot G(-1) - DEF$$

Fiscal Plan B: Budget Balancing

Under this fiscal package, every tax surcharge or reduction is accompanied by a corresponding change in the level of government expenditures so as to leave the deficit level intact; in particular, we assume that changes in tax revenues are entirely reflected in corresponding changes in the level of government purchases of investment goods. We implement this financing strategy in the context of our model by introducing the quantity of government purchases of investment goods, GI, and the level of government deficit DEF as endogenous variables and by adding the following two equations to the original form of our macroeconomic energy model:

$$\begin{aligned} \text{62B.}\quad p_{GI}\cdot GI = {} & TKE\cdot p_{KE}\cdot KE + TKN\cdot p_{KN}\cdot KN \\ & + TCE\cdot p_{EC}\cdot EC + TCN\cdot p_{NC}\cdot NC \\ & + TI\cdot p_{NI}\cdot NI + TL\cdot p_{L}\cdot L \\ & + TPE\cdot p_{I}(-1)\cdot KSE(-1) + TPN\cdot p_{I}(-1)\cdot KSN(-1) \\ & + TPH\cdot p_{I}(-1)\cdot [KRS(-1) + KCD(-1)] \\ & - [p_{LG}\cdot LG + p_{GE}\cdot GE + p_{GN}\cdot GN + MG\cdot p_{G}(-1)\cdot G(-1)] - DEF \end{aligned}$$

where

TPH - Effective tax rate on capital property, household sector

MG - Effective rate of return, government debt

and

$$\text{63B.}\quad p_{G}\cdot G = p_{G}\cdot G(-1) - DEF$$

TABLE 4-9: AVERAGE PERCENTAGE CHANGES IN ENDOGENOUS VARIABLES, 1948-1971, SIMULATIONS 1A THROUGH 5A

	Average percentage change in selected endogenous variables from historical simulation values 1948-1971				
Variable	Simulation 1A	Simulation 2A	Simulation 3A	Simulation 4A	Simulation 5A
p_{KE}	-0.969	-0.101	0.021	0.042	-0.385
EM	-0.937	0.097	0.003	-0.118	-0.293
p_{CN}	-0.461	0.094	0.002	-0.099	-0.140
p_{EMN}	-0.004	0.000	0.000	-0.000	-0.004
p_{KN}	-0.968	0.147	0.005	-0.164	-0.308
p_{EM}	-0.003	0.000	0.000	-0.000	-0.004
EC	-1.017	-0.197	0.021	0.127	-0.392
p_{EC}	-0.147	0.089	-0.002	-0.822	-0.035
RN	-1.157	0.274	-0.001	-0.227	-0.324
EMN	-0.945	0.140	0.001	-0.154	-0.285
NMN	-1.151	0.123	0.005	-0.149	-0.368
LN	-0.976	0.163	0.001	-0.175	-0.291
KN	-0.285	0.038	-0.001	-0.041	-0.075
NM	-1.143	-0.122	0.005	-0.147	-0.366
NI	-0.432	0.055	-0.001	-0.061	-0.121
p_{NI}	-0.245	0.023	0.001	-0.029	-0.080
NC	-1.008	0.165	0.001	-0.178	-0.302
p_{NC}	-0.360	0.075	0.000	-0.078	-0.106
RE	-1.115	-0.102	0.012	0.055	-0.287
NME	-0.863	0.062	0.005	-0.087	-0.281

TABLE 4-9 (cont.): AVERAGE PERCENTAGE CHANGES IN ENDOGENOUS VARIABLES, 1948-1971, SIMULATIONS 1A THROUGH 5A

	Average percentage change in selected endogenous variables from historical simulation values 1948-1971				
Variable	Simulation 1A	Simulation 2A	Simulation 3A	Simulation 4A	Simulation 5A
EME	-0.932	0.056	0.006	-0.084	-0.304
LE	-0.954	0.068	0.005	-0.095	-0.306
KE	-0.363	0.050	-0.002	-0.054	-0.097
p_{CE}	1.345	0.102	-0.002	-0.095	-0.048
CE	-1.168	-0.230	0.023	0.150	-0.443
CN	-1.242	0.202	0.002	-0.219	-0.373
p_I	-0.381	0.035	0.569	-0.045	-0.122
I	-0.683	0.089	-0.001	-0.098	-0.192
L	-0.774	0.129	0.001	-0.138	-0.232
p_L	0.0014	-0.212	0.000	0.212	0.212
ME	-0.960	-0.098	-1.371	4.712	-0.079
MN	-0.728	0.136	-0.776	-0.145	4.291
KSE	-0.394	0.054	-0.002	-0.058	-0.107
KSN	-0.312	0.041	-0.002	-0.045	-0.083
INE	-0.683	0.089	-0.001	-0.098	-0.192
INN	-0.683	0.089	-0.001	-0.098	-0.192
ICD	-0.683	0.089	-0.001	-0.098	-0.192
IMIN	-0.683	0.089	-0.001	-0.098	-0.192
IRS	-0.683	0.089	-0.001	-0.098	-0.192
S	1.390	-1.115	-0.682	2.491	12.612
LJ	0.293	-0.078	-0.016	0.134	0.567

TABLE 4-9 (cont.): AVERAGE PERCENTAGE CHANGES IN ENDOGENOUS VARIABLES, 1948-1971, SIMULATIONS 1A THROUGH 5A

	Average percentage change in selected endogenous variables from historical simulation values 1948-1971				
Variable	Simulation 1A	Simulation 2A	Simulation 3A	Simulation 4A	Simulation 5A
KCD	-0.555	0.072	-0.002	-0.078	-0.152
KRS	-0.324	0.043	-0.001	-0.047	-0.087
W	0.771	-0.636	-0.238	1.427	7.361
CK	-0.381	0.058	-0.002	-0.054	-0.102
p_{CK}	-2.049	-0.232	0.026	0.128	-0.664
MW	-0.715	0.110	-0.806	0.595	4.797
F	0.058	-0.041	-0.011	0.081	0.379

TABLE 4-10: AVERAGE PERCENTAGE CHANGE IN ENDOGENOUS VARIABLES 1948-1971, SIMULATIONS 1B THROUGH 5B

	Average percentage change in selected endogenous variables from historical simulation values 1948-1971				
Variable	Simulation 1B	Simulation 2B	Simulation 3B	Simulation 4B	Simulation 5B
P_{KE}	-1.821	-0.253	0.465	-0.889	-0.421
EM	-0.476	0.107	-0.240	0.392	0.182
P_{CN}	-0.866	-0.223	0.207	-0.534	-0.332
P_{EMN}	0.003	0.003	-0.003	0.007	0.004
P_{KN}	-1.825	-0.436	0.450	-1.095	-0.657
P_{EM}	0.003	0.003	-0.003	0.007	0.004
EC	-0.084	0.508	-0.461	1.154	0.749
P_{EC}	-0.471	-0.190	0.163	-0.432	-0.279
RN	-1.182	-0.210	0.000	-0.294	-0.280
EMN	-0.577	0.039	-0.194	0.254	0.085
NMN	-0.908	0.004	-0.125	0.122	0.025
LN	-0.715	-0.018	-0.140	0.117	0.000
KN	0.645	0.326	-0.481	0.975	0.517
NM	-0.890	0.009	-0.131	0.136	0.033
NI	0.271	0.227	-0.366	0.708	0.365
KSE	0.900	0.457	-0.667	1.361	0.724
KSN	0.702	0.356	-0.524	1.065	0.565
INE	1.499	0.787	-1.121	2.324	1.242
INN	1.499	0.787	-1.121	2.324	1.242
ICD	1.499	0.787	-1.121	2.324	1.242
IMIN	1.499	0.787	-1.121	2.324	1.242

TABLE 4-10 (cont.): AVERAGE PERCENTAGE CHANGE IN ENDOGENOUS VARIABLES, 1948-1971, SIMULATIONS 1B THROUGH 5B

	Average percentage change in selected endogenous variables from historical simulation values 1948-1971				
Variable	Simulation 1B	Simulation 2B	Simulation 3B	Simulation 4B	Simulation 5B
IRS	1.499	0.787	-1.121	2.324	1.242
S	0.731	0.586	-0.339	1.786	0.935
LJ	0.134	-0.003	0.067	-0.039	-0.010
KCD	1.211	0.625	-0.908	1.862	0.990
KRS	0.725	0.369	-0.541	1.102	0.584
W	0.079	0.205	0.127	0.665	0.334
CK	0.841	0.429	-0.628	1.280	0.679
P_{CK}	-1.549	0.356	-0.227	0.627	0.498
MW	-1.525	-0.379	-0.400	-0.266	-0.481
F	0.043	0.025	-0.003	0.065	0.038
P_{NI}	-0.413	-0.079	0.085	-0.209	-0.121
NC	-0.749	-0.020	-0.139	0.112	-0.004
P_{NC}	-0.677	-0.176	0.162	-0.420	-0.262
RE	0.139	0.440	-0.475	1.092	0.662
NME	-0.242	0.194	-0.319	0.598	0.311
EME	-0.375	0.174	-0.286	0.531	0.278
LE	-0.450	0.145	-0.260	0.463	0.235
KE	0.834	0.421	-0.617	1.256	0.667
P_{CE}	0.976	-0.214	0.182	-0.484	-0.314
CE	-0.102	0.584	-0.528	1.324	0.861
CN	-0.918	-0.023	-0.173	0.142	-0.002

TABLE 4-10 (cont.): AVERAGE PERCENTAGE CHANGE IN ENDOGENOUS VARIABLES, 1948-1971, SIMULATIONS 1B THROUGH 5B

	Average percentage change in selected endogenous variables from historical simulation values, 1948-1971				
Variable	Simulation 1B	Simulation 2B	Simulation 3B	Simulation 4B	Simulation 5B
P_{IN}	-0.707	-0.162	0.768	-0.415	-0.245
I	1.499	0.787	-1.121	2.324	1.242
L	-0.564	-0.013	-0.113	0.096	0.001
P_L	0.002	0.160	0.000	0.213	0.212
ME	-1.837	-0.152	-0.722		-0.275
MN	-1.397	-0.345	-0.441	-0.860	-0.418

TABLE 4-11: PERCENTAGE CHANGE IN SELECTED VARIABLES IN RESPONSE TO 1% REDUCTION IN TPE UNDER ALTERNATIVE FISCAL PACKAGES

		Historical G and DEF*	Fiscal Plan A Deficit Financing*	Fiscal Plan B Budget Balancing*
I	1948	0.0	0.0	0.883
	1958	0.065	-0.084	2.265
	1968	0.095	-0.173	4.251
KE	1948	0.0	0.0	0.0
	1958	0.051	-0.059	1.207
	1968	0.064	-0.100	2.386
KN	1948	0.0	0.0	0.0
	1958	0.034	-0.040	0.847
	1968	0.055	-0.085	2.037
P_{KE}	1948	-0.070	0.061	0.060
	1958	-0.172	0.090	-0.897
	1968	-0.210	-0.033	-1.531
P_{KN}	1948	0.146	-0.153	-0.154
	1958	0.085	-0.110	-0.902
	1968	0.075	-0.110	-2.076
EC	1948	-0.251	0.244	0.244
	1958	-0.232	0.130	1.033
	1968	-0.266	-0.083	2.237
EM	1948	0.060	-0.066	-0.067
	1958	0.050	-0.083	0.385
	1968	0.065	-0.146	0.878
NI	1948	0.0	0.0	0.0
	1958	0.038	-0.048	0.617
	1968	0.062	-0.117	1.533

*All figures represent percentage deviations from historical simulation values.

The impact of each of the two fiscal packages on the effectiveness of selected tax measures will be assessed by replicating the set of 5 simulations described in section 4-4 under each of the two financing schemes. That is, we simulate the effect of surcharges in the tax rates on energy consumption goods, labor income, investment goods, and reduction in capital property taxes in the energy and non-energy sector; the simulations were repeated for both Fiscal Plan A (1A through 5A) and Fiscal Plan B (1B through 5B).* Average percentage changes in selected endogenous variables for simulations 1A through 5A are shown in Table 4-9 and the corresponding values for simulations 1B through 5B are tabulated in Table 4-10. Tables 4-9 and 4-10 are directly comparable to Table 4-6 and thus, it is possible to observe the respective consequences of the deficit financing and budget balancing fiscal packages described above.

In what follows, we examine in some depth the implications of the two alternative budgetary schemes for one specific policy measure: a 1% reduction in the tax rate on capital property for the U.S. energy sector. We select this particular tax measure for detailed analysis both because it serves to illuminate the contrasts between the two budgetary strategies and also because it is representative of a variety of actual tax proposals frequently discussed in the context of stimulating investment expenditures.

From Table 4-11 we can observe the effect of the 1% reduction in TPE on key endogenous variables: gross private domestic investment, prices and quantities of capital services, supply of energy consumption and intermediate goods, and supply of investment goods. The most striking fea-

*2B corresponds to 0.05% surcharge on TL; 5B corresponds to a 1% proportional reduction in TPN. All other tax charges are the same as for the corresponding runs 1 through 5 in section 4-4.

ture of these results is the fact that the effect of the tax cut on investment and capital service levels is a slight increase under historical deficit and debt, a slight decline under deficit financing and a sharp increase under the budget-balancing fiscal plan. This seemingly conflicting outcome can be, in fact, explained by tracing the impact of the tax change through the model's equations.

The dynamic response to the tax reduction when deficit and debt levels are held at their historical values was already discussed in section 4.4. Under fiscal plan A, the immediate effect of the reduction in TPE shows up in equations 31 and 62A. The impact on Eq. 31 is the tendency for the after-tax rate of return ME to rise; this causes an increase in the portion of total wealth allocated to present consumption, through the intertemporal allocation equation 45. At the same time, the foregone revenues have the effect of increasing the government deficit through equation 62A; this increases the debt level according to equation 63A and reflects itself in higher values of national saving and end-of-period wealth, following equations 44 and 60. The corresponding increased demand for leisure and energy consumption goods, when reflected by the feedback effects throughout the model, cause the level of investment to decline. This result supports the conjecture made by Christensen [1] who used a simplified conceptual model to argue that investment incentives implemented in conjunction with a fiscal package based on deficit financing would lead to just such a counterintuitive outcome.

The initial effect of the tax reduction in the case of Fiscal Plan B is similar: through equation 31, the after-tax rate of return ME shows a tendency to rise. But in this instance, the foregone revenues are reflected in a smaller amount of investment goods purchased by the government, according to equation 62B. This leaves a larger portion of investment goods for gross private domestic investment: the funds for the purchase of these investment goods comes precisely from the increased after-tax property income originating from the tax cut. The conclusion is that investment incentives accompanied by reductions in government purchases of investment goods, result in significantly higher levels of gross investment and correspondingly higher equilibrium levels of capital services and energy.

