Key Characteristic Coupling and Resolving Key Characteristic Conflict

by

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ABSTRACT

Real complex assemblies have to deliver large number of customer requirements. Assemblies in general have many parts which work together to deliver those requirements. The involvement of many parts and presence of many requirements to be delivered, results in the involvement of a part in the delivery chains of more than one requirement. As a result most of the requirements are not delivered independently. Coupling among the requirements makes it hard to achieve all the requirements with in their respective tolerance limits.

The thesis gives classification of nature of relationships that can exist among various requirements. It discusses characteristic of each relationship and how it can affect the robustness of an assembly. When the requirements in the assembly are conflicting, i.e. reduction in variation in one of the requirements increases variation in conflicting requirement, it tends to become non-robust. Non-robust assemblies entail high manufacturing costs.

Aim of the thesis is to identify the scenarios of conflict in the assembly. Screw theory can be used to find the presence of coupling among requirements in the assembly. It can also be used to identify the nature of coupling. If coupling suggests that requirements are coupled, we analyze the intensity of the conflict. Not all conflicts need to be solved. Only the conflicts that will make assembly miss tolerance limits on its requirements need to be solved.

The thesis outlines some of the methods that can be used to either resolve conflict or reduce the amount of conflict in the assembly. Conflicts can be removed from the assembly by making suitable changes in design. Design changes will modify DFCs of the conflicting requirements. Use of appropriate assembly techniques can also remove

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conflicts from the assembly. An assembly without any conflicts is more robust and can be produced at a less cost as compared to the one having conflicts.

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Title: Senior Research Scientist

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Chapter 1. INTRODUCTION

1.1 Motivation

Assemblies have to deliver more than one requirement to meet the demands of the customer. The requirements are the product, process features that have significant bearing on the cost, functioning and safety of the assembly [Thornton, 1999]. Sometimes the requirements posed on an assembly are more than number of parts / subassemblies in the assembly. This leads to the conflict with in the requirements [Whitney et al., 1999]. The presence of conflict among the requirements in the assembly increases the risk of not meeting the requirements with in the specified tolerance limits. These conflicts make the product non-robust and lead to potentially high manufacturing costs [Söderberg and Johannesson, 1998]. [Whitney et al., 1999] have been focusing on these conflicts. They have given strategies to find out the conflicts that exist in the assembly, and to use the inherent freedom of parts in the assembly to resolve the conflicts.

Finding out the conflicts and resolving the critical conflicts will have tremendous impact on the industry. A systematic way to analyze and resolve the conflicts will result in more robust assemblies, which will bring down the production costs. The conflicts can be found out using Screw Theory [Konkar 1993; Konkar 1995], Datum Flow Chain (DFC) [Mantripragada and Whitney, 1998] and doing motion and constraint analysis on the assemblies [Shukla, 2001].

Many a times these conflicts arise due to negligence on the part of designers. A methodical approach to drive out most of the conflicts at the time of design will save lot of rework of design and will reduce the demand of high precision equipment to manufacture the product. The assurance of meeting the requirements even in presence of conflict in already existing design will also reduce the rework of design.

1.2 Goal of Research

The goal of this thesis is to assist people in industry,

- To find out the scenarios of Key Characteristic (KC) conflict in their products

- To focus on the scenarios in which the conflict will lead to non-robust assemblies

- To outline various methods that can be used to remove the conflicts from their products

If more than one KC is being delivered by an assembly, chances of KC conflict increases. Constraint and Motion Analysis [Shukla, 2002] using Screw Theory can be used to determine the nature of the relationship that exists among various KCs that are being delivered by the assembly. In some cases there exists assembly sequence which removes the conflict from the product. In other cases we can find alternative ways to remove or reduce the amount of conflict from the product.

All the conflicts are not equally important. The conflicts that make the assemblies non-robust are of higher priority, and need to be solved first. We will outline an algorithm

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which will distinguish the scenarios of KC conflicts that make the assembly non-robust from the ones that will not affect the robustness of the assembly significantly.

1.3 Organization of the Thesis

The thesis presents a way to determine the scenarios of KC conflicts in an assembly and outlines the approaches that can be used to remove or reduce the intensity of those conflicts from the assembly. Chapter 2 will introduce the notion of Key Characteristics and Datum Flow Chain. It will also outline the classification scheme of the constraint states of parts in an assembly. Screw theory and 4x4 matrices concepts will be highlighted. Algorithm of doing constraint and motion analysis using Screw Theory will also be discussed.

In chapter 3 we will see that there exist many other kinds of relationships among KCs in an assembly apart from KC conflict. KCs can either be independent or coupled. In this chapter we will discuss the classification of the KC relationships and will give examples of each kind of relationship. KC relationship is significantly different in case of Type-1 (assemblies that are constrained completely by feature relations between their parts) and Type-2 assemblies (assemblies that are under constrained by their features and need fixtures to add the additional constraint). We will present these differences for Type-1 and Type-2 assemblies.

Chapter 4 will give an algorithm for detecting the kind of KC coupling that exists in an assembly. Concepts of screws, wrenches, DFC will be used to determine the nature of

coupling. Constraint and Motion analysis using Screw theory, as mentioned in chapter 2 will be used in this chapter to analyze the nature of KC relationship in the assembly.

Chapter 5 will study the effect of KC conflict on the robustness of an assembly. It also outlines various methods that can used to find the variation stack-up in the KCs of the assembly. Use of Screw Theory to find the variations coming in closed loops will be presented. The result of the analysis will be compared with the results from Kinematic Analysis. After that we will seek an easy way to reduce the adverse effect of conflict on robustness by tightening the tolerances of parts involved in the delivery chains of the conflicting KCs. Variation synthesis for the parts involved in the delivery chains of conflicting KCs will also be presented.

Chapter 6 will introduce the reader to the various methods that can be used to resolve or reduce the intensity of KC conflict in an assembly. A case study will be given for each method. Chapter 7 will present the conclusion of the thesis. It will give algorithms to remove conflict for both existing assemblies and assemblies that are in their design phase. Finally it will present the areas for future work.

Chapter 2. KEY CHARACTERISTICS

2.1 Key Characteristics

"Key characteristics are the product, subassembly, part, and process features whose variation from nominal significantly impact the final cost, performance, or safety of a product" [Thornton, 1999]. Each assembly has to deliver customer level design objectives. Functional requirements (*FR*s) are set of requirements that characterize customer level design objectives [Magarb, 1997]. Design parameters (*DP*s) are the physical entities that are created by the design process to meet the *FR*s [Magarb, 1997]. Key characteristic is each specific measurable attribute, or characteristic, of each *FR*.

Key characteristic goes by different names, as Significant Characteristics, Key Product Characteristics, Functionally Important Topics, Engineering Characteristics, Critical-to-Function, and Critical-to-Quality.

A KC is said to be "delivered" if *FR* is achieved within some specified tolerance limit. If this characteristic is not delivered, the products' attractiveness reduces. KCs are those attributes that are critical, affected by variation-sensitive characteristic, and are worth controlling (cost effective).

KCs that are readily observed by a customer are called "customer-level KCs" or customer's requirements (*CR*s) [Magarb, 1997]. Anything that is important to a customer can be considered as KC. The examples of customer level KCs in an automobile

comprises of satisfactory sealing of the door, satisfactory door closing effort, and acceptable engine performance.

"Assembly-level KCs" refers to the requirements imposed on features and functions of the product. "Customer-level KCs" can be matched to "assembly-level KCs" using a formalized method called Quality function deployment (QFD) [Hauser and Clausing, 1988]. QFD begins with customer's requirements (customer-level KCs) and records the relative importance of those requirements. It then relates each of the customer-level KCs to corresponding assembly-level KCs.

Satisfactory sealing of door is achieved by having proper geometric relationship between parts of automotive frame and seal. Acceptable door closing effort depends upon the geometric relationship between door inner panel and body seal. For satisfactory engine performance, there are certain geometric relationships being imposed on the cylinder block, cylinder head and pistons. Using QFD we can do the flow-down to geometric assembly-level KCs from customer-level KCs.

Assembly-level KCs depend on manufactured parts making up delivery chain (delivery chain is a chain of parts that act together in a specific order in order to deliver KC) for each KC. These parts have "manufacturing-level KCs" associated with them. Manufacturing-level KCs are certain dimensions that parts must meet in order to deliver acceptable assembly-level KCs when put together. Such manufacturing-level KCs are commonly called as part tolerances.

In the remainder of the thesis, term KC will refer to geometric assembly-level KC, and will be denoted by a double red line. Figure 2.1 shows a diamond ring. The diameter of the ring is the customer-level KC, which also happens to be the geometric assembly-level KC for the ring.

Fig. 2.1: Diamond Ring

2.1.1 Measurement Points

Measurement Points of the geometric assembly-level KC are the points where variation effects can easily be observed. In case of fit and finish KC between outer part of door and body of an automobile, the variations at the corners are readily observable and give fair judgment about the "delivery" of the KC. Measurement Points are located at the places where variation effects are pronounced. For the fit and finish KC as mentioned above, the measurement points lie at the corners (or vertices) of the door (figure 2.2).

The delivery of KC can be ascertained by the variation coming on to the measurement points. If the variation on the measurement points is with in the tolerance limit on each point, KC is said to be delivered.

Measurement
Points
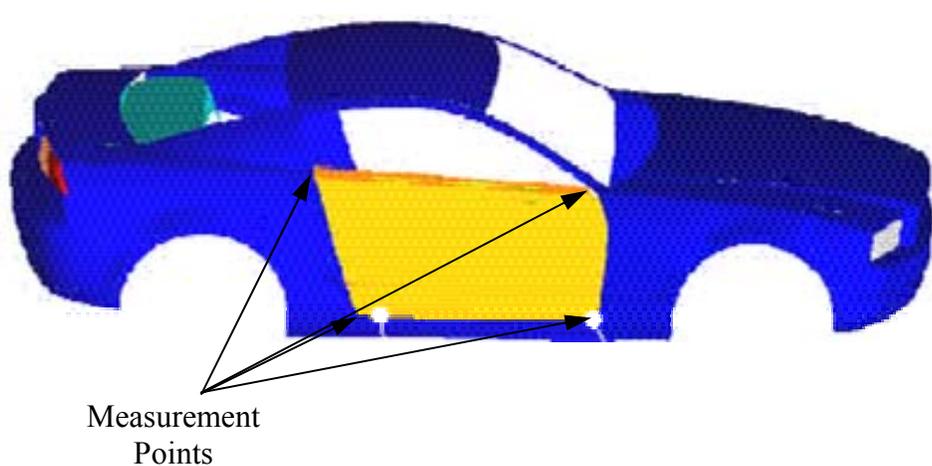
Fig. 2.2: Measurement Points for Fit and Finish KC

2.2 Datum Flow Chain (DFC)

During assembly various parts from different sources come together. Assembly can be considered as a dimensional and constraint relationship between those parts. Datum Flow Chain (DFC) is the concept that captures these dimensional and constraint relationships [Mantripragada and Whitney, 1998]. DFC helps in integrating system level design with

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initial stages of product design. DFC provides a set of tools and techniques for defining, documenting and evaluating the system level design decisions.

DFC is a useful tool in designing an assembly. To design an assembly we need fundamental structure of a top-down design process that shows how the assembly is supposed to deliver its KCs. The process comprises following steps [Whitney]

- Represent the top-level goals of the assembly (customer-level KCs)

- Link these goals to engineering requirements on the assembly and its parts (geometric assembly-level KCs). This process is known as Key Characteristics Flow-down [Thornton, 1999]

- Show how the parts will be constrained, and what features will be used to establish constraint, so that the parts will acquire their desired spatial relationships that achieve the KCs

- Show where the parts will be in space relative to each other both under nominal conditions and under variation

- Show how each part should be designed, dimensioned, and toleranced to support the plan

- Assure the robustness of the plan

A clear statement of these elements for a given assembly is called the *design intent* of the assembly. DFC captures the design intent of the assembly. It is a method of documenting a location strategy of the parts and relating the strategy to the delivery of product's KCs.

2.2.1 Background and Prior Work in DFC

Assemblies have been modeled systematically by [Lee and Gossard, 1985], [Sodhi and Turner, 1992], [Srikanth and Turner, 1990], and [Roy, Bannerjee and Liu, 1989] among others. Such methods are intended to capture relative part location and function, and enable linkage of design to functional analysis methods like kinematics, dynamics, and, in some cases, tolerances. Almost all of them need detailed descriptions of parts to start with, in order to apply their techniques. [Gui and Mäntylä, 1994] have attempted to apply a function-oriented structure modeling to visualize assemblies and represent them in varying levels of detail. DFC does not attempt to model assemblies functionally. DFC begins at the point where the functional requirements have been established and there is at least a concept sketch.

Top-down design of assemblies emphasizes the shift in focus from managing design of individual parts to managing the design of entire assembly in terms of mechanical "interfaces" between parts. [Hart-Smith, 1997] proposes eliminating or at least minimizing critical interfaces in the structural assembly rather than part-count reduction as a means of reducing costs. He emphasizes that, at every location in the assembly structure, there should only be one controlling element that defines location, and everything else should be designed to "drape to fit". In DFC the controlling element is a mate (a liaison that transfers location and dimensional constraint from one part to another) and the joints that drape to fit are contacts (liaisons that do not transfer dimensional or location constraints but have other important functions, such as attachment or reinforcement). [Muske, 1997] describes the application of dimensional

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management techniques on 747 fuselage sections. He describes a top-down design methodology to systematically translate key characteristics to critical features on parts and then to choose consistent assembly and fabrication methods.

Academic researchers have generated portions of the foundation. [Shah and Rogers, 1993] proposed an attributed graph model to interactively allocate tolerances, perform tolerance analysis, and validate dimensioning and tolerancing schemes at the part level. This model defines chains of dimensional relationships between different features on a part and can be used to detect over and under dimensioning (analogous to over- and under- constraint) of parts. [Wang and Ozsoy, 1990] provide a method for automatically generating tolerance chains based on assembly features in one-dimensional assemblies. [Shalon at al., 1992] show how to analyze complex assemblies, including detecting inconsistent tolerances datums, by adding coordinate frames to assembly features and propagating the tolerances by means of 4x4 matrices. [Zhang and Porchet, 1993] present the Oriented Functional Relationship Graph, which is similar to DFC, including the idea of a root node, propagation of location, checking of constrains, and propagation of tolerances. A similar approach is reported by [Tsai and Cutkosky, 1997] and by [Johannesson and Söderberg, 1998]. The DFC is extension of these ideas, emphasizing the concept of designing assemblies by designing DFC first, then defining the interfaces between parts at an abstract level, and finally providing detailed part geometry.

2.2.2 Properties of DFC

A datum flow chain is a directed acyclic graphical representation of an assembly with nodes representing the parts, and arcs representing the mates between them. Each node represents a part or fixture and every arc transfers dimensional and location constraint along one or more DOFs from the node at the tail to node at the head (figure 2.3). Cycles, or loops, in a DFC would mean that a part locates itself once the entire cycle is traversed and hence not permitted. From this, we can infer that there must be a single node in a DFC, a part that is not constrained by any other part in an assembly. This represents the part from which the assembly process begins, which is the base part or the base fixture. Every arc constrains certain degrees of freedom depending upon the type of mating conditions it represents. Each arc has an associated 4x4 transform matrix that represents mathematically how the part at the head of the arc is located with respect to the part at the tail of the arc. The DFC for ring in figure 2.1 is shown in figure 2.3.

Ring   Diamond

6
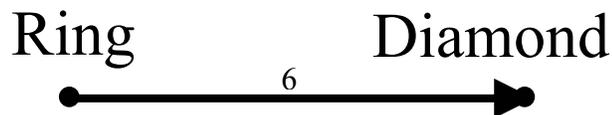
Fig. 2.3: DFC

Every arc is labeled to show which degrees of freedom it constrains, which depends upon on the type of mating conditions it represents. The sum of the unique degrees of freedom constrained by all the incoming arcs to a node in DFC should be equal to six (less if there are some kinematic properties in the assembly or designed mating conditions such as bearings or slip joints which can accommodate some amount of pre-determined motion; more if locked-in stress is necessary such as preloaded bearings). This is same as saying that each part should be properly constrained, except for cases where over- or under-constraint is necessary for a desired function.

There are certain assumptions being made to model the assembly process using a DFC. These are:

1. All parts in the assembly are assumed rigid. Rigid part is located completely once its position and orientation in 3-D is determined.

2. Each assembly operation completely locates a part being assembled with respect to existing parts in the assembly or an assembly fixture. After locating the part completely it is fastened to remaining parts in the assembly.

Assumption 1 states that each part is considered to be fully constrained once the three translations and three rotations are established. If an assembly, such as preloaded pair of ball bearings, must contain locked-in stress in order to deliver its KCs, the parts should still be sensibly constrained and located kinematically first, and then a plan should be developed for imposing the over-constraint in the desired way. Assumption 2 rationalizes the assembly process and it makes incomplete DFCs to make sense. An incomplete DFC

represents a partially completed assembly. If the parts in a partially completed assembly are not completely constrained, by each other or by fixtures, it is not reasonable to expect that they will be in proper condition for receipt of subsequent parts, in-process measurements, transport, or other actions that may require an incomplete assembly to be dimensionally coherent and robust.

DFC can be drawn for a number of scenarios. Part-level DFC represents the relationship between various locating features with in the same part. DFC can also be drawn for a subassembly or for given localized assembly problem. DFC can also be drawn for complex assemblies in their entirety. Figure 2.4 shows DFC of the KCs between door and body of an automobile.
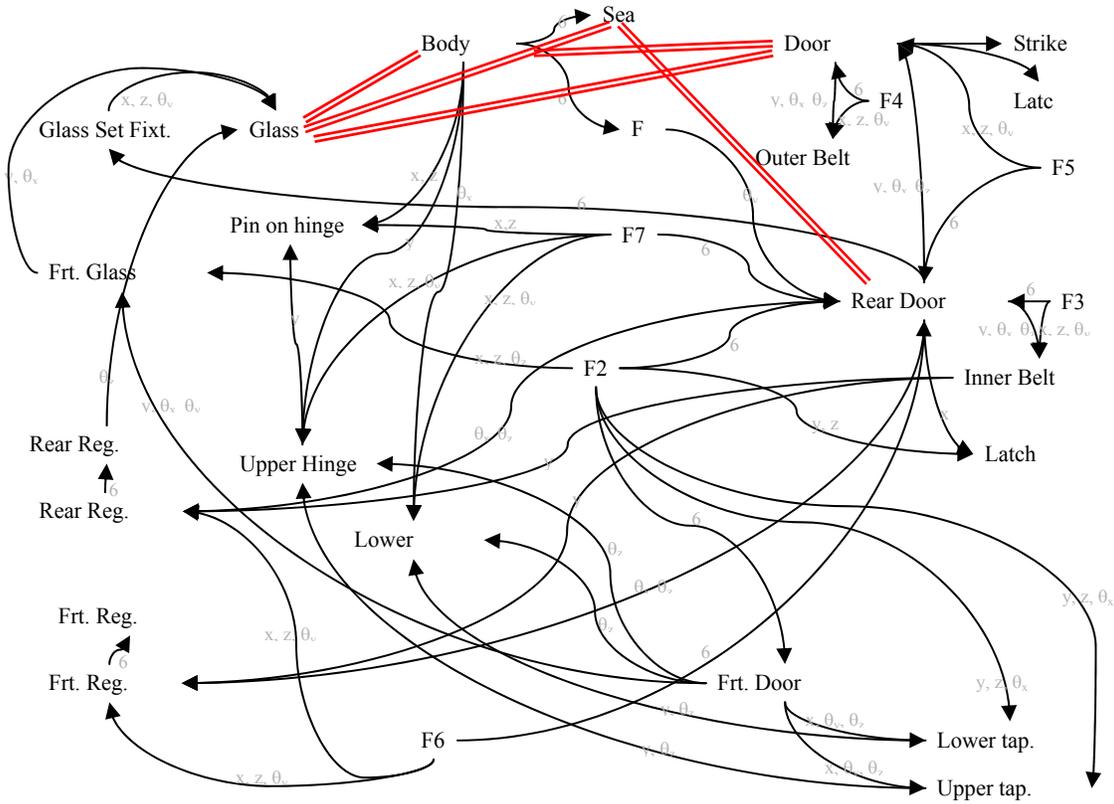
Fig. 2.4: DFC of the KCs between door and body of an
automobile. KCs are represented by double red lines,
mates by arrows (arrow represents transfer of
dimensional and constraint responsibility), and contacts
by dashed lines

DFC also depends upon the assembly sequence of an assembly. A single set of parts can
yield a number of distinct DFCs. Also a single DFC can potentially generate multiple
assembly sequences. Software developed at MIT uses DFC as an input and generates as
an output a set of possible assembly sequences, using the assumptions described above
[Baldwin et al., 1991].

2.3 Assembly Features: Mates, Contacts, and Hybrid Mate-Contacts

Parts are joined by assembly features in any assembly. A typical part has multiple assembly features joining it with other parts in the assembly. Not all of these features transfer location and dimensional constraint, and it is essential to distinguish the ones that transfer constraints from the ones that do not but provide other functions such as strength. Based on this distinction, assembly features can be split into three types: mates, contacts and hybrid mate-contacts.

2.3.1 Mates

Mates are assembly features that convey information about the transfer of location and dimensional constraint from one part to another. On a DFC diagram, a mate is represented by an arc with an arrow, with the direction of the arrow inferring the dimensional responsibility – the node (part) at the tail end of the arrow locates the node (part) at the head of the arrow. DFC represents KC delivery by tracing a chain of mates between the parts defining KC.

A mate can transfer constraint and location to any number and any combination of the six degrees of freedom (three rotational and three translational). For example, a square peg in square hole constrains all six degrees of freedom, while a round peg fitting into a round hole constrains five degrees of freedom and one rotational degree of freedom is left free. Total of seventeen distinct features have been catalogued in [Adams, 1998], with their constraint properties documented.

Assembly sequence determines the direction of the mate. Relative to a specific part mates can either be incoming or outgoing, based on the assembly sequence. The direction of the mate depends on the assembly sequence associated with the particular DFC being examined. A change in the assembly sequence can change the direction of the mate arrow.

2.3.2 Contacts

Assembly features that do not transfer dimensional or location constraint are called contacts. These features can have other important functions, such as attachment or reinforcement, but are not intended by the designer to convey dimensional and location constraint from one part to another. On a DFC diagram, a contact is represented by a dashed line.

2.3.3 Hybrid Mate-Contacts

Some assembly features can transfer dimensional and location constraint in some directions and can act as reinforcement in other directions. These assembly features are mates in some direction and contacts in other directions. Such assembly features are called Hybrid Mate-Contact. On a DFC diagram, a hybrid mate-contact is represented by a dashed arrow.

The process of assembly is not just of fastening parts together but should be thought of as a process that first defines the location of parts using the mates and then reinforces their location, if necessary, using contacts.

2.4 Constraint States

Constraint is the property of a design. Based on the mates involved in the assembly, a resulting product can be under-, over-, or properly constrained. A part has six degrees of freedom – three translational and three rotational. A part is considered properly constrained if each one of these degrees of freedom is constrained once and only once. Properly constrained assembly is composed entirely of properly constrained parts. Properly constrained assembly is robust to variations. In properly constrained assemblies delivery chains of KCs remain invariant even if there are variations coming on parts/fixtures/features. In over-constrained assemblies it is difficult to ascertain the delivery chains of KCs in presence of variation in parts/fixtures/features. The delivery chains of KCs in over-constrained assemblies may change depending upon which part/fixture/feature has significant amount of variation in its dimension.

Both under- and over-constraint may be required for functionality of the assembly, for example freely-rotating crankshaft of an engine (under-constraint), and preloaded ball bearings (over-constraint). Sometimes under- and over-constraint might be due to error while designing an assembly. Under these conditions, under-constraint will lead to random variations in the assembly that may cause non-delivery of some KCs. Over-constraint will either cause some random variation, or induce local stress in the parts or both.

Figure 2.5 shows cross sectional view of an inline-engine. In inline-engine some parts are properly constrained, crank shaft is under constrained and front cover is over constrained. Block is the base part and all other parts are located with respect to block.

Inline Engine

Sectional View   Front View

Properly Constrained
Over-Constrained
Under-Constrained
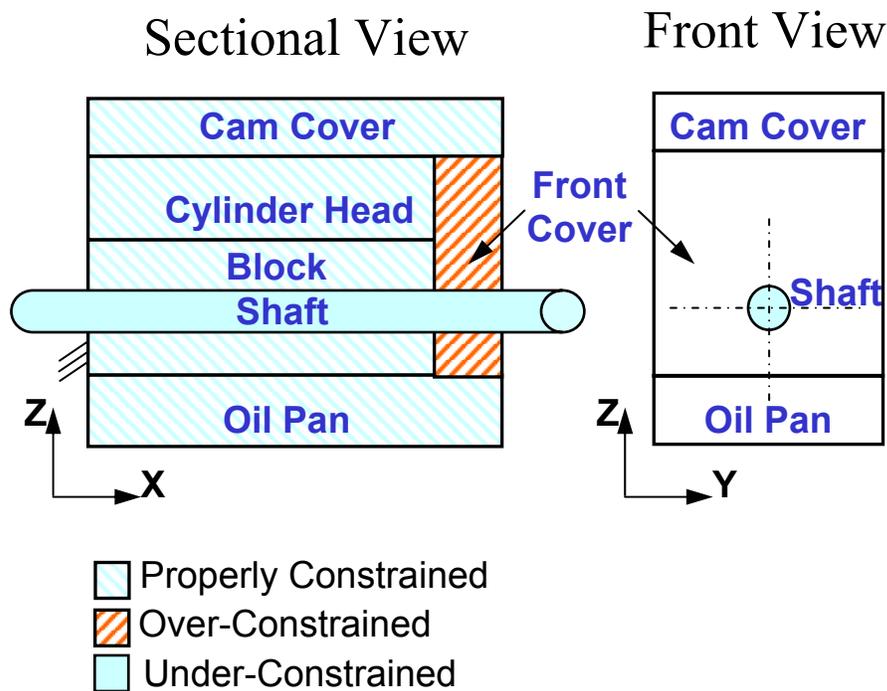
Fig. 2.5: Sectional View of Inline-Engine showing the constraint state the parts. It is also a Type-1 assembly

2.5 Assembly Architecture

All the parts in the assembly have to be located properly to meet KCs of the assembly. In certain assemblies parts get their location completely from other parts in the assembly, with base part defining the root of DFC. But in other assemblies, parts have relative degree of freedom with respect to other parts in the assembly during the process of assembly. To locate such parts completely until they are fastened together, we need fixtures. To clarify our approach of designing assemblies, we need to distinguish between two kind of assemblies, Type-1 assemblies and Type-2 assemblies.

2.5.1 Type-1 Assemblies

Type-1 assemblies are the assemblies that are constrained completely by feature relations between their parts. Type-1 assemblies are also called part-defined assemblies because the variation in the final assembly is determined completely by the variation contributed by each part in the assembly. The assembly process merely puts the parts together by joining their pre-defined mating features. Expressing Type-1 assembly in terms of DFC we can say that in Type-1 assembly every part has at least one mate with at least one other part in the assembly. The inline-engine shown in figure 2.5 is an example of Type-1 assembly.

