440 MHz RADAR OBSERVATIONS OF PLASMA TURBULENCE
IN THE AURORAL LOWER IONOSPHERE

by

CARLOS F. del POZO

B.S. Physics, Universidad Nacional de Ingeniería
(Lima-Perú, 1976)
Doctorat de 3ème Cycle in Physics, Université de Paris VI
(Paris-France, 1982)

SUBMITTED TO THE DEPARTMENT OF EARTH, ATMOSPHERE
AND PLANETARY SCIENCES IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR IN PHILOSOPHY

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

December, 1988

© Carlos del Pozo, 1988

The author hereby grants to M.I.T. permission to reproduce and to
distribute copies of this thesis document in whole or in part.

Signature of Author......................................

Department of Earth, Atmosphere and

Certified by........................................

Dr. J.C. Foster, Thesis Supervisor
M.I.T. HAYSTACK OBSERVATORY

Accepted by......................................

Chairman, Departmental Graduate Committee
Department of Earth, Atmosphere and
Planetary Sciences
ABSTRACT

In the ionosphere, during non-equilibrium conditions, electron density fluctuations may grow unstable, evolving to different turbulent states. The local state of the ionospheric plasma, in both stable and unstable situations, can be observed with the M.I.T.-Millstone Hill radar using the Thomson scattering technique. The radar measures the Fourier component of the density fluctuations spectrum with half of the operating wavelength (30-cm) along the observation direction (Bragg diffraction condition) and the projection of any organized plasma motions along this direction (Doppler-shift).

Depending on the space-time resolution and coverage (for a given experimental configuration) our system will allow the study of instability development as well as the large scale coupling between the unstable regions and the surrounding stable plasma. For the first time, this capability has been applied in the simultaneous determination of the instabilities' spectral properties and the local electric field. Turbulence in the auroral E-region is highly dependent on the magnitude and direction of the electric field.

The original contribution of our research has been the interpretation of the systematic observations of plasma turbulence at 30-cm, in the auroral lower ionosphere, made with this high sensitivity system acting as both a coherent and an incoherent backscatter. For this purpose we have performed the linear kinetic theory derivation of a number of instabilities associated with the most probable driving mechanisms in the auroral lower ionosphere (below 350 km altitude) and estimated their saturation amplitudes in the weak turbulence 'orbit-diffusion' approximation. We have also treated the radar measurement and the interpretation of the scattering cross-section and the turbulence power spectrum.

We have analysed data collected during magnetically disturbed periods and identified the frequent generation, in the lower E-region, of the modified two stream (type 1) waves and two types of secondaries (types 0 and 2) presumably from the turbulent saturation of two fluid-like instabilities: the low-frequency density-gradient drift and the ExB-gradient drift, respectively. We also observe the probable 'trace' of longer wavelength, ExB-gradient drift primaries.

We have determined that the unstable lower E-region is centered at 108 km altitude with a half-power spread (-3 dB 'thickness') of 8 km, that type 1 waves are generated for a threshold electric field between 20 to 25 mV/m and Kp-index greater than 3, with an absolute cross-section (saturation level) of 60 to 70 dB
above the stable fluctuating level, an aspect angle sensitivity of -7 to -11 dB/degree, and a flow angle sensitivity (power drop between the type 1 primaries and the secondaries) ranging between -15 and -20 dB. These values confirm and extend various previous observations. We have also observed the systematic increase of the saturation phase-velocity of the type 1 waves (the ion-sound speed) with the increase of the electric field strength beyond the instability threshold as is expected from the anomalous heating of the electron gas. Finally we have presented some experimental evidence of the presence of unstable density gradient drift waves in the upper E and lower F regions.

Thesis Supervisor: Dr. John C. Foster

Title: Assistant Director M.I.T. Haystack Observatory
# TABLE OF CONTENTS

ACKNOWLEDGEMENTS. i  
LIST OF SYMBOLS. ii  

1. INTRODUCTION.  
1. SCOPE OF THE PRESENT STUDY. 1  
2. RADAR DIAGNOSIS OF THE IONOSPHERE. 4  
3. NON-EQUILIBRIUM CONDITIONS. 6  
4. REVIEW OF THEORY AND RADAR OBSERVATION OF PLASMA INSTABILITY IN THE IONOSPHERE.  
   A. PRINCIPAL INSTABILITY MECHANISMS. 9  
   B. RADAR OBSERVATIONS OF IONOSPHERIC INSTABILITIES. 18  
   C. SUMMARY. 20  

2. 'ION-LINE' INSTABILITIES AT 30-cm.  
1. 'ION-LINE' KINETIC INSTABILITY. 26  
2. INSTABILITY MECHANISMS:  
   A. MODIFIED TWO-STREAM WAVES. 29  
   B. DENSITY GRADIENT-DRIFT INSTABILITIES 32  
   C. CURRENT-DRIVEN AND CURRENT CONVECTIVE ELECTROSTATIC ION-CYCLotron WAVES. 42  
   D. POST-ROSENBLUTH INSTABILITY. 54  
3. SATURATION SPECTRUM OF LINEAR KINETIC INSTABILITY IN
3. RADAR OBSERVATIONS OF PLASMA TURBULENCE.

1. THOMSON SCATTERING. 70

2. SPECTRAL DENSITY FUNCTION:
   A. TURBULENCE POWER-SPECTRUM. 74
   B. ASPECT ANGLE SENSITIVITY. 76

3. RADAR DETECTION OF PLASMA WAVES:
   A. SCATTERED POWER FROM A PLASMA. 78
   B. RADAR CROSS-SECTION. 80
   C. ANTENNA RADIATION PATTERN. 83
   D. MILLSTONE HILL RADAR OPERATION. 84

FIGURE CAPTIONS. 86

4. THE ANALYSIS OF THE SCATTERING CROSS-SECTION.

1. SCATTERING VOLUME:
   A. INCOHERENT SCATTERING. 90
   B. COHERENT SCATTERING. 91

2. INSTABILITY CROSS-SECTION AND EFFECTIVE SCATTERING VOLUME.
   A. CROSS-SECTION. 95
   B. LOG10 POWER UNITS (POL). 96
   C. EFFECTIVE SCATTERING VOLUME. 97

3. THIN LAYER MODEL. 99
4. OBSERVATIONS AND DATA ANALYSIS.

A. THE 'RADIO' AURORA.

B. OBSERVATION PERIODS.

C. ALTITUDE AND LOCAL TIME DISTRIBUTIONS OF THE INSTABILITY CROSS-SECTION.

D. MORPHOLOGY OF THE E-REGION 'CLUTTER' AT 30-cm.

E. ALTITUDE-AZIMUTH DISTRIBUTIONS.

F. HEIGHT AND ASPECT ANGLE DEPENDENCES.

FIGURE CAPTIONS.

5. THE ANALYSIS OF THE TURBULENCE SPECTRUM AT 30-cm.

1. AUTOCORRELATION FUNCTION (ACF).

2. SIGNAL-TO-NOISE RATIO.

3. SPECTRUM SEPARATION.

   A. SEPARATION ALGORITHM.

   B. INSTABILITY SPECTRUM.

4. ELECTRIC FIELD DETERMINATION.

5. DISCUSSION OF THE OBSERVED SPECTRAL PROPERTIES.

   A. ELECTRIC FIELD THRESHOLD.

   B. INSTABILITIES' SPECTRAL PEAK AND WIDTH.

   C. DISTRIBUTIONS OF THE SPECTRAL PARAMETERS.

      C.1. 'TYPE A' SPECTRUM.

      C.2. 'TYPE B' SPECTRUM.

   D. FLOW ANGLE DEPENDENCE.

   E. HEIGHT DEPENDENCE.

FIGURE CAPTIONS.
CONCLUSION.  211

APPENDICES.

APPENDIX 1. LINEAR KINETIC INSTABILITIES IN THE LOWER IONOSPHERE.  222

APPENDIX 2. NONLINEAR SATURATION AND 'ORBIT-DIFFUSION' APPROXIMATION.  241

APPENDIX 3. REGION OF FAVORABLE ASPECT ANGLE IN THE DIPOLE MAGNETIC FIELD APPROXIMATION.  262

REFERENCES.  274
ACKNOWLEDGEMENTS

First, I would like to thank my thesis advisor John Foster for his continuous support in all aspects of my research.

Next, I would like to thank Dr. J.P.St.Maurice for his encouragement, advice, and the interest he always showed in the development of this study.

Also, I would like to thank the members of my thesis committee Profs. R.Prinn, T.Madden, and Dr. T.Chang for the kindness in accepting to judge this work.

Finally, I like to express my recognition to the members of the Atmospheric Science Group of the MIT-Haystack Observatory for their assistance and friendship and, in particular, Drs. J.Holt and W.Oliver.
LIST OF SYMBOLS

\( \lambda_i, \lambda_e \)  
Debye length for the ions (i) and the electrons (e).

\( l_i, l_e \)  
Larmor radius for the ions and the electrons.

\( \omega_{pi}, \omega_{pe} \)  
ion-plasma and electron-plasma frequencies.

\( \Omega_i, \Omega_e \)  
ion's and electron's gyrofrequency.

\( \nu_{in}, \nu_{ii}, \nu_{ie} \)  
ion-neutral, ion-ion and ion-electron collision frequencies.

\( \nu_{en}, \nu_{ei}, \nu_e \)  
electron-neutral, electron-ion and effective electron collision frequencies.

\( \sqrt{\gamma} \)  
'Anomalous' collision frequency.

\( q_i, z_i \)  
ion charge and atomic number.

\( m_i, m_e \)  
ion and electron masses.

\( T_i, T_e, T_n \)  
ion, electron and neutral gas local temperatures.

\( n_0 \)  
ambient electron density.

\( E \)  
local electric field (external).

\( B \)  
ambient magnetic field.

\( \lambda_{in}, \lambda_{sc} \)  
wavelengths of the incident (transmitted) and the scattered radiation.

\( \omega, \lambda, k \)  
\( (k = 2\pi/\lambda) \) angular frequency, wavelength and wavevector
of the measured spectral component of the fluctuating plasma density.

\( \delta n_e, E_e, \phi_e \)  
fluctuating electron density, electric field and wave-energy
at wavevector $\mathbf{k}$.

$V_{\text{ph}}, D_{\text{sp}}$ phase velocity and spectral width of the unstable waves.

$v_j = \sqrt{T_j/m_j}$ ion's ($j = i$) and electron's ($j = e$) thermal speed.

c speed of light.

$C_S = \sqrt{(T_i + T_e)/m_e}$ ion-sound speed.

$\mathbf{V}_i, \mathbf{V}_e$ ion and electron drifts.

$\mathbf{V}_d = \mathbf{V}_e - \mathbf{V}_i$ electrons' differential drift.

$\mathbf{V}_E = \mathbf{E} \times \mathbf{B}/B^2$ cross-field drift.

$\mathbf{V}_{PJ} = \mathbf{V}_j E/B$ Pedersen drift.

$\mathbf{V}_{nj} = \mathbf{V}_j \frac{3}{2} \mathbf{B} \cdot \nabla \left( \frac{n}{n} \right) x B/B$ diamagnetic drift.

$L_n = |\nabla n/n|^{-1}$ density gradient scale-length.

$Z(\gamma_j)$ plasma function.

$\chi_j$ susceptibility function.

$I_n(\mu_j) = (\mu_j - k_0^2 \rho_j^2)$ modified Bessel function of order $n$.

$P_{nj} = P_n(\mu_j) = \mathbf{E}^\dagger \mathbf{H}_j I_n(\mu_j)$.

$S(\mathbf{k}, \omega)$ spectral density function.

$\sigma_0(\mathbf{k})$ or $\sigma_{IS}$ scattering cross-section of the stable fluctuations ('incoherent').

$\sigma(\mathbf{k})$ or $\sigma_{CH}$ scattering cross-section of the unstable waves ('coherent').

POWER = $P_S(\mathbf{R}, \omega)$ scattered power in the frequency range $\omega$ and $\omega + d\omega$

from a volume centered at the point $\mathbf{R}$.

$P_{IS}, P_{CH}$ received power from stable (IS) and unstable (CH) plasmas.

$R, AZ, EL$ range, azimuth and elevation of the observed point $\mathbf{R}$ in the radar frame.

$c_p$ transmitted pulse-length.

$\Delta R = c c_p$ range smearing.
\( f(\Theta) \) normalized antenna gain ('antenna pattern').

\( V_{\text{eff}} \) effective scattering volume.

\( N_e = n_o V_{\text{eff}} \) total number of electrons inside \( V_{\text{eff}} \).

\( \text{POL} = \text{LOG10}(\text{POWER}/K_R) \) 'logarithmic' power.

\( K_R \) radar constant at range \( R \).

\( \Theta_{\text{ASP}} \) aspect angle.

\( \text{ASP} \) aspect angle sensitivity.

\( \text{THETA} \) flow angle.
a mis padres
CHAPTER 1

INTRODUCTION

The ionosphere is a weakly ionized plasma in a weak magnetic field which supports a number of collective oscillations (normal modes). For our purposes, these modes are basically the electrostatic ion plasma (acoustic), the electron plasma (Langmuir) waves and the ion and electron Bernstein waves. Both the electron and ion plasma waves propagate obliquely to the magnetic field taking their energy from the parallel motion of the particles whereas the Bernstein modes propagate almost perpendicular to the magnetic field and are associated with the cyclotron motion of the particles.

When the system is stable, the energy content in each of those collective modes is balanced. In non-equilibrium conditions there arises a possibility of instability through coupling of these modes between themselves and a free energy source. In the ionosphere, non-equilibrium conditions are frequent and, in particular, the equatorial and auroral zones will contain irregularities (unstable density fluctuations and structures) with scale sizes ranging from hundreds of kilometers to centimeters. Irregularities play an active role in the ionosphere by producing heating, changing conductivities and introducing 'anomalous' transport effects (Huba and Ossakow [1981.b], Bernhardt et al. [1982], Sudan [1983. a], Robinson [1986], St.Maurice [1987, 1988]). Evidence of anomalous heating of the electron gas in the lower E-region has been presented (Schlegel and St.Maurice [1981], Nielsen and Schlegel [1985], St.Maurice and Laher [1985]). Unstable plasma waves may also disrupt transionospheric radio-communications channels.
('clutter', 'scintillations', see Fejer and Kelley [1980]).

Different observation systems and techniques (radars, rockets, satellites, etc.) are currently in use to study these phenomena and nowadays much has been achieved on their understanding. A detailed review of the development in the field can be found in Fejer [1979], Fejer and Kelley [1980], Kelley et al. [1980], and Ossakow [1981]. In-situ measurements using rockets are consistent with the radar observations (see Pfaff et al. [1985] for a review of equatorial results, and Pfaff et al. [1984], Prindahl and Bahnsen [1985] for recent auroral results). Of particular relevance for our present study are the short wavelength (kinetic) instabilities in the turbulent lower ionosphere and their observation with radar systems using the Thomson scattering technique.

1. SCOPE OF THE PRESENT STUDY.

The subject of this doctoral dissertation is the study of the 30-cm wavelength plasma instabilities in the auroral lower ionosphere. For this purpose we use the large set of systematic observations of coherent backscatter from the auroral E and F regions, made since 1983 with the MIT-Millstone Hill radar.

The Millstone Hill radar facility includes a fully steerable 46 m antenna operating at 440 MHz with power peak of 2.5 MW and main beam width of 1 degree.

Looking to the north at low elevation angles the radar beam is nearly perpendicular to the geomagnetic field at E-region altitudes and is sensitive to coherent backscatter from 30-cm unstable plasma waves propagating in this perpendicular direction (St.Maurice et al. [1988]). The high sensitivity of our system also allows the observation of microinstabilities generated in the F-region (Foster et al. [1988]).

Plasma instabilities are very large amplitude density fluctuations amplified
during non-equilibrium conditions by the presence of free energy sources. Unlike a normal fluid, a plasma may support a number of instabilities leading to different turbulence regimes. In the auroral ionosphere non-equilibrium conditions are frequent, they are the result of the exchange of energy and particles between the ionosphere, the magnetosphere and the solar wind.

In the lower E-region (electrojet) two sources of instability are dominant, (a) the modified two-stream (Farley-Buneman or type 1), covering the spectrum of both kinetic and fluid-like waves and, (b) the long-wavelength ExB-gradient drift instability. The turbulent saturation of this latter instability may amplify the short wavelength density fluctuations well above the thermal level (turbulence-generated type 2 'secondaries'). Also, the low-frequency density gradient drift turbulence (type 0 'secondaries'), induced by the type 1 waves outside the instability cone, may explain some of our observations.

Above this region, various types of instabilities may also be driven by the combined effects of field aligned currents, particle precipitation, transverse currents and electric fields.

Many theoretical and practical reasons justify the study of the unstable ionosphere and the Millstone Hill radar is an excellent tool to do so. At least three features of the Millstone Hill radar give a particular relevance to such observations, (1) its routine operation has led to a large database with good spatial coverage, (2) its very narrow antenna beam width and, (3) its simultaneous functioning as both, an incoherent and a coherent backscatter.

This system is routinely used in the diagnosis of the local state of the stable ionosphere, but detailed observations are made during turbulent conditions. The interpretation of these latter observations has been the main purpose of our research. From the experimental point of view, the original contribution of the
present study lies in the fact that, for the first time, the instabilities' spectral parameters together with the local electric field are simultaneously determined and compared. This is significant because auroral turbulence, particularly in the E-region, is highly dependent on both the strength and the orientation of the electric field.

The organization of the present study is as follows:

In this chapter we present a review of previous studies of ionospheric instabilities, in particular, those pertaining to the radar observations and their interpretation. Chapter 2, together with appendices 1 and 2, are dedicated to the linear kinetic theory derivation of various instabilities expected to be observed at 30-cm wavelength during disturbed ionospheric conditions and to the estimation of their corresponding saturation amplitudes in the frame of the weak turbulence 'orbit-diffusion' approximation. In chapter 3 we present the Thomson scattering technique and discuss the radar measurement of the instabilities' cross-section and the spectral function (turbulence power spectrum).

In chapter 4 we discuss the interpretation of the radar cross-section and simulate the radar response to the scattering from strongly field aligned irregularities in the lower E-region ('thin layer' model). At the end of the chapter we analyze the measured cross-section from two different types of experimental configurations: the first consisting of continuous 180°-azimuth radar scans (from west to north and north to east) for observation periods of more than 24 hours, and the second consisting of two hours of finer range resolution data, from elevation scans (from 2° to 5°) at a fixed azimuth (20° NE) in the local evening sector. The combined information from these two types of experiment leads to the determination of the geophysical properties of the unstable regions (location, extent, magnetic activity dependence, etc.) and shows the strong aspect-
angle dependence of the instability mechanisms as well as their basic concentration in the lower E-region.

In chapter 5 we discuss the determination of the instability spectral parameters (turbulence power spectrum) from the radar measurements. Using simultaneous observations of the F-region ion-drift we are able to estimate the local electric field, and to compare it with the instability spectral parameters.

In appendices 1 and 2 we review, respectively, the conditions for the linear generation of kinetic instability and its nonlinear saturation in the frame of the weak turbulence 'orbit-diffusion' approximation. Appendix 3 is an analytical exercise showing the geometry of the region of favorable aspect angle in the dipole magnetic field approximation. The actual magnetic field used in our analysis is the IGR (International Geomagnetic Reference) model for the year 1985.

We refer to appendices 1 and 2 for the definition of all the physical quantities and parameters listed in the following pages.

2. RADAR DIAGNOSIS OF THE IONOSPHERE.

Radar diagnosis of the ionosphere by the Thomson scattering technique (see Evans [1969] for a review) is a common procedure which measures a number of physical parameters describing the local state of the plasma for both the equilibrium and non-equilibrium conditions (Rosenbluth and Rostoker [1962], Sheffield [1975]). When the probing frequency is much greater than the electron plasma frequency (between 3 to 10 MHz in the ionosphere), the coupling between the plasma and the incident radiation may be assumed weak. In this case also, the observation wavelength is much greater than the Debye length and scattering is due to the collective plasma oscillations (electron density fluctuations).

The radar system acts as a spectrum analyser picking out only the component of
the density fluctuations spectrum satisfying the Bragg condition.

When only one radar is used (backscattering), this mode propagates along the observation direction (radar line of sight) with a wavelength half of the one emitted. The spectrum of the thermal equilibrium modes consists of two parts, the ion-line (ion-acoustic waves), in the central portion of the spectrum, covering a band of few KHz to some tens of KHz around the transmitted frequency and the narrow plasma lines (Langmuir waves) on both sides of the ion line and displaced from the center frequency by approximately the electron plasma frequency (Dougherty and Farley [1960]). There is no central maximum in the spectrum for a non isothermal yet stable plasma. In this case there are two maxima symmetric to the central frequency, and the 'sharpness' of those peaks is a measure of the temperature ratio (Salpeter [1960], Farley [1966]).

Instabilities may develop during non-equilibrium conditions where plasma oscillations may grow enormously by taking energy from a free energy source. We will call 'primary' waves the linear unstable modes which are characterized by a narrow, very large amplitude spectrum centered at the resonance frequency solution of the dispersion relationship (Ichimaru et al. [1962]). We will call 'secondary' waves those associated with the instability or instability-like processes induced by the primary instabilities either linearly or nonlinearly.

In the ionosphere, short-scale density gradients or intense electric fields associated with linearly unstable, long-wavelength modes may destabilize other linear short-wavelength instabilities (Sudan et al. [1973]). In these cases the spectral width is relatively broad. Also the most frequent mechanism of nonlinear generation of 'secondary' waves is given by the enhancement of the shorter scale density fluctuations well above the thermal level as a result of the strong turbulence saturation of longer wavelength instability ('cascading').
These waves have a much broader spectrum with center frequency statistically distributed around zero (Sudan and Keskinen [1979], Sudan [1983.b]).

Even if only collective modes are observed, it is a common practice to identify the 'incoherent spectrum' as the one associated with the equilibrium modes in opposition to the 'coherent spectrum' used to identify the unstable modes or instability-like signatures.

The first systematic observations of ionospheric irregularities were performed in the equatorial region and much of the initial effort to explain their physical nature has been concentrated in this zone. The strong field alignment of the irregularities together with the horizontal magnetic field at the equator makes it possible to observe a wide region and to identify various instability processes (electrojet turbulence, spread F phenomena, see Crochet [1981], Farley [1985], Kelley [1985]).

In recent years the study of plasma instability in the high latitude ionosphere has received a renewed interest (Keskinen and Ossakow [1983], Fejer et al. [1984]). For a review on the radar observations see for example Hanuise [1983].

The strong field alignment of instability imposes some geometric constraints to meet perpendicularity with a radar observing at this region (Hoftee and Forsyth [1971], Minkoff [1973]). Very sensitive radars may, furthermore, also detect unstable waves propagating away from the perpendicular plane in a large portion of the auroral ionosphere (Foster et al. [1988]). Keeping in mind the real physical differences between the two zones, equatorial results may be applied to the interpretation of the auroral observations.

3. NON-EQUILIBRIUM CONDITIONS.

All the physical processes occurring in the Earth's neutral atmosphere and
ionosphere are induced by solar energy entering both directly as electromagnetic radiation and indirectly through the complex electromagnetic interaction between the solar wind, the terrestrial and interplanetary magnetic fields and the Earth's generated plasma distributed in the ionosphere and magnetosphere.

Differential energy absorption over the globe leads to temperature and pressure gradients and the resulting wind systems in a wide range of altitudes. Above 80 km altitude the atmosphere is ionized and primarily controlled by the solar ultraviolet radiation, which, under quiet conditions, results in predictable diurnal, seasonal, and solar cyclic behavior. A more variable source of energy is the solar wind which penetrates and energizes the Earth's magnetosphere, where it is stored and impulsively released (magnetic 'substorms') generating complex electric potentials and currents. These phenomena map into the upper atmosphere, resulting in large transfers of corpuscular and electric energy. The consequences of this variable energy input are manifested by changes in ionization, electric conductivity, excitation, heating, dynamical behavior, composition and thermal structure. At high latitudes the ionosphere is the site of various physical processes leading to the generation of irregularities and plasma instabilities.

PLASMA INSTABILITY.

The onset of 'primary' instability as well of the linear 'secondaries' are predicted by the linear theory (chapter 2 and appendix 1). Here, we consider only nonlinear 'secondaries' generated by the amplification of shorter wavelength fluctuations (otherwise stable) due to the energy transfer from the longer wavelength turbulence. We arbitrarily classified the kinetic-like instabilities ($k \rho_i > 1$) into two classes, the short-wavelength ($\lambda < 10$ m, $k \rho_i > 1$) and the very short-wavelength ($\lambda < 1$ m, $k \rho_i >> 1$). And, we call fluid-like the
longer wavelength oscillations. For linear kinetic instability (particularly at very short wavelengths), the weak mode coupling assumption is generally true (Gary [1980], Gary and Sanderson [1981]) and saturation can be reached through the broadening of resonances by nonlinear wave-particle interactions in the 'orbit-diffusion' approximation (see appendix 2 and as far as 'Coulomb' collisions can be neglected (Dum [1975]) and a Gaussian wave-electric field amplitude may be assumed (Dubois and Pesme [1986])).

Large-scale turbulence in the non-equilibrium ionosphere is determined by the saturation of the fluid-like oscillations. In the lower E-region (electrojet), two instability mechanisms are dominant: the Farley-Buneman, covering the spectrum of both kinetic and fluid-like waves and, the long wavelength ExB-drift mode (Fejer et al. [1984.b]). In the F-region, the basic turbulent state is determined by the generalized Rayleigh-Taylor instability (a ExB-drift mode) at the longer wavelengths (Kelley et al. [1982]) and, by the low frequency density gradient drift modes (universal drift type waves) at the meter to ten of meters wavelengths (Kelley [1982]). Velocity shears (in the ExB flow) are also source of long wavelength Kelvin-Helmholtz instability (Kelley and Carlson [1977], Keskinen et al. [1988]). The generation of kinetic instability (ion-cyclotron and lower hybrid modes) by very localized shears in the ExB flow have recently been addressed by Ganguli et al. [1988].

A variety of kinetic instabilities in the auroral F-region, commonly present during disturbed conditions, are associated with field aligned currents, particle precipitation, transverse currents and electric fields. In the auroral lower ionosphere (below the F-region maximum) the most probable instability mechanisms are the density gradient drift including collisions and Pedersen currents (Gary et al. [1983], Gary and Cole [1983]), the collisional current-driven (Satyaraya-
current-driven (Satyarayana et al. [1985], Satyarayana and Chaturvedi [1986]) and the current-convective (Yamada and Hendel [1978]) ion-cyclotron and lower hybrid modes, and the Post-Rosenbluth type instability (Ott and Farley [1975], St.Maurice [1978], Lakhina and Bhatia [1984]).

Longer wavelength instability may provide the sharp density gradients needed for the linear generation of kinetic drift waves (Huba and Ossakow [1979.a, 1981.a], Gary et al. [1983]). Also the coupling between the various driving mechanisms may lower the threshold for the generation of some particular instabilities (Chatuverdi and Ossakow [1981], Gary and Cole [1983], Ganguli and Palmadeso [1987]).

4. REVIEW OF THEORY AND RADAR OBSERVATIONS OF PLASMA INSTABILITY IN THE LOWER IONOSPHERE.

A. PRINCIPAL INSTABILITY MECHANISMS.

(1). MODIFIED TWO-STREAM (FARLEY-BUNEMAN) AND \textit{E x B} DENSITY GRADIENT DRIFT WAVES.

In the lower E-region collisional effects are important and the presence of a transverse electric field is a source of perpendicular current. The relative streaming between electrons and ions perpendicular to the magnetic field may drive the plasma toward instability if it exceeds the ion-sound speed (Farley [1963], Buneman [1963]). This instability is known either as the modified two-stream or Farley-Buneman (F-B). It covers a wide range of wavelengths (cm to km) from hydrodynamic type to fully kinetic plasma oscillations (Lee et al. [1971], McBride et al. [1972], Fejer et al. [1984.b]).

In the equatorial and auroral regions between 90 km to 120 km, strong Hall currents (electrojets) provide the free energy source for instability. These currents are the result of the combined effects of collisions between charged
particles and neutrals and the ambient electric field. Above this region, the lower collisions rate does not allow the presence of a differential cross-field (ExB) drift and this instability mechanism does not occur.

During ionospheric disturbances the ambient electric field is generally above that required for linear instability (varying from 15 to 25 mV/m for different wavelengths, Farley [1963], Moorcroft [1978]). Farley-Buneman waves propagate with constant phase velocity, in a narrow angular region around the generating Hall current (also called instability cone, with an extent smaller than 60°, Schlegel [1980]). This phase velocity is the one of marginal instability and equal to the ion-sound speed (Sudan [1983.a]).

Shorter-scale (kinetic $k \rho_i > 1$) instabilities have frequencies and growth rates comparable to the lower-hybrid frequency and maximize slightly away from the perpendicular at wavelengths smaller than the ion gyroradius (Ossakow et al. [1975], $k \rho_e \approx 1$, $(k_\parallel / k_i)^2 \approx m_e / m_i$). Longer wavelength ($k \rho_e < 1$) instability has a lower electric field threshold and maximum growth rate for perpendicular wave propagation (Lee et al. [1971]).

In the presence of an electric field an ambient density gradient provides another source of free energy to drive a fluid-like instability. This linear mechanism is also called the ExB-gradient drift instability or Simon-Hoh instability (Rogister and D’Angelo [1971], Keskinen et al. [1979]). This mechanism is most efficient in regions where a moderate electric field has a component parallel to the ambient density gradient. Under typical ionospheric conditions, the minimum unstable wavelength is approximately of the order of ten of meters (Sudan and Keskinen [1979]). These large amplitude fluid-like waves may provide the sharp density gradients and the electric fields necessary for the linear generation of short wavelength secondaries either gradient-drift
or Farley-Buneman (Sudan et al. [1973]). In the electrojet region, however, turbulence is generally strong and the 'kinetic-like' secondaries are the result of nonlinear mode coupling (Sudan [1983.b]). In their saturation process the fluid-like primaries (either Farley-Buneman or ExB-drift waves) transfer energy to the shorter scales enhancing the shorter wavelength stable fluctuations well above their thermal background amplitudes.

Secondary waves are called 'type 0' waves if they are associated with the moderately strong turbulence of counter electrojet (local time post-midnight/ morning sector auroral lower E-region) or unfavorable ambient density gradient ($\nabla n_0 \cdot \mathbf{E} < 0$) conditions (Chrochet [1981], St.Maurice et al. [1986]). When the ambient density gradient is favorable ($\nabla n_0 \cdot \mathbf{E} > 0$) wave turbulence is stronger and secondaries are called 'type 2' waves (Sudan [1983.b]).

Linearly generated Farley-Buneman waves are also called 'type 1' waves. Fejer et al. [1986] have reported the observation of intermittent, two-stream type instabilities at 3-m with high phase velocities and high electron temperatures (of the surrounding stable plasma) and called them 'type 4' waves.

In the lower E-region turbulence is closely two-dimensional in the direction perpendicular to the ambient magnetic field and controlled by the saturation of the fluid-like instabilities. At the very short wavelengths, however, as long as the electric field may sustain linear instability, strong turbulence isotropy on the perpendicular plane is broken by the presence of a narrow cone about the ExB direction where 'type 1' waves may saturate by 'turbulent collisions' effects at much lower levels of turbulence (Sudan [1983.a]). It is, however, very possible that mode coupling effects play a role in the saturation of the F-B waves since this instability is non-dispersive, in particular in the auroral region where the driving current may be very strong. Nonlinear effects in the F-B instability
are not fully understood and may well explain the reported observation of unstable waves propagating at large angles away from perpendicular (Haldoupis et al. [1986], Moorcroft and Schlegel [1988], St.Maurice et al. [1988]) without invoking the unrealistic presence of current driven ion-acoustic turbulence (Volosevich and Liperovskiy [1975]).

(2). DENSITY GRADIENT DRIFT AND PEDERSEN GRADIENT DRIFT WAVES.

Above the electrojet region both electrons and ions experience the same cross field drifts due to the ambient electric field and only a weak Pedersen current may exist. At those altitudes, instabilities are mainly driven by density gradients, ion inertia and parallel currents. Kinetic waves (\(k p_i > 1\)) may be destabilized by the small scale density gradients perpendicular to both the magnetic field and the ambient electric field. In a weakly collisional plasma three types of such density gradient drift modes are possible:

(a). The universal density gradient drift which is generated by the electron diamagnetic drift with no density gradient scale length threshold. It is a low frequency mode \((\omega \ll \Omega_i)\) with maximum growth rate at \(k p_i \sim 1\) and propagating slightly away from the perpendicular direction \((k || \neq 0)\).

Weak ion-neutral and electron-neutral collisions reduce the maximum growth rate stabilizing the universal mode at sufficiently short wavelengths \(< \text{few meters}\).

Under typical ionospheric conditions this instability has appreciable growth above 250 km altitude and for plasma densities less than or equal to \(10^5\) cm\(^{-3}\) (Gary et al. [1983]).

Collisions are at the origin of another low frequency mode similar to the universal. This collisional universal mode may exist for shorter wavelengths
(< 1 m) but now the wave propagation is strictly perpendicular to the magnetic field. Growth rates are important only when electrons are sufficiently collisional ($v_e > \Omega_e$) and ions are sufficiently collisionless ($v_i < \Omega_i$). In the ionosphere these conditions are met at relatively high plasma densities ($n_0 > 10^5$ cm$^{-3}$) and above 250 km (Gary et al. [1983]).

(b). The ion cyclotron drift (ICD) is driven by the ion diamagnetic drift at frequencies equal to the ion cyclotron harmonics and with maximum growth rate at $k \Omega_e \sim 1$. Wave propagation is strictly perpendicular to the magnetic field. Ion-neutrals and ion-ion collisions demagnetize the ions stabilizing this mode (Gary and Sanderson [1978], Huba and Ossakow [1979.b]).

(c). The lower hybrid drift (LHD) is also driven by the ion diamagnetic drift at frequencies close to the lower hybrid frequency. Maximum growth rate occurs at $k \Omega_e \sim 1$ for perpendicular propagation. This mode is the unmagnetized counterpart of the ICD mode, has the largest growth rate (Huba [1981]) and is stabilized by electron collisions (Sperling and Goldman [1980]).

PEDERSEN DENSITY GRADIENT DRIFT MODES.

The ambient electric field together with the weak ion-neutral collisions which below 300 km are still important may sustain a weak Pedersen current. In the auroral F-region, the long wavelength (few 10 m to few km) ExB-gradient drift instability is a common feature (Huba et al. [1983]). As in the electrojet region, this instability is driven by the cross-field drift of the electrons parallel to a density gradient, but in this case the free energy source is the weak Pedersen current parallel to the ion diamagnetic drift. Large scale density
gradients (tens of km) necessary for the generation of this instability are associated with the low energy electron precipitation in the auroral F-region (Kelley et al. [1982]).

The equatorial counterpart of this situation is given by the spread F phenomena. Density gradients and electric fields needed for instability are generated at post-sunset conditions (Fejer and Kelley [1980]). The generalized Rayleigh-Taylor mechanism is responsible for instability. Ambient electric field, neutral winds and the usual gravitational term (upward vertical density gradient) are the destabilizing factors (Kelley et al. [1982]).

In both the equatorial and the auroral zones, free energy sources in the bottomside F-region are much weaker than those in the electrojet region and a strong turbulent state is not expected. Instead these long wavelength instabilities seem to provide the small scale (< 100 m) density gradients for the linear generation of secondary density gradient drift waves (Kelley [1982]).

In the auroral zone sharp horizontal density gradients may also occur at the edge of an arc or at the edge of the ionospheric trough.

Low-frequency short-wavelength instability can also be generated by the combined effects of small-scale density gradients and an electric field parallel to the diamagnetic drift. This mechanism is called the Pedersen drift instability (Gary and Cole [1983]).

A Pedersen current parallel to the electron diamagnetic drift (see appendix 1 for definition) favors the collisional universal mode. Stronger electric fields may also allow this low frequency mode to grow at typical ionospheric densities \( n_o \sim 10^5 \text{ cm}^{-3} \) and for altitudes about 200 km (Gary and Cole [1983]). This low frequency drift mode may exist for wavelengths smaller than 1 meter (see chapter 2). LHD waves are also favored by the presence of a Pedersen current parallel to
the ion diamagnetic drift. LHD modes are expected at very short wavelengths at altitudes as low as 200 km for typical ionospheric densities \( < 10^5 \text{ cm}^{-3} \) and moderate density gradient scale-lengths \( > 50 \text{ m} \).

Observations in the lower equatorial F-region have been reported (associated with the presumed universal drift and lower hybrid modes, Huba and Ossakow [1981.a]). ICD modes are heavily damped by ion-neutral or ion-ion collisions and they are not expected below 300 km for wavelengths smaller than one meter.

(3). CURRENT DRIVEN AND CURRENT CONVECTIVE ELECTROSTATIC ION CYCLOTRON WAVES.

Currents parallel to the magnetic field are a normal feature in the auroral ionosphere. They provide the free energy source to drive various instabilities: electrostatic ion-cyclotron, ion acoustic, and two-stream Buneman waves. Among these instabilities the electrostatic ion-cyclotron (EIC) waves have the lowest threshold under normal ionospheric conditions \( 0.1 < T_e / T_i < 10 \). In a highly non-isothermal plasma \( T_e / T_i > 10 \) current driven ion acoustic instability dominates (Kindel and Kennel [1971]).

Ion-cyclotron waves have been observed both at high (several earth radii) and low \( < 1000 \text{ km} \) altitudes along the auroral field lines. In particular evidence of short wavelength \( (k \rho_i \sim 1) \) EIC emissions at low altitudes \( <350 \text{ km} \) in the diffuse aurora have been presented (Bering [1984]). In the upper E-region \( >130 \text{ km altitude} \) VHF radar observations of EIC waves have been reported ('type 3' waves, Fejer et al. [1984.a], Haldoupis et al. [1985], Providakes et al. [1985], Prikeyl et al. [1987]). See for example D'Angelo and Merlino [1988] for a review of the observations. These waves have very narrow, resonance-type spectra which are centered at frequencies slightly greater than the \( \text{NO}^+, \text{O}_2^+ \) and \( \text{O}^+ \) harmonics. Frequent very broad spectral signatures also observed at 3-m in the
auroral zone by Basley and Ecklund [1972] could, possibly, be related with the ion-cyclotron instability in the upper auroral E-region (D'Angelo [1973]).

The transfer of the electrons' parallel energy into the ions perpendicular motion is at the origin of the EIC instability. In a non-collisional plasma ion-cyclotron waves grow towards instability when the electron growth rate exceeds the ion Landau damping. EIC waves propagate away from the perpendicular plane in all directions about the magnetic field line. Electron collisions (both with the neutrals and with the ions) are destabilizing and crucial for the excitation of EIC modes at low altitudes.

Ion-neutral collisions may stabilize these modes at E-region altitudes and ion-ion collisions in the F region. Electron collisions overcome the ion damping in the region between 130 km to 150 km of altitude and significantly enhance the instability growth rate in the region between 150 km to 300 km (Satyanarayana et al. [1985]). Growth rate maximizes for \(k_{\parallel} \sim 1\) and the lowest instability threshold lies between 150 km to 200 km (Satyanarayana et al. [1985]).

At the instability wavelength corresponding to our radar frequency (30-cm) EIC waves are not likely to be observed at altitudes below 250-300 km and for densities bigger than \(10^4\) cm\(^{-3}\) (ion-ion collisions destroy the ion gyromotions).

Above 250 km they are excited by relatively high parallel drifts (of the order of 100 km/sec, the electron thermal velocity at \(T_e = 1000^\circ\)K is equal to 170 km/sec).

The presence of a density gradient perpendicular to the magnetic field may reduce the parallel electron drift threshold for EIC instability. This mechanism is called the current convective instability and have been discussed by a number of authors (for recent studies see for example Chatuverdi and Ossakow [1982] for the fluid description and Yamada and Hendel [1978] for the kinetic
derivation). The current convective modes propagate parallel to the electron
diamagnetic drift. In the auroral F-region the current convective mechanism is
also a good candidate for the generation of long wavelength instabilities. They
may exist for relatively weak field-aligned currents and large-scale (> km)
horizontal density gradients even when the usual ExB gradient drift instability
is inoperative (Chaturvedi and Ossakow [1981]). Under these conditions the free
energy source is weak and long wavelength instability can't reach a strongly
turbulent state (Chaturvedi and Ossakow [1979]). They may provide the small
scale (> 100 m) density gradients needed for the linear generation of short
wavelength instabilities.

(4). POST-ROSENBLUTH INSTABILITY.

Above the electrojet region, weak ion-neutral collisions together with a strong
ambient electric field are at the origin of two unstable modes, an ion cyclotron
instability (Lakhina and Bhatia [1984]) and its unmagnetized high-frequency
counterpart (Ott and Farley [1975]). This high frequency mode is known as the
'loss cone' Post-Rosenbluth instability (Post and Rosenbluth, 1965).

At altitudes between 130 km to 300 km the ion-neutral collisions are weak
(\(\Omega_i > \nu_{in}\)) but still more important than the ion-ion collisions (\(\nu_{in} > \nu_{ii}^{+}\)).

For densities greater than 10^7 cm-3 ion-ion collisions destroy ion gyromotions
for scale-lengths smaller than 1 meter (condition \(\frac{k^2\rho^2\nu_i^+}{\nu_{ii}^{+}} > 1\)).

In the lower F-region, the competing effects of the ion-neutral collisions and
a perpendicular electric field will drive the ion distribution away from the
Maxwellian configuration into a double-humped distribution (St.Maurice [1978]).

Using the BGK collision approximation, instability arises once the ExB drift
exceeds a value close to 1.8 times the thermal speed of the neutral gas (Ott and
Farley [1975]). In the auroral ionosphere this condition corresponds to an electric field threshold of the order of 50 mV/m (from more realistic considerations this value may be as high as 80 mV/m, St.Maurice [1978]). In both cases instability maximum growth rate is for wave propagation away from the perpendicular direction. The ion-cyclotron modes have finite $k_{\parallel}$ and their perpendicular wavelengths fall into the range of $\lambda \sim$ 3 to 15 meters. They are stabilized by the ion-ion collisions (Lakhina and Bhatia [1984]).

Post-Rosenbluth instability maximizes for $k_{\parallel}/k_{\perp} \sim \sqrt{m_e/m_i}$, for frequencies of the order of the lower hybrid frequency and wavelengths shorter than 1 meter.

This later instability can be stabilized by the electron collisions both with the neutrals and the ions (Ott and Farley [1975]). In the region between 130 to 200 Km, it is a good approximation to consider that electrons are not collisional. This is also true in regions below 250 Km for densities smaller than $10^5$ cm$^{-3}$ and, below 300 Km for densities smaller than $10^4$ cm$^{-3}$. In those cases, electron-neutral collisions are rare but still more important than the electron-ion and the electron-electron collisions.

B. RADAR OBSERVATIONS OF IONOSPHERIC INSTABILITIES.

Radar systems with working frequencies above the ionospheric plasma frequency and used in the study of plasma waves ($\lambda > \lambda_e \lambda_i$) are classified as:

- High-frequency (HF from 3 to 30 MHz, decametric waves $\lambda < 100$ m), very high frequency (VHF from 30 to 300 MHz, metric waves $\lambda < 10$ m), ultra high frequency (UHF from 300 to 3000 MHz, decimetric waves $\lambda < 1$ m). Super high frequency systems (SHF, $\lambda < 10$ cm) may also be used in the study of ionospheric plasma processes.

Radar observations of the unstable auroral ionosphere, in particular the lower
E-region, have been performed in the VHF range at basically 50 MHz (Balsley and Ecklund [1972], Ecklund et al. [1977], Ogawa et al. [1980], Greenwald et al. [1975.a,b], Fejer et al. [1984], Providakes et al. [1985], Haldoupis et al. [1985, 1986]) and in the UHF range at 398 MHz (Tsunoda [1975, 1976], Moorcroft and Tsunoda [1978], Moorcroft [1978, 1979]) and at 140 MHz (Andre [1983], Nielsen et al. [1984], Haldoupis and Nielsen [1984], Moorcroft and Schlegel [1988]). The previous list of observations doesn’t pretend to be complete.

The first results of the study of E-region instability with the 440 MHz Millstone Hill system are presented in St.Maurice et al. [1988]. Some pioneer studies on the 'Radio-aurora' were also performed at Milltone Hill but using the 1298 MHz radar system (Abel and Newell [1969], Hagfors [1971]). We look forward for the combined use of the two Millstone Hill radars in the study of the turbulent auroral ionosphere.

For a review of the equatorial observations see for example Farley [1985] and Kelley [1985].

High frequency (HF) radars used in the study of ionospheric irregularities (Crochet [1981], Greenwald et al. [1983], Vilain et al. [1987]) have the advantage that perpendicularity to the magnetic field may be met in a large portion of the auroral ionosphere but their frequencies are very close to the plasma frequency and refraction effects become important. The physics of the radar/plasma coupling is different from the higher frequency cases and will be not treated here.

Single radar observations select the component of the density fluctuations spectrum with half of the transmitted wavelength and propagating along the radar line of sight. Depending on the accuracy of the measured spectrum, we may study the stationnary turbulence state (relaxation, two moments analysis: the
waves phase velocity, and the spectral width) or using higher order moments, to study processes away from saturation. From a number of observations and their relation to different geophysical parameters and conditions, the instability threshold, the waves propagation direction, the source location and extent, etc, may be determined.

The Millstone Hill UHF (440 MHz) radar is a high-sensitivity system primarily designed as an incoherent scattering diagnosis tool (measuring thermal fluctuations level). This characteristic provides access to the plasma state in the region surrounding the instability sources and in particular the local electric field (Holt et al. [1984]). Also, density and temperature gradients as well as velocity shears may be identified (Foster et al. [1985.a,b]).

Observed spectral width and turbulence regimes.

A qualitative classification of the plasma turbulence may be initiated from the measured spectral width. Calling $V_{ph}$ the waves phase velocity and $D_{sp}$ the spectral width, a very narrow ($D_{sp} \ll V_{ph}$) and a narrow ($D_{sp} < V_{ph}$) spectrum characterize, respectively, weak and moderate turbulent states. Strongly turbulent conditions are characterized by broad spectra ($D_{sp} \gg V_{ph}$) and an intermediate broad ($D_{sp} > V_{ph}$) spectrum is associated with moderately strong turbulence.

C. SUMMARY.

Summary of the radar-observation features of lower ionospheric turbulence.

In the following lines we summarize some of the reported or expected radar observation features of the various reviewed instabilities:

1. LOWER E-REGION (90-120 KM):

(a). Modified Two-stream (Farley-Buneman) instability.

Primary two-stream waves are generated for a threshold electric field between
20 to 25mV/m, their observed spectral width is narrow (Dsp < ion sound speed) and their phase velocity is constant (equal to the ion-sound speed). Wave propagation inside a narrow cone about the electrojet current (instability cone) and closely perpendicular to the magnetic field. There is, however, some evidence of instability generation at large angles between the wave-propagation direction and the ambient magnetic field (Haldoupis et al. [1986], Moorcroft and Schlegel [1988]).

(b). ExB-gradient-drift instability and secondary waves.

Long-wavelength instabilities are easily generated in the local evening sector where the ambient density gradient has a component parallel to a moderate electric field (\( \nabla n \cdot E > 0 \)) and the lower E-region is in a strongly turbulent state (type 2 waves, Keskinen et al. [1979]). When the ambient density gradient is not favorable (\( \nabla n \cdot E < 0 \), morning sector) long wavelength instability is basically of the Farley-Buneman type and the final state is less turbulent (possibly low frequency density-gradient drift or type 0 waves, St.Maurice et al. [1986]). These secondary waves move with the electrojet current and their phase velocities in the radar frame are expressed by \( \hat{V}_d \cos(\Theta) \), where \( \hat{V}_d \) is the lower E-region differential drift and \( \Theta \) is called the flow angle.

Turbulence is maximum in the plane transverse to the magnetic field and when observing perpendicular to the electrojet direction.

The enhanced short wavelength fluctuations present a broad spectrum (Dsp > ion sound speed) for the strongly turbulent conditions and an intermediate width spectrum for moderately strong turbulence.

(2). UPPER E AND LOWER F REGIONS (140-300 KM):

(a). Short-wavelength density gradient-drift instabilities.

Both the ion-cyclotron and lower hybrid gradient-drift modes have a threshold
phase velocity associated with the critical density gradient scale-length (smaller than 50 m for our wavelength at normal ionospheric conditions). They propagate strictly perpendicular to the magnetic field along the ion diamagnetic drift direction (Huba and Ossakow [1979.a], Huba [1981]). Generation of these instabilities and the low frequency universal type mode at very short wavelengths ($\lambda < 1$ m) may be helped by the presence of perpendicular electric field and weak collisions (Gary et al. [1983], Gary and Cole [1983]).

Density-drift modes probably saturate at weak to moderate turbulence conditions (Huba and Papadopoulos [1978], Huba and Ossakow [1981.a], Kelley [1982]) and their spectral width is expected to be narrow.

Ion-ion collisions may destroy the ion-cyclotron harmonics (Huba and Ossakow [1979.b]) and electron-ion collision stabilize the lower hybrid modes (Sperling and Goldman [1980]). Temperature gradients may also be source of short wavelength instability (Gary and Sanderson [1979]).

(b). Electrostatic ion-cyclotron (EIC) waves.

They are generated by strong field-aligned currents (Kindel and Kennel [1971], Satyanarayana et al. [1985]) and propagate almost perpendicular to the magnetic field with no preferential direction and have maximum growth rate for wavenumbers such that $k_{\perp} \sim 1$. There are, in the recent literature, many accounts of VHF radar observations of these modes above 150 Km in the auroral ionosphere (by the Cornell University and Canadian groups at 50 MHz, see Fejer et al. [1984], Haldoupis et al. [1986], Prikryl et al. [1987]). Those waves have frequencies close to the 1st and 2nd gyro-harmonics of the NO$^+$, O$^+$ and O$^+$ ions and a very narrow spectrum.

(c). Post-Rosenbluth (Ott-Farley) instability.

Electric field threshold ($>50$ mV/m), maximum growth rate for wavelength close
to our radar, high-frequency mode (of the order of the lower hybrid), no preferential phase propagation direction and expected narrow spectrum (Ott and Farley [1975], St.Maurice [1978]). The same unstable ion distribution exciting this instability may generate an ion-cyclotron mode at wavenumbers such that $k \rho_i \sim 1$ and similar electric field threshold (Lakhina and Bhatia [1984]).
CHAPTER 2

'ION-LINE' INSTABILITIES AT 30-cm

In the ionosphere, for the conditions which interest us, the particles' dynamics do not perturb the Earth's magnetic field (zero-beta hypothesis) and only electrostatic waves can be considered. During non-equilibrium conditions these waves may grow unstable, evolving to different turbulence states.

Linear instability is defined by the zeros of the dielectric function (dispersion relation) and by the Landau treatment of the singularities in the associated resonance function (see appendix 1). Resonant linear wave-particle interactions are responsible for the damping of the equilibrium modes, as well as for the growth of linear instability in a non-equilibrium plasma.