4.6 Summary

In this chapter we have presented the final form of our macroeconometric energy model. The results of historical simulation of the model throughout the period 1948-1971 were reported, together with error statistics for the simulated values of key endogenous variables. In terms of the accuracy of these *ex-post* forecasts, the performance of the model must be judged satisfactory. Next, we performed a set of simulations corresponding to the effects of surcharges and reductions in selected tax variables. An analysis of these simulation results highlighted the strongly interrelated nature of the variables in our general equilibrium model. An analysis of a tax reduction on the rate on capital property was conducted for a simplified partial equilibrium model and compared with the results obtained using the complete model. The conclusion was that a partial equilibrium analysis would lead to overpredict the effects of the tax cut because it neglects the stabilizing forces arising from feedback interrelationships among variables. Finally, we repeated the tax policy experiments under two alternative assumptions about government financing: specifically, we examined the implications of budget-balancing and deficit-financing fiscal packages. Detailed analysis of tax measures aimed at stimulating investment expenditures show that the impact on actual investment levels is critically dependent on the configuration of the entire budgetary scheme.

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CHAPTER V

OPTIMAL GROWTH POLICIES AND ENERGY RESOURCES

CHAPTER V

OPTIMAL GROWTH POLICIES AND ENERGY RESOURCES

"- Would you tell me, please, which way I ought to go from here?
- That depends a good deal on where you want to get to, said the Cat.
- I don't much care where -- said Alice.
- Then it doesn't matter which way you go, said the Cat."

- Lewis Carroll

5.1 Introduction

This chapter discusses the design of optimal policies for energy and economic growth. Our primary objective is to demonstrate that our macroeconomic model provides a suitable basis for optimization experiments that can yield insight into the structure and configuration of optimal fiscal plans, the relative degrees of controllability of alternative tax instruments, and the welfare gains accruing from alternative growth policies.

Our first goal is to arrive at a formulation that makes it possible to take advantage of the conceptual and algorithmic framework afforded by the theory of optimal control. To this end, we must construct a suitable objective function. Given such an index of performance and a set of available policy instruments, the problem of optimal growth can be posed in terms of the maximization of welfare subject to the behavioral and technological constraints embodied in our macroeconomic model.

The specification of a suitable measure of economic welfare is, of course, at the heart of such an endeavor. We postulate a social welfare function defined in terms of a discounted stream of utilities accruing to households from a sequence of full consumption flows. The arguments of this welfare measure -- i.e. the indices of full consumption -- are defined as aggregates of consumption levels of energy, non-energy, capital services and leisure, and are, therefore, directly tied to the variables in our macroeconomic model.

Two important properties of this welfare function must be emphasized. First, it constitutes a characterization of social preferences which is consistent with the preferences of consumers as revealed in the historical data. This follows from the fact that we have chosen both the social discount rate and the structure of inter-temporal preferences implicit in the welfare function to conform with the estimated model of inter-temporal allocation reported in Sec. 3.2.2. Second -- and this follows from much the same fact -- our welfare measure constitutes an objective function which is consistent with the behavioral assumptions underlying the equations of our macroeconomic model. Both of these characteristics of our welfare index are in contrast to the traditional practice in applications of optimal control to economic policy-making in which the objective function is chosen in an ad-hoc way, with the choice of target variables and reference paths guided more by conventional wisdom than by economic theory or empirical fact. Koopmans [1] has warned about the shortcomings of this practice in the context of theoretical models of optimal economic growth:

> "Our aim is to argue against the complete separation of the ethical or political choice of an objective function from the investigation of the set of technologically feasible paths. Our main conclusion will be that such a separation is not workable. Ignoring realities in adopting 'principles' may lead one to search for a non-existent optimum or to adopt an 'optimum' that is open to unanticipated objections."

We wish to point out that the formulation of optimal growth policies within the context of a welfare functional such as we have described, is susceptible of a descriptive, as well as a normative interpretation. In the former sense, our optimal control framework for the characterization of growth paths, can be viewed as an embodiment of the neoclassical paradigm: the economic system operating in a decentralized fashion attains a configuration of equilibrium magnitudes that results in the maximization of consumers' satisfactions. If, on the other hand, one were to adopt a normative interpretation, then the outcome of the optimization experiments might be regarded as a prescription for governmental action.

The possibility of this dual interpretation suggests that a characterization of growth paths in terms of an optimality criterion such as we have described, does not necessarily presuppose a definitive resolution of the controversies between active and passive or neutral fiscal policies discussed in Section 1.2. Indeed, in the descriptive interpretation, the optimal growth policies are not inconsistent with the views of Bauer [2] who -- according to Koopmans [1],

> "favors that balance between the welfare of present and future generations that is implied in the spontaneous and individual savings decisions of the present generation. A policy implementing this preference would merely seek to arrange for tax collection and other government actions affecting the economy in such a way as to distort or amend the individual savings preferences as little as possible."

This attitude, the reader will undoubtedly recognize, differs in small measure from the concepts behind the principle of fiscal neutrality studied by Phelps.

On the other hand, in the purely normative sense, the prescription of optimal growth policies can be argued to support the view of Allais [3], who proclaims that:

> "the balancing of the interests of different generations is an ethical or political problem, in which the competitive market solution has no valid claim to moral superiority over other solutions that depend for their realization on action by the state."

which is an argument for an active growth policy in the sense discussed in Chapter I.

In the next section, we briefly review the key contributions to the theory of optimal economic growth. In section 5.3, we derive the specific form for our welfare functional which characterizes preferences over infinite sequences of consumption programs. In 5.4, we recast the optimal growth problem as a finite horizon problem by constructing a valuation of the terminal capital stock that is consistent with the preferences of subsequent generations. In 5.5 we present the precise formulation of the optimal control problem and we discuss the computational algorithms used to compute the numerical solutions. Finally, in section 5.6 we present the results of the optimal control experiments and we discuss the properties and implications of the resulting growth paths.

5.2 The Theory of Optimal Economic Growth

That the subject of optimal economic growth had its origins with the study on optimal saving by Ramsey [4] is a proposition few would argue against. Britto [5] in a recent survey paper on the theory of economic growth, has stated that the "classic paper by the Cambridge philosopher F. P. Ramsey...set up a framework that has remained unaltered until the present day." Ramsey attempted to answer the question of how much a community should save by postulating a utility functional -- a social welfare function -- additive in the utility of different generations and depending on the level of consumption and negatively on the amount of work. (This latter dependence can be reinterpreted as a positive dependence on leisure). He posed the problem of finding the path of consumption that would maximize the utility functional subject to simplified constraints on technology and capital accumulation. Ramsey employed the techniques of the calculus of variations to solve the resulting infinite-horizon problem. An unusual feature of Ramsey's welfare function -- in the light of present-day practice -- is that utilities accruing to future generations were not discounted. Indeed, Ramsey pointedly stated that:

> "...it is assumed that we do not discount later enjoyments in comparison with earlier ones, a practice which is ethically indefensible and arises merely from the weakness of the imagination."

From a mathematical standpoint this assumption forced Ramsey to introduce a satiation level -- termed the bliss point -- to the utility of consumption in order to insure convergence of the integral over $[0,\infty]$. But the assumption of an effective discount rate of zero is most interesting because it relates to a question that lies at the center of controversies that concern both the proper role of government in growth as well as the characterization of intertemporal preferences in optimal growth models. Harrod [6], expressed himself no less forcefully on this question:

> "On the assumption...that a government is capable of planning what is best for its subjects, it will pay no attention to pure time preference, a polite expression for rapacity and the conquest of reason by passion."

On the opposite of this debate, arguments for a positive rate of social discount have been put forth, among others, by Böhm-Bawerk [7]:

> "...we feel less concerned about future sensations of joy and sorrow simply because they do lie in the future. Consequently we accord to goods which are intended to serve future ends a value which falls short of the true intensity of their future marginal utility."

and by Irving Fisher [8]:

> "In such an ideal loan market, therefore, where every individual could freely borrow or lend, the rates of preference or impatience for present over future income for all the different individuals would become, at the margin, exactly equal to each other and to the rate of interest."

Either a positive discount rate or the utility satiation level -- Ramsey's bliss point -- serve the purpose of eliminating well-posedness difficulties stemming from the infinite-horizon nature of the problem. But other approaches have also been suggested. Von Weizsäcker [9] introduced the so-called overtaking criterion according to which a feasible consumption path C_t is said to overtake another feasible consumption path $\underline{C}_t$ if there is some period in the future after which the utility yielded by the former path is not less than that yielded by the latter. The optimal path would be then the path that overtook all others. Hammond and Mirlees [10] introduced the notion of an agreeable plan as one that provides an arbitrarily close approximation to the optimum and which is constructed by disregarding the infinite nature of the horizon in a well-defined sense. Recently, Solow [11] has proposed the application of a max-min criterion to the problem of optimal capital accumulation; this max-min criterion, which also provides an alternative to the maximization of discounted utilities, constitutes an extension of the egalitarian principles espoused by Rawls [12] to the problem of intertemporal choice.

The final set of results concern the computation of optimal growth paths for finite planning horizons in which constraints on the terminal capital stock are specified; this leads to the family of results known as the turnpike theorems (Cass [13], Samuelson [14]).

Probably the single most dominant feature of the majority of past studies in optimal growth has been the absence of direct links to the processes underlying concrete economies. Solow's remark about the "stylized facts" in theoretical models of economic growth applies equally well to optimal economic growth studies: "I don't think that models like this lead directly to prescription for policy or even to detailed diagnosis. But neither are they a game. They are more like reconnaissance exercises."*

In the remainder of this chapter, we shall apply the principles of the theory of optimal economic growth to a relatively detailed macro-econometric energy model which is tied to historical data. Our approach is novel in that it will explore the feasibility of extending these same conceptual principles within a framework that is not entirely lacking in realism or empirical content.

5.3 An Intertemporal Measure of Aggregate Social Welfare

Our objective in this section is to construct an aggregate index of social welfare that will serve as the basis for the specification of the objective function in the optimal growth studies described in sections 5.5 and 5.6.

Our approach to the characterization of social preferences in the context of our growth model is based on a direct generalization of the theory of consumer behavior. We begin with a brief review of the sub-model of the household sector described in Chapter 2.

Recall that our model of consumer behavior was formulated starting from the assumption that households make their spending and working decisions on the basis of utility maximization. Specifically, we postulated the existence of a continuous, real-valued, positive, quasi-concave

*Solow, [15], p. 105.

utility function U(X), defined on the set of all feasible per capita consumption programs given by

$$X = [x_t, x_{t+1}, \ldots, x_{t+\tau}, \ldots]$$

where

$$x_t = (CE_t/P_t, CN_t/P_t, CK_t/P_t, LJ_t/P_t)$$

and

CE - Energy consumption goods purchased by households

CN - Non-energy consumption goods purchased by households

CK - Capital services supplied to households

LJ - Leisure time of the household sector

We assumed that preferences were stationary and that they were weakly homothetically separable with respect to the partition Π induced by the time ordering. It then followed from Theorem 2.4 that there existed aggregate price and quantity indices p_F^t and F_t such that an equivalent utility maximization problem could be posed in terms of $U(F^t)$, where

$$F^t = [F_t/P_t, F_{t+1}/P_{t+1}, \ldots, F_{t+\tau}/P_{t+\tau} \ldots]$$

is termed the per capita full consumption program.

By assuming stationary expectations about forward prices, the conditions for inter-temporal aggregation expressed in Theorem 2.8 were satisfied and, consequently, we were able to write

$$U(F^t) = \sum_{\tau=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{\tau} \mu_\tau(F_{t+\tau}/P_{t+\tau}) \qquad (5.1)^*$$

*Note that U is now defined on the set of F^t rather than X^t; we have maintained the same notation for both utility functions since there is little chance of confusion.

where δ is the social discount rate.

Finally, using the translog approximation to the direct intertemporal utility function we had

$$-\ln U(F^t) = \sum_{\tau=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{\tau} [\alpha_0 \ln \frac{F_{t+\tau}}{P_{t+\tau}} + \beta_{0t} \ln \frac{F_{t+\tau}}{P_{t+\tau}} \cdot t] \qquad (5.2)^*$$

The two expressions given by Eqs. (5.1) and (5.2) will be taken as the basis for the specification of an index of social welfare to be used in our optimal growth studies. Note that the utility index U is defined on the set of full consumption programs F^t; each intra-period index of full consumption $F_{t+\tau}$, is in turn a Divisia aggregate of CE, CN, CK and LJ which are all endogenously determined within our macroeconomic model. In effect, therefore, the welfare function is directly linked to the per capita consumption levels of energy, non-energy, capital services and leisure.

The utility index given by (5.1) introduces a partial order into the space of feasible consumption programs F^t since to each element

$$F^t = [F_t/P_t, F_{t+1}/P_{t+1}, \ldots, F_{t+\tau}/P_{t+\tau}, \ldots]$$

it assigns a real number $U(F^t)$. We emphasize that our ability to perform the intertemporal aggregation of preferences implicit in (5.1) hinged on the fulfillment of the following conditions:

(a) The selection of a <u>cardinal</u> utility index μ_τ for the characterization of intra-period preferences.

(b) Stationarity of preferences (cf. definition 2.10).

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*This approximation corresponds to a neutral, explicitly homothetic translog utility function.

(c) Weak separability of preferences (cf. definition 2.2).

(d) Stationary expectations of forward prices.

These conditions are entirely analogous, though slightly less restrictive, than those imposed by Koopmans [16] and Morishima [17] in their studies on intertemporal aggregation of utilities.

Since our measure of welfare given by Eq. 5.1 is an aggregate index, it is natural to ask how it relates to the utility functions of individual consumers. There are, it turns out, three possible interpretations:

(i) All consumers are assumed to have identical utility functions, equal to $U(F^t)$.

(ii) The welfare index is first defined as an aggregate utility function for the household sector as a whole, and then expressed in per capita terms.

(iii) The utility functions of individual consumers are different, but are then aggregated and expressed in per capita terms.

In both the second and third of the above interpretations $U(F^t)$ can be viewed as representing the utility of the "average" consumer. Conditions for the validity of utility aggregations from the standpoint of welfare economics are discussed below.*

Our measure of welfare given by the utility index (5.1), admits of both descriptive and normative interpretations. The descriptive interpretation, as we have already suggested, follows from regarding the utility function as an embodiment of the preferences of the household sector as revealed in the historical data.

The alternative interpretation is to view $U(F^t)$ as a social welfare

*For a detailed discussion of these conditions from the standpoint of the theory of functional aggregation, see Green [18].

function in the sense of Samuelson [19] and Bergson [20]. In this context, the relevance of Arrow's Impossibility theorem* to the specification of welfare functions in optimizing models of economic planning has been propounded by some. Julius Margolis has written:**

> "Despite many efforts to circumvent [Arrow's] contribution, the central theorem still holds. The typical response of the applied economist to the Arrow theorem is to acknowledge its existence with a footnote reference and to proceed to try the impossible."