2.5.2 Type-2 Assemblies

Type-2 assemblies are the assemblies that are under-constrained by their features and need fixtures to add the additional constraint. Most aircraft fuselage and automotive body assemblies are Type-2 assemblies and their body parts are usually given some or all of

the location constraint during the assembly process. The locating scheme for this kind of assemblies must include careful consideration of the assembly process. A different datum flow logic, assembly sequence, etc. will result in quite different assembly configurations, errors and quality. It is possible to build a perfect assembly out of imperfect parts and vice versa by choosing appropriate or inappropriate datum flow chain logic. Expressed in terms of DFC, Type-2 assembly is one where it is possible to have only contacts between all parts in the assembly. In such cases, the parts will have mates with fixtures, which are used to locate them. Type-2 assembly in general has mixture of contacts and mates. Figure 2.6 shows assembly sequence of a Type-2 assembly which happens to be underbody of car. Figure 2.7 shows DFC for the above assembly.
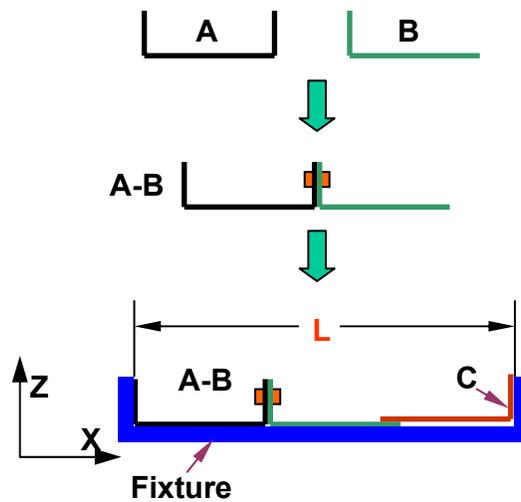
Fig. 2.6: Assembly Sequence of Underbody of Car (Type-2 Assembly)
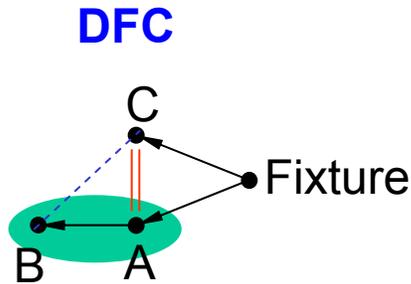
Fig. 2.7: DFC of
Underbody of Car

2.6 Mathematical methods used with DFC

2.6.1 4x4 Matrix Transformations

Matrix transformations provide mathematics needed to locate parts with respect to each other. Each part is assumed to have a base coordinate frame. The mathematical representation takes form of 4x4 matrices. This method of modeling spatial relations between objects in kinematic linkages was used by [Denavit and Hartenberg, 1955]. [Paul, 1981], [Simunovic, 1976], [Popplestone and Ambler, 1979], [Wesley, Taylor, and Grossman, 1980] used 4x4 matrices in assemblies and robotics. In CAD, researchers like Steven Coons used it to represents locations of objects in a computer in 1960s [Ahuja and Coons, 1968]. In 1980s, CAD researchers made assembly models of mechanical parts this way. [Lee and Gossard, 1985] The same mathematical model can be used for both chains of links in a linkage and chains of more general parts in an assembly.

The 4x4 matrix transformation permits representation of both the relative position of two objects and their relative orientation. The matrix represents the location and orientation of an entire coordinate frame. The mathematical form of the transformation is

$$T = \begin{bmatrix} R & p \\ 0^T & 1 \end{bmatrix} \qquad \qquad \ldots(2\text{-}1)$$

In the above equation, p is a 3x1 vector that indicates the position of the new frame relative to the old one, while R is 3x3 rotation matrix indicating the orientation of the new frame relative to the old one. (Superscript T indicates a vector or matrix transpose. By convention, all vectors are assumed to be column vectors, so a transposed vector is a row vector.)

The connective model of assembly defines a part as having a central coordinate frame and one or more assembly features. Each feature has its own coordinate frame. Assembly of two parts then consists of putting the frames of their features together. If two parts having the base coordinate frames as X and Y, being joined together by a mating feature with its coordinate frame F, is represented as

$$T_{XY} = T_{XF} * T_{FY} \qquad \qquad \ldots(2\text{-}2)$$

where T_{ij} is the 4x4 matrix transform between frame i and the frame j, expressed in the frame i coordinates.

[Whitney et al., 1994] extended the variation analysis based on 4x4 matrix transform approach to statistical analysis of GD&T. This approach of calculating accumulated variation in the part location due to variations in part-level dimensions combines the

variations in the location of assembly features by multiplying the matrix transforms representing the variations along the tolerance delivery chain. The tolerance delivery chain reflects the intent of the design team which is captured by DFC.

2.6.2 Screw Theory

Screw theory is used to determine the state of constraint of an assembly [Shukla and Whitney, 2001]. [Ball, 1900], [Waldron, 1966] used screw theory to determine the relative degrees of freedom (dof) between any two bodies in a mechanism. [Mason and Salisbury, 1985] have used screw theory to characterize the nature of different types of contacts between objects and robot gripper hands. [Ohwovoriole and Roth, 1981] Applied screw theory to come with two new types of screw systems. These new screw systems are called *repelling* and *contrary* screws. [Konkar, 1993], [Konkar 1995] developed screw system representation of assembly mating features and used the methods of screw theory to determine the number of relative degrees of freedom between any two parts in an assembly.

2.6.2.1 Screw

A screw is an ordered 6-tuple which can represent both twist and wrench. The first triplet represents a line vector associated with a unique line in space. The second triplet represents a free vector which is not confined to a specific line of action but having a specific direction associated with unique point in space.

2.6.2.2 Twist

Twist is a screw that denotes the instantaneous motion of a rigid body. Twist is of the form:

$$T = [\omega_x, \omega_y, \omega_z, v_x, v_y, v_z] \qquad \qquad \ldots(2\text{-}3)$$

The first triplet represents the angular velocity of the body with respect to a global reference frame. The second triplet represents the velocity, in the global reference frame, of a point on the body or its extension that is instantaneously located at the origin of the global reference frame. The line vector[1] represents the rotation vector, if any, of the body, and is called instantaneous spin axis (ISA). The free vector[2] represents the body's translation, whose magnitude may depend on the location of the unique point associated with it. In case body can undergo more than one independent motion, there is a separate twist for each degree of freedom, and the set of all these independent motions is represented by combination of all the twists as a stack of rows, and is called a twist matrix.

2.6.2.3 Wrench

Wrench is screw that denotes the resultant force and moment of a force system acting on a rigid body. Wrench is of the form:

$$W = [f_x, f_y, f_z, m_x, m_y, m_z] \qquad \qquad \ldots(2\text{-}4)$$

---

[1] Line vector is a vector with a fixed location in space. Rotation about an axis is a line vector.
[2] Free vector can float in the space. It has no fixed location. Translation in a direction is a free vector.

The first triplet represents the resultant force in a global reference frame. The second triplet represents the resultant moment of the force system about the origin of the global reference frame. In case body is acted on or can several independent forces and moments, there is a separate wrench for each one, and the set of all these independent forces and moments is represented by combination all the wrenches as a stack of rows, and is called a wrench-matrix.

Wrench matrix also denotes the direction of variation propagation. Wrench matrix represents resultant force and moment (constraint) of a force system acting on a body, so when ever there is a chance of small motion (due to variations in part dimensions), it is transferred to another part in the directions in which it is constrained by the part which has a variable dimension. Figure 2.8 shows part A and part B in 2-D. Part B can constrain the motion of part A only in z direction but not in x direction. Whenever there is a variation in the dimensions of part B, only z variation in part B will cause part A to move. If we assume that the feature between part A and part B is frictionless then variation in x dimension of part B will have no effect on the position of part A. Thus variation in part B can propagate to part A only in z direction, which is the constraint direction as given by the wrench matrix of the feature between part A and part B.
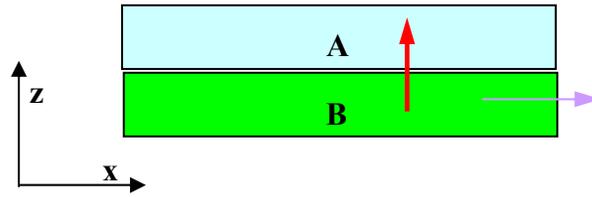
Fig. 2.8: 2-D Parts having
constraint in z direction

Wrench matrix for the feature between part A and part B is:

$$W_{AB} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$T_{AB} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

2.6.2.4 Relation between Twist and Wrench of a given feature

Rigid bodies can interact via features. [Adams 1998] has tabulated 17 different kinds of assembly features. Features allow rigid bodies to restrict each other's motions and exert forces and moments on each other. Twists express the motions and the wrenches express forces and moments. The wench cannot do any work along the direction of the twist.

Mathematically we can represent the situation as:

$$T = [\omega_x \ \omega_y \ \omega_z \ v_x \ v_y \ v_z]$$ ...(2-3)

$$W = [f_x \ f_y \ f_z \ m_x \ m_y \ m_z]$$ ...(2-4)

$$\Omega.M + v.F = 0$$ ...(2-5)

$$[\Omega{:}v]\begin{bmatrix} M \\ \\ F \end{bmatrix} = 0$$ ...(2-6)

Equation 2-6 suggests that wrench matrix and twist matrix for a given feature are null spaces of each other. (Null space of a given screw is its reciprocal). Thus from the given formulation we see that wrench and twist matrices are reciprocal of each other. If the rank of a twist is *n*, then the rank of its reciprocal wrench is *6-n*. The wrench-twist pair of a given feature form complementary spaces. If the twist describes directions along which motion is allowed, then the wrench matrix describes directions that can resist forces or moments.

2.6.3 Motion and Constraint Analysis using Screw Theory

[Shukla, 2001] gives the complete formulation of the algorithm for the motion and constraint analysis using screw theory. The method is to first convert DFC into Part-Feature diagram. (Part-Feature diagram is another representation of DFC). Assembly is represented as nodes. Both parts and features are represented as nodes. An assembly feature is typically between two parts. Each feature node is connected to the corresponding part nodes. There must be one and only one fixed part in Part-Feature diagram. This part will correspond to the root of DFC. Each assembly feature carries

some constraints. The constraints carried by an assembly feature can be represented as a wrench screw. Each feature node then has twist associated with it that represents the relative degrees of freedom between two parts. The reciprocal of twist-matrix is wrench-matrix that represents the constraints imposed by one part to another.

To do motion and constraint analysis, we need to identify all the paths [Shukla, 2001] from the part in question to the fixed part. A path is defined as a sequence of successive part and feature nodes starting from the part being analyzed and ending at the fixed part. For each path, we need to construct the twist-matrix for each assembly feature on the path, using the common (global) reference frame. All the twist-matrices associated with the feature nodes of the path need to be combined into one union twist-matrix (twist-union). Union of screws is given as:

$$\text{Union } (s_1 \; s_2 \; . \; . \; s_n) = \begin{pmatrix} s_1 \\ s_2 \\ . \; . \; . \\ s_n \end{pmatrix} \qquad \ldots(2.8)$$

To find the state of constraint of a part, form the intersection of all the effective twist-matrices representing different paths leading to the part under consideration from the fixed part. [Konkar, 1993] defines intersection of twist-matrices.

Intersection of screws is defined as:

Intersection (s_i) = Reciprocal {Union [Reciprocal $(s_i's)$]}

$$= \text{Reciprocal} \left\{ \begin{pmatrix} \text{Reciprocal } (s_1) \\ \text{Reciprocal } (s_2) \\ \dots\dots\dots \\ \text{Reciprocal } (s_n) \end{pmatrix} \right\} \qquad \dots(2\text{-}9)$$

If the intersection of effective twist-matrices results in a non-empty matrix it will represent under-constraints. The part will have as many degrees of freedom as the number of independent rows in the resultant intersection. An empty matrix (matrix having no rows and no columns) means that the part has no allowed motions. If the intersection of effective wrench-matrices results in a non-empty matrix it will represent over-constraints. The part will have as many degrees of freedom over-constrained as the number of independent rows in the resultant intersection. An empty matrix means that the part is not over-constrained in any degree of freedom.

2.7 Chapter summary

Key Characteristics (KCs) are the critical requirements that an assembly has to deliver in order to meet customer requirements. Key Characteristics are delivered by chain of parts, acting together to meet the requirements. Datum Flow Chain (DFC) is a graphical technique which captures part-to-part dimensional and location constraint relationships in the assembly. DFC is easy to put into practice because of its graphical nature.

DFC is a data structure which can store all the information about the assembly features that join one part to another in an assembly. It also gives direction of variation stack-up of each KC in an assembly.

Assembly can have many different kinds of features. Some pass both dimensional and location constraint from one part to another. Some features are present only to reinforce the strength of the constraint that has already been passed from one part to another. There also exists a class of features that do both of the above functions.

Based on the constraint on the degrees of freedom of a part, it can exist in three kinds of constraint states. Constraint state of the part in an assembly impacts its lifetime and function. It also dictates how that part passes the forces and variation. Some assemblies are features driven. The feature relationship between their parts completely determine the constrain state of each of the part. There also exists a class of assemblies that need extra fixtures to completely determine the constraint state of its parts. The assembly sequence has impact on the quality of the assembly for this class of assemblies.

4x4 matrix transformation and Screw Theory methods can be used with DFC to analyze assemblies. Screw Theory can be used to determine the constraint state of a part as well as to find the contribution of each part in the variation stack-up of a KC.

In the next chapter we will discuss classification of the nature of relationships that exist among various KCs in an assembly. We will also look at how feature relationships effect

the classification of KCs. Once we have classified the relationships, we will determine which sort of relationships we want to avoid in the assembly to make it more robust.

Chapter 3. CLASSIFICATION OF KEY CHARACTERISTIC RELATIONSHIP

3.1 Introduction

Automobiles, aircrafts and similar complex products have to meet a large number of customer requirements. These requirements can be quantitative like fit and finish, reliability or qualitative like appearance and comfort. These complex products have plethora of parts. The delivery of customer-level KCs (customer requirements) is accomplished by many parts acting together. The involvement of many parts and presence of more than one requirement sometimes leads to the involvement of a given part in the delivery of more than one requirement. Thus variation in a part has influence on more than one requirement. In this scenario it is difficult to deliver each of the requirements independently.

3.2 Prior Work

[Ulrich and Eppinger, 1995] have formulated product development in terms of determining customer requirements and converting them into engineering specifications, designs, and manufacturing plans [Hauser and Clausing, 1988]. [Lee and Thornton, 1996], [Thornton, 1999] Multiple KCs are addressed in the House of Quality [Hauser and Clausing, 1988]. Cunningham identified multiple conflicting KCs as a prime source of integration risk in complex products [Cunningham, 1998]. KC conflict in an integral architecture has been studied by [Whitney, Mantripragada, Adams and Rhee, 1999]. KC

conflict occurs when independence in the Datum Flow Chains of KCs cannot be achieved. KC conflict is undesirable, as it leads to a non-robust product and potentially high manufacturing costs [Söderberg and Johannesson, 1999]. An assembly exhibiting KC conflict is, in essence, coupling design parameters and functional requirements that is heavily discouraged in axiomatic design [Suh, 1990]. The disadvantages of this are clear – in order to deliver one KC properly, we have to impair the delivery of other conflicting KCs.

3.3 Classification

In the previous works only KC Conflict was talked about. But if we have more than one KC associated with an assembly then many combinations of KC relationship are possible. KCs can be independent, the case when the variation (statistically speaking variation means the standard deviation of the KC dimension from its nominal value) in one KC has no bearing on the variation in other KC in the assembly. We can control the variation in each of the KCs independently. If the variation in one KC has a bearing on the variation of other KC then in this case KC are said to be coupled. The variation in one KC can have both positive and negative influence on the variation in other KC. We need to classify this positive and negative influence of one variation on other. KC conflict is a special case of KC coupling when the variation in one KC has negative influence on the variation in the coupled KC.

3.3.1 Independent Key Characteristics

Key characteristics of an assembly are independent under two cases:

- KCs have independent DFCs. When the delivery chains of KCs pass through mutually exclusive parts, KCs are independent. In this case the DFCs of the KCs will not share any common node or arc (part/fixture/feature). Since DFCs do not share any part, thus variation on a part will have effect only on the KC it is associated with. Variation coming on each KC can be controlled independently in this scenario without affecting the variation in other KC.

- KCs share some common parts/fixtures/features but do not share any degree of freedom (dof). DFCs of the KCs can pass through the same part/fixture/feature but do not pass through the same degree of freedom. Figure 3.1 shows DFCs of two KCs, KC1 and KC2. DFCs of both KCs pass through parts B and C. But the dof of the feature between parts B and C that affects KC1 is different from the dof of the same feature that affects KC2.
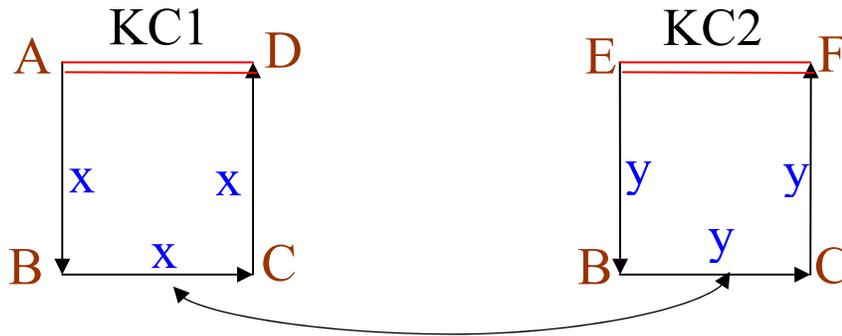
Fig. 3.1: Independent KCs, which share
common parts/feature but do not share
same degree of freedom

Figure 3.2 shows a scenario where KCs share common parts but do not share same dof. If we assume that part B is the base part which locates part C via two features, pin in a hole and pin in a slot. Part B can locate part C in all 6 dof. Parts A and D get their x location from parts B and C respectively. And parts E and F get their y location from parts B and C respectively. One of the KCs is the flushness in x directions of the positive x faces[3] of parts A and D. The other KC is the y distance between parts E and F. The DFC for this configuration is drawn in figure 3.1. We see that x variation in parts B and C will have effect on only flushness KC between parts A and D and no effect on the distance KC between parts E and F. Similarly y variation in parts B and C will have effect on only distance KC between parts E and F and no effect on flushness KC.

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[3] Positive face means that the outward normal from that face points in positive direction.
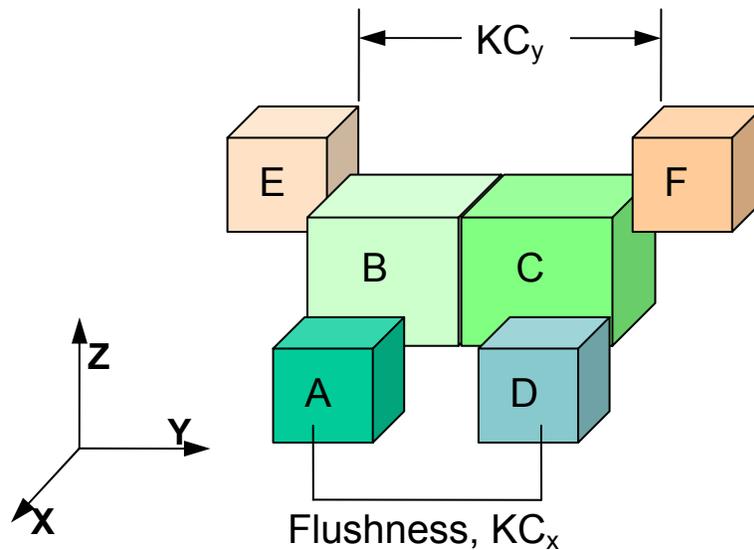
Fig. 3.2: Independent KCs. Flushness-KC between parts A and D, and Distance-KC between parts E and F are independent even though their DFCs pass though common parts B and C

3.3.2 Coupled Key Characteristics

Coupled Key Characteristics share some common parts/fixtures/features in their delivery chains in such a manner that controlling variation in one of KCs will have an effect on variation in other KC. The variations in coupled KCs can be either positively or negatively related. The variation relationship between KCs depends upon the effect of variation in common parts/fixtures/features on the variations of KCs. Based on this relation between the variations in KCs we can classify relationship among KCs as:

- Correlated Key Characteristics

- Conflicting Key Characteristics

- Correlated-Conflicting Key Characteristics

3.3.2.1 Correlated Key Characteristics

In an assembly if we try to reduce variation in one of KCs and it simultaneously reduces variation in other KC, then KCs are called correlated KCs. Correlated KCs share some common parts/fixtures/features in their delivery chains. And if we try to reduce the variation in the common parts it reduces the variations on all the correlated KCs. Figure 3.3 shows two parts joined together, part A and part B. One of the KCs is the overall length from part A to part B and other KC is the overall length of part B. Figure 3.4 and 3.5 show the DFCs for the two KCs mentioned above. DFC for KC1 goes from part A to the left end of part B and then finally to the right end of part B. KC1 is the distance from the left end of part A to the right end of part B. DFC for KC2 goes from left end of part B to right end of part B. KC2 is the distance from the left end of part B to the right end of part B.

From figure 3.4 and 3.5 we see that part B is common in the DFCs of the two KCs that are being studied. Therefore we will look at the effect of variation in the common part B on the variation in the two KCs. For the analysis we assume that there is no variation coming in the length of part A.

From figure 3.3 we can say that

$$KC1 = Length\_of\_part\_A + Length\_of\_part\_B \qquad \qquad \dots(3\text{-}1)$$

$$KC2 = Length\_of\_part\_B \qquad \qquad \dots(3\text{-}2)$$

If only the length of part B was allowed to vary, then on differentiating the above equations and denoting the differentials as the error coming on each of the variables (given the fact that dimension of A is not allowed to vary), we will get

$$error(KC1) = error(Length\_of\_part\_B) \qquad \qquad \dots(3\text{-}3)$$

$$error(KC2) = error(Length\_of\_part\_B) \qquad \qquad \dots(3\text{-}4)$$

On plotting (figure 3.6) the errors in the KCs as a function on error in the length of part B, we see that the error in the KCs increase with increase in error in the length of part B. Since the errors in the KCs are positively correlated, implying that if we try to reduce variation in one of the KCs, by reducing the variation in length of part B we will simultaneously reduce the variation in other KC. Thus KCs are correlated with respect to variation in part B. Here variation and error means the same thing. Statistically speaking variation/error implies the standard deviation of the KC/part/fixture/feature dimension from its nominal value.
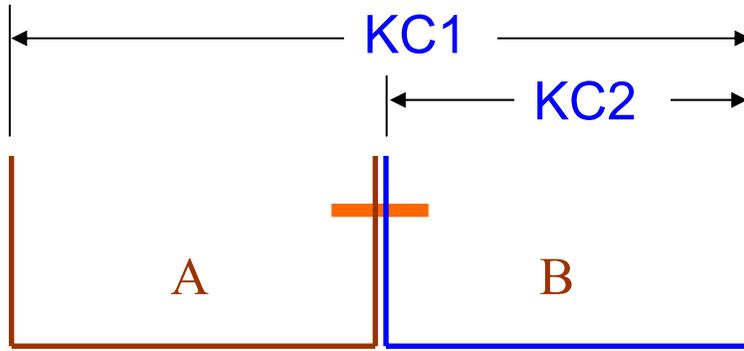
Fig. 3.3: KC1 is overall length of the assembly and KC2 is overall length of the part B. These KCs are correlated with respect to part B
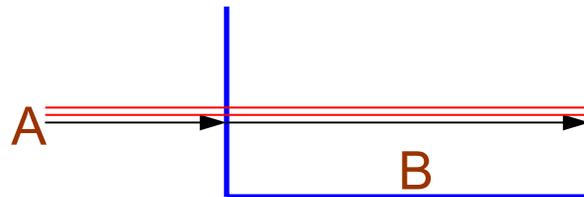
Fig. 3.4: DFC for KC1. For the assembly shown in figure 3.3
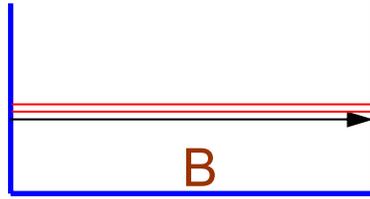
Fig. 3.5: DFC for KC2. For the assembly shown in figure 3.3
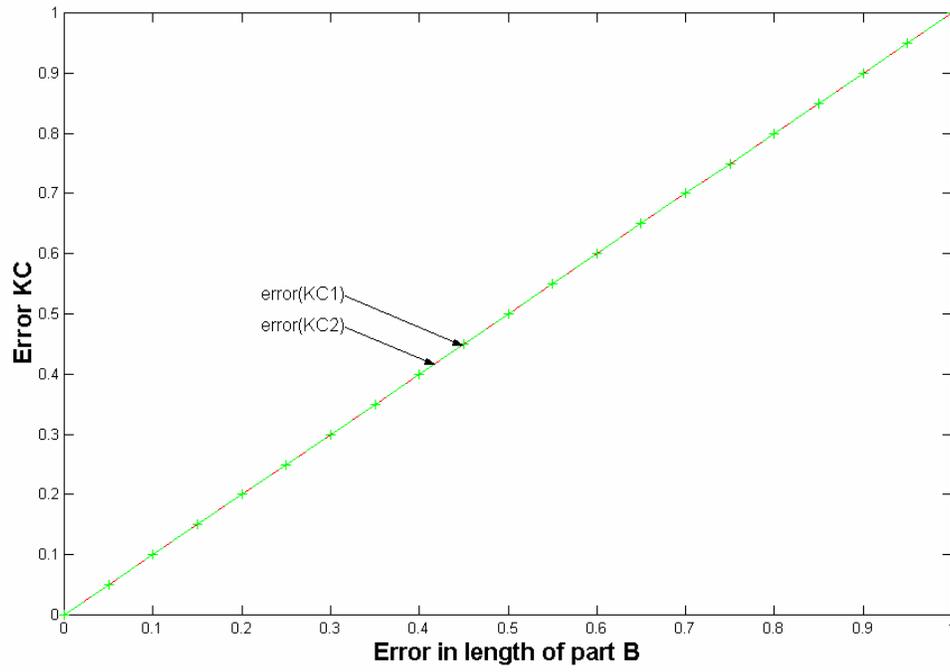
Fig. 3.6: Plot between errors in KCs vs. error in the length of part B

3.3.2.2 Conflicting Key Characteristics

In an assembly if we try to reduce variation in one of the KCs, and it increases variation in other KC, then KCs are said to be conflicting KCs. As is the case with correlated KCs, conflicting KCs share some common links (parts/fixtures/features) in their delivery chains. Many examples of conflicting KCs have already been found out by [Whitney et al., 1999]. Figure 3.7 shows a KC conflict scenario. For a square peg in a square hole one of the KCs (KC1) is the distance between upper surface of the hole to upper surface of the peg and other KC (KC2) is the distance between lower surface of the hole to lower surface of the peg. Assembly sequence consists of inserting the peg in the hole. Hence both the KCs are being made at the same assembly step.