Linear Landau growth rate may enhance modes in a certain phase velocity range by transferring energy from some free energy source of the plasma into wave energy. In linear theory, however, the fluctuating intensity grows steeply when approaching the instability domain; the spectral distributions of the plasma fluctuations and other electrodynamic quantities become infinite at frequencies and wavevectors that satisfy the dispersion relationship. This unlimited growth of the fluctuation intensity indicates that linear approximation is inadequate, and nonlinear effects must be taken into account to explain the finite saturation amplitude and the width of the observed instability spectrum.

If the plasma is dominated by one mode with frequency and wavenumber similar to those predicted by linear theory, nonlinear corrections do not radically alter
the qualitative features of the linear approximation, and at low levels of turbulence the saturation spectrum of a linearly unstable mode may be represented by a narrow Lorentzian distribution centered at the unstable frequency; in these cases the spectral peak and width can be calculated using the weak coupling 'orbit-diffusion' approximation (Dupree [1968]). This approximation is valid when the diffusion coefficients are weakly dependent on the velocity field and the fluctuating wave-field is a Gaussian-Markov process (Benford and Thomson [1972]). It will not be applied when 'Coulomb' collisions are important (Dum [1975]), or if self-consistency in the interaction between the particles and the wave-electric field has to be considered (DuBois and Pesme [1985]). Weak to moderate turbulence conditions are characterized by the existence of a relatively broad fluctuation spectrum and a relatively weak free-energy source or, equivalently, the fluctuations' autocorrelation time is much smaller than the instability's relaxation time, and the wave energy is much smaller than the particles' kinetic energy (Davidson [1972]). In kinetic theory such conditions may even arise from random particle motions without an initial diffusion of energy in wavenumber space (Dupree [1968]). Under these conditions, saturation of linear kinetic instability is achieved through the broadening of the wave-particle resonances by effects of the nonlinear interaction between the enhanced fluctuations and the particles. This saturation mechanism is also known as the 'turbulent collisions' or the 'orbit-diffusion' process (Dupree [1968], Weinstock [1970], Tsitovich [1972]): unstable waves gain energy from the resonant population by effects of linear Landau growth and are nonlinearly Landau-damped (resonance broadening) by the secular perturbation of the trajectories of the non-resonant particles (turbulent collisions).

In this chapter, together with appendices 1 and 2, we discuss the conditions
for the linear generation of 30-cm wavelength instabilities for frequencies smaller than both the electron plasma and the electron-cyclotron frequencies and evaluate their saturation amplitudes in the weak turbulence 'orbit-diffusion' approximation. For the determination of the saturation amplitudes we follow Dum and Dupree [1970], Gary [1980], and Gary and Sanderson [1981].

1. 'ION-LINE' KINETIC INSTABILITY.

The auroral ionosphere is a non-equilibrium plasma where several sources of free energy are available to drive various types of instabilities: currents (both parallel and perpendicular to the magnetic field), electric fields, velocity shears, density and temperature gradients.

In this chapter we discuss some short-wavelength (kinetic) instabilities expected to occur in the lower auroral ionosphere (from 90 km to 300 km altitude) and likely to be observed with our 440 MHz radar system. For this purpose we use linear kinetic theory in the local approximation.

Our system measures the 30-cm wavelength component of the density fluctuations spectrum in a frequency band of 50 KHz (the receiver bandwidth); therefore, only 'ion-line' instabilities ($\omega \ll \omega_p, \Omega_e$) can be observed.

Considering the effects of currents, electric fields and density gradients as possible driving sources, the unstable distribution functions are basically of two types:

(a) Drifting Maxwellians associated with currents carried by thermal particles (Hall, Pedersen, parallel and diamagnetic currents) and,

(b) Double-humped 'loss-cone' type distributions in the lower F-region.

In the lower auroral ionosphere, particle precipitation is a source of short wavelength 'ion-line' instabilities through the generation of short-scale densi-
ty and temperature gradients. Asymmetries in the electron distribution, induced by particle precipitation, may excite 'electron-line' instabilities (e.g. upper hybrid waves, Basu et al. [1982]). The ambient electric field and electron density gradient are the primary sources of free energy for instability growth in the lower E-region. Field aligned currents, density gradients, and transverse electric fields, together with the weak ion-neutral collisions, are the most common driving mechanisms in the upper E and the lower F regions (from 130 to 300 km of altitude).

Throughout the lower ionosphere at altitudes below 300 km, collisions between charged particles and neutrals are generally much more important than those between the charged particles themselves. In this region a simple close binary collision approximation (Bhatnagar, Gross and Krook model) is used. This model considers a velocity-independent effective collision frequency which is reciprocal of the relaxation time to a local Maxwellian distribution.

In the auroral ionosphere, for normal densities ($<10^5$ cm$^{-3}$) and altitudes below 300 km, the ion-neutral collisions dominates the ion-ion collisions. The ion-electron collision frequency is quite small and is generally ignored.

For the same densities and altitudes and very short wavelengths ($<1$ m), the electron collisions (with neutrals, with ions and themselves) can be represented by an effective collision frequency (Sperling and Goldman [1980]). In the ionosphere, electrons are generally magnetized. In the lower E-region (from 90 to 120 km), the strong ion-neutral collisions demagnetize the ions' motions. In the lower F-region, at very short wavelengths ($<1$ m) and densities greater than $10^4$ cm$^{-3}$, the ion-ion collisions destroy the ion gyromotions (condition $k^2n_i^2\frac{\omega_i}{\Omega_i} > 1$, Huba and Ossakow [1979]). In all other cases, ion-magnetization is important. In table 2.1 we present a list of collision frequencies as functions of
altitude for a model auroral ionosphere under disturbed magnetic conditions.

**TABLE 2.1.** Collision frequencies (from Banks and Kokarts [1975] and Gary and Cole [1983]). \( B = 0.5 \) Gauss, \( T_e \sim T_i \sim T_n \).

<table>
<thead>
<tr>
<th>ALTITUDE (km)</th>
<th>( T_e ) (( K ))</th>
<th>( n_e ) (cm(^{-3}))</th>
<th>( \nu_{ei}/\Omega_i )</th>
<th>( \nu_{vi}/\Omega_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>500</td>
<td>( 5.0 \times 10^9 )</td>
<td>100.00</td>
<td>1500.00</td>
</tr>
<tr>
<td>100</td>
<td>500</td>
<td>( 5.0 \times 10^9 )</td>
<td>30.00</td>
<td>400.00</td>
</tr>
<tr>
<td>110</td>
<td>500</td>
<td>( 5.0 \times 10^9 )</td>
<td>10.00</td>
<td>200.00</td>
</tr>
<tr>
<td>120</td>
<td>500</td>
<td>( 5.0 \times 10^9 )</td>
<td>0.95</td>
<td>40.00</td>
</tr>
<tr>
<td>130</td>
<td>500</td>
<td>( 5.0 \times 10^9 )</td>
<td>0.4</td>
<td>20.00</td>
</tr>
<tr>
<td>140</td>
<td>500</td>
<td>( 5.0 \times 10^9 )</td>
<td>0.12</td>
<td>5.60</td>
</tr>
<tr>
<td>150</td>
<td>690</td>
<td>( 4.6 \times 10^9 )</td>
<td>0.21</td>
<td>2.41</td>
</tr>
<tr>
<td>200</td>
<td>915</td>
<td>( 7.5 \times 10^9 )</td>
<td>0.041</td>
<td>0.547</td>
</tr>
<tr>
<td>250</td>
<td>975</td>
<td>( 2.1 \times 10^9 )</td>
<td>0.0134</td>
<td>0.171</td>
</tr>
<tr>
<td>300</td>
<td>990</td>
<td>( 7.5 \times 10^8 )</td>
<td>0.00507</td>
<td>0.061</td>
</tr>
<tr>
<td>350</td>
<td>1000</td>
<td>( 2.9 \times 10^8 )</td>
<td>0.00206</td>
<td>0.0241</td>
</tr>
<tr>
<td>400</td>
<td>1000</td>
<td>( 1.2 \times 10^8 )</td>
<td>0.00086</td>
<td>0.01</td>
</tr>
<tr>
<td>450</td>
<td>1000</td>
<td>( 5.2 \times 10^7 )</td>
<td>0.00038</td>
<td>0.0043</td>
</tr>
</tbody>
</table>

\( n_e \) (cm\(^{-3}\)) \hspace{1cm} \( \nu_{ei}/\Omega_i \) \hspace{1cm} \( \nu_{vi}/\Omega_i \) (\( T_e \sim T_i \sim 1000^\circ K \))

<table>
<thead>
<tr>
<th>( n_e ) (cm(^{-3}))</th>
<th>( \nu_{ei}/\Omega_i )</th>
<th>( \nu_{vi}/\Omega_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^3 )</td>
<td>0.00633</td>
<td>0.00004</td>
</tr>
<tr>
<td>( 10^4 )</td>
<td>0.0567</td>
<td>0.00038</td>
</tr>
<tr>
<td>( 10^5 )</td>
<td>0.533</td>
<td>0.00377</td>
</tr>
<tr>
<td>( 10^6 )</td>
<td>5.</td>
<td>0.0377</td>
</tr>
</tbody>
</table>
2. INSTABILITY MECHANISMS.

(A). MODIFIED TWO-STREAM AND ExB GRADIENT-DRIFT WAVES.

In the lower E region, short-wavelength Farley-Buneman waves can be generated when the differential drift between electrons and ions perpendicular to the magnetic field exceeds the ion-sound speed (Farley [1963], Buneman [1963]).

This is a relatively high frequency instability (\( \omega \gg \Omega \)) with maximum growth rate at \( k_e \sim 1 \) and slightly off-perpendicular wave propagation (\( k_i/k_e \sqrt{m_i/m_e} \)) (Mc Bride et al. [1972]). Instability driving sources are the combined effects of electron and ion inertia and the ambient electric field. Both the ion and electron collision frequencies are much bigger than the ion gyrofrequency.

A density gradient parallel to the electric field may help instability and, in particular, the ambient density gradient (vertical electron density profile) excites long wavelength modes (ExB-gradient drift instability).

In the following lines we derive the frequency and growth rate for the Farley Buneman instability in the very short wavelength limit (\( \lambda < 1 \text{ m} \)). To do so we use the results derived in appendix 1 for configuration 1 (figure (Al.a)).

Dispersion relation.

In the ionosphere below 120 km altitude, electrons are strongly collisional (\( v_e/k_i v_e \gg 1 \)) and the electron plasma function may be expanded in the large argument approximation (Clemmow and Dougherty [1969]):

\[
|\Im(\gamma_e)| = \left| \frac{\omega + i\gamma_e - k_i v_e}{k_i v_e} \right| \gg 1
\]

\[
z(\gamma_e) = -\frac{1}{\gamma_e} \left( 1 + \frac{\gamma_e}{\gamma_e^2} \right)
\]
From equation Al(30) the electrons susceptibility may be expressed by

\[
\chi_e = i \frac{T_e}{T_i} \frac{\omega_{pi}^2}{(\omega - k \cdot v_d)} \left[ \frac{\psi}{v_i} - i \frac{Z_i}{k \cdot v_i} \right]
\]

(2)

where \( \omega_{pi}^2 = \frac{n_e z_i^2 q_i^2}{\varepsilon_0 m_i} \) is the ion plasma frequency, and

\[
\psi = \frac{\alpha \cdot v_i}{\Omega_e \Omega_i} (1 + \frac{\omega^2 - k_i^2}{\omega^2 - k_{\perp}^2})
\]

(3)

Ions are unmagnetized (\( V_i \gg \Omega_i \)) and from equation Al(33) for the ions susceptibility, the dispersion relationship can be found:

\[
(\frac{\psi}{\psi_i} - i \frac{1}{k \cdot v_i}) = \frac{1 + Z_i(\psi_i)}{2ik^2c_s^2} \left[ \frac{1}{1 + i\Omega_\perp Z_i(\psi_i)/k \cdot v_i} \right] (\omega - k \cdot v_d)
\]

(4)

\[
\psi_i = (\omega + i\Omega_\perp - k \cdot v_i)/k \cdot v_i
\]

where \( c_s^2 = \frac{T_i}{m_i} \), \( c_s \) is the ion-sound speed and \( V_\perp \) the ion diamagnetic drift defined in appendix 1.

Equation (4) may still be simplified by using the two-pole approximation for the ion plasma function (Fried et al. [1968]):

\[
Z(\psi_i) = \frac{a^*}{(a - \psi_i)} - \frac{a}{(a^* + \psi_i)}
\]

(5)

with \( a = 0.51 - 0.81 \text{ i} \).

For short-wavelength instability \( \omega \gg k \cdot v_i \), \( \psi_i \ll k \cdot v_i \), and

\[ \omega \sim k \cdot c_s \], therefore:
Finally, the dispersion relation is expressed by:

\[
\frac{\dot{\psi}}{\partial t} - i \frac{\psi}{k l n \Omega_i} + i \frac{\omega - k \cdot V_d}{[\omega (\dot{\psi}_i - i \omega) + i k_i^2 c_s^2]} = 0 \tag{7}
\]

with \( \dot{\psi}_i = \dot{\psi}_i + 0.4 k V_d \) and \( \dot{\psi} \) is obtained from (3) replacing \( \dot{\psi}_i \) by \( \dot{\psi}_i \). Thus at very short wavelengths the ion's gyromotions increase the effective ion collisions. Equation (7) has the same functional dependence as that obtained from the fluid approximation (\( k_i^2 \rho_i^2 \ll 1 \), Fejer et al. [1984]).

Taking the real and imaginary parts of the dispersion relation when \( \omega = \omega_r + i \gamma \) and \( \gamma \ll \omega_r \),

\[
\omega_r = \frac{k \cdot V_d}{(1 + \psi)} \tag{8.a}
\]

\[
\gamma = \frac{1}{1 + \psi} \left[ \frac{\dot{\psi}^2}{\dot{\psi}_i} (\omega_r^2 - k_i^2 c_s^2) + \frac{\omega_r \psi_i}{k l n \Omega_i} \right] \tag{8.b}
\]

For short wavelengths the ambient density gradient may be neglected, and when \( \gamma = 0 \) (marginal stability condition):

\[
\omega_r = k_i c_s \tag{9.a}
\]

\[
k \cdot V_d - (1 + \psi) k_i c_s > k_i c_s \tag{9.b}
\]
Because $\psi > \phi$ the electron drift threshold is higher at shorter wavelengths. In our case, $\lambda \sim 30 \text{ cm}$ (if $k \sim 60$ if $\rho_e \sim 3 \text{ m}$) and the electron drift threshold is of the order of 400 to 450 m/s (electric field strength between 20 to 25 mv/m).

**B. DENSITY GRADIENT-DRIFT INSTABILITIES.**

Above 130 km for wavelengths such that $k \rho_e \sim 1$ and temperatures $T_e > T_i$, only three types of density gradient-drift modes are possible (Gary and Sanderson [1978]): the universal gradient-drift, the ion-cyclotron-drift (ICD) and the lower-hybrid-drift (LHD) modes.

In the following derivations we use the results of appendix 1 for the instability configuration 2.

**a. COLLISIONAL UNIVERSAL AND PEDERSEN GRADIENT DRIFT WAVES.**

Universal drift waves are low-frequency modes and non-collisional. They don't exist at wavelengths smaller than 1 meter or so. However, collisions and electric fields may excite two new universal-type drift modes at these wavelengths: the collisional universal and the low frequency Pedersen-drift waves (Gary et al. [1983] and Gary and Cole [1983]). Like the non-collisional universal drift instability, these two new modes have maximum growth rate at $k \rho_e \sim 1$, but now they propagate perpendicularly to the magnetic field. In this case, both electrons and ions are magnetized, and we must use the susceptibilities given by equations Al(29.b) and Al(30). Also for $k_\parallel = 0$ the plasma functions may be expanded in the large argument approximation (equation 1).

We are considering very short-wavelength modes with $k \rho_i \gg 1$, and in this case the terms $m \neq 0$ in the Bessel sums of the ions susceptibility (equation
Al(29.b)) may be neglected.

From equation Al(29.b),

\[
\chi_i = \frac{1}{k^2 \lambda_i^2} \left( \frac{\omega+i\gamma-k_i(V_{ni}+V_{pi})}{\omega+i\gamma} \Gamma_i(\mu_i) \right)
\]

From equation Al(30),

\[
\chi_e = \frac{1}{k^2 \lambda_e^2} \left( \frac{\omega+i\gamma-k_i(V_{ne}+V_{pe})}{\omega+i\gamma} \Gamma_e(\mu_e) \right)
\]

This low-frequency instability \((\omega \ll \Omega_i)\) requires sufficiently collisional electrons \((V_e > \Omega_i)\) and sufficiently non-collisional ions \((V_i \ll \Omega_i)\) (Gary et al. [1983]). \(V_i\) is the ion-neutral and \(V_e\) the total electron collision frequency respectively \((V_e = V_{en} + V_{ei})\). The condition \(V_e > \Omega_i\) is always true between 130 to 200 km altitude; above 200 km this is the case only if the densities are greater than or equal to \(10^5\) cm\(^{-3}\).

The electron susceptibility can now be written:

\[
\chi_e = \frac{1}{k^2 \lambda_e^2} \left( 1 - i \frac{k_i V_{ne}}{V_e} \frac{\Gamma_e}{1-\Gamma_e} \right)
\]

Therefore using the dispersion relationship for \(\omega = \omega_\gamma + i \gamma\) with \(\gamma \ll \omega_\gamma\), we find:

\[
\frac{\omega_\gamma}{\kappa_e} = - \frac{\Gamma_e}{\Gamma_i} \left( V_{ni} + V_{pi} \right) / \left[ 1 + \frac{\Gamma_e}{\Gamma_i} \left( 1 - \Gamma_i \right) \right]
\]

\[
\gamma = k_i V_{ni} \frac{\omega_\gamma \Gamma_e}{V_e(1-\Gamma_e)} - \left( 1 - \Gamma_i \right) V_i \left[ 1 - k_i \left( V_{ni} + V_{pi} \right) / \omega_\gamma \right]
\]
we are calling $\Gamma_{oe} = \Gamma_0(\mu_e)$ and $\Gamma_{oi} = \Gamma_0(\mu_i)$. 

In this case the phase velocity is parallel to the electrons' diamagnetic drift and $V_{ne}$ is negative in our reference frame (in configuration 2 the ion diamagnetic drift is positive).

$\psi > 0$ for instability and considering the case where $T_e = T_i$, 

$$\psi = k^2 V_i \frac{\Gamma_{oe}}{1 - \Gamma_{oe}} (V_i + V_e \frac{\Gamma_{oi}}{4V_e}) - \frac{V_e}{V_i}$$

(13)

In the lower F-region, the typical value for $P_i$ is 4 meters, and for our radar wavelength $kP_i \sim 80$ and $kP_e \sim 0.3$, therefore:

$$\Gamma_{oi} \sim 1/\sqrt{2\pi k_i P_i} << 1$$

$$\Gamma_{oe}/(1 - \Gamma_{oe}) \sim 1/k_i^2 P_i^2$$

Diamagnetic drift threshold for instability ($\psi = 0$),

$$\psi = \frac{\sqrt{V_i V_e}}{\sqrt{1 + 8\frac{2 \mu_e V_i^2}{V_i \mu_i V_e^2} k_i P_i - 1}}$$

(14)

Considering that ions are mainly atomic oxygen and because $V_i/V_e > 1$:

$$8\sqrt{2\pi} \frac{2 \mu_e V_i^2}{V_i \mu_i V_e^2} k_i P_i > 0.056 \frac{V_e}{V_i}$$

when no electric field is present we define $\left(\frac{V_{mi}/V_i}{V_i}\right)^2_{\text{min}} - \left(2 \frac{\mu_e}{m_i} \frac{2\pi}{k_i P_i}\right)^{1/2}$.

Because $V_{mi}/V_i = P_i / \sqrt{2} L_n$, we can define the minimum density scale
length for instability from equation (14). $L_n^0$ and $\tilde{L}_n$ are, respectively, the threshold values for the case where the electric field is zero and of the order of 10 to 15 mV/m. In table 2.2 we consider some diamagnetic drift thresholds and their equivalent density gradient scale-lengths in the region between 140 to 250 km altitude for typical ionospheric parameters. From this table, instability may be excited for density gradient scale-lengths ($L_n$) such that $L_n > L_n^0$ or $L_n > \tilde{L}_n$ at a given altitude.

**TABLE 2.2. Density gradient scale-length thresholds for the collisional universal drift mode ($L_n^0$) and the associated Pedersen mode ($\tilde{L}_n$, $E \approx 10$ mV/m).**

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>$\frac{\nu_e}{\nu_i}$</th>
<th>$\frac{\nu_e\nu_i}{\Omega_i^2}$</th>
<th>$\frac{\nu_e\nu_i}{\Omega_i^2} \frac{V_{ni}}{V_{ni}}$</th>
<th>$L_n^0(m)$</th>
<th>$\frac{\nu_e\nu_i}{\Omega_i^2} \frac{V_{ni}}{V_{ni}}$</th>
<th>$\tilde{L}_n(m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>50</td>
<td>1.4</td>
<td>0.14</td>
<td>20</td>
<td>0.05</td>
<td>56</td>
</tr>
<tr>
<td>150</td>
<td>13</td>
<td>0.5</td>
<td>0.084</td>
<td>34</td>
<td>0.02</td>
<td>140</td>
</tr>
<tr>
<td>200</td>
<td>26</td>
<td>0.044</td>
<td>0.025</td>
<td>113</td>
<td>0.008</td>
<td>354</td>
</tr>
<tr>
<td>250</td>
<td>75</td>
<td>0.013</td>
<td>0.0135</td>
<td>210</td>
<td>0.005</td>
<td>566</td>
</tr>
</tbody>
</table>

Above 140 km for moderate electric field, density gradient scale-lengths of the order of 50 m may excite short wavelength low frequency gradient drift modes.

b. ION-CYCLOTRON-DRIFT (ICD) WAVES.

In this paragraph we review the conditions for the excitation of short wavelength ion-cyclotron waves by the effects of small scale density gradients. This instability has maximum growth rate at $k \rho_e \sim 1$ and wave propagation perpendicular to the magnetic field. As before for $k_{\parallel} = 0$, the plasma functions for both electrons and ions may be expanded in the large argument approximation (equation 1). Also, electrons and ions are magnetized. In this case, however, a moderate electric field is not relevant for instability.
ICD modes are basically non-collisional, but they can still exist when electrons are weakly collisional. Above 200 km and for typical ionospheric densities ($< 10^5$ cm$^{-3}$), the effective electron collision frequency is smaller than the ion gyro-frequency ($\nu_e^0 + \nu_i^0 < \Omega_i$). From equations A1(30) and A1(29.b):

$$\chi_e = -\frac{1}{k^2 \lambda_e^2} \left[ \frac{1}{\omega} - \frac{(\omega - k_i V_i - i \nu_i)}{\omega} \right]$$

(15.a)

$$\chi_i = -\frac{1}{k^2 \lambda_i^2} \left( \frac{1 - \sum \Gamma_m(\mu_i)}{\omega - m \Omega_i + i \nu_i} \right)$$

(15.b)

Because $\omega \sim \Omega_i$, we may have (Kindel and Kennel [1971]),

$$\sum \frac{\Omega_i \Gamma_m(\mu_i)}{\omega - m \Omega_i + i \nu_i} \sim \frac{1 - \Gamma_{oi}}{\mu_i} + \frac{\Omega_i \Gamma_{oi}}{\omega - \Omega_i + i \nu_i}$$

and

$$\chi_i = -\frac{1}{k^2 \lambda_i^2} \left( 1 - \frac{(\omega - k_i V_i)}{\Omega_i} \right) \left[ \frac{1 - \Gamma_{oi}}{\mu_i} + \frac{\Omega_i \Gamma_{oi}}{\omega - \Omega_i + i \nu_i} \right]$$

(16)

Therefore using the dispersion equation for $\omega = \omega_f + i \gamma$, $\gamma < \omega_f$:

$$\left( \omega_f - \Omega_i \right) / \Omega_i = \frac{\Gamma_{ii}}{2(1 + \xi_0 \bar{G})} \sqrt{1 - 4 \frac{\nu_i^2}{\Omega_i^2} \left( 1 + 2 \xi_0 \right)^2}$$

(17)

with,

$$\Gamma_{ii} = (1 - k_i V_i / \Omega_i) \Gamma_{ii}$$

(18.a)

$$\bar{G} = 1 - \left[ \frac{1 - \Gamma_{oi}}{\mu_i} \right] \left( 1 - k_i V_i / \Omega_i \right) - (1 - k_i V_i / \Omega_i) / 2 \xi_0$$
where we are calling \( \tau_{\alpha} = T_{e} / T_{i} \).

Instability growth rate:

\[
\gamma = \frac{T_{i}}{T_{e}} \frac{\Gamma_{i e} \Omega_{i}^{2}}{\Omega_{i}^{2}} \left[ \frac{\left( \Omega_{i} \Gamma_{i e} \right)^{2} + \Omega_{i}^{2}}{\Omega_{i}^{2}} \right] \frac{v_{e}}{\Omega_{i}^{2}} \gamma_{i}.
\]

(19)

For instability \( \gamma > 0 \), and when \( \gamma = 0 \) we find the ion-diamagnetic velocity threshold (maximum density gradient scale-length):

\[
\left( \frac{v_{i}}{\gamma_{i}} \right)_{\text{min}} = \left( 1 - \frac{T_{i}}{T_{e}} \frac{\Gamma_{i e} \Omega_{i}^{2}}{\Omega_{i}^{2}} \right) \frac{v_{e}}{\gamma_{i}} / \sqrt{2 k_{\perp} \rho_{i}}.
\]

(20.a)

Applying relationship (18.c) one can define,

\[
\left( \frac{v_{i}}{\gamma_{i}} \right)_{\text{min}} = \left[ 1 - 2 \frac{T_{i}}{T_{e}} \frac{\Gamma_{i e} \Omega_{i}^{2}}{\Omega_{i}^{2}} \right] / \sqrt{2 k_{\perp} \rho_{i}}.
\]

(20.b)

and

\[
L_{\eta} = \sqrt{2} \left( \frac{v_{i}}{\gamma_{i}} \right) / \rho_{i}.
\]

(20.c)

For instability \( L_{\eta} < L_{\eta}^{0} \). \( L_{\eta}^{0} \) is defined by equations (20.b) and (20.c). Ion-cyclotron waves are destroyed by ion-ion collisions when \( k \rho_{i} \gamma_{i}^{2} / \Omega_{i}^{2} > 1 \). This is always the case for wavelengths such that \( k \rho_{i} > 50 \) and densities greater than \( 10^{14} \) cm\(^{-3}\). In our case \( k \rho_{i} \sim 80 \) (\( \lambda \sim 30 \) cm).
TABLE 2.3. Density gradient scale-length ($L_n^o$) and ion diamagnetic drift ($V_{n,min}$) thresholds for the ICD mode and when $n_e \approx 10^4$ cm$^{-3}$. Dashed lines correspond to the cases where $(V_{n,min})^o < 0$.

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>$\frac{\nu_e \nu_i}{\Omega_i^2}$</th>
<th>$(V_{n,min})^o$</th>
<th>$L_n^o$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.025</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>250</td>
<td>0.003</td>
<td>0</td>
<td>---</td>
</tr>
<tr>
<td>260</td>
<td>$2.5 \times 10^{-3}$</td>
<td>$8.8 \times 10^{-3}$</td>
<td>3000</td>
</tr>
<tr>
<td>280</td>
<td>$10^{-4}$</td>
<td>$5.6 \times 10^{-4}$</td>
<td>500</td>
</tr>
<tr>
<td>300</td>
<td>$5 \times 10^{-5}$</td>
<td>$7.2 \times 10^{-4}$</td>
<td>395</td>
</tr>
</tbody>
</table>

From table 2.3 using typical auroral parameters one can see that 30-cm ICD modes are not expected below 250 km. In the region above 250 km when densities are smaller than $10^4$ cm$^{-3}$, short-wavelength ($<1$ m) ICD waves are probably excited for density gradient scale-lengths of hundred of meters to few km. The auroral ionosphere is highly structured at scale-lengths of hundred of meters.

c.LOWER-HYBRID-DRIFT (LHD) WAVES.

Ion-ion collisions destroy the ion-cyclotron modes, but rather than drive the plasma to stability, they may allow a high frequency ($\omega >> \Omega_i$) unstable mode. LHD, like ICD waves, have maximum growth rate at $k \rho_e \approx 1$ and propagate perpendicular to the magnetic field. Electron collisions stabilize these waves and cause a density gradient threshold (Sperling and Goldman [1980]).

For densities greater than $10^4$ cm$^{-3}$ and wavelengths shorter than 1 meter, ion-ion collisions demagnetize the ions. From equation Al(33), considering only the diamagnetic drift, one finds:
\[ \chi_i = \frac{1}{k^2 \eta_i^2} \frac{1 + \frac{\omega_i z}{k_i v_i} z \left( \frac{\omega_i}{k_i v_i} \right)}{1 + i \frac{\omega_i}{k_i v_i} z \left( \frac{\omega_i}{k_i v_i} \right)} \]  

(21)

where \( \omega_i = \omega - k_i v_i + iv_i \).

For instability \( \omega \sim k_i v_i \gg \Omega_i > v_i \), and the plasma function can be expanded in the small argument approximation,

\[ Z(z) \sim z + i \sqrt{\pi} e^{z^2} \]  

(22.a)

\[ \chi_i = \frac{1}{k^2 \eta_i^2} \left[ 1 + i \sqrt{\pi} \left( \omega - k_i v_i \right) / k_i v_i \right] \]  

(22.b)

As in the previous cases, the electron susceptibility is given by

\[ \chi_e = \frac{1}{k^2 \eta_e^2} \left( \frac{1 - \left( \omega - k_i v_i + iv_e \right) \eta_e / \left( \omega + iv_e \right)}{1 - i \eta_e \eta_e / \left( \omega + iv_e \right)} \right) \]  

(23)

In this case \( \omega > \eta_e \) and,

\[ \chi_e = \frac{1}{k^2 \eta_e^2} \left( 1 - \eta_e \right) - k_i v_i \eta_e / \omega + i k_i v_i \eta_e / \omega^2 \]  

(24)

Finally, using the dispersion relation one can obtain

\[ \omega_r = k_i v_i \eta \left[ 1 + k^2 \eta_i^2 + T_i \left( 1 - \eta \right) / T_e \right] \]  

(25.a)

\[ \frac{\Delta}{\omega_r} = \frac{\left( \delta_r \right) \left( \sqrt{\pi} \left( k_i v_i - \omega_r \right) \right) \eta_e / T_e \omega_r \left( 1 - \eta \right) }{1 + k_i v_i \eta \left[ 1 + k^2 \eta_i^2 + T_i \left( 1 - \eta \right) / T_e \right] \omega_r \left( 1 - \eta \right) } \]  

(25.b)
And the minimum diamagnetic drift velocity for instability,

$$ \left( \frac{V_{\mu i}}{\mu_{\mu i}} \right)_{\text{min}} = \frac{V_e}{\Omega_{\mu i}} \left( \frac{m_e}{m_i} \right)^{1/2} \frac{V_i}{T_i} \frac{1 + k_{\mu i}^2}{1 + \frac{T_i}{T_e} \left( 1 - \Gamma_{\mu i} \right)} \left[ 1 + k_{\mu i}^2 \frac{\Gamma_{\mu i}}{k_{\mu i}^2 + \frac{T_i}{T_e} (1 - \Gamma_{\mu i})} \right]^2 \left[ 1 + k_{\mu i}^2 \right] \left[ 1 + \frac{T_i}{T_e} (1 + k_{\mu i}^2) \right] \left( \frac{2}{1 + k_{\mu i}^2} \right)$$

(26)

where $V_e = V_{en} + V_{ei}$ is the effective electron collision frequency.

In our case $k_{\mu i} \approx 80$, $k_{\mu i} \approx 0.3$, $k_{\mu i}^2 \ll 1$, and when $T_e \approx T_i$ we have

$$ \left( \frac{V_{\mu i}}{\mu_{\mu i}} \right)_{\text{min}} = \frac{V_e}{\Omega_{\mu i}} \frac{1}{2} \left( 1 - \Gamma_{\mu i} \right)^2 \left( 2 \frac{m_e}{m_i} \right)^{1/2} \left( \frac{V_i}{\Omega_{\mu i}} \right)^{1/2} \left( 1 + k_{\mu i}^2 \right) \left( \frac{V_i}{T_i} \right) \left( 1 + k_{\mu i}^2 \right) \left( 1 + \frac{T_i}{T_e} \right) \left( 1 - \Gamma_{\mu i} \right)$$

(27.a)

and

$$ \left( \frac{V_{\mu i}}{\mu_{\mu i}} \right)_{\text{min}} = 2 \left( \frac{m_e}{m_i} \right)^{1/2} \left( \frac{V_i}{\Omega_{\mu i}} \right)^{1/2} \left( \frac{V_i}{T_i} \right) \left( 1 + k_{\mu i}^2 \right) \left( 1 + \frac{T_i}{T_e} \right) \left( 1 - \Gamma_{\mu i} \right)$$

(27.b)

$L_{cr}^{\mu}$ is the critical density gradient scale-length for instability and is defined using equations (20.c) and (27.b).

**Lower hybrid Pedersen drift instability.**

We are now considering the effect of a moderate electric field ($V_E < V_{ei}$) in the generation of LHD modes. From equation A1(36), keeping the Pedersen drift term together with the ion-diamagnetic drift, for the case where $k_{\mu i} \gg 1$ and using the small argument approximation, we find:

$$ \chi_\mu = \frac{1}{k_{\mu i}^2} \left[ 1 + \frac{i}{\sqrt{\pi}} \left( \Omega_{\mu i} - k_{\mu i} (V_{ei} + V_{ei}) / k_{\mu i} V_{ei} \right) \right]$$

(28)

As before, the electron susceptibility may be expressed by equation (24); therefore, using the dispersion relation we find

$$ \omega_r = \frac{k_{\mu i} V_{ei} \Gamma_{\mu i}}{\left[ 1 + k_{\mu i}^2 \right] + \frac{T_i}{T_e} (1 - \Gamma_{\mu i})}$$

(29.a)
And the diamagnetic drift threshold in the case where \( T_e \gg T_i \),

\[
\left( \frac{V_i}{V_i \text{ min}} \right)^{\prime} = \frac{V_i \left( \omega_T \frac{\Omega_i}{k_i n_i V_i} \right)}{4 \omega_T \Omega_i (1 - \Gamma_{oe})} \sqrt{1 + \frac{16}{2 \pi^2 k_i \Omega_i} \left( 1 - \Gamma_{oe} \right) \frac{V_e \Omega_i}{V_i \Omega_i}} - 1
\]

When \( E = 0 \), equation (26) is recovered. Taking an electric field such that \( V_E \sim 0.3 \nu_i \) (\( E \sim 10 \text{ mV/m} \), \( \nu_i \sim 600 \text{ m/s} \)), and for \( k_i \Omega_i \sim 80 \),

\[
\left( \frac{V_i}{V_i \text{ min}} \right)^{\prime} = \frac{V_i \left( \omega_T \frac{\Omega_i}{k_i n_i V_i} \right)}{4 \omega_T \Omega_i (1 - \Gamma_{oe})} \sqrt{1 + 0.01 \frac{V_e \Omega_i}{V_i \Omega_i}} - 1
\]

In table 2.4 we take a model auroral ionosphere with \( T_e \sim T_i \sim T_n \sim 1000^\circ \text{K} \) and atomic oxygen as the main constituent above 150 km.

**TABLE 2.4. Density gradient scale length thresholds for the LHD mode.**

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>( n_e ) (cm(^{-3}))</th>
<th>( \frac{V_i}{\Omega_i} ) (m)</th>
<th>( \frac{V_e}{\Omega_i} ) (m)</th>
<th>( \frac{V_i}{V_i \text{ min}} )</th>
<th>( C_r ) (m)</th>
<th>( \frac{V_i}{V_i \text{ min}} \text{ min} )</th>
<th>( L_r ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>( 5 \times 10^4 )</td>
<td>0.2</td>
<td>2.5</td>
<td>0.16</td>
<td>18</td>
<td>0.055</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>( 10^5 )</td>
<td>3.0</td>
<td>0.17</td>
<td>17</td>
<td>0.065</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>( 10^4 )</td>
<td>0.05</td>
<td>0.6</td>
<td>0.08</td>
<td>35</td>
<td>0.04</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.1</td>
<td>0.1</td>
<td>28</td>
<td>0.066</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>( 3 \times 10^3 )</td>
<td>0.013</td>
<td>0.23</td>
<td>0.05</td>
<td>56</td>
<td>0.037</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.7</td>
<td>0.08</td>
<td>35</td>
<td>0.072</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>
Primed quantities in table 2.4 correspond to the case where a Pedersen drift is present. We see that the effect of a moderate electric field is to lower the altitude of the unstable region for density gradient scale-lengths of the order of 50 m. These density gradients could be the result of short-scale plasma structures (e.g. from precipitations) and/or unstable fluid-like waves.

(C). CURRENT-DRIVEN AND CURRENT-CONVECTIVE ELECTROSTATIC ION CYCLOTRON (EIC) WAVES.

Above 130 km, EIC waves are destabilized by field aligned currents (current driven). A density gradient perpendicular to the magnetic field may relax the conditions for instability (current-convective). Both current-driven and current convective instabilities have maximum growth rate at $k \rho_i \approx 1$ and for wave propagation slightly off-perpendicular. Ion-cyclotron harmonics are destroyed by ion-neutral collisions below 120 km altitude $(\nu_i > \Omega_i)$ and by ion-ion collisions above this altitude when $k^2 \rho_i^2 v_i / \Omega_i > 1$ (Huba and Ossakow [1978]). At our observation wavelength $k \rho_i \approx 80$ and this relationship is always true for densities greater than $10^4$ cm$^{-3}$.

Because $k \parallel \neq 0$, the electron susceptibility can be studied in the two limits, the large argument (strongly collisional electrons) and the small argument (weakly collisional electrons) approximations corresponding, respectively, to the E-region and the lower F-region (Satyarayana et al. [1985]).

Also, for instability, the ion mean free path must be much greater than the
parallel wavelength in order to avoid the neutralization of the perpendicular charge separation (associated with the EIC wave) by electrons flowing freely parallel to the magnetic field. This condition justifies the large argument approximation for the ion susceptibility and imposes a maximum limit for the parallel wavenumbers when the perpendicular wavelength is fixed:

\[
\frac{\lambda_i}{k_{\|}} > 1 \quad \text{or} \quad \frac{k_{\|}}{k_{\perp}} \ll \frac{\nu_i}{\nu_i} \sqrt{2} k_{\perp} x_i
\]  

(32)

From equation (32) the consideration of ion-neutral collisions imposes a maximum parallel wavenumber for instability:

\[
\left( \frac{k_{\|}}{k_{\perp}} \right)_{\text{max}} = \frac{\nu_i}{\nu_i} \sqrt{2} k_{\perp} x_i
\]  

(33)

For typical auroral conditions and for a perpendicular wavelength of 30 cm, \( (k_{\|}/k_{\perp})_{\text{max}} \leq 4 \times 10^{-3} \) above 130 km.

Ion susceptibility may, as in the case of ICD waves, be expressed by the following equation,

\[
\chi_i = \frac{1}{k^2 \rho_i^2} \left[ 1 - \left( 1 - \frac{k_{\|} \nu_i}{\nu_i} \right) \left( \frac{1 - \nu_i}{\nu_i} + \frac{\omega - \nu_i + i v_i}{\omega - \nu_i + i v_i} \right) \right]
\]  

(34)

Electron susceptibility.

1. Strongly collisional case, \( \nu_e >> k_{\|} \nu_e \) or:

\[
\frac{k_{\|}}{k_{\perp}} \ll \frac{\nu_e m_e}{\nu_i m_i} \sqrt{2} k_{\perp} x_e = \left( \frac{k_{\|}}{k_{\perp}} \right)_{\text{max}}^C
\]  

(35)

\( \left( \frac{k_{\|}}{k_{\perp}} \right)_{\text{max}}^C \) is the maximum parallel wavenumber for collisional electrons. This
ratio is smaller than \( \frac{k_\parallel}{k_\perp} \) for typical ionospheric conditions.

From equation Al(30) we can find

\[
\chi_e = \frac{1}{1 - 2i \nu_e \left( \frac{\omega - k_\parallel V_\parallel - k_\perp V_\perp}{k_\parallel \nu_e} \right) / \left( k_\parallel^2 \nu_e^2 \left( 1 + \frac{k_\perp^2 \nu_e^2}{k_\parallel^2 \Omega_e^2} \right) \right)}
\]

(36)

\( \nu_e \) is the effective electron collision frequency and for \( k \rho_i >> 1 \), \( k_\perp^2 \nu_e^2 / k_\parallel \Omega_e^2 \) << 1.

2. Non-collisional electrons, \( \nu_e << k_\parallel \nu_e \) or equivalently

\[
k_\parallel / k_\perp >> \frac{\nu_e}{\sqrt{2} k_\parallel \Omega_e} = \left( \frac{k_\parallel}{k_\perp} \right) \frac{c}{c_{\text{min}}} = \left( \frac{k_\parallel}{k_\perp} \right) \frac{c}{c_{\text{max}}}
\]

(37)

and,

\[
\chi_e = \frac{1}{k_\parallel^2 \nu_e^2} \left[ 1 + i \frac{k_\parallel}{k_\perp} \nu_e \left( \frac{\omega - k_\parallel V_\parallel - k_\perp V_\perp}{k_\parallel \nu_e} \right) / k_\parallel \nu_e \right]
\]

(38)

a. CURRENT-DRIVEN EIC WAVES.

In this case we are using the previous equations (34), (36), and (38) but considering only the parallel electron drift as the driving mechanism. Unstable waves have maximum growth rate at \( k \rho_i \) and propagate nearly perpendicular to the magnetic field in all directions.

a.1. Strongly collisional electrons.

Dispersion relation, \( \omega = \omega_r + i \gamma \), \( \gamma << \omega_r \):

\[
[1 - 2i \nu_e \left( \frac{\omega - k_\parallel V_\parallel}{k_\parallel \nu_e} \right) / k_\parallel^2 \nu_e^2 - 1] + \frac{T_e}{T_\parallel} \left[ 1 - \left( \frac{1 - \Gamma_i}{\mu_i} + \frac{\nu_i \Gamma_i}{\nu_e (\omega - \Gamma_i)} \right) \right] = 0
\]
real part

\[
\frac{\omega_r}{\Omega_i} = 1 + \frac{\tau_A \Gamma_i}{2(1 + 2G)} \left[ 1 + \sqrt{1 - 4\frac{v_i^2}{v_e^2} (1 + 2G) \frac{2}{\Omega_i^2} \frac{2}{\Omega_i^2} \frac{2}{\Omega_i^2}} \right]^{-1}
\]

imaginary part

\[
\frac{V}{\omega_r} = 2 \frac{v_e}{k_{\parallel} v_e \tau_A} \frac{(\Gamma_i \frac{v_e}{v_i} \omega_r - \omega_r)^2 + v_i^2}{\frac{\Gamma_i^2}{2} \frac{v_e^2}{v_i} \tau_A} - \frac{v_i}{\Omega_i}
\]

we are calling \( \tau_A = T_e / T_i \) and \( G = 1 - (1 - \Omega_{oi}) / \mu_i \).

From (39.b), the ion-cyclotron mode is only possible if the term inside the square root is bigger than or equal to zero; therefore

\[
\frac{v_i}{\Omega_i} \leq \frac{\tau_A \Gamma_i}{2} \left( 1 + 2G \right) \leq \frac{(\omega_r - \Omega_i^2) / \Omega_i^2}
\]

(40)

Instability occurs when \( \gamma > 0 \); for \( \gamma = 0 \) we will call \( V_{d_{\text{min}}} \) to the parallel velocity threshold, defined by:

\[
V_{d_{\text{min}}} = \frac{\omega_r}{k_{\parallel}} \left\{ 1 + \frac{\Gamma_i}{2} \frac{v_e^2}{v_i^2} \frac{\Omega_i^2}{\Omega_i^2} \right\}
\]

(41)

For \( \kappa_\parallel \gg 1 \) and \( T_e \sim T_i \), we have the approximation,

\[
V_{d_{\text{min}}} = \frac{\omega_r}{k_{\parallel}} \left[ 1 + \frac{\Gamma_i v_i^2}{v_e^2} \frac{\Omega_i^2}{\Omega_i^2} \right]
\]

Also, because \( \omega_r - \Omega_i < 2v_i^2 \), we have,

\[
V_{d_{\text{min}}} \geq \frac{\sigma_i}{k_{\parallel}} \left[ 1 + \frac{\Gamma_i}{2} \frac{v_e^2}{v_i^2} \frac{\Omega_i^2}{\Omega_i^2} \right] - V_{d_{\text{min}}}
\]

(42.a)
has its lowest value for \( \left( \frac{k_{\parallel}}{k_{\perp}} \right)_0 \) outside the limit of validity of the large argument approximation. In our case we will define,

\[
\frac{V_{d_{\text{min}}}}{V_{d_{\text{min}}}} = \frac{\Omega_i}{\Omega_{\text{max}} \chi} \left[ 1 + \left( \frac{k_{\parallel}}{k_{\perp}} \right)_0 \right]
\]

(42.b)

In table 2.5 we list the EIC instability threshold in the strongly collisional limit as a function of the altitude for typical auroral conditions. Parameter \( \chi \) is defined from equation (42.b):

\[
\chi = 1 + \frac{k_{\parallel}^2}{k_{\perpendicular}^2}
\]

Under this approximation, in the region between 130 to 150 km altitude, \( O_{2}^+ \) and \( N_{2}O^+ \) 30-cm cyclotron modes are expected to occur for moderate parallel velocity thresholds and almost perpendicular wave propagation. We must, however, accept this result with some caution until a more exact numerical analysis is performed. We consider ambient electron densities smaller than \( 10^4 \) cm\(^{-3} \); for larger values cyclotron motions are destroyed by the ion-ion collisions. Electron collisions destabilize while ion-neutral collisions stabilize the EIC modes (Satyarayana et al. [1985]). Above 150 km the thresholds are too high and instability is not possible.

**TABLE 2.5.** Parallel electron drift threshold \( (V_{d_{\text{min}}}) \) for EIC instability (for collisional electrons and \( n_e \sim 10^4 \) cm\(^{-3} \) ), and \( \Omega_i \sim 150 \) Hz.

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>( \frac{V_e}{\Omega_i} ) (km/s)</th>
<th>( \frac{\left( k_{\parallel}/k_{\perp} \right)<em>0}{\Omega</em>{\text{max}} \chi} )</th>
<th>( \frac{\left( k_{\parallel}/k_{\perp} \right)<em>{\text{max}}}{\Omega</em>{\text{max}}} )</th>
<th>( \alpha_1 )</th>
<th>( \frac{V_{d_{\text{min}}}}{V_{d_{\text{min}}}} ) (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>14.</td>
<td>3.8 10-3</td>
<td>1.8 10-3</td>
<td>4.2</td>
<td>1.62</td>
</tr>
</tbody>
</table>
a.2. Non-collisional electrons.

Dispersion relation:

\[ 1 + \sqrt{1 - \omega_r - \omega_i} \left( \frac{\omega - k \cdot v}{k \cdot v_e} \right) + \left[ 1 - \frac{\omega_i}{\omega - k \cdot v_i} \right] = 0 \]  

(43.a)

Real part, \[ \frac{\omega_r}{\omega_i} = 1 + \frac{\omega_i}{2} \left[ 1 + \sqrt{1 - 4\omega_i^2 (\omega - \omega_i)^2} \right] \]  

(43.b)

Imaginary part, \[ \frac{\omega_i}{\omega_i} = \sqrt{1 - (\omega - \omega_i)^2} \left( \frac{\omega - \omega_i}{\omega_i} + \frac{\omega_i}{\omega - \omega_i} \right) \]  

(43.c)

Parallel velocity threshold:

\[ v_{\text{dmin}} = \frac{\omega_r}{k_{\perp}} \left[ 1 + \frac{k_{\perp} v_{\text{dmin}} \omega_i}{\sqrt{1 - (\omega - \omega_i)^2}} \right] \]  

(44.a)

Using (39.b) and when \( T_e \approx T_i \),

\[ v_{\text{dmin}} = \frac{\omega_r}{k_{\perp}} \left[ 1 + \frac{2 k_{\perp} v_{\text{dmin}}}{\sqrt{1 - (\omega - \omega_i)^2}} \right] \]  

(44.b)

Because \( \omega_r - \omega_i \ll 2 \omega_i \), one can find

\[ v_{\text{dmin}} > \frac{\omega_i}{k_{\perp}} \left[ 1 + \frac{k_{\perp} v_{\text{dmin}}}{k_{\|} \omega_i} \left( \frac{2 m_i}{k_{\|} v_{\text{dmin}}} \right)^{1/2} \right] = v_{\text{dmin}}^0 \]  

(45)
In table 2.6 we list the parallel velocity threshold for EIC waves in the limit of non-collisional electrons. One can see that instability is not possible (thresholds are too strong). The two columns associated with parameters $\Omega_i/k_{\parallel}$, $\alpha_2$ and $v_{d\min}$ are obtained using the maximum (left) and the minimum (right) values of $k_{\parallel}$. We define $\alpha_2$ by equation (45):

$$\alpha_2 = 1 + \frac{k_{\parallel} \Omega_i}{v_{\parallel}} \left( \frac{2m_i}{\Omega_i} \right)^{1/2}$$

In this table we list the parallel velocity threshold for EIC waves in the limit of non-collisional electrons. One can see that instability is not possible (thresholds are too high). The reason is that even if the ion-ion collisions are not important for densities smaller than $10^4$ cm$^{-3}$, ion-neutral collisions limit the generation of 30-cm waves to almost perpendicular propagation (equation 33), increasing the instability threshold ($k_{\parallel} \rightarrow 0$, $v_{d\parallel} \Omega_i/k_{\parallel} \rightarrow \infty$). Above 300-350 km plasma is non-collisional and short wavelength EIC modes can be generated at much lower thresholds (Kindel and Kennel [1971]); in this case $\alpha_2 = 1$, $(k_{\parallel}/k_{\perp})_{\max} \sim 1/\sqrt{2}$, $(k_{\parallel}/k_{\perp})_{\min} = 0.01$, $(k_{\parallel}/k_{\perp})_{\min} = 0$ and $\Omega_i/k_{\parallel} > 3$ km/s for atomic oxygen ($O^{+}$).

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>$(k_{\parallel}/k_{\perp})_{\max}$</th>
<th>$(k_{\parallel}/k_{\perp})_{\min}$</th>
<th>$\Omega_i/k_{\parallel}$</th>
<th>$v_{d\min}$</th>
<th>$\alpha_2$</th>
<th>$\Omega_i/k_{\parallel}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>3.3 10^{-3}</td>
<td>1.8 10^{-3}</td>
<td>2.5 10^{-3}</td>
<td>37 21</td>
<td>93 95</td>
<td>NO$^+$</td>
</tr>
<tr>
<td>140</td>
<td>1.5 10^{-3}</td>
<td>1.1 10^{-3}</td>
<td>5. 7.</td>
<td>18 13</td>
<td>90 91</td>
<td>O$^+_2$</td>
</tr>
<tr>
<td>150</td>
<td>1.8 10^{-3}</td>
<td>2.5 10^{-4}</td>
<td>8. 60.</td>
<td>21 3</td>
<td>160 240</td>
<td>O$^+_2$</td>
</tr>
</tbody>
</table>
b. CURRENT-CONVECTIVE EIC WAVES.

