

A TEST OF THE CAPITAL ASSET PRICING MODEL
ON EUROPEAN STOCK MARKETS

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July 1973
667-73

Not for Quotation
Comments Invited

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An earlier draft of this paper was presented as part of a paper given
at the first International Congress on Stock Exchanges, Milan,
Italy, March 1972.

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I. Introduction

Recent developments in the theory of portfolio selection by Sharpe [1964] and Lintner [1965] have led to the formulation of the Capital Asset Pricing Model (CAPM). This equilibrium theory of the capital market claims that securities should be priced according to their systematic risk or covariance with the market. Tests of this theory on the U.S. market have been performed by many authors (Black, Jensen and Scholes (1972), Friend and Blume (1970), Jacob (1971)). This theory, however, has never been tested on other capital markets because of a lack of data.

The purpose of this paper is to test the asset pricing model on the eight major European stock markets. It is generally believed that European markets are less efficient than their U.S. equivalent. If this is the case, the pricing of risk for European securities might be less rational than for American securities. To draw a meaningful comparison, we replicated the same tests for U.S. stocks, using similar intervals and time periods.

The paper is organized as follows: Part II describes the asset pricing model and its testable form. The data base is presented in Part III. The rest of the paper consists of various tests of the model.

II. Specification of the Model

The Sharpe-Lintner Capital Asset Pricing Model (CAPM) relies on certain assumptions about investor behavior and capital markets. Stated

formally, these are

- a. The market is composed of risk-averting investors maximizing their one-period expected utility. They make all their decisions on the basis of only two parameters, the mean and standard deviation of the probability distribution of their terminal wealth.
- b. Expectations and portfolio opportunities are "homogenous"; that is, all investors have the same expectations and opportunities.
- c. Capital Markets are perfect in the sense that all assets are infinitely divisible; there are no transaction costs or taxes, and borrowing and lending rates are equal to each other and the same for all investors.

The result is the following relationship between the expected return on securities and their systematic risk levels,

$$E(\tilde{R}_j) = R_F + \beta_j (E(\tilde{R}_m) - R_F)$$

where

$$\begin{aligned} E(\tilde{R}_j) &= \text{the expected return on security } j \\ &= \frac{E(\tilde{P}_j) + E(\tilde{D}_j) - P_{oj}}{P_{oj}} \end{aligned}$$

$$E(\tilde{P}_j) = \text{the expected price of security } j \text{ at the end of the planning interval}$$

$$E(\tilde{D}_j) = \text{expected dividends paid on security } j \text{ during the planning interval}$$

$$P_{oj} = \text{the current price of security } j$$

$$R_F = \text{the riskless rate of interest available during the planning interval (e.g. Government bond rate)}$$

$$E(\tilde{R}_m) = \text{the expected return on the market portfolio during the planning interval}$$

$$\beta_j = \text{the systematic risk of security } j^1$$

¹The systematic risk is a relative risk measure. It is the non-diversifiable risk of a security normalized by the risk of the market portfolio. Formally,

$$\beta_j = \frac{\text{Covariance } (\tilde{R}_j, \tilde{R}_m)}{\text{Variance } (\tilde{R}_m)}$$

To empirically test the model we must first overcome the problem of the model being stated in terms of expected return rather than realized returns. The expected returns on securities are, of course, unobservable and additional assumptions have to be made about the stochastic characteristics of security prices. The following equation¹ is generally chosen as the stochastic version of the CAPM:

$$R_{jt} = R_{Ft} + \beta_j (R_{mt} - R_{Ft}) + \varepsilon_{jt} \dots 1(a)$$

$$= R_{Ft}(1 - \beta_j) + \beta_j R_{mt} + \varepsilon_{jt} \dots 1(b)$$

where

R_{jt} = the realized rate of return on security j during period t (e.g., a month)

R_{Ft} = the riskless rate during period t

R_{mt} = the realized return on the market index during period t

ε_{jt} = a residual term, which under the CAPM has a zero expected value.