The most penetrating analysis of this controversy is that by Leif Johansen [21]. Johansen emphasizes the distinction between Bergson's and Arrow's notion of a welfare function. In the latter interpretation, the welfare function is a function whose domain is an entire set of individual preference orderings and whose range is a set of aggregate social preferences measures: i.e. it is a mechanism for consistently synthesizing a set of social preference measures, given the set of individual orderings. It is this kind of mapping between <u>sets</u> of orderings whose existence is precluded by Arrow's theorem. In the Bergsonian interpretation, on the other hand -- as Johansen points out -- a social welfare function is taken to be a single aggregate measure of social preferences, defined consistently with <u>one particular</u> configuration of individual preferences. It is this single measure -- rather than the mechanism for synthesizing a set of such measures as in Arrow's sense -- which is required for the purposes of characterizing the objective function in our policy optimization studies.

We finally remark that in the context of measures of welfare defined

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*Arrow [22].
**Margolis, in Chase [23], p. 72.

in terms of per capita consumption levels, such as ours, there has been some argument as to whether each component of intra-period utility should be weighted by the population in that given year; Lerner [24] and Koopmans [1] have argued in favor of this approach and Arrow and Kurz [25] used it in their studies of optimal fiscal policy. Most writers, however, have chosen to use unweighted utility indices.

5.4 Finite Planning Horizon and Intergenerational Equity

The problem of finding optimal growth policies given the specification of the welfare function in the preceding section, can be posed as that of finding time paths for the policy instruments that will maximize $U(F^t)$ subject to the dynamic constraints of the macroeconomic model. An equivalent formulation* is to find policy variables so as to minimize $V(F^t) = -\ln U(F^t)$, where V is approximated by the translog form (5.2). By defining the per capita level of full consumption $HF_t = F_t/P_t$, we have:

$$V(F^t) = -\ln U(F^t) = \sum_{\tau=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{\tau} [\alpha_0 \ln HF_{t+\tau} + \beta_{0t} \ln HF_{t+\tau} \cdot t] \qquad (5.3)$$

This form of an objective function involves an infinite planning horizon, whereas for the purposes of computing numerical solutions to the optimal growth problem, we obviously require a formulation in terms of a finite horizon. Suppose that an optimal growth trajectory is to be computed for the planning period [0,T]. If the objective function for this planning problem is constructed by simply truncating (5.3), the result would be an optimal trajectory that would sharply shift towards consump-

*This follows from the monotonicity of the logarithm function.

tion and totally neglect investment during the last years of the period. This would be a reflection of the fact that the preferences of generations subsequent to the period [0,T] had been disregarded. There are two alternative approaches that can be adopted to resolve this difficulty. The first involves solving the optimal control problem over a period considerably longer than [0,T] and then discarding all but the first T periods of the solution. This method will eliminate the end-of-period effects described above, but since it adds substantially to the computational burdens, it is unattractive for large-scale problems. The alternative approach, which we shall adopt, consists of defining a new welfare measure by taking the truncation of (5.3) to [0,T] and adding to it a value function reflecting the summarized impact of variables in [0,T] on the outcome of the growth process for $\tau > T$. In particular, we attempt to define such a value function as a terminal cost on the state variables at period T, $\underline{x}(T)$. More precisely, we shall construct a value function V_K whose arguments are the components of capital stock at period T, i.e.:

$$V_K = V_K(KSN(T),\ KSE(T),\ KCD(T),\ KRS(T)) \tag{5.4}$$

The levels of capital stock are selected as the arguments for the terminal state value function because they reflect the levels of productive capacity and supply of capital services to households available to generations after the terminal time T. The function V_K can therefore be interpreted as a measure of the welfare value of the bequest of capital stock to future generations.

Our approach to the specification of a measure V_K consists in deriving a quantitative expression for the discounted value of utilities accruing to generations after T from the stream of consumption flows $HF_{t+\tau}$, $\tau > T$ consistent with a given bequest of capital stock at time T.

If we write

$$V(F^t) = -\ln U(F^t) = \sum_{\tau=0}^{T} \rho_\tau(HF_{t+\tau}) + \sum_{\tau=T+1}^{\infty} \rho_\tau(HF_{t+\tau}) \quad (5.5)$$

then we can view V_K as an approximation to the second term in Eq. (5.5) i.e.,

$$V(F^t) = \sum_{\tau=0}^{T} \rho_\tau(HF_{t+\tau}) + V_K(\underline{x}(T)) \quad (5.6)^*$$

In order to compute the approximation V_K, we make the following assumptions:

(a) Consumer preferences are stationary.

(b) Full consumption per capita grows at a constant exponential rate g for $\tau > T$.

(c) The ratios between the level of full consumption and the components of capital stock remain constant for $\tau \geq T$.

To summarize, our aim is to develop a valuation of the components of capital stock at time T commensurate with the underlying social welfare function; our approach is to equate the value of the capital stock at time T to the discounted sum or utilities of the consumption stream which it is capable of sustaining for $\tau > T$.

From assumption (b), it follows that

$$HF_{T+\tau} = (1 + g)^\tau \cdot HF_T \quad (5.7)$$

---

*Note that $V_K(\underline{x}(T))$ can be interpreted to be analogous to the "cost-to-go" function of dynamic programming; Bellman [26].

Define λ_{KSN}, λ_{KSE}, λ_{KCD}, λ_{KRS} such that

$$
\begin{aligned}
HF_T &= \lambda_{KSN} \cdot KSN(T) \\
HF_T &= \lambda_{KSE} \cdot KSE(T) \\
HF_T &= \lambda_{KCD} \cdot KCD(T) \\
HF_T &= \lambda_{KRS} \cdot KRS(T)
\end{aligned}
\tag{5.8}
$$

From (5.7) and assumption (c) it then follows that

$$
\begin{aligned}
HF_{T+\tau} &= (1 + g)^{\tau} \cdot \lambda_{KSN} \cdot KSN(T) \\
HF_{T+\tau} &= (1 + g)^{\tau} \cdot \lambda_{KSE} \cdot KSE(T) \\
HF_{T+\tau} &= (1 + g)^{\tau} \cdot \lambda_{KRS} \cdot KRS(T) \\
HF_{T+\tau} &= (1 + g)^{\tau} \cdot \lambda_{KCD} \cdot KCD(T)
\end{aligned}
\tag{5.9}
$$

We will assume that V_K is additively separable,* i.e.:

$$V_K = V_{KSN}(KSN(T)) + V_{KSE}(KSE(T)) + V_{KCD}(KCD(T)) + V_{KRS}(KRS(T)) \tag{5.10}$$

Each component of V_K will be constructed by equating it to the utility level of the consumption stream over (T+1, ∞) sustainable by the given level of capital stock at T.

We therefore have:

$$V_{KSN}(KSN(T)) = \sum_{\tau=T+1}^{\infty} \left(\frac{1}{1+\delta}\right)^{\tau} [\alpha_0 \ln HF_{t+\tau} + \beta_{0t} \ln HF_{t+\tau} \cdot t] \tag{5.11}$$

*It is not obvious whether much is to be gained by considering a more general structure.

which can be rewritten as follows by using (5.9):

$$V_{KSN}(KSN(T)) = (\frac{1}{1+\delta})^{T+1} \sum_{\tau=0}^{\infty} (\frac{1}{1+\delta})^{\tau} [\alpha_0 \ln[(1+g)^{\tau} \lambda_{KSN} \cdot KSN(T)]$$

$$+ \beta_{0t} \cdot \ln[(1+g)^{\tau} \lambda_{KSN} \cdot KSN(T)] \cdot t] \qquad (5.12)$$

Expanding, we obtain

$$V_{KSN}(KSN(T)) = (\frac{1}{1+\delta})^{T+1} [\alpha_0 \ln(1+g) \sum_{\tau=0}^{\infty} (\frac{1}{1+\delta})^{\tau} \tau$$

$$+ \alpha_0 \ln\lambda_{KSN} \sum_{\tau=0}^{\infty} (\frac{1}{1+\delta})^{\tau} + \alpha_0 \ln(KSN(T)) \sum_{\tau=0}^{\infty} (\frac{1}{1+\delta})^{\tau}$$

$$+ \beta_{0t} \ln(1+g) \cdot t \sum_{\tau=0}^{\infty} (\frac{1}{1+\delta})^{\tau} \cdot \tau + \beta_{0t} \ln \lambda_{KSN} \cdot t \sum_{\tau=0}^{\infty} (\frac{1}{1+\delta})^{\tau}$$

$$+ \beta_{0t} \ln(KSN(T)) \cdot t \sum_{\tau=0}^{\infty} (\frac{1}{1+\delta})^{\tau}] \qquad (5.13)$$

Using the identities

$$\sum_{\tau=0}^{\infty} (\frac{1}{1+\delta})^{\tau} = (1 - \frac{1}{1+\delta})^{-1} = \frac{1+\delta}{\delta}$$

and

$$\sum_{\tau=0}^{\infty} (\frac{1}{1+\delta})^{\tau} \tau = \frac{1}{1+\delta} (1 - \frac{1}{1+\delta})^{-2} = \frac{1+\delta}{\delta^2}$$

we obtain:

$$V_{KSN}(KSN(T)) = (\frac{1}{1+\delta})^{T} [(\alpha_0 + \beta_{0t}) \ln(1+g) \cdot \frac{1}{\delta^2}$$

$$+ (\alpha_0 + \beta_{0t}) \cdot \ln \lambda_{KSN} \cdot \frac{1}{\delta}$$

$$+ (\alpha_0 + \beta_{0t}) \cdot \ln(KSN(T)) \cdot \frac{1}{\delta}] \qquad (5.14)$$

The actual numerical values for the relevant parameters are:

α_0 = -0.07425*

β_{0t} = 0.0254*

g = 1.01**

λ_{KSN} = 1.60×10^{-5}

λ_{KSE} = 10.2×10^{-5}

λ_{KCD} = 3.79×10^{-5}

λ_{KRS} = 3.33×10^{-5}

δ = 0.0798

T = 23

Substituting the appropriate values into Eq. 5.14, we obtain:

$$V_{KSN}(KSN(T)) = 9.624 - 0.879\ln[KSN(T)] \tag{5.15}$$

Going through the derivation for the other components of V_K in analogous fashion, we obtain:

$$V_{KSE}(KSE(T)) = 8.000 - 0.879\ln[KSE(T)]$$
$$V_{KRS}(KRS(T)) = 9.041 - 0.879\ln[KRS(T)]$$
$$V_{KCD}(KCD(T)) = 8.866 - 0.879\ln[KCD(T)] \tag{5.16}$$

Finally we rewrite the constant terms in logarithmic form to obtain:

$$V_{KSN}(KSN(T)) = -\ln[KSN(T)/56622]$$

---

*These correspond to the explicitly homothetic translot inter-temporal utility function estimated in section 3.2.2.

**Full consumption per capita grew at an average annual rate of 1% over the period 1948-1971.

$$V_{KSE}(KSE(T)) = -\ln[KSE(T)/8328]$$

$$V_{KCD}(KCD(T)) = -\ln[KCD(T)/23968]$$

$$V_{KRS}(KRS(T)) = -\ln[KRS(T)/29193]$$

This latter form of the components of V_K makes it clear that they can be reinterpreted as penalty functions that are approximations to the following set of inequality constraints:

$$KSN(T) \geq 56622$$

$$KSE(T) \geq 8328$$

$$KCD(T) \geq 23968$$

$$KRS(T) \geq 29193$$

5.5 The Optimal Growth Problem: Formulation and Solution Algorithm

From the last section, it follows that the objective function for our optimal growth problem is given by the following welfare index:

$$V(F^t) = \sum_{\tau=0}^{T} \left(\frac{1}{1+\delta}\right)^{\tau} [\alpha_0 \ln HF_{t+\tau} + \beta_{0t} \ln HF_{t+\tau} \cdot t]$$
$$- \ln[KSN(T)/56622] - \ln[KSE(T)/8328]$$
$$- \ln[KCD(T)/23968] - \ln[KRS(T)/29193] \qquad (5.17)$$

with $\alpha_0 = -0.07425$, $\beta_{0t} = 0.0254$, $\delta = 0.0798$, $T = 23$.

The next step in the formulation of the optimal growth problem in a form suitable for the application of the techniques of optimal control theory is the characterization of our macroeconomic energy model in state-variable form. The detailed reformulation, which is presented in Appendix A, results in a set of implicit state equations of the form:

$$\underline{Q}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{w}(k),\underline{u}(k),k] = 0 \qquad (5.18)$$

where k is the time index, $\underline{x}$ is the state vector, $\underline{y}$ is the output vector, $\underline{w}$ is the vector of exogenous variables and $\underline{u}$ is the vector of control variables. The state-variable representation of our model involves 12 state variables and 49 output variables so that $\underline{Q}$ is a system of 61 simultaneous equations in the unknowns $\underline{x}(k+1)$ and $\underline{y}(k+1)$.

Given the objective function $V(F^t)$ in (5.17) and the state-space form of the macroeconomic model, (5.18), the optimal growth problem can be stated as follows:

Optimal Growth Problem

Find time-paths for the policy variables, $\{\underline{u}(k),\ k=1,\ \ldots,\ T\}$ such that the objective function

$$V(F^t)$$

given by (5.15) is minimized, subject to the dynamic constraints:

$$\underline{Q}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{w}(k),\underline{u}(k),k] = 0$$

This statement of the problem makes it clear that we are confronted with a large-scale optimal control problem with nonlinear state equations and a nonlinear objective function. The complexity of this problem is further increased by the fact that the state-space representation of our model is in implicit form. The implicit state equations arise, of course, because of the simultaneous interdependence among contemporaneous endogenous variables. All of the conceptual and algorithmic developments of the modern theory of optimal control, on the other hand, assume the existence of an explicit, fully recursive, state transition mapping of the form*:

$$\underline{x}(k+1) = \underline{f}(\underline{x}(k),\underline{u}(k),\underline{w}(k),k) \qquad (5.19)$$

Conventional applications of optimal control techniques to problems of economic policy have assumed a representation of the form (5.19) -- e.g. Pindyck [27], Kendrick and Taylor [28]. Alternatively, in those instances where the equations of the model could not be cast into an explicit state equation form, this question has been bypassed by recasting the dynamic optimization problem into a static problem and using the methods of static optimization rather than optimal control (Fair [29], Athans, Kuh, Pindyck [30]).

Our aim has been to preserve the dynamic decomposition properties of optimal control theory while at the same time maintaining the underlying realism of the economic model with its full range of interdependencies.

This objective has necessitated the development of new computational algorithms. These algorithms have been incorporated into OPCON, a general purpose software package for the solution of nonlinear optimal control problems with nonlinear objective functions. The algorithms in OPCON are generalizations of existing optimal control algorithms to the case of implicit state equations.

