

Imperfect Information and the Equitability  
of Competitive Prices

by

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In many markets, sellers have only imperfect information about the costs of sales to different buyers. The pattern of competitive pricing then depends on the information structure. Assuming aversion to cross-subsidy, new measures of horizontal and vertical pricing inequity are proposed, and their dependence on information is analyzed. Better information generally reduces vertical inequity, but if information about buyers is initially poor, additional low quality information may sharply increase horizontal inequity. If efficiency effects are small, this may justify banning the use of available information in setting competitive prices under some conditions.

## I. Introduction

The general policy question considered in this essay has arisen in debates about the structure of automobile insurance rates.<sup>1</sup> Under unregulated competition, any information that is available at negligible cost and that improves predictions of accident costs will affect insurance rates. Thus if married people have fewer (or cheaper) accidents than single people, all else equal, competition will produce lower rates for those who are married. Other variables that might similarly furnish imperfect information about accident costs and thus affect auto insurance rates under competition include accident record, place of residence, age, sex, and race. Insurance companies have nonetheless been prohibited from using some of these variables from time to time.

When confronted with an issue of this sort, an economist's natural response is to argue for the use of all available information on efficiency grounds. But if demand elasticities are low, as some surely are in the auto insurance context, deadweight losses from mispricing will generally be small relative to the associated wealth transfers, which matter to voters, politicians, and regulators.<sup>2</sup> Further, the usual argument justifying exclusive concern with net efficiency in terms of potential payments from winners to losers has less force than usual here, since imperfect information makes it impossible to identify the relevant winners and losers, thus ruling out compensation even in principle. Finally, the efficiency argument for the use of all available information encounters a second-best problem described in Section IV, below.

Public debate on questions of information suppression, as on most other questions, is at any rate generally much more concerned with equity than with efficiency. I thus focus on equity arguments for and against information suppression in what follows.<sup>3</sup> I do not adopt the traditional, utilitarian, welfare-economic approach of defining equity in terms of the distribution of utility or real income in the population.<sup>4</sup> This approach is not widely accepted in modern philosophical dis-

cussions of distributive justice.<sup>5</sup> More importantly, arguments about fairness made in policy debates cannot always be translated into utilitarian terms.<sup>6</sup> Economists have no special ability to define equity or justice, but we can and should explore the implications of definitions that are broadly accepted. The analysis below assumes aversion to cross-subsidy, a sentiment frequently expressed in regulatory contexts, not to real income inequality, and it borrows the notions of horizontal and vertical equity from discussions of tax policy.

Though the discussion below is often couched in terms of auto insurance for concreteness, the issues considered arise in many contexts. They are present to some extent whenever the gain (considered broadly) to one party to a transaction depends on ex ante unobservable characteristics of the other party, and the first party could productively gather and use imperfect information about the second. These issues thus arise in almost all fields of insurance.<sup>7</sup> To the extent that public utility rates do not exactly reflect buyer-specific costs, the rate structure is based on imperfect information about buyers, and some rate structure changes can be analyzed in terms of changing the information set used. The desirability of information suppression has been debated at length (though not always in those terms) in the contexts of hiring standards and credit availability (red-lining).<sup>8</sup> Finally, transfer programs that base payments on imperfect information about individuals' "needs" encounter similar issues, as do taxation systems that use imperfect information to estimate either "ability to pay" or "benefits".<sup>9</sup>

## II. Measures of Pricing Inequality

There are assumed to be  $T$  different types of potential buyers for some commodity, with  $\pi_t$  the known fraction of buyers of type  $t$ . The expected unit cost of selling to any buyer of type  $t$  is  $U_t$ , a constant assumed to be independent of total output and of seller identity. Under competition and perfect information, all buyers of type  $t$  would be charged  $U_t$  per unit. Conversely, all buyers that would

be charged the same price under these conditions are defined to be of the same type. Sellers cannot observe a buyer's type directly, and buyers cannot reliably signal their type to sellers.

Information freely available to sellers permits them to place buyers in one of  $C$  mutually exclusive and collectively exhaustive classes. The information structure is completely described by the matrix of conditional probabilities of classification:

$\pi_{tc}$  = probability that a buyer of type  $t$  will be placed in class  $c$  by available information

Since each buyer is assigned to one and only one class, we have

$$\sum_{c=1}^C \pi_{tc} = 1, \quad t=1, \dots, T. \quad (1)$$

The unconditional probability of being placed in any class is given by

$$\bar{\pi}_c = \sum_{t=1}^T \pi_t \pi_{tc}, \quad c=1, \dots, C. \quad (2)$$

Using (1), it is easy to verify that the  $\bar{\pi}_c$  sum to unity.<sup>10</sup>

All potential buyers in class  $c$  are charged the same price,  $P_c$ , since by assumption there is no way to distinguish among them. Under competition and constant costs,  $P_c$  must equal the expected cost of selling to a member of class  $c$ . Unless the information structure is perfect, so that each class contains one and only one type, buyers of the same type generally face different prices. The average price faced by buyers of type  $t$  is given by

$$\bar{P}_t = \sum_{c=1}^C \pi_{tc} P_c, \quad t=1, \dots, T. \quad (3)$$

Two sorts of equity arguments are excluded from consideration here. First, use of some kinds of information may be banned in some contexts because it would be immoral per se for such information to matter, regardless of the consequences. For many people, race fits this description. There is little room for analysis in such cases. Second, as Zeckhauser (1979) notes, there may be some commodities for which a consensus exists that it is socially optimal to charge the same price to all buyers, regardless of differences in buyer-specific costs or other attributes. If such a consensus exists, there is little more to be said. I want to concentrate on information (like marital status?) without overwhelming intrinsic moral properties and on commodities (like auto insurance?) where price equality is not morally compelling.