Additionally, in order to test an essentially single period model with time series data it is assumed that the β_j remains constant

¹The theoretical justification for this step follows one of two paths. The first is to assume a particular relation between security returns called the Market Model

$$\tilde{R}_j = E(\tilde{R}_j) + \beta_j (\tilde{R}_m - E(\tilde{R}_m)) + \tilde{\varepsilon}_j$$

This model was first proposed by Markowitz (1959), and extended by Sharpe (1963) and Fama (1968a). Substitution for $E(R_j)$ from the CAPM leads to the relationship given. The second approach is to assume R_j and R_m are distributed as bivariate normal. The result then follows (see Roll (1968)).

over time.¹ Thus ex post returns from several periods can be used to estimate and evaluate the parameters of the model.

The coefficients of the model are estimated by regressing a series of realized security returns on the corresponding returns on the market index. The fitted equation is given by

$$R_{jt} = \hat{\alpha}_j + \hat{\beta}_j R_{mt} + \hat{\mu}_{jt}$$

where $\hat{\beta}_j$ is an estimate of the systematic risk of security j . As can be seen from Equation 1(b) the intercept term, $\hat{\alpha}_j$, has an expected value under the CAPM assumptions of $\bar{R}_F(1 - \hat{\beta}_j)$, where \bar{R}_F is the average value of the risk free rate during the test period. Thus a direct test of the CAPM can be performed by estimating the model coefficients for a security over some time period and testing to see if $\hat{\alpha}_j$ is significantly different from $\bar{R}_F(1 - \hat{\beta}_j)$.

While it would be possible to test the model on a stock by stock basis, this would be an inefficient procedure which would not make maximum use of all the information available. Instead we have pooled the results from all of the individual stock regressions to provide an efficient cross sectional test of the model. For this test we regress the mean return of each security over the test period, \bar{R}_j , on the corresponding estimates of their systematic risk coefficients obtained from the individual stock regression, $\hat{\beta}_j$. The regression equation is

¹Evidence presented by Pogue and Solnik [1972] shows this assumption to be reasonable for the test periods considered in this paper.

$$\bar{R}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \mu_j. \quad \dots (2)$$

The CAPM can be used to provide theoretical values for γ_0 and γ_1 . If equation (1) holds as predicted by the CAPM then by averaging both sides of the equation over a series of periods we obtain

$$\bar{R}_j = \bar{R}_F + \beta_j (\bar{R}_m - \bar{R}_F) + \bar{\epsilon}_j$$

Therefore the CAPM can be tested by estimating the coefficients in equation (2) and testing whether the estimated coefficient γ_0 differs significantly from \bar{R}_F and $\hat{\gamma}_1$ from $(\bar{R}_m - \bar{R}_F)$.

We now proceed to the estimation of the γ_0 and γ_1 coefficients for each of the countries in our data base.

III. The Data¹

The data base used consisted of daily price and dividend data for 234 common stocks of eight major European countries. The data covered the period from March 1966 through March 1971. The data were corrected for all capital adjustments (splits, rights, etc.).

Security returns were computed on a bi-weekly basis, as follows²

$$r_t = \frac{P_t + d_t - P_{t-1}}{P_{t-1}}$$

¹We wish to thank Eurofinance for making their stock price tape available to us. Without this data the present study would have been impossible.

²The bi-weekly interval was chosen (as opposed to daily or monthly intervals) as a compromise between the problems of measurement error inherent in daily data and sampling inefficiencies associated with longer intervals. A discussion of the effects on the CAPM parameters of changes in the return measurement interval is contained in Pogue and Solnik (1972),

where r_t = the return during calendar interval t
 P_t = stock price at the end of the two week period
 P_{t-1} = stock price at the end of the previous two week period
 d_t = dividends paid during the two week period (assuming payment on ex-dividend dates)¹

The distribution of the sample by country is shown in Table 1. Within each country the companies in our sample tend to be the largest in terms of market value of shares outstanding. The 30 Italian stocks, for example, comprise 77 percent of the market value of all listed shares. For the United Kingdom, France and Germany the number is not as high but still in excess of 50 percent in each case.

For each country the rates of return on a market index were computed on a comparable basis --in particular the return on the index includes dividends.² The indexes chosen are listed in Table 1. They are either the only indexes available, or where choice existed, the most representative.