Figure 3.8 shows the DFC of the assembly shown in figure 3.7. A fixture is used to define the position of the peg with respect to the hole. Fixture locates the upper surface of the peg with respect to the upper surface of the hole. Fixture is capable of adjusting the distance between the upper surfaces of the peg and the hole. Hence we can vary the error/variation coming on KC1 by adjusting the position of the peg using fixture. During the manufacturing process the size of the square peg and square hole get fixed. Now for given dimensions of peg and hole, if we try to reduce the variation in KC1, i.e. accurately determining the clearance between the upper surfaces of hole and peg using the fixture, we will increase the variation in KC2. Figure 3.9 and 3.10 shows the DFCs of KC1 and KC2. From the DFCs it is clear that square hole is one of the common parts that are present in the delivery chains of both the KCs. We will see the relationship between KC1 and KC2 with respect to the square hole, which is a common node in the DFCs.
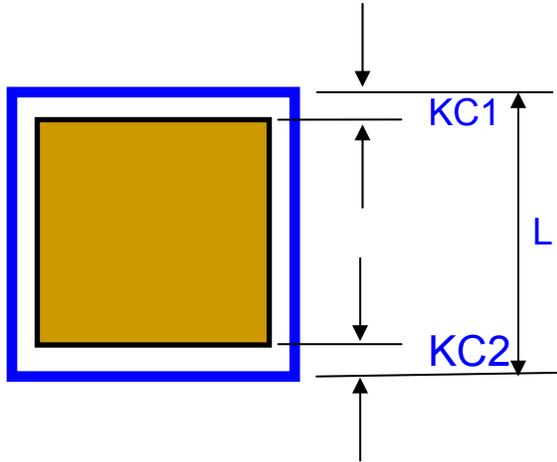
Fig. 3.7: Conflicting KCs. Square Peg in a Square Hole. KC1 if the distance between the upper surfaces of peg and hole, and KC2 is the distance between the lower surfaces of peg and hole. In this case KC1 and KC2 are conflicting
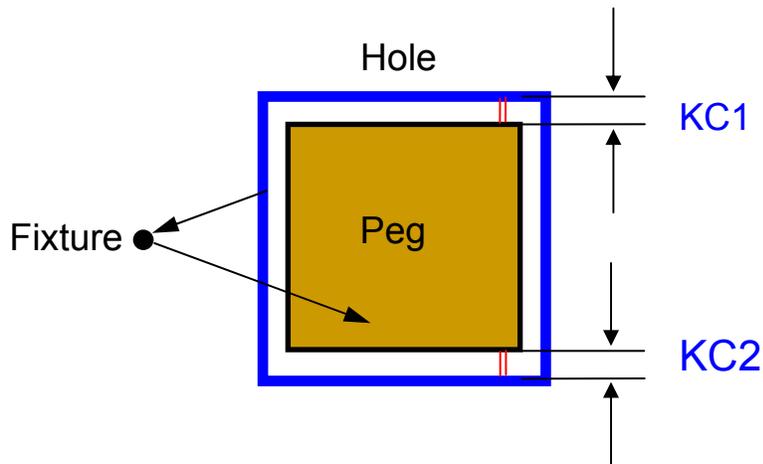
Fig. 3.8: DFC for the assembly shown in figure 3.7. Fixture is used to locate the position of upper surface of peg with respect to upper surface on hole
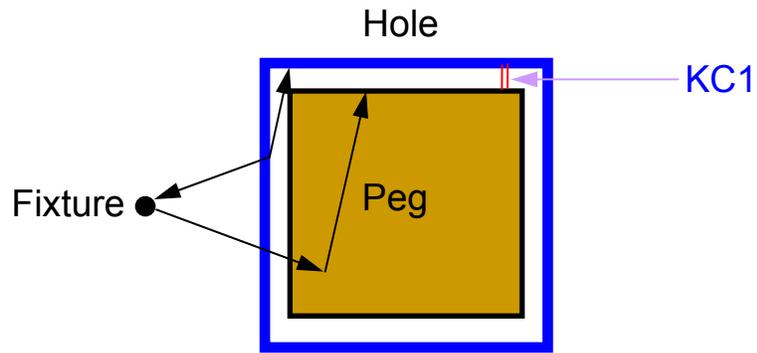
Fig. 3.9: DFC for the KC1 of the assembly shown in figure 3.7
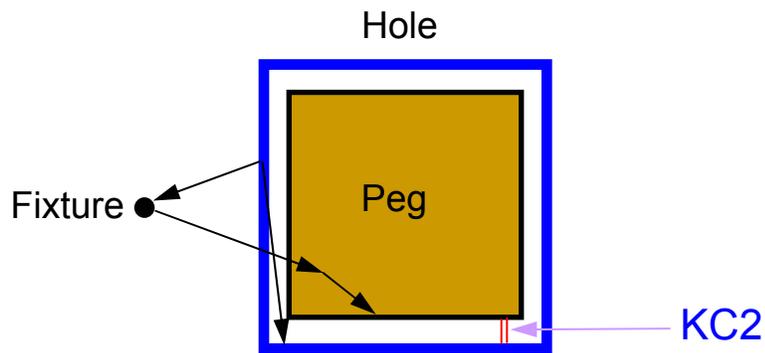
Fig. 3.10: DFC for the KC2 of the assembly shown in figure 3.7

If we assume in above case that dimension of square peg is fixed and it is not allowed to vary, but dimension of hole can vary and fixture can locate the peg in the desired position. For the above geometry:

$$KC1 + KC2 + Length\_of\_peg = L(length\_of\_hole) \qquad \ldots(3\text{-}5)$$

If we differentiate above equation, and denote the differential as error coming on each of the variables then differential of length of peg would be zero since it is not allowed to vary in this case but still fixture can be used to locate the peg in the desired position. By using fixture to locate the peg we can vary the error coming on KC1 and error on KC2 will adjust accordingly. Differential equation looks like:

$$error(KC1) + error(KC2) = error(L) \qquad \ldots(3\text{-}6)$$

If we denotes mean (Length of Hole) as $\overline{Length\_of\_hole}$

then $\overline{KC1} + \overline{KC2} + \overline{Length\_of\_peg} = \overline{Length\_of\_hole} \qquad \ldots(3\text{-}7)$

Error in KCs and in Length of hole is the difference between the original length and mean length, i.e.,

$$error(KC1) = KC1 - \overline{KC1} \qquad \ldots(3\text{-}8)$$

$$error(KC2) = KC2 - \overline{KC2} \qquad \ldots(3\text{-}9)$$

$$error(L) = Length\_of\_hole - \overline{Length\_of\_hole} \qquad ..(3\text{-}10)$$

and for the case being considered

$$Length\_of\_peg = \overline{Length\_of\_peg} \qquad ..(3\text{-}11)$$

since there is no variation in the length of the peg. If we subtract equation 3.7 from 3.5, we will get

$$\left(KC1 - \overline{KC1}\right) + \left(KC2 - \overline{KC2}\right) + \left(Length\_of\_peg - \overline{Length\_of\_peg}\right) = \left(Length\_of\_hole - \overline{Length\_of\_hole}\right) \quad ..(3\text{-}12)$$

Using equations from 3.8 to 3.11, we can rewrite equation 3.12 as

$$error(KC1) + error(KC2) = error(L)$$

which is same as equation 3.6.

On plotting error in KC verses error in Length (L), we see that for a given error in L, if we try to reduce the error one of the KCs, we will increase the error in other KC. This is equivalent of saying that if we try to reduce the variation in one of the KCs we will end up increasing the variation of the conflicting KC. Figure 3.11, shows the plot between the errors in KCs vs. error in L.
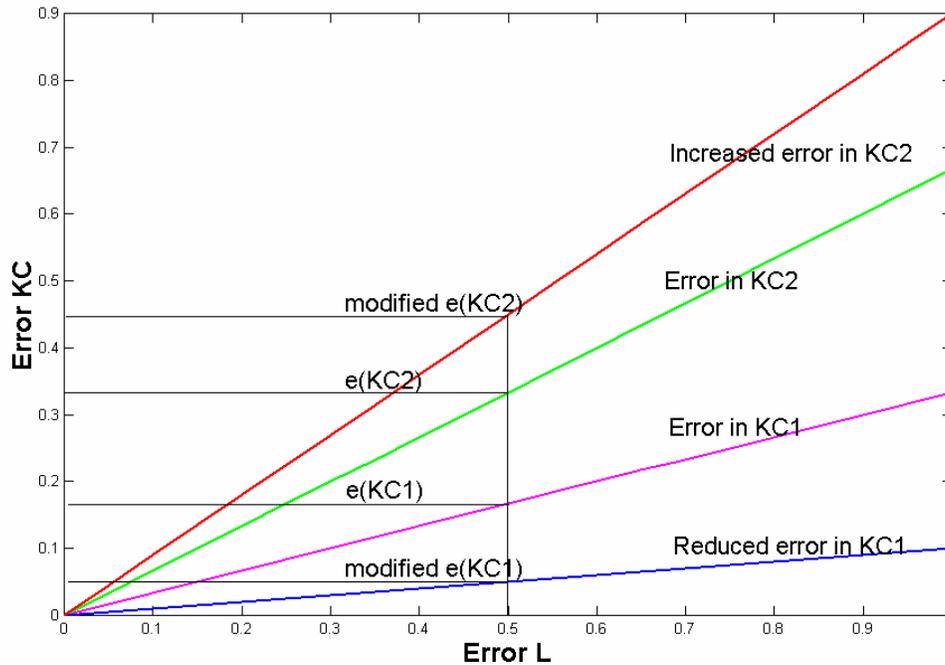
Fig. 3.11: Plot between errors in KCs vs. error in
L (length of the square hole)

For a given error in L and error in KC1, *e(KC1)*, we can find the error in KC2, *e(KC2)*. If

for the same error in L, we try to reduce the variation in KC1, i.e. we decrease *eKC1* to

modified e(KC1), error in KC2 will increase, i.e. error in KC2 would now have a new

value of *modified e(KC2)*. The analysis is done assuming that the dimension of square

peg is fixed.

Figure 3.12 shows the relation between the errors in KC1 and KC2 for a given error in

the length of the square hole. For the analysis purpose we have assumed that there is no

error coming on to the length of the peg. Once the error in the length of the square hole is fixed we can distribute the error between KC1 and KC2 by changing the positioning of the peg using fixture. If we try to locate the peg in such a way that we reduce the error in KC1, then for a given error in the length of square hole we will increase the error in KC2 and vice versa. Hence we see that with respect to a given variation in the length of the square hole, if we try to reduce the variation in KC1 we will be increasing the variation in KC2. Thus for the case shown in figure 3.7, KC1 and KC2 are conflicting with respect to the dimension of square hole, which is common in the delivery chain of both the KCs.
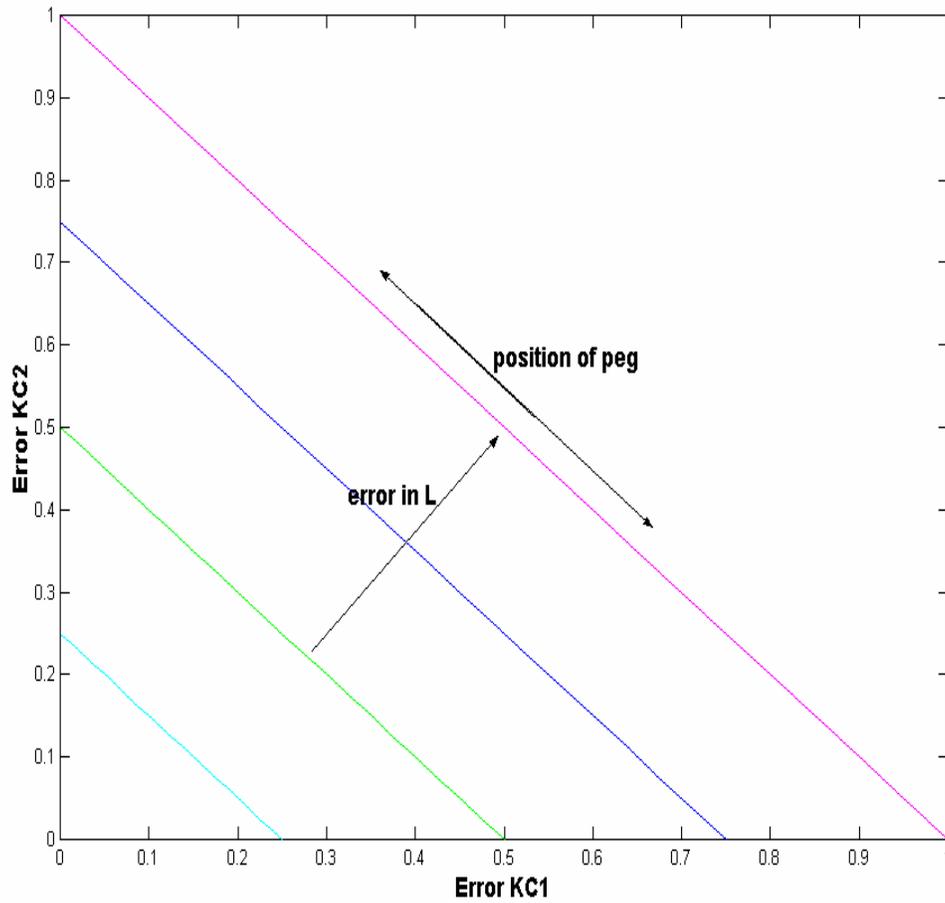
Fig. 3.12: Plot of error (KC2) vs. error (KC1) for
a given error in the dimension (L) of square hole

3.3.2.3 Correlated-Conflicting Key Characteristics

In an assembly a situation can arise when KCs are correlated in some directions and are
conflicting other directions. If we try to reduce variation in one of the KCs in some
directions, it may reduce the variation in other KC in some directions (which may be

different from the directions of variation reduction in first KC). In those directions KCs are correlated. And sometimes for the same KC pair, if we try to reduce the variation in one of the KCs in some other directions it may increase the variation in the other KC in some directions. KCs are conflicting in those directions.

Figure 3.13 shows example of a car door appearance KC. Appearance KC of Car Door is the measure of uniformity of the gap between the car door and the body of the car. Considering the appearance KC at the top and the bottom of the car door, we will find that appearance KC at the top and at the bottom are conflicting in up-down direction but are correlated in in-out direction. In up-down direction car door behaves in similar manner as square peg in square hole with KCs as the distances between upper and lower surfaces. And as we have seen before for the square peg in square hole case, that these KCs are conflicting.

Side View

Appearance KC Top

Up-down

Appearance KC Bottom

Fore-aft

Top View

Weather seal KC, placement DI

Appearance KC, placement DO

In-out

Appearance KC = uniformity of this gap
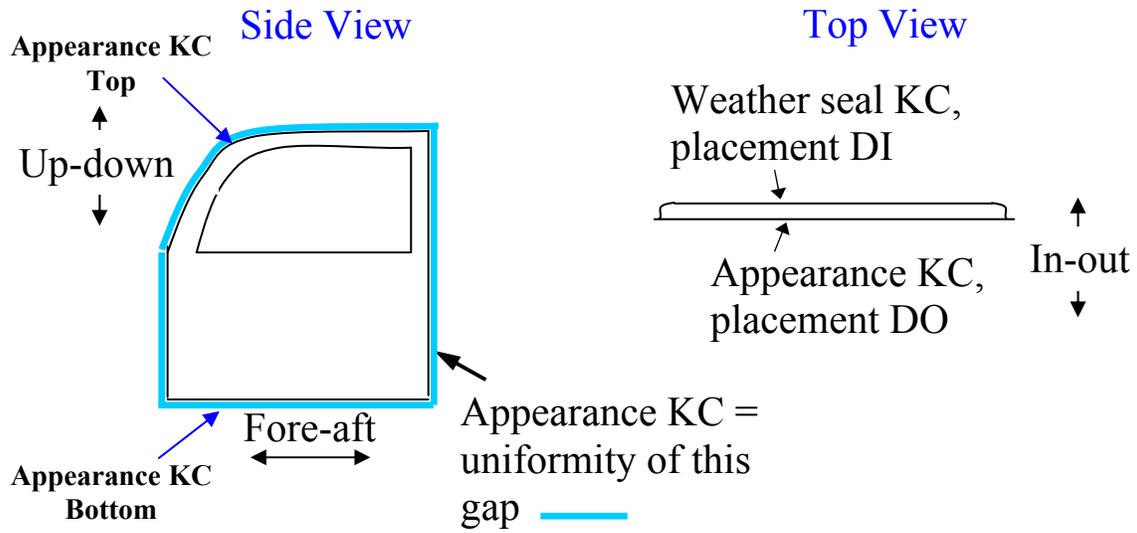
Fig. 3.13: Correlated-Conflicting Appearance KC of Car Door

Figure 3.14 shows Venn diagram representation of KC relationships.

Coupled KC's

Independent KC's

Correlated KC's

Conflicting KC's
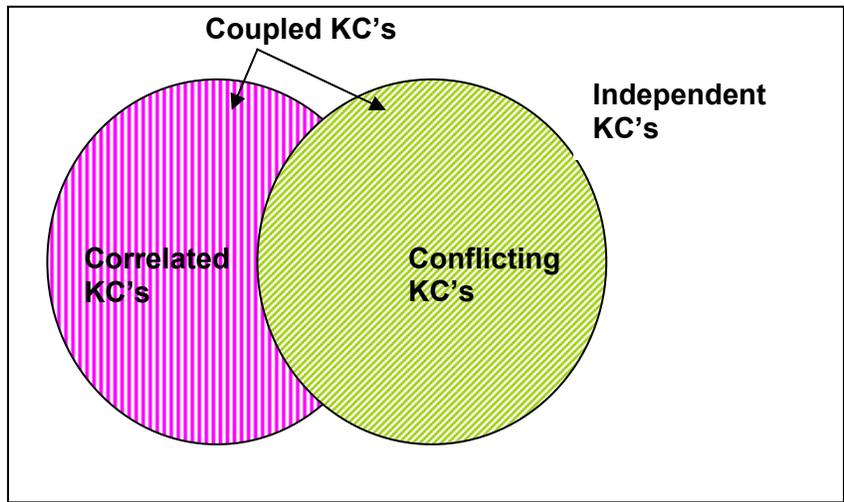
Fig. 3.14: Venn diagram representation

3.4 Effect of Assembly Sequence on KC Coupling

Assembly sequence has lot of bearing on KC Coupling in an assembly. There are certain kinds of assemblies which will have KC Coupling irrespective of the assembly sequence we choose. Type-1 assemblies, which are constrained completely by feature relations between their parts, are the assemblies where assembly sequence has no bearing on the KC Coupling. The assembly process merely puts the parts together by joining their pre-defined mating features.

In certain assemblies we can avoid situations of KC Coupling by properly choosing the assembly sequence. This is possible in case of Type-2 assemblies, which are the assemblies that are under-constrained by their features and need fixtures to add the additional constraint. If we consider the assembly sequence while determining the locating scheme of the parts in Type-2 assembly, we can decouple the KCs. Figure 3.15 shows a case of Type-2 assembly in which we can decouple the KCs by suitably choosing the assembly sequence. KC coupling can arise if more than one KC is delivered at single assembly step. In Type-2 assembly we have an option of decoupling KCs. If we choose an assembly sequence such that the previously coupled KCs are achieved at different assembly steps then we can decouple the KCs. This option is available only for Type-2 assemblies.

In some cases a subassembly is of Type-2 kind, but once fixture is used to locate all the parts of the subassembly it becomes properly constrained. Now this subassembly resembles a single part which can locate other parts of the complete assembly without

any need of extra fixtures. Thus subassembly with all other parts cannot be classified as Type-2 assembly. It was Type-2 assembly only till the subassembly was made. And after that it acquires all the properties of Type-1 assembly.
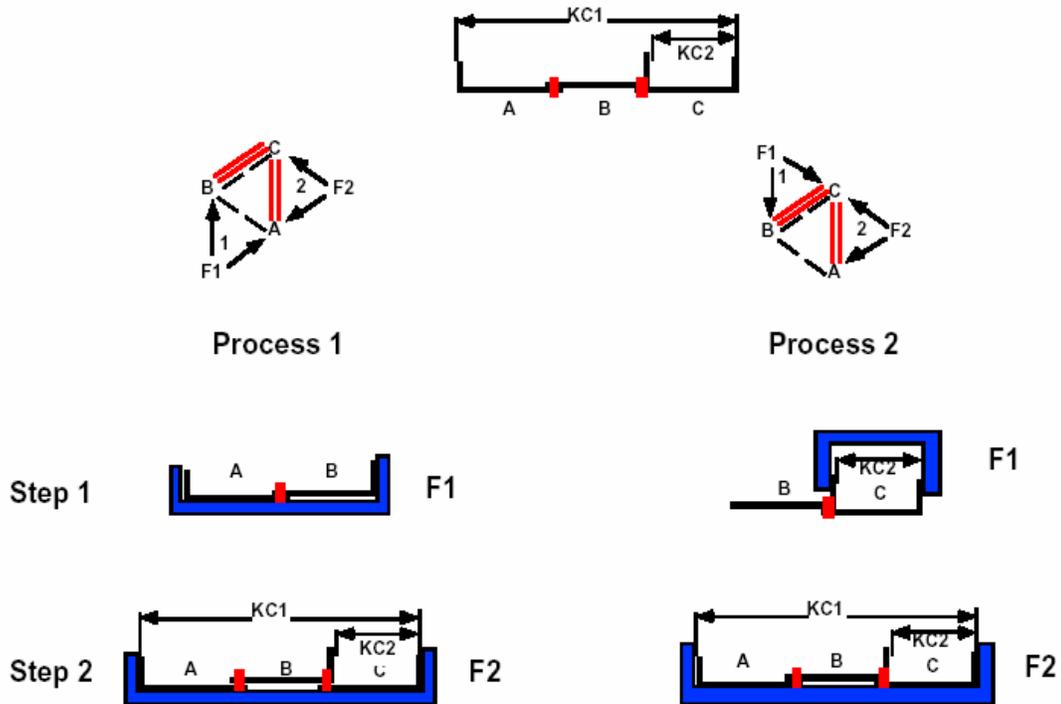
Fig. 3.15: In process 1 the KCs are coupled since both the KCs are achieved at the single assembly step. But in process 2 KCs are decoupled since they are achieved at different assembly steps

Sometimes the assembly sequence that would help avoiding KC coupling might not be feasible. For example, if fixture F1 of Process 2 in figure 3.15 were prohibitively expensive to build or maintain, Process 2 would not be feasible.

3.5 Chapter summary

Real assemblies have more than one KC that they have to deliver. Large number of parts act together to deliver those KCs. The presence of large number of KCs and parts result in the involvement of a part in the delivery chain of more than one KC. The involvement of a part in the delivery chain of more than one KC makes it difficult to achieve each of the KC independently.

Since we know that most of the KCs in an assembly bear relationship with respect to each other, to understand the nature of their relationship we classified the KCs based on their relationship. If the KCs can be met independently, the variation in one of the KCs has no bearing on the variation in other KC. If the variation in one KC has impact on the variation in other KC, they are said to be coupled KCs.

Coupled KCs can further be classified based on the relationship between the variations coming on each of them. If on reducing the variation in one of the KCs, variation in other KC also reduces then KCs are said to be correlated. The nature of the relationship between KCs is dependent on the part/fixture/feature which is common in the delivery chains of the KCs. Reducing variation in one of the KCs might increase variation in other KC. When such a thing happens, KCs are said to be conflicting. There also exist some KCs which are conflicting in some directions and correlated in other directions. Such KCs are called correlated-conflicting KCs.

KC coupling is also dependent on the architecture of the assembly. For Type-1 assemblies KC coupling does not depend on the assembly sequence. In case of Type-2 assemblies, assembly sequence plays important role in determining the nature of KC coupling. There is also a possibility of decoupling the KCs by making appropriate changes in the assembly sequence of Type-2 assemblies.

In the next chapter we will see how Screw Theory can be used to determine the nature of KC Coupling in an assembly. Constrain and Motion Analysis using Screw Theory can ascertain the exact relationship that exists among the KCs in the assembly. For the analysis purposes we will be dealing with geometric KCs and will denote them as wrench matrices.

Chapter 4. DETECTING KC-COUPLING USING SCREW THEORY

4.1 Introduction

An assembly may have to deliver more than one KC simultaneously. If the chains of parts delivering the KCs are not independent, then we might not achieve the variation in each KC individually. This scenario is called KC Coupling. As we have discussed before KC Coupling can be of three different type's viz. Correlated KCs, Conflicting KCs, and Correlated-Conflicting KCs.

Given an assembly we can draw DFC for that assembly, and can find the KCs of that assembly. Then we need to find which of those KCs are coupled and what the nature of the coupling is. It is essential to know the nature of the KC relationship, before starting the full scale manufacturing of the assembly. Specifically if the KCs are conflicting, then we need to do tolerance analysis on the conflicting KCs to make sure that both KCs satisfy the tolerance limit set on them.

Screw theory can be used to find the nature of KC relationship for an assembly which is not over-constrained. First we will discuss about the methods that have been used to detect the KC relationship and their shortcomings. Next we will look at an algorithm which can be used to predict relationship and its nature.

4.2 Previous work

[Whitney et al., 1999] have proposed an algorithm to find KC Conflict using Screw Theory. In the algorithm all the degrees of freedom that an assembly has, are used to form twist matrix of the assembly and then it is intersected with the twist matrix of each of the KC to determine if the assembly has sufficient degrees of freedom to meet each of the KC independently. [Goldenstheyn, 2002] has extended the use of screw theory to identify 'single-part' KC conflict. He treated KC as a requirement that represents a certain geometric relationship between parts in the assembly that must be met. Using this idea KC can be thought of as a 'virtual' constraint in the assembly. The idea of KC as a constraint is exploited for detection of KC conflict by screw theory.

'Single-part' KC conflict is very restrictive in its approach. In a complex assembly there are many parts involved in the delivery of KCs. To find the nature of KC relationship we need to take into account all the parts that are involved in the delivery chains of KCs. Furthermore as we have seen before that assembly sequence has bearing on the nature of KC relationship. By selecting suitable assembly sequence we can decouple the KCs. In the method suggest by [Whitney et al., 1999] assembly sequence has no role to play. The assemblies considered in the algorithm are under-constrained assemblies.

We will see here an algorithm that will take assembly sequence into account. Not only will it tell the presence of coupling among KCs but it will also be able to detect the nature of the coupling, i.e. whether KCs are conflicting, correlated or correlated-conflicting.

4.3 Algorithm

Type-1 assemblies are the assemblies that are constrained completely by feature relations between their parts. The assembly process merely puts the parts together by joining their pre-defined mating features. While finding the KC coupling in Type-1 assembly, assembly sequence has no role to play. On other hand, Type-2 assemblies are the assemblies that are under-constrained by their features and need fixtures to add the additional constraint. Assembly sequence in case of Type-2 has significant bearing on the locating scheme of parts. As we have seen before, that KC coupling in Type-2 assembly can arise when more than one KC is delivered at a same assembly step. This method looks at a single assembly step in Type-2 assembly to detect the presence of KC coupling. The algorithm that we are going to discuss will work for the case of under-constraint and properly-constraint assemblies.

In order to detect KC coupling in an assembly

- We need to find the pair of KCs that can be coupled

- Find the chain of parts involved in the delivery of each of the KC, which seems to be coupled or for which we need to detect coupling

- Infer the DFC of each of the KC from the chain of parts delivering KCs. Figure 4.1 shows the KCs formed between parts A and B and between parts C and D. To detect the nature of relationship between the KCs we draw the DFC for KC_{AB}

(KC between parts A and B) and for KC_{CD}. DFC from part A to B and from C to D may also contain other parts, which are involved in the delivery of KC_{AB} and KC_{CD}.

- Find the overall twist matrix between the two ends of DFC of the KC. Overall twist matrix is the union of the twist matrices of all the features that lie in DFC of the KC under consideration. For figure 4.1, overall twist matrices can be represented as TM_{AB} and TM_{CD}.

DFC$_{AB}$ **A** **B**

DFC$_{CD}$ **C** **D**
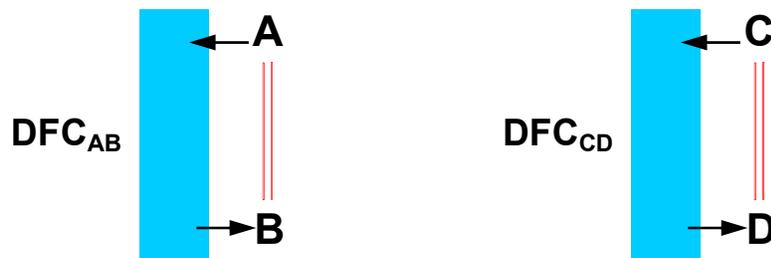
Fig. 4.1: DFCs of the KCs between parts A and B and between parts C and D

- Find the relative wrench matrices between the two ends of DFC of the KC. Relative wrench matrix is the reciprocal [Konkar, 1993], [Konkar, 1995],

[Adams, 1998] of overall twist matrix. For figure 4.1, relative wrench matrices can be represented as W_{AB} and W_{CD}.

$$W_{AB} = \mathrm{Re}\,ciprocal(TM_{AB}) \qquad \qquad \qquad \dots(4\text{-}1)$$

And,

$$W_{CD} = \mathrm{Re}\,ciprocal(TM_{CD}) \qquad \qquad \qquad \dots(4\text{-}2)$$

Relative wrench matrix between parts denotes the directions along which variations can propagate. Thus by knowing relative wrench matrix between the parts we can know the directions which can be affected by part variations. [Roy et al., 1991], [Salomons et al. 1996] discusses the effect of part variations on the variations coming at the two ends of KC.

- Extending the idea of 'single-part', and 'virtual' constraint KC [Goldenshteyn, 2002], we can represent the KCs between parts A and B and between parts C and D as wrench matrices. Wrench matrices denoting the KCs can be represented as KC_{AB} and KC_{CD}. KC wrench matrix denotes the directions in which KC is present. The variation in the direction of KC wrench matrix will produce variation in the KC.

- We then define another matrix called 'Constraint Matrix', which is the intersection of the relative wrench matrix and KC wrench matrix. We know that relative wrench matrix denotes the directions in which variation can propagate

from one part to another, between which KC is formed and KC wrench matrix denotes variation directions which can affect KC. On intersecting these two matrices we will get 'Constraint Matrix' which denotes the directions, along which variation can propagate from one part to another, between which KC is formed and can affect the delivery of KC. 'Constraint Matrix' for figure 4.1 can be represented as CM_{AB} and CM_{CD}.

$$CM_{AB} = W_{AB} \cap KC_{AB} \qquad \qquad \text{...(4-3)}$$

And,

$$CM_{CD} = W_{CD} \cap KC_{CD} \qquad \qquad \text{...(4-4)}$$

- Once we know the constraint matrix for each KC (CM_{AB} and CM_{CD}), to find KC coupling we need to intersect [Konkar, 1993] the constraint matrices of the KCs. We will call this intersection matrix as 'Coupling Matrix'. Constraint matrices give us the directions along which variation can propagate and can affect KCs. And intersection of these constraint matrices ('Coupling Matrix') gives us the common directions of variation that can affect both the KCs under consideration. If 'Coupling Matrix' turns out to be a null matrix then there is no common direction of variation that can affect both the KCs. For figure 4.1,

$$Coupling\_Matrix = CM_{AB} \cap CM_{CD} \qquad \qquad \text{...(4-5)}$$

- Once we know that KCs are coupled we can perform 'Variation and Contribution Analysis' [Fortini, 1967], [Greenwood and Chase, 1990], [Shukla, 2001] to know how the variation in common feature/part/fixture in the DFCs affects the variation in KCs. If on increasing the variation in the common feature/part/fixture in DFCs, (and rest of the other parts/fixtures/features are assumed to have fixed dimensions) the variation in both the KCs increases or decreases simultaneously, then KCs are said to be correlated. (If we try to reduce (or increase) the variation in one of KCs and it simultaneously reduces (increases) the variation in other KC, then KCs are called correlated KCs.) But if the variation in one of the KC increases and on the other it decreases then KCs are said to be conflicting. (If we try to reduce the variation in one of the KCs, and it increases the variation in other KC then KCs are said to be conflicting KCs.)