We take as an axiom in such cases that perfectly equitable pricing requires that each buyer be charged exactly the cost he imposes on sellers. In the notation above, perfect equity requires that all buyers of type  $t$  face a price of  $U_t$  for  $t=1, \dots, T$ . In regulatory language, this axiom defines perfect equity by the absence of cross-subsidy among individual buyers. Cross-subsidization is frequently alleged, investigated, and denounced in many regulatory arenas.<sup>11</sup> As in most partial equilibrium analyses of pricing, all buyers are here treated as equally deserving.<sup>12</sup> One can alternatively view our axiom as embodying the "benefit approach" to taxation.<sup>13</sup> Since all buyers are equally deserving, the alternative "ability to pay approach" is ruled out, and the assumed cost conditions provide a direct supply-price measure of benefit that does not depend on buyers' preference structures.

In order to construct measures of the importance of departures from perfect equity, let us assume for the moment that all buyers purchase one unit of the commodity in question, regardless of price. Let  $L(U,P)$  be the (non-negative) loss or inequity generated when a buyer with unit cost  $U$  is charged price  $P$ . We assumed above that (A)  $L(X,X) = 0$ , for all  $X$ . It seems reasonable to assume further that

(B)  $L(U,P)$  is strictly convex in  $P$  for any  $U$ . This means simply that large price-cost discrepancies are particularly inequitable. In order to fix the form of  $L$ , we make the additional assumption of competitive equitability.<sup>14</sup> That is, (C) if any class of buyers with different  $U$ 's must be charged a single price (perhaps because it is impossible to distinguish among them), the most equitable price is always the expected value of  $U$  for the group. Under constant costs, this is the (break-even) price that would be charged by a competitive industry, so that the assumption of competitive equitability is consistent with a general preference for market-determined outcomes.<sup>15</sup> It is shown in Appendix A that under assumptions (A) - (C), the loss function can be written as  $L(U,P) = (U - P)^2$ . With this loss function, normalization by the total number of buyers yields immediately our cross-subsidy measure of total inequity:

$$S = \sum_{t=1}^T \pi_t \left[ \sum_{c=1}^C \pi_{tc} (P_c - U_t)^2 \right] . \quad (4)$$

A somewhat more natural approach to inequity measurement begins by importing the notions of "horizontal equity" and "vertical equity" from discussions of taxation policy.<sup>16</sup> In this context, the principle of horizontal equity, that equals should be treated equally, is clearly violated to the extent that identical individuals, members of the same type, are charged different prices. For any one type, one can use the variance of the prices charged its members as a convenient measure of inequality of treatment that is consistent with the development above.

Taking a weighted sum across types yields our measure of horizontal inequity:

$$H = \sum_{t=1}^T \pi_t \left[ \sum_{c=1}^C \pi_{tc} (P_c - \bar{P}_t)^2 \right] . \quad (5)$$

In general, the notion of vertical equity seems to mean simply that those who are unequal should be treated in ways that "fairly" reflect their differences. Because

of the special structure of this problem, one can plausibly associate vertical inequity here specifically with cross-subsidies among buyers of different types. If it is fair for individuals to cover their costs, it is surely fair for groups of identical individuals to do so on average. Using the variance again as a measure of inequality, our measure of vertical inequity is given by:

$$V = \sum_{t=1}^T \pi_t (\bar{P}_t - U_t)^2 . \quad (6)$$

It is straightforward to show that as long as (3) holds,  $S = H + V$ .<sup>17</sup> My impression is that aversion to cross-subsidy stems from aversion to violations of horizontal and vertical equity, not the other way around. If this is true, H and V are of independent interest, but their sum may have no special ethical significance. The result that  $S = H + V$  is still useful, though, since S has convenient analytical properties.

The foregoing development was based on the assumption that all buyers purchase one unit each, regardless of price. The squared differences in units costs or prices in (4) - (6) were thus appropriately weighted by the number of buyers involved. If different buyers purchase different amounts, for whatever reasons, it may not seem appropriate to continue to give all buyers' squared per-unit differences equal weights.<sup>18</sup> One might, for instance, multiply those differences by the squares of the corresponding demands, thus working with squares of total cost or revenue differences. It seems reasonable and turns out to be most convenient to compromise between these two possibilities, to weight the squared per-unit differences by the corresponding quantities, not their squares. Thus if  $Q_{tc} = Q_t(P_c)$  is the average demand by a buyer of type t charged price  $P_c$ , S generalizes to

$$S = \sum_{t=1}^T \pi_t \sum_{c=1}^C \pi_{tc} Q_{tc} (P_c - U_t)^2 . \quad (4')$$



This approach preserves the relation  $S = H + V$ ; additional formal implications are given in Appendix B.

### III. Properties of Proposed Measures

We now use the assumption of competition to determine the prices charged to each class, the  $P_c$ . If all buyers demand one unit regardless of price, each class's price must equal expected costs:

$$P_c = \frac{\sum_{t=1}^T \pi_t \pi_{tc} U_t}{\bar{\pi}_c}, \quad c=1, \dots, C, \quad (7)$$

where the  $\bar{\pi}_c$ , defined by (2), give the fraction of buyers in each class.

Using equation (7), equations (4) - (6) can be re-written in a useful form.

Beginning with the total inequity measure,

$$\begin{aligned} S &= \sum_{t=1}^T \pi_t \left[ \sum_{c=1}^C \pi_{tc} (P_c^2 - 2P_c U_t + U_t^2) \right] \\ &= \sum_{c=1}^C P_c^2 \left[ \sum_{t=1}^T \pi_t \pi_{tc} \right] - 2 \sum_{c=1}^C P_c \left[ \sum_{t=1}^T \pi_t \pi_{tc} U_t \right] + \sum_{t=1}^T \pi_t U_t^2 \left[ \sum_{c=1}^C \pi_{tc} \right] \\ &= \sum_{t=1}^T \pi_t (U_t)^2 - \sum_{c=1}^C \bar{\pi}_c (P_c)^2. \end{aligned} \quad (8)$$

Proceeding in the same fashion, one can similarly decompose H and V:

$$H = \sum_{c=1}^C \bar{\pi}_c (P_c)^2 - \sum_{t=1}^T \pi_t (\bar{P}_t)^2, \quad (9)$$

$$V = \sum_{t=1}^T \pi_t (\bar{P}_t)^2 - 2 \sum_{c=1}^C \bar{\pi}_c (P_c)^2 + \sum_{t=1}^T \pi_t (U_t)^2 \quad (10)$$