The choice of risk free rates was more difficult, since for most countries short term government notes comparable to U.S. treasury bills do not exist. For these countries we used short term (prime) bank discount rate (see Table 1).

The United States data used for most of the comparison tests were taken from the University of Chicago CRSP file which contains monthly price relatives and dividend data for all securities listed on the New

¹Dividend data were not available for the Netherlands, Sweden and Switzerland; thus return is measured by the proportionate change in stock price.

²Except for Netherlands, Sweden and Switzerland where dividends were not included in the security returns.

Table 1

SUMMARY OF DATA BASE USED

Country	Number of Stocks in Sample	Market Index Used	Risk Free Rate Used
France	65	I.N.S.E.E.	Short term prime bank rate
Italy	30	24 ORE	" " " "
United Kingdom	40	Financial Times, Industrial Ordinary	31 day treasury notes
Germany	35	Herstatt Index	Short term prime bank rate
Netherlands	24	ANP/CBS	" " " "
Switzerland	17	Schweizerische Kreditanstalt	" " " "
Belgium	17	Indice de la Bourse de Bruxelles	" " " "
Sweden	6	Jacobson & Ponsbach	" " " "
United States	900	Standard & Poor's 500 Stock Composite Index	30 day U.S. Government Treasury Bills

York Stock Exchange in the period January 1926 - June 1970. The remaining tests were based on bi-weekly aggregations of daily NYSE data obtained from Standard and Poor's daily stock price tapes.

IV. Security Cross Sectional Results

The regression results for the eight European countries are given in Table 2. Comparative results for the American Market for 3 different time periods, are also contained in Table 3. The first row of American results are for a random sample of 100 NYSE stocks for the period March 1967 - March 1971. These results are also based on bi-weekly return data. The remaining results are based on monthly data and cover longer time periods. The 1956-1965 regression is taken from a paper by Jacob [1971] and includes 593 securities. The last result covers the 1960-1970 period and is based on 523 stocks.

We shall consider first the European results, using the U.K. as an illustrative example. On the U.K. market the relationship between mean return and risk is given by

$$\bar{R}_j = 0.12 + 0.32\hat{\beta}_j.$$

This relationship represents the average risk-return results experienced by our sample of British stocks. Higher risk stocks, as predicted by the CAPM, had higher rates of return, with return increasing 0.32 percent per two weeks (8.3 percent per year) for a one unit increase in $\hat{\beta}$. While the signs of $\hat{\gamma}_0$ and $\hat{\gamma}_1$ are in agreement with the CAPM predictions, the intercept term $\hat{\gamma}_0$ is smaller than the 0.20 percent predicted value (see table 2) and $\hat{\gamma}_1$ larger than the 0.20 theoretical value.

TABLE 2

SECURITY CROSS SECTIONAL RESULTS

$$\bar{R}_j = \gamma_0 + \gamma_1 \hat{b}_j + \mu_j$$

COUNTRY	No. of Stocks	Period	Return Measurement Interval	Regression Results			Theoretical Values	
				$\hat{\gamma}_0$	$\hat{\gamma}_1$	R^2	$\gamma_0 = \bar{R}_F$	$\gamma_1 = \bar{R}_M - \bar{R}_F$
FRANCE	65	Mar '66-Mar '71	bi-weekly	0.56 (0.10)*	-0.28 (0.16)	0.04	0.36	-0.10
ITALY	30	"	"	0.23 (0.31)	-0.20 (0.31)	0.01	0.24	-0.20
U.K.	40	"	"	0.12 (0.33)	0.32 (0.32)	0.02	0.20	0.20
GERMANY	35	"	"	0.70 (0.20)	-0.28 (0.23)	0.04	0.36	-0.05
NETHERLANDS	24	"	"	0.03 (0.30)	0.39 (0.30)	0.05	0.20	0.32
BELGIUM	17	"	"	0.05 (0.30)	0.30 (0.39)	0.05	0.20	0.16
SWITZERLAND	17	"	"	-0.31 (0.30)	0.70 (0.30)	0.28	0.12	0.70
SWEDEN	6	"	"	-1.02 (0.32)	1.40 (0.30)	0.79	0.20	0.50
UNITED STATES	100	Mar '67-Mar '71	"	0.172 (0.125)	0.300 (.101)	0.08	0.24	0.06
UNITED STATES	593	Jan '56-Dec '65	monthly	0.7 N.A.	0.3 (0.06)	0.03	0.21	0.79
UNITED STATES	523	Jan '60-June '70	Monthly	0.515 (0.08)	0.174 (0.07)	0.01	0.33	0.156