A small scale density gradient perpendicular to the magnetic field may help to excite the short-wavelength EIC modes for moderate field-aligned currents (Yamada and Hendel [1978]). In this case, however, wave propagation is parallel to the diamagnetic drift of the electrons for outgoing currents, or of the ions for incoming currents.

The following results provide, as in the case of current driven EIC modes, only rough estimates of instability thresholds at 30-cm wavelength. More accurate and reliable results have to be obtained by numerical analysis of the dispersion relation and for ionospheric models with different magnetic activity conditions.

As in the previous derivations we start with equations (34), (36) and (38), but keep the diamagnetic drift terms.

b.1. Strongly collisional electrons.

From the dispersion relation, we can find the frequency and growth rate of the unstable modes,

\[ \frac{\omega - \omega_i}{\Omega_i} = 1 + z_n \frac{\Gamma_i}{2} [ 1 + \sqrt{1 - 4 \nu_i^2 (1 + \overline{G})^2 / \beta_i^2 \Sigma_i^2} ] / (1 + \overline{G}) \]  

(46)

As in the case of the ICD modes \( \overline{\Gamma_i} \) and \( \overline{G} \) are defined by,

\[ \overline{\Gamma_i} = [ 1 - k_i \nu_i / \Sigma_i ] \Gamma_i \]  

(47.a)
we also have 
\[ \frac{\nu_i}{\Omega_i} \leq \left| \frac{\tau_A \sqrt{\nu_i}}{2(1 + \xi G)} \right| \] 
(47.c)

and 
\[ \Omega_r - \Omega_i \leq 2 \nu_i \]

Instability growth rate:

\[ \frac{\gamma}{\sigma_i} = 2 \frac{\nu_e}{k_{ll} V_e^2} \left( k_{ll} V_i + k_{ll} \nu_e - \Omega_r \right) \left[ \left( \Omega_r - \Omega_i \right)^2 + \nu_i^2 \right] / \tau_A \sqrt{\nu_i} \]
(48)

For \( \gamma = 0 \), the minimum parallel velocity threshold is given by,

\[ V_{d_{min}} = \frac{\Omega_r}{k_{ll}} \left[ 1 + \frac{1}{2} \left( 1 - k_{ll} \nu_i / \Omega_i \right) \frac{\tau_A \sqrt{\nu_i} k^2_{ll} V_e^2}{\nu_e \left[ \left( \Omega_r - \Omega_i \right)^2 + V_e^2 \right]} - k_{ll} \nu_i / \Omega_i \right] \] 
(49.a)

Considering that \( (\Omega_r - \Omega_i)^2 + \nu_i^2 = \Omega_i \tau_A \sqrt{\nu_i} (\Omega_r - \Omega_i)/(1 + \xi G) \), and for \( T_e \neq T_i \)

\[ V_{d_{min}} = \frac{\Omega_r}{k_{ll}} \left[ 1 - k_{ll} \nu_i / \Omega_i + k_{ll}^2 \nu_e^2 \sqrt{\nu_i} \Omega_i (\Omega_r - \Omega_i) \right] \] 
(49.b)

Also, because \( \Omega_r - \Omega_i \leq 2 \nu_i \),

\[ V_{d_{min}} \geq \frac{\Omega_i}{k_{ll}} \left[ 1 - k_{ll} \nu_i / \Omega_i + \frac{k_{ll}^2 \nu_i^2}{k_{ll}^2} \frac{m_i \nu_i^2 k_{ll} \Omega_i^2}{2m_e \nu_e} \right] \] 
(50)

\( V_{d_{min}} \) as a function of \( k_{ll} \) has a minimum value at

\[ \left( \frac{k_{ll}}{k_1} \right) = \frac{2m_e \nu_e}{\sqrt{m_i \nu_i^2 k_{ll} \Omega_i^2}} \frac{1 - k_{ll} \nu_i / \Omega_i}{(1 - k_{ll} \nu_i / \Omega_i)^{1/2}} \] 
(51.a)
As before, the condition $V_e \gg k_\perp \omega_e$ for strongly collisional electrons imposes a maximum limit for the parallel wavenumber,

$$\left( \frac{k_\parallel}{k_\perp} \right)_{\text{max}} = \frac{V_e}{\sqrt{2} k_\perp \omega_e} < \left( \frac{k_\parallel}{k_\perp} \right)_{\text{0}} (1 - k_\perp \omega_e / \Omega_i) \frac{1}{2}$$  \hspace{1cm} (51.b)

Equation (51.b) defines an upper limit for the density gradient scale-length ($L_n^\text{max}$):

$$\frac{k_\perp \omega_e / \Omega_i}{\Omega_i} \geq \left[ \left( \frac{k_\parallel}{k_\perp} \right)^2_0 - \left( \frac{k_\parallel}{k_\perp} \right)^2_{\text{max}} \right] / (k_\parallel / k_\perp)^2 = \frac{k_\perp \omega_e / \Omega_i}{\Omega_i} \min$$  \hspace{1cm} (52.a)

$$L_n^\text{max} = \frac{\Omega_i}{k_\parallel \omega_e / \Omega_i \min}$$  \hspace{1cm} (52.b)

For $O^+$ ions under typical auroral conditions, $\Omega_i = 300$ hz and $\rho_i \approx 4$ m.

Finally, we can define the minimum parallel drift threshold:

$$V_\parallel^\text{0} = \frac{2 \Omega_i}{k_\parallel \omega_e / \Omega_i \min} \left[ 1 - \left( k_\perp \omega_e / \Omega_i \right) \right] = \frac{2 \Omega_i}{k_\parallel \omega_e / \Omega_i \min}$$  \hspace{1cm} (53)

In table 2.7 we list the parallel velocity thresholds for EIC modes in the presence of density gradients with scale-lengths smaller than $L_n^\text{max}$. If we compare with table 2.5, instability is now possible for higher altitudes ($< 200$ km) and lower parallel velocity thresholds.

Equation (47.c), and its equivalent in the non-collisional electrons limit, imposes an upper limit to the diamagnetic drift or, correspondingly, a minimum value for the density gradient scale-length:

$$\left( \frac{V_\perp \omega_e / \Omega_i}{\Omega_i} \right)_{\text{max}} = \left( 1 + 4 \sqrt{2 \pi} k_\perp \rho_i \frac{V_}\parallel / \Omega_i }{\sqrt{2} k_\perp \rho_i} = \frac{\rho_i}{\sqrt{2} L_n^\text{min}}$$  \hspace{1cm} (54)
### TABLE 2.7. Thresholds for the current-convective EIC waves at $\Omega_e \approx 300 \text{ Hz}$, for collisional electrons and $n_e \approx 10^{4} \text{ cm}^{-3}$.

| Altitude (km) | $(k_{||}/k_{\perp})_{\max}^c$ | $(k_{||}/k_{\perp})_o$ | $L_{\min}$ (m) | $L_{\max}$ (m) | $(-\Omega_e/k_{||})_o$ | $v_{\text{dmin}}^c$ - $2(-\Omega_e/k_{||})_o$ | $v_{\text{dmin}}^c$ (km/s) | $v_{\text{dmin}}^c$ (km/s) |
|--------------|-----------------|-----------------|--------------|--------------|-----------------|------------------|-----------------|-----------------|
| 130          | 1.8 10-3        | 3.8 10-3        | 1.1          | 416          | 4               | 8                |                 |                 |
| 140          | 1.1 10-3        | 2.8 10-3        | 2.3          | 377          | 5               | 10               |                 |                 |
| 150          | 2.5 10-4        | 1.6 10-3        | 2.0          | 333          | 10              | 20               |                 |                 |
| 200          | 5.0 10-5        | 7.5 10-4        | 9.5          | 323          | 20              | 40               |                 |                 |
| 250          | 2.0 10-5        | 4.8 10-4        | 27           | 320          | 32              | 64               |                 |                 |
| 300          | 1.1 10-5        | 3.8 10-4        | 64           | 320          | 40              | 80               |                 |                 |
| 350          | 6.5 10-6        | 2.8 10-4        | 121          | 320          | 54              | 108              |                 |                 |

#### b.2. Non-collisional electrons.

The dispersion relation in this limit is obtained from the one in the current-driven case (a.2) replacing $\Gamma_{\perp i}^c$, $G$ by $\Gamma_{\perp i}^c$, $G$ defined in (47.a) and (47.b).

Taking the real part,

$$\frac{\omega_r}{\Omega_e} = \frac{1}{2(1 + \eta_{\perp i}^c)} \left[ 1 + \sqrt{1 - 4 \left( 1 + \frac{\eta_{\perp i}^c}{\Omega_e} \right)^2 \xi_{\| i}^c \xi_{\perp i}^c \xi_{\perp i}^c} \right]$$

From the imaginary part we can find the instability growth rate,

$$\gamma = \frac{\sqrt{\pi}}{\Omega_e} \left( k_{\| i} V_d + k_{\perp i} V_{\text{ne}} - \omega_r \right) \left[ (\omega_r - \Omega_e)^2 + \xi_{\perp i}^2 \right]^{1/2} \xi_{\perp i}^2$$

And the parallel velocity threshold,

$$v_{\text{dmin}}^c = \frac{\omega_r}{k_{\| i}} \left( 1 - k_{\perp i} V_{\text{ne}} / \Omega_e \right) + \frac{1}{\sqrt{\pi}} \left( 1 - k_{\perp i} V_{\text{ne}} / \Omega_e \right) \frac{\eta_{\perp i}^c}{(\omega_r - \Omega_e)^2 + \xi_{\perp i}^2}$$
Because \( \omega - \Omega_i < 2V_i \), when \( T_e T_i \) one can have

\[
V_{\text{min}} > \frac{\Omega_i}{k_{||}} \left[ 1 - k_{\perp} \sqrt{\nu_{\perp} \rho_i} + \frac{k_{||}}{k_{\perp}} \left( \frac{2m_i}{m_e} \right) \frac{\nu_{\parallel}}{\nu_{\perp}} \rho_i \right] = V_{\text{min}}^0
\]

(58)

As before \( (k_{||}/k_{\perp})_{\text{max}} = \frac{\nu_{\perp}}{\sqrt{2} \rho_i} \), \( (k_{||}/k_{\perp})_{\text{min}} = \frac{\nu_e}{\sqrt{2} \rho_i} \), and if we define

\[
k_{||0} = \frac{(k_{||\text{max}} + k_{||\text{min}})}{2}
\]

(59.a)

and

\[
V_{\text{min}}^0 = V_{\text{min}}^0 \frac{\alpha_3 \Omega_i}{k_{||0}}
\]

(59.b)

where,

\[
\alpha_3 = 1 - k_{\perp} \frac{\nu_{\perp}}{\Omega_i} + k_{||0} \rho_i \left( \frac{2 \sqrt{m_i \mu_0}}{m_e} \right)^{\frac{1}{2}}
\]

In our conditions for values of \( L_n \) ranging from 25 m to 200 m, \( k_{\perp} \nu_{\perp}/\Omega_i \) varies between 13 to 1.5. In table 2.8 we present the parallel velocity threshold \( V_{\text{min}}^0 \) for different density gradient scale-lengths as a function of altitude. Contrary to the current-driven case, incoming and outgoing field aligned currents together with small-scale density gradients may now excite 30 cm EIC waves. When wave propagation is parallel to the electron diamagnetic drift, \( k_{\perp} \nu_{\perp} \) is positive, and for outgoing currents, \( k_{\perp} V_d > 0 \). EIC modes can't exist for \( k_{\perp} \nu_{\perp} = \Omega_i (V_d = 0) \), because in this case the only source of free energy is the density gradient, and ICD modes are stabilized for \( k_{||} > 0 \).
TABLE 2.8. Threshold for the current-convective EIC instability in the non-collisional electrons limit (for \( n_e \sim 10^4 \text{ cm}^{-3} \)). Both incoming (-) and outgoing (+) currents may destabilize the plasma. The two columns under listed parameters give their values for \( k_{\text{max}} \) (left) and \( k_{\text{min}} \) (right).

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>( \frac{k_H}{k \perp} ) max</th>
<th>( \frac{k_H}{k \perp} ) min</th>
<th>( \frac{\Omega_i}{k \perp} )</th>
<th>( \frac{\Omega_i}{k \perp} ) wmax</th>
<th>( \Omega_i )</th>
<th>( L_i )</th>
<th>( \Delta )</th>
<th>( v_{\text{min}}^0 - \Delta \frac{\Omega_i}{k \perp} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>3.3 10^{-3}</td>
<td>1.8 10^{-3}</td>
<td>2.5 4.5</td>
<td>3</td>
<td>25</td>
<td>17</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>1.5 10^{-3}</td>
<td>1.1 10^{-3}</td>
<td>5 7</td>
<td>6</td>
<td>25</td>
<td>3</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>1.8 10^{-3}</td>
<td>2.5 10^{-4}</td>
<td>8 60</td>
<td>15</td>
<td>25 50</td>
<td>-0.3, 6.0</td>
<td>-4.5, 90</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>3.6 10^{-4}</td>
<td>5.0 10^{-5}</td>
<td>42 300</td>
<td>73</td>
<td>50 100</td>
<td>-2.4, 0.8</td>
<td>-175, 58</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>1.2 10^{-4}</td>
<td>2.0 10^{-5}</td>
<td>125 750</td>
<td>215</td>
<td>100 200</td>
<td>-1.4, 0.2</td>
<td>-300, 43</td>
<td></td>
</tr>
</tbody>
</table>

(D). POST-ROSENBLUTH (OTT-FARLEY) INSTABILITY.

Above the electrojet region (\( > 130 \text{ km} \)) the competing effects of the weak ion-neutral collisions and the ambient electric field drive the ion distribution towards a non-Maxwellian, marginally stable configuration (toroidal, 'loss-cone' type). A strong electric field (\( > 50 \text{ mv/m} \)) can destabilize the ion distribution generating a Post-Rosenbluth instability (Post and Rosenbluth [1962], Ott and Farley [1975], St.Maurice [1978]) with frequencies of the order of the lower hybrid frequency (\( \omega \sqrt{\frac{\Omega_i}{\Delta \Omega_i}} \gg \Omega_i \)). This instability has its maximum growth rate at \( k_{\text{r}} \perp e \perp 1 \) for wave propagation slightly off-perpendicular: \( k_{\text{r}}^2 / k_{\perp}^2 \omega e / m_i \).

In these conditions, \( \omega \gg v_e \) and \( \omega \gg k_{\perp} v_e \), and the electron susceptibility of equation A.1(30) may be expanded in the large argument approximation:

\[
\chi_e = \frac{1}{k_{\perp}^2} \left[ \chi_{\text{e}} - \frac{k_{\perp}^2 v_e^2}{(\omega + i v_e)^2} \right] \left[ 1 - i \frac{\gamma e \frac{\Delta \Omega_i}{\Delta \Omega_i}}{(\omega + i v_e)} \right]
\] 

\( (60. a) \)
Using equation Al(43) for the ion susceptibility, one can find the dispersion relation (Saadeggev and Galeev [1969]):

\[ 1 - k_{\|}^2 \omega_{pe}^2 / k_{\perp}^2 \omega_{ci}^2 + \frac{\omega_{pi}^2}{k_{\perp}^2 \omega_{ci}^2} \left( f_{i\perp}^{\tau}(0) + F(w) \right) = 0 \] (61)

and,

\[ \frac{\lambda}{\omega_r} = - \frac{\omega_r}{2} \left[ k_{\perp}^2 \omega_{ci}^2 + f_{i\perp}^{\tau}(0) + F_R(w) \right] - v_e / \omega_r \] (62.b)

Where \( v_n \) is the neutral thermal speed and, \( F_R(w) \), \( F_I(w) \) are the real and imaginary parts of the function \( F(w) \) defined by equations Al(42.a), Al(42.b). \( \lambda > 0 \) for instability or, equivalently,

\[ F_I(w) \leq - \frac{\sqrt{2} \omega_{pi}}{\omega_{pe} k_{\perp} \omega_{ci}^2} \] (63.a)

Replacing \( F_I(w) \) by the expression given by equation Al(42.b), we have

\[ g(x) = \left[ (2x - 1) I_0(x) - 2x I_1(x) \right] \exp(-x) > \text{Ai} \] (63.b)

Where \( x = \frac{\nu_n^2}{2} \), and \( \text{Ai} = 16 \left( \frac{2m_i}{m_e} \right)^{\frac{3}{2}} \left( \frac{k_{\perp} \sigma_i}{\omega_{pi}} \right)^{\frac{3}{2}} \frac{\nu_e}{\nu_i} \)

For instability \( \omega_r \leq \left( \frac{\nu_i}{m_i} \right)^{\frac{1}{2}} \sigma_i \) and \( k_{\perp} / k_{\|} \ll \left( \frac{m_e}{m_i} \right)^{\frac{1}{2}} \), therefore

\[ \text{Ai} > 16 \left( \frac{2m_i}{m_e} \right)^{\frac{3}{2}} \left( \frac{k_{\perp} \sigma_i}{\omega_{pi}} \right)^{\frac{3}{2}} \frac{\nu_e}{\nu_i} \] (64)
Considering atomic oxygen ions: \( A_i = 0.056(k_F \gamma) \frac{3 v_e}{\Omega_A} \frac{3/2}{n_o} \). \( n_o \) is the ambient electron density measured in \( \text{cm}^{-3} \). Finally, the electric field threshold for 30-cm instabilities may be found from the equation (where \( V_E = E/B \))

\[
g\left( \frac{V_E^2}{2}, \frac{v_n^2}{n_o^2} \right) = 3 \times 10^4 \frac{v_e}{\Omega_A} \frac{3/2}{n_o^3} - A_0
\]

In table 2.9 we list the values of \( g(x) \) for a few altitudes and typical auroral conditions. Because \( g(x = 0.81) = 0 \) and \( g(x = 1) = 0.92 \), electron collisions are not important; the ExB drift threshold is given by \( V_E \gtrsim 1.8 v_n \) (\( x \gtrsim 0.81 \)), or an electric field \( E \gtrsim 50 \text{ mV/m} \) for a neutral temperature of 1000 K. Our calculations are basically the same as Ott and Farley [1975] but including a BGK model for the electron neutral collisions. Under these conditions, instability may be possible from 130 to 300 km altitude where ion collisions are weak but dominant. More realistic calculations show, however, a higher threshold electric field (80 mV/m or greater, see St.Maurice [1978]).

**TABLE 2.9.** Post-Rosenbluth instability is excited for \( x \) such \( g(x) > A_0 \) (corresponding to \( V_E > 1.8 v_n \) or \( E > 50 \text{ mV/m} \) for \( O^+ \) and \( T_n \sim 1000 \text{ K} \)).

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>( \frac{v_e}{\Omega_A} )</th>
<th>( A_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>20.00</td>
<td>0.60</td>
</tr>
<tr>
<td>140</td>
<td>5.60</td>
<td>0.17</td>
</tr>
<tr>
<td>150</td>
<td>2.41</td>
<td>0.07</td>
</tr>
<tr>
<td>200</td>
<td>0.55</td>
<td>1.6 10-2</td>
</tr>
<tr>
<td>250</td>
<td>0.17</td>
<td>5.1 10-3</td>
</tr>
<tr>
<td>300</td>
<td>0.06</td>
<td>1.8 10-3</td>
</tr>
</tbody>
</table>
3. SATURATION SPECTRUM OF LINEAR KINETIC INSTABILITY.

We call \( < |E_{\mathbf{k}}|^2 > \) the power spectrum of the electric field fluctuations with wave vector \( \mathbf{k} \) at any given frequency \( \omega \). The power spectrum of a turbulent plasma may contain one or several peak structures in the frequency domain for a fixed wave vector \( \mathbf{k} \); each unstable mode is characterized by its spectral strength \( < |E_{\mathbf{k}}|^2 > \), the central frequency \( \omega_{c_k} \), and the spectral width \( \Delta \omega_{c_k} \).

The saturation spectrum in a homogeneous and stationary turbulent state is assumed to be Lorentzian (Ichimaru [1975]) which spectral width defines the instability relaxation time. In a weak-to-moderate turbulent state, kinetic instabilities saturate by the 'turbulent collisions' mechanism, and their saturation spectra can be calculated using the renormalized quasilinear theory, in the 'orbit-diffusion' approximation, reviewed in appendix 2.

In both the standard and renormalized quasilinear theories, stabilization is achieved when \( \gamma_k(t_s \gg \tau_D) = 0 \). \( \tau_D \) is the diffusion time.

Following the framework of appendix 2, the fluctuating electric field may be redefined by

\[
E_{\mathbf{k}} = E_{\mathbf{k}}(0) e^{-i \left[ \omega_{c_k} + i \left( \gamma_{c_k} + \Delta \omega_{c_k} \right) \right] t} \tag{66}
\]

\( t \) is called the faster time scale, and all the quantities \( \omega_{c_k}, \gamma_{c_k}, \Delta \omega_{c_k} \) and are functions of the slow time \( t_s \) alone. Therefore, the associated power spectrum must be expressed as

\[
< |E_{\mathbf{k}}|^2 > = \lim_{t_s \to \infty} \text{Re} \left\{ \int_0^{t_s} \int_0^{2\pi} \left< E_{\mathbf{k}}(t, \omega), E_{\mathbf{k}}(t') \right> e^{i \omega t'} d\omega \right\} \tag{67.a}
\]

where \( \tau = t - t' \) and \( E_{\mathbf{k}} = -i k \Phi_{\mathbf{k}} \).
Then,
\[
\langle |E_{\mathbf{k}}|^2 \rangle = \frac{\langle |E_{\mathbf{k}}|^2 \rangle}{(\omega - \omega_{\mathbf{k}})^2 + \Delta \omega_{\mathbf{k}}^2} 2 \Delta \omega_{\mathbf{k}}
\]  
(67.b)
and:
\[
\langle |E_{\mathbf{k}}|^2 \rangle = \frac{1}{2\gamma} \int_{-\infty}^{\infty} \langle |E_{\mathbf{k}}|^2 \rangle d\omega
\]  
(67.c)

Now the angular brackets also include an average over velocity space and an average over some space-time domain to allow comparison between theory and observations.

To obtain equation (67.b) we have used the fact that at saturation: \( \nu_{\mathbf{k}}(t_s \to \infty) = 0 \) and \( \Delta \omega_{\mathbf{k}} = \Delta \omega_{\mathbf{k}}(t_s \to \infty) \). \( \Delta \omega_{\mathbf{k}} \) is the average of the frequency broadening over the velocity space. The factor two in equations (67.a) and (67.b) accounts for the spectral symmetry between positive and negative frequencies.

The spectral strength \( \langle |E_{\mathbf{k}}|^2 \rangle \) and the autocorrelation function \( \langle E_{\mathbf{k}}(t) E_{\mathbf{k}}(t') \rangle \) are independent of time \( t \) by virtue of the stationarity of fluctuations on the time scale of the measurements. From equation (67.b) we can see that the saturation spectrum of the unstable mode \( \omega_{\mathbf{k}} \) is Lorentzian with half power width equal to \( \Delta \omega_{\mathbf{k}}/2\gamma \) and spectral peak \( \langle |E_{\mathbf{k}}|^2 \rangle \).

The reciprocal value of \( \Delta \omega_{\mathbf{k}}/2\gamma \) is called the correlation time of the turbulence at wavevector \( \mathbf{k} \). In a weak-to-moderate turbulent state, this time is equal to the diffusion time \( \tau_0 \) (see appendix 2). Because \( \frac{\tau_0}{\nu_{\mathbf{k}}} \geq \frac{2\pi}{\omega_{\mathbf{k}}^2} \) and \( \frac{\tau_0}{\nu_{\mathbf{k}}} \geq \frac{\Delta \omega_{\mathbf{k}}}{\omega_{\mathbf{k}}} \) spectral widths such that \( \Delta \omega_{\mathbf{k}} \leq \omega_{\mathbf{k}} \) may still characterize moderate turbulent states. Parameter \( \nu_{\mathbf{k}} \) may be estimated from the ratio between the energy density associated with the spectral peak and the plasma kinetic energy. A value of smaller than unity together with a narrow spectrum \( \Delta \omega_{\mathbf{k}} < \omega_{\mathbf{k}} \) characterize weak-to-moderate turbulence. Values \( \nu_{\mathbf{k}} > 1 \) characterize the strongly turbulent...
regime. Short wavelength ( \( k j > 1 \) ) instability-type spectra, with a broader width ( \( \omega k > \omega k \) ), are probably nonlinearly generated by mode coupling from the turbulent relaxation of the longer wavelength linearly unstable modes (Sudan and Keskinen [1979]).

4. WAVE-AMPLITUDE ESTIMATES (Instability absolute cross-sections).

Using the results from appendix 2 we may estimate the saturation amplitude of the linear kinetic instabilities reviewed before.

The nonlinearity in the dynamics of the non-resonant population (j) defines the resonance broadening and (equations (A2.38.a,b)):

\[
\Delta \omega_{k j} = \sum_{k'} k^2 \langle |\psi_{k'}|^2 \rangle \cos^2(\theta - \theta') \sum_{n=-\infty}^{\infty} \text{Im} \frac{g_{nj}^0}{F_{nj}} \tag{68.a}
\]

The overhead bars represent the average in the velocity space and \( v_{kj} - E_{kj}/B \), \( \cos(\theta - \theta') = k \cdot k' / k k' \).

For almost perpendicular wave propagation:

\[
\text{Im} \frac{g_{nj}^0}{F_{nj}} = \frac{\gamma_{kj} + \Delta \omega_{k j} + d c}{(\omega k - \omega_{kj} - \gamma_{d j} - n \gamma_j)^2 + (\omega k + \Delta \omega_{k j} + d c)^2} \tag{68.b}
\]

and

\[
d c \sim k_j^2 |P_j|^2 \gamma_j, \quad \Delta \omega_{k j} \sim k_j^2 |P_j|^2 \gamma_j
\]

\( \gamma_j \) is the 'anomalous' collision frequency and is independent of the wavenumber.

At saturation ( \( \gamma_k = 0 \) ), and from equations (A2.50.a,b) and (A2.51) (Dum and Dupree [1970], Gary [1980], Gary and Sanderson [1981])

\[
\langle |\xi_{kj}|^2 \rangle < \frac{B^2}{k_j^2 A_j E_A} \tag{69.a}
\]
where
\[
\bar{A}_{j,k_A} = \sum_{n=-\infty}^{\infty} \frac{\overline{F}_{n,j}(\nabla_{nj})}{(\omega_{nj} - \omega_{nk}, V_{d,j} - n\Omega_j)^2} \frac{k_A}{k_{n,j}}\] (69.b)
\[
\overline{F}_{n,j} = \frac{Q}{2} \left[ I_{n+1} + 2 I_n + I_{n-1} \right] \mu_{j} \] (69.c)

Calling \(k_0\) the wavenumber of maximum growth rate, \(k_A = k_0\) if \(k < k_0\) and \(k_A = k\) if \(k > k_0\) (Farley [1985]).

Equation (69.a) gives an estimate of the strength of the fluctuating electric field at wavevector \(k\) and replacing it in equation (67) an estimate of the scattering cross section may be found.

(a) When the electrons are the non-resonant population and \(k_0 \leq 1\), because \(\omega_{nj} \ll \Omega_e\), only the term with \(n = 0\) may be retained in the Bessel sums (68.b, 69.b). The reference frame moves with the ions electric drift, and above the electrojet region \(V_d = 0\). Therefore:
\[
\langle |E_{\omega_{nj}}|^2 \rangle \leq B^2 \Omega^2 \frac{k^2}{k_0^2} \overline{F}_o(\mu_{e0})
\]
and,
\[
\langle \frac{\delta n_{\omega_{nj}}^2}{n_{e0}^2} \rangle \leq \frac{1}{4} \omega_{nj}^2 \left( \frac{1}{k_0} \right)^2 \mu_{e0}^2 \Omega_e^2 \overline{F}_o(\mu_{e0})
\]

(b) When the non-resonant population are the ions and \(k_0 \gg 1\), we have consider two cases: the one corresponding to the universal drift waves \((\omega \ll \Omega_e)\) and the other to the current driven and current convective EIC modes \((\omega \sim n \Omega_e)\). In the first case (as in paragraph a), only the term with \(n = 0\) is retained. In the second case, the resonant harmonic \((\omega \sim n \Omega_e)\) dominates the Bessel sum.

When ion-neutral collisions are weak, the collisional broadening is determined by the ion-ion collisions: \(d_{\omega} = k^2 \overline{P}^2 \chi_{ij}^2\) (Dupree [1968]), and the anomalous ion collision frequency may be defined by \(\Delta \omega_{ij} = k^2 \overline{P}^2 \chi_{ij}^2\).
(A). MODIFIED TWO-STREAM WAVES.

In this instability electrons are the non-resonant population. From the dispersion relation, at saturation (Sudan [1983.a]), \( \omega_k = k_c S \) and

\[
(\omega_k - k_c \nu_d)^2 = (\psi + \psi^*) \frac{k^2_c}{S^2} C_s^2
\]

where \( \psi^* = \frac{\nu_k \nu_i}{S_c S_i} \).

Therefore from equation (68.a) taking \( \cos^2(\theta - \theta') = 1 \) and for

\[
\frac{\nu_k}{\nu_e} = \sum_{k' > k} <|\Psi_{k'}|^2> (1 + \frac{\nu_k}{\nu_e}) \frac{\Gamma_{oe} + \Gamma_{i}}{C_s^2 \psi^* (1 + \frac{\nu_k}{\nu_e})} \mid_{k'}
\]

or

\[
\frac{\nu_k}{\nu_e} = \sum_{k' > k} <|\Psi_{k'}|^2> \frac{\Gamma_{oe} + \Gamma_{i}}{C_s^2 \psi^* (1 + \frac{\nu_k}{\nu_e})} \mid_{k'}
\]

And because \( \frac{\nu_k}{\nu_e} \gg 1 \) we have

\[
\sum_{k' > k} <|\Psi_{k'}|^2> (\Gamma_{oe} + \Gamma_{i}) \mid_{k'} = C_s^2 \psi^* \psi^* \quad (72)
\]

Also, from Poisson's equation

\[
\frac{|\Delta n_{k}|^2}{n_0^2} = \frac{1}{4} \frac{k^2 |\Delta \nu_{k}|^2}{\mu_e^2 \sigma_e^2}
\]

Therefore

\[
\sum_{k' > k} k^2 (\Gamma_{oe} + \Gamma_{i}) \mid_{k'} \frac{|\Delta n_{k}|^2}{n_0^2} = \frac{1}{4} \frac{\Delta \nu_{k}^2}{\mu_e^2 \sigma_e^2} \left( \frac{\nu_i}{C_s} \right)^2
\]

(74)
Assuming that at saturation the maximum growth mode \( k_0 \) dominates the Fourier sum (72) and (74), we have

\[
\Gamma_{pe} + \Gamma_{e} \sim 2/3 \quad \text{and}
\]

\[
\left< \frac{|\delta n_{ke}|^2}{n_0^2} \right> \sim \frac{3}{8} \frac{\Delta \omega_{ke}}{\mu_{eo} s_e^2} \left( \frac{\nu_i}{\omega_{ke}} \right)^2
\]

where \( \mu_{eo} = \langle \frac{k_0^2 \rho_{e}^2}{e} \rangle \).

Furthermore, \( \Delta \omega_{ke} < \omega_{ke} \), then

\[
\max \left< \frac{|\delta n_{ke}|^2}{n_0^2} \right> = \frac{3}{8} \frac{\nu_i^2}{\mu_{eo} s_e^2}
\]

(75.a)

(75.b)

From equation (72) we also have

\[
\nu_i^2 \nu_x^2 \sim \frac{2}{3} \left( \frac{\Omega_{pe}}{c_s} \right)^2 \left< |\delta n_{ke}|^2 \right>
\]

(76.a)

and

\[
\nu_i^2 \nu_x \sim \omega_{LH}^2
\]

(76.b)

For \( \lambda \approx 30 \text{ cm} \), \( \mu_e = 0.1 \) and for a typical value of \( \nu_{pe} = 1 \text{ KHz} \), we can find that \( T_D \approx 600 \mu s \) (\( \omega_{LH} \approx 30 \text{ KHz} \)). From appendix 2 (equation (53.a)) because

\[
\frac{\Delta \omega_{pe}}{2\pi} = 1/T_D, \quad T_D = 2\pi / \mu_e \nu_x
\]

and \( \Delta \omega_{pe}/4\pi \sim 1 \text{ KHz} \).

Using the definition of the total cross-section from pag. 74 (chapter 3), for the thermal density fluctuations we find:

\[
\left< \frac{|\delta n_{ke}|^2}{n_0^2} \right>_{IS} = \frac{\sigma_0(k)}{n_0}
\]

(77)

with, \( \sigma_0(k) \sim 1/2 \) (m-3).

Therefore,

\[
\frac{\sigma_{CH}}{\sigma_{IS}} \sim \frac{3}{4} \left( \frac{k}{k_0} \right)^2 \frac{\nu_i^2 n_0}{\mu_e s_e^2}
\]

(78)
In the ionosphere, $\Omega_e \approx 10^7$ Hz, and in the electrojet region ($\sim 100$ km) $\nu_i^2 < 10$ KHz and for $n_o \lesssim 10^{12}$ m$^{-3}$:

$$\frac{\sigma_{ch}}{\sigma_{IS}} \lesssim 10^8$$

(79)

$\sigma_{ch}$ and $\sigma_{IS}$ are, respectively, the cross-sections of the unstable modes ('coherent') and the stable modes ('incoherent'); they are defined in chapter 3.

From equation (79), the backscattered power associated with the turbulent saturation of the modified two-stream instability is of the order of 80 dB above that of the stable fluctuations.

(B). DENSITY GRADIENT-DRIFT MODES.

The non-resonant population corresponding to the lower-hybrid and the ion cyclotron modes are the electrons, and their maximum growth rate occurs at $k_{\perp}^2 \approx 1$ for $k_{\parallel} = 0$. Short wavelength, low-frequency collisional drift modes have their maximum growth rate at $k_{\perp} \approx 1$ and $k_{\parallel} = 0$. Contrary to the universal drift waves (Gary [1980]), in this case the electrons are the non-resonant population.

(a) LHD WAVES.

From equation (69), because $\omega \ll \Omega_e$, only the term with $n = 0$ in the Bessel sum of (68) needs to be retained:

$$\bar{A}_{e,\perp 0} = \left( \frac{\Gamma_{0e} + \Gamma_{1e}}{\omega_{\perp e}^2} \right) \bar{E}_{e,\perp 0}$$

and

$$< |E_{e,\perp}|^2 > \approx \frac{\omega_{\perp e}^2 B^2}{k_{\perp 0}^2 (\Gamma_{0e} + \Gamma_{1e}) \nu_e}$$

(80.a)
where $k_0$ is the wavenumber of maximum growth rate ($k_0 = k - 1$).

From Poisson's equation,

$$< |S_{ne}^2| > = \frac{1}{4} \frac{k^2}{\mu_e^2 \sigma_e^2} \frac{< |E_{ne}|^2 >}{B^2}$$ \hspace{1cm} (81)

so,

$$< |S_{ne}^2| > \leq \frac{\omega_{ne}^2 (k/k_0)^2}{4(\Gamma_{oe} + \Gamma_{ie}) \omega_{eo} \mu_e^2 \sigma_e^2}$$ \hspace{1cm} (82)

with $\Gamma_{oe}(1) \approx 0.46$, $\Gamma_{ie}(1) \approx 0.21$.

And finally,

$$\frac{\sigma_{ch}}{\sigma_{IS}} \leq \frac{3}{8} \frac{(k/k_0)^2 \omega_{ne}^2}{\omega_{eo}^2 \mu_e^2 \sigma_e^2}$$ \hspace{1cm} (83)

all quantities are expressed in MKS units.

Taking $\omega_{ne} = 1$ to 40 KHz and $n_e < 10^{12}$ m$^{-3}$ we can find,

$$\frac{\sigma_{ch}}{\sigma_{IS}} \leq 10^4 - 10^7$$ \hspace{1cm} (84)

Or equivalently, the power level of the coherent backscatter (unstable waves) is between 40 to 70 dB above the thermal-fluctuations level.

(b) ICD WAVES.

Applying the same equation (83), but now for the frequency $\omega_{ne} \sim n \Omega_i$, with $\Omega_i \sim 300$ Hz for the atomic oxygen ions and $T_i \sim 1000$°K and $n_e < 10^4$ cm$^{-3}$, the coherent power level ranges from 6 to 30 dB above the thermal level for the ion cyclotron harmonics $n = 1$ and $n = 3$, respectively:
\[
\frac{\sigma_{eh}}{\sigma_{is}} \bigg|_{e} < 40 - 400
\]  \hspace{1cm} (85)

(c) **LOW-FREQUENCY SHORT-WAVELENGTH DENSITY-DRIFT WAVES.**

Both the collisional universal drift and the low-frequency Pedersen-drift modes have their maximum growth rate at \( k_{\parallel} \approx 1 \) for \( k_{\parallel} = 0 \), and the electrons are the non-resonant population. In this case we have

\[
\left\langle \frac{|\delta n_i|^2}{n_0^2} \right\rangle < \frac{\omega_e^2}{4 (\Gamma_{oe} + \Gamma_{ie}) \mu_e^2 \Omega_e^2}
\]  \hspace{1cm} (86)

Because \( k > k_0 \), \( k_0 \approx 1 \), the dominant mode in the sum (68) is the one with wavenumber \( k \), therefore \( \Gamma_{oe} (0.1) \approx 0.91 \), \( \Gamma_{ie} (0.1) \approx 0.045 \) and

\[
\left\langle \frac{|\delta n_i|^2}{n_0^2} \right\rangle \approx \frac{1}{4} \frac{\omega_e^2}{\mu_e^2 \Omega_e^2}
\]  \hspace{1cm} (87.a)

and,

\[
\frac{\sigma_{eh}}{\sigma_{is}} \bigg|_{e} \approx 25 \left( \frac{\omega_e}{\Omega_e} \right)^2 n_0
\]  \hspace{1cm} (87.b)

For a frequency smaller than \( 0.1 \Omega_i \) (\( \Omega_i \approx 300 \text{ Hz} \)), when \( n_0 \approx 10^{12} \text{ m}^{-3} \), the coherent power level is of the order of 25 dB above the thermal level.

(C). **EIC CURRENT-DRIVEN WAVES.**

In this case the ions are the non-resonant population, and from equations (68) and (69), only the term corresponding to the harmonic \( n \) needs to be retained in the Bessel sum. Choosing \( n = 1 \) we have:

\[
\bar{A}_i e = \frac{1}{2} \frac{\Gamma_{oe} + 2 \Gamma_{ie} + \Gamma_{2i}}{(\omega_e - \Omega_i)^2}
\]  \hspace{1cm} (88.a)
\[
\langle |E_{\parallel e}|^2 \rangle \leq \frac{(\omega_{ke} - \Omega_i)^2 B^2}{k^2 (\Gamma_{ei} + 2\Gamma_{ie} + \Gamma_{oi})} \tag{88.b}
\]

and,

\[
\langle \frac{\delta n_{\parallel e}^2}{n_0^2} \rangle \leq \left( \frac{T_i}{T_e} \right)^2 \frac{2(\omega_{ke} - \Omega_i)^2}{(k_{T_i} \rho_i)^4 \Omega_i^2 (\Gamma_{ei} + 2\Gamma_{ie} + \Gamma_{oi})} \tag{89}
\]

When \( k \rho_i \sim 1 \) we obtain the expression given by Gary [1980]:

\[
\frac{\partial E_{\parallel e}}{\partial t} \sim \frac{1}{8} \frac{T_i}{T_e} \frac{\Omega_i^2}{\omega_i^2} \frac{1}{(k_{T_i} \rho_i)^4} \tag{90}
\]

In our case \( k \rho_i^2 \gg 1 \) and \( \omega_{ke} - \Omega_i \ll \nu_i \), \( T_e \sim T_i \), therefore

\[
\frac{\delta \chi}{\delta \omega} \left|_{\omega_i} \right. \leq \sqrt{2\pi} \left( \frac{\nu_i}{\Omega_i} \right)^2 \frac{n_0}{(k_{T_i} \rho_i)^3} \tag{91}
\]

For \( n \leq 10^{10} \text{ m}^{-3}, \ \nu_i / \Omega_i < 0.1 \text{ (above 150 km)} \) and \( k \rho_i \sim 80 \), the coherent power level is of the order of 20 to 30 dB above the thermal level.

(D). POST-ROSENBLUTH WAVES.

In this case the non-resonant population are the electrons, and the instability has maximum growth rate for \( k \rho_e \sim 1 \). Using equation (68), as for the LHD modes we have

\[
\langle \frac{\delta n_{\parallel e}^2}{n_0^2} \rangle \leq \frac{1}{4} \frac{\omega_{ke} \left( k_e \right)^2}{\nu_e S_{ee} (\Gamma_{oe} + \Gamma_{le})} \tag{92}
\]

and the coherent power level is expected to be 40 to 70 dB above the thermal level for frequencies ranging between 1 to 40 KHz. As before we are using the maximum electron density \( (10^{12} \text{ m}^{-3}) \).
5. SUMMARY.

Expected 30-cm wavelength instabilities in the Auroral lower ionosphere:

(1) **Modified two-stream (Farley-Buneman).**

This instability is linearly generated by the electrojet current.

- **Altitude Range:** 90 to 120 km (lower E-region).
- **Threshold:** $E > 20$ to $25 \text{ mV/m}$ with maximum growth rate at $k \parallel 0$, $k \parallel \ll k \perp$.
- **Phase Velocity:** equal to $C_S$ (the ion-sound speed, typically between 350 to 500 m/s). Generation inside a narrow cone around the flow direction.
- **Spectrum:** narrow (spectral width such that $\Delta \omega /k < C_S$) associated with weak to moderate turbulence regimes. Expected amplitude < 80 dB above the stable fluctuations level.

(2) **$E \times B$ density gradient-drift.**

At 30-cm the otherwise stable plasma fluctuations may be enhanced by the 'cascading' of turbulent wave-energy from the longer wavelengths. The combined effects of a moderate electric field and the vertical ambient density gradient will linearly-destabilize the fluid-like modes ($\Lambda > 15$ to $30 \text{ m}$).

- **Altitude Range:** 90 to 140 km (E-region).
- **Threshold:** $E \omega 10 \text{ mV/m}$ for linear fluid instability with $\vec{E} \cdot \vec{\nabla} n_0 > 0$ (evening sector or 'counter-electrojet' conditions in the morning sector).
- **Phase Velocity:** statistically distributed around zero from $C_S$ to $-C_S$.
- **Spectrum:** broad ($\Delta \omega /k > C_S$) associated with the strong turbulence regime from the saturation of the fluid-like oscillations. From observations the amplitude is about 20 dB below the one of the two-stream instability.

(3) **Density gradient-drift waves.**

Short-scale density gradients (tens to hundreds of meter) associated with
particle-precipitation or fluid-like instabilities may linearly destabilize the collisional universal mode (CU), the ion-cyclotron drift mode (ICD) and the lower-hybrid drift (LHD) mode.

CU: excited above 170 km altitude for Lυ of the order of 50 m to few hundred of meters with maximum growth rate at kρcem 1 (λ ~ 20 m) and k∥ = 0. Phase velocity anti-parallel to the ion diamagnetic drift and close to zero in the frame moving with the ions. This mode is expected to saturate at weak to moderately strong turbulence levels (narrow to relatively broad spectral widths) and for wave amplitudes not greater than 30 dB above the stable-fluctuations level.

ICD: excited above 250 km altitude for Lυ of the order of 100 m and ambient densities (n0) smaller than 10^4 cm^-3. Maximum growth rate at kρcem 1 (λ ~ 15 cm) and k∥ = 0. In the ion's frame, the resonant frequency is close to the ion gyro-harmonics and the phase velocity parallel to the ion diamagnetic drift.

At 30-cm wavelength, these modes are expected to saturate in a weak turbulence regime for amplitudes not greater than 30 dB above the stable thermal level.

LHD: excited above 250 km altitude for Lυ of the order of 50 m to hundred of meters and n0 > 10^4 cm^-3. Maximum growth rate at kρcem 1 and k∥ = 0. Also, in the ion's frame the resonant frequency is of the order of the lower-hybrid frequency (Ω_i ~ Ω_e ~ Ω_i and the wave's phase velocity is parallel to the ion diamagnetic drift. They are expected to saturate at low turbulence regimes for amplitudes not greater than 70 dB above the stable-fluctuations level.

(4) Pedersen gradient-drift waves.

They are generated by the combined effects of a moderate electric field and the short-scale density gradients in the plane perpendicular to the magnetic field. When the ion-Pedersen and the ion-diamagnetic drifts are parallel, CU and LHD modes are now possible for altitudes above 140-150 km, E ~ 10 mV/m and
of the order of 50 to hundred of meters.

(5) **Current-driven electrostatic ion-cyclotron (EIC) waves.**

They can be excited at 30-cm and $\Omega_i \sim 150$ Hz for $n_e \lesssim 10^4$ cm$^{-3}$, helped by the electron-neutral collisions between 130 to 160 km altitude, for $k_{\parallel} / k_\perp < 2 \times 10^{-3}$ and for parallel electron drifts between 10 to 35 km/s. Wave-phase propagation has no preferential direction in the plane perpendicular to the magnetic field. Saturation is expected to occur in a weak turbulence regime and for amplitudes smaller than 30 dB above the stable-fluctuations level.

(6) **Current-convective EIC waves.**

The presence of small-scale density gradients transverse to a field aligned current will help EIC instability. For collisional electrons, 30-cm unstable waves are now possible between 130-140 to 200 km altitude for electron drifts between 5 to 40 km/s, $L_\parallel$ of the order of hundred of meters and $k_{\parallel} / k_\perp < 4 \times 10^{-3}$.

(7) **Post-Rosenbluth waves.**

They may be excited at 30-cm between 130 to 300 km altitude for $n_e > 10^4$ cm$^{-3}$ and frequencies of the order of the lower-hybrid frequency. The threshold electric fields are of the order of 50 mV/m (and as high as 80 mV/m) and the maximum growth rate occurs for $k_{\parallel} \sim 1$ and $k_{\parallel}^2 / k_\perp^2 \sim m_e / m_i$.

The phase velocity doesn't have a preferential propagation direction and saturation is also expected at low (weak) turbulence levels and for amplitudes smaller than 70 dB above the stable-fluctuations level.
CHAPTER 3

RADAR OBSERVATIONS OF PLASMA TURBULENCE

In the previous chapter we analysed the conditions for the linear generation of kinetic instabilities in the lower auroral ionosphere. Linear theory can predict the onset of an instability by defining the resonant frequency, the wave propagation direction and the instability threshold, set up by the corresponding free-energy driving sources. The predicted spectrum in both quasilinear and linear theories is singular at the resonance frequency in contradiction with observations. The finite width and amplitude of the measured spectrum are the result of nonlinear processes (turbulence) that saturate the instability. The saturation spectrum in an homogeneous and stationary turbulence state may be assumed to be a Lorentzian with its width proportional to the relaxation time (Tsitovich [1972], Ichimaru [1975]).

In this chapter we present the Thomson scattering technique and discuss the determination of the turbulence spectrum from radar measurements.

1. THOMSON SCATTERING.

An incident electromagnetic wave will be scattered by the interaction with a plasma. If the energy of the incident radiation is much smaller than the rest energy of the charges ($\hbar c^2$) this process is called Thomson scattering (Sheffield [1975]). It is actually a limiting case of Compton scattering where quantum effects may be neglected.

When the wavelength of the incident radiation is much longer than the Debye
length (shielding distance), the scattering is due to the plasma density fluctuations which are associated with the longitudinal modes in the plasma through Poisson's equation (transverse mode corrections are neglected for wave phase velocities much smaller than the speed of light).

The wavelength of the mode triggered by the geometry of the observation is given by the Bragg diffraction condition ('Bragg wavelength'). If only one radar is used for both transmission and reception (backscattering) the wavelength of the observed mode is half of the incident wavelength.

The spectrum of radiation scattered from an isothermal plasma has a central maximum caused by the incoherent scattering from electron density fluctuations (the Doppler width of this maximum is governed by the ion thermal speed), and two widely separated narrow peaks, symmetrically placed on each side, associated with the scattering by the Langmuir modes. There is no central maximum in the spectrum for a non-isothermal plasma. There are two maxima, symmetric relative to the center frequency, which are associated with the ion-acoustic oscillations and two peaks associated with the electron plasma modes.

In a non-equilibrium plasma, the cross-section of the scattering of the electromagnetic waves by collective oscillations may grow enormously when approaching the kinetic instability threshold (the mode with frequency $\omega_k$ and wave-vector $k$ is a solution of the dispersion equation). The saturation peak and the width of the instability spectrum are determined by the turbulence level.

2. SPECTRAL DENSITY FUNCTION.

The fundamental quantity characterizing the scattering process is the spectral density function $S(k, \omega)$, associated with the electron density fluctuations.

Function $S(k, \omega)$ is the Fourier transform of the electron density correlation
function. In non-equilibrium conditions this function is proportional to the
turbulence power spectrum.

The differential cross-section for the transfer of momentum \( \tilde{\mathbf{k}} \) (corresponding
to scattering into a solid angle \( d\Omega \)) and energy \( \pi \omega \) from an electromagnetic
wave to the electrons in the plasma is given by (Ichimaru et al. [1962]):

\[
\frac{d^2\sigma}{d\Omega d\omega} = r_o^2 \left(1 - \frac{1}{2} \sin^2 \psi \right) S(\tilde{\mathbf{k}}, \omega) \tag{1}
\]

where \( r_o \) is the classical electron radius, \( r_o = \frac{e^2}{m_e c^2} \approx 2.82 \times 10^{-15} \text{ m} \), and \( \psi \)
is the angle between the incident and scattered wave directions. For the back-scattering case \( \psi = \pi \).

Actually the differential cross-section corresponding to the total number of
electrons (\( N_e \)) inside the scattering volume, is equal to \( N_e \frac{d^2\sigma}{d\Omega d\omega} \).

\( S(\tilde{\mathbf{k}}, \omega) \) is defined by:

\[
S(\tilde{\mathbf{k}}, \omega) = \lim_{\nu \to \infty} \frac{1}{n_0} \int_{-\infty}^{\infty} \int_{-T}^{T} \tilde{e}(\nu, x_i-\omega t) \hat{e}(\nu, x_i t) n_e(x_i, t) \, dt \, dx_i \tag{2}
\]

Or, equivalently:

\[
S(\tilde{\mathbf{k}}, \omega) = \sqrt{\left\langle \frac{|\delta n_e(\tilde{\mathbf{k}}, \omega)|^2}{N_e} \right\rangle} \tag{3}
\]

Where we have called,

\[
\delta n_e(\tilde{\mathbf{k}}, \omega) = \frac{1}{\sqrt{\nu}} \int \int \tilde{e}(\nu, x_i-\omega t) \hat{e}(\nu, x_i t) n_e(x_i, t) \, dt \, dx_i
\]

\( n_e(x, t) = n_o(x, t) + n_e(x, t) \) is the electron density and \( n_e(x, t) \)
its fluctuating part with Fourier amplitude $\mathcal{S} n_e (k, \omega) . \]

In kinetic theory the description of a plasma is essentially statistical and the angular brackets define a theoretical ensemble average: $n_e$ is a stochastic process and the average is usually performed over the initial conditions.

In practice the averaging volume and time are not arbitrarily large. The volume $V$ is defined by the observation process: (a) by the antenna radiation pattern and, (b) by the spatial distribution of the scattering sources.