It is easy to show that

$$\sum_{c=1}^C \bar{\pi}_c (P_c) = \sum_{t=1}^T \pi_t (\bar{P}_t) = \sum_{t=1}^T \pi_t (U_t) , \quad (11)$$

so that variances may be substituted for the sums of squares in (8) - (10):

$$S = \text{Var}(U_t) - \text{Var}(P_c) , \quad (8')$$

$$H = \text{Var}(P_c) - \text{Var}(\bar{P}_t) , \quad (9')$$

$$V = \text{Var}(\bar{P}_t) - 2\text{Var}(P_c) + \text{Var}(U_t) . \quad (10')$$

From (7), the  $P_c$  are weighted averages of the  $U_t$  under competition. If sellers have no information, all buyers pay the same price, and  $\text{Var}(P_c) = 0$ . If sellers have perfect information, all buyers pay exactly the unit cost they impose on sellers. Then  $\text{Var}(P_c) = \text{Var}(U_t)$ , and  $S = 0$ . In between these extremes, additional information tends to drive the  $P_c$  farther apart, as the members of each class become more nearly alike in cost terms.

Using (8), one can prove formally that use of an informative, independent pass/fail test always reduces S when demands are perfectly inelastic. Consider a test that every buyer either passes or fails and let  $\theta_t$  be the probability that a buyer of type  $t$  passes. Since these probabilities do not depend on a buyer's class, this test is independent of other available information sources. Buyers in class  $c$  pay  $P_c$  before this independent pass/fail test is available; afterwards they are charged  $P_c^p$  if they pass and  $P_c^f$  if they fail.<sup>19</sup> The development in Appendix B implies that one can set all demands equal to unity with no loss of generality in the perfectly inelastic case. Doing so, the fraction of buyers in each class who pass the test is given by

$$\bar{\theta}_c = \frac{\sum_{t=1}^T \pi_t \pi_{tc} \theta_t}{\bar{\pi}_c}, \quad c=1, \dots, C. \quad (12)$$

Using the breakeven constraint to evaluate the  $P_c^P$  and  $P_c^f$ , one obtains after a bit of algebra

$$Z_c \equiv P_c^P - P_c^f = \frac{\sum_{t=1}^T \pi_t \pi_{tc} (\theta_t - \bar{\theta}_c) U_t}{\bar{\pi}_c \bar{\theta}_c (1 - \bar{\theta}_c)}, \quad c=1, \dots, C, \quad (13)$$

$$P_c^P = P_c + (1 - \bar{\theta}_c) Z_c, \quad P_c^f = P_c - \bar{\theta}_c Z_c, \quad (14)$$

where  $P_c$  is the price charged all members of class C before the availability of the test being considered. If the test is of any value at all, some of the  $Z_c$  are non-zero, and the test is informative.

Using (8) and (14), the change in S brought about by use of an independent pass/fail test can be written as follows:

$$\begin{aligned} \Delta S &= \sum_{c=1}^C \bar{\pi}_c (P_c)^2 - \sum_{c=1}^C \bar{\pi}_c [\bar{\theta}_c (P_c^P)^2 + (1 - \bar{\theta}_c) (P_c^f)^2] \\ &= \sum_{c=1}^C \bar{\pi}_c [(P_c)^2 - \bar{\theta}_c (P_c + (1 - \bar{\theta}_c) Z_c)^2 - (1 - \bar{\theta}_c) (P_c - \bar{\theta}_c Z_c)^2] \\ &= - \sum_{c=1}^C \bar{\pi}_c \bar{\theta}_c (1 - \bar{\theta}_c) (Z_c)^2 \leq 0. \end{aligned}$$

As long as the test is informative, so that its outcome affects expected costs in at least one class, use of the test causes S to fall, and the proof is complete.

Now consider the components of S: H and V. When there is no information and all buyers pay the same price,  $V = S = \text{Var}(U_t)$ . At the other extreme,  $V = S = 0$

under perfect information. Thus  $V$  falls on average as one moves from zero information to perfect information. It is easy to see that if  $C = 1$  initially (no information), a test of the sort analyzed above must lower  $V$ . Similarly, if  $V$  is positive, any new information that takes  $S$  to zero (perfect information) must lower  $V$ .<sup>20</sup> Thus one's general expectation is that increases in information tend to reduce vertical inequity.<sup>21</sup> If one is mainly concerned with reducing cross-subsidies among different types of buyers, this analysis thus provides little support for restricting the use of information about buyers in setting competitive prices.

Our measure of horizontal inequity,  $H$ , is also zero when information is perfect. But its behavior differs qualitatively from that of  $S$  and  $V$  because it alone is zero when there is no information and  $C = 1$ . Thus on average  $H$  does not change as information is made better. Since  $H$  is non-negative by construction and is positive when buyers of at least one type pay different prices, it follows that some increases in information increase horizontal inequity.<sup>22</sup>

When  $C = 1$  (no information), it is clear that any pass/fail test for which  $0 < \theta_t < 1$  for at least one  $t$  causes  $H$  to increase. At the other extreme, making information perfect must drive  $H$  to zero. This comparison suggests that increases in information tend to raise  $H$  when information is poor and to lower  $H$  when information is good. This suggestion is confirmed for two special cases in Sections V and VI, below.

The general picture of the behavior of  $S$ ,  $V$ , and  $H$  that emerges from this discussion is illustrated by Figure 1. (This Figure is in fact exact for the example of Section VI, if "information" is taken to be the squared correlation coefficient between predicted and actual  $U$ 's.)