Using this algorithm we can find KC coupling in an assembly and using the variation and contribution analysis we can classify the nature of KC coupling.

4.4 Example

We will consider the example of underbody of car, which is made of three parts. It is a Type-2 assembly since fixture is used in locating the parts. Figure 4.2 shows the assembly sequence and the KCs that are to be delivered by the underbody. KC1 is the overall length of the underbody, i.e. the length from one end of part A to the opposite end of part C. And KC2 is the overall length of the combination of parts B and C. The important thing to note in this example is that both the KCs are delivered at the same

assembly step; hence there is a chance of KC coupling (we will see later that these two KCs are coupled). Figure 4.3 shows DFCs of the assembly and of the individual KCs. In the first assembly step part A is joined to part B (a mate exists between these two parts). And in the next assembly step part C is joined to the properly constrained subassembly of parts A and B with help of the fixture.
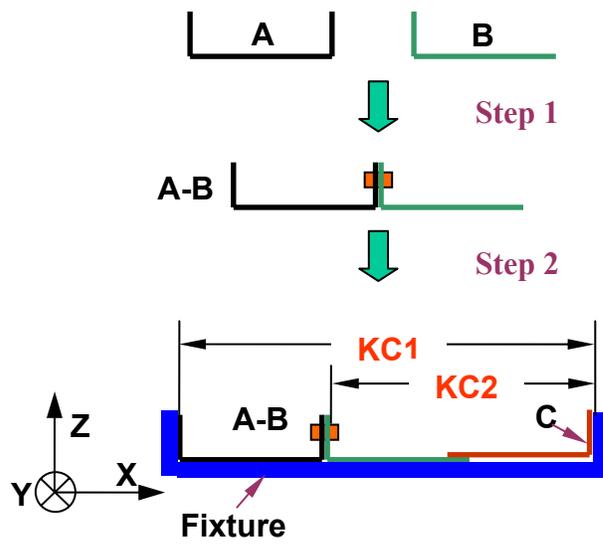
Fig. 4.2: Assembly sequence of underbody of car, showing the KCs to be delivered
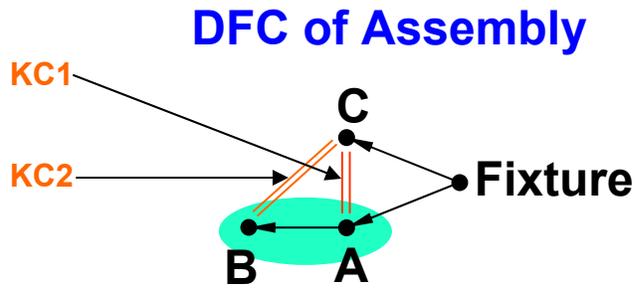
DFC of Assembly
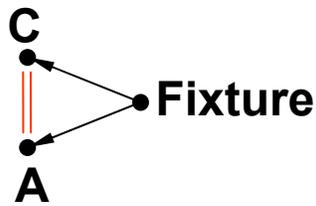
DFC of KC1

DFC of KC2
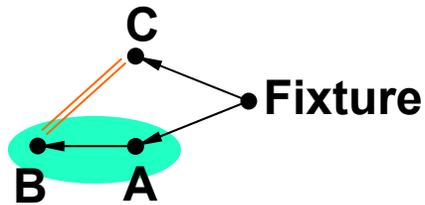
Fig. 4.3: DFCs of assembly and of the KCs

From figure 4.2:

$$KC_{AC} = KC_1 \qquad \qquad \text{…(4-6)}$$

$$KC_{BC} = KC_2 \qquad \qquad \text{…(4-7)}$$

$$T_{FA} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \ldots(4\text{-}8)$$

where F stands for fixture. T_{FA} has a non zero entry in the column of v_y. This is because part A can move relative to fixture in y direction.

$$T_{AB} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots(4\text{-}9)$$

T_{AB} has non zero entries in the columns of θ_x, v_y, and v_z. This is because part B can move relative to part A in y and z directions and can rotate relative to part A in θ_x direction.

$$T_{FC} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad ..(4\text{-}10)$$

T_{FC} has non zero entries in the columns of θ_x, v_y, and v_z. This is because part C can move relative to fixture in y and z directions and can rotate relative to fixture in θ_x direction.

$$T_{BC} = T_{FC} \cup T_{FA} \cup T_{AB} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad ..(4\text{-}11)$$

from DFC.

$$\Rightarrow W_{BC} = \text{Re}\,ciprocal(T_{BC}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad ..(4\text{-}12)$$

T_{BC} has non zero entries in the columns of θ_x, v_y, and v_z. This is because part C can move relative to part B in y and z directions and can rotate relative to part B in θ_x direction. W_{BC} has non zero entries in the columns of f_x, m_y, and m_z. This is because part C is restrained by part B in x direction and also in θ_y and θ_z rotation directions.

$$T_{AC} = T_{FC} \cup T_{FA} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad ..(4\text{-}13)$$

from DFC.

$$\Rightarrow W_{AC} = \operatorname{Re}ciprocal(T_{AC}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad ..(4\text{-}14)$$

T_{AC} has non zero entries in the columns of θ_x, v_y, and v_z. This is because part C can move relative to part A in y and z directions and can rotate relative to part A in θ_x direction. W_{AC} has non zero entries in the columns of f_x, m_y, and m_z. This is because part C is restrained by part A in x direction and also in θ_y and θ_z rotation directions.

$$KC_{AC} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad ..(4\text{-}15)$$

$$KC_{BC} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad ..(4\text{-}16)$$

The KCs are being represented as wrench matrices. KC_{AC} and KC_{BC} have non zero entry in the column of f_x. This is because the both the KCs exist in x direction.

$$CM_{AC} = KC_{AC} \cap W_{AC} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad ..(4\text{-}17)$$

$$CM_{BC} = KC_{BC} \cap W_{BC} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad ..(4\text{-}18)$$

CM_{AC} and CM_{BC} have non zero entry in the column of f_x. This implies that x direction is the direction along which variation can propagate from part A to part C, and from part B to part C, and the variation affect the delivery of KC_{AC} and KC_{BC} in x direction.

$$Coupling\_Matrix = CM_{AC} \cap CM_{BC} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \neq [\phi] \qquad ..(4\text{-}19)$$

Since $Coupling\_Matrix \neq [\phi]$, the above two KCs are coupled.

$Coupling\_Matrix$ has a non zero entry in the column of f_x. This implies that the KCs are coupled in x direction.

Now we need to perform variation analysis on the KCs in order to characterize the nature of coupling that exists between the two. From DFCs of the KCs we can see that fixture is common in the delivery chains of both the KCs. To see the nature of coupling between the two with respect to the common part in their DFCs, we will fix the dimensions of all other parts except the fixture and see the effect of variation in fixture dimension on the variation coming on to the KCs.

From figure 4.2 we can say that

$$KC1 = Length\_of\_Fixture \qquad\qquad ..(4\text{-}20)$$

$$KC2 + Length\_of\_A = Length\_of\_Fixture \qquad\qquad ..(4\text{-}21)$$

If only length of fixture was allowed to vary, then on differentiating the above equations and denoting the differentials as the error coming on each of the variables (given the fact that dimension of A is not allowed to vary), we will get

$$error(KC1) = error(Fixture) \qquad\qquad ..(4\text{-}22)$$

$$error(KC2) = error(Fixture) \qquad\qquad ..(4\text{-}23)$$

On plotting (figure 4.4) the errors in the KCs as a function of error in the length of fixture, we see that the error in the KCs increases with increase in error in the length of the fixture. Since the errors in KCs are positively correlated, implying that if we try to reduce variation in one of the KCs, by reducing the variation in fixture we will simultaneously reduce the variation in other KC. Thus KCs are correlated with respect to variation in fixture.
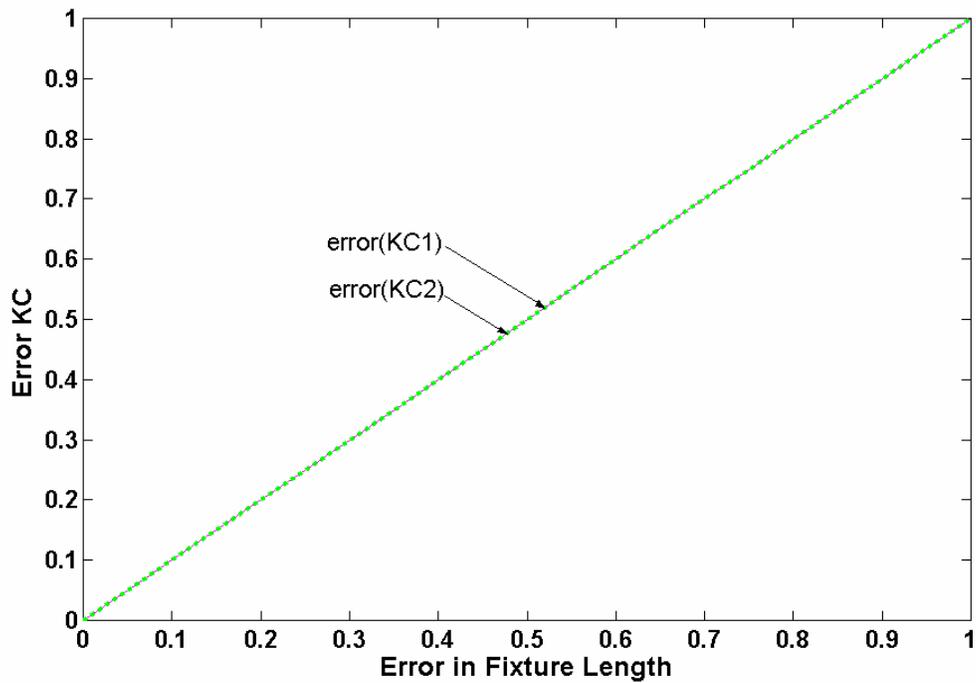
Fig. 4.4: Plot between errors in KCs vs. error in
the length of the fixture

The utility of this method will be enhanced if it is to be used with current CAD and
assembly sequence analyzing software packages. This algorithm can detect the presence
of KC coupling at each assembly step and will also tell us the nature of the coupling. If
the coupling is a correlation then we are in a good shape, since if we try to reduce the
variation in one of correlated KCs, we will simultaneously reduce the variation in other
KC. But it the coupling is a conflict, we need to look for the seriousness of the situation.
If we try to reduce the variation in one of the conflicting KCs and find out that the other

KC does not meet its tolerance limit then we need to look for alternative assembly sequence which can resolve the conflict.

4.5 Chapter summary

Screw Theory can be used to analyze the nature of relationship that exists among various KCs in an assembly. KCs in the assembly can be independent, or coupled. Nature of coupling among KCs can further be broken into conflict, correlation and their combination. An algorithm was presented that can find the nature of relationship among KCs for a properly constrained assembly.

For the analysis purposes KCs were being taken as wrench matrices. If the variation comes in the direction of KC, it will displace the KC from its nominal value. The nature of variation, coming on various KCs determine the relationship among them. Constraint analysis is used to determine if there exists a possibility of KC coupling. It is an efficient way to disregard all the KCs which are independent at first place. Once we have determined that KCs are coupled, we can apply Motion analysis to find the exact nature of coupling.

An example was shown to illustrate the working of the algorithm. Underbody of car with three parts was taken as an example. Constraint analysis showed the presence of a coupling. Motion analysis provided information about the exact nature of coupling.

The algorithm technique shown here can easily be clubbed as a subroutine to commonly used CAD packages. This will help designers in detecting the presence of KC coupling in early stages of design. Also they will know the exact nature of coupling. If the coupling is harmful for the quality of the assembly, it can be removed in early stages.

In the next chapter we will probe into the scenarios which can compel us to remove KC conflict from the assembly. Not all conflicts are harmful. The ones which threaten the robustness of the assembly are to be removed to achieve good quality product.

Chapter 5. NEED TO RESOLVE KC CONFLICT

5.1 Introduction

In a mechanical assembly, a part is generally involved in the delivery chains of more than one KC. This leads to the situation of KC Coupling. As we have seen in the last chapter we can use Screw Theory to find information about KC Coupling. If the KCs are correlated then our design is good since on reducing the variation in one of correlated KCs (by reducing variation in parts common in the delivery chains of the KCs), the variation in other KC simultaneously goes down. But on the other hand if the KCs are conflicting then on reducing variation in one of the conflicting KCs, variation in other KC will increase.

We need not resolve all the situations of KC conflict in designs. We should pick out the designs having KC conflict, for which resolving KC conflict is imperative. For the conflicting KCs, if the variations coming on to the KCs are less than the tolerance limits set on to the KCs, then we need not redesign the mechanical assembly. But if the variation in one of the conflicting KCs surpasses the tolerance limit imposed on that KC, we need to resolve the existing KC conflict.

In order to ascertain the situations where we need to resolve KC conflict, we should first perform analysis on the assembly using Screw theory to find the nature of coupling between the KCs of the assembly. Once we have selected a set of conflicting KCs, we

should then look for the tolerance limits set on to those KCs by designers and manufacturers. The next step is to find the variations that will come on to the KCs.

5.2 Variation Analysis

The aim of variation analysis is to estimate the variation coming on to the KCs of the assembly, from the naturally occurring variations in part dimensions and features. The variation analysis methods are divided into two distinct categories based on the type of input variations analyzed [Hong and Chang, 2002]; *dimensional variations* and *geometric variations*.

One of the most widely accepted techniques in variation analysis is the tolerance chain technique. 'Dimensional tolerance chain' is used to represent the chain in which a conventional (plus/minus) tolerance is given for each arc in the chain. The various methods that are based on dimensional tolerance chain can be classified into three broad approaches:

- Linear/linearized accumulation models,

- Statistical tolerance analysis, and

- Monte Carlo simulation methods.

Linear/linearized accumulation model for variation analysis is one of the oldest techniques used in variation analysis. [Fortini, 1967] presents the two most common

models for variation accumulation; worst case and statistical (root sum square, or RSS for short):

$$T_{AMS} = \sum_{i=1}^{n} T_i \qquad \qquad ...(5\text{-}1)$$

and

$$T_{ASM} = \sqrt{\sum_{i=1}^{n} T_i^2} \qquad \qquad ...(5\text{-}2)$$

respectively, where T_{ASM} is the variation coming on to the KC and T_i is variation coming on to the part involved in the delivery chain of the KC. [Greenwood and Chase, 1987] proposed mean shift model to take into account set-up errors and drifts due to time varying factors such as tool wear, which was extension of [Fortini, 1967] framework. [Greenwood and Chase, 1988, 1990] showed application of these techniques to nonlinear problems. [Zhang and Wang, 1993c] published an analysis done for cam mechanism. [de Pennington et al., 1987] and [Juster et al., 1992] have introduced the concepts of triplex arithmetic and tolerance graph to provide a theoretical framework for the worst-case tolerance chain analysis.

Statistical tolerance analysis methods come from dimensional tolerance chain technique. These methods characterize the sum dimension Y of the design equation $Y = f (X_1, X_2... X_n)$ in statistical terms, starting from an assumption, of a certain kind of distribution of X_i's. It is based on the methods of moments explained in [Evans, 1974], i.e. depends upon the estimation of moments of Y such as mean, variance and standard deviation. [Bjørke, 1989] contains one of the most representative works in this field. Based on the classification of chain links, [Bjørke, 1989] derived various cases of spans and gaps.

[Treacy et al., 1991] extended [Bjørke, 1989] work using the development in automatic tolerance chain generation and analysis technique. [Varghese et al., 1996a] used a numerical convolution to derive an efficient method to deal with the finite range of probability density function. [Lin et al., 1997] work uses the beta distribution, instead of the Gaussian (normal) distribution, for its flexibility.

The derivation of the statistical moments of a function of random variables in an analytical manner is a difficulty, especially when the functional form is complicated. To overcome this challenge *Monte Carlo simulation method* is frequently used for the statistical analysis of the problems in real world. [Turner and Wozny, 1987] showed the extensive use of Monte Carlo method in the development of variational geometry system. Application of this method requires an explicit definition of the design function. It is usually difficult to define such a function in a real mechanical assembly. The usage of Monte Carlo simulation is quite limited, due to lack of explicitly derived equations. [Skowronski and Turner, 1997] showed a method of variance reduction in the Monte Carlo tolerance analysis.

Variation analysis is based on variation chain models. The variation chain model is a special case of variational geometry in which only the dimension (size) can vary [Shah and Mäntylä, 1995]. Variational geometry is a dimension-driven, constraint-based approach to the definition of geometry of an object. Exact geometry is determined from part topology and a set of dimensions. The system maintains the whole set of

simultaneous equations, which is adapted to the dimension variations while maintaining the topology of the object.

First concept of variational geometry was developed by [Hillyard and Braid, 1978], who introduced the shape model with stiffeners. It provided a method to check whether a part is under-, over- or exactly dimensioned. In addition, it was also shown that it is possible to find the resulting tolerance for the un-dimensioned parts. [Light and Gossard, 1982] and [Gossard et al., 1988] outlined the more detailed theory of variational geometry. Variation dimension models are particularly suited to variation sensitivity analysis, since the designer is interested in finding out how sensitive a dimension or KC is, to changes in other dimensions which come in the delivery chain of the KC [Shah and Mäntylä, 1995].

Manufacturers and designers set the tolerance limits on KCs, parts, fixtures and features in an assembly. Once we know the tolerance limits on parts, fixtures and features we can simulate the variations coming on the parts and features, by using any of the above mentioned analysis methods. Monte Carlo simulation method is most commonly used. Most popular commercial package is Variation Simulation Analysis, or VSA, which is based on Monte Carlo simulation. The background and application of the software is given in [Craig, 1988], [Peelman-Fuscaldo, 1991], [Sitko, 1991], [Hutchings, 1999] and [Roy et al., 1991]. The inputs to Variation Simulation Analysis are assembly sequence, assembly methods, output relationships and measurements, and component tolerances and datum schemes. We can use 4x4 matrix transformations to analyze variations in open loops [Denavit and Hartenberg, 1955], [Paul, 1981], [Simunovic, 1976], [Popplestone

and Ambler, 1979], [Wesley, Taylor, and Grossman, 1980]. The variation coming on to parts and features can also be modeled as 4x4 matrices. If the 4x4 matrix for a part or feature is given by:

$$T = \begin{bmatrix} R & p \\ 0^T & 1 \end{bmatrix}$$
...(5-3)

then the variation for that particular part or feature can be modeled as:

$$dT = \begin{bmatrix} dR & dp \\ 0^T & 1 \end{bmatrix}$$
...(5-4)

where **dR** is the variation coming on to the angular orientation, and **dp** is the variation coming on to the spatial location of the part, fixture or feature. The above mentioned methods can be used to open loop analysis. For closed loop analysis, [Chase et al., 1995, 1996, 1997, 1998, Gao et al. 1998a, b] used linearized accumulation models. Using accumulation models they studied more complex 2D/3D mechanical assemblies. [Gao et al., 1995] compared the results of the direct linearization method with those obtained from Monte Carlo simulation. [Gao et. al, 1996] proposed a method for application of Monte Carlo method to the implicit assembly constraints.

5.3 Screw Theory to find variation in Closed Loop

Some of the methods mentioned above can be used to find the variations coming in closed loops. We can also use Screw Theory [Konkar, 1993], [Konkar, 1995] to find the variations coming in closed loops in a generalized three dimensional case. The assembly has to be either under- or properly constrained to apply this analysis method. The method consists of finding the part-feature diagram [Shukla, 2001] of the DFC for the KC, whose variation we want to model. For the variation analysis, variation of a part, fixture or

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feature in each degree of freedom can be modeled as a twist matrix. The twist matrix for the variation in a part, fixture or feature is represented as:

$$T = [\omega; v] \qquad \qquad ...(5\text{-}5)$$

The variation in each degree of freedom is modeled as unit motion in that degree of freedom.

Once we know the part-feature diagram for a KC, which is a modified version of DFC for the KC, and all the variations that can arise in the parts, fixtures and features, we can find the variation coming on to the KC. For small variations we can apply Principle of Superposition, which says that the effect of all variations in parts, fixtures and features on KC, is just the sum of the affect of each individual part/fixture/feature variation on the KC.

Mathematically we can say,

$$Variation\_KC = \sum_i Variation\_KC_i \qquad \qquad ...(5\text{-}6)$$

where *Variation_KC*, is the overall variation coming on to the KC, and *Variation_KC$_i$*, is the variation coming on to the KC because of the variation in i^{th} part, fixture or feature involved in the delivery chain of the KC.

For a part in a closed loop, if we know the variation coming at one end of the part, we need to first evaluate how the variation propagates to the other end of the same part in a closed loop, before we can find the effect of that variation on the KC. Figure 5.1 shows a

rigid body 1-2. The rigid body 1-2 is any body which is a part of closed loop chain. If the parts of the closed chain have variations coming on to their dimensions, it will change the location of the rigid body 1-2. In the given analysis we will assume that there is no variation in the dimension of the rigid body 1-2. We will analyze the effect of variation in other parts of the closed loop on the location of rigid body 1-2. We can evaluate the twist matrix of the rigid body 1-2, once we know the part-feature diagram of the DFC, which passes through the rigid body and the directions of the variations that can arise in the parts or features constituting the delivery chain of the KC. The twist matrix for the rigid body 1-2 can be represented as:

$$T_{12} = [\omega_{12}, v_{12}] \qquad \qquad ...(5\text{-}7)$$

where $\boldsymbol{\omega_{12}}$ represents the instantaneous angular velocity of the rigid body in global co-ordinate frame and $\mathbf{v_{12}}$ represents the instantaneous linear velocity of the rigid body in global co-ordinate frame.
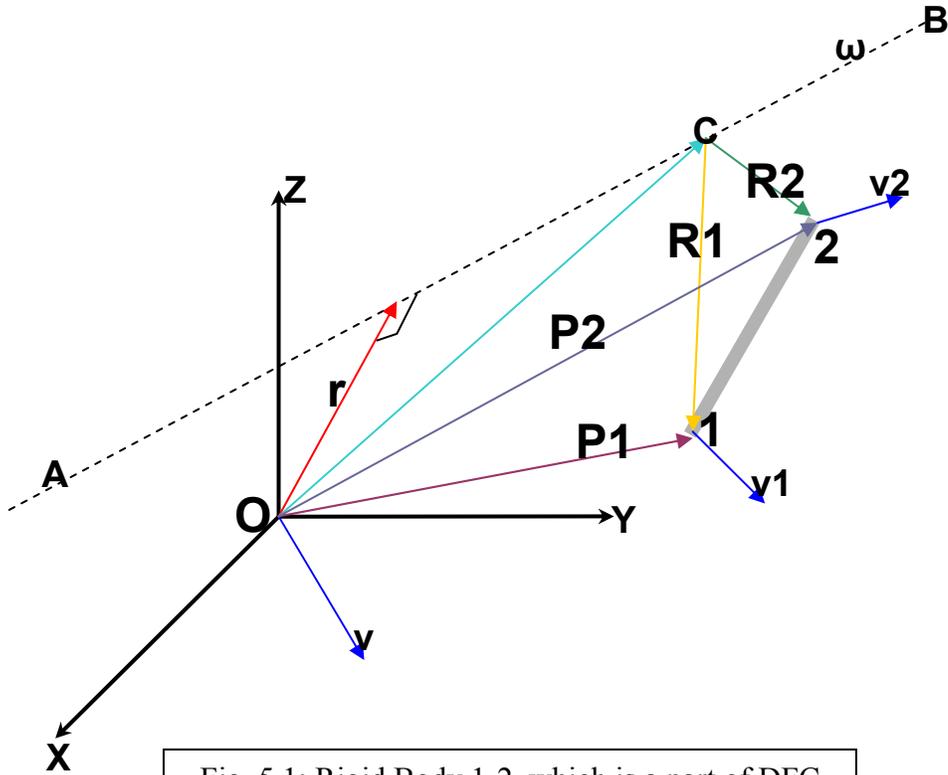
Fig. 5.1: Rigid Body 1-2, which is a part of DFC
of a KC. The variation propagation from one end
of rigid body to other end is being studied

Next step is to ascertain the location of the axis of instantaneous rotation of the rigid body

1-2. Let A-B is the instantaneous axis of rotation (figure 5.1). If **r** is the vector from the

origin of global co-ordinate frame and is perpendicular to instantaneous axis of rotation

of the rigid body, then **r** can be determined from the following relations:

$$\vec{r} \times \vec{\omega}_{12} = \vec{v}_{12} \qquad \qquad ...(5\text{-}8)$$

and,

$$\vec{r} \bullet \vec{\omega}_{12} = 0 \qquad \qquad ...(5\text{-}9)$$

ω₁₂ and **v₁₂** are given by the twist matrix of the rigid body.

The equation of instantaneous axis of rotation is given by:

$$\overrightarrow{OC} = t\vec{\omega}_{12} + \vec{r} \qquad\qquad ..(5\text{-}10)$$

where $\boldsymbol{\omega_{12}}$ vector denotes the direction of angular velocity of the rigid body, **r** denotes the perpendicular distance of the instantaneous axis of rotation from the origin of global co-ordinate frame and t is a scalar which can take any value from $-\infty$ to $+\infty$. This will give us location of point C, which can be any point lying of the axis of instantaneous rotation A-B of the rigid body 1-2. Variable t parameterizes the location of point C.

P1 is the vector from the origin of the global co-ordinate frame to the point **1** on the rigid body, and **P2** is the vector from the origin of the global co-ordinate frame to the point **2** on the rigid body. Assuming **C** is any point lying on the instantaneous axis of rotation, the vectors from point **C** to points **1** and **2** are represented as:

$$\overrightarrow{R1} = \overrightarrow{P1} - \overrightarrow{OC} \qquad\qquad ..(5\text{-}11)$$

and since

$$\overrightarrow{OC} = t\vec{\omega}_{12} + \vec{r} \qquad\qquad ..(5\text{-}10)$$

$$\Rightarrow \overrightarrow{R1} = \overrightarrow{P1} - t\vec{\omega}_{12} - \vec{r} \qquad\qquad ..(5\text{-}12)$$

Similarly

$$\overrightarrow{R2} = \overrightarrow{P2} - \overrightarrow{OC} = \overrightarrow{P2} - t\vec{\omega}_{12} - \vec{r} \qquad\qquad ..(5\text{-}13)$$

where **R1** is vector from any point **C** on the axis of instantaneous rotation to point **1** on the rigid body and **R2** is vector from any other point **C** which also lies on the axis of instantaneous rotation, to point **2** on the rigid body.