We also need to consider a plasma state stationary and ergodic (on the time scale $T$) in order to equate the time-average value of the fluctuating density with the theoretical ensemble average. This condition is verified when: $T >> t_{\text{obs}} \gg \tau_{\text{ac}}$ (Sheffield [1975]). $t_{\text{obs}}$ is the duration of an individual observation, $T$ is the averaging time (or integration time) and $\tau_{\text{ac}}$ is the autocorrelation time of the plasma fluctuations.

In the stable case, the observation time for fluctuations with wavenumber $k$ is of the order of $1/\kappa v_i$, where $v_i$ is the ion thermal speed. The autocorrelation time is of the order of the ion-plasma oscillation period. In the unstable case, $\tau_{\text{ac}} > 2 \pi / \omega_c$ and $t_{\text{obs}} > 2 \pi / \overline{\Delta \omega}_k$, where $\omega_c$ is the central frequency and $\overline{\Delta \omega}_k$ the width of the instability spectrum.

A 'normalized' total cross-section is defined by:

$$\sigma (k) = \frac{1}{2 \pi V} \int_{-\infty}^{+\infty} d\omega \ \mathcal{S} (k, \omega)$$  \hspace{1cm} (4)

where $V$ is the scattering volume. $r_o^2 \sigma (k)$ is called the volume reflectivity at wavevector $k$. The mean square of the relative fluctuating electron density or wave turbulence level at wavevector $k$ is, therefore, equal to:
For a stable plasma the electron density fluctuations are due to the thermal random motion of the particles and their total cross-section is given by (Evans [1969]):

$$\langle \left( \frac{n_e}{n_o} \right)^2 \rangle = \sigma(\kappa)/n_o$$

Actually equation (5) holds when $T_e/T_i < 3$ and $\alpha^2 = 1/k^2 r^2 e^2 << 1$.

$r_o^2 \sigma(\kappa)$ and $r_o^2 \sigma(\kappa)$ are the 'volume reflectivities' of the unstable and stable ionospheric plasma, respectively.

We will use the convention of calling incoherent scattering that associated with the stable plasma fluctuations and coherent scattering that produced by the unstable plasma modes. Actually, both scattering processes are produced by coherent plasma motions (collective modes).

**A. TURBULENCE SPECTRAL DENSITY FUNCTION.**

In non-equilibrium conditions, the spectral density function is a measure of the turbulence level of the unstable density fluctuations. The power spectrum of a turbulent plasma may contain one or several peak structures in the frequency domain for a fixed wavevector $k$; each unstable mode is characterized by its spectral strength $< |E_k|^2 >$, the central frequency $\omega_k$, and the spectral width $\Delta \omega_k$. In a homogeneous and stationary turbulence state these peaks have Lorentzian shapes (Ichimaru [1975]) and their widths define the instabilities' relaxation times. Thus, we will have:
In the previous chapter, together with appendix 2, we have discussed the mechanisms for the turbulent saturation of linear kinetic instabilities in the 'orbit diffusion' approximation. Replacing the result of (6.a) in Poisson's equation, we obtain the power spectrum of the fluctuating electron density (in MKS units and the electron temperature is expressed in Joules):

\[
\langle |\delta n_e|^2 \rangle \propto \frac{2|\xi|^2}{\epsilon^2} \frac{1}{k^2 \lambda_e^2} \frac{\bar{\Delta \omega_k}}{(\omega - \omega_k)^2 + \Delta \omega_k^2} \]

(6.b)

where \( \chi_e(\omega) \) is the linear electron susceptibility function (appendix 1).

From equation A1(30) for the electron susceptibility,

\[
|\chi_e| = \frac{1}{k^2 \lambda_e^2} \left[ 1 + O(|E_k|^1) \right] \]

(7.a)

to we will take:

\[
|\chi_e| = \frac{1}{k^2 \lambda_e^2} = \frac{\lambda_e \epsilon}{k^2 \epsilon \omega_e} \]

(7.b)

From equation (3), the spectral density function may be obtained:

\[
S(\omega) = \frac{2N_e \epsilon^2}{k^2 T_e^2} \frac{<|E_k|^2> \bar{\Delta \omega_k}}{(\omega - \omega_k)^2 + \Delta \omega_k^2} \]

(8.a)

Consequently the total cross-section is:
\[ \sigma(k) = \frac{\eta_0 e^2}{k^2 - l^2} \langle |E_k|^2 \rangle \] (8.b)

And the instability total cross section is proportional to the strength of the fluctuating electric field.

(B). ASPECT ANGLE SENSITIVITY.

Instability growth is generally dependent on the angle between the wave propagation direction and the ambient magnetic field (aspect angle \( \Theta_{\text{ASP}} \)).

For the class of instabilities with maximum growth rate at perpendicular or nearly perpendicular wave propagation, the final turbulence state ('saturation') is almost 'two dimensional' with a small diffusion in the \( k_{||} \)-space (Sudan [1983.a and b], Rosenbluth and Sudan [1986]). This finite spread in the parallel wavenumber defines the aspect angle sensitivity (range of angles and wave amplitude attenuation for off-perpendicular propagation) of the instability.

For fully developed turbulence, \( \delta n_e(k, \omega) \) is a random function and the ensemble averaged of its correlation function has the form:

\[ \left< \frac{\delta n_e(k, \omega) \delta n_e(k', \omega')}{\eta_0^2} \right> = \frac{I_{k, \omega}}{\eta_0^2} \delta(\omega - \omega') \delta(k - k') \] (9)

where \( I_{k, \omega} \) is the turbulence power spectrum and equivalent to the 'spectral function'. In general, a homogeneous and stationnary turbulent state is characterized by \( k^a I_{k, \omega} \) = constant (Sudan and Pfirsch [1985]). For the case of the linearly unstable Farley-Buneman waves (type 1) this constant is the total wave-energy and \( a = 2 \) (Rosenbluth and Sudan [1986]) whereas the wavelength spectrum of the strong turbulence-related type 2 waves in the lower E-region is given by \( a \sim 8/3 \) (Sudan [1983.b]).
From the previous section (4.A) (equations (8.a), (8.b)) and (9)), for instabilities with $k_{\parallel} \ll k_{\perp}$ and maximum growth rate (at $k_{\parallel}^2 - k_{\perp}^2 + k_{\parallel 0}^2$) close to the perpendicular ($k_{\parallel 0} \sim 0, \Theta_{\text{ASP}} \sim 90^\circ$), we can find that:

$$\frac{I_{k_0}}{I_{k_{\parallel 0}}} = \left[ \frac{1}{1 + \frac{\langle k_{\parallel}^2 \rangle}{2k_{\parallel 0}^2}} \right]^{|k_{\parallel}|^2} - e^{-\frac{\Delta^2}{2}} \right.$$  \hspace{0.2cm} (10.a)

where $\langle k_{\parallel}^2 \rangle/2$ is the square of the rms value of the parallel wavenumber fluctuations, corresponding to the average aspect angle deviation $\delta\Theta_{\text{ASP}}$:

$$\delta\Theta_{\text{ASP}} = |\Theta_{\text{ASP}} - \Theta_{\text{ASP}}^0| \hspace{0.2cm} \sin^2\delta\Theta_{\text{ASP}} = \langle k_{\parallel}^2 / k_{\perp}^2 \rangle$$

and the variance $\Delta^2 = 2/a$.

We will approximate this Gaussian distribution by a decimal-exponential distribution of the measured power:

$$\text{POWER} \left| \frac{I_{k_0}}{I_{k_{\parallel 0}}} \right. = \frac{I_{k_0}}{I_{k_{\parallel 0}}} \sim 10^{\frac{\text{ASP}}{10} \delta\Theta_{\text{ASP}}}$$ \hspace{0.2cm} (10.b)

$\text{PMAX}$ is the turbulence power level for $k_{\parallel} \sim 0$ ($\Theta_{\text{ASP}} \sim 90^\circ$).

Distributions (10.a) and (10.b) have the same standard deviation $\Delta$ and median $\delta\Theta_{\text{ASP}}$. The median defines the '50 % value' such that $\delta\Theta_{\text{ASP}} < \delta\Theta_{\text{ASP}}$ for half of all cases. Thus,

$$\Delta = \sqrt{-\frac{10}{\text{ASP}}} \hspace{0.2cm} \delta\Theta_{\text{ASP}} = \log_{10}(2) \Delta$$

and $\text{ASP} = -10(a/2)$ is called the 'macroscopic' aspect angle sensitivity and is measured in dB/degree (10 dB corresponds to one 10-folding). $\text{ASP} = -10$ for the
case where \( a = 2 \).

From the previous discussion, the aspect sensitivity is a macroscopic, average quantity associated with the turbulent diffusion in the \( k_\parallel \)-space.

3. RADAR DETECTION OF PLASMA WAVES.

(A). SCATTERED POWER FROM A PLASMA.

The net scattering in the radar beam direction is due to the charge density fluctuations. At high frequencies (higher than the plasma frequency) the plasma can be considered optically thin (no significant losses in crossing it) and we may ignore multiple scattering. It can therefore be assumed that each charge 'sees' the same incident field and the plasma as a whole is not disturbed by the wave (Sheffield [1975]).

The scattered power for non relativistic plasmas \((v_e << c)\), in the frequency range \( \omega \) and \( \omega + d\omega \) and the solid angle \( d\Omega \) centered in \( R \) (the 'observed point', \( R \) is the distance from the radar to the scattering region) is given by:

\[
d P_s = P_s(R, \omega) d\Omega d\omega = P_i L n_0 d^2 \sigma
\]  

where \( n_0 \) is the local electron density, \( P_i \) is the average incident power and \( L \) is the linear dimension of the scattering volume in the radar beam direction (range smearing). For pulsed radars, if \( \tau_p \) is the emitted pulse length \( L = c \tau_p \), where \( c \) is the speed of light. \( d^2 \sigma/d\Omega d\omega \) is the differential cross-section defined by equation (1). Assuming that the incident wave is plane monochromatic, the wave electric field is given by:
\[ E_i = E_{i0} \cos(\mathbf{k}_{in} \cdot \mathbf{x} - \omega_{in} t) \]  

(12)

The incident power \( P_i \) may be expressed as,

\[ P_i = \frac{E_{i0}^2 \mathcal{A}}{2Z_0} \]  

(13)

\( \mathcal{A} \) is the beam cross-section, \( Z_0 = 1/\varepsilon_0 \sim 377 \) ohms is the impedance of the free space.

The radar acts as a spectrum analyser picking out only the plasma mode with wavevector \( \mathbf{k}_p \) and Doppler frequency \( \omega \).

From momentum and energy conservation:

\[ \mathbf{k} = \mathbf{k}_{sc} - \mathbf{k}_{in} \]  

(14.a)

\[ \omega = \omega_{sc} - \omega_{in} \]  

(14.b)

subscripts \( \text{in} \) and \( \text{sc} \) identify the incident and scattered waves respectively.

\( \omega \) will be called \( \omega_k \) if it satisfies the dispersion relation \( \varepsilon(\mathbf{k}, \omega) = 0 \).

The Doppler frequency \( \omega \) is the result of an organized motion in the plasma; for the stable case, in the ionosphere, \( \omega/k \) is actually the Doppler-shift of the ion-line and equal to the component of the ion-drift velocity along the observation direction. For non-equilibrium situations \( \omega/k \) is the phase velocity of the unstable mode.

Equation (14.a) is equivalent to the Bragg diffraction condition for a regular lattice with interplane separation \( \lambda = 2\pi/k \). In the backscattering experiments
\[ k = 2 \frac{k_{n}}{\lambda_{m}} \quad \text{and} \quad \lambda = \frac{\lambda_{m}}{2} \quad \text{where} \quad \lambda_{m} \quad \text{is the radar wavelength.} \]

For the Millstone Hill radar \( \omega_{m}/2\pi = 440 \text{ MHz} \) and \( \lambda_{m} \approx 68 \text{ cm} \). The maximum Doppler frequency to be detected is limited by the receiver bandwidth (BF).

In Table I we list the Millstone Hill radar characteristics.

**Table 3.1. Millstone Hill Radar.**

| Geographic Coordinates: 42°36' LAT. N., 71°30' LON. W. |
| Geomagnetic Coordinates: 56° INVLAT. and, 14.5° N.W. MAG. DECLINATION |
| Frequency: 440 MHZ (\( \lambda_{m} \approx 68 \text{ cm} \)) |
| Peak Power: 2.3 MWatts |
| Pulse Length: 20 to 2000 \( \mu \text{sec.} \) |
| Receiver Bandwidth: 50 KHZ |
| Antenna Gain: 47.1 dB |
| Antenna Diameter: 46 m (fully steerable) |
| Main Beam Half Power Width: 1 degree |

**B. Radar Cross-Section.**

The differential radar cross-section is defined by (Murdin [1980]):

\[ dP_{s} = \left( \frac{P_{t} G_{t} A_{2}}{4 \pi R_{2}^{2}} \right) \ln_{0} d \frac{\sigma^{2}}{\sigma} \]  

\( (15) \)
Comparing with equation (11) the average incident power $P_i$ is:

$$P_i = \frac{1}{4\pi} P_t G_1 \frac{A_2}{R_2^2}$$ \hspace{1cm} (16)

$dP_s$ is the power received by the receiver antenna at the distance $R_2$ from the scattering volume $dV = R_1 L d\Omega$, in the frequency range $\omega$ and $\omega + d\omega$.

$R_1$ is the distance from the transmitter antenna and like before $L = c \tau_p$.

$P_t$ is the transmitter power and $G_1$ the transmitter antenna gain. $A_2$ is the receiver antenna effective aperture.

Since we are using the same antenna to transmit and receive we will drop the subscripts 1 and 2. The distance $R$ is called the radar range and $L$ is the range smearing. The antenna gain is defined by:

$$G = 4\pi \frac{A}{\lambda^2_{in}}$$ \hspace{1cm} (17)

the effective aperture $A$ is also the beam cross section of equation (13) and is equal to:

$$A = \eta \chi \frac{D^2}{\lambda^2} f(\Theta)$$ \hspace{1cm} (18)

$D$ is the antenna diameter, $\eta$ is the antenna efficiency, and $f(\Theta)$ is the relative antenna power pattern ($\Theta$ is the azimuthal angle away from the observation direction).

Integrating over the solid angle we can define the effective scattering volume for a monostatic, single pulsed radar observation, when the scattering source is uniformly distributed:


\[ P_{SW} = \frac{P_t G_0 A_0 \zeta_0^2}{4 \pi R^4} \eta_0 S(k, \omega) V_{eff} d\omega \]  

(19)

where,

\[ G_0 = \eta \left( \frac{\kappa D}{\lambda_{in}} \right)^2 = 4 \pi \frac{A_0}{\lambda_{in}^2} \]  

(20)

\[ V_{eff} = 2\pi L R^2 \int_0^{\pi/2} d\theta \sin \theta f(\theta) \]  

(21)

In general,

\[ P_{SW} = \frac{P_t G A}{4 \pi R^4} \int_0^{2\pi} d\theta \int_0^{\pi/2} d\phi S(k, \omega) f(\theta) d\omega \]  

(22)

\[ S(k, \omega) \] is also a function of \( R \), and \( V_s \) is the scattering volume. \( f(\theta) = G/G_0 \) is also known as normalized antenna gain pattern.

For UHF radar systems (\( \lambda_{in} < 1 \text{ m} \)) the theoretical antenna pattern is given by (Murdin [1980]):

\[ f(\theta) = \left[ \frac{2 J_1 (\kappa D \sin \theta / \lambda_{in})}{\kappa D \sin \theta / \lambda_{in}} \right]^2 e^{-p_A \theta^2} \]  

(23.a)

where,

\[ p_A = \frac{1}{4} \left( \frac{\kappa D}{\lambda_{in}} \right)^2 \]  

and,

\[ V_{eff} = \frac{\kappa C \zeta_0 R^2}{4 p_A} \]  

(23.b)

For the Millstone Hill radar \( D/\lambda_{in} = 67.65 \) and \( p_A = 1.24 \times 10^4 \).

Also from equation (13) we can find the incident rms electric field at a given range \( R \) from the antenna:

\[ P_i = \frac{1}{4\pi} P_t G \frac{A}{R^2} = \frac{E_{io}^2 A}{2 \pi \varepsilon_0} \]  

(24.a)
and
\[ E_{\text{rms}}^2 = \frac{1}{2} E_{\text{rms}}^2 = \frac{P_t G_0 \xi_0}{4\pi R^2} \quad (24.b) \]

along the main beam direction \( \theta = 0 \), then:
\[ E_{\text{rms}}^2 = \frac{P_t G_0 \xi_0}{4\pi R^2} \quad (24.c) \]

For the Millstone Hill radar, \( P_t = 2.3 \) MWatts and \( G_0 = 47.1 \) dB. Choosing, for example, \( R = 1000 \) km, one can find \( E_{\text{rms}} = 1.8 \) V/m.

(C) ANTENNA RADIATION PATTERN

From equation (19) and (21) one can see that the radar cross-section is a function of the square of the normalized antenna gain pattern \( f(\theta) \).

In figures (3.1a) and (3.1b) we show the decimal logarithm of \( f(\theta) \) multiplied by ten (dB units). The actual three dimensional pattern is the revolution figure around the y-axis.

Figure 1.a shows the total 90° pattern and figure 1.b gives a finer angular resolution of this pattern up to the first side-lobe (\( \approx 3 \) degrees). A power level of -x dB represents a measured power 10\(^{-0.1x}\) times smaller than a reference level.

For an uniformly distributed scattering region filling the radar field of view, if we neglect any power losses, the power at the receiver closely follows the \( f(\theta) \) dependence. Equivalently, the power from the radar sweeping of localized scattering sources also present a clear \( f^2(\theta) \) modulation. Generally, \( f^2(\theta) \) gives a good estimate of the power drop (in # dB) between two observations of the unstable region: one along the main beam direction (\( \theta = 0 \)) and another at
an angle \( \theta \) away from it.

From figure 1.b we can see that the corresponding two-way dB levels for angles 0.5\( ^\circ \), 1\( ^\circ \), and 2\( ^\circ \) away from the main beam direction are respectively -8, -25, and -55 dB. The two-way power drop associated with the first side lobes (at \( \pm 3^\circ \)) is -45 dB.

(D) MILLSTONE HILL RADAR OPERATION.

A number of transmitted pulses with different lengths and interseparations defines an observation cycle. In the single pulse technique, the emitter is on (emission phase) only during a time \( T_p \) (pulse length) at the beginning of the observation cycle, the rest of the time (sampling phase) only the receiver is on. An observation cycle is repeated at the so-called pulse repetition frequency (PRF) and the time between the center of one emitted pulse and the next is called the interpulse period (IPP = 1/PRF). IPP is limited by the radar duty cycle (time fraction of transmitter operation \( \sim 5 \% \) and IPP \( \sim 20 T_p \)).

The sampling period (\( T = 20 \mu \text{sec} \), the inverse of the receiver bandwidth BF) is also the minimum time resolution. \( T_p \) is the correlation time of the observations and the range resolution \( \Delta R \) is equal to \( c T_p / 2 \).

Observations separated by a time greater than the pulse length are uncorrelated (both in space and time). For time-scales smaller than \( T_p \), the time resolution and range discrimination are not independent (Farley [1969], Rino [1972]). Moreover, assuming stationarity over times equal or greater than the pulse length, an estimate of the position of a localized scatter with an apparent range resolution as small as \( c T \) can be found.

As we shall discuss in chapter 5, with a number of \( N = T_p / T \) samples, an experimental estimate of the autocorrelation function of the scattering process
may be calculated. The frequency resolution of the associated power spectrum is

$$\delta f = 1/T_p$$ (twice the Nyquist frequency).

In figure 2 we present a sketch of the single pulse technique.
FIGURE CAPTIONS

CHAPTER 3.

FIGURE (3.1.a). This figure displays the normalized antenna gain pattern (in dB) for the Millstone Hill 440 MHz radar as a function of the azimuthal angle away from the main beam direction and for the total 90° pattern.

FIGURE (3.1.b). Same as figure (a) but for a finer angular resolution, up to the first-side lobe at 3°.

FIGURE (3.2). It gives a sketch of the single pulse technique. The duration of an observation cycle is defined by the IPP (interpulse period); at the beginning of the cycle only the emitter is on during a time $T_p$ (the transmitted pulse length) the rest of the time only the receiver is on, sampling the incoming signal every $\tau = 20 \mu$sec (reciprocal of the receiver bandwidth).
FIGURE 3.1
SINGLE PULSE TECHNIQUE

TIME

RANGE

\[ \Delta R = c \tau_p \]

transmit

receive

PULSE SCHEME

sampling

IPP

FIGURE 3.2
CHAPTER 4

THE ANALYSIS OF THE SCATTERING CROSS-SECTION

The spectral function defined in chapter 3 is the fundamental quantity to be measured in all the scattering experiments. The total scattering cross-section is obtained by integrating this function over the frequency domain and is also proportional to the volume reflectivity (see chapter 3, section 2).

In the incoherent scatter case this quantity is practically equal to the number of electrons inside the scattering volume, and in the coherent case it is proportional to the electric field strength of the unstable fluctuations at half of the radar wavelength (backscattering).

The analysis of the radar cross-section as a function of various geophysical parameters leads to the determination of the main characteristics of the instability sources: their spatial extent and location, their distributions with local time and magnetic activity and their aspect angle dependence.

The unambiguous identification of different driving mechanisms and instability thresholds requires the complementary spectral information and will be discussed in the next chapter.

Because our radar system is calibrated to study the thermal plasma fluctuations we have access to both coherent and incoherent backscattering data from adjacent regions during the same observation period. This characteristic together with the very narrow antenna beam width (1 degree) will allow a fine resolution study of the plasma turbulence in the auroral lower ionosphere. Radar convolution
effects (antenna pattern 'contamination') may, however, be very important when observing the unstable density fluctuations. In this chapter we discuss this problem and present the analysis of the measured radar cross-section of the unstable fluctuations for a number of observation periods.

1. SCATTERING VOLUME

At any given time \( t \), the receiver power is the sum of all the contributions coming from the volume enclosed by the two wavefronts of radius \( c(t-t_0) \) and \( c(t+\tau_{\rho}-t_0) \), where \( t_0 \) is the starting time of the observations.

The range gate of an individual observation is defined by \( R = c(t+\tau_{\rho}/2-t_0) \) and two consecutive, independent observed points are separated by the range smearing \( c\tau_{\rho} \). Different power contributions add linearly and are weighted by two-way antenna pattern; an effective scattering volume may be defined by the weighted volume of the total scattering region by this two-way pattern.

In non-equilibrium conditions, the scattering is also strongly dependent on the angle between the wave propagation direction and the magnetic field (aspect angle) and, often, some preferential perpendicular direction (e.g. flow angle).

(A). INCOHERENT SCATTERING.

The amplitude of the stable density fluctuations in the ionospheric plasma is proportional to the total number of electrons inside the scattering volume and, in consequence, smaller than \( 10^6 \) \( \text{cm}^{-3} \) (F-region maximum); the ratio between the densities in two different points does not exceed 4 to 5 orders of magnitude (and is usually much lower). In radar-observation jargon this is equivalent to saying that, during stable conditions, the relative power level between two different radar-illuminated regions is not greater than 40 to 50 dB (10 dB is equal...
to one order of magnitude or a 10-folding).

The received power is modulated by the square of the antenna pattern (equation (3.22)) and from figure (3.1), contributions from regions outside the one degree beam width are negligible. The scattering volume is determined by the antenna pattern and the transmitted pulse length and is equal to the effective scattering volume defined in the previous chapter \( \sim 10^2 \text{ m}^3 \), for \( R \sim 500 \text{ km} \) and \( T_p = 2 \text{ msec} \), equation (3.21)). All the physical quantities associated with the measured cross-section and the spectrum will characterize the local state of the stable plasma inside this volume centered at the observed point \( R \) and for the time \( t \). Vector \( R \) is given by the range gate \( R \) and the direction of the radar main beam (line of sight), and \( t \) is the time of the observation.

(B) COHERENT SCATTERING.

In a non-equilibrium, unstable ionosphere the situation is much more complicated: plasma instabilities are very large amplitude density fluctuations randomly distributed in space and time for which, given the high sensitivity of our system, radar convolution effects are very important (antenna pattern 'contamination'). Only when the instability source is intercepted by the radar main beam is the backscattered signal generated inside the effective volume centered on vector \( R \). In all the other cases, the coherent backscattering comes from off-main beam intersections of the unstable region and because of the strong antenna pattern modulation, an estimate of the position of the effective volume, where most the scattering is coming from, can be found. This estimate is, however, subject to ambiguity since our knowledge of the instabilities' space-time distribution and propagation direction is incomplete. The received power at any range gate \( R \) will be 'contaminated' by the scattering coming from instabilities
located somewhere between the two spherical wavefronts at \( R \pm c \frac{Zp}{2} \). Figure (4.1) shows the case where a main beam observation of the F-region (at \( R_a \)) is contaminated by the off-main beam intersections of the unstable E-region (at \( R_b \)). Both \( R_a \) and \( R_b \) lie inside the region between \( R \pm c \frac{Zp}{2} \) (\( R \leq R_a, R_b \leq R \)).

Signal from \( R \) is reduced in intensity by \( f(\theta) \) with \( \cos \theta = \frac{R_a - R_b}{R_a R_b} \) and the backscatter power from irregularities can be strong (above the stable fluctuations level) even for far side-lobes contributions.

An additional observational constraint also exists, because in order to observe both the incoherent and the coherent fluctuations, our receiver imposes an upper limit to the measured power level ('receiver saturation'); this limit is typically of the order of 30 to 40 dB above that corresponding to the stable fluctuations. The 'true' saturation level can always be measured by manually tuning the receiver.

E-region instabilities may exist in a widely extended spatial region for time scales on the order of tens of minutes to a few hours (diffuse aurora). Above this region, instabilities are generally more localized and short-lived.

If instabilities are distributed in narrow horizontal layers, the determination of the location and extent of different instability sources from the radar observations is possible.

At every particular observation direction, the radar defines a power-range profile strongly modulated by the two-way antenna pattern \( f(\theta) \). The range \( R \) is associated with an apparent main-beam altitude \( h \) when the antenna pattern intersects the unstable layer (at altitude \( h_0 \)) at an angle away from the main-beam direction. Relative maxima in the power profile may in some circumstances be associated with different unstable regions at different altitudes.
2. INSTABILITY CROSS-SECTION AND EFFECTIVE SCATTERING VOLUME.

Assuming that the right conditions for the onset of instability are met, the received power from an instability region is a function of (Oskman et al., 1986): (1) the saturation level of the instabilities, (2) the angle between the line of sight and the magnetic field (aspect angle), (3) the antenna radiation pattern and, often, (4) the angle between a driving perpendicular current and the line of sight (flow angle).

In order to estimate the effective scattering volume we shall assume that the instabilities are uniformly distributed and had reached saturation some time before our observation. Under these conditions, using equation (3.22), the instability effective scattering volume is equal to the weighted volume of the unstable region where aspect and flow angles are favorable. The weighting function is the two-way antenna pattern \( f^2(\Theta) \).

We shall call \((R, \Theta, \Phi)\) the main beam spherical coordinate system where the axis \( z \) is along the main beam direction. Coordinates \((R, \pi/2-EL, AZ)\) define the radar spherical system (appendix 3), \( EL \) is the elevation angle and \( AZ \) the azimuth, \( AZ = 0 \) towards the geographic north. Calling \((AZ0, EL0)\) the line of sight direction, we have:

\[
\cos \Theta = \cos(AZ-AZ0)\cosEL0\cosEL + \sinEL0\sinEL
\]

(1)

Considering that the geometric dependences may be factored out of the scattering cross-section (Farley et al., 1981), the backscattered radiation from the instability region follows the distribution \( G_R(\Theta, \Phi) \):

\[
G_R(\Theta, \Phi) = f^2(\Theta)u_R(\Theta, \Phi)
\]

(2)
$U_R(\theta, \varphi)$ characterize both the aspect and flow angle dependences.

Considering only the aspect angle dependence and calling $\Theta_{ASP}$ the aspect angle of maximum wave-amplitude, one may define (Minkoff, 1973):

$$U_R(\theta, \varphi) = 1 \text{ if } |\Theta_{ASP} - \Theta_{ASP}| < \delta$$  \hspace{1cm} (3.a)

and zero otherwise. For a large class of instabilities $\Theta_{ASP} \approx 90^\circ$.

Usually $\delta$ is smaller than a few degrees; aspect angle $\Theta_{ASP}$ is a function of $R, \Theta$, and $\varphi$. The following expression is often used in the literature (Oskman et al., 1986) to model the aspect angle dependence in the E-region:

$$U_R(\theta, \varphi) = \exp(0.1 \text{ ASP} |\Theta_{ASP} - 90|)$$  \hspace{1cm} (3.b)

ASP is a constant called the aspect sensitivity and is somehow related to the turbulent diffusion in the parallel wavenumber space. In linear theory this parameter is associated with the growth rate dependence on $k_\parallel$ for slightly off-perpendicular wave propagation (less than 1 degree, see for example Ossakow et al. [1975], Wang and Tsunoda [1975], Schlegel and St.Maurice [1982]). Wave propagation up to 2 to 3 degrees off-perpendicular may also be explained by the linear theory when large values of the electron-neutral collision frequency are considered (Ogawa et al. [1980]). From the observations, a value of ASP close to $-10$ dB/degree is obtained (Ecklund et al. [1975], McDiarmid [1976], and later in this chapter). The angular extent $\delta$ where equation (3.b) applies is not well defined but in any case its value is not greater than $4^\circ$ (Williams, 1980).
(A). INSTABILITY CROSS-SECTION.

The absolute cross-section of the backscattering process from the stable plasma fluctuations is a measure of the local electron density. From equation (3.19), after integration over the frequency domain, we have:

\[ P_{IS} = K \rho N_e \sigma_o(k) \]  \hspace{1cm} (4)

where \( P_{IS} \) is the received power ('incoherent'), \( N_e \) is the total number of electrons in the effective scattering volume and \( \sigma_o(k) \) is the total cross section of the stable plasma modes with wave vector \( k \) parallel to the line of sight. Factor \( K \rho \) is a function of the range and the transmitted pulse length alone:

\[ K \rho = \frac{P_G A_r^2 V_{eff}}{4 \pi R^4} \]  \hspace{1cm} (5.a)

or equivalently, from equation (3.23.b),

\[ K \rho = \frac{P_G A_r^2 c z_p}{16 \pi R^2} \]  \hspace{1cm} (5.b)

Therefore from equation (3.5) we found:

\[ P_{IS} = K \rho \eta_0 / (1 + T_e / T_i) \]  \hspace{1cm} (6.a)

and,

\[ n_o = P_{IS} (1 + T_e / T_i) / K \rho \]  \hspace{1cm} (6.b)

For non-equilibrium conditions, the scattered power is expressed by (equation
(3.22): 

$$PCH = K R n_o \int_{R - \frac{1}{2} c T_P}^{R + \frac{1}{2} c T_P} \int_{R - \frac{1}{2} c T_P}^R d \Omega(R) \sigma(k, \theta, \phi; R) \int_{0}^{2} d \Omega(\theta) U_R(\theta, \phi)$$  \hspace{1cm} (7)

Actually \( \sigma(k, \theta, \phi; R) \) is the total cross section of the instability process at wavenumber \( k \).

Various plasma processes associated with different driving mechanisms may be responsible for the observed coherent backscattering from the lower auroral ionosphere. Their common feature is that their wave propagation direction is almost perpendicular to the magnetic field. Identification of any flow angle dependence needs spectral information and an estimate of the flow direction.

In the next chapter we will discuss this identification for the E-region ExB-current driven instabilities.

(B). LOG10 POWER UNITS (POL).

In order to facilitate the analysis of the measured cross section, we define the 'logarithmic' power, POL:

$$POL = \text{LOG10}( \text{POWER} / K R)$$  \hspace{1cm} (8)

POWER is either PCH or PIS and \( K R \) is defined by equations (5.a) and (5.b).

Expressing the ambient electron density \( n_o \) in cm\(^{-3}\), \( 10^{POL} \) is the equivalent density associated with the measured power. In the mid-latitude ionosphere this value does not exceed \( 10^6 \text{ cm}^{-3} \) (F-region maximum) and, thus, \( POL < 6 \) for a stable plasma. In non-equilibrium, unstable conditions the quantity \( 10^{(POL - 6)} \) gives the dB turbulence power level above the thermal fluctuations. The associated
volume reflectivities for the stable and the unstable E-region are respectively of the order of \(10^{-17}\) m\(^{-1}\) (corresponding to an electron density of \(10^5\) cm\(^{-3}\)) and \(10^{-9}\) m\(^{-1}\). The maximum measured power, with a system such as ours (operating as both, an incoherent and a coherent backscatter), is generally limited by the receiver saturation level. \text{POL} is a function of the range \(R\) (equations (5.b) and (8)) and when \text{POWER} is the same at two different range gates (receiver saturation), \text{POL} is apparently greater at the greater range. Tuning the receiver above the real instability saturation level is, in principle, possible. This will, however, destroy the dual-mode (incoherent/coherent) radar operation. In any case an a-posteriori estimate can be always found using the antenna side-lobe effects.

In the lower auroral E-region the average turbulence level appears to be about 60 to 70 dB above that corresponding to the stable density-fluctuations level (Williams [1980], Moorcroft [1987]). This value is in good agreement with our own estimates (later in this chapter). Farley et al. [1981] also found a value of the order of 70 dB for the type 1 unstable waves (Farley-Buneman) in the equatorial electrojet region.

For all reviewed instabilities in the auroral lower ionosphere (chapter 2), the saturation amplitudes estimated in the frame of the 'orbit-diffusion' approximation ranges between 30 and 80 dB above the thermal background.

\textbf{(C). EFFECTIVE SCATTERING VOLUME OF THE INSTABILITY REGION.}

In order to estimate the total scattering cross-section one can integrate equation (7) over the potentially unstable region defined by the aspect angle condition of equation (3.b). In appendix 3, as an exercise showing the global geometric constraints of our observations, we describe the determination of this re-
gion in the dipole magnetic field approximation. The actual magnetic field model used in our calculations and data analysis is much more accurate (the International Geomagnetic Reference Field model for the year 1985).

In this section we evaluate the integral of equation (7) considering two different distributions of coherent scatterers inside the potentially unstable region defined by condition (3.b). In the first case (1), the instability is uniformly distributed and fills the region, and in the second case (2), the instability occurs only in a thin layer of constant altitude in the lower E-region. Equation (7) may also be written as:

\[ \text{PCH} - \text{PSAT} \left( \frac{R_0}{R} \right)^2 A(EL_0, AZ_0, R) \]  (9)

with

\[ \text{PSAT} = K R_0 N e \sigma(k) \]  (10.a)

\[ A(EL_0, AZ_0, R) = \frac{\text{Ve}ff}{\text{Ve}ff} \]  (10.b)

and,

\[ \text{Ve}ff = c_{zp} R^2 \int_{EL_1}^{EL_2} dEL \int_{AZ_1}^{AZ_2} dAZ \sin \theta \frac{2}{r} U_R(\theta, \psi) \]  (11)

Veff is the effective volume defined by equation (3.21), PSAT is the instability saturation power at range \( R_0 \), \( K R_0 \) the radar constant at this range and Veff is the instability effective scattering volume. EL1, EL2 and AZ1, AZ2 are the angular limits of the unstable region and functions of R (assumed constant over \( \Delta R = c_{zp} \)). When the radar main beam intersects the instability region and \( U_R(\theta, \psi) = 1 \), Veff = Veff or \( A(EL_0, AZ_0, R) = 1 \), using equation (3.8.b):

\[ \text{PCH} = K R_0 N e n_0 e^2 < |E_0|^2 > / k^2 T^2 e \]  (12)
In all other cases, the function \( A(EL0,AZ0,R) \) must be estimated from equations (10.b) and (11).

Figures (4.2.a and b), (4.3.a and b) show the results of the integration of equation (7) (or equation (9)) for two different elevation angles (4° and 6°) and for the 'distributed' instability model and the 'thin layer' model (frames (a) and (b) respectively). In these two models we have assumed that \( \log_{10}(PSAT) = 12 \). Also, in the thin layer model, the instability layer was placed at 110 km altitude. We do not show all the calculated altitudes because the additional information is redundant with the only exception that for the thin layer model and for both elevation angles, a secondary maximum in the altitude power profile appears: about 160 km (for 4°) and 190 km (for 6°) altitude. The last altitudes (without considering range smearing) at which the effects of this unstable layer are felt are 193 km at 4° and 231 km at 6° elevation. These altitudes correspond to the range defined by the intersection between the tangent plane at Millstone Hill and the spherical shell at 110 km altitude ('horizon' for \( R \approx 1189 \) km).

3. THIN LAYER MODEL OF THE E-REGION.

E-region instabilities are concentrated in the electrojet region in horizontal layers associated with the ionospheric ionization layering (Fejer and Kelley [1980]). Statistically, these layers may be considered uniform and very narrow; their thickness are the result of the turbulent diffusion in the parallel wave-number space. When the spatial resolution of our measurements can't resolve the fine structure of the lower E-region, the thin layer model gives a very good representation of the observations helping in the analysis of the radar convolution effects (antenna pattern 'contamination'). In any case the actual 'thick-
ness' of the unstable region must be smaller than the radar height-resolution.

In the logarithmic scale, function \( A(EL0, AZ0, R) \) defined by equation (10.a), is practically equal to the \( \log10 \) of the calculated power (figures (4.2.a) and (4.3.a)) minus 12 (the \( \log10 \) of PSAT) (see equation (9)). The antenna pattern (figure (3.1)) is strongly dependent on the azimuthal angle away from the observation direction and we may define a function \( a(EL*, AZ*, R) \) such that:

\[
PCH = a(EL*, AZ*, R) \text{ PMXI} \quad (13.a)
\]

\( \text{PMXI is the power corresponding to the maximum value of the integrand in equation (7) as a function of } \theta \text{ and } \varphi:
\]

\[
\text{PMXI} = \max_{\theta, \varphi} \left( \sigma(k) f^2(\theta) U(\theta, \varphi) \text{ Veff} \right) \quad (13.b)
\]

\((R, EL*, AZ*)\) can be considered the most probable center of the effective instability region and from equation (5) we may define \( \theta^* - \theta^*(EL*, AZ*; EL0, AZ0, R) \) the azimuthal angle of the most probable region away from the main beam direction.

Table 4.1 gives a summary of the parameters used in the simulation of the radar response to the scattering from an unstable thin layer. Using equation (13.b), in the region where \( U_R(\theta, \varphi) = 1 \) we have:

\[
A(EL0, AZ0, R) = a(EL*, AZ*, R) f^2(\theta) \quad (14)
\]

Figures (4.4.a, b, c and d) for \( EL0 = 4^\circ \) and figures (4.5.a, b, c and d) for \( EL0 = 6^\circ \) show respectively the distributions of function \( a(EL*, AZ*, R) \) and the angles \( \theta^*, EL* \) and \( AZ* \) as functions of the main beam azimuth (AZ0) for dif-
different apparent altitudes in the thin layer model. As we can see, when $290^\circ < \text{AZO} < 410^\circ$, the apparent power-altitude profiles are basically due to off-beam intersections of the unstable layer at an azimuth equal to the main-beam azimuth (elevation-antenna pattern contamination). Outside this region or for higher altitudes, side-lobe/azimuth effects become important.

**TABLE 4.1.** Definition of some parameters used in the simulation of the backscattered power from an unstable layer.

- **ELO, AZO**: main beam elevation and azimuth.
- **EL*, AZ***: elevation and azimuth of the most probable instability source location.
- **$V_{\text{eff}}$**: effective volume of the incoherent scattering ($\sim 10^{12} \text{ m}^3$ for $\tau_p = 2 \text{ msec and } R \sim 500 \text{ Km}$).
- **$V_{\text{eff}}$**: effective volume of coherent backscattering.
- **$P_{\text{IS}}$**: received power from stable plasma fluctuations.
- **$P_{\text{CH}}$**: received power from unstable plasma fluctuations.
- **$P_{\text{SAT}}$**: instabilities’ saturation power.
- **$P_{\text{MAXI}}$**: maximum contribution to $P_{\text{CH}}$ from elementary region enclosed by an effective volume $V_{\text{eff}}$, centered at $(R, AZ*, EL*)$.

**DISCUSSION OF NUMERICAL RESULTS.**

In these calculations, $\text{AZO} = 360^\circ$ defines the geographic meridian passing through Millstone Hill whereas $\text{AZO} = 345.5^\circ$ identifies the direction along the magnetic meridian. The numbers labeling the curves in figures (4.4) and (4.5) are the altitudes expressed in km. Those corresponding to the thin layer model are only apparent (off-main beam intersections of the 110 km altitude layer).
(1). 'Distributed' instability results.

In both figures (4.2.b) and (4.3.b) the backscattered power shows the strong dependence on the geometry of the region of close-to-perpendicular aspect angle (see appendix 3): above 140 km altitude, maximum power is received when the line of sight lies on the magnetic meridian plane. Below 140 km, perpendicularity may be reached for azimuths between 15 to 35 degrees on both sides of the same plane.

(2). Thin layer model.

The results from the thin layer model are the most relevant in the interpretation of our observations and will be discussed in detail in the next lines. Figures (4.4.a) and (4.5.a) show the LOG10 of the ratio between the total backscattered power (PCH or POWER in the figures) and PMXI. POWER is the sum of all the contributions from the elementary volumes (of dimension Veff) covering the semi-spheric shell defined by the range R and the transmitted pulse length.

PMXI is the maximum value inside this sum. For almost all the line of sight directions and altitudes, this value lies between 0.5 and 1.5 and, therefore, PMXI is a good approximation of the total power for a power drop smaller than 15 dB. Also, the point (R, EL*, AZ*) is a good estimate of the position of the effective scattering region.

In figures (4.4.b) and (4.5.b) angle ANG* or θ*. We can see that the contributions to the coherent power, when observing inside the region between 300 to 400 degrees azimuth, come from the main beam and near side-lobes (smaller than 10°). In particular at 140 km they are mainly from the first side lobe (at 3°) for ELO = 6° (and from EL* = 2° for ELO = 4°) and from the main beam at 110 km.

Outside this azimuthal region, contributions are from far side lobes and the
asymmetry between west (AZO = 270°) and east (AZO = 450°) is because the region of favorable aspect angle is symmetric to the magnetic meridian plane which is tilted 14.5 degrees to the west.

Finally, figures (4.4.c and d) and (4.5.c and d) display the distributions of azimuth and elevation angles of the most probable position of the effective instability volume on the 110 km altitude layer.

In case (c) the 'range contamination' effects are clearly shown: the apparent power-altitude profile, in a given observation direction (AZO, AZ* AZO), is due to the off-main beam intersection of the unstable layer (at EL*). Apparent altitudes correspond to the ranges of these intersections and elevation ELO (the main-beam elevation). Figure (d) shows that the main beam azimuth (AZO) and the effective azimuth (AZ*) are practically equal for the entire field of view.

4. OBSERVATIONS AND DATA ANALYSIS.

Looking to the north at low elevation angles, the radar beam is nearly perpendicular to the geomagnetic field at E-region altitudes and is sensitive to coherent backscatter from strongly field aligned 30-cm irregularities (unstable plasma waves at close to perpendicular propagation). Observation of plasma process occurring at F-region altitudes is also possible due to either off-perpendicular instability generation or the high sensitivity of the Millstone Hill 440 MHz antenna. The geometry of the observations is sketched in figures (A.3.1 and 2) from appendix 3.

(A). THE 'RADIO' AURORA.

High latitude density irregularities in both the E and F regions cover a wide spatial range and are generally associated with the visual aurora and for this
reason are also called 'radio-aurora' and sometimes auroral 'clutter'.

Depending on their spatial extent and lifetime both visual and radio auroras are labeled diffuse (or continuous) and discrete. Discrete auroras are localized and short-lived events while diffuse auroras exhibit a large spatial extent (thousand of km) and lifetime of the order of tens of minutes (Tsunoda et al. [1976]).

In this study we discuss observations pertaining to the diffuse radio-aurora conditions. The auroral zone is co-located with the region of precipitating electrons and protons into the earth's atmosphere and its dynamics are controlled by the ionosphere/magnetosphere coupling (see for example Whalen [1981], Sharber [1981]). A very important global mechanism for instability in this zone is provided by the magnetospheric convection electric field (Foster [1984], Holt et al. [1985]). Other sources of instability are the localized energetic particle precipitation and parallel currents (see review in chapter 1).

During disturbed ionospheric conditions ('substorms'), an enhanced electric field penetrates into the lower latitudes at all local times and the associated ExB flow is a source of instability. In the lower E-region, by effects of collisions between charged particles and neutrals, this flow is indeed a Hall current ('electrojet') and two main types of instability become possible. The first one may be linearly generated for electric fields above 20 mV/m (Farley-Buneman waves). The second one is the fluid-like ExB-gradient drift instability excited at much lower electric fields (10 mV/m). Enhanced fluctuations at 30-cm (secondary waves) resulting from the turbulent 'cascading' of the long wavelength instability seems to explain the observations away from the F-B instability conditions. These two mechanisms are manifested in our data and will be clearly determined using the spectral information in the next chapter. Evidence of other
instability processes above the electrojet region is also noticed.

1. **AVERAGE PATTERNS OF ExB-FLOW AND HALL CONDUCTIVITY AS FUNCTIONS OF PARTICLE PRECIPITATION.**

   **FIGURE (4.6).**

   This figure shows the superposition of the average convection equipotential contours and the enhanced Hall conductivity (color coded) for four different precipitation levels (index p). Values of p = 1, 3, 7 and 9 correspond to plots (a), (b), (c) and (d) respectively. These averages were obtained by Foster et al. [1986.a,b] using 6 years (1978-84) of coincident observations of convection velocity (F-region ion-drift) with the Millstone Hill radar and particle precipitation (electrons and protons with energies between 300 eV and 20 KeV) from the NOAA/TIROS satellite. Index p is proportional to the decimal logarithm of the total precipitating power input to a single hemisphere (Foster et al. [1986.b]).

   The ExB current follows the equipotential contours, sunward outside the polar cap (∝ the circle of 75° invariant latitude) and anti-sunward inside it.

   Invariant latitude is in fact the magnetic field line apex latitude. For us it will mean the same as magnetic latitude unless stated otherwise.

2. **ELECTRIC FIELD STRENGTH vs. LOCAL TIME.**

   **FIGURE (4.7).**

   This figure shows the average electric field strength from the previous figure, as a function of local time and at 63° invariant latitude. E-region altitudes as observed from Millstone Hill at low radar elevation angles ( < 6°) map between 60° to 64° invariant latitude. Farley-Buneman instability may indeed be generated for p > 7 (on average, the electric field strength is about 20 mV/m or greater).

3. **INSTABILITY OCCURRENCE vs. MAGNETIC ACTIVITY.**
FIGURE (4.8).

Disturbed ionospheric conditions are usually characterized by the kp-index. This index is associated with the substorm activity and quantifies the geomagnetic field perturbation (as measured on the ground) produced by the variations in the intensity of the ionospheric electrojet current. From Foster et al. [1986.a], Kp = 1-, 1o, 1+, 2-, 2+, 3-, 3+, 4o, 5-, 6- correspond to p = 1 to 10, respectively.

In this figure we present the Kp-distribution of approximately 20,000 coherent 'echoes' (maximum power for altitudes below 300 km and such that POL = LOG10(POWER) > 6) observed during the 1983-86 period with the Millstone Hill 440 MHz radar. The coherent backscatter is concentrated in the E region.

We may conclude that irregularity occurrence is highly probable for kp greater than 3 (and also corresponding to p > 7).

(B). OBSERVATION PERIODS.

In this study we analyse the data from two types of experimental configurations corresponding to two different spatial resolutions (transmitted pulse lengths).

The first type consists of long-pulse (2 msec), 180° azimuth-scans at two different elevations (4° the July 23-25/83 period and 6° the April 20-21/85 periods). The azimuth resolution is 5° (corresponding to an integration time of 30 sec) and every 180°-scan is completed in 20 minutes. The range resolution of the long-pulse observations is 300 km or 20 to 30 km altitude for 4° to 6° main beam elevation.

The second experimental configuration will be discussed at the end of this chapter. It consists of a relatively short data set of finer resolution observations (0.64 msec) and used to complement the results of the long pulse
measurements.

**JULY 23-25/83 AND APRIL 20-21/85 PERIODS.**

**FIGURES (4.9a), (4.9b) and (4.10).**

These figures display the POL parameter at 120 km altitude, the line of sight component of the F-region ion-drift at about 250 km altitude (VO) and the Kp index for the complete observation periods as a function of universal time (UT, UT = local time (LT) + 5 hours). The vertical bars at the left give the variation scales of three parameters. Numbers on the far right hand side of the panels are the corresponding baseline values.

Increasing values of POL are correlated with increased Kp and F-region ion-drift velocity (for POL > 6 and Kp > 3). The ZIG-ZAG structure of the radar data is the result of the individual 20 minutes 180° AZ scans (West-to North-to East).

Maximum POL corresponds to azimuths about the magnetic meridian. The maximum power level observed was, however, imposed by 'receiver saturation'.

**(C). ALTITUDE AND LOCAL TIME DISTRIBUTIONS OF THE INSTABILITY CROSS-SECTION.**

In this paragraph we discuss the effects of the receiver saturation on our observations and the way to, nevertheless, obtain some estimates of the real location and turbulence level of the unstable region.

**a. Receiver saturation effect.**

When observing the unstable E-region, the receiver saturation often occurs at power levels below that of the instabilities. This level is, yet, few orders of magnitude above that corresponding to the stable (incoherent scatter) density fluctuations. This effect arises because, in order to use our system in both the
incoherent and the coherent 'operation modes', the receiver has to be kept within the dynamic range of the incoherent scattering. In any case, the real saturation level can always be determined by tuning the receiver manually.

The receiver saturation increases the uncertainty in the determination of the location and extent of the unstable region. Calling $\Delta_{\text{RSAT}}$ the associated range smearing, if $R_0$ and $R$ are the first and the last observed ranges in receiver saturation, $R = R_0 + \Delta_{\text{RSAT}}\;$. From equation (8), the 'logarithmic' power (POL) used in our analysis, is proportional to $R^2$ and the LOG10(POWER) at $R$ (POL) is apparently greater than that at $R_0$ (POL0), due to the applied range correction. From equation (8):

$$\text{POL} = \text{POL0} + 2 \log_{10}(R/R_0)$$  \hspace{1cm} (15)

In order to better estimate the value of $\Delta_{\text{RSAT}}$, we use an 'apparent' range resolution. The transmitter pulse length imposes the minimum correlation distance between any two observations. However, in chapter 3 (paragraph 3.D), we mentioned that an estimate of the position of a localized 'scatter' may be obtained within a range resolution smaller than this correlation distance (range smearing $\Delta R$); if the source is stationary for times greater than the pulse length, the 20 $\mu$sec time-step sampling defining the spectrum can be used to estimate the power-range distribution inside $\Delta R$ and the position of the maximum.

In figures (4.11) and (4.12) we use an 'apparent' range resolution of 75 km (one-fourth of the actual resolution).

1. COHERENT ECHO POWER AS A FUNCTION OF ALTITUDE AND LOCAL TIME.

FIGURE (4.11).