#### IV. A Digression on Efficiency

In order to consider the efficiency implications of information use, we must treat the general case of price-dependent demands. Define average demand function

slopes as follows:

$$S_{tc} = [Q_t(P_c) - Q_t(U_t)]/[U_t - P_c] , \quad t=1, \dots, T ; c=1, \dots, C .$$

One can then approximate the deadweight loss from charging price  $P_c$  to a member of class  $t$  instead of  $U_t$ , the relevant marginal cost, by

$$-(1/2)(P_c - U_t)[Q_t(P_c) - Q_t(U_t)] = (S_{tc}/2)(P_c - U_t)^2 .$$

Adding across individuals, we obtain an approximation to the standard measure of deadweight loss:

$$D = (1/2) \sum_{t=1}^T \pi_t \sum_{c=1}^C \pi_{tc} S_{tc} (P_c - U_t)^2 . \quad (15)$$

which may be compared to the total inequity measure defined by (4'). (In the context of insurance,  $D$  may be a very rough approximation, as it doesn't deal explicitly with moral hazard, adverse selection, disappearance of markets, efficiency of risk-bearing, and related considerations.)

In this model,  $D$ ,  $S$ , and any ordering of information structures provide three different measures of the distance of equilibrium from perfection. While the arguments of Section III may suggest that the latter two measures move together in some cases of interest, there is no reason to suppose that the first and third do so. Since  $D$  is positive with no information and zero with perfect information, use of additional information is efficiency enhancing on average. But it does not follow, as is sometimes implicitly assumed in policy discussions, that improved information always increases efficiency.

The point here is similar to other second-best arguments about comparisons between distorted equilibria. For instance, it is well-known that in a public enterprise monopoly with constant returns to scale, optimal pricing involves zero

excess profits. On average, one thus expects that price changes that lower initially positive profits tend to enhance welfare. But one cannot claim that every profit-reducing price change is welfare-enhancing.

#### V. A Discrete (Two-Type) Example

Consider a situation with  $T = 2$  in which demands are perfectly inelastic. Following Appendix B, let all buyers purchase one unit each without loss of generality. Let us consider these types as good and bad drivers, so that we can associate with them expected auto insurance claims costs of  $U_g$  and  $U_b$ , respectively, with  $\Delta \equiv U_b - U_g > 0$ . Let  $\pi$  be the fraction of drivers who are good. Consider the simplest non-trivial information structure, a test designed to detect good drivers that has an error rate of  $\alpha$ , with  $0 < \alpha < 1/2$ . That is, the probability that good drivers fail the test and the probability that bad drivers pass it both equal  $\alpha$ . The lower is  $\alpha$ , the better is information in this example.<sup>23</sup> When  $\alpha = 1/2$ , the test provides no information; when  $\alpha = 0$ , information is perfect.

Use of the test divides buyers into two classes: those who pass and pay  $P_p$  for their insurance, and those who fail and are charged  $P_f$ . Let  $\bar{P}_g$  and  $\bar{P}_b$  be the type-average prices paid by good and bad drivers, respectively. It is useful to define the following function:

$$g(\pi, \alpha) = [\pi(1 - \alpha)] / [\alpha(1 - \alpha) + \pi(1 - \alpha)(1 - 2\alpha)^2] . \quad (16)$$

Note that  $g(\pi, 0) = 1$  for any  $\pi$ . Over the range  $0 < \alpha < 1/2$ ,  $g$  is non-increasing in  $\alpha$  for any  $\pi$ ; it is strictly decreasing unless  $\pi = 1/2$ .

Direct computation, using (7), yields

$$P_f - P_p = (1 - 2\alpha)g(\pi, \alpha)\Delta . \quad (17)$$

It is straightforward to show that as  $\alpha$  falls from  $1/2$  to zero,  $(P_f - P_p)$  rises monotonically from zero to  $\Delta$ . Thus a lower error rate increases the difference

between the two class prices. This means that as the probability of an individual driver's being mis-classified falls, the consequences of being mis-classified become more severe. Using (3), one obtains directly

$$\bar{P}_b - \bar{P}_g = (1 - 2\alpha)(P_f - P_p) , \quad (18)$$

so that  $(\bar{P}_b - \bar{P}_g)$  also rises monotonically from zero to  $\Delta$  as  $\alpha$  falls from 1/2 to zero. A lower error rate also increases the difference between the two type-average prices. Note that unless information is perfect, good drivers on average subsidize bad drivers, since the difference in average prices is less than the difference in expected costs.

The three inequity measures proposed in Section II can be shown with sufficient algebra to be equal to the following expressions:

$$S = \alpha(1 - \alpha)\Delta^2 g(\pi, \alpha) , \quad (19)$$

$$H = \alpha(1 - \alpha)(1 - 2\alpha)^2 \Delta^2 g(\pi, \alpha)^2 , \quad (20)$$

$$V = \alpha^2(1 - \alpha)^2 \Delta^2 g(\pi, \alpha)^2 / \pi(1 - \pi) = S^2 / \Delta^2 (1 - \pi) . \quad (21)$$

The derivative of  $S$  with respect to  $\alpha$  has the sign of  $(1 - 2\alpha)$ , so that over the relevant range, a lower error rate reduces  $S$ . From the second equality in (21), it is immediate that a lower error rate also reduces  $V$ . Thus better information in this example always reduces both vertical and total inequity.

The behavior of  $H$  is somewhat more complex, though the general pattern is broadly consistent with Figure 1. Reductions in  $\alpha$  lower the probability of mis-classification but raise the consequences thereof. The first effect dominates for  $\alpha$  near zero, but the second is more important for  $\alpha$  near 1/2.

The simplest case arises when  $\pi = 1/2$ , so that  $g = 1$  for all  $\alpha$ . In this case,  $H$  is a strictly concave function of  $\alpha$  with a unique maximum at  $\alpha = \alpha^* \equiv (2 - \sqrt{2})/4 \approx .1464$ . Since  $\partial g / \partial \alpha < 0$  for  $\pi \neq 1/2$ ,  $H$  is maximized at a lower value of  $\alpha$  in all

other cases. Since  $g < 1$  at any such maximum and the other factor of  $H$  is less than its maximum value, the maximum value of  $H$  must be lower when  $\pi \neq 1/2$  than when  $\pi = 1/2$ . (Indeed, since  $\partial g/\partial \pi > 0$  for  $0 \leq \pi < 1/2$ , the maximum value of  $H$  is an increasing function of  $\pi$  over this range.) To summarize,  $H(\alpha)$  is maximized at  $\alpha = \alpha^* \approx .15$  when  $\pi = 1/2$ ; for  $\pi \neq 1/2$ ,  $H(\alpha)$  attains a smaller maximum value at some  $\alpha < \alpha^*$ .