One set of algorithms in OPCON belong to the family of <u>differential dynamic programming</u> (DDP) methods introduced by Jacobson and Mayne [31]. The DDP algorithms are based on the application of the principle of

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*It might be noted that the distinction between implicit and explicit state equations is analogous -- and in some cases, identical -- to the distinction between the <u>structural</u> and the <u>reduced</u> forms of an econometric model.

optimality (Bellman [26]) within a neighborhood of a nominal trajectory. This leads to a successive approximation procedure involving first- and second-order expansions of the value function. The Taylor series coefficients serve locally as "sufficient statistics" for the cost-to-go function and consequently eliminate the "curse of dimensionality" usually associated with dynamic programming. The cost of this feature is, of course, that one no longer has a guarantee of a global, but only of a local optimum.

The second family of algorithms implemented in OPCON is of the Min-H type, studied by Kelley [32]. These algorithms also use a successive approximation technique, but involve repeated minimizations of the Hamiltonian function at every time stage. These methods can be derived by a direct application of the discrete-time minimum principle (Halkin [33]).

The necessity of modifying these algorithms for the case of implicit systems arises from the fact that they all require the evaluation of the Jacobian and Hessian matrices of the state transition mapping f and the gradients of the Hamiltonian function. Our approach consisted in utilizing the implicit function theorem in order to derive functional relationships that relate the first- and second-order derivatives of f to the corresponding derivatives of Q. For more details on the OPCON computer program, the reader is referred to Appendix B.

5.6 Welfare Gains from Optimal Tax Policies

In this section we present computational results for a set of numerical experiments based on the optimal growth problem described above. The main purpose of these optimization experiments is to investigate the relative degrees of controllability afforded by the various tax instruments and, in addition, to determine the relative welfare gains accruing from alternative optimal growth paths.

The experiments were conducted over the period 1948 to 1971. This means that in addition to conveying information on the aspects just mentioned, our results allow an evaluation of historical tax policy relative to our aggregate measure of social welfare. Since our welfare index is, by construction, consistent with the historical data and with the behavioral assumptions underlying our macroeconomic model, a superficial examination of our optimal growth problem might lead to the suggestion that we are embarking upon a tautological exercise. That this is not so follows from the observation that, although it is true that our welfare function constitutes an embodiment of the preferences of consumers as revealed in the historical data, the particular configuration of equilibrium values throughout the period were attained _given_ a specific set of paths for the tax variables. What we are attempting to do is, in fact, to lift this latter restriction and, while maintaining consumer preferences unchanged, to ask whether a different set of fiscal policy variables might have resulted in the attainment of higher overall welfare. It would be quite legitimate, in fact, to argue that our particular formulation constitutes a search for a neutral fiscal policy in the sense of Phelps [34]: indeed, given a representation of the intertemporal preferences of consumers, we seek to determine those paths for the tax instruments leading to an outcome of the growth process that will best conform with these preferences. In this sense, the significance of our optimal control exercises is that they provide the basis for a quantitative appraisal of the extent to which historical tax policies have deviated from a policy of fiscal neutrality.

We have computed seven different optimal growth paths, each corresponding to a different set of controllable policy variables. We refer to a given set of controllable policy variables as a strategy. The seven strategies considered were defined by taking a single tax instrument to be controllable in each case:

Strategy 1: Controllable tax rates: TCE, effective tax rate on energy consumption goods.

Strategy 2: Controllable tax rates: TI, effective tax rate on investment goods.

Strategy 3: Controllable tax rates: TL, effective tax rate on labor income.

Strategy 4: Controllable tax rates: ITCE, investment tax credit, energy sector.

Strategy 5: Controllable tax rates: ITCN, investment tax credit, non-energy sector.

Strategy 6: Controllable tax rates: TKE, effective tax rate on capital income, energy sector.

Strategy 7: Controllable tax rates: TKN, effective tax rate on capital income, non-energy sector.

The optimal paths for the policy instruments are tabulated against the historical values in Tables 5-1 through 5-7. We observe that the optimal tax rate for TCE is an average of 10% below the historical values with the sharpest decline in the early part of the period. The optimal rate for TI is approximately 14% above the historical rates, with the increase concentrated in the early years and gradually declining towards the end of the period. The optimal rate for TL is approximately 14% below the historical rate. The optimal rates for ITCE and ITCN are approximately 6% and 28% above the historical rates, respectively. Finally, TKE and TKN have optimal values which are 0.1% and 1.0% below their historical values, respectively.

In Table 5-8, we present values of the welfare index under historical tax rates and under optimal tax rates for each one of the seven strategies. V is the value of the objective function, used in the optimal control computations. $U = \exp[-V]$ is the value of the underlying utility index. J is a scaled utility index obtained by weighting U by the population of the U.S. in 1948 expressed in thousands. We have also tabulated the values of $\Delta J = J_{opt} - J_{hist}$ which give the welfare gains accruing from the use of an optimal tax path as compared with the historical tax rates. We can regard the value of ΔJ for a given strategy as a measure of the degree of controllability afforded by that particular policy variable. Using this criterion of controllability, we observe that the greatest degree of control is obtained with TCE and TI. We also note from Table 5-8 that ΔV for TKE and ITCE are of the same order of magnitude; the same holds for ΔV corresponding to TKN and ITCN. These two results are as expected since the tax variables involved have their effects through the price of capital services in the energy and non-energy sectors, respectively.

The final set of tables, i.e. 5-9 through 5-13, present the optimal paths for full consumption per capita, as well as the three commodity groups and leisure, under strategies 1 through 5. We summarize below the average direction of change of each optimal path relative to the historical values under each of the five strategies:

	HCE	HCN	HCK	HLJ	HF
Strategy 1 (TCE)	-	+	+	-	+
Strategy 2 (TI)	-	-	-	+	+
Strategy 3 (TL)	+	-	-	+	+
Strategy 4 (ITCE)	0	0	0	0	+
Strategy 5 (ITCN)	-	-	-	+	+

The results of our optimal growth experiments lead us to the following set of conclusions:

1. Optimal growth paths show a tendency to have higher consumption levels towards the initial part of the planning period at the expense of consumption in the later years of the period.
2. The most effective tax instruments in terms of their effect on the level of welfare are TCE and TI. This is not surprising since TCE affects CE which is effectively an argument in the utility function and TI also has direct effect on the relative prices of consumption and investment.
3. Optimal paths of full consumption are higher than historical paths under each of the seven strategies; however the shifts in the com-

position of full consumption -- i.e. the changes in leisure and the three commodity groups -- are in general, in different directions under alternative strategies.

4. The fact that the optimal for TCE is below the historical values and TI is above its historical value suggests an overcapitalization of the economy and a tendency towards a higher consumption-investment ratio.
5. The changes in the various tax rates are sharpest in the early part of the planning period and decline gradually toward the end. If we view the growth turnpike as the utility-maximizing steady-state path, then this result can be interpreted as being indicative of the fact that the initial capital stock position of the economy is far from the turnpike capital stock whereas the terminal capital stock position is close to the turnpike capital stock.

We must emphasize that these observations are only suggestive of the properties of optimal growth paths and that definitive conclusions must await further investigation.

5.7 Summary

In this chapter, we have presented a framework of analysis that enabled us to compute optimal growth policies within the context of our macroeconomic energy model. We have discussed the specification of a social welfare function which is consistent with the preferences of consumers as revealed in the historical data; this index of aggregate welfare was susceptible of both normative and descriptive interpretations. The resulting infinite-horizon optimization problem was reduced to a finite-horizon problem by constructing a valuation of the terminal capital stock that was commensurate with the underlying welfare measure.

Given the objective function and the behavioral and technological constraints embodied in our macroeconomic model, the problem of optimal growth can be formulated as a nonlinear optimal control problem with implicit state equations. The numerical solution of this optimal control problem demanded the development of new solution algorithms that are generalizations of existing algorithms in the theory of optimal control. These algorithms have been incorporated into OPCON, a general purpose software package for the solution of nonlinear optimal control problems with nonlinear objective functions.

Finally, we present the results of a set of numerical optimization experiments that yield insight into the structure of optimal growth paths and the relative welfare gains accruing under alternative policies.

	TCE HISTORICAL	TCE OPTIMAL
1948	0.08520	0.00849
1949	0.09069	0.02680
1950	0.08843	0.03155
1951	0.08095	0.04235
1952	0.09054	0.05476
1953	0.08599	0.05374
1954	0.08919	0.06032
1955	0.08413	0.05926
1956	0.08780	0.07644
1957	0.10130	0.08232
1958	0.10664	0.08995
1959	0.10993	0.09453
1960	0.12709	0.11384
1961	0.14485	0.13295
1962	0.12399	0.11225
1963	0.12489	0.11497
1964	0.12674	0.11793
1965	0.12993	0.12196
1966	0.12065	0.11327
1967	0.11430	0.10773
1968	0.12442	0.11883
1969	0.12384	0.10765
1970	0.12276	0.11784
1971	0.11865	0.11864

Table 5-1: <u>OPTIMAL AND HISTORICAL RATES FOR TCE, EFFECTIVE TAX RATE ON ENERGY CONSUMPTION, 1948-1971</u>

	TI HISTORICAL	TI OPTIMAL
1948	0.04960	0.12614
1949	0.05317	0.11008
1950	0.05084	0.09932
1951	0.04870	0.09337
1952	0.05082	0.09119
1953	0.05091	0.08686
1954	0.04785	0.07859
1955	0.04762	0.07429
1956	0.04846	0.07252
1957	0.04810	0.06966
1958	0.04714	0.06620
1959	0.04813	0.06553
1960	0.05073	0.06651
1961	0.04954	0.06269
1972	0.04955	0.06239
1963	0.04997	0.06157
1964	0.04917	0.05973
1965	0.04814	0.05766
1966	0.04484	0.05351
1967	0.04588	0.05379
1968	0.04834	0.05547
1969	0.04855	0.05501
1970	0.05084	0.05062
1971	0.05157	0.05157

Table 5-2: OPTIMAL AND HISTORICAL TAX RATES FOR TI, EFFECTIVE TAX RATE ON INVESTMENT GOODS, 1948-1971

	TL HISTORICAL	TL OPTIMAL
1948	0.08119	0.04971
1949	0.06825	0.03977
1950	0.06493	0.03130
1951	0.09163	0.07395
1952	0.10305	0.08518
1953	0.10104	0.08415
1954	0.08860	0.07035
1955	0.09044	0.07374
1956	0.09393	0.07872
1957	0.09545	0.08047
1958	0.09428	0.07837
1959	0.09484	0.07928
1960	0.09896	0.08383
1961	0.09805	0.08360
1962	0.10097	0.08527
1963	0.10179	0.08847
1964	0.09043	0.07692
1965	0.09351	0.08027
1966	0.09850	0.08410
1967	0.10184	0.08743
1968	0.11092	0.09588
1969	0.12099	0.09296
1970	0.11159	0.09491
1971	0.10282	0.10282

Table 5-3: OPTIMAL AND HISTORICAL RATES FOR TL, EFFECTIVE TAX RATE ON LABOR INCOME, 1948-1971

	ITCE HISTORICAL	ITCE OPTIMAL
1948	0.0	0.01081
1949	0.0	0.00292
1950	0.0	0.02566
1951	0.0	0.00509
1952	0.0	0.00560
1953	0.0	0.00488
1954	0.0	0.00524
1955	0.0	0.00546
1956	0.0	0.00491
1957	0.0	0.00433
1958	0.0	0.00488
1959	0.0	0.00392
1960	0.0	0.00360
1961	0.0	0.00314
1962	0.05845	0.06196
1963	0.07261	0.07606
1964	0.07449	0.07772
1965	0.08417	0.08721
1966	0.08298	0.08600
1967	0.08218	0.08488
1968	0.09000	0.09237
1969	0.06540	0.06809
1970	0.02960	0.03154
1971	0.05460	0.05405

Table 5.4: OPTIMAL AND HISTORICAL RATES FOR ITCE, INVESTMENT TAX CREDIT, U.S. ENERGY SECTOR, 1948-1971

	ITCN HISTORICAL	ITCN OPTIMAL
1948	0.0	0.08304
1949	0.0	0.01956
1950	0.0	0.19000
1951	0.0	0.03128
1952	0.0	0.03168
1953	0.0	0.02143
1954	0.0	0.03070
1955	0.0	0.03241
1956	0.0	0.03052
1957	0.0	0.02669
1958	0.0	0.02391
1959	0.0	0.02360
1960	0.0	0.02202
1961	0.0	0.01930
1962	0.34709	0.46363
1963	0.04311	0.06379
1964	0.04423	0.06421
1965	0.04998	0.06927
1966	0.04926	0.06896
1967	0.04879	0.06674
1968	0.05348	0.06838
1969	0.03887	0.05637
1970	0.01759	0.03059
1971	0.03210	0.03209

Table 5.5: OPTIMAL AND HISTORICAL RATES, FOR ITCN, INVESTMENT TAX CREDIT, U.S. NON-ENERGY SECTOR, 1948-1971

	TKE HISTORICAL	TKE OPTIMAL
1948	0.3122	0.3080
1949	0.2656	0.2644
1950	0.2234	0.2147
1951	0.4168	0.4144
1952	0.2534	0.2513
1953	0.2679	0.2660
1954	0.2444	0.2429
1955	0.2558	0.2542
1956	0.3816	0.3799
1957	0.3757	0.3743
1958	0.2447	0.2432
1959	0.3670	0.3658
1960	0.3957	0.3946
1961	0.3904	0.3894
1962	0.3724	0.3714
1963	0.3792	0.3783
1964	0.3852	0.3844
1965	0.3784	0.3776
1966	0.3712	0.3705
1967	0.3592	0.3585
1968	0.4035	0.4029
1969	0.3964	0.3957
1970	0.3602	0.3597
1971	0.3600	0.3599

Table 5-6: OPTIMAL AND HISTORICAL RATES FOR TKE, EFFECTIVE TAX RATE ON CAPITAL PROPERTY, U.S. ENERGY SECTOR, 1948-1971

	TKN HISTORICAL	TKN OPTIMAL
1948	0.5098	0.3721
1949	0.3486	0.3393
1950	0.5109	0.4303
1951	0.5470	0.5264
1952	0.4873	0.4688
1953	0.5016	0.4856
1954	0.5059	0.4917
1955	0.5053	0.4908
1956	0.5008	0.4876
1957	0.4931	0.4821
1958	0.4956	0.4814
1959	0.5210	0.5113
1960	0.5194	0.5103
1961	0.5124	0.5045
1962	0.4887	0.4840
1963	0.4978	0.4904
1964	0.5056	0.4984
1965	0.4966	0.4904
1966	0.4873	0.4809
1967	0.4714	0.4657
1968	0.5296	0.5224
1969	0.5202	0.5141
1970	0.4728	0.4684
1971	0.4725	0.4724

Table 5-7: OPTIMAL AND HISTORICAL RATES FOR TKN, EFFECTIVE TAX RATE ON CAPITAL PROPERTY, U.S. NON-ENERGY SECTOR, 1948-1971