The analysed data is the superposition of the two long-pulse observation
periods (July 23-25/83 and April 20-21/85). In frame (a) we display the distribution of the main-beam altitude of the maximum coherent echo power (for POL > 6) as a function of local time. This is the maximum value in the power-range profile for every single radar line of sight measurement. Frames (b) and (c) present the distribution of the number of echo-occurrences as a function of altitude and local time respectively.

Between 100 to 150 km altitude, the receiver was saturated and the altitude of the 'true' maximum power level is expected to be lower and closer to the first range gate at saturation (110 km altitude).

From this figure we may conclude that,

1. There is almost no coherent echo occurrences between 05 to 15 hours LT and its number is maximum about dusk (18 hours LT).

Also, frames (a) and (b) show a very good correlation with the electric field distribution of figure (4.7) when p > 7. This implies that the observed instabilities were basically ExB driven.

2. During all our observations Kp was greater than 3 and from the discussion of figures (4.7) and (4.8) we expect this measured coherent backscatter power in the lower E-region to be the signature of the Farley-Buneman waves as well as the ExB-drift turbulence-related secondaries at 30-cm.

3. From the statistics of frame (c), the range spread associated with the receiver saturation seems to be of the order of 1.5 times the range resolution. In this figure the range separation between two plotted points is equal to \( \Delta R/2 \) for \( \Delta R = 300 \) km (the range resolution) and the 'width' of the distribution is of the order of 3(\( \Delta R/2 \)) (\( \sim 450 \) km, \( \Delta h \sim 40 \) km).

4. Above 150 km, coherent backscatter can be related to other instability mechanisms. Even if in this figure we only display the maximum coherent power
for every single observation, non systematic antenna pattern contamination from lower E-region instability, occurring at greater latitudes, may account for most of the observations. From figures (4.9) and (4.10) Kp > 3, and from the Foster-Evans model the average particle-precipitation during these periods is important. Also, the presence of strong, localized electric fields (above 50 mV/m) is highly probable, and the generation of 30-cm density gradient-drift waves as well as the Post-Rosenbluth instability can't be discarded (see chapter 2). We have no information about field-aligned currents during these periods, but in chapter 2 we found that generation of current-driven instabilities at 30 cm in the lower ionosphere is rather exceptional.

2. RELATIVE POWER AS A FUNCTION OF ALTITUDE.

FIGURE (4.12).

In order to better estimate the extent of the receiver-saturation region, we are displaying, for the same data as figure (4.11), the decimal logarithm of the relative power as a function of altitude but for the two observation periods separately. On July 23-25/83 (EL = 4°) the biggest value of POL was 9.25 and the altitude spread at saturation of the order of 30 km corresponding to ΔRSAT = 1.5 ΔR (ΔR = 300 km). On April 20-21/85 (EL = 6°) the altitude spread is of the order of 45 km and the maximum POL = 8.14.

As we found in equation (15), the value of parameter POL, inside the region of receiver saturation, is greater for the greatest range (or higher altitude). The first altitude at receiver saturation was close to 110 km in both cases and the last one close to 140 km in case (a) and close to 160 km in case (b).

b. Estimate of the instability saturation level.

The power-range profile closely follows the antenna pattern (power in # dB drops, at least, by twice the level given by this pattern, see chapter 3
paragraph 3.C) and knowing $\Delta_{RSAT}$ an estimate of the average turbulence level may be obtained. Below 150 km altitude, the maximum measured power corresponds to the range (let say RO) where the radar main beam intersects the unstable region. This value is of the same order as the receiver-saturation power plus the absolute value of the two-way antenna pattern power-drop at an angle $\delta \theta$ away from the main beam direction (we will consider that the aspect angle attenuation is of the order of 10 dB or less). If EL is the radar elevation, the angle $\delta \theta$ is defined by the range spread at receiver saturation as follows:

$$\delta \theta = \frac{\tan(EL)}{1 + RO/\Delta_{RSAT}}$$

(16)

For the altitude of 110 km, RO = 900 km (at EL = $4^\circ$) and RO = 600 km (at EL = $6^\circ$). Using a value of $\Delta_{RSAT} = 450$ km, we may find:

(1) For EL = $4^\circ$, $\delta \theta \sim 1.4^\circ$ corresponding to a two-way power drop of the order of -40 dB (figure (3.1)).

(2) For EL = $6^\circ$, $\delta \theta \sim 2.5^\circ$ corresponding closely to the -50 dB two-way power drop.

Finally the estimate of the average turbulence level is practically the same for the two cases and close to POL = 13, if any attenuation effect other than the antenna pattern distribution is considered (or POL = 12 if an aspect angle attenuation of -10 dB/degree is assumed).

(D). MORPHOLOGY OF THE E-REGION 'CLUTTER' AT 30-cm.

In this section we discuss the global dependences of the observed instability region. From now on, the analyzed data will consist of two 24-hour segments of the two corresponding observation periods with a range resolution of 300 km.
The first 24-hour set, for the July 23-24/83 period, starts at 16 UT (11 LT) and the second one, for the April 20-21/85 period, starts at 18 UT (13 LT).

1. LATITUDE-LOCAL TIME DISTRIBUTION.

FIGURES (4.13a) and (4.13b).

These figures show the latitudinal (magnetic latitude) extent of the instability region. In both figures, the displayed parameter is the maximum POL (> 6) for every single observation. The local time is displayed in the usual way: midnight-noon (00-12 LT), dawn-dusk (06-18 LT).

E-region altitudes (between 90 to 150 km) map into the 58° to 63° magnetic latitude region. Notice that for the April 20-21/85 period where magnetic activity was higher, echo occurrence became possible at lower latitudes (even south of our station, at 55-56 degrees). In these figures we also include data from a few additional higher elevation angles (up to 88°, going from north to south along the line defined by AZ = 360° and 180°) measured at the end of every 20 minutes, 180°-azimuth scan.

2. ALTITUDE-ASPECT ANGLE BOUNDARIES.

FIGURES (4.14) and (4.15).

These figures show the distributions of all values of POL > 6 for the two 24-hour periods (July 23-24/83 and April 20-21/85) including all the antenna pattern contamination effects above the POL = 6 level. Contamination effects below this level can't be distinguished from stable density fluctuations on the grounds of the cross-section (power) information alone. They can only be filtered out during the analysis of the spectrum (next chapter).

It is interesting to notice that the continuous overlapping of the individual observations, for the two different 24-hour periods, leads to rather well organized patterns (both in intensity and geometrically). In these figures, we
are plotting the projections of the observed points into the Earth's surface along their corresponding Earth radii. The x-axis is positive towards the geographic east (right) and is centered at Millstone Hill. The y-axis points towards the geographic north.

Full-line contours circling the data correspond to the boundaries of the lower E-region (semi-circles of 95 and 125 km altitude, centered at (0,0)) and to the boundaries of the favorable aspect angle region (quasi-circular contours at 89° and 91° aspect angle, symmetric to the magnetic meridian plane through (0,0) at 14.5° NW, see appendix 3). The intersection between these two regions defines the most probable unstable region.

Inner broken-line contours correspond to the 'apparent' 125 km altitude and the 91° aspect angle boundaries, including the range smearing effects. 'Apparent' means that these contours correspond to the 125 km altitude and the 91° aspect angle but calculated at a lower elevation angle. This angle difference is associated with the half of the range smearing.

Outer broken-line contours correspond to the intersection of the tangent plane at Millstone Hill and the spherical shell at (125 + Δh/2) km altitude (this boundary will be called the lower E-region 'horizon') and to the 91° aspect angle for zero-degree elevation. Δh = ΔR sin(EL) is the altitude uncertainty associated with the range smearing. The outer 91° aspect angle boundary encloses the region where any side-lobe intersection of the instability region below (125 + Δh/2) km altitude can still make an angle smaller or equal than 91° with the magnetic field. The E-region 'horizon' altitudes corresponding to the elevations of 4° and 6° are, respectively, equal to 220 km and 250 km and equal to ~250 and 300 km considering a range smearing of 300 km (corresponding to the 2 msec pulse length). Finally, from the analysis of these two figures we may conclude:
(1) All the coherent backscattering coming from altitudes below 220 km (EL = 4°) or 250 km (EL = 6°) may well be associated with lower E-region, strongly field aligned instabilities.

(2) Above the region limited by the E-region horizon, the effects of the unstable electrojet are not felt. In this region also, aspect angles are always greater than 91°.

FIGURES (4.16) and (4.17).

These figures correspond to the same data as before but now we only display the power maxima (at the smallest range at which receiver saturation occurs), from every measured power-range profile. Different maxima may be related with different instability mechanisms at different altitudes. The power profile is, however, strongly modulated by the antenna pattern and secondary maxima associated with side-lobe observations of instabilities generated in the electrojet region but at higher latitudes can't be discarded.

From the results of the thin layer model simulations (paragraph 4.3), off-main beam contributions up to the first-side lobe (at 3° away from the main beam direction) are at the origin of a secondary maximum in the power profile at altitudes close to the instability layer 'horizon'. This effect is clearly shown in the EL = 6° observations. In figure (4.17) secondary maxima presumed from first-side lobe contributions follow the E-region outer boundary.

On the EL = 4° data (figure (4.16)) the not so clear presence of the 3° side lobe contributions is explained by the one-degree or so elevation shielding due to the topography. The more suitable elevation to study the unstable auroral ionosphere with our radar is, therefore, of the order of 4° or smaller. Only in these cases we may try to separate the different driving mechanisms, corresponding to secondary maxima, by using a better range resolution measurements.
Also, figure (4.17), for the April 20-21/85 period, shows some cases of instability generation at a few degrees away from perpendicularity (presumably less than 3 to 4) when the main-beam intersects the lower E-region.

(E). ALTITUDE-AZIMUTH DISTRIBUTIONS.

Figures (4.18), (4.19) and (4.20) display the altitude-coded distributions of the decimal logarithm of the average power as functions of the main beam azimuth. The analysed periods are the same as figures (4.14) to (4.17) and the average is calculated at every range and azimuth for values of POL > 5, over the entire observation periods. Receiver saturation is responsible for the superposition of E-region altitude contours (between 100 to 150 km).

Instabilities are basically concentrated in a relatively narrow horizontal layer in the lower E-region and main-beam altitudes are only apparent; in this layer the actual location of the coherent source is defined by the magnetic latitude and the azimuth of the observation. The magnetic latitudes corresponding to main-beam altitudes between 100 and 300 km, range between: (a) at 4° elevation, 61° to 67° for AZ \( \sim 360° \), 59.5° to 63.5° for AZ \( \sim 310° \) and 60° to 65° for AZ \( \sim 380° \). (b) at 6° elevation, 59.5° to 66° for AZ \( \sim 360° \), 58° to 62° for AZ \( \sim 310° \) and 59° to 64° for AZ \( \sim 380° \).

For a given azimuth and altitude the average considers all the measured powers such that POL > 5. On July 23-24/83 this average was performed over the entire 24-hour length for the same data presented in figure (4.14) whereas for the highly-disturbed April 20-21/85 period (same as figure (4.15)) the average was performed over two different time-sectors, the local evening and the local morning sectors. These figures show the good agreement between the qualitative results of the thin layer model (figures (4.2a) and (4.3b)) and this averaged
data, in particular, the similarity with the July 23-24/83 observations is remarkable (the magnetic activity during this period was moderately disturbed).

An average turbulence level of POL - 13 may be more suitable than the value of 12 used in the model to represent the observations.

Near the magnetic meridian (AZ $\sim 345^\circ$) the power on the altitude contours drops much faster than in the thin layer model. This fact may be explained, as we will discuss in the next chapter, by the presence of secondary waves and 'flow angle' effects not considered by this simple model. In this region, the radar line of sight is, generally, closely perpendicular to the flow direction and the primary type 1 instability is not possible. We thus only observe secondary waves for which the saturation level is nearly -20 dB lower than that of the primaries (this result agrees with those reported by Farley et al. [1981] and, Andre [1982]). Observations corresponding to the April 20-21/85 period show a less striking similarity with the model predictions probably because this was a highly disturbed and variable magnetic-activity period.

Finally, given the poor range resolution of this data set, we may only conclude that, on statistical grounds, the lower E-region instability is dominant and uniformly distributed in a layer of vertical spread ('thickness') much smaller than height-resolution given by the combination of the radar-pulse length and the receiver saturation effect (40 km) inside this region. The altitude where the maximum power (turbulence level) occurs as well as the aspect angle dependence, will be more accurately determined using a better altitude resolution data set (without instrumental saturation) in the next section.

(F). HEIGHT AND ASPECT ANGLE DEPENDENCES.

In this final section we analyse a relatively short (two-hours length), finer
height-resolution data set. The particular interest of these observations is that during this entire period the receiver was tuned to avoid instrumental saturation and the observation direction may have been closely parallel to the ExB direction (as expected from the observed average convection field pattern). If so, the maximum received power (slightly above POL = 12) was presumably that of the primary-instability (type 1) saturation.

1. THE OBSERVATION PERIOD.

_Figure (4.21)._ This figure introduces the two-hour observation period on July 31/84 (from 16:30 to 18:30 LT). The transmitted pulse length of 0.64 msec corresponds to 100 km range-resolution. The radar operation consisted of elevation angle scans (from 2 to 5 degrees back and forth, integrating over 30 secs. at each elevation) at a fixed azimuth (AZ = 380° or 20°NE). This azimuth corresponds to the region where coherent echoes are normally the strongest. The data was taken during very disturbed magnetic conditions (Kp ~ 6). The figure displays parameter POL at three different main-beam altitudes and the index Kp as functions of universal time; greater values of POL were observed between 110 and 120 km altitude. The numbers on the right side of the listed parameters give the variation scale between the baselines (on the far right hand side of panel) and the top horizontal segments close to these names. The maximum received power was 12.03 for an altitude of 116 km (or 12 and ~110 km correcting the R^2 dependence of POL for a range smearing of 100 km).

2. ALTITUDE-ASPECT ANGLE DISTRIBUTIONS.

_Figure (4.22)._ This figure shows the distribution of the LOG10 of the relative power (POWER/PMAX), as function of the altitude (plot (a)) and of the aspect angle (plot (b)). PMAX is the maximum measured power and LOG10(Power/PMAX) = POL - 12.03. We only plot values of POL greater than 6. The altitude of this maxi-
mum is 116 km (the center altitude of the unstable region may, however, be statistically determined to minimize the observational incertainties). The full-line curves superimposed on the data represent the \( \log_{10} \) of \( PAV/PMAX \), where \( PAV \) is the average power at each altitude.

The best estimates of the center altitude (ALTO) and the aspect angle \( \theta_{ASP}^\circ \) of the unstable lower E-region are determined by averaging the points inside the 'half-power' drop level (-3 dB level or -0.3 in our figures):

\[
\text{ALTO} = \frac{\sum_{i=1}^{N} ALTi}{N} = 108 \text{ km}
\]

\[
\text{THICKNESS} = \sqrt{\frac{\sum_{i=1}^{N} (ALTi - ALTO)^2}{N}} = 8 \text{ km}
\]

\[
\Delta \text{ALT} = \sqrt{\frac{\sum_{i=1}^{N} (\Delta R \sin (ELi)/2)^2}{N}} = 2 \text{ km}
\]

\[
\theta_{ASP}^\circ = \frac{\sum_{i=1}^{N} \theta_{ASP}^i}{N} = 90.2 \text{ degrees}
\]

\[
\Delta \theta_{ASP} = \sqrt{\frac{\sum_{i=1}^{N} (\theta_{ASP}^i \cdot \theta_{ASP}^\circ)^2}{N}} = 0.45 \text{ degree}
\]

\( N \) is the total number of points inside the -3 dB region and equal to 70. The thickness of the unstable layer is in fact the half-power altitude spread, \( \Delta \text{ALT} \) is given by the average of the different altitude resolutions for the different elevation angles (ELi). The calculated thickness is, however, comparable to the vertical spread of our 1° main-beam width.

3. 'MACROSCOPIC' ASPECT ANGLE SENSITIVITY.

In chapter 3 (section 2) we mentioned that the 'macroscopic' aspect angle sensitivity (ASP) is a measure of the turbulent diffusion in the \( k_n \) space for uns-
table waves propagating closely perpendicular to the magnetic field. The experimental determination of the aspect angle sensitivity is strongly biased by the radar convolution effects and we will call this estimate the 'radar' aspect angle sensitivity. If the antenna main beam intersects the unstable region, the received power will decrease exponentially with $\Delta \theta_{\text{ASP}} - |\theta_{\text{ASP}} - \theta_{\text{ASP}}^\circ| \) (see chapter 3, equation (10.b)), where $\theta_{\text{ASP}}$ is the aspect angle of the observation and $\theta_{\text{ASP}}^\circ$ that of maximum linear instability growth. A good estimate of $\theta_{\text{ASP}}$ may be obtained by considering data from a region where it can be assumed that no attenuation other than aspect angle effects are important and hence, the $\text{LOG}10$ of the relative power level is proportional to the aspect angle deviation. From equation (3.12b), $\Delta \text{POL} = \text{LOG}10(\text{POWER}/\text{PMAX}) = 0.1 \theta_{\text{ASP}}$.

In this region we must also assume a stationary turbulence level (any real time variability is not expected to be systematic). In the statistical sense, the determination of the aspect angle sensitivity is also a test of the linear correlation between $\Delta \text{POL}$ and $\Delta \theta_{\text{ASP}}$. The extent of the region where this test applies is given by the maximum error in the determination of the aspect angle and is defined by the antenna gain pattern, the range smearing, and the radar elevation angle. Considering the 100 km range resolution and the maximum radar elevation of 5° for the observation period here analysed, this error is of the order of 0.5° plus the statistical uncertainty in the determination of $\theta_{\text{ASP}}^\circ$ (≈ 0.5°) and, therefore, the lower power limit of the correlation region is the -20 dB level (close to the two-way antenna power drop at 1° away the main beam direction, see figure (3.1b)). Figure (4.22a) show clearly the convolution of the antenna-beam shape with the power-altitude profile; from figure (4.22b) this effect does not seems to strongly bias our aspect angle sensitivity analysis.

'GAUSSIAN CONDITION'.
We may impose another condition for the estimation of ASP by assuming that inside the -3 dB region (where the average $\Theta_{ASP}^o$ is calculated) the variables $\delta_{POL}$ and $\delta_{ASP}$ are normally distributed with variances $\Delta_{POL}^2 \sim (0.3)^2$ and $\Delta_{ASP}^2 \sim \Delta_{ASP}^2$ respectively. In this case, $\Delta_{POL} < \Delta_{ASP}$, and values of $\delta_{ASP} < 0.45^o$ are correlated only with values of $\delta_{POL}$ such that $(\delta_{POL})^2 < 11 \Delta_{ASP}^2$ (or $\delta_{POL} < 1.5$ with a confidence close to 100%, F-distribution test, Bendat and Piersol [1971]). Therefore, in this case points with $\delta_{ASP} < 0.45^o$ and $\delta_{POL} > 1.5$ do not need to be considered.

**ESTIMATION OF ASP.**

**FIGURES (4.23a) and (4.23b).** In these figures we present the determination of the aspect angle sensitivity; in figure (a) we consider all the measured points (192) inside the -20 dB region and the full-line gives the equation $\delta_{POL} = -0.38 - 0.72 \delta_{ASP}$, for a correlation coefficient of 0.52 (99% confidence interval) and corresponding to a value of ASP = -7.2 dB/degree. After applying the 'Gaussian condition' to the data set of figure (a) we obtained the 162 points displayed in figure (b) (points on the left-side of the arrows in figure (a) were excluded). The correlation coefficient for the equation $\delta_{POL} = 0.04 - 1.07 \delta_{ASP}$ is equal to 0.62 (also 99% confidence) and the corresponding aspect angle sensitivity is equal to -10.7 dB/degree.

As we mentioned already, the determination of the aspect angle sensitivity and the thickness of the unstable region are biased by the effects of radar convolution; this is true for ours as well as other radar estimates of ASP. The values of ASP given by our analysis ($\sim$ -7 and -11 dB/degree) are in good agreement with previous determinations (Ecklund et al. [1975], McDiarmid [1976]).

A better estimate of ASP will need a number of observations with finer height resolution at different conditions (in order to test its possible variation).
CHAPTER 4.

FIGURE (4.1). The main beam observation of the stable F-region (at $\mathbf{R}_a$) is 'contaminated' by the strong backscatter from the unstable E-region (at $\mathbf{R}_b$), observed out of the main beam but at the same range ($\mathbf{R}_a = \mathbf{R}_b$). In the bottom of this figure we display the typical spectral signatures corresponding to the main beam observations at the points $\mathbf{R}_a$ (right hand side) and $\mathbf{R}_b$ (left).

FIGURE (4.2). Displays the total power resulting from the simulation of the backscattering from an unstable region with aspect angle sensitivity of $-10$ dB/degree. In case (a), instability is constraint to a thin layer of fixed altitude (110 km) whereas in case (b) instability is distributed over the whole region where the aspect angle is less than $\approx 4^\circ$ away from perpendicular to the ambient magnetic field. The radar is looking to the north at an elevation angle of $4^\circ$.

The horizontal axis displays the main beam azimuths (AZ), the geographic east is defined by AZ = 270°, the west by AZ = 450°. The vertical axis gives the LOG10 of the backscattered power expressed in cm-3 (parameter POL defined in paragraph 2.B, chapter 4). The numbers labeling the curves are the main beam altitudes in km. For case (a) these altitudes are apparent (the backscattering is actually coming from the off-beam intersections of the unstable layer).

FIGURE (4.3). Same as figure (4.2) but for a main beam elevation of $6^\circ$. 
FIGURE (4.4). Here we show the results of the calculations with the thin layer model (figure (4.2.a)) for a main beam elevation of $4^\circ$. The horizontal axis in all the presented cases is the same as figure (4.2).

(a) POWER is the total power and PMXI is the maximum contribution to it from the region enclosed by an equivalent effective scattering volume. The vertical axis displays the relative power in 'logarithmic' units and indeed its value can be greater than one. We can see that for almost all observation directions (AZ), PMXI is a good approximation of the total power within a power drop smaller than 15 dB.

(b) ANG* is the azimuthal angle away from the main beam direction where contribution PMXI is generated.

(c) and (d): the point $(R, \xi/2 - EL*, AZ*)$, in the spherical system centered at Millstone Hill, defines the position of the region where PMXI is coming from. This point is given by the intersection between the arc of radius $R$ (the range), in the plane containing the radar line of sight, and the unstable layer (because $AZ* \sim AZ$).

FIGURE (4.5). Same as figure (4.4) but for a main beam elevation of $6^\circ$.

FIGURE (4.6). Here we show the superposition of the average convection equipotential contours and the enhanced Hall conductivity (color-coded) for four different precipitation levels (index $p$) from the Foster-Evans model (Foster et al. [1986.a, b]). Cases (a), (b), (c) and (d) correspond to values of $p = 1, 3, 7$ and 9, respectively. Index $p$ is proportional to the decimal logarithm of the total precipitating power input to a single hemisphere.
FIGURE (4.7). This figure shows the average electric field strength corresponding to the figure (4.6) as a function of local time and at 63° magnetic latitude. E-region altitudes as observed from Millstone Hill at low elevation angles (< 6°) map between ~58° to 64°. Farley-Buneman instability is expected for average electric field strength of the order of 20 mV/m or greater (and for p > 7).

FIGURE (4.8). Here we show the correlation between the magnetic activity (Index Kp) and the number of coherent 'echo' occurrences for approximately 20,000 cases during the years 1983-85. We have only considered the maximum power at every particular observation direction for altitudes between 100 to 300 km and such that LOG10(POWER) > 6.

FIGURE (4.9). Displays the parameter POL - LOG10(POWER) (on the left hand side) between 110 to 130 km altitude, the line of sight component of the ion-drift velocity at F-region altitudes (~250 km) identified by VO, and the index Kp for the July 23-25/83 observation period. Numbers on the right hand side of the parameter names give the variation scale between the zero-line (numbers on the far right hand side of panel) and the horizontal segment close to these names. The step-function increase in POL denotes the interval of coherent-echo observation. This is correlated with increased Kp and ion velocity. See section (4.B) (chapter 4) for the description of the experimental configuration.

FIGURE (4.10). Same as figure (4.9) but for the April 20-21/85 period.
FIGURE (4.11). Here we display the distribution of the maximum value of \( \text{POL} > 6 \) (for every observation direction) as a function of altitude and local time (a), for the two measurement periods put together. In (b) and (c) we show, respectively, the distribution of echo-occurrence as a function of altitude and local time.

FIGURE (4.12). For the same data as figure (4.11) we plot the LOG10 of the relative power as a function of altitude for each observation period (cases (a) and (b)). \( \text{PMAX} \) is the receiver saturation power, the first altitude at saturation is close to 105-110 km for both periods, and the last altitude is close to 140 km in case (a) and to 160 km in case (b).

FIGURE (4.13). Here we show the latitudinal (magnetic) extent of two 24-hour periods (cases (a) and (b)) of coherent 'echo' occurrences. For elevation angles between 4° and 6°, E-region altitudes (\( \sim 90 \) to 140 km) map into the 58° to 63° magnetic latitude circles. The plotted parameter is the maximum \( \text{POL} > 6 \) at every radar line of sight. Case (a), for the July 23/83 period, starts at 11:00 LT and case (b), for the April 20/85, starts at 13:30 LT.

FIGURE (4.14). This figure shows the distribution of all values of \( \text{POL} > 6 \) for the same 24-hour period of figure (4.13a) and for main elevation = 4°. The observed points are projected along their corresponding Earth's radii to the Earth's surface. Axis x is centered at Millstone Hill and positive towards the geographic east (to the right), axis y is positive towards the geographic north. Full line contours encircling the data points correspond to the boundaries of the lower E-region as seen by the radar experiment (semi-circles at 95 and 125
km altitude) and to the boundaries of the region of favorable aspect angle (quasi-circular contours at 89° and 91°, symmetric to the magnetic meridian plane through Millstone Hill). Inner broken-line contours correspond to the 125 km altitude and 91° aspect angle boundaries considering range smearing due to the radar pulse length. Outer broken-line contours define the intersection between the tangent plane at Millstone Hill and the spheric shell at 125 km altitude ('horizon') and the 91° aspect angle contour at zero elevation.

**FIGURE (4.15).** Same as before but for the April 20-21/85 period at 6° elevation.

**FIGURE (4.16).** This figure is similar to the (4.14) but now we only display the maxima of POL > 6 for every measured power-range profile. If the effects of the antenna side-lobes are discriminated, secondary maxima may correspond to the signatures of different instability mechanisms at different altitudes. We can see that instability is concentrated in the lower E-region; at EL = 4° the first side-lobe (at \(\sim 3°\)) is practically shielded by the Earth's topography.

**FIGURE (4.17).** Same as the previous figure but for the April 20-21/85 at EL = 6°. In this case the sum of the backscattered power from the 3° side-lobe intersections of the unstable lower E-region down to zero elevation produce an 'image' of this region at greater ranges (close to the 'horizon').

**FIGURE (4.18).** This figure displays the altitude-coded distribution of the averaged power, in 'logarithmic' units, as a function of the main beam azimuth (AZ). Receiver saturation is responsible for the superposition of the 100 to 150
km altitude contours. The average was performed over the entire 24-hour period on July 23-24/83. See text for discussion.

FIGURES (4.19) and (4.20). Show similar distributions to figure (4.18) but now the average over the 24-hour on the April 20-21/85 period has been performed on two parts, one covering the local morning-noon sector (4.19) and the other the local afternoon-evening sector (4.20).

FIGURE (4.21). This figure provides an overview of the two-hour observation period on July 31/84 consisting of a number of elevation scans (from 2° to 5°) at a fixed azimuth (≈ 20° NE). It displays parameter POL at three different altitudes (≈ 95, 115, and 135 km) and index Kp as functions of the universal time (UT = local time + 5). Numbers on the right hand side of the parameter names give the variation scale between the zero-lines (numbers on the far right hand side of panel) and the top horizontal segment close to these names.

The receiver was tuned to avoid instrumental saturation and the maximum observed power (POL ≈ 12) is, presumably, that of the type 1 waves.

FIGURE (4.22). Here we show the distribution of the LOG10 of the relative power (for July 31/84) as a function of altitude (a) and aspect angle (b). The full-line curves superimposed on the data-points are the average values. The best estimates of the center altitude of the unstable layer (108 km) and its corresponding aspect angle (90.2°) are determined by averaging the data-points inside the - 3 dB power level ('half-power' drop). The vertical spread of this layer ('thickness') is equal to 8 km.
FIGURE (4.23). This figure shows the determination of the aspect angle sensitivity (ASP) of the type 1 waves, the full-lines are the result of the linear correlation between the decimal logarithm of the relative power and the aspect angle deviation.

In case (a) we consider all the data points inside the -20 dB region and find a value of ASP $\sim -7$ dB/degree. In case (b) we excluded the points on the left side of the arrows showed in figure (a) ('Gaussian' condition, paragraph (4.F.3) chapter 4) and found that ASP $\sim -11$ dB/degree.
FIGURE 4.1
THIN LAYER ALT = 110 KM, MAIN BEAM EL = 4°

(a)

DISTRIBUTED INSTABILITY, MAIN BEAM EL = 4°

(b)

FIGURE 4.2
THIN LAYER ALT=110 KM, MAIN BEAM EL=6°

DISTRIBUTED INSTABILITY, MAIN BEAM EL=6°

FIGURE 4.3
THIN LAYER ALT = 110 KM, MAIN BEAM EL = 4°

(a)

(b)

FIGURE 4.4
THIN LAYER ALT=110 KM, MAIN BEAM EL=4°

FIGURE 4.4

(c)

(d)
THIN LAYER ALT = 110 KM, MAIN BEAM EL = 6°

(a)

(b)

FIGURE 4.5
ELECTRIC FIELD AT 63 deg

MAGNETIC LOCAL TIME (hours)

FIGURE 4.7
ALL COHERENT ECHOES YEARS 1983 TO 1986

FIGURE 4.8
FIGURE 4.9
FIGURE 4.9

(a)

(b)
FIGURE 4.10
FIGURE 4.11

DISTRIBUTION OF \( \log_{10}(\text{POWER}) > 6 \)
FIGURE 4.12
MILLSTONE HILL AZIMUTH SCAN

1983 7 23 - 24

16 0 12 16 6 23 UT

LOG POWER (POPL)*1000

9000
8700
8400
8100
7800
7500
7200
6900
6600
6300
6000

(a)

1985 4 20 - 21

18 29 36 18 3 39 UT

LOG POWER (POPL)*1000

8000
7800
7600
7400
7200
7000
6800
6600
6400
6200
6000

(b)

FIGURE 4.13
MILLSTONE HILL AZIMUTH SCAN

LOG POWER (POPL) * 1000

GROUND DISTANCE FROM MILLSTONE HILL (km)

FIGURE 4.14
FIGURE 4.15
MILLSTONE HILL AZIMUTH SCAN

LOG POWER (POPL) * 1000

GROUND DISTANCE FROM MILLSTONE HILL (km)

FIGURE 4.16
FIGURE 4.17
MILLSTONE HILL AZIMUTH SCAN

1983 7 23 - 24

16 0 12 16 6 23 UT

LOG10(POWER)

ALTITUDE (KM)

FIGURE 4.18
MILLSTONE HILL AZIMUTH SCAN

1985 4 20

18 29 36

4 0 35 UT

LOG10(POWER)

ALTITUDE (KM)

AZIMUTH

FIGURE 4.19
FIGURE 4.20
FIGURE 4.21

1984 7 31

AZ = 375.385
EL = 0.5
ALT = 130,140

AZ = 375.385
EL = 0.5
ALT = 110,120

AZ = 375.385
EL = 0.5
ALT = 90,100

Kp 5

UT (hours)
FIGURE 4.22

(a) 

(b)
FIGURE 4.23
CHAPTER 5

THE ANALYSIS OF THE TURBULENCE SPECTRUM AT 30-cm

A wide variety of physical parameters may be deduced from the accurate measurement of the frequency spectrum. In the backscatter observations the radar acts as a spectrum analyser picking out only the component of the fluctuations spectrum with wavelength half of the radar wavelength and propagating along the line of sight (Bragg diffraction condition). The spectrum of the stable plasma modes ('incoherent') is the superposition of two components associated respectively with the damped ion-acoustic and Langmuir oscillations. In a turbulent plasma, the power spectrum may consist of one or more very large amplitude peaks associated with the unstable modes ('coherent'). The displacement of every peak from the transmitted frequency (Doppler-shift) measures the waves' phase velocity.

The spectral shape is in general arbitrary, but, during saturation conditions, only the two first moments determine the presumed Lorentzian spectrum. In these cases, the power peak and the spectral width define, respectively, the saturation amplitude and the turbulence relaxation time.

In this chapter we present the determination of the instability spectrum and discuss its properties for a number of cases corresponding to the observation periods introduced in the previous chapter. Because of the unique capability of the Millstone Hill system to measure both the stable and the unstable density fluctuations we are able to estimate (from a number of measured radar line of
The transmitter frequency (440 MHz) is too high for common radar signal amplifiers and it is usual to multiply it by a 440 - $\Delta \omega$ frequency in order to obtain a resultant intermediate frequency (IF) signal. This new signal carries all the information unchanged and at Millstone Hill $\Delta \omega = 30$ MHz.

The incoming IF signal is stochastic and the quantity of interest is the ACF defined by:

$$\rho_\lambda (R, \tau) = \langle \eta(R, t) \eta^*(R, t + \tau) \rangle / \langle |\eta(R, t)|^2 \rangle \quad (1)$$

$\eta(R, t)$ is the complex signal (containing both amplitude and phase information) associated with the scattering process and $\rho_\lambda$ is proportional to the ACF of the fluctuating electron density (during both stable and unstable conditions) and the angular brackets denote an ensemble average. $R$, is the center of the effective scattering volume and $\tau$ the time lag. Fluctuations with time scales smaller than the pulse length ($\tau_p$) are undistinguishable from the time variations $\tau$. As we mentioned in chapter 3, time resolution and range discrimination are not independent. Roughly speaking the time lag resolution is defined by the pulse length or the inverse of the receiver bandwidth, whichever is smaller, and the range resolution by whichever is larger (Rino [1972]).

In addition to the scattered signal, cosmic and receiver noise are present in each output channel and:
\[ \tilde{z}(t) = \tilde{z}(t) + \eta(t) \]  

\( \tilde{z}(t) \) is the scattered signal and \( \eta(t) \) is the total noise. They are assumed to be uncorrelated and, a separate estimate of the noise ACF must be subtracted from the equation (1) to obtain the ACF of the signal alone.

At Millstone Hill the noise is subtracted at every measured range gate using a dual frequency scheme (440 and 440.2 MHz): noise and calibration data are taken at greater range without reducing the duty cycle. In addition to the random errors systematic instrumental errors also occur which are due to the finite length of the transmitted pulse and the finite receiver bandwidth.

The correlation function of the signal \( \tilde{z}(t) \) is the result of the product of the 'true' plasma correlation function \( \rho \) with the ACF of the transmitted pulse \( F_p \) and the convolution with the gating function \( F \) (Farley [1969]):

\[ < \tilde{z}(t) \tilde{z}^*(t + \tau) > = K \int d\omega \rho(\omega, \tau - \tau'; R)F_p(\tau - \tau_1)F_q(\tau_1) \]  

\( K \) is a proportionality factor, \( \rho(\omega, \tau, R) \) is the plasma-waves correlation time (the spatial Fourier transformation of the plasma ACF). The receiver bandwidth is large and \( F_q(\tau) \) is approximated by a delta function. Also in the single pulse technique the transmitted pulse is closely square and \( F_p \) has a triangular shape and is zero for \( |\tau - \tau_1| > \tau_p \) (Fadaray rotation effects can be neglected).

**ESTIMATE OF THE ACF.**

Experimental estimates of the autocorrelation function \( \rho_7 \) must be formed from a finite number of samples of the radar IF signal. The sampling frequency is equal to the receiver bandwidth (BF) and the total number of
samples at a given range gate is equal to \( N = BFz_p \).

Calling \( z_i \) \((1 \leq i \leq N)\) a sample of the complex signal \( \tilde{z}(t) \), an estimate of the ACF at lag \( z \) is given by:

\[
\hat{\rho}_{zz}(z) = \frac{1}{N-z} \sum_{i=1}^{N-z} z_i z_{i+z}^{*} \quad 0 \leq z \leq N - 1 \tag{4}
\]

At Millstone Hill, \( N \) is fixed at 64 for the longer pulses \((z_p \geq 1.28 \text{ msec})\).

From the sampling theorem (Blackman and Tukey [1958]) a stationary random process \( \tilde{z}(t) \) can be arbitrarily closely approximated in the interval \( T \) by a number \( N \) of equally spaced samples of length \( \delta z \) \((T = N \delta z, \delta z = 1/BF)\). The frequency resolution in the associated power spectrum is equal to \( 1/T = BF/N \) (twice the Nyquist frequency).

From equations (1), (2), and (3) the expected value of the observed ACF at lag \( z \) is expressed by (Hagfors [1977]):

\[
\langle \hat{\rho}_{zz}(z) \rangle = \hat{\rho}(z)F_p(z) + \langle \hat{\rho}_n(z) \rangle \tag{5}
\]

The 'hats' over all variables represent the 'best' estimate values. In the actual data reduction, a large number of pulses are averaged before the noise substraction in order to minimize the statistical uncertainty in the estimate of \( \langle \rho_{zz}(z) \rangle \). This estimate will improve as the integration time is increased provided the process is ergodic. We currently use an integration time of 30 sec. This time, however, can be much smaller if we are only interested in the instability spectral information (and not in the surrounding stable state plasma).

After the substraction of the noise ACF estimate and the correction for pulse
shape $F_p$, equation (5) yields an estimate of the plasma ACF. The spectral density function is the temporal Fourier transform of the plasma ACF:

$$S(k, \omega) = 2\text{Re}\left(\int_0^\infty d\tau \exp(-i\omega \tau) f(k, \tau; R)\right)$$

(6)

Where $\omega = 2\pi f (l = 1, N)$ and $f$ is of the order of $1/\tau_p$.

2. SIGNAL-TO-NOISE RATIO (SNR).

Most radar applications are concerned with the detection and filtering of a signal embodied in the noise. The spectral power of the signal carrying the relevant physical information is usually smaller than the one of the noise.

The ratio between the signal and the noise powers (SNR) quantifies the limits of the filtering procedure. If both signal and noise are Gaussian stationary processes with zero mean and variances $\sigma_s^2, \sigma_n^2$, the ratio between the signal and noise powers is expressed by:

$$\text{SNR} = \frac{\sigma_s^2}{\sigma_n^2}$$

(7)

The statistical uncertainty in the measurement of a property $A$ from a broadband signal in presence of noise and systematic errors is expected to be of the form (Baron [1977]):

$$\frac{\sigma_A}{m_A} = \frac{1}{\sqrt{N_S}} \left( a + \frac{b}{\text{SNR}} \right)$$

(8)

$m_A$ and $\sigma_A$ are respectively the mean and the variance of the derived normal.
process from the measured signal. \( N_s \) is the number of samples, \( a \) and \( b \) are coefficients dependent on the details of the signal processing. \( N_s = N_{BF} N_{P} \), where \( N_{P} \) is the number of pulses during the integration time. In the single pulse technique, the uncertainty in the measurement of the power of the scattered signal is, within a good approximation, given by (Farley [1969]):

\[
\frac{\Delta P}{P} = \frac{1}{\sqrt{N_s}} \left( 1 + \frac{1}{\text{SNR}} \right)
\]  (9)

The last relationship is derived from equation (5) neglecting the instrumental errors. From equation (2), \( P = \sigma^2_{\text{SNR}} \) and \( \Delta P = \sigma^2_{\text{SNR}} / \sqrt{N_s} \) is the variance of the new process consisting of \( N \) independent observations of the Gaussian signal \( \sigma^2_{\text{SNR}} \).

\( P \) is a measure of \( \sigma^2_{\text{SNR}} (k) \) the scattering total cross section (chapter 4).

3. SPECTRUM SEPARATION.

Under normal ionospheric conditions (stable plasma), the signal to noise ratio obtainable with the single pulse technique is generally not greater than unity. Moreover, the minimum SNR for which our radar system may still recover any IS signal ('incoherent scattered' or stable) embedded in the noise is 0.1.

When unstable fluctuations are 'picked out' by off main beam backscattering, the received signal is broadband and its spectrum is the superposition of an IS component from the stable plasma fluctuations (in the region intersected by the main beam) and one or more peaks associated with the unstable oscillations from elsewhere (figure (4.1)). Main beam observations of instabilities as well as 'hard target' returns (from satellites or other) are characterized by \( \text{SNR} \gg 1 \).

In cases where \( \text{SNR} < 10 \) the IS spectrum and the coherent 'contamination' may be separated. Because generally \( \text{SNR} < 1 \), the coherent component may be
considered as the 'noise' \((N_{CH})\) thus, \(S_{IS}/N_{CH} > 0.1\). When \(SNR > 10\), only coherent backscattering is measured, in this case the IS 'contamination' from stable regions is completely screened. In the E and the lower F regions, the coherent radar returns are basically due to unstable plasma processes. Higher in the F-region 'false' instability returns from satellites are a common feature. Radar study of a whole class of current driven instabilities in the top side ionosphere is however a new exciting possibility (Foster et al. [1988]).

(A). SEPARATION ALGORITHM.

For a wide range of non equilibrium conditions, the ionospheric plasma may be stable but the spectrum of the thermal fluctuations is not necessarily symmetric (after substraction of the Doppler-shift). The presence of field aligned currents induce an asymmetry in the ion-line spectrum (Rosenbluth and Rostoker [1962], Rino [1972]). As in the 'normal' non-equilibrium 'symmetric' cases \((T_e/T_i < 10\), moderate perpendicular currents, etc), the symmetric part of the stable spectrum leads to the determination of the plasma state inside the scattering volume. The use of a dual IF frequency (28 and 32 MHz) at Millstone Hill eliminates any instrumental asymmetries in the measured spectrum. True spectral asymmetries will be treated in the same way as the coherent contamination. A good estimate of the F-region ion-drift velocity (ion-line Doppler shift for a symmetric spectrum) yields to a good determination of the local electric field (as we discuss in the next section 4).

Situations where \(SNR > 10\) and \(POL > 6\) are unambiguously related with the presence of plasma instabilities (if they are not satellites).

The Rosenbluth-Rostoker ion-line asymmetry may be associated with the onset of current driven instabilities (ion-cyclotron, ion-sound and, Buneman) in the top-
side ionosphere (Kindel and Kennel [1972], Foster et al. [1988]). We have no
evidence of bottomside F-region current-driven related processes at 30-cm. This
is not surprising due to the screening effects (antenna pattern contamination)
from the statistically dominant lower E-region instability. We have, however,
some evidence of the generation of other types of instabilities, but the unambi-
guous identification of the driving mechanisms needs complementary information.

The measured spectrum of the ionospheric plasma fluctuations during both stable
and unstable conditions may be classified as follows:

1. Symmetric stable spectrum (IS): generally $0.1 < \text{SNR} < 1$ and $\text{POL} < 6$.

2. Asymmetric stable or coherent contamination: $\text{SNR} < 10$, $\text{POL}$ is smaller or
not much greater than 6. Two spectral components IS and CH may be obtained.

3. Unstable, $\text{SNR} < 10$ and $\text{POL} > 6$: two peaks one narrow and one broad. The
power, in 'logarithmic' units, below these two peaks is also greater than 6,

4. Unstable, $\text{SNR} > 10$ and $\text{POL} > 6$: only one coherent peak.

(B). INSTABILITY SPECTRUM.

Let be $Y_{x}(i = 1,N)$ the power spectrum from the measured ACF at the spectral
gate $i \int f \cdot \delta f = BF/N$, $BF = 50$ KHz and $\delta f = 0.78$ or 1.56 KHz for $N = 64$ or
32 respectively. Assuming that instabilities saturate some time before our
observation, the turbulence power spectrum may be considered Lorentzian and, a
spectral peak $Y_{pk}$ is a 'true' instability signature (of either main beam inter-
section of the unstable region or antenna pattern contamination) if:
\[
Y_{pk} - Y_M = \max_{i=1,N} (Y_{i}) 
\] (10)

and, calling \( \Delta f \) the half power width of the Lorentzian relaxation spectrum,

\[
\frac{Y_{M+1}}{Y_M} = \frac{\Delta f}{\Delta f^2 + \delta f^2} 
\] (11)

any of the following conditions is verified:

(a) If \( \frac{Y_{M+2}}{Y_M} < 0.8 \) then \( \Delta f < 2 \delta f \).

(b) If \( \frac{Y_{M+1}}{Y_M} < 0.8 \) then \( \Delta f < \delta f \).

Also, calling SIGMA the estimate of the statistical error (noise fluctuation level) after the substraction of the noise,

\[
\text{SIGMA} = \left( \sum_{i=1}^{N} Y_{i}^2 \right)^{1/2} / N 
\] (12)

we should have:

(c) \( |Y_M - Y_{M+2}| > 2 \text{ SIGMA} \).

\( N = i_2 - i_1 \), is the number of noise samples. The ion-line maximum half-width is not greater than 2.5 to 3 \( C_s \) (Evans [1969]), where \( C_s \) is the ion-sound speed and, therefore, the maximum spectral width is of the order of 18 \( v_i \) when \( T_e / T_i < 10 \) (\( v_i \) is the ion thermal speed and \( C_s \) \( \approx \sqrt{T_e / T_i} \)). This value is generally smaller than 40 KHz and because of our 50 KHz receiver bandwith we may
choose $N_N = 8$.

Therefore, if conditions (a) and (c) are verified, the instability spectrum consists of five measured points and the actual spectral width is not greater than $2 \delta f$. If conditions (b) and (c) hold together, the spectrum is defined by three points and the instability spectral width is smaller than $\delta f$. In these cases we will take as the estimate of this width either $\delta f$ or $\overline{\delta f} = A_{CH}/2 Y_M$ ($A_{CH}$ is the area below the peak) whichever is smaller.

Generally for ranges falling inside the E-region, the maximum observed spectral peak is imposed by the receiver saturation level and the real instability spectral width is not accessible. In any case, when the measured spectral width is smaller than the waves phase velocity the spectrum is narrow and often associated with the weak to moderate turbulence saturation of the linear kinetic instability. Broader spectra are the signature of strongly turbulent conditions.

4. ELECTRIC FIELD DETERMINATION.

The Millstone Hill radar acts as both an incoherent and a coherent backscatter providing simultaneous information about the unstable E-region (wave phase velocity) and the ion motion at F-region altitudes (see figure (4.1)).

After filtering out the possible spectral asymmetries from antenna pattern contamination a good estimate of the F-region ion-drift may be obtained. In the F-region (above 150-160 km of altitude), ions and electrons experience the same ExB drift and from a number of measured line of sight components of this velocity field (azimuth scan) the electric field may be recovered (Holt et al. [1984], Foster et al. [1985]).

Assuming the equipotentiality of the magnetic field lines ($E \cdot B = 0$) and the electric field stationarity ($\nabla \cdot \mathbf{E} = 0$), the local electric field vector can be
estimated from a potential function \((\phi)\) with only two independent variables (i.e., the magnetic latitude and longitude). F-region electric fields are mapped into the E-region along the magnetic field lines.

At Millstone Hill the ensemble of line of sight velocities from F-region ranges (200 to 500 km altitude) observed during a single azimuth scan usually can be fit with an electrostatic potential function developed in some base functions (B-splines) with the condition \(\phi = 0\) at the latitude of our radar station. The time resolution (universal/local time uncertainty) is of the order of the duration of the azimuth scan used in the calculation (equal to 20 minutes for the 180° azimuth scan).

(a). WEST-TO-NORTH-TO-EAST AZIMUTH SCAN.

FIGURE (5.0a).

This figure shows the calculated electric potential contours for a 180° azimuth scan (at EL = 4°) on July 23/83 (from 15:08 to 15:30 LT). These contours were plotted at 2 KV separation and superimposed on the measured coherent echo power (POL from 5 to 10 correspond to the color-coded intensities from gray to black).

This scan clearly shows the two regions of maximum coherent backscatter, coincident with the regions where the radar beam is nearly perpendicular to the magnetic field at E-region altitudes. These strong coherent echoes occurred at a time when the region of northward-directed auroral convection electric field extended equatorward to the latitudes where E-region perpendicularity is possible. The strength of the electric field in these regions was above the threshold for the generation of the Farley-Buneman instability (20 to 25 mV/m).

(b). TWO INSTABILITY-TYPE SPECTRUM.

FIGURE (5.0b).
Using the algorithm presented earlier in this chapter (section 3), a program was written to automatically separate sharply peaked spectra into two components which are displayed. This figure shows the spectra measured along the direction defined by \( AZ = 20^\circ \text{NE} \) (line '1' in the scan of figure (5.0a)); the region where the received power is maximum is centered at \( \text{LON} \approx 300^\circ \) and \( \text{LAT} \approx 61^\circ \). In this direction and below 250 km altitude, all the range gates show the presence of coherent backscatter. In this figure \( V_{P1} \) represents the phase velocity of the broader spectrum, \( V_{P2} \) that of the narrower spectrum (both measured in m/s) and, \( z \) is the altitude in km. \( V_{P1} \) rises systematically with \( z \) and \( V_{P2} \) is basically constant between 99 to 159 km altitude (\( V_{P2} \approx -250 \) m/s) and for \( z \) between 182 to 232 (\( V_{P2} \approx -450 \) m/s). \( V_{IS} \) is the line of sight projection of the ion-drift velocity and \( V_{PK} \) the phase velocity of the coherent spectral component. \( \text{POL} \) is the \( \log_{10} \) of the received power expressed in \( \text{cm}^{-3} \).

The vertical scale of the individual plots have been normalized to unity and the horizontal scale goes from -25 kHz to 25 kHz with a frequency resolution equal to 780 Hz (64 sampled points). Between 100 to 206 km altitude, SNR was greater than 10 and no incoherent component can be recovered.

Full-line spectra are associated with the cases where two spectral components are present. All the other spectra (crosses) are the incoherent scattering signatures from stable plasma fluctuations.

Narrow spectra at E-region altitudes are generally associated with the Farley Buneman waves. In the same altitude region, broader spectra are associated with the ExB turbulence-generated secondaries. Instability-like spectral signatures at 232 and 286 km altitude are basically due to the antenna pattern contamination from the unstable lower E-region. We can't rule out completely the possibility of other in-situ instability mechanisms (see chapter 2).
(c) **SPECTRUM 'CONTAMINATION'**.

**FIGURE (5.0c).**

For the same scan displayed in figure (5.0a) and for AZ = 270° (along line '2') far side-lobe 'contamination' effects from presumed type 1 waves are responsible for the spectral signatures between 180 to 205 km altitude as is clearly shown by the superposition of the narrow peak and a typical F-region spectrum. The narrower spectral component at 138 km may well be the signature of a side-lobe intersection of the unstable lower E-region. A case-by-case study of the measured spectra goes, however, beyond the space-time resolution of the actual observations. In this study we propose a statistical analysis of the measured spectra in order to define their main regularities and to identify the most probable sources of instability in the auroral lower ionosphere.

5. DISCUSSION OF THE OBSERVED SPECTRAL PROPERTIES.

In this section we present the results of the analysis of the measured power spectrum corresponding to the same set of observations introduced in the previous chapter. The spectral resolution of the long pulse observations is equal to 780 Hz (260 m/s) and equal to 1.5 KHz (520 m/s) for the shorter pulse.