The effects of reductions in the error rate on  $H$  in this example depend on both the initial conditions and the magnitude of the reduction. If  $\pi = 1/2$  and  $\alpha = .10$ , for instance, any reduction in  $\alpha$  lowers both horizontal and vertical inequity. If information is initially less perfect, however, the effect on  $H$  of decreasing  $\alpha$  depends on the size of the decrease. If  $\pi = 1/2$  and  $\alpha = .25$ , for instance, reductions of less than .10 in  $\alpha$  definitely increase  $H$ . Only much larger reductions can lower  $H$ . The farther  $\pi$  is from  $1/2$ , the lower  $\alpha$  must be before one can be certain that further reductions decrease  $H$ . The larger is the initial error rate, the smaller the decrease in it, and the farther  $\pi$  is from  $1/2$ , the more likely it is that  $H$  is increased. Conversely, the smaller is  $\alpha$ , the larger the drop in  $\alpha$ , and the closer  $\pi$  is to  $1/2$ , the more likely it is that the improvement in information enhances both dimensions of pricing equity.<sup>24</sup>

#### VI. A Continuous (Gaussian) Example

In the auto insurance context, suppose that all drivers purchase one unit of insurance regardless of price, that the expected cost of insuring a driver of type  $t$  is just  $\$t$ , and that the distribution of drivers across types is normal with mean  $\mu_t$  and standard deviation  $\sigma_t$ .<sup>25</sup> Assume that all information about any particular driver is summarized by a single observation of a random variable  $c$ , and suppose that drivers face a price schedule that varies continuously with  $c$ . Let the marginal distribution of  $c$  be normal with mean  $\mu_c$  and standard deviation  $\sigma_c$ . Finally, assume that the pair  $(t,c)$  follows a bivariate normal distribution with correlation



coefficient  $\rho$ .

We can evaluate our inequity measures most easily under these assumptions by using equations (8') - (10') in Section III, rather than going back to the obvious modifications of the original discrete-case definitions. It follows immediately from the assumptions above that

$$\text{Var}(U_t) = \sigma_t^2 \quad (22)$$

Every driver is assumed to be charged the expected cost corresponding to his individual value of  $c$ . Using bivariate normality, this implies that for all  $c$ ,

$$P_c = E[t|c] = \mu_t + \rho(\sigma_t/\sigma_c)(c - \mu_c) .$$

Evaluating the variance of the quantity on the right, we have

$$\text{Var}(P_c) = \rho^2 \sigma_t^2 . \quad (23)$$

The average (or expected) price paid by a driver of type  $t$  is obtained as follows:

$$\bar{P}_t = E[P_c|t] = \mu_t + \rho(\sigma_t/\sigma_c)(E[c|t] - \mu_c) = \mu_t + \rho^2(t - \mu_t) ,$$

which immediately implies

$$\text{Var}(\bar{P}_t) = \rho^4 \sigma_t^2 . \quad (24)$$

From (23) and (24), better information (higher  $\rho$ ) increases the variance of both class and type-average prices. As in Section V, unless information is perfect, above-average drivers subsidize below-average drivers, since prices vary less than costs.

Substituting (22) - (24) into (8') - (10') yields the following simple expressions for the three inequity measures in the bivariate normal case:

$$S = \sigma_t^2(1 - \rho^2) , \quad (25)$$

$$H = \sigma_t^2 \rho^2(1 - \rho^2) , \quad (26)$$

$$V = \sigma_t^2(1 - \rho^2)^2 = S^2/\sigma_t^2 . \quad (27)$$

Figure 1 is drawn for this example with  $\rho^2$  measuring "information".

It is clear from (25) and (27) that improving information by increasing  $\rho$  lowers both S and V. Similarly, (26) implies that H( $\rho$ ) is maximized by  $\rho = \rho^* \equiv 1/\sqrt{2} \approx .707.$

As in the example of Section V, if information is initially good enough ( $\rho \geq \rho^*$ ), making it better (raising  $\rho$ ) always lowers both H and V. If information is initially not very good, so that the  $R^2$  on a regression explaining individual cost data is less than  $(\rho^*)^2 = 1/2$ , small increases in information (i.e., in  $\rho$ ) always increase horizontal inequity, and only large increases can lower it. The better is initial information and the larger the increase in  $\rho$ , the more likely it is that H will fall along with V.

## VII. Conclusions and Implications

Under constant costs, perfect equity in pricing is here taken to require charging each individual buyer exactly the cost he imposes on sellers. The approach here is not of the standard utilitarian sort. An attempt is made to formalize aversion to cross-subsidy, a non-utilitarian principle of fairness commonly asserted in the context of regulation. I hope that this exercise at least serves to call attention to the possibility of formal, non-utilitarian analysis of equity issues that arise in real policy debates.

If sellers have imperfect information about the costs that buyers impose on them, some cross-subsidy among buyers is inevitable even under competition. Measures of pricing inequity proposed in Section II decompose the total deviation

of competitive pricing from subsidy-free pricing into components, denoted H and V, that measure the departures from horizontal equity and vertical equity, respectively. On average, better information reduces both V and  $S = H + V$ , but it may well increase H. There exist situations in which suppression of imperfect information about buyers reduces horizontal inequity while increasing vertical inequity. Such suppression may reduce efficiency but need not do so, and reductions in inequity may in any case be judged to outweigh efficiency losses.

In the examples of Sections V and VI, improvements in information always lower V. They are more likely to reduce H as well (i) the more important the improvement and (ii) the better the initial information. If new information reduces both H and V, there is no equity case (under our normative assumptions) for restricting its use. Some discussion of information use in setting insurance rates has considered the first of these two elements, but the importance of initial conditions seems not to have been explicitly noted before. If information about buyer-specific costs is initially poor, the use of additional low-quality information may raise H while lowering V. Depending on the magnitudes of the effects and the relative importances attached by society to horizontal and vertical equity, it may be desirable to ban the use of some available information in some such cases. Of course, it may be even better to subsidize production of substantial improvements in available information.