Control Variable	V_{hist}	V_{opt}	U^{*}_{hist}	U_{opt}	J^{**}_{hist}	J_{opt}	ΔJ
TCE	-2.683789	-2.687329	14.64047	14.69238	2146746.	2154358.	7612.0
TI	-2.683789	-2.685293	14.64047	14.66250	2146746.	2149971.	3231.0
TL	-2.683789	-2.684070	14.64047	14.64458	2146746.	2147350.	604.0
TKE	-2.683789	-2.683790	14.64047	14.64048	2146746.	2146748.	2.0
TKN	-2.683789	-2.683935	14.64047	14.64259	2146746.	2147058.	312.0
ITCE	-2.683789	-2.683792	14.64047	14.64052	2146746.	2146753.	7.0
ITCN	-2.683789	-2.684194	14.64047	14.64640	2146746.	2147616.	870.0

* $U = \ln[-V]$ ** $J = 146632.\ U$ $\Delta J = J_{opt} - J_{hist}$

Table 5-8: <u>WELFARE GAINS FROM OPTIMAL TAX POLICIES UNDER ALTERNATIVE STRATEGIES</u>

Table 5-9: OPTIMAL PATHS OF ENERGY CONSUMPTION PER CAPITA UNDER ALTERNATIVE STRATEGIES, 1948-1971

	HCE HISTORICAL SIMULATION	HCE OPTIMAL STRATEGY 1 (TCE)	HCE OPTIMAL STRATEGY 2 (TI)	HCE OPTIMAL STRATEGY 3 (TL)	HCE OPTIMAL STRATEGY 4 (ITCE)	HCE OPTIMAL STRATEGY 5 (ITCN)
1948	0.04896	0.05063	0.04898	0.05166	0.04896	0.04891
1949	0.06007	0.06176	0.05959	0.06287	0.06007	0.06006
1950	0.06701	0.06898	0.06668	0.07049	0.06701	0.06659
1951	0.06150	0.06253	0.06120	0.06343	0.06150	0.06149
1952	0.06504	0.05313	0.06484	0.06761	0.06504	0.06502
1953	0.07018	0.05641	0.06993	0.07262	0.07018	0.07016
1954	0.08070	0.05497	0.08034	0.08291	0.08070	0.08067
1955	0.08528	0.05137	0.08492	0.08710	0.08528	0.08524
1956	0.09120	0.05302	0.09088	0.09287	0.09120	0.09116
1957	0.09952	0.06333	0.09919	0.10131	0.09952	0.09949
1958	0.10133	0.06834	0.10102	0.10325	0.10132	0.10129
1959	0.10372	0.07347	0.10343	0.10561	0.10372	0.10369
1960	0.10729	0.07793	0.10703	0.10913	0.10729	0.10726
1961	0.10888	0.08093	0.10862	0.11058	0.10888	0.10885
1962	0.11907	0.09171	0.11883	0.12114	0.11907	0.11778
1963	0.12027	0.09925	0.12002	0.12199	0.12027	0.12024
1964	0.12773	0.10181	0.12748	0.12947	0.12773	0.12768
1965	0.13075	0.10472	0.13051	0.13242	0.13075	0.13069
1966	0.13949	0.11857	0.13925	0.1415	0.13949	0.13943
1967	0.14578	0.12970	0.14553	0.14794	0.14578	0.14571
1968	0.14986	0.13829	0.14961	0.15215	0.14985	0.14979
1969	0.15574	0.15360	0.15550	0.16124	0.15574	0.15567
1970	0.17234	0.17307	0.17207	0.17537	0.17234	0.17228
1971	0.15972	0.15557	0.15957	0.15839	0.15971	0.15968

Table 5-10: OPTIMAL PATHS OF NON-ENERGY CONSUMPTION PER CAPITA UNDER ALTERNATIVE STRATEGIES, 1948-1971

	HCN HISTORICAL SIMULATION	HCN OPTIMAL STRATEGY 1 (TCE)	HCN OPTIMAL STRATEGY 2 (TI)	HCN OPTIMAL STRATEGY 3 (TL)	HCN OPTIMAL STRATEGY 4 (ITCE)	HCN OPTIMAL STRATEGY 5 (ITCN)
1948	1.0466	1.1308	1.0467	1.0191	1.0466	1.0460
1949	1.0875	1.1491	1.0835	1.0629	1.0875	1.0874
1950	1.0877	1.1380	1.0858	1.0635	1.0877	1.0853
1951	1.2225	1.2767	1.2178	1.1985	1.2225	1.2223
1952	1.3610	2.0812	1.3531	1.3160	1.3609	1.3604
1953	1.3836	2.2098	1.3763	1.3420	1.3836	1.3832
1954	1.2417	2.8466	1.2394	1.2213	1.2417	1.2415
1955	1.1607	3.3248	1.1596	1.1489	1.1608	1.1606
1956	1.1730	3.5550	1.1722	1.1629	1.1730	1.1729
1957	1.2451	3.5340	1.2442	1.2329	1.2451	1.2450
1958	1.2552	3.5919	1.2544	1.2428	1.2552	1.2551
1959	1.2794	3.6126	1.2786	1.2669	1.2794	1.2793
1960	1.2998	3.7136	1.2992	1.2975	1.2999	1.2998
1961	1.3171	3.8826	1.3164	1.3058	1.3171	1.3170
1962	1.3522	3.9617	1.3517	1.3396	1.3522	1.3500
1963	1.4401	3.8013	1.4393	1.4263	1.4401	1.44
1964	1.4142	4.2706	1.4137	1.4033	1.4143	1.4141
1965	1.4214	4.4442	1.4209	1.4112	1.4214	1.4213
1966	1.4998	4.5267	1.4991	1.4872	1.4997	1.4996
1967	1.5997	4.4965	1.5989	1.5843	1.5996	1.5994
1968	1.6336	4.6063	1.6327	1.6174	1.6335	1.6333
1969	1.7234	4.5796	1.7226	1.6896	1.7234	1.7231
1970	1.8264	4.9084	1.8255	1.8043	1.8265	1.8263
1971	1.7088	5.4549	1.7083	1.7079	1.7088	1.7087

Table 5-11: OPTIMAL PATHS OF CAPITAL SERVICES TO HOUSEHOLDS PER CAPITA UNDER ALTERNATIVE STRATEGIES, 1948-1971

	HCK HISTORICAL SIMULATION	HCK OPTIMAL STRATEGY 1 (TCE)	HCK OPTIMAL STRATEGY 2 (TI)	HCK OPTIMAL STRATEGY 3 (TL)	HCK OPTIMAL STRATEGY 4 (ITCE)	HCK OPTIMAL STRATEGY 5 (ITCN)
1948	0.25639	0.25639	0.25639	0.25639	0.25639	0.25639
1949	0.27307	0.27307	0.27307	0.27307	0.27307	0.27307
1950	0.28934	0.28993	0.28934	0.28920	0.28934	0.28934
1951	0.31308	0.31435	0.31305	0.31275	0.31309	0.31308
1952	0.33119	0.33314	0.33112	0.33067	0.33119	0.33115
1953	0.34073	0.34326	0.34060	0.34001	0.34073	0.34068
1954	0.34978	0.35742	0.34958	0.34873	0.34978	0.34973
1955	0.35572	0.36954	0.35546	0.35435	0.35572	0.35566
1956	0.37388	0.40112	0.37358	0.37228	0.37388	0.37382
1957	0.38790	0.43217	0.38756	0.38621	0.38789	0.38784
1958	0.39549	0.45699	0.39514	0.39379	0.39549	0.39543
1959	0.39571	0.47250	0.39535	0.39399	0.39571	0.39565
1960	0.40605	0.50230	0.40567	0.40424	0.40604	0.40598
1961	0.41688	0.53127	0.41649	0.41500	0.41688	0.41681
1962	0.42485	0.55707	0.42445	0.42289	0.42485	0.42478
1963	0.43750	0.59112	0.43707	0.43542	0.43749	0.43742
1964	0.45174	0.62751	0.45130	0.44954	0.45174	0.45162
1965	0.46787	0.66538	0.46741	0.46552	0.46787	0.46774
1966	0.49101	0.71712	0.49052	0.48849	0.49101	0.49087
1967	0.51472	0.77195	0.51420	0.51204	0.51472	0.51458
1968	0.53557	0.82534	0.53502	0.53270	0.53557	0.53543
1969	0.55793	0.88312	0.55735	0.55479	0.55793	0.55777
1970	0.57805	0.93891	0.57743	0.57464	0.57804	0.57788
1971	0.58672	0.97544	0.58609	0.58290	0.58672	0.58655

Table 5-12: OPTIMAL PATHS OF LEISURE PER CAPITA UNDER ALTERNATIVE STRATEGIES, 1948-1971

	HLJ HISTORICAL SIMULATION	HLJ OPTIMAL STRATEGY 1 (TCE)	HLJ OPTIMAL STRATEGY 2 (TI)	HLJ OPTIMAL STRATEGY 3 (TL)	HLJ OPTIMAL STRATEGY 4 (ITCE)	HLJ OPTIMAL STRATEGY 5 (ITCN)
1948	11.2943	11.1289	11.2921	11.3414	11.2944	11.3039
1949	11.2111	11.1086	11.2851	11.2497	11.2111	11.2115
1950	11.2993	11.2248	11.3432	11.3329	11.2998	11.3555
1951	11.4117	11.3264	11.4664	11.4482	11.4117	11.4135
1952	10.8381	9.7481	11.8964	10.9102	10.8383	10.8415
1953	10.9035	9.7518	11.9597	10.9650	10.9035	10.9059
1954	11.2111	9.1018	11.2522	11.2381	11.2112	11.2136
1955	11.3221	8.6321	11.3553	11.3367	11.3221	11.3250
1956	11.3264	8.5291	11.3889	11.3752	11.3624	11.3651
1957	11.3070	8.6684	11.3326	11.3240	11.3070	11.3092
1958	11.4487	8.7493	11.4702	11.4662	11.4487	11.4050
1959	11.4370	8.7751	11.4924	11.4911	11.4730	11.4750
1960	11.5212	8.7898	11.5378	11.5389	11.5212	11.5230
1961	11.7755	8.9270	11.7909	11.7912	11.7755	11.7770
1962	11.6664	8.9343	11.6789	11.6838	11.6664	11.7635
1963	11.7244	8.1843	11.7371	11.7437	11.7244	11.7256
1964	11.9242	8.9842	11.9352	11.9387	11.9241	11.9262
1965	12.0637	8.9970	12.0738	12.0770	12.0637	11.0657
1966	12.2027	9.2244	12.2117	12.2192	12.2029	12.2052
1967	12.3303	9.5265	12.3392	12.3499	12.3304	12.3328
1968	12.5240	9.663	12.5323	12.5444	12.5241	12.5258
1969	12.6913	9.9834	12.6989	12.7329	12.6913	12.6939
1970	12.8415	10.0161	12.8487	12.8671	12.8414	12.8429
1971	13.3164	9.9084	13.3180	12.3180	13.3164	13.3166

Table 5-13: OPTIMAL PATHS OF FULL CONSUMPTION PER CAPITA UNDER ALTERNATIVE STRATEGIES, 1948-1971

	HF* HISTORICAL SIMULATION	HF OPTIMAL STRATEGY 1 (TCE)	HF OPTIMAL STRATEGY 2 (TI)	HF OPTIMAL STRATEGY 3 (TL)	HF OPTIMAL STRATEGY 4 (ITCE)	HF OPTIMAL STRATEGY 5 (ITCN)
1948	11.5230	11.4698	11.5207	11.5428	11.5230	11.5302
1949	11.5385	11.5160	11.5945	11.5543	11.5385	11.5388
1950	11.6461	11.6375	11.6797	11.6597	11.6465	11.6891
1951	11.8764	11.8610	11.9164	11.8885	11.8764	11.8777
1952	11.5625	11.3118	11.6025	11.5857	11.5626	11.5647
1953	11.6605	11.4543	11.6993	11.6777	11.6606	11.6621
1954	11.817	11.5267	11.8475	11.8244	11.817	11.8187
1955	11.8504	11.5916	11.8755	11.8544	11.8504	11.8525
1956	11.9293	11.7603	11.9492	11.9329	11.9294	11.9313
1957	11.9871	11.9144	12.0061	11.9924	11.9871	11.9886
1958	12.1258	12.0743	12.1416	12.1315	12.1258	12.127
1959	12.1746	12.1445	12.1887	12.1806	12.1746	12.176
1960	12.2535	12.2943	12.2654	12.2591	12.2535	12.2547
1961	12.4944	12.6039	12.5053	12.4989	12.4944	12.4954
1962	12.4715	12.6451	12.4802	12.4771	12.4715	12.5459
1963	12.6180	12.8498	12.6266	12.6231	12.618	12.6187
1964	12.7921	13.1697	12.7996	12.7958	12.7921	12.7934
1965	12.9385	13.3906	12.9452	12.9416	12.9385	12.9398
1966	13.1729	13.7451	13.1786	13.1771	13.173	13.1745
1967	13.4126	14.0523	13.4179	13.4168	13.4126	13.4139
1968	13.6362	14.3462	13.6411	13.6405	13.6362	13.6371
1969	13.8966	14.6853	13.9009	13.9095	13.8966	13.8981
1970	14.1796	15.1284	14.1833	14.1839	14.1796	14.1803
1971	14.4381	15.5465	14.4370	14.4312	14.4381	14.4379

*HF = F/P

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APPENDIX A: STATE-SPACE FORM OF MACROECONOMIC ENERGY MODEL

APPENDIX A

STATE-SPACE FORM OF MACROECONOMIC ENERGY MODEL

A.1 Introduction

In this appendix we formulate the state-space representation of our macroeconomic energy model. Because of the simultaneous interdependence among contemporaneous endogenous variables, a fully recursive formulation in the conventional state transition mapping form is not feasible and we must adopt a representation in terms of implicit state equations.

The implicit state equation representation of an econometric model is a system of simultaneous equations of the form:

$$Q[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),w(k),k] = 0 \qquad \text{A.1}$$

or, equivalently,

$$q_1[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k] = 0$$
$$q_2[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k] = 0$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$q_{NQ}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k] = 0 \qquad \text{A.2}$$

where

$$\underline{x}(k) = (x_1(k),x_2(k),\ldots,x_{NS}(k))$$

is the state vector at time k

$\underline{x}(k+1) = x_1(k+1), x_2(k+1, \ldots, x_{NS}(k+1))$

is the <u>state vector</u> at time k+1

$\underline{y}(k+1) = (y_1(k+1), y_2(k+1), \ldots, y_{NY}(k\_1))$

is the <u>output vector</u> at time k+1

$\underline{u}(k) = (u_1(k), u_2(k), \ldots, u_{NC}(k))$

is the <u>control vector</u> at time k

$\underline{w}(k) = (w_1(k), w(2), \ldots, w_{NW}(k))$

is the <u>exogenous input vector</u> at time k

The implicit state equation system A.1 should be viewed as a simultaneous equation system in the unknown vectors $\underline{x}(k+1)$ and $\underline{y}(k+1)$ with the quantities $\underline{x}(k), \underline{u}(k)$ and $\underline{w}(k)$ regarded as predetermined variables. It follows that one must have dim Q = dim $\underline{x}$ + dim $\underline{y}$, i.e. NQ = NS + NY.