We are limiting our discussion to the local evening sector and the premidnight hours. The reasons: (1) instability is more frequent and (2) in these times the auroral region is closer to the radar field of view.

**TIME PERIODS.**

(1) July 23/83 (EL = 4°, long pulse, 180° AZ scan) from 09 to 21 LT and in
particular from 13 to 20 LT,

(2) April 20-21/85 (EL = 6°, long pulse, 180 AZ scan) from 13 to 01 LT and particularly between 13 to 18 LT.

(3) July 31/84 (short pulse, elevation scan < 5°, AZ = 380°) from 16:30 to 18:30 LT. As before this finer altitude resolution data set is used to complement the results from the analysis of the other two periods.

(A). ELECTRIC FIELD THRESHOLD.

1. ELECTRIC FIELD AND LOG10(POWER) DISTRIBUTIONS.

23 JULY 83 (09-21 hours LT).

FIGURES (5.1a) and (5.1b) display the local time distributions of the calculated electric field (EE) and parameter POL (proportional to the LOG10 of the area below the spectrum) when instability is present. Both distributions are color-coded for altitudes between 100 to 250 km. Blue color corresponds to main beam observations at E-region altitudes. Higher altitudes are usually associated with the antenna pattern contamination effects.

The electric field was calculated as a function of the magnetic latitude and longitude in the F region and mapped into the E-region along the magnetic field lines. The apparent altitude dependence of the electric field is the effect of this real latitude-longitude variation (EE is greater at higher latitudes which are sampled at higher main-beam altitudes). The opposite effect is observed in POL; lower altitudes (inside the E-region) correspond to greater values of POL (closer to main beam). Because instability is basically concentrated in the electrojet region, smaller POL at higher apparent altitudes are generally the
result of off beam center intersections of the unstable layer. Real instability at higher altitudes (much less frequent) can't be discarded.

The altitude spread of both POL and EE is the result of the antenna pattern contamination and the radar sweeping from west-to north-to east over every 20 minutes. Figures (a) and (b) clearly show the correlation between high values of EE and times of instability generation. In the region where EE > 20 mV/m (about 13 to 14 LT) instability develops. Moorcroft [1980] using the 398 MHz Homer-Alaska radar found an instability threshold of the order of 23 mV/m.

20-21 APRIL 85 (13-01 hours LT).

FIGURES (5.2a) and (5.2b).

These figures basically show the same features discussed in the previous case but for the April 20-21/85 period. In this case even for electric fields of the order of 15 mV/m, instability is present in a large range of azimuths. The dispersion in the values of EE and particularly POL are, as before, the result of the 180° azimuth scanning every 20 minutes. The electric fields calculated as discussed for figure (5.0a) are in fact an average over a few degrees in latitude and longitude and we may expect greater values for the actual local electric field (and probably above instability threshold). This period was highly magnetically disturbed and the electric field was possibly miss-determined by neglecting the expected strong E-region neutral wind. This activity may also explain the high dispersion in parameter POL as compared with the results of July 23/83.

The correlation between high values of the electric field (EE > 20 mV/m) and instability-occurrence (higher values of POL) is also noticed.

2. CORRELATION BETWEEN THE ELECTRIC FIELD AND LOG10(POWER).

FIGURES (5.3a) and (5.3b).

The correlation between EE and POL becomes clear when only points below 160 km
and for times where EE was above 20 mV/m (13-20 LT on July 23/83 and 13-18 LT on April 20/85) are considered. The uncorrelated smaller POL values are the result of either the side-lobe contamination effects or the observation of secondary waves.

(B). INSTABILITIES' SPECTRAL PEAK AND WIDTH.

In section (5.4) we presented the algorithm applied in the determination of the instability spectrum in three possible situations, (1) when only one instability peak is present, (2) when two simultaneous instability peaks occur and, (3) the stable incoherent spectrum is contaminated by the instability spectrum picked-out by a side-lobe.

We call PWK the 'logarithmic' power associated with the 'narrow' spectral peaks after separation (displayed in figure (5.4)) and POL is the usual total power in 'logarithmic' units which combines all contributions. We call DSP to the the spectral width of the coherent spectrum measured in m/s (displayed in figure (5.5)). When the receiver is saturated the true spectral width is smaller than DSP and the instability saturation level is greater than PWK. In our measurements the frequency resolution largely overlaps the effect of the receiver saturation on the determination of the spectral width and in the cases where the spectrum separation algorithm applies, the receiver is never saturated.

As we will discuss in the following lines, the dispersion plot between the 'power of the peak' (PWK) and the total power (POL) is an indicator of the relative importance of the main-beam and side-lobe observations of both stable and unstable fluctuations in the mixed spectra. Also, the distribution of the spectral width (DSP) with POL and the main beam azimuth will allow the identification of two different classes of spectral signatures.
1. **Dispersion Plot Between the Instability Power and the Total Power.**

Figures (5.4a) and (5.4b) present the dispersion plots of PWK with POL for the two observation periods (left-hand side, color-coded). On the right-hand side we plot (full-lines) the equation:

\[ \text{PWK} = \log_{10} \left( 10^{\text{POL}} - n_0 \right) \]  

(13)

\( n_0 \) is the ambient density measured in cm\(^{-3}\). In figure (a) \( 5 \times 10^3 < n_0 < 10^5 \) and in figure (b) \( 10^3 < n_0 < 10^5 \). Furthermore, the minimum SNR for the analysis of the incoherent spectrum is equal to 0.1 and when applied to the separation of two instability peaks (case (4) in paragraph (3.A)):

\[ \text{PWK}_1 = \log_{10} \left( 10^{\text{POL} - \text{PWK}_2} - 10^{\text{n}_0} \right) \]

The separation algorithm only applies if S/N < 10; the broken-lines in both figures represent the limit POL - PWK = 1 when \( n_0 < 10^5 \) cm\(^{-3}\) and PWK > 5.

The dispersion plots are in good agreement with the linear plots for reasonable ambient densities. This figure together with figure (5.5) are presented in order to show some of the conditions and outputs of our spectrum separation algorithm.

2. **Dispersion Plots Between the Instability Spectral Width, the Total Power and Azimuth.**

Observations of intense coherent backscattering can be explained by the existence of various plasma turbulence states at 30-cm wavelength. As we show in the following figures, our observations of E-region clutter give strong evidence of the possible linear generation of modified two-stream instability (type 1) and two types of long-wavelength turbulence-related 'secondaries' in the elec-
trojet region. Evidence supporting our identification of type 1 waves includes the following (expected type 1 characteristics are enclosed in parantheses): the type 1 waves are linearly generated for a given strength of the ExB current (electric field threshold) inside a limited angular region (instability cone) around the flow direction (flow angle dependence) in a plane closely perpendicular to the local magnetic field (aspect angle dependence). They are associated with narrow spectral signatures and Doppler-shift equal to the ion-sound speed.

The secondary waves (either type 0 or type 2, see introduction) are likely the result of the enhancement of short-wavelength fluctuations well above the thermal level by the transfer of turbulent wave-energy from linearly generated long wavelength primaries (either type 1 or ExB-gradient drift). They are associated with relatively broad spectral signatures and Doppler-shifts smaller than the ion-sound speed and statistically distributed around zero. The turbulence level of the type 0 waves is lower than that of the type 2 secondaries and consequently their spectral widths are narrower.

Finally, when type 1 waves are also generated, secondary waves are preferently observed in the direction perpendicular to the electrojet current (because their amplitudes are a few orders of magnitude smaller than that of the primaries).

Figures (5.5a) and (5.5b) for July 23/83 and figures (5.6c) and (5.6d) for April 20/85 show, respectively, the dispersion plots of POL with DSP and DSP with azimuth (AZ). This spectral information corresponds to all the cases where instability signatures were present: (a) only one peak (with POL > 6), (b) two peaks (with both POL and PWK greater than 6) and (c) the spectral contamination in the stable IS component (when POL < 6). DSP-POL ('TYPE A' and 'TYPE B' spectra).

When two presumed instability peaks are separated, they clearly range in two
classes of spectral widths, the 'narrower' peaks with DSP between 200 and 400 m/s ('type A') and the 'broader' ones with DSP greater than 600 m/s ('type B') and both POL and PWK > 6. The 'narrower' spectra correspond to cases (a), (c) and to the narrow component in case (b). These spectra are widely spread in POL and altitude showing the expected effects of the antenna pattern contamination in the observation of the Farley-Buneman (type 1) instability (when radar main beam direction is outside the unstable region). As we shall see later part of the type A spectra are presumably the signature of type 0 secondaries and the type B spectra those of type 2 waves. Broader peaks with POL < 6 may correspond to incoherent spectra and are considered as such (and not shown in the figures).

Spectral widths greater than or equal to 1000 m/s are concentrated at E-region altitudes with almost no apparent altitude dispersion (associated with antenna pattern contamination effects) implying that their corresponding saturation amplitudes are smaller than those of the other instability signatures.

DSP-AZ.

From figures (5.5b) and (5.5d), the narrower spectral widths show only a weak dependence on the azimuth around AZ ~ 345° as is expected if the type 1 instability is dominant (see Moorcroft and Tsunoda [1976]). In this region the spectral widths are consistently larger at E-region altitudes (blue color). As we will show later, in this region, the ExB flow is nearly perpendicular to the radar line of sight and type 1 instability is not possible. The direction defined by AZ ~ 345° corresponds to the magnetic meridian plane through Millstone Hill (for the magnetic declination angle of 14.5° NW) and the ExB flow in the local evening sector is closely perpendicular to this plane (Foster et al. [1986a,b]).

The distribution of the broader spectral width is, on the contrary, strongly dependent on the azimuth. Spectral width clearly peaks about the two regions of
favorable aspect angle (at E-region altitudes). This latter fact is not surprising if the observed waves are the ExB-gradient drift turbulence-generated secondaries (type 2 waves). Strong turbulence develops in the transverse plane to the magnetic field (Sudan [1983.b]); if the broadest spectra are associated with lower power levels, the aspect angle attenuation may significantly screen their observation away from perpendicularity.

Moorcroft and Tsunoda [1976] using a 398 MHz radar (at Homer, Alaska), found a distribution of DSP with AZ similar to ours. Their data, however, present only few observations of our type B spectra.

To summarize we found, other than the signature of primary Farley-Buneman instability, evidence of the presence of two types of 'secondary' waves and their characteristics will be discussed in the following lines. Greenwald [1975] using a 50 MHz radar system have also reported the observation of two types of non two-stream, secondary waves in the auroral electroject. It is also important to recall that most of the type B spectral signature are associated with narrow 'primary-like' spectra. This and other facts to be discussed later (section C.2 pag. 178) seem to indicate some sort of two-step mechanism in the generation of Farley-Buneman waves in the direction perpendicular to the ExB current (Sudan et al. [1973], Kudeki et al. [1985]).

(C). DISTRIBUTION OF THE SPECTRAL PARAMETERS.

In the next four figures we are presenting the main results of the analysis of the spectrum.

Figures (5.6) and (5.7) show the correlations between POL and AZ (a), the waves phase velocity (VPH) and AZ (b), between POL and VPH (c) and between DSP and VPH (d) for spectral widths smaller than 600 m/s (type A spectrum) and for
the two observation periods. Parameter VPH is positive when the waves propagate away from the radar and negative in the opposite direction.

Figures (5.8) and (5.9) display the same parameters as before but now for the broader spectral widths (DSP > 600 m/s). As was discussed in the previous chapter, between 100 to 140 km altitude (blue color) the maximum power is that of the receiver saturation and consequently the actual dependences of the turbulence level, on the azimuth and the phase velocity are not directly accessible but, nevertheless, off-main beam observations (color-coded) give us an indication of what these dependences should be.

(C.1). 'TYPE A' SPECTRUM (DSP < 600 m/s).

1. DISTRIBUTION OF THE TOTAL POWER WITH AZIMUTH.

As we mentioned before, even when the type B spectra are not considered (figures (5.6a) and (5.7a)) we still notice the presence of unstable waves when looking almost perpendicular to the flow (magnetic meridian at AZ \( \sim 345^\circ \), see figure (5.11) for the correspondence between AZ and the flow angle). These type A waves have phase velocities distributed around zero and other characteristics of the 'secondaries' including the fact that in this range they have the broadest spectral widths (from figures (5.6b,d) and (5.7b,d)). Figures (5.6a) and (5.7a) also show a very good agreement with the results of the thin layer instability for the lower E-region as it was noticed in the analysis of the cross-section (chapter 4, figures (4.19) and (4.20)). The 5 to 10 dB asymmetry in the power levels between east (AZ > 360\(^\circ\)) and west (AZ < 360\(^\circ\)) may be due to the fact that, for the same range, the magnetic latitude is higher when observing to the east and hence the electric field is greater. The saturation amplitude of type 1 waves seems to increase with the electric field strength beyond the instability threshold (see for example St.Maurice and Schlegel [1982]).
2. DISTRIBUTION OF THE INSTABILITIES' PHASE VELOCITY WITH AZIMUTH.

Figures (5.7b) and (5.8b) show the clear cut between the presumed type I primaries and the type A secondaries. This effect is similar to the 'plateau-formation' found with other UHF radars (Abel and Newell at 1298 MHz, Tsunoda 1976 and Moorcroft and Tsunoda 1978 at 398 MHz). When the radar is directed nearly perpendicular to the flow direction (for AZ = 340° ± 10° in our case), the phase velocity varies rapidly with azimuth and the observations are concentrated inside the E-region (blue color). Outside this region the phase velocity remains relatively constant. These regions are referred as 'plateaus' and are a general property of the type I irregularities (Greenwald 1978).

The points around AZ = 280° and AZ = 430° where VPH abruptly changes sign may correspond to far side-lobe intersections (see antenna pattern in figure (3.1a)) of instability sources in regions where the line of sight projection of the ExB flow can change of sign.

The average phase speed on the 'plateaus' shows an east (negative value)-west (positive value) asymmetry. This asymmetry is, in absolute value, of the order of 100 m/s on July 23/83 and of the order of 200 m/s on April 20/85, as can be seen in figures (5.7c) and (5.8c) respectively.

Furthermore, phase-velocities on the 'plateaus' are spread over 100 to 150 m/s around their average values and parameter VPH is, in absolute value, systematically greater at higher apparent altitudes (color-coded).

The 'plateau' formation is the result of the nonlinear saturation of the Farley Buneman waves' phase velocity at the sound-speed of the medium (this fact may be explained by using the weak turbulence 'orbit-diffusion' approximation, see Sudan 1983.a, b). Primdahl and Bansen 1985 have reported rocket measurements of lower E-region electric field perturbations which are consistent with Sudan's
predictions on the saturation of type 1 waves.

**East-West asymmetry.**

The East-West asymmetry in VPH is very likely the effect of neutral winds in the auroral lower E-region, blowing eastward with magnitudes ranging from 50 to 100 m/s (below 110 km altitude and between 14 to 20 hours LT, Wand [1983.a and b]). Tsunoda [1976] and Moorcroft [1980] also noticed this phenomenon: the wave's phase velocity, in the radar frame of reference, would be Doppler shifted by the component of the ion-drift along the observation direction (the ions move with the neutrals in the lower E-region). Correcting this asymmetry, an estimate of the average absolute 'plateau' velocities of the order of 400 m/s and 500 m/s for the July 23/83 and April 20/85 periods can, respectively, be found.

In the presence of type 1 waves, secondaries are observed outside the primary instability cone, in the region between AZ ~ 310° and 370°, with phase velocities ranging from zero to ± C_s. For both observation periods VPHAJO at AZ ~ 345° (the magnetic north direction at Milltone Hill). A typical value of C_s (ω \sqrt{2T_e / m_e}) in the auroral lower E-region is of the order of 400 m/s (for NO+ or O_2^+ and temperatures close to 300°K). Values of C_s between 400 and 500 m/s correspond to electron temperatures between 300°K and 500°K (for the same constituents). The electric field threshold for the generation of the type 1 instability is, in these conditions, of the order of 20 to 25 mV/m. Moorcroft [1979, 1980], for practically the same wavelength (ω ~ 30-cm, 398 MHz), found similar results.

**Increase of the ion-sound speed with the electric field.**

The altitude-spread of VPH on the 'plateaus' is also explained by the nonlinear theory. This phenomenon results from the anomalous heating of the electron gas by the unstable waves in their saturation process (Schlegel and St.Maurice [1981], St.Maurice and Laher [1985], Robinson [1986]). In the electrojet region
larger phase velocities (in absolute value) are associated with higher latitudes, larger electric field strength, and higher electron temperature.

The spread of the 'plateau' velocities at apparent higher altitudes (color-coded) is the result of the off-main beam intersections of the unstable E-region at higher latitudes where the electric field is greater; the electron temperature is expected to increase with the electric field beyond the type 1 instability threshold (see for example Nielsen and Schlegel [1985]).

3. DISPERSION PLOT BETWEEN THE TOTAL POWER AND THE INSTABILITIES' PHASE VELOCITY.

The magnitude of the phase velocity of the presumed type 1 waves is concentrated around two different average values near the E-region ion-sound speed (~ 500 m/s, figures (5.6c) and (5.7c)). The east (negative)-west (positive) asymmetry in either plot can be explained by the presence of a neutral wind blowing eastward. The spread of parameter POL in the apparent altitude (color-coded) is the result of off-main beam intersections of the unstable region.

During both observation periods, no important off-main beam backscattering from type A secondaries are noticed (when looking closely perpendicular to the ExB flow) meaning that their saturation level must be few orders of magnitude below that of the primaries. A remarkable difference between figures (5.6c) and (5.7c) is that for the highly disturbed April 20/85 period, parameter POL shows almost no altitude-dispersion when observing the type A secondaries (for VPH around zero); we also observed some type A spectra at E-region altitudes (blue and green colors) with relatively high phase velocities (from 800 to 1100 m/s).

4. DISPERSION PLOT BETWEEN THE INSTABILITIES' SPECTRAL WIDTH AND PHASE VELOCITY.

As we mentioned before, the 'type A' secondaries have the broadest spectral widths in the range DSP < 600 m/s. The dispersion plots of figures (5.6d) and (5.7d) don't show, however, any sharp transition between the spectral widths of
the primary and the secondary waves. Type A secondaries have maximum width around $V_{PH} \sim 0$. These results can be compared with those from Moorcroft and Tsunoda [1976] where they reported the observation of mainly type 1 waves. The cases which they didn't expect to be signatures of Farley-Buneman primaries are similar to our type A secondaries.

(C.2). 'TYPE B' SPECTRUM ($DSP > 600$ m/s).

Figures (5.8) and (5.9) for the July 23/83 (13 to 20 hours LT) and the April 20/85 (13 to 18 hours LT) periods, respectively, display the dispersion plots of parameters $POL$ with $AZ$ (figures (5.8a) and (5.9a)), $V_{PH}$ with $AZ$ (figures (5.8b) and (5.9b)), $POL$ with $V_{PH}$ (figures (5.8c) and (5.9c)) and $DSP$ with $V_{PH}$ (figures (5.8d) and (5.9d)). In our experimental configuration, type B spectra are mainly observed together with the narrower type A spectral signatures. The area below each spectral component, in 'logarithmic' units (chapter 4 section 2), is greater than 6 and neither of them can be mistaken as the signatures of 'stable' fluctuations.

1. DISTRIBUTION OF THE TOTAL POWER WITH AZIMUTH.

Figures (5.8a) and (5.9a) display the dispersion plots of $POL$ with $AZ$ for the cases where type B spectra were observed (either alone or mixed with a type A component). These figures reproduce the same features as the cases where $DSP < 600$ and for $POL > 6$ (figures (5.6a) and (5.7a)).

2. DISTRIBUTION OF THE INSTABILITIES' PHASE VELOCITY WITH AZIMUTH.

The type 2 'secondary' nature of the type B waves become clear in figures (5.8b and 5.9b) and (5.8c and 5.9c). The dispersion plot of $V_{PH}$ with $AZ$ shows that the phase velocity closely follows the 'cos($THETA$) law' where $THETA$ is the angle between the radar line of sight and the $ExB$ direction. Around the directions where $AZ = 310^\circ$ and $AZ = 380^\circ$, the radar main beam direction reaches perpendi-
cularity with the magnetic field at E-region altitudes and the received power is more intense as compared to that from main beam observations of the lower E region at directions closely perpendicular to the flow (aspect angle effect).

Moreover, in these regions the ExB-gradient drift turbulence is expected to be stronger and hence the spectral width dispersion to larger.

In the region where the radar line of sight is nearly perpendicular to the flow direction (AZ $\sim 345^\circ$), our observations are concentrated at E-region altitudes (blue color) meaning that saturation amplitudes are smaller due possibly to aspect angle attenuation effects alone (off center beam observations corresponding to power levels below POL = 6 and are not plotted).

3. **DISPERSION PLOT BETWEEN THE TOTAL POWER AND THE INSTABILITIES' PHASE VELOCITY.**

Figures (5.8c) and (5.9c) show the same east-west asymmetry noticed when analyzing the narrower spectral signatures (DSP < 600 m/s). In these cases, however, the phase velocities are concentrated closer to $V_{PH} = 0$.

4. **DISPERSION PLOT BETWEEN THE INSTABILITIES' SPECTRAL WIDTH AND PHASE VELOCITY.**

Figures (5.8d) and (5.9d) also show evidence of the possible type 2 nature of the 30-cm 'Type B' secondaries (the ExB-gradient drift turbulence-related waves, discussed by Sudan [1983.b]). Type 2 turbulence is concentrated in the transverse plane to the magnetic field and the waves' phase velocity follow the electrojet current ($\mathbf{V}_d = \mathbf{V}_e - \mathbf{V}_i$); in the lower E-region $\mathbf{V}_e$ (the ExB drift) and $\mathbf{V}_n$ (the neutral wind). Figures (5.5b) and (5.5d) showed that the spectral width of type B waves is maximum (thus turbulence level) when the radar line of sight is closely perpendicular to the magnetic field at E-region altitudes (AZ $\sim 310^\circ$ and $380^\circ$).

In the frame moving with the ions, the phase velocity of both the 'type A' and the 'type B' secondaries are equal to $V_{PH2} = k \cdot \mathbf{V}_d - \mathbf{V}_d \cos(\Theta)$ where $k$ is
the radar observation direction and \( \Theta \) the flow angle, moreover inside the instability cone (if type 1 waves are also present) the phase velocities of the primary two-stream waves is limited by the ion-sound speed and by \( V_{\perp} \) for the Type 2 turbulence spectral signatures.

During local evening hours the ExB flow is nearly perpendicular to the magnetic meridian at Millstone Hill (AZ &lt; 345°) and considering an E-region neutral wind (\( V_n \)) blowing eastward (geographic), we have in the radar frame:

\[
V_{PH2} \sim V_E \cos(\Theta) + V_n \cos(35°)
\]

therefore, when \( \Theta \sim 90° (AZ \sim 345°) \) one finds \( V_{PH2} \sim V_n \), the eastward asymmetry.

The \( \cos(\Theta) \) dependence (\( \Theta \) is proportional to AZ, see figures (5.10a,b)) and the Eastward asymmetry are clearly shown in figures (5.8b) and (5.9b).

Another interesting feature that seems to confirm the ExB-gradient drift nature of the type B secondaries is showed in figures (5.8d) and (5.9d). The bottom-side of the distribution of the spectral width with the phase velocity presents a 'triangular' wave structure: the first 'triangle' is formed by \( V_{PH} \) ranging between 0 to \( \pm 130 \) m/s with maximum spectral width at \( V_{PH} \sim 0 \). This pattern is repeated three times in the July 23/83 period and five times in the April 20/85 data. The distributions are nearly symmetric to \( V_{PH} = 0 \) and the 'periodicity' in \( V_{PH} \) is of the order of 250 to 300 m/s in absolute value.

We believe that this feature can be the result of a modulation of the turbulence power by the long-wavelength ExB-gradient drift primaries (\( \lambda > 15 \) to 30 m).

Our estimate of the width of the type B spectrum is equal to half of the spectral power divided by the peak amplitude. In section (B.2) we mentioned that the type B waves were generally observed together with a narrow type A spectrum.
that may well be the signatures of two-stream waves propagating outside the primary instability cone. All these considerations are strongly speculative and more work need to be done to happily explain this feature. We should mention, however, that a somehow similar modulation was reported by Kudek\'{i} et al. [1982] and Pfaff et al. [1982] from co-located observations of the equatorial electro-jet with the Jicamarca 50 MHz radar and sounding rockets respectively. Farley-Buneman waves outside the instability cone may be excited by the wave-electric field of the long-wavelength ExB-gradient drift primaries (two-step process, Sudan et al. [1973]). Using quasilinear theory Kudek\'{i} et al. [1985] could explain why, basically, only kilometer-length primaries with phase-velocities of the order of 100 to 150 m/s will participate in this two-step process.

From Keskinen and Ossakow [1983], the minimum wavelength for the linear ExB-gradient drift instability is given by:

$$\frac{\sqrt{\nu_e} V_d}{\nu_e L n_o (1+\psi)} > \frac{k^2 C_s^2}{\nu_\lambda^2}$$

(14)

$L n_o$ is the ambient density gradient scale-length (for definition of the other parameters see chapter 2). From equation (14) we can have:

$$\lambda > 2 \sqrt{\frac{\nu_e}{\nu_\lambda}} L n_o C_s \frac{C_s}{\nu_e}$$

at 110 km altitude $\nu_\lambda/\nu_e \sim 10$, $\nu_e/\nu_\lambda \sim 200$ and, $\lambda > 20$ m if $L n_o \sim 10$ km for $C_s \sim 400$ m/s. In the long-wavelength limit these waves are dispersive and their dispersion relation is modified (Kudek\'{i} et al. [1982]). Now their phase velocity is given by

$$\nu_d \sim \frac{C_s}{(1+\psi)(1+\frac{k^2}{k_0^2})}$$
where \( k_\circ = \frac{\mathcal{V}_e}{[\Omega_i L_n (1 + \psi^2)]} \). Waves with \( k \sim k_\circ \) are basically those that can exist inside the primary instability cone and generate the two-stream primaries, outside this region. At typical auroral ionosphere conditions, at 105-110 km altitude \( \mathcal{V}_e \sim 20 \), \( \psi \sim 0.2 \) and for \( k_\circ / k \sim 1.2 \), \( \mathcal{V}_d \sim 135 \text{ m/s} \) for \( C_s \sim 400 \text{ m/s} \), also \( \mathcal{V}_o \sim 0.26 \text{ m/s} \), \( \mathcal{V}_n \sim 1 \text{ to } 3 \text{ km} \) (for \( L_n \sim 4 \text{ to } 10 \text{ km} \)).

The observed phase velocities in figures (5.8d) and (5.9d) range from \( -\mathcal{V}_d \) to \( \mathcal{V}_d \) \( (\mathcal{V}_d > C_s) \) and present a 'triangular' shaped distribution symmetric to \( V_{PH} \sim 0 \) with half period of the order to 130 m/s. On July 23/83 \( \mathcal{V}_d \sim 400 \text{ m/s} \) and we can count three maxima \( (\mathcal{V}_d / \mathcal{V}_o \sim 3) \). On April 20/85 \( \mathcal{V}_d \sim 600 \text{ m/s} \) and we have, roughly, five maxima \( (\mathcal{V}_d / \mathcal{V}_o \sim 5) \). We may conclude that we probably observe the 'trace' of the kilometer-length ExB-gradient drift primaries.

The 'type A' secondaries may be the result of a moderately strong density gradient drift turbulence, induced by the Farley-Buneman waves outside the instability cone (and similar to the type 0 signatures discussed by St.Maurice et al. [1986]), whereas the 'type B' secondaries may correspond to the strong turbulence regime associated with the ExB-gradient drift instability.

(D). FLOW ANGLE DEPENDENCE.

The flow angle (THETA) is the angle between the wave-propagation direction (defined by the radar line of sight) and the ExB-flow direction. The electric field is determined from our single radar observations by applying the algorithm presented in paragraph (5.4). The maximum flow angle for the type 1 waves (instability cone) seems to be smaller than 60° (Schlegel [1980]). The flow direction, around dusk (18:00 LT) is generally perpendicular to the magnetic meridian plane (following closely the circles of constant invariant latitude) and radar line of sights greater than 30° away from this plane generally fall inside the...
instability cone.

1. DISTRIBUTION OF THE TOTAL POWER WITH THE FLOW ANGLE.

Figures (5.10a) and (5.10c) present the distribution of parameter POL with flow angle. The 7/23/83 period, the direction of the magnetic meridian also corresponds to the 90° flow angle direction, the 4/20/85 period where the magnetic conditions were highly disturbed, the flow direction moves southerward, away from perpendicularity to the magnetic meridian plane, as much as 30°.

The flow angle 'sensitivity' of the backscattered power can be easily inferred from figure (5.10a). Assuming the instabilities are concentrated in the electrojet region (and centered at ~110 km altitude) for a given apparent altitude (off beam intersection of the unstable layer) the aspect angle is nearly independent of the azimuth of the observation and aspect angle attenuation effects don't need to be considered in the determination of the power drop between the direction perpendicular to the flow (AZ ~345°) and that of maximum power (AZ ~310° or 380°). This power drop is close to -20 dB for the July 23/83 period.

During the April 20/85 period (figure 5.11c) this 'sensitivity' is less well defined (it may be near to -15 dB however). Andre [1983] for the 1-m wavelength (140 MHz) auroral electrojet instabilities has reported the same range of values for the flow angle 'sensitivity'.

2. DISTRIBUTION OF THE INSTABILITIES' PHASE VELOCITY WITH FLOW ANGLE.

Figures (5.10b) and (5.10d) present practically the same characteristics of the 'plateau' distributions in azimuth. As can be expected, on July 23/83, VPHA ~ 0 corresponds to the flow angle THETA close to 90°. On April 20/85, however, VPHA ~ 0 corresponds to THETA ~70°. This 20° off-set of the flow pattern may be explained if we assume the presence of a relatively strong poleward component of the neutral wind in the lower E-region during highly magnetic-disturbed periods.
The presence of neutral winds in the lower E-region will induce the miss determination of the flow direction because now, in the radar frame, the ExB drift velocity and the electrojet current are not the same.

During very disturbed conditions, poleward neutral winds of the order of 100 to 150 m/s may be expected and shall account for this 20° out-phasing.

3. DISPERSION PLOT BETWEEN FLOW ANGLE AND AZIMUTH.

In figures (5.11a) and (5.11b) we present the correlation plots between the flow angle and the azimuth. After the analysis of the distributions of POL and VPH with AZ and THETA, it is not surprising to find a close correlation between angles THETA and AZ. Under moderately disturbed conditions, the ExB-flow in the local evening sector, follows the circles of constant magnetic latitude. During more disturbed conditions the flow is enhanced and displaced as a whole towards the lower latitudes. The linear plots on the right hand side of figure (5.12a) show the cases where the flow follows the circle of 63° magnetic latitude (full-line) and when it follows the direction perpendicular to the magnetic meridian plane at Millstone Hill (broken-lines). This latter case corresponds to the penetration of the convection electric field, of strength above the type I instability threshold, into the Millstone Hill latitude or lower.

As we can see, during the July 23/83 period, the average ExB flow is well represented by the linear plots. Dispersion is associated with the off-main beam intersections of the unstable flow at various latitudes. For the highly disturbed April 20/85 period, the unstable ExB flow was above the Millstone Hill radar and rotating as much as 30° clockwise as compared with the July 23/83 period.

(E). HEIGHT DEPENDENCE.

In this final section we will discuss the short pulse observations on July 31/
84 (from 16:30 to 18:30 LT). During this period the maximum observed power was,
presumably, that associated with the real saturation amplitude of the unstable
waves (and of the order of POL~12). In the color-coded distributions, discussed
in the following lines, color violet corresponds to values of POL between 10 to
12. The good spatial resolution of this data set allows the analysis of the
height dependence of the spectral parameters. The spectral resolution is rather
poor (~500 m/s), and we can't differentiate between the spectral widths corres-
ponding to either the type I primaries or the 'type A' secondaries. This has to
be done using the phase velocity information.

1. DISTRIBUTIONS OF THE TOTAL POWER WITH ALTITUDE, ASPECT ANGLE AND LOCAL TIME.

FIGURES (5.12a) and (5.12b) show the POL color-coded distributions of the ob-
served data as functions of the altitude, aspect angle (both as defined by the
main beam direction) and local time. Two clearly differentiated features are
shown in these plots at each half of the observation period (the yellow spots in
the first half and the extended violet spot in the second half). Before discuss-
ing figure (5.13) we will present the electric field data.

2. ELECTRIC FIELD.

In figure (5.14) we display the best estimate of the electric field using the
Foster-Evans model (Foster et al. [1986.a,b]). This model predicts the electric
field as a function of the local time and latitude for different auroral condi-
tions as functions of the particle precipitation level. This electric field has
been calibrated by using a linear fit between the model predictions and the mea-
sured line of sight projections of the ion-drift at F-region altitudes (160 to
250 km). For the first half of the observation period the electric field was
below the Farley-Buneman instability threshold. The two 'spikes' in the electric
field may be real but we can not say much about that by lack of complementary
information. In the second half of this period the electric field was greater than 20 mV/m most of the time.

3. DISTRIBUTIONS OF THE INSTABILITIES' PHASE VELOCITY AND SPECTRAL WIDTH.

FIGURE (5.13). Figure (a) displays the altitude-phase velocity distribution of parameter POL (color-coded) during the 21:30 to 22:30 UT period (16:30 to 17:30 LT) and figure (b) shows the corresponding altitude-spectral width distribution. Figures (c) and (d) show the same distributions but for the 22:30 to 23:30 UT period (17:30 to 18:30 LT).

16:30-17:30 LT.

There is no evidence of Farley-Buneman instability during this period. The sharp transition (at 140 km) of the distribution of the phase velocities with altitude seems to indicate that instabilities are generated at different altitudes and are moving with the ions. In the E-region, ions move with the neutral wind and, follow the ExB drift in the F-region (above ∼140 km altitude). The distribution of the spectral widths also shows the probable presence of three different driving mechanisms (density gradient drift at two turbulence regimes and the turbulence-related ExB gradient drift); particle precipitation level was high during this period (see figure (5.15)) and it may explain the presence of the narrow (DSP < 500 m/s) and moderately broad spectral signatures (600 < DSP < 900 m/s). They may be the result of the density-gradient instability at two turbulence regimes: the narrower widths could correspond to the linear generation of 30-cm waves and the moderately broad widths be the result of mode coupling between longer-wavelength density gradient primaries. The other spectral signatures concentrated around 110 to 130 km altitude seem the signatures of type 2 turbulence-generated secondaries. At these altitudes and for the local evening hours, the ExB-gradient drift instability is favored ($E \cdot \nabla n_\sigma > 0$).
The ambient density gradient $\nabla n_e$ is associated with the vertical electron density profile and is typically negative northward. Calling $L_{\nabla n_e}$ the vertical electron density scale-length and $L_{n_e}$ to the component of this length in the direction of the local electric field, we have that $L_{\nabla n_e} = L_{n_e} \cos(\chi)$ where $\chi$ is the magnetic dip angle (Greenwald [1974]). At Millstone Hill $\chi \sim 71^\circ$ and for $L_{n_e} > 6$ km we have that $L_{\nabla n_e} > 2$ km.

In our analysis we don't consider data-points below 100 to 105 km altitude because they are the result of the off-main beam, higher elevation, intersections of the unstable region for very low main beam elevations ($\sim 2^\circ$).

17:30-18:30 LT.

The electric field was above threshold for Farley-Buneman instability and the presence of type 1 waves is clearly shown in our data. The waves' phase velocity variation with altitude seems the result of the variation of the ion drift from zero to $E/B$ (along the upper E-region up to the lower F-region).

The phase velocity of the presumed Farley-Buneman waves varies from 300 to 550 m/s. Because of the poor spectral resolution, the transition between the type 1 waves to the 'type A' secondaries can't not be identified. Broader spectra (type 2 waves) are concentrated in the region (between 120 to 140 km altitude) where the ambient density gradient may lead to a strongly turbulent ExB-gradient drift state. As in the previous analysis (16:30 to 17:30 LT), we are not considering the data-points at altitudes below 100 km.

Finally, in figure (5.15) we show the particle-precipitation activity (expressed by index $p$, see chapter 4 section A) for the July 31-August 01/84 period (in universal time) and the corresponding east-west and north-south components of the ExB drift at 64° magnetic latitude from the Foster-Evans model (Foster et al. [1986.a,b]).
CHAPTER 5.

FIGURE (5.0). (a) This plot shows the calculated electric field potential contours (every 2 kV) corresponding to a 180° AZ scan (at EL = 4°) during July 23/83 (from 15:08 to 15:30 LT). Intensity variations from gray to black are associated with values of observed POL from 5 to 10.

(b) Here we show the measured spectrum for different ranges and along AZ = 20° NE (defined by the line '1' in figure (a)). The vertical scale of the individual spectra have been normalized to the unity and the horizontal scale goes from -25 to 25 kHz; the frequency resolution is equal to 780 Hz. VP1 is the phase velocity (in m/s) of the 'broad' spectral component, VP2 that of the 'narrower' one and z is the main beam altitude in km. VIS is the line of sight projection of the ion-drift velocity, VPK the phase velocity of the coherent spectral component, and POL the LOG10 of the received power in equivalent electron density units (cm-3). Line-related spectra are associated with cases where two spectral components are identified whereas the other spectra (crosses) are the signature of stable plasma fluctuations.

(c) This figure is similar to that of figure (b) but for AZ = 270° (direction defined by the line '2' in (a)). In this case we only observe stable plasma fluctuations either alone or 'contaminated' by side-lobe antenna pattern effects (VPK is the velocity of the 'unstable' spectral component).
FIGURES (5.1a) and (5.1b). They display the local time distributions of the calculated electric field (using algorithm described in paragraph 4 in chapter 5) (a), and parameter POL (b) during July 23/83 when instability is present. The color scale gives the apparent (main beam) altitude of the observed points; the altitude spread is the result of the experimental configuration (180° AZ scans every 20 minutes) and the antenna pattern 'contamination'. Here higher altitudes correspond to higher latitudes and greater electric fields, because instability is basically concentrated in the lower E layer, higher apparent altitudes also correspond to smaller values of POL (from off beam intersections of the unstable region).

FIGURES (5.2a) and (5.2b). The format of figure (5.1) is followed for the April 20-21/85 period. Magnetic conditions were more disturbed during this period which may explain the stronger dispersion in the distribution of POL as compared with the July 23/83 data.

FIGURES (5.3a) and (5.3b). These figures show the correlation between the electric field strength (EE) and POL for the two experimental periods. The presumed type 1 instabilities are associated with the sharp transition in POL for values of EE > 20 mV/m. In these observations, when instability is present, the receiver saturates at main beam altitudes between 105 to 150 km. The uncorrelated values of POL and EE are the result of either side-lobe 'contamination' or the observation of secondary waves.

FIGURE (5.4). This figure displays correlation plots (color-coded by altitu-
between LOG10 of the power associated with the 'unstable' spectral components (PWK) and parameter POL (total power). On the right hand side we have plotted (full-line) the equation $\text{PWK} = \text{LOG10}(10^{\text{POL}})$ for $n_0 \sim 5 \times 10^3$ and $10^5$ cm$^{-3}$ (a) and $n_0 \sim 10^3$ and $10^5$ cm$^{-3}$ (b). Broken-lines represent the limit $\text{POL} - \text{PWK} = 1$ (see paragraph (4.B), chapter 5).

**FIGURE (5.5).** Here we show correlation plots between POL and the width of the 'unstable' spectral components (DSP), and between DSP and the main beam azimuth AZ for the July 23/83 ((a), (b)) and the April 20/85 ((c), (d)) periods.

We clearly see the presence of two classes of spectral width: 'narrower' spectra (DSP < 400 m/s) are associated with both the type 1 primaries and presumed type 0 secondaries whereas 'broader' spectra (DSP > 600 m/s) may correspond to type 2 secondaries.

**FIGURE (5.6).** This figure show the correlation plots between POL and AZ (a), the waves phase velocity (VPH) and AZ (b), POL and VPH (c) and DSP and VPH (d) for the 'narrower' spectral signatures during the July 23/83 observations. The generation of Farley-Buneman instability at 30-cm (type 1 primaries) and presumed type 0 secondaries (low-frequency density-gradient drift turbulence) can explain these observations (see paragraph (5.C.1) chapter 5).

**FIGURE (5.7).** Same as figure (5.6) but for the April 20/85 period.

**FIGURE (5.8).** This figure shows the properties of the 'broader' spectral signatures measured during the July 23/83 period. As before we display the correlation plots between: (a) POL and AZ, (b) VPH and AZ, (c) POL and VPH and
(d) DSP and VPH. These plots show the type 2 nature of the 'broader' secondaries (EXB-gradient drift turbulence), moreover, the closely periodic structure in the bottom of distribution (d) appears to be the 'trace' of the kilometer-length ExB-gradient drift primaries. See text for discussion (pag. 179).

FIGURE (5.9). Same as figure (5.8) but for the April 20/85 period.

FIGURE (5.10). Plots (a) and (c) show the distribution of POL with flow angle (angle between the radar line of sight and the ExB flow direction) for the two observation periods. A flow angle 'sensitivity' (power drop between the type 1 primaries and the secondaries in the direction perpendicular to the flow) of -20 to -15 dB can be inferred. In figure (b), as expected, values of VPH close to zero correspond to flow angles close to $90^\circ$. The $\sim 20^\circ$ clockwise miss determination of the flow angle during the April 20/85 period (d), seems the result of a $20^\circ$ out-phasing between the ExB flow pattern at F-region altitudes and the electron drift in the lower E-region by effects of a relatively strong poleward component (of the order of 100 to 150 m/s) of the neutral wind blowing at electrojet altitudes (see paragraph (5.D), chapter 5).

FIGURE (5.11). The altitude color-coded plots in (a) and (b) show the correlation between flow angle and azimuth for the two observation periods. Linear plots on the right hand side show the limits where the ExB flow follows the direction perpendicular to the magnetic meridian plane through Millstone Hill (broken-line) and when it follows the circle of $63^\circ$ magnetic latitude (full-line).
FIGURE (5.12). This figure shows the distributions of POL (color-coded) with altitude and local time (a) and that with aspect angle and local time (b) for the finer height resolution (5 to 10 km) data set introduced in figure (4.21).

FIGURE (5.13). Here we display the distributions of POL with altitude and phase velocity and with altitude and spectral width for the July 31/84 data, but divided into two periods: from 16:30 to 17:30 LT (plots (a) and (b)) and from 17:30 to 18:30 LT (plots (c) and (d)). Plots (a) and (b) show the presence of two probable density-gradient drift turbulence states associated with high particle precipitation activity: narrower spectra could be the signature of linearly unstable 30-cm waves and intermediate width spectrum the result of mode-coupling at longer wavelengths. The much broader spectra may correspond to type 2 waves. Plots (c) and (d) show the signatures of type 1 and type 2 waves; data points at altitudes below ~105 km are not considered in our analysis.

FIGURE (5.14). This figure displays the best estimate of the electric field for the July 31/84 period from the fit between the Foster-Evans model and the measured line of sight components of the ion-drift velocity in the F-region (> 150 km altitude). Between 16:30 to 17:30 LT (21:30-22:30 UT) the electric field was below the Farley-Buneman instability threshold whereas for the 17:30 to 18:30 LT (22:30-23:30 UT) period it was above it (> 20 mV/m).

FIGURE (5.15). Here we show the particle precipitation activity (index p) during the July 31/84 period (top panel) and the magnetic East-west and North-south components of the ExB flow at 64° magnetic latitude (from the Foster-Evans model). Precipitation activity was important during the whole period (p > 7).
JULY 23, 1983  22:20 UT
APEX MAGNETIC LATITUDE & LONGITUDE

FIGURE 5.0a
FIGURE 5.0b
FIGURE 5.0c
FIGURE 5.2
FIGURE 5.3
FIGURE 5.4
FIGURE 5.6
FIGURE 5.7
FIGURE 5.9
FIGURE 5.11

23 JULY 83 13-20 LT

20 APRIL 85 13-18 LT

FLOW ANGLE (degrees) vs. AZIMUTH (degrees)
FIGURE 5.12
FIGURE 5.13
FIGURE 5.14
CONCLUSION

The M.I.T.-Millstone Hill radar facility is an excellent tool to probe the auroral ionosphere (up to 1000 km) during both, stable and unstable conditions. For the first time, a high sensitivity system acting as both an incoherent and a coherent backscatter has been used to systematically study plasma turbulence in the auroral lower ionosphere. The basic goal of this dissertation is the analysis and the interpretation of these dual-mode observations.

The unique capabilities of the Millstone Hill radar system allowed us to reproduce and extend a number of previous observations made with different radar systems in various conditions.

The main original contribution of this study is the simultaneous determination of the instabilities' spectral properties and their relation to the local electric field. We identified two linear instability mechanisms (the Farley-Buneman and, probably, the 'trace' of the kilometer-length ExB-gradient drift primaries) and two turbulence-related waves (the type 0 and type 2 'secondaries') in the lower E-region. We have also initiated a study of other types of instabilities developing in the upper E and lower F regions and, in particular, some of our analysed data seems to indicate the presence of low-frequency density-gradient drift waves in two turbulence regimes.

Millstone Hill radar unique capabilities.

The research program pursued has concentrated on the unique capabilities of
this system. The Millstone Hill radar was primarily designed to measure stable density fluctuations ('incoherent') of the ionospheric plasma but, in addition, it can provide information about instability development ('coherent' modes) as well as the large-scale coupling between the unstable regions and the surrounding stable plasma. Routine radar operation with the completely steerable antenna has generated a large database which has been used in our investigation.

(a) Looking to the north at low elevation angles, the radar beam is nearly perpendicular to the Earth's magnetic field at E-region altitudes and is sensitive to intense coherent backscattering from 30-cm strongly field aligned instabilities. The high sensitivity of our system may also allow the observation of micro-instabilities generated in the F-region.

(b) The simultaneous determination of the instabilities' spectral parameters and the local electric field over large spatial regions and extended observation periods allows the study of the relationship between instability generation and electric field at E-region altitudes.

(c) The narrow antenna beam (1°) combined with shorter transmitted pulse lengths (defining a relatively small effective scattering volume) also allows the fine spatial resolution study of the unstable regions. We use this characteristic for the analysis of the aspect angle dependence of the E-region 'clutter'.

(d) The high sensitivity of our radar (i.e. its calibration to measure the thermal fluctuations level), allows the determination of the scattering cross-section of the 30-cm unstable waves in absolute units as well as their latitudinal extent (through the off-beam and side-lobe intersections of the unstable region). It was not an easy task to define the instabilities' effective scattering volume because of the important radar convolution effects (e.g. antenna pattern 'contamination'). In chapters 4 and 5 this problem was treated.
Main results of our study.

Our study was divided into two parts: theoretical and experimental.

(1) In the theoretical part (chapters 1, 2 and appendices 1, 2) we have reviewed the conditions for the linear generation of kinetic instabilities expected to occur under auroral lower ionosphere conditions and likely to be observed with our 440 MHz antenna. We have also estimated their corresponding saturation amplitudes in the framework of the weak turbulence 'orbit-diffusion' approximation. The originality of this review dwells on the fact that we have completed some calculations presented in the literature and extended their application to the 30-cm wavelength instabilities. In particular, we have derived the collisional kinetic current-convective mode in the lower ionosphere and included the effects of neutral collisions (the BGK approximation) and of a moderate transverse electric field in the derivation of the density-gradient drift and of the Post-Rosenbluth instabilities.

Considering the effects of currents (both parallel and transverse to the magnetic field), transverse electric fields and density gradients as well as the collisions between the charged particles and the neutrals, we have found that the main linear instability mechanisms at 30-cm, are (1) the modified two-stream in the electrojet region, (2) the density-gradient drift (the collisional universal and the lower hybrid modes), and (3) the collisional ion-cyclotron current-driven and current-convective modes, in both the upper E and the lower F regions. In the lower F-region (4) the 30-cm Pedersen-drift and (5) the Post-Rosenbluth modes can also be excited.

At this wavelength, analytical approximations in the dispersion relation as we have used are always possible (because $k^2 \rho_i^2 \gg 1$ and $k^2 \rho_e^2 \ll 1$). In the future,
we plan to solve the dispersion relation numerically in order to verify our approximations and study the generation of the different instabilities in a range of more realistic ionospheric conditions.

(2) In the **experimental part** we first discuss the Thomson scattering technique and the radar convolution effects in the observation of the unstable plasma fluctuations and then the interpretation of the measured scattering cross section and the power spectrum (chapters 3, 4 and 5).

(A) **Antenna pattern convolution.**

Because our radar system is calibrated to measure the stable (thermal) density fluctuations, convolution effects between the antenna radiation pattern and the unstable plasma waves can be very important (the extent of the scattering volume is a function of the relative power level between the unstable and the stable fluctuations, the antenna pattern, and the aspect angle and flow angle sensitivities). Yet, in the cases where the radar main beam intercepts an instability source, the backscattering is basically generated inside a relatively well defined effective volume (given by the central radiation pattern and the transmitted pulse length alone) centered at the range \( R \) along the observation direction.

In order to study the effects of the radar convolution on the statistical distributions of the measured cross-section, we have calculated the radar response to a thin (i.e., its vertical spread is smaller than our altitude resolution) uniform, unstable layer placed in the lower E-region (electrojet).

Our long-pulse (2 msec) observations with an equivalent height-resolution greater than the vertical extent of the unstable electrojet are well represented by such model. This fact facilitates the analysis of the antenna side-lobe contribution to the received signal and provides a good estimate of the location of the effective volume enclosing the instability source.
The main results of this comparison are:

(i) The intensity of the backscatter from irregularities in the main-beam and the observed aspect angle sensitivity lead to the conclusion that the lower E-region instabilities are strongly field aligned, and concentrated in a layer with vertical extend smaller than our height resolution. Their occurrence will account for most of the E-region coherent backscatter.

(ii) All the coherent backscattering coming from ranges smaller than the one corresponding to the intersection of the tangent plane at Millstone Hill and the outer boundary of the unstable lower E-region at 125 km ( 'horizon') are generally due to the side-lobe backscatter from this unstable region. The main beam apparent altitudes corresponding to this range are equal to 220 km at $EL = 4^\circ$ and 250 km at $EL = 6^\circ$ (considering the range smearing of our long-pulse observations these altitudes can be as high as 300 km). Beyond this range, and for a given elevation, the effects of the principal 'clutter' region are not felt.

(iii) For our radar system and elevation angles smaller than or equal to $4^\circ$, secondary maxima in the power-range profile may correspond either to the latitudinal structure (penetration of the enhanced convection electric field) in the electrojet region or to different instability sources at different altitudes.

Using the spectral information and a finer range resolution we might resolve, for ranges inside the lower E-region 'horizon', possible instability mechanisms other than the electrojet.

(B) Analysis of the scattering cross-section and the turbulence power spectrum.

Observations of intense coherent backscattering can be explained by the exist-
ence of various plasma turbulence states at 30-cm wavelength. In particular from our present study, the frequent generation of the modified two-stream instability (type 1) and two types of turbulence-related 'secondaries' ranging into two classes of spectral width ('narrower' and 'broader' spectrum), in the disturbed lower E-region, are clearly established.