APPENDICES

Appendix A

In this Appendix, we establish that if the non-negative loss function  $L(U,P)$  associated with charging price  $P$  when unit cost  $U$  satisfies (A)  $L(X,X) = 0$  for all  $X$ , (B)  $L(U,P)$  is strictly convex in  $P$  for all  $U$ , and (C) competitive equitability, then  $L(U,P)$  can be written as  $(U - P)^2$ . (The converse is obviously correct as well.)

Consider a class of buyers such that the (conditional) probability that a randomly-selected buyer is of type  $t$  is  $\pi_t^0$ . The average inequity produced by charging all members of this group price  $P$  is then given by

$$I = \sum_t L(U_t, P) \pi_t^0 .$$

(We stick with discrete distributions to avoid inessential complications caused by possible non-existence of expected loss.) The assumption of competitive equitability asserts that  $I$  is always minimized (for any set of probabilities and type-specific unit costs) by setting price equal to expected cost:

$$P = \mu^0 = \sum_t U_t \pi_t^0 .$$

Because of assumption (B), this is equivalent to the statement that  $P = \mu^0$  always satisfies the first-order condition for minimizing  $I$ :

$$I_P = \sum_t L_P(U_t, P) \pi_t^0 = 0 \tag{28}$$

Let  $\{\pi_0^1\}$  be any other vector of (conditional) probabilities, and let the associated mean cost be  $\mu^1$ . Let  $\gamma$  be any number between zero and one, and let  $\mu^\gamma = \gamma\mu^1 + (1 - \gamma)\mu^0$ . Competitive equitability then implies that the following

first-order condition must be satisfied for all  $\gamma$ :

$$\sum_t L_P(U_t, \mu^\gamma) [\gamma \pi_t^1 + (1 - \gamma) \pi_t^0] = 0 .$$

Differentiating this relation with respect to  $\gamma$ , evaluating the derivative at  $\gamma = 0$ , and using (28) with  $P = \mu^0$ , we obtain

$$(\mu^0 - \mu^1) \sum_t L_{PP}(U_t, \mu^0) \pi_t^0 = \sum_t L_P(U_t, \mu^0) \pi_t^1 .$$

Since the left-hand side depends only on the mean of  $U$  with probabilities  $\pi_t^1$ , the right-hand side cannot involve any higher moments of  $U$ . This means that  $L_P$  must be linear in  $U$ :

$$L_P(U_t, \mu^0) = a(\mu^0) + b(\mu^0) U_t ,$$

where  $a(\cdot)$  and  $b(\cdot)$  are (at this point) arbitrary functions. Since  $L_P(\mu^0, \mu^0) = 0$  for all  $\mu^0$  from (A) and (B), the preceding equation implies that in general,

$$L_P(U, P) = b(P)(U - P) .$$

Differentiating this expression with respect to  $P$ , we obtain

$$L_{PP}(U, P) = b'(P)(U - P) - b(P) .$$

In order for assumption (B) to be satisfied for all  $U$  and  $P$ ,  $b'(P)$  must be zero, and  $b$  must be a negative constant. (If  $L$  were required to depend only on the difference between  $U$  and  $P$ , this result would also be implied thereby.) But this means that we can integrate and obtain

$$L(U, P) = k(U - P)^2 + h(U) ,$$

where  $k$  is a positive constant and  $h(\cdot)$  is an arbitrary function. Without loss of generality, we can set  $k = 0$  and  $h(U) = 0$ , since the function  $h(\cdot)$  cannot affect comparisons of alternative pricing arrangements. We can thus write  $L(U,P) = (U - P)^2$ , as asserted in the text.

Appendix B

To see the implications of weighting squared price-cost differences by quantities purchased when buyers' demands vary, let us proceed in two steps. First, suppose that demands are all perfectly inelastic, with  $Q_t$  the expected quantity demanded by a buyer of type  $t$ . With large numbers, we can treat  $Q_t$  as non-stochastic. Then (3) still provides the natural definition of type-average prices. One can thus maintain the form of definitions (4) - (6) by simply replacing the  $\pi_t$  with demand-weighted probabilities:

$$\pi'_t = \pi_t Q_t / \sum_{s=1}^T \pi_s Q_s, \quad t=1, \dots, T. \quad (29)$$

(In all that follows, primes denote demand-weighted quantities.) It is easy to show that  $S = H + V$  with this change.

Now suppose that the demand functions of at least some types are not perfectly inelastic. Let  $Q_{tc} = Q_t(P_c)$  be the average demand by a buyer of type  $t$  charged price  $P_c$ . If quantity weighting is sensible in computing inequity measures, it is natural to use quantity weights in computing type-average prices as well. Let us thus generalize (3) to

$$\bar{P}'_t = \sum_{c=1}^C (\pi_{tc} Q_{tc} / \bar{Q}_t) P_{c,t}, \quad t=1, \dots, T, \quad (3')$$

where the type-average demands are given by

$$\bar{Q}_t = \sum_{c=1}^C \pi_{tc} Q_{tc}, \quad t=1, \dots, T. \quad (30)$$

The quantity-weighted generalizations of (4) - (6) can then be written as follows:

$$S = \sum_{t=1}^T (\pi_t \bar{Q}_t) \left[ \sum_{c=1}^C (\pi_{tc} Q_{tc} / \bar{Q}_t) (P_c - U_t)^2 \right], \quad (31)$$

$$H = \sum_{t=1}^T (\pi_t \bar{Q}_t) \left[ \sum_{c=1}^C (\pi_{tc} Q_{tc} / \bar{Q}_t) (P_c - \bar{P}'_t)^2 \right], \quad (32)$$

$$V = \sum_{t=1}^T (\pi_t \bar{Q}_t) (\bar{P}'_t - U_t)^2. \quad (33)$$

It is easy to show that  $S = H + V$  for these generalizations.