* Standard Error

Thus the rate of increase of realized return with risk is higher than predicted by the CAPM. However, the uncertainty associated with the measured coefficients, as expressed by the standard errors, is sufficiently large that we can not conclude at a high level of confidence that the actual (as opposed to measured) values of the coefficients differ from the theoretical values.

In the case of France, Italy and Germany the average return on the market was less than the risk free rate, i.e. $\bar{R}_m - \bar{R}_F$ was negative as shown in the last column of Table 2, and therefore the slope coefficient γ_0 should be negative. In other words, riskier stocks should have even more negative returns than less risky ones. When the results for these countries are examined, the coefficients are generally comparable in sign and magnitude with the CAPM a priori estimates. The Italian results are virtually identical with the predicted values. For France and Germany the slope coefficients $\hat{\gamma}_1$ are more negative than the theoretical values, indicating that riskier stocks declined more than predicted by the CAPM. Similar statements can be made for the remaining four smaller markets, although the consistency of the results declines somewhat.

Before attempting to draw conclusions from these results, the major limitations of our test/^{design} should be reviewed. First, the time period (5 years) is short. Many random factors, which may cancel out in the longer run, can profoundly distort results for shorter time periods. Second, our security samples are relatively small and not randomly selected. These factors can lead to inefficient and/or biased results. Thirdly, the proportion of cross-section mean return variation accounted for by the risk coefficients is low, approximately 4 percent on average.

While these limitations prevent us from reaching definitive conclusions at this point, the European results, on the whole, are consistent with the hypothesis that securities prices on the average reflect systematic risk, in the way predicted by the CAPM.¹

When compared with the American results, the following observations can be made. First, the percentage of cross-sectional return variation attributable to risk differences is roughly comparable. For the three U.S. regressions the percentage averaged 4 percent, identical to the European average. Second, the standard errors for the U.S. coefficients are smaller, but this is primarily the result of much larger sample sizes. The smaller uncertainty regarding the $\hat{\gamma}_0$ and $\hat{\gamma}_1$ estimates allows more definitive tests of results. For the 1956-1965 period the slope of the regression line ($\hat{\gamma}_1$) substantially understates the theoretical value. The reverse is true for the results from the 1960-1970 period. On the whole, the European results are comparable with those for the U.S.

V. Portfolio Grouping of Securities

In general, the proportion of mean return differences accounted for by the regression lines are low but comparable to those found on the

¹This conclusion is supported by additional tests conducted by Solnik (1972) which show that the unique or residual security risk is relatively unimportant in explaining differences in realized returns. For these tests an additional term was added to Equation 2.

$$\bar{R}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \gamma_2 (\hat{SE}_j)$$

where SE_j is a measure of the unique risk of security j ($SE_j = \text{Total Risk} - \text{Systematic Risk}$). For the eight markets tested the SE_j factor was found to be relatively unimportant in explaining differences in the R_j .

U.S. market. If the assumptions of the least squares regression procedure were met, then the estimates of γ_0 and γ_1 obtained from the security regressions would be the most efficient possible. However, this is not the case, since the independent variable in the cross-section regressions, $\hat{\beta}_j$, is measured with error. As long as $\hat{\beta}_j$ contains measurement error, the estimates of $\hat{\gamma}_0$ and $\hat{\gamma}_1$ will be subject to the well known errors in the variable bias and will be inconsistent. The result of this bias is that $\hat{\gamma}_1$ will be understated and $\hat{\gamma}_0$ correspondingly overstated. Hence tests of relationship between $\hat{\gamma}_0$ and $\hat{\gamma}_1$ and their theoretical values \bar{R}_F and $\bar{R}_m - \bar{R}_F$ will be misleading.