For any given ω_{12} of the rigid body, the velocities at two ends of the rigid body are given by:

$$\vec{v1} = \vec{\omega}_{12} \times \vec{R1} = \vec{\omega}_{12} \times (\vec{P1} - t\vec{\omega}_{12} - \vec{r}) \qquad ..(5\text{-}14)$$

$$\Rightarrow \vec{v1} = \vec{\omega}_{12} \times \vec{P1} - t(\vec{\omega}_{12} \times \vec{\omega}_{12}) - \vec{\omega}_{12} \times \vec{r}, \quad \because \vec{\omega}_{12} \times \vec{\omega}_{12} = 0$$

$$\Rightarrow \vec{v1} = \vec{\omega}_{12} \times \vec{P1} - \vec{\omega}_{12} \times \vec{r} \qquad ..(5\text{-}15)$$

Similarly,

$$\vec{v2} = \vec{\omega}_{12} \times \vec{P2} - \vec{\omega}_{12} \times \vec{r} \qquad ..(5\text{-}16)$$

where **v1** is the velocity which will come at point **1** due to angular velocity ω_{12} and **v2** is the velocity which will come at point **2**. **v1** and **v2** give the direction of variations that can arise at point **1** and **2** respectively. As we mentioned before that **R1** and **R2** are the vectors from the axis of instantaneous rotation to point **1** and **2** respectively, such that **R1** and **R2** are also perpendicular to the direction of instantaneous rotation. Now **R1** and **R2** have to satisfy the relations that:

$$\vec{v1} = \vec{\omega}_{12} \times \vec{R1}$$

and

$$\vec{\omega}_{12} \bullet \vec{R1} = 0$$

Similarly

$$\vec{v2} = \vec{\omega}_{12} \times \vec{R2}$$

and

$$\vec{\omega}_{12} \bullet \vec{R2} = 0$$

We can use above relations to find **R1** and **R2**. radius.m Matlab® file given in appendix gives radius vector satisfying above relations. Now if we know the variation coming at point **1**, we can find the variation coming at point **2**. In a closed loop if **dP1** denotes the variation coming at point **1** of the rigid body due the variations in other parts/fixtures/features of the closed loop. We can then say:

$$\vec{dP1} = a\vec{\omega}_{12} \times \vec{R1} \qquad\qquad\qquad ..(5\text{-}17)$$

where *a* is a scalar.

Since we know **dP1**, **ω₁₂**, and **R1**, we can determine the value of the scalar *a*:

$$a = \frac{norm(\vec{dP1})}{norm(\vec{\omega}_{12} \times \vec{R1})} \qquad\qquad\qquad ..(5\text{-}18)$$

where *norm* is a function which finds the effective magnitude of the vector on which it operates.

Once we know *a*, we can find the value of variation coming at point **2** of the rigid body, because of variations in other parts/fixtures/features of the closed loop, which is given by:

$$\vec{dP2} = a\vec{\omega}_{12} \times \vec{R2} = \frac{\left|\vec{dP1}\right|}{\left|\vec{\omega}_{12} \times \vec{R1}\right|}\left(\vec{\omega}_{12} \times \vec{R2}\right) \qquad\qquad\qquad ..(5\text{-}19)$$

Using the above analysis we can find the new location of rigid body 1-2 (which is a part of closed loop chain) because of variation coming on other parts/features in the closed loop. For the above analysis we assumed that there is no variation in the rigid body itself. Now if we know the variation in the first link in part-feature diagram, we can use the above method to calculate the variation coming at the ends of each part/fixture/feature involved in the delivery chain of the KC, and finally the variation coming on to the KC.

5.4 Example

We will show the application of Screw Theory to find the variations in a closed loop, with an example of slider crank. The slider crank is shown in figure 5.2. The crank angle is assumed to fixed at 45˚ from the vertical axis (z-axis in the figure 5.2), for the analysis purposes. The example is taken from [Gao et al., 1998b].
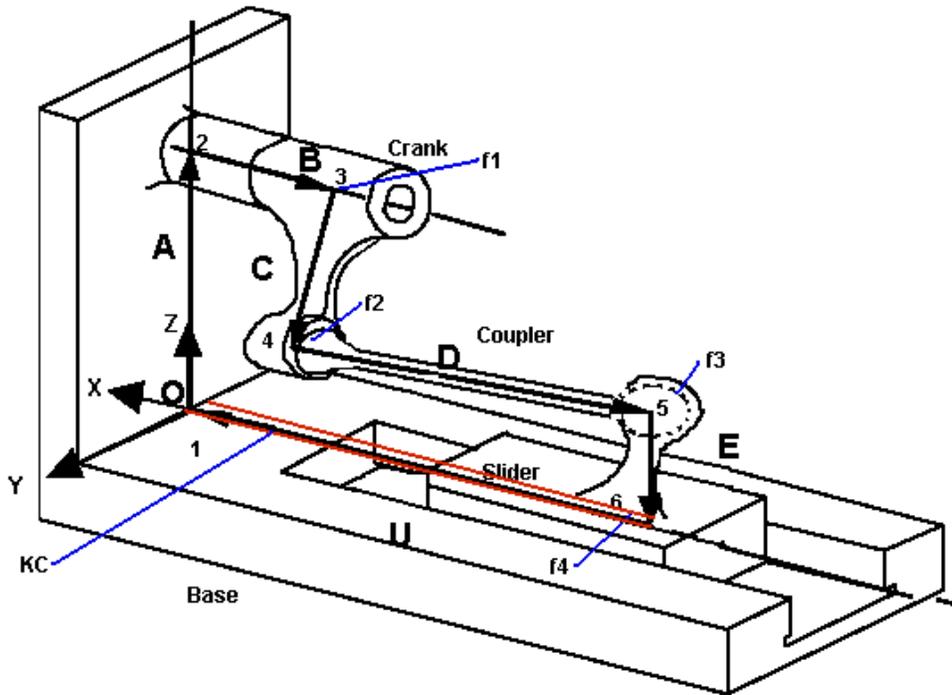
Fig. 5.2: 3-D Slider Crank Mechanism,
with features and parts marked on the
diagram

Let the KC be the length of the dimension **U**, which is the distance of the slider from the
origin of global co-ordinate frame **O**. **f1**, **f2**, **f3**, **f4** denotes the features in the assembly
and **A**, **B**, **C**, **D**, **E** denotes the parts involved in the delivery chain of the KC. Table 5.1
shows the location of the features in the nominal case and table 5.2 denotes the tolerance
limits on the parts.

Table 5.1: Location of Assembly Features

Feature	Location in Global Co-ordinate frame (x, y, z)
f1	(-12, 0, 20)
f2	(-12, 10.6066, 9.3934)
f3	(-39.7164, 0, 5)
f4	(-39.7164, 0, 0)

Table 5.2: Tolerance limits on the linear dimensions of the Parts

Part	Tolerance Limit ($\pm$) on the dimension of part	Direction of tolerance in global co-ordinate system
A	0.025	$1\hat{k}$
B	0.0125	$1\hat{i}$
C	0.0125	$1\hat{j} - 1\hat{k}$
D	0.03	$-1\hat{i} - 0.3827\hat{j} - 0.1585\hat{k}$
E	0.0025	$1\hat{k}$

Figure 5.3 shows the part-feature diagram of the slider crank mechanism.
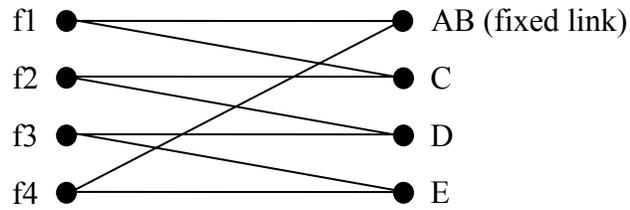
Fig. 5.3: Part-Feature Diagram of Slider
Crank Mechanism

For fixed crank angle, the twist matrices of the features are:

$$f1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$f2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 9.3934 & -10.6066 \\ 0 & 1 & 0 & -9.3934 & 0 & -12 \\ 0 & 0 & 1 & 10.6066 & 12 & 0 \end{bmatrix},$$

$$f3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 5 & 0 \\ 0 & 1 & 0 & -5 & 0 & -39.7164 \\ 0 & 0 & 1 & 0 & 39.7164 & 0 \end{bmatrix}, \text{ and}$$

$$f4 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

i).   In case of no variation, and crank angle fixed, from the part-feature diagram we get:

$Dof\_of\_C = reciprocal([reciprocal(f1); reciprocal([f2; f3; f4])])$,

$\Rightarrow Dof\_of\_C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

$Dof\_of\_D = reciprocal([reciprocal([f1; f2]); reciprocal([f3; f4])])$,

$\Rightarrow Dof\_of\_D = \begin{bmatrix} 1 & 0.3827 & 0.1585 & -1.9134 & 11.2956 & -15.1988 \end{bmatrix}$,

$Dof\_of\_E = reciprocal([reciprocal([f1; f2; f3]); reciprocal(f4)])$, and

$\Rightarrow Dof\_of\_E = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

where Dof means degree of freedom of a part.

From the **Dof_of_D** we can get the direction of rotation axis of the coupler, which is:

$1\hat{i} + 0.3827\hat{j} + 0.1585\hat{k}$

The direction of the coupler in global co-ordinate frame is given by:

Point(5) – Point(4), where point(**5**) is at the location of **feature 3** and point(**4**) is at the location of **feature 2**.

$= \begin{pmatrix} -39.7164 & 0 & 5 \end{pmatrix} - \begin{pmatrix} -12 & 10.6066 & 9.3934 \end{pmatrix}$

$= -27.7164\hat{i} - 10.6066\hat{j} - 4.3934\hat{k}$

On normalizing the above vector with respect to its *x* value we get,

$$= 1\hat{i} + 0.3827\hat{j} + 0.1585\hat{k}$$

Thus we see that the coupler will rotate in the direction of vector **D**, which is along the coupler. Since link is free to rotate in a direction which coincides with the direction of the coupler, its rotation will not produce any variation in Key Characteristic **U**.

ii).   In case of linear variation in part **A** (Table 5.2), only the twist matrix for feature **f1** changes

$$f1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and}$$

$$\Rightarrow Dof\_of\_C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

Hence **C** will move as a rigid body in z-direction along with **A** due to variation in **A**.

$$\Rightarrow Dof\_of\_D = \begin{bmatrix} 1 & 0.3827 & 0.1585 & -1.9136 & 11.2956 & -15.1995 \\ 0 & 1 & 0 & -9.3934 & 0 & -39.7164 \end{bmatrix}$$

Part **D** will have two degrees of freedom due to variation in part **A**.

$$\Rightarrow Dof\_of\_E = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Hence part **E** will move in x-direction (direction of slot for the slider) due to variation in part **A**.

Looking at **Dof_of_D** we see that there are two directions of rotation possible for the coupler.

$\vec{\omega}_1 = 1\hat{i} + 0.3827\hat{j} + 0.1585\hat{k}$, and

$\vec{\omega}_2 = 0\hat{i} + 1\hat{j} + 0\hat{k}$

We have already seen that ω_1 does not produce any variation in the assembly. So if we can find ω_3 such that no component of it lies along ω_1, then we can get the variation coming on to the coupler from ω_3.

We need to search for ω_3 such that ω_3 is perpendicular to ω_1, and

$$\vec{\omega}_2 = \alpha\vec{\omega}_1 + \beta\vec{\omega}_3 \qquad ..(5\text{-}20)$$

where α and β are some scalar constants. On finding the dot product of the above equation with ω_1,

$$\vec{\omega}_2 \bullet \vec{\omega}_1 = \alpha(\vec{\omega}_1 \bullet \vec{\omega}_1) + \beta(\vec{\omega}_2 \bullet \vec{\omega}_1), \qquad ..(5\text{-}21)$$

since $\vec{\omega}_3 \bullet \vec{\omega}_1 = 0$

$$\Rightarrow \alpha = \frac{\vec{\omega}_1 \bullet \vec{\omega}_2}{\vec{\omega}_1 \bullet \vec{\omega}_1} = 0.3267$$

$$\Rightarrow \beta\vec{\omega}_3 = \vec{\omega}_2 - \alpha\vec{\omega}_1 = \begin{bmatrix} -0.3267 & 0.875 & -0.0518 \end{bmatrix}$$

On normalizing ω_3 we get

$$\vec{\omega}_3 = 1\hat{i} - 2.6787\hat{j} + 0.1585\hat{k}$$

Now since we know α, we can modify **Dof_of_D** matrix accordingly,

$Row_2 = Row_2 - \alpha.Row_1$, and writing Row_2 in row reduced echelon form.

$$\text{mod}\mathit{ified\_Dof\_of\_D} = \begin{bmatrix} 1 & 0.3827 & 0.1585 & -1.9136 & 11.2956 & -15.1995 \\ 1 & -2.6787 & 0.1585 & 26.8430 & 11.2956 & 21.5373 \end{bmatrix}$$

From 2nd row in the modified matrix,

$\vec{\omega} = \begin{bmatrix} 1 & -2.6787 & 0.1585 \end{bmatrix}$, and

$\vec{v} = \begin{bmatrix} 26.843 & 11.2956 & 21.5373 \end{bmatrix}$

To find the location of instantaneous axis of rotation, we will use the above mentioned relations:

$$\vec{r} \times \vec{\omega} = \vec{v} \qquad \qquad \qquad \text{...(5-8)}$$

And,

$$\vec{r} \bullet \vec{\omega} = 0 \qquad \qquad \qquad \text{...(5-9)}$$

We can calculate **r** using radius.m Matlab® file, which is given in appendix. From radius.m we get:

$\vec{r} = \begin{bmatrix} -7.2534 & -2.1075 & 10.1456 \end{bmatrix}$

From this we can get the equation of instantaneous axis of rotation:

$$\overrightarrow{OC} = t\vec{\omega} + \vec{r} \qquad \qquad \qquad \text{..(5-10)}$$

$$\overrightarrow{OC} = t\begin{pmatrix} 1 & -2.6787 & 0.1585 \end{pmatrix} + \begin{pmatrix} -7.2534 & -2.1075 & 10.1456 \end{pmatrix}$$

For coupler, the location co-ordinates of extreme ends are:

P1 = (-12   10.6066   9.3934)

P2 = (-39.7164   0   5)

We need to find the radius vectors from the axis of instantaneous rotation to the end points of the coupler, such that the radius vectors are perpendicular to the direction of axis of instantaneous rotation. Let the radius vectors be denoted by **R1**, **R2**.

$$\overrightarrow{R1} = \overrightarrow{P1} - t_1\vec{\omega} - \vec{r}\,, \qquad\qquad\qquad ..(5\text{-}12)$$

$$\because \vec{\omega} \bullet \overrightarrow{R1} = 0\,,$$

$$\Rightarrow t_1 = \frac{\overrightarrow{P1} \bullet \vec{\omega}}{\vec{\omega} \bullet \vec{\omega}} = -4.7464$$

$$\Rightarrow \overrightarrow{R1} = \left(-0.2089 \times 10^{-3} \quad -0.0578 \times 10^{-3} \quad 0.1030 \times 10^{-3}\right)$$

Similarly,

$$\overrightarrow{R2} = \overrightarrow{P2} - t_2\vec{\omega} - \vec{r}\,, \qquad\qquad\qquad ..(5\text{-}13)$$

$$\because \vec{\omega} \bullet \overrightarrow{R2} = 0\,,$$

$$\Rightarrow t_2 = \frac{\overrightarrow{P2} \bullet \vec{\omega}}{\vec{\omega} \bullet \vec{\omega}} = -4.7465$$

$$\Rightarrow \overrightarrow{R2} = \left(-27.7165 \quad -10.6069 \quad -4.3933\right)$$

Since we know the variation coming on to part **C**, we also know the variation coming on point **4** of part **D**. Let the variations at two ends of the coupler be denoted by **dP1** and **dP2**.

$$\overrightarrow{dP1} = \begin{pmatrix} 0 & 0 & 0.025 \end{pmatrix}$$

$$\because \overrightarrow{dP1} = a\vec{\omega} \times \overrightarrow{R1} \qquad\qquad ..(5\text{-}17)$$

$$\therefore a = \frac{\left|\overrightarrow{dP1}\right|}{\left|\vec{\omega} \times \overrightarrow{R1}\right|} \qquad\qquad ..(5\text{-}18)$$

$$\overrightarrow{d\theta} = a\vec{\omega} = \begin{pmatrix} 0 & -0.0510 \times 10^{-3} & 0.1232 \times 10^{-3} \end{pmatrix}$$

$$\overrightarrow{dP2} = \overrightarrow{d\theta} \times \overrightarrow{R2} = \begin{pmatrix} 0.0039 & 0 & 0 \end{pmatrix} \qquad\qquad ..(5\text{-}19)$$

theta.m Matlab® is used to calculate **dθ** from **dP1** and **R1**, and the file is given in appendix.

Since part **E** is attached at point **P2** to part **D**, thus the variation in **E** in x-direction would be same as the variation in point **P2** in x-direction. The variation in location of part **E** is the variation coming on the KC **U**.

Variation in **U** due to variation in **A** = **0.0040**

iii). In case of linear variation in part **B** (Table 5.2), only the twist matrix of feature **f1** changes

$$f1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \text{ and}$$

$$\Rightarrow Dof\_of\_C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

Hence **C** will move as a rigid body in x-direction along with **B** due to variation in **B**.

$$\Rightarrow Dof\_of\_D = \begin{bmatrix} 1 & 0.3827 & 0.1585 & 0 & 11.2956 & -15.1995 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Part **D** will have two degrees of freedom due to variation in part **B**, and the extra degree of freedom is in x-direction due to variation.

$$\Rightarrow Dof\_of\_E = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Hence part **E** will move in x-direction (direction of slot for the slider) due to variation in part **B**. Thus due to variation in part **B** all the parts **C**, **D**, and **E** in x-direction and the movement in x-direction is of same amount as the variation coming on part **B**. The variation coming on the KC **U** is same as the variation coming on the location of part **E** in x-direction, which is same the variation in part **B**.

Variation in **U** due to variation in **B** = **0.0125**

iv).   In case of linear variation in part **C** (Table 5.2), only the twist matrix of feature **f2** changes

$$f2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 9.3934 & -10.6066 \\ 0 & 1 & 0 & -9.3934 & 0 & -12 \\ 0 & 0 & 1 & 10.6066 & 12 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \text{ and}$$

$$\Rightarrow Dof\_of\_C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Rightarrow Dof\_of\_D = \begin{bmatrix} 1 & 0.3827 & 0.1585 & -1.9134 & 11.2956 & -15.1995 \\ 0 & 1 & 1 & 1.2132 & 39.7164 & -39.7164 \end{bmatrix}$$

Part **D** will have two degrees of freedom due to variation in part **C**.

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$\Rightarrow Dof\_of\_E = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

Hence part **E** will move in x-direction (direction of slot for the slider) due to variation in part **C**.

Looking at **Dof_of_D** we see that there are two directions of rotation possible for the coupler.

$\vec{\omega}_1 = 1\hat{i} + 0.3827\hat{j} + 0.1585\hat{k}$, and

$\vec{\omega}_2 = 0\hat{i} + 1\hat{j} + 1\hat{k}$

We have already seen that ω_1 does not produce any variation in the assembly. So if we can find ω_3 such that no component of it lies along ω_1, then we can get the variation coming on to the coupler from ω_3.

We need to search for ω_3 such that ω_3 is perpendicular to ω_1, and

$$\vec{\omega}_2 = \alpha\vec{\omega}_1 + \beta\vec{\omega}_3 \qquad\qquad ..(5\text{-}20)$$

where α and β are some scalar constants. On finding the dot product of the above equation with ω_1,

$$\vec{\omega}_2 \bullet \vec{\omega}_1 = \alpha\left(\vec{\omega}_1 \bullet \vec{\omega}_1\right) + \beta\left(\vec{\omega}_2 \bullet \vec{\omega}_1\right), \qquad\qquad ..(5\text{-}21)$$

since $\vec{\omega}_3 \bullet \vec{\omega}_1 = 0$

$$\Rightarrow \alpha = \frac{\vec{\omega}_1 \bullet \vec{\omega}_2}{\vec{\omega}_1 \bullet \vec{\omega}_1} = 0.4619$$

$$\Rightarrow \beta\vec{\omega}_3 = \vec{\omega}_2 - \alpha\vec{\omega}_1 = \begin{bmatrix} -0.4619 & 0.8232 & 0.9268 \end{bmatrix}$$

On normalizing **ω₃** we get

$$\vec{\omega}_3 = 1\hat{i} - 1.7821\hat{j} - 2.0063\hat{k}$$

Now since we know α, we can modify **Dof_of_D** matrix accordingly,

Row₂ = Row₂ – α.Row₁, and writing Row₂ in row reduced echelon form.

$$\mathrm{mod}ified\_Dof\_of\_D = \begin{bmatrix} 1 & 0.3827 & 0.1585 & -1.9136 & 11.2956 & -15.1995 \\ 1 & -1.7821 & -2.0063 & -4.5397 & -74.6812 & 70.7780 \end{bmatrix}$$

From 2ⁿᵈ row in the modified matrix,

$$\vec{\omega} = \begin{bmatrix} 1 & -1.7821 & -2.0063 \end{bmatrix}, \text{ and}$$

$$\vec{v} = \begin{bmatrix} -4.5397 & -74.6812 & 70.7780 \end{bmatrix}$$

To find the location of instantaneous axis of rotation, we will use the above mentioned relations:

$$\vec{r} \times \vec{\omega} = \vec{v} \qquad \qquad \text{...(5-8)}$$

And,

$$\vec{r} \bullet \vec{\omega} = 0 \qquad \qquad \text{...(5-9)}$$

From radius.m we get:

$$\vec{r} = \begin{bmatrix} -39.7164 & 0.6066 & -0.6066 \end{bmatrix}$$

From this we can get the equation of instantaneous axis of rotation:

$$\overrightarrow{OC} = t\vec{\omega} + \vec{r} \qquad \qquad \text{..(5-10)}$$

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$$\overrightarrow{OC} = t\begin{pmatrix} 1 & -1.7821 & -2.0063 \end{pmatrix} + \begin{pmatrix} -39.7164 & 0.6066 & -0.6066 \end{pmatrix}$$

For coupler, the location co-ordinates of extreme ends are:

P1 = (-12   10.6066   9.3934)

P2 = (-39.7164   0   5)

We need to find the radius vectors from the axis of instantaneous rotation to the end points of the coupler, such that the radius vectors are perpendicular to the direction of axis of instantaneous rotation. Let the radius vectors be denoted by **R1**, **R2**.

$$\overrightarrow{R1} = \overrightarrow{P1} - t_1\vec{\omega} - \vec{r} , \qquad\qquad ..(5\text{-}12)$$

$$\because \vec{\omega} \bullet \overrightarrow{R1} = 0 ,$$

$$\Rightarrow t_1 = \frac{\overrightarrow{P1} \bullet \vec{\omega}}{\vec{\omega} \bullet \vec{\omega}} = 10$$

$$\Rightarrow \overrightarrow{R1} = \begin{pmatrix} 27.7164 & 0 & 0 \end{pmatrix}$$

Similarly,

$$\overrightarrow{R2} = \overrightarrow{P2} - t_2\vec{\omega} - \vec{r} ,$$

$$\because \vec{\omega} \bullet \overrightarrow{R2} = 0 ,$$

$$\Rightarrow t_2 = \frac{\overrightarrow{P2} \bullet \vec{\omega}}{\vec{\omega} \bullet \vec{\omega}} = 2.5$$

$$\Rightarrow \overrightarrow{R2} = \begin{pmatrix} 0 & -3.1066 & -3.1066 \end{pmatrix}$$

Since we know the variation coming on to part **C**, we also know the variation coming on point **4** of part **D**. Let the variations at two ends of the coupler be denoted by **dP1** and **dP2**.

$$\overrightarrow{dP1} = \left(0 \quad \frac{0.0125}{\sqrt{2}} \quad -\frac{0.0125}{\sqrt{2}}\right) = (0 \quad 0.0088 \quad -0.0088)$$

$$\because \overrightarrow{dP1} = a\vec{\omega} \times \overrightarrow{R1} \qquad\qquad ..(5\text{-}17)$$

$$a = \frac{\left|\overrightarrow{dP1}\right|}{\left|\vec{\omega} \times \overrightarrow{R1}\right|} \qquad\qquad ..(5\text{-}18)$$

$$\overrightarrow{d\theta} = a\vec{\omega} = \left(0 \quad 0.3175 \times 10^{-3} \quad 0.3175 \times 10^{-3}\right)$$

$$\overrightarrow{dP2} = \overrightarrow{d\theta} \times \overrightarrow{R2} = (0.002 \quad 0 \quad 0) \qquad\qquad ..(5\text{-}19)$$

Since part **E** is attached at point **P2** to part **D**, thus the variation in **E** in x-direction would be same as the variation in point **P2** in x-direction. The variation in location of part **E** is the variation coming on the KC **U**.

Variation in **U** due to variation in **C** = **0.0020**

v).   In case of linear variation in part **D** (Table 5.2), only the twist matrix of feature **f3** changes

$$f3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 5 & 0 \\ 0 & 1 & 0 & -5 & 0 & -39.7164 \\ 0 & 0 & 1 & 0 & 39.7164 & 0 \\ 0 & 0 & 0 & 1 & 0.3827 & 0.1585 \end{bmatrix}, \text{ and}$$

$\Rightarrow Dof\_of\_C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

$\Rightarrow Dof\_of\_D = \begin{bmatrix} 1 & 0.3827 & 0.1585 & -1.9134 & 11.2956 & -15.1995 \\ 0 & 1 & -2.4145 & -35.0632 & -28.9741 & -12 \end{bmatrix}$

Part **D** will have two degrees of freedom due to variation in part **D**.

$\Rightarrow Dof\_of\_E = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

Hence part **E** will move in x-direction (direction of slot for the slider) due to variation in part **D**.

Looking at **Dof_of_D** we see that there are two directions of rotation possible for the coupler.

$\vec{\omega}_1 = 1\hat{i} + 0.3827\hat{j} + 0.1585\hat{k}$, and

$\vec{\omega}_2 = 0\hat{i} + 1\hat{j} - 2.4145\hat{k}$

We have already seen that ω_1 does not produce any variation in the assembly. So if we can find ω_3 such that no component of it lies along ω_1, then we can get the variation coming on to the coupler from ω_3.

We need to search for ω_3 such that ω_3 is perpendicular to ω_1, and

$$\vec{\omega}_2 = \alpha\vec{\omega}_1 + \beta\vec{\omega}_3 \qquad ..(5\text{-}20)$$

where α and β are some scalar constants. On finding the dot product of the above equation with ω_1,

$$\vec{\omega}_2 \bullet \vec{\omega}_1 = \alpha(\vec{\omega}_1 \bullet \vec{\omega}_1) + \beta(\vec{\omega}_2 \bullet \vec{\omega}_1), \qquad ..(5\text{-}21)$$

since $\vec{\omega}_3 \bullet \vec{\omega}_1 = 0$

$$\Rightarrow \alpha = \frac{\vec{\omega}_1 \bullet \vec{\omega}_2}{\vec{\omega}_1 \bullet \vec{\omega}_1} = 0$$

$$\Rightarrow \beta\vec{\omega}_3 = \vec{\omega}_2 - \alpha\vec{\omega}_1 = \begin{bmatrix} 0 & 1 & -2.4145 \end{bmatrix}$$

On normalizing ω_3 we get

$$\vec{\omega}_3 = 0\hat{i} + 1\hat{j} - 2.4145\hat{k}$$

From 2nd row of matrix of degree of freedom on part **D**,

$$\vec{\omega} = \begin{bmatrix} 0 & 1 & -2.4145 \end{bmatrix}, \text{ and}$$

$$\vec{v} = \begin{bmatrix} -35.0632 & -28.9741 & -12 \end{bmatrix}$$

To find the location of instantaneous axis of rotation, we will use the above mentioned relations:

$$\vec{r} \times \vec{\omega} = \vec{v} \qquad\qquad ...(5\text{-}8)$$

And,

$$\vec{r} \bullet \vec{\omega} = 0 \qquad\qquad ...(5\text{-}9)$$

From radius.m we get:

$$\vec{r} = \begin{bmatrix} -12 & 12.3957 & 5.1338 \end{bmatrix}$$

From this we can get the equation of instantaneous axis of rotation:

$$\overrightarrow{OC} = t\vec{\omega} + \vec{r} \qquad\qquad ..(5\text{-}10)$$

$$\overrightarrow{OC} = t\begin{pmatrix}0 & 1 & -2.4145\end{pmatrix} + \begin{pmatrix}-12 & 12.3957 & 5.1338\end{pmatrix}$$

For coupler, the location co-ordinates of extreme ends are:

P1 = (-12   10.6066   9.3934)

P2 = (-39.7164   0   5)

We need to find the radius vectors from the axis of instantaneous rotation to the end points of the coupler, such that the radius vectors are perpendicular to the direction of axis of instantaneous rotation. Let the radius vectors be denoted by **R1**, **R2**.

$$\overrightarrow{R1} = \overrightarrow{P1} - t_1\vec{\omega} - \vec{r} , \tag{5-12}$$

$$\because \vec{\omega} \bullet \overrightarrow{R1} = 0 ,$$

$$\Rightarrow t_1 = \frac{\overrightarrow{P1} \bullet \vec{\omega}}{\vec{\omega} \bullet \vec{\omega}} = -1.7678$$

$$\Rightarrow \overrightarrow{R1} = \begin{pmatrix}0 & 0 & 0\end{pmatrix}$$

Similarly,

$$\overrightarrow{R2} = \overrightarrow{P2} - t_2\vec{\omega} - \vec{r} , \tag{5-13}$$

$$\because \vec{\omega} \bullet \overrightarrow{R2} = 0 ,$$

$$\Rightarrow t_2 = \frac{\overrightarrow{P2} \bullet \vec{\omega}}{\vec{\omega} \bullet \vec{\omega}} = -1.7676$$

$$\Rightarrow \overrightarrow{R2} = \begin{pmatrix}-27.7164 & -10.6281 & -4.4018\end{pmatrix}$$

Since there is a variation in the size of part **D**, so the length of part **D** will change and the vector **R** which points along **D** will be modified to vector **r**.

$$\vec{R} = \begin{pmatrix} 27.7164 & -10.6066 & -4.3934 \end{pmatrix}$$

$$\vec{r} = \frac{30.03}{30} \times \vec{R} = \begin{pmatrix} -27.7441 & -10.6172 & -4.3978 \end{pmatrix}$$

The direction of variation **t** for the point **5** on the coupler, due to angular velocity **ω** will be given by:

$$\vec{t} = \vec{\omega} \times \vec{r} = \begin{pmatrix} -30.0330 & 66.9882 & 27.7441 \end{pmatrix}$$

After variation point **5** will move to a point given by:

$$\vec{r} + a.\vec{t} = \begin{pmatrix} -27.7441 & -10.6172 & -4.3978 \end{pmatrix} + a.\begin{pmatrix} -30.0330 & 66.9882 & 27.7441 \end{pmatrix}$$

$$..(5\text{-}22)$$

where a is any scalar. Now since **Dof_of_E** is along x-direction, this means that point **5** can move only in x-direction, thus even after variation its y and z co-ordinates will not change. This condition will give us the value of a, which is then used to find the amount of variation coming in location of point **5**.

$$y\_component(\vec{r} + a.\vec{t}) = y\_component(\vec{R}) \qquad ..(5\text{-}23)$$

$$\Rightarrow a = 1.5834 \times 10^{-4}$$

$$\Rightarrow Variation\_Point\,5 = (\vec{r} + a.\vec{t} - \vec{R}) = \begin{pmatrix} -0.0325 & 0 & 0 \end{pmatrix} \qquad ..(5\text{-}24)$$

Since part **E** is attached at point **P2** to part **D**, thus the variation in **E** in x-direction would be same as the variation in point **P2** in x-direction. The variation in location of part **E** is the variation coming on the KC **U**.