To achieve notational symmetry with the perfectly inelastic case, one can replace  $(\pi_t \bar{Q}_t)$  in (31) - (33) with demand-weighted probabilities given by the obvious generalization of equation (29):

$$\pi'_t = \pi_t \bar{Q}_t / \sum_{s=1}^S \pi_s \bar{Q}_s, \quad t=1, \dots, T, \quad (29')$$

and define quantity-weighted conditional probabilities by

$$\pi'_{tc} = \pi_{tc} Q_{tc} / \bar{Q}_t, \quad t=1, \dots, T; \quad c=1, \dots, C. \quad (34)$$

Note that the  $\pi'_t$  sum to one, and the  $\pi'_{tc}$  sum to one over  $c$  for any  $t$ . The  $\pi'_{tc}$  give the fractions of each type's total demand occurring in the various classes.

If demands are not identical, one must generalize (7) in the text to equate expected revenues and expected costs for each class:

$$\begin{aligned}
 P_c &= \frac{\sum_{t=1}^T \pi_t \pi_{tc} Q_{tc} U_t}{\sum_{t=1}^T \pi_t \pi_{tc} Q_{tc}} = \frac{\sum_{t=1}^T \pi'_t \pi'_{tc} U_t}{\sum_{t=1}^T \pi'_t \pi'_{tc}} , \\
 &= \frac{\sum_{t=1}^T \pi'_t \pi'_{tc} U_t}{\bar{\pi}'_c} , \qquad c=1, \dots, C . \qquad (7')
 \end{aligned}$$

where the  $\pi'_t$  and  $\pi'_{tc}$  are the quantity-weighted probabilities defined by above, and the last equality defines the  $\bar{\pi}'_c$  by generalization of (2). Note that the  $\bar{\pi}'_c$  give the fraction of total demand in each class.

Using (7') instead of (7), the development in the text leading to (8') - (10') goes through essentially unchanged. Quantity-weighted probabilities replace un-weighted probabilities, and the  $\bar{P}'_t$  replace the  $\bar{P}_t$ . Thus the ordinary variances in (8') - (10') are replaced by quantity-weighted variances in this generalization.



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Footnotes

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1. See, for instance, National Underwriter (1978, 1979), Ferreira (1978), Findlay (1979), and Zeckhauser (1979). Zeckhauser's paper provides an illuminating discussion of some of the broader issues addressed in this essay, though our basic approaches differ.
2. Recent studies finding transfers to be larger relative to efficiency gains from deregulation include Friedlaender and Spady (1981) and Kalt (1981).
3. Difficulties involved in actually suppressing information (by requiring insurance firms to sell at a loss in high-cost areas, for instance) are assumed away in what follows. Our concern is with measures of desirability, not questions of feasibility.
4. For interesting examples of the utilitarian approach to the analysis of the equity effects of pricing, see Willig and Bailey (1981) and Hoy (1981).
5. Though John Rawls' important work is often interpreted in utilitarian terms, Rawls (1974, esp. p. 644) himself explicitly rejects that interpretation. Robert Nozick (1974, esp. ch. 7) forcefully rejects utilitarianism along with all other end-state approaches to defining justice.
6. See, for instance, the discussions of aversion to "unfair" exogenous shocks as a principle of equity in economic regulation and energy policy by Owen

and Braeutigam (1978, ch. 1) and Schmalensee (1980), respectively. I do not mean to assert that policy arguments couched in terms of general principles of equity are not often self-serving, but I would argue that any such principles that are frequently employed in persuasive arguments deserve to be taken seriously by economists interested in informing policy debates.

7. Cummins (1980) has a good discussion of the use of information on sex in life insurance; see also Fisher (1980).
8. On the use of imperfectly informative testing in hiring, see Smith (1978) and Borjas and Goldberg (1978). Discrimination in mortgage lending is considered by Benston (1978) and Black, Schweitzer, and Mandell (1978). To the extent that the relevant decisions are discrete (hire/don't hire) rather than continuous (choose a wage), these situations differ formally from those analyzed in the text.
9. Consider, for instance, Stern's (1980) recent analysis of lump-sum taxation with errors in classification.
10. Extension of all this to continuous or mixed distributions is straightforward; see Section VI, below, for an example.
11. Our cost assumption avoids a number of problems with the notion of subsidy-free prices when economies of scale and scope are present; see Zajac (1978, chs. 7-8) for a general discussion and Faulhaber and Levinson (1980) for interesting recent developments.
12. We are thus ruling out the use of pricing or information suppression to redistribute income; see Zeckhauser (1979) in this context and, more generally, Posner (1971) and Schmalensee (1979, ch. 2). In Nozick's (1974, ch. 7) terms, what I have in mind here is a "process principle" of justice: transactions made at equitable prices produce equitable changes in individuals' utilities, regardless of the original or final utility distributions in the population.

13. Musgrave (1959, ch. 4) presents the historical evolution of this approach. His later (pp. 176-78) discussion of "benefit taxation" relates more closely to the private good case considered here.
14. I am indebted to Severin Borenstein for suggesting this assumption.
15. In general, the loss functions proposed by Ferreira (1978) and Zeckhauser (1979, p. 9) do not satisfy competitive equitability; for some within-class distributions, they call for cross-subsidies among classes to minimize pricing inequity. (Ferreira considers a measure like eq. (4), below, but rejects it.)
16. For expositions of these concepts, see Musgrave (1959, ch. 8) and, especially, Atkinson and Stiglitz (1980, sect. 11-4). These discussions mainly follow the ability to pay approach to tax policy, however, while the partial equilibrium treatment here reflects the benefit approach. In terms of the Atkinson-Stiglitz (1980, pp. 354-55) alternatives, horizontal equity is taken here to be an independent principle of justice, not an implication of utilitarian assumptions, nor simply a constraint likely to be socially useful.
17. This decomposition is important in the analysis of variance. On income inequality measures with this same sort of decomposition property, see Shorrocks (1980) and the recent literature cited there.
18. It is assumed that buyers reveal demands only after sellers announce prices, so that sellers cannot obtain valuable information from quantities demanded.
19. This is obviously only one, illustrative way of adding new information. One can also increase information by improving an existing test; this is explored further in the examples in Sections V and VI, below. A great deal of additional work, which would carry us well beyond the scope of this essay, would be required to analyze more generally the effects of new information on our measures of inequity.
20. Proofs are as follows. (i) If there is no information,  $H = 0$  initially. A test lowers  $S$ ,  $H$  can't fall, so  $V = S - H$  must fall. (ii)  $V = 0$  under perfect

information, so that last bit of information must lower  $V$  if it was positive.