Evidence supporting our identification of type 1 waves includes the following (expected type 1 characteristics are enclosed in parentheses): the type 1 waves we identify in our observations are linearly generated for a given strength of the electrojet current (electric field threshold) inside a limited angular region (instability cone) around the flow direction (flow angle dependence) in a plane closely perpendicular to the local magnetic field (aspect angle dependence). They are associated with narrow spectral signatures and relative constant Doppler-shift (the ion-sound speed).

The secondary waves (either type 2 or type 0) seem to result from the enhancement of short-wavelength density fluctuations well above the thermal level by the transfer of turbulent wave-energy from the fluid-like primaries (either ExB gradient drift or type 1). They are associated with broad spectral signatures and Doppler-shifts smaller than the ion-sound speed and statistically distributed around zero.

From the analysis of the radar observations we have concluded:

(a) Coherent echo-occurrence is concentrated in the lower E-region for local times between 15 to 05 hours and is maximum around 18-20 LT (from the Foster-Evans model the average electric field strength is maximum in this sector, see figure (4.7)). The apparent lack of echo-occurrences in the morning-noon sector is due to the fact that, during these hours, the auroral oval (and thus the region of enhanced electrojet) is not intersected by our radar line of sight at
E-region altitudes. Also this occurrence is most probable for values of Kp greater than 3 and average electric fields of the order of 20 mV/m or greater (corresponding to a precipitation index greater than 7). The very good correlation between the average electric field distribution in local time (from the Foster-Evans model, figures (4.7) and (4.8)) and echo-occurrence, at E-region altitudes, implies that in this region instabilities are basically driven by the ExB (Hall) current associated with the convection electric field and the particle-precipitation activity (regulating the auroral E-region conductivity).

(b) In chapter 4 we determined a radar aspect angle sensitivity of the order of -7 to -11 dB per degree as well as a maximum turbulence power level of 60 to 70 dB above the stable fluctuation background (volume reflectivity for E-region coherent backscatter \( \sim 10^{-9} \) m-1) for an unstable lower E-region centered at 108 km altitude and with a vertical spread of 8 km (-3 dB 'thickness').

(c) We have determined a threshold electric field of the order of 20 to 25 mV/m (see figures (5.2), (5.3) and (5.4) and discussions) for the generation of the type 1 instabilities. Also, a flow angle 'sensitivity' of the order of -20 to -15 dB (for the attenuation between the maximum power in the direction perpendicular to the ExB flow as compared to that in the edge of the instability cone) has been found (see figures (5.11) and (5.12)).

(d) The phase velocity of type 1 waves is observed to saturate at the presumed ion-sound speed of the medium ('plateau' formation, figures (5.7b) and (5.8b)).

We also noticed that the spread of the phase velocity on the 'plateaus' is associated with the systematic increase in the apparent altitude of our observations. Here 'altitudes' correspond to the off-beam intersections of the unstable E-layer and are associated with both higher latitudes and greater electric fields. This 'plateau' spread is explained by the increase of the waves phase
velocity with the electric field strength beyond the Farley-Buneman instability threshold (previously observed at 1-m wavelength by Nielsen and Schlegel [1985]) and presumably due to the anomalous heating of the electron-gas by the unstable waves (Robinson [1986]).

(e) The 'narrower' ('type A') secondaries seem the result of the turbulence induced by the type 1 waves either through mode-coupling or by a two-step process (low-frequency density gradient drift waves induced by the enhanced type 1 wave-amplitudes) and are presumably the same as type 0 waves reported by St.Maurice et al. [1986]. The 'broader' ('type B') secondaries are very likely the result of the turbulent 'cascading' from the fluid-like ExB-gradient drift modes (and also called type 2 waves). Associated with the type B spectra we also observe the probable 'trace' of kilometer-length ExB-gradient drift primaries (see discussion on figures (5.8) and (5.9) in chapter 5).

(f) We have also discussed some evidence of instability generation in the upper E and the lower F regions: during the finer height-resolution experiment (on July 31/84) we identified the presence of two separate classes of spectra (the 'narrow' and 'intermediate width' spectral signatures) moving with the apparent ExB drift velocity above 130-140 km altitude and when no type 1 waves were registered in the lower E-region (chapter 5 paragraph E, figure (5.13)). The more plausible driving mechanism explaining these observations is the low frequency density-gradient drift instability reviewed in chapter 2 (associated with reasonably sharp density gradients from particle precipitation) at, probably, two different turbulence regimes (weak and moderately strong, see discussion on figure (5.13)). High particle-precipitation levels were measured in this period.

(g) The identification of other possible instability mechanisms explaining the coherent backscattering during the periods discussed in chapter 4 and observed
at ranges outside the E-region 'horizon' or when no systematic side-lobe contamination inside it is expected (figures (4.11), (4.12), (4.16) and (4.17)) can be done and needs a better resolution in both range and frequency. The fact that the Millstone Hill radar measures the stable and the unstable plasma fluctuations will also help (as in the case of the electric field's determination) to define other parameters related with various instability thresholds (density and temperature gradients, ExB-flow shears, etc).

3) Recommendations for future studies.
(a) A frequent limitation of our observations was that at E-region altitudes the measured saturation level, for main-beam intersections of the unstable layer, was imposed by the receiver. This effect can be easily overcome in the future in order to initiate a more detailed study of the evolution of the instability.

Indirectly, the saturation level can always be estimated from off-beam and side lobe observations.
(b) Integration times much smaller than that used in the presented observations (30 sec), may be applied in the study of short-lived events and the growth of instability through saturation. Our system may also allow the finer study of the instability development as well as the large scale coupling between the unstable regions and the surrounding stable plasma associated with anomalous heating and transport processes.
(c) The identification of various instability mechanisms at altitudes inside the unstable E-region horizon is possible and needs more systematic observations at finer spatial and spectral resolutions (this might be done with a multipulse transmission scheme actually operational in our 440 MHz system).
(d) Finally, the use in the near-future, of a multifrequency system is possible
by combining the 440 MHz and the 1298 MHz completely steerable radar antennas located both at the Millstone Hill site. This configuration will allow the comparative study of the ionospheric turbulence at two wavelengths, the better determination of the scattering volume and a very fine spatial resolution.

Collaboration with other groups in order to use a bistatic radar configuration is also possible. In this case, instability can be triggered for a range of wavelengths function of the angle between the two intersecting radar beams.
APPENDICES
APPENDIX 1

LINEAR KINETIC INSTABILITIES IN THE LOWER IONOSPHERE

1. INITIAL ASSUMPTIONS.

(a) For instabilities with wavelengths such that $k_l > 1$ ($\lambda \lesssim 20$ m for $l > 3$ m) the use of kinetic theory is required. In this case Larmor radius effects are important. $\rho_i, \rho_e$ are the ion and electron Larmor radius and in the ionosphere $\rho_e \sim 1.5$ cm.

Debye length. The potential field around a point charge is effectively screened out by the induced space-charge field in the electron gas for distances greater than the Debye length. In the MKS units,

$$\lambda_D = \sum_j \lambda_j$$

and

$$\lambda_j^2 = \frac{e^2 T_j}{k_B^2 n_j}$$

(b) Weak density gradients perpendicular to the magnetic field:

$$\varepsilon = \left[ \frac{1}{n} \frac{dn}{dx} \right]^4, \quad \varepsilon \rho_i \ll 1$$

(1)

(c) Collisional effects are well represented by the BGK (Bhatnagar, Gross, and Krook) model. Such is the case when collisions between charged particles and neutrals are dominant. This is generally true for ions below 300 km for typical ionospheric densities ($\lesssim 10^5$ cm$^{-3}$).

When $k \rho_e \ll 1$ total electron collisions may be represented by an effective
collision frequency: \[ \nu_e = \nu_{en} + \nu_{ei} \cdot \left(1 + \left(\frac{k_i^2}{\nu_e^2}\right) /3\right) \], in a BGK model for a large set of ionospheric conditions and altitudes (Sperling and Goldman [1980]).

(d) Wave propagation is perpendicular \((k_i = 0)\) or almost perpendicular to the magnetic field \((k_i << k_\parallel)\).

(e) Electric drifts.

In the presence of an uniform electric field electrons and ions experience the Pedersen \(\nu_P\) and Hall \(\nu_H\) drifts perpendicularly to the magnetic field:

\[\nu_{H,j} = \frac{\Omega_j^2}{\nu_j^2 + \Omega_j^2} \frac{E \times B}{B^2} \]  \hspace{1cm} (2.a)

\[\nu_{P,j} = \frac{\nu_j \Omega_j^2}{\nu_j^2 + \Omega_j^2} \frac{E}{B} \]  \hspace{1cm} (2.b)

\(\nu_i\) and \(\nu_e\) are respectively the effective ion-neutral and electron-neutral collision frequencies.

In the ionosphere \(\nu_e << \Omega_e\) and electrons move with the ExB drift. Below 120 km the ions are unmagnetized \((\nu_i >> \Omega_i)\) and move with the neutrals.

Above 130 km \(\Omega_i >> \nu_i\) and both electrons and ions move with the ExB (cross-field) drift.

(f) Reference frame.

In all our calculations we use the frame moving with the ions' electric drift.

In the lower E-region this frame is practically the same than the one moving with the neutrals. In the F-region this frame is the one moving with the cross field drift. The radar reference frame is in a good approximation the same than the neutrals frame and the neutral atmosphere will be assumed to rigidly rotate with the Earth. This assumption must be taken with some reserve since the
strength of the neutral winds in the auroral ionosphere may be as high as 100 to 200 m/s.

2. ELECTROSTATIC WAVES IN THE LOWER AURORAL IONOSPHERE.

We derive the linear kinetic dispersion relationship for two different configurations corresponding to the lower E-region and the lower F-region conditions. The ionosphere is assumed to be a two species plasma in an uniform magnetic field where only longitudinal oscillations are possible (zero-beta).

1) Configuration 1 (figure Al.a).

Instability driving sources are the ambient electric field, the ambient density gradient, and the electron and ion inertia.

In the region between 90 to 120 km of altitude (electrojet), the modified two stream (Farley-Buneman), and the ExB density gradient drift instabilities may be generated (Schlegel [1982], Fejer et al. [1984]).

For simplicity the plasma is considered inhomogeneous only in the direction parallel to the electric field. In any case only the component of the ambient density gradient parallel to the electric field favors instability.

This configuration is also used to derive the EIC current-driven and the current-convective instabilities above the electrojet region ( > 130 km). In these cases a parallel electron drift (field-aligned current) together with a short-scale density gradient are the driving terms. A moderate electric field (associated with a cross field drift $V_E$ smaller than the ion thermal speed) does not affect these instabilities. For the case of a stronger transverse electric field see Ganguli and Palmadeso [1987].

2) Configuration 2 (figure Al.b).

Instability driving sources are the ambient electric field, the short-scale density gradients, and the ion-neutral collisions (region between 130 to 300
km). In these conditions the density-gradient-drift (universal-drift, the ion cyclotron-drift, and the lower-hybrid-drift) instabilities may exist (Gary et al. [1983]). Short wavelength instabilities are also favored by the presence of a moderate electric field (Pedersen-drift mechanism), the most unstable situation is the one where the short-scale gradient is perpendicular to the electric field (Gary and Cole [1983]).

Stronger electric fields ($V_E > V_c$ or $|E| > 50$ mV/m) may also induce an unstable loss-cone type ion distribution (for the 0 ) in the weakly collisional lower F-region (Ott and Farley [1975], Lakhina and Bhatia [1984]).

In figures 1.a and 1.b, axis x is positive towards the magnetic north, axis y towards the east and axis z parallel to $\mathbf{B}$.

We also have:

\[
\begin{align*}
V_{e\perp} &= \left( V_{e} - \frac{1}{2} \mathbf{v}_i \right)_\perp \\
V_{e\parallel} &= V_{e}\mathbf{i} \\
E &= \text{the ambient electric field,} \\
\nabla n &= \text{the density gradient,} \\
E_E &= \text{the cross-field drift,} \\
V_nj &= \text{the diamagnetic drift associated with the density gradient and,} \\
V_{P_i} &= \text{the ions Pedersen drift.}
\end{align*}
\]

Boltzmann-Vlasov equation and BGK collision term.

In the frame moving with the neutrals:

\[
\frac{\partial f_j}{\partial t} + \nabla \cdot \left( v_j f_j + \frac{1}{2} z_j q_j (E + v_x B) \right) \cdot \nabla \frac{f_j}{n_j} = C_j
\]

where $C_j = - V_j \left( \frac{f_j}{n_j} \cdot \nabla f^{(m)}_j \right)$ is the BGK collision term and
\( f_{0j}^{(M)} = \frac{n_{0j}}{(\gamma \nu_j^2)^{3/2}} \exp\left( - \frac{\nu^2}{\gamma \nu_j^2} \right) \)  

\( f_{0j}^{(M)} \) is the Maxwellian distribution in the neutral frame toward which collisions drive the electron and ion distributions. The effective collision frequency \( \gamma \nu_j \) is the reciprocal of the relaxation time to this local equilibrium configuration.

**Poisson equation.**

\[ \nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \sum_j z_j q_j n_j \]  

and \( n_j = \int d\nu f_{0j}(\mathbf{x}, \nu_j, t) \)

**Linearization.**

In linear kinetic theory plasma oscillations are considered small perturbations (first-order) about a zeroth-order state defined by the initial and the boundary conditions:

\[ f_j' = f_j + f_{ij}, \quad n_j' = n_j + n_{ij}, \quad \mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1 \quad \text{and} \quad \mathbf{E}_1 = - \nabla \phi_1 \]

**Zeroth-order equation.**

\[ \bar{\nu} \cdot \frac{\partial f_{0j}}{\partial \mathbf{x}} + \frac{z_j q_j}{m_j} ( \mathbf{E}_0 + \bar{\nu} \times \mathbf{B} ). \frac{\partial}{\partial \nu} f_{0j} = 0 \]  

The density gradient is antiparallel to the x direction.

\[ f_{0j}(\mathbf{x}, \bar{\nu}) = \frac{n_{0j}(\nu_j)}{(\gamma \nu_j^2)^{3/2}} \exp\left( - \frac{(\bar{\nu} - \nu_j)^2}{\nu_j^2} / \nu_j^2 \right) \]  

\( \bar{\nu}_j = \nu_{dj} + \nu_{nj} \)
Where \( n_{ij}(x) = (1 + \varepsilon x)n_{ij} \), \( \nu_{ji} = \frac{eB_{j}n_{i}E_{y}}{\sqrt{2}} \).

In configuration 1 in the frame moving with the ions \( \nu_{de} = \nu_{a} \) and \( \nu_{di} = 0 \), so we have:

\[
\hat{f}_{ij}(x, \nu) = \frac{n_{ij}}{(\pi\nu_{j}^{2})^{3/2}} (1 + \varepsilon x + \varepsilon \frac{\nu_{y}}{\nu_{j}})^{\exp(-\frac{\nu^{2}}{\nu_{j}^{2}})} (9.a)
\]

In configuration 2 in the frame moving with \( \nu \), taking into account the weak ion-neutral collisions (\( \nu_{i} \ll \Omega_{i} \)):

\[
\nu_{e} = \nu_{ne}
\]

\[
\nu_{i} = \nu_{pi} + \frac{\nu_{i}v_{i}}{\nu_{i}} \nu_{i} x B /B + \nu_{ni} \text{ with } \nu_{pi} = \frac{\nu_{i}}{\nu_{i}} E /B
\]

\[
\hat{f}_{ij}(x, \nu) = \frac{n_{ij}}{(\pi\nu_{j}^{2})^{3/2}} (1 + \varepsilon x + \varepsilon \frac{\nu_{y}}{\nu_{j}} - 2 \frac{\varepsilon^{2} \nu_{j}^{2}}{\nu_{j}^{2}} x + 2 \nu_{y} \nu_{j} \nu_{y})^{\exp(-\frac{\nu^{2}}{\nu_{j}^{2}})} (9.b)
\]

First-order equation.

Calling \( \nu \) the velocity measured in the frame moving with the ions electric drift,

\[
\frac{d}{dt} \hat{f}_{ij} = -\frac{\varepsilon_{ij} \nu_{i}}{\mu_{j}} \hat{E} + \frac{\nu_{i}}{\nu_{j}} \hat{f}_{ij} + \nu_{ij} \hat{f}_{ij} (10)
\]

The total time derivative is performed over an unperturbed orbit. The plasma is inhomogeneous along the \( x \) direction and all fluctuating quantities can be Fourier transform in the \( y \) and \( z \) directions. Taking \( k = k_{y} \hat{E}_{y} + k_{z} \hat{E}_{z} \) (local approximation) and considering elementary harmonic oscillations (plasma modes):

\[
\hat{f}_{ij}(x, \nu, t) = \hat{f}_{ij}(x, \nu) \exp(i(k \cdot x - \omega t)) (11.a)
\]
\[
E_\lambda = E_\mu \exp i(k \cdot x - \omega t)
\]

\[
n_{ij}(x, \kappa, \omega) = \int \mathcal{N} f_{ij}(x, \nu)
\]

(11.b)

(11.c)

calling \( n \cdot = n_{ij}(x, \kappa, \omega) \) from equation (10) one can find

\[
n_{ij} = \frac{\int \mathcal{N} f_{ij}^{(A)}}{1 - \frac{\mathcal{N}}{n_{ij}} \int \mathcal{N} f_{ij}^{(B)}}
\]

(12)

where

\[
f_{ij}^{(A)} \exp i(k \cdot x - \omega t) = -\frac{\mathcal{N}}{n_{ij}} \int_{-\infty}^{t} dt' E_{\mu} \cdot \frac{\partial f_{ij}}{\partial \nu}
\]

(13.a)

\[
f_{ij}^{(B)} \exp i(k \cdot x - \omega t) = \int_{-\infty}^{t} dt' f_{ij}^{(M)} \exp i(k \cdot x' - \omega t')
\]

(13.b)

and \( \omega_j - \omega + i \nu_j \), \( x' = x(t') \), \( \nu' = \nu(t') \). The initial time has been set-up at \( t_0 = -\infty \) in order to neglect any initial condition.

For kinetic waves moreover local homogeneity may be assumed inside the volume defined by the density gradient scale-length \( (\lambda < \rho_1 \ll L_\eta) \) and the vector \( k_y \hat{e}_y \) may be called \( \kappa_\perp \) (the perpendicular wave-vector).

Unperturbed orbits:

\[
x' = x - \frac{\mathcal{V}}{\mathcal{N}} [\sin(\omega_j \frac{\tau}{2} + \varphi) - \sin \varphi] - V_d x_j \tau
\]

(14.a)

\[
y' = y + \frac{\mathcal{V}}{\mathcal{N}} [\cos(\omega_j \frac{\tau}{2} + \varphi) - \cos \varphi] - V_d y_j \tau
\]

(14.b)

\[
z' = z - (\nu_1 + V_{d11} \tau)
\]

(14.c)

\[
v'_x = v_\perp \cos(\omega_j \frac{\tau}{2} + \varphi) + V_d x_j
\]

(15.a)

\[
v'_y = v_\perp \sin(\omega_j \frac{\tau}{2} + \varphi) + V_d y_j
\]

(15.b)

\[
v'_z = v_{11} + V_{d11}
\]

(15.c)

and \( \tau = t - t' \)

(a) Particles move freely parallel to the magnetic field and generally the ions parallel drift may be neglected.
(b) By assuming weak collisions \( \nu_j << \Omega_j \) one can ignore the effects of collisions on the unperturbed orbits.

(c) In the case where collisions are strong \( \nu_j \gg \Omega_j \), cyclotron motions are destroyed and the unperturbed orbits become straight lines.

3. DISPERSION RELATION.

After all the previous definitions we can proceed to calculate the electrons and ions susceptibilities

\[
\chi_j = - \frac{m_j}{z_j q_j \phi_k} n_{lj} \tag{16}
\]

where \( \lambda_j^2 = n_j z_j^2 q_j^2 / \epsilon_0 T_j \) and \( \lambda_j \) is the Debye length.

For wavelengths such that \( k^2 \lambda_j^2 << 1 \) the plasma quasineutrality holds:

\[
n_{oe} = n_{oi} = n_o .
\]

From Poisson's equation the dispersion relationship is expressed by,

\[
D(\vec{x}, \vec{\omega}, k) = 1 + \chi_e + \chi_i = 0 \tag{17}
\]

Using equations (12), (13), the orbits (14), (15) and equation (16) the dispersion equation (17) may be calculated.

In the following lines we are listing some key steps for its derivation in the two potentially unstable configurations described before.

From equations (14) the orbit function may be expressed by:

\[
k \cdot (\vec{x}' - \vec{x}) = k \cdot \frac{\phi}{\Omega_j} \left[ \cos(\Omega_j \tau + \varphi) - \cos \varphi \right] - \left( k \parallel v_\parallel + k \cdot \vec{V}_d_j \right) \tau \tag{18}
\]

and in equations (13) one can replace:
\[
\frac{\partial^2}{\partial t^2} \phi_{j} \left|_{\text{mix}} \right. = -i \frac{1}{\omega_j} \left( (1 + \varepsilon \frac{v}{v_j} \sin \varphi \cdot \frac{2k \varepsilon v \sin(\frac{\varphi}{v_j} + \frac{3}{2} \frac{v}{v_j} \omega_j^2) \right) F_{0j} \right.
\]

(19.a)

\[
\frac{\partial^2}{\partial t^2} \phi_{j} \left|_{\text{mix}} \right. = -i \frac{1}{\omega_j} \left( (1 + \varepsilon \frac{v}{v_j} \sin \varphi \cdot \frac{2k \varepsilon v \sin \varphi \cdot \frac{3}{2} \frac{v}{v_j} \omega_j^2) \right) F_{0j} \right.
\]

(19.b)

\[F_{0j}\] represents the normalized Maxwellian distribution function in the frame moving with the ions electric drift. Subscripts (1) and (2) identify the two configurations considered.

We also need to consider some identities between the Bessel functions (Abramovitz and Stegun [1965]):

\[
\exp iz\cos(\omega_j z + \varphi) = \sum_{m=-\infty}^{\infty} J_m(z) \exp -im(\frac{\pi}{2} + \omega_j z - \varphi) \]

(20.a)

\[
\int_0^{2\pi} d\varphi \exp i[(n-m)\frac{\pi}{2} - (n-m+1)\varphi] = 2\pi i \sum_{n+1}^{\infty} \delta_{n+1, m} \]

(20.b)

\[
J_{m+1}(z) + J_{m-1}(z) = 2m J_m(z)/z \]

(20.c)

\[
J_{m+1}(z) - J_{m-1}(z) = 2 J'_m(z) \]

(20.d)

With \[J_m(z) = \frac{d}{d\omega} J_m(z)\]

Calling \(f_{1j}\), \(f_{2j}\), \(f_{1j}\), \(f_{2j}\) the respective expressions of \(f_{1j}\) and \(f_{2j}\) in the configurations 1 and 2, and using identities (20) one can find:
We have called

\[ \overline{\omega}_j = \omega + \mathbf{v}_j \cdot \mathbf{k} \mathbf{v}/\mathbf{s}_j \quad \text{and} \quad \mu_j = k^2 \rho_j^2, \]

where \( \rho_j = \mathbf{v}_j/\mathbf{s}_j \) is the Larmor radius.

In equation (13.6) distribution \( f_{0j} \) have to be expressed in the frame moving with the ions electric drift.

In configuration 1 is a good approximation to take:

\[
\left| f_{0j}^{(1)} \right| = \frac{m_0}{\mathbf{m}_j^{(1)/2}} \exp - \frac{\mathbf{v}^2}{\mathbf{v}_j^2} 
\]

and in configuration 2,

\[
\left| f_{0j}^{(2)} \right| = \frac{m_0}{\mathbf{m}_j^{(2)/2}} \exp - \frac{2\mathbf{v}_E}{\mathbf{v}_j^2} \mathbf{v} \exp - (\mathbf{v}^2 + \mathbf{v}_E^2)/\mathbf{v}_j^2 
\]
Also because \( V_E / v_e \ll 1 \) only the effects of the electric field on the ion distribution need to be considered:

\[
\begin{align*}
\mathbf{f}_{1i}^{(8)} & = \exp - \frac{\mathbf{V}_E}{v_e^2} \left\{ \int_0^\infty d\tau F_{1}^{(1)}(\mathbf{v}_i') \exp i\left[ k_i(\mathbf{x}_i' - \mathbf{x}_i) + i2V_{E}\frac{\mathbf{J}_{\mathbf{v}_i}}{v_e^2} \right] \right\} \\
(2) & \end{align*}
\]

And, after some algebra:

\[
\begin{align*}
\mathbf{f}_{1j}^{(8)} & = -2 \pi \sum_{m=\infty}^{\infty} 2 J_2^2(z) F_{ij}(\mathbf{v}_j)/(\mathbf{\omega}_j - \mathbf{\Omega}_j - k_i v_i) \\
(1) & \\
\mathbf{f}_{1j}^{(8)} & = -2 \pi \sum_{m=\infty}^{\infty} [ 2 J_2^2(z) F_{ij}(\mathbf{v}_j)/(\mathbf{\omega}_j - \mathbf{\Omega}_j - k_i v_i) ] \\
(2) & \\
F_{ij}(\mathbf{v}_j)/(\mathbf{\omega}_j - \mathbf{\Omega}_j - k_i v_i) & \end{align*}
\]

with \( a_E = 1 + i \sqrt{2} \mathbf{V}_E/(k_{\mathbf{v}_i} v_{\mathbf{v}_i}) \).

Finally integrating equations (22) and (26) over the velocity space and replacing in equation (16), the susceptibilities are determined:

\[
\chi_j = \frac{1 + \sum_{l} \mathbf{Z} (\mathbf{\omega}_j - k_i \mathbf{v})_{\mathbf{v}_j} + \mathbf{j}_{\mathbf{v}_j} \mathbf{\Omega}_j \mathbf{v}_{\mathbf{v}_j})/(k_i v_{\mathbf{v}_j})}{1 + i \mathbf{v}_j \sum_{l} \mathbf{Z} /k_i v_{\mathbf{v}_j}}
\]

(27.a)
where, $Z(\tilde{\gamma}_j)$ is the plasma function, $\tilde{\gamma}_j^e = (\bar{\omega}_j \cdot \bar{k}_j^e)/k || v_j$, and when $k || v_j >> 1$. Equations 27.a and 27.b have been also derived by Gary et al. [1983] and Gary and Cole [1983] respectively. The additional feature in our calculations is the inclusion of the effects of a moderate to strong electric field ($V_E \leq v_j$).

In the last derivations (equations (19) and following) we have applied some additional definitions:

\[
\int_0^\infty dz z \exp(-cz^2) J_m(az) J_m(bz) = \frac{1}{2a} \Gamma_m(\frac{ab}{2c}) \exp(-a^2 + b^2)/4c
\]

(28.a)

\[
\Gamma_m(\mu_j') = I_m(\mu_j') \exp - \mu_j, \quad \Gamma'_m = \frac{d}{d \mu_j} \Gamma_m
\]

(28.b)

\[
\int_0^\infty dv \frac{2}{v_j^2} J_m(k v_j/\bar{s}_j) \exp(-v_j^2/v_j^2) = v_j^2 \Gamma_m(\mu_j')/2
\]

(28.c)

\[
Z(\tilde{\gamma}_j^e)/k || = \frac{1}{\tilde{\gamma}_j} \int_{-\infty}^{\infty} dv || \exp(-v_j^2/v_j^2)/(\bar{\omega}_j \cdot \bar{k}_j^e - k || v_j)
\]

(28.d)

4.**Approximations** (magnetized particles).

Under normal ionospheric conditions for weak density gradients ($\rho_i << L_n$) and short wavelengths ($k \rho_i > 1$) it is a good approximation to consider (Gary et al. [1983], Gary and Cole [1983]):
Above 130 km \( \frac{V_e}{\Omega} \ll 1 \) and for moderate electric fields (\( V_E < V'_e \)),

\[
\chi_i \mid = \frac{1}{2k^2 \lambda_i^2} \left( \frac{1 + \sum \hat{\Gamma}_m (\mu_i) Z \left( \frac{\hat{\omega}_i}{\lambda_i} \right) \left( \frac{\hat{\omega}_i - k \cdot \hat{V}_i}{\lambda_i} \right) / k_i \nu_i}{1 + i \nu_i \sum \hat{\Gamma}_m (\mu_i) Z / k_i \nu_i} \right) \tag{29.a}
\]

\[
\chi_i \mid = \frac{1}{k^2 \lambda_i^2} \left( \frac{1 + \sum \hat{\Gamma}_m (\mu'_i) Z \left( \frac{\hat{\omega}_i - k \cdot \hat{V}_i}{\lambda_i + k \cdot \hat{n}_i} \right) / k_i \nu_i}{1 + i \nu_i \sum \hat{\Gamma}_m (\mu'_i) Z / k_i \nu_i} \right) \tag{29.b}
\]

when \( k_{\nu} \rho_{\nu} \gg 1 \) one can take \( \mu'_i - \mu_i \).

Also in the electron susceptibilities one can ignore all the \(| m | > 1\) terms in the Bessel sums:

\[
\chi_e \mid = \frac{1}{2k^2 \lambda_e^2} \left( \frac{1 + \sum \hat{\Gamma}_m (\mu_e) Z \left( \frac{\hat{\omega}_e}{\lambda_e} \right) \left( \frac{\hat{\omega}_e - k \cdot \hat{n}_e}{\lambda_e} \right) / k_i \nu_e}{1 + i \nu_e \sum \hat{\Gamma}_m (\mu_e) Z \left( \frac{\hat{\omega}_e}{\lambda_e} \right) / k_i \nu_e} \right) \tag{30}
\]

where \( \hat{\omega}_e = \hat{\omega} + i \nu_e \), \( \hat{\omega}_d = \hat{\omega}_e + k \cdot \hat{n}_d \).

In our case \( \rho_e \sim 60 \) to \( 80 \) (\( \rho_e \sim 3 \) to \( 4 \)), \( k_{\nu} \rho_{\nu} \sim 0.3 \) and further approximations are still possible.

5. SUSCEPTIBILITY FOR UNMAGNETIZED IONS.

In the ionosphere electrons are generally magnetized but ions may well be unmagnetized. In the electrojet region ion-neutral collisions destroy the ions gyromotions and the unperturbed orbits become straight lines.

Above 130 km, ion-ion collisions (\( \nu_{ii} \)) may also demagnetize the ions when \( k^2 \lambda_i^2 \nu_{ii}/\Omega_i > 1 \). The last condition is verified in the auroral zone for \( k_{\nu} \rho_{\nu} > 50 \) (\( \lambda_i < 50 \) cm if \( \rho_e \sim 4 \)) and electron densities bigger than \( 10^4 \) cm\(^{-3}\).

The orbit function is now expressed as:
Electrojet region \((< 120 \text{ km})\),

\[
\begin{align*}
\text{f}^{(1)}_{oi} & = \frac{2\pi i q_i}{2\pi m_i} \int_0^{2\pi} \int_0^{\infty} d\varphi \, dz \, [2k_y \frac{\partial}{\partial z} f_{oi}] + 2k_y \frac{\partial}{\partial z} f_{oi} | \exp -i(\omega_i - k_y v_y - k_y v_y) z \ (31.\text{a}) \\
\text{f}^{(2)}_{oi} & = \frac{2\pi i q_i}{2\pi m_i} \int_0^{2\pi} \int_0^{\infty} d\varphi \, dz \, [2k_y \frac{\partial}{\partial z} f_{oi}] + 2k_y \frac{\partial}{\partial z} f_{oi} | \exp -i(\omega_i - k_y v_y - k_y v_y) z \ (31.\text{b})
\end{align*}
\]

with

\[
\begin{align*}
\text{f}^{(1)}_{oi} & = \frac{\mathcal{M}_0}{(\pi v_i^2)^{3/2}} \exp -\frac{(\chi^2 - \dot{v}_i \omega_i)^2}{v_i^2} \\
\text{f}^{(2)}_{oi} & = \frac{\mathcal{M}_0}{(\pi v_i^2)^{3/2}} \exp -\frac{\chi^2}{v_i^2}
\end{align*}
\]

Assuming that \( k_{\parallel} << k_{\perp} \) from equation (16) it can be found:

\[
\begin{align*}
\chi_{\perp} & = \frac{1}{k_{\perp}^2} \left( \frac{\omega_i}{v_i^2} \right) & \chi_{\parallel} & = \frac{1}{k_{\parallel}^2} \left( \frac{\omega_i}{v_i^2} \right) \\
\end{align*}
\]

where \( \omega_i = \bar{\omega}_i - \frac{\omega}{n} \hat{V}_{\parallel}, \quad \omega_i = \omega + i \chi_i \), and \( V_{\parallel} = 0 \).

Above 130 km of altitude in the frame moving with \( V_{E} \) (parallel to the x direction):

\[
\begin{align*}
\text{f}^{(1)}_{oi} & = \frac{\mathcal{M}_0}{(\pi v_i^2)^{3/2}} \exp -\frac{(\chi^2 - \dot{v}_i \omega_i)^2}{v_i^2} \\
\text{f}^{(2)}_{oi} & = \frac{\mathcal{M}_0}{(\pi v_i^2)^{3/2}} \exp -\frac{2v_E \chi v_i}{v_i^2} \exp -\frac{\chi^2 + \chi_i^2}{v_i^2}
\end{align*}
\]
The orbit function is now expressed by:

\[
i[ k \cdot (x' - x) - \bar{\omega}_i \tau ] - 2 \frac{V_E}{v_i^2} v'_x = i[\bar{\omega}_i - k_v v_i (1 + i\sqrt{2} \frac{V_E}{k_v v_i}) - k_v v_i] \tau + \frac{-2 V_E}{v_i^2} v_x
\]  

(35)

and using the fact that along the x direction,

\[
\frac{1}{\sqrt{2} \pi v_i^2} \int_{-\infty}^{\infty} dv_x \exp \left(-\frac{(v_x + V_E)^2}{v_i^2}\right) = 1
\]

one can obtain the susceptibility function:

\[
\chi_i = \frac{1 + \bar{\omega}_i Z(\bar{\omega}_i/k_v v_i)/v_i}{k_v^2 v_i^2 + i \sqrt{2} \pi Z(\bar{\omega}_i/k_v v_i)/v_i} \quad (36)
\]

with \( k_v = k_v [1 + i\sqrt{2} \frac{V_E}{k_v v_i}] \), \( \bar{\omega}_i = \bar{\omega}_i - k \cdot (V_m + V_p) \). This last expression generalizes that obtained by Gary and Cole [1983] when a moderate electric field is considered and for \( k^2 v_i^2 \gg 1 \).

In the cases considered \( V_E \ll V_i \) and the electric field does not affect the stability of the ions distribution. However if in the lower F-region the cross field drift is bigger than 1.8 to 2 times the neutral thermal speed, the ion distribution (O^+) becomes unstable (Ott and Farley [1975]).

**Unmagnetized ions in a strong electric field.**

This situation is relevant in the lower F-region. The ions susceptibility is of the loss-cone type (Post and Rosenbluth [1966]) and potentially unstable.

Weak ion-neutral collisions demagnetize the ions and compete with a destabilizing electric field to drive the distribution function to a marginally stable distribution. In the following lines we are presenting the main points
in the derivation of the ions susceptibility.

Due to the collisions, the ion distribution function is independent of the azimuthal angle \( \varphi \), therefore:

\[
\tilde{f}(\tilde{v}) = \frac{1}{2\pi} \int d\varphi \frac{\mathcal{N}_0}{(\pi \mathcal{V}_n^2)^{3/2}} \exp \left(-\frac{\tilde{v}^2}{2\mathcal{V}_n^2}\right) \tag{37.a}
\]

\( \tilde{v} \) is measured in the neutral frame, moving with the cross-field drift \( \mathcal{V}_E \). 

one can have,

\[
\tilde{v}_x = \tilde{v}_x - \mathcal{V}_E, \quad \tilde{v}_y = \tilde{v}_y - \mathcal{V}_n, \quad \tilde{v}_\parallel = \text{const}
\]

and:

\[
f = \frac{\mathcal{N}_0}{(\pi \mathcal{V}_n^2)^{3/2}} \exp \left(-\frac{\tilde{v}^2 + \mathcal{V}_E^2 + \mathcal{V}_n^2}{2\mathcal{V}_n^2} \right) \sum_{m=-\infty}^{\infty} J_m(b \tilde{v}_y) J_m(c \tilde{v}_x) \exp \left(i \frac{\mathcal{V}_n}{2} m \right) \tag{37.b}
\]

with \( b = 2 \mathcal{V}_E /\mathcal{V}_n^2 \), \( c = 2 \mathcal{V}_n /\mathcal{V}_n^2 \).

Using the following identity (Abramovitz and Stegun [1965]):

\[
I_\nu(z) \exp i \nu \psi = \sum_{m=-\infty}^{\infty} (-1)^m (\exp i \phi) I_m(x) I_m(y) \tag{38}
\]

\[
z^2 = x^2 + y^2 - 2xy \cos \phi
\]

\[
\exp i \psi = (x - y \exp -i \phi)/z
\]

The zeroth-order ion distribution is finally expressed by,

\[
f_{oi}(\tilde{v}) = \frac{\mathcal{N}_0}{(\pi \mathcal{V}_n^2)^{3/2}} \exp \left[-\left(\frac{\mathcal{V}_E}{2} + \frac{\mathcal{V}_n^2}{2\mathcal{V}_n^2}\right) \right] \sum_{m=-\infty}^{\infty} \int (2a \mathcal{V}_n \mathcal{V}_y /\mathcal{V}_n^2) \tag{39}
\]

where \( a = \left(1 + \frac{\mathcal{V}_n^2}{\Omega_i^2}\right)^{1/2} \). Above 130 km \( \mathcal{V}_n^2 < \Omega_i^2 \) and \( a \sim 1 \).

This zeroth-order distribution function is unstable to both, a lower-hybrid
susceptibility function.

Functions \( f^{(A)}_{\perp \perp} \) and \( f^{(B)}_{\perp \perp} \) to be replaced in equation (16) are of the form:

\[
\begin{align*}
  f^{(A)}_{\perp \perp} &= \frac{-i}{\omega - k_{\perp} v_{\perp} \sin \phi} \int_{0}^{2\pi} d\phi \frac{2k_{\perp} v_{\perp} \sin \phi}{\omega} \frac{\partial}{\partial v_{\perp}} \left( \frac{f_{\perp}}{\partial v_{\perp}} \right) \\
  f^{(B)}_{\perp \perp} &= -i \int_{0}^{2\pi} d\phi \frac{f_{\perp}}{\omega - k_{\perp} v_{\perp} \sin \phi} 
\end{align*}
\]

Integrating over the velocity space,

\[
\int d\nu \ f^{(A)}_{\perp \perp} = \frac{-i}{\omega - k_{\perp} v_{\perp} \sin \phi} \left[ f^{(A)}_{\perp \perp}(0) + F(\nu) \right]
\]

\[
F(\nu) = \int_{0}^{\infty} d\nu^{-2} \left[ \frac{\nu}{\nu - \nu_{\perp}^2} \right] \frac{1}{2} \frac{\partial^2}{\partial \nu^{-2}} \frac{f_{\perp}}{\partial \nu^{-1}}
\]

where

\[
\begin{align*}
  f^{(A)}_{\perp \perp}(\nu_{\perp}) &= \frac{1}{\nu_{\perp}^2} \left( \frac{2\nu_{\perp}}{v_{\perp}^2} \right) \exp \left( -\nu_{\perp} \nu_{\perp}^2 / \nu_{\perp}^2 \right) \\
  w &= \nu_{\perp}^2 / k_{\perp} v_{\perp}^2
\end{align*}
\]

and

\[
F(w) \text{ is a complex function, } F(w) = F_{R}(w) + i F_{I}(w) :
\]

\[
F_{R}(w) = \sqrt{w} \exp \left( -\frac{\nu_{\perp}^2}{4} \right) \int_{0}^{\infty} d\nu \nu^{-2} \left[ \left( \frac{\nu_{\perp}^2}{4} - 1 \right) I_0(\alpha x) - \frac{\nu_{\perp}^2}{4} L_2(\alpha x) \right] / \sqrt{w - \nu_{\perp}^2}
\]

with \( \alpha = 2 \frac{V_{E}}{v_{\perp}} \).

Using the analytical properties of \( f^{(A)}_{\perp \perp} \), the imaginary part of \( F(w) \) can be put in the following form:

...
\[ F_1(w) = 2 \sqrt{w} \int_0^\infty dx \frac{\partial}{\partial x^2} f_i \perp \]

and,

\[ F_1(w) = -\sqrt{w} \frac{\pi^{1/2}}{2} \exp\left(-\frac{\alpha^2}{8}\right) \left[ \left( \frac{\alpha^2}{4} - 1 \right) I_0\left(\frac{\alpha^2}{8}\right) - \frac{\alpha^2}{4} I_1\left(\frac{\alpha^2}{8}\right) \right] \quad (42.b) \]

Finally, the only relevant case for us is the 'Ott-Farley' instability \( (\Omega >> \Omega_i >> \nu_i) \) and the ions susceptibility is expressed by:

\[ \chi_i = \frac{1}{k^2 \lambda_i^2} \left[ f_i(0) + F(w) \right] \quad (43) \]
FIGURE A1.a

FIGURE A1.b
1. BASIC ASSUMPTIONS.

A plasma is in a turbulent state when a large number of collective oscillations are excited through the presence of an instability (Davidson [1972]).

Plasma turbulence can be characterized by two parameters, $\varepsilon_1 = \frac{E}{nT}$ and $\varepsilon_2 = \frac{\tau_{ac}}{\tau_{tr}}$. $\varepsilon_1$ defines the ratio of the wave energy ($\frac{E}{nT}$) to the thermal energy of the particles ($nT$) and $\varepsilon_2$ defines the ratio of the autocorrelation time ($\tau_{ac}$) of the waves to the trapping time ($\tau_{tr}$).

The trapping time ($\tau_{tr} = \sqrt{\frac{1}{q\kappa E_{\kappa}}}$) is the characteristic bounce period of a resonant particle of mass $m$ and charge $q$ in the trough of the wave of amplitude $E_{\kappa}$ and wavenumber $\kappa$.

Weak turbulence is defined as the state where $\varepsilon_1 << 1$ and $\varepsilon_2 << 1$. Moderate turbulence when still $\varepsilon_1 << 1$ but $\varepsilon_2 \sim 1$.

All other regimes with $\varepsilon_1 > 1$ are characterized as strong turbulence.

In a weakly turbulent state condition $\varepsilon_2 << 1$ requires the presence of a broad spectrum of small amplitude modes verifying the linear dispersion relationship and which produce relatively small random deviations of the particles from their unperturbed orbits.

Situations with $\varepsilon_2 \sim 1$ where only one mode is present (discrete spectrum) or instability is non-dispersive (continuous spectrum) 'kinetic' turbulence may arise from random particles motions without a corresponding diffusion of energy.
in the wave number space (Dupree [1968]).

The plasma is considered to be composed of two species, a resonant component which drives the instability and a second species which is non-resonant. A plasma species is called resonant with the instability if its thermal velocity is comparable to the wave-phase velocity and non-resonant if its thermal velocity is larger. Resonance is assumed to be broad, i.e. such that the waves autocorrelation time is smaller or equal than the instability period (Gary [1980]).

In a weak-to-moderate turbulence state a particle is unable to decide whether it is trapped or not (τ_{ac} < τ_{tr}) and it undergoes a random walk in the velocity space diffusing with a characteristic time scale τ_D (τ_D > τ_{tr}). Diffusion produces the broadening of the wave-particle resonance by allowing an increased number of non-resonant particles to exchange energy with the unstable mode.

Stabilization may be achieved if the resonance is broadened to an extent such as to include a sufficient number of particles absorbing energy from the wave (Dum and Dupree [1970]). If the diffusion time is much greater than the trapping time, the condition for moderate turbulence becomes \( \xi_2' = \frac{\tau_{ac}}{\tau_D} \ll 1 \).

In this situation the diffusion process is Markovian (statistical perturbation of the trajectories is independent of the past history; Benford and Thomson [1972]). The existence of parameter \( \xi_2' \ll 1 \) yields to a separation between two time scales, the fast time associated with the fluctuations and the slow time related to the diffusion process.

As we shall see the Markovian condition is equivalent to neglect the fast time dependance in the orbit's perturbation function.

The Markovian hypothesis is strictly valid when \( \xi_2' \ll 1 \). In any case one must be aware of the possible complications when \( \xi_2' \ll \xi_2 \ll 1 \) (\( \tau_{ac} \ll \tau_{tr} \ll \tau_D \)).
where non-Markovian effects could be important (Dupree and Tetreault [1978]).

To summarize, when the wave energy at the shorter scales is relatively weak ($\xi_{\ell} \ll 1$) linear kinetic instabilities will saturate through the broadening of the wave-particle resonances by the nonlinear orbit-diffusion process also known as 'turbulent collisions' (Tsitovich [1972]). At higher levels of turbulence ($\xi_{\ell} \gtrsim 1$) saturation is achieved by the coupling between the waves themselves (wave-wave interactions). This is always the case for long-wavelength (fluid like) instabilities.

2. RENORMALIZED QUASILINEAR THEORY.

Starting with the Vlasov equation the only two sources of resonance broadening are the wave-particle and wave-wave interactions. If wave-particle interactions are more important than wave-wave coupling processes, the energy density of the fluctuating field will have a relative maximum corresponding to the wave-vector of the linearly unstable mode (Gary [1980]).

In a weak-to-moderate turbulence state, the effects of wave-wave interactions on the stabilization of kinetic instabilities may be neglected (weak mode coupling approximation). In this situation, wave saturation is achieved through the 'turbulent collisions' between the waves and the non-resonant particles.

Unstable waves grow up linearly by absorbing energy from the resonant particles (linear Landau growth) and saturate by transferring it to the non-resonant population through the secular perturbation of their trajectories (nonlinear Landau damping). In weakly collisional plasmas 'turbulent collisions' are generally much more important than collisions between particles (Dupree [1968]).

The correct treatment of the orbit-diffusion or 'turbulent collisions' saturation mechanism is given by the renormalized quasilinear theory (Tsitovich [1972], Benford and Thomson [1972], Dupree and Tetreault [1978]). As in the
usual quasilinear theory the exact solution of the Vlasov equation is expanded in a perturbation serie of small parameter $\varepsilon_1 \ll 1$, but now the first order solution is obtained integrating over the statistically averaged perturbed trajectories and accounts for all the wave-particle interactions. Higher order terms are exclusively associated with wave-wave interactions.

In the following lines we are presenting the main points of the renormalized quasilinear calculation of the saturation spectrum of a linearly unstable mode in a weak-to-moderate turbulence state. We shall start with the exact Vlasov equation for a magnetized plasma. To include a BGK collisions model in the calculation is straightforward.

**Vlasov Equation**

\[
\frac{\partial}{\partial t} f_j + \mathbf{v}_j \cdot \frac{\partial}{\partial \mathbf{x}_j} f_j + \frac{q_j}{m_j} (\mathbf{E} - \mathbf{v}_j \times \mathbf{B}) \cdot \frac{\partial}{\partial \mathbf{v}_j} f_j = 0
\]  

the electric field is electrostatic $\mathbf{E} = -\nabla \phi$ and,

**Poisson Equation**

\[
\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \sum_j m_j q_j \int d\mathbf{v}_j f_j(\mathbf{x}_j, \mathbf{v}_j, t)
\]

As is usual in turbulence theory the distribution function and the electric field are divided into two parts, a statistically averaged (zeroth-order) component and a fluctuating (stochastic) part:

\[
f_j(\mathbf{x}_j, \mathbf{v}_j, t) = \langle f_j(\mathbf{x}_j, \mathbf{v}_j, t) \rangle + \tilde{f}_j(\mathbf{x}_j, \mathbf{v}_j, t)
\]

\[
\mathbf{E}(\mathbf{x}_j, t) = \mathbf{E}_0 + \tilde{\mathbf{E}}(\mathbf{x}_j, t)
\]

The fluctuating parts verify $\langle \tilde{f}_j(\mathbf{x}_j, \mathbf{v}_j, t) \rangle = 0$ and $\langle \tilde{\mathbf{E}}(\mathbf{x}_j, t) \rangle = 0$. The
regular electric field $E_0$ is assumed uniform and we can choose an inertial reference frame where $E_{0\perp} = 0$. $E_{0\parallel}$ is always taken equal to zero.

Defining the Vlasov operator $L_j(t)$ by:

$$L_j(t) = \frac{\partial}{\partial \tilde{x}_j} + \frac{q_j}{m_j} (\tilde{E} - \tilde{\gamma} \times \tilde{B}) \cdot \frac{\partial}{\partial \tilde{v}_j}$$  \hspace{1cm} (4)$$

It may also be divided into two parts, a regular part and a stochastic part (Weinstock [1969]):

$$L_j(t) = \overline{L}_j(t) + \overline{\tilde{L}}_j(t)$$  \hspace{1cm} (5)$$

with

$$\langle L_j(t) \overline{f}_j \rangle = \overline{L}_j(t) \overline{f}_j$$ \hspace{1cm} (6.a)$$

and

$$\overline{\tilde{L}}_j(t) = \frac{q_j}{m_j} \tilde{E} \cdot \frac{\partial}{\partial \tilde{v}_j}$$ \hspace{1cm} (6.b)$$

Condition (6.a) is equivalent to assume that the statistical average also replaces the exact orbit $[x(t), y(t), t]$ by an 'average' one, then formally:

$$\langle f_j(\tilde{x}, \tilde{y}, t) \rangle = \overline{f}_j(\overline{x}, \overline{y}, t)$$ \hspace{1cm} (7)$$

Vlasov equation (1) is therefore equivalent to two coupled equations one for the average distribution and another for the stochastic part:

$$\left[ \frac{\partial}{\partial t} + \overline{L}_j(t) \right] \overline{f}_j = - \frac{q_j}{m_j} \langle \tilde{E} \cdot \frac{\partial}{\partial \tilde{v}_j} \overline{f}_j \rangle$$ \hspace{1cm} (8.a)$$

$$\left[ \frac{\partial}{\partial t} + \overline{\tilde{L}}_j(t) \right] \tilde{f}_j = - \frac{q_j}{m_j} \tilde{E} \cdot \frac{\partial}{\partial \tilde{v}_j} \overline{f}_j - \frac{q_j}{m_j} \left[ \tilde{E} \cdot \frac{\partial}{\partial \tilde{v}_j} \langle \overline{f}_j \rangle - \langle \tilde{E} \cdot \frac{\partial}{\partial \tilde{v}_j} \overline{f}_j \rangle \right]$$ \hspace{1cm} (8.b)$$
Assuming a spatially homogeneous turbulence, the fluctuating quantities may be decomposed in Fourier series:

\[ \hat{f}_j = \sum_{k \in k} \hat{f}_{k, j}(\mathbf{r}, t) e^{ik \cdot \mathbf{r}} \]  
\[ \hat{\xi} = \sum_{k \in k} \xi_{k, j}(t) e^{ik \cdot \mathbf{r}} \]  

(9.a)  
(9.b)

In the ionosphere, plasma is 'infinite' and the sums over \( k \) are in fact Fourier integrals.