In order to go from the standard econometric model form to the implicit state equation form, it is sufficient to replace each unlagged endogenous variable by an output variable y_i and each lagged endogenous variable by a state variable x_i.

In the next section we list the implicit state equations for our macroeconomic energy model. These state equations can be found to be in precise correspondence with the 61 equations in standard econometric form listed in Section 4.2.

In the remaining sections of this Appendix we list the definitions of the state, output and exogenous variables in terms of the original variables in the model.

A.2 Implicit State Equations*

$$q_1[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= w_{47}(k)*x_3(k) - y_1(k+1) = 0$$

$$q_2[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= w_{48}(k)*x_4(k) - y_2(k+1) = 0$$

$$q_3[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= [1 - w_{50}(k)]*x_3(k) + y_{10}(k+1) - x_3(k+1) = 0$$

$$q_4[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= [1 - w_{51}(k)]*x_4(k) + y_{11}(k+1) - x_4(k+1) = 0$$

$$q_5[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= y_{10}(k+1) + y_{11}(k+1) + y_8(k+1) + y_{12}(k+1) - y_6(k+1) - y_g(k+1) = 0$$

$$q_6[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= y_6(k+1) + w_{13}(k) + w_g(k) - y_7(k+1) = 0$$

$$q_7[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= [1 - w_4(k)]*x_1(k) + y_8(k+1) - x_1(k+1) = 0$$

$$q_8[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= [1 - w_5(k)]*x_2(k) + y_{12}(k+1) - x_2(k+1) = 0$$

$$q_9[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= w_{22}(k)/w_{39}(k) - y_3(k+1) = 0$$

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*Note that control variables $\underline{u}$ do not appear explicitly in the state equations because they are defined subsequently as a subset of the exogenous input vector $\underline{w}$.

$$q_{10}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= w_6(k)/w_{39}(k) - y_4(k+1) = 0$$

$$q_{11}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= y_{13}(k+1)/w_{39}(k) - y_5(k+1) = 0$$

$$q_{12}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= w_{69}(k) + 0.35*\ln[x_6(k)] + 0.366*\ln[x_7(k)] + 0.281705*\ln[y_7(k+1)]$$
$$+ 0.2211*\ln[x_8(k)] + 0.370572*\ln[w_{37}(k-1)]$$
$$+ 0.18266*\ln[x_9(k)] + 0.363098*\ln[w_{46}(k-1)]$$
$$+ 0.026949*\ln[w_{34}(k-1)] - \ln[x_5(k)] \qquad = 0$$

$$q_{13}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= w_{64}(k)*y_6(k+1) - y_{10}(k+1) = 0$$

$$q_{14}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= w_{65}(k)*y_6(k+1) - y_{11}(k+1) = 0$$

$$q_{15}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= w_{70}(k)*y_6(k+1) - y_8(k+1) = 0$$

$$q_{16}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$w_{71}(k)*y_6(k+1) - y_{12}(k+1) = 0$$

$$q_{17}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= w_{72}(k)*[x_1(k) + x_2(k)] - y_{13}(k+1) = 0$$

$$q_{18}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= y_{35}(k+1)*y_{34}(k+1) + y_{32}(k+1)*y_{31}(k+1) - y_{29}(k+1) = 0$$

$q_{19}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$

$= [y_{39}(k+1)*x_6(k+1) + y_{25}(k+1)*x_7(k+1) - x_5(k+1)/[-y_7(k)]$

$- y_{40}(k+10 = 0$

$q_{20}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$

$= p_3 + p_{28}* \ln[y_{37}(k+1)/w_{33}(k)] + p_{29} \ln[w_{35}(k)/w_{33}(k)]$

$+ p_{27}* \ln[w_{45}(k)/w_{33}(k)] + p_{30}* \ln(y_{26}(k+1)/w_{33}(k)]$

$+ p_{92}* \ln[y_{34}(k+1)/y_{31}(k+1)] + p_{32}*w_{56}(k)$

$- y_{37}(k+1)*y_1(k+1)/y_{29}(k+1) = 0$

$q_{21}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k+1),\underline{w}(k),k]$

$= p_4 + p_{29}* \ln[y_{37}(k+1)/w_{33}(k)] + p_{34}* \ln[w_{35}(k)/w_{33}(k)]$

$+ p_{33}* \ln[w_{45}(k)/w_{33}(k)] + p_{35}* \ln[y_{26}(k+1)/w_{33}(k)]$

$+ p_{93}* \ln[y_{34}(k+1)/y_{31}(k+1)] + p_{37}*w_{56}(k)$

$- w_{35}(k)*y_{23}(k+1)/y_{29}(k+1) = 0$

$q_{22}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$

$= p_2 + p_{27}* \ln[y_{37}(k+1)/w_{33}(k)] + p_{33}* \ln[w_{35}(k)/w_{33}(k)]$

$+ p_{23}* \ln[w_{45}(k)/w_{33}(k)] + p_{24}* \ln[y_{26}(k+1)/w_{33}(k)]$

$+ p_{91}* \ln[y_{34}(k+1)/y_{31}(k+1)] + p_{26}*w_{56}(k)$

$- w_{45}(k)*y_{30}(k+1)/y_{29}(k+1) = 0$

$q_{23}[\underline{x}(k\_,\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$

$= p_6 + p_{30}* \ln[y_{37}(k+1)/w_{33}(k)] + p_{35}* \ln[w_{35}(k)/w_{33}(k)]$

$+ p_{30}* \ln[w_{45}(k)/w_{33}(k)] + p_{39}* \ln[y_{26}(k+1)/w_{33}(k)]$

$+ p_{94}* \ln[y_{34}(k+1)/y_{31}(k+1)] + p_{41}*w_{56}(k)$

$- y_{26}(k+1)*y_{27}(k+1)/y_{29}(k+1) = 0$

$$q_{24}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$

$$= p_{14} + p_{70}*\ \ln[x_8(k+1)/w_{34}(k)] + p_{71}*\ \ln[w_{37}(k)/w_{34}(k)]$$
$$+ p_{69}*\ \ln[x_9(k+1)/w_{34}(k)] + p_{72}(\ \ln[w_{46}(k)/w_{34}(k)]$$
$$+ p_{96}*\ \ln[x_6(k+1)/y_7(k+1)] + p_{100}*\ \ln[x_7(k+1)/y_7(k+1)]$$
$$+ p_{74}*w_{56}(k) - x_8(k+1)*y_2(k+1)/x_5(k+1) = 0$$

$$q_{25}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$

$$= p_{15} + p_{71}*\ \ln[x_8(k+1)/w_{34}(k)\ \ *\ p_{76}*\ \ln[w_{37}(k)/w_{34}(k)]$$
$$+ p_{75}*\ \ln[x_9(k+1)/w_{34}(k)]\ *\ p_{77}*\ \ln[w_{46}(k)/w_{34}(k)]$$
$$+ p_{97}*\ \ln[x_6(k+1)/y_7(k+1)] + p_{101}*\ \ln[x_7(k+1)/y_7(k+1)]$$
$$+ p_{79}*w_{56}(k) - w_{37}(k)*y_{24}(k+1)/x_5(k+1) = 0$$

$$q_{26}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$

$$= p_{12} + p_{69}*\ \ln[x_8(k+1)/w_{34}(k)]\ *\ p_{75}*\ \ln[w_{37}(k)/w_{34}(k)]$$
$$+ p_{62}*\ \ln[x_9(k+1)/w_{34}(k)] + p_{63}*\ \ln[w_{46}(k)/w_{34}(k)]$$
$$+ p_{95}*\ \ln[x_6(k+1)/y_7(k+1)] + p_{99}*\ \ln[x_7(k+1)/y_7(k+1)]$$
$$+ p_{65}*w_{56}(k) - x_9(k+1)*y_{33}(k+1)/x_5(k+1) = 0$$

$$q_{27}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$

$$= p_{17} + p_{72}*\ \ln[x_8(k+1)/w_{34}(k)] + p_{77}*\ \ln[w_{37}(k)/w_{34}(k)]$$
$$+ p_{63}*\ \ln[x_9(k+1)/w_{34}(k)] + p_{84}*\ \ln[w_{46}(k)/w_{34}(k)]$$
$$+ p_{98}*\ \ln[x_6(k+1)/y_7(k+1)] + p_{102}*\ \ln[x_7(k+1)/y_7(k+1)$$
$$+ p_{86}*w_{56}(k) - w_{46}(k)*y_{28}(k+1)/x_5(k+1) = 0$$

$$q_{28}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$

$$= p_1 + p_{92}*\ \ln[y_{37}(k+1)/w_{33}(k)] + p_{93}*\ \ln[w_{35}(k)/w_{33}(k)]$$
$$+ p_{91}*\ \ln[w_{45}(k)/w_{33}(k)] + p_{94}\ \ln[y_{26}(k+1)/w_{33}(k)]$$
$$+ p_{21}*\ \ln[y_{34}(k+1)/y_{31}(k+1)] + p_{22}*w_{56}(k)$$
$$- y_{35}(k+1)*y_{34}(k+1)/y_{29}(k+1) = 0$$

$$q_{29}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$

$$= p_{11} + p_{96}* \ln[x_8(k+1)/w_{34}(k)] + p_{97}* \ln[w_{37}(k)/w_{34}(k)]$$
$$+ p_{95}* \ln[x_9(k+1)/w_{34}(k)] + p_{98}* \ln(w_{46}(k)/w_{34}(k)]$$
$$+ p_{58}* \ln[x_6(k+1)/y_7(k+1)] + p_{80}* \ln[x_7(k+1)/y_7(k+1)]$$
$$+ p_{60}*w_{56}(k) - y_{39}(k+1)*x_6(k+1)/x_5(k+1) = 0$$

$$q_{30}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$

$$= p_{16} + p_{100}* \ln[x_8(k+1)/w_{34}(k)] + p_{101}* \ln[w_{37}(k)/w_{34}(k)]$$
$$+ p_{99}* \ln[x_9(k+1)/w_{34}(k)] + p_{102}* \ln[w_{46}(k)/w_{34}(k)]$$
$$+ p_{80}* \ln[x_6(k+1)/y_7(k)] + p_{81}* \ln[x_7(k+1)/y_7(k+1)]$$
$$+ p_{82}*w_{56}(k) - y_{25}(k+1)*x_7(k+1)/x_5(k+1) = 0$$

$$q_{31}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$

$$= y_{19}(k+1) + w_{53}(k) + w_{12}(k) + w_7(k) - x_6(k+1) = 0$$

$$q_{32}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$

$$= y_{27}(k+1) + y_{28}(k+1) - x_7(k+1) = 0$$

$$q_{33}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$

$$= y_{36}(k+1) + w_{52}(k) + w_{11}(k) + w_8(k) - y_{34}(k+1) = 0$$

$$q_{34}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$

$$= y_{33}(k+1) + y_{30}(k+1) - y_{31}(k+1) = 0$$

$$q_{35}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$

$$= [w_{41}(k)*w_{52}(k) + w_{30}(k)*w_{11}(k) + w_{27}(k)*w_8(k)$$
$$- (1 + w_{54}(k))*y_{35}(k+1)*y_{34}(k+1)]/[-y_{36}(k+1)] - y_{41}(k+1) = 0$$

$$q_{36}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= [w_{31}(k)*w_{13}(k) + w_{28}(k)*w_{9}(k)$$
$$- (1 + w_{55}(k))*y_{40}(k+1)*y_{7}(k+1)]/[-y_{6}(k+1)] - x_{10}(k+1) = 0$$

$$q_{37}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= w_{45}(x)*y_{30}(k+1) + x_{9}(k+1)*y_{33}(k+1) - y_{32}(k+1)*y_{31}(k+1) = 0$$

$$q_{38}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= y_{26}(k+1)*y_{27}(k+1) + w_{46}(k)*y_{28}(k+1) - y_{25}(k+1)*x_{7}(k+1) = 0$$

$$q_{39}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= w_{66}(k)*y_{32}(k+1) - x_{9}(k+1) = 0$$

$$q_{40}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= w_{63}(k)*y_{25}(k+1) - y_{26}(k+1) = 0$$

$$q_{41}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= y_{23}(k+1) + y_{24}(k+1) + w_{21}(k) + w_{23}(k) + w_{24}(k) - y_{22}(k+1) = 0$$

$$q_{42}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= [1 - w_{59}(k)]*[w_{35}(k)*y_{23}(k+1) + w_{37}(k)*y_{24}(k+1)$$
$$+ w_{36}(k)*w_{21}(k) + w_{38}(k)*w_{23}(k)] - y_{20}(k+1)*y_{22}(k+1) = 0$$

$$q_{43}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= [(y_{37}(k+1)*w_{47}(k) - x_{10}(k+1)*w_{61}(k))*(1 - w_{57}(k))/$$
$$(1 - w_{57}(k)*w_{43}(k) - w_{19}(k)) + w_{1}(k) - w_{50}(k)*x_{10}(k+1)]/$$
$$x_{10}(k) - y_{42}(k+1) = 0$$

$$q_{44}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= [(x_8(k+1)*w_{48}(k) - x_{10}(k+1)*w_{62}(k))*(1 - w_{58}(k))/$$
$$(1 - w_{58}(k)*w_{44}(k) - w_{20}(k)) + w_1(k) - w_{51}(k)*x_{10}K+1)]/$$
$$x_{10}(k) - y_{43}(k+1) = 0$$

$$q_{45}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= [1 + y_{14}(k+1)]*x_{11}(k)*[p_{104} + p_{90}*w_{56}(k)]*[1 +$$
$$(p_{19}*y_{20}(k+1)*y_3(k+1) + p_{19}*y_4(k+1))/(1 + y_{14}(k+1))*x_{11}(k)]/$$
$$[p_{103} + p_{52}*w_{56}(k)] - y_{16}(k+1) = 0$$

$$q_{46}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= -[p_8 + p_{45}* \ln(y_{21}(k+1)) + p_{47}* \ln(y_{17}(k+1)) + p_{46}*\ln(y_5(k+1))$$
$$+ p_{44}* \ln(y_{38}(k+1)) + p_{48}*w_{56}(k) + y_{20}(k+1)*y_{21}(k+1)/y_{16}(k+1)]$$
$$= 0$$

$$q_{47}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= -[p_{1o} + p_{47}* \ln(y_{21}(k+1)) + p_{83}* \ln(y_{17}(k+1))$$
$$+ p_{68}* \ln(y_5(k+1)) + p_{61}* \ln(y_{38}(k+1)) + p_{89}*w_{56}(k)$$
$$+ w_{25}(k)*y_{17}(k+1)/y_{16}(k+1)] = 0$$

$$q_{48}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= -[p_9 + p_{46}* \ln(y_{21}(k+1)) + p_{68}* \ln(y_{17}(k+1)) + p_{50}* \ln(y_5(k+1))$$
$$+ p_{49} \ \ln(y_{38}(k+1)) + p_{51}*w_{56}(k) + y_{15}(k+1)*y_5(k+1)/y_{16}(k+1)] = 0$$

$$q_{49}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= y_{38}(k+1)*w_{39}(k) - y_{36}(k+1) = 0$$

$$q_{50}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k+1),\underline{w}(k+1),k]$$
$$= y_{17}(k+1)*w_{39}(k) - y_{18}(k+1) = 0$$