Variation in **U** due to variation in **D** = **-0.0325**

vi).   In case of linear variation in part **E** (Table 5.2), only the twist matrix of feature **f4** changes

$$f4 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and}$$

$$\Rightarrow Dof\_of\_C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Rightarrow Dof\_of\_D = \begin{bmatrix} 1 & 0.3827 & 0.1585 & -1.9134 & 11.2956 & -15.1995 \\ 1 & -2.6782 & 0.1585 & 26.8390 & 11.2963 & 21.5314 \end{bmatrix}$$

Part **D** will have two degrees of freedom due to variation in part **D**.

$$\Rightarrow Dof\_of\_E = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & -6.3086 \end{bmatrix}$$

Hence part **E** will have coupled motion in *x*- and *z*-directions. When the variation in *z*-direction is 0.0025, corresponding variation in *x*-direction is given by:

$$x\_variation = \frac{1}{-6.3086} \times 0.0025 = -3.963 \times 10^{-4}$$

The variation in *x*-location of part **E** is the variation coming on the KC **U**.

Variation in **U** due to variation in **E** = **-3.963x10^{-4}**

These are the results that we will get when we will perform Kinematic Analysis on the Slider Crank. Appendix shows the Matlab® files VariationA.m, VariationB.m, VariationC.m, VariationD.m and VariationE.m, in which the effect of variation in parts on the KC **U** is found out using Kinematic Analysis. The results from the above analysis

using Screw Theory matches with those from Kinematic Analysis. The objective of Kinematic Analysis is to determine the kinematic quantities such as displacement, variation, velocity etc. in an under-constrained or properly constrained assembly [Ghosh and Mallik, 1994].

To see the affect of all the variations together, we can invoke Principle of Superposition. Principle of Superposition states that if the relationship between various variations is linear then affect of all the variations is the sum of the affects of each individual variation studied one at a time.

5.5 Intensity of KC Conflict

We can use any of the above mentioned methods to find the variations coming on the KCs in an assembly. Using Screw Theory we can also find the relationship between various KCs. We need to watch out for KC conflicts arising in the assembly. If the assembly already exists, and has some KCs which are conflicting, we need to redesign the assembly only if the effect of the conflict is serious. Once we know the variations coming onto conflicting KCs, we can compare the variations on each KC to the tolerance limit of that KC. If

a. $$Variation_i \leq Tolerance_i \qquad ..(5\text{-}25)$$

where $Variation_i$ is the variation coming on i^{th} conflicting KC, and $Tolerance_i$ is the tolerance limit of ith KC. If the variation on each of the conflicting KC is less than the tolerance limit, then we need not make any changes in the design. Even

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though there is KC Conflict scenario in the assembly, but the variations on each of the conflicting KC is less than the tolerance limit set on to those KCs.

b. $Variation_i > Tolerance_i$ ..(5-26)

where Variation$_i$ is the variation coming on ith conflicting KC, and Tolerance$_i$ is the tolerance limit of ith KC. If the variation on *any* of the conflicting KCs is more than the tolerance limit on that KC, we need to make some changes in the assembly. The proposed changes are:

  i). Reduce the variation coming on the KC, by reducing the variations in the parts/fixtures involved in the delivery chain of that KC

  ii). Redesign the assembly to eliminate KC conflict

Since it is easier to tighten the variation coming in a part/fixture, rather than redesigning the assembly, (which is more tedious to do) we seek strategies to reduce the variations in parts/fixtures. The variation in KC can be reduced by reducing the variations in parts, fixtures or features which are present in the delivery chain of that KC. This method of reducing variation in KC by reducing variation on parts, fixtures or features is known as *Variation Synthesis*.

5.5.1 Variation Synthesis

Variation synthesis is a method to allocate the variation in the KCs to the individual part, fixture or feature variations, thus ensuring that the variation in the KCs fall with in the tolerance limits [Ngoi and Ong, 1998]. Variation synthesis

can be interpreted as assigning an optimal set of variations, $X_i s$, so that $Y = f(X_1, X_2, ..., X_n)$ is satisfied, where Y is the variation coming on the KC and the $X_i s$ are the variations coming on parts, fixtures and features involved in the delivery chain of the KC. The existing variation synthesis models can be broadly classified into three categories:

- Optimization methods

- Quality Engineering methods, and

- Methods based on genetic algorithm

Most of the optimization methods use cost-tolerance models. These methods are mainly concerned with the dimensional variations. [Speckhart, 1972] and [Spotts, 1973] are the initial attempts to assign optimal set of tolerances that will guarantee the predetermined design specifications. Both the methods utilize Lagrange multipliers to solve the nonlinear programming problems. [Wilde, 1975] shows that the formulation of the variation assignment problem can be simplified by using pseudo-boolean programming. [Ostwald and Huang, 1977] formulated the problem as a linear integer program to deal with large-scale design problems.[Michael and Siddall, 1981], [Michael and Siddall, 1982] extended the conventional design optimization problem, in which the nominal values of the design variables are of interest, to include the optimal allocation of manufacturing tolerances. The proposed non-linear optimization scheme creates an optimal region of interest rather than a single point. [Lee and Woo, 1989], [Lee and Woo,

1990] formulated the variation synthesis as a probabilistic optimization problem in which a random variable is associated with a dimension and its variation. With the aid of a notion called the reliability index, they transformed the problems into the deterministic optimization and provided the relevant solution procedures.

Quality engineering methods [Taguchi et al., 1989] seeks for an integrated system of overall quality control in which every activity involved in production is controlled to produce products whose KCs deviate minimally from their target values.

Genetic Algorithms have also been applied to come with optimal set of variation allocation. [Lee and Johnson, 1993] used genetic algorithm and Monte Carlo method for the first time to come up with a scheme of variation allocation. The genetic algorithm method has been adopted by many other researchers such as [Iannuzz and Sandgren, 1995], [Kanai et al., 1995], [Carpinetti and Chetwynd, 1995], [Al-Ansary and Deiab, 1997], [Chen and Fischer, 2000], and [Li et al., 2000].

If the result of variation synthesis suggests that the variation in conflicting KCs can be brought under the tolerance limit, then we need not make any changes in the design of the assembly. But if the results are not positive, then we should strive for the methods that will resolve or reduce the amount conflict in the assembly.

5.6 Chapter summary

KC conflict is the situation we need to avoid in our assemblies. Presence of a conflict in assemblies tends to make them non-robust. To meet the quality requirements, assembly has to deliver it's KCs with in some given tolerance limits. If the conflicting KCs meet the requirements set on them, we need not remove the conflict from the assembly. But if the KCs meet the tolerance requirement very closely or altogether miss the requirements then we need to either resolve the conflict or reduce the intensity of the conflict.

There are many different methods that exist in the literature which can be used to find the variation coming on the KCs of the assembly. 4x4 matrices method, Kinematic analysis, Variational analysis etc. can be used to find the variations coming on the KCs in both open and closed loops. It was also shown, how we can apply Screw Theory to find the variations coming on KCs in a closed loop for a general assembly in three dimensions. The method will work for under- and properly-constrained assemblies. For over-constrained assemblies we need to know the constitutive relationships of the over-constrained parts in order to apply Screw Theory.

An example was solved using Screw Theory to illustrate the method. The analysis results for Slider-Crank mechanism by Screw Theory were compared to that from Kinematic analysis. Superposition principle can be applied to Screw Theory method in the same way as it is applied in Kinematic analysis method.

By knowing the variation coming on each of the conflicting KCs we can determine the intensity of conflict. If the variations in conflicting KCs are less than the tolerance limits set on the KCs then conflict is not strong enough to have an impact on the quality of the product. But if the variation overshoots the tolerance limit then we need to seek methods to either reduce the amount of conflict or resolve the conflict from the assembly.

The amount of conflict can be reduced by tightening the tolerances of the parts, fixtures that are involved in the delivery chains of conflicting KCs. This process is named as variation synthesis. But sometimes it is neither practically feasible nor economically viable to reduce the tolerance limit on the parts/fixtures. It is when we resort to other methods for resolving conflicts in the assembly.

In the next chapter we will look into the methods that can be used to either reduce the amount of conflict in the assembly or altogether remove the conflict from the assembly. There exists many ways to resolve the conflict. We will outline the methods that are either commonly being used in industry or have potential of being used with very less effort.

Chapter 6. RESOLVING KC CONFLICT

6.1 Introduction

Most of the real assemblies have to deliver more than one KC with in their respective tolerance limits. In many situations the KCs that the assembly has to deliver are not independent. The KCs might be related to each other. The relationship can be of two types, correlation, and conflicting. If the KCs in an assembly are conflicting, then we are in jeopardy of not meeting all of the conflicting KCs with in their tolerance limit. Since for the conflicting KCs if we try to reduce the variation in one of the KCs the variation in other conflicting KC will increase, and it might miss the tolerance limit set on its variation.

We need to strive for a robust design which is free from any kind of conflicts. As we saw in the last chapter that we need not resolve all the KC conflicts but only those which have high probability of not meeting their tolerance limit. It is at this point where this algorithm seeks a different approach as from axiomatic design theory [Suh, 1990]. The different ways to resolve or reduce amount of KC conflict in the assembly are:

- Change in assembly sequence

- Selective assembly for Type-1 assemblies

- Redefining DFCs of conflicting KCs

- Use of compliant parts

- Prioritizing KCs

The list given in this text is not exhaustive. There can be many other ways to resolve or reduce the amount of KC conflict.

We will provide an example for each of the methods of resolving KC conflict. These methods can be used alone or in combination to resolve or reduce the amount of KC conflict in an assembly.

6.2 Change in assembly sequence

We can resolve certain kinds of KC conflicts in the assembly by making suitable changes in the assembly sequence. This technique will work only for the Type-2 kind of assemblies. Since Type-1 assemblies are constrained completely by feature relations between their parts. The assembly process for Type-1 assemblies merely puts the parts together by joining their pre-defined mating features.

Figure 6.1 shows the assembly of a peg in a hole. This is similar to the example of square peg in square hole, except for the fact that here in this example the hole is made out of two parts, **part A** and **part B**.
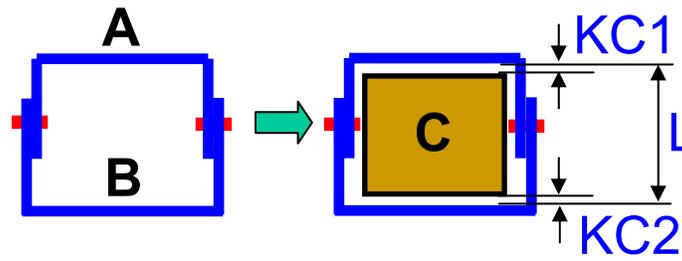
Fig. 6.1: Peg in a Hole. Assembly
Sequence does not resolve KC
Conflict

The two KCs are the distance between upper surface of the peg to upper surface of the hole and that between lower surfaces of the peg and the hole. In chapter on classification of KC relationship, we have seen that in this kind of configuration the two KCs are conflicting. The graph in figure 3.5 showed us that for a given L if we try to reduce the variation coming on one of the KCs we will increase the variation coming on other KC. Figure 6.1 represents the assembly sequence in which **parts A** and **B** are mated first to make subassembly of a hole, and in the next assembly step **part C** is inserted into the so formed hole. In above scenario both the KCs are achieved at the same assembly step.

By making suitable changes in the assembly sequence we can resolve the KC conflict. If the two KC are achieved at different assembly steps then there is a chance of resolving the conflict. Figure 6.2 shows one of the feasible assembly sequences by which we can resolve the conflict.
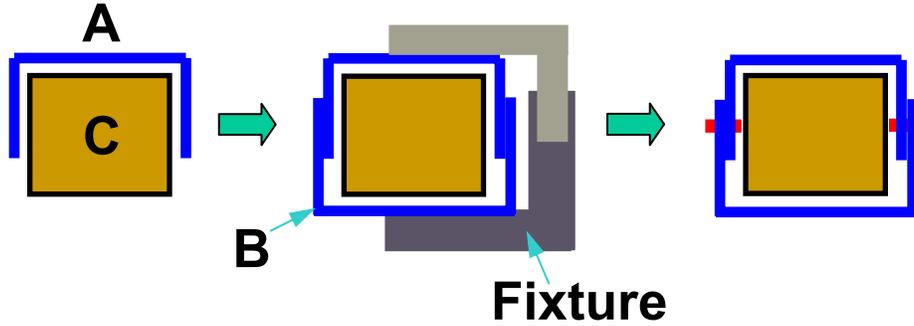
Fig. 6.2: Peg in a Hole.
Assembly Sequence resolves
KC conflict

In the first assembly step mate **part C** to **part A** such that we achieve **KC1** with in its tolerance limits. Thus in the first assembly step only **KC1** is achieved. And in the next step we use a flexible fixture to attach **part B** to the subassembly of **part A** and **C**, in such a manner that we achieve **KC2** with in tolerance limit. **KC2** is formed in the second assembly step. Finally we can secure the *contact liaison* formed between **part A** and **B**. In this assembly sequence we are achieving the KCs at different assembly steps, and there are sufficient degrees of freedom in the assembly to meet the two KCs independently.

6.3 Selective Assembly

Selective assembly is a very useful method of meeting the conflicting KCs with in their tolerance limits especially in case of Type-1 assembly. We try to select a part, which is

common in the delivery chains of both the KCs, having a dimension which will make sure that both the KCs lie with in their tolerance zone. Figure 6.3 shows the case of a peg in a hole, with the KCs as described in the case of figure 6.1. This is a Type-1 assembly, so we cannot resolve the conflict between KCs by making adjustments in the assembly sequence. For a given dimension L of hole, we select the peg of a dimension by which we can achieve both **KC1** and **KC2** with in their tolerance limit.
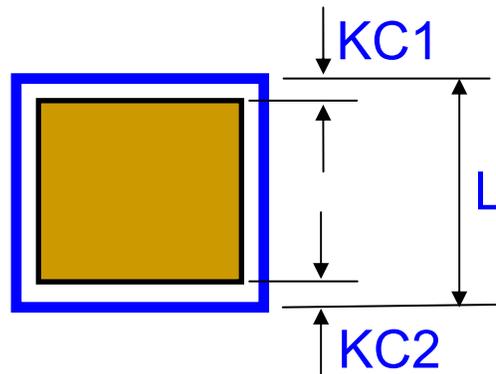
Fig. 6.3: Selective
Assembly for a Type-1
assembly. Peg in a hole

Selective assembly is also practiced for selecting a suitable size piston for a given size of bore in an engine. The KC for this case is the uniform gap between piston and bore through out the periphery of the piston. This KC exists in all the directions in that plane perpendicular to the central axes of bore and piston. Gap sizes at the diametrically

opposite ends of the piston give rise to KC conflict. During assembly of the engine, pistons are placed in different bins according to the measurement of their diameter. For each bore, first its diameter is determined and then a piston is selected from the suitable bin so that there is uniform gap between piston and bore and the gap is with in the tolerance limit. Figure 6.4 shows the KC conflict in the assembly of piston-cylinder in an engine.
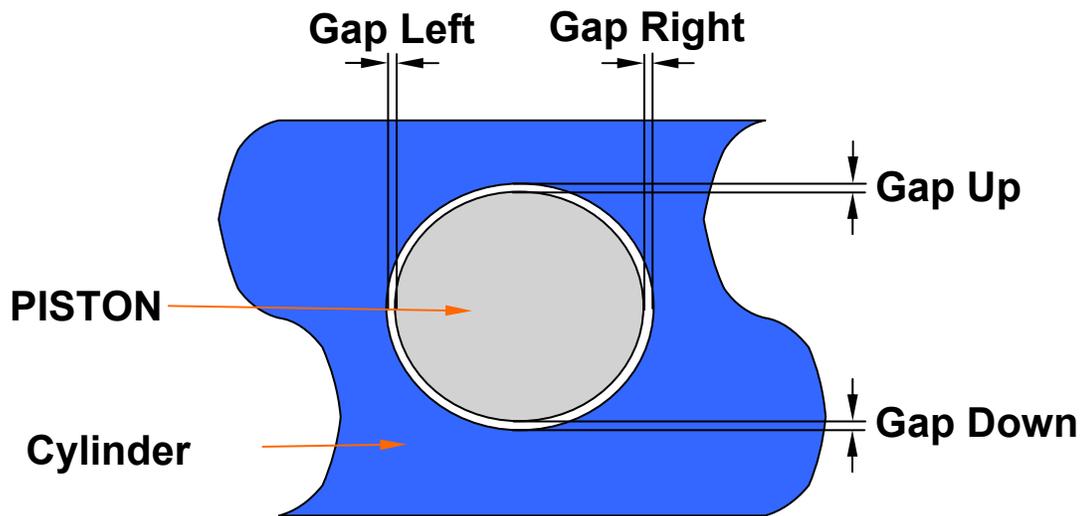
Fig. 6.4: Selective Assembly of Piston and KC conflict at diametrically opposite ends of Piston

For piston-cylinder assembly KCs **Gap Up** and **Gap Down** are in conflict with each other, and so are the KCs **Gap Left** and **Gap Right**.

We cannot solve KC Conflict using selective assembly method, but by using it we make sure that we achieve the conflicting KCs with in their tolerance limits.

6.4 Redefining DFCs of Conflicting KCs

We can remove the conflict in KCs by making appropriate changes in their DFCs. DFCs can be changed in following ways:

- Changes in the assembly features (location strategy of parts)

- Eliminating certain KCs by redesigning the assembly

- Adding or removing parts or fixtures to resolve conflict

- Converting certain Type-1 assemblies to Type-2 assemblies

We will use the example of T-Joints in a V-Engine, to show how above methods can be applied to resolve conflicts in an assembly. T-Joint is a joint where three parts come together in the form of a **T**. Figure 6.5 shows a T-Joint, in which **part A** and **B** forms the base of the T-Joint and **part C** forms the tee of the T-Joint. The KC in the T-Joint is achieving perfect step-size. Step-size is the distance between the upper surfaces of the base parts. In figure 6.5, it is the distance between the upper surfaces of **part A** and **B**. In nominal case (for perfect T-Joint) the step-size should be zero. In engines non-zero step-sizes in T-Joints are responsible for the oil leaks, which are directly related to customer-level KCs.
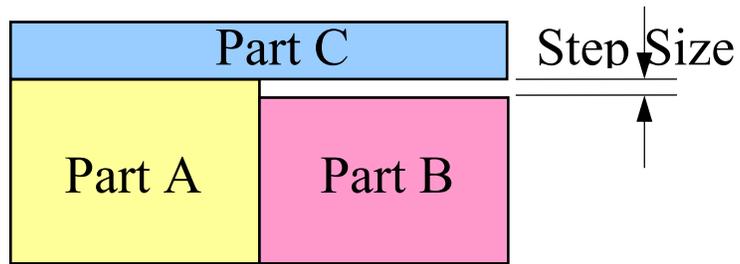
Fig. 6.5: T-Joint. Part A and B are the base parts, and part C forms the tee of the T-Joint

Figure 6.6 shows a T-Joint formed between Oil-Pan, Cylinder-Block Housing and Front Cover of an engine. In this T-Joint, Oil-Pan and Housing forms the base parts of T-Joint and Front Cover forms the tee of the T-Joint.
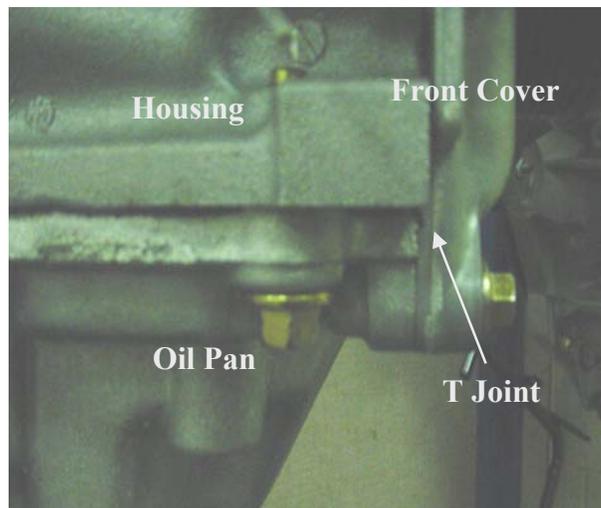
Fig. 6.6: T-Joint between Oil Pan, Cylinder Block Housing and Front Cover

Figure 6.7 shows cross section of a typical V-Engine. Cylinder Block is the base part of the V-Engine. Bulk Head Housing, Front Cover and Cylinder Head are attached to Cylinder Block. Oil Pan is attached to the Bulk Head Housing. Cam Cover is secured to Cylinder Head. Transmission is located by Cylinder Block, Bulk Head Housing and Oil Pan. There are six T-Joints in total, in the cross section shown. These are, T-Joints between Oil Pan-Bulk Head Housing-Transmission, Bulk Head Housing-Cylinder Block-Transmission, Oil Pan-Bulk Head Housing-Front Cover, Front Cover-Bulk Head Housing-Cylinder Block, Cylinder Block-Front Cover-Cylinder Head, and Cylinder Head-Front Cover-Cam Cover.
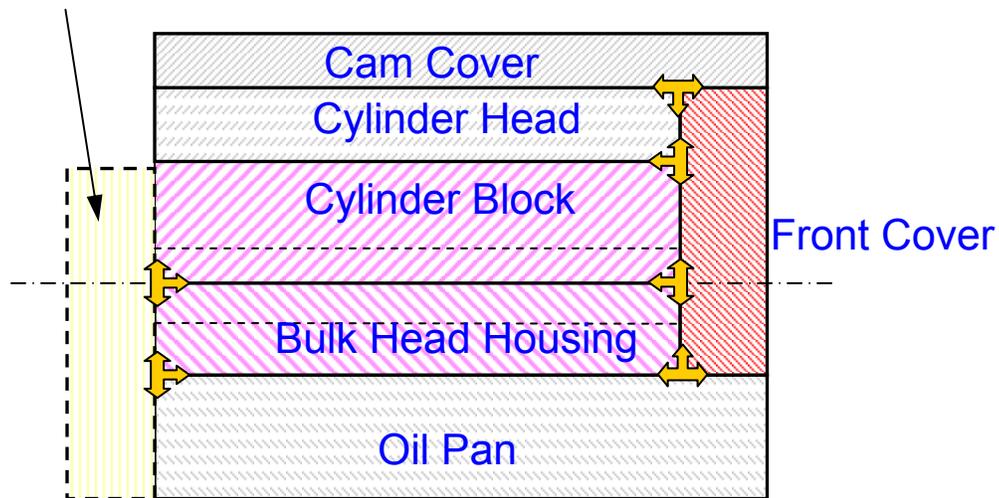
Fig. 6.7: Cross Section of V-Engine with T-Joints

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For the T-Joints step size is the KC. By applying Screw Theory analysis to find the relationship among step sizes of various T-Joints, we will find that, step sizes of T-Joints between Cam Cover-Cylinder Head-Front Cover and Front Cover-Oil Pan-Bulk Head Housing are conflicting. Step sizes of T-joints between Cylinder Block-Bulk Head Housing-Transmission and Front Cover-Cylinder Block-Bulk Head Housing are also conflicting. Step sizes of T-Joints between Transmission-Cylinder Block-Bulk Head Housing and Transmission-Bulk Head Housing-Oil pan are correlated, and so are the step sizes of T-Joints between Front Cover-Cylinder Block-Cylinder Head and Front Cover-Cylinder Head-Bulk Head Housing.

By doing constraint analysis on the parts we find that Cylinder Head, Front Cover and Bulk Head Housing are over-constrained. When ever a part is involved in more than one T-Joint in the same direction, on opposite sides of the part, it is over-constrained. Cylinder Block, Bulk Head Housing and Front Cover are involved in two T-Joints each in the same direction on their opposite sides.