Replacing (9.a) and (9.b) in equations (8.a) and (8.b), and neglecting the wave-wave interactions (weak mode-coupling hypothesis) one may find:

\[ \left[ \frac{\partial}{\partial t} + \mathcal{L}_j(t) \right] \hat{f}_j = -\frac{q_j}{m_j} \sum_k \left< e^{ik \cdot \mathbf{r}} \frac{\partial}{\partial \mathbf{r}} \hat{f}_{j, k} \right> \]  
\[ e^{ik \cdot \mathbf{r}} \left[ \frac{\partial}{\partial t} + \mathcal{L}_j(t) \right] \hat{f}_{j, k} e^{ik \cdot \mathbf{r}} = -\hat{\mathcal{V}}_{k, j} \hat{f}_j - \frac{q_j}{m_j} \xi_{k, j} \frac{\partial}{\partial \mathbf{r}} \hat{f}_j \]  

(10.a)  
(10.b)

\( \hat{\mathcal{V}}_{k, j} \) is the 'turbulent collisions' operator (Tsitovich [1972]) accounting for all the nonlinear wave-particle interactions (nonlinear Landau damping). \( \hat{\mathcal{V}}_{k, j} \) is a function of \( \mathbf{r} \) and \( t \). To neglect mode coupling implies that \( k \) and \( k_1 \) are parallel.

Introducing the evolution operator \( U(t', t) \) (Benford and Thomson [1972]):

\[ U(t', t) f(\mathbf{x}, \mathbf{r}, t) = f(\mathbf{x}, \mathbf{r}, t') \]  
\[ f(\mathbf{x}', \mathbf{r}', t') = \int d\mathbf{x} d\mathbf{r} \mathcal{G}(\mathbf{x}', \mathbf{r}', t; \mathbf{x}, \mathbf{r}, t) f(\mathbf{x}, \mathbf{r}, t) \]  

(11.a)  
(11.b)
Where $G(x',v',t';x,v,t)$ is the Green function of the Vlasov equation (1) with initial condition:

$$G(x',v',t';x,v,t) = \int (x-x') \delta(v-v')$$

(12)

and

$$f'_j \equiv U(t',t) \bar{f}_j$$

(13.a)

$$f'_j \equiv U(t',t) f'_j$$

(13.b)

Primed quantities will represent functions of the time $t'$.

Equation (10.b) can be formally integrated over the 'average' orbit $(x',v',t')$ to obtain:

$$f_j(t) = f_j(t_0) - \frac{q_j}{m_j} \int_{t_0}^{t} \int_{t_0}^{t'} \sum_{l} d\tau' \int_{t_0}^{t} \frac{d\tau}{m_j} e^{i(k \cdot x - \omega \tau')} \bar{f}_j$$

(14)

Replacing (14) in equation (10.a) a diffusion equation for the background distribution $f_j$ may be found:

$$\left[ \frac{\partial}{\partial t} + \mathbb{L}_j(t) \right] \bar{f}_j = \left< \frac{\partial}{\partial v_j} \cdot \hat{D}_j \cdot \frac{\partial}{\partial v_j} \right> \bar{f}_j$$

(15)

where

$$\mathbb{D}_j = \frac{q_j^2}{m_j^2} \sum_{k1,k2} \int_{t_0}^{t} dt' \left< E_{k1} (t) E_{k2} (t') \right> \hat{A}_{j,k1,k2} e^{i(k \cdot x)}$$

(16.a)

$$\hat{A}_{j,k1,k2} \equiv e^{i(k \cdot (x' - x))} \int_{t_0}^{t} \frac{d\tau}{\lambda_{k1}} \bar{U}(t',t) U(t,t')$$

(16.b)

$\mathbb{D}_j$ is the diffusion operator. The 'turbulent collisions' operator $\hat{A}_{j,k1,k2}$ may also be expressed by:
All the previous operators are average quantities.

When turbulence is stationnary and spatially homogeneous, because $E$ is parallel to $k$ (longitudinal waves) we may have (Tsitovich [1972]):

$$\langle E(x,t) E(x,t') \rangle - \langle |E(x,t)|^2 \rangle \frac{k_x}{k_t^2} \delta (k_x)$$

(18.a)

and called $\tau = t - t'$.

We are moreover interested in stationary, adiabatic Markovian situations where two time scales may be considered, one associated with the electric field fluctuations ($t$) and the other associated with the secular changes in the background distribution and averages ($t_s$) due to the orbits diffusion:

$$\langle E(x,t,t_s) E(x,t',t_s) \rangle = \langle |E(x,t_s)|^2 \rangle \frac{k_x}{k_t^2} \delta (k_x)$$

(19)

In any case, the system is assumed ergodic (ensemble average = time average) for times much greater than the diffusion time ($t_s \rightarrow \infty$).

3. Resonance Broadening.

At this point we are going to make a few more assumptions in order to obtain some results which can be compared with observations.

(a) Like in the standard quasilinear theory one shall assume that instability is sufficiently well developed such that only growing modes are retained (free
streaming terms may be neglected, Davidson [1972]). Also setting \( \tau_0 = -\infty \), the plasma is initially in thermal equilibrium.

Therefore:

\[
\begin{align*}
f_{j,k}^{(\tau',t)} &= f_{j,k}^{(\tau)} e^{-i \int_{\tau}^{t} (\omega_{j}^* + i \nu_{j}^*) d\tau'} \\
E_{j,k}^{(\tau)} &= E_{j,k}^{(\tau)} e^{-i \int_{\tau}^{t} (\omega_{j}^* + i \nu_{j}^*) d\tau'}
\end{align*}
\]  

(20.a) (20.b)

\( \omega_{j}^* \) and \( \nu_{j}^* \) are functions of time through the variations of the background distribution.

Reference frame. \( \omega_{j}^* - \omega_{k}^* - k \cdot V_{j} \) is the Doppler shifted frequency for electrons and ions measured in the reference frame where \( E_0 = 0 \). Like in the linear case (appendix 1) this frame is, within a good approximation, the one moving with the ions' drift (which, of course, must be smaller than the ion thermal speed).

(b) The existence of parameter \( \xi_2' = \frac{L_{ac}}{L_{D}} \ll 1 \) in weak-to-moderate turbulence yields to the separation of two time scales: the slow time \( (t > L_{ac}/L_{D}) \) related to the orbit-diffusion process and the fast time \( (t > L_{ac}) \) to the time scale of the particles random walk due to the electric field fluctuations ('turbulent collisions').

Condition \( \xi_2' \ll 1 \) is also equivalent to assume that diffusion is a Markovian process (Benford and Thomson [1972]) where the average quantities vary slowly with time \( t_s' \):

\[
\begin{align*}
f_{j,k}^{(\nu, t, t_s')} &= f_{j,k}^{(\nu)} \exp -i[\omega_{j}^* (t_s') + i \nu_{j}^* (t_s')] t_s' + \text{higher order terms} \quad (21.a)
\end{align*}
\]
\[ E_j(t,t_s) = \widehat{E}_j \exp \left( -i \left( \omega_j(t_s) + i \gamma_j(t_s) \right) t_s \right) \]  

(21.b)

and

\[ \widehat{\mathbf{v}}_{j\mathbf{k}} = \widehat{\mathbf{v}}_{j\mathbf{k}} \left( \overline{\mathbf{v}}_{\perp}, \overline{\mathbf{v}}_{\parallel}, t_s \right) \]  

(21.c)

The process have been assumed spatially homogeneous in the presence of a magnetic field and \( \widehat{\mathbf{v}}_{j\mathbf{k}} \) is a function of the magnitude of the average velocity in the perpendicular and parallel directions.

(c) Average perturbed orbits.

Exact orbits:

\[
\frac{d\mathbf{z}}{dt} = \mathbf{z} \tag{22.a}
\]

\[
\frac{d\mathbf{v}}{dt} = -\mathbf{v} \times \mathbf{z}_j + \frac{q_i}{m_j} \mathbf{E}(\mathbf{z}, t) \tag{22.b}
\]

Average orbits:

\[
\frac{d\overline{\mathbf{z}}}{dt} = \overline{\mathbf{z}} \tag{23.a}
\]

\[
\frac{d\overline{\mathbf{v}}}{dt} = -\overline{\mathbf{v}} \times \overline{\mathbf{z}}_j \tag{23.b}
\]

\[
\langle [\mathbf{v}(t_s) - \mathbf{v}(t_s)] [\mathbf{v}(t_s) - \mathbf{v}(t_s)] \rangle \bigg|_{t_s=-\infty}^{t_s} \overset{\mathbf{v}}{D}_{j0} \tag{23.c}
\]

\[
\overset{\mathbf{v}}{D}_{j0} = \frac{q_i^2}{m_j^2} \int_0^\infty \mathbf{E}(\mathbf{z}, t_s) \mathbf{E}(\mathbf{z}, t_s, t) d\zeta \tag{23.d}
\]

\( \overset{\mathbf{v}}{D}_{j0} \) is the velocity space diffusion tensor for the population \( j \).

'Average' orbits are in fact zeroth-order orbits in the fast time scale \( t \) but whose velocity magnitude changes adiabatically due to the diffusion process: the
the mean square velocity variation is proportional to $t_s$ through the diffusion coefficients, and:

$$\langle \Delta^2 \rangle (t', t_s) = \langle \Delta \phi (t', t_s) \rangle + \langle \Delta \phi (t, t_s) \rangle + \langle \Delta n \rangle \hat{b}$$  \hspace{1cm} (24.a)

$$\langle \Delta \phi \rangle = \langle \Delta \phi (t_s) \rangle \left[ \cos (\varphi_0 + \varphi) \hat{\zeta}_1 + \sin (\varphi_0 + \varphi) \hat{\zeta}_2 \right]$$  \hspace{1cm} (24.b)

$$\langle \hat{\zeta}_0 \cdot (\Delta \phi) \rangle = -\frac{k_e}{\sqrt{q_i}} \left[ \sin (\varphi_0 + \varphi) - \sin \varphi \right] + \langle \Delta \phi \rangle + \langle \Delta n \rangle \hat{b}$$  \hspace{1cm} (24.c)

where

$$\langle \Delta \phi (t_s) \rangle - 1 \langle \Delta \phi (t_s, t_s) \rangle$$

$$k = k_\perp + k_\parallel$$

$$\hat{\zeta}_0 = \hat{k}_\parallel$$

$$\hat{k}_\perp = -\hat{\zeta}_0 \cos \varphi$$

$$t = t - t'$$

$k_\perp$ is the unit vector parallel to $k_\parallel$ and $\hat{b}$ the one parallel to $B$. $E_{\parallel} = 0$ and particles move freely parallel to the magnetic field.

Also:

$$\frac{k_\perp U(t', t)}{\delta \phi} = [1 + \cos (\varphi_0 + \varphi) + \sin (\varphi_0 + \varphi)] \frac{k_\perp^2}{\delta \phi_\perp} + k_\parallel \frac{2}{\delta \phi_\parallel}$$  \hspace{1cm} (25.a)

$$\frac{k_\parallel}{\delta \phi} = (1 + \cos \varphi + \sin \varphi) k_\perp \frac{k_\perp}{\delta \phi_\perp} + k_\parallel \frac{2}{\delta \phi_\parallel}$$  \hspace{1cm} (25.b)

Finally, taking into account all the previous definitions, the diffusion equation (15) may be written:
\[ \frac{\partial \vec{f}_j}{\partial t} = 0 \]  
\[ \frac{\partial \vec{f}_j}{\partial t} = \frac{\partial}{\partial \vec{r}_j} \cdot \frac{\partial}{\partial \vec{r}_j} \vec{f}_j \]  

(26.a) 

(26.b) 

We are calling \( \vec{\nu} = \vec{\nu}(\vec{r}_s, t) = \vec{\nu}(t) \), \( \vec{v} \), and \( \vec{b} \), and

\[ \frac{\partial \vec{\nu}}{\partial \vec{r}_j} = -\frac{q_j}{\mu_j^2} \sum_{i \neq j} \left< |E_{\vec{k}_1}^i(t)\right|^2 \right>_{\vec{r}_j} \exp i k \cdot \vec{x} \]  

(27.a) 

\[ B_{\vec{k}_1 \vec{k}_2} = -\int d\Omega \int_0^\infty \left< \sum_{\vec{k}_2} [(1 + \cos \varphi + \sin \varphi)k_2 + k_1] \exp i k \cdot \vec{x} \right> \exp -i \left( \Omega_2 + i \varphi + i \frac{\varphi}{\Omega_2} \right) \left[ \sin(\Omega_2 \varphi + \varphi) \right] \sum_{\vec{k}_1} \left< \vec{x} \cdot \vec{F}_{\vec{k}_1} \right> \sum_{\vec{m}_1} \left< \vec{F}_{\vec{m}_1} \right> \sum_{\vec{m}_2} \left< \vec{F}_{\vec{m}_2} \right> \]  

(27.b) 

Using the relationships:

\[ k \cdot (\vec{x} - \vec{x}') = -k \vec{\nu}_{\vec{k}_1} \left[ \sin(\Omega_2 \varphi + \varphi) - \sin \varphi \right] - \Omega_2 \vec{k}_{\vec{m}} \]  

(28.a) 

\[ \exp i k \cdot (\vec{x} - \vec{x}') = \sum_{M} J(k \vec{\nu}_{\vec{k}_1}, J(k \vec{\nu}_{\vec{m}}) \exp -i[(m-n) \varphi + \Omega_2 \varphi] \]  

(28.b) 

One can find after integration over and : 

\[ B_{\vec{k}_1 \vec{r}_s - \vec{x}_1} \left< \frac{k_1 k_{\vec{m}_1} k_{\vec{m}_2} \sum_{k_{\vec{m}_3}} \left< \frac{k_{\vec{m}_3} k_{\vec{m}_4}}{k_{\vec{m}_3}^2} \right> \right>_{\vec{r}_s - \vec{x}_1} \]  

(29.a) 

\[ B_{\vec{k}_1 \vec{r}_s - \vec{x}_1} \left< \frac{k_1 k_{\vec{m}_1} k_{\vec{m}_2} \sum_{k_{\vec{m}_3}} \left< \frac{k_{\vec{m}_3} k_{\vec{m}_4}}{k_{\vec{m}_3}^2} \right> \right>_{\vec{r}_s - \vec{x}_1} \]  

(29.b) 

\[ B_{\vec{k}_1 \vec{r}_s - \vec{x}_1} \left< \frac{k_1 k_{\vec{m}_1} k_{\vec{m}_2} \sum_{k_{\vec{m}_3}} \left< \frac{k_{\vec{m}_3} k_{\vec{m}_4}}{k_{\vec{m}_3}^2} \right> \right>_{\vec{r}_s - \vec{x}_1} \]  

(30.a)
The resonance operator $\hat{R}_n$ is defined by,

$$\hat{R}_n = -\frac{1}{\omega_k^n + i k - n \nu_j - k || \nu_j + i \nu_j}$$

and calling $z = k \nu_j / \omega_j$:

$$F_n(z) = \frac{J_n(z)}{J_{n+1}(z)} + 2J_n(z) + 2J_{n-1}(z)$$

$J_n$ is the Bessel function of order $n$ (Abramovitz and Stegun [1965]).

Also the 'turbulent collisions' operator can be written:

$$\hat{V}_n = -\frac{g^2}{\nu j} \sum_{\epsilon} \langle |E_{\nu j}(t)|^2 \rangle \hat{E}_{\nu j} \hat{E}_{\nu j}$$

Integration over $\nu$ filters out any dependence on the initial orientations perpendicular to the magnetic field (random phase hypothesis).

We are mainly interested in the case where wave propagation is closely perpendicular to the magnetic field ($k_|| \ll k_\perp$) and only perpendicular terms need to be kept.

Following Weinstock [1970] we consider three cases for the representation of the 'turbulent collisions' operator and the 'resonance' operator:

(a) the weakly magnetized case (Dupree [1968], Dum and Dupree [1970]),
(b) the strongly magnetized case (Weinstock [1970]) and,
(c) the unmagnetized case (Dupree [1967], Tsitovich [1972]).

(a) WEAKLY MAGNETIZED CASE.

This situation is the most relevant in the ionosphere. In this case both the 'turbulent collisions' and the resonance operators are diagonal and may be simultaneously represented by their eigenvalues: the 'turbulent collisions' damping and the resonance function.

When the diffusion time scale $\tau_D$ for the non-resonant population is greater than the particles gyroperiod, the velocity space diffusion due to the fluctuating electric field is equivalent to the diffusion in the configuration space with fluctuating cross-field velocity $\mathbf{\tilde{v}}\cdot\mathbf{E} \times \mathbf{B} / B^2$. This is always true in the ionosphere when electrons are the non-resonant population. In practice we need not to consider the nonlinearity in the resonant population because the nonlinear damping due to the 'turbulent collisions' between the finite amplitude waves and the non-resonant particles determines, almost completely, the broadening of the resonances at saturation (Gary [1980], Gary and Sanderson [1981]).

From condition $\epsilon_2' \ll 1$ (weak-to-moderate turbulence) we have that $\omega_k^{-1} \sim \tau_{ac}$ and, when the ions are the non-resonant population the diffusion in the velocity space is equivalent to that in the configuration space only if the unstable frequency $\omega_k$ is not much bigger than the ions gyrofrequency. This is the case for the low frequency gradient-drift and EIC instabilities considered in our study. The 'turbulent collisions' operators obtained from the Vlasov equation with either a fluctuating velocity field or a fluctuating electric field are the same.

Considering a 'transverse' Fourier development of the fluctuating part of the
distribution function for perpendicular propagation:

\[
\tilde{f}_j = \sum_{k*} \tilde{f}_{jk*} e^{i\kappa_1 x} = \sum_{k*} \tilde{f}_{jk*} e^{i\kappa_1 x}
\]  
(33)

Wave vector \( \kappa \) is parallel to \( E \) and the transverse vector \( \kappa* \) is defined by

\[ \kappa* = \nabla \times \kappa \] 

Therefore, operator \( \hat{\nabla}_{jk*} \) from equation (32) must be equal to the one obtained replacing \( \frac{q_i}{m_j} \) by \( -i \kappa* \cdot \delta \)

\[
\hat{\nabla}_{jk*} = \sum_{\kappa*} \frac{1}{B^2} \langle |E_{k*}(t_s)|^2 \rangle \kappa* \cdot B \delta_{jk*} \exp i\kappa* \cdot x
\]  
(34)

Furthermore, averaging all the nonfluctuating quantities over a spatial domain defined by the density gradient scale length (or any other inhomogeneity scale length):

\[
\tilde{f}_j = \frac{1}{L^3} \int d^3 \tilde{f}_j
\]  
(35.a)

\[
\hat{\nabla}^0_{jk*} = \sum_{\kappa*} \frac{\langle |E_{k*}(t_s)|^2 \rangle}{B^2} \kappa* \cdot B \delta_{jk*} \cdot \kappa*
\]  
(35.b)

\[
\hat{B}^0_{j0} = \frac{q_i}{m_j} \sum_{\kappa*} \langle |E_{k*}(t_s)|^2 \rangle \kappa* \cdot B \delta_{jk*}
\]  
(35.c)

also

\[
\kappa* \cdot B \delta_{jk*} \kappa* = \langle \delta \times \frac{R}{m_j} \rangle \cdot \kappa
\]  
(36.a)

\[
\hat{\nabla}^0_{jk*} = \sum_{\kappa*} \frac{\langle |E_{k*}(t_s)|^2 \rangle}{B^2} \kappa* \langle \delta \times \frac{R}{m_j} \rangle \cdot \kappa
\]  
(36.b)

or equivalently

\[
\hat{\nabla}^0_{j0} = \kappa* \cdot \hat{B}^0_{j0} \cdot \kappa* / \Omega_j^2
\]  
(36.c)
Therefore using equations (35.b), (29.a) and (31) when $k_\parallel \ll k_\perp$:

$$\hat{\nu}^0_{k_\parallel k_\perp} = 2\pi \sum_{k_{\perp 1}} \frac{\langle |E_{k_\parallel 1}|^2 \rangle (k_{\perp 1} k_\perp)^2}{B^2} \sum_n F_{n_j} \hat{R}^0_{n_j}$$  (37.a)

$$[\omega_{k_j} - nS_j - k_{\parallel 1} \nu_{\parallel 1} + i (\nu_{k_j} + \dot{\nu}_{k_j}^0)] \hat{R}^0_{n_j} = - \hat{1}$$  (37.b)

Both operators $\hat{\nu}^0_{k_j}$ and $\hat{R}^0_{n_j}$ may be replaced by their eigen values $\Delta \omega_{k_j}$ and $\delta_{n_j}^0$ respectively:

$$\Delta \omega_{k_j} = 2\pi \sum_{k_{\perp 1}} \frac{\langle |E_{k_\parallel 1}|^2 \rangle (k_{\perp 1} k_\perp)^2}{B^2} \sum_n F_{n_j} \text{Im} g_{n_j}^0$$  (38.a)

$$\text{Im} g_{n_j}^0 = \frac{\nu_{k_j} + \Delta \omega_{k_j}}{(\omega_{k_j} - nS_j - k_{\parallel 1} \nu_{\parallel 1})^2 + (\nu_{k_j} + \Delta \omega_{k_j})^2}$$  (38.b)

The apparent difference between equation (38.a) and equation (36) in Farley [1985] is that in fact, the cross-field velocity fluctuations determine the nonlinear damping in the direction perpendicular to the wave propagation.

The resonance function $\text{Im} g_{n_j}^0$ is Lorentzian and the 'turbulent collisions' damping $\Delta \omega_{k_j}$ is real and positive.

Only in this situation (weakly magnetized plasma) the resonance function has a Lorentzian shape leading to a straightforward generalization of the quasilinear theory. The 'turbulent collisions' damping $\Delta \omega_{k_j}$ multiplied by the imaginary unit is added to the resonant frequency $\omega_{k_j}$ like a BGK particles collision frequency.

In the strongly magnetized as well as in the unmagnetized situations, diffusion in the velocity space and that in the configuration space are not equivalent.

Operators $\hat{\nu}^0_{k_j}$ and $\hat{R}^0_{n_j}$ do not commute and therefore cannot be simultaneously
replaced by their eigenvalues. In these cases the resonance function is not Lorentzian.

(b) STRONGLY MAGNETIZED CASE.

Operators \( \hat{\gamma} \) and \( \hat{\kappa} \) verify as before the following relationship:

\[
(\omega - i \gamma - n \Delta j - k_1 I_i - i \gamma - \Delta j) \hat{R}_{nj} = -1
\]

This equation can be solved by replacing \( \hat{R}_{nj} \) by its eigenvalue \( \delta_{nj} \), operator \( \hat{\gamma} \) is not diagonal in this representation and is given by the equations (32) and (35):

\[
\hat{\gamma} = \frac{\partial}{\partial y} \cdot \sum_{\Delta j} \frac{\partial y}{\partial x}
\]

Considering perpendicular propagation only, equation (39) may be written:

\[
\left[ \chi + i \frac{\partial}{\partial y} \frac{d_0 k}{\partial y} \right] \delta_{nj} = -1
\]

Where \( \chi = \omega_{kj} + i \gamma - n \Delta j \), \( y - k_1 y_1 \) and \( d_0 k \):

\[
d_0 k = 2\pi \frac{q^2}{m_j^2} \sum_{\Delta j} k^2 \langle |E_{kj}|^2 \rangle \sum_\eta \text{Im} \delta_{\eta j} F_{\eta j}
\]

Away from resonance, \( \delta_{nj} \) is the same as one given by the quasilinear theory so one only needs to consider the solution of equation (41) near resonance \( \eta_n = n \delta_{nj} \) (Tsitovich [1972]): \( |x - \eta_n| < \epsilon \), \( \epsilon \) is positive and arbitrarily small. Assuming \( d_0 (y + \epsilon) - d_0 (y - \epsilon) = O(\epsilon^2) \) (weak velocity dependence):

\[
\left[ \chi + i \frac{d_0 k}{\partial y} (\eta) \frac{\partial^2}{\partial x^2} \right] \delta_{nj} = -1
\]

Performing a Fourier transformation:
\[ f_{\eta j}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\delta^{\infty}}{\delta z} e^{ixz} \, dz \]  

(43.a)

\[ \frac{\partial}{\partial z} \delta_{\eta j} + z^2 \delta_{\eta j} (\gamma) \delta_{\eta j} = 2\pi i \delta(z) \]  

(43.b)

Finally

\[ \delta_{\eta j} = 2\pi i e^{-i \frac{1}{3} \delta_{\eta j} z^3} \]  

(44.a)

\[ \delta_{\eta j}(x) = - \int_{0}^{\infty} e^{ixt - \frac{1}{3} \delta_{\eta j} z^3} \, dz \]  

(44.b)

\[ d_{\omega k} = k \cdot \mathbf{b} \cdot \mathbf{k} \] and the diffusion time is equal to \[ \tau_D = \left( \frac{d_{\omega k}}{3} \right)^{-1/3} \] .

(C) UNMAGNETIZED CASE.

The velocity space diffusion tensor is expressed by (Tsitovich [1972]):

\[ V_{\eta j} = \frac{q_j^2}{\hbar^2} \sum_{\kappa_1} \frac{\kappa_1 \kappa_2}{\kappa_1^2} \langle \mathbf{E}_{\kappa_1} \mathbf{E}_{\kappa_2} \rangle \text{Im} \delta_{\eta j} \kappa_1 \]  

(45.a)

where the resonance function is given by

\[ \delta_{\eta j} \kappa_1 = - \int_{0}^{\infty} dt \, e^{i [\omega_{\kappa_1} - \kappa_1 \mathbf{b} \cdot \mathbf{k} + i \delta_{\eta j} \kappa_1 \frac{z^3}{3}]} \]  

(45.b)

and \[ d_{\omega k} = k \cdot \mathbf{b} \cdot \mathbf{k} \] , \[ \tau_D = \left( \frac{d_{\omega k}}{3} \right)^{-1/3} \] .

(4) WAVE-ENERGY DENSITY AT SATURATION.

Back to the weakly magnetized case, using equations (38) one can find an estimate of the wave-energy density \( \mathcal{E}_k \) at saturation (Dum and Dupree [1970]).

The wave-energy density of the electrostatic fluctuations with wavevector \( k \) is defined by (MKS units):

\[ \mathcal{E}_k = \frac{1}{2} \varepsilon_0 \langle |\mathbf{E}_k|^2 \rangle \]  

(46)
\( \varepsilon_0 \) is the permittivity of the free space.

From equations (38), because the non-resonant population verified (\( \omega_{ej} - n_0 \omega_j \)), and for almost perpendicular wave propagation one may find that at saturation (\( V' = 0 \)):

\[
\frac{\Delta \omega_{ej}}{\hbar^2} = \frac{4\pi}{\varepsilon_0 B^2} \sum_{\kappa_1} \phi_{\kappa_1} \ \Delta \omega_{ej} \ A_{j\kappa_1} \ \cos^2(\theta - \theta_j) \quad (47.a)
\]

where

\[
A_{j\kappa} = \sum_{n=-\infty}^{+\infty} \frac{F_{nj}}{(\omega_{ej} - n_0 \omega_j - n \omega_j)^2} \quad (47.b)
\]

Effective angular extend

\( \theta_c = (\theta - \theta_j)_{\text{max}} \)

Resonance condition in more than one dimension (\( \omega_{ej} = k \cos \theta \)) is a function of the angle between the particle's resonant velocity and the wave propagation direction. An 'instability cone' about the direction of the maximum growth rate, can be defined by the resonance broadening at saturation in the following way:

\[
(\kappa + \Delta \kappa) \cos \theta_c = k \quad (48.a)
\]

where,

\[
\Delta \kappa / k = \Delta \omega_{ej} / \omega_{ej} \quad (48.b)
\]

The critical angle \( \theta_c \) in weak-to-moderate turbulence conditions is smaller than 60° (\( \Delta \omega_{ej} / \omega_{ej} < 1 \)).

In all the following estimates we are taking \( \cos^2(\theta - \theta_j) \sim 1 \).

Back to equation (47.a) one may find (Dum and Dupree [1970]):

\[
\frac{2k^2}{\varepsilon_0 B^2} \ \sum_{\kappa_1} k_1^2 \ \phi_{\kappa_1} \ A_{j\kappa_1} = 1 \quad (49)
\]
Averaging over the velocity space and assuming that at saturation the faster growing mode \( \omega_{k_o} \) dominates the Fourier sum, one can estimate the threshold rms electric field amplitude, for saturation:

\[
\int \frac{d\nu}{\nu} f_o(\nu) A_{j \nu} = \sum_n \frac{\bar{F}_n(M_j)}{(\omega_{k_o} - k \nu_j - n\omega_j)^2} = \bar{A}_{j k_o} \tag{50.a}
\]

\[
\bar{F}_n(M_j) = \frac{\delta n}{2} \left[ I_{n+1}(M_j) + 2 I_n(M_j) + I_{n-1}(M_j) \right] \tag{50.b}
\]

\( M_j = \frac{k^2 \rho_j^2}{2} \), \( \rho_j \) is the particle's gyroradius and \( I_n \) the Bessel function of imaginary argument and order \( n \) (Abramovitz and Stegun [1965]). \( f_o(\nu) \) is the normalized Maxwellian distribution.

Finally:

\[
\frac{\varepsilon_0}{2} \langle E_{\nu} \rangle^2 \leq \frac{\varepsilon_0}{2} \frac{E^2}{k^2} \bar{A}_{j k_o} - \Phi k_o \tag{51}
\]

The saturation amplitude threshold for the faster growing mode will impose an upper limit for the electrostatic energy of the other unstable modes.

The Fourier sum of the equation (47.a) is in fact performed over the wave vector region where \( |k_\perp| > k_o \) (Farley [1984]). \( k_o \) is the wave number of maximum growth rate and in situations where \( k \rho_i < 1 \) and \( k \rho_e > 1 \) the Fourier sum may be replaced by the term with \( k_\perp \sim k_o \) if \( k_o \rho_e \sim 1 \) (equation (51)).

For instabilities with \( k_o \rho_i \sim 1 \), the Fourier sum may be replaced by the term with \( k_\perp \sim k \) alone.

(5) FREQUENCY BROADENING AT SATURATION.

For almost perpendicular wave propagation, the renormalized quasilinear theory in the weakly magnetized limit (Dupree [1968], Weinstock [1970]) is equivalent to replace the linear susceptibility functions the resonant frequency \( \omega_z \) by
the new one $\omega_j + i(\Delta \omega_j + \nu_j)$ if a BGK collisions frequency $\nu_j$ is also considered. The linear growth rate $\gamma_{\text{le}}^L$ is now replaced by $\gamma_{\text{le}}^{NL} + \Delta \omega_j$, where $\gamma_{\text{le}}^{NL}$ or simply is the nonlinear growth rate and $\Delta \omega_j$ is the total frequency broadening which is a function of $\Delta \omega_j$ and $\nu_j$ for both the resonant and the nonresonant particles. $\Delta \omega_j$ is determined from the dispersion relation.

Perpendicular to the magnetic field, the collisional broadening associated with the BGK binary collisions is expressed by (Dupree [1968]):

$$d_{\text{c},j} = \frac{1}{k^2} \frac{p_j^2}{j_j} \nu_j$$  \hspace{1cm} (52)

Also, because the wave-diffusion coefficient $D_{\perp,j}$ is independent of $k$, an anomalous collision frequency may be defined (Sudan [1983.a]):

$$\Delta \omega_{\text{c},j} = \frac{1}{k^2} \frac{p_j^2}{j_j} \nu_j^*$$  \hspace{1cm} (53.a)

and

$$\nu_j^* = \frac{1}{p_j^2} D_{\perp,j}$$  \hspace{1cm} (53.b)

Generally the nonlinear broadening is larger than the collisional one, and also in practice only the nonlinearity in the non resonant population determines the 'turbulent collisions' broadening (Gary [1980]). Gary [1980] and Gary and Sanderson [1981] give a self-consistent set of equations (second order development of the Vlasov-Poisson equations in parameter $\xi_j$) to calculate the anomalous transport processes associated with the 'orbit diffusion' saturation mechanism.
APPENDIX 3

REGION OF FAVORABLE ASPECT ANGLE IN THE DIPOLE MAGNETIC FIELD APPROXIMATION

In this appendix we present a graphic method to estimate the scattering volume of strongly aspect angle dependent radar backscattering.

The purpose of this exercise is to give a global view of the observation geometry. The use of the dipole magnetic field approximation responds only to this analytical need. As a matter of fact, a more sophisticated magnetic field model is used in the data interpretation.

The geometry of the observation is presented in figures (A3.1) and (A3.2). Axis x and y are positive toward the geographic north and east respectively. The geomagnetic meridian plane passing through Millstone Hill makes an angle (magnetic declination) of 14.5 degrees with the x-z plane. The axis z is perpendicular to the earth surface. We are considering the local spheric Earth approximation with geoid radius at Millstone Hill equal to 6368.4 km.

As we mentioned in chapter 4 the potentially unstable region contributing to the backscatter power, at any given time \( t \), is defined by the intersection between the two wavefronts of radius \( c(t - t_o) \) and \( c(t + \tau_p - t_o) \) and the regions where the aspect angle is favorable. \( t_o \) is the observation starting time and \( \tau_p \) the radar pulse length. The weighted volume of this region by the antenna radiation pattern will give an estimate of the instability effective scattering volume.

1. RANGE-ALTITUDE-GROUND DISTANCE.
The radar line-of-sight or main beam direction is defined by two angles:
the azimuth (AZ) and the elevation (EL) angles. AZ is measured clockwise away
from the y axis and EL away from the tangent plane at Millstone Hill (figure
(A3.1) and (A3.2)). The radar range R is equal to:

\[ R = c \left( t - t_o + \frac{1}{2} c \varepsilon_p \right) \quad (1) \]

The corresponding main beam altitude is

\[ h = \sqrt{R^2 + R^2 + 2R_e R \sin \theta} = R_e \quad (2) \]

and the ground distance

\[ d = R_e \Delta \theta \quad (3) \]

\[ \Delta \theta \] is the differential latitude between the observation point and the radar
station,

\[ \Delta \theta = \cos^{-1} \left[ \frac{R^2 + (R_e + h)^2 - R_e^2}{2(R_e + h) R} \right] \quad (4) \]

The axis y in figure (A3.2) is tangent to the curve CC' representing the earth
surface. The angle EL is now measured clockwise away from the y axis. Segment
RA is perpendicular to the earth surface and \( h = |\overline{RA}| \) and \( G_d = |\overline{OA}| \).

2. SURFACES OF CONSTANT ASPECT ANGLE.

In the dipole magnetic field approximation, the intersection between the
magnetic meridian plane at Millstone Hill and a surface of constant magnetic
field strength defines a magnetic 'field line':
\[ \cos^2 \theta = \left(1 + \frac{h}{R_e}\right) \cos^2 \bar{\theta} \] \hspace{1cm} (5)

\( \bar{\theta} \) is the invariant magnetic latitude, \( h \) and \( \theta \) are the altitude and the magnetic latitude of the observation point.

The magnetic inclination angle \( I \) is the angle between the magnetic field \( B \) and the horizontal at the observation point:

\[ \tan I = 2 \tan \theta \] \hspace{1cm} (6)

The aspect angle \( \chi \) is the angle between the radar line-of-sight and the magnetic field:

\[ \chi = I + EL + (\theta - \bar{\theta}_o) \] \hspace{1cm} (7)

\( \bar{\theta}_o \) - 56 degrees is invariant latitude at Millstone Hill.

The intersection between the magnetic meridian plane and the surface of 90 degrees aspect angle is represented in figure (A3.3). The curves of constant aspect angle at a given elevation are circles tangent to the magnetic parallel at Millstone Hill (figure (A3.5)). Also, perpendicularly can only be achieved for latitudes northward from our station (\( \bar{\theta} > \bar{\theta}_o \)).

From equations (6) and (7) we can find:

\[ \tan (\chi - EL + \bar{\theta}_o - \theta) = 2 \tan \theta \] \hspace{1cm} (8)

Calling \( \chi_o = \chi - EL + \bar{\theta}_o - \pi/2 \), the point of aspect angle \( \chi \) in the intersection of the magnetic meridian plane and the plane of elevation \( EL \), is
defined only by its magnetic latitude $\Theta$:

$$\tan \Theta = \frac{3}{4} \tan \chi_0 + \frac{1}{2} \sqrt{\frac{9}{4} \tan^2 \chi_o + 2}$$

(9)

From equation (9) when $\Theta - \bar{\Theta}_o$ the minimum elevation angle for which perpendicularity ($\chi - \pi/2$) is still possible is equal to $ELO = 18.64^\circ$.

2.a. CONTOURS OF CONSTANT ASPECT ANGLE IN THE MAGNETIC MERIDIAN PLANE.

Figure (A3.3) represents the intersection of the surface of 90 degrees aspect angle with the magnetic meridian plane. Axis z is perpendicular to the Earth's surface at Millstone Hill. In this figure we are also presenting the definitions of the magnetic inclination and the aspect angles.

Calling $\Delta \Theta = \Theta - \bar{\Theta}_o$, from equation (4) we may have:

$$\tan \Delta \Theta = \frac{RCOS \theta / Re}{1 + \frac{R}{Re} \sin \theta}$$

(10)

and from equation (10),

$$\tan \Delta \Theta = \sqrt{A^2 - B} - A$$

(11)

with

$$A = \frac{3}{2} \frac{\tan \bar{\Theta}_o + \tan \bar{\theta}}{2 - \tan \bar{\theta} \tan \bar{\Theta}_o}$$

(12.a)

$$B = \frac{(2 \tan \bar{\theta} \tan \bar{\Theta}_o - 1)}{2 - \tan \bar{\theta} \tan \bar{\Theta}_o}$$

(12.b)

also $\bar{\theta} = \theta + \chi - \pi/2$.

Using equations (10) and (11) we can obtain:
\[ R = R_e \left( \sqrt{c^2 - d} - c \right) \]  

(13)

where

\[ C = \frac{A \cos EL + B \sin EL}{K} \]  

(14.a)

\[ D = \frac{B}{K} \]  

(14.b)

and

\[ K = 1 + A \sin 2EL + (B - 1) \sin^2 EL \]  

(14.c)

The range \( R \) identifies the point of aspect angle \( \chi \) on the magnetic meridian plane at the latitude \( \theta \) and elevation angle \( EL \). In table (A3.1) we list the values of the range \( R \) corresponding to different elevation angles.

**TABLE (A3.1).** Range (R) of the intersection point between the surfaces of constant aspect angle (at 89°, 90°, and 91°), the plane of constant elevation angle (EL) and the magnetic meridian plane at Millstone Hill.

<table>
<thead>
<tr>
<th>EL (deg.)</th>
<th>R (km, ( \chi = 89° ))</th>
<th>90°</th>
<th>91°</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>958</td>
<td>1032</td>
<td>1106</td>
</tr>
<tr>
<td>5.0</td>
<td>888</td>
<td>962</td>
<td>1036</td>
</tr>
<tr>
<td>6.0</td>
<td>818</td>
<td>892</td>
<td>966</td>
</tr>
<tr>
<td>7.0</td>
<td>748</td>
<td>822</td>
<td>896</td>
</tr>
<tr>
<td>8.0</td>
<td>678</td>
<td>751</td>
<td>825</td>
</tr>
<tr>
<td>9.0</td>
<td>608</td>
<td>681</td>
<td>755</td>
</tr>
<tr>
<td>10.0</td>
<td>537</td>
<td>610</td>
<td>684</td>
</tr>
<tr>
<td>12.0</td>
<td>397</td>
<td>469</td>
<td>543</td>
</tr>
<tr>
<td>14.0</td>
<td>256</td>
<td>328</td>
<td>401</td>
</tr>
<tr>
<td>15.0</td>
<td>186</td>
<td>258</td>
<td>330</td>
</tr>
</tbody>
</table>
2.b. CONTOURS OF CONSTANT ASPECT ANGLE AT A GIVEN ELEVATION.

In order to proof that the contours of constant aspect angle and fixed elevation are circles tangent to the magnetic parallel at Millstone Hill we are using the fact that the inclination angle \( I \) is constant along the circles of invariant latitude and for a fixed altitude. Figure (A3.4) represents the magnetic meridian plane and \( \chi_o \) is the aspect angle at the point \((\vec{\theta}, h)\) and elevation EL. Another point, in the same plane, with same aspect angle must have a different elevation EL:

\[
\chi_o = \chi(\vec{\theta}, h, EL) = \chi(\theta, \theta', EL')
\] (15)

This new point can be mapped into the point lying on the curve of aspect angle \( \chi_o \) at the intersection of the plane of elevation EL with the strip of constant altitude \( h \) along the circle of latitude \( \Theta \). \( h \) is the height of the point on the plane of elevation EL such that: \( h' = h + \delta h \) and \( \delta \Theta - \theta' - \Theta - \delta \theta' - \Theta \), where \( \theta' \) is the latitude of the point of altitude \( h' \) and elevation EL (figure (A3.4)). By construction, the angles \( \Theta, \Theta', \Theta \) are the invariant latitudes corresponding to \( \vec{\Theta}, \vec{\Theta}', \vec{\Theta} \) respectively. From equation (5) one can have:

\[
\kappa - h - \delta h, \quad \theta' - \Theta - \delta \Theta, \quad \Theta' - \Theta - \delta \theta \quad \text{and} \quad \theta' = (\Theta + \Theta')/2
\]

Therefore, in figure (A3.5) \( \Omega - \chi + \beta = \kappa/2 \). If we call \( \mathcal{G}_\kappa \) the ground distance associated with the point, on the magnetic meridian plane at Millstone Hill, with aspect angle \( \chi_o \) we find for the curve of constant aspect angle:
\[ G_d = \overline{G_d} \cos \overline{\alpha Z} \] (16)

\[ \overline{\alpha Z} = AZ + AZM - \frac{\pi}{2} \], AZ is the azimuth defined in figure (A3.1) and AZM is the magnetic declination at Millstone Hill. In table (A3.2), \( \overline{G_d} \) (km) is the ground distance of the intersection between the magnetic meridian plane at Millstone Hill, the surfaces of constant aspect angle and the plane of constant elevation angle; \( \overline{\Theta} \) (deg.) is the apex latitude corresponding to this point and I (deg.) is the inclination angle of the field line defined by \( \overline{\Theta} \) and EL.

**TABLE (A3.2).** The contours of constant aspect at a given elevation are circles tangent to the magnetic parallel at Millstone Hill with diameter \( \overline{G_d} \) in the Earth's surface projection.

<table>
<thead>
<tr>
<th>( \overline{\Theta} )</th>
<th>( \overline{G_d} )</th>
<th>I</th>
<th>EL - 0°</th>
<th>3°</th>
<th>4°</th>
<th>5°</th>
<th>6°</th>
</tr>
</thead>
<tbody>
<tr>
<td>74.00</td>
<td>2000</td>
<td>81.84</td>
<td>97.84</td>
<td>100.84</td>
<td>101.84</td>
<td>102.84</td>
<td>103.84</td>
</tr>
<tr>
<td>71.74</td>
<td>1750</td>
<td>80.63</td>
<td>96.38</td>
<td>99.38</td>
<td>100.38</td>
<td>101.38</td>
<td>102.38</td>
</tr>
<tr>
<td>69.49</td>
<td>1500</td>
<td>79.40</td>
<td>92.90</td>
<td>95.90</td>
<td>96.90</td>
<td>97.90</td>
<td>98.90</td>
</tr>
<tr>
<td>67.24</td>
<td>1250</td>
<td>78.15</td>
<td>89.40</td>
<td>92.40</td>
<td>93.40</td>
<td>94.40</td>
<td>95.40</td>
</tr>
<tr>
<td>65.00</td>
<td>1000</td>
<td>76.87</td>
<td>85.87</td>
<td>88.87</td>
<td>89.87</td>
<td>90.87</td>
<td>91.87</td>
</tr>
<tr>
<td>62.74</td>
<td>750</td>
<td>75.55</td>
<td>82.30</td>
<td>85.30</td>
<td>86.30</td>
<td>87.30</td>
<td>88.30</td>
</tr>
<tr>
<td>60.50</td>
<td>500</td>
<td>74.20</td>
<td>78.70</td>
<td>81.70</td>
<td>82.70</td>
<td>83.70</td>
<td>84.70</td>
</tr>
<tr>
<td>58.25</td>
<td>250</td>
<td>73.00</td>
<td>75.05</td>
<td>78.05</td>
<td>79.05</td>
<td>80.05</td>
<td>81.05</td>
</tr>
<tr>
<td>56.00</td>
<td>0</td>
<td>71.36</td>
<td>71.36</td>
<td>74.36</td>
<td>75.36</td>
<td>76.36</td>
<td>77.36</td>
</tr>
</tbody>
</table>

3. POTENTIALLY UNSTABLE REGION.

For strongly field aligned irregularities and for a given range R, the volume
enclosing the potentially unstable plasma is defined by the intersection between the semiespheres of radius \( R \pm c z_p/2 \) and, the surfaces of constant aspect angle \( 90^\circ \pm 1^\circ \).

Placing the lower boundary of the E-region at 90 km (curve DD' in figure (A3.2), parallel to CC'), the angle (measured from the axis y) at which the semicircle of range R intersects the curve DD' defines the minimum elevation angle (EL1) of the scattering region. The intersection between the magnetic meridian plane, the semisphere of radius \( R - c z_p/2 \) and the surface of 91 degrees aspect angle, defines the maximum elevation angle (EL2). This three-dimensional geometry can be projected into the earth surface yielding to the 'ruler and compass' determination of the boundaries of the potentially unstable region.

This is done by mapping along the earth radius the intersection between the instability volume and the cones defined by the z-axis (at Millstone Hill) and different elevation angles. Two different altitudes corresponding to the same projected point are identified by their elevation angles. For the case where \( R = 770 \) km and \( z_p = 2 \) msec, the unstable region is limited by the elevations EL1 = 3.5° and EL2 = 10°. Table (A3.3) lists the parameters defining the points where the surfaces of constant aspect angle intersect the cones of EL = 4°, 5° and 6°, and the sem-spheres of constant range. The broken-lines represent the cases where intersection is not possible. The altitudes and ground distances correspond to the ranges \( R = 770 \) km and \( R \pm c z_p/2 \) (620 and 920 km), the azimuthal angles are measured away from the magnetic meridian plane (+ to the west, - to the east). The intersection between the surface of constant aspect angle and the cone of constant elevation is a circle poleward and tangent to the magnetopause at Millstone Hill with radius \( R_A \).
TABLE (A3.3). Lists the azimuthal angles of the intersections between the surfaces of constant aspect angle and the cones of constant elevation for a given altitude (h) and ground distance.

<table>
<thead>
<tr>
<th>ELEVATION (deg)</th>
<th>ALTITUDE (km)</th>
<th>GROUND DISTANCE (km)</th>
<th>R_A (km)</th>
<th>AZIMUTH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>R-cθ/2</td>
<td>R+cθ/2</td>
</tr>
<tr>
<td>4</td>
<td>100±10</td>
<td>609</td>
<td>758</td>
<td>906</td>
</tr>
<tr>
<td>5</td>
<td>112±13</td>
<td>607</td>
<td>757</td>
<td>903</td>
</tr>
<tr>
<td>6</td>
<td>126±15</td>
<td>605</td>
<td>753</td>
<td>900</td>
</tr>
</tbody>
</table>
\[ \chi = I + EL + (\theta - \bar{\theta}) \]
REFERENCES


ANDRE D., The dependence of the relative backscatter cross section of 1-m density fluctuation in the auroral electrojet on the angle between electron drift and radar wave vector, J.G.R., 88, 8043 (1983).


- The current convective instability as applied to the auroral ionosphere, J.C.R., 86, 4811 (1981).


D'ANGELO N., Type 3 spectra of the radar aurora, J.C.R., 78, 3587 (1973).


FOSTER J.C., J.M.HOLT, G.B.LORIOT, and W.L.Oliver, World day ionospheric observations at Millstone Hill, EOS, 21, 457 (1985.a).

FOSTER J.C., J.M.HOLT, J.D.KELLEY, and V.B.WICKWAR, High-resolution observations of electric fields and F-region plasma parameters in the cleft ionosphere, J.Holtet and A.Egeland (eds.), The Polar cap, 349 (1985.b)


- Radar observations of auroral electrojet currents, J.G.R., 80, 3642 (1975.b).


HAGFORS T., Some properties of radar auroral echoes as observed at a
frequency of 1295 MHz, AGARD Technical Meeting, Lindau-Germany (1971).


- On 11-cm irregularities during equatorial spread F, J.G.R., 86, 829
Diffusion of small-scale density irregularities during equatorial spread F, 

HUBA J.D., Physical mechanism of the lower-hybrid-drift instability in a 

HUBA J.D., S.L.OSSAKOW, P.SATYANARAYANA, and P.W.GUZDAR, Linear theory of the 

ICHIMARU S., D.PINES, and N.ROSTOKER, Observation of critical fluctuations 


KELLEY M.C., and C.W.CARLSON, Observations of intense velocity shear and 

KELLEY M.C., K.D.BAKER, C.RINO, and J.C.ULWICK, Simultaneous rocket probe 
scintillation and incoherent scatter observations of irregularities in the 

KELLEY M.C., J.F.VICKREY, C.W.CARLSON, and R.TORBERT, On the origin and 

KELLEY M.C., Nonlinear saturation spectra of electric fields and density 


KESKINEN M.J., R.N.SUDAN, and R.L.FERCH, Temporal and spatial power spectrum 
studies of numerical simulations of type II gradient drift irregularities 


PRIKRYL P., J.A. KOEHLER, G.J. SOFKO, D.J. McEWEN, and D. STEELE, Ionospheric ion cyclotron wave generation inferred from coordinated Doppler radar, optical, and magnetic observations, J.G.R., 92, 3315 (1987).


ROBINSON T.R., Towards a self-consistent nonlinear theory of radar auroral


SHARBER J. R., The continuous (diffuse) aurora and auroral-E ionization, T. S. Chang, B. Coppi and J. R. Jasperse (eds.), Physics of the space plasmas, 115


St. MAURICE J. P., On a mechanism for the formation of VLF electrostatic emissions in the high latitude F-region, Planet. Space Sci., 26, 801 (1978).


St. MAURICE, and R. LAHER, Are observed broadband plasma wave amplitudes large enough to explain the enhanced electron temperatures of the high-latitude E region?, J. G. R., 90, 2843 (1985).


St. MAURICE J. P., J. C. FOSTER, J. M. HOLT, and C. del POZO, First results on the observation of 440 MHz high-latitude coherent echoes with the Millstone Hill radar, submit to J. G. R. (1988).


TSUNODA R.T., Electric field measurements above a radar scattering volume producing 'diffuse' auroral echoes, J.G.R., 80, 4297 (1975).


VOLOSEVICH A.V., and V.A.LIPEROVSKIY, Generation of small-scale inhomogeneities in a turbulent plasma and radio auroras, Geomagn.i aeronomiya, 15, 58 (1975).
WAND R.H., Seasonal variations of lower thermospheric winds from the Mills-
- Geomagnetic activity effects on semidiurnal winds in the lower thermosphere,

WANG T.N.C., and R.T.TSUNODA, On a crossed field two-stream plasma instability
  in the auroral plasma, J.G.R., 80, 2172 (1975).

WEINSTOCK J., Formulation of a statistical theory of strong plasma turbulence,
- Turbulent plasmas in a magnetic field: A statistical theory, Phys.Fluids,

WHALEN J.A., General characteristics of the auroral ionosphere, T.S.Chang,

WILLIAMS P.J., Clutter and interference in EISCAT data, S.Westerlund (ed.),
  Proceedings EISCAT annual review meeting, 51 (1980).

YAMADA M., and H.W.HENDEL, Current-driven instabilities and resultant
  anomalous effects in isothermal, inhomogeneous plasmas, Phys.Fluids, 21,
  1555 (1978).