There are a variety of methods available to attempt to correct for this bias.¹ One particularly effective method involves classifying the observations into groups (portfolios) and fitting the group means. To obtain consistent estimates from the grouping process two conditions are required. First, the securities must be classified into portfolios independently of the values of the errors in the $\hat{\beta}_j$ estimates, since otherwise the law of large numbers would not apply to reduce the error in the portfolio β . Second, the beta values of the different portfolios (the averages of the component securities) should differ as much as possible if good estimates of γ_0 and γ_1 are to be obtained.

These requirements are jointly fulfilled in the following way. The securities were ranked by their estimated beta values in the period March 1966 - February 1967. Portfolios were then formed by grouping the securities into portfolios. The first portfolio contained the first subgroup of stocks with the highest beta values and so on until all securities were

¹See Malinvaud [1970]. Black, Jensen and Scholes (1972) have applied these methods to NYSE data.

included in portfolios. Mean return and beta coefficients were then estimated for each portfolio during the March 1967 - March 1971 period. The previous cross-sectional regressions were then rerun by regressing the mean portfolio returns on the corresponding estimated portfolio betas to obtain revised estimates of γ_0 and γ_1 .¹

The advantage of this approach is that it yields unbiased (for large portfolios) and efficient (for many portfolios) estimates of γ_0 and γ_1 . The trade-off between these two properties becomes very important, particularly in our case where the number of securities in each European sample is small. If the portfolios contain too few stocks, measurement error will not be significantly reduced. If we form too few portfolios we will obtain inefficient estimates of the coefficients. To obtain the major benefit of the law of large numbers to reduce measurement error, we have included at least ten securities in each portfolio. A second alternative was to dichotomize the population into portfolios containing the highest and lowest β securities as suggested by Wald. Tests will now be conducted using these sets of portfolios.

VI. Portfolio Cross-Sectional Results

Portfolios were constructed for the four major European markets, France, the United Kingdom, Italy, and Germany. The results are reported in Table 3, along with comparative results for the United States market. Two U. S. regressions are reported, one for a comparable period (March 1967 - June 1970) and one for a longer period (1931-1965). The former results were computed for the purposes of this paper, the latter are presented in a paper by Black, Jensen and Scholes (1972).

¹The initial period ranking procedure tends to eliminate the correlation between the errors of measurement of the second period betas of the securities classified into each portfolio, thus meeting the first grouping requirement.

TABLE 3
Portfolio Cross Sectional Results

$$\bar{R}_j^P = \gamma_0 + \gamma_1 \hat{\beta}_j^P + \mu_j$$

Country	10 STOCKS PORTFOLIOS					DICHOTOMIZATION					THEORETICAL VALUES		
	Test Period	No. Port.	No. of Stocks Pr. Port.	$\hat{\gamma}_0$	$\hat{\gamma}_1$	R^2	No. Port.	No. of Stocks Pr. Port.	$\hat{\gamma}_0$	$\hat{\gamma}_1$	R^2	$\gamma_0 - \bar{R}_f$	$\gamma_1 - \bar{R}_m - \bar{R}_f$
FRANCE	Mar. '67-Mar. '71	5	12	0.15 (0.28)	0.45 * (0.33)	0.39	2	30	0.01	0.71	1.0	0.25	0.30
ITALY	Mar. '67-Mar. '70	3	10	0.40 (0.33)	0.13 (0.19)	0.22	2	15	0.42	0.10	1.0	0.24	0.25
U.K.	Mar. '67-Mar. '71	4	10	0.02 (0.22)	0.28 (0.22)	0.44	2	20	0.10	0.18	1.0	0.20	0.13
GERMANY	Mar. '67-Mar. '71	3	11	2.50 (0.73)	-1.81 (0.69)	0.87	2	17	5.00	-3.75	1.0	0.36	0.21
UNITED** STATES	Mar. '67-June '70	10	100	0.407 (0.31)	-0.924 (0.30)	0.54	-	--	--	--	--	0.48	-0.23
UNITED** STATES	Jan. '31-Dec. '65	10	75	0.359 (0.06)	1.08 (0.06)	0.98	-	--	--	--	--	0.16	1.42