DFC of the V-Engine as shown in figure 6.7 is given in figure 6.8. Arrows denotes the direction of location. Arrow originates from the part which acts as locating part and its head points to the part which is being located. The double red lines denote the step size KCs that exists at the T-Joints which are being formed in the given architecture.
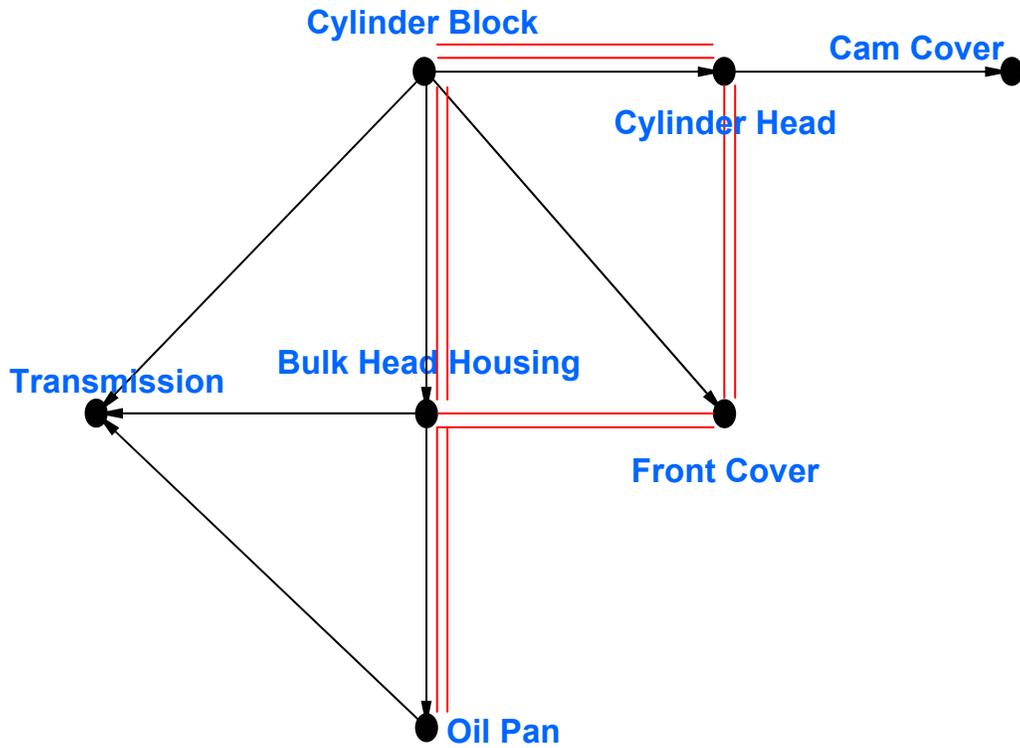
Fig. 6.8: DFC of the architecture
shown in figure 6.7

In the above architecture there exists two pairs of T-Joints whose step size KCs are conflicting. If we make suitable changes in the architecture, i.e. redefine DFC of the above architecture we can remove the conflicts from the design. One such architecture is shown in figure 6.9.
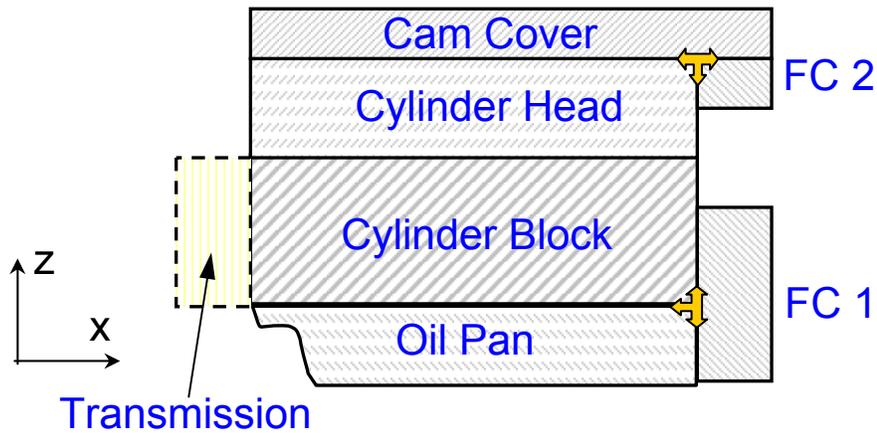
Fig. 6.9: Improved
Architecture of Engine with
no conflicts

In the above architecture we have made one part out of Cylinder Block and Bulk Head Housing, which we call Cylinder Block in figure 6.9. Front Cover is split into two parts, FC1 (Front Cover 1) and FC2 (Front Cover 2). FC1 gets it x and z location from Cylinder Block and in turn locates Oil Pan in x direction. Oil Pan gets its z location from Cylinder Block. Transmission is now located solely by Cylinder Block. FC2 gets its x location from Cylinder Head and z location from Cam Cover. There are only two T-Joints in the above architecture, one is between FC1-Oil Pan-Cylinder Block, and other is between Cam Cover-Cylinder Head-FC2. And since the two T-Joints have no part in common, i.e. since they have independent DFCs, there is no coupling between them. By making these changes in the architecture we have eliminated several T-Joints as compared to previous architecture. DFC for the above architecture is shown in figure 6.10.
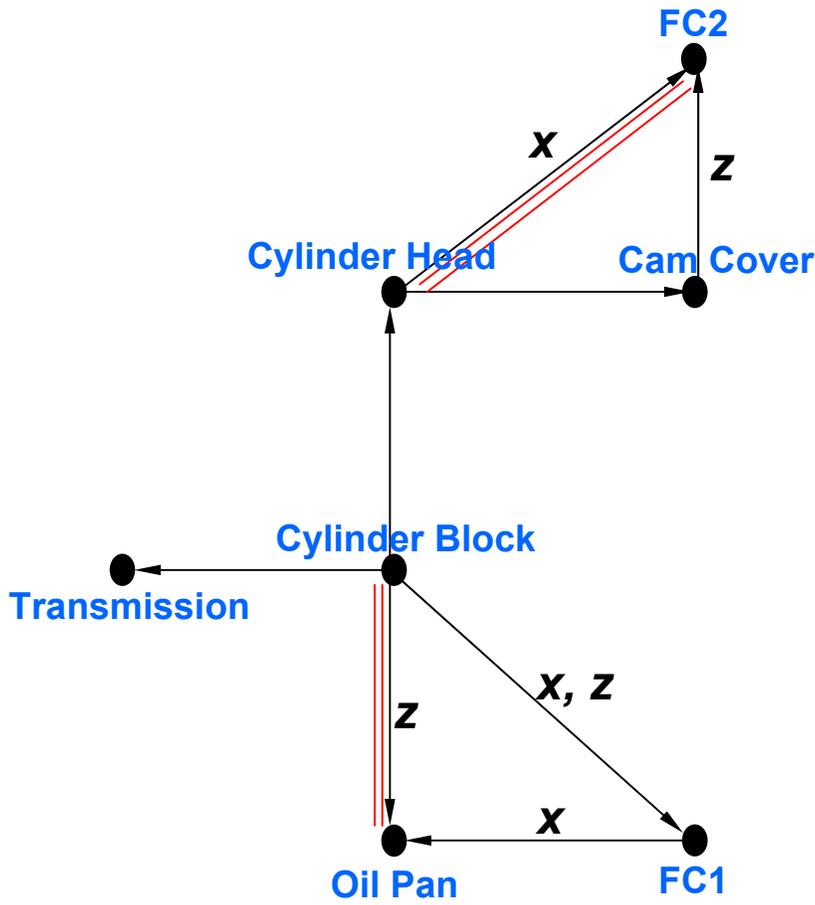
Fig. 6.10: DFC of improved architecture of Engine which has no conflicting T-Joints

Thus by making appropriate changes in DFC of the assembly we can resolve KC conflicts in the assembly.

6.4.1 Resolving Conflict by changing architecture

We can remove the conflict from the Type-1 assembly, by converting it to Type-2 assembly. Type-2 assembly provides us with alternative assembly sequences which can remove the conflict from the assembly. Taking the example of a square peg in square hole, figure 6.3 shows the situation when the assembly is Type-1 assembly. There exists KC conflict for this case. But if we can change the architecture of the assembly such that hole is made of two parts (as in figure 6.2), we can select an assembly sequence that can remove the conflict from the assembly.

6.5 Use of Compliant Parts

KC Conflict can also be solved using compliant parts in an assembly. Compliant parts deform easily on application of force. When ever there is situation of conflict in the assembly because of over-constrain in a compliant part, it deforms suitably to relieve over-constrain in the assembly and hence reduces the amount of conflict in the assembly. We can also use compliant parts of varying stiffness to further reduce the amount of conflict.

Figure 6.11 shows the assembly of Cylinder Head and Intake Manifold on Cylinder Block in a V-Engine. There are four KCs in the cross section shown. First is the uniform gap KC between Left Cylinder Head and Cylinder Block, second is the uniform gap KC between Right Cylinder Head and Cylinder Block, third is the uniform gap KC between Left Cylinder Head and Intake Manifold and last is the uniform gap KC between Right Cylinder Head and Intake Manifold. There are three types of gaskets being used in the

assembly shown in figure 6.11. The gaskets have different stiffness. Gasket 1 between Cylinder Heads and Cylinder Block has the maximum stiffness. Gasket 2 between Cylinder Heads and Intake Manifold has medium stiffness and gasket 3 between Intake Manifold and Cylinder Block has the least stiffness. Once we achieve KC1 and KC2 with in their tolerance limits, we can either reduce the variation in KC3 or KC4 if no gaskets are used. But by the use of the gaskets of varying stiffness, once we achieve KC1 and KC2, we try to adjust the gaps between Cylinder Heads and Intake Manifold so that we can achieve KC3 and KC4 with in their tolerance limits. Since deformable gaskets are present between the parts hence it is easy to achieve the suitable gap size by varying the forces coming on each gasket. The force on each gasket is varied by changing the maximum torque applied on the bolts, which are used to secure the parts with respect to each other.
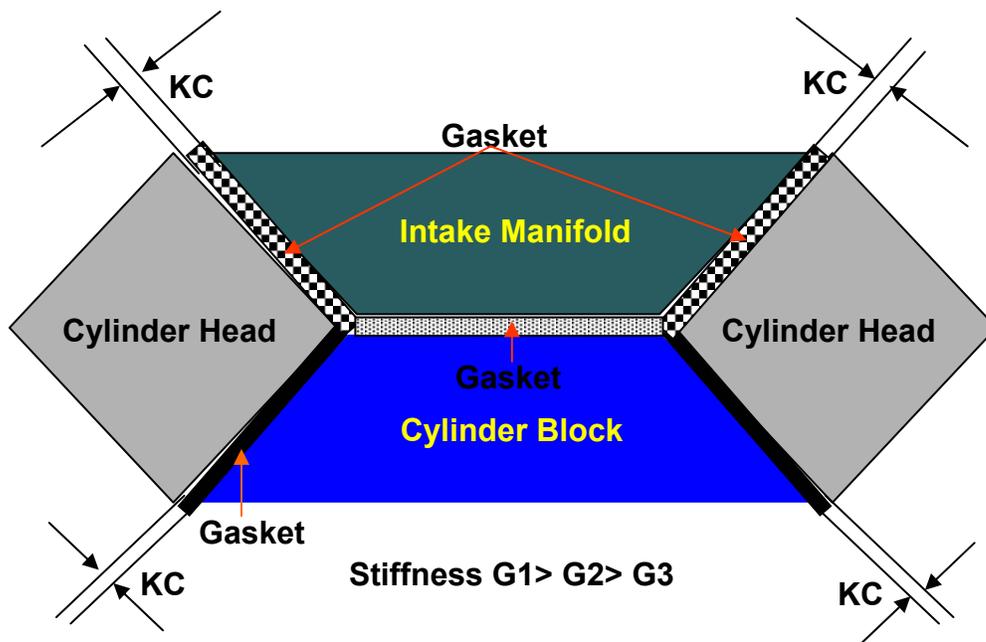
Fig. 6.11: Use of Compliant parts of
varying stiffness to reduce the amount
of conflict in V-Engine

6.6 Prioritizing KCs

In some assemblies it is not practically possible to eliminate KC conflict. Some of the
KCs in the assembly are more important than others. We want to meet important KCs
with higher confidence. In case of KC conflict, we can prioritize the KCs based on their
relative importance, and then try to develop DFCs of those KCs in such a way that the
variation stack-up is least for the KC having the highest priority.

In the assembly of car door to body side there are two important KCs. These are Weather Sealing KC and Appearance KC. Weather Seal KC is dictated by the gap between door inner and body side and Appearance KC is governed by the uniformity of the gap between door outer and body side. These two KCs are conflicting, and there is no pragmatic way to decouple these KCs. The only way to decouple these KCs is to attach both door inner and door outer separately to body side to achieve these KCs with in their tolerance limits and in the last assembly step secure door inner to door outer. But this assembly sequence is infeasible. Figure 6.12 shows the two KCs in side and top view.
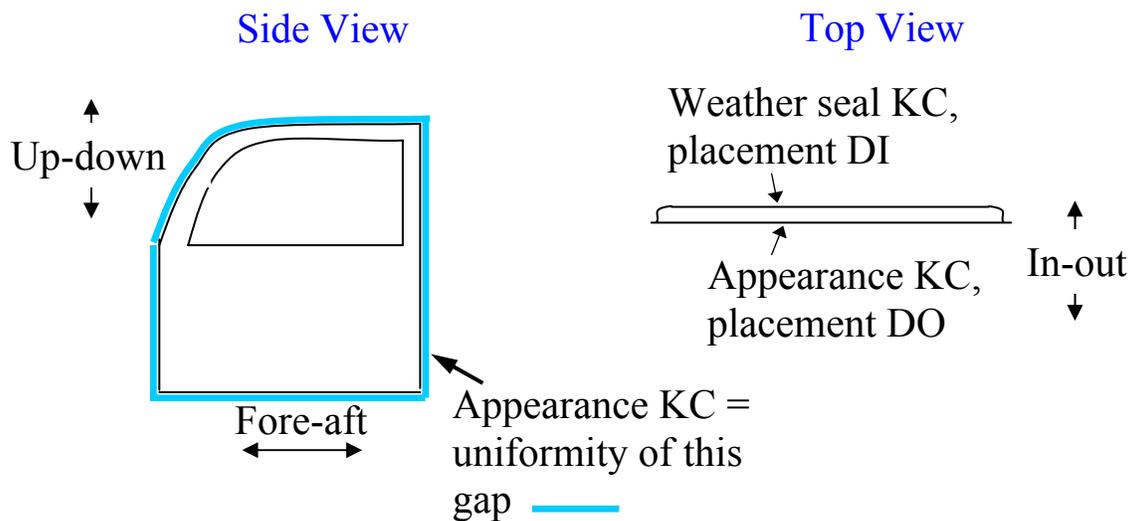
Fig. 6.12: Weather Seal and Appearance KC in Side and Top View

The first assembly step is the formation of door subassembly. Fixture is used to attach door inner, door outer and hinge leading to the formation of door subassembly. In the second assembly step door subassembly is attached to the body side. There are two possible ways to attach door subassembly to the body side, either by using door outer to locate the subassembly or using door inner. If the Appearance KC has higher priority than Weather Seal KC, then door outer is used to locate door subassembly to the body side. This assembly sequence reduces the variation stack-up between body side and door outer and hence Appearance KC is achieved with higher confidence. But it increases the variation stack-up between the sealing on the body side and door inner. Figure 6.13 shows DFC for this assembly sequence.
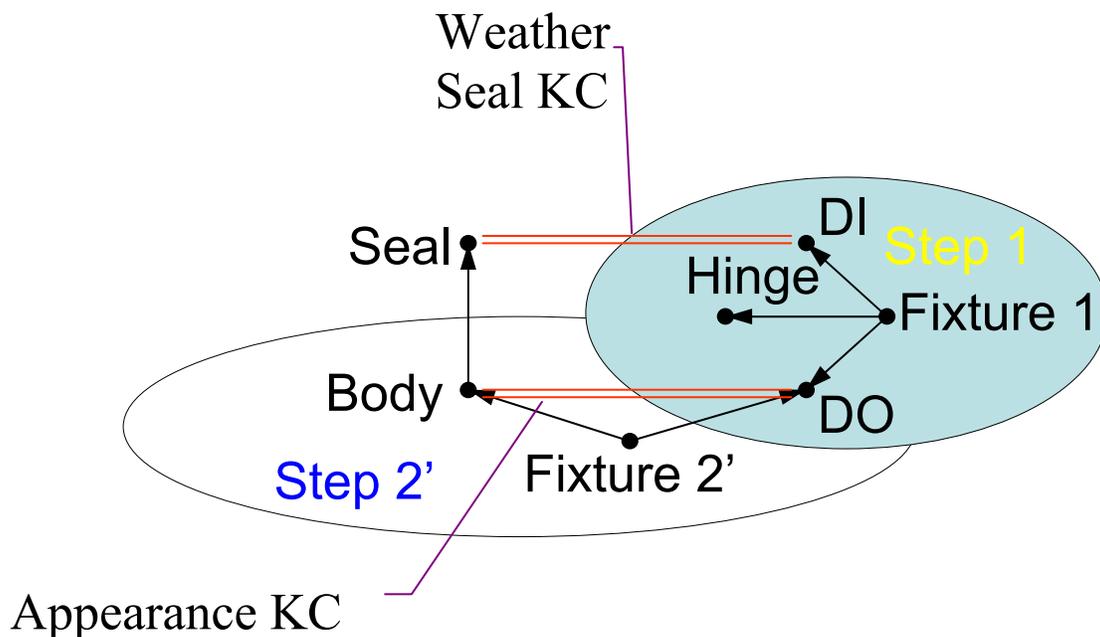
Fig. 6.13: DFC of assembly sequence when Appearance KC has higher priority than Weather Sealing KC

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If the Weather Seal KC has higher priority than Appearance KC, then door inner is used to locate door subassembly to the body side. This assembly sequence reduces the variation stack-up between the sealing on the body side and door inner and hence Weather Seal KC is achieved with higher confidence. But it increases the variation stack-up between the body side and door outer. Figure 6.14 shows DFC for this assembly sequence.
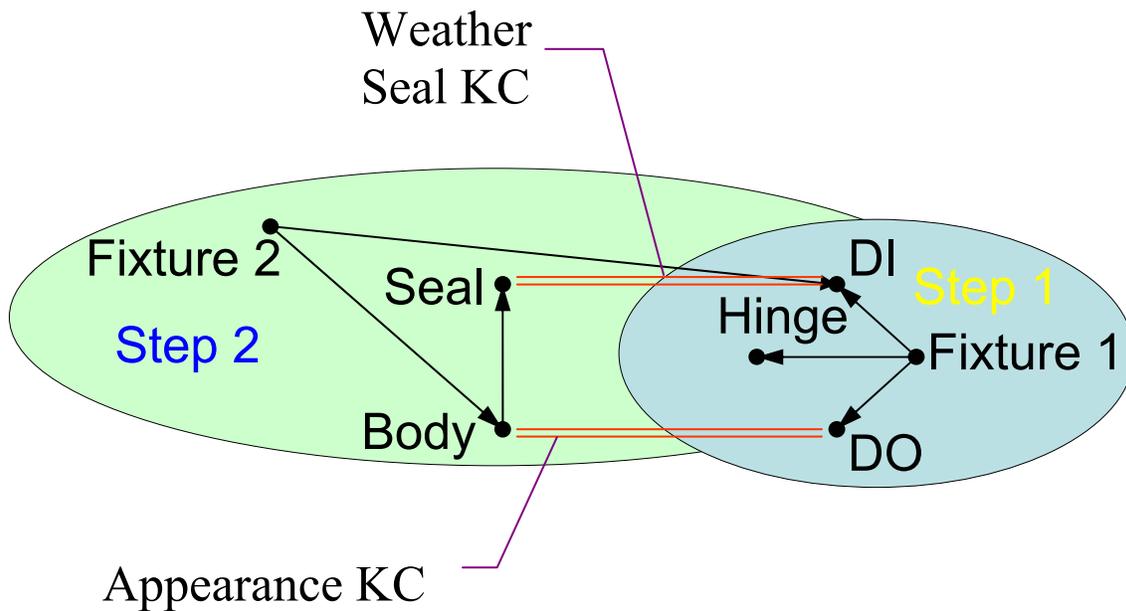
Fig. 6.14: DFC of assembly sequence when Weather Seal KC has higher priority than Appearance KC

There can be many other methods which can be used to resolve KC conflict in an assembly and to make sure that KCs are achieved with in their respective tolerance limits. After making any change in the assembly architecture or its DFC, we should find out the new KCs in the assembly and the nature of the relationship among various KCs using Screw Theory. If the assembly is free of conflicts we are in good shape, but if there are some conflicts still existing in the assembly we should seek one of the above mentioned methods or their combination to resolve the conflict or reduce its intensity. Once we have resolved all the conflicts in the assembly or have made sure that all the KCs are met with in their tolerance zones we can stop this iterative process.

6.7 Chapter summary

The chapter mentioned different methods that can be used to either reduce the amount of KC conflict in an assembly or remove it altogether. We can use each of the methods separately or in combination to meet all the specifications imposed on the assembly. The methods given in this thesis are not the only ones that can be used. There can be many other ways to remove or reduce conflict in the assembly.

In Type-2 assemblies' assembly sequence has major impact on the quality of the product. Certain KC conflicts in Type-2 assembly can be removed by suitably changing the assembly sequence. Type-2 assemblies have inherent degrees of freedom in the parts of the assembly which get fixed during the assembly. This makes it possible to adjust the fixation of degrees of freedom of the parts suitably, so as to remove certain conflicts from the assembly.

For the Type-1 assemblies, selective assembly can be used for the parts that are common in the delivery chains of conflicting KCs. The parts are selected for each assembly, having dimensions that will make the variation stack-up on the conflicting KCs with in the tolerance limits imposed on them.

KC conflict can be removed from the assembly by making apposite changes in DFCs of the KCs. There are many different ways to change DFCs. Changing location strategy of parts, removing some of the KCs by redesign, adding or removing certain parts or fixtures, and converting certain Type-1 assemblies to Type-2 assemblies are some of the ways to change DFCs of the conflicting KCs.

The intensity of the conflict can be reduced by using compliant parts of varying stiffness in the assembly. Compliant parts are used in place of rigid parts which are common in the delivery chains of conflicting KCs. The inherent tendency to deform in case of compliant parts reduces the amount of conflict in the assembly.

Sometimes not all the conflicting KCs are of same importance. We can prioritize the conflicting KCs. After that we need to make sure that we meet the KC with higher priority with greater confidence. We can do this either by reducing the variation stack-up in DFC for that KC or by changing the location strategy of the parts, to make sure that variation coming on higher priority KC is significantly less than the variation coming on less important KC.

Once we have resolved the conflict in the assembly, we should again do the Screw Theory analysis on the assembly to make sure that we have not created new conflicts in the assembly. Thus design of assemblies is an iterative process.

In the next chapter we will summarize the findings of the thesis. We will also look at the algorithms that we can use for both existing and new assemblies to drive away KC conflict from them.

Chapter 7. CONCLUSION AND FUTURE WORK

7.1 Summary

Key Characteristic conflicts result in a non-robust assembly. Non-robust assembly leads to increase in manufacturing costs of the assembly. KC conflicts sometimes arise because of an error during design of the product or during design of assembly sequence. We can bear some conflicts in the assembly provided that the product meets all the requirements imposed on it. The first step in resolving KC conflict in the assembly is to find out the intensity of the conflict. If the intensity of the conflict is less, i.e. even with the presence of the conflict, the assembly does not fall out of specification; we need not resolve the conflict. But if the conflict makes the assembly fall out of specification then we need to resolve the conflict. We can choose among various methods to resolve or reduce the amount of conflict.

In chapter 1 we saw that detecting KC conflict and eliminating crucial conflicts from the assembly will have a significant impact on the performance of the product. Not only will it improve the quality of the product but will also help the manufacturers in bringing down the cost of the product. There exists many assemblies that have KC conflict present in them but we need not redesign all of them. We need to redesign only the ones which will either give us significant improvement in costs or quality or both.

Chapter 2 presented the concept of Key Characteristics as is used in industry. It outlined a way to select measurement points in the assembly. DFC as a concept for capturing

dimensional and location constraint in the assembly was discussed next. Assembly features can be of different types. After dealing with DFC, a light has been shed on the types of assembly features. A part can exist in three different constraint states based on the degrees of freedom that are being constrained for that part. In many assemblies assembly sequence has significant bearing on the quality of the assembly. These assemblies have undetermined degrees of freedom that are being fixed by the use of fixtures, and are called Type-2 assemblies. In Type-1 assemblies, feature relationship completely determines the constraint states of the parts. The assembly process merely puts the parts together by joining their pre-defined mating features. There are many mathematical methods that can be used with DFC. The prominent ones are 4x4 matrix transformation method and Screw Theory. 4x4 matrix methods can be used to determine the variation stack-up in open loops. Screw Theory can be used for determining the constraint state of part in the assembly as well in doing motion analysis. It can used to find the variation stack-up in both closed and open loops.

Chapter 3 outlined the nature of relationship that can exist among various KCs in the assembly. Whenever there is more than one KC that is being delivered by the assembly, chances of KC Coupling increases. KCs in the assembly can be independent if their DFCs are independent or if they do not share any common degree of freedom in case they have a common node in their DFCs. KC coupling can be of three different type's viz., KC conflict, KC correlation, and KC correlation-conflict. The nature of relationship depends upon the affect that is being produced on the variation of a KC when there is an effort of variation reduction in other KC. If the variation reduction in one KC leads to

variation reduction in other one also, they are correlated. If the variation reduction in one KC leads to increase in the variation in others KC, they are conflicting. But if it results in decrease in variation in some directions but increase in other then they are correlated in some directions and conflicting in other directions. The effect of assembly sequence on the nature of KC coupling was also being presented.

Chapter 4 presented a method to detect KC coupling in the assembly using Screw Theory. The algorithm that was discussed will work with under- or properly-constrained assemblies. KCs were being taken as wrench matrices to find the existence of coupling among the KCs. Once we know that the KCs were coupled, we can perform variation and contribution analysis to determine the exact nature of the coupling. The algorithm was followed by an example which showed the application of the method in real situations. The nature of KC coupling in the underbody of car was being verified. It also discussed how this algorithm can detect the presence of KC coupling at each assembly step, and the edge it will give to assembly sequence analyzing software packages, if used with those packages.

Chapter 5 discussed situations in which we need to resolve KC conflict in the assembly. Many existing assemblies have KC conflicts in them. But it is not profitable to resolve each one of the conflict in the assembly. Only the conflicts that have tendency to make assembly non-robust must be resolved. To ascertain the situations in which we need to resolve the conflict, we should first do the variation analysis on the conflicting KCs to find the variation stack-up coming on the KCs. To determine the robustness of the

assembly we will compare the results of variation analysis with the tolerance limits set on the KCs. There are many different methods that exist in literature, which can be used to do variation analysis. A screw theory method of doing variation analysis for under- or properly constrained assembly in three dimensions was being presented. Slider crank example was discussed next to illustrate the working of the algorithm. Once we know the variation coming on each of the conflicting KCs, we compare it to the tolerance limit on the KCs to find the intensity of the conflict. If the variation overshoots the tolerance limit, we seek for the ways to reduce the variation stack-up on the KCs. One of the ways to reduce the variation stack-up is by doing variation synthesis on the parts involved in the delivery chains of the KCs. This is the simplest of all the techniques which entails no design change. But it might not be possible in all situations to built parts with high precision and without affecting the profitability of the product. In those scenarios we seek alternative ways to either eliminate or reduce the amount of conflict in the assembly.

Chapter 6 outlined various methods that can be used to either resolve or reduce the amount of KC conflict in the assembly. For Type-2 assemblies the conflict can sometimes be resolved by changing the assembly sequence. For Type-1 assemblies amount of conflict can be reduced by practicing selective assembly of the parts which are common in the delivery chains of conflicting KCs. KC conflict can be removed from the assembly by redefining DFCs of the conflicting KCs. DFC can be redefined by changing the locating strategy of parts, eliminating certain KCs by suitable redesign of assembly, adding or removing parts or fixtures and in some situations converting Type-1 assemblies to Type-2 assemblies and taking advantage of the assembly sequence to resolve the

conflict. Compliant parts can also be used in the assembly to reduce the amount of conflict in the assembly. In many cases of conflict, not all the conflicting KCs have the same priority. We try to meet the conflicting KC having higher priority with more confidence as compared to the conflicting KC having lower priority. The examples of how each of these methods can be applied in real assemblies were presented in the chapter.

7.2 Approach for Existing Assemblies

The approach suggested in this thesis can be used for both existing assemblies as well as for the assemblies that are in their conceptual or design phase. This will make the assemblies more robust and will also bring down the manufacturing costs of the assemblies.

If an assembly already exists the first step is to identify all the KCs of the assemblies. Then next step is to draw DFCs for each of the KC. Once we have information about all the KCs, we can find the relationships that exist among various KCs using the algorithm outlined in chapter 4. We can determine the pairs of conflicting KCs from the analysis. After that we will utilize the method presented in chapter 5, to determine the affect of KC conflict on the robustness of the assembly. If we can meet all the conflicting KCs with in their tolerance limits we need not make changes in the design. In case the analysis results show that assembly is on the verge of becoming non-robust, we can try to tighten the tolerance limits on the parts that are common in the delivery chains of conflicting KCs. If the tightening of the tolerance limits on parts is practically feasible and economically

viable, we can use this strategy to produce our assemblies with in the specifications. But if such a feat is neither practically feasible nor economically viable, we need to seek for other methods to resolve the conflict in the assembly or to reduce their effect. Some of the methods to resolve or reduce the amount of KC conflict are outlined in chapter 6. After implementing these methods we need to check our assembly again for other conflicts which might make our assembly non-robust or which might provide directions of further improvements in future designs.

7.3 Designing New Product

The approach mentioned in this thesis not only improves the existing assemblies but can also have tremendous impact in the design phase of the assembly. It makes sure that there are no significant conflicts in the assembly. And even if there are certain conflicts which are deliberately left out, they do not impact the quality of the product.

The first step in designing an assembly without KC conflicts is to first identify all the KCs that the assembly is supposed to deliver. These are "customer-level KCs". Using Quality function deployment (QFD) approach we can break "customer-level KCs" to "assembly-level KCs". After identifying all the "assembly-level KCs" that assembly has to deliver, we need to make DFC for each of the KC, such that all the KCs are independent. The result of this design step would be an assembly with lot of parts, since all the KCs are achieved independently. On DFC parts will show up as nodes. Thus after the first design step we will have DFC of the assembly having many nodes.

The next step would be to reduce the number of nodes of DFCs by combining the functioning of various parts into single one. After each reduction of node we need to check for the presence of conflict in the assembly. If the reduction of node leads to conflict in the assembly we need to undo the previous step and seek for an alternative way to reduce the number of nodes. We need to iterate the above step until there is no possible way of reducing number of nodes in DFC without producing a conflict in the assembly.