$q_{51}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$

$= y_{18}(k+1) - w_2(k) - y_{19}(k+1) = 0$

$q_{52}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$

$= w_{67}(k)*[x_3(k)*y_{42}(k+1) + x_4(k)*y_{43}(k+1)]/[x_3(k) + x_4(k)$
$+ x_1(k) + x_2(k)] - y_{14}(k+1) = 0$

$q_{53}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$

$= [y_{16}(k+1) - y_{38}(k+1)*y_{41}(k+1) - y_5(k+1)*y_{15}(k+1)$
$- y_{21}(k+1)*y_{20}(k+1)]/y_{17}(k+1) - w_{25}(k+1) = 0$

$q_{54}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$

$= [y_{37}(k+1)*y_1(k+1) + w_{35}(k)*y_{23}(k+1) + w_{45}(k)*y_{30}(k+1)$
$+ y_{26}(k+1)*y_{27}(k+1) - y_{35}(k+1)*y_{34}(k+1) - y_{32}(k+1)*y_{31}(k+1)]/$
$[-w_{33}(k)] - y_{44}(k+1) = 0$

$q_{55}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$

$= [x_8(k+1)*y_2(k+1) + w_{37}(k)*y_{24}(k+1) + x_9(k+1)*y_{33}(k+1)$
$+ w_{46}(k)*y_{28}(k+1) - y_{39}(k+1)*x_6(k+1) - y_{25}(k+1)*x_7(k+1)$
$- y_{40}(k+1)*x_7(k+1)*y_7*k+1)]/[-w_{34}(k)] - y_{45}(k+1) = 0$

$q_{56}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$

$= [w_{42}(k)*w_{53}(k) + w_{32}(k)*w_{12}(k) + w_{26}(k)*w_7(k)$
$- (1 + w_{60}(k))*y_{39}(k+1)*x_6(k+1)]/[-y_{19}(k+1)] - y_{46}(k+1) = 0$

$q_{57}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$

$= x_{10}(k+1)*[x_3(k+1) + x_4(k+1) + x_1(k+1) + x_2(k+1)]*w_{68}(k)$
$+ w_{29}(k)*w_{10}(k) + w_{40}(k)*w_{49}(k) - y_{47}(k+1)$

$$q_{58}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= y_{47}(k+1)/w_{39}(k) - x_{11}(k+1) = 0$$

$$q_{59}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= y_{21}(k+1)*w_{39}(k) - y_{48}(k+1) = 0$$

$$q_{60}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= x_{12}(k+1) - (0.822514*y_{21}(k+1) + 1.07866*y_{5}(k+1)$$
$$+ 0.934999*y_{17}(k+1) + 2.49568*y_{38}(k+1) + 0.855918 = 0$$

$$q_{61}[\underline{x}(k),\underline{x}(k+1),\underline{y}(k+1),\underline{u}(k),\underline{w}(k),k]$$
$$= x_{10}(k+1)*y_{6}(k+1) + w_{29}(k)*[w_{10}(k) - w_{10}(k-1)] + w_{40}(k)*[w_{49}(k)$$
$$- w_{49}(k-1)] - y_{49}(k+1) = 0$$

A.3 State Variables

x_1 = KCD

x_2 = KRS

x_3 = KSE

x_4 = KSN

x_5 = VN

x_6 = NC

x_7 = NM

x_8 = p_{KN}

x_9 = p_{EMN}

x_{10} = p_I

x_{11} = HW

x_{12} = F

A.4 Output Variables

y_1 = KE

y_2 = KN

y_3 = HLH

y_4 = HEL

y_5 = HCK

y_6 = I

y_7 = NI

y_8 = ICD

y_9 = IMIN

y_{10} = INE

y_{11} = INN

y_{12} = IRS

y_{13} = CK

y_{14} = MW

y_{15} = p_{CK}

y_{16} = VF*

y_{17} = HCND

y_{18} = CND

y_{19} = CNQ

y_{20} = p_L

y_{21} = HLJ

y_{22} = L

y_{23} = LE

y_{24} = LN

---

*VF = $p_F \cdot F$

$y_{25} = p_{NM}$

$y_{26} = p_{NME}$

$y_{27} = NME$

$y_{28} = NMN$

$y_{29} = VE$

$y_{30} = EME$

$y_{31} = EM$

$y_{32} = p_{EM}$

$y_{33} = EMN$

$y_{34} = EC$

$y_{35} = p_{EC}$

$y_{36} = CE$

$y_{37} = p_{KE}$

$y_{38} = HCE$

$y_{39} = p_{NC}$

$y_{40} = p_{NI}$

$y_{41} = p_{CE}$

$y_{42} = ME$

$y_{43} = MN$

$y_{44} = RE$

$y_{45} = RN$

$y_{46} = p_{CN}$

$y_{47} = W$

$y_{48} = LJ$

$y_{49} = S$

A.5 Exogenous Variables

w_1 = CAPG

w_2 = CIM

w_3 = CK

w_4 = DCD

w_5 = DRS

w_6 = EL

w_7 = EXN

w_8 = EXE

w_9 = EXI

w_{10} = G

w_{11} = GEQ

w_{12} = GNC

w_{13} = GNI

w_{14} = DUM1*

w_{15} = DUM2*

w_{16} = DUM3*

w_{17} = DUM4*

w_{18} = DUM5*

w_{19} = ITCE

w_{20} = ITCN

w_{21} = LG

w_{22} = LH

w_{23} = LR

---

*These are dummy variables used internally in the OPCON model subroutine.

w_{24} = LU

$w_{25} = p_{CND}$

$w_{26} = p_{EXC}$

$w_{27} = p_{EXE}$

$w_{28} = p_{EXI}$

$w_{29} = p_{G}$

$w_{30} = p_{GE}$

$w_{31} = p_{GI}$

$w_{32} = p_{GNC}$

$w_{33} = p_{RE}$

$w_{34} = p_{RN}$

$w_{35} = p_{LE}$

$w_{36} = p_{LG}$

$w_{37} = p_{LN}$

$w_{38} = p_{LR}$

$w_{39} = p_{R}$

$w_{40} = p_{STE}$

$w_{41} = p_{STN}$

$w_{42} = p_{VDN}$

$w_{43} = p_{XEE}$

$w_{44} = p_{XNN}$

w_{45} = QKE

w_{46} = QKN

w_{47} = R

w_{48} = REPE

w_{49} = REPN

w_{50} = STE

w_{51} = STN

w_{52} = TEC

w_{53} = TI

w_{54} = t (TIME)

w_{55} = TKE

w_{56} = TKN

w_{57} = TL

w_{58} = TNC

w_{59} = TPE

w_{60} = TPN

w_{61} = ZEN

w_{62} = ZINE

w_{63} = ZINN

w_{64} = ZNE

w_{65} = ZRN

w_{66} = ZW

w_{67} = ALAG

w_{68} = ZICD

w_{69} = ZIRS

w_{70} = QCK

A.6 Parameters

p_1 = AEC

p_2 = AEE

p_3 = AEK

p_4 = AEL

p_5 = AEM

p_6 = AEN

p_7 = AER

p_8 = AJ

p_9 = AK

p_{10} = AN

p_{11} = ANC

p_{12} = ANE

p_{13} = ANI

p_{14} = ANK

p_{15} = ANL

p_{16} = ANM

p_{17} = ANN

p_{18} = ANR

p_{19} = A_1

p_{20} = A_2

p_{21} = BECC

p_{22} = BECT

p_{23} = BEEE

p_{24} = BEEN

p_{25} = BEER

p_{26} = BEET

p_{27} = BEKE

p_{28} = BEKK

p_{29} = BEKL

p_{30} = BEKN

p_{31} = BEKR

p_{32} = BEKT

p_{33} = BELE

p_{34} = BELL

p_{35} = BELN

p_{36} = BELR

p_{37} = BELT

p_{38} = BEMC

p_{39} = BENN

p_{40} = BENR

p_{41} = BENT

p_{42} = BERR

p_{43} = BERT

p_{44} = BJE

p_{45} = BJJ

p_{46} = BJK

p_{47} = BJN

p_{48} = BJT

p_{49} = BKE

p_{50} = BKK

p_{51} = BKT

p_{52} = BNCC

p_{53} = BNCM

p_{54} = BMJ

p_{55} = BMK

p_{56} = BMN

p_{57} = BMT

p_{58} = BNCC

p_{59} = BNCM

p_{60} = BNCT

p_{61} = BNE

p_{62} = BNEE

p_{63} = BNEN

p_{64} = BNER

p_{65} = BNET

p_{66} = BNIC

p_{67} = BNIM

p_{68} = BNK

p_{69} = BNKE

p_{70} = BNKK

p_{71} = BNKL

p_{72} = BNKN

p_{73} = BNKR

p_{74} = BNKT

p_{75} = BNLE

p_{76} = BNLL

p_{77} = BNLN

p_{78} = BNLR

p_{79} = BNLT

p_{80} = BNMC

p_{81} = BNMM

p_{82} = BNMT

p_{83} = BNN

p_{84} = BNNN

p_{85} = BNNR

p_{86} = BNNT

p_{87} = BNRR

p_{88} = BNRT

p_{89} = BNT

p_{90} = BO

p_{91} = CECE

p_{92} = CECK

p_{93} = CECL

p_{94} = CECN

p_{95} = CNCE

p_{96} = CNCK

p_{97} = CNCL

p_{98} = CNCN

p_{99} = CNME

p_{100} = CNMK

p_{101} = CNML

p_{102} = CNMN

p_{103} = GM

p_{104} = GO

APPENDIX B: OPCON -- A GENERAL PURPOSE COMPUTER CODE FOR THE SOLUTION OF NONLINEAR TIME-VARYING OPTIMAL CONTROL PROBLEMS

APPENDIX B

OPCON -- A GENERAL PURPOSE COMPUTER CODE FOR THE SOLUTION OF NONLINEAR TIME-VARYING OPTIMAL CONTROL PROBLEMS

B.1 Introduction

This appendix presents a brief summary of the structure and capabilities of OPCON, a computer code designed for the solution of nonlinear, time-varying optimal control problems. Several features of the program are specifically designed for use in connection with econometric models. The code is entirely written in FORTRAN IV and its design is such that it places minimum demands on the software operating environment. Because of the use of object-time dimensioning, the size of the internal array spaces and therefore the overall storage requirements are at the control of the user. There are, therefore, no hard limits on the dimensionality of the systems that can be handled.

A popular approach to the solution of large-scale optimal control problems for econometric models has been the reformulation of the problem as a static optimization problem which involves "stacking up" the time sequence of control variables into one large vector. One of the arguments often made in favor of this approach is that it bypasses the apparently bewildering problem of computing implicit Hamiltonians in models for which contemporaneous endogenous variables are mutually interdependent in a simultaneous fashion. It turns out that this difficulty is more apparent than real. The static optimization approach ignores the central principle of optimal control theory: i.e., the decomposition of a dynamic problem

TABLE B1: SYNOPSIS OF OPCON

- Purpose: To solve optimal control problems of nonlinear time-varying systems in discrete-time with nonlinear cost functionals.

- Method: There are four optional solution methods:
  - (i) First-order differential dynamic programming, small variations method
  - (ii) Second-order differential dynamic programming, small variations method
  - (iii) Min-H first variation method
  - (iv) Differential dynamic programming, global variations method*

- Scope: There are no hard limits on the dimensionality of the systems that can be handled by OPCON.

- Input: The user, in addition to providing a subroutine that specifies the state equations and the cost functional, must specify an initial guess for the optimal control path and must give the value of the initial state vector. The user has to select from the various optional features provided for a given solution method.

- Output: After convergence, the program will print out the optimal control path and the optimal state trajectory; if convergence is not achieved within the prescribed number of iterations, then the best current approximations to the optimal control path and state trajectory will be printed. More detailed printouts of individual iterations as well as plotting of any system variables are included as optional features.

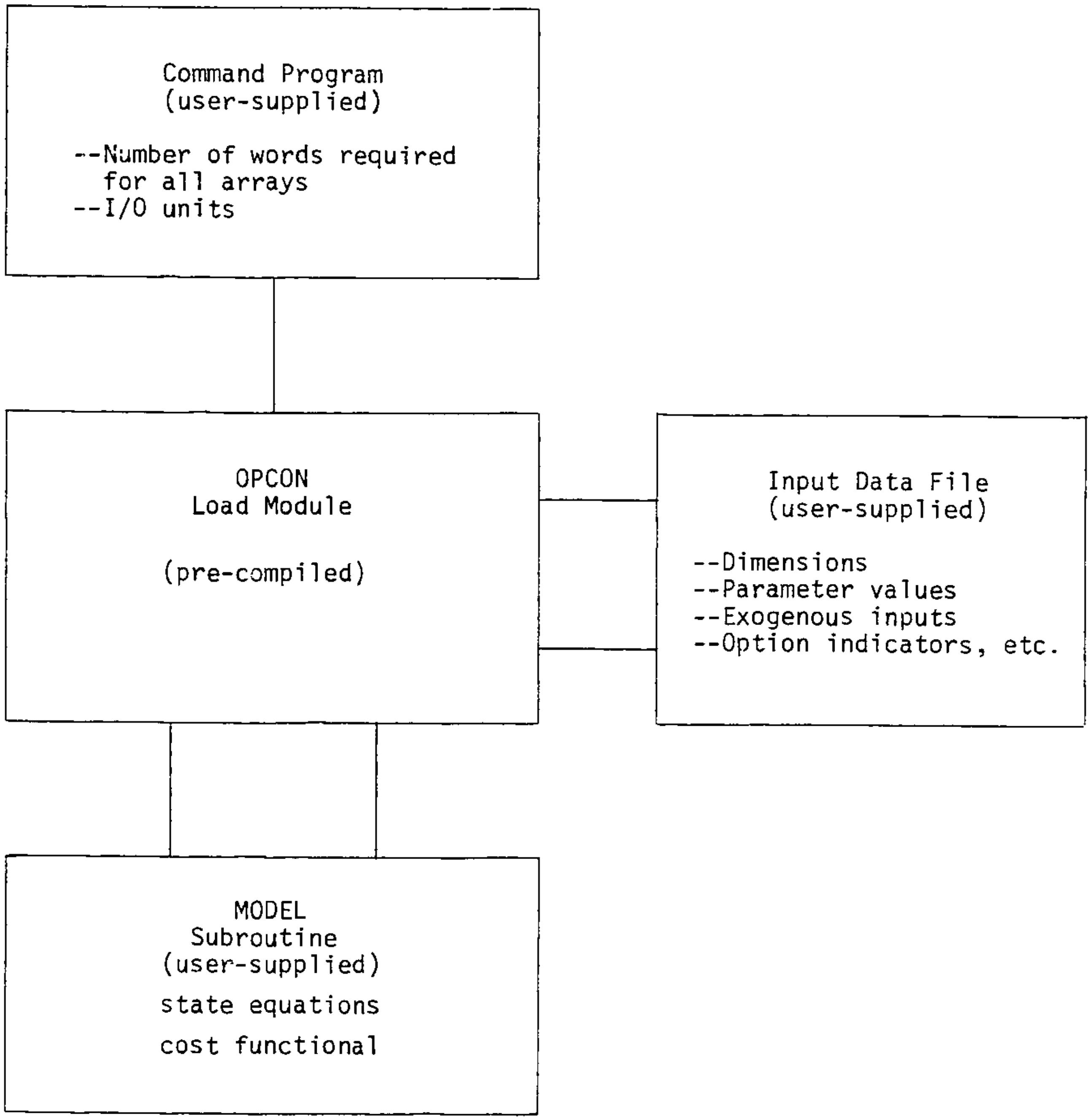

Fig. B1: Information flow between OPCON and user's sub-program into a sequence of static problems and, in addition, fails to provide the conceptual insights derived from the state-space formulation -- e.g., the information provided by the optimal co-state vectors and local feedback matrices.