21. I conjecture that informative, independent pass/fail tests always lower  $V$  when demands are perfectly inelastic. I have been unable to prove this, however, or to produce a counterexample by simulation. (See note 24, below.)
22. Some of Hoy's (1980) results have a similar flavor, though our basic approaches differ.
23. Everything that follows in this Section is symmetric around  $\alpha = 1/2$ , so that increases in  $\alpha$  above  $1/2$  also improve information. There is also symmetry about the point  $\pi = 1/2$ .
24. As an extension of this example, I investigated, via simulation, the consequences of adding a second independent pass/fail test, with error rate  $\beta$ . In all cases examined, adding the second test lowered  $V$  and  $S$ , as did reducing  $\beta$  for fixed  $\alpha$  and  $\pi$ . Addition of the second test was more likely to lower  $H$  the smaller was  $\beta$ , the closer  $\pi$  was to  $1/2$ , and the smaller was  $\alpha$ . For all values of  $\alpha$  and  $\pi$  examined,  $H$  seemed either to be monotone increasing in  $\beta$  or to attain an internal maximum for  $0 < \beta < 1/2$ . In general, the effects on  $H$  of adding a second test seemed to depend on both initial conditions and the magnitude of the change in information in much the same qualitative way as the effects of lowering  $\alpha$ .
25. With some reinterpretation, this same analysis can be made to apply when demands are unequal but the quantity-weighted density of buyers across unit costs is normal, using the sort of transformation discussed in Appendix B.

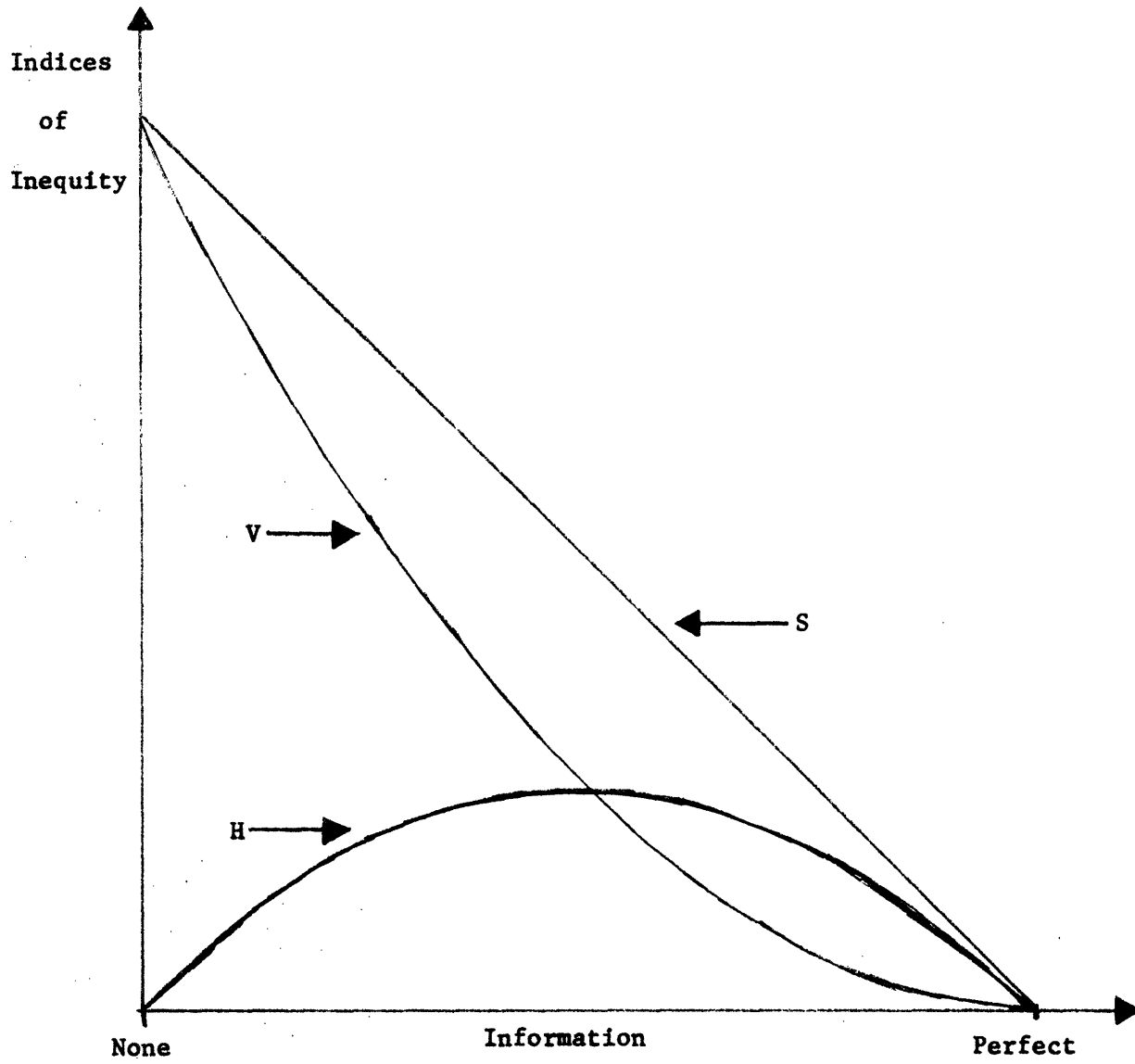


Fig. 1 -- Behavior of Inequity Indices