The process of reducing nodes one by one can be tedious. We can look for alternative algorithms by which we can reduce a number of nodes in batches. This technique resembles optimization techniques which have been used at number of places.

7.4 Scope for Future Research

The effectiveness of above algorithms can be tremendously increased by the development of software that will support the functionality of DFC. Apart from that it can also perform motion analysis, variation analysis, constraint analysis, contribution analysis, variation synthesis etc. This software coupled with assembly sequence analyzer software will have potential of detecting KC conflict at each assembly step. If some of the conflict resolving techniques are present in software as subroutines, which software can call when it sees the presence of a conflict will enhance the effectiveness of the package.

To determine if the KC is achieved with in its tolerance limits we mark some points on an assembly as measurement points. The amount of variation on the measurement points is

taken as guide for the variation coming on the KC. Generally the measurement points are chosen where the effect of variation is significant in the assembly. We need to further explore this issue to come up with a generic algorithm by which we can determine the location of effective measurement points. The measurement points so found should be such that the variation on measurement points is actual representation of the variation coming on the KC.

There is still scope of research in the area of over-constrain assemblies. Most of the methods and algorithms presented here work for under- or properly-constrained assemblies. It is because under- and properly-constrained assemblies have unique DFC. But in case of over-constrain in an assembly it becomes difficult to ascertain the variation-stack up chain. The problem is aggravated by the presence of parts which are not completely rigid. This calls for an analysis of the stack-up chains using first principle approach. The method to detect KC coupling using Screw Theory can be extended to over-constrain assemblies if we know the constitutive relationships for the over-constrain parts in the assembly.

In most of the case studies given in this thesis, relationship among two KCs had been studied. When we have more than two KCs which are all coupled and there exist some parts in the assembly that are being shared by the delivery chains of all the KCs and the complexity of the problem increases. The analysis process of the problem still remains the same. Software which can analyze the relationship among various KCs can easily deal with these kinds of scenarios.

The list of alternatives to eliminate or reduce the amount of KC conflict is not all exhaustive. These are the ways that are most commonly being used in industry or can be used very easily in industry. There can be many other ways which can eliminate or reduce the amount of conflict. A thorough study is needed to document all the methods which could be used to reduce the amount of conflict. If these methods can be integrated with already existing assembly analyzing software packages, it will help the designer to see the conflicts in the design and will help them in taking remedial steps in early stages.

Affect of conflict reduction in the assembly on the cost of production of assembly is also an interesting area to be looked into. It will give us the list of conflicts in the assembly, whose removal from the assembly will have significant affect on the cost of production. We can then run an optimization technique to come up with a balance between the cost reduction and quality improvement. Based on the cost and tolerance constraints we can select the conflicts that will result in significant cost reduction as well as in considerable quality improvement.

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Appendix: MATLAB® Files

There are many MATLAB® files (MATLAB® is a trademark of The MathWorks, Inc.) that are being used throughout the text.

```matlab
%
%     File:          Conflict.m
%     Purpose:       To Plot the error in Conflicting KCs
%     Author:        Jagmeet Singh
%     Date:          2nd Feburary 2003
%
%

i=1;

for x = 0:0.05:1;

   % Error in Length of the Peg
   eL(i)= x;

   % Error in KC1 (assumed some fraction of error in
     Length)
   ekc1(i)= x/3;

   % Error in KC2 (calculated from the error equation)
   ekc2(i)= ey(i) - ekc1(i);



   % Modifying the error in KC1 to see effect of reducing
     variation
   aekc1(i)= x/10;

   % Modified error in KC2 corresponding to modified error
     in KC1
   aekc2(i)= ey(i) - aekc1(i);

   i=i+1;
end


% Plotting the results to see the effect of changing
% variation in one of conflicting KCs on the other KC

plot(eL,ekc1,'m',eL,ekc2,'g',eL,aekc1,'b',eL,aekc2,'r');
```

```
ylabel('Error KC');
xlabel('Error L');
```

```
%
%    File:        Conflict1.m
%    Purpose:     To Plot the errors of Conflicting KCs
%    Author:   Jagmeet Singh
%    Date:        23nd Feburary 2003
%
%


i=1;
eL(1)=1;
eL(2)=0.75;
eL(3)=0.5;
eL(4)=0.25;

for x = 0:0.05:1;


    % Error in KC1 (assumed some fraction of error in Length)
    ekc1(i)= x*eL(1);

    % Error in KC2 (calculated from the error equation)
    ekc2(i)= eL(1) - ekc1(i);




    % Modifying the error in KC1 to see effect of reducing
      variation
    aekc1(i)= x*eL(2);

    % Modified error in KC2 corresponding to modified error
      in KC1
    aekc2(i)= eL(2) - aekc1(i);




    % Modifying the error in KC1 to see effect of reducing
      variation
    bekc1(i)= x*eL(3);

    % Modified error in KC2 corresponding to modified error
      in KC1
    bekc2(i)= eL(3) - bekc1(i);

    % Modifying the error in KC1 to see effect of reducing
variation
    cekc1(i)= x*eL(4);

    % Modified error in KC2 corresponding to modified error
      in KC1
    cekc2(i)= eL(4) - cekc1(i);
```

```matlab
    i=i+1;
end


    %  Plotting the results to see the effect of modifying of
       changing
    %  variation in one of conflicting KCs on the other KC
       given a
    %   constant error in the dimension of common part

plot(ekc2,ekc1,'m',aekc2,aekc1,'b',bekc2,bekc1,'g',cekc2,cek
c1,'c');

ylabel('Error KC2');
xlabel('Error KC1');
```

```matlab
%
%    File:        Coupling.m
%    Purpose:     To Plot the error in Correlated KCs
%    Author:      Jagmeet Singh
%    Date:        23nd Feburary 2003
%
%

i=1;

for x = 0:0.05:1;

    % Error in Length of fixture
    eL(i)= x;

    % Error in KC1
    ekc1(i)= x;

    % Error in KC2
    ekc2(i)= x;


    i=i+1;
end


% Plotting the results to see the effect of changing
% variation in one of the coupled KCs on the other KC

plot(eL,ekc1,'m',eL,ekc2,'g');

ylabel('Error KC');
xlabel('Error in length of fixture');
```

```
%
%   File:       Radius.m
%   Purpose:    Solving For the Radius Vector for DOF
Matrix, to calculate the location of Instantaneous center
of   rotation of coupler
%   Author:     Jagmeet Singh
%   Date:       3rd April 2002
%
%


% We can get w, v vector from dof matrix

w=[wx wy wz];
v=[vx vy vz];

% to get the direction of "r"
dir_r=cross(w,v);

% Let r= t*dir_r, where 't' is any scalar
% to calculate t

V=cross(dir_r,w);

% 't' is calulated from non-zero entries
t=norm(v)/norm(V);

% Location of Instantaneous Center
r= t*dir_r
```

```matlab
%
%   File:       Theta.m
%   Purpose:    Solving For the Theta Vector, Once Radius
%   Vector and Small perturbation vector is known
%   Author:     Jagmeet Singh
%   Date:       4th April 2002
%
%

% We are given R, S vector
R=[rx ry rz];
S=[sx sy sz];

% to get the direction of "theta"
dir_theta=cross(R,S);

% Let theta= t*dir_theta, where 't' is any scalar
% to calculate t
s=cross(dir_theta,R);

% 't' is calulated from non-zero entries
t=norm(S)/norm(s);

% Value of Theta
theta= t*dir_theta
```

```
%
%   File:       VariationA.m
%   Purpose:    To find the effect of Variation in part A
on Variation in KC U, using Kinematic Analysis
%   Author:     Jagmeet Singh
%   Date:       7th April 2002
%
%




% To calculate the location of the slider when there is no
variation in Nominal Case


% To calculate the location of point 4 on the Mechanism in
nominal case

m1=roty(dtr(-90));
m2=[1 0 0 20;0 1 0 0;0 0 1 0;0 0 0 1];
m3=roty(dtr(-90));
m4=[1 0 0 12;0 1 0 0;0 0 1 0;0 0 0 1];
m5=roty(dtr(-90));
m6=rotz(dtr(45));
m7=[1 0 0 15;0 1 0 0;0 0 1 0;0 0 0 1];



% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by in nominal case
t=m1*m2*m3*m4*m5*m6*m7;
a=t(1,4);
b=t(2,4);
c=t(3,4);




% To calculate the z-location of point 5 (-U,0,zee) in
nominal case
m13=[1 0 0 5;0 1 0 0;0 0 1 0;0 0 0 1];
t1=m13;
zee=t1(1,4);



% To find the length(len)of the coupler in nominal case
m10=[1 0 0 30;0 1 0 0;0 0 1 0;0 0 0 1];
t2=m10;
```

```matlab
len=t2(1,4);
```


```matlab
% To calculate the location of the slider based on the
physical constraint that loop is closed.

% This means that the distance between point 4 and point 5
is equal to the length of coupler "len"
% (a+U)^2 + (b-0)^2 + (c-zee)^2 = len^2"
% x-location of slider is defined to be "-U"


var1= len^2 - (((c-zee)^2) + (b^2));
var2=var1^(0.5);

% Location of the slider in nominal case
U=var2-a;

%
%

% To calculate the location of point 4 on the Mechanism in
case of variation

m1=roty(dtr(-90));
m2=[1 0 0 20;0 1 0 0;0 0 1 0;0 0 0 1];
m3=roty(dtr(-90));
m4=[1 0 0 12;0 1 0 0;0 0 1 0;0 0 0 1];
m5=roty(dtr(-90));
m6=rotz(dtr(45));
m7=[1 0 0 15;0 1 0 0;0 0 1 0;0 0 0 1];
dm2=[1 0 0 0.025;0 1 0 0;0 0 1 0;0 0 0 1];


% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by
%t=m1*m2*dm2*m3*m4*dm4*m5*m6*m7*dm7;
%a=t(1,4);
%b=t(2,4);
%c=t(3,4);
```

```matlab
% To calculate the z-location of point 5 (-U1,0,zee)
m13=[1 0 0 5;0 1 0 0;0 0 1 0;0 0 0 1];


% To find the length(len)of the coupler including variation
m10=[1 0 0 30;0 1 0 0;0 0 1 0;0 0 0 1];

%dm10=[1 0 0 0.03;0 1 0 0;0 0 1 0;0 0 0 1];
%t2=m10*dm10;
%len=t2(1,4);



%--------------------------
% To check for all possible combinations
%--------------------------

i=0;

for a1=1:2


            if a1==2
                dm2(1,4)=-1*dm2(1,4);
            end


            i=i+1;

% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by
t=m1*m2*dm2*m3*m4*m5*m6*m7;
a=t(1,4);
b=t(2,4);
c=t(3,4);

% To calculate the z-location of point 5 (-U1,0,zee)
t1=m13;
zee=t1(1,4);


% To find the length(len)of the coupler including variation
t2=m10;
len=t2(1,4);


% To calculate the location of the slider based on the
physical constraint
```

```matlab
% that loop is closed.

% This means that the distance between point 4 and point 5
is equal to the length of coupler "len"
% (a+U)^2 + (b-0)^2 + (c-zee)^2 = len^2"
% x-location of slider is defined to be "-U1"


var1= len^2 - (((c-zee)^2) + (b^2));
var2=var1^(0.5);

% U1 denotes the x-location of the slider when the
variation is included in analysis
U1=var2-a


%
%


% The variation (Var) in the location of slider based on
variation in manufacturing level variables

Var(i)= (U1-U)

end



max(Var);


%U1=39.7124
%Var=-0.0040
%U1=39.7203
%Var=-0.0040    0.0040

% The Result from Screw Theory was -0.0039 which matches
with this result
```

```
%
%   File:       VariationB.m
%   Purpose:    To find the effect of Variation in part B
on Variation in KC U, using Kinematic Analysis
%   Author:     Jagmeet Singh
%   Date:       7th April 2002
%
%

% To calculate the location of the slider when there is no
variation in Nominal Case


% To calculate the location of point 4 on the Mechanism in
nominal case

m1=roty(dtr(-90));
m2=[1 0 0 20;0 1 0 0;0 0 1 0;0 0 0 1];
m3=roty(dtr(-90));
m4=[1 0 0 12;0 1 0 0;0 0 1 0;0 0 0 1];
m5=roty(dtr(-90));
m6=rotz(dtr(45));
m7=[1 0 0 15;0 1 0 0;0 0 1 0;0 0 0 1];


% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by in nominal case
t=m1*m2*m3*m4*m5*m6*m7;
a=t(1,4);
b=t(2,4);
c=t(3,4);




% To calculate the z-location of point 5 (-U,0,zee) in
nominal case
m13=[1 0 0 5;0 1 0 0;0 0 1 0;0 0 0 1];
t1=m13;
zee=t1(1,4);


% To find the length(len)of the coupler in nominal case
m10=[1 0 0 30;0 1 0 0;0 0 1 0;0 0 0 1];
t2=m10;
len=t2(1,4);
```

```
% To calculate the location of the slider based on the
physical constraint that loop is closed.

% This means that the distance between point 4 and point 5
is equal to the length of coupler "len"
% (a+U)^2 + (b-0)^2 + (c-zee)^2 = len^2"
% x-location of slider is defined to be "-U"


var1= len^2 - (((c-zee)^2) + (b^2));
var2=var1^(0.5);

% Location of the slider in nominal case
U=var2-a;

%
%

% To calculate the location of point 4 on the Mechanism in
case of variation

m1=roty(dtr(-90));
m2=[1 0 0 20;0 1 0 0;0 0 1 0;0 0 0 1];
m3=roty(dtr(-90));
m4=[1 0 0 12;0 1 0 0;0 0 1 0;0 0 0 1];
m5=roty(dtr(-90));
m6=rotz(dtr(45));
m7=[1 0 0 15;0 1 0 0;0 0 1 0;0 0 0 1];

dm4=[1 0 0 0.0125;0 1 0 0;0 0 1 0;0 0 0 1];


% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by
%t=m1*m2*dm2*m3*m4*dm4*m5*m6*m7*dm7;
%a=t(1,4);
%b=t(2,4);
%c=t(3,4);



% To calculate the z-location of point 5 (-U1,0,zee)
```

```matlab
m13=[1 0 0 5;0 1 0 0;0 0 1 0;0 0 0 1];


% To find the length(len)of the coupler including variation
m10=[1 0 0 30;0 1 0 0;0 0 1 0;0 0 0 1];


%-------------------------
% To check for all possible combinations
%-------------------------

i=0;

for a1=1:2


            if a1==2
                dm4(1,4)=-1*dm4(1,4);
            end


            i=i+1;

% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by
t=m1*m2*m3*m4*dm4*m5*m6*m7;
a=t(1,4);
b=t(2,4);
c=t(3,4);

% To calculate the z-location of point 5 (-U1,0,zee)
t1=m13;
zee=t1(1,4);


% To find the length(len)of the coupler including variation
t2=m10;
len=t2(1,4);


% To calculate the location of the slider based on the
physical constraint that loop is closed.

% This means that the distance between point 4 and point 5
is equal to the length of coupler "len"
% (a+U)^2 + (b-0)^2 + (c-zee)^2 = len^2"
% x-location of slider is defined to be "-U1"
```

```matlab
var1= len^2 - (((c-zee)^2) + (b^2));
var2=var1^(0.5);

% U1 denotes the x-location of the slider when the
variation is included in analysis
U1=var2-a



%
%



% The variation (Var) in the location of slider based on
variation in manufacturing level variables

Var(i)= (U1-U)

end


max(Var);


%U1=39.7289
%Var=0.0125
%U1=39.7039
%Var=0.0125   -0.0125
```

```
%
%   File:       VariationC.m
%   Purpose:    To find the effect of Variation in part C
on Variation in KC U, using Kinematic Analysis
%   Author:     Jagmeet Singh
%   Date:       7th April 2002
%
%

% To calculate the location of the slider when there is no
variation in Nominal Case


% To calculate the location of point 4 on the Mechanism in
nominal case

m1=roty(dtr(-90));
m2=[1 0 0 20;0 1 0 0;0 0 1 0;0 0 0 1];
m3=roty(dtr(-90));
m4=[1 0 0 12;0 1 0 0;0 0 1 0;0 0 0 1];
m5=roty(dtr(-90));
m6=rotz(dtr(45));
m7=[1 0 0 15;0 1 0 0;0 0 1 0;0 0 0 1];


% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by in nominal case
t=m1*m2*m3*m4*m5*m6*m7;
a=t(1,4);
b=t(2,4);
c=t(3,4);




% To calculate the z-location of point 5 (-U,0,zee) in
nominal case
m13=[1 0 0 5;0 1 0 0;0 0 1 0;0 0 0 1];
t1=m13;
zee=t1(1,4);



% To find the length(len)of the coupler in nominal case
m10=[1 0 0 30;0 1 0 0;0 0 1 0;0 0 0 1];
t2=m10;
len=t2(1,4);
```

```
% To calculate the location of the slider based on the
physical constraint that loop is closed.

% This means that the distance between point 4 and point 5
is equal to the length of coupler "len"
% (a+U)^2 + (b-0)^2 + (c-zee)^2 = len^2"
% x-location of slider is defined to be "-U"


var1= len^2 - (((c-zee)^2) + (b^2));
var2=var1^(0.5);

% Location of the slider in nominal case
U=var2-a;


%
%
%

% To calculate the location of point 4 on the Mechanism in
case of variation

m1=roty(dtr(-90));
m2=[1 0 0 20;0 1 0 0;0 0 1 0;0 0 0 1];
m3=roty(dtr(-90));
m4=[1 0 0 12;0 1 0 0;0 0 1 0;0 0 0 1];
m5=roty(dtr(-90));
m6=rotz(dtr(45));
m7=[1 0 0 15;0 1 0 0;0 0 1 0;0 0 0 1];

dm7=[1 0 0 0.0125;0 1 0 0;0 0 1 0;0 0 0 1];


% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by
%t=m1*m2*dm2*m3*m4*dm4*m5*m6*m7*dm7;
%a=t(1,4);
%b=t(2,4);
%c=t(3,4);
```

```matlab
% To calculate the z-location of point 5 (-U1,0,zee)
m13=[1 0 0 5;0 1 0 0;0 0 1 0;0 0 0 1];



% To find the length(len)of the coupler including variation
m10=[1 0 0 30;0 1 0 0;0 0 1 0;0 0 0 1];

%dm10=[1 0 0 0.03;0 1 0 0;0 0 1 0;0 0 0 1];
%t2=m10*dm10;
%len=t2(1,4);



%------------------------
% To check for all possible combinations
%------------------------

i=0;

for a1=1:2


            if a1==2
                dm7(1,4)=-1*dm7(1,4);
            end


            i=i+1;

% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by
t=m1*m2*m3*m4*m5*m6*m7*dm7;
a=t(1,4);
b=t(2,4);
c=t(3,4);

% To calculate the z-location of point 5 (-U1,0,zee)
t1=m13;
zee=t1(1,4);


% To find the length(len)of the coupler including variation
t2=m10;
len=t2(1,4);
```

```matlab
% To calculate the location of the slider based on the
physical constraint that loop is closed.

% This means that the distance between point 4 and point 5
is equal to the length of coupler "len"
% (a+U)^2 + (b-0)^2 + (c-zee)^2 = len^2"
% x-location of slider is defined to be "-U1"


var1= len^2 - (((c-zee)^2) + (b^2));
var2=var1^(0.5);

% U1 denotes the x-location of the slider when the
variation is included in analysis
U1=var2-a


%
%


% The variation (Var) in the location of slider based on
variation in manufacturing level variables

Var(i)= (U1-U)

end


max(Var);


%U1=39.7144
%Var=-0.0020
%U1=39.7184
%Var=-0.0020     0.0020
```

185

```
%
%   File:       VariationD.m
%   Purpose:    To find the effect of Variation in part D
on Variation in KC U, using Kinematic Analysis
%   Author:     Jagmeet Singh
%   Date:       7th April 2002
%
%


% To calculate the location of the slider when there is no
variation in Nominal Case


% To calculate the location of point 4 on the Mechanism in
nominal case

m1=roty(dtr(-90));
m2=[1 0 0 20;0 1 0 0;0 0 1 0;0 0 0 1];
m3=roty(dtr(-90));
m4=[1 0 0 12;0 1 0 0;0 0 1 0;0 0 0 1];
m5=roty(dtr(-90));
m6=rotz(dtr(45));
m7=[1 0 0 15;0 1 0 0;0 0 1 0;0 0 0 1];


% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by in nominal case
t=m1*m2*m3*m4*m5*m6*m7;
a=t(1,4);
b=t(2,4);
c=t(3,4);




% To calculate the z-location of point 5 (-U,0,zee) in
nominal case
m13=[1 0 0 5;0 1 0 0;0 0 1 0;0 0 0 1];
t1=m13;
zee=t1(1,4);


% To find the length(len)of the coupler in nominal case
m10=[1 0 0 30;0 1 0 0;0 0 1 0;0 0 0 1];
t2=m10;
len=t2(1,4);
```

```
% To calculate the location of the slider based on the
physical constraint that loop is closed.

% This means that the distance between point 4 and point 5
is equal to the length of coupler "len"
% (a+U)^2 + (b-0)^2 + (c-zee)^2 = len^2"
% x-location of slider is defined to be "-U"


var1= len^2 - (((c-zee)^2) + (b^2));
var2=var1^(0.5);

% Location of the slider in nominal case
U=var2-a;



%
%

% To calculate the location of point 4 on the Mechanism in
case of variation

m1=roty(dtr(-90));
m2=[1 0 0 20;0 1 0 0;0 0 1 0;0 0 0 1];
m3=roty(dtr(-90));
m4=[1 0 0 12;0 1 0 0;0 0 1 0;0 0 0 1];
m5=roty(dtr(-90));
m6=rotz(dtr(45));
m7=[1 0 0 15;0 1 0 0;0 0 1 0;0 0 0 1];


% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by
%t=m1*m2*dm2*m3*m4*dm4*m5*m6*m7*dm7;
%a=t(1,4);
%b=t(2,4);
%c=t(3,4);




% To calculate the z-location of point 5 (-U1,0,zee)
m13=[1 0 0 5;0 1 0 0;0 0 1 0;0 0 0 1];
```

```matlab
%dm13=[1 0 0 0.0025;0 1 0 0;0 0 1 0;0 0 0 1];
%t1=m13*dm13;
%zee=t1(1,4);


% To find the length(len)of the coupler including variation
m10=[1 0 0 30;0 1 0 0;0 0 1 0;0 0 0 1];
dm10=[1 0 0 0.03;0 1 0 0;0 0 1 0;0 0 0 1];

%t2=m10*dm10;
%len=t2(1,4);


%---------------------------
% To check for all possible combinations
%---------------------------

i=0;

for a1=1:2


                if a1==2
                    dm10(1,4)=-1*dm10(1,4);
                end


                i=i+1;

% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by
t=m1*m2*m3*m4*m5*m6*m7;
a=t(1,4);
b=t(2,4);
c=t(3,4);

% To calculate the z-location of point 5 (-U1,0,zee)
t1=m13;
zee=t1(1,4);


% To find the length(len)of the coupler including variation
t2=m10*dm10;
len=t2(1,4);
```

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```matlab
% To calculate the location of the slider based on the
physical constraint that loop is closed.

% This means that the distance between point 4 and point 5
is equal to  the length of coupler "len"
% (a+U)^2 + (b-0)^2 + (c-zee)^2 = len^2"
% x-location of slider is defined to be "-U1"


var1= len^2 - (((c-zee)^2) + (b^2));
var2=var1^(0.5);

% U1 denotes the x-location of the slider when the
variation is included in analysis
U1=var2-a


%
%


% The variation (Var) in the location of slider based on
variation in manufacturing level variables

Var(i)= (U1-U)

end


max(Var);


%U1=39.7489
%Var=0.0325
%U1=39.6839
%Var=0.0325    -0.0325
```

```
%
%   File:       VariationE.m
%   Purpose:    To find the effect of Variation in part E
on Variation in KC U, using Kinematic Analysis
%   Author:     Jagmeet Singh
%   Date:       7th April 2002
%
%

% To calculate the location of the slider when there is no
variation in Nominal Case


% To calculate the location of point 4 on the Mechanism in
nominal case

m1=roty(dtr(-90));
m2=[1 0 0 20;0 1 0 0;0 0 1 0;0 0 0 1];
m3=roty(dtr(-90));
m4=[1 0 0 12;0 1 0 0;0 0 1 0;0 0 0 1];
m5=roty(dtr(-90));
m6=rotz(dtr(45));
m7=[1 0 0 15;0 1 0 0;0 0 1 0;0 0 0 1];


% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by in
% nominal case
t=m1*m2*m3*m4*m5*m6*m7;
a=t(1,4);
b=t(2,4);
c=t(3,4);




% To calculate the z-location of point 5 (-U,0,zee) in
nominal case
m13=[1 0 0 5;0 1 0 0;0 0 1 0;0 0 0 1];
t1=m13;
zee=t1(1,4);


% To find the length(len)of the coupler in nominal case
m10=[1 0 0 30;0 1 0 0;0 0 1 0;0 0 0 1];
t2=m10;
len=t2(1,4);
```

```matlab
% To calculate the location of the slider based on the
physical constraint that loop is closed.

% This means that the distance between point 4 and point 5
is equal to the length of coupler "len"
% (a+U)^2 + (b-0)^2 + (c-zee)^2 = len^2"
% x-location of slider is defined to be "-U"


var1= len^2 - (((c-zee)^2) + (b^2));
var2=var1^(0.5);

% Location of the slider in nominal case
U=var2-a;



%
%

% To calculate the location of point 4 on the Mechanism in
case of variation

m1=roty(dtr(-90));
m2=[1 0 0 20;0 1 0 0;0 0 1 0;0 0 0 1];
m3=roty(dtr(-90));
m4=[1 0 0 12;0 1 0 0;0 0 1 0;0 0 0 1];
m5=roty(dtr(-90));
m6=rotz(dtr(45));
m7=[1 0 0 15;0 1 0 0;0 0 1 0;0 0 0 1];

%dm2=[1 0 0 0.025;0 1 0 0;0 0 1 0;0 0 0 1];
%dm4=[1 0 0 0.0125;0 1 0 0;0 0 1 0;0 0 0 1];
%dm7=[1 0 0 0.0125;0 1 0 0;0 0 1 0;0 0 0 1];


% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by
%t=m1*m2*dm2*m3*m4*dm4*m5*m6*m7*dm7;
%a=t(1,4);
%b=t(2,4);
%c=t(3,4);
```

```matlab
% To calculate the z-location of point 5 (-U1,0,zee)
m13=[1 0 0 5;0 1 0 0;0 0 1 0;0 0 0 1];
dm13=[1 0 0 0.0025;0 1 0 0;0 0 1 0;0 0 0 1];

%t1=m13*dm13;
%zee=t1(1,4);


% To find the length(len)of the coupler including variation
m10=[1 0 0 30;0 1 0 0;0 0 1 0;0 0 0 1];

%dm10=[1 0 0 0.03;0 1 0 0;0 0 1 0;0 0 0 1];
%t2=m10*dm10;
%len=t2(1,4);


%--------------------------
% To check for all possible combinations
%--------------------------

i=0;

for a1=1:2


                if a1==2
                    dm13(1,4)=-1*dm13(1,4);
                end


                i=i+1;

% Location of point 4 (a,b,c) in the Global Co-ordinate
frame is given by
t=m1*m2*m3*m4*m5*m6*m7;
a=t(1,4);
b=t(2,4);
c=t(3,4);

% To calculate the z-location of point 5 (-U1,0,zee)
t1=m13*dm13;
zee=t1(1,4);
```

```matlab
% To find the length(len)of the coupler including variation
t2=m10;
len=t2(1,4);



% To calculate the location of the slider based on the
physical constraint that loop is closed.

% This means that the distance between point 4 and point 5
is equal to  the length of coupler "len"
% (a+U)^2 + (b-0)^2 + (c-zee)^2 = len^2"
% x-location of slider is defined to be "-U1"


var1= len^2 - (((c-zee)^2) + (b^2));
var2=var1^(0.5);

% U1 denotes the x-location of the slider when the
variation is included in analysis
U1=var2-a


%
%


% The variation (Var) in the location of slider based on
variation in manufacturing level variables

Var(i)= (U1-U)

end

max(Var);


%U1=39.7168
%Var=3.9617e-004
%U1=39.7160
%Var=1.0e-003 *(0.3962   -0.3964)
```