There are three optional solution methods in OPCON, all of which are generalizations of well-known algorithms in optimal control theory to the case of implicit state transition mappings and implicit Hamiltonians:

(a) First-order differential dynamic programming, small variations method (DDP1)

(b) Second-order differential dynamic programming, small variations method (DDP2)

(c) First-order Min-H method (MINH1)

The various solution algorithms differ as to their relative efficiency, robustness and convergence properties. The versatility thus afforded is imperative when dealing with nonlinear dynamic optimization problems. OPCON incorporates special-purpose numerical differentiation subroutines for the efficient computation of first- and second-order derivatives for the case of implicit systems.

In order to use OPCON, the user must provide a subroutine in FORTRAN that gives the model's equations and structure of the cost functional, he must also write a command program consisting of two instructions that define the overall array space and the input and output devices to be used. The <u>model</u> subroutine and the <u>command</u> program constitute the <u>user sub-program</u>. The user sub-program is then compiled and linked with the pre-compiled OPCON load module. In addition, the user must provide an <u>input data file</u> that gives information on dimensionality of the variables, exogenous inputs, selection of options, etc. The standard option in OPCON is numerical computation of derivatives; if the user wishes to by-pass this option and enter analytical derivatives, he must include them in appropriate sub-routines as part of the user's sub-program.

B.2 Program Highlights

The following are some of the main provisions and optional features of OPCON:

- The program can handle systems with implicit state equations and algebraic constraints. This provision is especially important for applications involving econometric models, where it is often not possible to recast even the simplest systems in explicit state equation form.
- Special provisions are included for the treatment of block-recursive systems; the user must specify the number and composition of the blocks.
- Each optional solution algorithm may be used individually or sequentially in a sequence specified by the user; in the case of sequential use, the control variables for each succeeding optimization will be initialized to the values resulting from the preceding run.
- In subroutine DDP1, step-size adjustment can be determined by any one of three optional methods: if no direct search is desired, then the step-size will be adjusted uniformly across time-stages; if it is desired to search for an optimum step-size at each time-stage, the user can select between two linear search methods: golden section search and quadratic interpolation.
- Subroutines DDP2 and DDPG include various optional features in

addition to the standard options associated with second-order differential dynamic programming. For instance, the local feedback gain matrices may be set equal to the identity matrix, or may be recomputed only every k_1 time-stages or every k_2 iterations. In addition, several provisions are included to correct for the cases where the Hessian matrix is ill-conditioned or is indefinite and fails to generate descent directions.

- After convergence occurs in DDP2 or DDPG, there is the option of printing out a sequence of matrices that correspond to a <u>time-varying linearization</u> about the optimal trajectory; the option includes printouts of the linearized system matrix A(t), the linearized input matrix B(t), the Ricatti matrix corresponding to the linearized system, K(t), the linearized gain matrix G(t) and the linearized closed loop system matrix ACL(t).
- The derivatives required by the selected solution method can be pre-computed analytically and entered into the program as part of the user sub-program. Alternatively, the user has the option of requesting that all required derivatives be computed internally by appropriate numerical differentiation subroutines included as part of OPCON.
- Standard output of the program includes a listing of all relevant input data and parameters and the optimal control path and optimal state trajectory after convergence; the user can suppress part of this standard output if desired. In addition, the user can request more detailed printouts of the solution process, including values of the current control paths and state trajectories at each iteration. A plotting subroutine will plot graphs of any single

variable or pair of variables as a function of time.

° Special provisions are included that allow for recursive simulations and optimizations. These options facilitate the performance of:

-- Sensitivity studies

-- Use of penalty function methods

-- Convexification of singular or ill-conditioned problems

B.3 - Algorithms

The two differential dynamic programming methods available in OPCON (DDP1, DDP2) are generalizations of the first- and second-order algorithms proposed by Jacobson and Mayne [B1]; the development of these algorithms is based on the application of the principle of optimality about a local neighborhood of a nominal trajectory, using a Taylor series expansion of the cost-to-go function. The MINH1 algorithm is a straightforward generalization of Kelley's method [B2] and is most easily derived by a direct application of the Minimum Principle. The four algorithms, when considered jointly, span a fairly wide range in terms of their relative efficiency, robustness and convergence properties. The first-order differential dynamic programming method with small variations (DDP1) is the least expensive of the methods and, correspondingly, is the least attractive from the standpoint of robustness and convergence properties; MINH1 and DDP2 are approximately equal in robustness but DDP2 has better convergence properties and is more expensive. A short description of each method follows:

(a) DDP1 - First-order differential dynamic programming, small variations

The first-order small variations version of differential dynamic programming is a successive approximation algorithm in which at every iteration a new control sequence is constructed which minimizes a linearized expansion of the value function about the current nominal trajectory. It has a good deal of common structure with various gradient-type algorithms and differs from them primarily in the way the step-size is controlled so as to remain within the region of linearization.

The main steps in the algorithm are as follows:

Step 1.- Given an initial state vector $\underline{x}_0$ and an initial guess for the control variables $[\underline{u}_k]$, compute the corresponding nominal trajectory $[\underline{x}_k]$.

Step 2.- Starting at the terminal time NT, compute the sequence of first-order approximations to the value function $[V_x^k(\underline{x}_k)]$ about the nominal trajectory. Compute the expected change in cost a_0.

Step 3.- Compute a new control sequence $[\overset{new}{u_k}]$ as follows:

$$\overset{new}{u_k} = \overset{old}{u_k} - \varepsilon \cdot H_u^k$$

where ε is the step-size and H_u^k is the gradient of the Hamiltonian with respect to u.

Step 4.- Using the updated control sequence, compute the corresponding state trajectory and the new value of the cost function V; compare the change in cost $V - \underline{V}$ with the expected change in cost a_0. If $V - \underline{V}$ is negative and $|V - \underline{V}|/a_0 \geq c$, accept the new trajectory. Otherwise, reduce ε and repeat Step 3.

Step 5.- Reset the current nominal control path and state trajectory, reset the step-size and go to Step 2.

Step 6.- Stop when $| a_0 | < \eta$.

In subroutine DDP1, the user is required to provide the following input parameters:

EPSIL - Initial value for the step-size.

DELMIN - Lower bound on the ratio of actual to expected cost change for acceptance of new trajectory (c in the above description). Typical value is 0.5.

ETA - Parameter to be used as a convergence criterion (η).

ITRM - Maximum number of iterations to be performed.

ITRCHM - Maximum number of step-size reductions to be performed in any one given iteration.

Step-size adjustment will be performed by one of three available methods according to the value of the indicator ISERCH:

If ISERCH=0 Step size will be set equal to predetermined fixed value uniformly across time stages

If ISERCH=1 A search for the optimum step-size will be performed by the golden section search method (Murray [B3], p. 9)

If ISERCH=2 A search for the optimum step-size will be performed by the quadratic interpolation method (Murray [B3], p. 9)

The two search methods provided, based on function comparison and function approximation, respectively, can be used either separately or jointly to synthesize virtually any arbitrary search method.

(b) DDP2 - Differential Dynamic Programming; Second-Order Small Variations

The successive approximations algorithm implemented in DDP2 can be derived by formally applying the principle of optimality to a second-order approximation of the value function about the current nominal trajectory.

Special conditions are applied to insure that at every iteration, the updated control path and corresponding trajectory will not go outside the region where the quadratic approximation is valid. In the case of linear systems and quadratic criteria, the algorithm reduces to the solution of the discrete-time Riccati equation and thus converges in one iteration. Structurally, the algorithm bears resemblance to the second-variation successive sweep method (McReynolds and Bryson [B4]) and other Newton-type algorithms but is computationally more efficient and does not impose the restriction of positive definiteness on the inverse Hessian H_{uu}. The major steps of the second-order differential dynamic programming algorithm are the following:

Step 1.- Same as DDP1

Step 2.- Calculate the sequence of second-order approximations to the value function backwards in time: $[V_x^k(\underline{x}_k)]$ and $[V_{xx}^k(\underline{x}_k)]$; store the sequence of local gain matrices.

Step 3.- Compute a new control sequence using

$$u_k^{new} = u_k^{old} + \varepsilon \cdot P_k + Q_k \cdot (x_k - \underline{x}_k)$$

where $[P_k]$ and $[Q_k]$ are sequences of local gain matrices computed in Step 2.

Steps 4-6.- Same as DDP1

The set of parameters required from the user are the same as those listed for DDP1. In addition, there exist the following provisions:

° The matrices P_k and Q_k in Step 3 are computed in terms of the inverse of the extended Hessian matrix C_k. In the event that C_k is either singular, ill-conditioned or indefinite, the basic method must be modified in order to generate descent directions. OPCON includes the following options to deal with this situation:

(i) Replace C_k by the identity matrix I for those time-stages for which it is ill-conditioned or indefinite.

(ii) Substitute the inverse of C_k by the generalized Penrose inverse (Wilkinson [B5]).

(iii) Use Greenstad's variant of Newton's method in order to improve the direction of search when C_k is indefinite. This can be costly since it involves an eigenvector analysis (Greenstadt [B6]).

(iv) Modify C_k by using Murray's variation of the Marquardt-Levenberg method. This involves the Cholesky factorization of C_k suitably modified so as to insure numerical stability even when it is not positive-definite (Murray [B3]).

Since these modifications of the basic method -- especially (iii) and (iv) -- can become computationally costly, it is recommended that they be used only in the event that the simple variation indicated in (i) has been shown to be ineffective.

° If the user so specifies, the matrices P_k and Q_k will only be recomputed every k_1 time-stages or every k_2 iterations. This may result in substantial computational savings and may not hinder the overall effectiveness of the algorithm especially when restricted to the early set of iterations.

° After convergence occurs in DDP2, the user has the option of requesting printouts of the following time-varying matrices linearized about the optimal trajectory:

- Linearized system matrices A(k)

- Linearized input matrices B(k)
- Riccatic matrix corresponding to the linearized system K(k)
- Linearized gain matrix G(k)
- Linearized closed-loop system matrix ACL(k)

(c) MINH1 - First-Order Min-H Method

The first-order Min-H method is an iterative scheme wherein each successive control sequence is selected after minimizing the Hamiltonian at each stage with respect to the control inputs, where the Hamiltonian is computed using a first-order expansion of the value function. The basic steps are:

Step 1.- Same as DDP1

Step 2.- Same as DDP1

Step 3.- Find u_k^* to minimize the Hamiltonian function

$$H^k\ (x_k,\ u_k^{old},\ P_k)$$

Step 4.- For $\lambda = 1$, define the new control sequence

$$u_k^{new} = u_k^{old} - \lambda\ (u_k^{old} - u_k^*)$$

Step 5.- If cost does not decrease, reduce λ , $0 \leq \lambda \leq 1$, repeat Step 4.

Step 6.- Stop if no improvement for $\lambda < \eta$

The crucial step, of course, is the minimization of the Hamiltonian in Step 3. Three options are available to perform this functional minimization:

- Newton (indirect)

- Davidon-Fletcher-Powell
- Gauss-Newton

B.4 - Implicit State Equations, Algebraic Constraints

A dynamical system in discrete time can be characterized by means of the state equation

$$x(k+1) = f[x(k), u(k), w(k), k] \qquad (1)$$

where x is the state vector, u is the control vector and w is a vector of exogenous variables -- i.e., uncontrollable inputs.

If the state transition mapping f is given by an analytical expression, then the system is fully recursive and the state equations are said to be of the explicit form. It may occur, on the other hand, that the dynamical behavior of the system is given by a set of equations of the form

$$\emptyset[x(k+1), x(k), u(k), w(k), k] = 0 \qquad (2)$$

and that an expression for x(k+1) in closed form cannot be obtained. In this case, the state equations are said to be of the implicit type. If the state representation of a dynamical system includes equations of both forms (1) and (2), the state equations are said to be of the mixed type.

OPCON has provisions that allow it to handle systems of the explicit, implicit or mixed types. All the required first and second-order derivatives will be computed internally using special-purpose implicit differentiation subroutines. If the numerical differentiation option is not selected in the case of implicit systems, then the user will be required to provide appropriate analytical derivatives of the implicit state equations $\emptyset$ which will be used internally to compute the implicit derivatives of the state transition mapping f.

It can be seen that the implicit state formulation is sufficiently general to include any set of algebraic constraints involving state, control or exogenous variables as a special case.

The ability to handle implicit state equations is crucial for applications involving econometric models. The implicit state equation corresponds to the structural form of an econometric model, whereas an explicit state equation can only be obtained if a reduced form of the model exists. An analytical reduced form often does not exist for the simplest kinds of models.

The solution of the implicit system of equations is performed by a nonlinear algebraic equation solving subroutine based on Newton's method. Frequently it occurs that econometric models can be partitioned into blocks of equations such that an equation in any given block does not depend on variables in any of the succeeding blocks. Considerable computational savings can be achieved by exploiting this block-recursive structure. OPCON provides the user with the option of specifying a block-recursive structure when entering the model equations; this structure is utilized internally by the program both to solve the state equations at each time period and to compute the numerical derivatives.

B.5 Summary

We have presented an overview of OPCON, a user-oriented, self-contained computer code whose capabilities include:

- Solution of nonlinear, time-varying optimal control problems with nonlinear cost functionals
- Simulation of nonlinear implicit systems
- Performance of sensitivity studies
- Computation of local optimal feedback matrices

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