* Standard Error

** Monthly Return Interval

The U.S. portfolios were constructed by grouping all available NYSE stocks into ten portfolios, where the grouping criteria was based on estimated β values in the previous five-year period. The portfolios were updated each year to include new securities and thus contain increasing numbers of securities during the test periods. For the longer period the portfolios contained approximately 75 securities on average, and 100 securities each for the March 1967 - June 1970 period.

The results for France, Italy¹ and the United Kingdom are again consistent with the model.

The German results, however, are another story. The values are inconsistent with any reasonable expectations. The intercept term, which is an estimator of the riskless rate, has an annualized value of 60 percent while the slope coefficient $\hat{\gamma}_1$, which is supposed to be $\bar{R}_m - \bar{R}_F$, implies a negative rate of approximately -60 percent. These results on the surface would question the rationality of the German market. The reason for these results, however, can be seen from an examination of Table 4. Table 4 shows the mean portfolio returns and beta values for the non-overlapping portfolios. As can be seen, our grouping procedure worked well for France, Italy and the United Kingdom in producing a dispersion of the portfolio beta values. This was not the case for Germany, however, where the three portfolios have beta values of 1.11, 1.06 and 1.02 and sufficiently large standard errors that the estimated values cannot be considered to be significantly different. Thus one of the requirements

¹For Italy, the test was performed on the period 1967-70 rather than 1967-71 because the market return in the latter period was close to zero leading to a poor testing power.

Table 4
RISK-RETURN CHARACTERISTICS OF EUROPEAN PORTFOLIOS

Country	No. Port.	No. Stocks PER PORT.	Period	Average		$\hat{\beta}_p$	Standard Error $\hat{\beta}_p$
				Bi-Weekly Return (%)			
France	5	12	March '67 -	0.738		1.08	0.061
			March '71	0.493		0.88	0.049
Italy	3	10	March '67 -	0.399		0.89	0.048
				0.568		0.86	0.044
			March '70	0.470		0.58	0.057
				0.562		1.10	0.058
U.K.	4	10	March '67 -	0.682		1.01	0.045
				0.321		0.81	0.060
			March '71	0.424		1.23	0.043
0.213		1.06		0.051			
Germany	3	11	March '67 -	0.286		0.85	0.043
				0.235		0.73	0.056
			March '71	0.501		1.11	0.059
0.539		1.06		0.054			
				0.668		1.02	0.067

of our grouping procedure has not been met for this country, resulting in an inability to obtain estimates¹ of γ_0 and γ_1 .

When the European results are compared with the American tests shown in Table 3 (or with the work of other U.S. researchers as Jacob (1971)) they are found to be similar. The proportion of cross-sectional portfolio returns that can be explained by variation in portfolio β values is roughly equivalent when similar time horizons are considered. Whereas the $\hat{\gamma}_1$ value in the last and most comprehensive tend to understate the theoretical values in the U.S. market, no pattern of this type seems to exist in the European data. However, the test period for the European markets is short and firm support for this conclusion must await better data resources and additional research.

VII. Concluding Remarks

Our research provides some support to the hypothesis that systematic risk is an important factor in the pricing of European securities. A positive relationship between realized return and risk has been shown for each market except Germany. No evidence of a lesser rationality or efficiency of the European stock markets has been found. However this is not the only dimension of market efficiency. Even if the pricing of risk is rational, institutional factors or thin markets might create market inefficiencies which are not revealed by these tests.

¹The reason for the lack of difference between the portfolio β results from a lack of stability in the beta coefficients for German stocks. Thus beta rankings in the March 1966 - March 1967 period gave little information about betas during the next four years. Thus, the β coefficients for the portfolios will differ only by chance. See Pogue and Solnik (1972,b) for evidence on the stability of the European beta coefficients.

An obvious word of caution is in order. The test period was short and the sample limited. More definite conclusions must await better data resources.

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